Title: Defining the AdS/CFT Y-system and Solving it Using a Finite Set of Equations

Date: Aug 19, 2011 10:50 AM

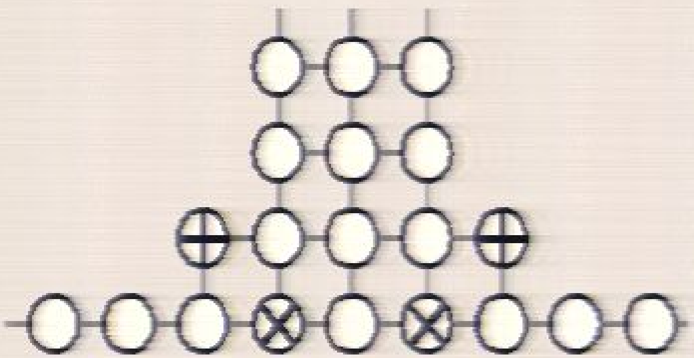
URL: http://pirsa.org/11080060

Abstract: I will show how to solve the AdS/CFT Y-system in terms of a finite set of nonlinear integral equations (FiNLIE). To uniquely define the solution we impose the set of constraints on the Y- and T-functions which can be summarized as: symmetry (PSU(2,2|4) + Z_4) + analyticity + large volume asymptotics. Some of these constraints describe previously unknown properties of the Y-system. As an important check of our approach, we showed that the proposed constraints can be also used to derive the infinite set of the TBA equations. We also successfully checked FiNLIE numerically for the case of Konishi operator.

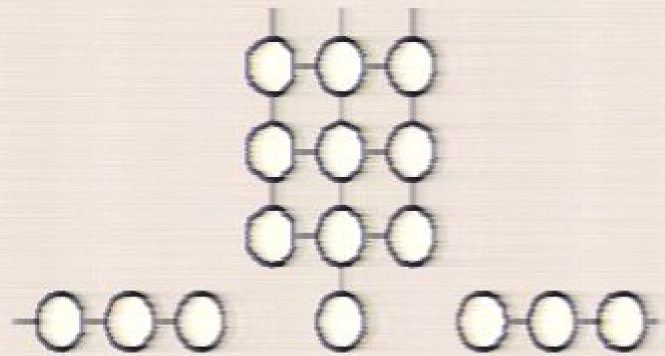
Defining the AdS/CFT Y-system and solving it by a finite set of equations

Dmytro Volin
Penn State University

Perimeter, 2011



Mirror



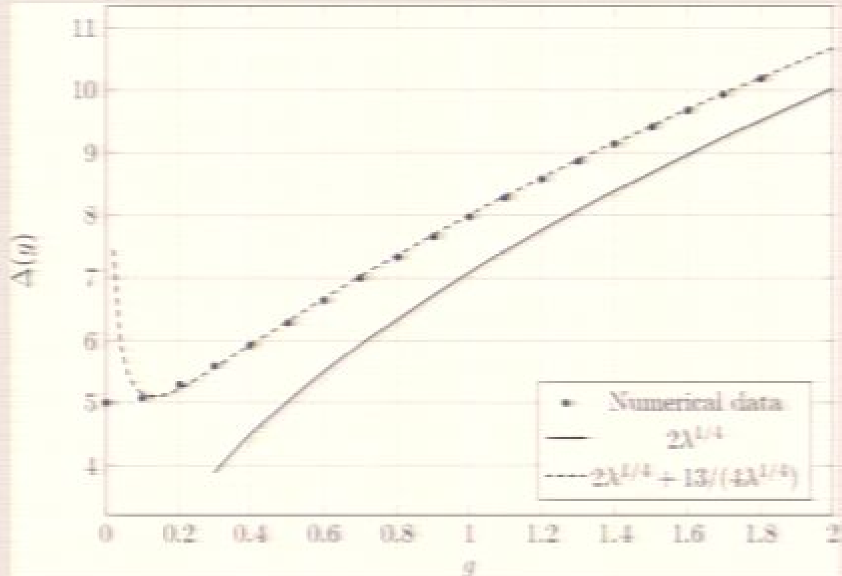
Magic

110*.****
N.Gromov,
V.Kazakov,
S.Leurent.
D.V.

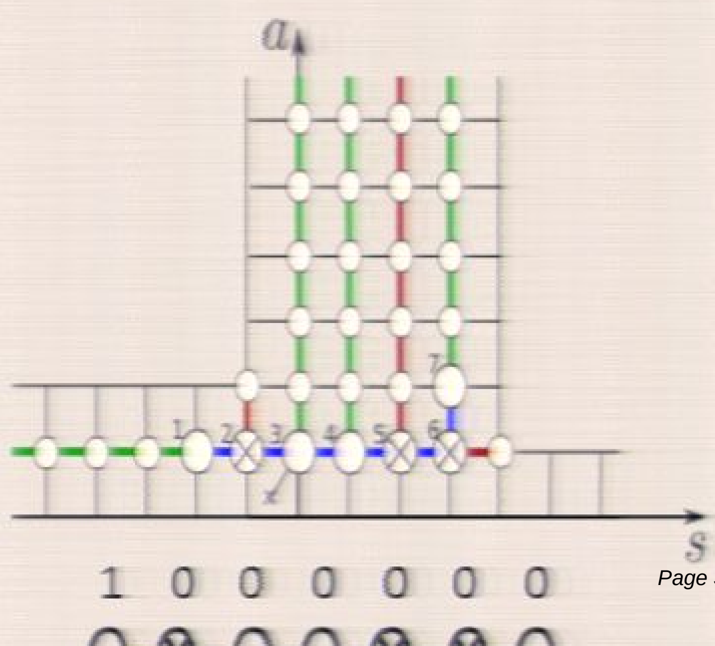
U(2,2|4) + Det=1 + $\mathbb{Z}_4$

+ analyticity + large volume explicitly = solution of the spectral problem ther papers discussed in this talk:

1102.1040, N.Gromov, D.Serban, I.Shenderovich, D.V.]
hort operators in Sl(2) sector (Konishi et al)
nalytically at strong coupling from quasiclassics



1012.3453, D.V]
atterns in arbitrary T-hooks



$\mathcal{N}=4$ SYM = IIB, $AdS_5 x S^5$

0 ————————————————————→ ∞

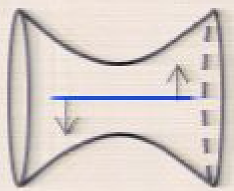
$$g^2 = \frac{g_{YM}^2 N_c}{16\pi^2}$$

Spectral problem:

Conformal dimension of local operators = Energy of string states = ?

Infinite volume: cusp anomalous dimension

$\boldsymbol{Tr\, Z\, D^S Z}$



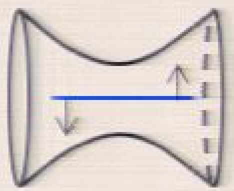
$$\Delta - S = f[g] \log S + \ldots, \quad S \to \infty$$

Finite Volume: dimension of Konishi operator

$$\boldsymbol{Tr\, XZXZ - Tr\, XXZZ}$$

Infinite volume: cusp anomalous dimension

$Tr\, Z\, D^S Z$



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Solved by bootstrap for factorized scattering matrix

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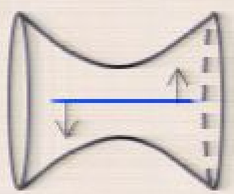

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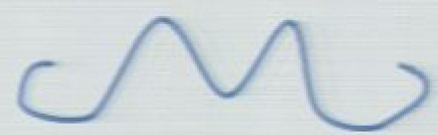

$$Tr\, XZXZ - Tr\, XXZZ$$

Should be solvable by bootstrap for transfer matrices

Bootstrap, infinite volume case: [Zamolodchikov & Zamolodchikov,78]

Object	Constraints	Equations for spectrum	Explicit solution
Factorized S-matrix	Symmetry (SU(2|2),YBE,crossing) Analyticity, Particle structure	Asymptotic Bethe Ansatz	Cusp anomalous dimension

Staudacher, 04]

[Beisert, 05]
[Janik, 06]
[Beisert, Hernandez, Lopes, 06]
[Beisert, Eden, Staudacher, 06]
....
[Arutyunov, Frolov, 09]
[D.V., 09]

[Beisert, Staudacher, 03]
[Beisert, 03-04]

usp anomalous dimension $\quad Tr\, Z\, D^S Z$

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Weak coupling:

[Moch, Vermaseren, Vogt, 04]
[Lipatov et al., 04]
[Bern et al., 06]
[Cachazo et al., 06]

$$f[g] = 8g^2 - \frac{8}{3}\pi^2 g^4 + \frac{88}{45}\pi^4 g^6 - \left(\frac{584}{315}\pi^6 + 64\zeta(3)^2\right) g^8 + \ldots$$

[Beisert, Eden, Staudacher, 06]

Numerics:

[Benna, Benvenuti, Klebanov, Scardicchio, 06]

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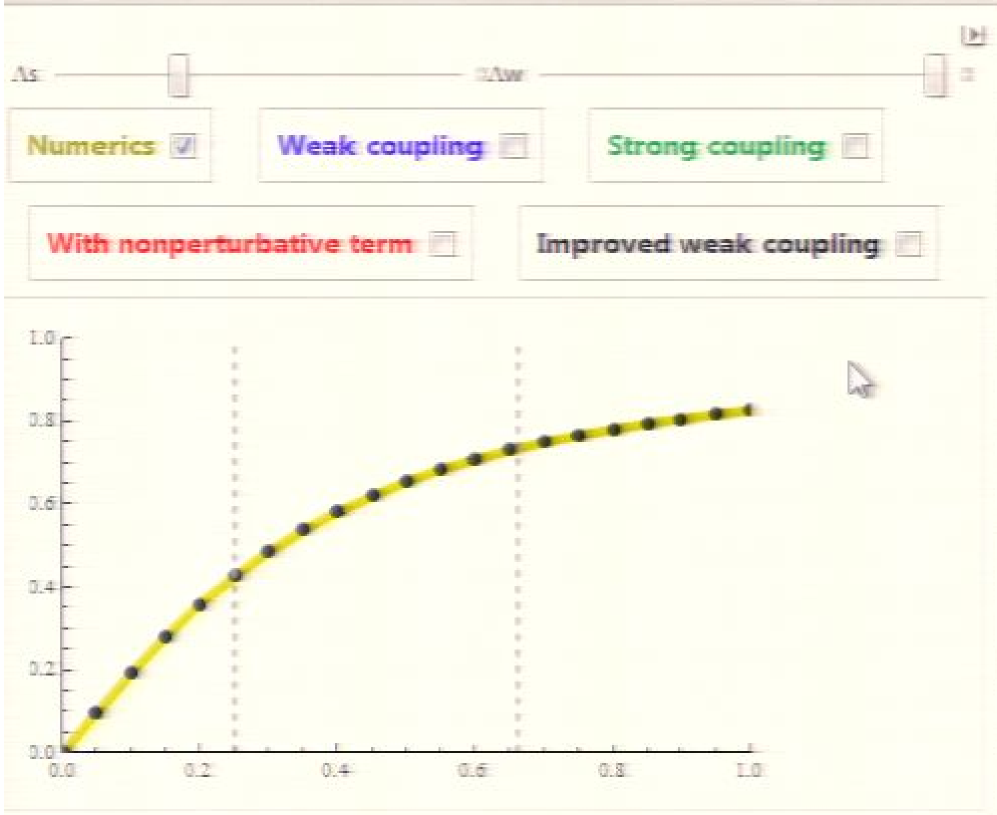

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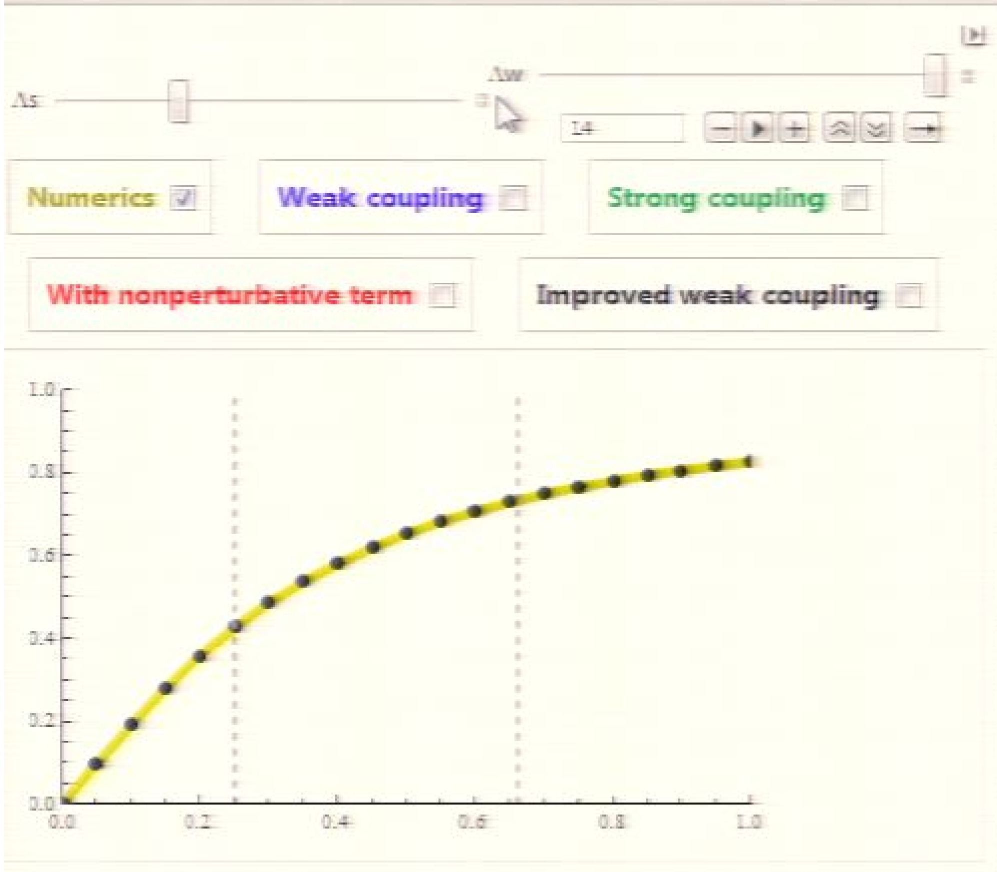

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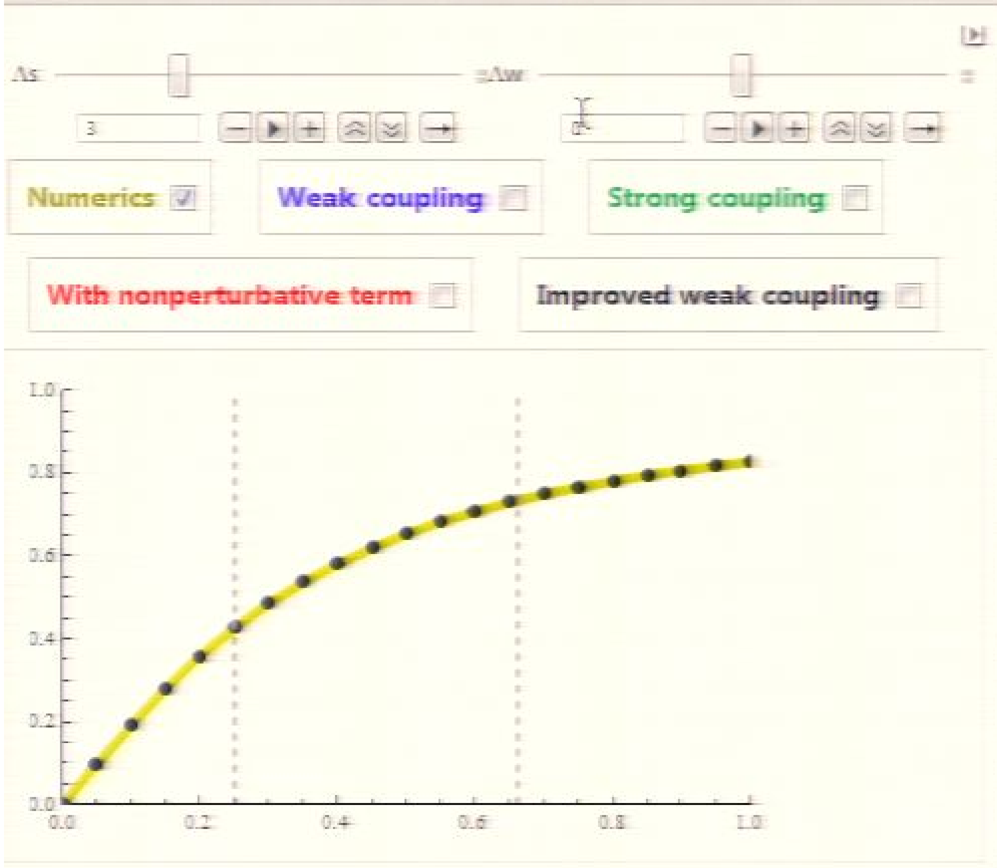

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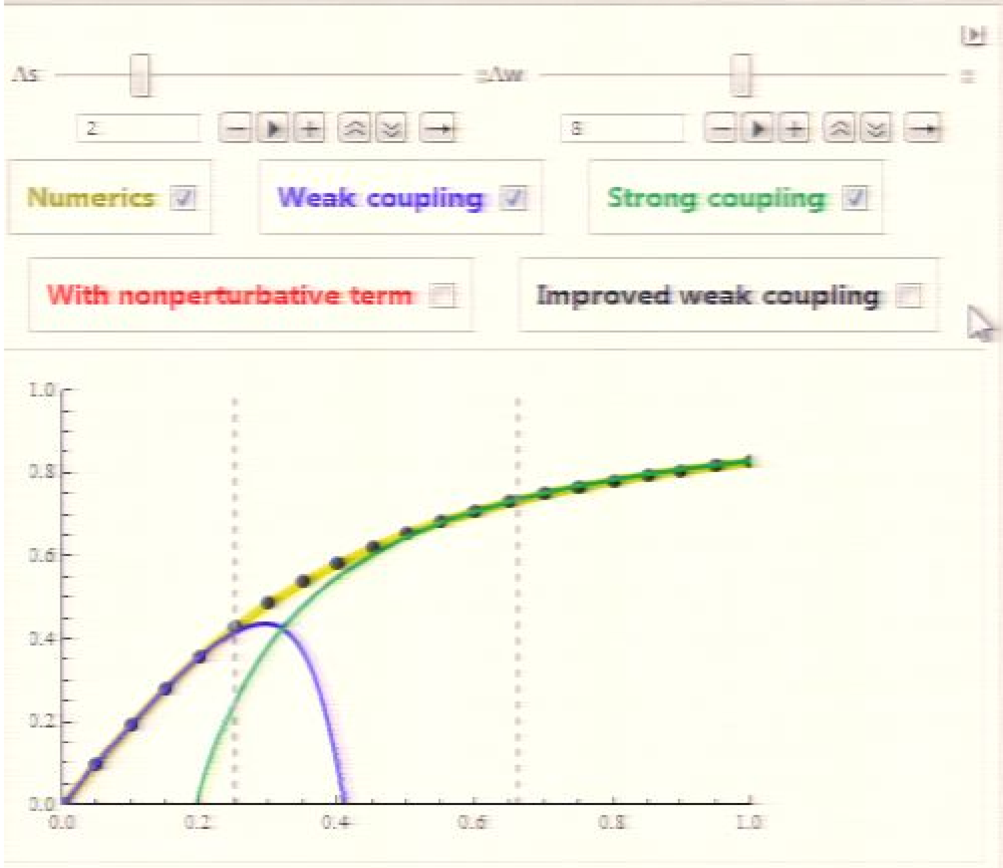

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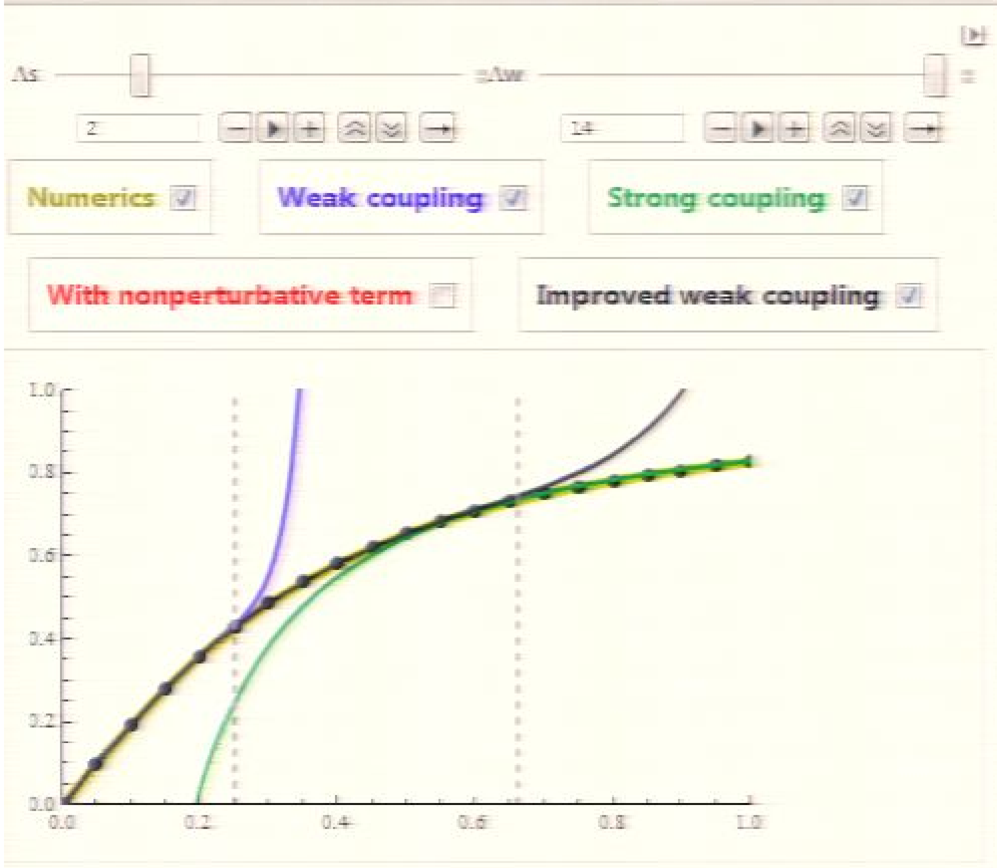

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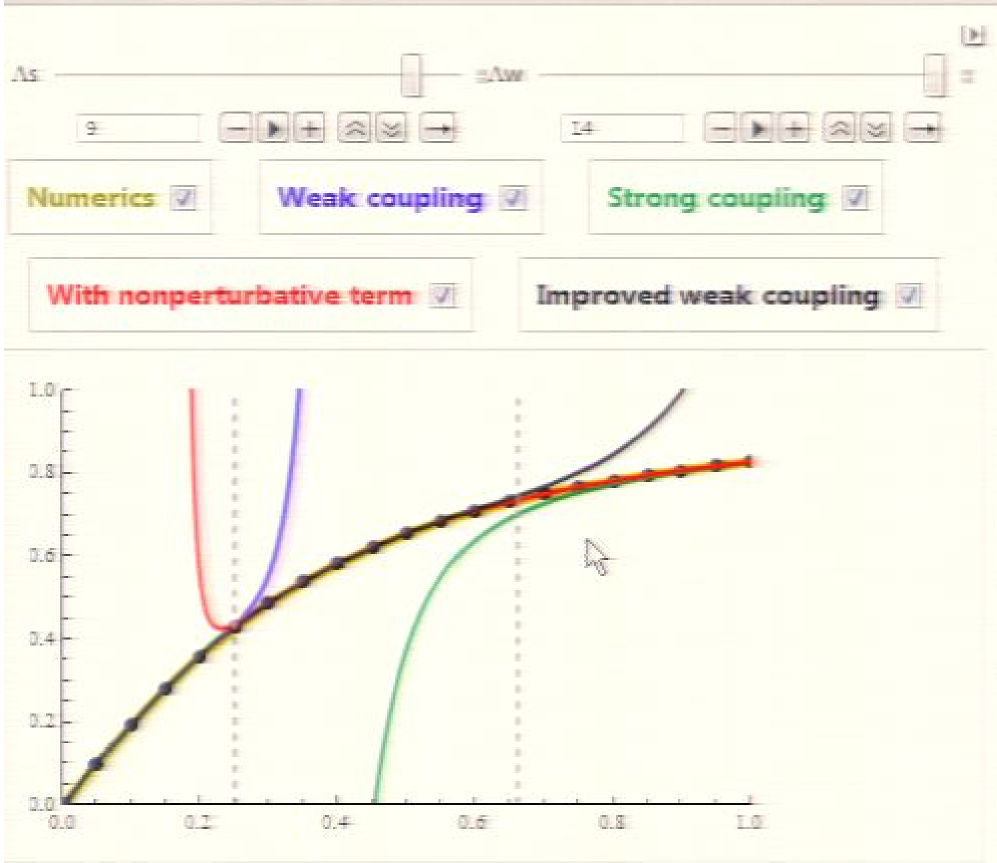

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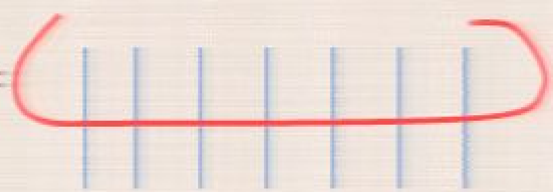
What it is expected to be (from experience in other integrable systems):

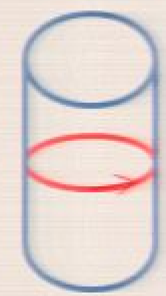
Objects | Constraints | Equations for spectrum | Explicit solution

$$T = Tr(L \dots L) =$$



spin chains

$$T = Tr\, Pe^{\int A}$$



field theory

[Bazhanov, Lukyanov, Zamolodchikov, 09]
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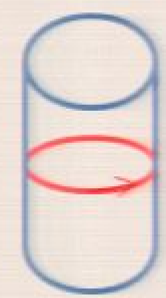
$T = Tr\, P e^{\int A}$ 

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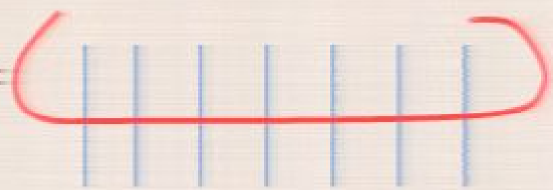
$T = Tr(L \dots L) =$ 

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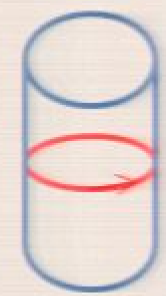
$T = Tr\, P e^{\int A}$ 

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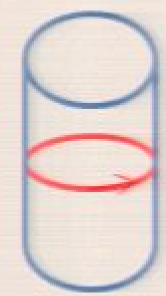

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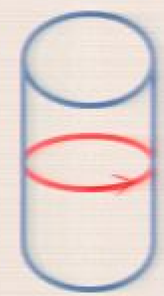
$T = Tr\, P e^{\int A}$ 

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Objects	Constraints	Equations for spectrum	Explicit solution
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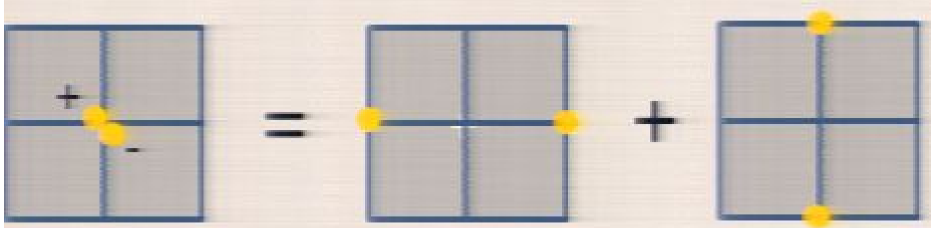
Hirota equation:

$$T^{+}_{a,s}T^{-}_{a,s} = T_{a,s+1}T_{a,s-1} + T_{a+1,s}T_{a-1,s}$$

$$f^{\pm} = f(u \pm \tfrac{i}{2})$$



Hirota is a quantum version of relation between characters

Objects	Constraints	Equations for spectrum	Explicit solution
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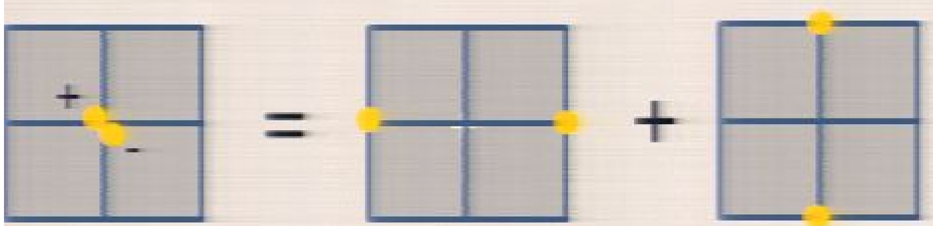

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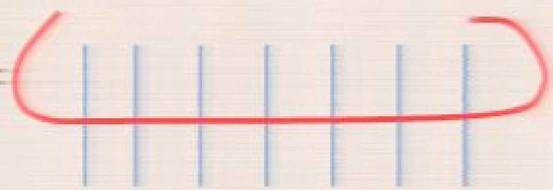
$T = Tr(L \dots L) =$ 

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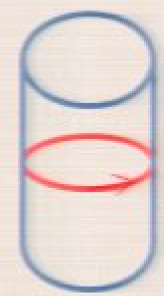
$T = Tr\, P e^{\int A}$ 

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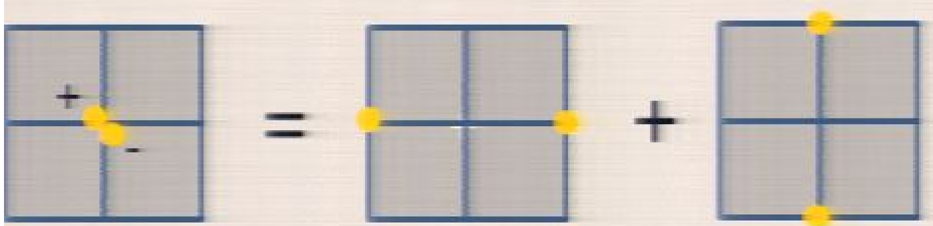

Hirota is a quantum version of relation between characters

Objects | Constraints | Equations for spectrum | Explicit solution

Transfer matrices → Symmetry + ... → →

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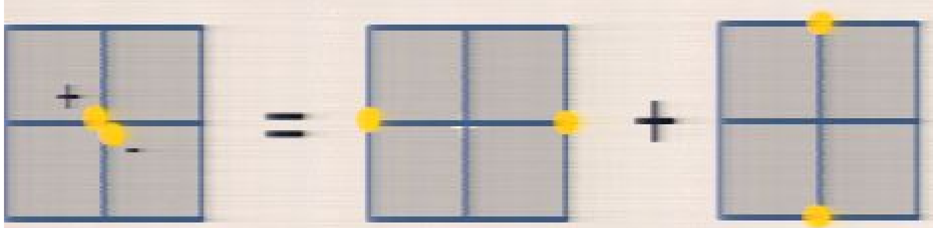

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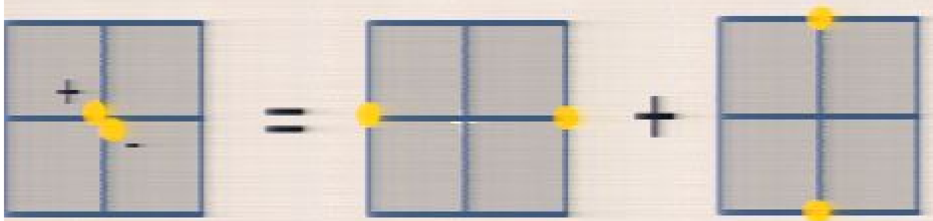

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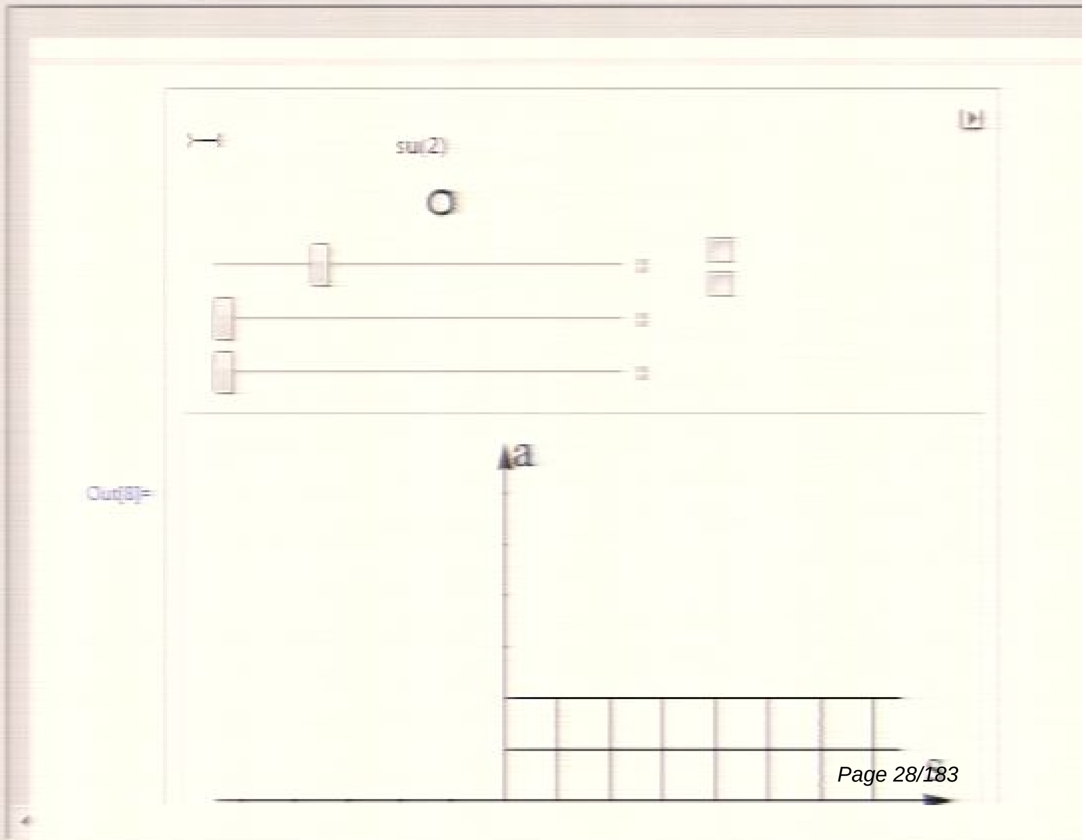

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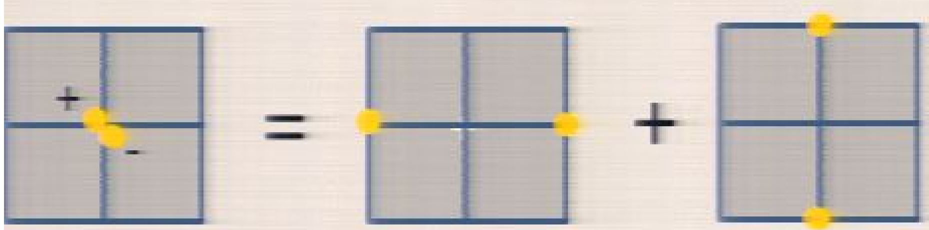

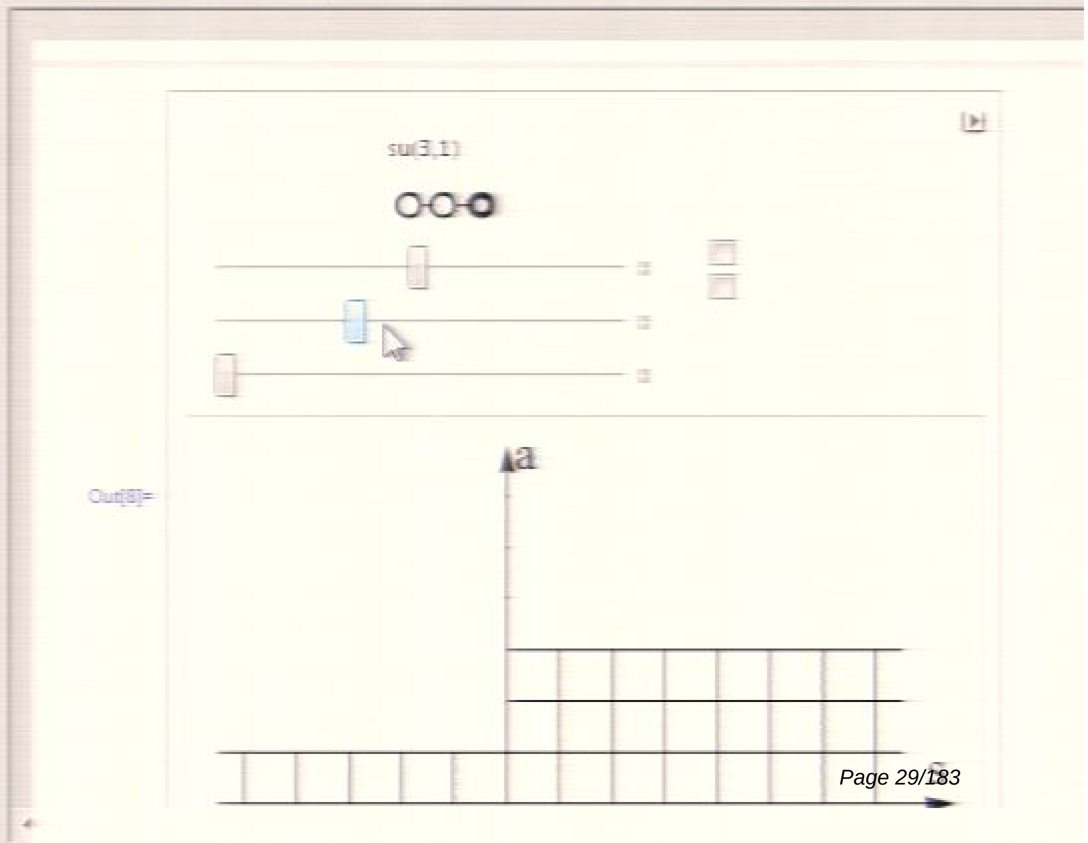

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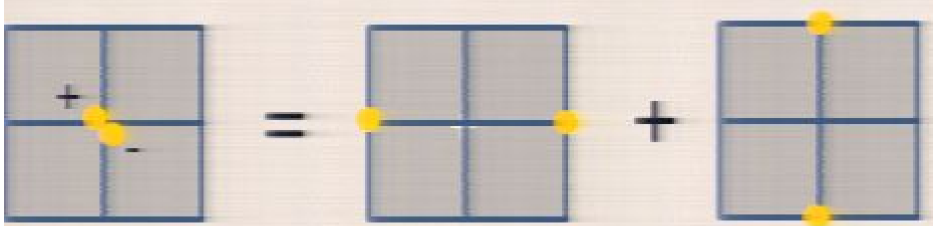

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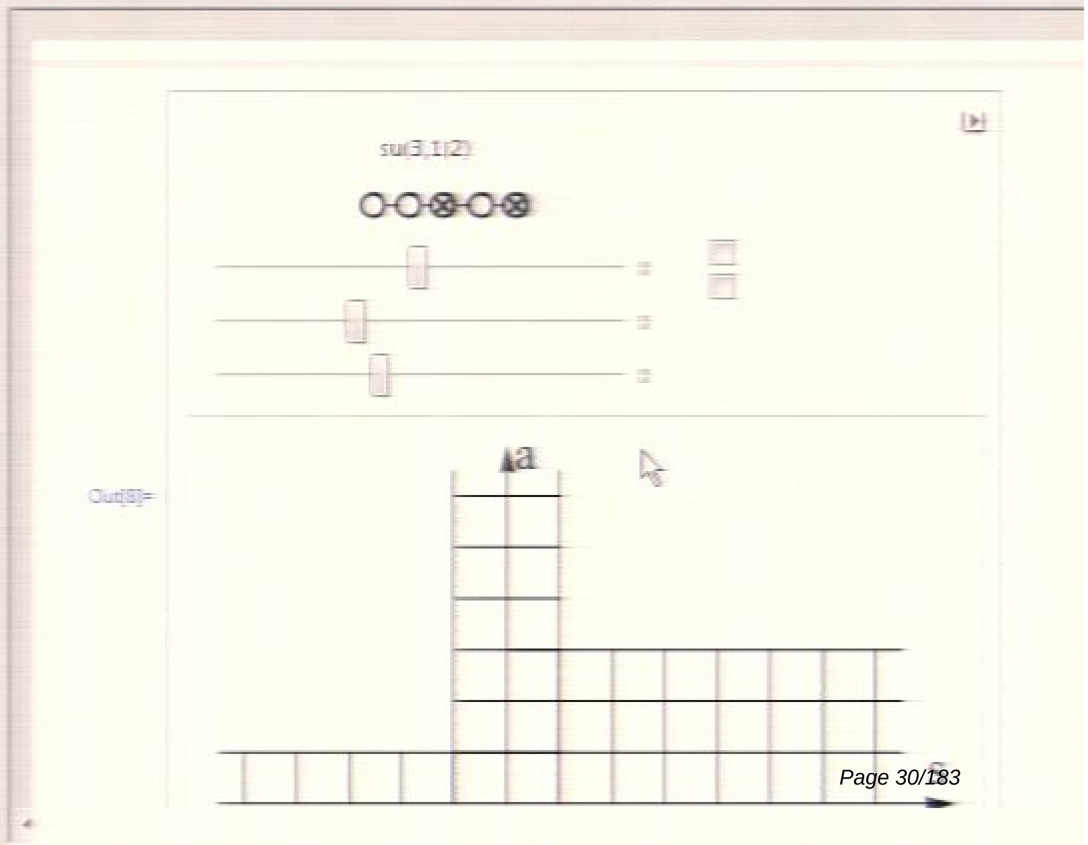

