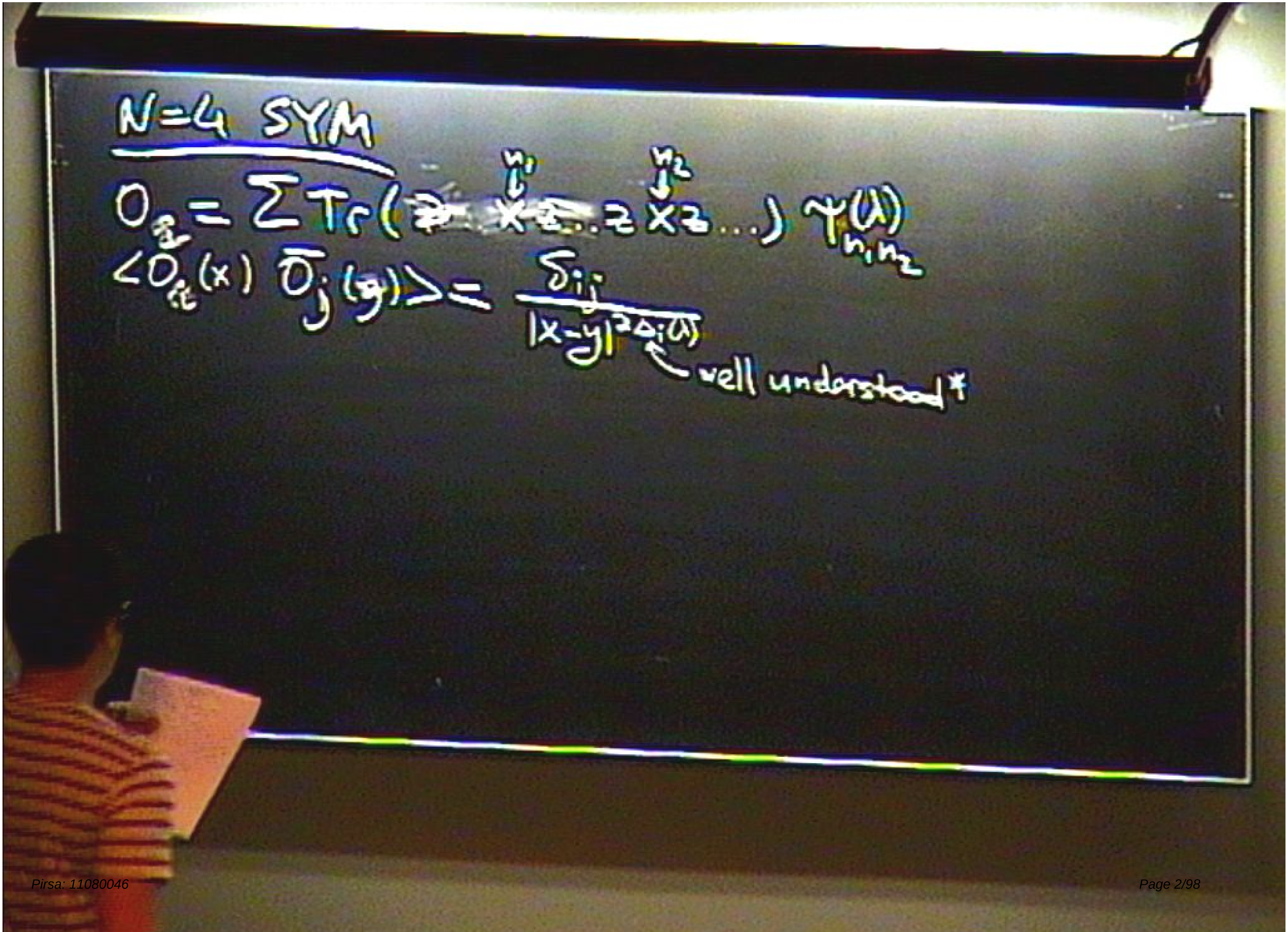
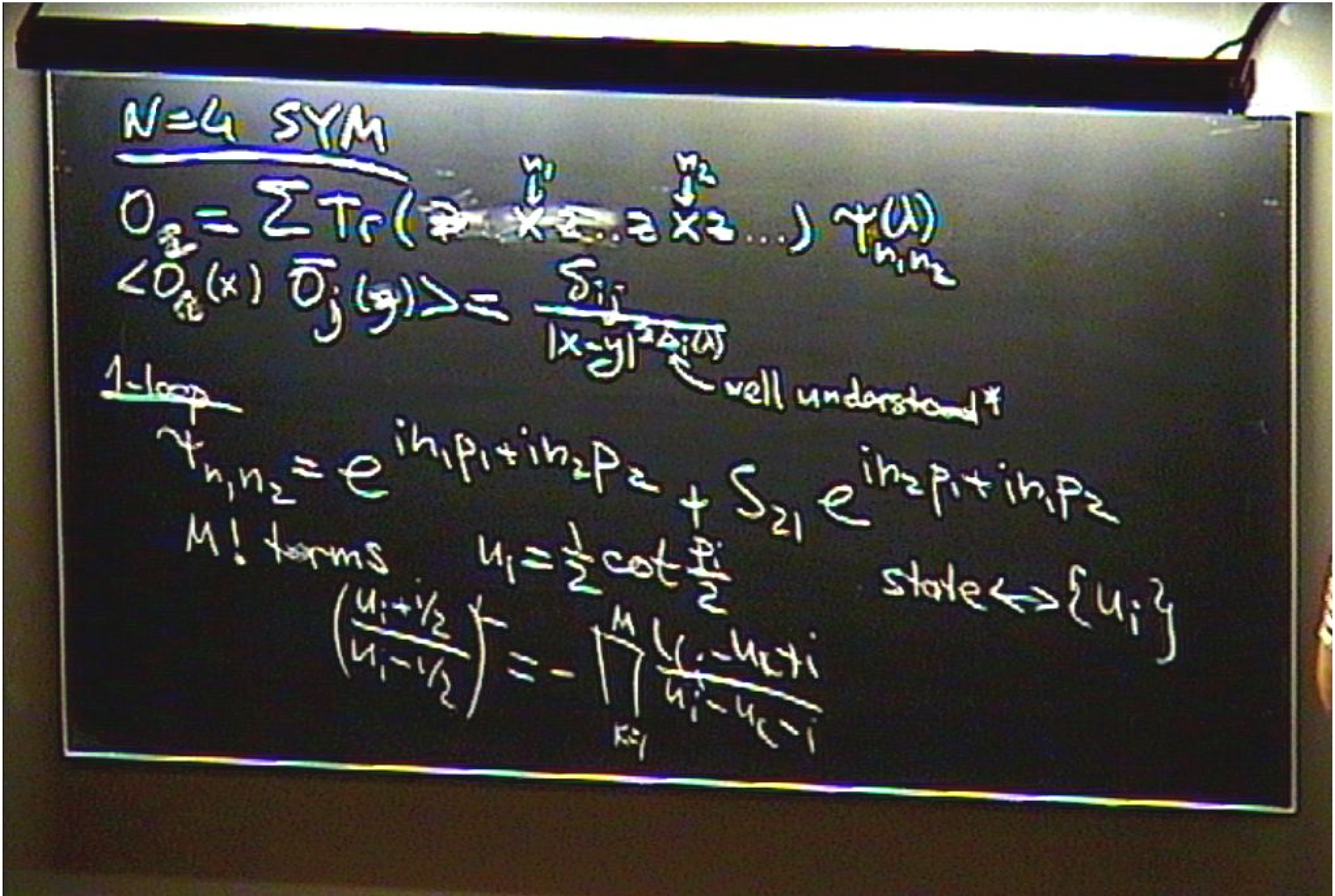
Title: Quasiclassical 3pt Functions

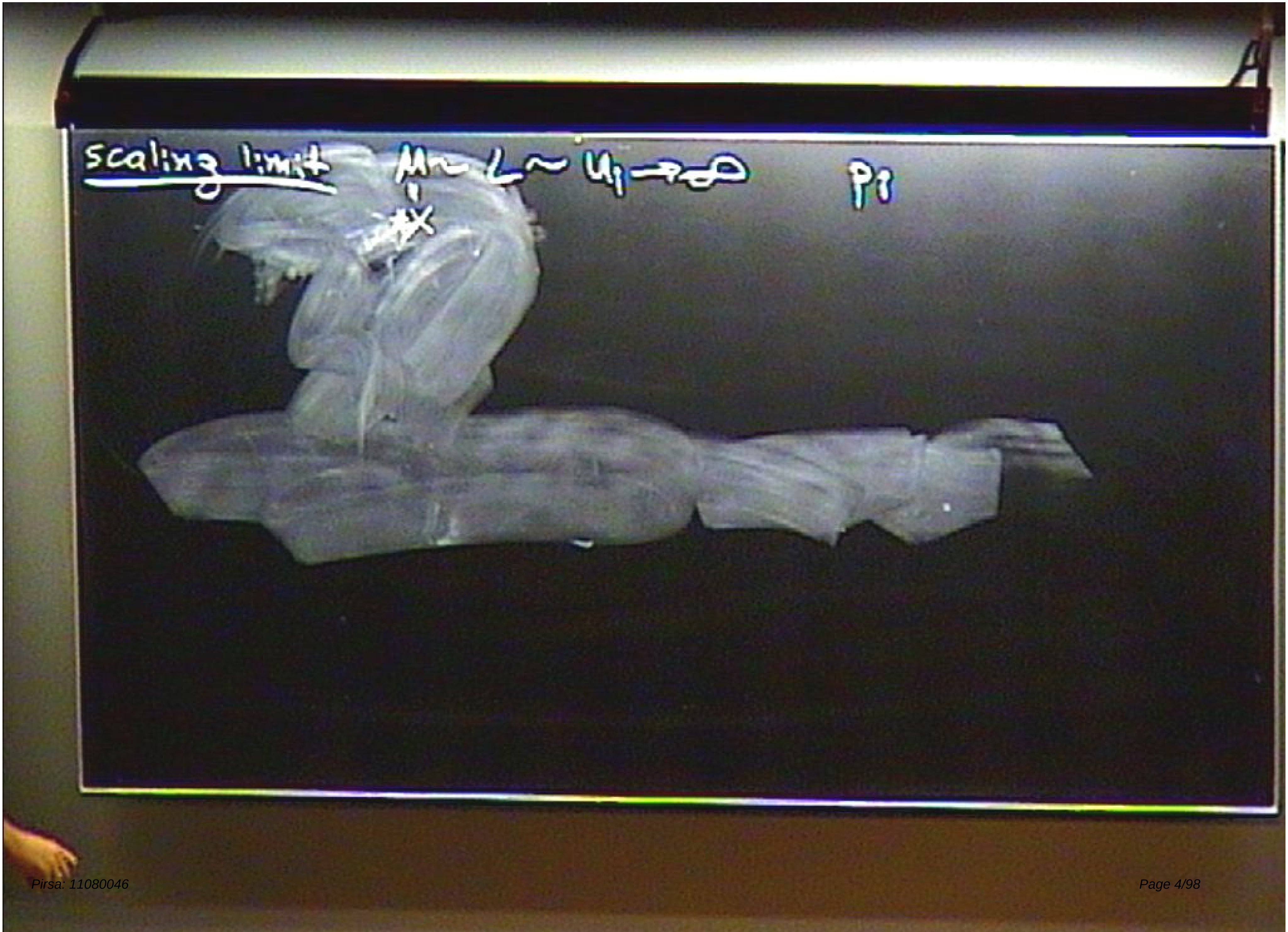
Date: Aug 15, 2011 11:40 AM

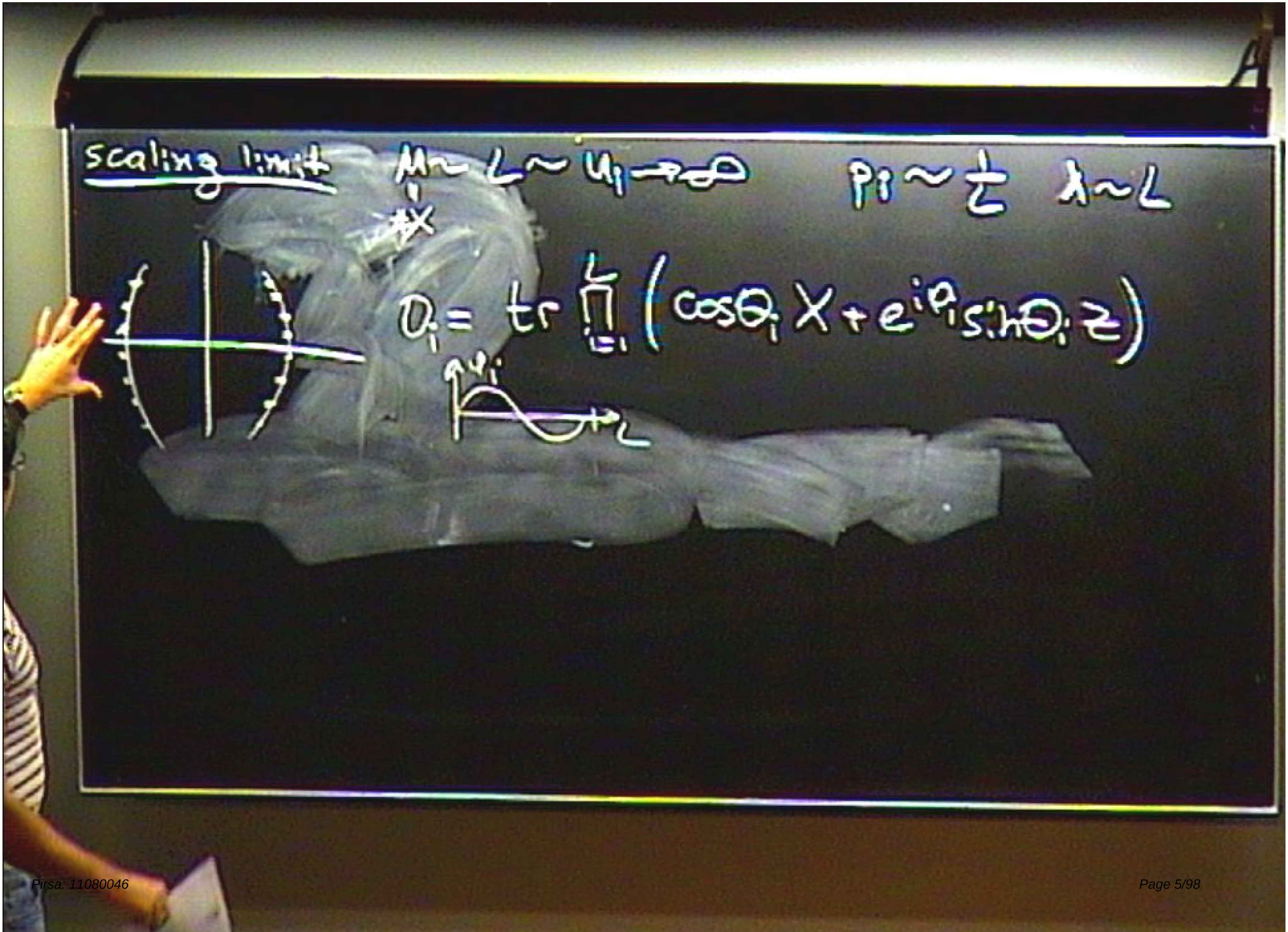
URL: http://pirsa.org/11080046

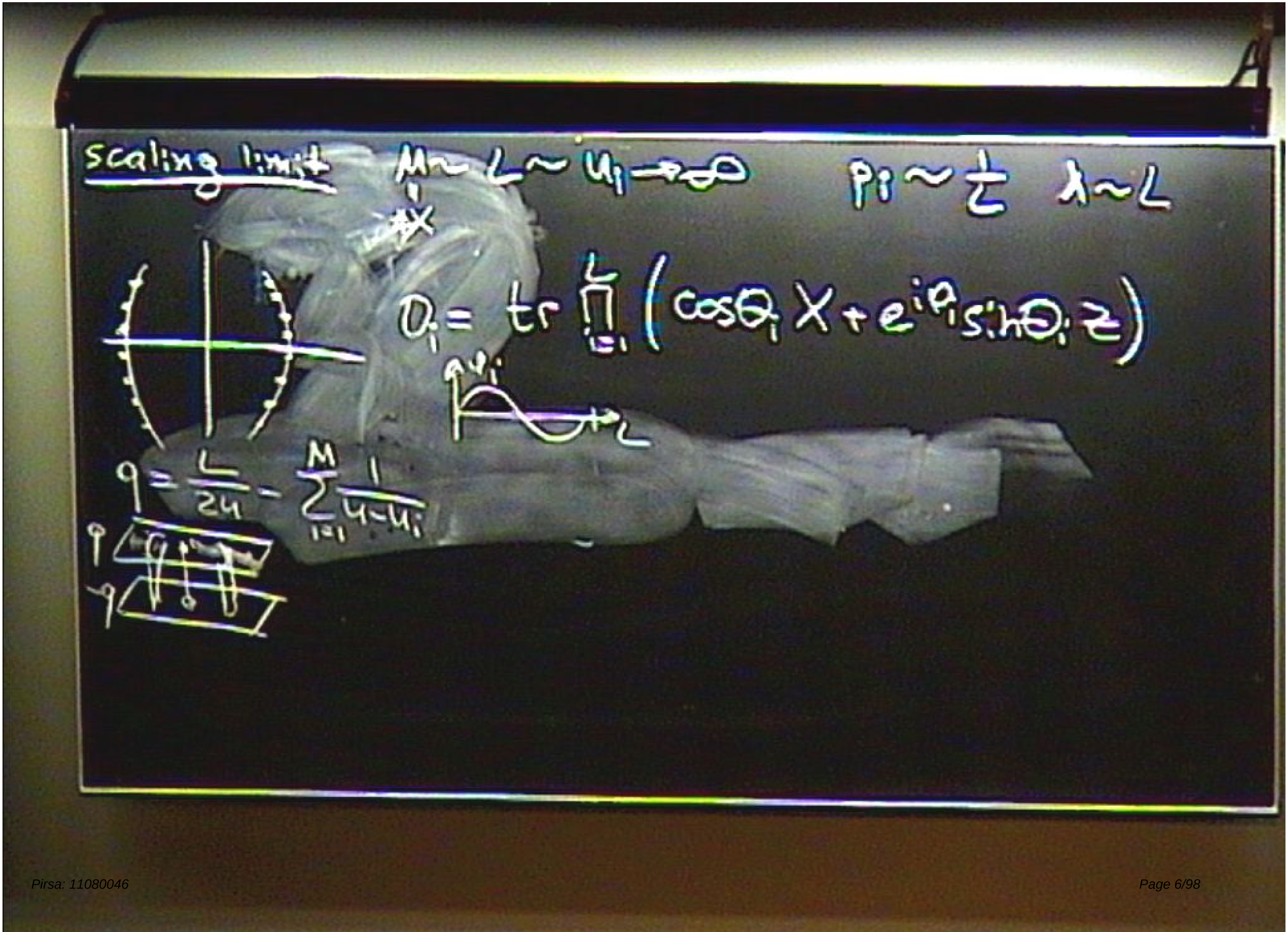
Abstract: We compute three-point functions of single trace operators in planar N = 4 SYM. We consider the limit where one of the operators is much smaller than the other two. We find a precise match between weak and strong coupling in the Frolov-Tseytlin classical limit for a very general class of classical solutions. To achieve this match we clarify the issue of back-reaction and identify precisely which three-point functions are captured by a classical computation.


N=4 SYM
O_I = Σ Tr(… X Z … Z X Z …) Ψ^(λ)_{n1 n2}
<O_I(x) Ō_J(y)> = δ_IJ / |x-y|^{2Δ_I(λ)}
well understood*


N=4 SYM
O = Σ Tr(… X Z … Z X Z …) Ψ(λ) n1 n2
<O(x) Ōj(y)> = δij / |x-y|^2Δi(λ)
well understood*
1-loop
Ψn1n2 = e^(in1p1+in2p2) + S21 e^(in2p1+in1p2)
M! terms
u1 = ½ cot pi/2
state ↔ {ui}
((ui+½)/(ui-½))^L = - ∏ k=1 M (ui-uk+i)/(ui-uk-i)


Scaling limit


Scaling limit
M ~ L ~ u_i → ∞
p_i ~ 1/L
λ ~ L
O_i = tr ∏ (cosθ_i X + e^{iφ_i} sinθ_i Z)


scaling limit
M ~ L ~ u_i → ∞
p_i ~ 1/L
λ ~ L
O_i = tr ∏_{i=1}^{L} (cosθ_i X + e^{iφ_i} sinθ_i Z)
q = L/2u − Σ_{i=1}^{M} 1/(u − u_i)

N=4 SYM

$$O_I = \sum \mathrm{Tr}(z \ldots \overset{n_1}{X} z \ldots z \overset{n_2}{X} z \ldots)\, \psi^{(\lambda)}_{n_1 n_2}$$

$$\langle O_I(x)\, \bar{O}_J(y) \rangle = \frac{\delta_{IJ}}{|x-y|^{2\Delta_I(\lambda)}}$$

← well understood!

