Title: Cosmic Strings

Date: Aug 09, 2011 01:30 PM

URL: http://pirsa.org/11080071

Abstract:

Cosmic Strings

Patrick Chan
Supervisor: Louis Leblond

August 9, 2011

Outline

Introduction and Motivation

String Dynamics

Cusp Formations

Extra Dimensions

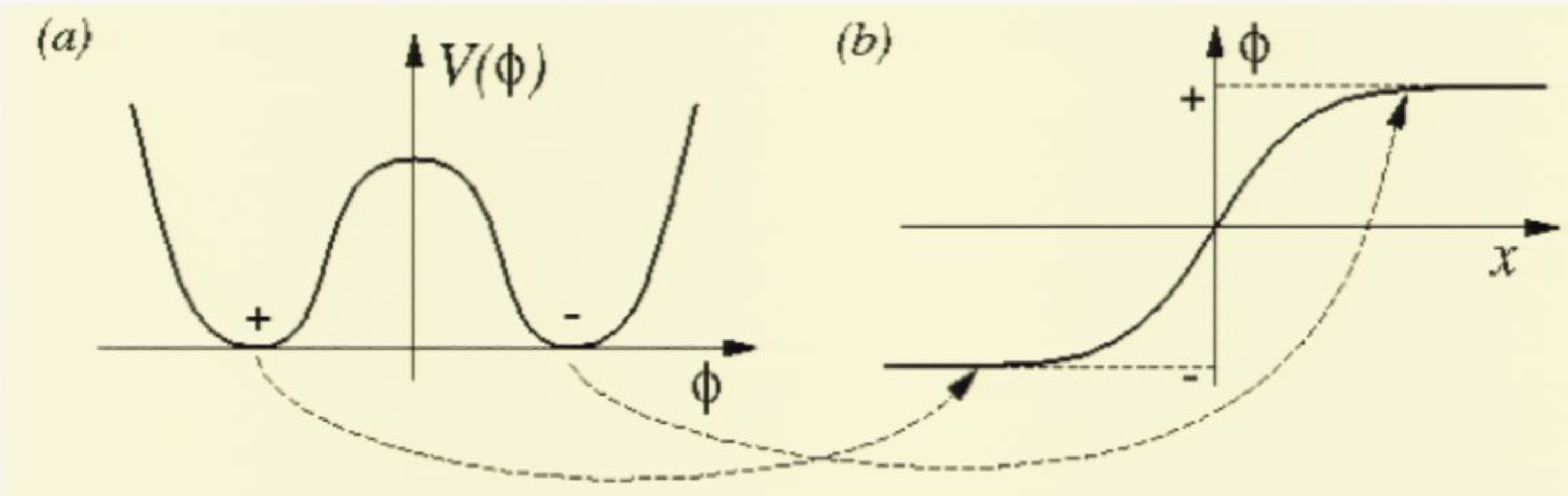
A New Approach

Conclusions

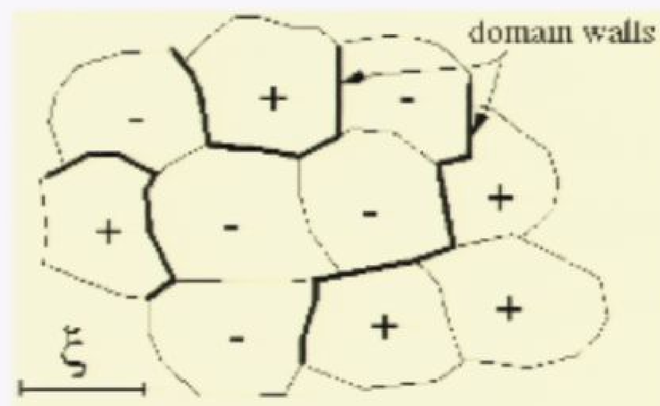
Phase Transitions and Topological Defects

- Phase transitions can have a wide variety of important implications, including the formation of topological defects - cosmic strings, domain walls, and monopoles.
- The type of defect formed is determined by the symmetry properties of the matter and the nature of the phase transition.

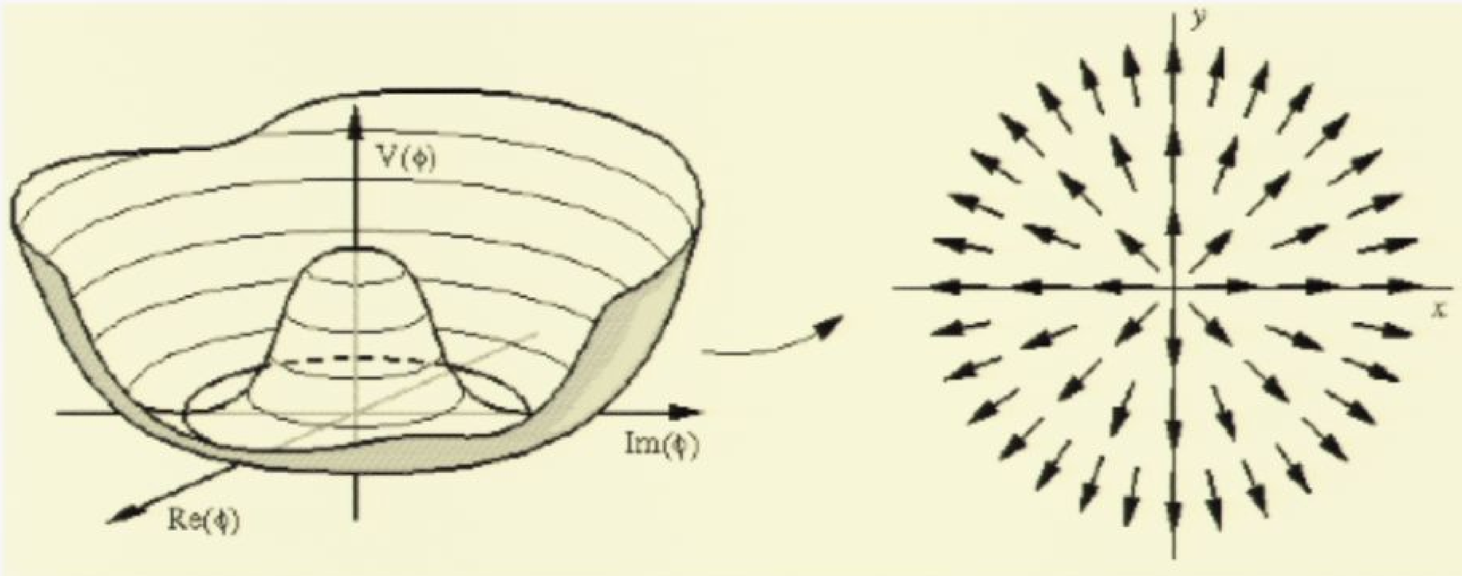
Domain Walls - Discrete Symmetry



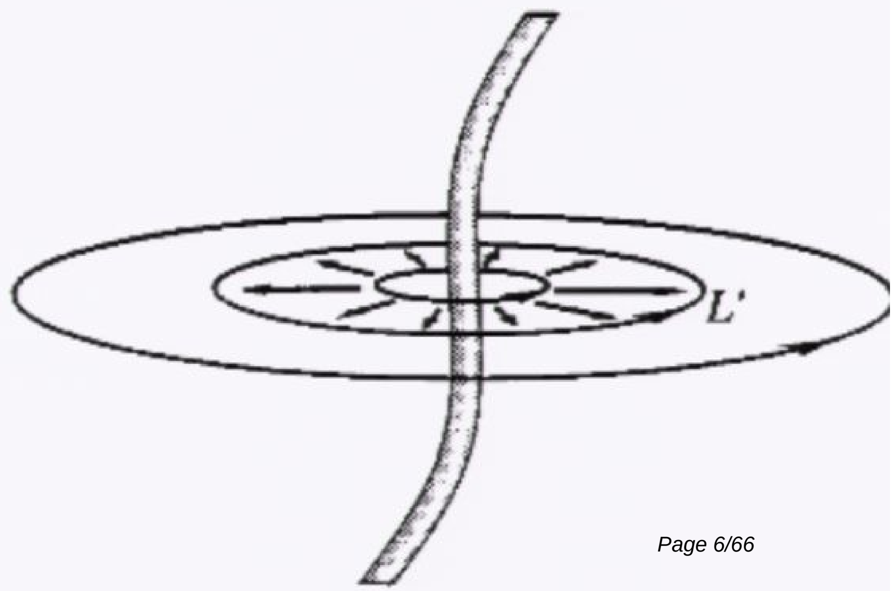
Topological defects are the “boundaries” between these regions with different choices of minima.



Cosmic Strings - $U(1)$ Symmetry



The "Mexican hat" potential has a "hole" in the vacuum manifold.

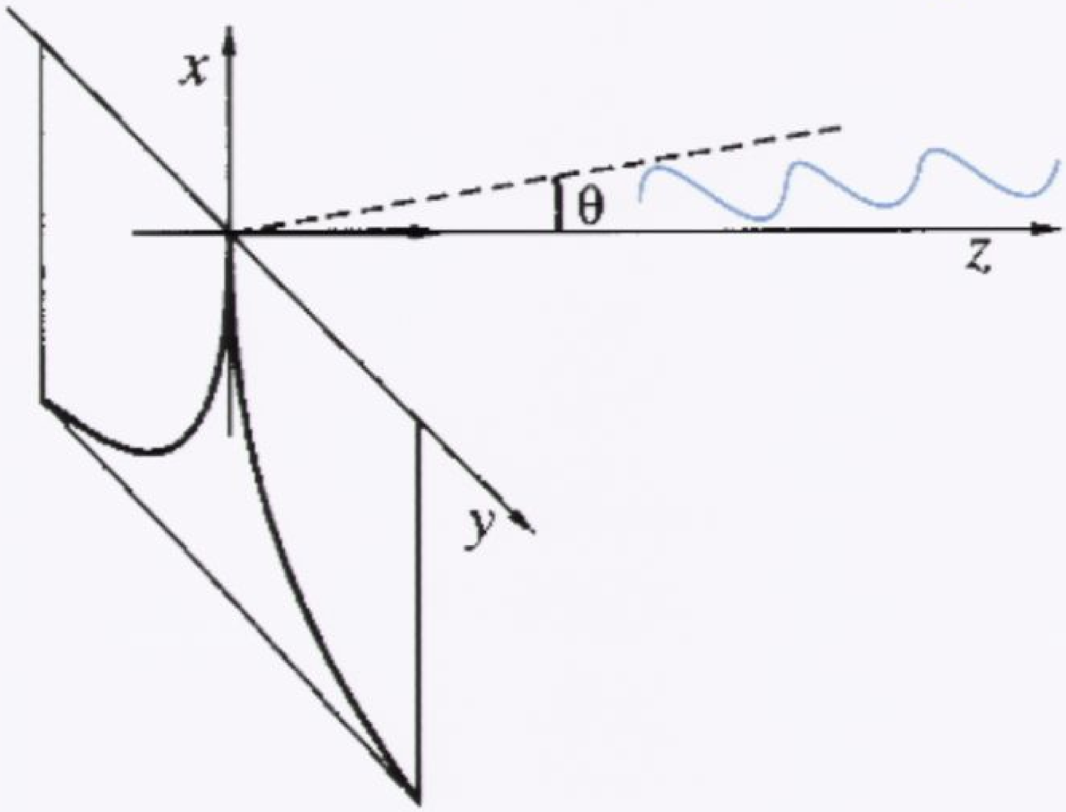


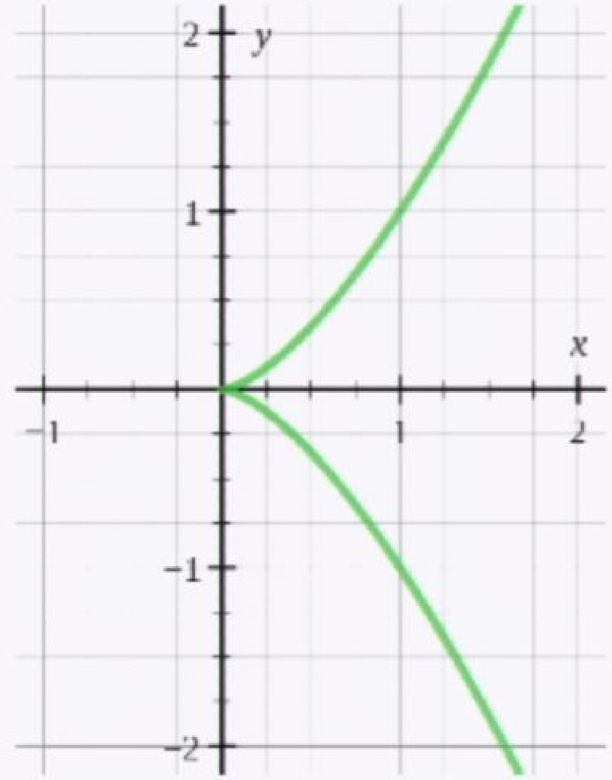
Why do we care?

- Link to the physics of the very early universe
- Cosmic strings could be the "seeds" that led to the formation of the large-scale structures
- String evolution corresponds to minimal surfaces

How do we look for a cosmic string?

Gravitational wave bursts from cusps





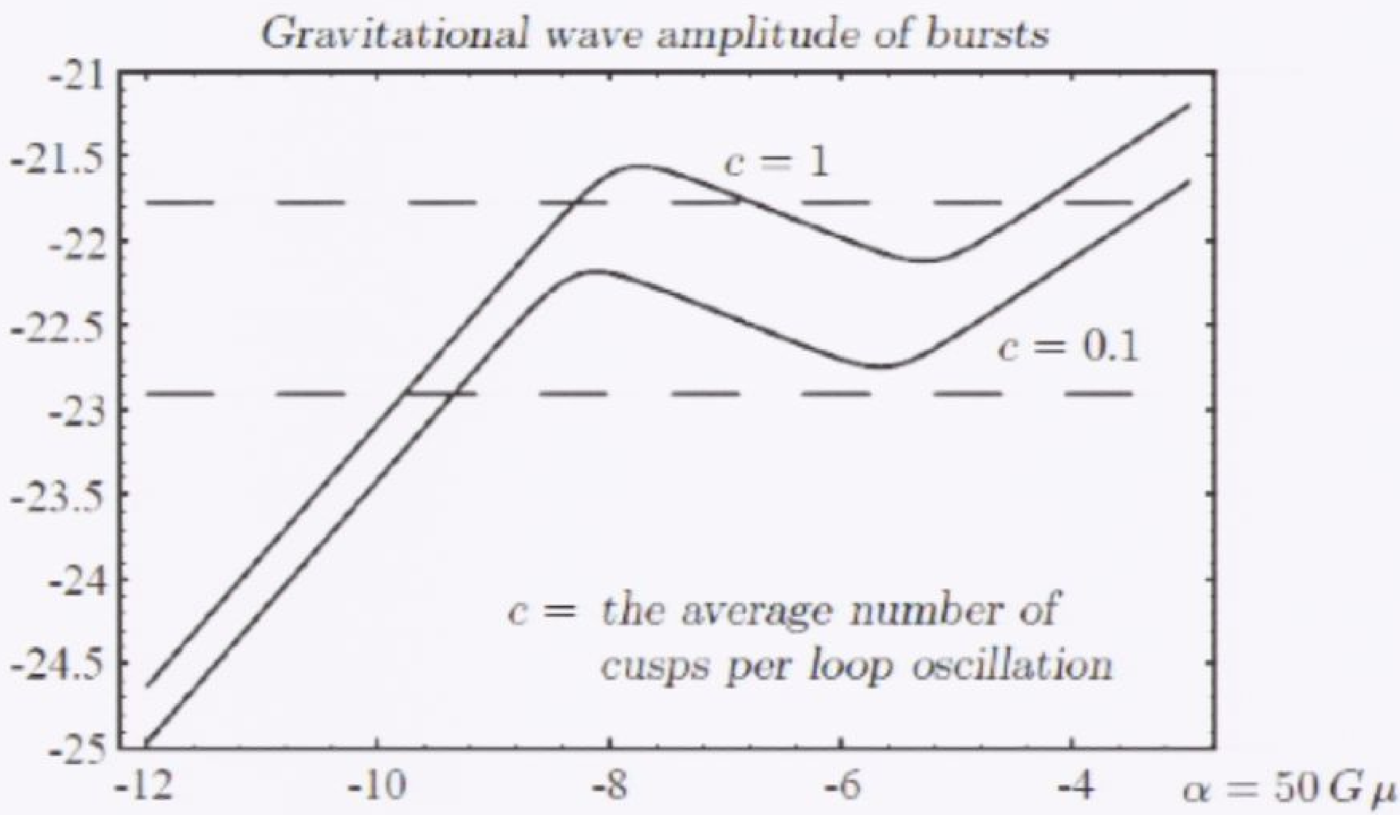
Typical cusp:

$$y^2 = x^3$$

$$\mathbf{r}(s) = (s^2, s^3)$$

Have we seen a cosmic string?

- Nope
- but LIGO is working hard



Question:

How do we (better) model cusp formation in cosmic strings?

