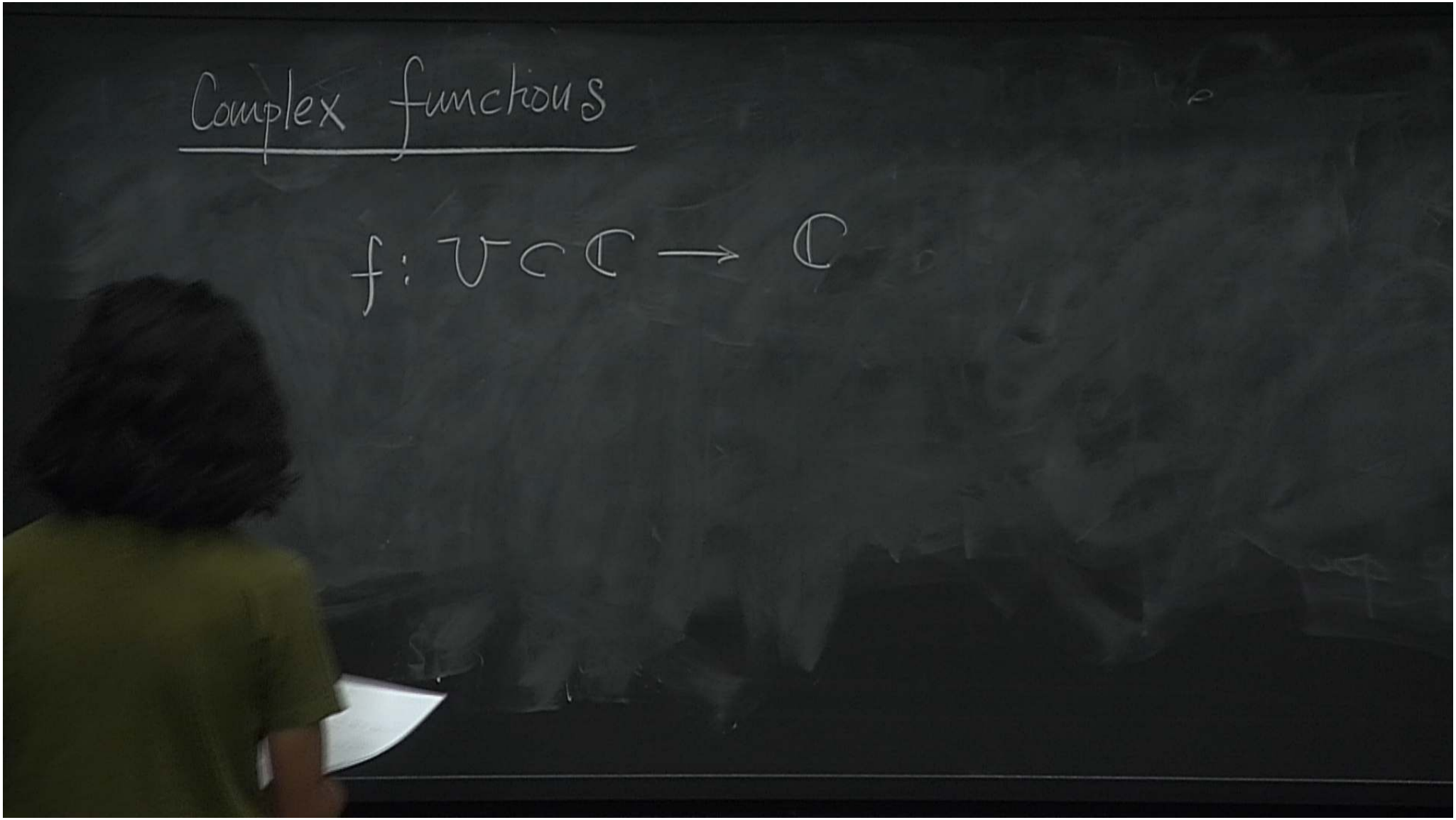
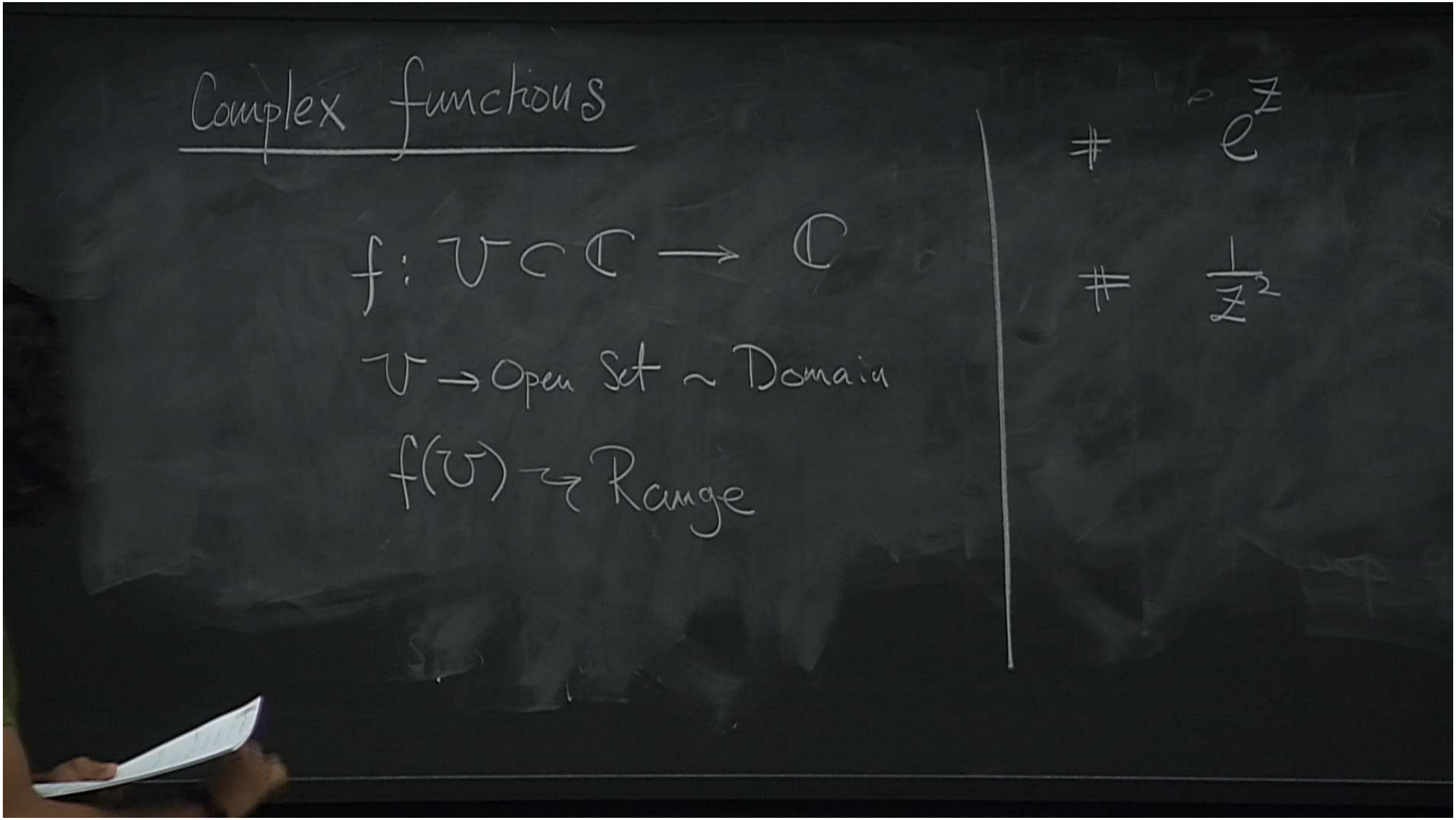
Title: Complex Analysis - Lecture 2a

Date: Aug 17, 2011 09:00 AM

URL: http://pirsa.pi.local/11080078

Abstract:

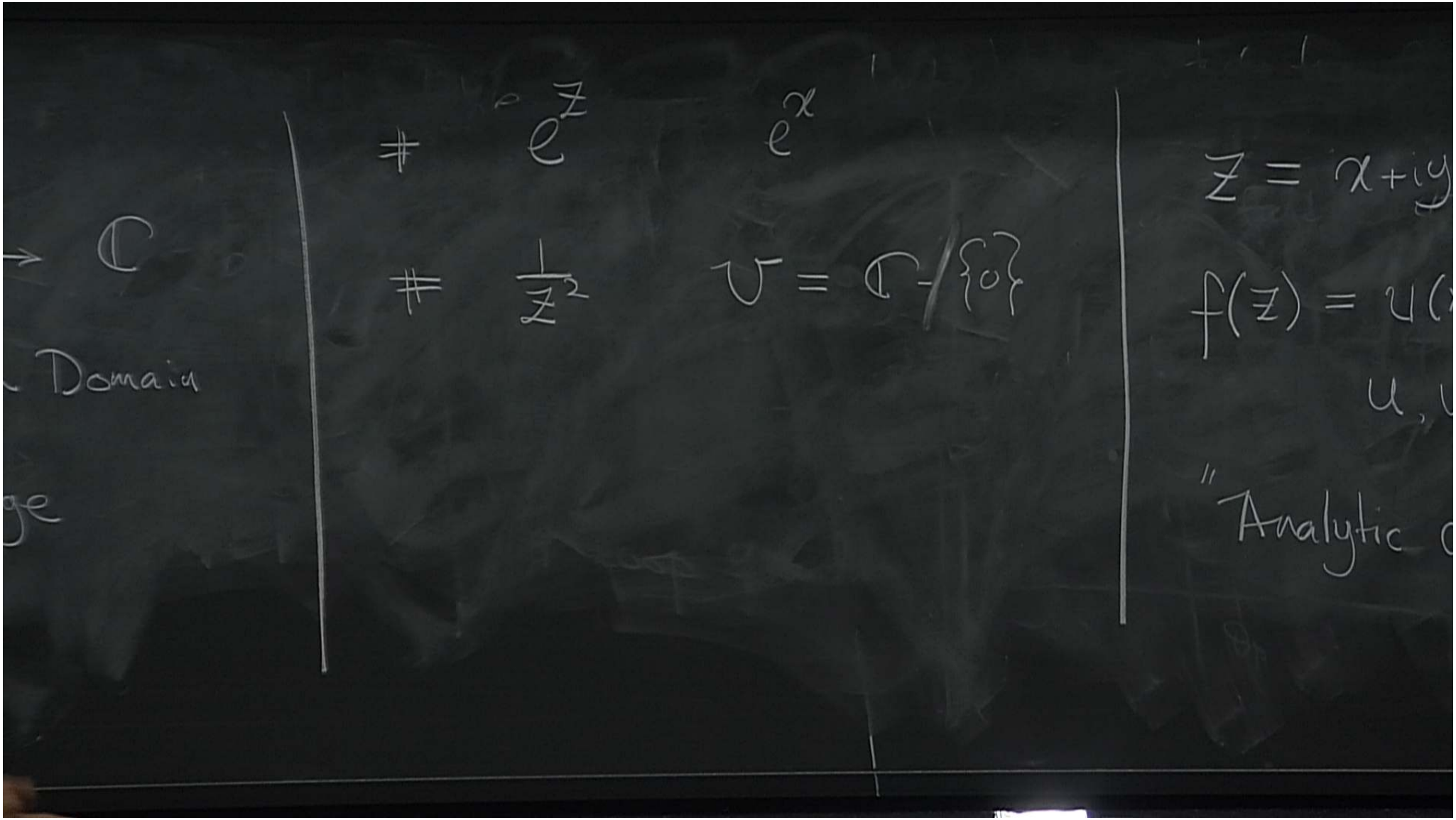

Complex functions
$f: V \subset \mathbb{C} \to \mathbb{C}$


Complex functions
$f: U \subset \mathbb{C} \to \mathbb{C}$
$U \to$ Open Set $\sim$ Domain
$f(U) \to$ Range
$\neq e^z$
$\neq \frac{1}{z^2}$

$z = x + iy$

$f(z) = u(x,y) + i\,v(x,y)$

$u, v \rightarrow$ real functions of $x \& y$...


