Title: Gong Show

Date: Aug 18, 2011 05:00 PM

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Abstract: <div id="Cleaner">Holographic Three Point Functions of Giant Gravitons<div id="Cleaner">Living on the Edge<div id="Cleaner">Hybrid Nonlinear Integral Equations for AdS_5 x S^5<div id="Cleaner">Exotic Y-systems via Wall-Crossing<div id="Cleaner">Lifting Asymptotic Degeneracies<div id="Cleaner">Finite-Size Corrections in AdS4 x CP3: vs. Bethe-Ansatz</div></div></div></div></div></div>

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Gong Show

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Pünktlich sind die Gewissenhaften und die Neugierigen.
–Arthur Schopenhauer

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Punctual are the conscientious and curious.
–Arthur Schopenhauer

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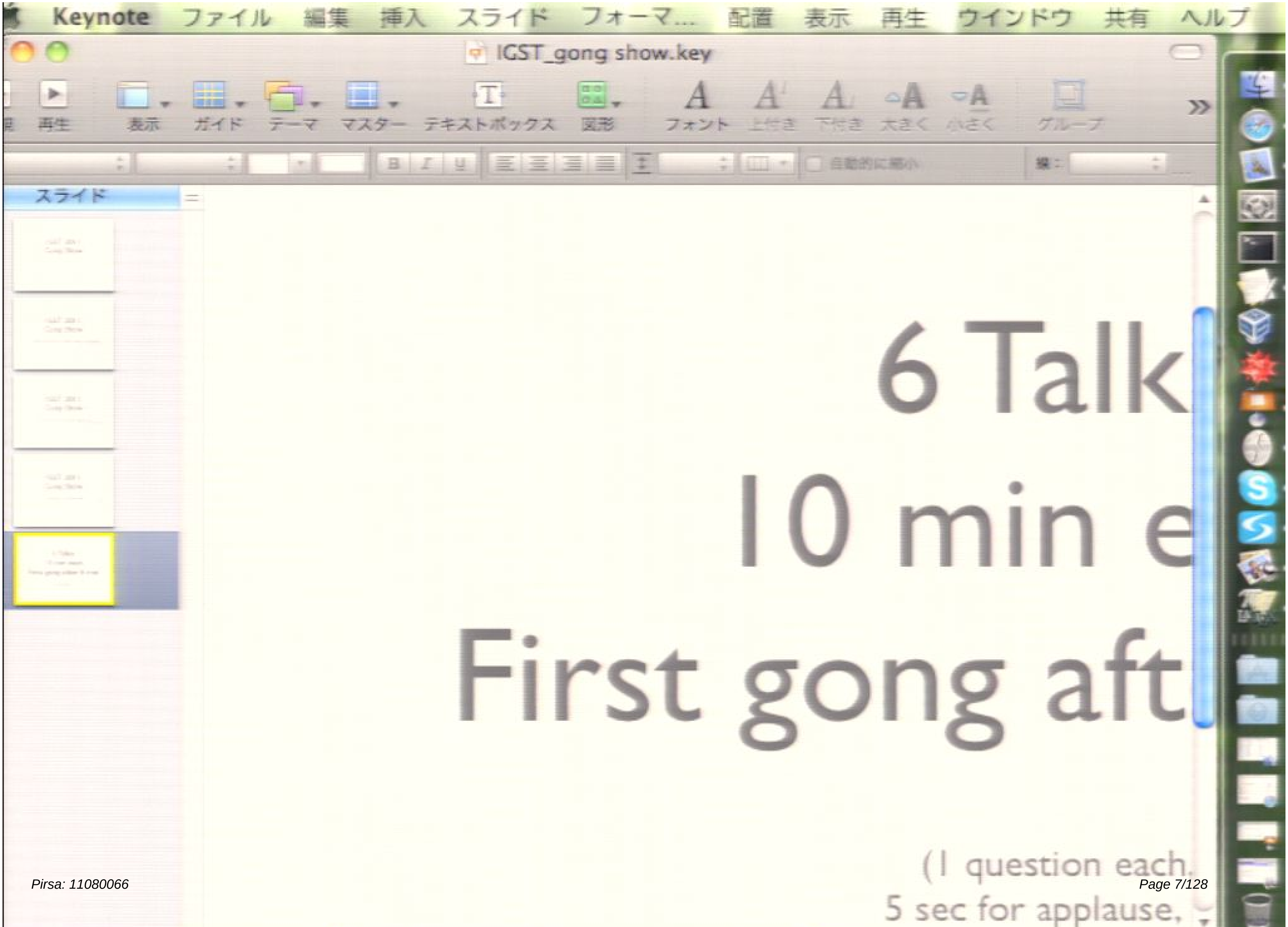
Punctuality is the thief of time.

–Oscar Wilde

6 Talks
10 min each
First gong after 8 min

(1 question each,
5 sec for applause, ...)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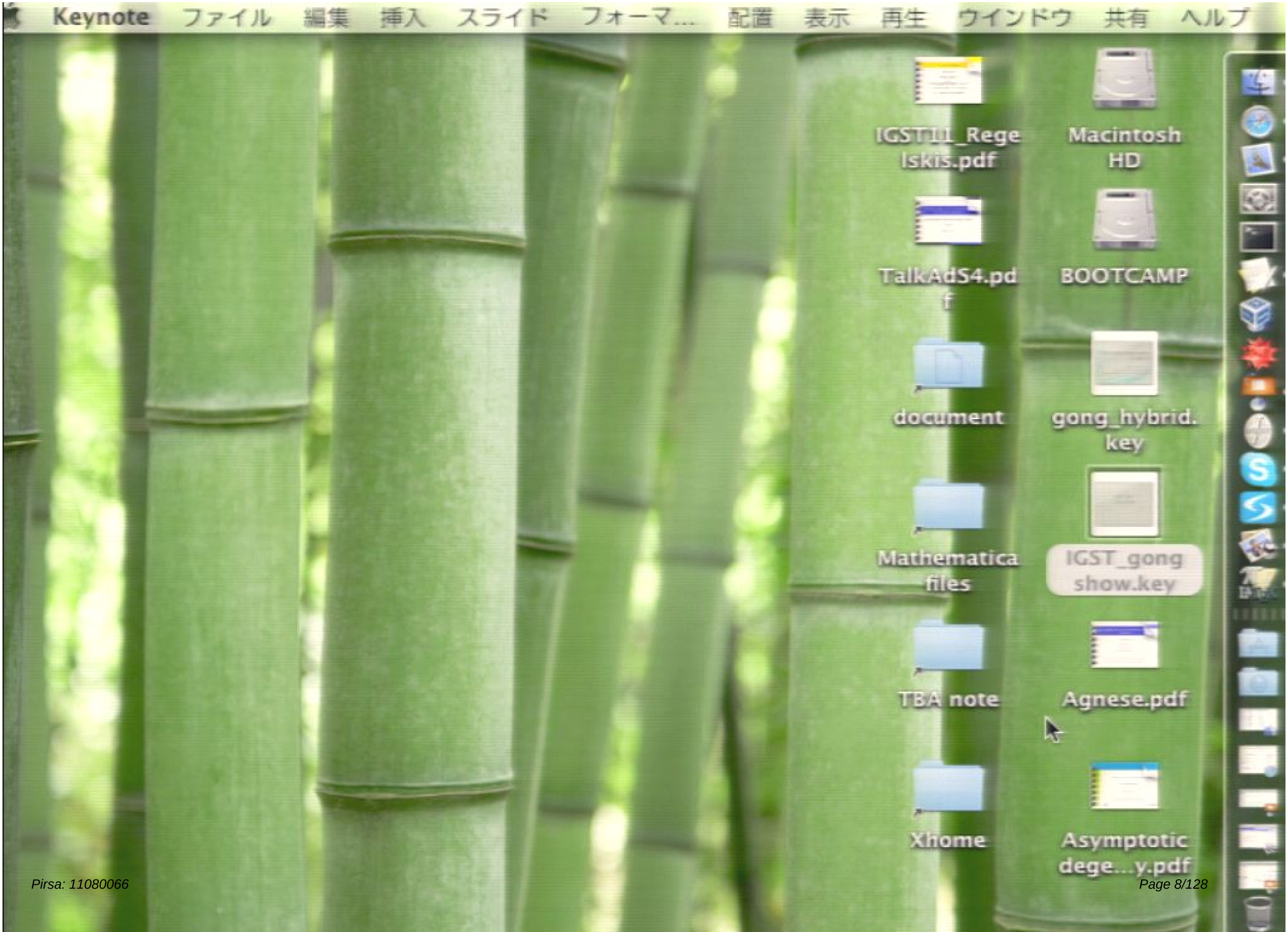
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Holographic three-point functions of giant gravitons

Agnese Bissi

Niels Bohr Institute

August 18, 2011
Perimeter Institute

based on hep-th 1103.4079 (JHEP 1106 (2011) 085)
with C. Kristjansen, D. Young and K. Zoubos.

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Introduction

- Building blocks in conformal field theories are 2-point and 3-point functions of local gauge invariant operators.

(See Janik's talk and Gromov's talk)

$$\langle \mathcal{O}_i(x)\,\mathcal{O}_j(y)\rangle = \frac{\delta_{ij}}{|x-y|^{2\Delta_i}}$$

$$\langle \mathcal{O}_i(x)\mathcal{O}_j(y)\mathcal{O}_k(z)\rangle = \frac{C_{ijk}}{|x-y|^{\Delta_i+\Delta_j-\Delta_k}|y-z|^{\Delta_j+\Delta_k-\Delta_i}|z-x|^{\Delta_i+\Delta_k-\Delta_j}}$$

- We compute the structure constant for the 3 pt function of
  1. 2 giant gravitons and 1 light operator
  2. 2 Schur polynomials and 1 chiral primary

Introduction

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- We compute the structure constant for the 3 pt function of
  1. 2 giant gravitons and 1 light operator
  2. 2 Schur polynomials and 1 chiral primary

Giant gravitons & light operator

- Giant gravitons have dimensions $\sim \sqrt{\lambda}$ and the light operator has dimension $\sim 1 \Rightarrow$ semiclassical method.

[Zarembo, 2010]

Strategy

- Two point function: D-brane solution continued to the Euclidean Poincaré patch
- Vary the Euclidean D-brane action in accordance with the supergravity fluctuations
- Evaluate the fluctuations on the Wick rotated giant graviton solution

Giant graviton on S^5

- Giant graviton with worldvolume $\mathbb{R}(\subset AdS_5) \times S^3(\subset S^5)$.
- The action for the D3-brane is

$$S_{D3} = -\frac{N}{2\pi^2}\int d^4\sigma\left(\sqrt{-g} - P[C_4]\right)$$

where $g_{ab} = \partial_a X^M \partial_b X_M$, $a, b = 0, \ldots, 3 \rightarrow$ worldvolume coordinates, $X^M \rightarrow$ embedding coordinates and $C_{\phi\chi_1\chi_2\chi_3} = \cos^4\theta\,\mathrm{Vol}(\Omega_3)$

[Grisaru Myers Tafjord, 2000]

- The three point function structure constant (for giant gravitons on S^5) is

$$C^A_{k,k-J,J} = \sqrt{J}\frac{k}{N}\left(1 - \frac{k}{N}\right)^{J/2}$$

Giant graviton on AdS_5

- Consider a giant graviton with worldvolume $\mathbb{R} \times S^3 (\subset AdS_5)$
- The action is

$$S_{D3} = -\frac{N}{2\pi^2} \int d^4\sigma \left(\sqrt{-g} + P[C_4] \right)$$

where

$$C_{t\tilde{\chi}_1\tilde{\chi}_2\tilde{\chi}_3} = -\sinh^4 \rho \, \mathrm{Vol}(\tilde{\Omega}_3)$$

[Grisaru Myers Tafjord, 2000]

- The three point function structure constant (for giant gravitons on AdS_5) is

$$C^S_{k,k-J,J} = \frac{1}{\sqrt{J}} \left(\left(1 + \frac{k}{N}\right)^{J/2} - \left(1 + \frac{k}{N}\right)^{-J/2} \right)$$

Dual objects

- Giant gravitons are dual to Schur polynomial operators:

[Corley Jevicki Ramgoolam, 2002]

$$\chi_{R_n}(Z) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_{R_n}(\sigma)\, Z_{i_1}^{i_{\sigma(1)}} \dots Z_{i_n}^{i_{\sigma(n)}}$$

where R_n is an irreducible representation of $U(N)$ described in terms of a Young tableau with n boxes and $\chi_{R_n}(\sigma)$ is the character of the element σ in the representation R_n.

  - S^5 giant gravitons $\Rightarrow$ Antisymmetric representation
  - AdS_5 giant gravitons $\Rightarrow$ Symmetric representation

- Light operator dual to single trace chiral primary
- Note: There is no limit in which the Schur polynomial reduces to a chiral primary.

Three point function

$$C_{k,k-J,J} \equiv \frac{\langle \chi_k(\bar{Z})\chi_{k-J}(Z)\mathrm{Tr}Z^J\rangle}{\sqrt{\langle \chi_k(\bar{Z})\chi_k(Z)\rangle\langle \chi_{k-J}(\bar{Z})\chi_{k-J}(Z)\rangle\langle \mathrm{Tr}\bar{Z}^J\mathrm{Tr}Z^J\rangle}}$$

$$\langle \mathrm{Tr}Z^J\,\mathrm{Tr}\bar{Z}^J\rangle = \frac{1}{J+1}\left\{\frac{\Gamma(N+J+1)}{\Gamma(N)} - \frac{\Gamma(N+1)}{\Gamma(N-J)}\right\}$$

[Kristjansen Plefka Semenoff Staudacher, 2002]

$$\langle \chi_k^S(\bar{Z})\chi_k^S(Z)\rangle = \prod_{j=1}^{k}(N-1+j)$$

$$\langle \chi_k^A(\bar{Z})\chi_k^A(Z)\rangle = \prod_{i=1}^{k}(N-i+1)$$

Numerator

- Expand $\mathrm{Tr} Z^J$ in the basis of Schur polynomials

$$\mathrm{Tr}\, Z^J = \sum_{R_J} \chi_{R_J}(\sigma_0)\chi_{R_J}(Z)$$

$$C^S_{k,k-J,J} = \frac{\sqrt{\prod_{p=k-J+1}^{k}(N+p-1)}}{\sqrt{JN^J(1+c(J)\,\frac{1}{N^2}+\ldots)}}$$

$$C^A_{k,k-J,J} = (-1)^{(J-1)}\,\frac{\sqrt{\prod_{p=k-J+1}^{k}(N-p+1)}}{\sqrt{JN^J(1+c(J)\,\frac{1}{N^2}+\ldots)}}$$

Limits

- Schur polynomials $\Rightarrow$ large Young tableaux
- Chiral primary $\Rightarrow$ small operator

$$N \to \infty, \qquad k \to \infty, \qquad \frac{k}{N} \text{ finite}, \qquad J \ll k, \qquad J \ll \sqrt{N}$$

$$C^S_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(1 + \frac{k}{N}\right)^{J/2}$$

$$C^A_{k,k-J,J} = (-1)^{(J-1)}\frac{1}{\sqrt{J}}\left(1 - \frac{k}{N}\right)^{J/2}$$

Limits

Giant gravitons & light operator

$$C^{A}_{k,k-J,J} = \sqrt{J}\frac{k}{N}\left(1-\frac{k}{N}\right)^{J/2} \xrightarrow[\frac{k}{N}\to 0]{} \sqrt{J}\frac{k}{N}$$

$$C^{S}_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(\left(1+\frac{k}{N}\right)^{J/2} - \left(1+\frac{k}{N}\right)^{-J/2}\right) \xrightarrow[\frac{k}{N}\to 0]{} \sqrt{J}\frac{k}{N}$$

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Comparison

Giant gravitons & light operator

$$C^A_{k,k-J,J} = \sqrt{J}\frac{k}{N}\left(1-\frac{k}{N}\right)^{J/2} \xrightarrow[\frac{k}{N}\to 1]{} J\left(\frac{1}{\sqrt{J}}\left(1-\frac{k}{N}\right)^{J/2}\right)$$

$$C^S_{k,k-J,J} = \frac{1}{\sqrt{J}}\left(\left(1+\frac{k}{N}\right)^{\frac{J}{2}}-\left(1+\frac{k}{N}\right)^{-\frac{J}{2}}\right) \xrightarrow[\frac{k}{N}\to\infty]{} \frac{1}{\sqrt{J}}\left(1+\frac{k}{N}\right)^{J/2}$$

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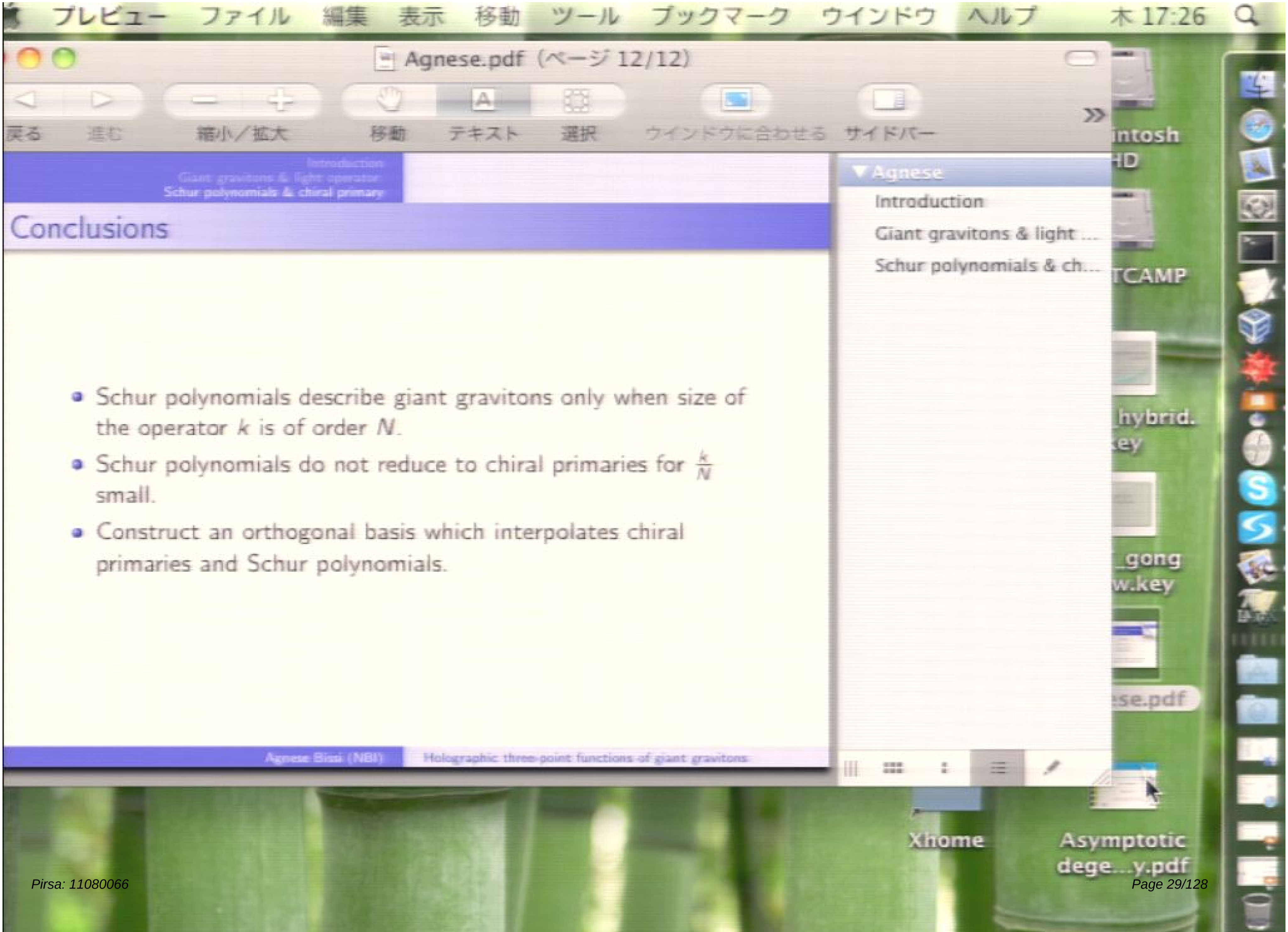
Conclusions

