Title: Physics in Nature Presentation: The Endurance of Tress in Wind

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Abstract:

The Bending of Trees in Wind

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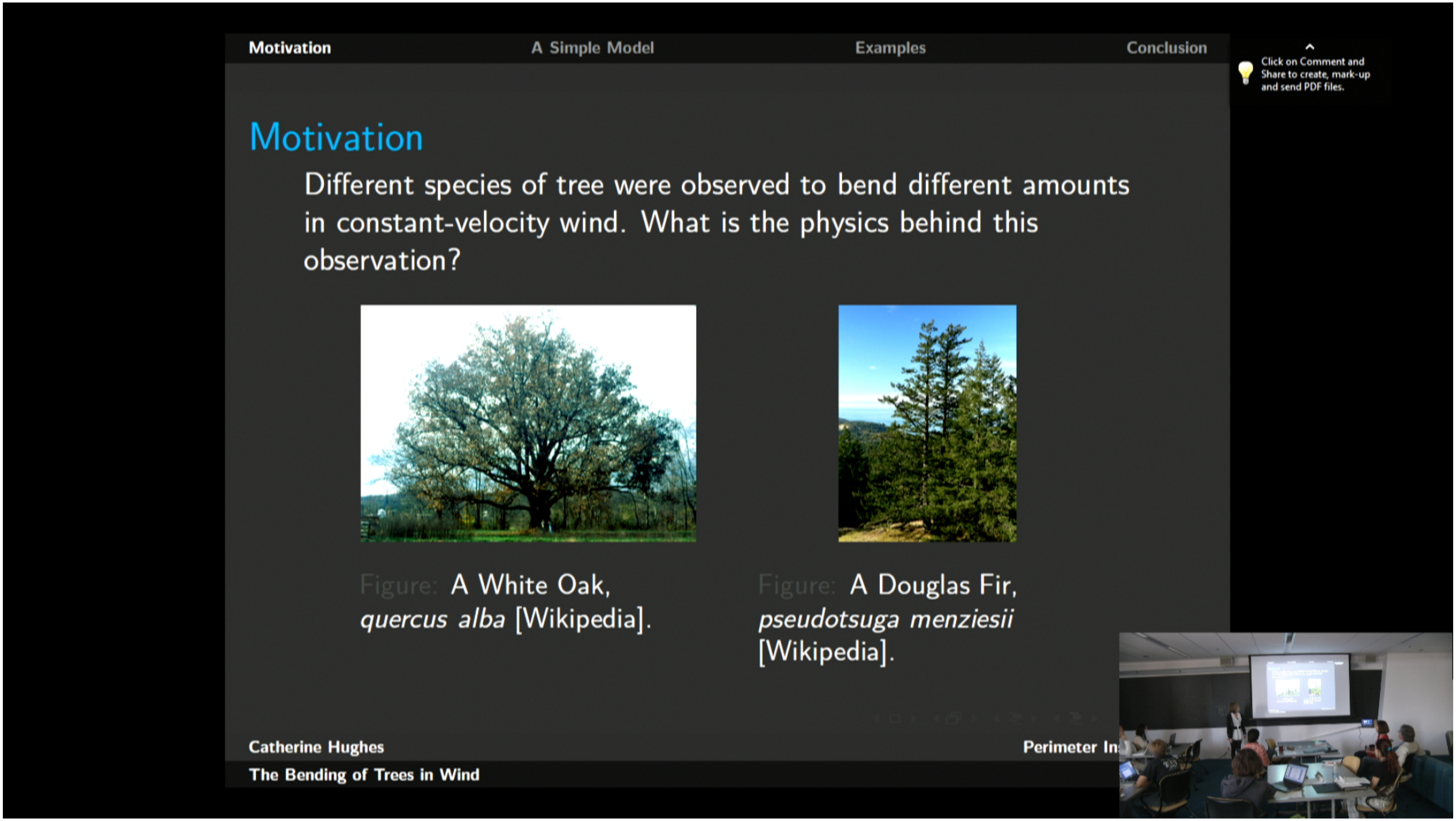
The Bending of Trees in Wind



Motivation A Simple Model Examples Conclusion

Motivation

Different species of tree were observed to bend different amounts in constant-velocity wind. What is the physics behind this observation?

