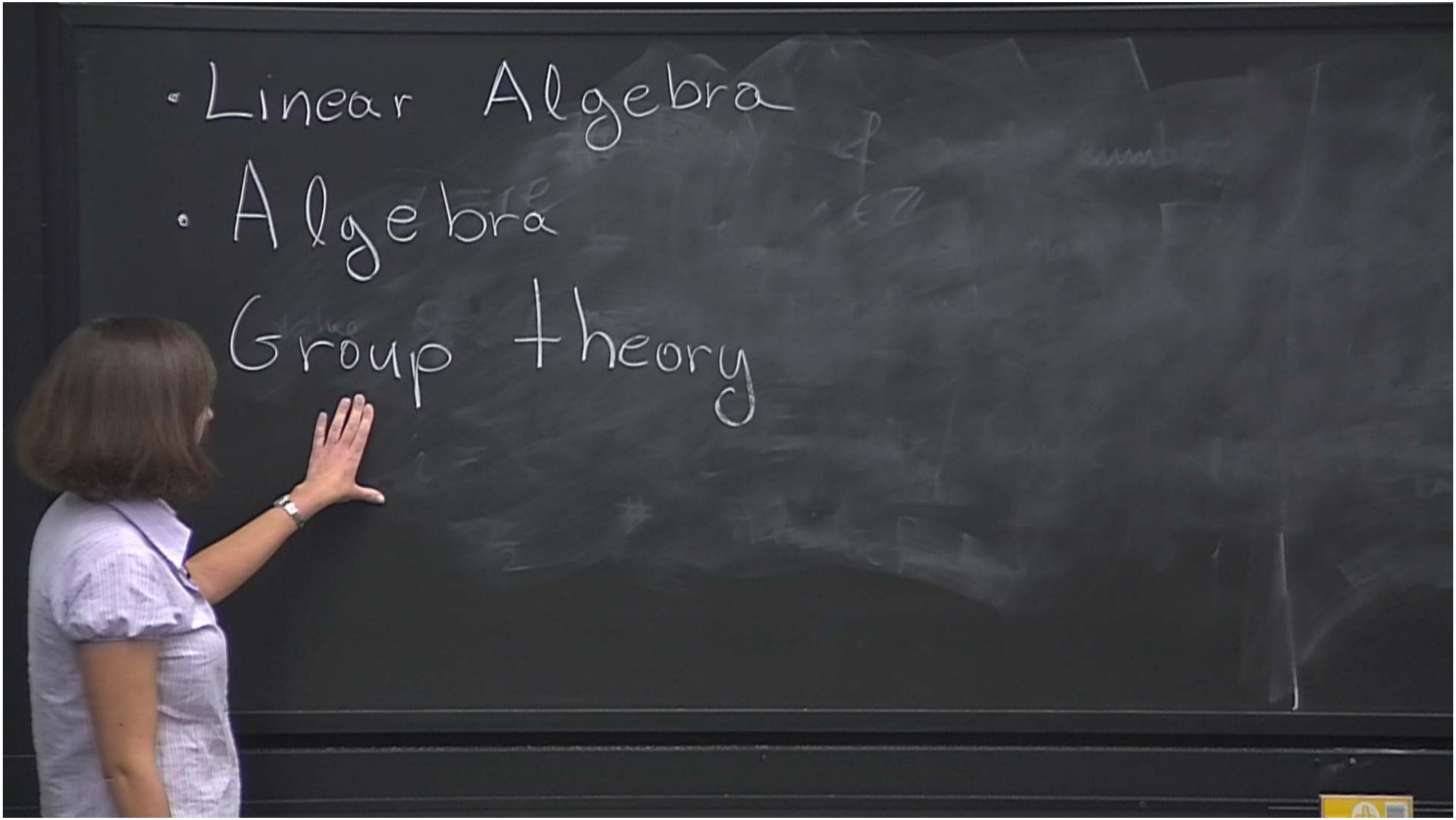
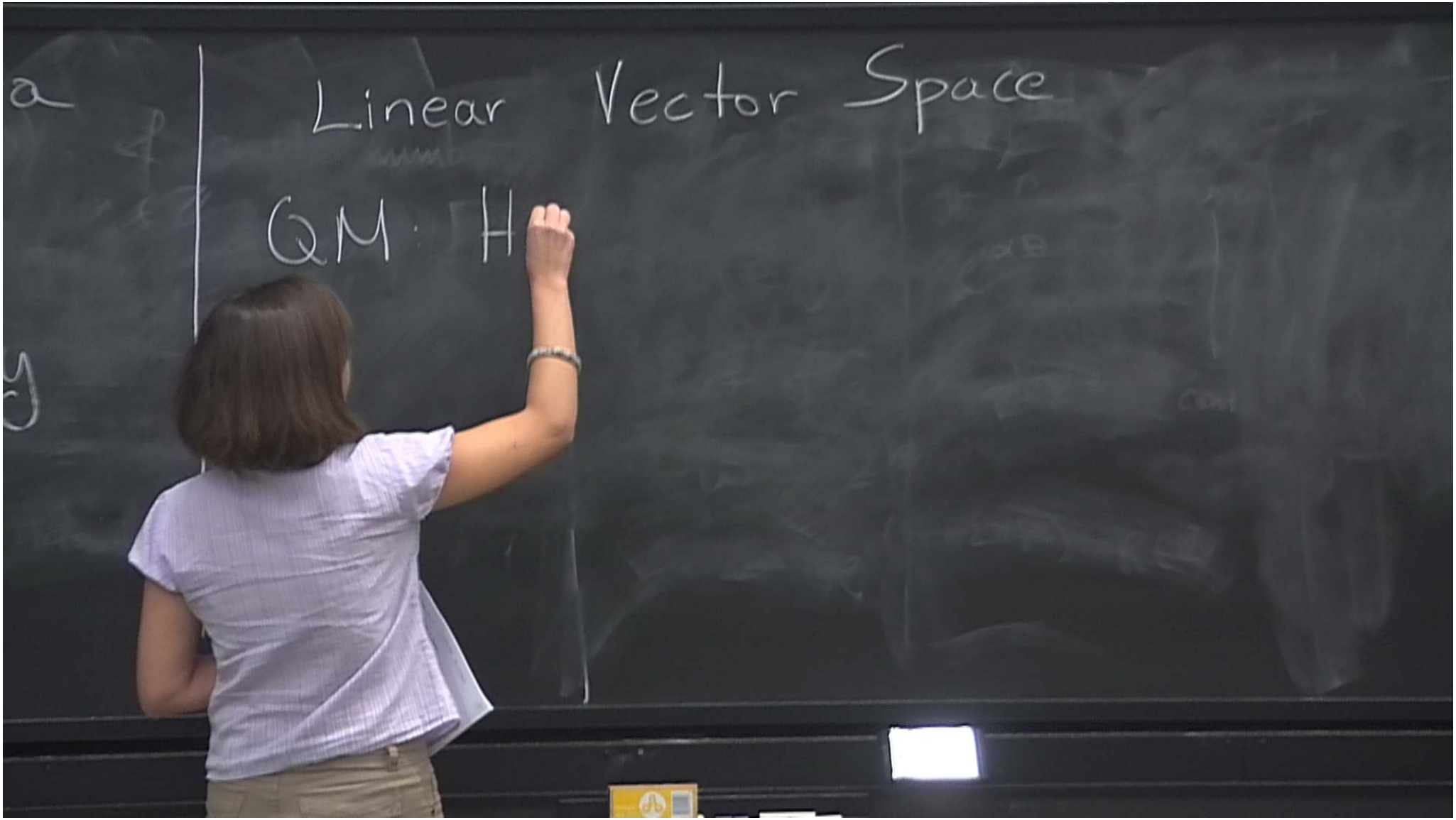
Title: Algebra - Lecture 1

Date: Aug 16, 2011 11:30 AM

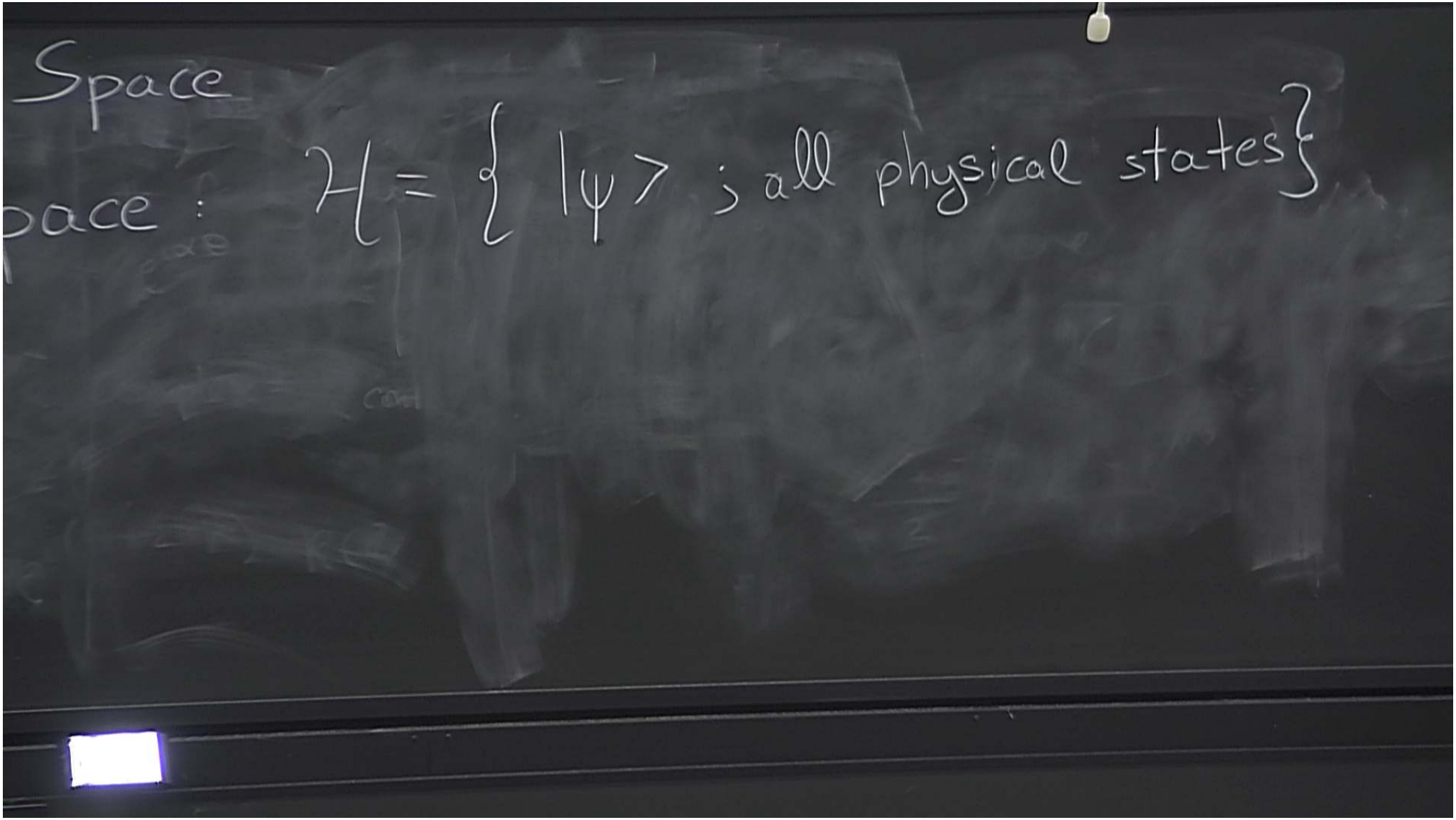
URL: http://pirsa.pi.local/11080064

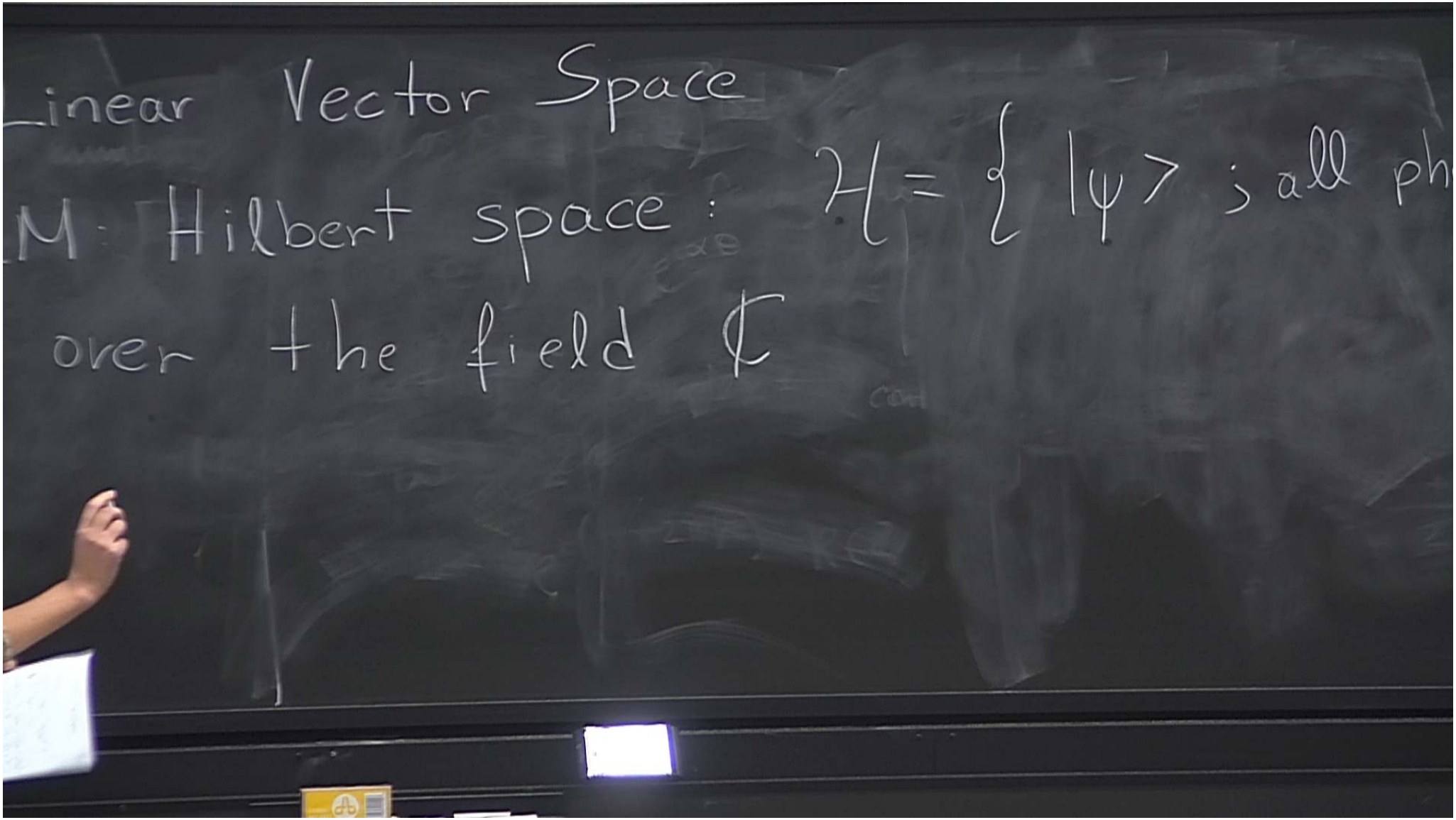
Abstract:

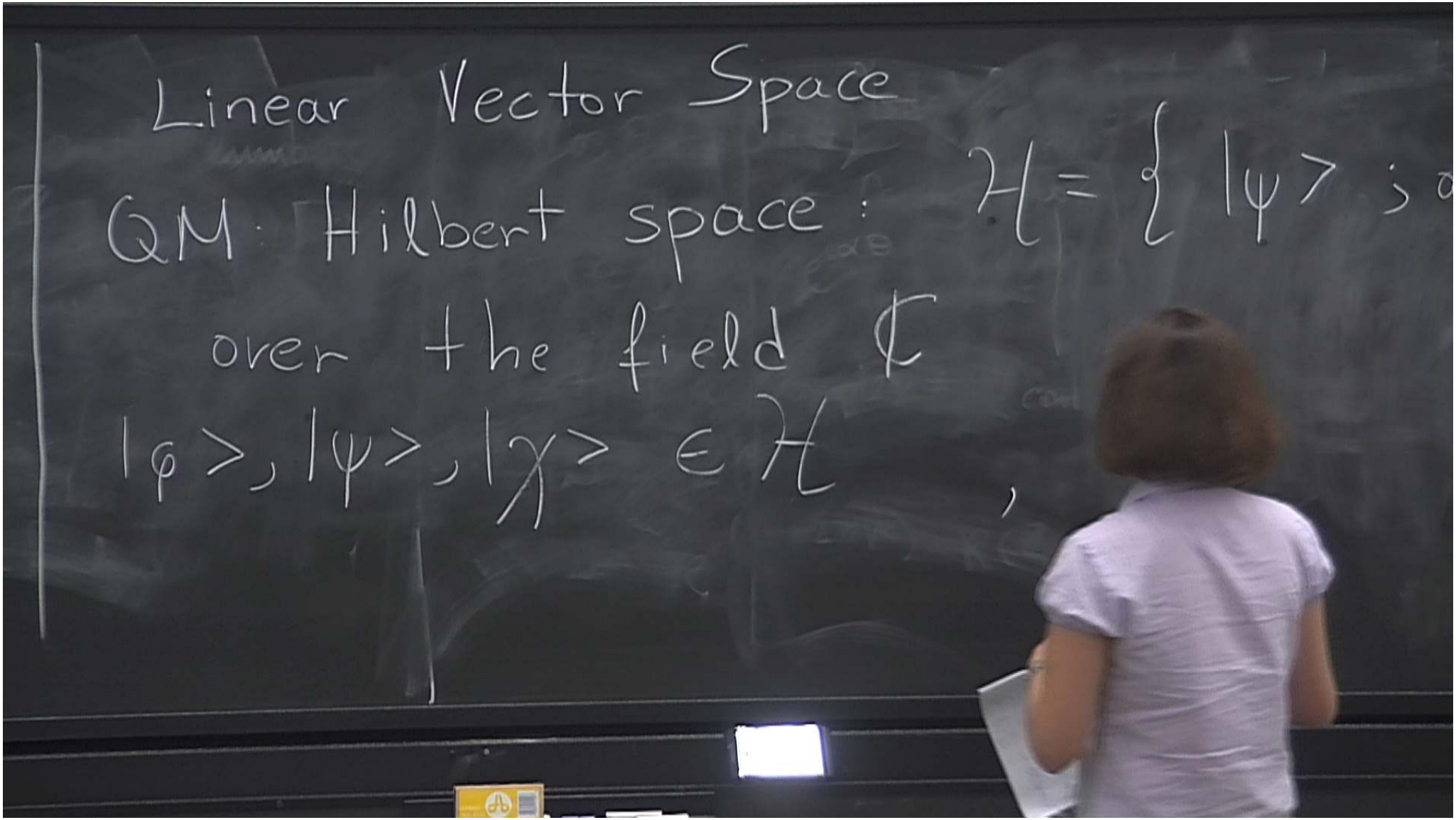

• Linear Algebra
• Algebra
Group theory

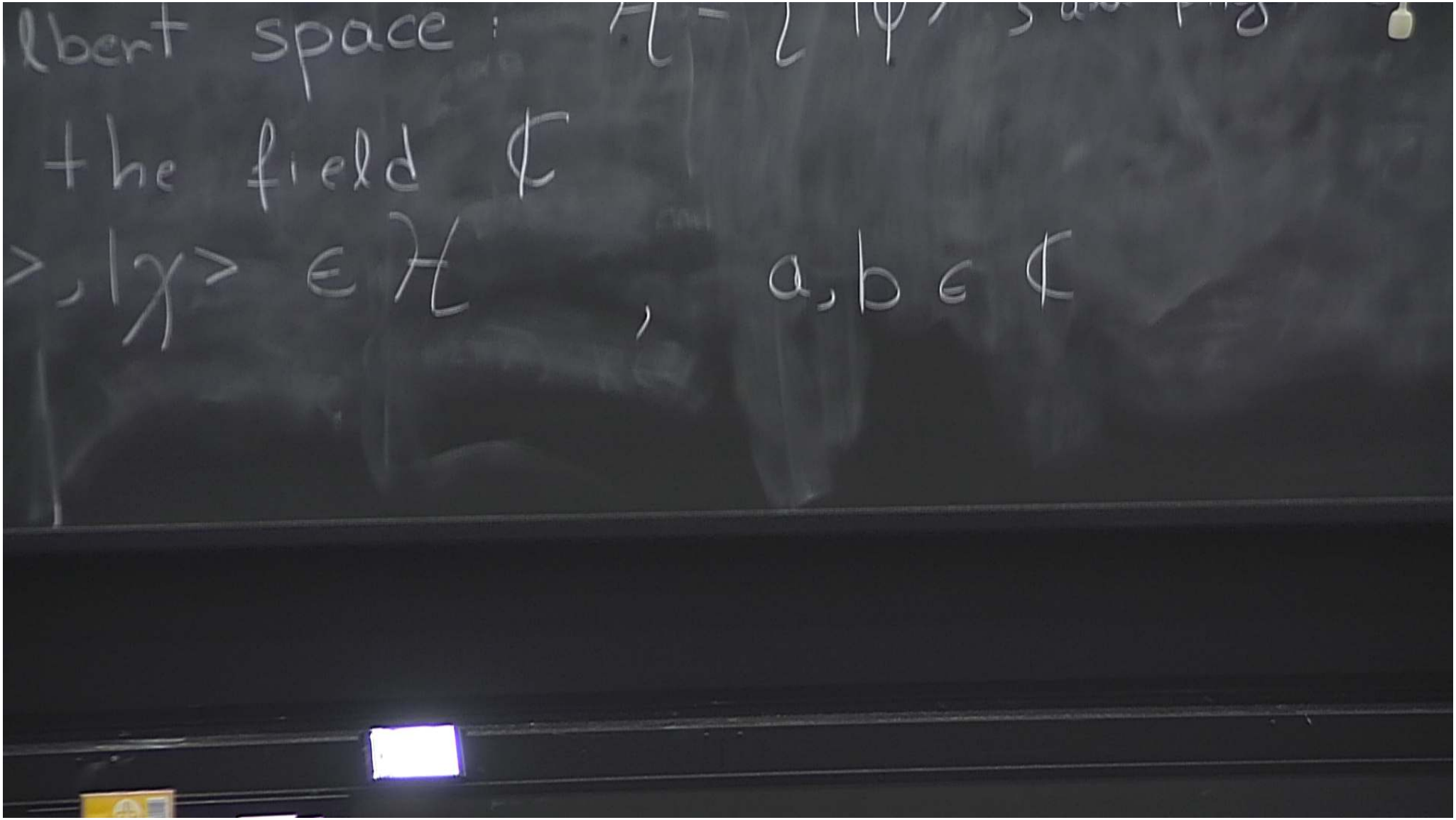

Linear Vector Space
QM: H

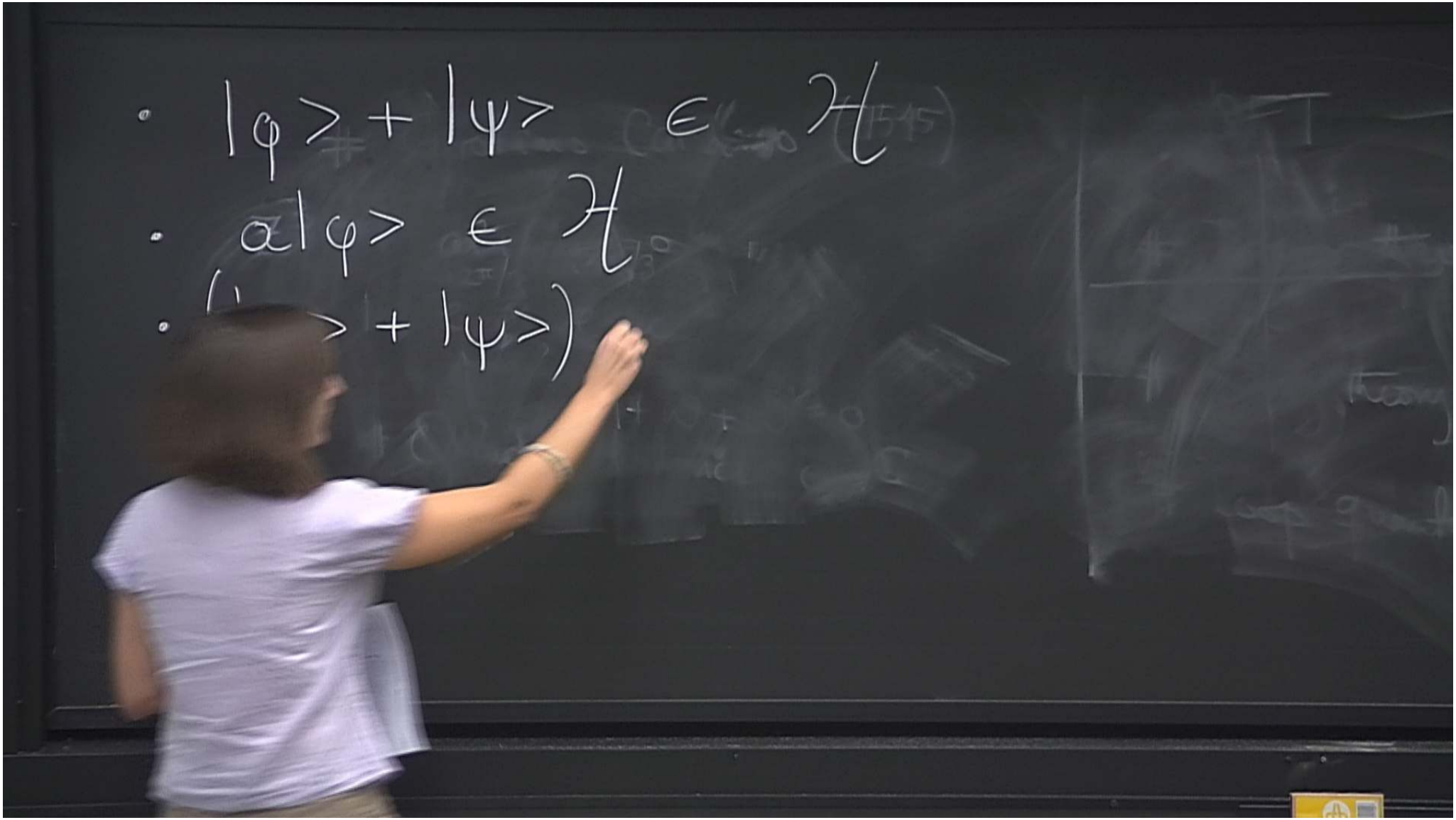
Linear Vector Space
QM: Hilbert space: $\mathcal{H} = \{ |\psi\rangle ;$

