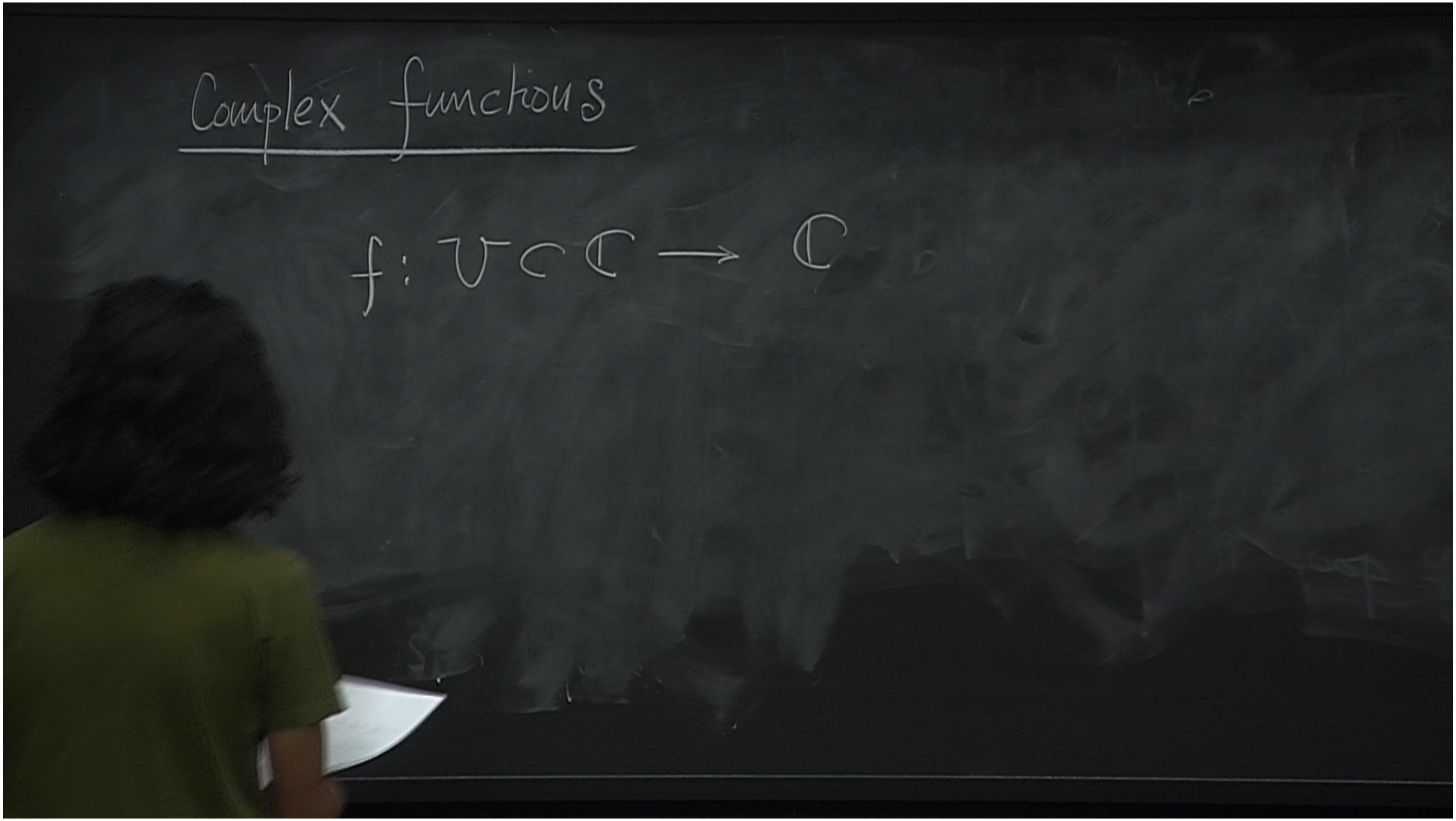
Title: Complex Analysis - Lecture 2a

Date: Aug 17, 2011 09:00 AM

URL: http://pirsa.pi.local/11080078

Abstract:


Complex functions
$f : U \subset \mathbb{C} \longrightarrow \mathbb{C}$

# Complex functions

$$f : U \subset \mathbb{C} \longrightarrow \mathbb{C}$$

$U \rightarrow$ Open Set ~ Domain

$f(U) \rightarrow$ Range

\# $e^{z}$

\# $\frac{1}{z^{2}}$

$\{0\}$

$$z = x + iy$$

$$f(z) = u(x,y) + i\,v(x,y)$$

$u, v \rightarrow$ real functions of $x$ & $y$.

$\to \mathbb{C}$

Domain

ge

$\# \quad e^{z} \qquad e^{x}$

$\# \quad \frac{1}{z^2} \qquad U = \mathbb{C} - \{0\}$

$z = x + iy$

$f(z) = u($

$u, v$

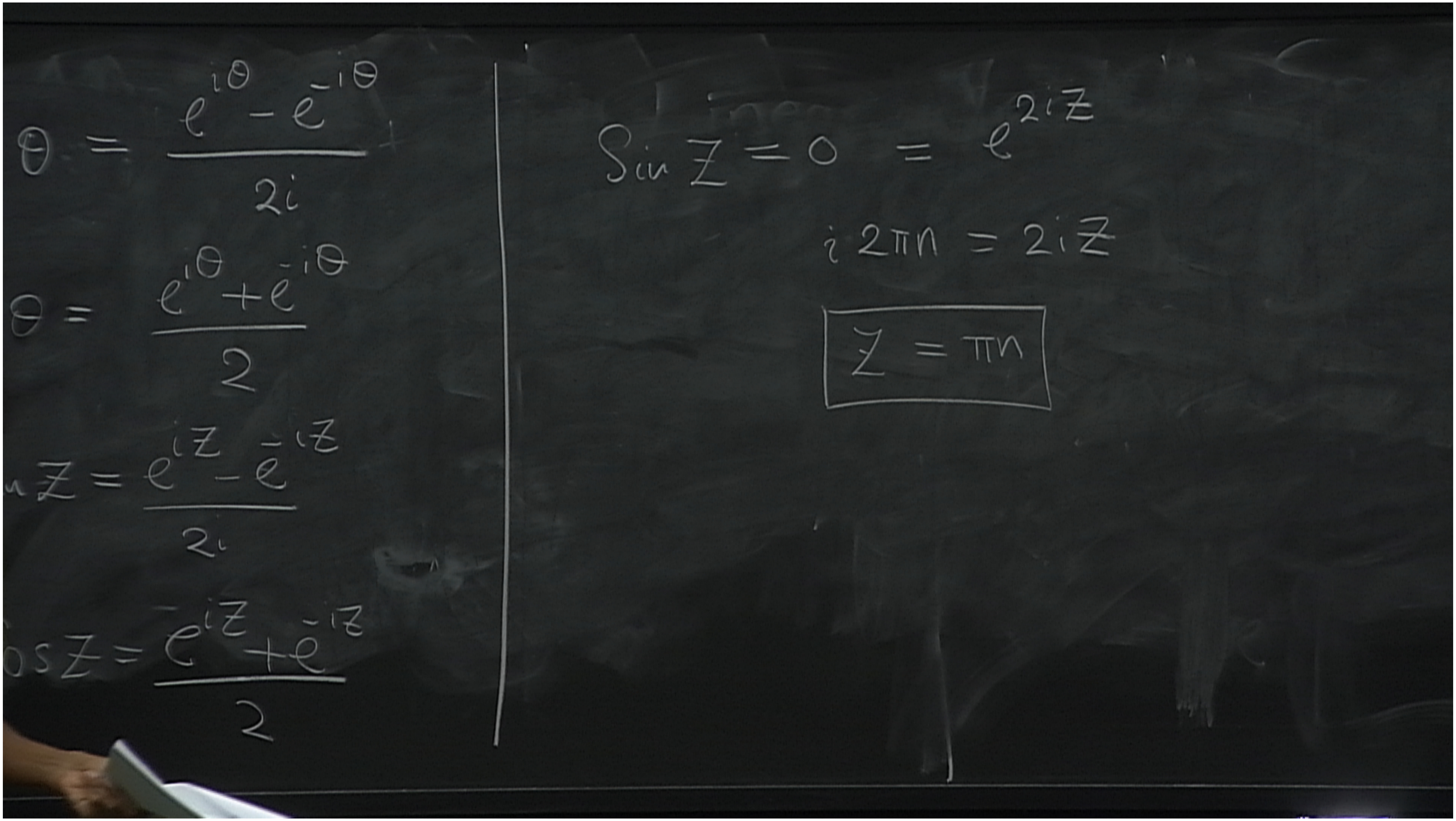
"Analytic

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}$$

$$\cos z = \frac{e^{iz} + e^{-iz}}{2}$$


$\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$
$\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$
$n Z = \frac{e^{iZ} - e^{-iZ}}{2i}$
$os Z = \frac{e^{iZ} + e^{-iZ}}{2}$
$\sin Z = 0 = e^{2iZ}$
$i 2\pi n = 2iZ$
$\boxed{Z = \pi n}$

