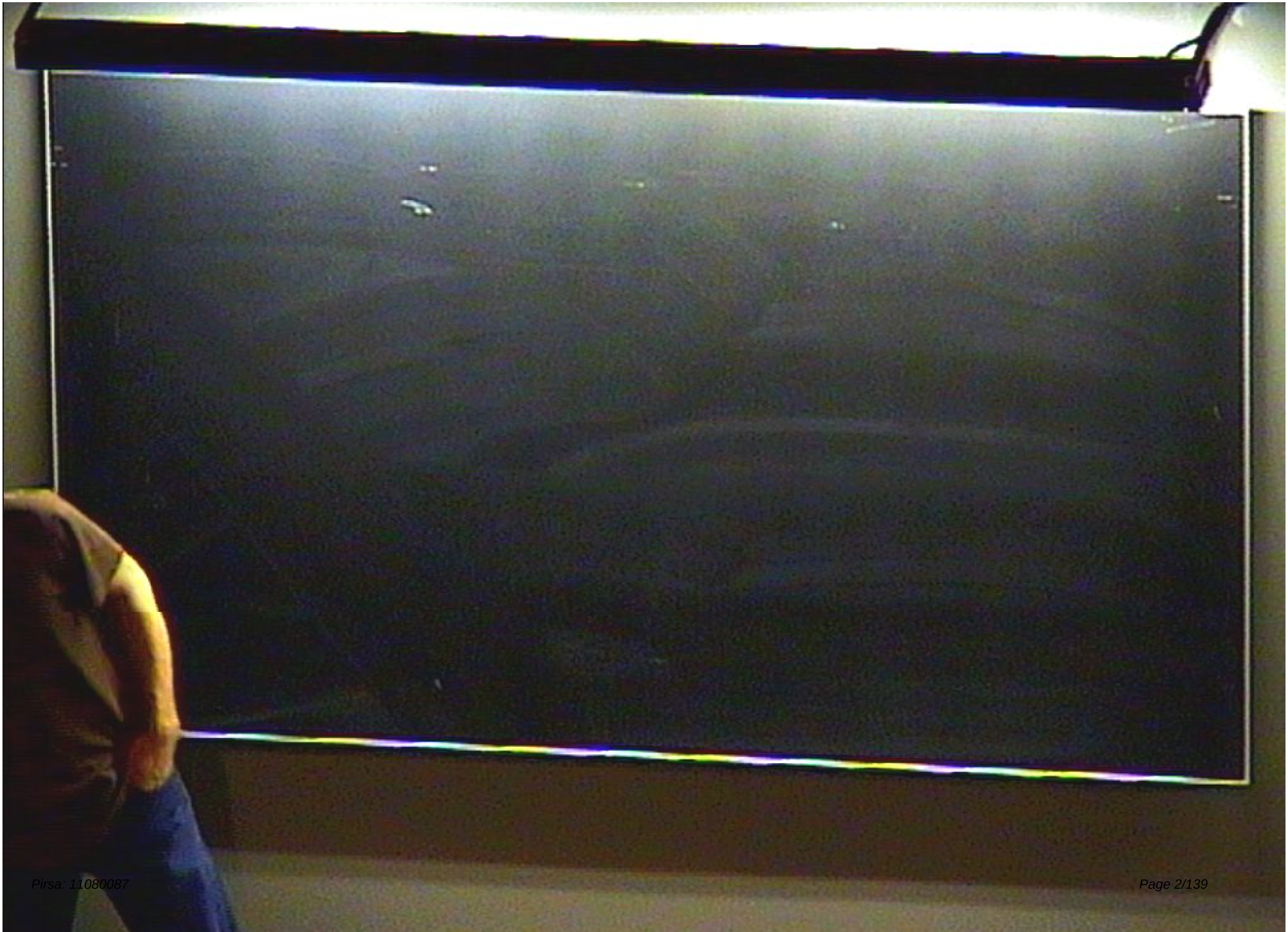
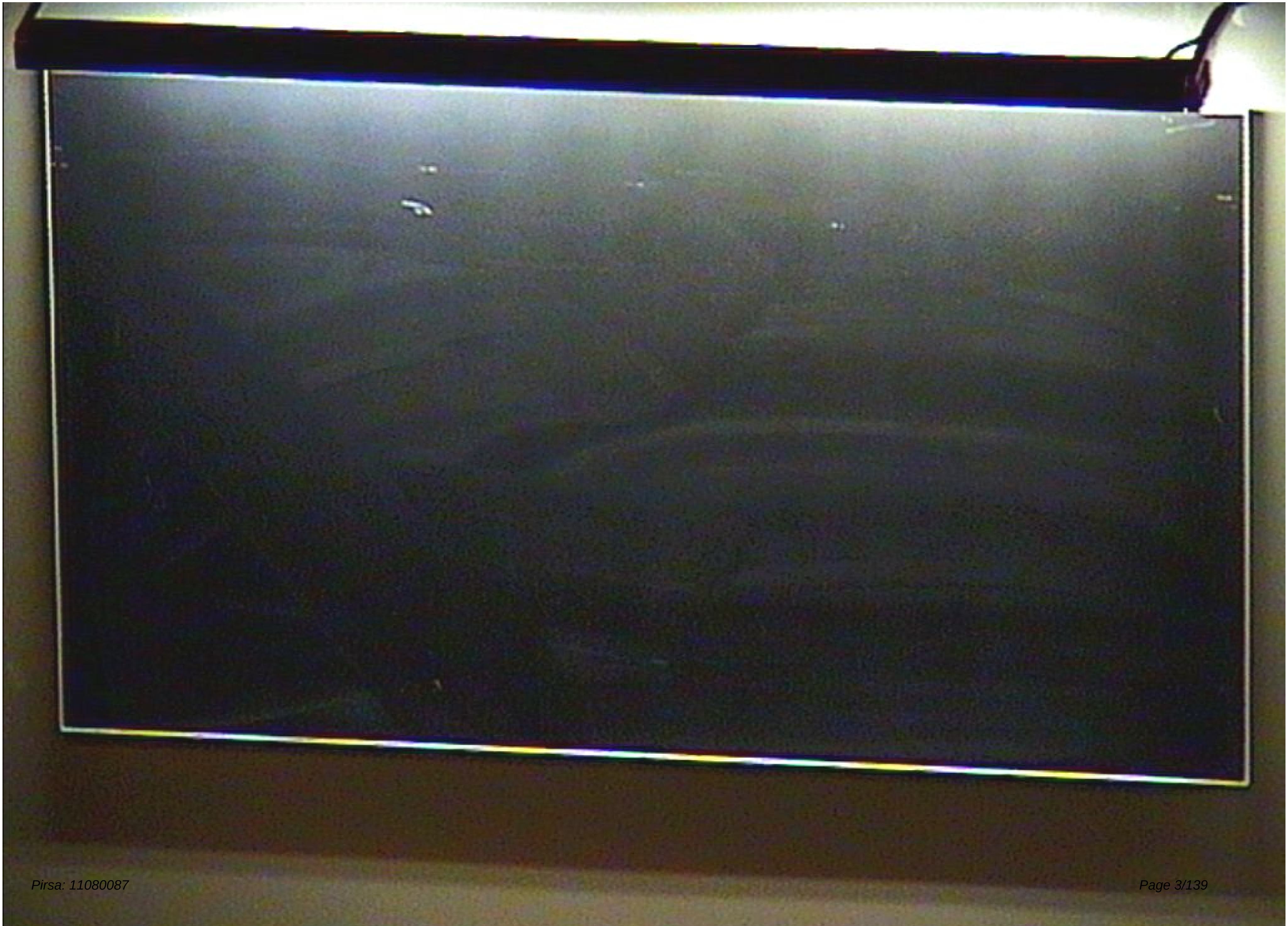
Title: Scattering Amplitudes from (super) Wilson Loops

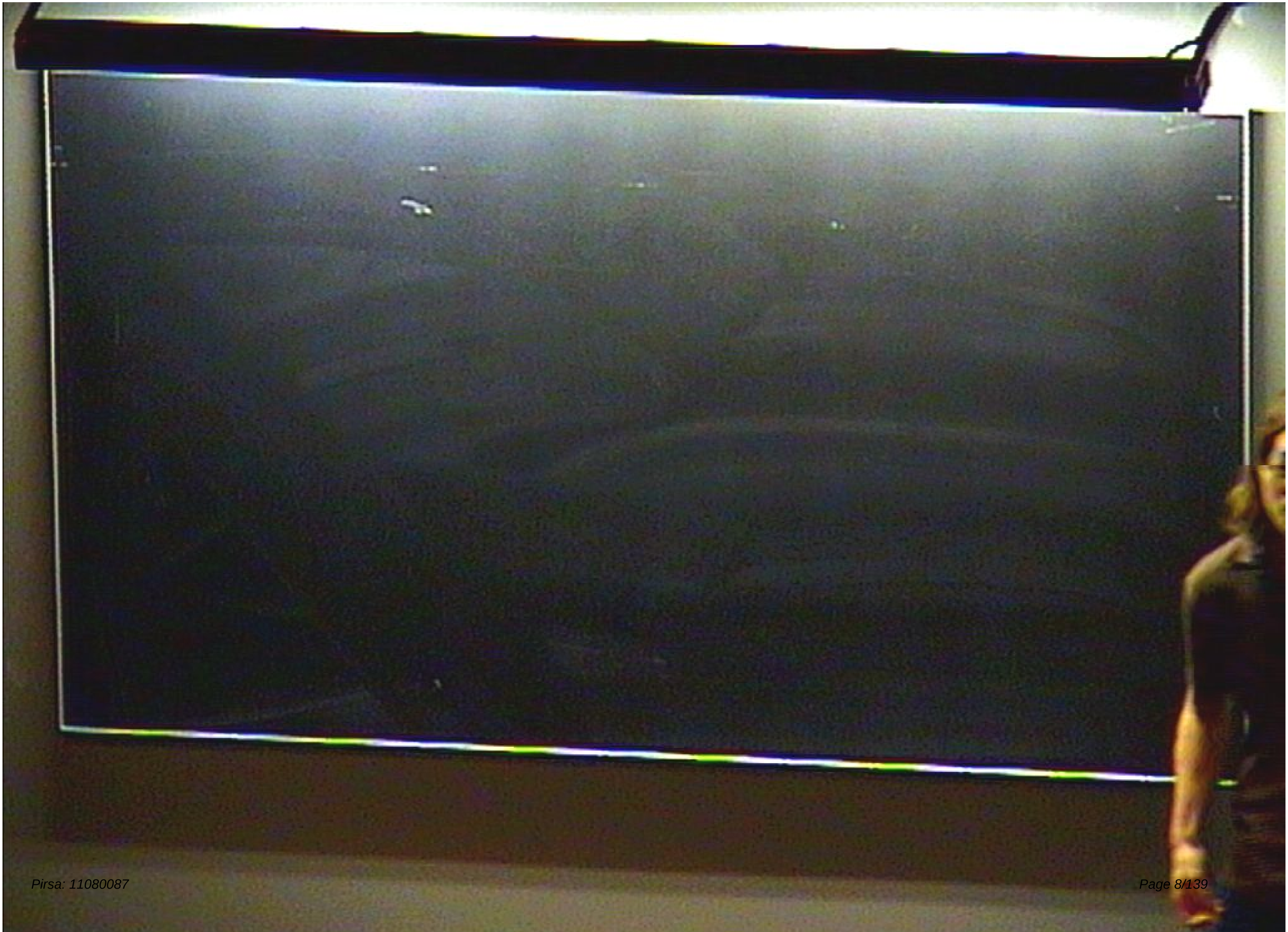
Date: Aug 18, 2011 10:50 AM

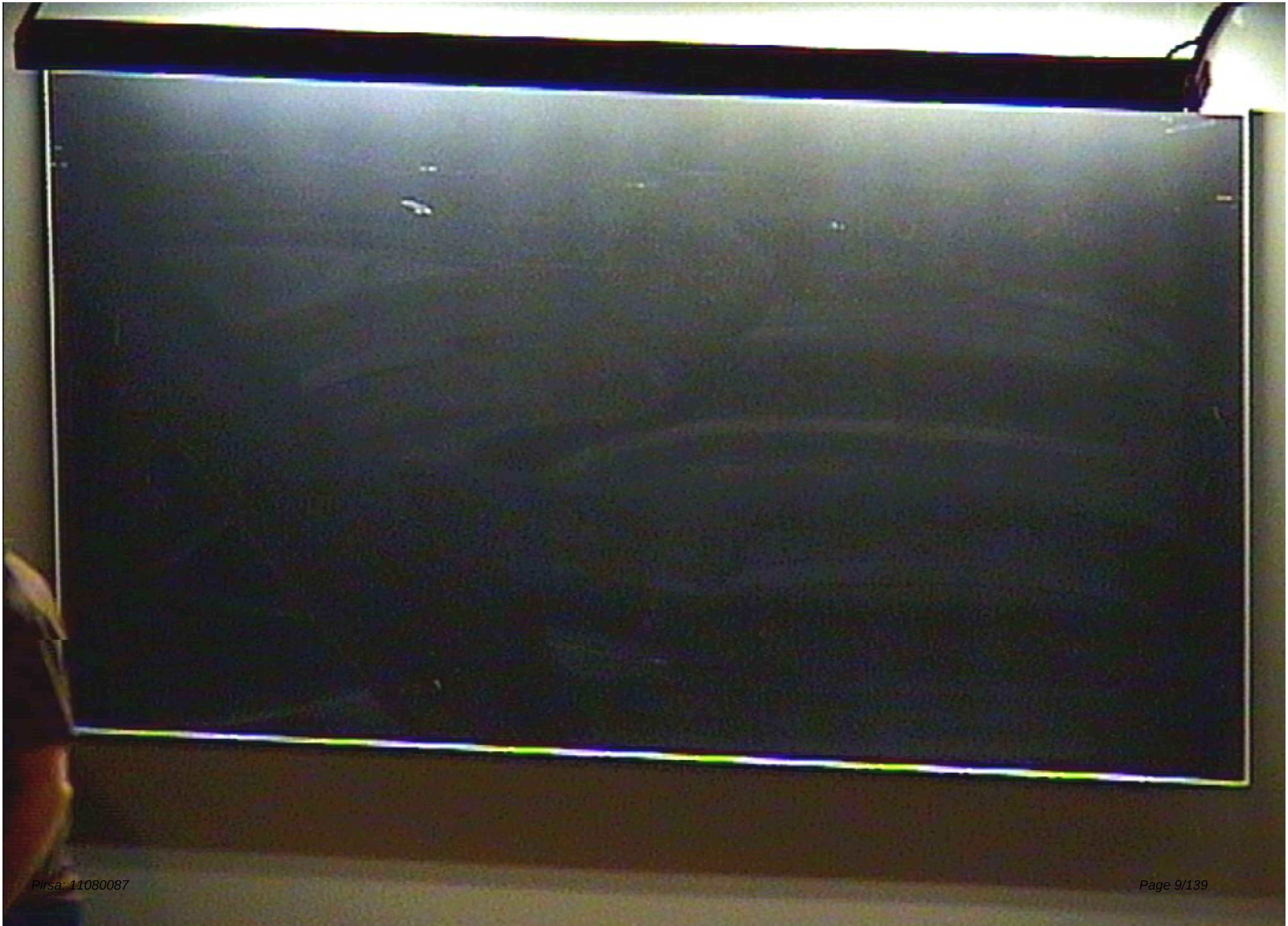
URL: http://pirsa.org/11080087

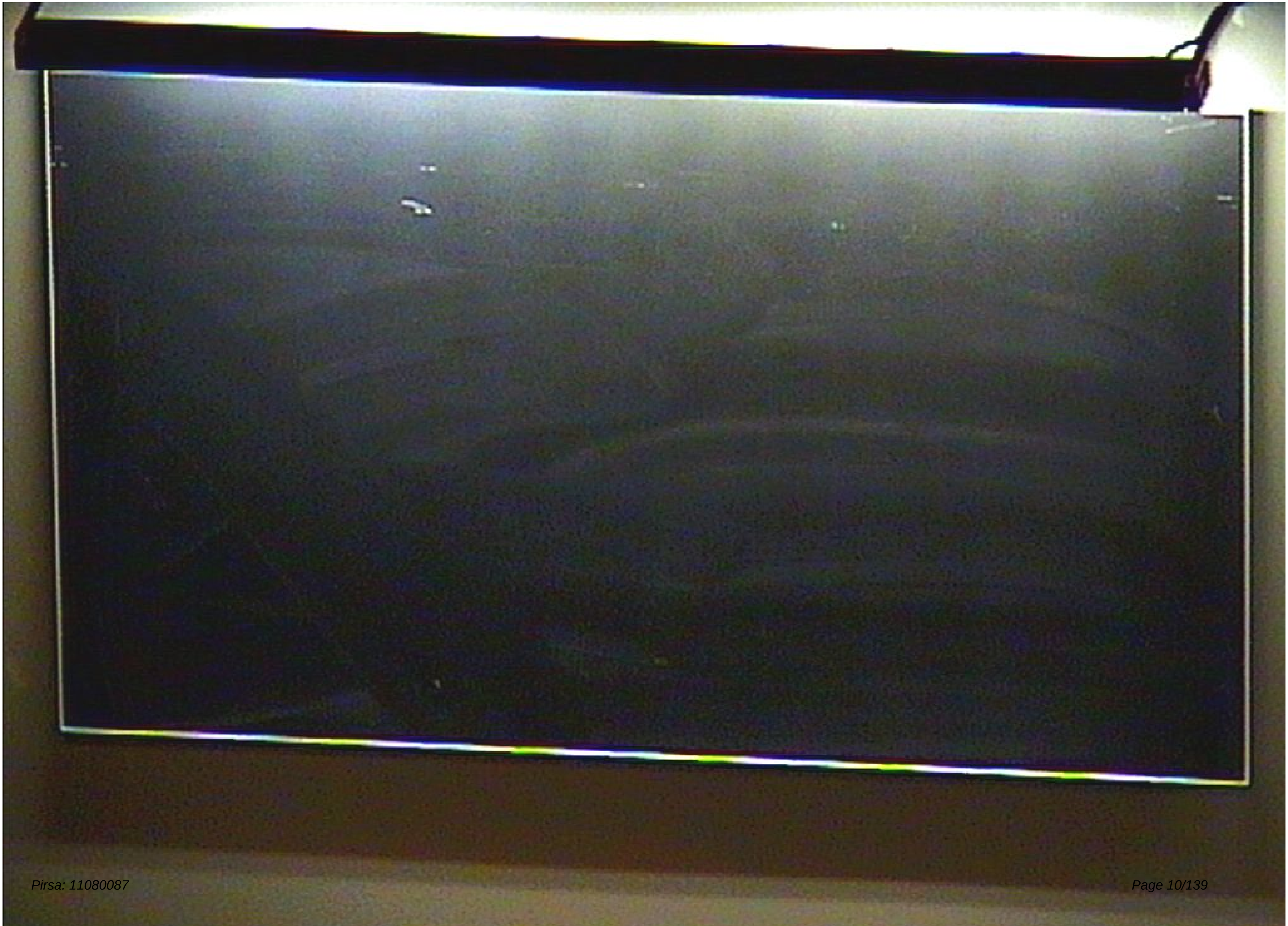
Abstract: The S-matrix of N=4 super Yang-Mills in the planar limit enjoys a remarkable duality with non-BPS null polygon Wilson loops in the same theory, but with the role of momenta and position interchanged. I will attempt to explain how such a duality works, by stressing how familiar notions such as factorization limits, unitarity and the loop integrand translate into simple and verifiable statements about Wilson loops. This requires a suitable supersymmetric extension of Wilson loops, which I will describe. I will then discuss recent progress in using the full supersymmetry of the theory on the Wilson loop side to simplify scattering amplitude computations

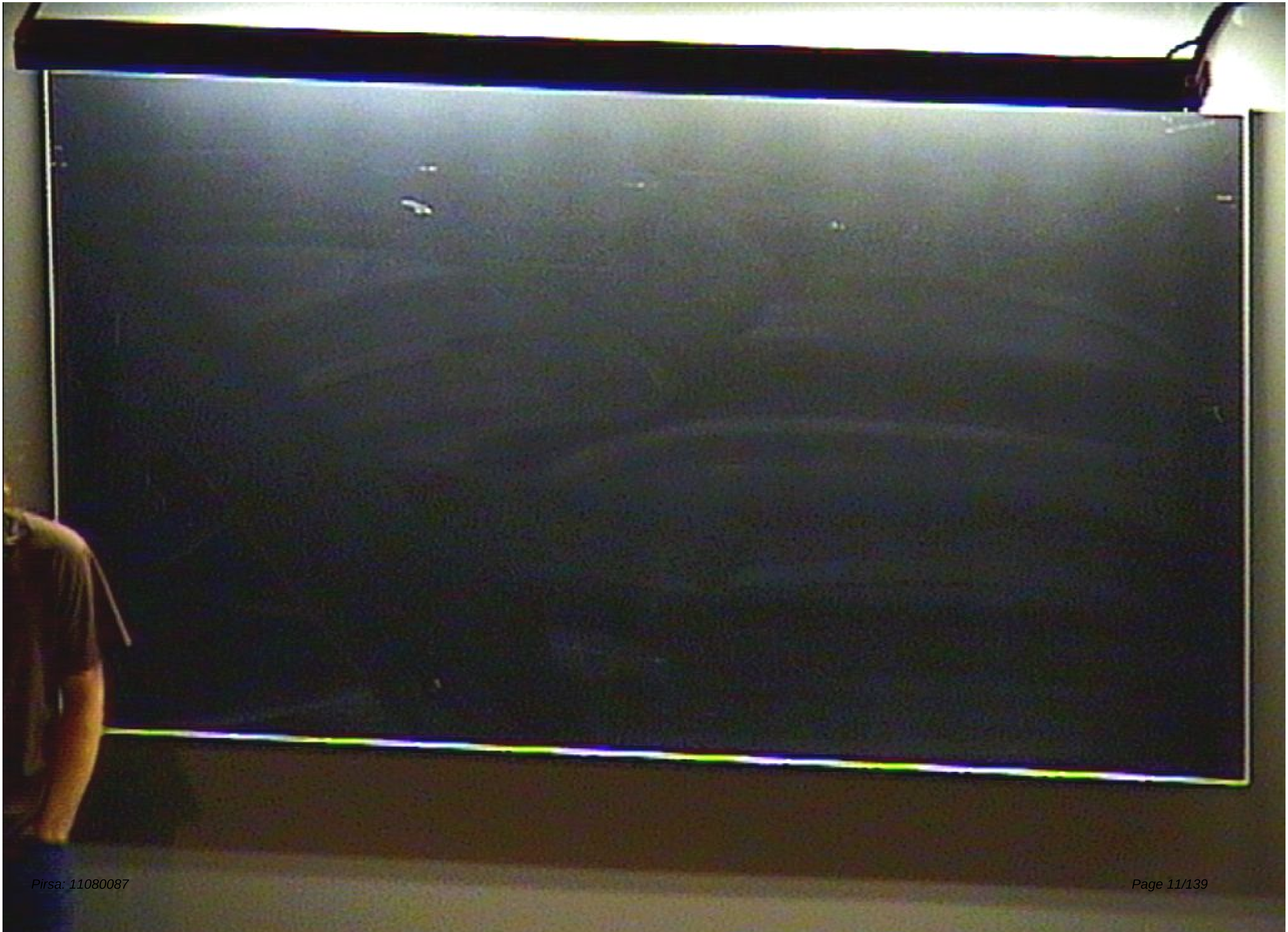


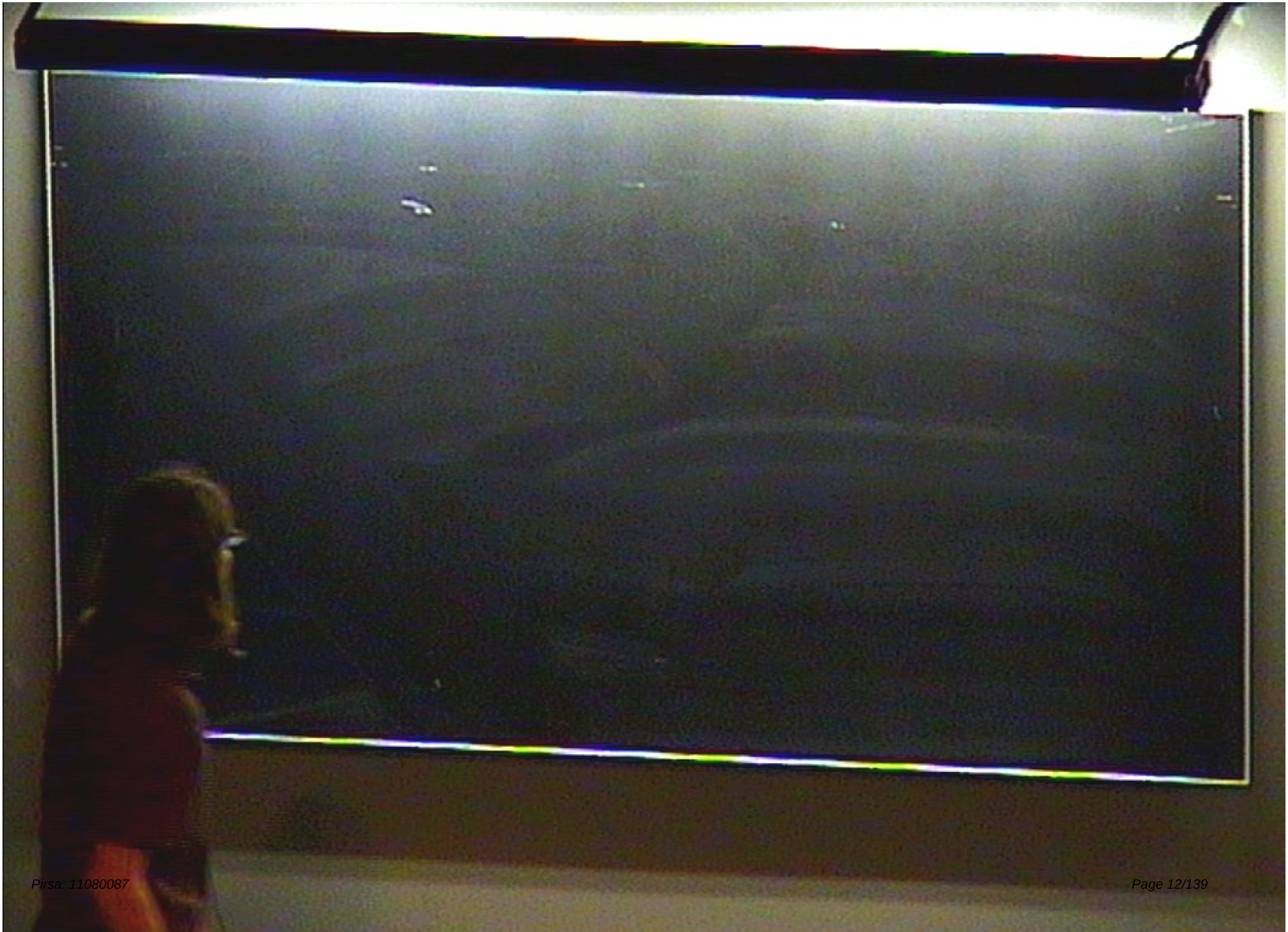


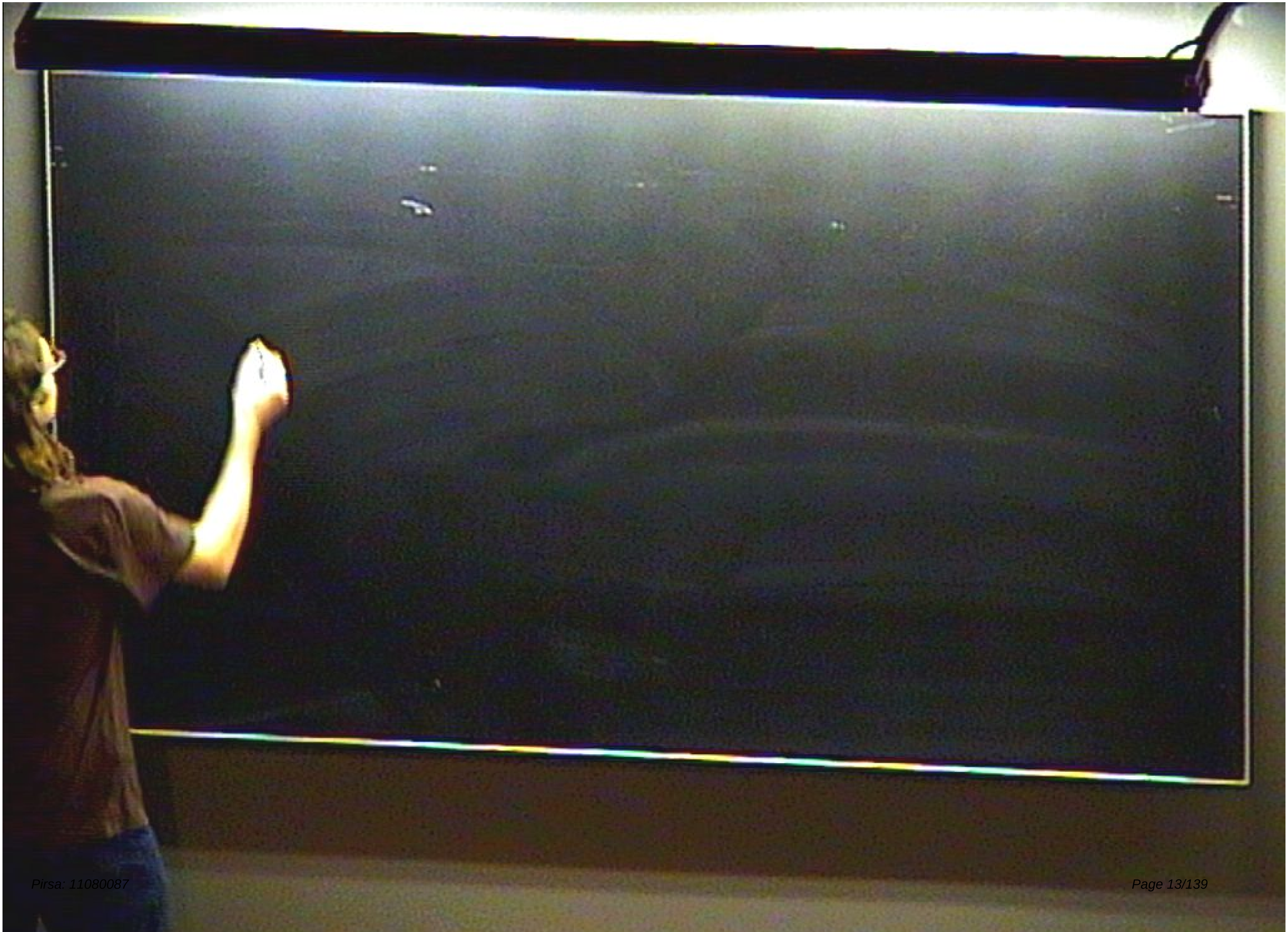


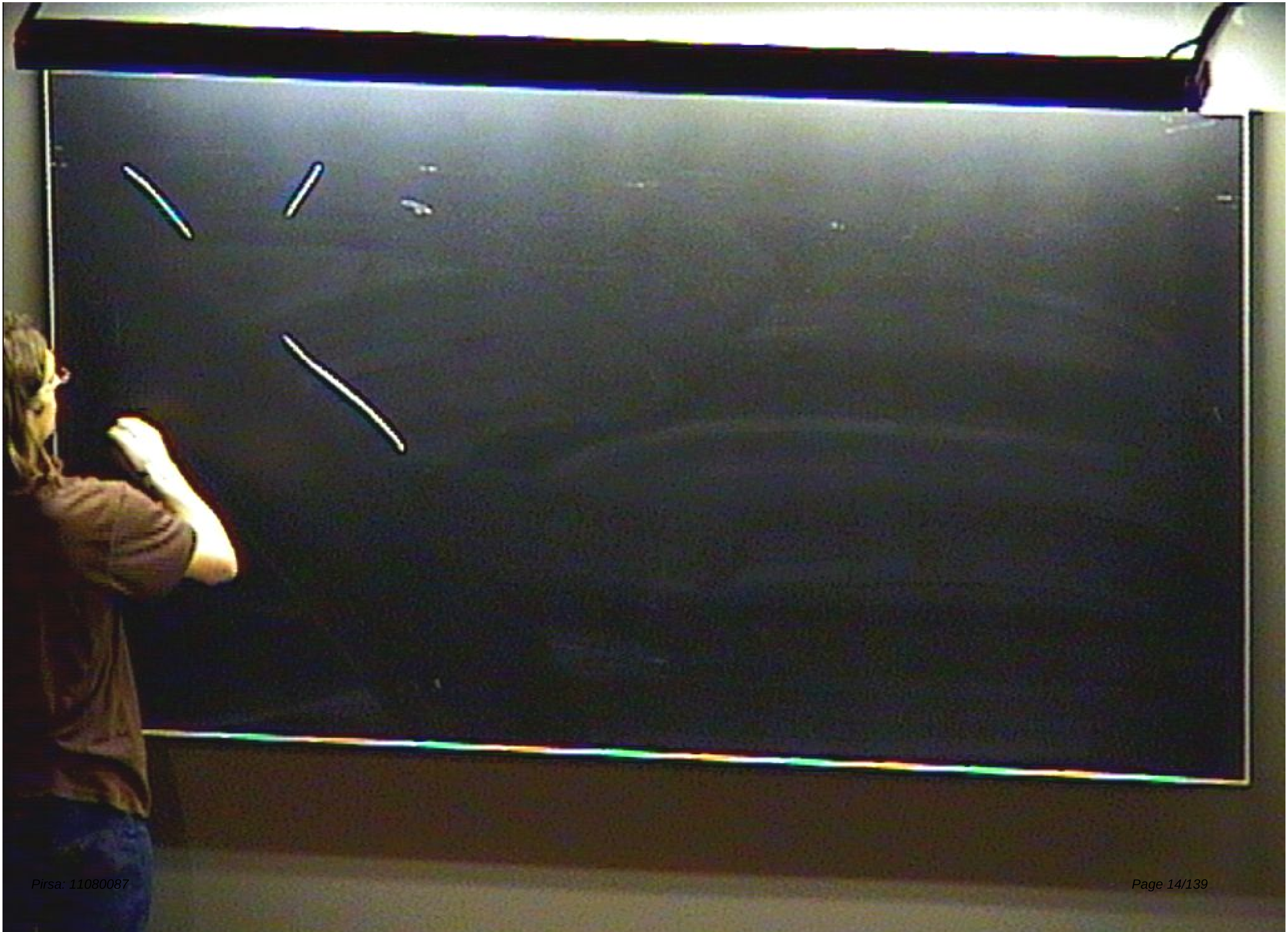


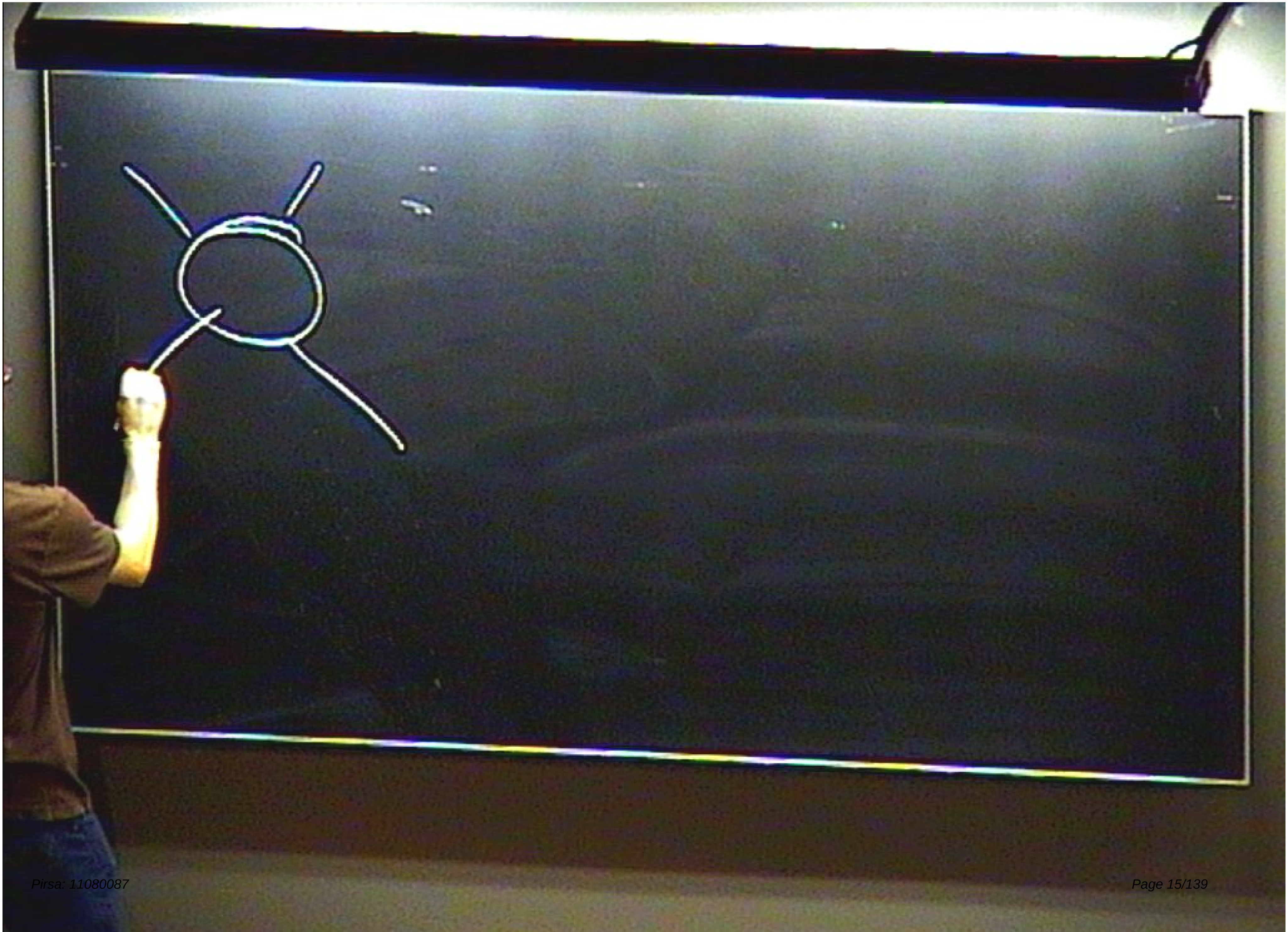


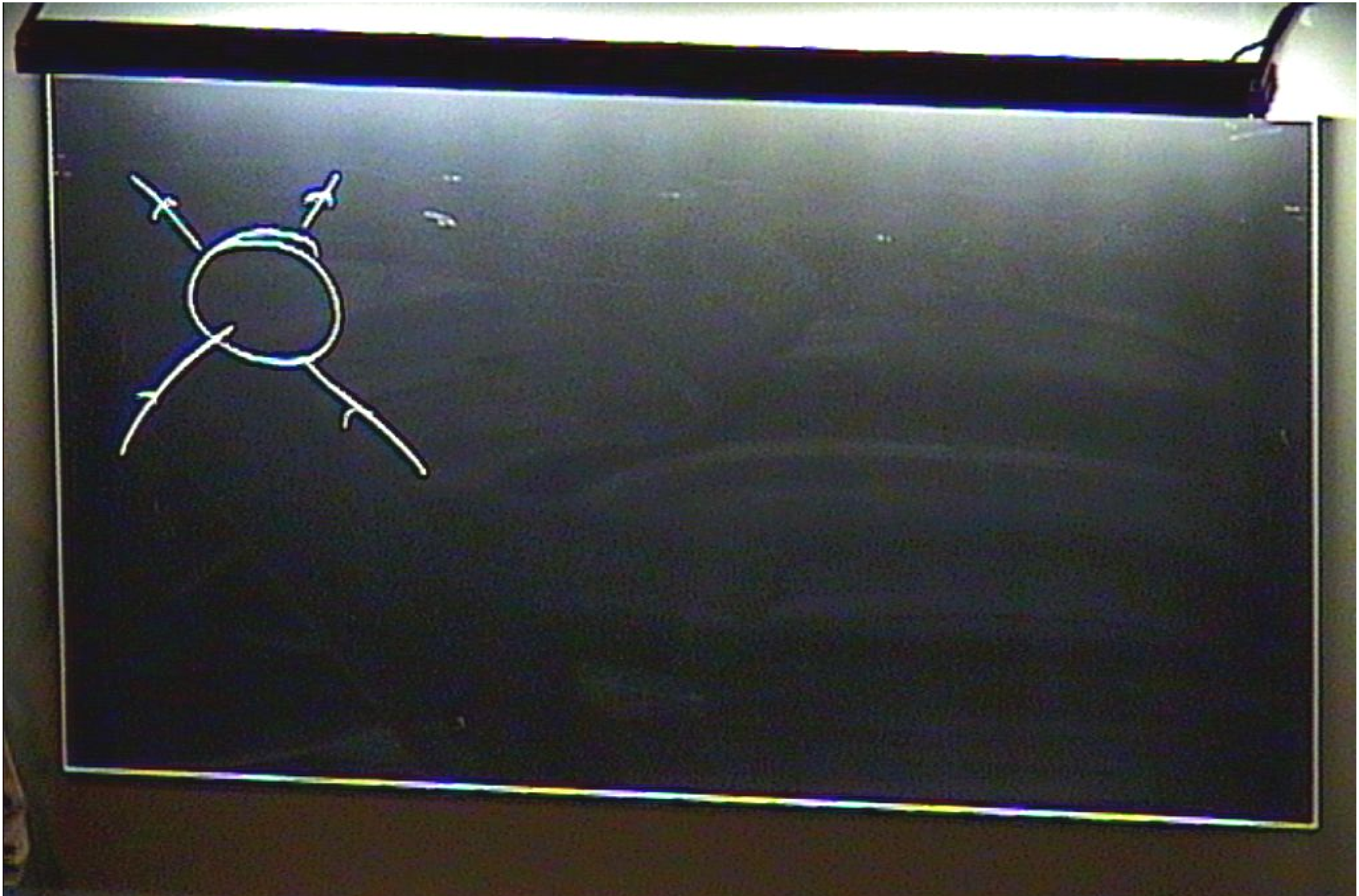


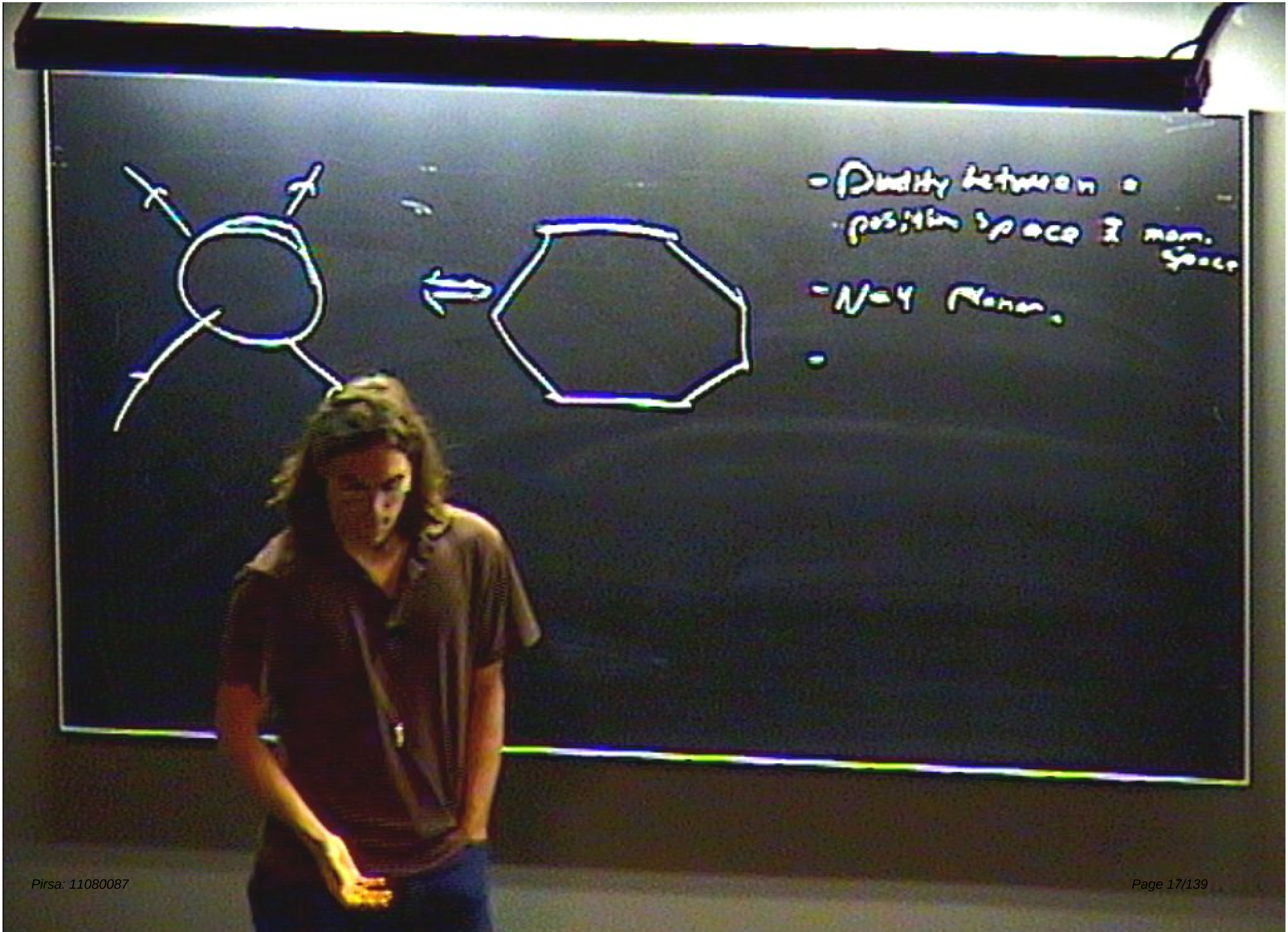


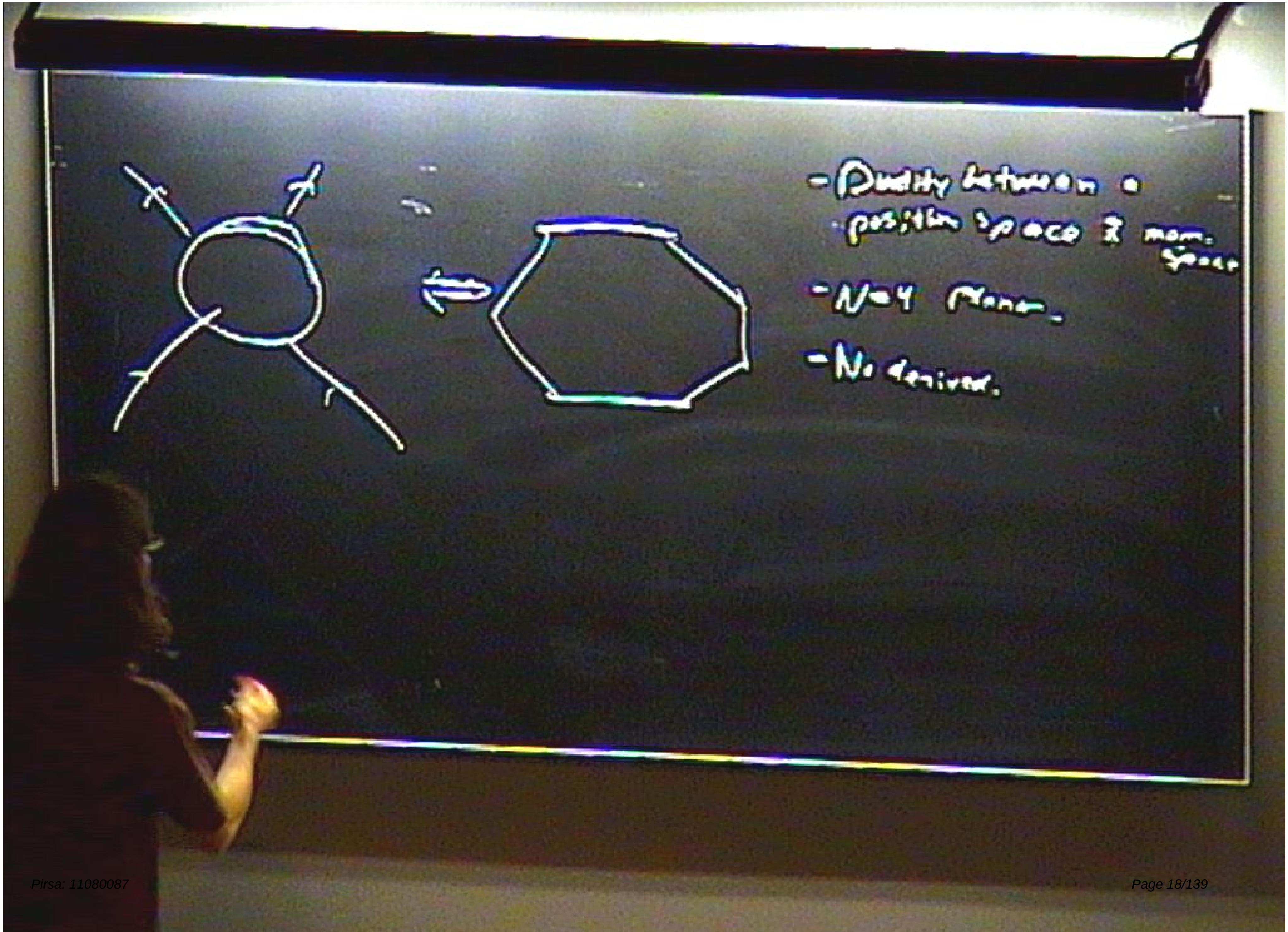




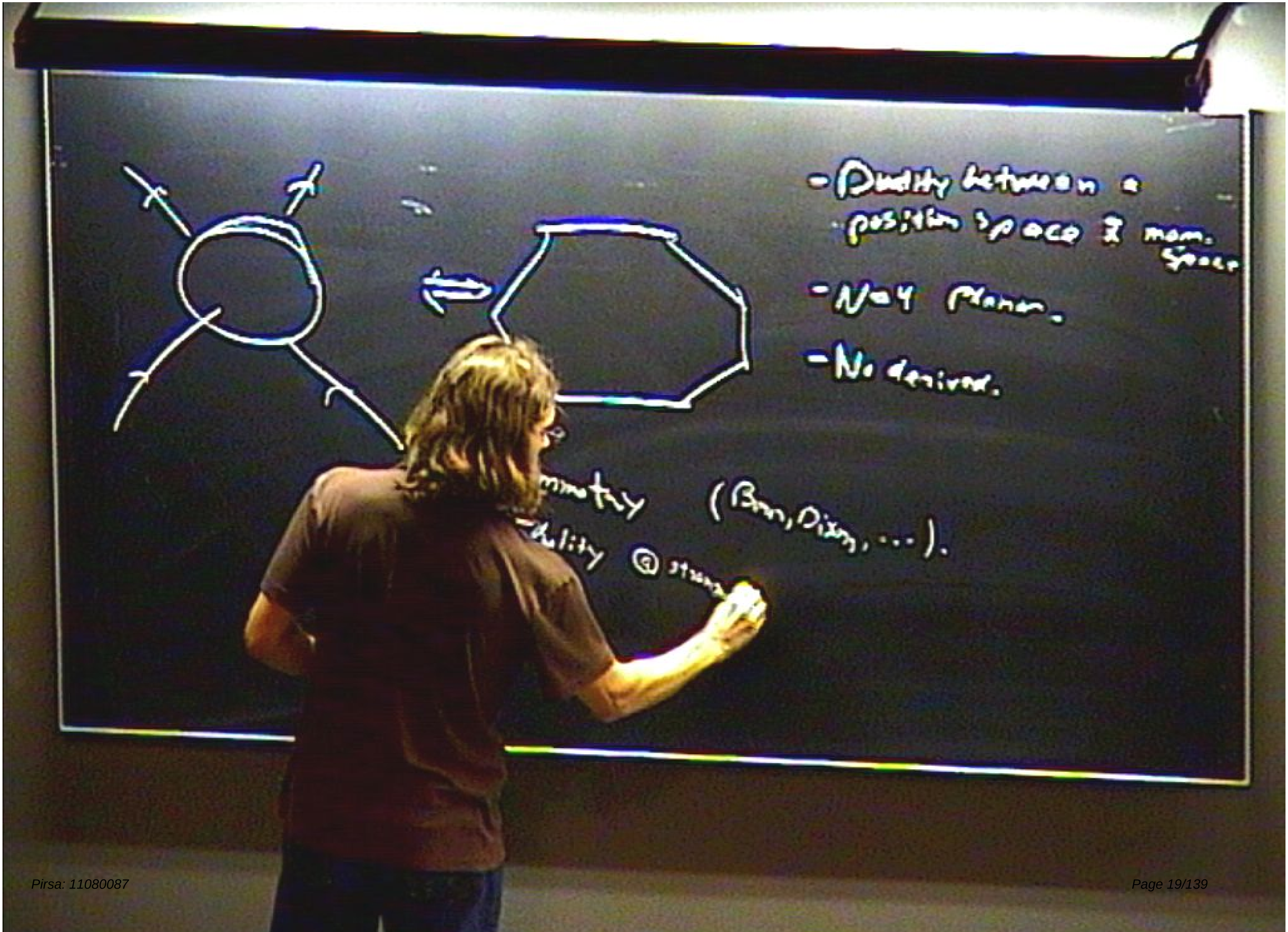


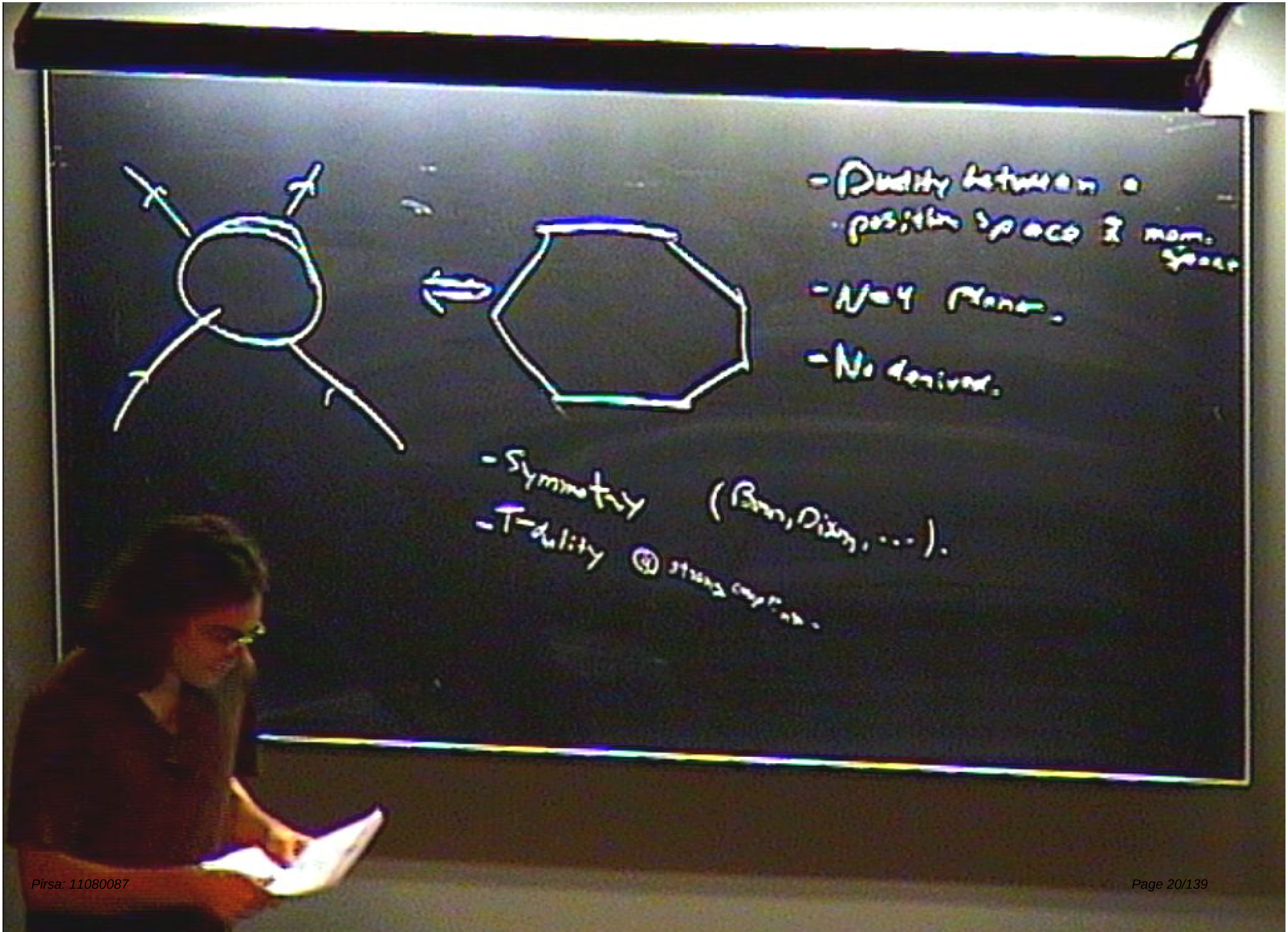


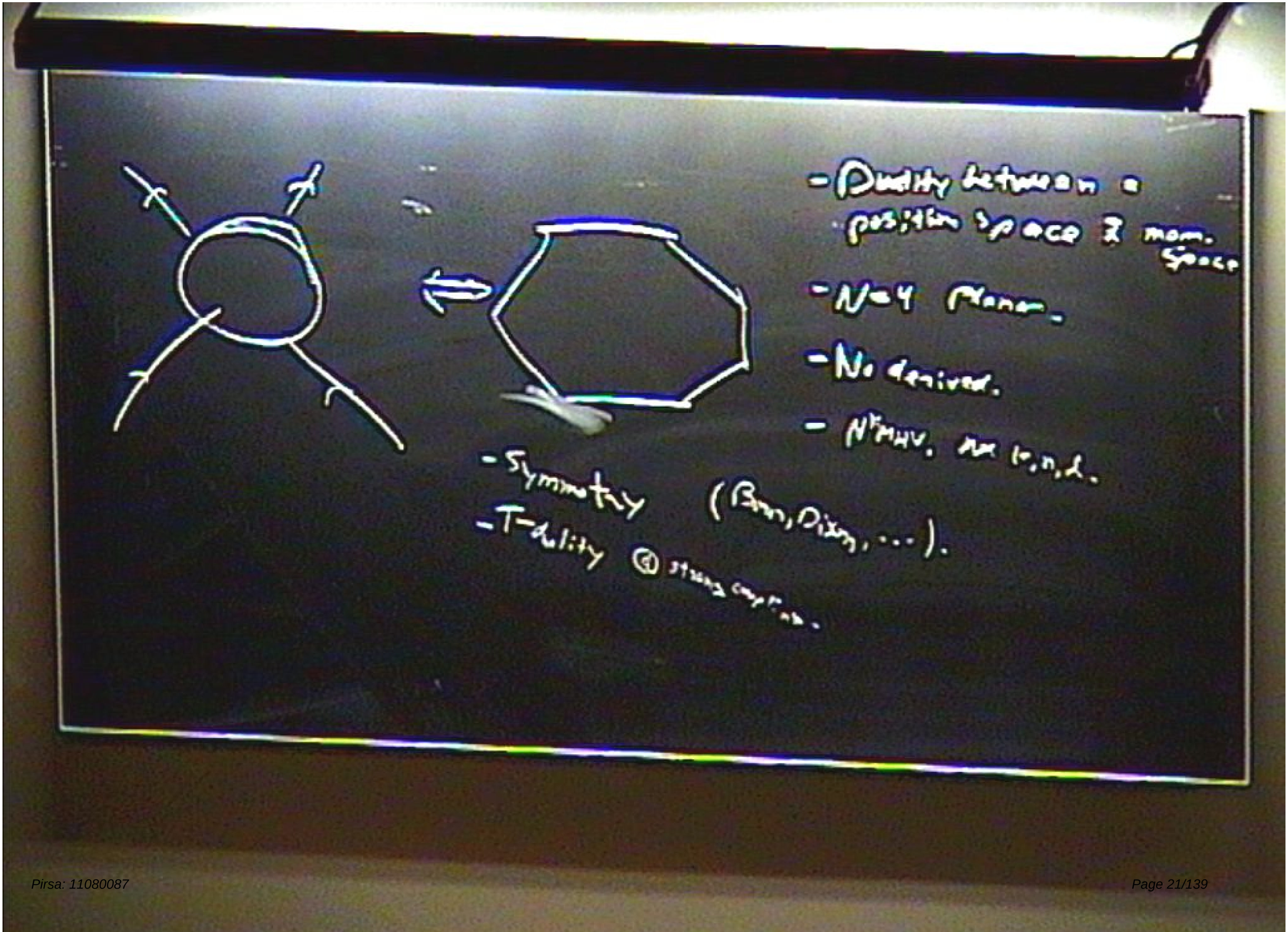

- Duality between
position space & mom. space
- N=4 Planar.