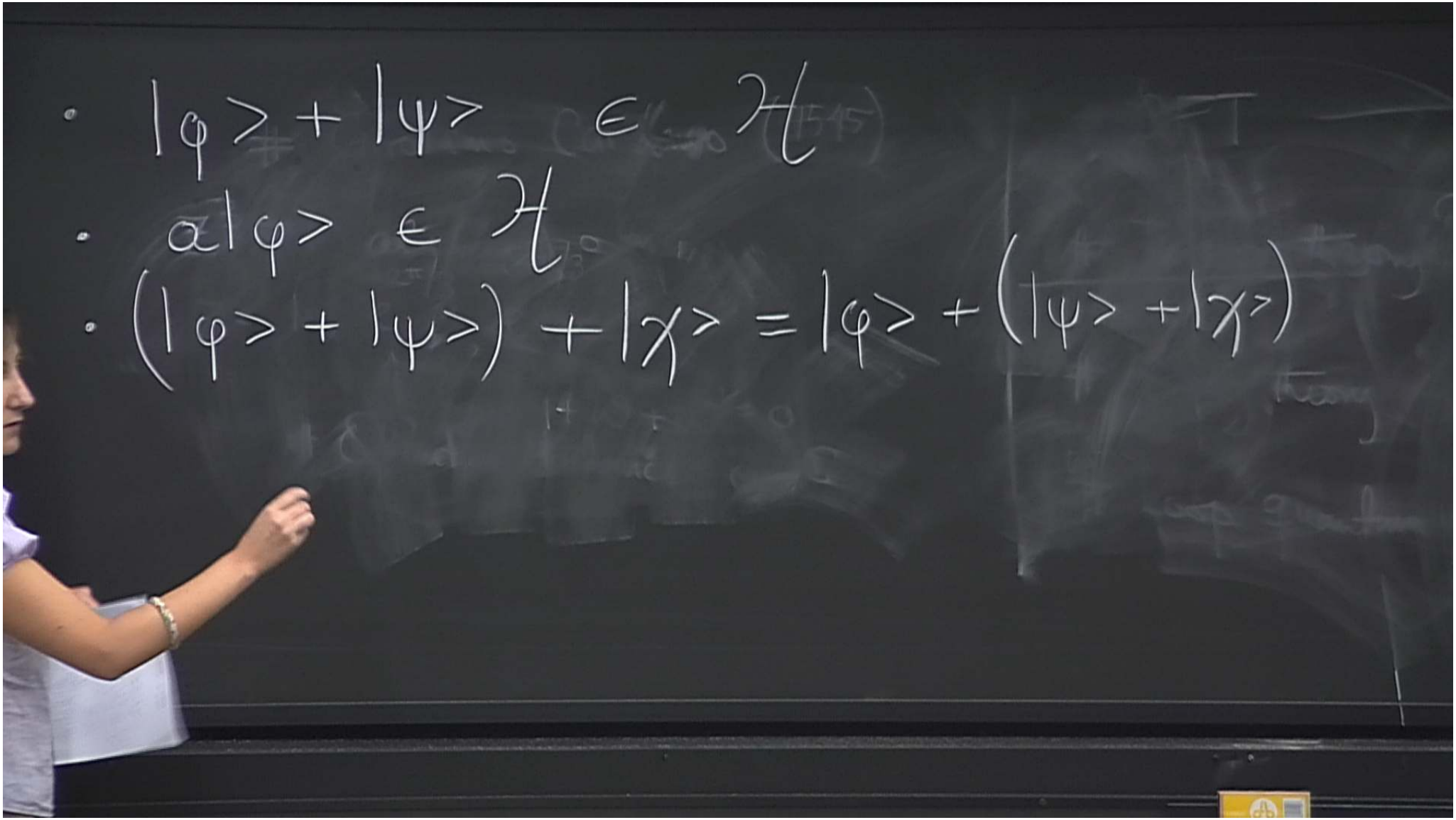

Space
pace : $\mathcal{H} = \{ |\psi\rangle ; \text{all physical states} \}$

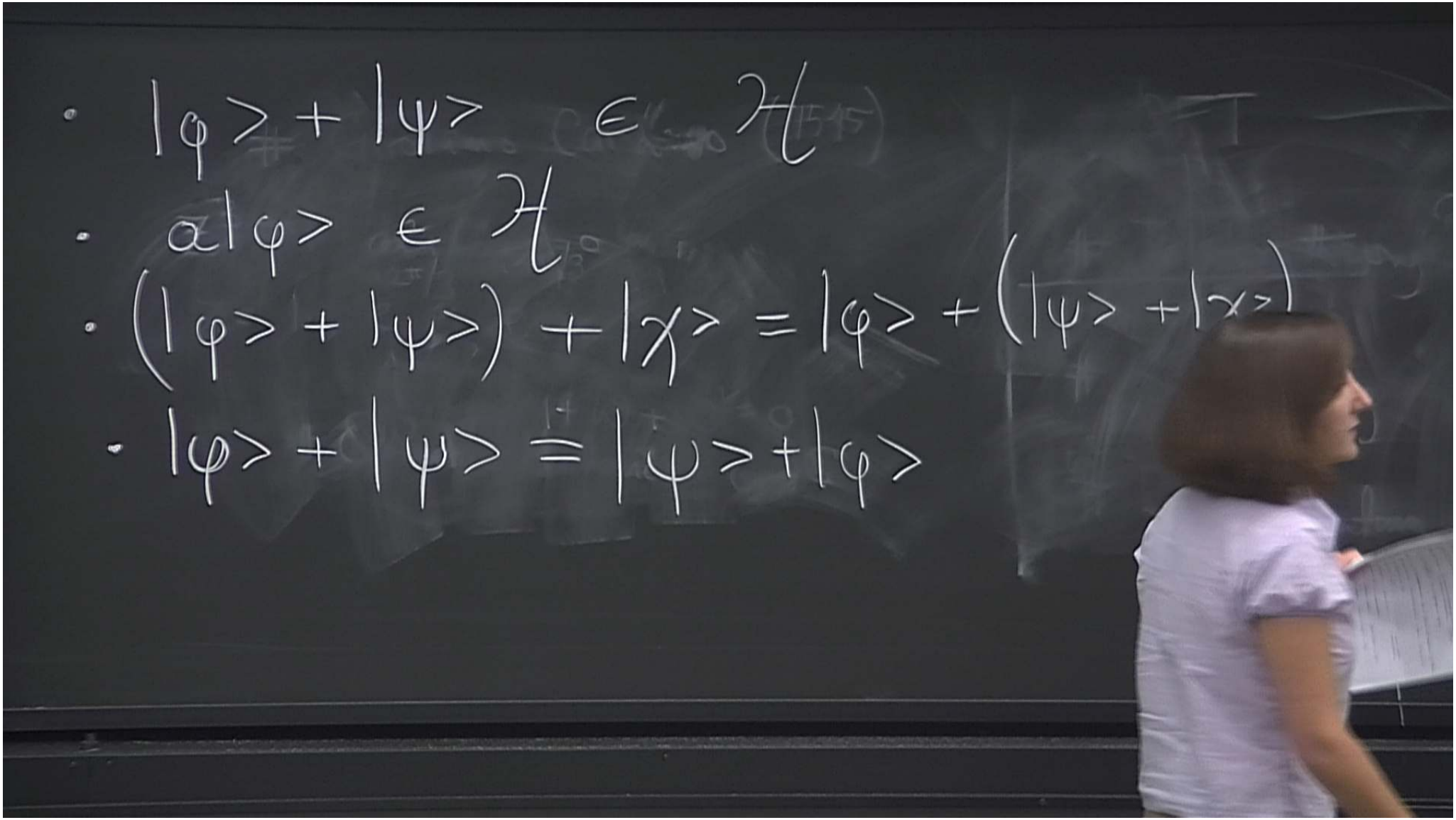

inear Vector Space
M: Hilbert space: $\mathcal{H} = \{ |\psi\rangle$; all ph
over the field $\mathbb{C}$

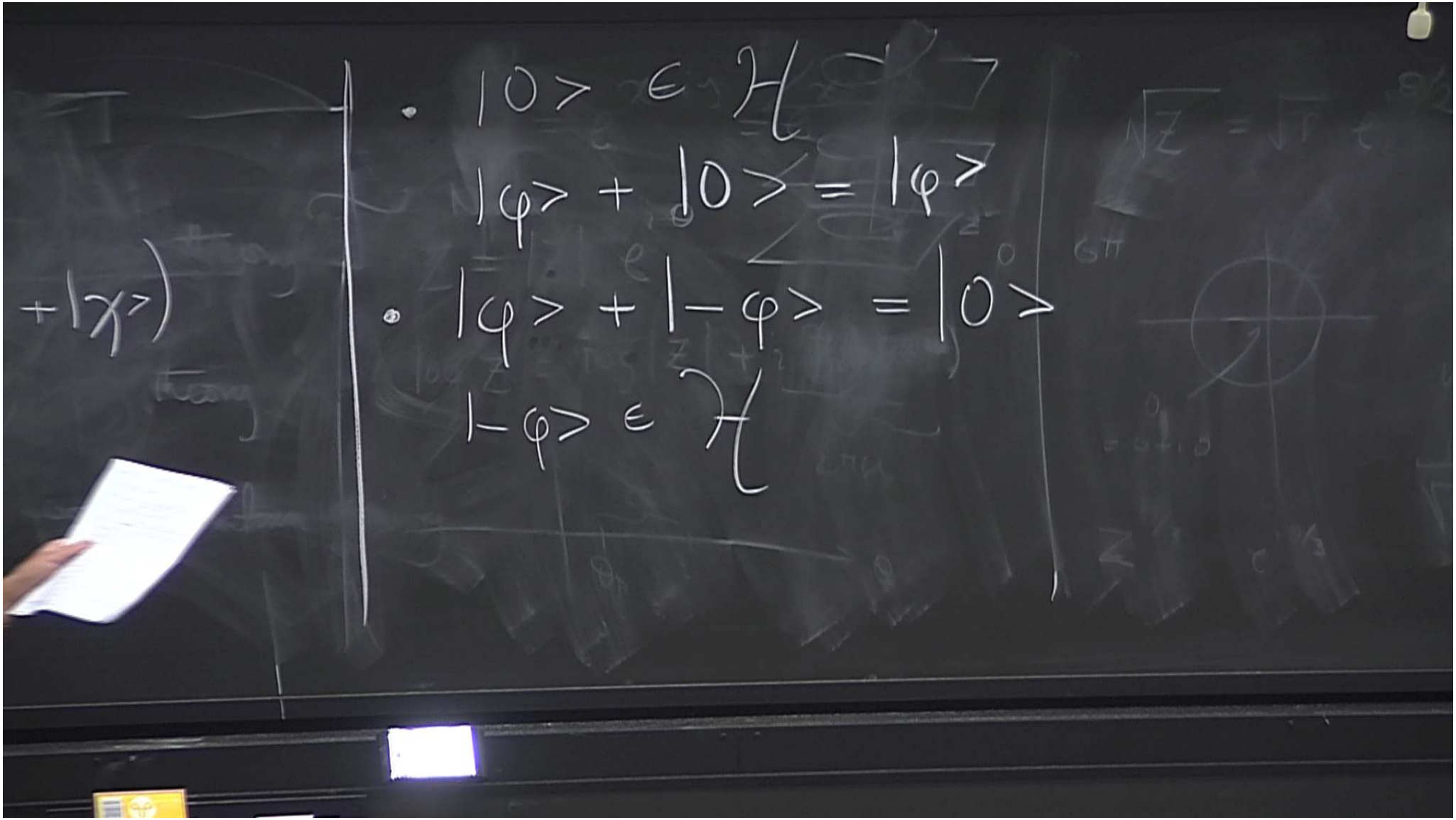

Linear Vector Space
QM: Hilbert space: $\mathcal{H} = \{ |\psi\rangle ; \alpha$
over the field $\mathbb{C}$
$|\varphi\rangle, |\psi\rangle, |\chi\rangle \in \mathcal{H}$,

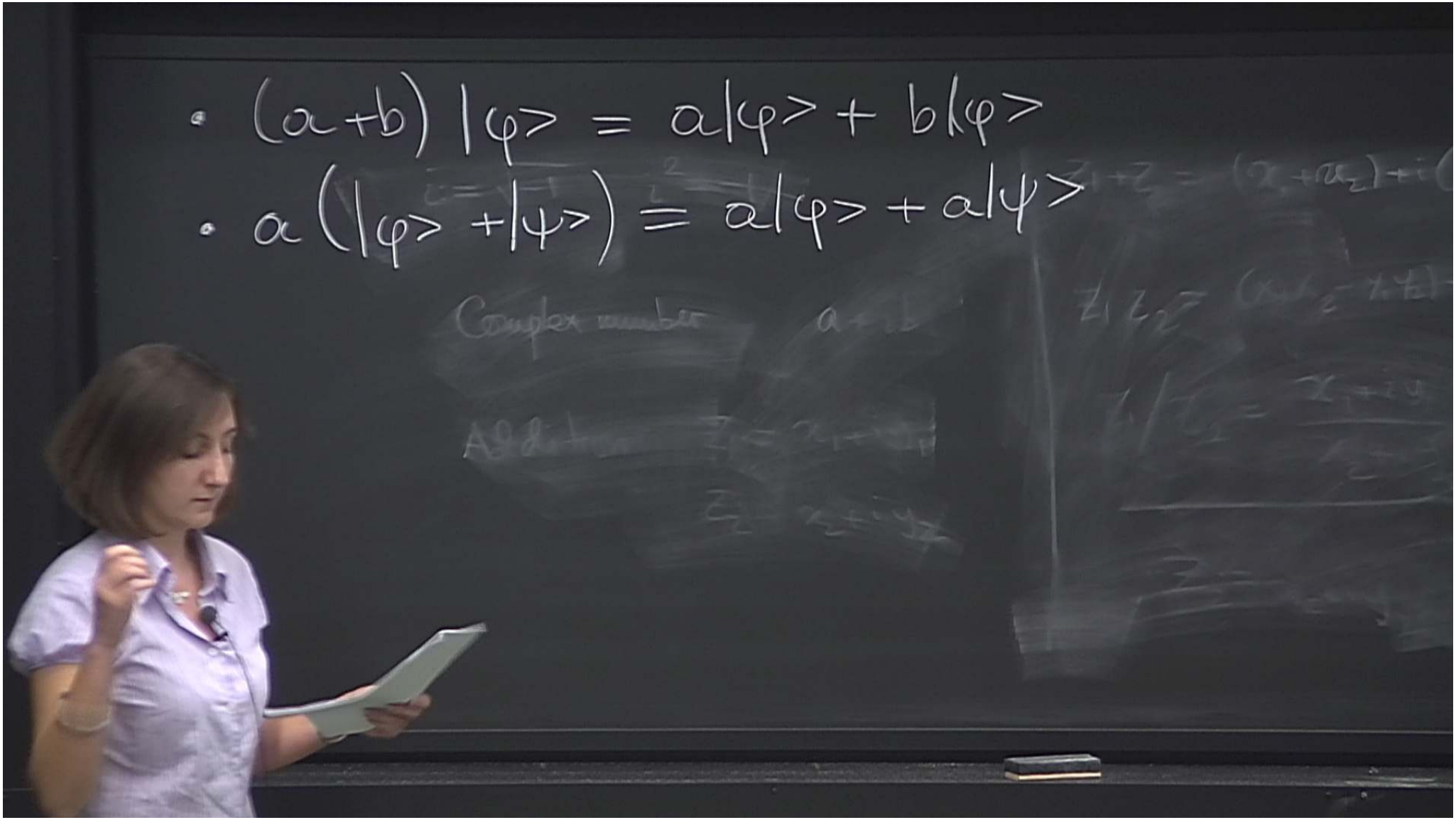

lbert space :
the field ℂ
, |χ> ∈ ℋ , a, b ∈ ℂ

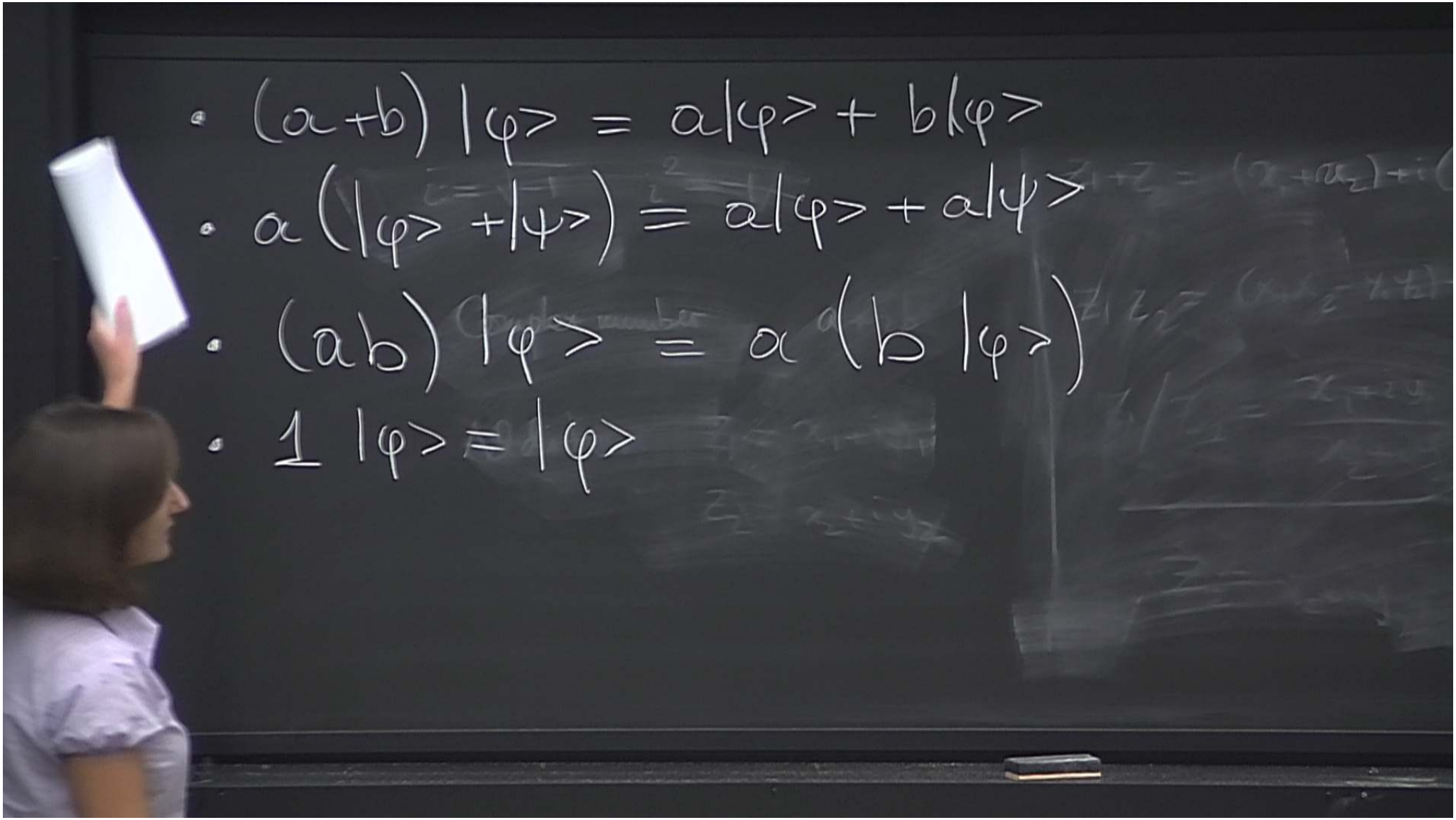

$|\varphi\rangle + |\psi\rangle \in \mathcal{H}$
$\alpha|\varphi\rangle \in \mathcal{H}$
$+ |\psi\rangle)$

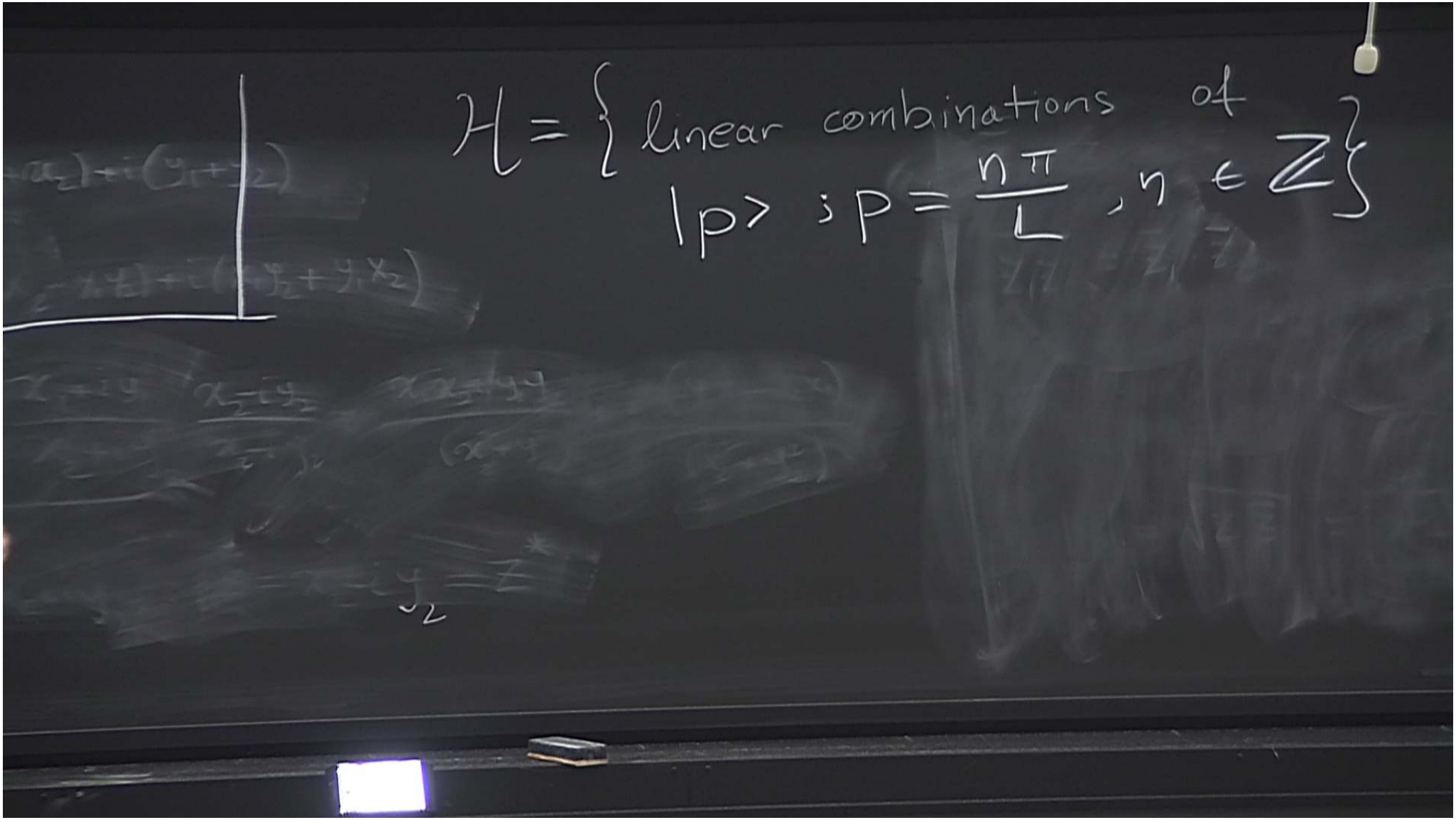

$|\varphi\rangle + |\psi\rangle \in \mathcal{H}$
$a|\varphi\rangle \in \mathcal{H}$
$(|\varphi\rangle + |\psi\rangle) + |\chi\rangle = |\varphi\rangle + (|\psi\rangle + |\chi\rangle)$