- Schur polynomials describe giant gravitons only when size of the operator k is of order N.
- Schur polynomials do not reduce to chiral primaries for $\frac{k}{N}$ small.
- Construct an orthogonal basis which interpolates chiral primaries and Schur polynomials.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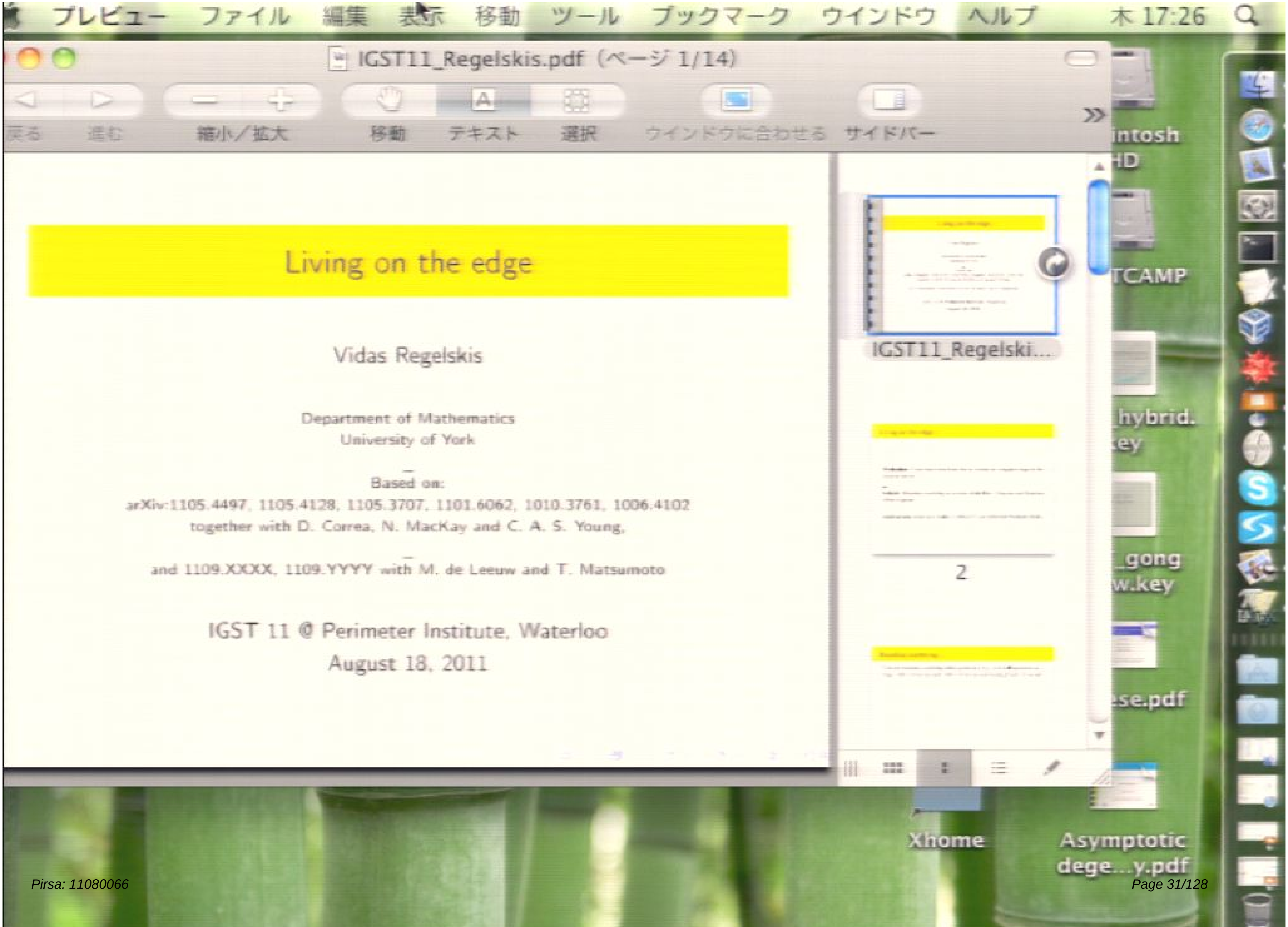
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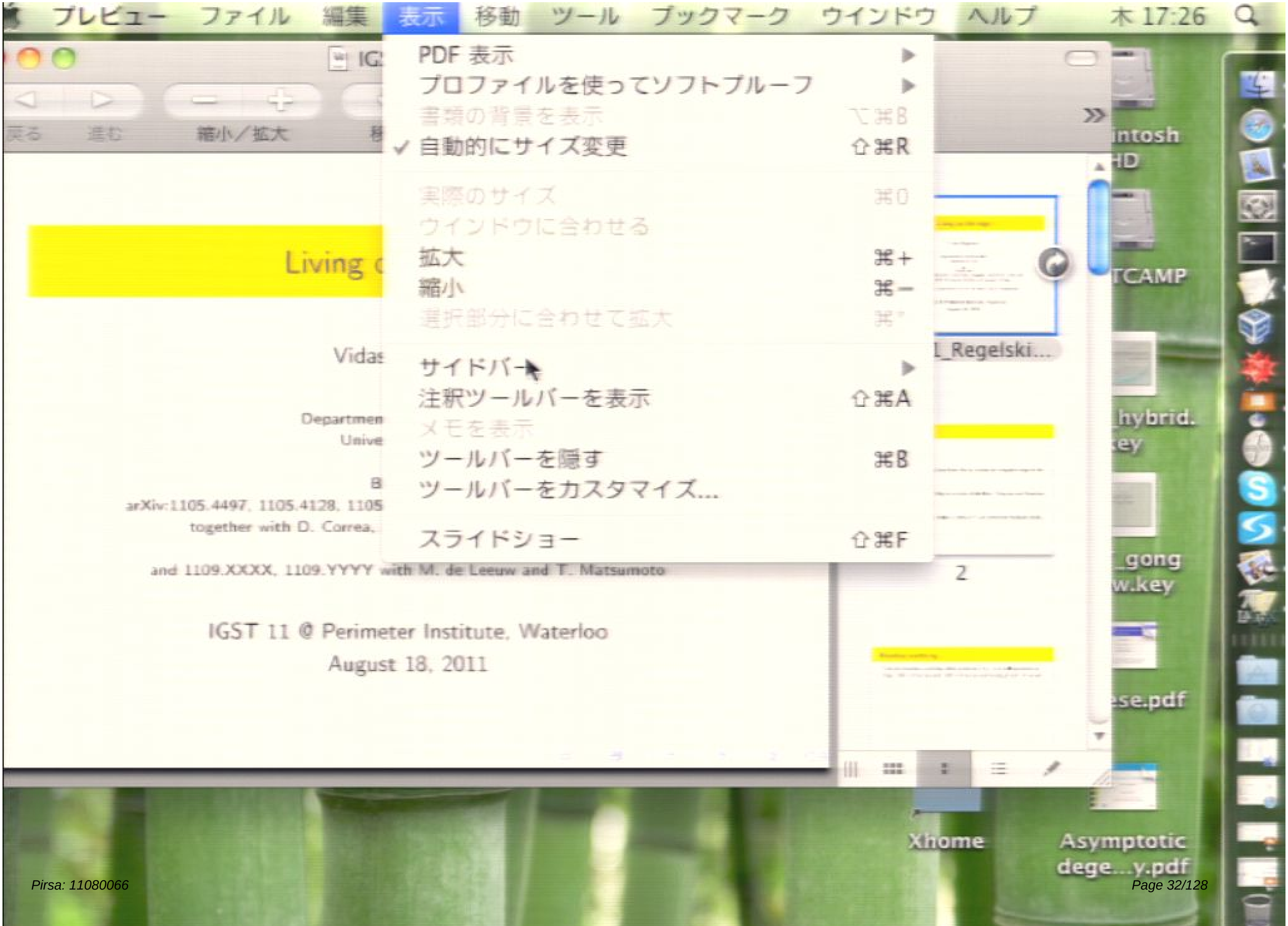
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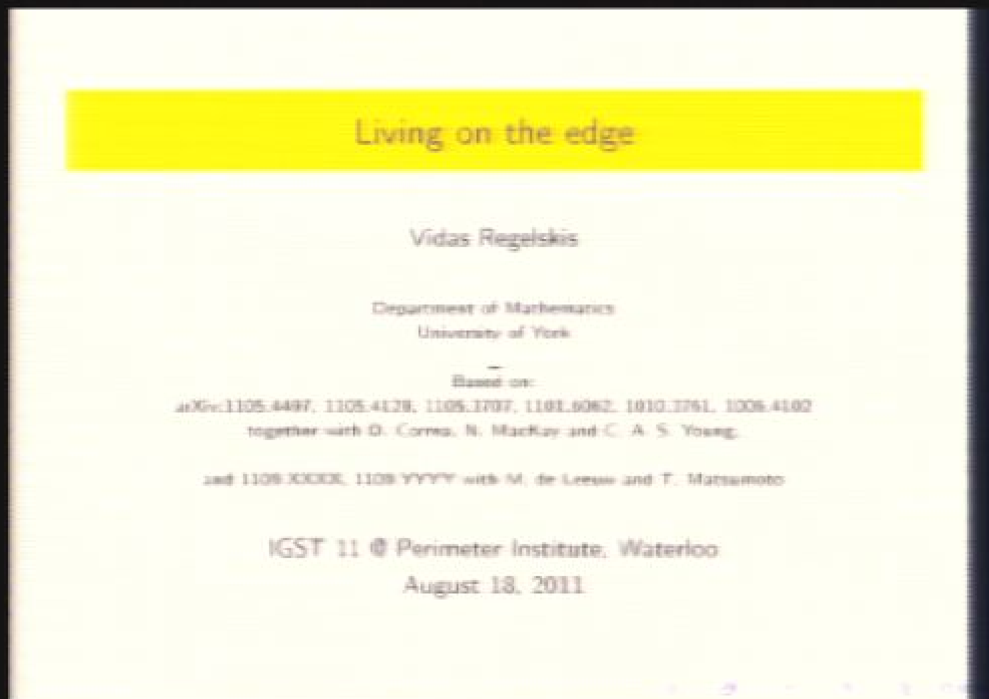
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Living on the edge

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Motivation: If you would ever have choice, choose an integrable edge of the world to live on.

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Subject: Boundary scattering on a semi-infinite line - Yangians and Quantum Affine Algebras.

—

Applications: Open spin-chains in AdS/CFT and Deformed Hubbard Model.

Boundary scattering

Consider boundary scattering which preserves $\mathfrak{h} \subseteq \mathfrak{g}$, with bulk governed by $Y(\mathfrak{g}): \Delta \mathbb{J}^a = \mathbb{J}^a \otimes 1 + 1 \otimes \mathbb{J}^a$, $\Delta \widehat{\mathbb{J}}^a = \widehat{\mathbb{J}}^a \otimes 1 + 1 \otimes \widehat{\mathbb{J}}^a + \frac{1}{2} f^a_{\ bc} \mathbb{J}^b \otimes \mathbb{J}^c$, $\widehat{\mathbb{J}}^a \mapsto u \mathbb{J}^a$

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—

A. Let $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}, \quad [\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h},\ [\mathfrak{h},\mathfrak{m}] \subset \mathfrak{m},\ [\mathfrak{m},\mathfrak{m}] \subset \mathfrak{m}$, and $\mathbb{J}^{p,q} \in \mathfrak{m},\ \mathbb{J}^{i,j} \in \mathfrak{h}$.

$\Rightarrow Y(\mathfrak{g},\mathfrak{h}): \quad \widetilde{\mathbb{J}}^p = \widehat{\mathbb{J}}^p + \frac{1}{4} f^p_{\ qi} (\mathbb{J}^q \mathbb{J}^i + \mathbb{J}^i \mathbb{J}^q),\ \Delta\widetilde{\mathbb{J}}^p = \widetilde{\mathbb{J}}^p \otimes 1 + 1 \otimes \widetilde{\mathbb{J}}^p + f^p_{\ qi} \mathbb{J}^q \otimes \mathbb{J}^i$

[G.Delius, N.MacKay, B.Short '01]

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[G.Delius, N.MacKay, B.Short '01]

B. Let $\mathfrak{g}_L \oplus \mathfrak{g}_R = \mathfrak{g}_+ \oplus \mathfrak{g}_-$ and $\mathbb{J}^a_+ = \mathbb{J}^a_L + \alpha(\mathbb{J}^a_R) \in \mathfrak{g}_+,\ \mathbb{J}^a_- = \mathbb{J}^a_L - \alpha(\mathbb{J}^a_R) \in \mathfrak{g}_-$.
$\Rightarrow Y(\mathfrak{g} \times \mathfrak{g}, \mathfrak{g}): \quad \widetilde{\mathbb{J}}^a_- := \widehat{\mathbb{J}}^a_- + \frac{1}{8} f^a_{cb} (\mathbb{J}^c_- \mathbb{J}^b_+ + \mathbb{J}^b_+ \mathbb{J}^c_-)$

[N.MacKay, V.R '11]

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$\Rightarrow Y(\mathfrak{g}, \mathfrak{h}): \quad \widetilde{\mathbb{J}}^p = \widehat{\mathbb{J}}^p + \frac{1}{4} f^p_{\ qi} (\mathbb{J}^q \mathbb{J}^i + \mathbb{J}^i \mathbb{J}^q), \; \Delta \widetilde{\mathbb{J}}^p = \widetilde{\mathbb{J}}^p \otimes 1 + 1 \otimes \widetilde{\mathbb{J}}^p + f^p_{\ qi} \mathbb{J}^q \otimes \mathbb{J}^i$

[G.Delius, N.MacKay, B.Short '01]

B. Let $\mathfrak{g}_L \oplus \mathfrak{g}_R = \mathfrak{g}_+ \oplus \mathfrak{g}_-$ and $\mathbb{J}^a_+ = \mathbb{J}^a_L + \alpha(\mathbb{J}^a_R) \in \mathfrak{g}_+, \; \mathbb{J}^a_- = \mathbb{J}^a_L - \alpha(\mathbb{J}^a_R) \in \mathfrak{g}_-$,
$\Rightarrow Y(\mathfrak{g} \times \mathfrak{g}, \mathfrak{g}): \quad \widetilde{\mathbb{J}}^a_- := \widehat{\mathbb{J}}^a_- + \frac{1}{8} f^a_{\ cb} (\mathbb{J}^c_- \mathbb{J}^b_+ + \mathbb{J}^b_+ \mathbb{J}^c_-)$

[N.MacKay, V.R '11]

C. Let $\mathfrak{g} = \mathfrak{h}, \quad \Rightarrow Y(\mathfrak{g}, \mathfrak{g}): \quad \widetilde{\widetilde{\mathbb{J}}}^{cb} = [\widehat{\mathbb{J}}^c, \widehat{\mathbb{J}}^b] + \frac{1}{2} f^c_{\ de} \mathbb{J}^d [\widehat{\mathbb{J}}^b, \mathbb{J}^e] + \frac{1}{2} f^b_{\ de} [\mathbb{J}^e, \widehat{\mathbb{J}}^c]$,
$\Delta \widetilde{\widetilde{\mathbb{J}}}^{cb} = \widetilde{\widetilde{\mathbb{J}}}^{cb} \otimes 1 + 1 \otimes \widetilde{\widetilde{\mathbb{J}}}^{cb} + f^a_{\ bc} \left(f^c_{\ de} [\mathbb{J}^d, \widehat{\mathbb{J}}^b] \otimes \mathbb{J}^e + f^b_{\ de} [\widehat{\mathbb{J}}^c, \mathbb{J}^d] \otimes \mathbb{J}^e \right) + \mathcal{O}(\mathbb{J}^3)$

[N.MacKay, V.R '11]

Integrable Boundaries

The well known integrable boundaries in AdS/CFT are:

1. D3 'Giant graviton' brane wrapping maximal $S^3 \subset AdS_5 \times S^5$
   1.1 $Z = 0$ "Giant graviton"
   1.2 $Y = 0$ "Giant graviton"
2. D7 brane wrapping entire AdS_5 and maximal $S^3 \subset S^5$
   2.1 $Z = 0$ D7 brane
   2.2 $Z = 0$ D7 brane
3. D5 brane wrapping $AdS_4 \times S^2 \subset AdS_5 \times S^5$
   3.1 "Vertical" D5
   3.2 "Horizontal" D5

here S^5 is defined as $X^2 + Y^2 + Z^2 = R^2 \equiv 1$

C.Ahn, D.Bak, Z.Bajnok, H.Chen, D.Correa, N.Dorey, D.Hofman, J.Maldacena, R.Murgan, R.Nepomechie, K.Okamura, L.Palla, S.Rey, C.Shi, C.Young, N.MacKay & V.R.

Boundary scattering

Consider boundary scattering which preserves $\mathfrak{h} \subseteq \mathfrak{g}$, with bulk governed by $\mathrm{Y}(\mathfrak{g}):\ \Delta\mathbb{J}^a = \mathbb{J}^a \otimes 1 + 1 \otimes \mathbb{J}^a,\ \Delta\widehat{\mathbb{J}}^a = \widehat{\mathbb{J}}^a \otimes 1 + 1 \otimes \widehat{\mathbb{J}}^a + \frac{1}{2} f^a_{\ bc} \mathbb{J}^b \otimes \mathbb{J}^c,\ \widehat{\mathbb{J}}^a \mapsto u\mathbb{J}^a$

—

A. Let $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}, \quad [\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h},\ [\mathfrak{h},\mathfrak{m}] \subset \mathfrak{m},\ [\mathfrak{m},\mathfrak{m}] \subset \mathfrak{m}$, and $\mathbb{J}^{p,q} \in \mathfrak{m},\ \mathbb{J}^{i,j} \in \mathfrak{h}$.

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[N.MacKay, V.R '11]

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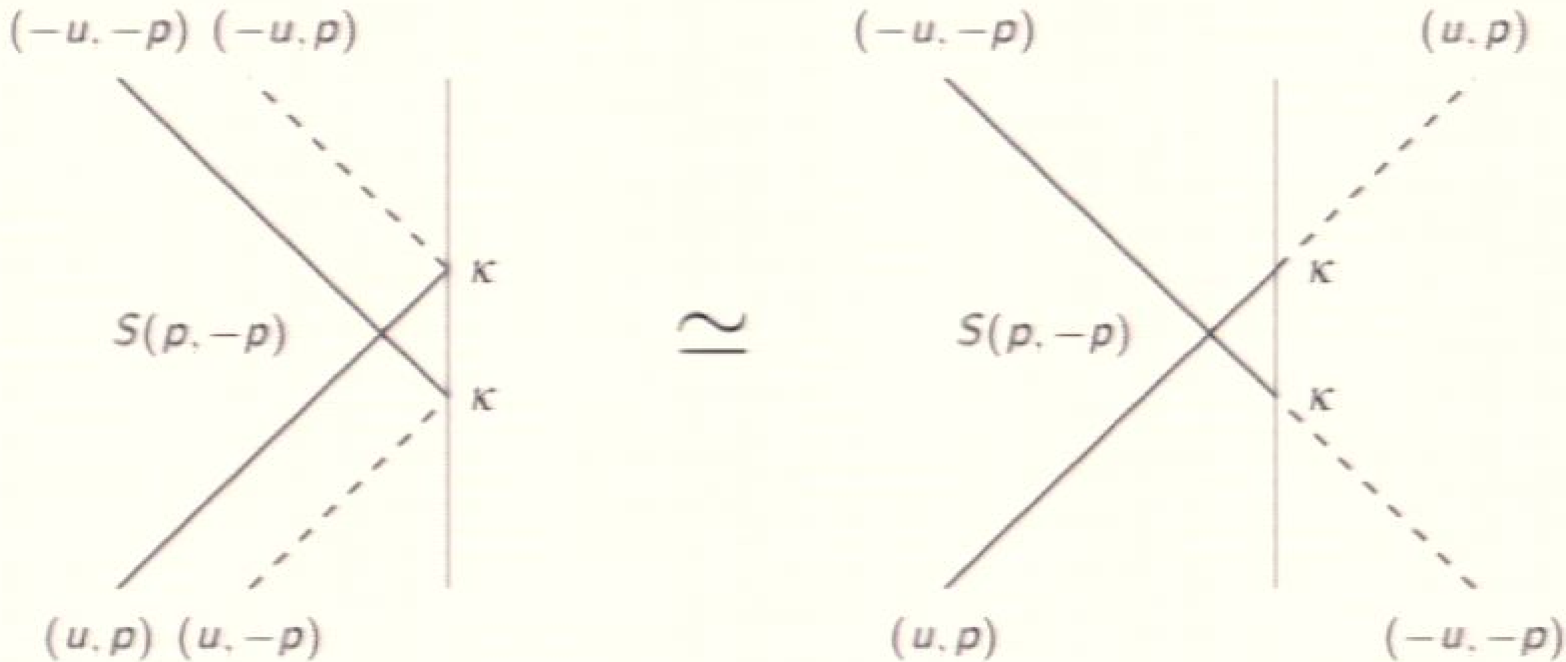
[N.MacKay, V.R '11]

Integrable Boundaries - Scattering Theory

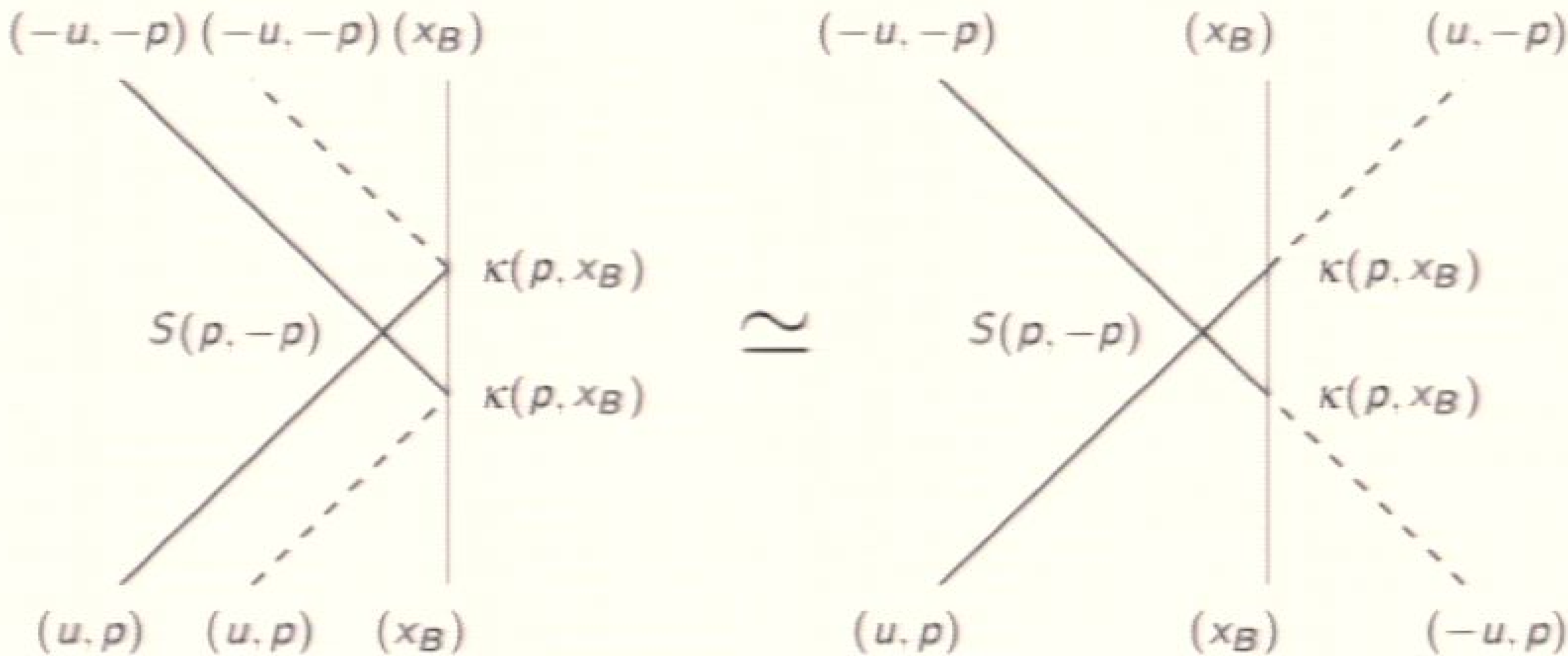
Brane	Boundary algebra	Boundary rep
$D3h/D7h$	$\mathfrak{psu}(2\|1) \times \widetilde{\mathfrak{psu}}(2\|1)$	$1 \otimes 1$
$D3v$	$\mathfrak{psu}(2\|2) \times \widetilde{\mathfrak{psu}}(2\|2) \ltimes \mathbb{R}^3$	*vector* $\otimes$ *vector*
$D7v$	$\mathfrak{su}(2) \times \mathfrak{su}(2) \times \widetilde{\mathfrak{psu}}(2\|2) \ltimes \mathbb{R}^3$	$1 \otimes$ *vector*
$D5h$	$\mathfrak{psu}(2\|2)_+ \ltimes \mathbb{R}^3$	1
$D5v$	$\mathfrak{psu}(2\|2)_+ \ltimes \mathbb{R}^3$	*vector*

Bulk algebra - $\mathfrak{psu}(2|2) \times \widetilde{\mathfrak{psu}}(2|2) \ltimes \mathbb{R}^3$

Reflection from the D5h brane



Reflection from the D5v brane



Outlook

- Boundaries are rich in integrable structures.
- They have secret symmetries originating from those of $Y(\mathfrak{g})$.
- Reflection from D5 is equivalent to bulk scattering with periodic and periodic twisted* bcs.
- Boundary scattering is closely related to the scattering in bulk (we have obtained some fusion rules).
- Bulk and boundary scattering in AdS/CFT in closely related to the Deformed Hubbard Model.

