Title: Complex Analysis 1

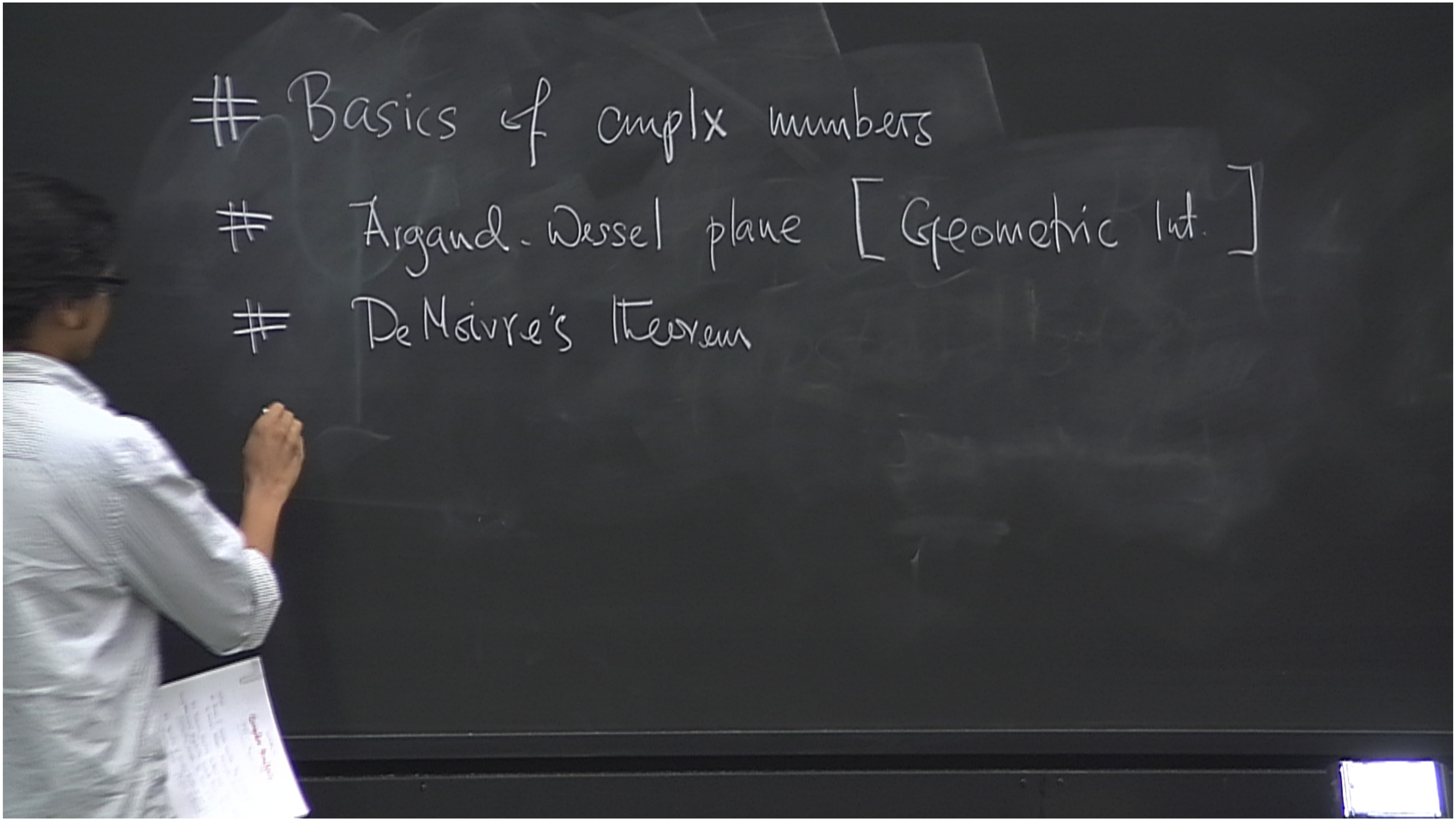
Date: Aug 16, 2011 10:15 AM

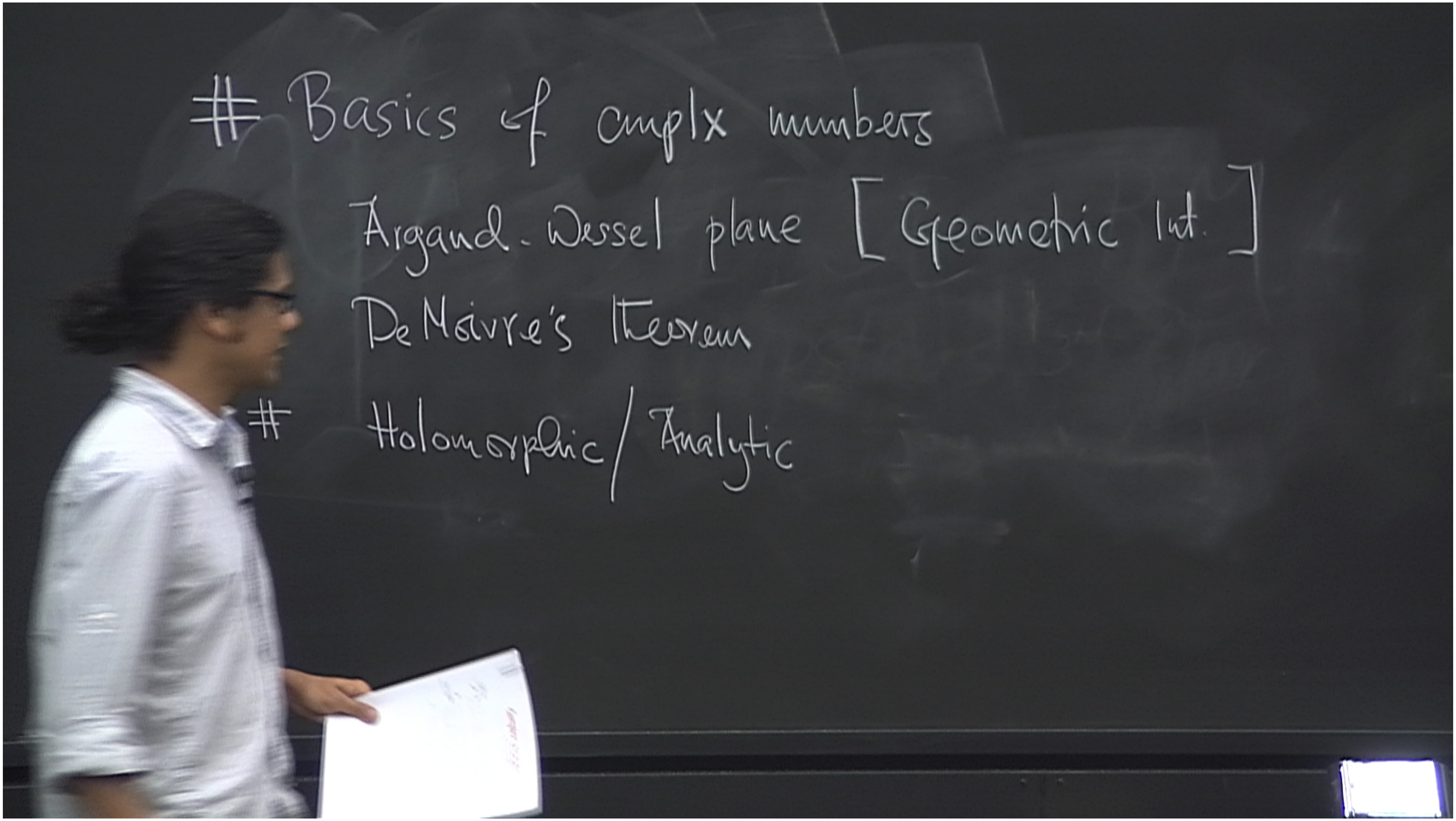
URL: http://pirsa.pi.local/11080063

Abstract:

Basics of cmplx numbers

Argand - Wessel plane


Basics of cmplx numbers
Argand-Wessel plane [Geometric Int.]
De Moivre's Theorem



Basics of cmplx numbers
Argand-Wessel plane [Geometric Int.]
De Moivre's Theorem
Holomorphic/Analytic

Basics of cmplx numbers

Argand-Wessel plane [Geometric Int.]

De Moivre's Theorem

Holomorphic/Analytic functions

Harmonic functions

sics of cmplx numbers

Argand-Wessel plane [Geometric Int.]

De Moivre's theorem

Holomorphic / Analytic functions [Taylor's theorem]

Harmonic functions

Isolated singular

nt.]

's Theorem]

Isolated singularities – Laurent's Series

Cauchy's Theorem

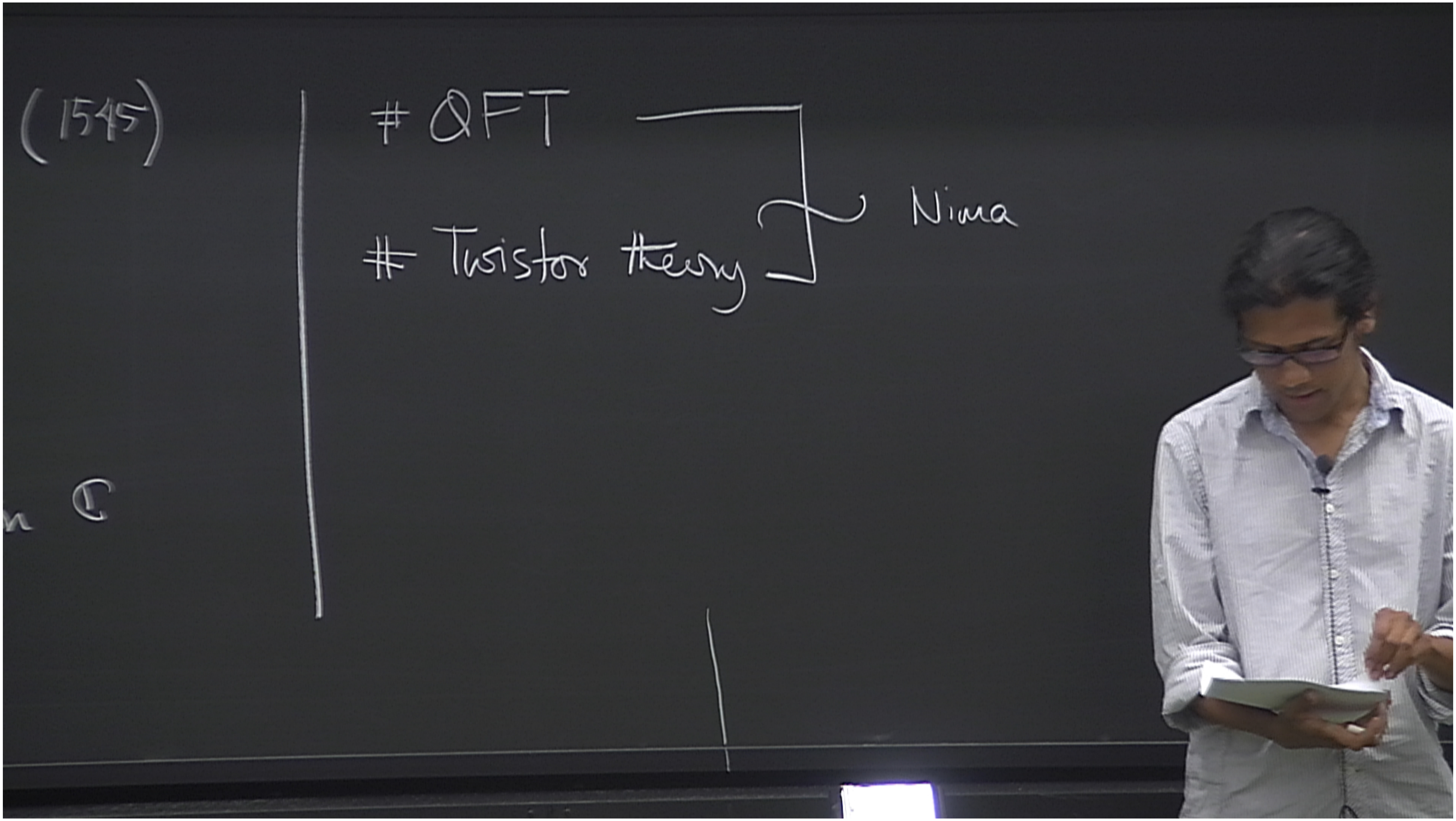
Applications in evaluating integrals

Analytic continuation.

