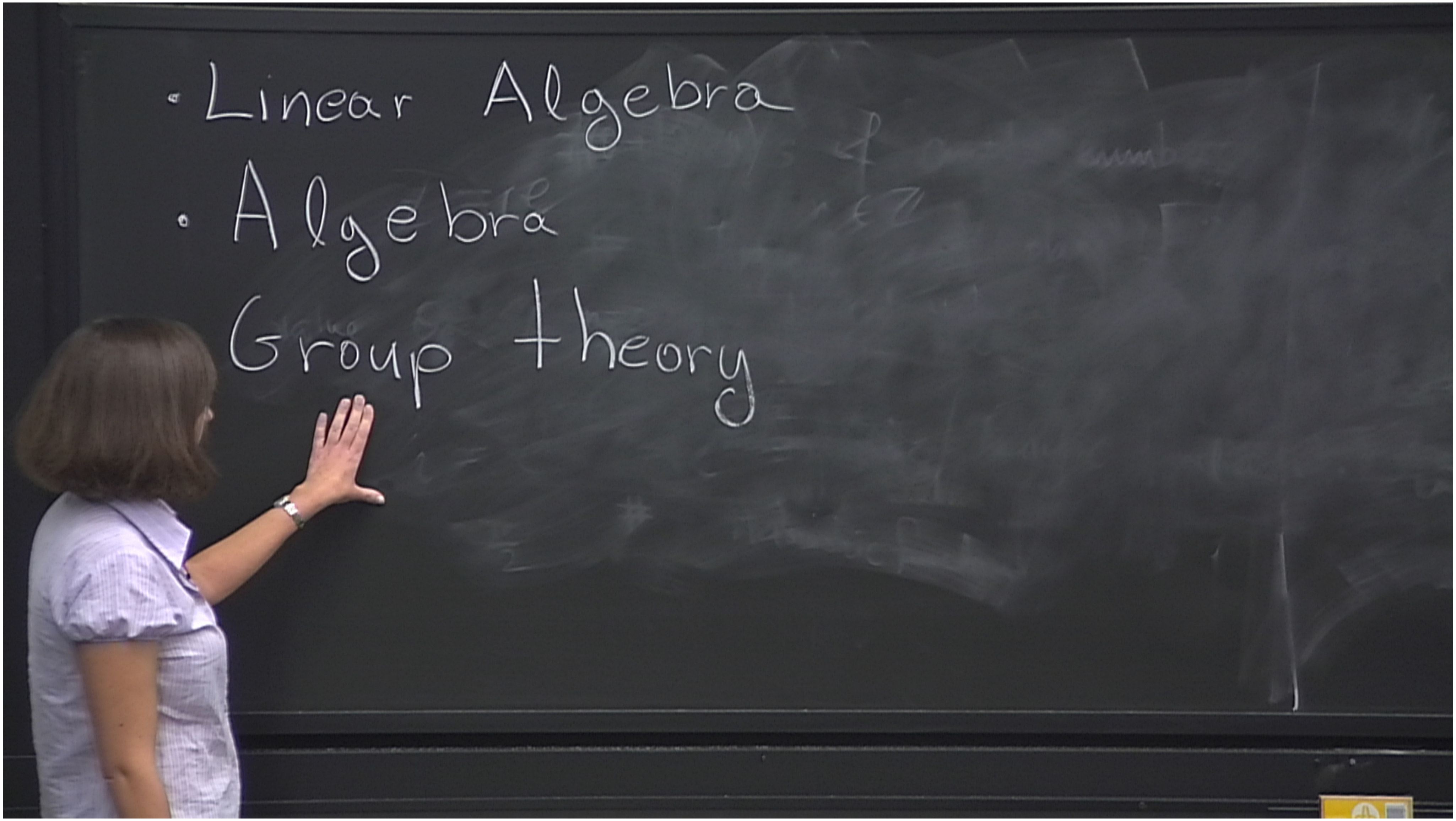
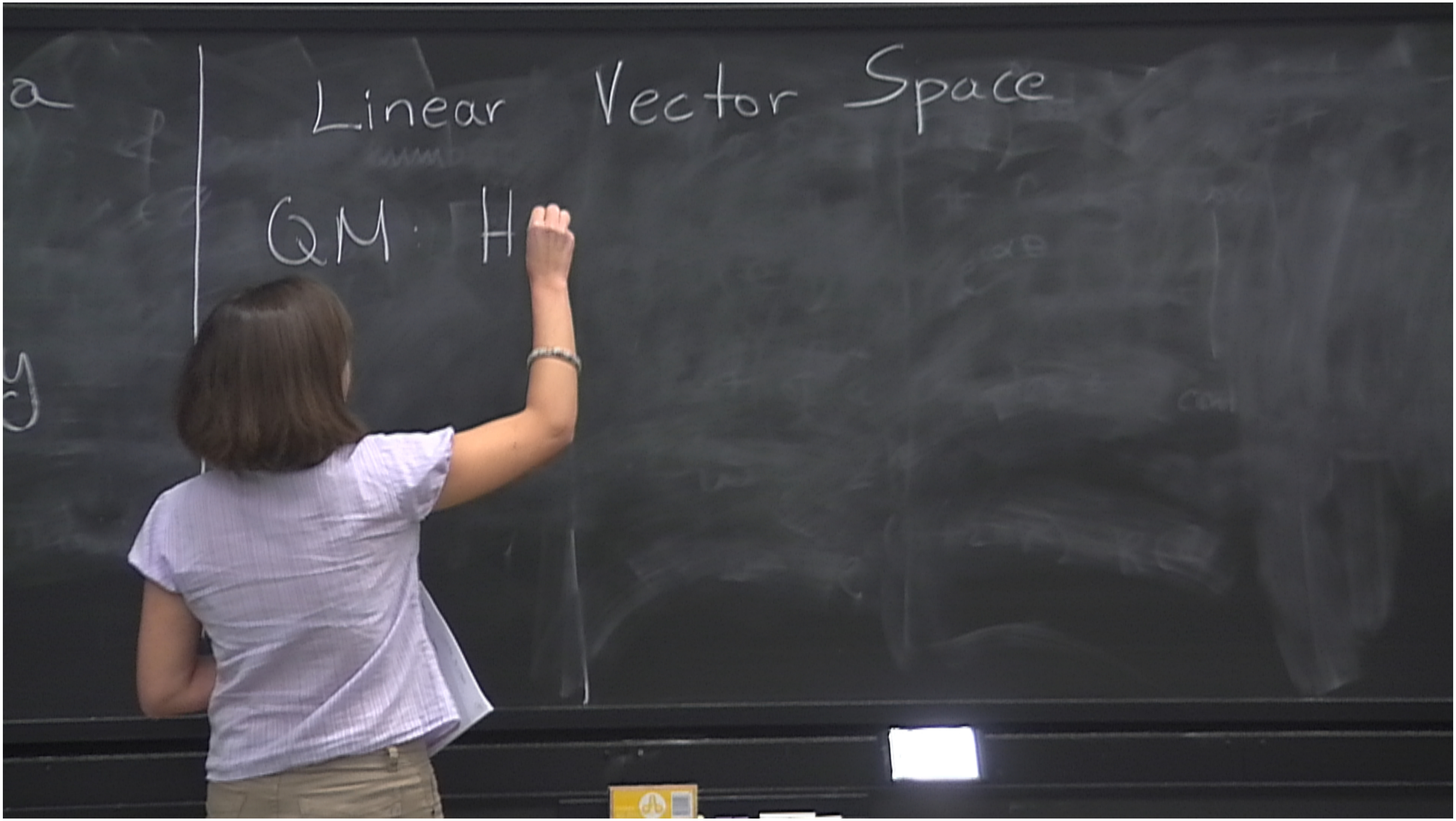
Title: Algebra - Lecture 1

Date: Aug 16, 2011 11:30 AM

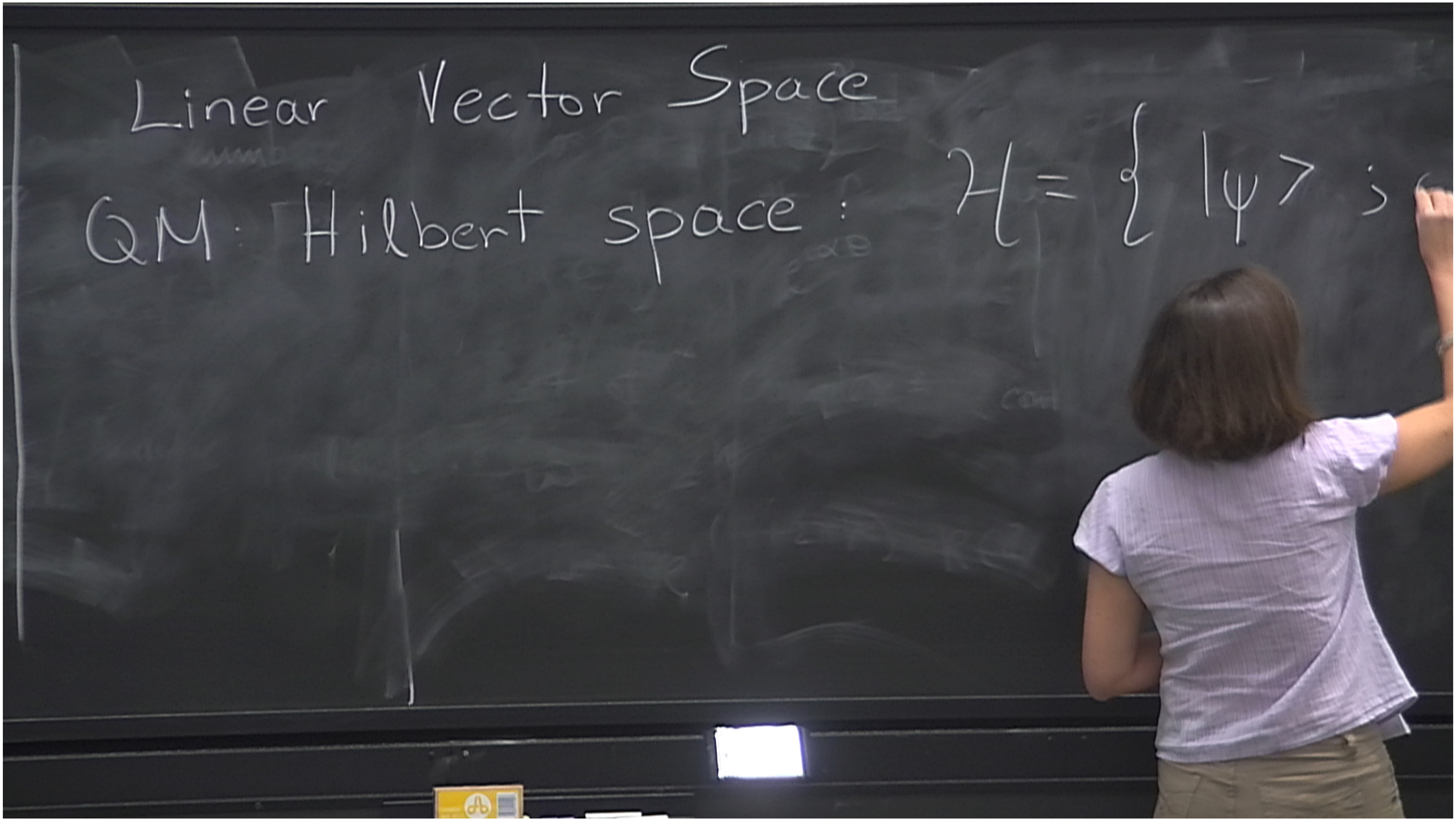
URL: http://pirsa.pi.local/11080064

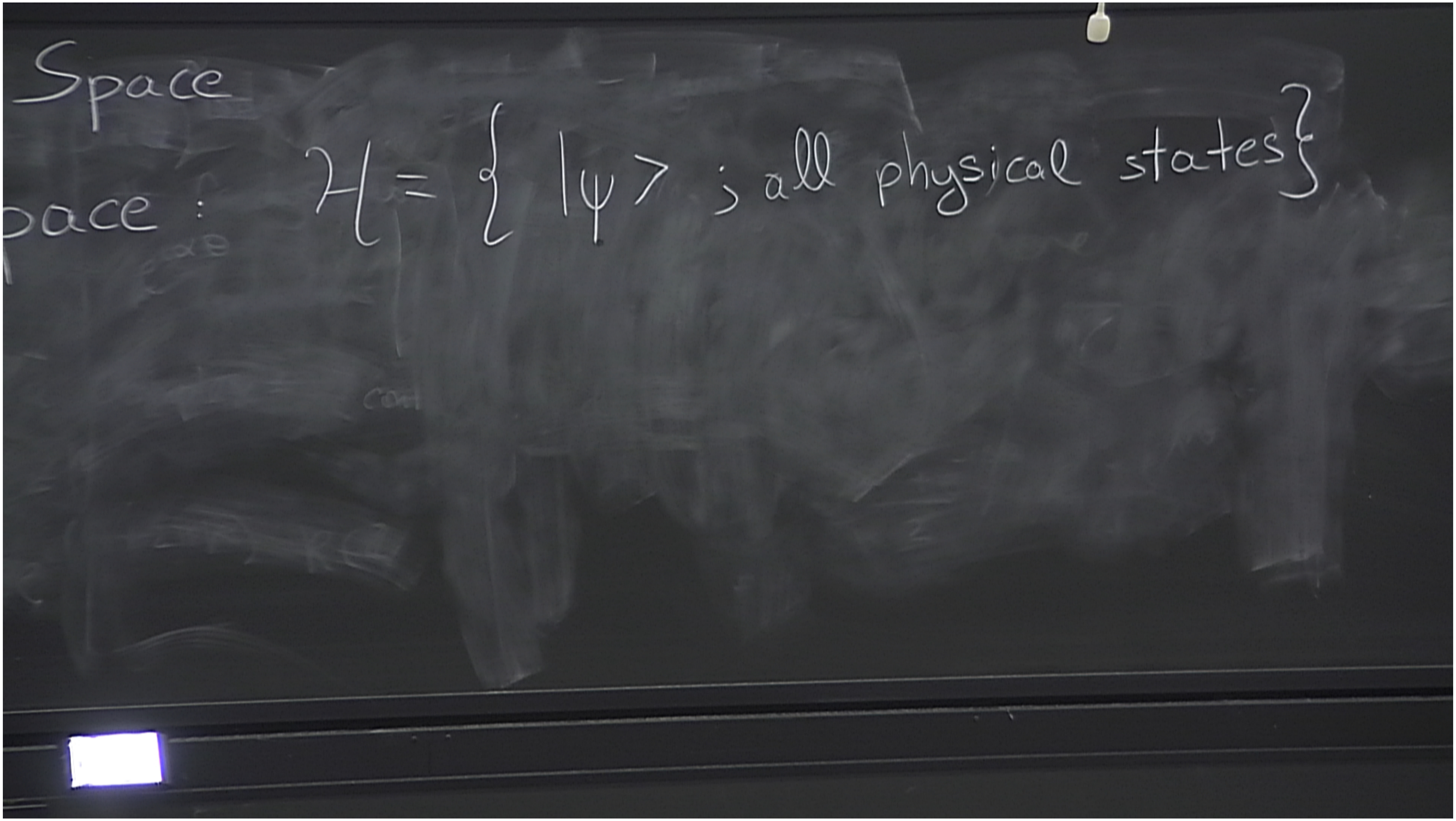
Abstract:

