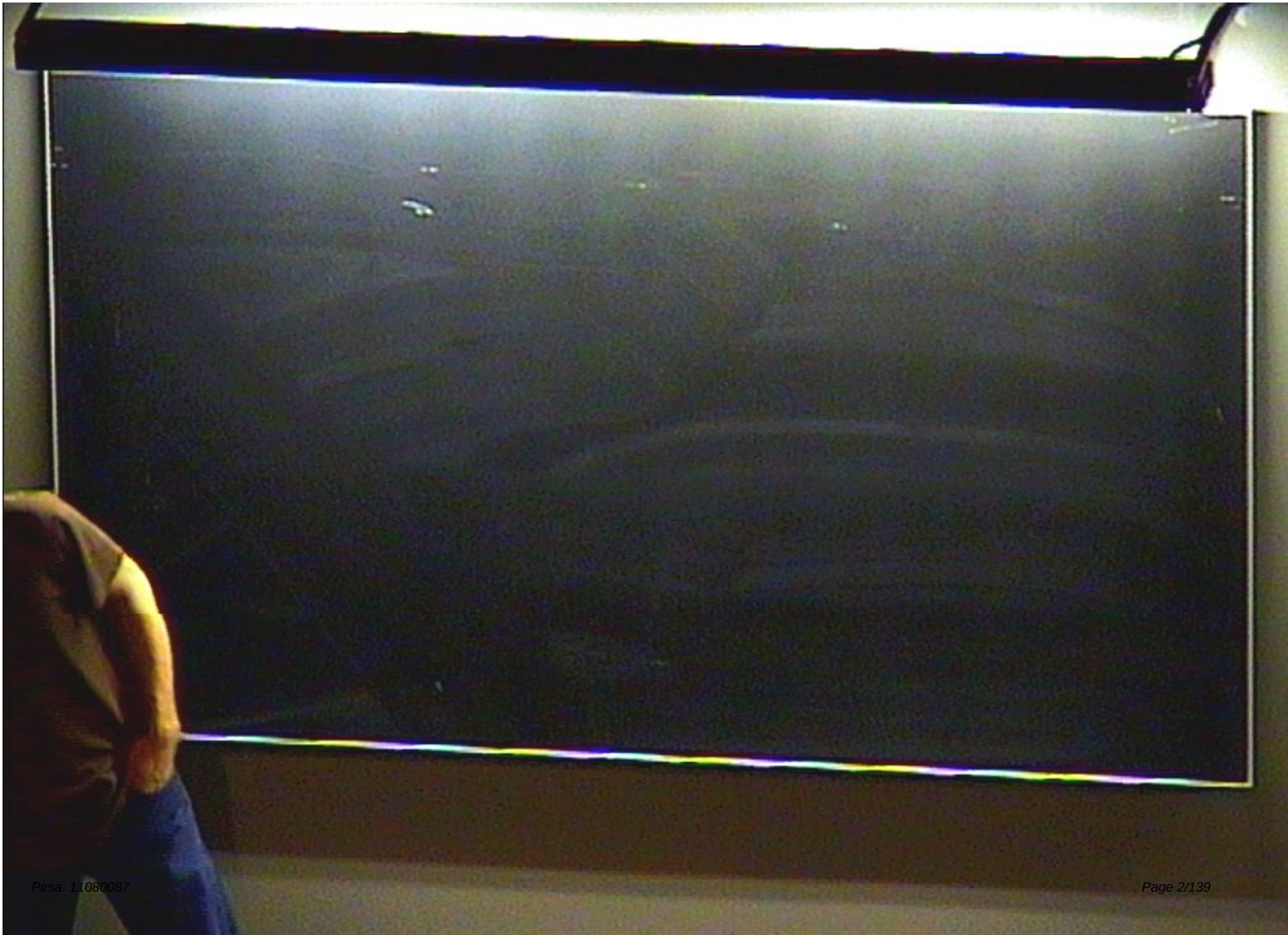


Title: Scattering Amplitudes from (super) Wilson Loops

Date: Aug 18, 2011 10:50 AM

URL: <http://pirsa.org/11080087>

Abstract: The S-matrix of $N=4$ super Yang-Mills in the planar limit enjoys a remarkable duality with non-BPS null polygon Wilson loops in the same theory, but with the role of momenta and position interchanged. I will attempt to explain how such a duality works, by stressing how familiar notions such as factorization limits, unitarity and the loop integrand translate into simple and verifiable statements about Wilson loops. This requires a suitable supersymmetric extension of Wilson loops, which I will describe. I will then discuss recent progress in using the full supersymmetry of the theory on the Wilson loop side to simplify scattering amplitude computations