Girolamo Cardano (1545)

$$ax^2 + bx + c = 0$$

$$b^2 < 4ac$$


(1545)
QFT
Twistor theory
Nima

(1545)

n ℂ

QFT

Twistor theory

Nima

String theory

Loop quantum gravity

$i := \sqrt{-1} \qquad i^2 = -1$

Complex number $\qquad a + ib$

$i := \sqrt{-1}$ $i^2 = -1$

Complex number $a + ib$

Addition: $z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 \cdot z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2)$$

$$z_1 / z_2 = \frac{x_1 + i y_1}{x_2 + i y_2} \, \frac{x_2 - i y_2}{x_2 - i y_2} = \frac{x_1 x_2 + y_1 y_2}{(x_2^2 + y_2^2)} + i \, \frac{(y_1 x_2 - y_2 x_1)}{(x_2^2 + y_2^2)}$$

$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 \cdot z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + y_1 x_2)$$

$$z_1 / z_2 = \frac{x_1 + i y_1}{\underline{x_2 + i y_2}} \, \frac{x_2 - i y_2}{x_2 - i y_2} = \frac{x_1 x_2 + y_1 y_2}{(x_2^2 + y_2^2)} + i \, \frac{(y_1 x_2 - y_2 x_1)}{(x_2^2 + y_2^2)}$$

$$z_2 = x_2 + i y_2 \qquad \bar{z} = x_2 - i y_2 = z^*$$

$$\frac{\ldots y_1 y_2}{\ldots + y_2^2)} + i\,\frac{(y_1 x_2 - y_2 x_1)}{(x_2^2 + y_2^2)}$$

$$\ldots z^*$$

$$\bar{\bar{z}} = z$$

$$\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$$

$$\ldots y_1 y_2 \over \ldots + y_2^2) + i \, \frac{(y_1 x_2 - y_2 x_1)}{(x_2^2 + y_2^2)}$$

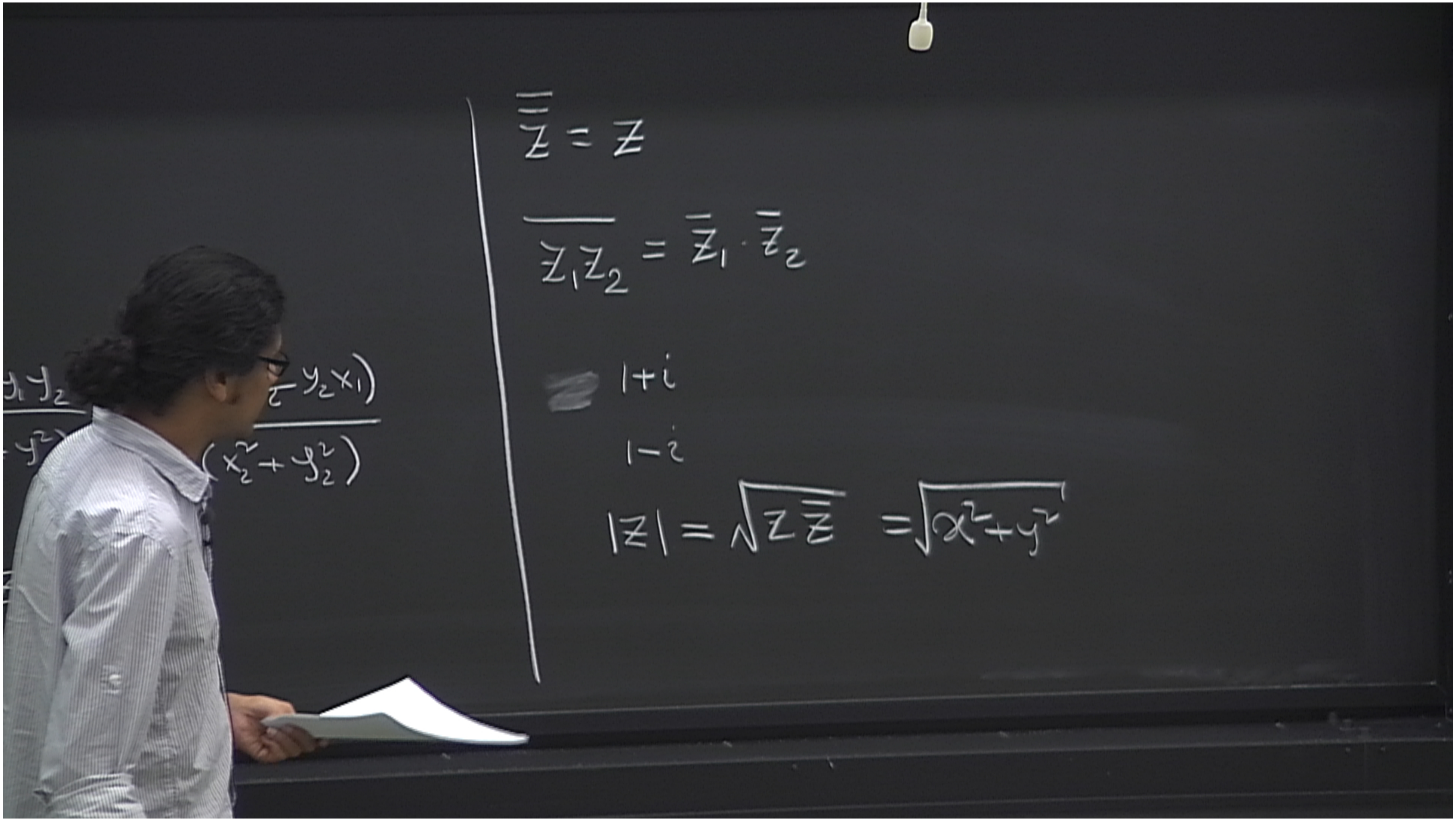
z^*

$$\bar{\bar{z}} = z$$

$$\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$$

$1+i$

$1-i$


$\bar{\bar{z}} = z$
$\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$
$1+i$
$1-i$
$|z| = \sqrt{z\bar{z}} = \sqrt{x^2+y^2}$
$(x_2^2+y_2^2)$

$$\bar{\bar{z}} = z$$

$$\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$$

$1+2i \quad \sqrt{1+4} = \sqrt{5}$

$1+i$

$1+i$

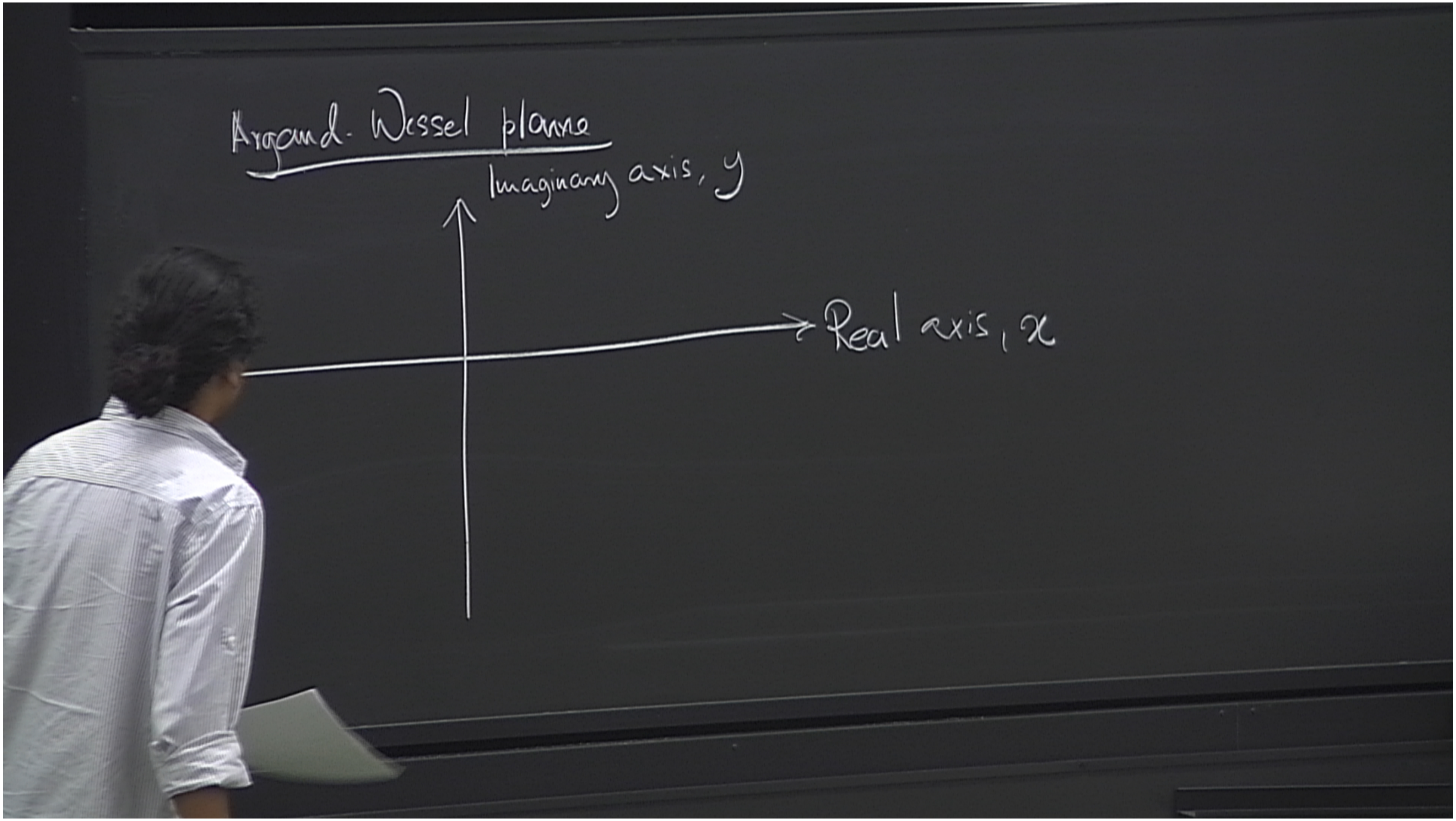
$1-i$

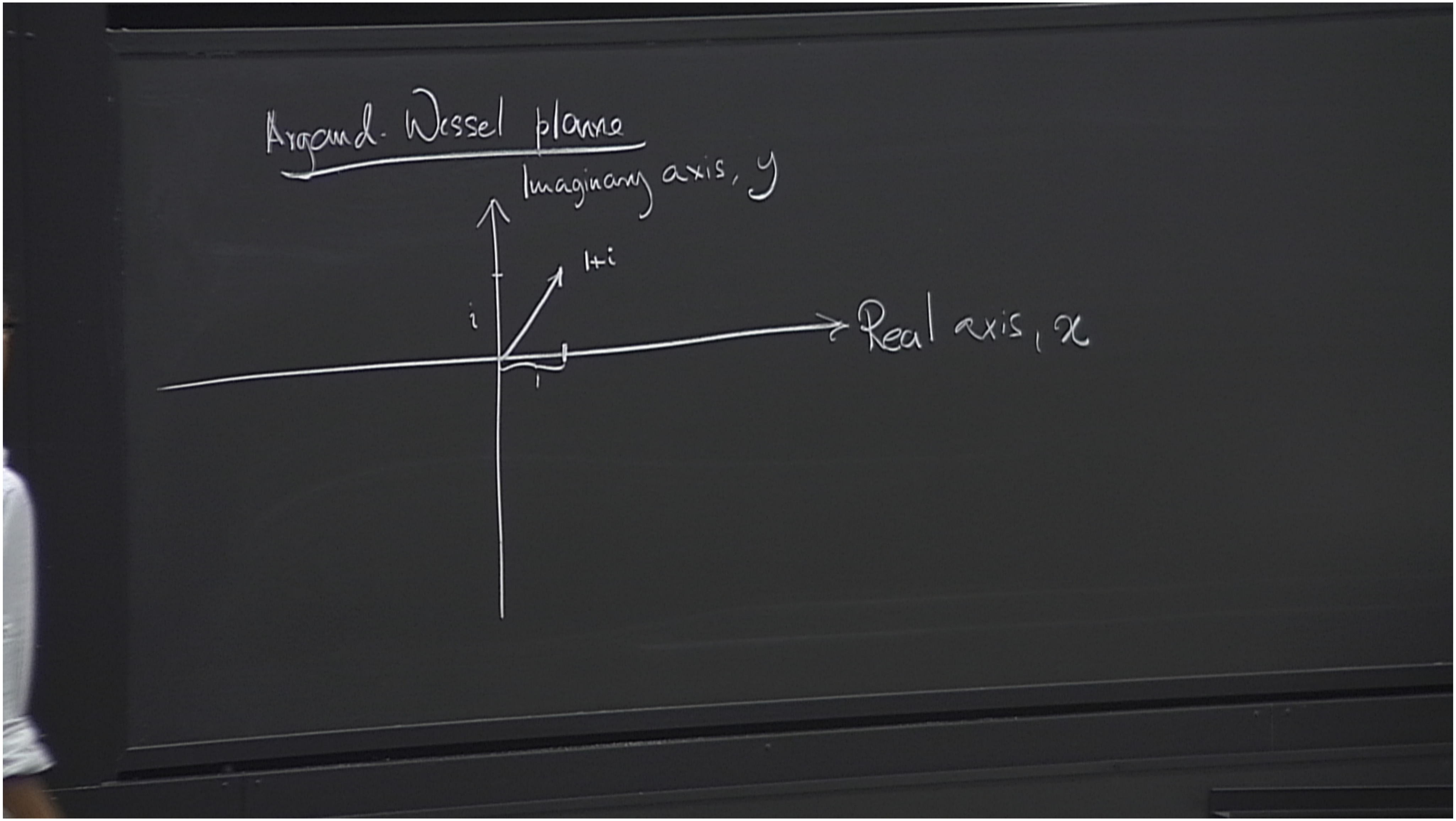
$\frac{\ldots y_1 y_2}{\ldots + y_2^2)} + i \frac{(y_1 x_2 - y_2 x_1)}{(x_2^2 + y_2^2)}$

