Title: Physics in Nature Presentation: Sunflowers, (Magnetic) Cacti, and the Golden Ratio

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Abstract:

Sunflowers, (magnetic) Cacti, and the Golden Ratio

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Perimeter Institute for Theoretical Physics

PSI Student Presentations – August 19th, 2011

Outline

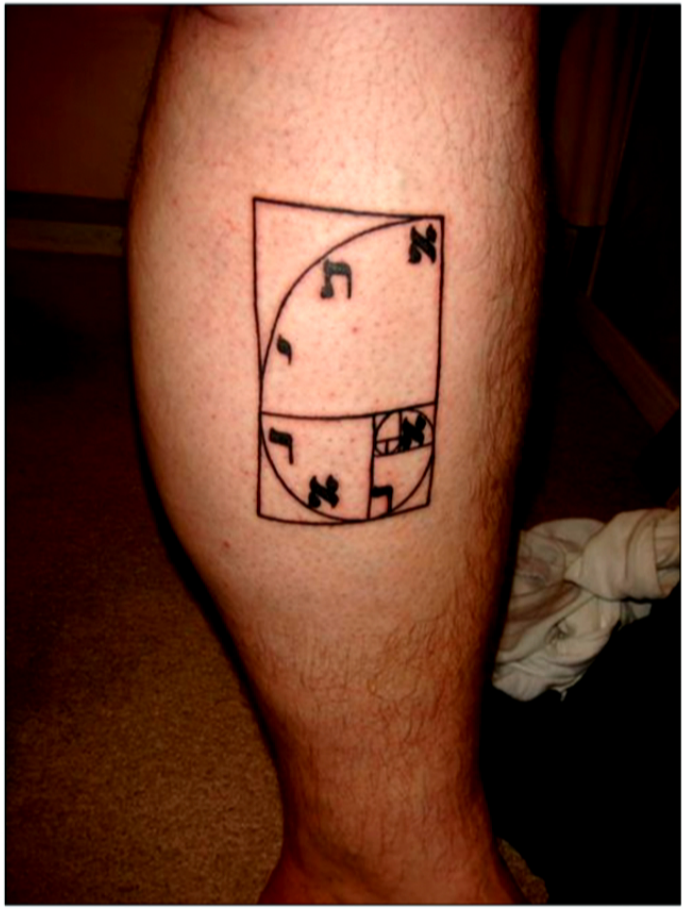
- Mathematical properties
- In nature (outdoors and in)