Figure: A White Oak, *quercus alba* [Wikipedia].

Figure: A Douglas Fir, *pseudotsuga menziesii* [Wikipedia].

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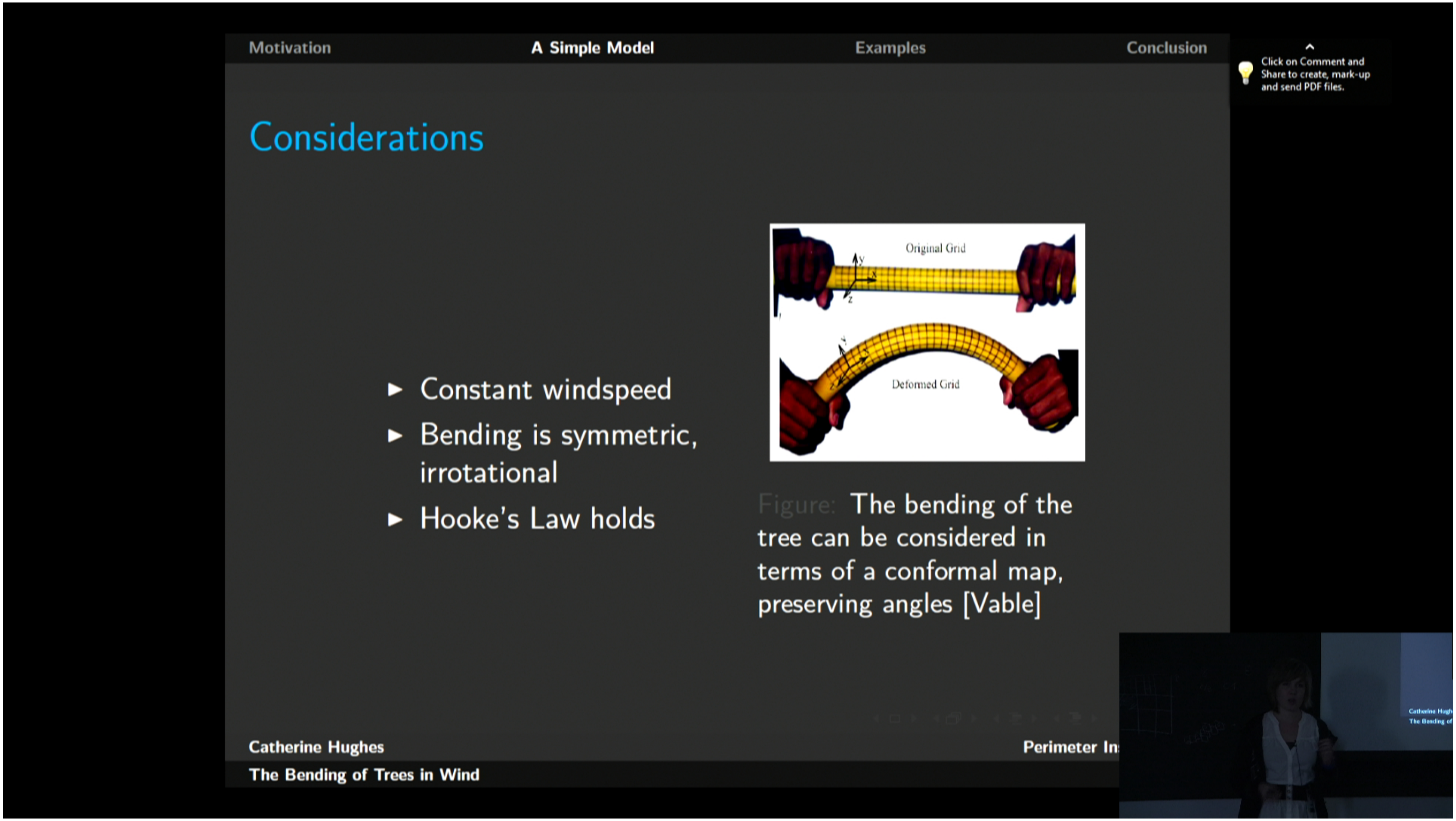