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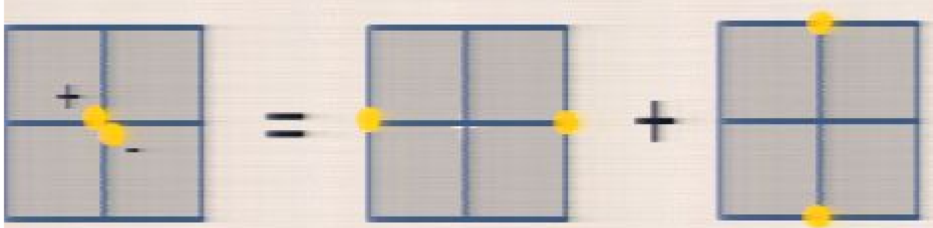

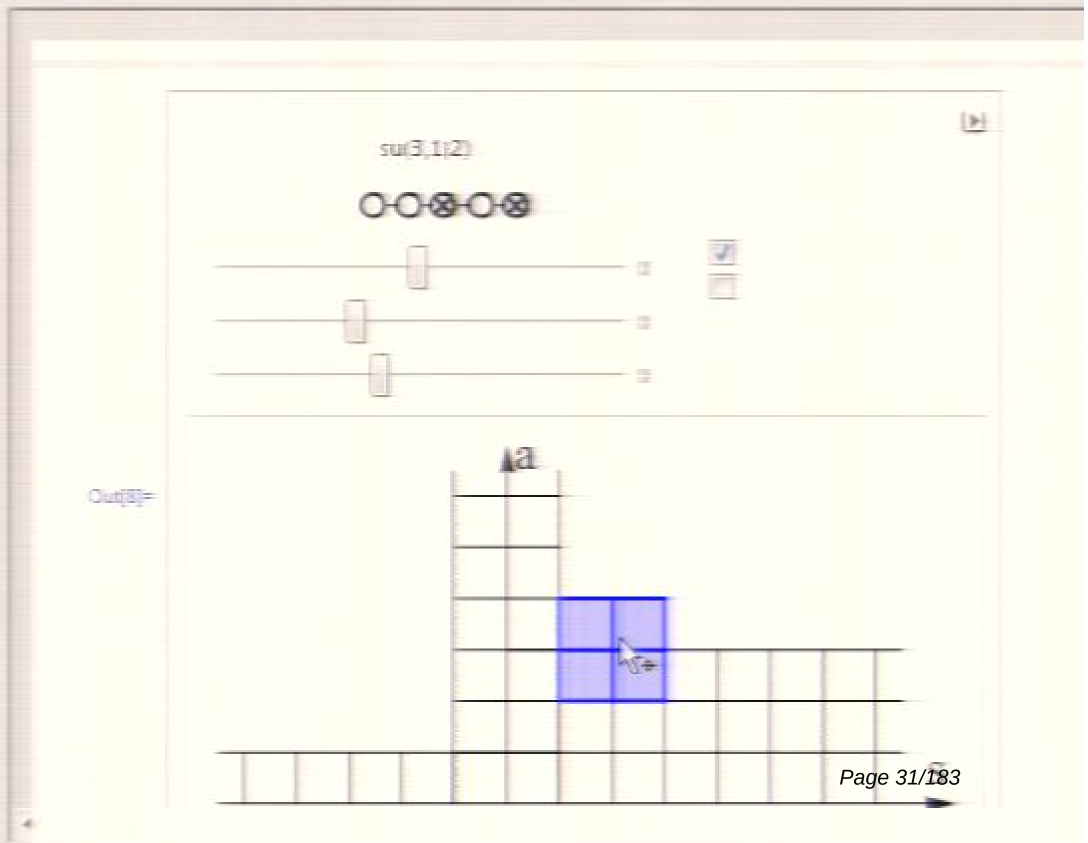

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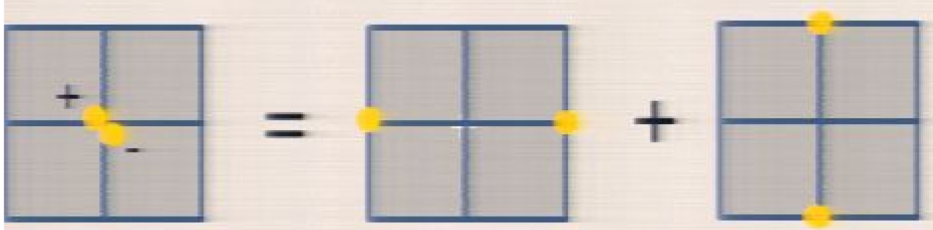

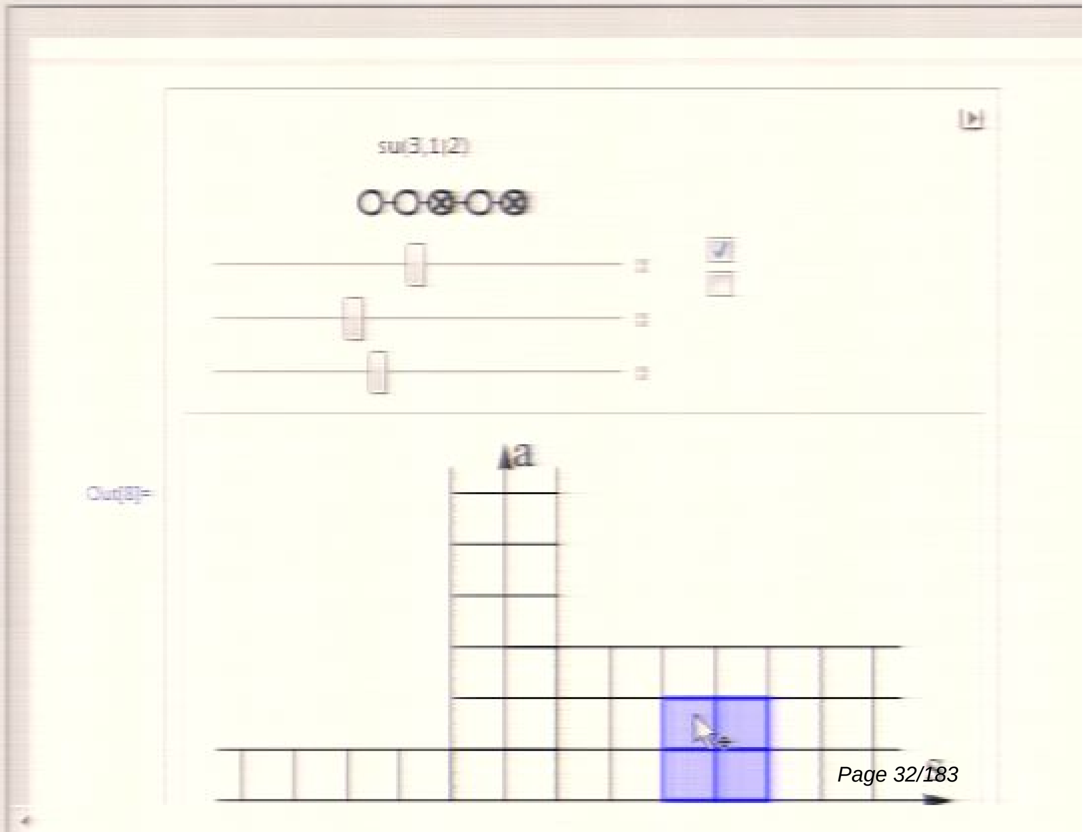

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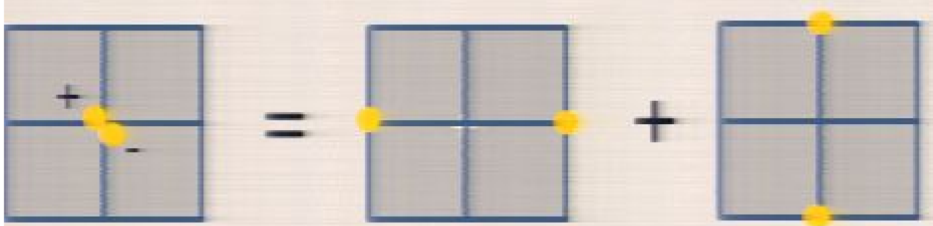

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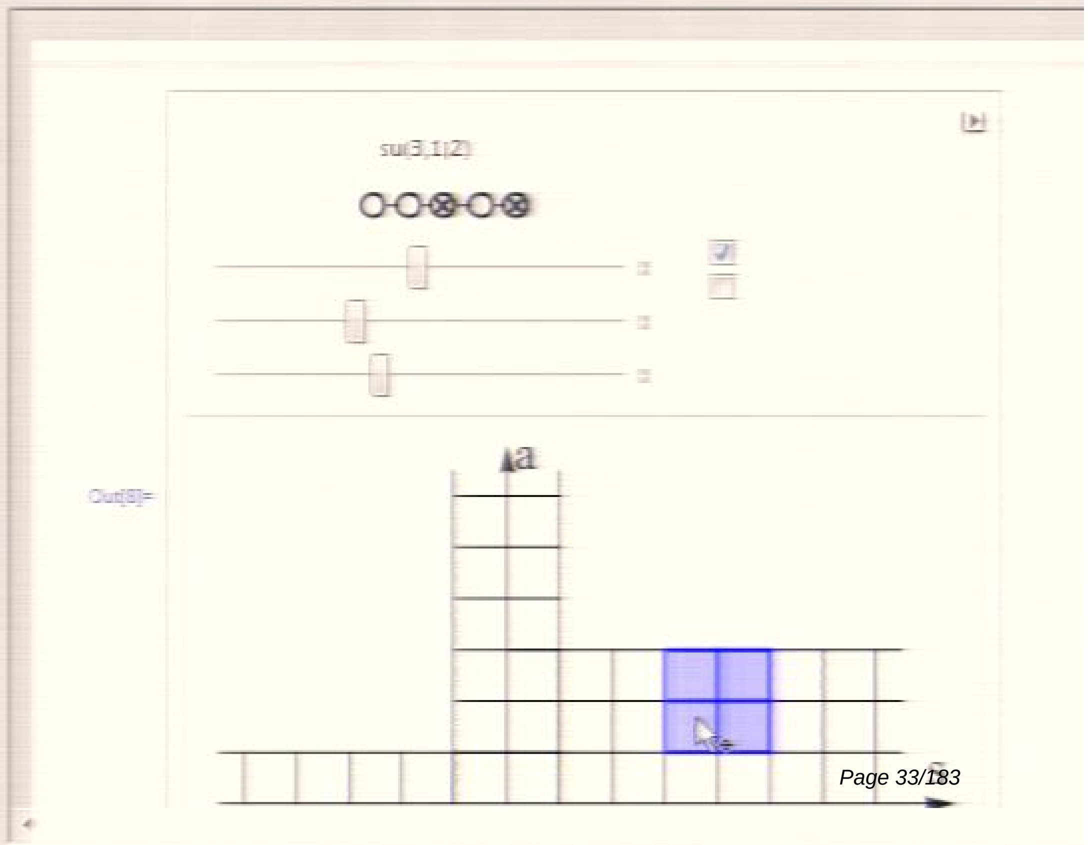

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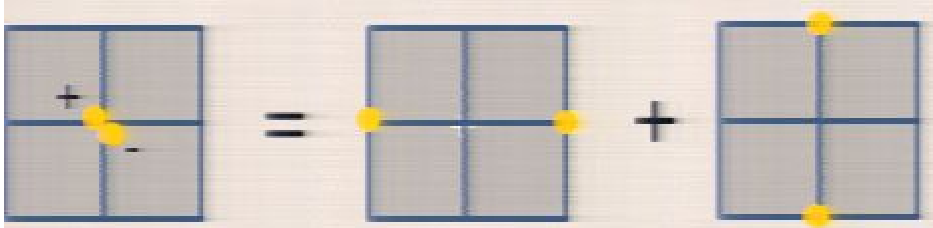

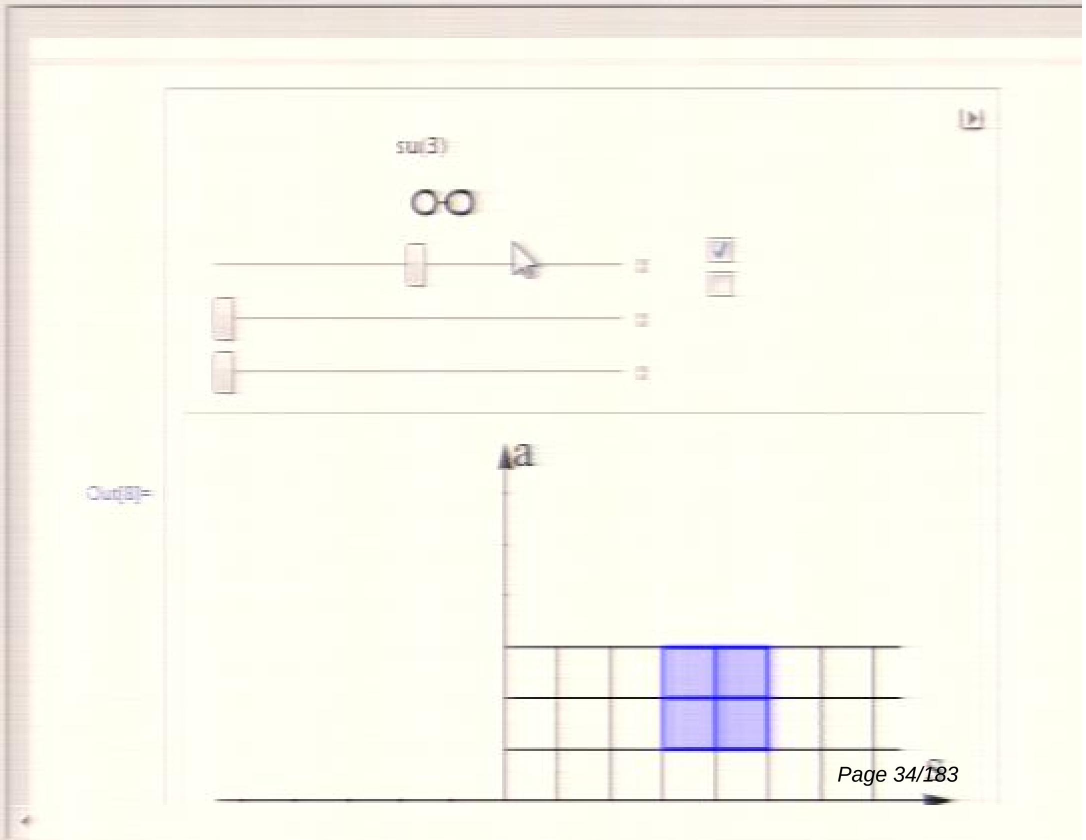

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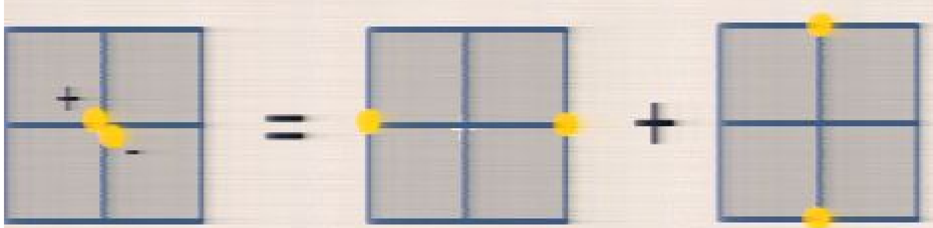

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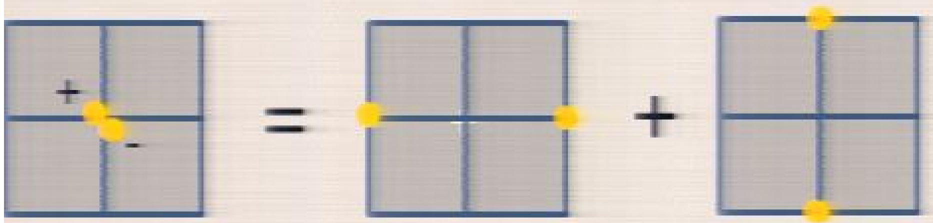

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Some facts about Hirota system

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(for generic T-hook is still a conjecture, true for PSU(2,2|4) case)

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- A hope: T-functions are quantization of monodromy matrices

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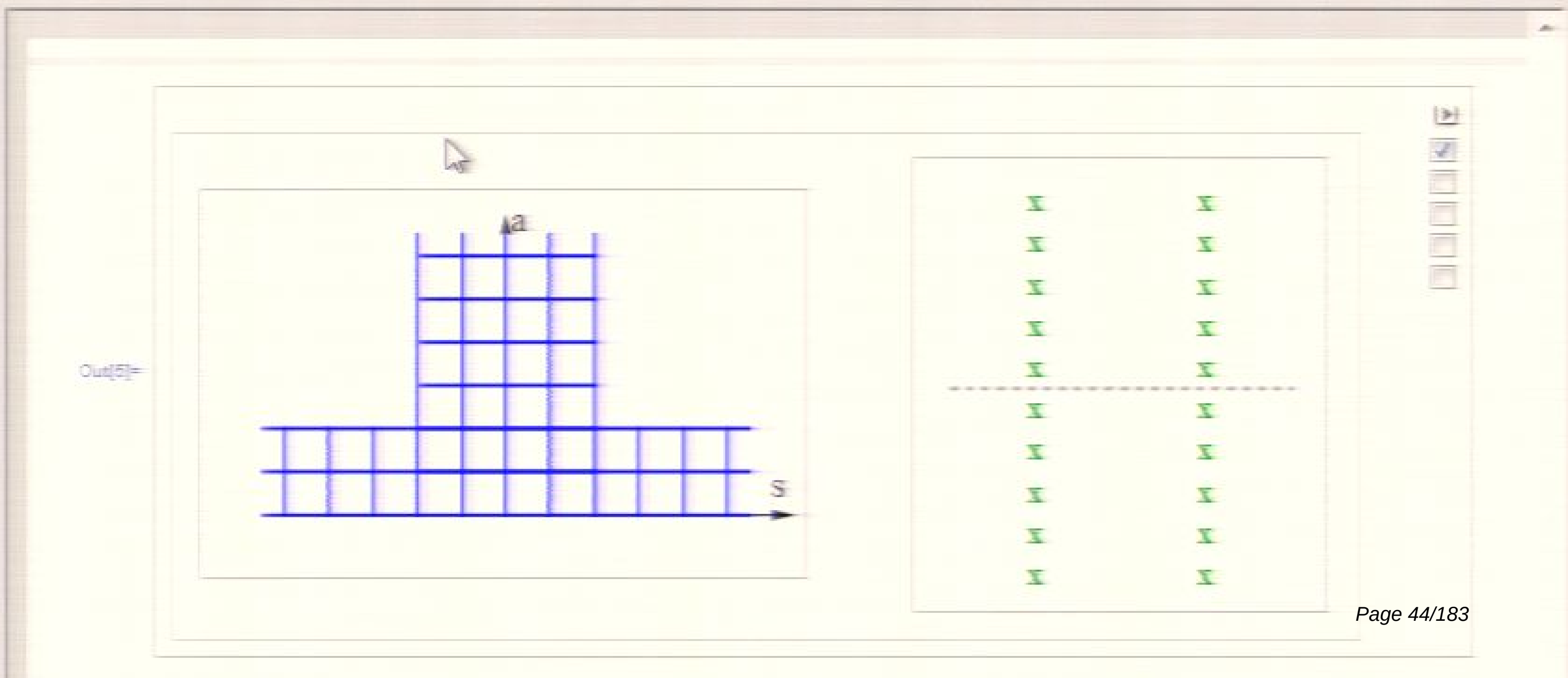

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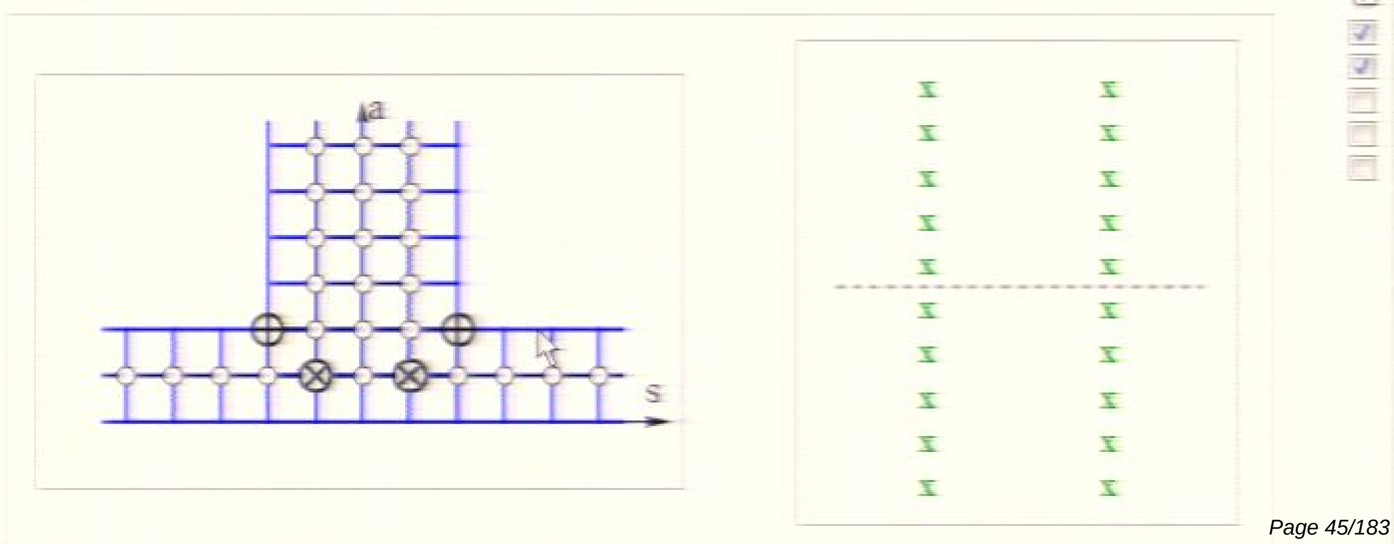

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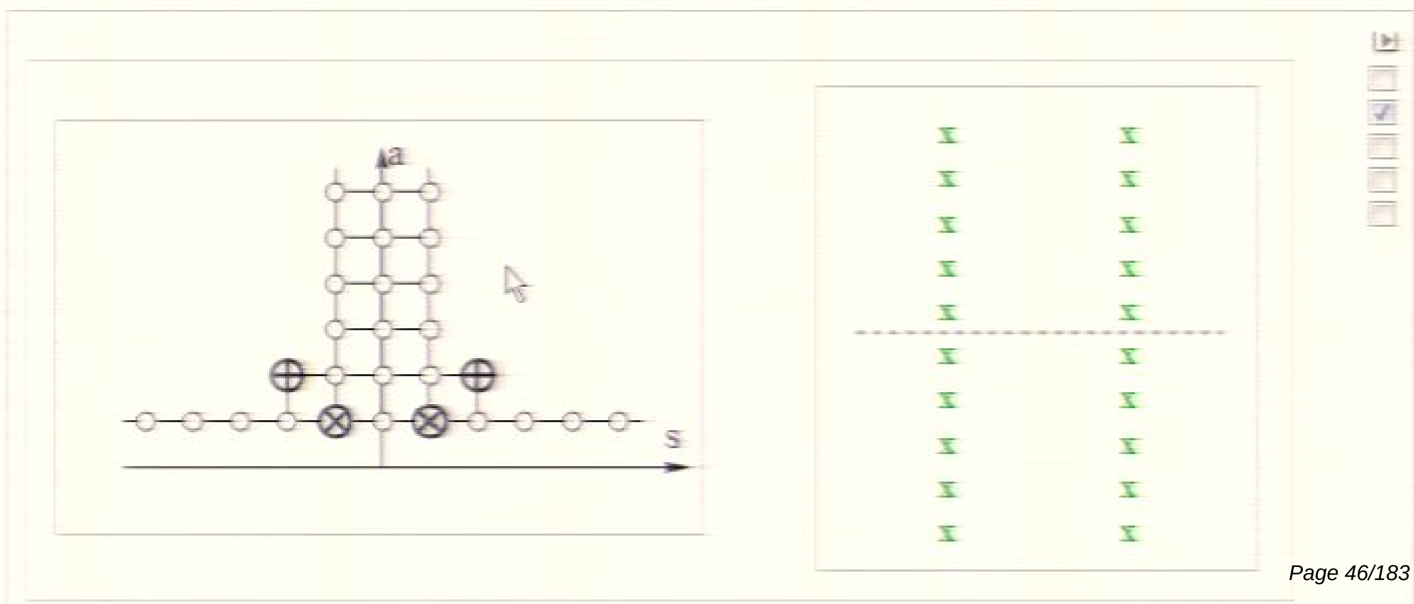

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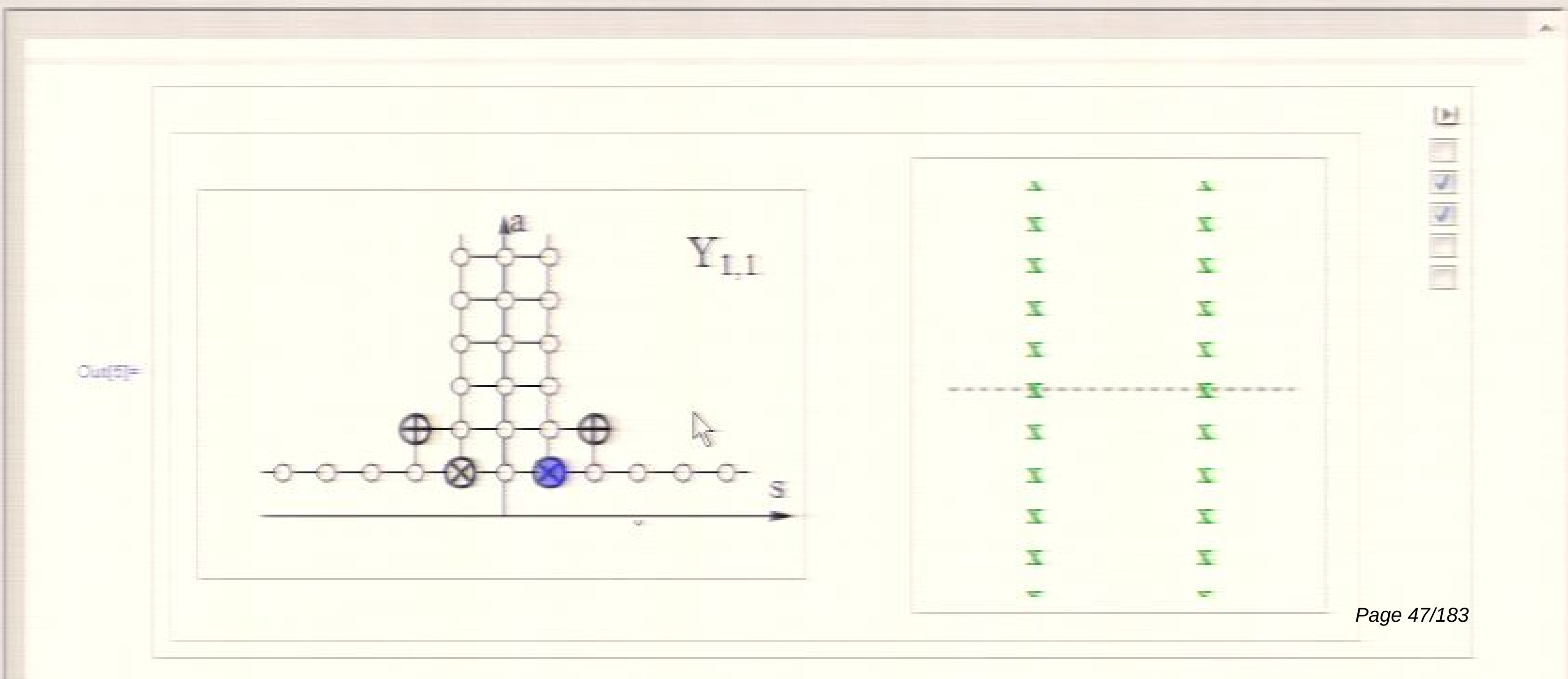

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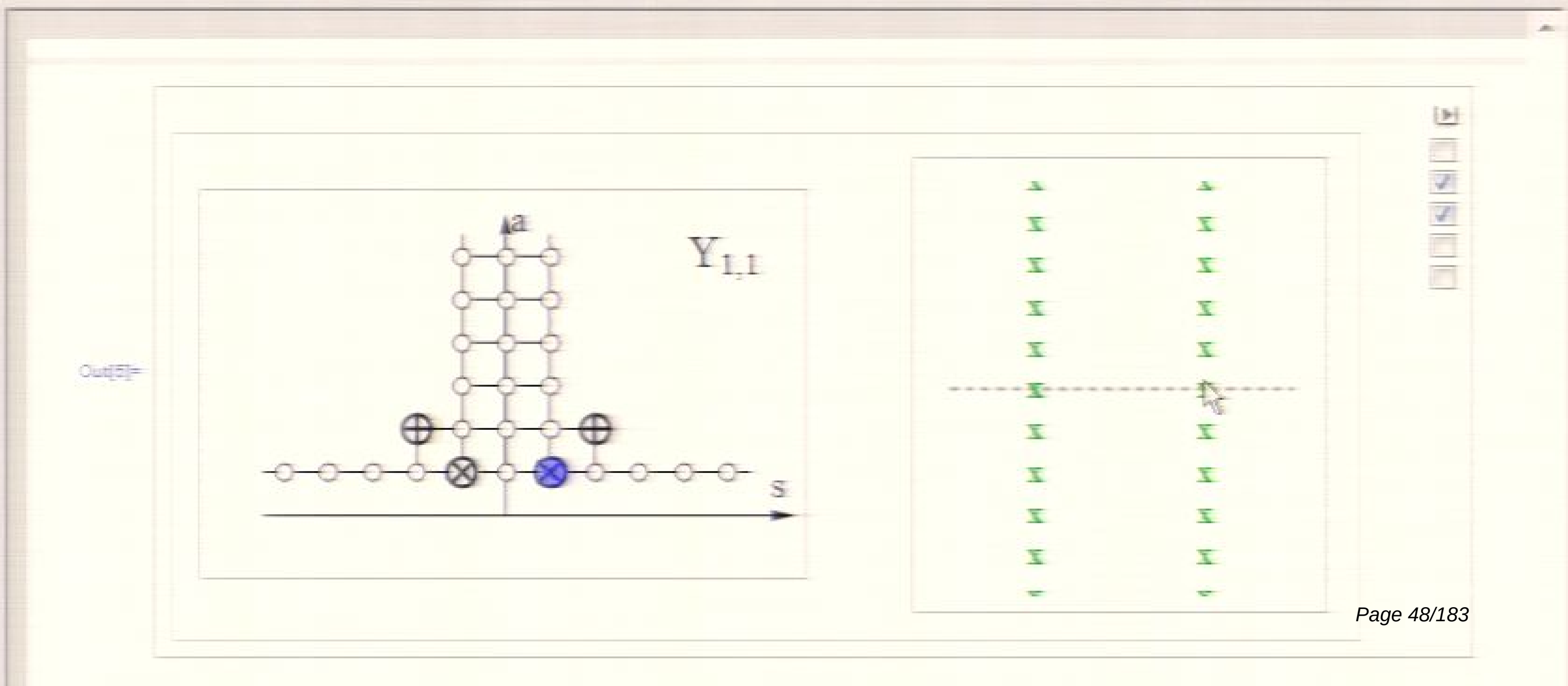

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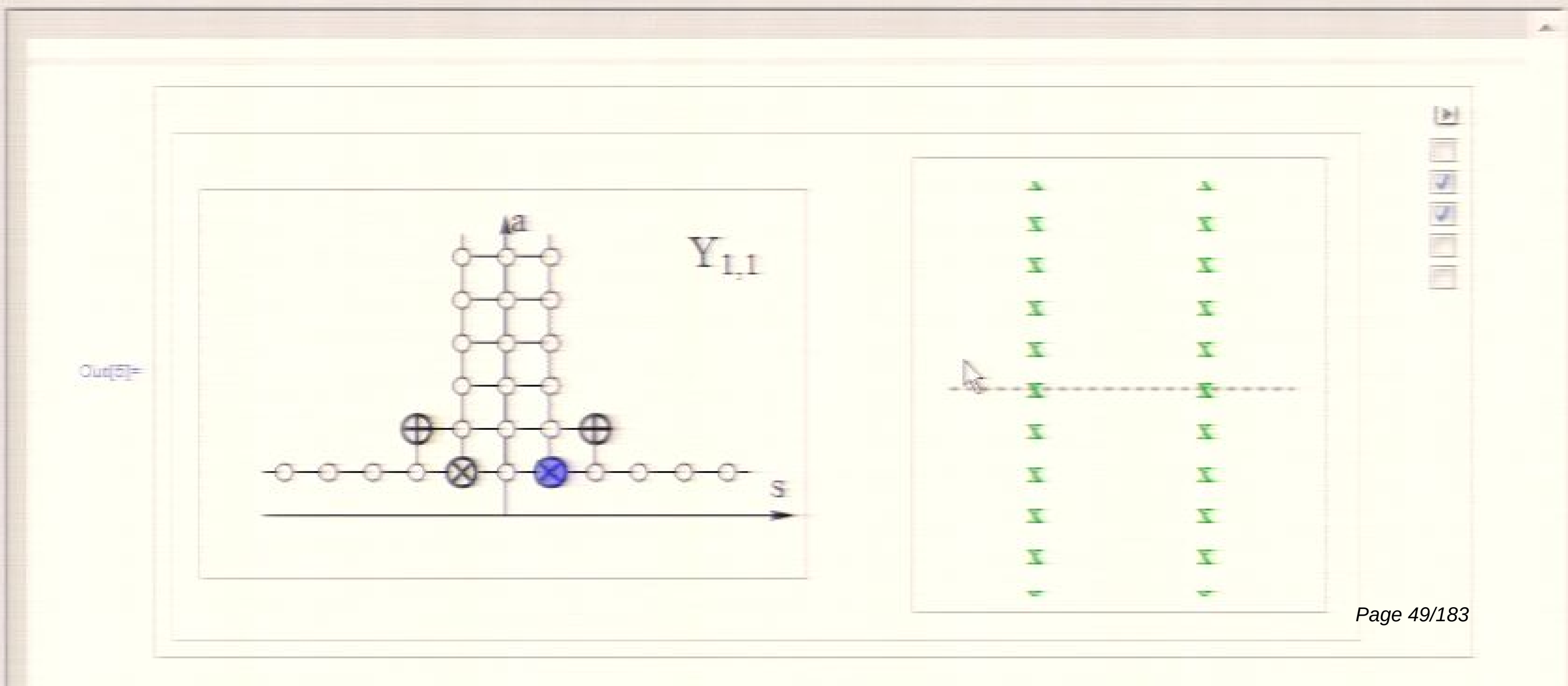

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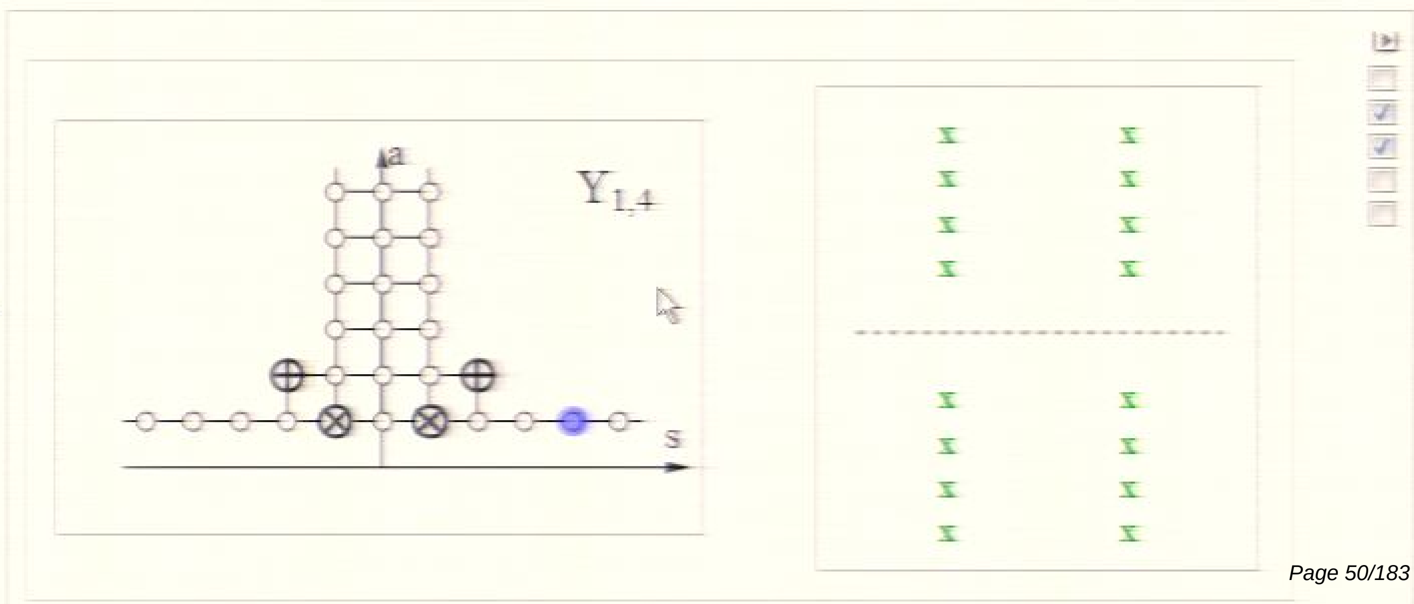

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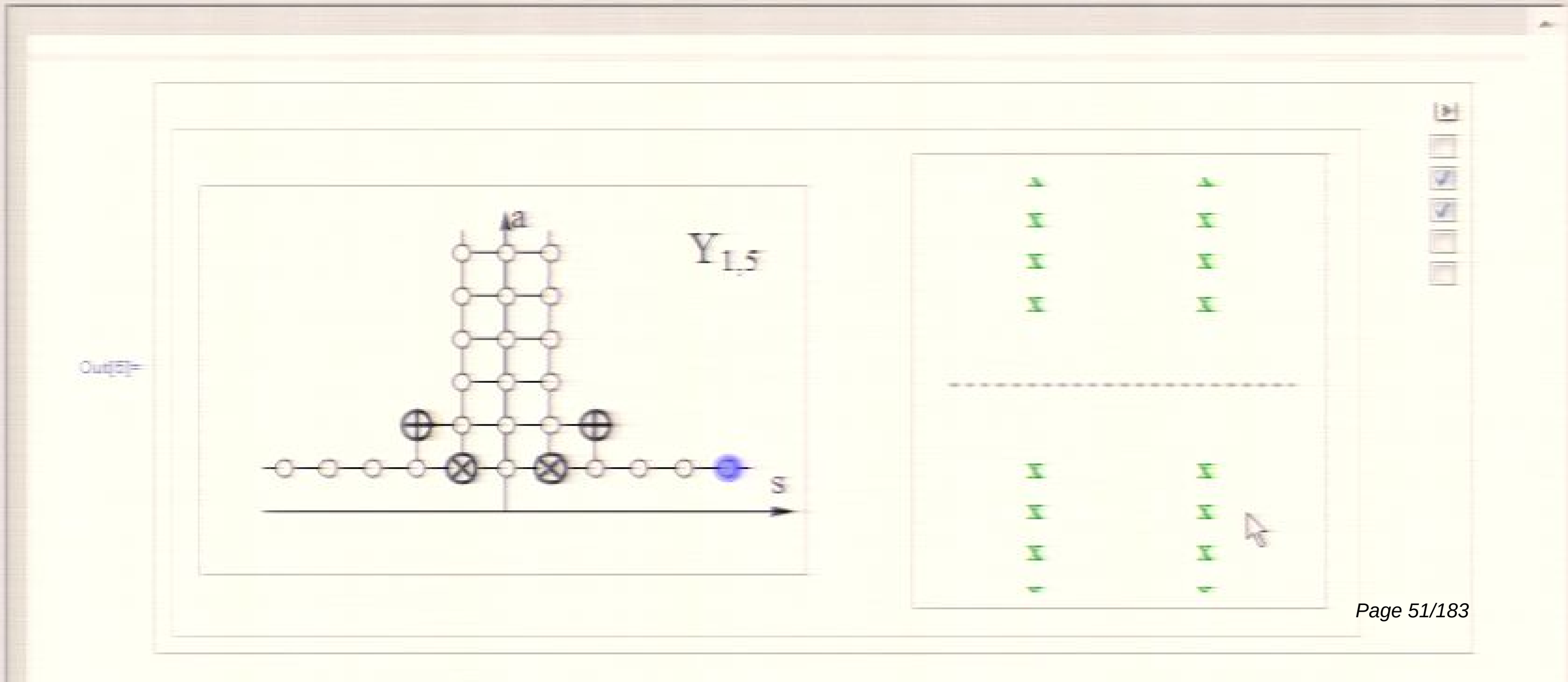