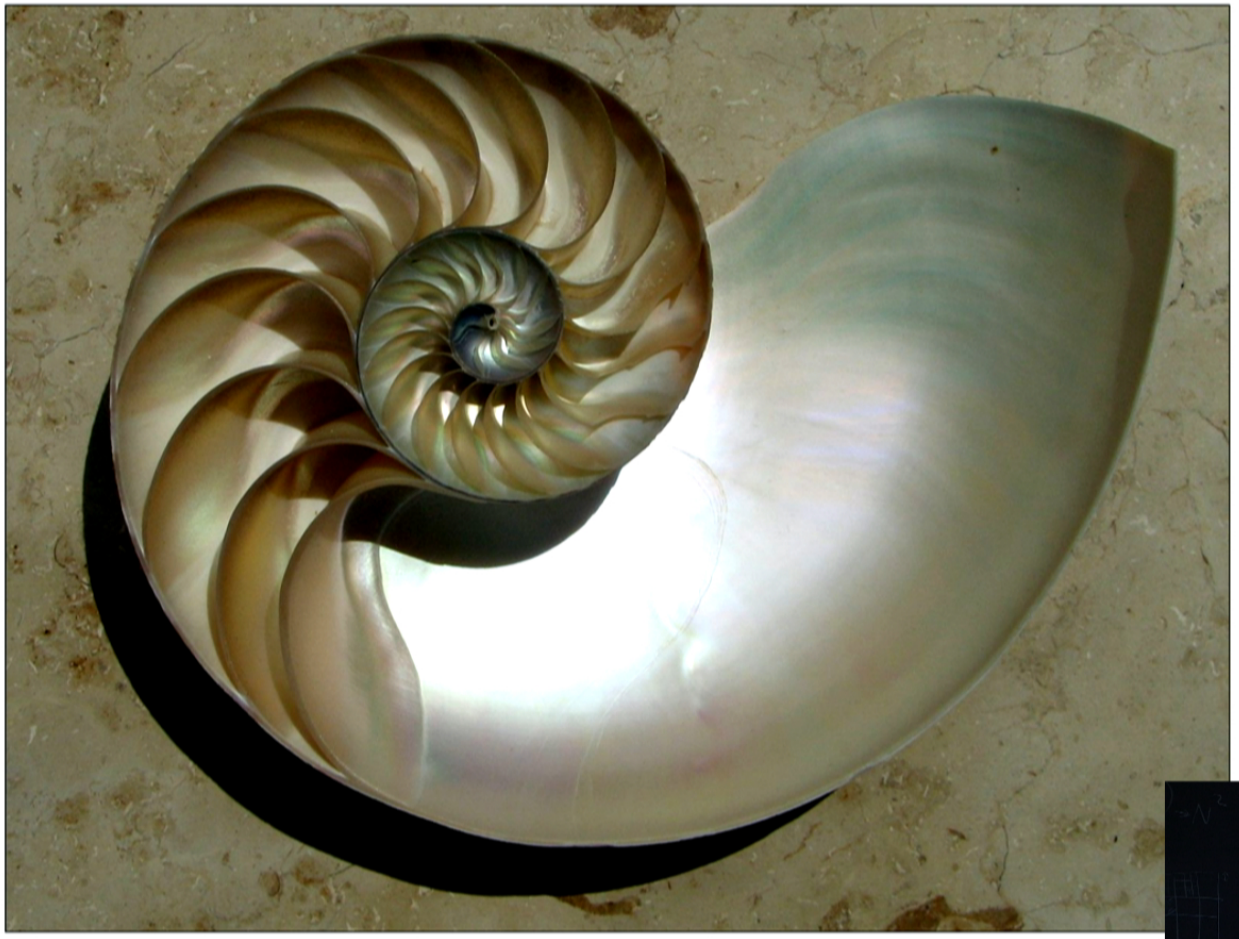
Some facts

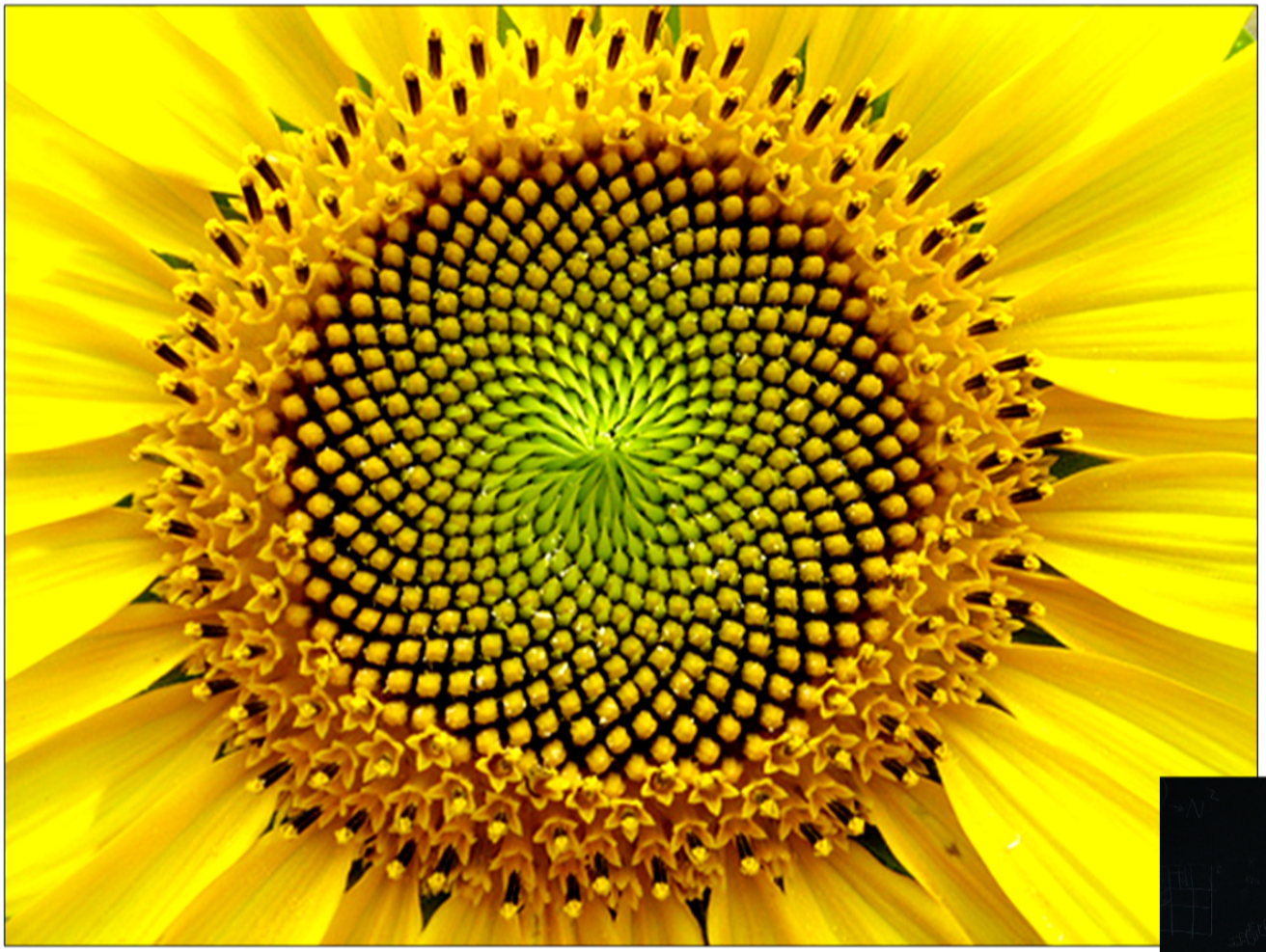
- Golden ratio solves $\Gamma^2 = \Gamma + 1$
  - $\Gamma = \dfrac{1+\sqrt{5}}{2}$
- Related to Fibonacci $F_{n+2} = F_n + F_{n+1}$