→ ℂ
Domain
ge
e^z
$\frac{1}{z^2}$
e^x
$U = \mathbb{C} - \{0\}$
$z = x + iy$
$f(z) = u($
$u,$
"Analytic

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

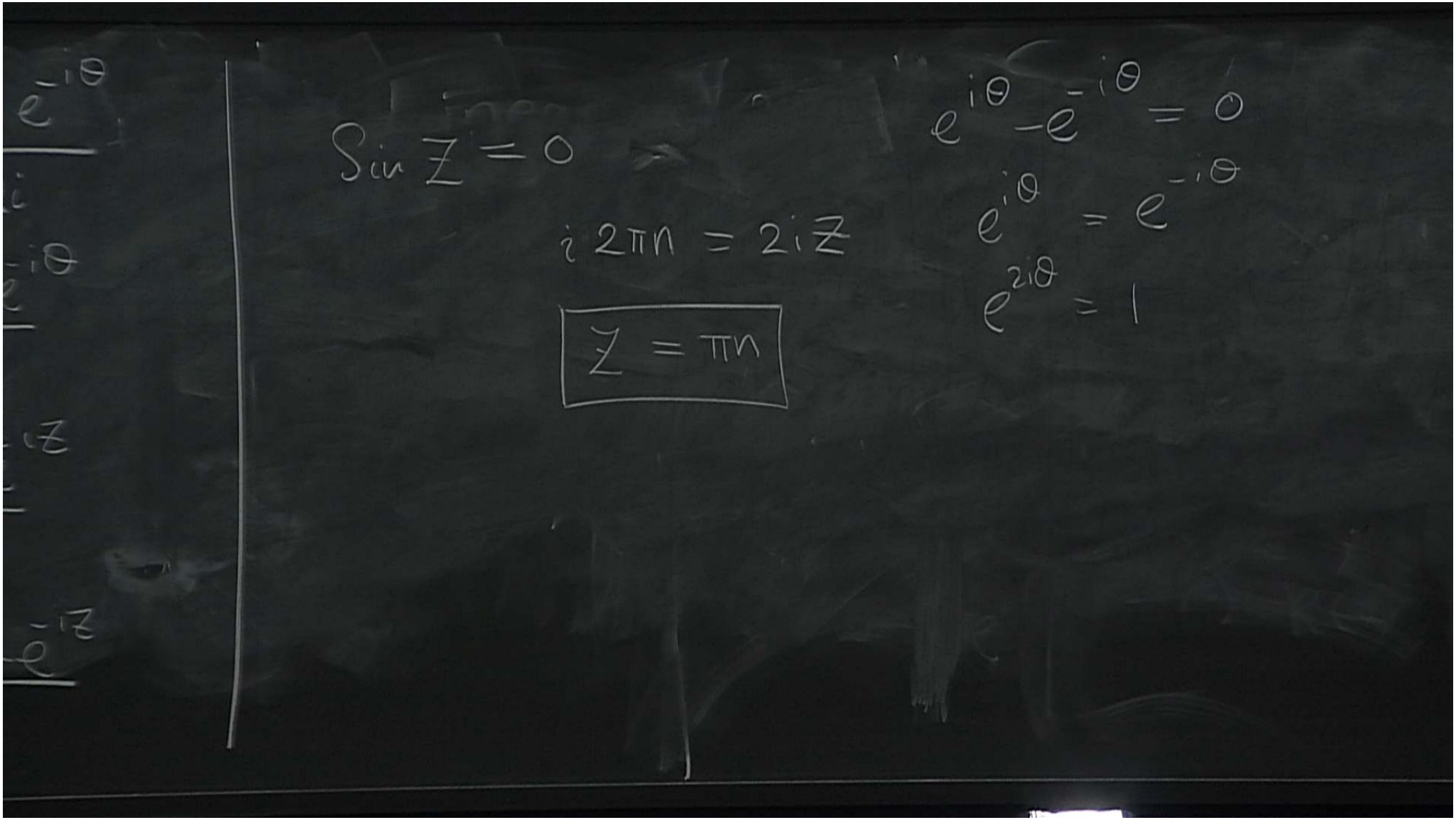
$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$