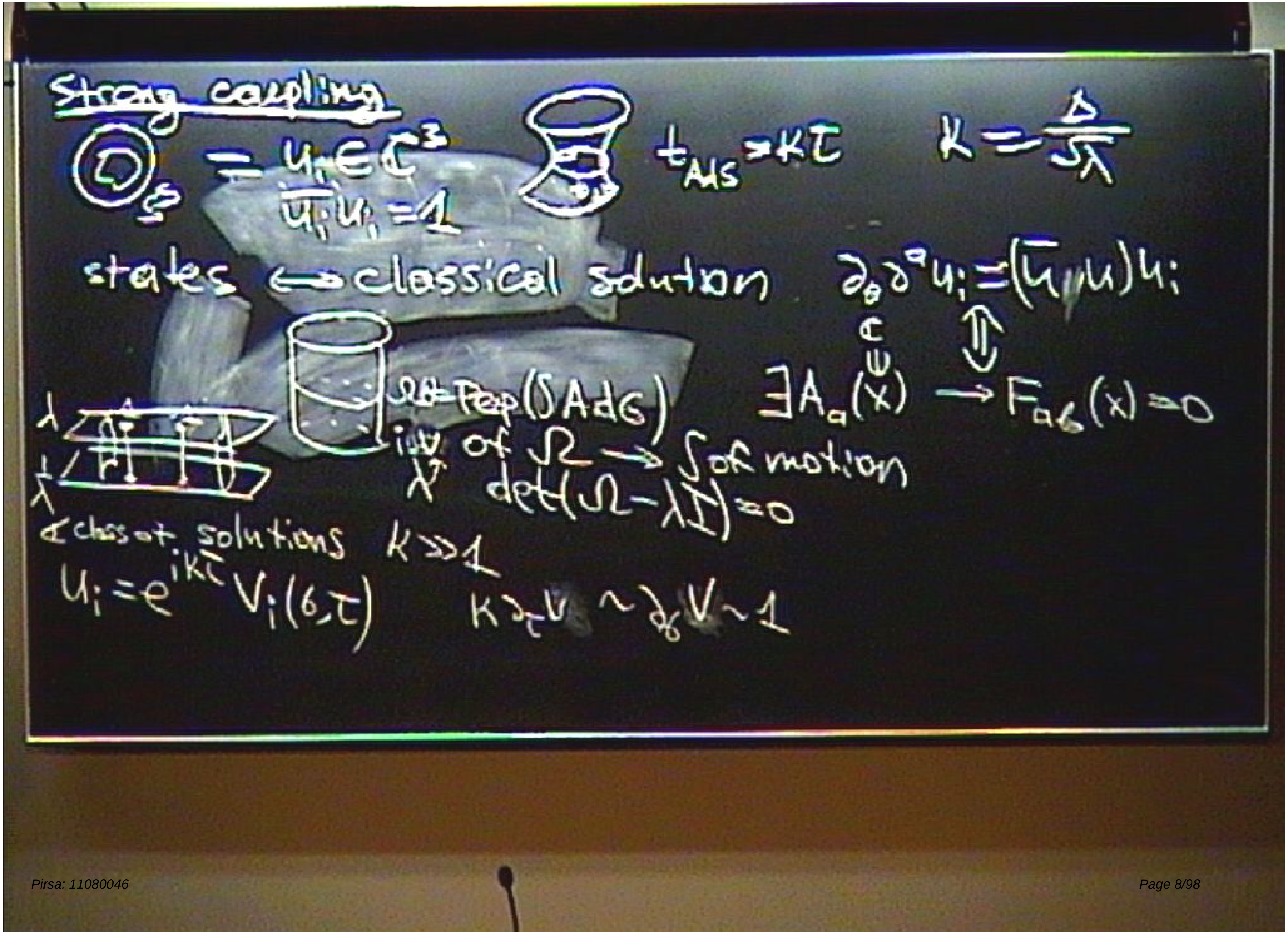
1-loop

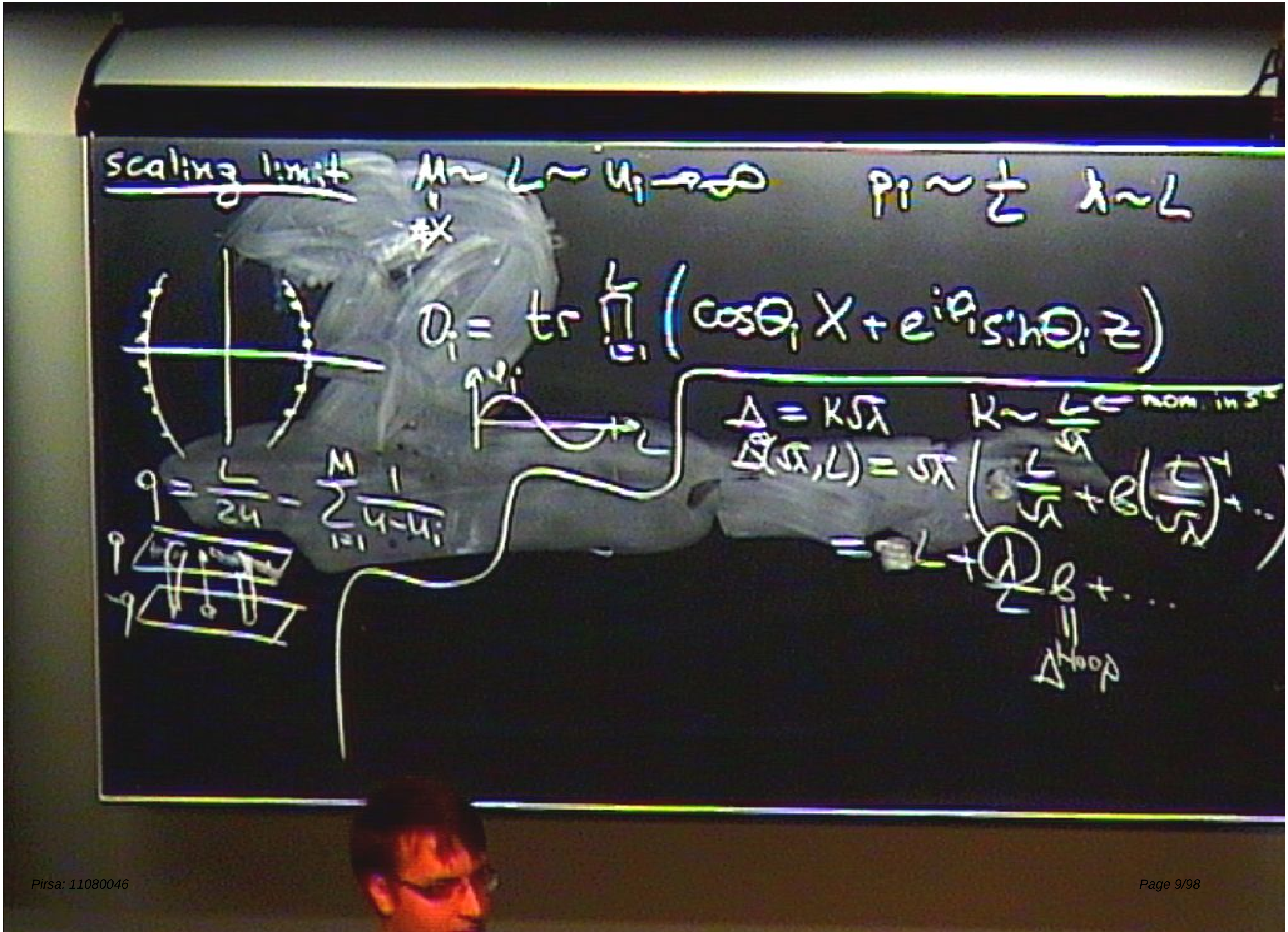
$$\psi_{n_1 n_2} = e^{i n_1 p_1 + i n_2 p_2} + S_{21}\, e^{i n_2 p_1 + i n_1 p_2}$$

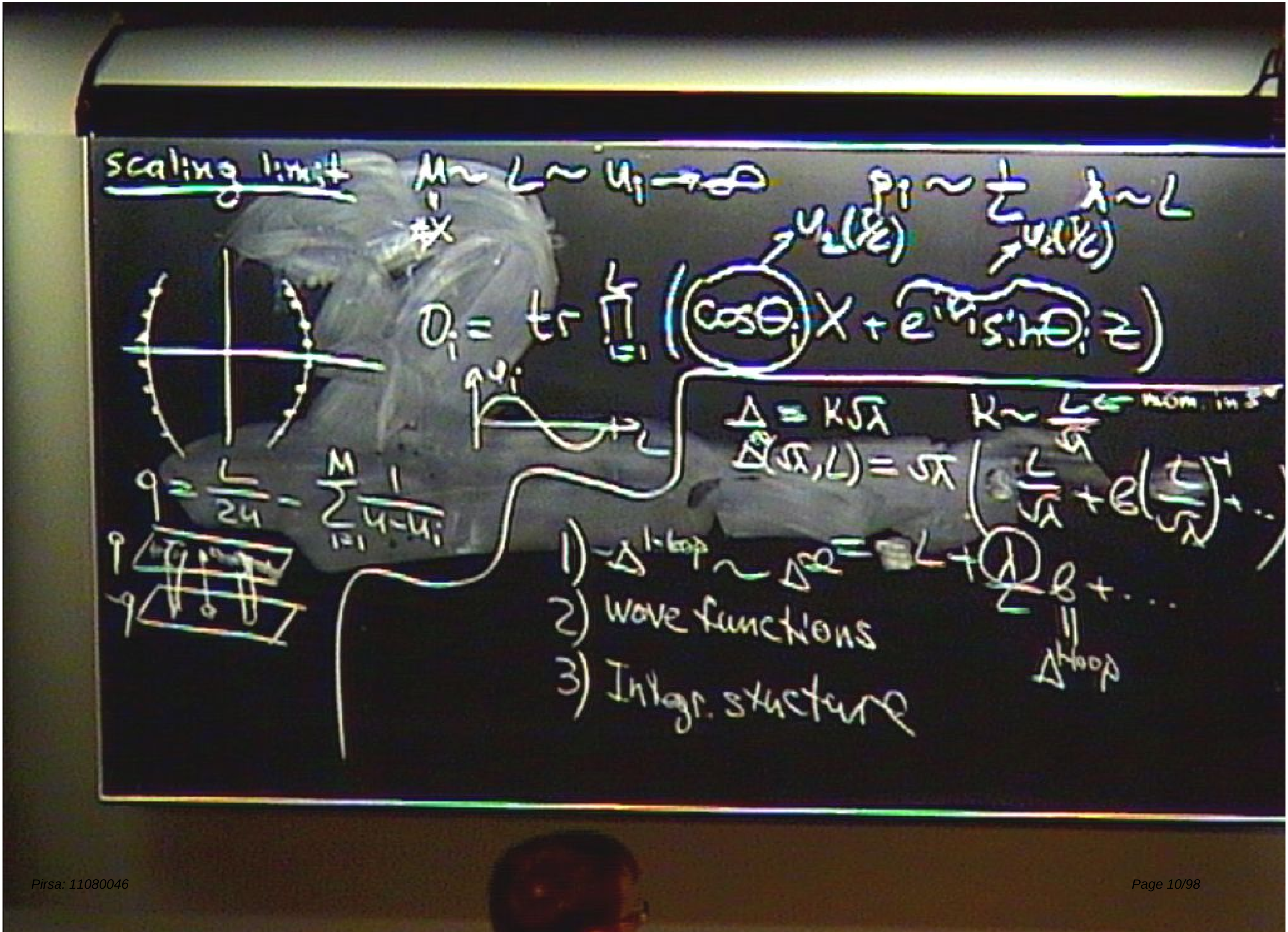
M! terms $\quad u_i = \frac{1}{2}\cot\frac{p_i}{2} \quad$ state $\leftrightarrow \{u_i\}$

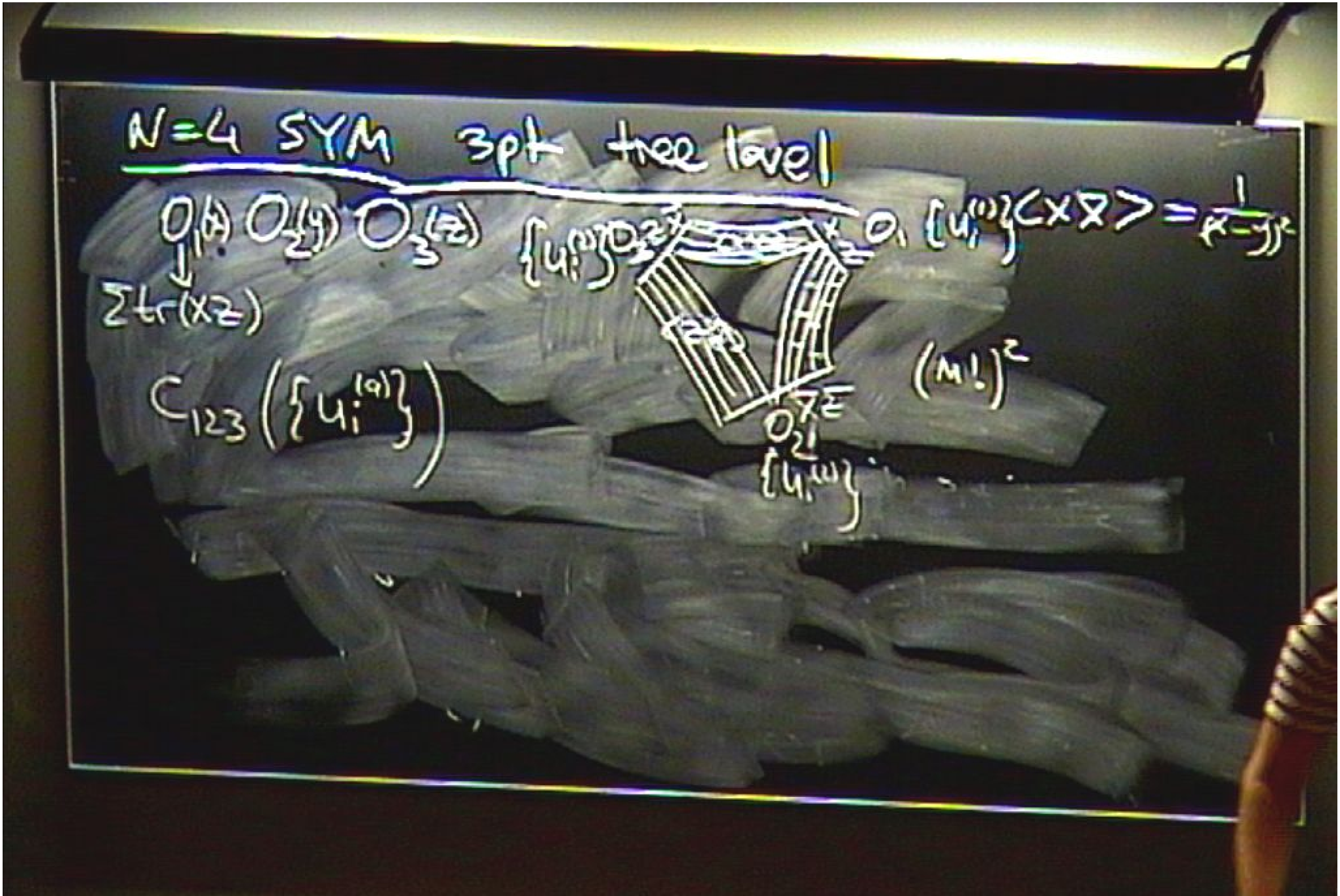
$$\left(\frac{u_i + i/2}{u_i - i/2}\right)^L = -\prod_{k=1}^{M} \frac{u_i - u_k + i}{u_i - u_k - i}$$

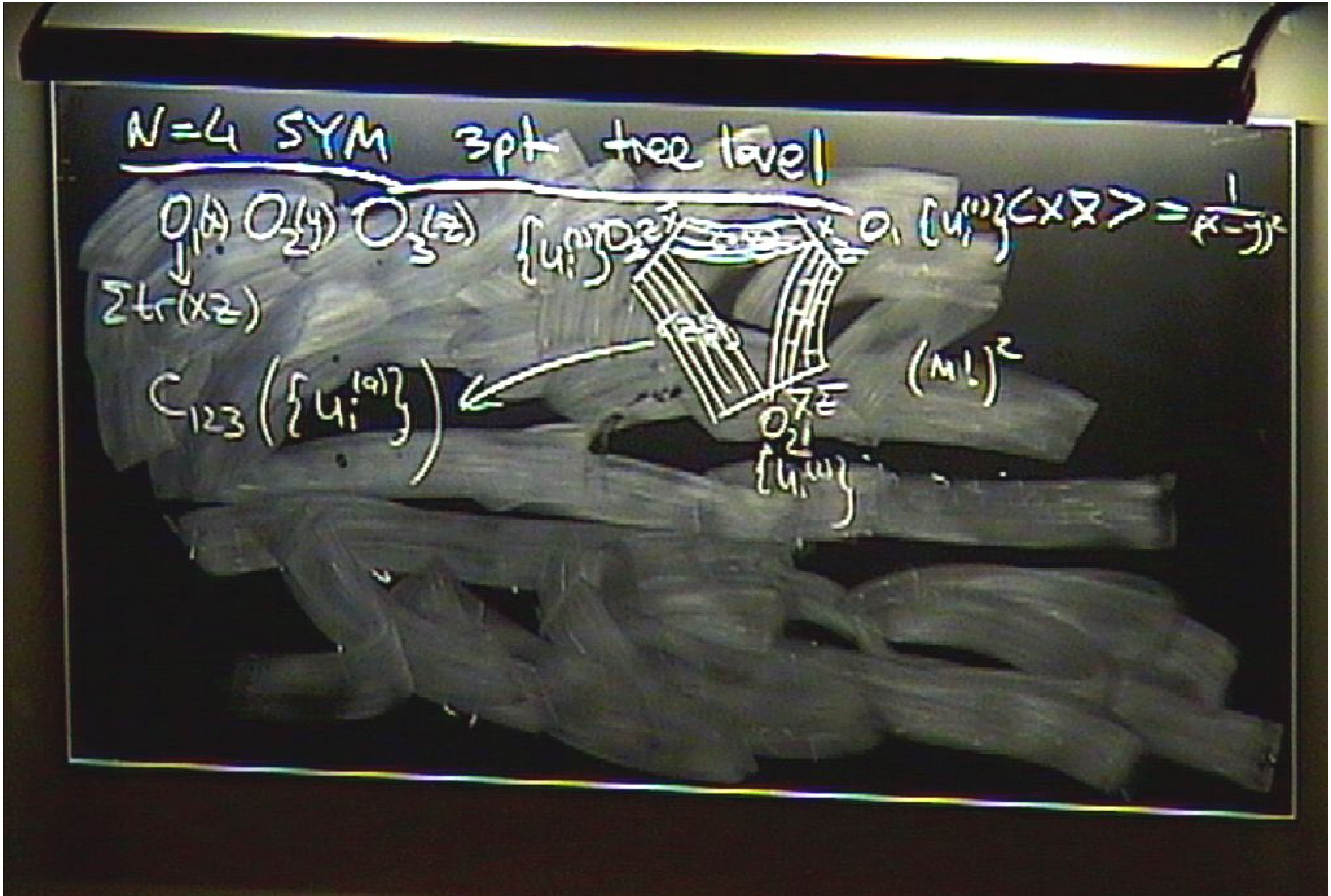
$$\Delta = L + \lambda \sum_i \sin^2 p_i/2 + \ldots$$


Strong coupling
$\mathcal{O}_S = u_i \in \mathbb{C}^3$, $\bar{u}_i u_i = 1$
$t_{AdS} = \kappa \tau$
$\kappa = \frac{\Delta}{\sqrt{\lambda}}$
states ⟷ classical solution
$\partial_a \partial^a u_i = (\bar{u}, u) u_i$
$\exists A_a(x) \rightarrow F_{ab}(x) = 0$
Ω Pop(S AdS)
i.v. of Ω → $\int$ of motion
λ $\det(\Omega - \lambda I) = 0$
class of solutions $\kappa \gg 1$
$u_i = e^{i\kappa\tau} V_i(\sigma, \tau)$
$\kappa \partial_\tau V \sim \partial_\sigma V \sim 1$


Scaling limit
M ~ L ~ u_i → ∞
p_1 ~ 1/L   λ ~ L
O_i = tr ∏ (cos θ_i X + e^{iφ_i} sin θ_i Z)
Δ = K√λ
K ~ L/√λ ← mom. in S^5
Δ^{1-loop}


Scaling limit
M ~ L ~ u_1 → ∞
p_1 ~ 1/L
λ ~ L
Δ = K√λ
Δ(√λ, L) = √λ
1) Δ^1-loop
2) wave functions
3) Integr. structure
Δ^1-loop


N=4 SYM 3pt tree level
O1(x) O2(y) O3(z)
Σ tr(XZ)
C123({ui(a)})
(M!)2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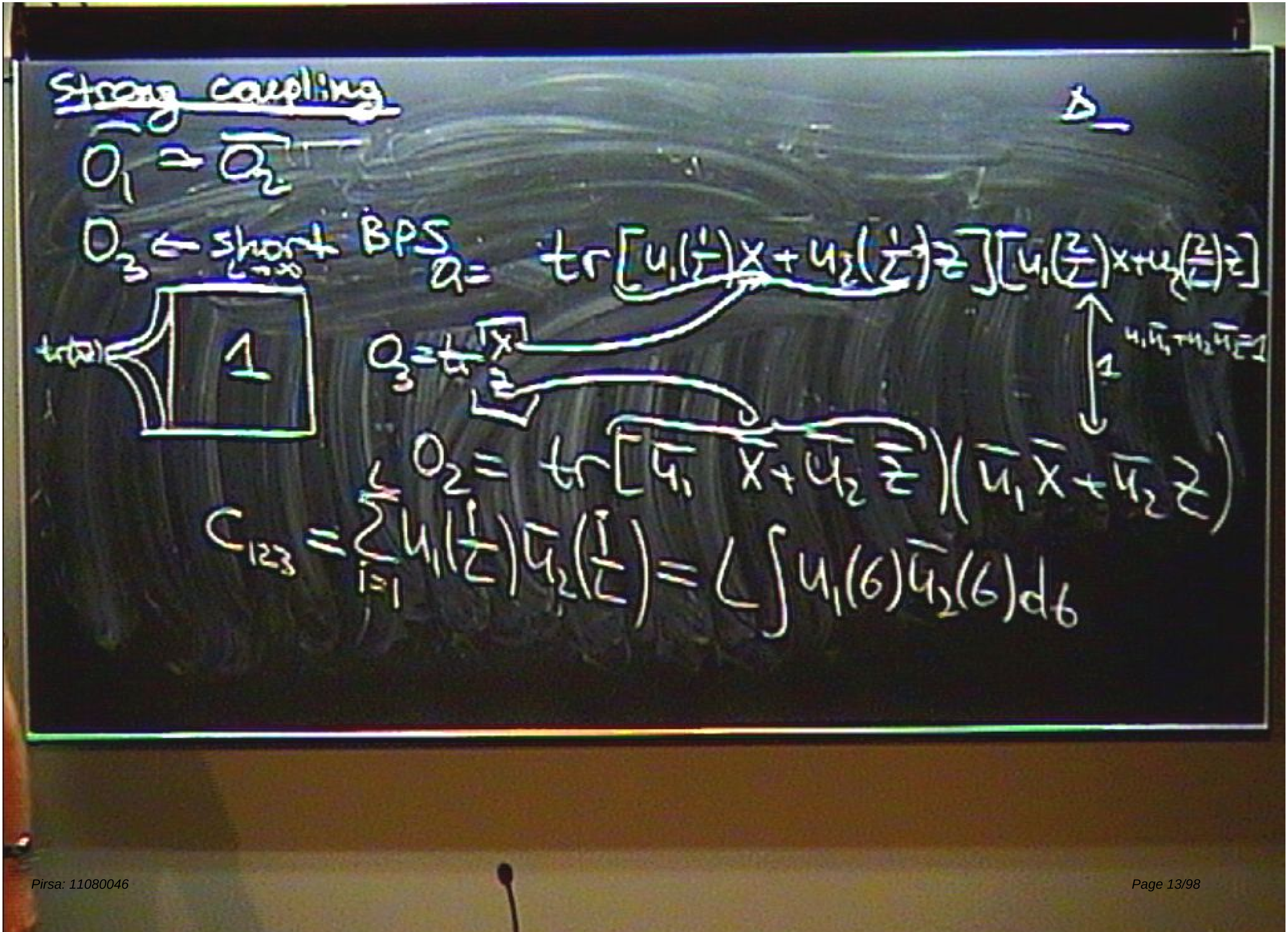
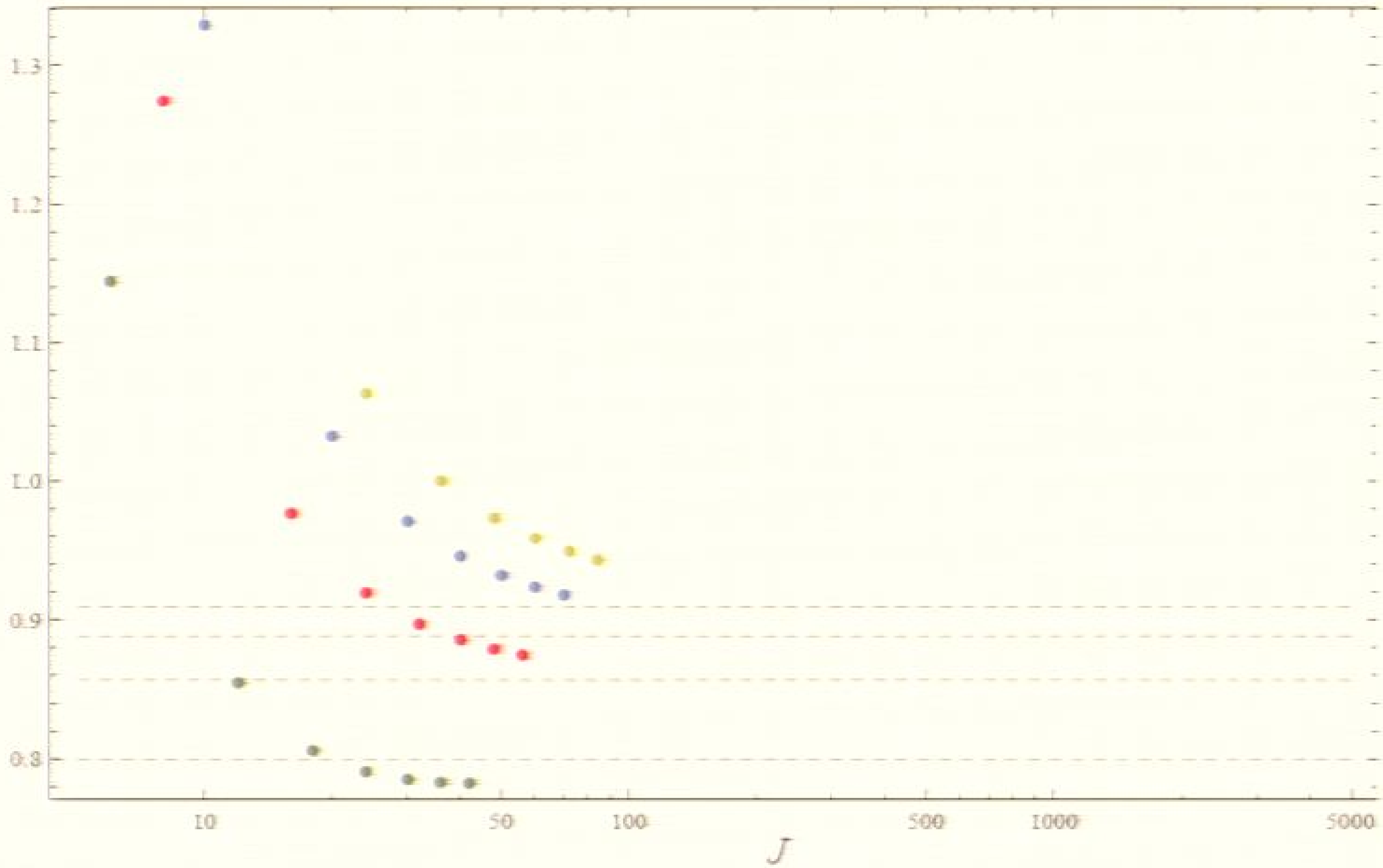
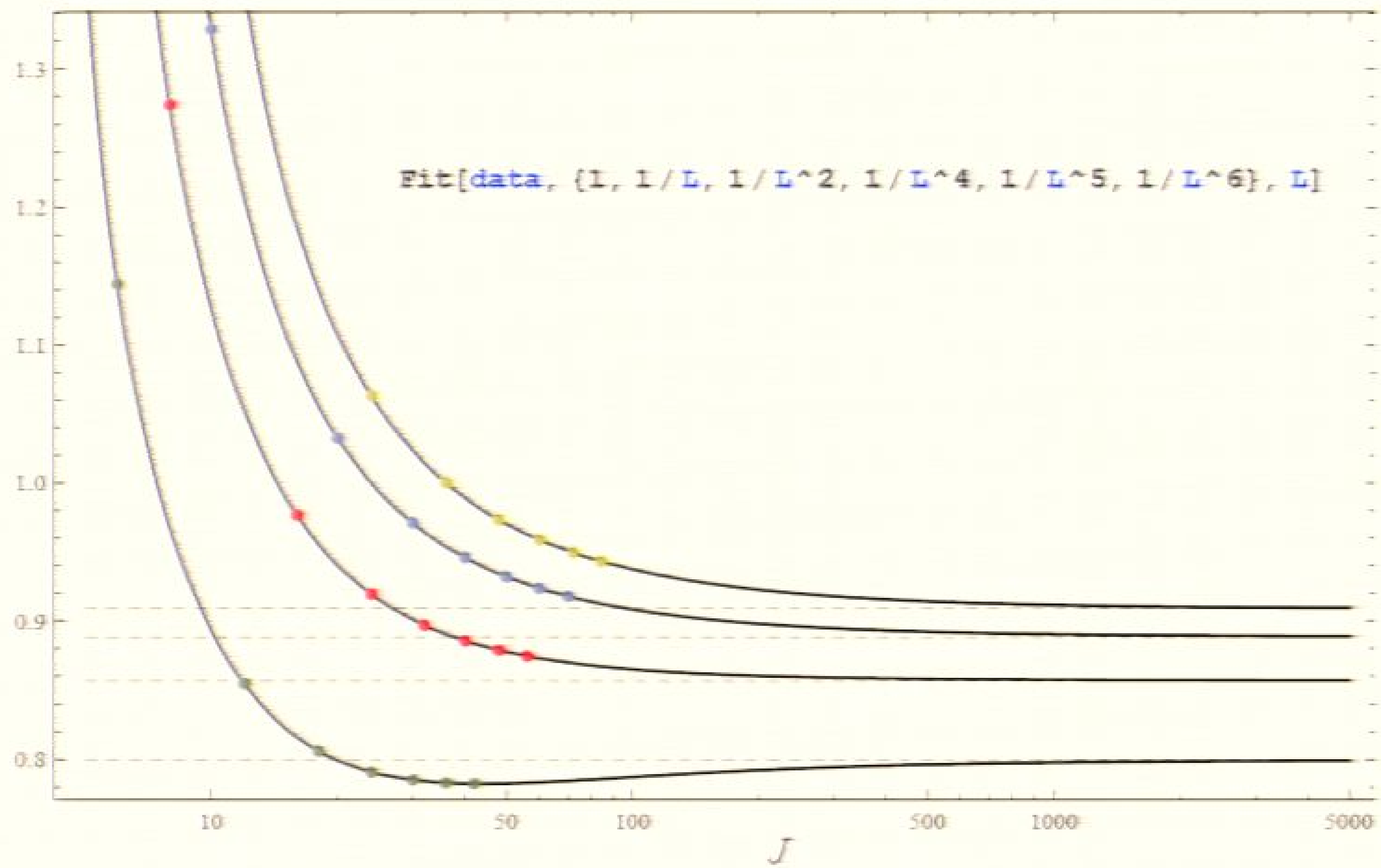
<X X̄> = 1/(x-y)²
(M!)²