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Considerations

- Constant windspeed
- Bending is symmetric, irrotational
- Hooke's Law holds

Figure: The bending of the tree can be considered in terms of a conformal map, preserving angles [Vable]

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A Geometric Argument

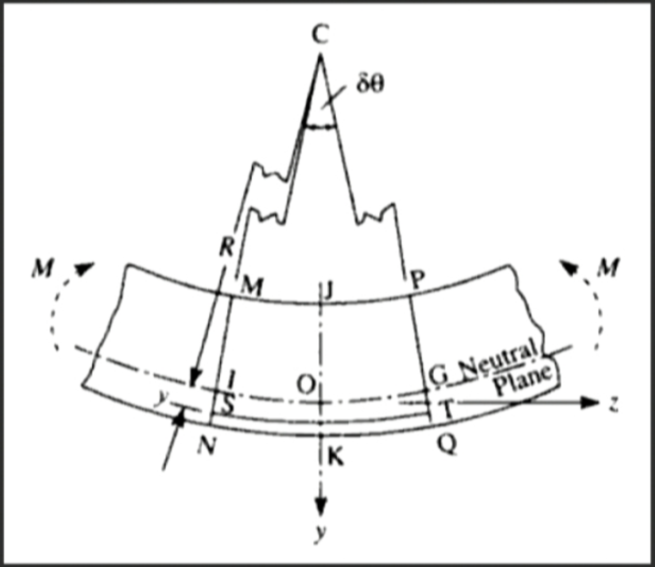
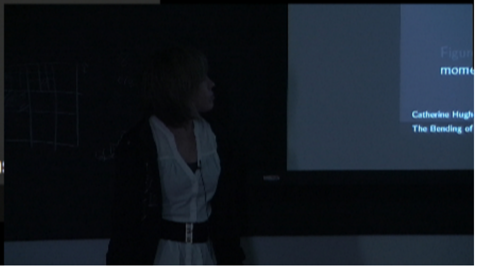
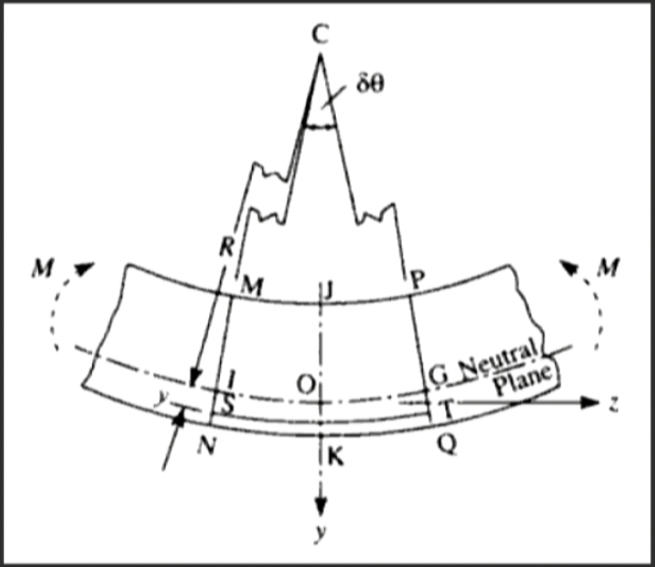


Figure: A schematic of a beam bent symmetrically under an applied moment M. The neutral plane undergoes no stretching [Megson].



