

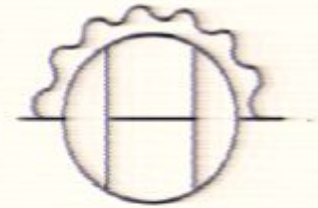
Title: New Applications for Supergraphs

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Abstract:

New Applications for Supergraphs



Joseph Minahan

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C. Sieg, JM in progress

15 August 2011

IGST2011 at Perimeter Institute

Introduction

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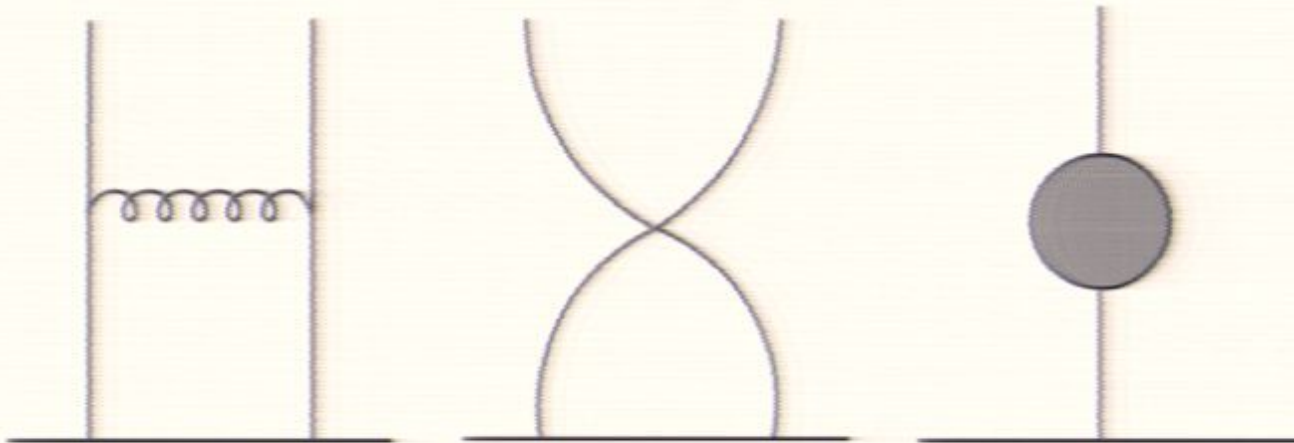
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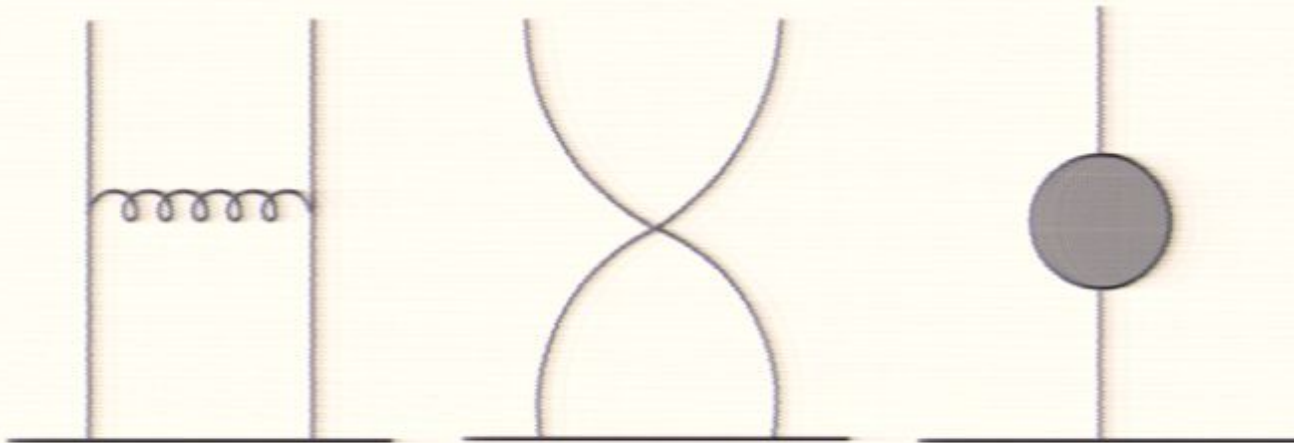
Zarembo, JM (2002)



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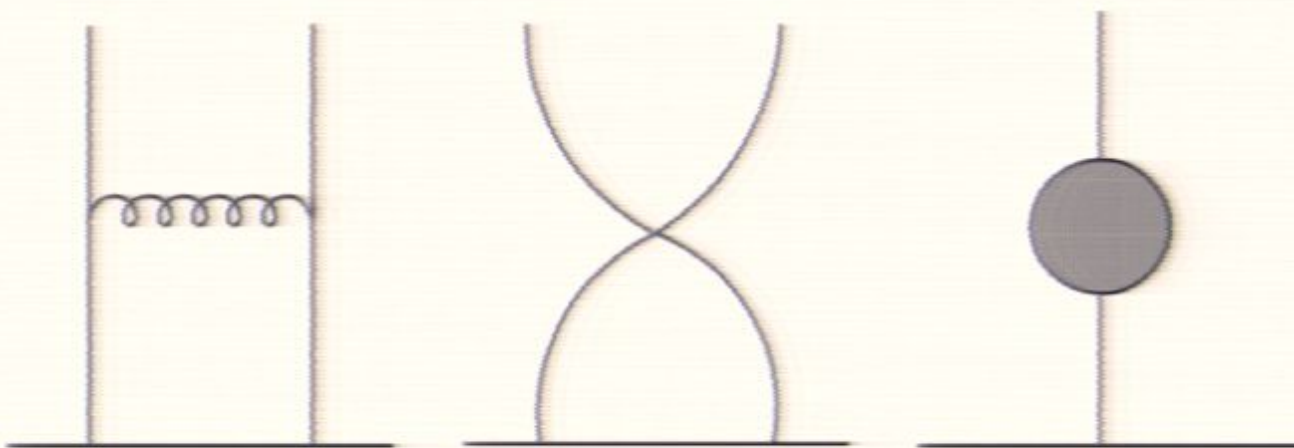
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$$g^2 \equiv \frac{g_{\text{YM}}^2 N}{16\pi^2}$$



$$= 2g^2 \chi(1)$$

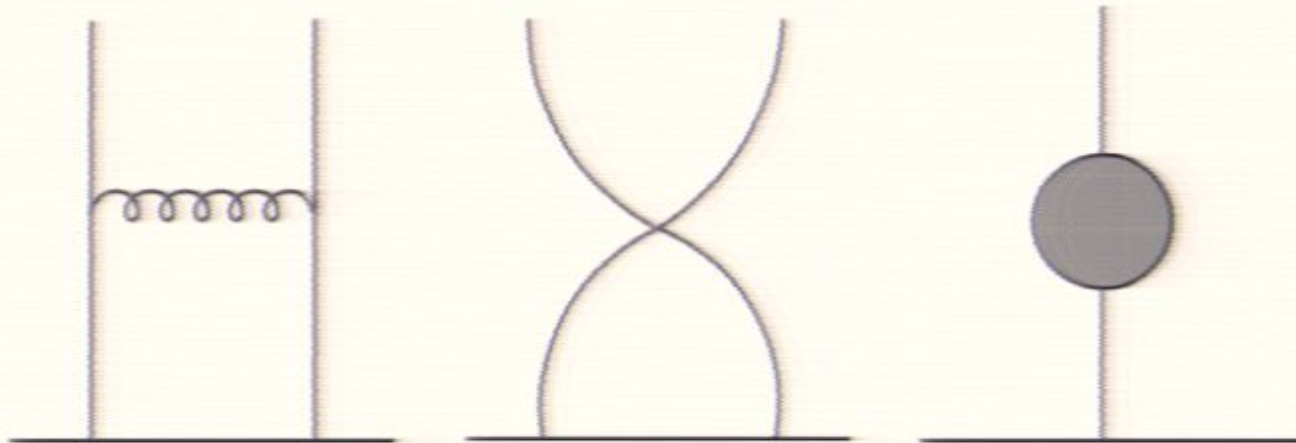
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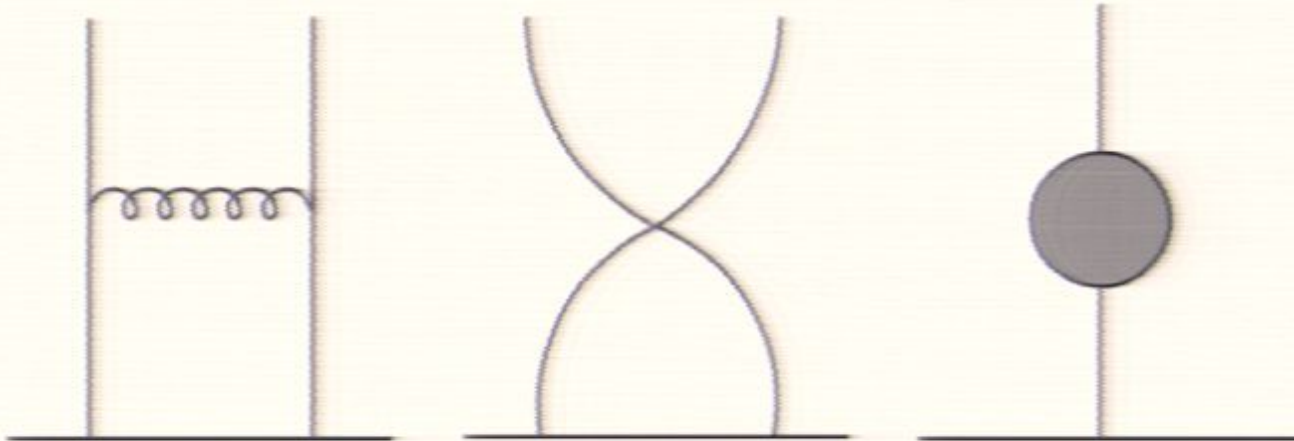
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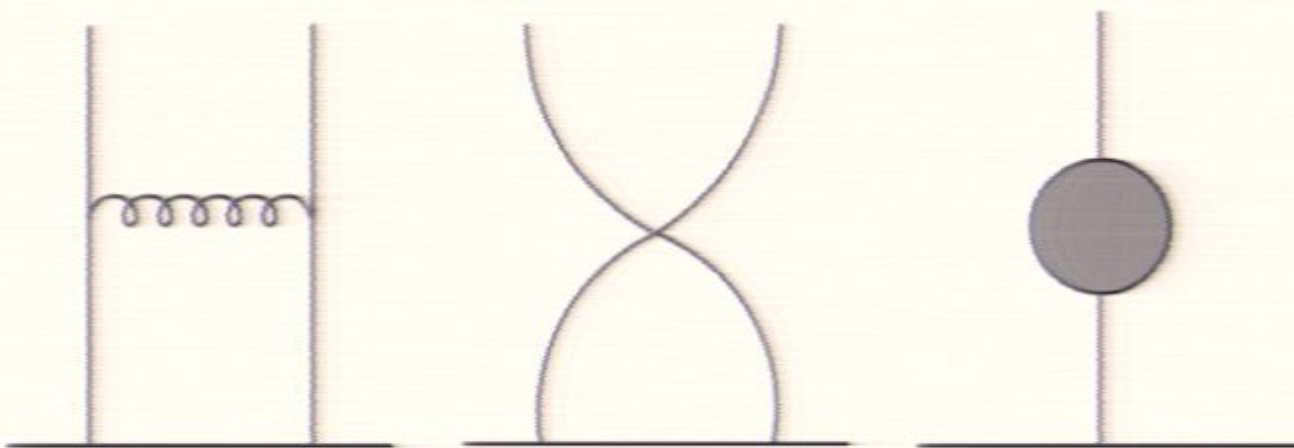
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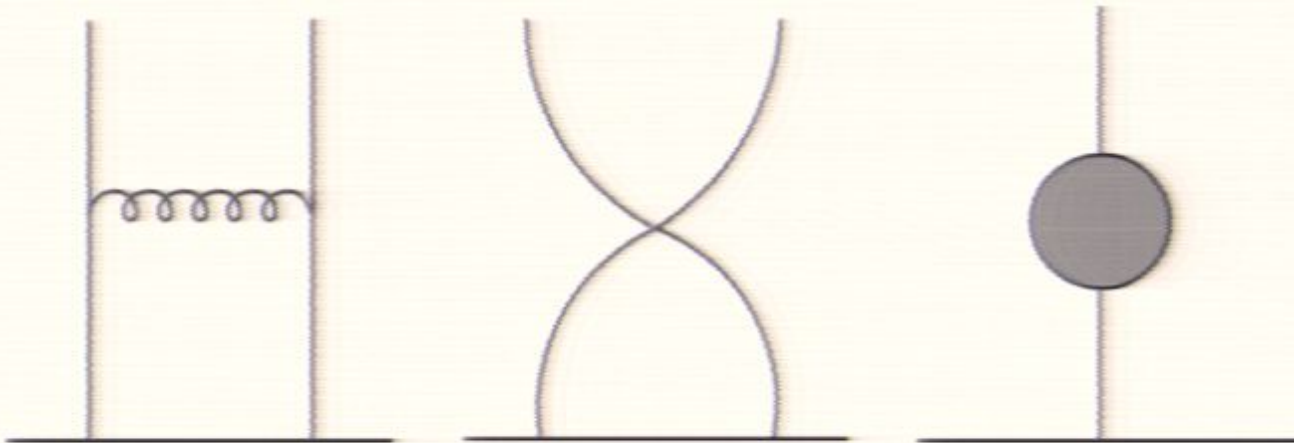
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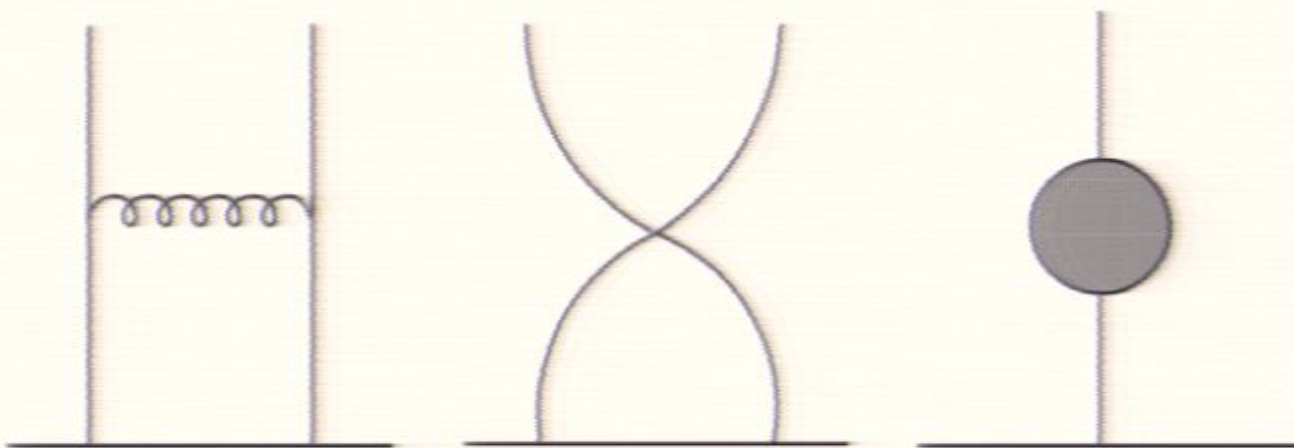
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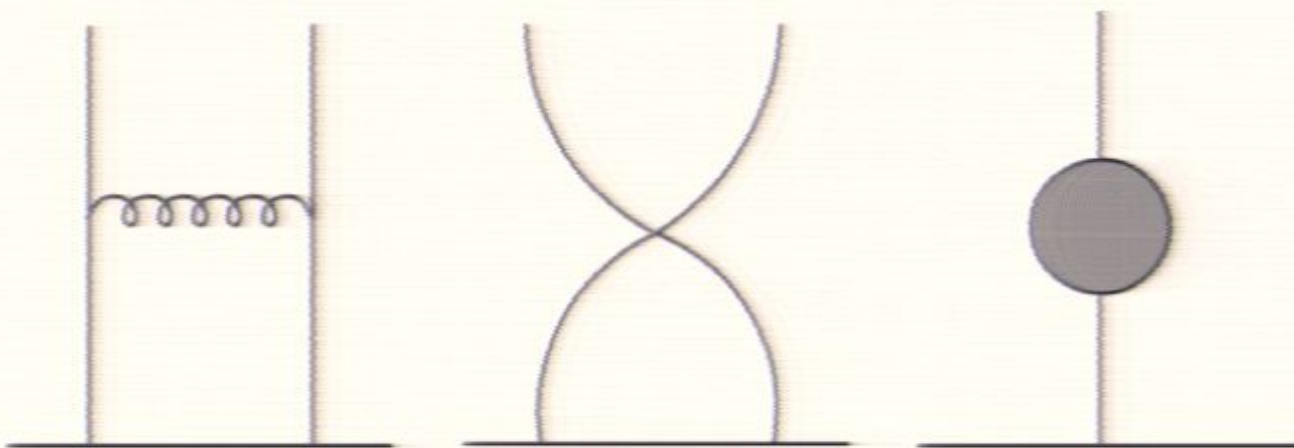
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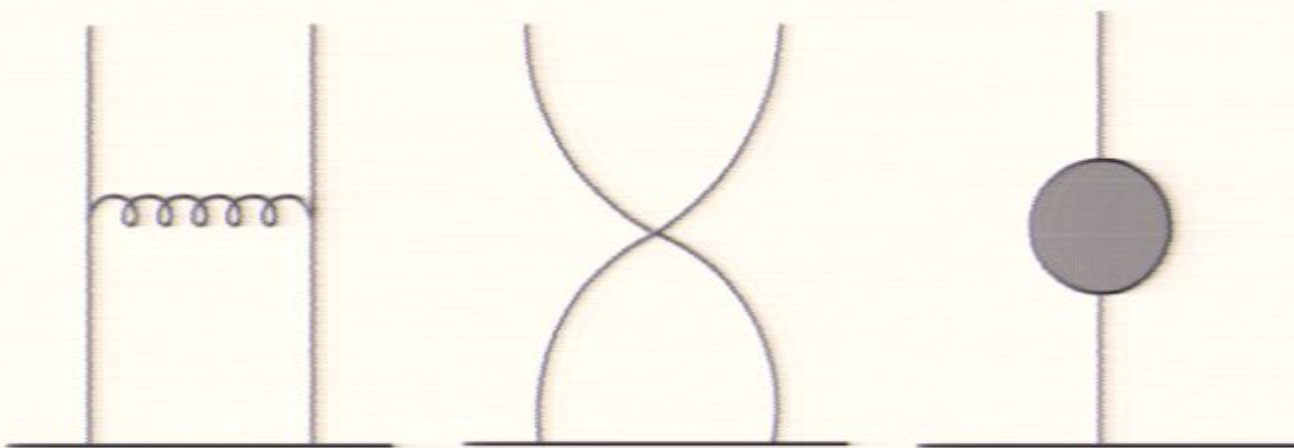
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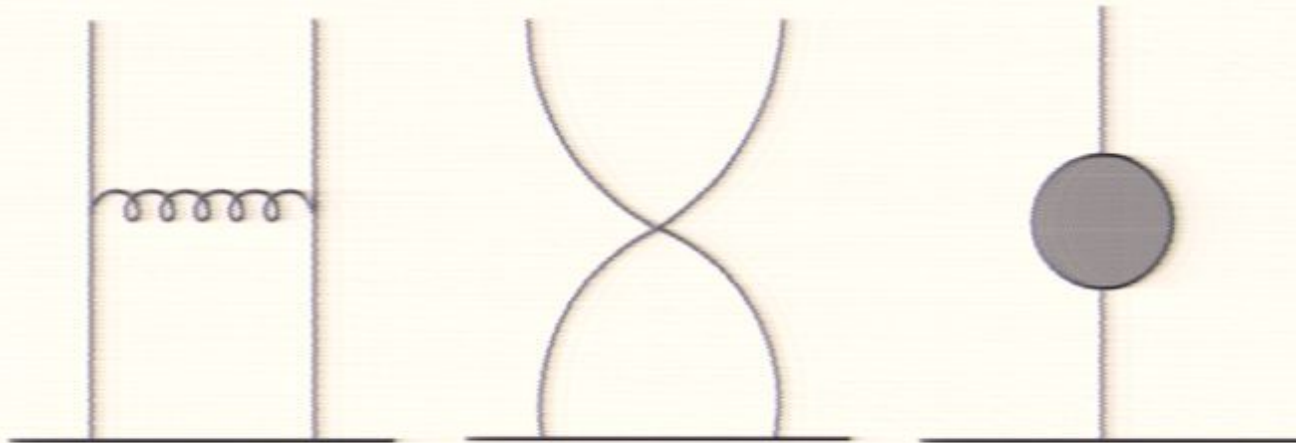
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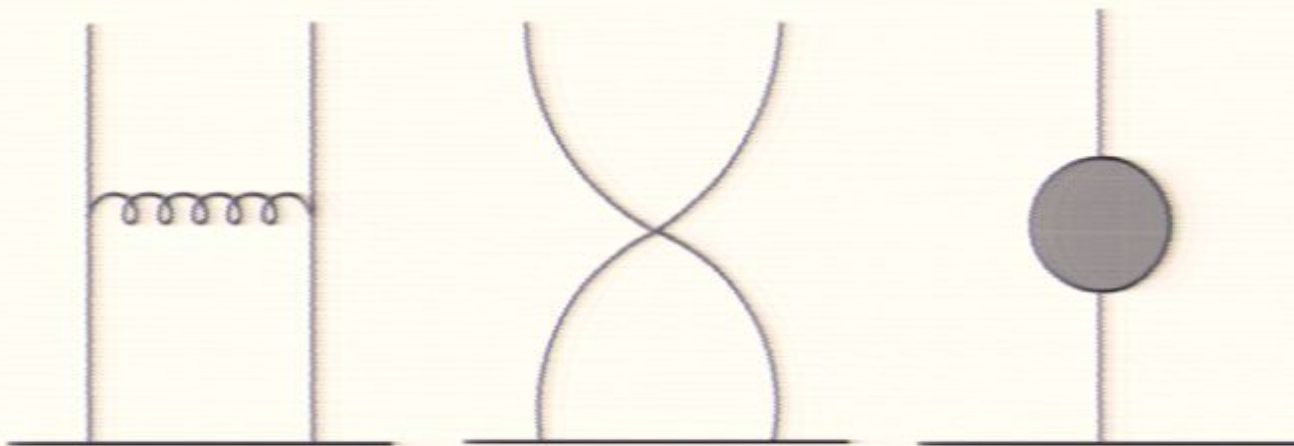
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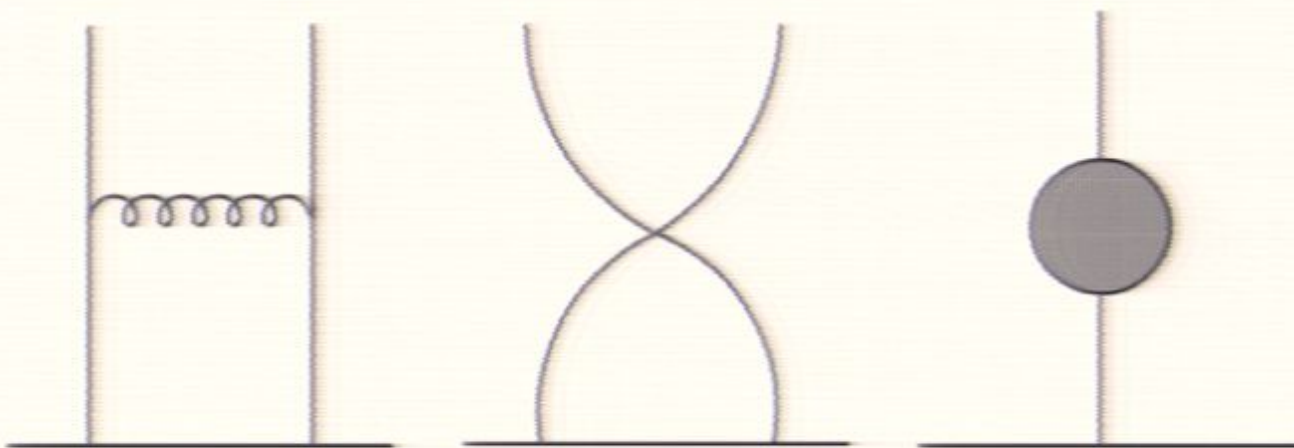
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Why use superspace?: One-loop in $\mathcal{N} = 4$ SYM