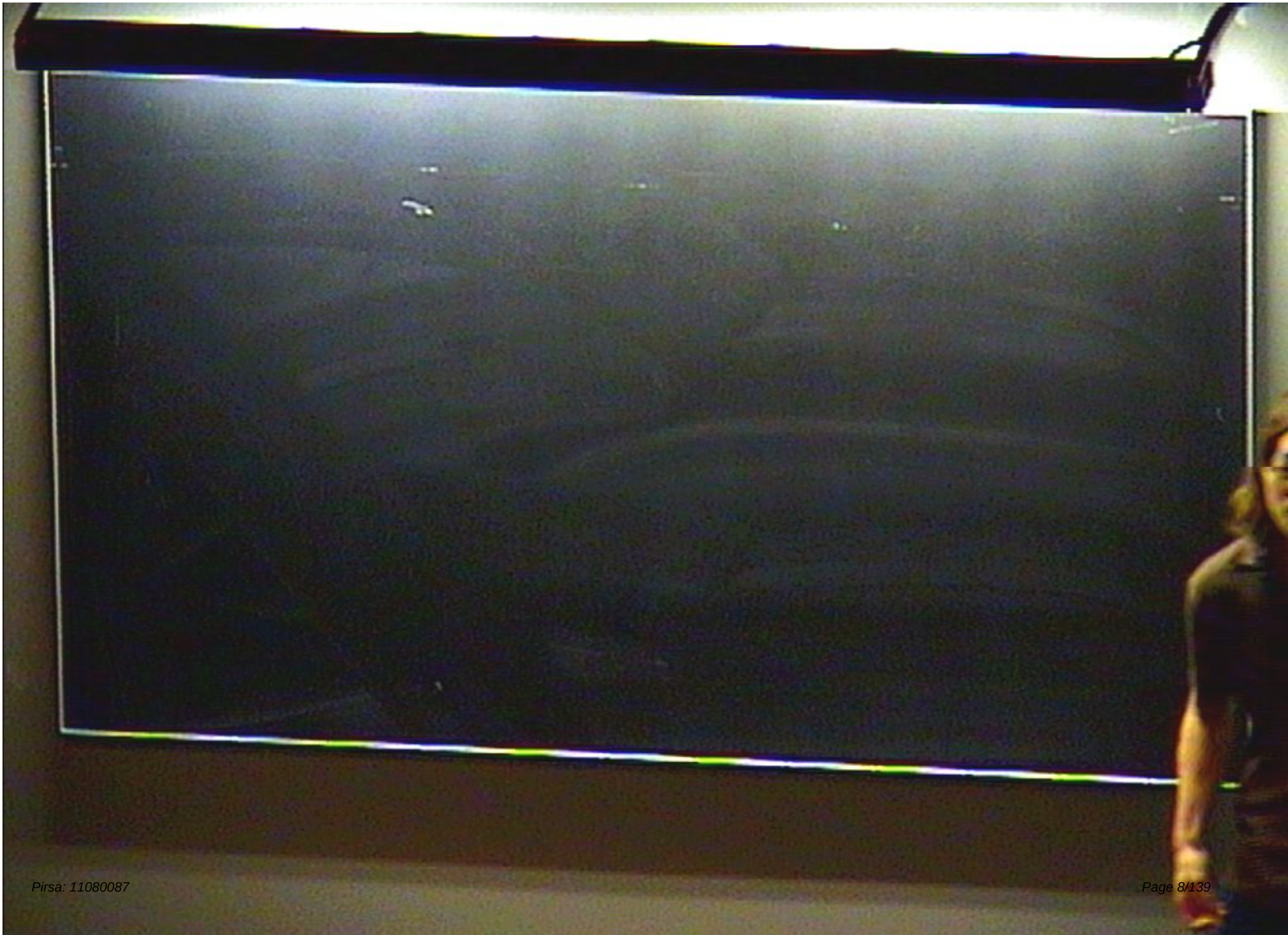




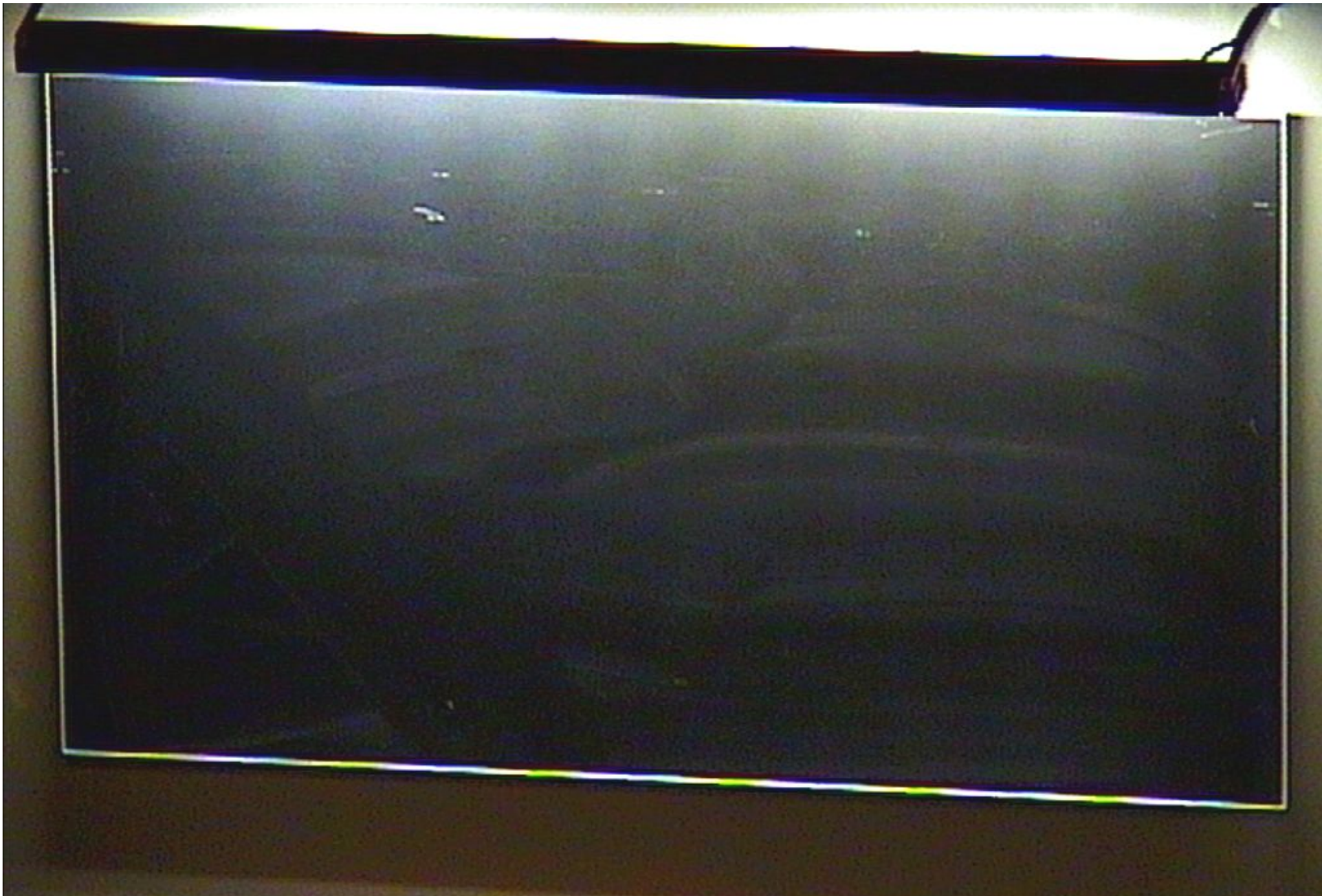




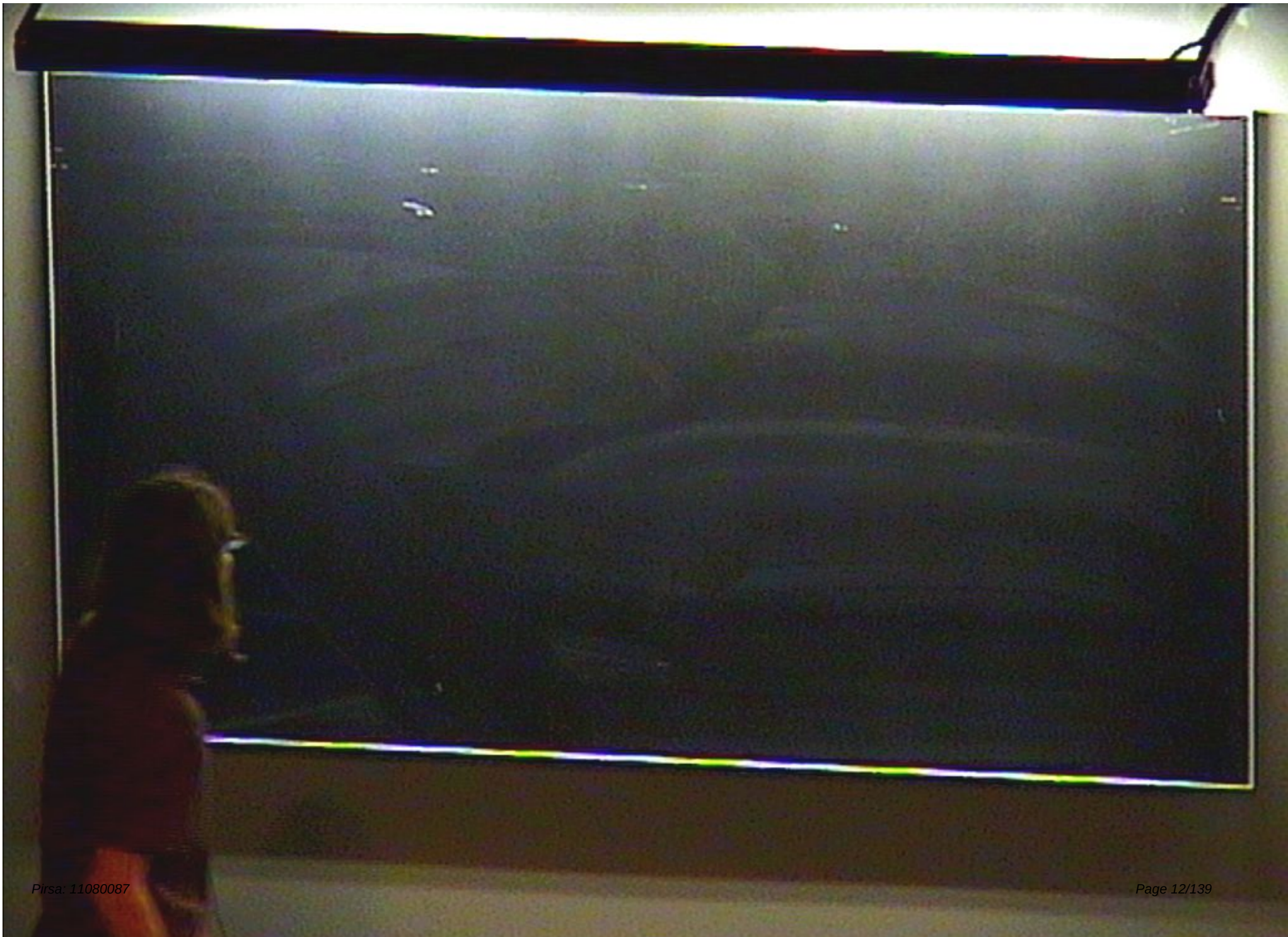




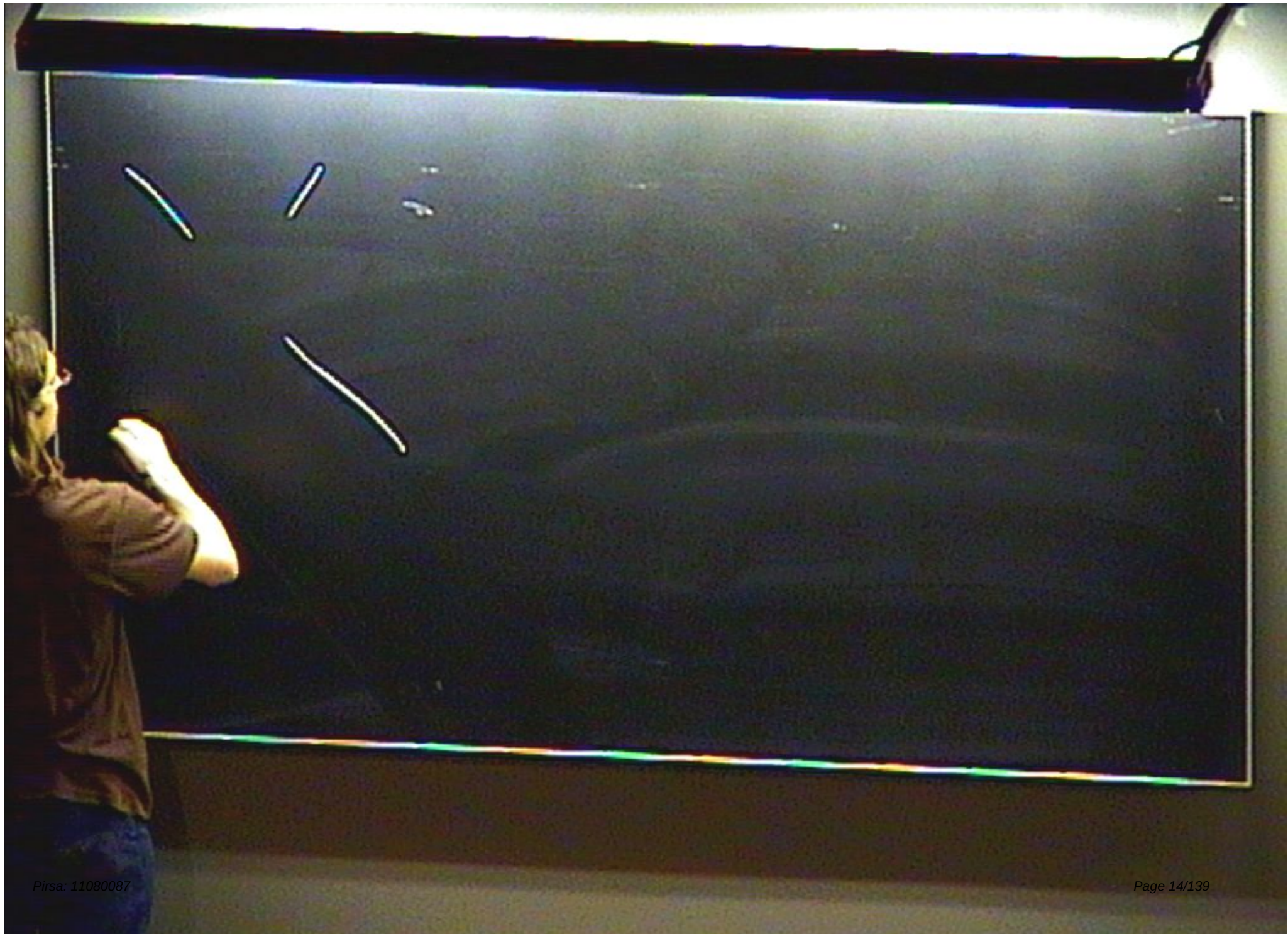




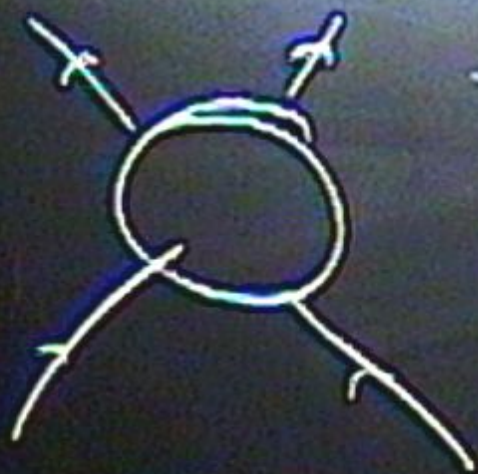






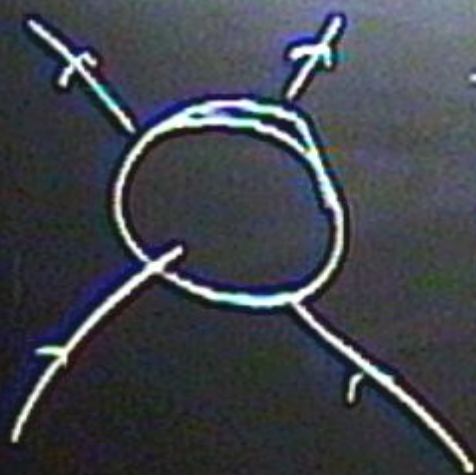




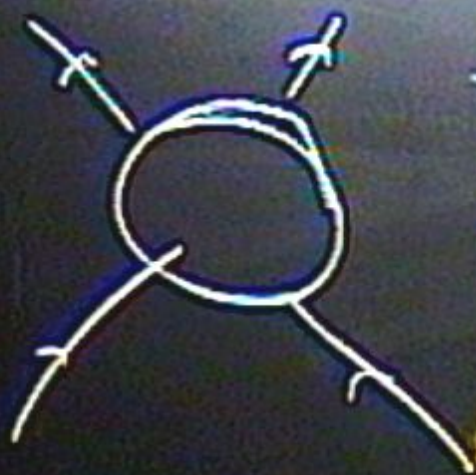




- Duality between a position space & mom. space
- $N=4$ Planar.

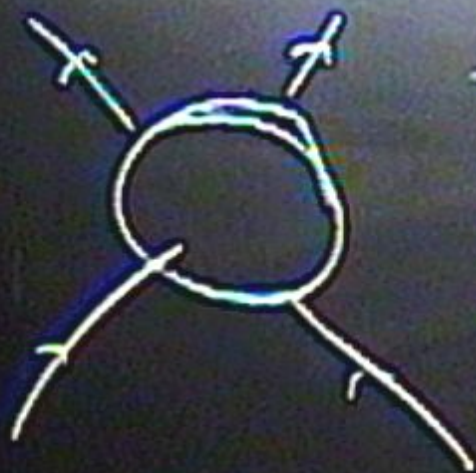


- Duality between a position space & mom. space
- $N=4$ Planar.
- No derivatives.



- Duality between a position space & mom. space
- $N=4$ Planar.
- No derived.

symmetry (Bran, Dixon, ...).



- Duality between a
position space & mom.
space

- $N=4$ Planar.

- No derivative.

- Symmetry (Bianchi, Dixon, ...).

- T-duality @ string comp. Plan.



- Duality between n position space & mom. space

- $N=4$ Planar.

- No derived.

- N^k MHV, not 10, n, d.

- Symmetry (Barn, Dixon, ...).

- T-duality @ string comp. time.

- Trees and loops.

$$A_{\text{int}} = \frac{\delta^{\text{th}}(\epsilon p) \delta^{\text{th}}(\epsilon n)}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$



- Symmetry (Spin, Color, ...).
- T-duality (1 string coupling).

- Duality between a position space & mom. space.

- $N=4$ Planar.

- No derivation.

- NIMV, not to, n, d.

$$A_n = \frac{\delta^{\text{as}}(\varepsilon_P) \delta^{\text{as}}(\varepsilon_M)}{\langle 123 \dots n \rangle}$$

$$A_n^{\text{tree}}$$

$$A_n = \frac{\delta^{\text{IR}}(\epsilon_F) \delta^{\text{IR}}(\epsilon_M)}{\langle 123 \dots n \rangle}$$

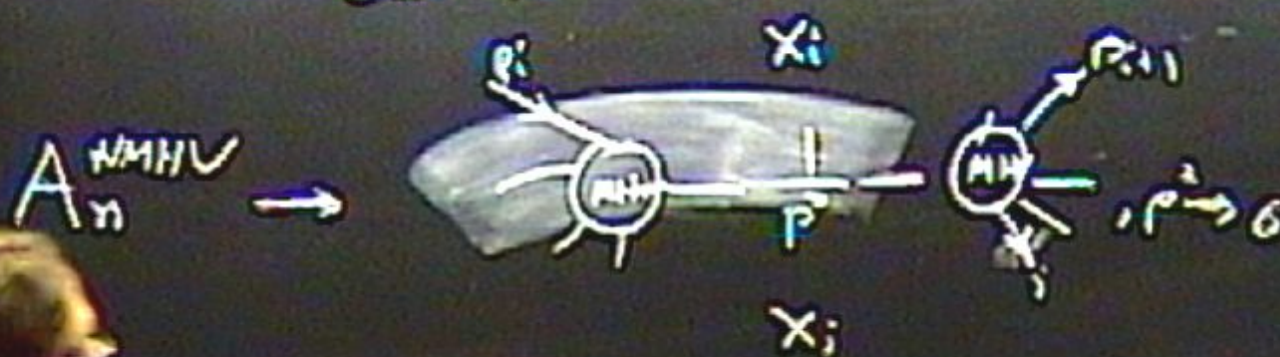
$$A_n^{\text{MHV}} \rightarrow$$



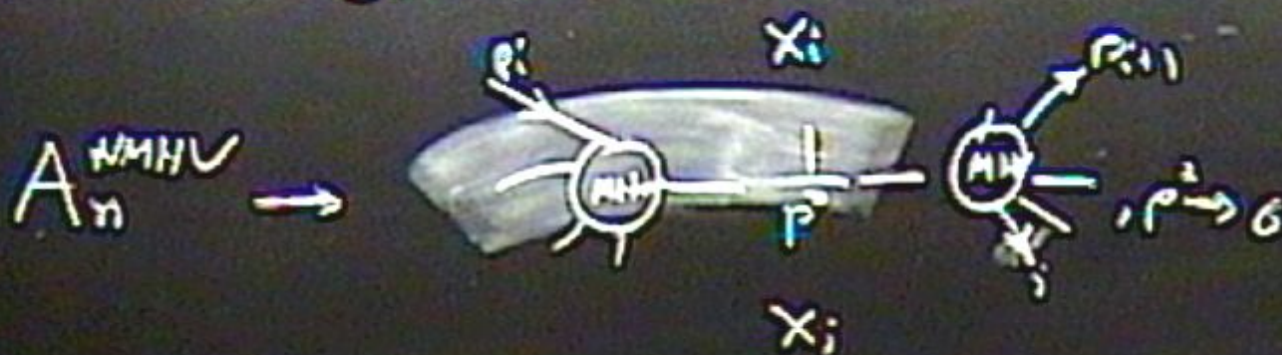
$$A_{\text{MHV}} = \frac{\delta^4(\Sigma p) \delta^{4\text{D}}(\Sigma \mathcal{M})}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$



$$A_n = \frac{\delta^{\mu\nu}(\xi p) \delta^{\alpha\beta}(\xi m)}{\langle 123 \dots n \rangle}$$



$$A_n^{\text{MHV}} = \frac{\delta^4(\sum p) \delta^{12}(\sum \lambda)}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$



- Traces and loops.



- Trees and Loops.



$$\int \frac{dx}{(x-s)^2 + d}$$

- Trees and Loops.



$$\int d^4x \int d^4y \frac{d^4k}{(x-y)^2 + i0} \left((x-y)^2 + i0 \right)^{-1} (x-y)^2 + i0 \dots$$

$$\int \frac{d^4k}{k^2 + i0} \sim \epsilon \ln \epsilon + \text{analytic}$$

- Trees and Loops.



$$\begin{aligned}
 & \int d^4x \\
 & \int d^4x \frac{1}{(x-x_1)^2 + (x-x_2)^2 + (x-x_3)^2 + \dots} \\
 & \sim \int_0^\epsilon \frac{dx}{x + \epsilon + \epsilon} \sim \epsilon \ln \epsilon + \text{analytic} \\
 & \sim (x-x_1)^2 \ln(x-x_1)^2 + \dots
 \end{aligned}$$

- Tree and Loops.



Diagram illustrating a tree diagram (left) and a loop diagram (right) in a quantum field theory context.

The tree diagram (left) shows a vertex x_i connected to a vertex p_i via a wavy line. A dashed line connects p_i to x_i .

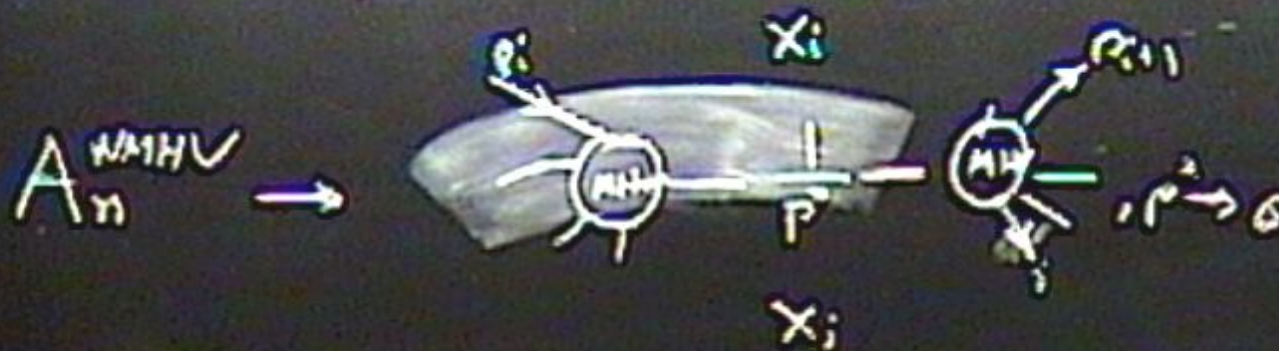
The loop diagram (right) shows a vertex x_i connected to a vertex p_i via a wavy line. A dashed line connects p_i to x_i . The loop is labeled $\int d^4x$.

The loop diagram is associated with the following mathematical expressions:

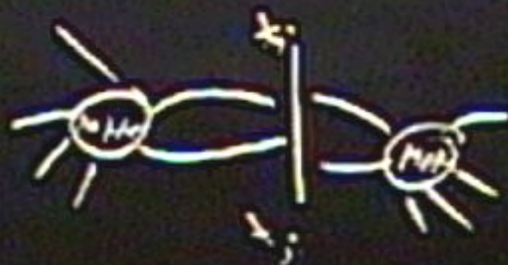
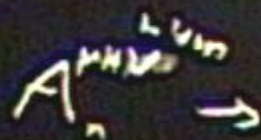
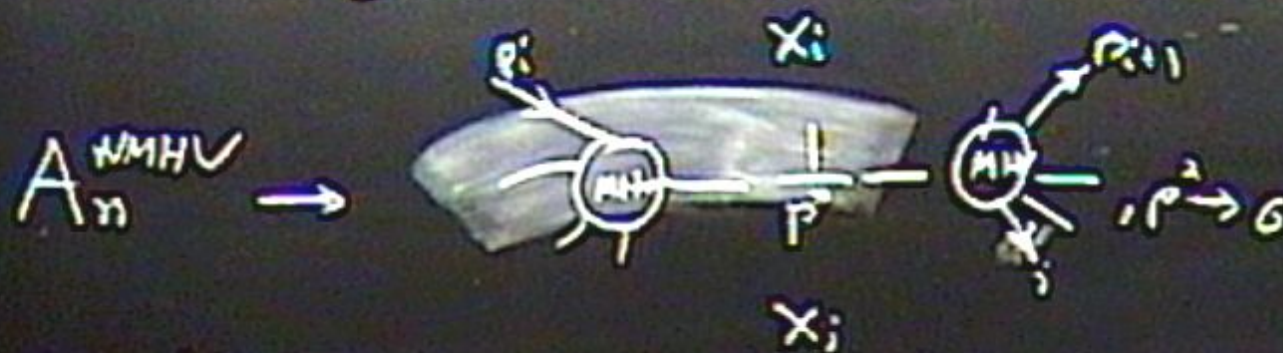
$$\int d^4x \frac{d^4x}{\epsilon + s + t} \sim \epsilon \ln \epsilon + \text{analytic}$$

$$\int d^4x \frac{d^4x}{(x_i - x_j)^2} \sim \frac{1}{(x_i - x_j)^2}$$

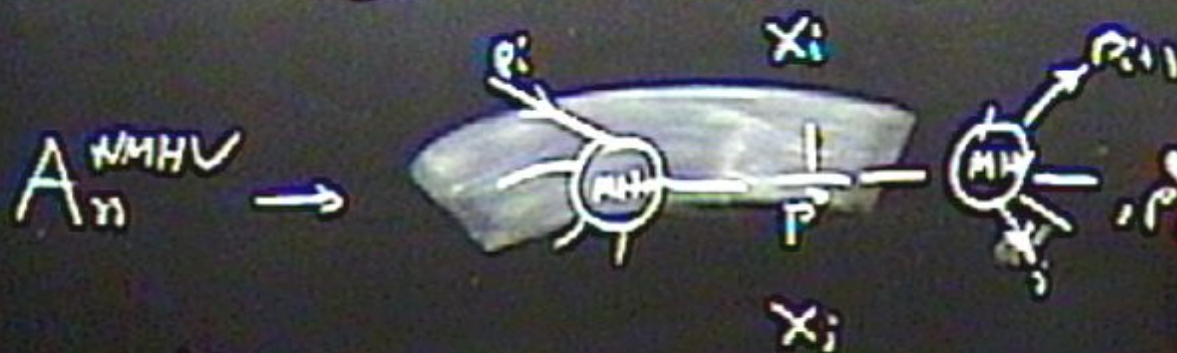
$$A_{\text{MHV}} = \frac{\delta^4(\sum p) \delta^{40}(\sum \lambda)}{\langle 123 \dots n \rangle}$$



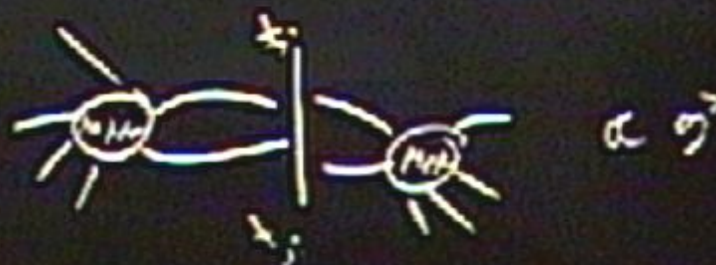
$$A_n^{\text{MHV}} = \frac{\delta^4(\sum p) \delta^{40}(\sum \lambda)}{\langle 123 \rangle \dots \langle n1 \rangle}$$



$$A_{MHV} = \frac{\delta^4(p) \delta^{(4)}(\Sigma M)}{\langle 12 \rangle \langle 23 \rangle \dots \langle n1 \rangle}$$



$A_{MHV} \rightarrow$



- Trees and Loops.