$$\Gamma = \lim_{n\to\infty} F_n / F_{n-1}$$

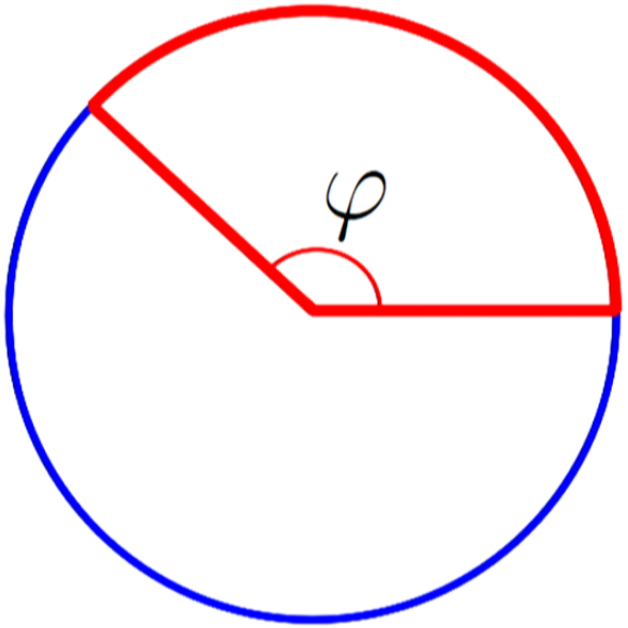

1.6180339
887498948
482045868
343656381
177203091
798057628
621354486
227052604
628189024







$$\varphi = \frac{2\pi}{\Gamma + 1}$$



Sunflower seeds

- Form on cylindrical stem

Sunflower seeds

- Form on cylindrical stem

Sunflower seeds

- Form on cylindrical stem
  - One on top of another; fixed rate

In the lab

- The magnetic cactus[1]

[1]C. Nisoli *et al*., PRL **102**, 186103 (2009)

In the lab

- The magnetic cactus[1]

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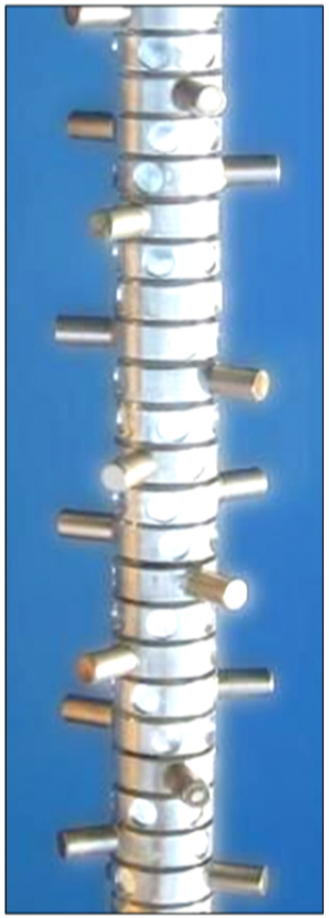
In the lab

- The magnetic cactus[1]
  - Dipoles on a cylindrical shell; radius r
  - Axial spacing a
  - Only azimuthal movement allowed

[1]C. Nisoli *et al.*, PRL **102**, 186103 (2009)



In the lab

- The magnetic cactus[1]
  - Dipoles on a cylindrical shell; radius r
  - Only azimuthal movement allowed
  - Axial spacing a

- Success! Golden-angle spacing

[1] C. Nisoli *et al.*, PRL **102**, 186103 (2009)