Components

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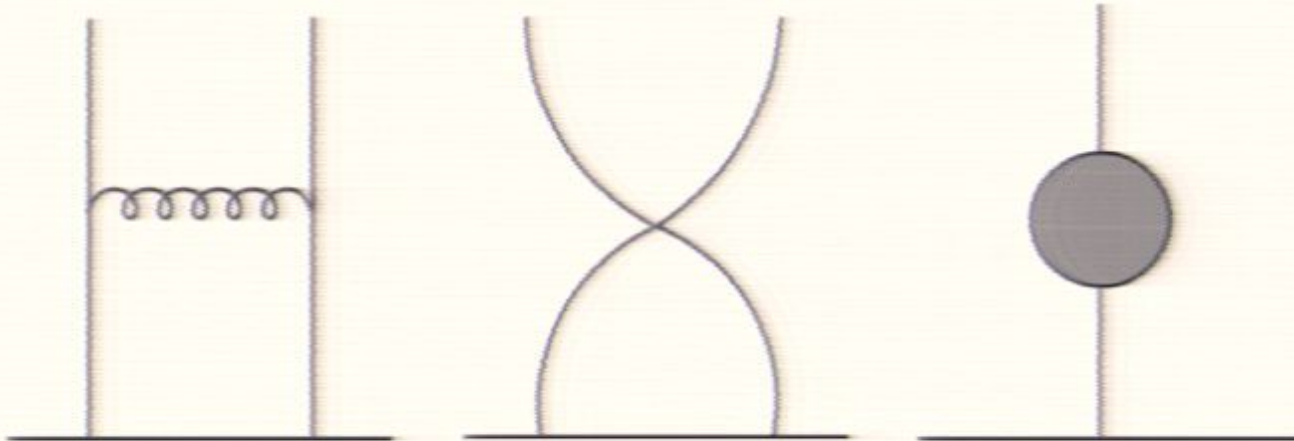
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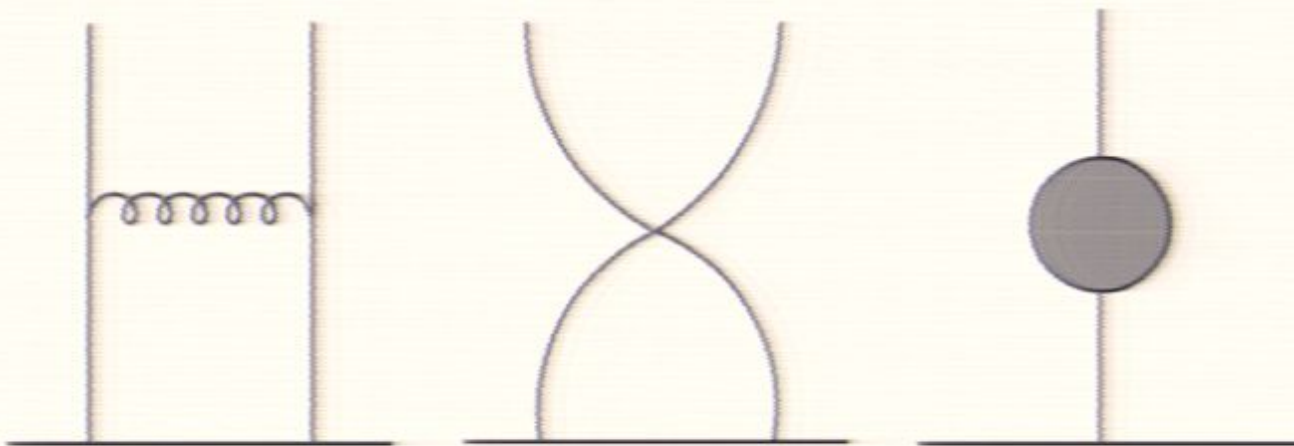
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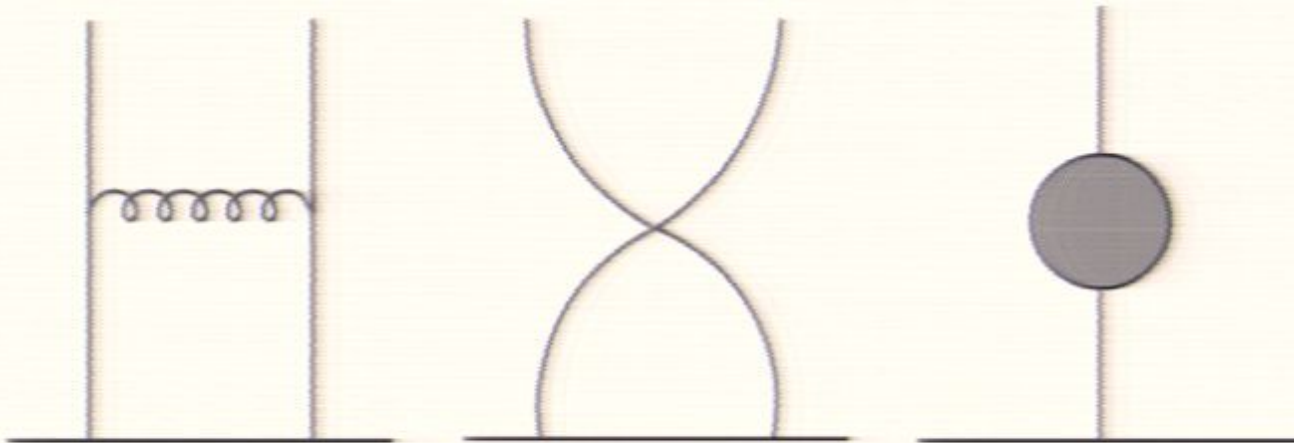
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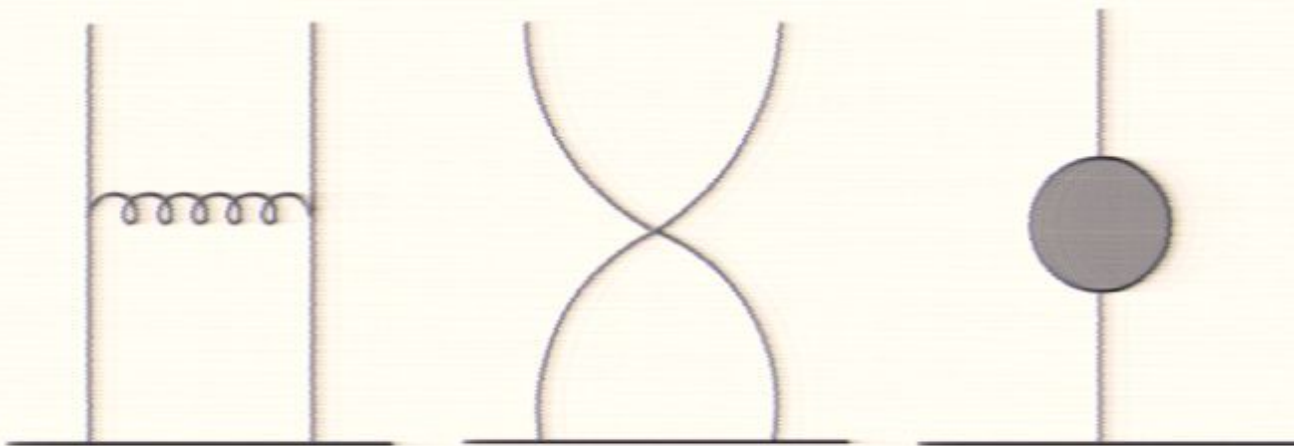
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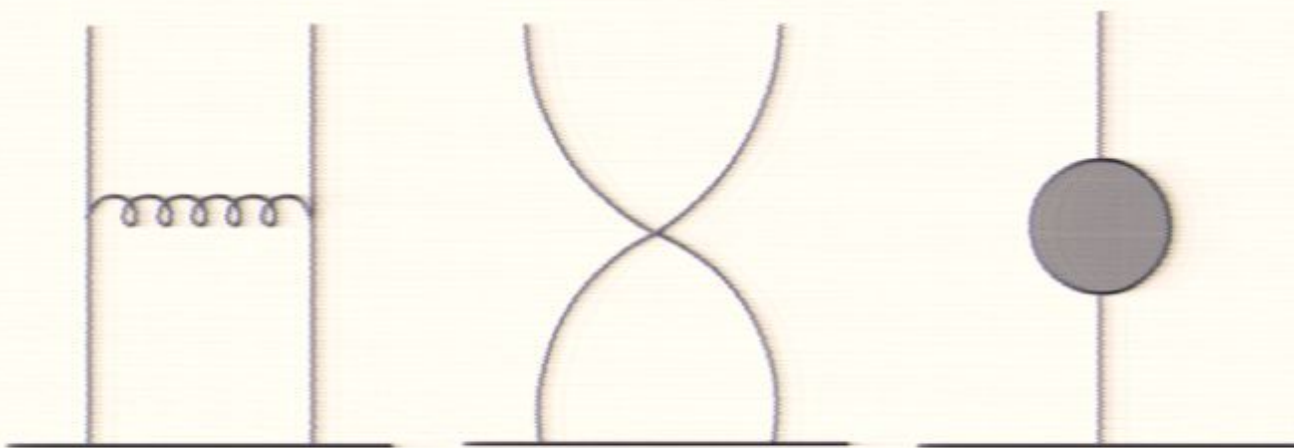
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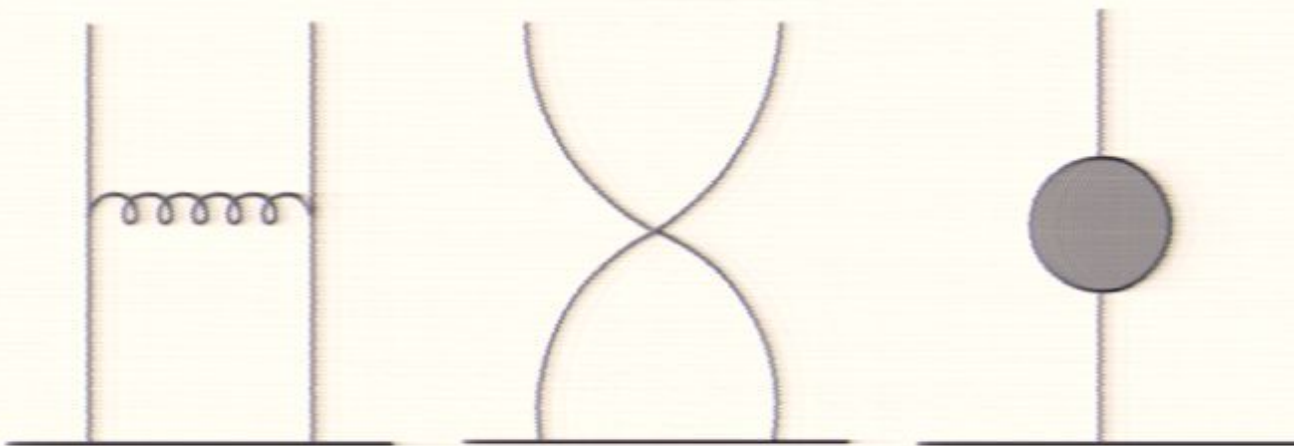
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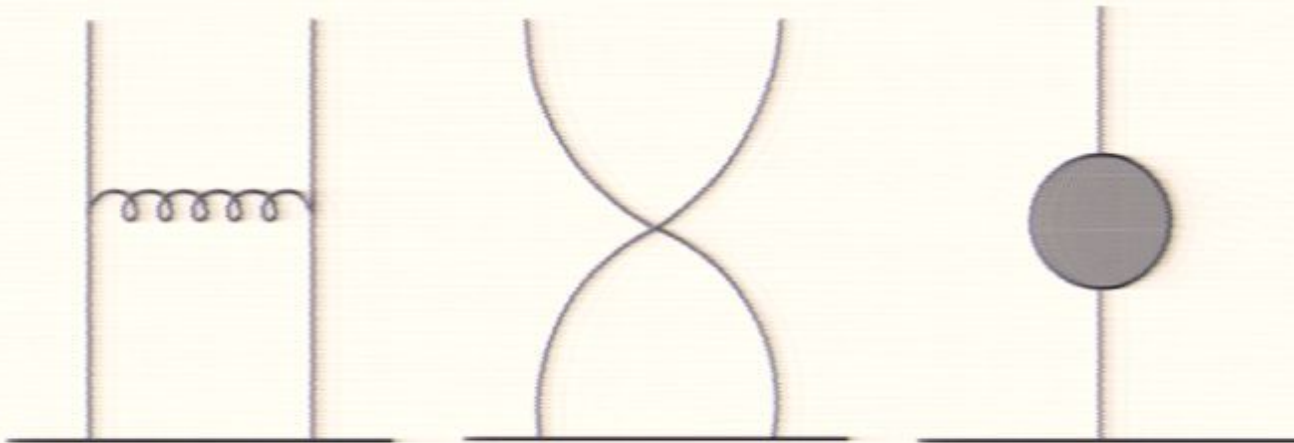
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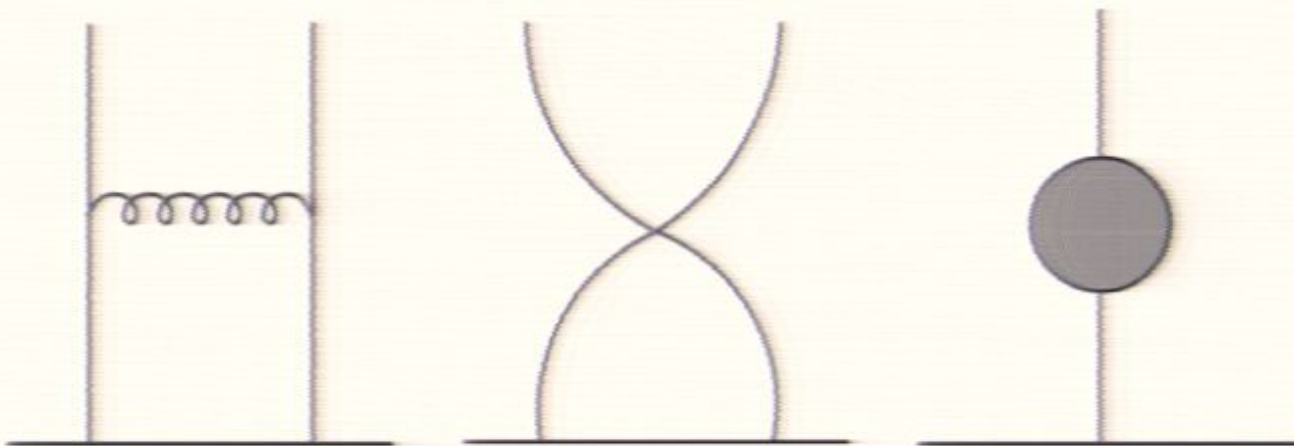
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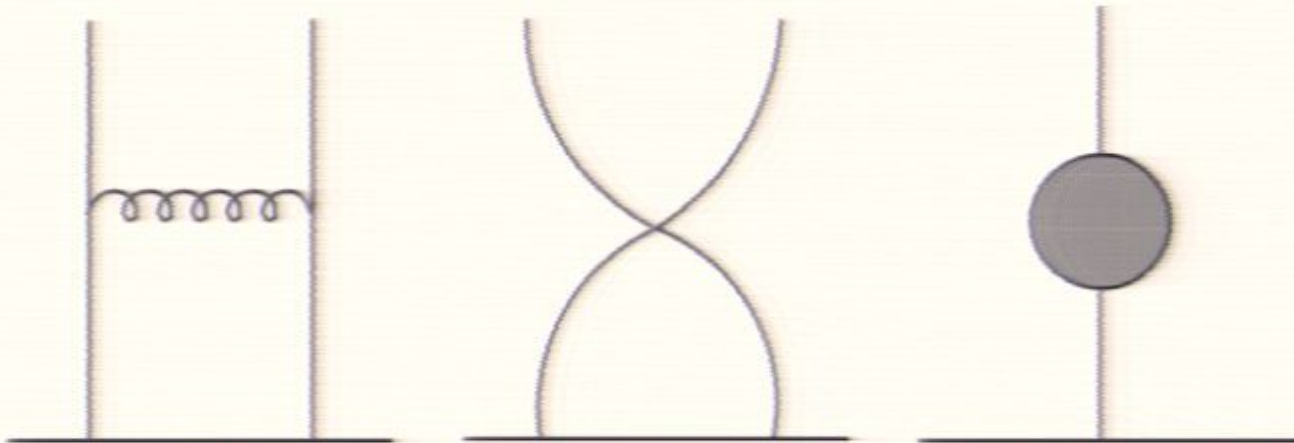
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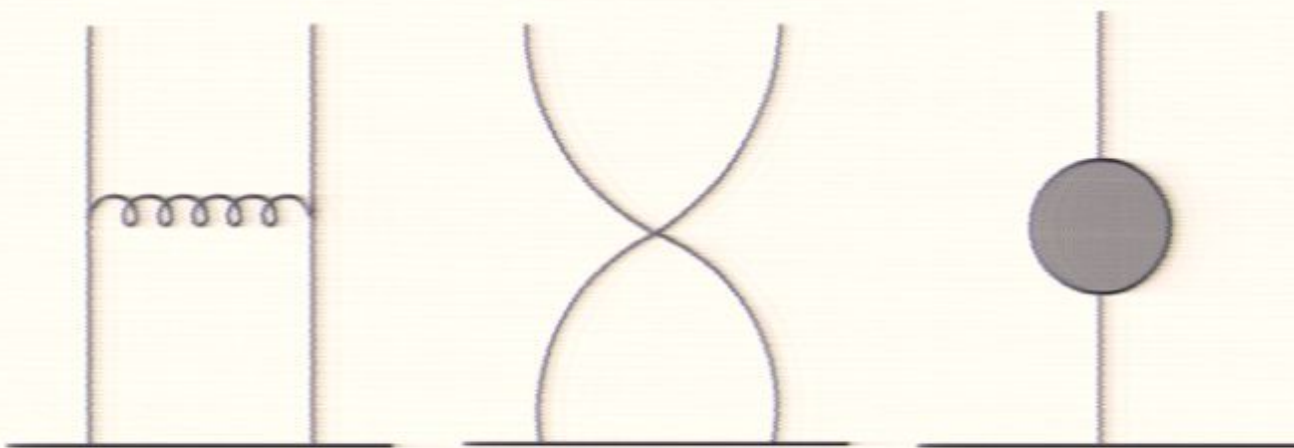
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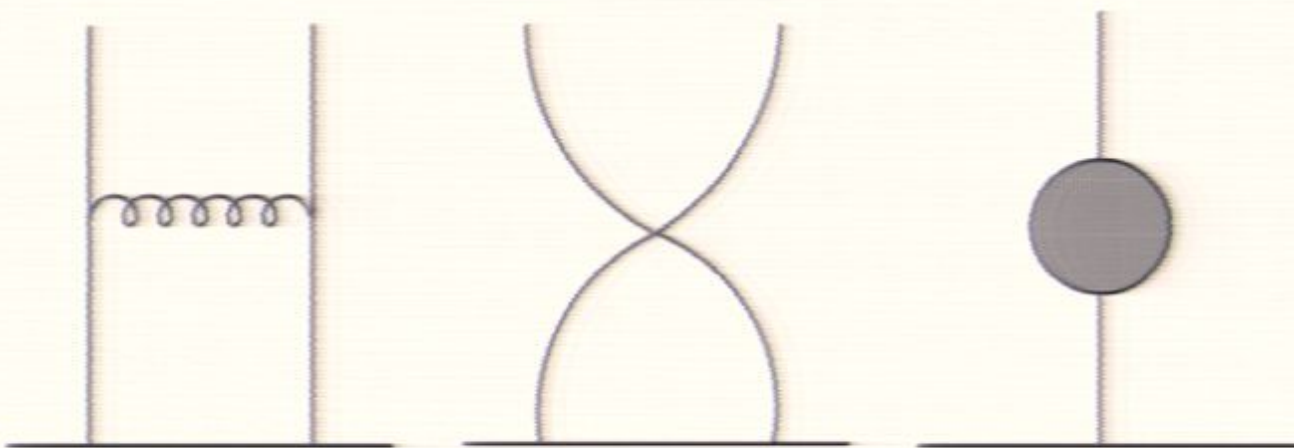
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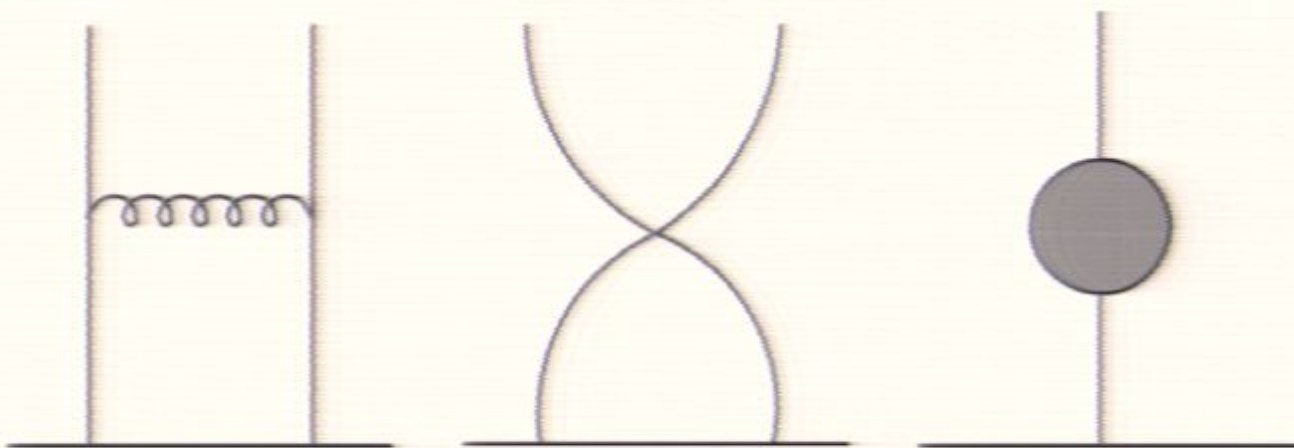
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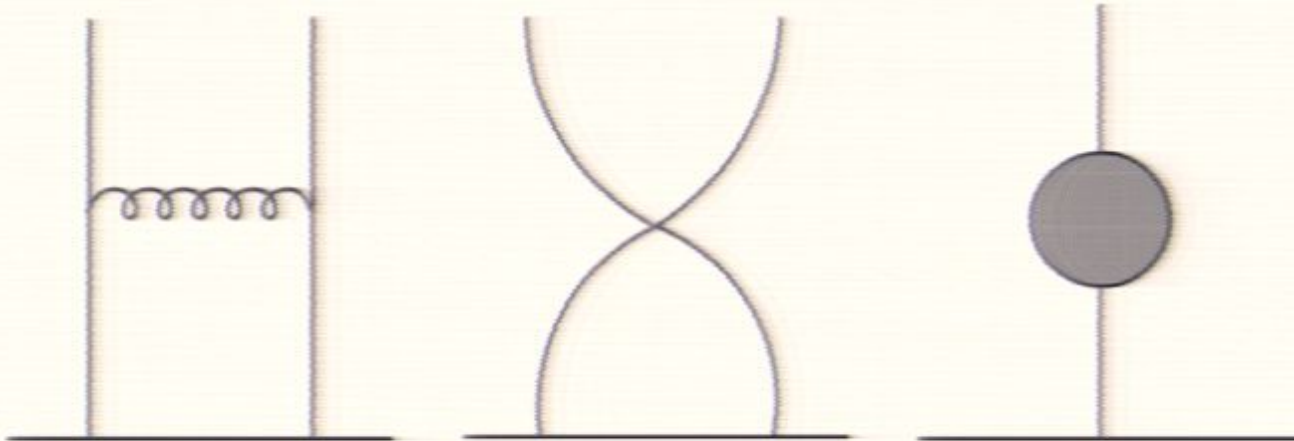
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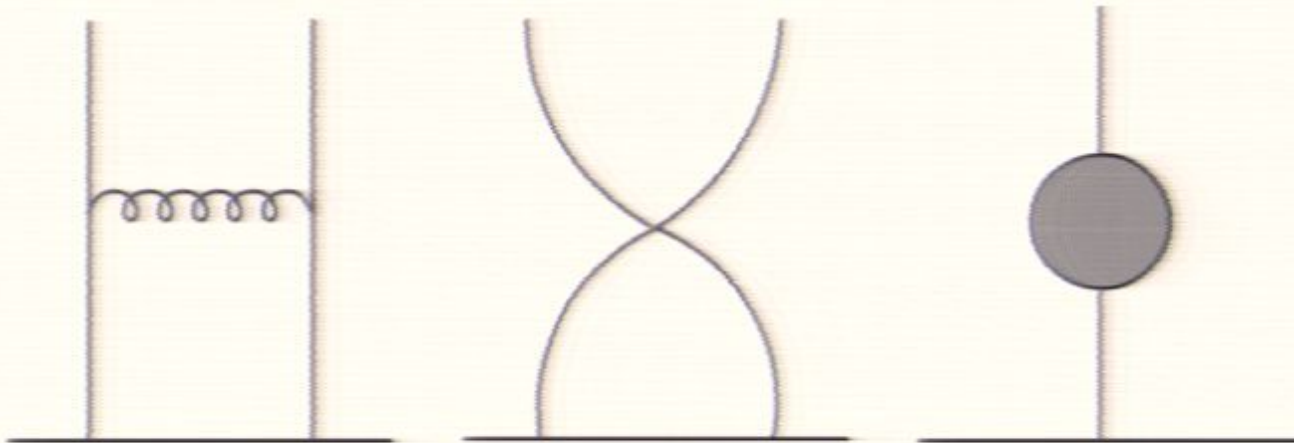
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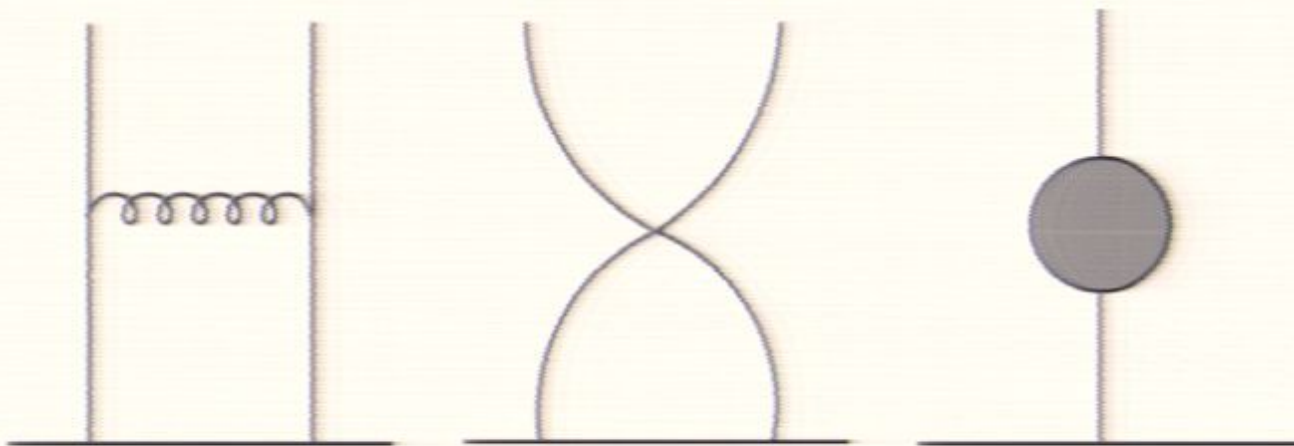
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Connected: $\chi(a_1, \dots, a_j \dots a_n)$ if $|a_j - a_{j+1}| = 1 \quad \forall \quad 1 \leq j < n.$