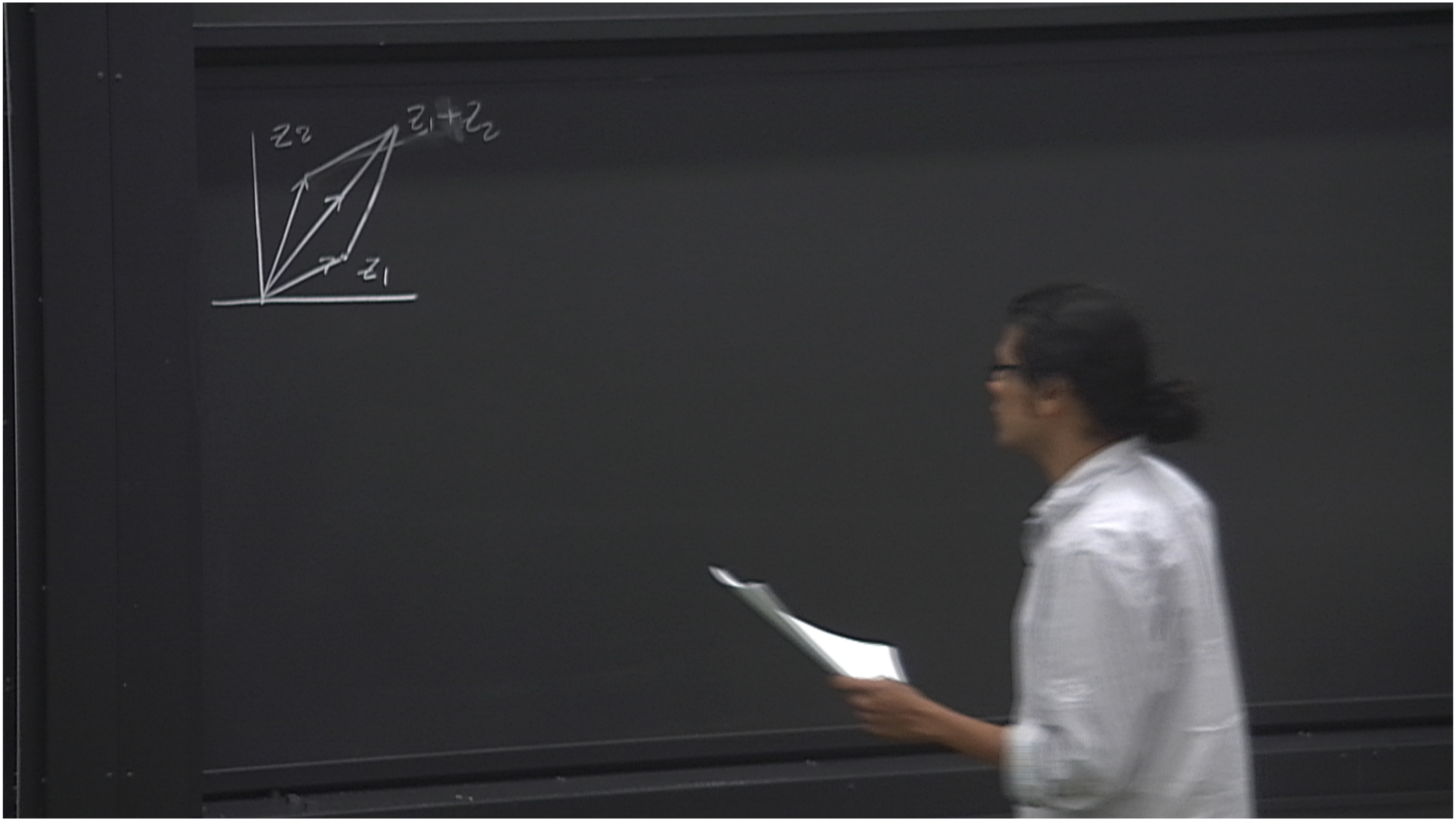
z^*

$$|z| = \sqrt{z\bar{z}} = \sqrt{x^2+y^2} \sim \text{Real, positive}$$

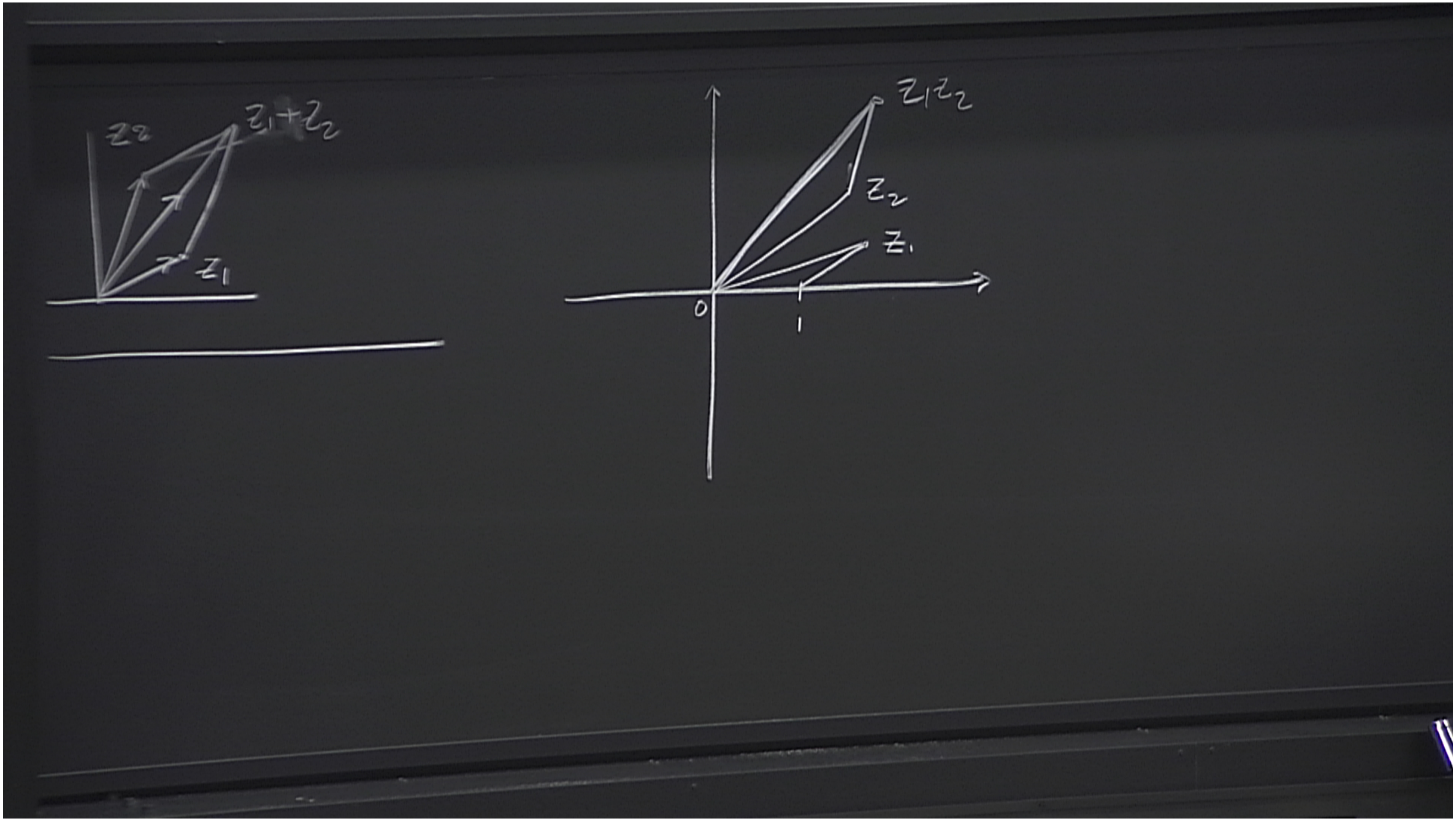
$|\sqrt{z}|$

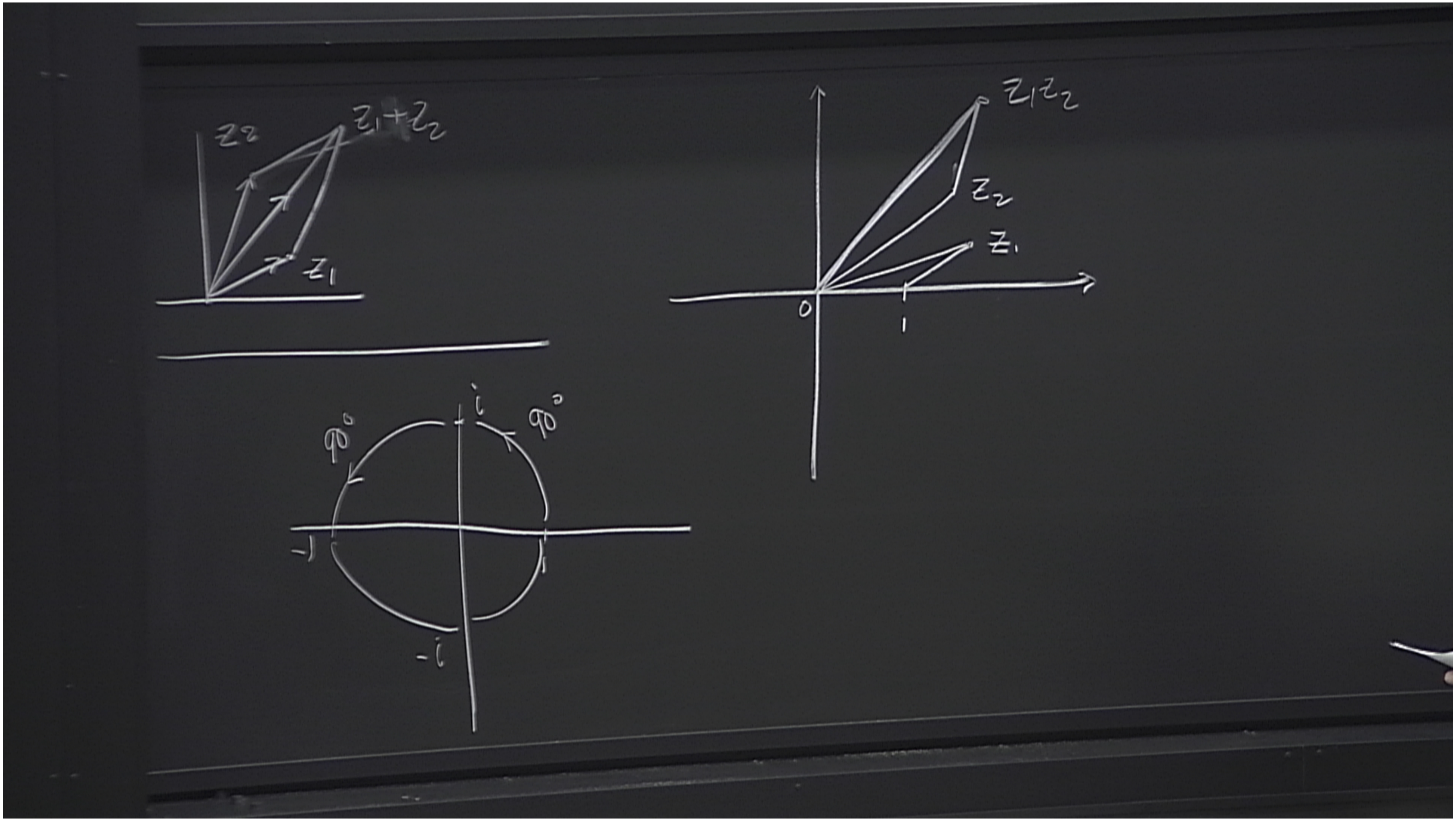

Argand-Wessel plane
Imaginary axis, y
Real axis, x


Argand-Wessel plane
Imaginary axis, y
1+i
i
1
Real axis, x



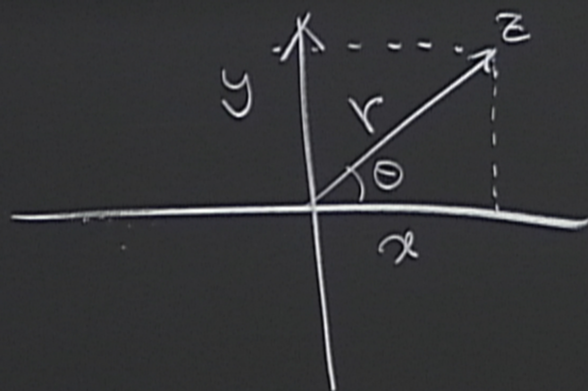
z_2
z_1+z_2
z_1


z2
z1+z2
z1
z1z2
z2
z1
0
1



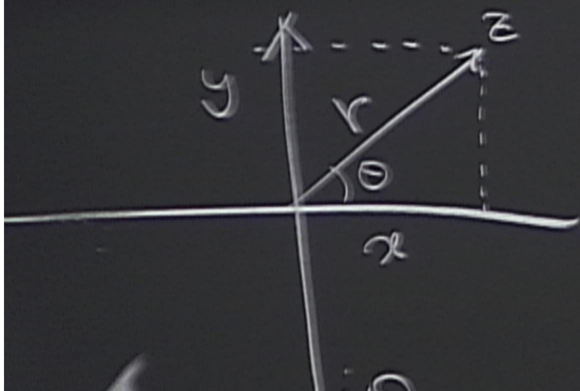
z_2
z_1+z_2
z_1
z_1z_2
z_2
z_1
0
1
i
$90°$
$90°$
-1
1
$-i$

$$Z = r\cos\theta + i\,r\sin\theta$$



$$r = |Z|$$

$$Z = r\cos\theta + i\, r\sin\theta$$



$$r = |Z|$$

$$Z = r e^{i\theta}$$

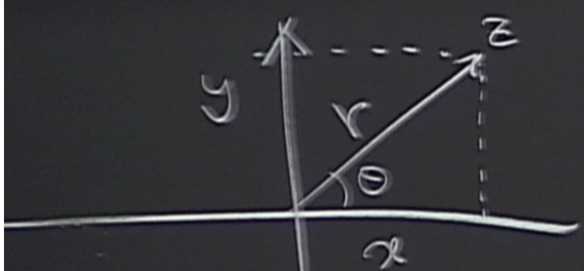
$$\theta = \mathrm{Arg}\, Z$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

$$\sin\theta = \theta - \frac{1}{3}\theta^3 + \frac{\theta^5}{5!}$$

