Title: Effects of Disorder on Superfluids

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Abstract:

Effects of Disorder on Superfluids

Nilesh Tripuraneni

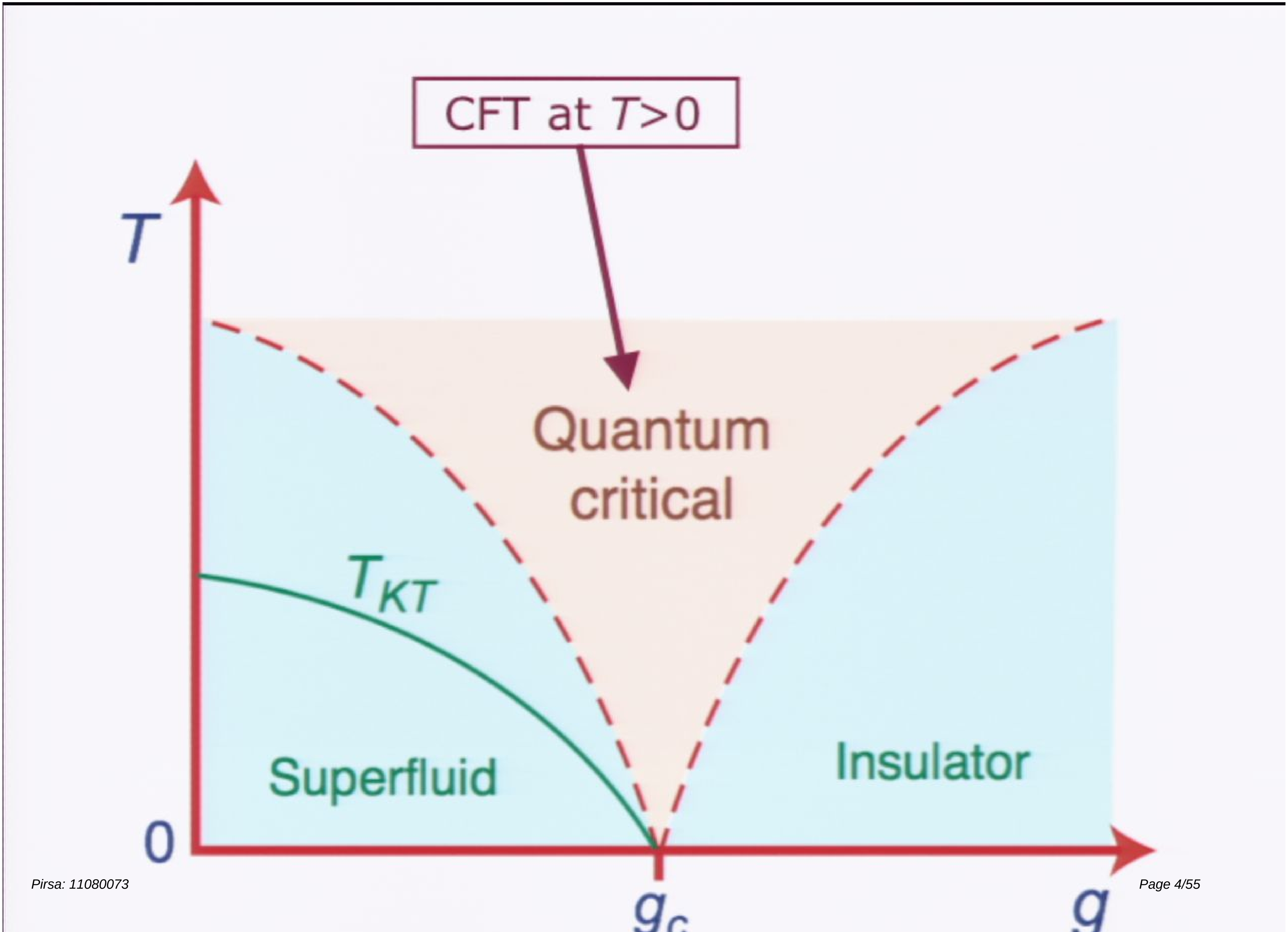
Advisors:Janet Hung and Yanwen Shang

Superconductivity/ Superfluidity

Landau/Ginzburg $\Leftrightarrow F = \alpha(T - T_c)|\varphi|^2 + \beta|\varphi|^4 + ..$

Broken U(1) symmetry

B.C.S. Theory $\Longrightarrow$ **fermions** $\Longrightarrow$ **Cooper pair** $\Longrightarrow$ **condense into boson superfluid**


CFT at $T>0$
T
Quantum critical
T_{KT}
Superfluid
Insulator
0
g_c
g

AdS/CFT Correspondence

N=4 SYM in 3+1 $\Leftrightarrow$ IIB on $AdS_5 \times S^5$

The precise statement:

$$\left\langle e^{\int d^4x \phi_0(\vec{x}) O(\vec{x})} \right\rangle_{\text{CFT}} = Z_{\text{String}}\left[\phi(\vec{x}, z=0) \equiv \phi_0(\vec{x})\right]$$

Generalized to "Gauge/Gravity" Duality

The Dictionary

trongly Coupled Field Theory $\Leftrightarrow$ Weak Gravity

$$_{\text{String}}\left[\phi(\vec{x},z=0)\equiv\phi_0(\vec{x})\right]\approx e^{-S_{classical}}\,[B.C.\rightarrow\phi_0(\vec{x})]$$

Fields in AdS $\Leftrightarrow$ Local Operators in CFT
Spin $\Leftrightarrow$ Spin
Mass $\Leftrightarrow$ Scaling Dimension
Black Holes $\Leftrightarrow$ Thermodynamics
Gauged Symmetries $\Leftrightarrow$ Global Symmetries

Gravity Dual to Superconductor/ Superfluid

- **Einstein-Maxwell-charged scalar**

$$= \int \sqrt{-g}\, d^4x \left(R + \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \left| \nabla_\mu \psi - iqA_\mu \psi \right|^2 - m^2 |\psi|^2 \right.$$

$m^2{}_{eff} = m^2 + q^2 g^{tt} A_t{}^2 \Leftrightarrow$ **Lower temperature** $\Longrightarrow$
g^{tt} **closer to double horizon**
$\Longrightarrow$ **more instability**