Strong coupling
O1 → O2
O3 ← short BPS
Q=tr[u1(1/L)X+u2(1/L)Z][u1(2/L)X+u2(2/L)Z]
u1ū1+u2ū2=1
1
O3=tr XZ
O2=tr[ū1X+ū2Z)(ū1X+ū2Z)
C123=Σ u1(i/L)ū2(i/L)=L∫u1(σ)ū2(σ)dσ


1.3
1.2
1.1
1.0
0.9
0.8
10
50
100
500
1000
5000
J


Fit[data, {1, 1/L, 1/L^2, 1/L^4, 1/L^5, 1/L^6}, L]
1.3
1.2
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0.9
0.8
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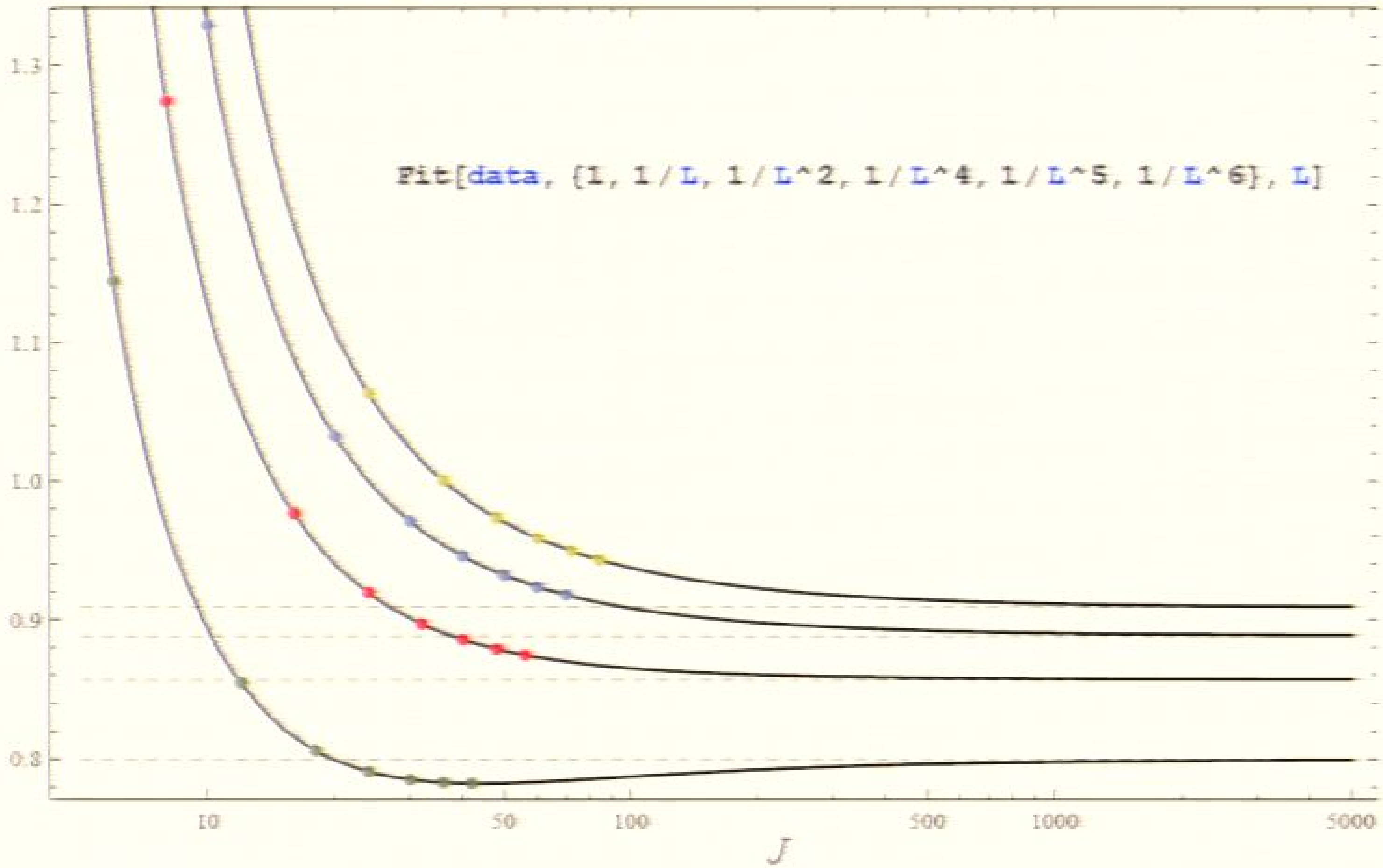
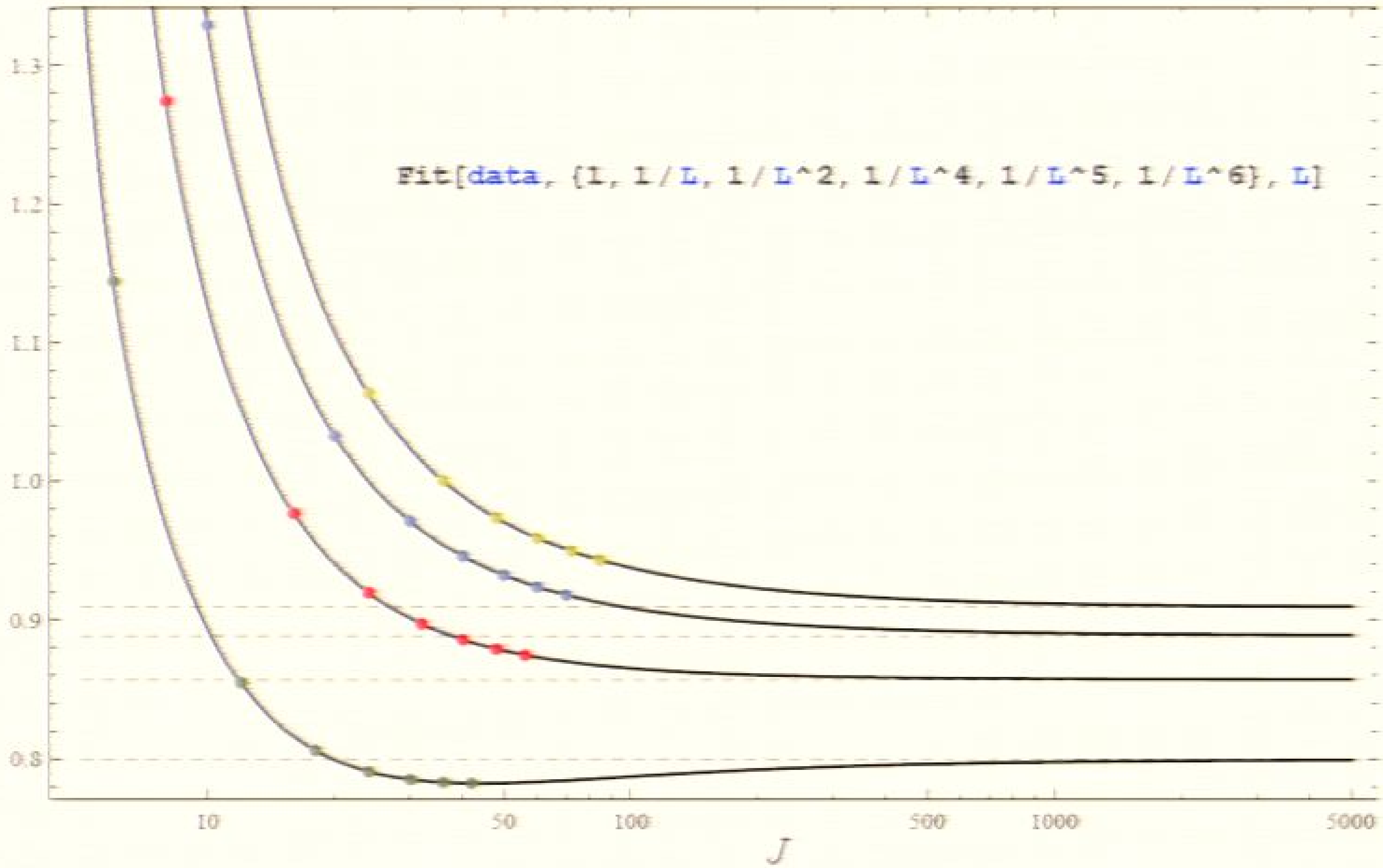


α	Prediction	Extrapolation	Error
$\frac{1}{3}$	0.7994912	0.7992237	3×10^{-4}
$\frac{1}{4}$	0.8568976	0.8568895	8×10^{-6}
$\frac{1}{5}$	0.8887595	0.8887601	6×10^{-7}
$\frac{1}{6}$	0.9090155	0.9090167	1×10^{-6}



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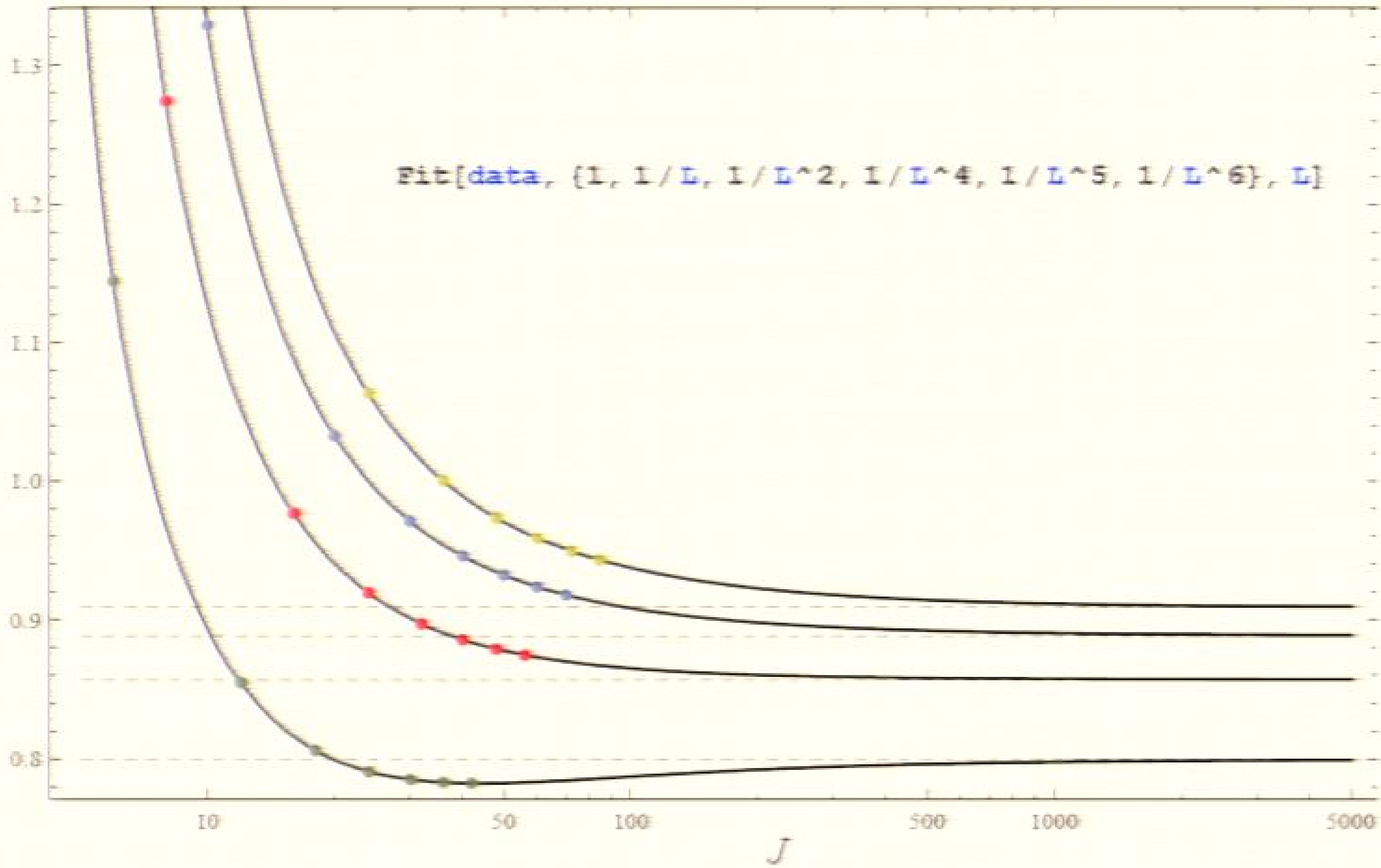

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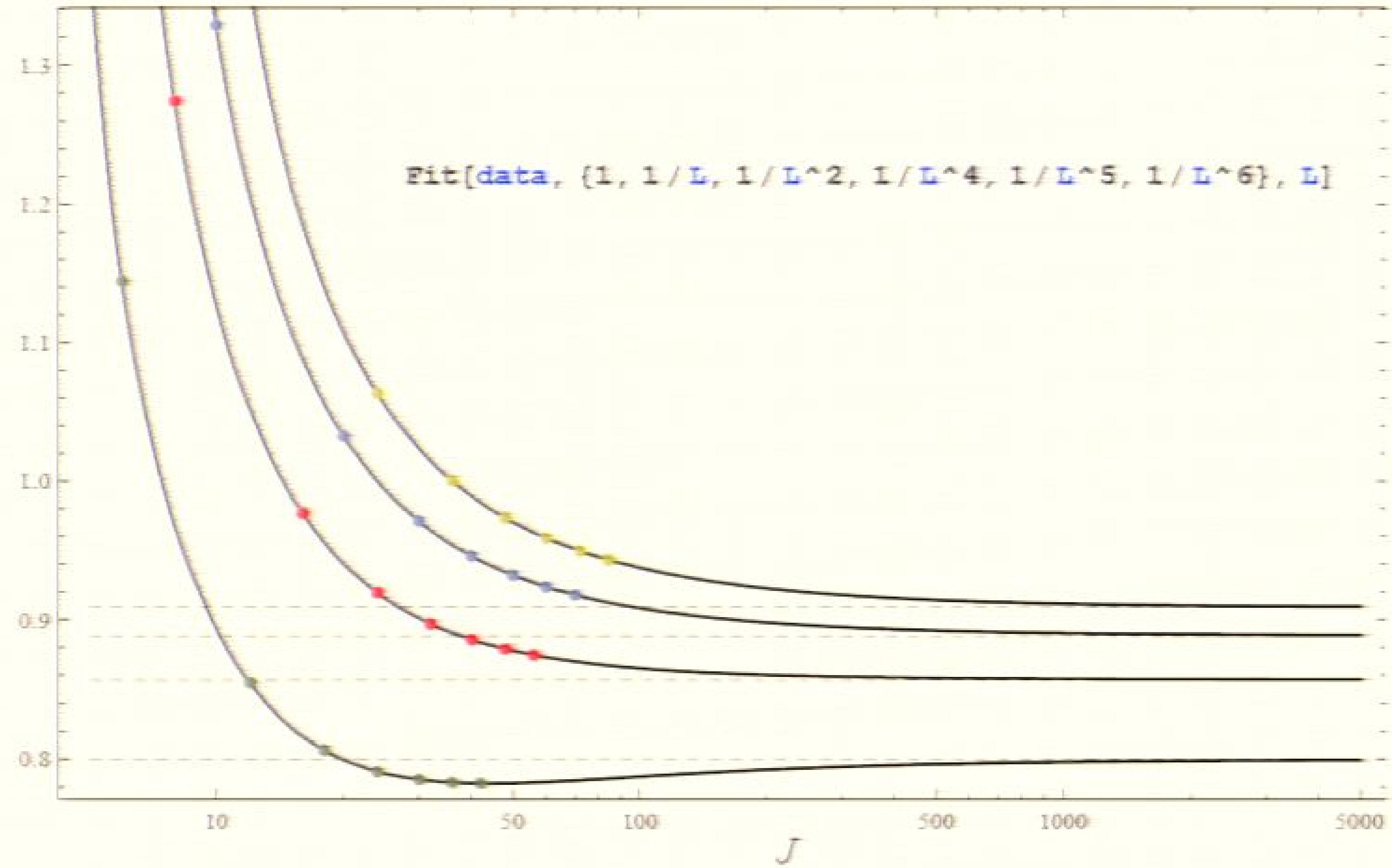

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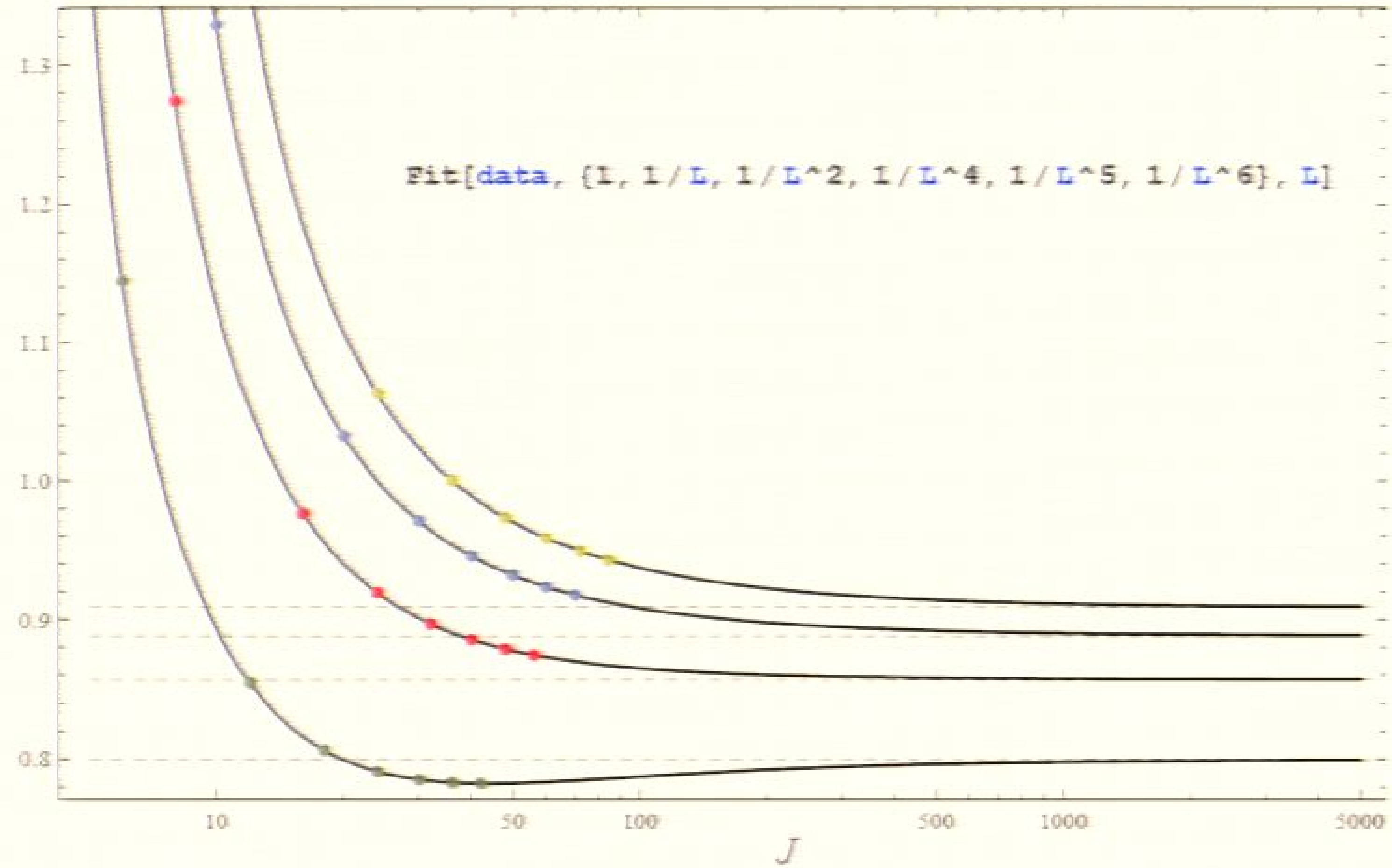