MATLAB model

- Again, repulsive particles on a cylinder
- Vary the axial spacing a

MATLAB model

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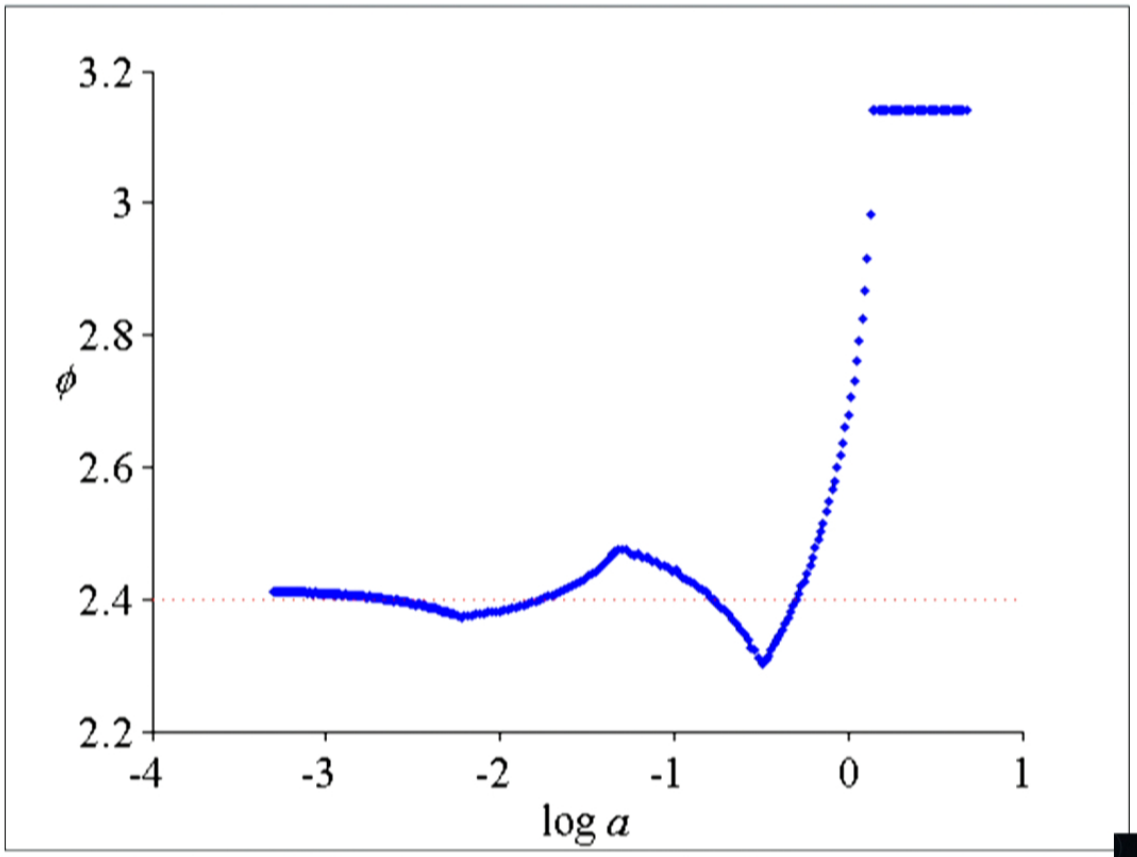
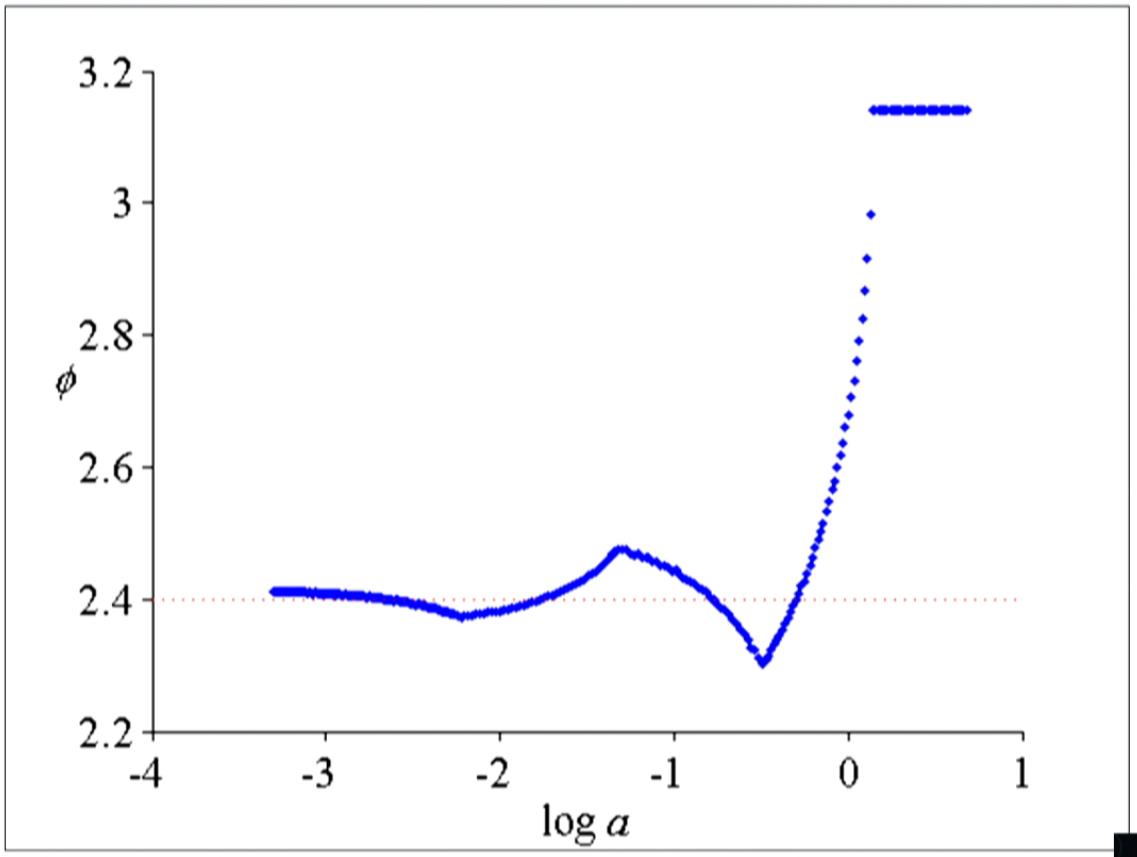
- Calculate the optimal angle $\phi(a)$

MATLAB model

- Again, repulsive particles on a cylinder
- Vary the axial spacing a
- Calculate the optimal angle $\phi(a)$
  - Maximize nearest-neighbour distance d_{nn}