$$i^2 = -1$$

$$Z = r\cos\theta + i\,r\sin\theta$$



$$r = |Z|$$

$$\boxed{Z = r e^{i\theta}}$$

$$e^{i\theta} = \sum_{n=0}^{\infty} \frac{(i\theta)^n}{n!}$$

$$\theta = \mathrm{Arg}\, Z$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$$

$$\sin\theta = \theta - \frac{1}{3}\theta^3 + \frac{\theta^5}{5!}$$

$$i^2 = -1$$

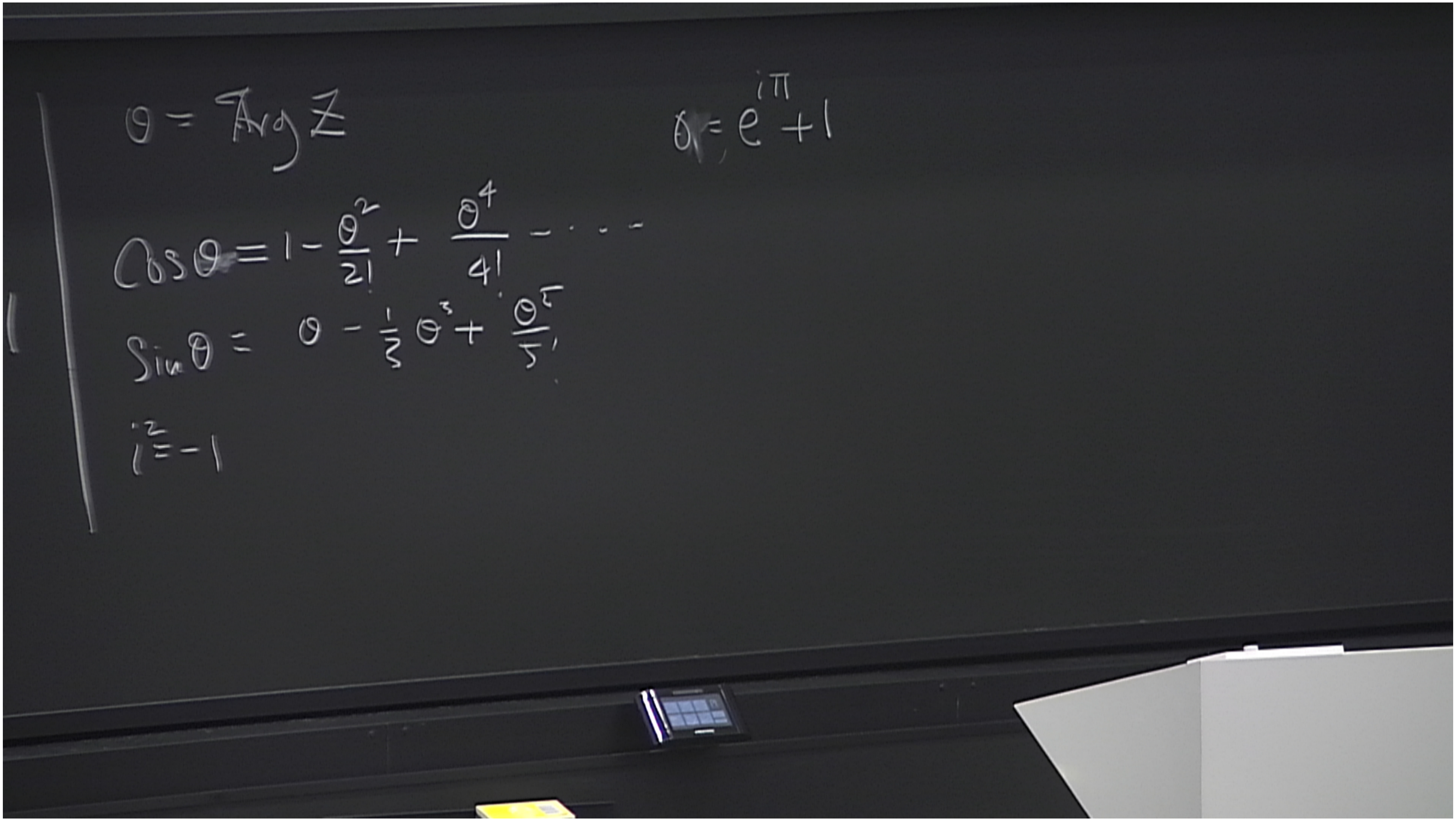
$\theta = \operatorname{Arg} Z$

$-1 = e^{i\pi}$

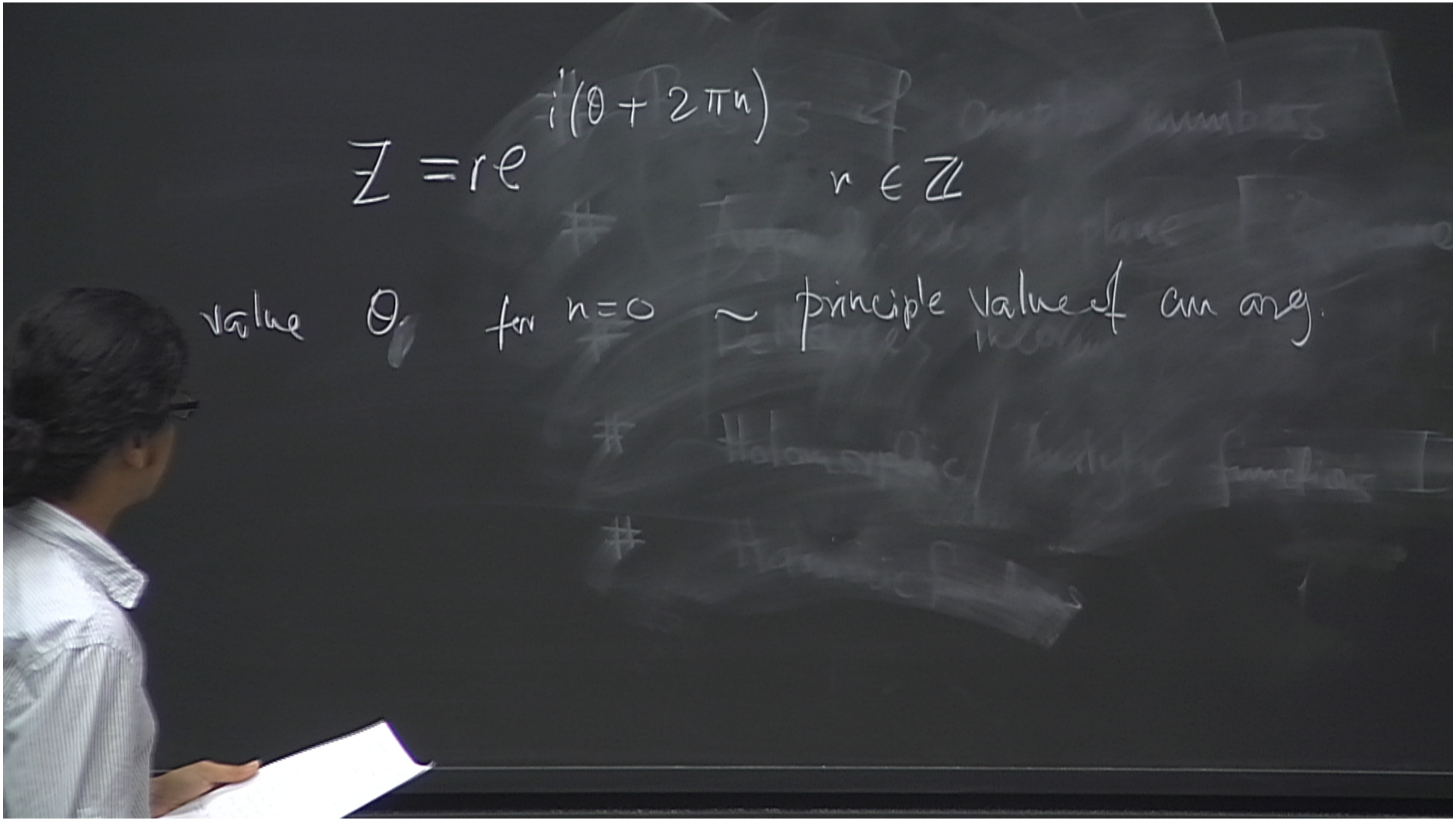
$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$

$\sin\theta = \theta - \frac{1}{3}\theta^3 + \frac{\theta^5}{5!}$

$i^2 = -1$



$\theta = \mathrm{Arg}\, Z$
$0 = e^{i\pi} + 1$
$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots$
$\sin\theta = \theta - \frac{1}{3}\theta^3 + \frac{\theta^5}{5!}$
$i^2 = -1$