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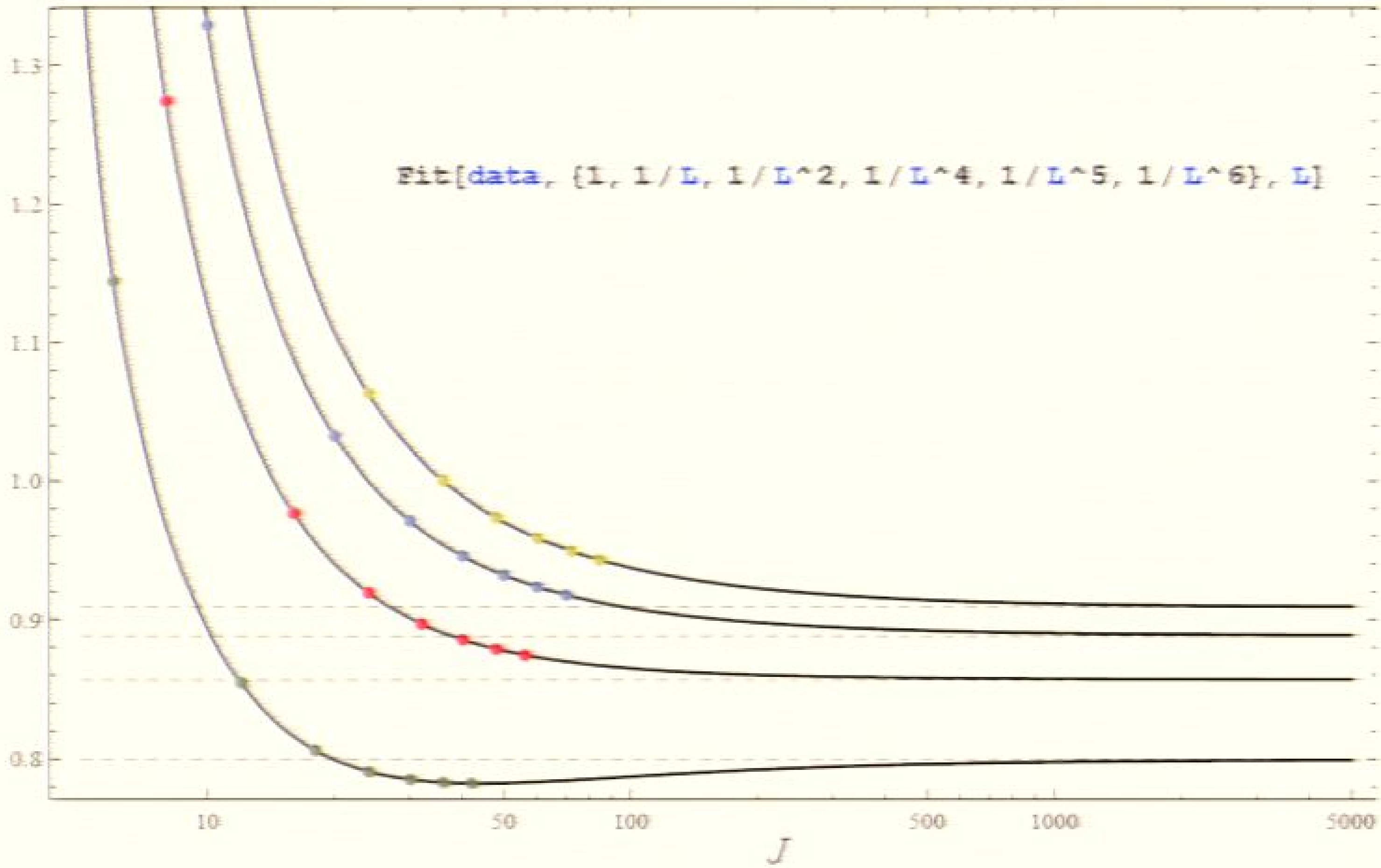

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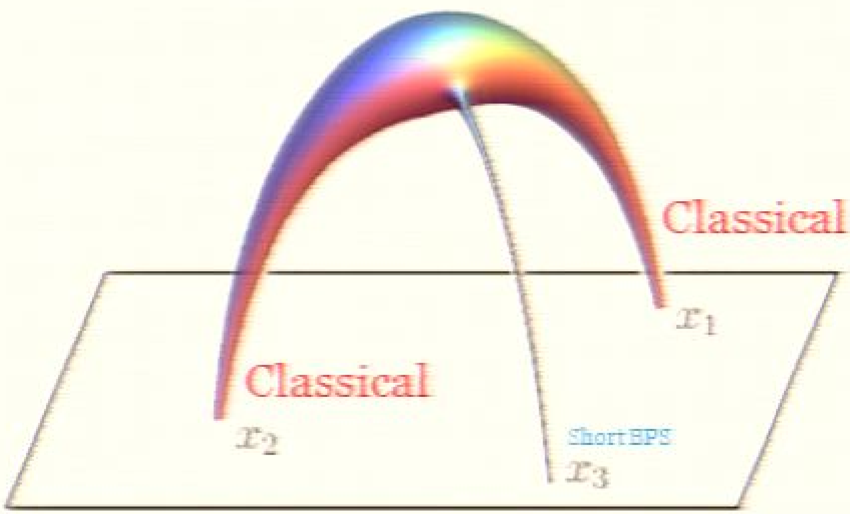

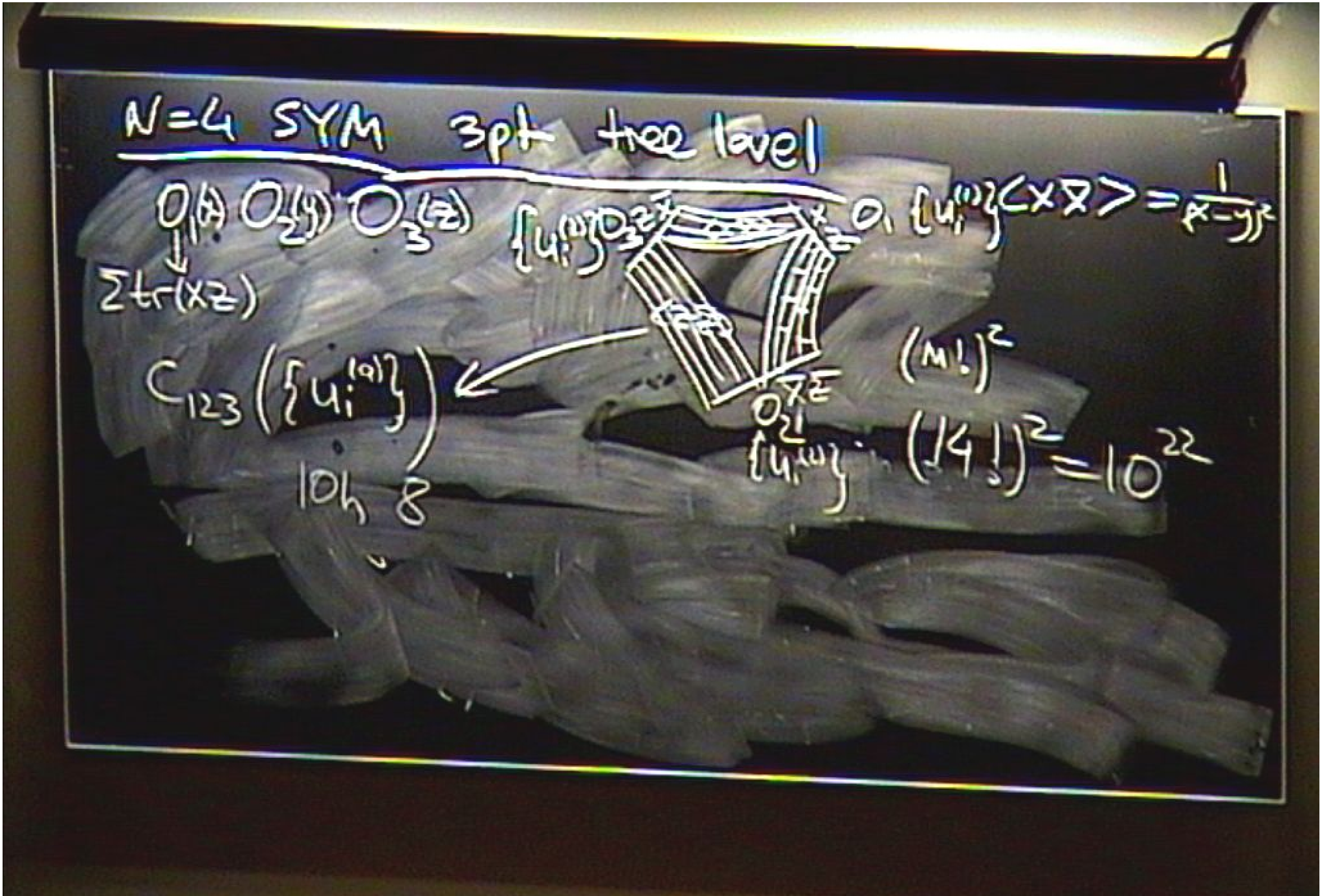
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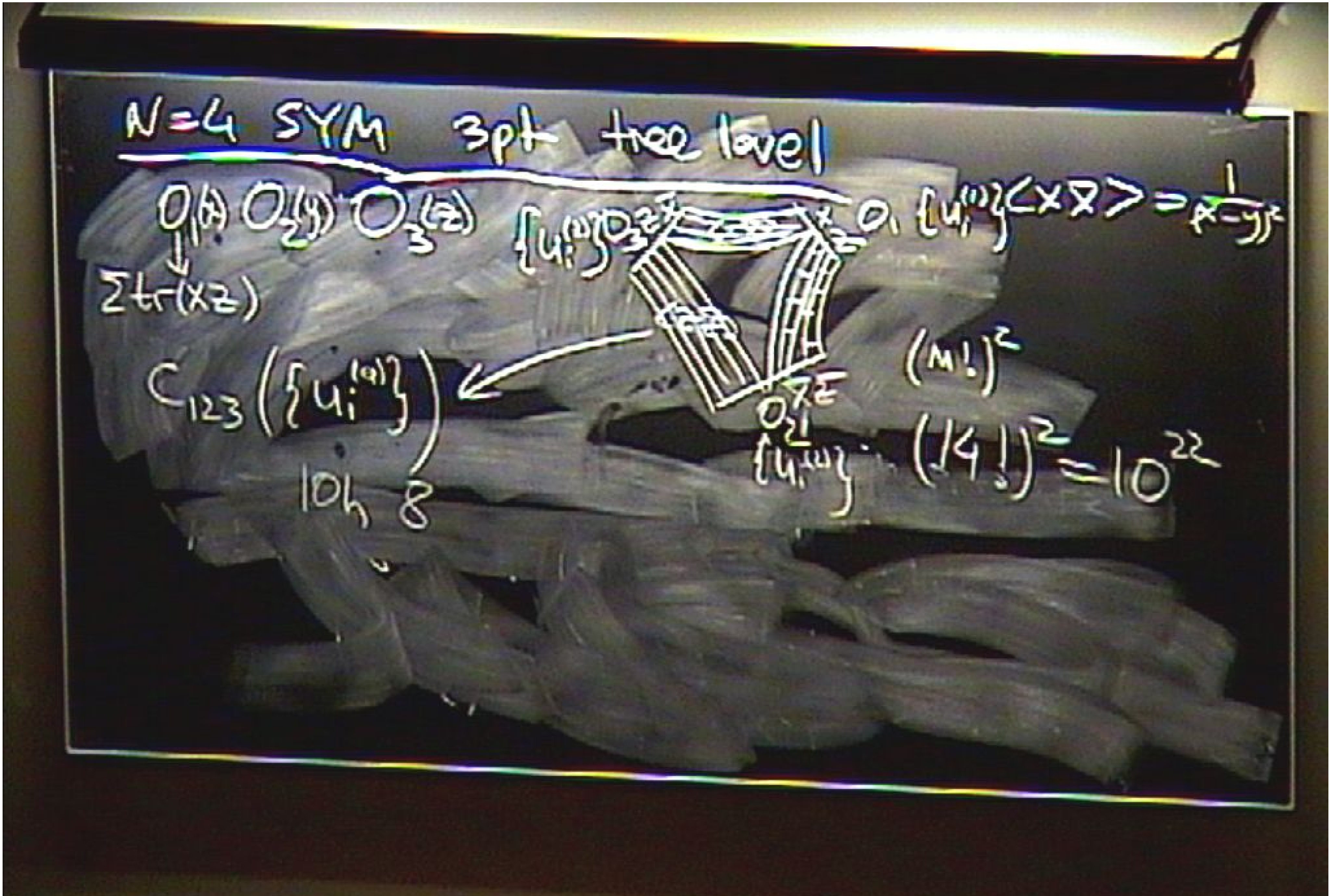
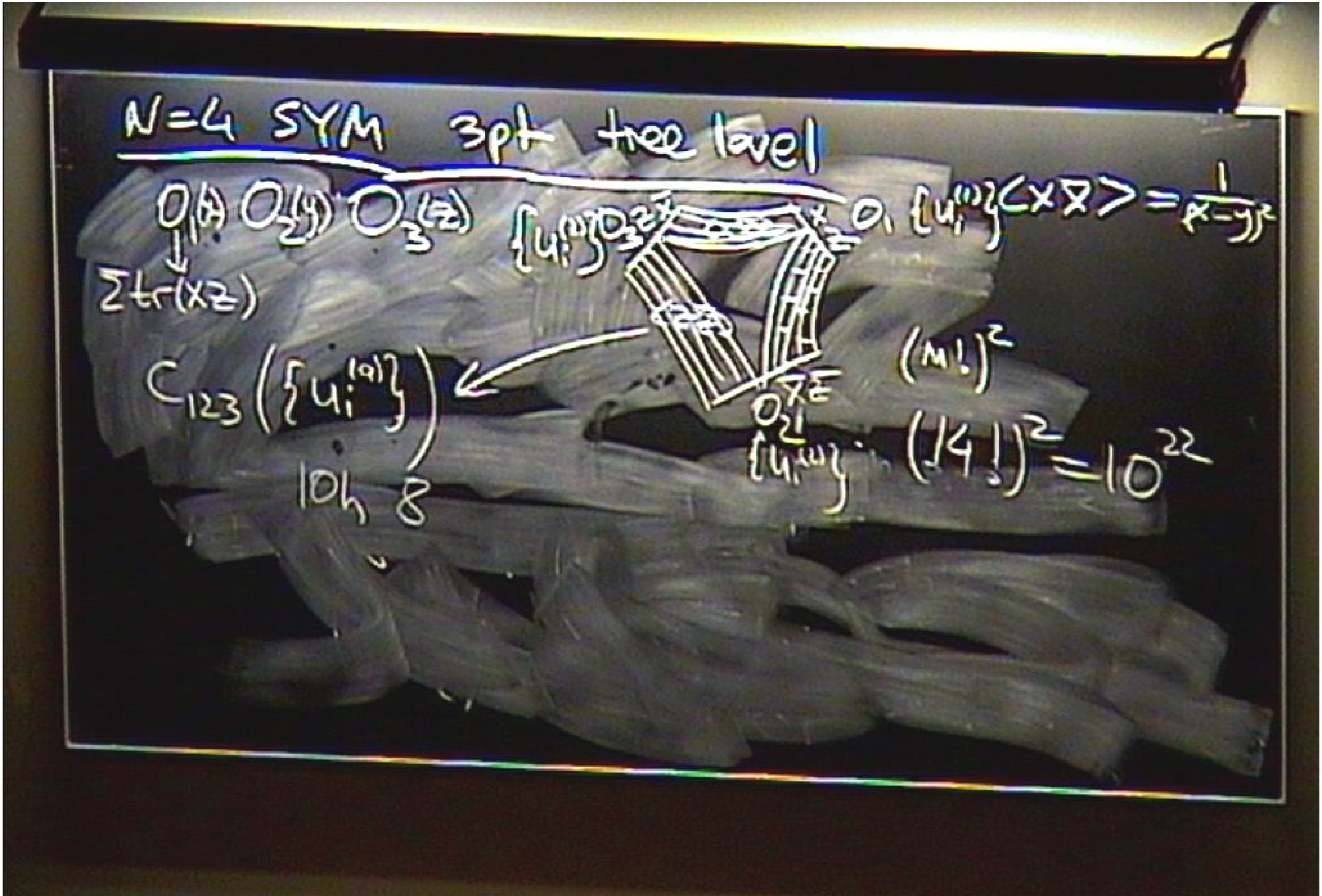
C123({ui(a)})
(M!)2
(14!)2 = 1022


N=4 SYM 3pt tree level
O1(x) O2(y) O3(z)
Σ tr(XZ)
C123({ui(a)})
{ui(3)} O3
O1 {ui(1)}
<X X̄> = 1/(x-y)²
O2
{ui(2)}
(M!)²
(14!)² = 10²²
10h 8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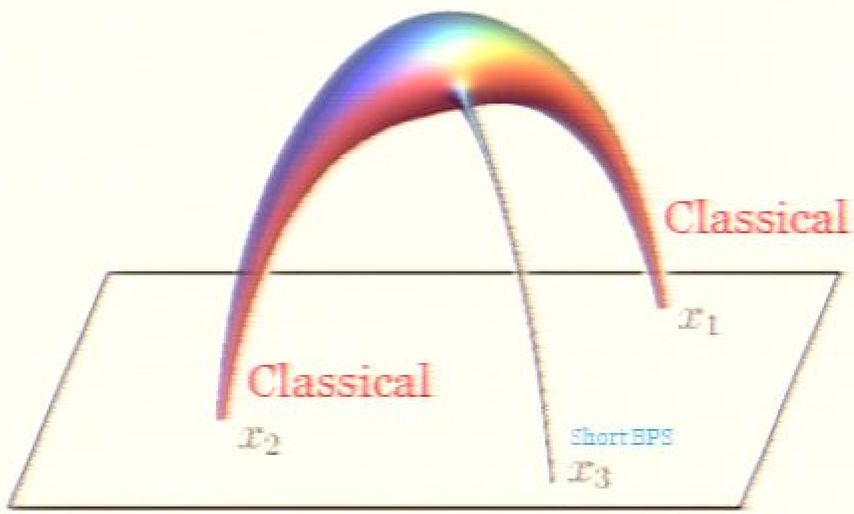
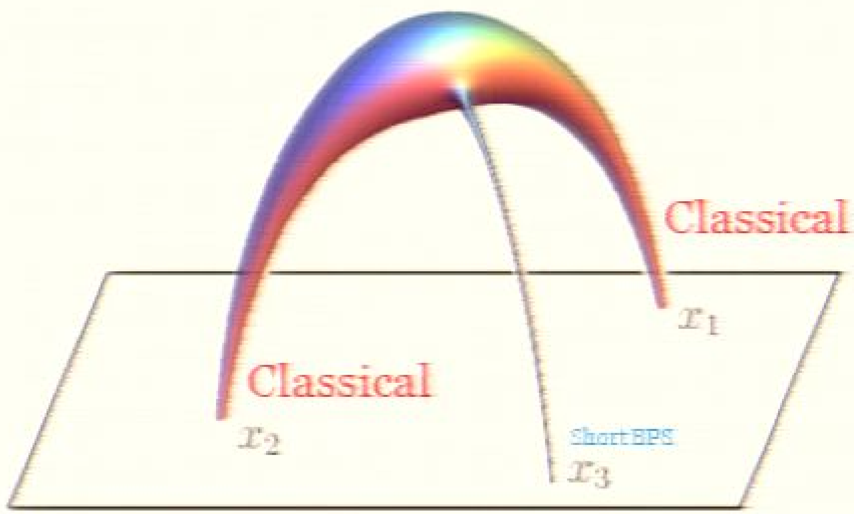
Strong coupling



Strong coupling

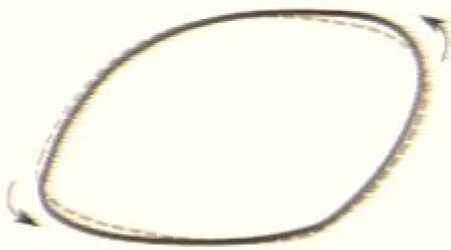


[Costa, Monteiro, Santos, Zoakos `10; Zarembo `10]

$$C = \int_{-\infty}^{\infty} d\tau_e \int_0^1 d\sigma \frac{u_1 \bar{u}_2}{\cosh(\kappa\tau_e)} \left[\frac{2\kappa^2}{\cosh^2(\kappa\tau_e)} - 2\partial_\sigma \bar{u} \partial_\sigma u \right]$$

Back-reaction

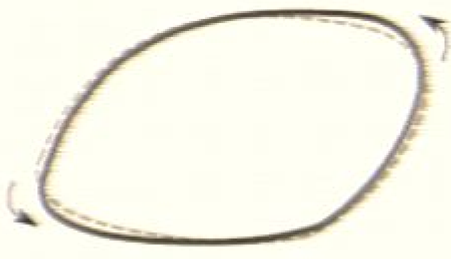
Back-reaction



(A)

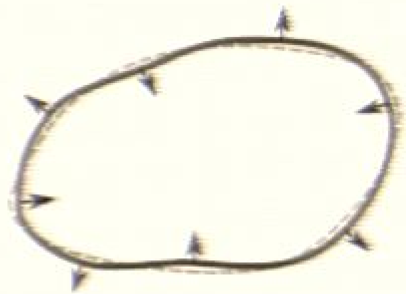
Global symmetry
transformation

Back-reaction



(A)

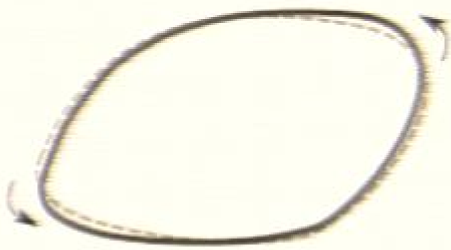
Global symmetry transformation



(B)

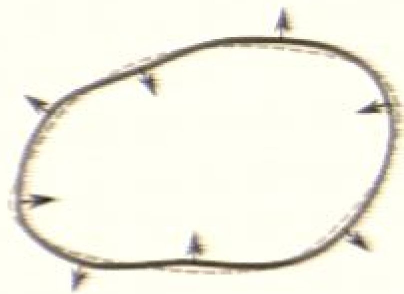
A quantum fluctuation

Back-reaction



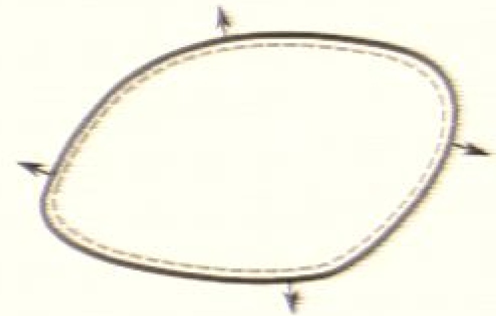
(A)

Global symmetry transformation



(B)