$$\phi_n = \frac{2\pi}{\frac{F_n}{F_{n-1}} + 1}$$



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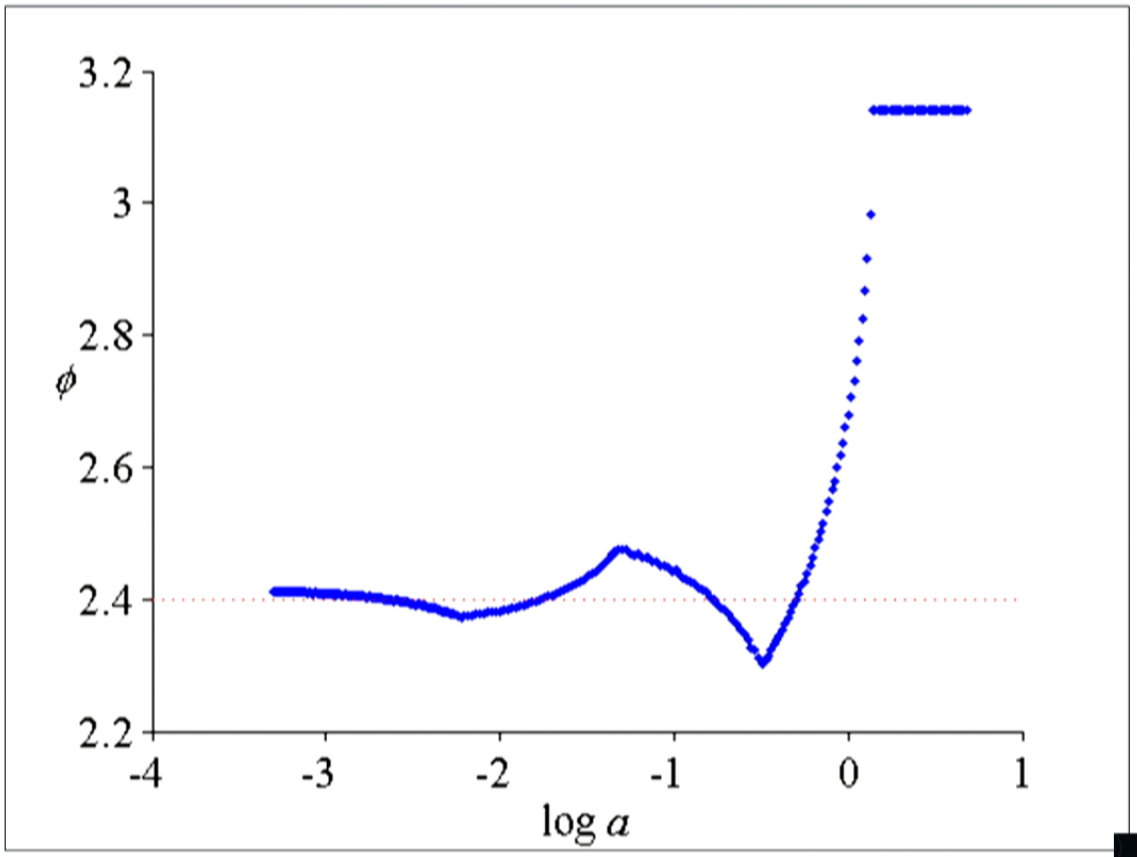