-Duality between
position space & mom. space
-N=4 Planar
-No derivat.

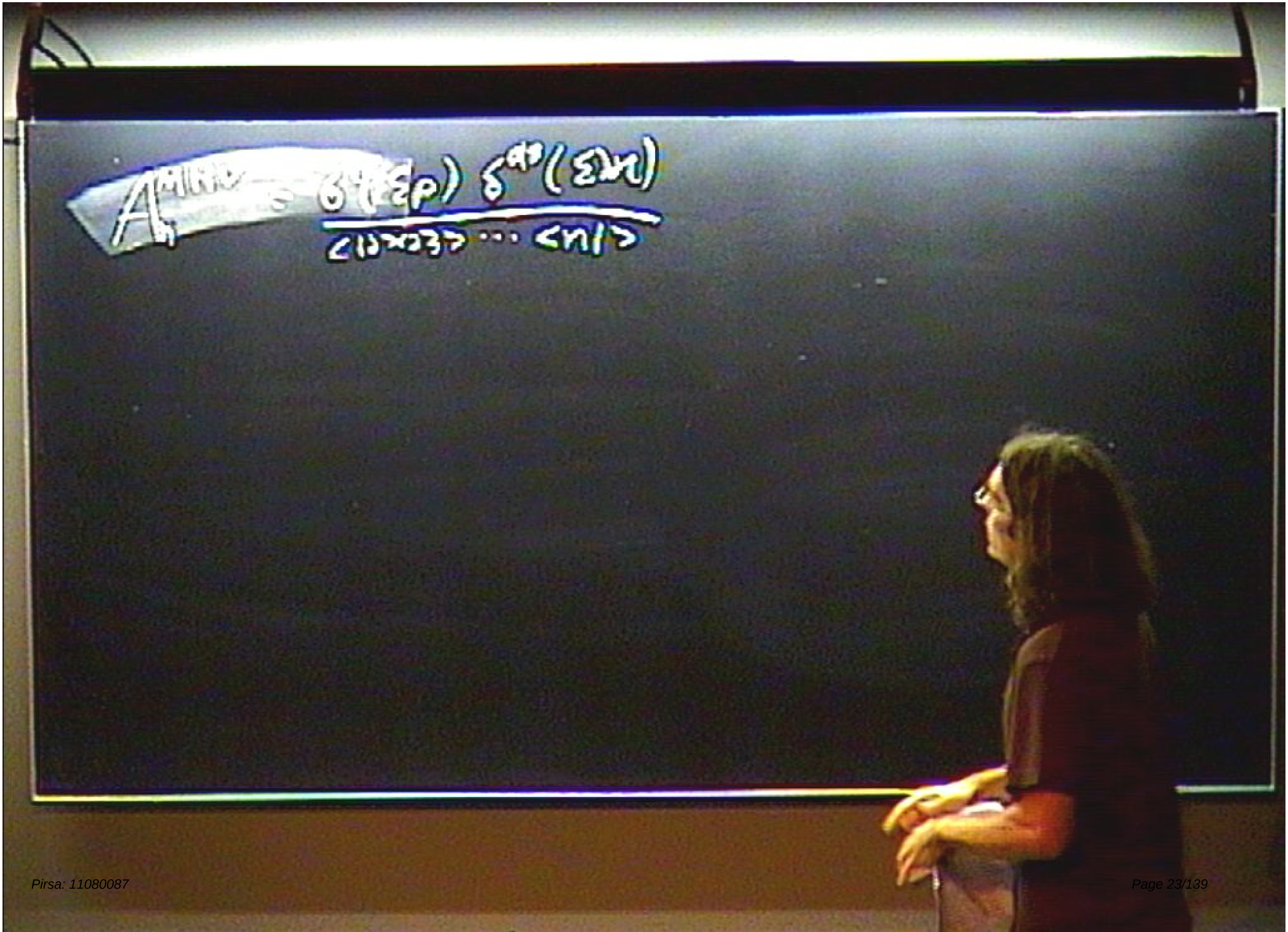

- Duality between position space & mom. space
- N=4 Planar.
- No derivat.
(Born, Dirac, ...).

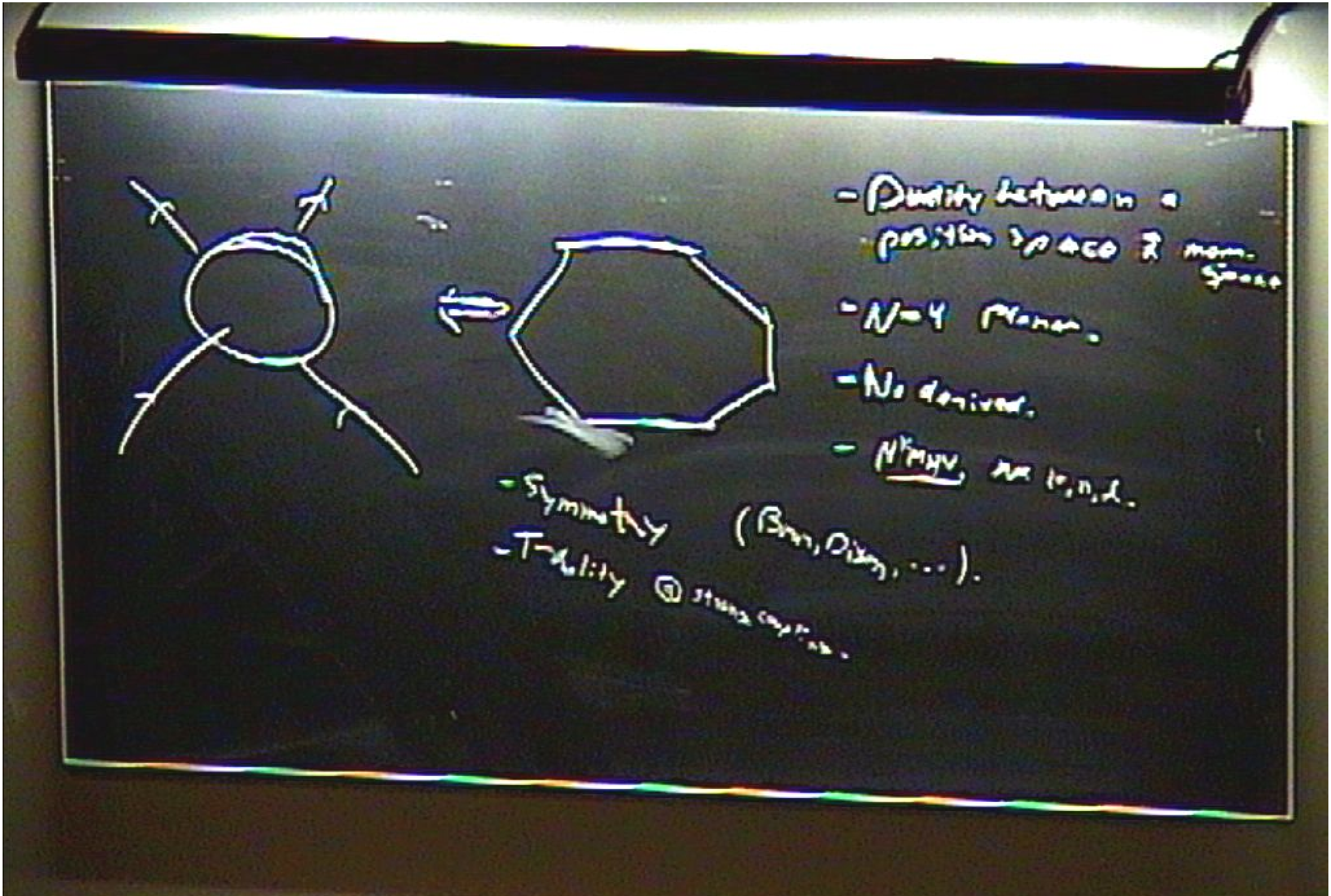

- Duality between position space & mom. space
- N=4 Planar.
- No derivat.
- Symmetry (Born, Dixon, ...).
- T-duality

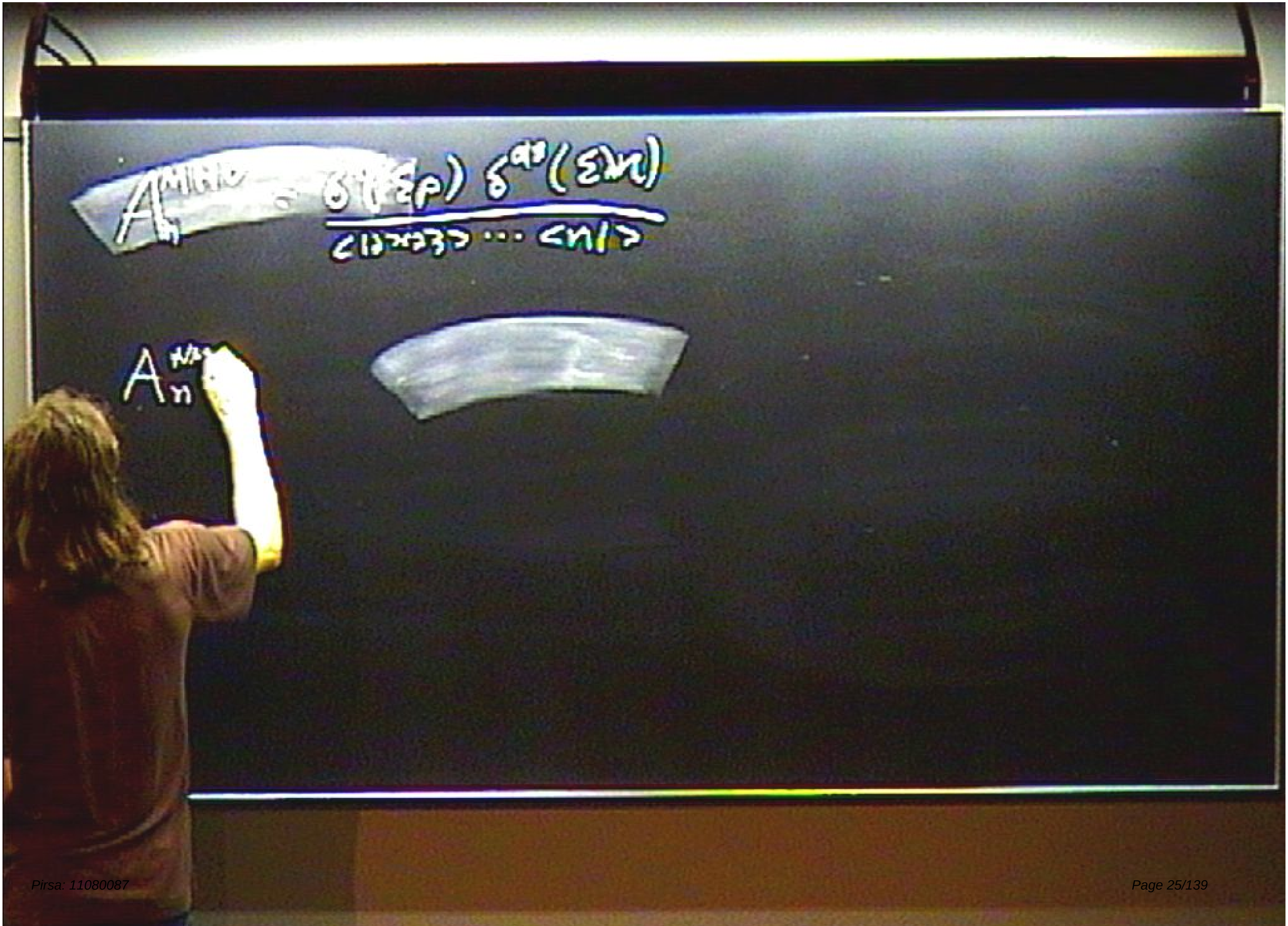


- Duality between position space & mom. space
- N=4 Planar.
- Symmetry
- T-duality @ strong coupling.

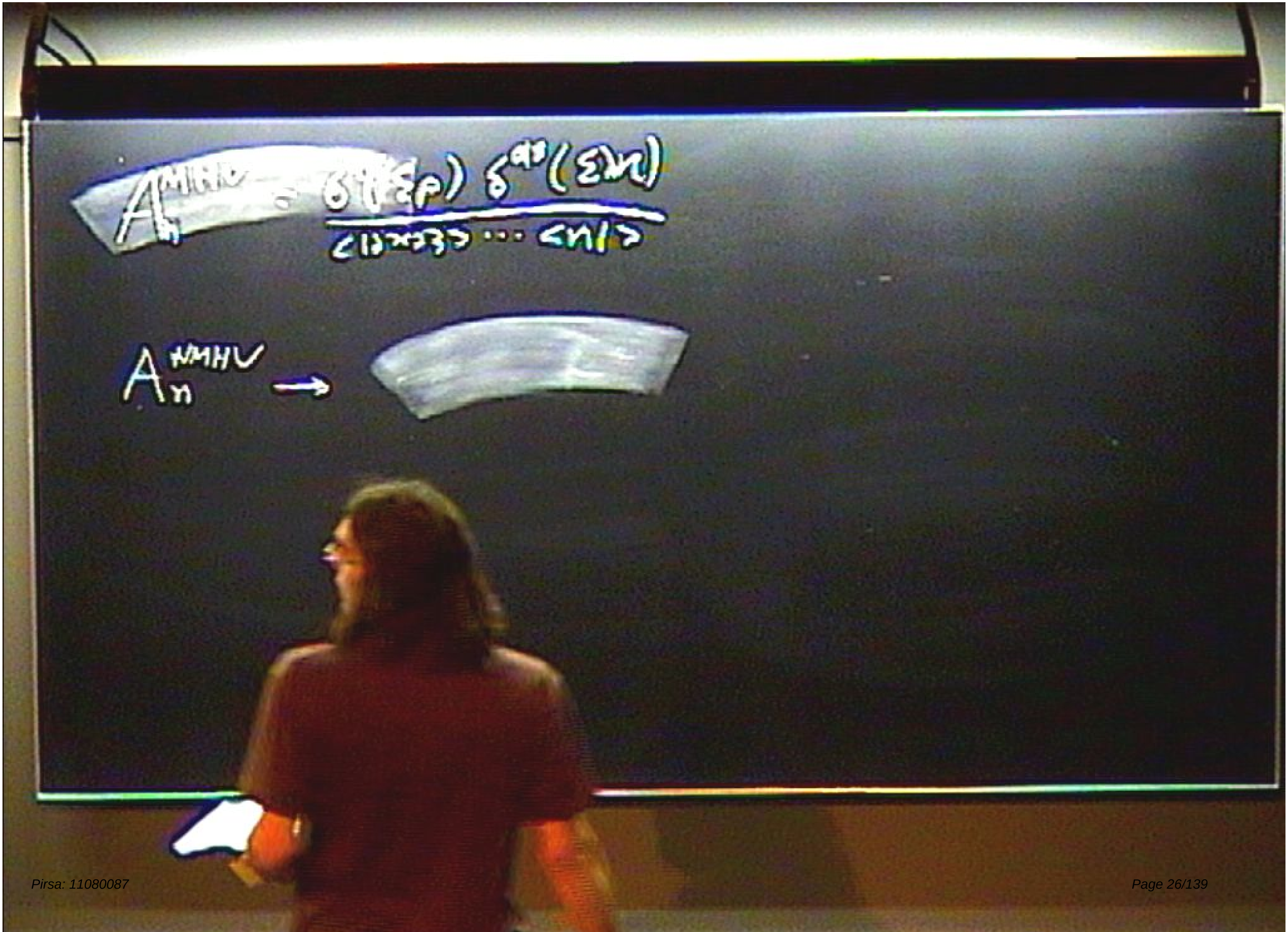
- Trees and Loops.



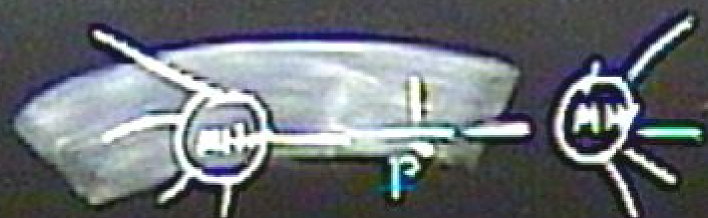

- Duality between a
position space & mom. space
- N=4 Planar
- No derivat.
- Symmetry (Bm, Dim, ...).
- T-duality @ strong coupling.

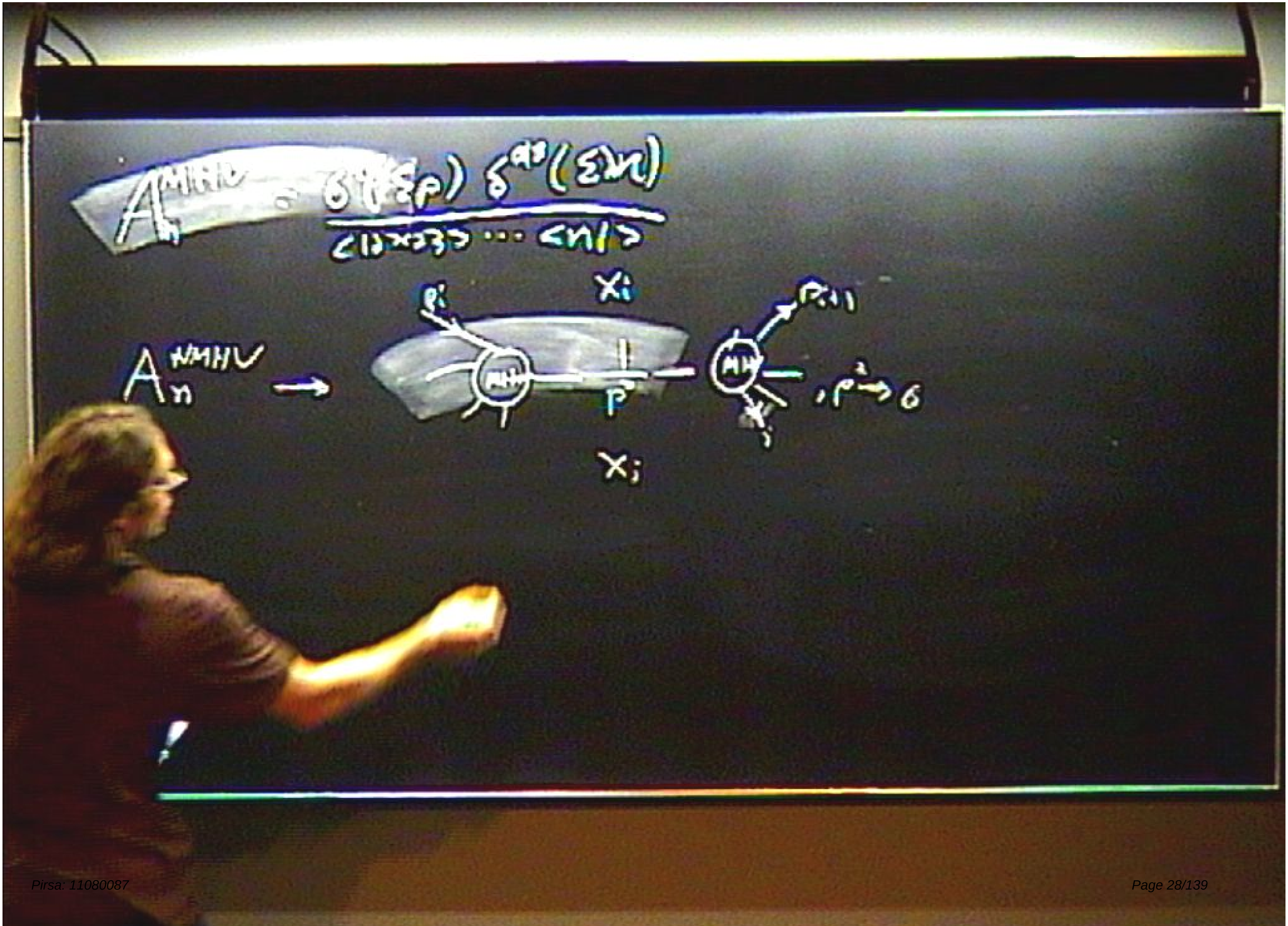


$A_n^{MHV} = \frac{\delta^4(\Sigma p)\,\delta^8(\Sigma \eta\lambda)}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$
A_n


$A_n^{MHV} = \frac{\delta^4(\Sigma p)\,\delta^8(\Sigma \eta\lambda)}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$
$A_n^{NMHV} \rightarrow$

$$A_n^{MHV} = \frac{\delta^4(\Sigma p)\,\delta^8(\Sigma \lambda\eta)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

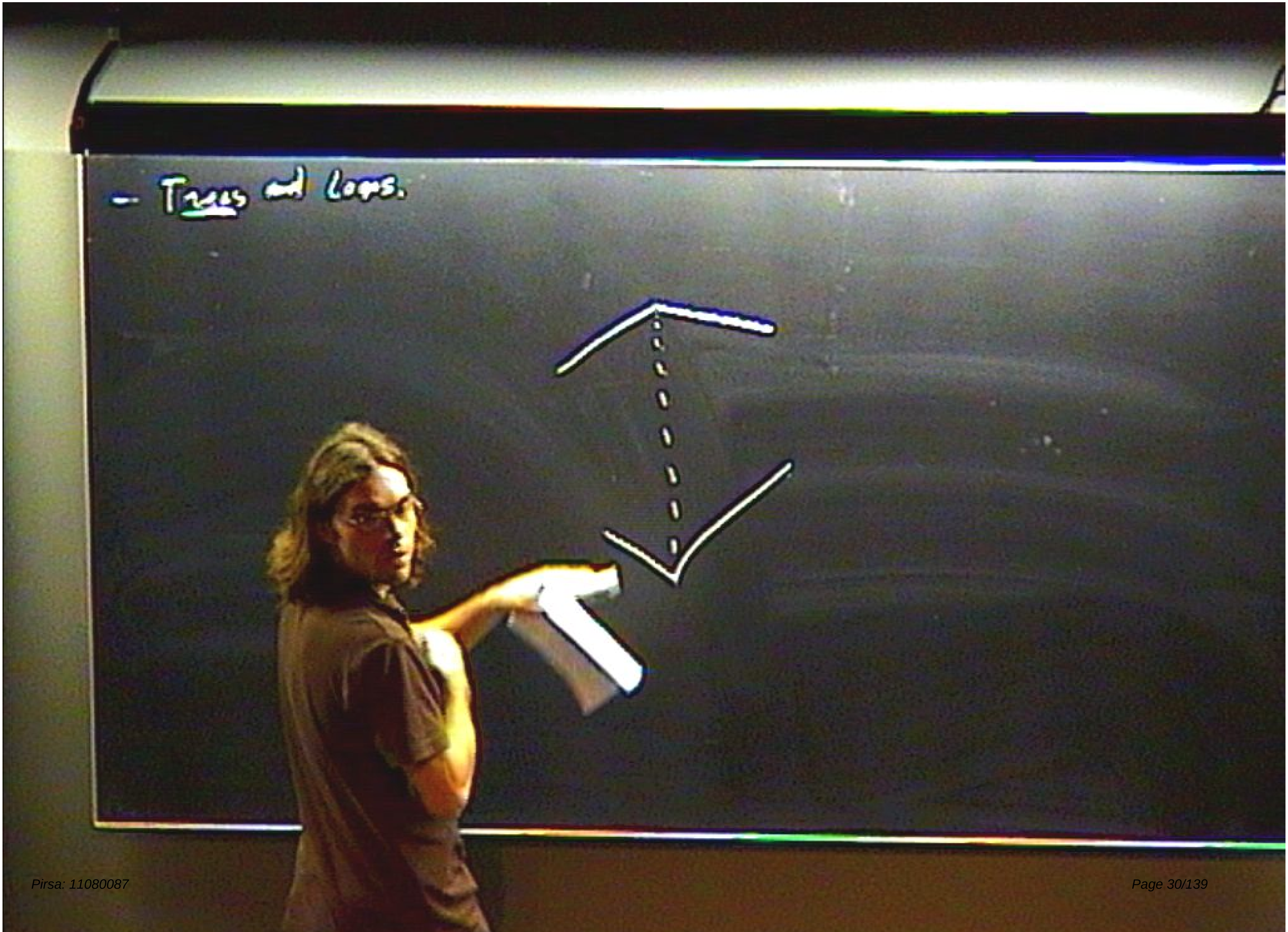
$A_n^{NMHV} \longrightarrow$  $,\ P^2 \to 0$

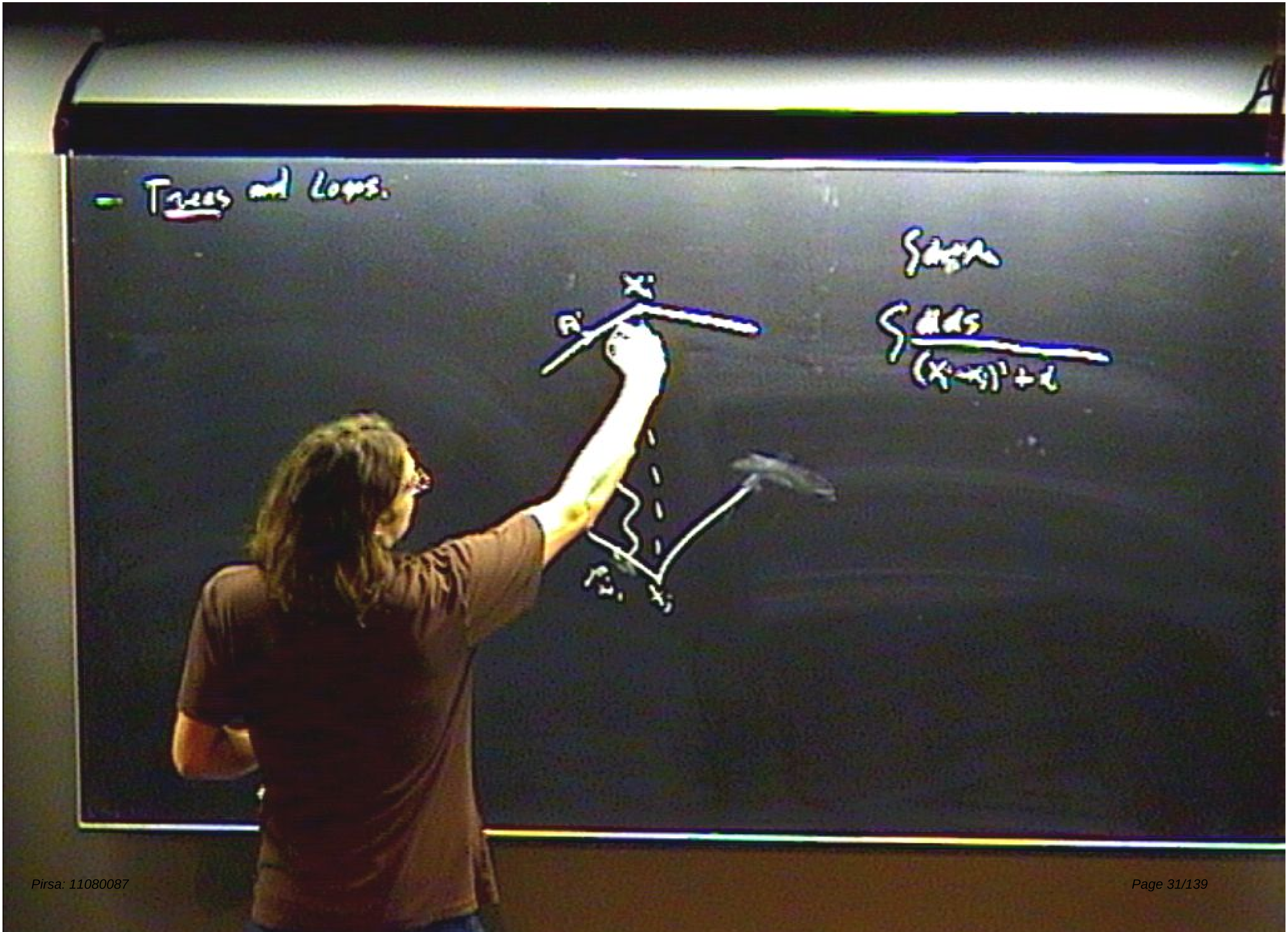

A_n^{MHV}
$\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle$
$A_n^{NMHV} \longrightarrow$
x_i
x_j
$P^2 \rightarrow 0$

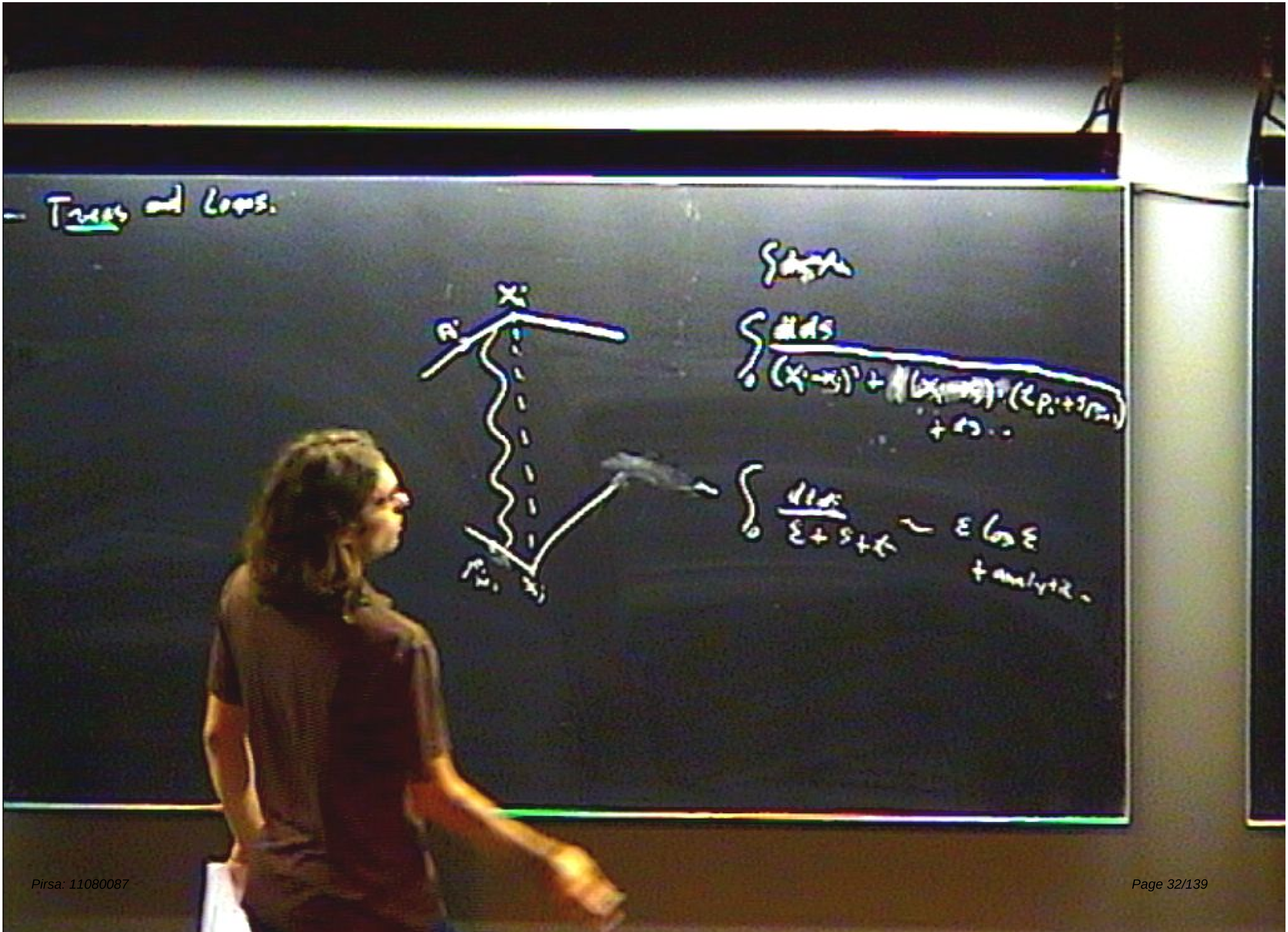
$$A_n^{MHV} = \frac{\delta^4(\sum p)\,\delta^{8}(\sum \lambda\eta)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

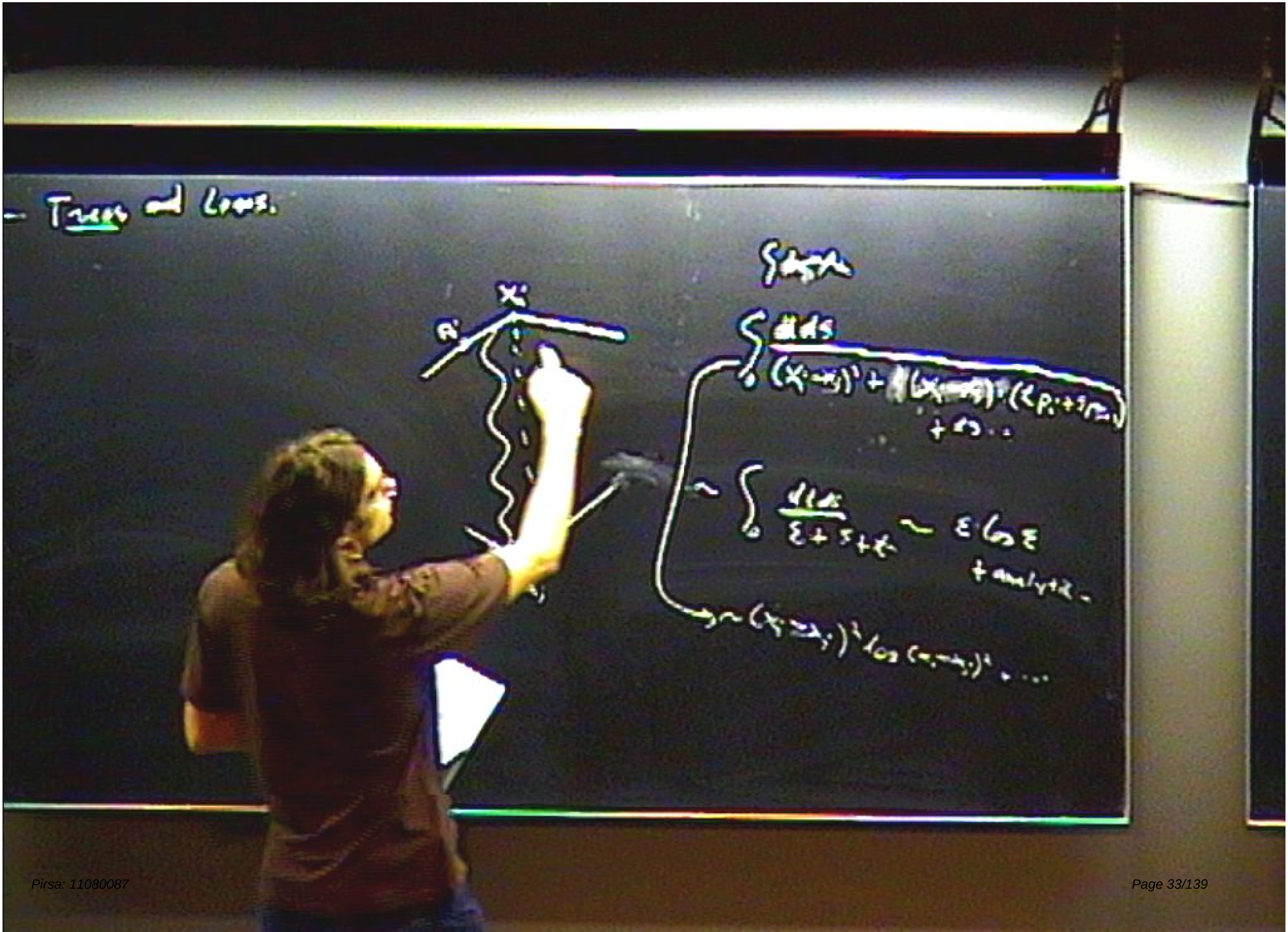
$$A_n^{NMHV} \longrightarrow$$

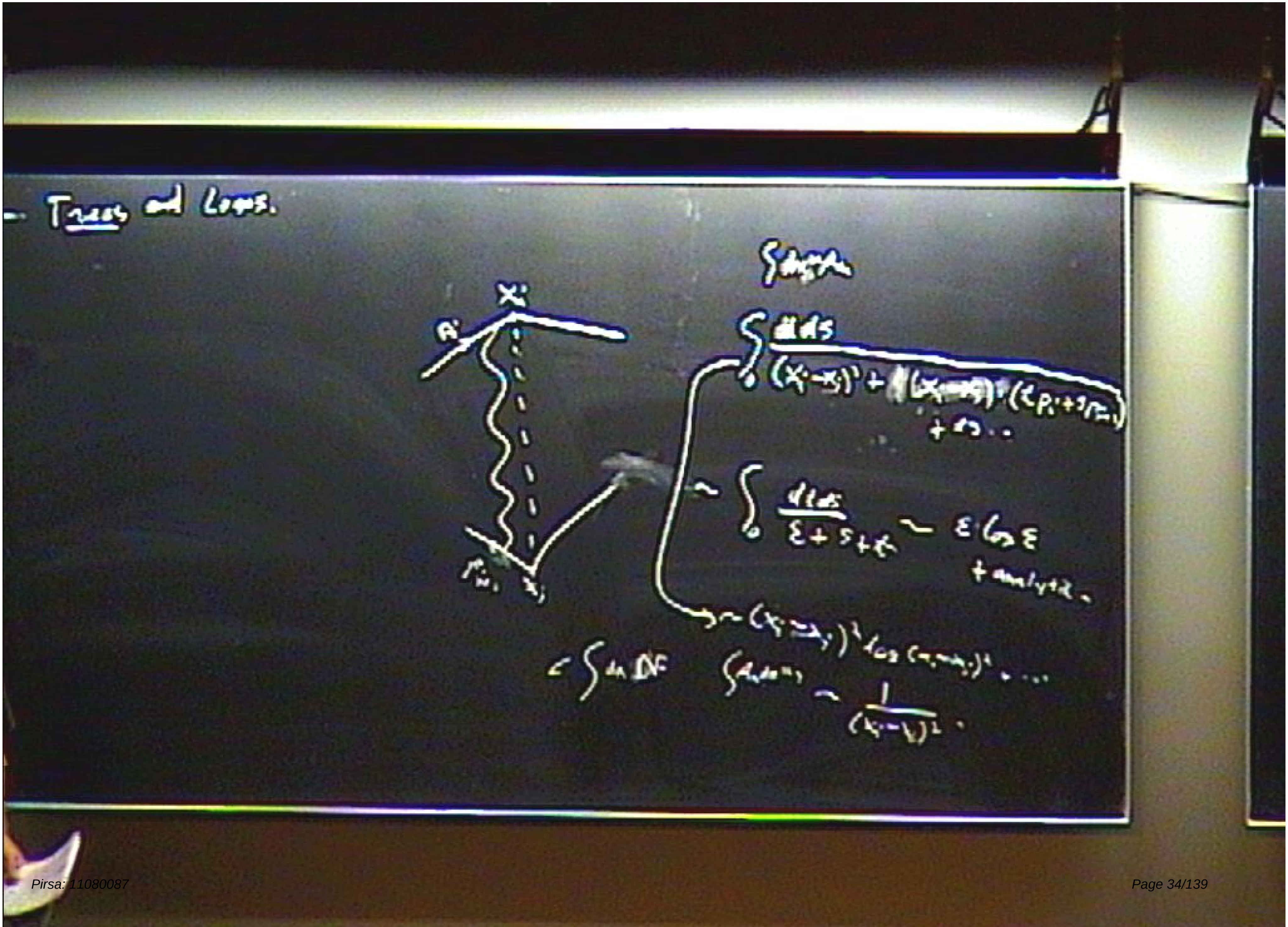
$$,\ p^2 \to 0$$

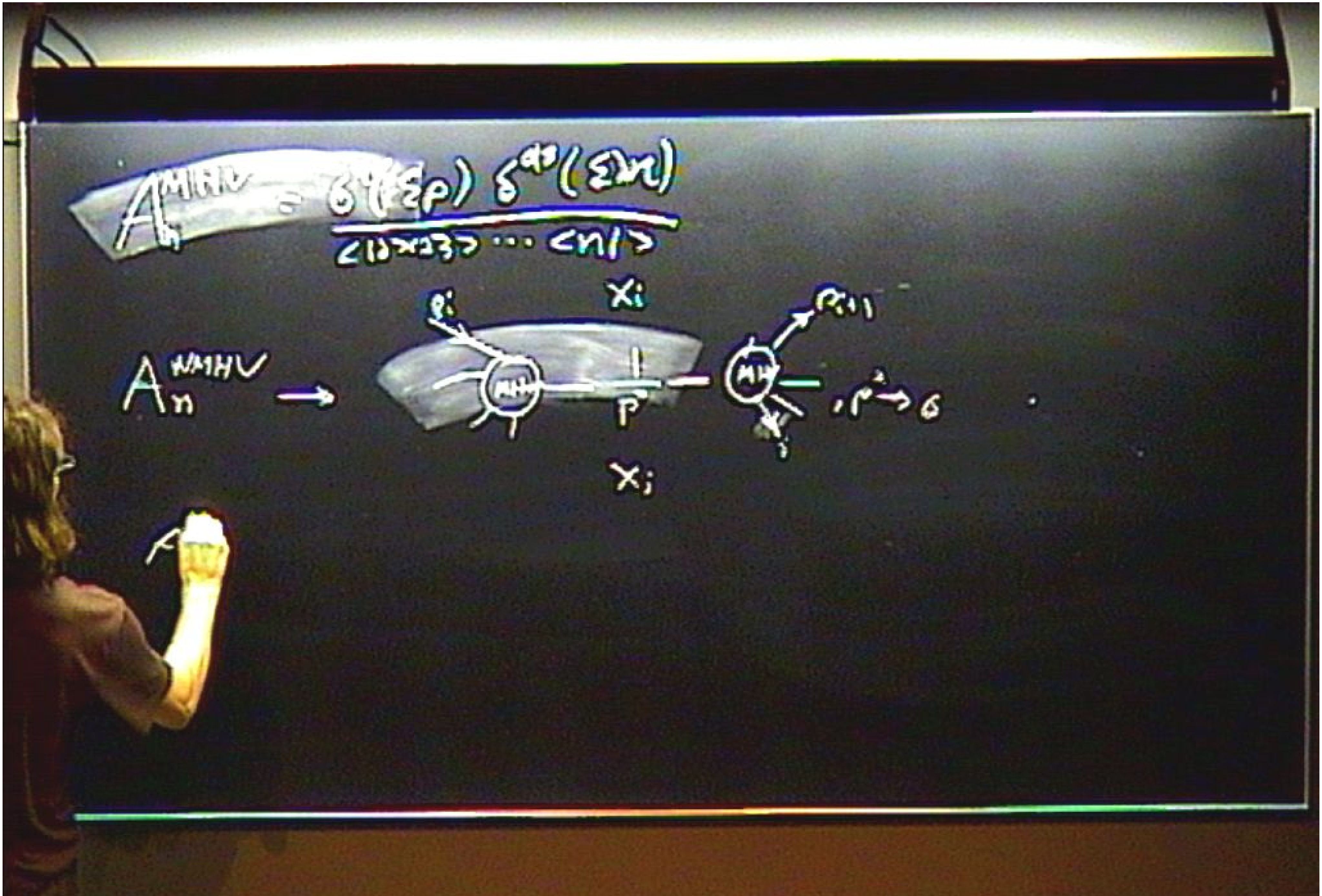

- Trees and loops.

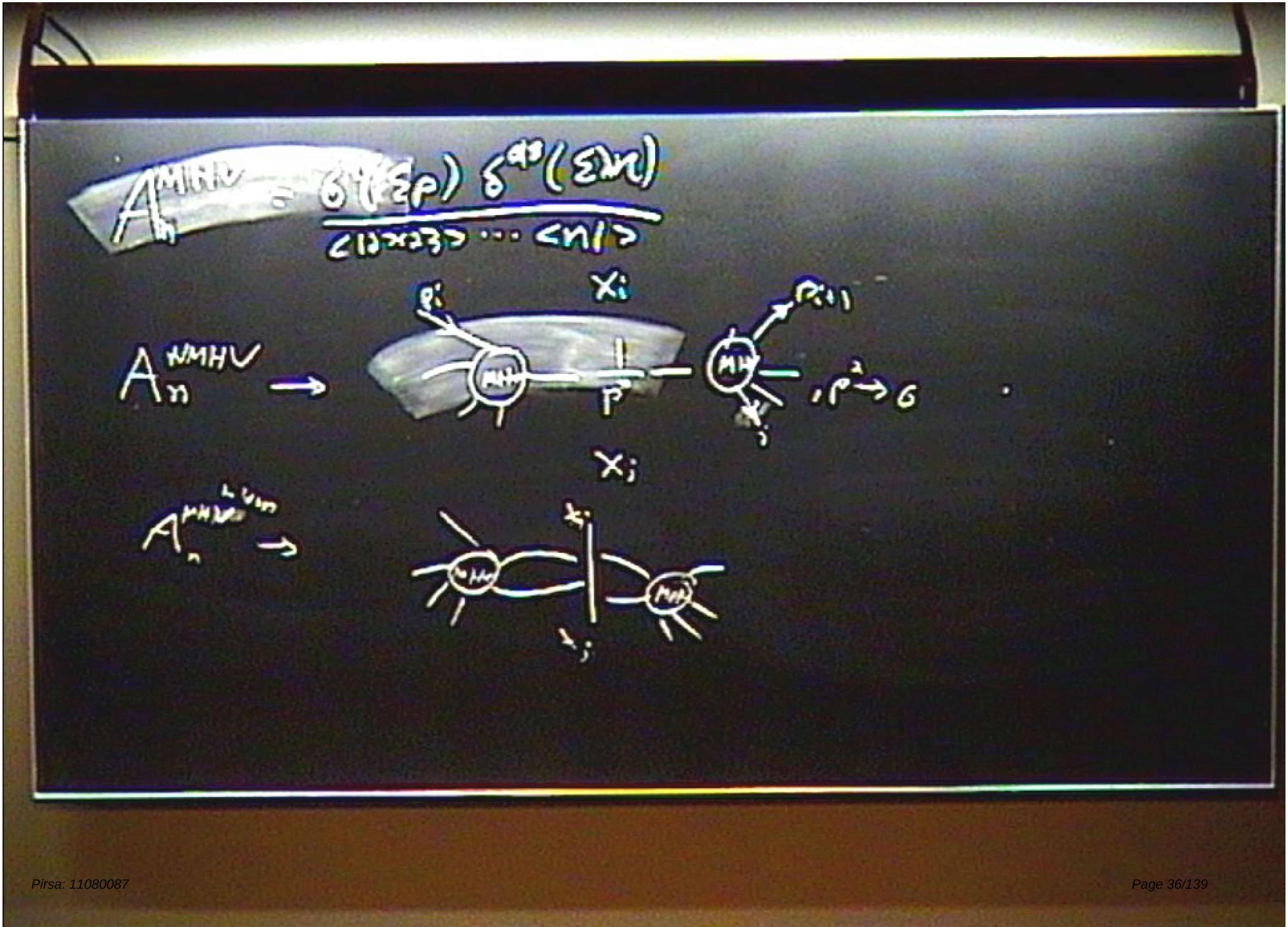

– Trees and Loops.