Surface

$\int d\vec{s}$

$$\frac{1}{(x_i - x_j)^2 + (x_i - x_j)^2 (L_P + 3r) + \dots}$$

$$\sim \int \frac{d\vec{s}}{\epsilon + 3 + \epsilon} \sim \epsilon \log \epsilon + \text{analytic}$$

$$\sim (x_i - x_j)^2 \log (x_i - x_j)^2 + \dots$$

$$\sim \int d\vec{s} \frac{dF}{dN} \sim \frac{1}{(x_i - x_j)^2}$$

- Traces and Loops.

- $(H_{loop})_{imp} = (H_{tree} + k)_{wc}$



Single

$$\int d^4s$$

$$\frac{1}{(x_i - s)^2 + i\epsilon} + \frac{1}{(x_f - s)^2 + i\epsilon} + \dots$$

$$\sim \int \frac{d^4s}{s^2 + i\epsilon} \sim \epsilon \log \epsilon + \text{analytic}$$

$$\sim (x_i - x_f)^2 \log(x_i - x_f)^2 + \dots$$

$$\int \frac{d^4s}{s^2 + i\epsilon} \sim \frac{1}{(x_i - x_f)^2}$$

- Traces and Loops.

- $(H \text{ loops})_{\text{imp}} = (H \text{ loops} + K) w_2$

~~$H \text{ loops} = 4K$~~



Sigma

$\int ds$

$\int_0^1 \frac{ds}{(x_i - s)^2 + (x_j - s)^2 + (x_i - x_j)^2 + s^2 + \dots}$

$\sim \int_0^1 \frac{ds}{\epsilon + s + x} \sim \epsilon \log \epsilon + \text{analytic}$

$\sim (x_i - x_j)^2 \log(x_i - x_j)^2 + \dots$
 $\sim \frac{1}{(x_i - x_j)^2}$

$\sim \int \frac{dN}{2N}$



x_i

N.M.H.V. 1-22



Logos.

~~Logos~~

$$= (w_{\text{new}} + k)_{\text{new}}$$

Sample

Sales

$$(x_1 - x_2)^2 + (x_1 - x_2) \cdot (x_1 + x_2) + x_3 \dots$$

- Trees and loops.

- $(\text{H loops})_{\text{rep}} = (\text{H loops} + k)_{\text{wc}}$

~~# of~~ $= 4k$

- A



- Trees and Loops.

- $(H_{\text{loops}})_{\text{reg}} = (H_{\text{loops}} + k)_{\text{wc}}$

- $A_n(p, \pi) = \underline{\delta^4(\epsilon p) \delta^{013}(\Sigma \lambda n)}$

- Trees and Loops.

- $(H_{\text{loops}})_{\text{mp}} = (H_{\text{trees}} + k)w_c$

- $A_n(p, \pi) = \frac{\delta^q(\varepsilon p) \delta^{q+1}(\varepsilon \lambda n)}{c_1 c_2 c_3 \dots c_n} W_n(x, \frac{1}{\varepsilon})$

$N \text{ MHU } 1-2$



$N \text{ MHU } 1-2$



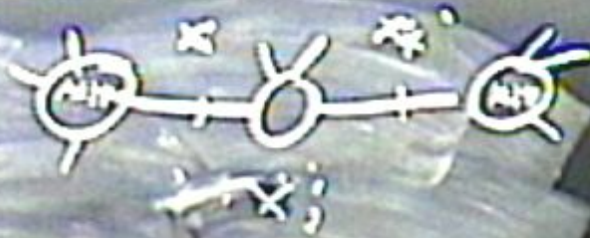
$\text{MHU} \cdot W_n = 0$

x_i



$\alpha \beta$

~~NMHV tree~~



~~NMHV 1-loop~~



$$\frac{d}{d\ln s} \langle W_n \rangle = 0$$

$$\frac{d}{d\ln s} W_n \propto g V_n$$

x_i



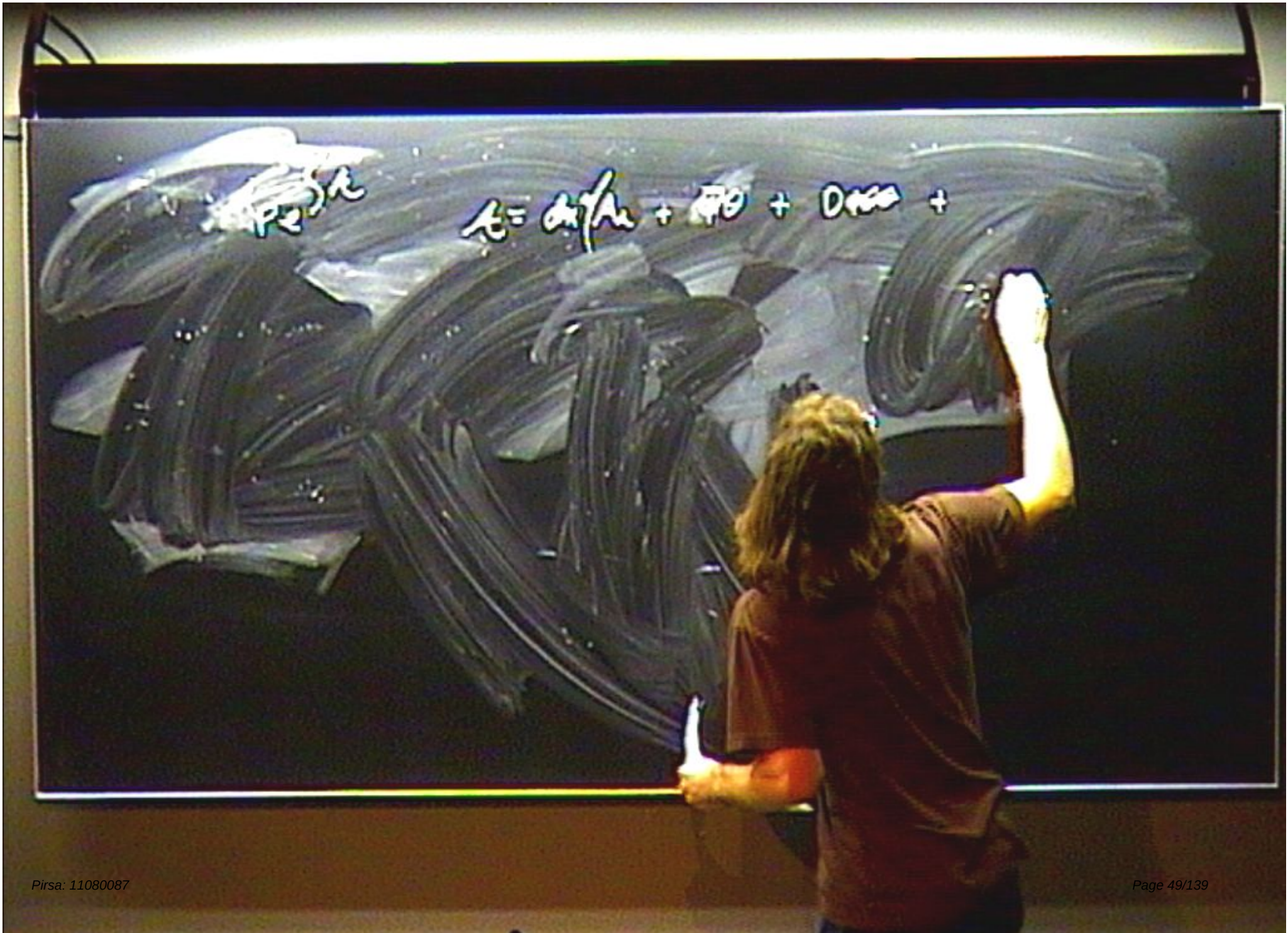
$$\propto g^2$$

- Trees and Loops.

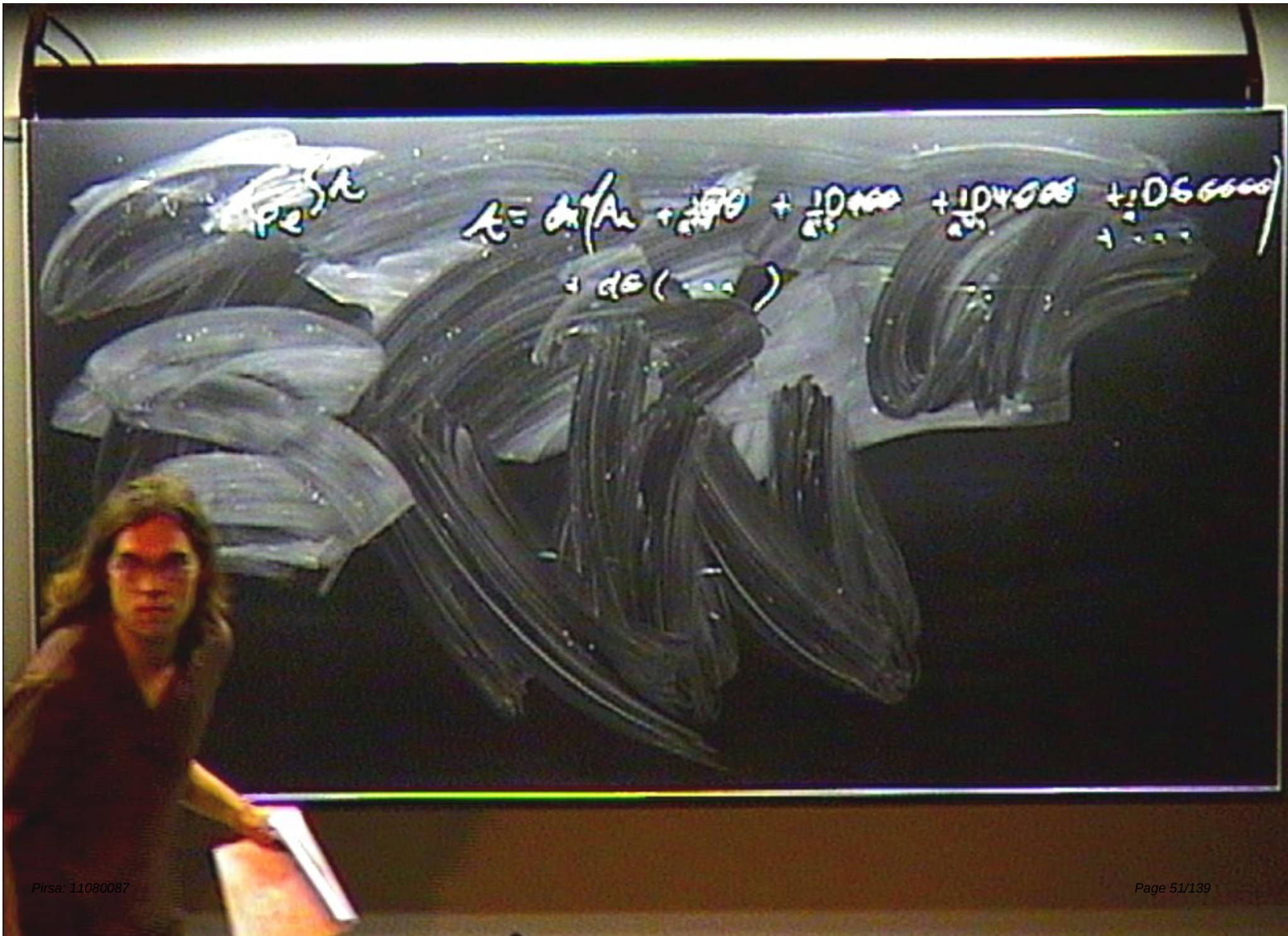
- $(H_{\text{loops}})_{\text{imp}} = (H_{\text{loops}} + k)w_c$

- $A_n(p, \pi) = \frac{\delta^q(\varepsilon p) \delta^{q+1}(\varepsilon \lambda n)}{c_1 c_2 \dots c_n} \text{Val}(x, \frac{q}{q+1})$, $c = \frac{2k}{4\pi}$

- $\{q_{\text{ren}} +$







SA
PE

$$K = \frac{d}{A} \left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4} + \frac{1}{A_5} + \dots \right) + dG(\dots)$$

$$S = \frac{1}{2} \left(\frac{F^2}{4} + \frac{1}{2} \frac{F^2}{4} + \frac{(F^2)^2}{2} + \frac{1}{2} \frac{F^2}{4} + \frac{1}{2} \frac{F^2}{4} + \dots \right)$$

Pe SA

$$E = \sigma_f A_f + \sigma_s A_s + \sigma_{s2} A_{s2} + \sigma_{s3} A_{s3} + \dots$$

$$S = \frac{1}{2} \int d^4x \left(F^2 + \bar{\psi} \not{\partial} \psi + \frac{(\phi \phi^\dagger)^2}{2} + \psi \phi \phi^\dagger + \bar{\psi} \psi \phi + \text{c.c.} \right)$$

$\mathcal{L}_{\text{eff}}$
 $\mathcal{L}_{\text{eff}}$

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \not{D} \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 + \dots$$

$$S = \int d^4x \left(\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \not{D} \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 + \dots \right)$$

$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $\phi \rightarrow \phi$

S_A
 P_e

$$L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \bar{\psi} \not{D} \psi + \frac{1}{2} D_\mu \phi D^\mu \phi + \frac{1}{2} D_\mu \phi D^\mu \phi + \frac{1}{2} D_\mu \phi D^\mu \phi + \dots$$

$$S = \frac{1}{4} \int d^4x \left(\frac{F^2}{4} + \bar{\psi} \not{D} \psi + \frac{(\phi \not{D} \phi)^2}{2} + \dots \right)$$

$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $F \rightarrow F$

SA
pe

$$\mathcal{L} = \partial_\mu \bar{\psi} \gamma^\mu \psi + \frac{1}{2} \bar{\psi} \psi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D_\mu \phi D^\mu \phi + \frac{1}{2} D_\mu \psi D^\mu \psi + \dots$$

Feynman

$$S = \int d^4x \left(\frac{F^2}{4} + \bar{\psi} \gamma^\mu \psi + \frac{(\partial \phi)^2}{2} + \dots \right)$$

$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $F_{\mu\nu} \rightarrow F_{\mu\nu}$
 $\phi \rightarrow \phi$
 $\psi \rightarrow \psi$
 Gauge Field

Fiducial

$\int_{\mathcal{P}_2} \mathcal{L}$

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \not{D} \psi + \frac{1}{2} D_\mu \phi D^\mu \phi + \frac{1}{2} D_\mu \chi D^\mu \chi + \frac{1}{4} D_\mu F_{\nu\rho} F^{\nu\rho} + \dots$$

$$S = \int d^4x \left(\frac{F^2}{4} + \bar{\psi} \not{D} \psi + \frac{(\partial\phi)^2}{2} + \psi \psi + \bar{\psi} \psi + \mathcal{L}_{\text{gauge}} \right)$$

$A_\mu \rightarrow A_\mu$

$F_{\mu\nu} \rightarrow F_{\mu\nu}$

$$S \rightarrow \int d^4x \left[F^2 + \bar{\psi} \not{D} \psi + \frac{(\partial\phi)^2}{2} + \bar{\psi} \psi \right] + \lambda \left[G^2 + \bar{\psi} \psi + \mathcal{L}_{\text{gauge}} \right]$$

$\int d^4x$
 $\int d^4x$

$$L = \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \lambda \phi^4 + \dots$$

Feynman

$$S = \int d^4x \left(\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \lambda \phi^4 \right)$$

$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $F_{\mu\nu} \rightarrow F_{\mu\nu}$
 gauge field.

$$S = \int d^4x \left(F_{\mu\nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \lambda \phi^4 \right)$$

Fiducial

$\mathcal{L}_E$
 $\mathcal{L}_E$

$$\mathcal{L} = \mathcal{L}_E + \mathcal{L}_G + \mathcal{L}_H + \mathcal{L}_I + \mathcal{L}_J + \dots$$

$$S = \int d^4x \left(\frac{F^2}{4} + \bar{\psi} \not{D} \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \dots \right)$$

$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $F_{\mu\nu} \rightarrow F_{\mu\nu}$
Fierz

$$S \rightarrow \int d^4x \left[FG + \bar{\psi} \not{D} \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \dots \right]$$

Fiducial

$\mathcal{L}_{\text{eff}}$

$$\mathcal{L} = \text{tr} \left(\frac{1}{2} \partial_\mu A_\nu \partial^\mu A^\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \gamma^\mu \partial_\mu \psi + \bar{\psi} \psi + \dots \right)$$

$$S = \int d^4x \left(-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi + \bar{\psi} \psi + \dots \right)$$

$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $F_{\mu\nu} \rightarrow \lambda(G_{\mu\nu})$
 gauge field.

$$S \rightarrow \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi + \bar{\psi} \psi + \lambda(G_{\mu\nu}^2 + \bar{\psi} \psi + \dots) \right] \text{SDYM}$$

Fiducial

$\int_{\mathcal{P}} \mathcal{L}$

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \bar{\psi} \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \mathcal{L}_{\text{int}} + \dots$$

$$\mathcal{L} = \frac{1}{4} \left(\frac{F^2}{4} + \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \bar{\psi} \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \mathcal{L}_{\text{int}} \right)$$

$A_\mu \rightarrow A_\mu$

$F_{\mu\nu} \rightarrow F_{\mu\nu}$

$$\mathcal{L} \rightarrow \left[F_{\mu\nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi + \frac{1}{2} (\partial_\mu \phi)^2 + \bar{\psi} \psi \right]_{\text{SDYM}} + \lambda (G^2 + \bar{\psi} \psi + \mathcal{L}_{\text{int}})$$

- Trees and Loops.