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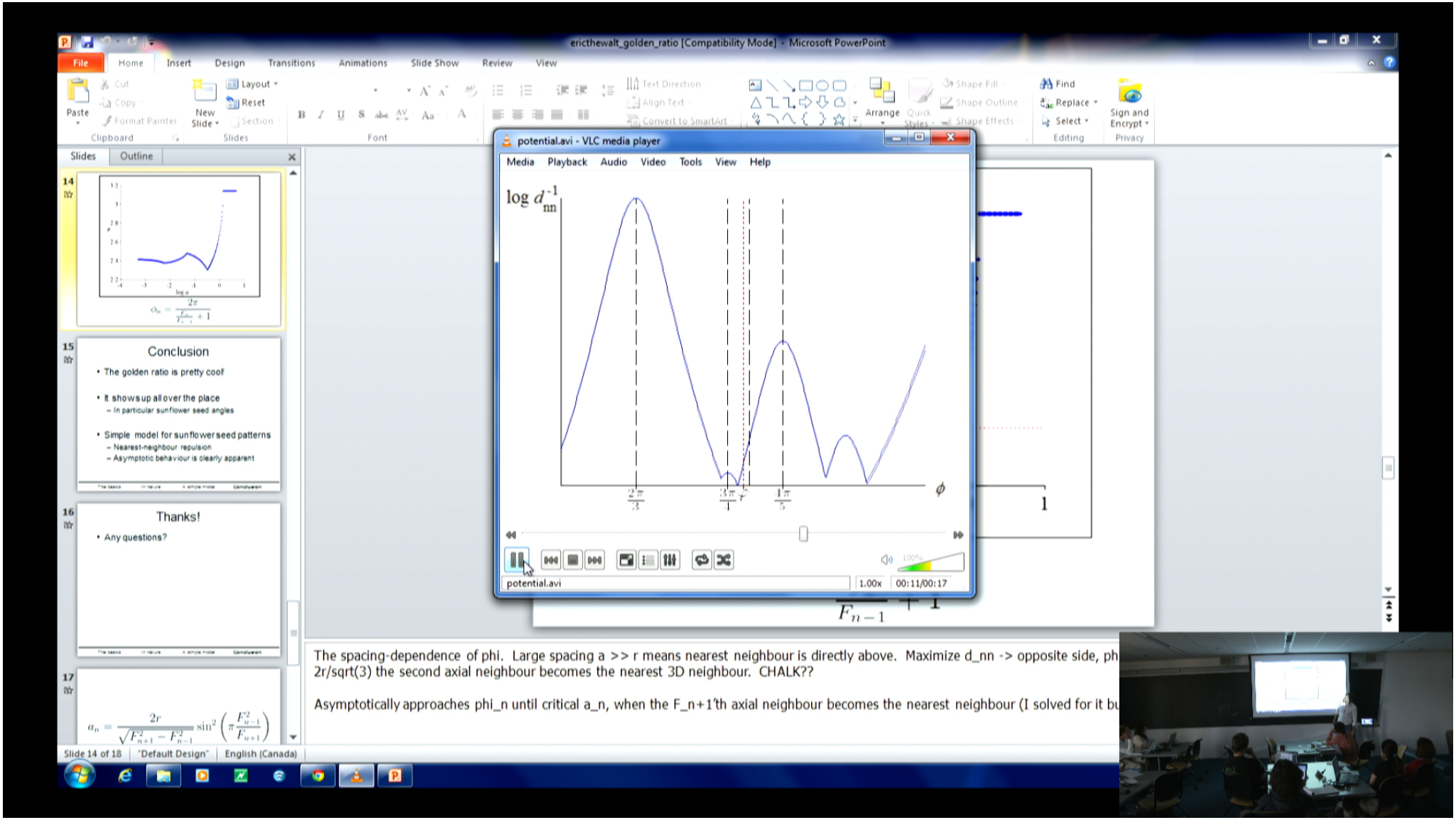
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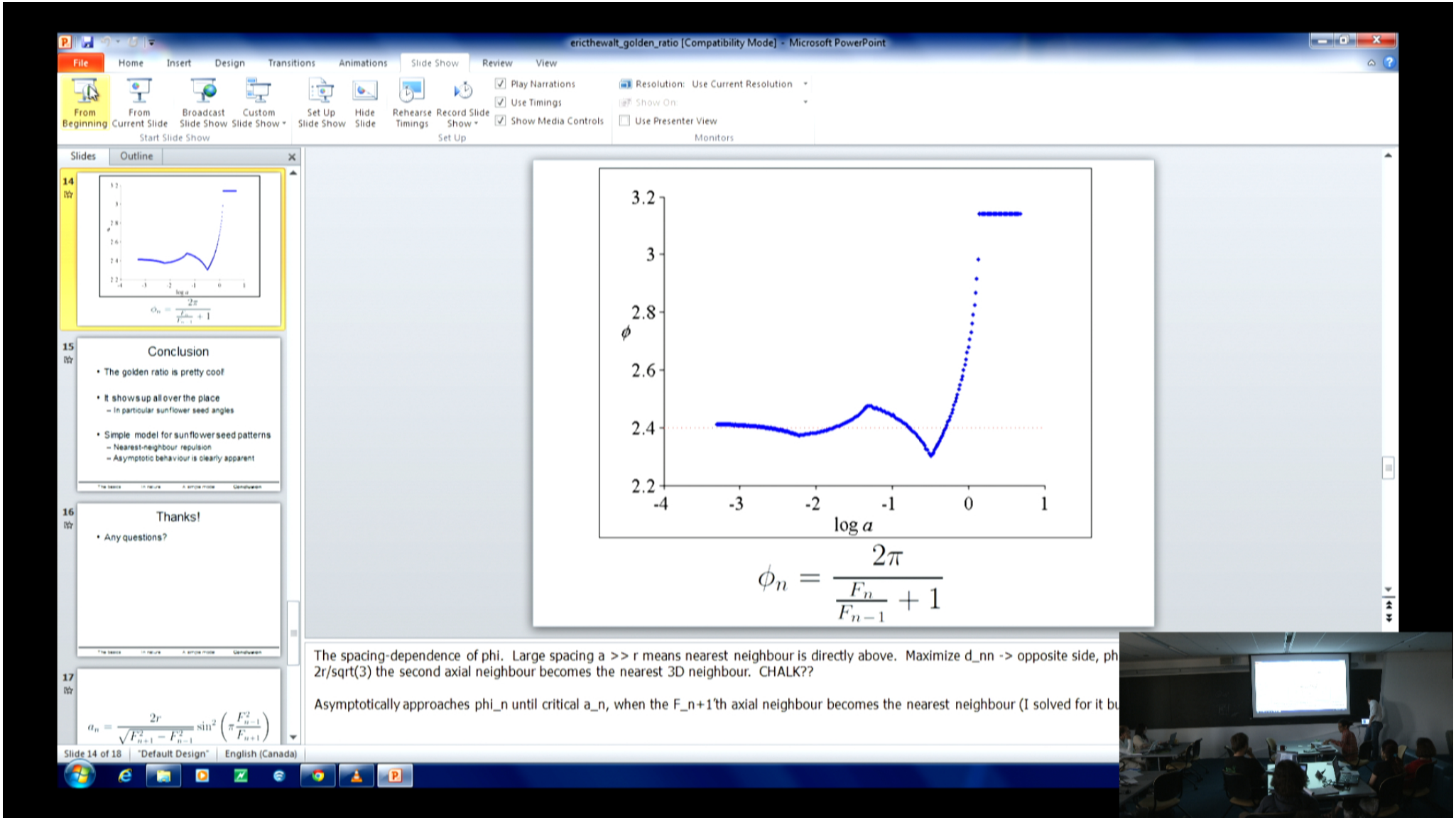
Conclusion
The golden ratio is pretty cool
It shows up all over the place
In particular sunflower seed angles
Simple model for sunflower seed patterns
Nearest-neighbour repulsion
Asymptotic behaviour is clearly apparent
Thanks!
Any questions?
The spacing-dependence of phi. Large spacing a >> r means nearest neighbour is directly above. Maximize d_nn -> opposite side, ph
2r/sqrt(3) the second axial neighbour becomes the nearest 3D neighbour. CHALK??
Asymptotically approaches phi_n until critical a_n, when the F_n+1'th axial neighbour becomes the nearest neighbour (I solved for it bu
Slide 14 of 18
"Default Design"
English (Canada)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Record Slide Show
Play Narrations
Use Timings
Show Media Controls
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Use Presenter View
Start Slide Show
Set Up
Monitors
Slides
Outline
14
15
Conclusion
• The golden ratio is pretty cool
• It shows up all over the place
– In particular sunflower seed angles
• Simple model for sunflower seed patterns
– Nearest-neighbour repulsion
– Asymptotic behaviour is clearly apparent
16
Thanks!
• Any questions?
17
3.2
3
2.8
2.6
2.4
2.2
-4
-3
-2
-1
0
1
log a
ϕ
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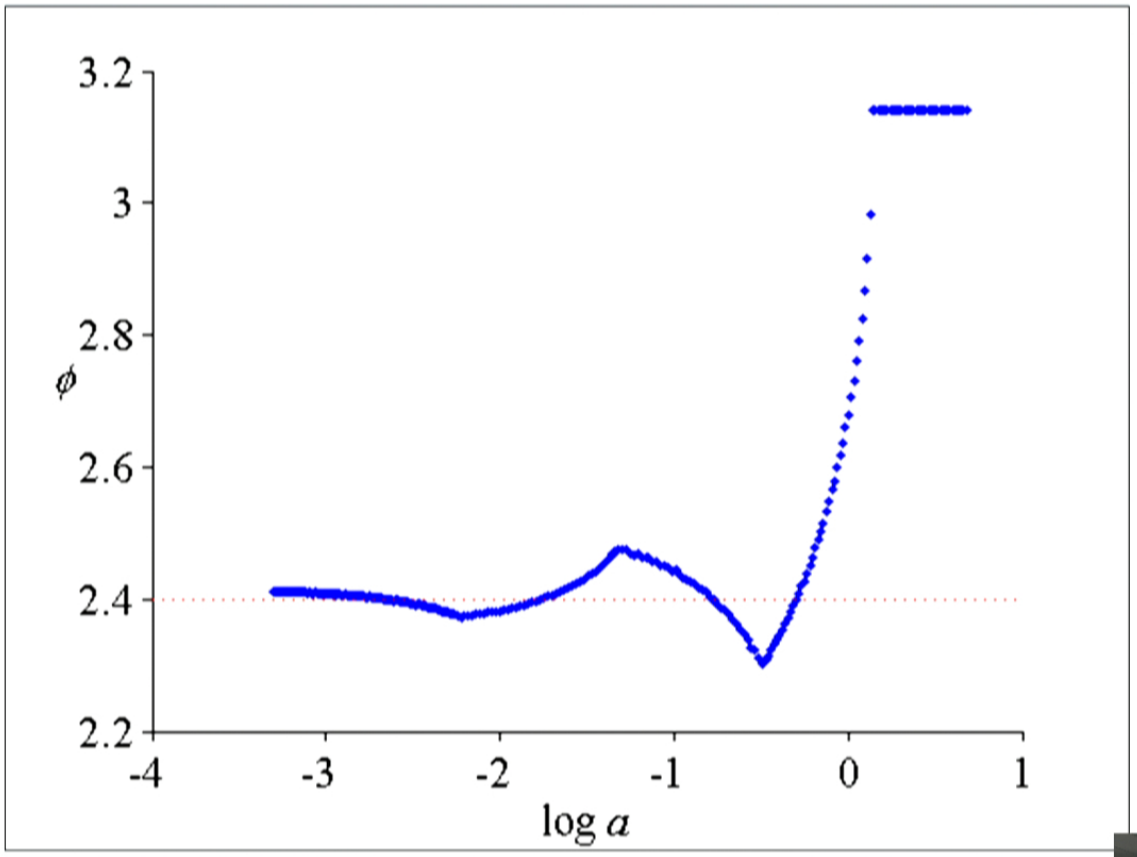