A Geometric Argument



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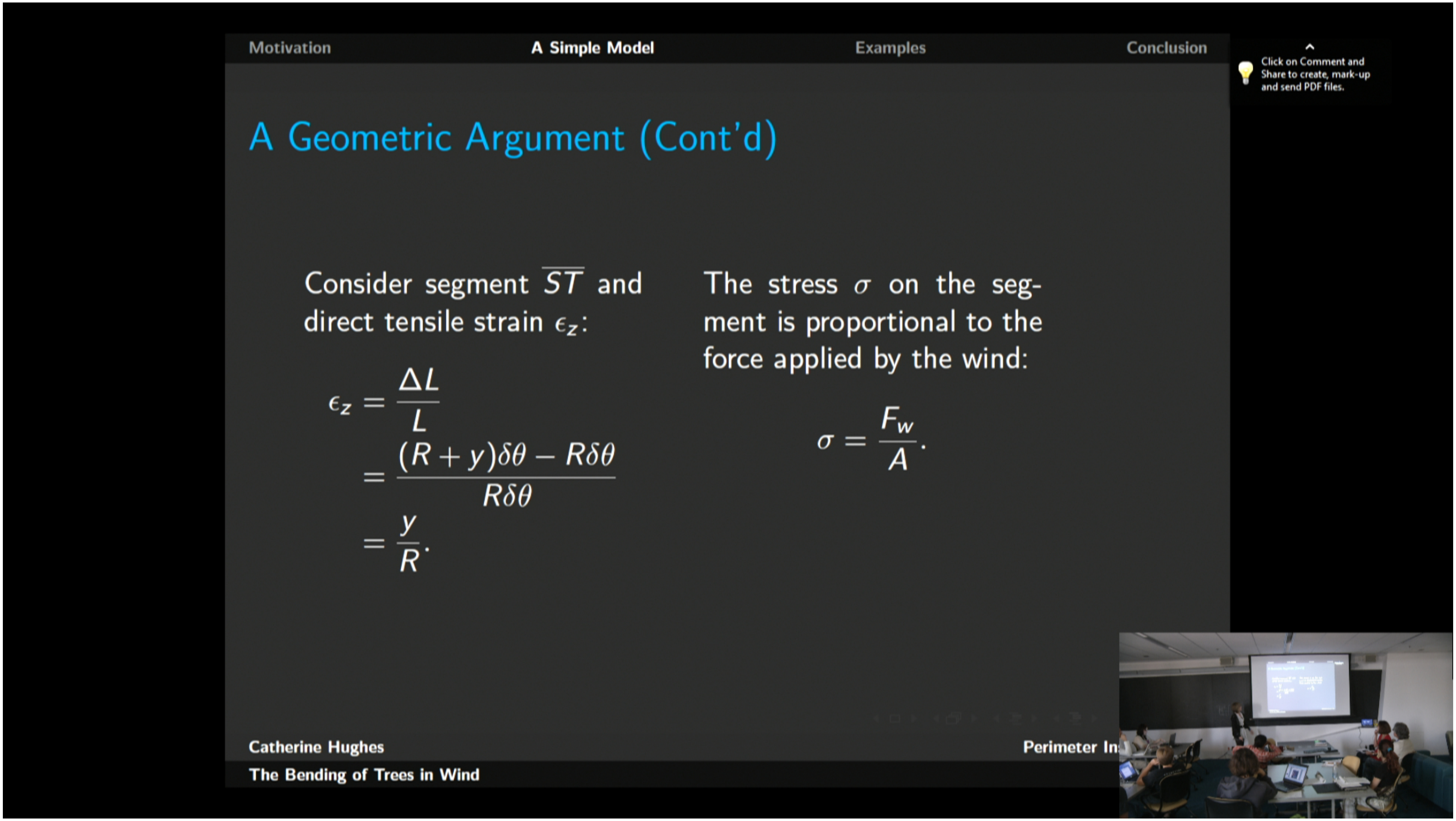