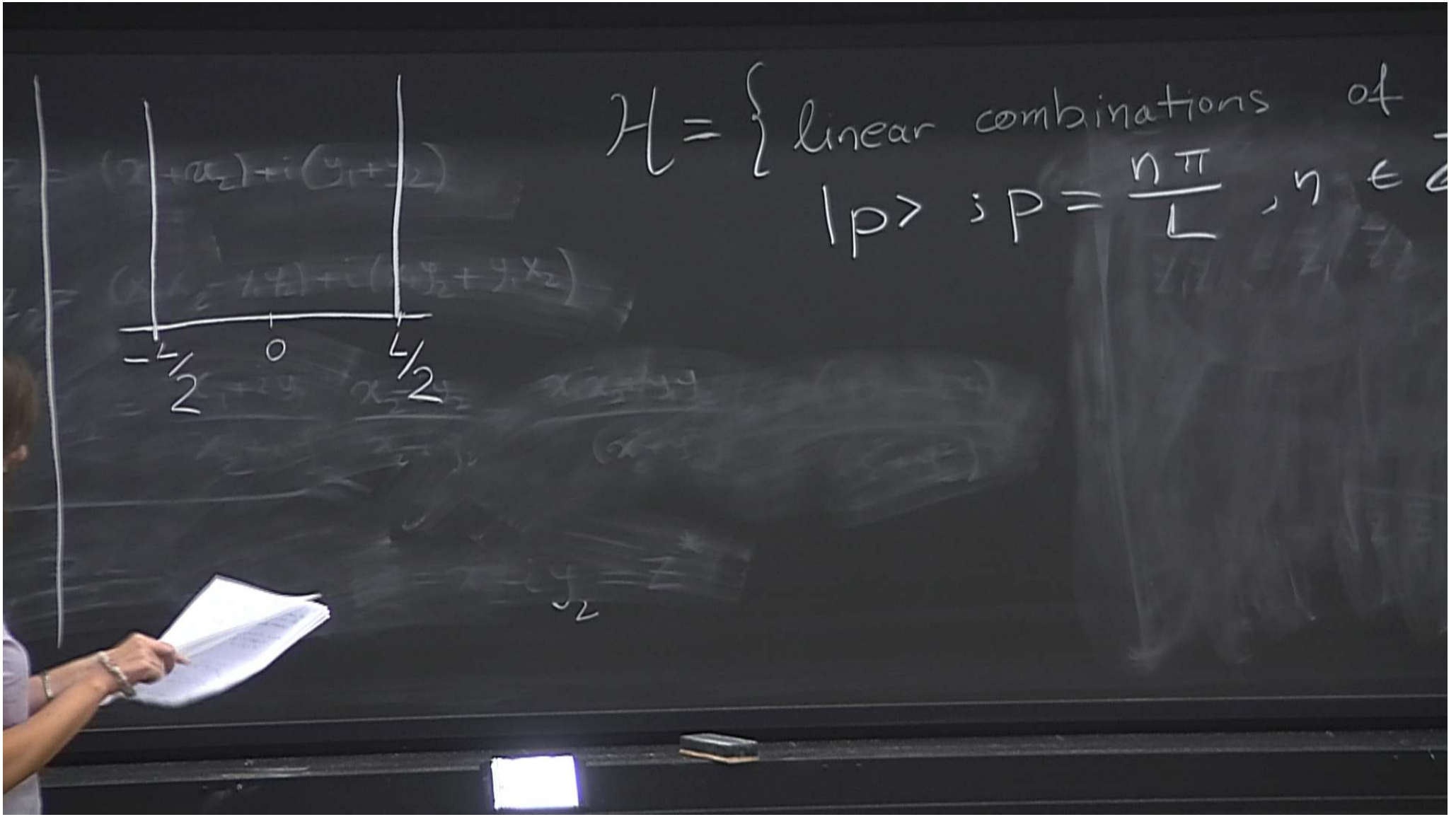

$|\varphi\rangle + |\psi\rangle \in \mathcal{H}$
$a|\varphi\rangle \in \mathcal{H}$
$(|\varphi\rangle + |\psi\rangle) + |\chi\rangle = |\varphi\rangle + (|\psi\rangle + |\chi\rangle)$
$|\varphi\rangle + |\psi\rangle = |\psi\rangle + |\varphi\rangle$

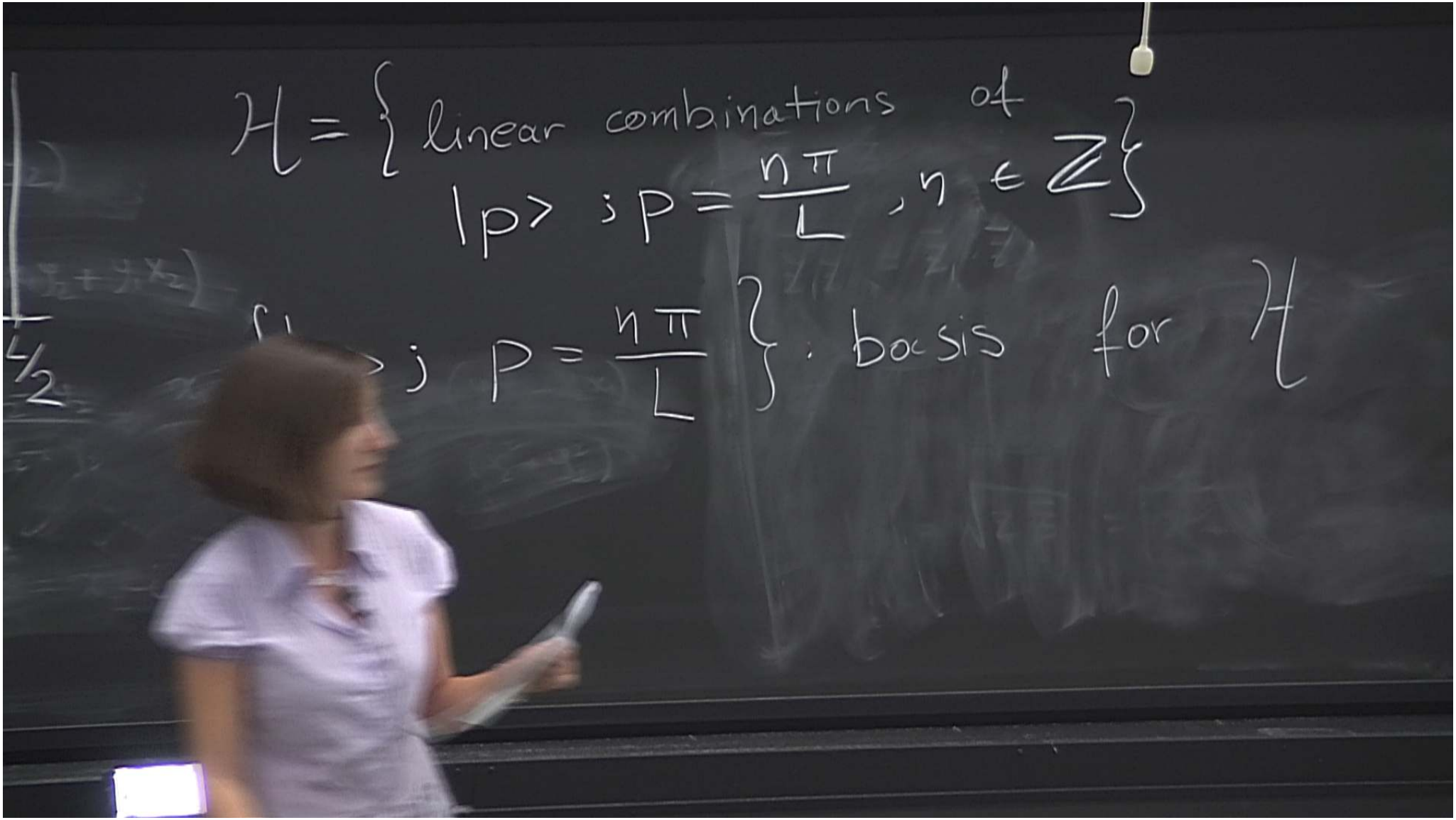

$|0\rangle \in \mathcal{H}$
$|\varphi\rangle + |0\rangle = |\varphi\rangle$
$|\varphi\rangle + |1-\varphi\rangle = |0\rangle$
$|1-\varphi\rangle \in \mathcal{H}$
$+|\chi\rangle)$


• $(a+b)\,|\varphi\rangle = a|\varphi\rangle + b|\varphi\rangle$
• $a\,(|\varphi\rangle + |\psi\rangle) = a|\varphi\rangle + a|\psi\rangle$


(a+b)|φ> = a|φ> + b|φ>
a(|φ> + |ψ>) = a|φ> + a|ψ>
(ab)|φ> = a(b|φ>)
1|φ> = |φ>

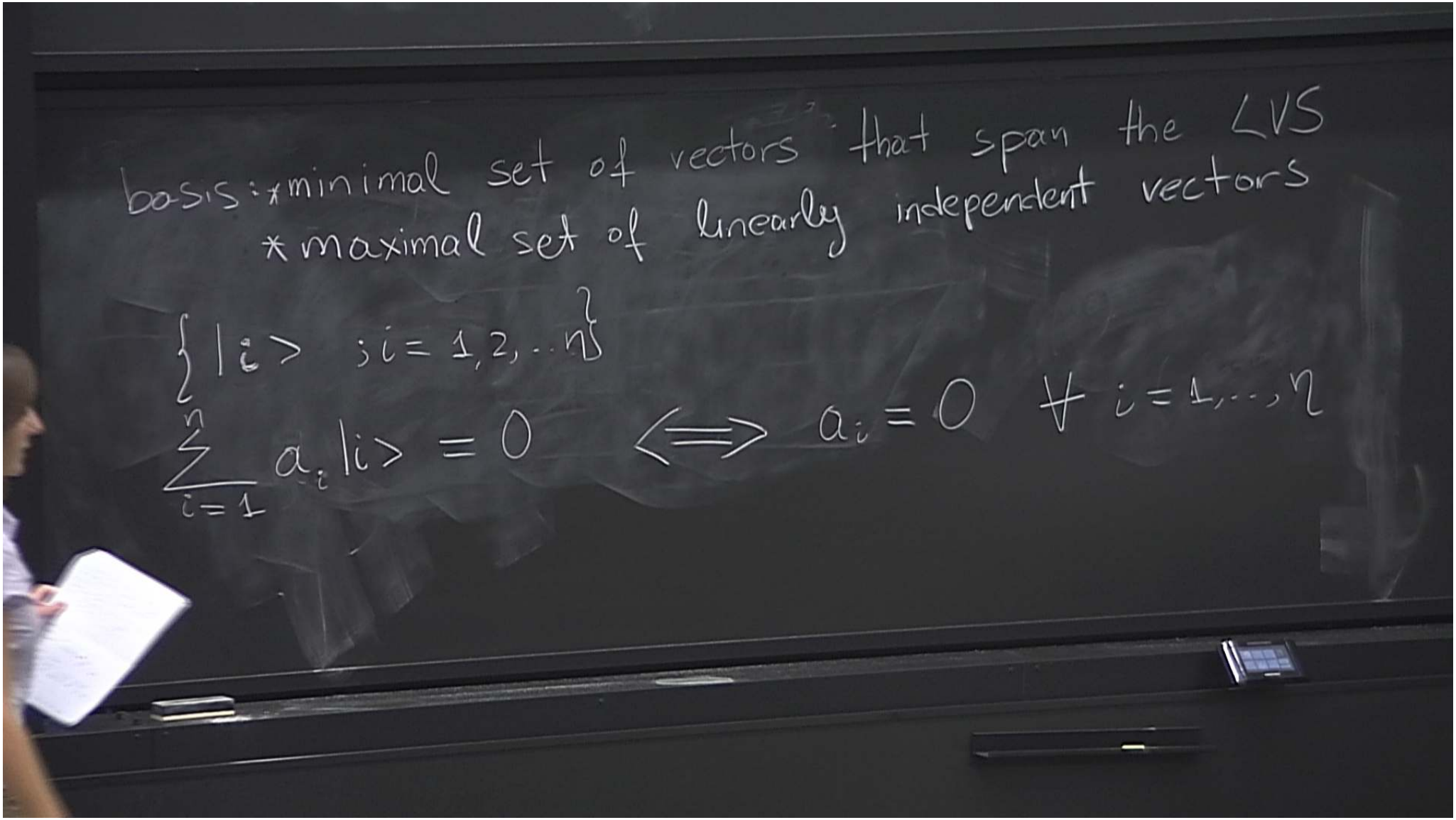

$\mathcal{H} = \{$ linear combinations of
$|p\rangle \; ; p = \frac{n\pi}{L}, n \in \mathbb{Z}\}$

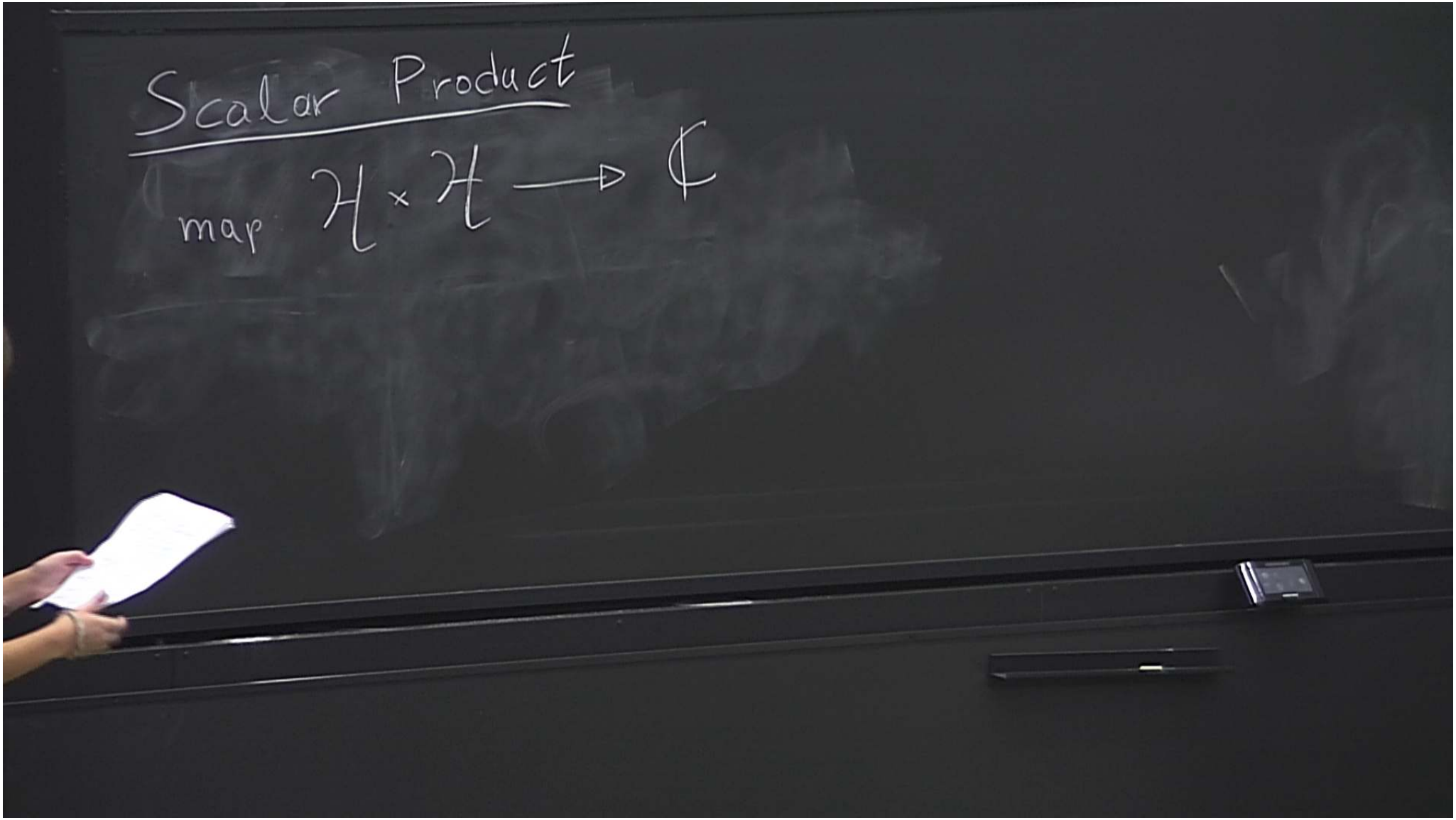

$\mathcal{H} = \{$ linear combinations of
$|p\rangle$; $p = \frac{n\pi}{L}$, $n \in$
$-L/2$
0
$L/2$

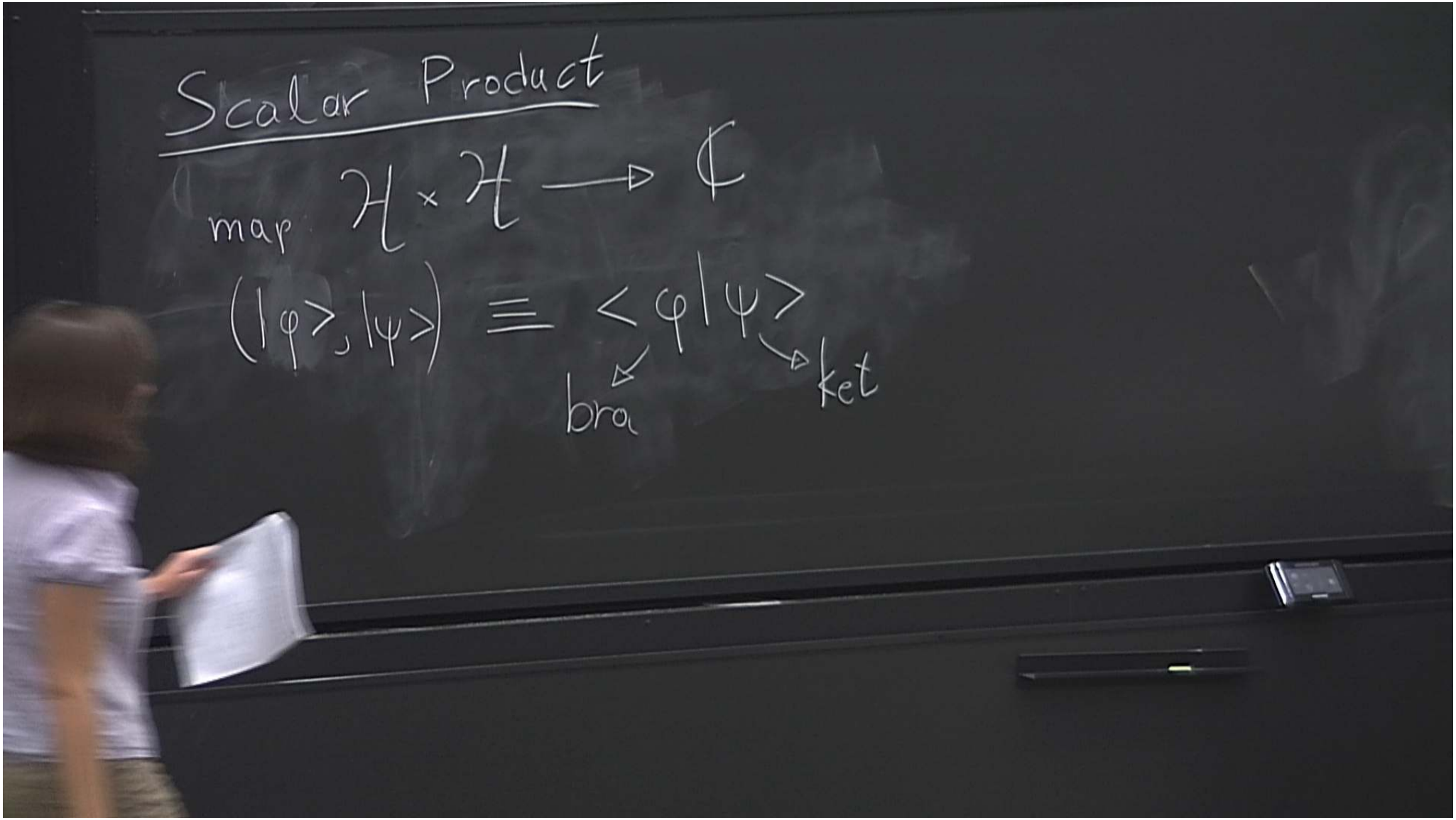

$\mathcal{H} = \{$ linear combinations of
$|p\rangle \; ; p = \frac{n\pi}{L}, n \in \mathbb{Z}\}$
$\{|p\rangle \; ; p = \frac{n\pi}{L}\}$: basis for $\mathcal{H}$

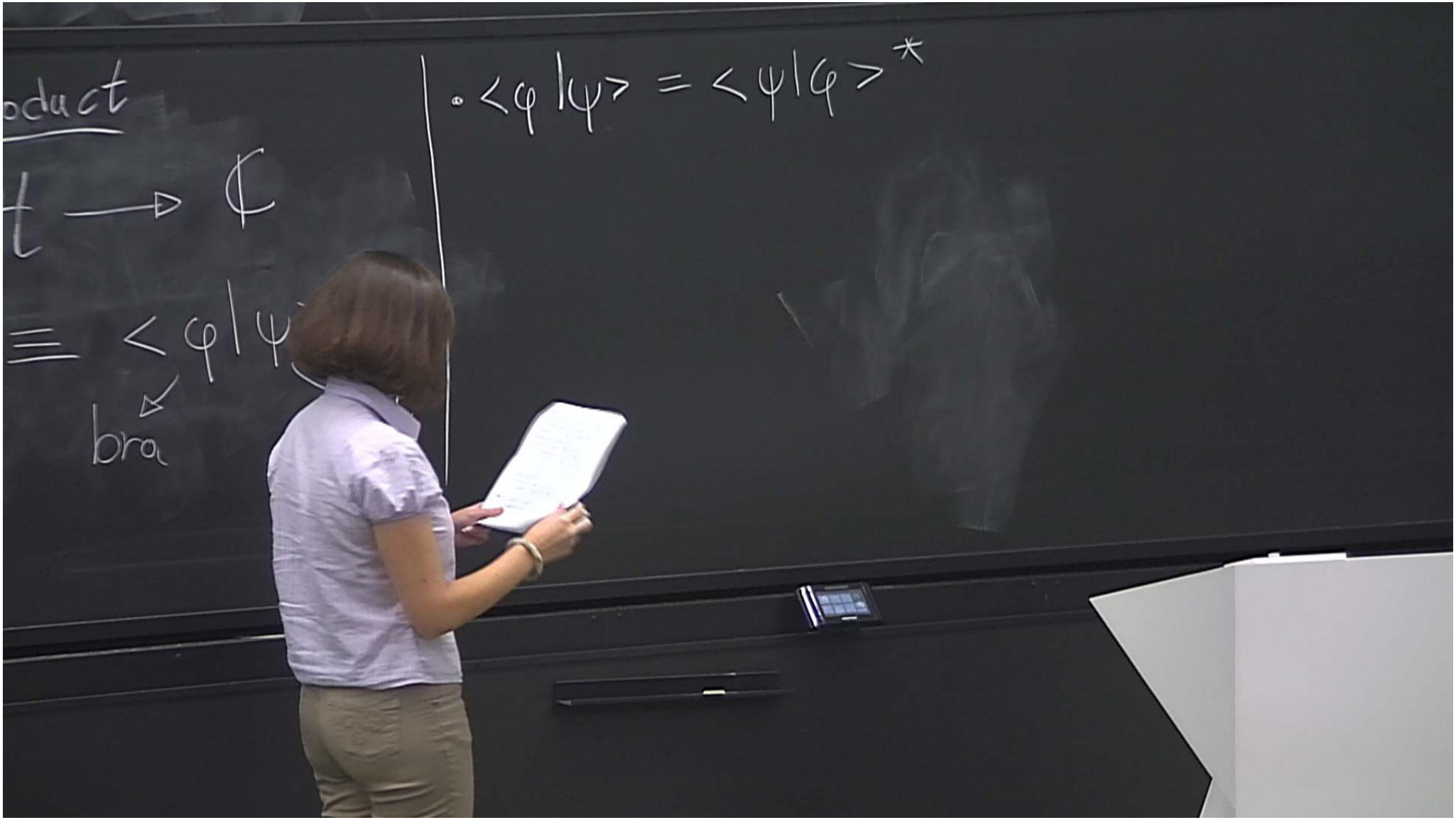
basis: * minimal set of vectors that span the LVS
* maximal set of linearly independent vectors