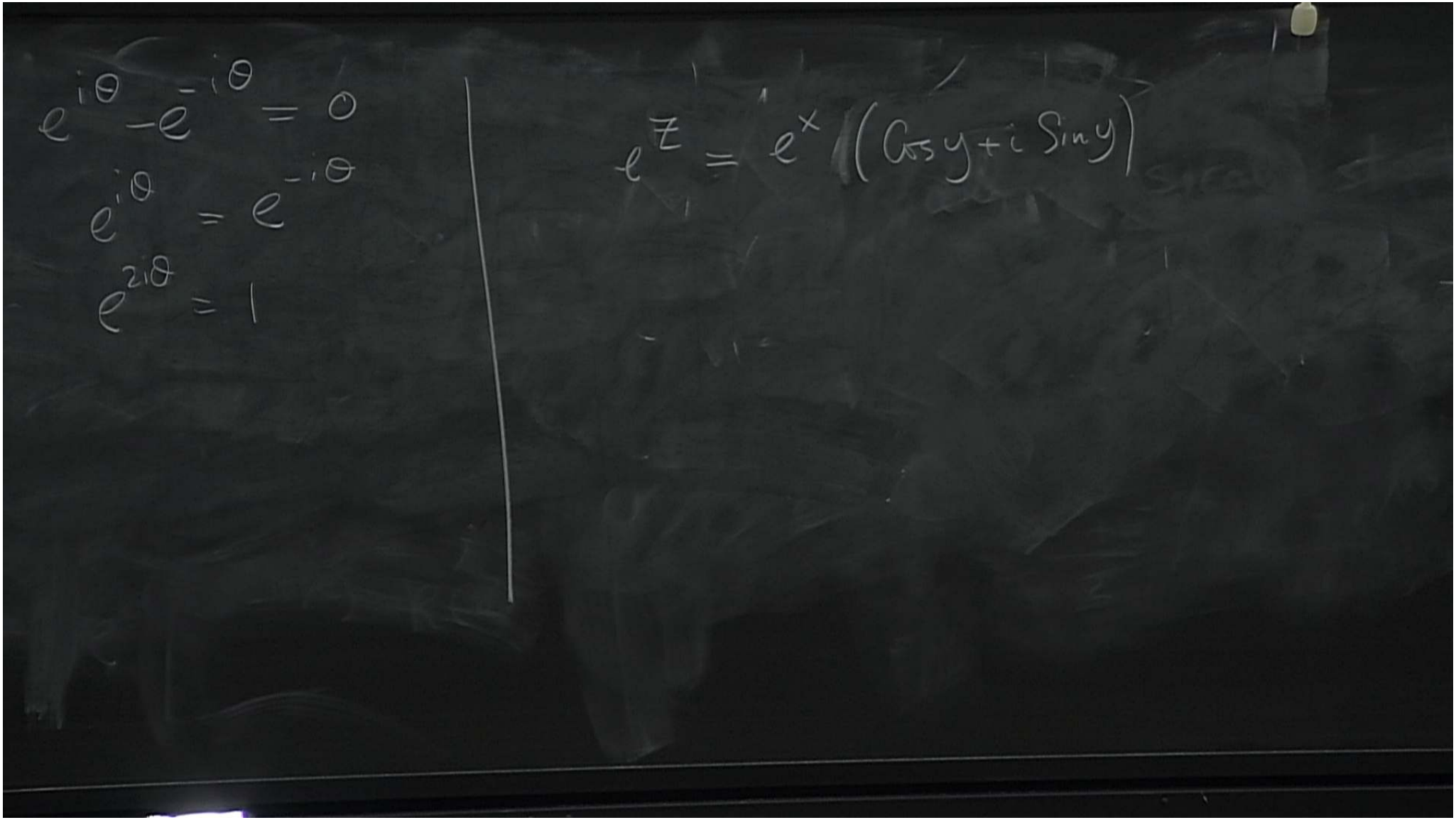
$$\sin z = 0 = e^{2iz}$$

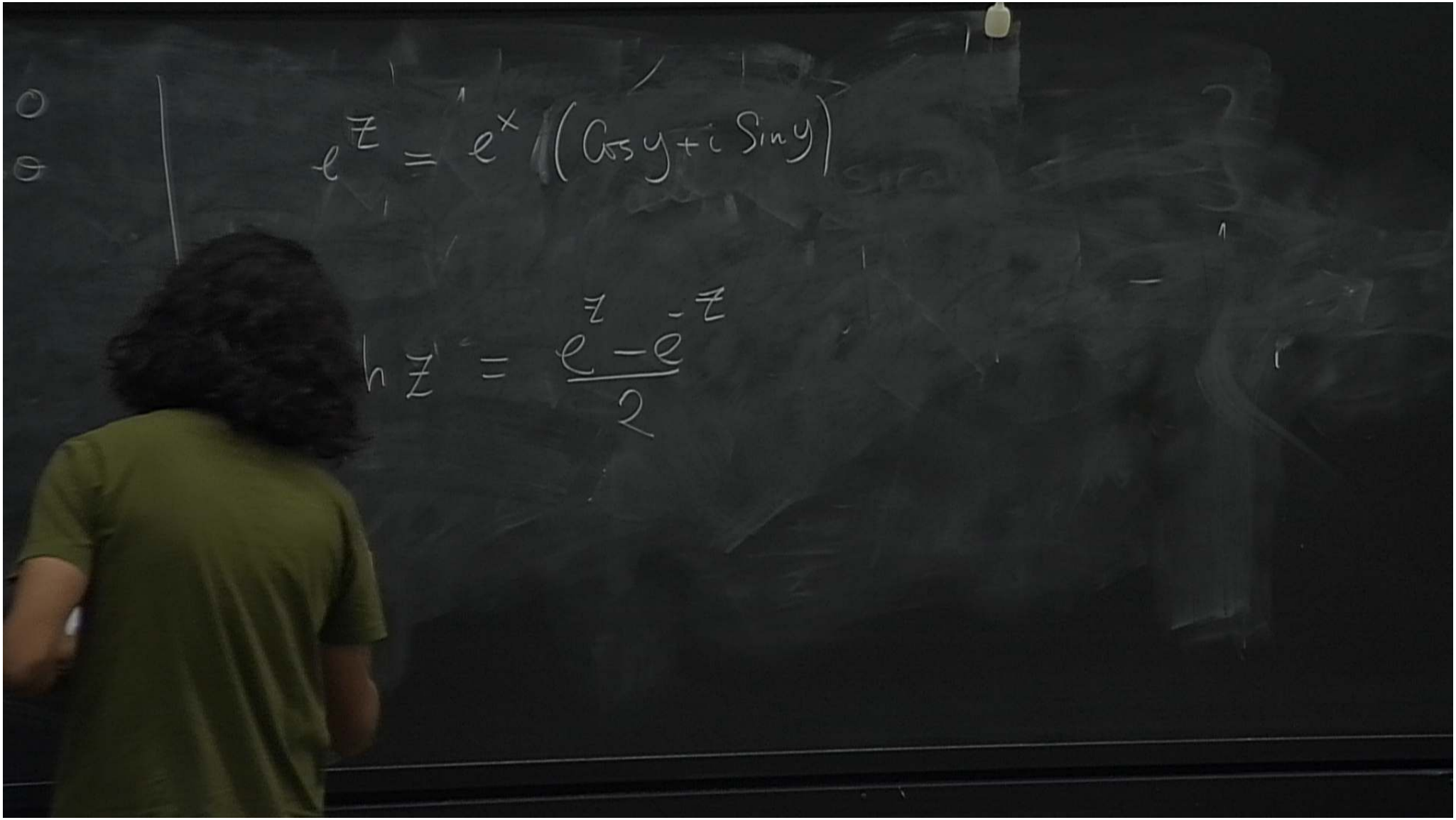
$$i 2\pi n = 2iz$$

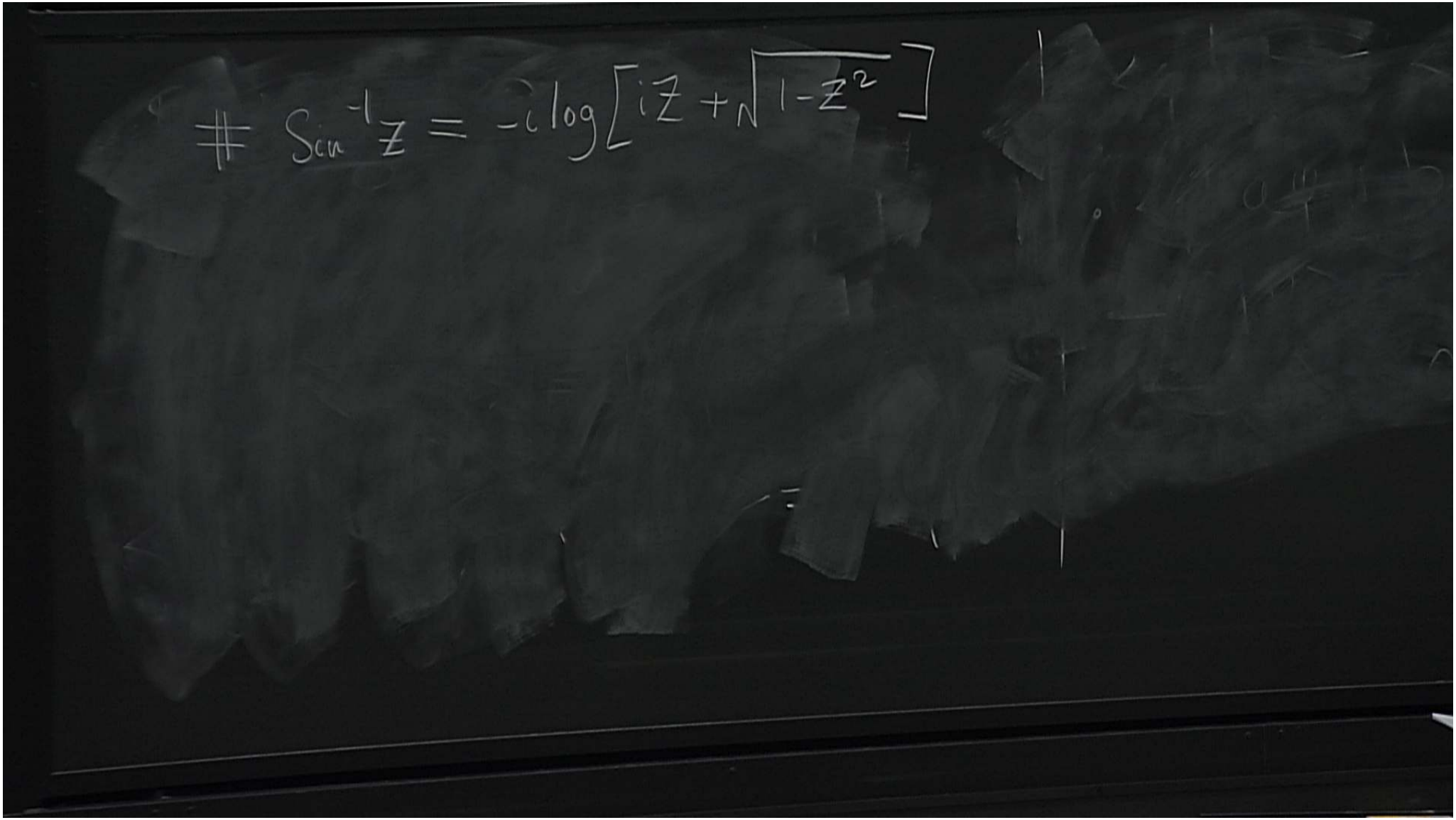
$$\boxed{z = \pi n}$$


$\sin z = 0$
$e^{i\theta} - e^{-i\theta} = 0$
$e^{i\theta} = e^{-i\theta}$
$e^{2i\theta} = 1$
$i\,2\pi n = 2iz$
$z = \pi n$

Analytic continuation
Notice: Your books will be ready on Monday (not Fri


$e^{i\theta} - e^{-i\theta} = 0$
$e^{i\theta} = e^{-i\theta}$
$e^{2i\theta} = 1$
$e^{z} = e^{x}(\cos y + i \sin y)$