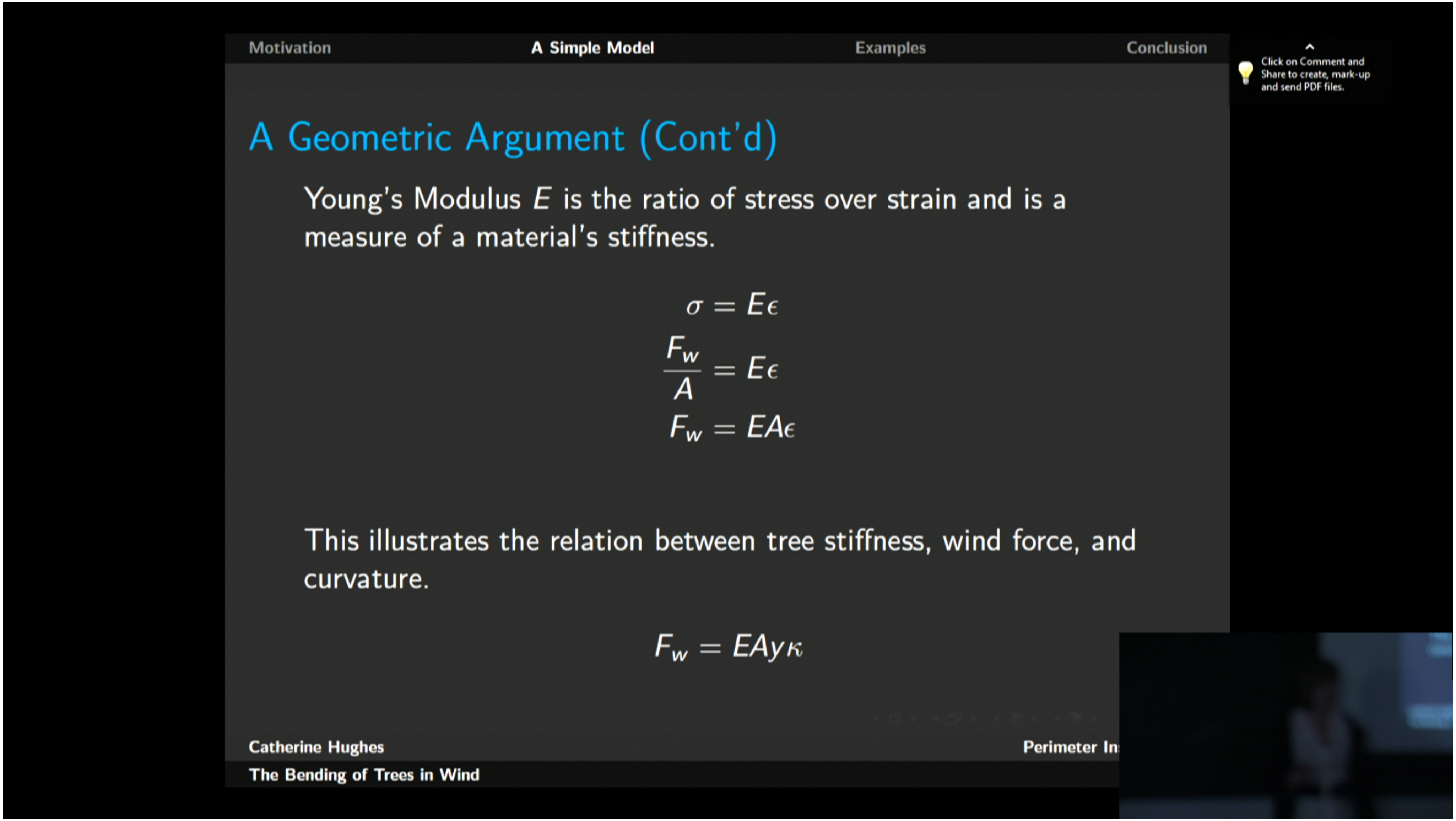
A Geometric Argument (Cont'd)