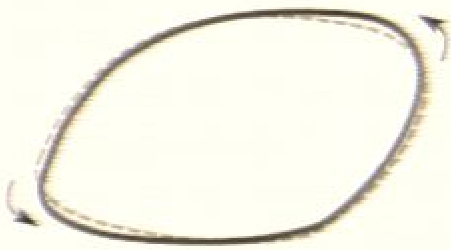
A quantum fluctuation



(C)

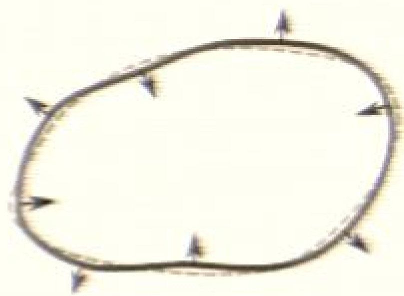
“Zero mode”

Back-reaction



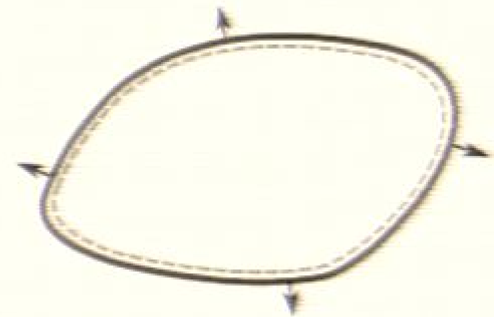
(A)

Global symmetry transformation



(B)

A quantum fluctuation



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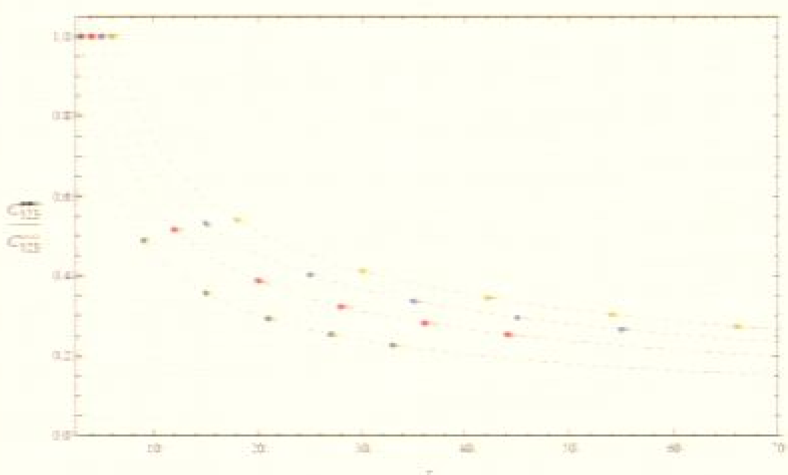
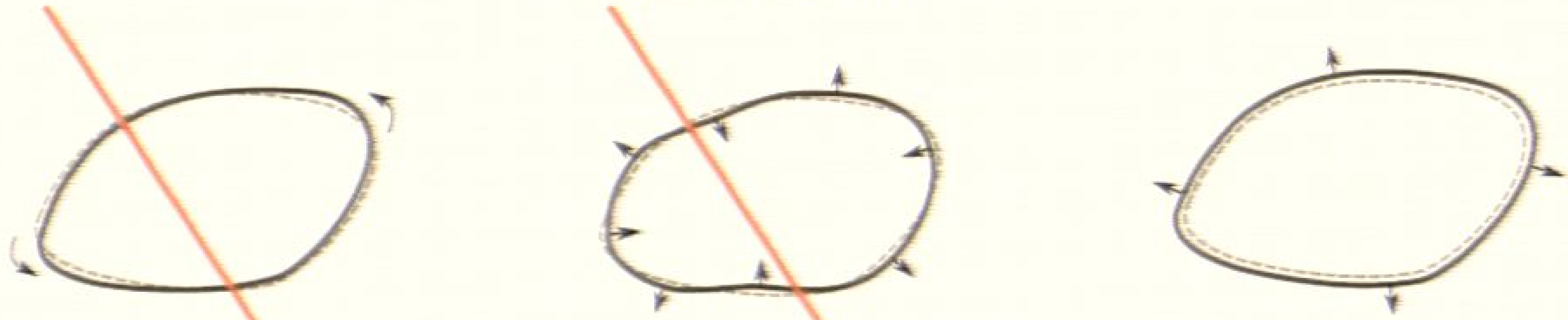


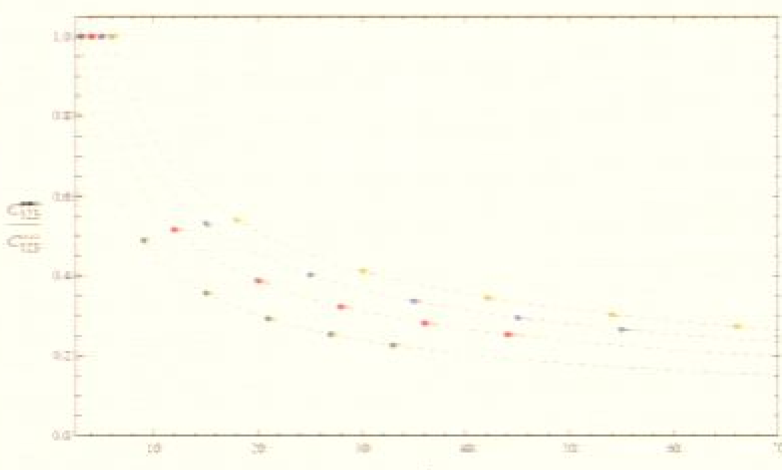
Back-reaction



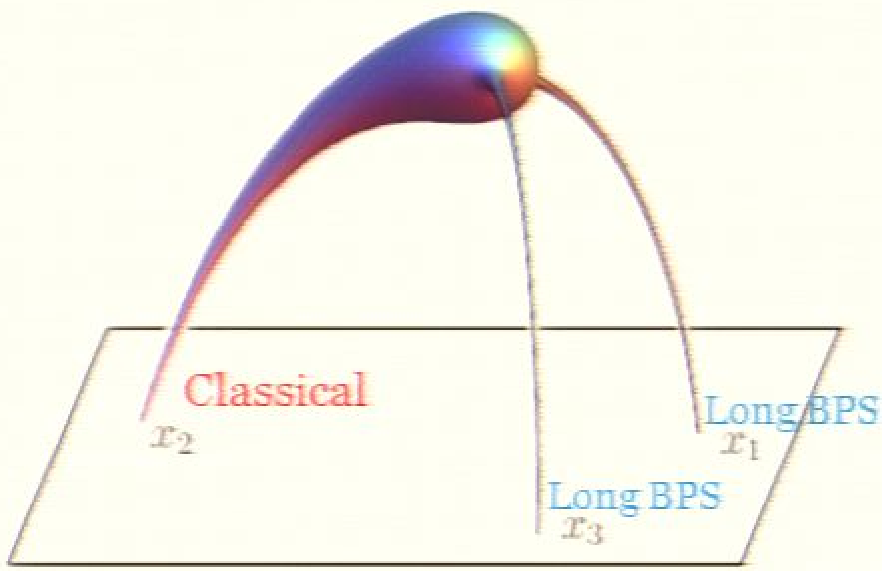
(A) Global symmetry transformation

(B) A quantum fluctuation

(C) "Zero mode"



Heavy-Light-Light



$$C_{123} \simeq \mathscr{A}(l, \{u_k\})$$

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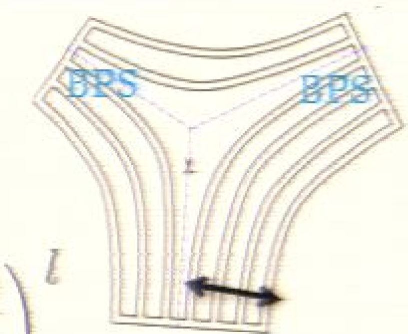
$$\mathscr{A}(l, \{u_k\}) = \sum_{\alpha \cup \bar{\alpha} = \{u_k\}} \prod_{u \in \alpha} \prod_{v \in \bar{\alpha}} \left(1 + \frac{i}{v - u}\right) \left(\frac{v + i/2}{v - i/2}\right)^l$$

$$C_{123} \simeq \mathscr{A}(l, \{u_k\})$$



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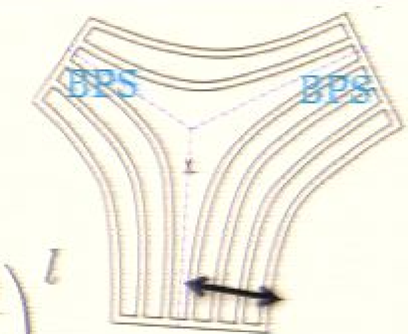

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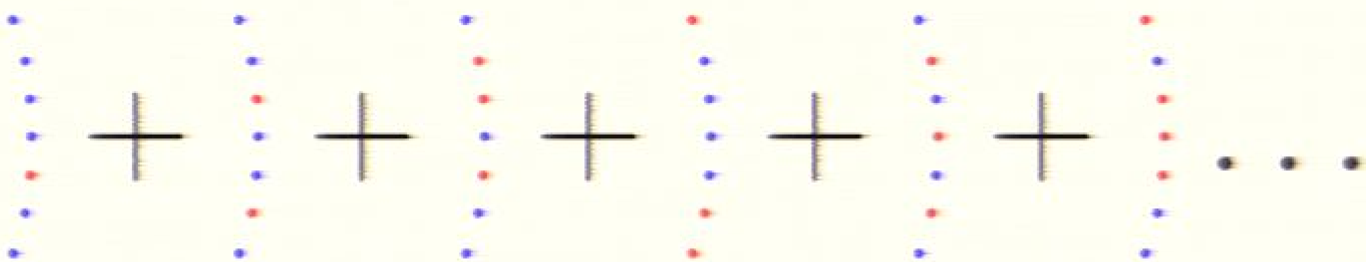

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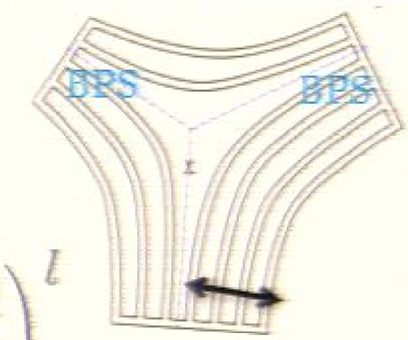

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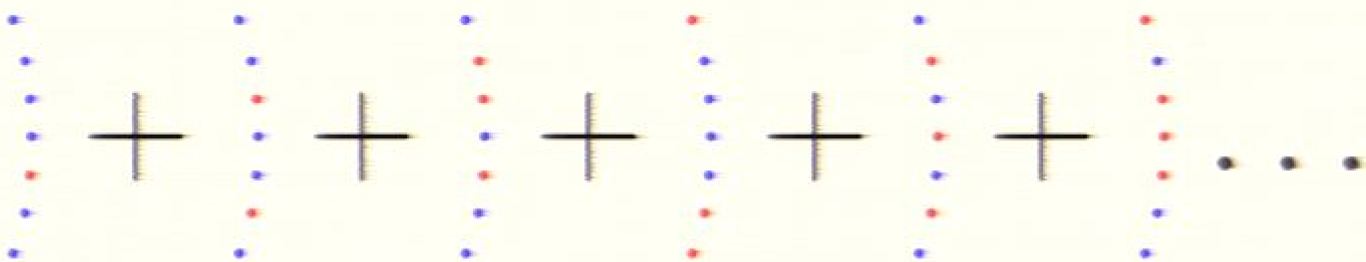
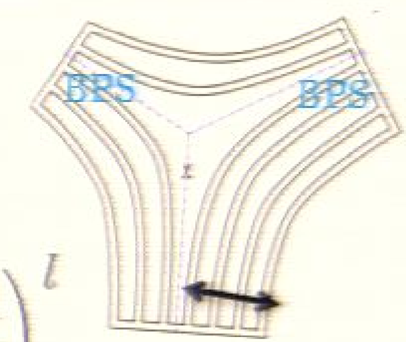


$$\mathscr{A} \simeq \int \mathcal{D}\rho_a \mathcal{D}\rho_b \delta(\rho_a + \rho_b - \rho) \mu(\rho_a, \rho_b) \exp\left[\int \frac{il}{2} \int du \frac{\rho_b}{u} + \ldots\right]$$



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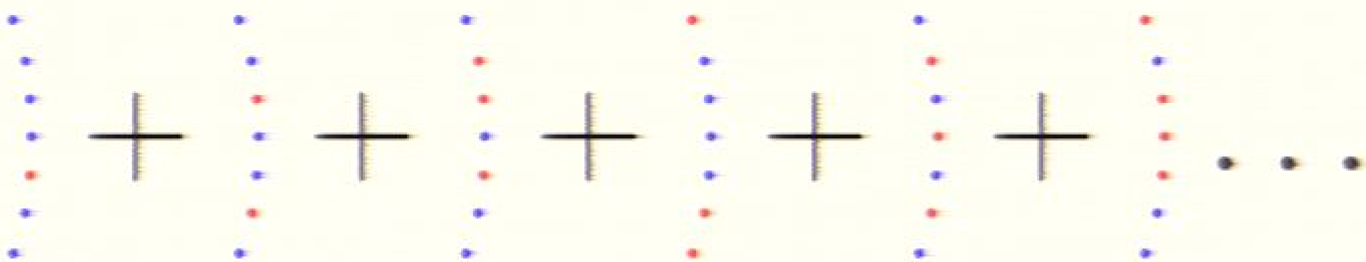

$$\mathscr{A} \simeq \int \mathcal{D}\rho_a \mathcal{D}\rho_b \delta(\rho_a + \rho_b - \rho) \mu(\rho_a, \rho_b) \exp\left[\int \frac{il}{2} \int du \frac{\rho_b}{u} + \dots\right]$$

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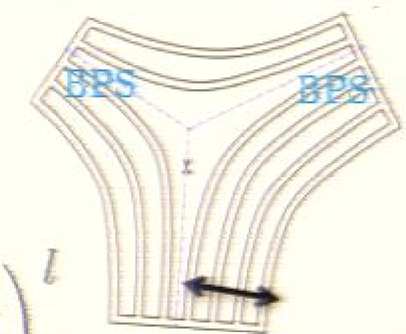

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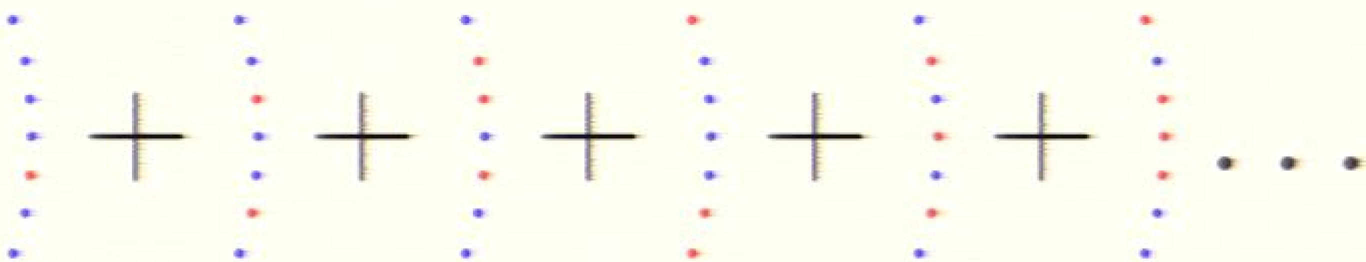

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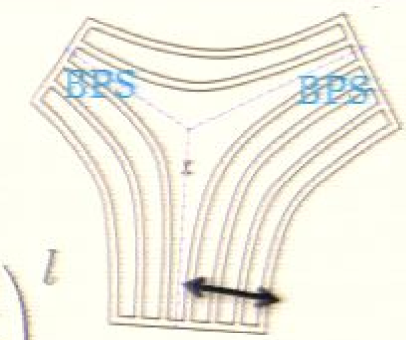

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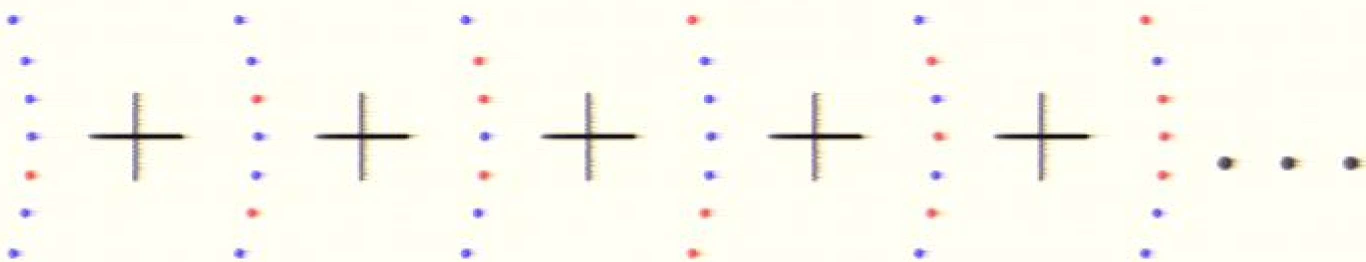

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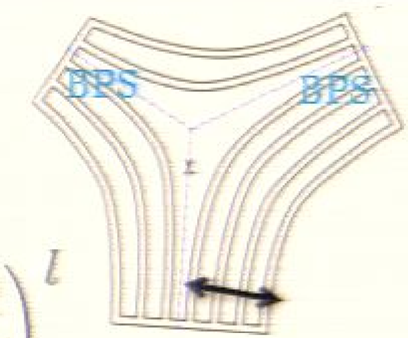

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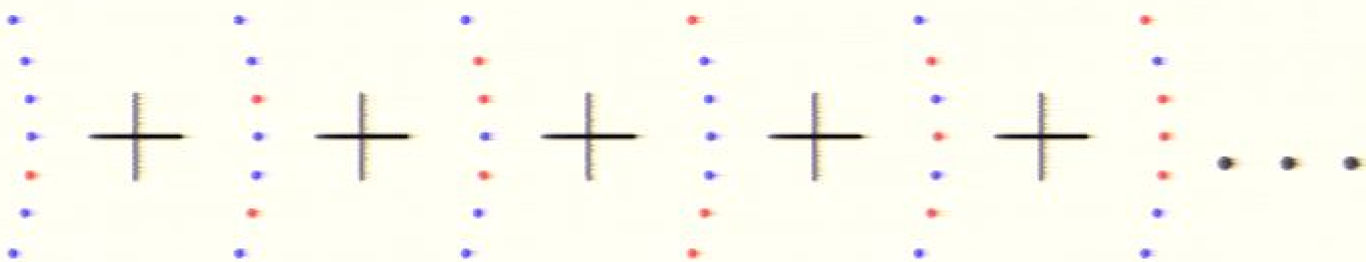

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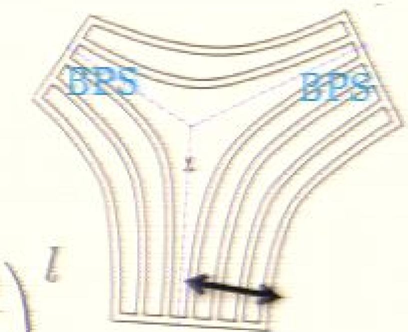

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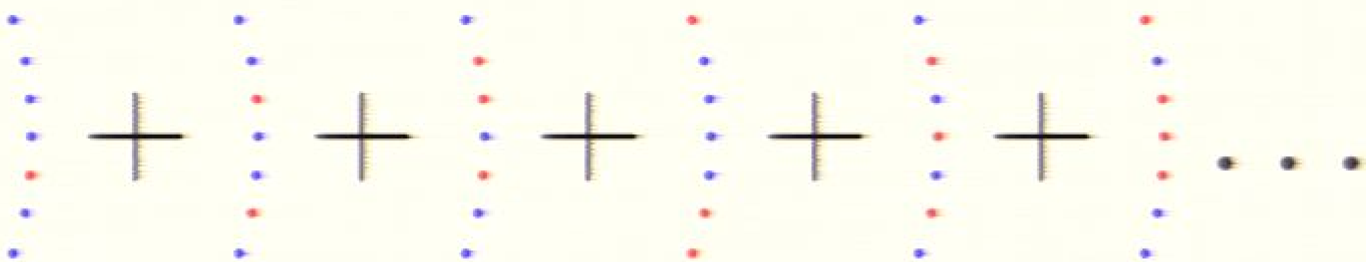

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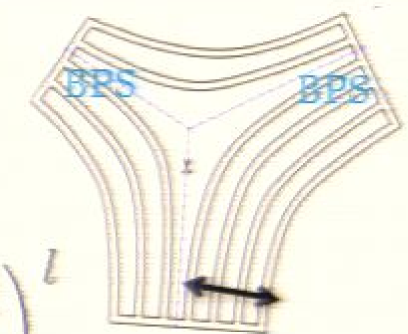

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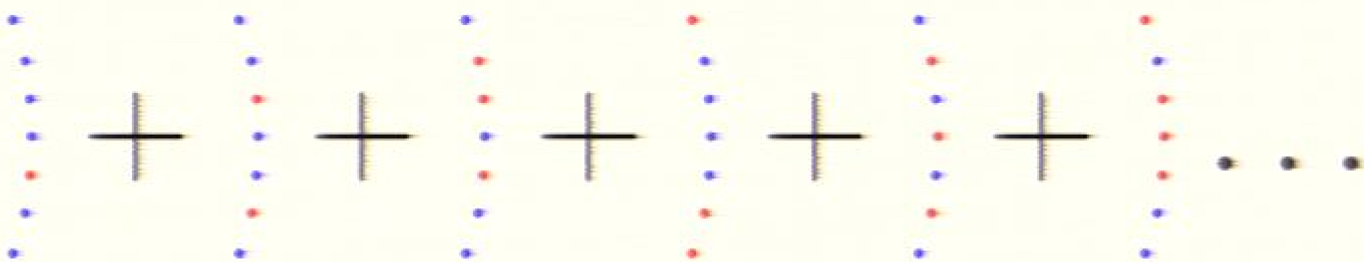

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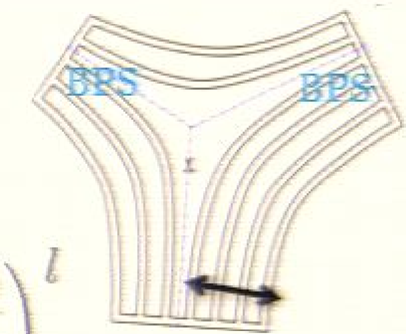

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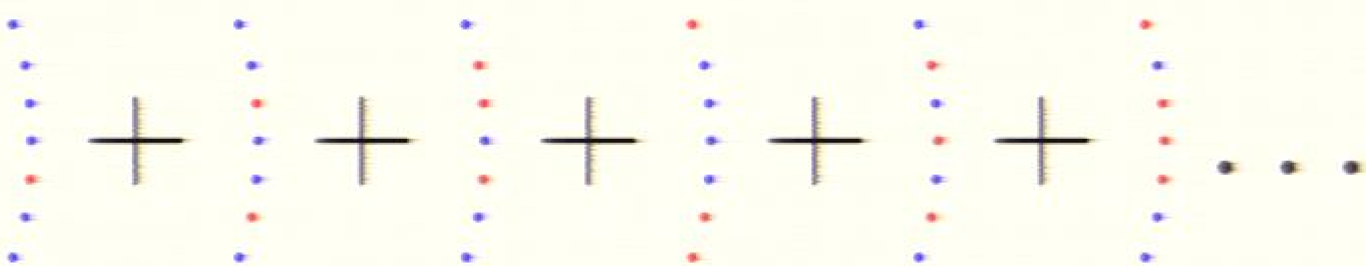

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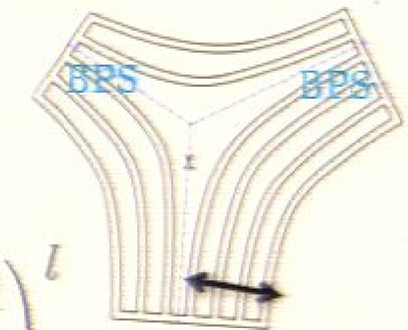