Outline

- Introduction and Motivation
- String Dynamics
- Cusp Formations
- Extra Dimensions
- A New Approach
- Conclusions

Nambu-Goto Action

- We can parametrize a cosmic string as

$$X^\mu = X^\mu(\tau, \sigma),$$

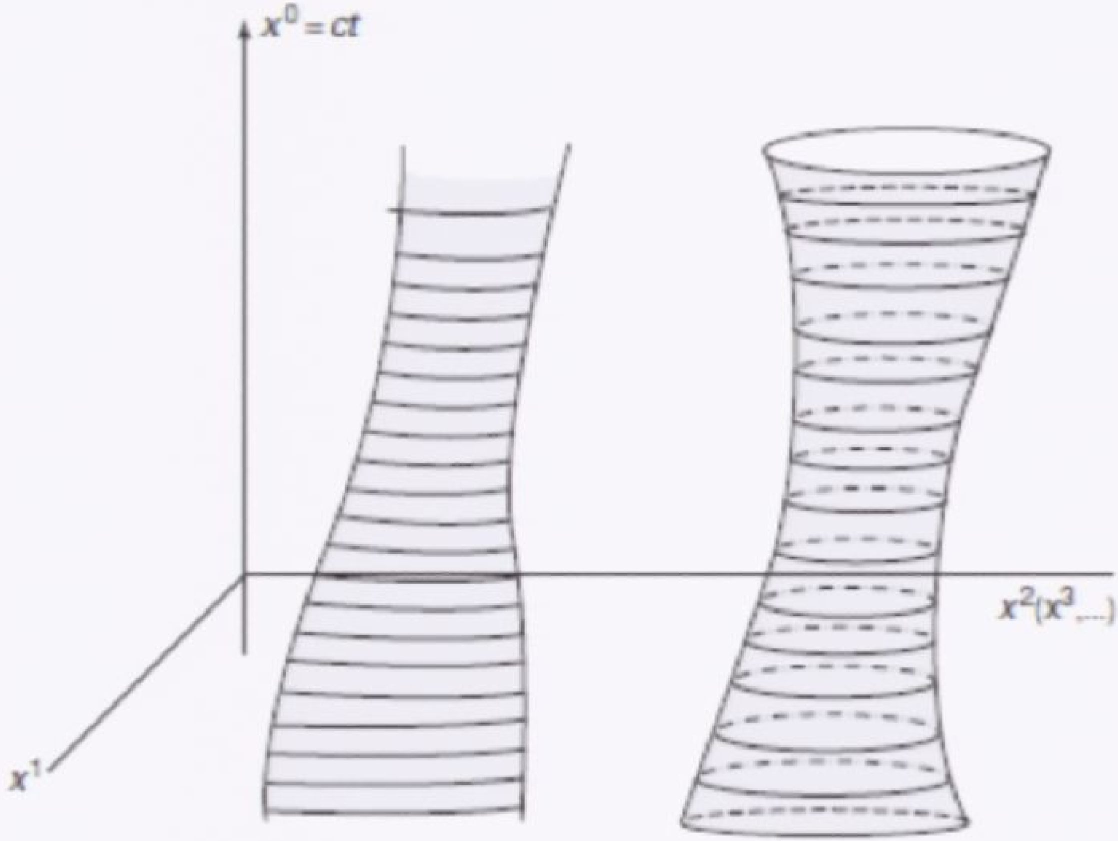
where $\mu = 0, 1, 2, 3$.

- The Nambu-Goto action is given by

$$\mathcal{S} = -\mu \int d\tau \int d\sigma \sqrt{-\gamma}$$

where $\gamma_{\alpha\beta} = \eta_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^\alpha} \frac{\partial X^\nu}{\partial \xi^\beta}$ is the pull-back metric.

String Evolution as Minimal Surface



- String moving in spacetime traces out a minimal surface
- Action is proportional to area, and hence *reparametrization invariant.*

Equation of Motion

- First Variation

$$\delta S = -\mu \int_{\tau_f}^{\tau_i} d\tau \int_0^{\sigma_0} d\sigma \quad \delta X^\mu \left(\frac{\partial}{\partial \tau} \left(\frac{\partial \mathcal{L}}{\partial \dot{X}^\mu} \right) + \frac{\partial}{\partial \sigma} \left(\frac{\partial \mathcal{L}}{\partial X'^\mu} \right) \right)$$

- Equation of Motion

$$\frac{\partial}{\partial \tau} \left(\frac{(\dot{X} \cdot X')X' - (X')^2 \dot{X}}{\sqrt{(\dot{X} \cdot X')^2 - (\dot{X})^2 (X')^2}} \right) + \frac{\partial}{\partial \sigma} \left(\frac{(\dot{X} \cdot X')\dot{X} - (\dot{X})^2 X'}{\sqrt{(\dot{X} \cdot X')^2 - (\dot{X})^2 (X')^2}} \right) = 0$$

Temporal Gauge

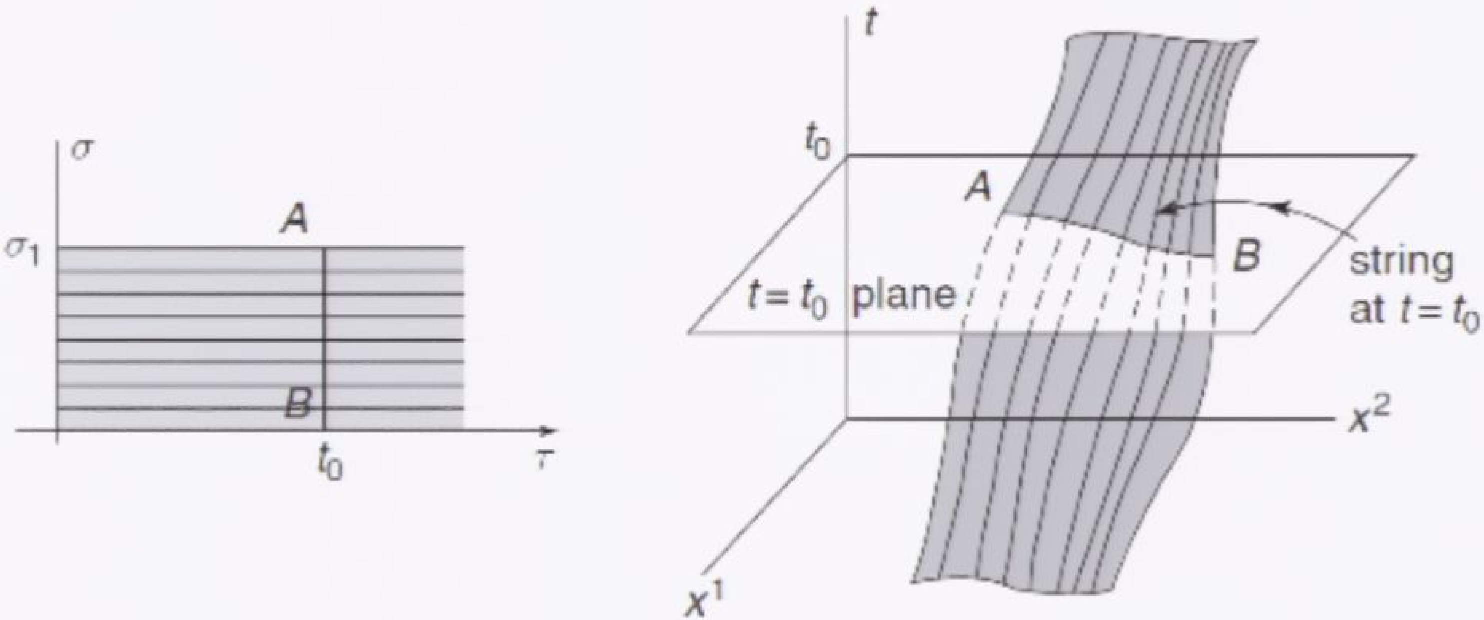


Figure: $X^\mu(\tau,\sigma) = \Big(\tau, \mathbf{r}(\tau,\sigma)\Big)$

Orthogonal Gauge

- How do we get

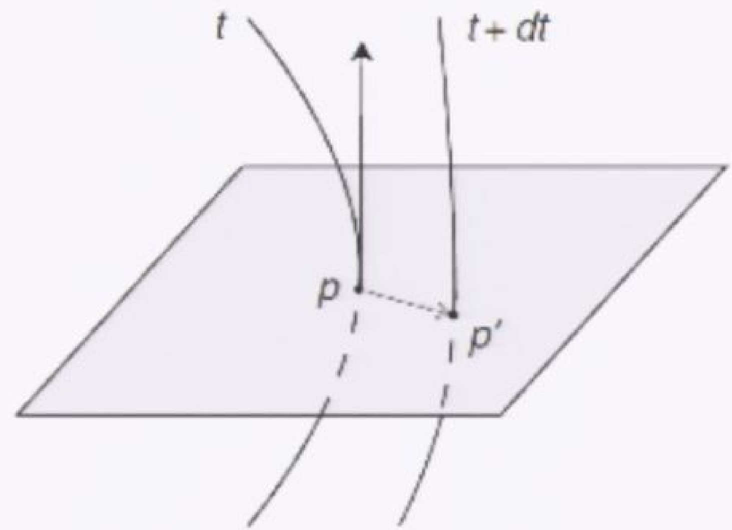
$$\dot{\mathbf{r}} \cdot \mathbf{r}' = 0?$$

- Reparmetrize

$$(\tau, \sigma) \mapsto \mathbf{r}(\tau, f(\tau, \sigma)),$$

where f solves the linear transport equation

$$f_t = -\frac{\mathbf{r}_\tau \cdot \mathbf{r}_\sigma}{|\mathbf{r}_\sigma|^2} f_\sigma$$



Transverse velocity

$$\mathbf{v}_\perp = \dot{\mathbf{r}} - (\dot{\mathbf{r}} \cdot \mathbf{r}')\mathbf{r}'$$

Wave Equation and Constraints

- Wave Equation

$$\frac{\partial^2 \mathbf{r}}{\partial \tau^2} - \frac{\partial^2 \mathbf{r}}{\partial \sigma^2} = 0$$

- Constraint can be written as

$$\left(\frac{\partial \mathbf{r}}{\partial \tau}\right)^2 + \left(\frac{\partial \mathbf{r}}{\partial \sigma}\right)^2 = 1$$

- Periodic boundary conditions

Representation of Solutions

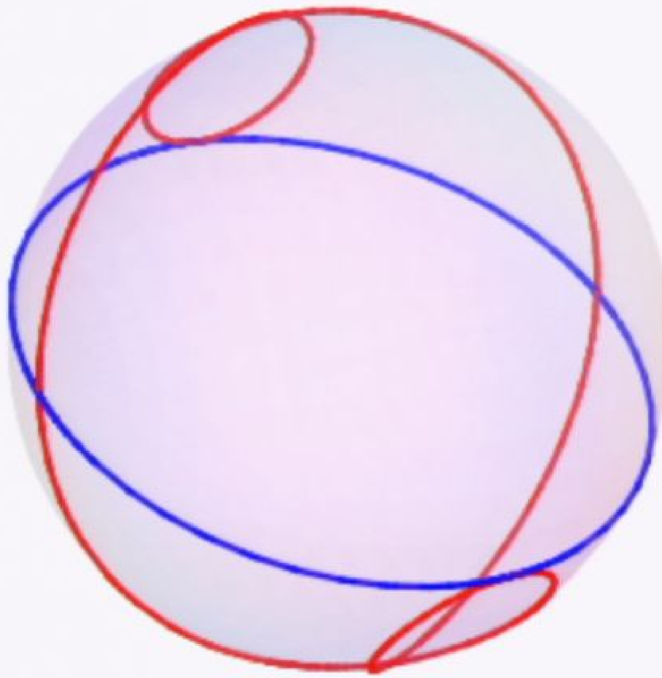
Using the d'Alembert formula, we have

$$\mathbf{r}(\tau,\sigma) = \frac{1}{2}\Big(\mathbf{a}(\tau+\sigma) + \mathbf{b}(\tau-\sigma)\Big)$$

where

$$|\mathbf{a}'|^2 = |\mathbf{b}'|^2 = 1$$

Kibble-Turok Sphere



Left- and Right-movers on the Kibble-Turok sphere.

Kibble-Turok Strings (1982)

$$\mathbf{r}_{KT}(s,t) = \frac{1}{2}\begin{pmatrix} (1-\alpha)\sin u + \frac{1}{3}\alpha \sin 3u \\ -(1-\alpha)\cos u - \frac{1}{3}\alpha\cos 3u \\ -\sqrt{\alpha(1-\alpha)}\cos u \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \sin v \\ -\cos v \\ 0 \end{pmatrix},$$

where $u = t + s$, $v = t - s$, and α is an arbitrary parameter between 0 and 1.

Preview File Edit View Go Tools Bookmarks Window Help Tue Aug 9 1:56 PM

talk.pdf (page 18 of 63)

evious Next Zoom Move Text Select Annotate Sidebar

ibble-Turok Strings (1982)

$$r_{KT}(s,t) = \frac{1}{2}\begin{pmatrix} (1-\alpha)\sin u + \frac{1}{3}\alpha\sin 3u \\ -(1-\alpha)\cos u - \frac{1}{3}\alpha\cos 3u \\ -\sqrt{\alpha(1-\alpha)}\cos u \end{pmatrix} + \frac{1}{2}\begin{pmatrix} \sin v \\ -\cos v \\ 0 \end{pmatrix},$$

where $u = t + s$, $v = t - s$, and α is an arbitrary parameter between 0 and 1.

ibble-Turok Strings (1982)

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

mathematica_slides.nb

Slide 1 of 5

Kibble-Turok Strings

T.W.B Kibble and Neil Turok. Self-intersection of Cosmic Strings. Phys. Lett., B116:141, 1982.

```
Manipulate[
 a[u_] = {(1 - α) Cos[u] + α Sin[3 u] / 3, - (1 - α) Cos[u] - α Cos[3 u] / 3,
   -Sqrt[α (1 - α)] Cos[u]};
 b[v_] = {Sin[v], -Cos[v], 0};
 ParametricPlot3D[(a[t + s] + b[t - s]) / 2, {s, 0, 2 Pi}, PlotRange → 1],
 {{α, 0.5}, 0, 1}, {t, 0, 2 π}]
```

Slide 2 of 5

Turok Strings

Neil Turok. Grand Unified Strings and Galaxy Formation. Nucl. Phys., B242:520, 1984.

```
Manipulate[
 a[u_] = {(1 - α) Cos[u] + α Sin[3 u] / 3, - (1 - α) Cos[u] - α Cos[3 u] / 3,
   -Sqrt[α (1 - α)] Cos[u]};
```

100%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

mathematica_slides.nb

```
ParametricPlot3D[(a[t + s] + b[t - s]) / 2, {s, 0, 2 Pi}, PlotRange → 1],
{{α, 0.5}, 0, 1}, {t, 0, 2 π}]
```

Slide 2 of 5

Turok Strings

Neil Turok. Grand Unified Strings and Galaxy Formation. Nucl. Phys., B242:520, 1984.

```
Manipulate[
 a[u_] = {(1 - α) Cos[u] + α Sin[3 u] / 3, - (1 - α) Cos[u] - α Cos[3 u] / 3,
   -Sqrt[α (1 - α)] Cos[u]};
 b[v_] = {Sin[v], -Cos[ϕ] Cos[v], -Sin[ϕ] Cos[v]};
 ParametricPlot3D[(a[t + s] + b[t - s]) / 2, {s, 0, 2 Pi}, PlotRange → 1],
 {{α, 0.5}, 0, 1}, {{ϕ, π / 2}, 0, π}, {t, 0, 2 π}]
```

Slide 3 of 5

Chen, DiCarlo, and Hotes (CDH) Strings

This is a text cell.

```
Manipulate[CC = Cos[θ] - Cos[η];
```

100%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Running...mathematica_slides.nb

```
ParametricPlot3D[(a[t + s] + b[t - s]) / 2, {s, 0, 2 Pi}, PlotRange → 1],
{{α, 0.5}, 0, 1}, {t, 0, 2 π}]
```

Slide 2 of 5

Turok Strings

Neil Turok. Grand Unified Strings and Galaxy Formation. Nucl. Phys., B242:520, 1984.

```
In[1]:= Manipulate[
  a[u_] = {(1 - α) Cos[u] + α Sin[3 u] / 3, - (1 - α) Cos[u] - α Cos[3 u] / 3,
    -Sqrt[α (1 - α)] Cos[u]};
  b[v_] = {Sin[v], -Cos[ϕ] Cos[v], -Sin[ϕ] Cos[v]};
  ParametricPlot3D[(a[t + s] + b[t - s]) / 2, {s, 0, 2 Pi}, PlotRange → 1],
  {{α, 0.5}, 0, 1}, {{ϕ, π / 2}, 0, π}, {t, 0, 2 π}]
```

Slide 3 of 5

Chen, DiCarlo, and Hotes (CDH) Strings

This is a text cell.

```
Manipulate[CC = Cos[θ] - Cos[η];
```

100%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

mathematica_slides.nb

Turok Strings

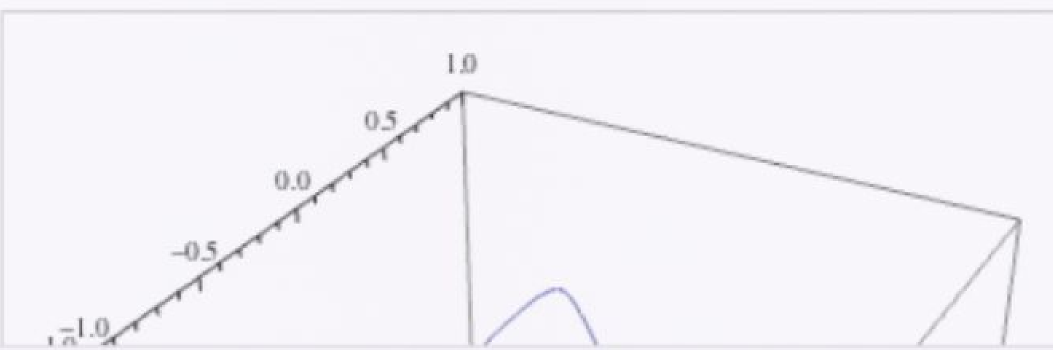
Neil Turok. Grand Unified Strings and Galaxy Formation. Nucl. Phys., B242:520, 1984.

