

Title: Fermionic Basis of Local Fields in the Sine-Gordon Model

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Abstract: In this talk we give a survey of recent developments concerning the fermionic structure in the sine-Gordon model. For the lattice counterpart (6 vertex model), we introduce fermions acting on the space of (quasi) local operators. The main theorem is a determinant formula for the expectation values of fermionic descendants of primary fields. In the continuum limit this construction gives rise to a basis of the space of all descendant fields, whose expectation values take a very simple form. Unexpectedly, it turns out that the action of our fermions on form factors coincides with yet another fermions which have been introduced some time ago by Babelon, Bernard and Smirnov.

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Michio Jimbo (Rikkyo University, Japan)

Integrability in Gauge/String Theories  
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- ▶ 6 vertex model and expectation values

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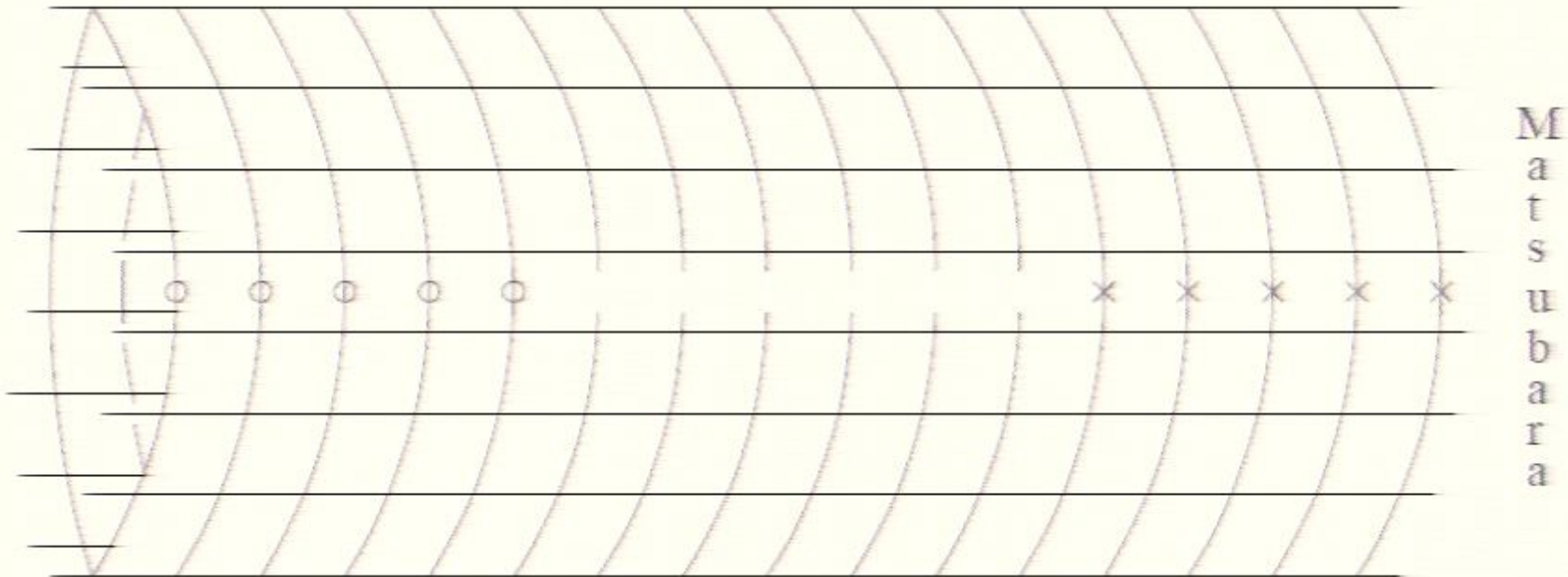
## Fermionic basis on the lattice

We consider a 6 vertex model on a cylinder

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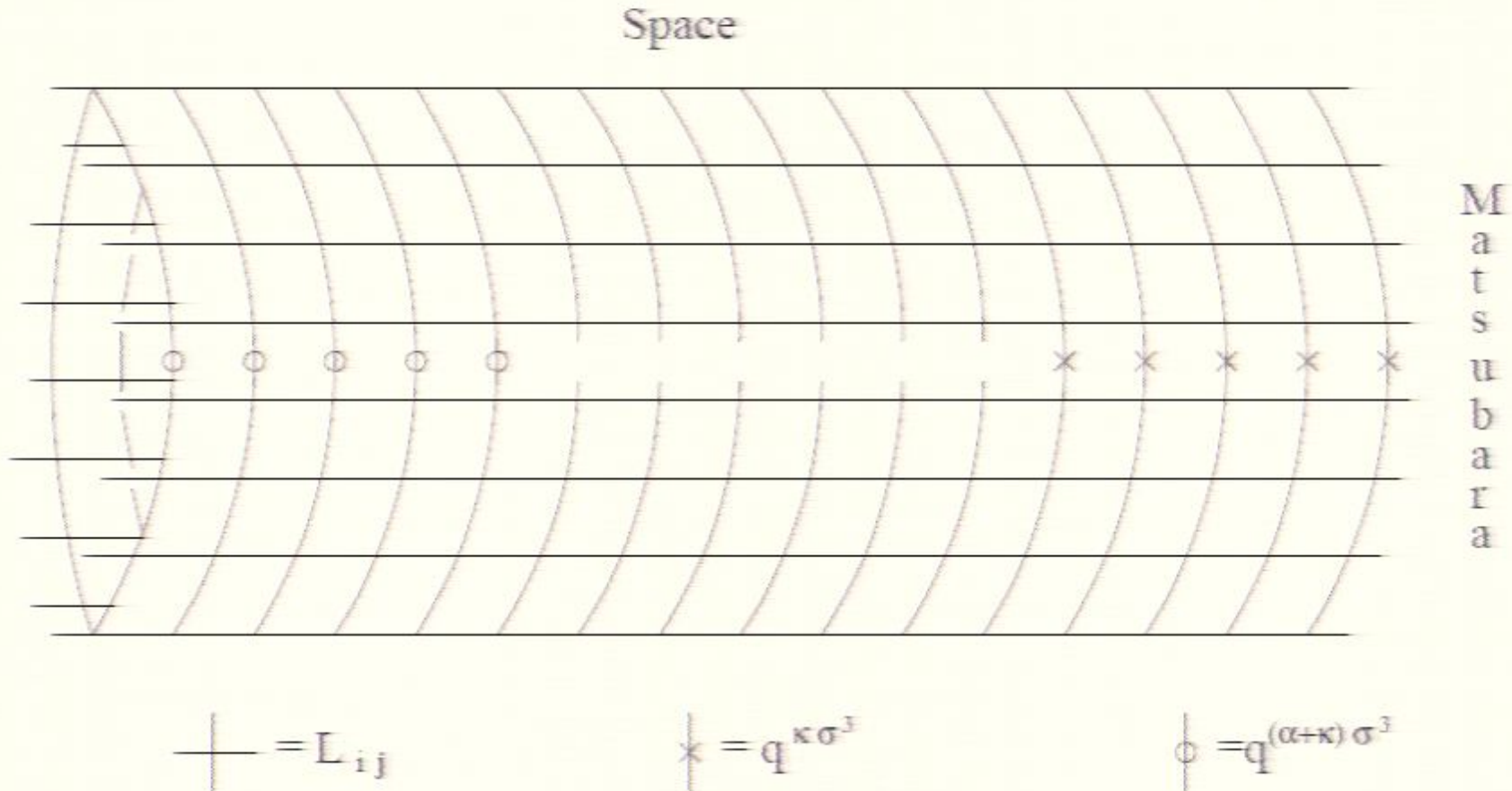
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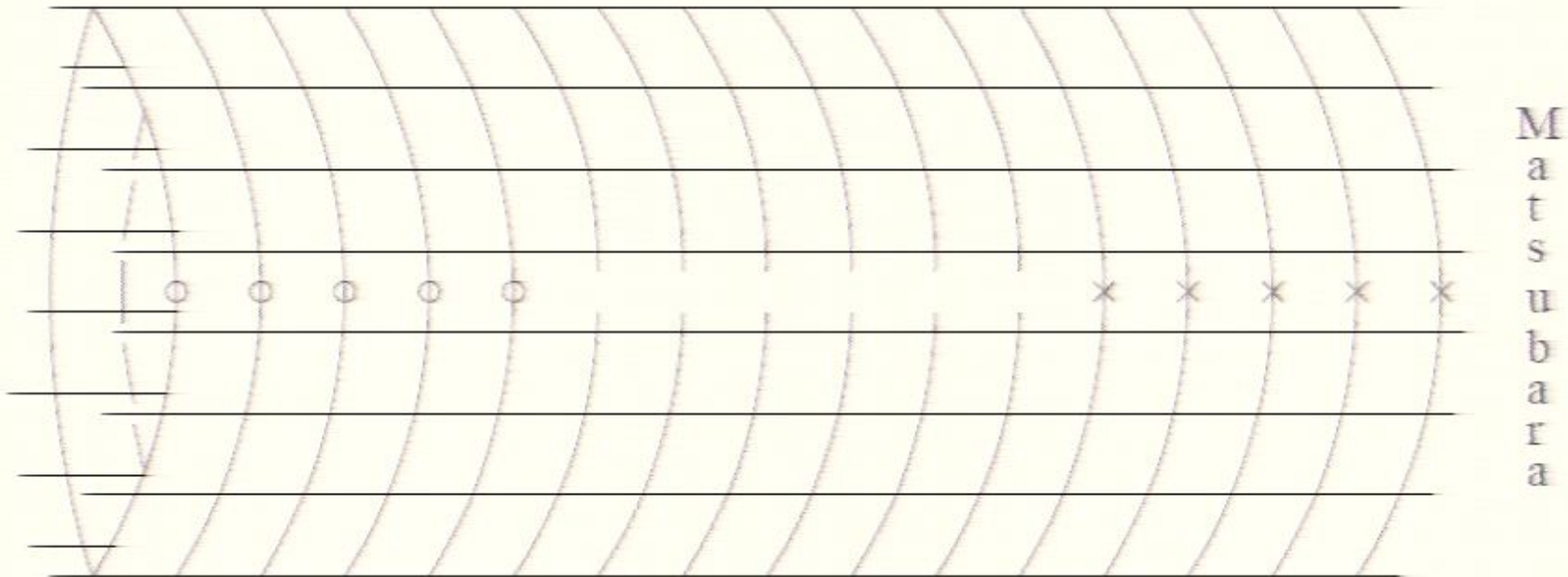