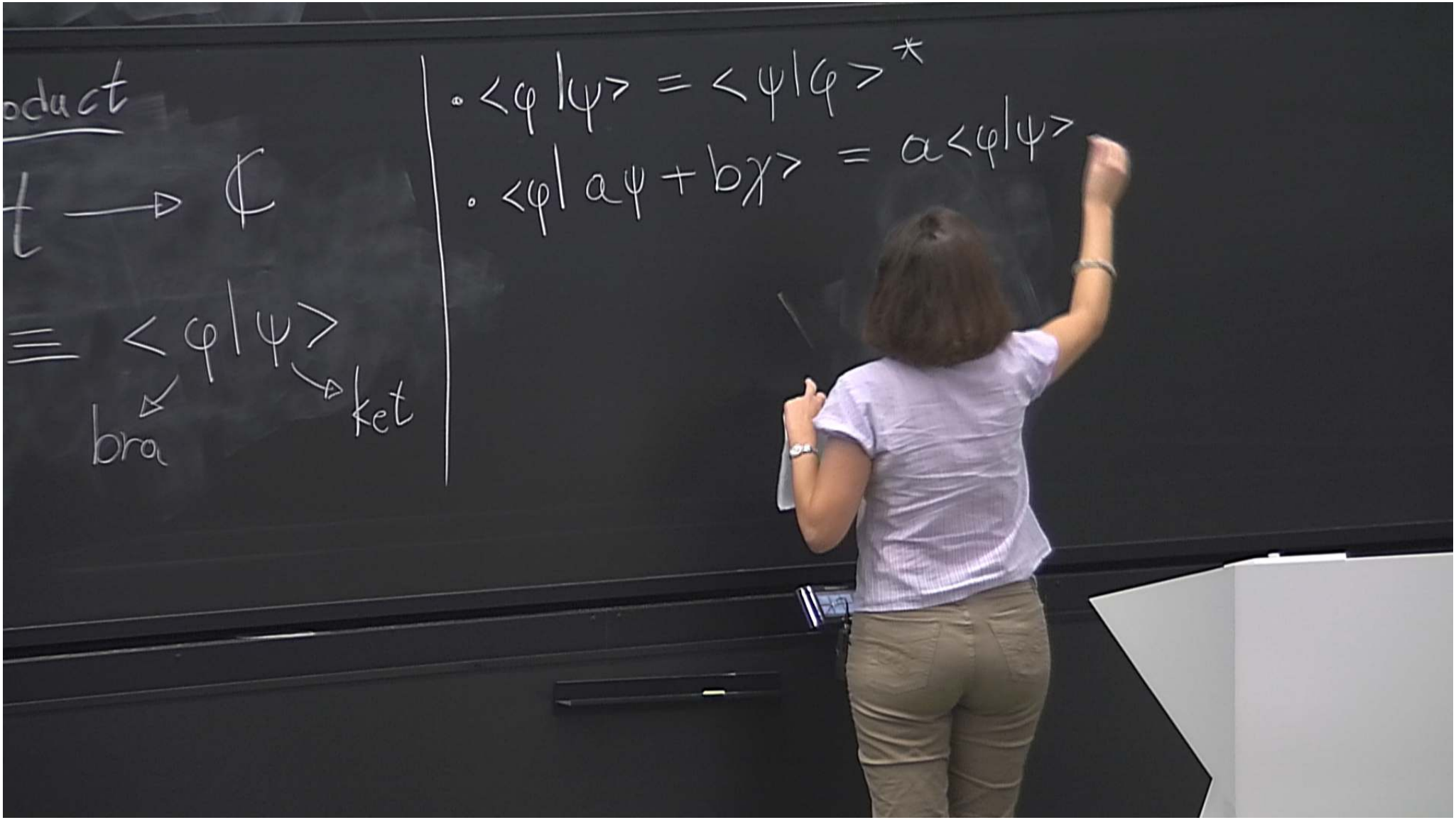
$$\{|i\rangle \;; i = 1, 2, \ldots n\}$$

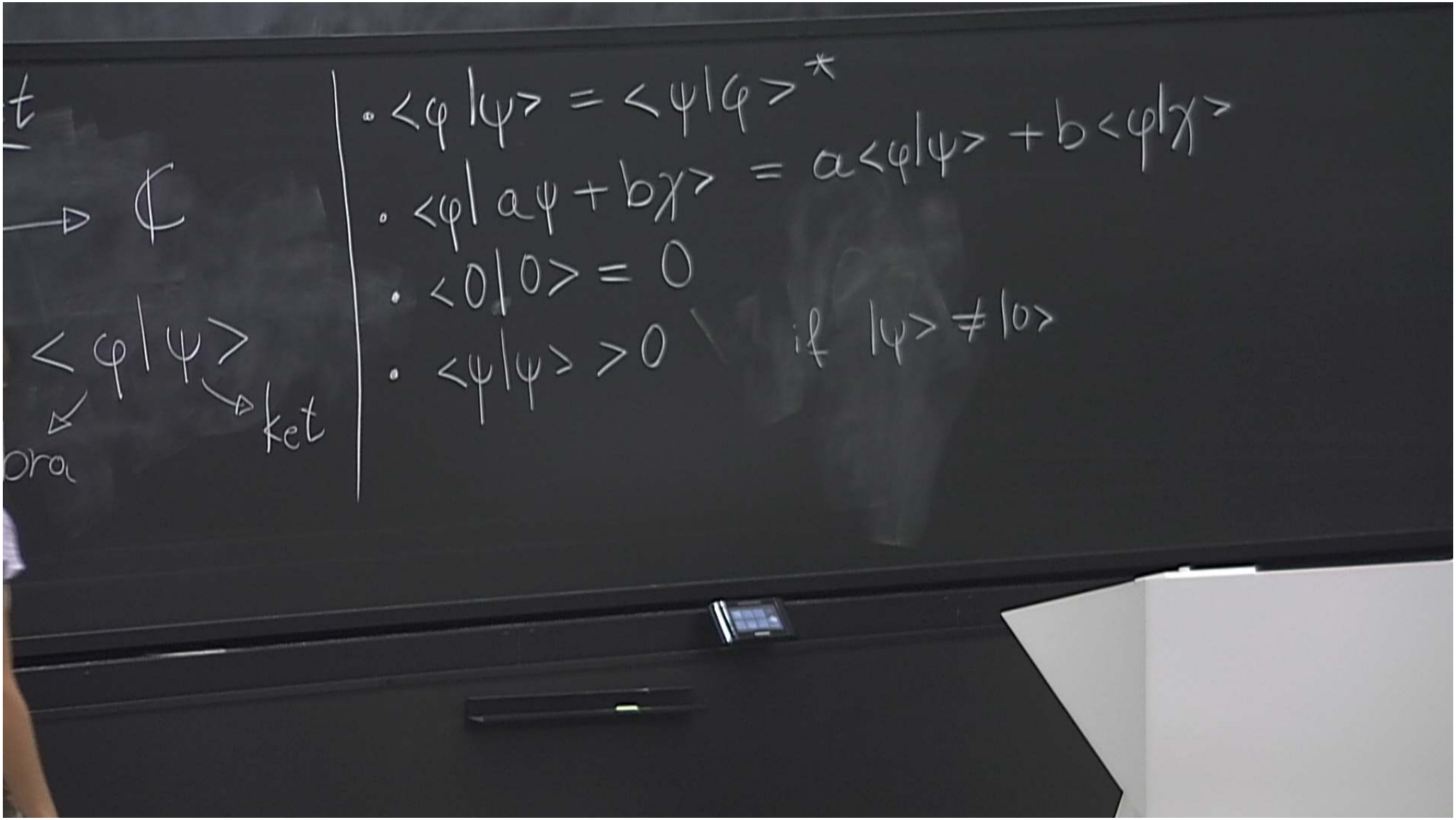

basis: * minimal set of vectors that span the LVS
* maximal set of linearly independent vectors
$\{|i\rangle \; ; i = 1, 2, .. n\}$
$\sum_{i=1}^{n} a_i |i\rangle = 0 \iff a_i = 0 \;\; \forall \; i = 1, .., n$

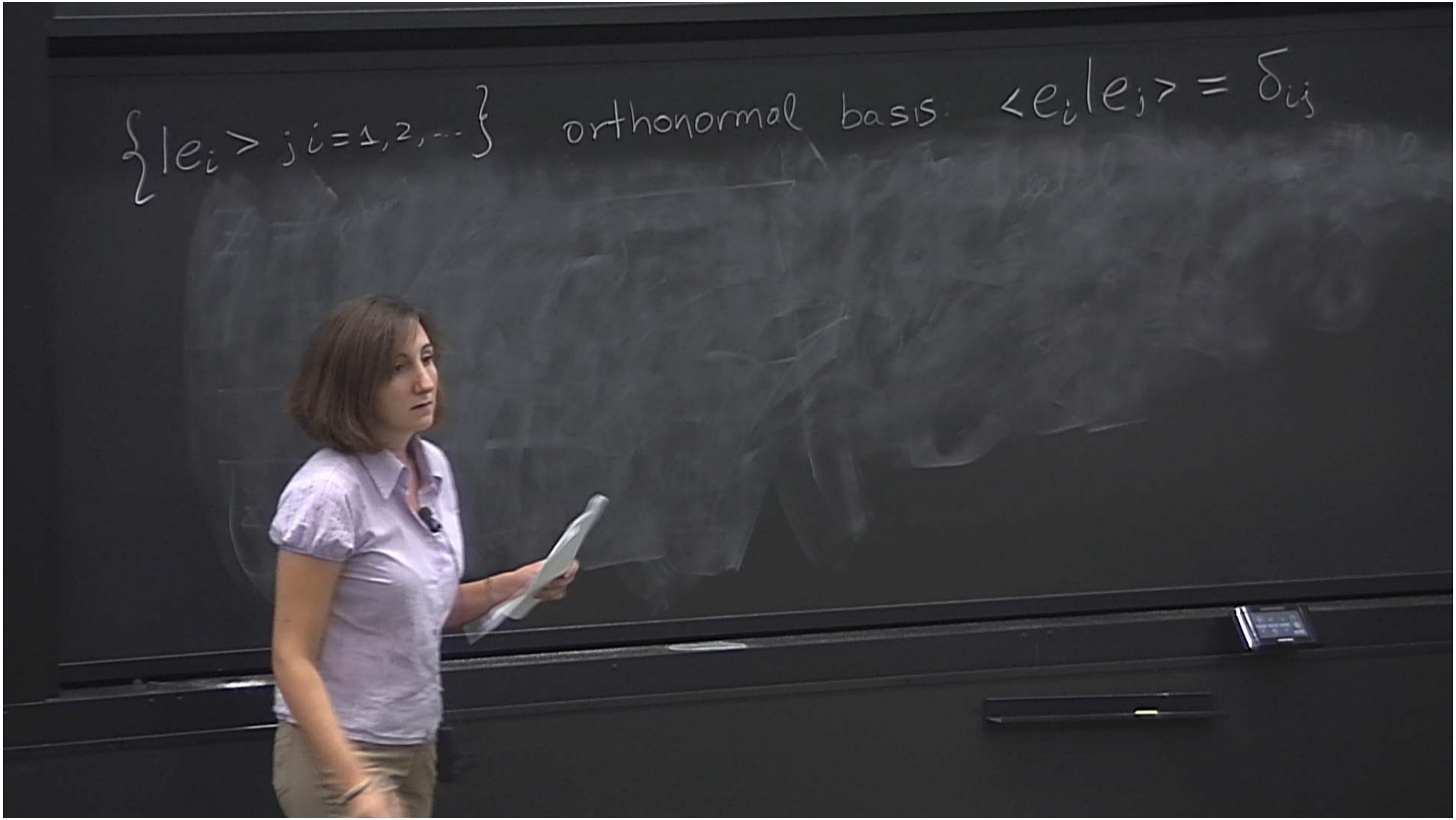

Scalar Product
map $\mathcal{H} \times \mathcal{H} \longrightarrow \mathbb{C}$

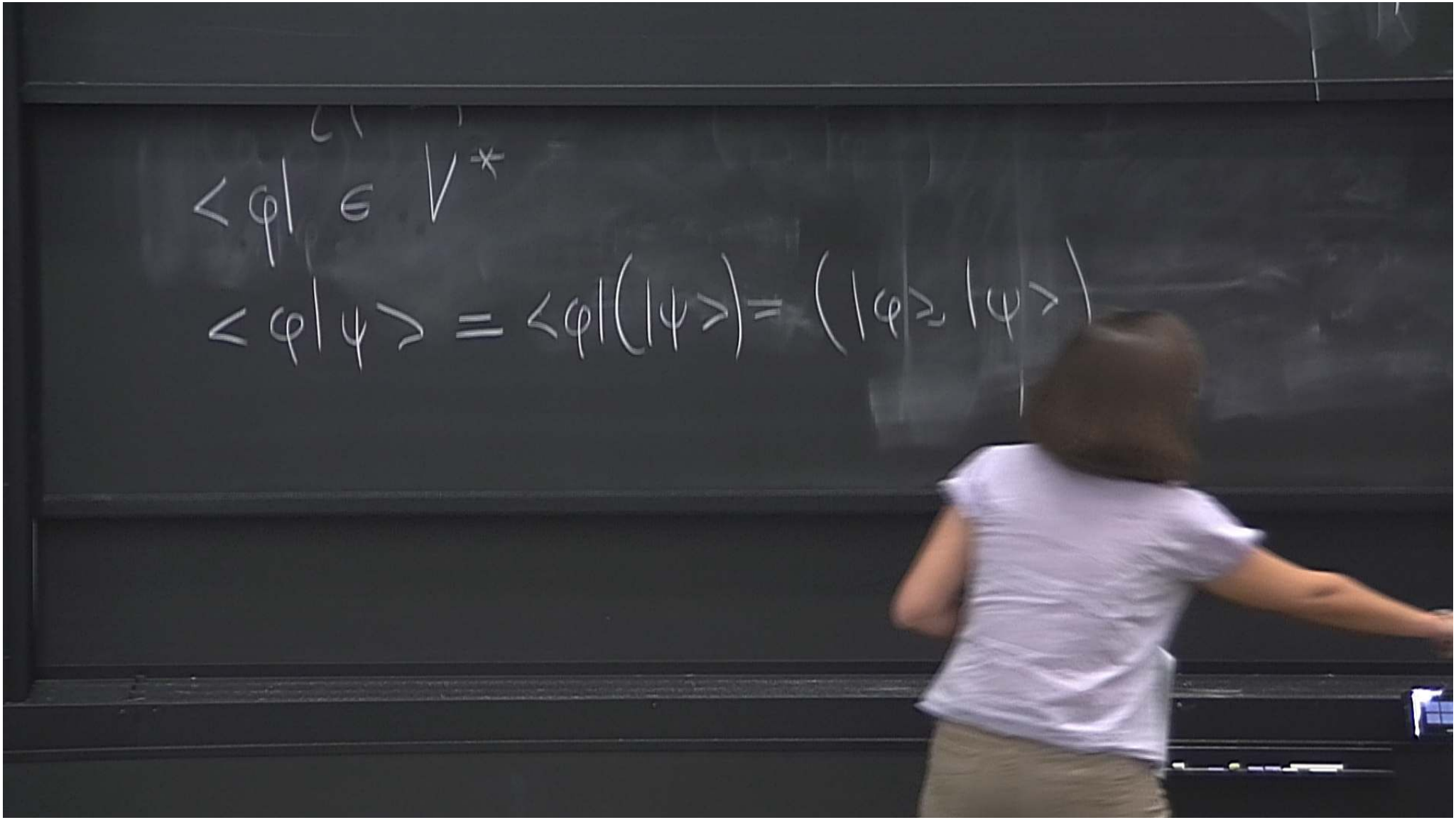

Scalar Product
map $\mathcal{H} \times \mathcal{H} \longrightarrow \mathbb{C}$
$(|\varphi\rangle, |\psi\rangle) \equiv \langle\varphi|\psi\rangle$
bra
ket

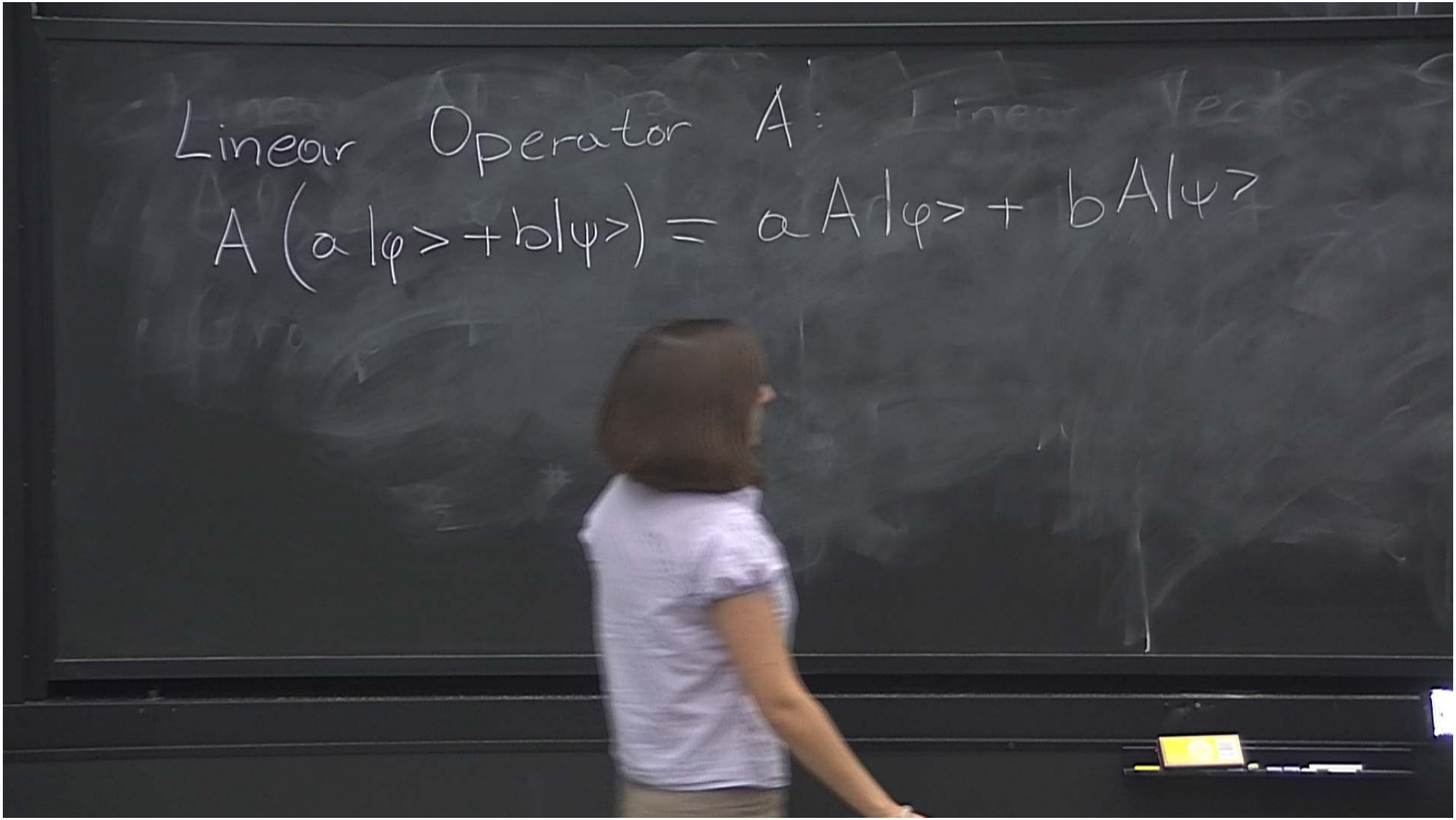

oduct
$\mathbb{C}$
$\equiv \langle \varphi | \psi$
bra
$\cdot \langle \varphi | \psi \rangle = \langle \psi | \varphi \rangle^*$

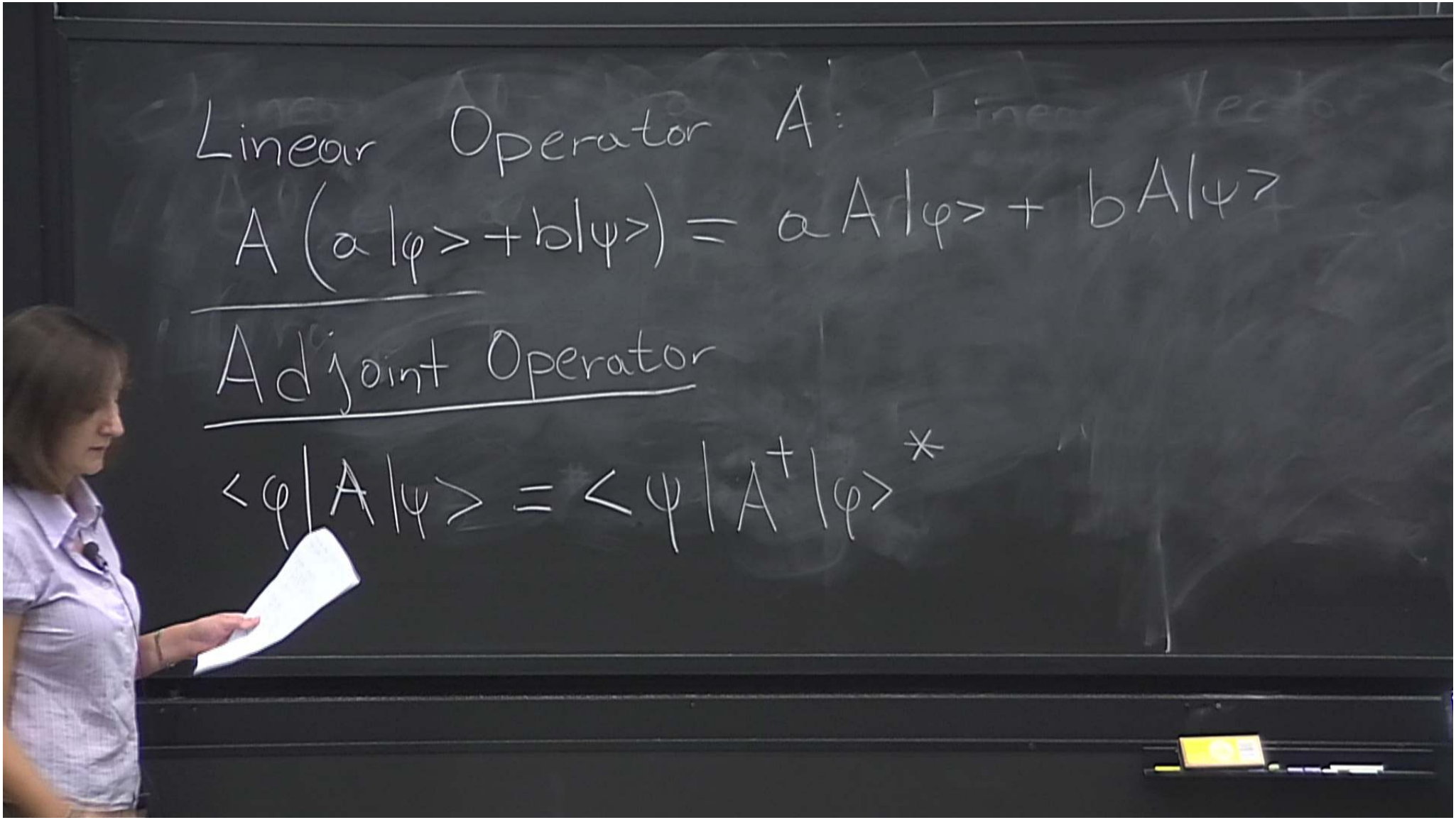

oduct
$\mathbb{C}$
$\equiv \langle\varphi|\psi\rangle$
bra
ket
$\cdot \langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^*$
$\cdot \langle\varphi|a\psi + b\chi\rangle = a\langle\varphi|\psi\rangle$