In[1]:=
```
Manipulate[
 a[u_] = {(1 - α) Cos[u] + α Sin[3 u] / 3, - (1 - α) Cos[u] - α Cos[3 u] / 3,
   -Sqrt[α (1 - α)] Cos[u]};
 b[v_] = {Sin[v], -Cos[ϕ] Cos[v], -Sin[ϕ] Cos[v]};
 ParametricPlot3D[(a[t + s] + b[t - s]) / 2, {s, 0, 2 Pi}, PlotRange → 1],
 {{α, 0.5}, 0, 1}, {{ϕ, π / 2}, 0, π}, {t, 0, 2 π}]
```

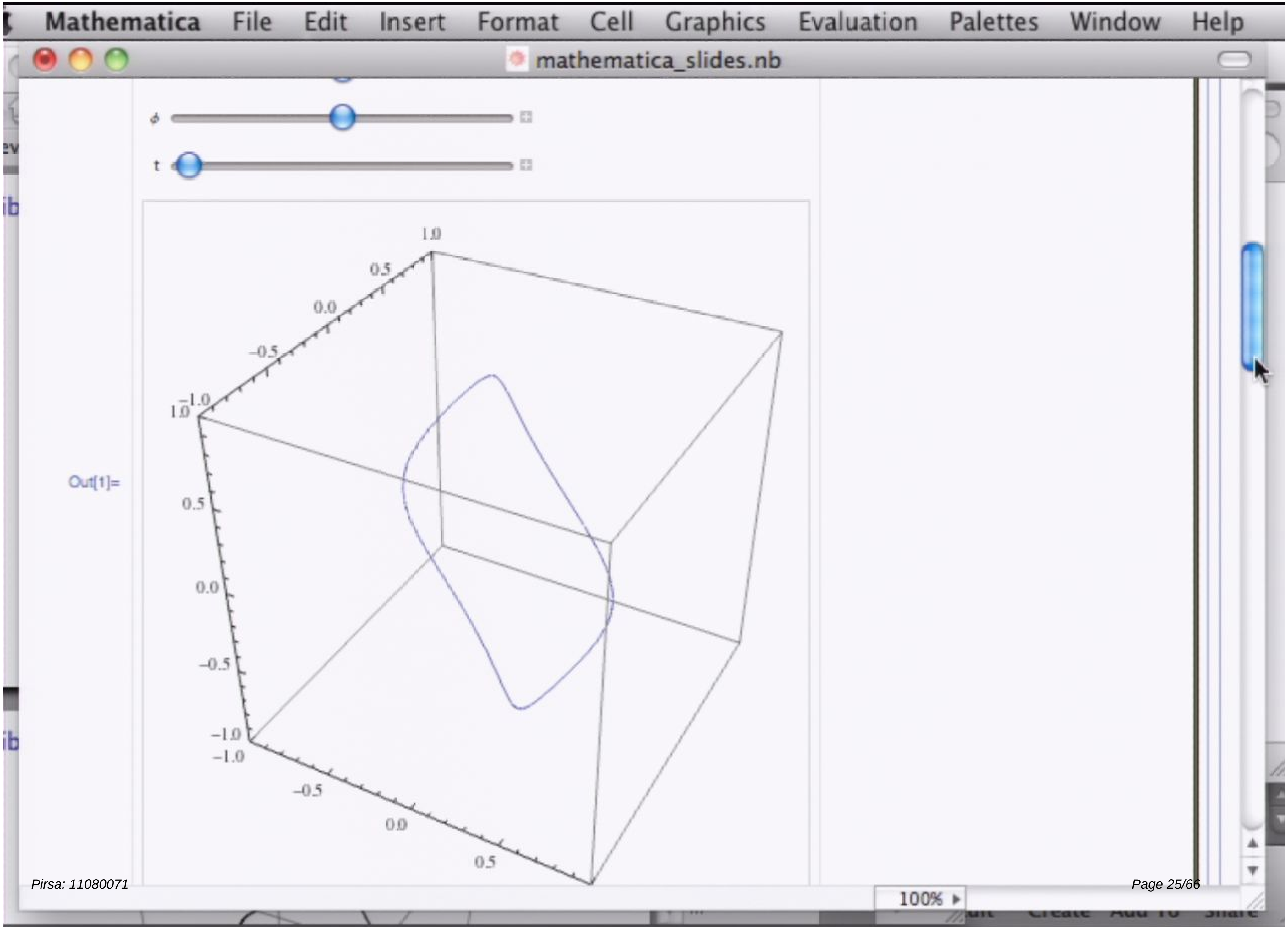
α

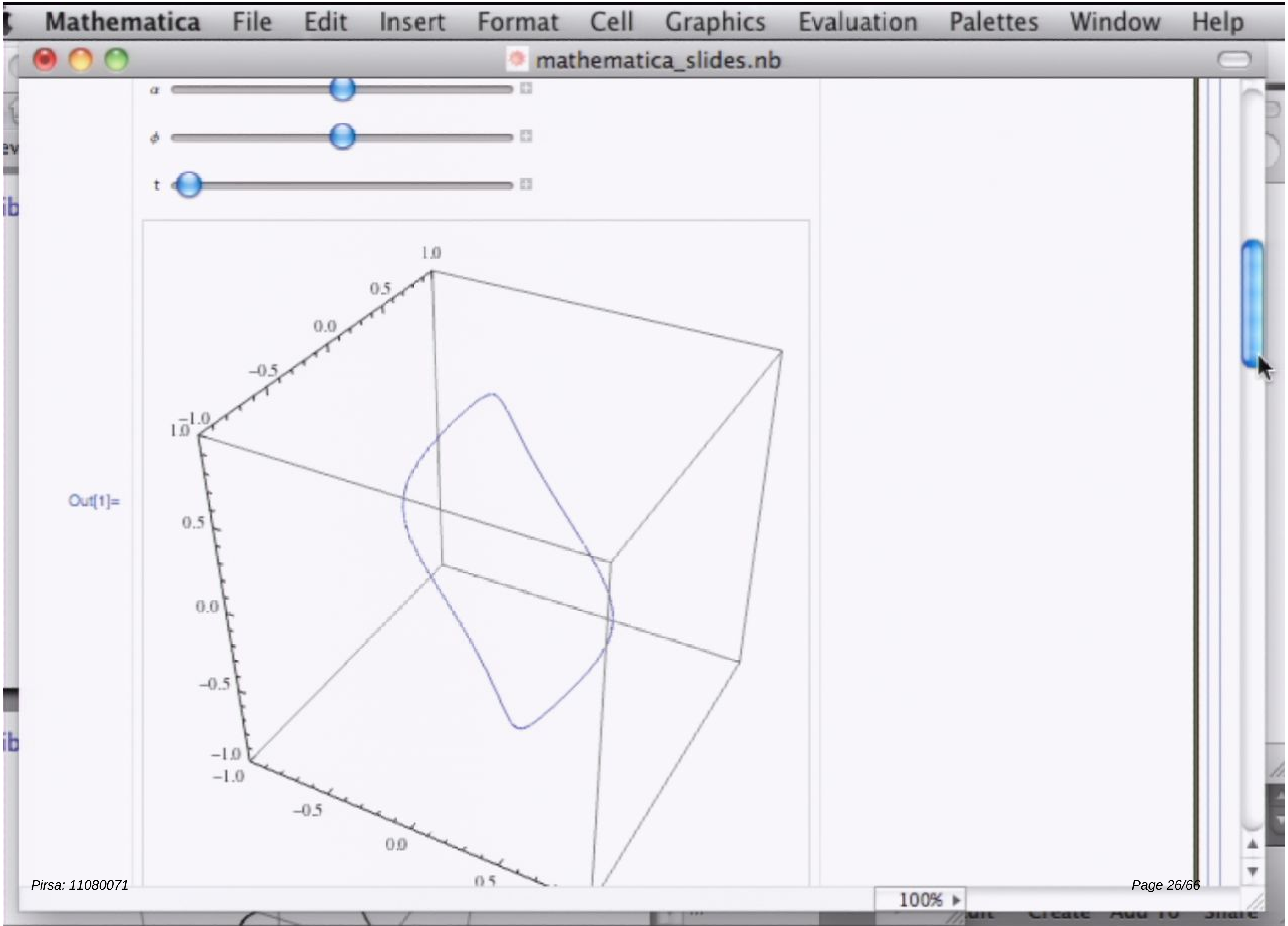
ϕ

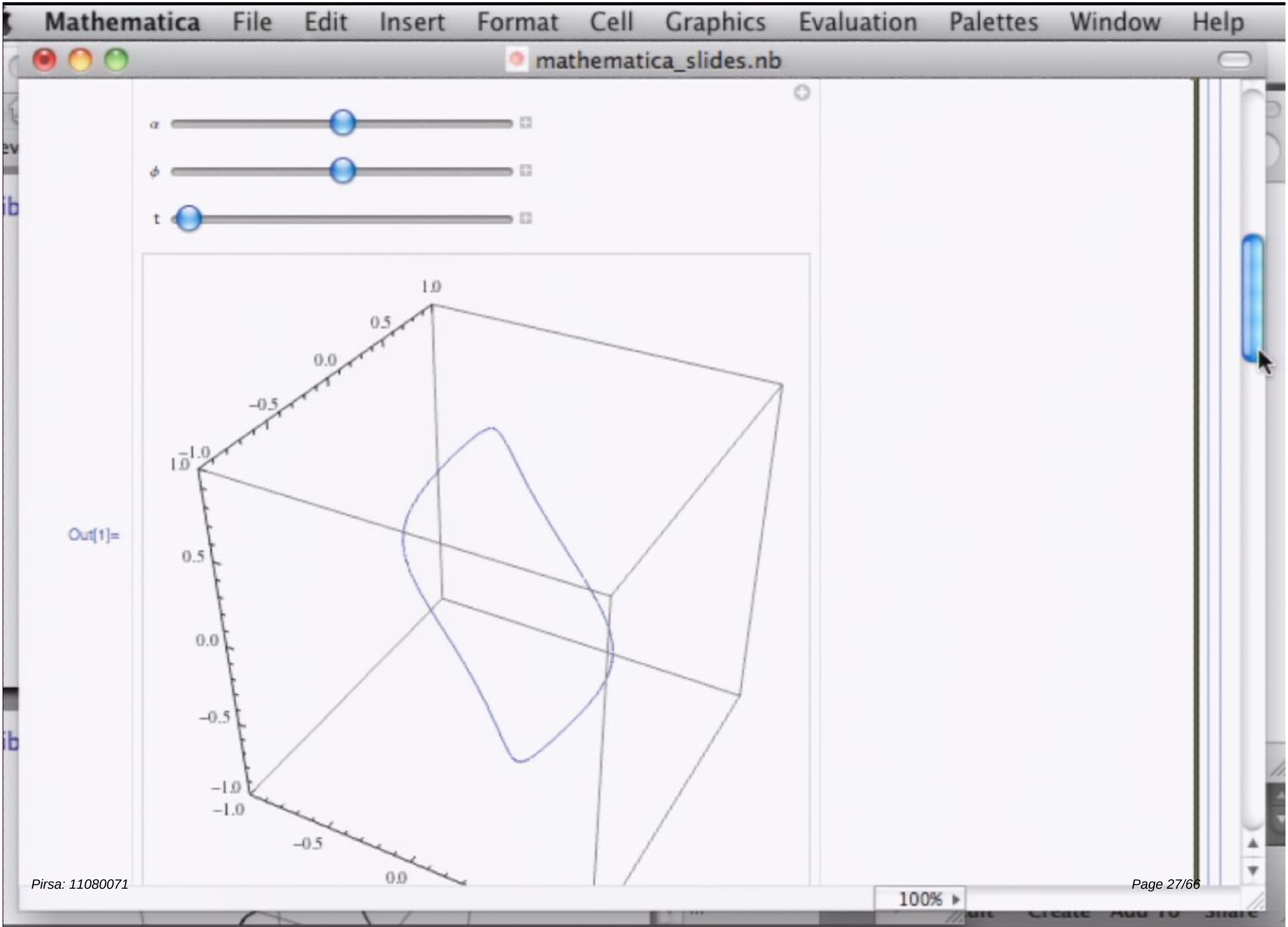
t

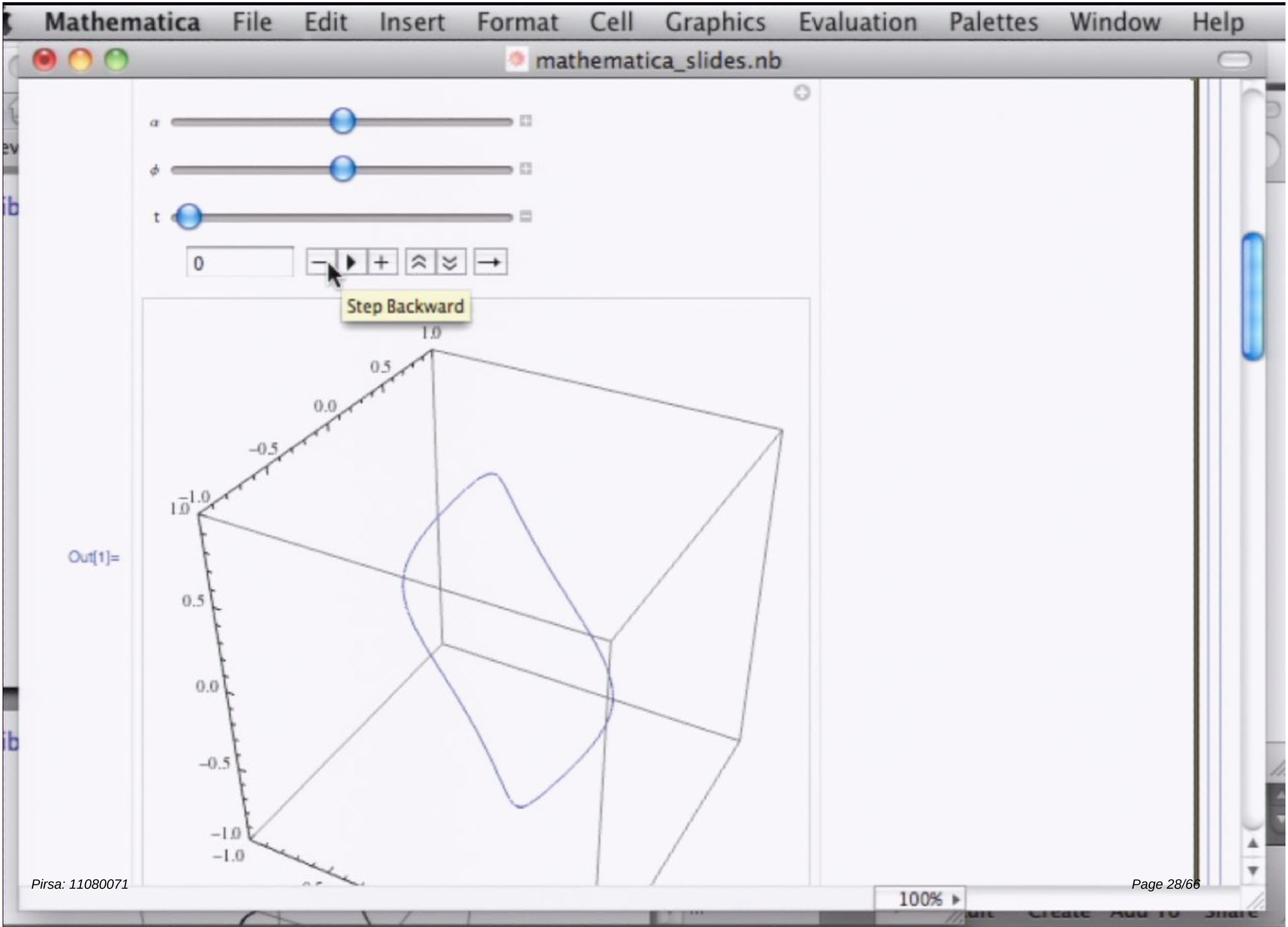


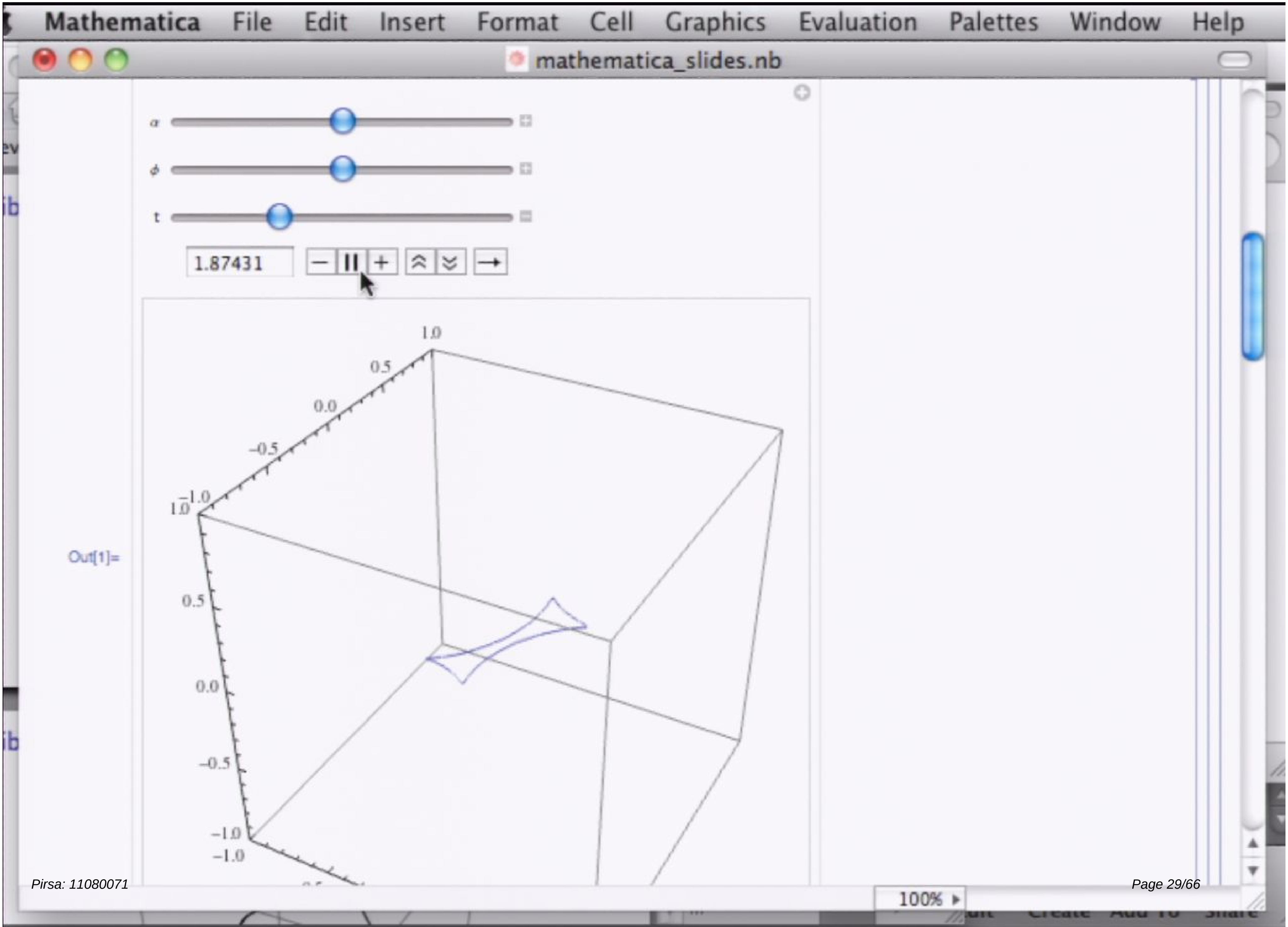
100%

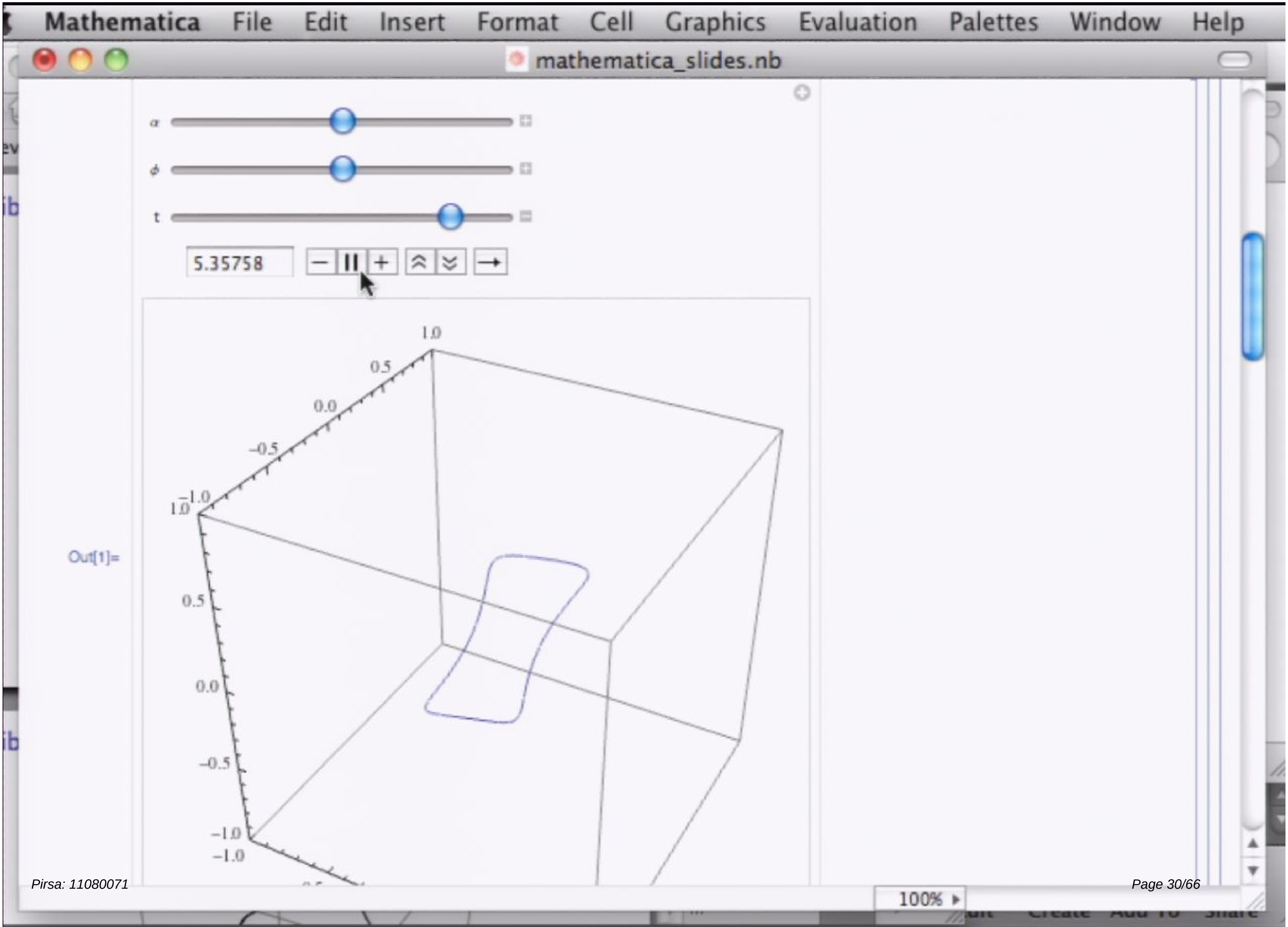

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
ϕ
t
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
100%

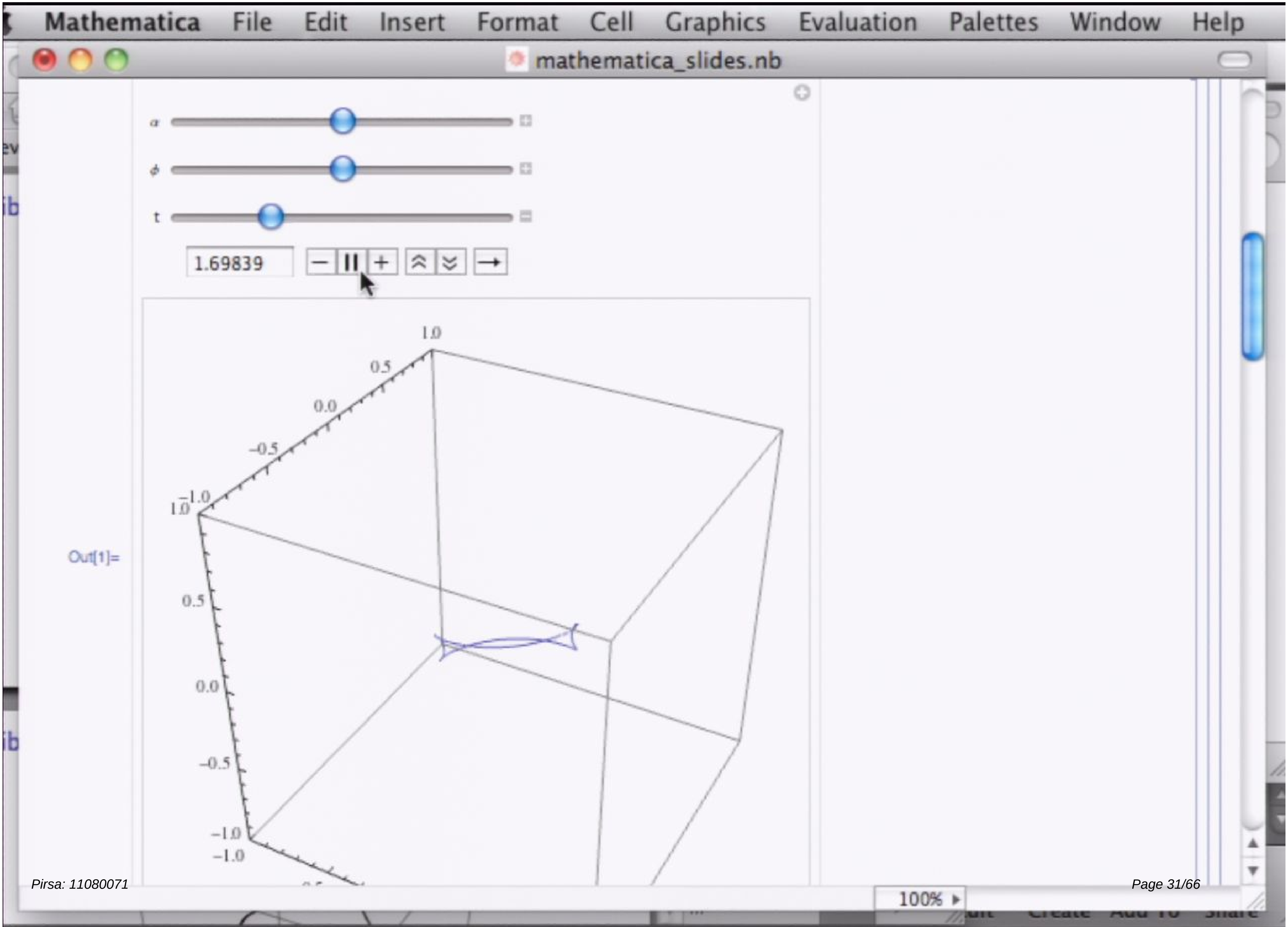