$e^z = e^x(\cos y + i \sin y)$
$\sinh z = \frac{e^z - e^{-z}}{2}$

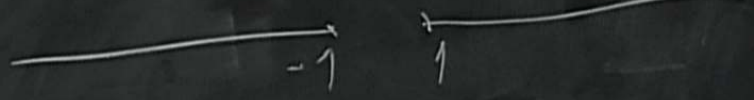

$\# \sin^{-1} z = -i \log\left[iz + \sqrt{1-z^2}\right]$

$\# \sin^{-1} z = -i \log\left[iz \pm \sqrt{1-z^2}\right]$

$w = n\pi$

$\sin w = z$

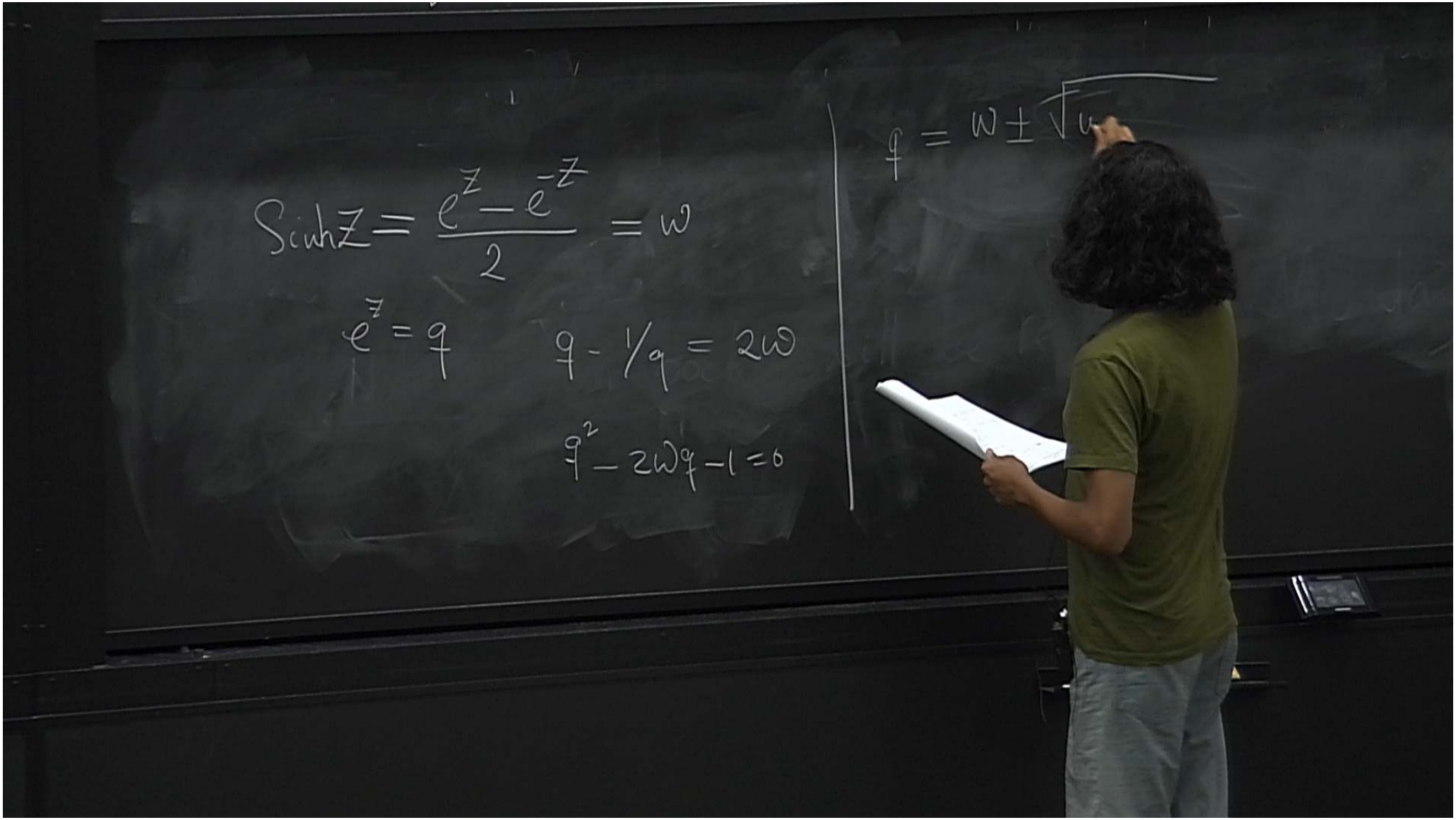
$\frac{e^{iw} - e^{-iw}}{2i} = z$

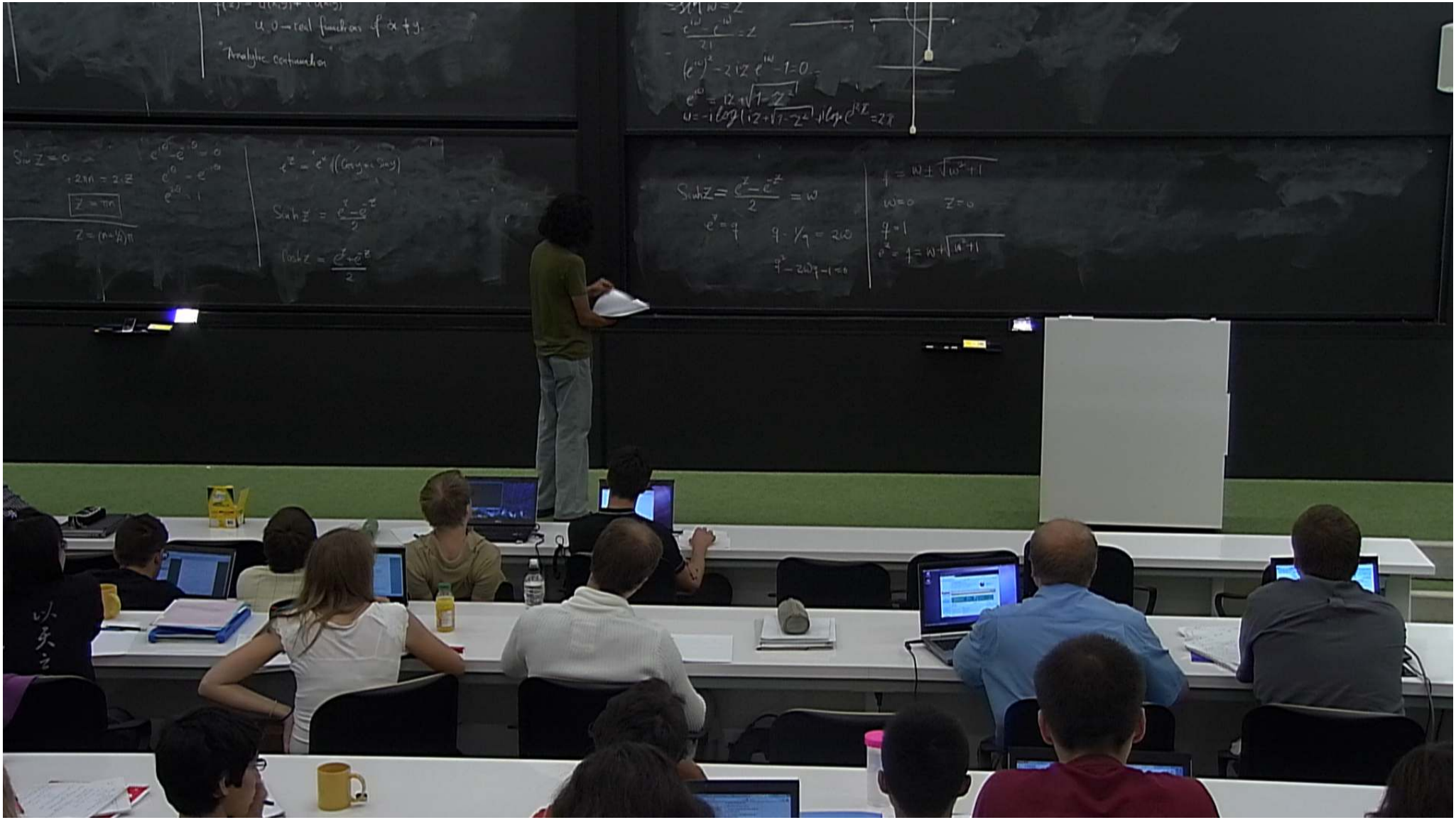


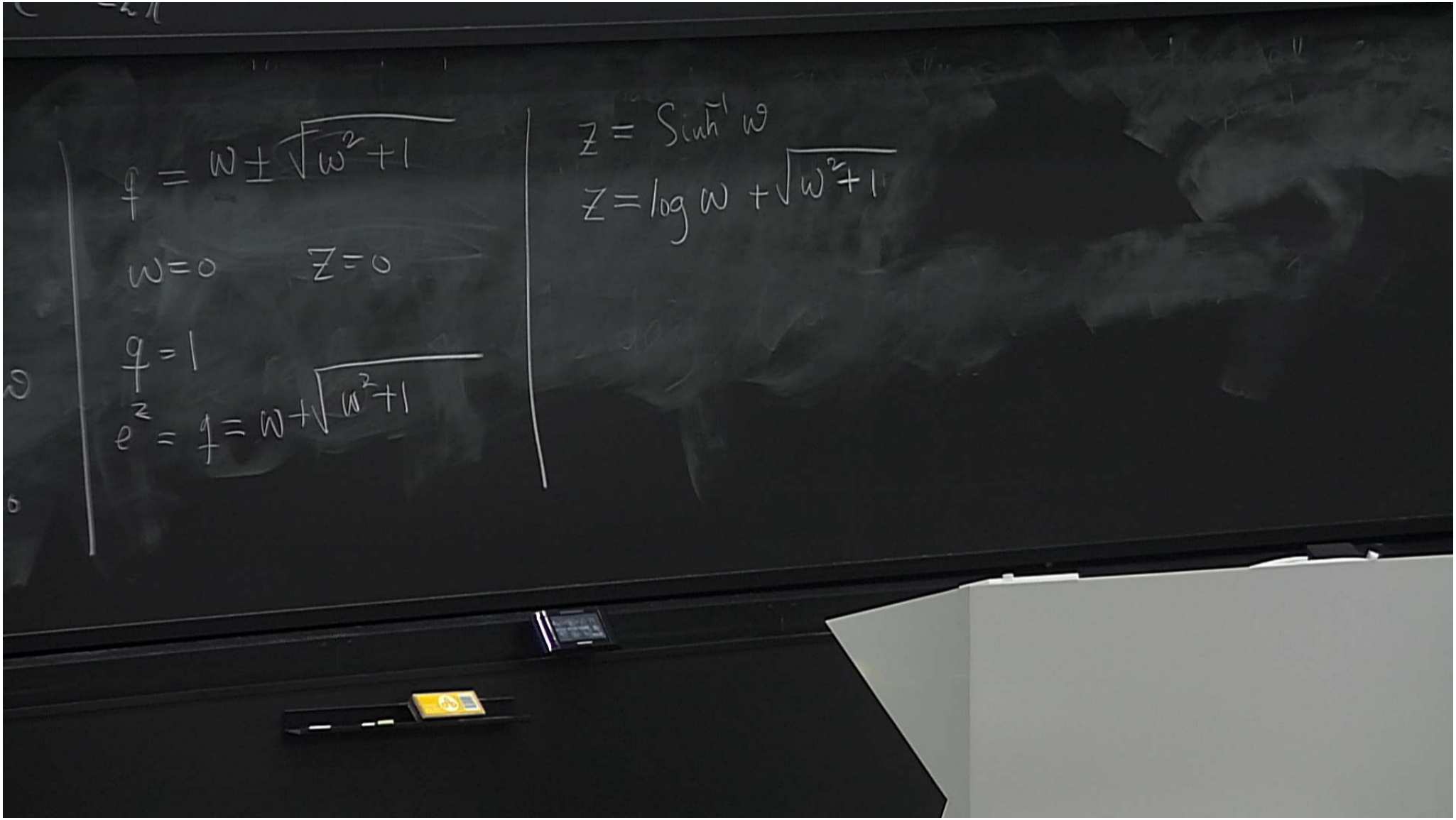
$(e^{iw})^2 - 2iz e^{iw} - 1 = 0$

$e^{iw} = iz + \sqrt{1-z^2}$

$w = -i \log\left(iz + \sqrt{1-z^2}\right) + i \log_e e^{i2\pi} = 2\pi$


$\sinh z = \frac{e^z - e^{-z}}{2} = w$
$e^z = q$
$q - 1/q = 2w$
$q^2 - 2wq - 1 = 0$
$q = w \pm \sqrt{}$


u, v → real functions of x & y.
"Analytic continuation"
Sinh z = (e^z − e^−z)/2 = w
q = w ± √(w² + 1)
w = 0 z = 0
q = 1


$w = z + \sqrt{z^2+1}$
$w=0 \qquad z=0$
$q=1$
$e^z = q = w+\sqrt{w^2+1}$
$z = \sinh^{-1} w$
$z = \log w + \sqrt{w^2+1}$

$$\cos^{-1} z = -i \log\left(z + i\sqrt{1 - z^2}\right)$$

$$\cosh^{-1} z = \log\left(z + \sqrt{z^2 - 1}\right)$$

Complex differentiability

$-z^2$)