Every supergraph is proportional to a definite irreducible chiral function.
[$a_j \neq a_{j+1}$]

Chiral functions

Definition ($\mathcal{N} = 4$ SYM):

$$\chi(a_1, a_2 \dots a_n) \equiv \sum_{j=0}^{L-1} (1 - P_{j+a_1, j+a_1+1})(1 - P_{j+a_2, j+a_2+1}) \dots (1 - P_{j+a_n, j+a_n+1})$$

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One-magnon state

$$|p\rangle \equiv \sum_{j=1}^L e^{ipj} |XXX \dots \overset{j}{\downarrow} XYX \dots XX\rangle, \quad \varepsilon(p) = \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}} - 1, \quad g = \frac{\sqrt{\lambda}}{4\pi}$$

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$$\text{Resum: } \sum_{m=1}^{\infty} \frac{\Gamma(3/2)}{m! \Gamma(3/2 - m)} g^{2m} [\chi(1)^m]_c = F_1(g^2) \chi(1) + F_2(g^2) [\chi(1,2) + \chi(2,1)] \\ + F_3(g^2) [\chi(1,2,3) + \chi(3,2,1) + \chi(1,2,1) + \chi(2,1,2)] + \dots$$

$$F_n(g^2) = \frac{(-1)^{n+1} \Gamma(n-1/2)}{2\sqrt{\pi} n!} (2g)^{2n} {}_2F_1(n-1/2, n; n+1; -8g^2) \quad (\text{algebraic})$$

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One-magnon state

$$|p\rangle \equiv \sum_{j=1}^L e^{ipj} |XXX \dots \overset{j}{XYX} \dots XX\rangle, \quad \varepsilon(p) = \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}} - 1, \quad g = \frac{\sqrt{\lambda}}{4\pi}$$

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$$[\chi(1)^2]_c = \left[\begin{array}{c} \text{Y} \\ \text{X} \end{array} \times \begin{array}{c} \text{Y} \\ \text{X} \end{array} \right]_c = \begin{array}{c} \text{Y} \\ \text{X} \end{array} + \begin{array}{c} \text{Y} \\ \text{O} \\ \text{X} \end{array} + \begin{array}{c} \text{Y} \\ \text{Y} \\ \text{X} \end{array} = \chi(2,1) + 2\chi(1) + \chi(1,2)$$

$$\text{Resum: } \sum_{m=1}^{\infty} \frac{\Gamma(3/2)}{m! \Gamma(3/2 - m)} g^{2m} [\chi(1)^m]_c = F_1(g^2) \chi(1) + F_2(g^2) [\chi(1,2) + \chi(2,1)] \\ + F_3(g^2) [\chi(1,2,3) + \chi(3,2,1) + \chi(1,2,1) + \chi(2,1,2)] + \dots$$

$$F_n(g^2) = \frac{(-1)^{n+1} \Gamma(n-1/2)}{2\sqrt{\pi} n!} (2g)^{2n} {}_2F_1(n-1/2, n; n+1; -8g^2) \quad (\text{algebraic})$$

One-magnon state

$$|p\rangle \equiv \sum_{j=1}^L e^{ipj} |XXX \dots \overset{j}{XYX} \dots XX\rangle, \quad \varepsilon(p) = \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}} - 1, \quad g = \frac{\sqrt{\lambda}}{4\pi}$$

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Caveat: $\chi(a_1 \dots a_j, a_j \pm 1, a_j \dots a_n) = \chi(a_1 \dots a_j \dots a_n)$ ($SU(2)$ sector):

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One-magnon state

$$|p\rangle \equiv \sum_{j=1}^L e^{ipj} |XXX \dots \overset{j}{XYX} \dots XX\rangle, \quad \varepsilon(p) = \sqrt{1 + 16g^2 \sin^2 \frac{p}{2}} - 1, \quad g = \frac{\sqrt{\lambda}}{4\pi}$$

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Computing supergraphs: Feynman rules

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Fields (all in adjoint):

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Propagators:

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$$\langle V^a_b V^c_d \rangle = \frac{\text{---} \xrightarrow{p} \text{---}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}^j} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta^j_i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i, \quad V_{V \bar{\phi}^i} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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$$V_{\bar{\phi}_i} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta_i^i, \quad V_{V^2 \bar{\phi}_i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^i, \quad V_{V \bar{\phi}_i} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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$$V_{\bar{\phi}^j} V \phi^i = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta^j_i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i, \quad V_{V \bar{\phi}^i} V \phi^i = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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$$V_{\bar{\phi}^j} V_{\phi^i} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} g_{\text{YM}} \delta^j_i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i, \quad V_{V \bar{\phi}^i} V_{\phi^i} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\text{---} \xrightarrow{p} \text{---}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\text{---} \xrightarrow{p} \text{---}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}^j V \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta^j_i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i, \quad V_{V \bar{\phi}^i V \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}^i} V \phi^j = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta_i^j, \quad V_{V^2 \bar{\phi}^i \phi^j} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^j, \quad V_{V \bar{\phi}^i} V \phi^j = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^j$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}^j} V \phi^i = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta^i_j, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^i_i, \quad V_{V \bar{\phi}^i} V \phi^i = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^i_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^j) Vector superfields: V

Propagators:

$$\langle \phi^j_a \bar{\phi}_i^c \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}_j} V \phi^j = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} g_{\text{YM}} \delta_i^j, \quad V_{V^2 \bar{\phi}_i \phi^i} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^j, \quad V_{V \bar{\phi}_i} V \phi^i = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^j$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_{\bar{1}d} \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overrightarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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$$V_{\bar{\phi}^j} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta^j_i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i, \quad V_{V \bar{\phi}^i} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}_j^c \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ \bar{D}_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

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Propagators:

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ \bar{D}_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

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$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}^j} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta^j_i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i, \quad V_{V \bar{\phi}^i} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^j_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i{}^a{}_b \bar{\phi}^j{}^c{}_d \rangle = \frac{\overrightarrow{p}}{p} = \delta^i{}_j \frac{\delta^a{}_d \delta^c{}_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a{}_b V^c{}_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a{}_d \delta^c{}_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}^i} V_{\phi^i} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} g_{\text{YM}} \delta_i^i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^i, \quad V_{V \bar{\phi}^i} V_{\phi^i} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\text{---} \xrightarrow{p} \text{---}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\text{---} \xrightarrow{p} \text{---}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \text{---} \begin{array}{l} \nearrow D_2 \\ \searrow D_2 \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \text{---} \begin{array}{l} \nearrow D_2 \\ \searrow D_2 \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}_j^c \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

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$$V_{\bar{\phi}_j} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta_i^j, \quad V_{V^2 \bar{\phi}_i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^i, \quad V_{V \bar{\phi}_i} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}_j^c \rangle = \frac{\text{---} \xrightarrow{p} \text{---}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \text{---} \xrightarrow{D_2} \\ | \\ \text{---} \xrightarrow{D_2} \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} \text{---} \xrightarrow{D_2} \\ | \\ \text{---} \xrightarrow{D_2} \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}_j V \phi^i} = \begin{array}{c} \text{---} \xrightarrow{D_2} \\ | \\ \text{---} \xrightarrow{D_2} \end{array} g_{\text{YM}} \delta_i^j, \quad V_{V^2 \bar{\phi}_i \phi^i} = \begin{array}{c} \text{---} \xrightarrow{D_2} \\ | \\ \text{---} \xrightarrow{D_2} \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^i, \quad V_{V \bar{\phi}_i V \phi^i} = \begin{array}{c} \text{---} \xrightarrow{D_2} \\ | \\ \text{---} \xrightarrow{D_2} \end{array} \frac{g_{\text{YM}}^2}{2} \delta_i^i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D^2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D^2} \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} \overrightarrow{D^2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D^2} \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}^i} V_{\phi^i} = \begin{array}{c} \overrightarrow{D^2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D^2} \end{array} g_{\text{YM}} \delta^i_i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} \overrightarrow{D^2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D^2} \end{array} \frac{2 g_{\text{YM}}}{2} \delta^i_i, \quad V_{V \bar{\phi}^i} V_{\phi^i} = \begin{array}{c} \overrightarrow{D^2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D^2} \end{array} \frac{2 g_{\text{YM}}}{2} \delta^i_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^j) Vector superfields: V

Propagators:

$$\langle \phi^j_a \bar{\phi}^c_d \rangle = \frac{\overrightarrow{p}}{p} = \delta^j_i \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \circlearrowleft + G_{ikj} \circlearrowleft), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \circlearrowleft + G^{ikj} \circlearrowleft)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

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Propagators:

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$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^j{}^a{}_b \bar{\phi}_j{}^c{}_d \rangle = \frac{\overrightarrow{p}}{p} = \delta^i{}_j \frac{\delta^a{}_d \delta^c{}_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G_{ijk} \circlearrowleft + G_{ikj} \circlearrowright), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G^{ijk} \circlearrowleft + G^{ikj} \circlearrowright)$$

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Computing supergraphs: Feynman rules

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$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

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Computing supergraphs: Feynman rules

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Computing supergraphs: Feynman rules

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$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

$$V_{\bar{\phi}^i} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta^i_i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^i_i, \quad V_{V \bar{\phi}^i} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^i_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{SYM}} \epsilon_{ijk}$

$$V_{\bar{\phi}^i} V_{\phi^j} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} g_{\text{SYM}} \delta^j_i, \quad V_{V^2} \bar{\phi}^i \phi^j = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} \frac{g_{\text{SYM}}^2}{2} \delta^j_i, \quad V_V \bar{\phi}^i V_{\phi^j} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} \frac{g_{\text{SYM}}^2}{2} \delta^j_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^i) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}_j^c \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

$$\langle V^a_b V^c_d \rangle = \frac{\overleftarrow{p}}{p} = -\frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} \overleftarrow{D_2} \\ | \\ \text{---} \\ | \\ \overrightarrow{D_2} \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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$$V_{\bar{\phi}_j} V_{\phi^i} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} g_{\text{SYM}} \delta_i^j, \quad V_{V^2 \bar{\phi}_i \phi^j} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} \frac{g_{\text{SYM}}^2}{2} \delta_i^j, \quad V_{V \bar{\phi}_i} V_{\phi^j} = \begin{array}{c} \overleftarrow{D_2} \\ | \\ \text{---} \\ | \\ \overrightarrow{D_2} \end{array} \frac{g_{\text{SYM}}^2}{2} \delta_i^j$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

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Propagators:

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ \bar{D}_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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$$V_{\bar{\phi}^i} V_{\phi^i} = \begin{array}{c} \bar{D}_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} g_{\text{YM}} \delta^i_i, \quad V_{V^2 \bar{\phi}^i \phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ \bar{D}_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^i_i, \quad V_{V \bar{\phi}^i} V_{\phi^i} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} \frac{g_{\text{YM}}^2}{2} \delta^i_i$$

Computing supergraphs: Feynman rules

Fields (all in adjoint):

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Propagators:

$$\langle \phi^i_a \bar{\phi}_j^c \rangle = \frac{\overline{\xrightarrow{p}}}{p} = \delta^j_i \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overline{D_2} \\ | \\ \text{---} \\ | \\ \underline{D_2} \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} \underline{D_2} \\ | \\ \text{---} \\ | \\ \overline{D_2} \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

$\mathcal{N} = 4$: $G_{ijk} = g_{\text{YM}} \epsilon_{ijk}$

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Computing supergraphs: Feynman rules

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Propagators:

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

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Propagators:

$$\langle \phi^i_a \bar{\phi}_j^c \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

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$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

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Vertices:

$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G_{ijk} \circlearrowleft + G_{ikj} \circlearrowleft), \quad V_{\bar{\phi}_i \bar{\phi}_j \bar{\phi}_k} = \begin{array}{c} \overrightarrow{D_2} \\ | \\ \text{---} \\ | \\ \overleftarrow{D_2} \end{array} i(G^{ijk} \circlearrowleft + G^{ikj} \circlearrowleft)$$

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Computing supergraphs: Feynman rules

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$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \bigcirc + G_{ikj} \bigcirc), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \bigcirc + G^{ikj} \bigcirc)$$

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

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$$V_{\phi^i \phi^j \phi^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G_{ijk} \odot + G_{ikj} \odot), \quad V_{\bar{\phi}^i \bar{\phi}^j \bar{\phi}^k} = \begin{array}{c} D_2 \\ | \\ \text{---} \\ | \\ D_2 \end{array} i(G^{ijk} \odot + G^{ikj} \odot)$$

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Computing supergraphs: Feynman rules

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Computing supergraphs: Feynman rules

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Computing supergraphs: Feynman rules

Fields (all in adjoint):

Chiral superfields: X, Y, Z (or ϕ^j) Vector superfields: V

Propagators:

$$\langle \phi^i_a \bar{\phi}^c_j \rangle = \frac{\overrightarrow{p}}{p} = \delta^i_j \frac{\delta^a_d \delta^c_b}{p^2} \delta^4(\theta_1 - \theta_2)$$

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Vertices:

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Computing supergraphs: Chiral Operators

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Single trace operators made with ϕ^i or W_α

Scalar operators: $\mathcal{O} = \text{tr}[XYYXZZZYXXXX]$

Scalar operators as chiral vertices:



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Putting everything together:

Compute:

$$\mathcal{D} = \sum_{\ell=0}^{\infty} g^{2\ell} \mathcal{D}_\ell$$

$\mathcal{D}_0 = L,$  $\mathcal{D}_1 = 2\chi(1), \quad \mathcal{D}_2 = -2[\chi(1,2) + \chi(2,1)] - 4\chi(1)$