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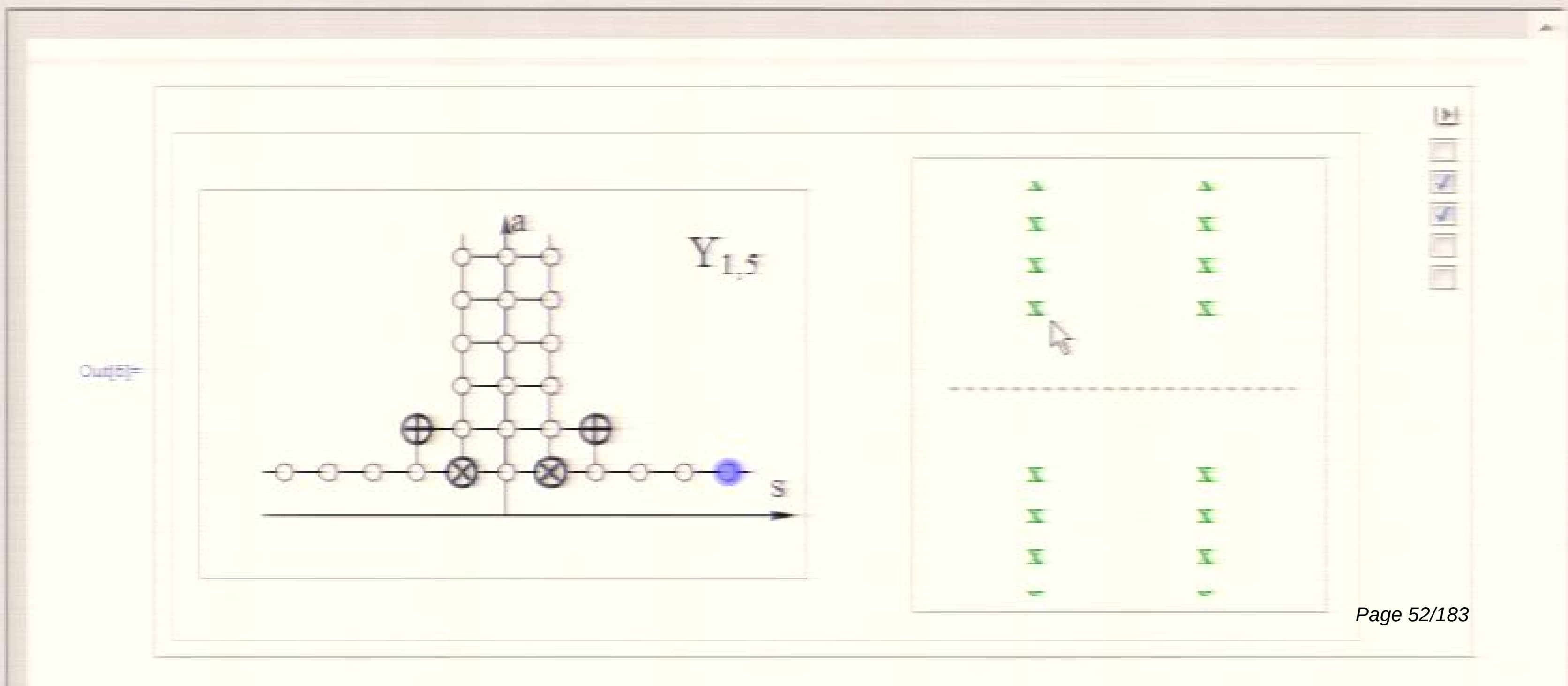

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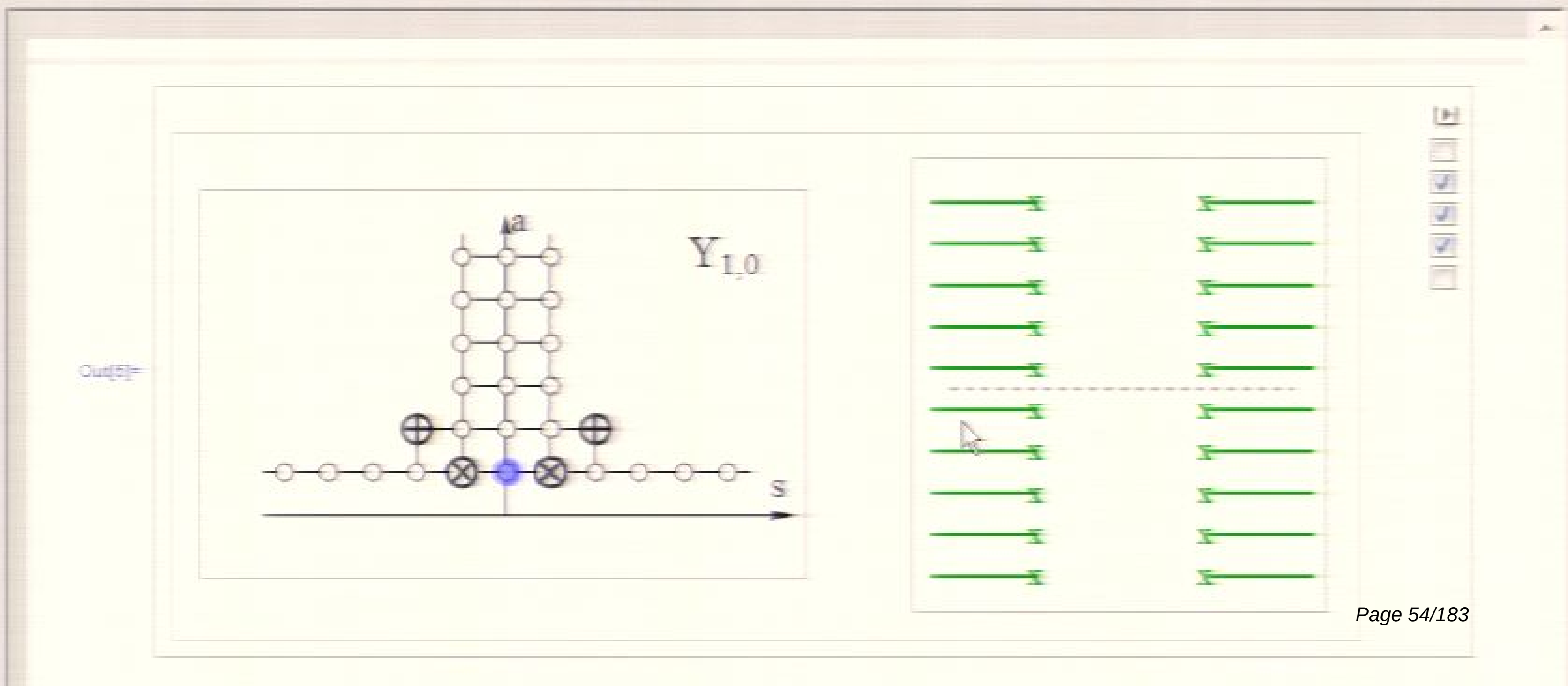
$$^{[a]} = f(u + \tfrac{ia}{2})$$

$$^{\pm} = f(u \pm \tfrac{i}{2})$$

$$_{a,s} \in \mathcal{A}_{|a-|s||}$$



Objects	Constraints	Equations for spectrum	Explicit solution
Transfer matrices	Symmetry (Hirota) + +Analyticity +Poloes/zeroes/ asymptotics		

$$Y_{a,s}=\frac{T_{a,s+1}T_{a,s-1}}{T_{a+1,s}T_{a-1,s}}$$

$$\frac{Y^{+}_{a,s}Y^{-}_{a,s}}{Y_{a+1,s}Y_{a-1,s}}=\frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}$$

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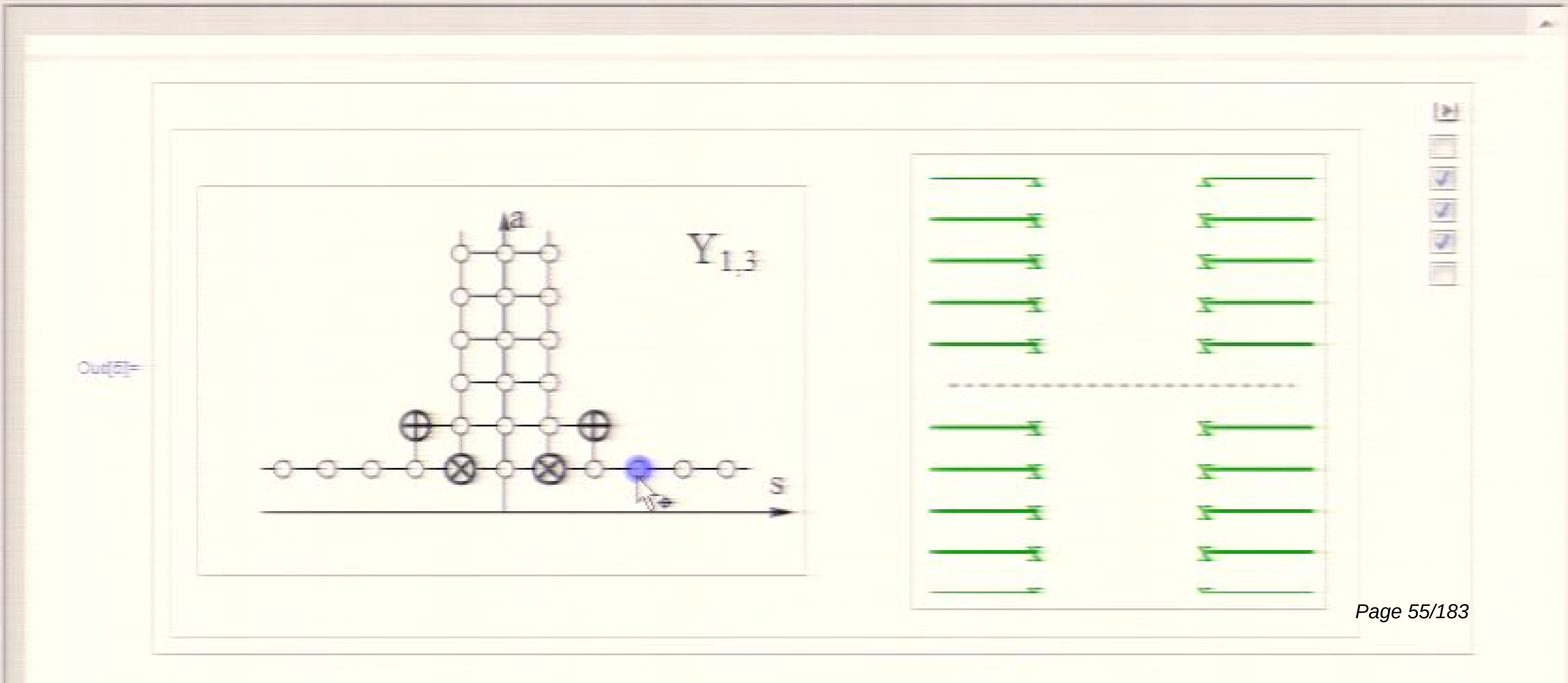

Objects | Constraints | Equations for spectrum | Explicit solution

Transfer matrices → **Symmetry (Hirota) + +Analyticity +Poloes/zeroes/ asymptotics** → →

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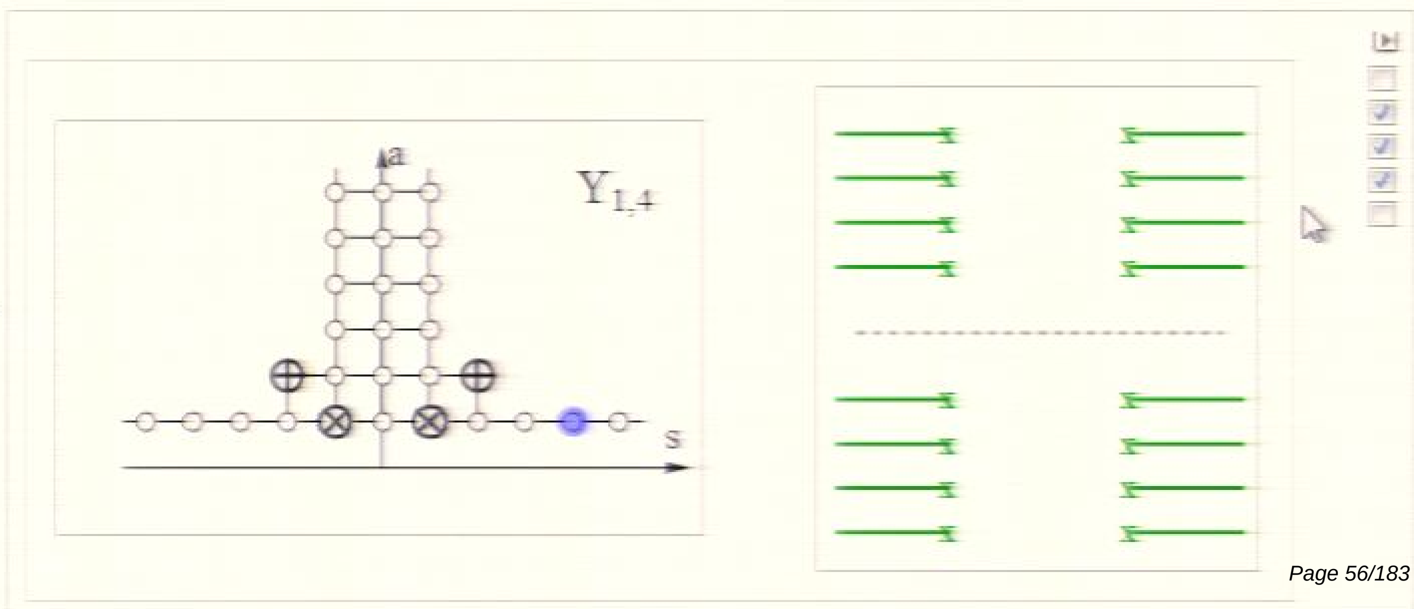
$^{\pm} = f(u \pm \frac{i}{2})$

$a,s \in \mathcal{A}_{|a-|s||}$

Out[5]=



Objects | Constraints | Equations for spectrum | Explicit solution

Transfer matrices → **Symmetry (Hirota) + +Analyticity +Poloes/zeroes/ asymptotics** → →

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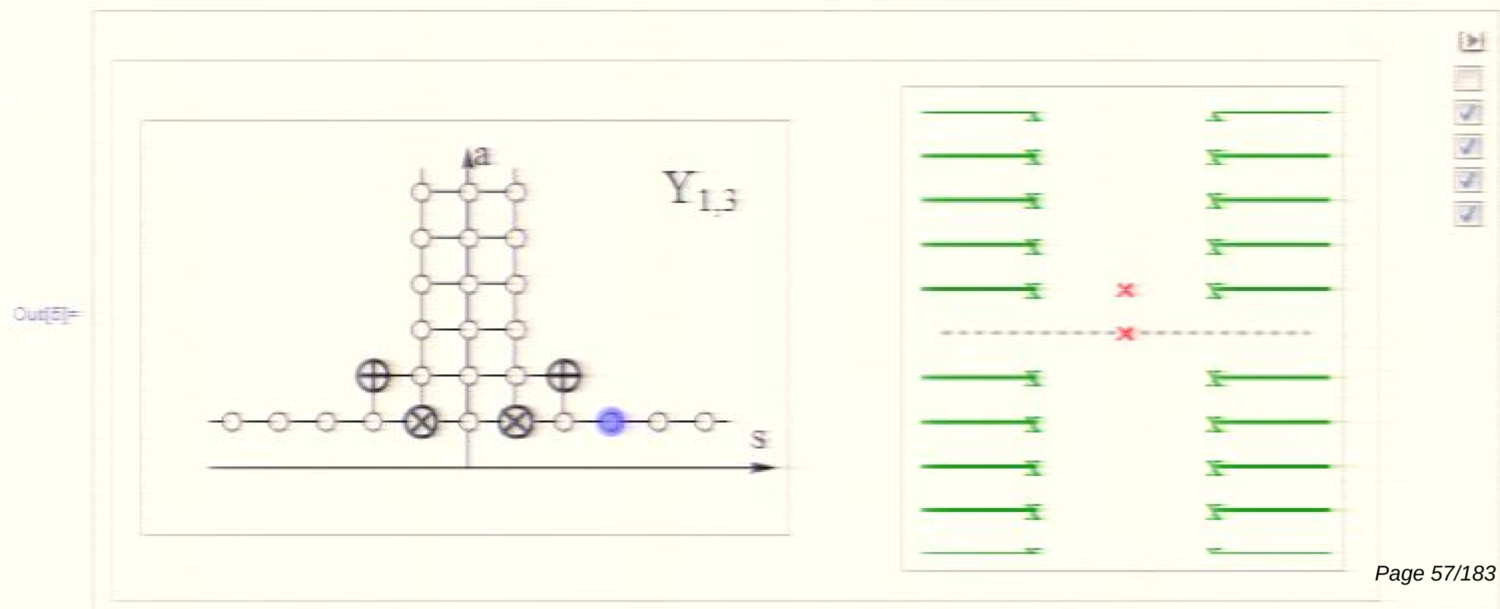

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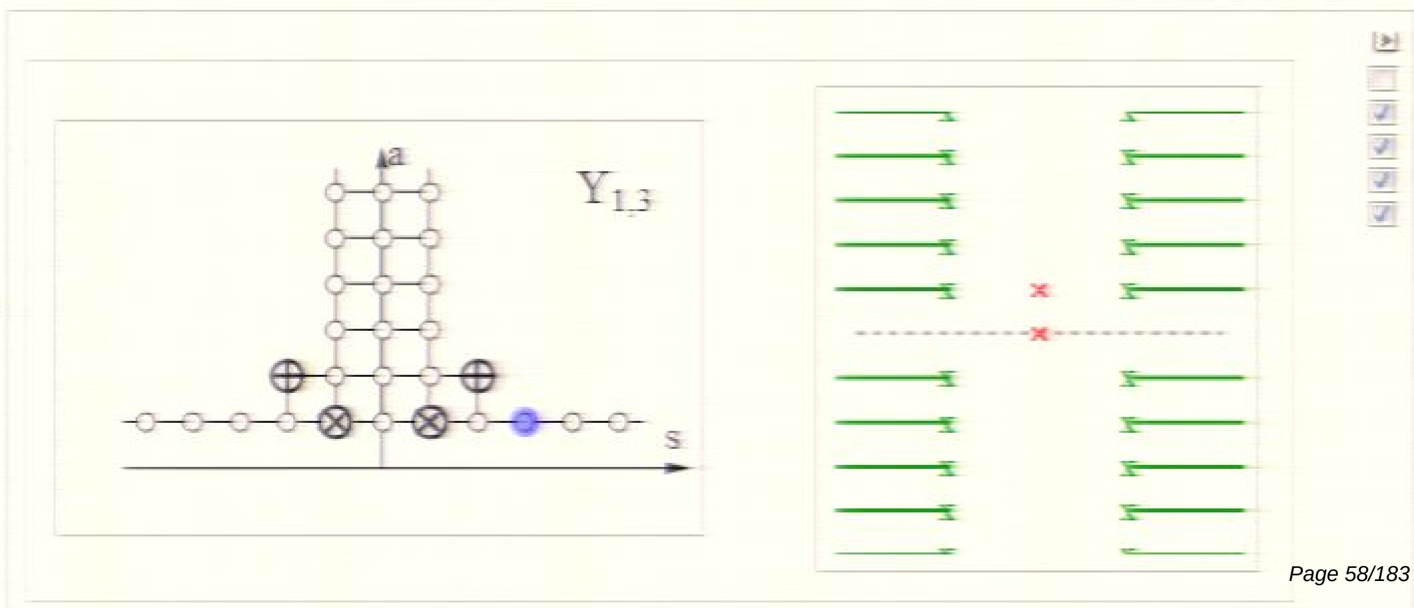

Objects | Constraints | Equations for spectrum | Explicit solution

Transfer matrices

→

Symmetry (Hirota)
+ Discontinuity conditions
+Analyticity
+Poloes/zeroes/ asymptotic

→

→

[Cavaglia,Fioravanti,Tateo, 10]
[Balog,Hegedus 11]

From mirror TBA:
[Bombardelli,Fioravanti,Tateo, 09]
[Gromov, Kazakov, Kozak, Vieira, 09]
[Arutyunov, Frolov, 09]

Objects

Transfer matrices

Constraints

Symmetry
(Hirota)
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Equations for spectrum

TBA
(infinite set)

Explicit solution

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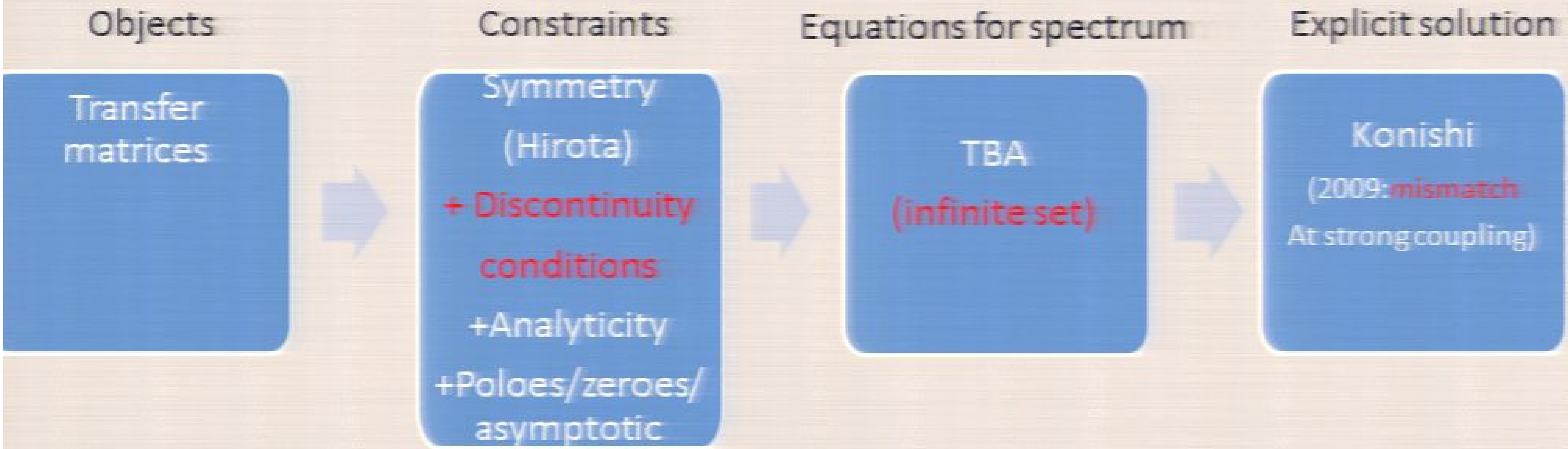

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Objects

Transfer matrices
(classical level only)

Constraints

Symmetry
(Hirota)
+ Discontinuity conditions
+Analyticity
+Poloes/zeroes/asymptotic

[Cavaglia,Fioravanti,Tateo, 10]
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Equations for spectrum

TBA
(infinite set)

Explicit solution

Konishi
(2009:mismatch
At strong coupling)

From mirror TBA:
[Bombardelli,Fioravanti,Tateo, 09]
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Objects	Constraints	Equations for spectrum	Explicit solution
Transfer matrices (classical level only)	Symmetry (Hirota) + Discontinuity conditions +Analyticity +Poloes/zeroes/ asymptotic	TBA (infinite set)	Konishi (2009: mismatch At strong coupling)

[Cavaglia,Fioravanti,Tateo, 10]
[Balog,Hegedus 11]

This was the situation one year ago…

trong coupling of the sl(2) sector (Konishi et al):

Konishi

(2009: mismatch

At strong coupling)

:009: $\Delta[g] = 2\sqrt{4\pi g} + \frac{2 \text{ or } 1}{\sqrt{4\pi g}}$ [Gromov, Kazakov, Vieira, 09] [Roiban, Tseytlin, 09]

!2/2011: Analytical derivations (using yet to be proved ıssumptions):

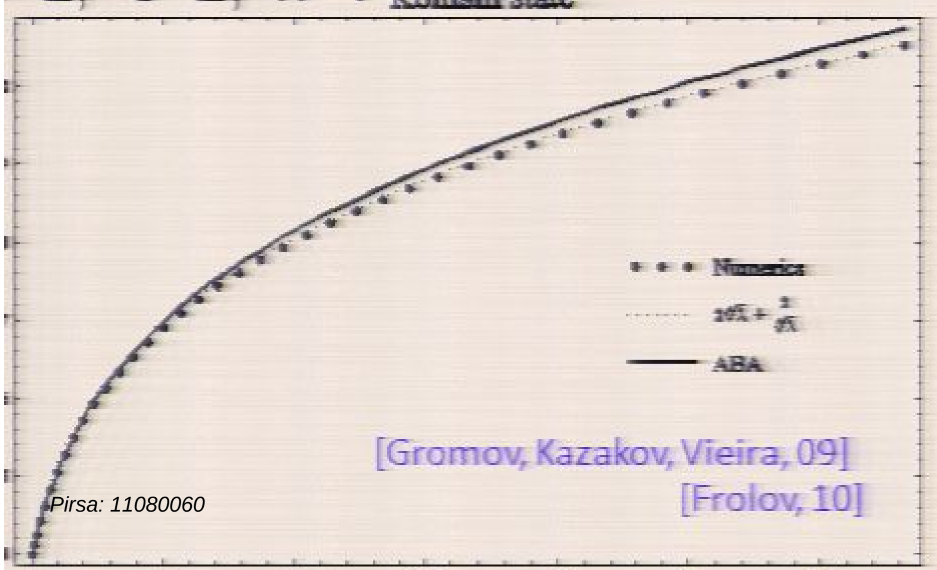
Gromov,Shenderovich,Serban, D.V.]

Roiban,Tseytlin]

Masuccato,Valilio]

$$\Delta - J - S = \lambda^{1/4}\sqrt{2nS} + \frac{1}{\lambda^{1/2}}\frac{2J^2 + S(3S-2)}{4\sqrt{2nS}} + \mathcal{O}(\lambda^{-3/4})$$

J=2, S=2, n=1 Konishi state



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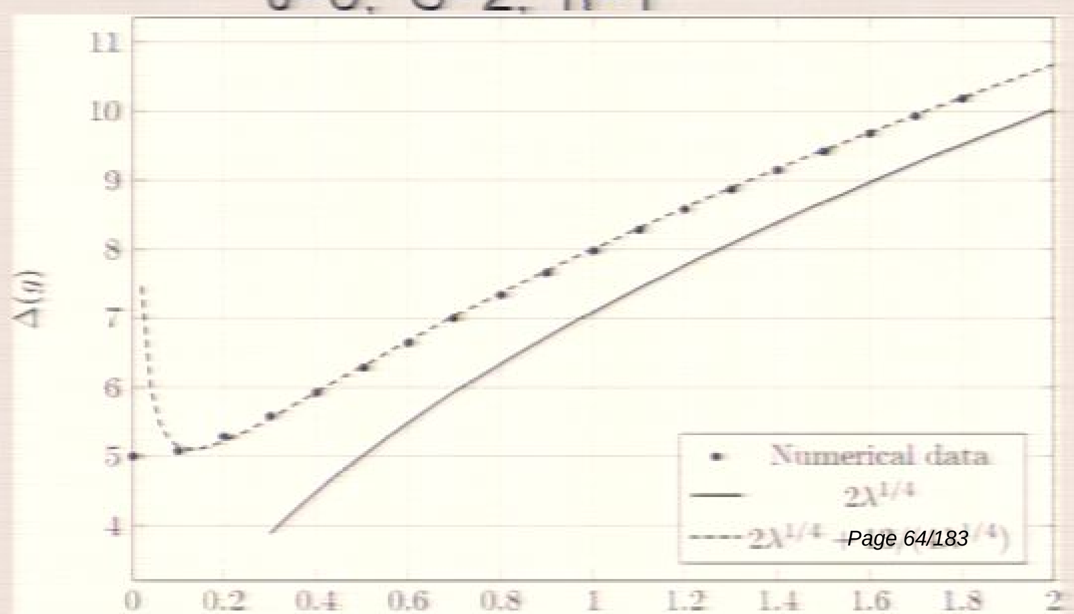

trong coupling of the sl(2) sector (Konishi et al):

Konishi
(2011:agreement)

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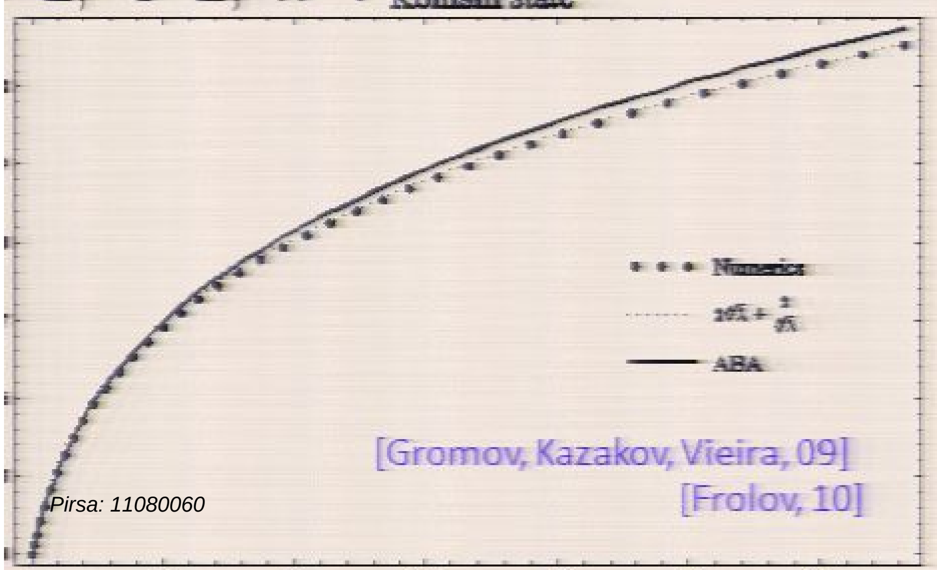
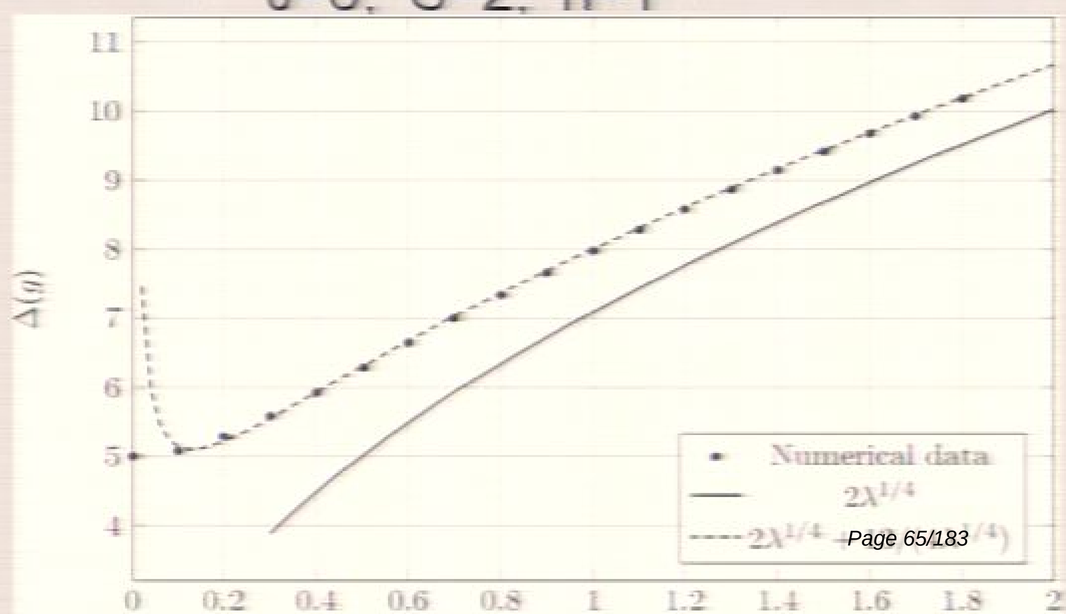


[Gromov, Kazakov, Vieira, 09]
[Frolov, 10]

J=3, S=2, n=1



N.Gromov,V.Kazakov, S.Leurent,D.V. 110?.????]

Discontinuity conditions

$Y_{1,s} \in \mathcal{A}_{s-1} \longrightarrow T_{1,s} \in \mathcal{A}_s$ in some gauge

Mirror:

N.Gromov,V.Kazakov, S.Leurent,D.V. 110?.????]

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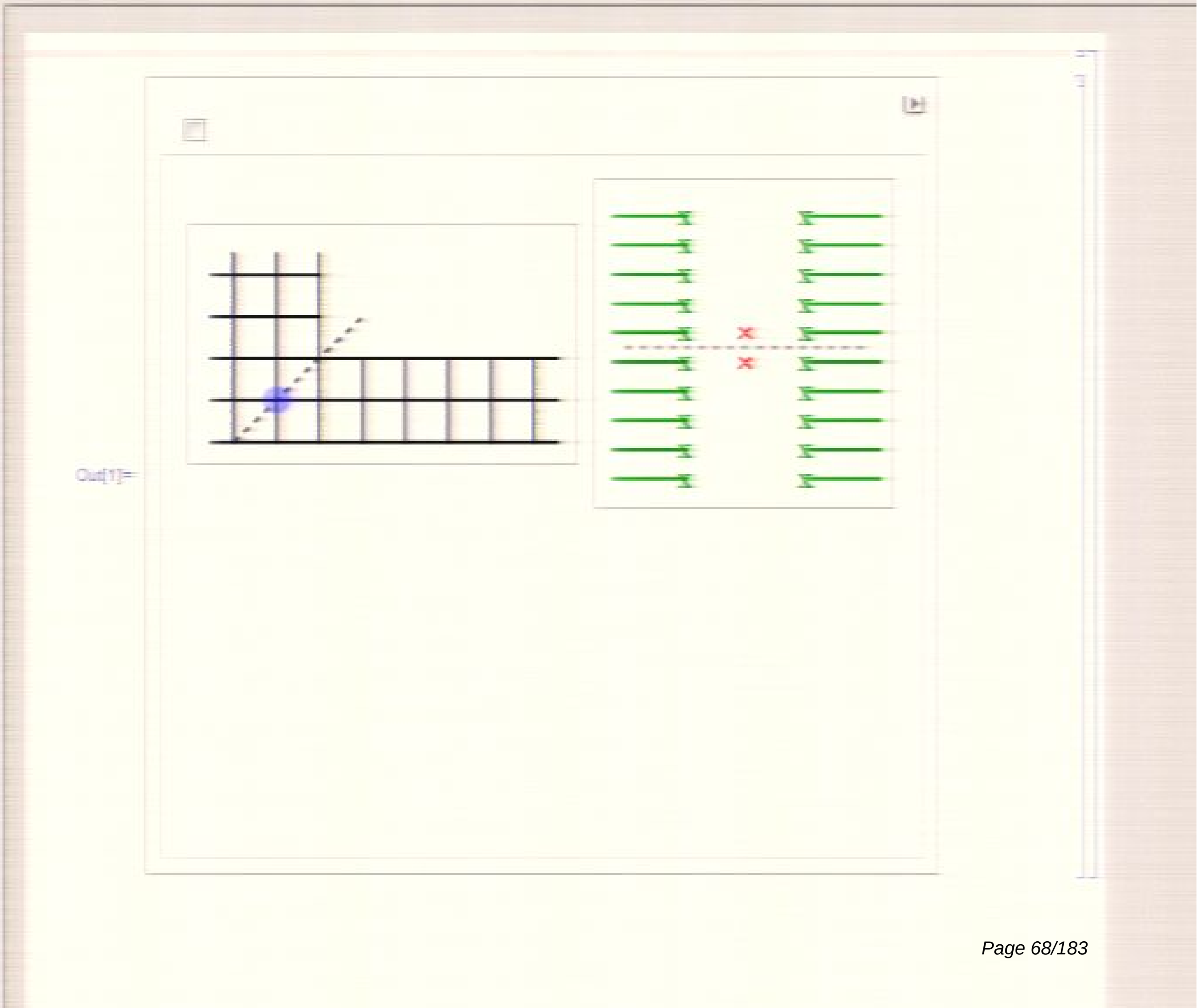

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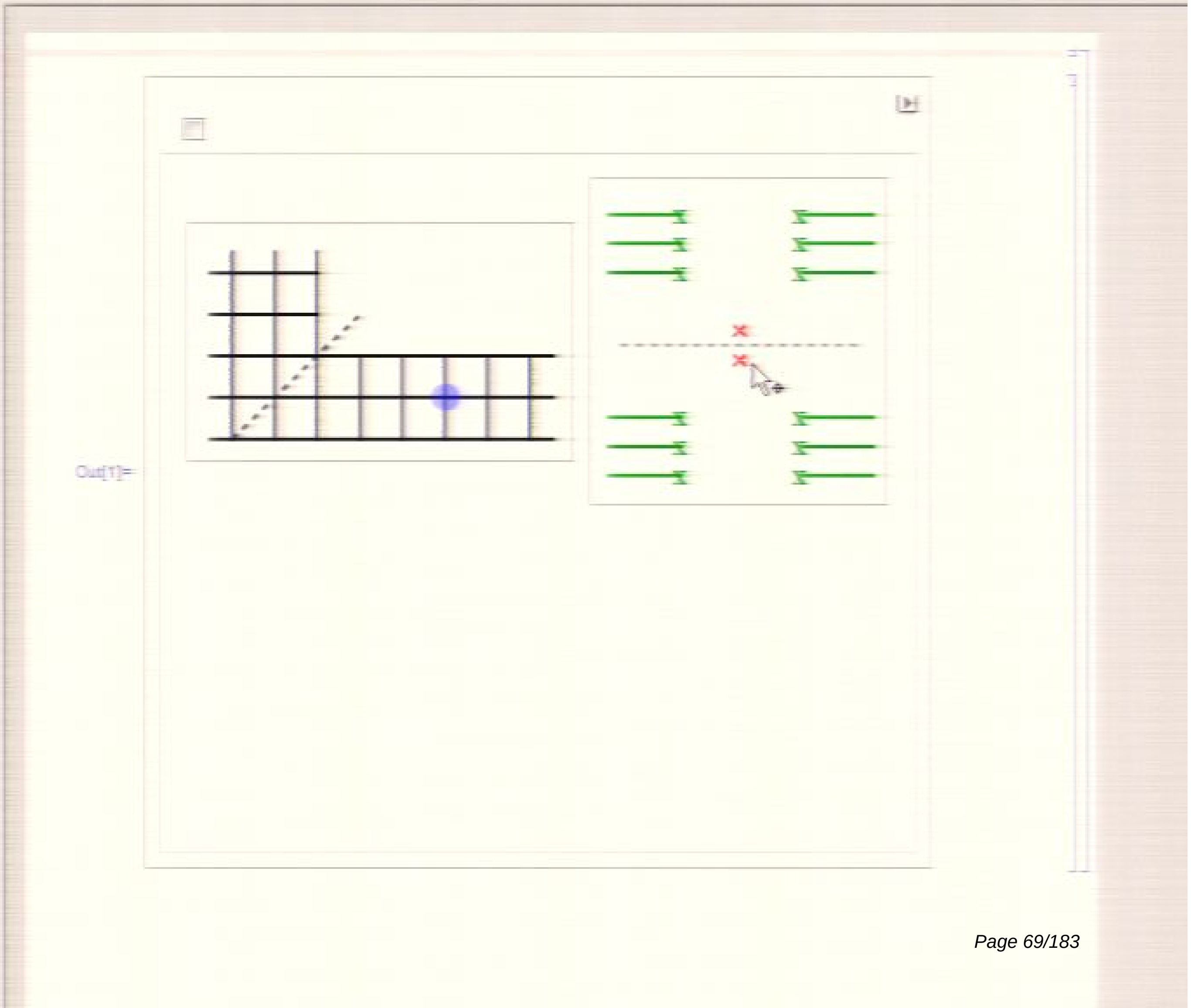