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
100%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
100%

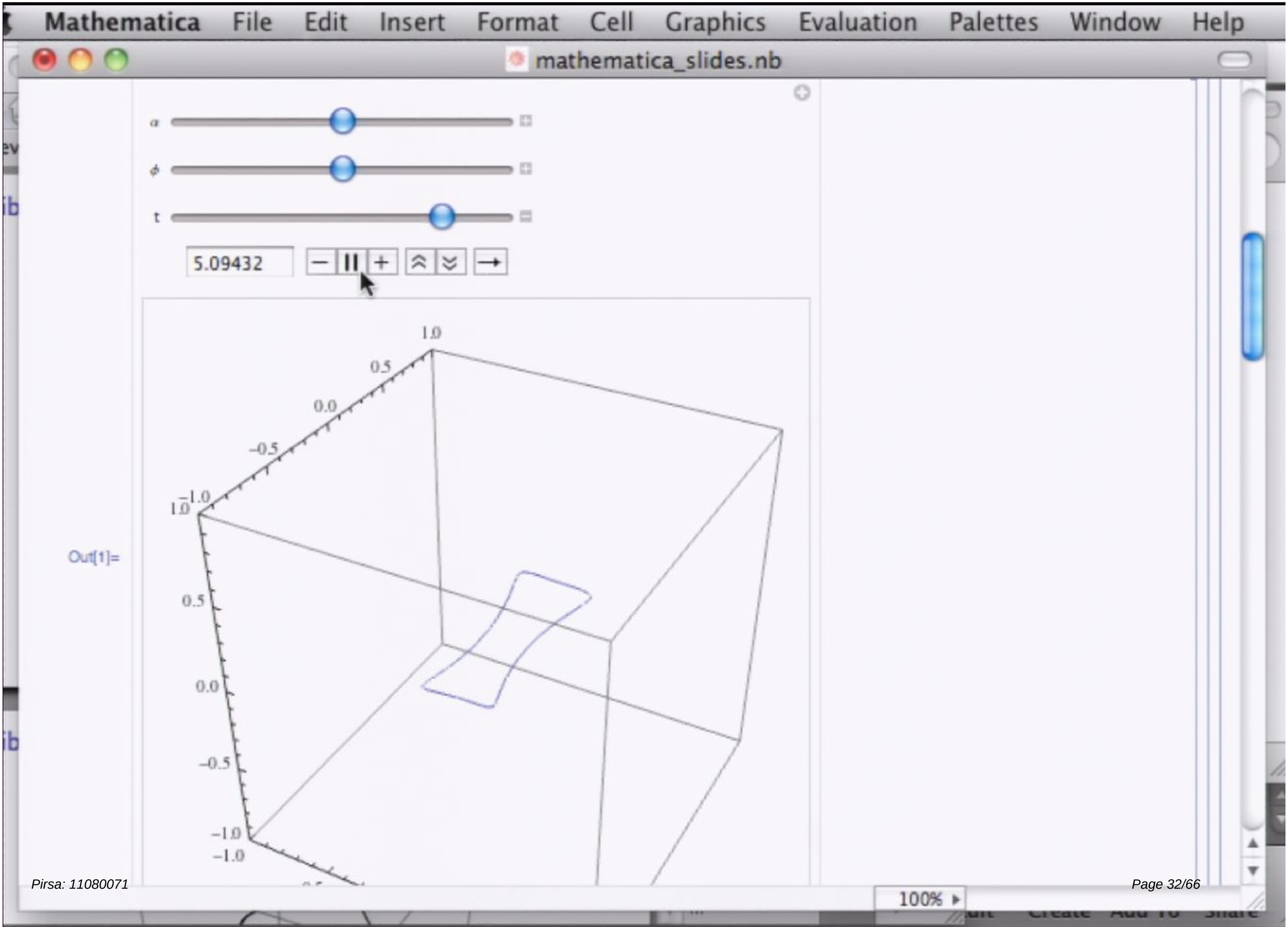

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
0
Step Backward
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
1.0
0.5
0.0
−0.5
−1.0
−1.0
100%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
1.87431
Out[1]=
1.0
0.5
0.0
–0.5
–1.0
1.0
0.5
0.0
–0.5
–1.0
–1.0
100%

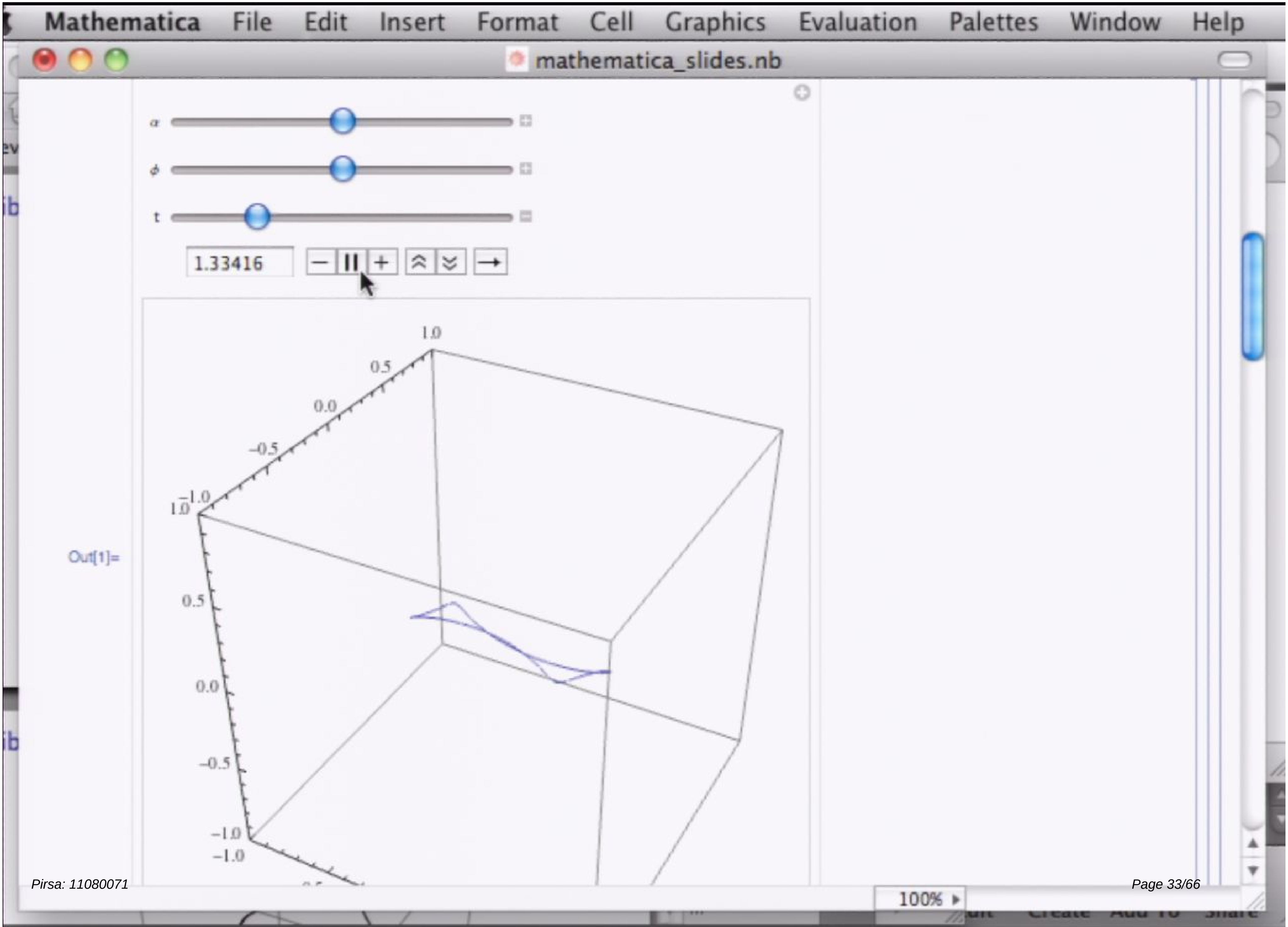

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
5.35758
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
−1.0
100%



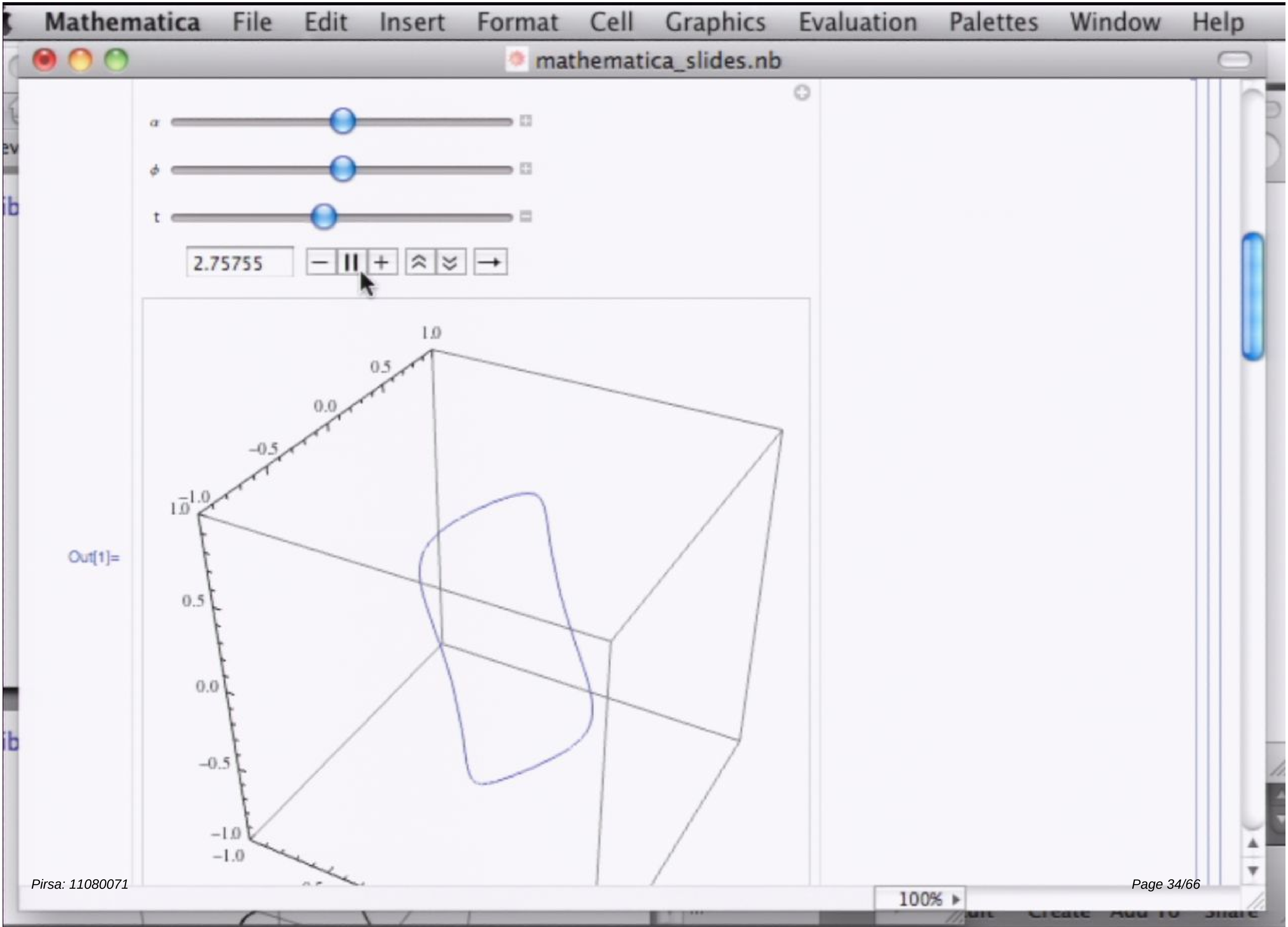
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
1.69839
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
100%

