Title: Topological Order in 4D

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Abstract:

Topological order in 4D

(Dalimil Mazáč + Aliosha Hamma)

- order: emergence, robustness, universality
- phase transitions

Topological order in 4D

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- order: emergence, robustness, universality
- phase transitions

  classical: vary T, F singular

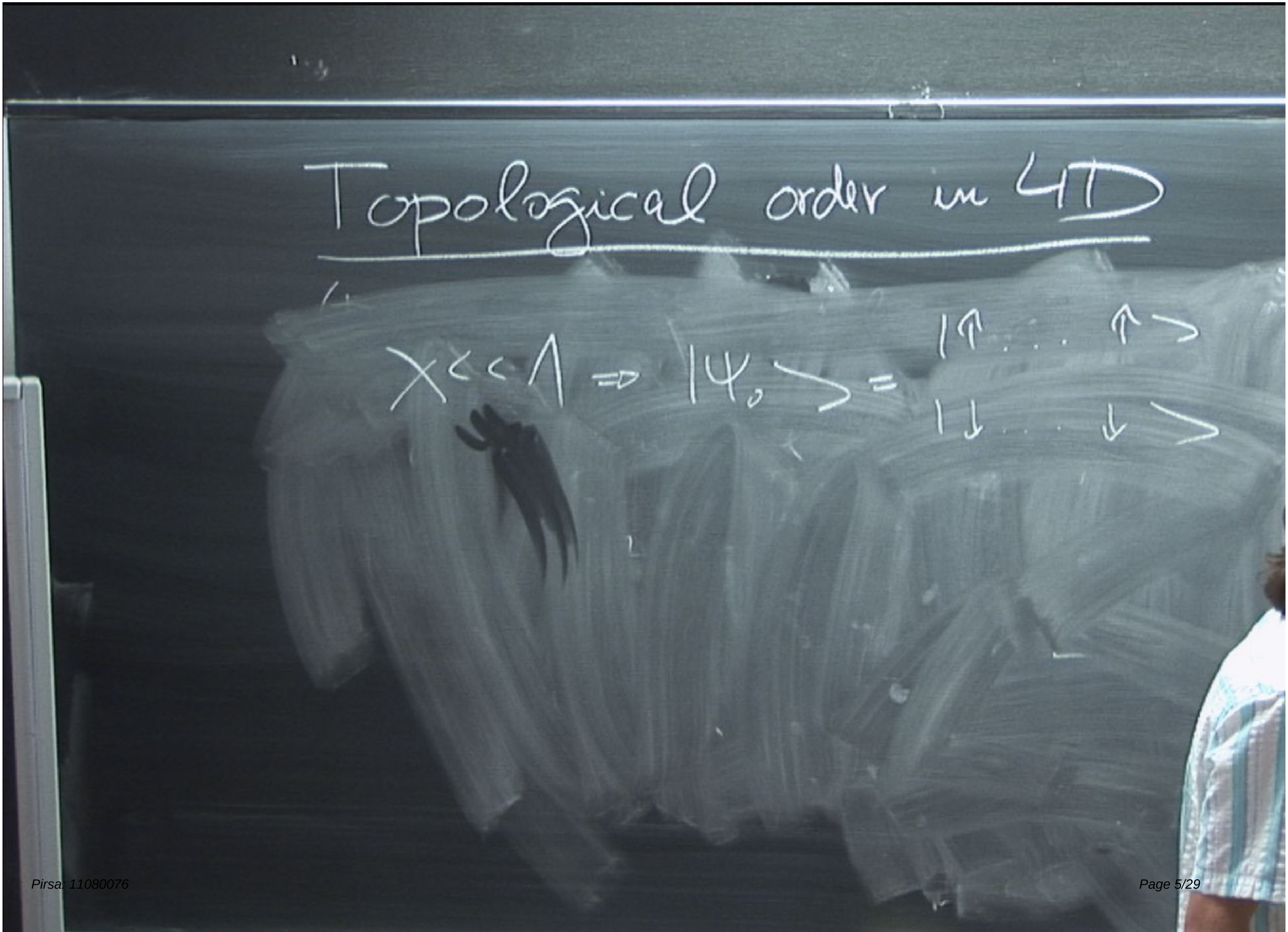
  quantum: $H(\lambda)$, gap $= 0$

Q: What characterizes a phase?

A: Symmetry ?

$$H(\lambda) = -\sum_i Z_i Z_{i+1} - \lambda \sum_i X_i$$

$$Z_i = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad X_i = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$