$e^{-i\theta}$

$i$

$e^{-i\theta}$

$iz$

$e^{-iz}$

$\sin z = 0$

$i\,2\pi n = 2iz$

$$\boxed{z = \pi n}$$

$e^{i\theta} - e^{-i\theta} = 0$

$e^{i\theta} = e^{-i\theta}$

$e^{2i\theta} = 1$

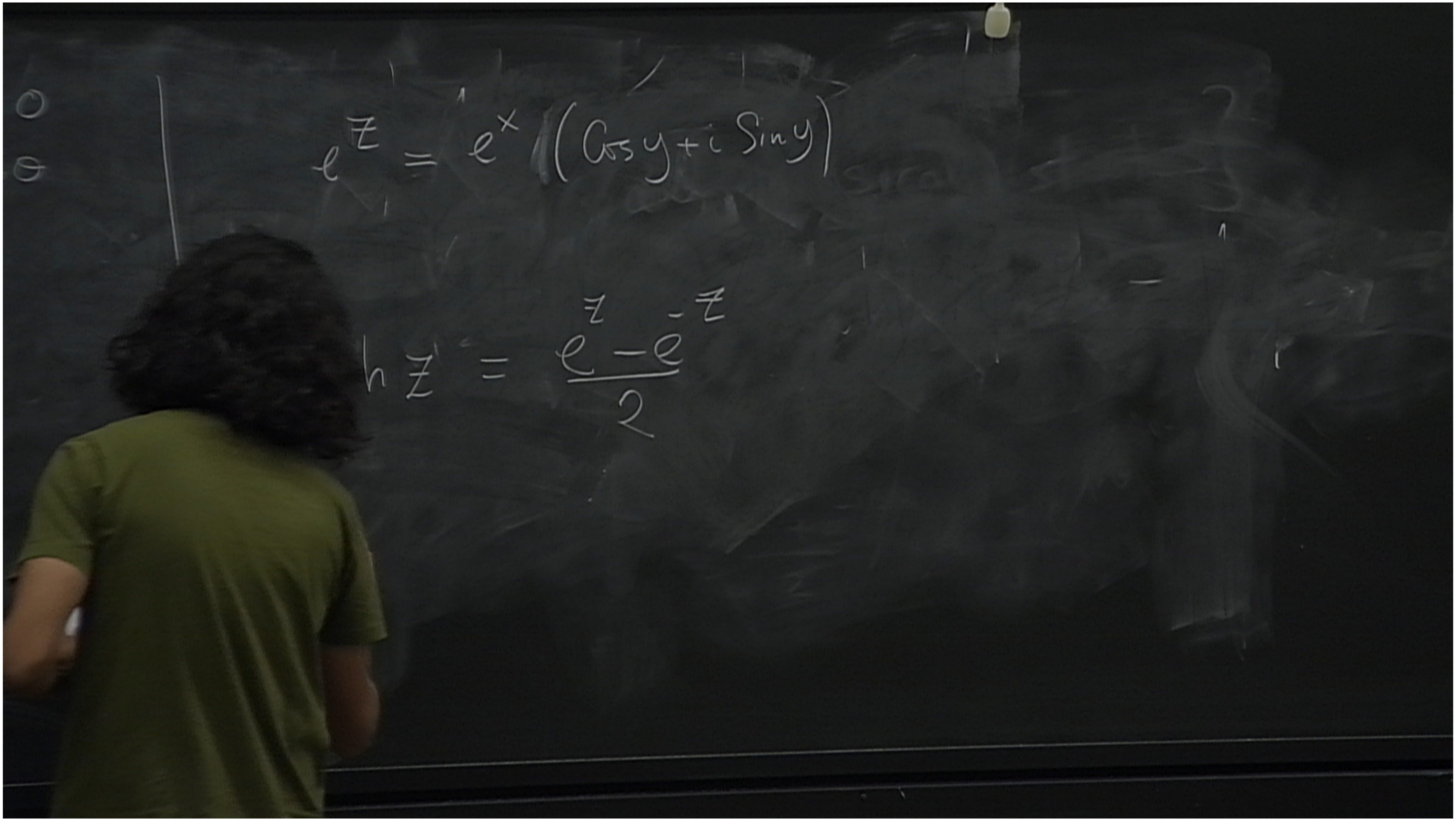

Notice: Your books will be ready on Monday (not
Analytic continuation

$$e^{i\theta} - e^{-i\theta} = 0$$

$$e^{i\theta} = e^{-i\theta}$$

$$e^{2i\theta} = 1$$

$$e^{z} = e^{x}(\cos y + i \sin y)$$


$e^{z} = e^{x}(\cos y + i \sin y)$
$h z = \frac{e^{z} - e^{-z}}{2}$

$$\# \quad \sin^{-1} z = -i \log\left[ i z + \sqrt{1 - z^2} \right]$$