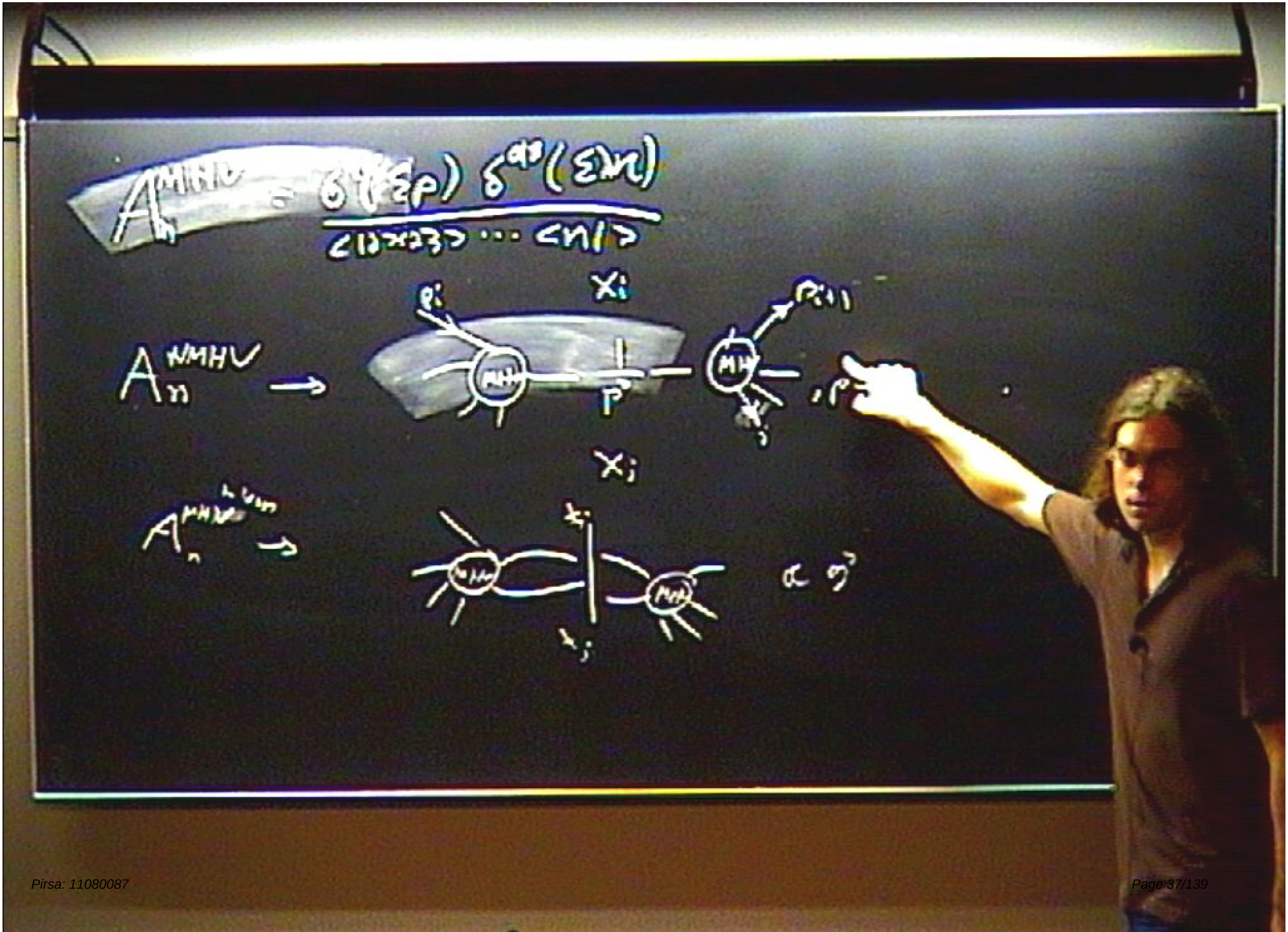

Trees and Loops.
+ analytic.


Trees and Loops.

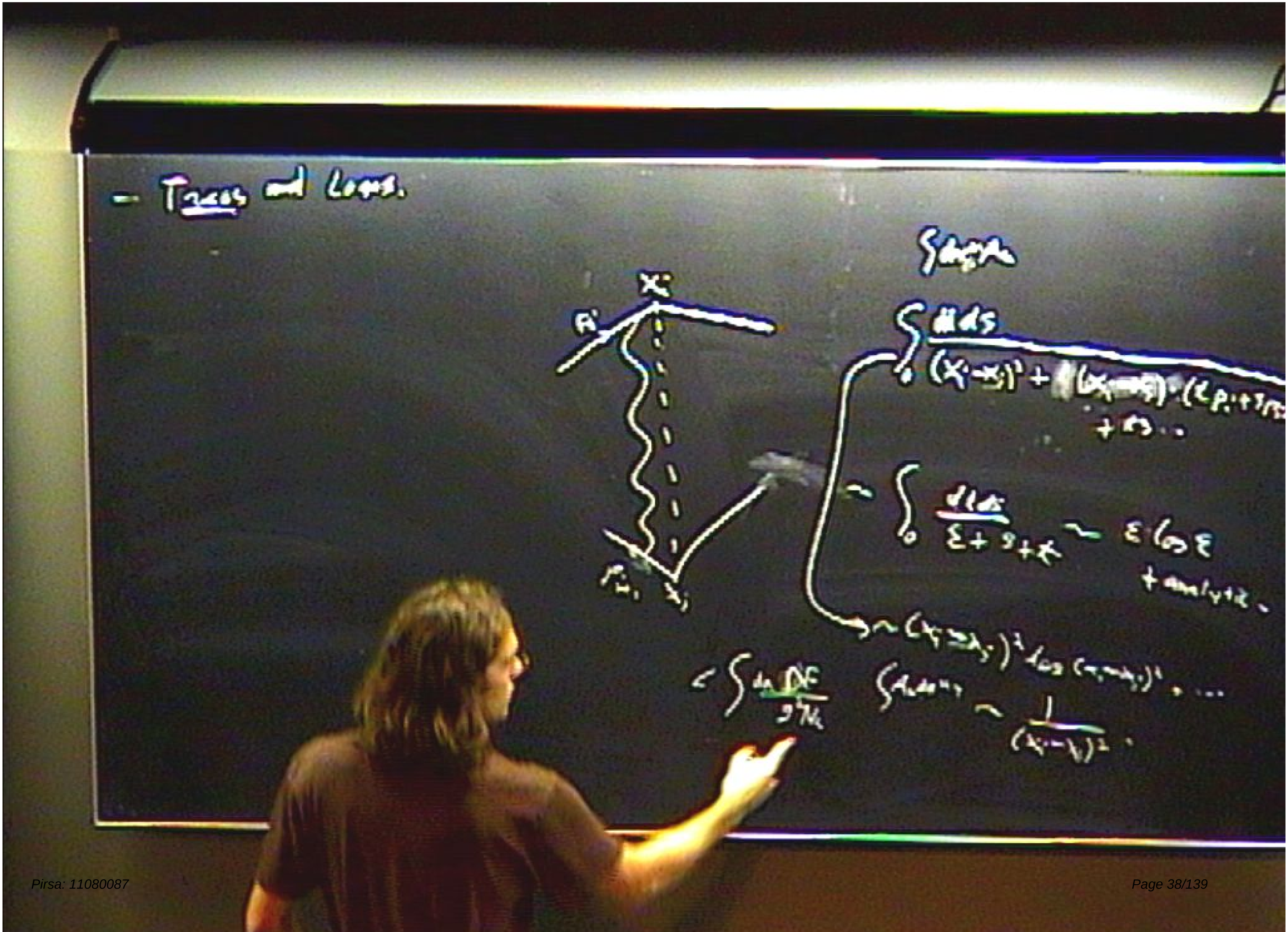

Trees and Loops.
+ analytic

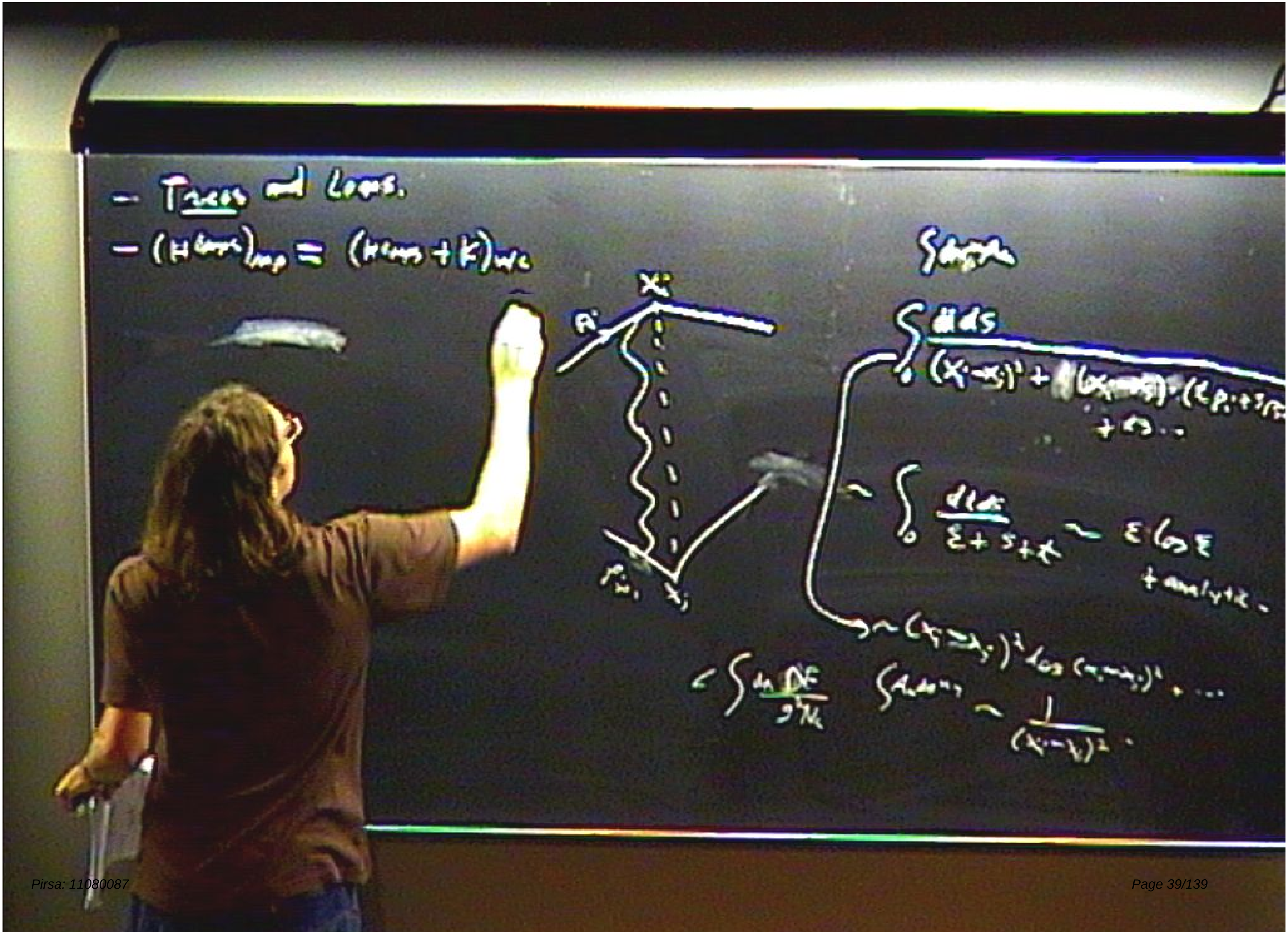

$A_n^{MHV} = \frac{\delta^4(\Sigma p)\,\delta^{8}(\Sigma \lambda\eta)}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$
x_i
$A_n^{NMHV} \rightarrow$
MHV
$\frac{1}{p^2}$
MHV
$p^2 \to 0$
x_j

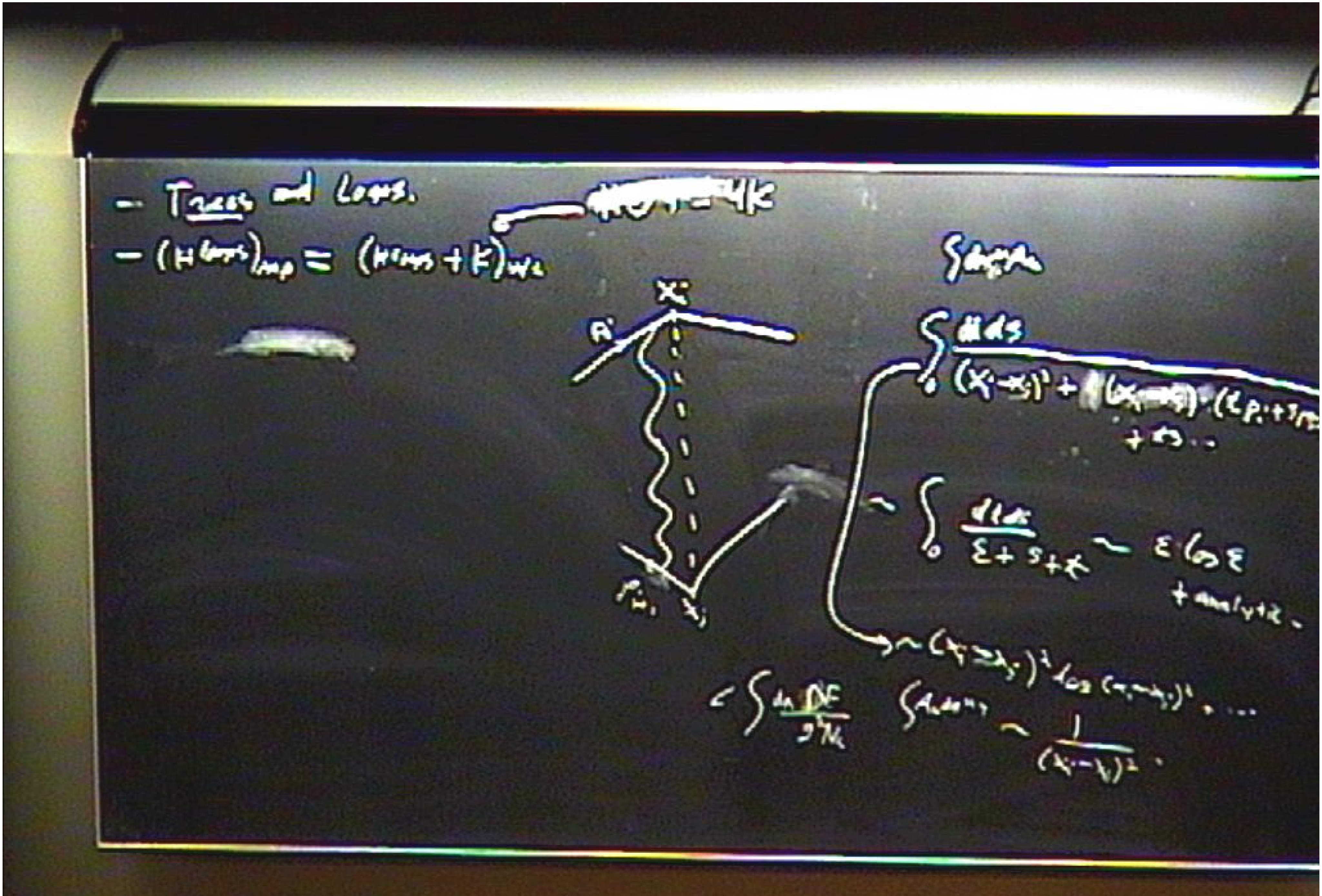

A_n^MHV = δ^4(Σp) δ^8(Σλη) / ⟨12⟩⟨23⟩ ··· ⟨n1⟩
x_i
p_i
p_{i+1}
A_n^NMHV →
MHV
1/P^2
P^2 → 0
x_j
A_n^MHV 1-loop →
MHV
x_i
x_j

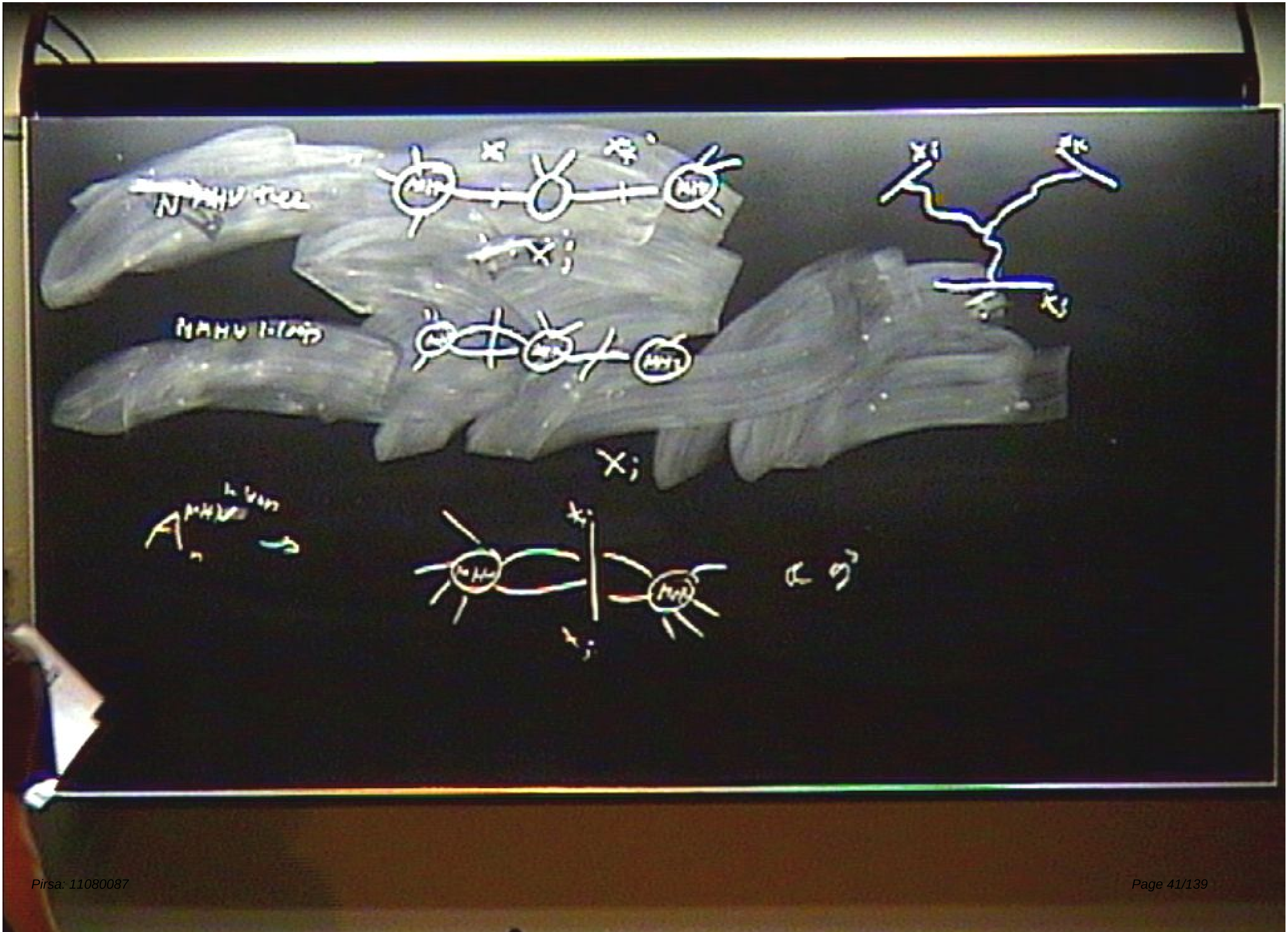


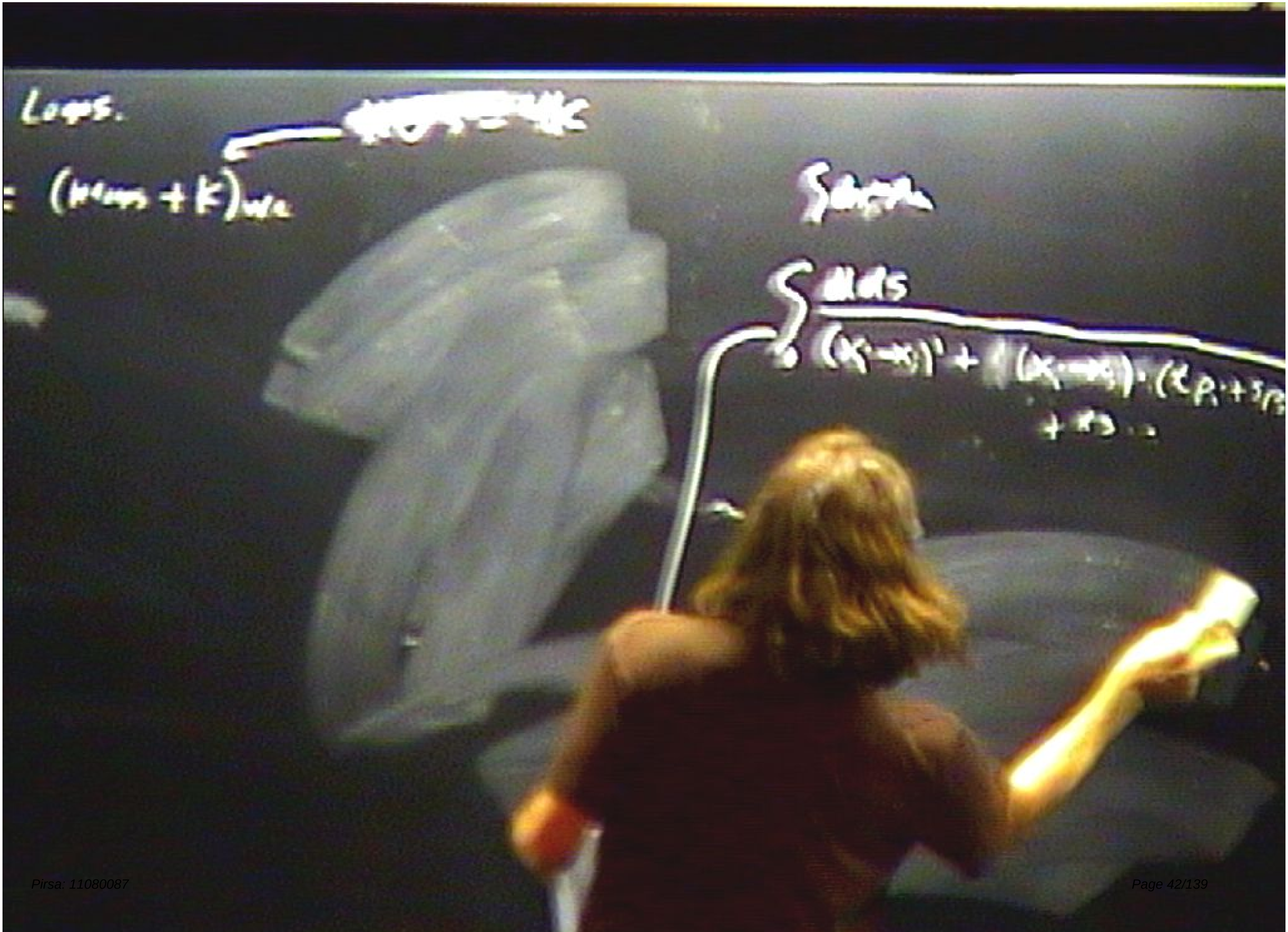
<12><23> ⋯ <n1>
NMHV
MHV
P
x_i
x_j


— Traces and Loops.
+ analytic.


— Traces and Loops.
+ analytic

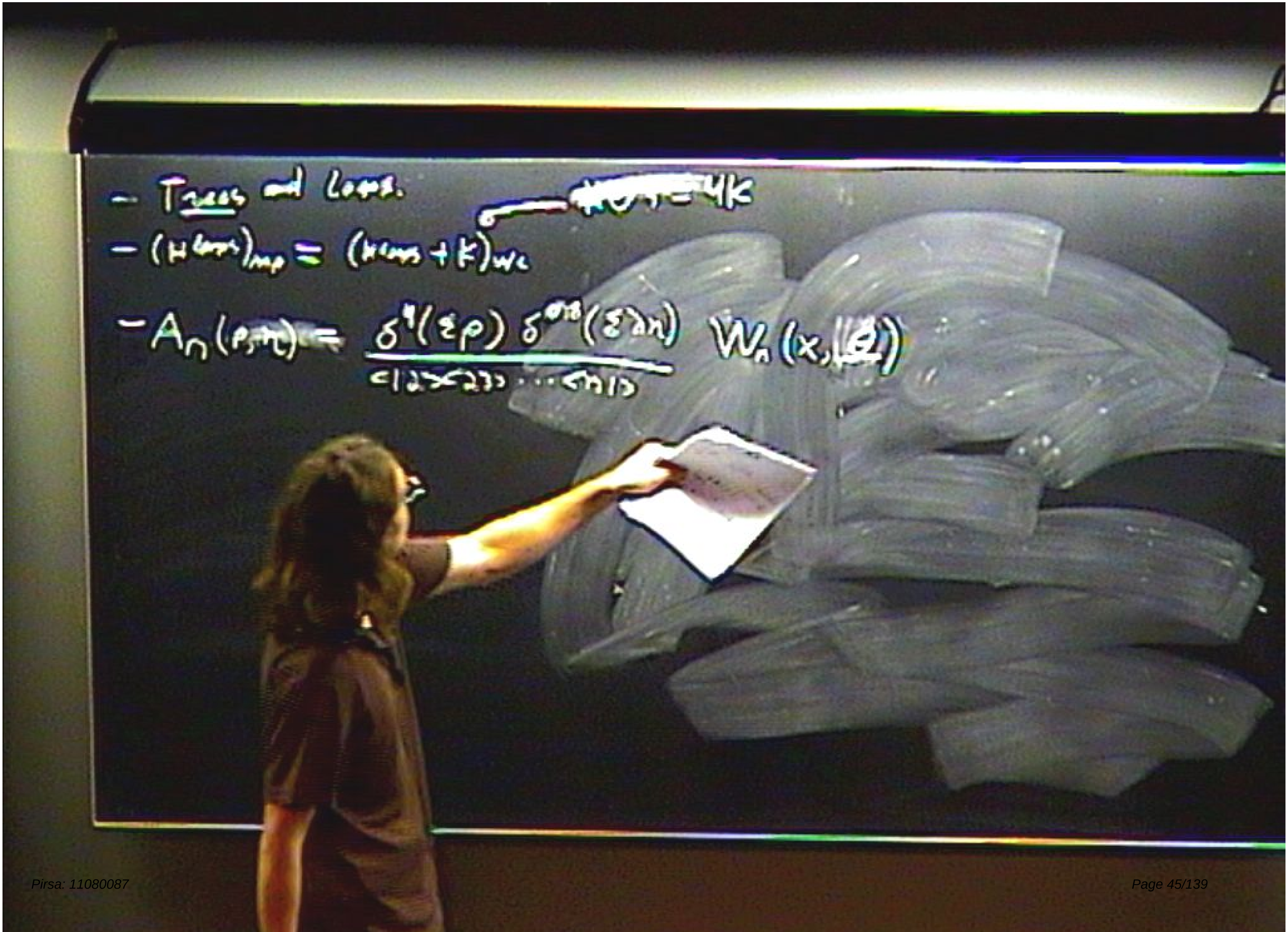

— Trees and Loops.
+ analytic.

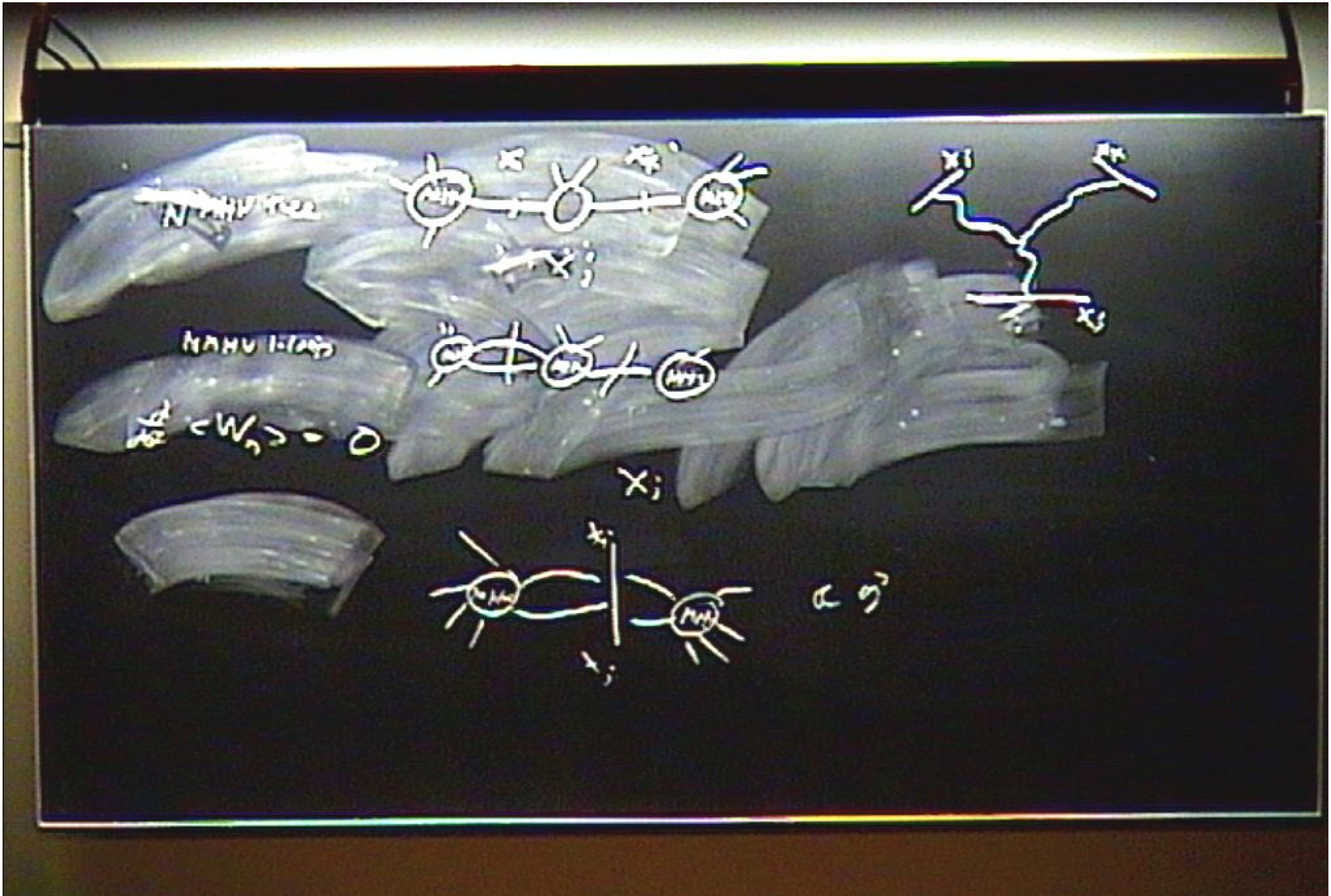



Loops.

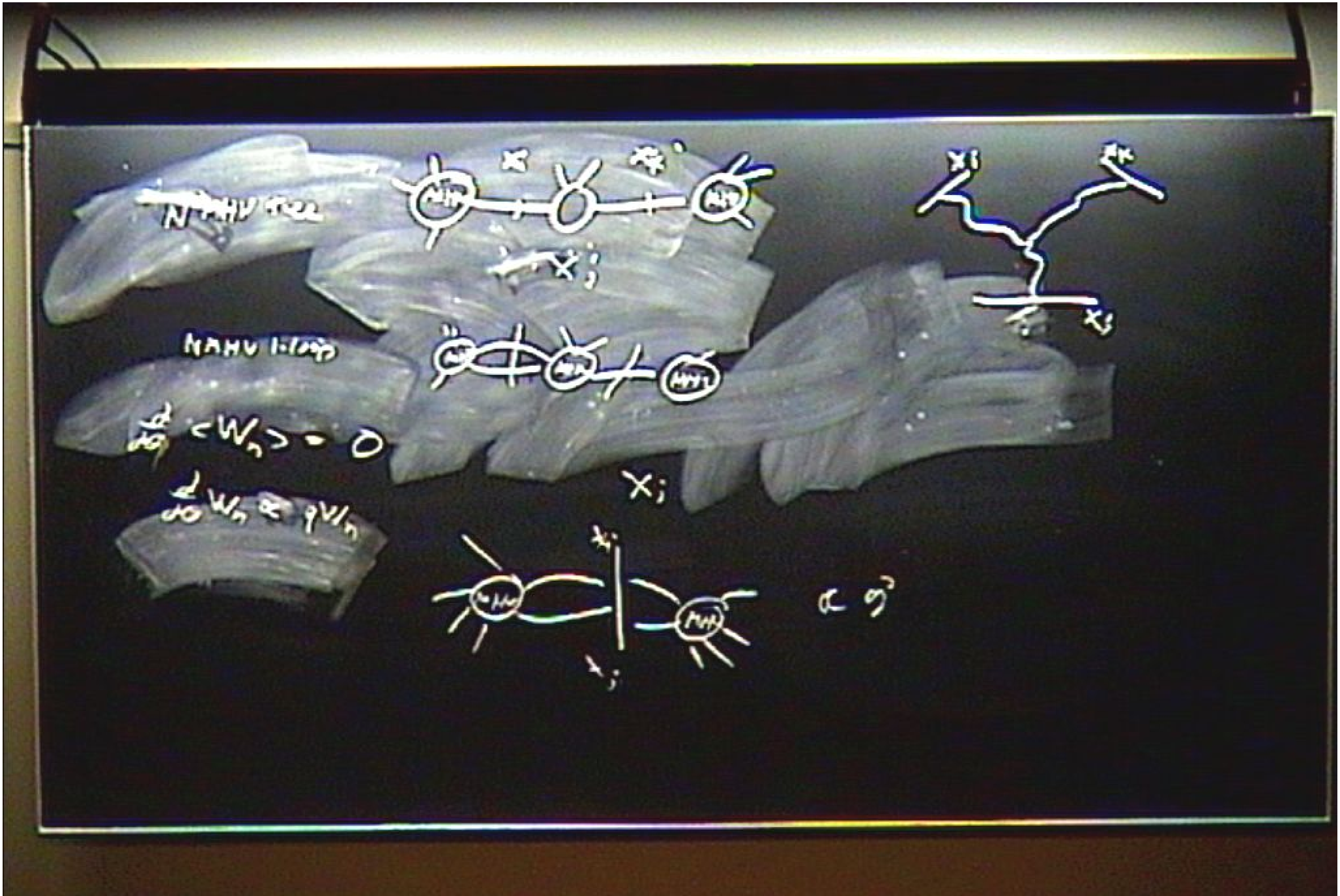

– Trees and loops.
– A

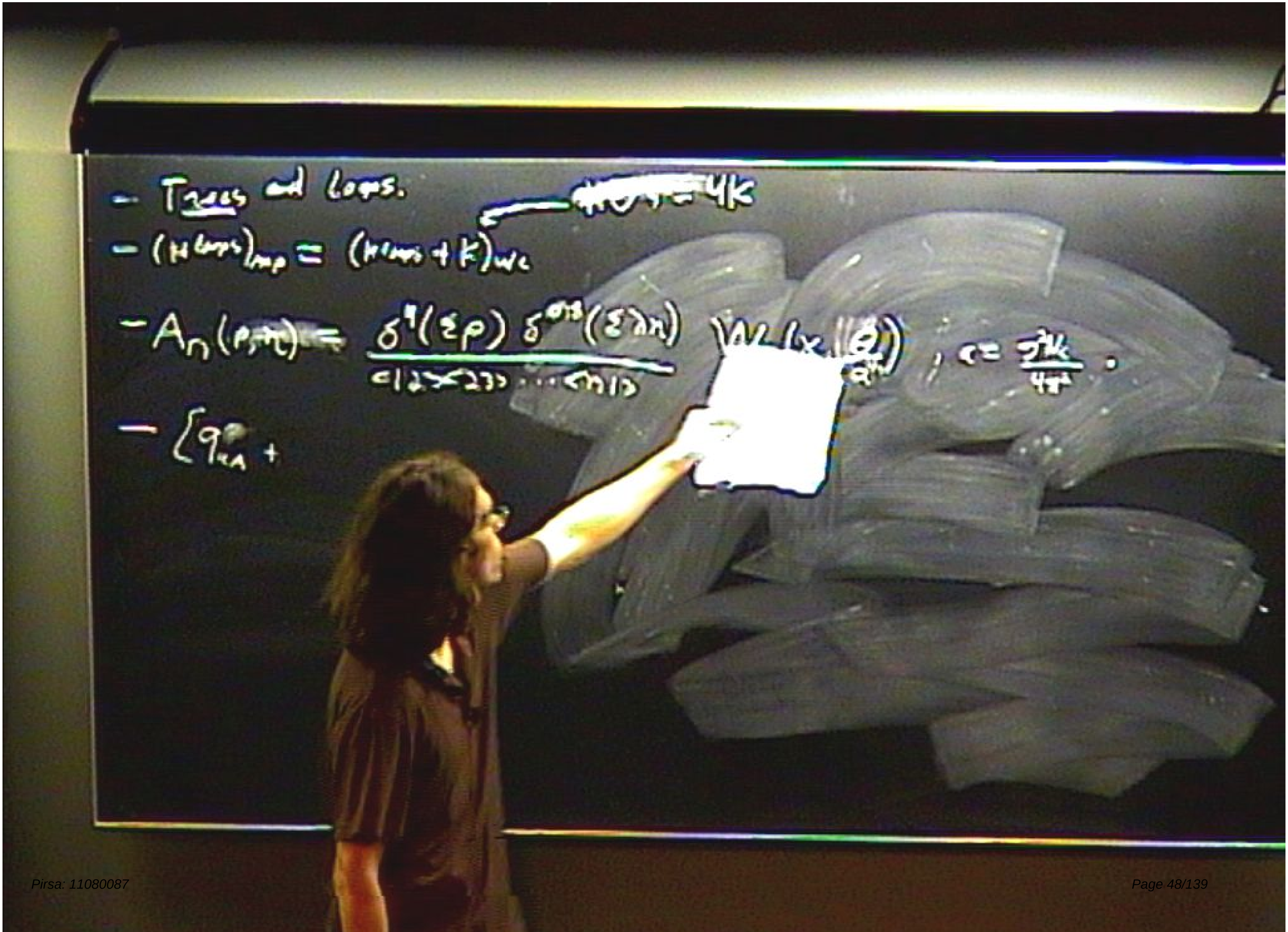
- Trees and loops.
- $(n \text{ loops})_{MP} = (n \text{ loops} + K)_{WC}$ $\quad = 4K$
- $A_n(p, \eta) = \underline{\delta^4(\Sigma p)\, \delta^{0|8}(\Sigma \lambda \eta)}$

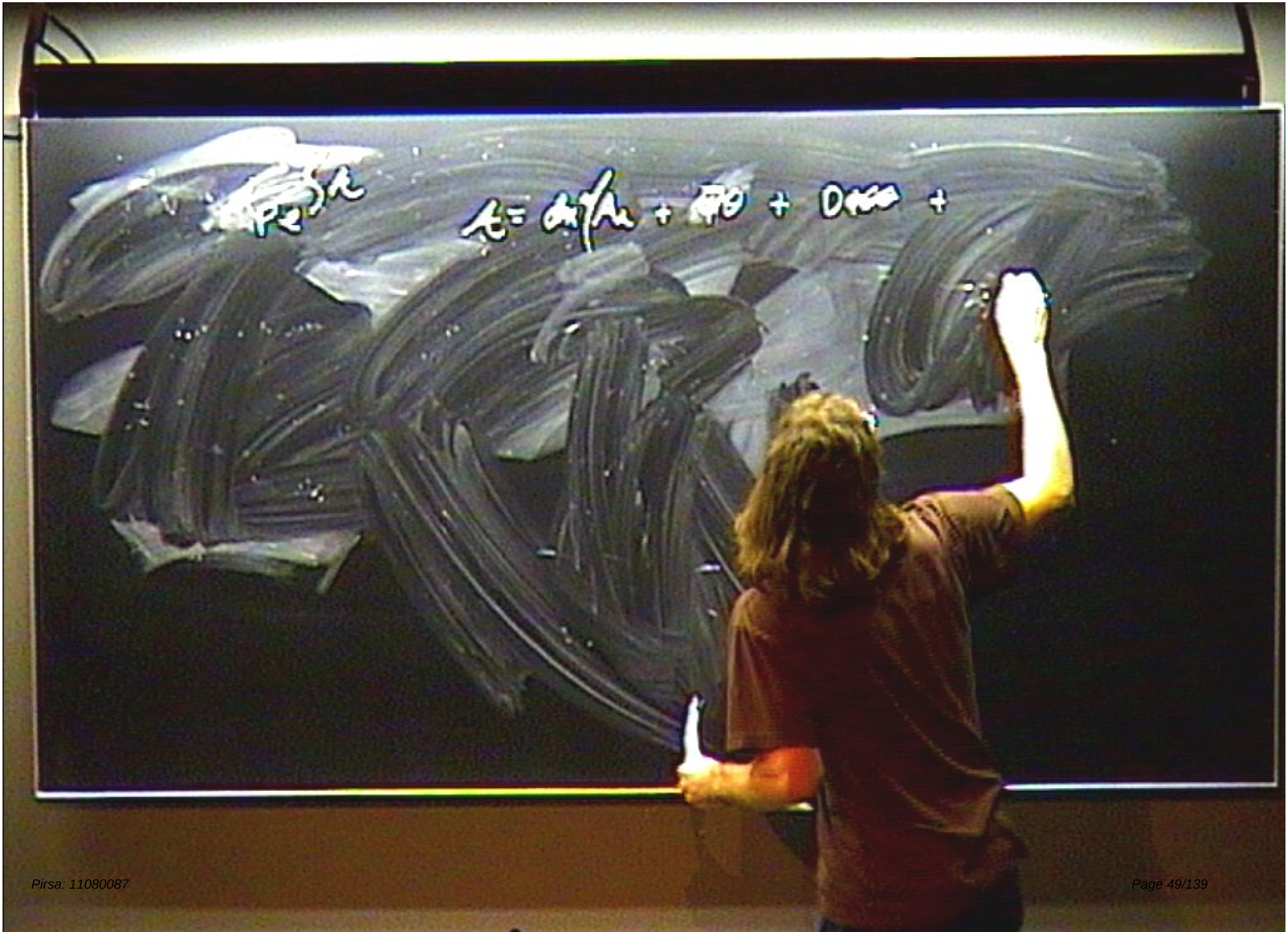

- Trees and Loops.
- (N^k loops)_mp = (N loops + k)_wc
- A_n(p,η) = δ^4(Σp) δ^0|8(Σλη) / ⟨12⟩⟨23⟩···⟨n1⟩ W_n(x,θ)

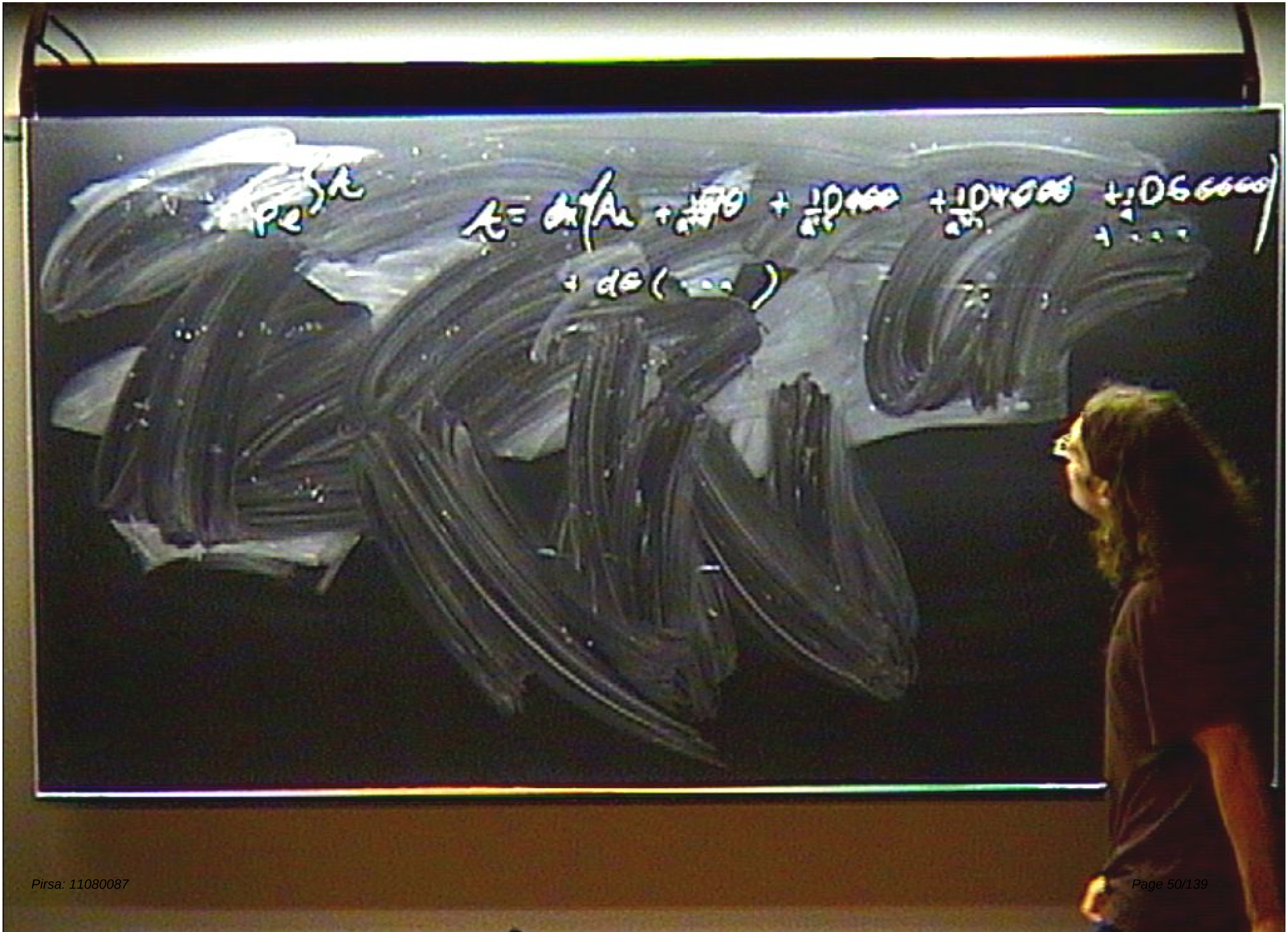


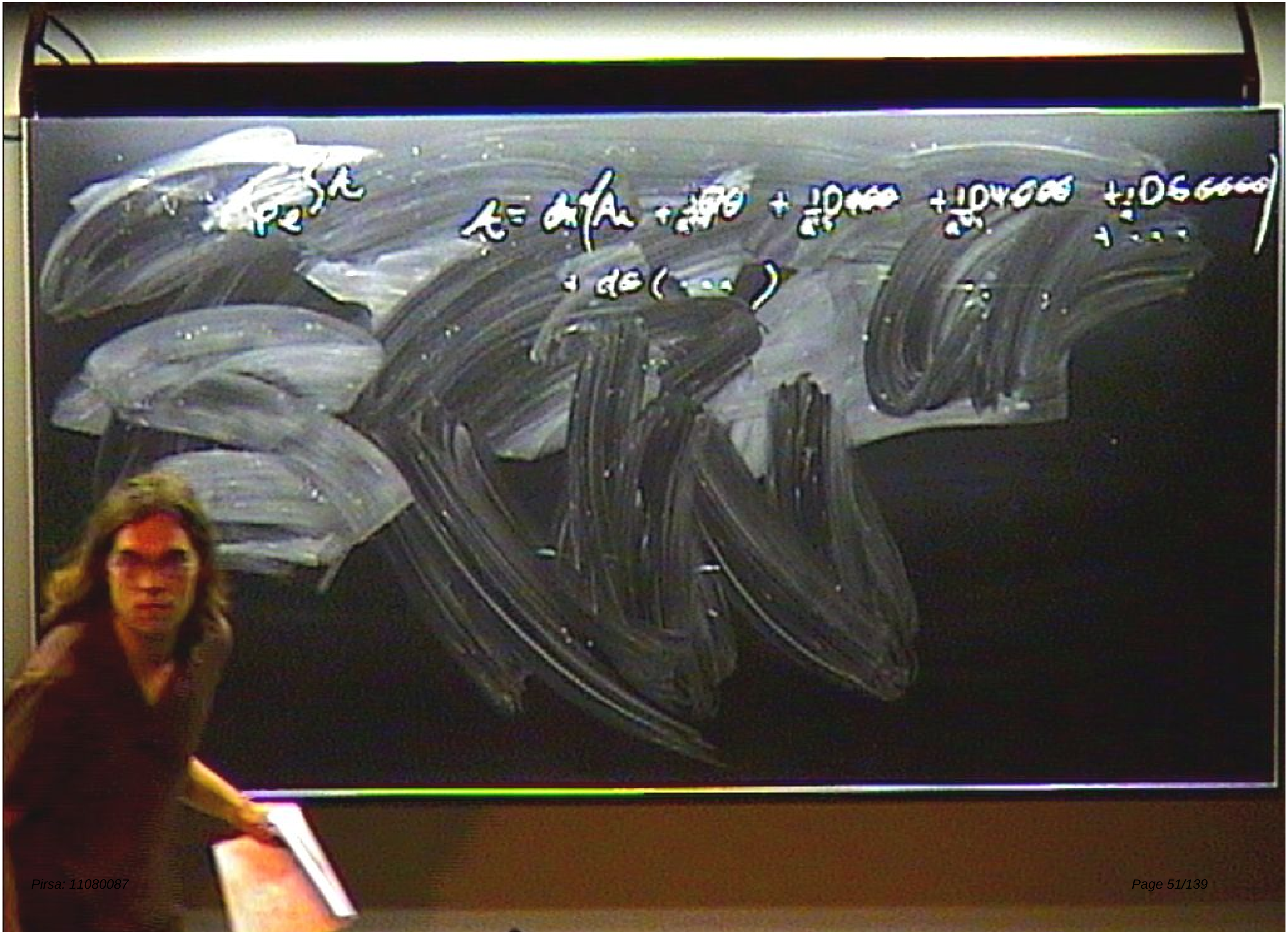
<W_n> = 0
x_j


NMHV tree
NMHV 1-loop
$\frac{d}{dg}\langle W_n\rangle = 0$
$\frac{d}{dg} W_n \propto g V_n$
x_i
x_j
$\propto g^3$