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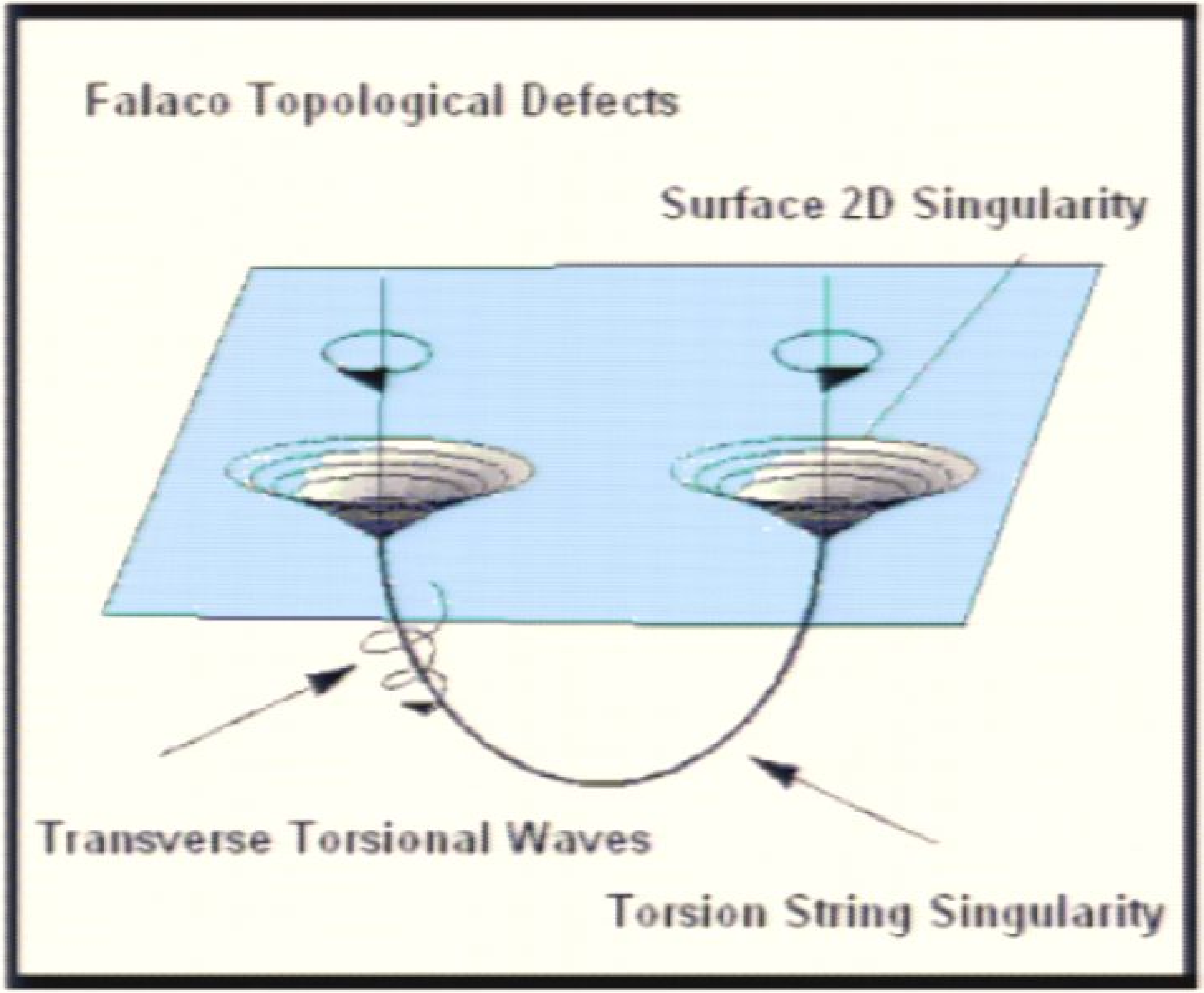
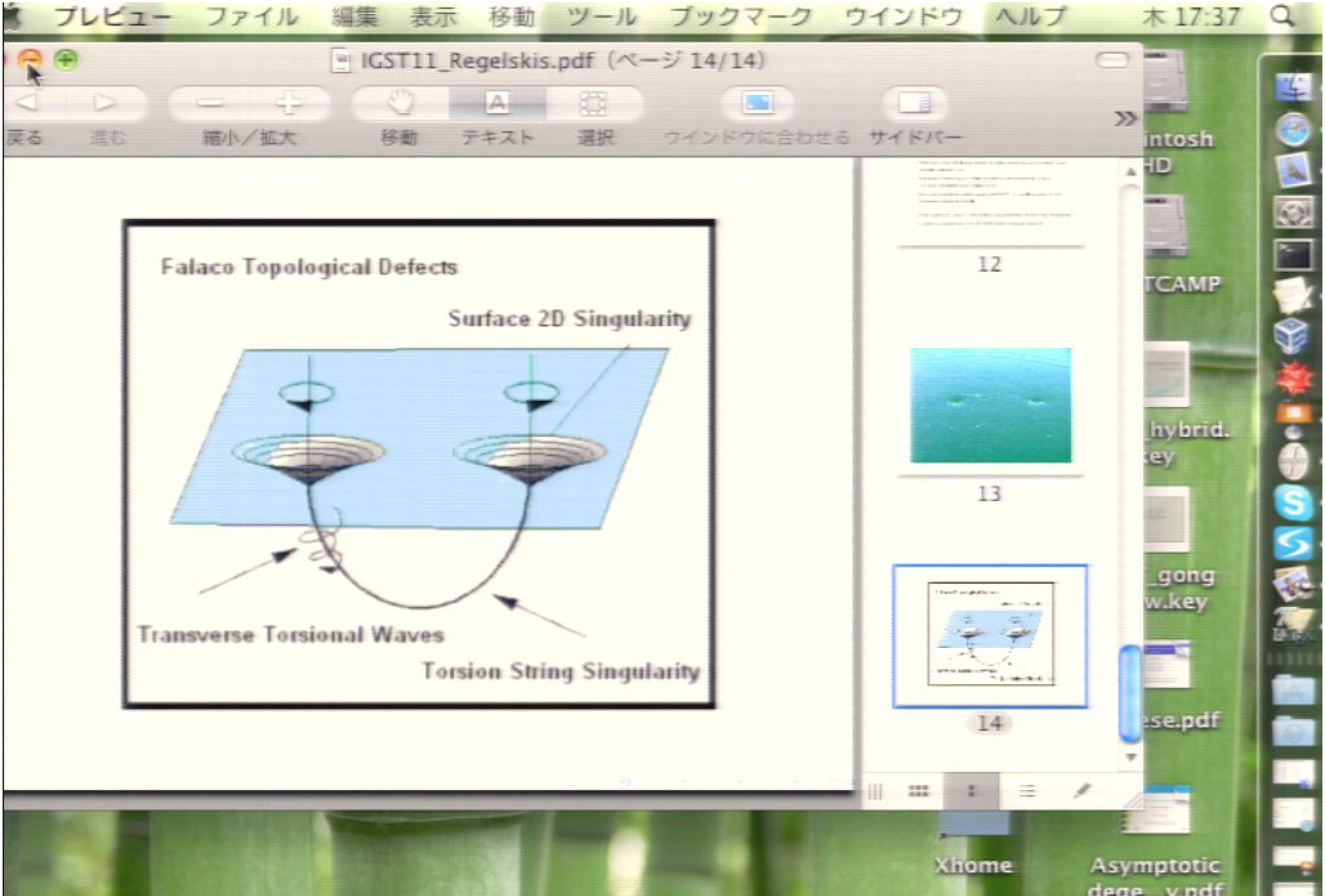
- Boundary scattering is closely related to the scattering in bulk (we have obtained some fusion rules).
- Bulk and boundary scattering in AdS/CFT in closely related to the Deformed Hubbard Model.

- Open question: what is the underlying symmetry of non-susy boundary?
- Is there a connection with the Pohlmeyer reduced theories?


Falaco Topological Defects
Surface 2D Singularity
Transverse Torsional Waves
Torsion String Singularity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戻る　進む　縮小／拡大　移動　テキスト　選択　ウインドウに合わせる　サイドバー
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Surface 2D Singularity
Transverse Torsional Waves
Torsion String Singularity
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Hybrid nonlinear integral equations for $AdS_5 \times S^5$

Ryo Suzuki (Utrecht University)

arXiv:1101.5165, J.Phys.A44 (2011) 235401

Gong Show @IGST2011, Perimeter Institute, August 2011

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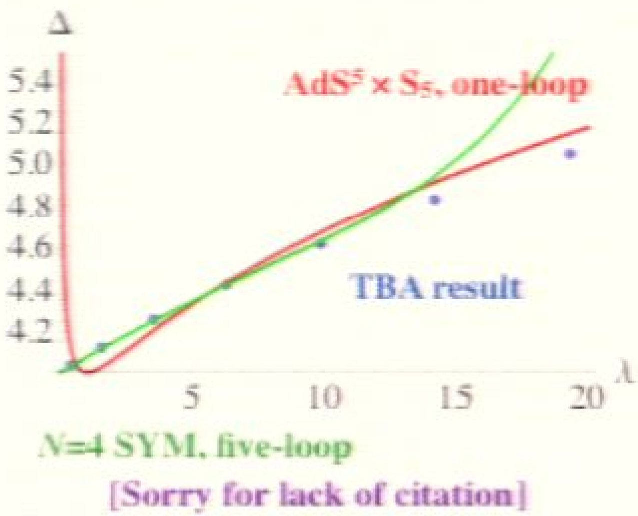
Motivation

The exact energy of AdS/CFT/integrability can be computed by Thermodynamic Bethe Ansatz (TBA) equations.



TBA gives us the precise dictionary between the states of N=4 SYM and $AdS_5 \times S^5$ superstring via Δ=E

$O_{\rm SYM} \overset{?}{\longleftrightarrow}$ 

However, in practice, it is difficult to solve TBA numerically.

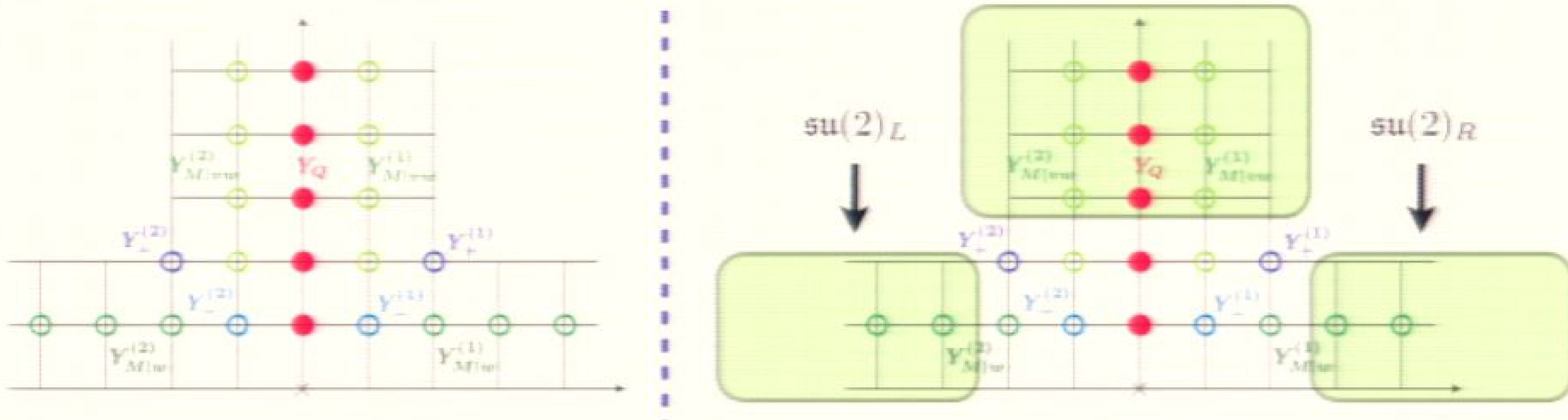
[Problem 1] TBA is too complicated.

[Problem 2] TBA depends on the value of coupling constant.

$$\mathrm{TBA}(\lambda < \lambda_{\rm cr}^{(i)}) \neq \mathrm{TBA}(\lambda > \lambda_{\rm cr}^{(i)}), \quad (i = 1, 2, 3, \dots)$$

From TBA to NLIE

Rewrite TBA into nonlinear integral equations (NLIE) for finitely many variables.



TBA at horizontal wings: $\log Y_{M|w} = \log(1+Y_{M-1|w})(1+Y_{M+1|w}) \star s$

Need to know $Y_{M+1|w}$ to compute $Y_{M|w}$ $\Rightarrow$ Cannot truncate at finite M !

NLIE: $\log \mathfrak{a}_s = \log(1+\mathfrak{a}_s) \star K_f - \log(1+\bar{\mathfrak{a}}_s) \star K_f^{[+2-2\gamma]} + \log(1+Y_{s-2|w}^{[-\gamma]}) \star s + \dots$

Closed equations for $(\mathfrak{a}_s\,,\bar{\mathfrak{a}}_s)$ and $Y_{M|w}$ $(M \leq s-2)$!

Simpler equations may relieve the problems of TBA

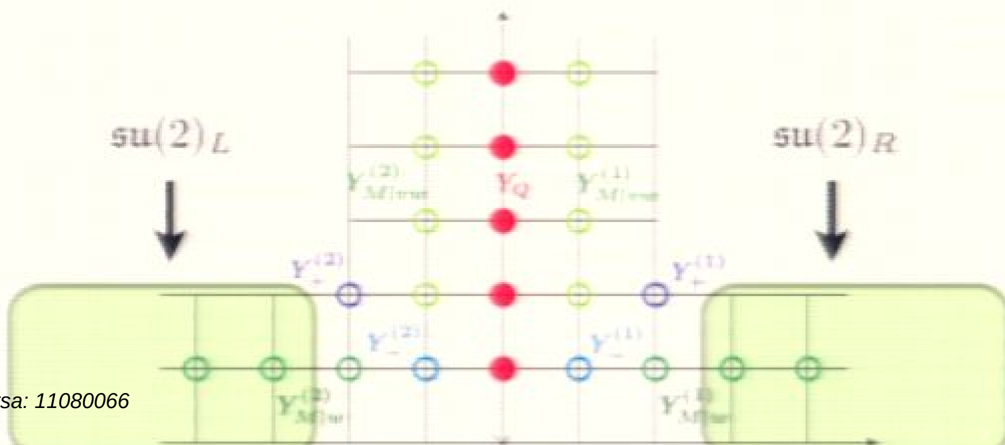
Construction of NLIE from A_1 T-system

TBA in general: Formulated by the mirror string hypothesis

NLIE in general (not just AdS/CFT): How to formulate?

This "how-to" was known in cond-mat people (A. Klümper, J. Suzuki, ...)
But their methods were not general enough for the application to AdS/CFT

I "generalized" their construction of *hybrid* NLIE, applicable for any integrable models with A_1 T-system.



Application to AdS/CFT;
horizontal wings go away
(This NLIE is hybrid because TBA is used in the vertical wing)

Key ideas of hybrid NLIE in general

Recall that 1. TBA = Y-system + discontinuity relations

2. Y-system = T-system by change of variables

T-system: $T^-_{a,s}\, T^+_{a,s} = T_{a-1,s}\, T_{a+1,s} + T_{a,s-1}\, T_{a,s+1}$

The trick is

3. Linearize T-system (= TQ-relations of [Krichever Lipan Wiegmann Zabrodin])

4. Choose the "proper" gauge invariant variables

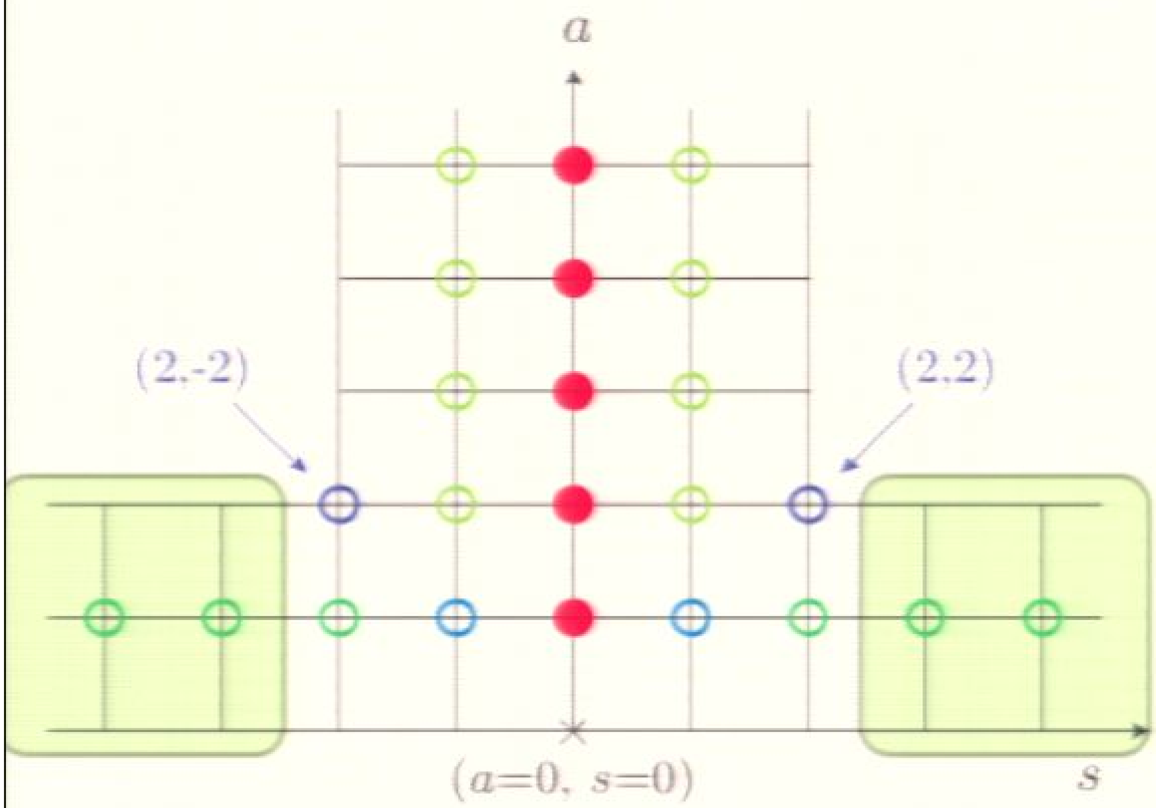
$$1 + \mathfrak{a}_s \equiv \frac{Q_{1,s-1}\, T^+_{1,s}}{\overline{Q}_{1,s-1}\, L^+_{1,s}}, \quad 1 + \overline{\mathfrak{a}}_s \equiv \frac{\overline{Q}_{1,s-1}\, T^-_{1,s}}{Q_{1,s-1}\, \overline{L}^-_{1,s}}$$

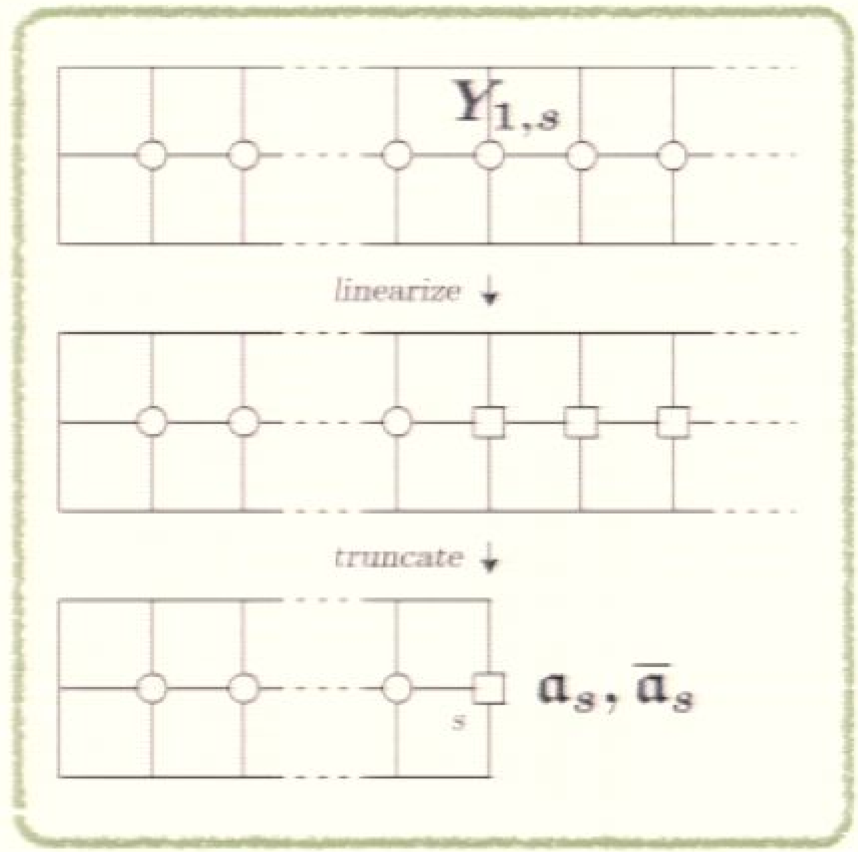
$T_{1,s}$, $L_{1,s}$, $\overline{L}_{1,s}$ are Wronskians of **Q**

Remark on generality:

We only need the Wronskian formula for T by Q's, and good analyticity of Q.