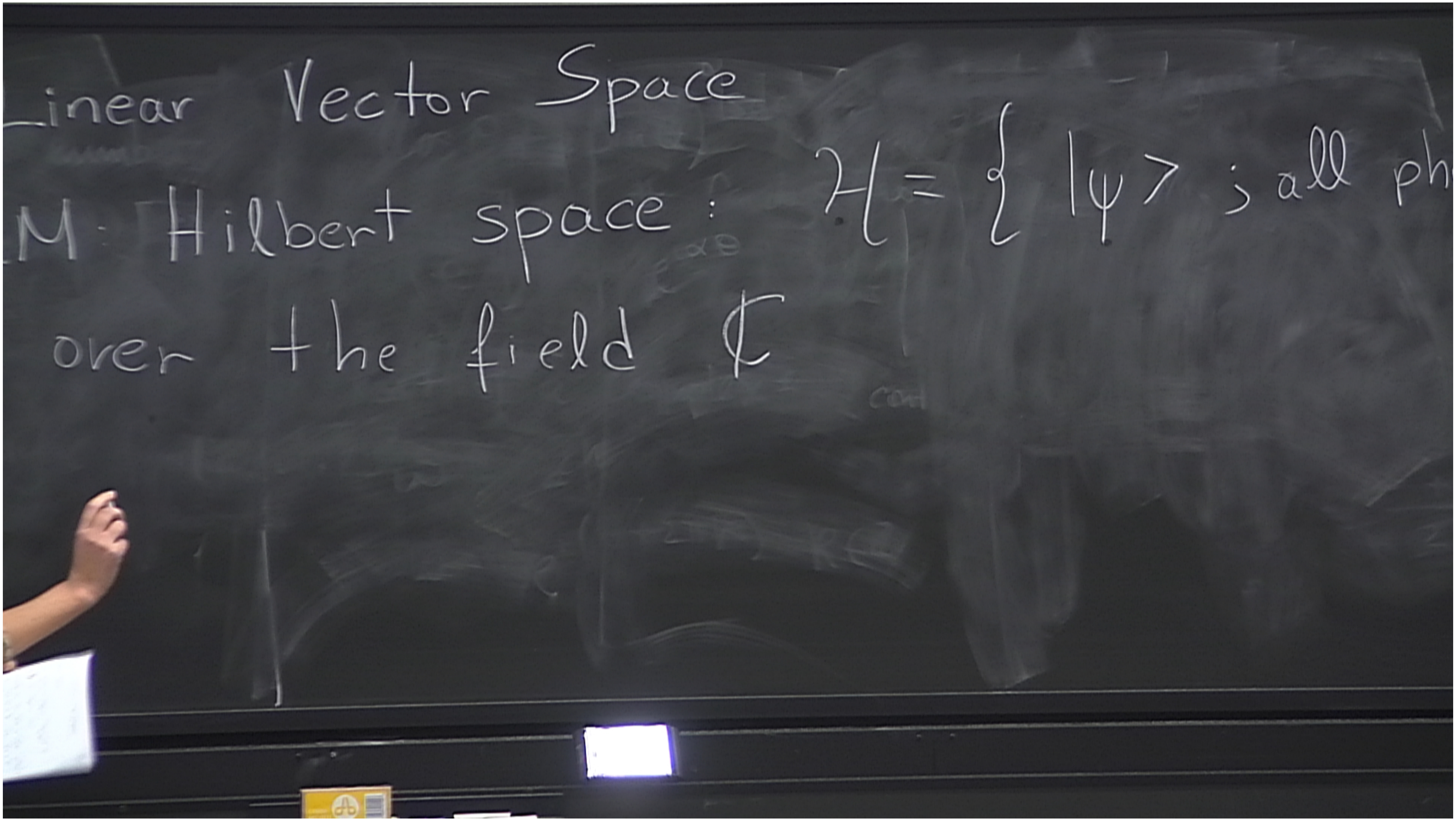

• Linear Algebra
• Algebra
Group theory

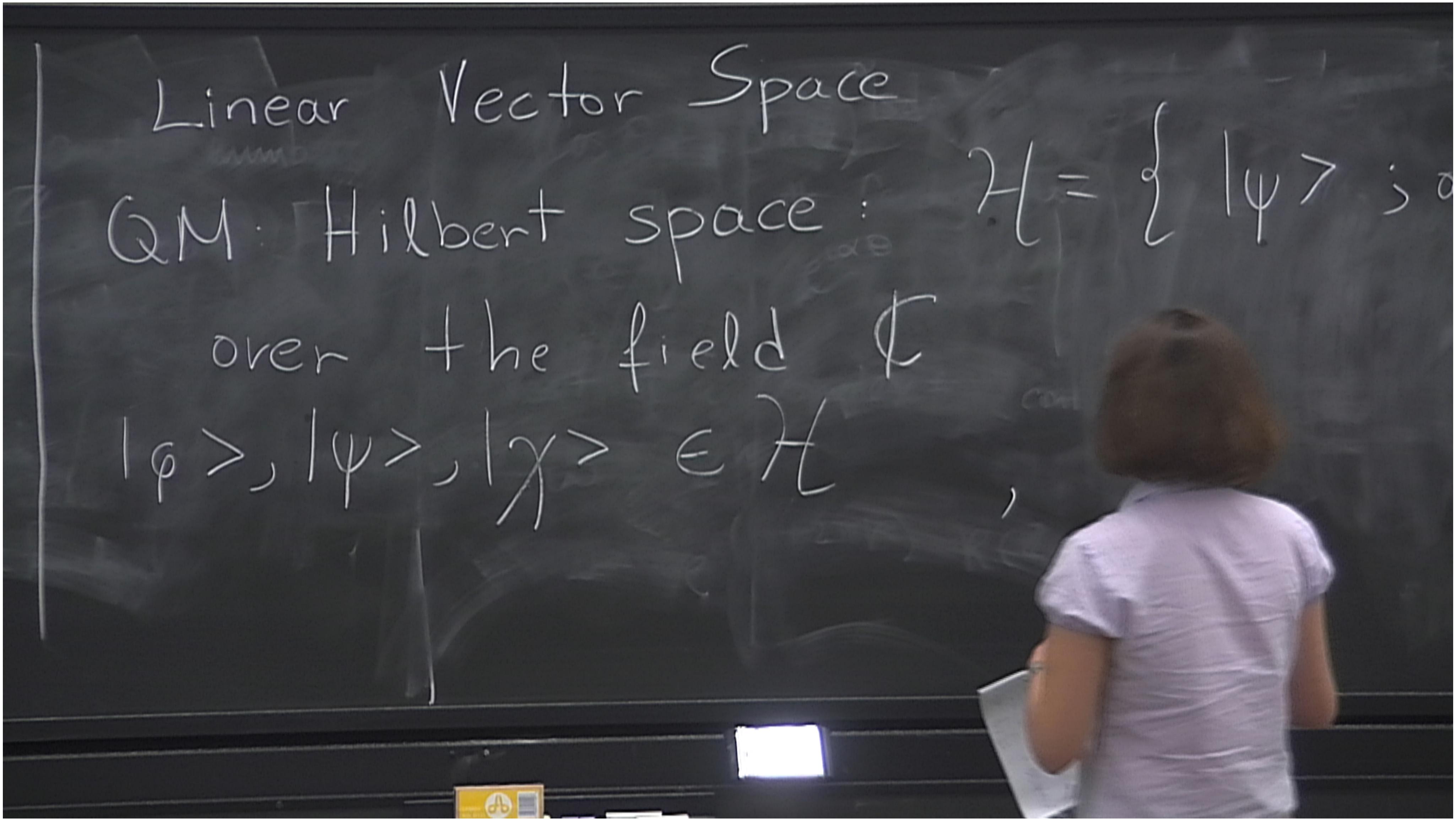


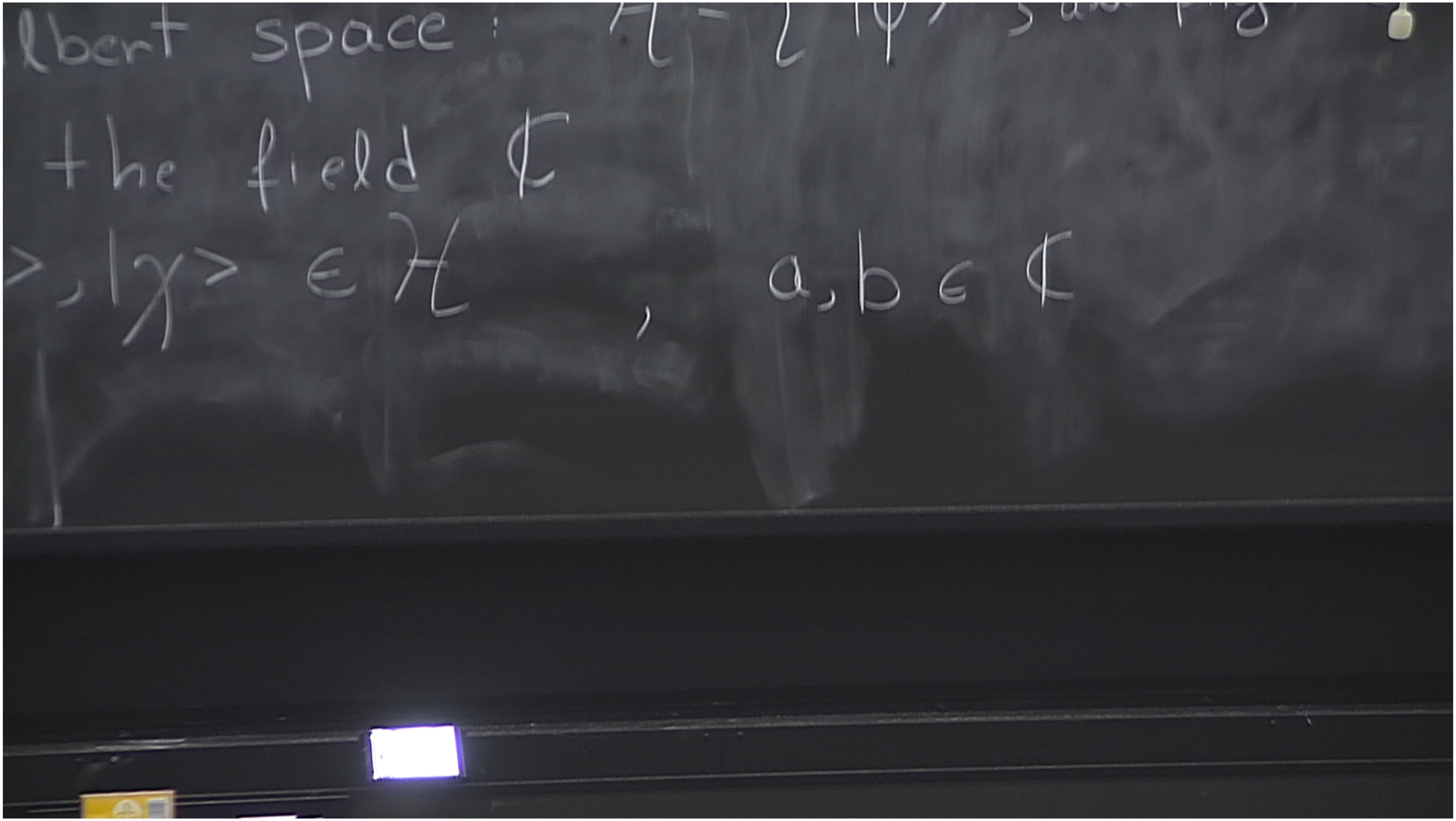
Linear Vector Space
QM: H

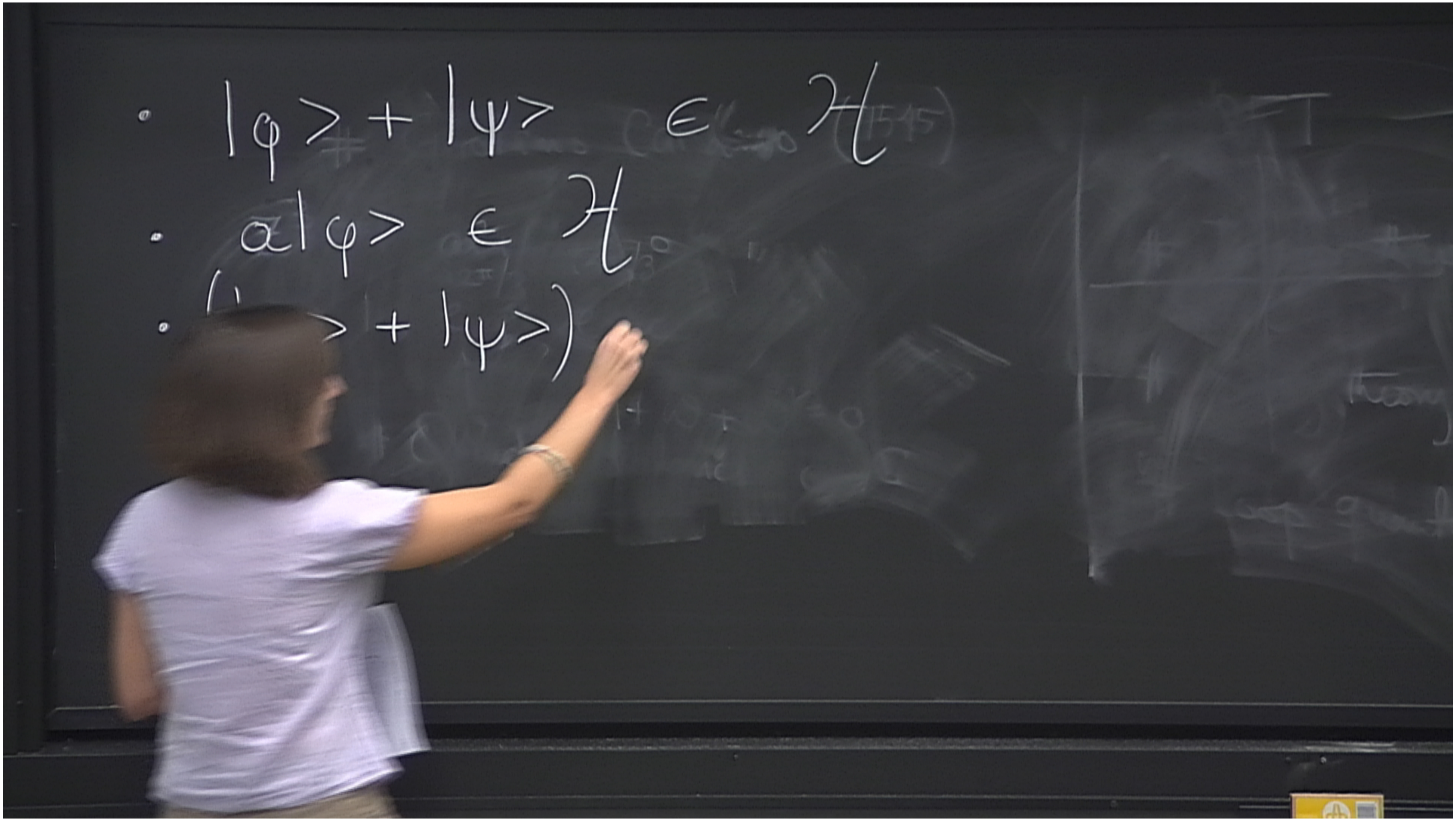

Linear Vector Space
QM: Hilbert space:
ℋ = { |ψ> ;

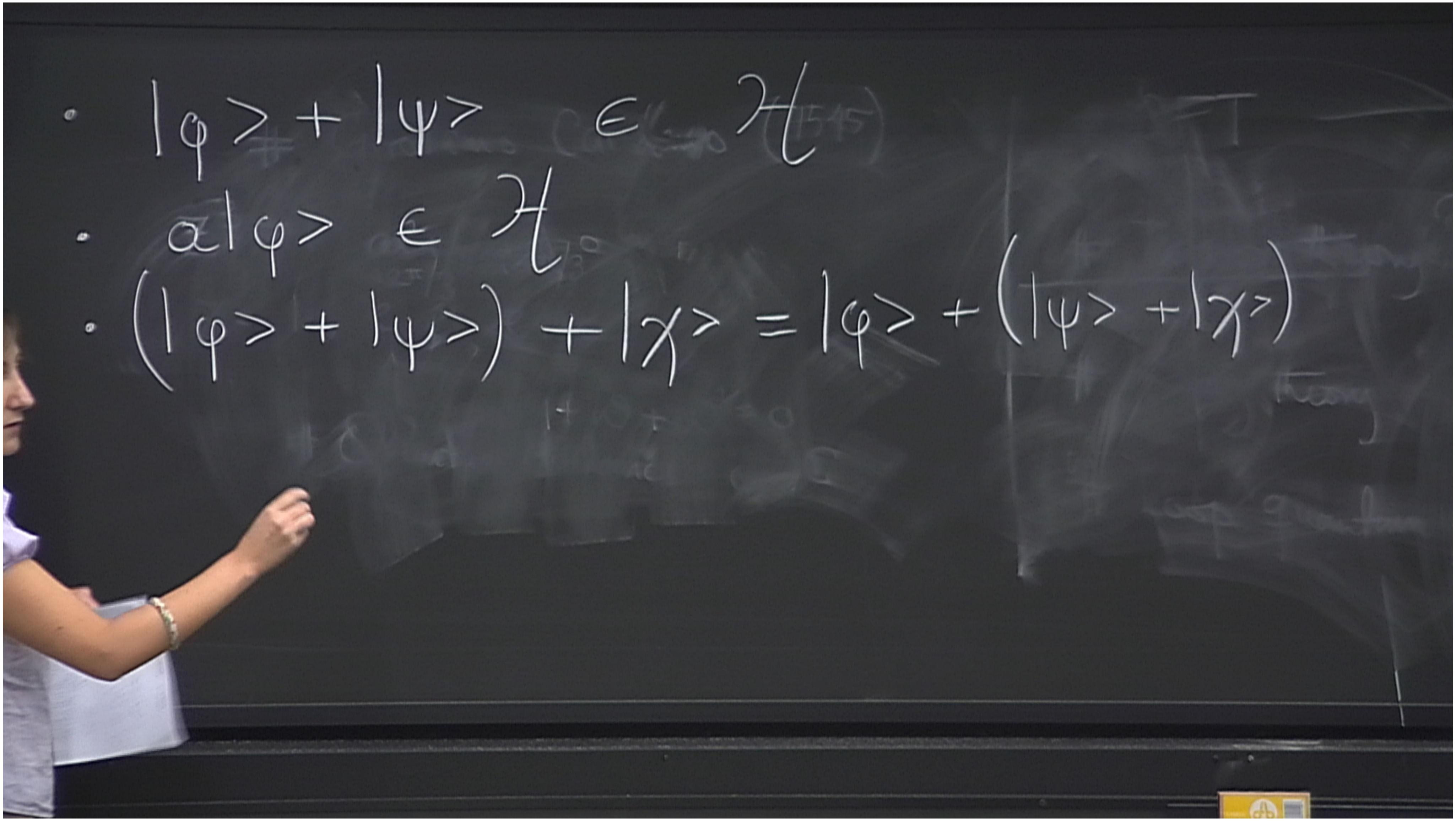

Space
pace :
$\mathcal{H} = \{ |\psi\rangle ; \text{all physical states} \}$

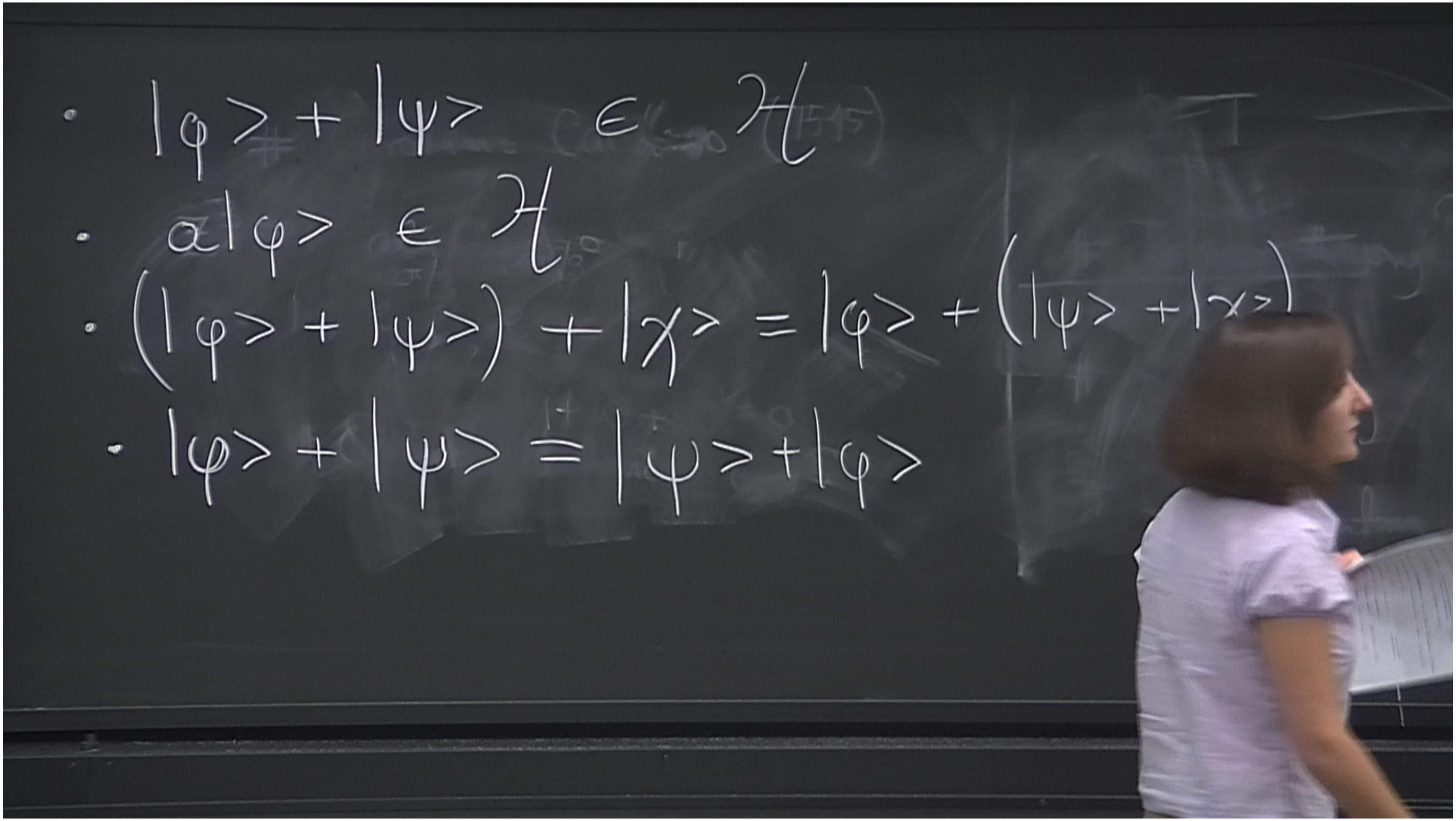

inear Vector Space
M: Hilbert space:
over the field ℂ
$\mathcal{H} = \{ |\psi\rangle$; all ph

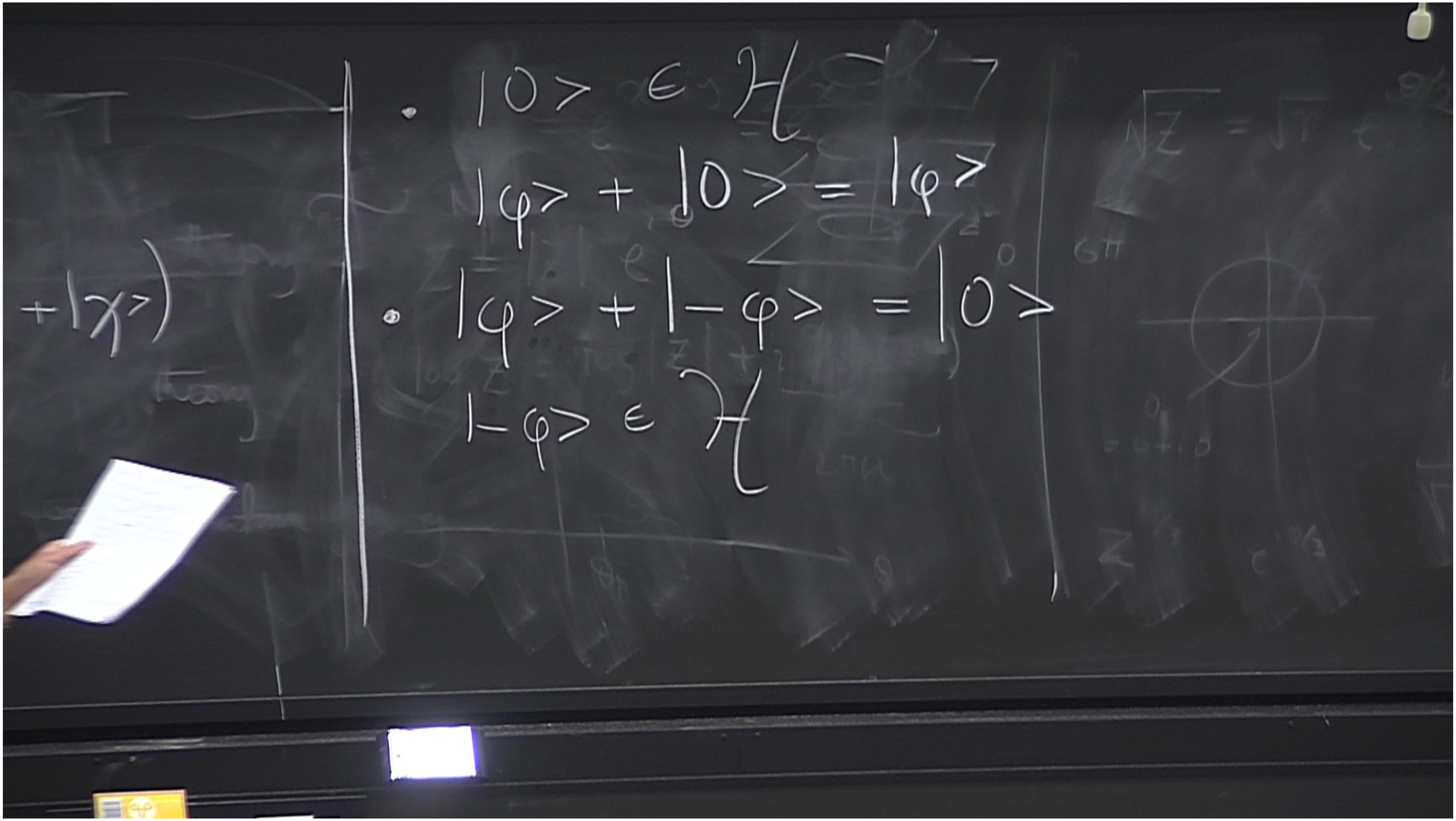

Linear Vector Space
QM: Hilbert space:
over the field ℂ
|φ>, |ψ>, |χ> ∈ ℋ
ℋ = { |ψ> ;

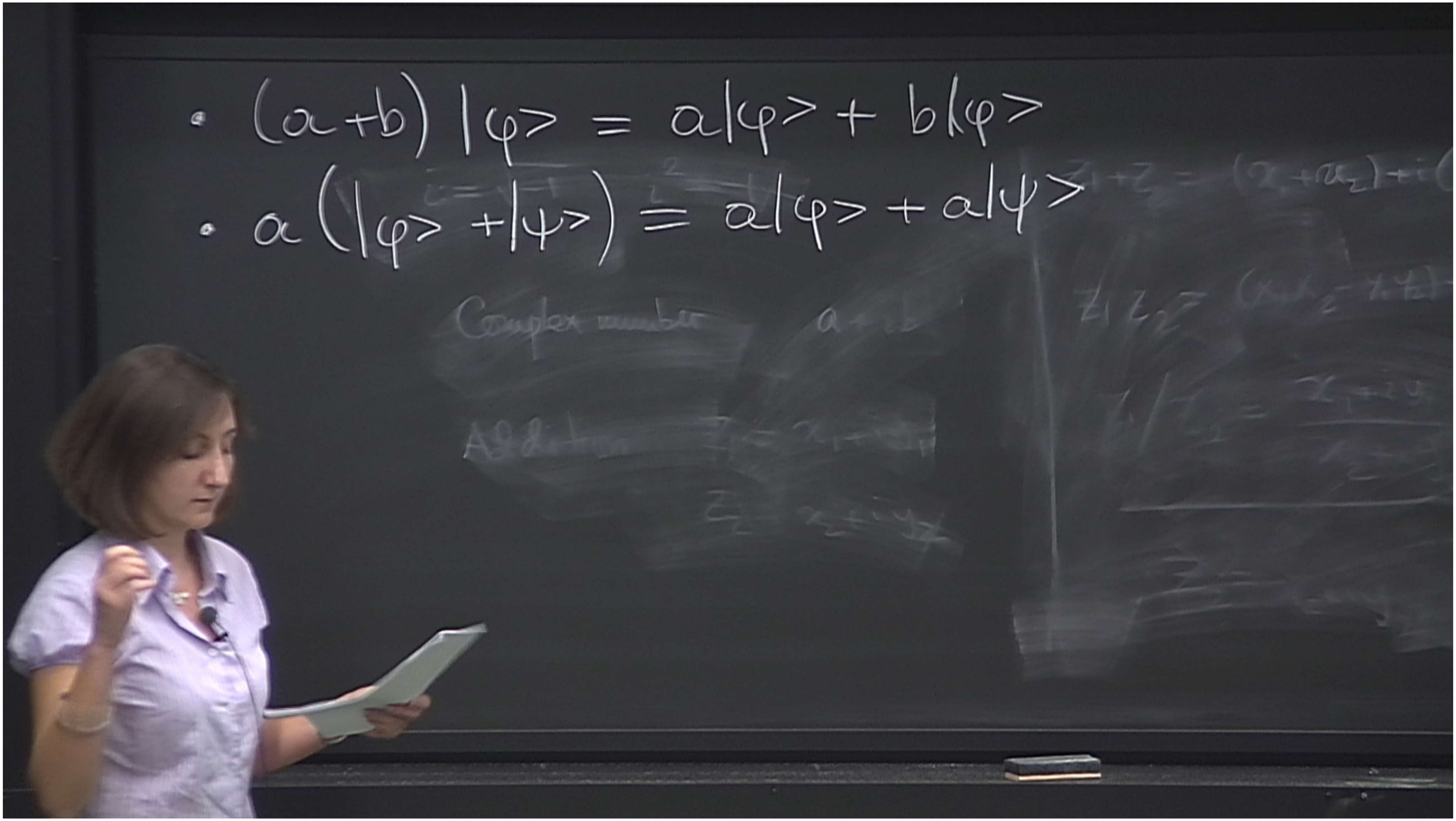

lbert space:
the field ℂ
>, |χ> ∈ H , a, b ∈ ℂ

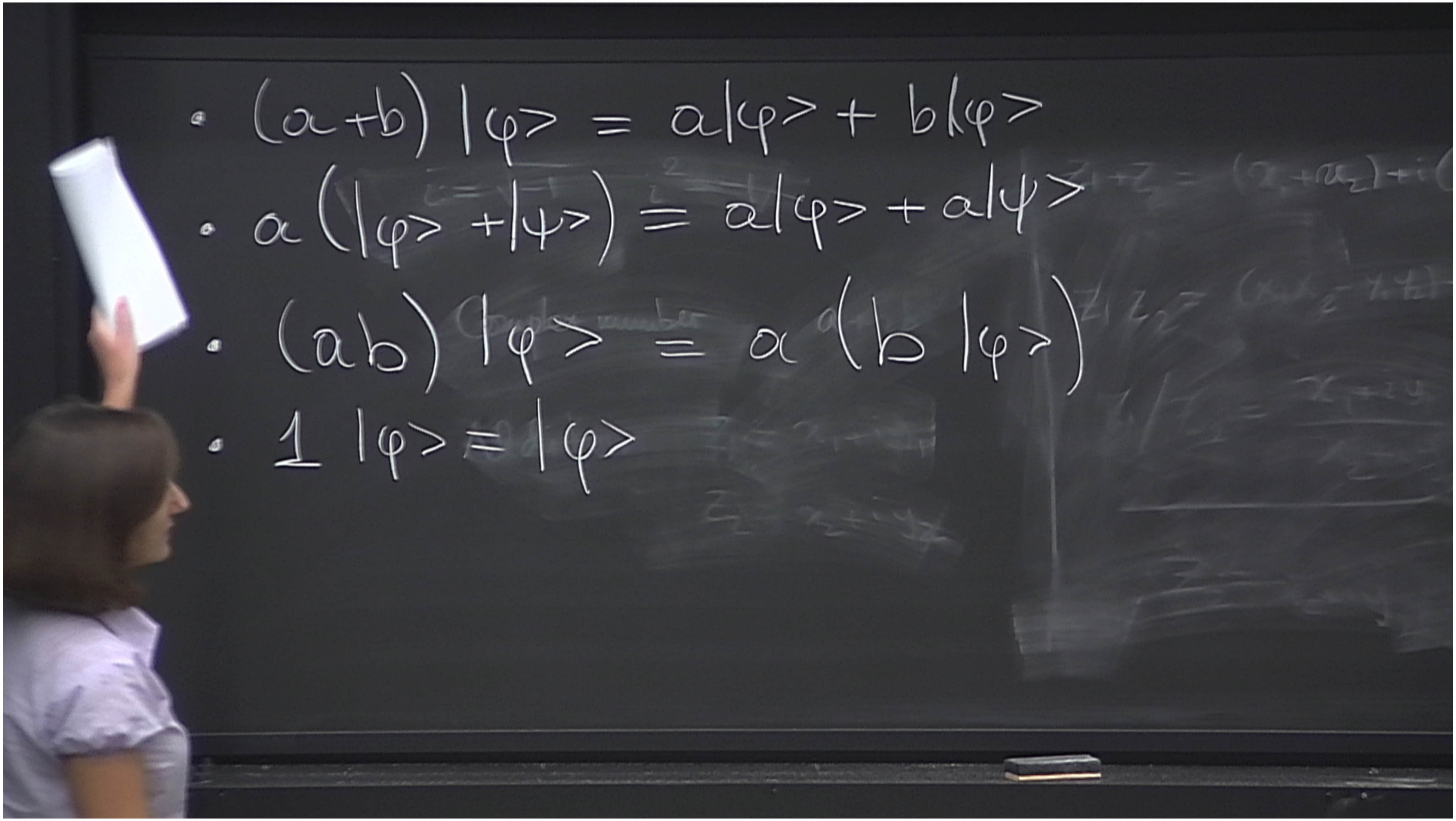

|φ> + |ψ> ∈ H
α|φ> ∈ H
> + |ψ>)