$$\# \quad \sin^{-1} z = -i \log\left[ iz \pm \sqrt{1-z^2} \right]$$

$$w = n\pi$$

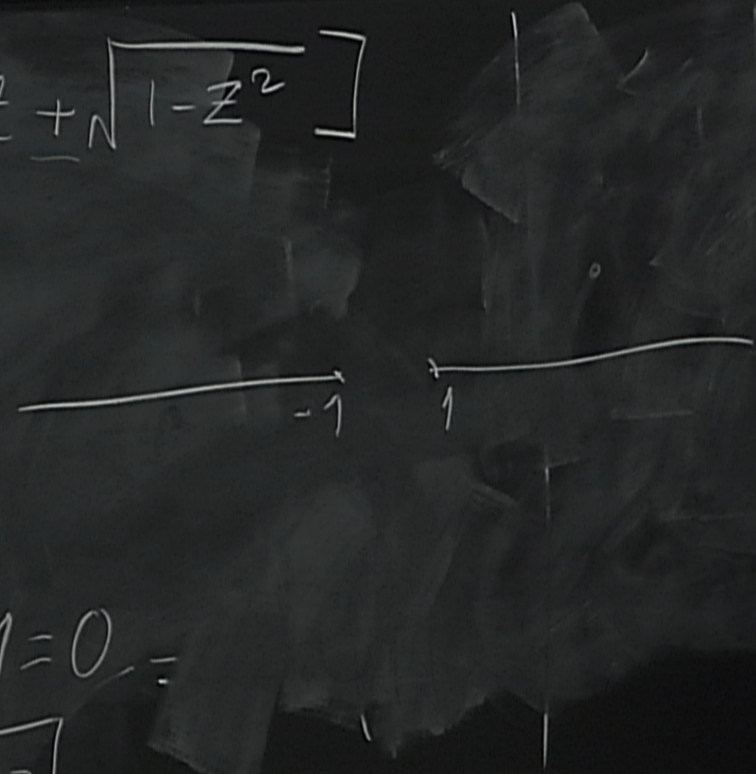
$$\sin w = z$$

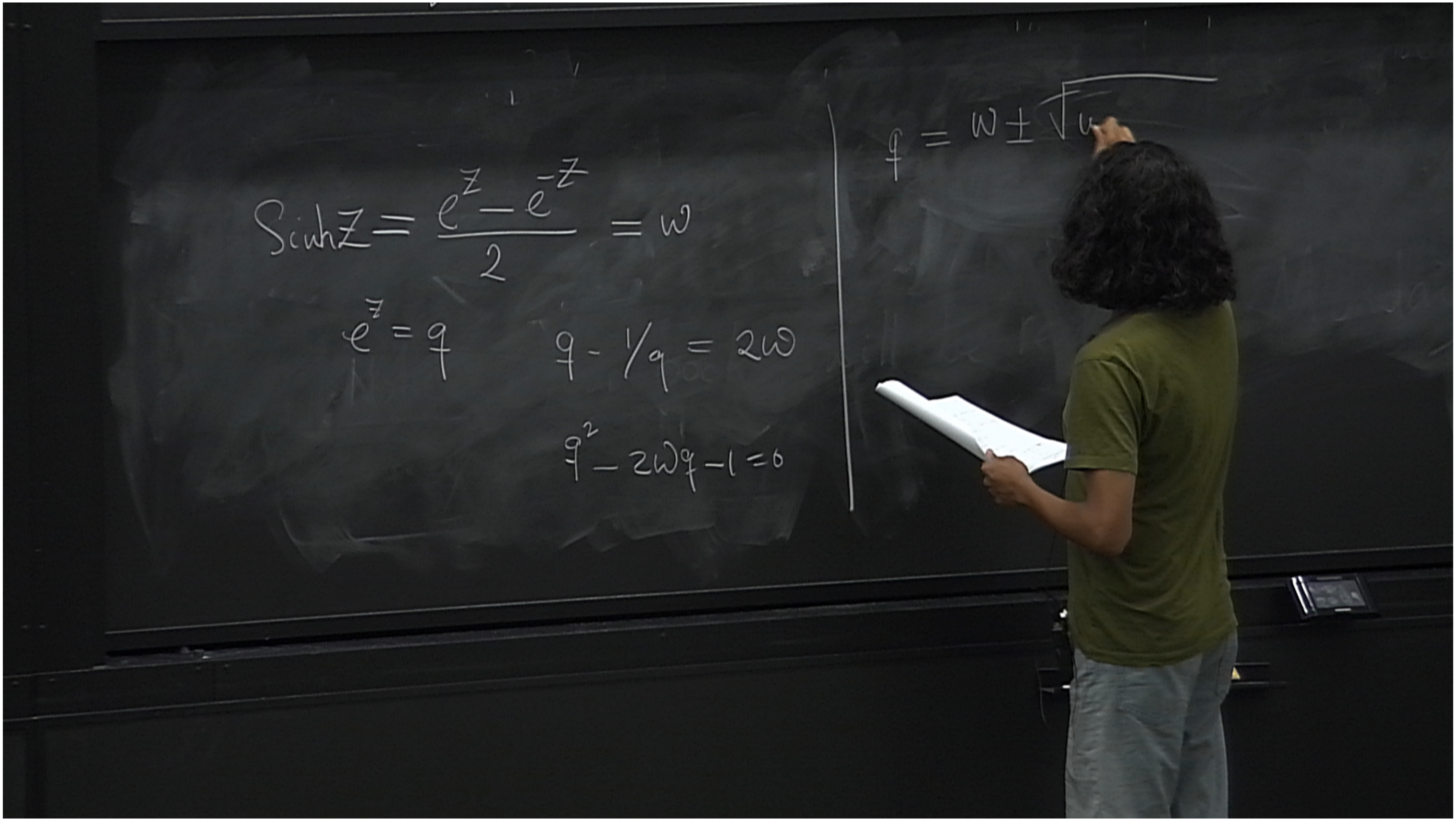
$$\frac{e^{iw} - e^{-iw}}{2i} = z$$

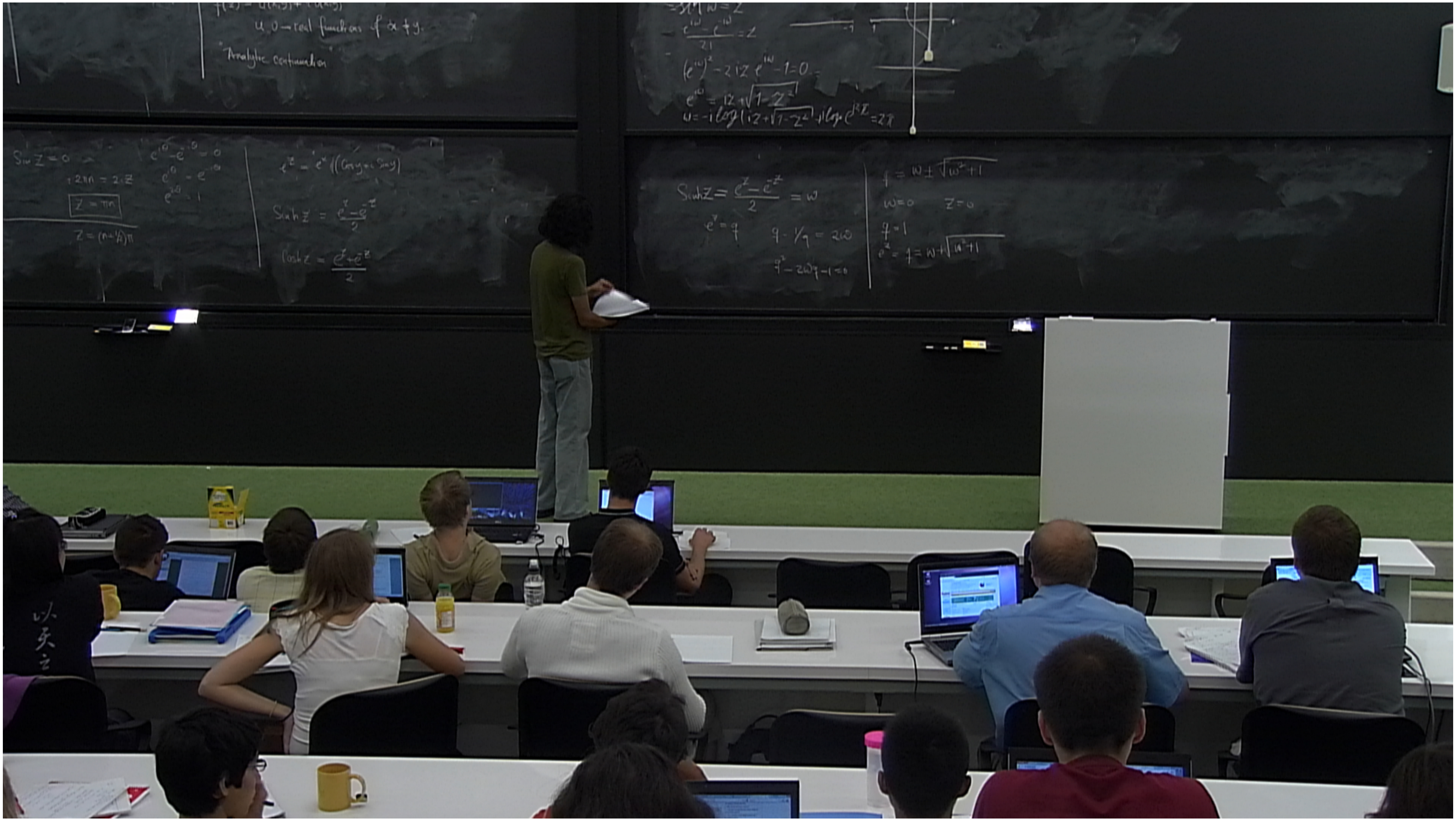
$$(e^{iw})^2 - 2iz\, e^{iw} - 1 = 0$$

$$e^{iw} = iz + \sqrt{1-z^2}$$

$$w = -i \log\left( iz + \sqrt{1-z^2} \right) + i \log e^{i2\pi} = 2\pi$$




$\sinh Z = \frac{e^{Z} - e^{-Z}}{2} = w$
$e^{z} = q$
$q - 1/q = 2w$
$q^2 - 2wq - 1 = 0$
$q = w \pm$