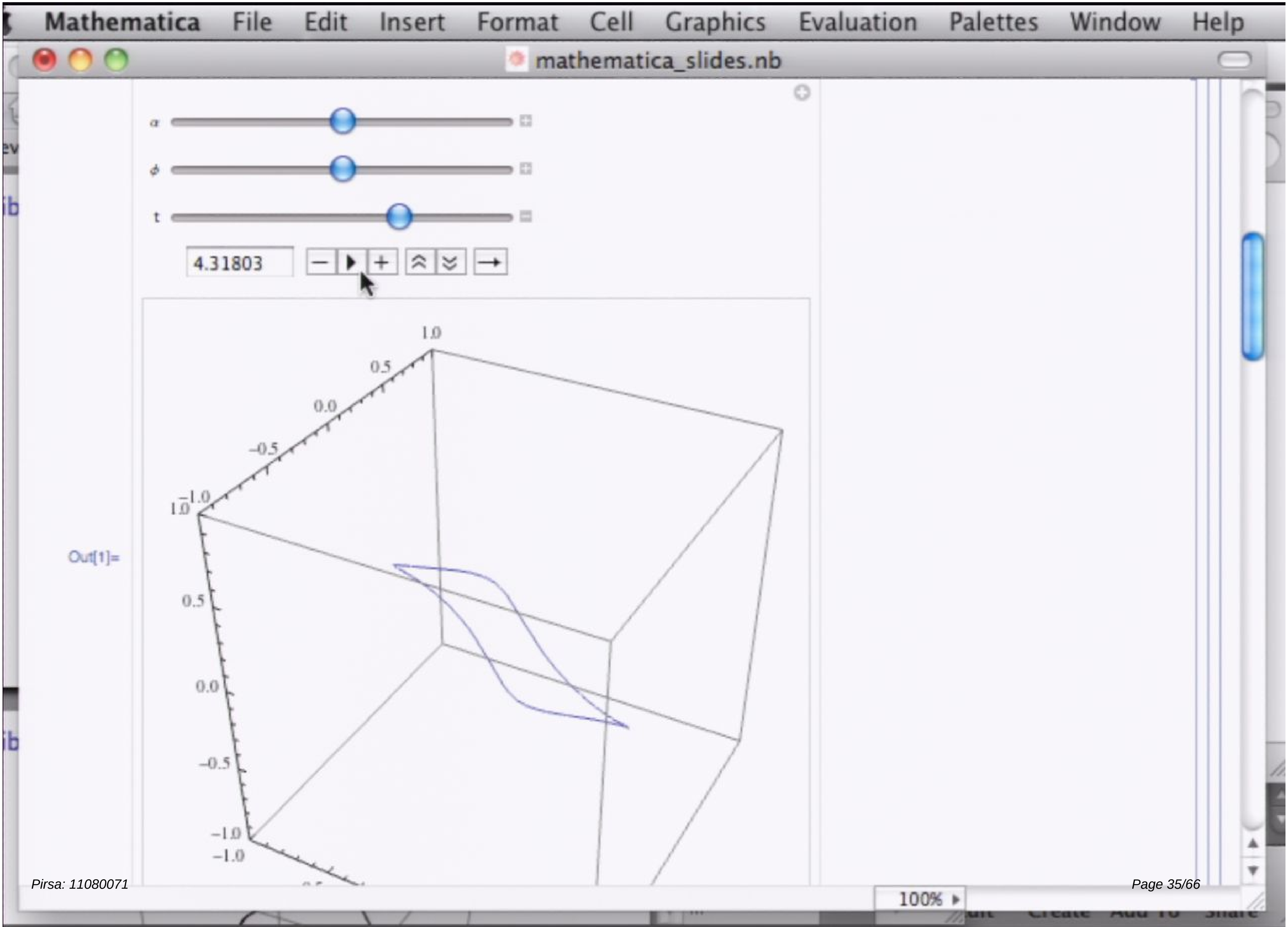


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
5.09432
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
100%

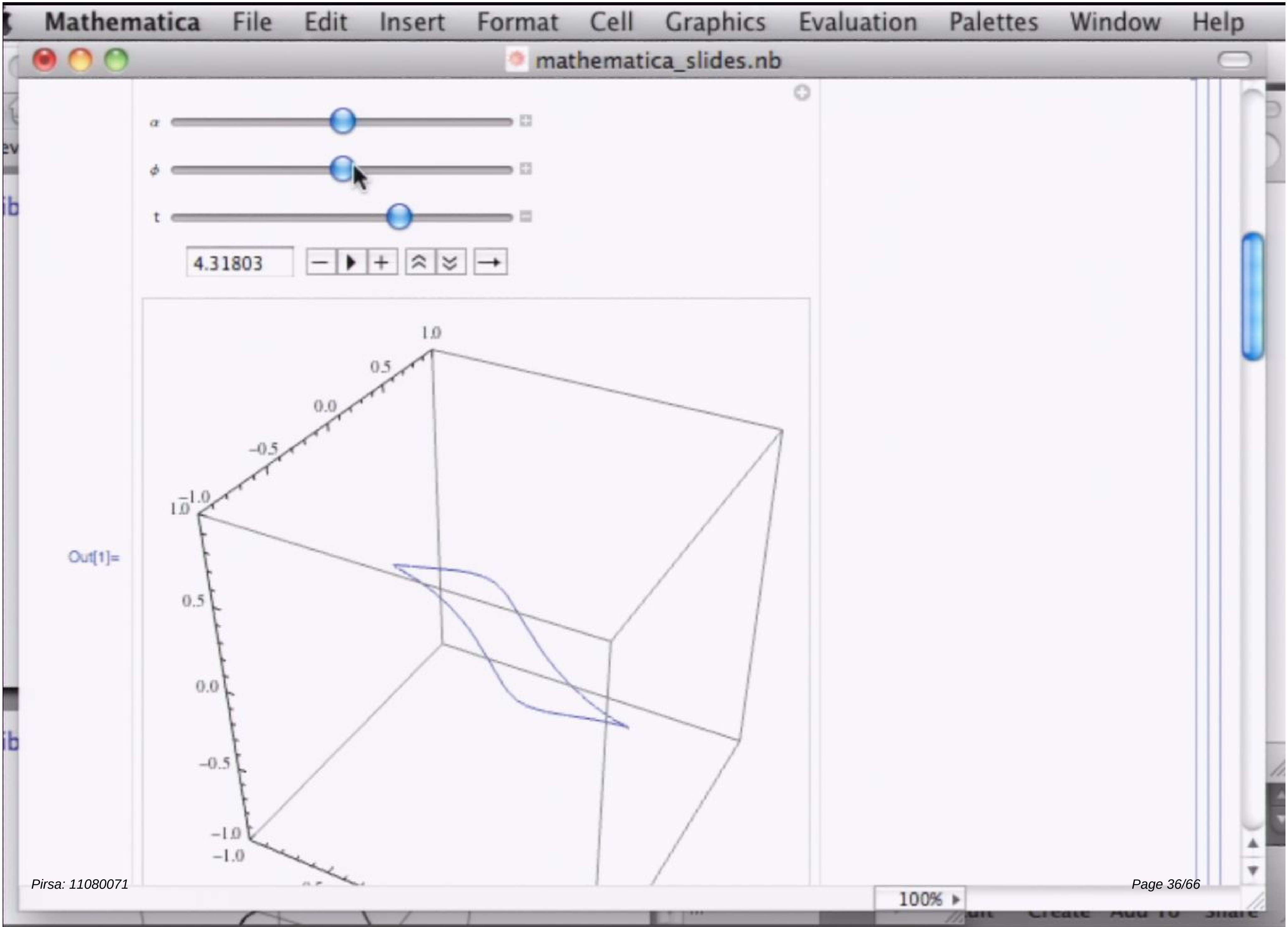


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
1.33416
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
100%

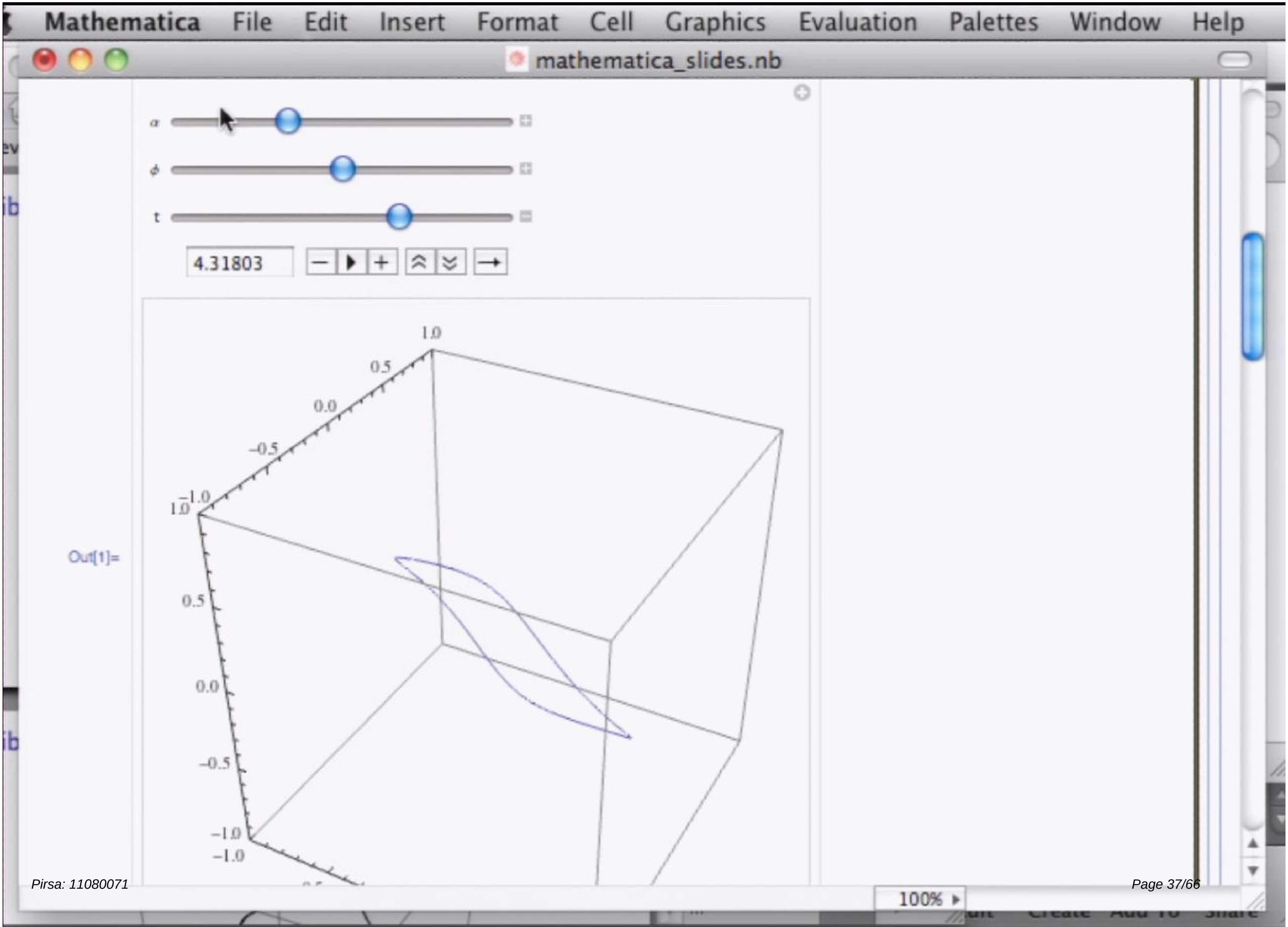

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
φ
t
2.75755
Out[1]=
1.0
0.5
0.0
-0.5
-1.0
1.0
0.5
0.0
-0.5
-1.0
-1.0
100%



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
4.31803
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
1.0
0.5
0.0
−0.5
−1.0
−1.0
100%



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
4.31803
1.0
0.5
0.0
−0.5
−1.0
1.0
Out[1]=
0.5
0.0
−0.5
−1.0
−1.0
100%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
α
ϕ
t
4.31803
Out[1]=
1.0
0.5
0.0
−0.5
−1.0
1.0
0.5
0.0
−0.5
−1.0
−1.0
100%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

mathematica_slides.nb

Chen, DiCarlo, and Hotes (CDH) Strings

This is a text cell.

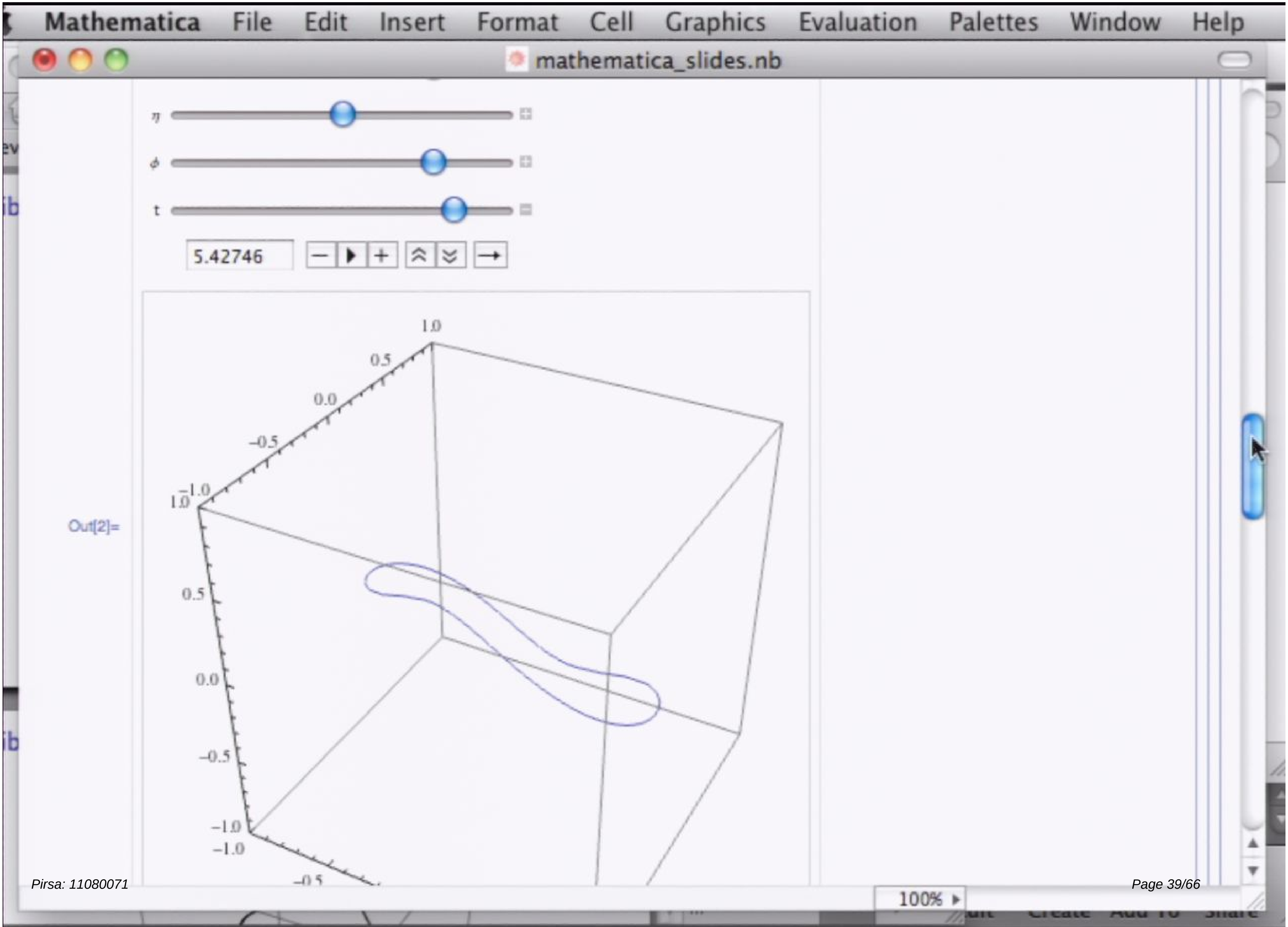
```
Manipulate[CC = Cos[θ] - Cos[η];
 S = Sin[θ] Sin[η];
 a[u_] := {CC Cos[θ], S, -CC Sin[θ]} Sin[3 u] / 6 - {-S Cos[θ], CC, S Sin[θ]} Cos[3 u] / 6 +
    {2 - CC Cos[θ], -S, CC Sin[θ]} Sin[u] / 2 -
    {3 S Cos[θ], Cos[θ] + Cos[η], -Sin[η] (1 - 3 Cos[θ]^2)} Cos[u] / 2;
 b[v_] := {1, 0, 0} Sin[v] - {0, Cos[ϕ], Sin[ϕ]} Cos[v];
 plot1 = ParametricPlot3D[(a[t + s] + b[t - s]) / 2, {s, 0, 2 Pi}, PlotRange → 1],
 {{θ, 2 π / 5}, 0, π / 2}, {{η, π / 2}, 0, π}, {{ϕ, 4 π / 5}, 0, π}, {{t, 0}, 0, 2 π}]
```