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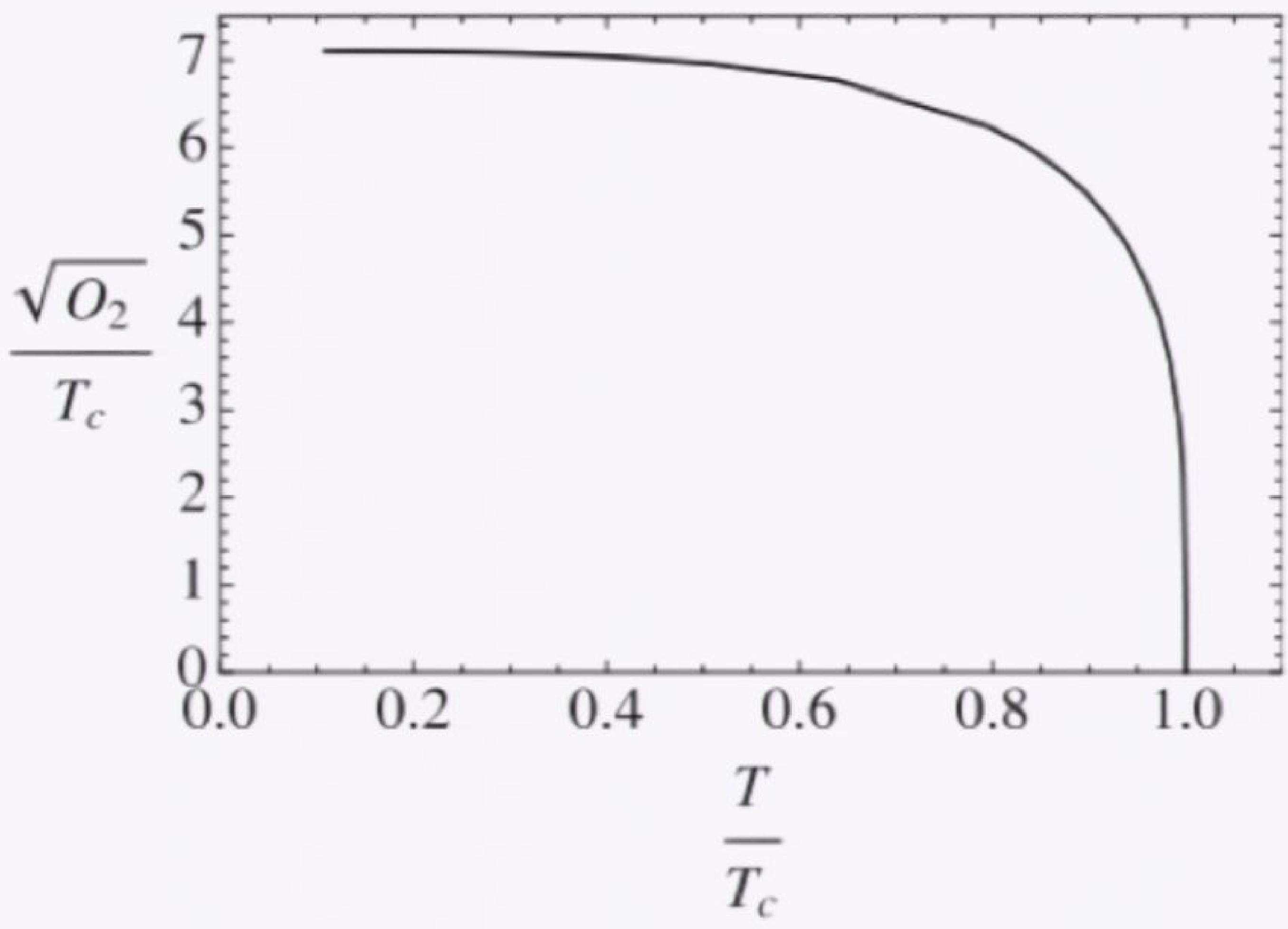
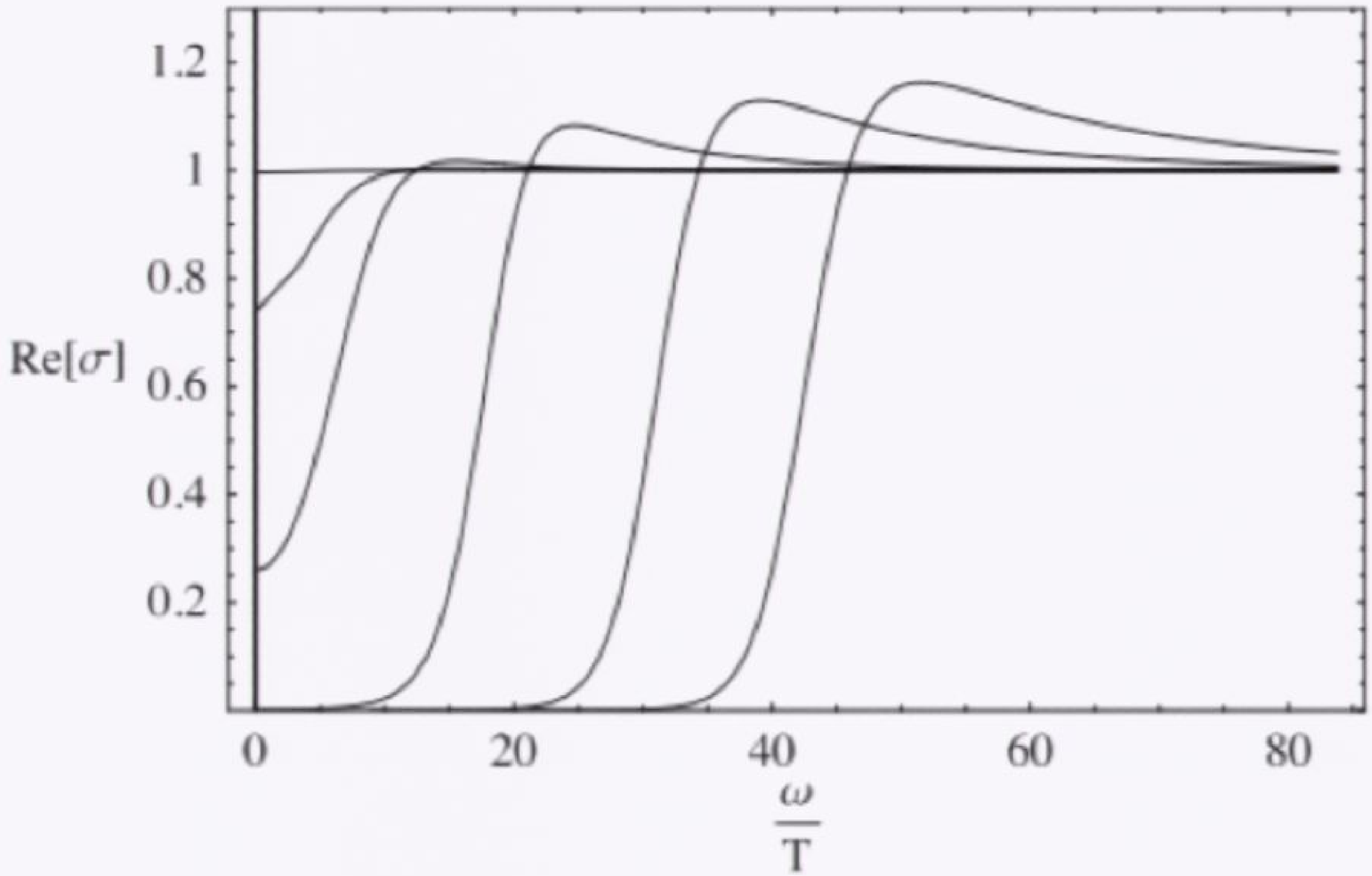
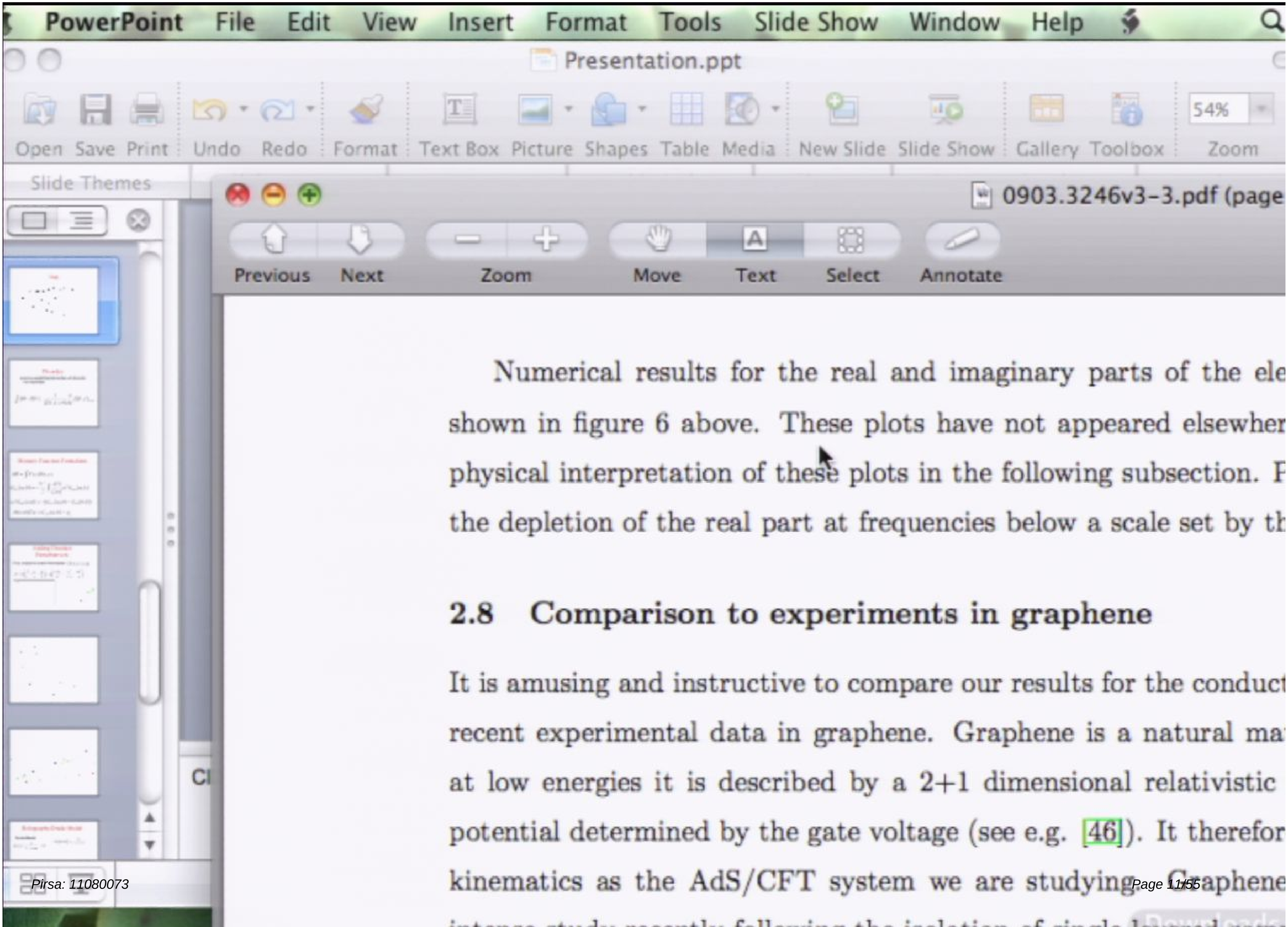
Condensation



Gap

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Numerical results for the real and imaginary parts of the
shown in figure 6 above. These plots have not appeared
physical interpretation of these plots in the following subsection.
the depletion of the real part at frequencies below a scale set by
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It is amusing and instructive to compare our results for the
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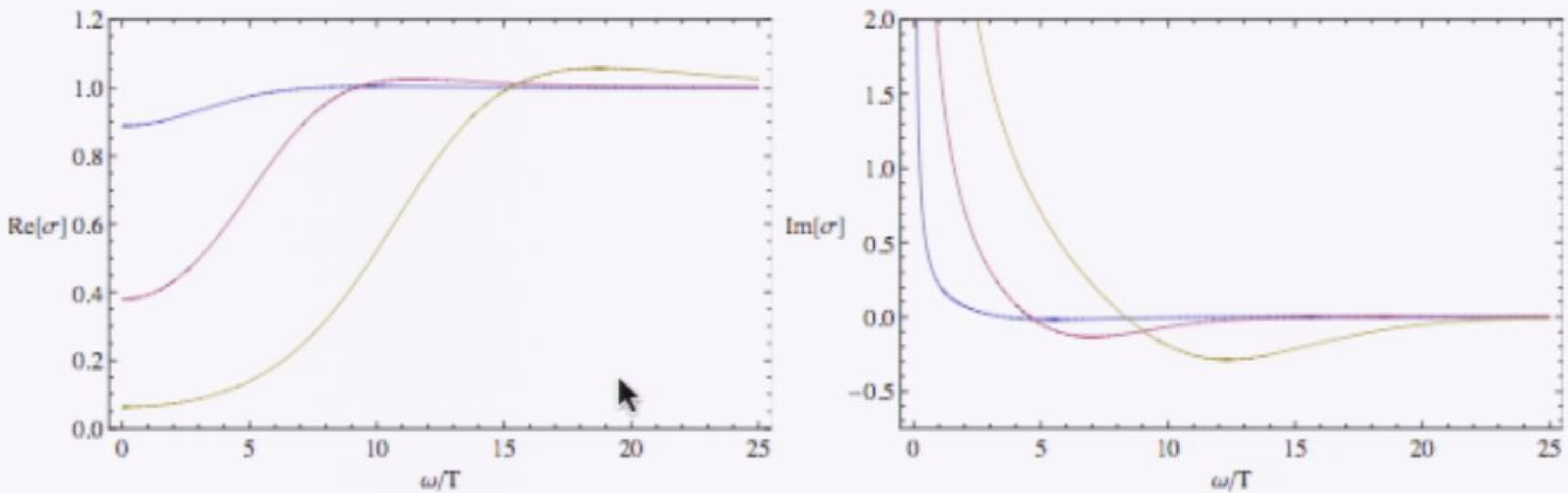

Figure 6: The real (left) and imaginary (right) parts of the electrical conductivity computed via AdS/CFT as described in the text. The conductivity is shown as a function of frequency. Different curves correspond to different values of the chemical potential at fixed temperature. The gap becomes deeper at larger chemical potential. We have set $g = 1$ in (115).

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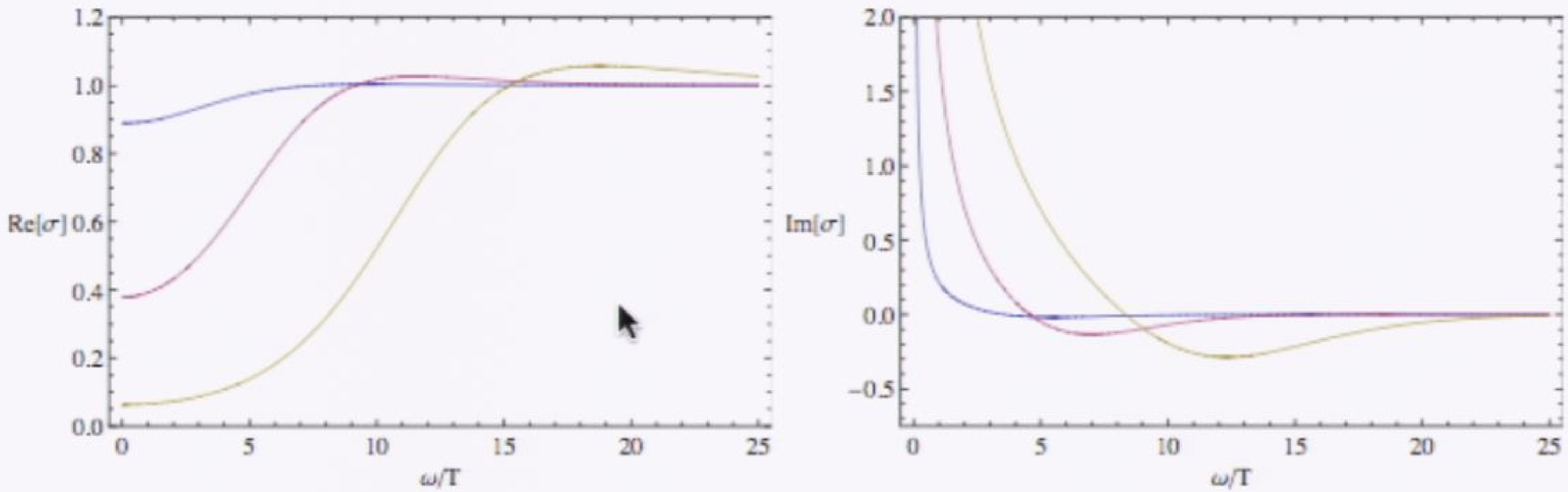