- $(N_{\text{loops}})_{\text{reg}} = (N_{\text{loops}} + k) w_c$

- $A_n(p, \pi) = \frac{\delta^q(\varepsilon p) \delta^{\sigma_1 \sigma_2}(\varepsilon \lambda n)}{c_1 \lambda^{\sigma_1} c_2 \lambda^{\sigma_2} \dots c_n \lambda^{\sigma_n}} W_n(x, \frac{\partial}{\partial x})$, $c = \frac{\sigma^2 k}{4\pi^2}$.

- $\{q_{\mu A}^{\sigma}\} W_n(x, \frac{\partial}{\partial x}) = 0$

- Trees to SDYM

- Trees and Loops.

- $(H_{\text{loops}})_{\text{prop}} = (H_{\text{trees}} + k) w_c$

- $A_n(p, \pi) = \frac{\delta^q(\varepsilon p) \delta^{\text{ons}}(\varepsilon \lambda n)}{= 1 2 \dots 2 3 \dots \dots n 1 \dots} W_n(x, \frac{\partial}{\partial x}) ; c = \frac{2\pi k}{4\pi} .$

- $\{q_{\mu\nu} + a^{\mu\nu} \frac{\partial}{\partial \sigma^{\mu\nu}}\} W_n(x, \frac{\partial}{\partial x}) = 0$

- Trees are dual to SDYM we.

Fiducial

$\mathcal{L}_{\text{eff}}$

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \not{D} \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 + \dots$$

$$S = \int d^4x \left(\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \not{D} \psi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \right)$$

$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $F_{\mu\nu} \rightarrow \partial_\mu A_\nu - \partial_\nu A_\mu$
 gauge field.

$$S \rightarrow \int d^4x \left(\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \right) \text{ SDYM}$$

Fiducial

$\mathcal{L}_{\text{eff}}$

$$\mathcal{L} = \mathcal{L}_{\text{eff}} + \frac{1}{\Lambda^2} + \frac{1}{\Lambda^4} + \frac{1}{\Lambda^6} + \frac{1}{\Lambda^8} + \dots$$

$$S = \int d^4x \left(\frac{F^2}{4} + \bar{\psi} \not{D} \psi + \frac{(\phi^\dagger)^2}{2} + \psi \psi + \bar{\psi} \bar{\psi} + \mathcal{L}_{\text{eff}} \right)$$

$\bar{\psi} \rightarrow \bar{\psi} \gamma_5$
 $\psi \rightarrow \psi \gamma_5$
 $\phi \rightarrow \phi$
 $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}} + \mathcal{L}_{\text{eff}}$
 $\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff}} + \mathcal{L}_{\text{eff}}$

$$S \rightarrow \int d^4x \left(\frac{F^2}{4} + \bar{\psi} \not{D} \psi + \frac{(\phi^\dagger)^2}{2} + \psi \psi + \bar{\psi} \bar{\psi} \right) \text{SDYM}$$

$$gA = (dA)$$

Fiducial

$\int d^4x$

$$\mathcal{L} = \det(A) + \frac{1}{2} \bar{\psi} \psi + \frac{1}{2} \bar{\psi} \psi + \frac{1}{2} \bar{\psi} \psi + \frac{1}{2} \bar{\psi} \psi + \dots$$

$$S = \frac{1}{4} \int d^4x \left(F_{\mu\nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi + \bar{\psi} \psi \right)$$

$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $\lambda \rightarrow \lambda$
 $\lambda(G_{\mu\nu})$
 λ is a scalar field.

$$S \rightarrow \int d^4x \left(F_{\mu\nu}^2 + \bar{\psi} \gamma^\mu \partial_\mu \psi + \bar{\psi} \psi \right) + \lambda \left(G^2 + \bar{\psi} \psi + F_{\mu\nu}^2 \right)$$

SDYM

$$qA = (\partial W + A \wedge W)$$

$$\bar{q}A = \mathcal{O}(\lambda)$$

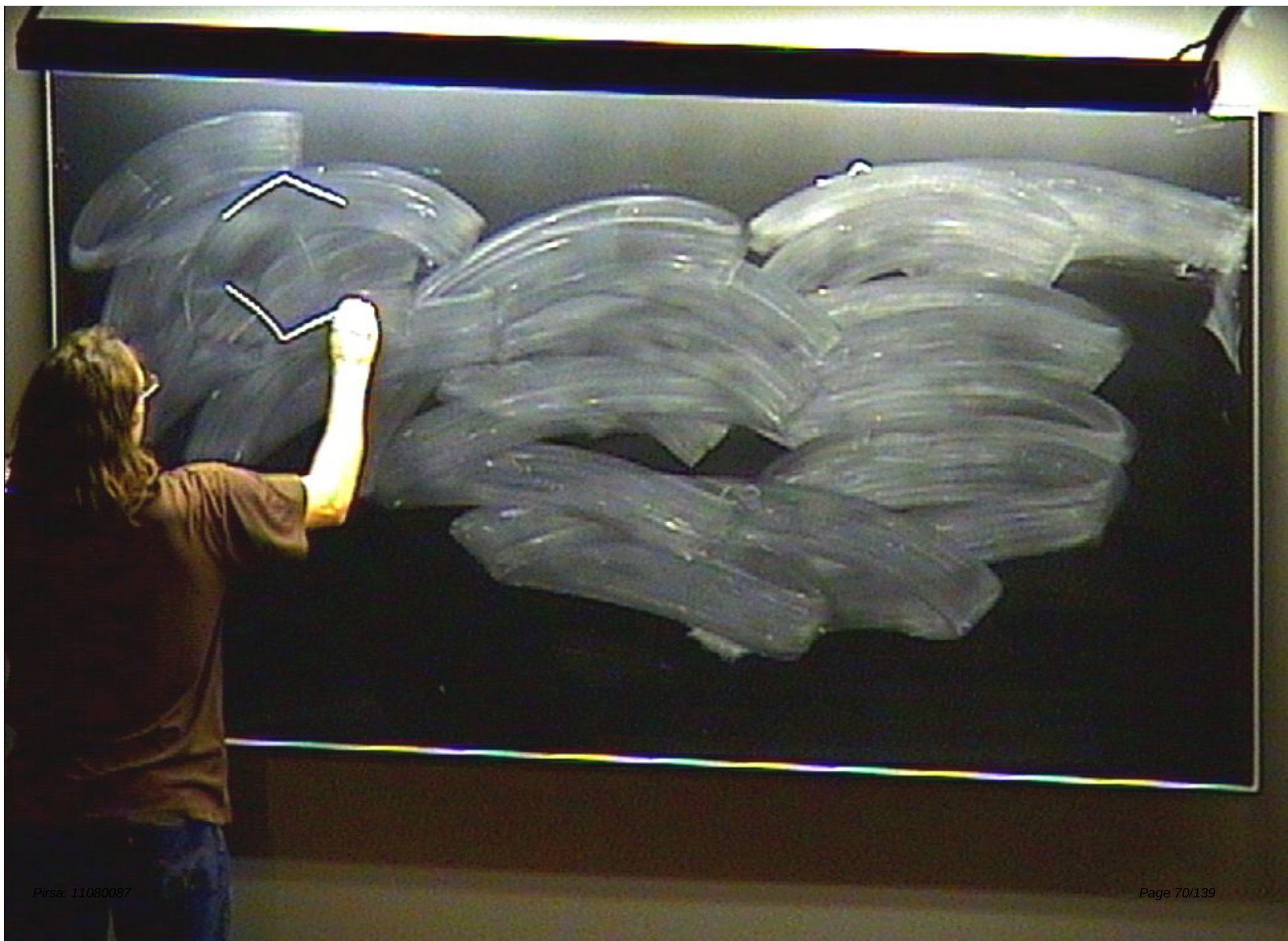
- Trees and Loops.

- $(H_{\text{loops}})_{\text{reg}} = (H_{\text{loops}} + k) w_c$

- $A_n(p, \pi) = \frac{\delta^4(\epsilon p) \delta^{\text{orb}}(\epsilon \lambda n)}{= 1 2 \times 2 3 \times \dots \times n 1} W_n(x, \frac{\epsilon}{\alpha^n})$, $c = \frac{2^2 w_c}{4\pi}$.

- $\left[g_{\mu\nu} + \alpha^{\mu\nu} \frac{\partial}{\partial \sigma^{\mu\nu}} \right] W_n(x, \frac{\epsilon}{\alpha^n}) = 0$

- Trees are dual to SDYM w_c



- Trees and Loops.

- $(N_{\text{loops}})_{\text{reg}} = (N_{\text{loops}} + K)_{\text{reg}}$

- $A_n(p, m) = \frac{\delta^4(\sum p) \delta^{03}(\sum \lambda n)}{1 \cdot 2 \cdot \dots \cdot n!} W_n(x, \frac{1}{a^n})$, $c = \frac{g^2 k}{4\pi}$

- $\left[q_{\mu n} + a^{\mu n} \frac{\partial}{\partial a^n} \right] W_n(x, \frac{1}{a^n}) = 0$

- Trees are dual to SDYM $W \Rightarrow \tilde{q} = 0 \Leftrightarrow$ Yangian Symmetry

- Trees and Loops.

$$- (H_{\text{loops}})_{\text{mp}} = (H_{\text{trees}} + K)_{\text{wc}}$$

$$- A_n(p, n) = \frac{\delta^4(\epsilon p) \delta^{05}(\epsilon \lambda n)}{c_1 c_2 c_3 \dots c_n} W_n(x, \frac{\partial}{\partial n}) ; c = \frac{2\pi k}{4\pi}$$

$$- \left[q_{\text{en}} + a_{\text{en}} \frac{\partial}{\partial n} \right] W_n(x, \frac{\partial}{\partial n}) = 0$$

- Trees are dual to SDYM $\text{wc} \Rightarrow \bar{q} = 0 \Leftrightarrow$ Yangian Symmetry

$$- (x_i - x_{i+1})^2 = 0$$

$$(x_i - x_{i+1})(\theta_i - \theta_{i+1}) = 0$$

$\{A, \psi\}$

$$L = \frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \not{D} \psi + \frac{1}{2} (\phi^\dagger \phi)^2 + \frac{1}{2} (\phi^\dagger \phi) + \frac{1}{2} (\phi^\dagger \phi)^2 + \frac{1}{2} (\phi^\dagger \phi)^2 + \dots$$

$$S = \frac{1}{4} \left(\frac{F^2}{4} + \bar{\psi} \not{D} \psi + \frac{1}{2} (\phi^\dagger \phi)^2 + \frac{1}{2} (\phi^\dagger \phi) + \frac{1}{2} (\phi^\dagger \phi)^2 + \frac{1}{2} (\phi^\dagger \phi)^2 \right)$$

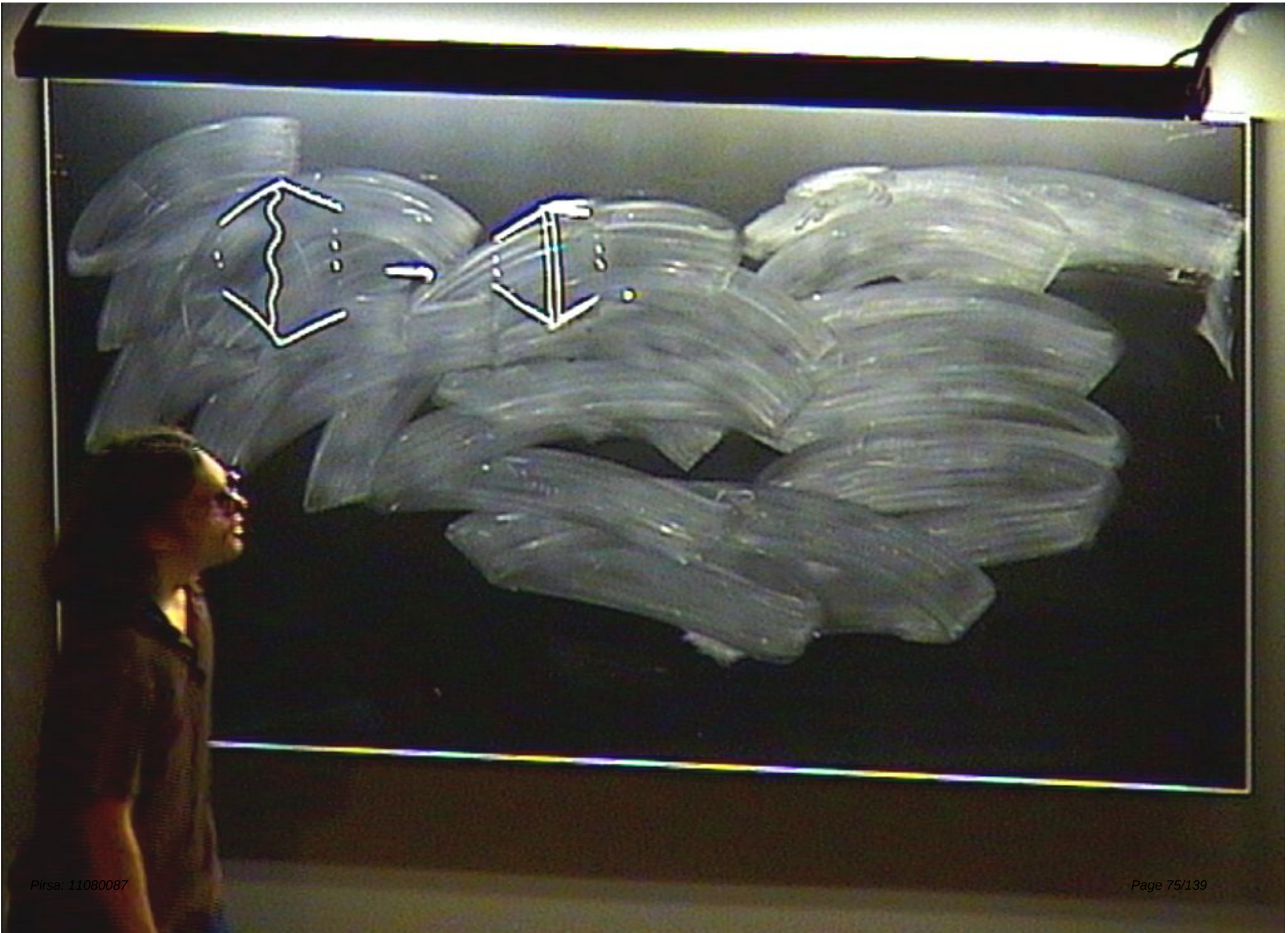
$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $F_{\mu\nu} \rightarrow \lambda(G_{\mu\nu})$
 gauge field.

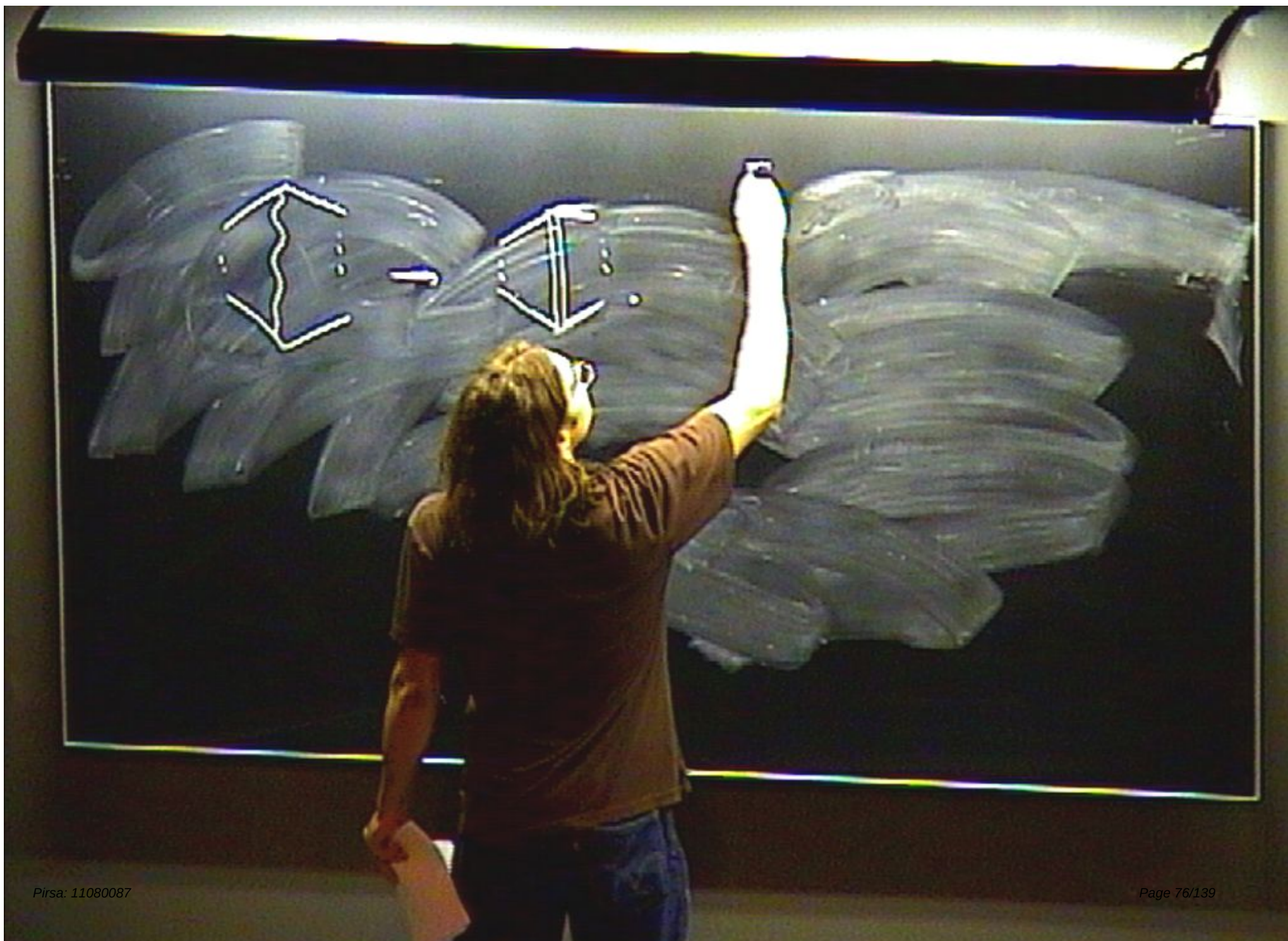
$$S \rightarrow \left[\frac{F^2}{4} + \bar{\psi} \not{D} \psi + \frac{1}{2} (\phi^\dagger \phi)^2 + \frac{1}{2} (\phi^\dagger \phi) \right] \text{SDYM}$$