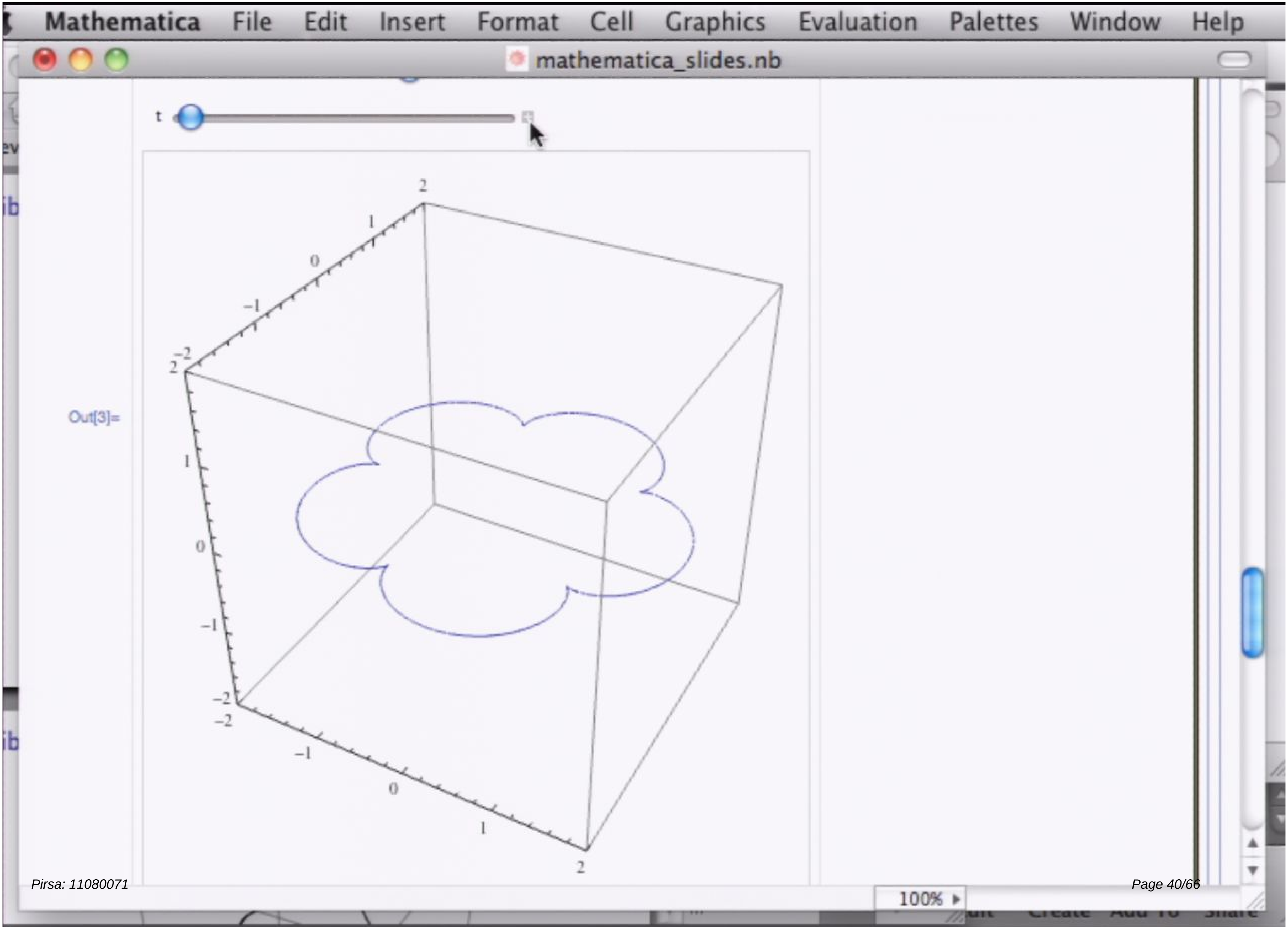

Rigidly Rotating Strings

This is a text cell.

```
Manipulate[
 AA = (1 - y^2 + x^2)^2 - 4 x^2;
```


100%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
η
ϕ
t
5.42746
Out[2]=
1.0
0.5
0.0
-0.5
-1.0
1.0
0.5
0.0
-0.5
-1.0
-1.0
-0.5
100%

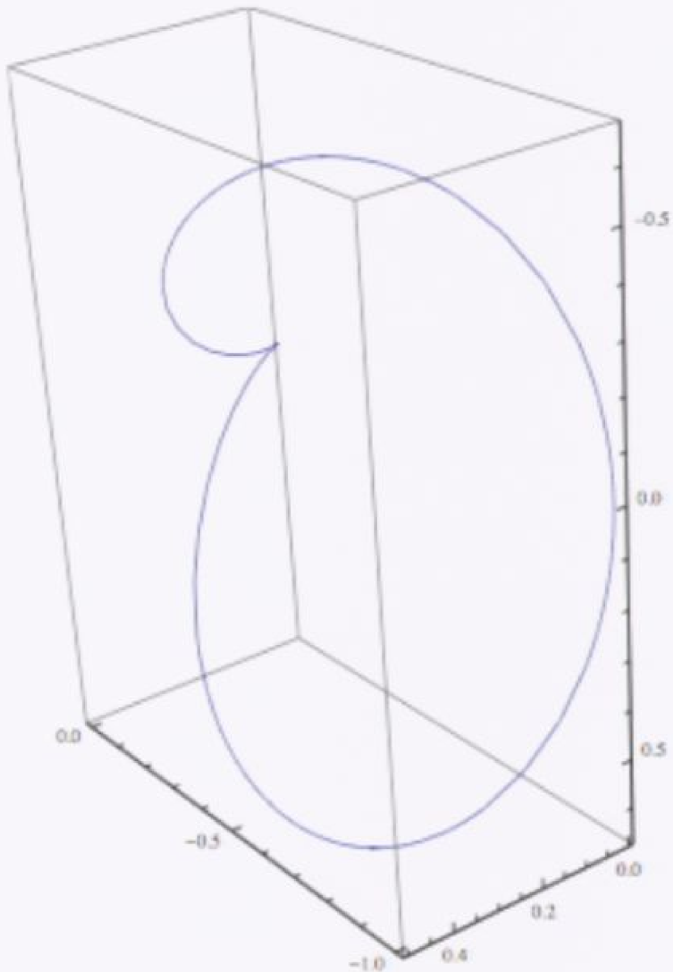


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
mathematica_slides.nb
t
Out[3]=
2
1
0
−1
−2
2
1
0
−1
−2
−2
−1
0
1
2
100%

Outline

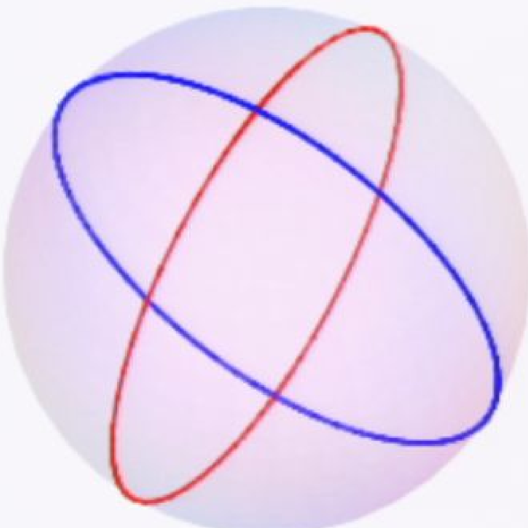
Outline

Vhat is a Cusp?



- Points at which the string reaches the speed of light.
- Extrinsic curvature diverges

Cusps as Intersections



- At intersection,

$$\frac{\partial \mathbf{r}}{\partial t}(t_0, \sigma_0) = a'(u_0) = b'(v_0)$$
$$\frac{\partial \mathbf{r}}{\partial \sigma}(t_0, \sigma_0) = \Big(a'(u_0) - b'(v_0)\Big)/2 = 0$$

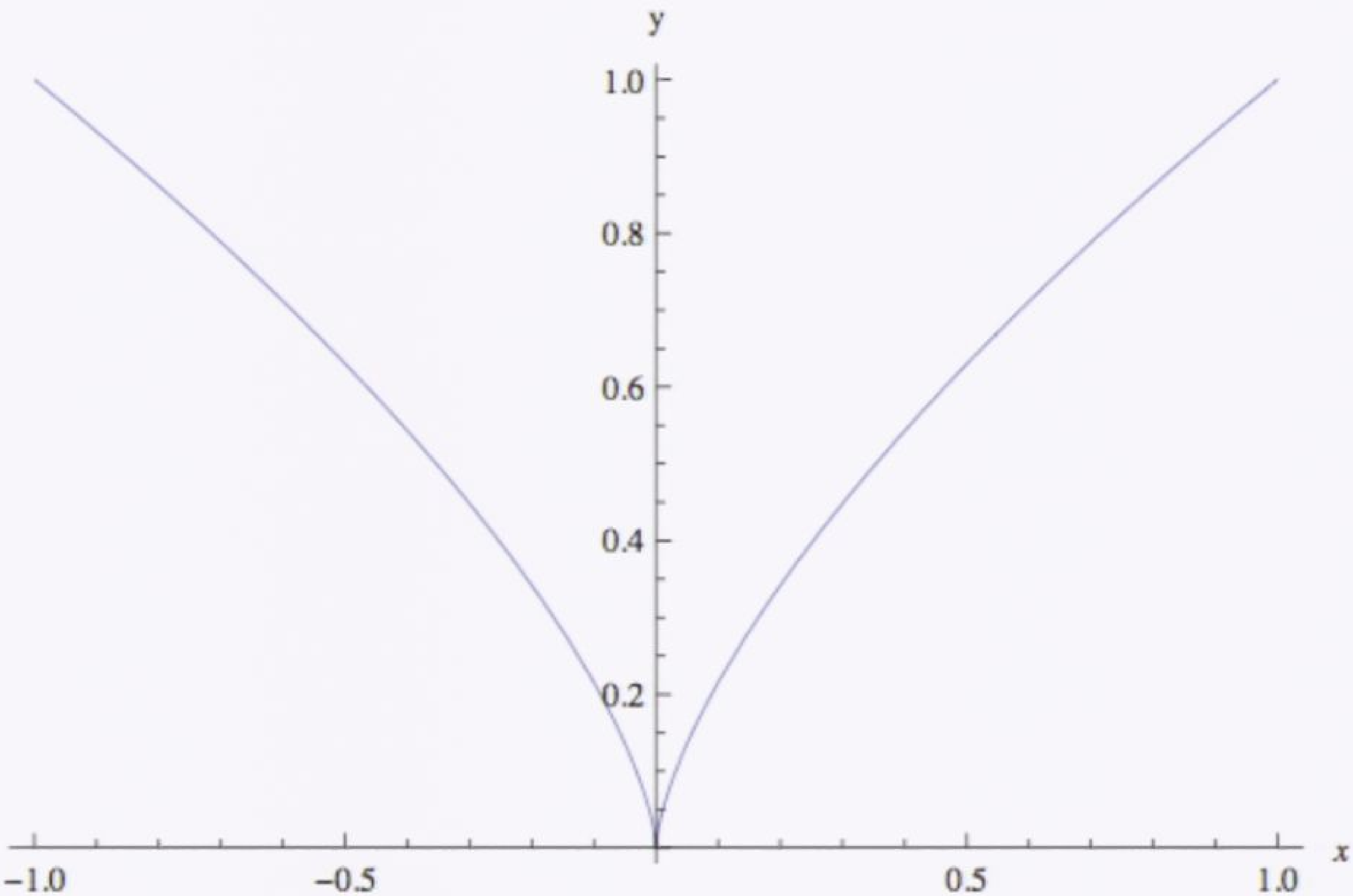
- Local Expansion:

$$\mathbf{r}(t_0, \sigma) - \mathbf{r}_0 = \frac{(\sigma - \sigma_0)^2}{2}\frac{\partial^2 \mathbf{r}}{\partial \sigma^2} + \frac{(\sigma - \sigma_0)^3}{6}\frac{\partial^3 \mathbf{r}}{\partial \sigma^3} + \dots$$

$$a'(u_0) = b'(v_0)$$

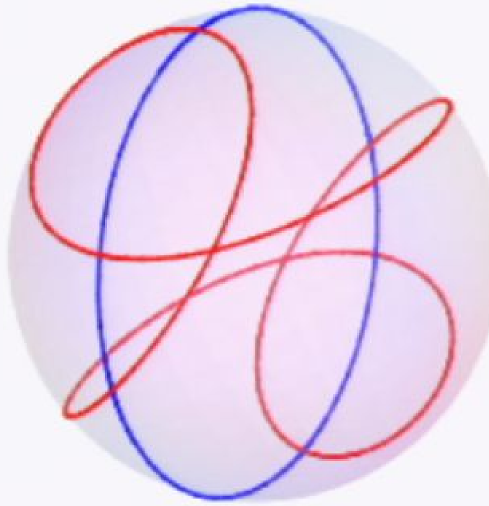
Prototype of all Cusps

$$\mathbf{r}(s) = (s^3, s^2) \implies y = x^{2/3}$$



How many cusps?

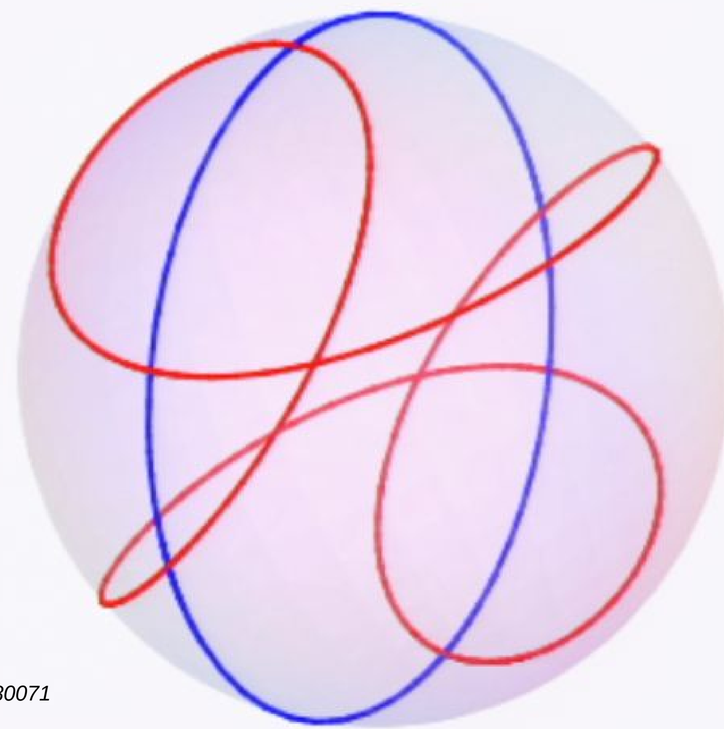
- It's important to model the number of cusps per oscillation
- String evolution is fairly complicated, we have to resort to numerical methods
- Classical solutions already exhibit a variety of behavior

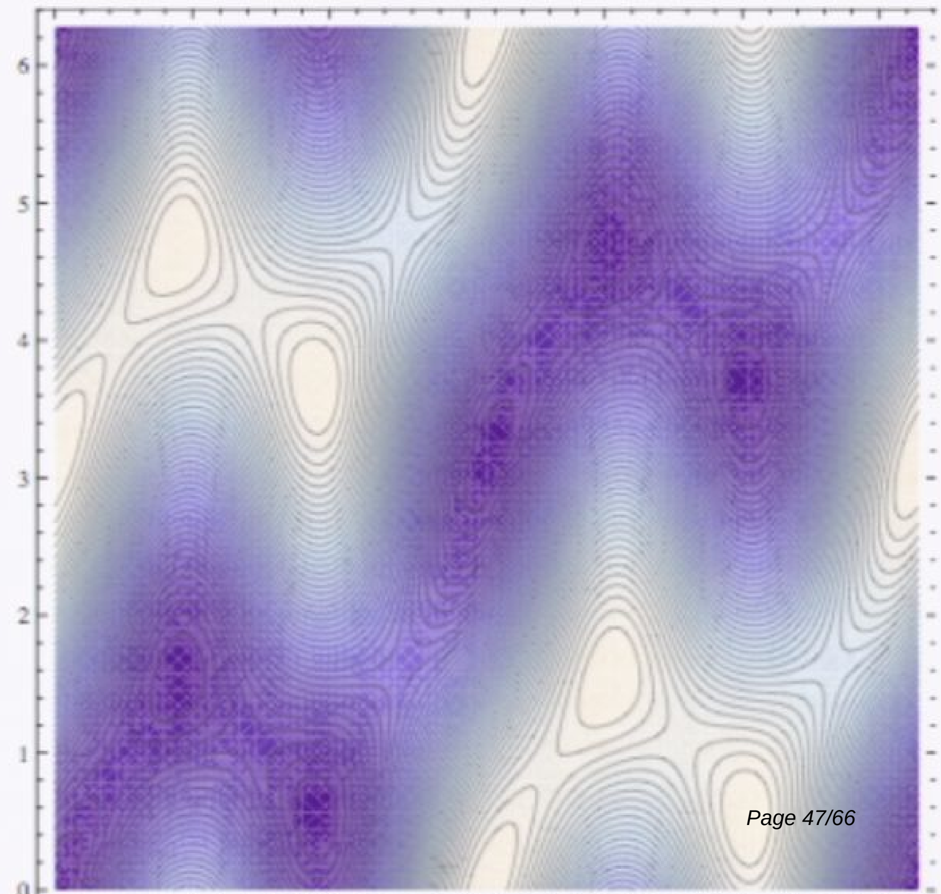


Cusp Counting as Minimization

Want to minimize

$$f(u,v) = |\mathbf{a}'(u) - \mathbf{b}'(v)|^2$$





Minimization Techniques

- Gradient Descent:

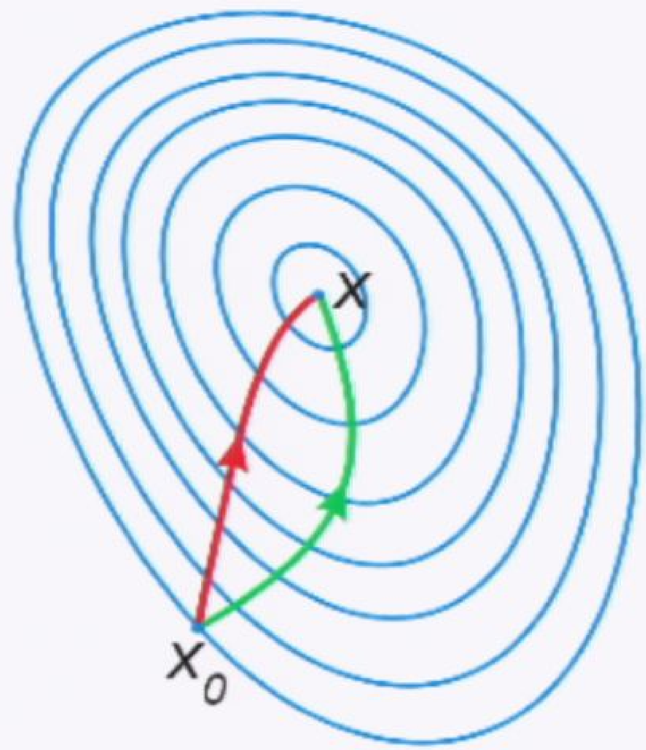
$$x_{n+1} = x_n - \gamma_n \nabla F(x_n)$$

for some small enough γ_n

- Newton Method:

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$

- If we start close enough to a minimum, the sequence will converge.