$$q = w \pm \sqrt{w^2+1}$$

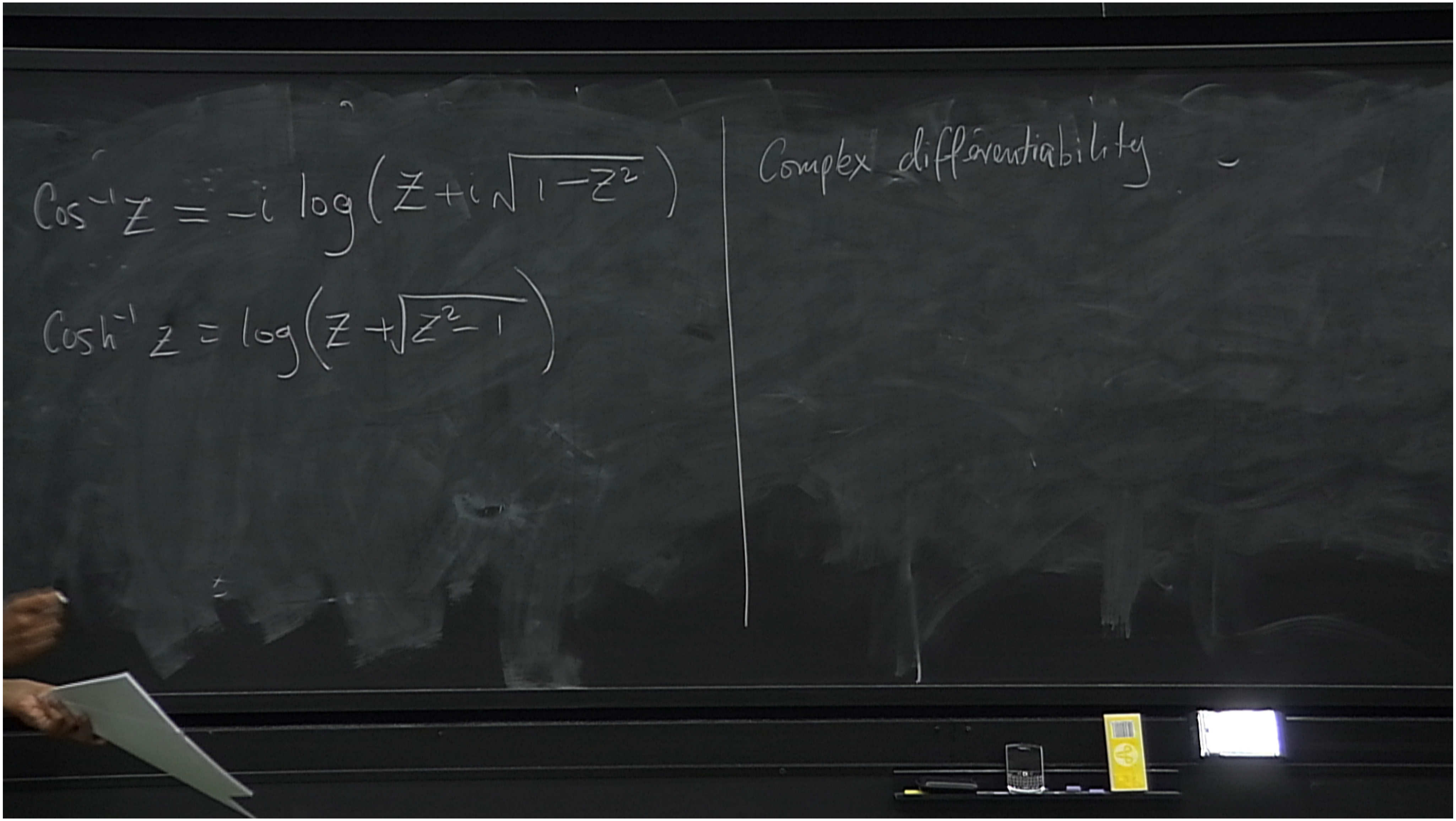
$w=0 \qquad z=0$

$q=1$

$$e^z = q = w + \sqrt{w^2+1}$$

$$z = \sinh^{-1} w$$

$$z = \log w + \sqrt{w^2+1}$$



$\cos^{-1} z = -i \log\left(z + i\sqrt{1-z^2}\right)$
$\cosh^{-1} z = \log\left(z + \sqrt{z^2-1}\right)$
Complex differentiability

