Title: Physics in Nature Presentation: The Way of Walking, or the Physics of Gait

Date: Aug 19, 2011 09:30 AM

URL: http://pirsa.pi.local/11080090

Abstract:

Overview

- Motivation
- Energy vs stability
- A simple model – perturbed motion stability

Motivation

- Rehabilitation
- Robots
- Curiosity?

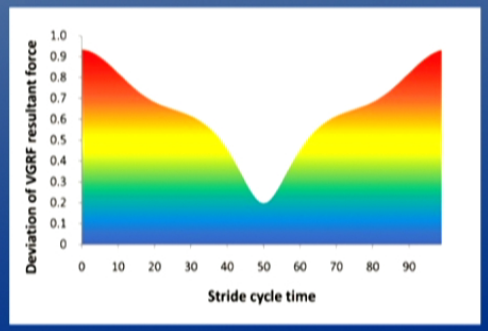
Energy

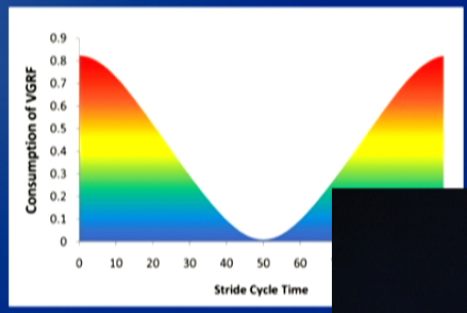
Consideration of vertical ground reaction forces (VGRF) leads one to define the action function:

$$\Psi(t_0) = \sum_{t=1/f}^{T} \left(F_Z(t) + F_Z(t+t_0) - mg\right)^2$$

In a whole stride gravitational potential energy = 0, the total vertical mechanical energy is the kinetic energy $E_k = \frac{1}{Tf}\sum \frac{1}{2}mv_t^2 > 0$

It turns out that we walk according to the Principle of Least Action, as defined above.





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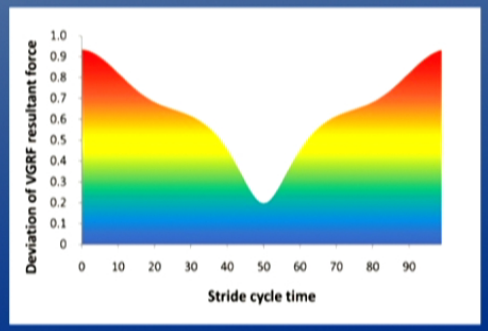
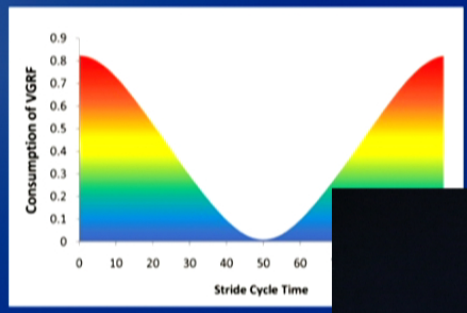




So where's stability?

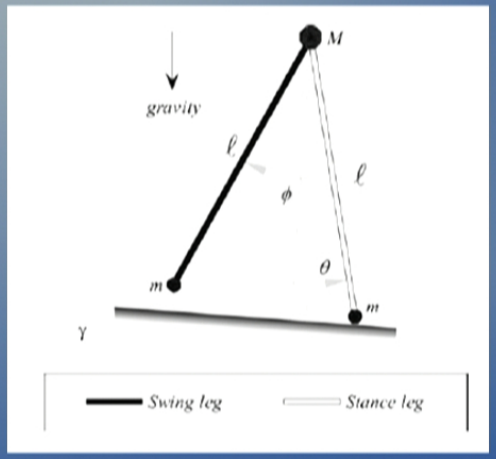
Apart from minimizing energy, we must also maintain stability in gait.

Not clear how relatively important the two notions are, so introduce destabilizing effects to study this.

A simple choice is walking downhill.

Experiments – a more stable and energetically more costly gait ensues.