Topological order in 4D
X << Λ ⇒ |Ψ₀> = |↑ . . . ↑>
|↓ . . . ↓>

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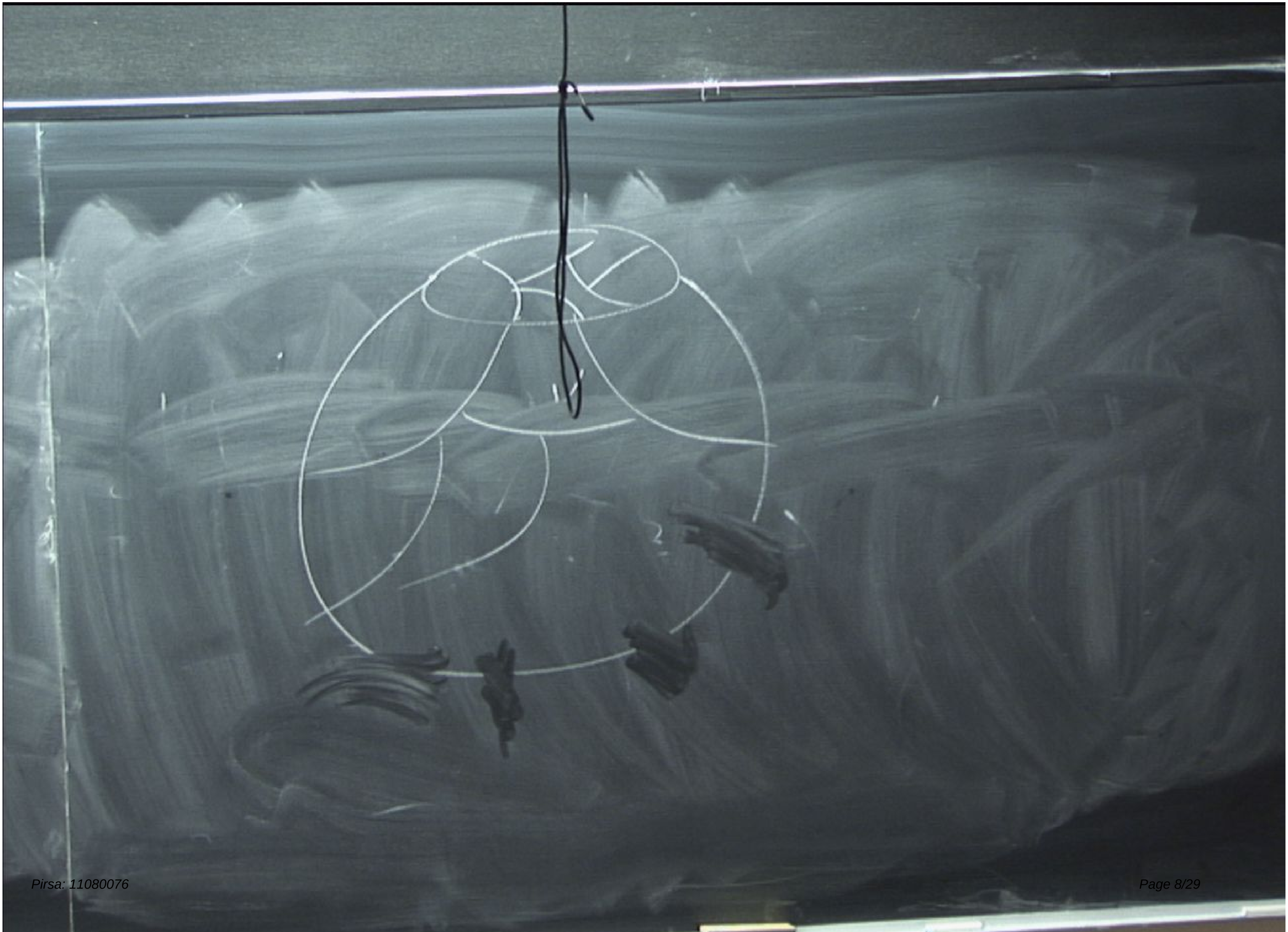
$$S = \prod_i X_i \qquad SHS^{-1} = H$$

Topological order in 4D

$$\lambda \ll \Lambda \Rightarrow |\Psi_0\rangle = \begin{matrix} |\uparrow \ldots \uparrow\rangle \\ |\downarrow \ldots \downarrow\rangle \end{matrix}$$

$$\lambda \gg \Lambda \Rightarrow |\Psi_0\rangle = |\rightarrow \quad \rightarrow\rangle$$

$$|\rightarrow\rangle = \frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}}$$



Topological order in 4D

2D toric code

N vertices, 2N , $\mathcal{H} = \bigotimes_{i=1}^{2N} \mathcal{H}_i$ $\quad \mathcal{H}_i = \mathbb{C}$

$$H = -\lambda \sum_s A_s - \mu \sum_p B_p$$

$$A_s = \prod_{i \in s}^{4} X_i \qquad B_p = \prod_{i \in p}^{4} Z_i$$

$$[A_s, A_{s'}] = [B_p, B_{p'}] = [A_s, B_p] = 0$$

$$\mathcal{H}_g = \left\{ |\psi\rangle \in \mathcal{H} \;\middle|\; \begin{array}{l} \forall s \; A_s|\psi\rangle = |\psi\rangle \\ \forall p \; B_p|\psi\rangle = |\psi\rangle \end{array} \right\}$$

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$$\mathcal{H}_g = \{ |\psi\rangle \in \mathcal{H} \mid \forall s\ A_s|\psi\rangle = |\psi\rangle,\ \forall p\ B_p|\psi\rangle = |\psi\rangle \}$$

$$G = \langle A_s | s \rangle \cong B_1(\bar{\Lambda})$$

$$|G| = 2^{N-1}$$

Topological order in 4D

2D toric code

N vertices, 2N

$$|\Psi_0\rangle = \frac{1}{\sqrt{|G|}} \sum_{g \in G} g\,|\uparrow \ldots \uparrow\rangle$$