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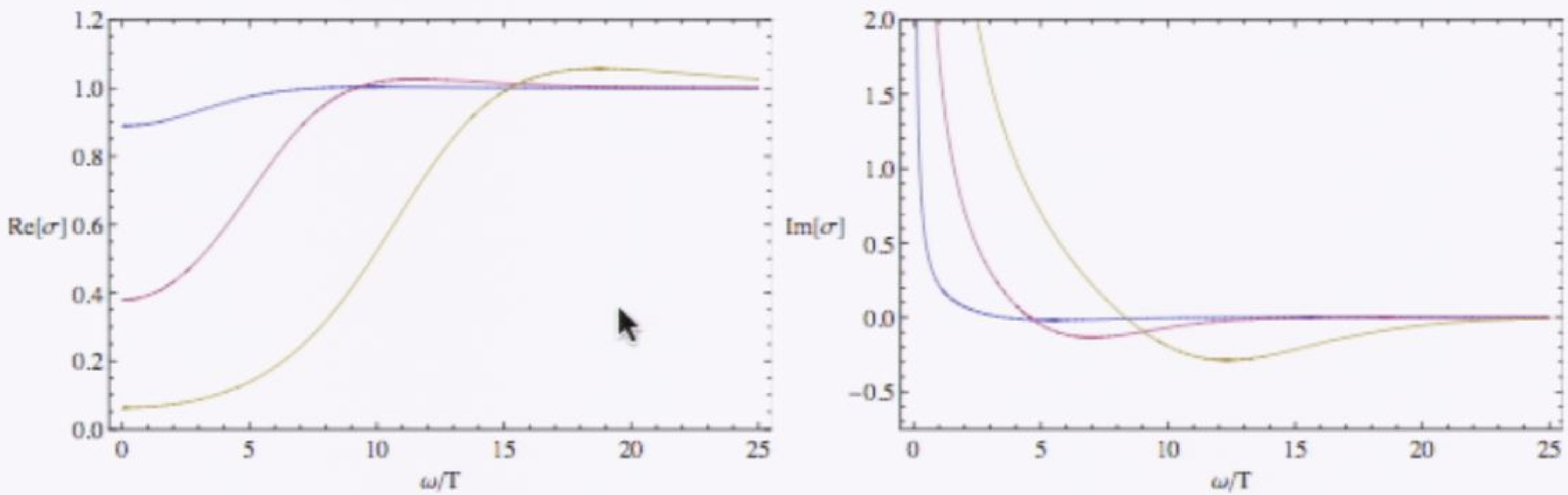

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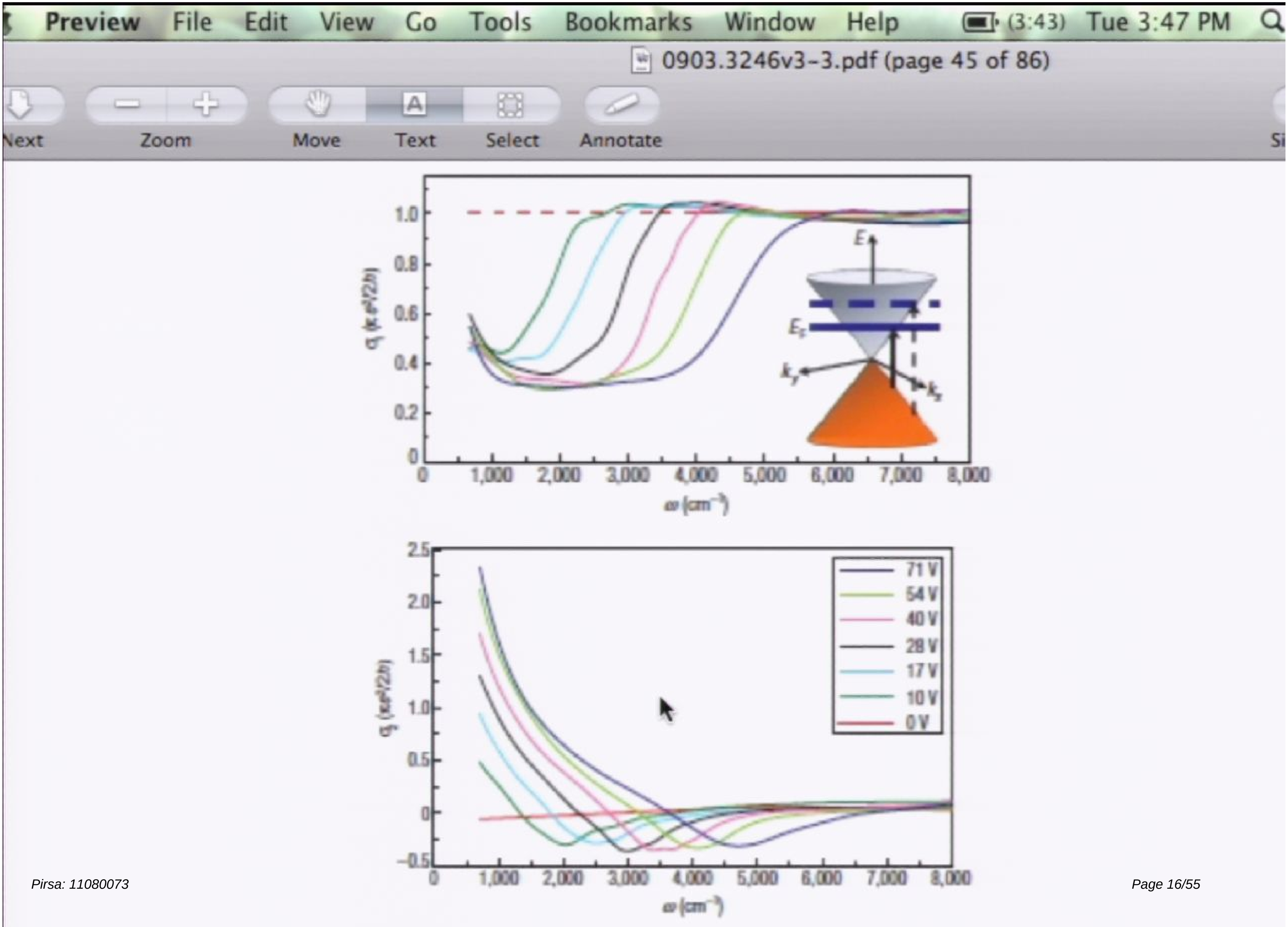
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k_x
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2.0
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1.0
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0
−0.5
71 V
54 V
40 V
28 V
17 V
10 V
0 V

Fields

$$\psi = \frac{O_1}{r} + \frac{O_2}{r^2} + \cdots \quad \phi = \mu - \frac{\rho}{r} + \cdots$$

$$g = r^2 + a + \frac{b}{r} \ldots \quad e^{-\chi} g = r^2 - \frac{c}{2r} \ldots$$

$$e^{\chi} \rightarrow e^{\chi} a^2, t \rightarrow at, \phi \rightarrow \phi / a$$

$$(\psi_H, E_H) \mapsto (\mu, \rho, O_1, O_2, M)$$

Disorder

precise weighting/interaction of disorder not important

$$\int DV \;\; P[V] \;\; \frac{1}{Z[V,J=0]} \frac{\delta}{\delta J} Z[V,J]_{J=0}$$

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Memory Function Formalism

$$\delta H = \int V(y) O(t,y)$$

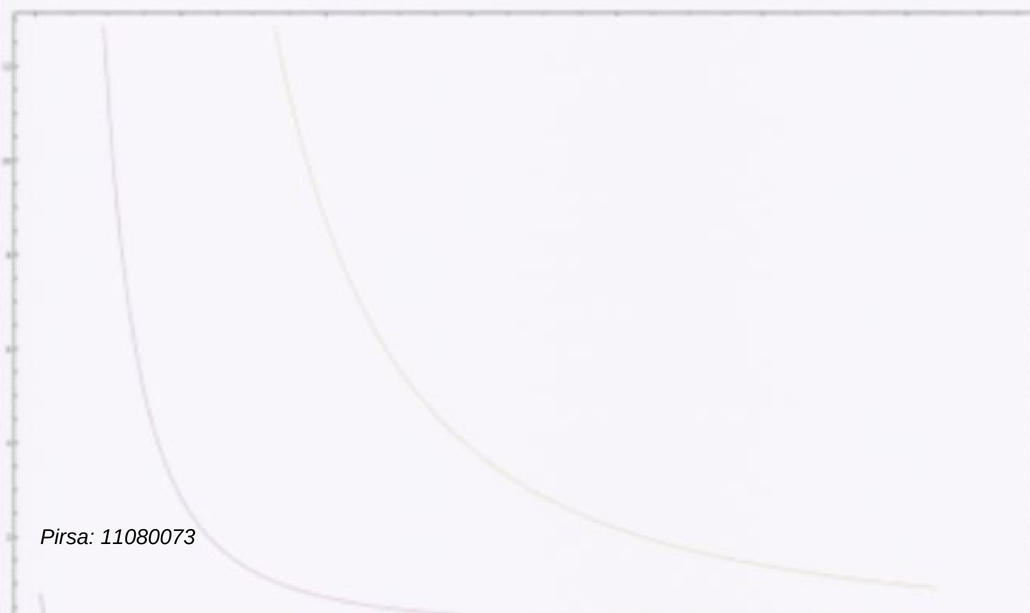
$$G_{FF}(\omega,0) = -\frac{\bar{V}^2}{2} \int \frac{d^2k}{(2\pi)^2} k^2 G_{OO}(\omega,k)$$

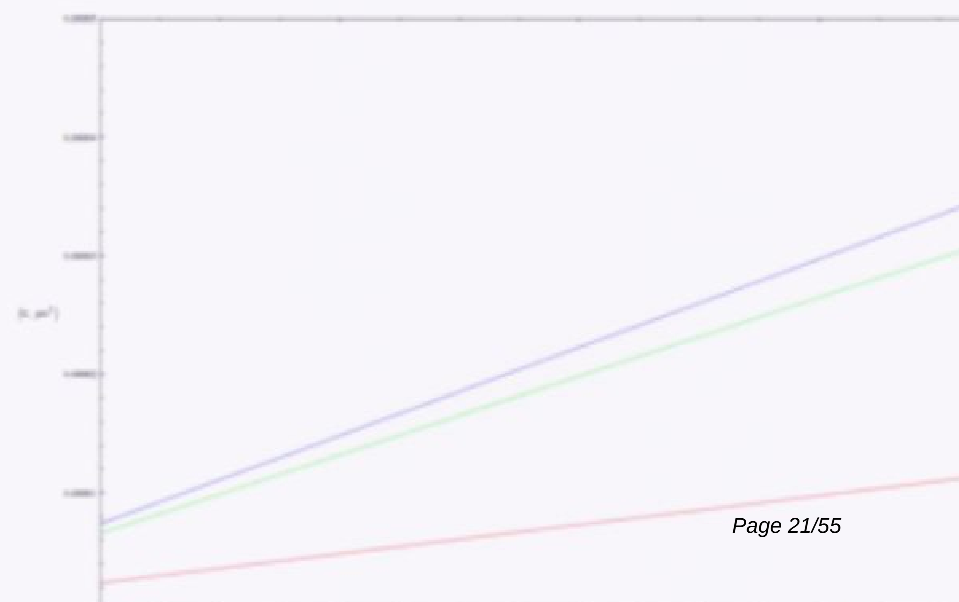
$$\omega^2 G_{PP}(\omega,0) = -\left(G_{FF}(\omega,0) - G_{FF}(0,0)\right)$$

Adding Disorder Perturbatively

Free, massive scalar field matter $O(t,y) = \phi$

$$\phi'' - \phi'\left(\frac{2}{r} + \frac{g'}{g} - \frac{\chi'}{2}\right) + \phi\left(\frac{e^{\chi}\omega^2}{g^2} - \frac{k^2}{r^2 g} - \frac{m^2}{g}\right)$$