A simple model – dynamic walker



Two phases of motion:

- Continuous leg swing
- Discrete foot-strike

The Hamiltonian for swing phase:

$$H(\mathbf{s}) = (1+\beta)\cos(\gamma-\theta) - \beta\cos(\gamma-\theta+\phi) + \frac{p_\theta^2 + (2-2\cos\phi)\,p_\theta p_\phi + (2-2\cos\phi+\beta^{-1})\,p_\phi^2}{2+2\beta\sin^2\phi}$$

$$\mathbf{s} = \begin{bmatrix}\theta & \phi & p_\theta & p_\phi\end{bmatrix}^{\mathrm{T}}$$

Foot-strike: inelastic collision and angular momentum conservation

$$\phi - 2\theta = 0$$

Resultant EoM's complicated and not enlightening.

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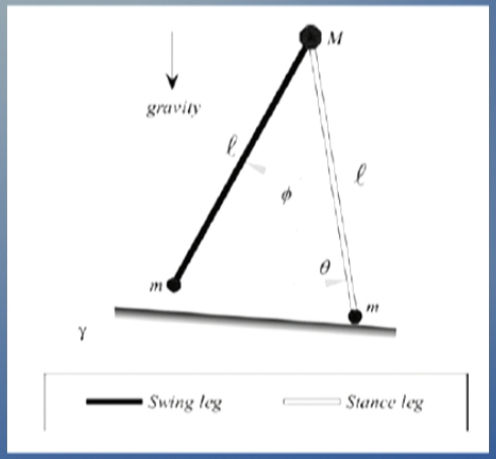

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Perturbations and Stability

Stability problem of the solutions to walking simplified by Poincare maps and Poincare sections

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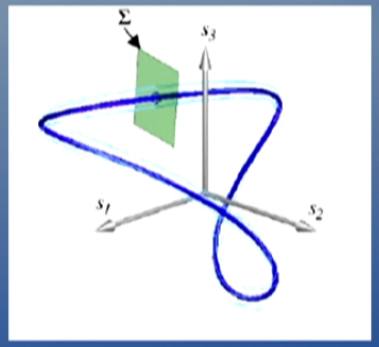
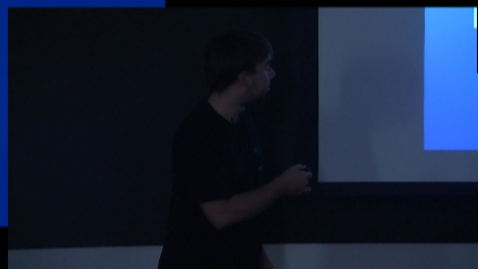


Allows to consider stability discretely instead of continuously, reduces dimensionality

Simplest solution: $\mathbf{s}_{eq}^{\Sigma} = \mathbf{P}\left(\mathbf{s}_{eq}^{\Sigma}\right)$

Need a criterion for stability!



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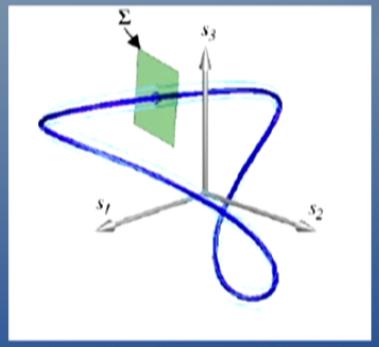

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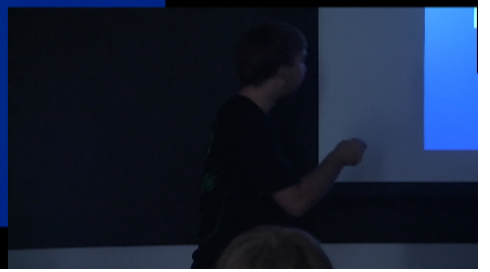

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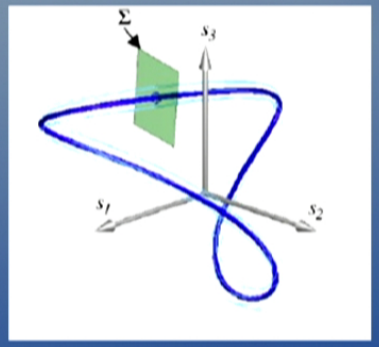

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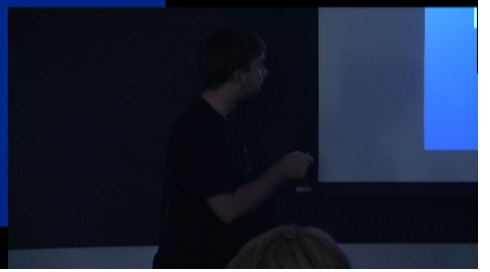
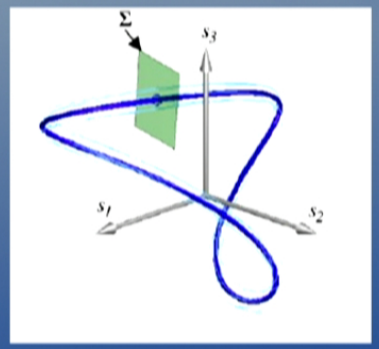