$\Gamma'_1 =$

$\Gamma'_2 =$

$\Gamma_1 =$

$\Gamma_2 =$

$$|\psi_1\rangle = \Gamma_1 |\psi_0\rangle$$

$$|\psi_2\rangle = \Gamma_2 |\psi_0\rangle$$

$$|\psi_3\rangle = \Gamma_1 \Gamma_2 |\psi_0\rangle$$

Topological order in 4D

$O_1 = \cancel{z \; z \; z \; z}$ $\qquad \Delta_2 =$

$\{\Gamma_{1,2}, O_{1,2}\} \overset{1}{\longleftrightarrow} \{\sigma^x_{1,2}, \sigma^z_{1,2}\}$

$A_{1\to 2} \triangleq \Theta(\exp(-L))$

$$X = A \cup B$$

$$\rho = |\psi\rangle\langle\psi|$$

$$\rho_A = \mathrm{tr}_B\left(|\psi\rangle\langle\psi|\right)$$

$$S = -\mathrm{tr}_A\left(\rho_A \log \rho_A\right)$$

$$S = (N_{AB} - 1)\log 2$$

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$$S = (N_{AB} - 1)\log 2$$

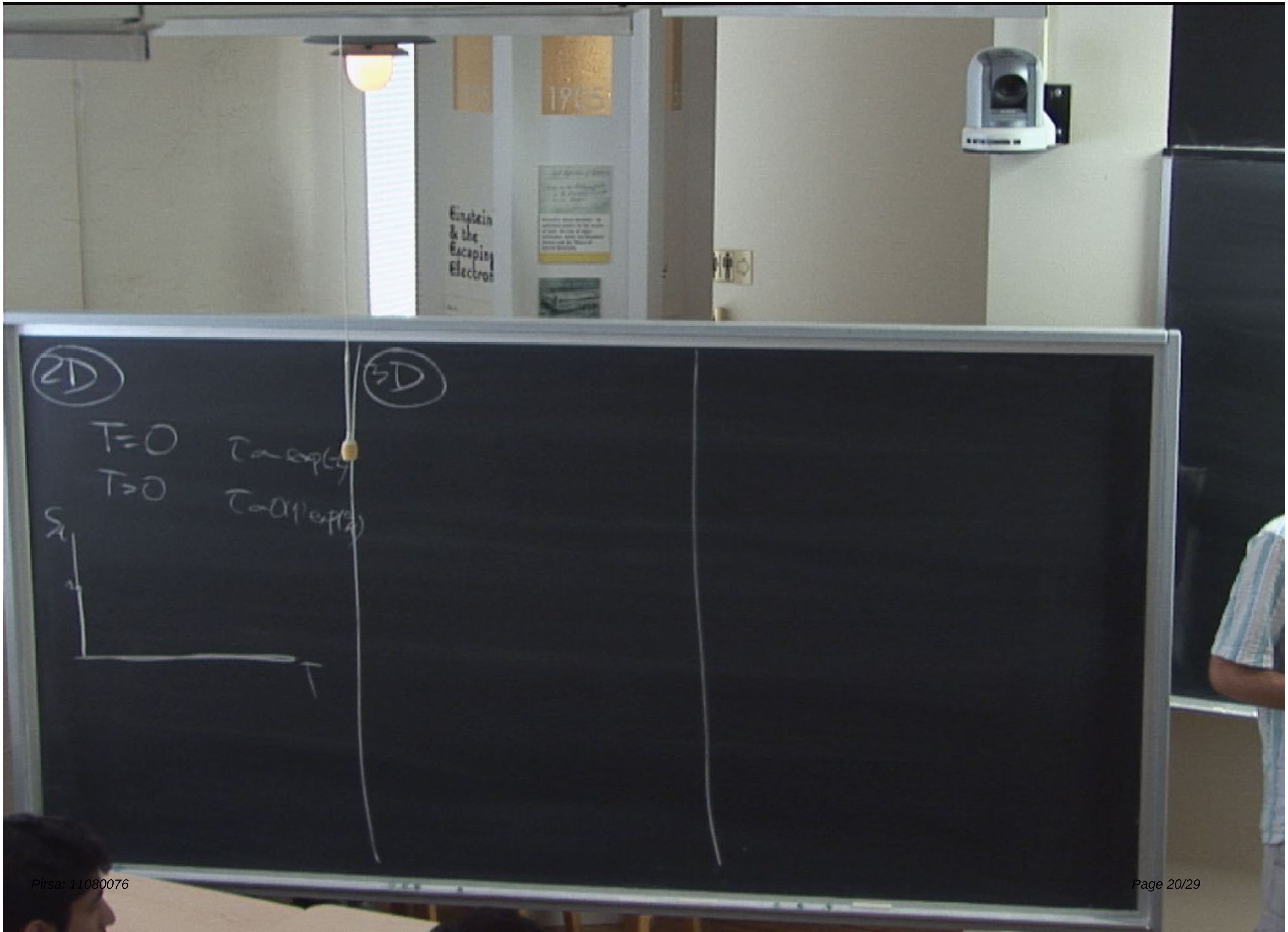
$$S = \beta|\partial| - \gamma$$ universal