The rest of derivation is simple math, involving no approximation, no ansatz.

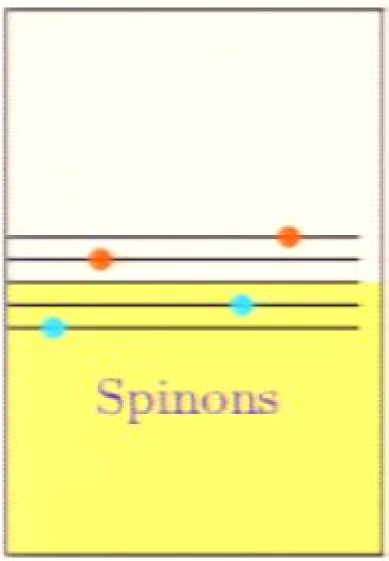

a
(2,-2)
(2,2)
(a=0, s=0)
s


$Y_{1,s}$
linearize
truncate
s
$\mathfrak{a}_s, \bar{\mathfrak{a}}_s$

Physical rôle of new variables

In the $XXX_{\frac{s}{2}}$ model, $(\mathfrak{a}_s\,,\bar{\mathfrak{a}}_s)$ represent the spinon excitations

Here *spinons* refer to elementary excitations over the AF vacuum.



Antiferromagnetic vacuum

$$(1+Y_{1,s}) = (1+\mathfrak{a}_s)(1+\bar{\mathfrak{a}}_s)$$

"mesons" or magnons — "quarks" or spinons

Why do we identify $(\mathfrak{a}_s\,,\bar{\mathfrak{a}}_s)$ as spinons?

$$\mathfrak{a}_s(v) \sim e^{iZ(v)}\,, \quad Z(v) = \text{ counting function}$$

Z counts **Bethe roots** and **holes** on an equal footing, which is useful to study excitations over AF vacuum.

Key ideas of hybrid NLIE in general

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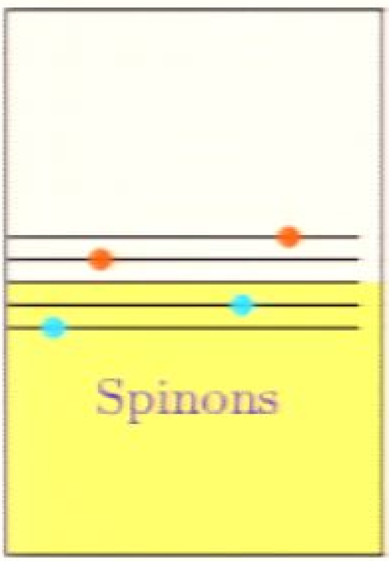

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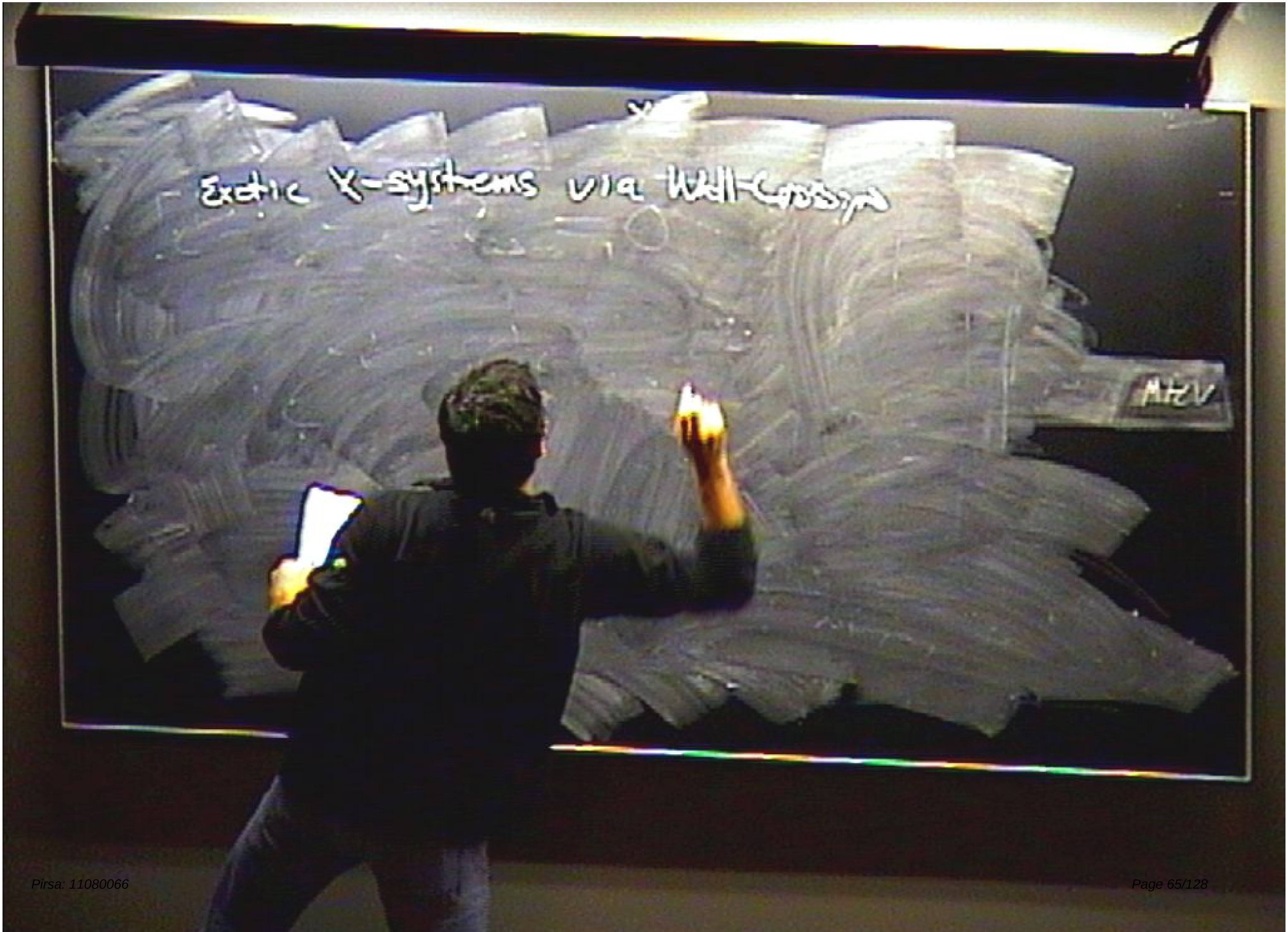
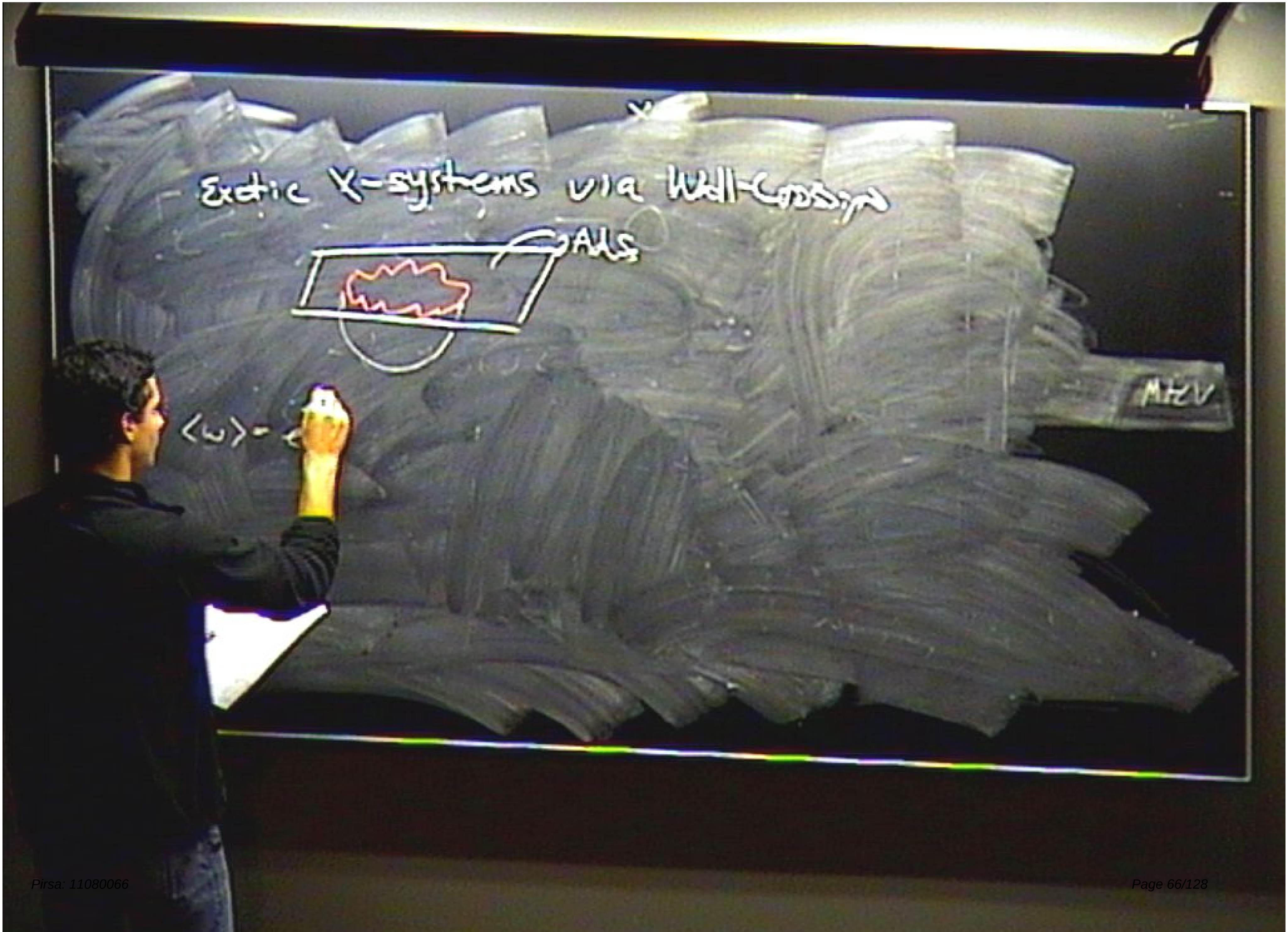
More Questions

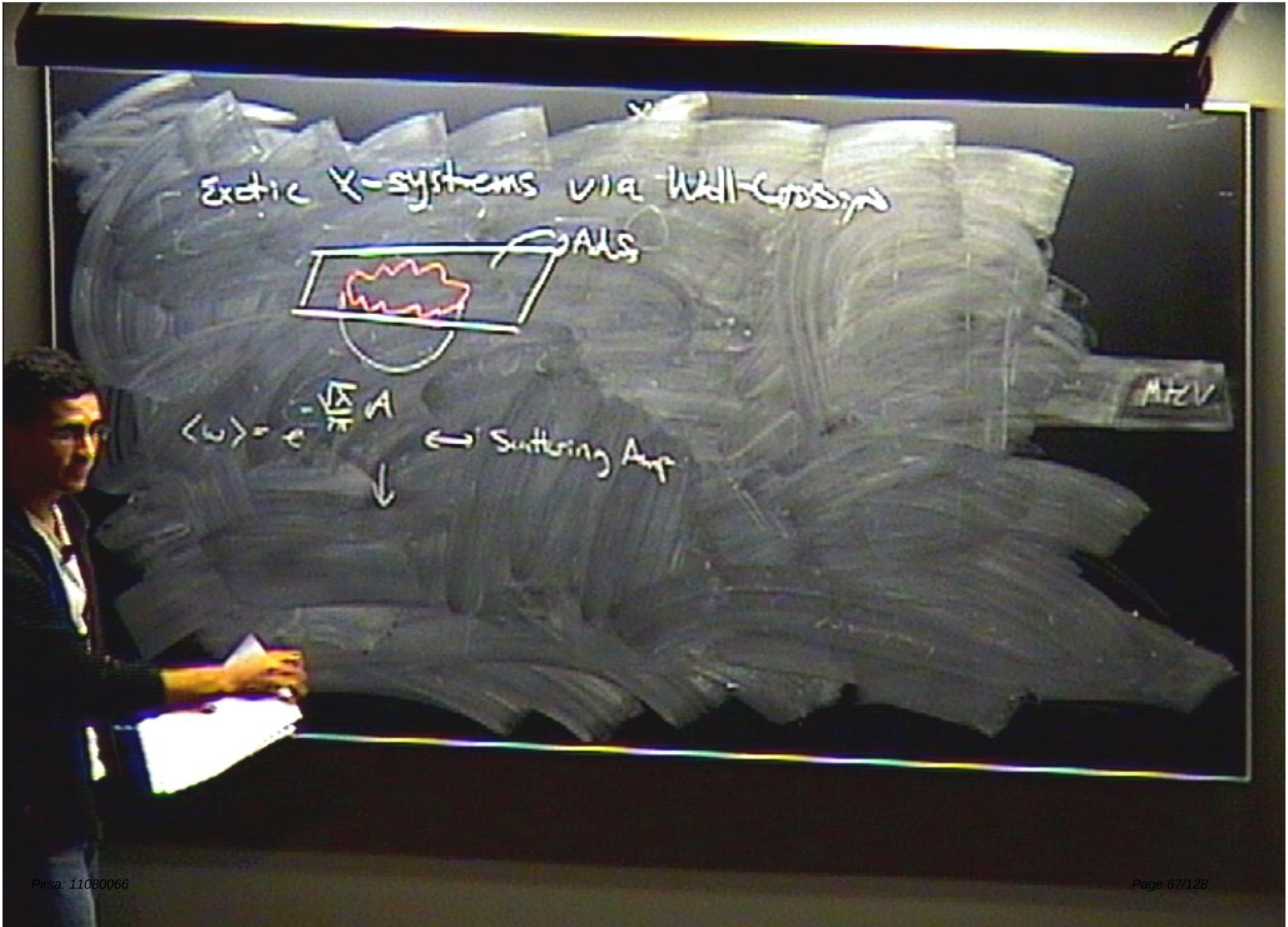
- Physical rôles of new variables in AdS/CFT ?
- General construction of NLIE, and application to the vertical direction?
- How many critical coupling constants in NLIE?
- Any implication for "integrable lattice regularization" of $AdS_5 \times S^5$ superstring?

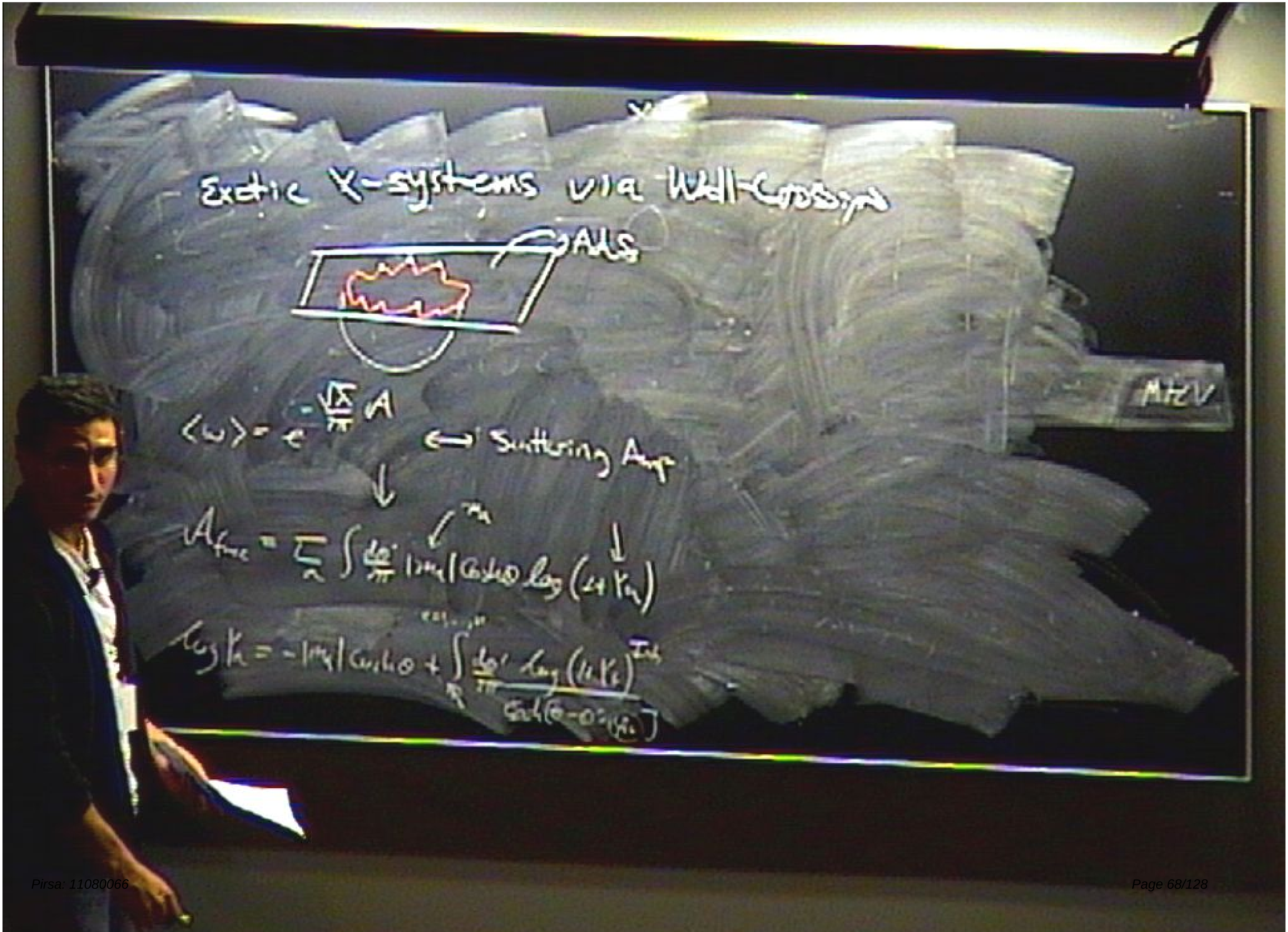
Thank you for attention


Exotic X-systems via Wall-Crossing
MHV



Exotic X-systems via Wall-Crossing
AdS
MHV
⟨w⟩ =


Exotic Y-systems via Wall-Crossing
AdS
$\langle w \rangle = e^{-\frac{\sqrt{\lambda}}{\pi} A}$ ⟷ Scattering Amp
MtcV