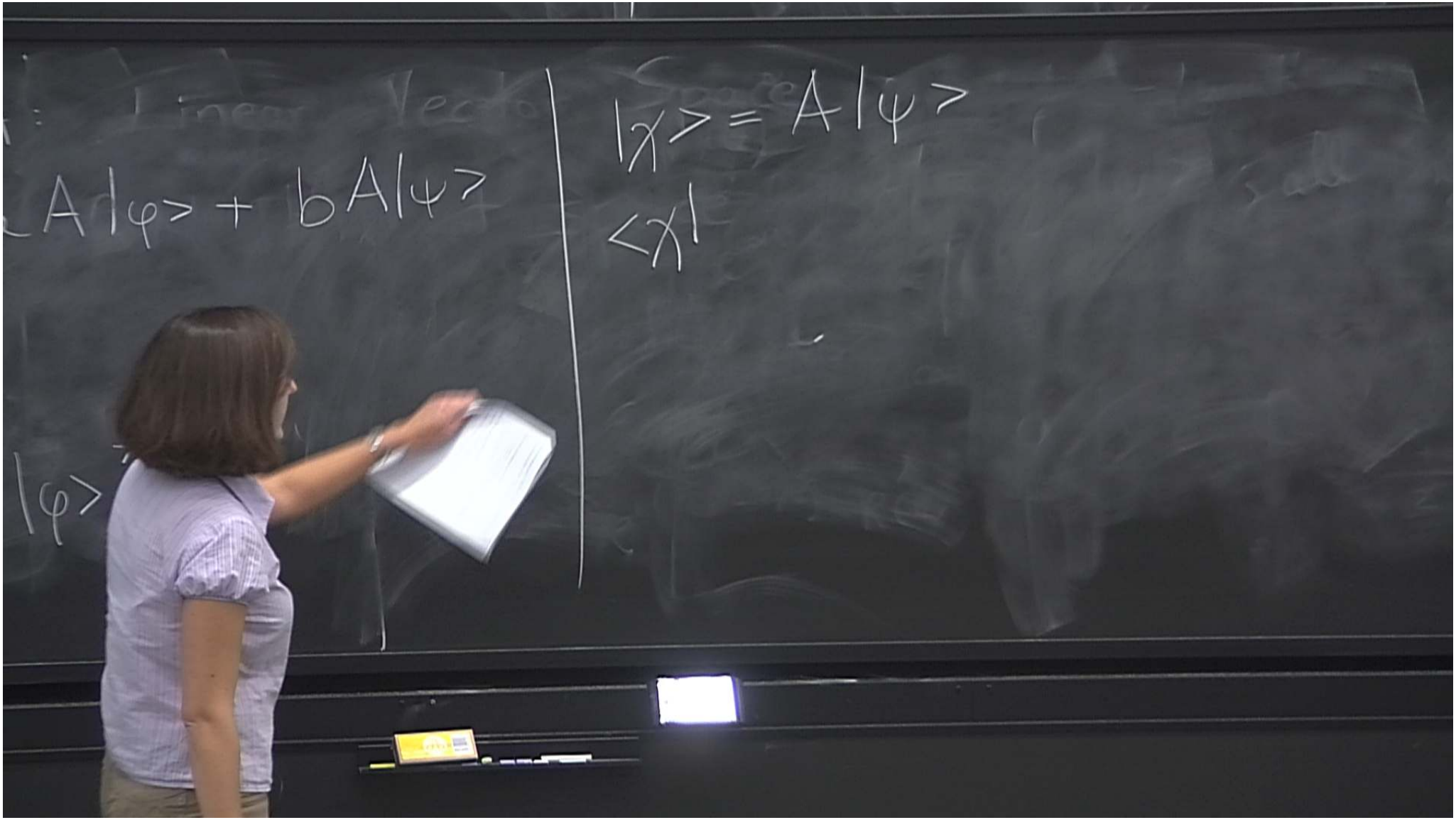

t
→ ℂ
⟨φ|ψ⟩
ket
ora
• ⟨φ|ψ⟩ = ⟨ψ|φ⟩*
• ⟨φ| aψ + bχ⟩ = a⟨φ|ψ⟩ + b⟨φ|χ⟩
• ⟨0|0⟩ = 0
• ⟨ψ|ψ⟩ > 0 if |ψ⟩ ≠ |0⟩

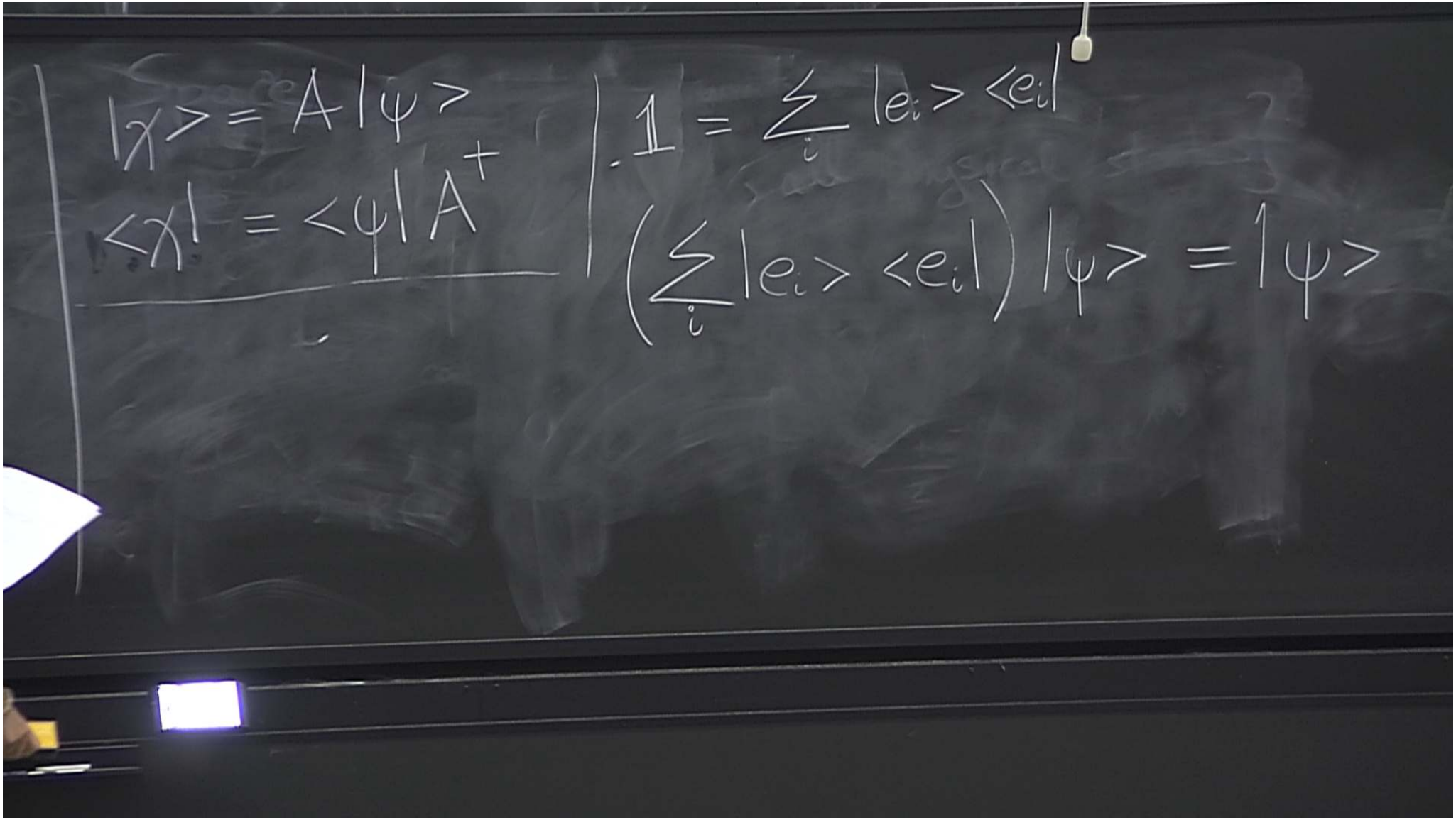

$\{|e_i\rangle \; j \; i=1,2,\ldots\}$ orthonormal basis: $\langle e_i|e_j\rangle = \delta_{ij}$

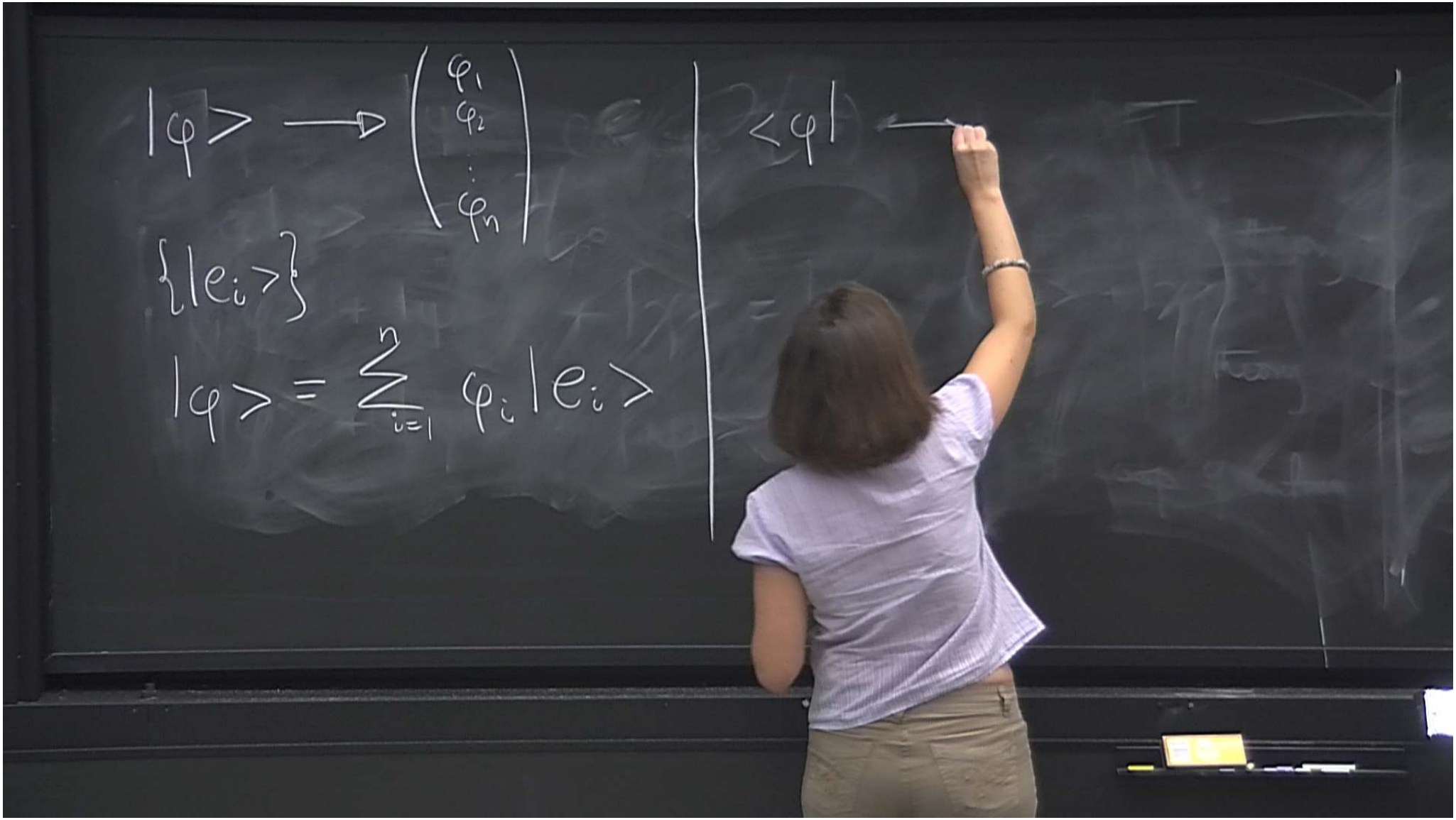

$\langle\varphi| \in V^*$
$\langle\varphi|\psi\rangle = \langle\varphi|(|\psi\rangle) = (|\varphi\rangle, |\psi\rangle)$

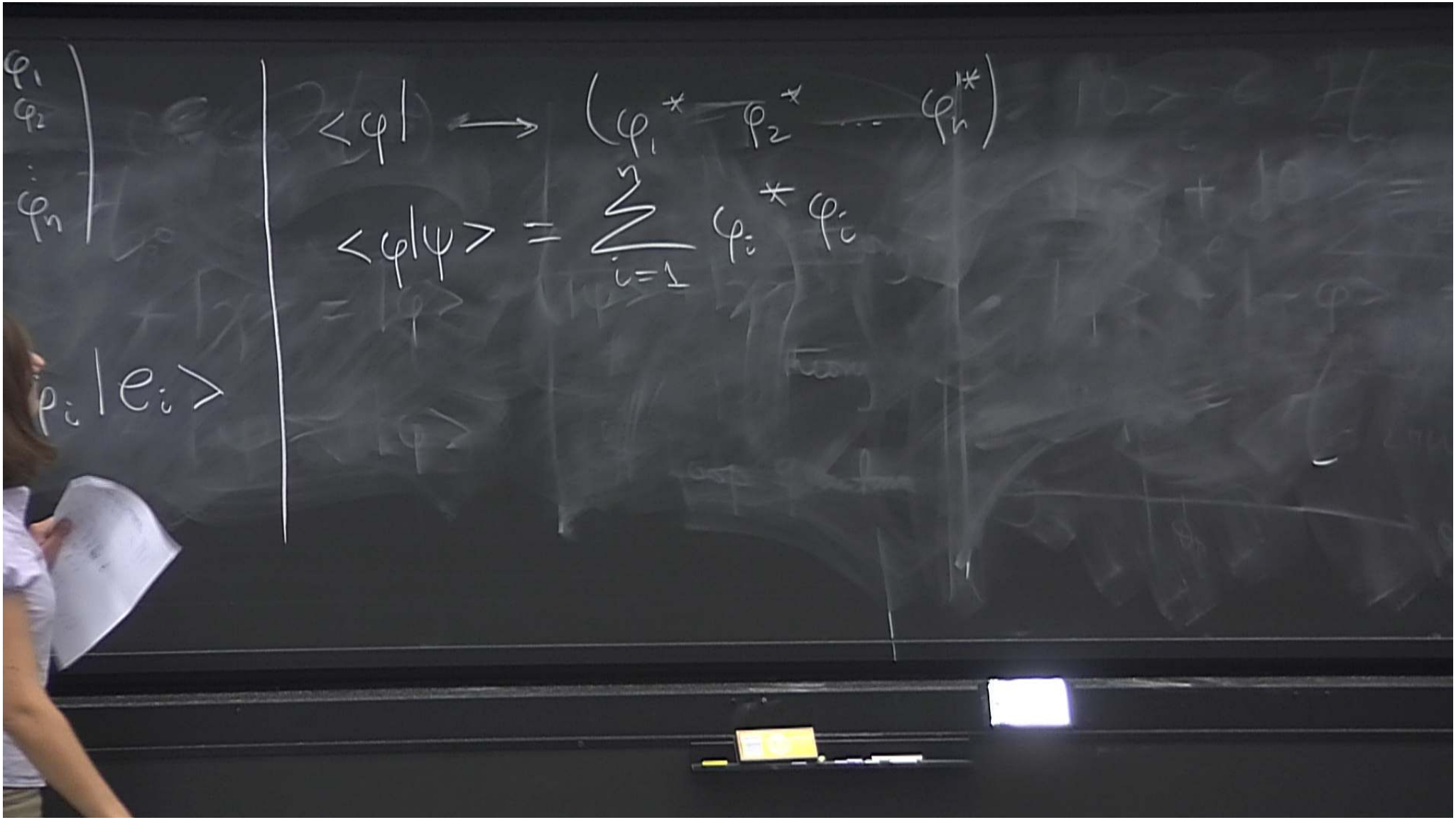

Linear Operator A:
A(a|φ> + b|ψ>) = aA|φ> + bA|ψ>

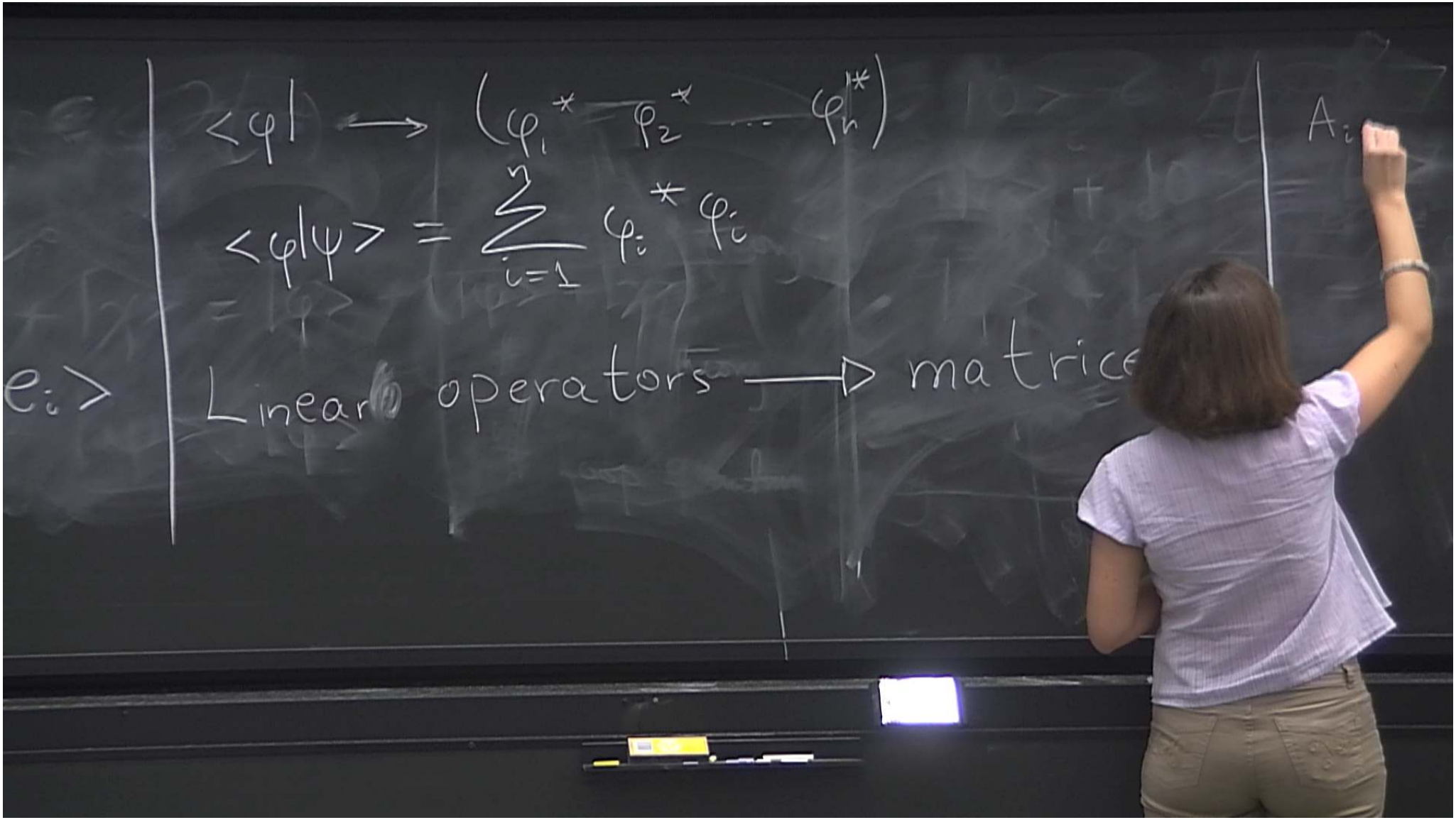

Linear Operator A:
$A(a|\varphi\rangle + b|\psi\rangle) = aA|\varphi\rangle + bA|\psi\rangle$
Adjoint Operator
$\langle\varphi|A|\psi\rangle = \langle\psi|A^{\dagger}|\varphi\rangle^{*}$