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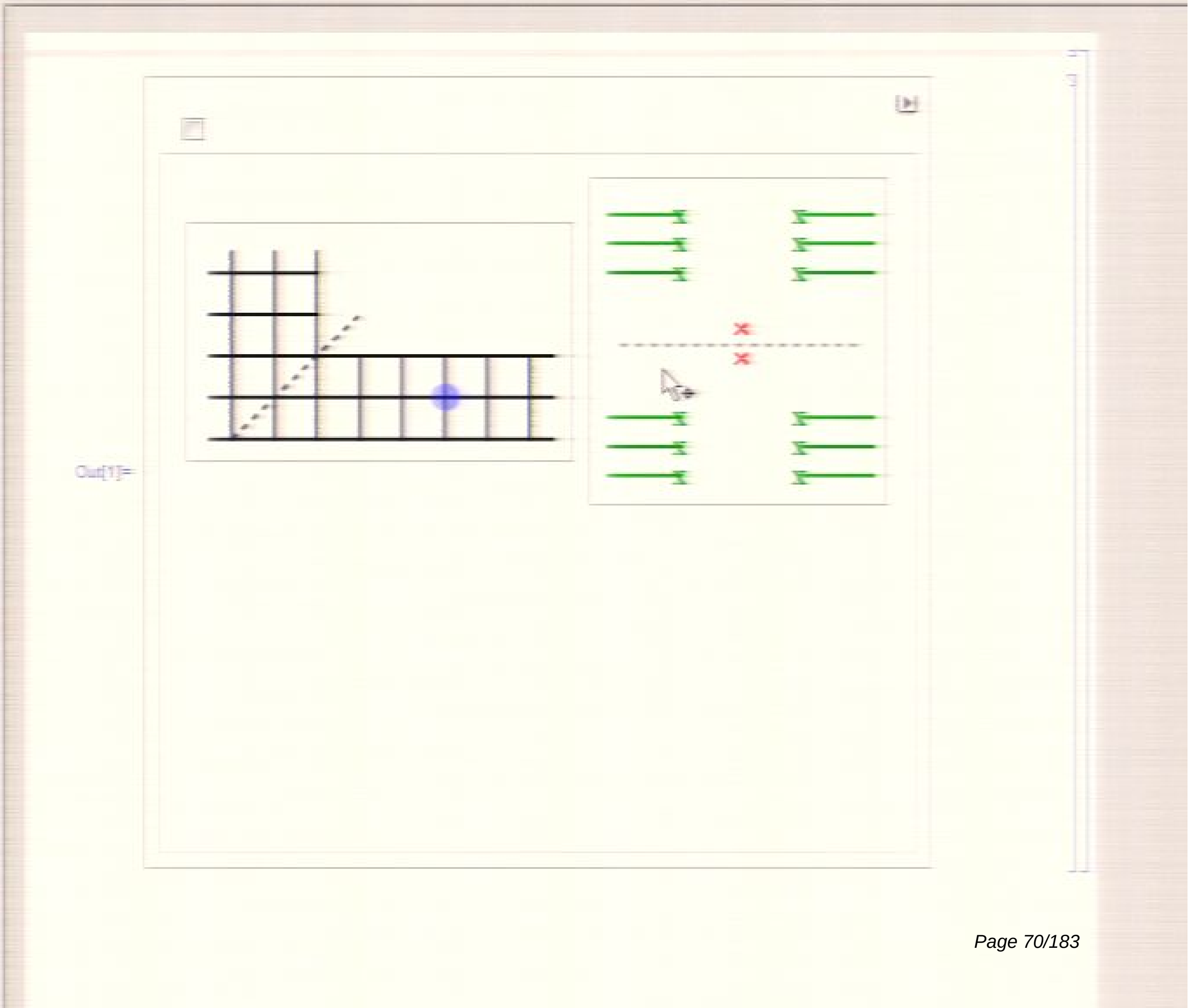

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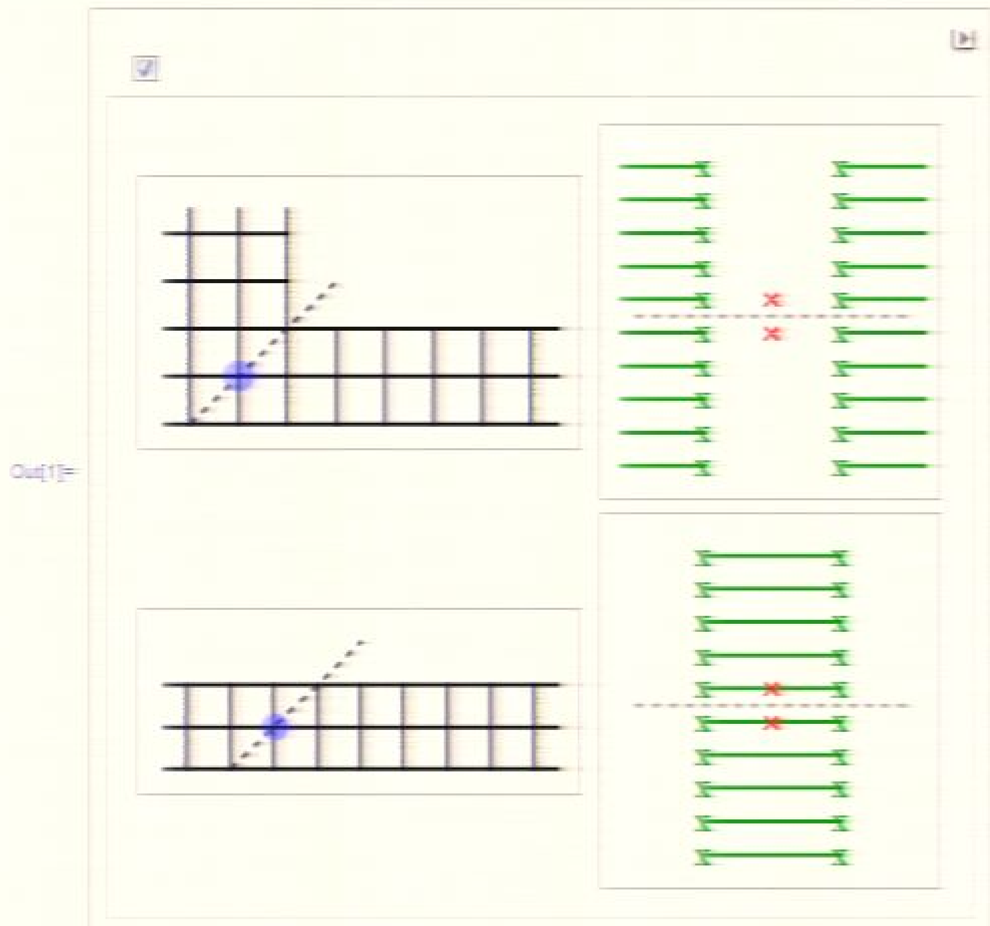

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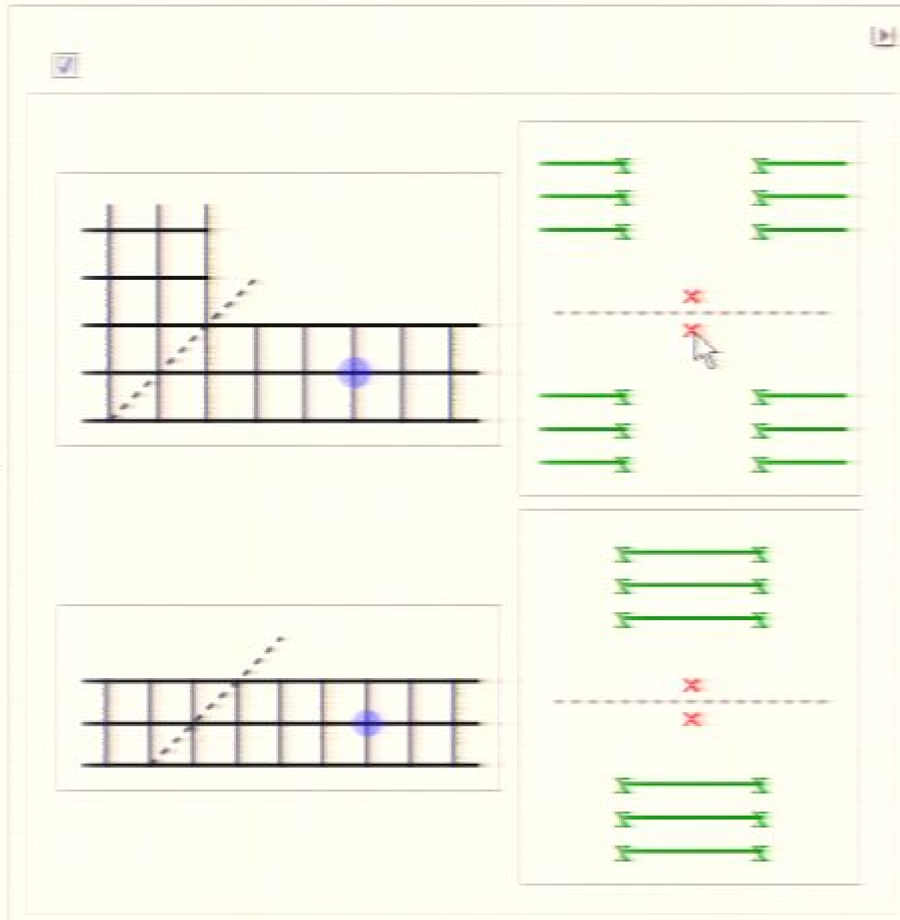

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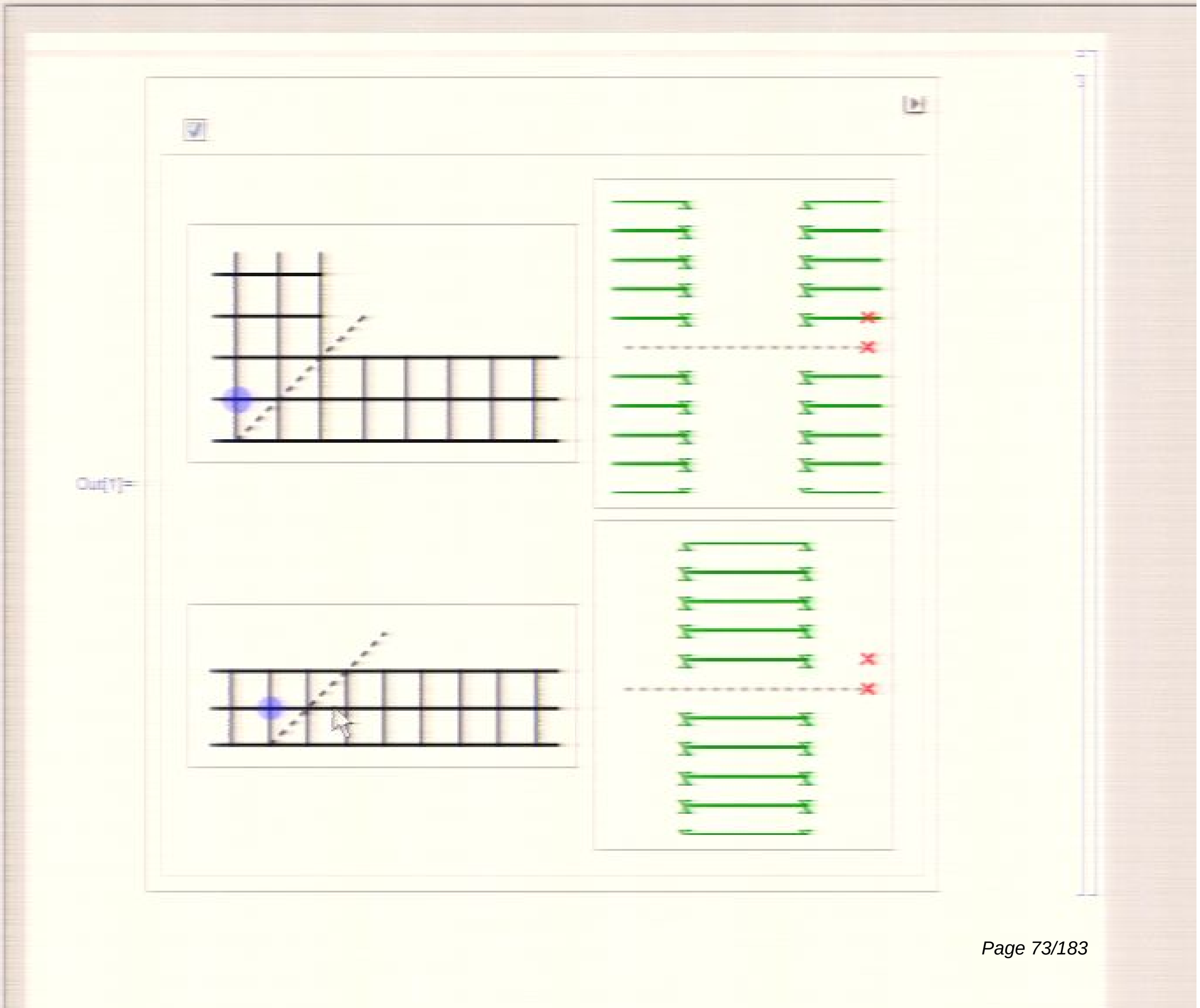

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N.Gromov,V.Kazakov, S.Leurent,D.V. 110?.????]

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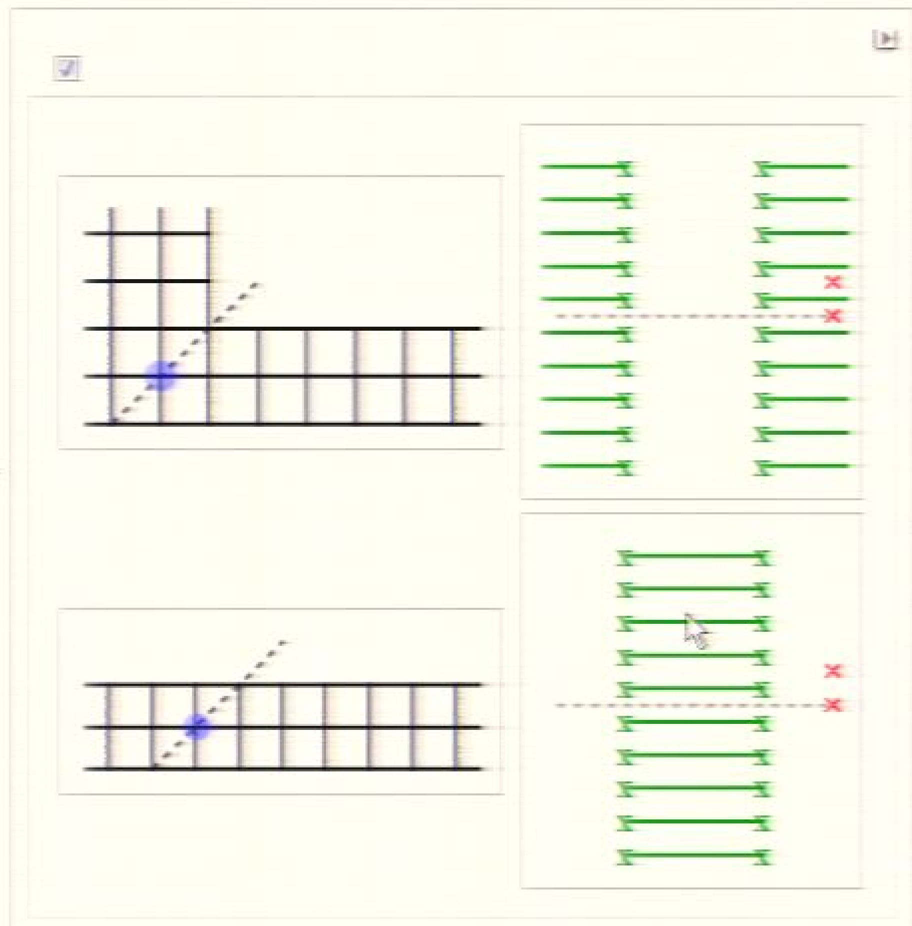

N.Gromov,V.Kazakov, S.Leurent,D.V. 110?.????]

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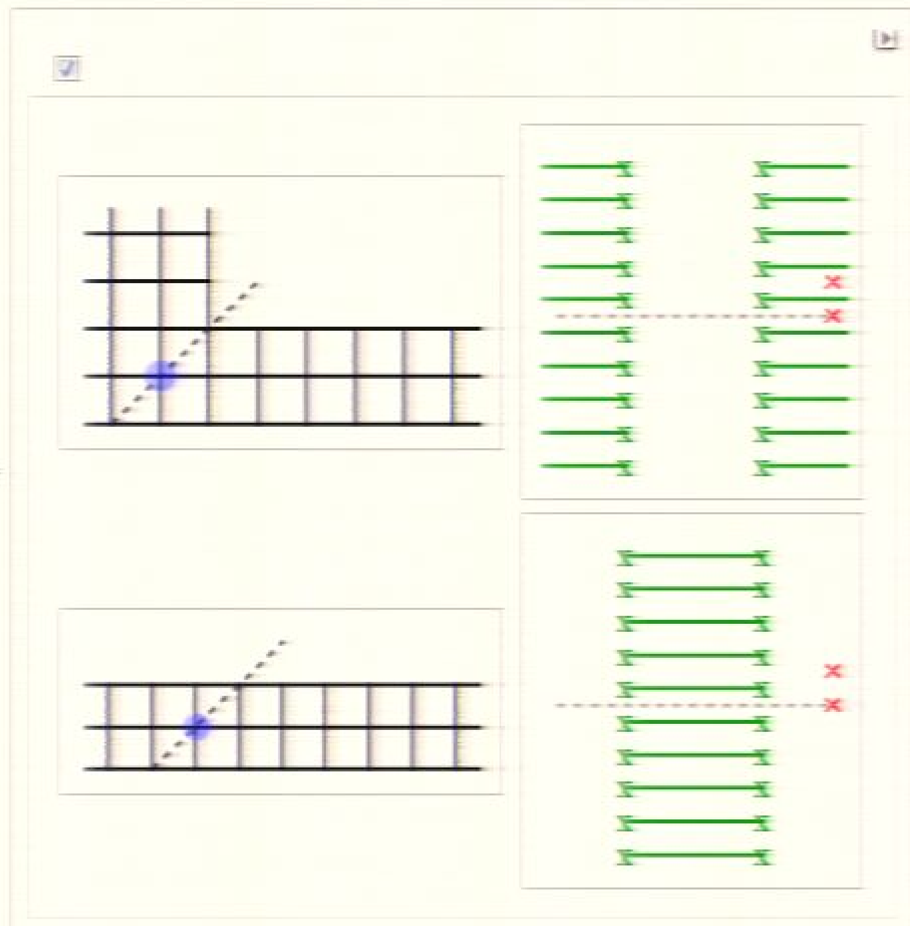
Mirror:

Out[1]=



Magic:

$\hat{T}_{a,s},\ \hat{Y}_{a,s}$

[N.Gromov,V.Kazakov, S.Leurent,D.V. 110?.????]

Discontinuity conditions

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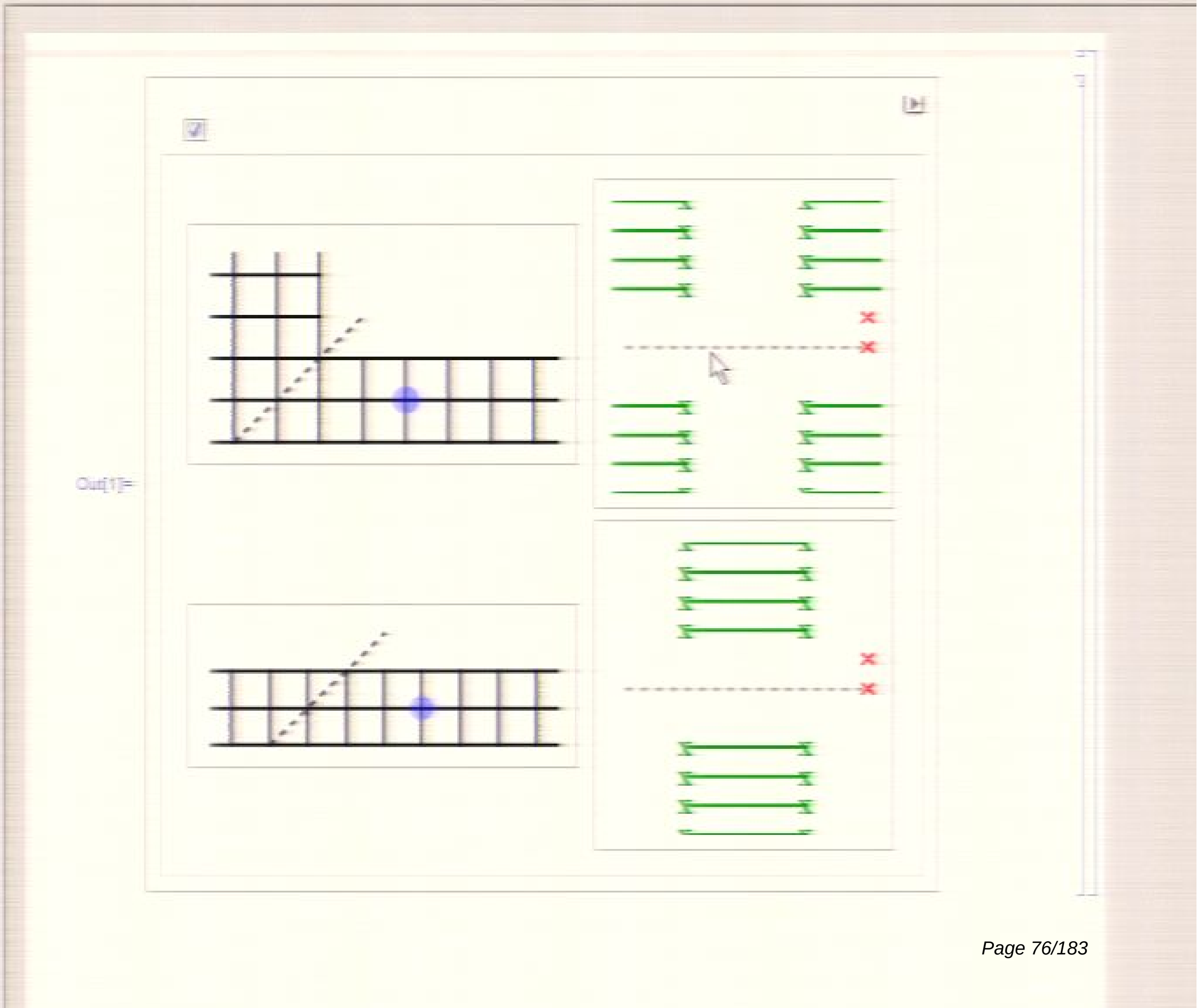
Mirror:



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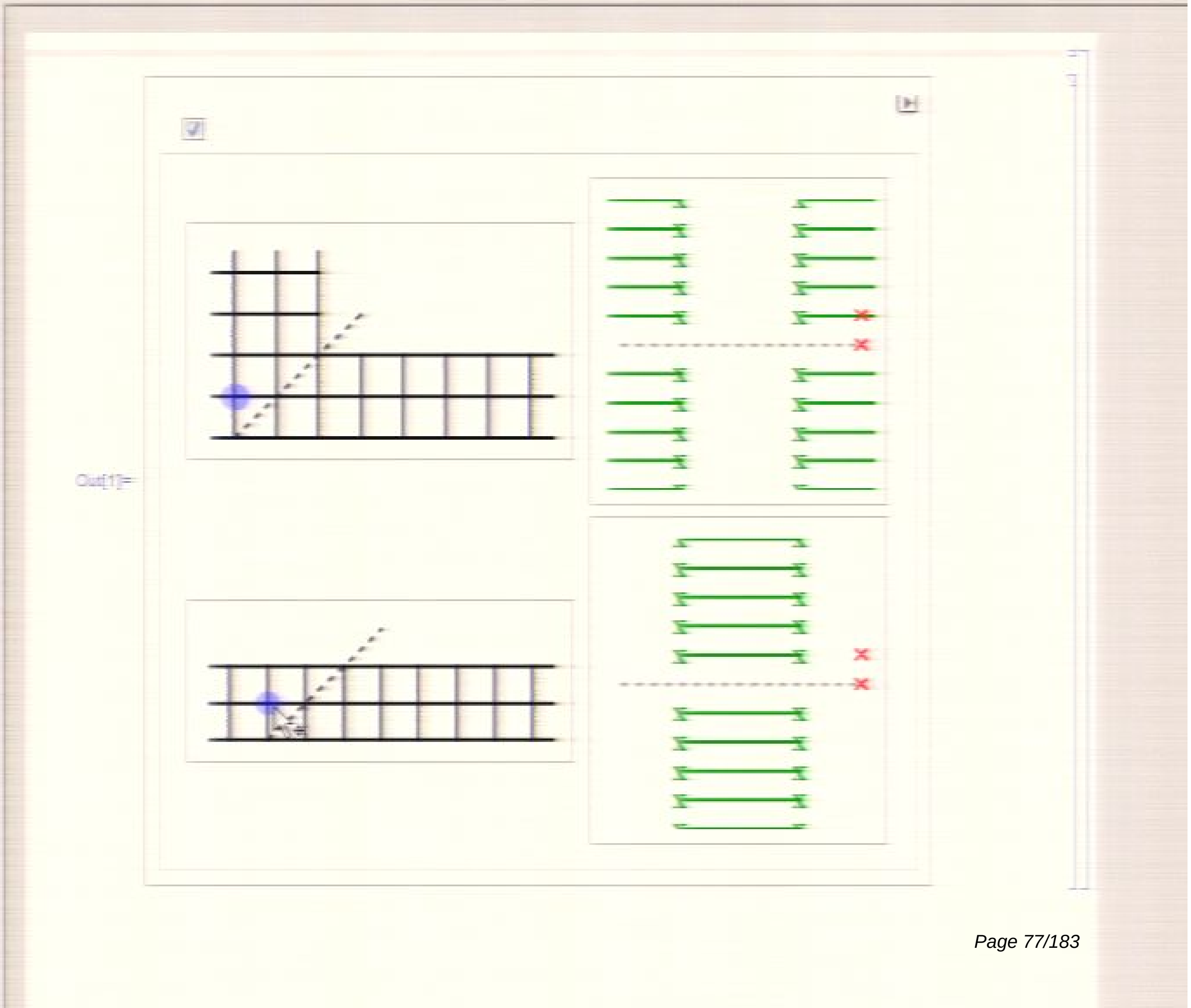


$\hat{T}_{a,s},\ \hat{Y}_{a,s}$

$\hat{T}_{1,0} = 0$

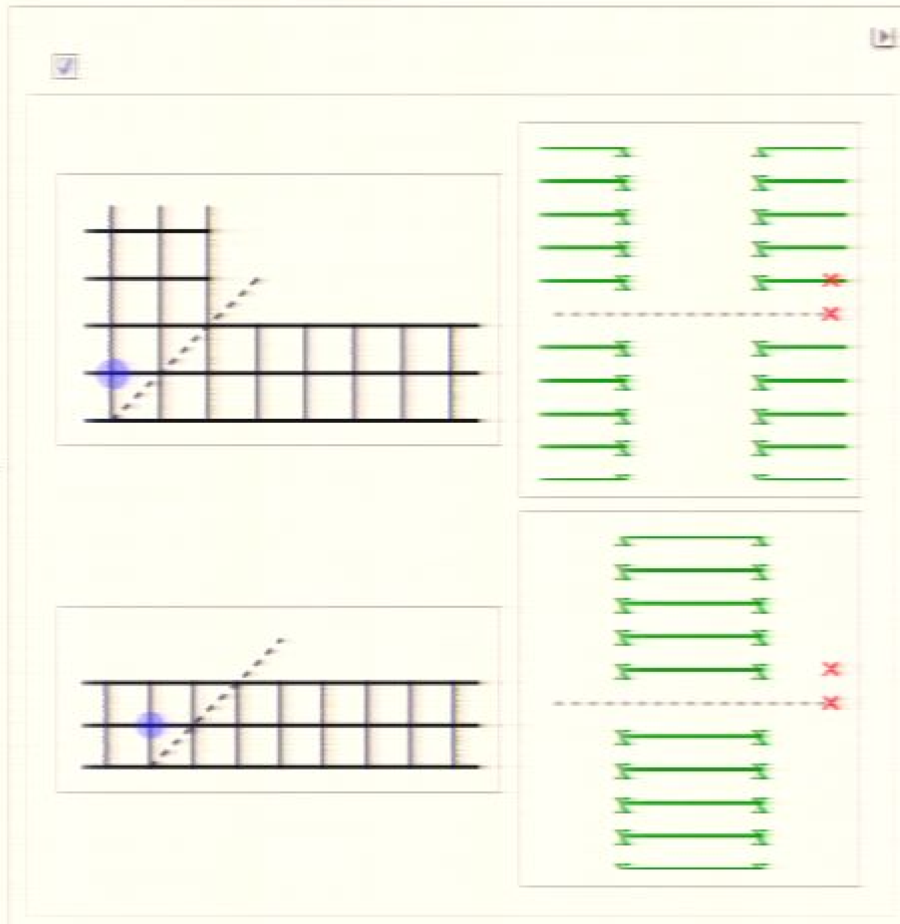
N.Gromov,V.Kazakov, S.Leurent,D.V. 110?.????]

Discontinuity conditions

$Y_{1,s} \in \mathcal{A}_{s-1} \longrightarrow T_{1,s} \in \mathcal{A}_s$ in some gauge

Mirror:

$$Y_{1,2}^{+}Y_{1,2}^{-} = (1+Y_{1,3})\frac{1+Y_{1,1}}{1+1/Y_{2,2}}$$



Magic:

$\hat{T}_{a,s},\ \hat{Y}_{a,s}$

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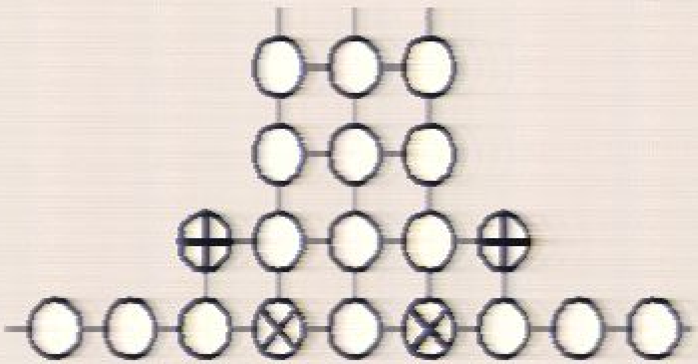
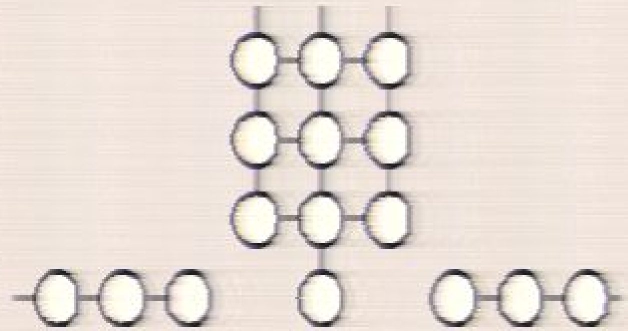




Mirror

Magic

$$Y^+_{1,2}Y^-_{1,2} = (1+Y_{1,3})\frac{1+Y_{1,1}}{1+1/Y_{2,2}}$$

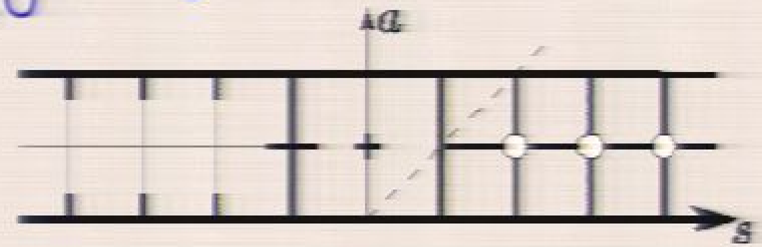
$$\hat{Y}^+_{1,2}\hat{Y}^-_{1,2} = 1+Y_{1,3} \qquad \hat{\mathbb{T}}_{1,0} = 0$$

$$Y^+_{2,\pm1}Y^-_{2,\pm1} = \frac{1+Y_{2,0}}{1+1/Y_{3,1}}\frac{1+Y_{2,2}}{1+1/Y_{1,1}}$$

$$\hat{Y}^+_{2,\pm1}\hat{Y}^-_{2,\pm1} = \frac{1+Y_{2,0}}{1+1/Y_{3,1}} \qquad \hat{\mathbb{T}}_{0,1} = 0$$

The magic of the Magic sheet

$$\hat{\mathbb{T}}_{1,0} = 0$$



$$\hat{\mathbb{T}}_{0,s} = 1$$

$$\hat{\mathbb{T}}_{1,s} = Q_1^{[+s]}\bar{Q}_2^{[-s]} - Q_2^{[+s]}\bar{Q}_1^{[-s]}$$

$$\mathbb{T}_{1,s} = \frac{\hat{\mathbb{T}}_{1,s}}{Q_1^{[+s]}\bar{Q}_1^{[-s]}} = \bar{Q}_2^{[-s]} - Q_2^{[+s]}$$

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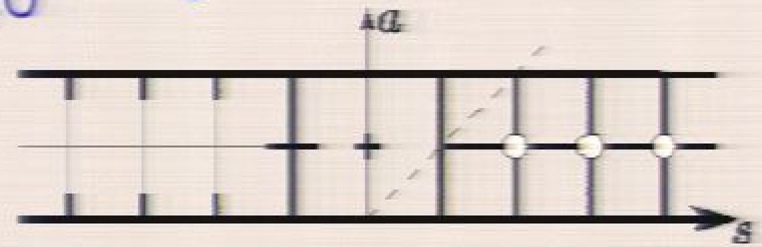

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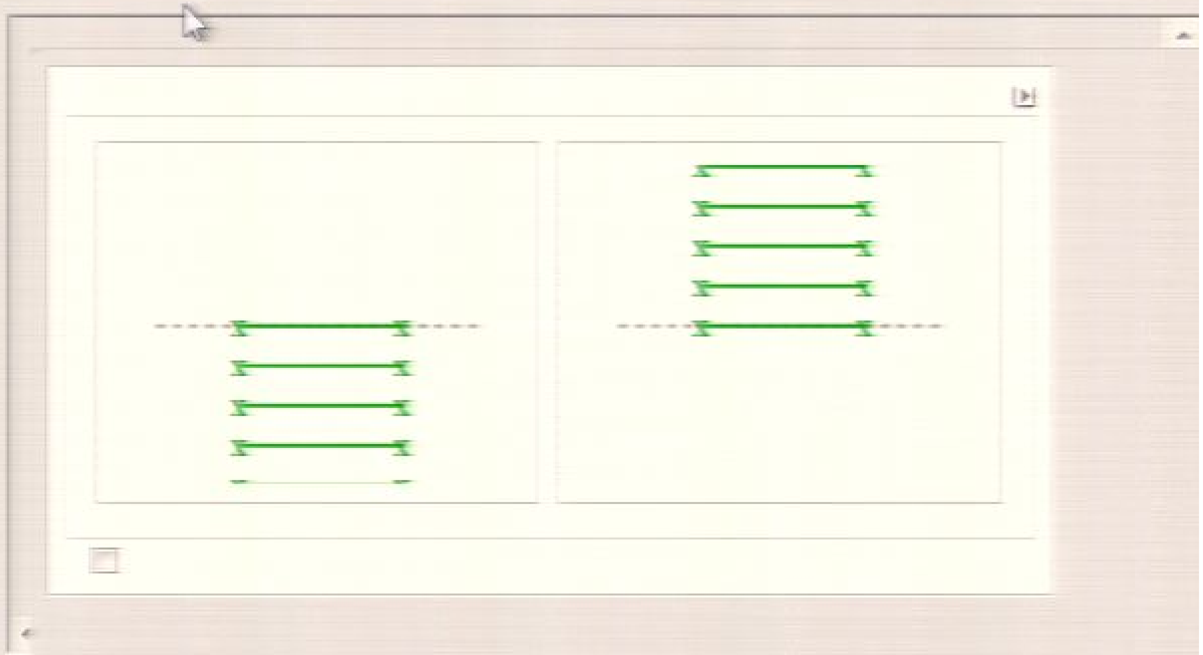

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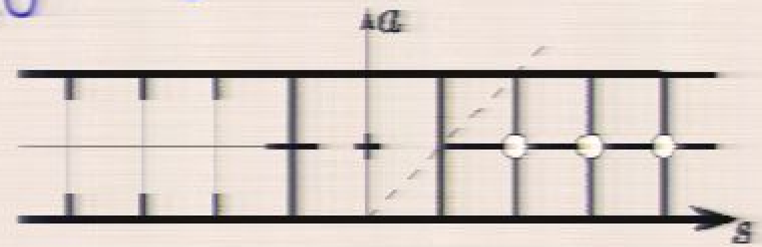

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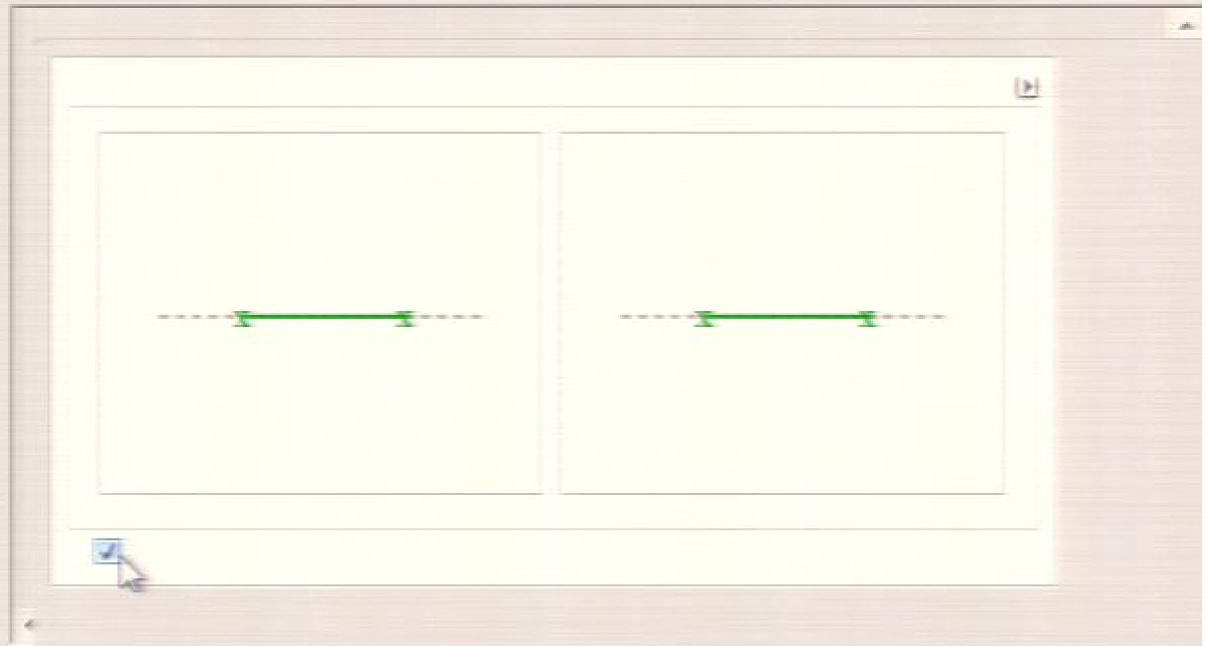
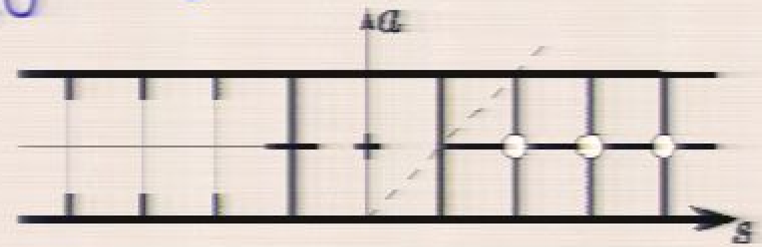


$$\bar{Q}_2 = Q_2$$

The magic of the Magic sheet

$$\hat{\mathbb{T}}_{1,0} = 0$$



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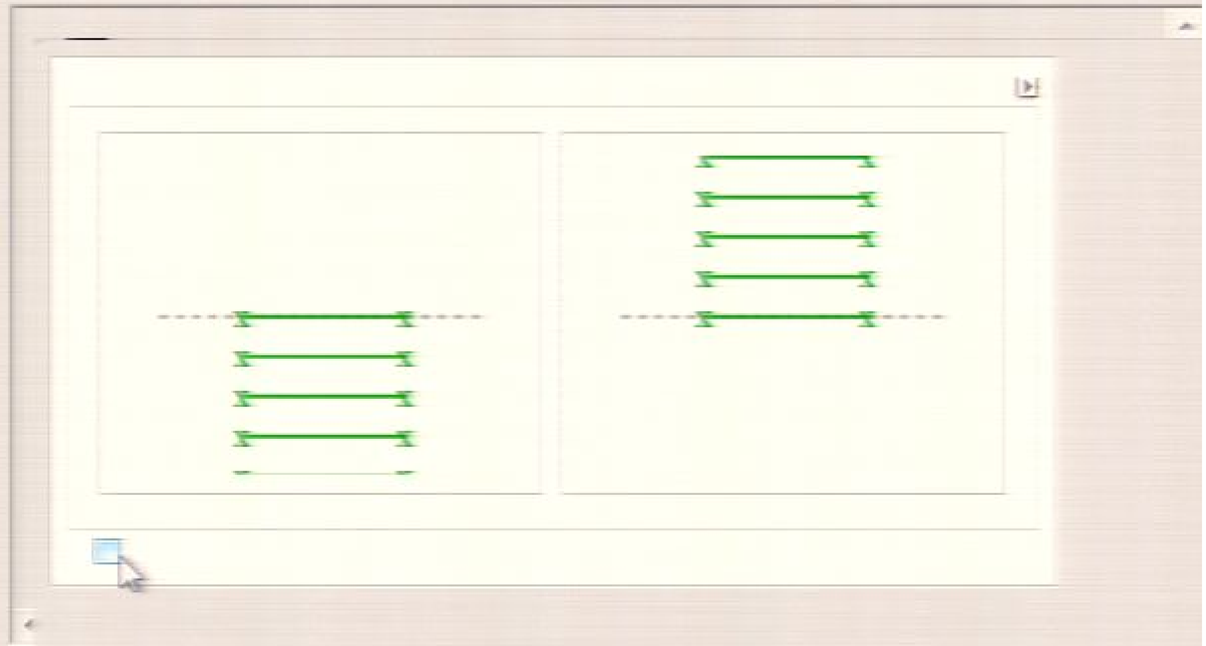

The magic of the Magic sheet

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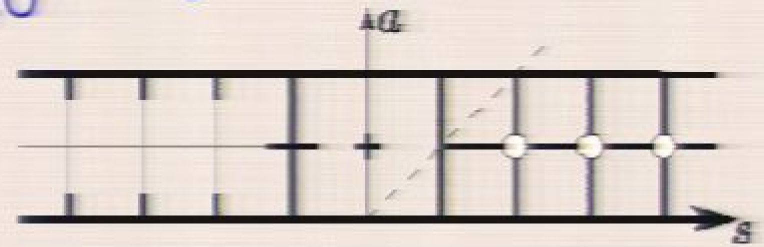

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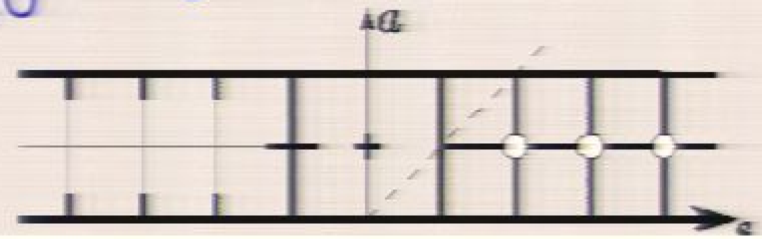

$\hat{\mathcal{T}}_{1,s}$ functions have only two cuts!