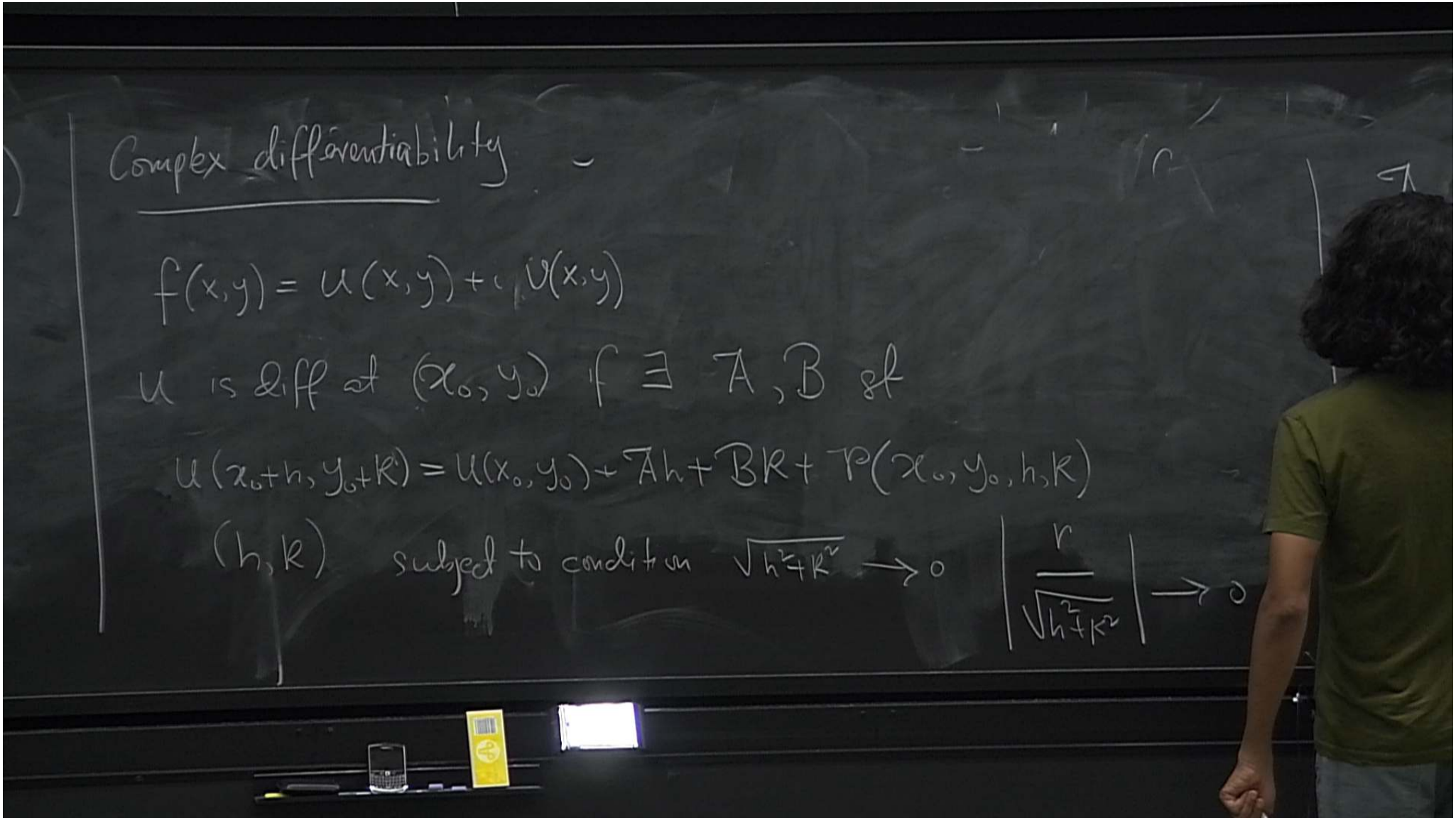
Complex differentiability

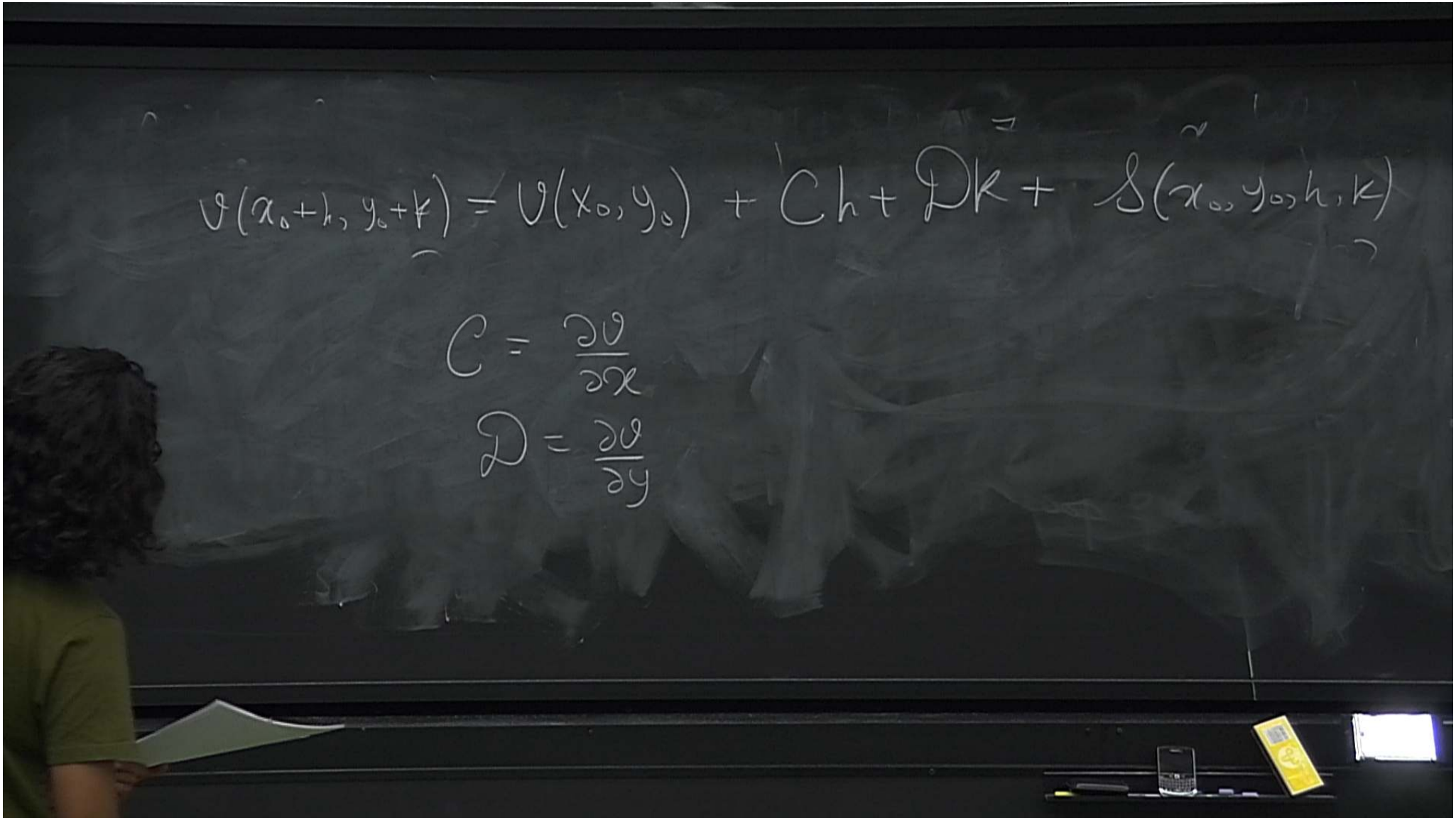
$f(x,y) = u(x,y) + i\,v(x,y)$

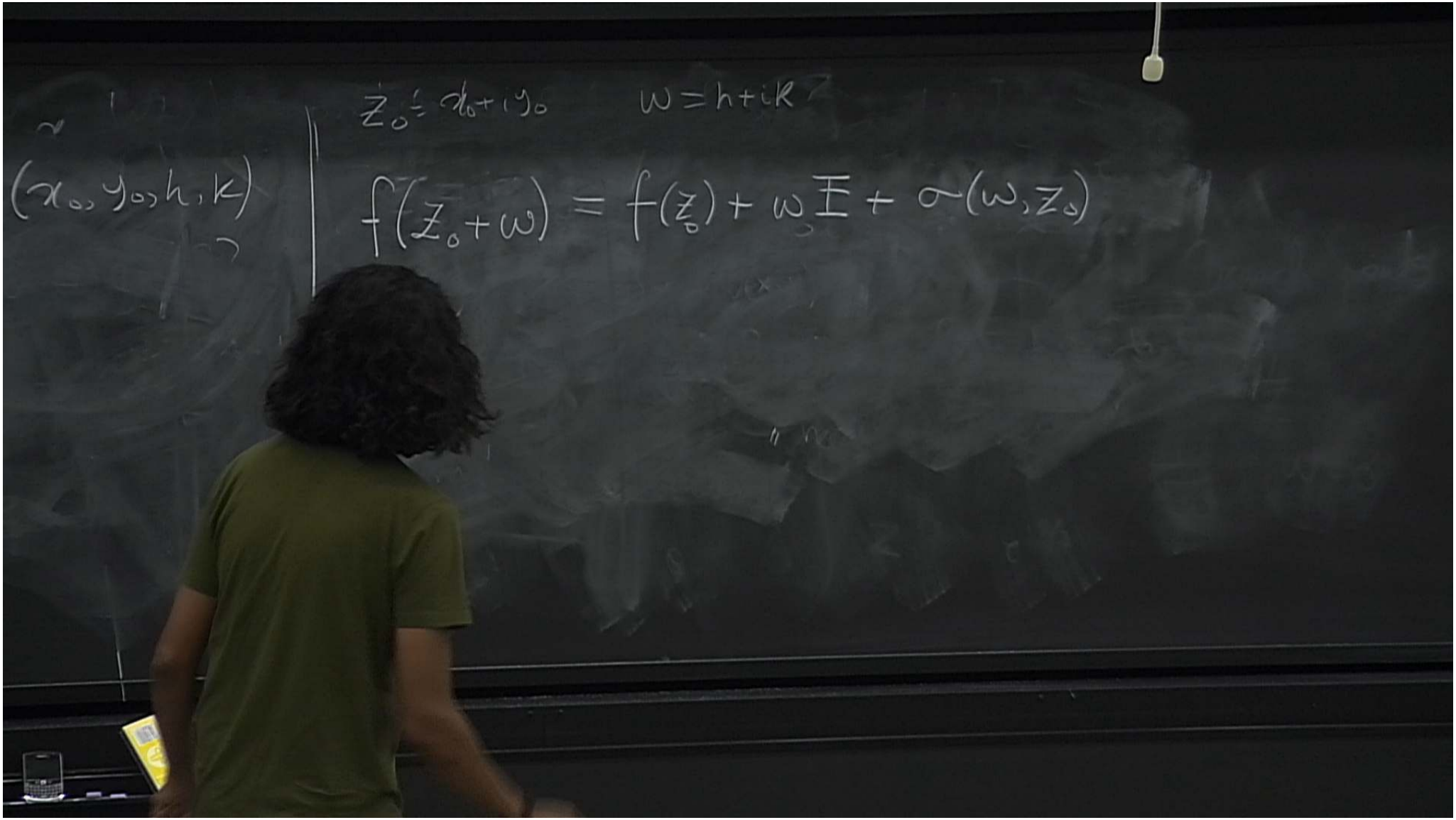
u is diff at (x_0, y_0) if $\exists\ A, B$ s.t.

$u(x_0+h, y_0+k) = u(x_0,y_0) + Ah + Bk + r(x_0, y_0, h, k)$

(h,k) subject to condition $\sqrt{h^2+k^2} \to 0$ $\left|\dfrac{r}{\sqrt{h^2+k^2}}\right| \to 0$


Complex differentiability
$f(x,y) = u(x,y) + i\,v(x,y)$
u is diff at (x_0, y_0) if $\exists\ A, B$ st
$u(x_0+h, y_0+k) = u(x_0,y_0) + Ah + Bk + P(x_0, y_0, h, k)$
(h,k) subject to condition $\sqrt{h^2+k^2} \to 0$
$\left|\frac{r}{\sqrt{h^2+k^2}}\right| \to 0$


$v(x_0+h, y_0+k) = v(x_0, y_0) + Ch + Dk + \delta(x_0, y_0, h, k)$
$C = \frac{\partial v}{\partial x}$
$D = \frac{\partial v}{\partial y}$


(x_0, y_0, h, k)
$z_0 = x_0 + i y_0$
$w = h + ik$
$f(z_0 + w) = f(z_0) + w_1 \Xi + \sigma(w, z_0)$