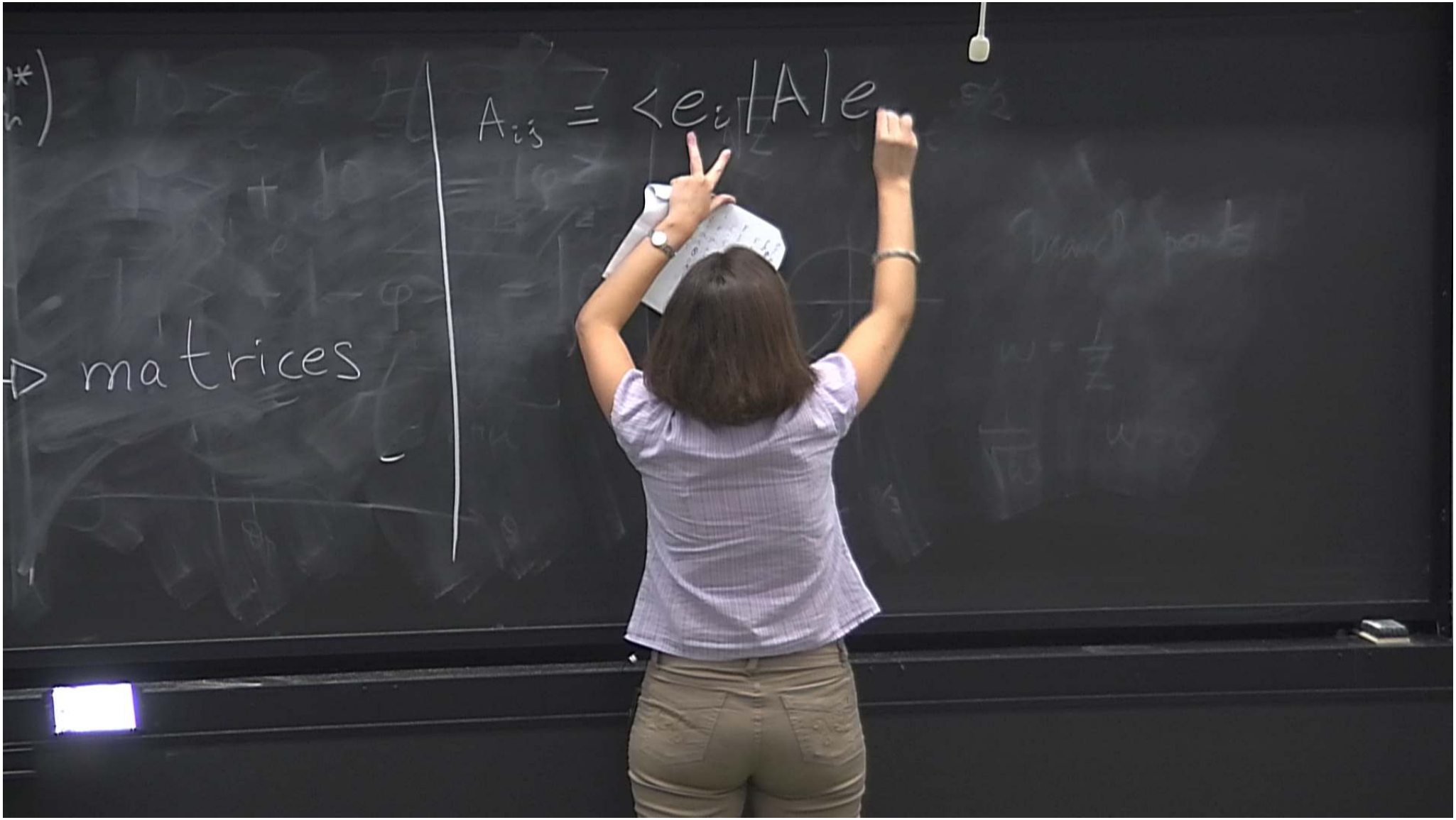

A|φ> + bA|ψ>
|φ>
|χ> = A|ψ>
<χ|

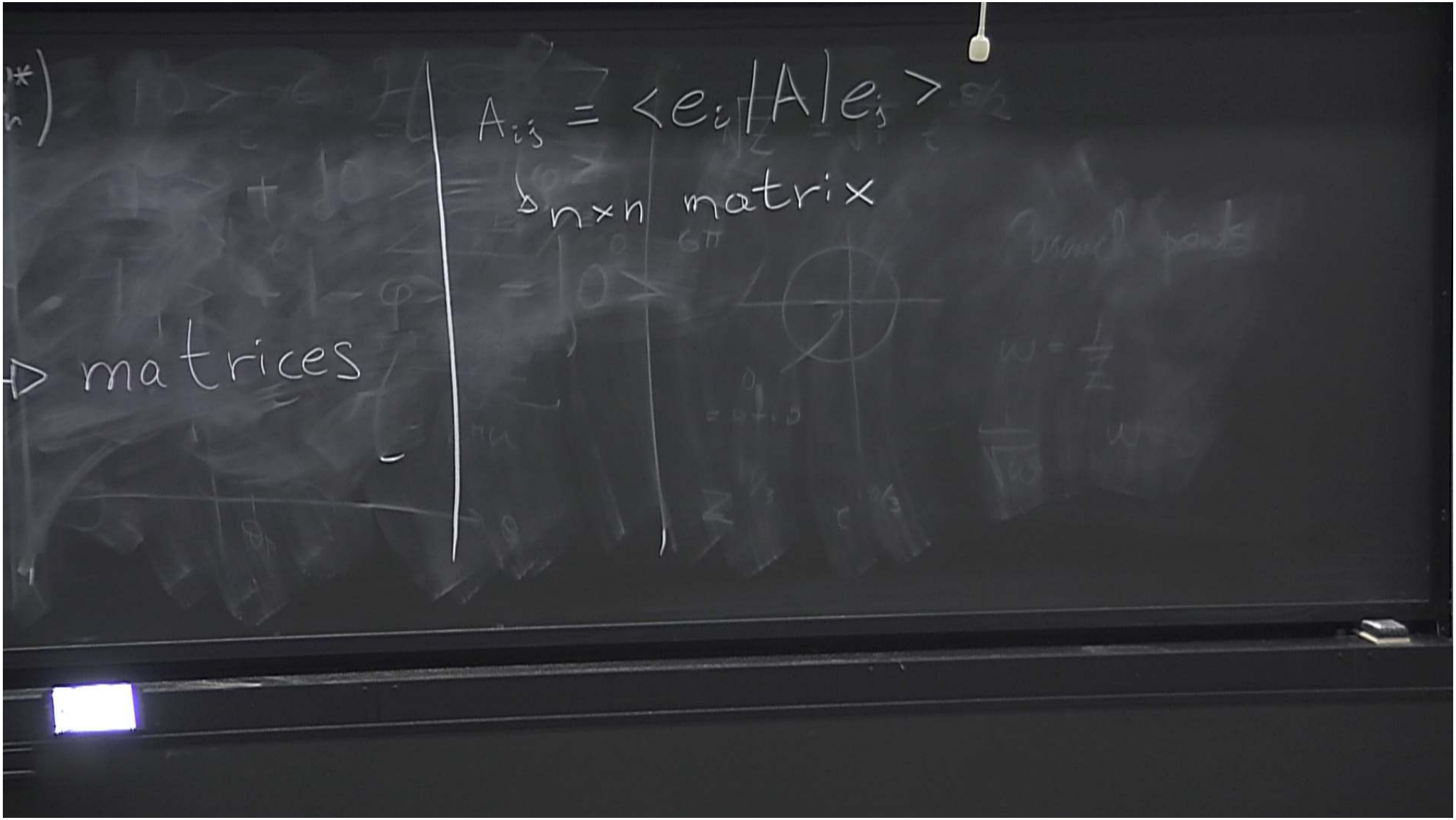

$|\chi\rangle = A|\psi\rangle$
$\langle\chi| = \langle\psi|A^{+}$
$\mathbb{1} = \sum_i |e_i\rangle\langle e_i|$
$\left(\sum_i |e_i\rangle\langle e_i|\right)|\psi\rangle = |\psi\rangle$

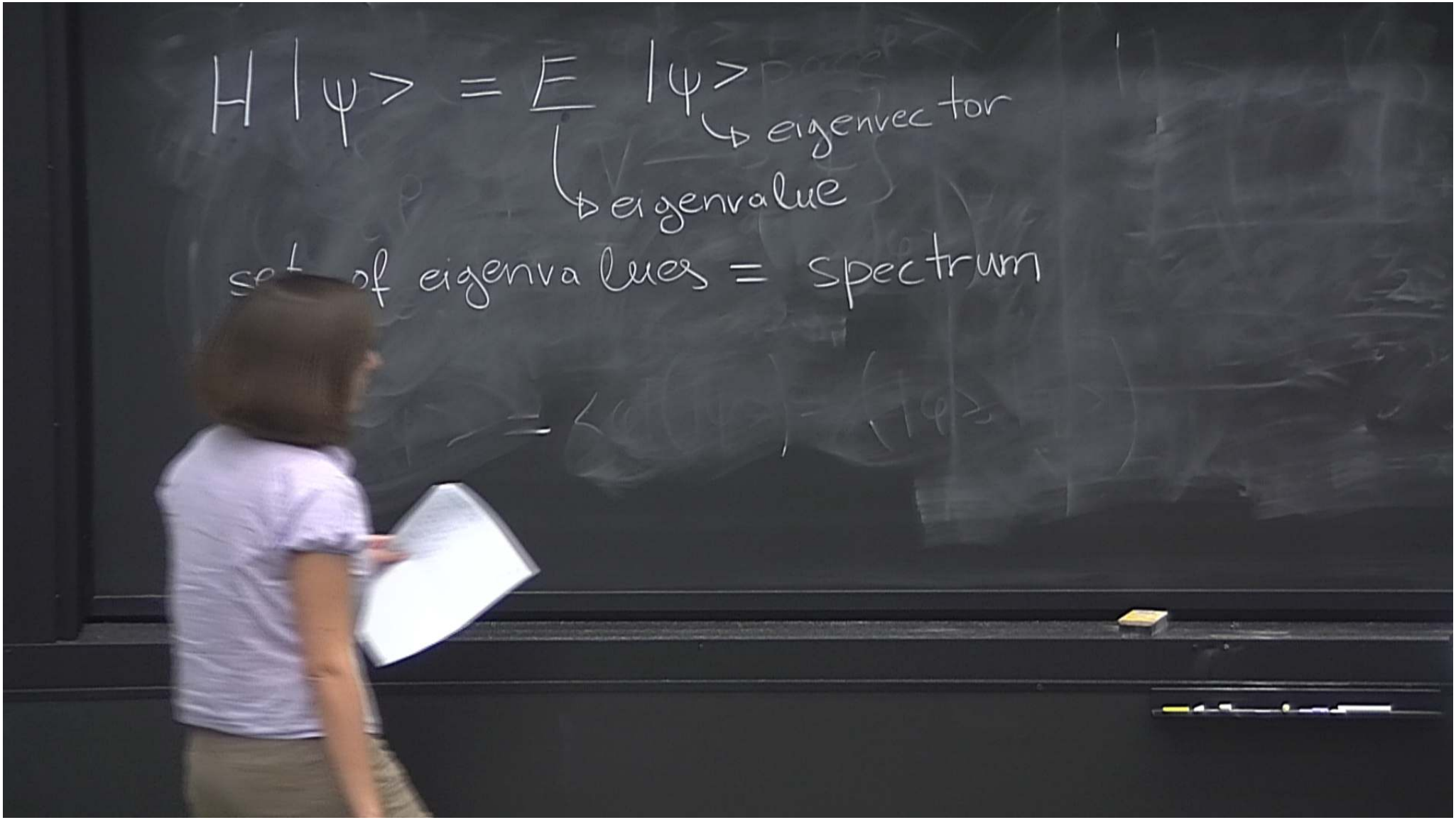

$|\varphi\rangle \longrightarrow \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_n \end{pmatrix}$
$\langle\varphi| \longrightarrow$
$\{|e_i\rangle\}$
$|\varphi\rangle = \sum_{i=1}^{n} \varphi_i |e_i\rangle$

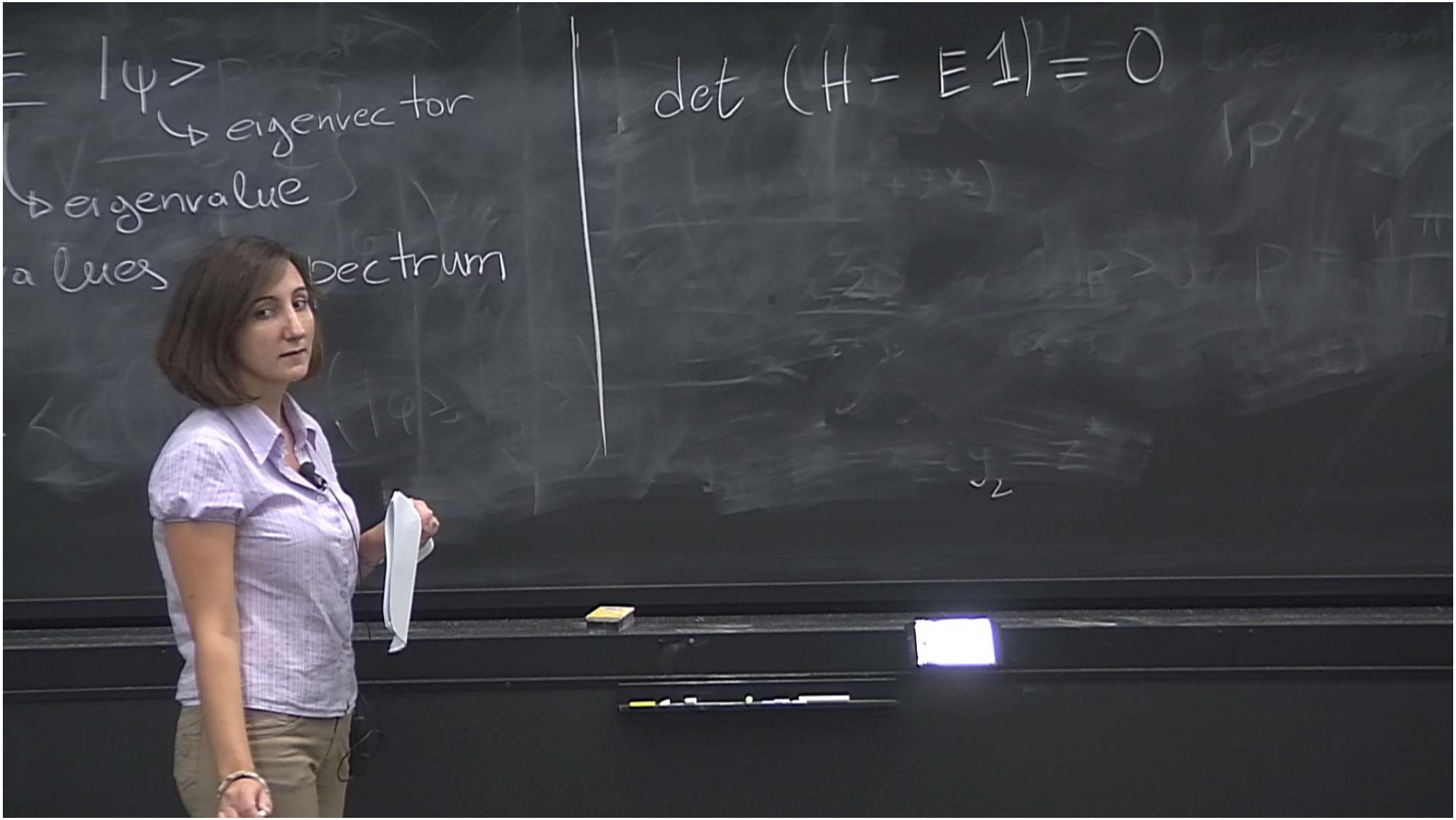

φ_1
φ_2
φ_n
$\langle\varphi| \longrightarrow (\varphi_1^* \quad \varphi_2^* \quad \cdots \quad \varphi_n^*)$
$\langle\varphi|\psi\rangle = \sum_{i=1}^{n} \varphi_i^* \varphi_i$
$= \langle\varphi|$
$\varphi_i |e_i\rangle$

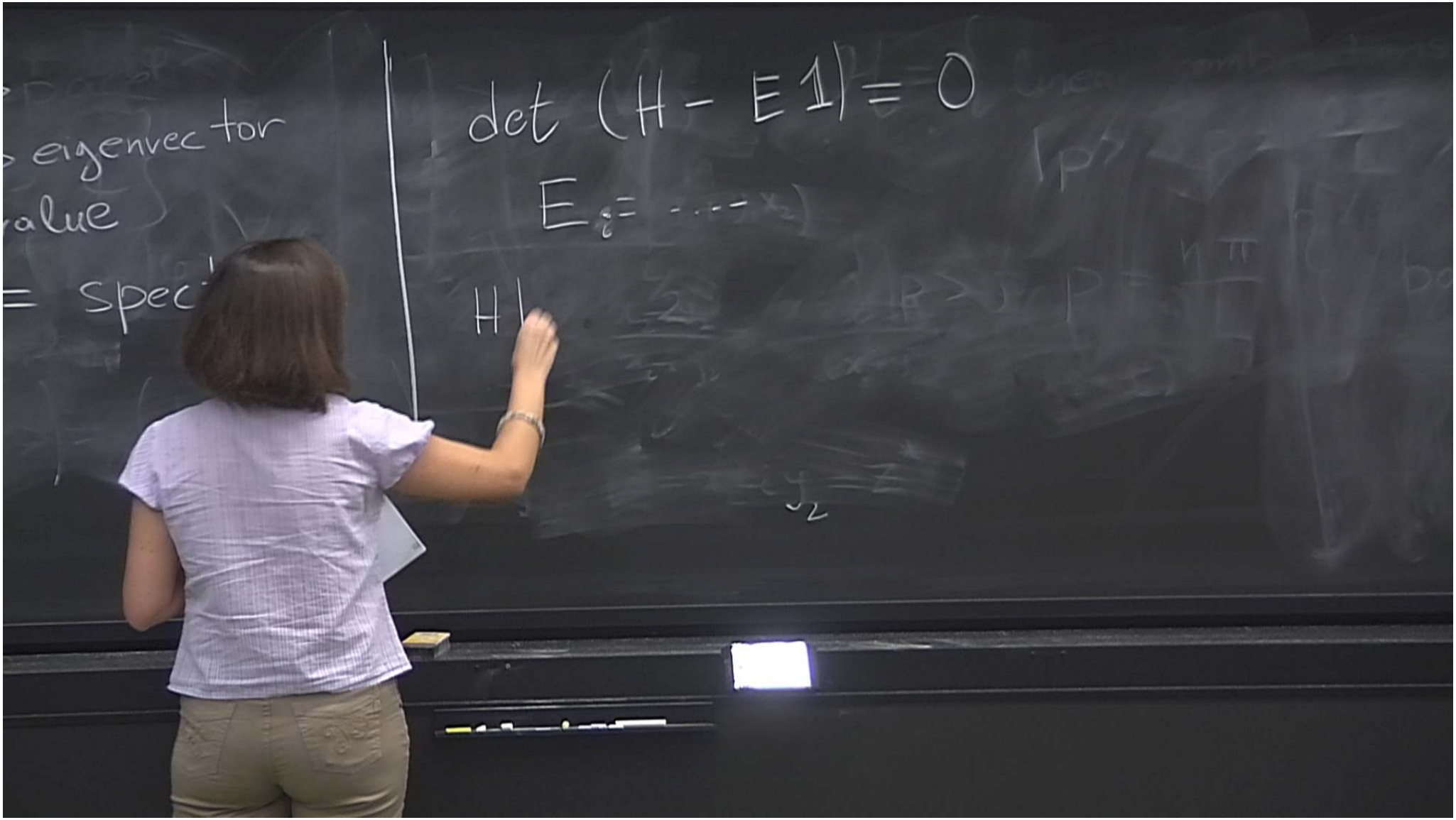

$\langle\varphi| \longrightarrow (\varphi_1^* \quad \varphi_2^* \quad \cdots \quad \varphi_n^*)$
$\langle\varphi|\psi\rangle = \sum_{i=1}^{n} \varphi_i^* \varphi_i$
$= \langle\varphi\rangle$
$e_i\rangle$
Linear operators $\longrightarrow$ matrice
A_i