$\overline{-z^2})$

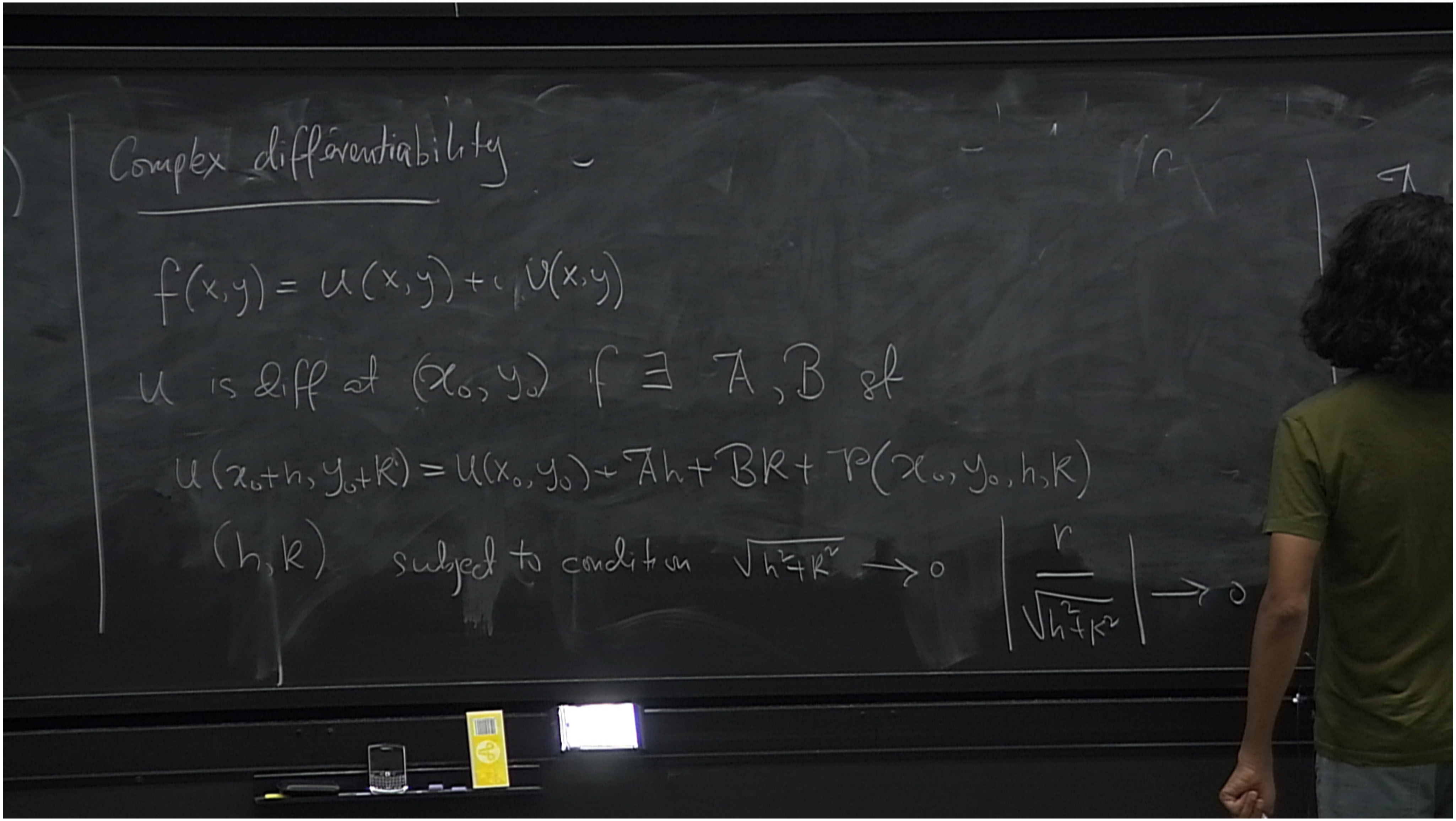
Complex differentiability

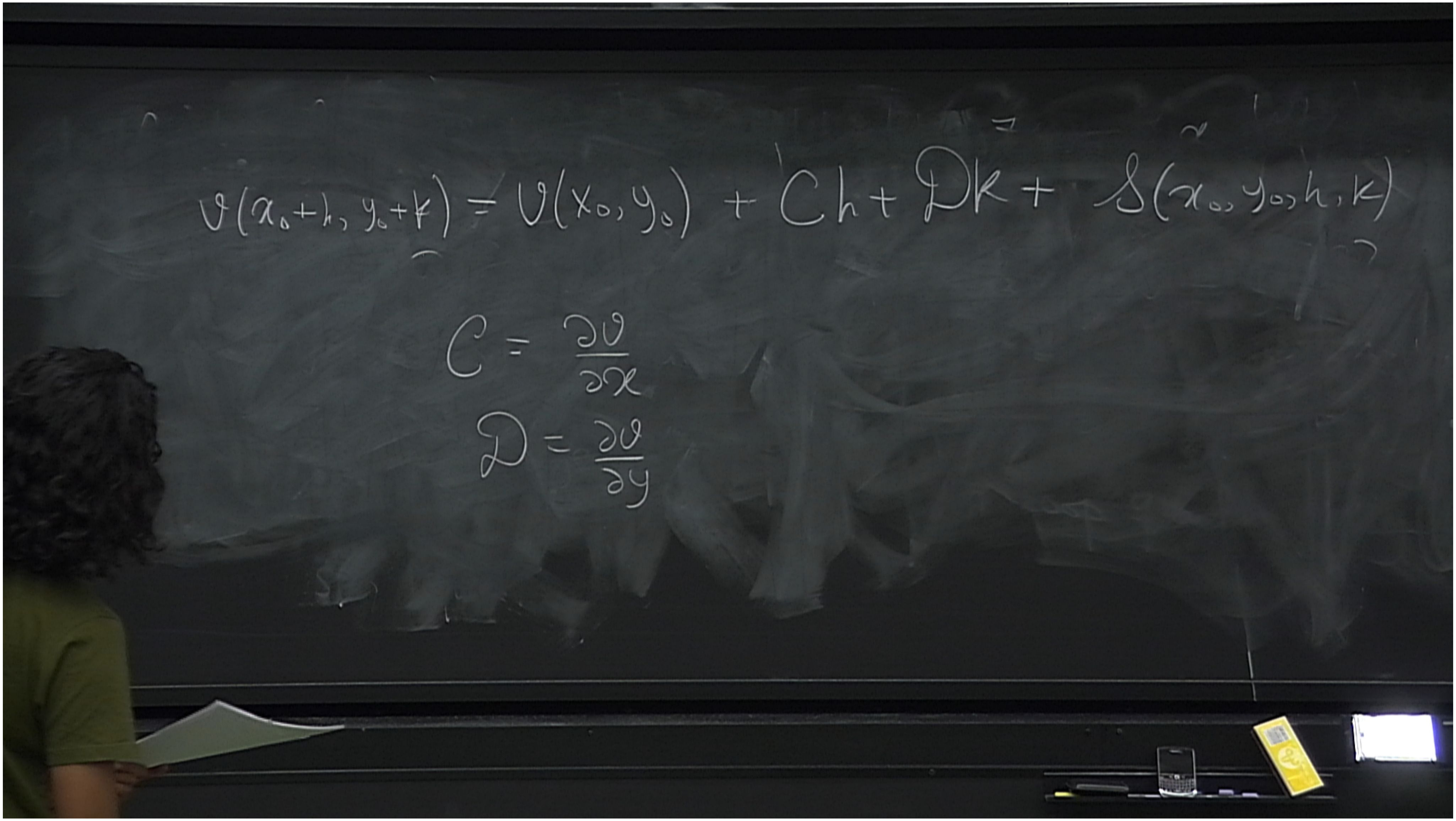
$$f(x,y) = u(x,y) + i\,v(x,y)$$

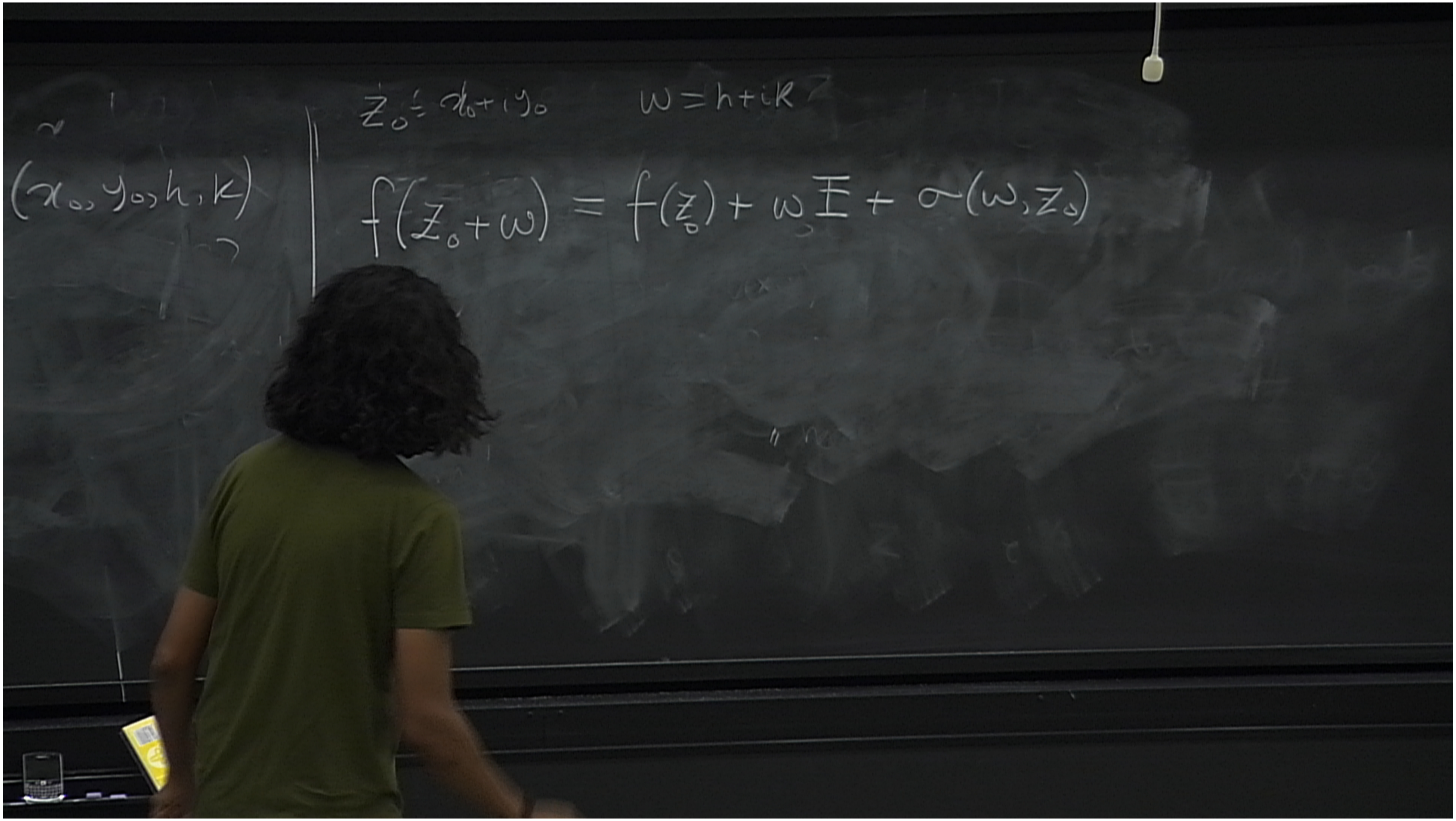
$u$ is diff at $(x_0, y_0)$ if $\exists$ $A, B$ st

$$u(x_0+h, y_0+k) = u(x_0,y_0) + Ah + Bk + r(x_0, y_0, h, k)$$

$(h,k)$ subject to condition $\sqrt{h^2+k^2} \to 0$ $\quad \left| \dfrac{r}{\sqrt{h^2+k^2}} \right| \to 0$


Complex differentiability
$f(x,y) = u(x,y) + i\,v(x,y)$
$u$ is diff at $(x_0, y_0)$ if $\exists$ $A, B$ st
$u(x_0+h, y_0+k) = u(x_0, y_0) + Ah + Bk + r(x_0, y_0, h, k)$
$(h,k)$ subject to condition $\sqrt{h^2+k^2} \to 0$
$\left|\frac{r}{\sqrt{h^2+k^2}}\right| \to 0$


$v(x_0+h, y_0+k) = v(x_0, y_0) + Ch + Dk + \delta(x_0, y_0, h, k)$
$C = \frac{\partial v}{\partial x}$
$D = \frac{\partial v}{\partial y}$


$z_0 = x_0 + iy_0$
$w = h + ik$
$(x_0, y_0, h, k)$
$f(z_0 + w) = f(z_0) + w E + \sigma(w, z_0)$