– Trees and loops.
4K
– (n loops)mp = (n loops + K)WL
– An
– {

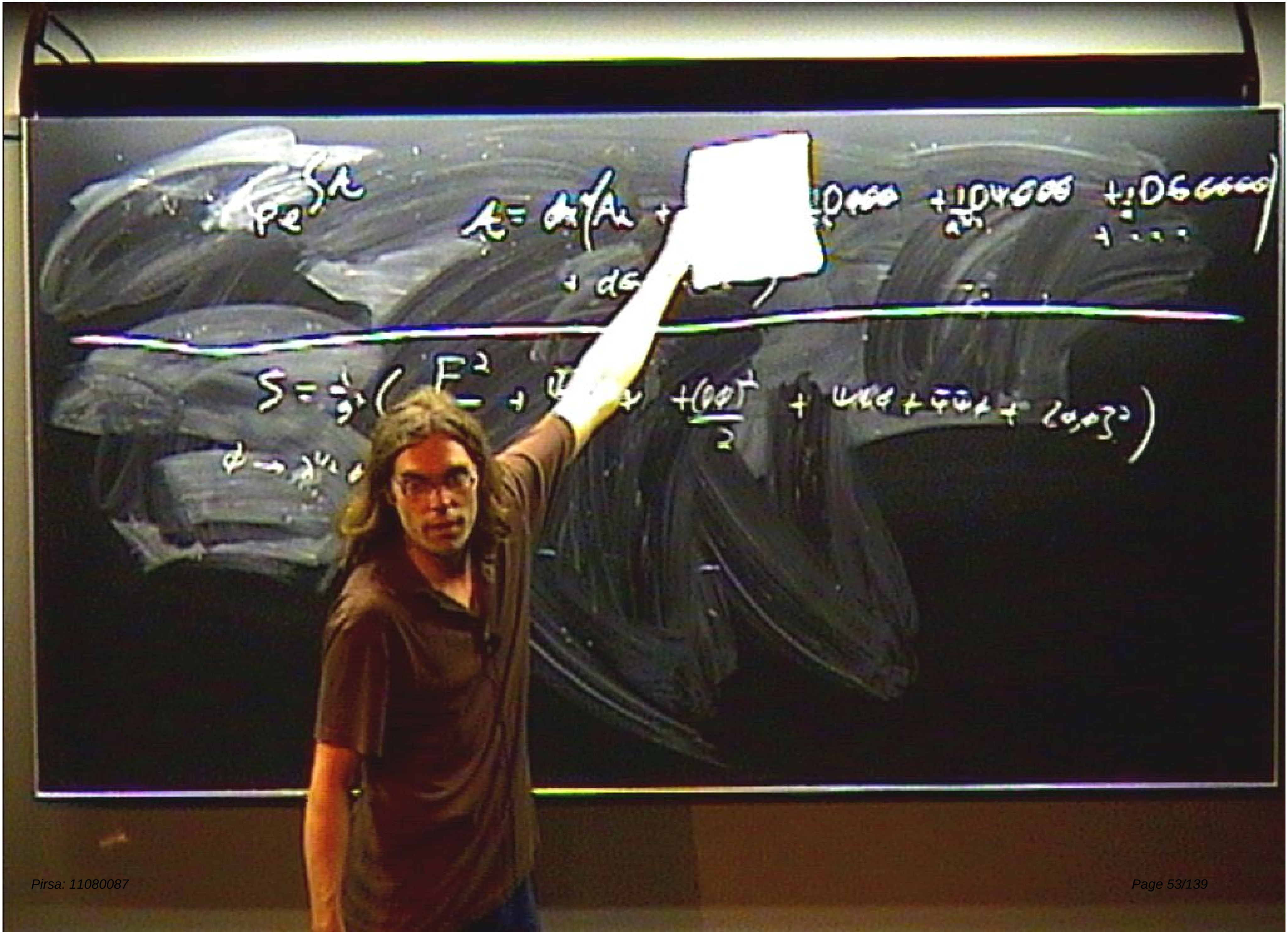


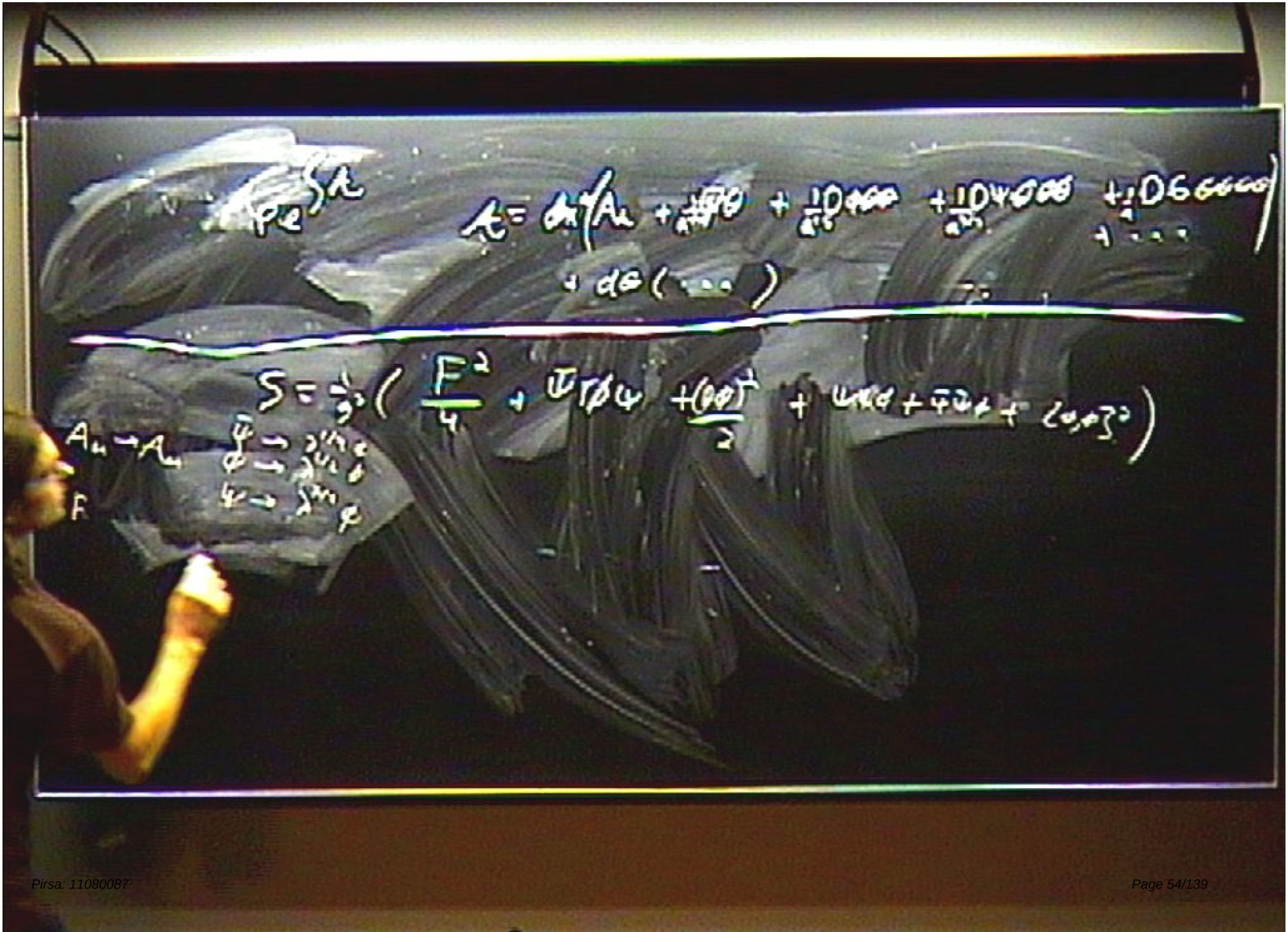


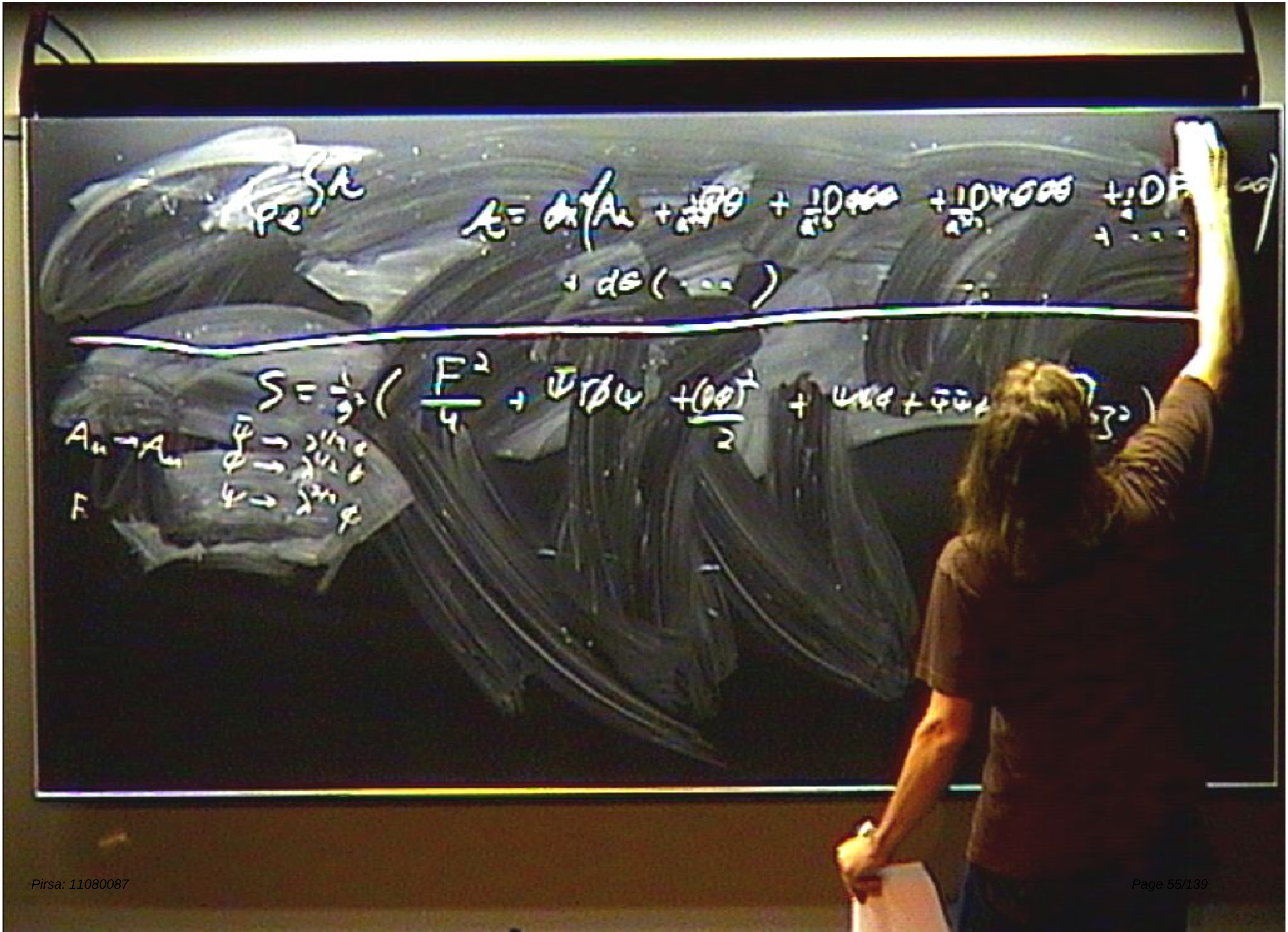


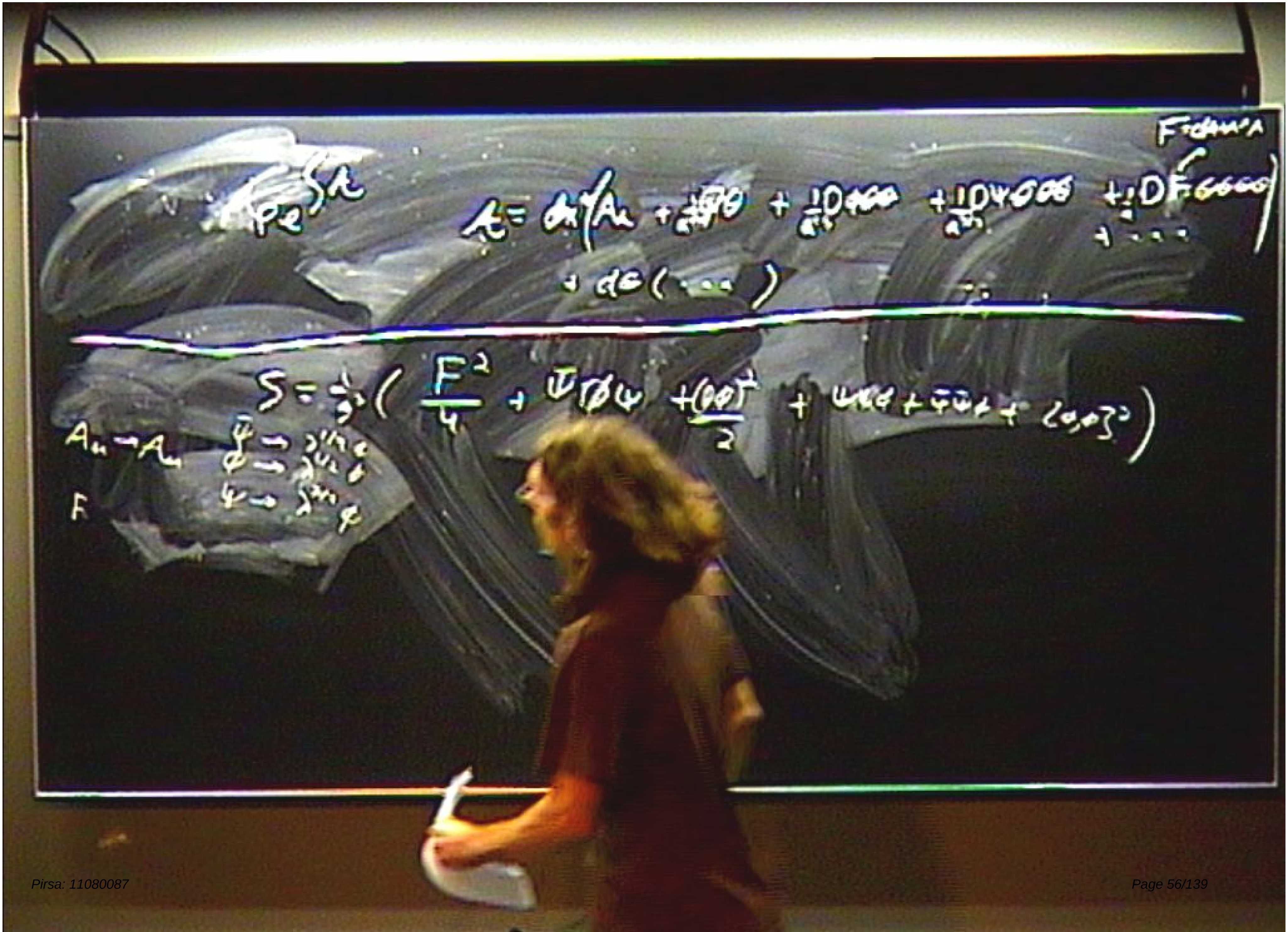
$$\mathcal{L} = \sigma^\mu A_\mu + \tfrac{1}{2!}\sigma\sigma + \tfrac{1}{3!}D\sigma\sigma\sigma + \tfrac{1}{4!}D\psi\sigma\sigma\sigma + \tfrac{1}{6!}D6\sigma\sigma\sigma\sigma + \cdots + d\sigma(\cdots)$$

$$S = \frac{1}{g^2}\left(\frac{F^2}{4} + \bar\psi \not{D} \psi + \frac{(D\phi)^2}{2} + \psi\psi\phi + \bar\psi\bar\psi\phi + [\phi,\phi]^2\right)$$









$$A = \int d^4x \left(A_\mu + \tfrac{1}{\Lambda}\bar{\psi}\psi\phi + \tfrac{1}{\Lambda^2} D\phi\phi\phi + \tfrac{1}{\Lambda^3} D\psi\phi\phi\phi + \tfrac{1}{\Lambda} D F\phi\phi\phi\phi + \cdots \right)$$

$$+ \, d\theta \, (\cdots)$$

$$S = \frac{1}{g^2} \left(\frac{F^2}{4} + \bar{\psi}\Gamma \not{D} \psi + \frac{(D\phi)^2}{2} + \psi\psi\phi + \bar{\psi}\bar{\psi}\phi + [\phi,\phi]^2 \right)$$

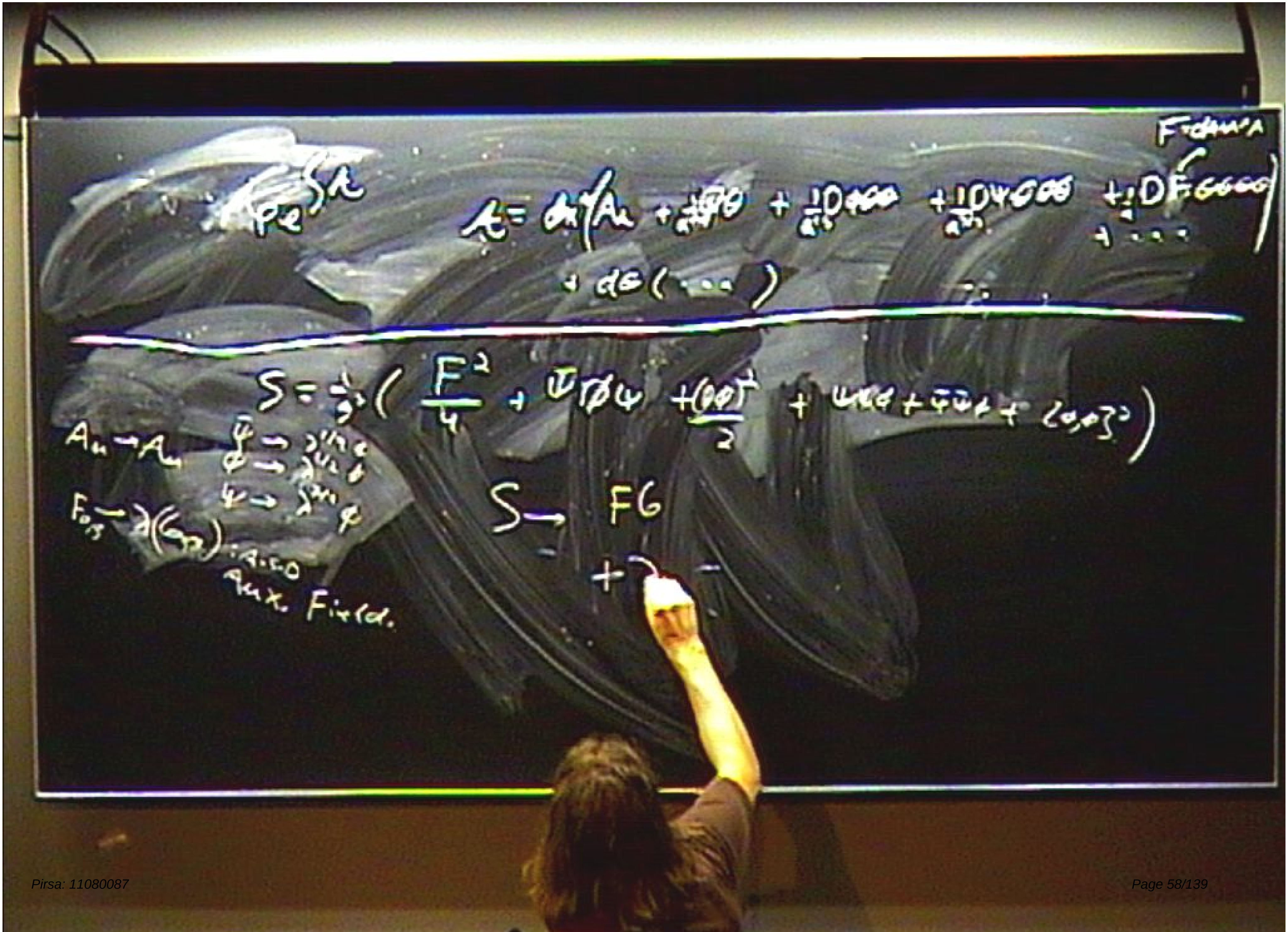
$A_\mu \to A_\mu$ $\quad \bar{\psi} \to \lambda^{1/2} \bar{\psi}$

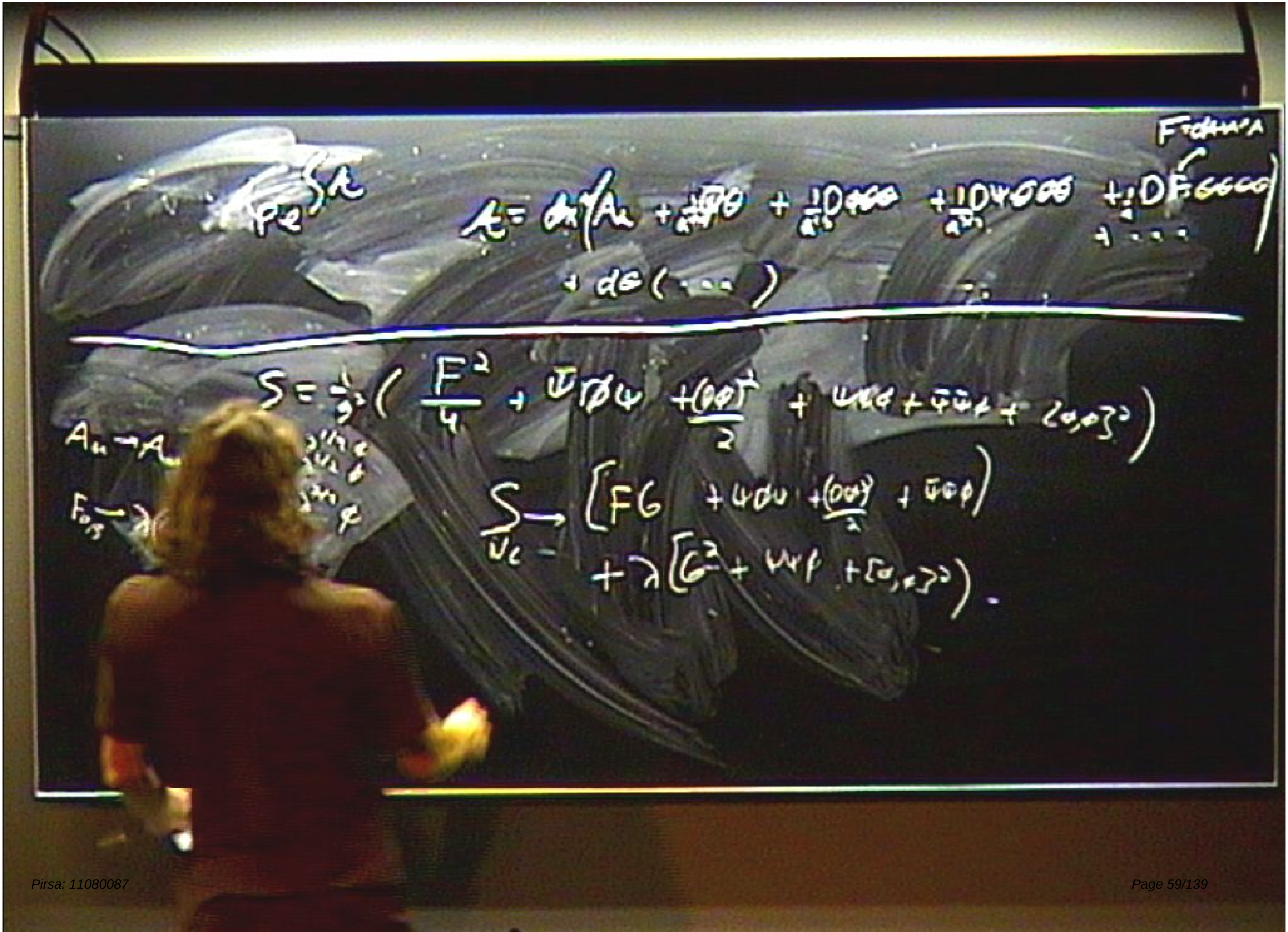
$\phi \to \lambda^{1/2} \phi$

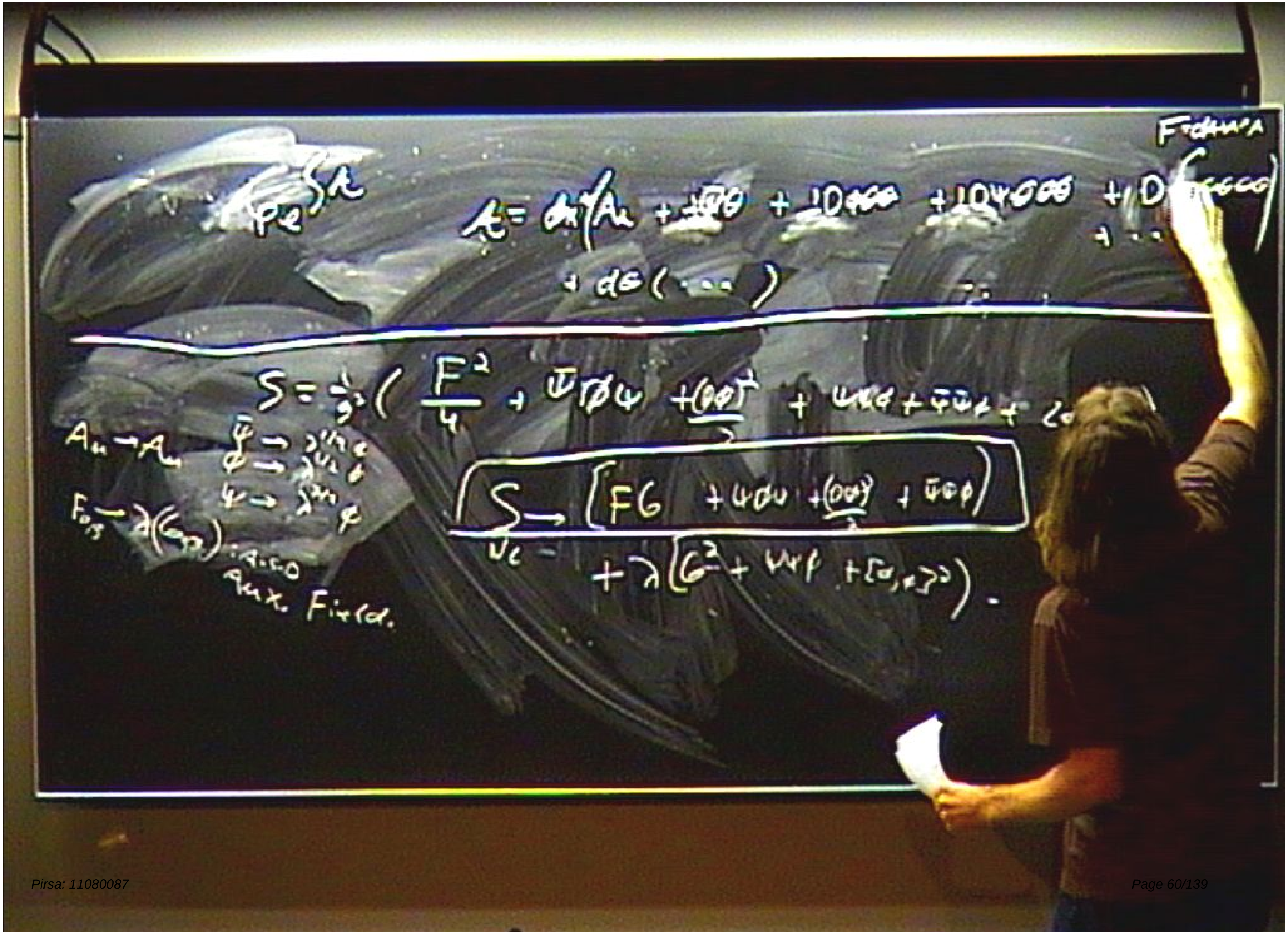
$\psi \to \lambda^{3/2} \psi$

$F_{\alpha\beta} \to \lambda(\cdots)$: a.c.o

quant. Fixed.


S → FG
aux. Field.




+ dG(...)
S = 1/g² (F²/4
Aμ → Aμ
Fixed.

Fierma

$\sim \phi_e^{\,S_E}$

$\mathcal{L} = d\,\mathrm{tr}\Big(A_\mu + \bar\psi\theta + D\psi\ldots + D\psi\ldots + D\ldots$
$+ \ldots\Big)$
$+ d\theta(\ldots)$

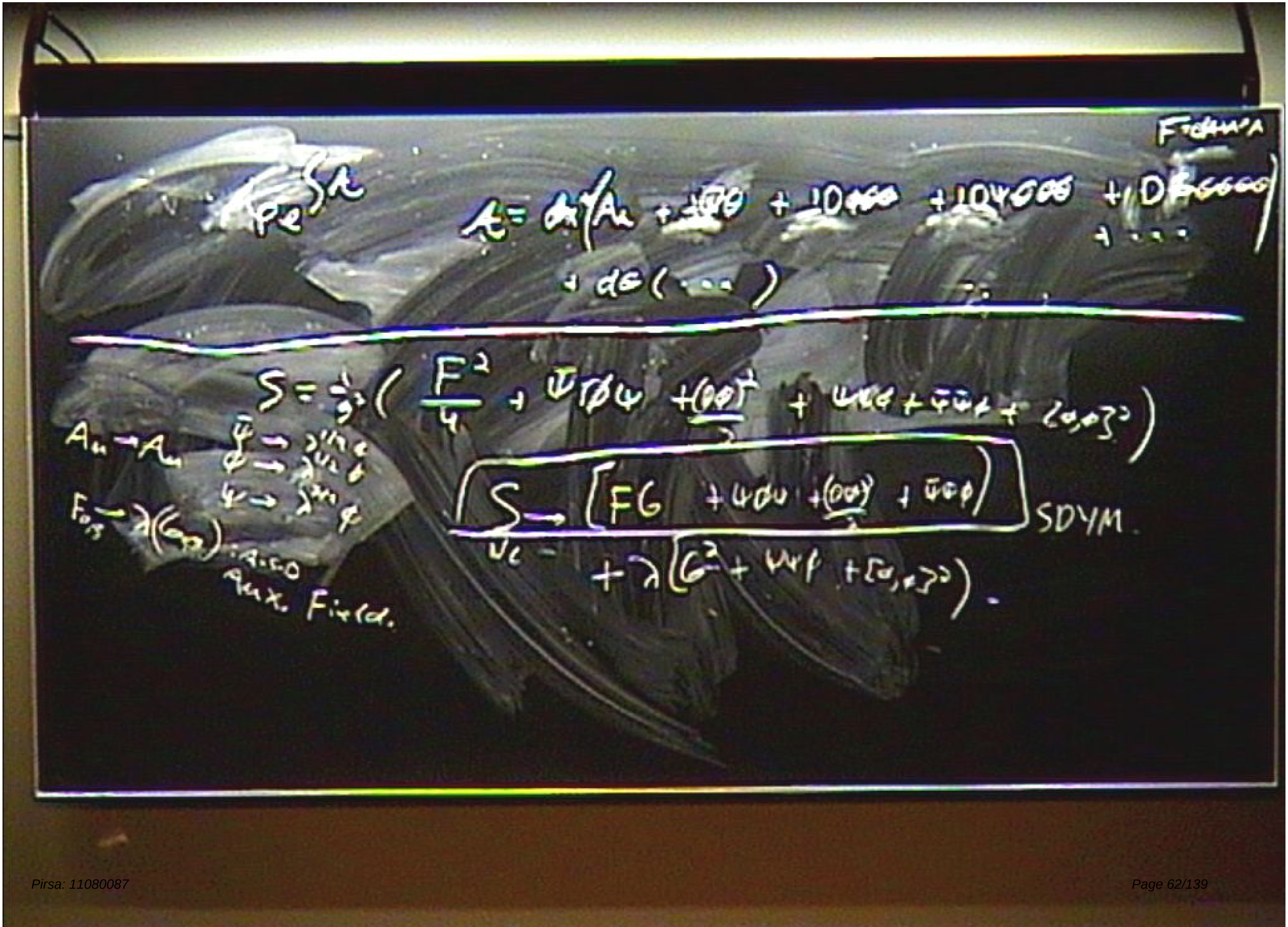
$$S = \frac{1}{g^2}\left(\frac{F^2}{4} + \bar\psi D\!\!\!/\,\psi + (D\phi)^2 + \psi\psi\phi + \bar\psi\bar\psi\phi + [\phi,\phi]^2\right)$$

$A_\mu \to A_\mu$ $\quad \bar\psi \to \lambda^{1/2}\bar\psi$ $\quad \phi \to \lambda^{1/2}\phi$ $\quad \psi \to \lambda^{1/2}\psi$

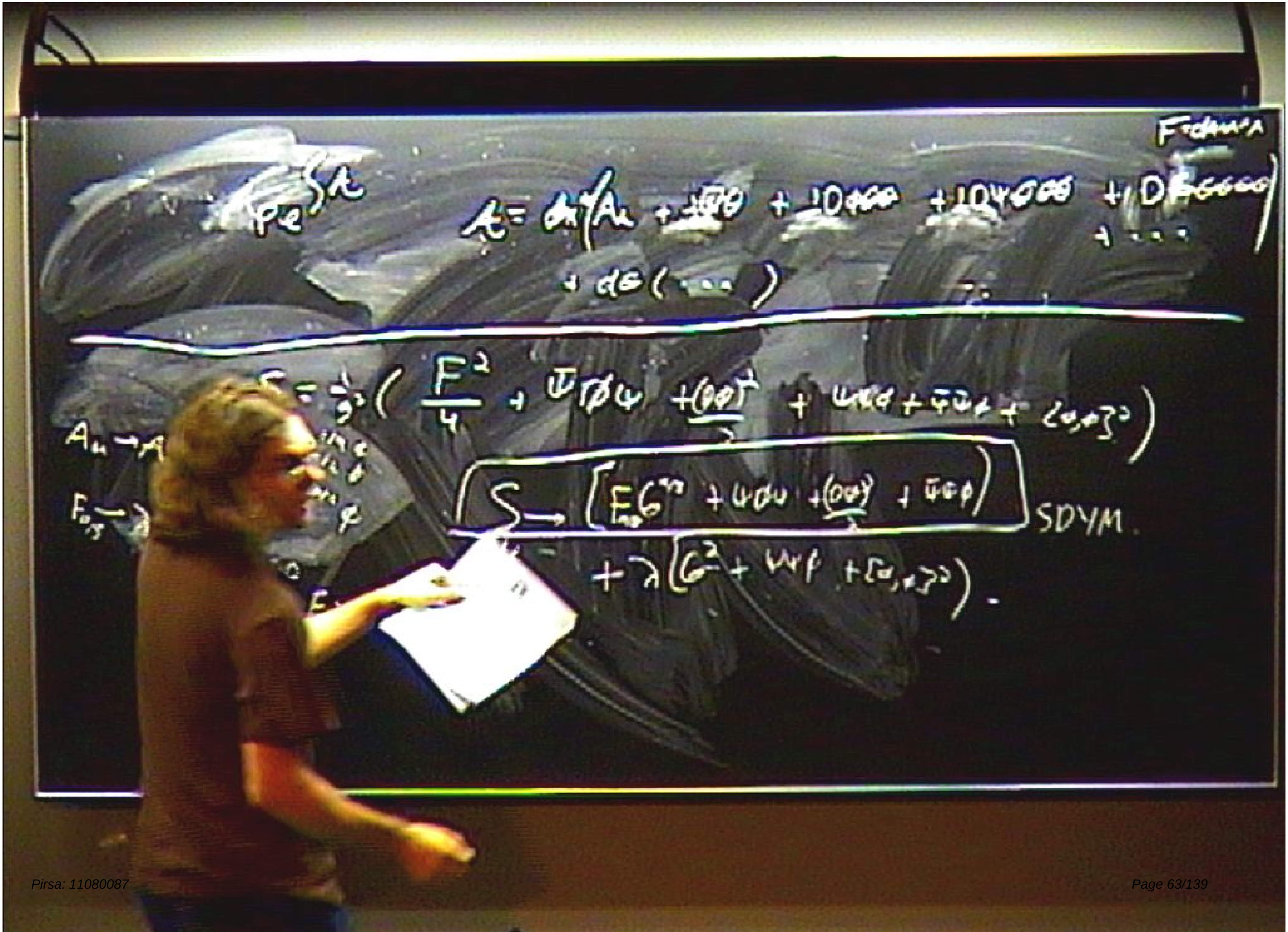
$F_{\mu\nu} \to \lambda(G_{\mu\nu})$: A.S.D.

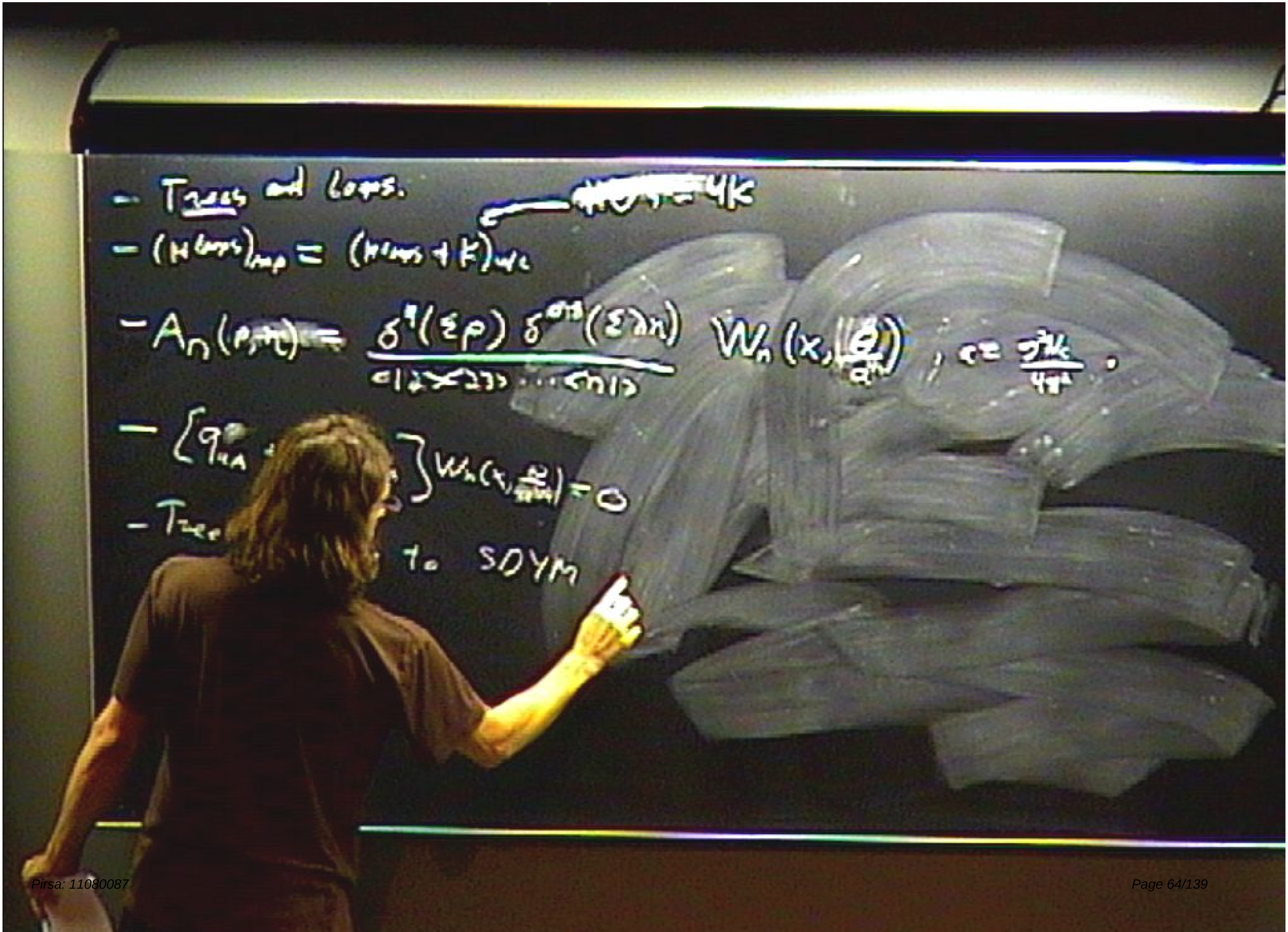
aux. Field.

$$\boxed{S_{UL} \to \int\left(FG + \psi D\psi + (D\phi)^2 + \bar\psi\psi\phi\right)}$$
$$+ \lambda\left(G^2 + \psi\psi\phi + [\phi,\phi]^2\right).$$

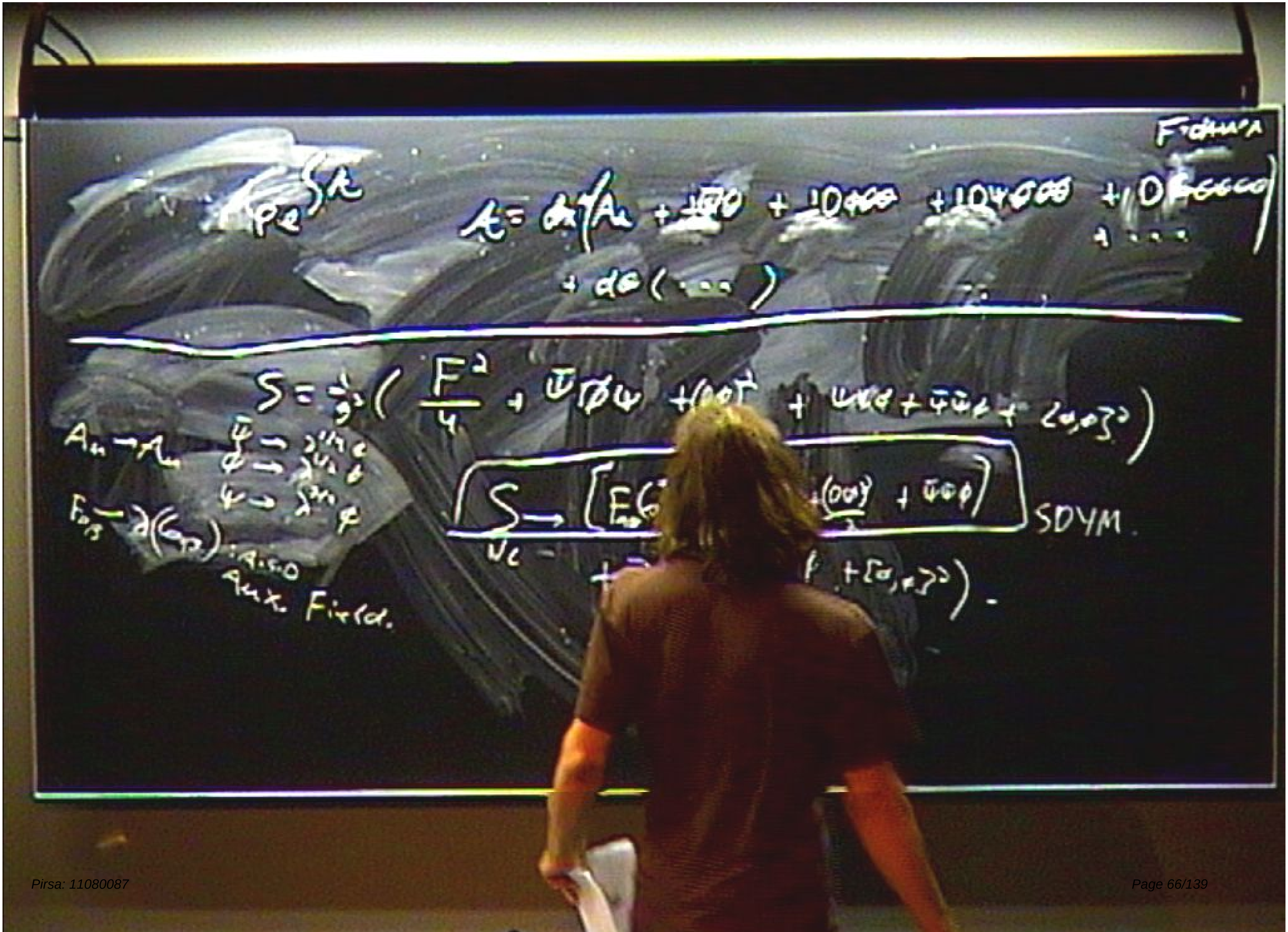


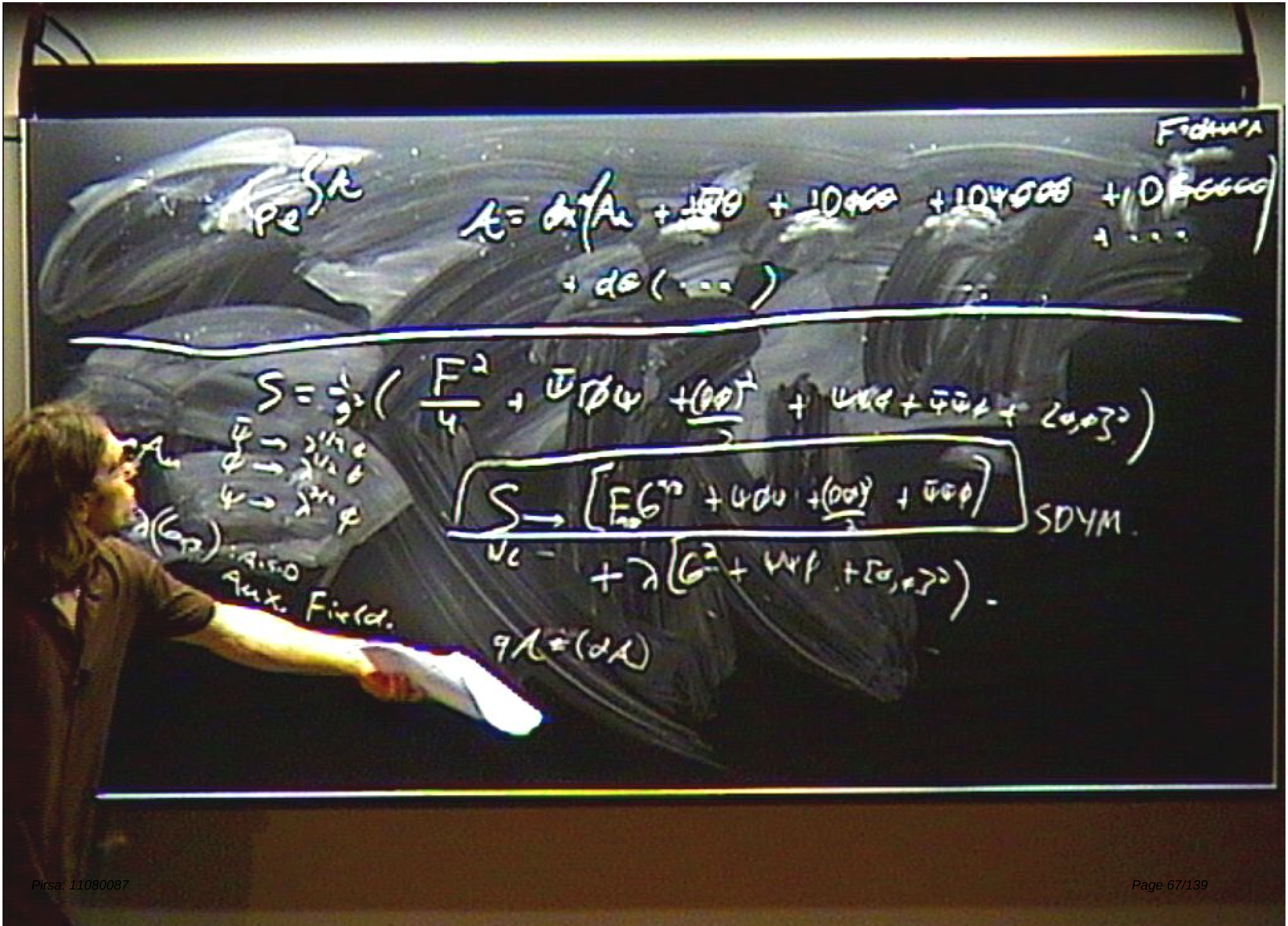
+ dG(...)
S = 1/g² (F²/4
SDYM
A_μ → A_μ
Aux. Field.

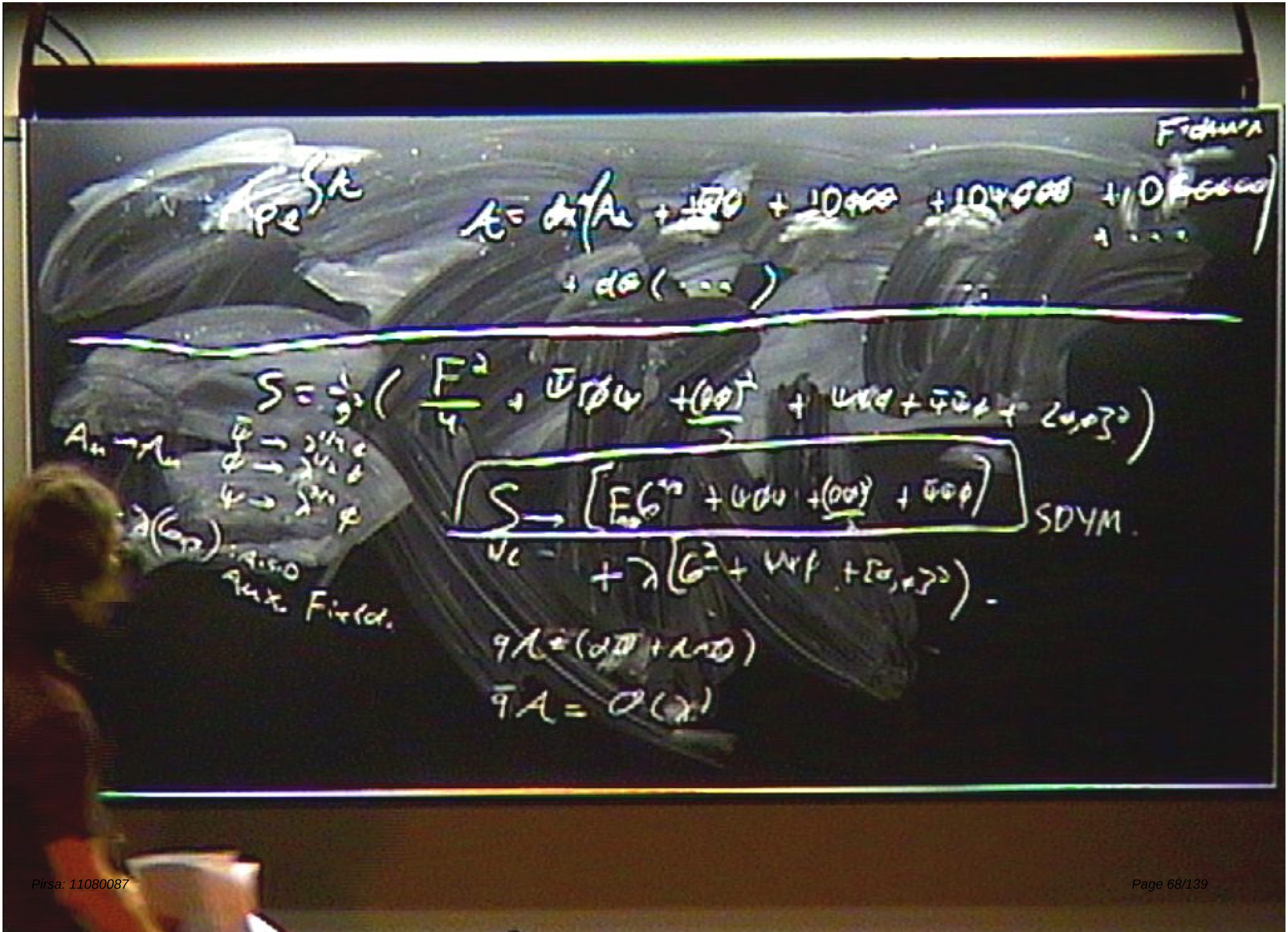

SDYM

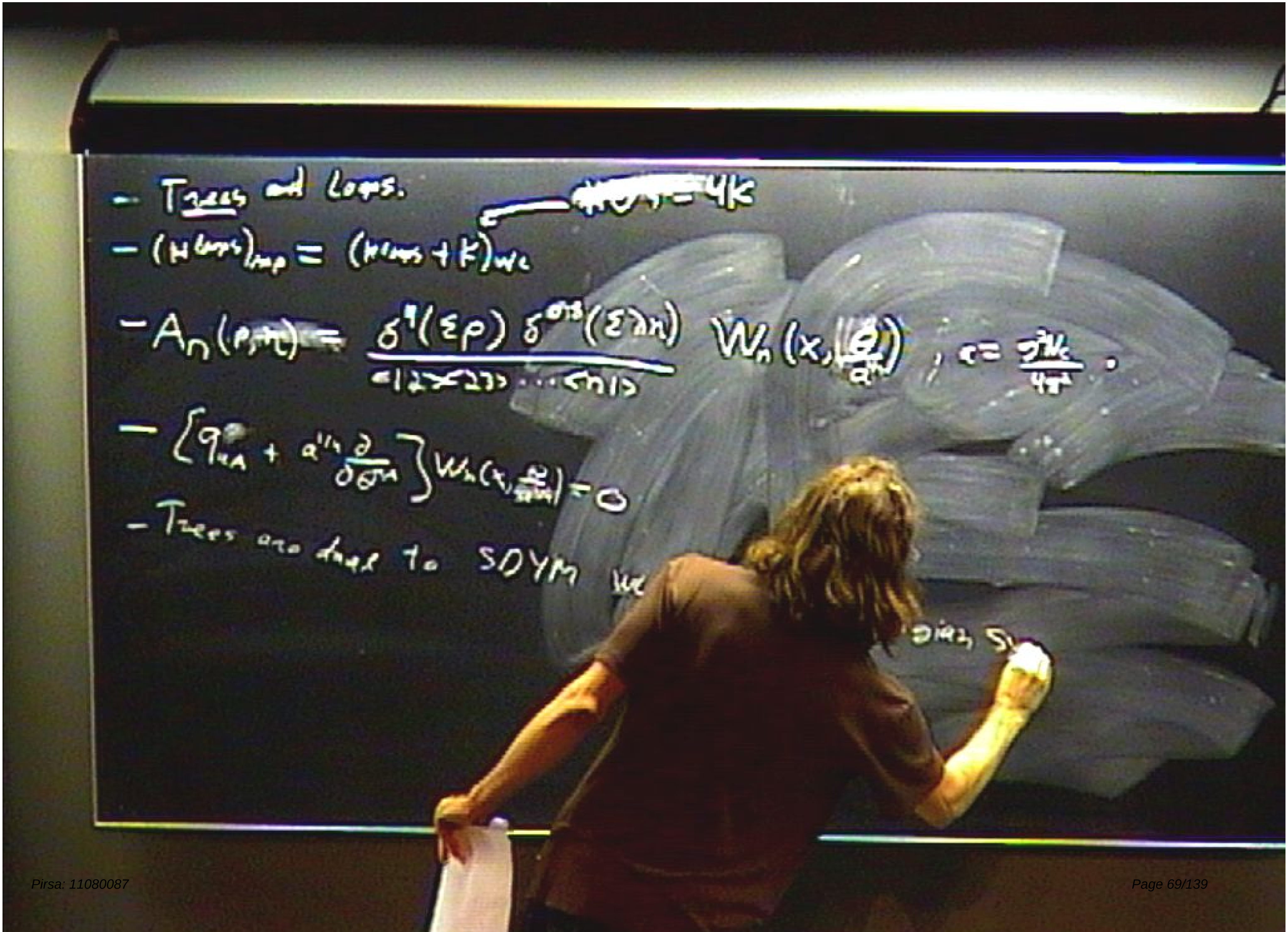

- Trees and loops.
- Trees
to SDYM

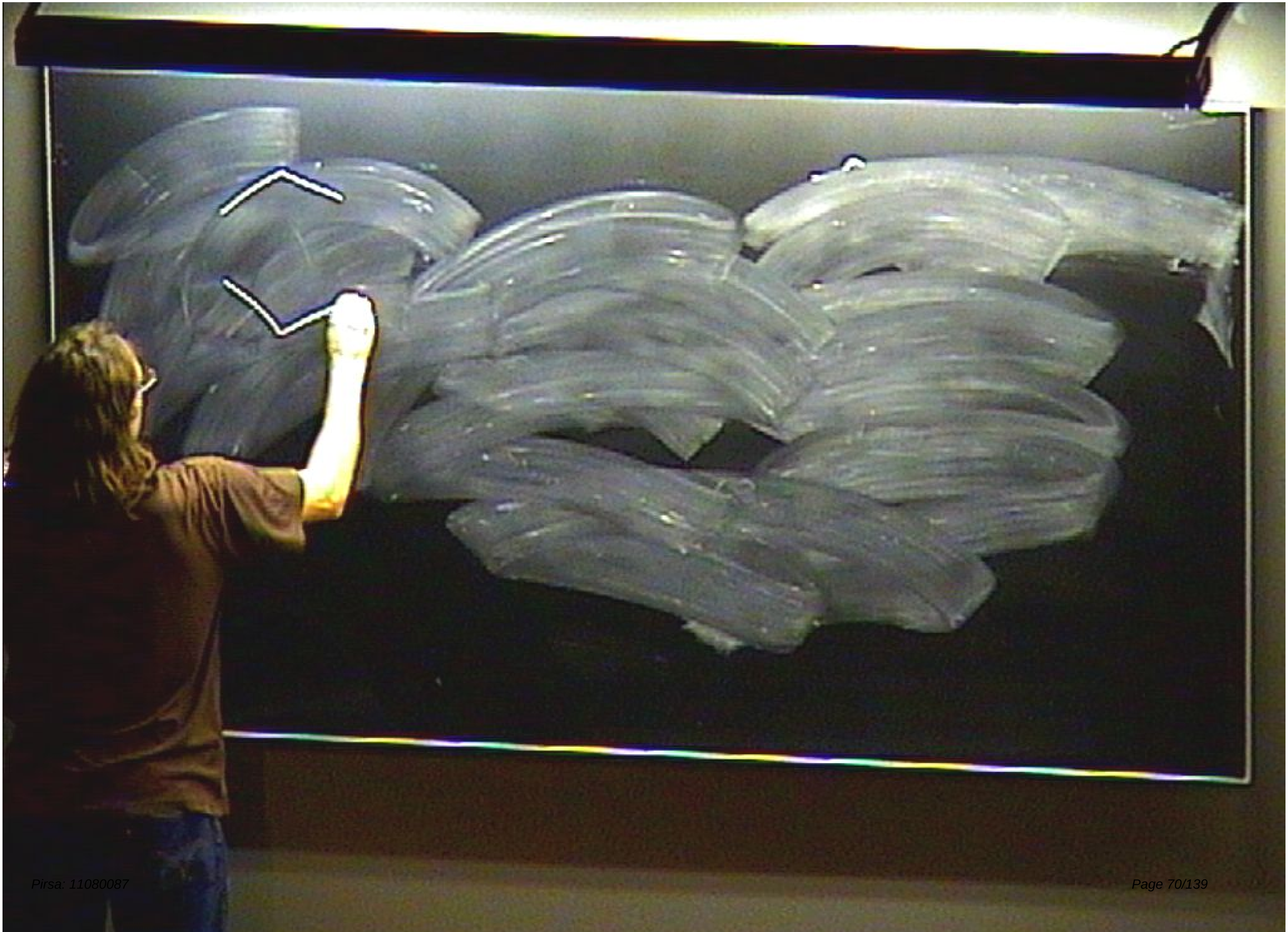
- Trees and Loops.
- $(H\ \text{loops})_{\text{amp}} = (H\ \text{loops} + K)_{WL}$ $= 4K$
- $A_n(p,\eta) = \dfrac{\delta^4(\Sigma p)\,\delta^{0|8}(\Sigma \lambda\eta)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}\, W_n(x, \theta)$, $\epsilon = \dfrac{g^2 N}{4\pi^2}$.
- $\left\{ q_{\alpha A} + a^{1/2}\dfrac{\partial}{\partial\theta^{\alpha A}} \right\} W_n(x,\theta) = 0$
- Trees are dual to SDYM WL

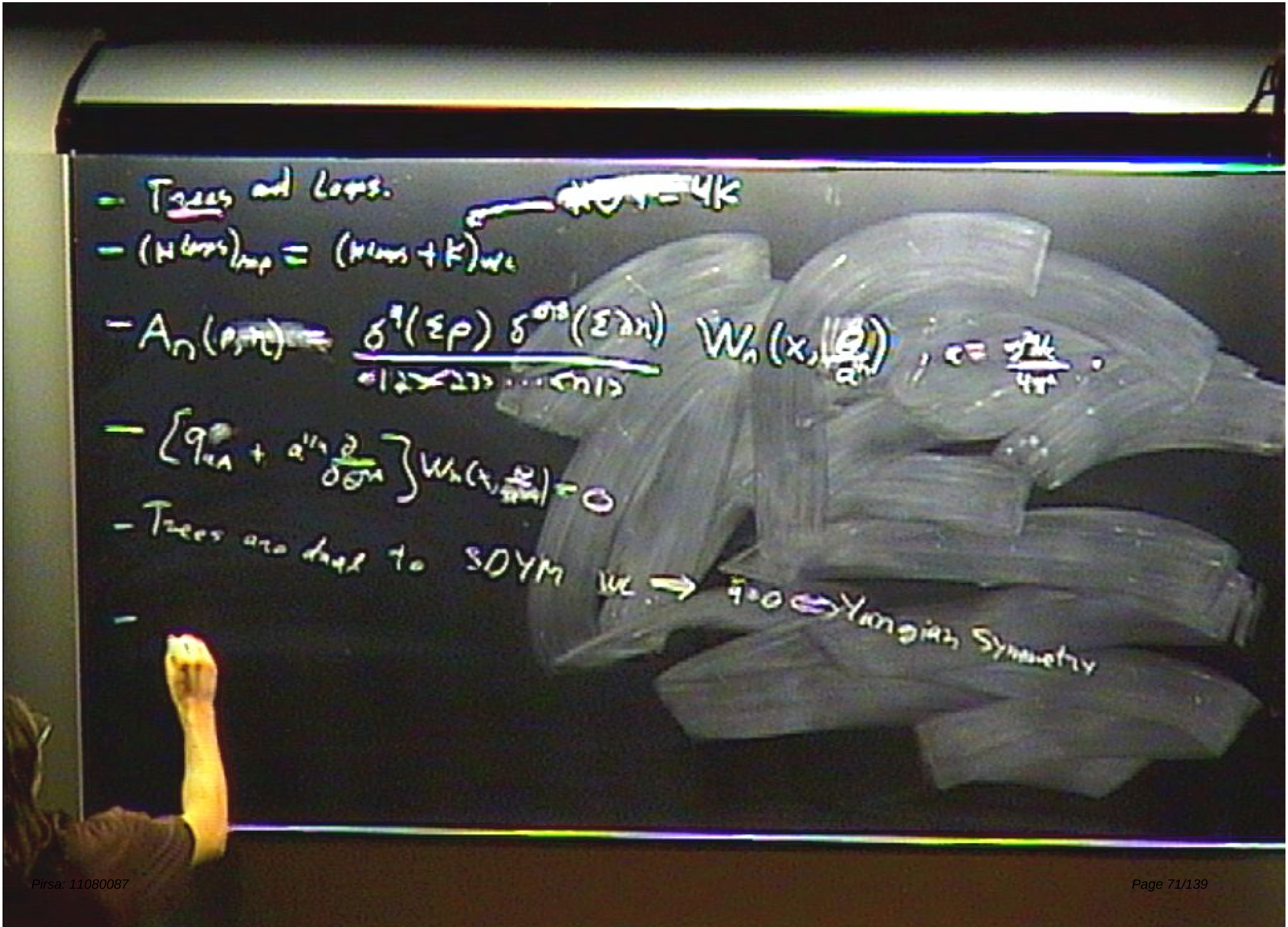

SDYM
Aux. Field.