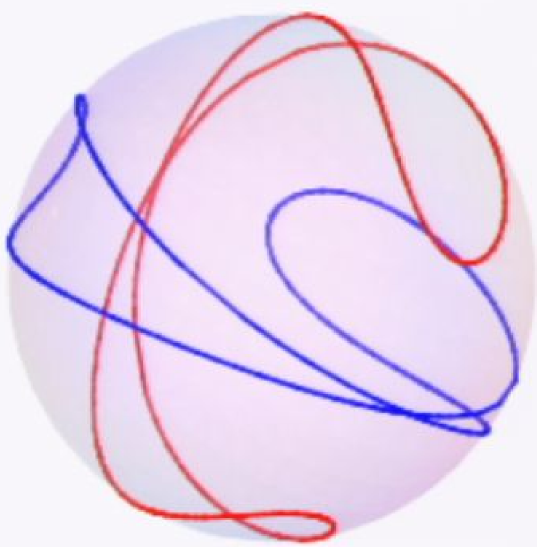


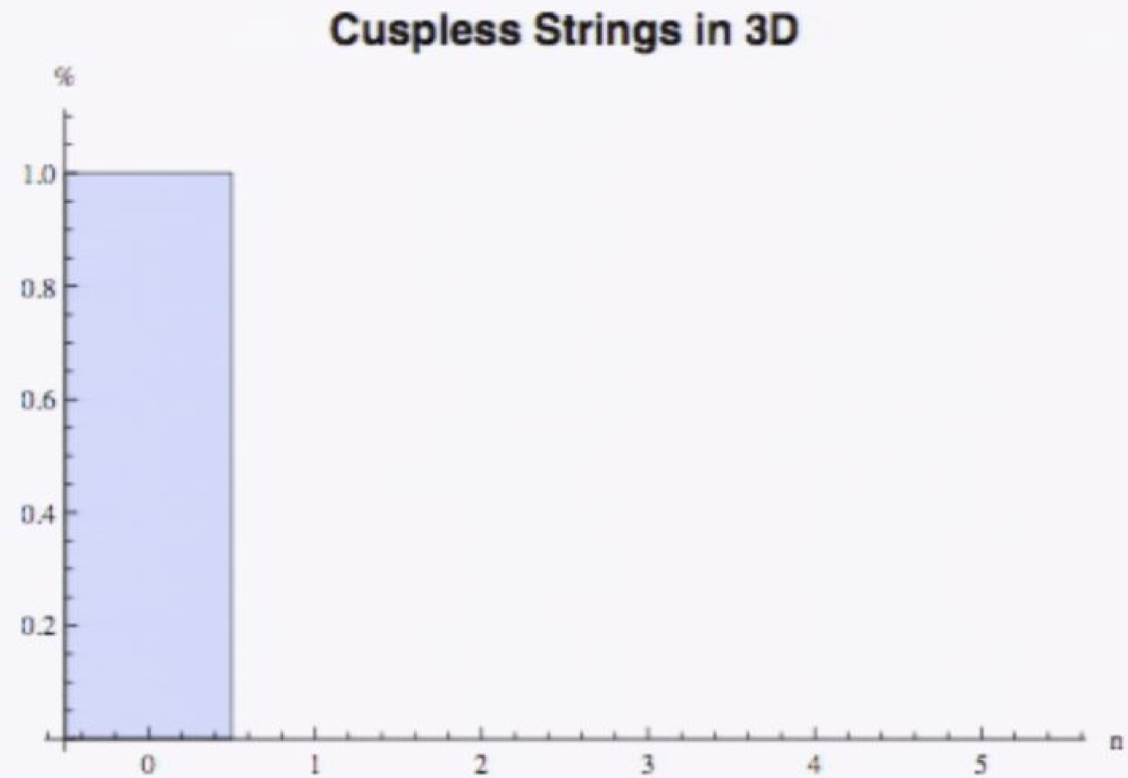
Our Strategy

1. Simulate a number of string solutions from a particular family
2. For each string,
   2.1 Throw a random points on the (u, v) space
   2.2 Run our minimization procedure, record (in increasing order) any new local minimum we find
   2.3 Keep trying until we don't get any more new minima
   2.4 Count the number of minima below a cutoff
3. Average over all string solutions

Numerical Results - checking the simple cases

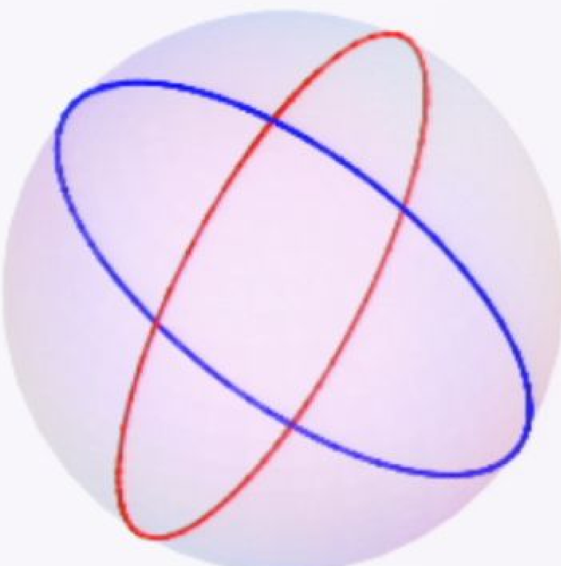
Cuspless Strings:

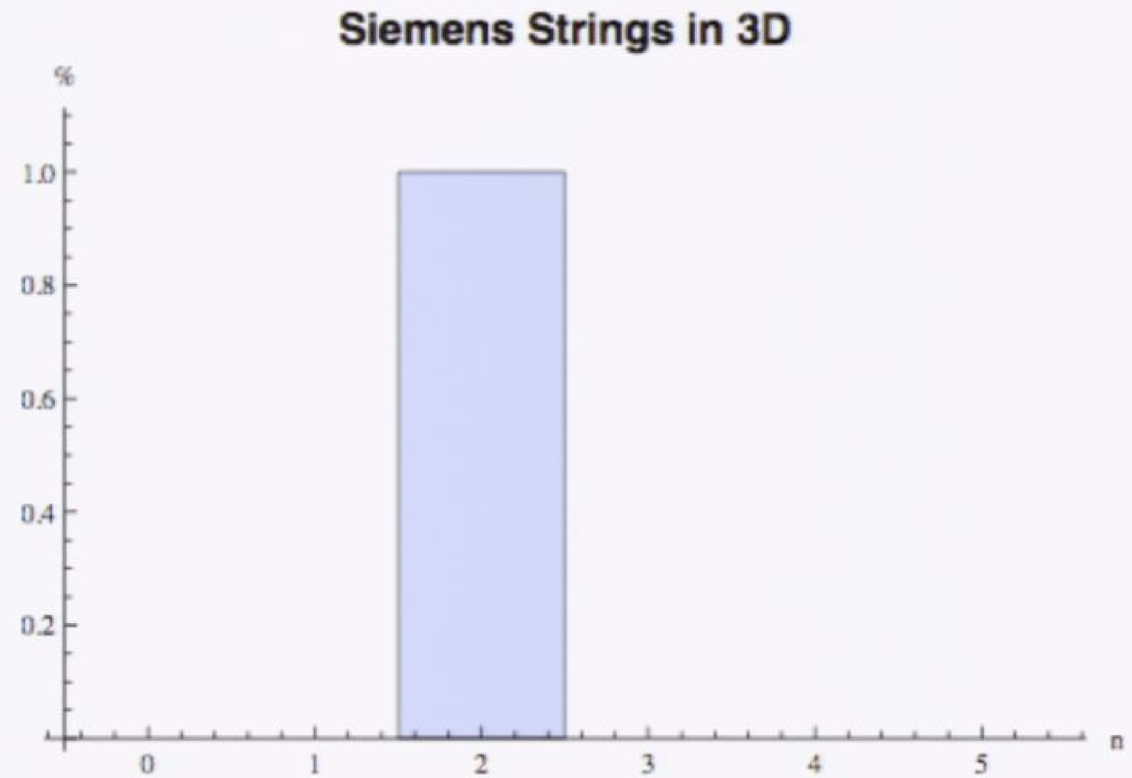




Numerical Results - checking the simple cases

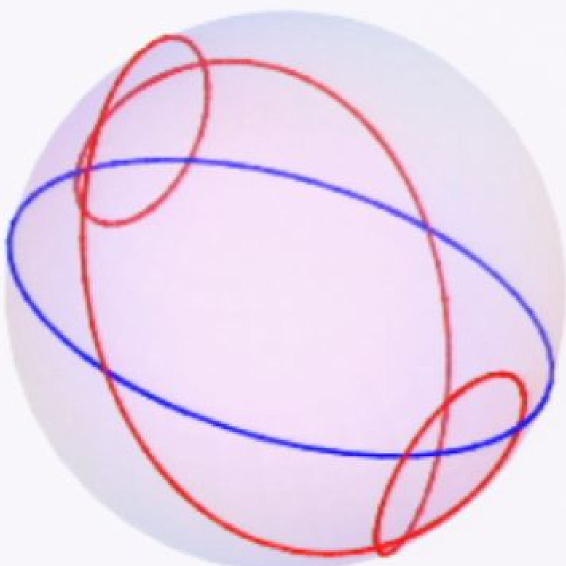
Simplest Siemens Strings:

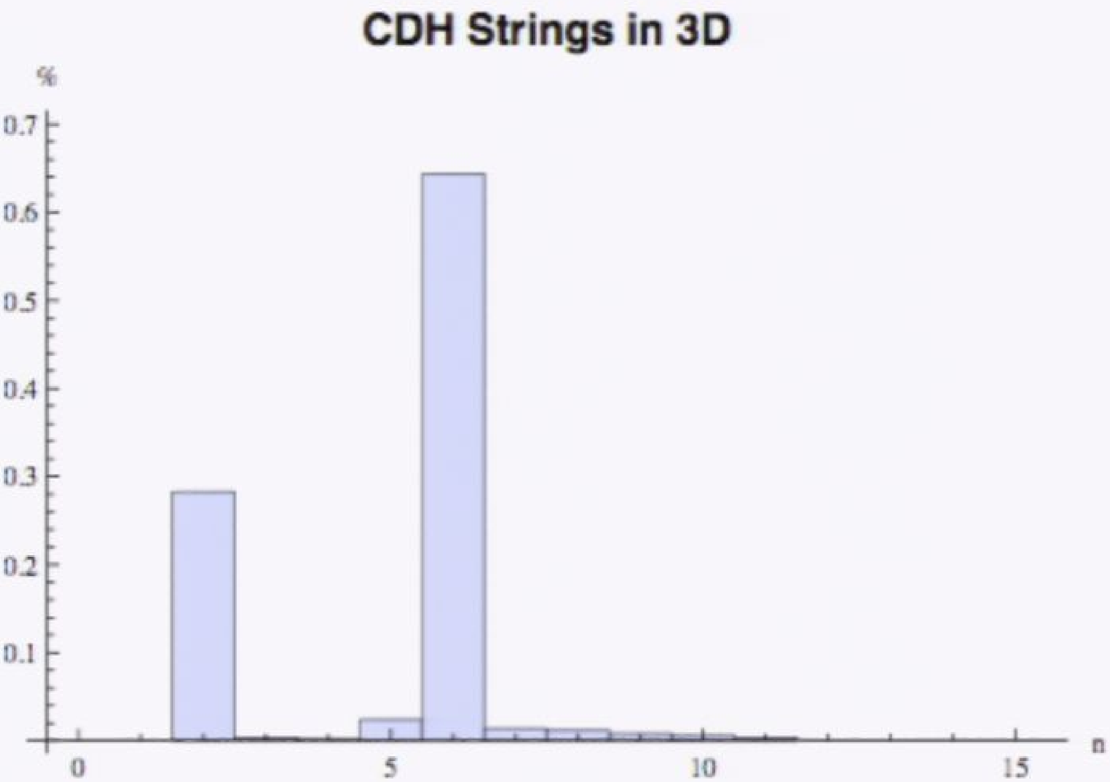




Numerical Results

CDH Strings:

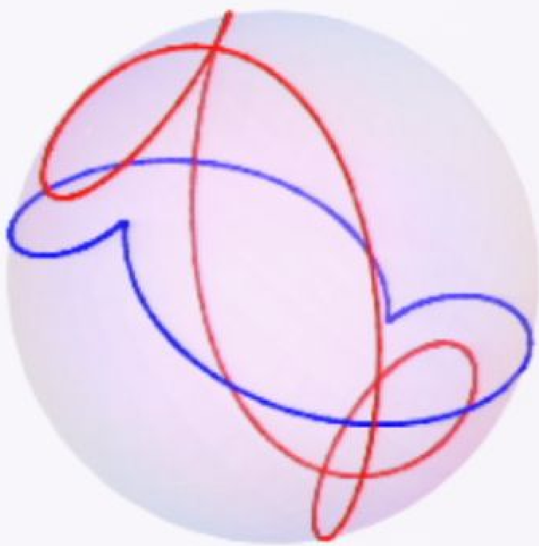


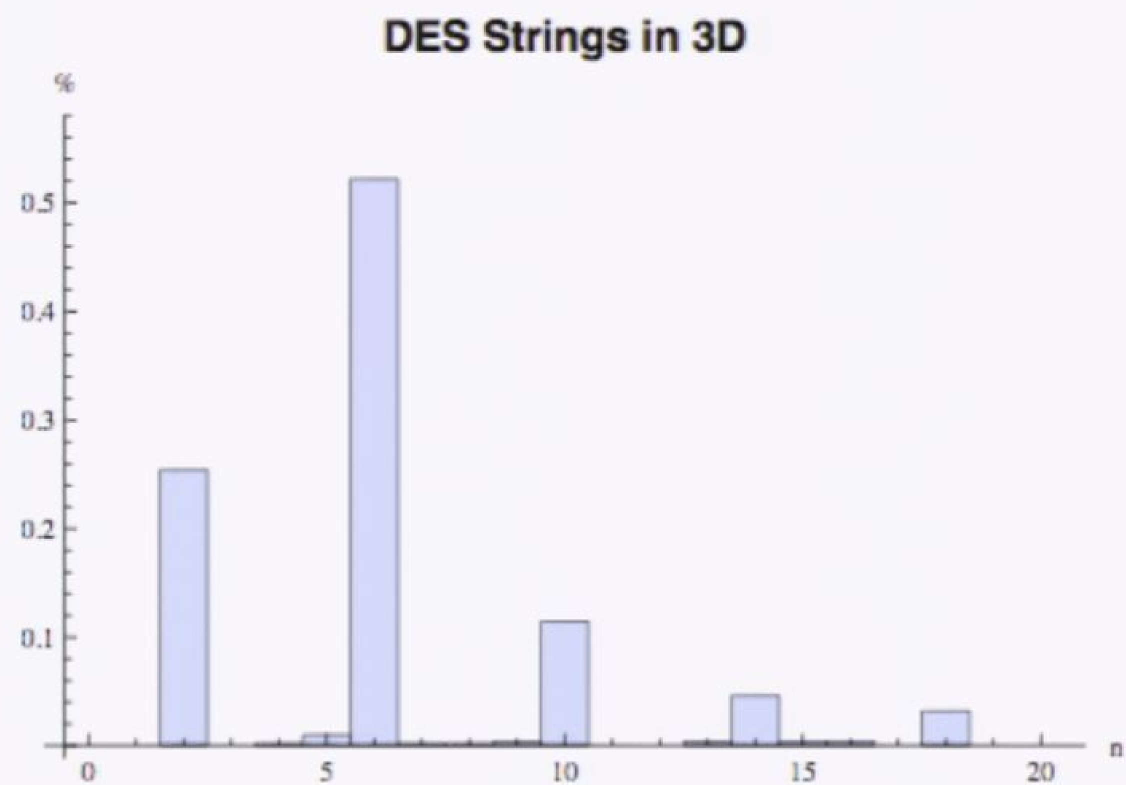


Average $= 4.96$

Numerical Results

DES Strings:





Average = 6.3

Outline

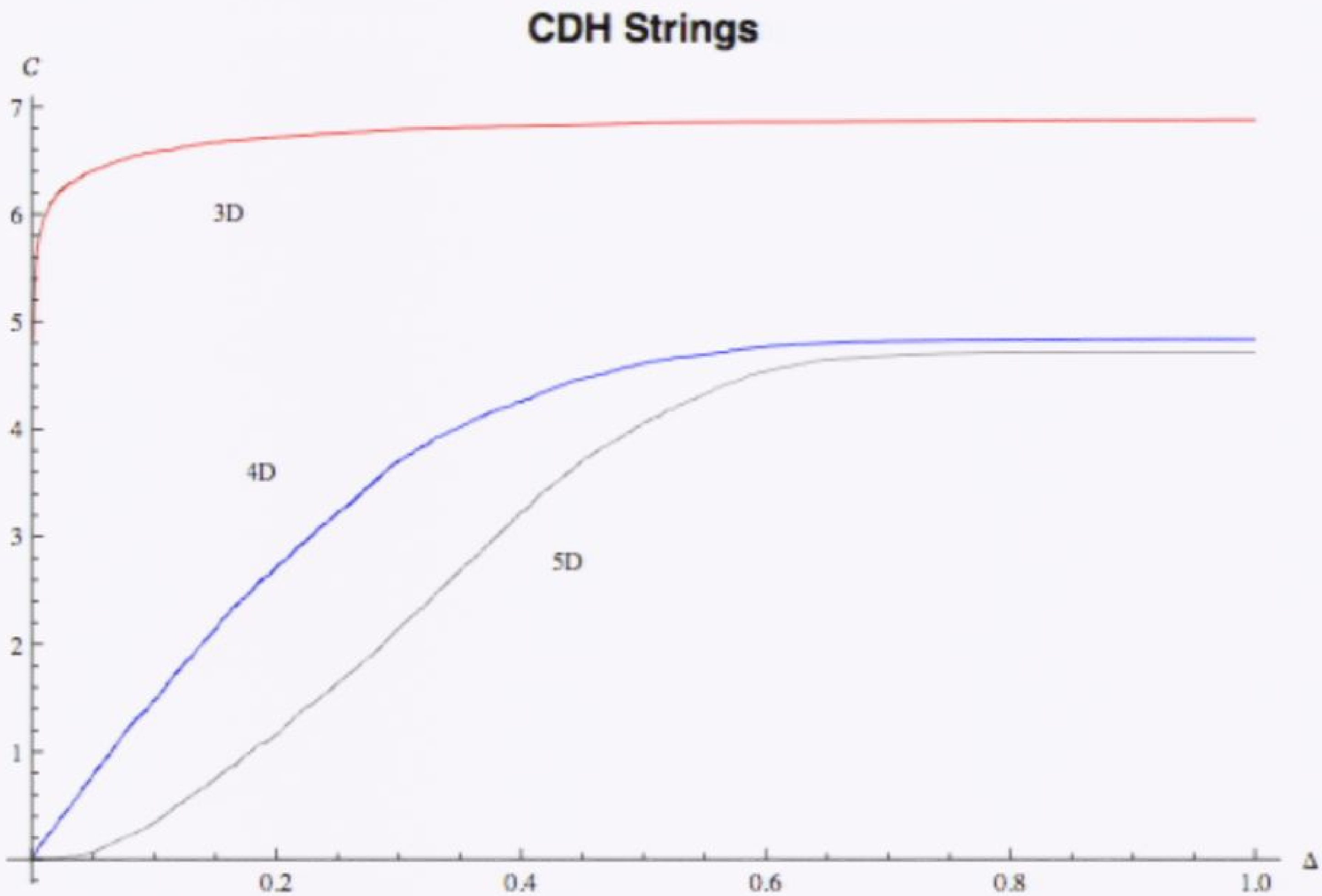
No More Cusps

- Two smooth curves in S^3 don't intersect. (Need to solve three equations but only have two degrees of freedom.)
- In the presence of extra dimensions, the probability of exact cusp is strictly zero.
- Near Cusp Events