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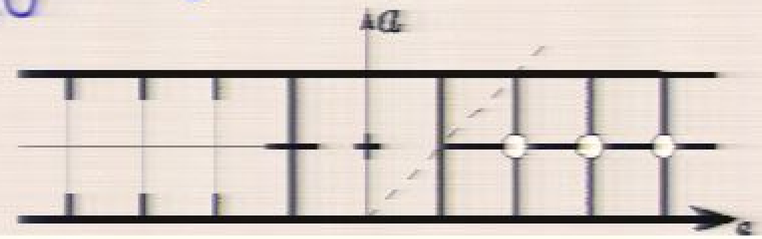

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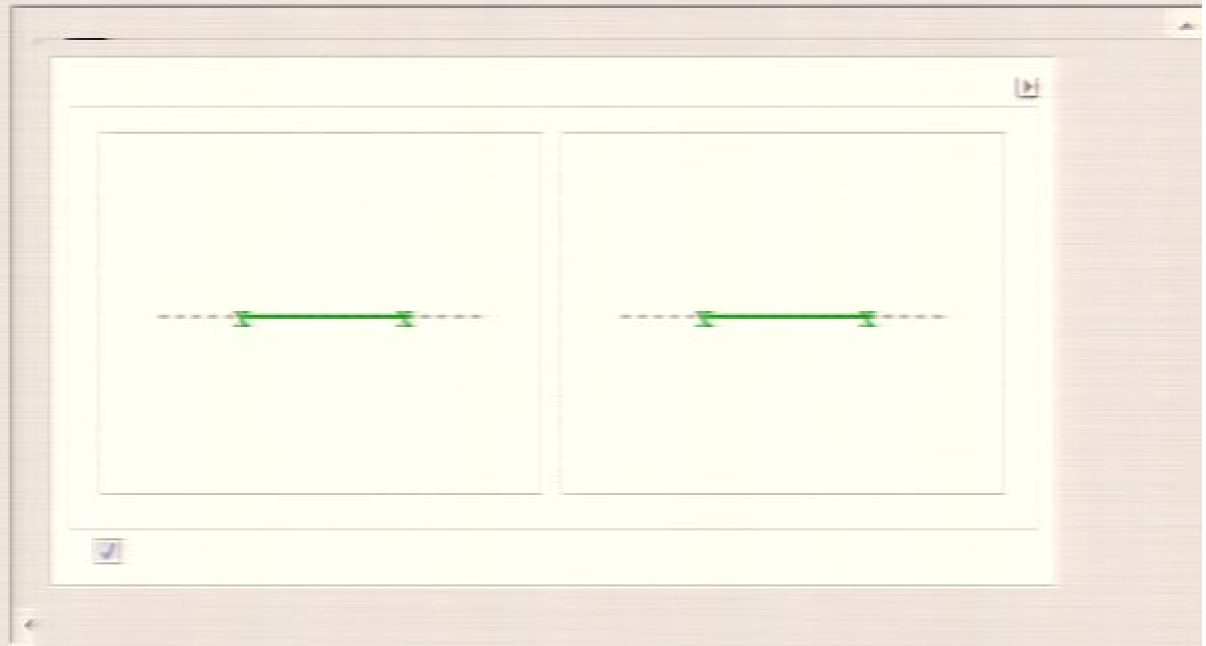

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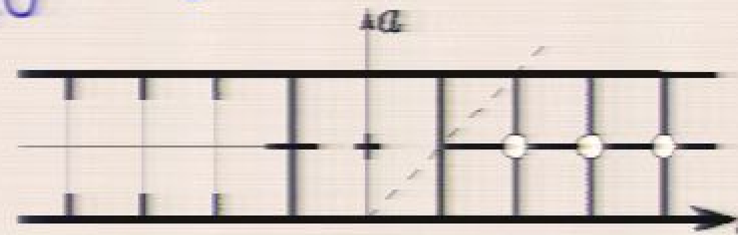

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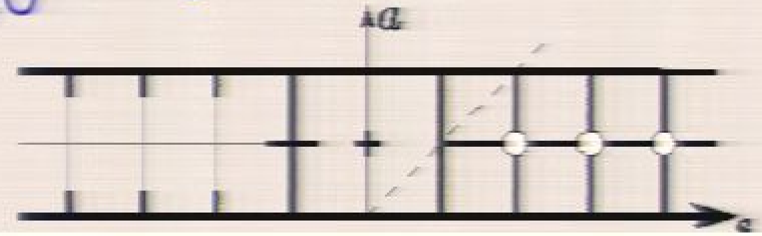

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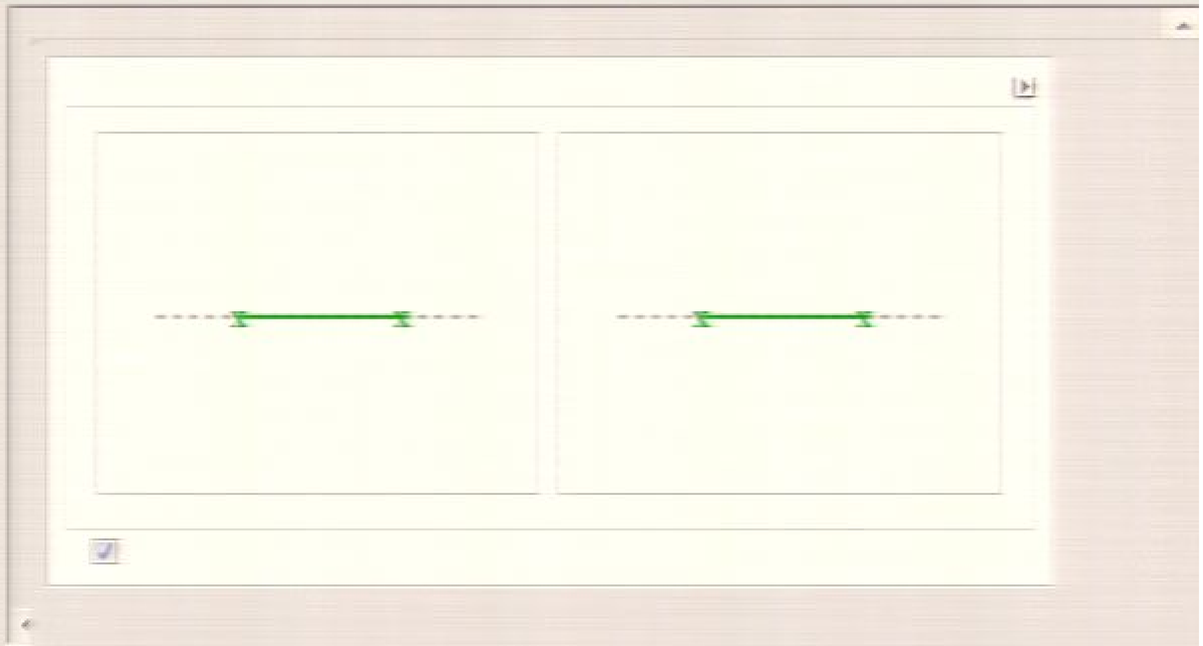

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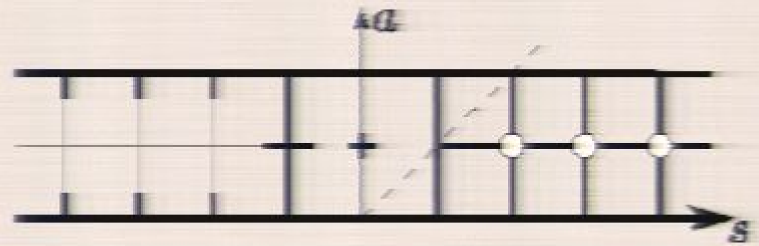

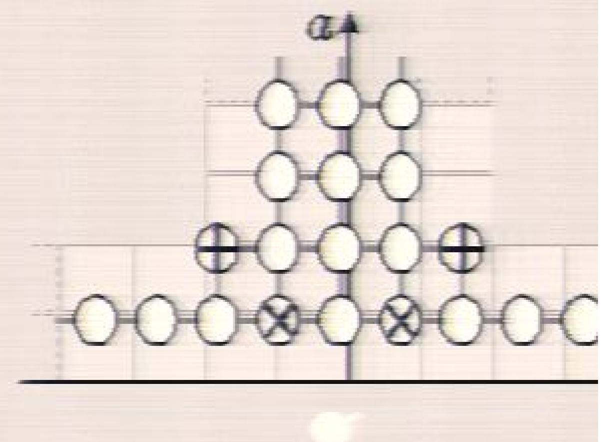

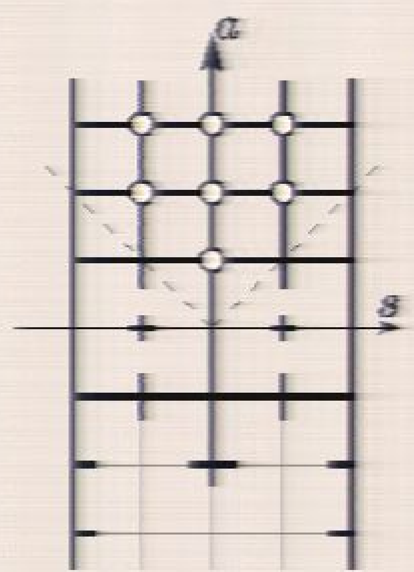

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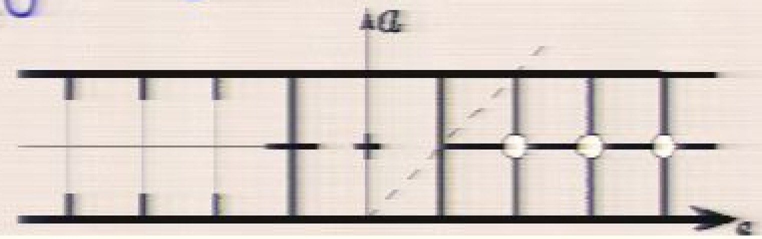

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N.Gromov,V.Kazakov, S.Leurent,D.V. 110?.????]

Discontinuity conditions

$Y_{1,s} \in \mathcal{A}_{s-1} \longrightarrow T_{1,s} \in \mathcal{A}_s$ in some gauge

Mirror:

$$Y_{1,2}^+ Y_{1,2}^- = (1+Y_{1,3}) \frac{1+Y_{1,1}}{1+1/Y_{2,2}}$$

Magic:

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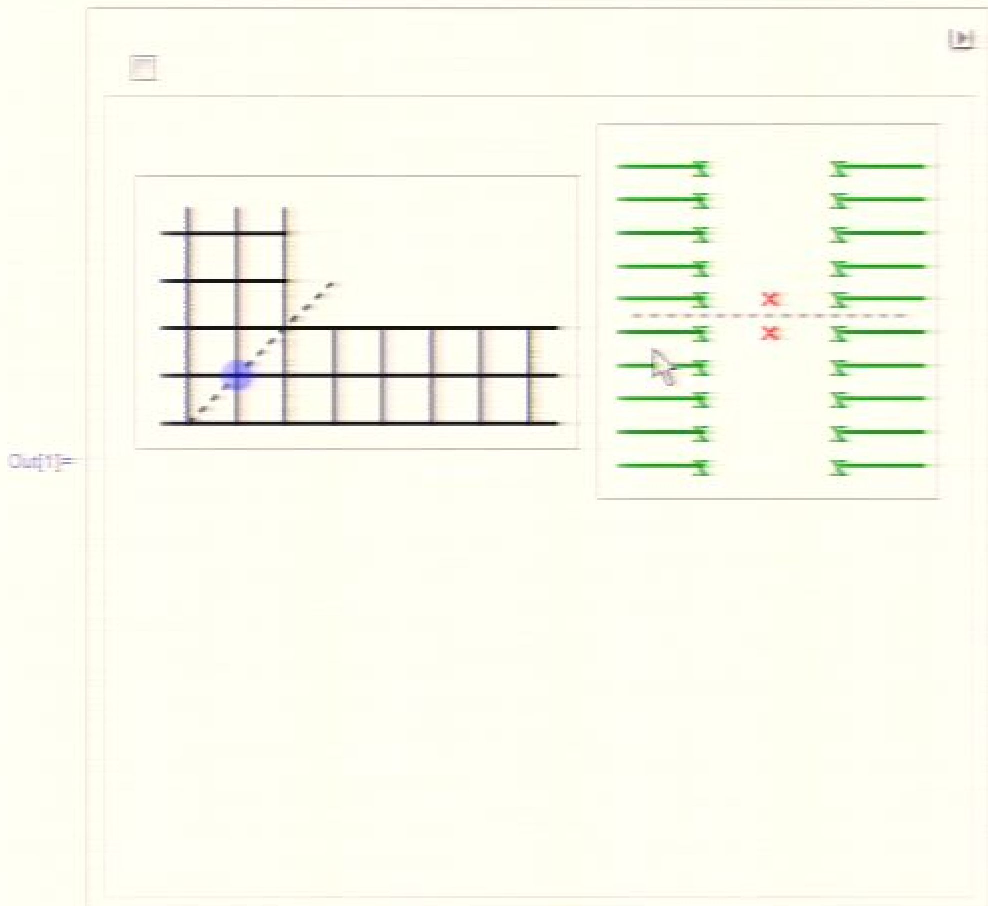

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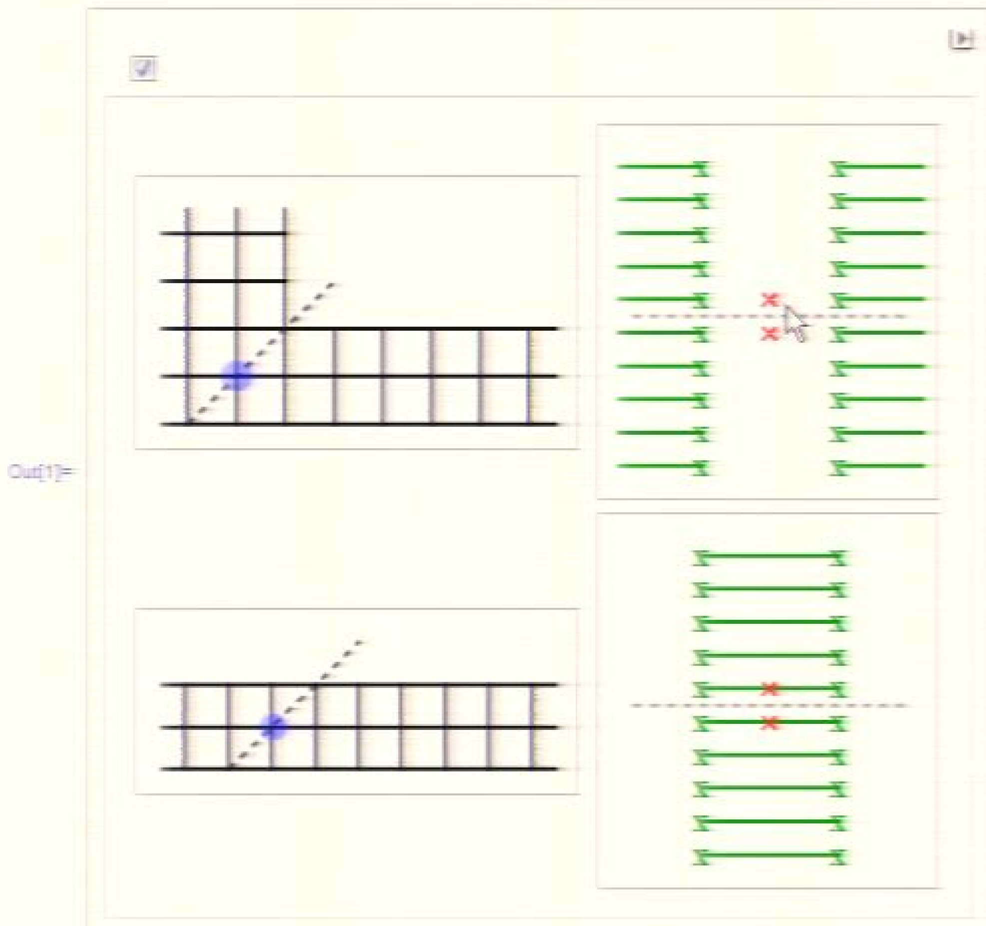

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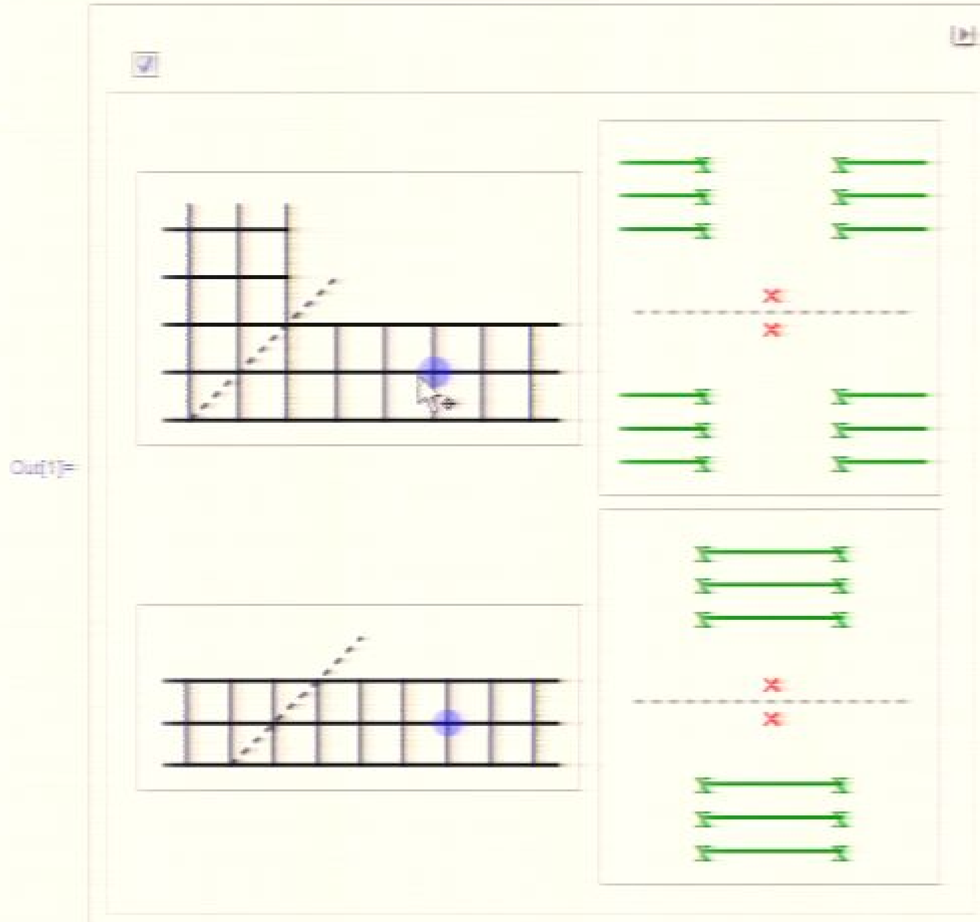

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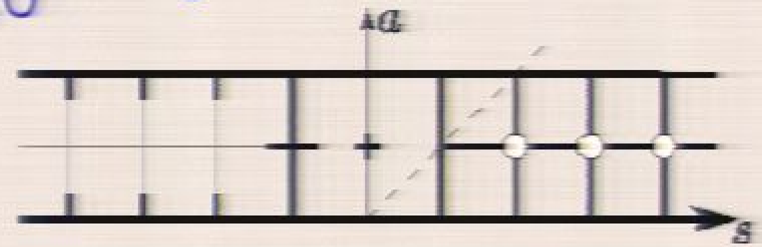

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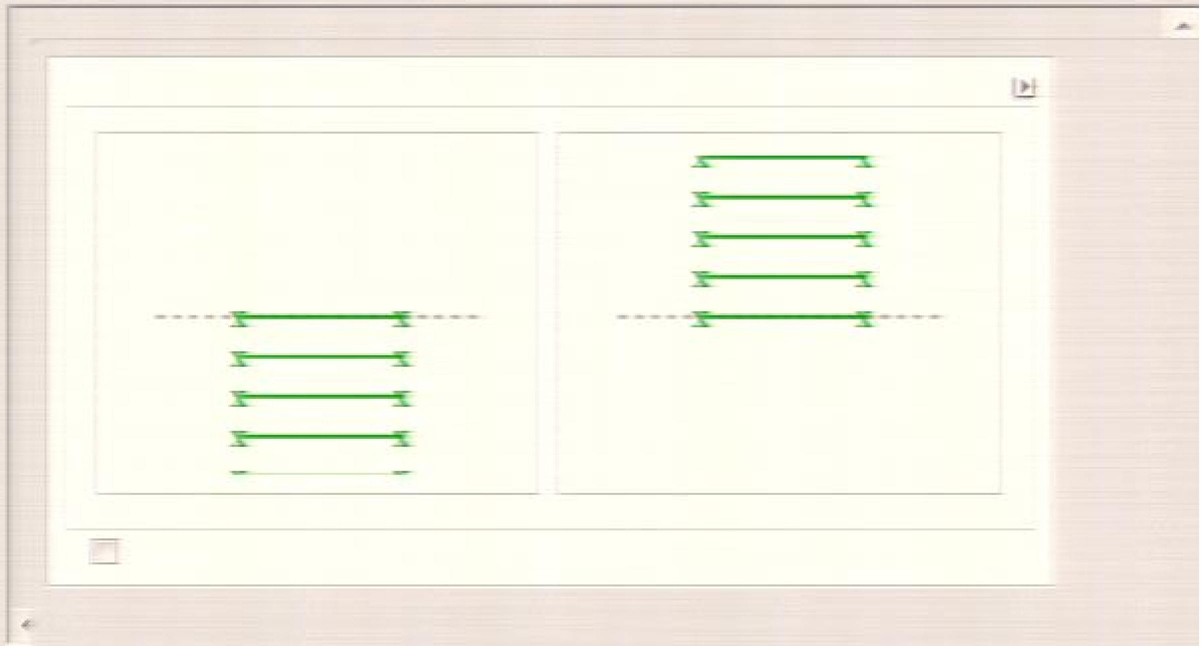

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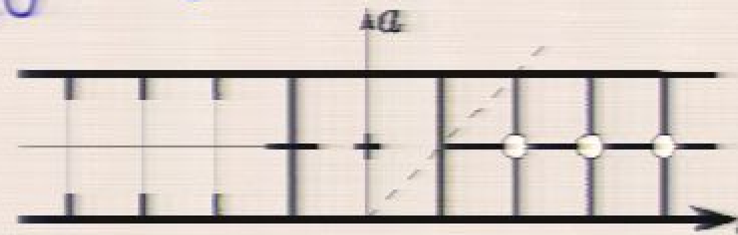

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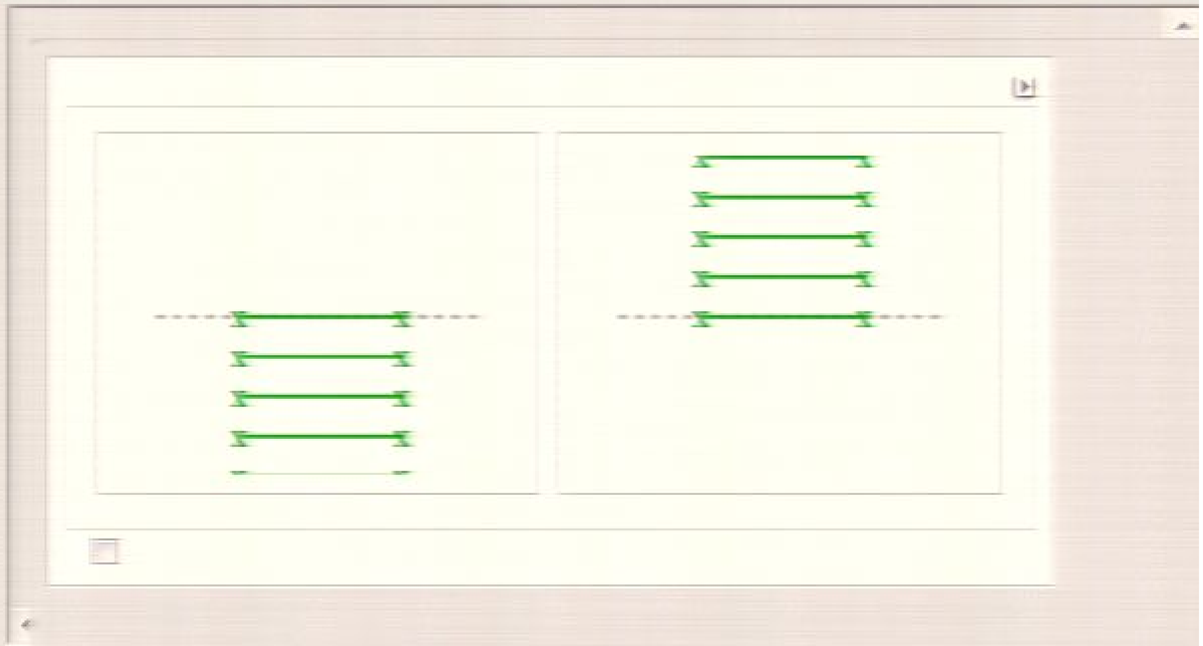

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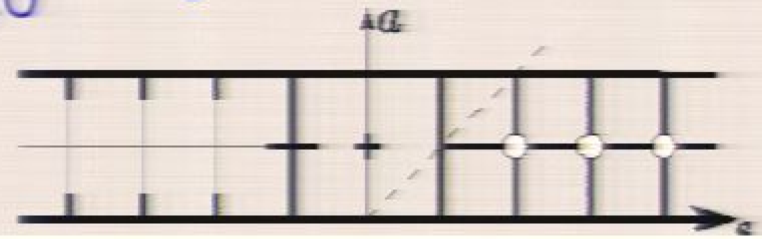

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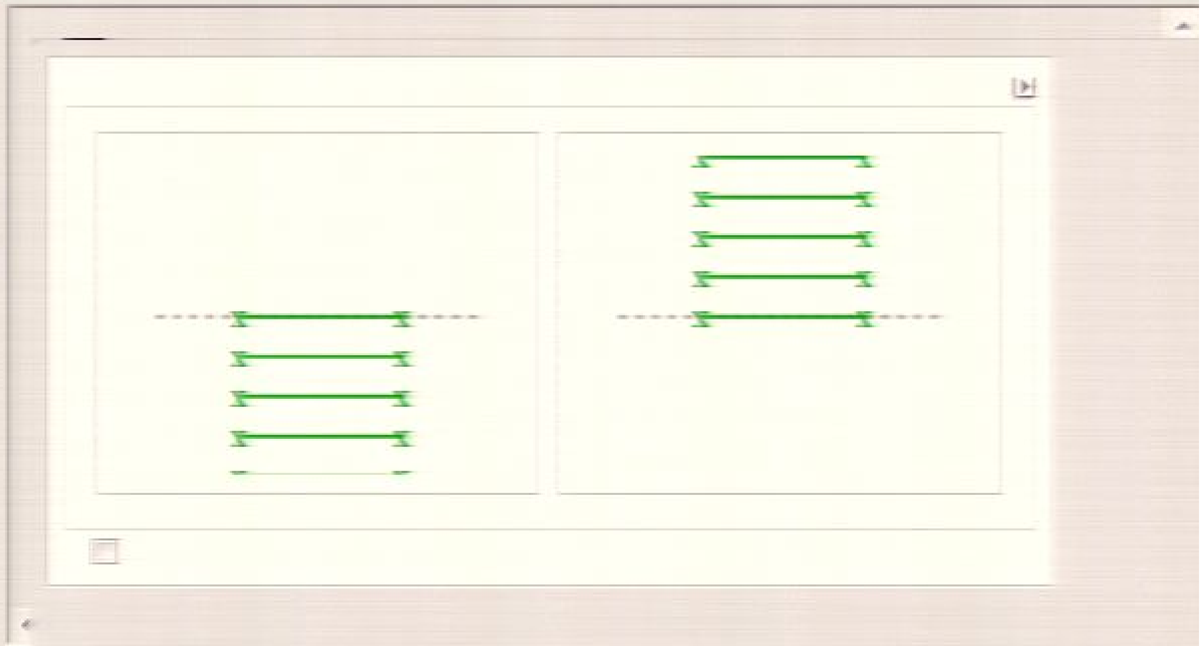

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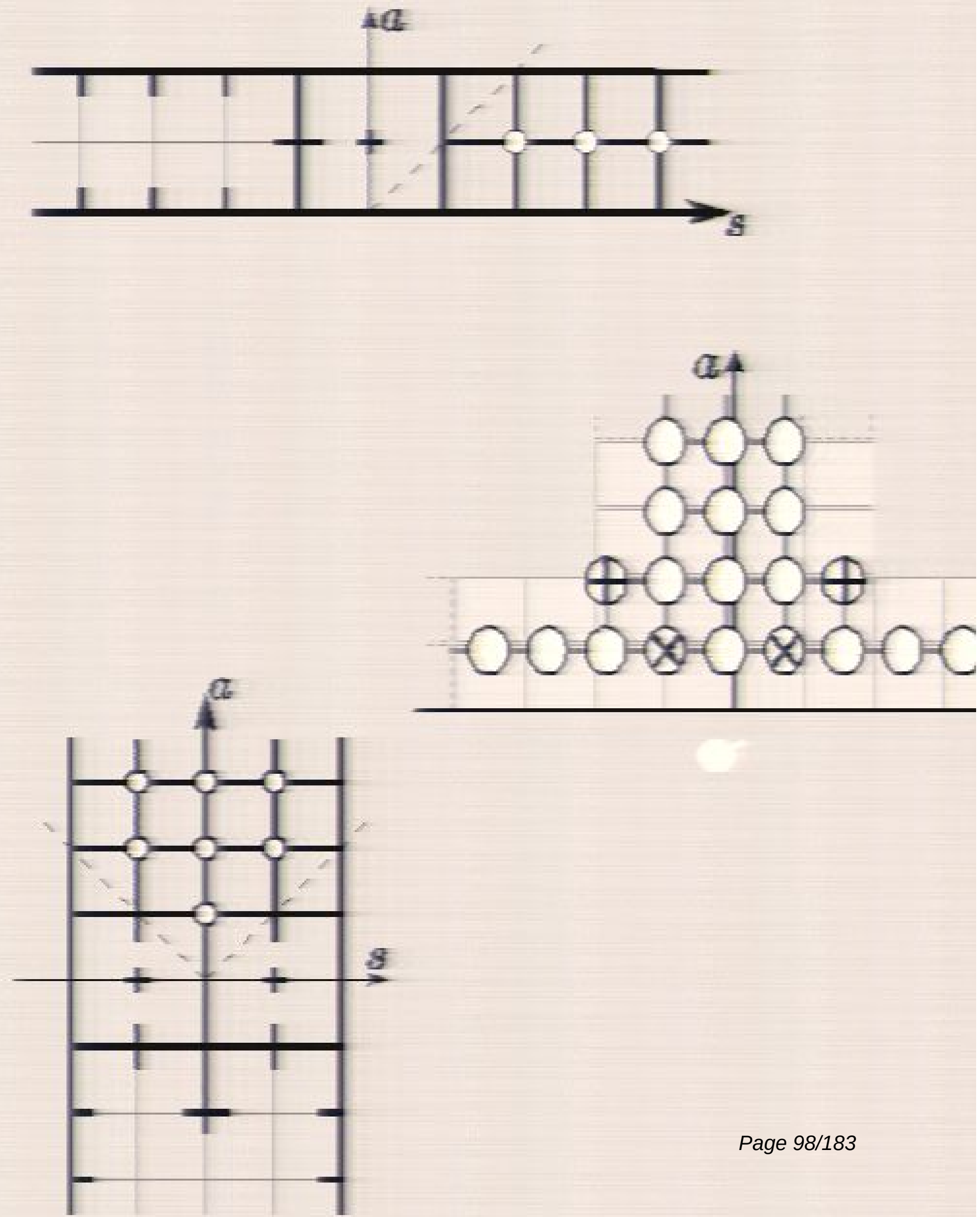

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[Gromov, Kazakov, Leurent, Tsuboi, '10]

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Complete set of properties of $\mathbf{T}$ and $\mathbb{T}$ gauges:

Symmetry

$$\mathrm{T}_{n,2}=\mathrm{T}_{2,n},\ \mathrm{T}_{n,-2}=\mathrm{T}_{2,-n},\ n\geq 2$$

$$\mathrm{T}^{+}_{0,0}=\mathrm{T}^{-}_{0,0}$$ (Unimodularity)

$$\hat{\mathrm{T}}_{a,s}=(-1)^{s}\hat{\mathrm{T}}_{-a,s} \qquad (\mathbb{Z}_4)$$

$$\mathbb{T}_{a,s}=\mathrm{T}_{a,s}(\mathcal{F}^{[a+s]})^{a-2},\ \mathcal{F}\equiv\sqrt{\mathrm{T}_{0,0}}$$

$$\hat{\mathbb{T}}_{a,s}=(-1)^{a}\hat{\mathbb{T}}_{a,-s} \qquad (\mathbb{Z}_4)$$

Analyticity

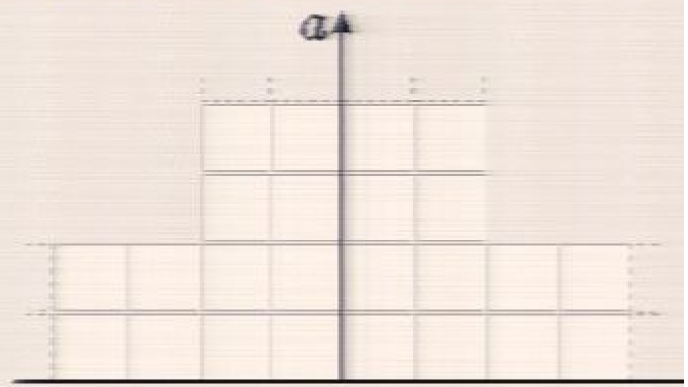
$$\mathrm{T}_{a,0}\in\mathcal{A}_{a+1}$$

$$\mathrm{T}_{a,\pm 1}\in\mathcal{A}_{a}$$

$$\mathrm{T}_{a,\pm 2}\in\mathcal{A}_{a-1}$$

No poles

Minimal # of zeroes



$$\mathbb{T}_{0,\pm s}=1$$

$$\mathbb{T}_{1,\pm s}\in\mathcal{A}_{s}$$

$$\mathbb{T}_{2,\pm s}\in\mathcal{A}_{s-1}$$

Two cuts for $\mathbb{T}_{1,\pm s}$

No poles

- Listed above properties (symmetry+analyticity) + correct large volume (asymptotic Bethe Ansatz) behavior uniquely fix solution of the Y-system = Hirota system

- In particular, these properties are equivalent to the TBA equations

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Symmetry
(Hirota)
+ Discontinuity
conditions
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+Poles/zeres/asymptotics

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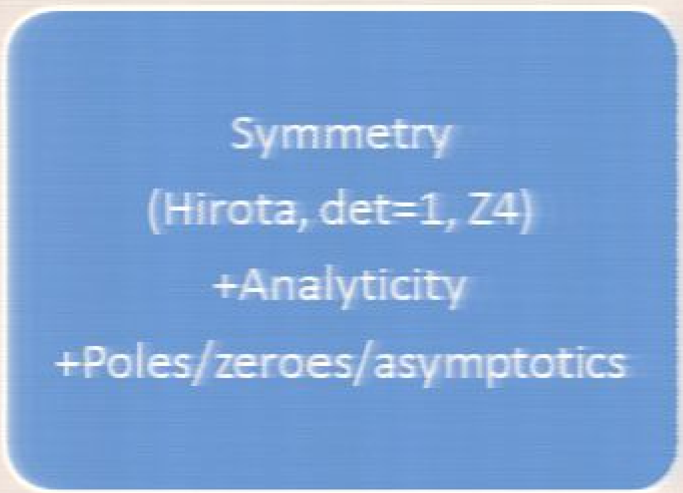

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Symmetry
(Hirota, det=1, Z4)
+Analyticity
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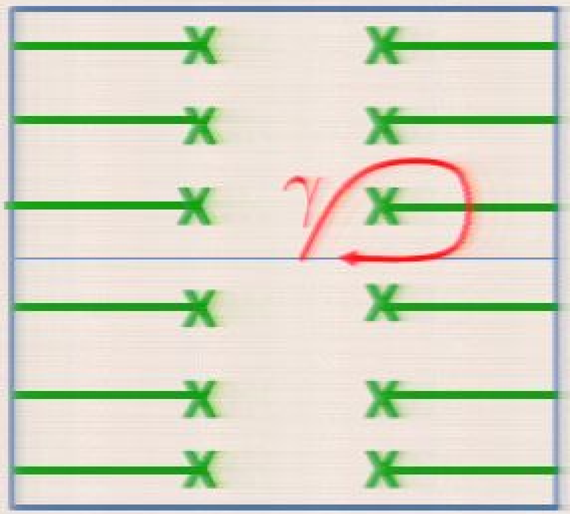
- What did I forget?

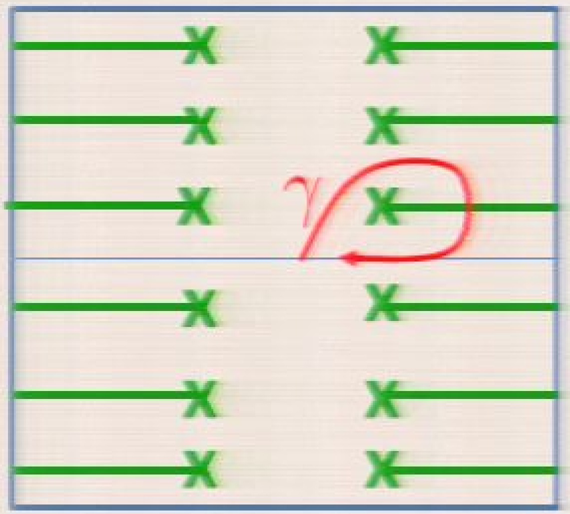
Exact Bethe equations

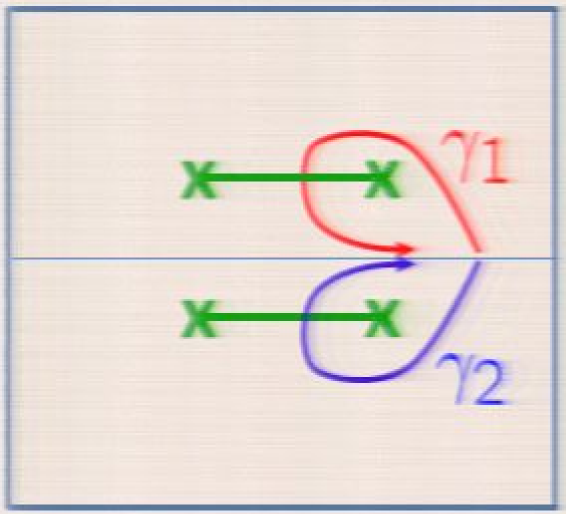


$$Y_{1,0}^{\gamma}(u_j) = -1$$

This is a condition for absence of singularities
In the physical **T**-gauge

Exact Bethe equations





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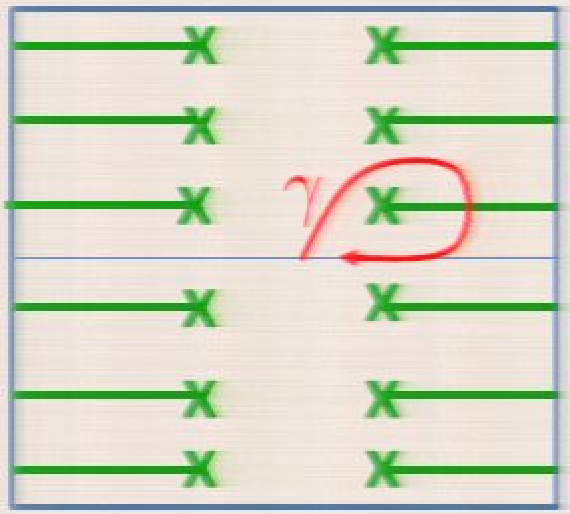

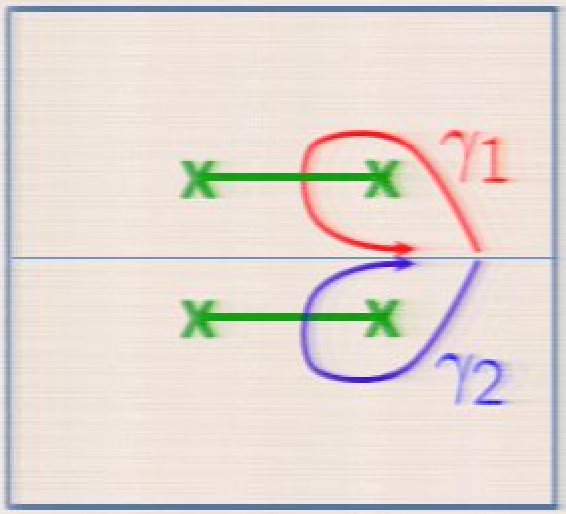

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New formula for the energy

$$\partial_u \log \mathrm{T}_{1,0} \simeq \frac{2E}{u}, \quad u \to \infty$$

TBA

(infinite set)

TBA

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FiNLIE !