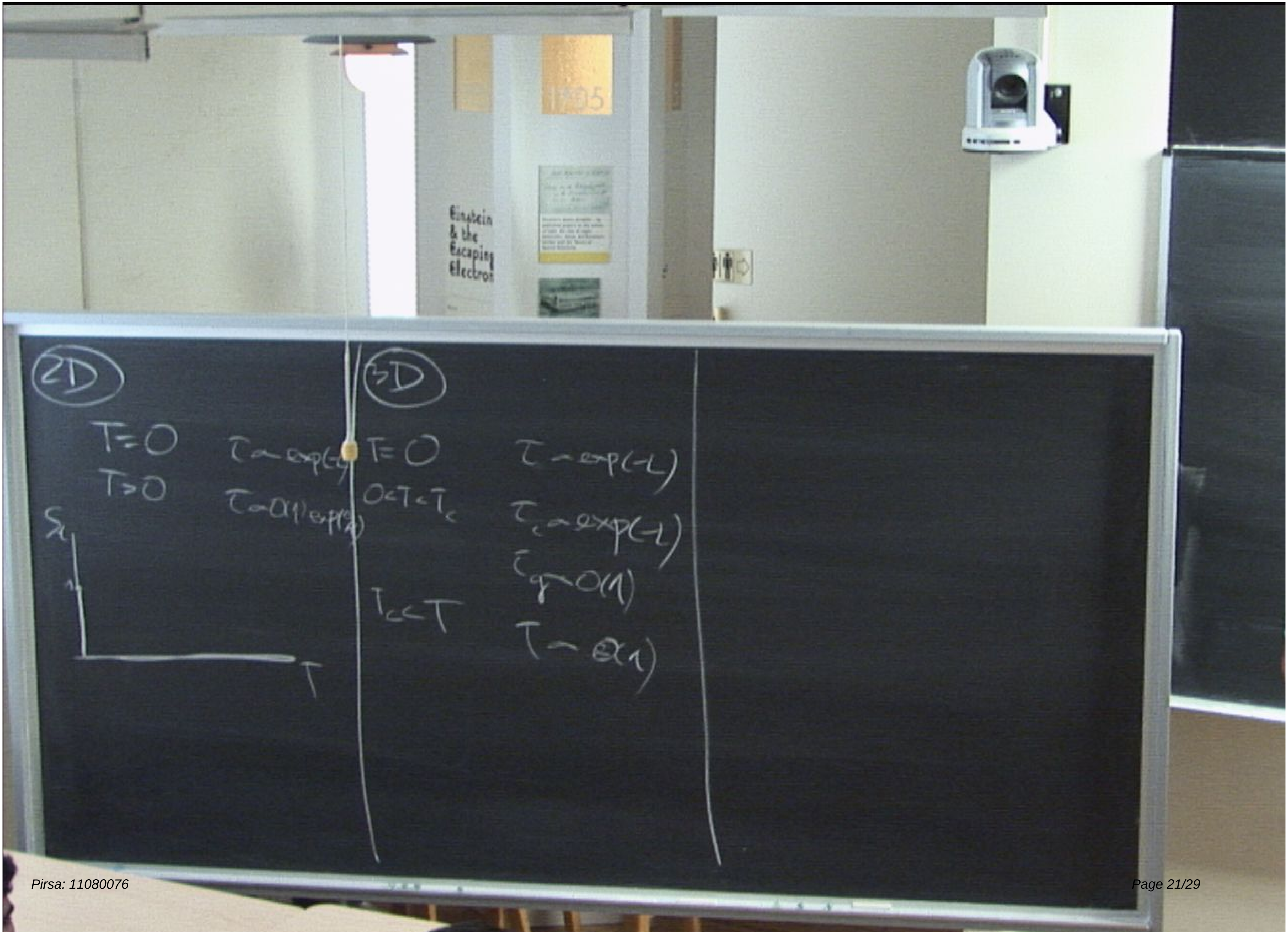
Topological order in 4D

$T > 0$ $\langle \Psi | \Psi(t) \rangle$

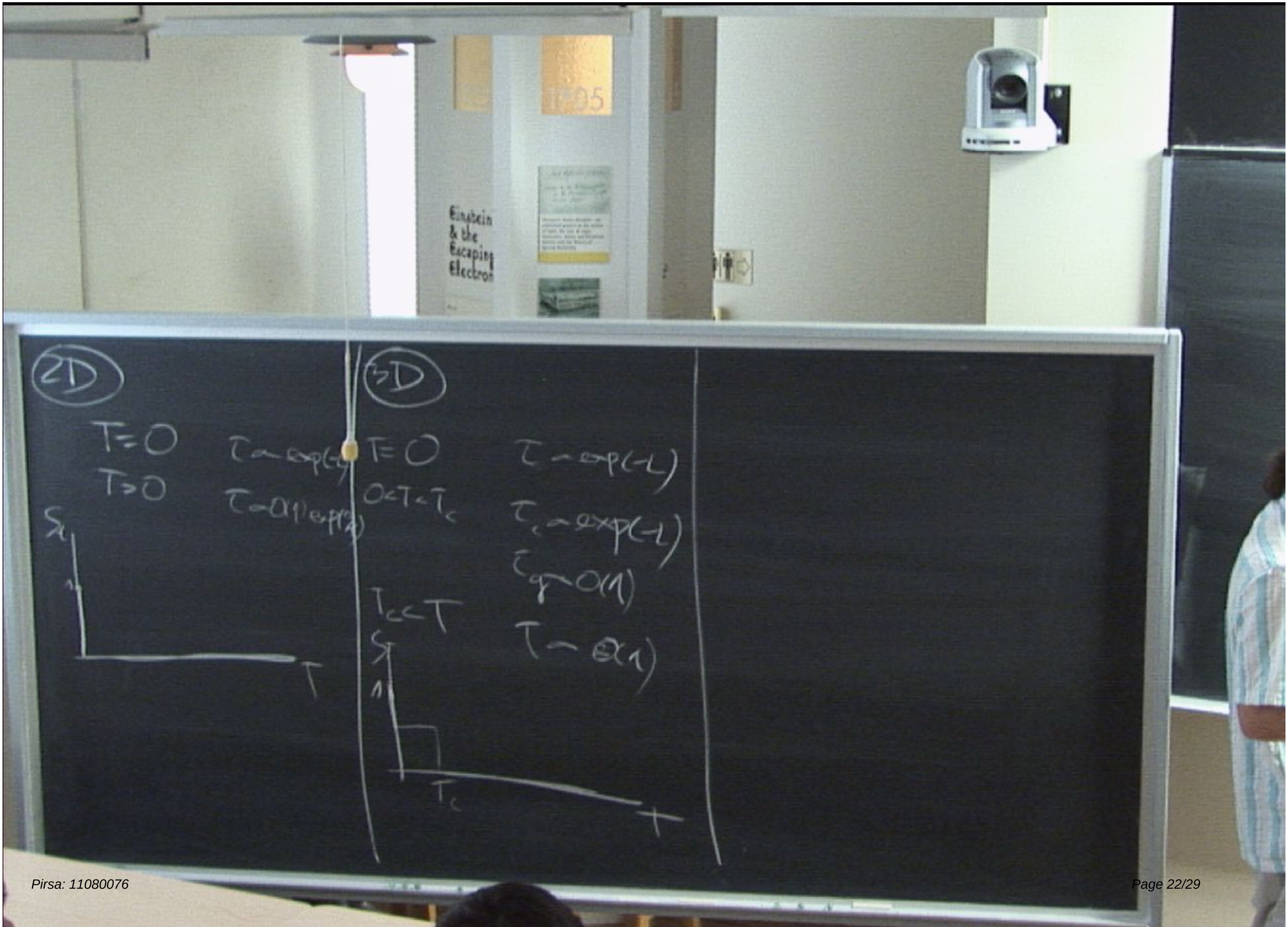
Topological order in 4D