Computing supergraphs: Chiral Operators

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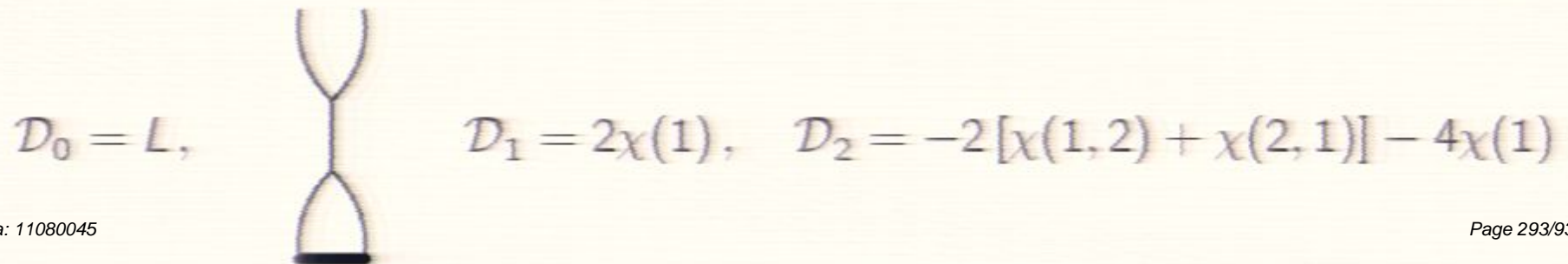
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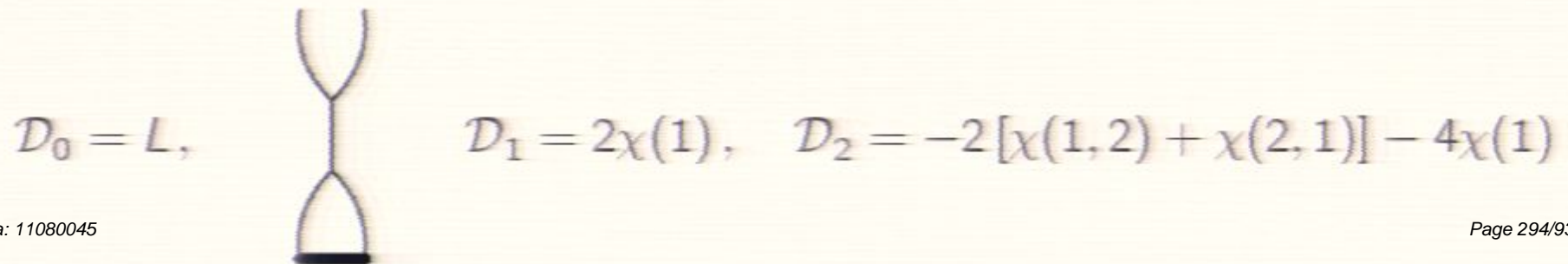
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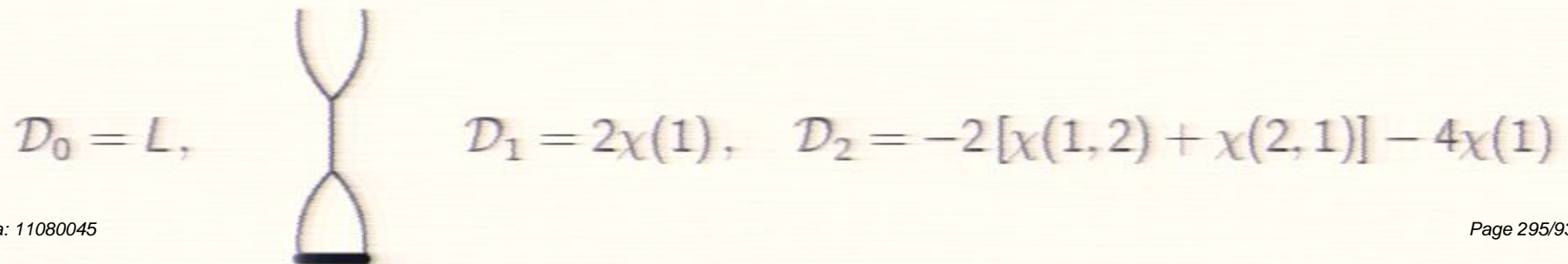
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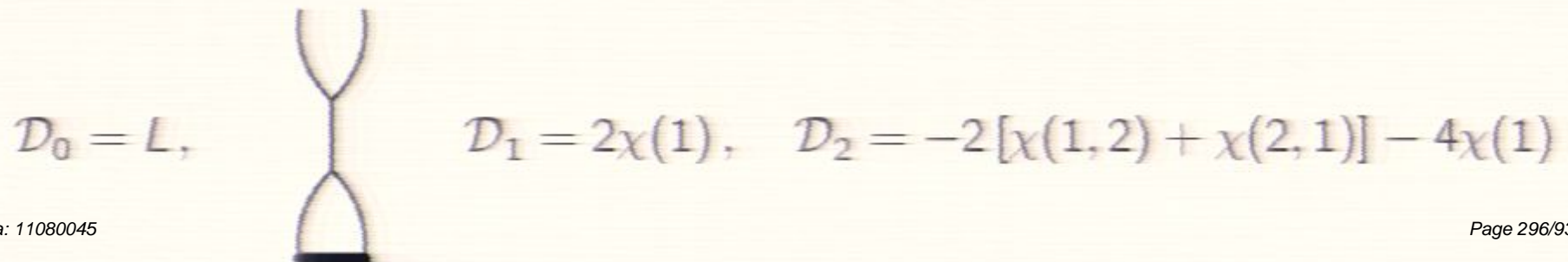
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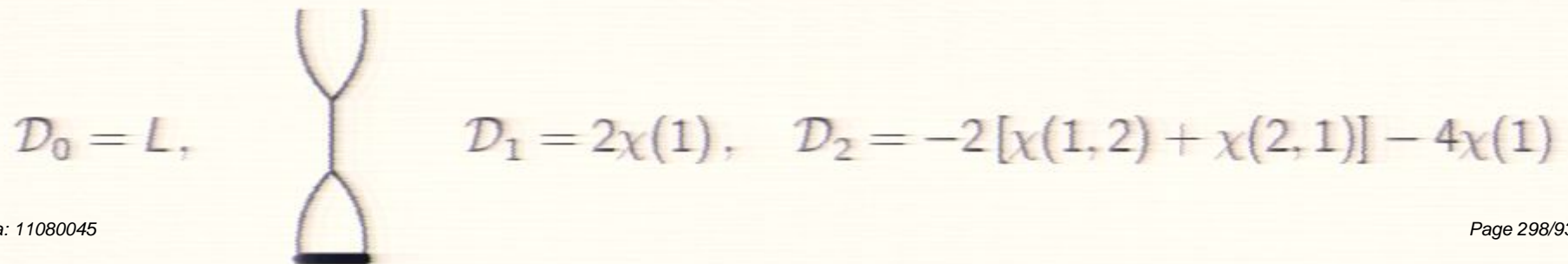
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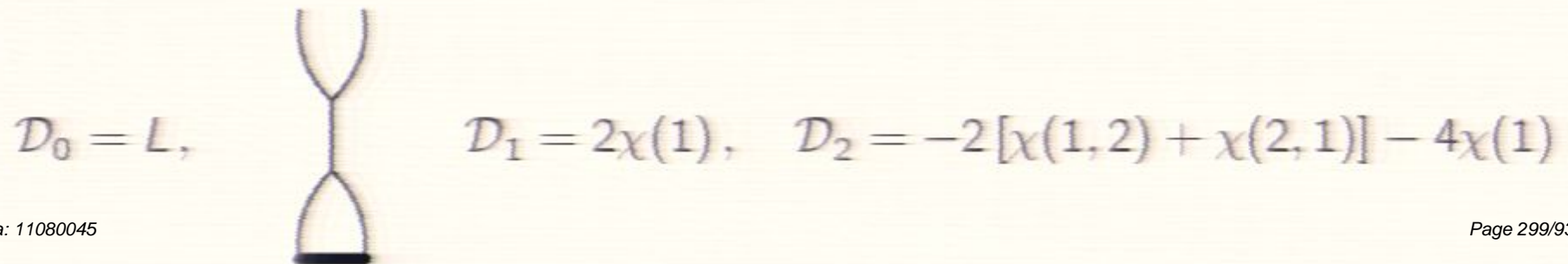
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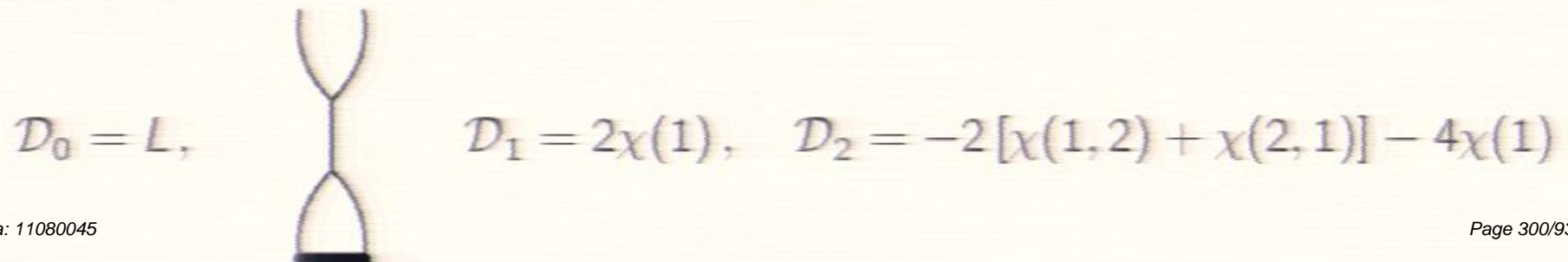
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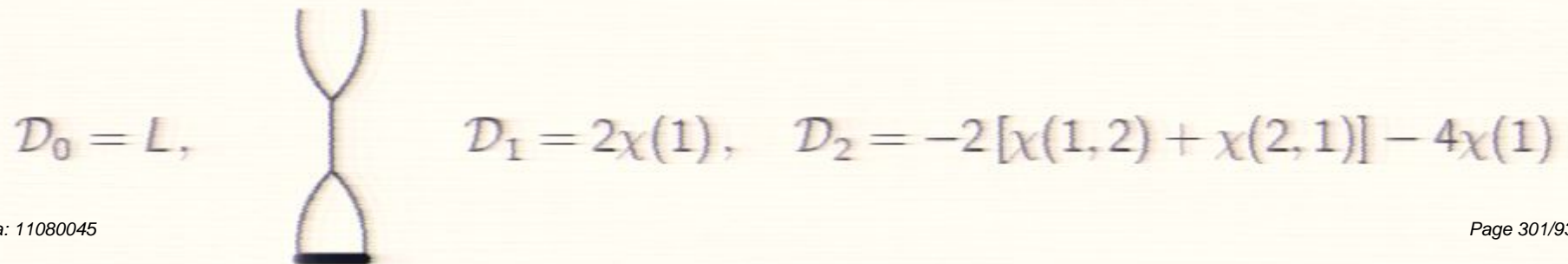
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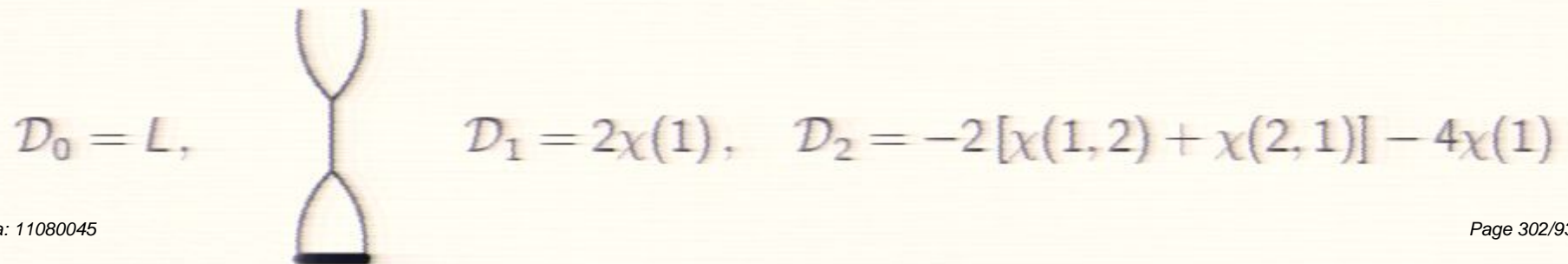
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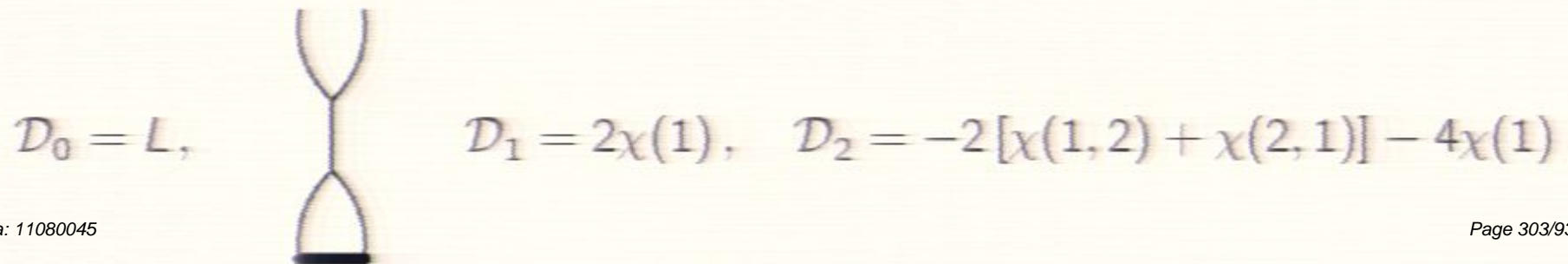
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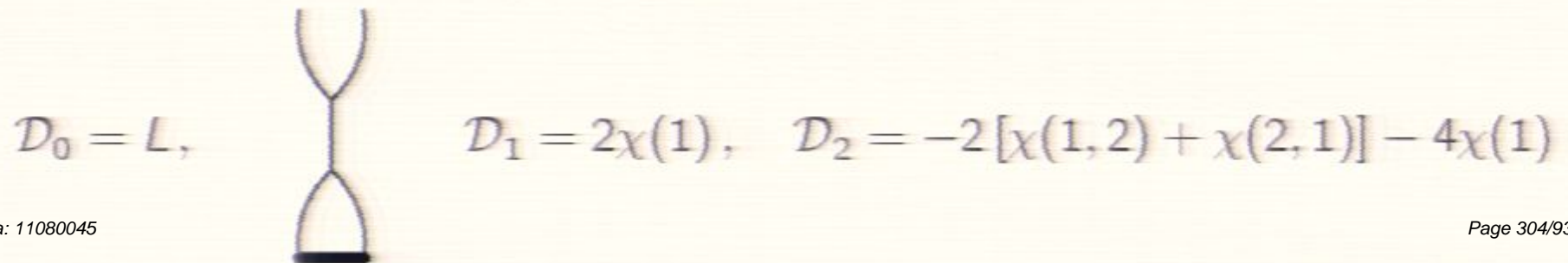
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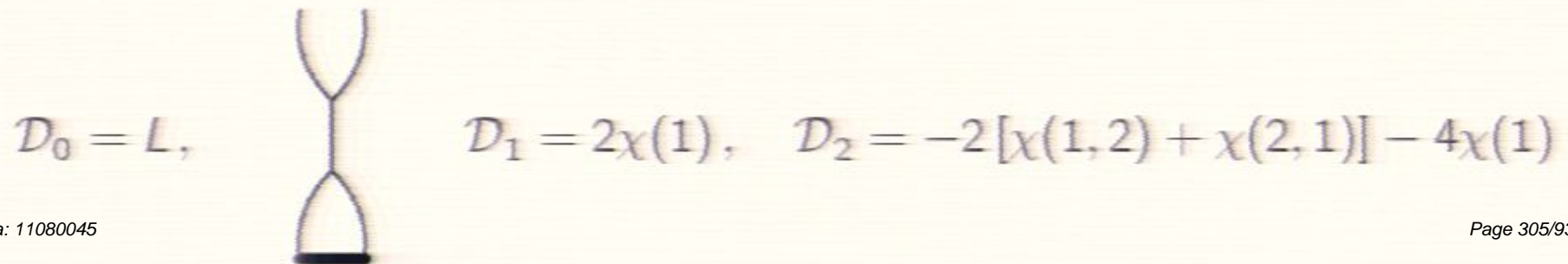
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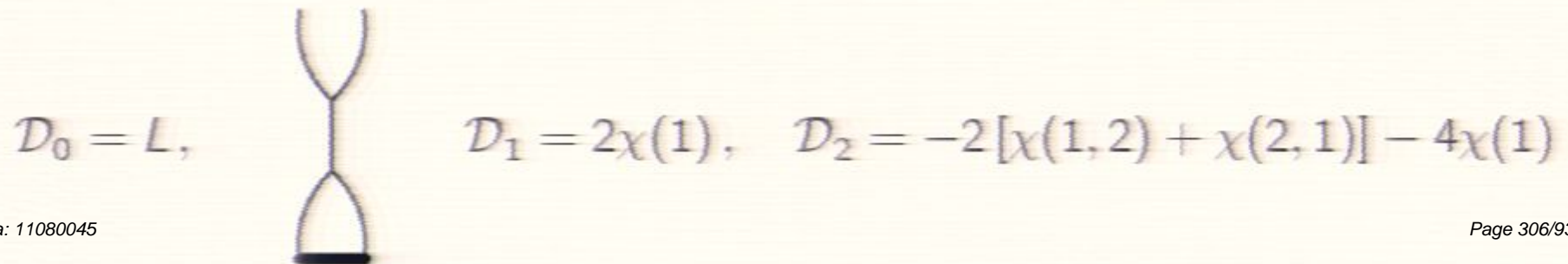
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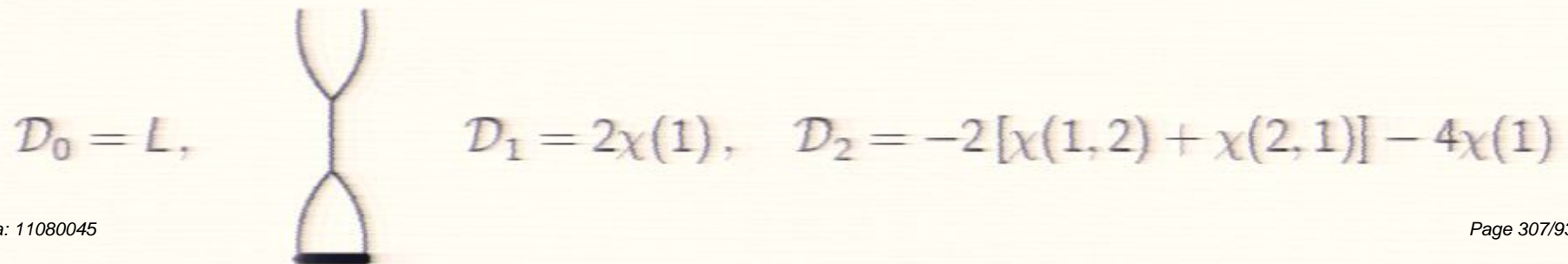
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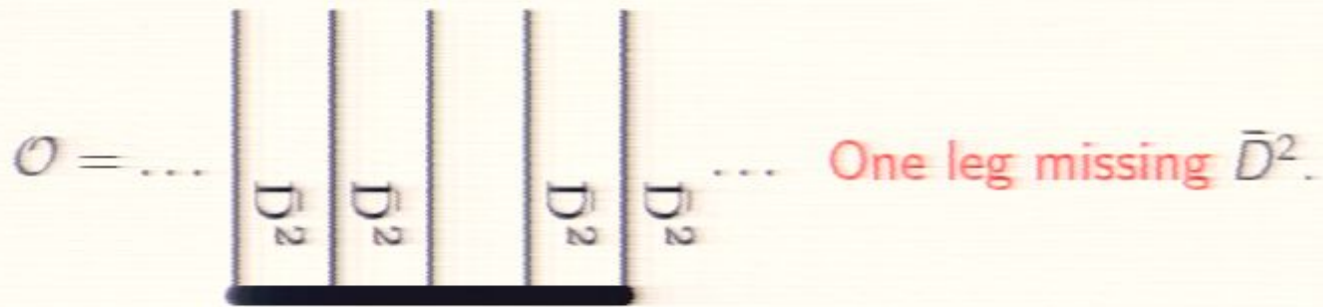


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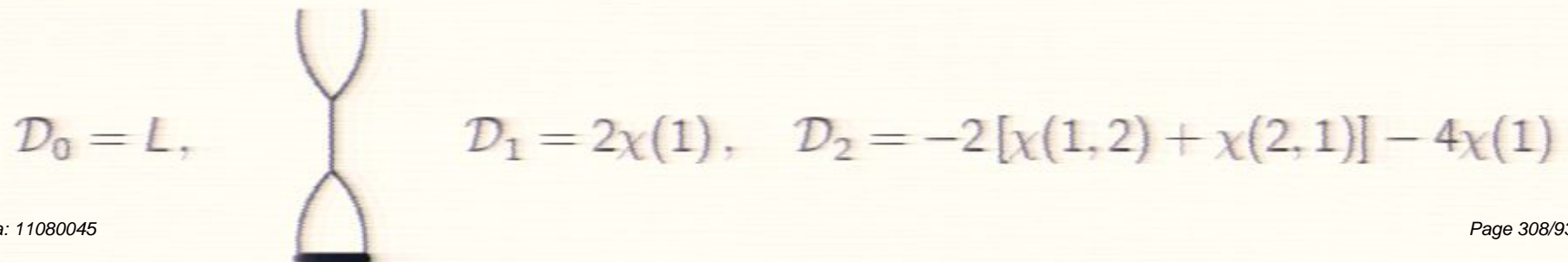
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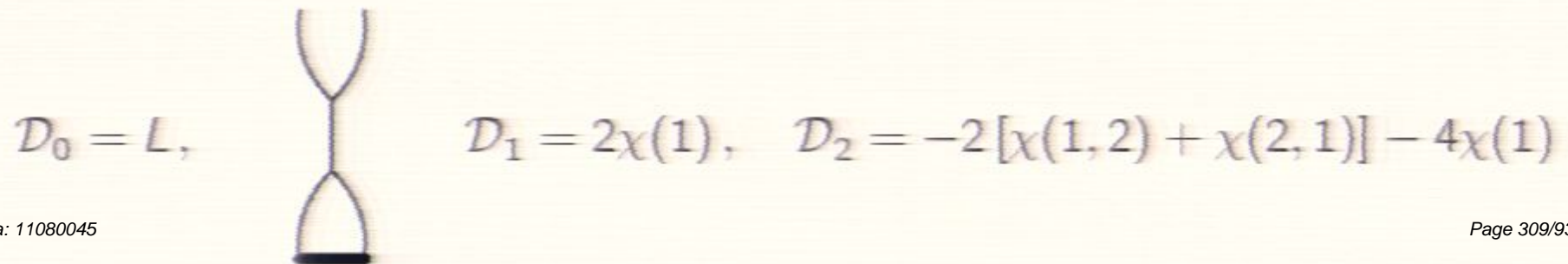
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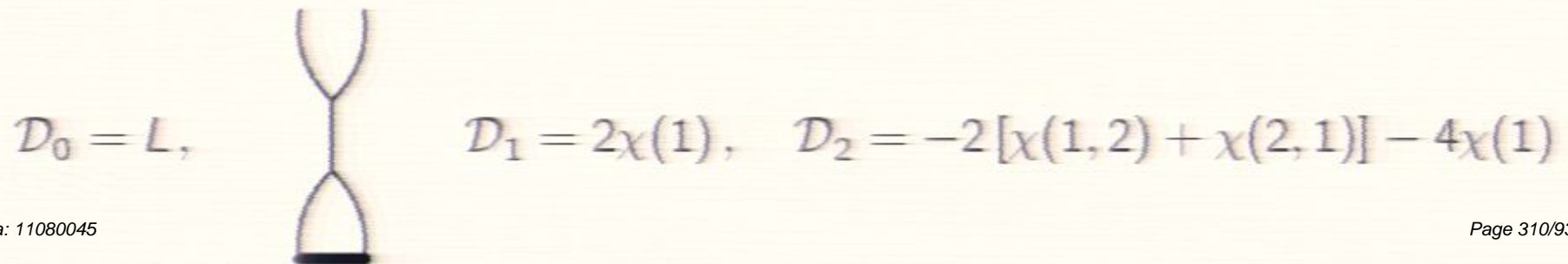
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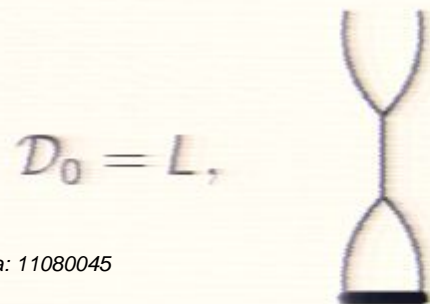
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Three loops ($SU(2)$ sector)

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$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

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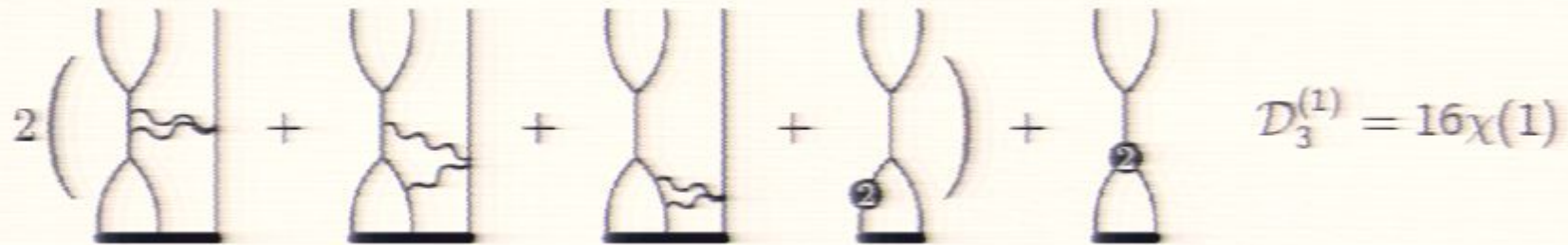
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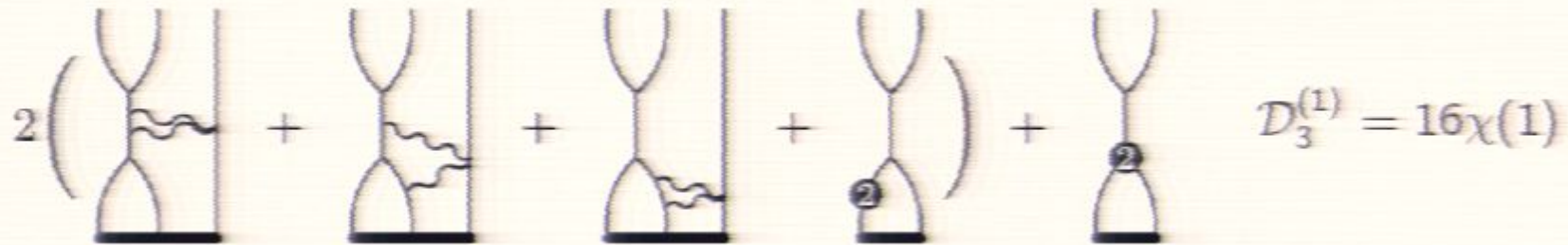
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The diagram shows the expansion of the supergraph $\mathcal{D}_3^{(1)}$. It consists of five terms:

- A bracketed sum of four terms, each representing a different configuration of wavy lines (representing fermion loops) between two vertices.
- A term with a loop labeled '2'.