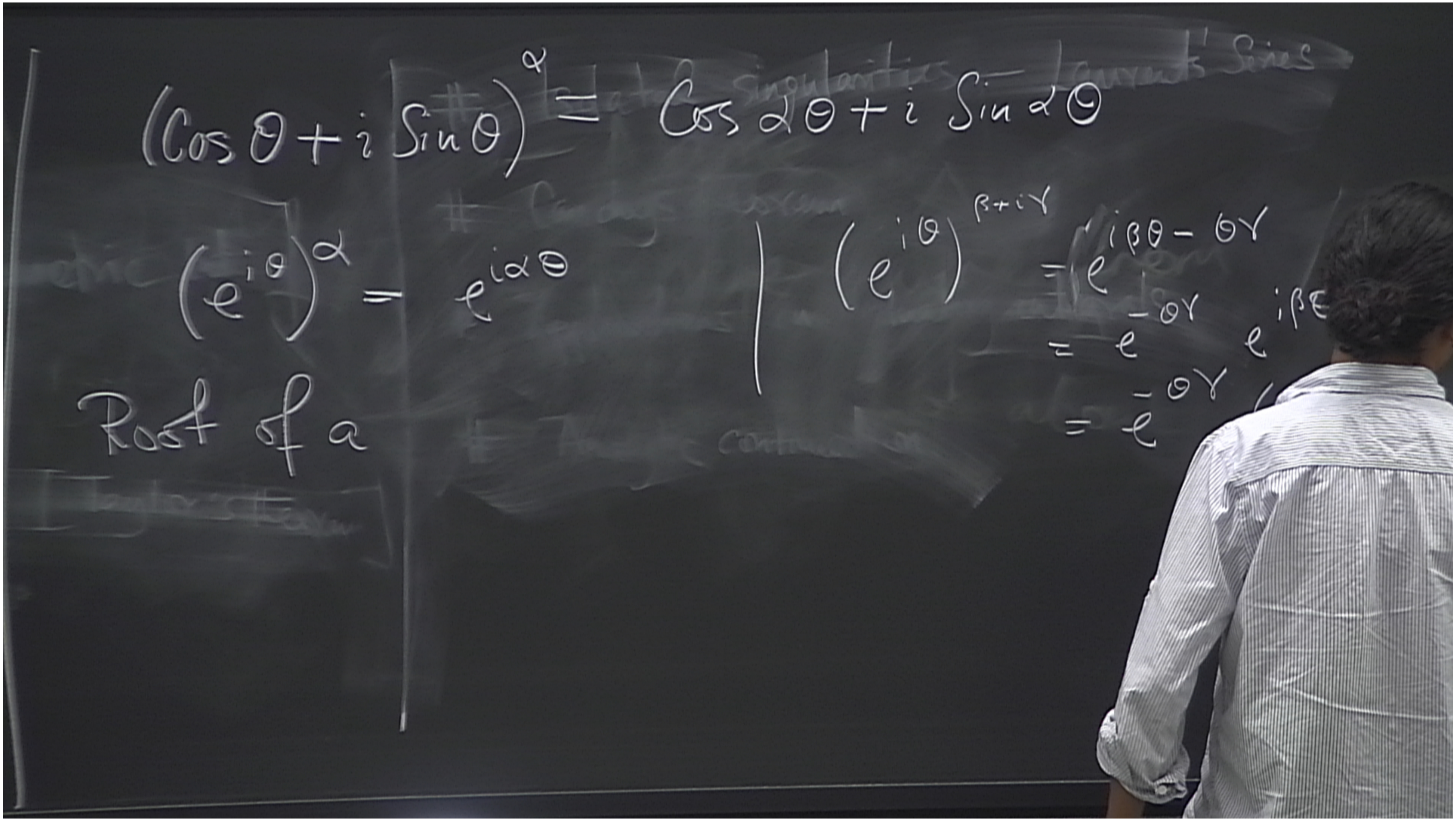
$Z = re^{i(\theta + 2\pi n)}$
$n \in \mathbb{Z}$
value θ for $n=0$ ~ principle value of an arg.

$$z = re^{i(\theta + 2\pi n)} \qquad n \in \mathbb{Z}$$

value θ for $n=0$ ~ principle value of an arg

$$i = e^{i\pi/2} = e^{i5\pi/2} = \dots$$

$$\frac{\pi}{2}$$



$(\cos\theta + i\sin\theta)^{\alpha} = \cos\alpha\theta + i\sin\alpha\theta$
$(e^{i\theta})^{\alpha} = e^{i\alpha\theta}$
Root of a
$(e^{i\theta})^{\beta+i\gamma} = e^{i\beta\theta - \theta\gamma}$
$= e^{-\theta\gamma}\, e^{i\beta\theta}$
$= e^{-\theta\gamma}$

$$\ldots i\sin\theta)^{\alpha} = \cos\alpha\theta + i\sin\alpha\theta$$

$$= e^{i\alpha\theta}$$

$$\left(e^{i\theta}\right)^{\beta+i\gamma} = e^{i\beta\theta-\theta\gamma}$$

$$= e^{-\theta\gamma}\, e^{i\beta\theta}$$

$$= e^{-\theta\gamma}\left(\cos\beta\theta + i\sin\beta\theta\right)$$

$$(\cos\theta + i\sin\theta)^{\alpha} = \cos\alpha\theta + i\sin\alpha\theta$$

$$= e^{i\alpha\theta}$$

$$\left(e^{i\theta}\right)^{\beta+i\gamma} = e^{i\beta\theta-\theta\gamma}$$

$$= e^{-\theta\gamma}\, e^{i\beta\theta}$$

$$= e^{-\theta\gamma}\left(\cos\beta\theta + i\sin\beta\theta\right)$$

$$\uparrow$$

$$\frac{e^{i\beta\theta}+e^{-i\beta\theta}}{2} + \cancel{i}\,\frac{e^{i\beta\theta}-e^{-i\beta\theta}}{2\cancel{i}}$$