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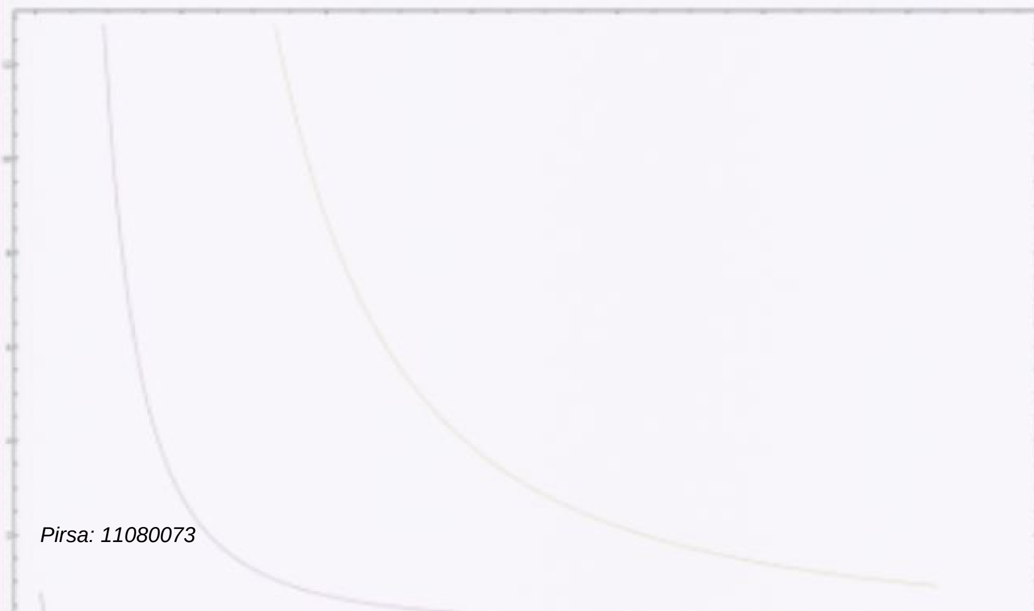

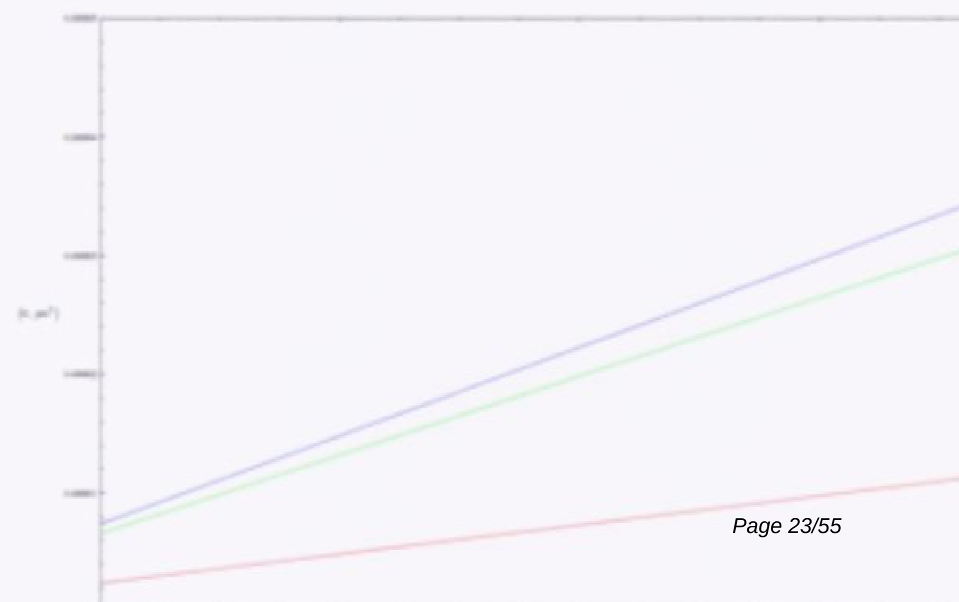

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Schwarzschild-AdS Black Brane

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2)$$

$$\psi = \psi(r) \qquad\qquad A_\mu = (\phi(r), 0, 0, 0)$$

Equations of Motion

$$\psi''+\left(\frac{g'}{g}-\frac{\chi'}{2}+\frac{2}{r}\right)\psi'+\frac{q^2\phi^2e^{\chi}}{g^2}\psi-\frac{m^2}{g}\psi=0$$

$$\phi''+\left(\frac{\chi'}{2}+\frac{2}{r}\right)\phi'-\frac{2q^2\psi^2}{g}\phi=0$$

$$\chi'+r\psi'^2+\frac{rq^2\phi^2\psi^2e^{\chi}}{g^2}=0$$

$$g'+\left(\frac{1}{r}-\frac{\chi'}{2}\right)g+\frac{r\phi'^2e^{\chi}}{4}-3r+\frac{rm^2\psi^2}{2}=0$$

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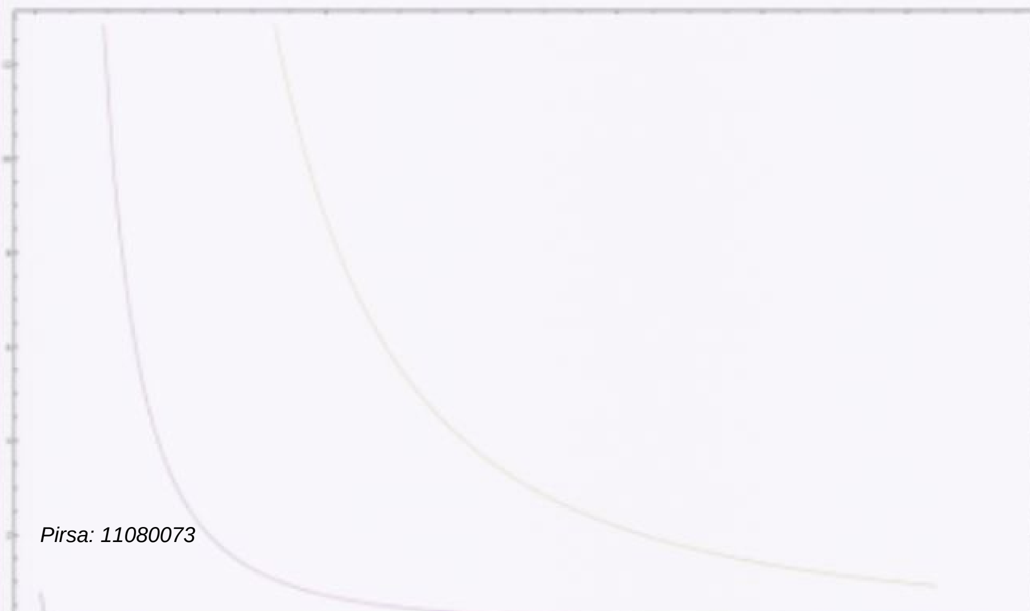

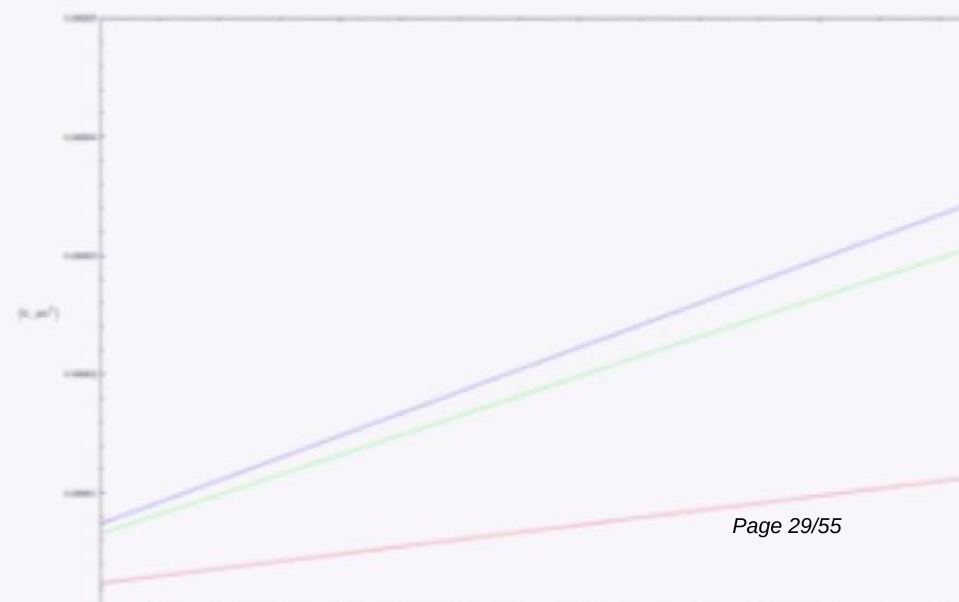

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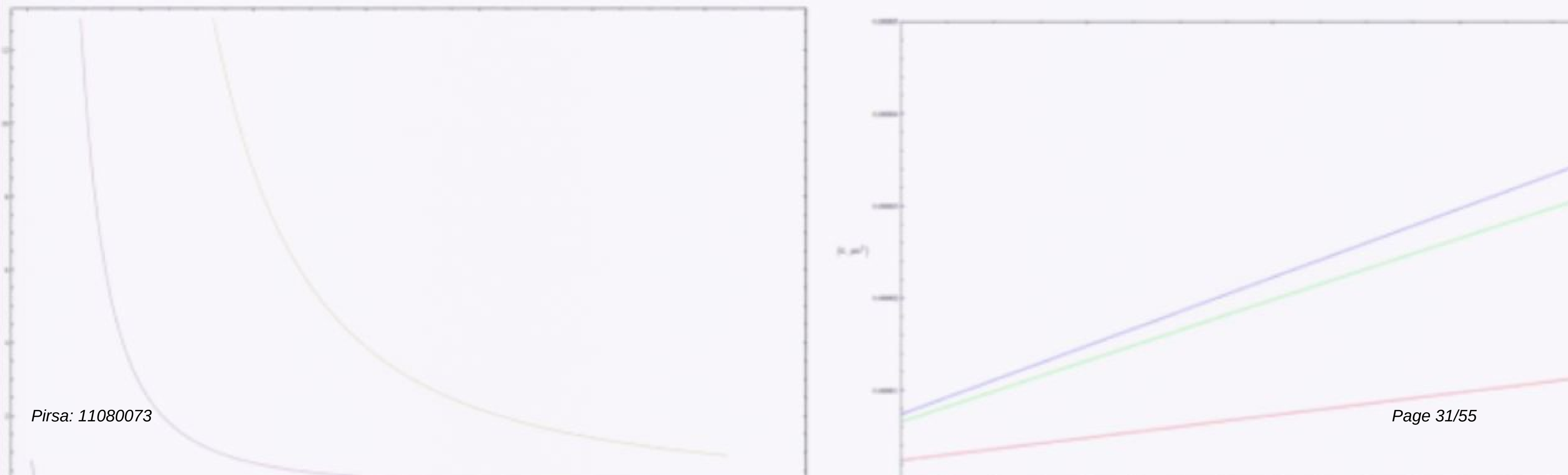