 The equation is: $2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) + \text{diagram 5} = \mathcal{D}_3^{(1)} = 16\chi(1)$

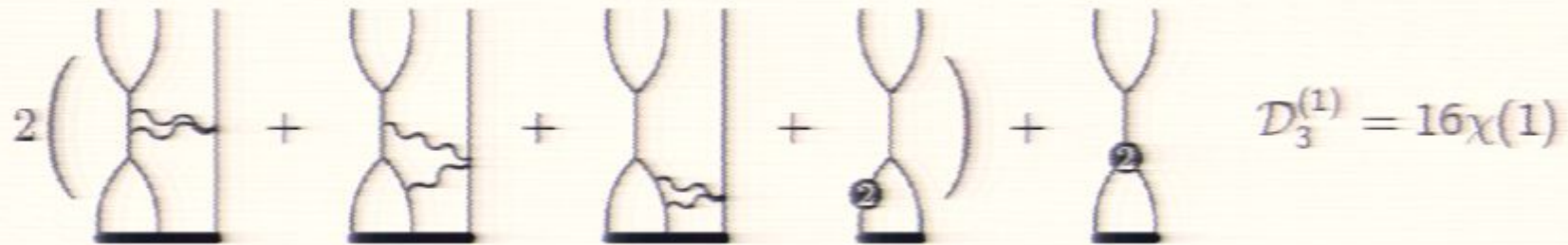
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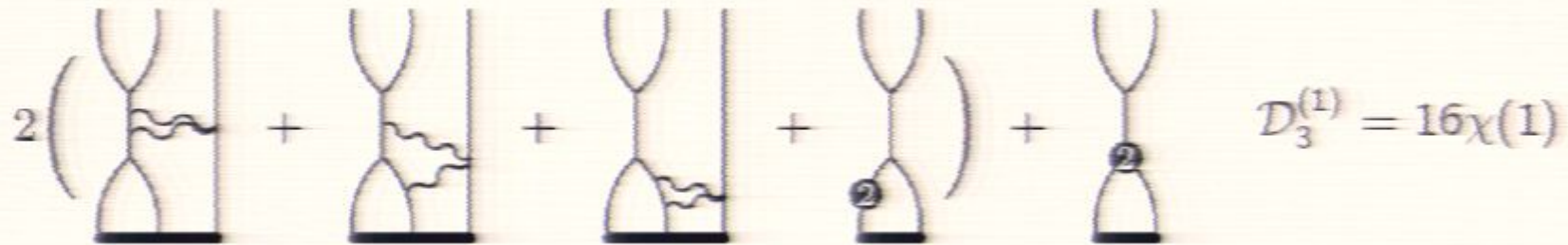
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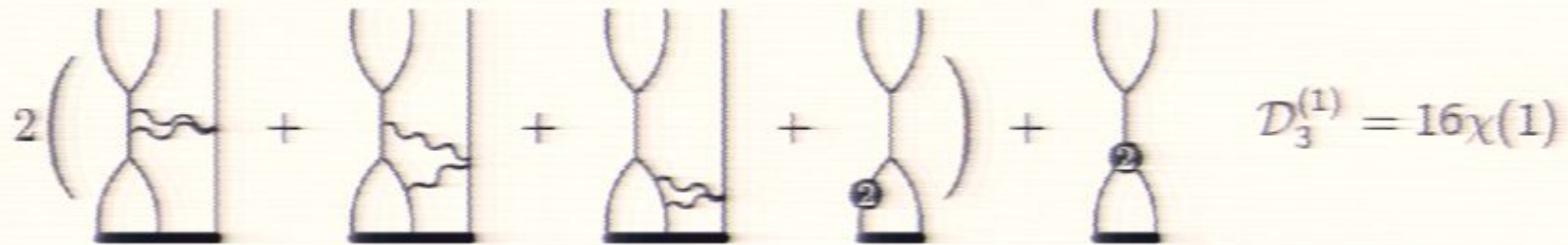
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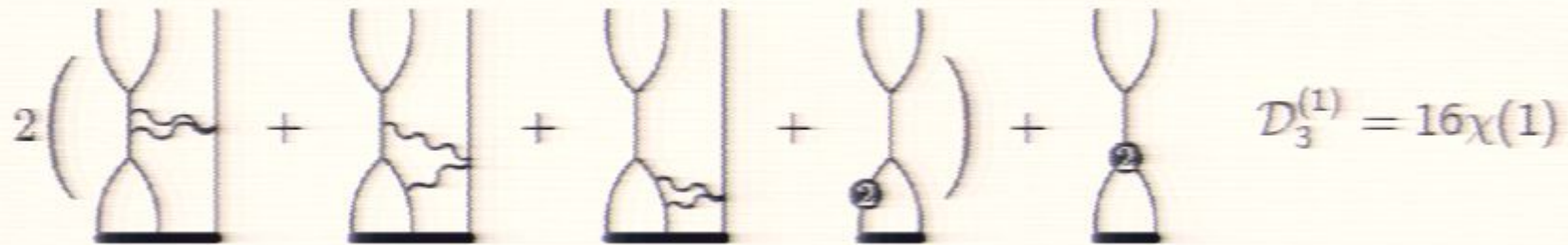
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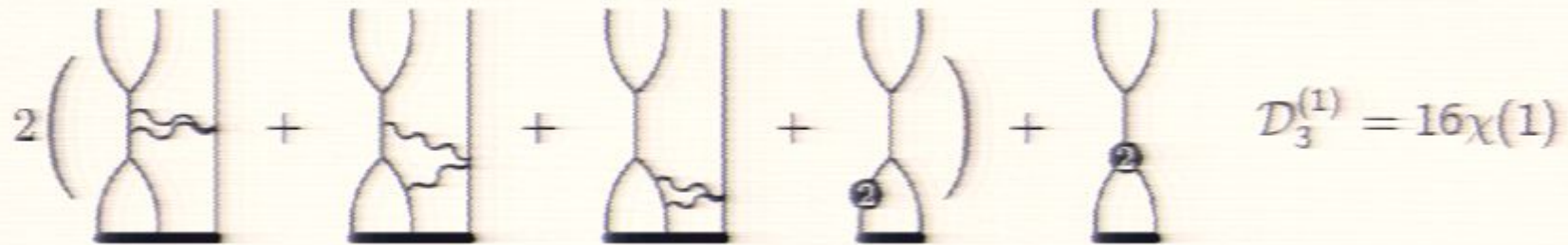
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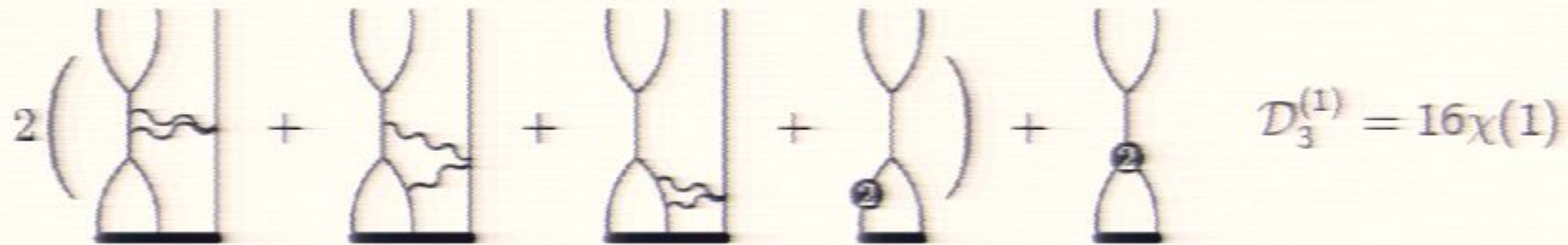
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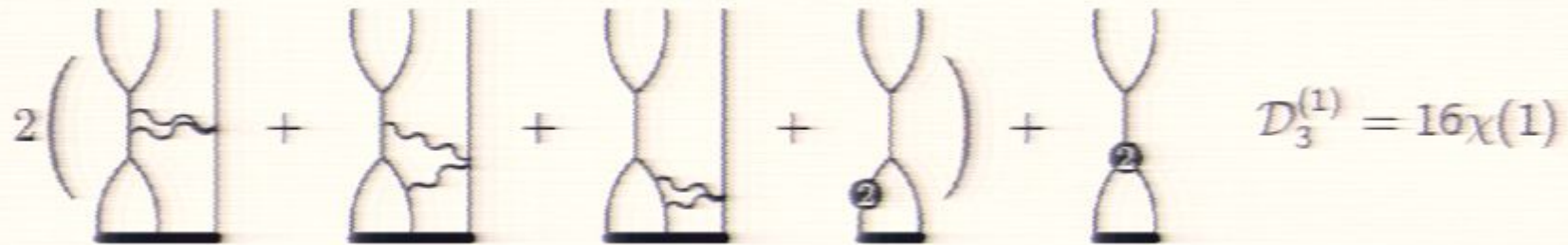
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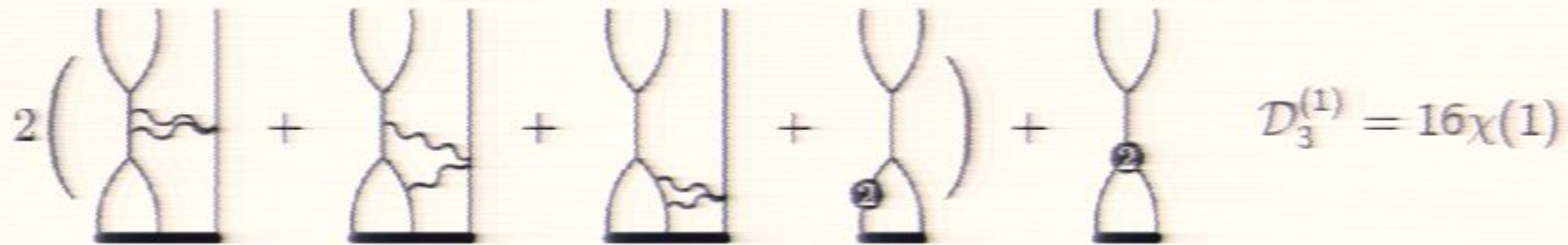
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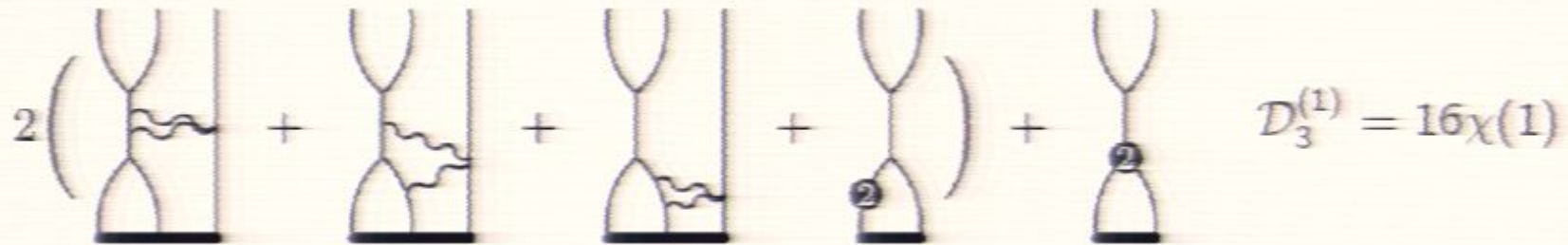
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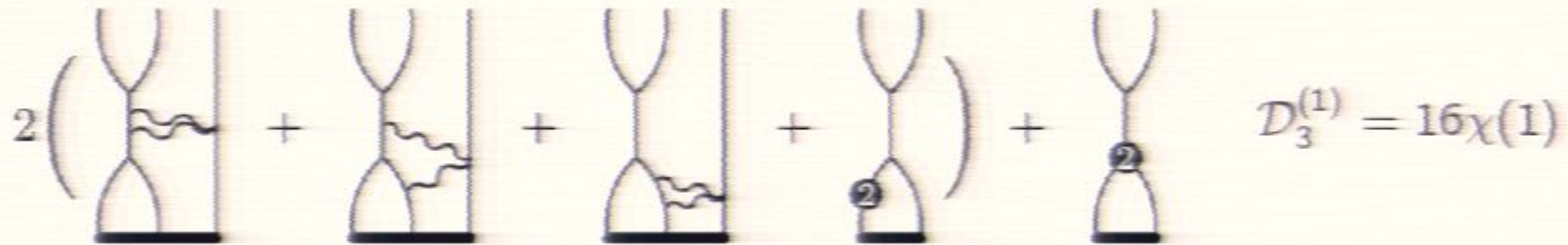
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Supergraphs: Sieg (2010)

Diagrammatic representation of the supergraph expansion for $\mathcal{D}_3^{(1)}$. The diagram shows five terms: a sum of four terms in parentheses multiplied by 2, and a fifth term. The terms are diagrams of two vertical lines connected at the top and bottom, with various internal wavy lines and a small circle labeled '2'.

$$2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) + \text{diagram 5} \quad \mathcal{D}_3^{(1)} = 16\chi(1)$$

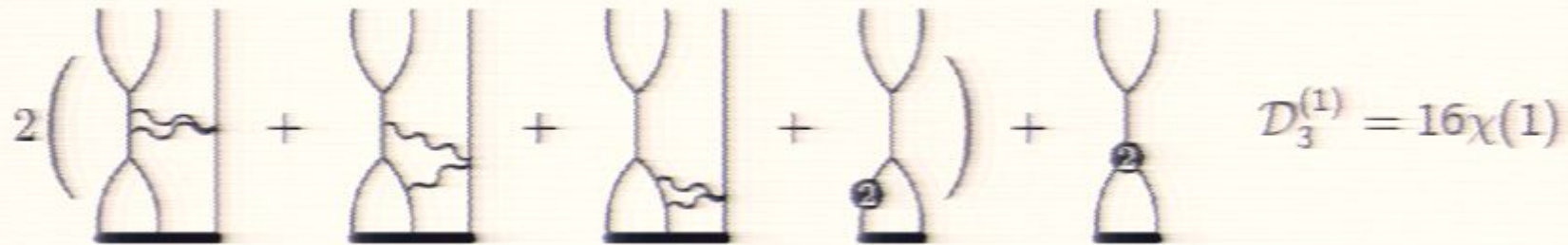
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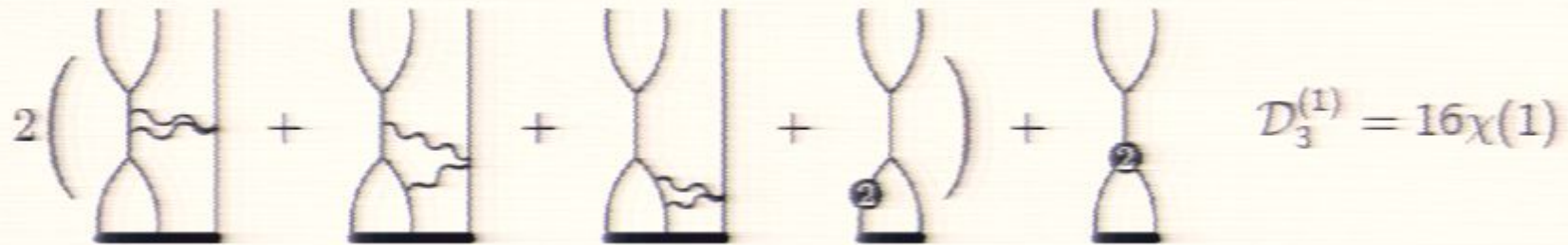
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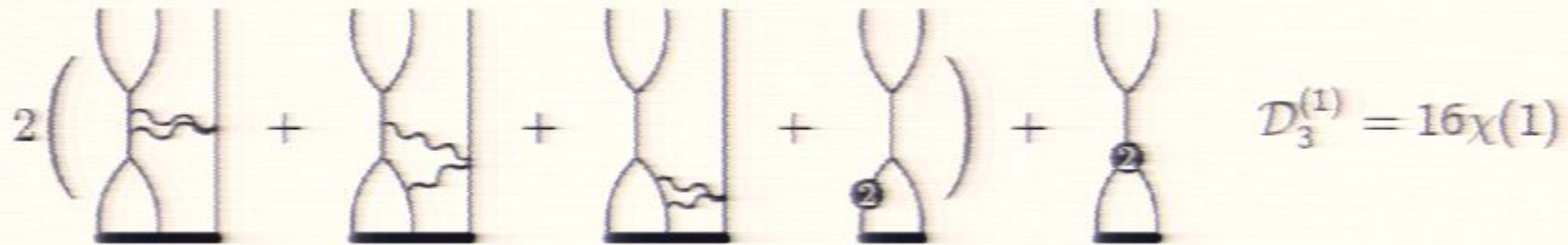
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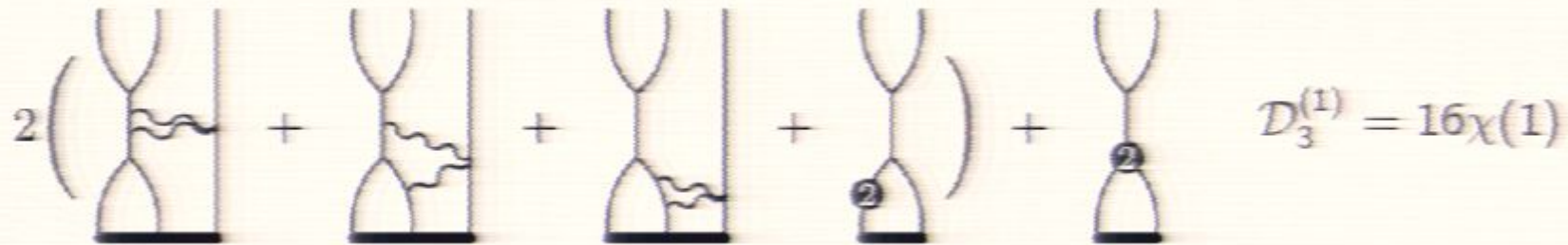
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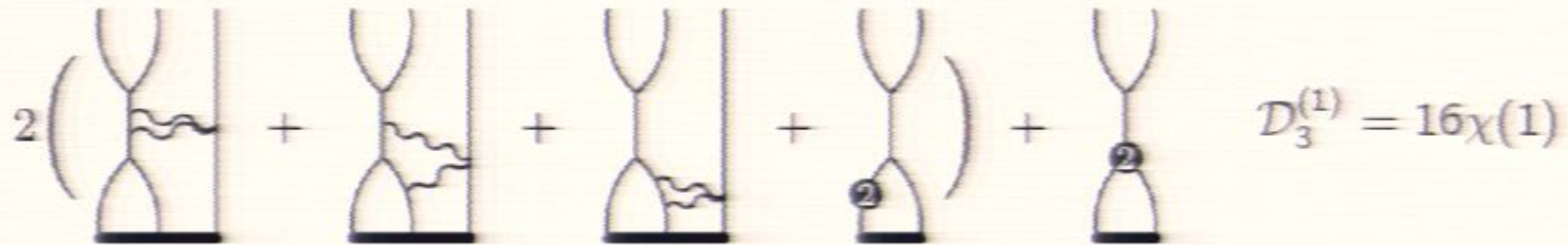
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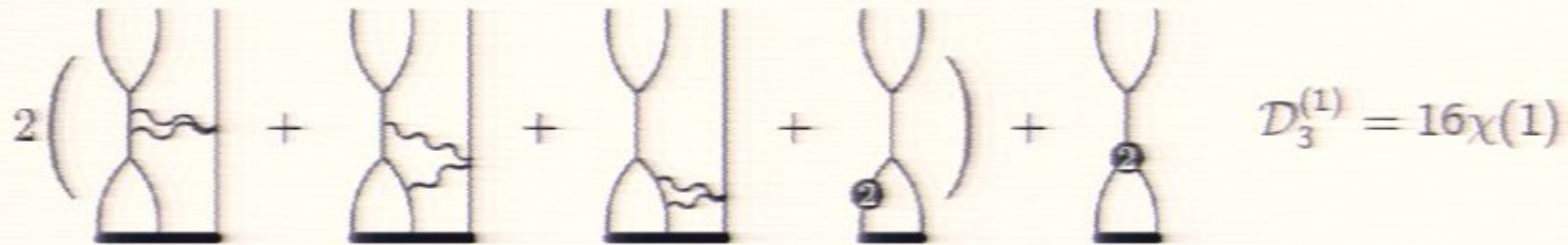
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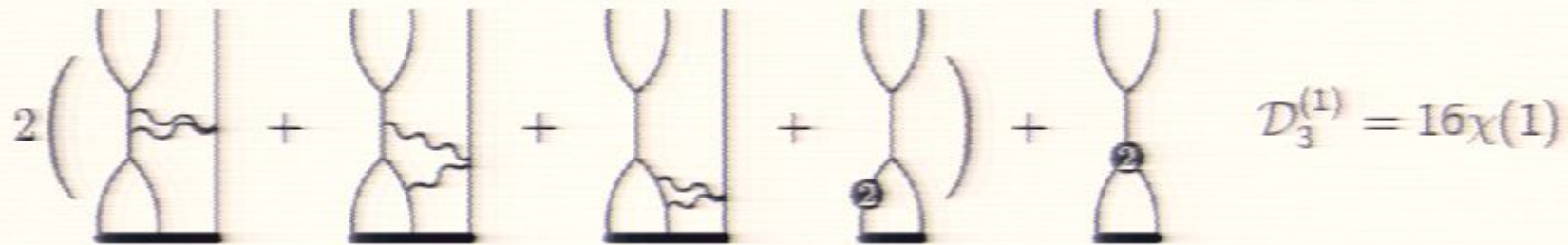
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Supergraphs: Sieg (2010)



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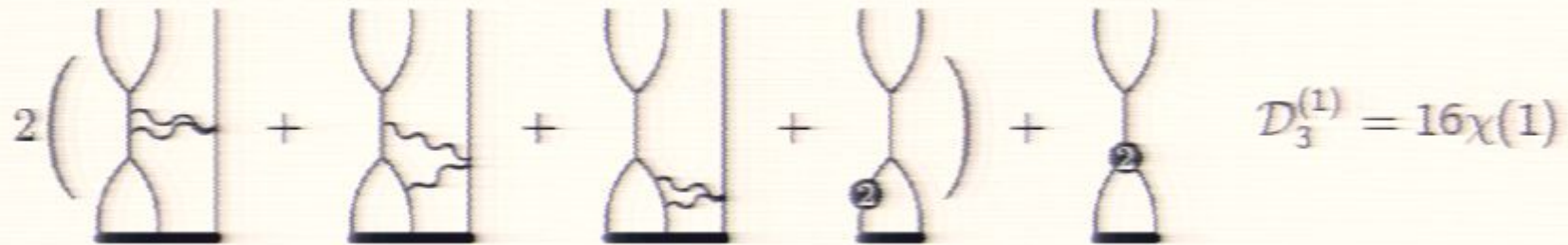
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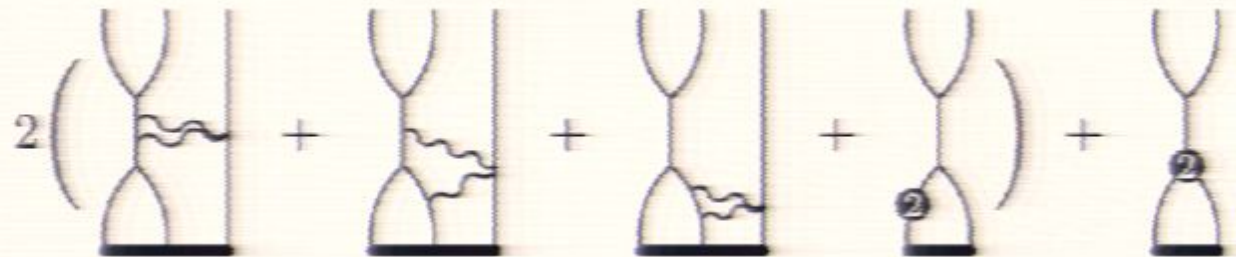
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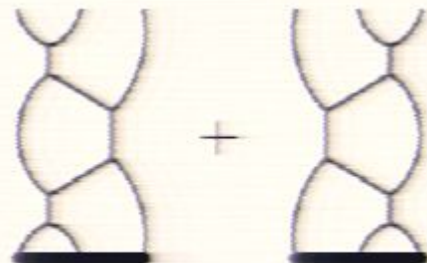
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Supergraphs: Sieg (2010)



$$2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) + \text{diagram 5} \quad \mathcal{D}_3^{(1)} = 16\chi(1)$$



$$\text{diagram 1} + \text{diagram 2} \quad \mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$$

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

Diagrammatic equation for $\mathcal{D}_3^{(1)}$. On the left, a large parenthesis is multiplied by 2. Inside the parenthesis are five diagrams: 1) a vertical line with a wavy line connecting two vertices on the line; 2) a vertical line with a wavy line connecting two vertices on the line, with a small circle containing the number 2 at the bottom vertex; 3) a vertical line with a wavy line connecting two vertices on the line, with a small circle containing the number 2 at the top vertex; 4) a vertical line with a wavy line connecting two vertices on the line, with a small circle containing the number 2 at the top vertex; 5) a vertical line with a small circle containing the number 2 at the bottom vertex. To the right of the parenthesis is a plus sign followed by a diagram of a vertical line with a small circle containing the number 2 at the bottom vertex. To the right of this is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