$z_0 = x_0 + iy_0 \qquad w = h + ik$

$$f(z_0 + w) = f(z_0) + w\,\Xi + \sigma(w, z_0) \quad *$$

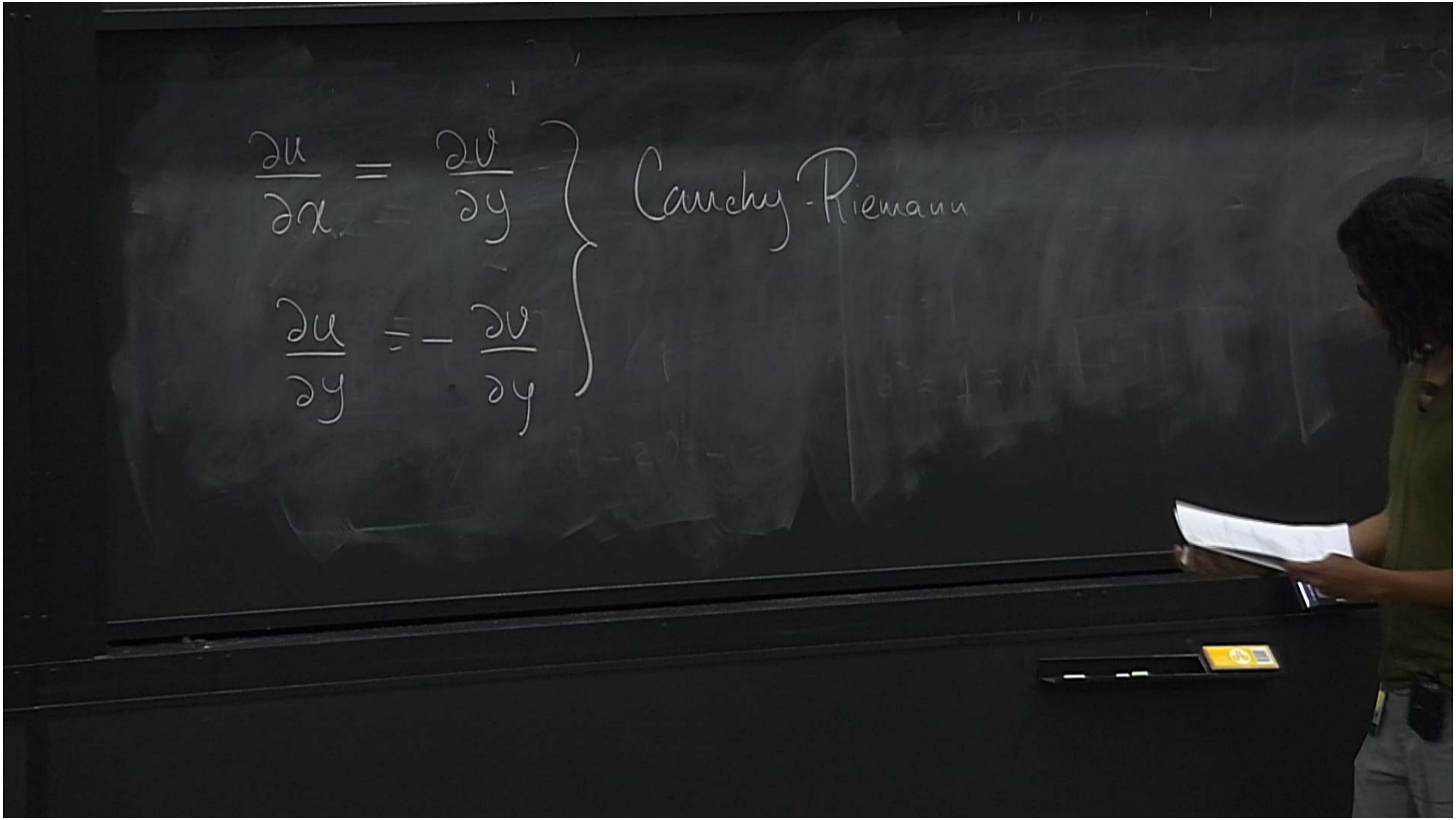
$$\Xi = \alpha + i\beta$$

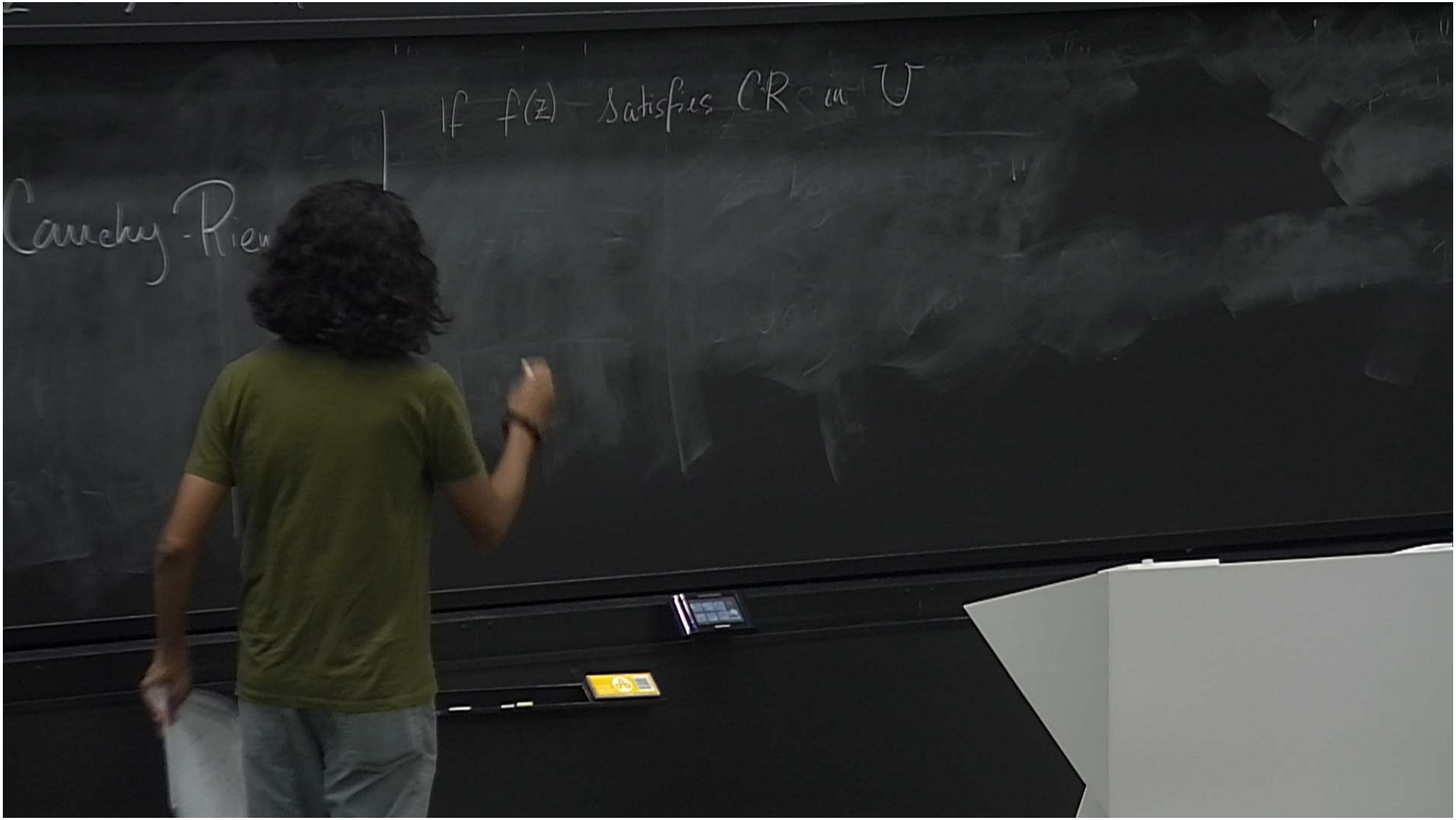
as $w \to 0$ $\left|\frac{\sigma}{w}\right| \to 0$

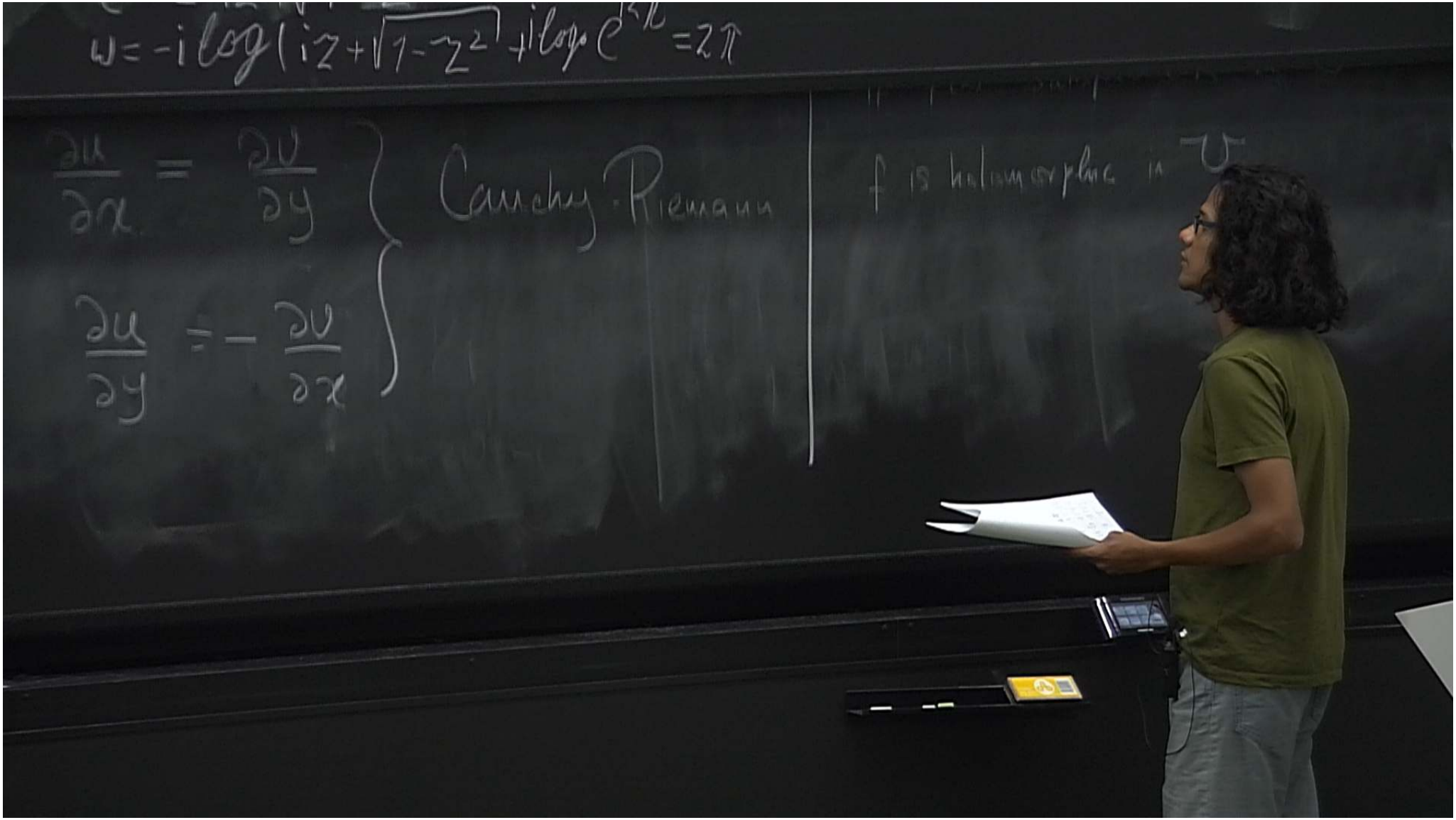
$\mathrm{Re}: \; u(x_0 + h, y_0 + k) = u(x_0, y_0) + h\alpha - k\beta + \cdots$

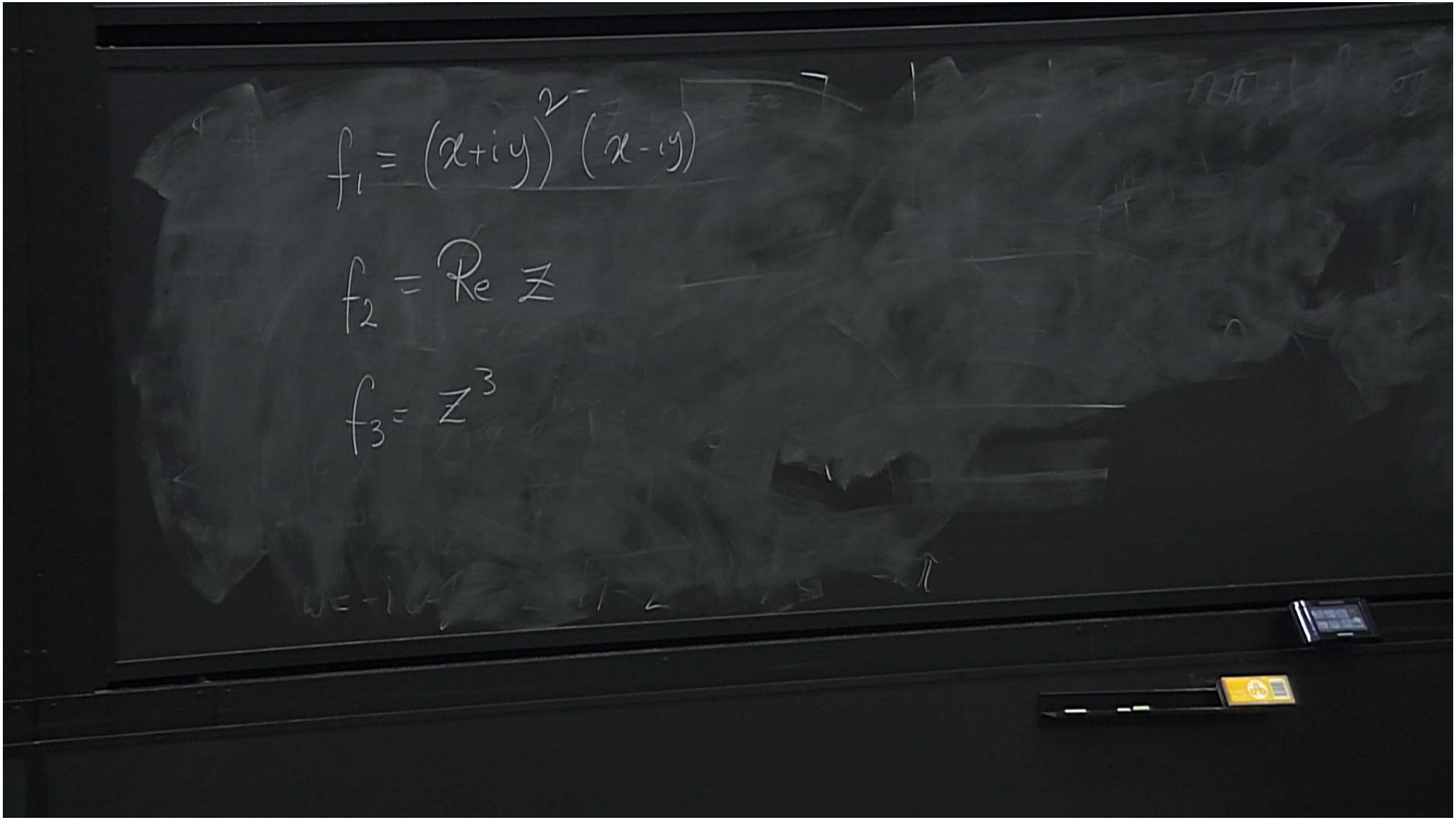
$\mathrm{Im}: \; v(\;\;\;) = v(\;\;) + h\beta + k\alpha + \cdots$