SDYM
aux. Field.


SDYM


- Trees and Loops.
= 4K
- (N loops)MP = (N loops + K)WL
- An(p,η) = δ4(Σp) δ0|8(Σλη) / <12><23>···<n1> Wn(x, ∂/∂η) , ε = g2Nc/4π2 .
- {qαA + ε1/2 ∂/∂θA} Wn(x, ∂/∂η) = 0
- Trees are dual to SDYM




- Trees and Loops.
- (N loops)amp = (N loops + K)WL
- An = δ4(Σp) δ0|8(Σλη) / ⟨12⟩⟨23⟩ ... ⟨n1⟩ Wn(x, θ)
- Trees are dual to SDYM WL ⇒ q=0 ⇔ Yangian Symmetry

– Trees and Loops.

– $(L\text{ loops})_{MP} = (L\text{ loops} + K)_{WL}$ $= 4K$

– $A_n(p,\eta) = \dfrac{\delta^4(\Sigma p)\,\delta^{0|8}(\Sigma \lambda\eta)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}\, W_n\!\left(x, \frac{\partial}{\partial\eta}\right)$, $c = \frac{g^2 N}{4\pi^2}$.

– $\left\{ q_{\alpha A} + a^{1/2}\dfrac{\partial}{\partial\theta^A} \right\} W_n\!\left(x, \frac{\partial}{\partial\eta}\right) = 0$

– Trees are dual to SDYM WL $\Rightarrow$ $\bar{q} = 0 \Leftrightarrow$ Yangian Symmetry

– $(x_i - x_{i+1})^2 = 0$
$(x_i - x_{i+1})(\theta_i - \theta_{i+1}) = 0$

Fermion

e^{S_R}

$A = dx(A_\mu + \psi\theta + \psi\theta\theta + \psi\theta\theta\theta + D\theta\theta\theta\theta) + \ldots$

$+ d\theta(\ldots)$

$S = \frac{1}{g^2}\left(\frac{F^2}{4} + \bar{\psi}\not{D}\psi + \frac{(D\phi)^2}{2} + \psi\psi\phi + \bar{\psi}\bar{\psi}\phi + [\phi,\phi]^2\right)$

$A_\mu \to A_\mu$

$\bar{\psi} \to \lambda^{1/2}\bar{\psi}$

$\phi \to \lambda^{1/2}\phi$

$\psi \to \lambda^{3/4}\psi$

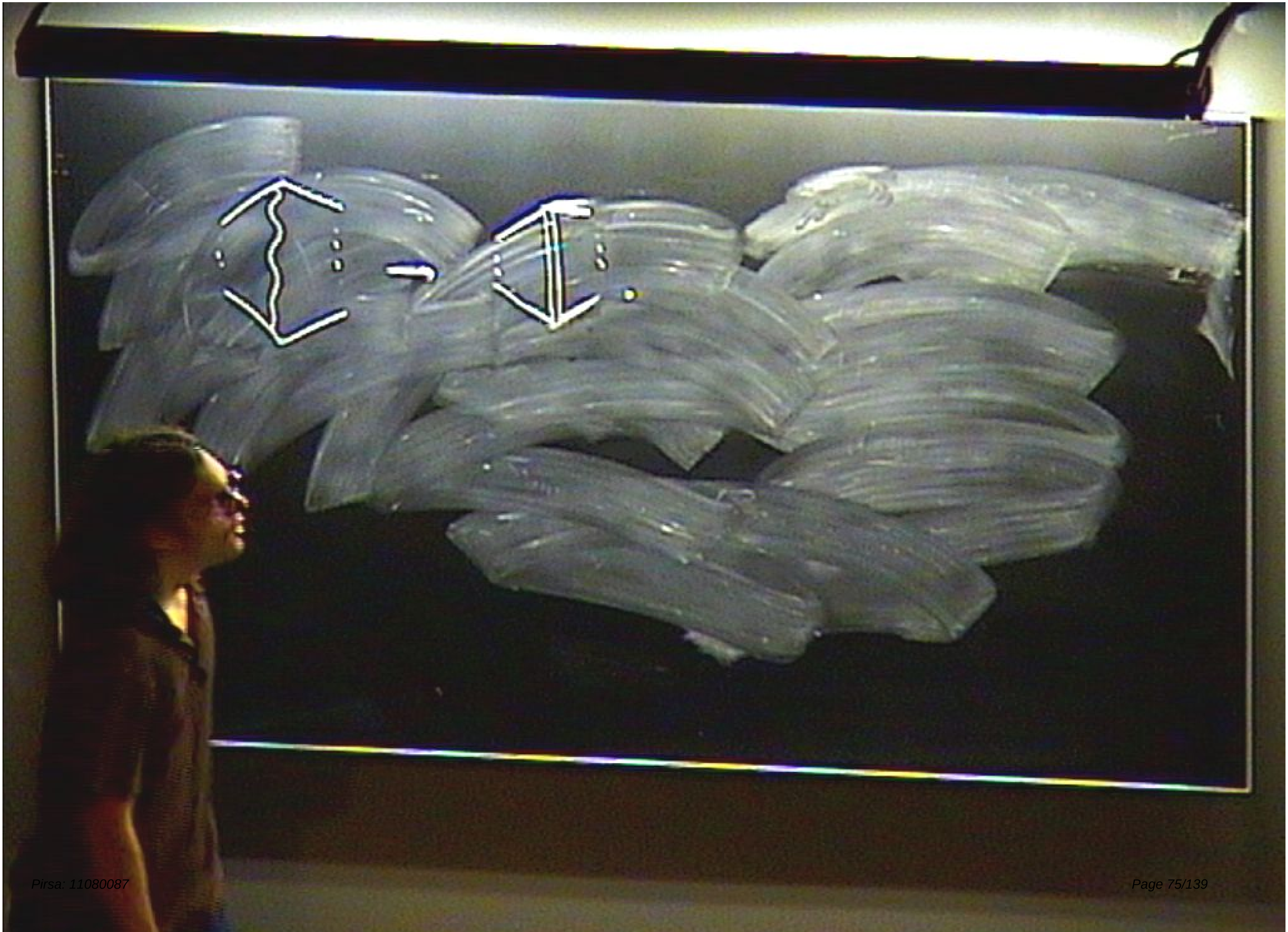
$F_{\alpha\beta} \to \lambda(G_{\alpha\beta})$ Aux. Field.

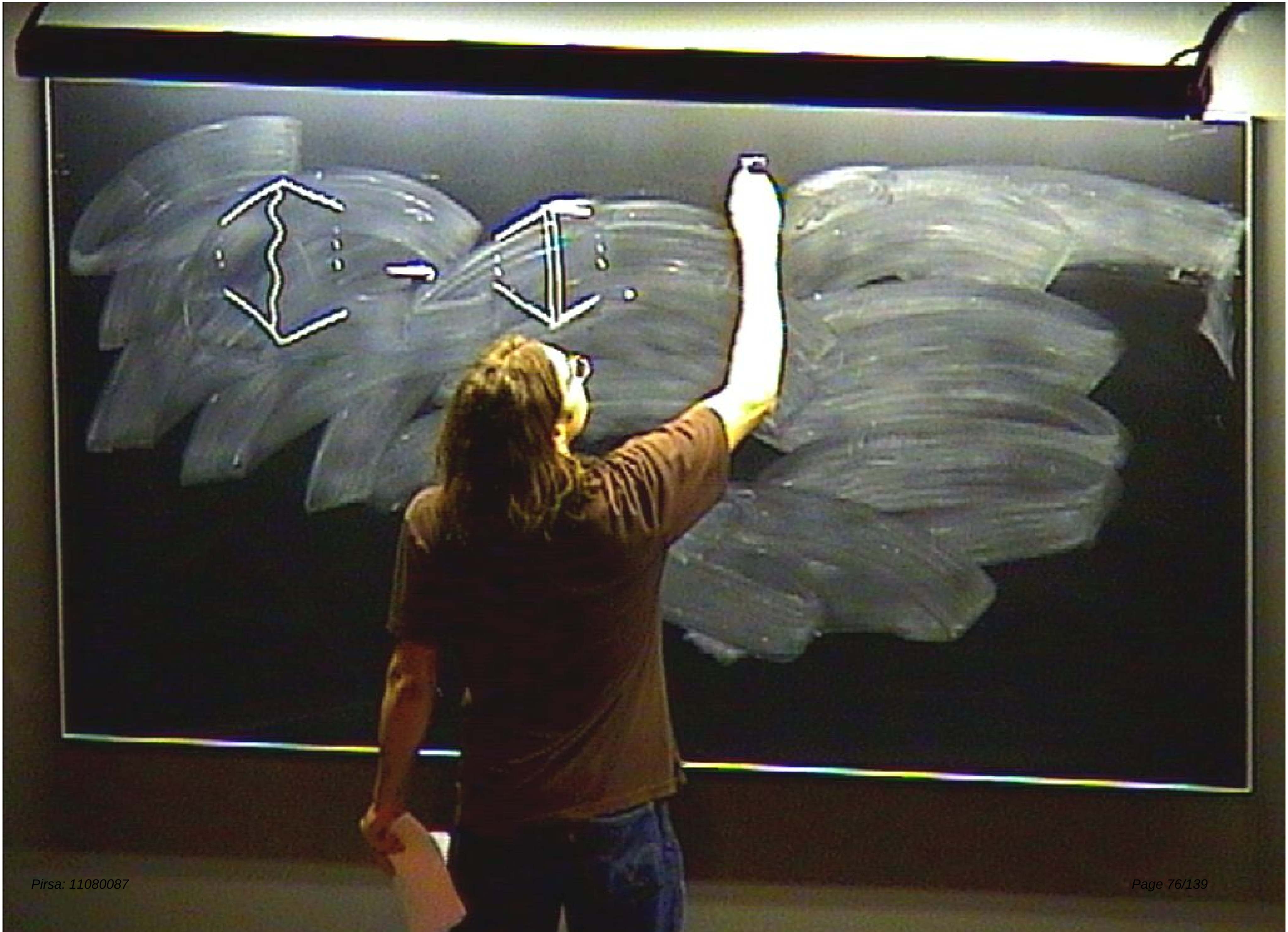
$\boxed{S \to \left[E\,G^{\alpha\beta} + \psi D\psi + \frac{(D\phi)^2}{2} + \bar{\psi}\bar{\psi}\phi\right]}$ SDYM

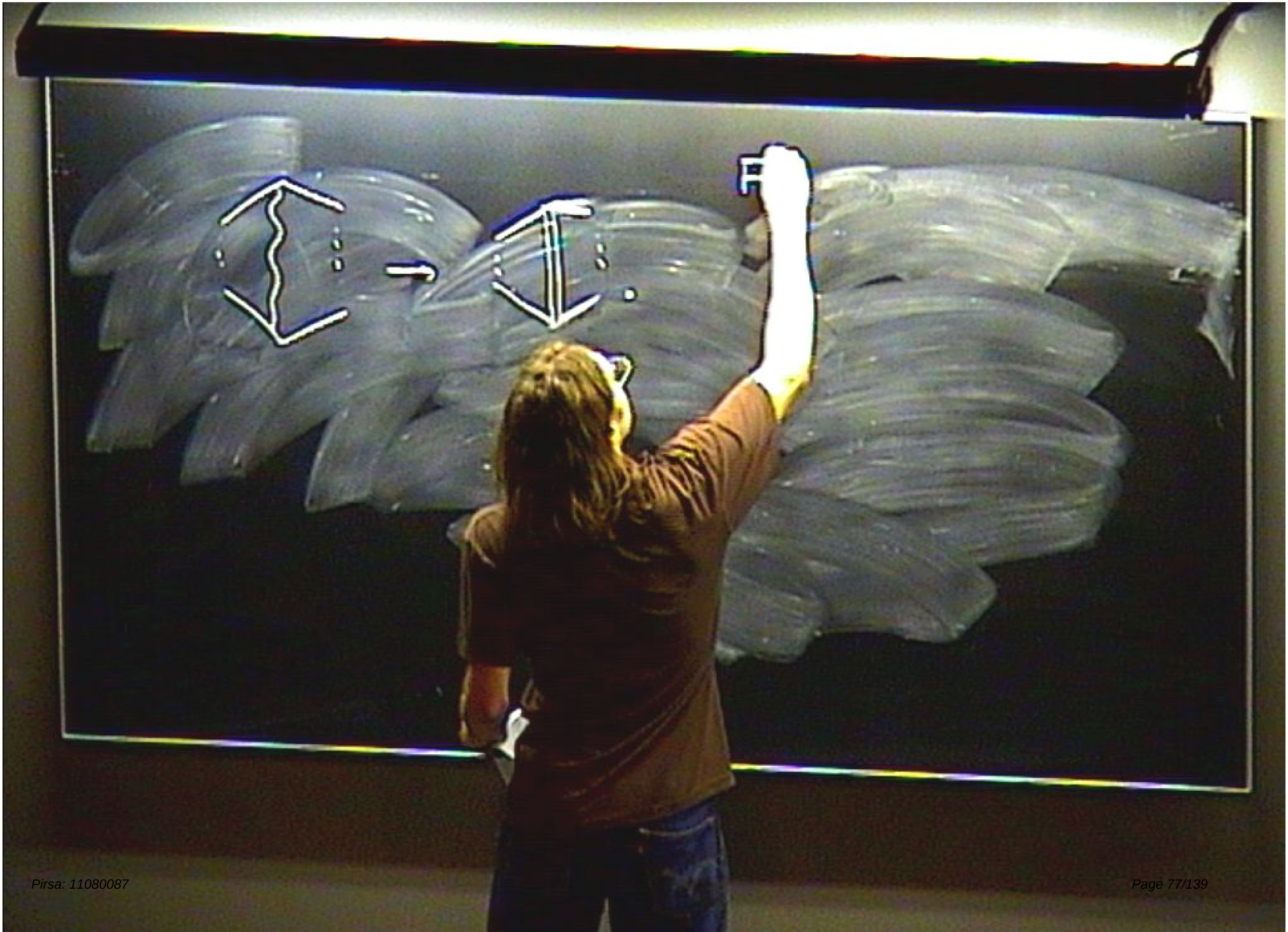
$+ \lambda\left(G^2 + \psi\psi\phi + [\phi,\phi]^2\right)$.

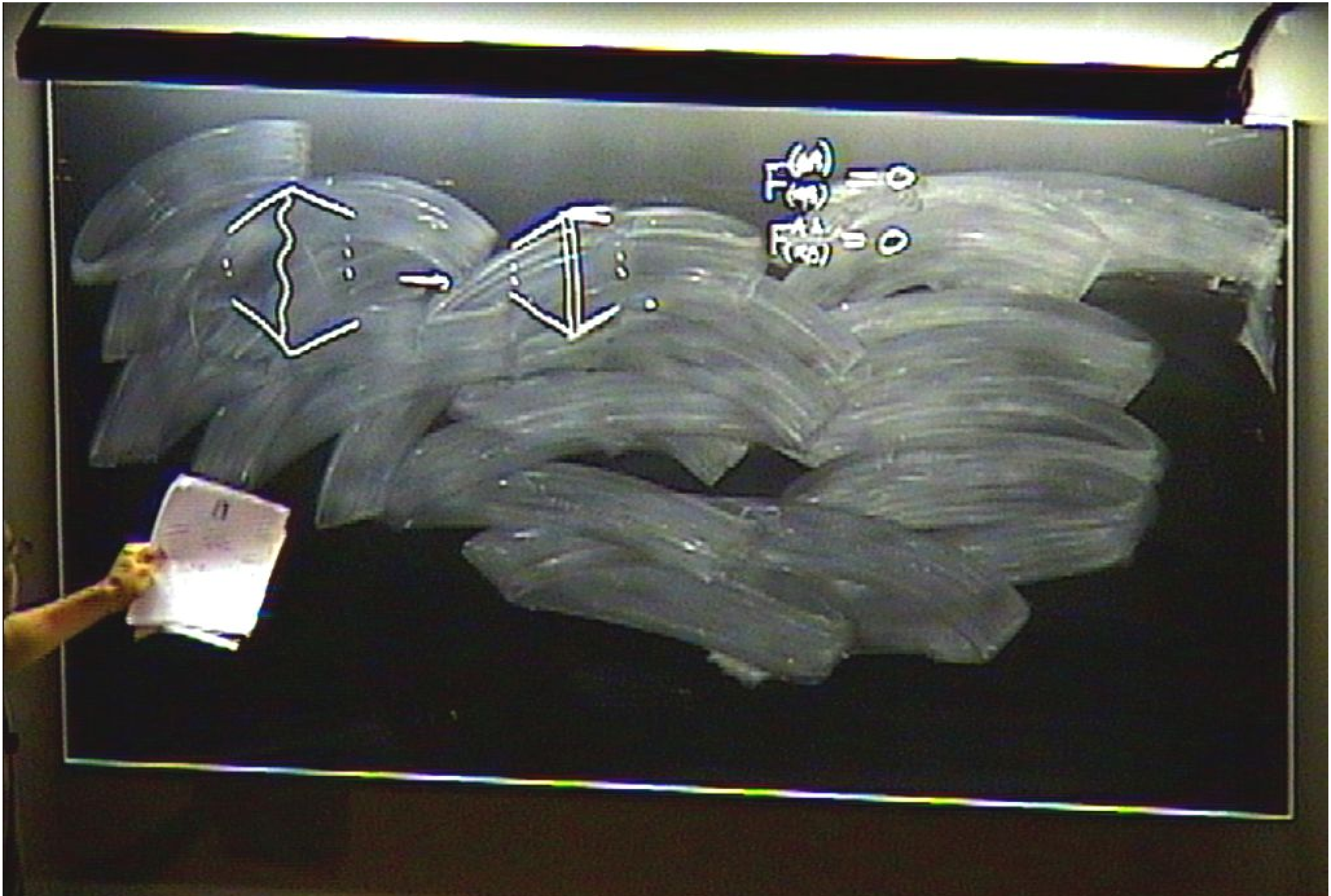
$q A = (d\bar{\theta} + \lambda\bar{\theta})$