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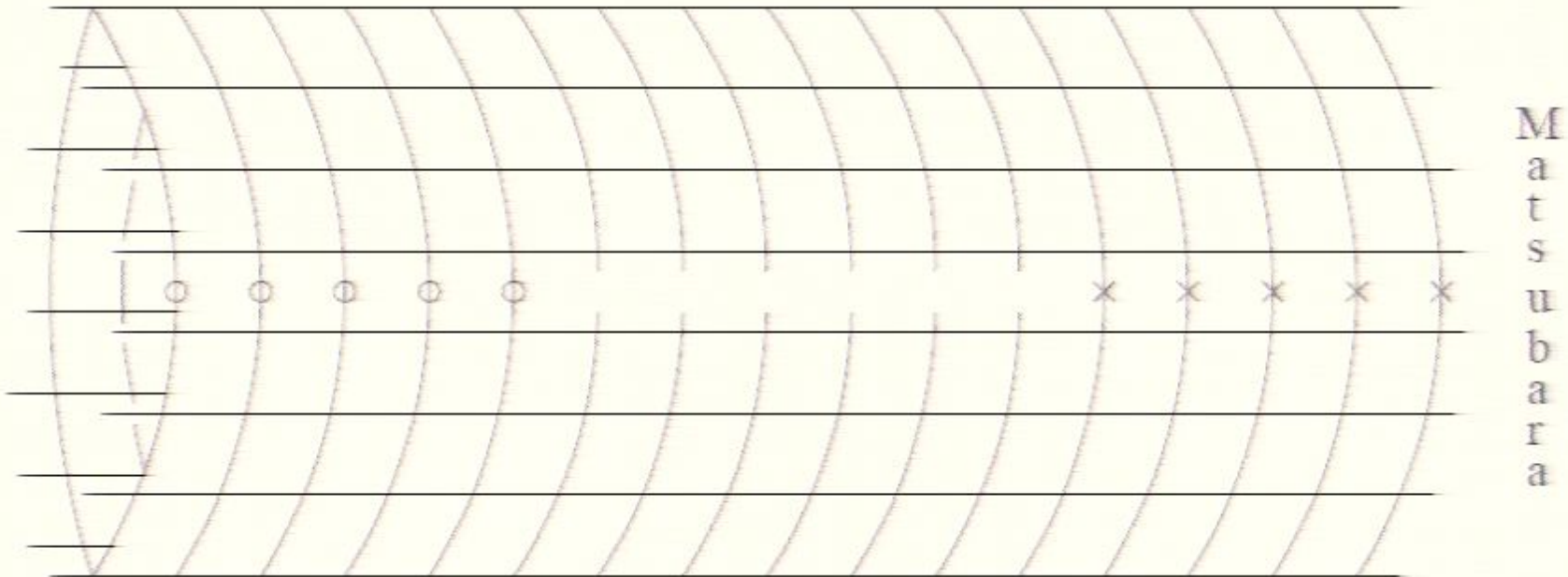
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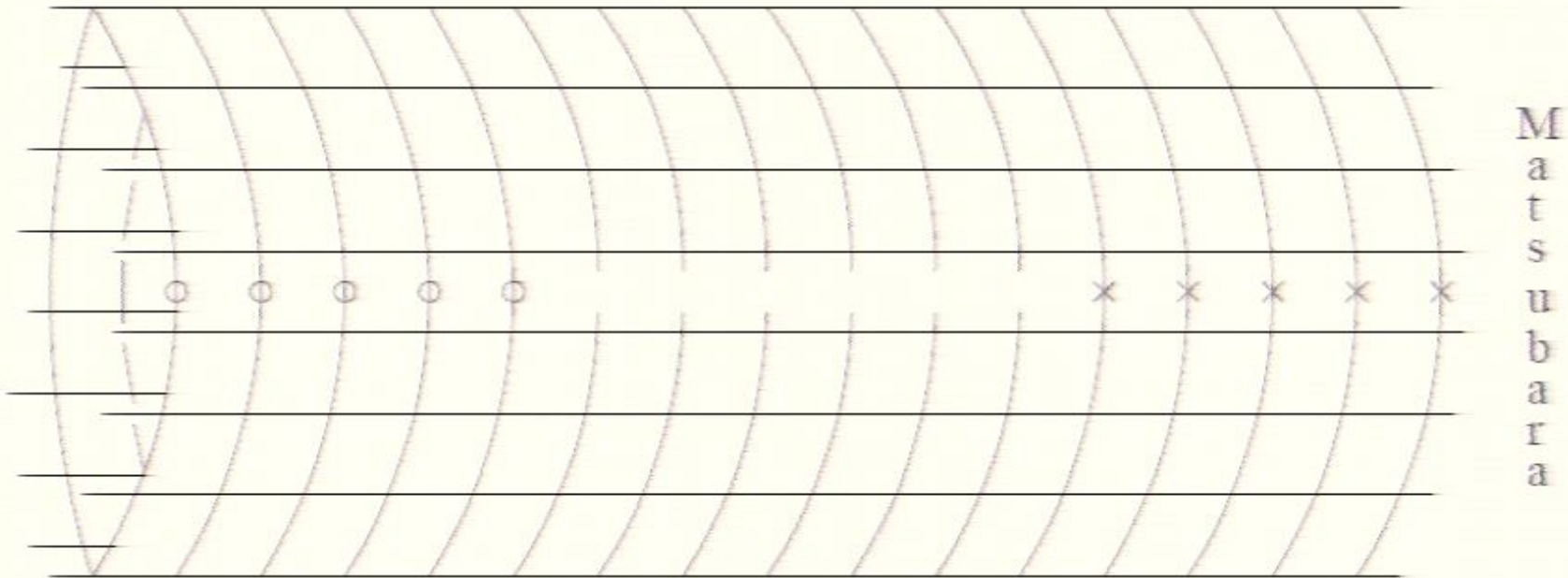
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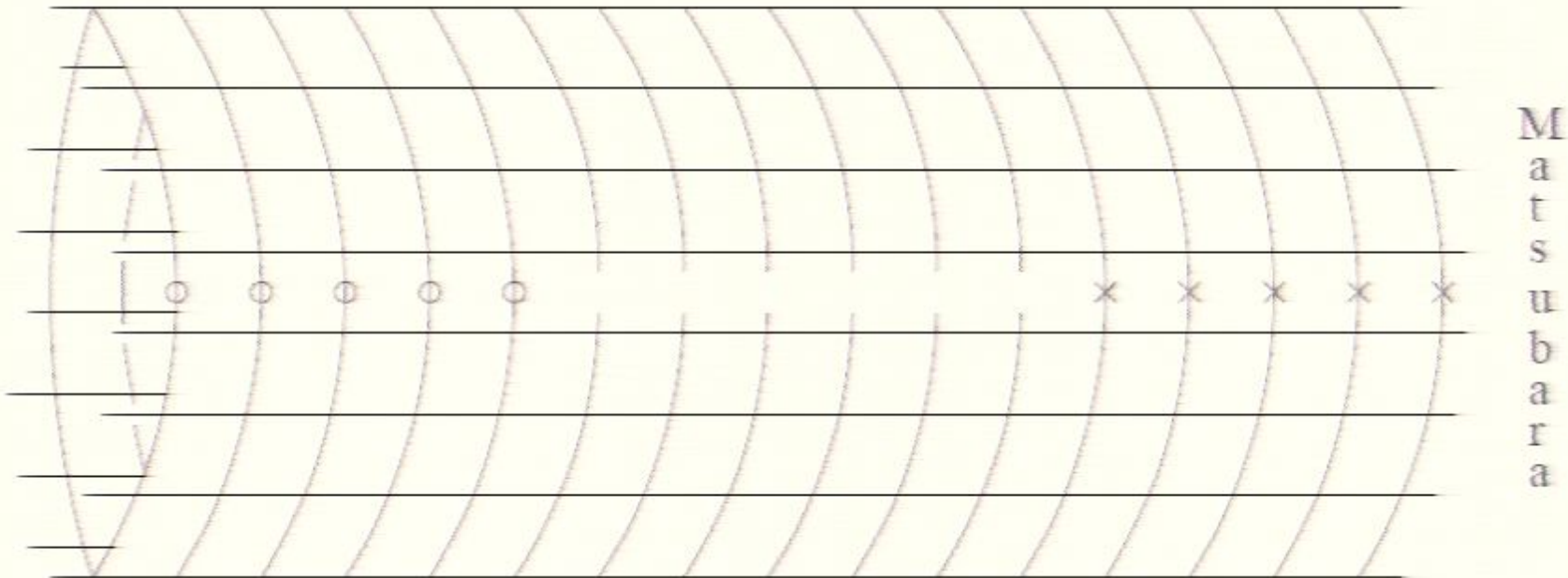
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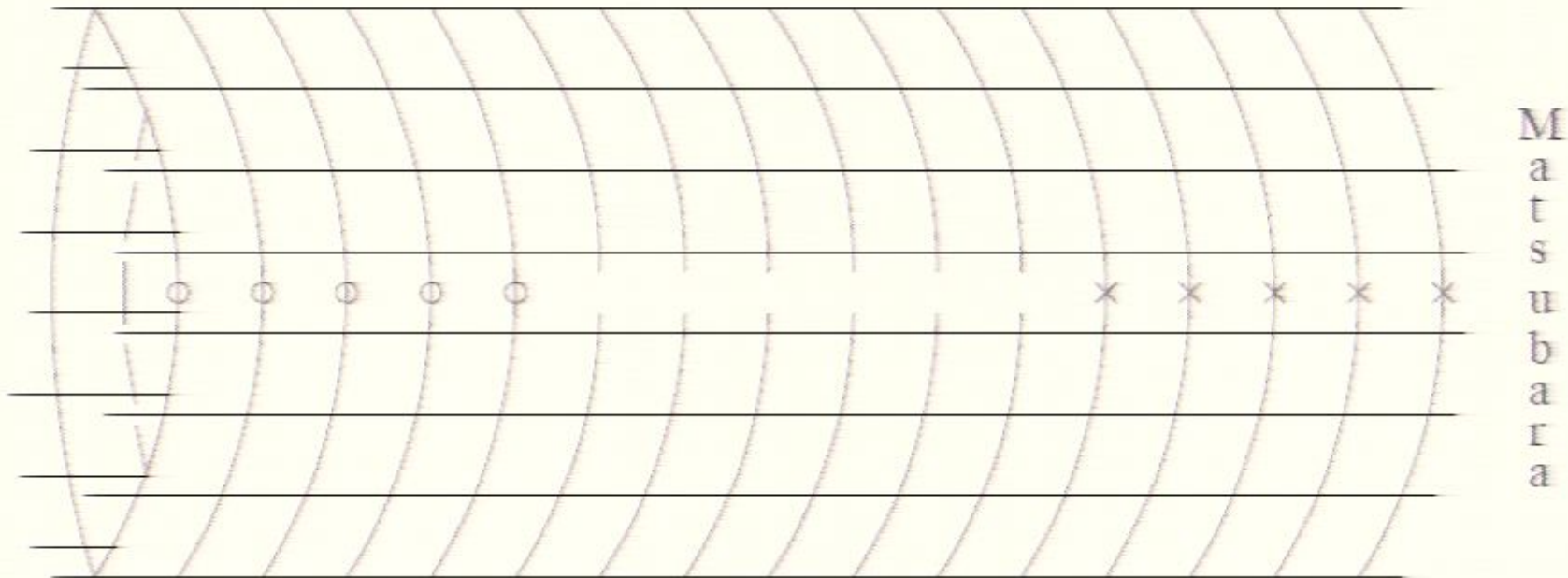