$$\Delta = \frac{1}{2} \min |\mathbf{a}'(u) - \mathbf{b}'(v)|$$

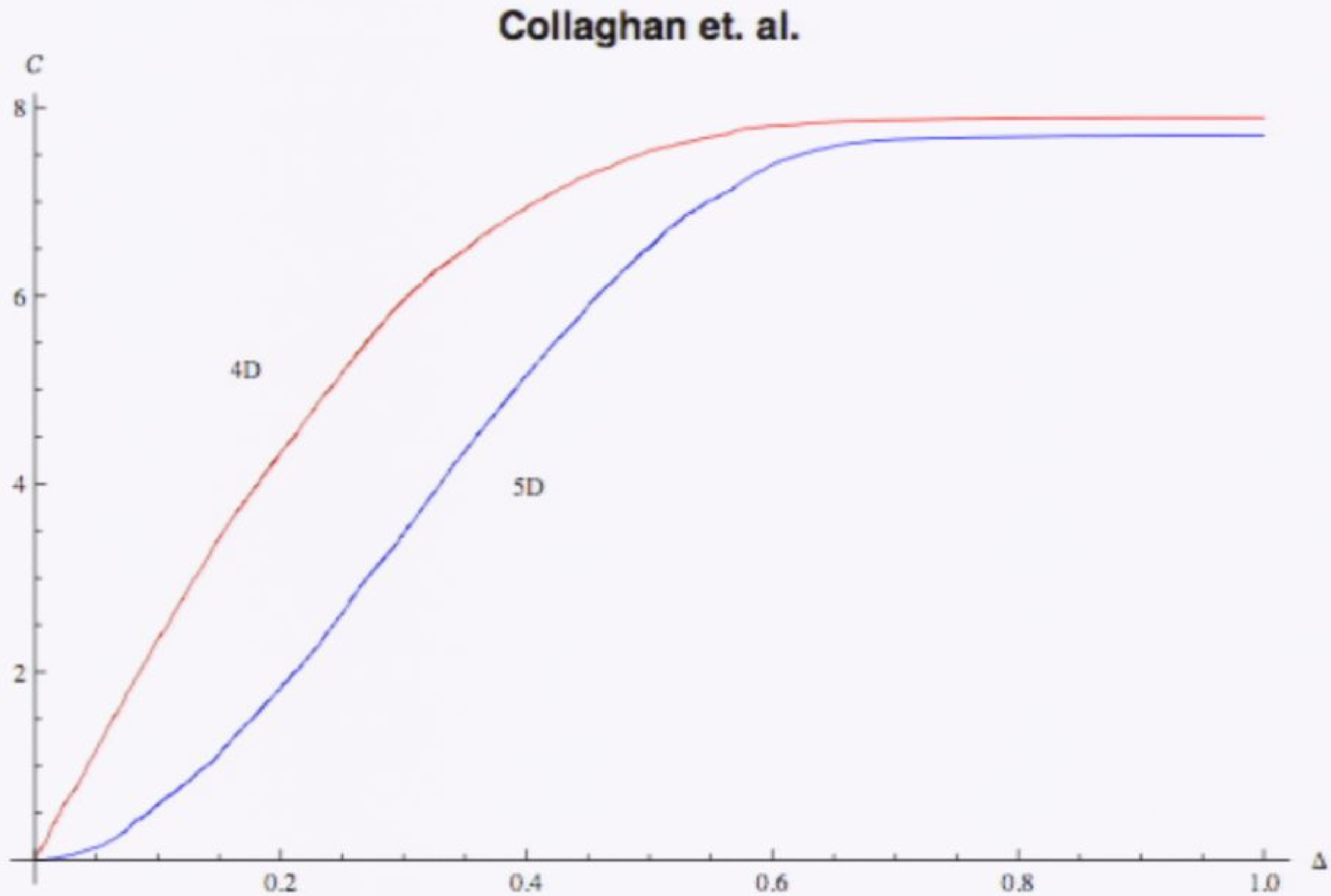
Results - number of dimensions

CDH Strings in 3, 4, and 5 spatial dimensions

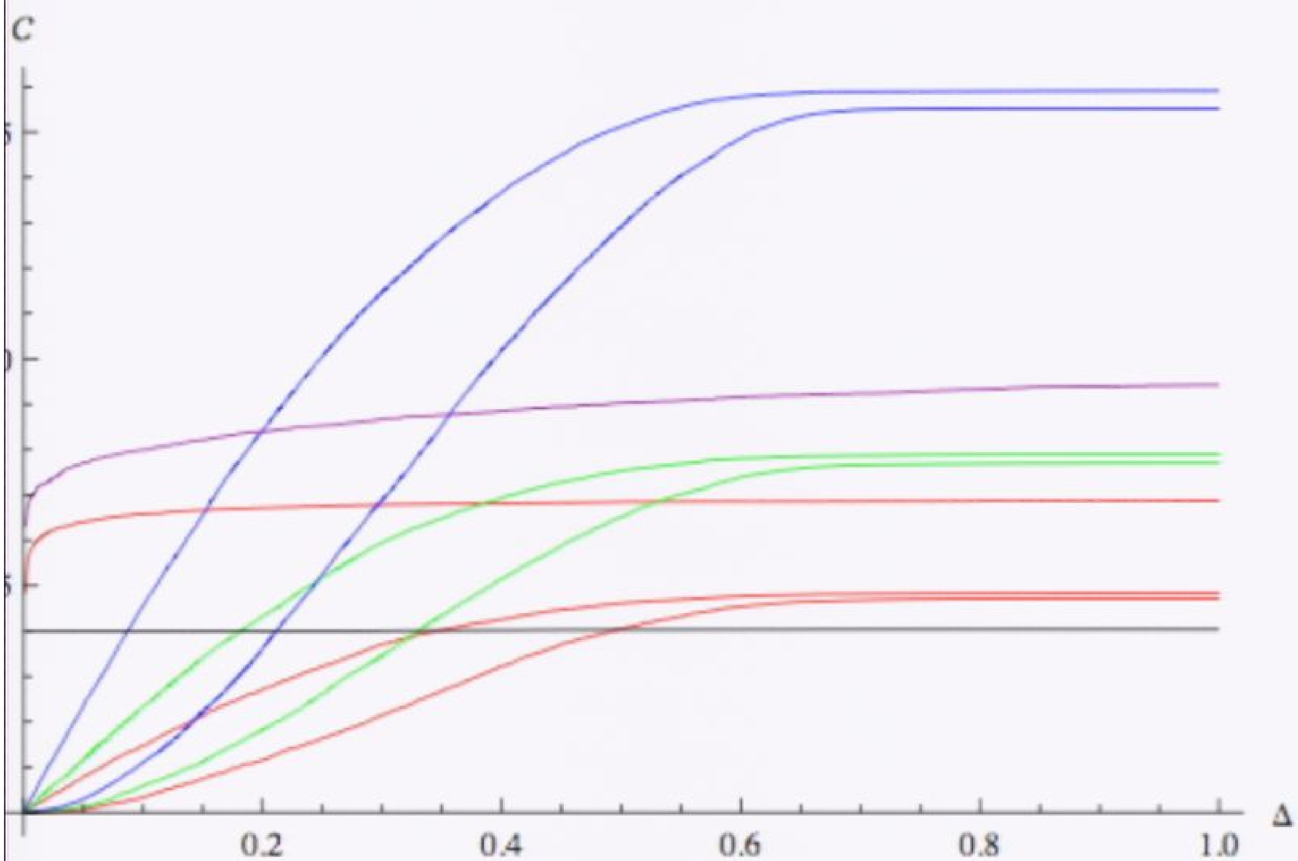


Results - number of dimensions

Collaghan *et al.* proposed the 1-5/1 Strings



Results - overview

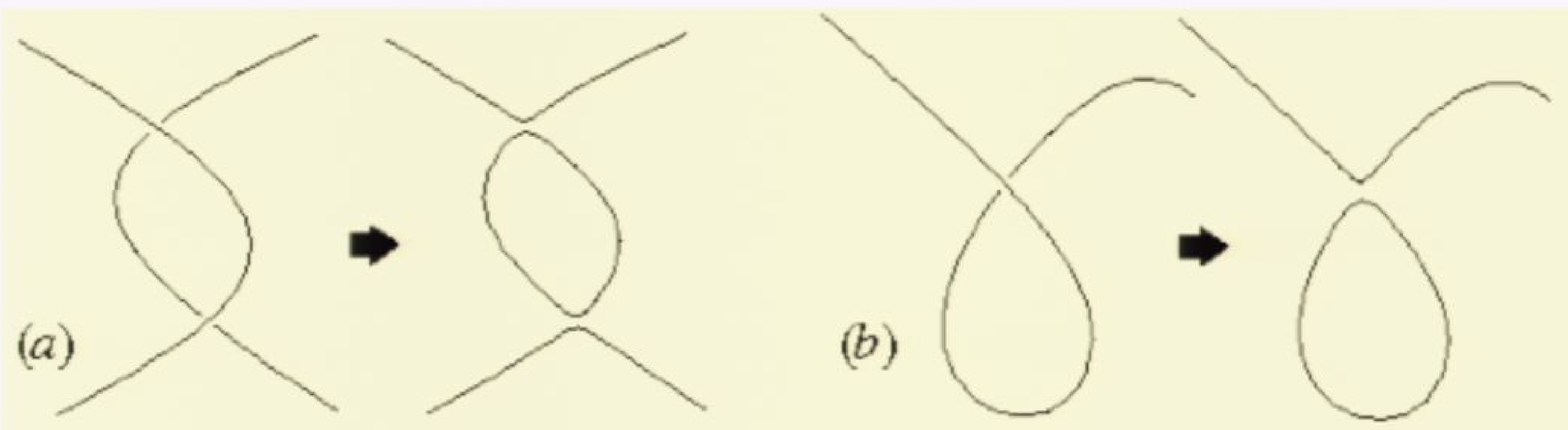


- CDH Strings
- 1-10/1 Strings
- DES Strings
- Siemens Strings
- Collaghan Strings

Outline

String Intercommutation

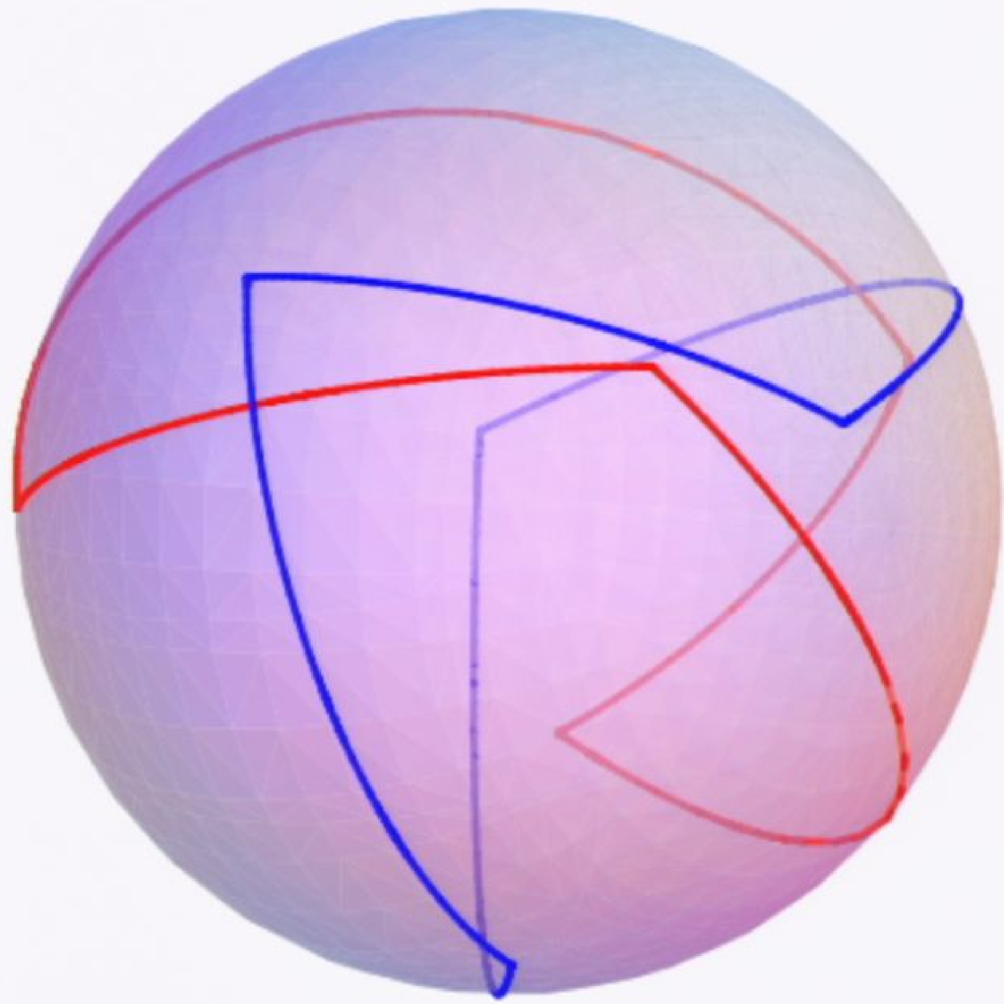
- Whenever two long strings cross each other, they exchange ends, or "intercommute".
- A long string can intercommute with itself, in which case a loop will be produced



Network of Strings

From numerical simulation, we model loops as piecewise linear.

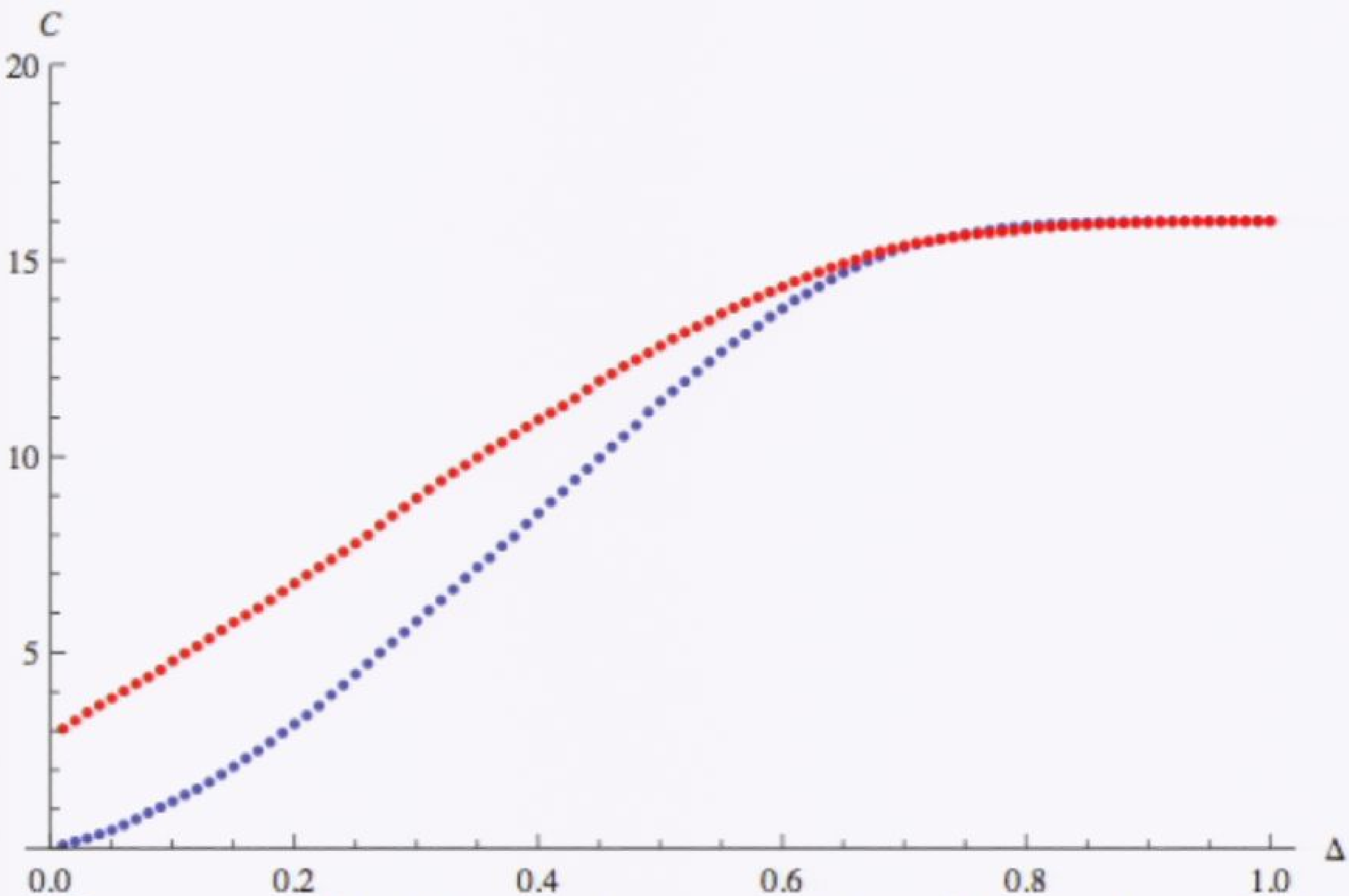
Random Curves on Kibble-Turok Sphere



Numerical Results - 3D

Cusps are generic in 3D

Numerical Results - Extra Dimensions



- 3D Strings
- 4D Strings

Same qualitative features, extra dimensions kill cusps.

Outline

Introduction and Motivation

String Dynamics

Cusp Formations

Extra Dimensions

A New Approach

Conclusions

Conclusions

- In 3D, LIGO may have been too conservative with their search.
- But with extra dimensions, it may be much harder to detect a cosmic string than previously thought.

Other parts of the story:

- Consider the evolution of cosmic strings in an expanding universe.
- Add a Randall-Sundrum type potential in the extra dimensions.