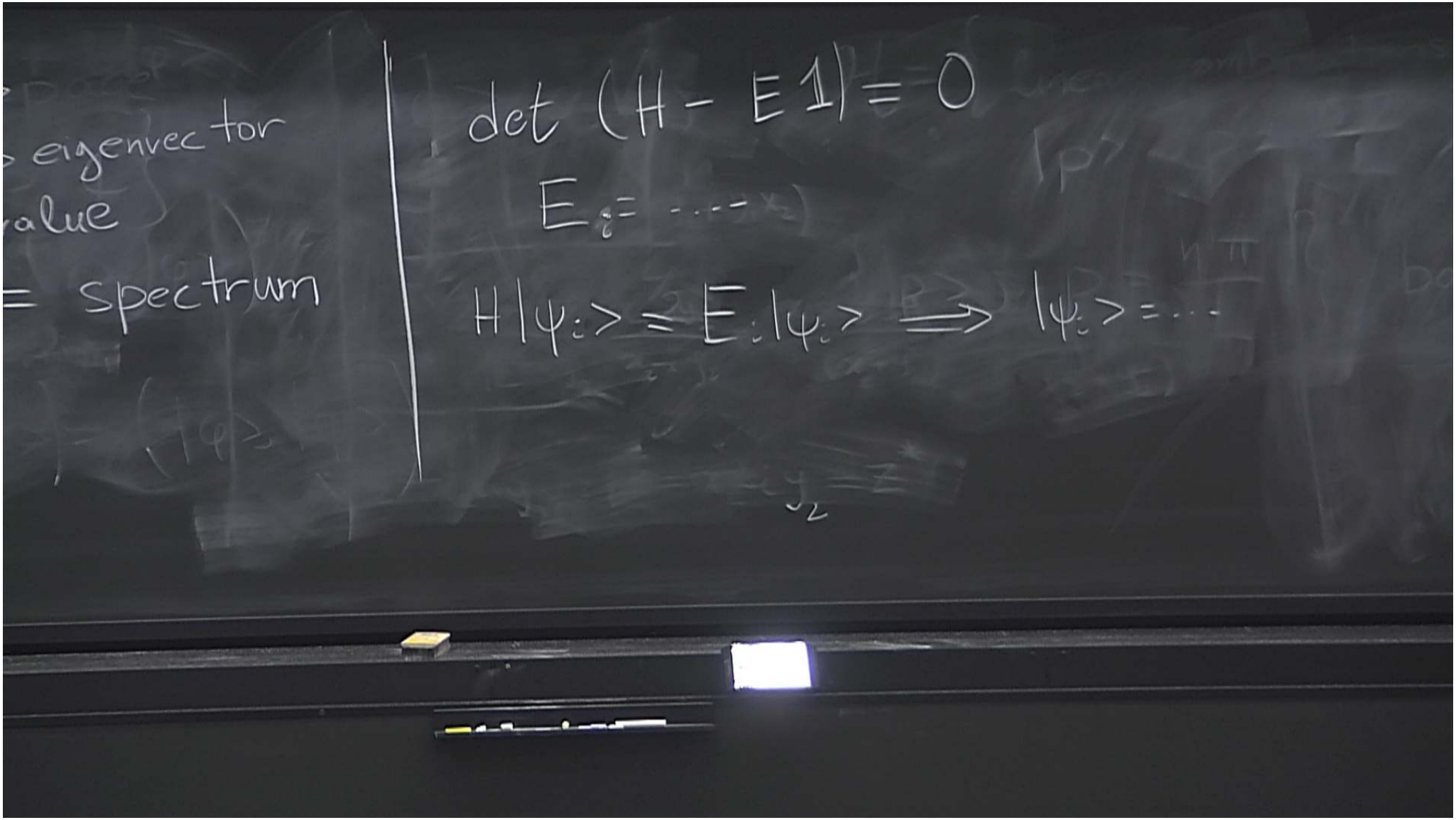

matrices
$A_{ij} = \langle e_i | A | e_j \rangle$

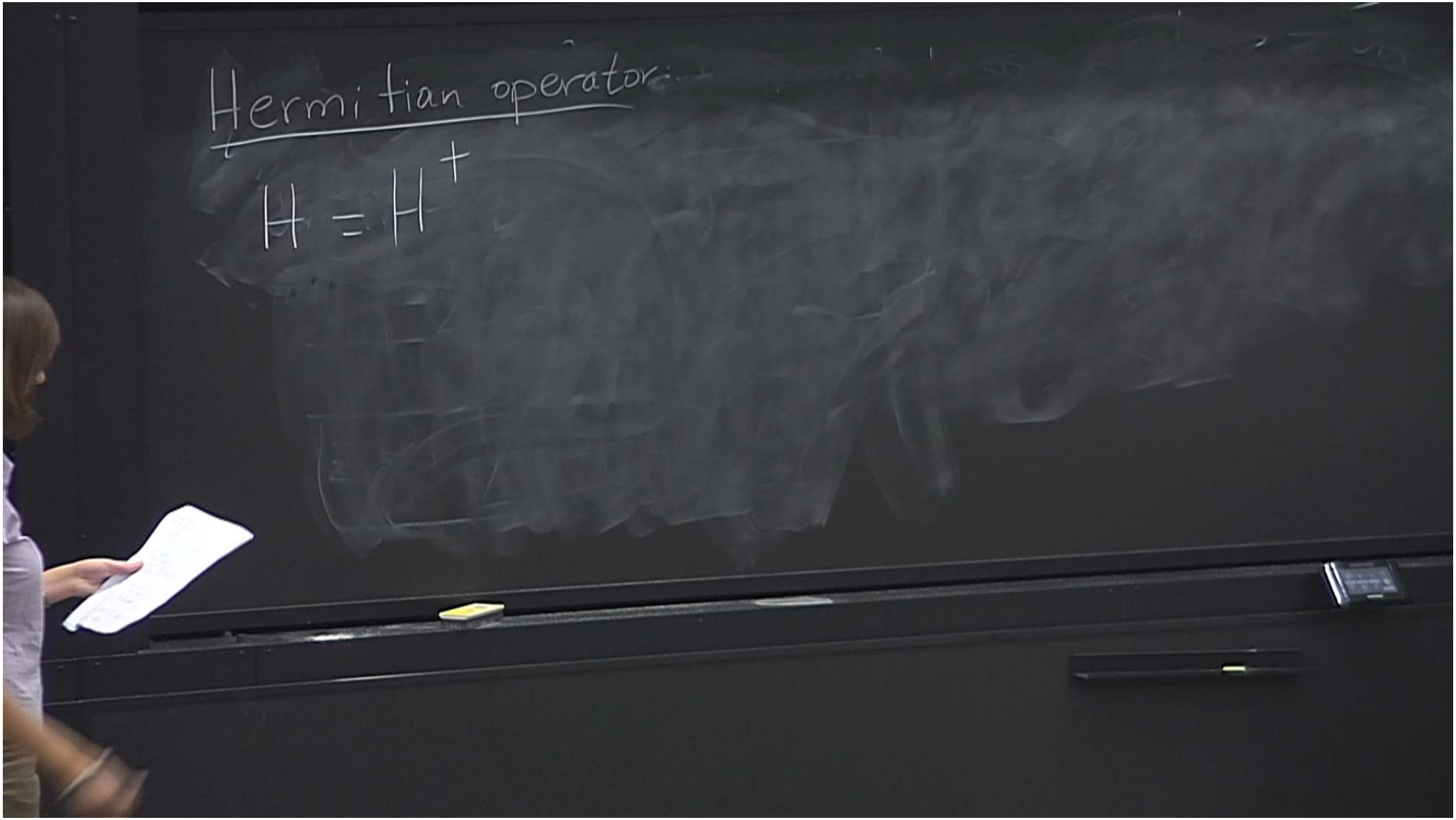

matrices
$A_{ij} = \langle e_i | A | e_j \rangle$
$n \times n$ matrix

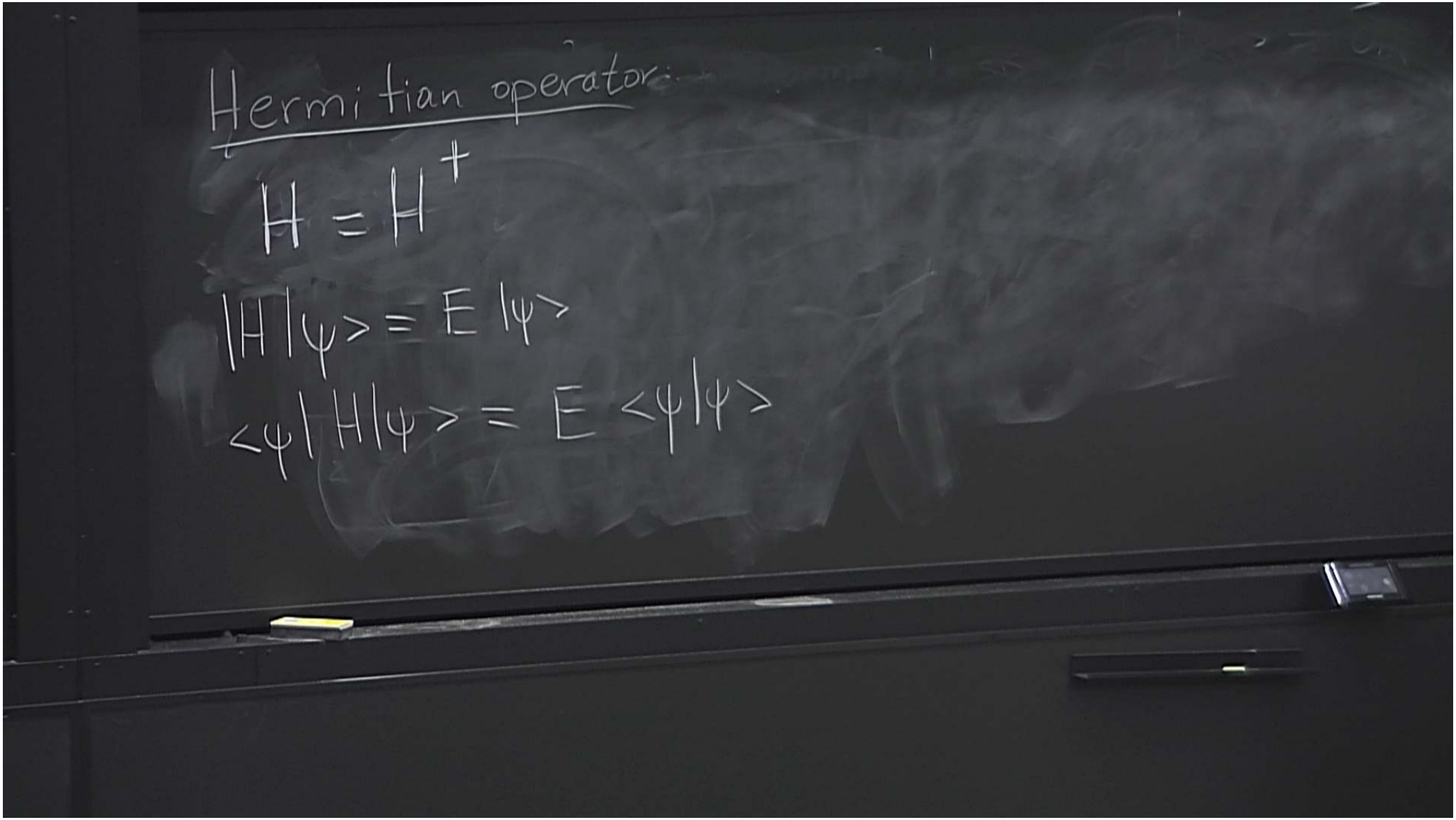

H|ψ> = E |ψ>
eigenvector
eigenvalue
set of eigenvalues = spectrum

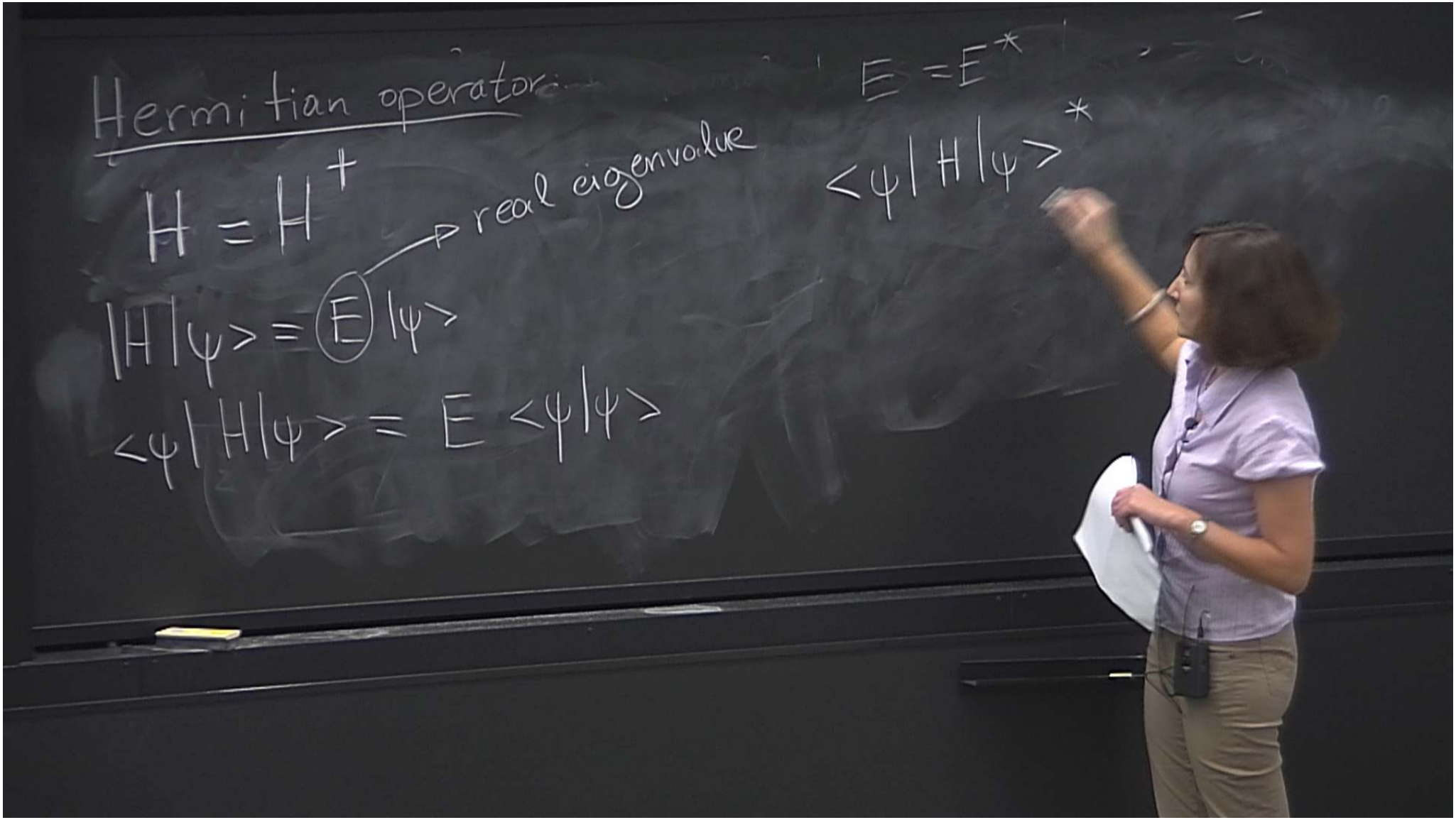

|ψ>
eigenvector
eigenvalue
spectrum
det (H - E 1) = 0

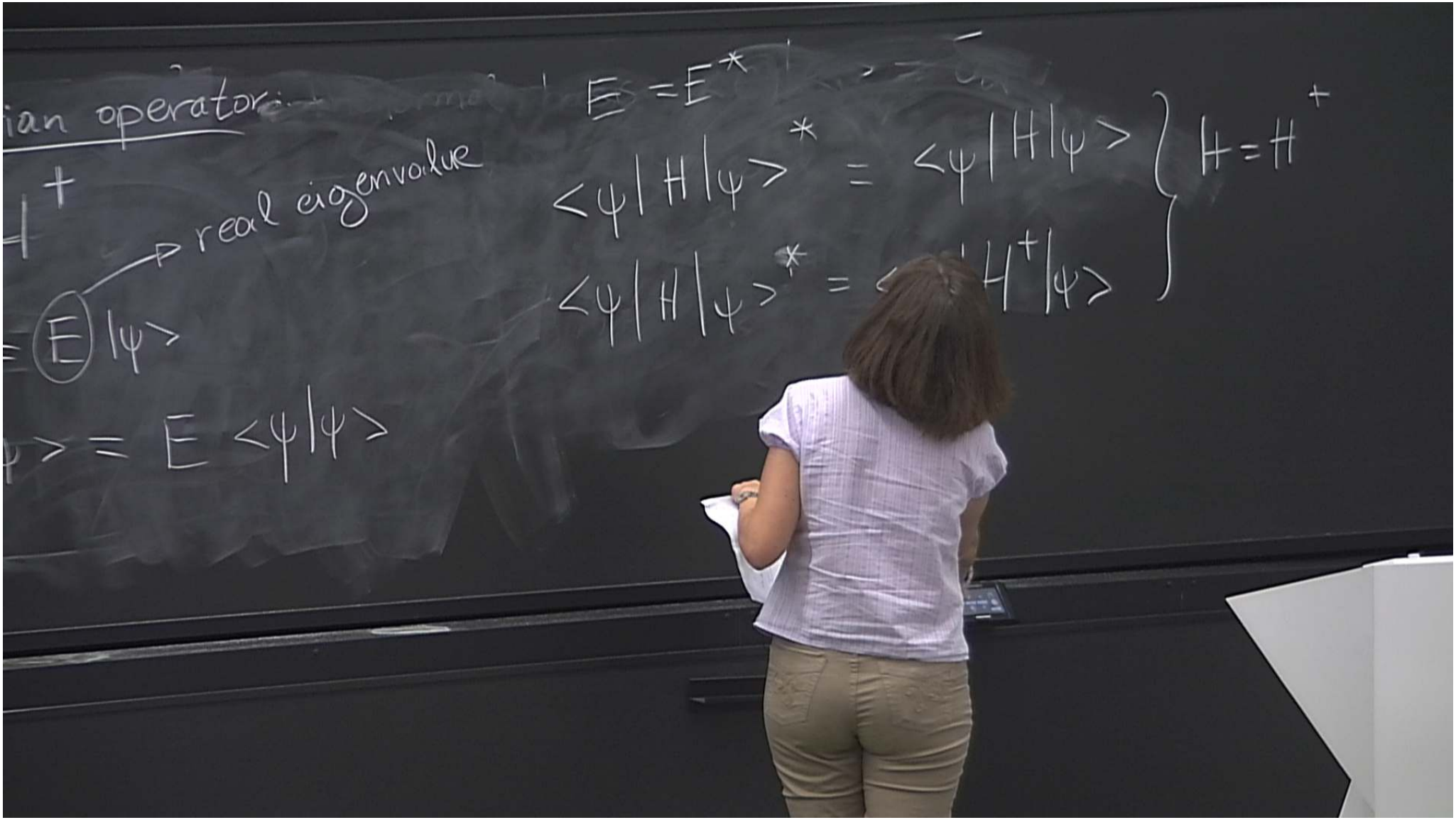

eigenvector
value
= spec
det (H - E 1) = 0
H

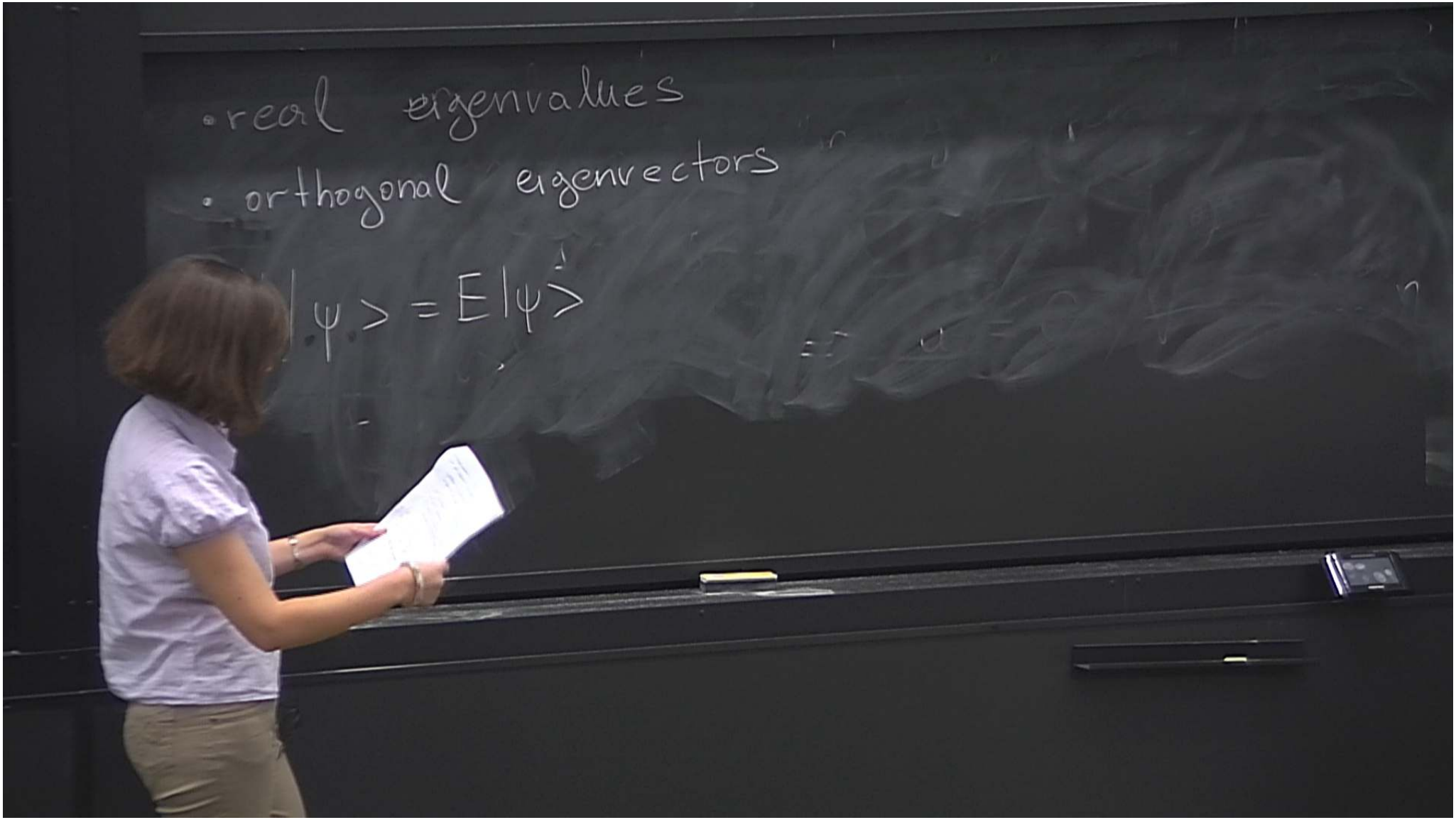

eigenvector
value
= spectrum
det (H − E 1) = 0
E_i = ...
$H|\psi_i\rangle = E_i|\psi_i\rangle \Longrightarrow |\psi_i\rangle = \ldots$

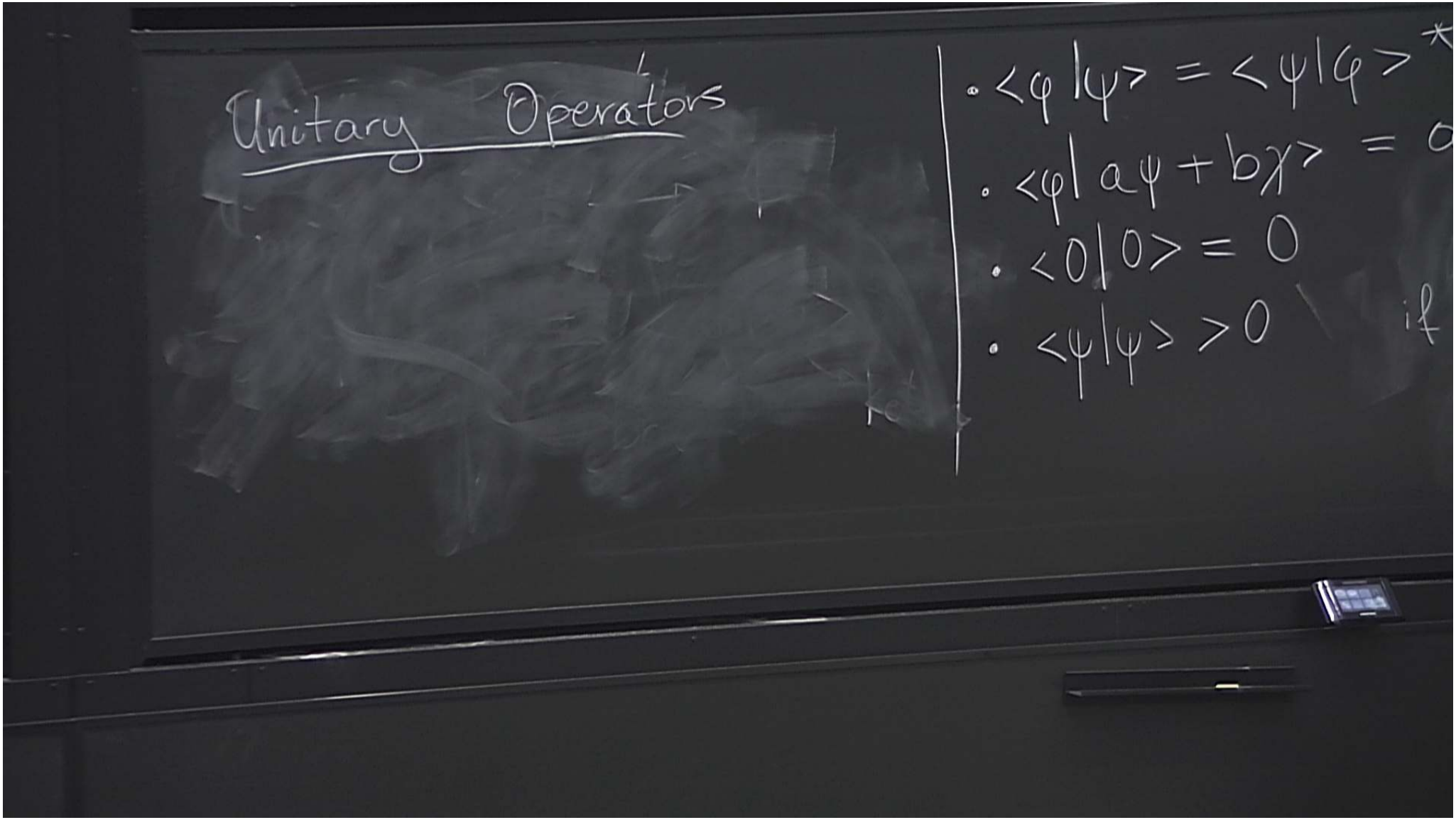

Hermitian operator:
$H = H^{\dagger}$

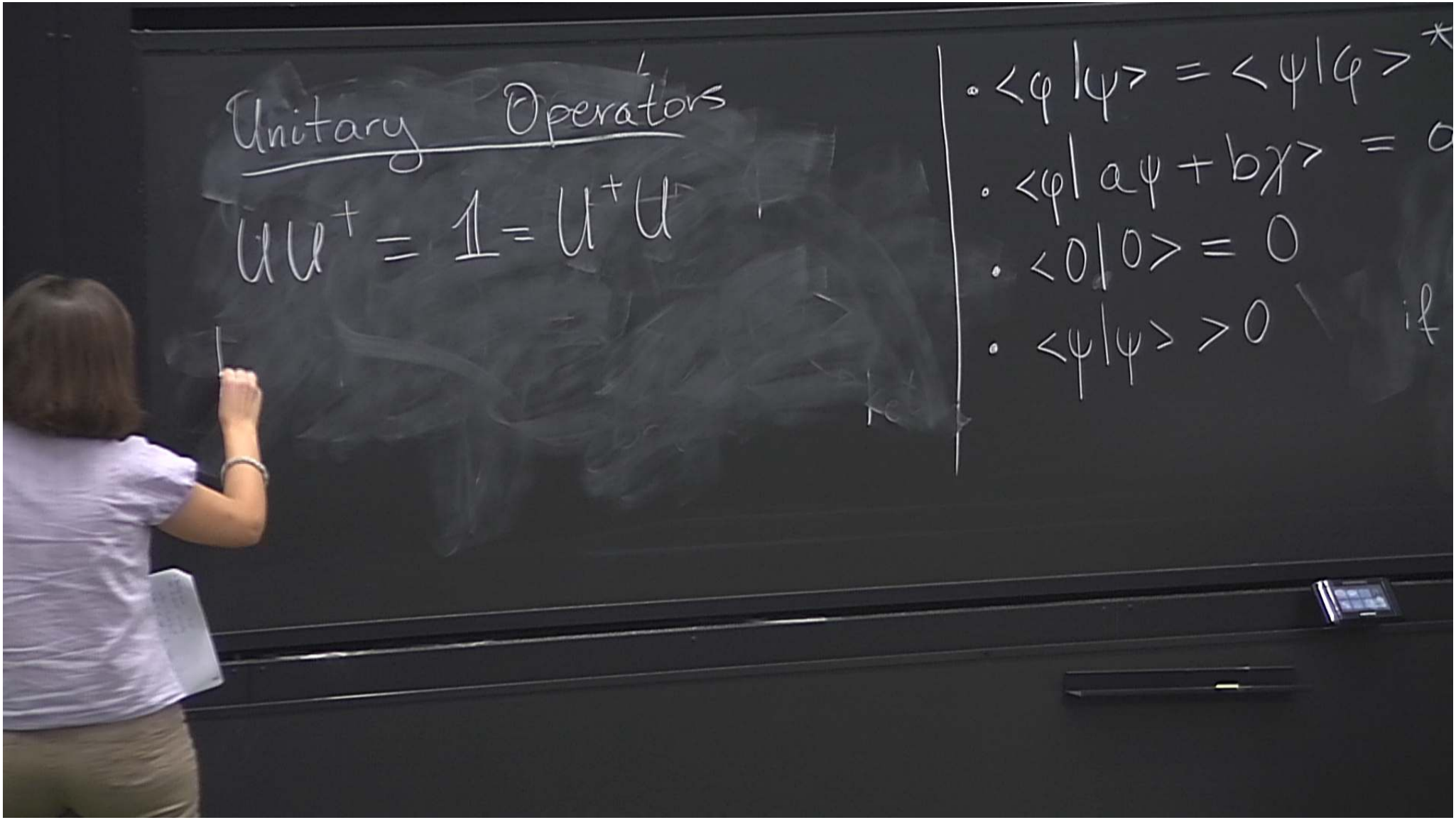

Hermitian operator:
$H = H^{\dagger}$
$|H|\psi\rangle = E|\psi\rangle$
$\langle\psi|H|\psi\rangle = E\langle\psi|\psi\rangle$