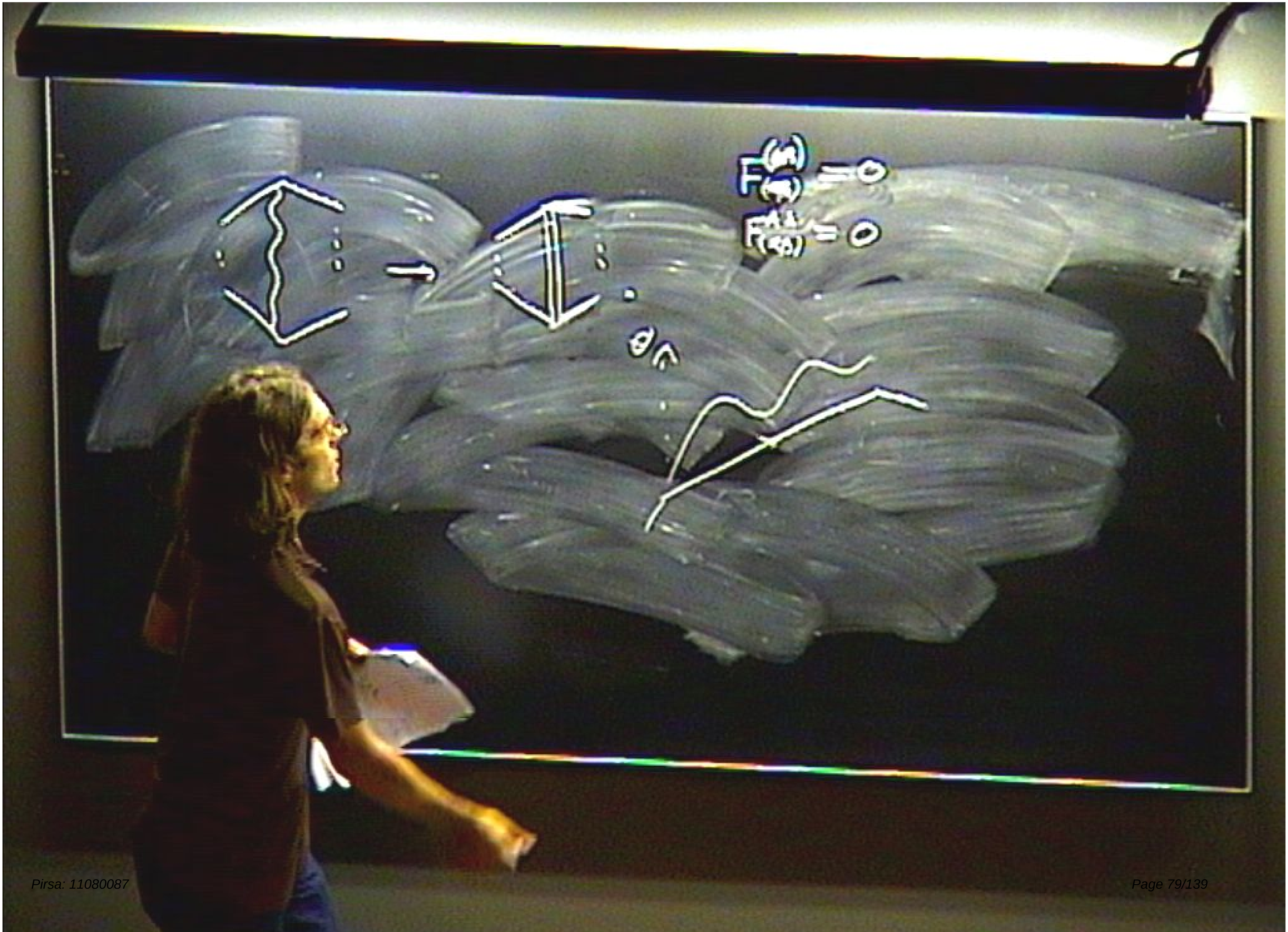
$\bar{q} A = O(\lambda)$

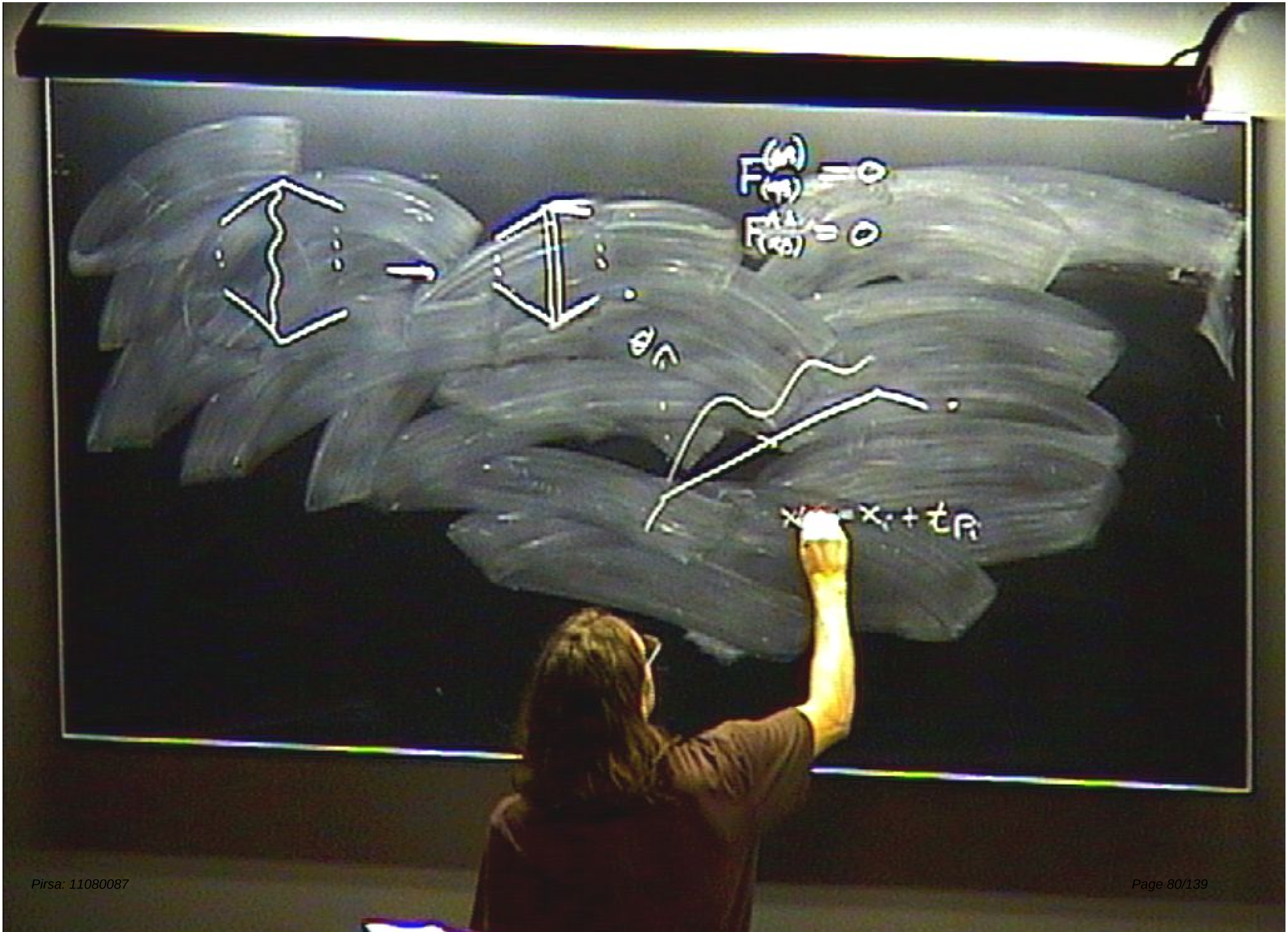


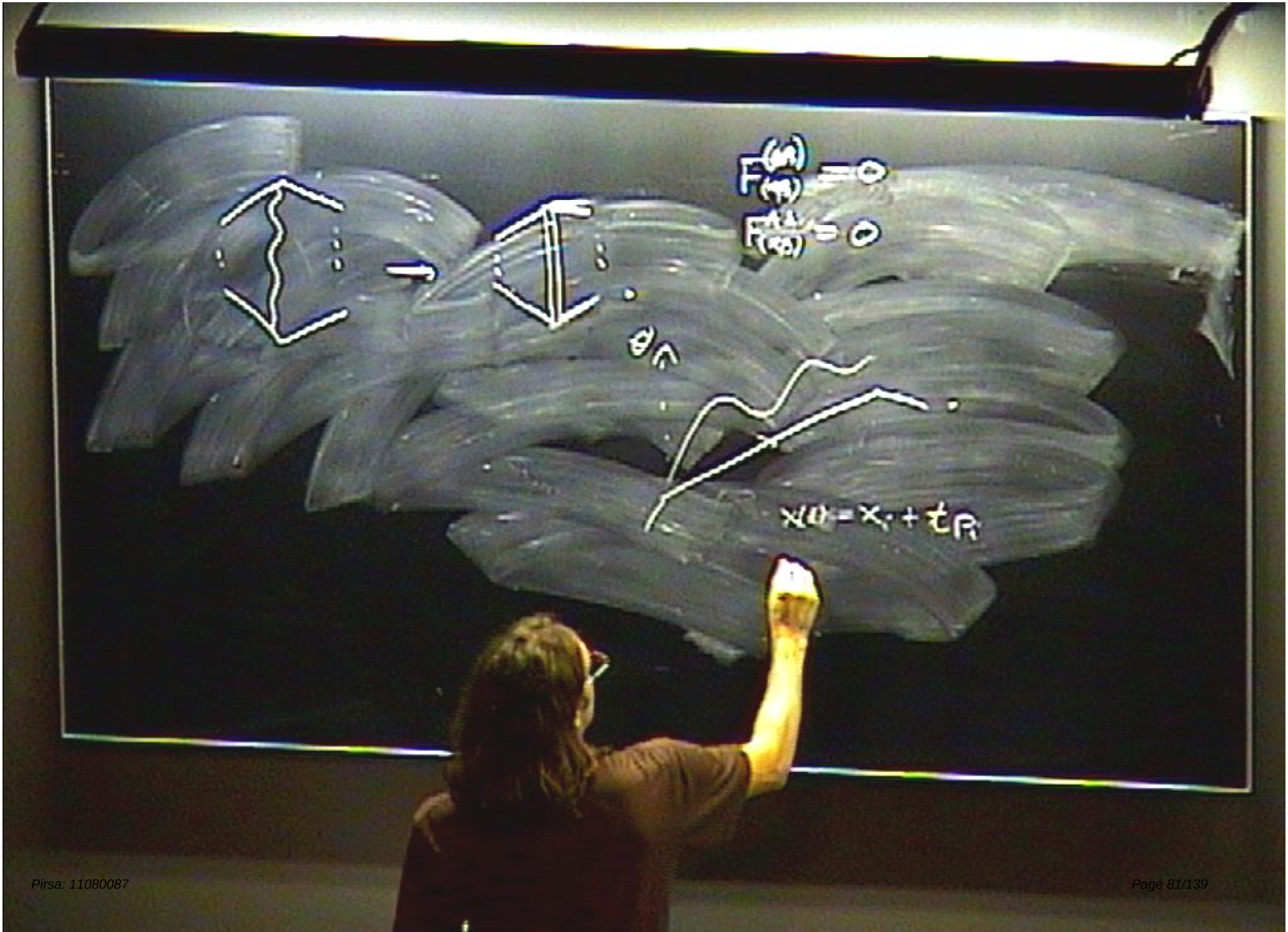


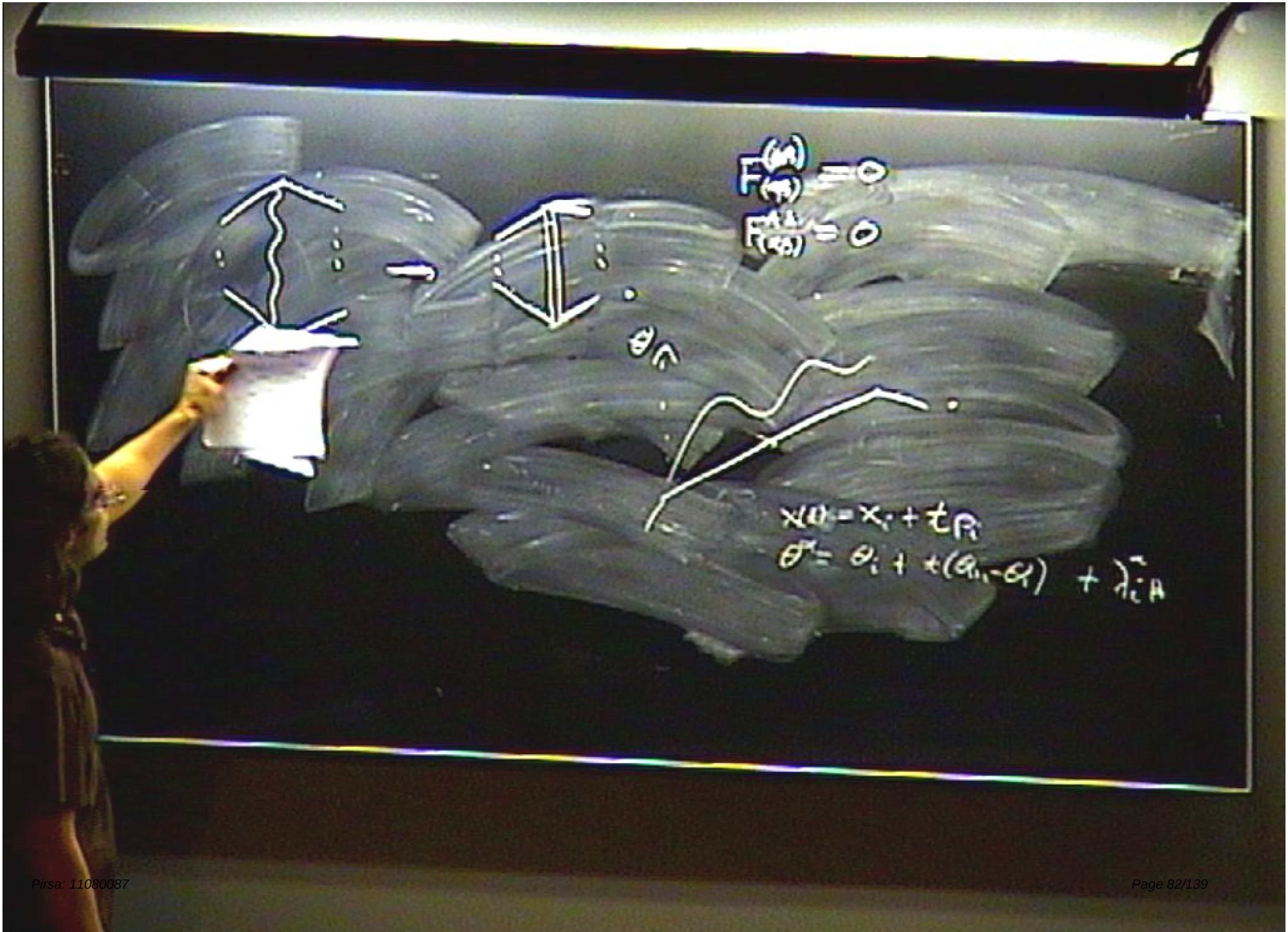


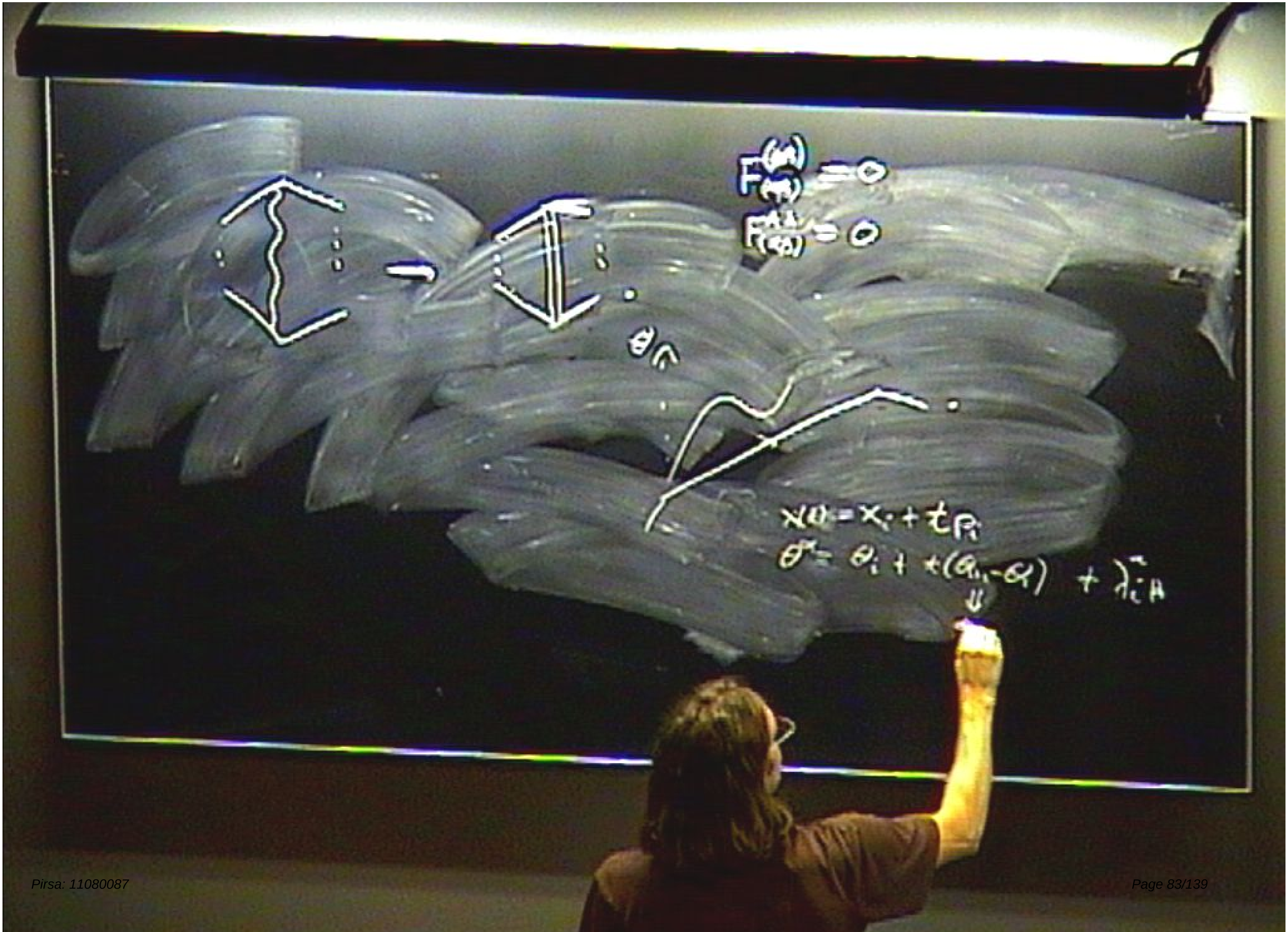


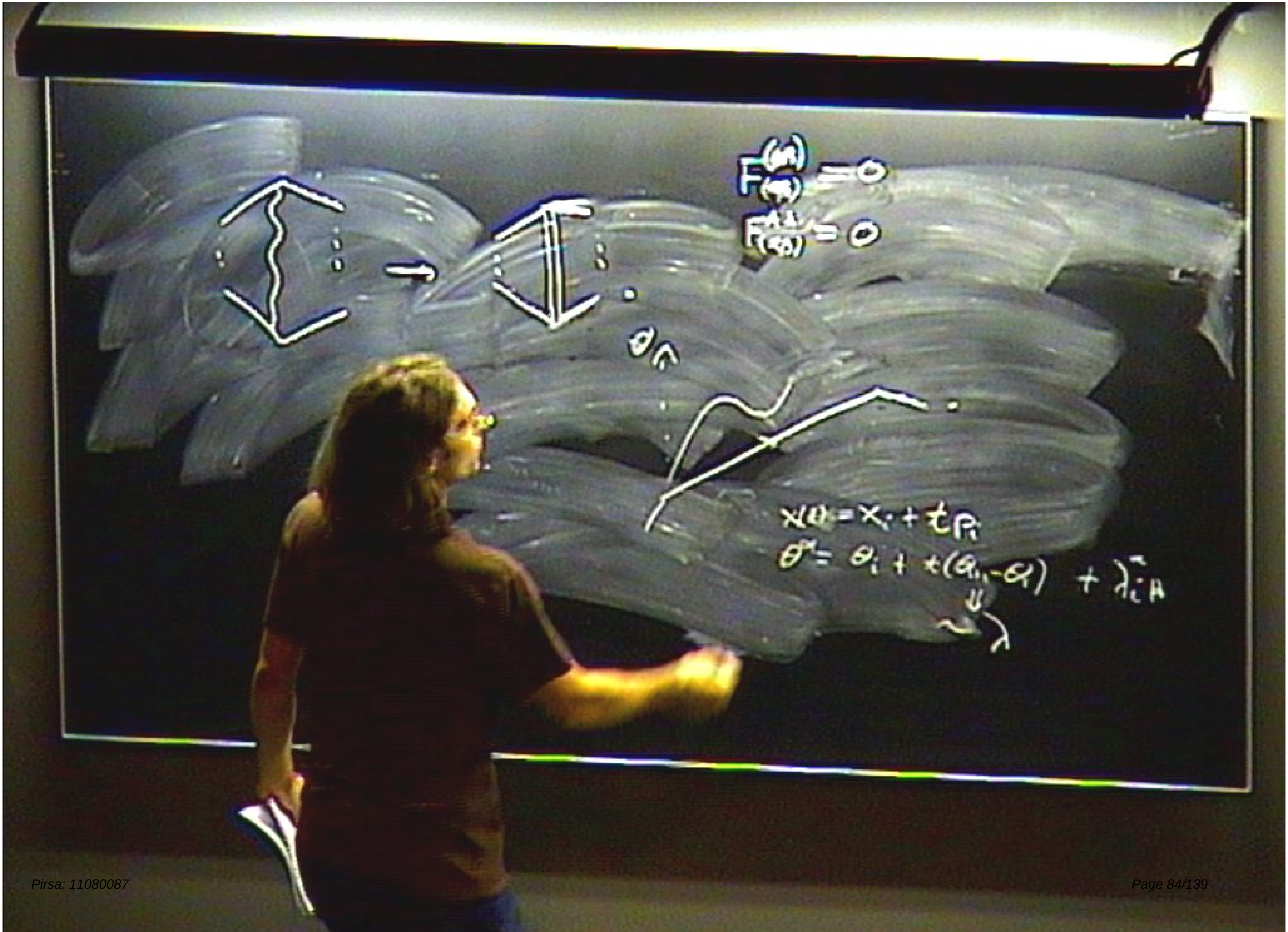


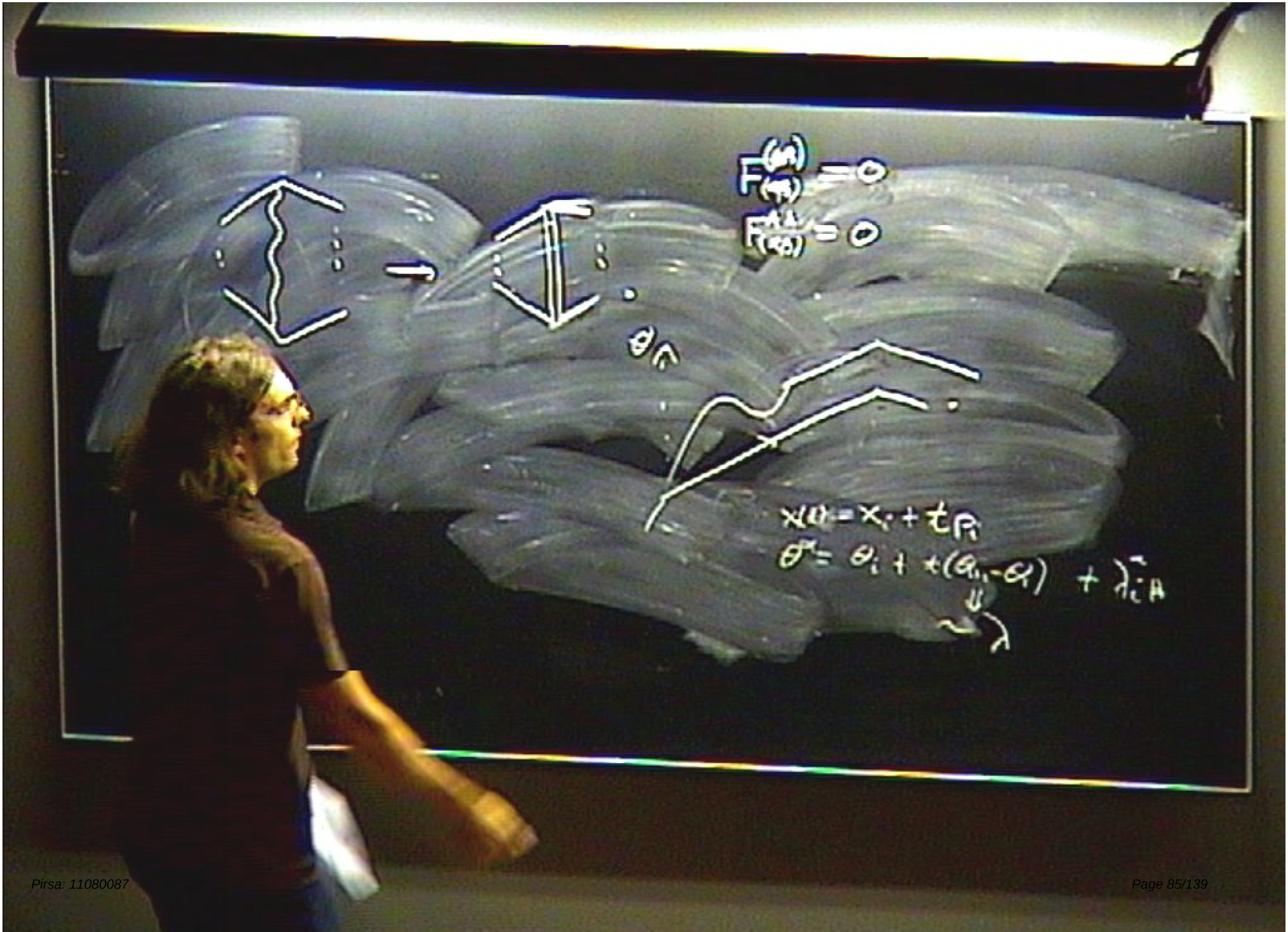


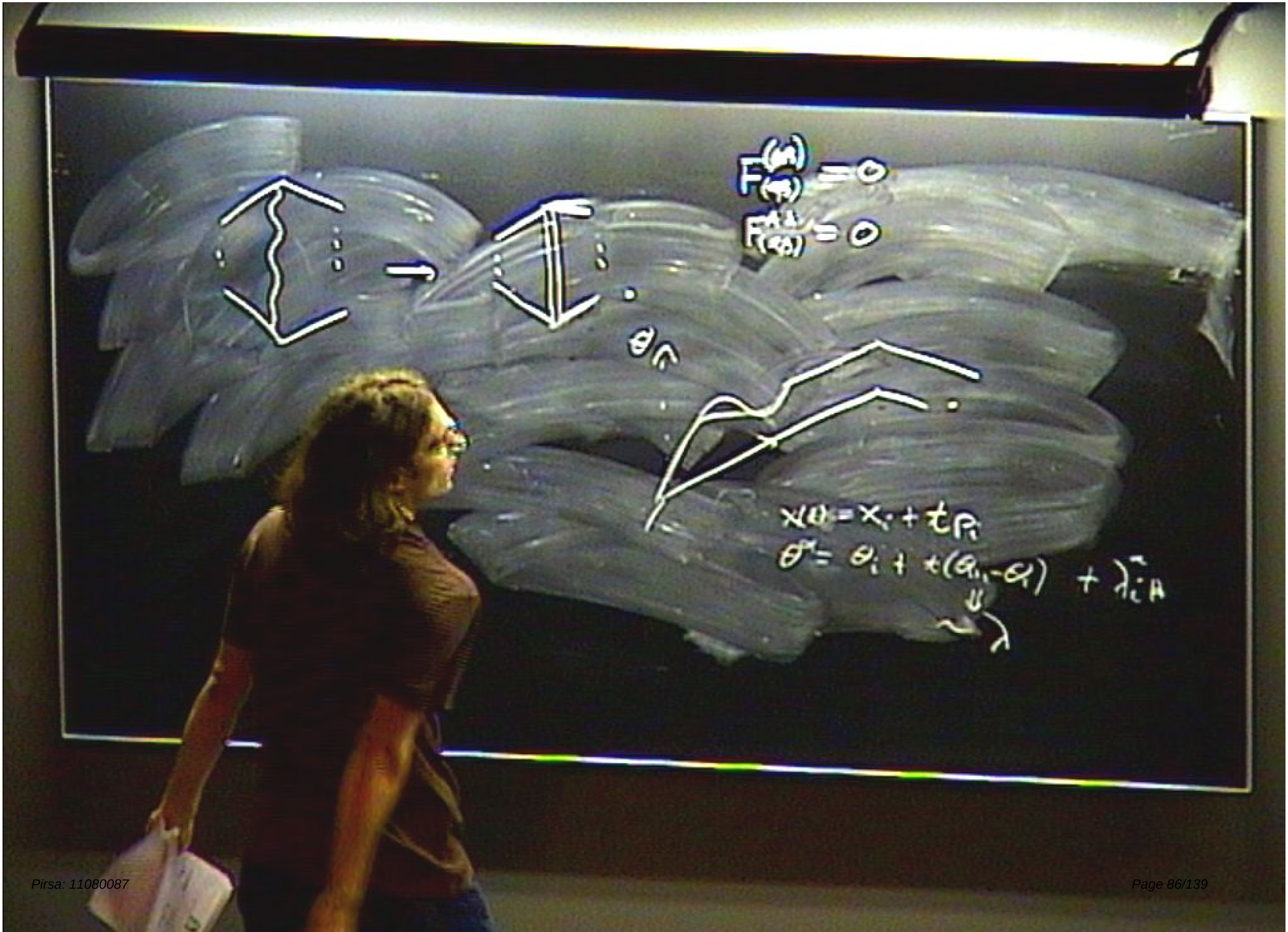


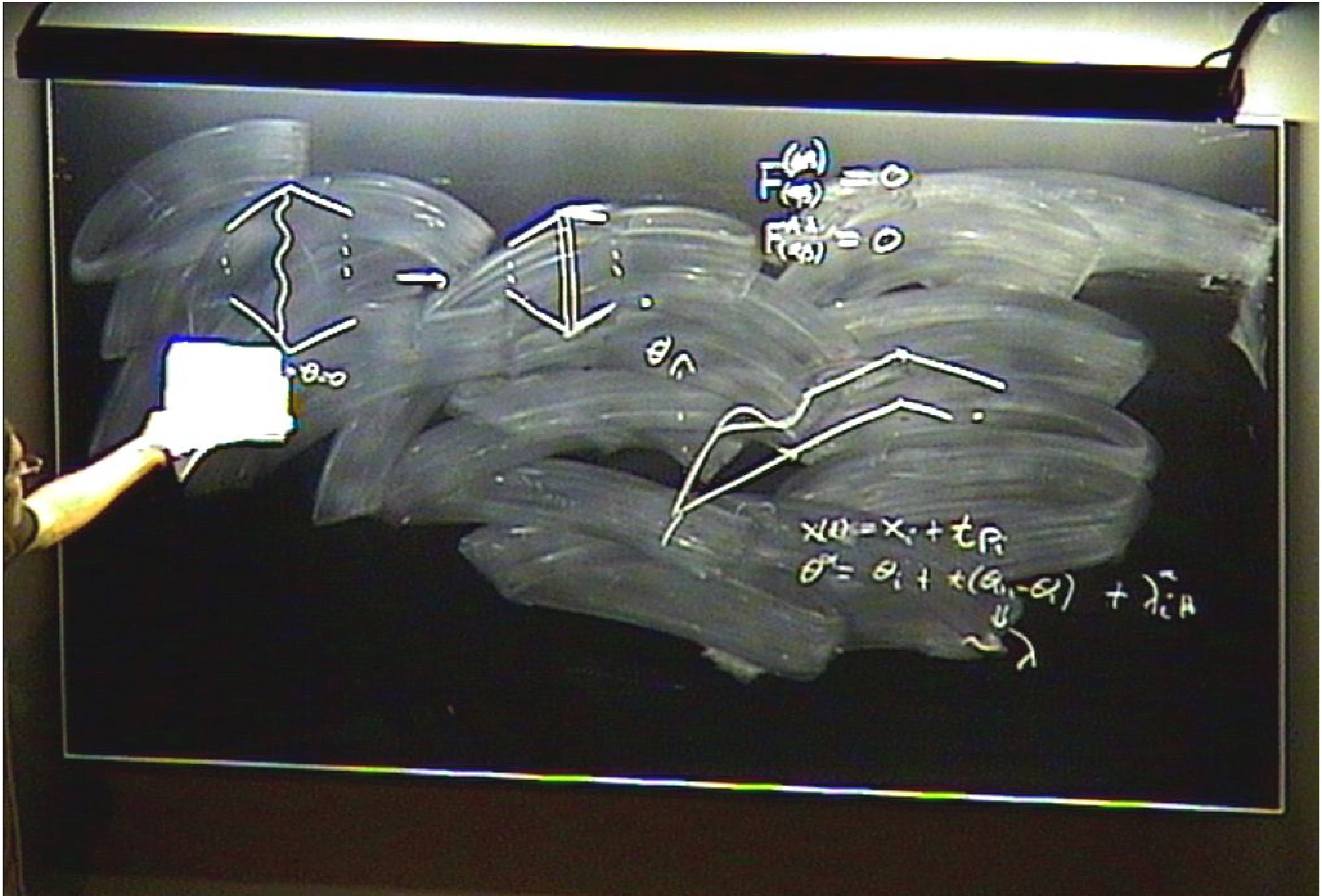


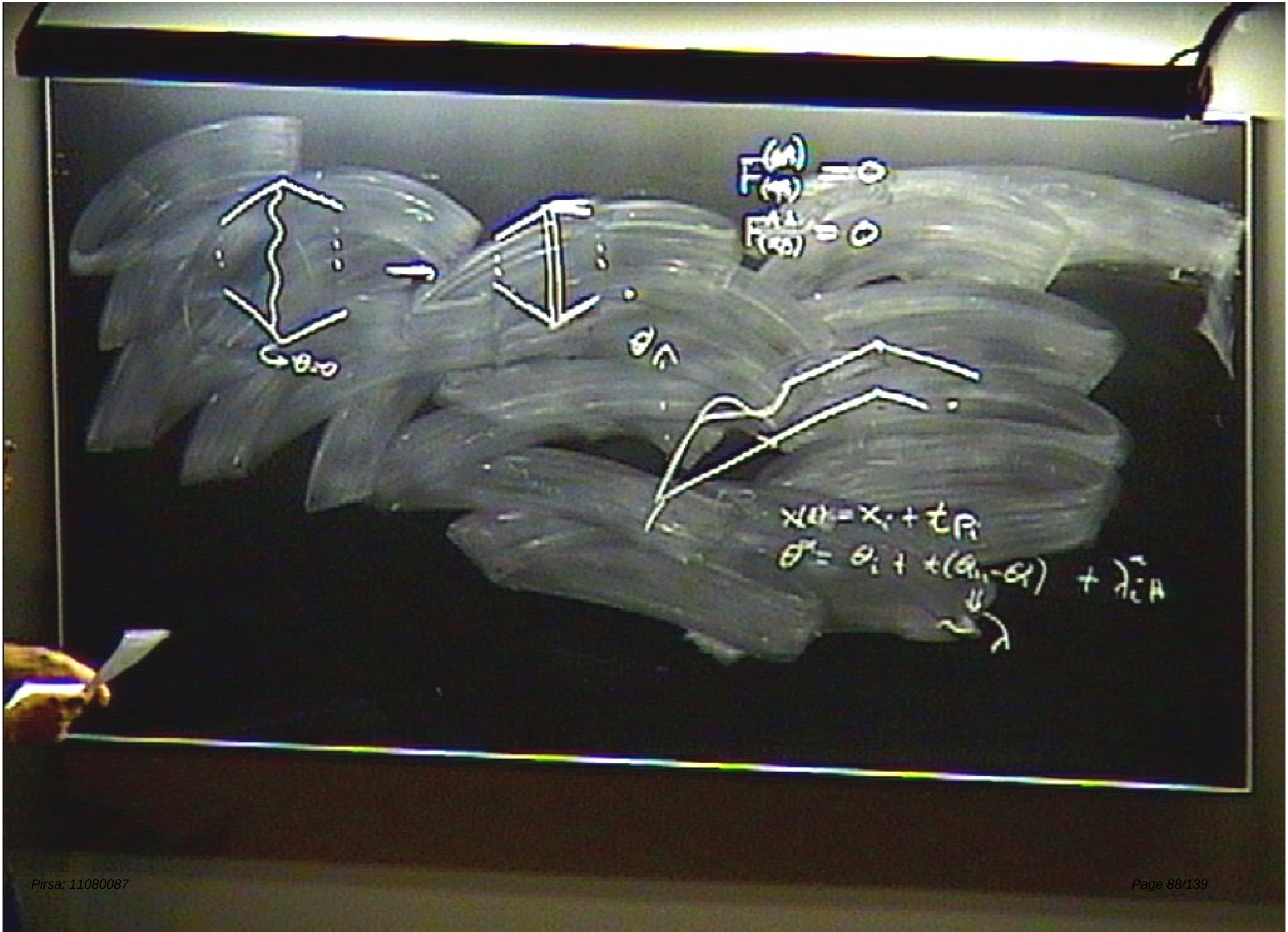


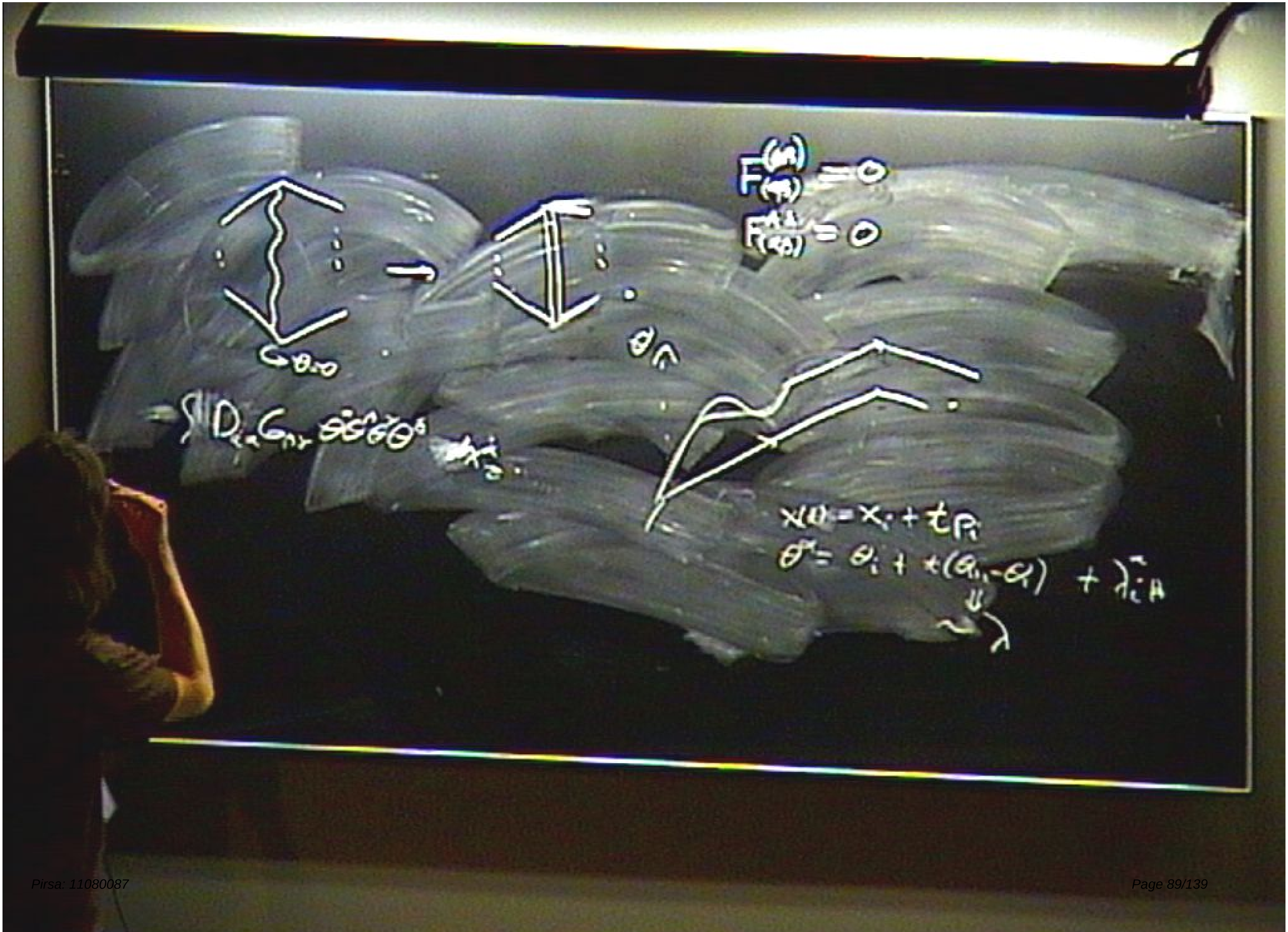


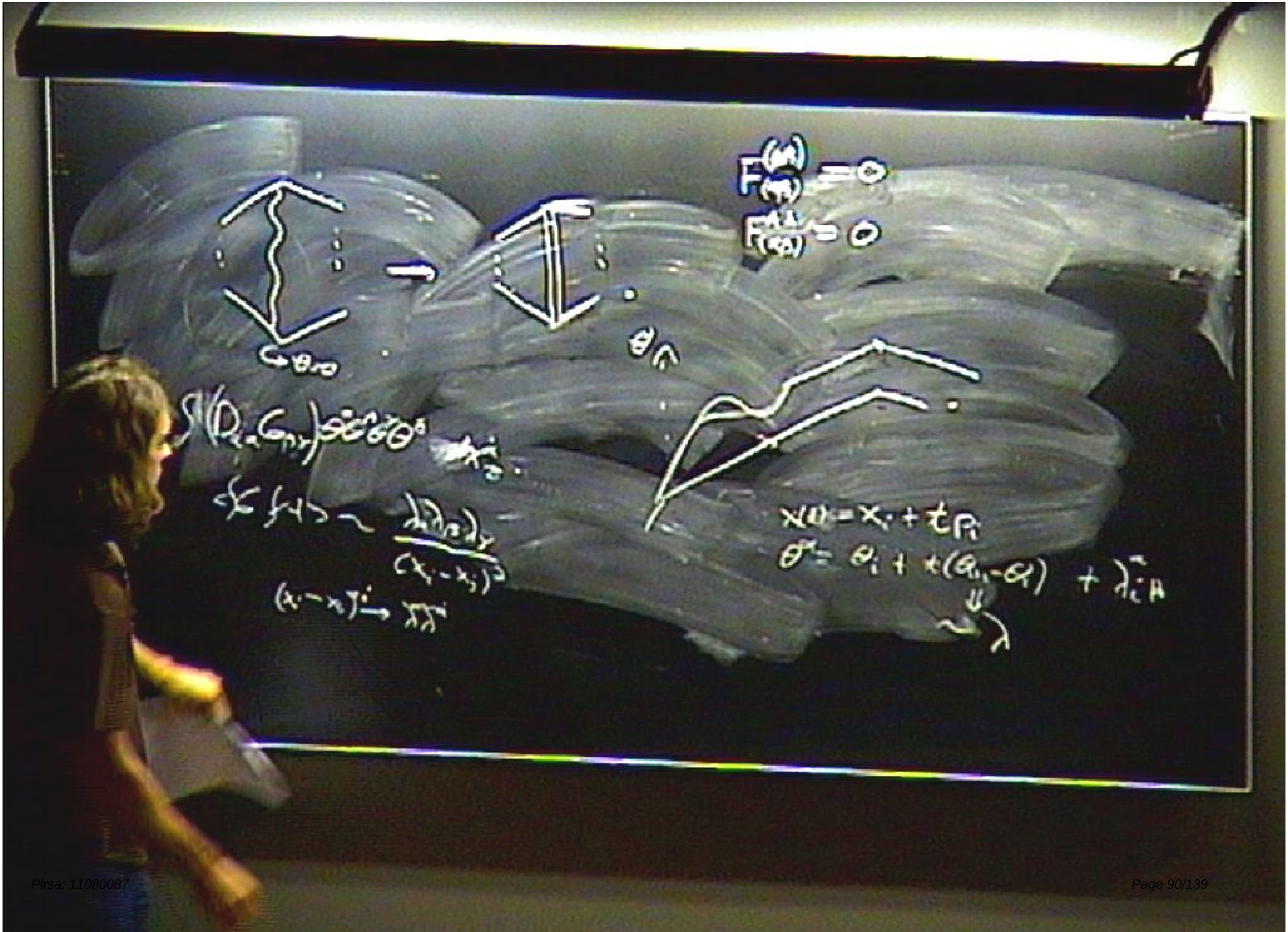


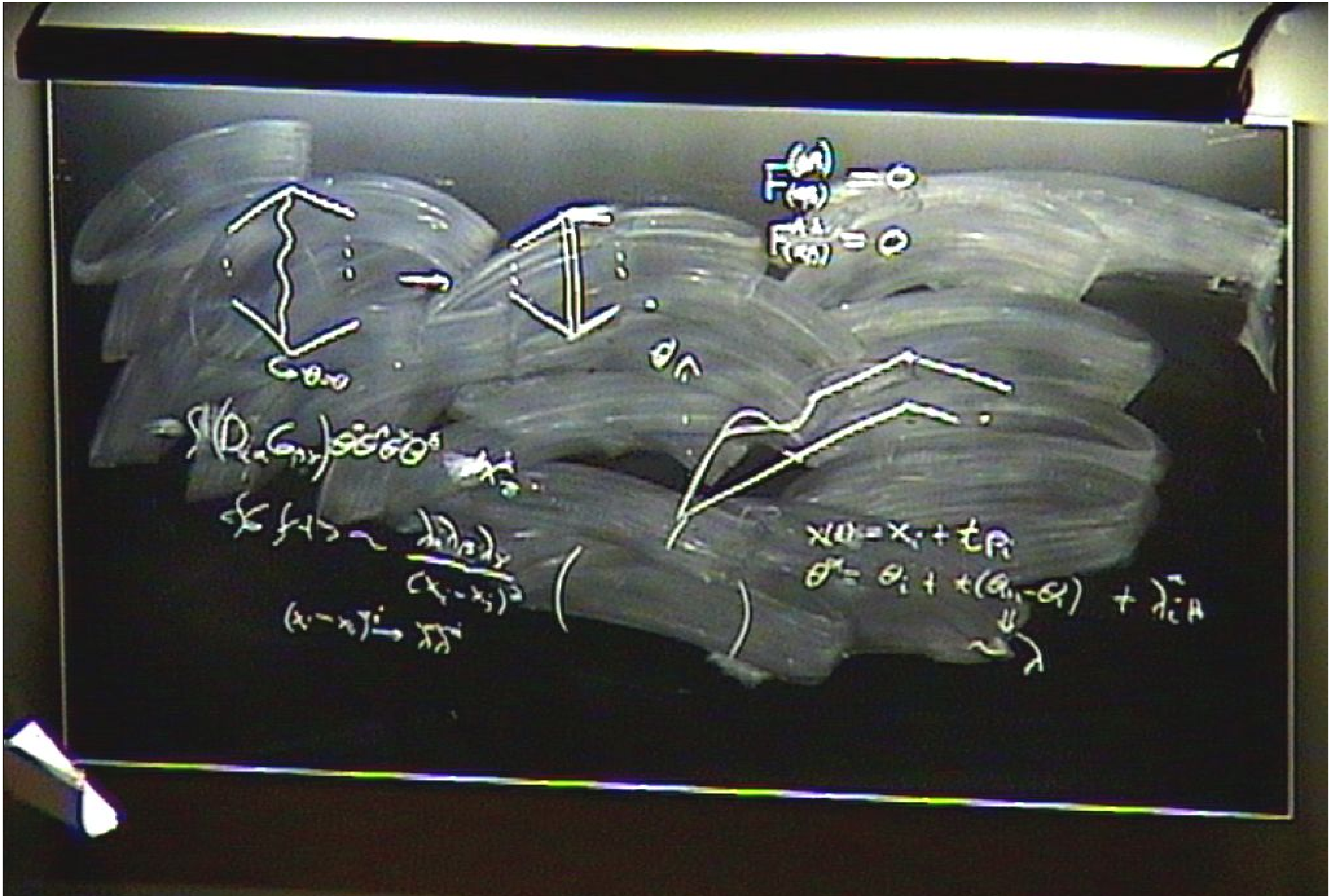


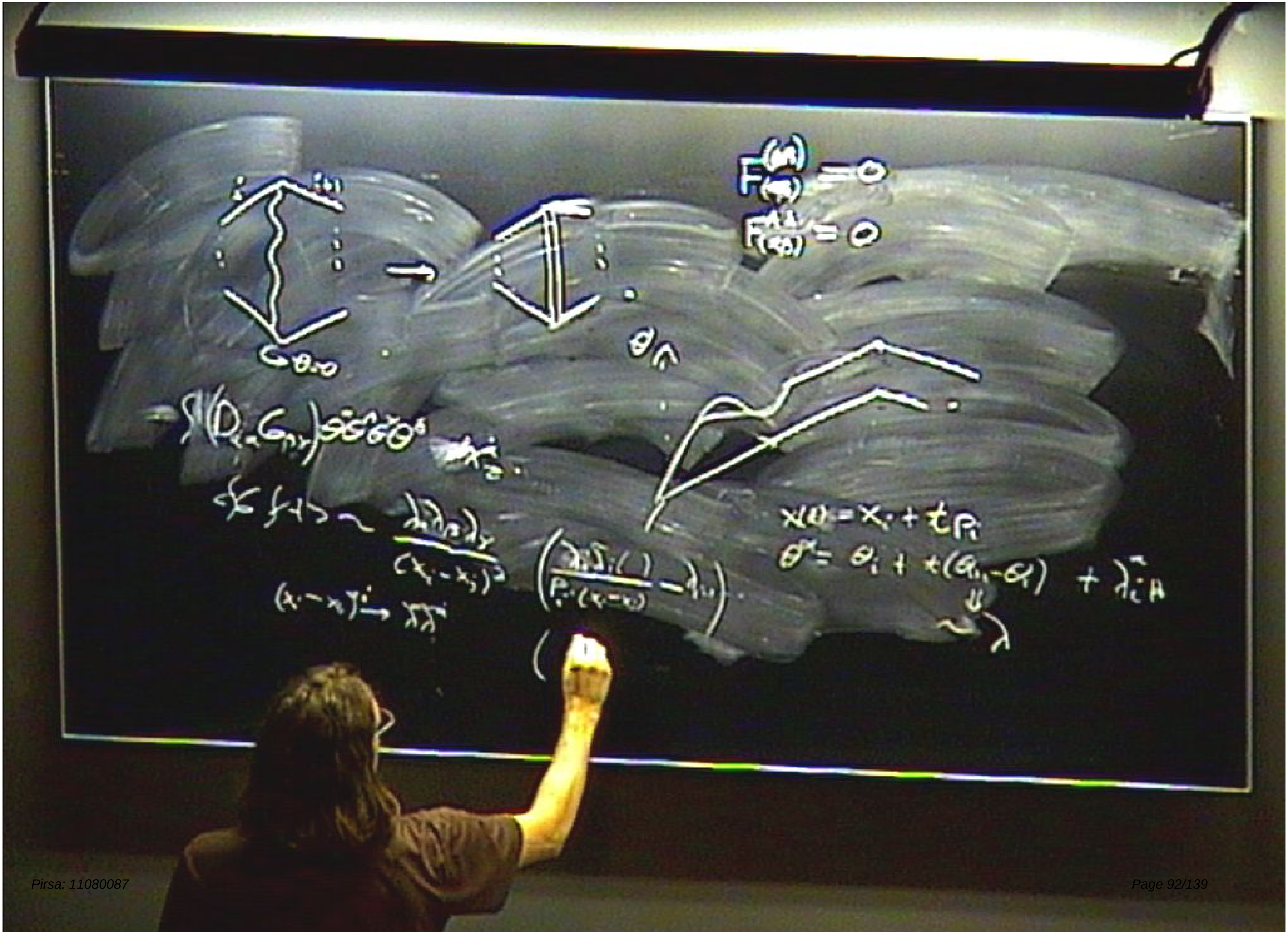


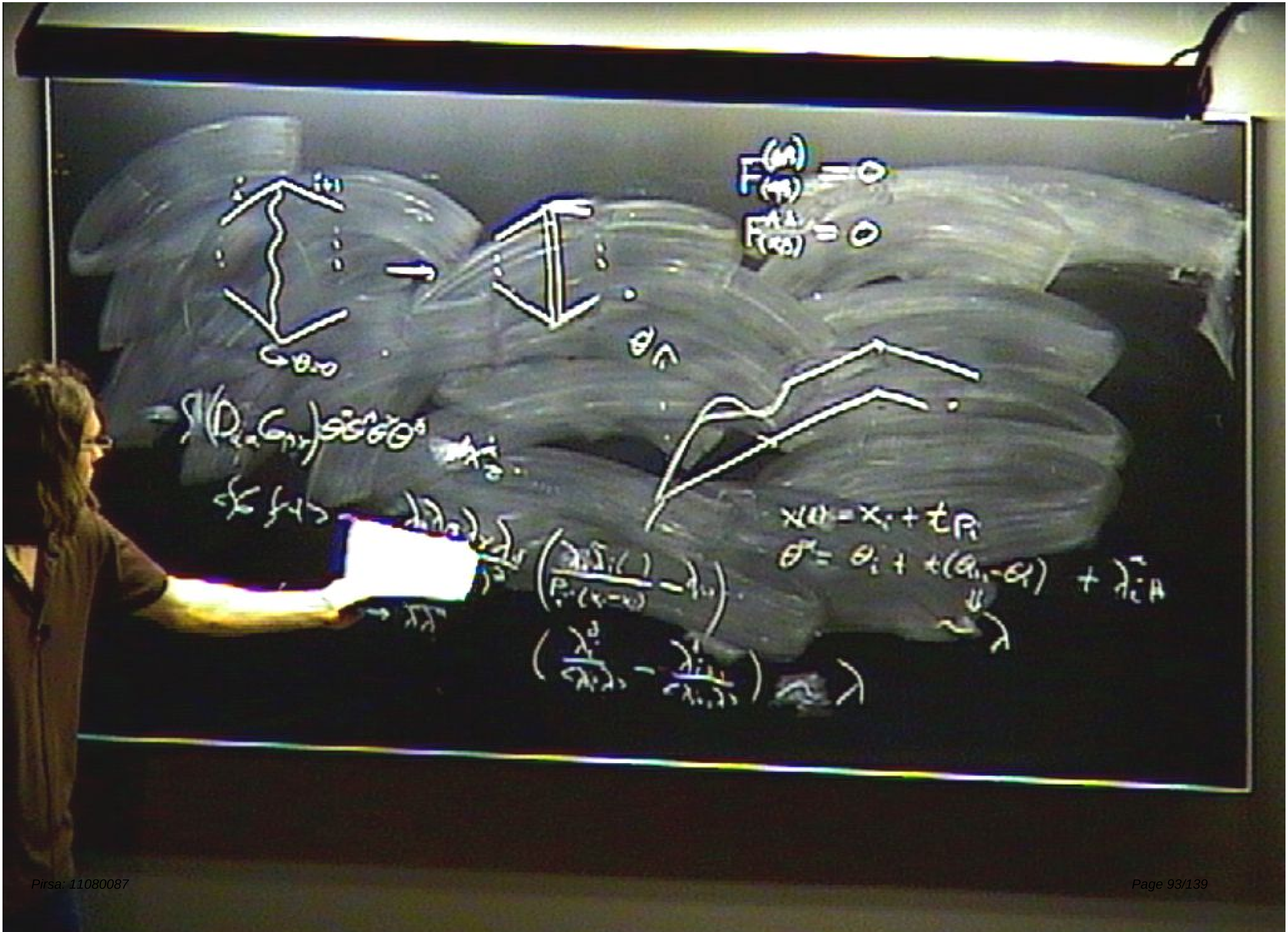


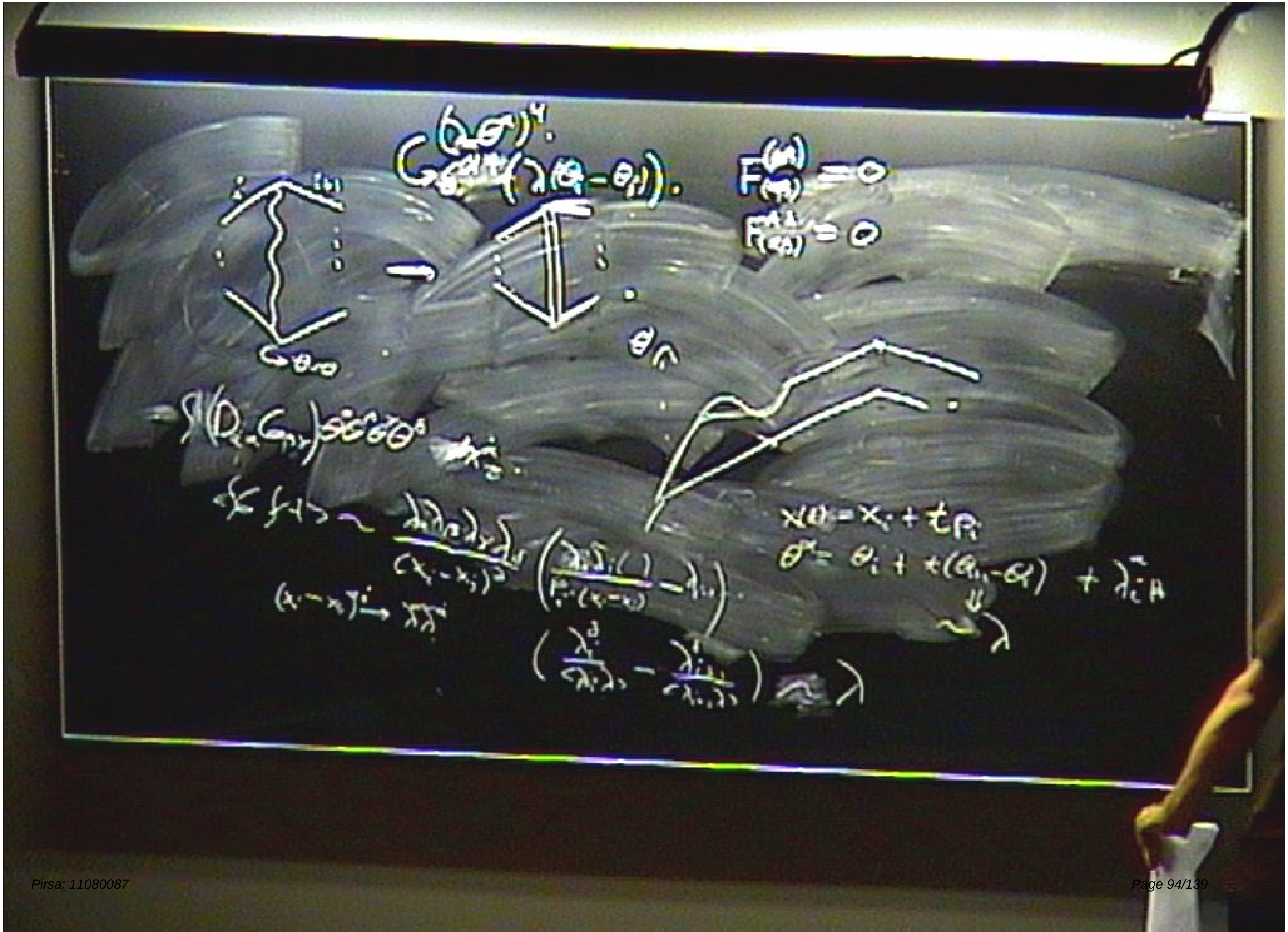

x(t) = x_i + t p_i
= 0
= 0

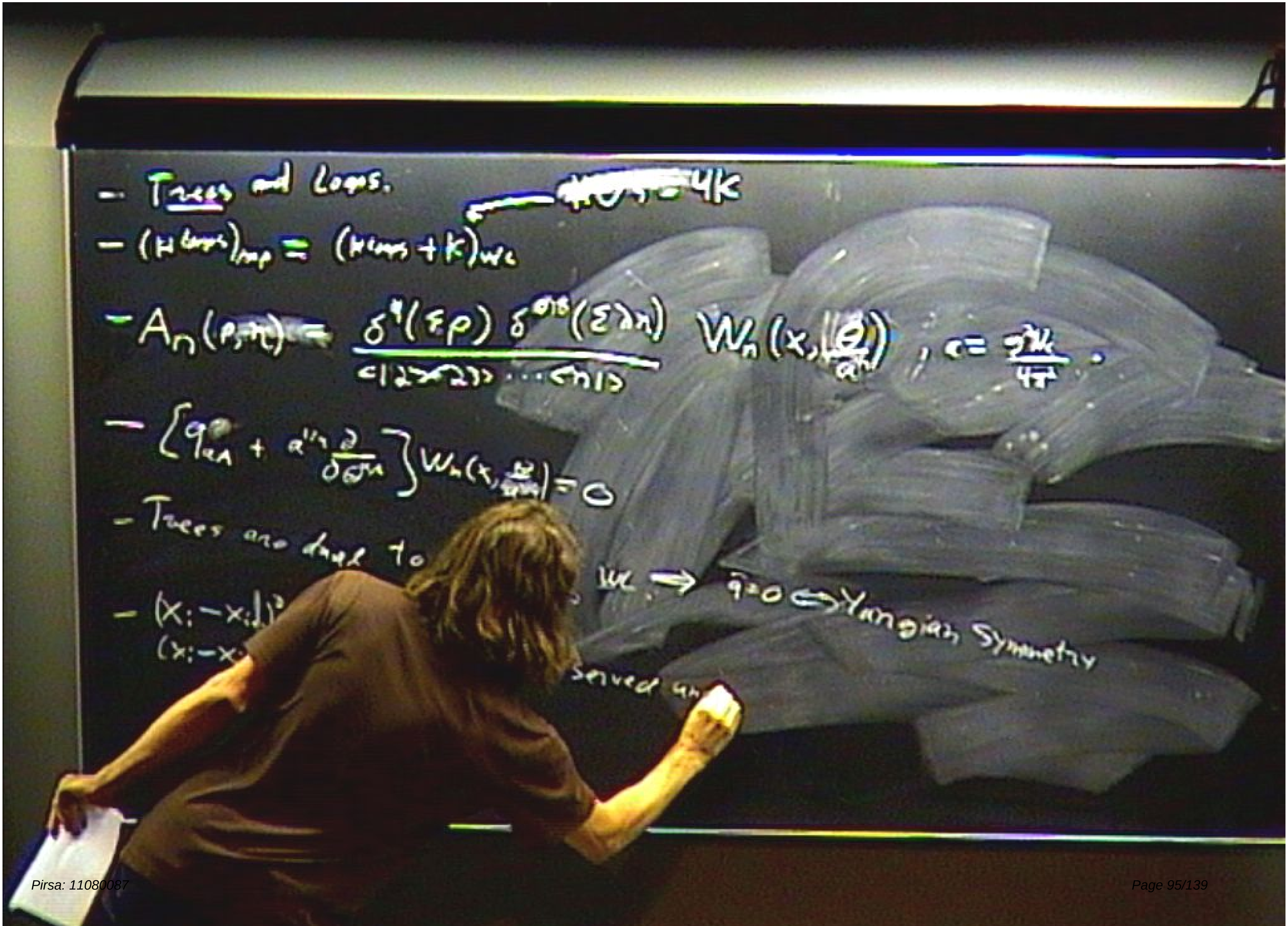



x(t) = x_i + t p_i




x(t) = x_i + t P_i
θ^a = θ_i + t(θ_{i+1} − θ_i) + λ_i^a




- Trees and Loops.
4K
- (# loops)mp = (# loops + K)WL
- An(p,η) = δ4(Σp) δ0|8(Σλη) / ⟨12⟩⟨23⟩ ··· ⟨n1⟩ Wn(x, θ), a = g²Nc/4π².
- {qαA + a ∂/∂θ} Wn(x, θ) = 0
- Trees are dual to
WL ⇒ q=0 ⟺ Yangian Symmetry
- (xi − xi)²
(xi − x
served an

– Trees and Loops.

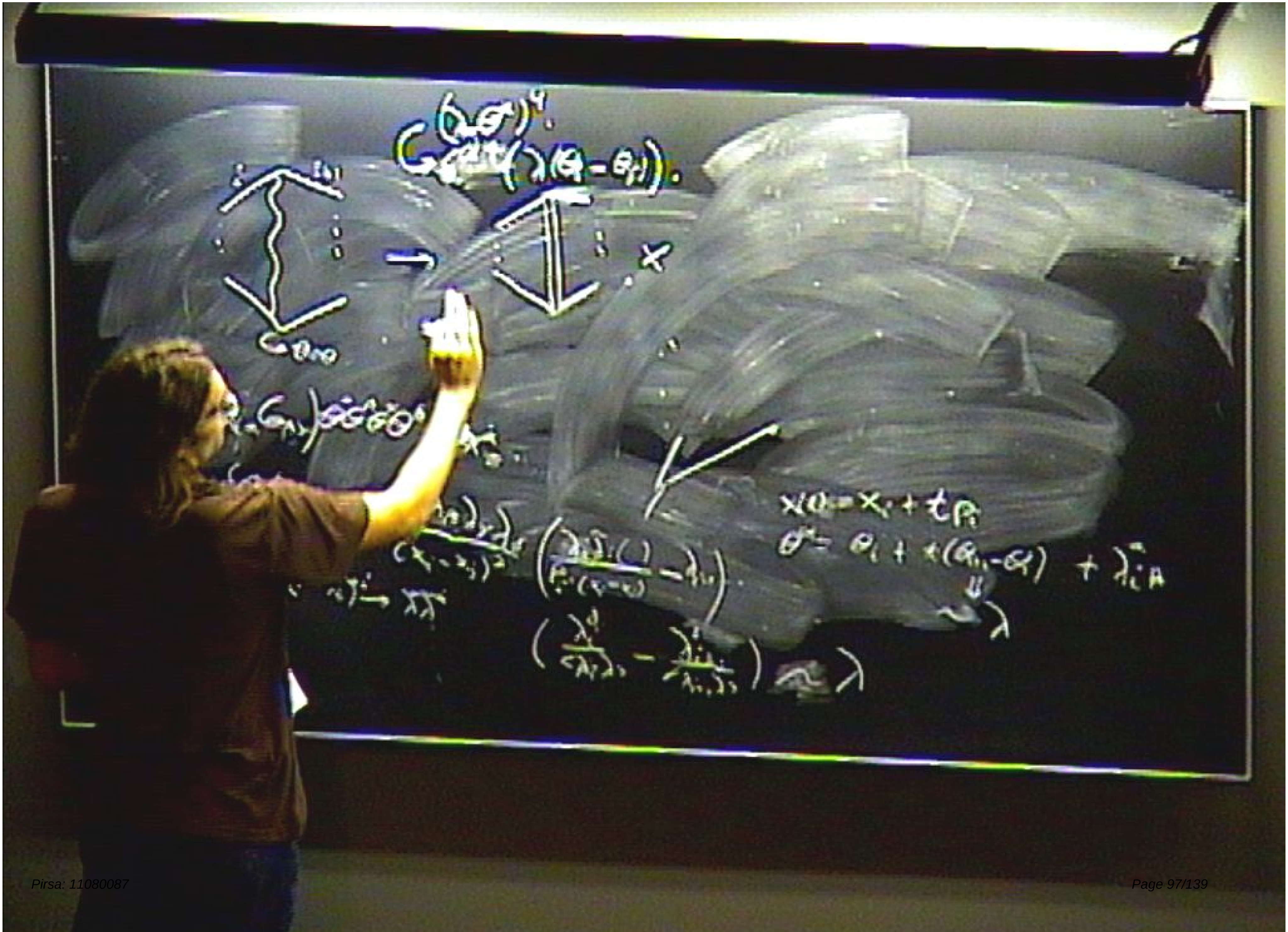
– $(H^{\text{loops}})_{mp} = (H^{\text{loops}} + K)_{WL}$ ~~...~~ $= 4K$

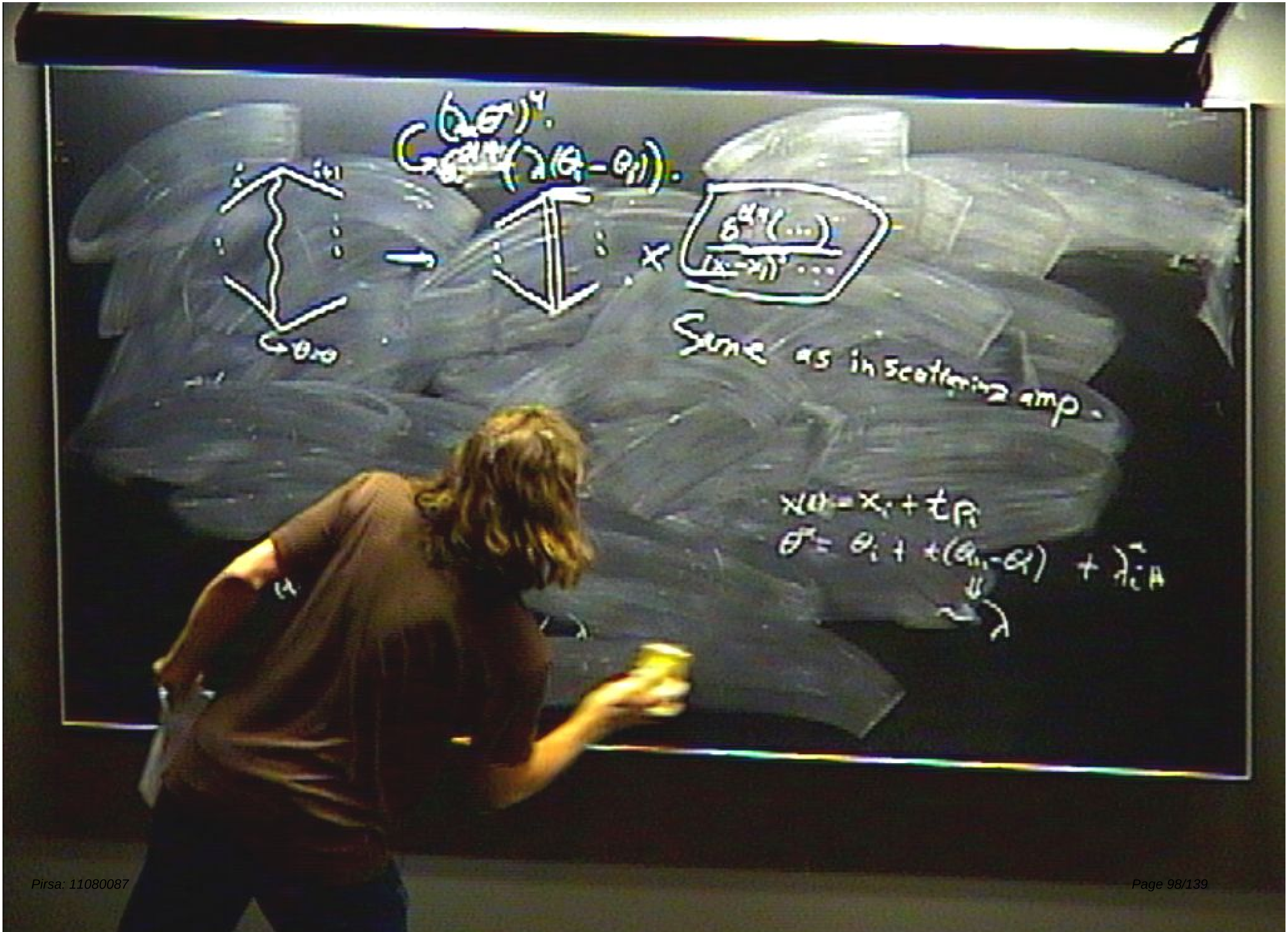
– $A_n(p, \lambda\eta) = \dfrac{\delta^4(\Sigma p)\, \delta^{0|8}(\Sigma \lambda\eta)}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle} W_n\left(x, \dfrac{\theta}{a^{1/2}}\right), \quad a = \dfrac{g^2 N}{4\pi}.$

– $\left\{ q_{\alpha A} + a^{1/2} \dfrac{\partial}{\partial \theta^{\alpha A}} \right\} W_n\left(x, \dfrac{\theta}{a^{1/2}}\right) = 0$

– Trees are dual to SDYM WL $\Rightarrow$ $\bar{q} = 0 \Leftrightarrow$ Yangian Symmetry

– $\left.\begin{array}{l} (x_i - x_{i+1})^2 = 0 \\ (x_i - x_{i+1})(\theta_i - \theta_{i+1}) = 0 \end{array}\right\}$ preserved under factorization.




Same as in scattering amp.

Fidelman

$$\mathcal{A} = dx^{\mu}\left(A_\mu + \ldots\theta + \ldots + \ldots + D\ldots\right) + \ldots$$

$$+\, d\bar\theta(\ldots)$$

$$S = \frac{1}{g^2}\left(\frac{F^2}{4} + \bar\Psi \not{D} \Psi + \frac{(D\phi)^2}{2} + \ldots + [\phi,\phi]^2\right)$$

$A_\mu \to A_\mu$

$\bar\Psi \to \lambda^{1/2}\bar\Psi$

$\phi \to \lambda^{1/2}\phi$

$\Psi \to \lambda^{3/4}\Psi$

$F_{\alpha\beta} \to \lambda(G_{\alpha\beta})$

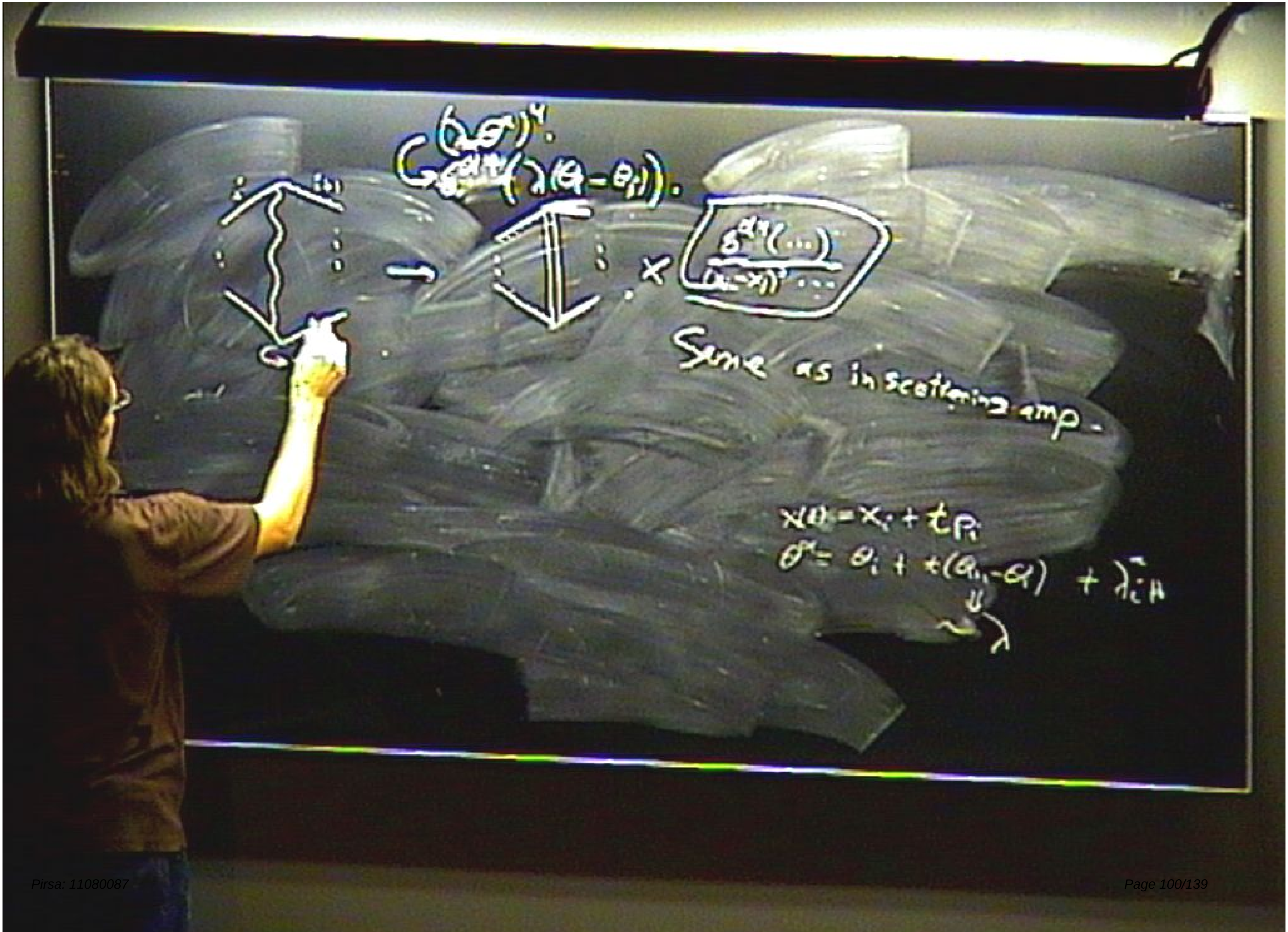
aux. Field.

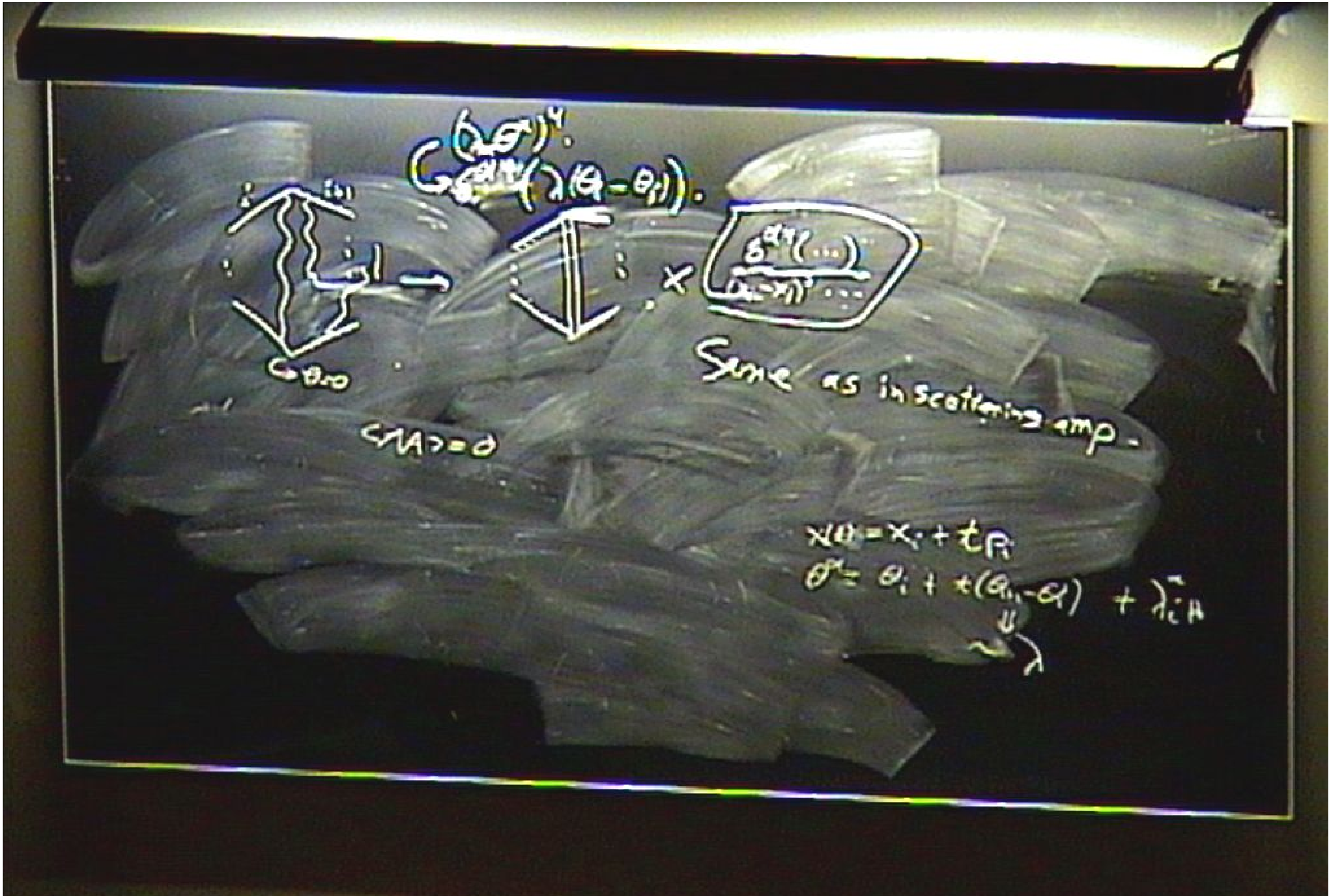
$$\boxed{S \to \left[EG^{\alpha\beta} + \Psi D\Psi + \frac{(D\phi)^2}{2} + \bar\Psi\phi\phi\right]}$$ SDYM.