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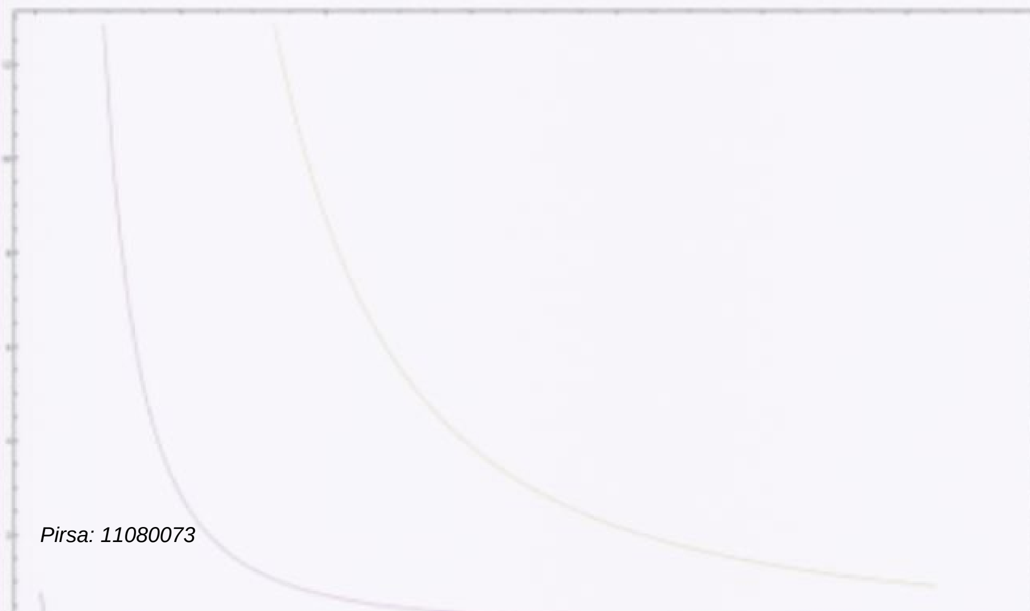

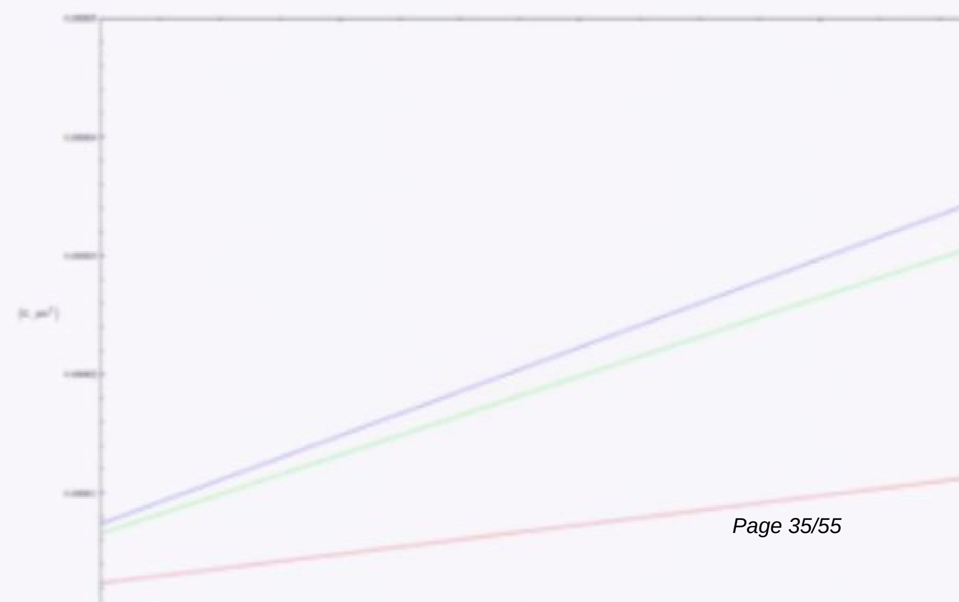

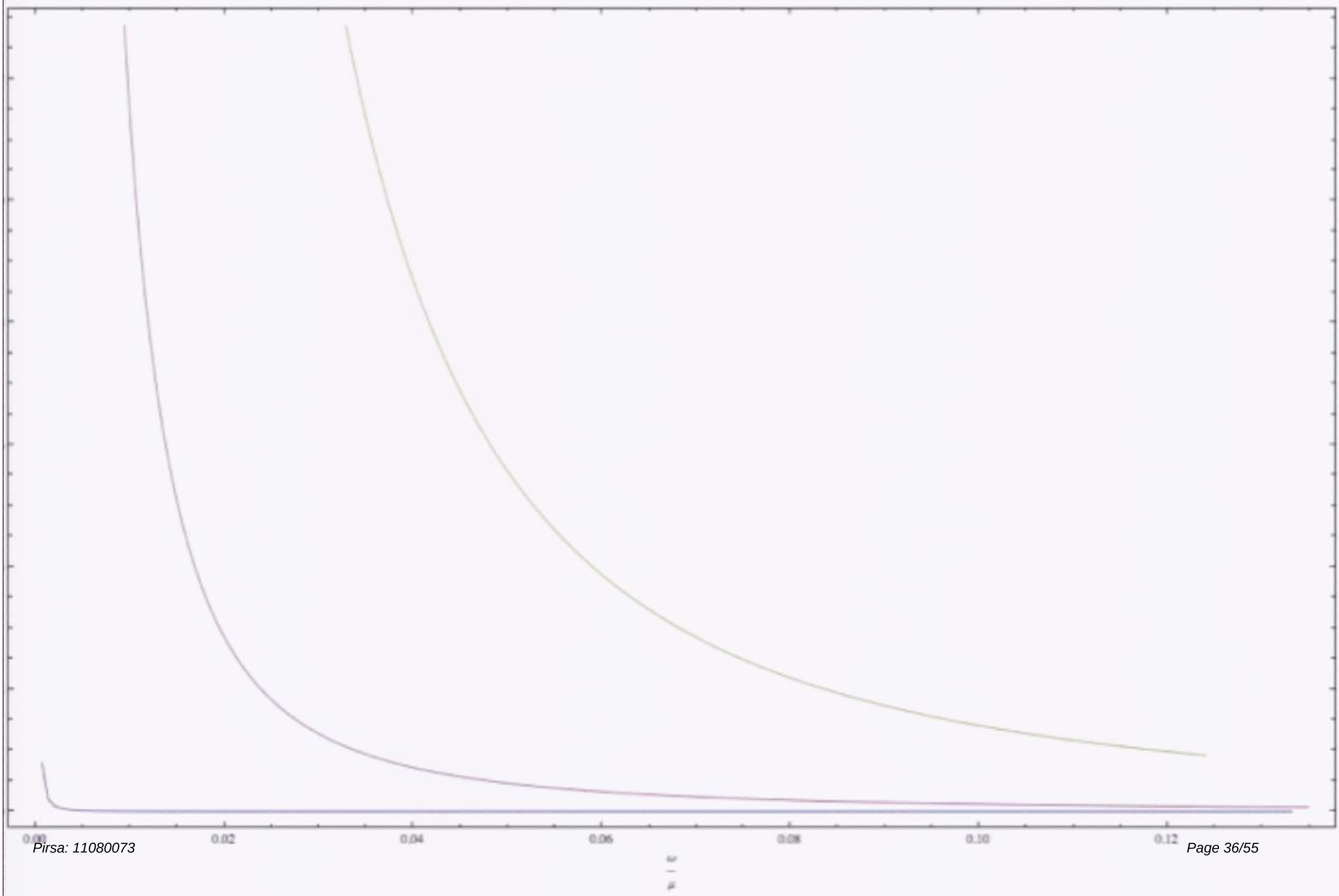
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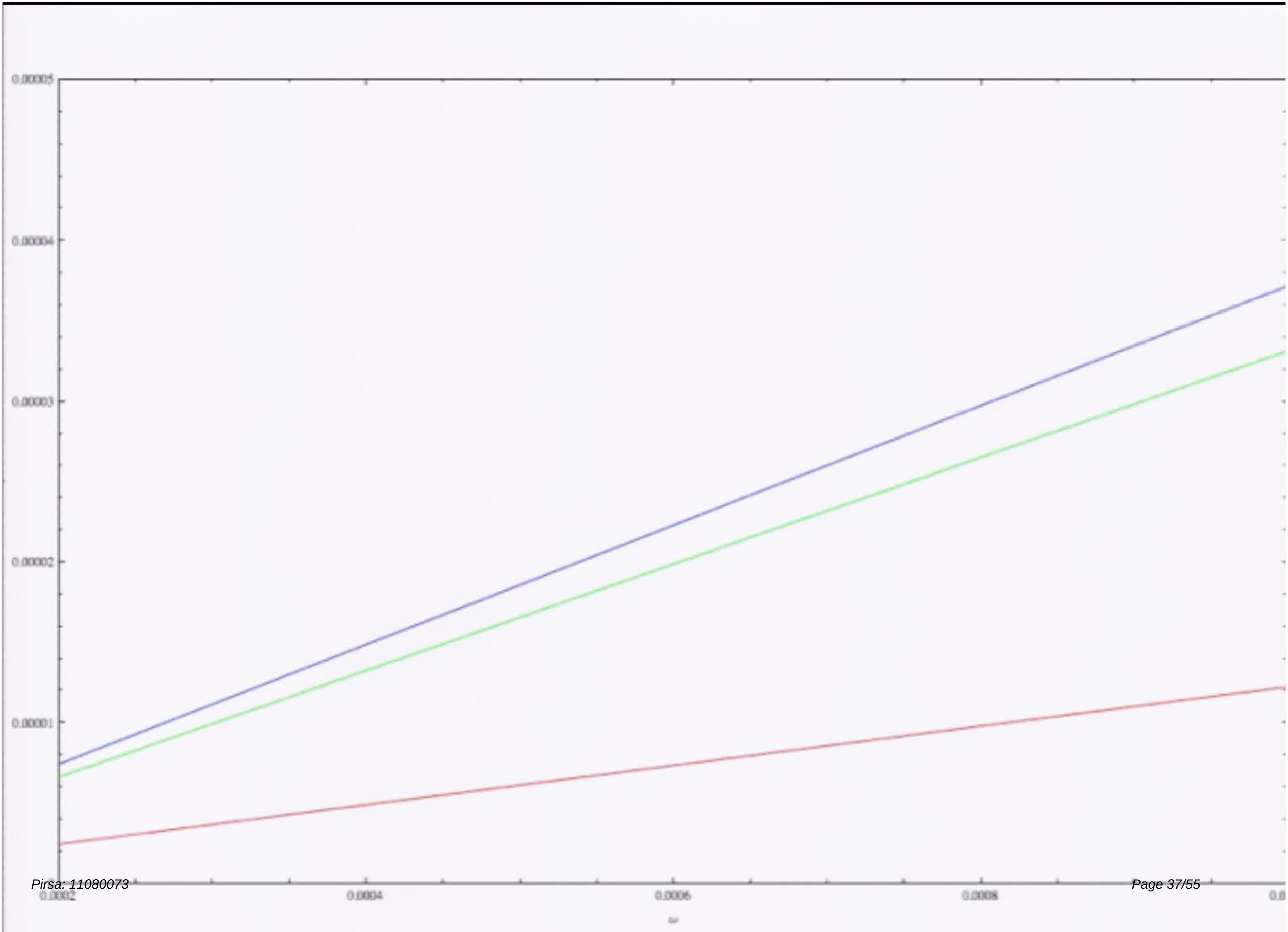