Consider segment $\overline{ST}$ and direct tensile strain ϵ_z:

$$\begin{aligned}\epsilon_z &= \frac{\Delta L}{L} \\ &= \frac{(R+y)\delta\theta - R\delta\theta}{R\delta\theta} \\ &= \frac{y}{R}.\end{aligned}$$

The stress σ on the segment is proportional to the force applied by the wind:

$$\sigma = \frac{F_w}{A}.$$



A Geometric Argument (Cont'd)

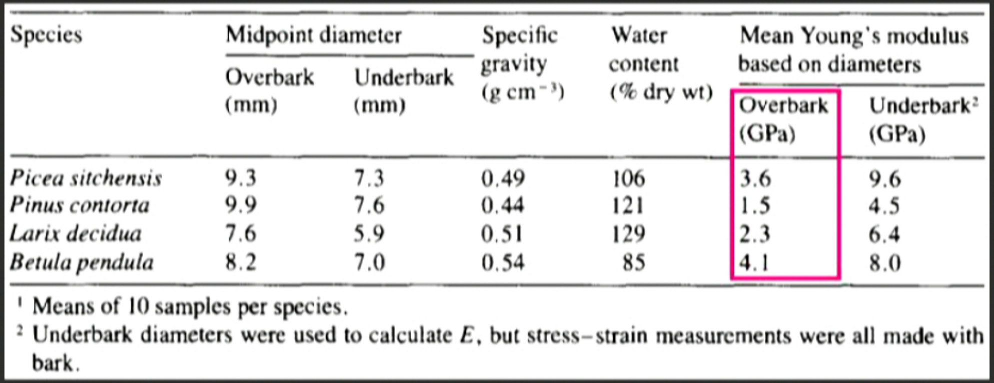
Young's Modulus E is the ratio of stress over strain and is a measure of a material's stiffness.

$$\begin{aligned} \sigma &= E\epsilon \\ \frac{F_w}{A} &= E\epsilon \\ F_w &= EA\epsilon \end{aligned}$$

This illustrates the relation between tree stiffness, wind force, and curvature.