Exotic X-systems via Wall-Crossing
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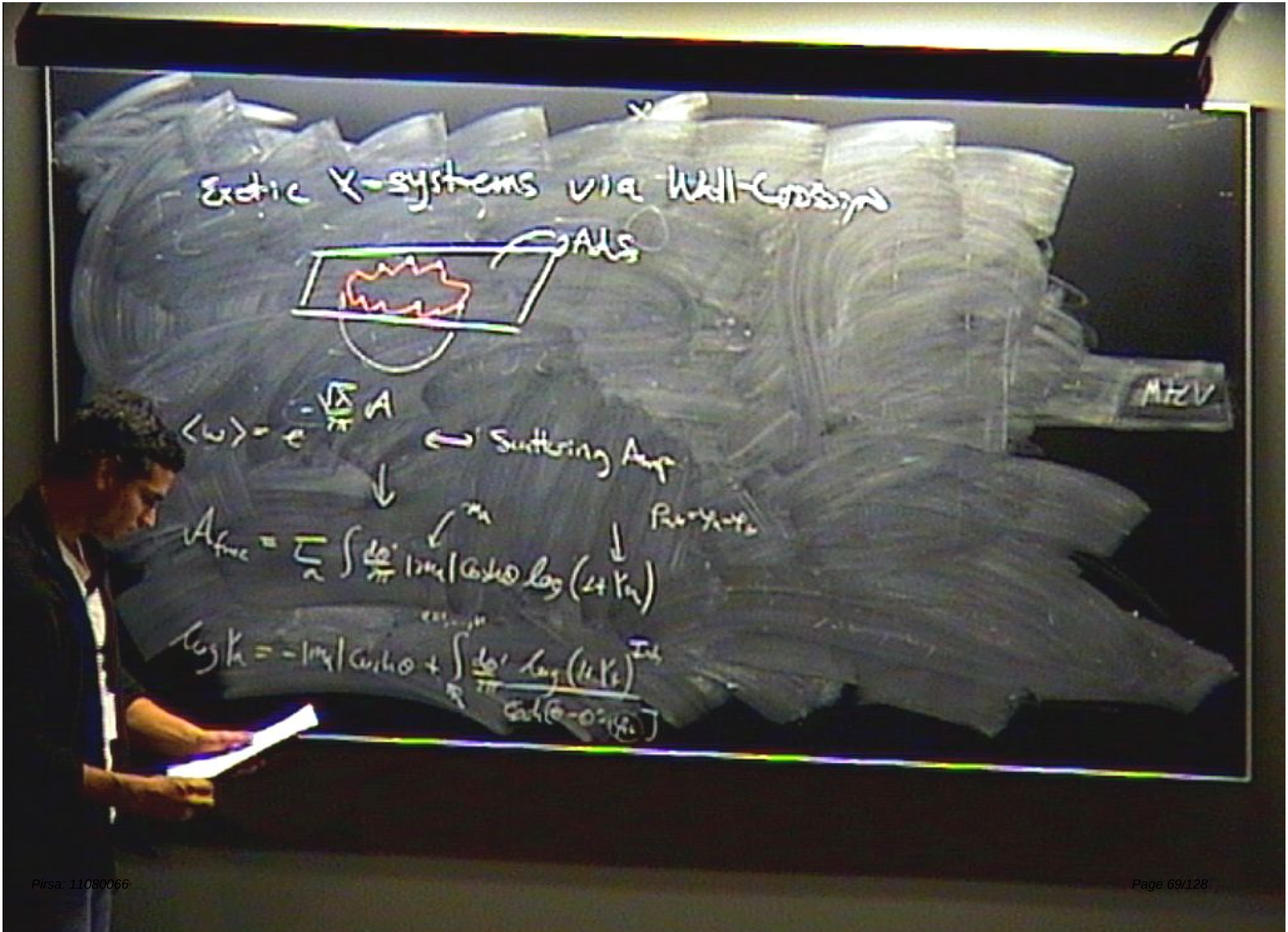
MHV
Scattering Amp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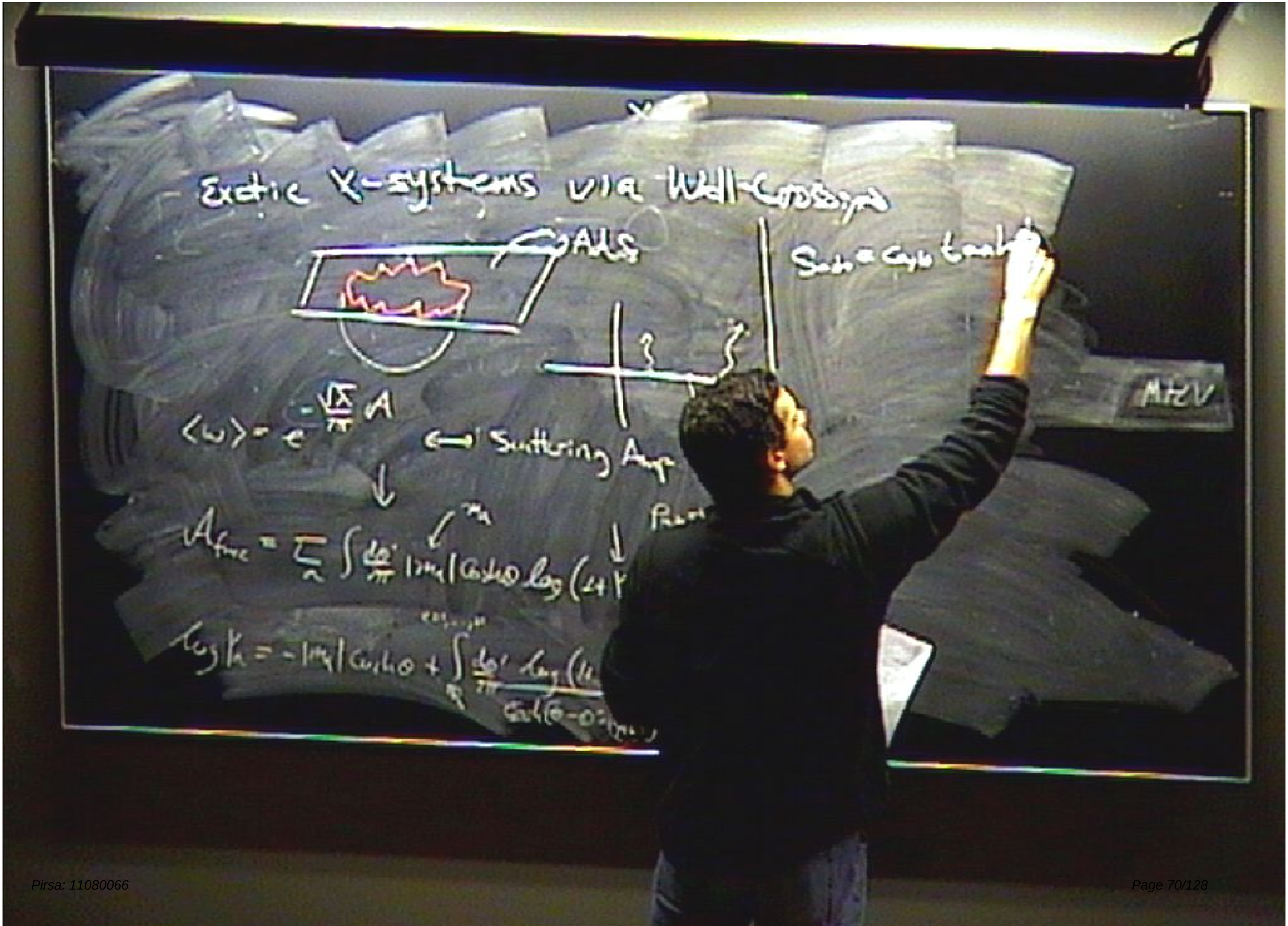
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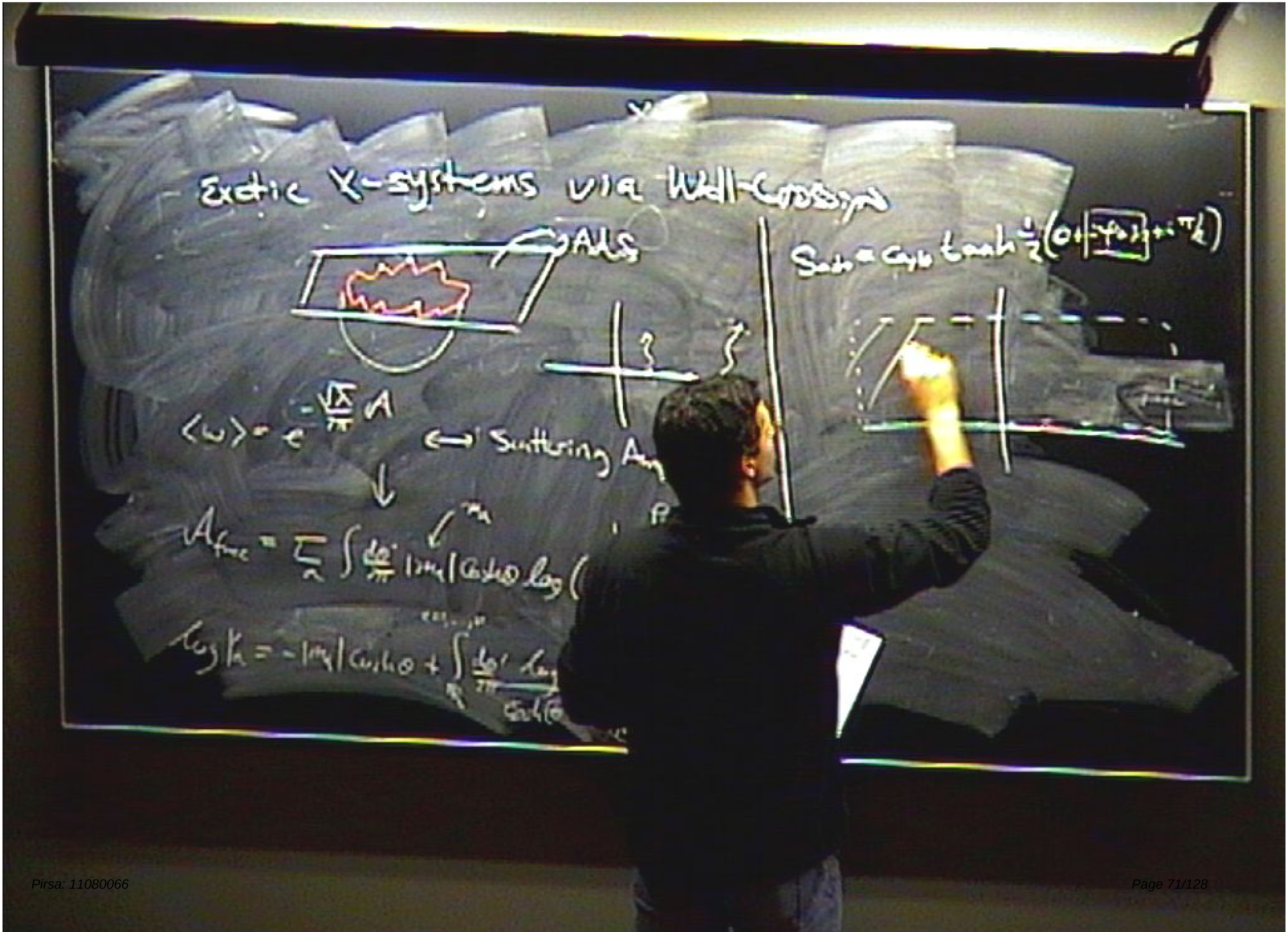
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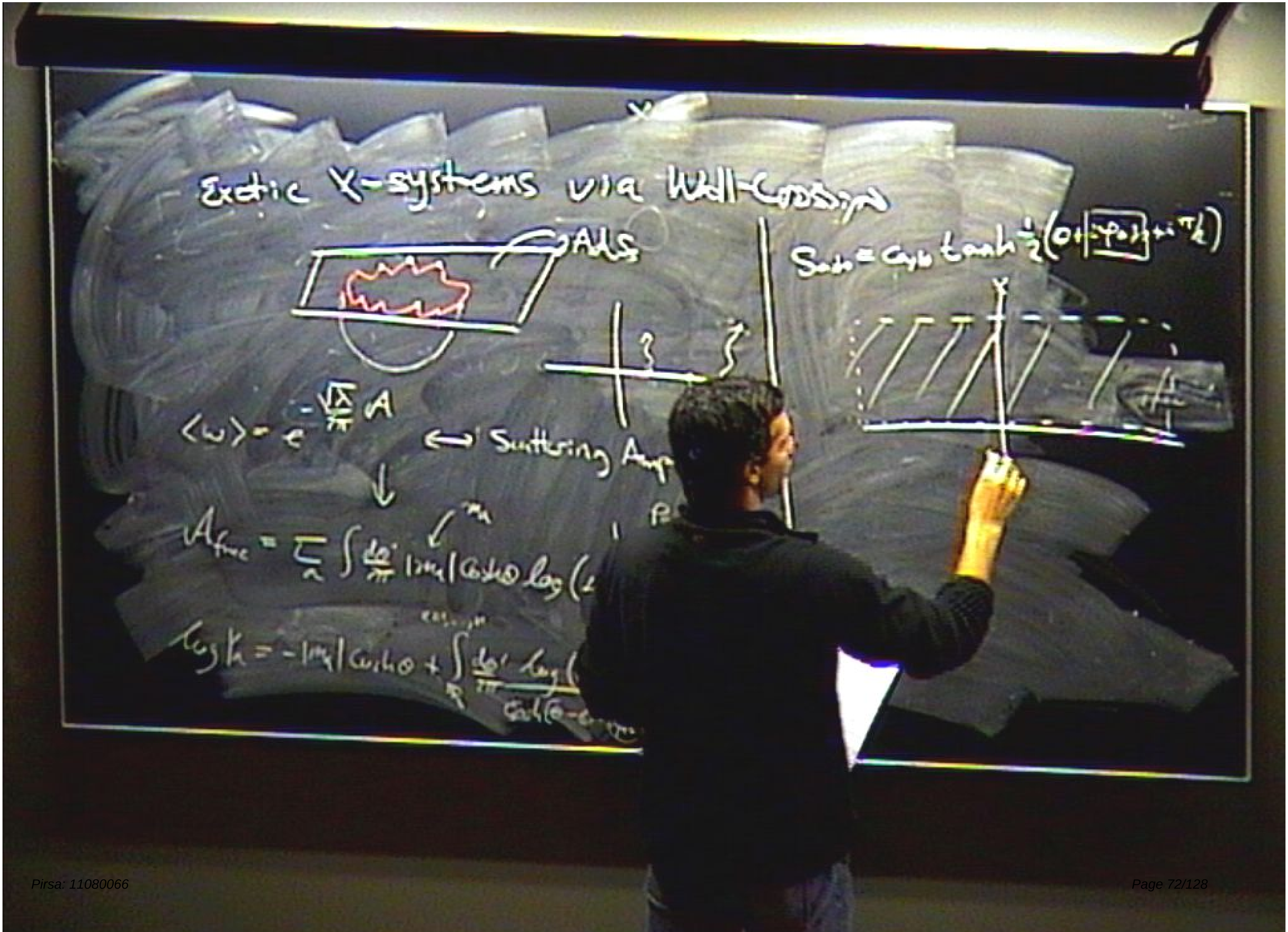
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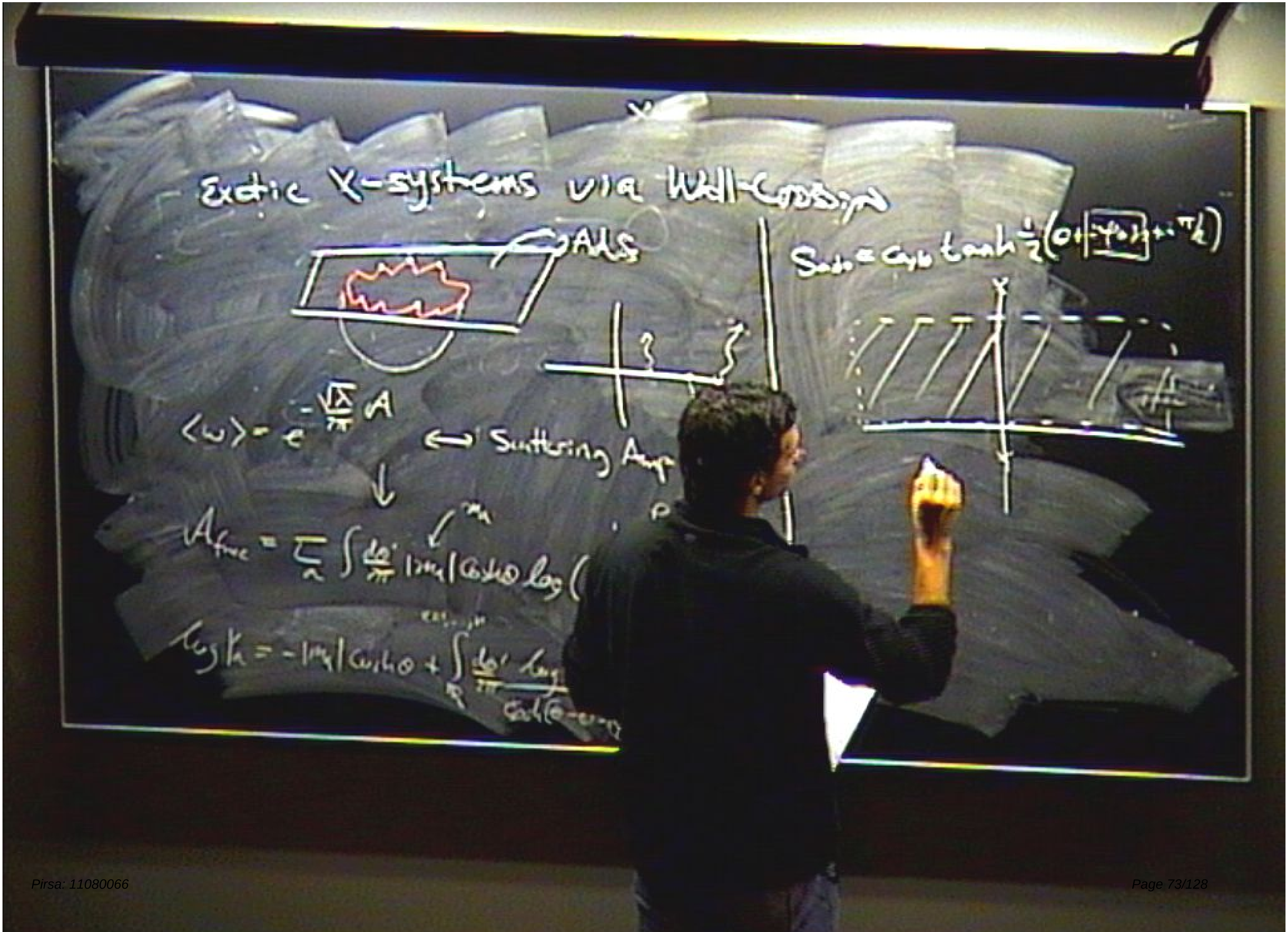