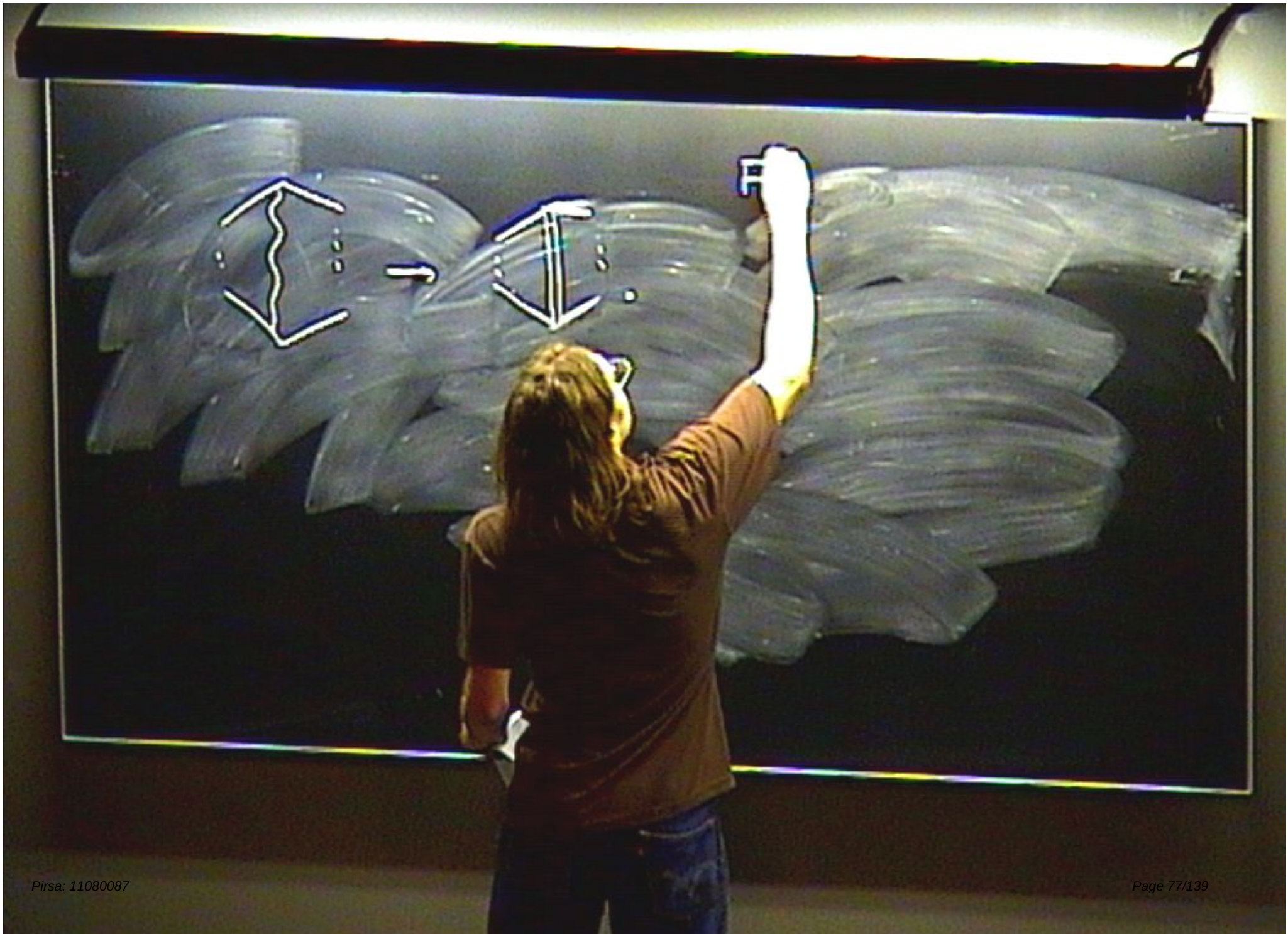
$$qA = (2\psi + \lambda \psi)$$

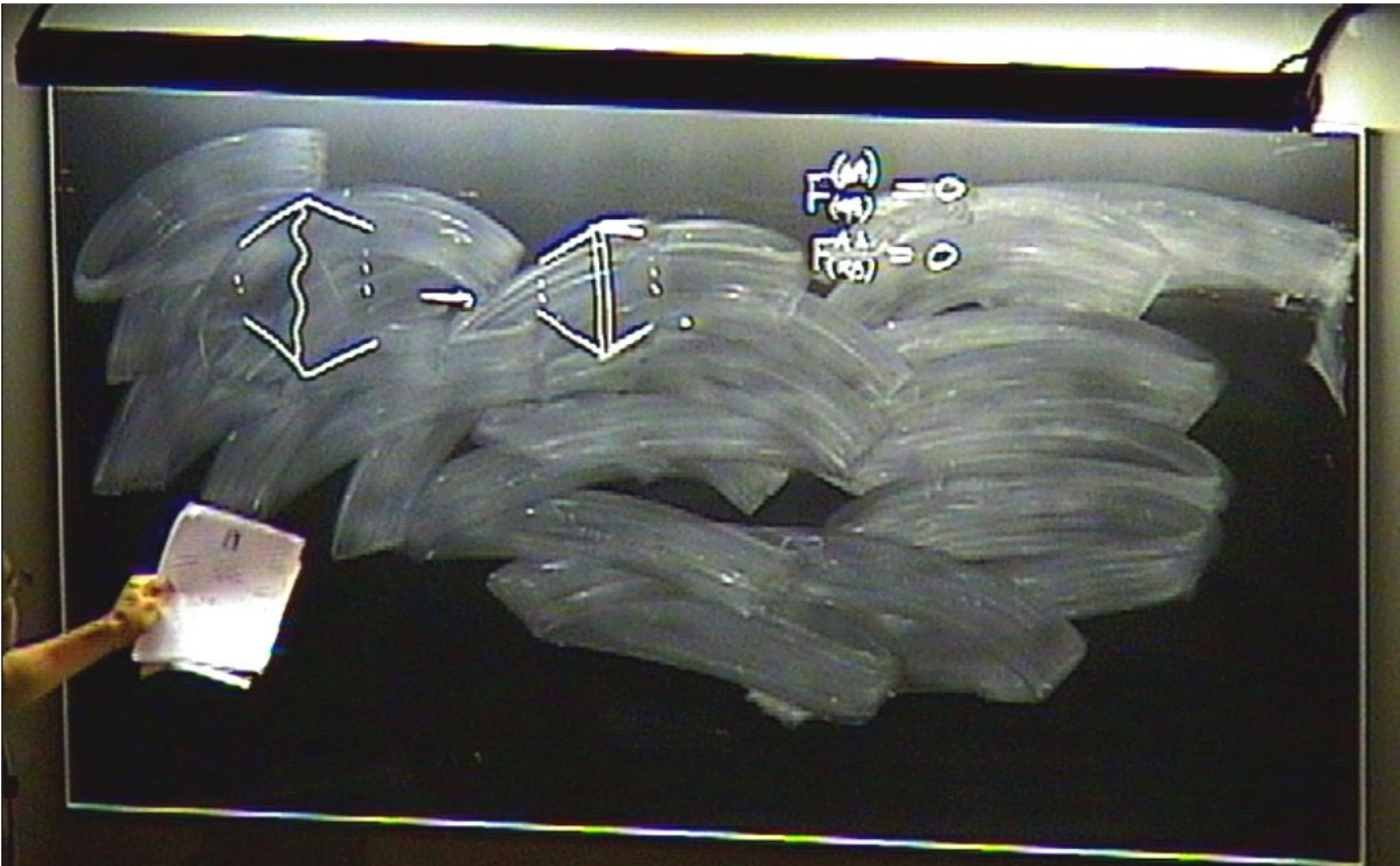
$$\bar{q}A = \mathcal{O}(\lambda)$$

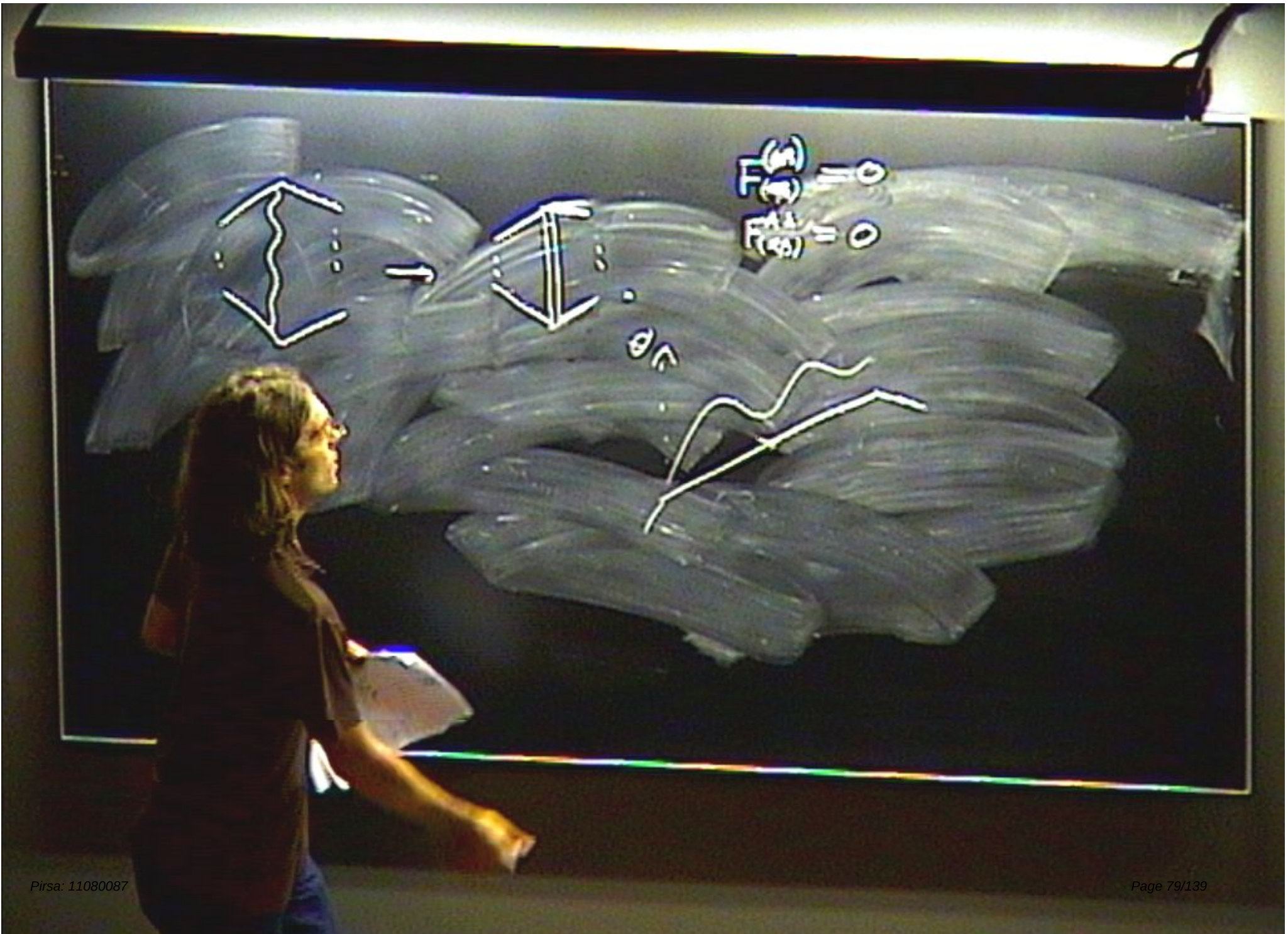


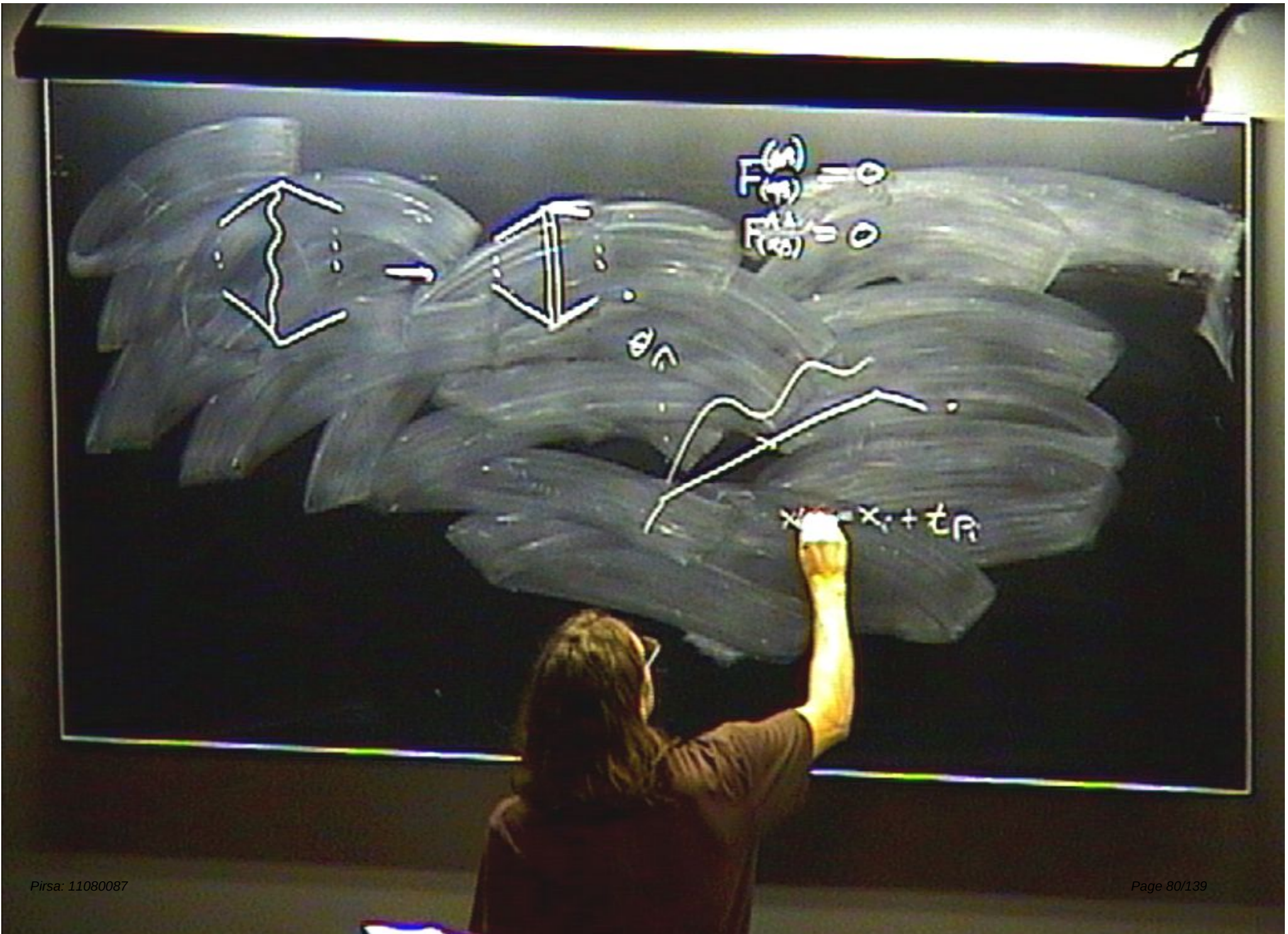


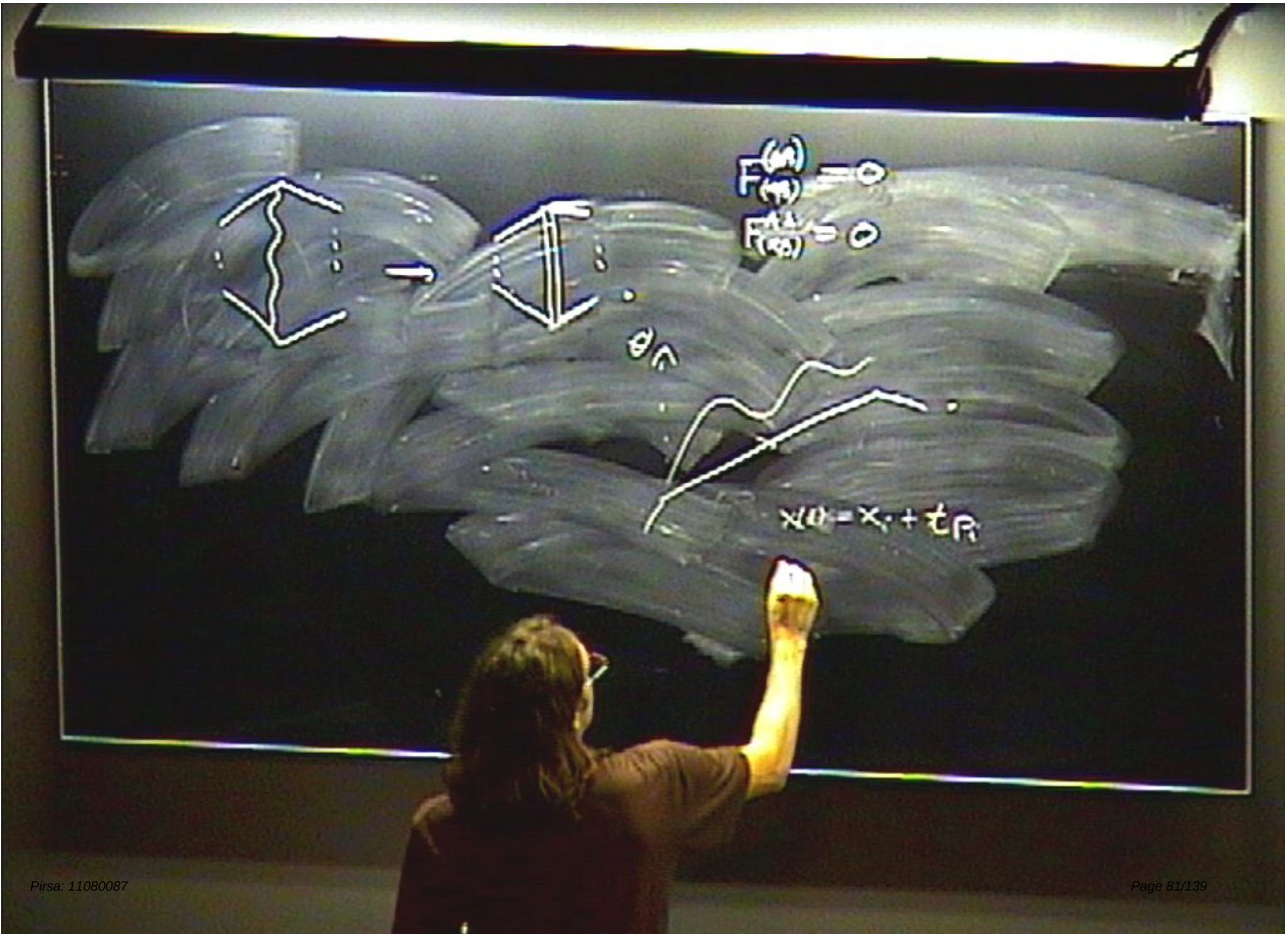


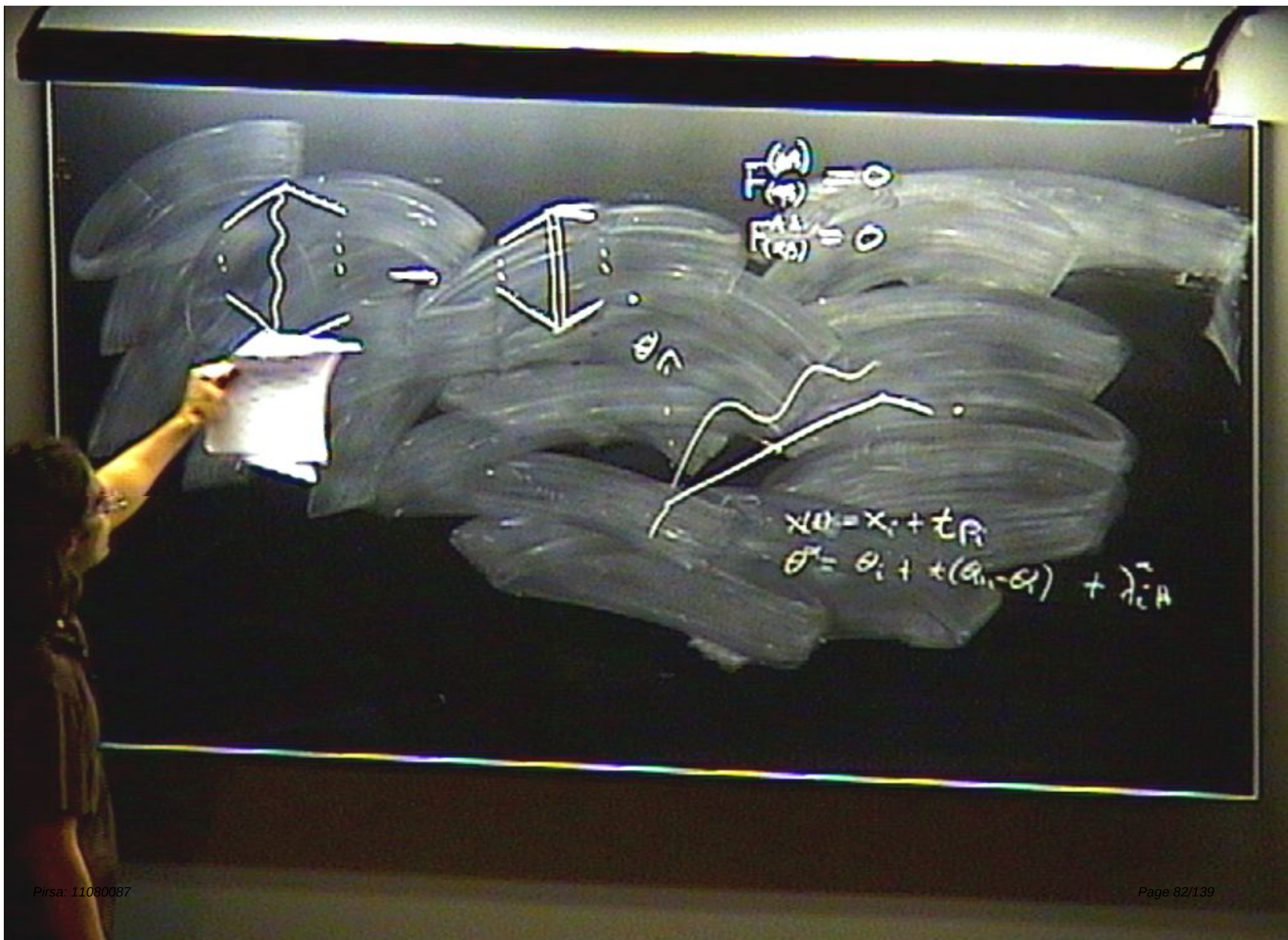


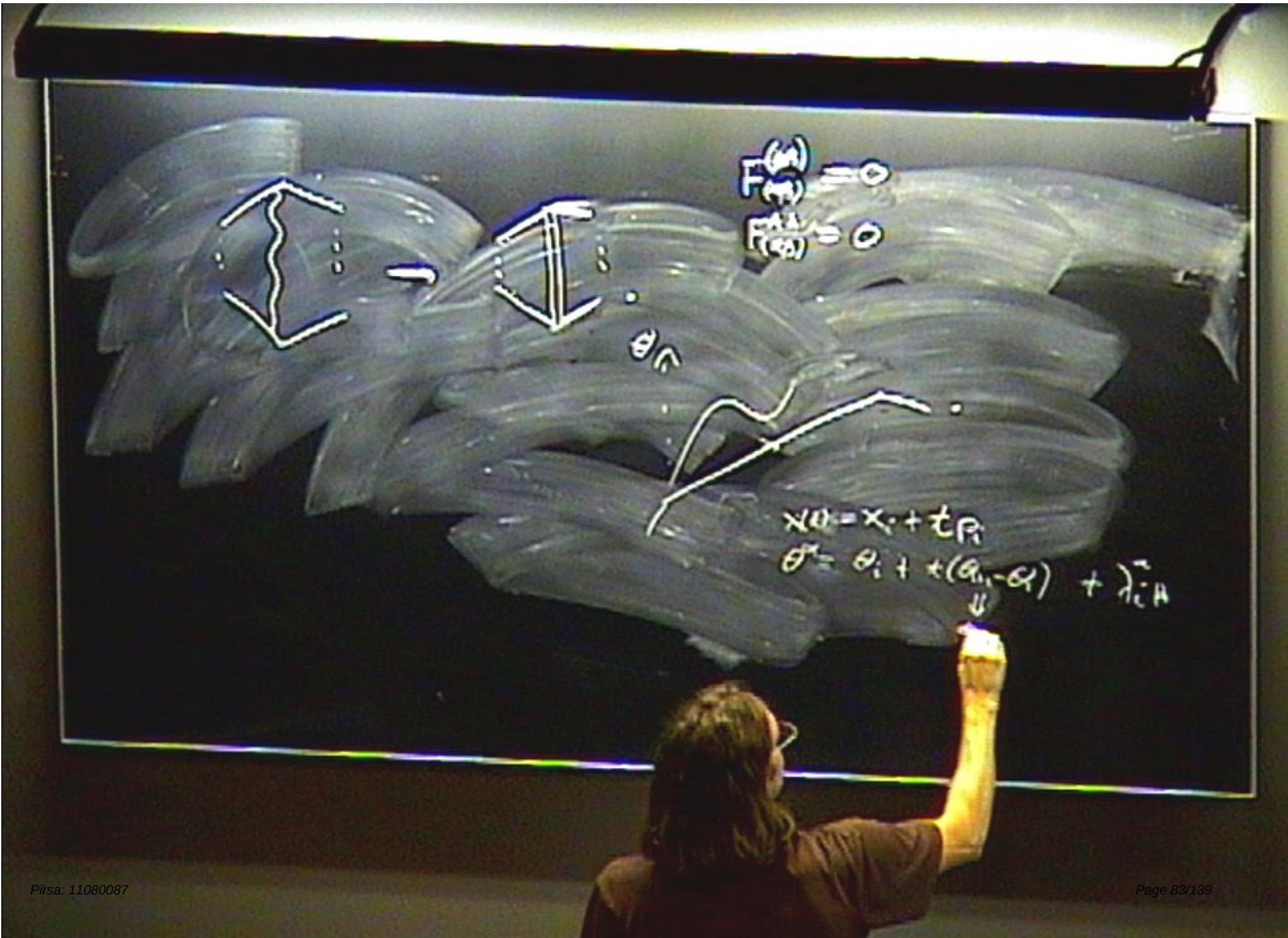


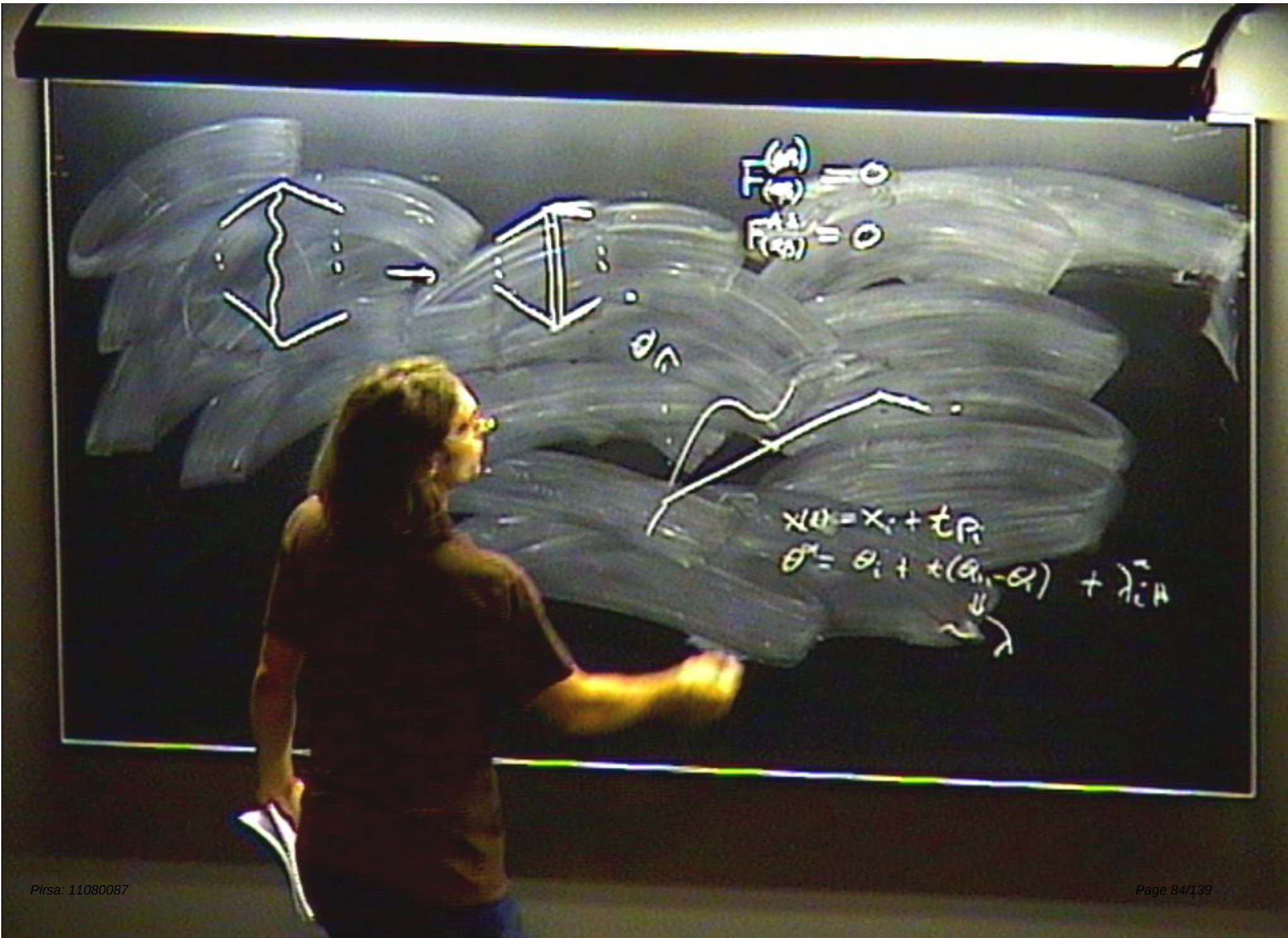














θ_i

$$F(\theta) = 0$$

$$F'(\theta) = 0$$



$$x(t) = x_i + tR$$

$$\theta^* = \theta_i + t(a_{i+1} - \theta_i) + \lambda_i A$$

$\downarrow$
 $\sim$



$\rightarrow$



θ_n

$$F(x) = 0$$

$$F'(x) = 0$$



$$x(t) = x_i + t p_i$$

$$\theta^* = \theta_i + t(a_{i+1} - \theta_i) + \lambda_i^* \tilde{A}$$

$$F(x) = 0$$

$$F'(x) = 0$$



θ_0

$$x(t) = x_i + t p_i$$

$$\theta^x = \theta_i + t(a_{i1} - \theta_i) + \lambda_i \tilde{A}$$



-



θ

$$F(x) = 0$$

$$F'(x) = 0$$



$$x(t) = x_i + tR$$

$$\theta^* = \theta_i + t(a_{i+1} - \theta_i) + \lambda_i^* A$$



$$F(\mathbf{r}) = 0$$

$$F(\mathbf{r}) = 0$$

$$\sum D_{ij} G_{ij} \theta_i \theta_j$$

θ_i



$$x(t) = x_i + t R_i$$

$$\theta^* = \theta_i + t (a_{i+1} - a_i) + \lambda_i H$$



$$\begin{aligned} F(x) &= 0 \\ F'(x) &= 0 \end{aligned}$$

$$S(D_n, G_{n,r}) \circ \circ \circ \circ$$

$$f(x) \sim \frac{h(x)}{(x-x_i)^2}$$

$$(x-x_i)^2 \rightarrow \frac{1}{\lambda^2}$$



$$\begin{aligned} x(t) &= x_i + t R_i \\ \theta^* &= \theta_i + t(a_{i+1} - a_i) + \lambda_i^* H \end{aligned}$$



$$F(\vec{x}) = 0$$