$z_0 = x_0 + i y_0 \qquad w = h + ik$

$$f(z_0 + w) = f(z_0) + w\,\Xi + \sigma(w, z_0) \quad *$$

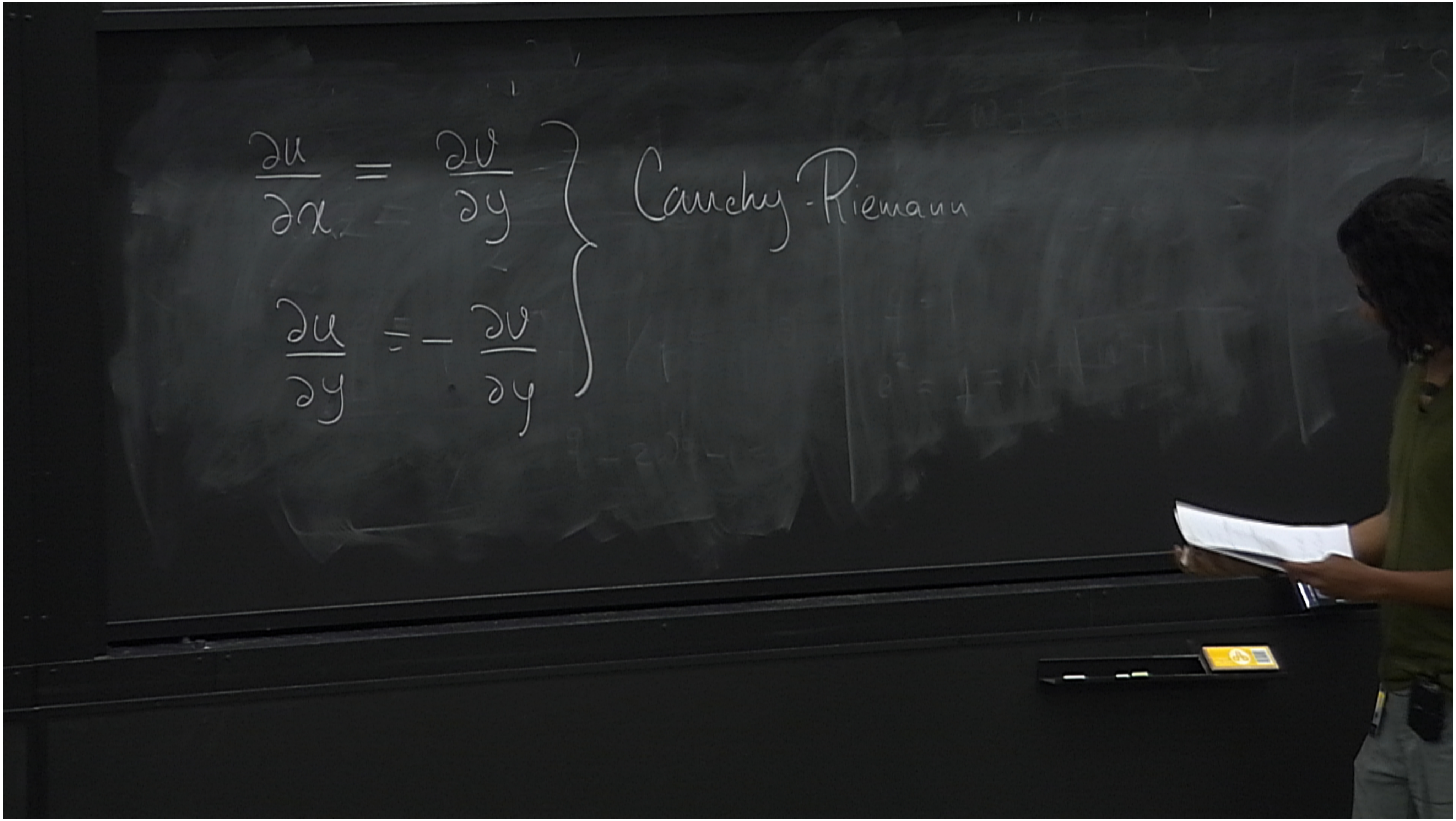
$$\Xi = \alpha + i\beta$$

as $w \to 0$ $\qquad \left|\frac{\sigma}{w}\right| \to 0$

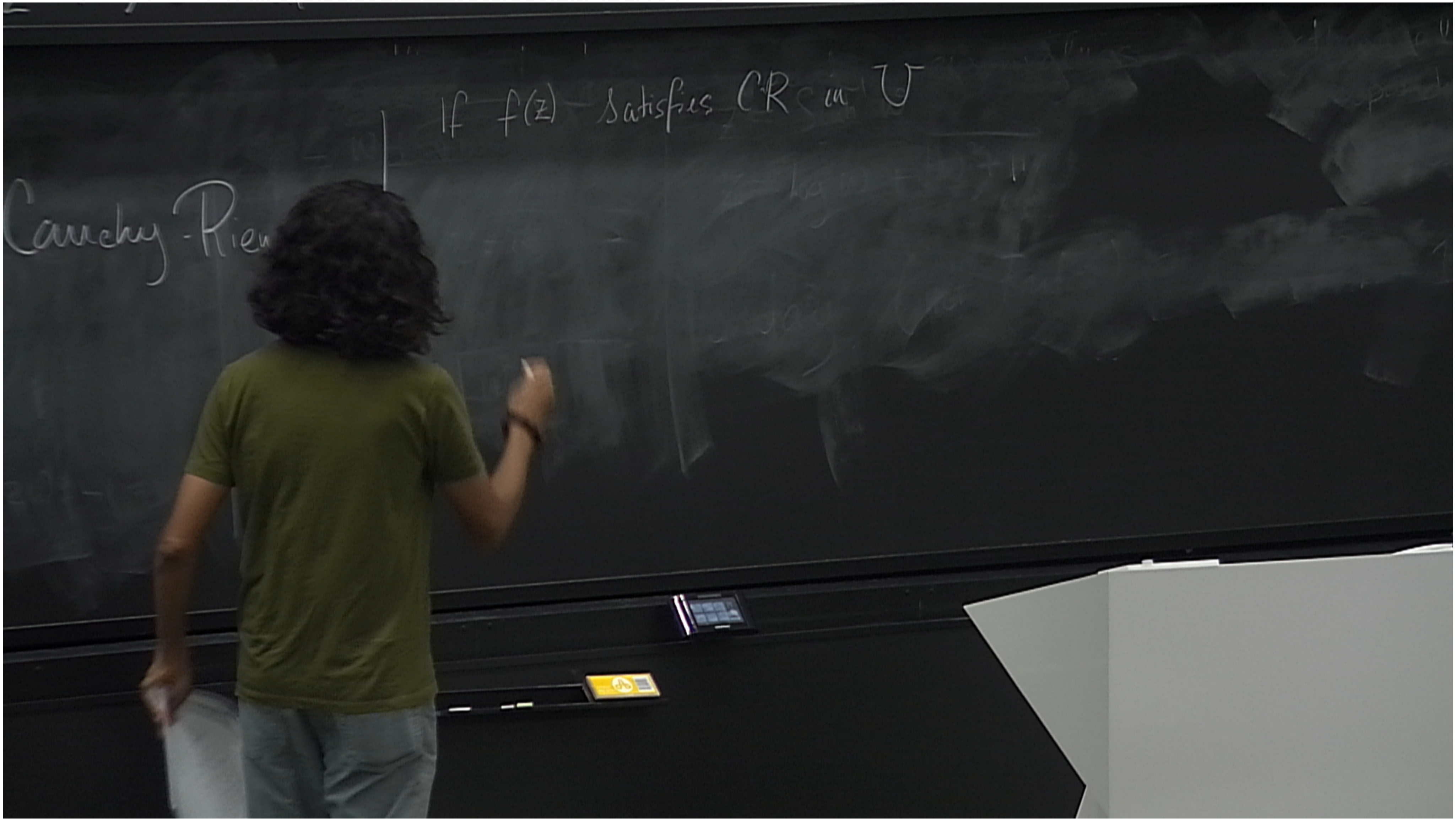
Re: $u(x_0+h, y_0+k) = u(x_0, y_0) + h\alpha - k\beta + \dots$

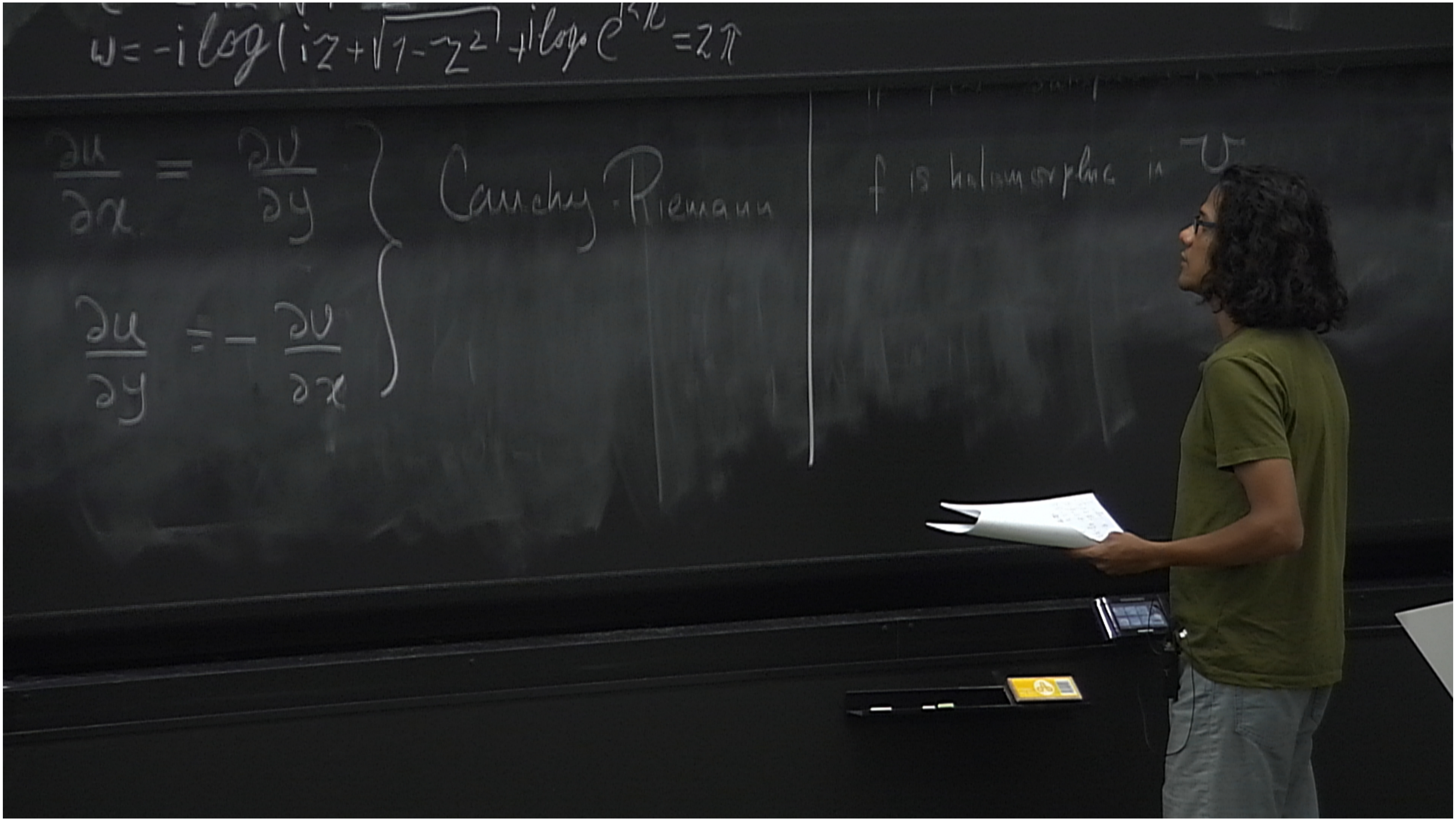
Im $v(\quad) = v(\quad) + h\beta + k\alpha + \dots$

$$\alpha = \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \qquad \beta = -\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$