$$=$$

$$(\cos\theta + i\sin\theta)^{\alpha} = \cos\alpha\theta + i\sin\alpha\theta$$

$$\left(e^{i\theta}\right)^{\alpha} = e^{i\alpha\theta}$$

Root of a complex #

$$w = z^{n}$$

$\theta)^{\alpha} = \cos \alpha\theta + i \sin \alpha\theta$

$i\alpha\theta$

plex #

$\pi k)$ $\quad k \in \mathbb{Z}$

$$Z = r^{1/n} e^{i\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)}$$

$k = 0, 1, \ldots, n-1$

n distinct values of Z

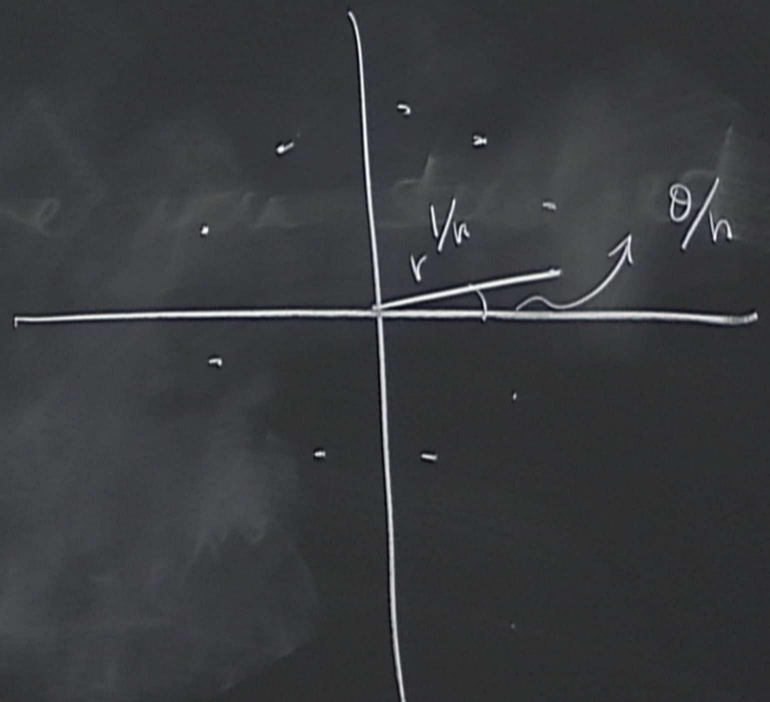
=

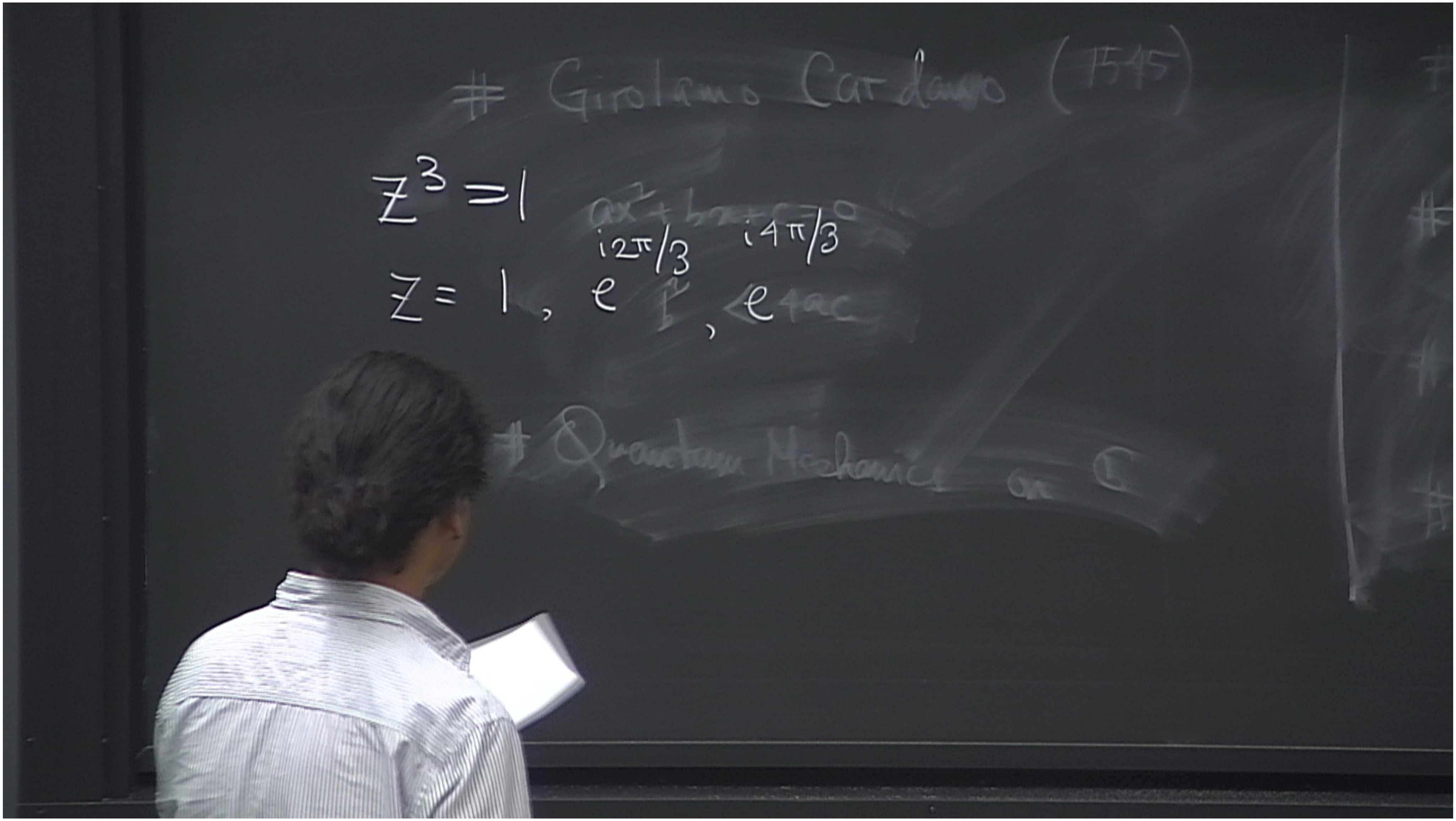
$\theta + i \sin \alpha\Theta$

$$Z = r^{1/n} e^{i\left(\frac{\theta}{n} + \frac{2\pi k}{n}\right)}$$

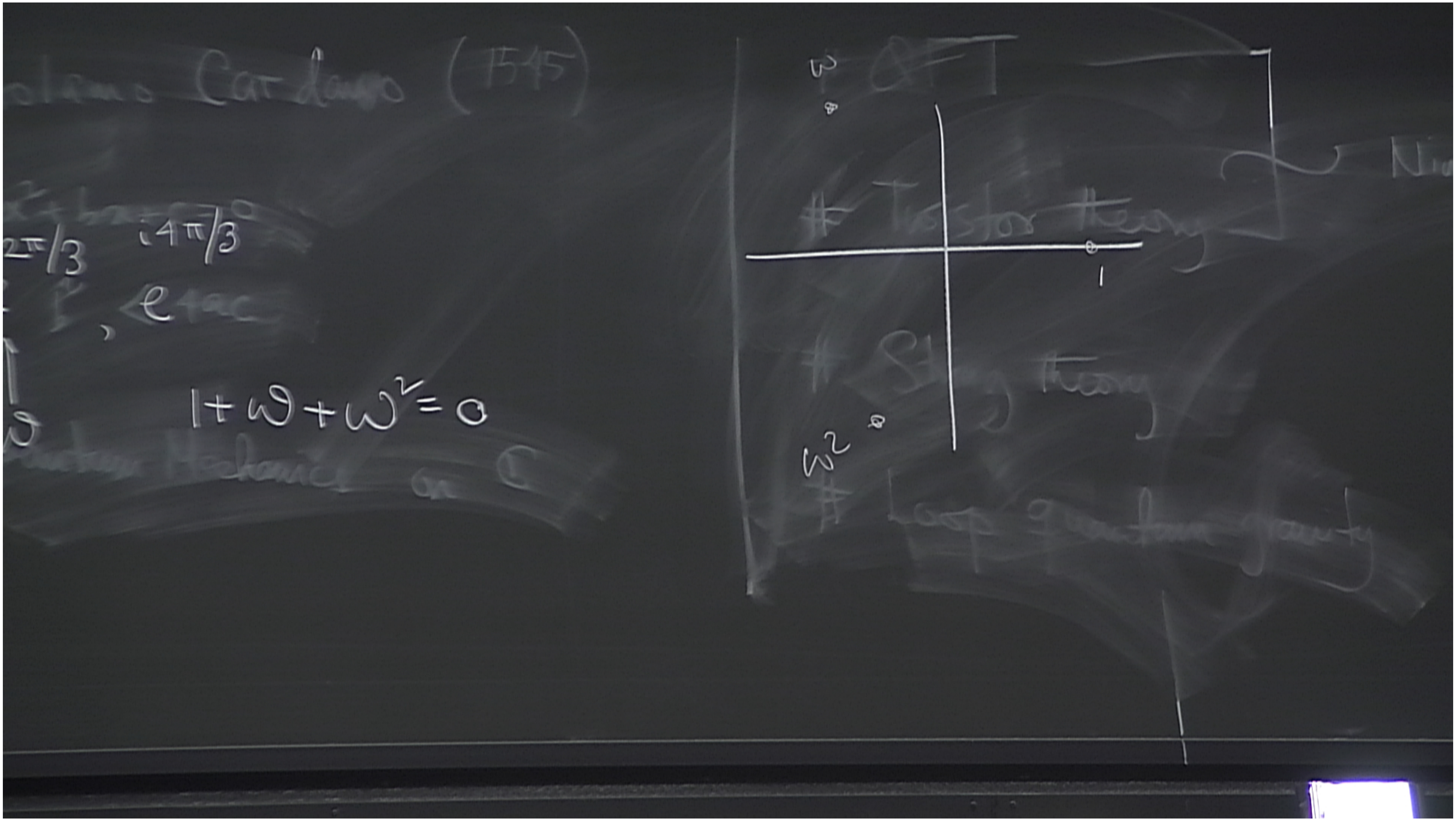
$k = 0, 1, \ldots, n-1$

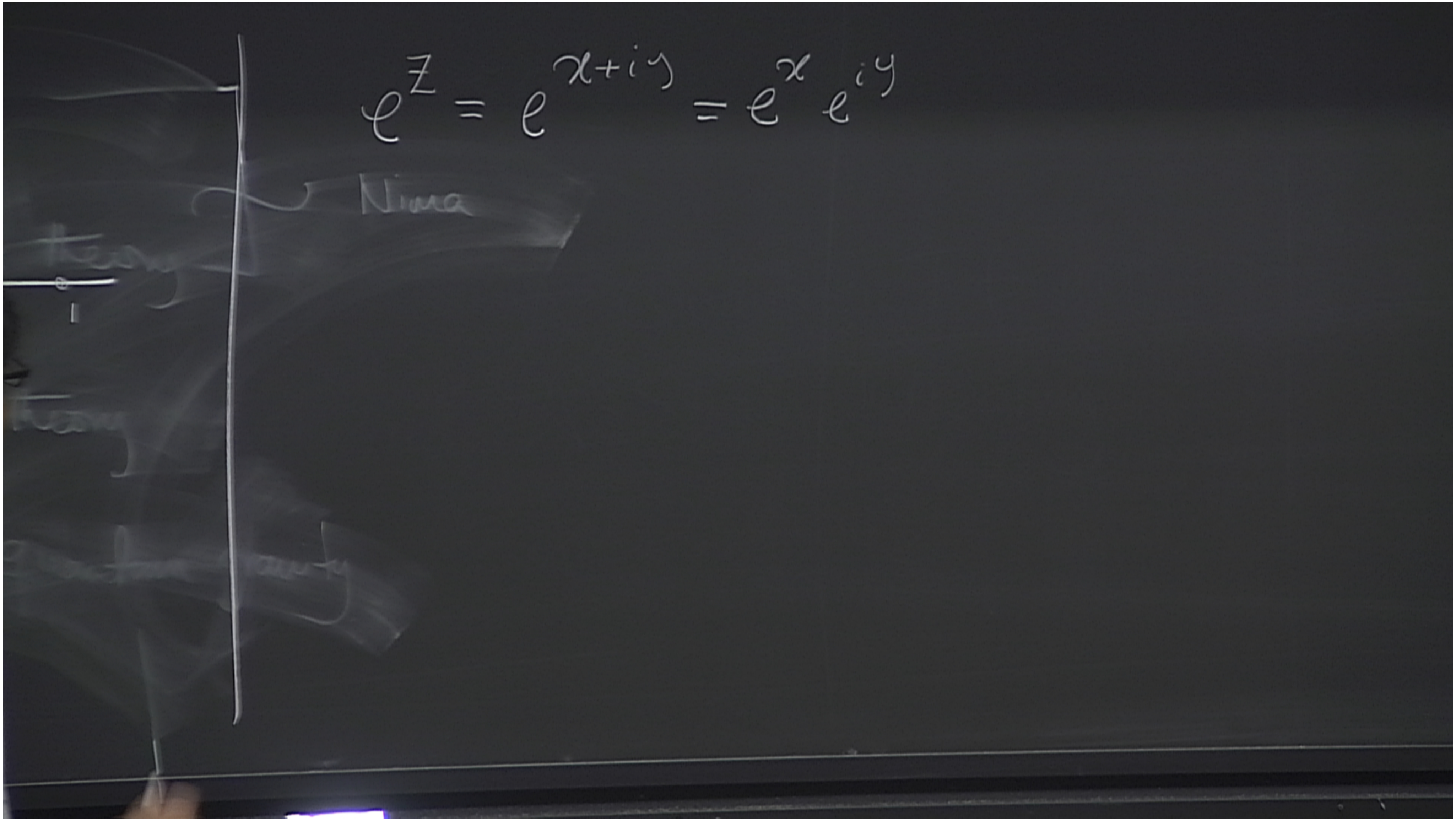
n distinct values of Z

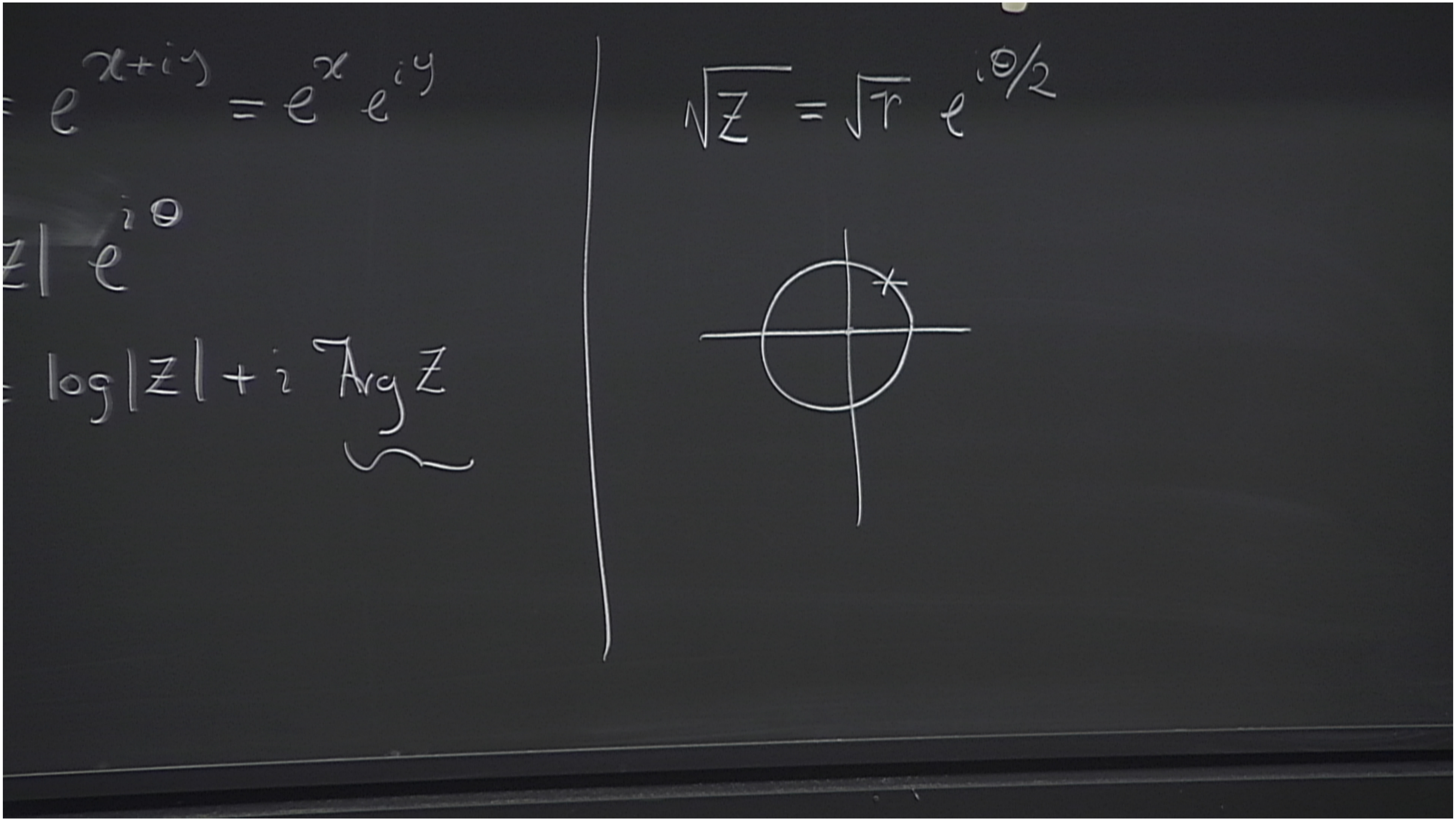


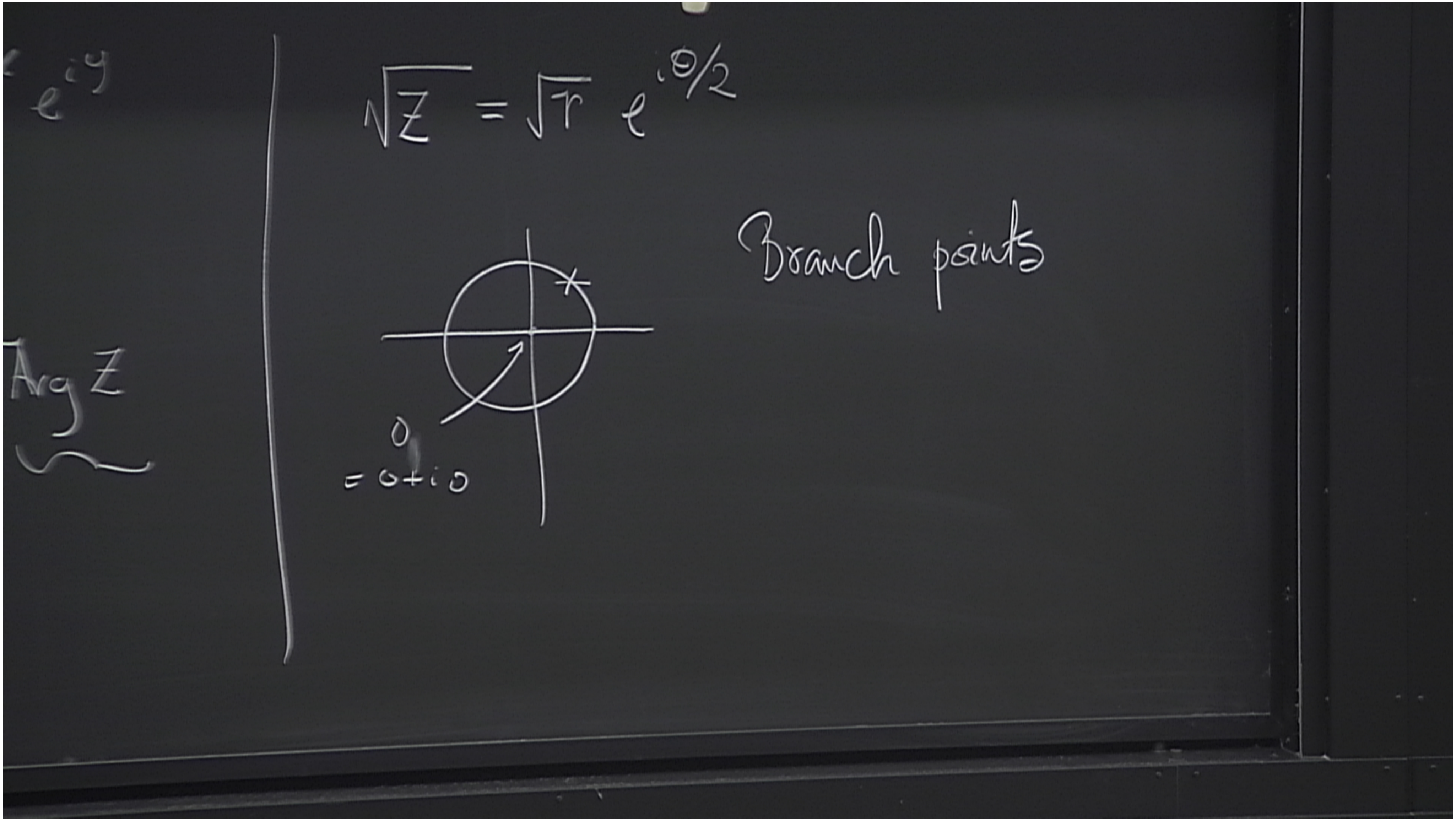


Girolamo Cardano (1545)
z^3 = 1
z = 1, e^{i2π/3}, e^{i4π/3}
# Quantum Mechanics on C