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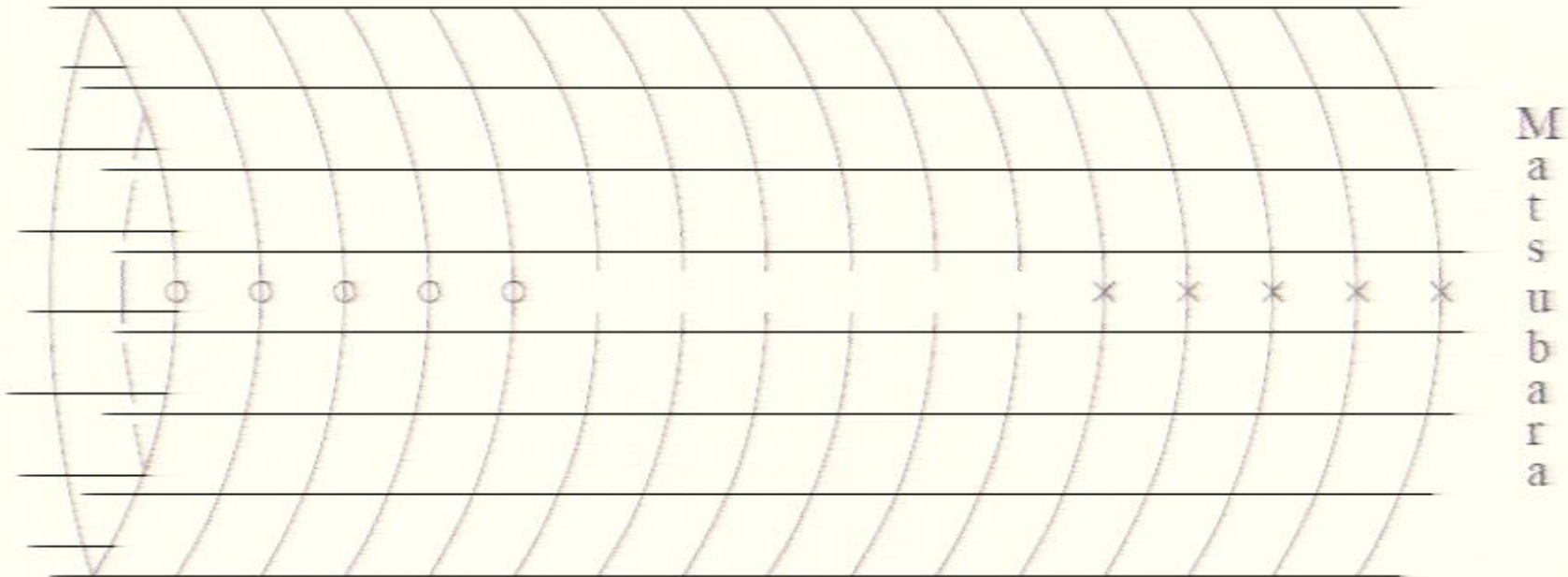
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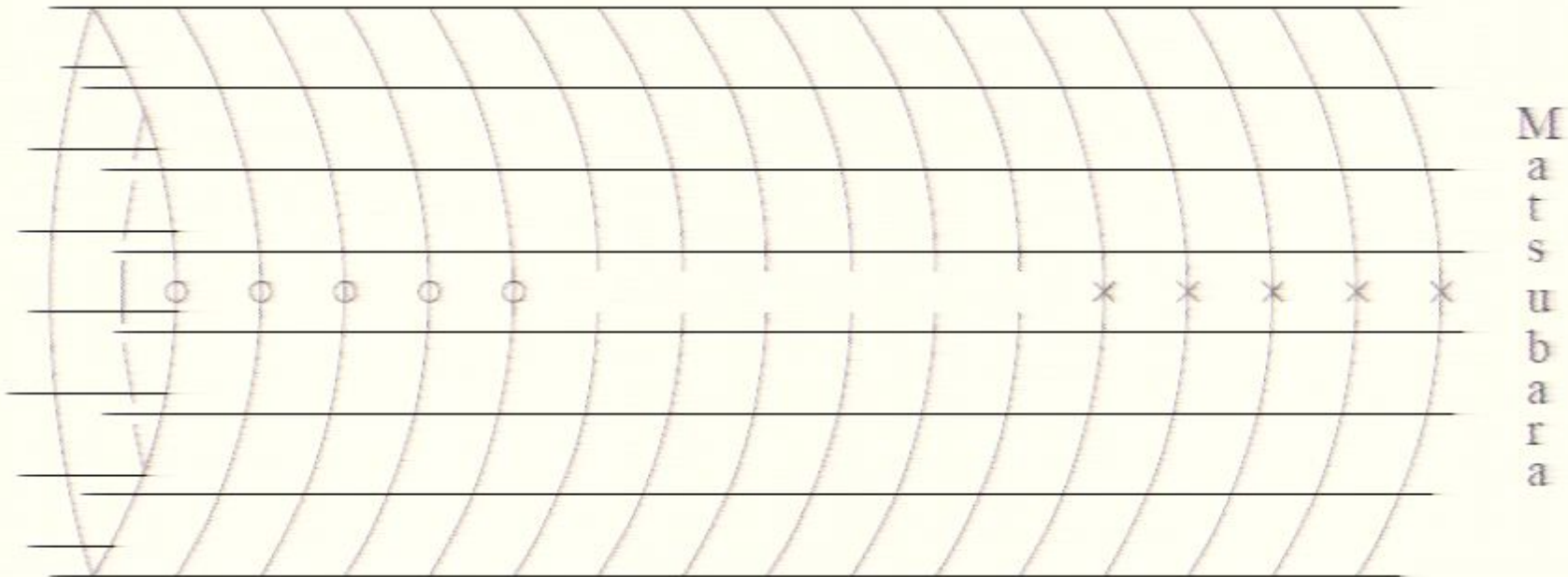
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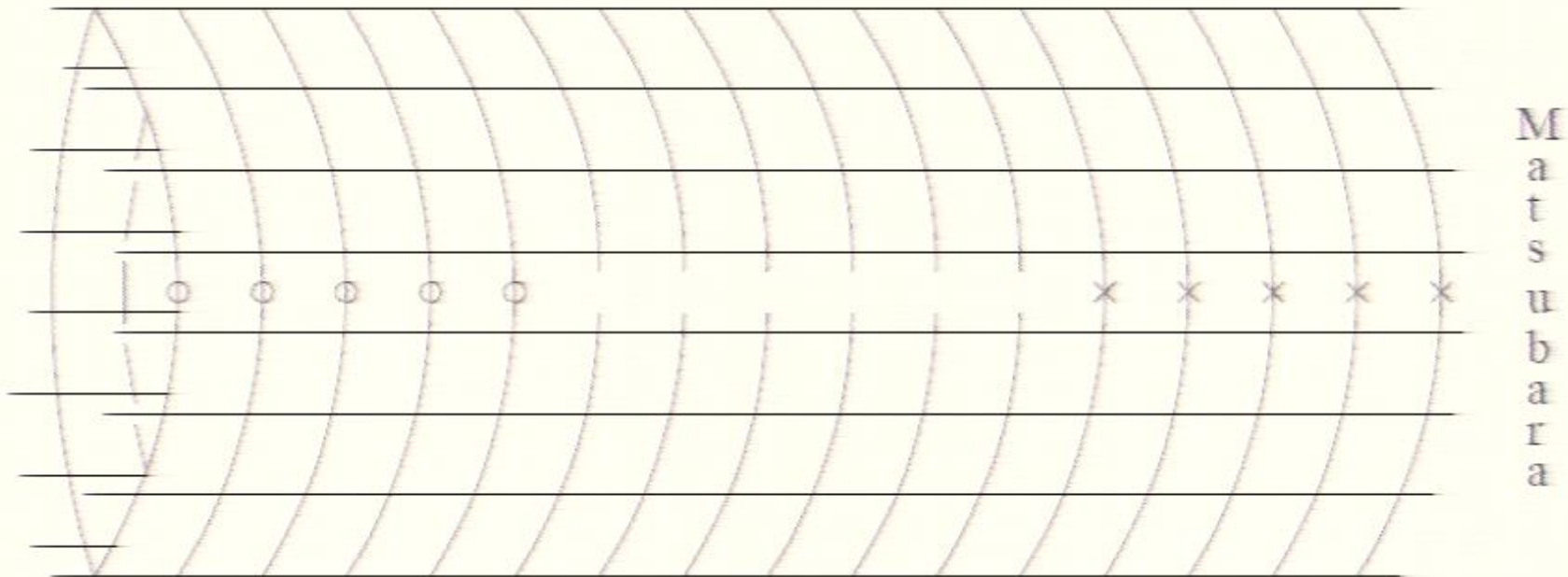
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and  $|\kappa\rangle$ ,  $\langle \kappa|$  are eigen(co)vector of the transfer matrix