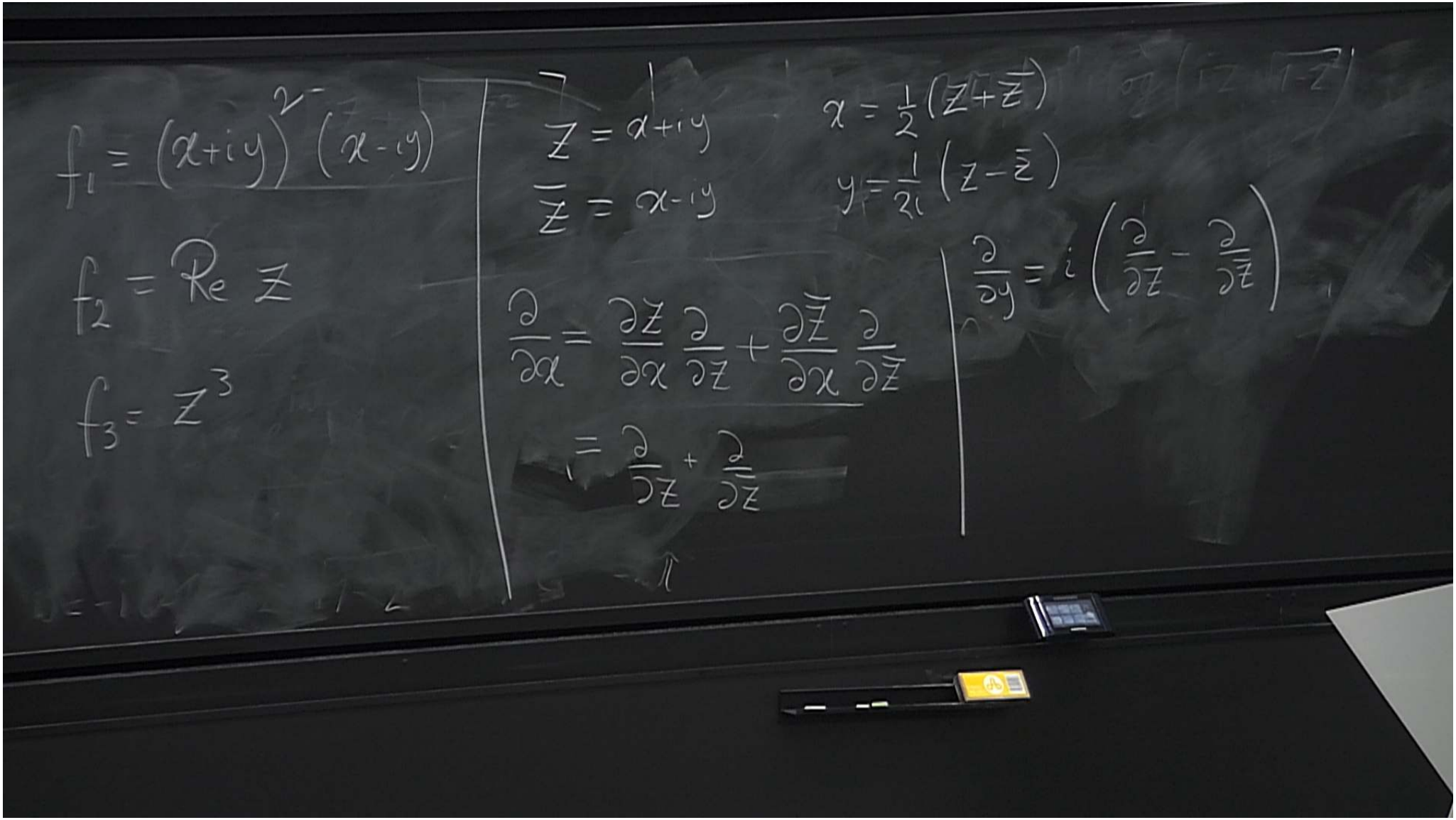
$$\alpha = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \beta = -\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$


$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$
$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial y}$
Cauchy-Riemann


If f(z) satisfies CR in U
Cauchy-Rie


$w = -i \log\left(iz + \sqrt{1-z^2}\right) + i \log e^{2\pi} = 2\pi$
$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$
$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
Cauchy-Riemann
f is holomorphic in U


$f_1 = (x+iy)^2 (x-y)$
$f_2 = \mathrm{Re}\, z$
$f_3 = z^3$

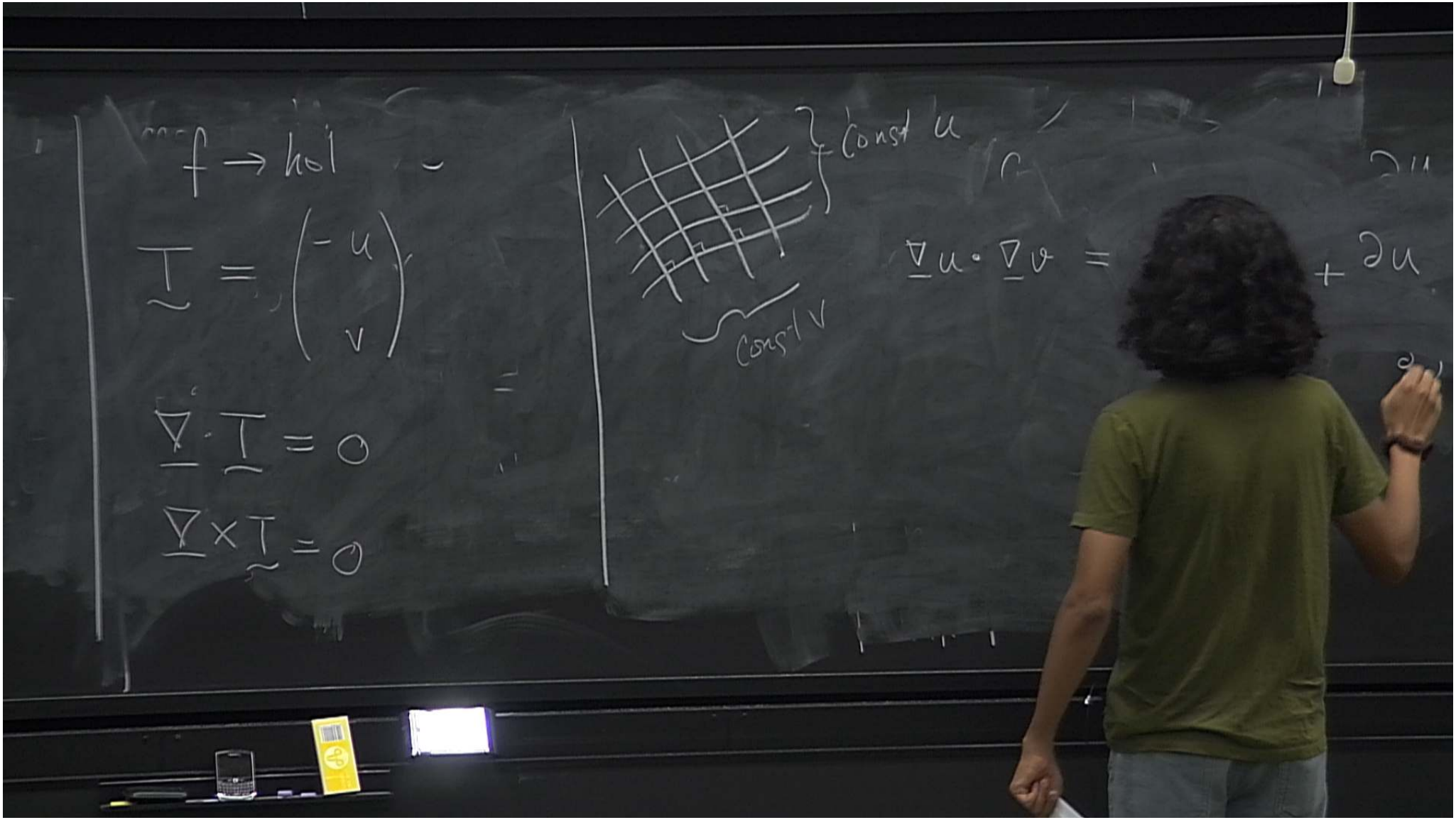

$f_1 = (x+iy)^2 (x-y)$
$f_2 = \mathrm{Re}\, z$
$f_3 = z^3$
$z = x+iy$
$\bar{z} = x-iy$
$x = \frac{1}{2}(z+\bar{z})$
$y = \frac{1}{2i}(z-\bar{z})$
$\frac{\partial}{\partial x} = \frac{\partial z}{\partial x}\frac{\partial}{\partial z} + \frac{\partial \bar{z}}{\partial x}\frac{\partial}{\partial \bar{z}}$
$= \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}$
$\frac{\partial}{\partial y} = i\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}\right)$