Diagrammatic equation for $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)}$. On the left, two diagrams are added: 1) a diagram with two vertices on a vertical line, each having a loop attached to it; 2) a diagram with two vertices on a vertical line, each having a loop attached to it, with a small circle containing the number 2 at the bottom vertex. To the right of the plus sign is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are grouped by a large left parenthesis and a right parenthesis. Each term consists of a vertical line with a loop structure. The first three terms have a wavy line connecting two loops. The fourth term has a small circle with the number '2' inside, connected to the loop structure. The fifth term is a vertical line with a single loop and a small circle with the number '2' inside. To the right of the diagram is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms separated by a plus sign. Each term consists of a vertical line with two loops connected by a horizontal line. The first term has the horizontal line above the loops, and the second term has it below. To the right of the diagram is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

Diagrammatic equation for $\mathcal{D}_3^{(1)}$. On the left, a large parenthesis contains five diagrams: a vertical line with a wavy line connecting two vertices, a vertical line with a wavy line connecting two vertices, a vertical line with a wavy line connecting two vertices, a vertical line with a wavy line connecting two vertices, and a vertical line with a wavy line connecting two vertices. To the right of the parenthesis is a vertical line with a wavy line connecting two vertices. To the right of this is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

Diagrammatic equation for $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)}$. On the left, two diagrams are shown: a vertical line with two vertices connected by a wavy line, and a vertical line with two vertices connected by a wavy line. To the right of these is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

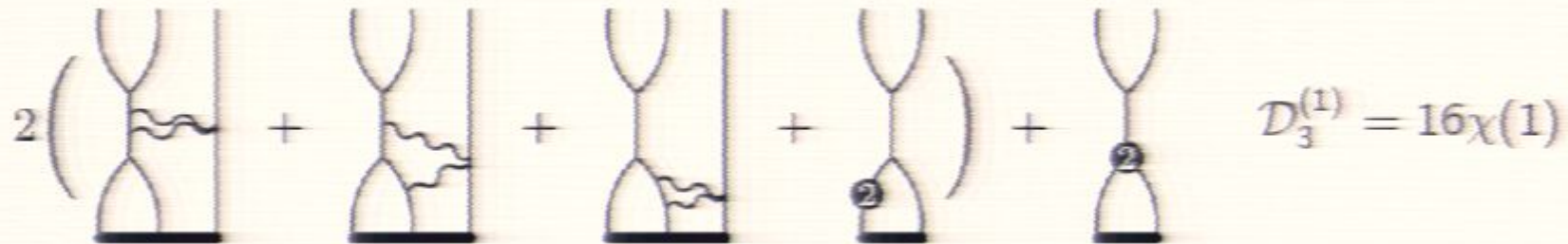
Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

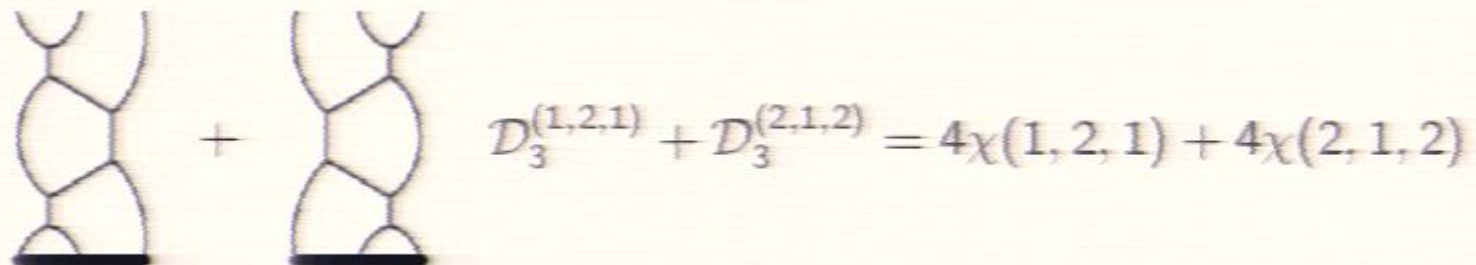
$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)



$$2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) + \text{diagram 5} \quad \mathcal{D}_3^{(1)} = 16\chi(1)$$



$$\text{diagram 1} + \text{diagram 2} \quad \mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$$

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

Diagrammatic equation for $\mathcal{D}_3^{(1)}$. On the left, a large parenthesis is preceded by a factor of 2. Inside the parenthesis are four diagrams representing different ways to connect three vertical lines (representing fermions) with wavy lines (representing gluons). The first three diagrams show a wavy line connecting the top two lines, the middle two, and the bottom two, respectively. The fourth diagram shows a wavy line connecting the top two lines, with a small circle containing the number 2 on the bottom line. To the right of the parenthesis is a plus sign followed by a fifth diagram: a vertical line with a small circle containing the number 2 on the bottom line. To the right of this entire expression is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

Diagrammatic equation for $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)}$. On the left, two diagrams are shown separated by a plus sign. Each diagram consists of two vertical lines connected by two wavy lines, forming a figure-eight shape. The first diagram has the wavy lines connecting the top two lines and the bottom two lines. The second diagram has the wavy lines connecting the middle two lines and the bottom two lines. To the right of these diagrams is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

Diagrammatic equation for $\mathcal{D}_3^{(1)}$. On the left, a large parenthesis is preceded by a factor of 2. Inside the parenthesis are four diagrams representing different ways to connect three vertical lines with a wavy line. The first three diagrams show the wavy line connecting the top two lines, and the fourth shows it connecting the bottom two lines. The fourth diagram has a small circle with the number 2 inside. To the right of the parenthesis is a plus sign followed by a diagram of a vertical line with a small circle with the number 2 inside. To the right of this is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

Diagrammatic equation for $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)}$. On the left are two diagrams representing the exchange of two lines. The first diagram shows two lines crossing, with the top line going to the right and the bottom line going to the left. The second diagram shows the opposite crossing. To the right of these diagrams is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are enclosed in large parentheses and multiplied by a factor of 2. Each of these four terms consists of a vertical line with a loop on the left side. The first three terms have a wavy line connecting the top and bottom of the loop. The fourth term has a small circle with the number '2' inside, located at the bottom of the loop. The fifth term is a vertical line with a loop at the bottom, also containing a small circle with the number '2' inside. To the right of this sum is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms added together. Each term consists of two vertical lines connected by two loops, one above the other. The first term has the loops connected in a specific way, and the second term has them connected differently. To the right of this sum is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are grouped by a large left parenthesis and a right parenthesis. Each term consists of a vertical line with a loop at the top and a loop at the bottom. The first three terms have a wavy line connecting the two loops. The fourth term has a small circle with the number '2' inside, connected to the bottom loop. The fifth term is a vertical line with a loop at the top and a loop at the bottom, with a small circle with the number '2' inside connected to the bottom loop. To the right of the diagram is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms separated by a plus sign. Each term consists of a vertical line with a loop at the top and a loop at the bottom, with a loop in the middle. The first term has the middle loop on the left side, and the second term has the middle loop on the right side. To the right of the diagram is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are grouped by a large left parenthesis and a right parenthesis. Each term consists of a vertical line with a loop structure. The first three terms have a wavy line connecting the two sides of the loop. The fourth term has a small circle with the number '2' inside. The fifth term is a vertical line with a small circle with the number '2' inside. To the right of the diagram is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms added together. Each term consists of a vertical line with a loop structure. The first term has a loop on the left side, and the second term has a loop on the right side. To the right of the diagram is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are grouped by a large parenthesis with a coefficient of 2. Each term consists of two vertical lines connected at the top and bottom by a loop. A wavy line connects the two vertical lines at different heights. The heights of the wavy line increase from left to right. The fifth term is a single vertical line with a loop at the bottom, with a small circle containing the number 2 next to it. To the right of the diagram is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms separated by a plus sign. Each term consists of two vertical lines connected at the top and bottom by two loops. The loops are arranged in a chain. The first term has the loops arranged such that the top loop is on the left and the bottom loop is on the right. The second term has the loops arranged such that the top loop is on the right and the bottom loop is on the left. To the right of the diagram is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are enclosed in large parentheses and multiplied by a factor of 2. Each term consists of a vertical line with a loop structure. The first three terms have a wavy line connecting two vertices on the vertical line. The fourth term has a small circle with the number '2' inside, connected to the vertical line. The fifth term is a vertical line with a small circle with the number '2' inside. To the right of the diagram is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms added together. Each term consists of a vertical line with two loops. The first term has a loop on the left side, and the second term has a loop on the right side. To the right of the diagram is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

Diagrammatic equation for $\mathcal{D}_3^{(1)}$. On the left, a large parenthesis contains five diagrams: a vertical line with a wavy line connecting two vertices, a vertical line with a wavy line connecting two vertices, a vertical line with a wavy line connecting two vertices, a vertical line with a wavy line connecting two vertices, and a vertical line with a wavy line connecting two vertices. To the right of the parenthesis is a vertical line with a wavy line connecting two vertices. To the right of this is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

Diagrammatic equation for $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)}$. On the left, two diagrams are shown: a vertical line with two vertices connected by a wavy line, and a vertical line with two vertices connected by a wavy line. To the right of these is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

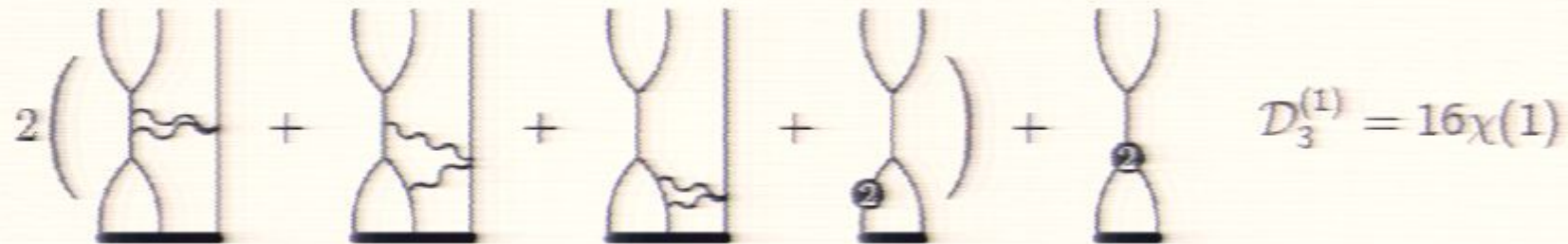
Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

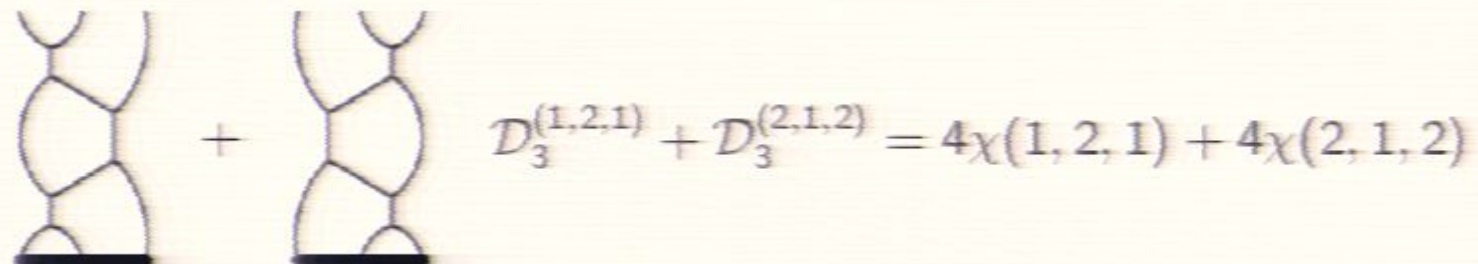
$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)



$$2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) + \text{diagram 5} = \mathcal{D}_3^{(1)} = 16\chi(1)$$



$$\text{diagram 6} + \text{diagram 7} = \mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$$

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure


Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

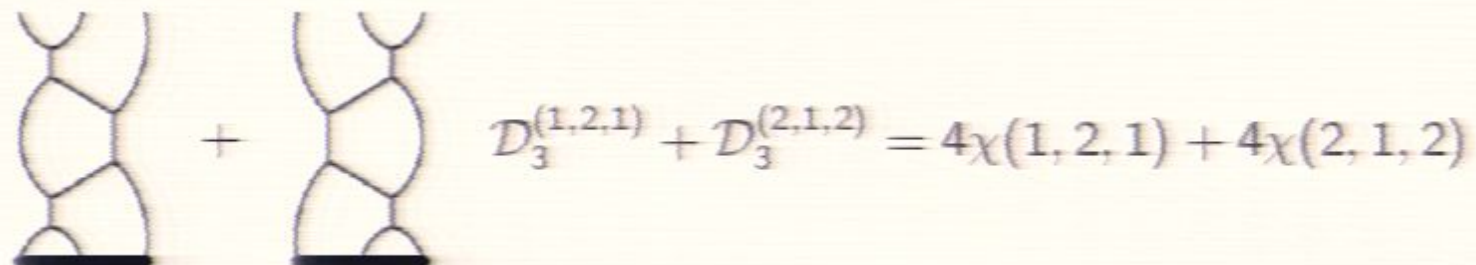
$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)



$$2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) + \text{diagram 5} \quad \mathcal{D}_3^{(1)} = 16\chi(1)$$



$$\text{diagram 1} + \text{diagram 2} \quad \mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$$

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are grouped in large parentheses and multiplied by a factor of 2. Each term consists of a vertical line with a loop structure. The first three terms have a wavy line connecting two vertices on the vertical line. The fourth term has a vertex labeled '2' on the vertical line. The fifth term is a vertical line with a vertex labeled '2' on the vertical line. To the right of the diagram is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms added together. Each term consists of a vertical line with two loops. The first term has a loop on the left side, and the second term has a loop on the right side. To the right of the diagram is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure

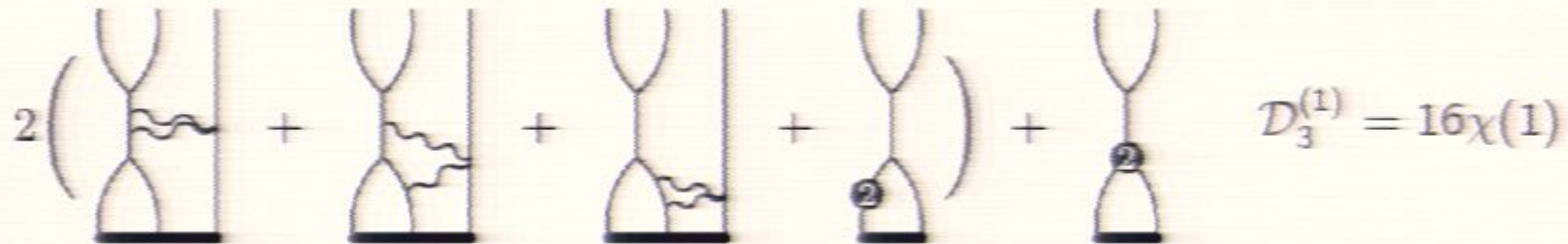
Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

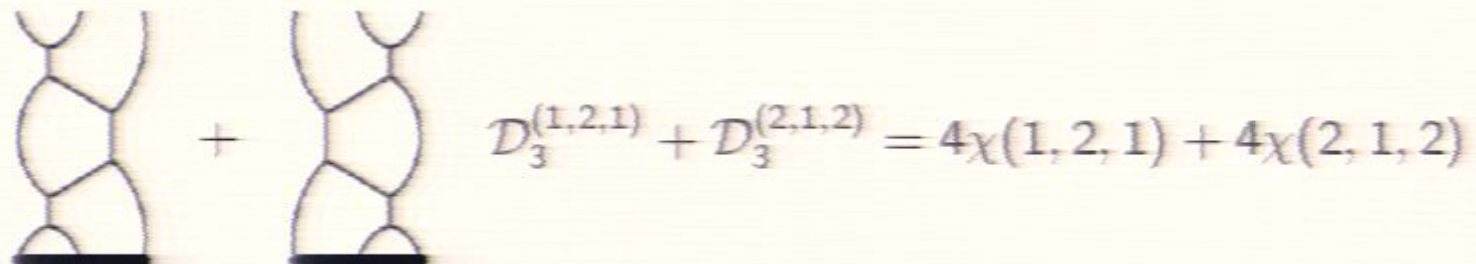
$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)



$$2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) + \text{diagram 5} \quad \mathcal{D}_3^{(1)} = 16\chi(1)$$



$$\text{diagram 1} + \text{diagram 2} \quad \mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$$

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are grouped in large parentheses and multiplied by a factor of 2. Each term consists of a vertical line with a loop structure. The first three terms have a wavy line connecting two vertices on the vertical line. The fourth term has a vertex labeled '2' on the vertical line. The fifth term is a vertical line with a vertex labeled '2' on the vertical line. To the right of the diagram is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms added together. Each term consists of a vertical line with a loop structure. The first term has a vertex labeled '2' on the vertical line. The second term has a vertex labeled '2' on the vertical line. To the right of the diagram is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

$$2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) + \text{diagram 5} \quad \mathcal{D}_3^{(1)} = 16\chi(1)$$

$$\text{diagram 6} + \text{diagram 7} \quad \mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$$

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure


Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

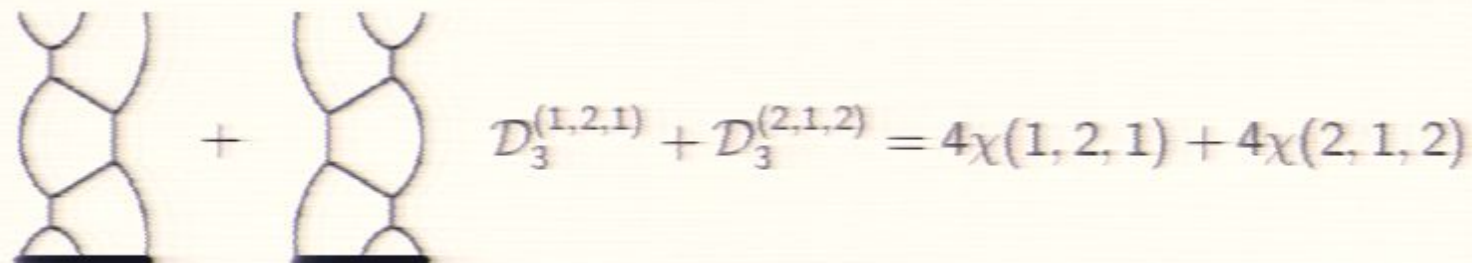
$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)



$$2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} + \text{diagram 5} \right) = \mathcal{D}_3^{(1)} = 16\chi(1)$$



$$\text{diagram 1} + \text{diagram 2} = \mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$$

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure

Three loops ($SU(2)$ sector)

Integrability: Beisert, Kristjansen, Staudacher (2003)

$$\mathcal{D}_3 = 4[\chi(1, 2, 3) + \chi(3, 2, 1)] - 4\chi(1, 3) + 16(\chi(1, 2) + \chi(2, 1)) + 24\chi(1)$$

Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are grouped in large parentheses and multiplied by a factor of 2. Each term consists of a vertical line with a loop structure. The first three terms have a wavy line connecting the two sides of the loop. The fourth term has a small circle with the number '2' inside. The fifth term is a simple loop with a small circle with the number '2' inside. To the right of the diagram is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms added together. Each term consists of a vertical line with a loop structure. The first term has a loop with a wavy line connecting the two sides. The second term has a loop with a wavy line connecting the two sides. To the right of the diagram is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

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Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

The diagram shows a sum of five terms. The first four terms are grouped in large parentheses and multiplied by a factor of 2. Each term consists of two vertical lines connected at the top and bottom by a loop. A wavy line connects the two vertical lines at different heights. The first three terms have the wavy line at different positions, and the fourth has a small circle with the number '2' next to it. The fifth term is a single vertical line with a small circle with the number '2' next to it. To the right of the diagram is the equation $\mathcal{D}_3^{(1)} = 16\chi(1)$.

The diagram shows two terms added together. Each term consists of two vertical lines connected at the top and bottom by a loop. The first term has a loop connecting the two vertical lines in the middle. The second term has a loop connecting the two vertical lines in the middle, but the loop is oriented differently. To the right of the diagram is the equation $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$.

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure

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Integrability: Beisert, Kristjansen, Staudacher (2003)

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Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

Diagrammatic equation for $\mathcal{D}_3^{(1)} = 16\chi(1)$. The left side shows a sum of five diagrams: a pair of diagrams with a wavy line, a pair with a zigzag line, a pair with a dashed line, a pair with a solid line, and a single diagram with a circle labeled '2'. The right side is $\mathcal{D}_3^{(1)} = 16\chi(1)$.

Diagrammatic equation for $\mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$. The left side shows two diagrams representing the supergraphs. The right side is the sum of the corresponding characters.

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure

Three loops ($SU(2)$ sector)

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Consistent with $F_n(g^2)$ using $\chi(1, 2, 1) = \chi(2, 1, 2) = \chi(1)$.

Supergraphs: Sieg (2010)

$$2 \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) + \text{diagram 5} \quad \mathcal{D}_3^{(1)} = 16\chi(1)$$

$$\text{diagram 6} + \text{diagram 7} \quad \mathcal{D}_3^{(1,2,1)} + \mathcal{D}_3^{(2,1,2)} = 4\chi(1, 2, 1) + 4\chi(2, 1, 2)$$

$$[\chi(1)^3]_c = 4\chi(1) + 4\chi(1, 2) + 4\chi(2, 1) + \chi(1, 2, 3) + \chi(3, 2, 1) + \chi(1, 2, 1) + \chi(2, 1, 2)$$

Supergraphs capture the $[\chi(1)^3]_c$ structure

Conjecture

The all-loop sum of supergraphs with the connected chiral function $\chi(a_1, a_2 \dots a_j, \dots a_n)$ is given by

$$F_n(g^2)\chi(a_1, a_2 \dots a_j, \dots a_n)$$

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- ▶ This is a statement about supergraphs and can be applied outside the $SU(2)$ sector where

$$\chi(a_1 \dots a_j, a_j \pm 1, a_j \dots a_n) \neq \chi(a_1 \dots a_j \dots a_n)$$

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- ▶ Can be applied it to $\mathcal{N} = 1$ superconformal theories with different chiral functions.