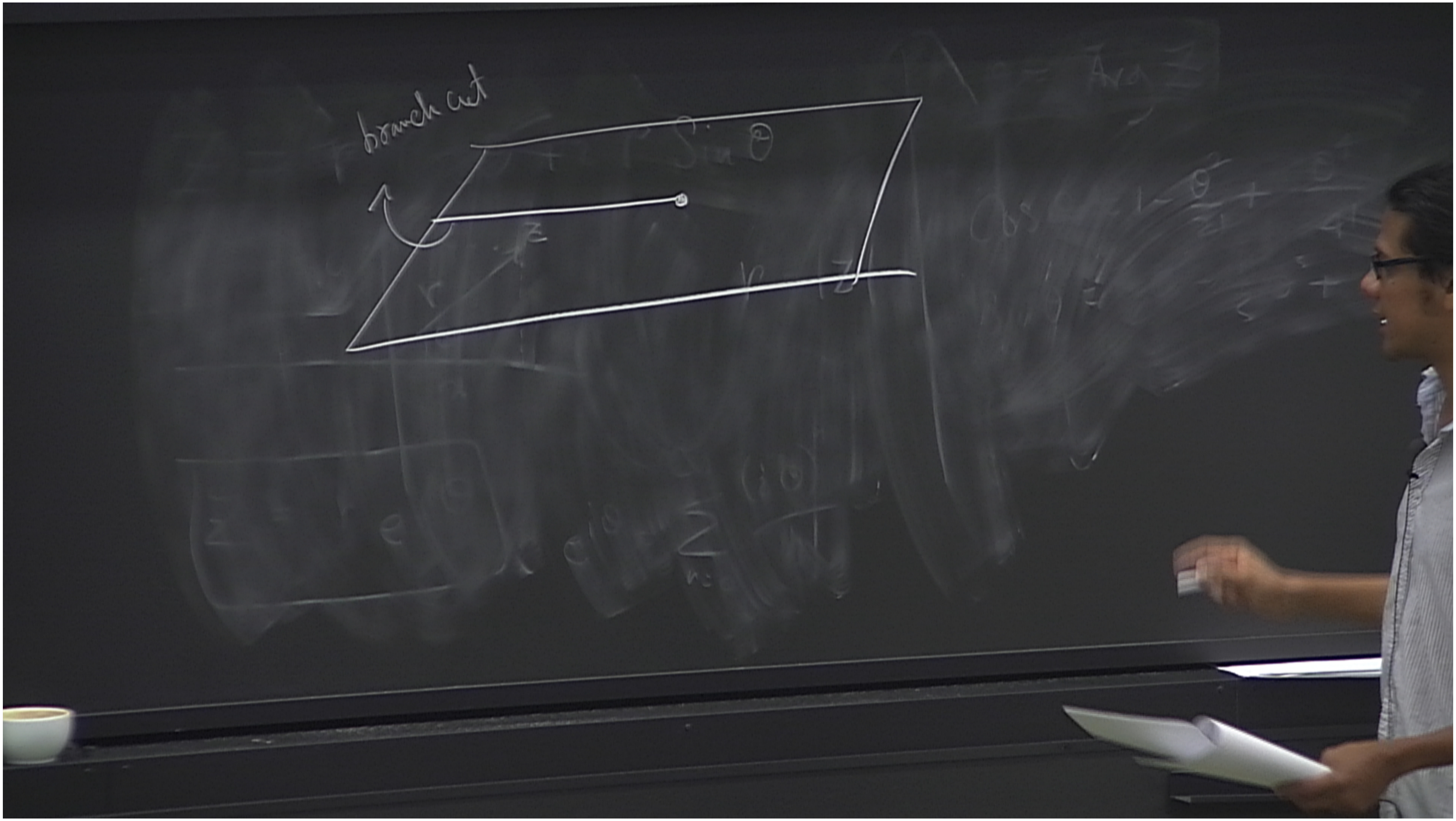

Cardano (1545)
i4π/3
2π/3
1+ω+ω²=0
ω
ω²
1

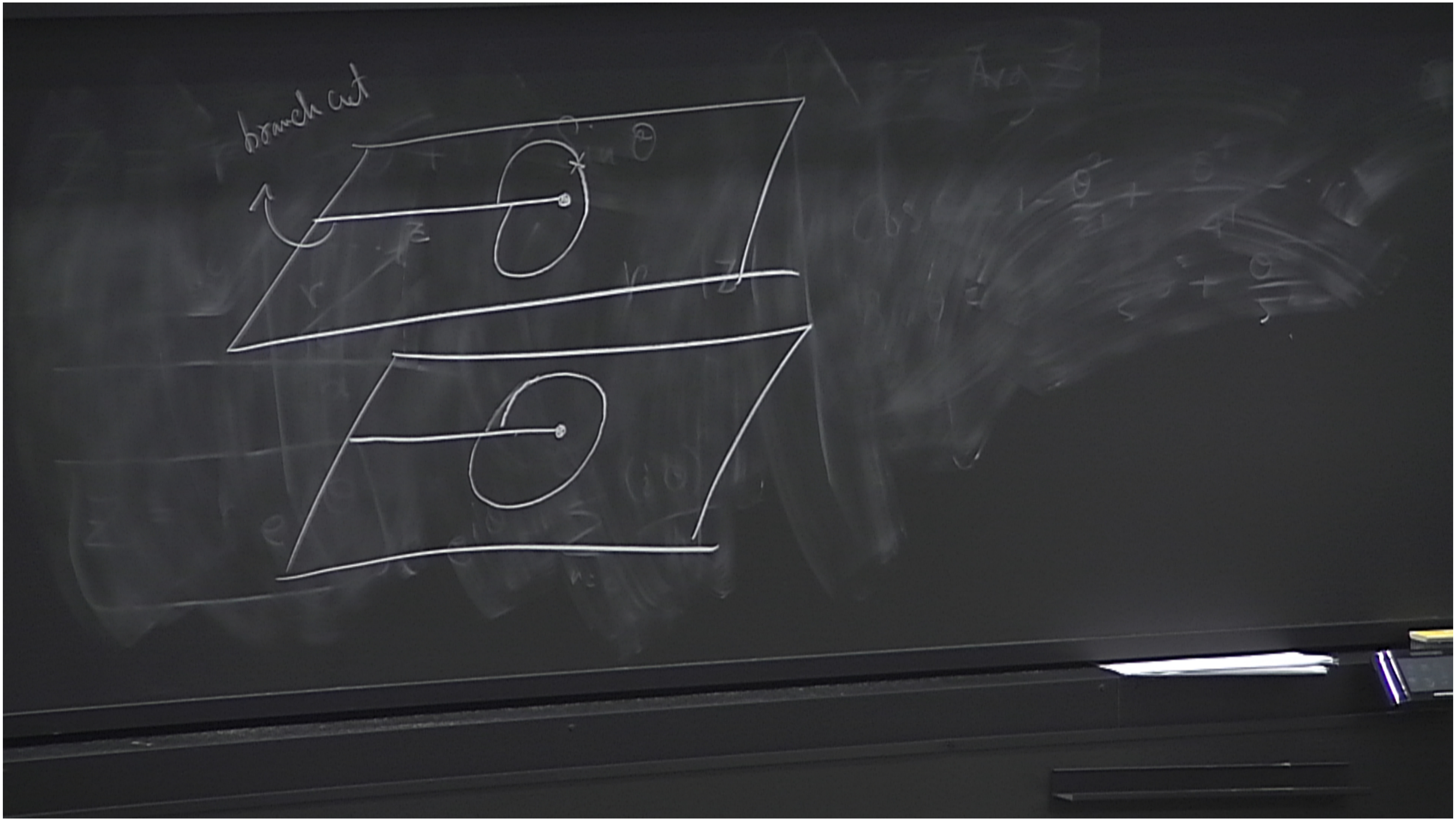

$e^{z} = e^{x+iy} = e^{x} e^{iy}$
Nima

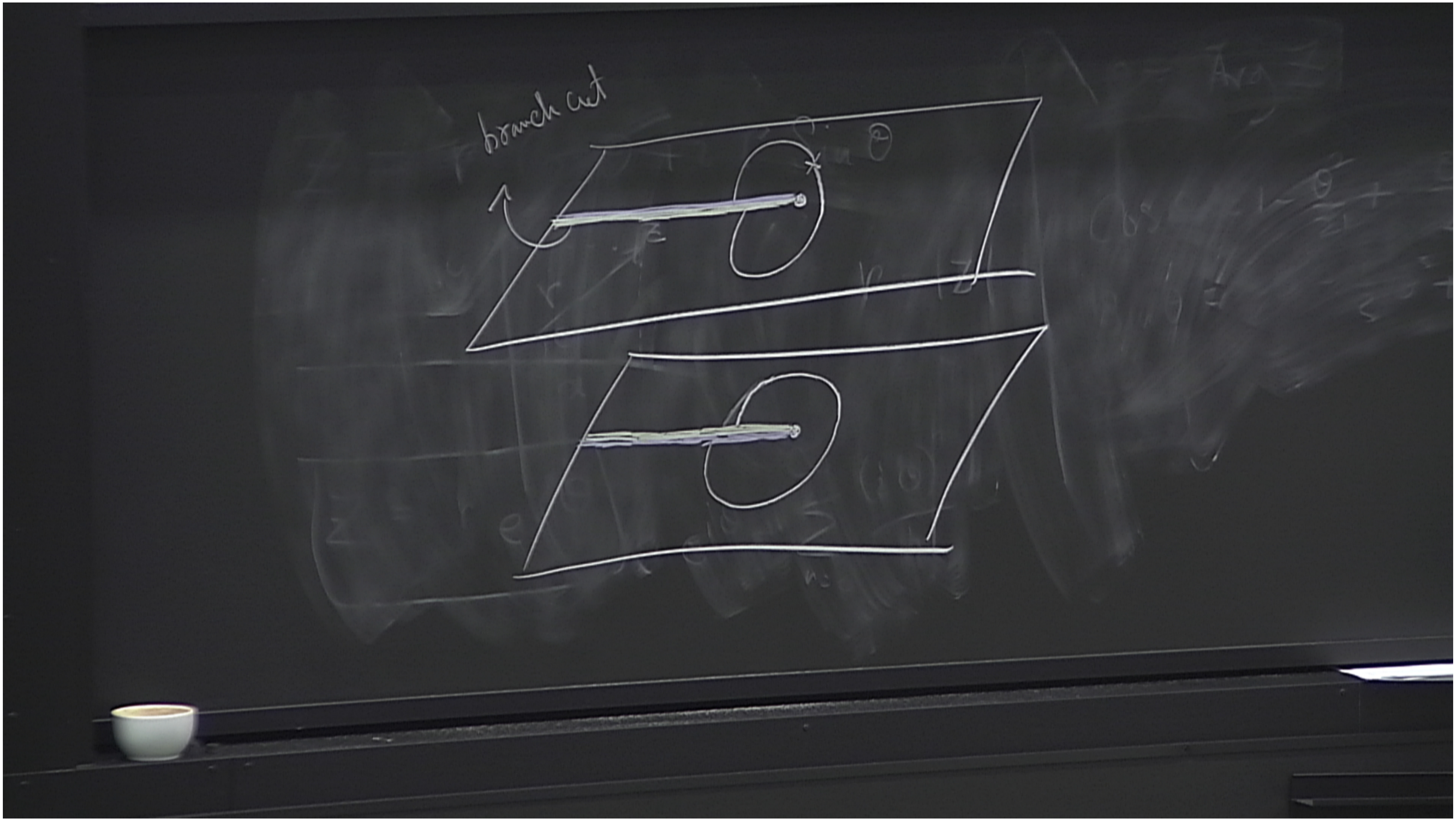

$= e^{x+iy} = e^x e^{iy}$
$|z| e^{i\theta}$
$= \log|z| + i \operatorname{Arg} z$
$\sqrt{z} = \sqrt{r}\, e^{i\theta/2}$