$$S(T) = \alpha|A| + \beta|\partial| - \gamma$$


Einstein & the Escaping Electron
1905
2D
3D
T=0
T>0

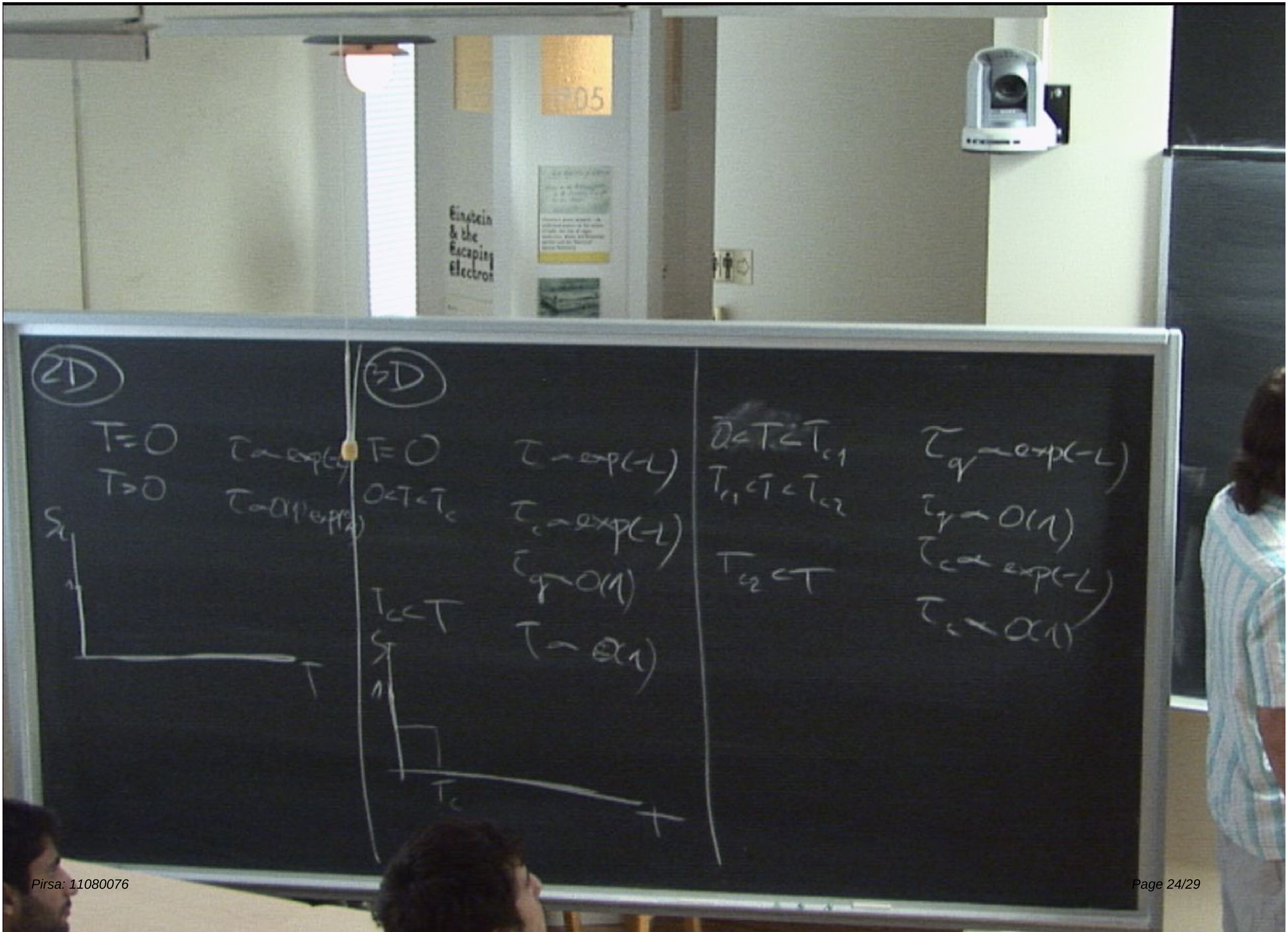