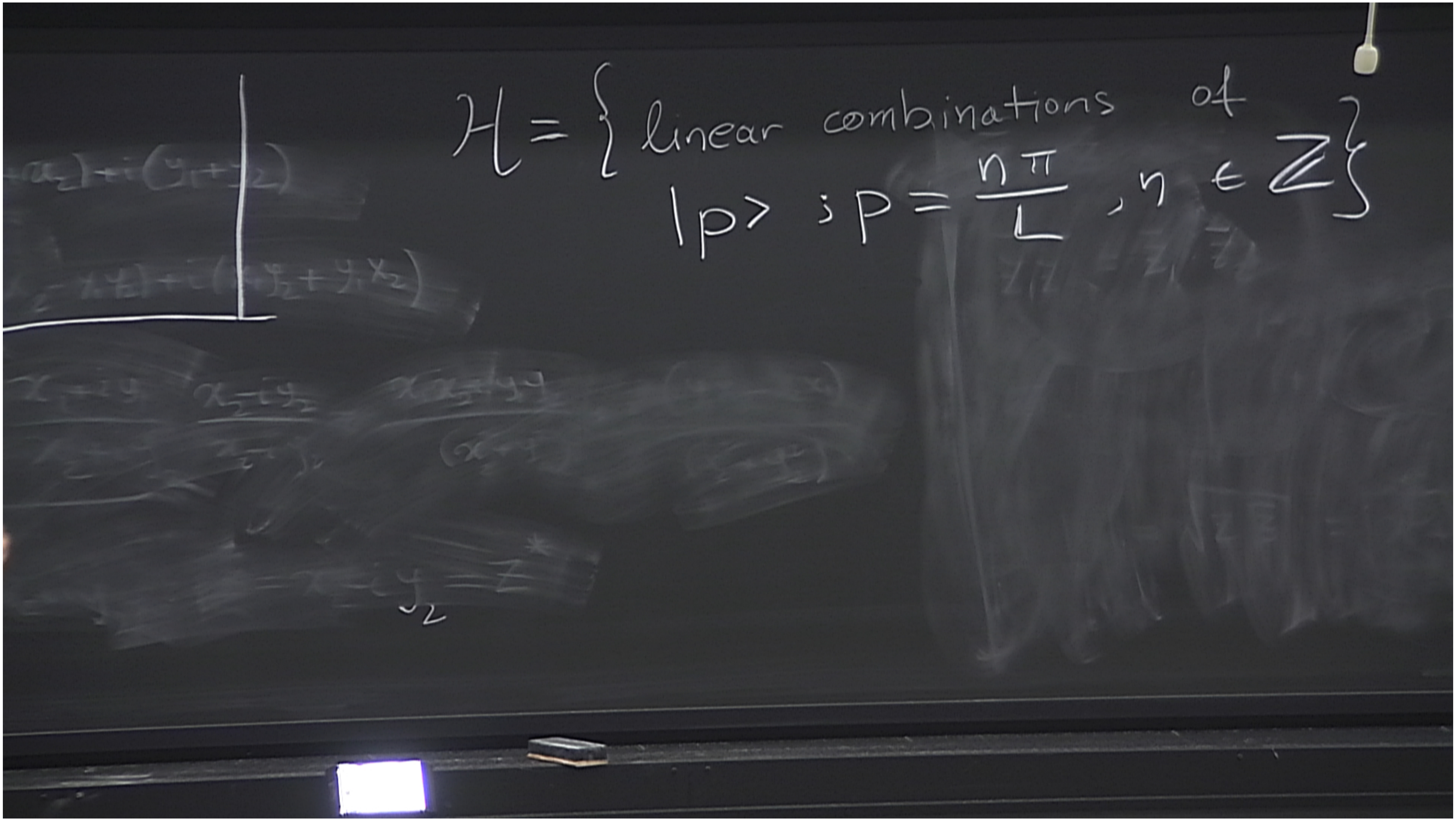

• |φ> + |ψ> ∈ ℋ
• α|φ> ∈ ℋ
• (|φ> + |ψ>) + |χ> = |φ> + (|ψ> + |χ>)

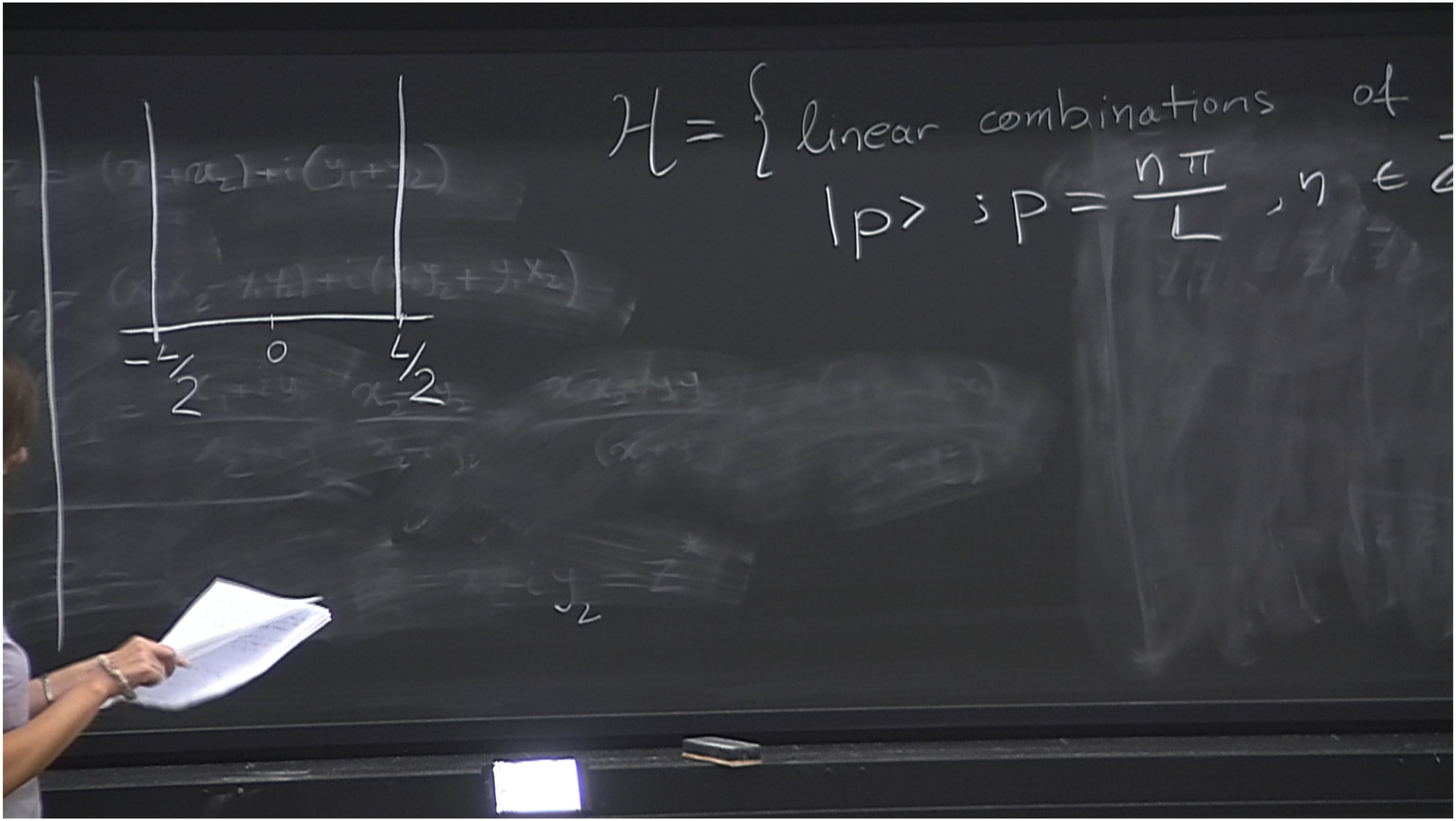

$|\varphi\rangle + |\psi\rangle \in \mathcal{H}$
$\alpha|\varphi\rangle \in \mathcal{H}$
$(|\varphi\rangle + |\psi\rangle) + |\chi\rangle = |\varphi\rangle + (|\psi\rangle + |\chi\rangle)$
$|\varphi\rangle + |\psi\rangle = |\psi\rangle + |\varphi\rangle$

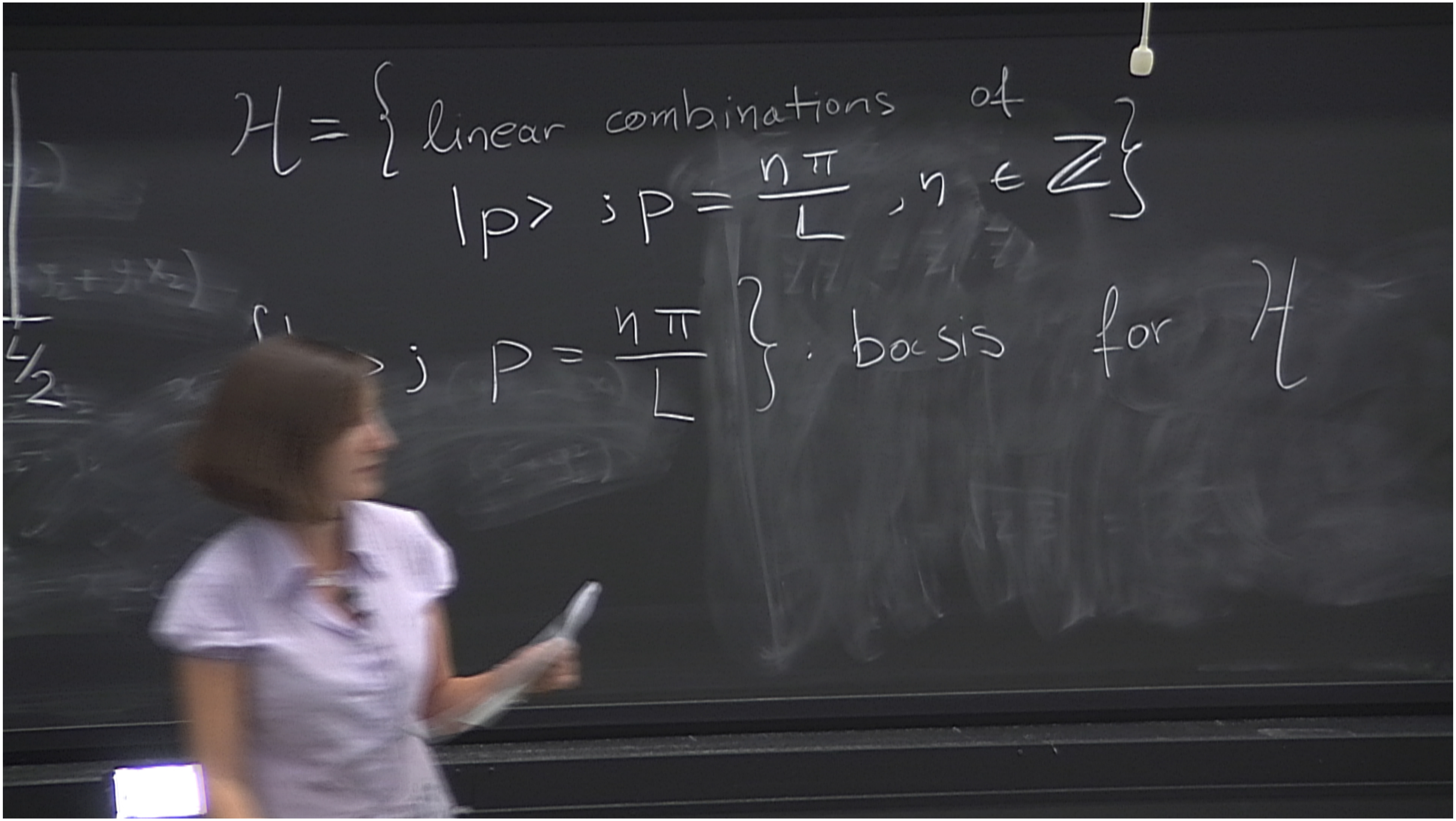

+|χ>)
|0> ∈ H
|φ> + |0> = |φ>
|φ> + |−φ> = |0>
|−φ> ∈ H

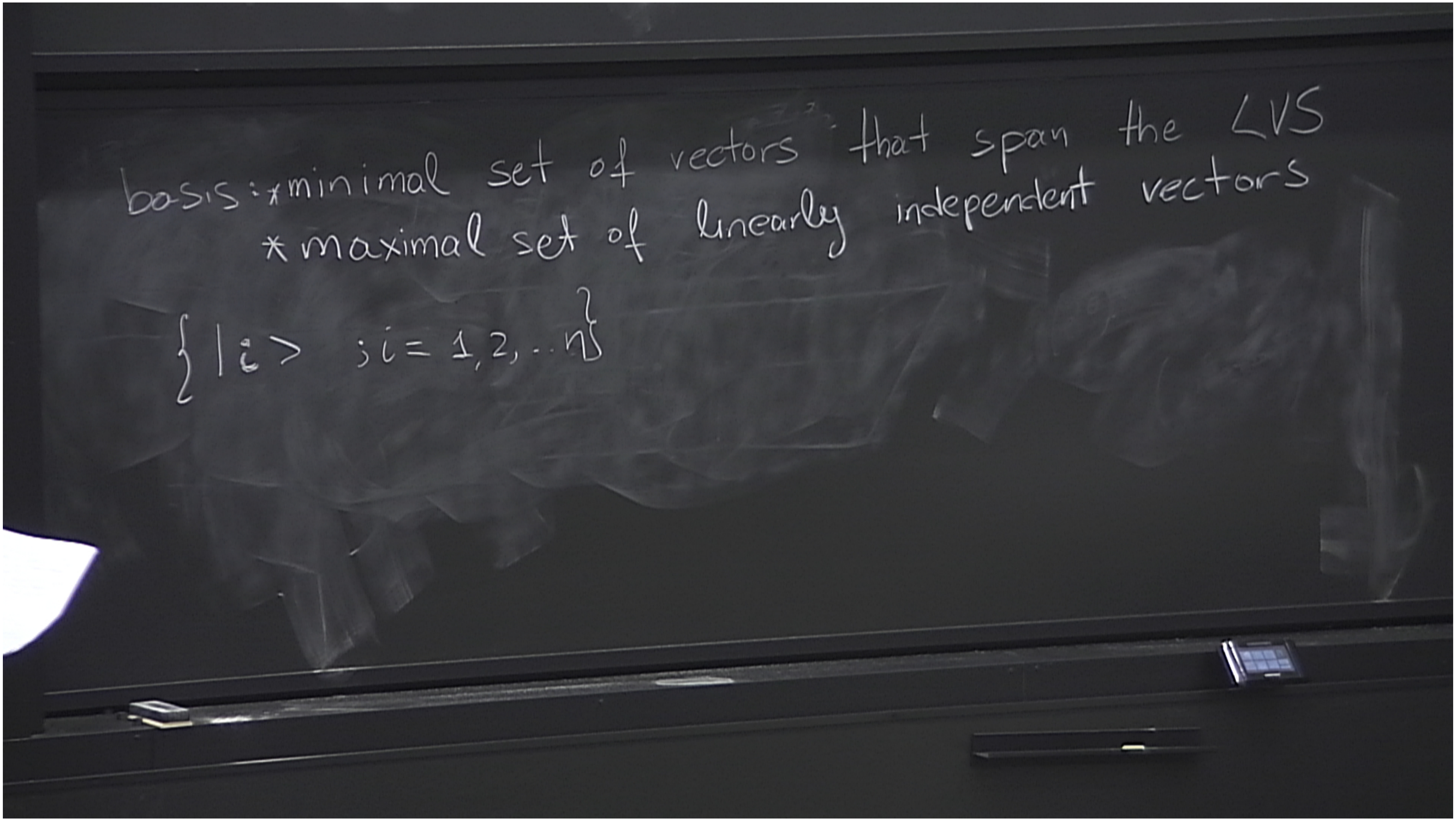

• (a+b)|φ> = a|φ> + b|φ>
• a(|φ> + |ψ>) = a|φ> + a|ψ>

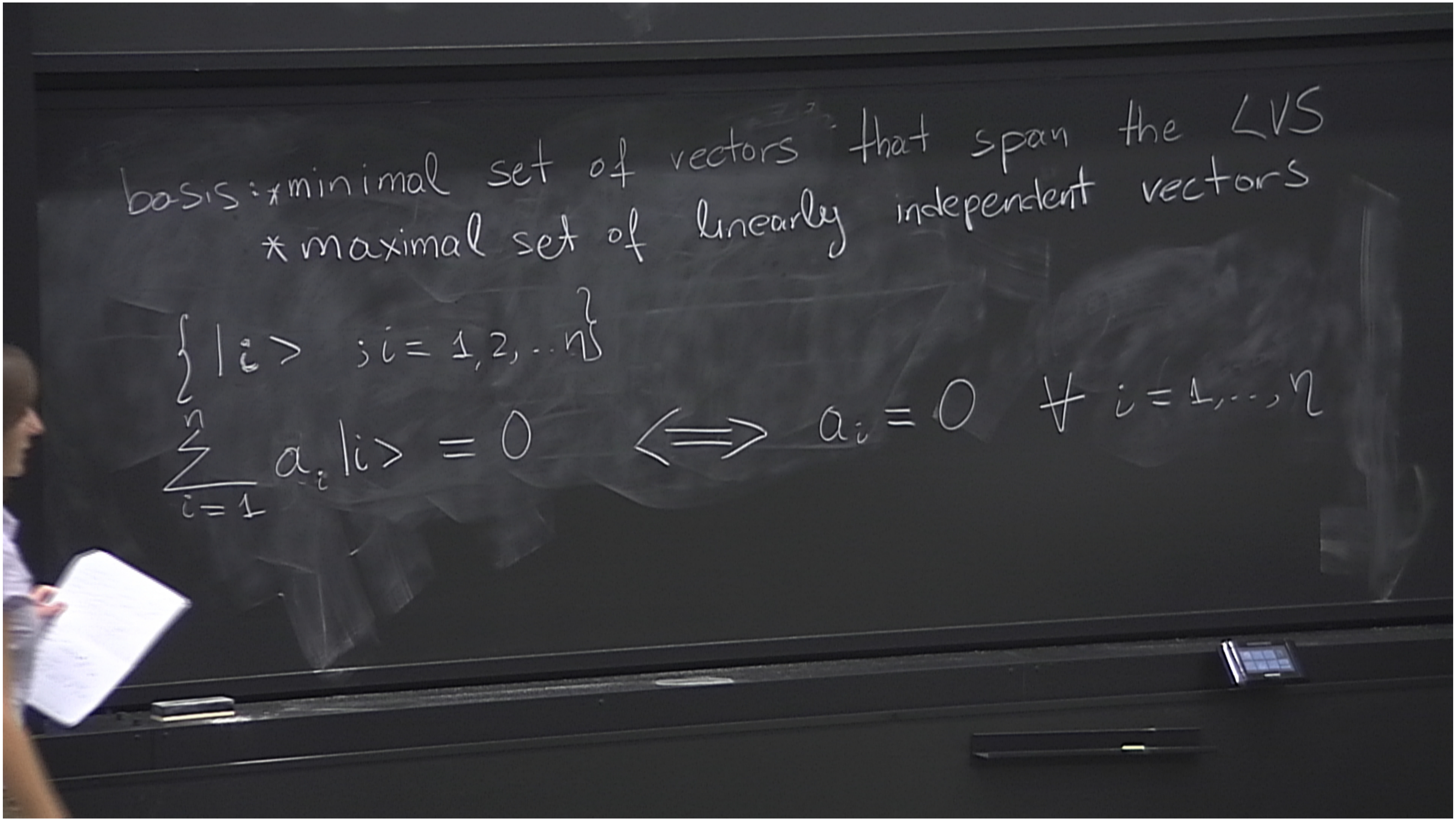

• (a+b)|φ> = a|φ> + b|φ>
• a(|φ> + |ψ>) = a|φ> + a|ψ>
• (ab)|φ> = a(b|φ>)
• 1|φ> = |φ>

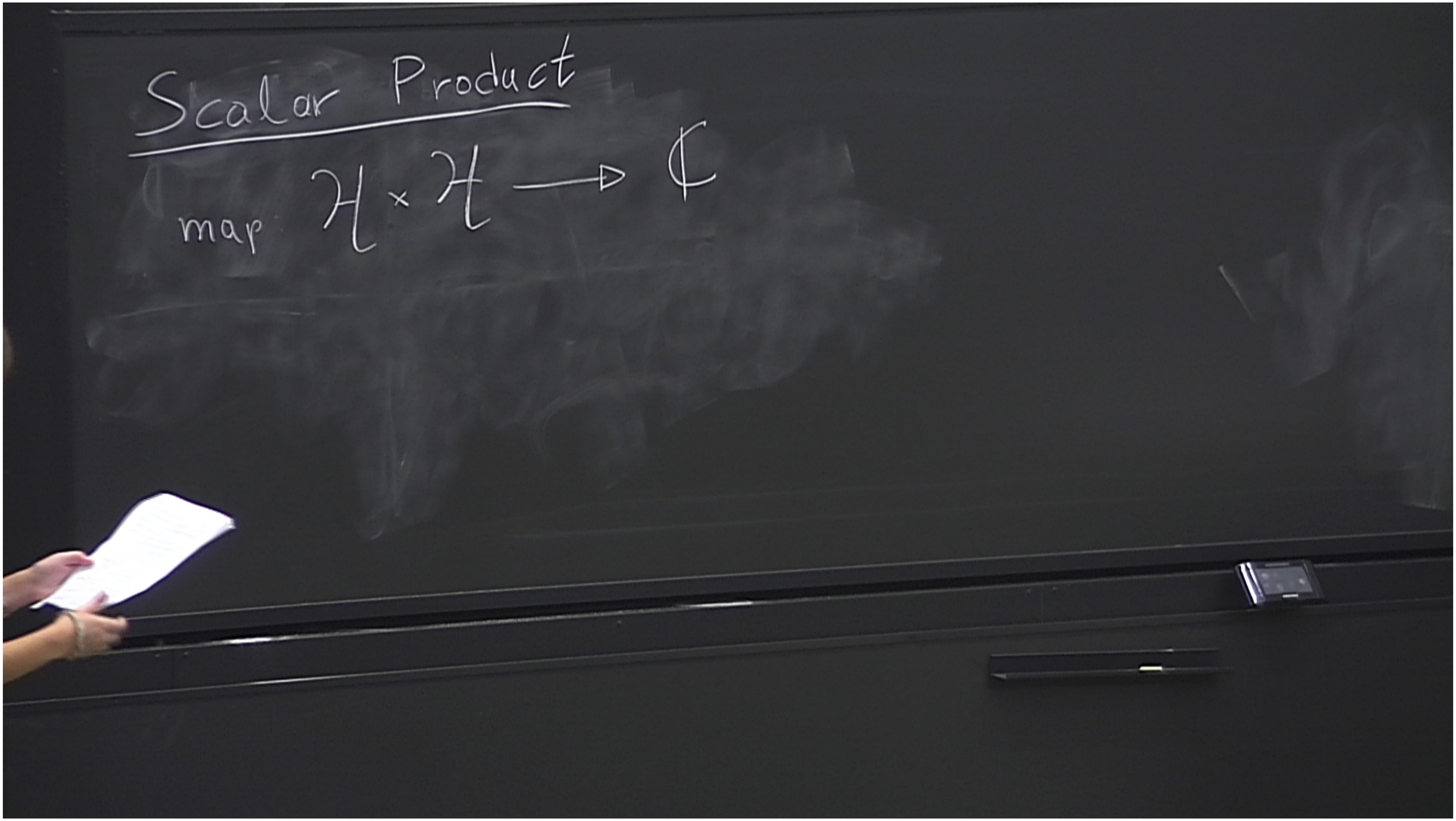

$\mathcal{H} = \{$ linear combinations of
$|p\rangle$; $p = \frac{n\pi}{L}$, $n \in \mathbb{Z}\}$