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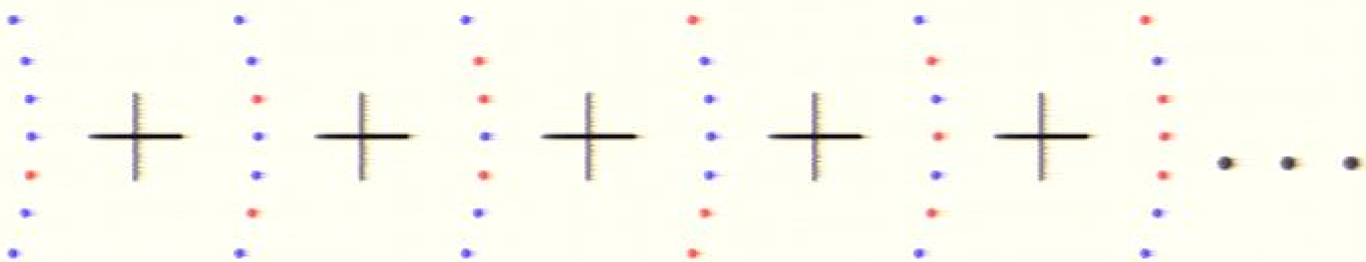

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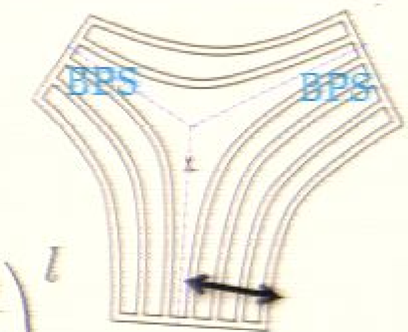

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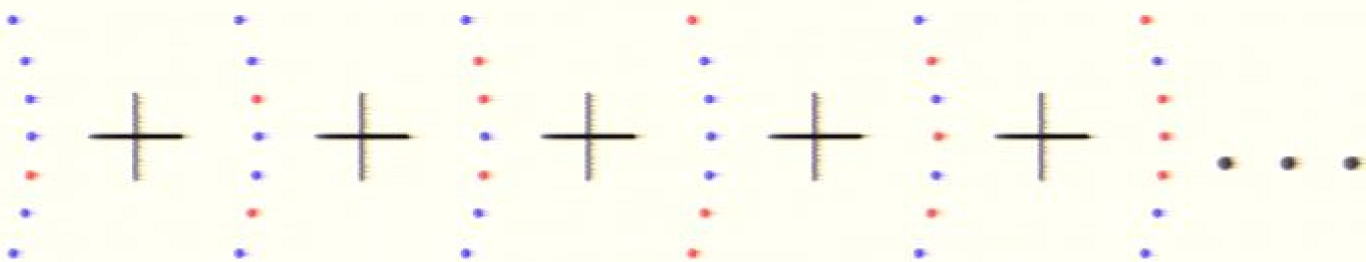

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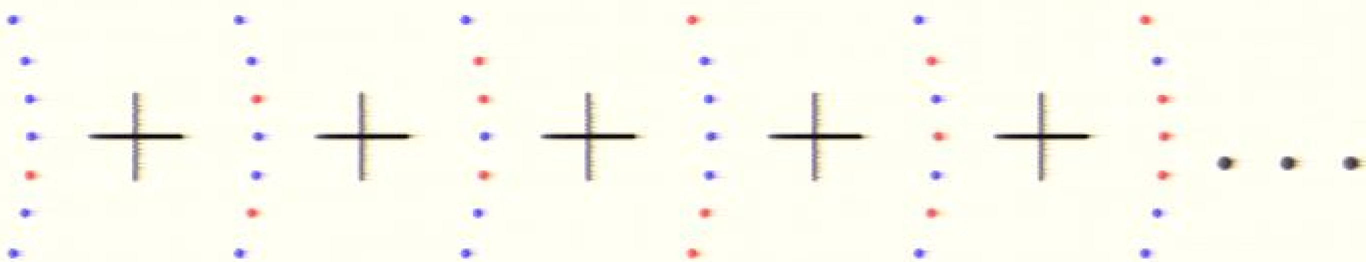

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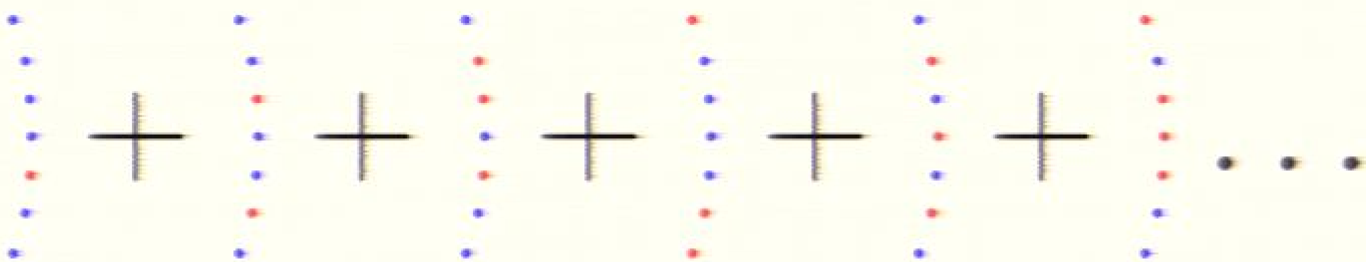

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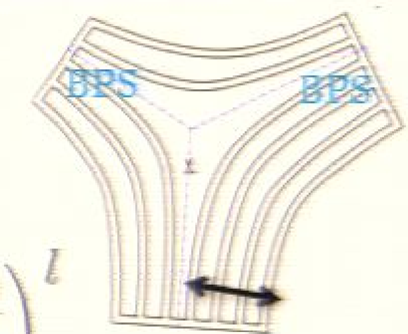

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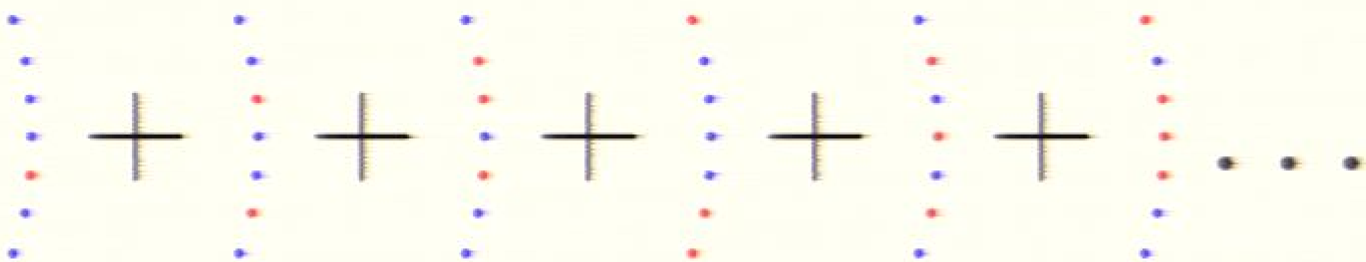

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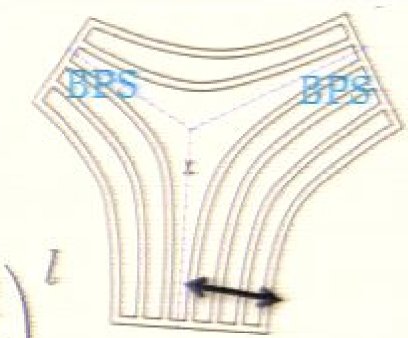

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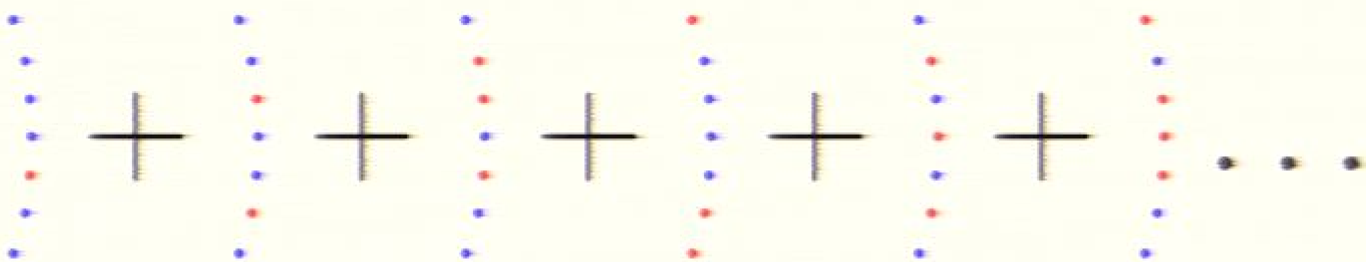

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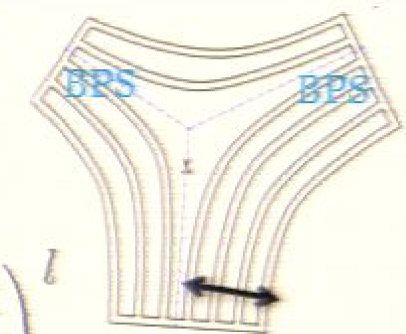

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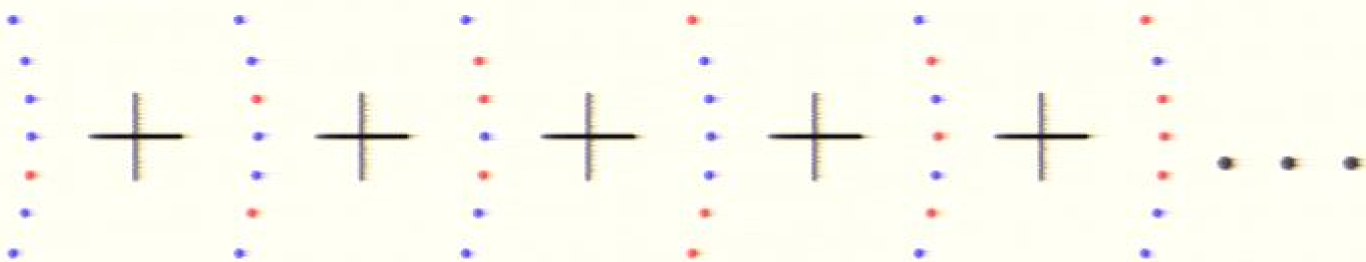

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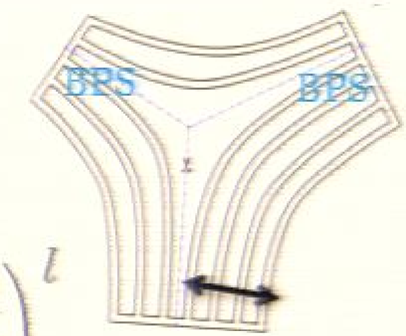

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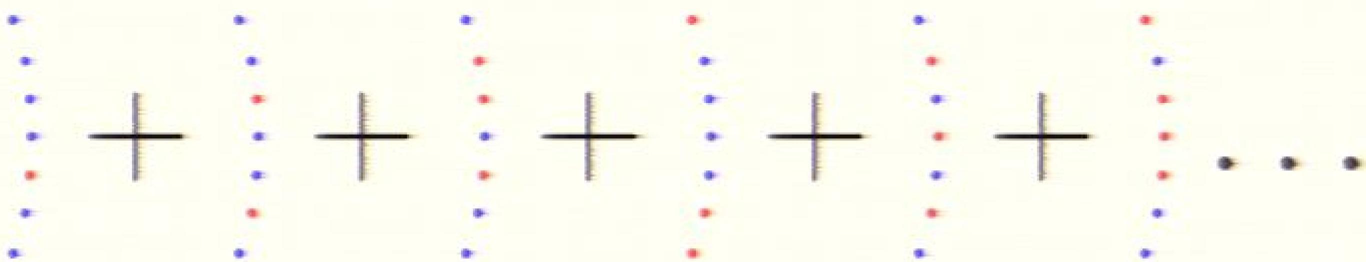

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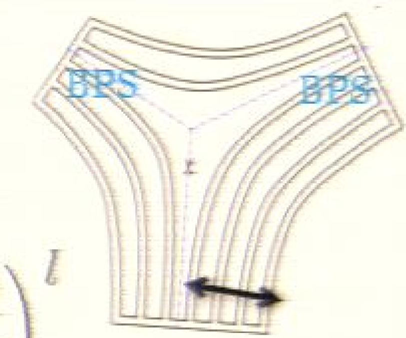

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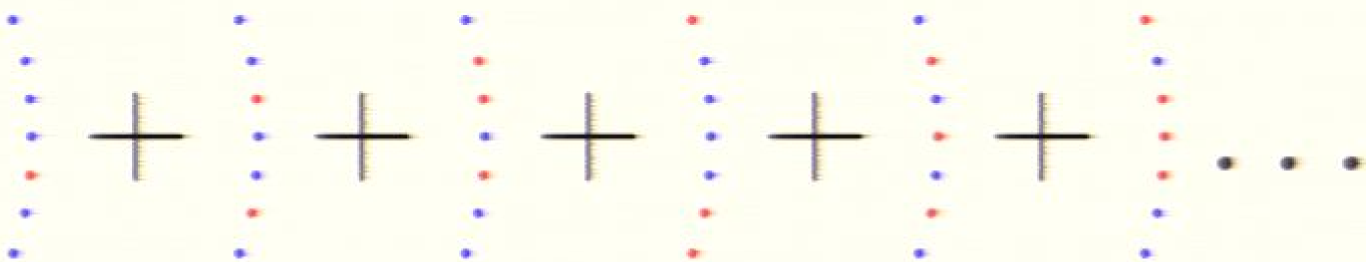

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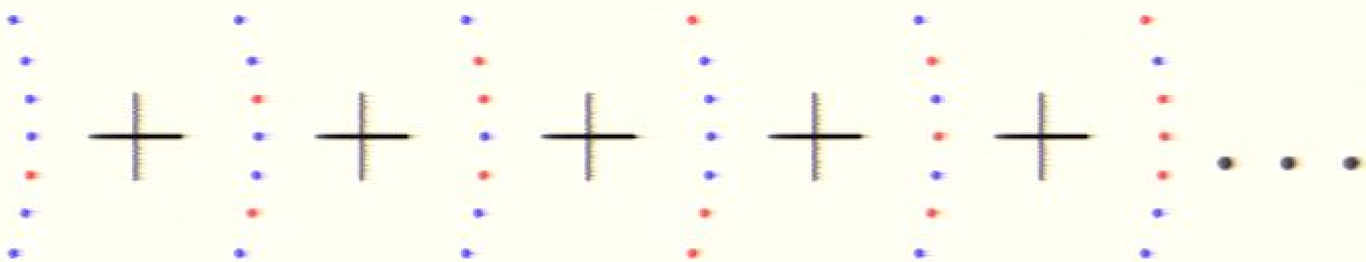

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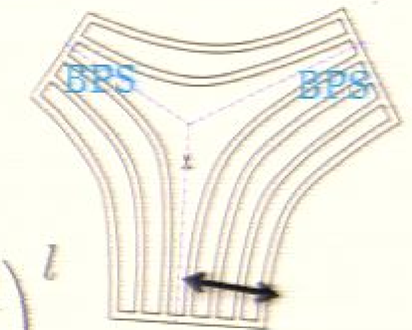

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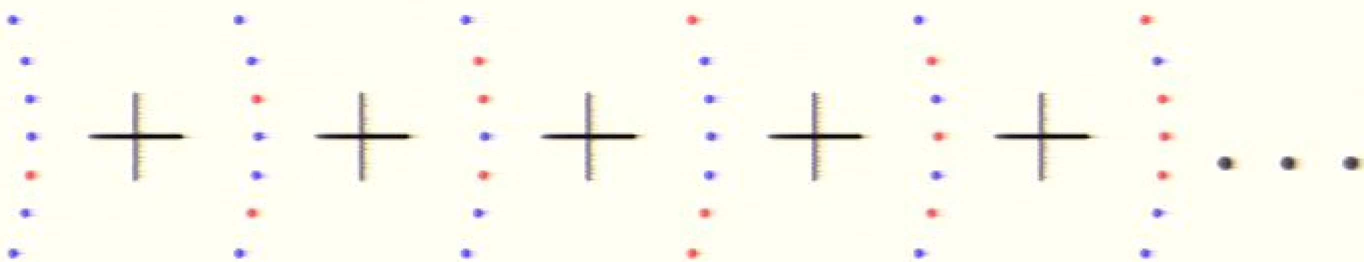

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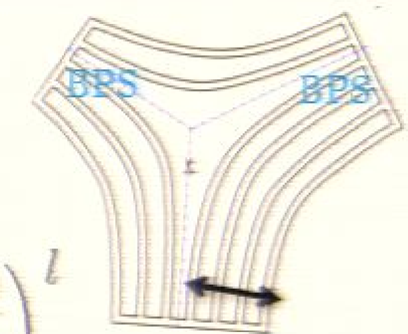

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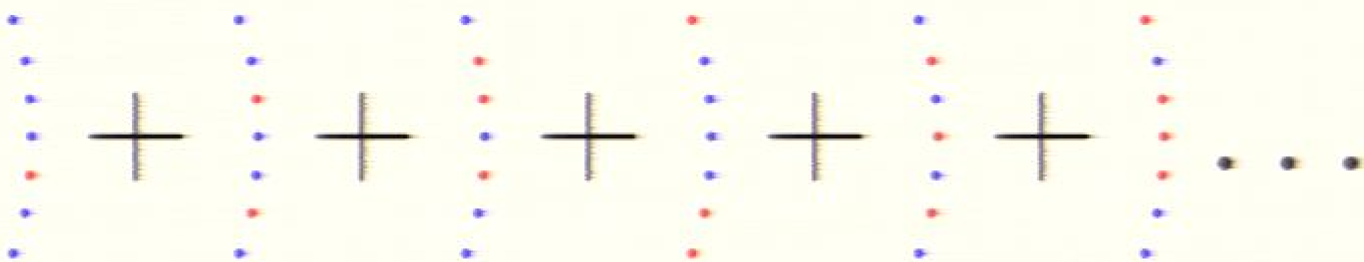

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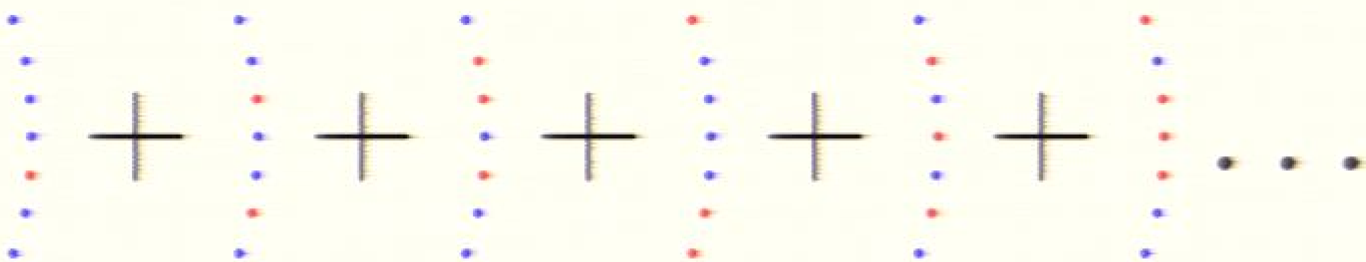

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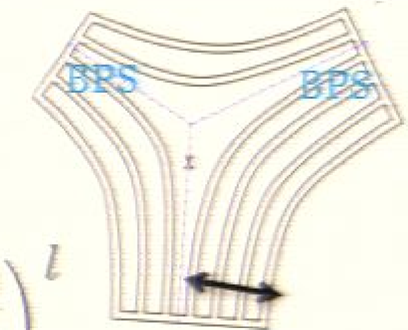
$$\frac{\pi}{12} - \frac{\pi\rho^2}{2} + \rho \log(2\pi\rho) - \frac{\mathrm{Li}_2(e^{-2\pi\rho})}{2\pi}$$



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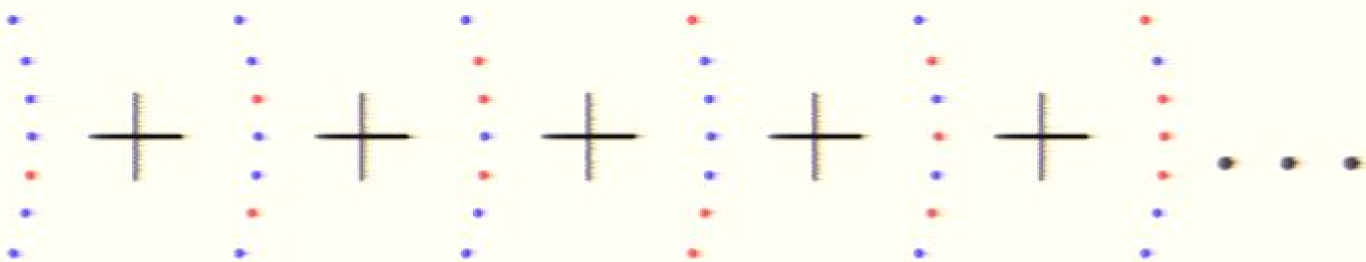

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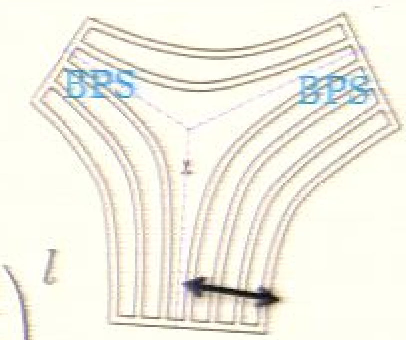

$$C_{123} \simeq \mathscr{A}(l, \{u_k\})$$

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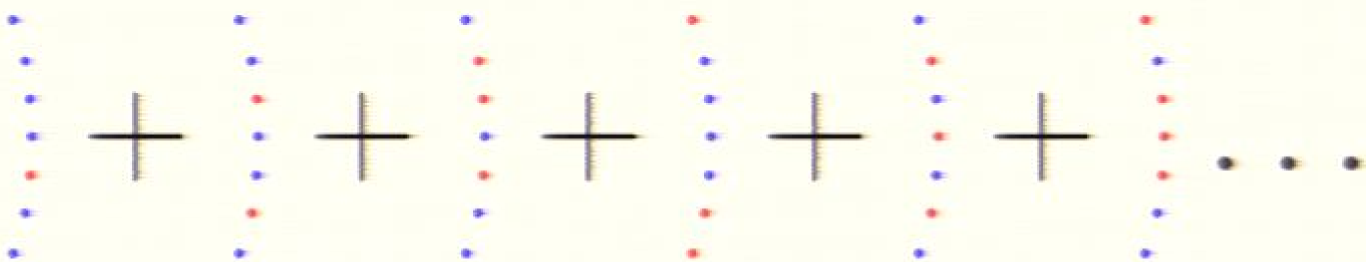

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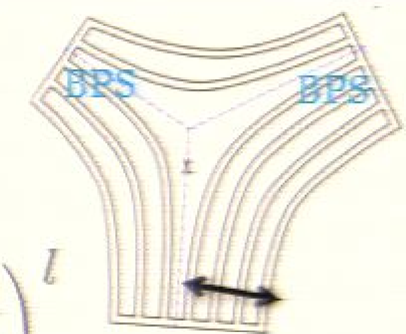

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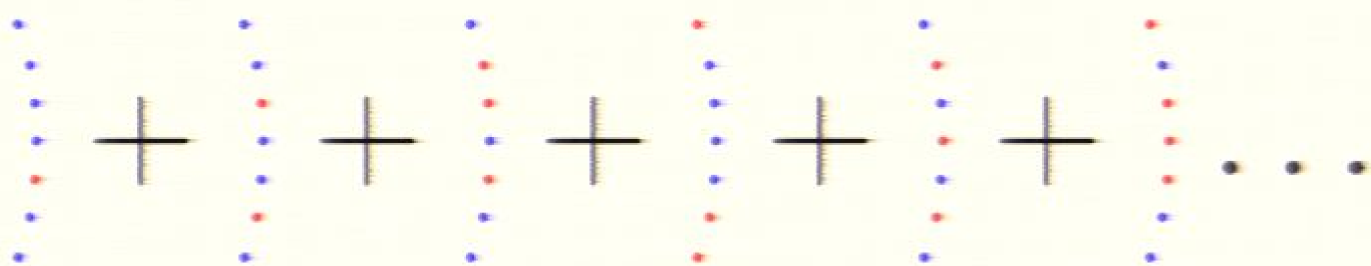