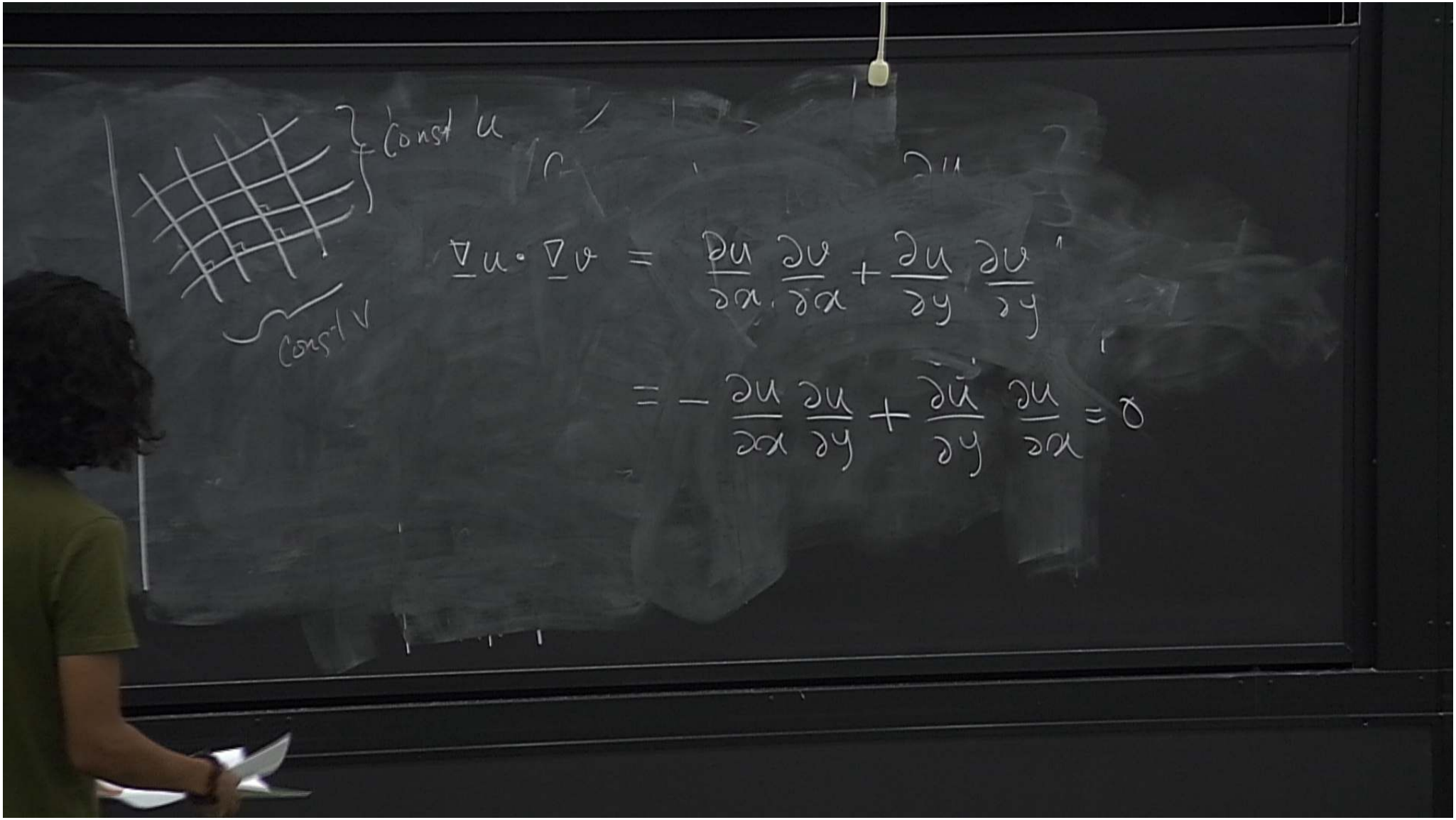
$$\left(\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0$$

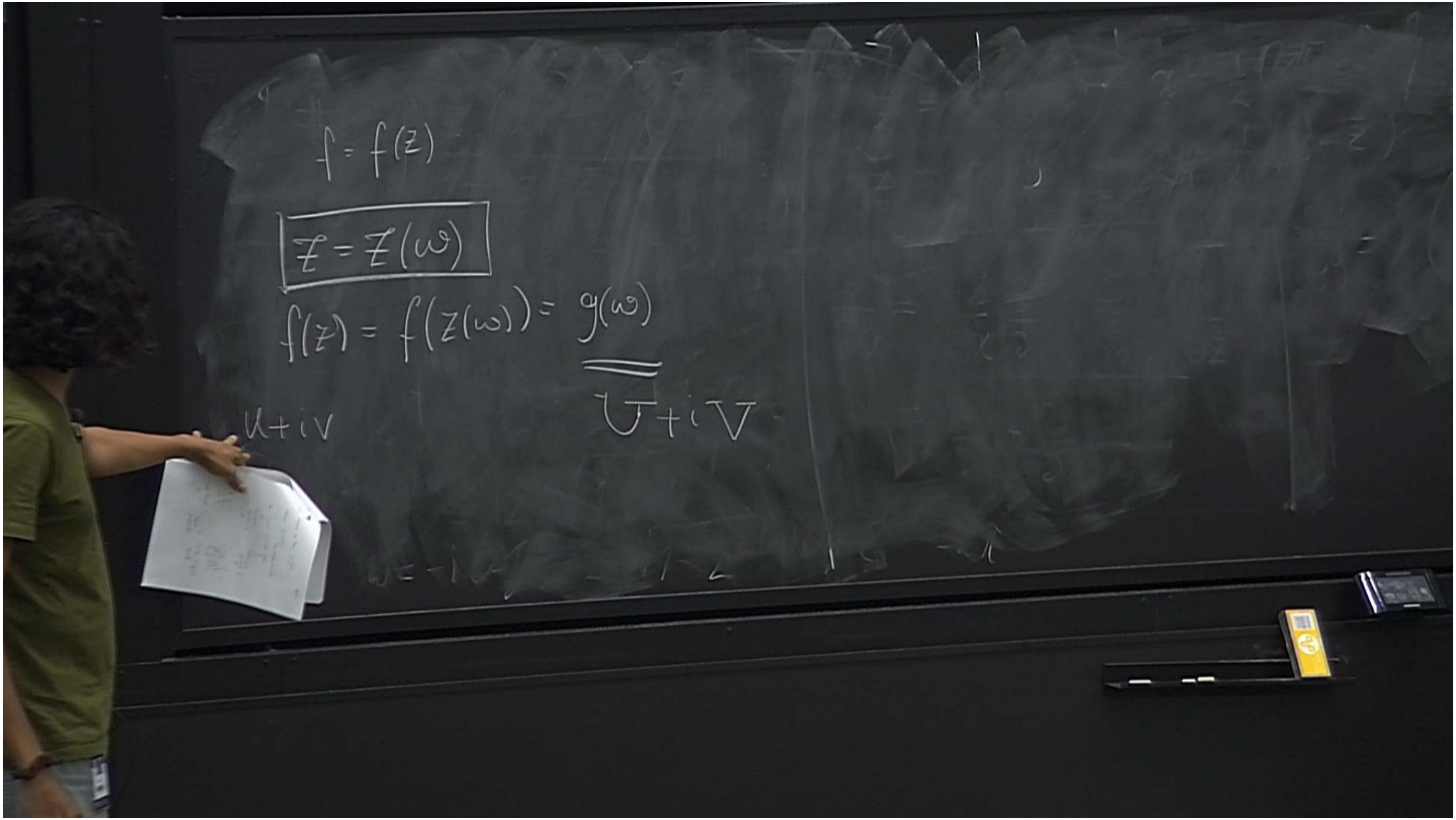
$$2\frac{\partial}{\partial \bar{z}}(u+iv) = 0$$

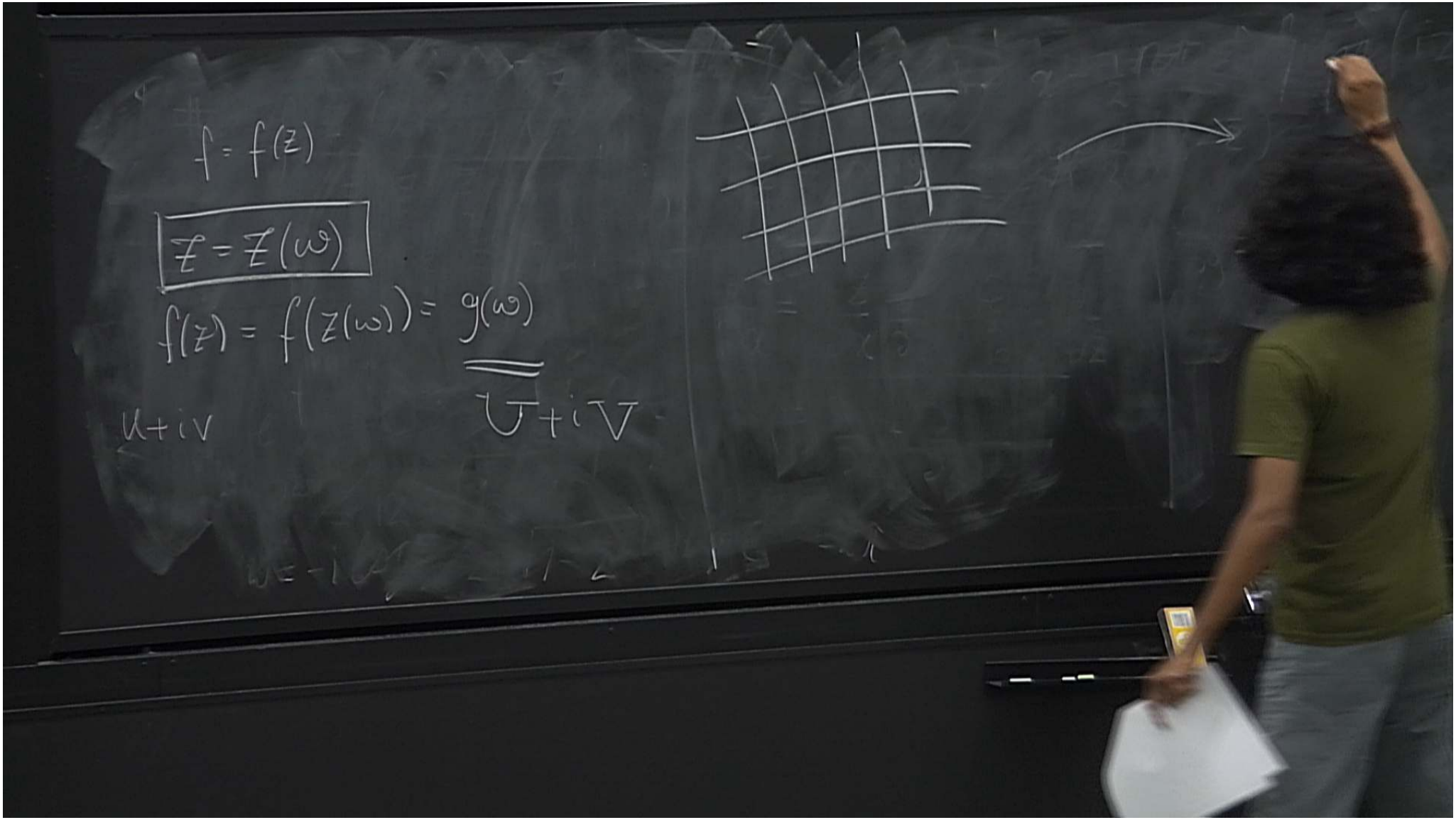
$$\frac{\partial f}{\partial \bar{z}} = 0$$

$$f = f(z)$$


f → hol
T = (−u, v)
∇ · T = 0
∇ × T = 0
Const u
Const v
∇u · ∇v =
+ ∂u


Const u
Const v
$\nabla u \cdot \nabla v = \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y}$
$= -\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial u}{\partial x} = 0$


f = f(z)
z = z(w)
f(z) = f(z(w)) = g(w)
u+iv
U+iV


f = f(z)
z = z(w)
f(z) = f(z(w)) = g(w)
u+iv
U+iV