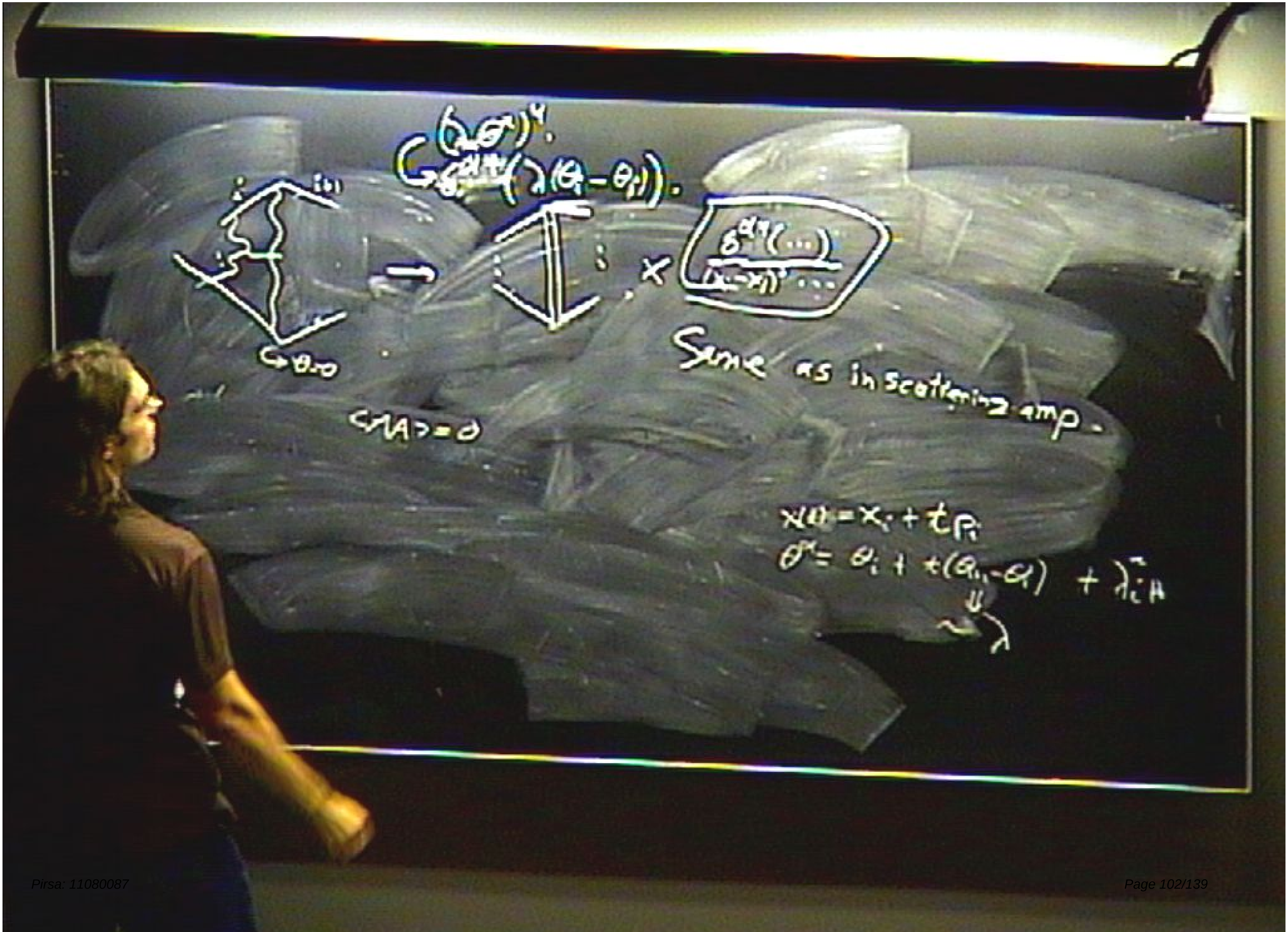
$$+\,\lambda\left(G^2 + \Psi\Psi\phi + [\phi,\phi]^2\right).$$

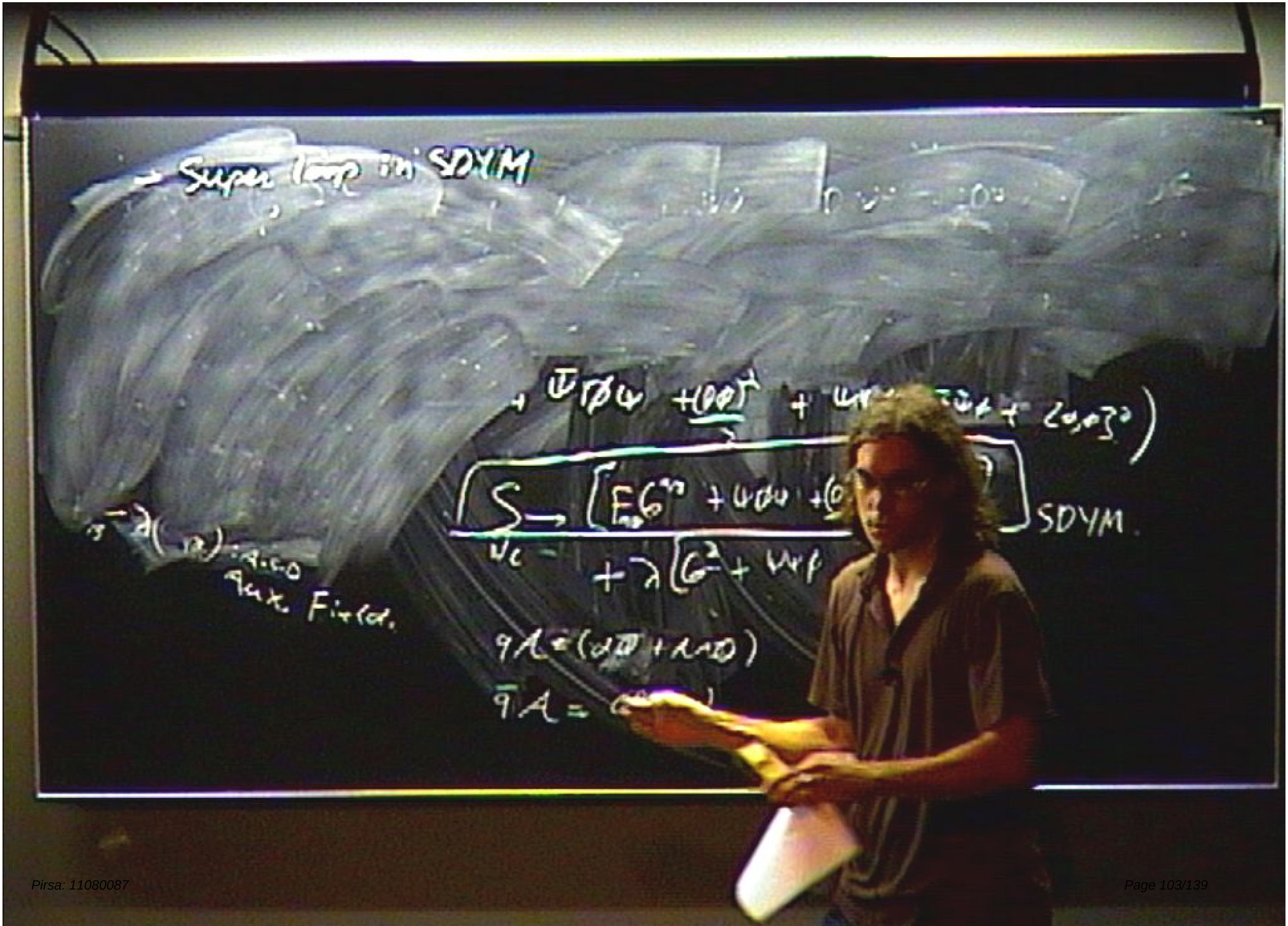
$$q\mathcal{A} = (d\bar D + \lambda\wedge D)$$

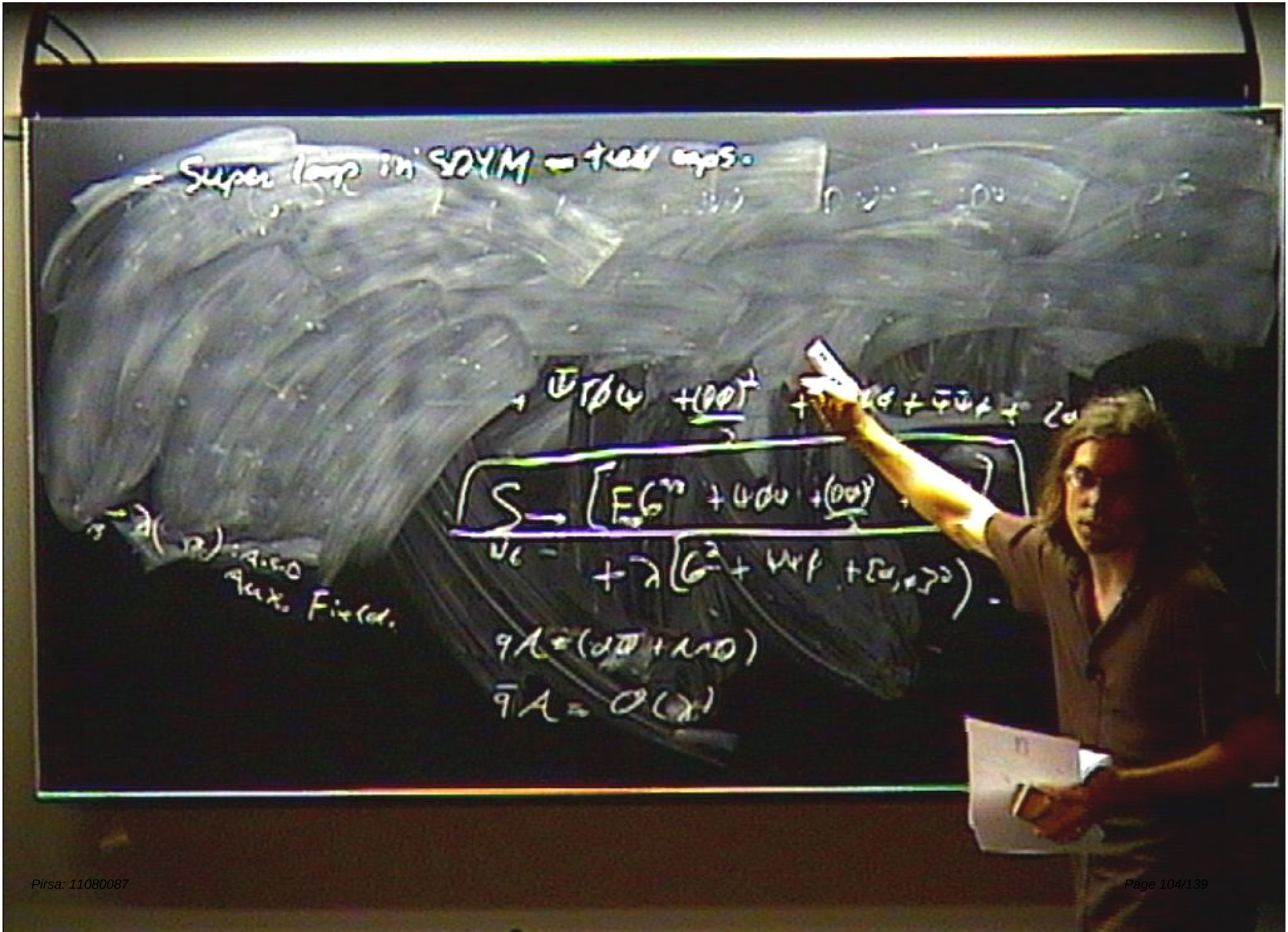
$$\bar q\mathcal{A} = \mathcal{O}(\lambda)$$


Same as in scattering amp.


Same as in scattering amp.
<M>=0


Same as in scattering amp.
<A>=0


SDYM
SDYM.
Aux. Field.


Super loop in SDYM


Super Loop in SDYM = tree amps.
Field.

– Trees and Loops.

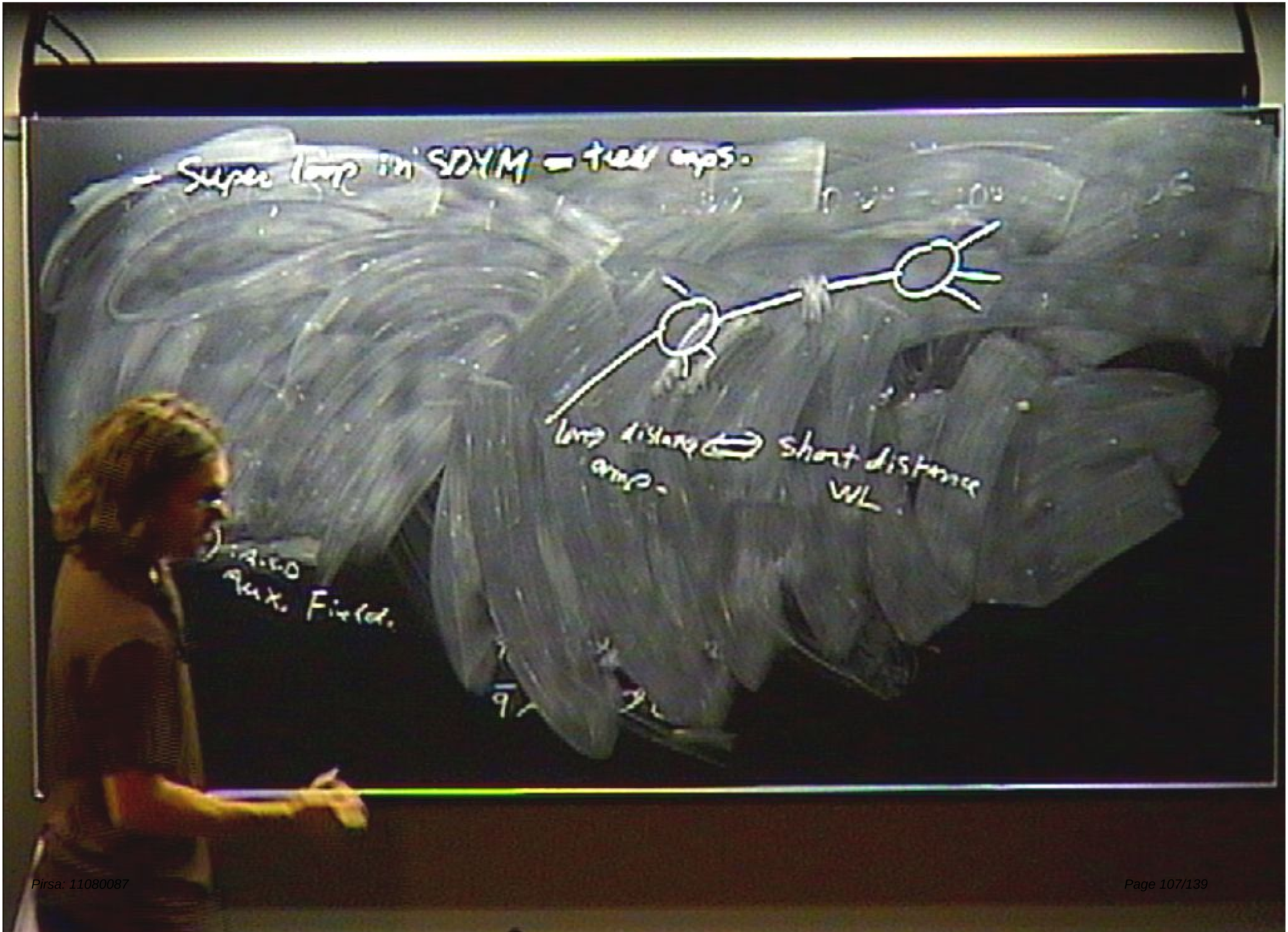
– $(H\text{ loops})_{mp} = (H\text{ loops} + K)_{WL}$ $\quad = 4K$

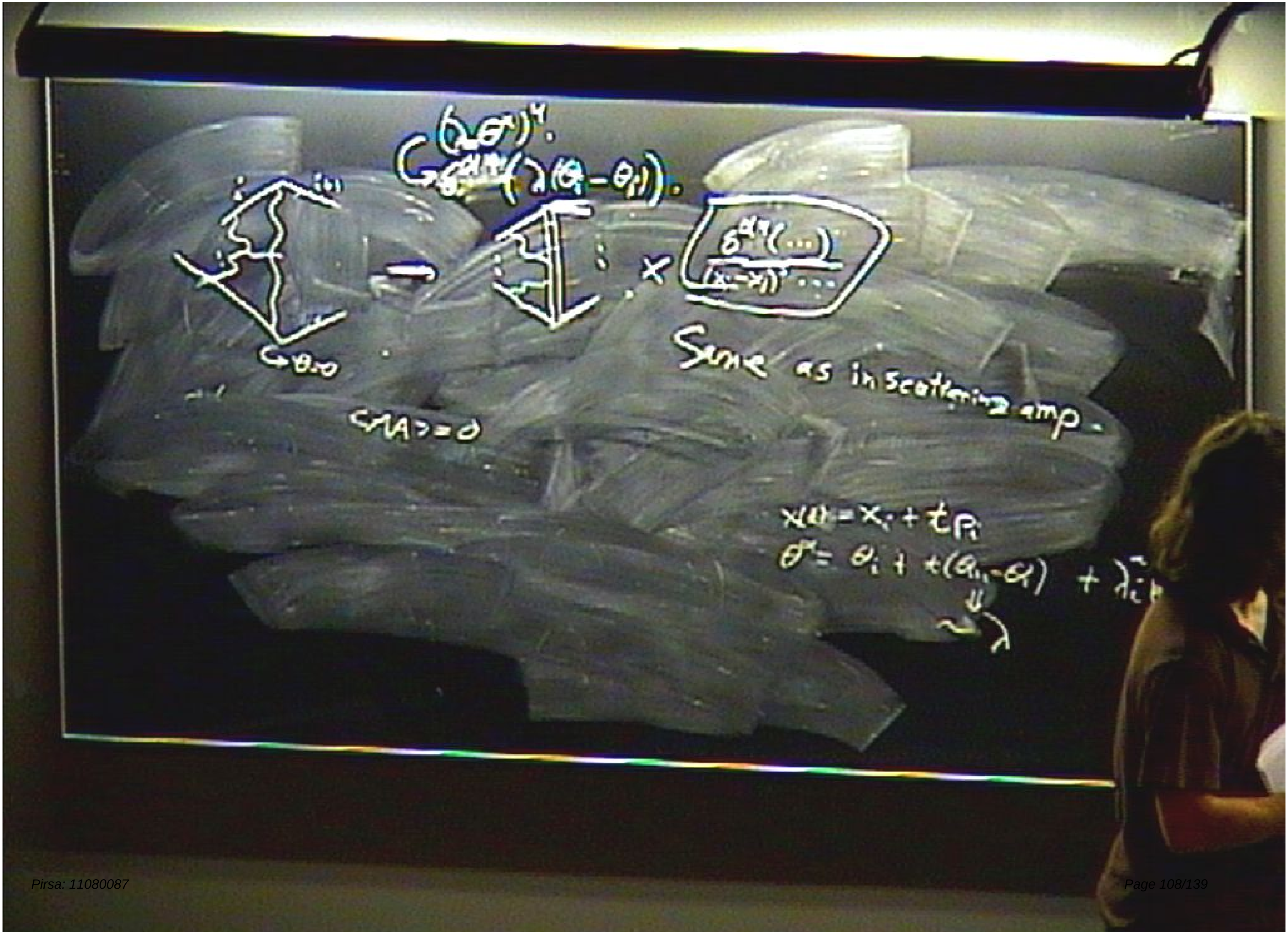
– $A_n(p,\eta) = \dfrac{\delta^4(\Sigma p)\,\delta^{0|8}(\Sigma\lambda\eta)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}\, W_n\!\left(x, \dfrac{\theta}{a^{1/2}}\right), \quad a = \dfrac{g^2 N}{4\pi^2}$.

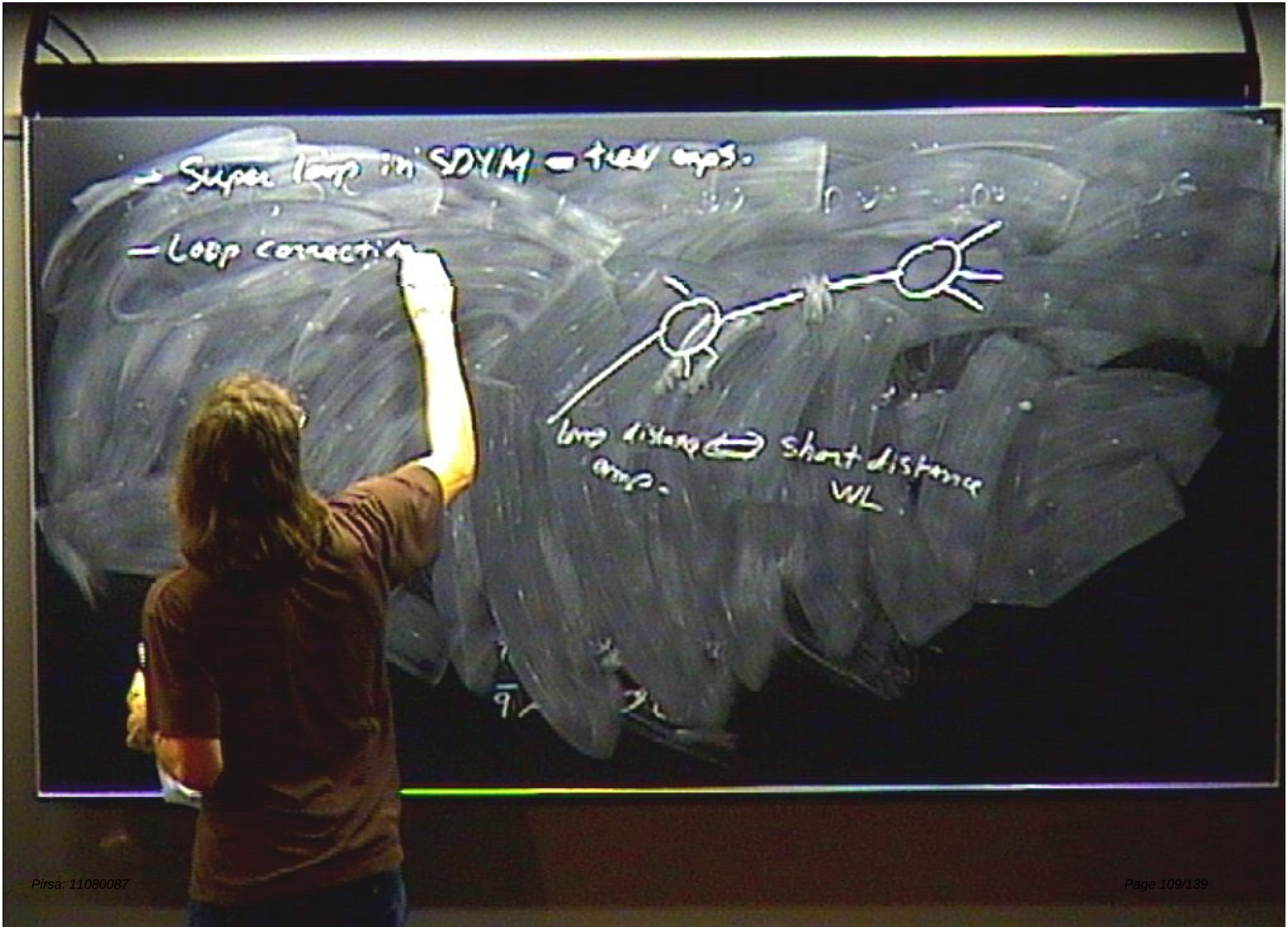
– $\left\{ q_{kA} + a^{1/2}\dfrac{\partial}{\partial\theta^A} \right\} W_n\!\left(x, \dfrac{\theta}{a^{1/2}}\right) = 0$

– Trees are dual to SDYM WL $\Rightarrow$ $\bar{q}=0 \Leftrightarrow$ Yangian Symmetry

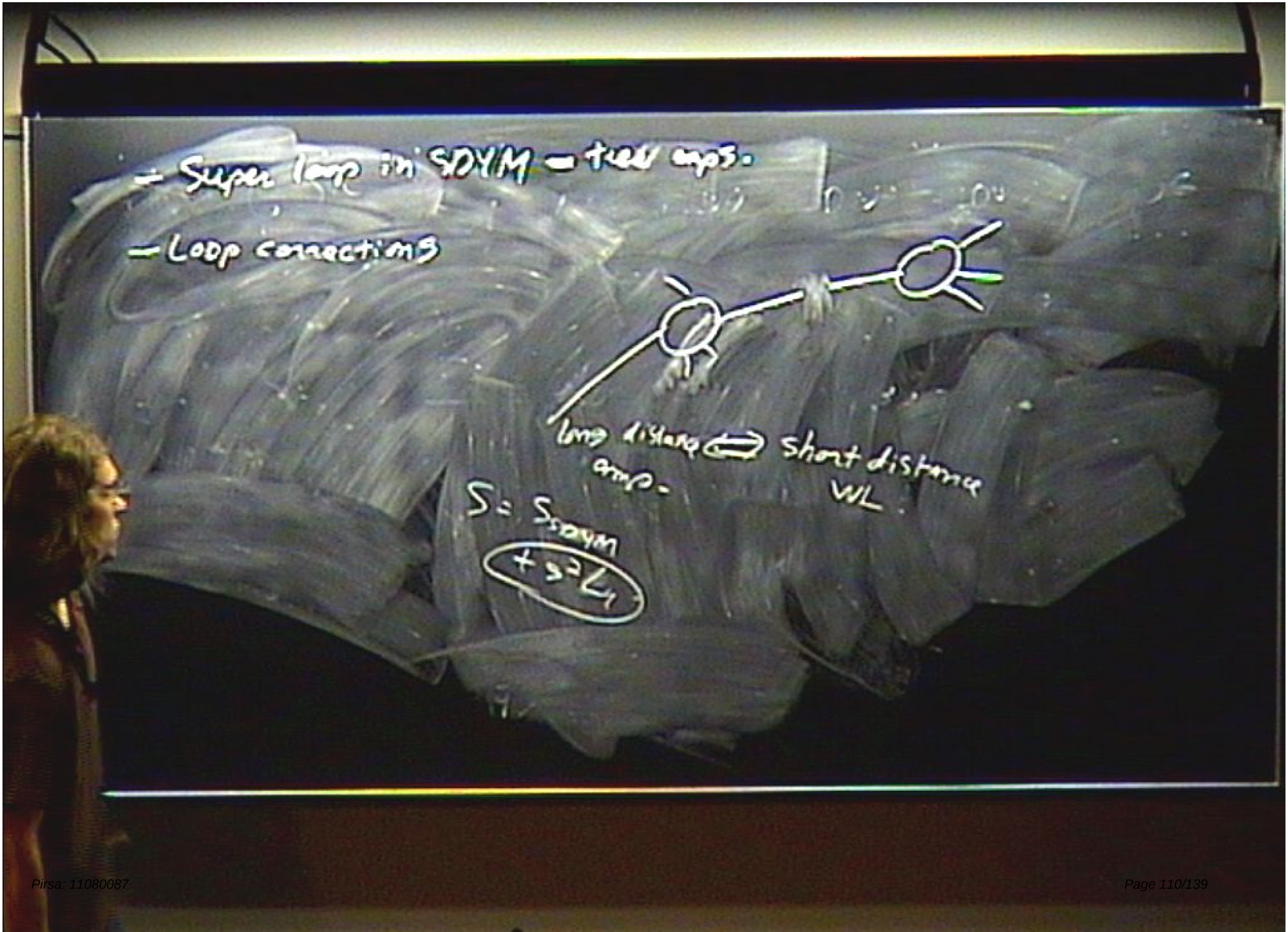
– $\left.\begin{array}{l}(x_i - x_{i+1})^2 = 0 \\ (x_i - x_{i+1})(\theta_i - \theta_{i+1}) = 0\end{array}\right\}$ preserved under factorization.

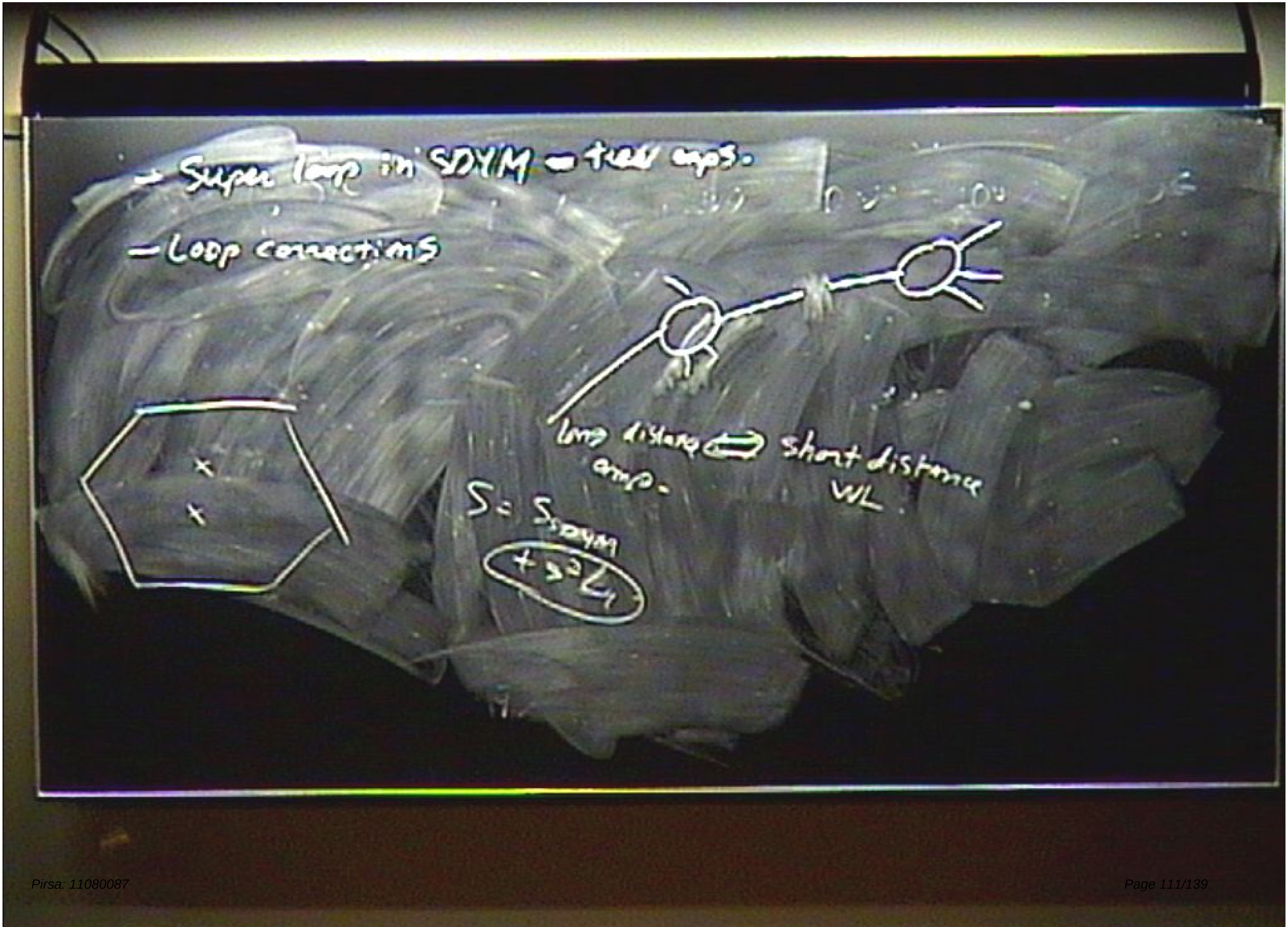

Super loop in SDYM = tree amps.
long distance ⟺ short distance
amps.
WL
aux. Field.


Same as in scattering amp.
<MA>=0
x(t) = x_i + t p_i



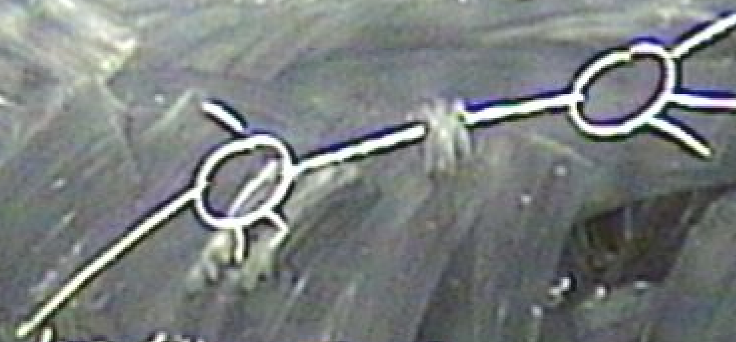
Super loop in SDYM = tree amps.
Loop connection
long distance amp.
short distance WL


- Super loop in SDYM = tree amps.
- Loop corrections
long distance ⟺ short distance
amp.
WL
S = S_SDYM
+ g² L₁


→ Super loop in SDYM = tree amps.
– Loop corrections
long distance ⟺ short distance
amp.
WL
S = S_SDYM
+ s²L₁

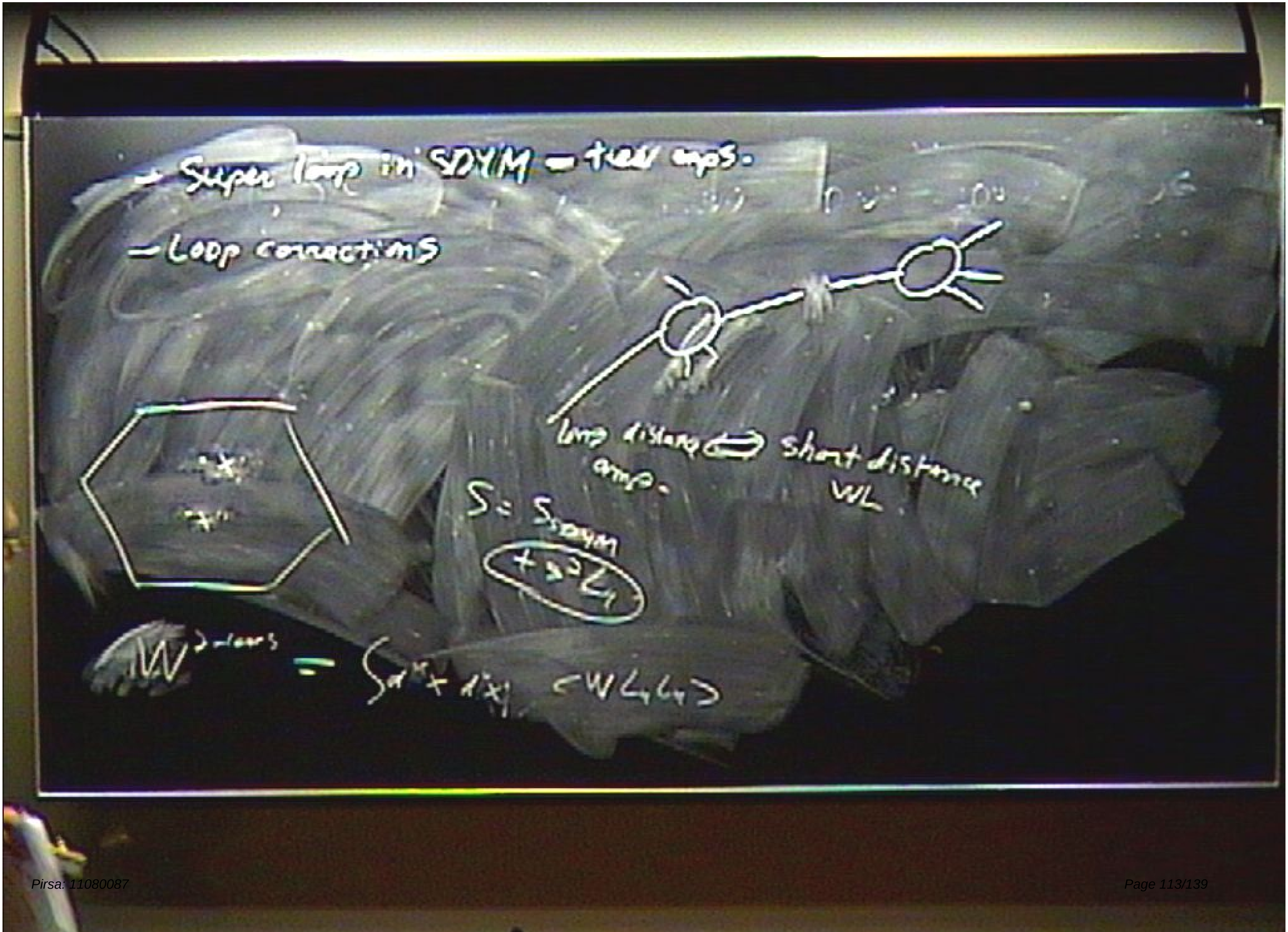
- Super loop in SDYM = tree amps.

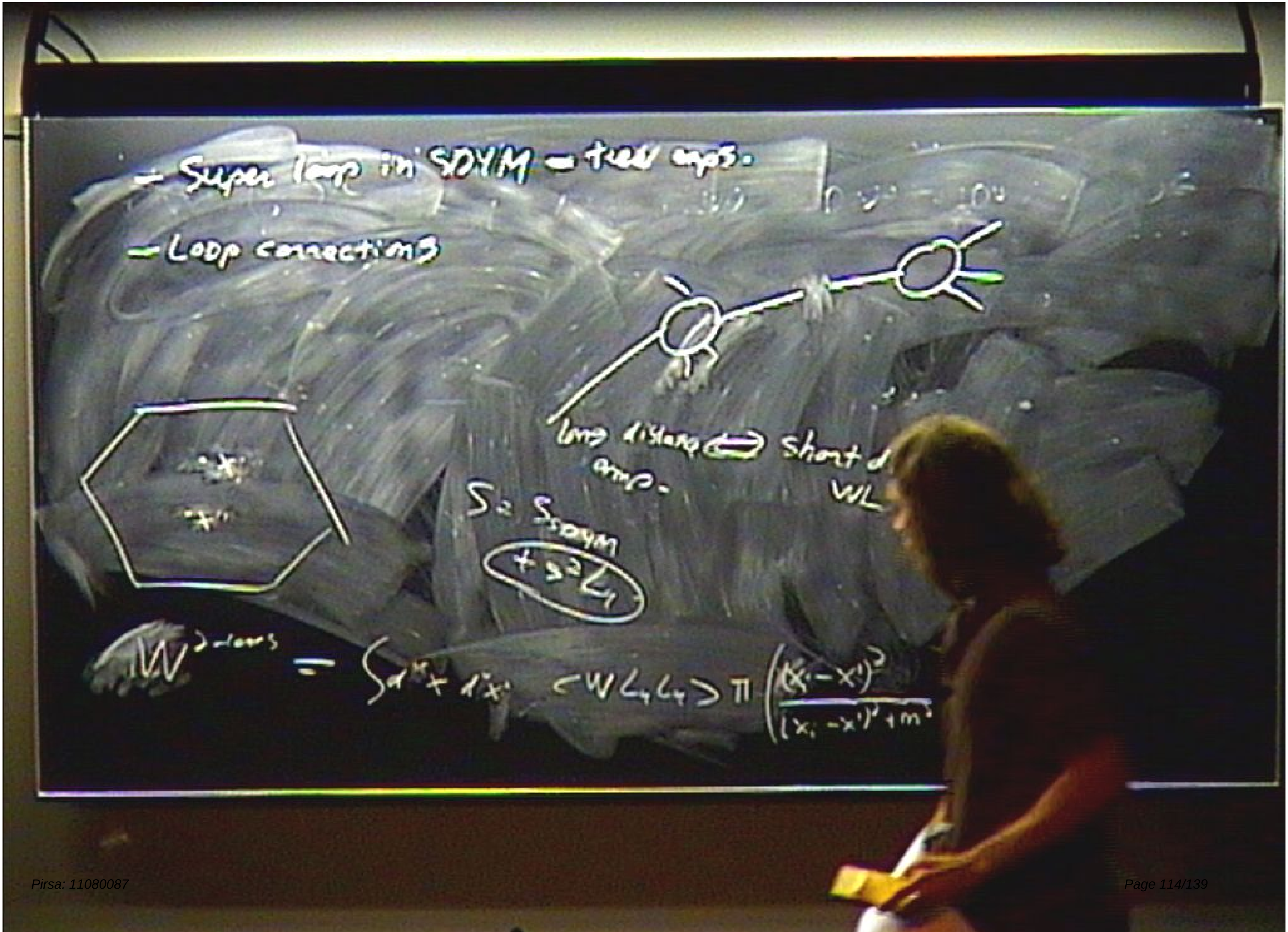
- Loop corrections



long distance $\Longleftrightarrow$ short distance
amp. WL

$S = S_{SDYM} + g^2 L_1$


- Super loop in SDYM – tree amps.
- Loop corrections
long distance ⟺ short distance
amp.
WL
S = S_SDYM
+ g² L₁
W^2-loops = ∫ d⁴x d⁴y ⟨W L₁ L₁⟩


- Super loop in SDYM – tree amps.
- Loop connections
long distance ⟺ Short d
amp.
WL
S = S_SDYM + g²L_I
W^2-loops = ∫d⁴x d⁴x' <W L_I L_I> ∏ (x_i − x')² / ((x_i − x')² + m²)

– Super loop in SDYM – tree amps.

– Loop corrections

long distance $\Longleftrightarrow$ short distance
amp. WL

$S = S_{SDYM} + g^2 L_1$

$$W^{2\text{-loops}} = \int d^4x \, d^4x' \, \langle W L_1 L_1 \rangle \prod \left(\frac{(x_i - x')^2}{(x_i - x')^2 + m^2} \right)$$

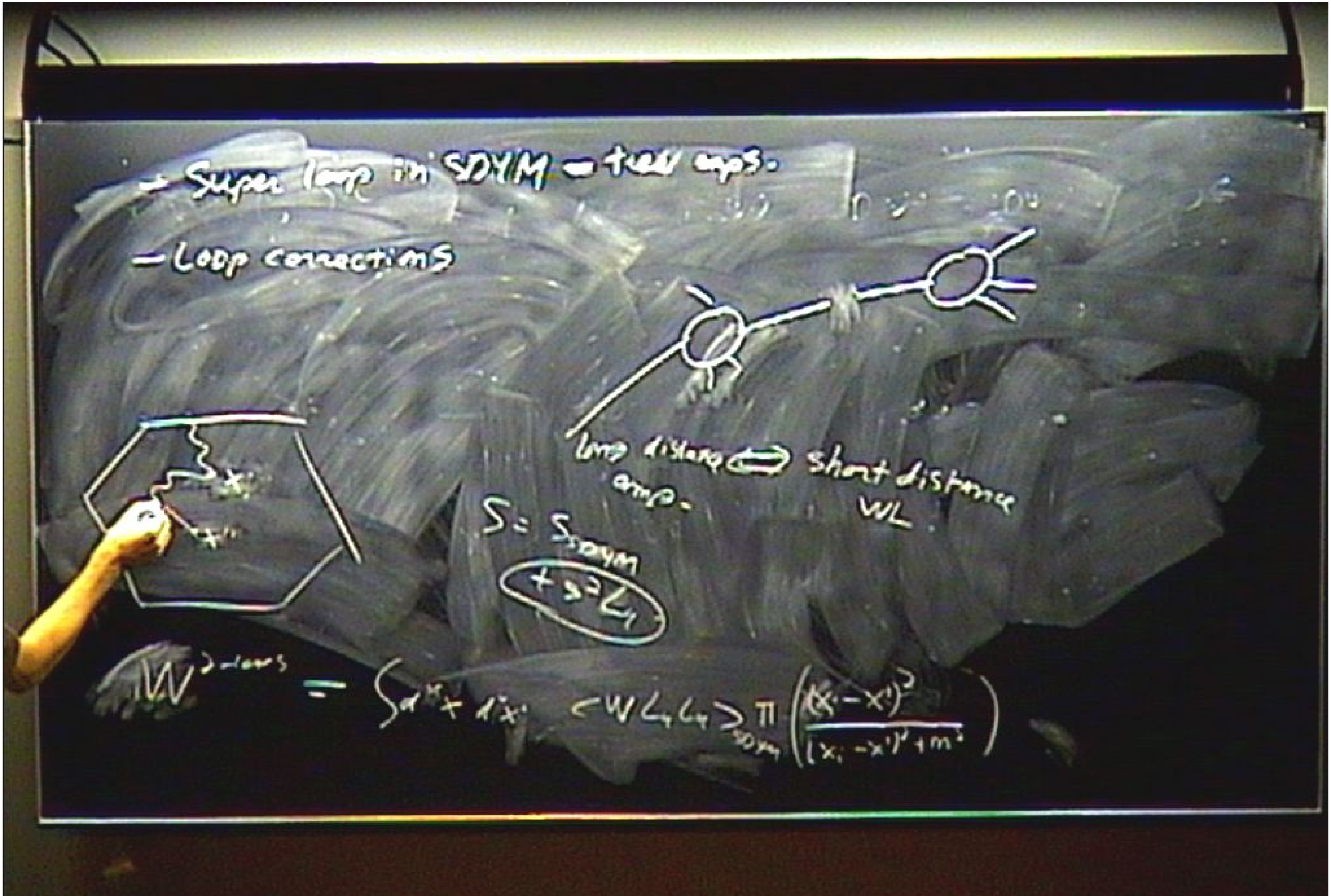
– Super loop in SDYM = tree amps.

– Loop corrections

long distance amp. $\Longleftrightarrow$ short distance WL

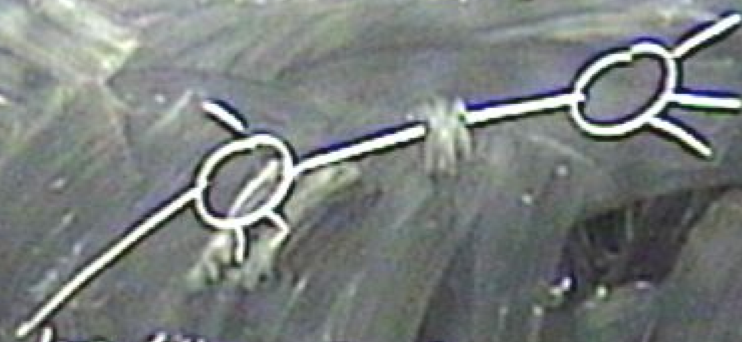
$$S = S_{SDYM} + g^2 L_1$$

$$W^{2\text{-loops}} = \int d^4x\, d^4x' \, \langle W L_1 L_1 \rangle_{SDYM} \prod \left(\frac{(x_i - x')^2}{(x_i - x')^2 + m^2} \right)$$


- Super loop in SDYM = tree amps.
- Loop corrections
long distance ⟺ short distance
amp.
WL
S = S_SDYM
+ g² L_I
W^2-loops
= ∫ d⁴x d⁴x' <W L_I L_I>_SDYM Π

– Super loop in SDYM = tree amps.

– Loop corrections



long distance $\Longleftrightarrow$ short distance
amp. WL

$$S = S_{SDYM} + g^2 L_1$$

$$W^{2\text{-loops}} = \int d^4x \, d^4x' \, \langle W L_1 L_1 \rangle_{SDYM} \prod \left(\frac{(x_i - x')^2}{(x_i - x')^2 + m^2} \right)$$

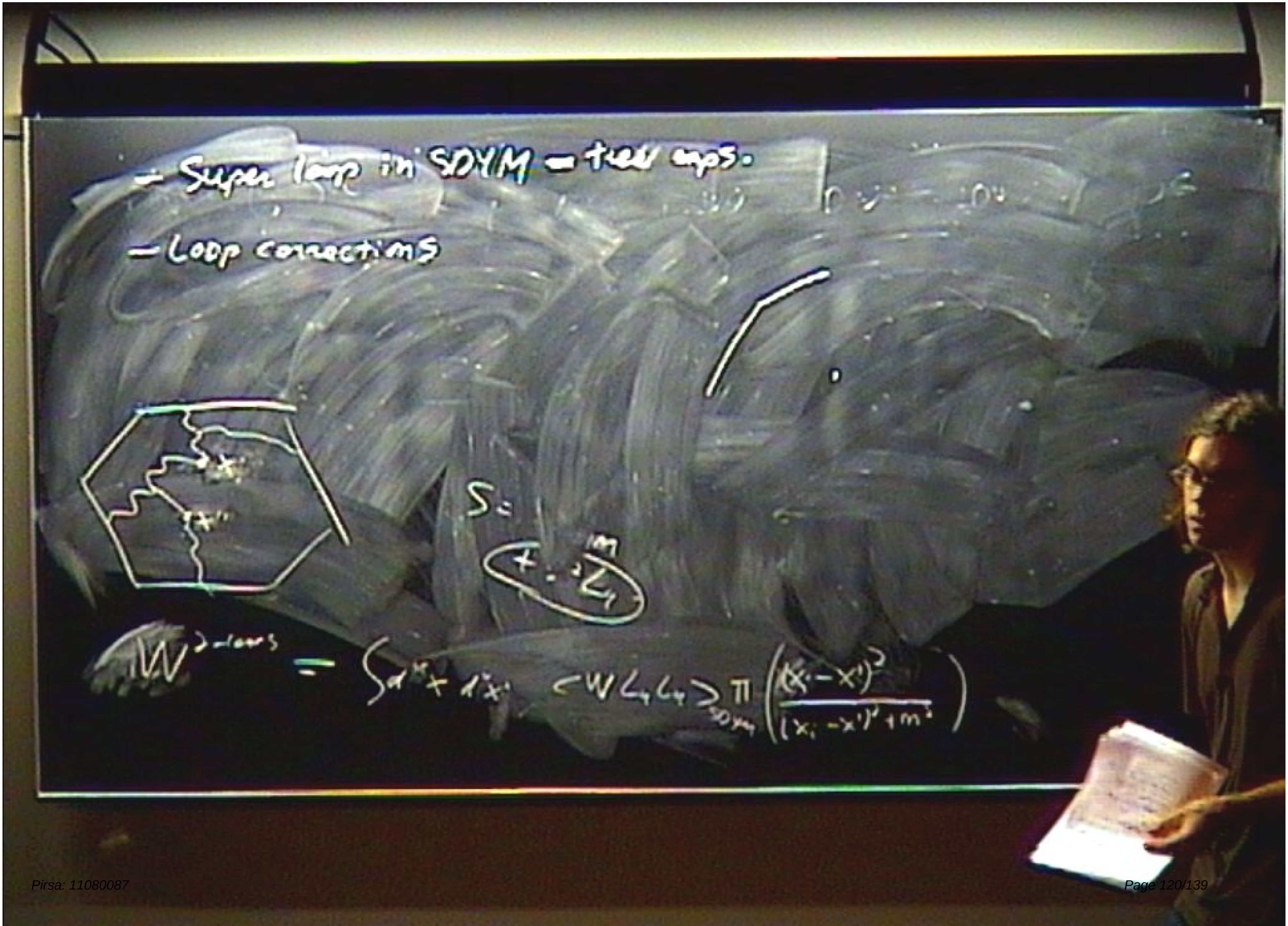
- Super loop in SDYM = tree amps.

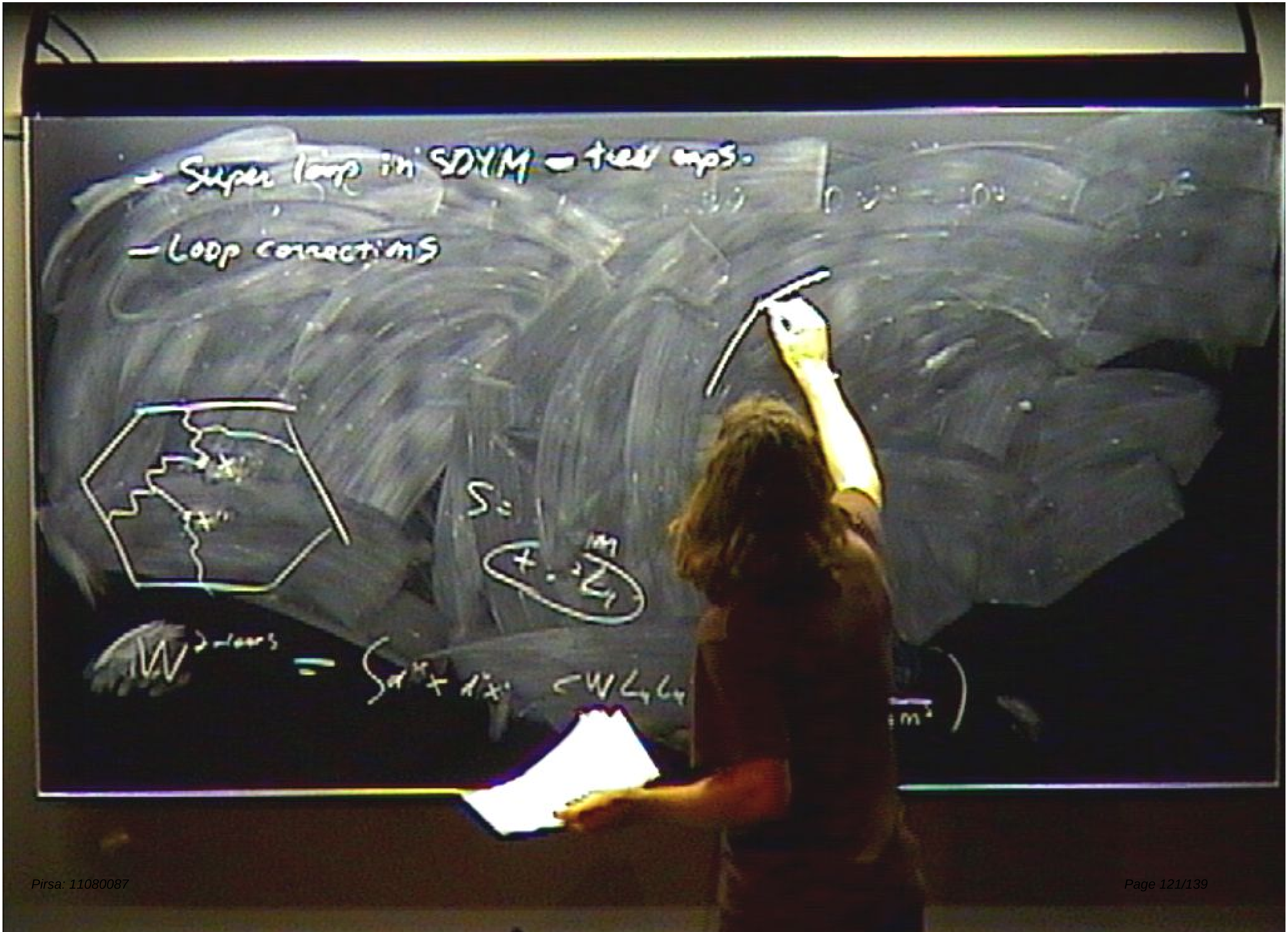
- Loop corrections

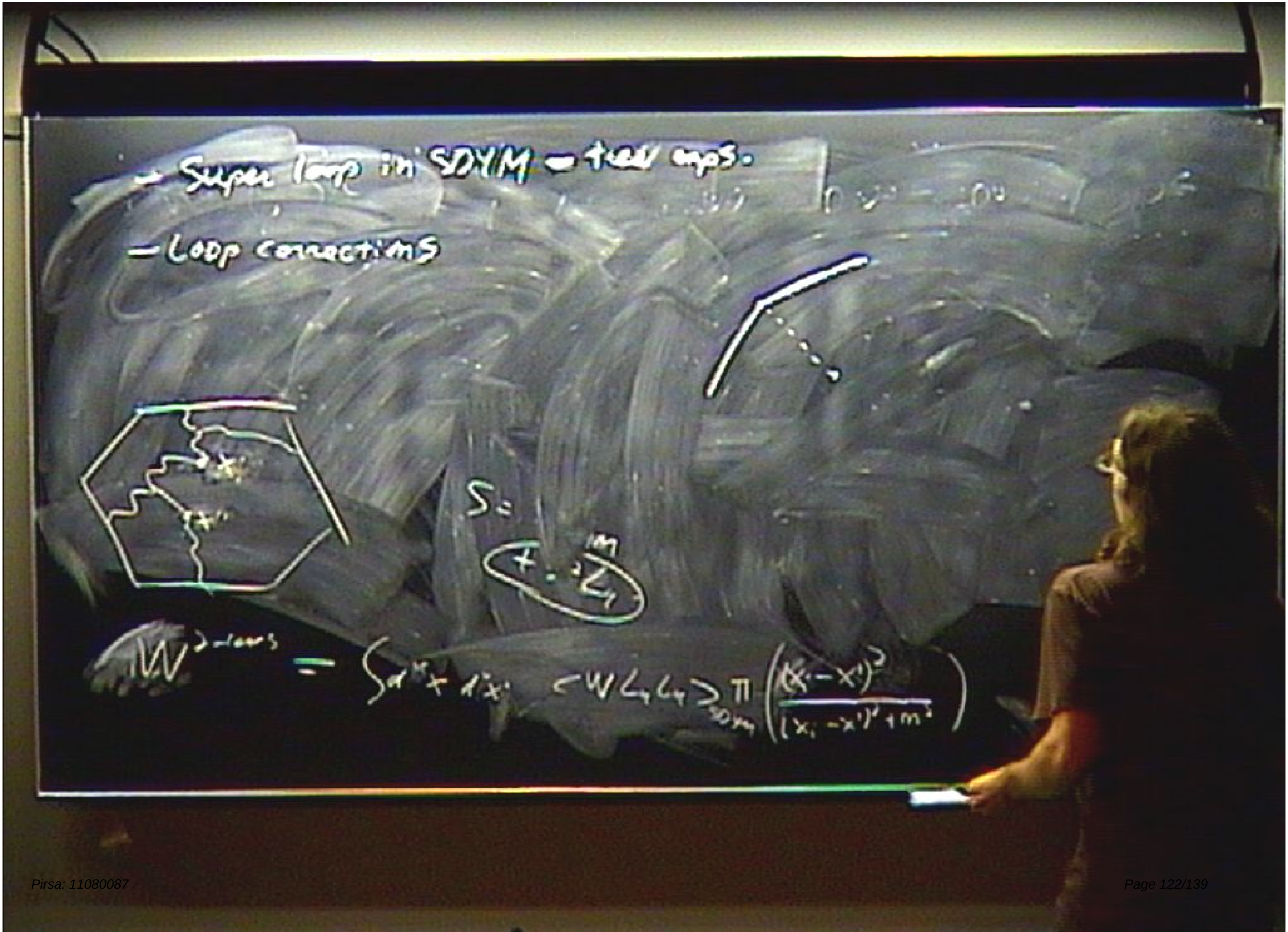
loop distance $\Longleftrightarrow$ short distance
amp. WL