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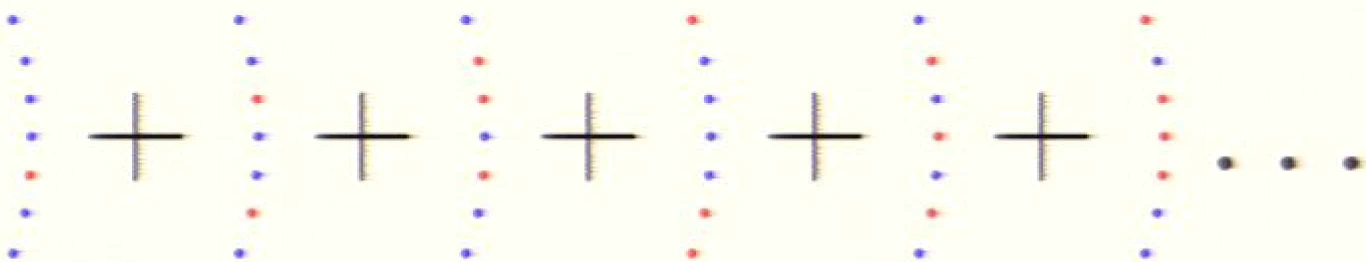

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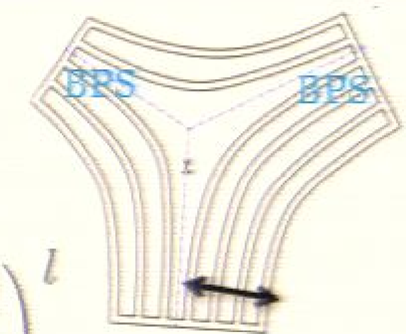

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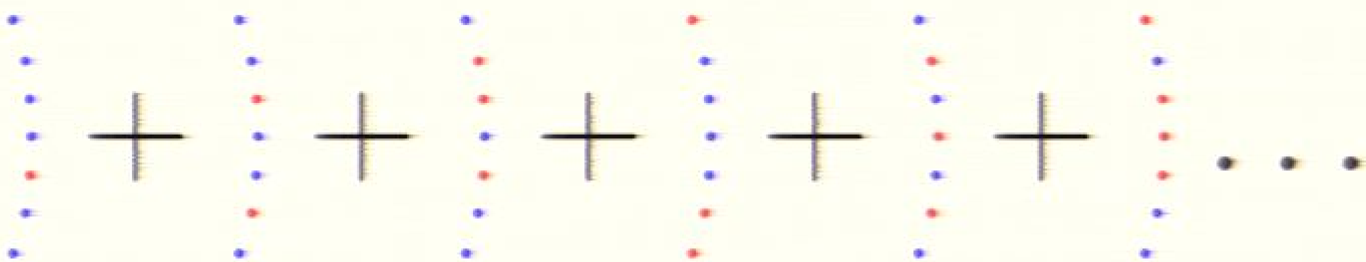

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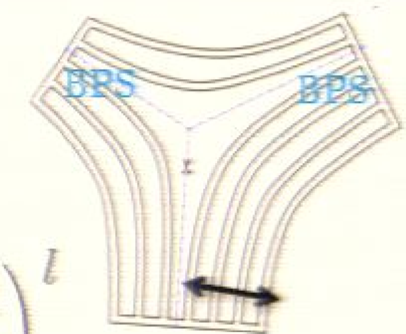

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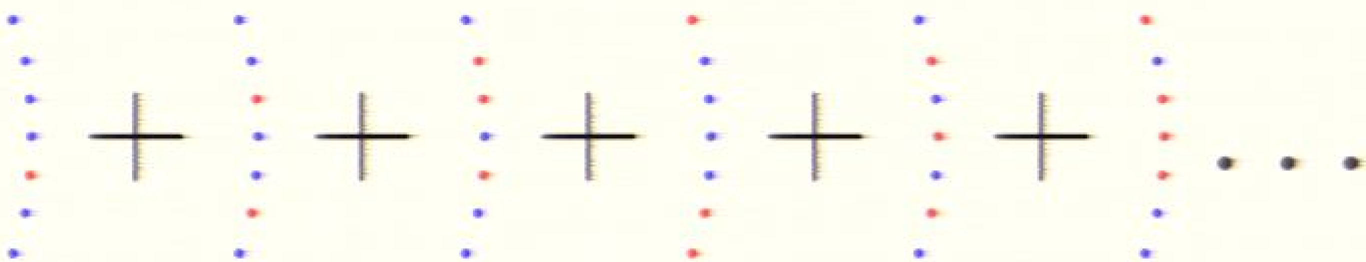

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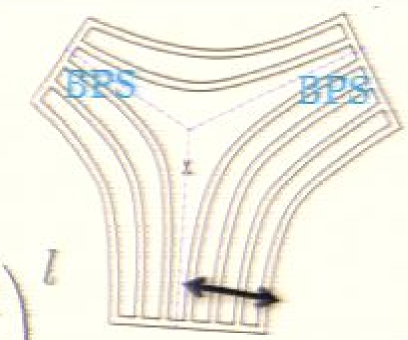

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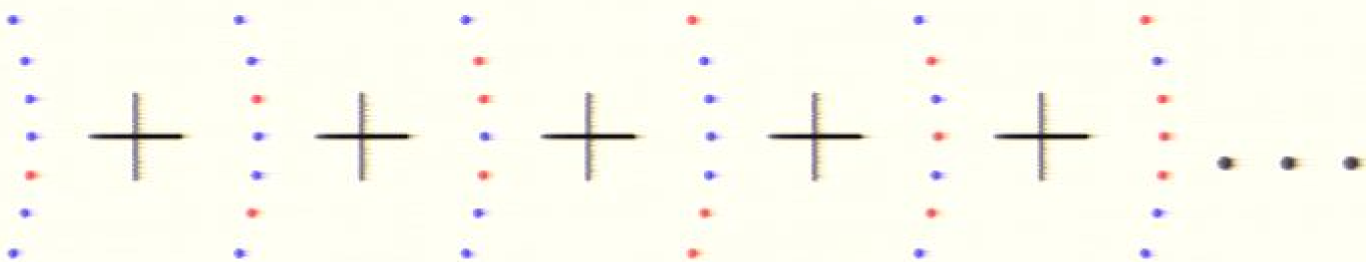

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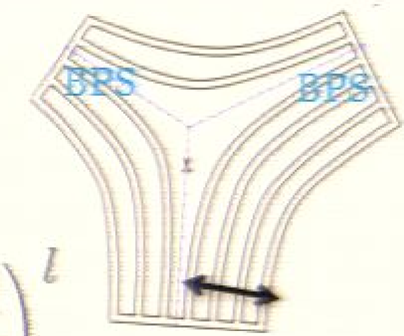

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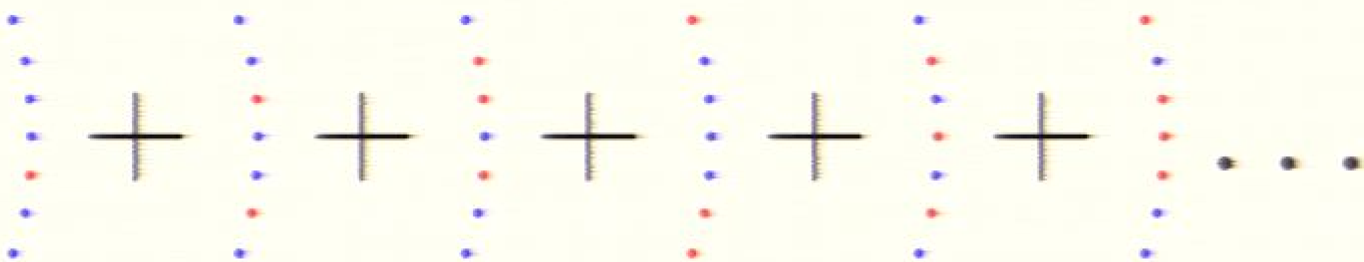

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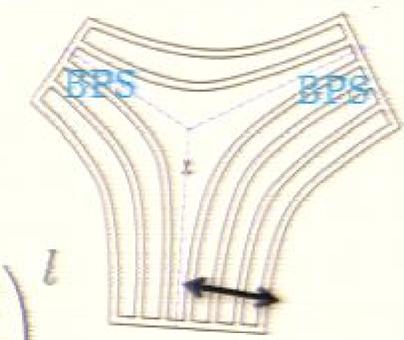

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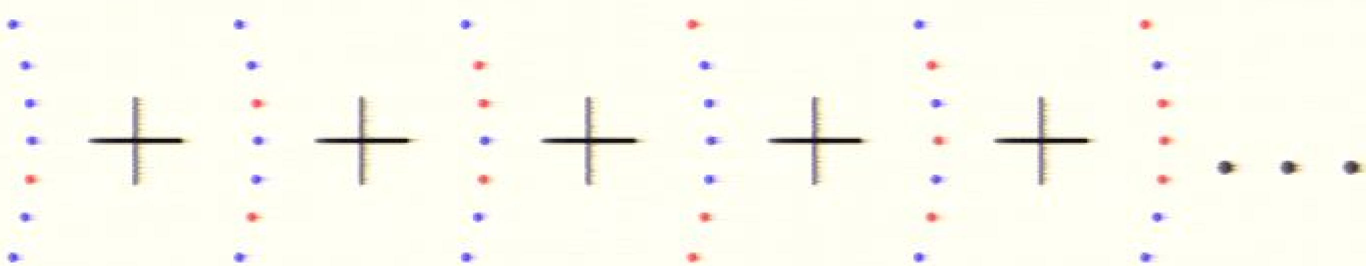

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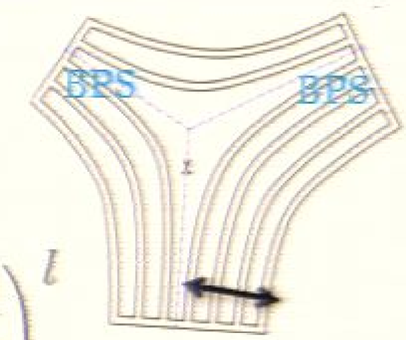

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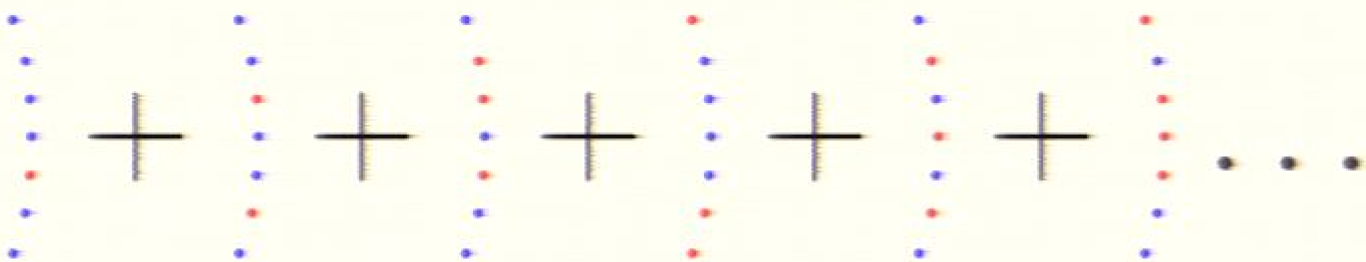

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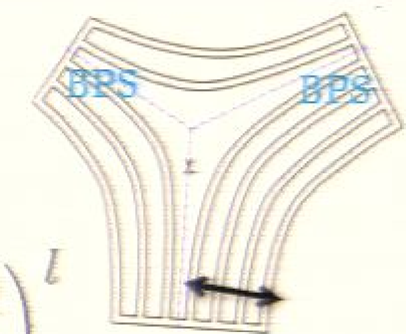

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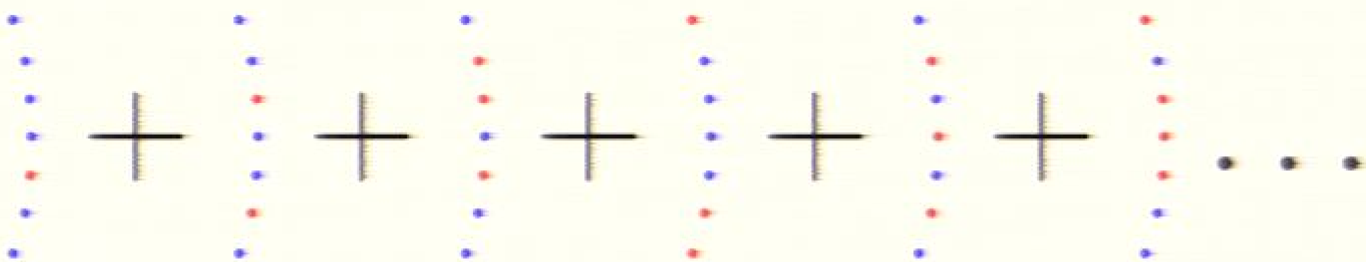

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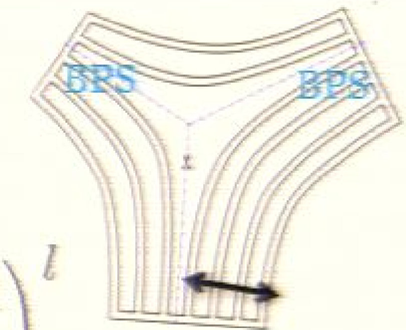

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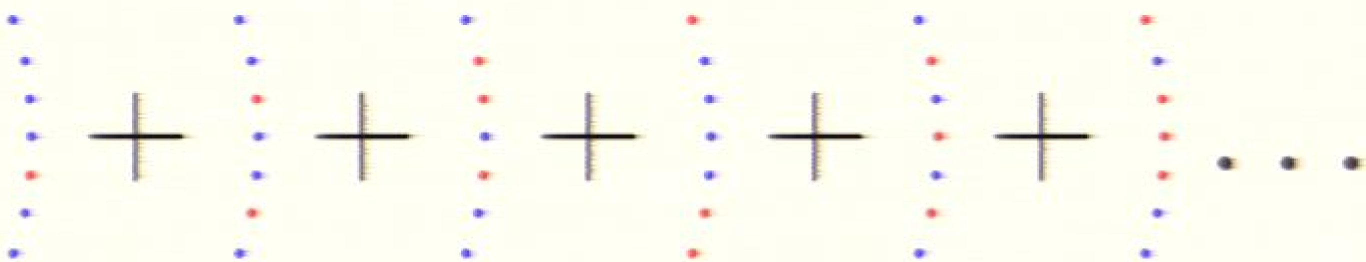

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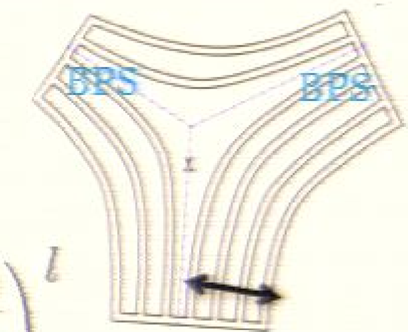

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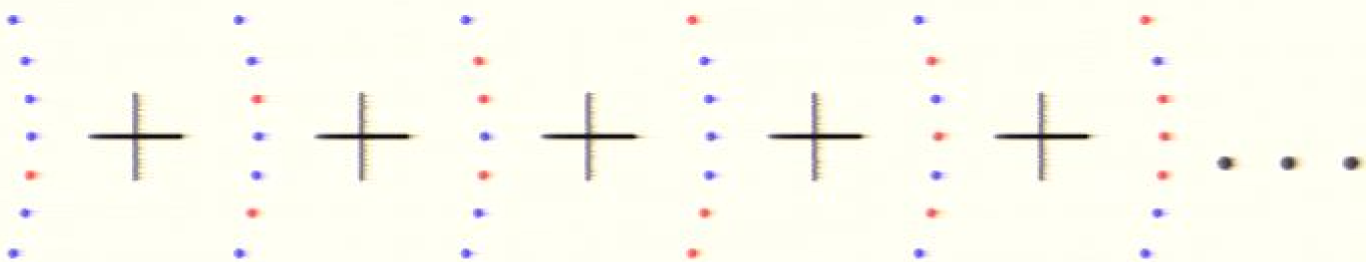

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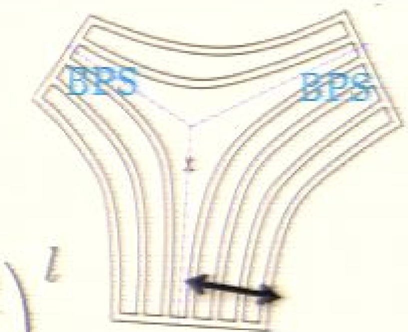

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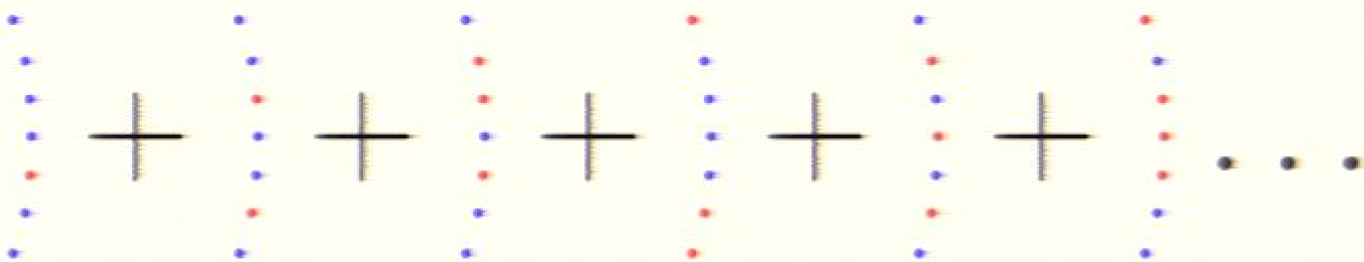

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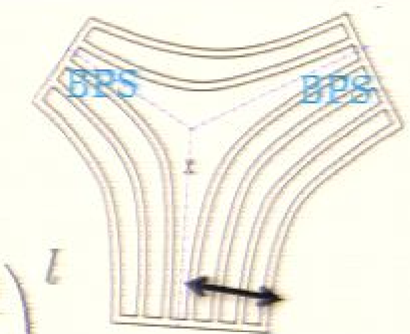

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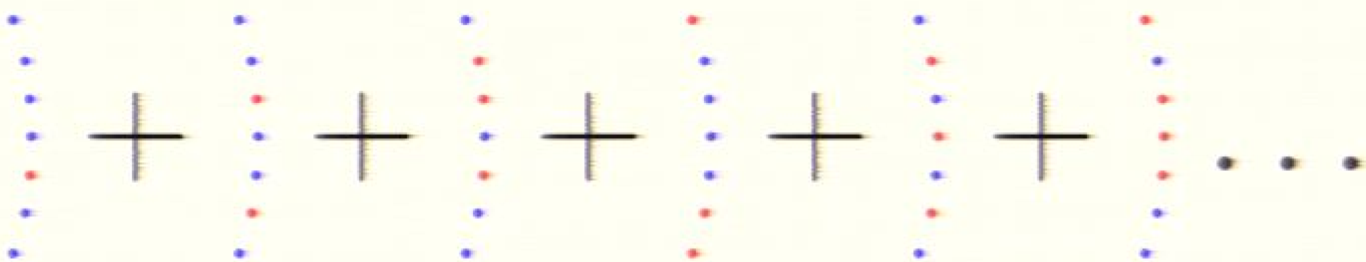

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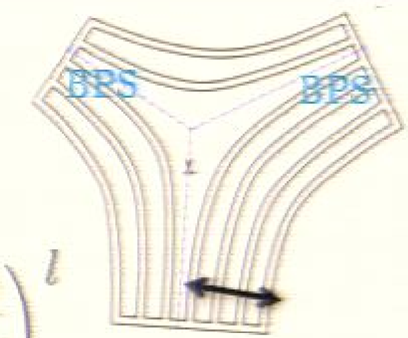

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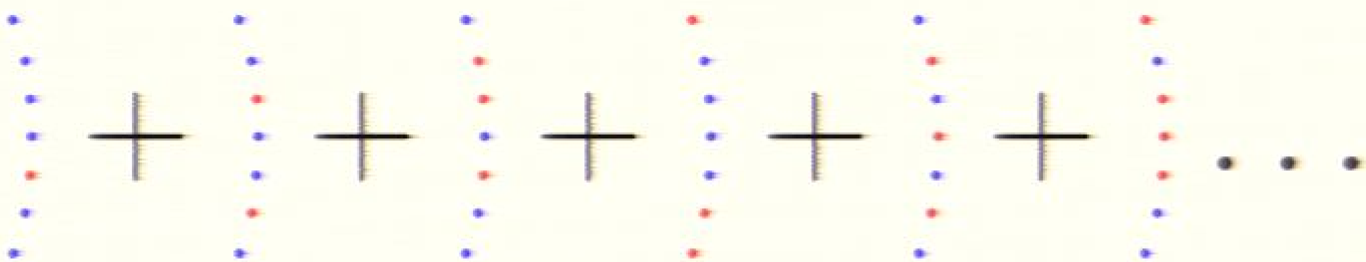

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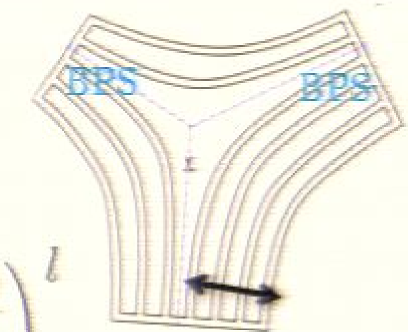

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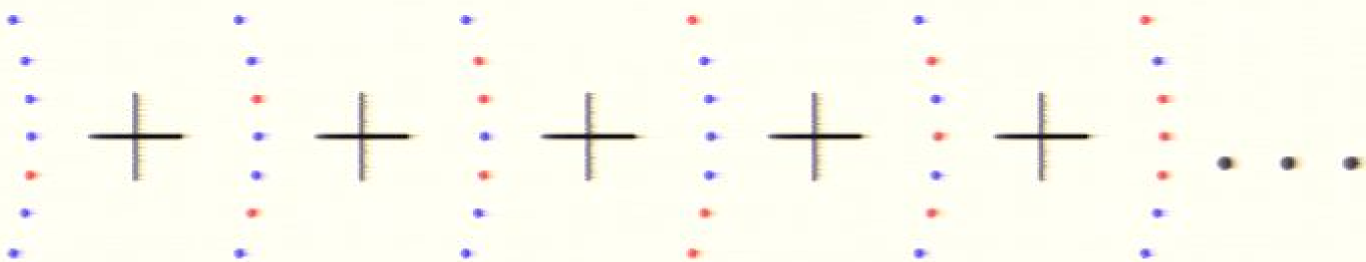

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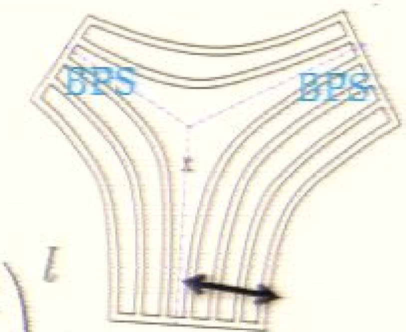