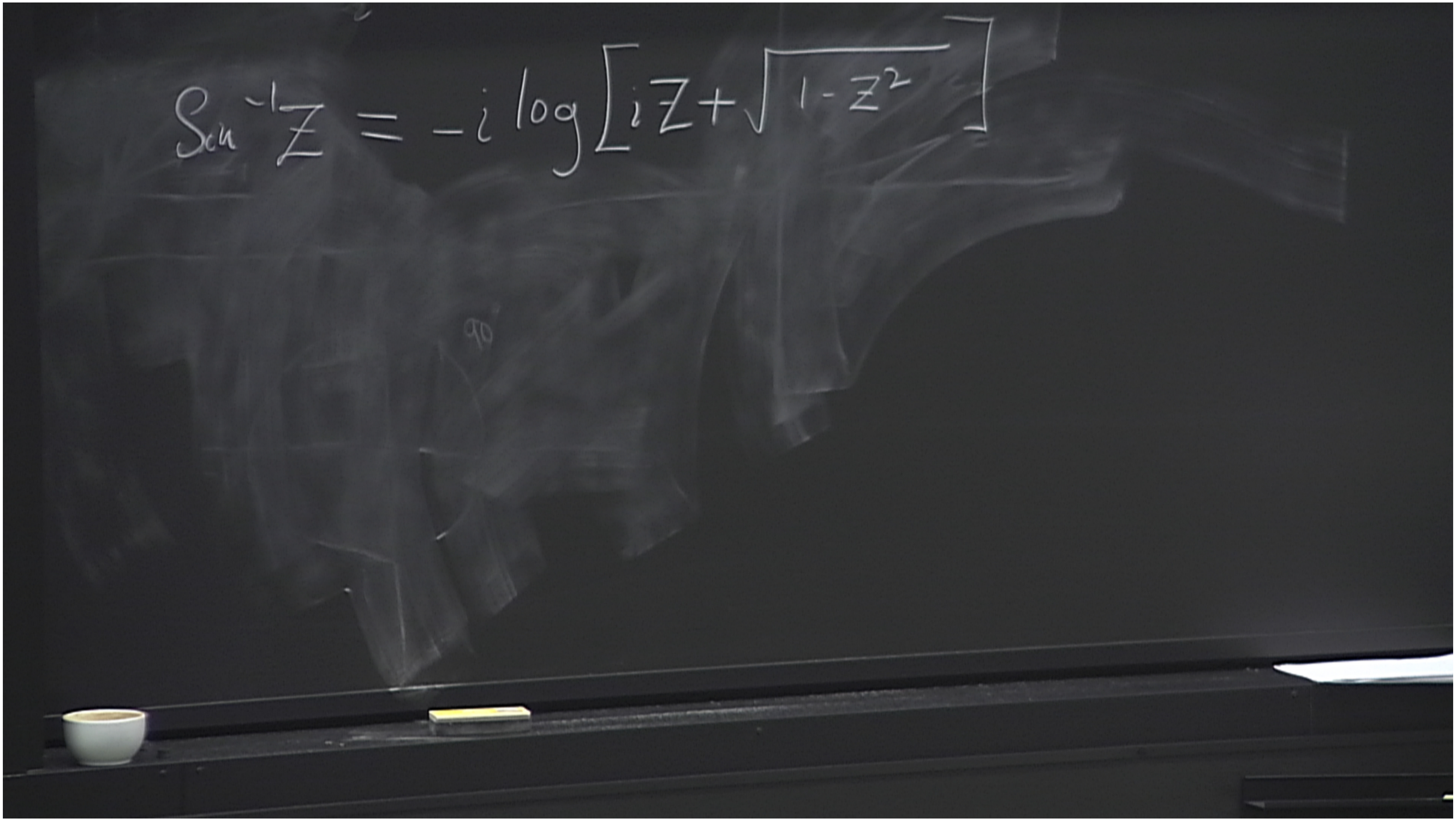
e^{iy}
$\sqrt{Z} = \sqrt{r}\, e^{i\theta/2}$
Branch points
Arg Z
0
$= 0 + i0$

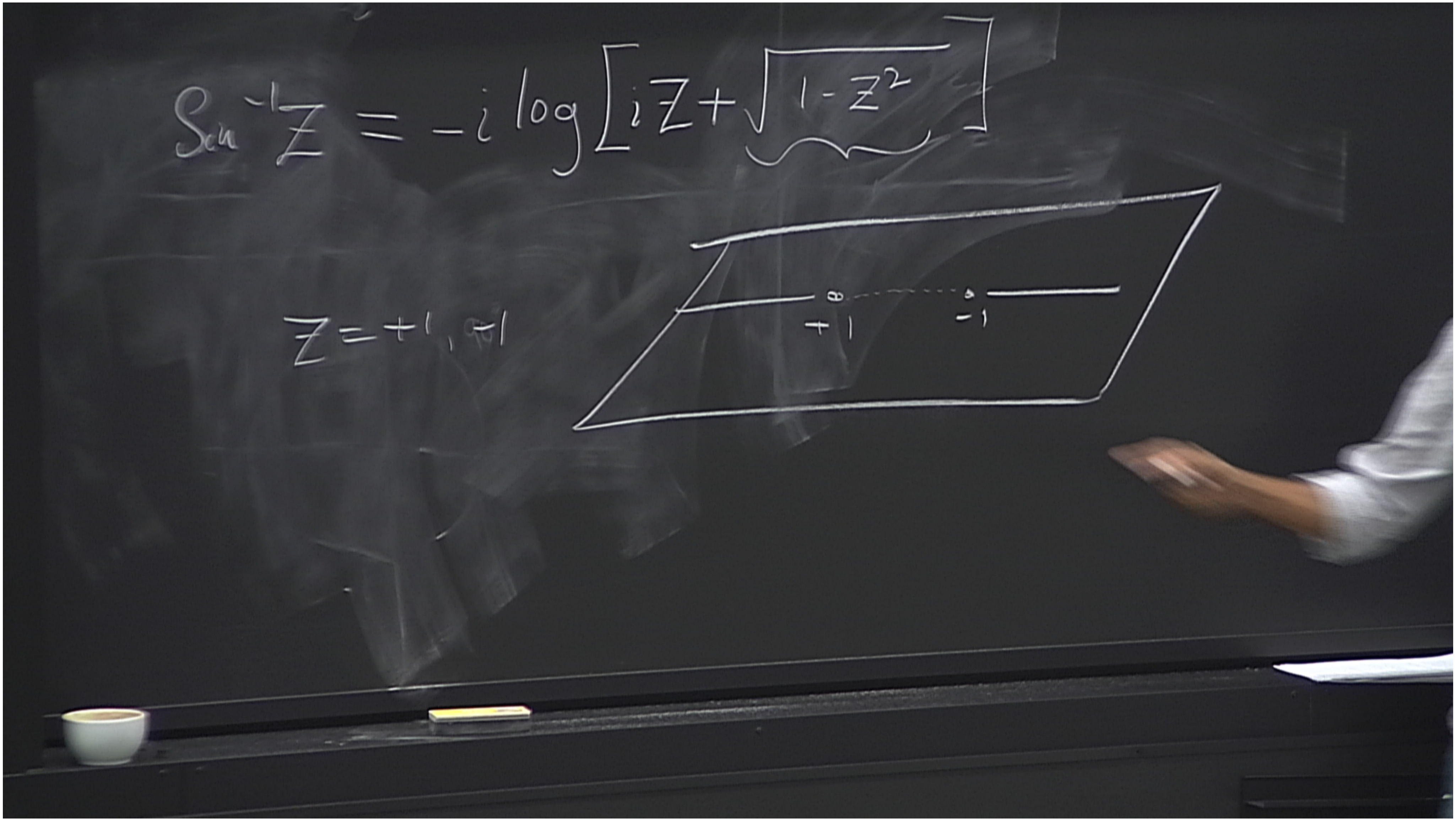

branch cut
r sin θ

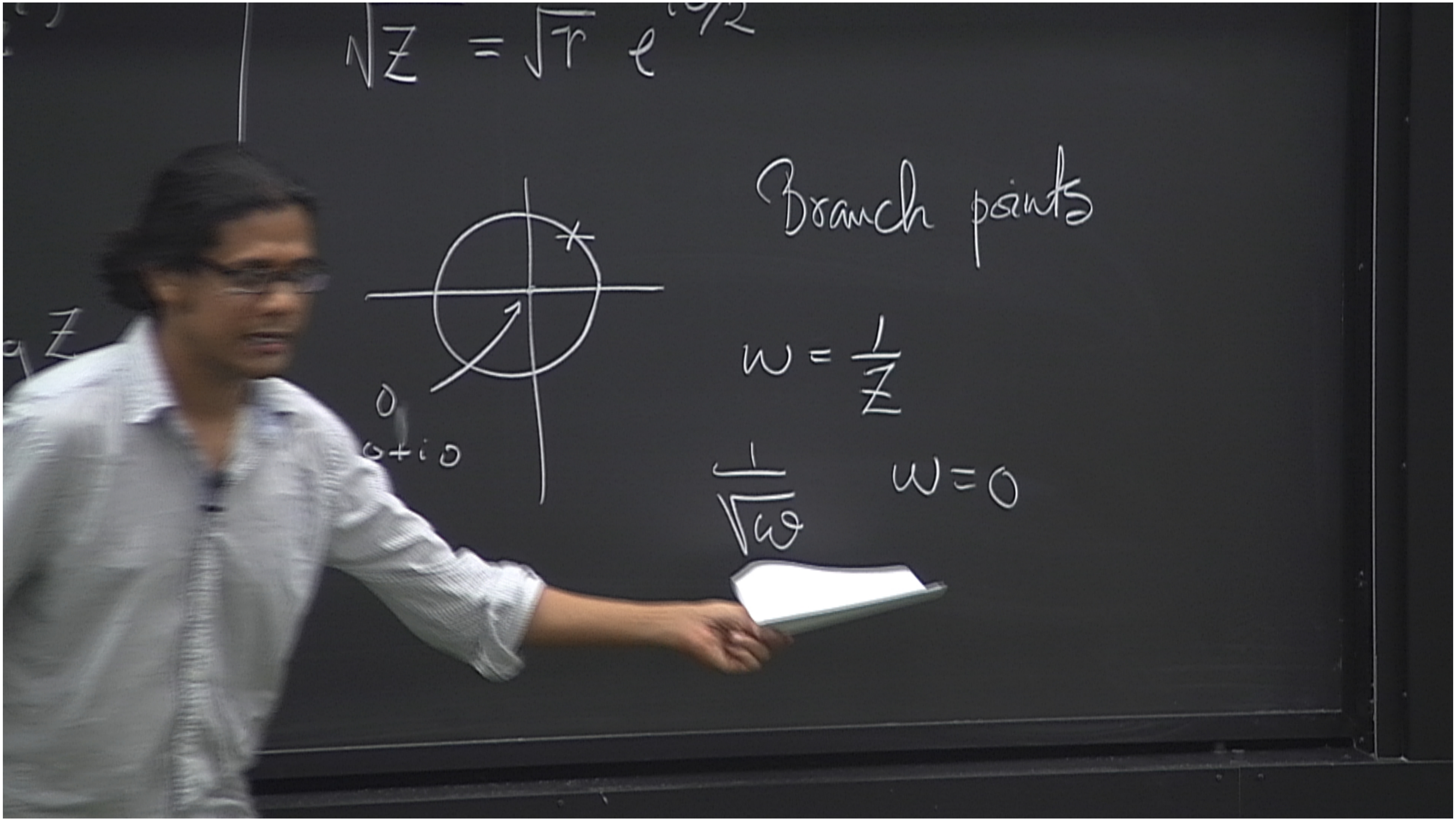

branch cut


branch cut


2 Sheeted Riemann Surface


$\sin^{-1} z = -i \log\left[iz + \sqrt{1 - z^2}\right]$


Sin⁻¹ Z = −i log [iZ + √(1 − Z²)]
Z = +1, −1
+1
−1


√Z = √r e
Branch points
w = 1/Z
1/√w
w = 0