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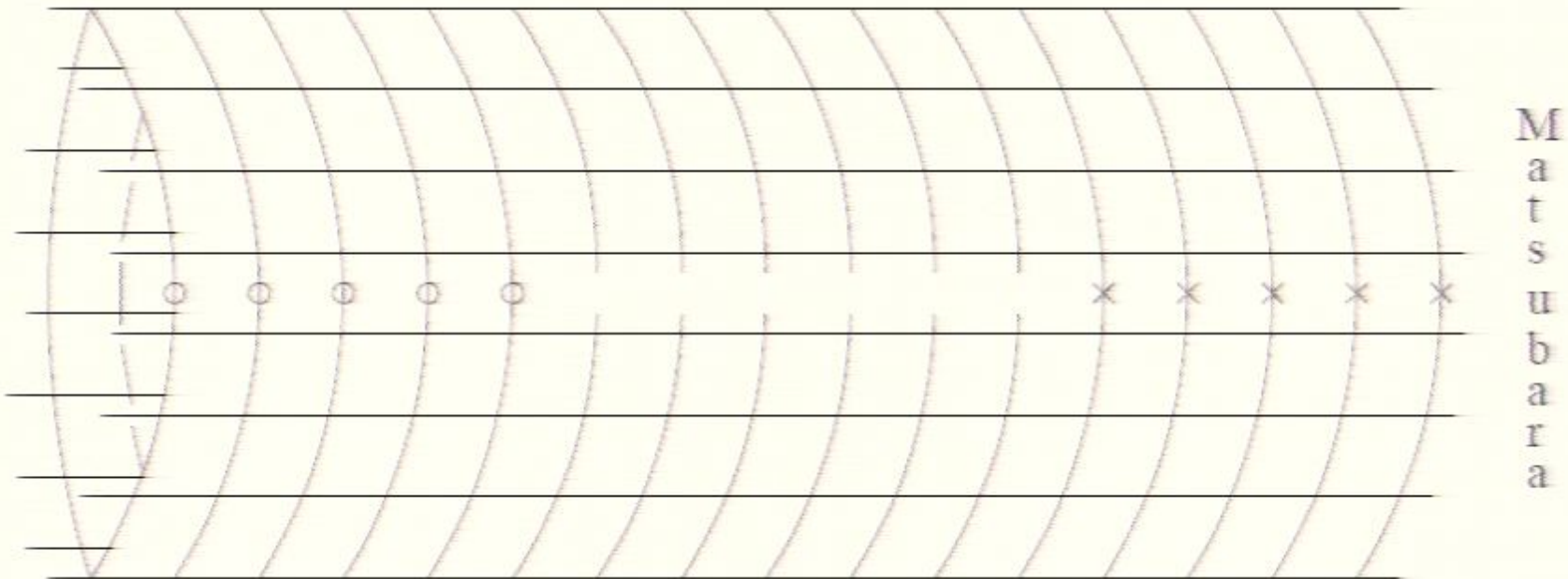
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## Fermionic basis on the lattice