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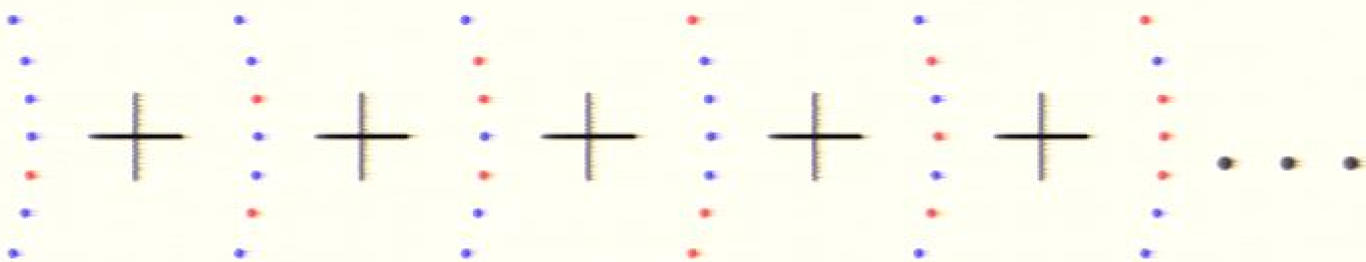

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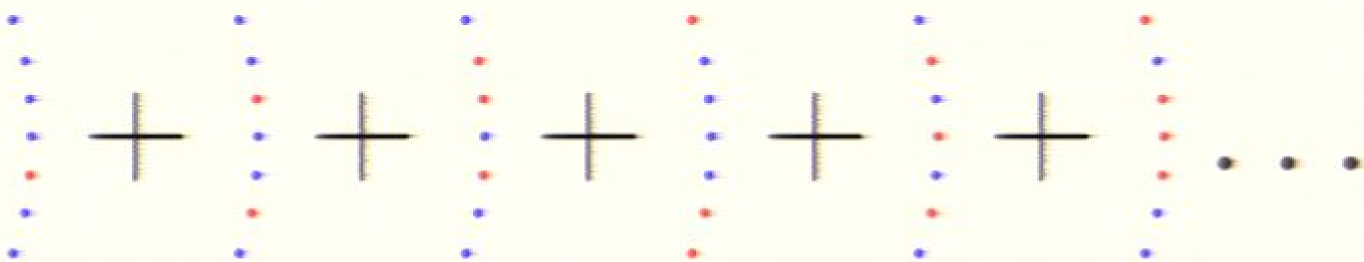

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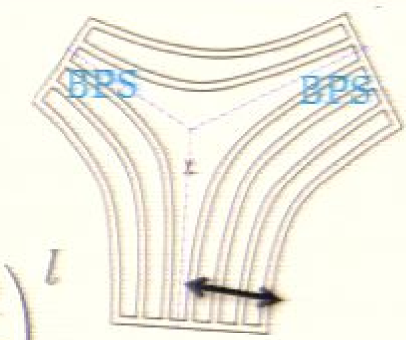

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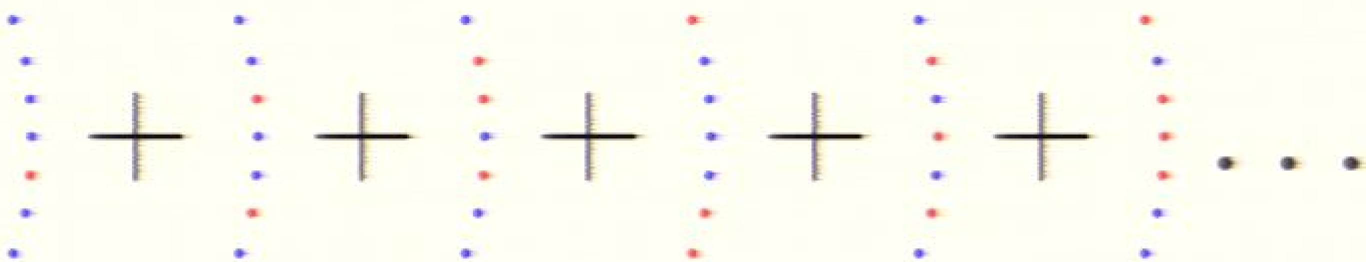

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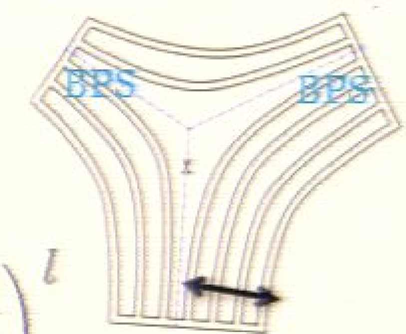

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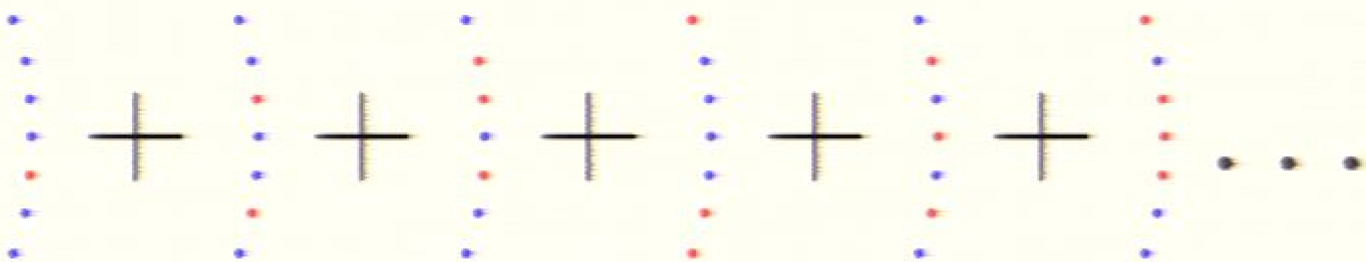

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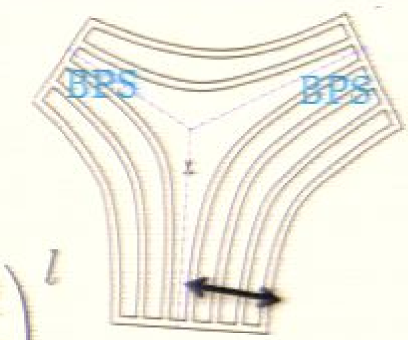

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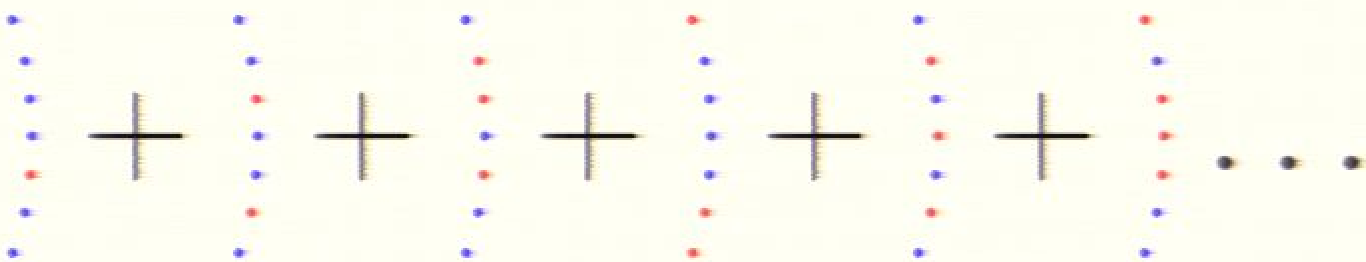

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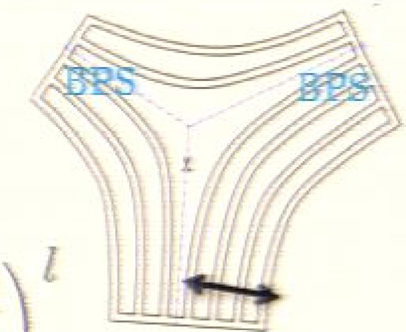

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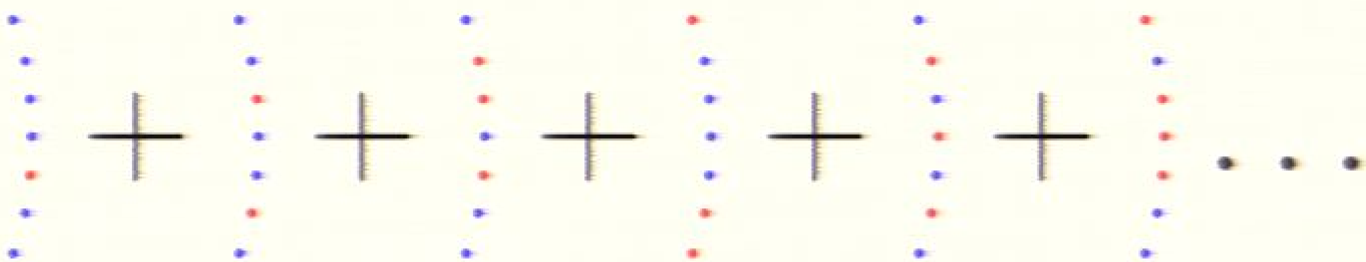

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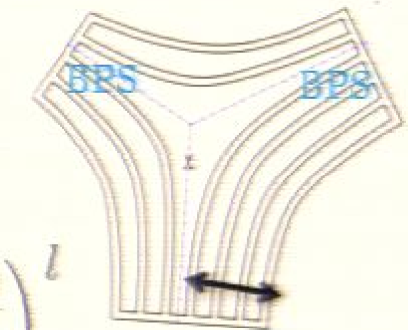

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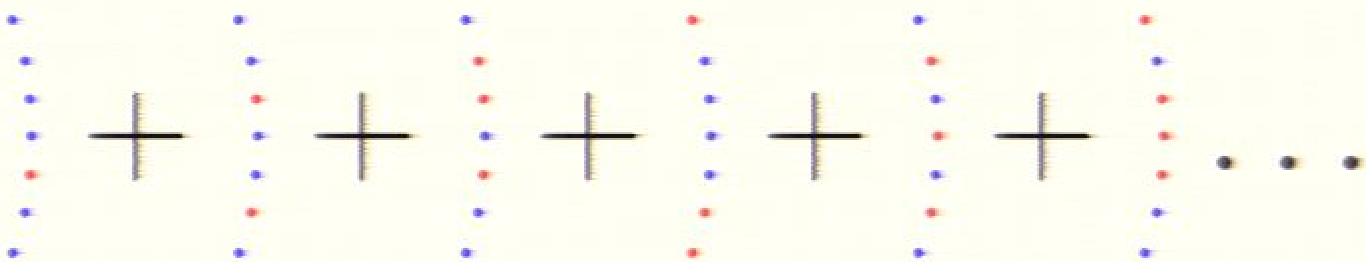

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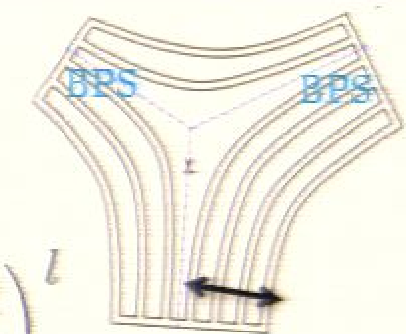

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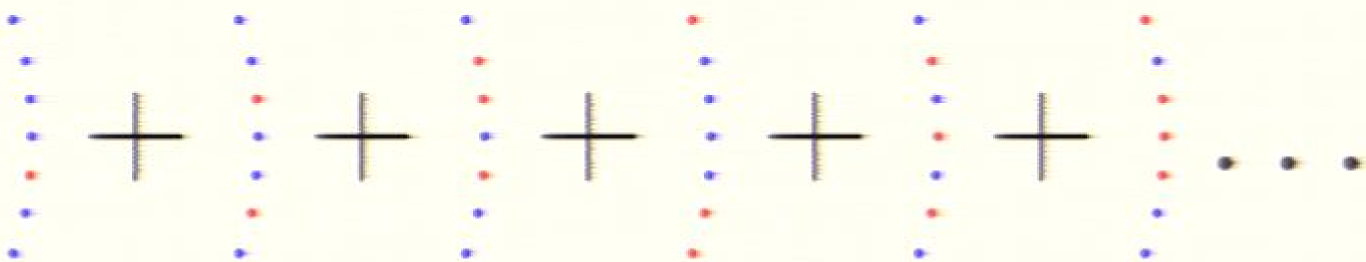

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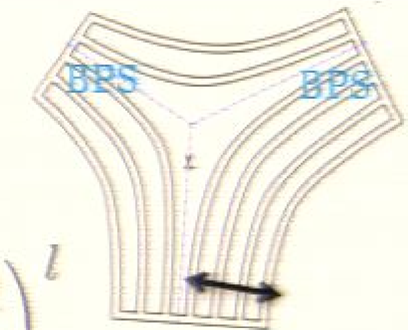

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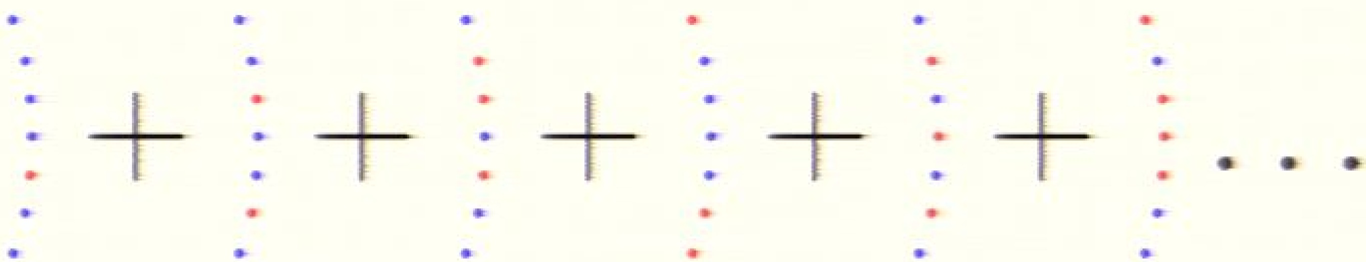

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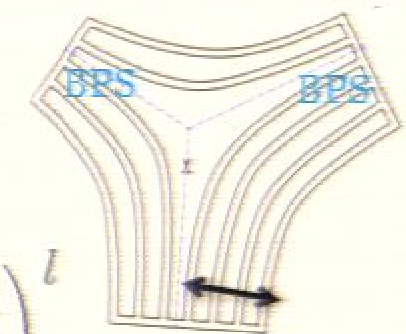

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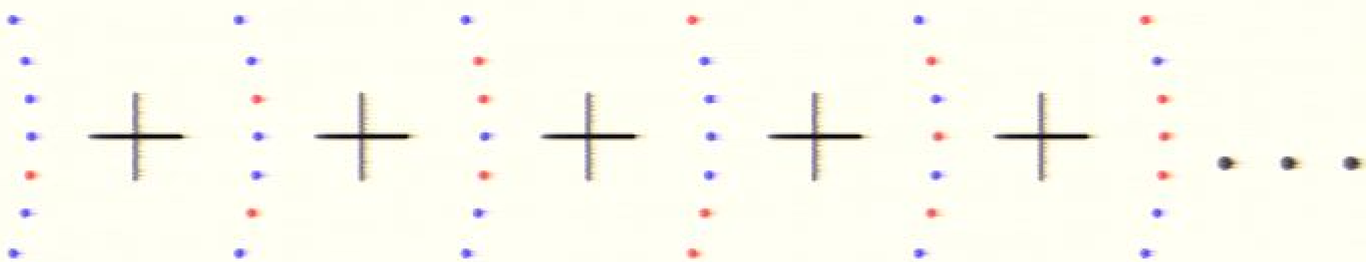

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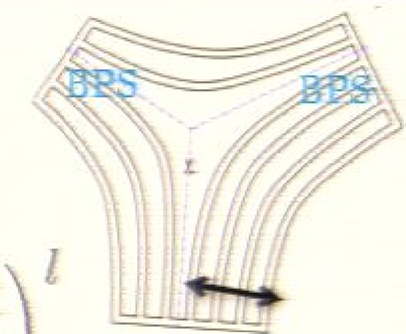

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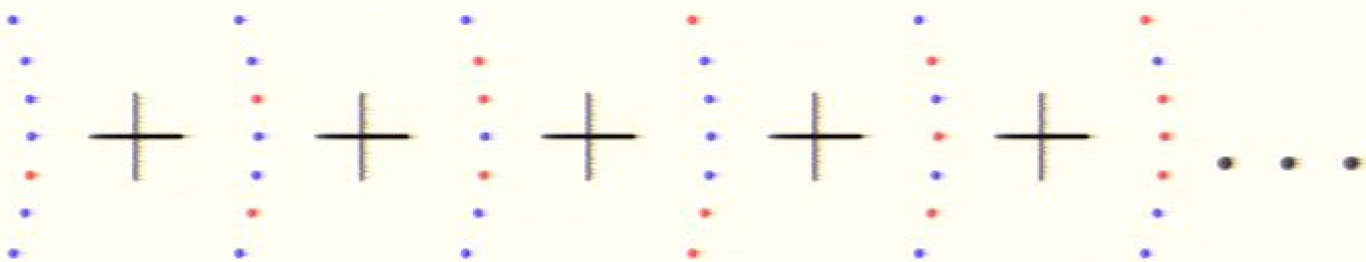

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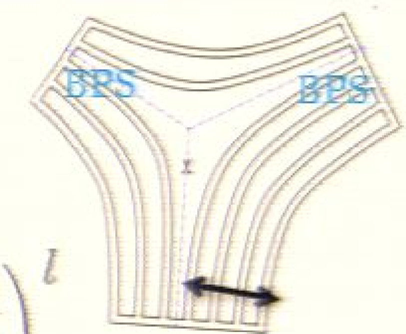

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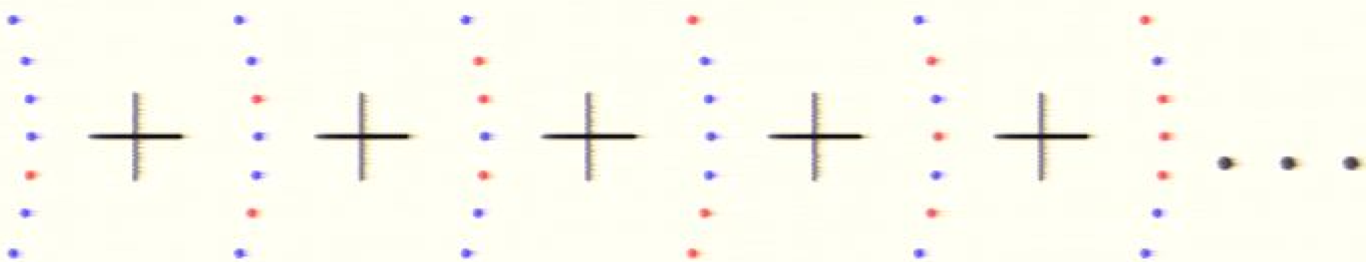

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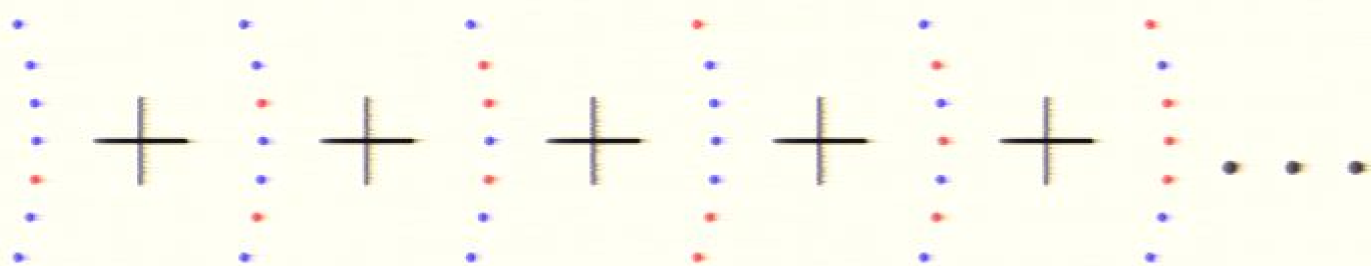

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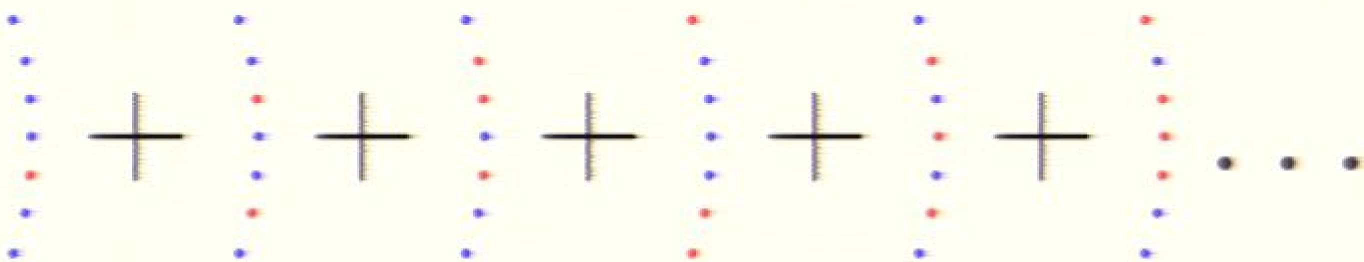

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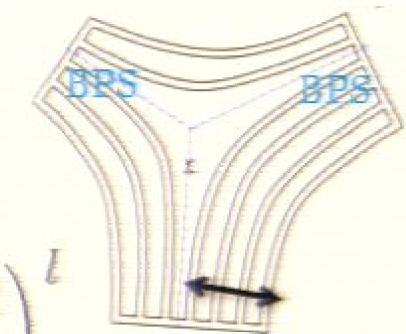

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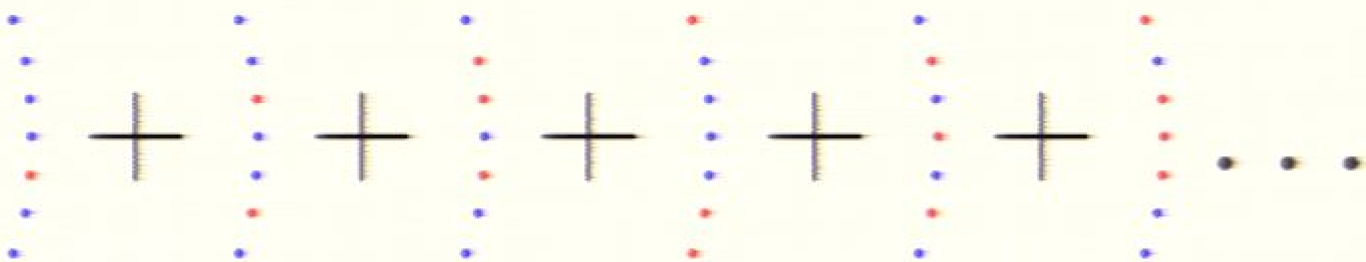

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References

- **Weak coupling:**
  - Minahan, Zarembo `02
  - Frolov, Tsetlin `03
  - Beisert, Minahan, Staudacher, Zarembo `03
  - Beisert, Frolov, Staudacher, Tseytlin `03
  - Staudacher `04
- **Coherent states:**
  - Kruczenski `03
  - Kruczenski, Ryzhov, Tseytlin `04
- **Quasiclassical limit:**
  - Sutherland `95; Dhar, Sriram, Shastry `00
  - Kazakov, Marshakov, Minahan, Zarembo `04
  - Arutyunov, Frolov, Staudacher `04
- **3pt functions old:**
  - **3BPS** Lee, Minwalla, Eangamani, Seiberg `98
  - **1loop** Okuyama, Tseng `04; Alday, David, Gava, Narain `05
  - **Integrability** Roiban, Volovich `04
  - **Near BMN** Beisert, Kristjansen Plefka, Semenoff Staudacher `02
  - **Extremal** Eden, Howe, Schubert, Sokatchev, West `99; D`Hoker, Freedman, Mathur, Matusis, Rastelli` 99;
- **Correlators recent:**
  - **Strong coupling:** Janik `10; Costa, Monteiro, Santos, Zoakos `10; Zarembo `10; Russo, Tseytlin `10;
  - **Strong coupling (case study):** Hernandz `11; Georgious `10; Buchbinder, Tseytlin `11; Bissi, Kristjansen, Young, Zoubos `11; Arnaudov, Rashkov, Vestsov` 11;...

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