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➢ Wronskian parameterization

Right band, paremeterization

(azakov, (IGST2010 talk)]
elated:[Suzuki, 11]

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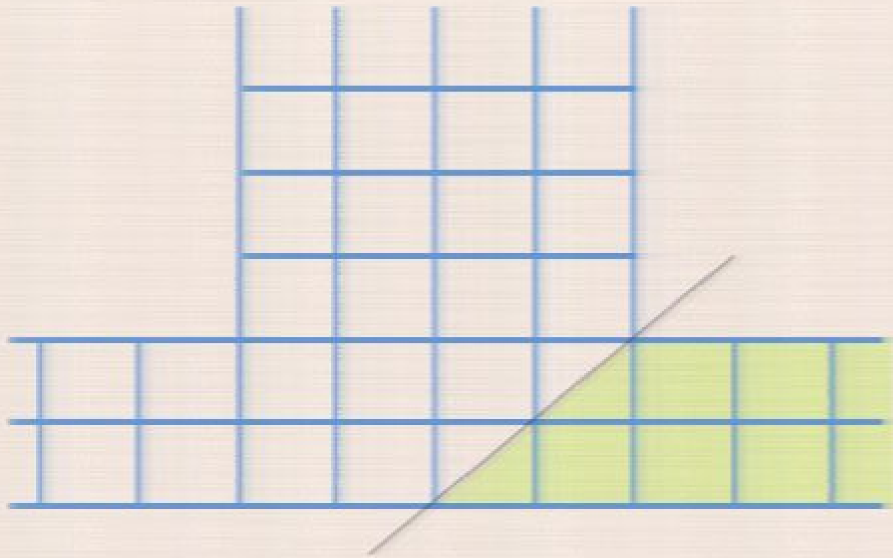

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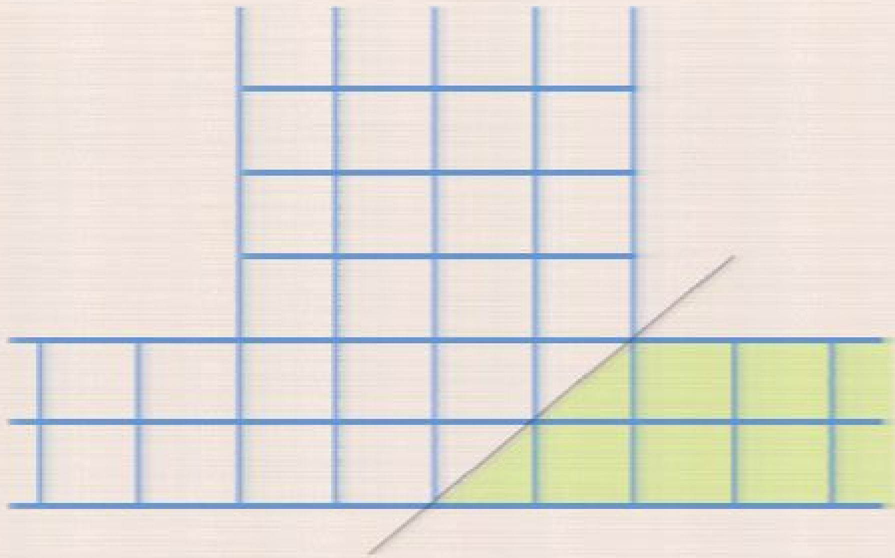

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$$\rho = Q^{[+0]} - Q^{[-0]}, \quad u^2 < 4g^2$$

Right band, paremeterization

Kazakov, (IGST2010 talk)]
related:[Suzuki, 11]

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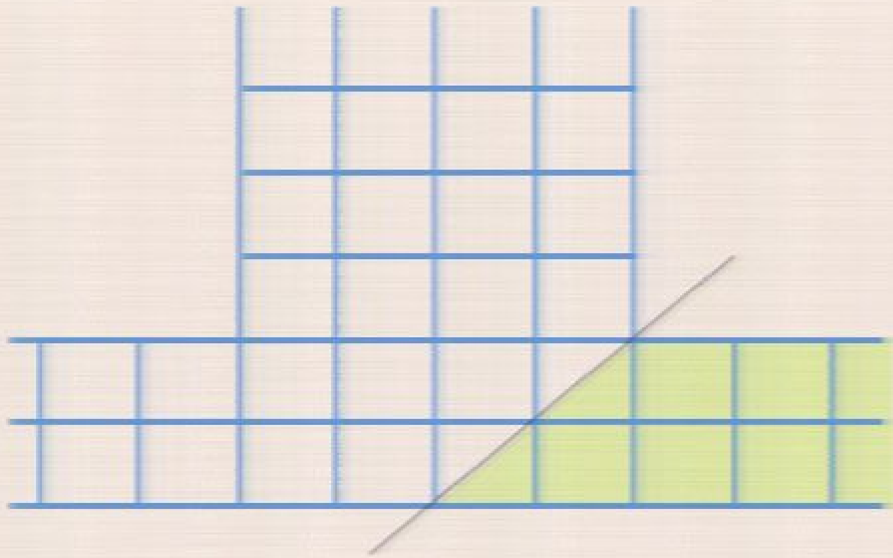

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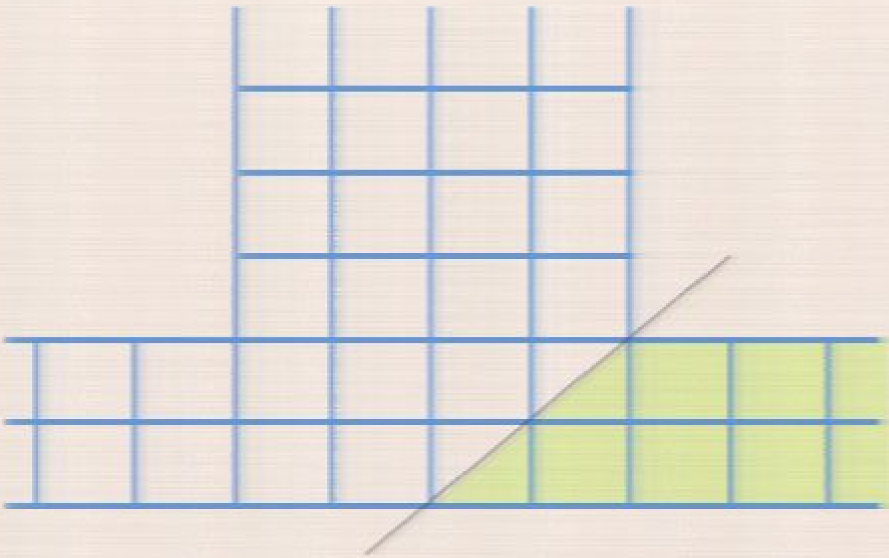

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- Cannot change at finite volume (topological argument). Therefore:

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elated:[Suzuki, 11]

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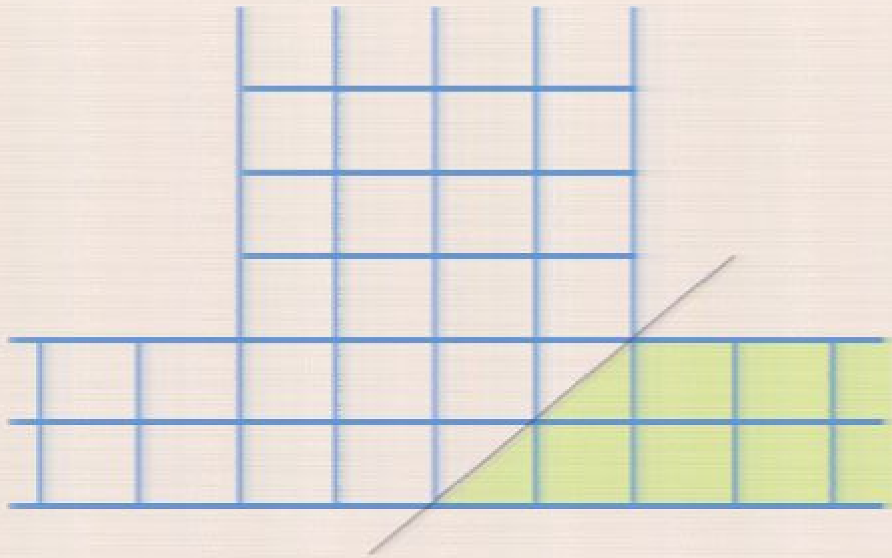


- Parameterized by a single function (with a single cut even), but need to know it everywhere in the complex plane

$$\rho = Q^{[+0]} - Q^{[-0]}, \quad u^2 < 4g^2$$

- From large volume solution (BS Bethe Ansatz, Sl(2) sector):

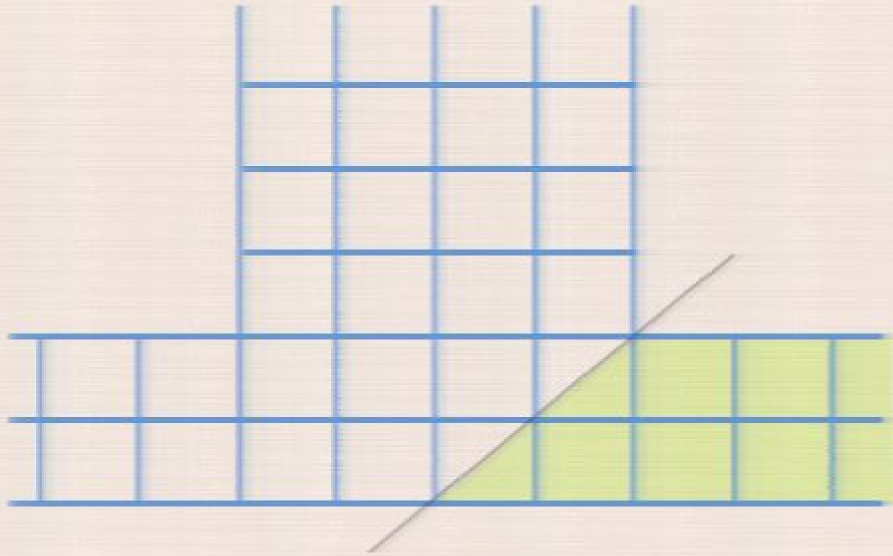
$$Q = -iu + \mathcal{O}(1), \quad u \to \infty$$

- Cannot change at finite volume (topological argument). Therefore:

$$Q(u) = -iu + \int_{-2g}^{2g} \frac{dv}{2\pi i} \frac{\rho(v)}{v-u}$$

Right band, solution

$$\mathcal{T}_{0,s} = 1 \qquad \mathcal{T}_{1,s} = Q^{[+s]} - Q^{[-s]}$$



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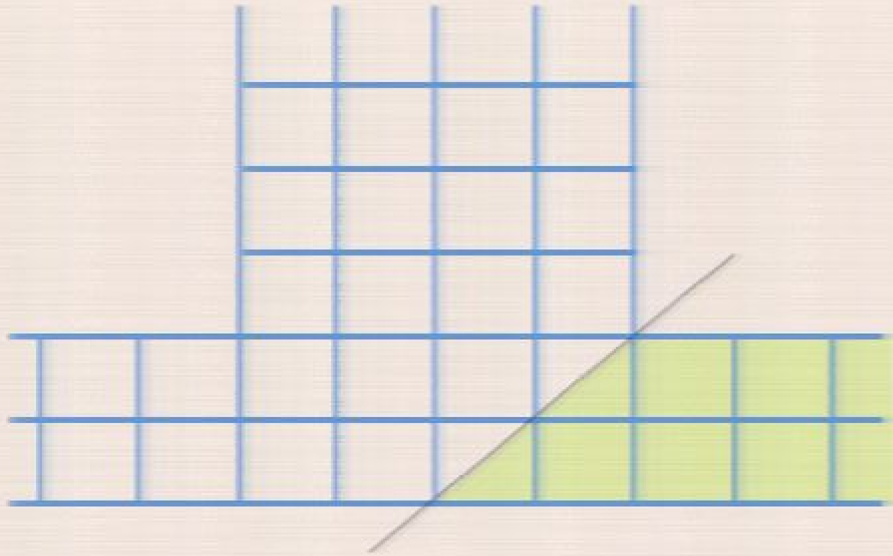

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$$\frac{1+Y_{1,1}}{1+\frac{1}{Y_{2,2}}} = \frac{\mathcal{T}_{1,1}^{+}\mathcal{T}_{1,1}^{-}\mathcal{T}_{2,3}}{\mathcal{T}_{2,2}^{+}\mathcal{T}_{2,2}^{-}} = \frac{(Q^{[+2]} - Q^{[-0]})(Q^{[-2]} - Q^{[+0]})}{(Q^{[+2]} - Q^{[+0]})(Q^{[-2]} - Q^{[-0]})}$$

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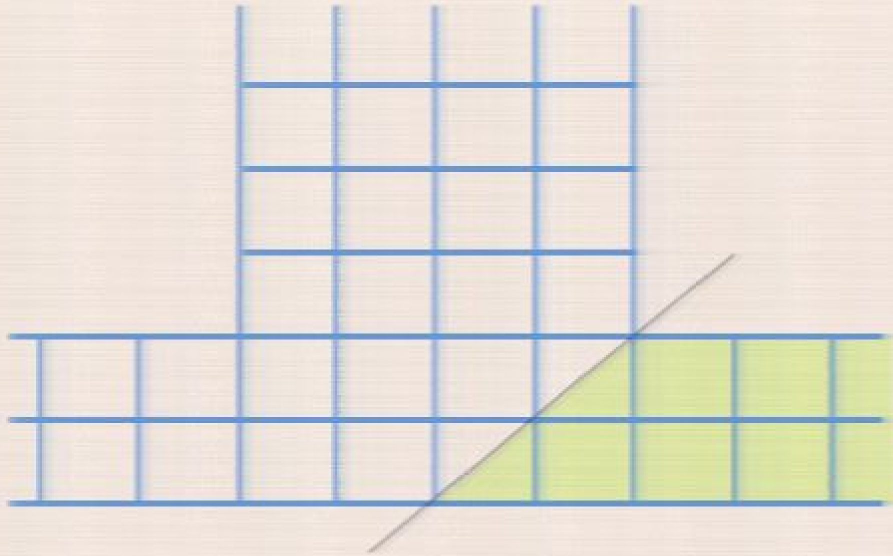

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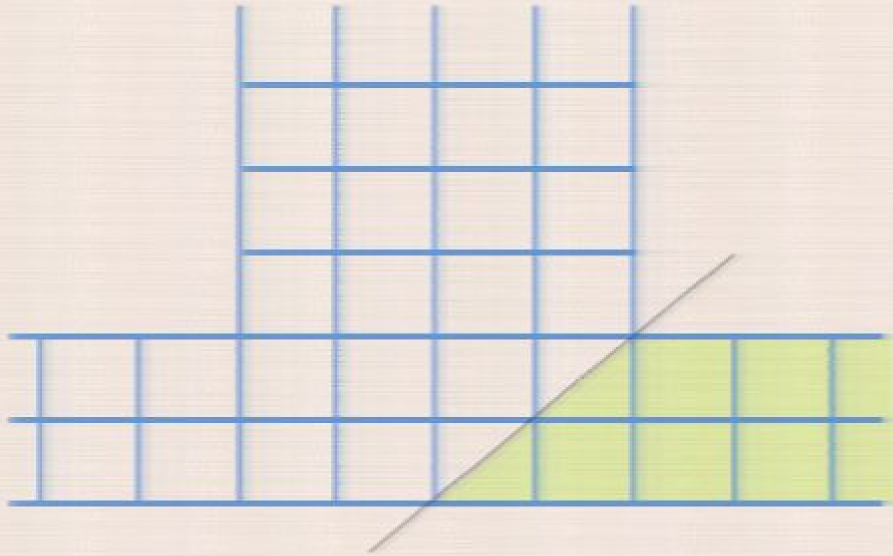

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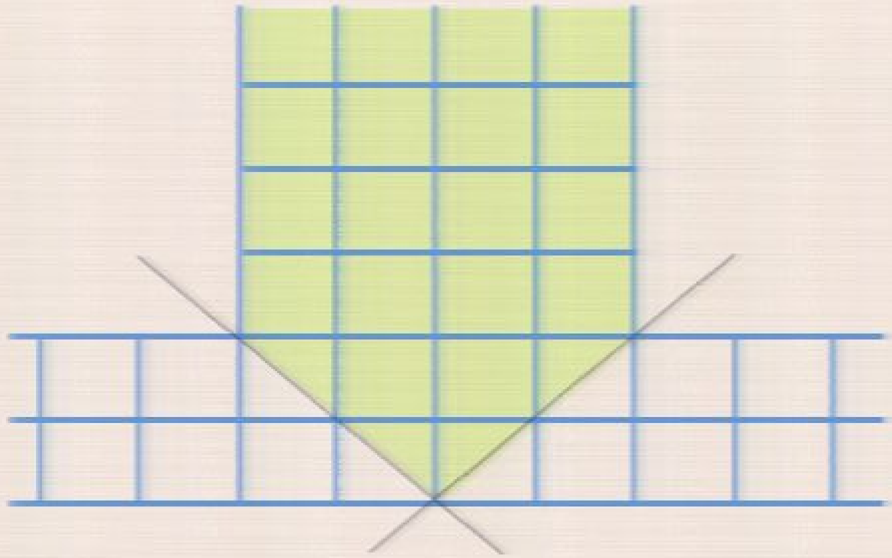
$$Q(u \pm i0) = -iu + \frac{\rho(u)}{2} + \fint_{-2g}^{2g} \frac{dv}{2\pi i} \frac{\rho(v)}{v-u}$$

- The density for the right band can be found from the knowledge of $Y_{1,1}$ and $Y_{2,2}$

Upper band, paremeterization

$q_\emptyset \quad q_1 \quad q_2 \quad q_3 \quad q_4$

$p_\emptyset \quad p_1 \quad p_2 \quad p_3 \quad p_4$



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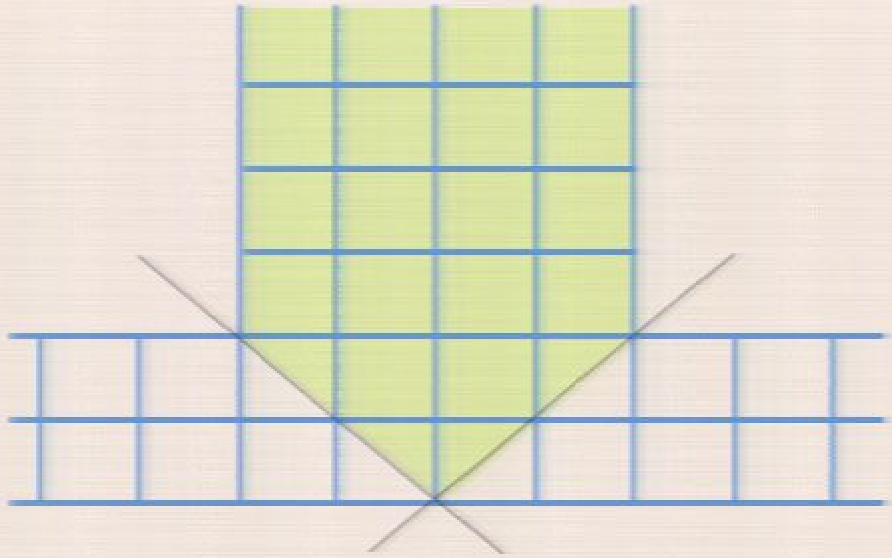
$$q \equiv q_i \psi^i \qquad p \equiv p_i \psi^i$$



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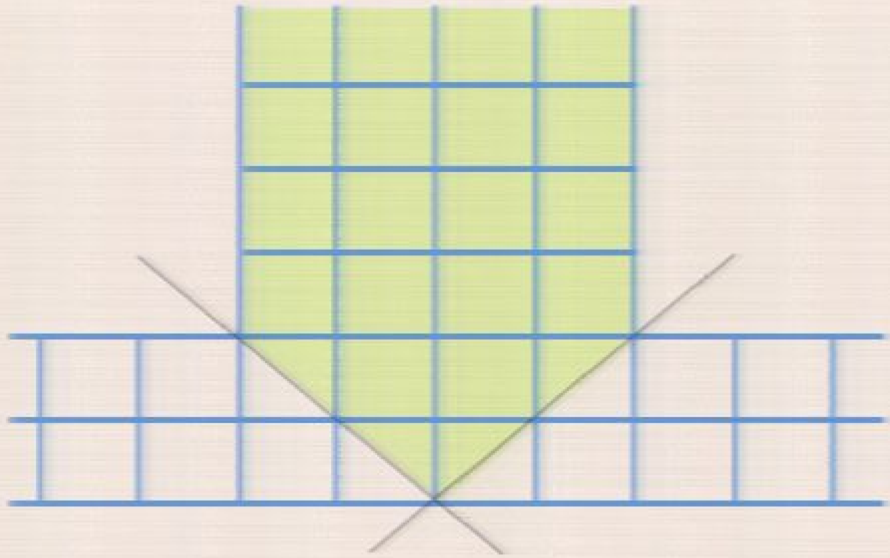
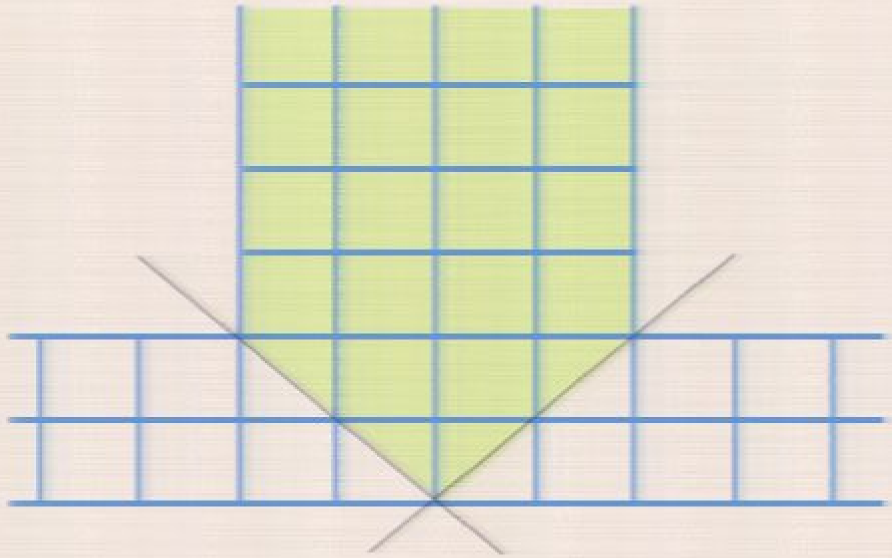


$$q_{(2)} \equiv \frac{q^+ \wedge q^-}{q_\emptyset} \qquad q_{ij} q_\emptyset = q_i^+ q_j^- - q_i^- q_j^+$$

Upper band, paremeterization



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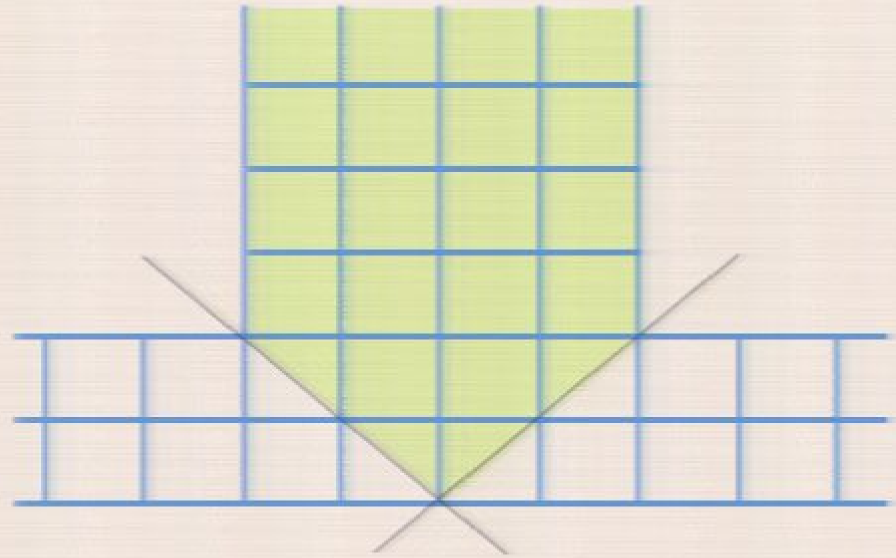

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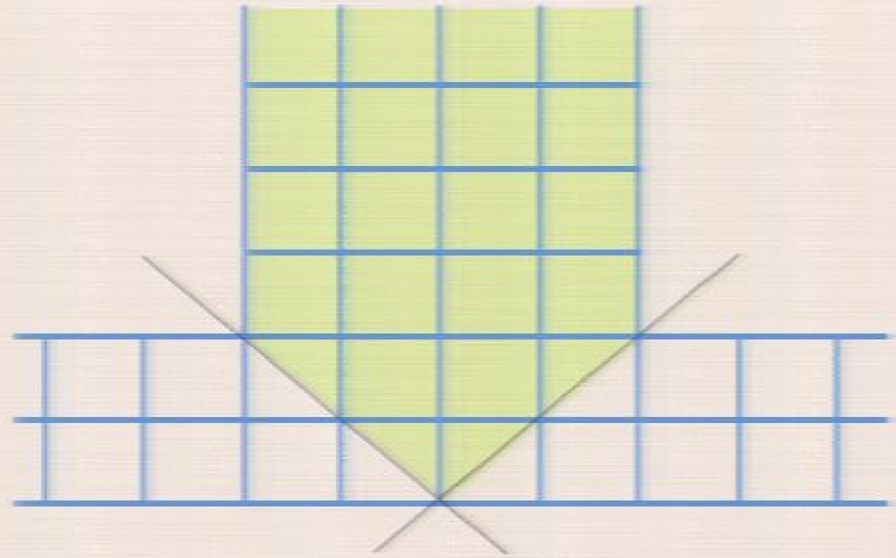
Upper band, paremeterization



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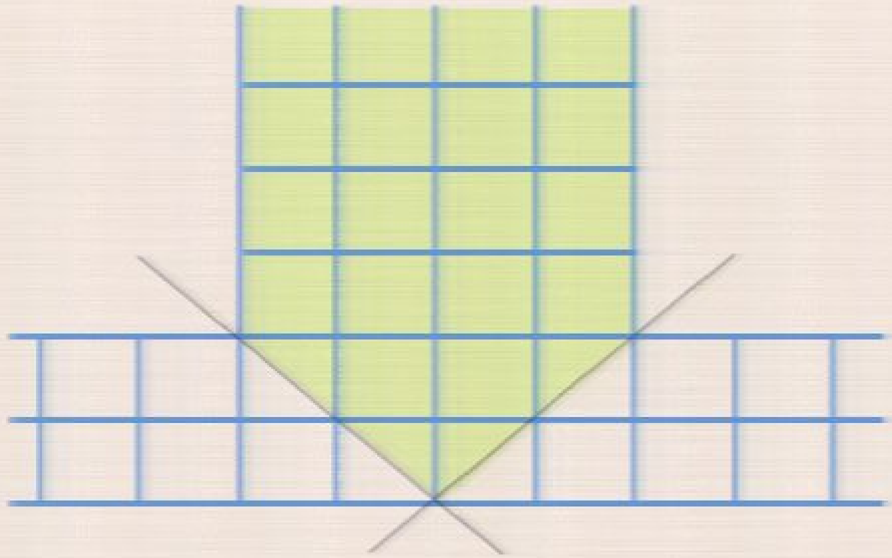
p - the same

Upper band, paremeterization



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p - the same

$$T_{a,s} = q_{(2-s)}^{[+a]} \wedge p_{(2+s)}^{[-a]}$$

- Due to reality of T-functions p-s can be expressed through complex conjugation of q-s
- Need to define q-s

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$$
\begin{array}{ccccc}
 & & q_{12} & & \\
 & q_1 & & q_{123} & \\
 & & q_{13} & & \\
 & q_2 & & q_{124} & \\
q_{\emptyset} & & q_{14} & & q_{1234} \\
 & q_3 & q_{23} & q_{134} & \\
 & q_4 & q_{24} & q_{234} & \\
 & & q_{34} & &
\end{array}
$$

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$q_\emptyset$

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q_{12} q_{13} q_{14} q_{23} q_{24} q_{34}

q_{123} q_{124} q_{134} q_{234}

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q_{34}

- Use gauge freedom, use LR symmetry

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$$\prod_{k=1}^{M}(u-\tilde{u}_k)$$

	1		q_{123}	
		q_{13}		
	q_2	q_{14}	q_{124}	
$q_\emptyset$				q_{1234}
	q_3	q_{23}	q_{134}	
	q_4	q_{24}	q_{234}	
		q_{34}		

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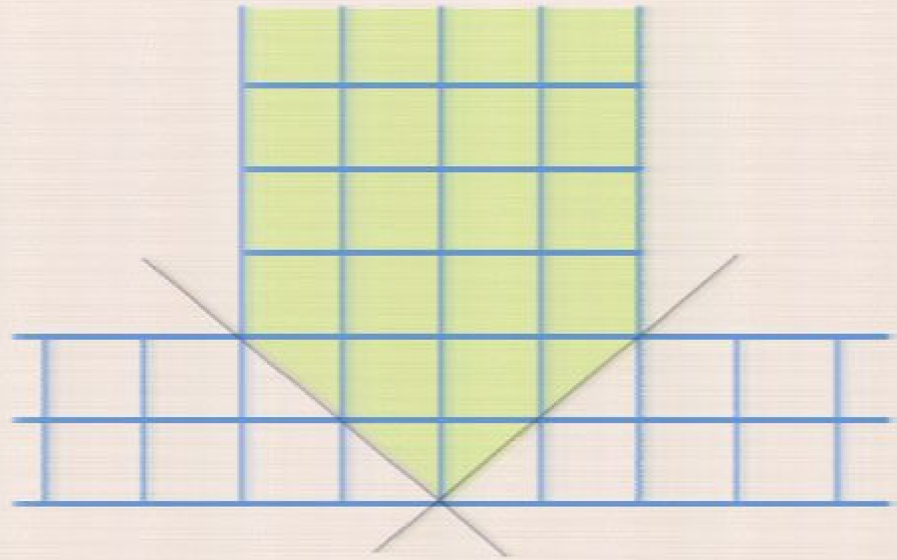
Upper band, paremeterization

- In a particular gauge $\mathscr{T}$-gauge

upper band is parameterized by two functions:

q_2 q_{123}



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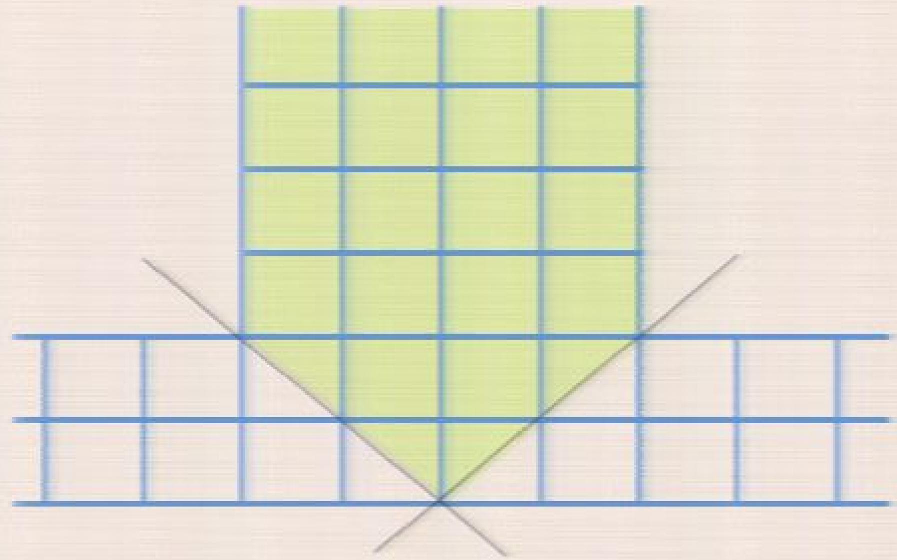
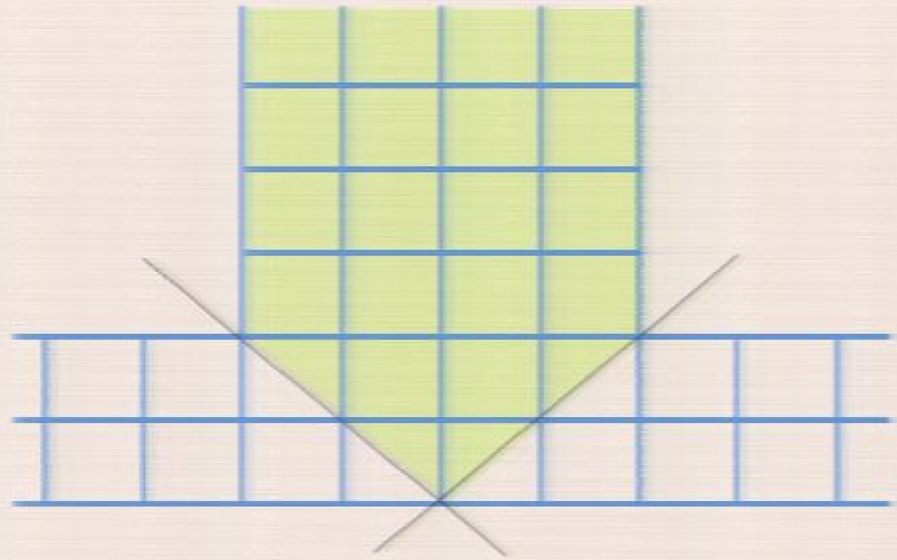


- q_2 is a direct analog of Q for the upper band:

Upper band, paremeterization



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$$-q_2 = P_{M-1} + \int_{-2g}^{2g} \frac{dv}{2\pi i} \frac{\rho_2(v)}{v-u} + \int_{-\infty}^{\infty} dv \left(q_3^{[+0]} \bar{q}_4^{[-0]} + q_4^{[+0]} \bar{q}_3^{[-0]}\right)$$

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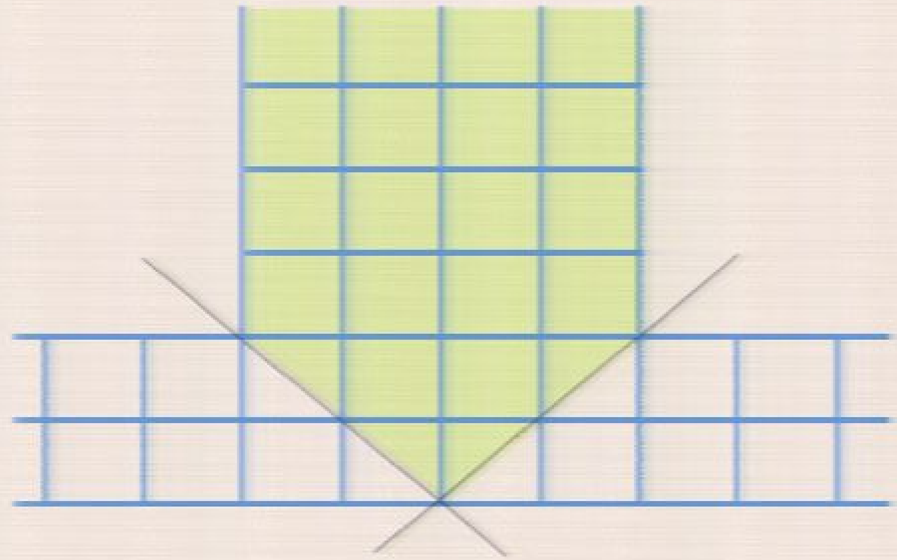

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- q_2 is fixed again from the knowledge of $Y_{1,1}$ and $Y_{2,2}$

$$\frac{1+Y_{2,2}}{1+\frac{1}{Y_{1,1}}} = \frac{\mathcal{T}_{2,2}^{+}\mathcal{T}_{2,2}^{-}\mathcal{T}_{1,0}}{\mathcal{T}_{1,1}^{+}\mathcal{T}_{1,1}^{-}\mathcal{T}_{3,2}}$$

Closing system of equations

$$q_{123} \quad Y_{1,1} \quad Y_{2,2}$$

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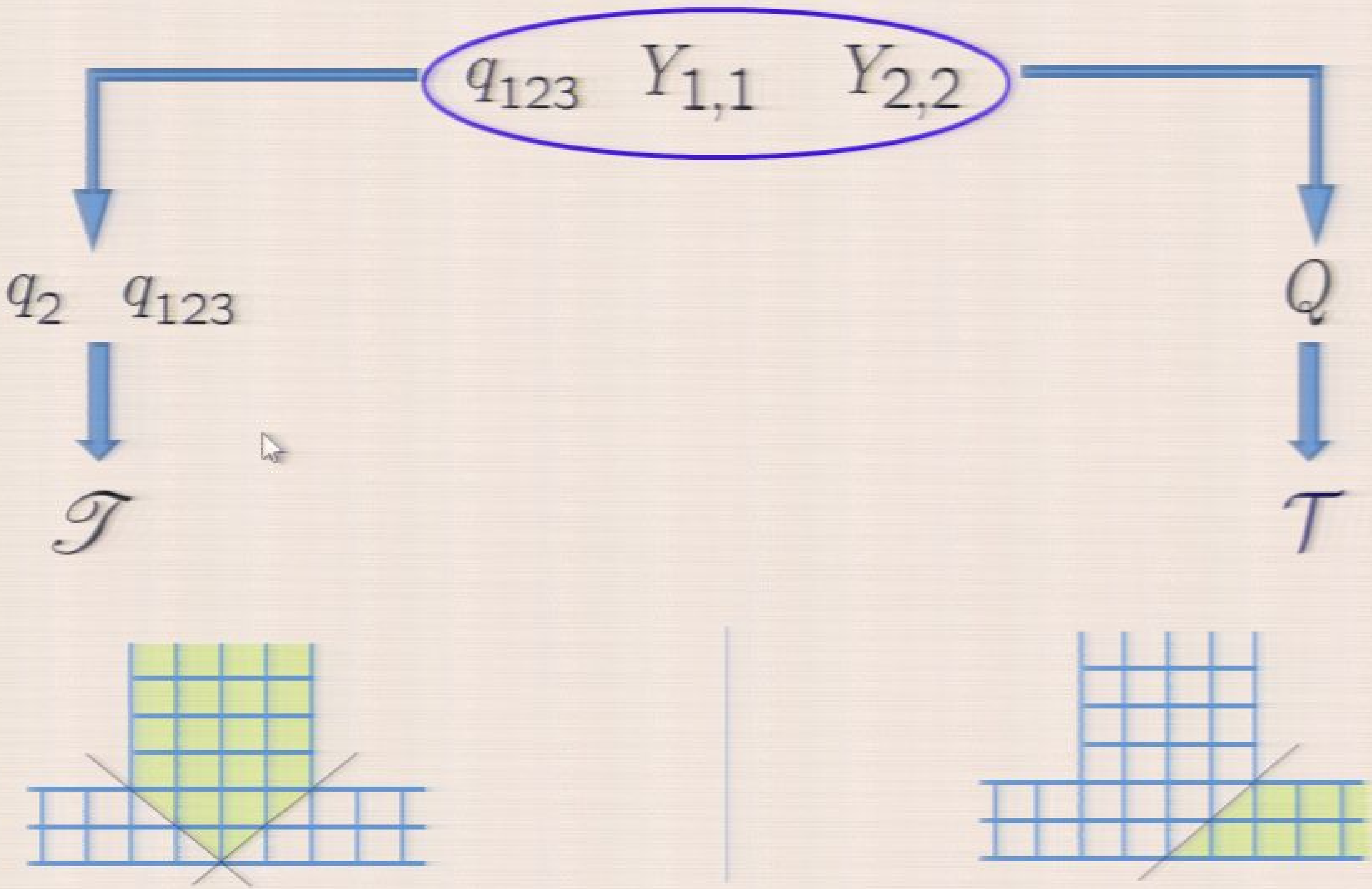
$q_{123} \quad Y_{1,1} \quad Y_{2,2}$