We consider a 6 vertex model on a cylinder

Space



$$\text{---} = L_{ij}$$

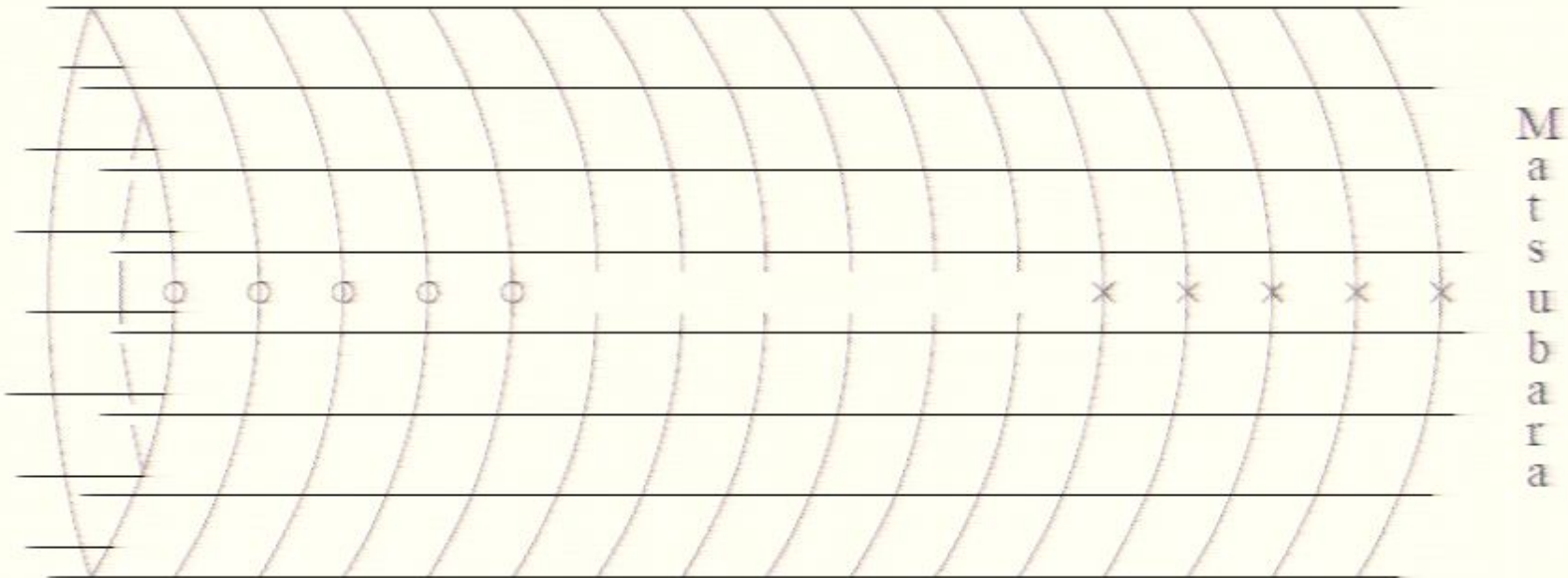
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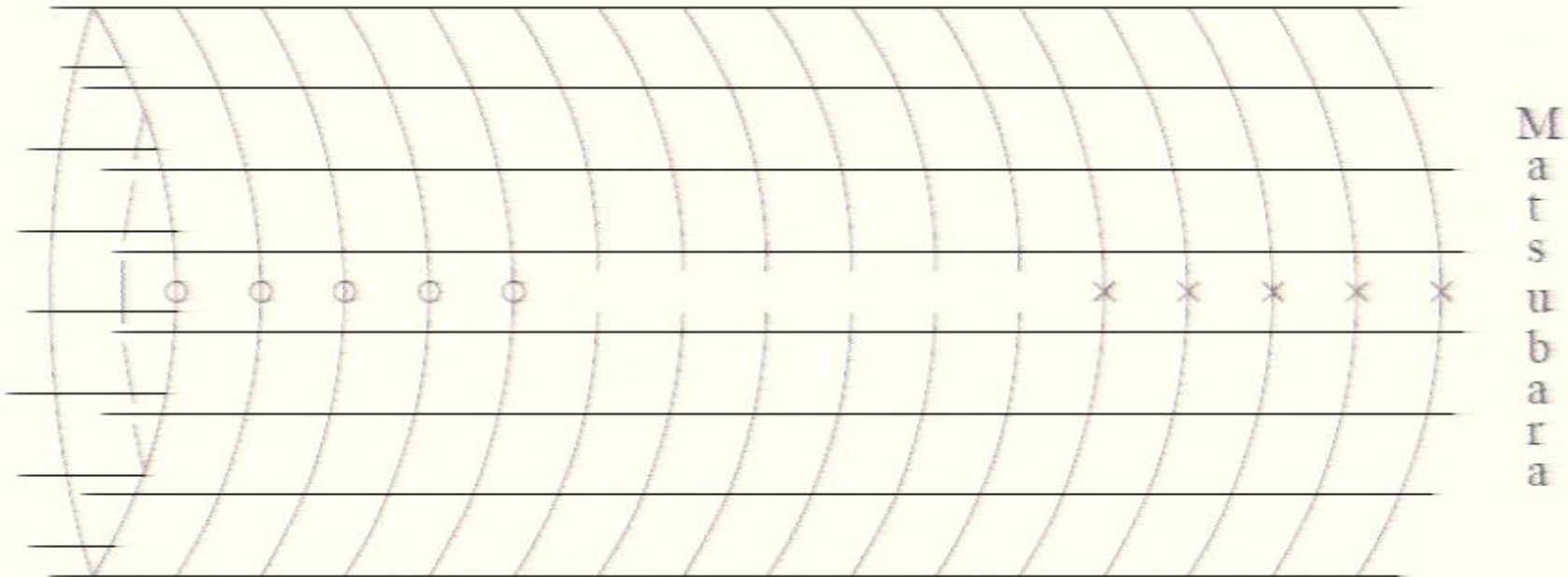
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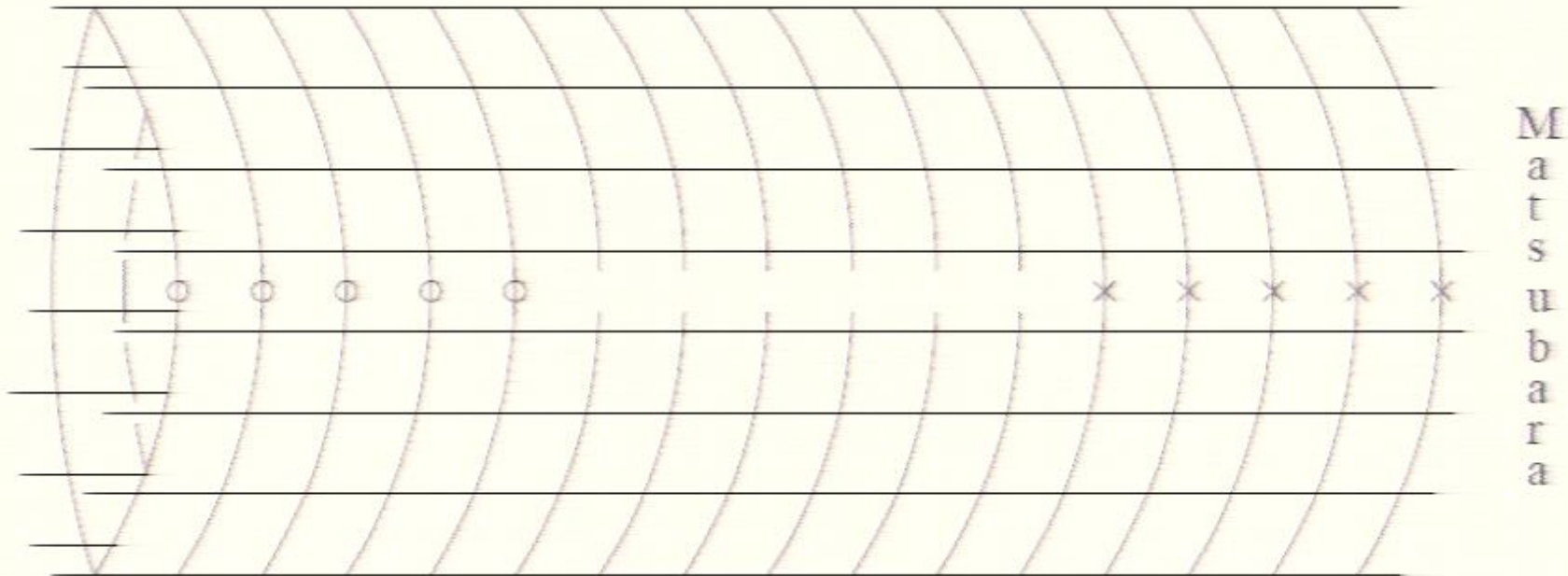
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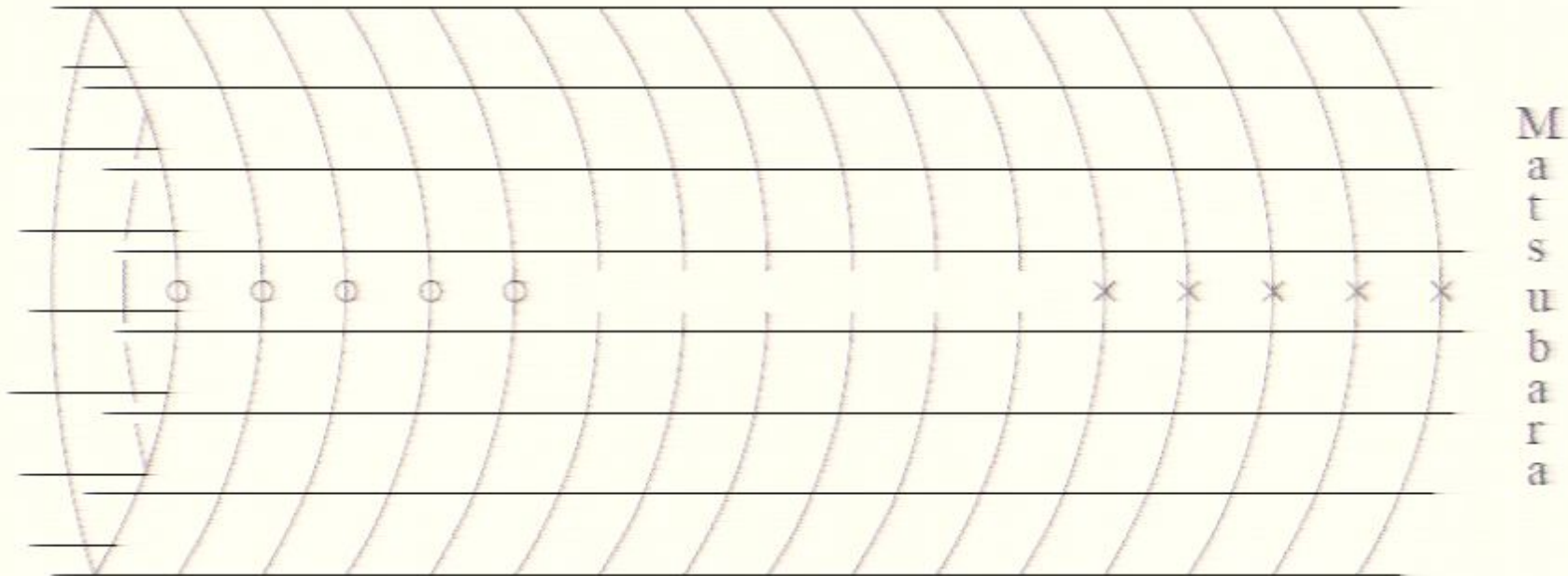