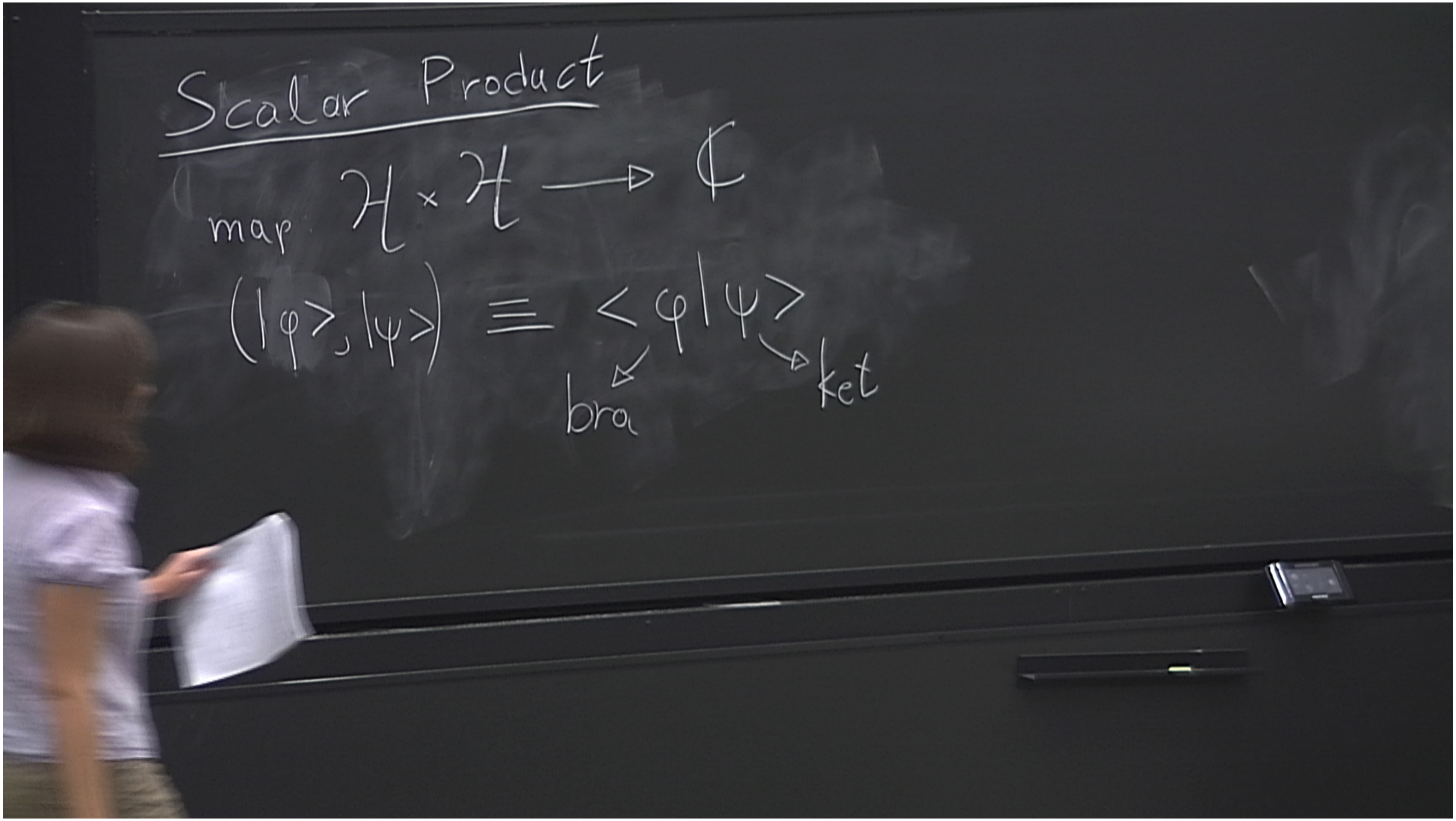

H = { linear combinations of
|p> ; p = nπ/L , n ∈
-L/2
0
L/2

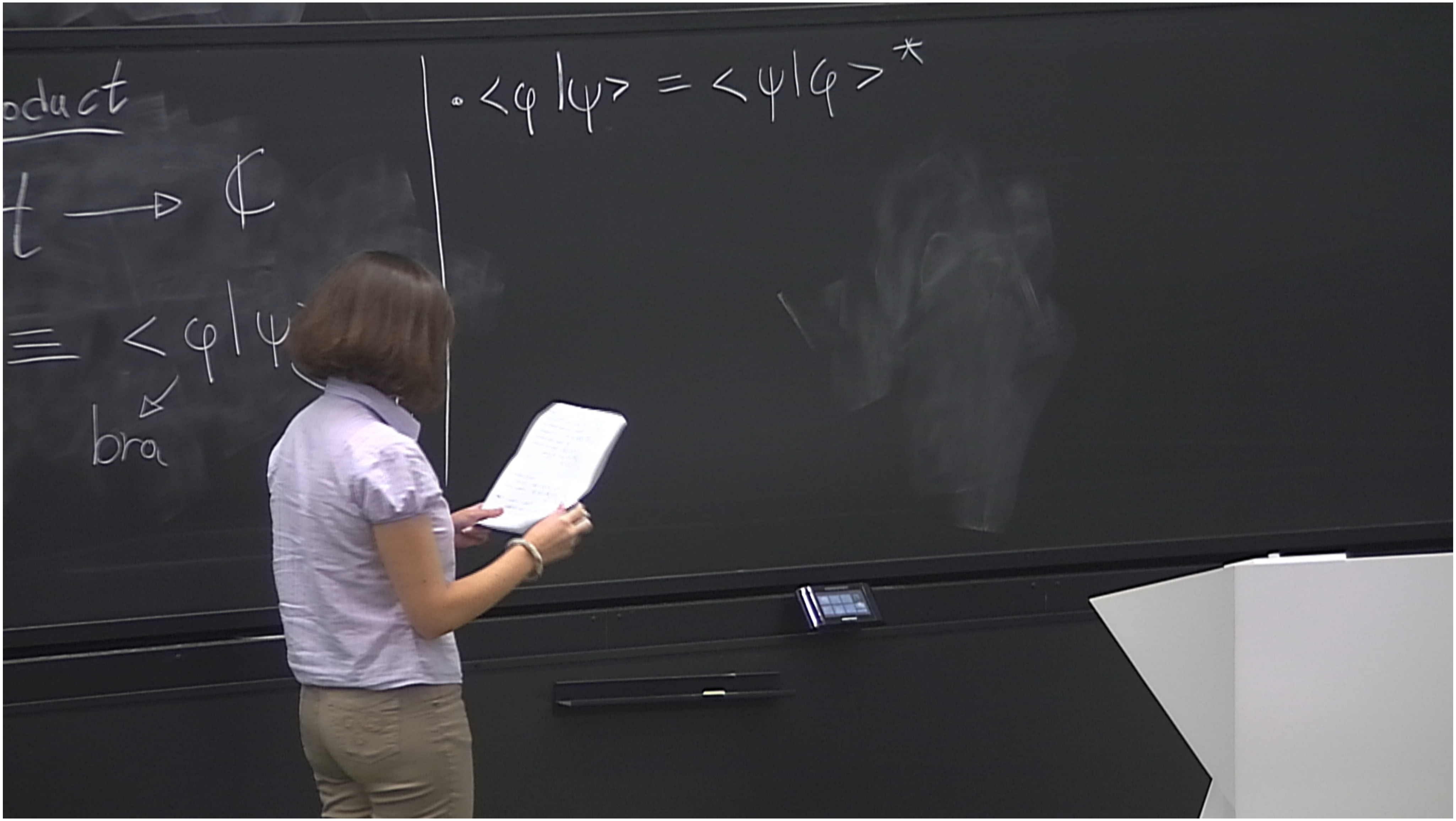

$\mathcal{H} = \{$ linear combinations of $|p\rangle ; p = \frac{n\pi}{L}, n \in \mathbb{Z} \}$
$; \; p = \frac{n\pi}{L} \}$: basis for $\mathcal{H}$

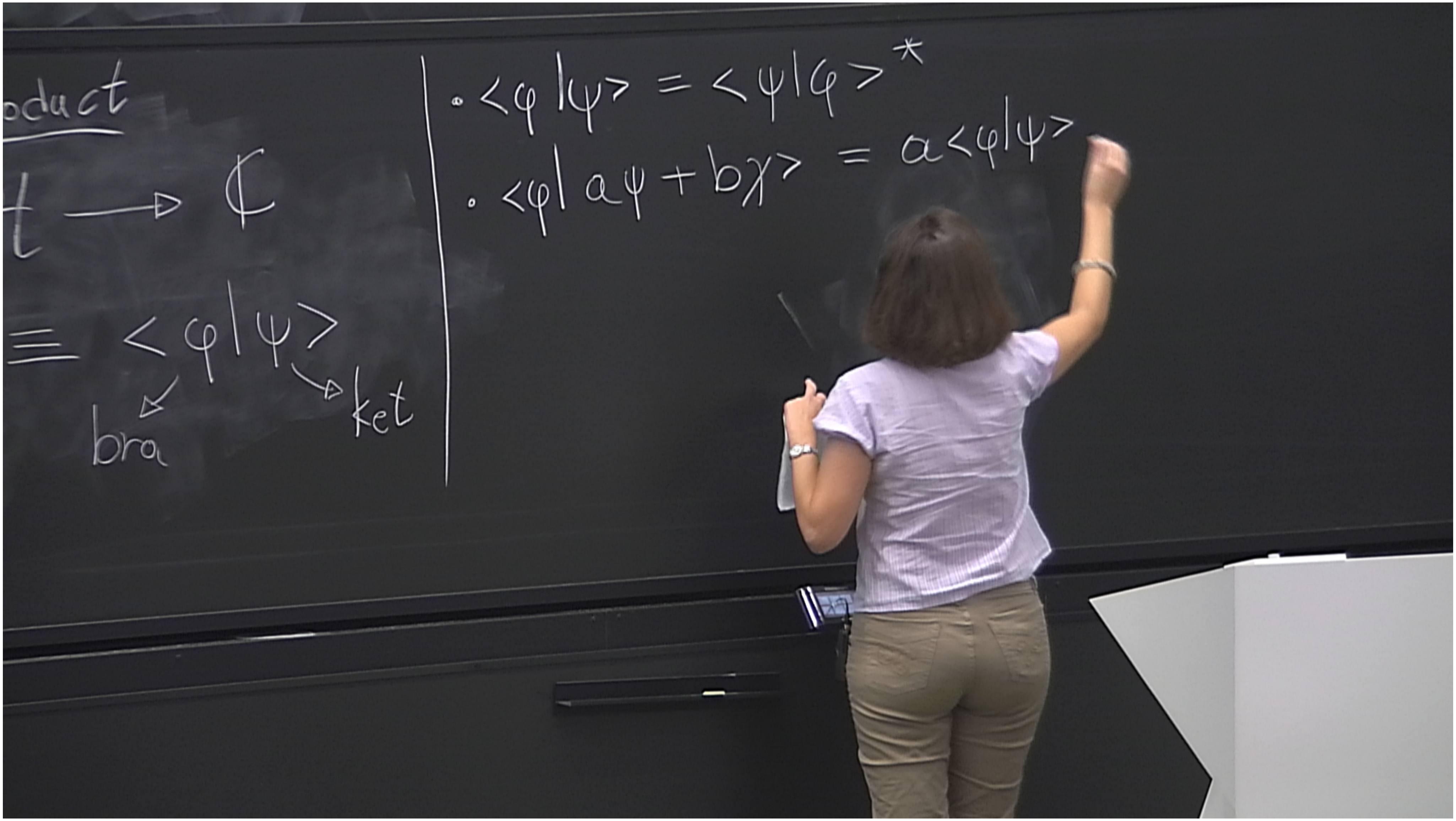

basis: * minimal set of vectors that span the LVS
* maximal set of linearly independent vectors
{ |i> ; i = 1, 2, .. n }

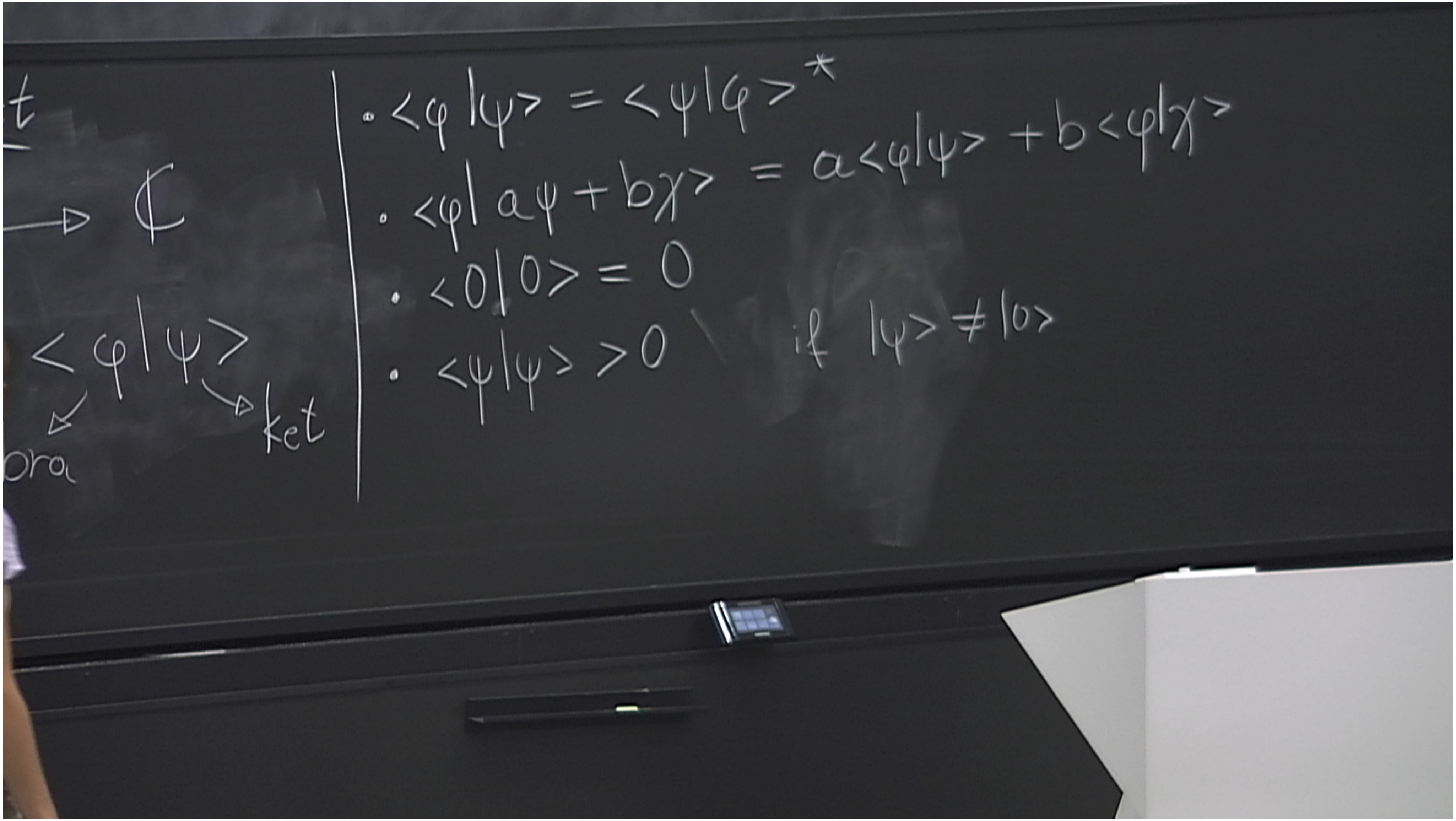

basis: * minimal set of vectors that span the LVS
* maximal set of linearly independent vectors
$\{ |i\rangle \; ; i = 1, 2, \ldots n \}$
$\sum_{i=1}^{n} a_i |i\rangle = 0 \iff a_i = 0 \;\; \forall \, i = 1, \ldots, n$


Scalar Product
map $\mathcal{H} \times \mathcal{H} \longrightarrow \mathbb{C}$

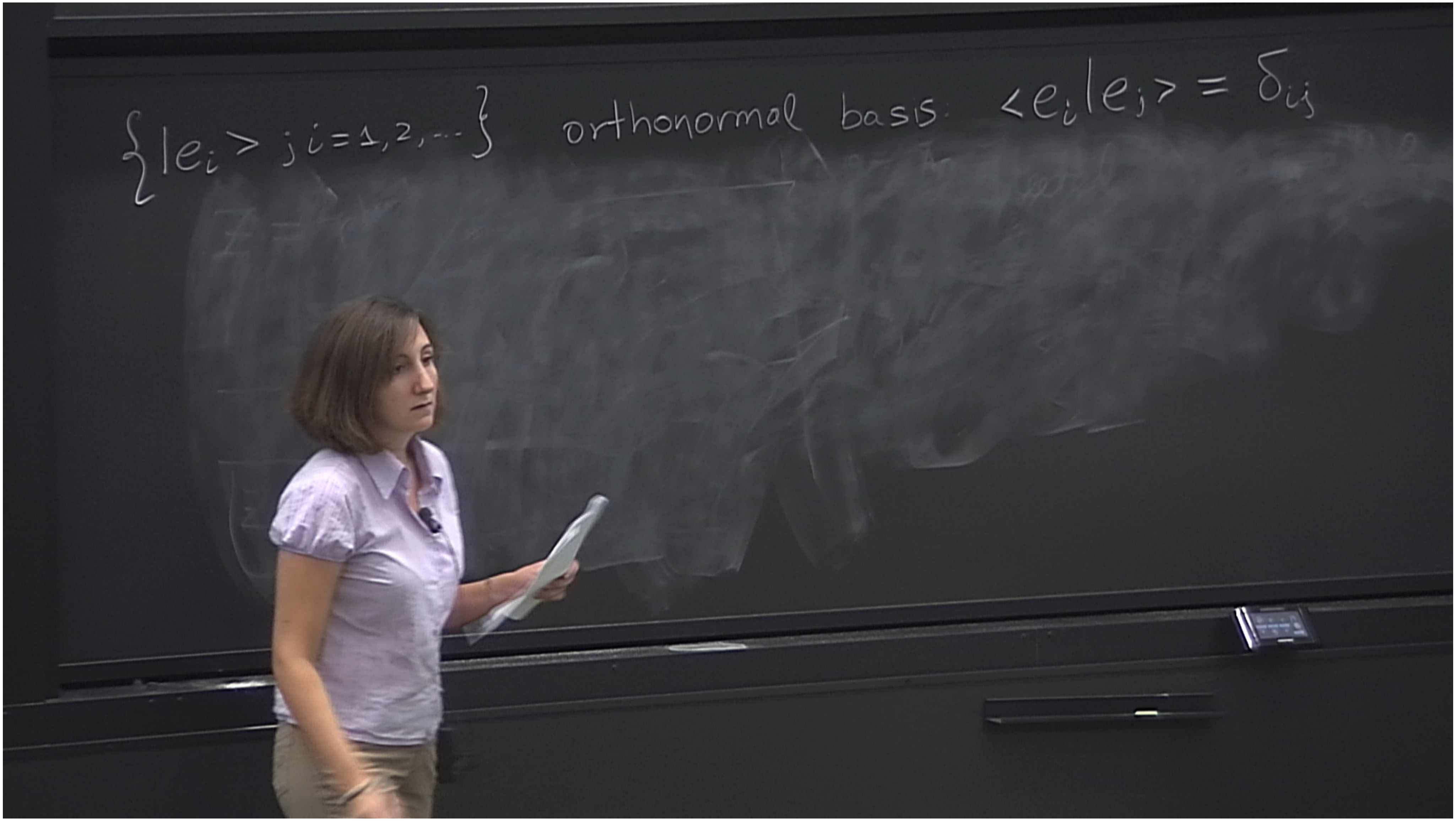

Scalar Product
map $\mathcal{H} \times \mathcal{H} \longrightarrow \mathbb{C}$
$(|\varphi\rangle, |\psi\rangle) \equiv \langle\varphi|\psi\rangle$
bra
ket