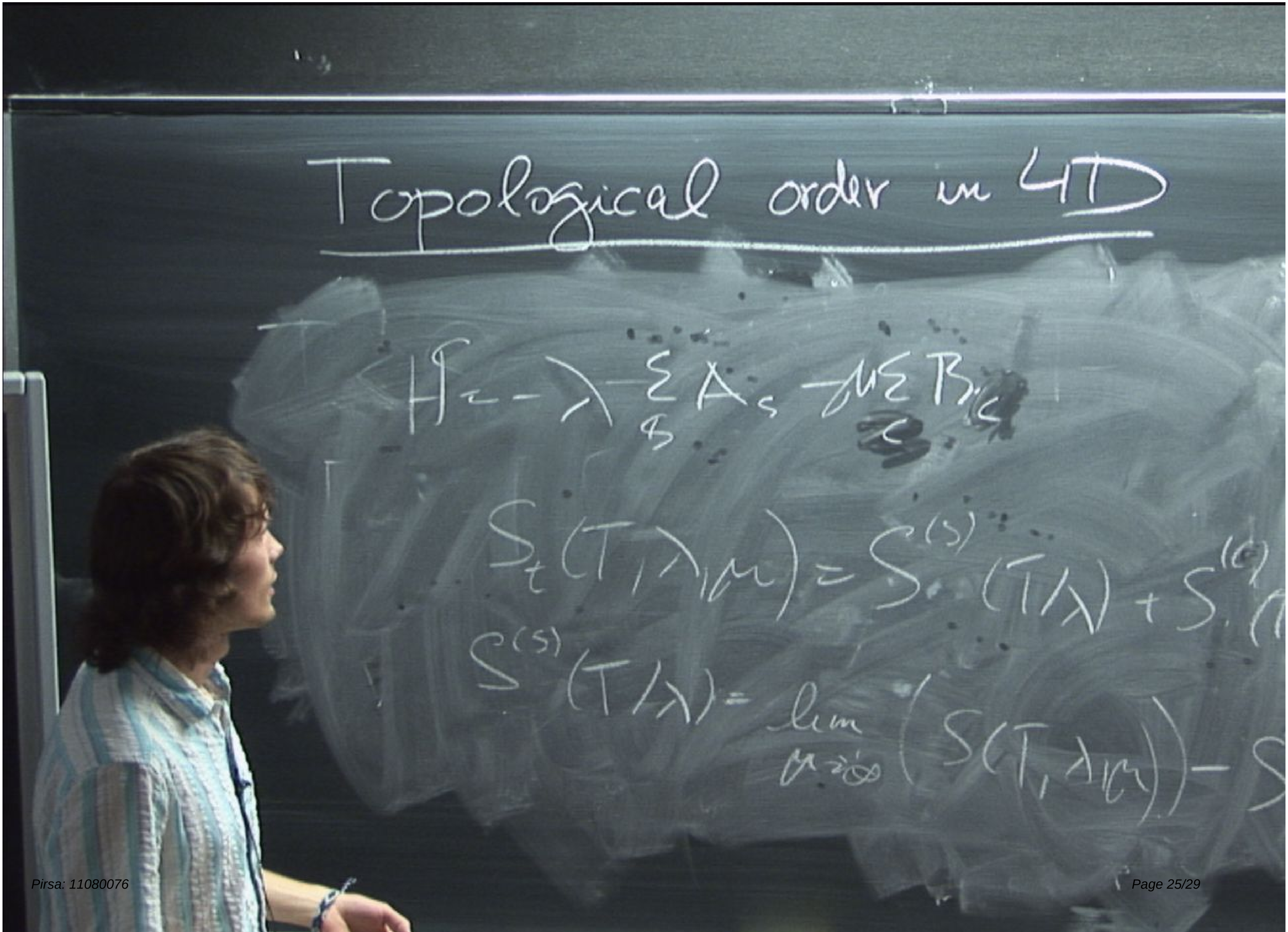
Einstein & the Escaping Electron
2D
T=0
τ ~ exp(-L)
T>0
τ ~ O(1)
S_L
T
3D
T=0
τ ~ exp(-L)
0<T<T_c
τ_c ~ exp(-L)
τ_q ~ O(1)
T_c<T
τ ~ O(1)