$$\frac{\partial F}{\partial x_i} = 0$$

$$S(D_{x_i} G_{0,0}) \circ \dots \circ \circ$$

$$f(x) \sim \frac{f(x_i) - f(x_j)}{(x_i - x_j)^2}$$



$$x(t) = x_i + t \vec{p}_i$$

$$\theta^* = \theta_i + t(\theta_{i+1} - \theta_i) + \lambda_i \vec{a}$$



θ_n

$$F(x) = 0$$

$$F'(x) = 0$$



$$S(D_{i,j} G_{i,j}) \circ \circ \circ \circ \circ$$

$$f(x) \sim \frac{\lambda(x)}{(x-x_i)^2}$$

$$\left(\frac{\lambda(x)}{P_i(x-x_i)} - q_i \right)$$

$$x(t) = x_i + t P_i$$

$$\theta^* = \theta_i + t(a_{i+1} - \theta_i) + \lambda_i H$$



$\rightarrow$



θ_i

$$F_{ij}^{(n)} = 0$$

$$F_{ij}^{(n)} = 0$$

$$S(D_{ij} G_{ij}) \theta_i \theta_j$$

$\langle f, f \rangle$

$\lambda_i \lambda_j$

$$\left(\frac{\lambda_i \lambda_j}{P_{ij}(x-y)} - \lambda_i \right)$$

$$\left(\frac{\lambda_i}{\lambda_i \lambda_j} - \frac{\lambda_j}{\lambda_i \lambda_j} \right)$$

$$x(t) = x_i + t R$$

$$\theta^t = \theta_i + t(a_{ij} - \theta_i) + \lambda_i A$$



$$G(\sigma)^4 \cdot G(\sigma)^4 (\lambda(\sigma - \sigma_i))$$

$$F(\sigma) = 0$$

$$F'(\sigma) = 0$$

$$S(D, G_{ij}) \text{ } \sigma \sigma \sigma \sigma$$

$$f(x) \sim \frac{\lambda_i \lambda_j \lambda_k \lambda_l}{(x_i - x_j)^2} \left(\frac{\lambda_i \lambda_j (1)}{1 - (x_i - x_j)} - 1 \right)$$

$$(x_i - x_j) \rightarrow \lambda_i \lambda_j$$

$$\left(\frac{\lambda_i^2}{\lambda_i \lambda_j} - \frac{\lambda_{i+1}^2}{\lambda_{i+1} \lambda_j} \right) \sim \lambda$$

$$x(t) = x_i + t p_i$$

$$\theta^* = \theta_i + t(a_{i+1} - a_i) + \lambda_i^* H$$

- Trees and Loops.