$$F_w = EAy\kappa$$

Table of Young's Moduli for Different Tree Species

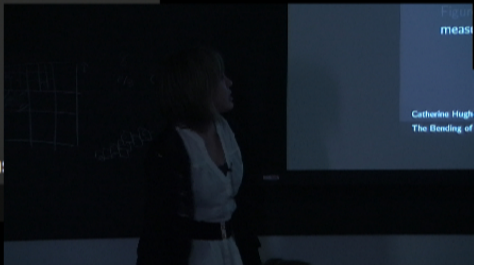


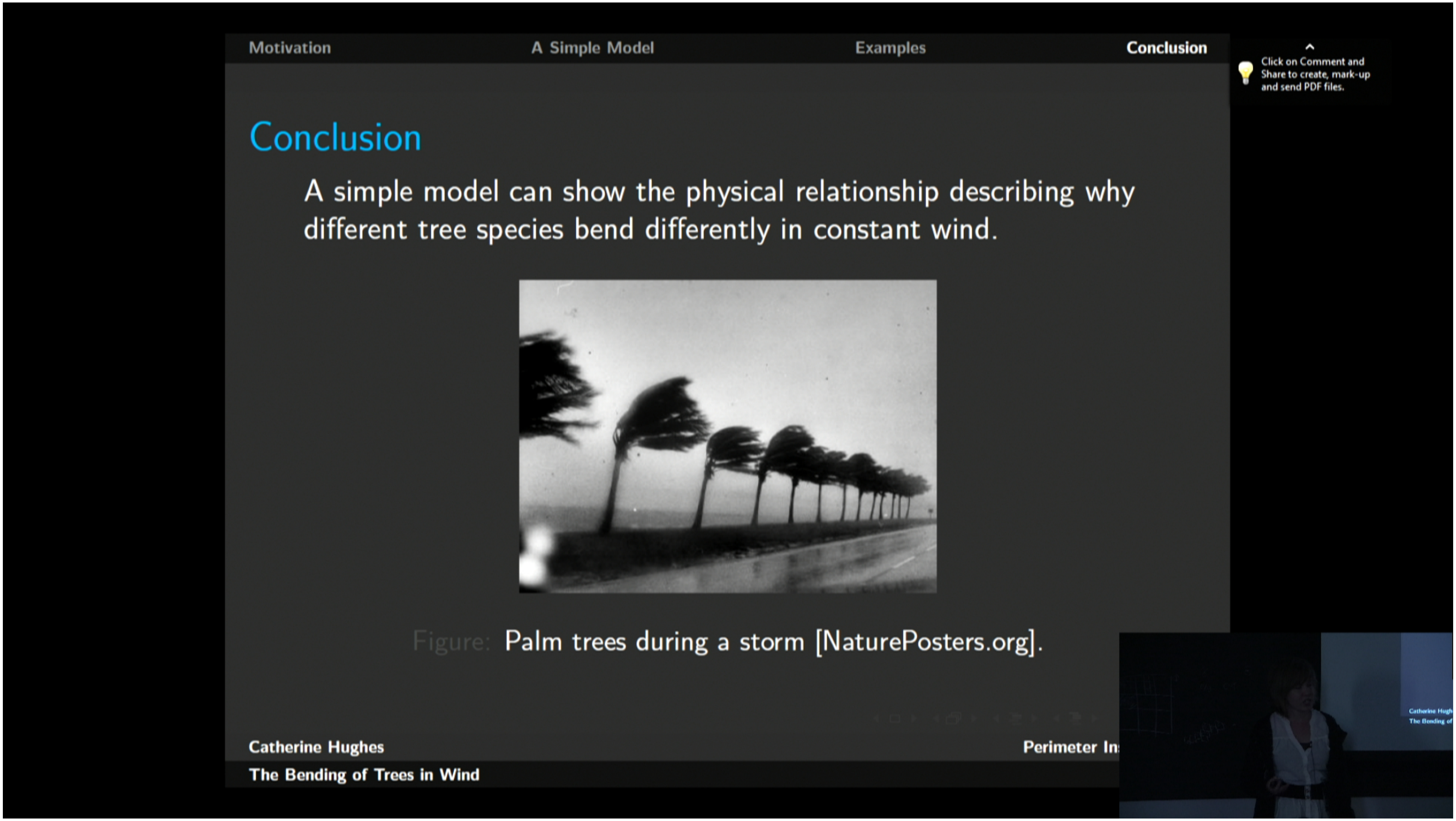
Species	Midpoint diameter		Specific gravity (g cm^{-3})	Water content (% dry wt)	Mean Young's modulus based on diameters	
	Overbark (mm)	Underbark (mm)			Overbark (GPa)	Underbark[2] (GPa)
Picea sitchensis	9.3	7.3	0.49	106	3.6	9.6
Pinus contorta	9.9	7.6	0.44	121	1.5	4.5
Larix decidua	7.6	5.9	0.51	129	2.3	6.4
Betula pendula	8.2	7.0	0.54	85	4.1	8.0

[1] Means of 10 samples per species.

[2] Underbark diameters were used to calculate E, but stress–strain measurements were all made with bark.

Figure: Mean Young's Modulus in bending of branches in four species, measured with bark [Cannell & Morgan].