Einstein & the Escaping Electron
2D
T=0
τ ~ exp(-L)
T>0
3D
T=0
τ ~ exp(-L)
0<T<T_c
τ_c ~ exp(-L)
τ_q ~ O(1)
T_c<T
τ ~ O(1)
T
T_c
T

Topological order in 4D

$$H = -\lambda \sum_s A_s - \sum_c B_c$$


2D
T=0
T>0
3D
T=0
0<T<Tc
Tc<T
0<T<Tc1
Tc1<T<Tc2
Tc2<T
Einstein & the Escaping Electron


Topological order in 4D
$H = -\lambda \sum_s A_s - \mu \sum_c B_c$
$S_t(T,\lambda,\mu) = S^{(s)}(T/\lambda) + S^{(c)}($
$S^{(s)}(T/\lambda) = \lim_{\mu\to\infty} \left(S(T,\lambda,\mu) - S \right.$

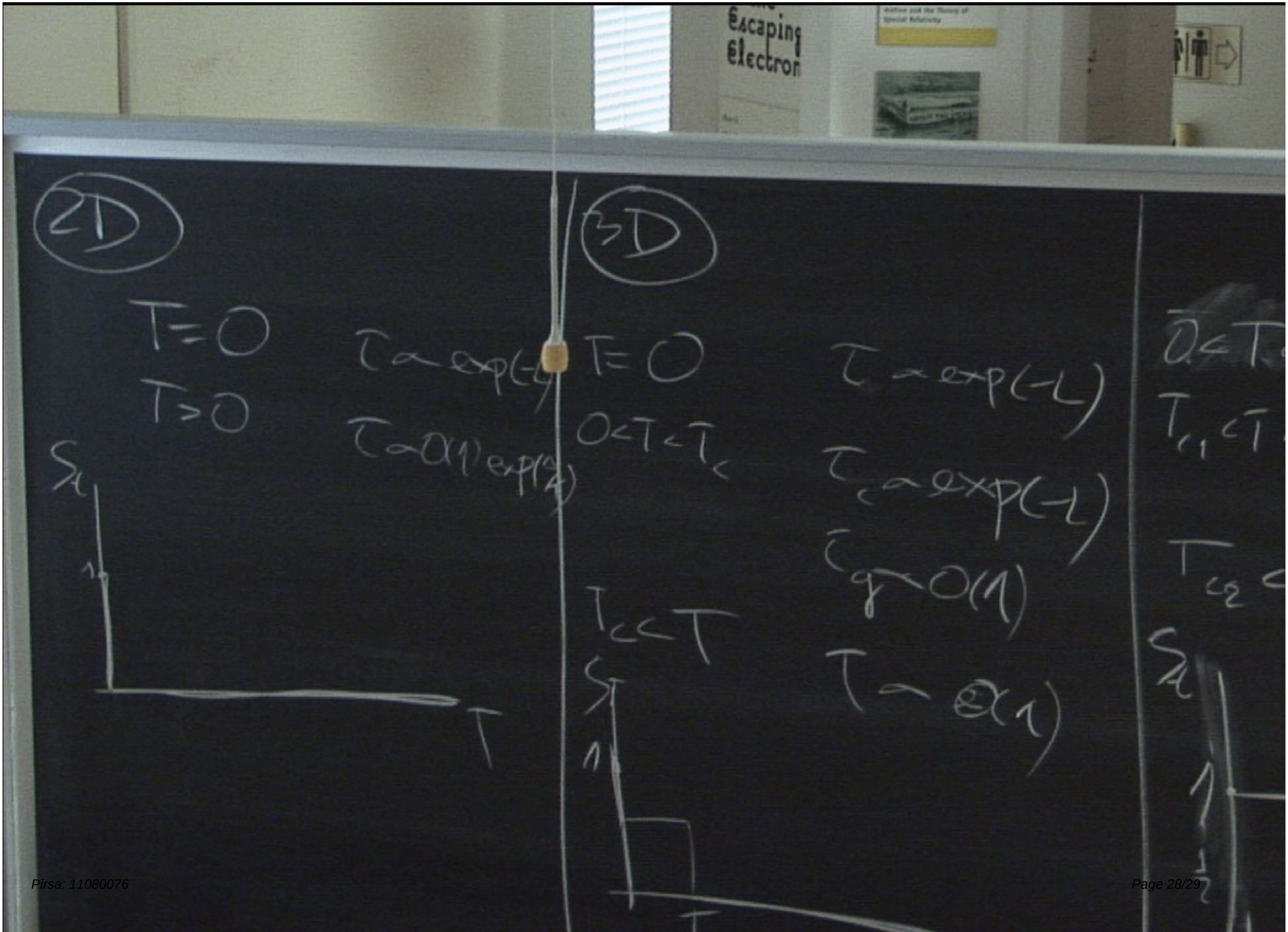