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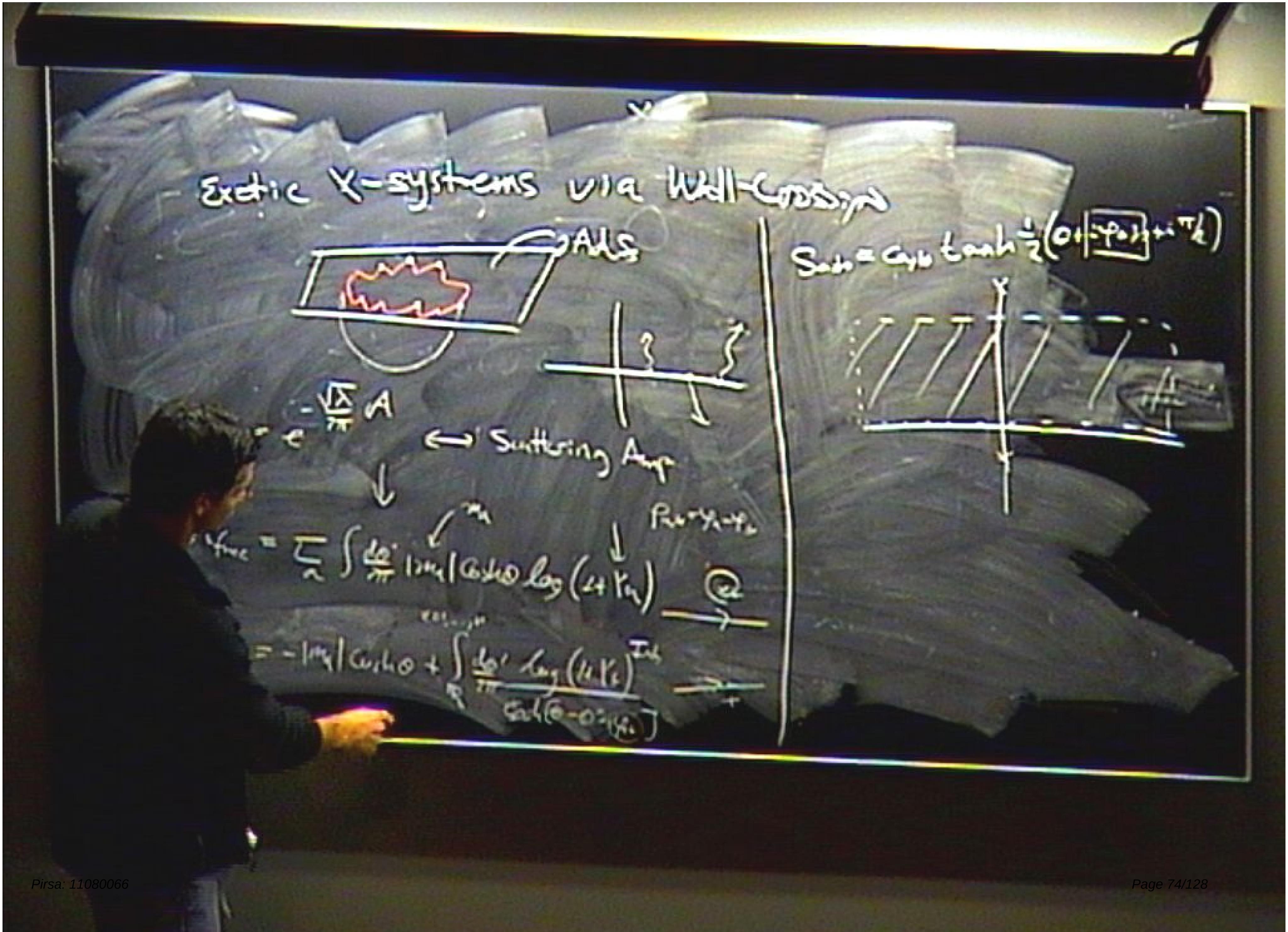
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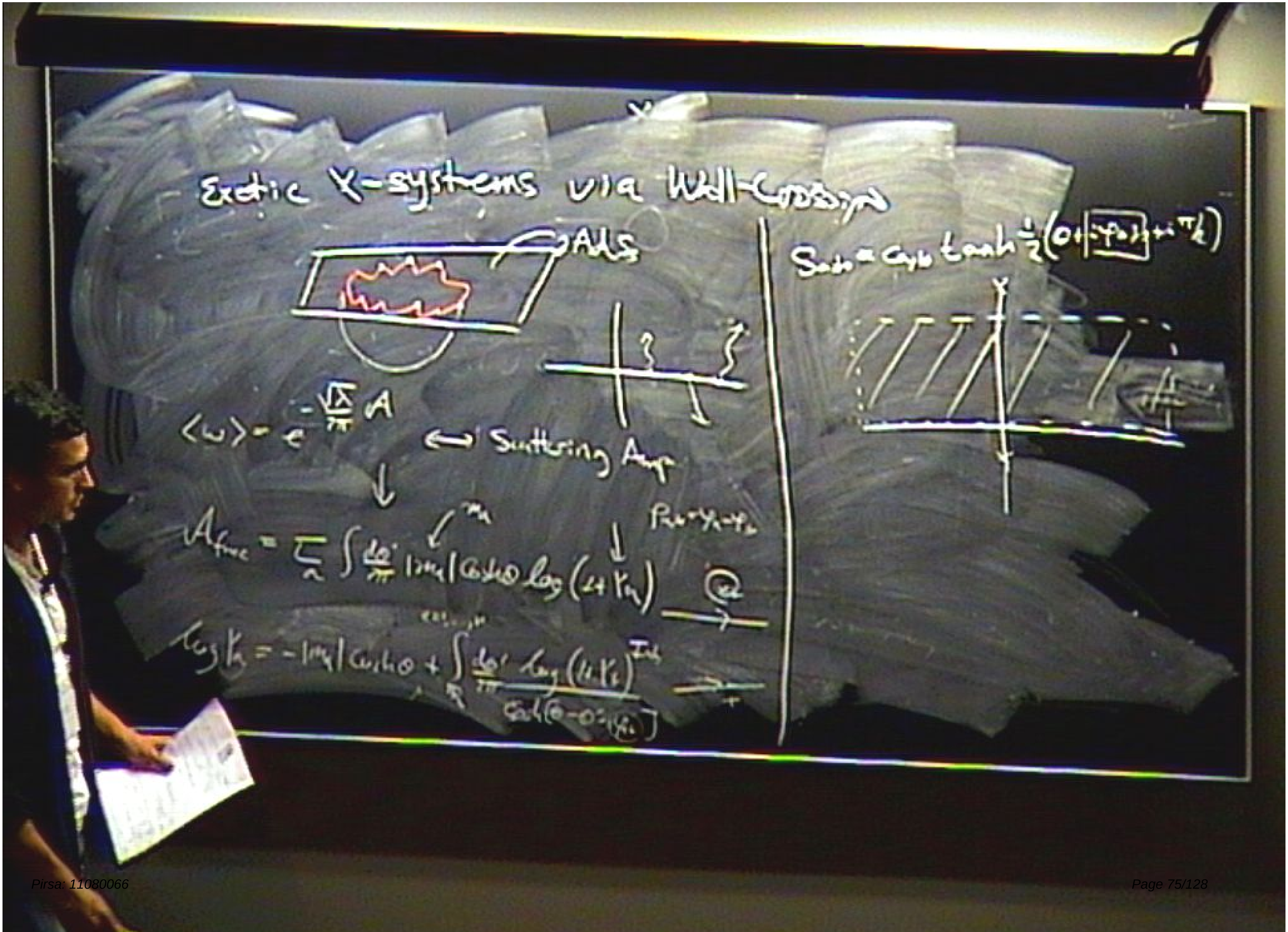
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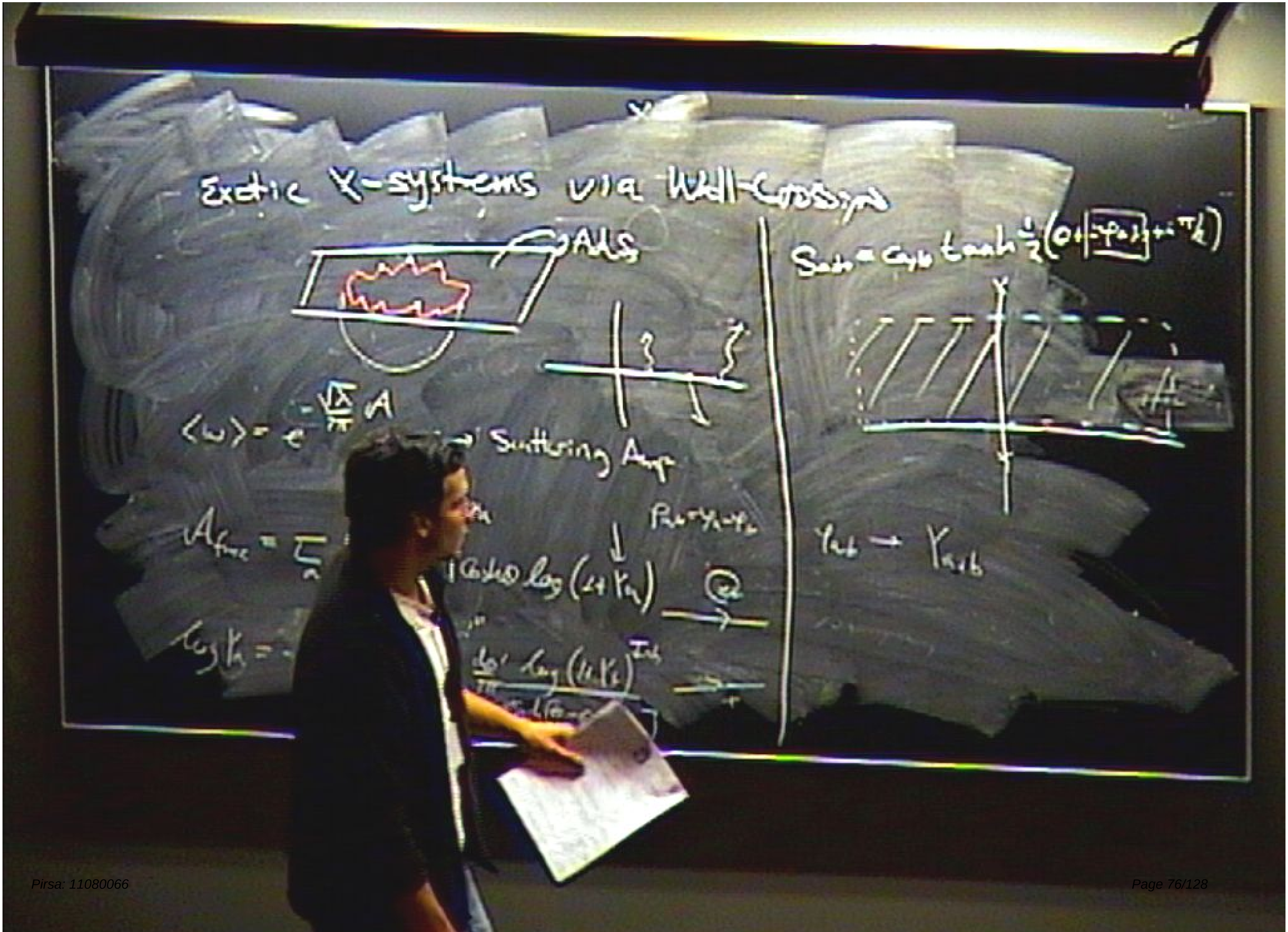
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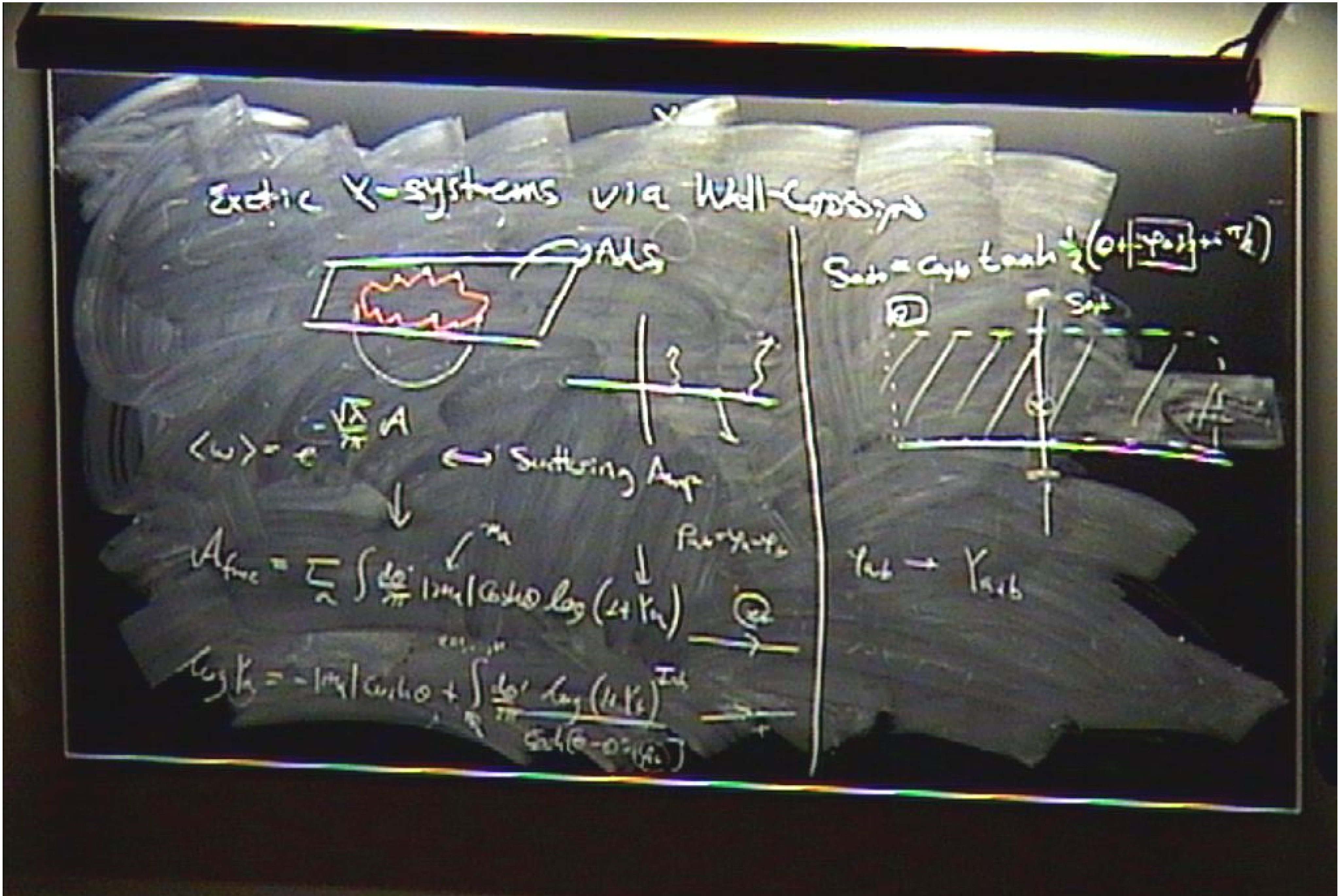
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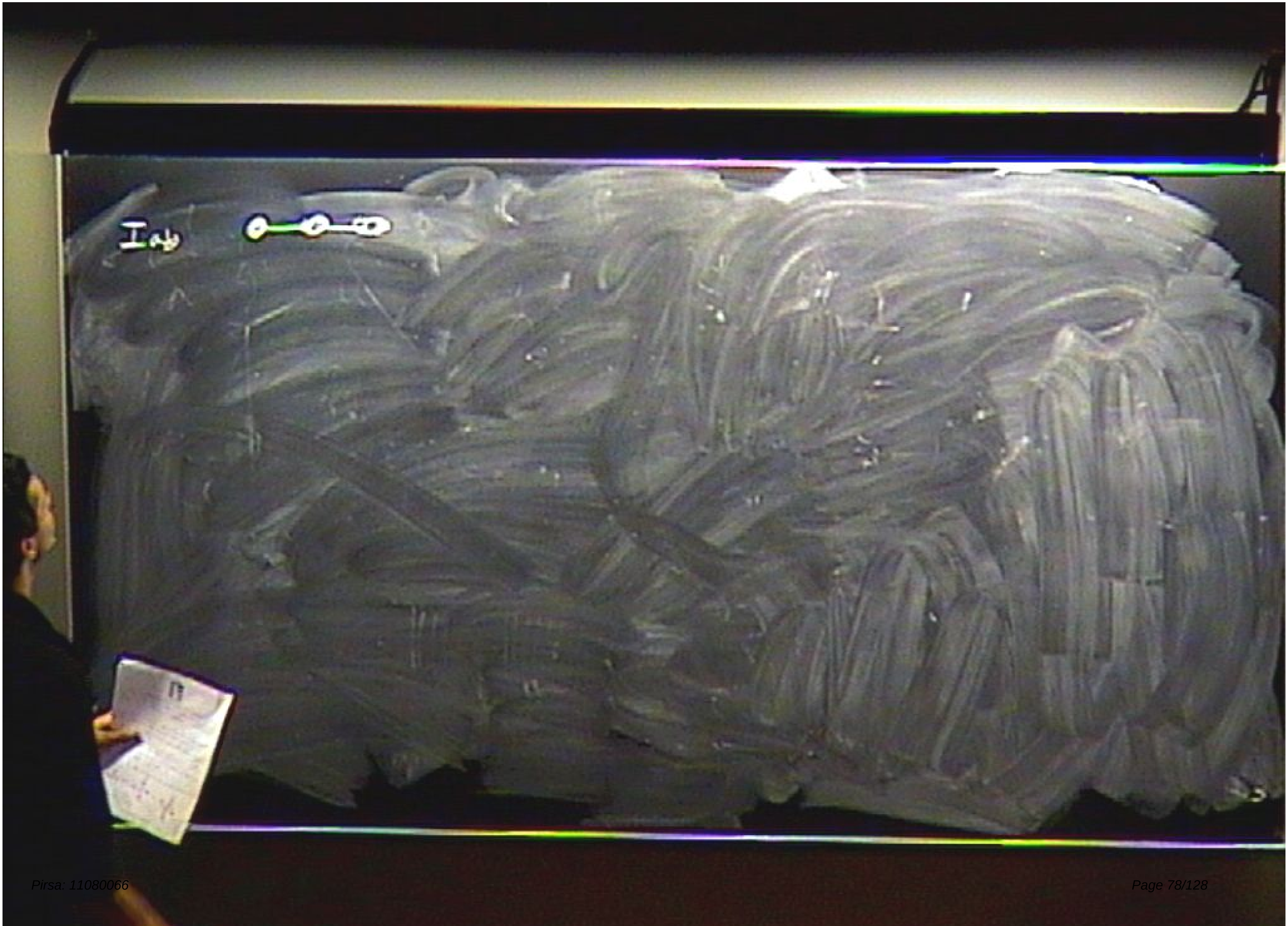