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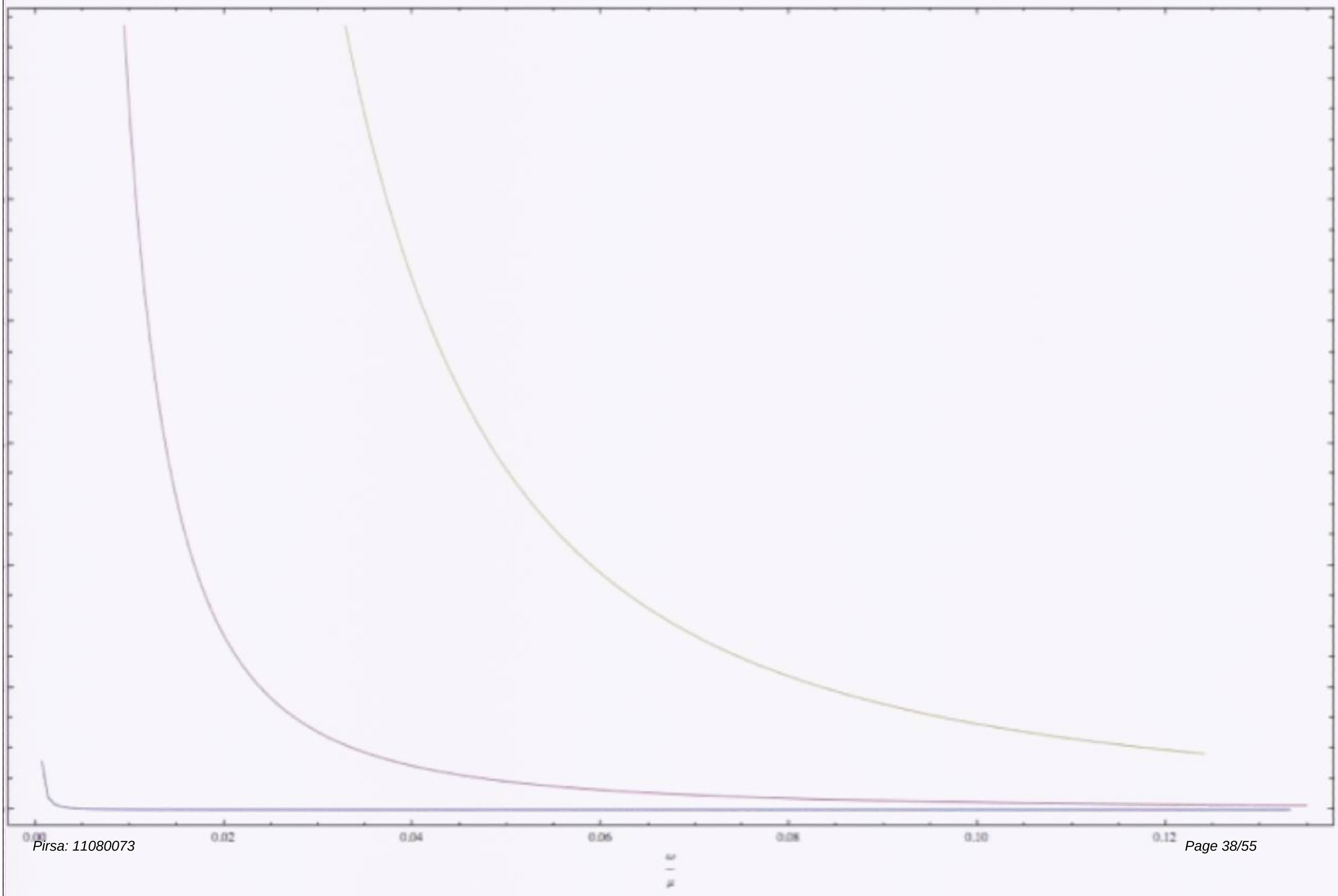
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Holography/Drude Model

Drude Model

$$\kappa(\omega) = \frac{\tau}{1 - i\omega\tau} \implies \mathrm{Re}\left[\kappa(\omega)\right] \propto \frac{k\tau}{1+\omega^2\tau^2}$$

Holography+Perturbative Disorder

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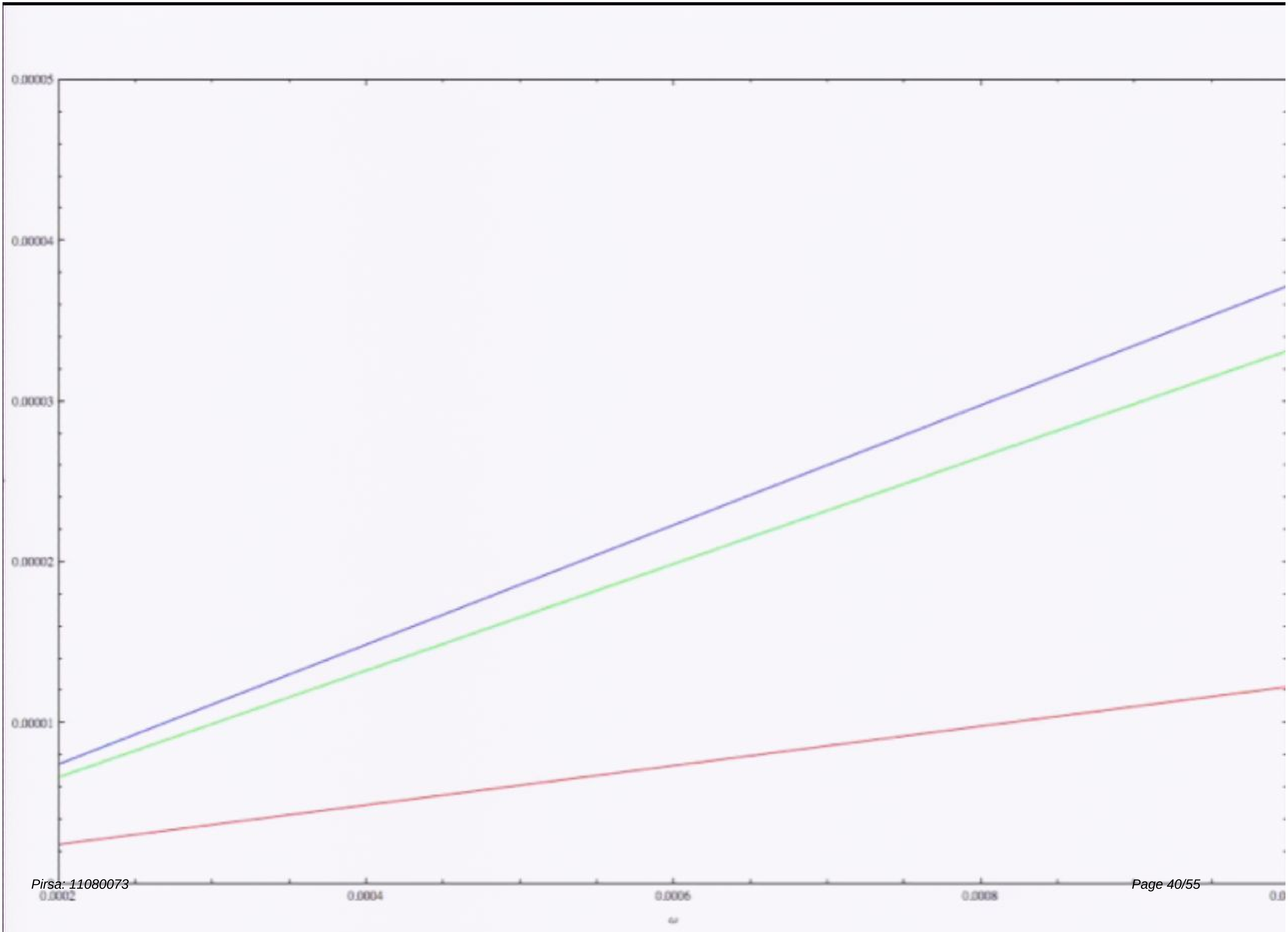
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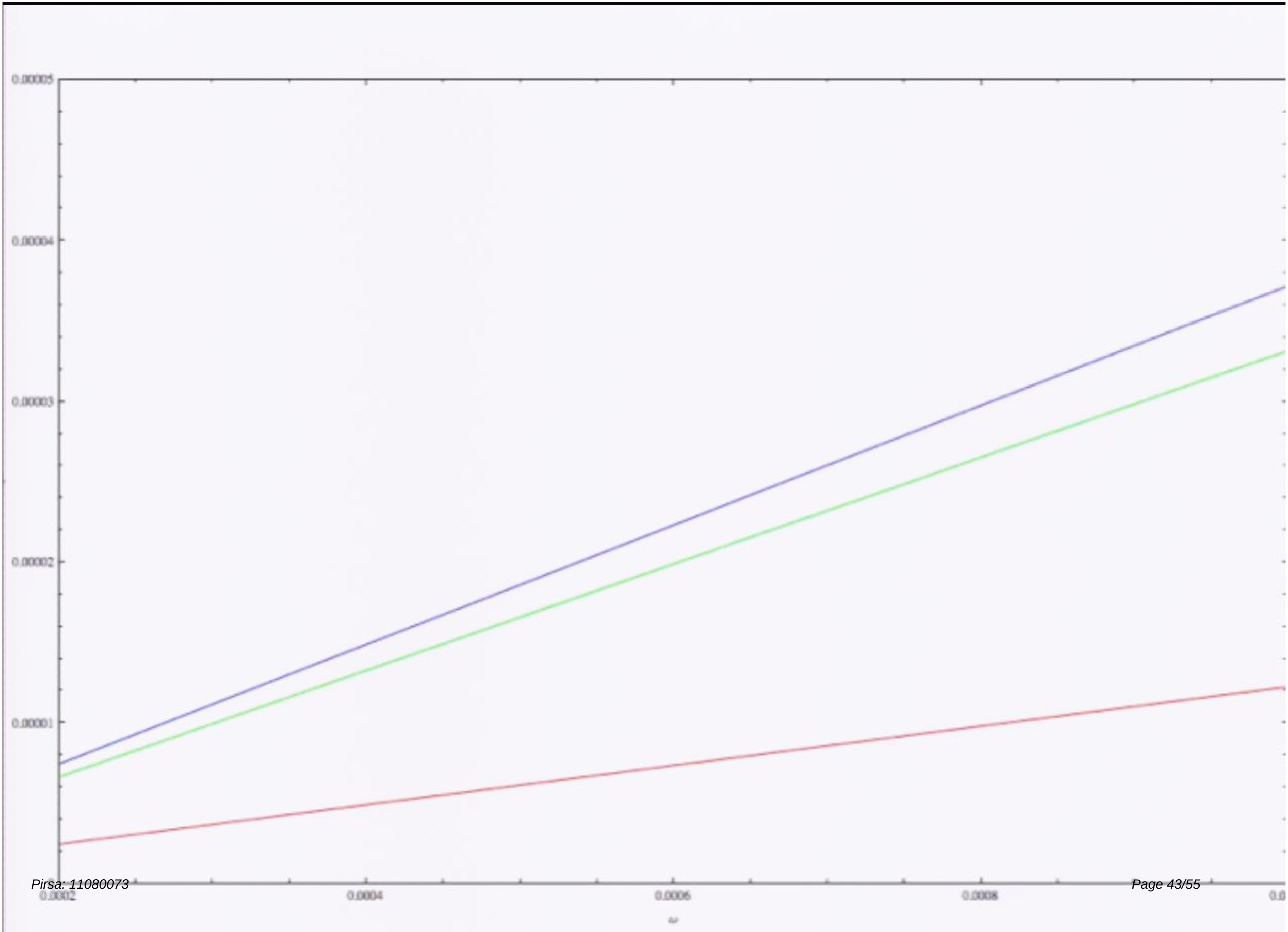
Holography+Perturbative Disorder – agrees with crude Drude model