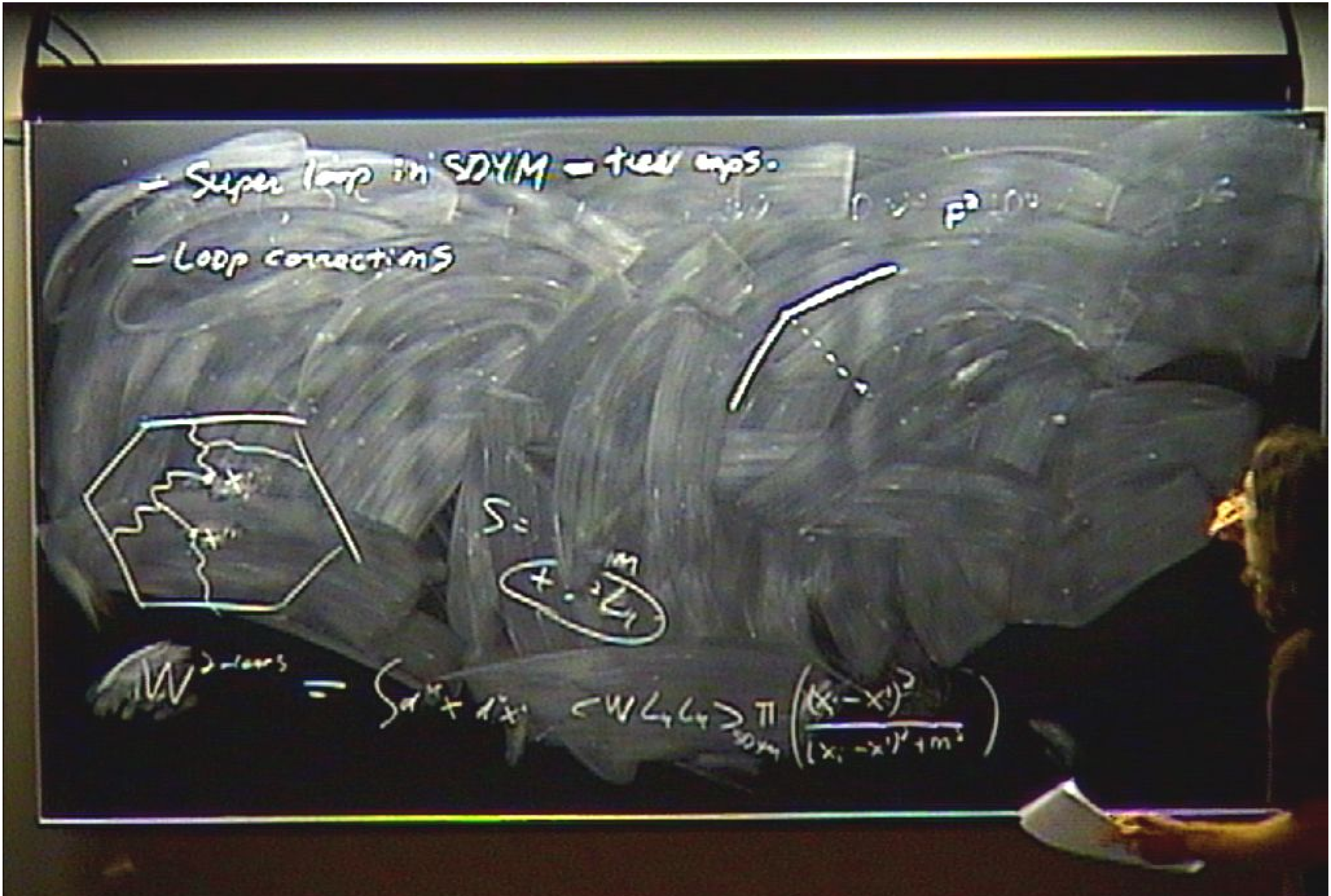
$$S = S_{SDYM} + g^2 L_4$$

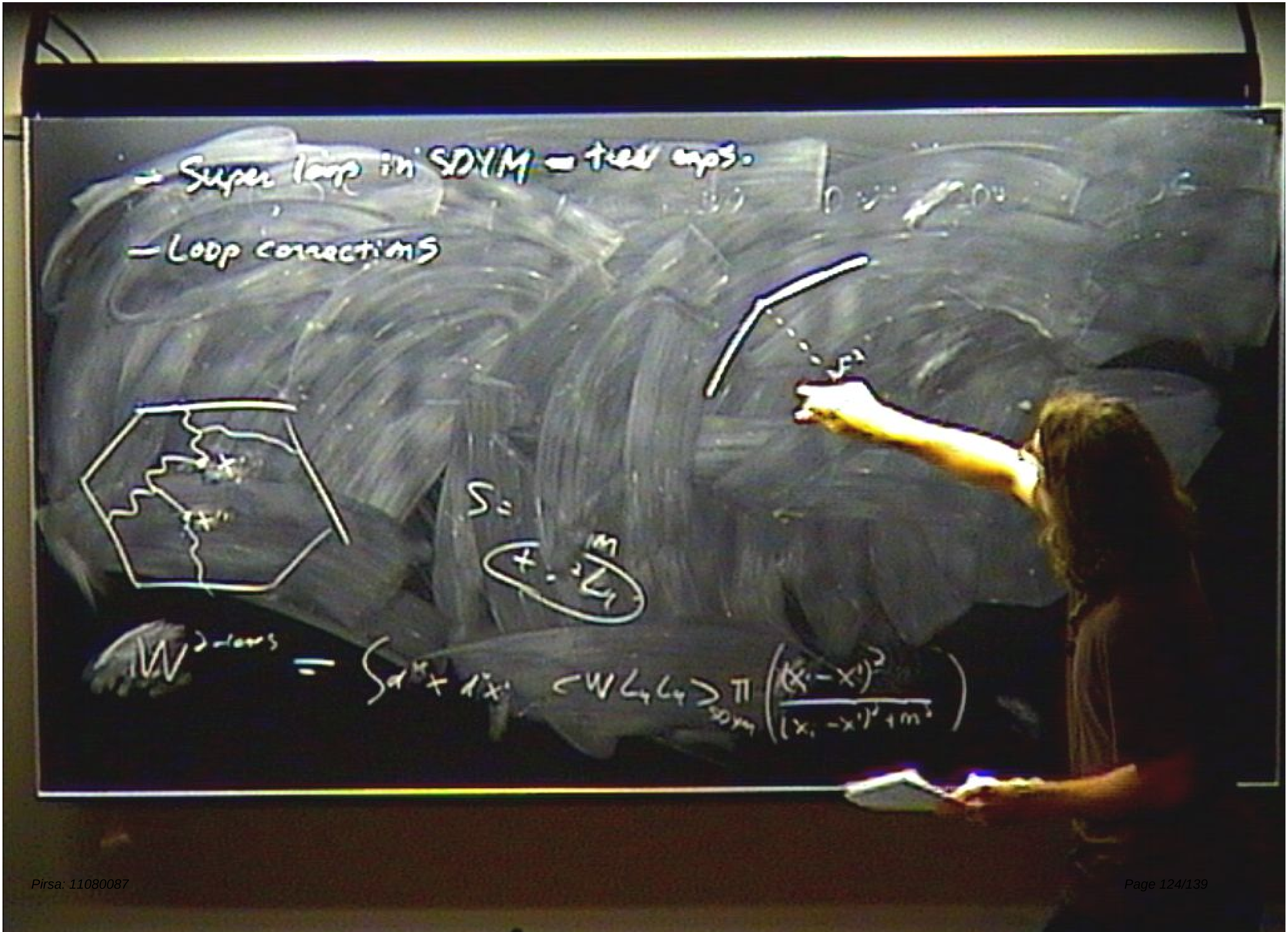
$$W^{2\text{-loops}} = \int d^4x\, d^4x' \, \langle W L_4 L_4 \rangle_{SDYM} \prod \left(\frac{(x_i - x')^2}{(x_i - x')^2 + m^2} \right)$$


- Super loop in SDYM = tree amps.
- Loop corrections
S:
W^2-loops = ∫d^4x d^4x' <W L_4 L_4>_SDYM ∏ ((x_i - x')^2 / ((x_i - x')^2 + m^2))


- Super loop in SDYM - tree amps.
- Loop corrections


- Super loop in SDYM
- Loop connections
S =
m


- Super loop in SDYM = tree amps.
- Loop corrections
S=
W 2-loops = ∫ d4x d4x' <W L4 L4>SDYM Π ((xi - x')2 / (xi - x')2 + m2)



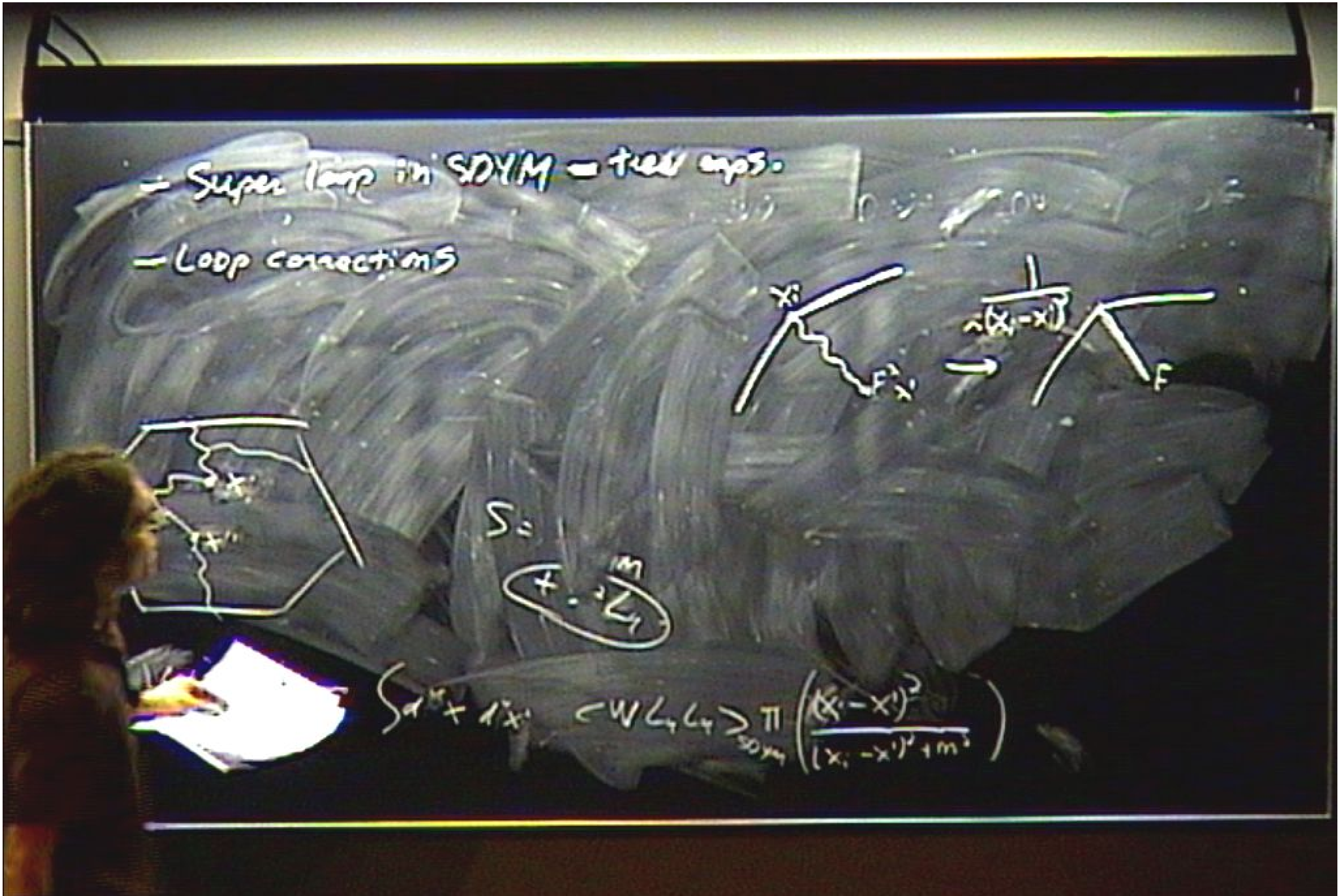
– Super loop in SDYM – tree amps.
– Loop corrections

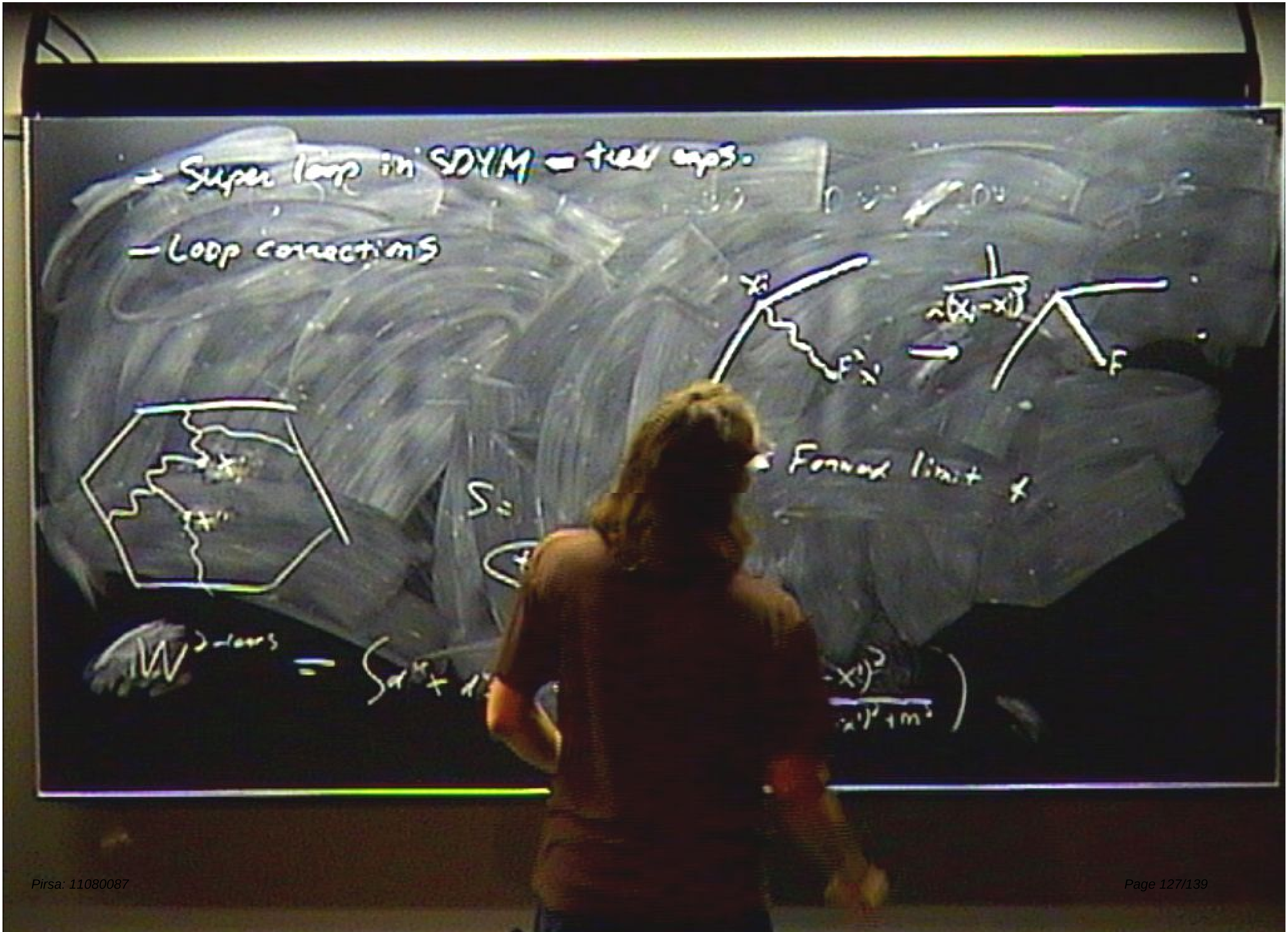
- Super loop in SDYM = tree amps.

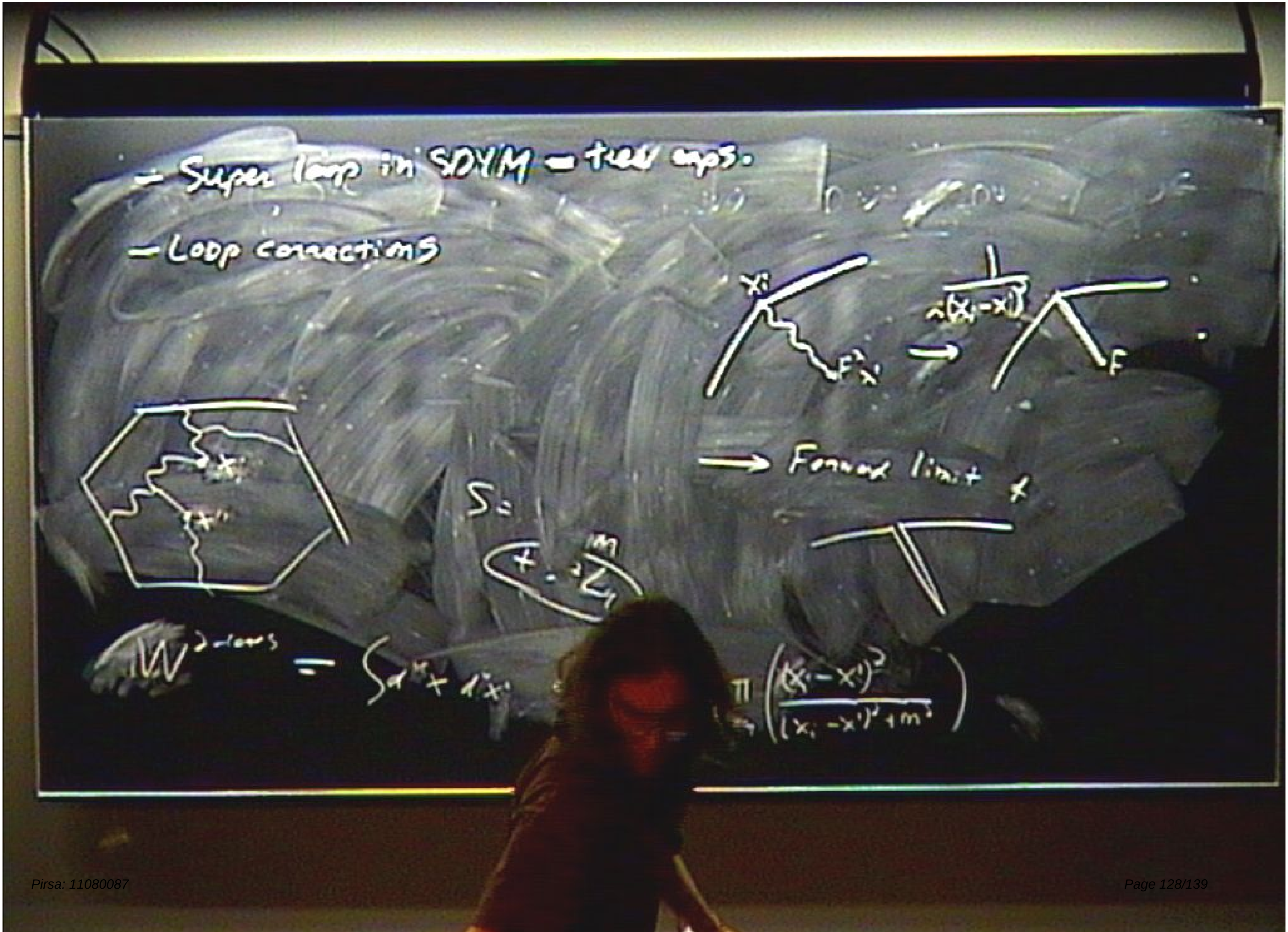
- Loop corrections

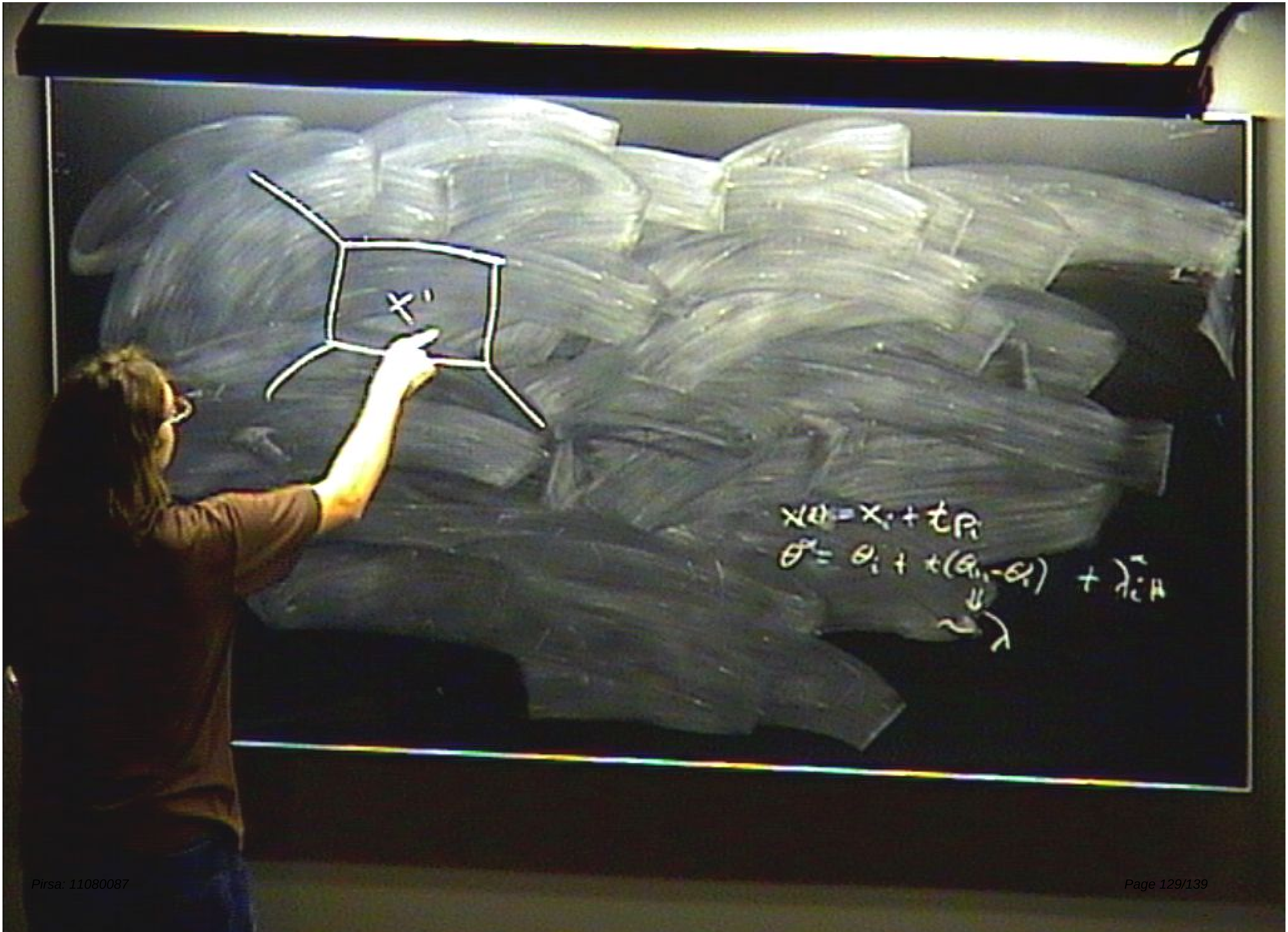
$S = \quad x' \cdot \mathcal{L}_1 \quad m$

$$W^{2\text{-loops}} = \int d^4x' \, d^4x'' \, \langle W \mathcal{L}_1 \mathcal{L}_1 \rangle_{SDYM} \prod \left(\frac{(x_i - x')^2}{(x_i - x')^2 + m^2} \right)$$


- Super loop in SDYM = tree amps.
- Loop corrections
x'
x''
S =
X · ∂L_1
m
∫ d^{4m}x d^4x' ⟨W L_1 L_1⟩_SDYM Π ((x_i − x')^2 / ((x_i − x')^2 + m^2))


- Super loop in SDYM
- Loop corrections
Forward limit


- Super loop in SDYM = tree amps.
- Loop corrections
Forward limit +


x'
x(t) = x_i + t p_i
θ^a = θ_i + t(θ_{i+1} − θ_i) + λ_i^a
λ

– Super loop in SDYM = tree amps.

– Loop corrections

$\longrightarrow$ Forward limit x

$S = \int d^4x \, d^4\theta \ \mathcal{L}_1$

$$W^{2\text{-loops}} = \int d^4x \, d^4x' \, \langle W \mathcal{L}_1 \mathcal{L}_1 \rangle_{SDYM} \prod \left(\frac{(x_i - x')^2}{(x_i - x')^2 + m^2} \right)$$

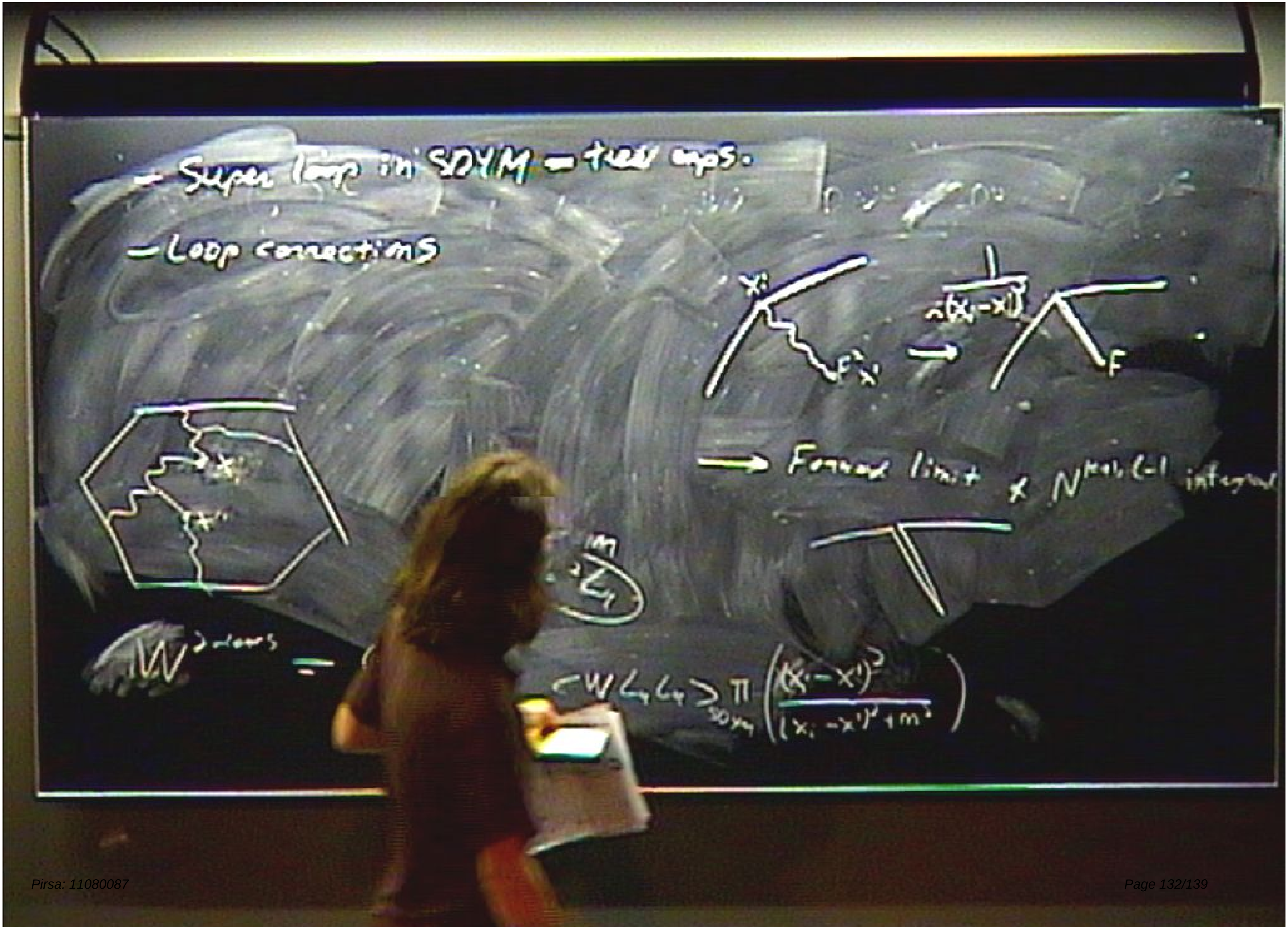
— Super loop in SDYM = tree amps.

— Loop corrections

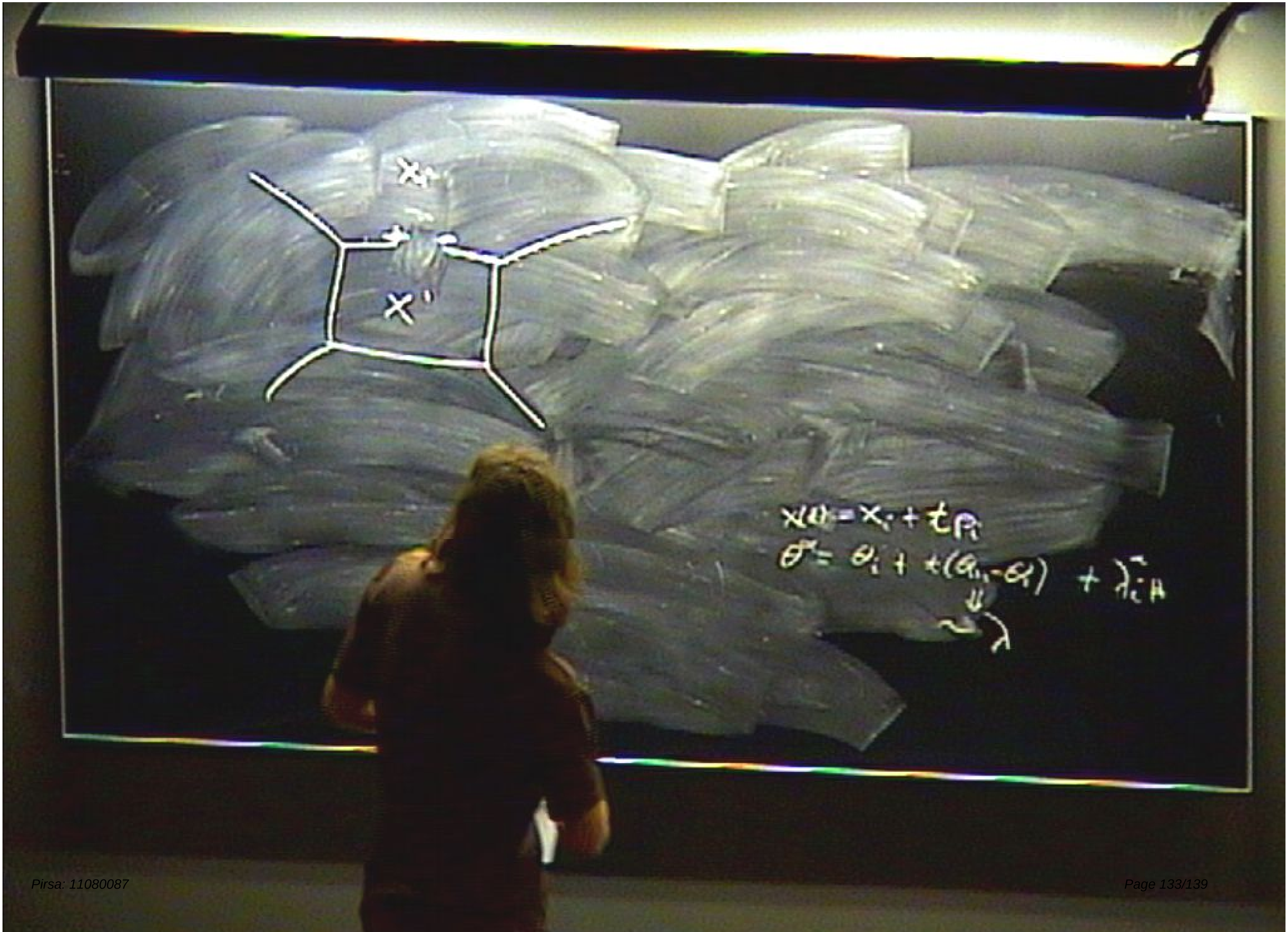
→ Forward limit ×

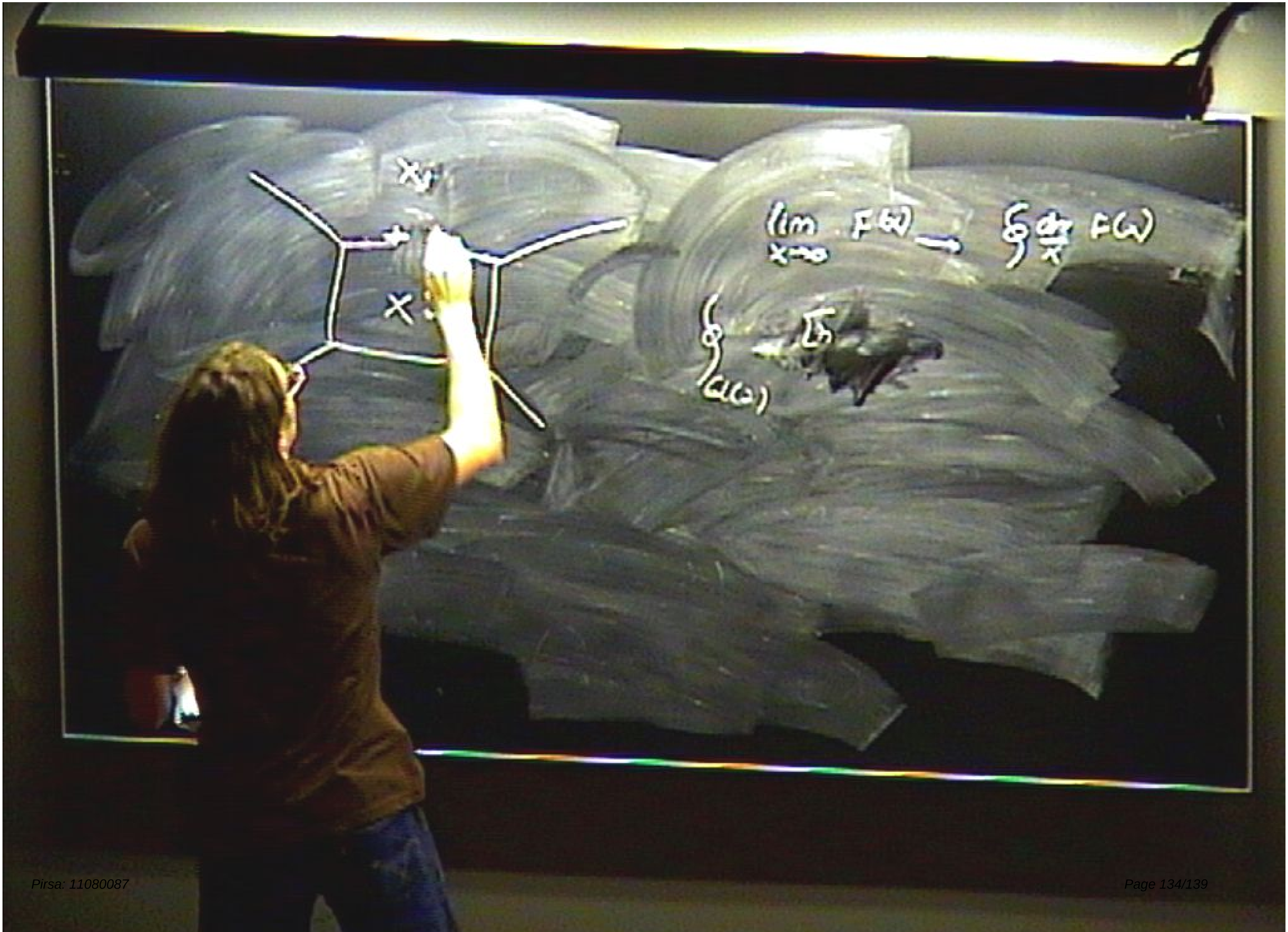
$S = \dots + \dots \int \mathcal{L}_1$ (m)

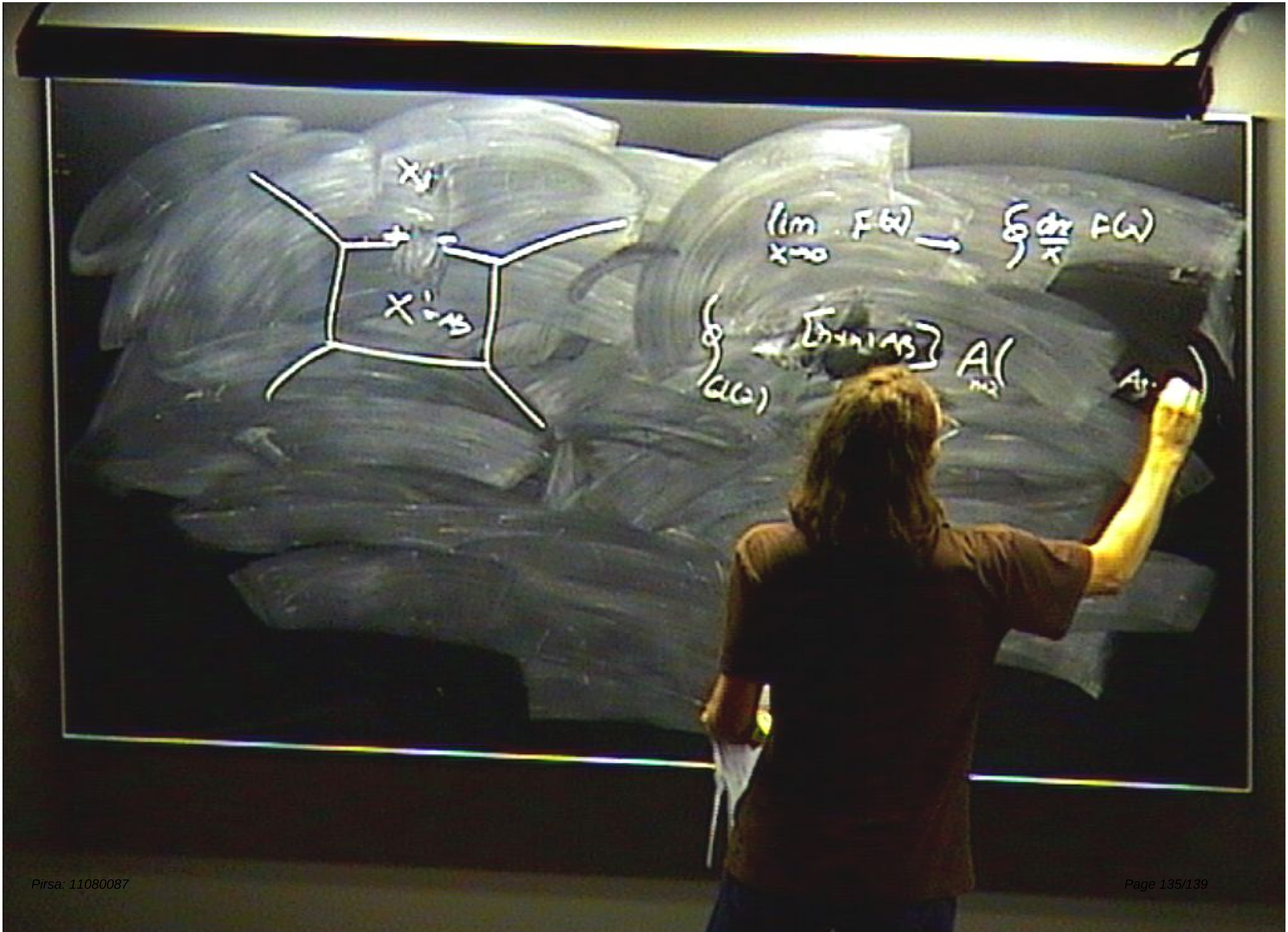
$$W^{2\text{-loops}} = \int d^4x\, d^4x' \,\langle W \mathcal{L}_1 \mathcal{L}_1 \rangle_{SDYM} \prod \left(\frac{(x_i - x')^2}{(x_i - x')^2 + m^2} \right)$$

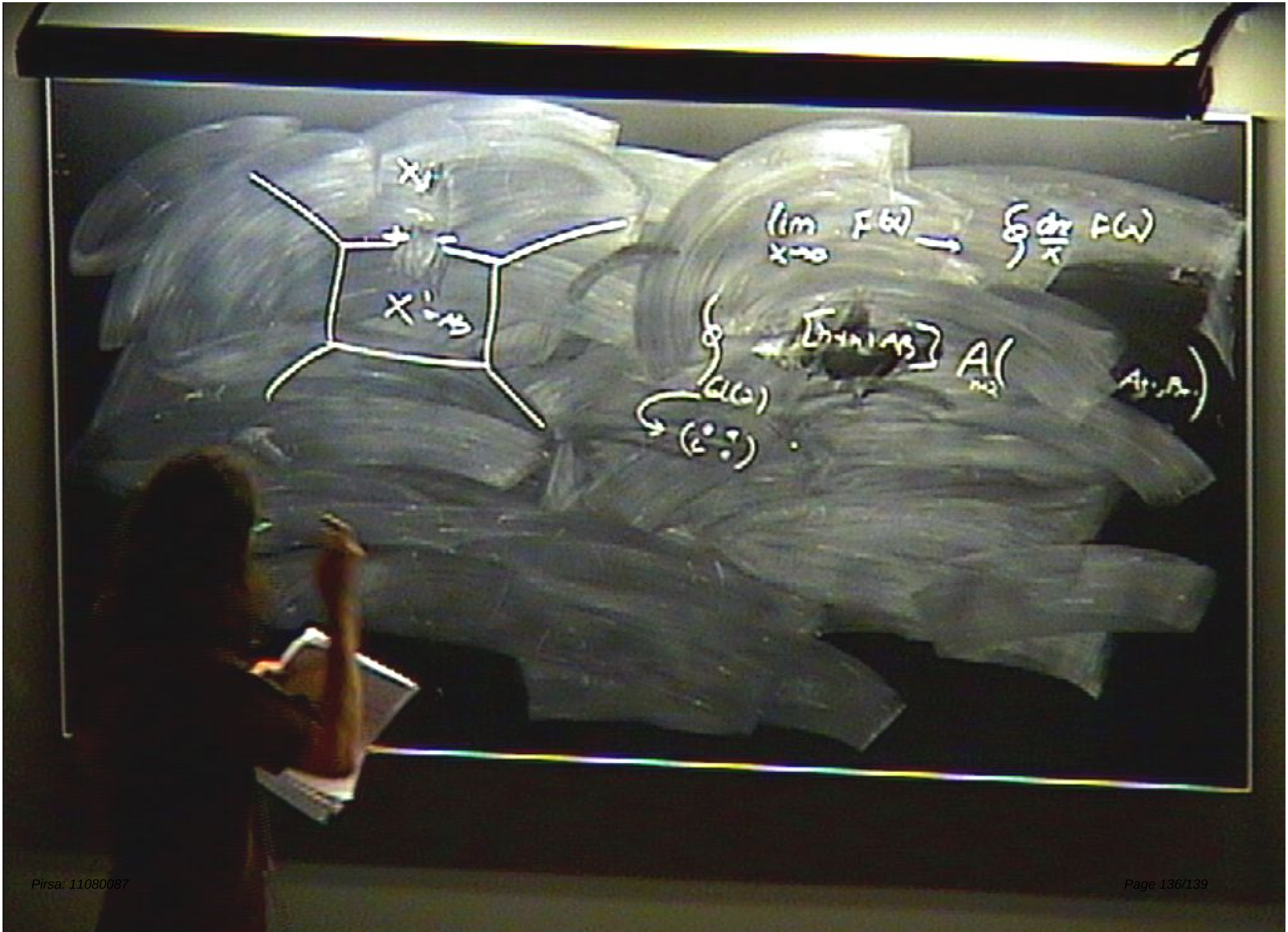


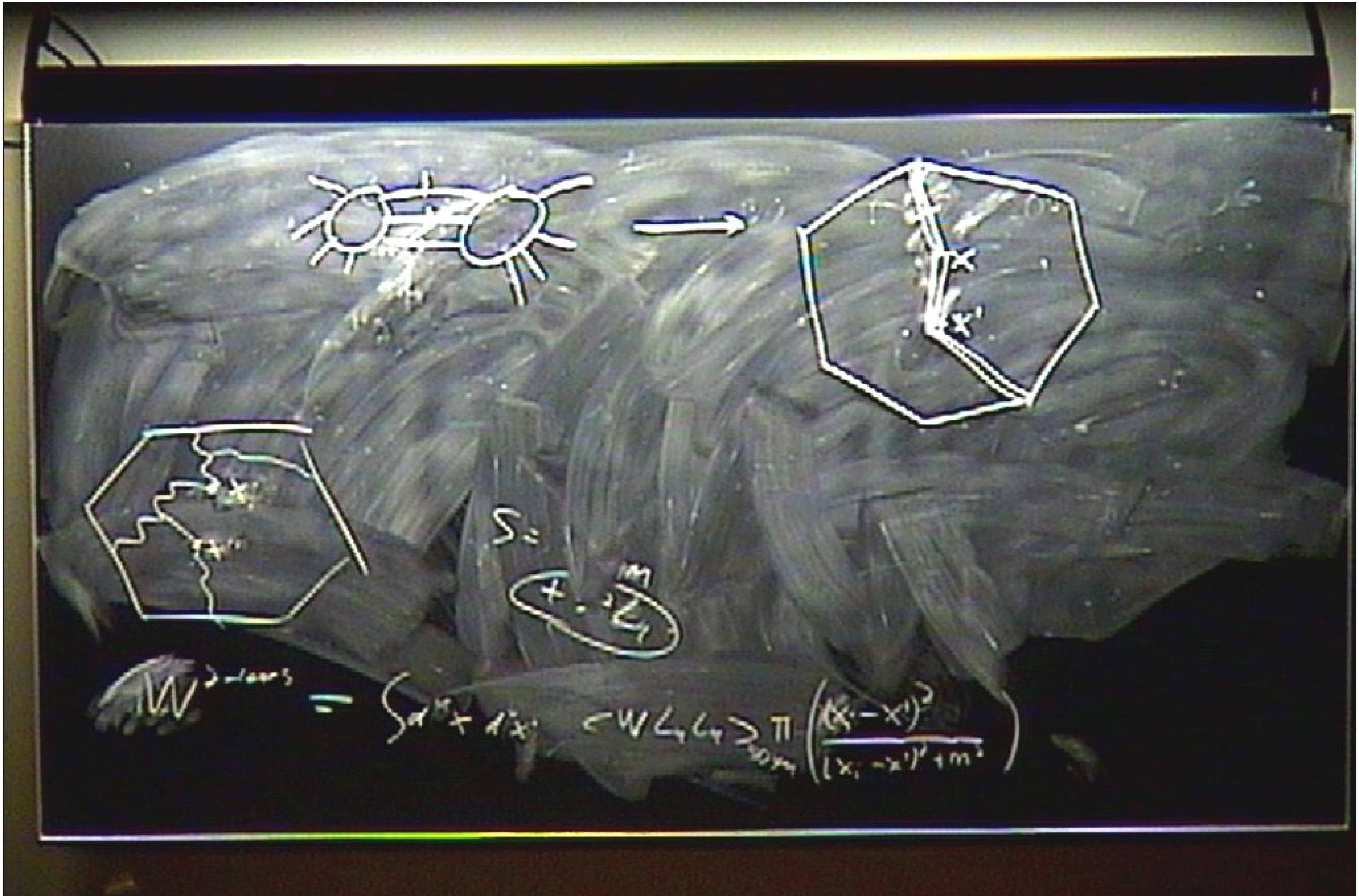
– Super loop in SDYM – tree amps.
– Loop corrections
Forward limit
integral
2-loops
$\langle W \mathcal{L}_4 \mathcal{L}_4 \rangle_{SDYM} \prod \left(\frac{(x_i - x')^2}{(x_i - x')^2 + m^2} \right)$

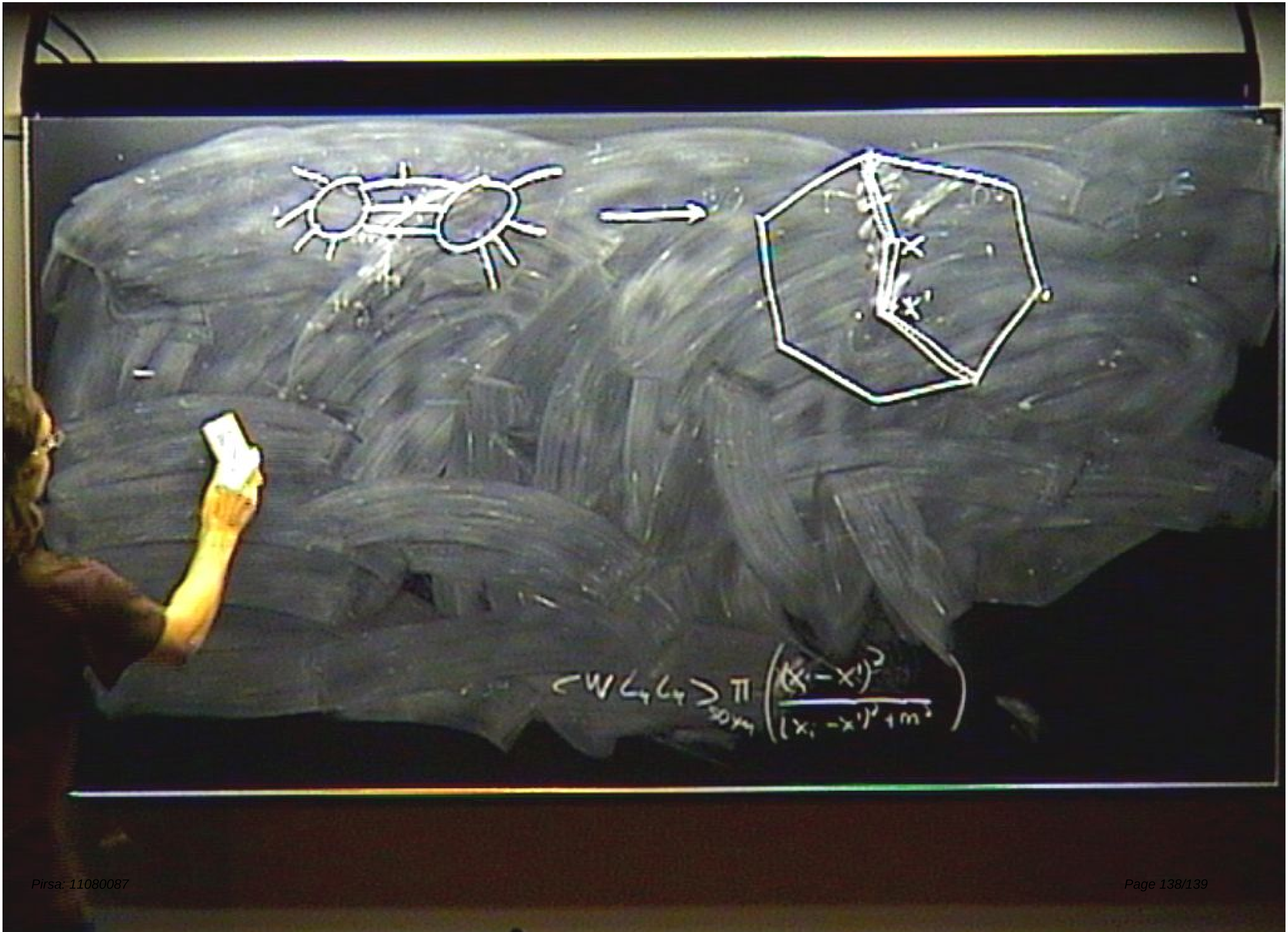

x'
x(t) = x_i + t p_i

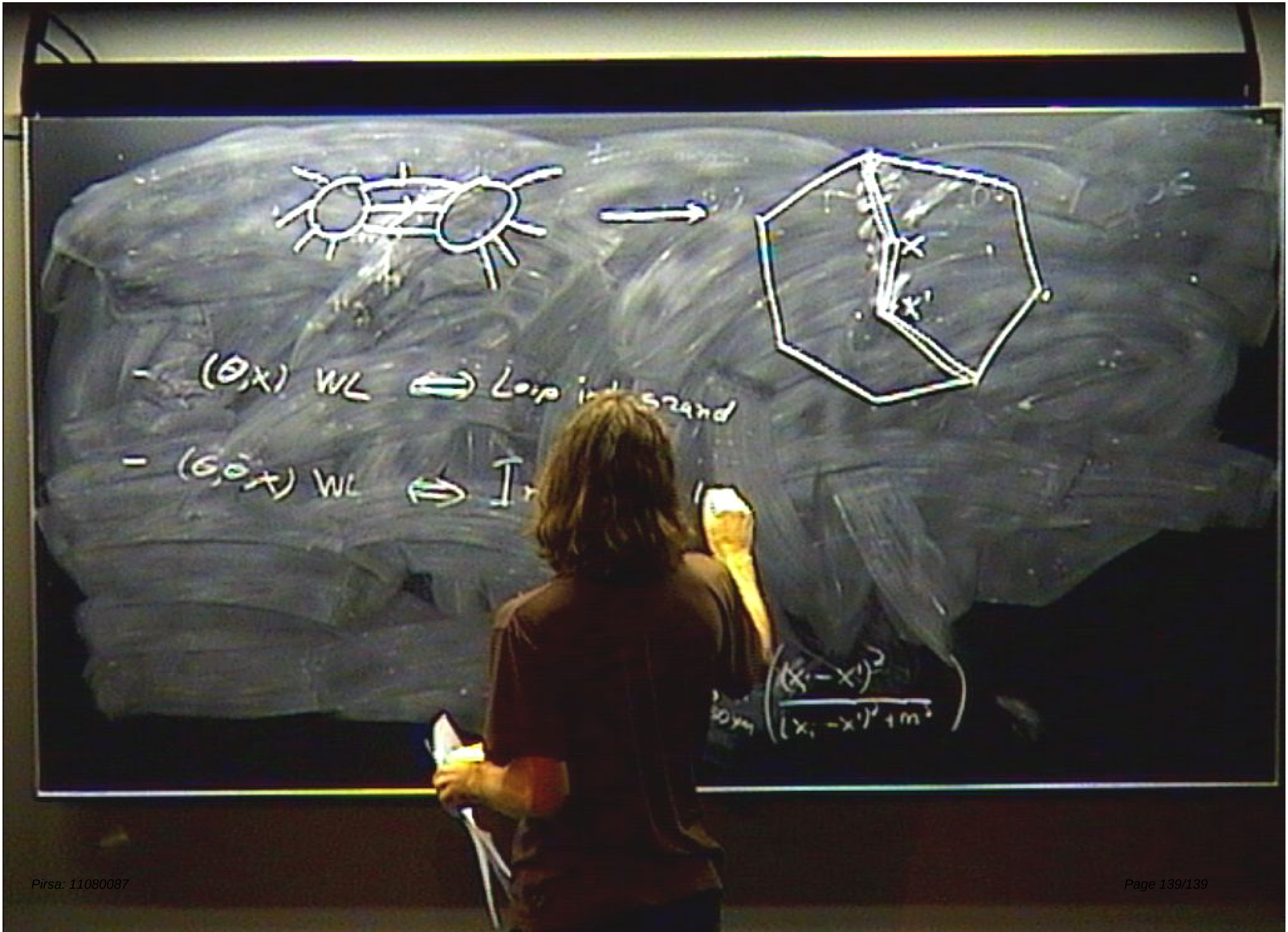

lim F(w)
F(w)








x
x'


x
x'
- (θ,x) WL ⟺ Loop in
strand
- (σ,σ̄,x) WL ⟺