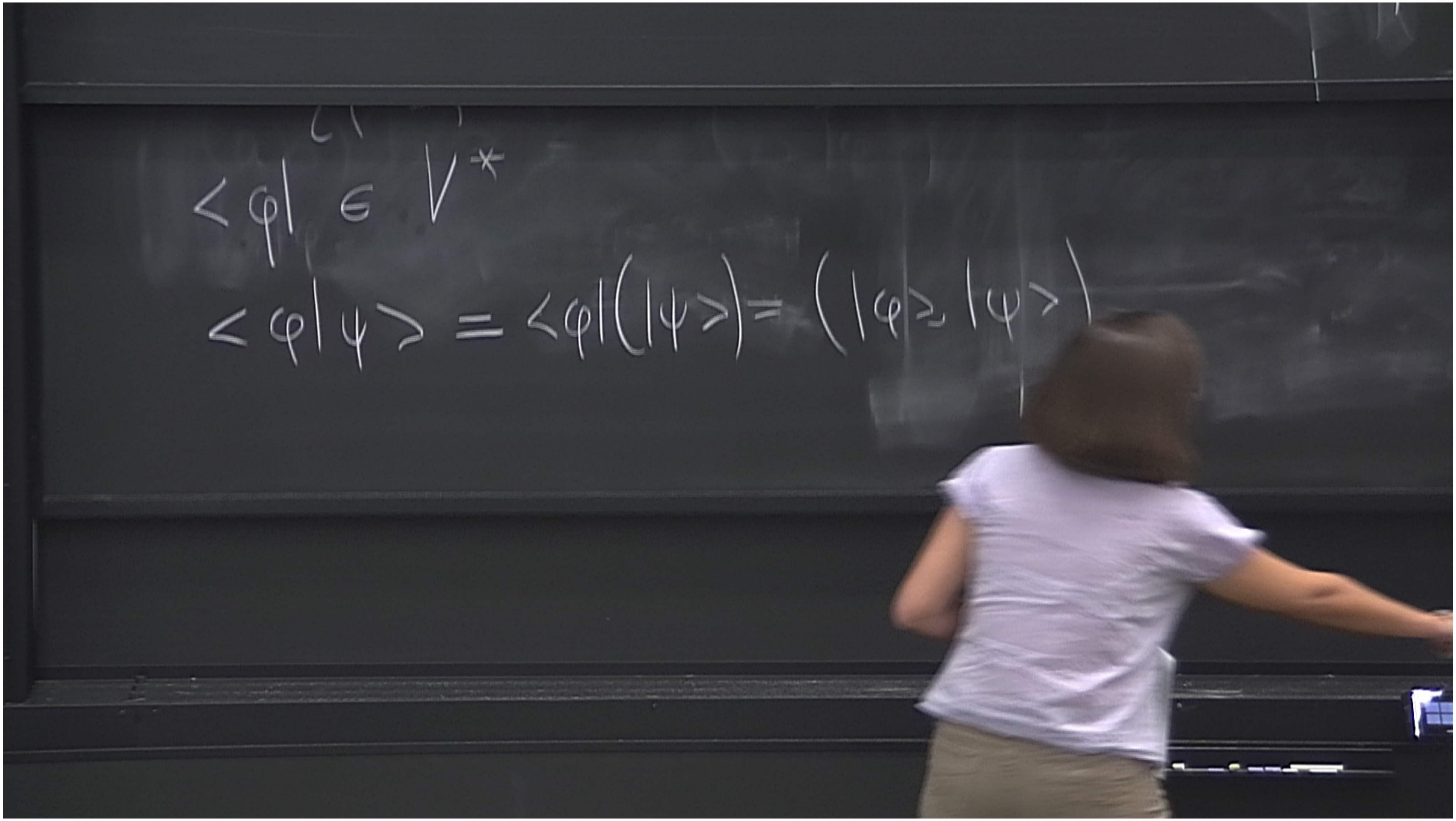

oduct
≡ <φ|ψ
bra
• <φ|ψ> = <ψ|φ>*

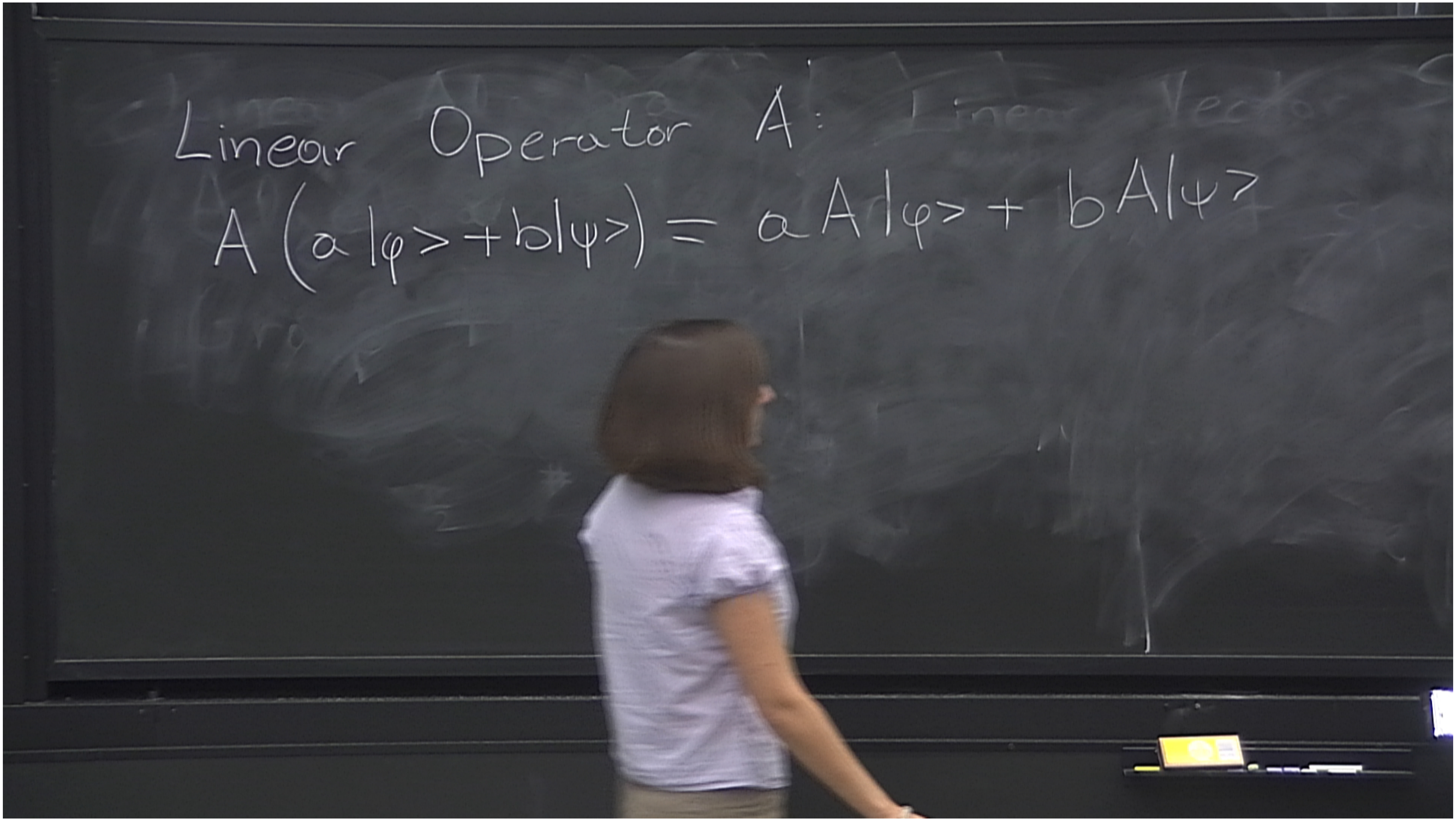

oduct
≡ <φ|ψ>
bra
ket
• <φ|ψ> = <ψ|φ>*
• <φ|aψ + bχ> = a<φ|ψ>

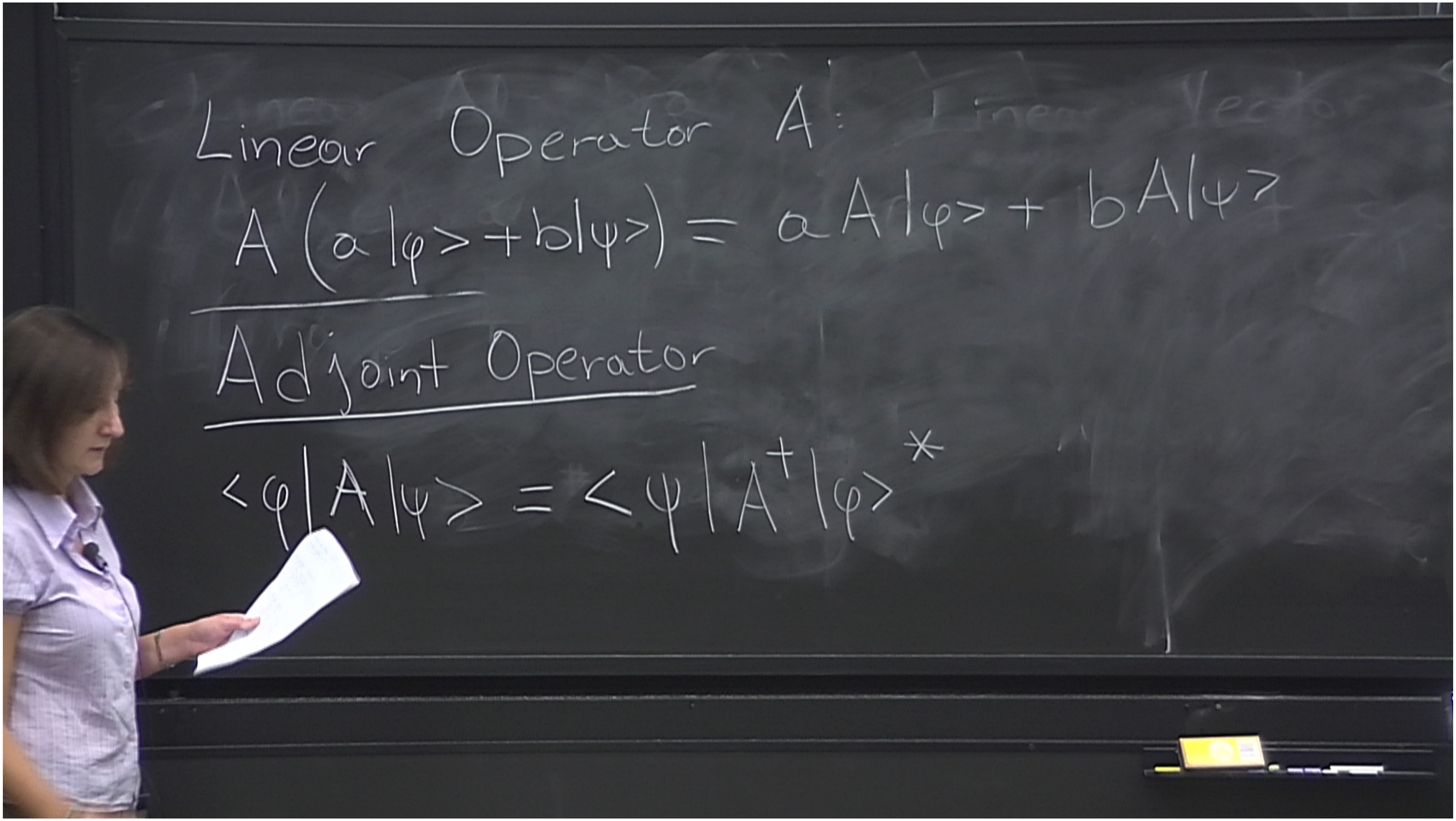


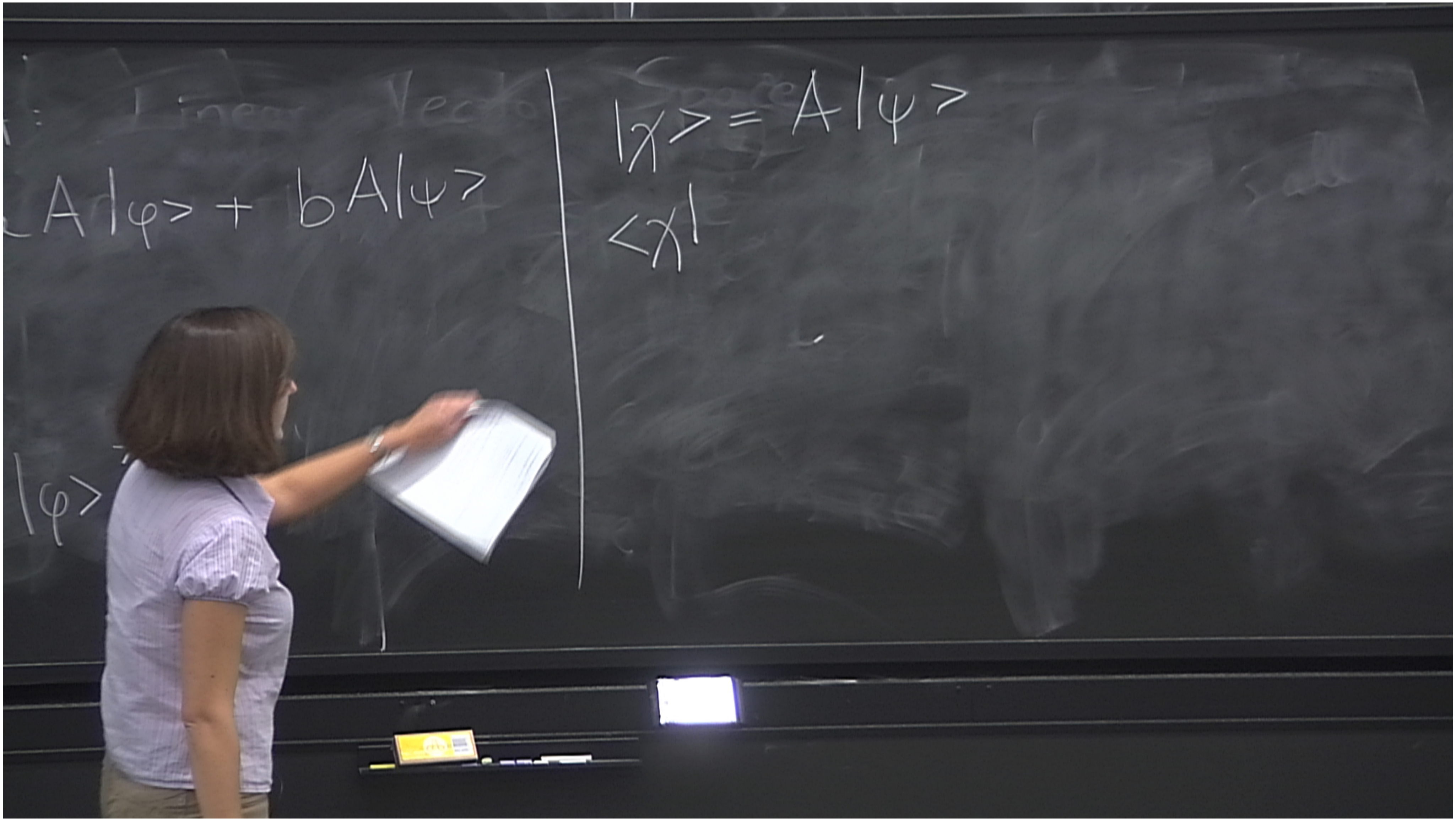
t
→ ℂ
< φ | ψ >
bra
ket
• <φ|ψ> = <ψ|φ>*
• <φ|aψ + bχ> = a<φ|ψ> + b<φ|χ>
• <0|0> = 0
• <ψ|ψ> > 0 if |ψ> ≠ |0>

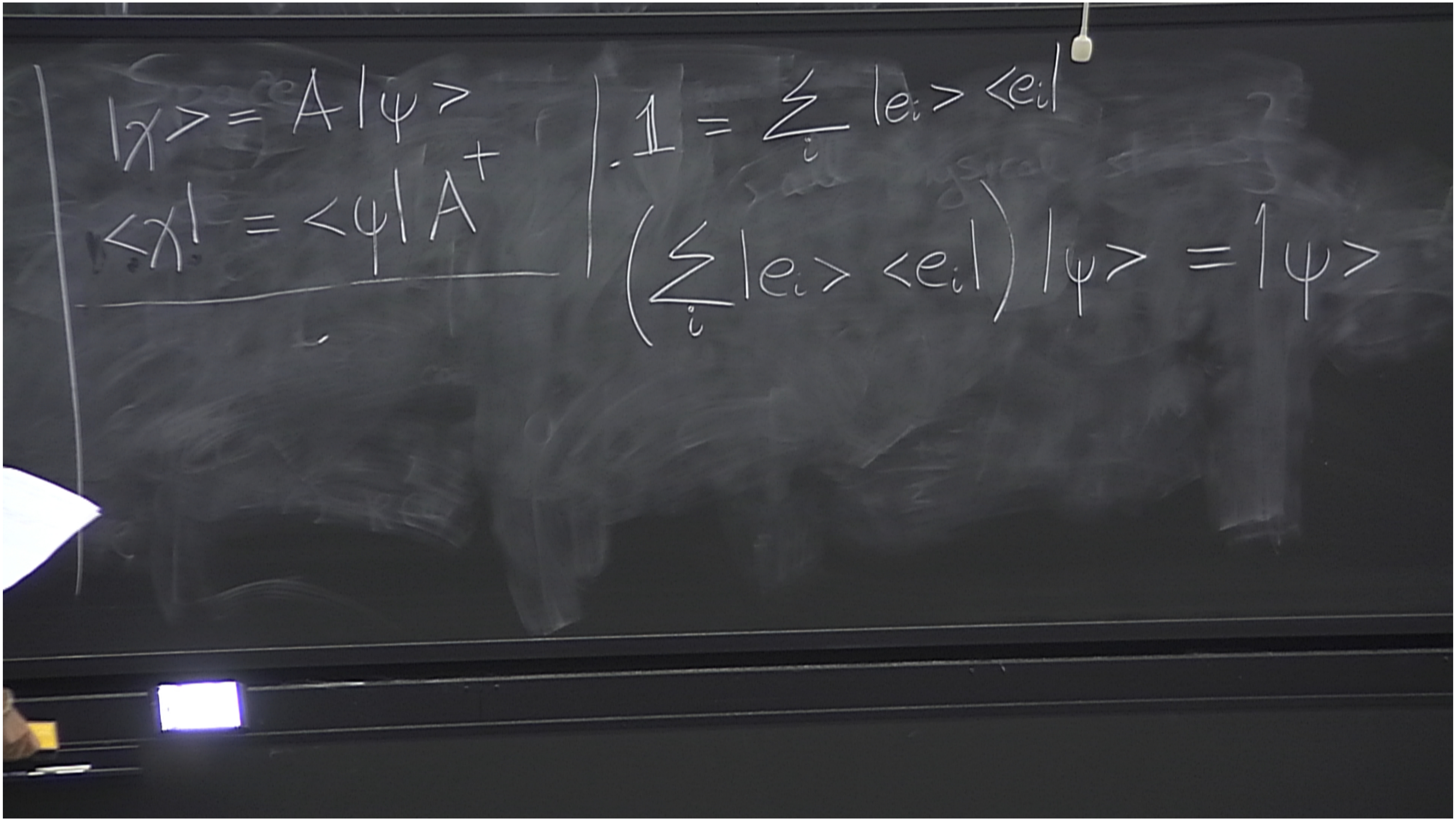

$\{|e_i\rangle ; i=1,2,\dots\}$ orthonormal basis: $\langle e_i|e_j\rangle = \delta_{ij}$

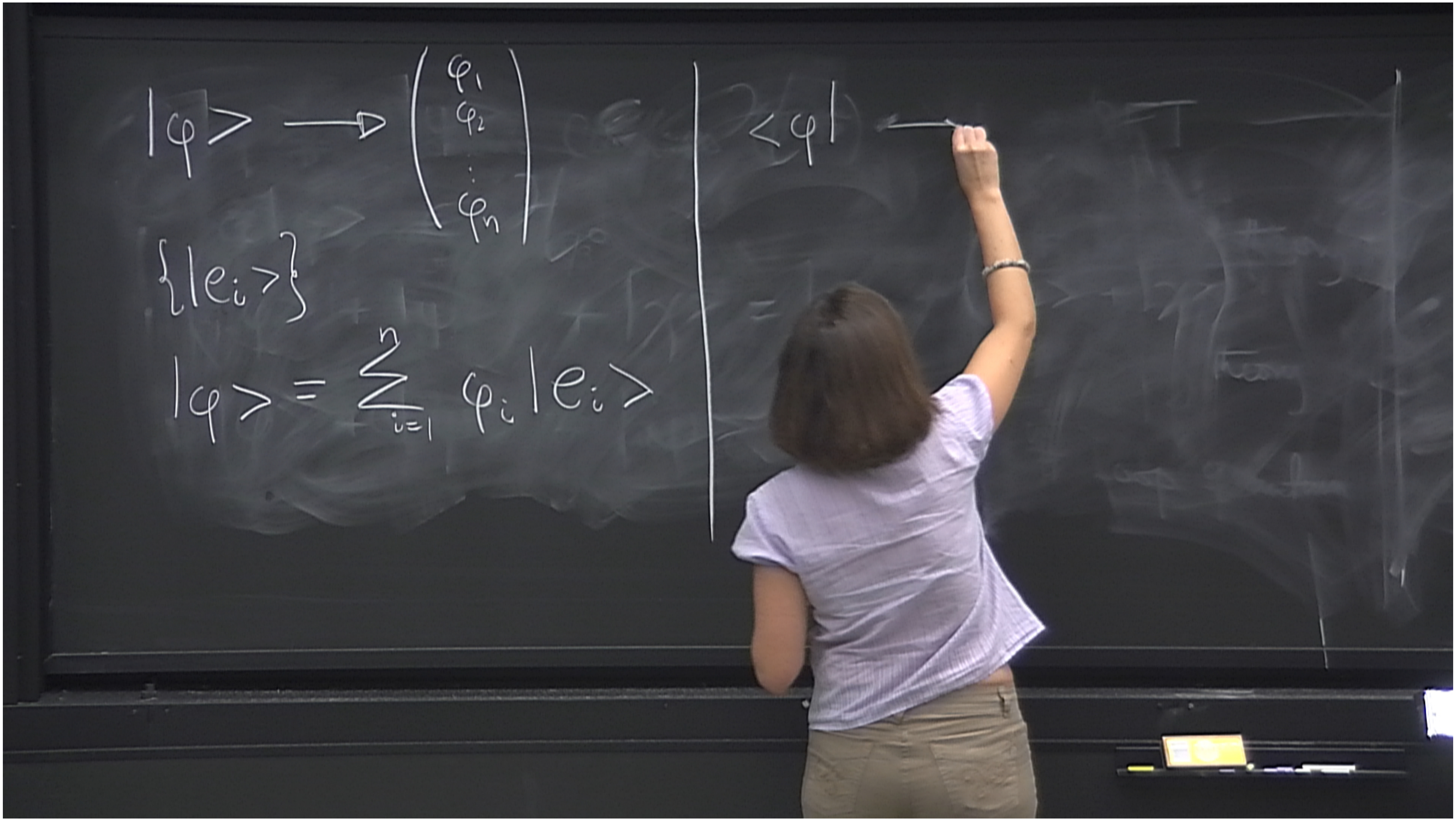

$\langle \varphi | \in V^*$
$\langle \varphi | \psi \rangle = \langle \varphi | (| \psi \rangle) = (| \varphi \rangle , | \psi \rangle)$

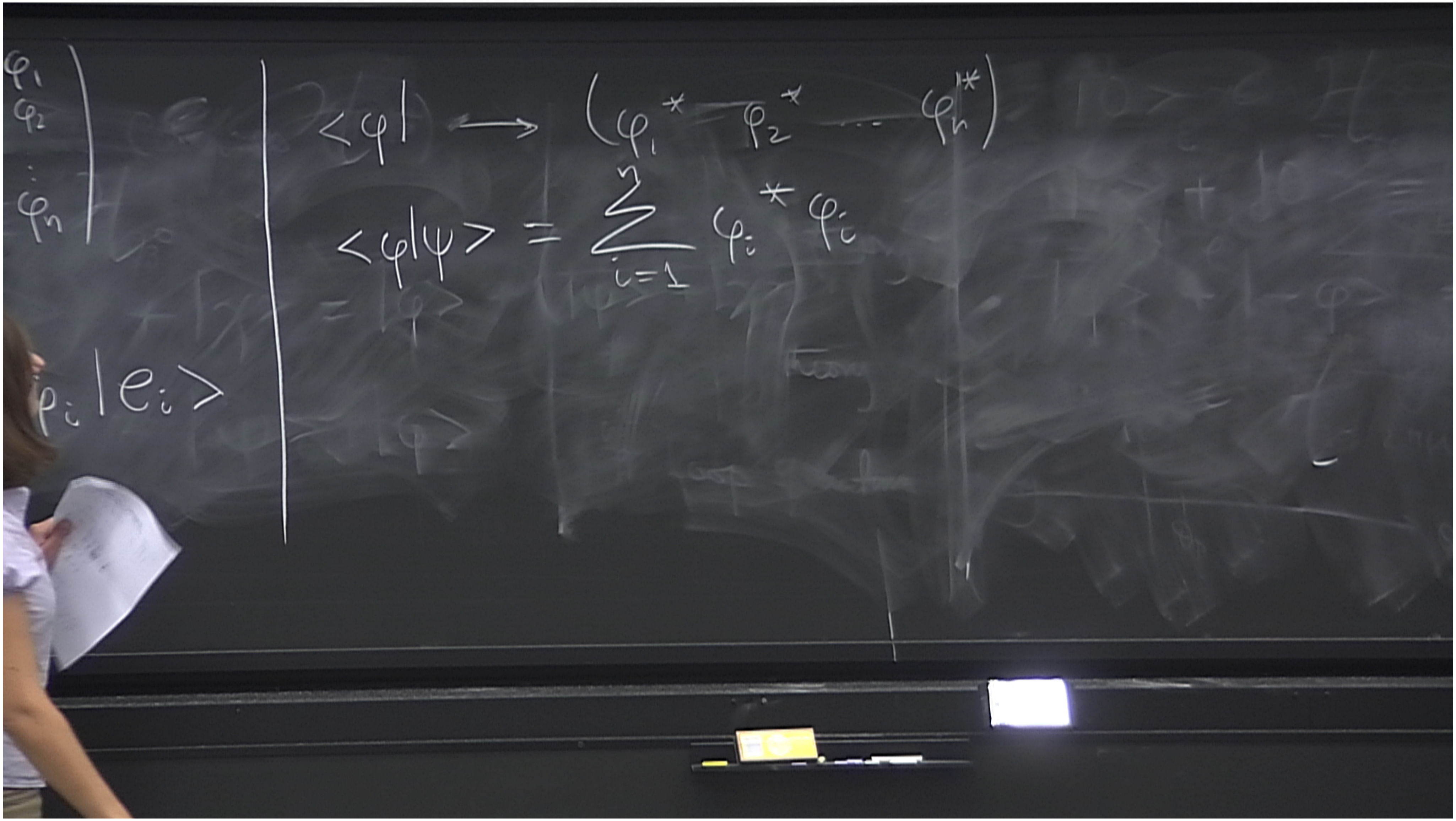

Linear Operator A:
A(a|φ> + b|ψ>) = aA|φ> + bA|ψ>

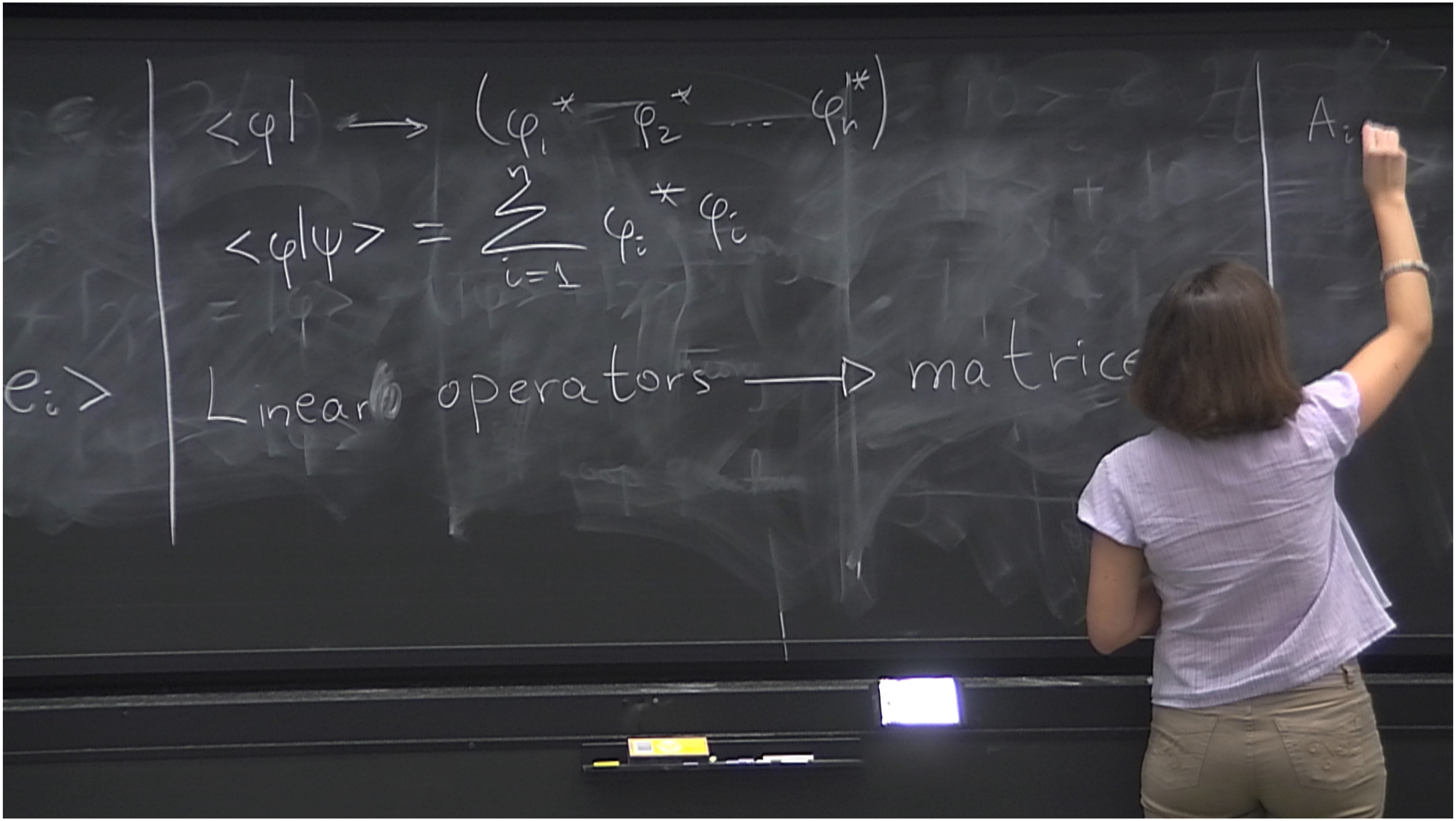

Linear Operator A:
A(a|φ> + b|ψ>) = aA|φ> + bA|ψ>
Adjoint Operator
<φ|A|ψ> = <ψ|A†|φ>*