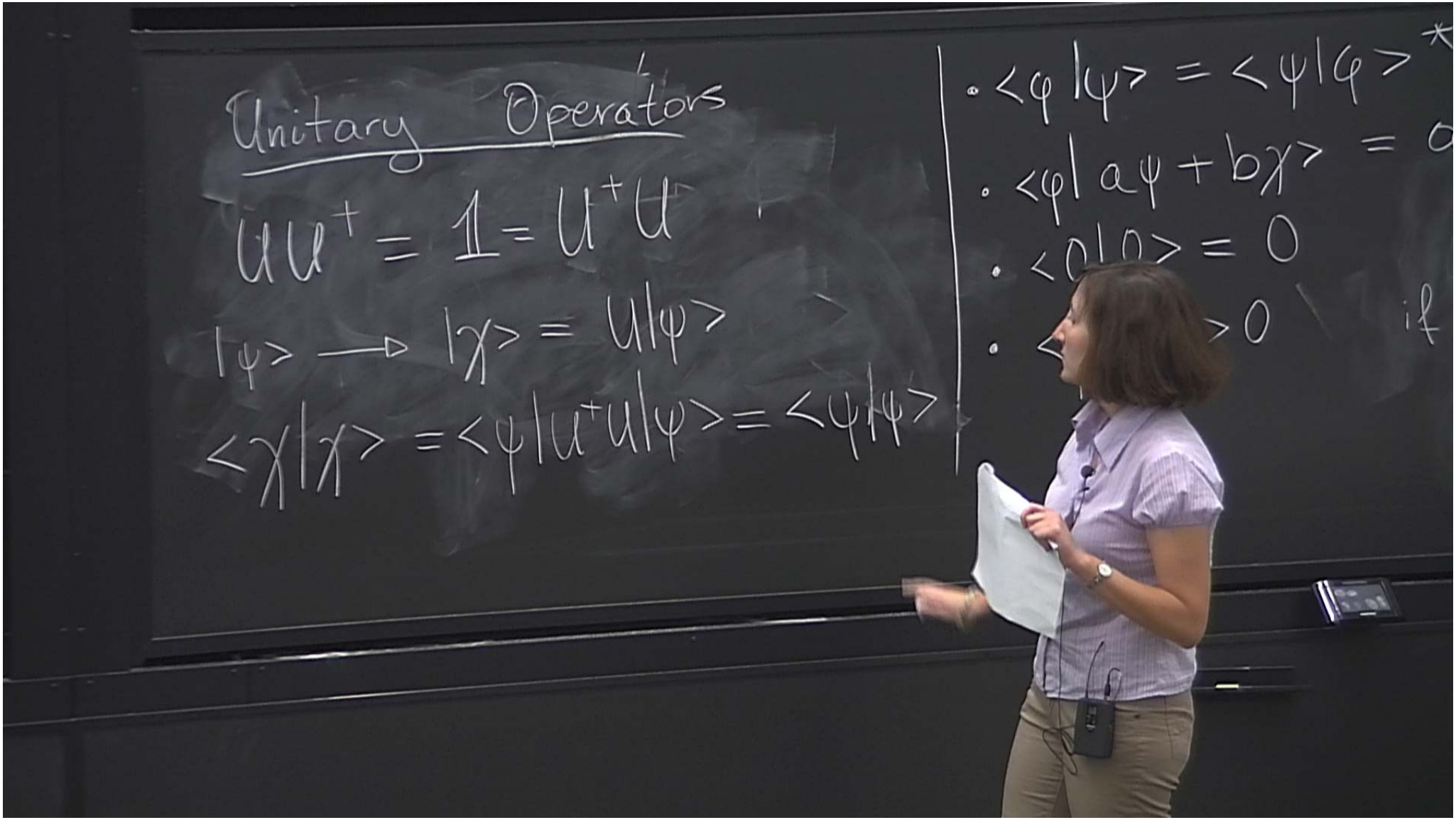

Hermitian operator:
$H = H^\dagger$
$H|\psi\rangle = E|\psi\rangle$
real eigenvalue
$\langle\psi|H|\psi\rangle = E\langle\psi|\psi\rangle$
$E = E^*$
$\langle\psi|H|\psi\rangle^*$

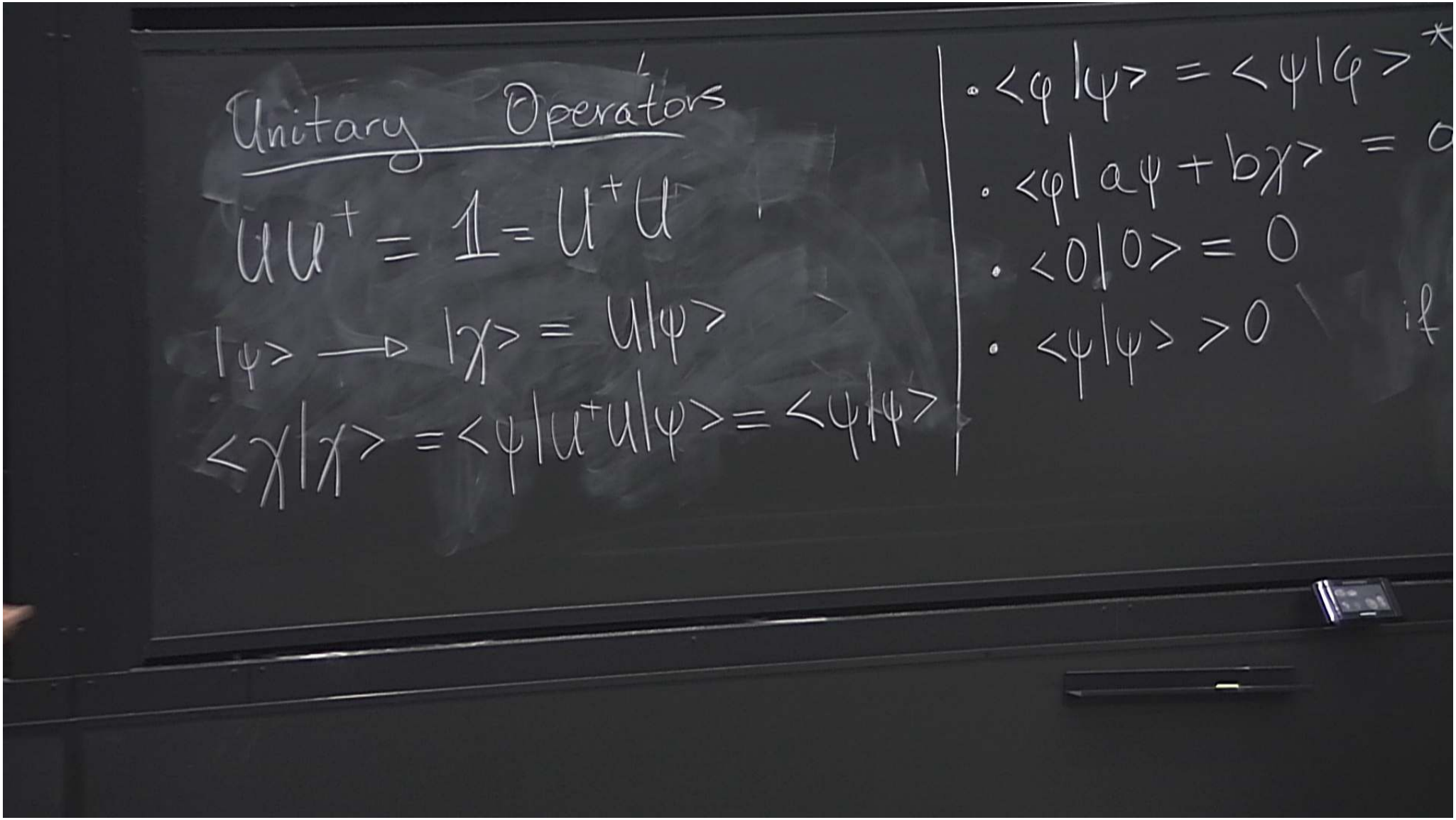

ian operator
real eigenvalue
E |ψ>
ψ> = E <ψ|ψ>
E = E*
<ψ|H|ψ>* = <ψ|H|ψ>
H = H+


• real eigenvalues
• orthogonal eigenvectors
|ψ> = E|ψ>


Unitary Operators
• ⟨φ|ψ⟩ = ⟨ψ|φ⟩*
• ⟨φ| aψ + bχ⟩ = a
• ⟨0|0⟩ = 0
• ⟨ψ|ψ⟩ > 0 if


Unitary Operators
$UU^\dagger = 1 = U^\dagger U$
$\langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^*$
$\langle\varphi|a\psi + b\chi\rangle =$
$\langle 0|0\rangle = 0$
$\langle\psi|\psi\rangle > 0$
if


Unitary Operators
$UU^\dagger = 1 = U^\dagger U$
$|\psi\rangle \longrightarrow |\chi\rangle = U|\psi\rangle$
$\langle\chi|\chi\rangle = \langle\psi|U^\dagger U|\psi\rangle = \langle\psi|\psi\rangle$
$\langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^*$
$\langle\varphi|a\psi + b\chi\rangle$
$\langle 0|0\rangle = 0$
if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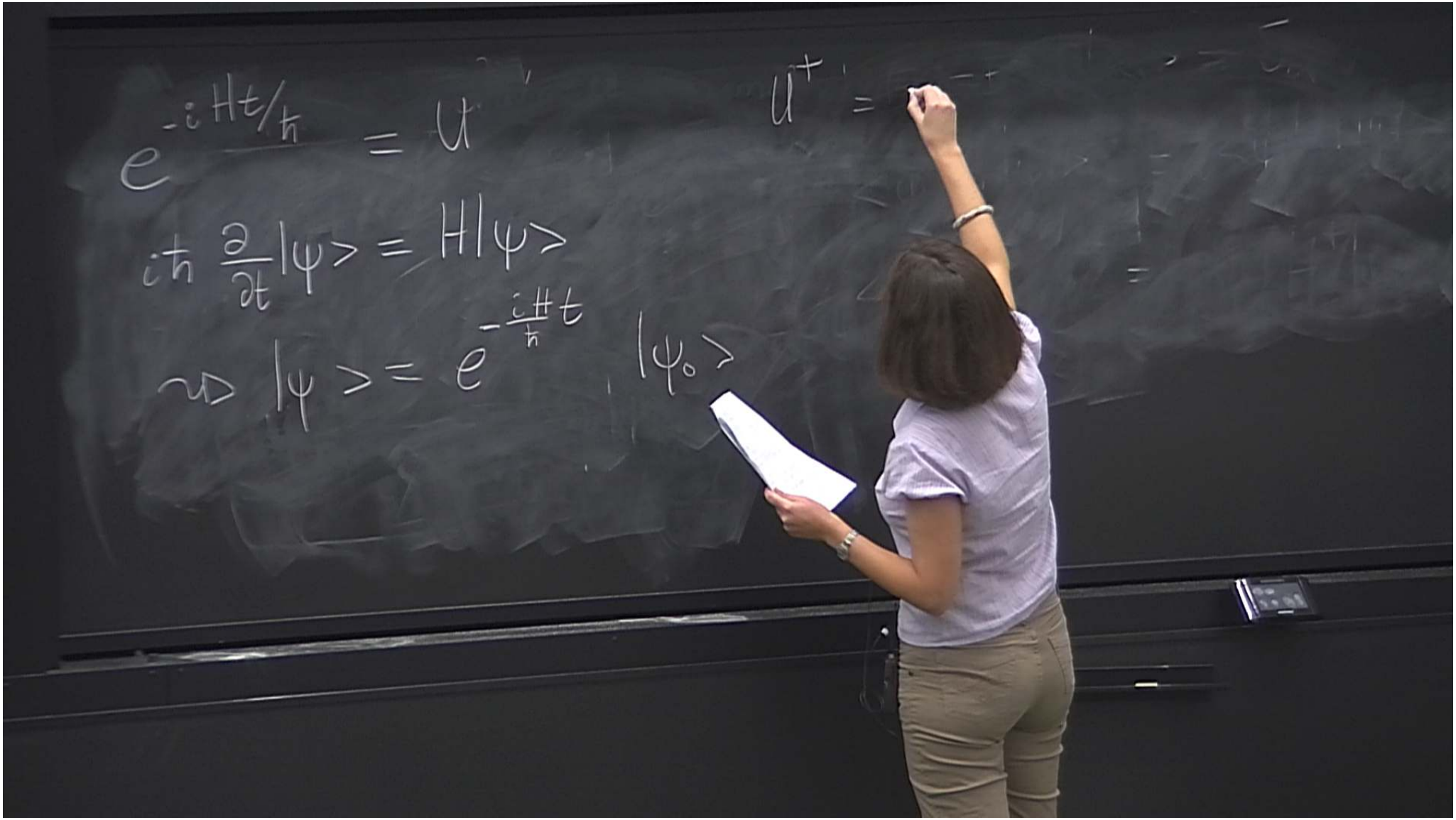
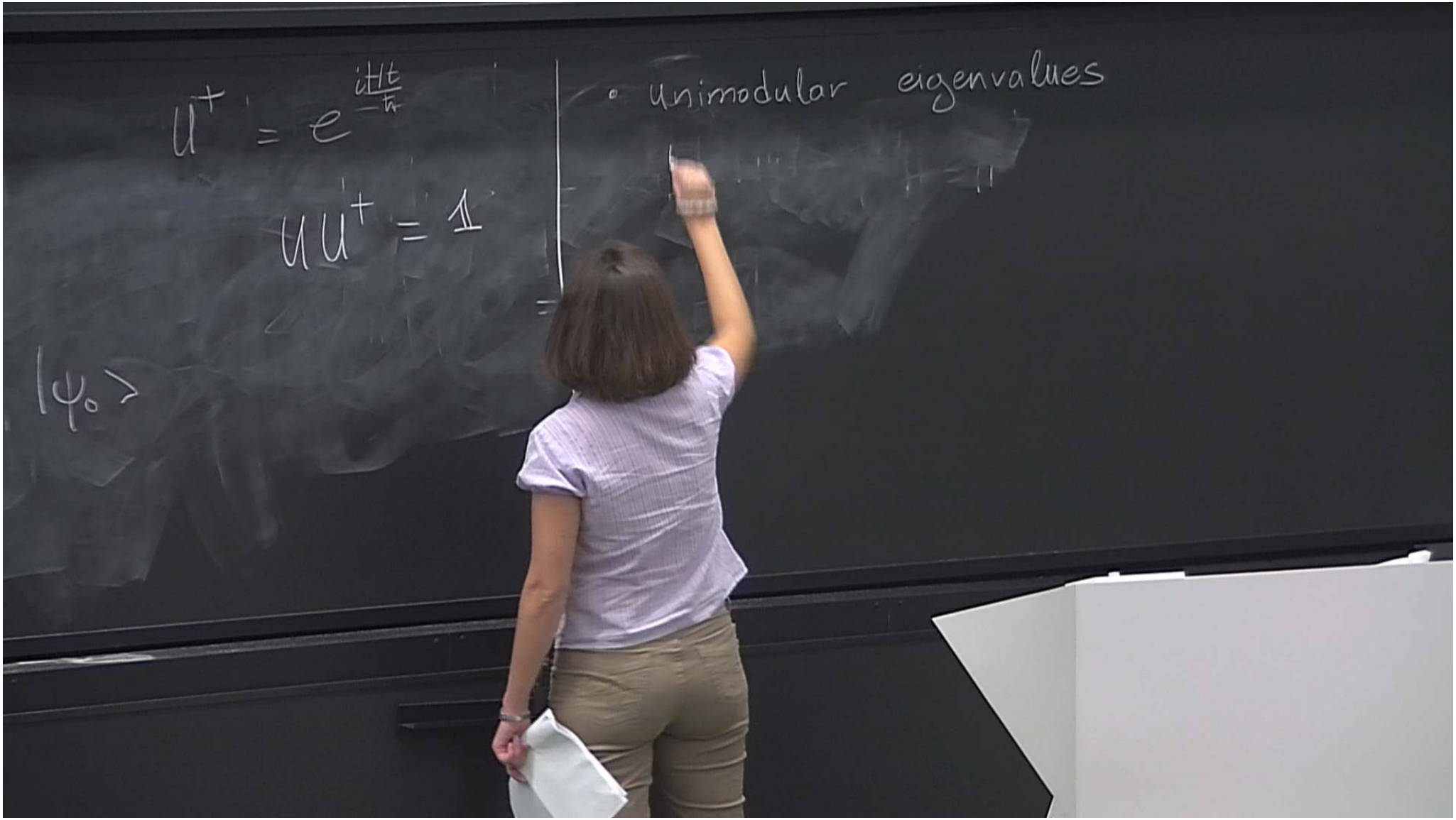
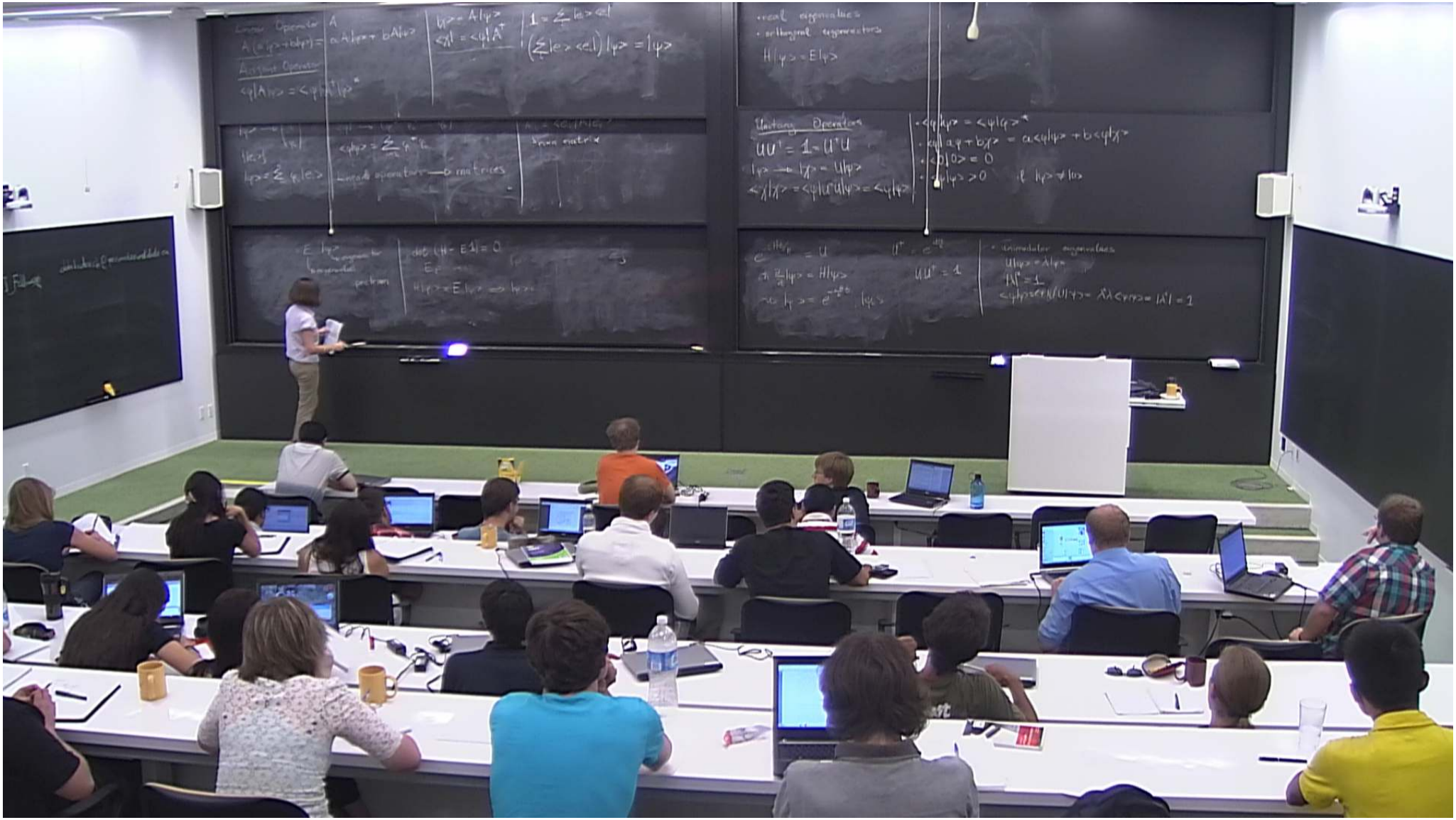
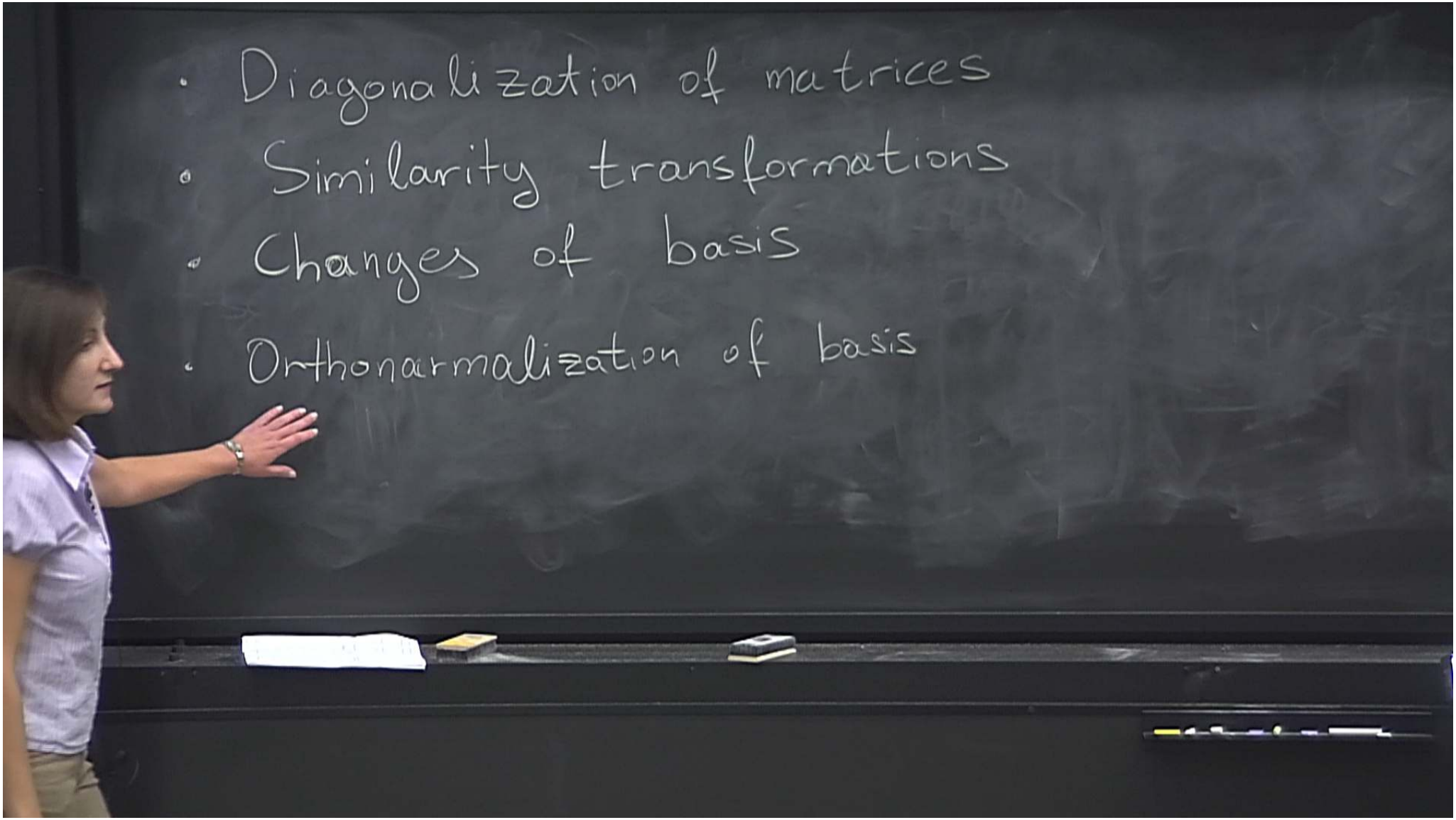

$e^{-iHt/\hbar} = U$
$U^{\dagger} =$
$i\hbar \frac{\partial}{\partial t}|\psi\rangle = H|\psi\rangle$
$\rightsquigarrow |\psi\rangle = e^{-\frac{iHt}{\hbar}} |\psi_0\rangle$


$U^\dagger = e^{\frac{iHt}{\hbar}}$
$UU^\dagger = \mathbb{1}$
$|\psi_0\rangle$
• unimodular eigenvalues


Linear Operator
Unitary Operators
real eigenvalues
orthogonal eigenvectors
Unimodular eigenvalues


• Diagonalization of matrices
• Similarity transformations
• Changes of basis
• Orthonormalization of basis