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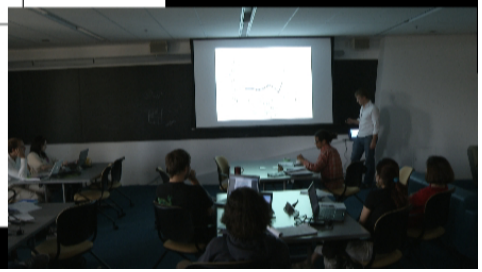

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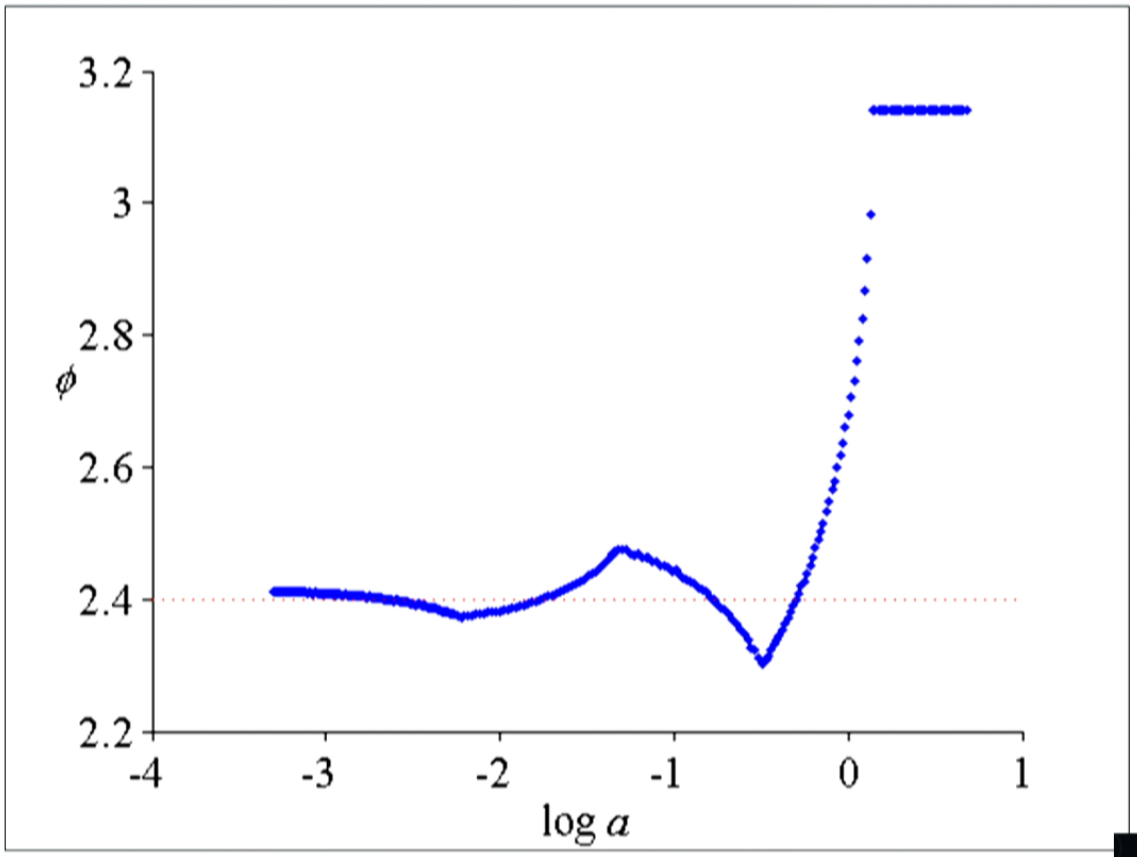