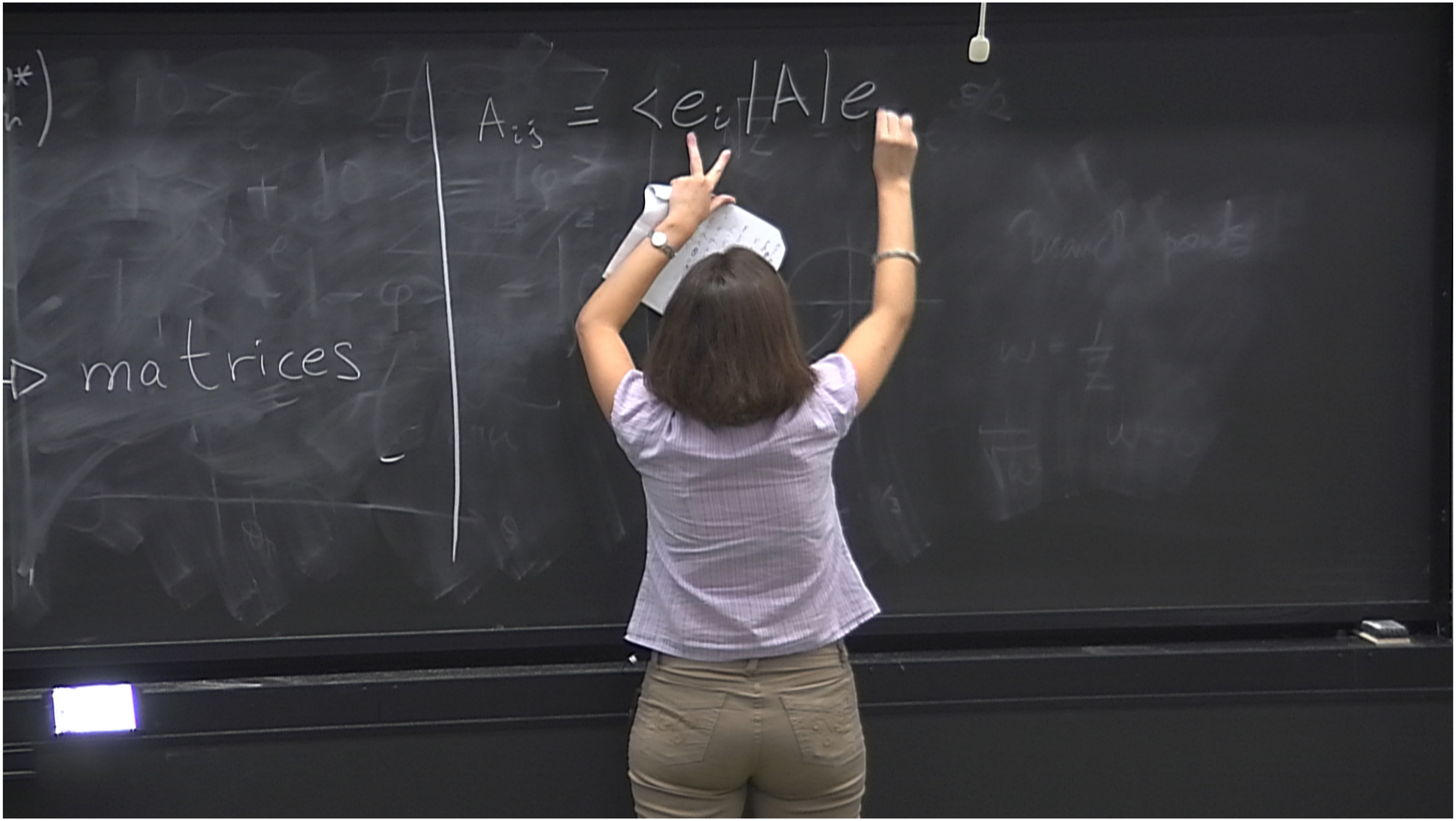

|χ> = A|ψ>
<χ|
A|φ> + bA|ψ>
|φ>


|χ> = A|ψ>
<χ| = <ψ|A†
𝟙 = Σ_i |e_i><e_i|
(Σ_i |e_i><e_i|) |ψ> = |ψ>

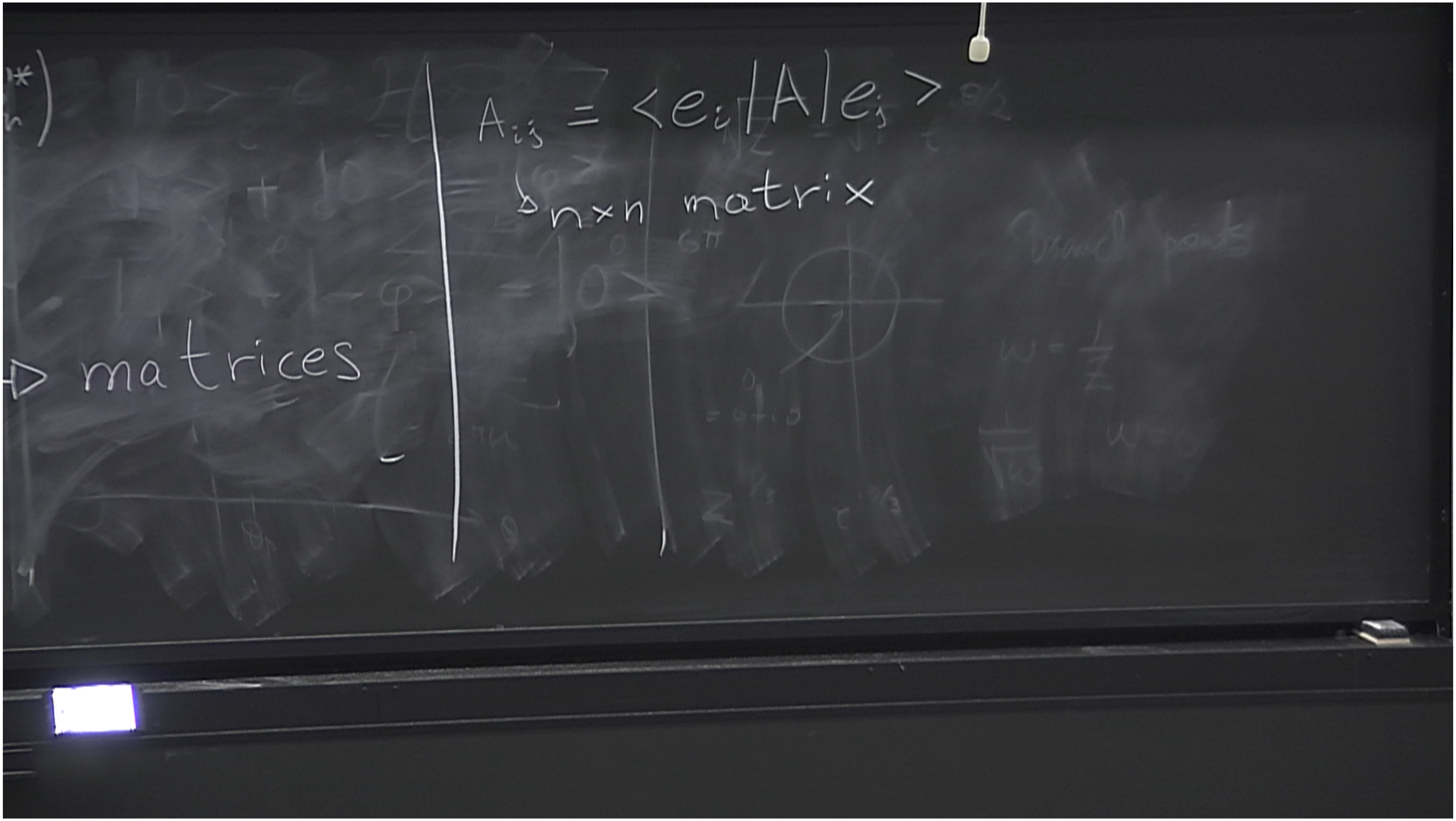

$|\varphi\rangle \longrightarrow \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_n \end{pmatrix}$
$\{|e_i\rangle\}$
$|\varphi\rangle = \sum_{i=1}^{n} \varphi_i |e_i\rangle$
$\langle\varphi|$


⟨φ| ⟷ (φ1* φ2* ... φn*)
⟨φ|ψ⟩ = Σ i=1 to n φi* ψi

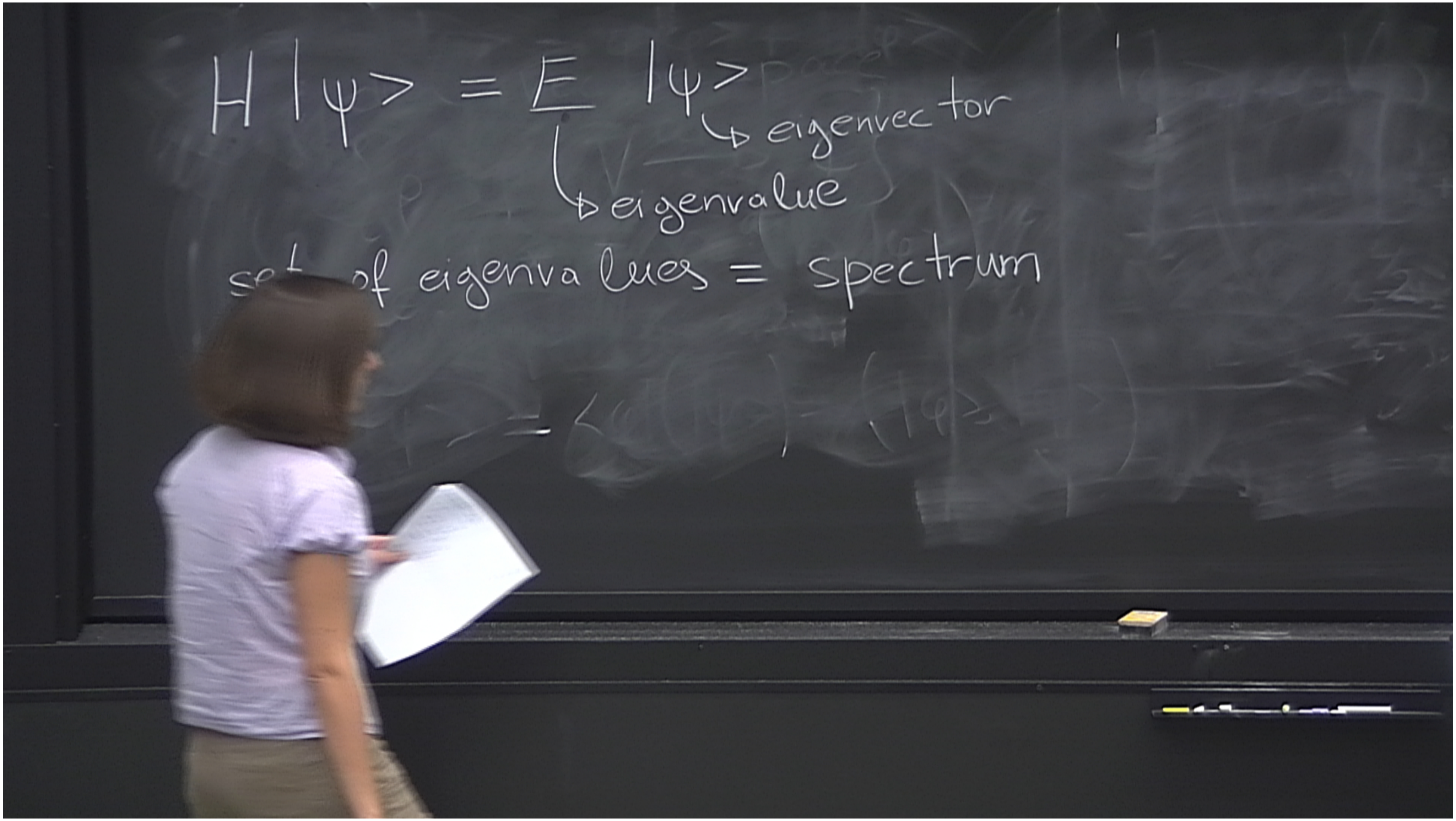

$\langle\varphi| \longrightarrow (\varphi_1^* \quad \varphi_2^* \quad \cdots \quad \varphi_n^*)$
$\langle\varphi|\psi\rangle = \sum_{i=1}^{n} \varphi_i^* \psi_i$
$e_i\rangle$
Linear operators ⟶ matrice
A_i