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Four loops ($SU(2)$ sector)

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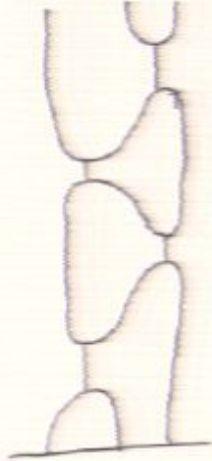
$\chi(1, 2, 1, 2):$



$$\Delta^{(1,2,1,2)} \mathcal{D}_4 = -10 \chi(1, 2, 1, 2) \text{ Consistent with } F_4(g^2)$$

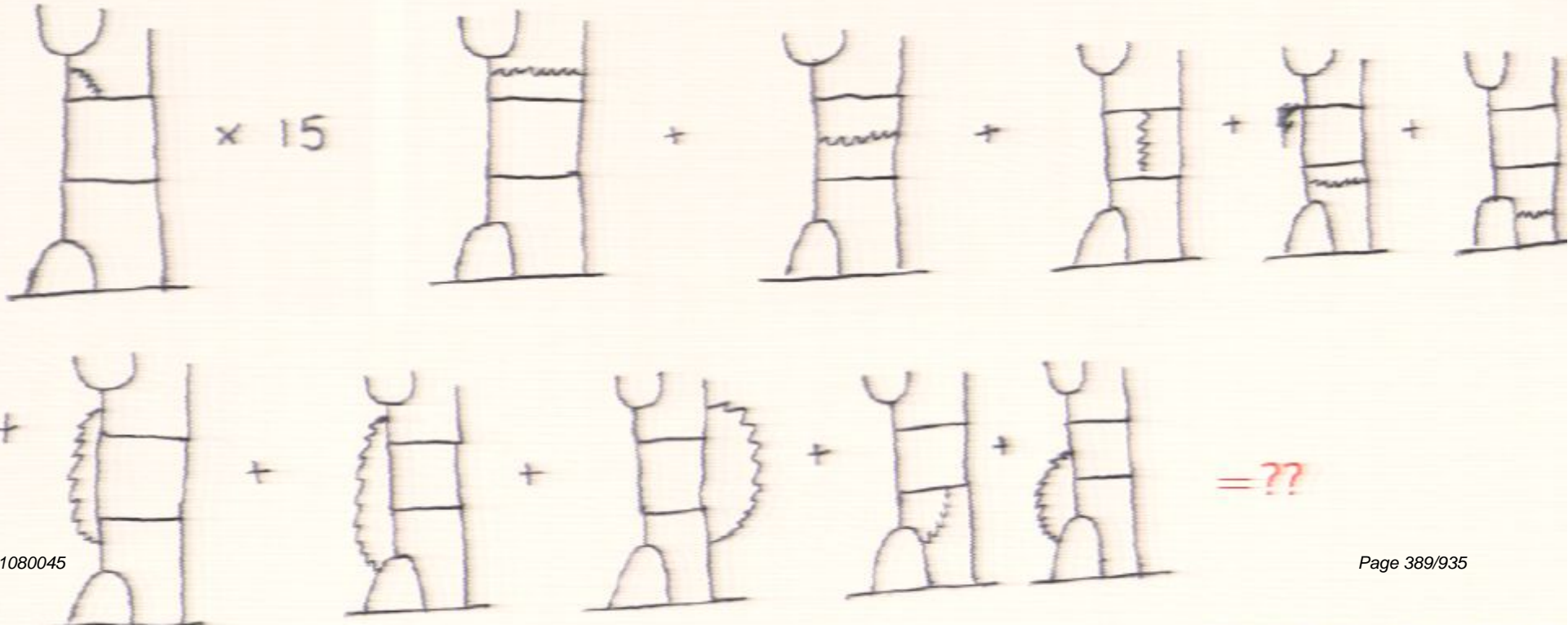
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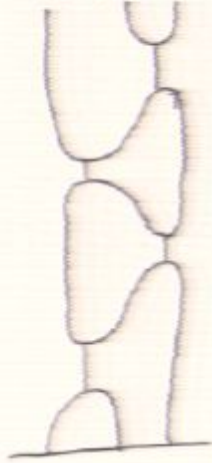
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$\chi(1, 2, 1)$:



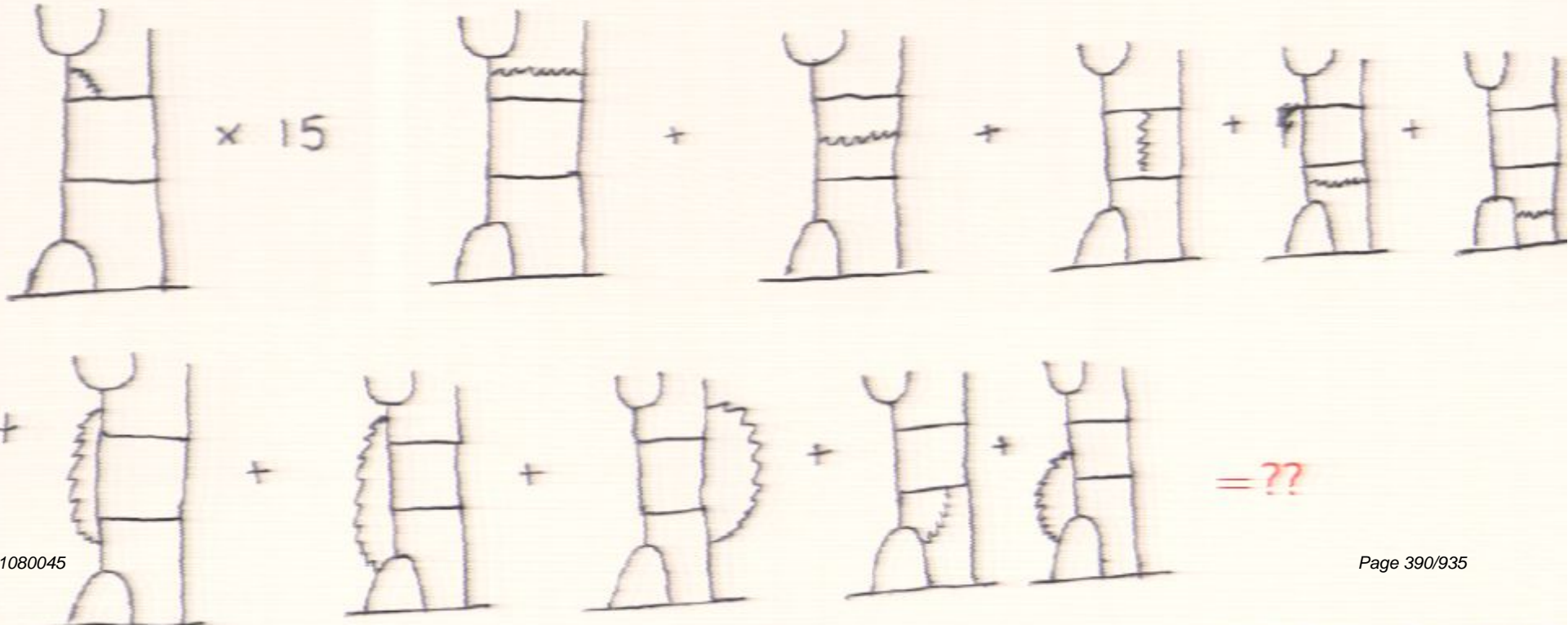
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$\Delta^{(1,2,1,2)} \mathcal{D}_4 = -10 \chi(1, 2, 1, 2)$ Consistent with $F_4(g^2)$

$\chi(1, 2, 1)$:



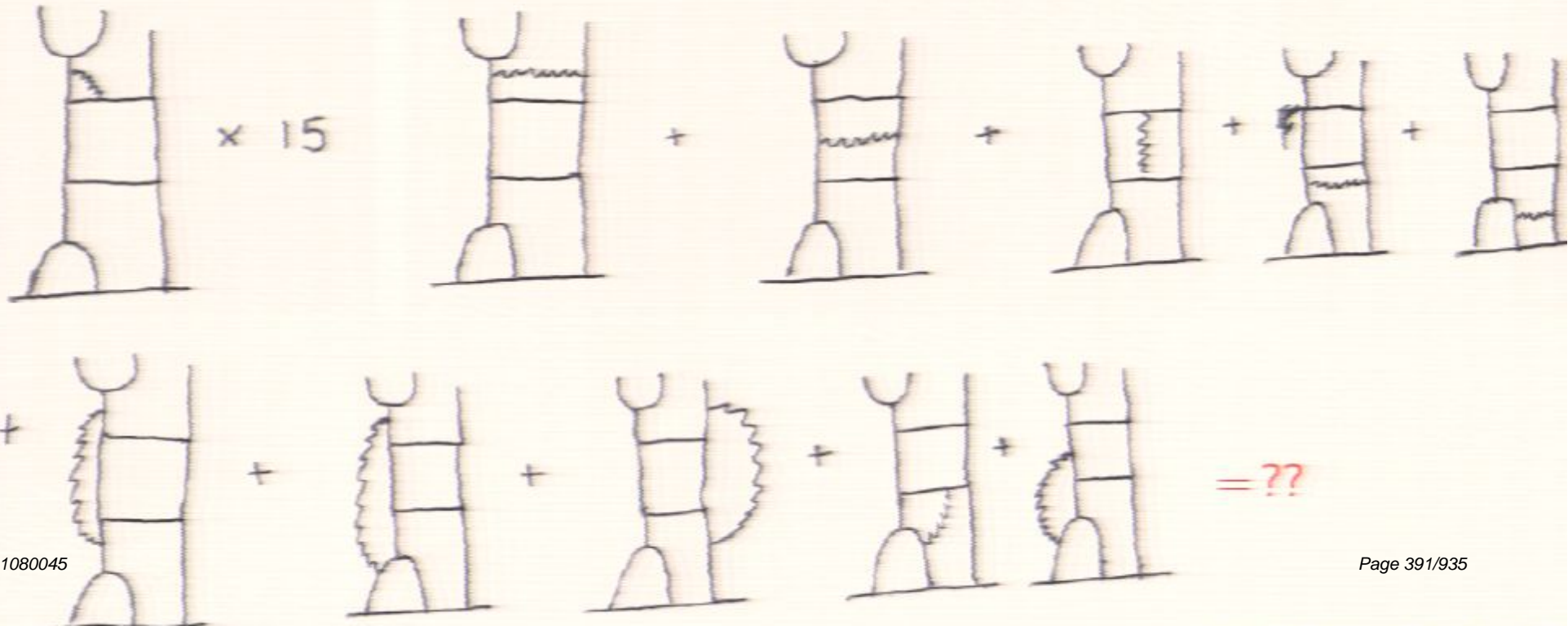
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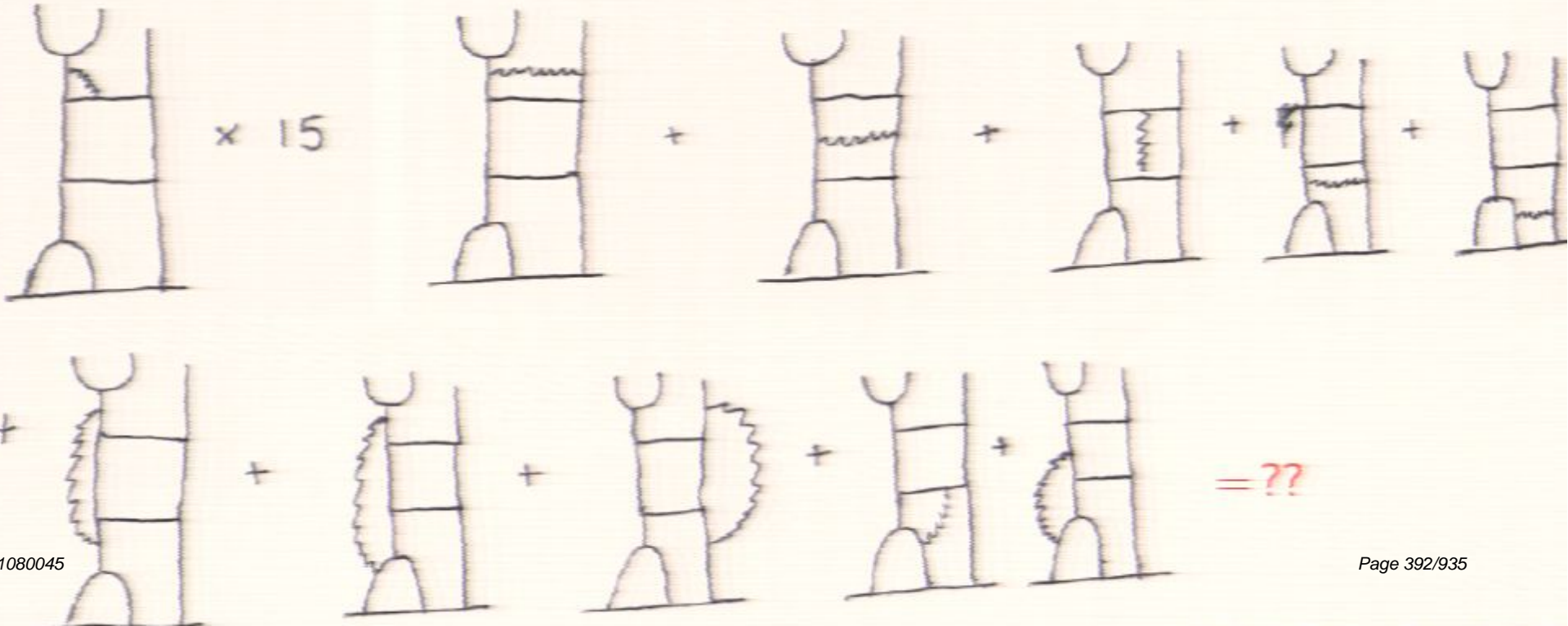
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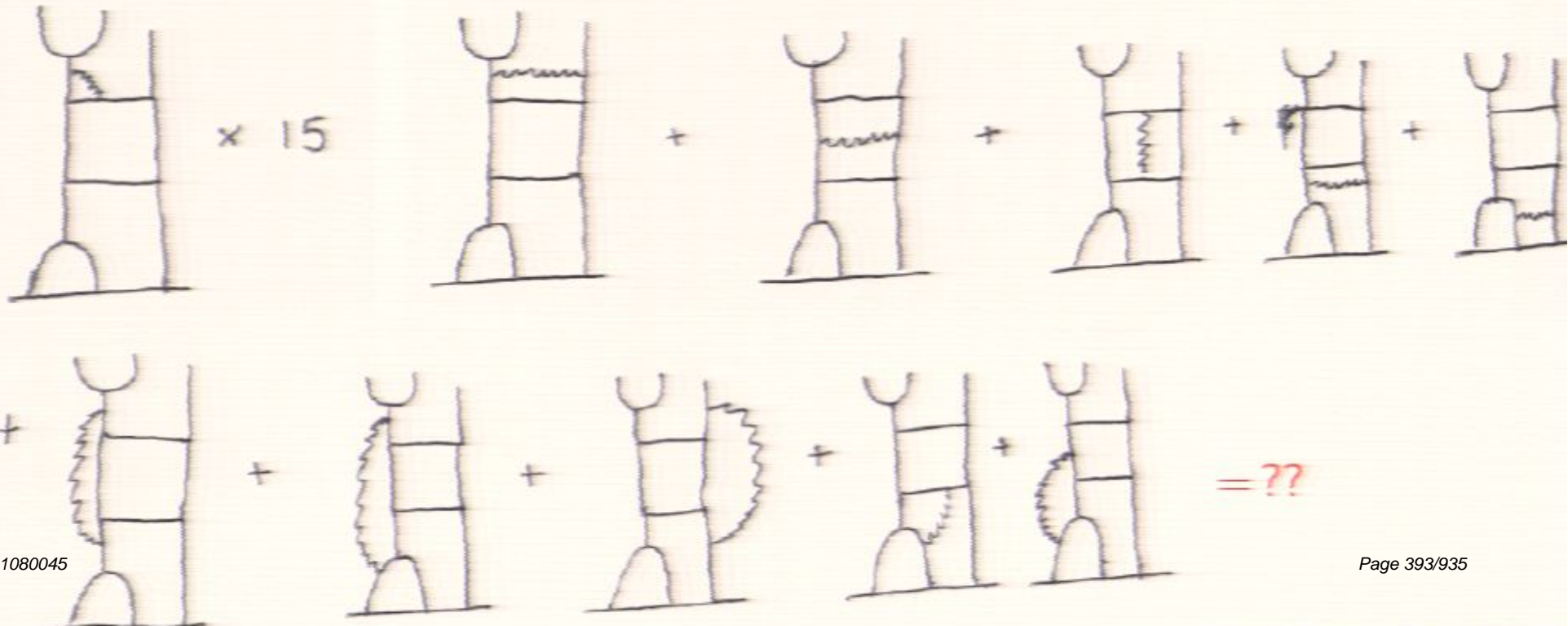
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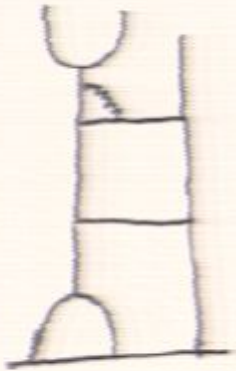
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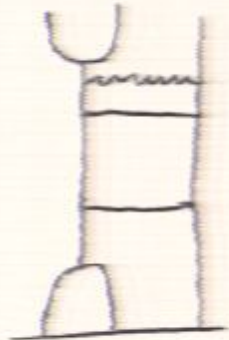


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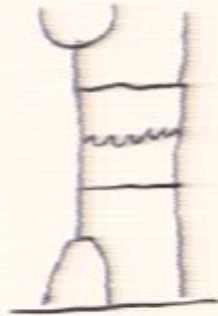
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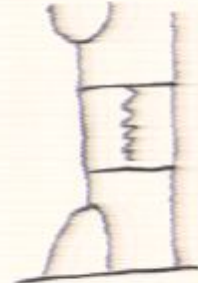
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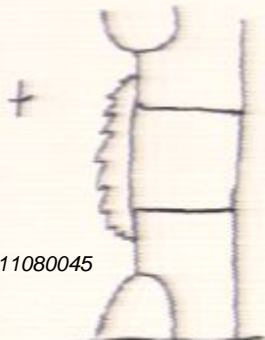
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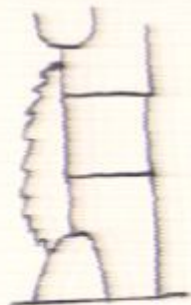
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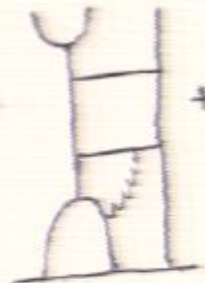
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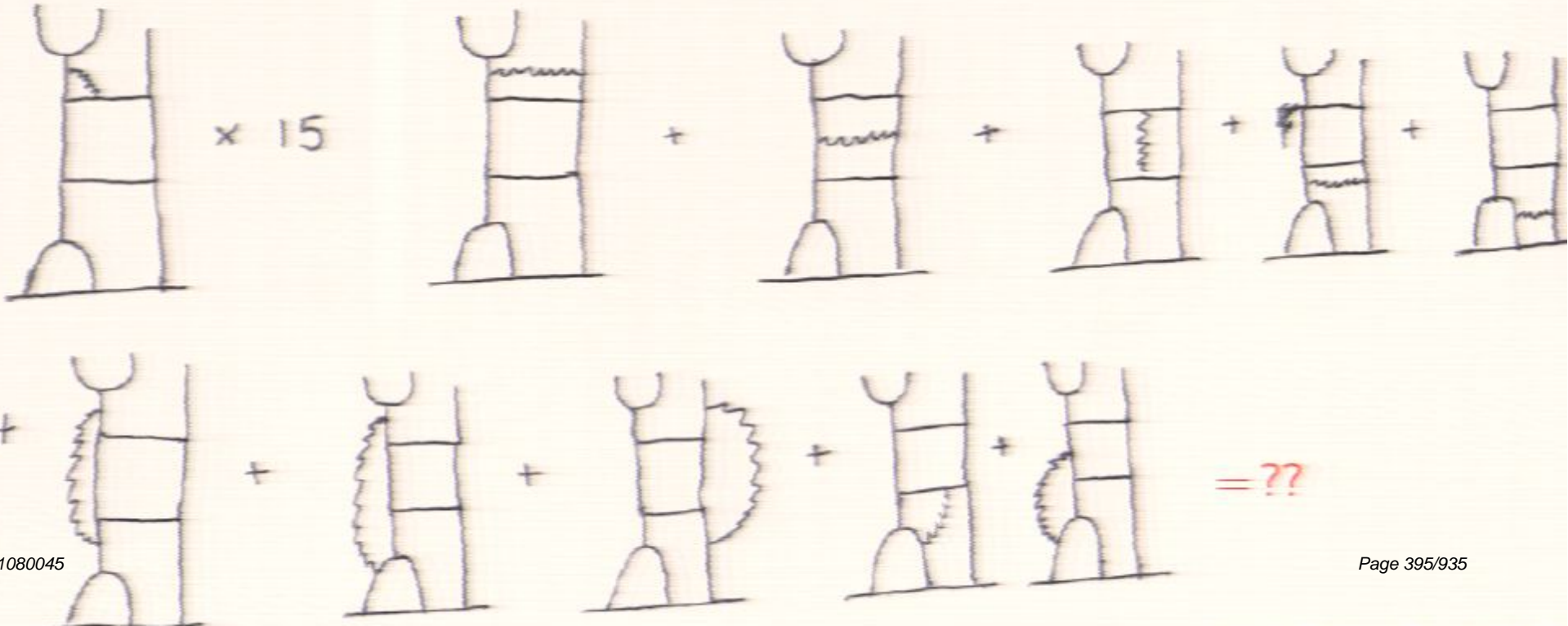
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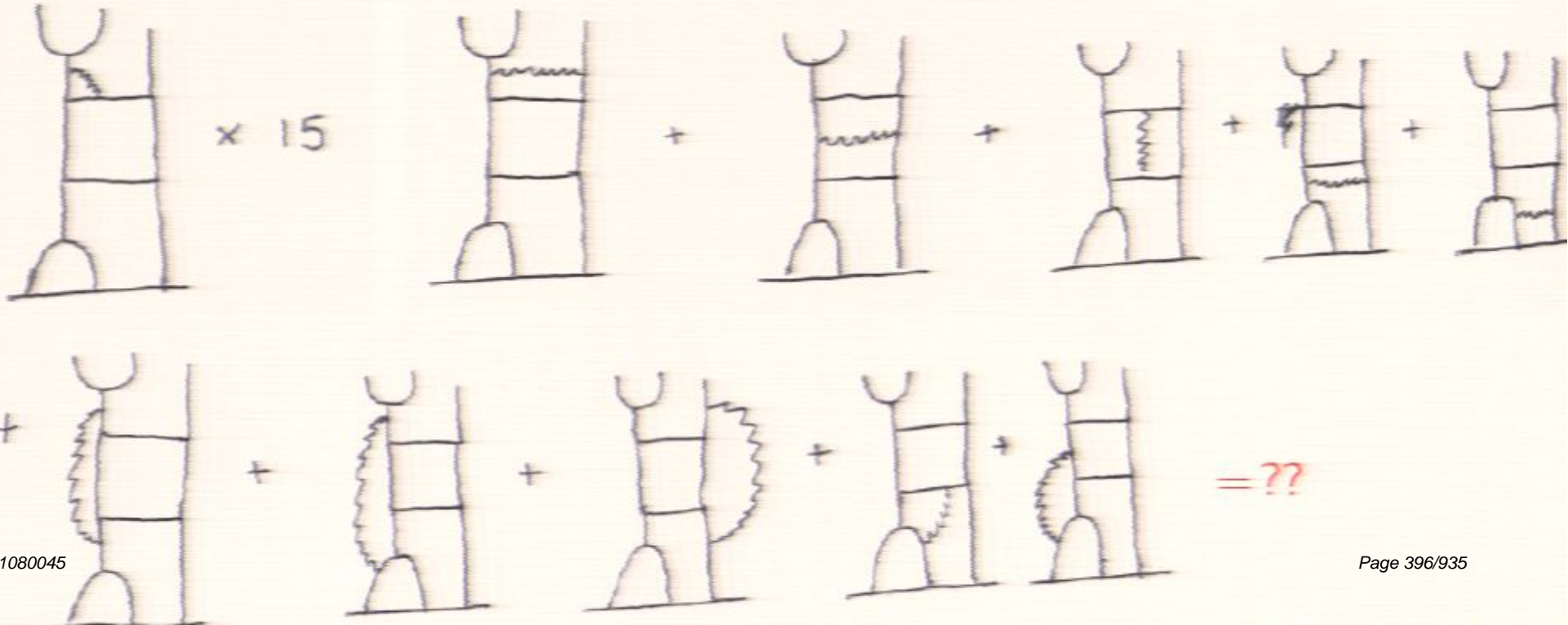
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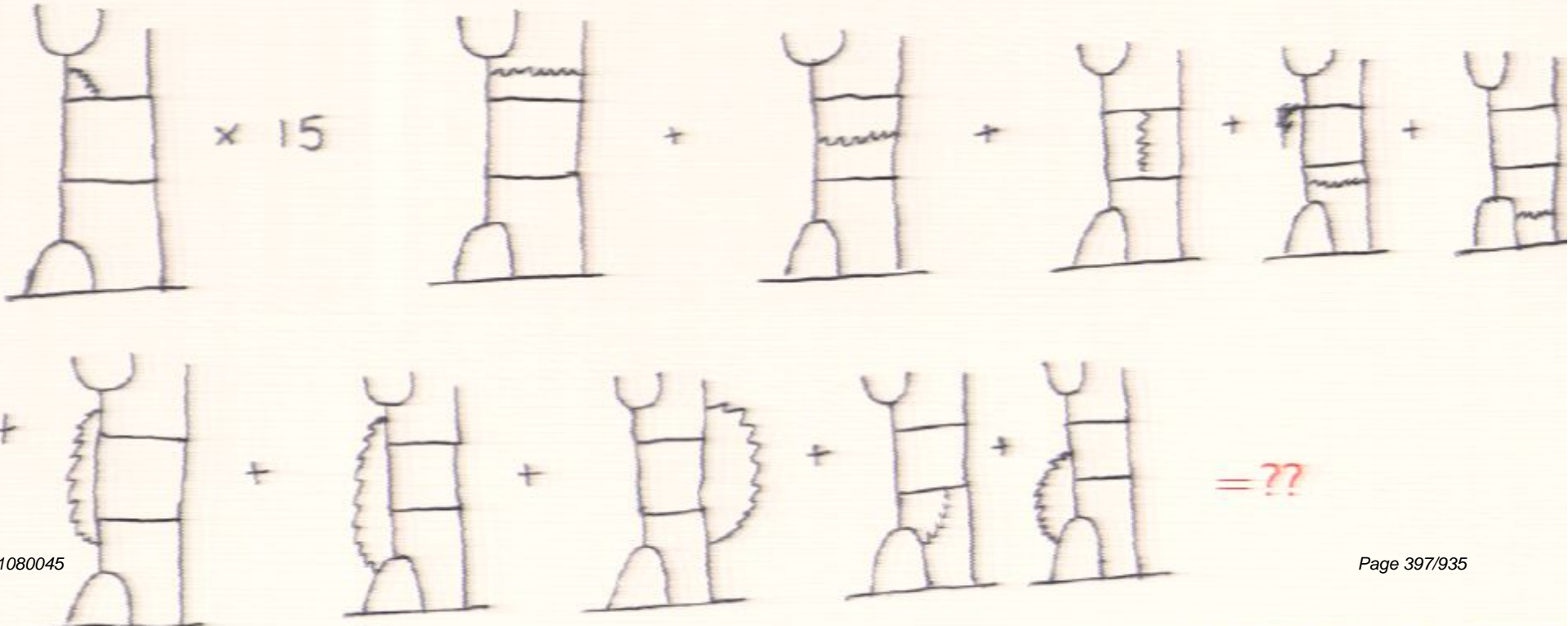
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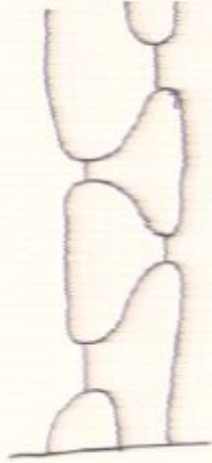
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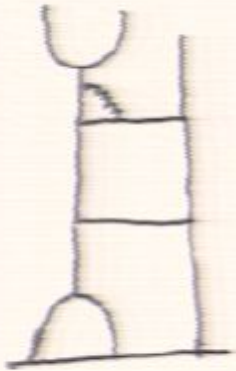
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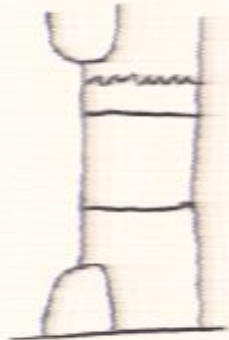