Disorder Weak in some dimensionless units

Extract temperature dependence from holography +Perturbative Disorder

Consider direct coupling of disorder to order parameter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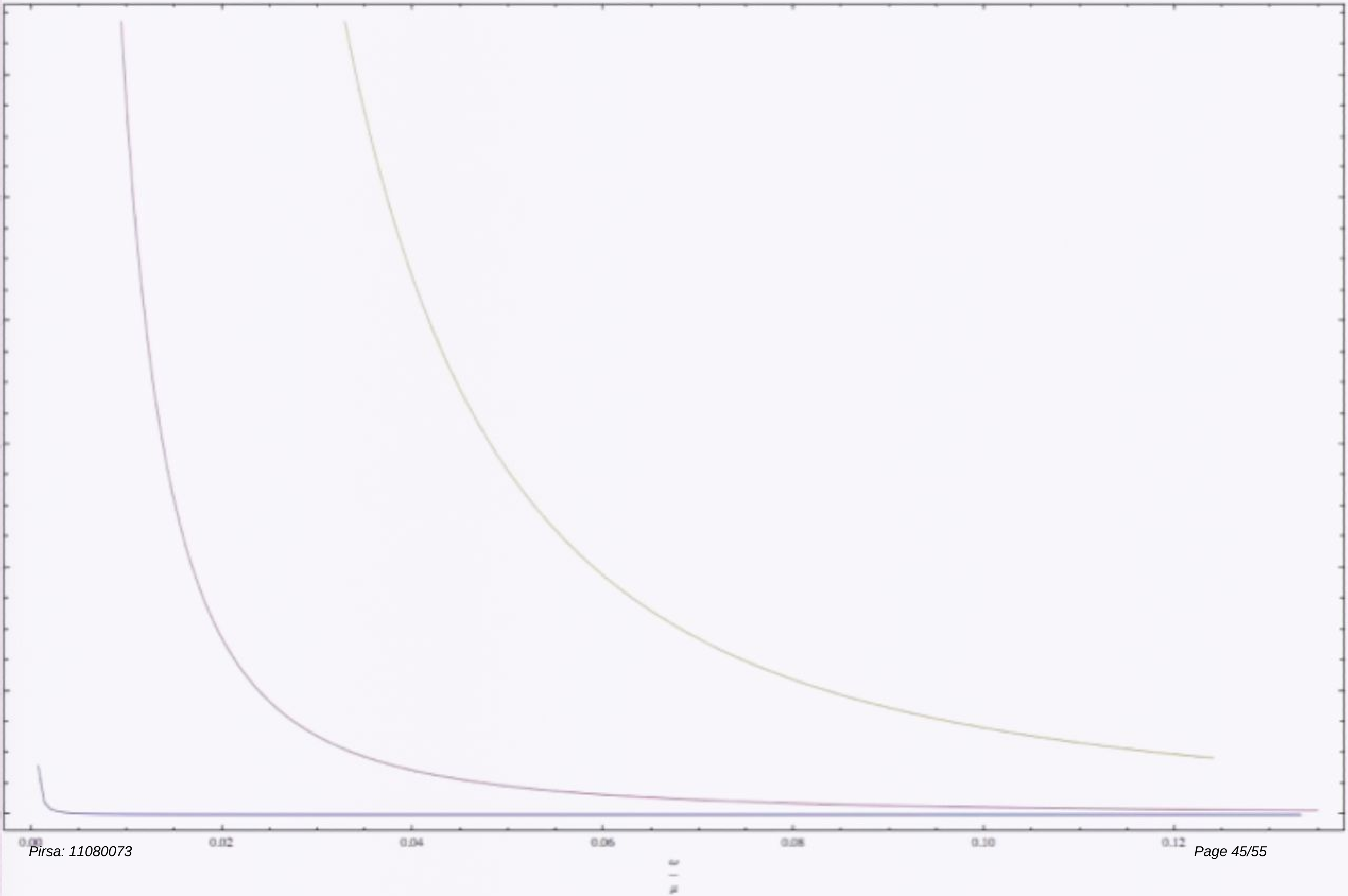
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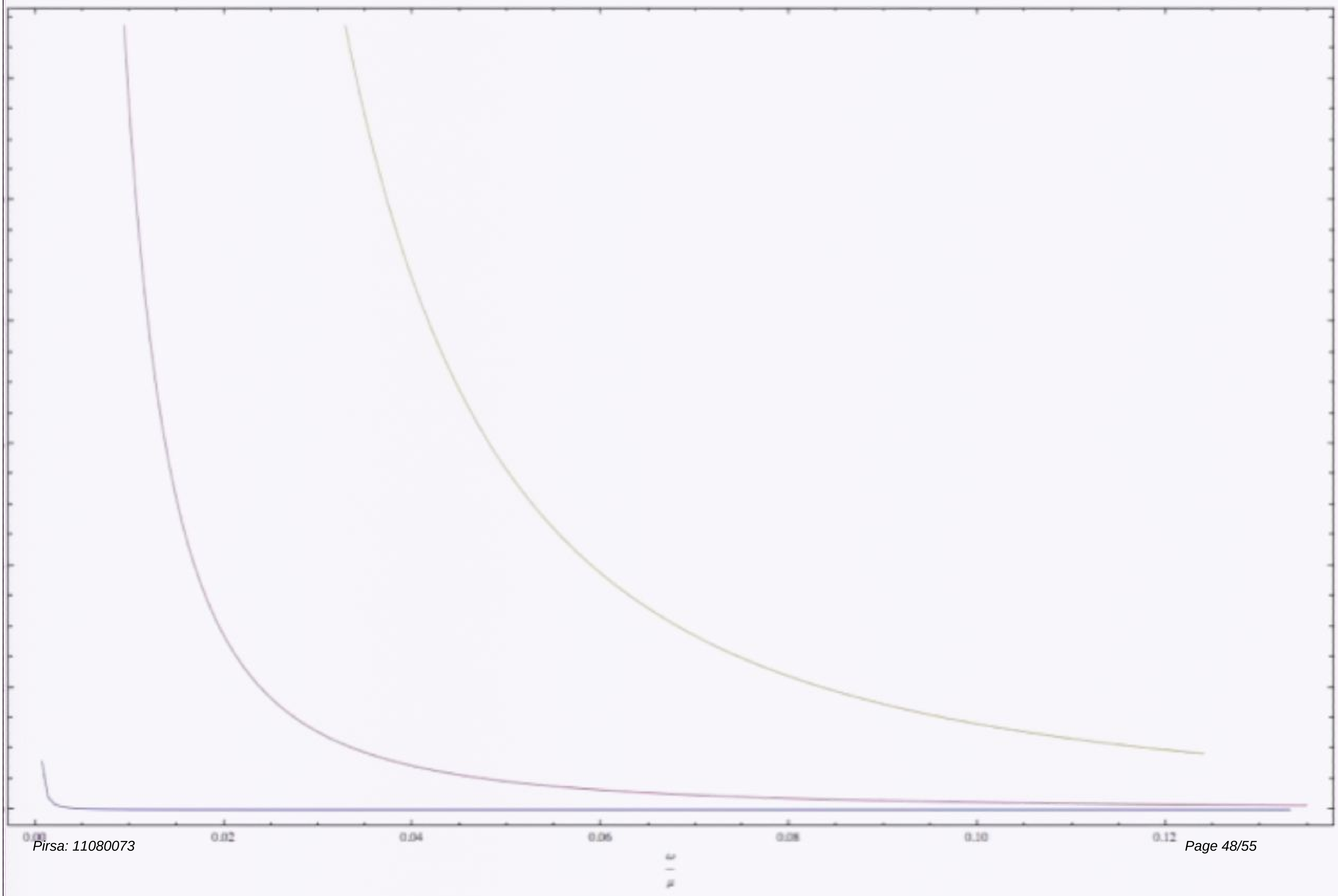
Perimeter Institute



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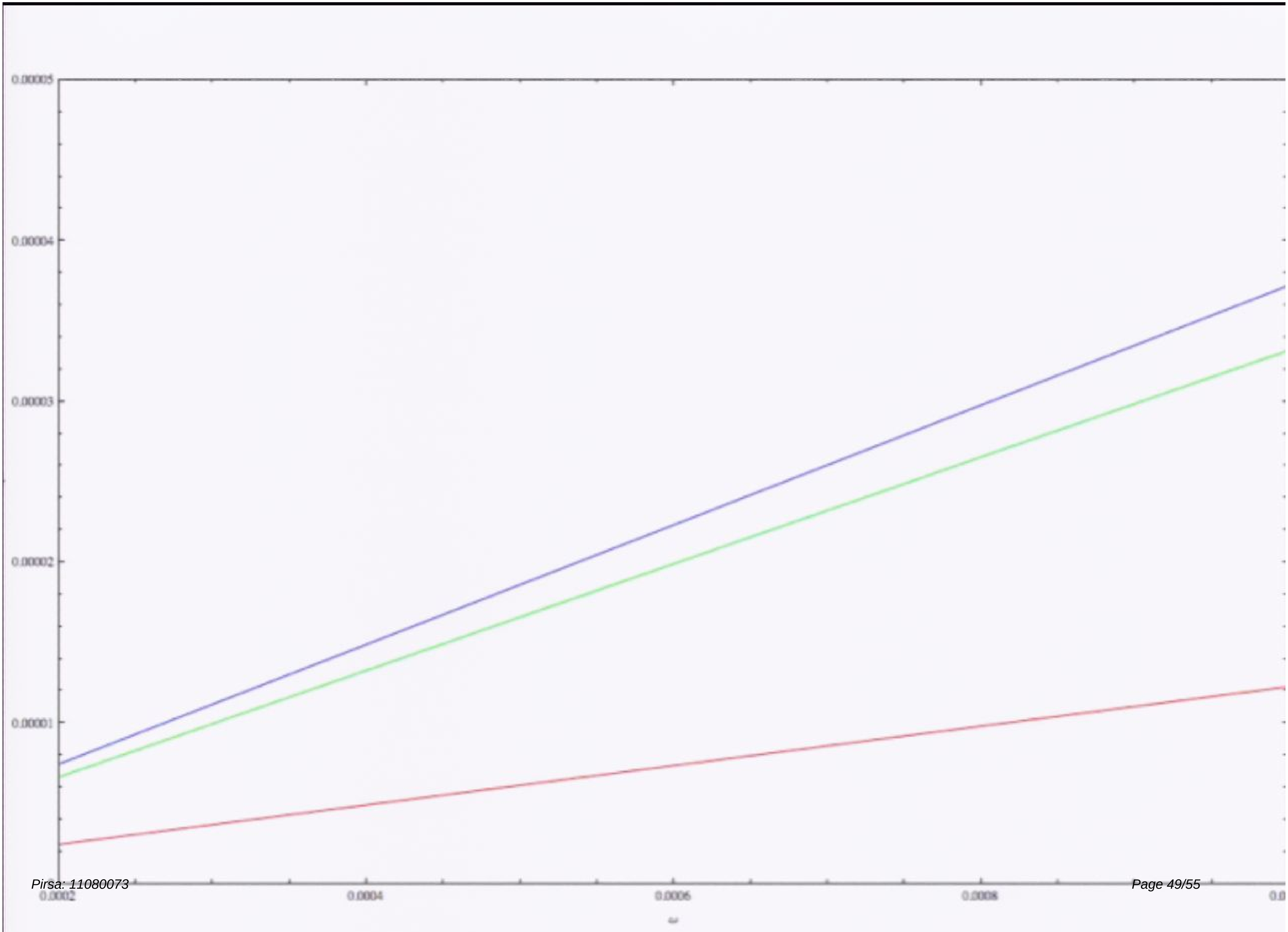
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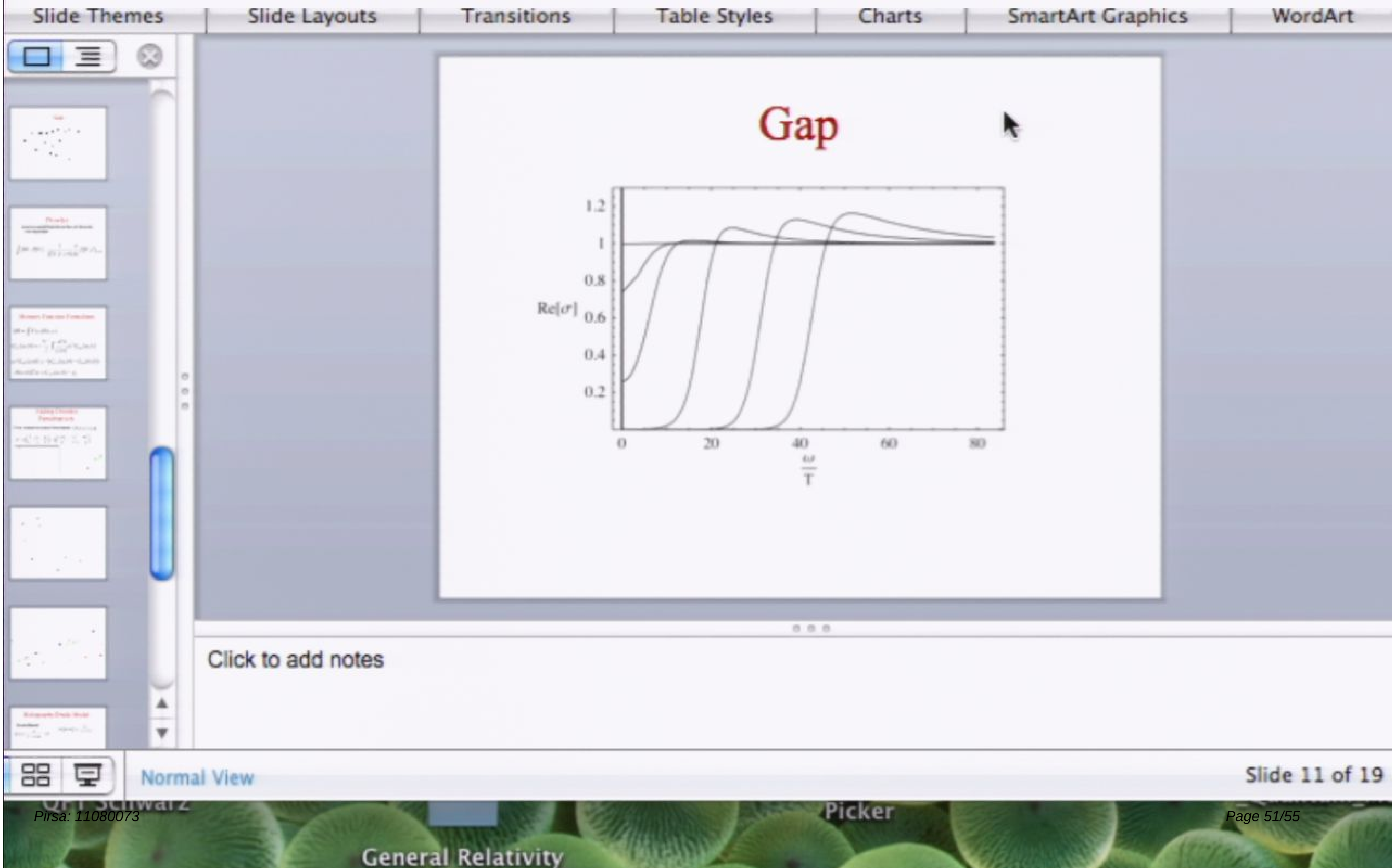