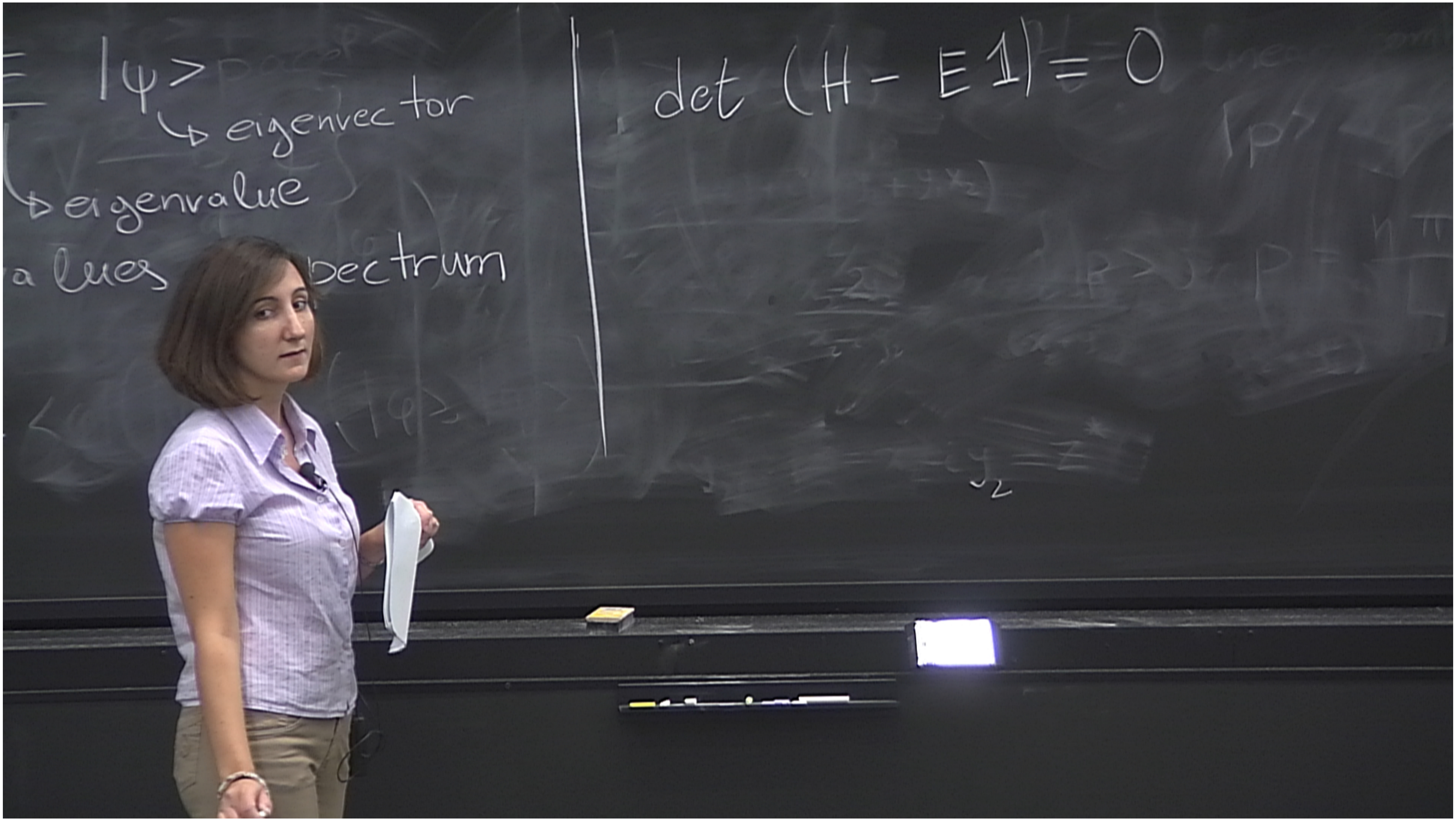


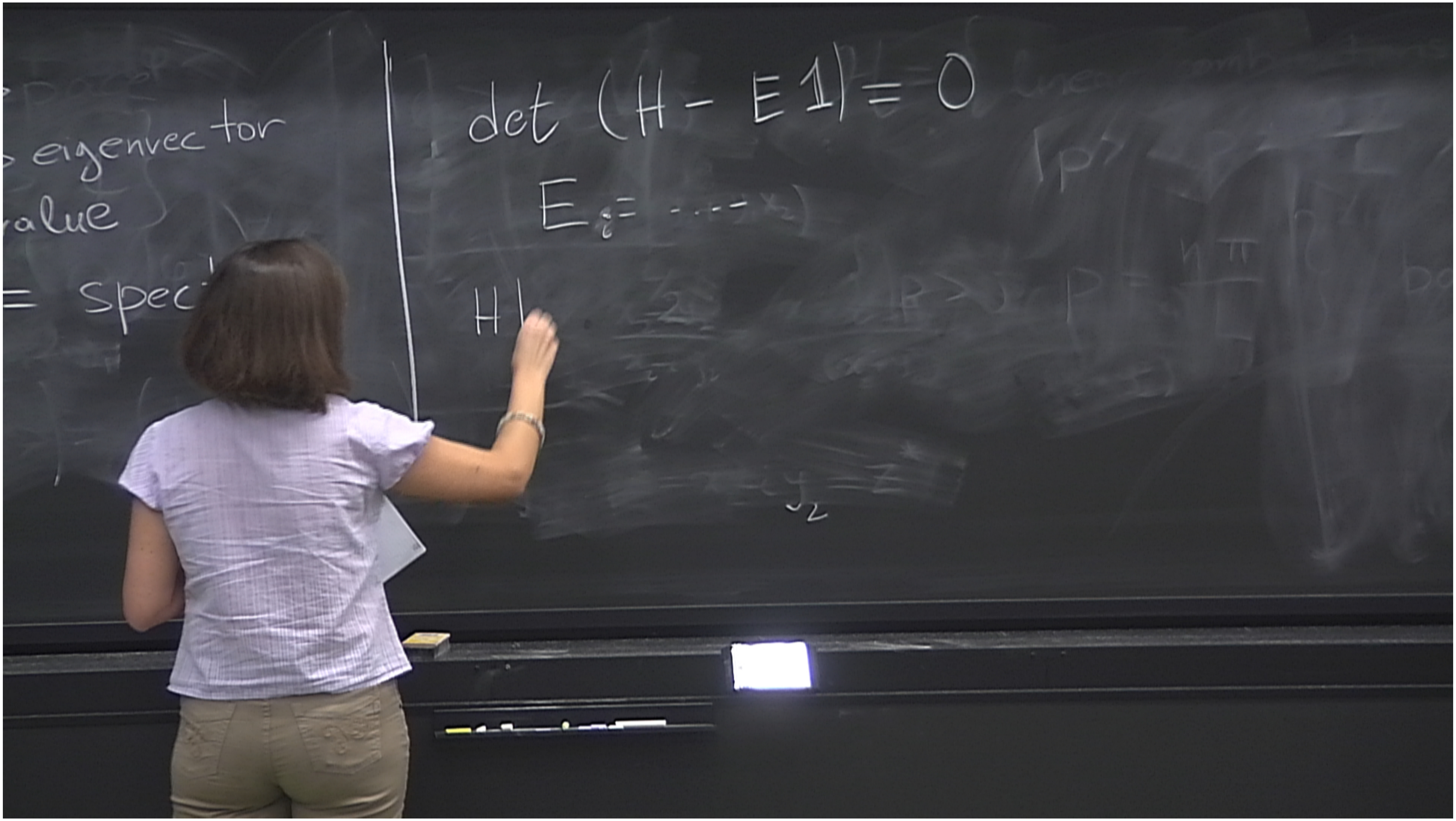
$A_{ij} = \langle e_i | A | e$
matrices

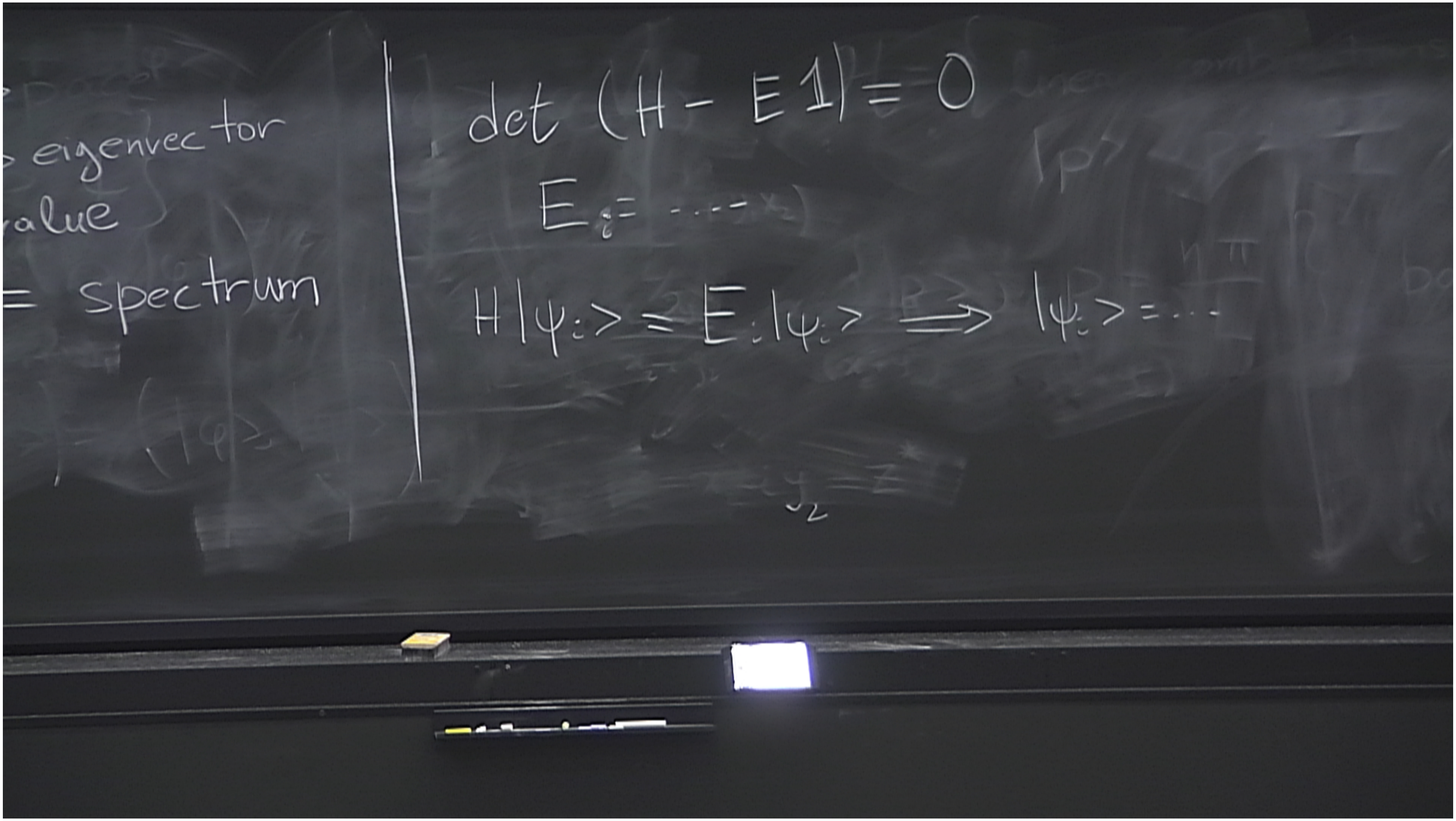


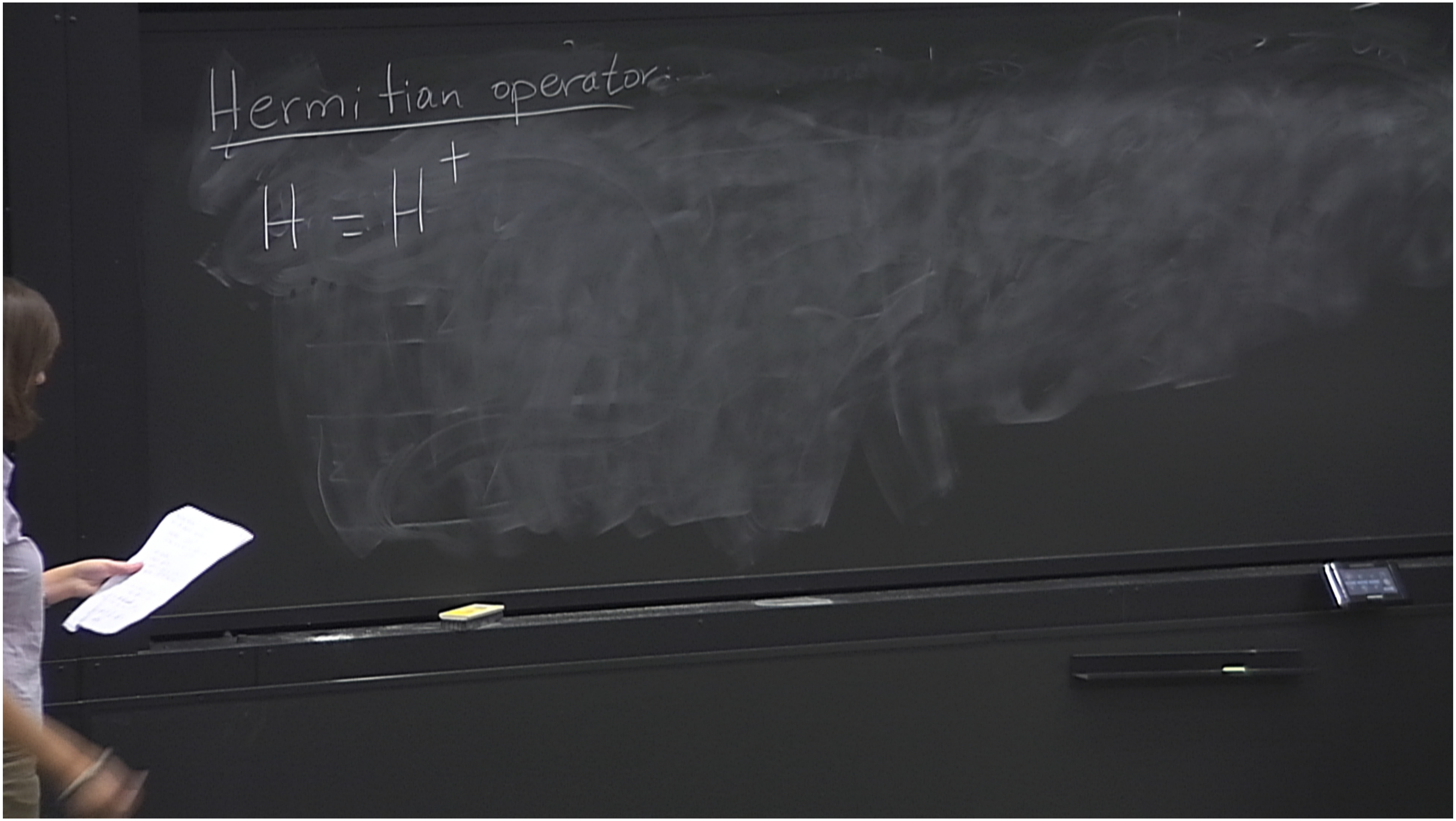
$A_{ij} = \langle e_i | A | e_j \rangle$
$\rightarrow n \times n$ matrix
$\rightarrow$ matrices

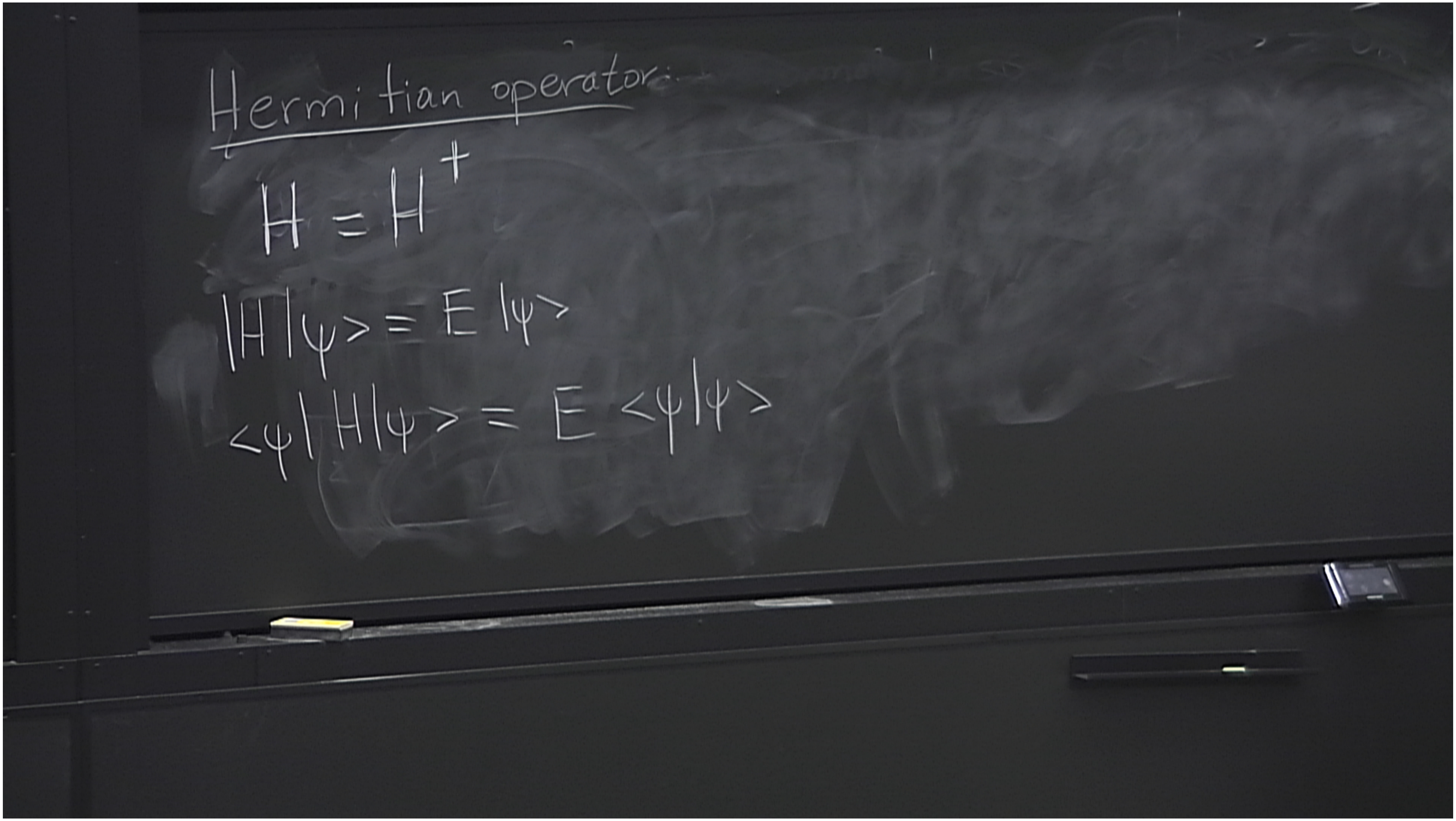

H|ψ> = E |ψ>
eigenvector
eigenvalue
of eigenvalues = spectrum

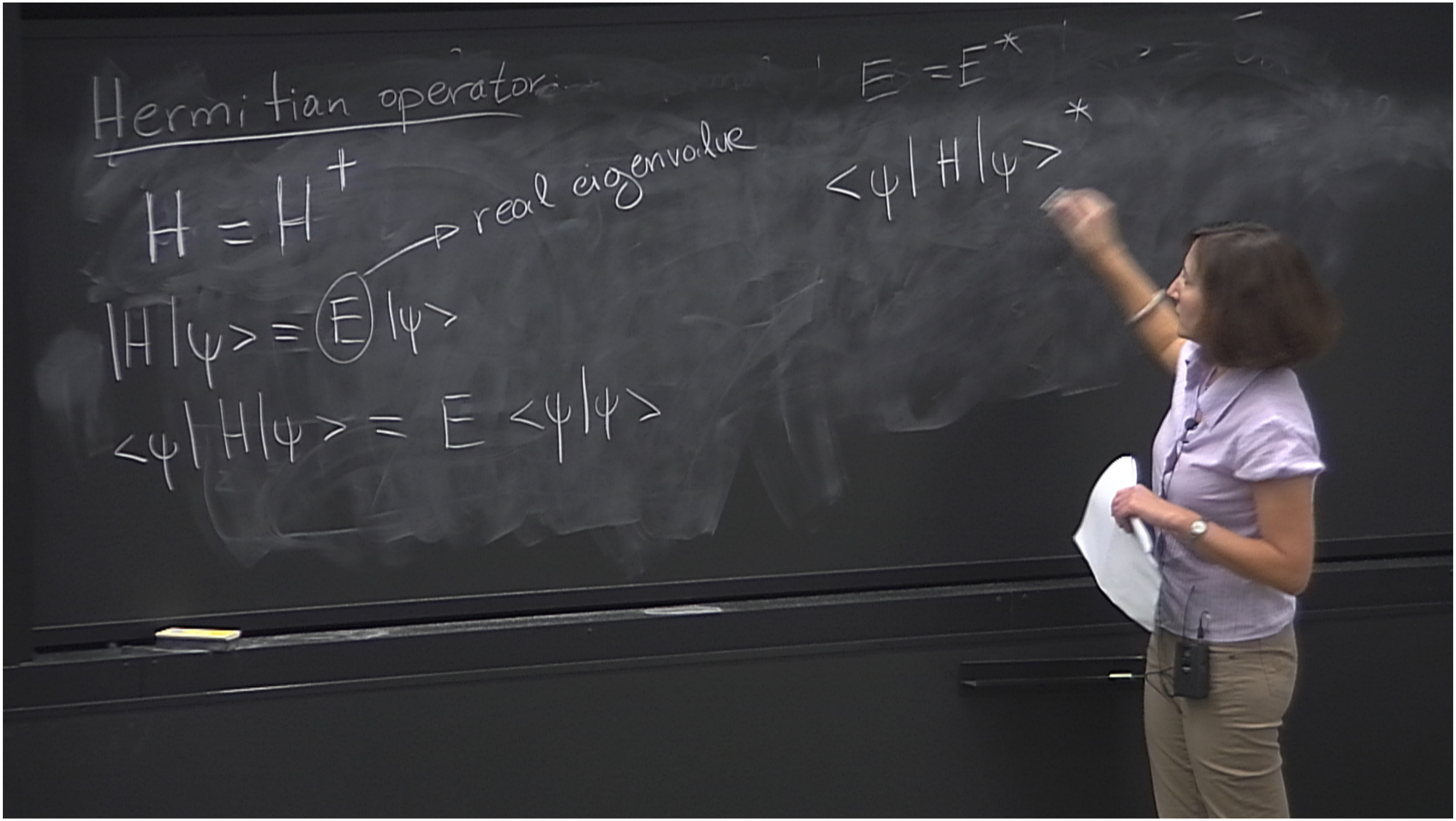

|ψ>
eigenvector
eigenvalue
spectrum
det (H - E 1) = 0

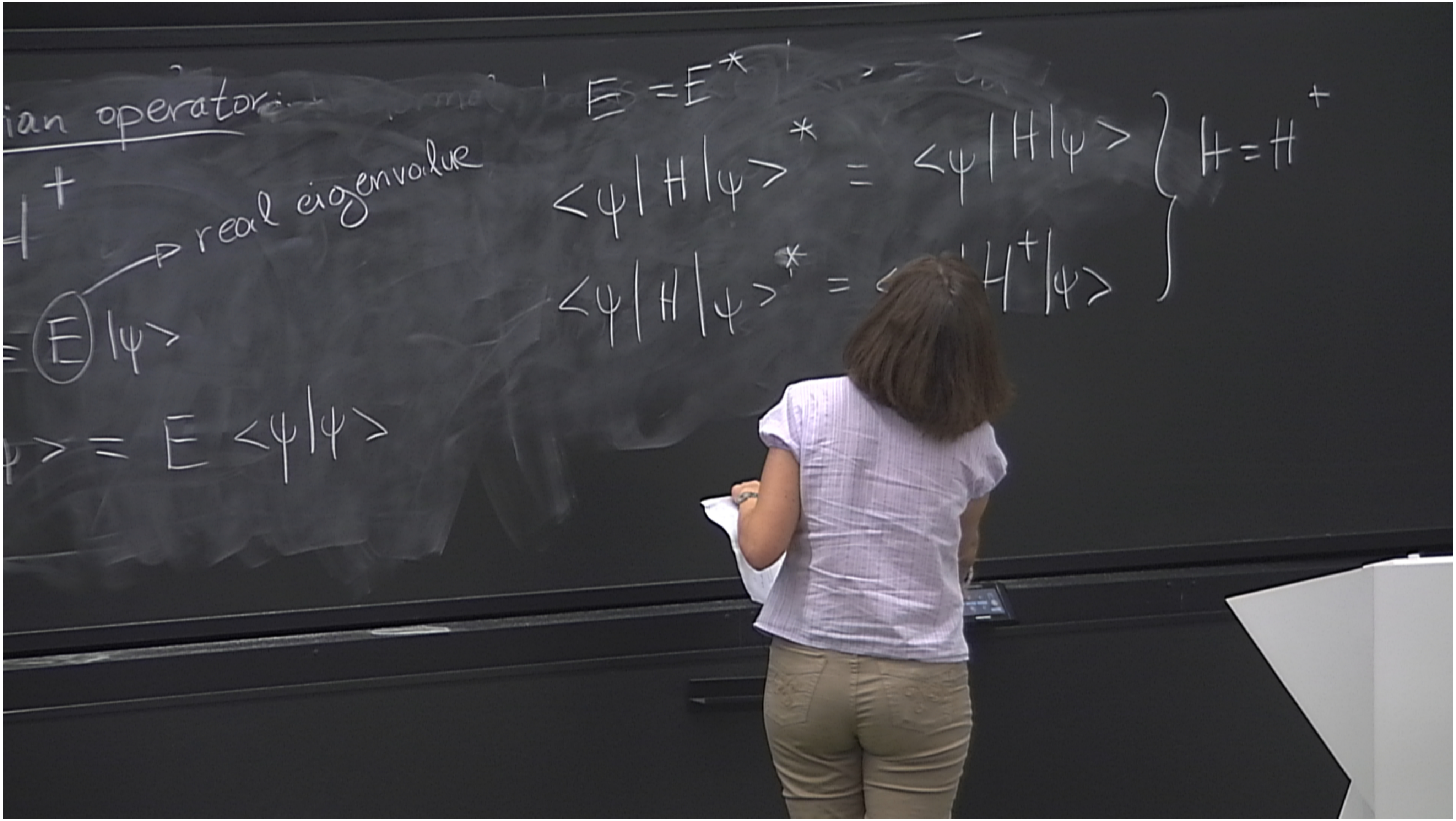

eigenvec tor
value
= spec
det (H − E 1) = 0
E_i = ...
H |

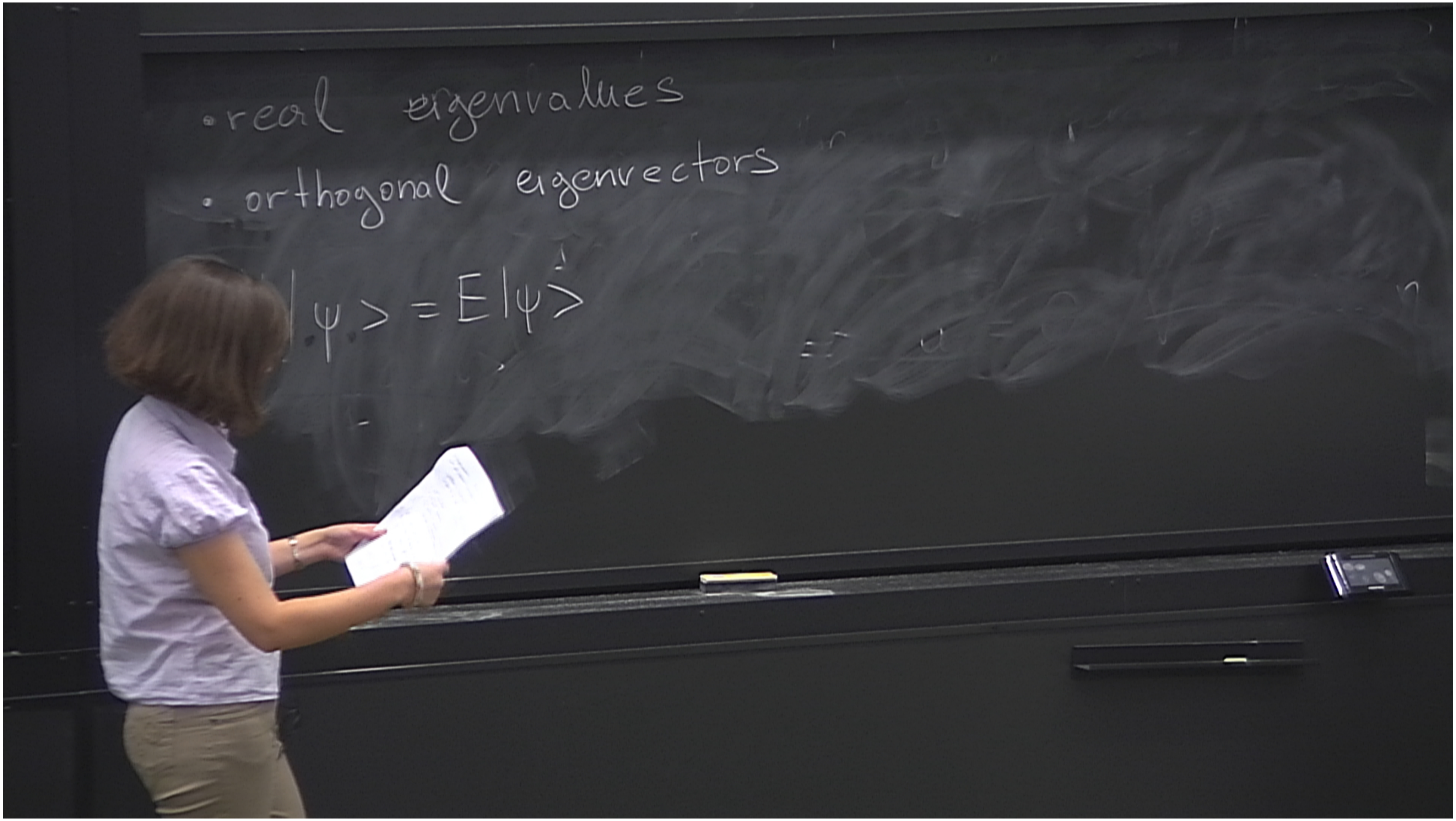

eigenvector
alue
= spectrum
det (H − E 1) = 0
E_i = ...
H|ψ_i> = E_i|ψ_i> ⟹ |ψ_i> = . . .