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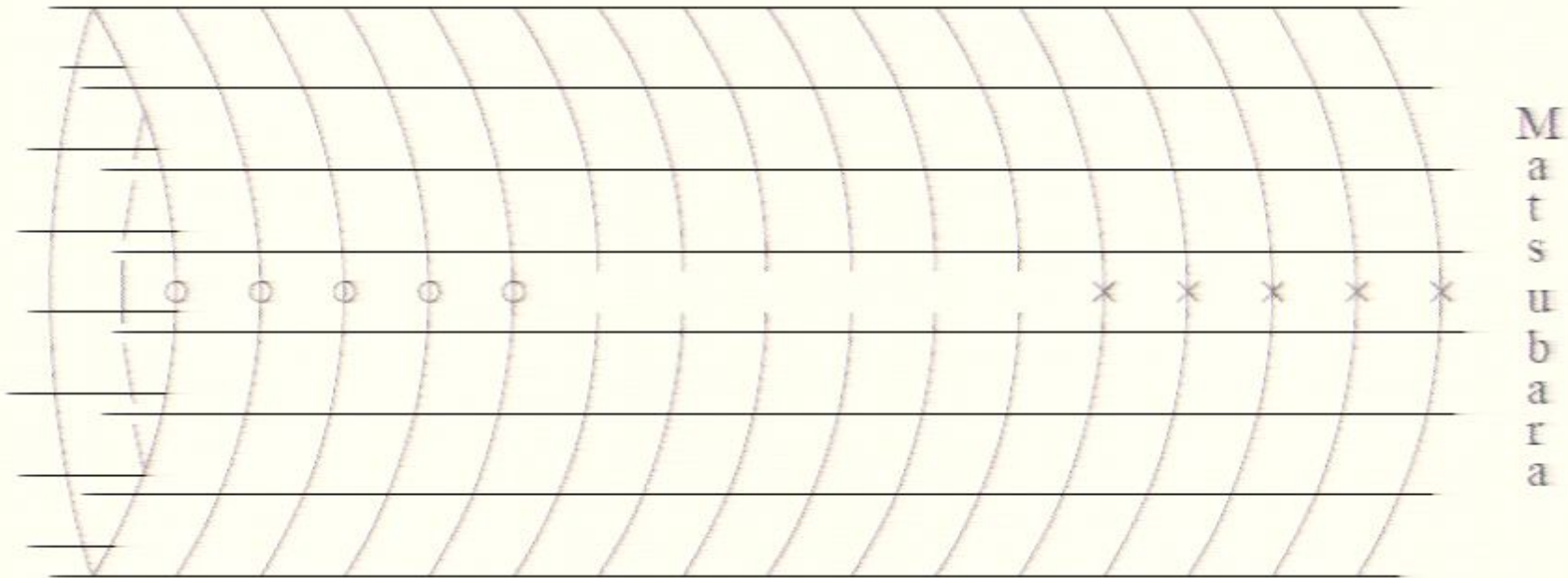
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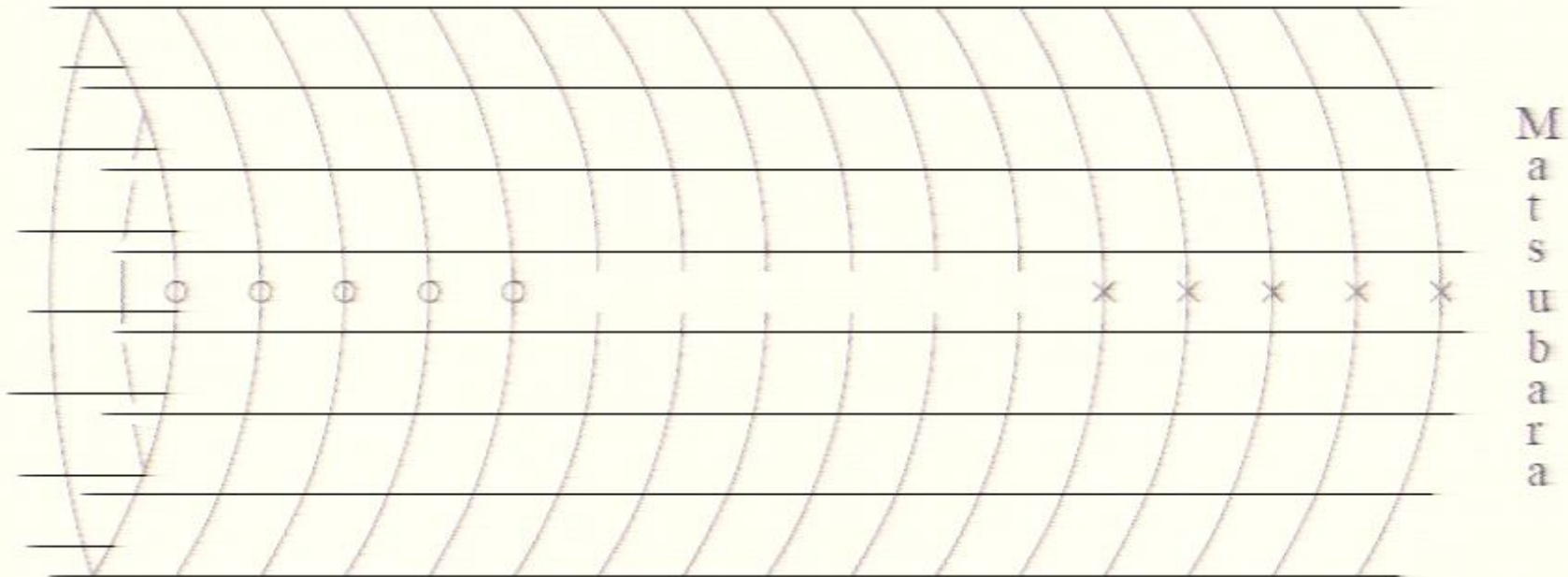
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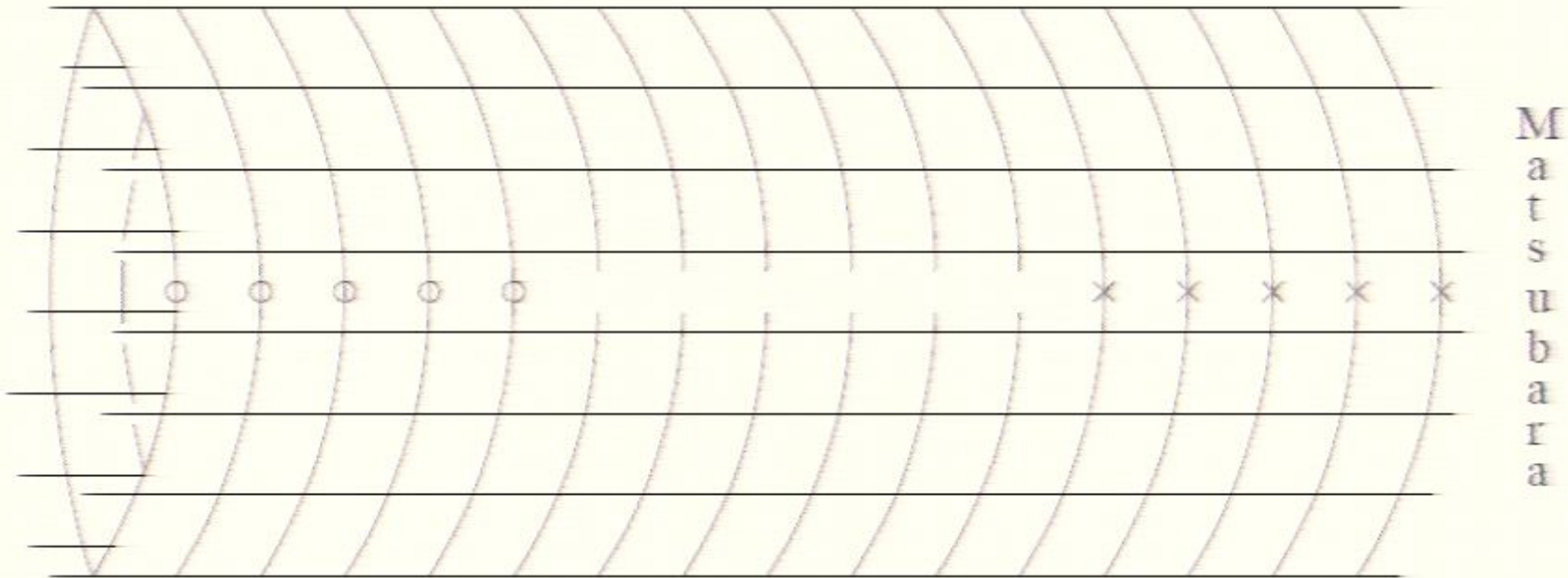