- $(H_{loops})_{reg} = (H_{loops} + k)w_c$

- $A_n(p, \pi) = \frac{\delta^4(\epsilon p) \delta^{10}(\epsilon \lambda \pi)}{c(12 \times 2) \dots c(n12)} W_n(x, \frac{\partial}{\partial \pi})$, $c = \frac{2\pi k}{4\pi}$

- $\left\{ q_{\mu\nu} + \alpha' \frac{\partial}{\partial \pi} \right\} W_n(x, \frac{\partial}{\partial \pi}) = 0$

- Trees are dual to

- $(x_i - x_j)^2$
 $(x_i - x_j)$

$W_c \Rightarrow \bar{q} = 0 \Leftrightarrow$ Yangian Symmetry
 served as

- Trees and Loops.

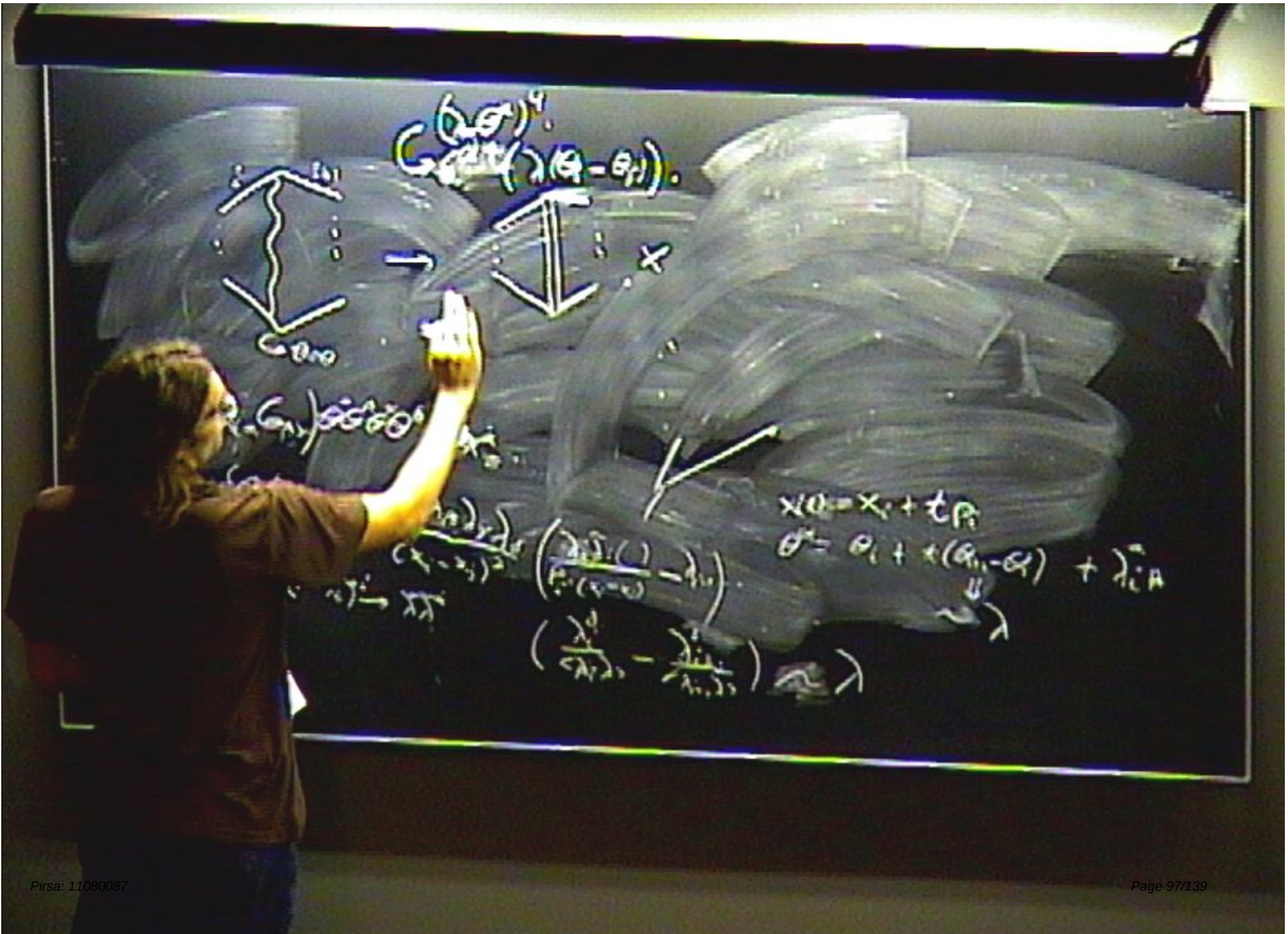
- $(H_{loop})_{np} = (H_{tree} + k)w_L$

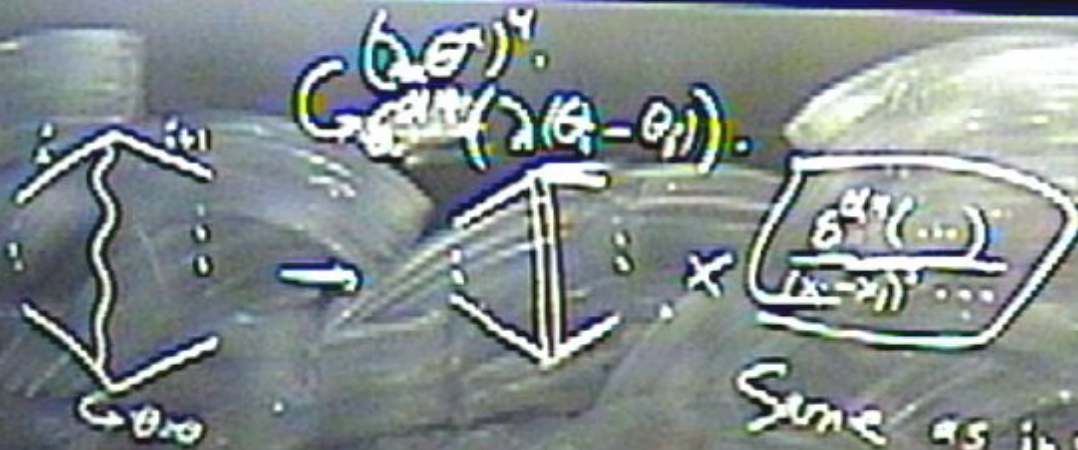
- $A_n(p, \pi) = \frac{\delta^4(\epsilon p) \delta^{015}(\epsilon \lambda \pi)}{\langle 12 \times 23 \rangle \dots \langle n-1 n \rangle} W_n(x, \frac{\epsilon}{\alpha})$, $\epsilon = \frac{2\pi k}{4\pi}$

- $\left[q_{\mu n} + \alpha' \frac{\partial}{\partial \sigma n} \right] W_n(x, \frac{\epsilon}{\alpha}) = 0$

- Trees are dual to SYM $W_L \rightarrow \bar{q} q \leftrightarrow$ Yangian Symmetry

- $\left. \begin{aligned} (x_i - x_{i+1})^2 &= 0 \\ (x_i - x_{i+1})(\theta_i - \theta_{i+1}) &= 0 \end{aligned} \right\}$ preserved upon factorization.





$$\begin{aligned}
 x(t) &= x_i + tR \\
 \theta^* &= \theta_i + t(\theta_{i+1} - \theta_i) + \lambda_i^{-1} \vec{A}
 \end{aligned}$$

Formula

$$L = d(A_0 + \frac{1}{2} \theta + \frac{1}{2} \phi + \frac{1}{2} \psi + \frac{1}{2} \chi + \frac{1}{2} \dots) + dS(\dots)$$

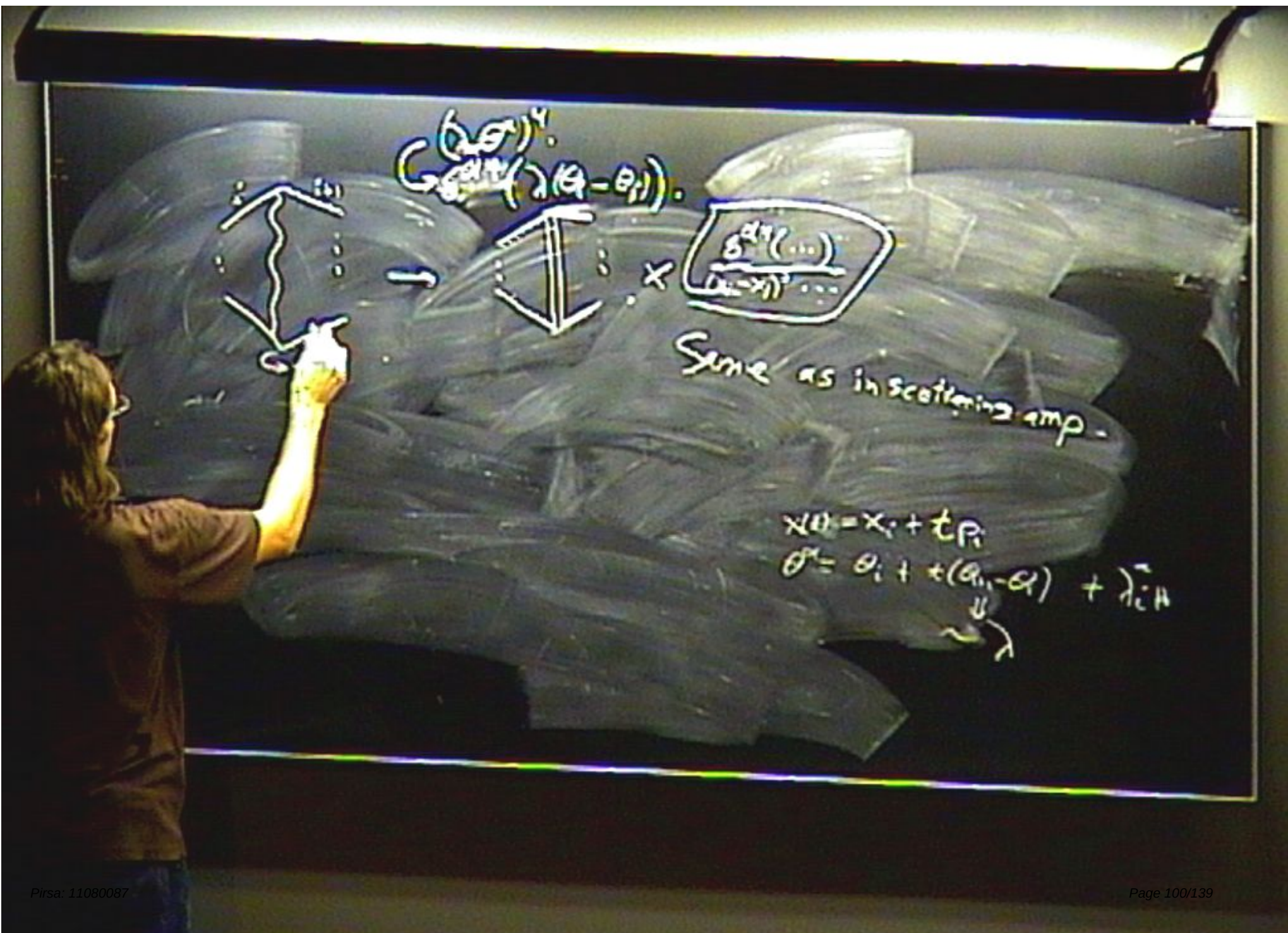
$$S = \frac{1}{4} \left(\frac{F^2}{4} + \bar{\psi} \gamma \partial \psi + \frac{(\partial \phi)^2}{2} + \psi \psi + \bar{\psi} \bar{\psi} + \langle \phi, \partial^2 \phi \rangle \right)$$

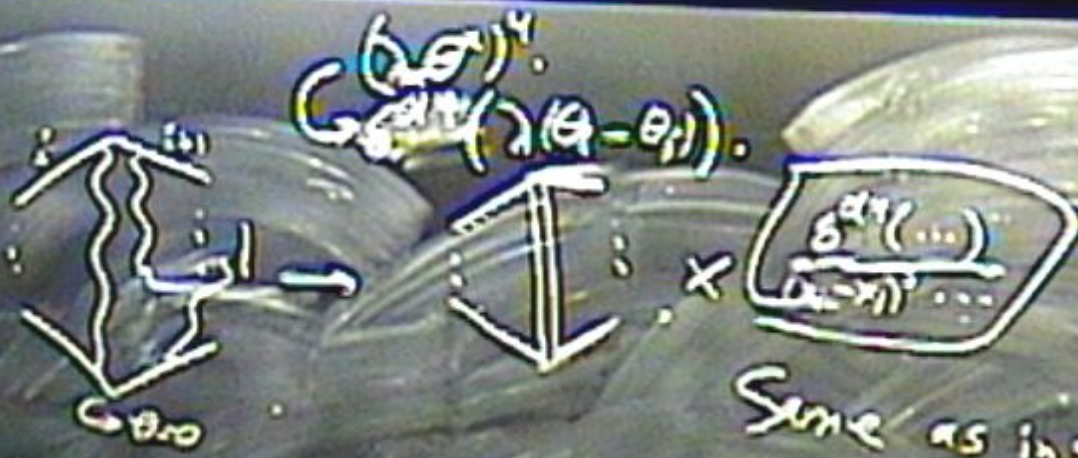
$A_\mu \rightarrow A_\mu$
 $\bar{\psi} \rightarrow \bar{\psi}$
 $\psi \rightarrow \psi$
 $F_{\mu\nu} \rightarrow \partial(G_{\mu\nu})$
 gauge field.

$$S \rightarrow \left[E G^2 + \psi \psi + \frac{(\partial \phi)^2}{2} + \bar{\psi} \bar{\psi} \right]_{SDYM} + \lambda \left(G^2 + \psi \psi + \langle \phi, \partial^2 \phi \rangle \right)$$

$$qA = (d\theta + \lambda d\phi)$$

$$\bar{q}A = O(\lambda)$$

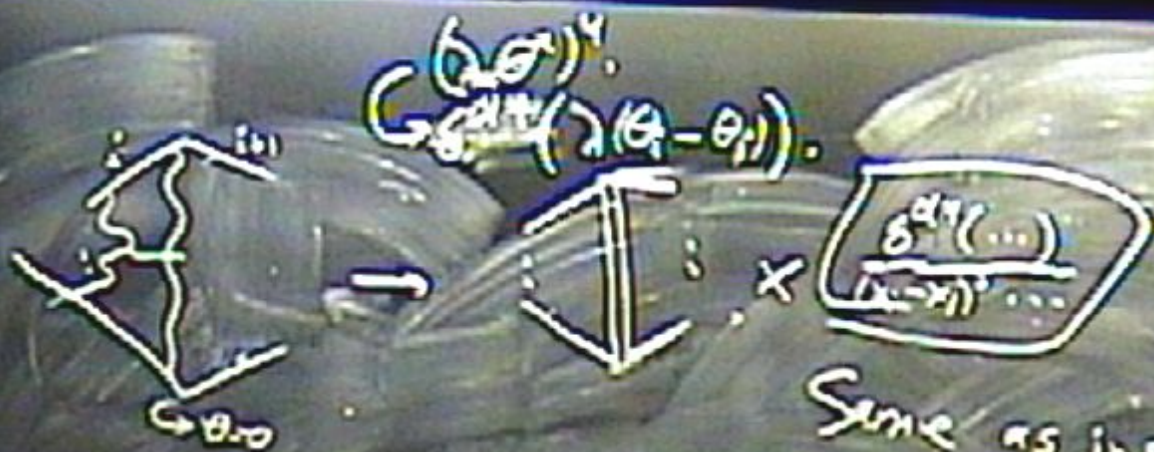




Same as in scattering amp.

$$\langle \gamma | A | 0 \rangle = 0$$

$$\begin{aligned}
 x_f &= x_i + tR \\
 \theta_f &= \theta_i + t(q_{f,i} - q_i) + \lambda_{i,f}^{\sim}
 \end{aligned}$$



$$G_{\theta=0} = 0$$

Same as in scattering amp.

$$x_f = x_i + t p_i$$

$$\theta_f = \theta_i + t(q_{i1} - q_i) + \lambda_{i1} \vec{A}$$

→ Super loop in SYM

$$+ \bar{\psi} \not{D} \psi + \frac{1}{2} \text{Tr} F^2 + \frac{1}{4} \text{Tr} F^2 + \frac{1}{4} \text{Tr} F^2 + \frac{1}{4} \text{Tr} F^2$$

$$\boxed{S \rightarrow \int d^4x \left[\frac{1}{4} \text{Tr} F^2 + \frac{1}{2} \text{Tr} \bar{\psi} \not{D} \psi + \frac{1}{4} \text{Tr} F^2 + \frac{1}{4} \text{Tr} F^2 \right]} \text{ SYM}$$

$\mathcal{N}=2$ (12) = 4.00
 9 ext. Fields

$$+ \lambda \left(G^2 + \bar{\psi} \not{D} \psi \right)$$

$$qA = (\omega \not{D} + \lambda \not{D})$$

$$\bar{q}A = \bar{\psi} \not{D} \psi$$

→ Super loop in SYM = tree exps.

$$+ \bar{\psi} \not{D} \psi + \frac{1}{2} (\not{D} \psi)^2 + \frac{1}{2} (\not{D} \bar{\psi})^2 + \dots$$

$$\boxed{S \rightarrow \int d^4x \left[-\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi} \not{D} \psi + \frac{1}{2} (\not{D} \psi)^2 + \frac{1}{2} (\not{D} \bar{\psi})^2 \right]}$$

$\lambda = 2(12) = 24$
 aux. Field.

$$qA = (\alpha \psi + \lambda \psi)$$

$$\bar{q}A = \mathcal{O}(\lambda)$$

→ Super loop in SYM = tree exps.