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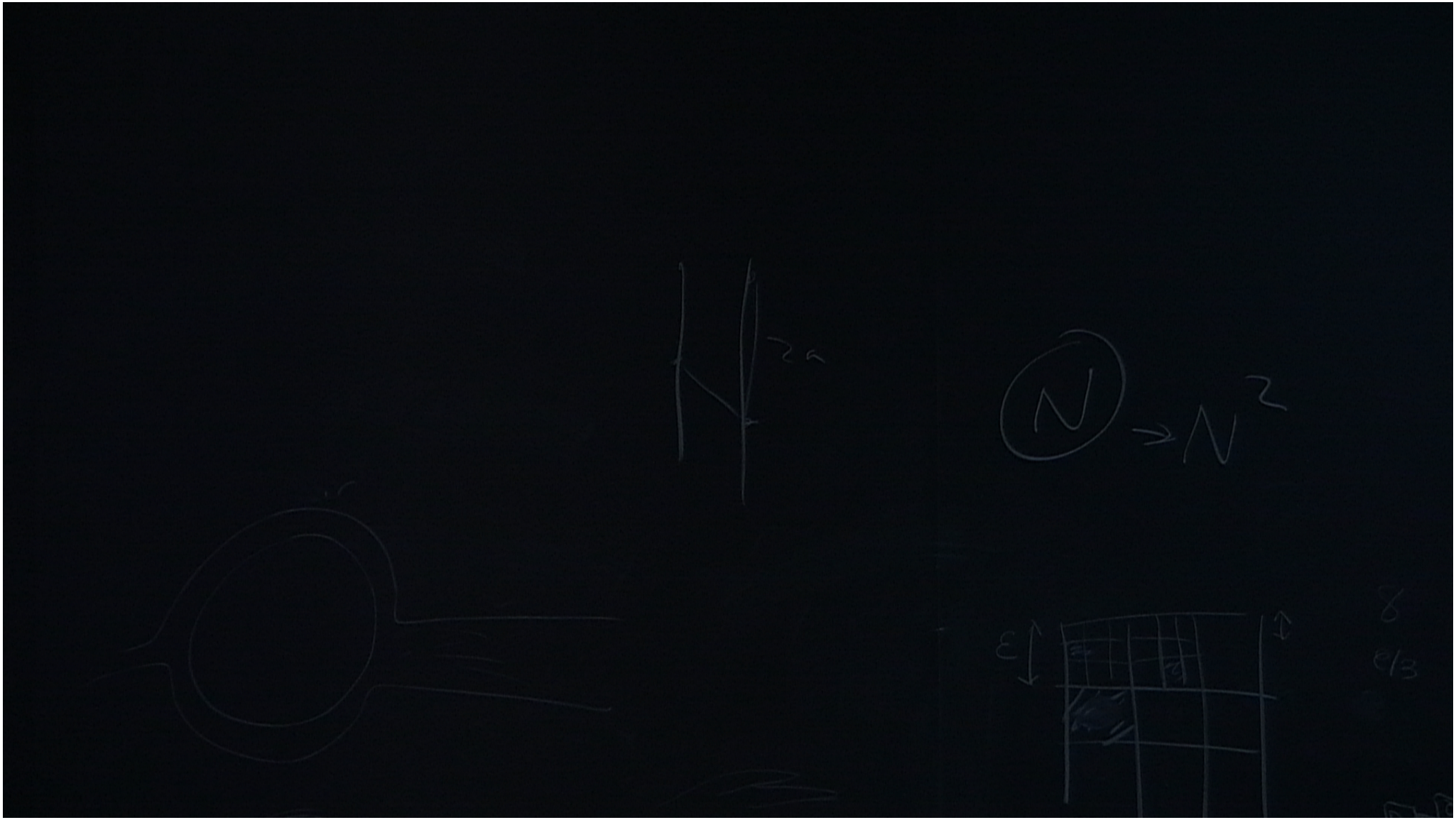
N^{2a}
$N \to N^2$
ε
δ
$\varepsilon/3$

Conclusion

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  - In particular sunflower seed angles
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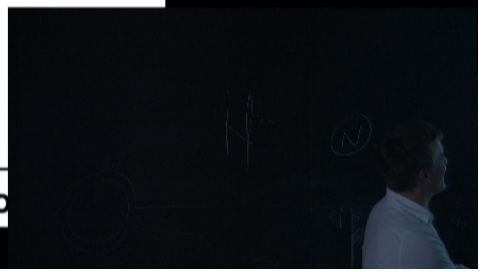

$$q\left|\alpha - \frac{p}{q}\right|$$

p/q better than p'/q' when $q\,|\alpha - p/q| < q'\,|\alpha - p'/q'|$