$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$
$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial y}$
Cauchy-Riemann


If f(z) satisfies CR in U
Cauchy-Rie


∂u/∂x = ∂v/∂y
∂u/∂y = −∂v/∂x
Cauchy-Riemann
f is holomorphic in U

$$f_1 = (x+iy)^2 (x-iy)$$

$$f_2 = \mathrm{Re}\, z$$

$$f_3 = z^3$$

$$f_1 = (x+iy)^2 (x-iy)$$

$$f_2 = \mathrm{Re}\, z$$

$$f_3 = z^3$$

$$z = x+iy$$

$$\bar{z} = x-iy$$

$$x = \frac{1}{2}(z+\bar{z})$$

$$y = \frac{1}{2i}(z-\bar{z})$$

$$\frac{\partial}{\partial x} = \frac{\partial z}{\partial x}\frac{\partial}{\partial z} + \frac{\partial \bar{z}}{\partial x}\frac{\partial}{\partial \bar{z}}$$

$$= \frac{\partial}{\partial z} + \frac{\partial}{\partial z}$$

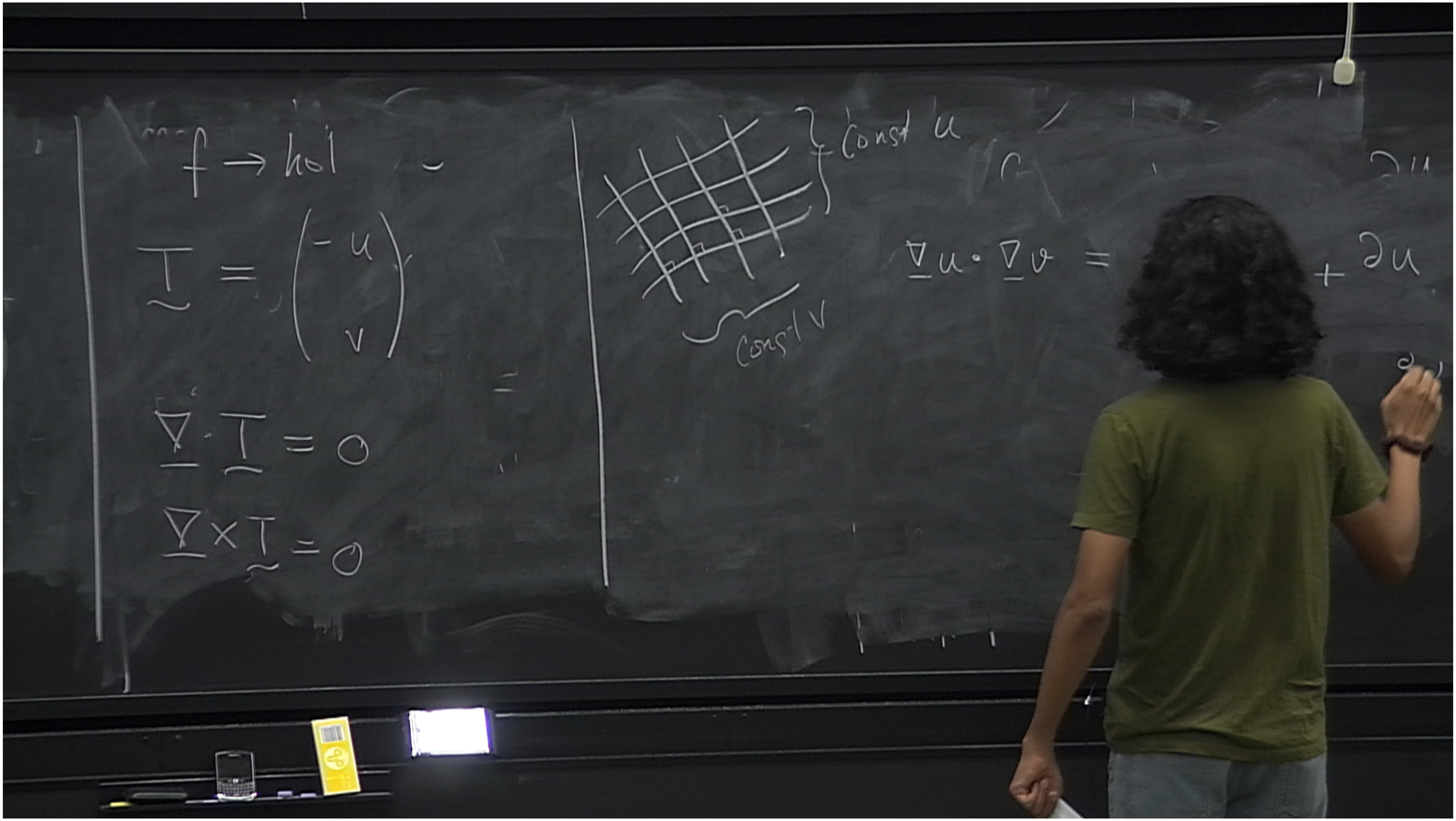
$$\frac{\partial}{\partial y} = i\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}}\right)$$

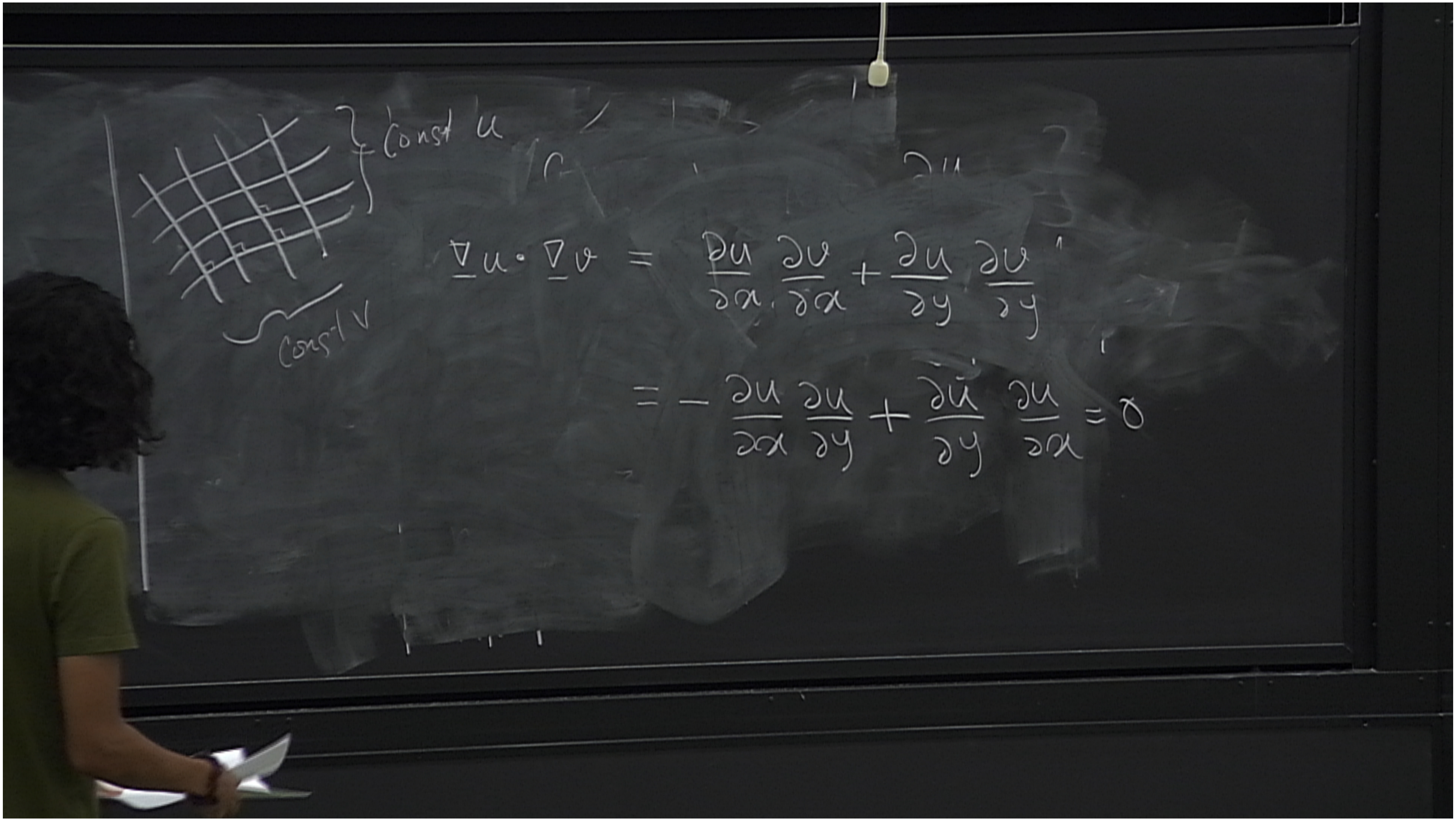
$$\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) + i\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0$$