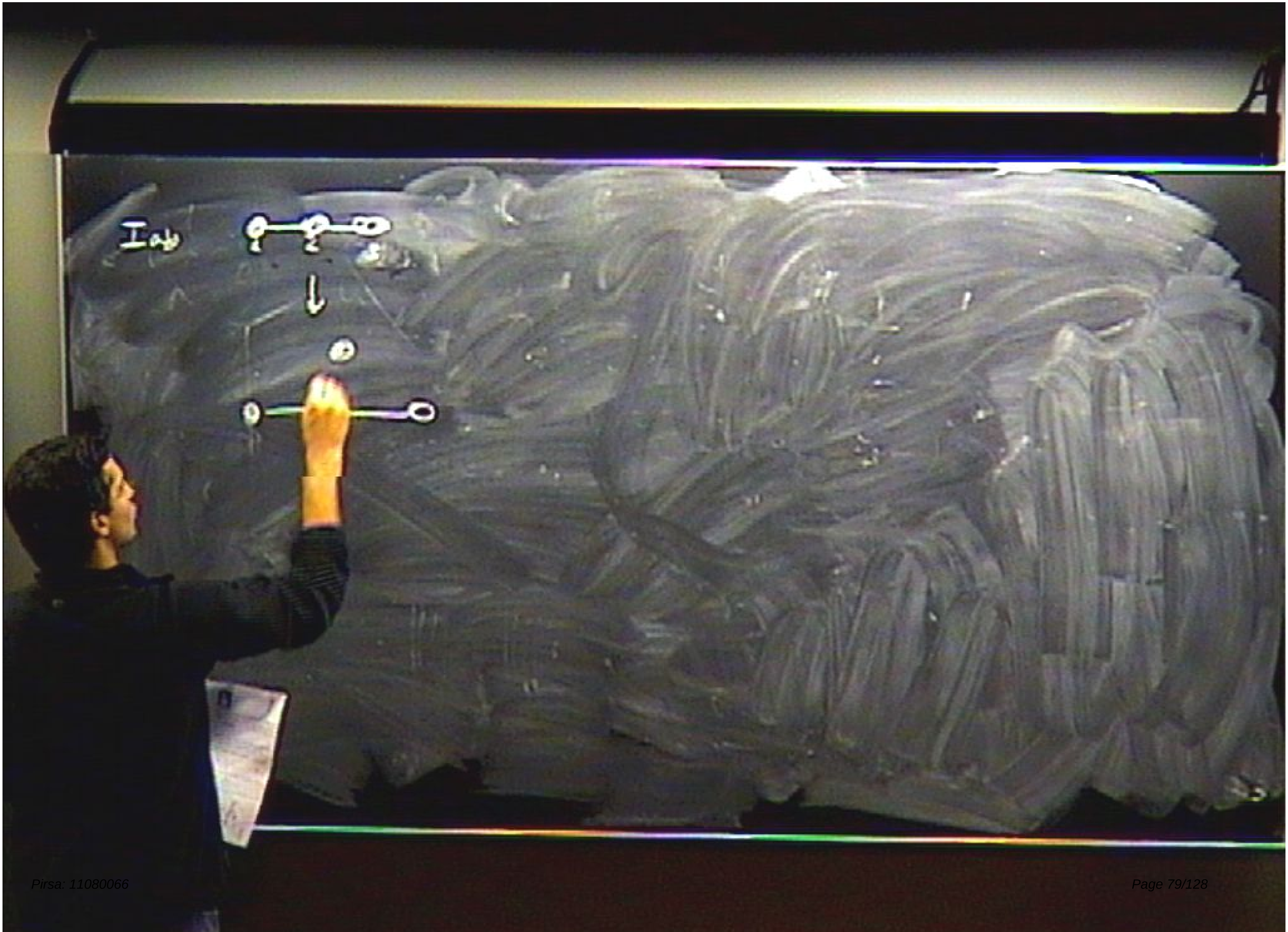

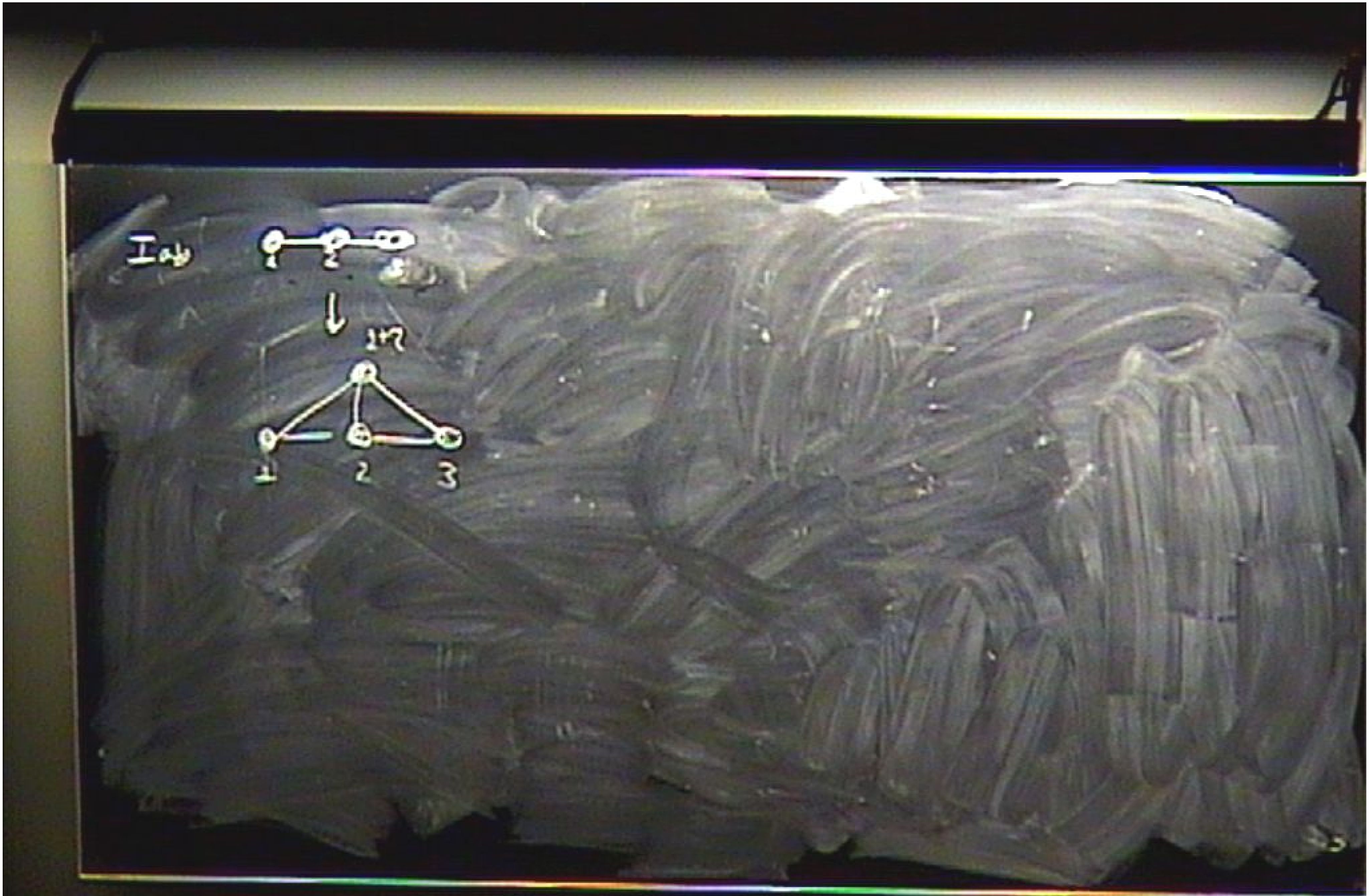

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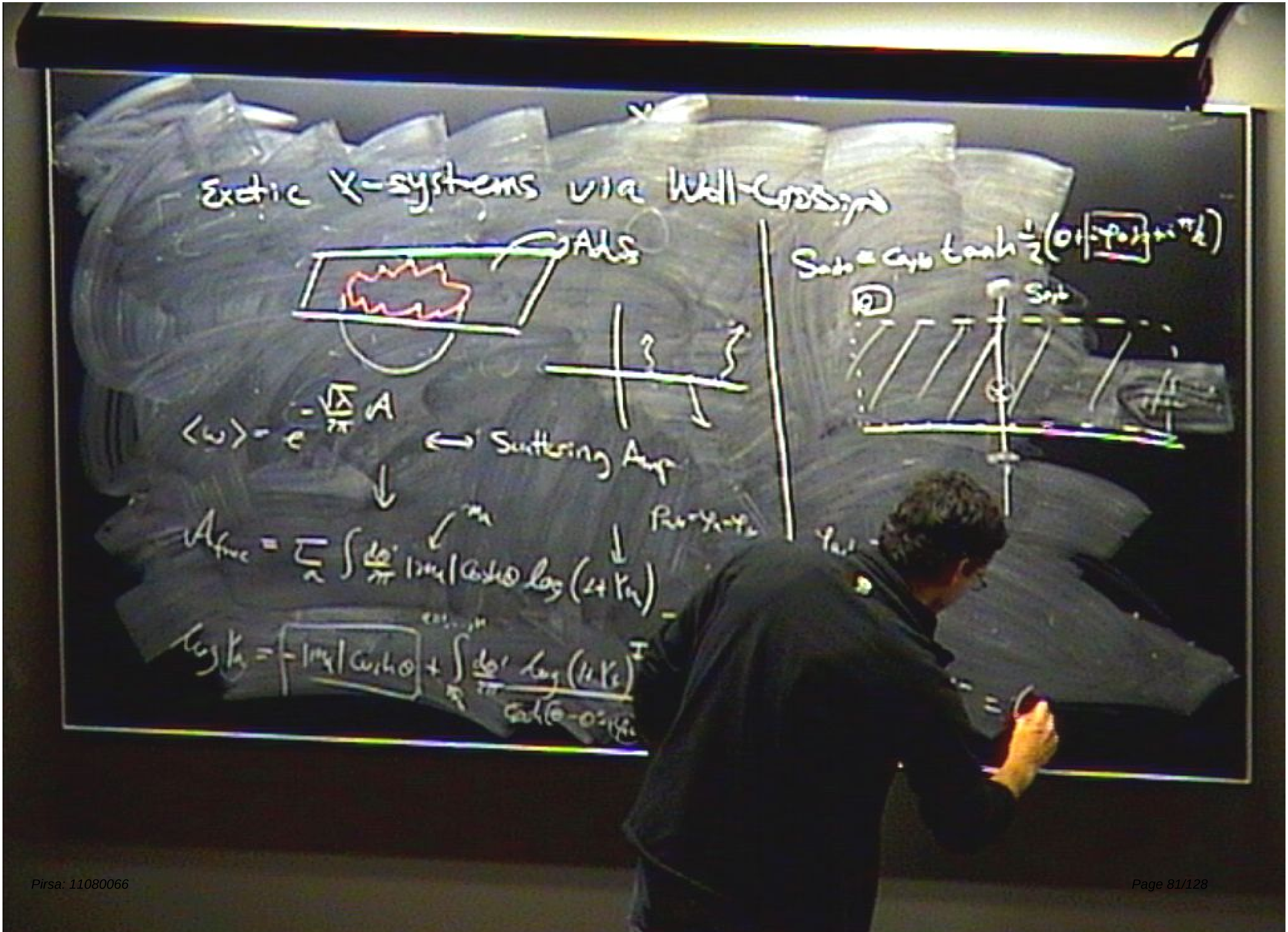
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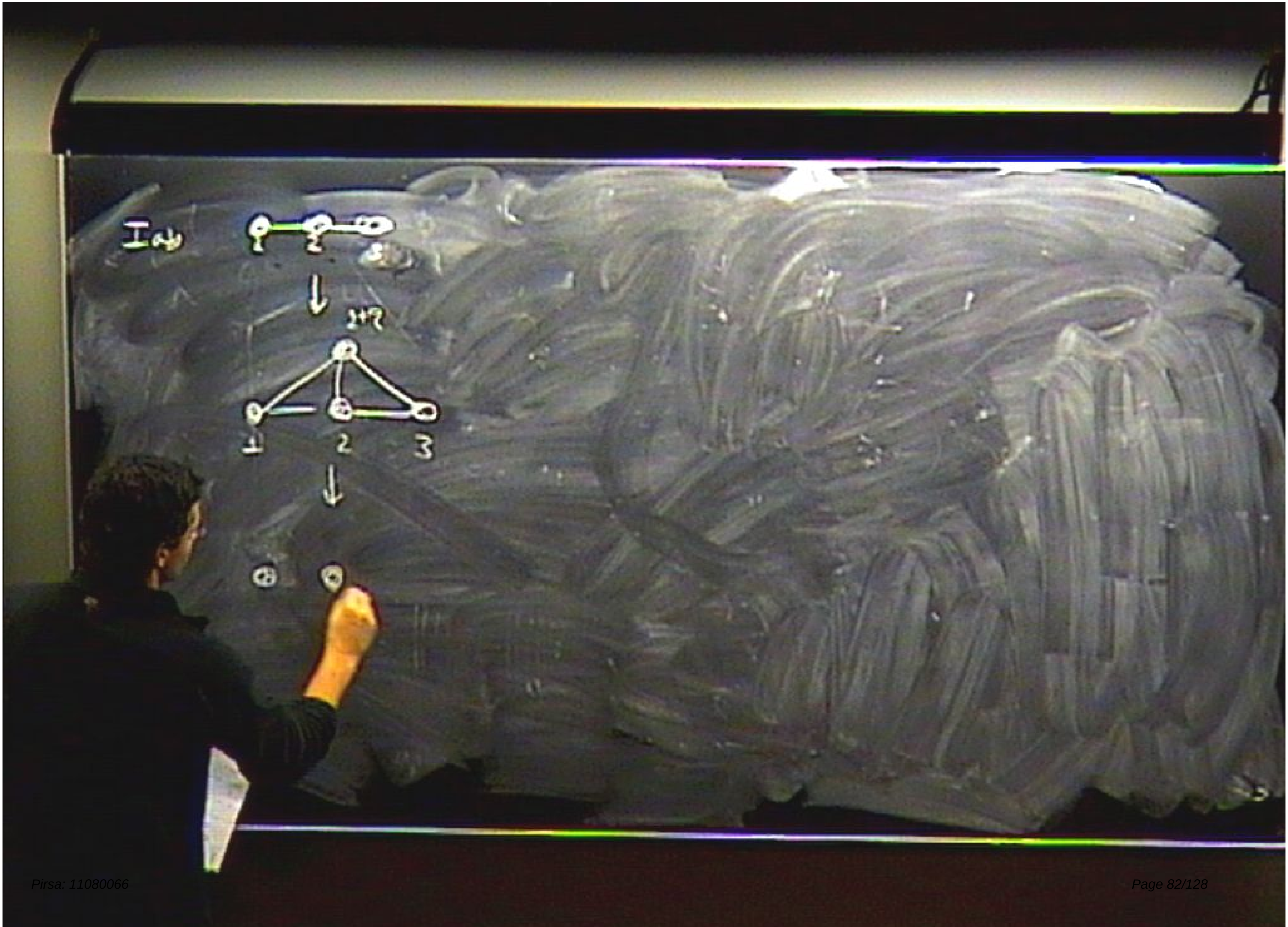
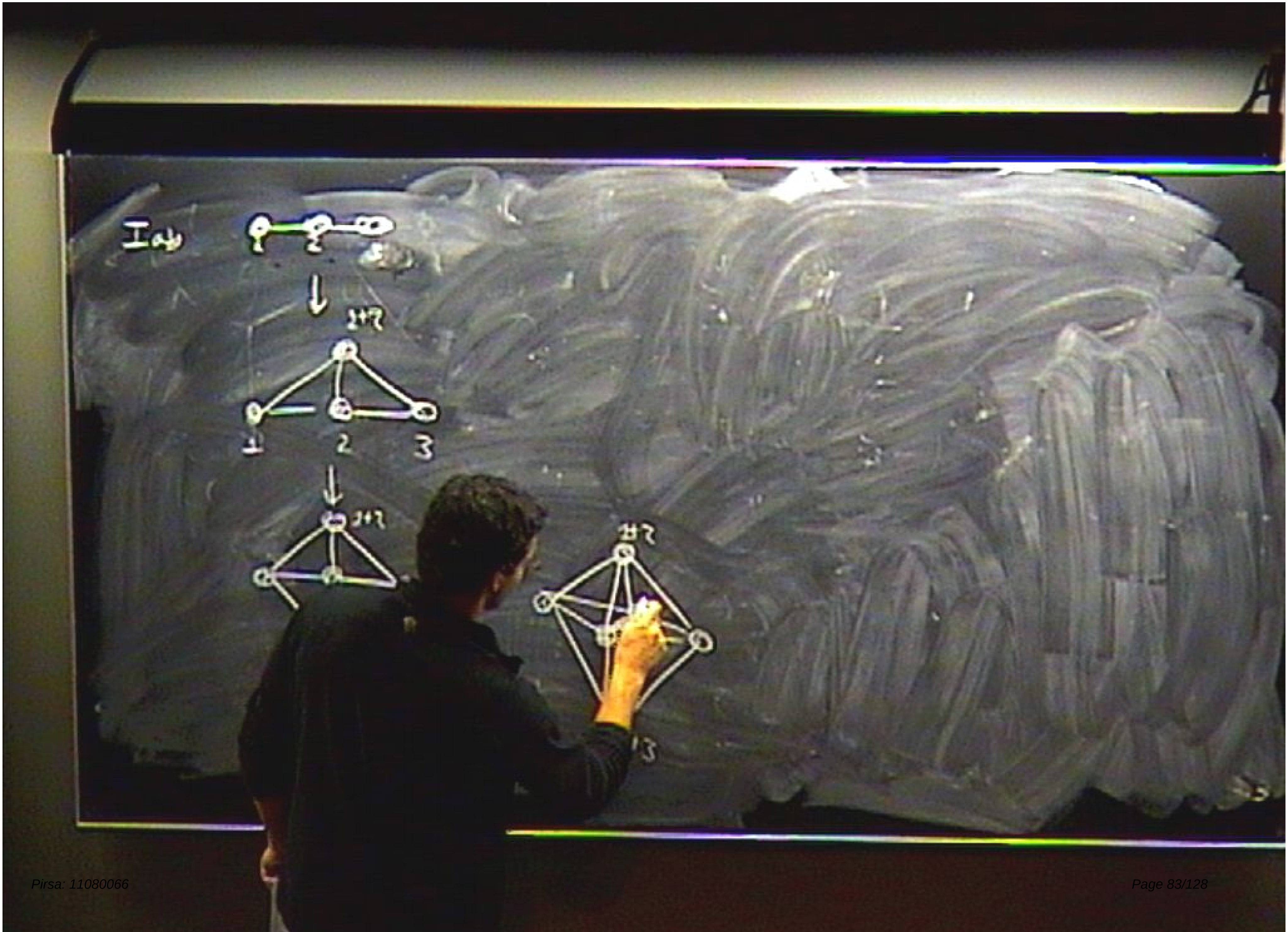
Scattering Amp

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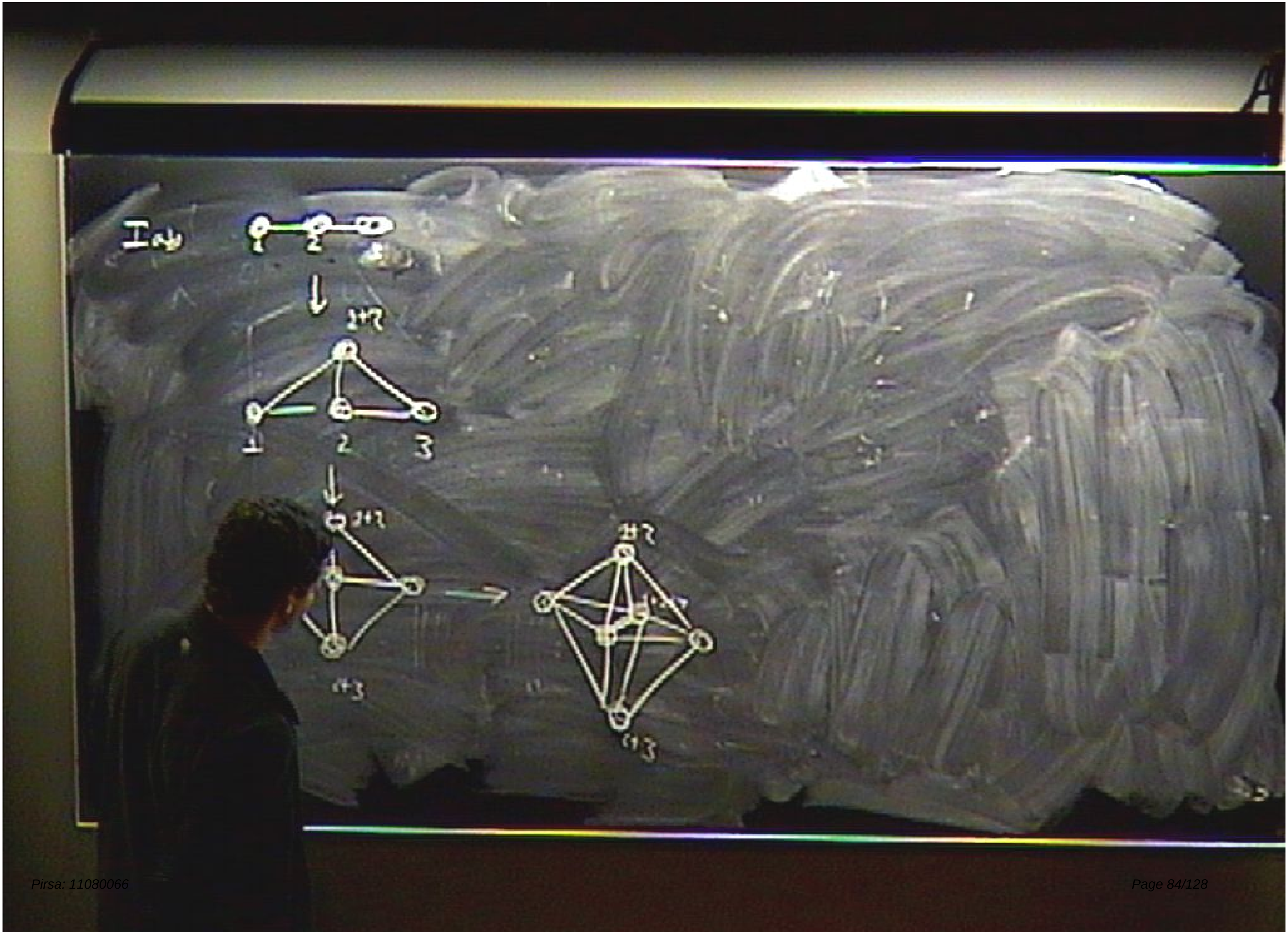

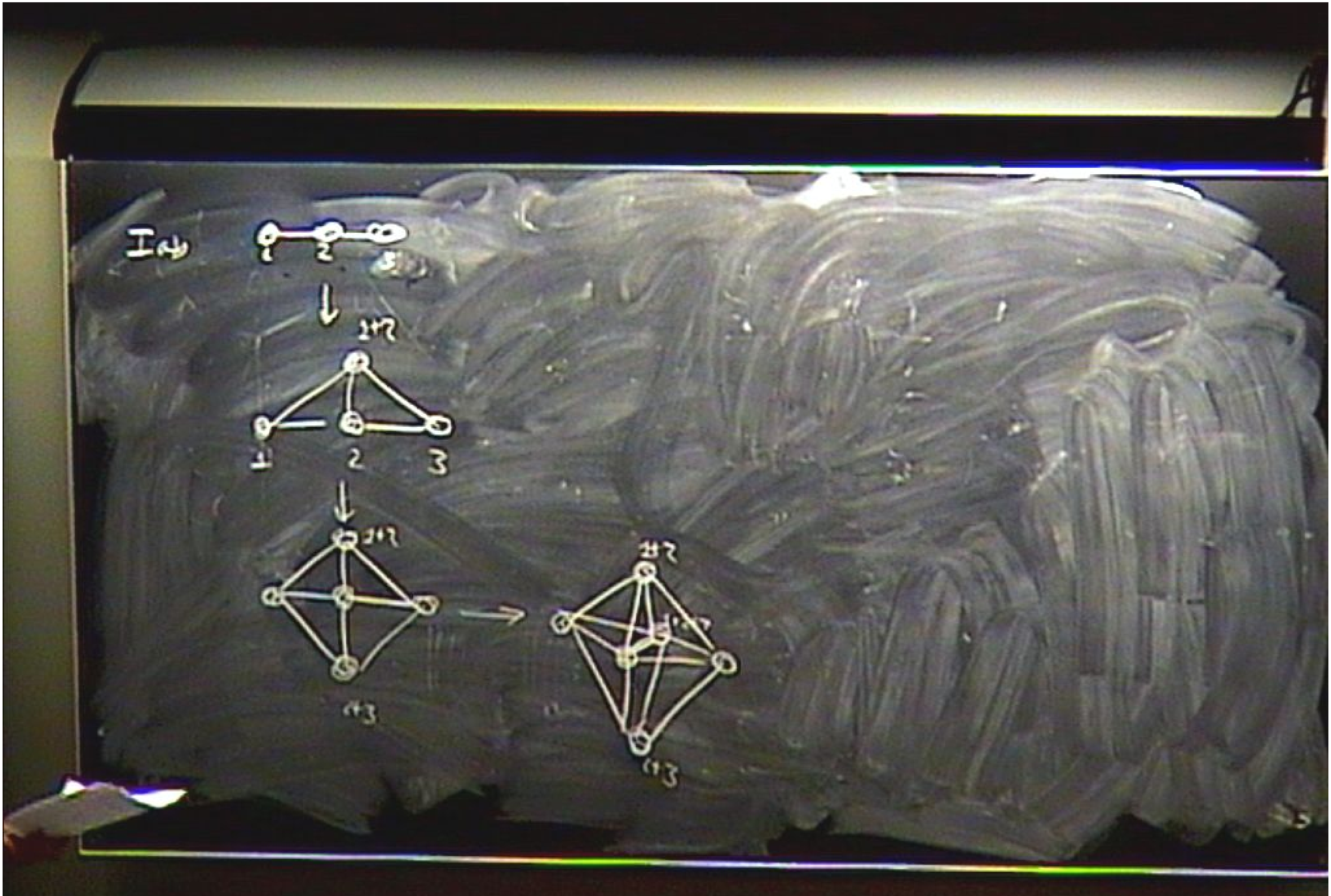
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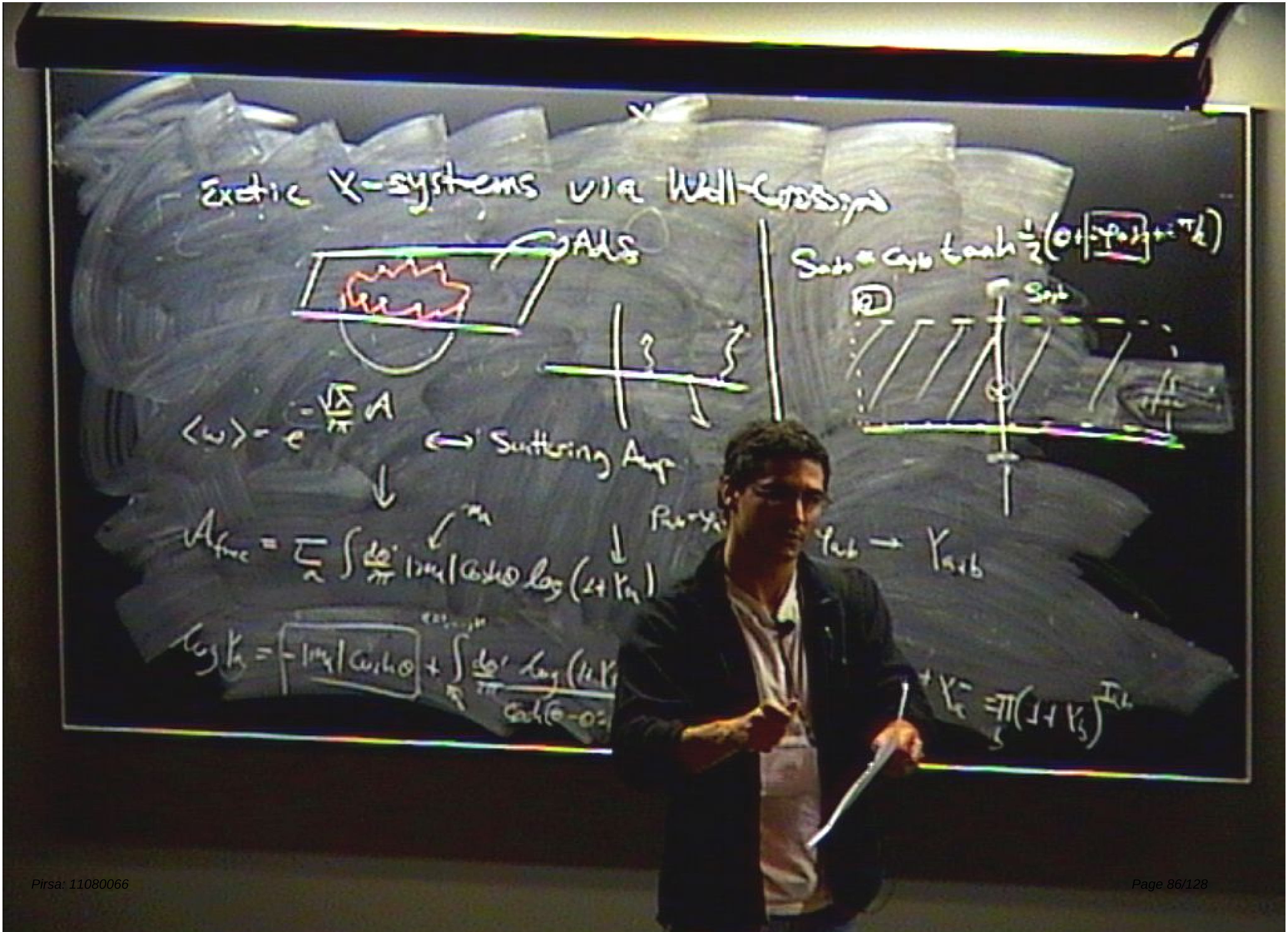
Exotic Y-systems via Wall-Crossing
AdS
Scattering Amp

Lifting asymptotic degeneracies

Stijn J. van Tongeren

Lifting asymptotic degeneracies

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August 18, 2011

Alessandro Sfondrini and ST [1106.3909]
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Universiteit Utrecht

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A qualitative feature of the $AdS_5 \times S^5$ string spectrum not captured by the asymptotic Bethe ansatz

My talk

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A qualitative feature of the $AdS_5 \times S^5$ string spectrum not captured by the asymptotic Bethe ansatz

Finite size effects are not simply quantitative

Outline

- Asymptotic degeneracies
- Lifting via finite size effects
- Two concrete states

Asymptotic degeracies I

Stijn J. van
Tongeren

Asymptotic degeracies I

- Expected: degenerate superconformal multiplets

Asymptotic degeracies I

Lifting
asymptotic
degeneracies

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Tongeren

- Expected: degenerate superconformal multiplets
- Manifest $\mathfrak{su}_L(2|2) \oplus \mathfrak{su}_R(2|2)$: $\mathbb{Z}_2$ symmetry in spectrum

Asymptotic degeracies I

Lifting asymptotic degeneracies

Stijn J. van Tongeren

- Expected: degenerate superconformal multiplets
- Manifest $\mathfrak{su}_L(2|2) \oplus \mathfrak{su}_R(2|2)$: $\mathbb{Z}_2$ symmetry in spectrum
- Asymptotically there can be more!