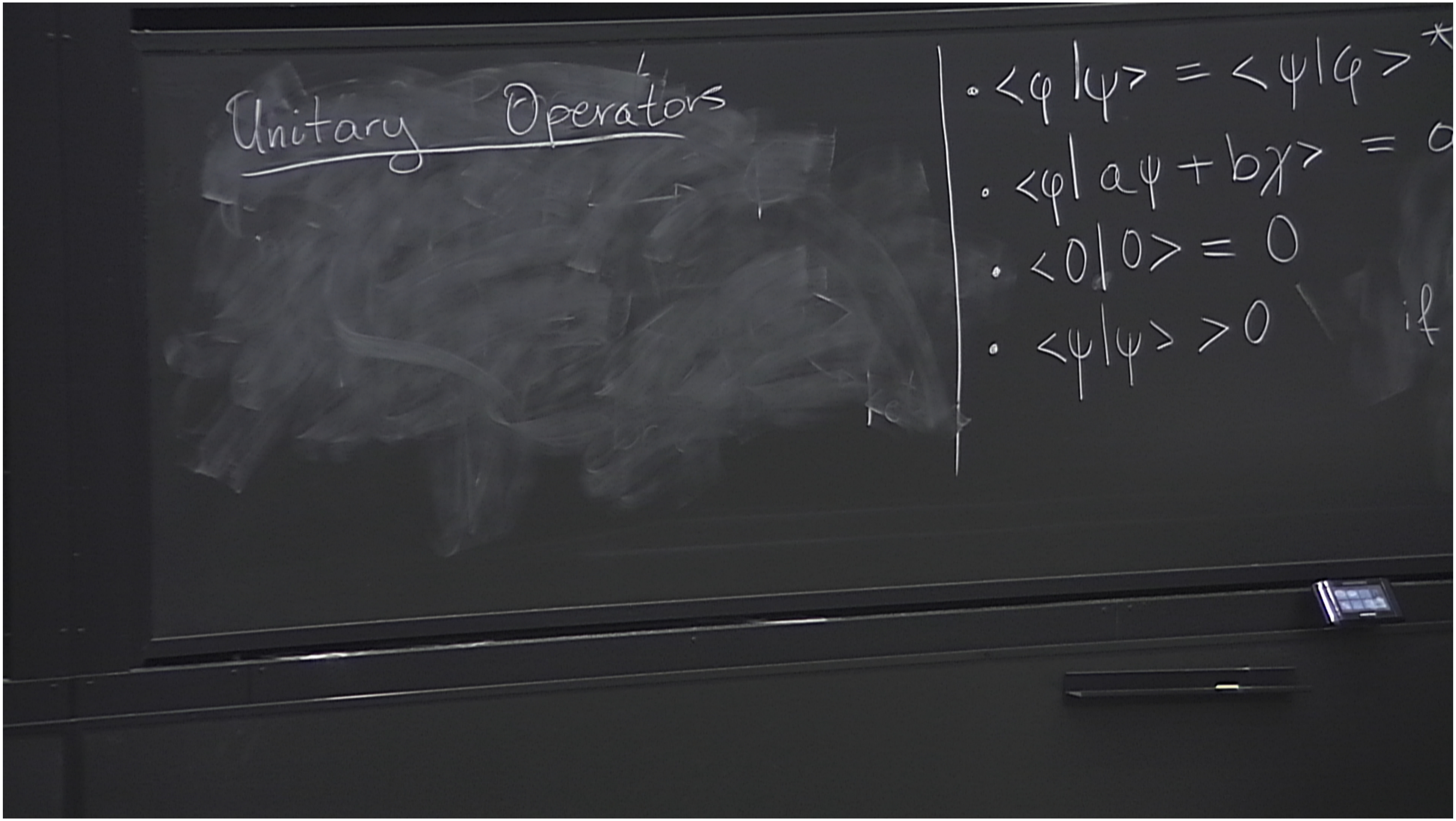

Hermitian operator:
H = H†

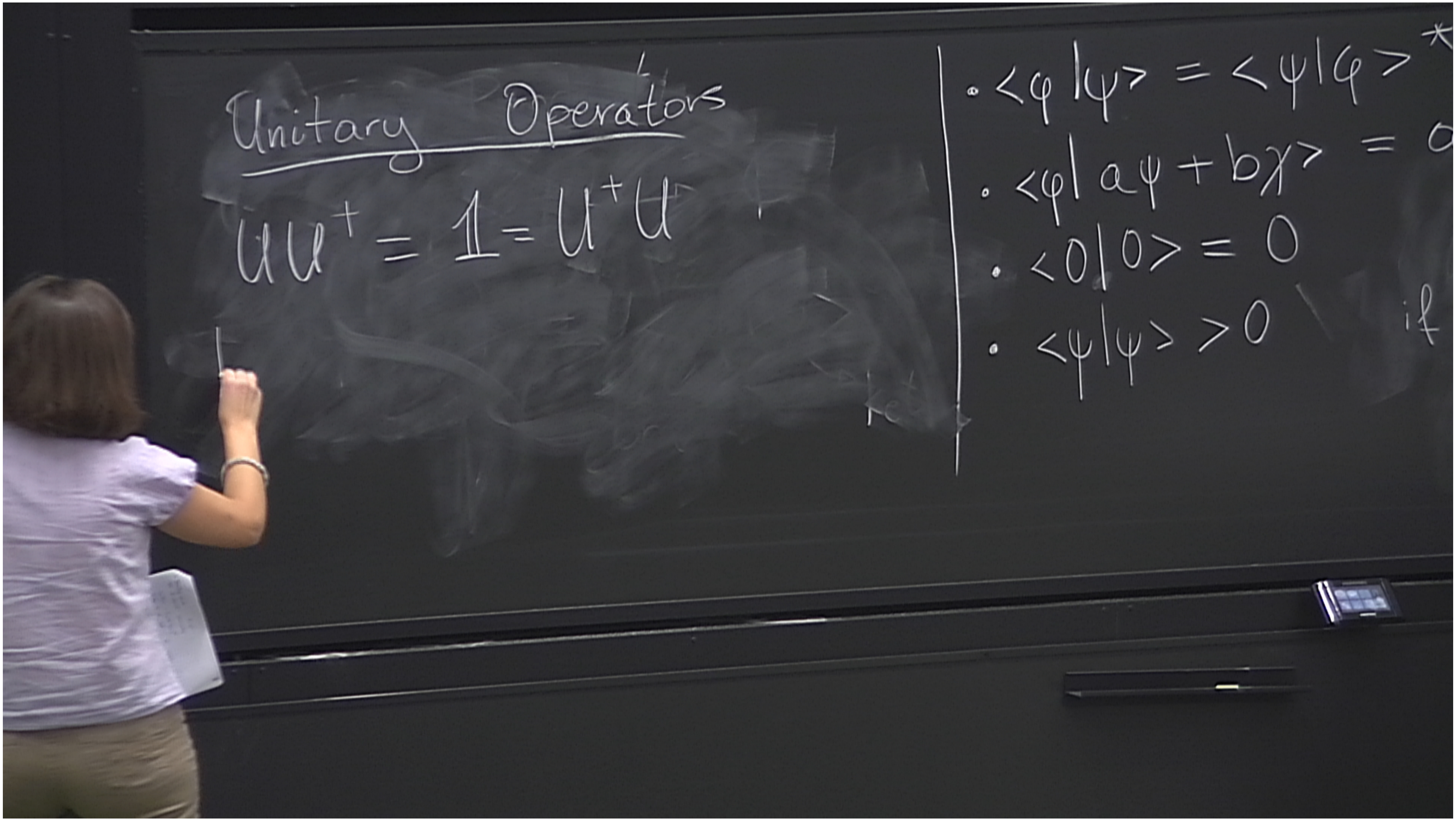

Hermitian operator:
H = H†
H|ψ> = E|ψ>
<ψ|H|ψ> = E <ψ|ψ>


Hermitian operator
H = H†
real eigenvalue
H|ψ> = E|ψ>
<ψ|H|ψ> = E <ψ|ψ>
E = E*
<ψ|H|ψ>*

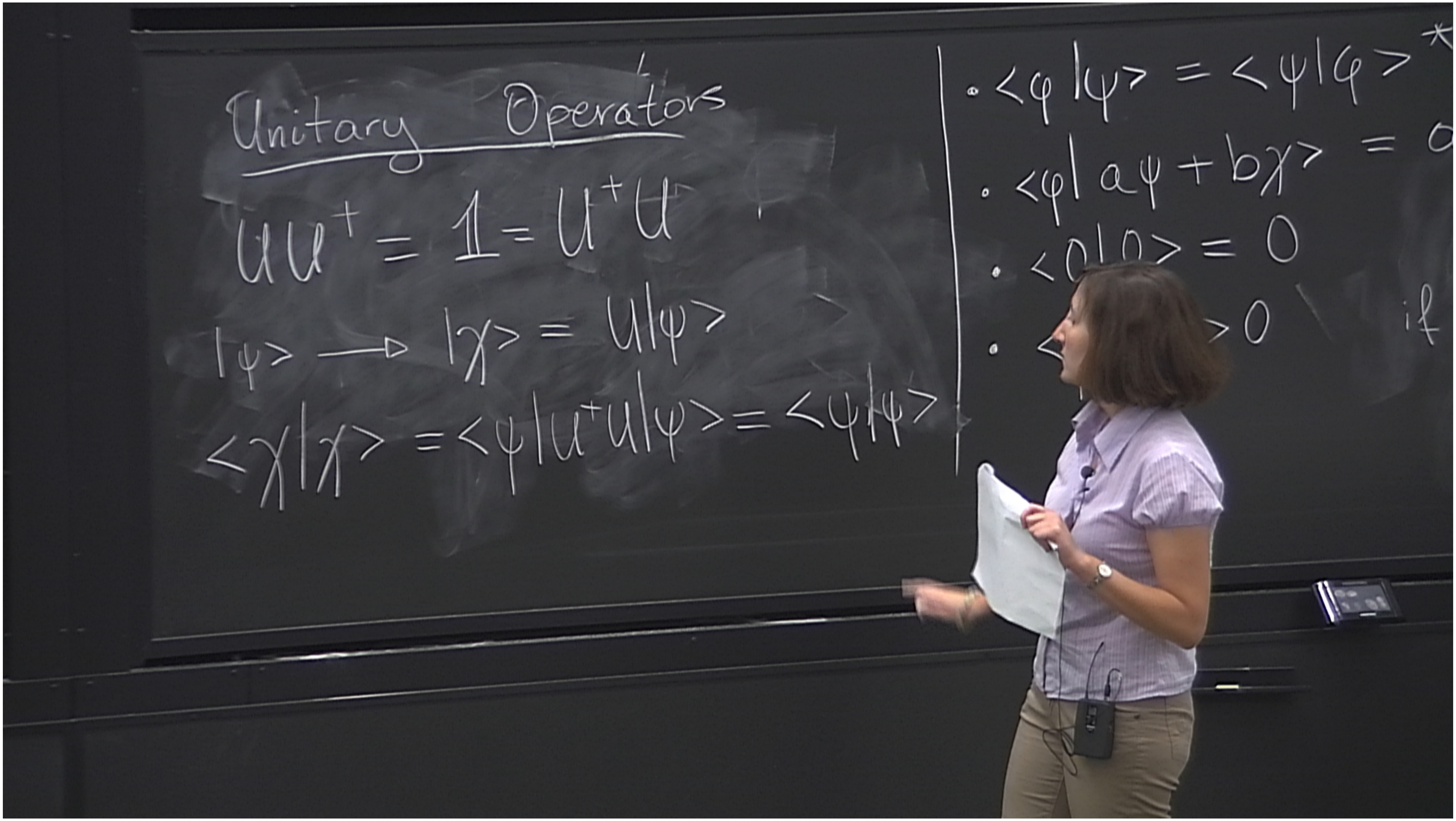

ian operator
real eigenvalue
E = E*
<ψ|H|ψ>* = <ψ|H|ψ>
H = H+
<ψ|H|ψ>* =
H+|ψ>
E |ψ>
ψ> = E <ψ|ψ>

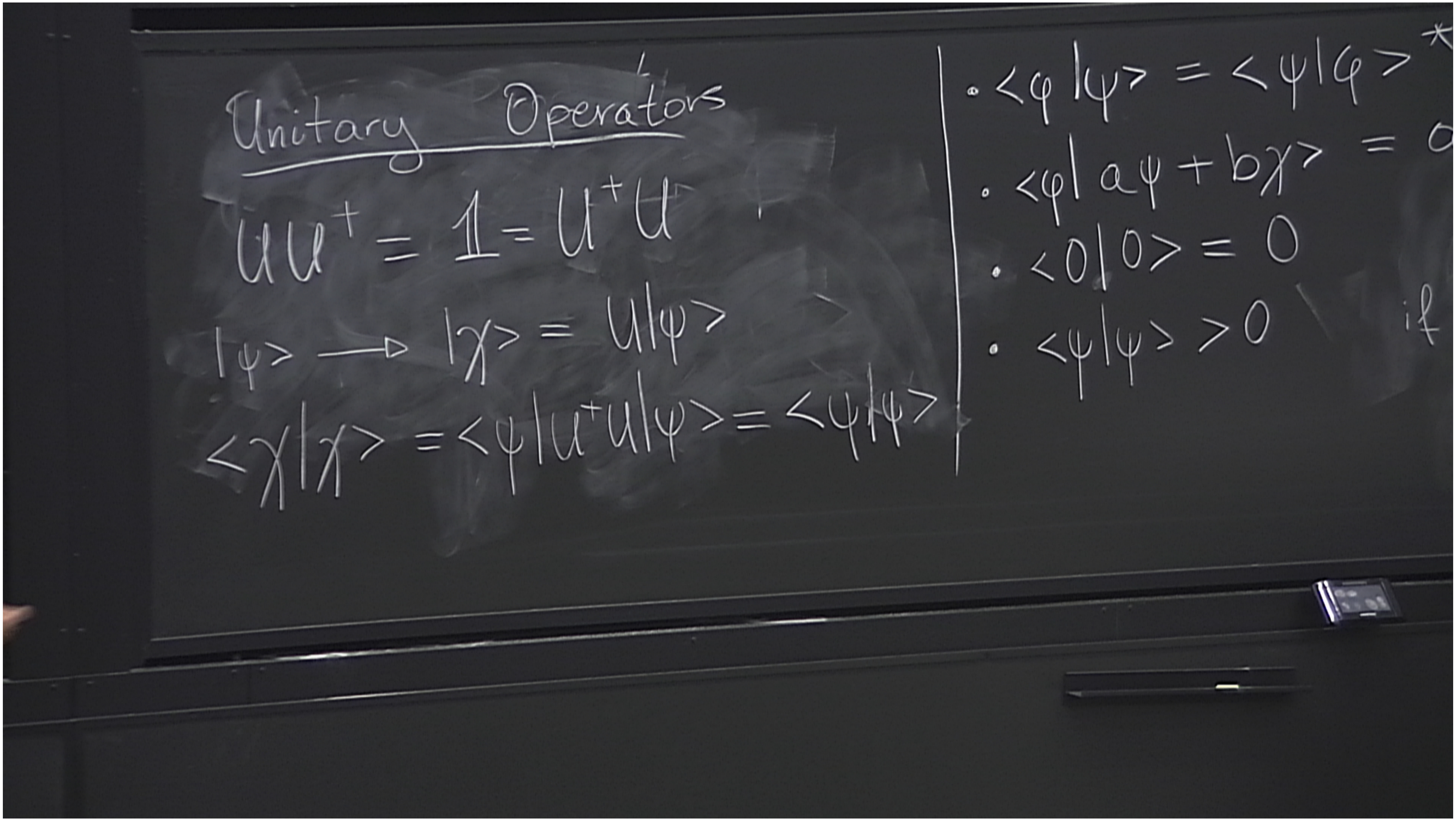

• real eigenvalues
• orthogonal eigenvectors
ψ> = E|ψ>

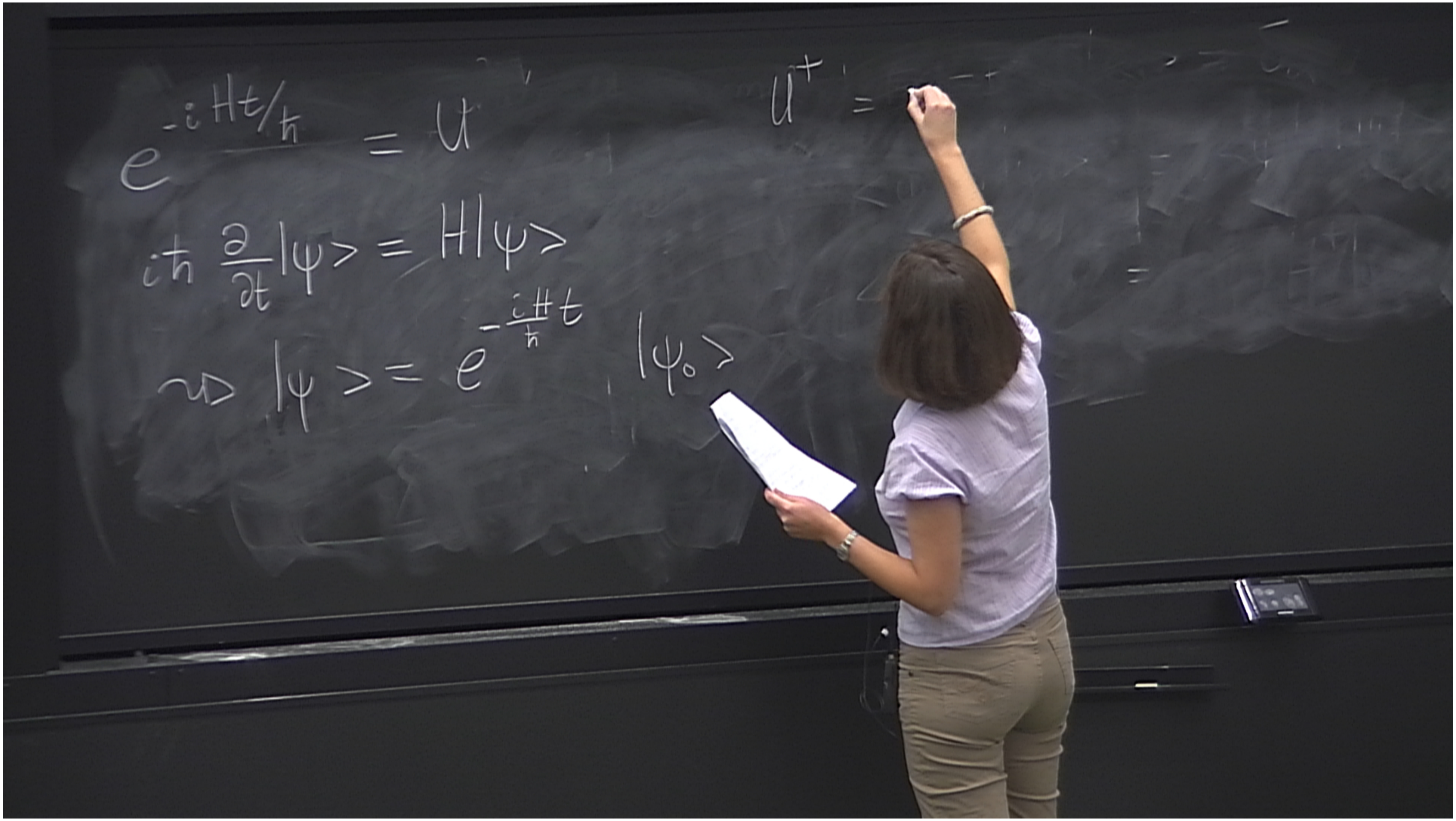

Unitary Operators
• <φ|ψ> = <ψ|φ>*
• <φ| aψ + bχ> = a
• <0|0> = 0
• <ψ|ψ> > 0
if

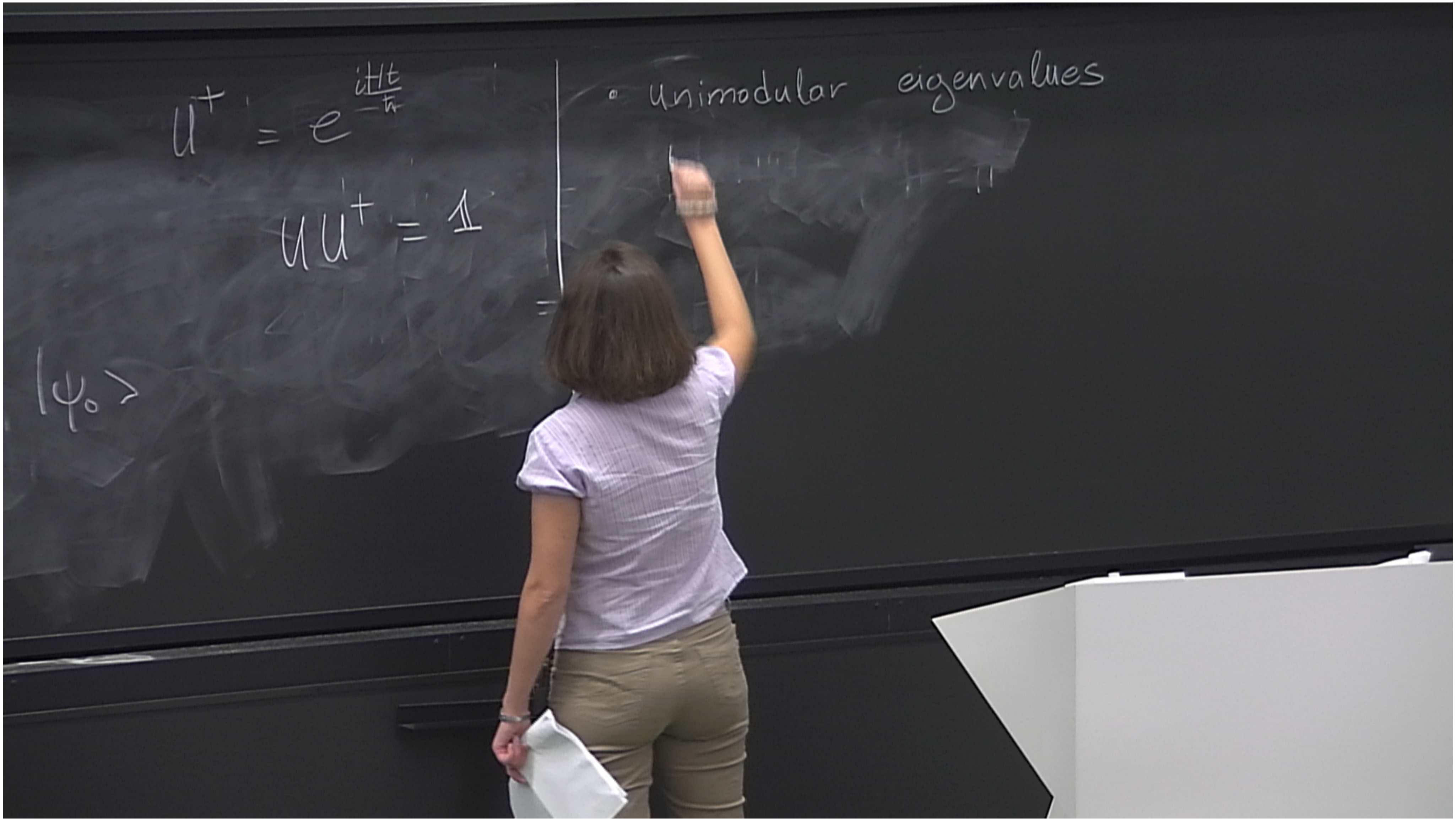


Unitary Operators
$UU^{\dagger} = \mathbb{1} = U^{\dagger}U$
$\langle \varphi | \psi \rangle = \langle \psi | \varphi \rangle^{*}$
$\langle \varphi | a\psi + b\chi \rangle =$
$\langle 0 | 0 \rangle = 0$
$\langle \psi | \psi \rangle > 0$
if


Unitary Operators
UU† = 1 = U†U
|ψ> ⟶ |χ> = U|ψ>
<χ|χ> = <ψ|U†U|ψ> = <ψ|ψ>
<φ|ψ> = <ψ|φ>*
<φ|aψ + bχ> = a
<0|0> = 0
> 0
if

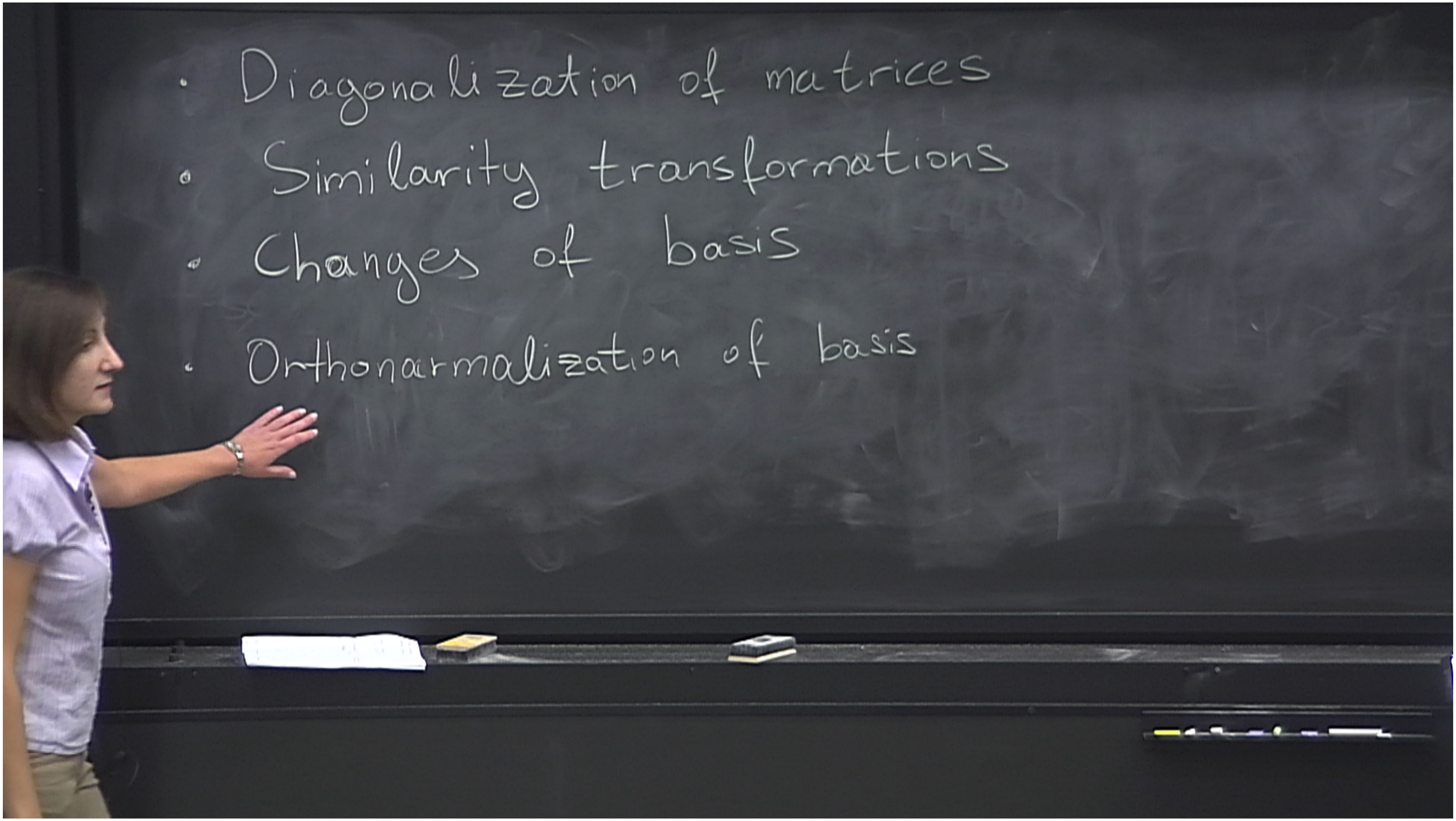

Unitary Operators
UU† = 𝟙 = U†U
|ψ> ⟶ |χ> = U|ψ>
<χ|χ> = <ψ|U†U|ψ> = <ψ|ψ>
• <φ|ψ> = <ψ|φ>*
• <φ| aψ + bχ> =
• <0|0> = 0
• <ψ|ψ> > 0 if


e^-iHt/ħ = U
U† =
iħ ∂/∂t |ψ> = H|ψ>
|ψ> = e^-iHt/ħ |ψ0>



• Unimodular eigenvalues
UU+ = 1


real eigenvalues
orthogonal eigenvectors
Unitary Operators
matrices



• Diagonalization of matrices
• Similarity transformations
• Changes of basis
• Orthonarmalization of basis