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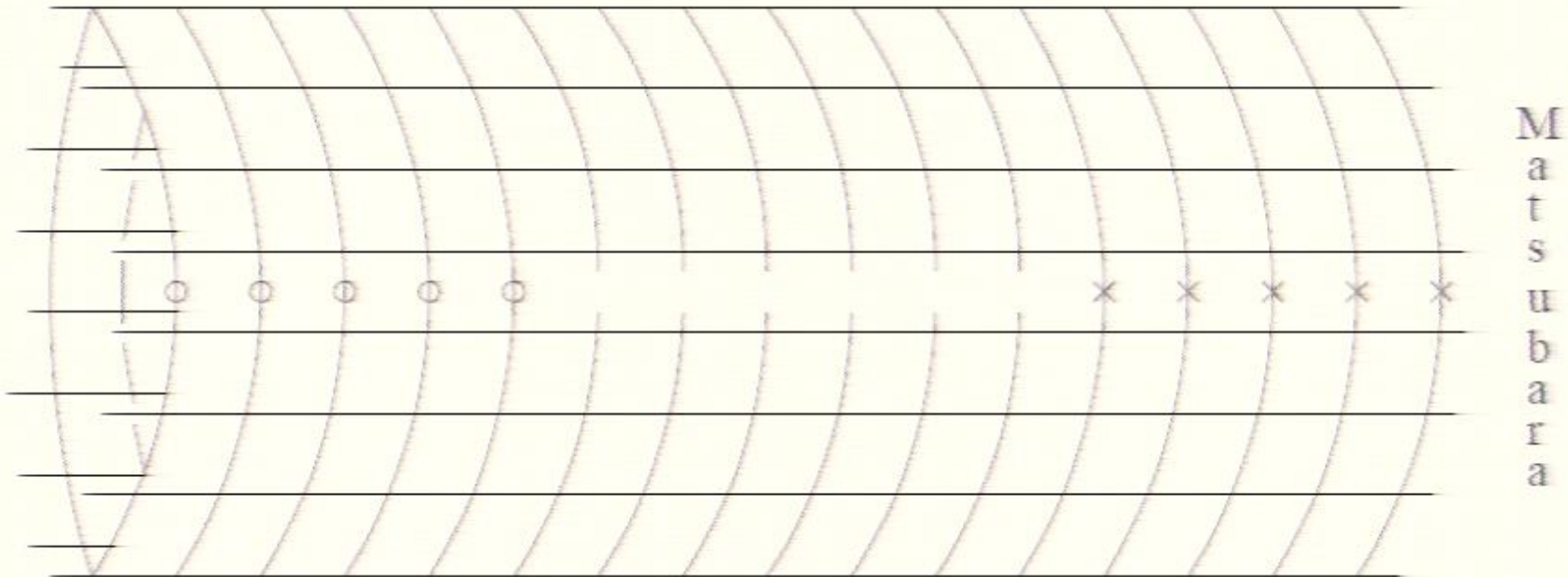
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$$Z_n \left\{ \mathbf{t}^*(\eta_1) \cdots \mathbf{t}^*(\eta_s) \mathbf{b}^*(\zeta_1) \cdots \mathbf{b}^*(\zeta_r) \mathbf{c}^*(\xi_r) \cdots \mathbf{c}^*(\xi_1) (q^{2\alpha S(0)}) \right\} \\ = \prod_{i=1}^s 2\rho(\eta_i) \cdot \det(\omega_n(\zeta_j, \xi_k)),$$