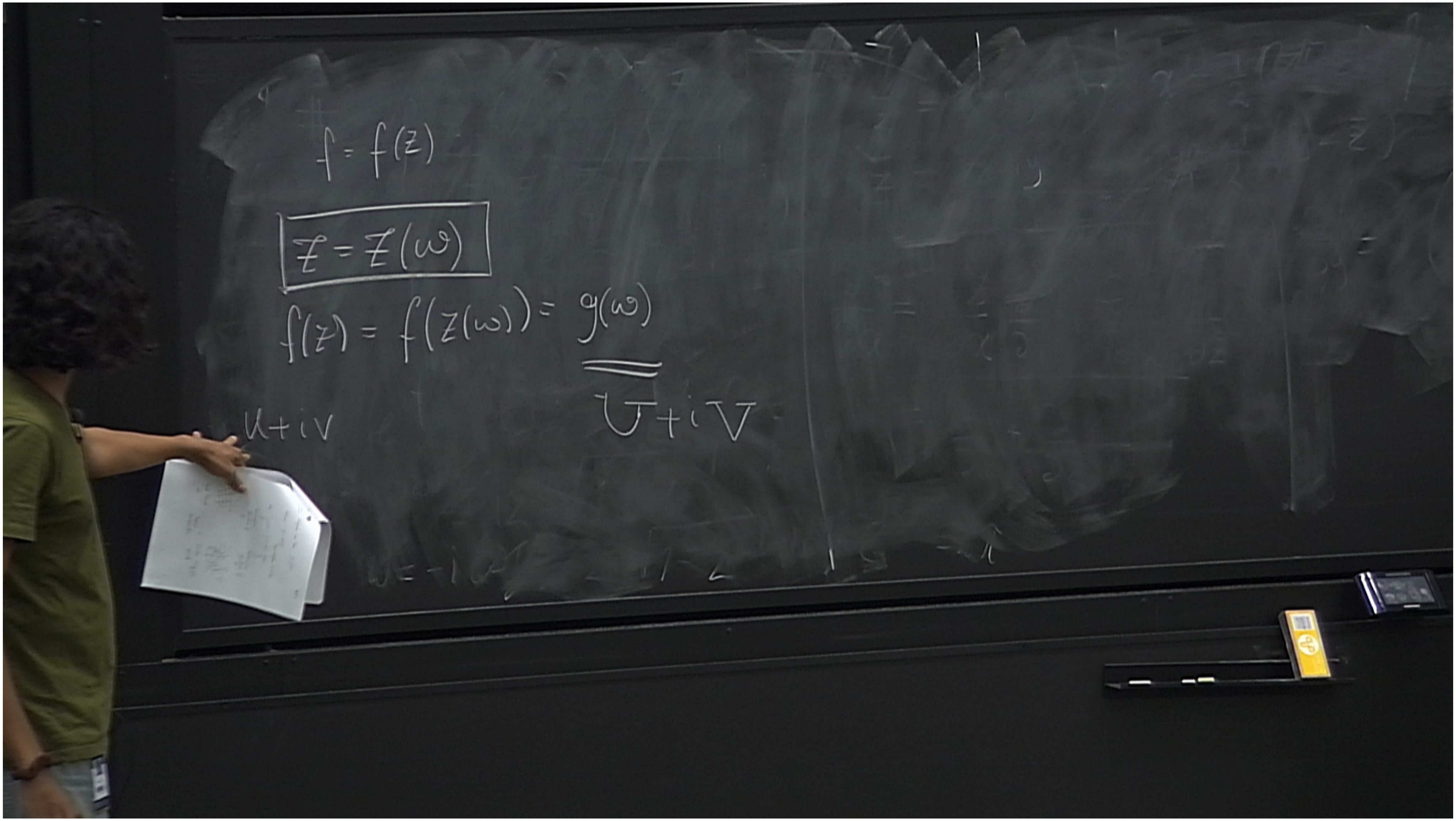
$$2\frac{\partial}{\partial \bar{z}}(u + iv) = 0$$

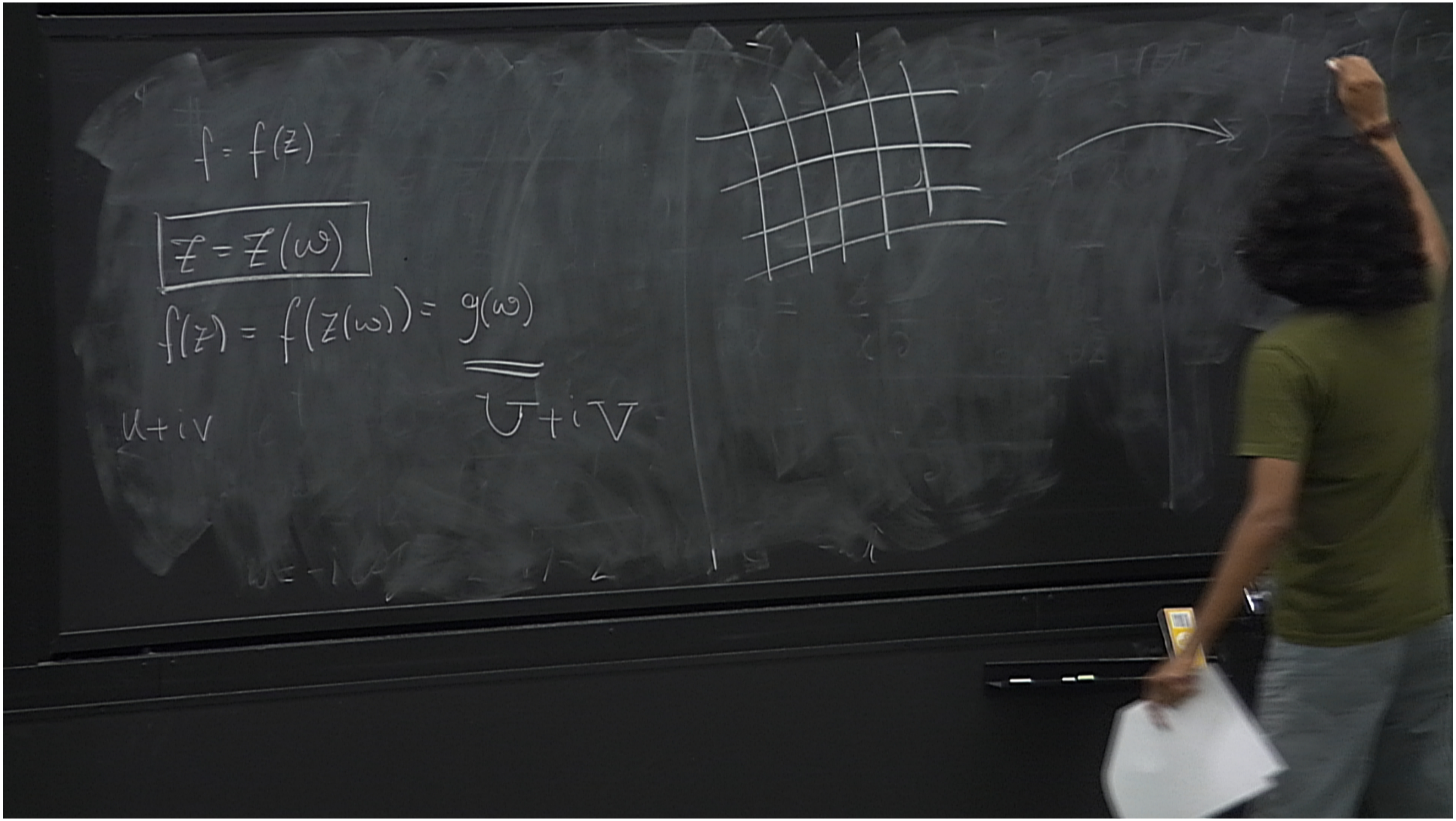
$$\frac{\partial f}{\partial \bar{z}} = 0$$

$$f = f(z)$$


f → hol
I = (-u, v)
∇ · I = 0
∇ × I = 0
const u
const v
∇u · ∇v =
+ ∂u


const u
const v
$\underline{\nabla} u \cdot \underline{\nabla} v = \frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial y}$
$= -\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial u}{\partial x} = 0$


f = f(z)
Z = Z(ω)
f(z) = f(Z(ω)) = g(ω)
u+iv
U+iV


f = f(z)
z = z(w)
f(z) = f(z(w)) = g(w)
u+iv
U+iV