$3-2(12) = 4.00$
9ext. Field.

9/ 20

- Trees and Loops.

- $(H_{\text{loops}})_{\text{mp}} = (H_{\text{trees}} + k) w_c$

~~$4k$~~ $\rightarrow 4k$

- $A_n(p, \pi) = \frac{\delta^4(\epsilon p) \delta^{0123}(\epsilon \lambda \pi)}{s_{12} s_{23} \dots s_{n1}} W_n(x, \frac{\theta}{a^2})$, $c = \frac{g^2 k}{4\pi}$.

- $\left\{ q_{\mu\nu} + a^{\mu\nu} \frac{\partial}{\partial \sigma^{\mu\nu}} \right\} W_n(x, \frac{\theta}{a^2}) = 0$

- Trees are dual to SDYM $w_c \Rightarrow \bar{q} = 0 \Leftrightarrow$ Yangian Symmetry

- $\left. \begin{aligned} (x_i - x_{i+1})^2 &= 0 \\ (x_i - x_{i+1})(\theta_i - \theta_{i+1}) &= 0 \end{aligned} \right\}$ preserved under factorization.

→ Super loop in SDYM = tree ops.



long distance $\Rightarrow$ short distance
amp. WL

Radio
Fied.

9/2

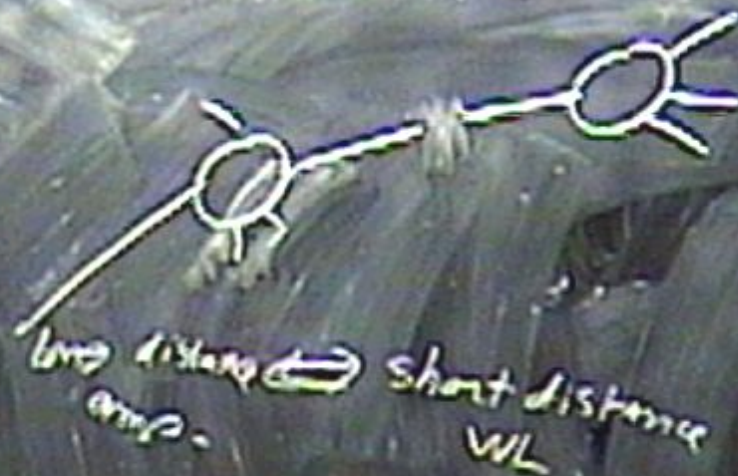
$$\begin{aligned}
 & \text{Diagram 1: } \text{triangle with } i, f \text{ and } \theta \rightarrow 0 \\
 & \text{Diagram 2: } \text{triangle with } i, f \text{ and } \theta \rightarrow \theta_1 \\
 & \text{Equation: } \frac{\delta^{(4)}(\dots)}{(x-x_1)^4} \dots
 \end{aligned}$$

Same as in scattering amp.

$$\begin{aligned}
 x(t) &= x_i + t P_i \\
 \theta^* &= \theta_i + t(a_{i1} - \theta_i) + \lambda_i t
 \end{aligned}$$

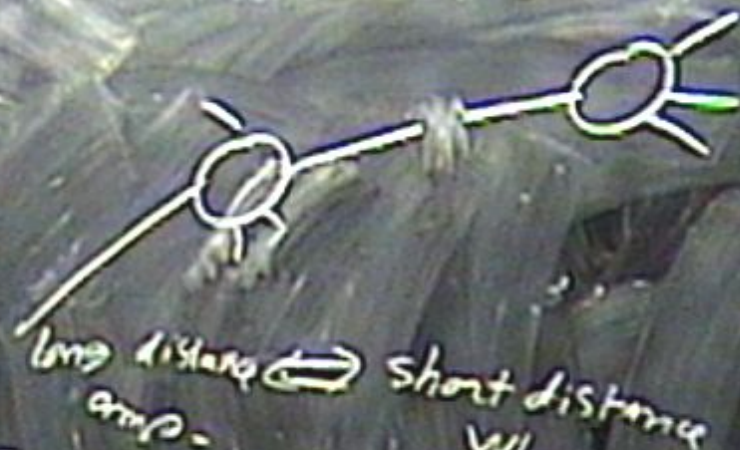
→ Super loop in SDYM = two exps.

— Loop connection



→ Super loop in SDYM = tree ops.

→ Loop corrections

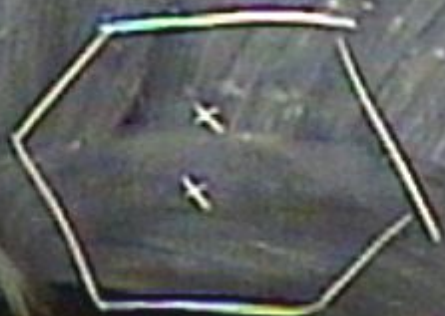


long distance $\Leftrightarrow$ short distance
comp. WL

$$S = S_{\text{SDYM}} + g^2 L_1$$

→ Super loop in SDYM = tree exp.

— Loop corrections



long distance $\Rightarrow$ short distance
exp. WL

$$S = S_{SDYM} + g^2 L$$

→ Super loop in SDYM = tree ops.

→ Loop corrections



long distance $\Rightarrow$ short distance
amp. WL

$$S = S_{SDYM} + g^2 L_1$$

→ Super loop in SDYM = tied ends.

— Loop connections



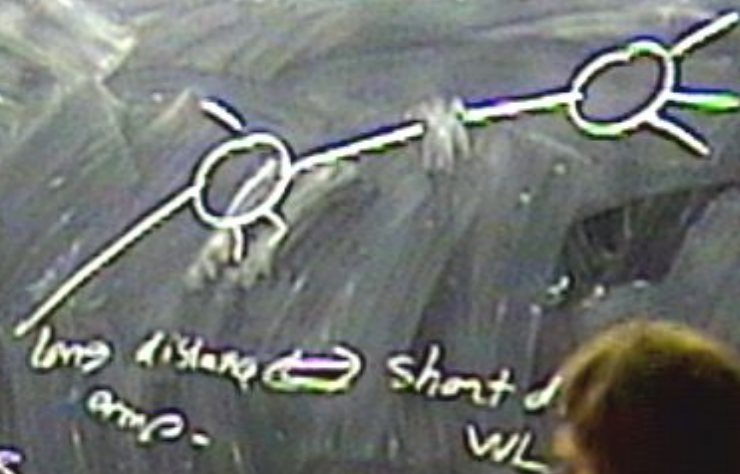
long distance $\Leftrightarrow$ short distance
amp. WL

$$S = S_{SDYM} + g^2 L_4$$

$$W^{2-loop} = \int d^4x + d^4x_1 \langle WL_4 L_4 \rangle$$

→ Super loop in SYM = tree exp.

→ Loop connections



$$S = S_{\text{SYM}} + g^2 L_4$$

$$W^{2\text{-loop}} = \int d^4x + d^4x' \langle W L_4 L_4 \rangle \pi \frac{(x - x')^2}{(x - x')^2 + m^2}$$

→ Super loop in SYM = tree ops.

— Loop corrections



long distance $\Leftrightarrow$ short distance
amp. WL

$$S = S_{SYM} + g^2 L_4$$

$$W^{2-loop} = \int d^4x + d^4x' \langle WL_4 L_4 \rangle \pi \left(\frac{(x - x')^2}{(x - x')^2 + m^2} \right)$$

→ Super loop in SDYM = tree eps.

— Loop corrections



long distance $\Leftrightarrow$ short distance
loop WL

$$S = S_{SDYM} + g^2 L_1$$

$$W^{2-loop} = \int d^4x + d^4x' \langle WL_1 L_1 \rangle_{SDYM} \pi \left(\frac{(x - x')^2}{(x - x')^2 + m^2} \right)$$

→ Super loop in SDYM = tree exp.

— Loop corrections



long distance $\Leftrightarrow$ short distance
exp. WL

$$S = S_{SDYM} + g^2 L_1$$

$$W^{2-loop} = \int d^4x + d^4x' \langle WL_4 L_4 \rangle_{SDYM} \pi \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

→ Super loop in SDYM = tree ops.

— Loop corrections



long distance $\Leftrightarrow$ short distance
amp. WL

$$S = S_{\text{SDYM}} + g^2 L_1$$

$$W^{2\text{-loop}} = \int d^4x + d^4x' \langle W L_4 L_4 \rangle_{\text{SDYM}} \prod \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

→ Super loop in SDYM - tree ops.

- Loop corrections



long distance $\Leftrightarrow$ short distance
amp. WL

$$S = S_{SDYM} + g^2 L_4$$

$$W^{2-loop} = \int d^4x + d^4x' \langle WL_4 L_4 \rangle_{SDYM} \pi \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

= Super loop in SDYM = tree eps.

- Loop corrections

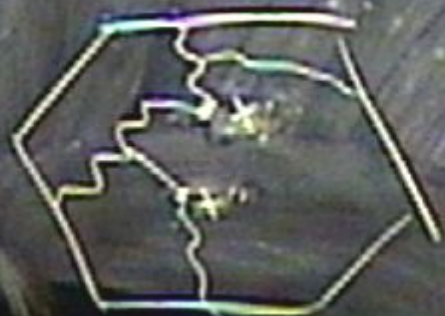


$$S = \frac{1}{2} \sum_i \dot{x}_i^2$$

$$W^{2\text{-loops}} = \int d^4x + d^4x' \langle W L_4 L_4 \rangle_{\text{SDYM}} \pi \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

→ Super loop in SDYM - tree exp.

- Loop connections

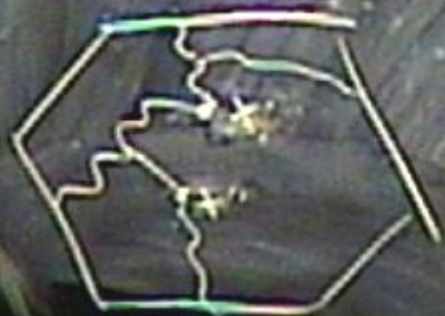


$$S = \frac{1}{2} \sum_i \dot{x}_i^2$$

$$W^{2\text{-loops}} = \int d^4x + d^4x' \quad \leftarrow W L_4 L_4$$

→ Super loop in SYM = tree ops.

— Loop corrections



$$S = \frac{1}{2} \sum_i \dot{x}_i^2$$

$$W^{2\text{-loop}} = \int d^4x + d^4x' \langle W L_4 L_4 \rangle_{\text{SYM}} \pi \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

→ Super loop in SDYM = tree exps.

— Loop corrections



$$S = \frac{1}{2} \sum_i \dot{x}_i^2$$

$$W^{2\text{-loop}} = \int d^4x + d^4x' \langle W L_4 L_4 \rangle_{\text{SDYM}} \prod \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

→ Super loop in SDYM = tree exps.

— Loop corrections



$$S = \frac{1}{2} \int d^4x \, \text{Tr} \, L_\mu L_\mu$$

$$W^{2\text{-loop}} = \int d^4x + i \int d^4x \, \langle W L_\mu L_\mu \rangle_{\text{SDYM}} \prod \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

→ Super loop in SDYM = tree ops.

→ Loop corrections



$$S = \oint \frac{1}{2} L_1$$

$$W^{2\text{-loop}} = \int d^4x + d^4x' \langle W L_1 L_1 \rangle_{\text{SDYM}} \pi \left(\frac{(x_i - x_{i+1})^2}{(x_i - x_{i+1})^2 + m^2} \right)$$

→ Super loop in SDYM = tree exps.

→ Loop corrections



$$S = \int d^4x \, \frac{1}{2} \bar{\psi} \gamma_\mu \psi$$

$$\int d^4x \, d^4x' \, \langle W L_q L_q \rangle_{SDYM} \pi \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

→ Super loop in SDYM = tree eps.

— Loop corrections



→ Forward limit $\neq$

$S =$

$$W^{2\text{-loops}} = \int d^4x + d^4y \left(\frac{(x-y)^2}{(x-y)^2 + m^2} \right)$$

→ Super loop in SYM = tree exp.

→ Loop corrections

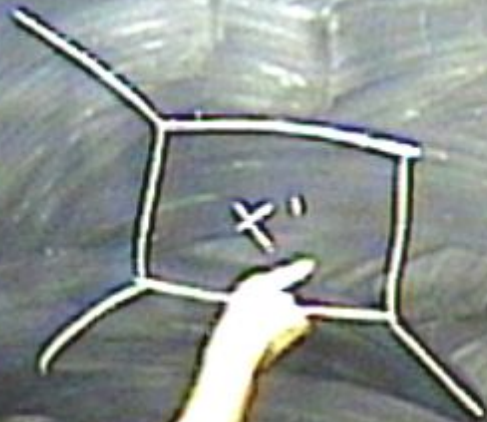


→ Fermion limit $\neq$



$$S = \frac{1}{2} \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \dots \right)$$

$$W^{2-loop} = \int d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \dots \right) \rightarrow \frac{\pi}{(x_i - x_j)^2 + m^2}$$



$$\begin{aligned}x_{i+1} &= x_i + \epsilon R_i \\ \theta_{i+1} &= \theta_i + \epsilon (a_{i+1} - \theta_i) + \tilde{\lambda}_i H\end{aligned}$$

$\downarrow$
 $\sim \lambda$

→ Super loop in SYM = tree exps.

— Loop corrections



→ Forward limit ϵ



$$S = \int d^4x \int d^4x' \frac{1}{L_1 L_2}$$

$$W^{2\text{-loop}} = \int d^4x \int d^4x' \langle W L_1 L_2 \rangle_{\text{SYM}} \pi \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

→ Super loop in SYM = tree ops.

— Loop connections



→ Forward limit $\neq$



$$S = \left(+ \frac{3}{2} L_4 \right)$$

$$W_{2\text{-loop}} = \int d^4x + d^4x' \langle W L_4 L_4 \rangle_{\text{SYM}} \pi \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$

→ Super loop in SDYM = tree ops.

→ Loop connections



→ Forward limit $\times$ $N^{k+1} (L-1)$ integral



$$\langle W L_4 L_4 \rangle_{\text{SDYM}} = \pi \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$



$$\begin{aligned}
 x(t) &= x_i + t p_i \\
 \theta^x &= \theta_i + t(a_{i1} - \theta_i) + \lambda_i H
 \end{aligned}$$

$\downarrow$
 $\sim$



$$\lim_{x \rightarrow 0} F(x) \rightarrow \int \frac{dF}{dx} F(x)$$

$$\left\{ \begin{array}{l} \mathcal{L} \\ \mathcal{G}(x) \end{array} \right.$$



$$\lim_{x \rightarrow 0} F(x) \rightarrow \xi \frac{d}{dx} F(x)$$

$$\left\{ \begin{array}{l} \text{[...]} \end{array} \right\} A(n)$$

$$A_j$$



$$\lim_{x \rightarrow 0} F(x) \rightarrow \oint \frac{dx}{x} F(x)$$

$$\left\{ \begin{array}{l} A(x) \\ A(x) \end{array} \right\} \rightarrow A(x) \quad A(x)$$



$$S = \sum_{i=1}^M$$

$$W_{2\text{-loop}} = \int d^4x + d^4x' \langle W L_4 L_4 \rangle_{\text{1-loop}} \pi \left(\frac{(x - x')^2}{(x - x')^2 + m^2} \right)$$



$$\langle W L_q L_q \rangle_{50\mu m} \propto \pi \left(\frac{(x_i - x'_i)^2}{(x_i - x'_i)^2 + m^2} \right)$$



- $(0, x)$ WL $\Leftrightarrow$ Loop in strand

- $(\vec{e}, x)$ WL $\Leftrightarrow$ In

$$\frac{(\vec{x} - \vec{x}')^2}{(\vec{x} - \vec{x}')^2 + m^2}$$