The asymptotic Bethe equations

Stijn J. van Tongeren

$$1 = e^{ip_k J} \prod_{l=1, l\neq k}^{K^{\mathrm{I}}} S_{\mathfrak{sl}(2)}(p_k, p_l) \prod_{\alpha=L,R} \prod_{l=1}^{K^{\mathrm{II}}_{(\alpha)}} \frac{x_k^- - y_l^{(\alpha)}}{x_k^+ - y_l^{(\alpha)}} \sqrt{\frac{x_k^+}{x_k^-}} \,,$$

$$1 = \prod_{l=1}^{K^{\mathrm{I}}} \frac{y_k^{(\alpha)} - x_l^-}{y_k^{(\alpha)} - x_l^+} \sqrt{\frac{x_l^+}{x_l^-}} \prod_{l=1}^{K^{\mathrm{III}}_{(\alpha)}} \frac{\nu_k^{(\alpha)} - w_l^{(\alpha)} + \frac{i}{g}}{\nu_k^{(\alpha)} - w_l^{(\alpha)} - \frac{i}{g}} \,,$$

$$1 = \prod_{l=1}^{K^{\mathrm{II}}_{(\alpha)}} \frac{w_k^{(\alpha)} - \nu_l^{(\alpha)} + \frac{i}{g}}{w_k^{(\alpha)} - \nu_l^{(\alpha)} - \frac{i}{g}} \prod_{l=1, l\neq k}^{K^{\mathrm{III}}_{(\alpha)}} \frac{w_k^{(\alpha)} - w_l^{(\alpha)} - \frac{2i}{g}}{w_k^{(\alpha)} - w_l^{(\alpha)} + \frac{2i}{g}} \,.$$

The asymptotic Bethe equations with $K^{III} = 0$

Lifting asymptotic degeneracies

Stijn J. van Tongeren

$$1 = e^{ip_k J} \prod_{l=1, l\neq k}^{K^{\mathrm{I}}} S_{\mathfrak{sl}(2)}(p_k, p_l) \prod_{\alpha=L,R} \prod_{l=1}^{K^{\mathrm{II}}_{(\alpha)}} \frac{x_k^- - y_l^{(\alpha)}}{x_k^+ - y_l^{(\alpha)}} \sqrt{\frac{x_k^+}{x_k^-}}\,,$$

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$$1 = \prod_{l=1}^{K^{\mathrm{I}}} \frac{y_k^{(\alpha)} - x_l^-}{y_k^{(\alpha)} - x_l^+} \sqrt{\frac{x_l^+}{x_l^-}}\,.$$

The same equation for *all* y roots!

Asymptotic degeracies II

Redistributing y roots between left and right sectors gives the same momenta, hence degenerate energies

Lifting degeneracies

Lifting degeneracies

Stijn J. van Tongeren

- Finite size energy:

$$E = \sum_{k=1}^{K^{\mathrm{I}}} \mathcal{E}_k - \frac{1}{2\pi} \sum_{Q=1}^{\infty} \int du \, \frac{d\tilde{p}^Q}{du} \log(1 + Y_Q)$$

Lifting degeneracies

Lifting asymptotic degeneracies

Stijn J. van Tongeren

- Finite size energy:

$$E = \sum_{k=1}^{K^1} \mathcal{E}_k - \frac{1}{2\pi} \sum_{Q=1}^{\infty} \int du \, \frac{d\tilde{p}^Q}{du} \log(1 + Y_Q)$$

- Y_Q from TBA: different eqs $\rightarrow$ different Y_Q $\rightarrow$ different E

Lifting degeneracies

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- Y_Q from TBA: different eqs $\rightarrow$ different Y_Q $\rightarrow$ different E

- Lüscher corrections are already manifestly different

Two states

State	K^{I}	$K^{\mathrm{II}}_{(L)}$	$K^{\mathrm{II}}_{(R)}$	$K^{\mathrm{III}}_{(\alpha)}$	Weights
Θ	4	2	0	0	$[2, J-1, 0]_{(2,4)}$
Ψ	4	1	1	0	$[1, J-1, 1]_{(3,3)}$

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Lifting asymptotic degeneracies

Stijn J. van Tongeren

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Explicit different TBA equations

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Asymptotically identical energies

Explicit different TBA equations

Leading order finite size correction ($J = 4$):

$$E^{\Theta}_{LO} = -\left(\frac{231}{32}\,\zeta(11) + \frac{21}{32}\,\zeta(9) - \frac{259}{32}\,\zeta(7) - \frac{113}{16}\,\zeta(5) + \frac{161}{32}\,\zeta(3) + \frac{1887}{1024}\right) g^{14}$$
$$\approx -0.2761\, g^{14},$$
$$E^{\Psi}_{LO} = -\left(\frac{231}{32}\,\zeta(11) + \frac{105}{64}\,\zeta(9) - \frac{553}{64}\,\zeta(7) - \frac{589}{64}\,\zeta(5) + \frac{49}{8}\,\zeta(3) + \frac{2269}{1024}\right) g^{14}$$
$$\approx -0.1889\, g^{14}.$$

The main point

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The ABA misses qualitative information on the string spectrum.

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The ABA misses qualitative information on the string spectrum.

Finite size effects contain this information.

Two states

Lifting asymptotic degeneracies

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Strings on $AdS_4 \times CP^3$ vs. Bethe Ansatz: the All-Order Equivalence

Gianluca Grignani, Davide Astolfi, Andrew V. Zayakin

INFN Perugia,
Italy

August 18, 2011

Summary

- The one-loop correction to the single-magnon dispersion law is finite (and exponentially small in J), supporting the evidence for

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- The one-loop correction to the single-magnon dispersion law is finite (and exponentially small in J), supporting the evidence for $a_1 = 0$ rather than for $a_1 = -\frac{\log 2}{2\pi}$. Thus the Gromov–Vieira asymptotic Bethe Ansatz is confirmed rather than semiclassical calculations.
- In the two-particle sector the finite-size ($1/J$) corrections to magnon interaction energies on the string side and on the Bethe-Ansatz side agree up to the 4^{th} degree in $\lambda' \equiv \frac{\lambda}{J^2}$.

Thus we conjecture that the asymptotic all-loop Gromov–Vieira Bethe Ansatz agrees with strings in all orders in at strong coupling.

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- The one-loop correction to the single-magnon dispersion law is finite (and exponentially small in J), supporting the evidence for $a_1 = 0$ rather than for $a_1 = -\frac{\log 2}{2\pi}$. Thus the Gromov–Vieira asymptotic Bethe Ansatz is confirmed rather than semiclassical calculations.
- In the two-particle sector the finite-size $(1/J)$ corrections to magnon interaction energies on the string side and on the Bethe-Ansatz side agree up to the 4^{th} degree in $\lambda' \equiv \frac{\lambda}{J^2}$.

Thus we conjecture that the asymptotic all-loop Gromov–Vieira Bethe Ansatz agrees with strings in all orders in λ' at strong coupling.

String Hamiltonian in the Penrose Limit

The plane-wave fermionic Hamiltonian is

$$\mathcal{H}_{2,F} = \frac{i}{4c^2}(c^2\psi_+\psi'_+ - 4\rho_+\rho'_+ + 2c^2\psi_-\psi'_- - 2\rho_-\rho'_-) - \frac{i}{2}\psi_+\rho_+ + i\psi_-\rho_- + \frac{1}{2}\psi_-\Gamma_{56}\rho_-$$

where the conjugate momenta are $\rho \equiv \frac{\delta\mathcal{L}_2}{\delta\psi} = -\frac{ic}{2}(2\mathcal{P}_- + \mathcal{P}_+)\psi^*$ and $\rho_\pm = \mathcal{P}_\pm\rho$.
The quartic purely fermionic Hamiltonian is

$$\mathcal{H}_{4,F} = -\frac{i}{12}\left(\bar\theta\Gamma_{11}\Gamma_-\mathcal{M}^2\theta' + \bar\theta\Gamma_-\mathcal{M}^2\Gamma_{11}\theta'\right) - \frac{1}{2c}(A^2_{+,\sigma} - \bar{A}^2_{+,\sigma})$$
$$-\frac{1}{4}A_{+,\sigma}(\bar{C}_{+-} + \bar{B}_{+56} + \bar{B}_{+78}) + \frac{1}{4}\bar{A}_{+,\sigma}(C_{+-} - C_{++} + B_{+56} + B_{+78})$$
$$-\frac{c}{8}\sum_{i=1}^{4}C^2_{+i} - \frac{c}{32}\sum_{i=5}^{8}\left[2C_{+i} - s_iB_{+4i} + \frac{1}{2}\sum_{j=5}^{8}\epsilon_{ij}B_{+-j}\right]^2$$

The mixed cubic Hamiltonian is

$$\mathcal{H}_{3,BF} = \frac{i}{2}\sum_{i=1}^{8}(C_{+i}p_i + \bar{C}_{+i}X'^i) - \frac{ic}{4}(B_{+56} - B_{+78})u_4 - \frac{ic}{4}B_{+-4}u_4$$
$$-\frac{i}{4}\sum_{i=5}^{8}s_i(B_{+4i}p_i + \bar{B}_{+4i}X'^i) - \frac{i}{8}\sum_{i,j=5}^{8}\epsilon_{ij}(B_{+-i}p_j + \bar{B}_{+-i}X'^j) \quad (1)$$

The mixed quartic Hamiltonian

$$
\begin{aligned}
\mathcal{H}_{4,BF} = & \frac{i}{c^2} \sum_{i=1}^{8} \left(p_i^2 + (X'^i)^2 \right) \left[\bar{A}_{+,\sigma} + \frac{c}{4}(B_{+56} + B_{+78} - C_{++} + C_{+-}) \right] - i\bar{A}_{+,\sigma} \left[\sum_{i=1}^{3} u_i^2 - u_4^2 \right] \\
& + \frac{2i}{c^2} \sum_{i=1}^{8} p_i X'^i \left[A_{+,\sigma} + \frac{c}{4}(\bar{B}_{+56} + \bar{B}_{+78}) + \frac{c}{4}\bar{C}_{+-} \right] + \frac{ic}{2} \sum_{i=1}^{3} u_i^2 C_{++} - \frac{ic}{4} \sum_{i=1}^{4} u_i^2 (B_{+56} + B_{+78}) \\
& + \frac{i}{2} u_4 \sum_{i=5}^{8} s_i \left[C_{+i} p_i - \bar{C}_{+i} X'^i \right] - \frac{i}{c} \sum_{i,j=1}^{8} \left[C_{ij}(X'^i X'^j - p_i p_j) + 2\bar{C}_{ij} X'^i p_j \right] - i \sum_{i,j=1}^{3} u_i' u_j \bar{B}_{+ij} \\
& - \frac{i}{8} u_4 \sum_{i,j=5}^{8} s_i \epsilon_{ij} (3B_{+-i} p_j + \bar{B}_{+-i} X'^j) + \frac{i}{4}(B_{+56} p_{x_1} y_1 + \bar{B}_{+56} x_1' y_1 + B_{+78} p_{x_2} y_2 + \bar{B}_{+78} x_2' y_2) \\
& - \frac{i}{2} \sum_{i=1}^{4} \sum_{j=1}^{8} u_i \left[B_{-ij} p_j - \bar{B}_{-ij} X'^j \right] + \frac{i}{2c} \sum_{i=1}^{8} \sum_{j=5}^{8} s_j \left[(p_i p_j - X'^i X'^j) B_{4ij} + (p_i X'^j - X'^i p_j) \bar{B}_{4ij} \right] \\
& - \frac{i}{2} \sum_{i=1}^{3} \sum_{j=4}^{8} u_i \left[B_{+ij} p_j - \bar{B}_{+ij} X'^j \right] - \frac{i}{4} u_4 \sum_{i=5}^{8} (B_{+4i} p_i + 3\bar{B}_{+4i} X'^i) + \frac{i}{2} u_4 \sum_{i=1}^{3} (B_{+4i} p_i - \bar{B}_{+4i} u_i') \\
& - \frac{i}{4c} \sum_{i=1}^{8} \sum_{j,k=5}^{8} \epsilon_{jk} \left[(B_{+ij} - B_{-ij})(p_i p_k - X'^i X'^k) + (\bar{B}_{+ij} - \bar{B}_{-ij})(p_i X'^k - X'^i p_k) \right] \\
& + \frac{i}{2c^2} \sum_{i,j=1}^{8} (p_i p_j' + X'^i X''^j) \bar{E}_{ij} - \frac{i}{2c^2} \sum_{i,j=1}^{8} (X'^i p_j' + p_i X''^j) E_{ij} - \frac{3i}{4c} \sum_{i,j=1}^{8} (p_i p_j - X'^i X'^j) C_{i+;j} \\
& + \frac{3i}{4c} \sum_{i,j=1}^{8} (X'^i p_j - p_i X'^j) \bar{C}_{i+;j} - \frac{i}{4c} \sum_{i,j=1}^{8} (p_i p_j + X'^i X'^j) C_{+i;j} - \frac{i}{4c} \sum_{i,j=1}^{8} (X'^i p_j + p_i X'^j) \bar{C}_{+i;j} \\
& + \frac{iu_4}{2} \sum_{i=1}^{8} (p_j \mathcal{B}_{+-4;i} - X'^j \bar{\mathcal{B}}_{+-4;i}) + \frac{i}{2c} \sum_{i=5}^{8} \sum_{j=1}^{8} s_i \left[(p_i p_j - X'^i X'^j) \mathcal{B}_{+4i;j} + (X'^i p_j - p_i X'^j) \bar{\mathcal{B}}_{+4i;j} \right] \\
& + \frac{i}{4c} \sum_{i,j=5}^{8} \sum_{k=1}^{8} \epsilon_{ij} \left[(p_i p_k + X'^i X'^k)(\mathcal{B}_{+-i;k} + E_{jk}) + (X'^i p_k + p_i X'^k)(\bar{\mathcal{B}}_{+-i;k} - \bar{E}_{jk}) \right].
\end{aligned}
$$

Technical gamma-matrix stuff

The following combinations of gamma-matrices have been introduced for brevity:

$$A_{a,A} = \bar{\theta}\Gamma_a \partial_A \theta \,, \quad \tilde{A}_{a,A} = \bar{\theta}\Gamma_{11}\Gamma_a \partial_A \theta$$

$$B_{abc} = \bar{\theta}\Gamma_a \Gamma_{bc}\theta \,, \quad \tilde{B}_{abc} = \bar{\theta}\Gamma_{11}\Gamma_a \Gamma_{bc}\theta$$

$$C_{ab} = \bar{\theta}\Gamma_a P \Gamma_{0123}\Gamma_b \theta \,, \quad \tilde{C}_{ab} = \bar{\theta}\Gamma_{11}\Gamma_a P \Gamma_{0123}\Gamma_b \theta$$

$$\mathcal{B}_{abc;d} = \bar{\theta}\Gamma_{abc}(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_d\theta \,, \quad \tilde{\mathcal{B}}_{abc;d} = \bar{\theta}\Gamma_{11}\Gamma_{abc}(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_d\theta$$

$$\mathcal{C}_{ab;c} = \bar{\theta}\Gamma_a P \Gamma_{0123}\Gamma_b(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_c\theta \,, \quad \tilde{\mathcal{C}}_{ab;c} = \bar{\theta}\Gamma_{11}\Gamma_a P \Gamma_{0123}\Gamma_b(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_c\theta$$

$$E_{ab} = \bar{\theta}\Gamma_a(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_b\theta \,, \quad \tilde{E}_{ab} = \bar{\theta}\Gamma_{11}\Gamma_a(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_b\theta$$

The projector P is defined as

$$P = \frac{3 - J}{4} \tag{2}$$

with

$$J = \Gamma_{0123}\Gamma_{11}(-\Gamma_{49} - \Gamma_{56} + \Gamma_{78}) = \Gamma_{5678} - \Gamma_{49}(\Gamma_{56} - \Gamma_{78}) \tag{3}$$

Here we assume that θ obeys $P\theta = \theta$. The projectors $\mathcal{P}_\pm$ are defined as

$$\mathcal{P}_+ = \frac{I + \Gamma_{5678}}{2}\frac{I + \Gamma_{4956}}{2} \,, \quad \mathcal{P}_- = \frac{I - \Gamma_{5678}}{2}\frac{I - \Gamma_{09}}{2} \tag{4}$$

and the relation among P and $\mathcal{P}_\pm$ is

$$P = \mathcal{P}_+ + \mathcal{P}_- + \mathcal{P}'_- \,, \quad I = \mathcal{P}_+ + \mathcal{P}_- + \mathcal{P}'_+ + \mathcal{P}'_-$$

$$\mathcal{P}'_+ = \frac{I + \Gamma_{5678}}{2}\frac{I - \Gamma_{4956}}{2} \,, \quad \mathcal{P}'_- = \frac{I - \Gamma_{5678}}{2}\frac{I + \Gamma_{09}}{2} \,.$$

String calculation: Dispersion Laws

Bosons

For the heavy bosonic states

$$E_n^{heavy\ boson} = \Omega_n + \frac{n^2}{cR^2\Omega_n} \sum_z \left(\frac{1}{\Omega_z} - \frac{1}{\omega_z} \right).$$

Fermions

For light states

$$E_n^{light\ fermion} = \omega_n + \frac{n^2}{2cR^2\omega_n} \sum_z \left(\frac{1}{\Omega_z} - \frac{1}{\omega_z} \right).$$

For heavy states

$$E_n^{heavy\ fermion} = \Omega_n + \frac{n^2}{cR^2\Omega_n} \sum_z \left(\frac{1}{\Omega_z} - \frac{1}{\omega_z} \right).$$

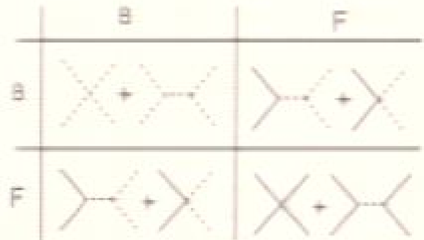
In the strong-coupling limit $h(\lambda)$ has an expansion

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} + a_1 + \dots,$$

from our dispersion laws we can see that $a_1 = 0$.

Mixing Matrix for Two-Magnon Sector from Strings

There are 24 states, 8 boson-boson and 16 fermion-fermion that are tree-level degenerate and could mix in the NLO. We have checked by the explicit calculation of the mixing matrix which is depicted here



that there are no mixings, however, different states acquire different energy corrections in the $\frac{1}{J}$ order and thus degeneracy is partially lifted. Here I show the energy shifts for the whole string spectrum in terms of power expansion

$$E^{(finite-size)} = \frac{8}{J} \sum_i a_i \left(\lambda' \pi^2 n^2\right)^i .$$

a_1	a_2	a_3	a_4	a_5	Multiplicity
1	−12	96	−768	6144	2
0	−4	32	−256	2048	10
−1	4	−32	256	−2048	10
−2	12	−96	768	−6144	2

Let us proceed to Bethe Ansatz now.

Mixing Matrix from Bethe Ansatz

Boson-boson spectrum from Bethe Ansatz, $E^{\text{finite-size}} = \frac{8\varepsilon}{J}$

state					ε			
K_4	$K_{\bar{4}}$	K_3	K_2	K_1				
1	1	1	1	1	$-n^2\pi^2\lambda'$	$+4n^4\pi^4(\lambda')^2$	$-32n^6\pi^6(\lambda')^3$	$+256n^8\pi^8(\lambda')^4$
2	0	1	1	1		$-4n^4\pi^4(\lambda')^2$	$+32n^6\pi^6(\lambda')^3$	$-256n^8\pi^8(\lambda')^4$
1	1	1	1	1	$-2n^2\pi^2\lambda'$	$+12n^4\pi^4(\lambda')^2$	$-96n^6\pi^6(\lambda')^3$	$+768n^8\pi^8(\lambda')^4$
2	0	1	1	1	$-n^2\pi^2\lambda'$	$+4n^4\pi^4(\lambda')^2$	$-32n^6\pi^6(\lambda')^3$	$+256n^8\pi^8(\lambda')^4$

Fermion-fermion spectrum from Bethe Ansatz

state					ε			
K_4	$K_{\bar{4}}$	K_3	K_2	K_1				
1	1	2	2	0		$-4n^4\pi^4\lambda'^2$	$+32n^6\pi^6\lambda'^3$	$-256n^8\pi^8\lambda'^4$
2	0	2	2	0	$n^2\pi^2\lambda'$	$-12n^4\pi^4\lambda'^2$	$+96n^6\pi^6\lambda'^3$	$-768n^8\pi^8\lambda'^4$
1	1	2	1	0	$-n^2\pi^2\lambda'$	$+4n^4\pi^4\lambda'^2$	$-32n^6\pi^6\lambda'^3$	$+256n^8\pi^8\lambda'^4$
2	0	2	1	0		$-4n^4\pi^4\lambda'^2$	$+32n^6\pi^6(\lambda')^3$	$-256n^8\pi^8\lambda'^4$
1	1	2	1	0	$-n^2\pi^2\lambda'$	$+4n^4\pi^4\lambda'^2$	$-32n^6\pi^6\lambda'^3$	$+256n^8\pi^8\lambda'^4$
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Comparison of the Two-Magnon Sector

Finite-size correction is given as λ'-series:

$$E^{(finite-size)} = \frac{8}{J} \sum_i a_i \left(\lambda' \pi^2 n^2\right)^i$$

Boson-boson spectrum comparison

Expansion coefficient					Multiplicity	Corresponding BA states				
a_1	a_2	a_3	a_4	a_5		K_4	$K_{\bar{4}}$	K_3	K_2	K_1
0	−4	32	−256	2048	2	2	0	1	1	$1_{\text{branch 1}}$
−1	4	−32	256	−2048	4	2	0	1	1	$1_{\text{branch 2}}$
						1	1	1	1	$1_{\text{branch 1}}$
−2	12	−96	768	−6144	2	1	1	1	1	$1_{\text{branch 2}}$

Fermion-fermion spectrum comparison

Expansion coefficient					Multiplicity	Corresponding BA states				
a_1	a_2	a_3	a_4	a_5		K_4	$K_{\bar{4}}$	K_3	K_2	K_1
1	−12	96	−768	6144	2	2	0	2	2	0
0	−4	32	−256	2048	8	1	1	2	2	0
						2	0	2	1	$0_{\text{branch 1}}$
						2	0	2	1	$0_{\text{branch 2}}$
						2	0	2	0	0
−1	4	−32	256	−2048	6	1	1	2	1	$0_{\text{branch 1}}$
						1	1	2	1	$0_{\text{branch 2}}$
						1	1	2	0	0

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- Not necessarily, since we know e.g. about three-loop discrepancies at weak coupling, problems with wrappings etc.

Thus a nontrivial test of the Gromov–Vieira Bethe Ansatz has been performed in the one- and two-magnon sectors by comparison with exact string perturbation theory at strong coupling

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