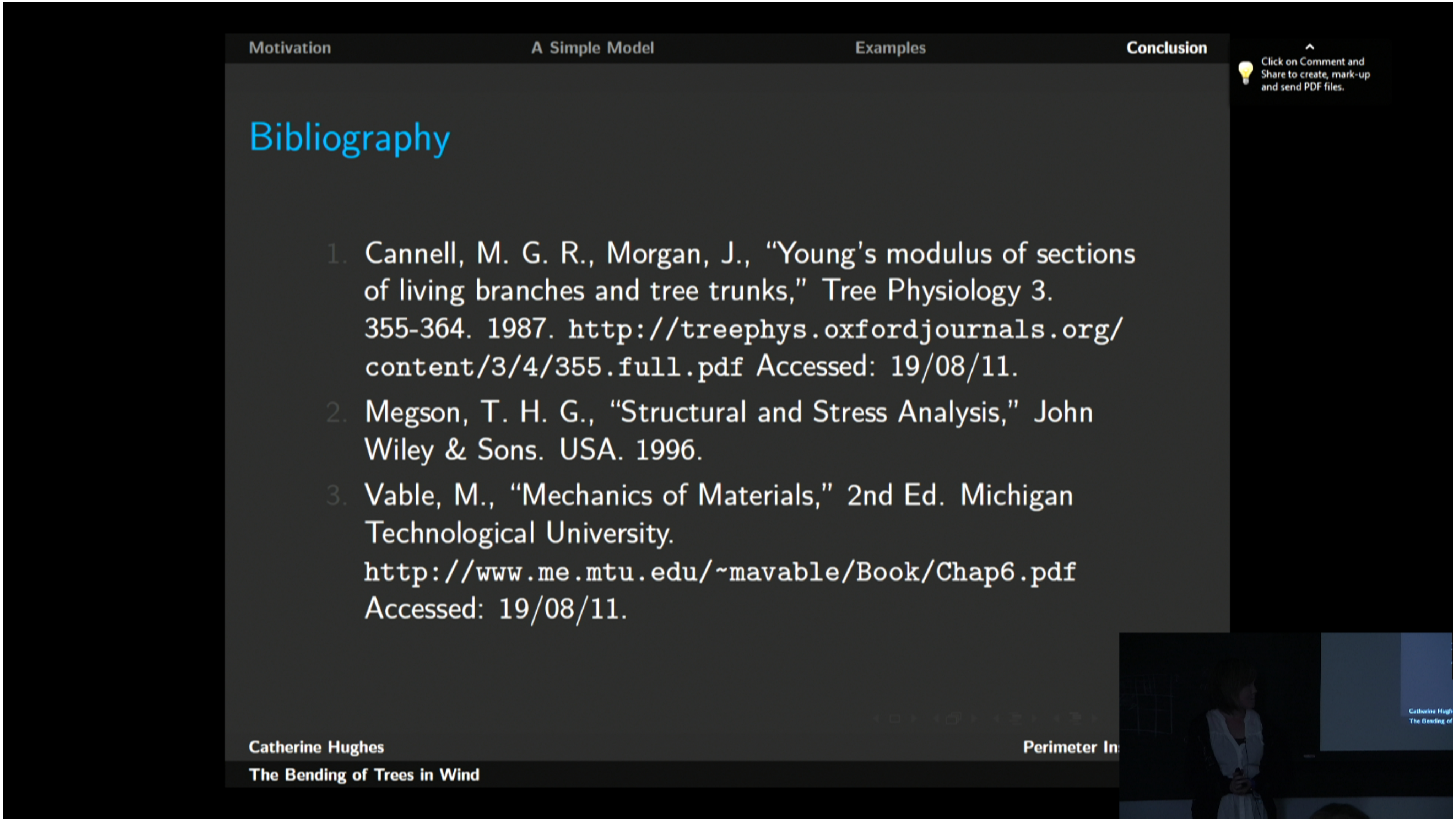




Conclusion

A simple model can show the physical relationship describing why different tree species bend differently in constant wind.

Figure: Palm trees during a storm [NaturePosters.org].



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