where

$$\rho(\eta) = \frac{T(\eta, \kappa + \alpha)}{T(\eta, \kappa)}, \quad T_{\mathbf{M}}(\eta, \kappa) |\kappa\rangle = T(\eta, \kappa) |\kappa\rangle,$$

and  $\omega_n(\zeta, \xi)$  is defined through the TBA data in the Matsubara space.



# TBA data

## TBA data

- ▶ auxiliary function characterizing  $|\kappa\rangle$

$$\log a(\zeta, \kappa) = -2\pi i\nu\kappa + \log \frac{a(\zeta)}{d(\zeta)} - \int_{\gamma} K_0(\zeta/\xi) \log(1 + a(\xi, \kappa)) \frac{d\xi^2}{\xi^2},$$

where  $\gamma$  encircles the Bethe roots clockwise, and

$$a(\zeta) = (1 - q\zeta^2)^n, \quad d(\zeta) = (1 - q^{-1}\zeta^2)^n,$$

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► resolvent

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$$\frac{1}{4} \omega_n(\zeta, \xi) = f_{\text{left}} \star (I + \mathcal{R}_{\text{dress}}) \star f_{\text{right}}(\zeta, \xi) - \omega_0(\zeta, \xi),$$

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## TBA data

- ▶ auxiliary function characterizing  $|\kappa\rangle$

$$\log a(\zeta, \kappa) = -2\pi i\nu\kappa + \log \frac{a(\zeta)}{d(\zeta)} - \int_{\gamma} K_0(\zeta/\xi) \log(1 + a(\xi, \kappa)) \frac{d\xi^2}{\xi^2},$$

where  $\gamma$  encircles the Bethe roots clockwise, and

$$a(\zeta) = (1 - q\zeta^2)^n, \quad d(\zeta) = (1 - q^{-1}\zeta^2)^n,$$

$$K_\alpha(\zeta) = \Delta_\zeta \psi(\zeta, \alpha), \quad \psi(\zeta, \alpha) = \frac{\zeta^\alpha}{\zeta^2 - 1},$$

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## The main formula

The following is the key result (JMS 2009).

$$Z_{\mathbf{n}} \left\{ \mathbf{t}^*(\eta_1) \cdots \mathbf{t}^*(\eta_s) \mathbf{b}^*(\zeta_1) \cdots \mathbf{b}^*(\zeta_r) \mathbf{c}^*(\xi_r) \cdots \mathbf{c}^*(\xi_1) (q^{2\alpha S(0)}) \right\} \\ = \prod_{i=1}^s 2\rho(\eta_i) \cdot \det(\omega_{\mathbf{n}}(\zeta_j, \xi_k)),$$

where

$$\rho(\eta) = \frac{T(\eta, \kappa + \alpha)}{T(\eta, \kappa)}, \quad T_{\mathbf{M}}(\eta, \kappa) | \kappa \rangle = T(\eta, \kappa) | \kappa \rangle,$$

and  $\omega_{\mathbf{n}}(\zeta, \xi)$  is defined through the TBA data in the Matsubara space.

► resolvent

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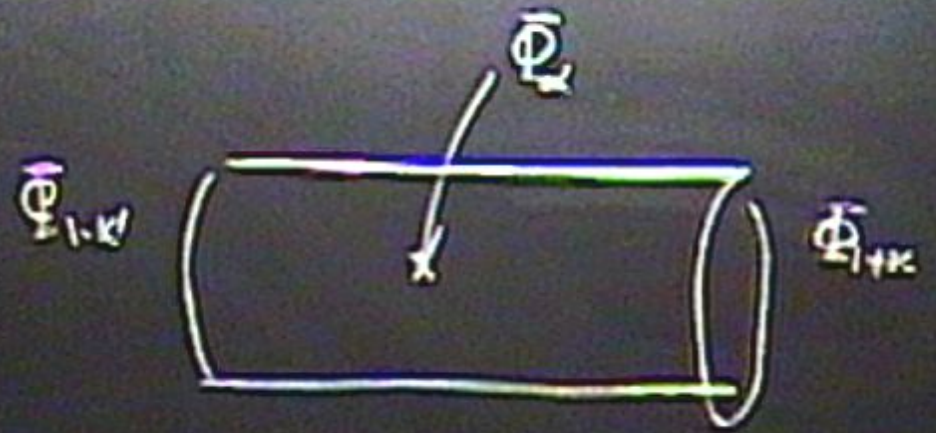
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# Conjecture (screening operators)



Example:

For the function  $\omega_n(\zeta, \xi)$ :

$$\omega^{\text{sc}}(\lambda, \mu) = \lim \frac{1}{4} \omega_n \left( (Ca)^\nu \lambda, (Ca)^\nu \mu \right)$$

has the asymptotics of the form

$$\begin{aligned} \omega^{\text{sc}}(\lambda, \mu) &\simeq \sum_{j,k=1}^{\infty} \omega_{j,k}(\kappa, \alpha) \lambda^{-\frac{2j-1}{\nu}} \mu^{-\frac{2k-1}{\nu}} \quad (\lambda^2, \mu^2 \rightarrow \infty), \\ &\simeq \sum_{j,k=1}^{\infty} \tilde{\omega}_{j,k}(\kappa, \alpha) \lambda^{-\frac{2j-1}{\nu}} \mu^{-\alpha+2k} \quad (\lambda^2 \rightarrow \infty, \mu^2 \rightarrow 0). \end{aligned}$$

We postulate that the coefficients in these expansions are some three point functions in CFT.

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Start with the 6 vertex model with alternating inhomogeneity parameters  $\zeta_0, \zeta_0^{-1}$  in both directions. The sine-Gordon model is obtained as a scaling limit

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# Form factors

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It is possible to generate 'all' towers  $\{L_{\mathcal{O}}^{(n)}\}$  by acting with certain fermions (Babelon-Bernard-Smirnov, 1997)

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