$q_2 \quad q_{123}$

Q

Closing system of equations



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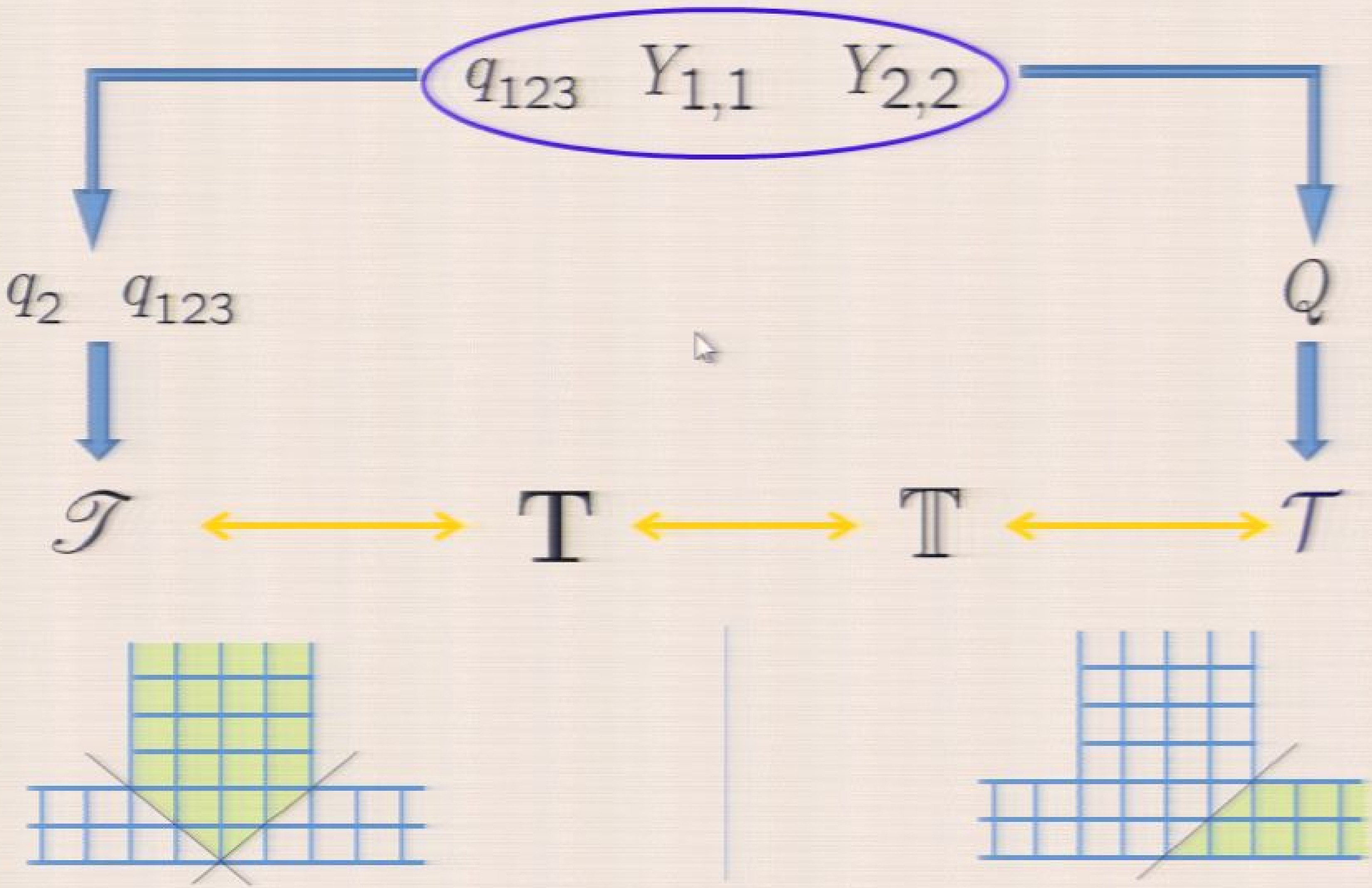

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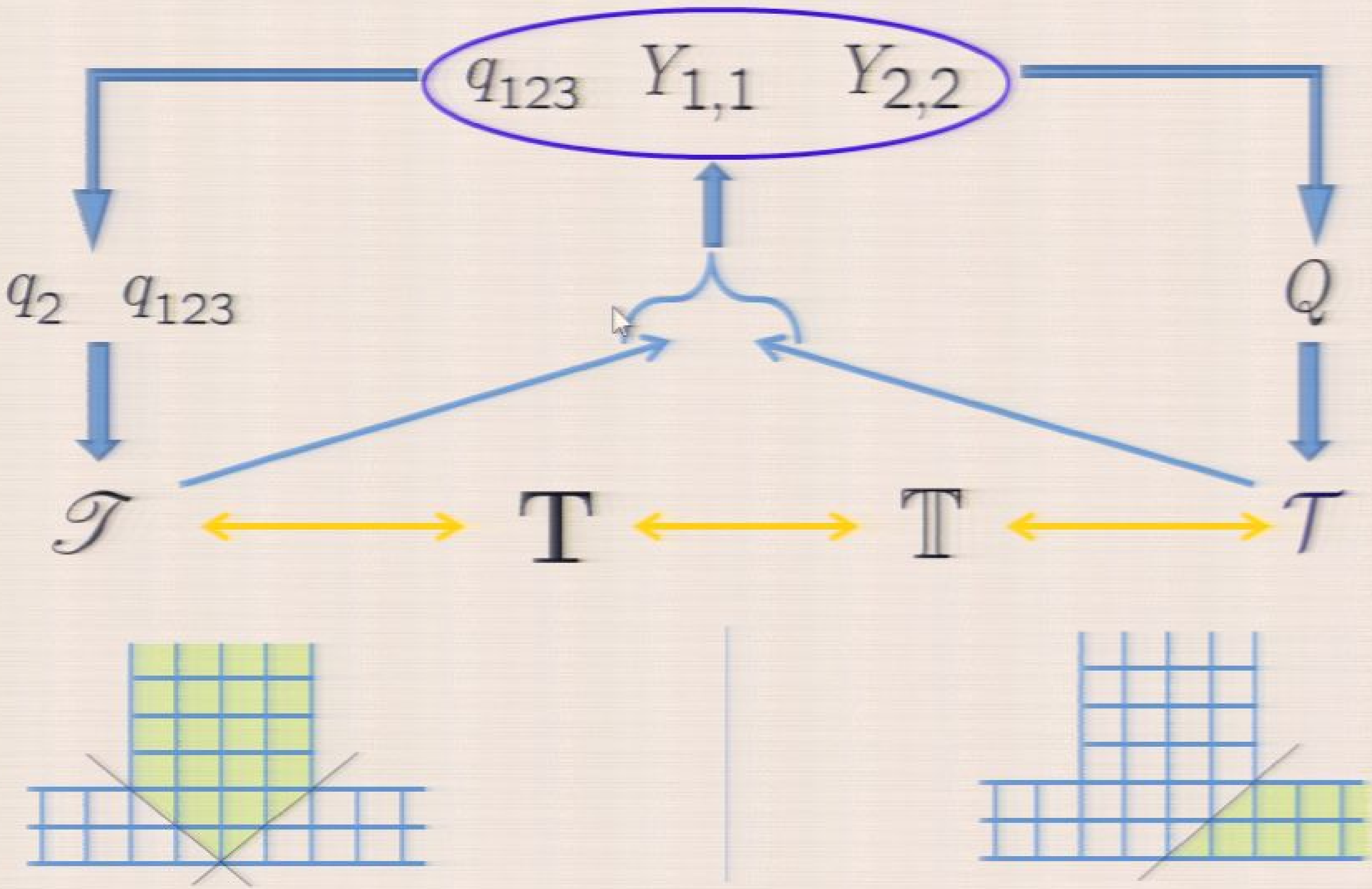


Complete system of equations:

light band:

$$Q(u) = -iu + \int_{-2g}^{2g} \frac{dv}{2\pi i}\frac{\rho(v)}{v-u}$$

$$\mathcal{T}_{0,s} = 1 \quad \mathcal{T}_{1,s} = Q^{[+s]} - Q^{[-s]}$$

$$\mathcal{T}_{2,s} = (Q^{[+s+1]} - Q^{[+s-1]})(Q^{[-s+1]} - Q^{[-s-1]})$$

$$\frac{1+Y_{1,1}}{1+\frac{1}{Y_{2,2}}} = \frac{\mathcal{T}_{1,1}^{+}\mathcal{T}_{1,1}^{-}\mathcal{T}_{2,3}}{\mathcal{T}_{2,2}^{+}\mathcal{T}_{2,2}^{-}}$$

Upper band:

$$T_{a,s} = q_{(2-s)}^{[+a]} \wedge p_{(2+s)}^{[-a]}$$

$$q_1 = 1 \quad -q_2 = P_{M-1} + \int_{-2g}^{2g} \frac{dv}{2\pi i}\frac{\rho_2(v)}{(v-u)} + \int_{-\infty}^{\infty} dv \left(q_3^{[+0]} q_4^{[-0]} + q_4^{[+0]} q_3^{[-0]}\right) \quad q_{12} = \prod_{k=1}^{M}(u - \tilde{u}_k)$$

$$q_\emptyset q_{ij} = q_i^+ q_j^- - q_j^+ q_i^-$$
$$q_i q_{ijk} = q_{ij}^+ q_{ik}^- - q_{ik}^+ q_{ij}^-$$

$$\frac{1+Y_{2,2}}{1+\frac{1}{Y_{1,1}}} = \frac{\mathcal{T}_{2,2}^{+}\mathcal{T}_{2,2}^{-}\mathcal{T}_{1,0}}{\mathcal{T}_{1,1}^{+}\mathcal{T}_{1,1}^{-}\mathcal{T}_{3,2}}$$

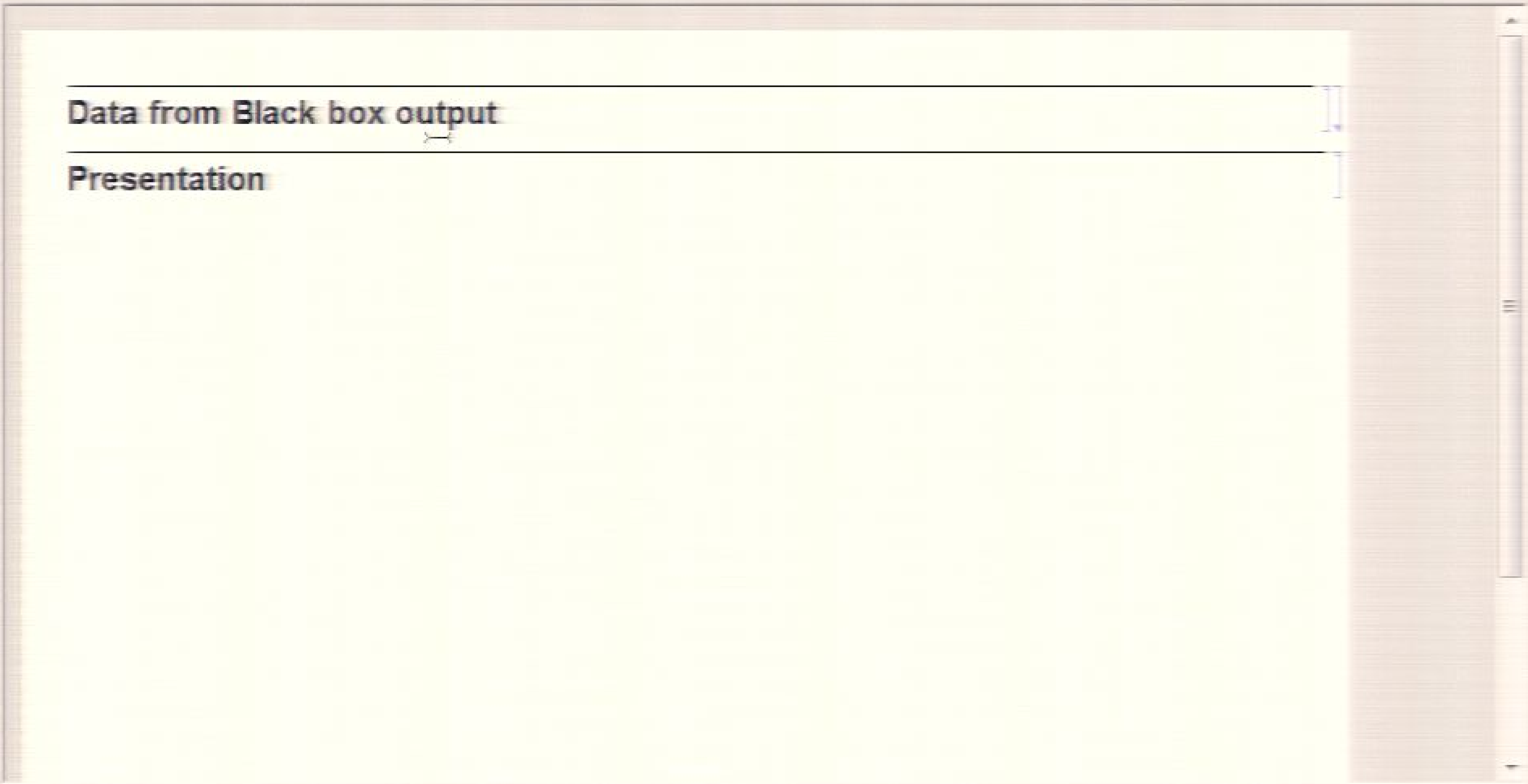
Gluing equaitons:

$$\log Y_{1,1} = \log\left(-\frac{R^{(+)}T_{1,2}}{R^{(-)}\mathcal{T}_{2,1}}\right) + \mathcal{Z} *_{\mathcal{Z}} \log\frac{\mathcal{T}_{1,0}}{Q^+Q^-} - \frac{1}{2}(\mathcal{Z}_1 + \mathcal{K}_1) * \log\frac{\mathcal{T}_{0,0}}{Q^2} - \mathcal{K}_1 * \log\frac{T_{1,1}}{\mathcal{T}_{1,1}}$$

$$\log q_{123} = \log\Lambda + \log\frac{\hat{h}}{f^+} + \frac{1}{2}\Psi * \rho_c \quad \log\hat{h} = -\frac{L+2}{2}\log\hat{x}(u) + \mathcal{Z}\hat{*}\log\left(\frac{\left(f\bar{f}\sqrt{\mathcal{T}_{0,0}}\right)^+ (Y_{1,1}Y_{2,2}-1)}{\rho}\right) \quad \log f^2 = \Psi^+ * \rho_b$$

$$\rho(v) = \begin{cases} \log\frac{\mathcal{T}_{1,0}^2}{\mathcal{T}_{0,0}^+\mathcal{T}_{0,0}^- Y_{1,1}^2 Y_{2,2}^2}, & |v| < 2g \\ \log\frac{\mathcal{T}_{1,0}^2}{\mathcal{T}_{0,0}^+\mathcal{T}_{0,0}^-}, & |v| > 2g \end{cases} \qquad \rho_c = \log\frac{\mathcal{T}_{0,0}^-}{\mathcal{T}_{0,0}^+}\left(\frac{\mathcal{T}_{1,1}^+ T_{1,1}^-}{T_{1,1}^+ \mathcal{T}_{1,1}^-}\right)^2$$

- We explicitly checked equation numerically for the Konishi state and got a complete agreement with the TBA approach.



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Data from Black box output

Presentation

In[94]:= `SolutionForKonishi[] // FullSimplify`

Out[93]= `No way!`

- We explicitly checked equation numerically for the Konishi state and got a complete agreement with the TBA approach.

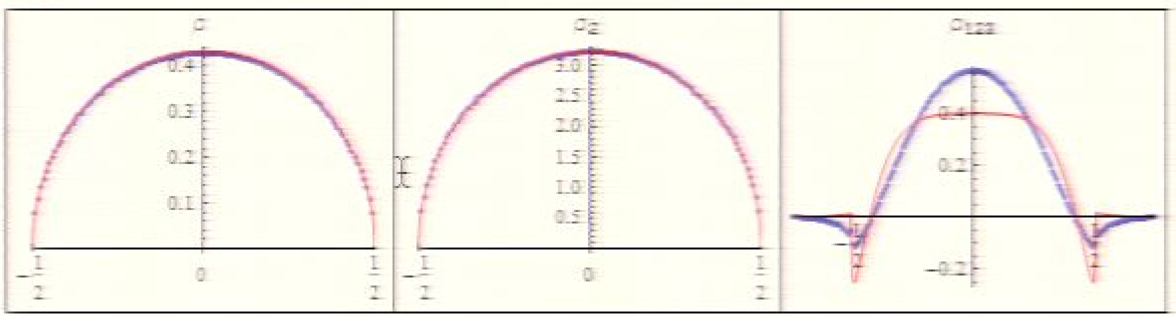
Data from Black box output

Presentation

In[94]:= SolutionForKonishi[] // FullSimplify

g=0.25

Out[94]=



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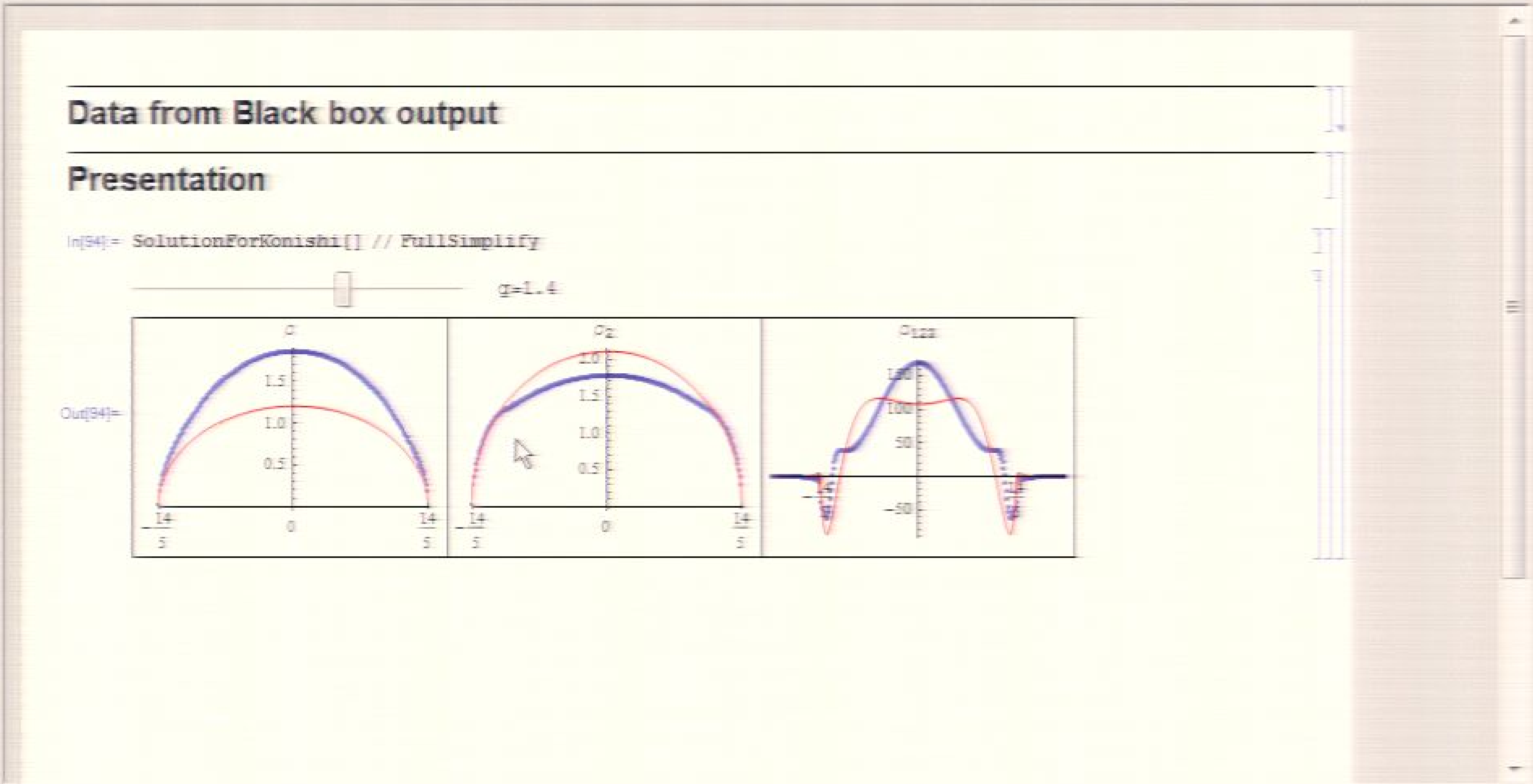

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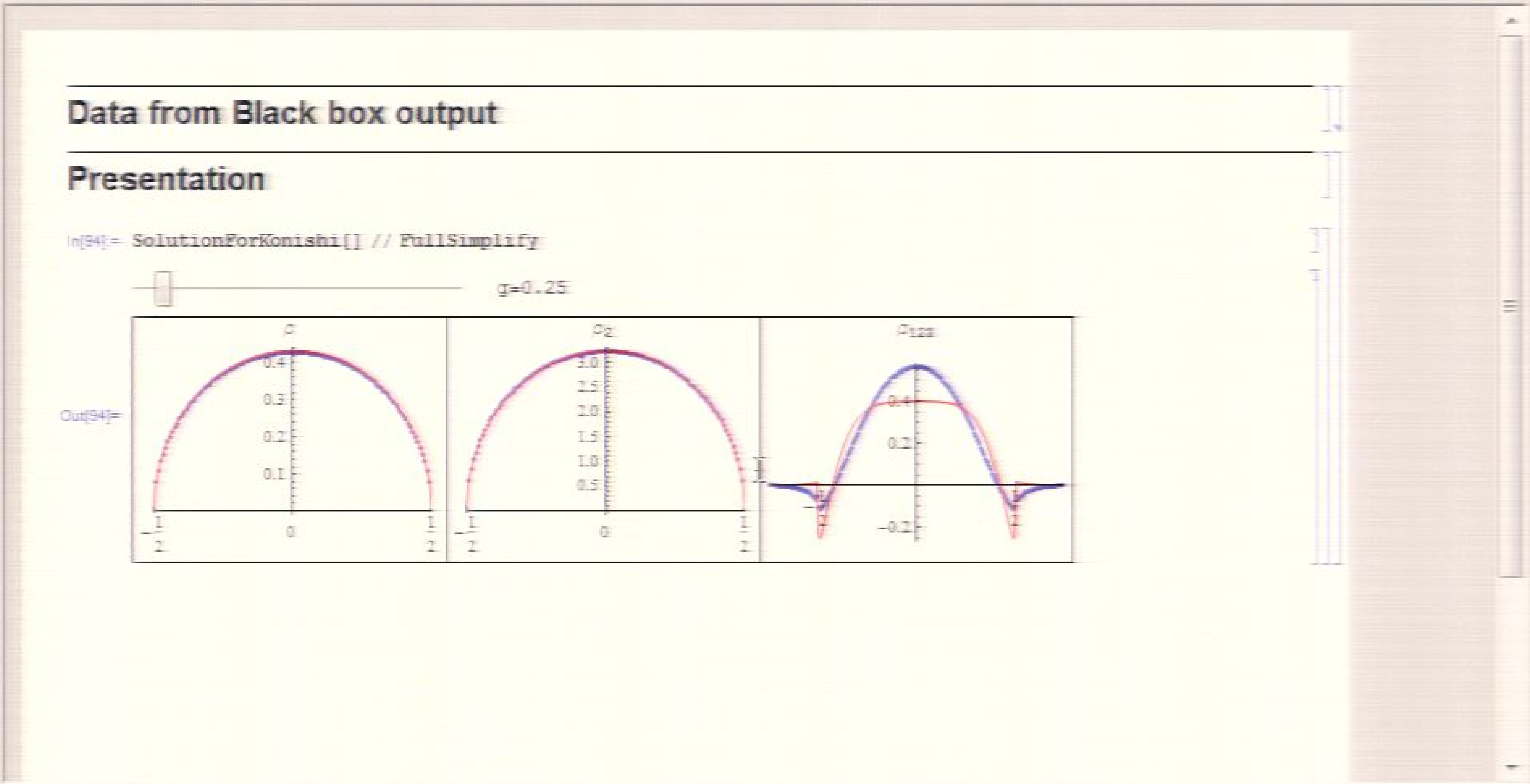

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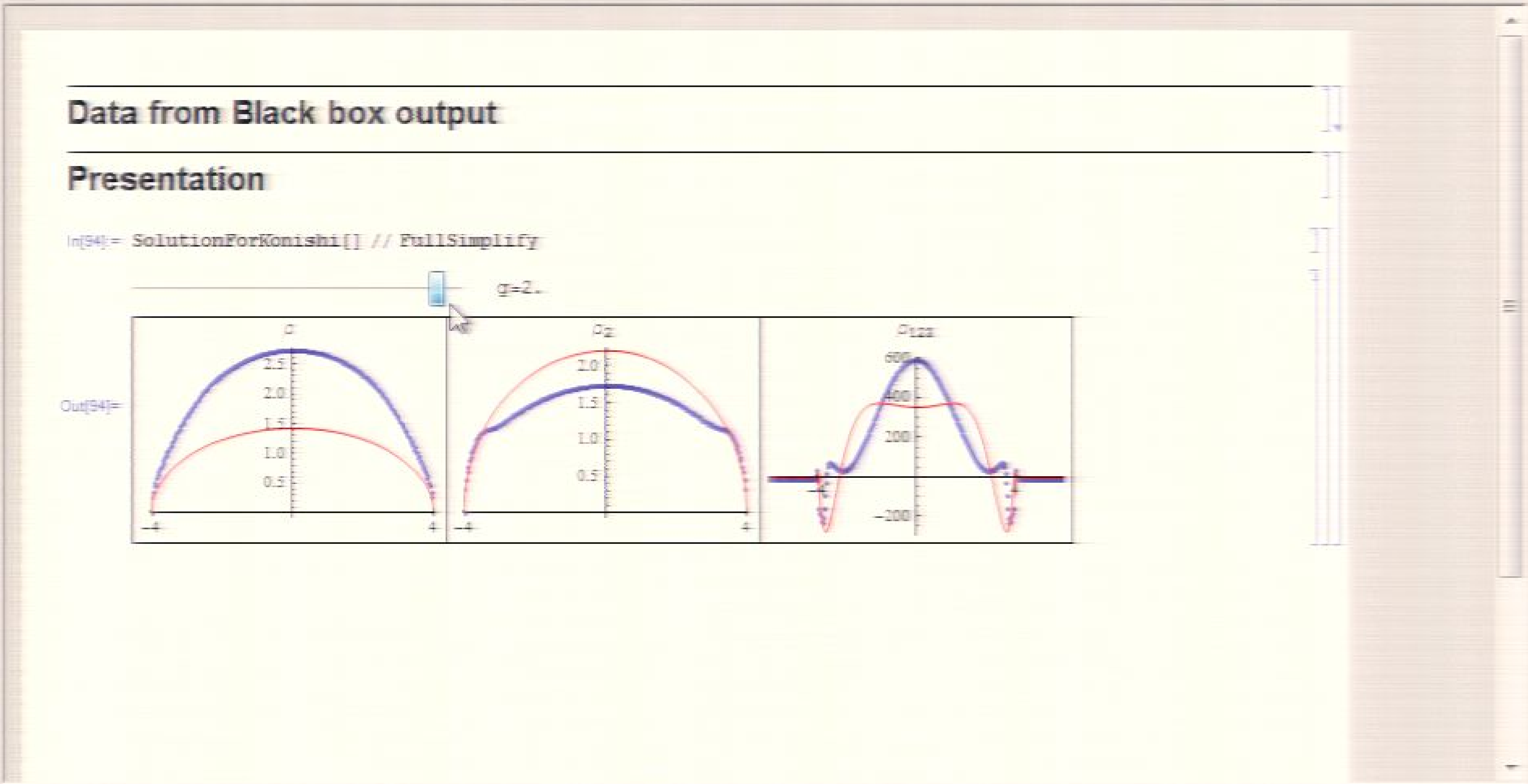

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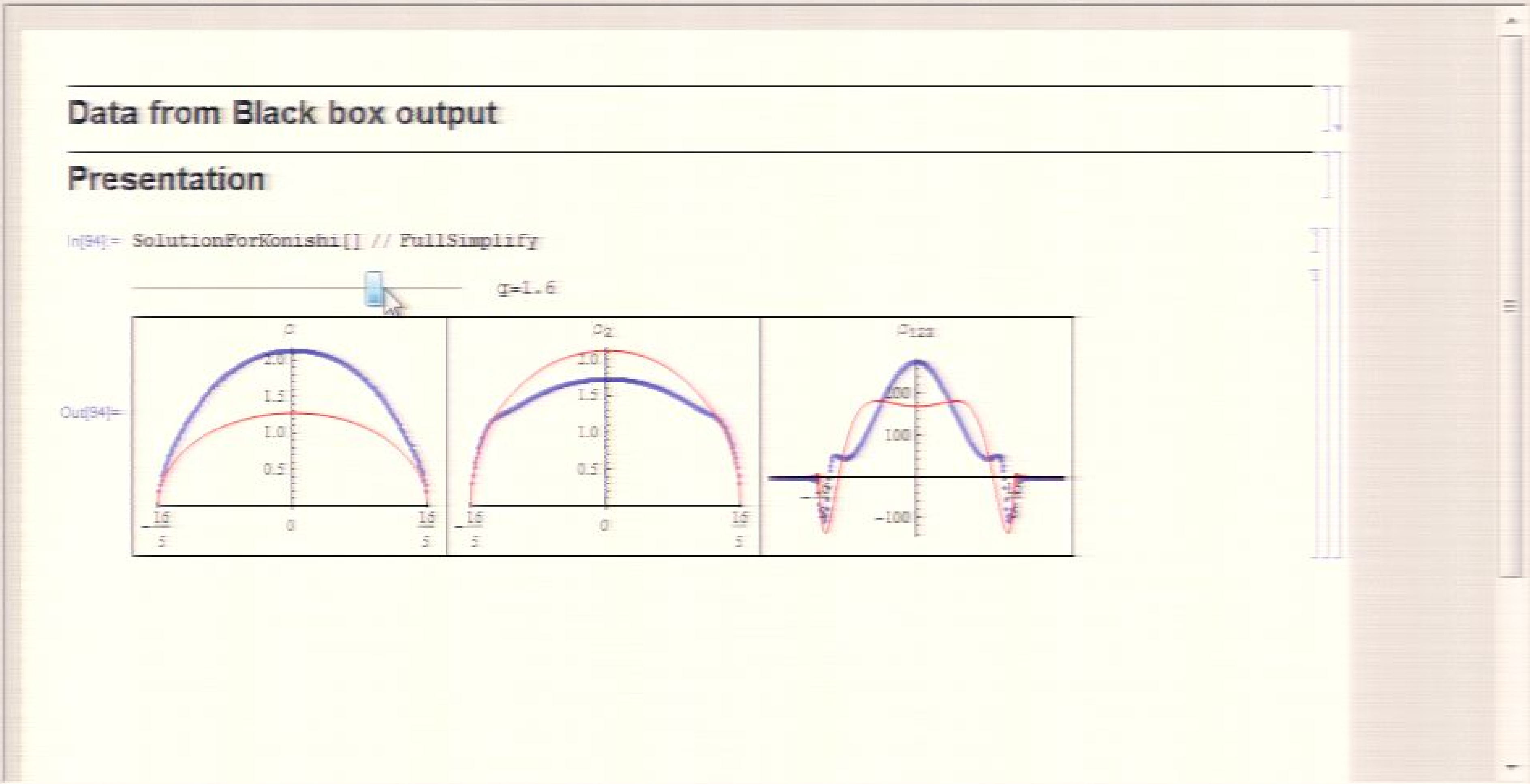

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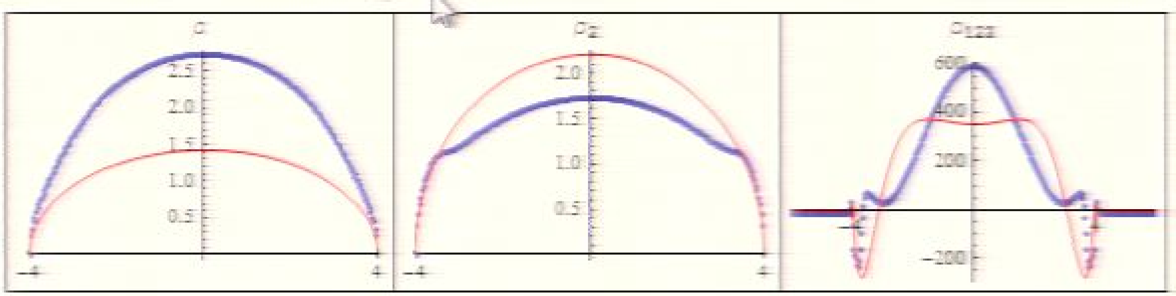
Data from Black box output

Presentation

In[94]:=
```
SolutionForKonishi[] // FullSimplify
```

q=2.

Out[94]=



Discussion

Transfer matrices
(classical level only)

→

Symmetry
(Hirota+det=1+Z4)
+Analyticity
+Poles/Zeroes
/Asymptotics

→

FiNLIE

→

Konishi – ok
Some systematics now?

Discussion

Transfer matrices (classical level only) → Symmetry (Hirota+det=1+Z4) +Analyticity +Poles/Zeroes /Asymptotics → FiNLIE → Konishi – ok Some systematics now?

Approaching now to the systematic study:

- Weak coupling (e.g transcendentality structure)
- Strong copuling (asymptotic? Borel summable?)
- BFKL?

Discussion

Transfer matrices (classical level only) → Symmetry (Hirota+det=1+Z4) +Analyticity +Poles/Zeroes /Asymptotics → FiNLIE → Konishi – ok Some systematics now?

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- Weak coupling (e.g transcendentality structure)
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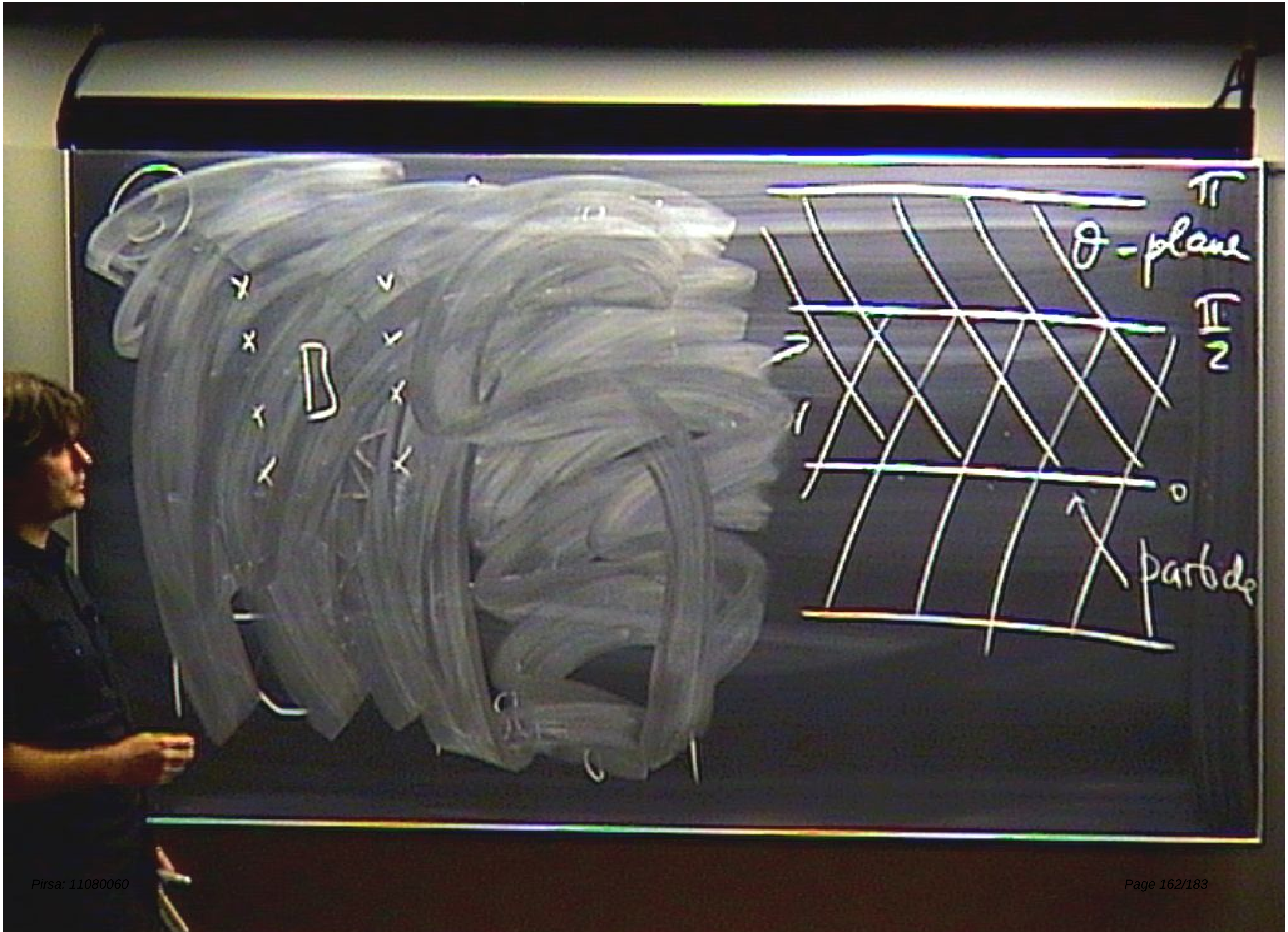
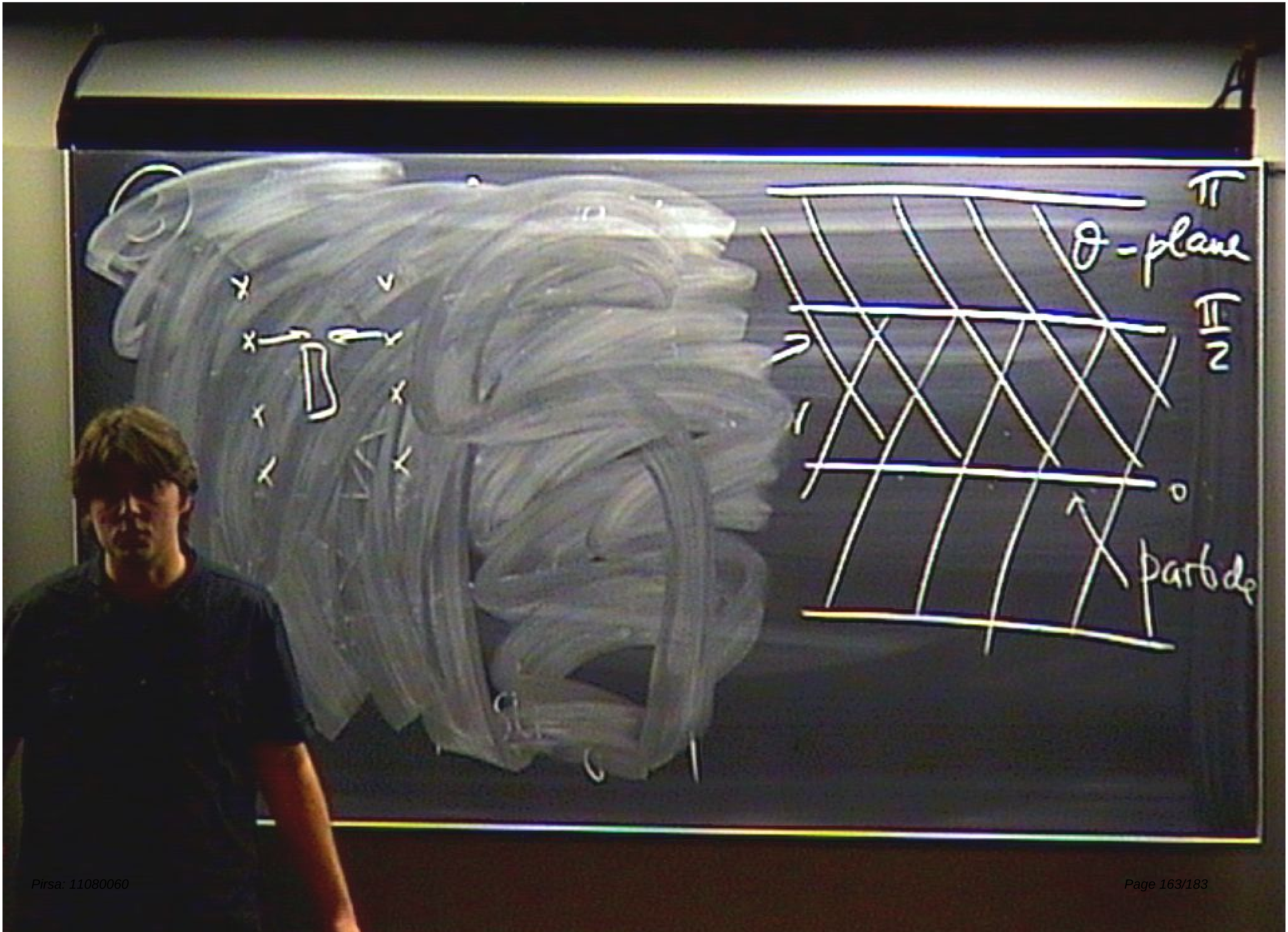
Need to define transfer matrices at weak coupling!