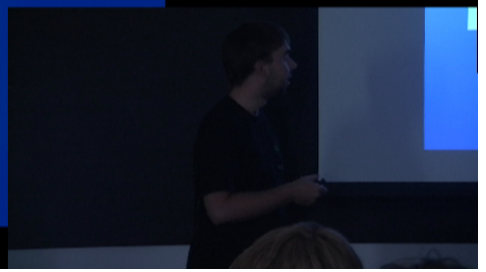
Orbital Stability

Look at the Poincare intersection – if points stay close to equilibrium then stable, otherwise unstable



Quantified way: estimate the Jacobian of the map and find eigenvalues

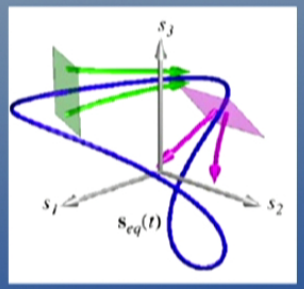
If the absolute value of any eigenvalue is > 1 then the equilibrium is unstable.



Local Dynamic Stability

If the system is described by

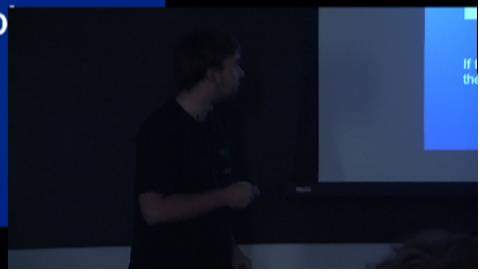
$$\dot{\mathbf{s}} = \mathbf{F}(\mathbf{s})$$



Then we want to look at eigenvalues of the Jacobian of F evaluated along the equilibrium solution

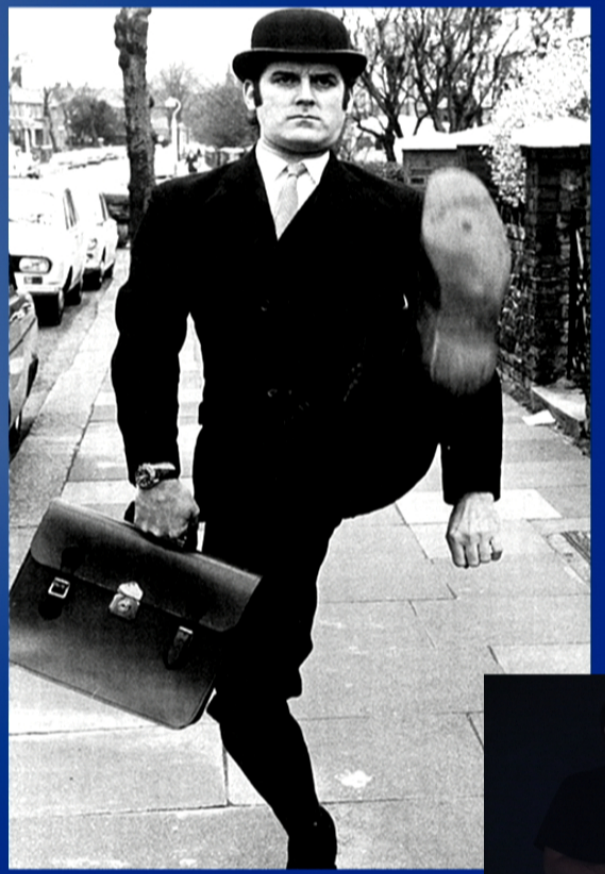
$$D_{\mathbf{s}}\mathbf{F}|_{\mathbf{s}_{eq}(t)}$$

If the real parts of all eigenvalues of the Jacobian are < 0 at time t, then the solution is locally stable at t. Otherwise it is locally unstab



Summary

- Human gait is governed by minimizing the energy cost and maximizing stability.
- Relative importance of the two is not known, so study perturbed systems.
- Experimental results support in most cases the predictions of the simplified models



http://www.youtube.com/user/BostonDynamics#p/a/u/0/ja_UsmXVPVk