$S^{(s)}(T/\lambda) = \frac{1}{2}$, 0

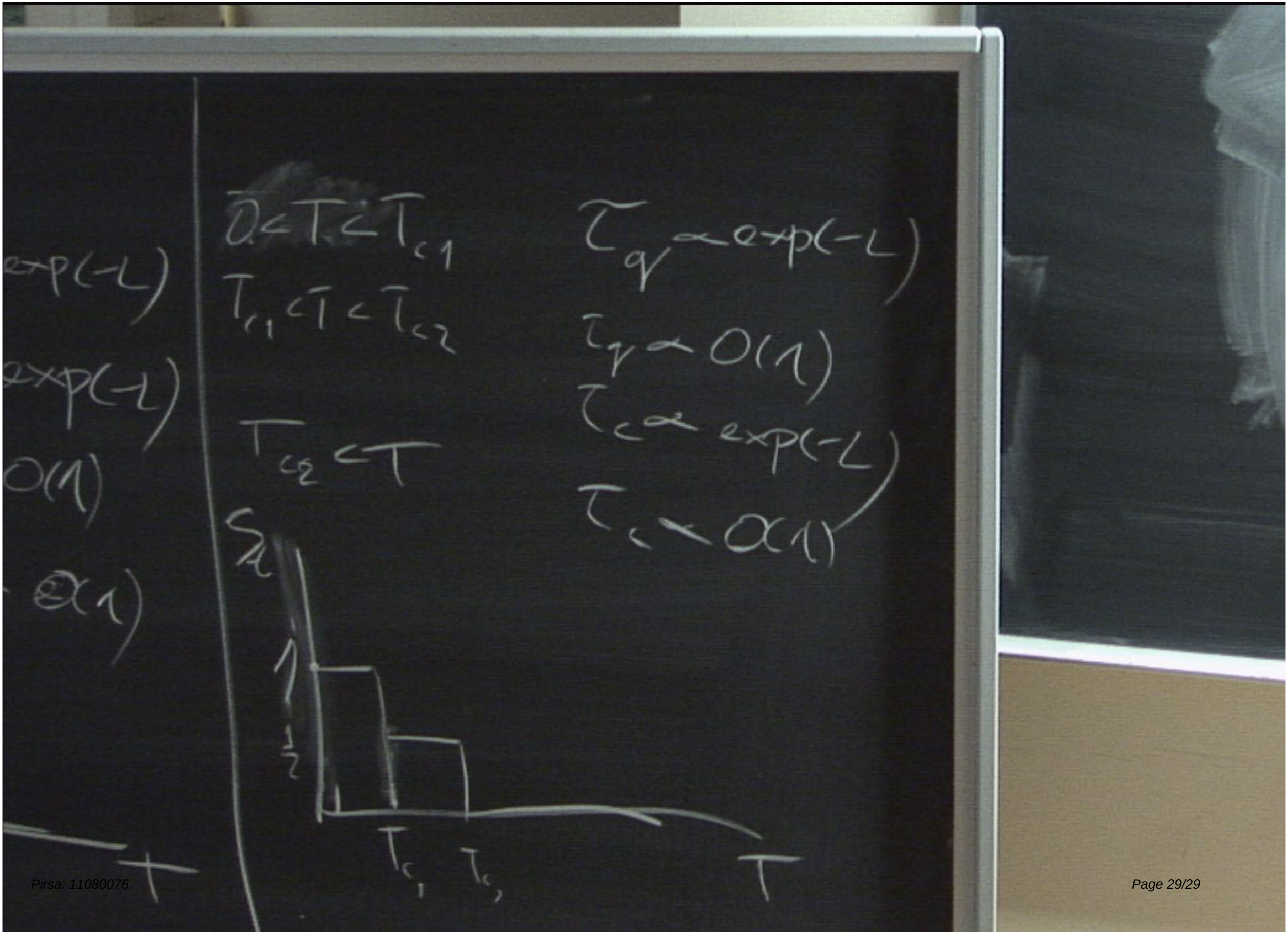
$T_{c,1} = 2.6\lambda$

(T/μ)

$S^{(s)}(0)$


2D
T=0
T>0
S
T
3D
T=0
τ ~ exp(-L)
0<T<T_c
τ_c ~ exp(-L)
τ_q ~ O(1)
T_c<T
τ ~ O(1)
0<T<T_c1
τ_q ~ exp(-L)
T_c1<T<T_c2
τ_q ~ O(1)
τ_c ~ exp(-L)
T_c2<T
τ_c ~ O(1)


Escaping
Electron
2D
T=0
T>0
S
T
3D
T=0
τ ∝ exp(-L)
0<T<T_c
τ_c ∝ exp(-L)
τ_g ∝ O(1)
T_c<T
τ ∝ O(1)
0<T
T_c1<T
T_c2


exp(-L)
exp(-L)
O(1)
0 < T < Tc1
Tc1 < T < Tc2
Tc2 < T
τq ∝ exp(-L)
τq ∝ O(1)
τc ∝ exp(-L)
τc ∝ O(1)
1
1/2
T