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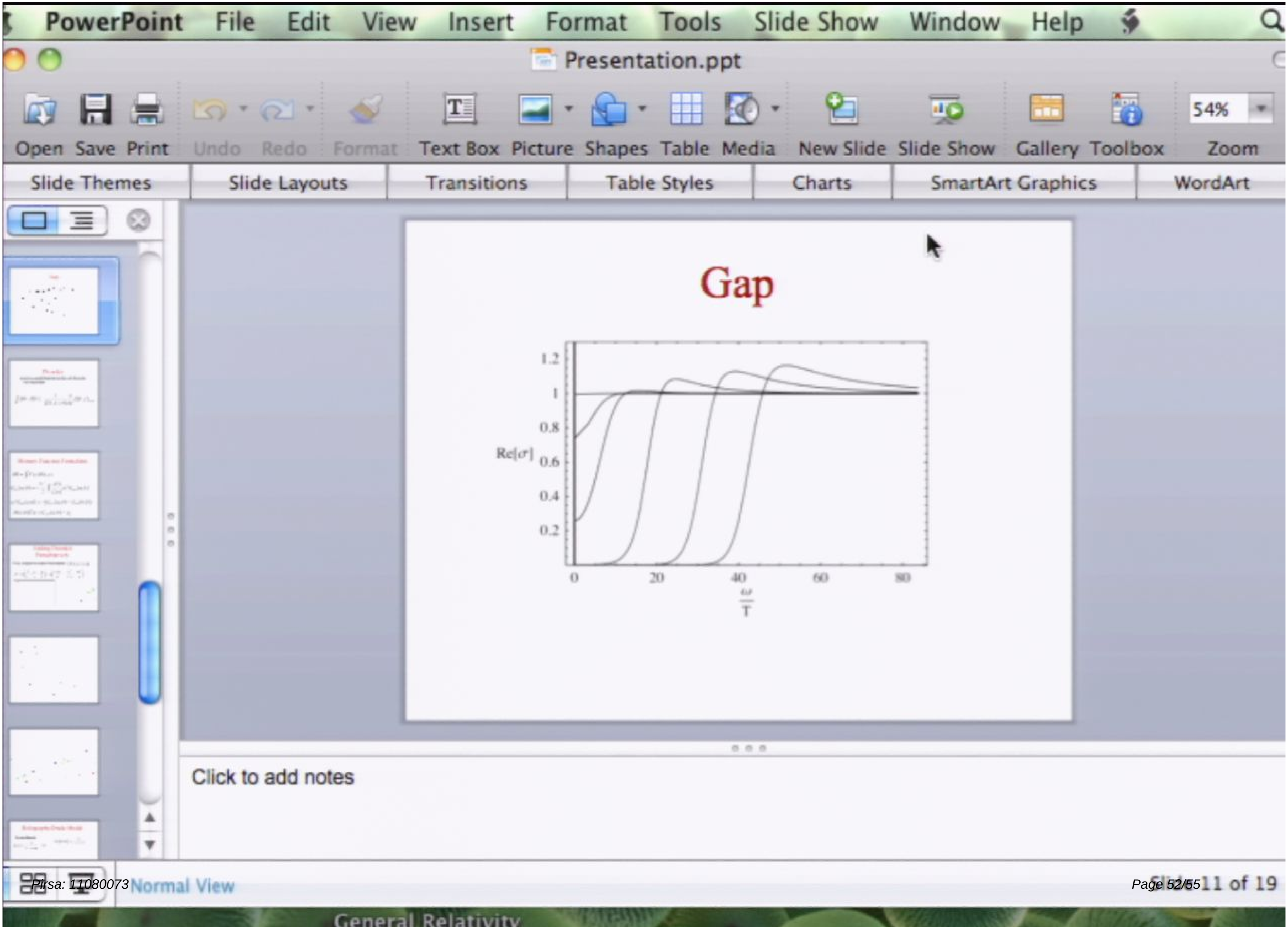
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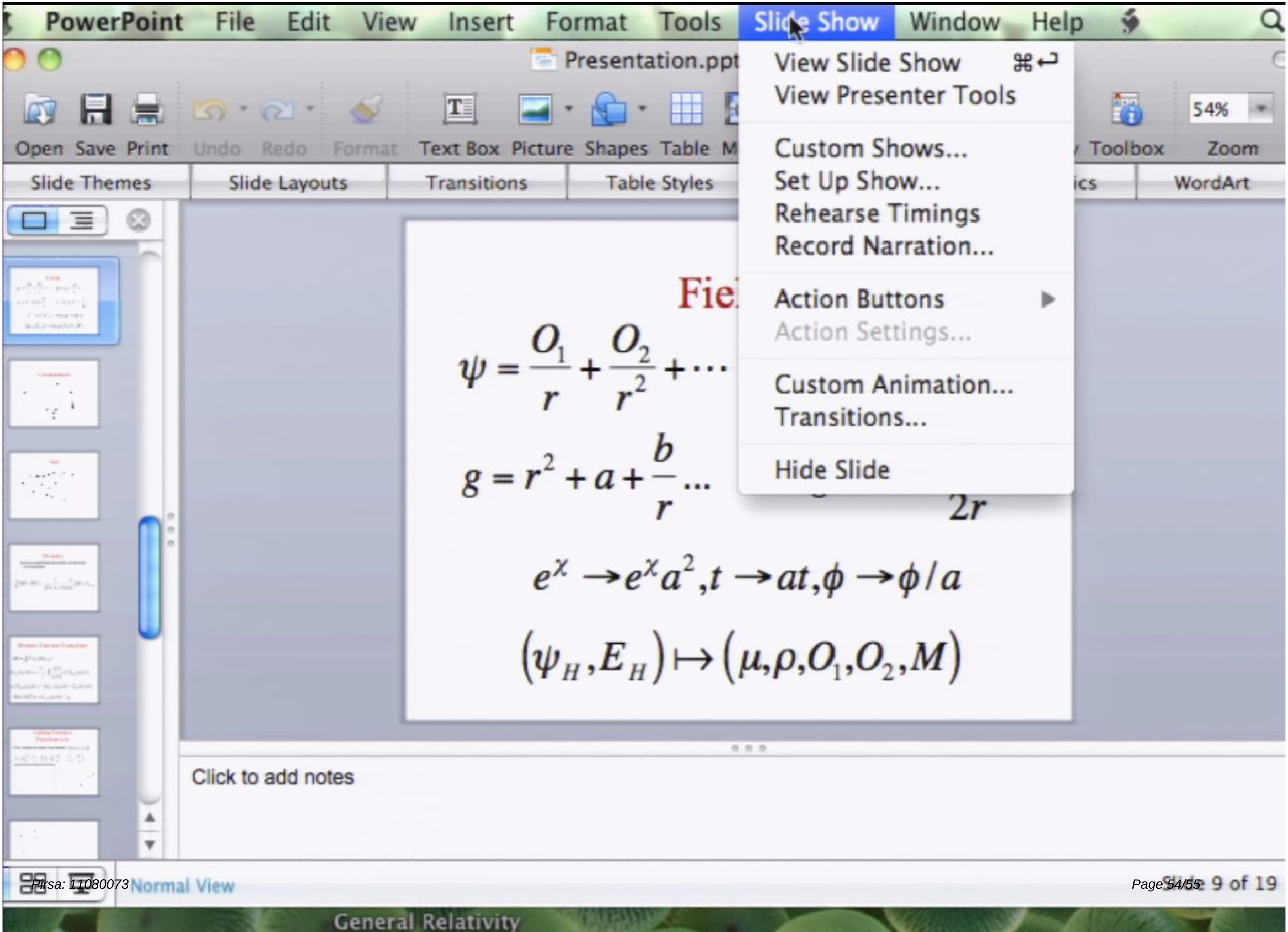
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Custom Shows...
Set Up Show...
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Record Narration...
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Hide Slide
$\psi = \frac{O_1}{r} + \frac{O_2}{r^2} + \cdots$
$g = r^2 + a + \frac{b}{r} \ldots$
$2r$
$e^\chi \to e^\chi a^2, t \to at, \phi \to \phi / a$
$(\psi_H, E_H) \mapsto (\mu, \rho, O_1, O_2, M)$
Click to add notes
Normal View
General Relativity

Effects of Disorder on Superfluids

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