π
θ-plane
π/2
0
particle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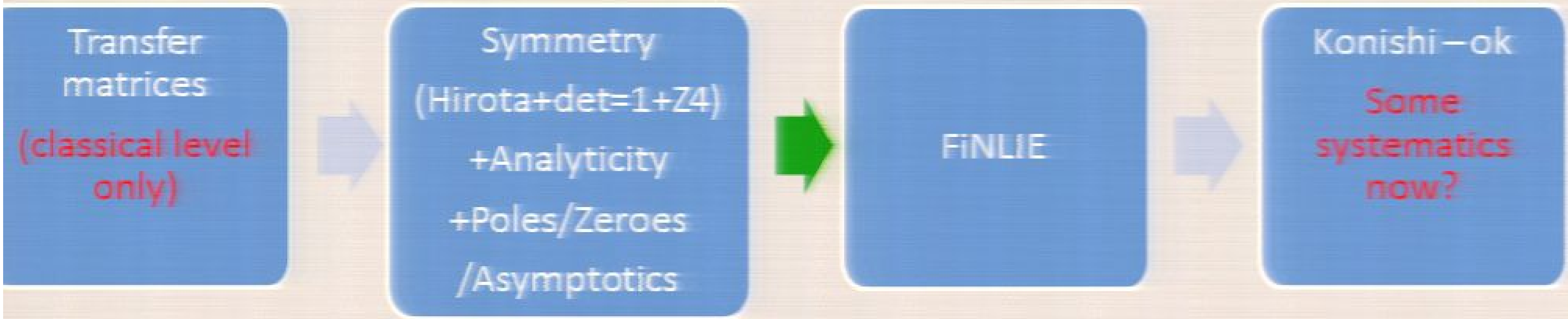

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- Weak coupling (e.g transcendentality structure)
- Strong copuling (asymptotic? Borel summable?)
- BFKL?

Need to define transfer matrices at weak coupling!

Need to quantize transfer matrices at strong coupling (see next talk)

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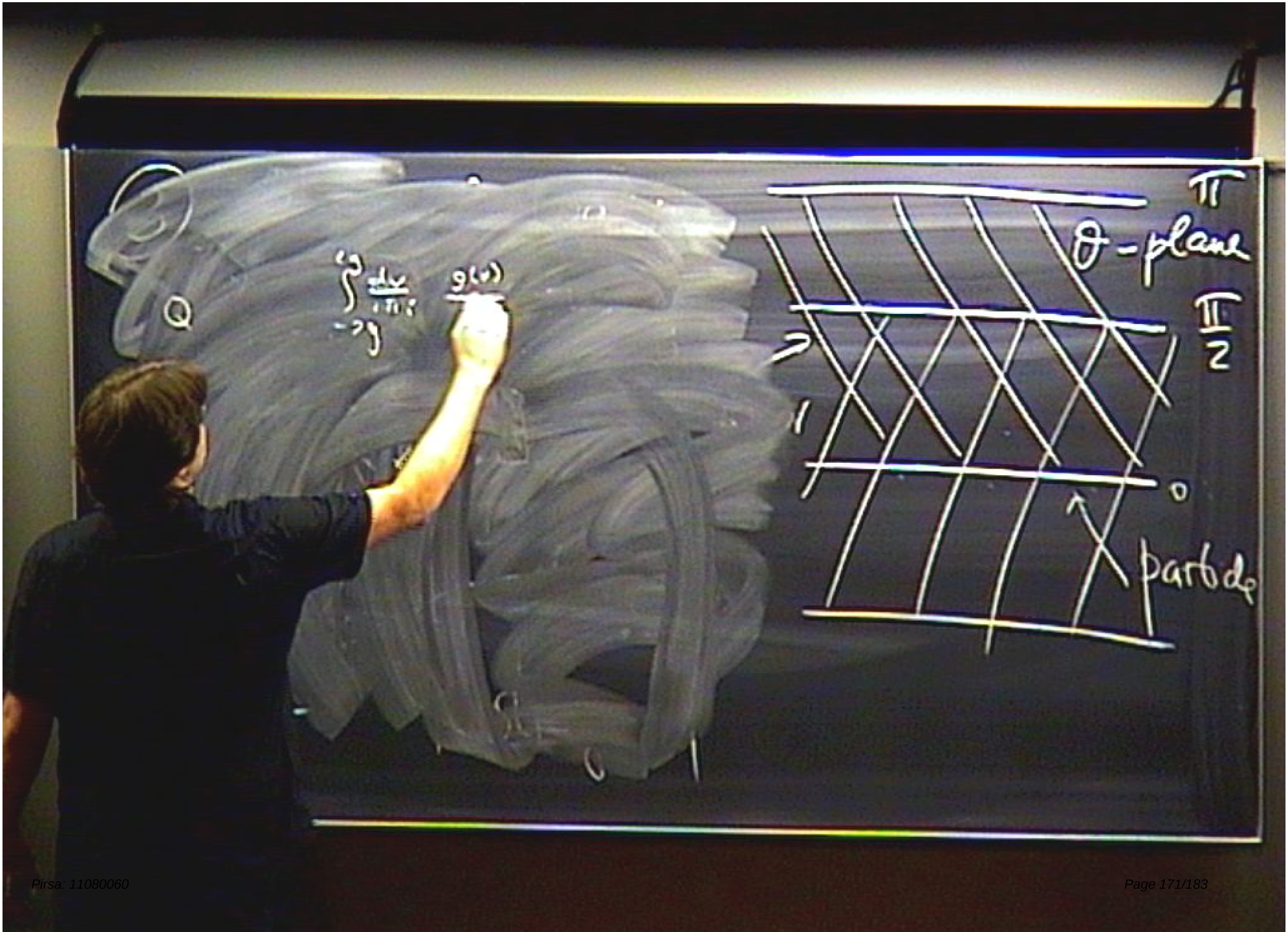
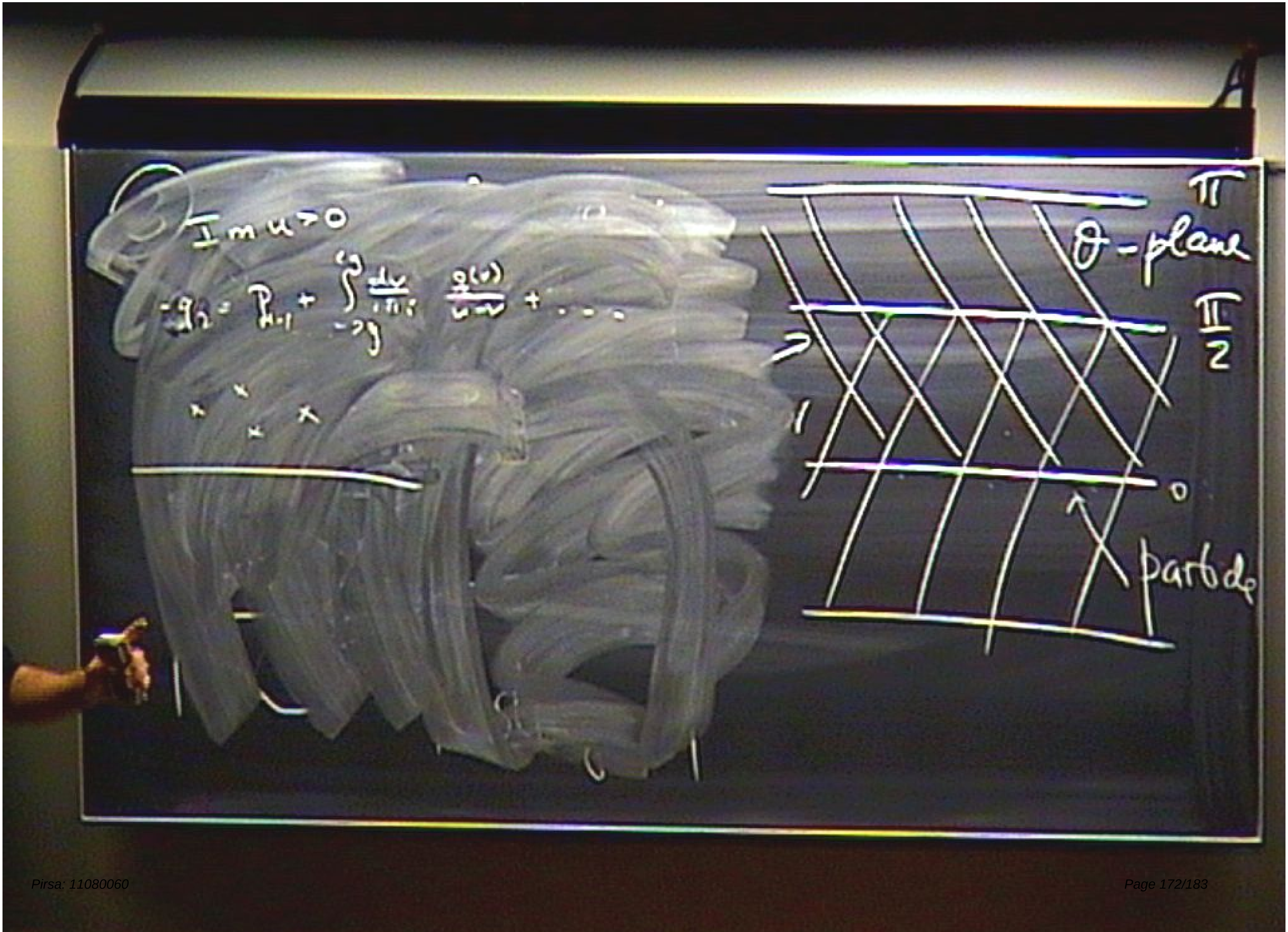

π
θ-plane
π/2
0
particle
Q


Im u > 0
θ-plane
π
π/2
0
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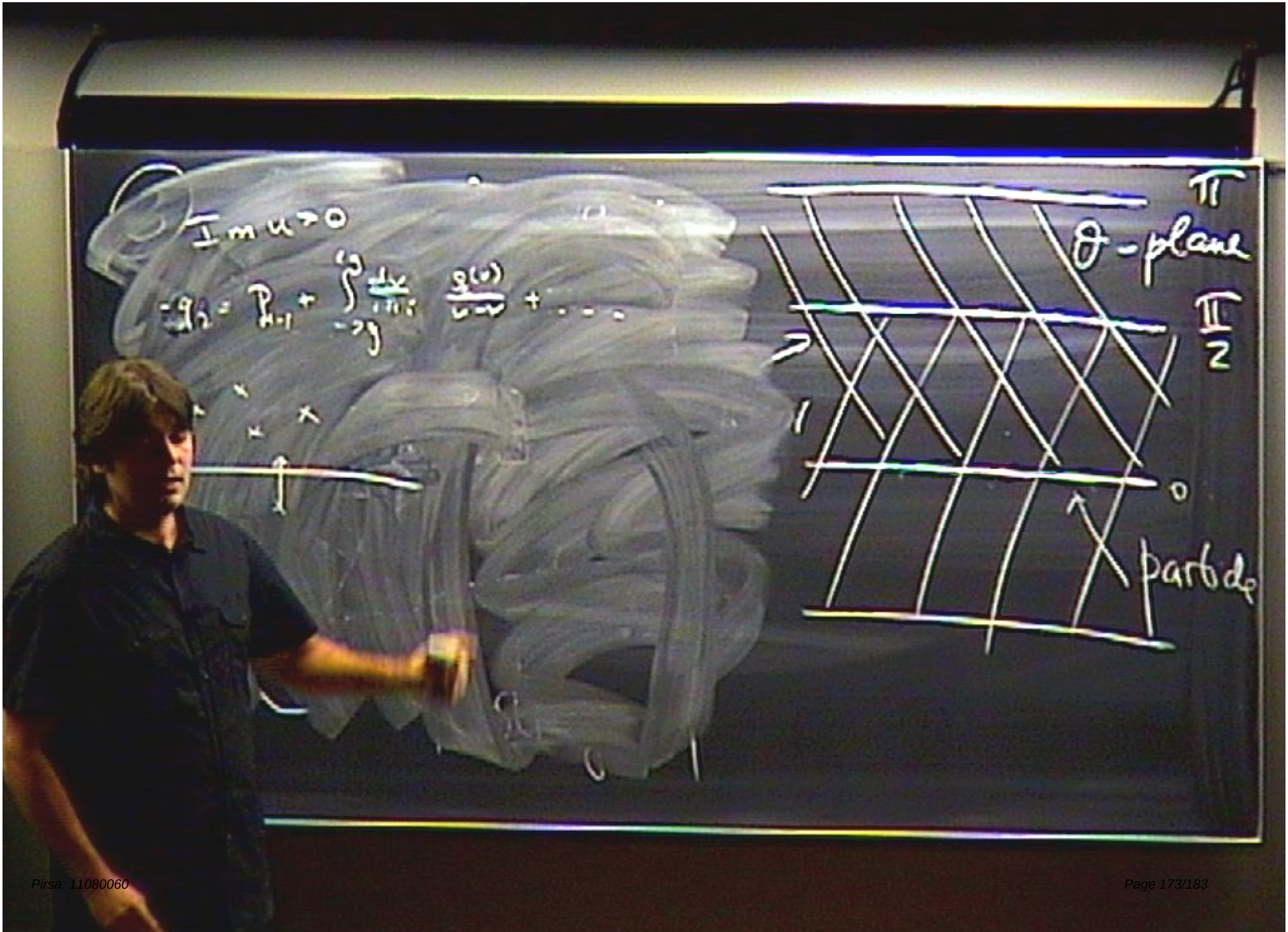
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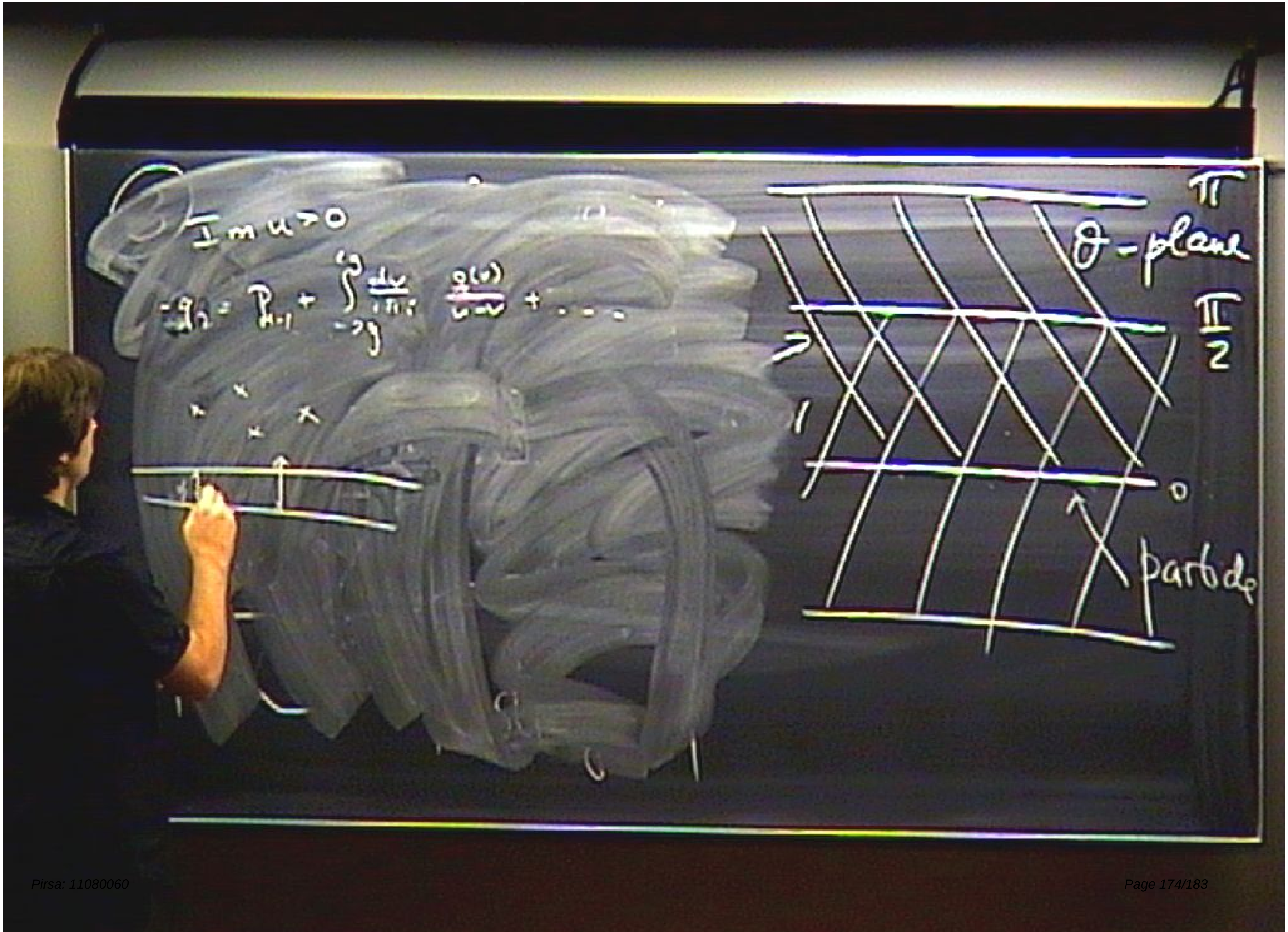
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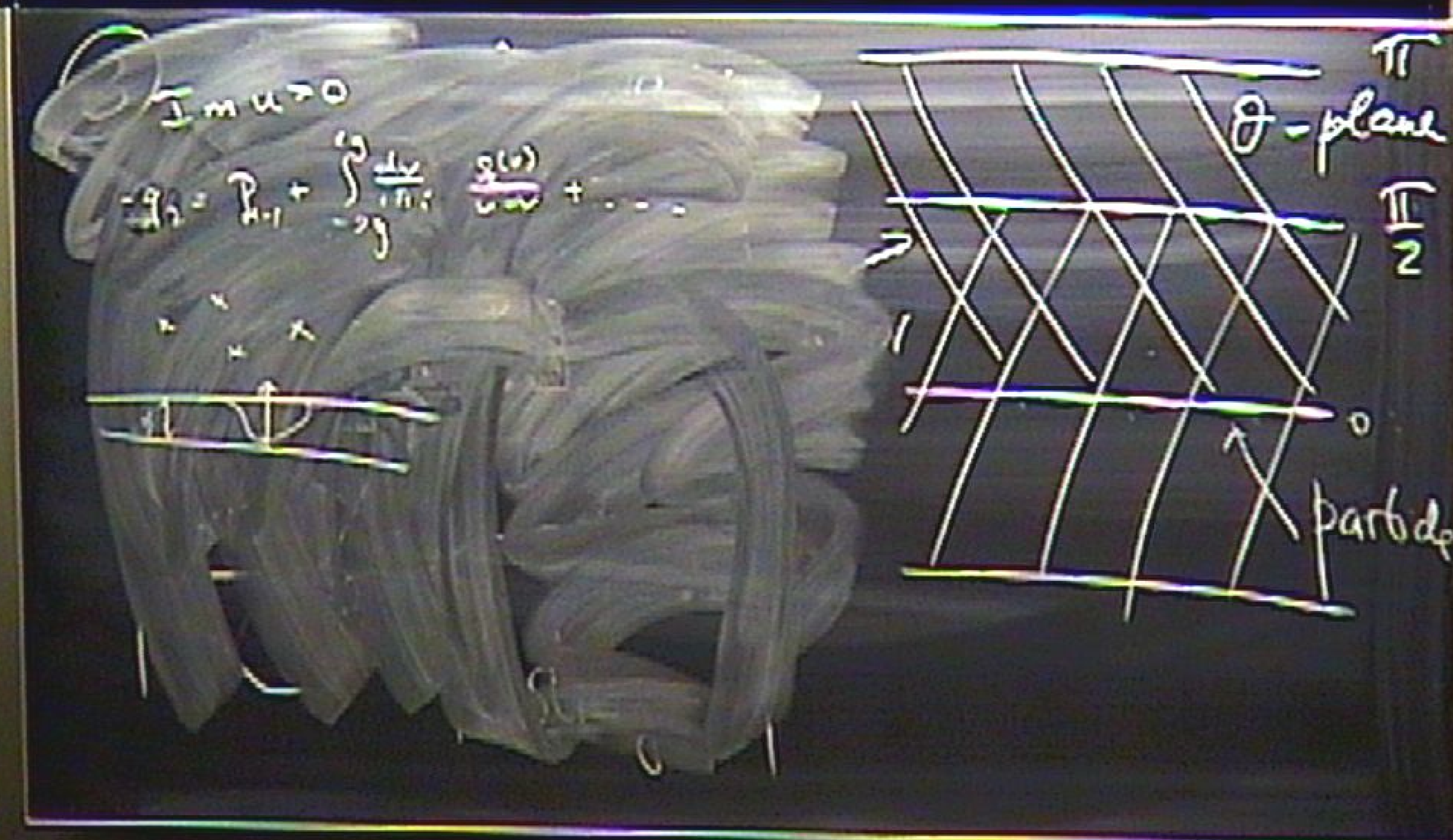
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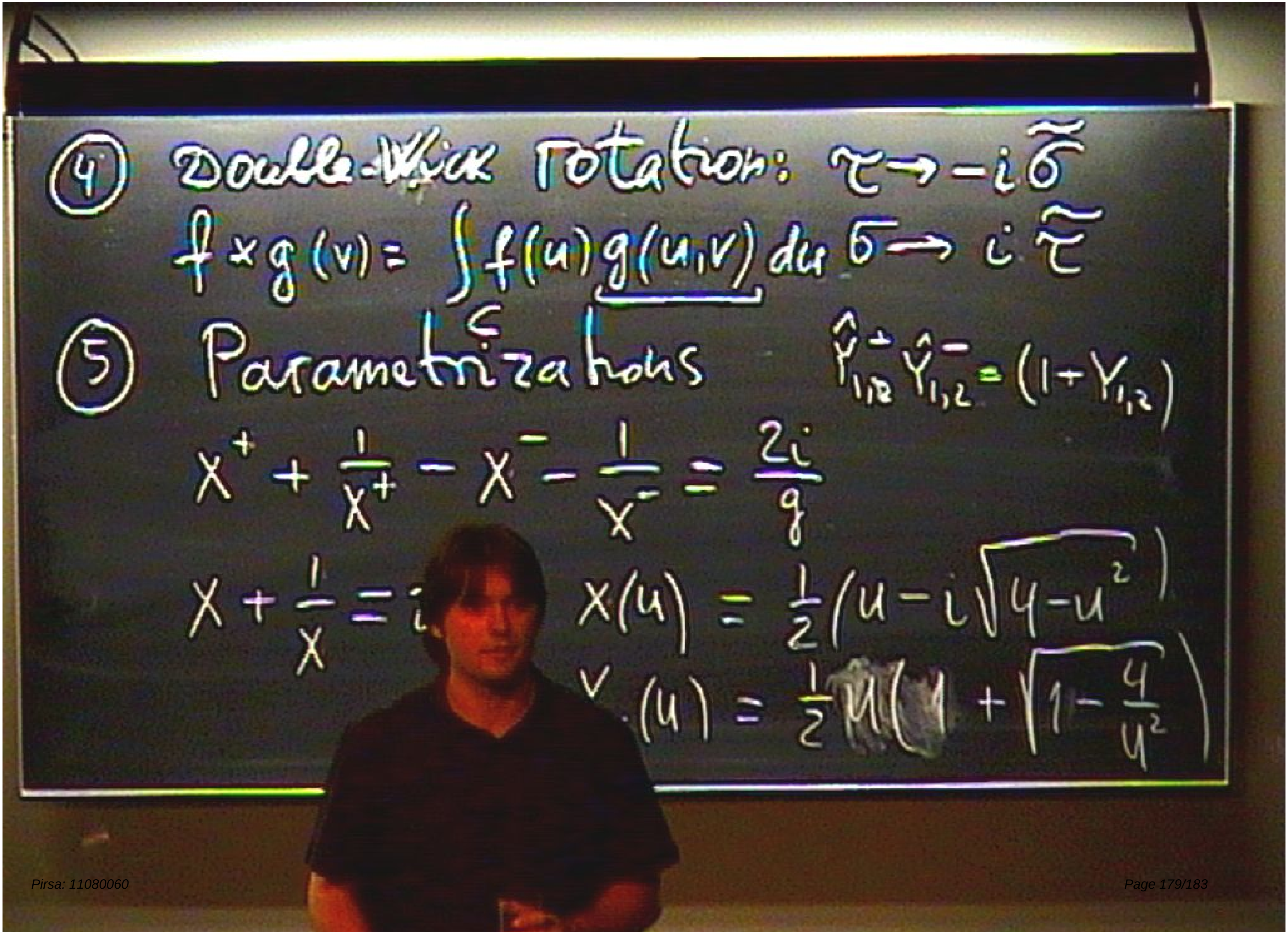
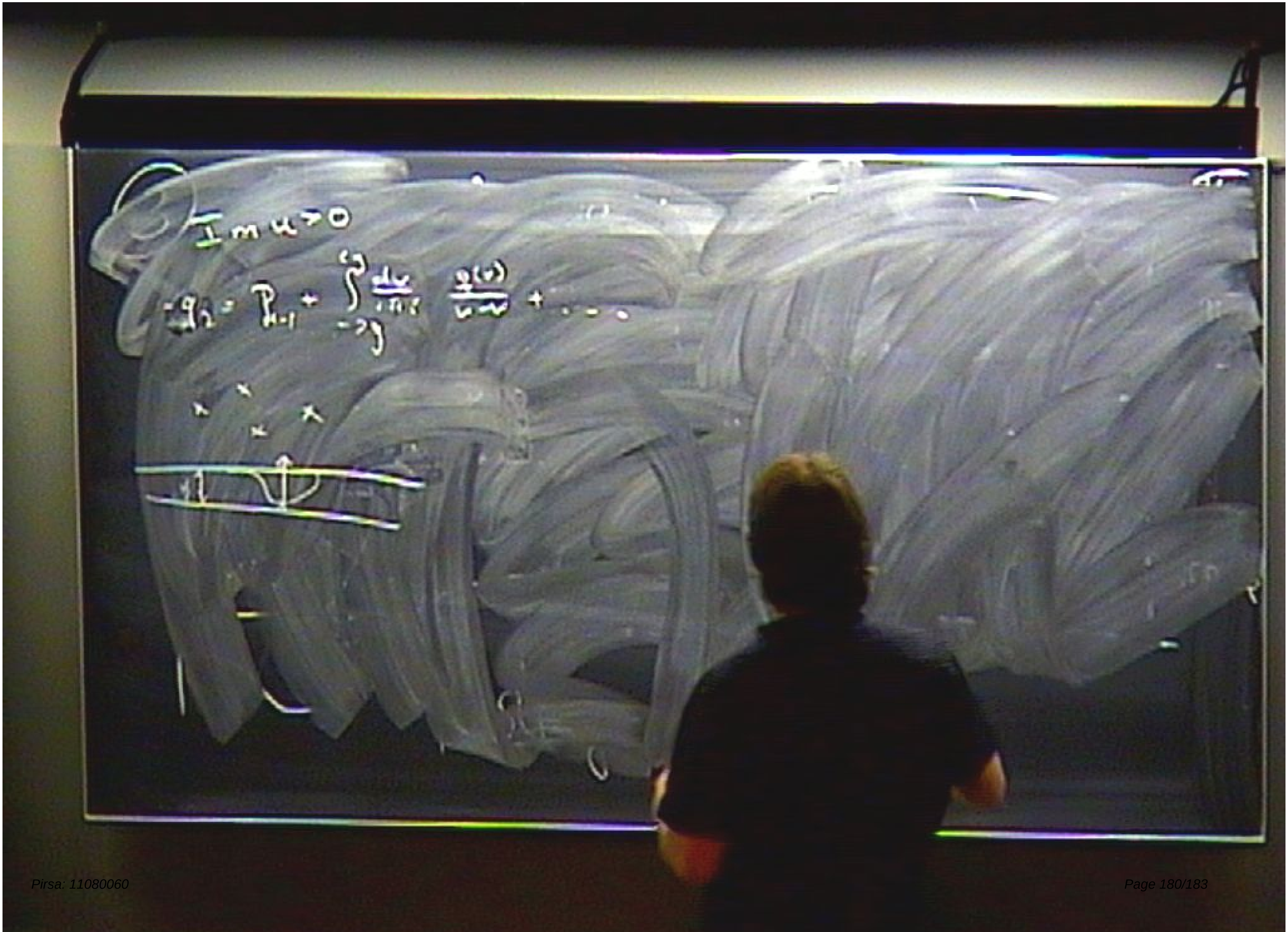
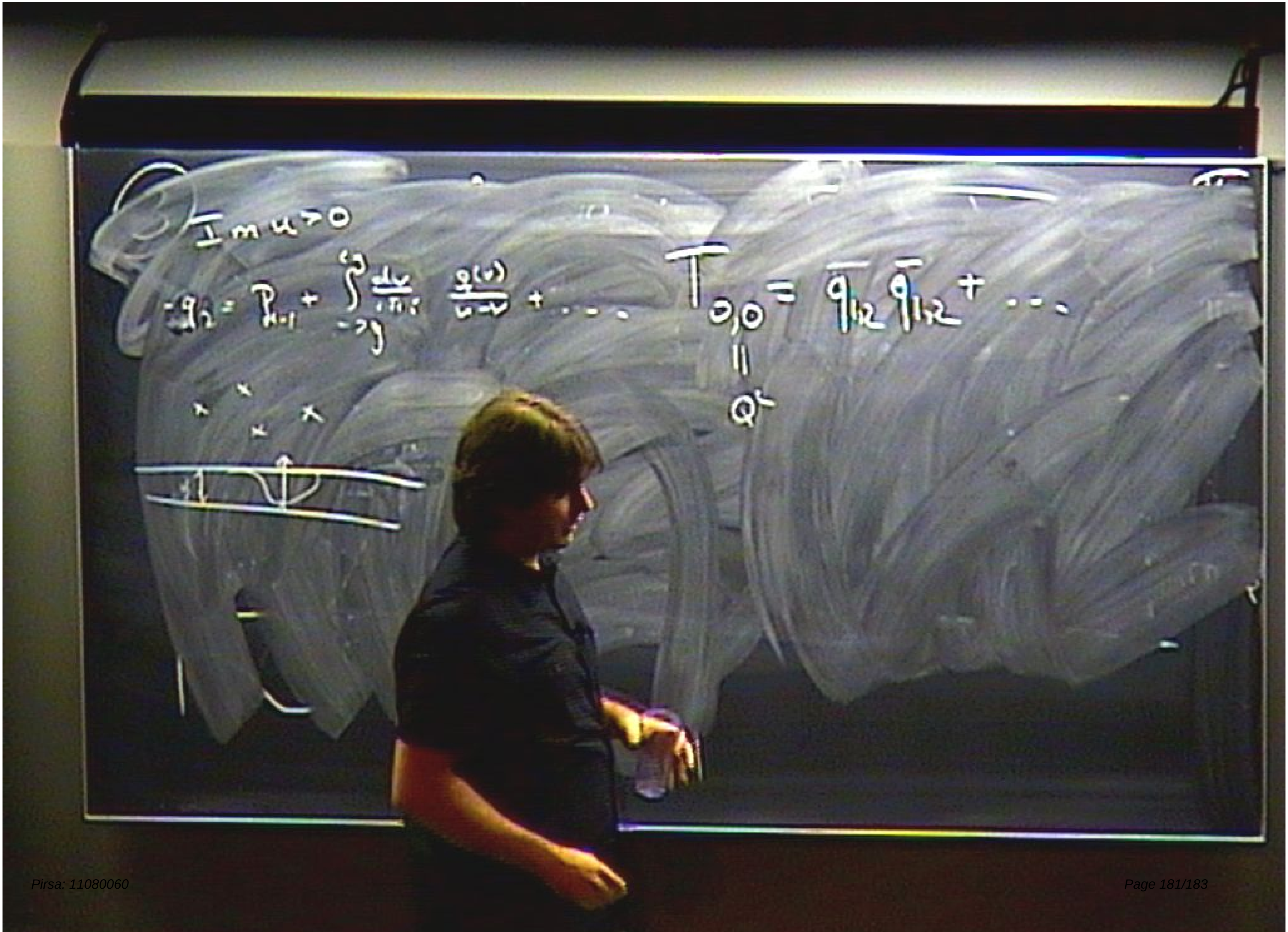
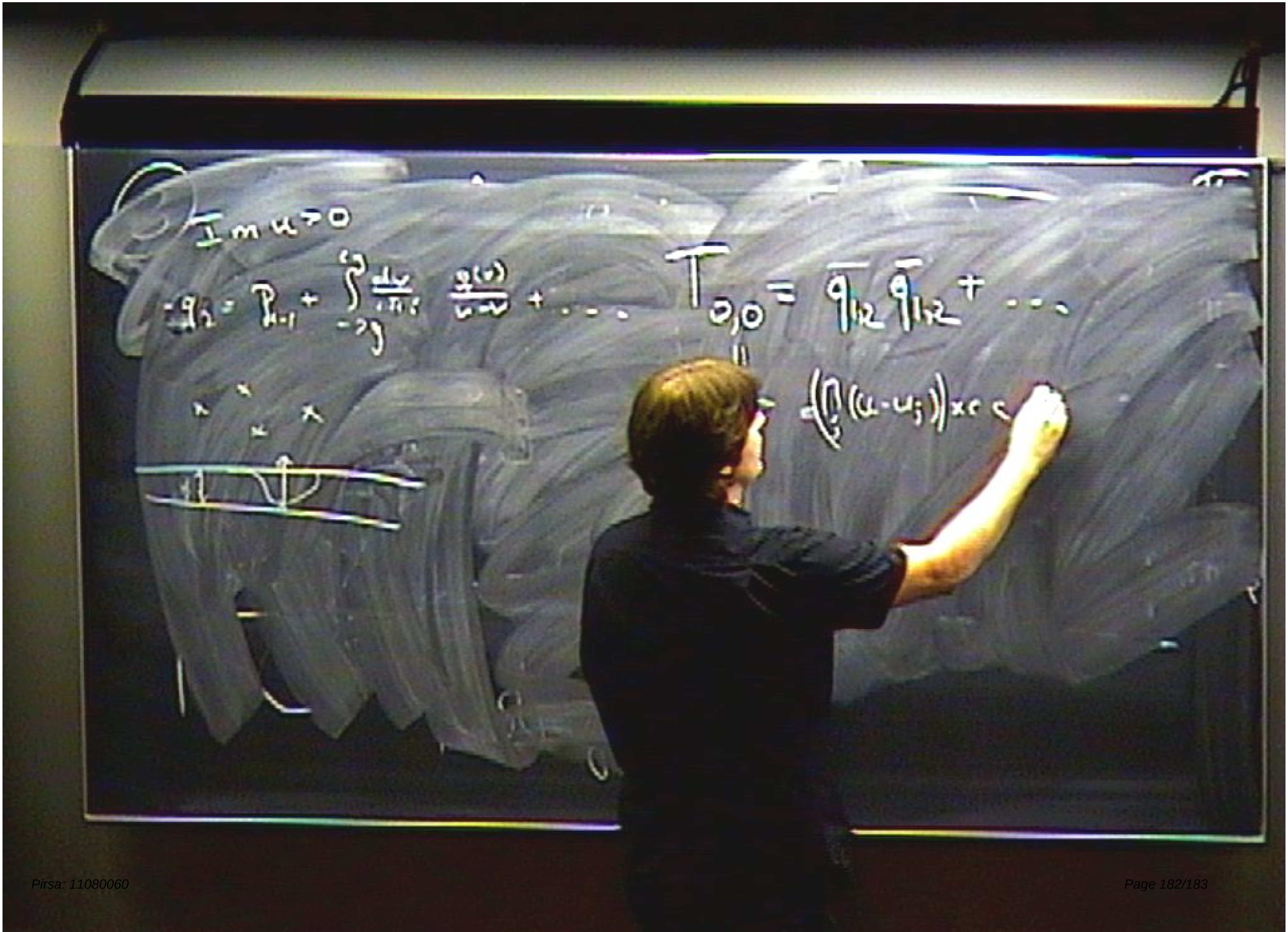

④ Double-Wick rotation: τ → −i σ̃
f * g(v) = ∫ f(u) g(u,v) du   σ → i τ̃
⑤ Parametrizations
X⁺ + 1/X⁺ − X⁻ − 1/X⁻ = 2i/g
X + 1/X = i
x(u) = ½ (u − i √(4 − u²))
(u) = ½ u (1 + √(1 − 4/u²))


Im u > 0


Im u > 0
T0,0 = q1/2 q1/2 + . . .



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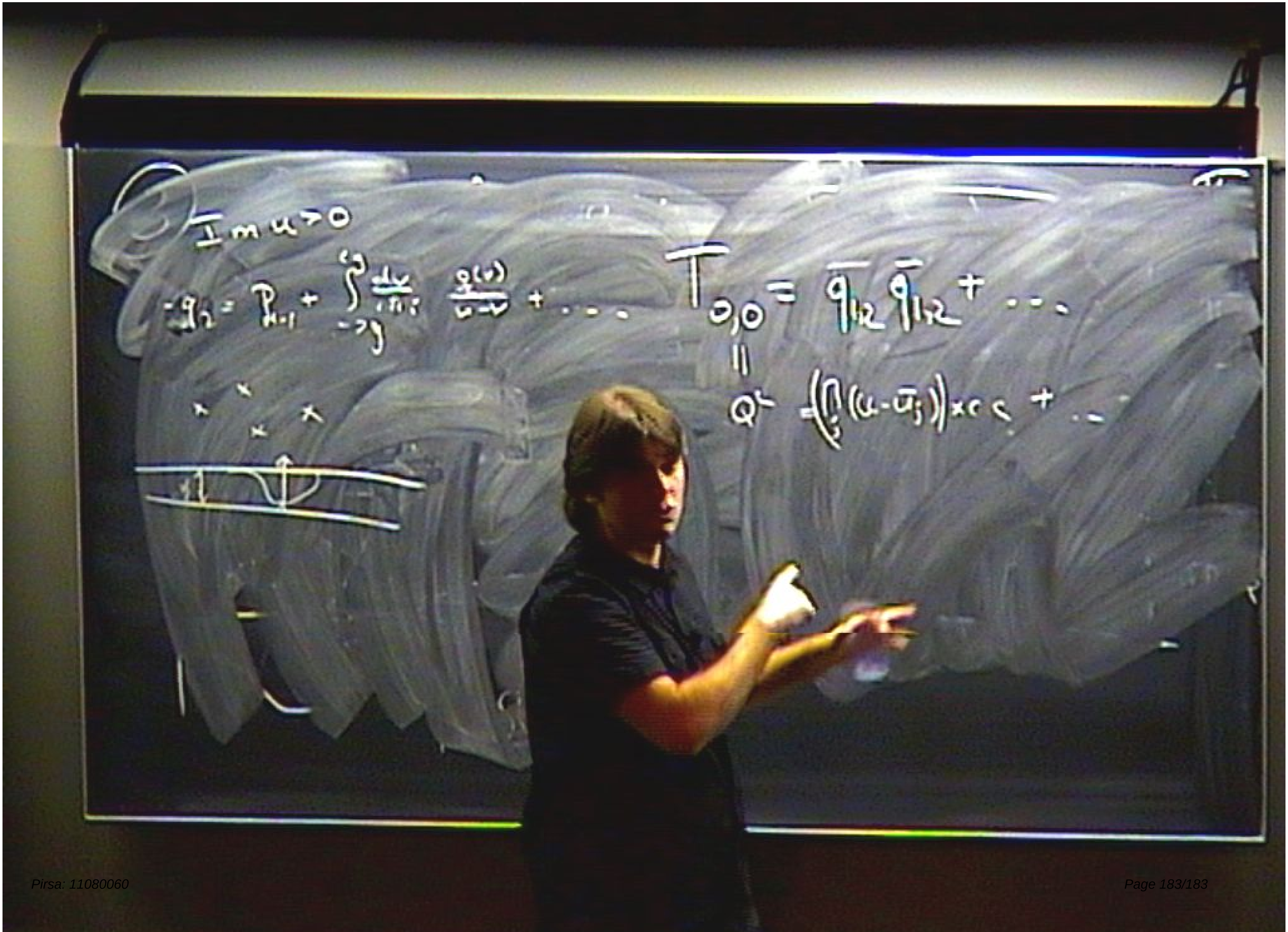
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