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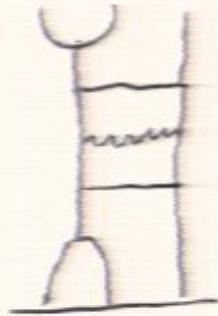
$\chi(1, 2, 1)$:



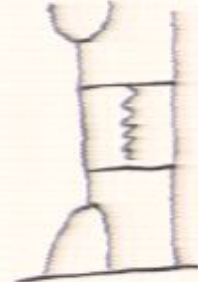
$\times 15$



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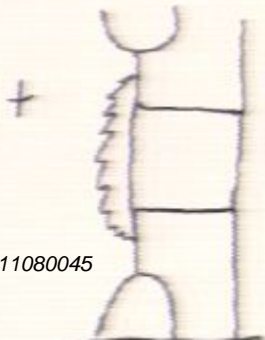
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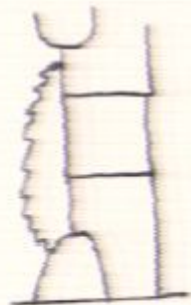
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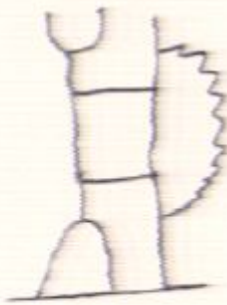
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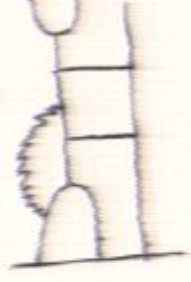
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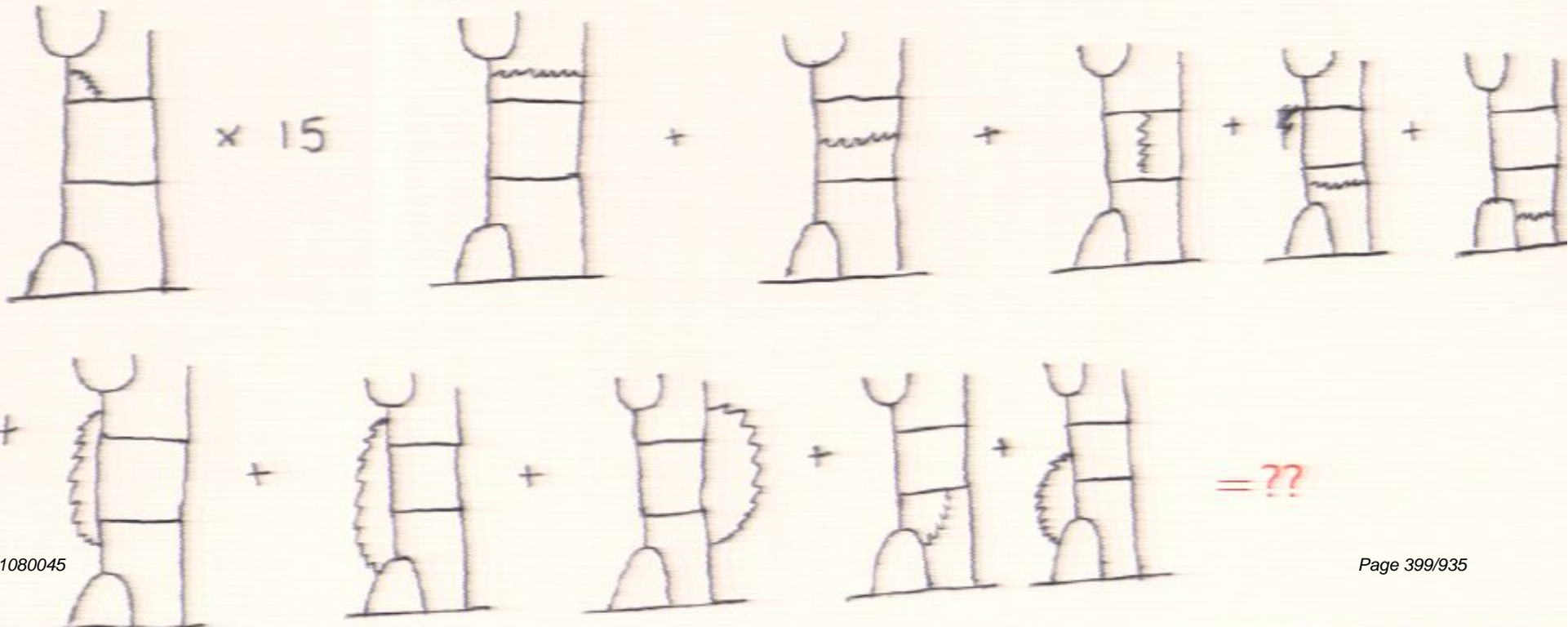
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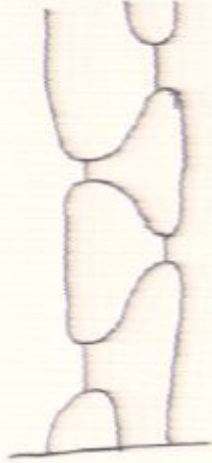
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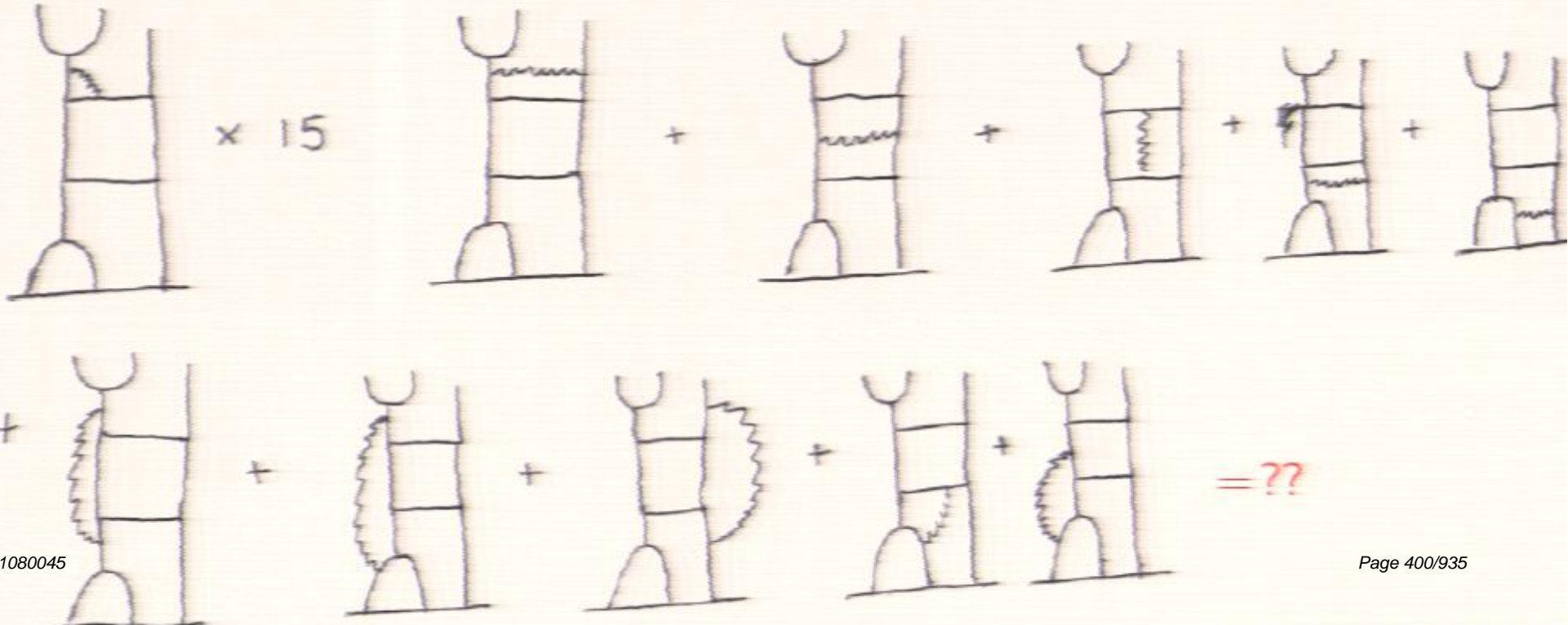
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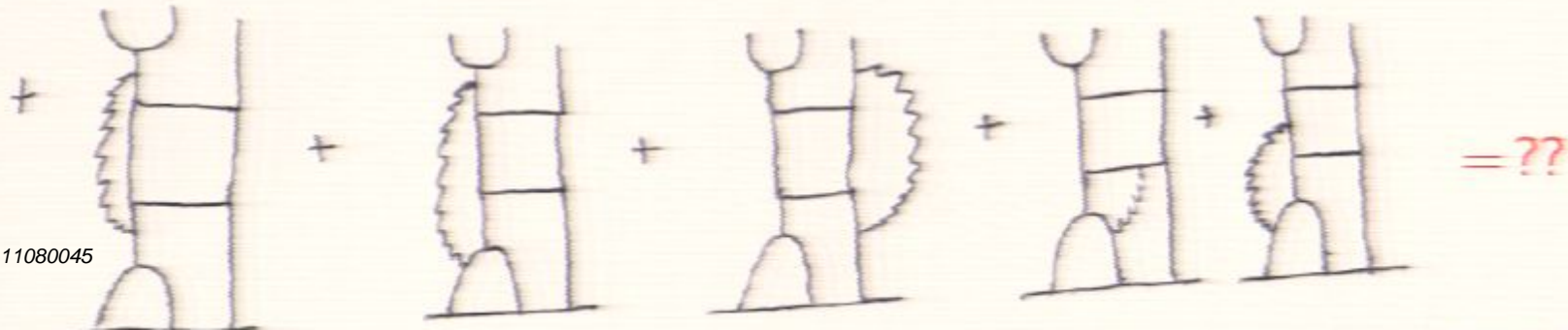
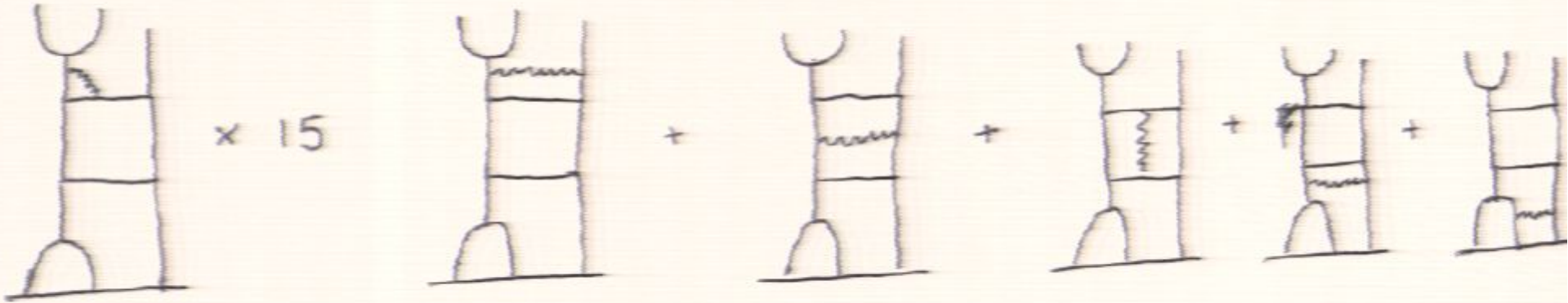
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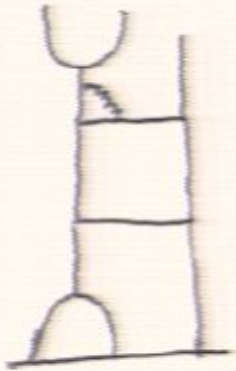
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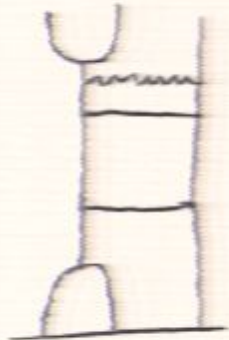


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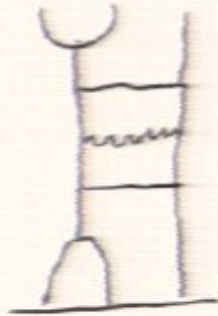
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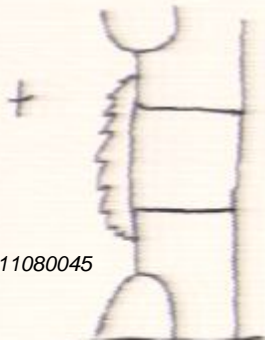
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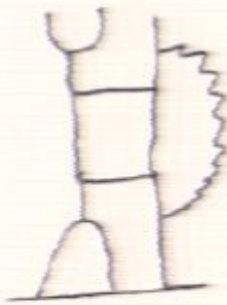
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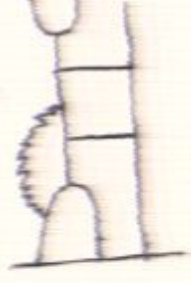
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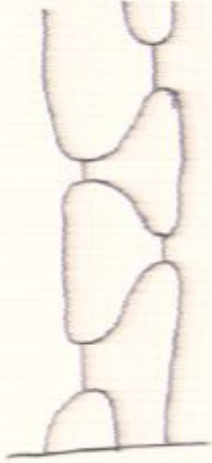
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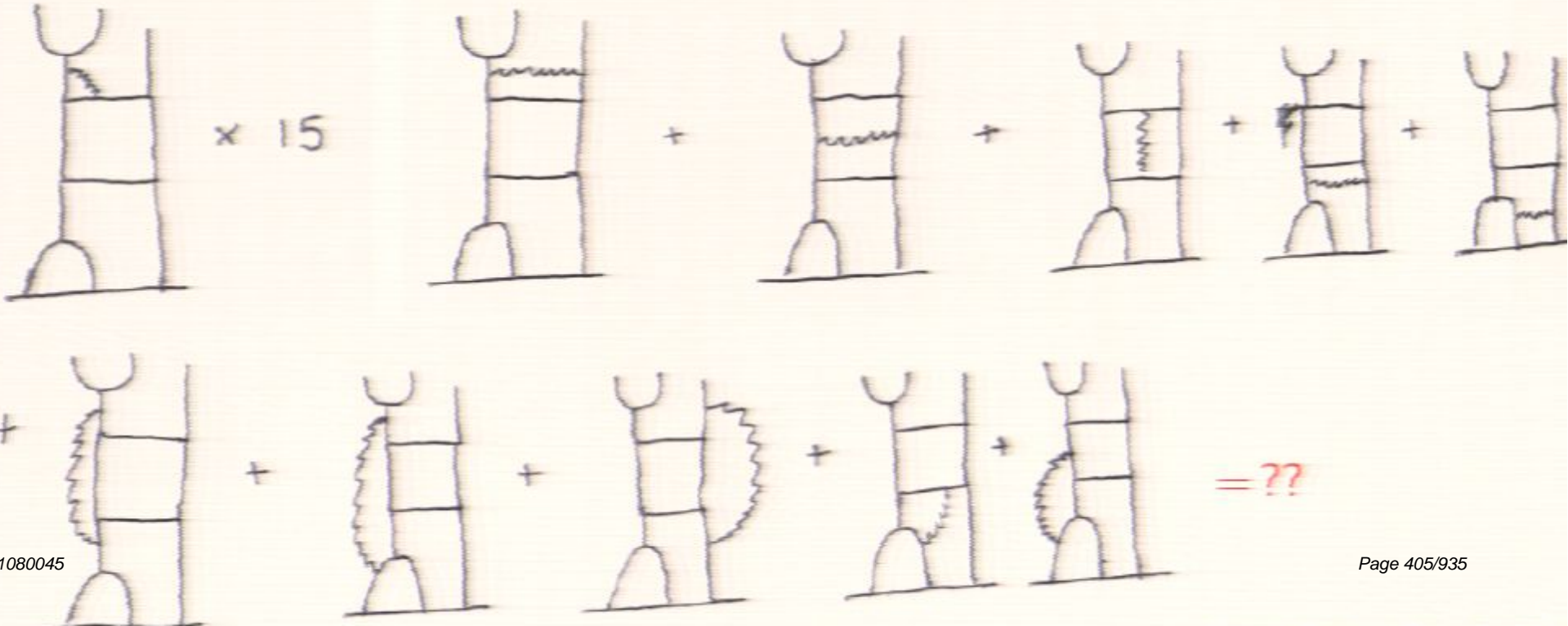
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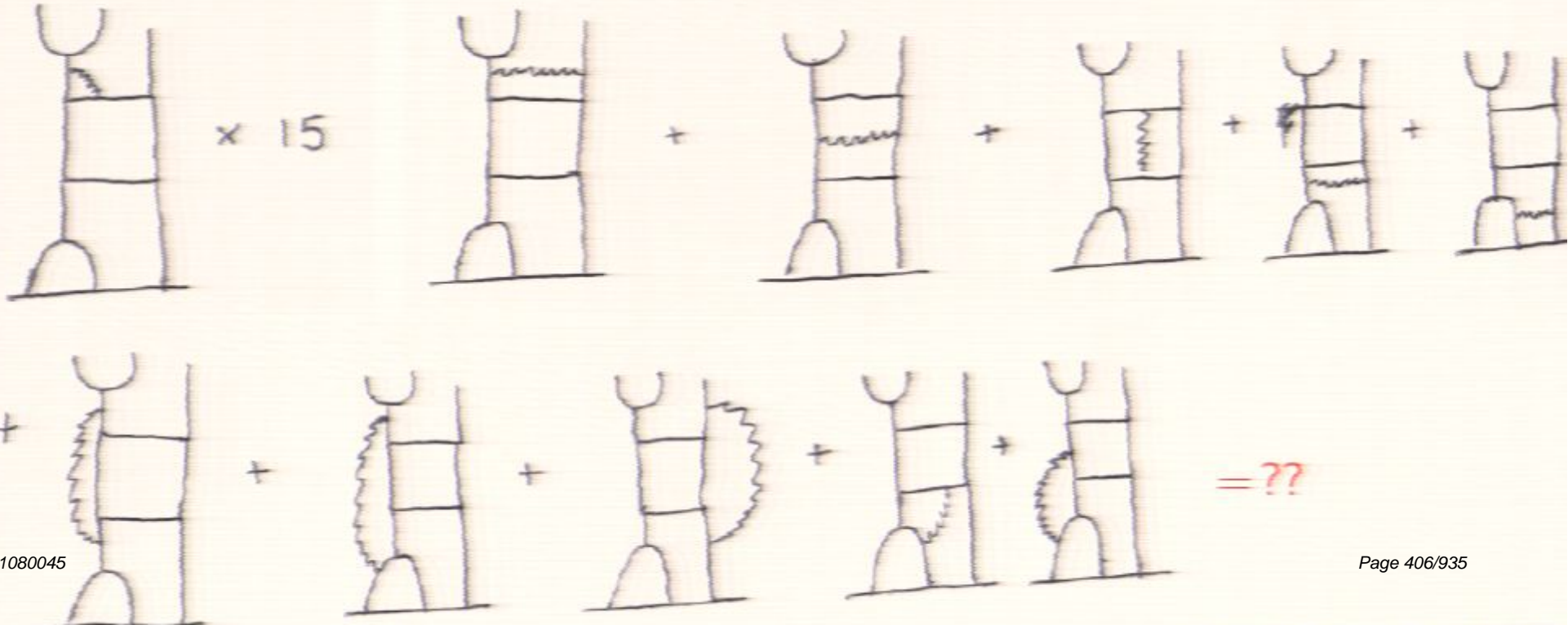
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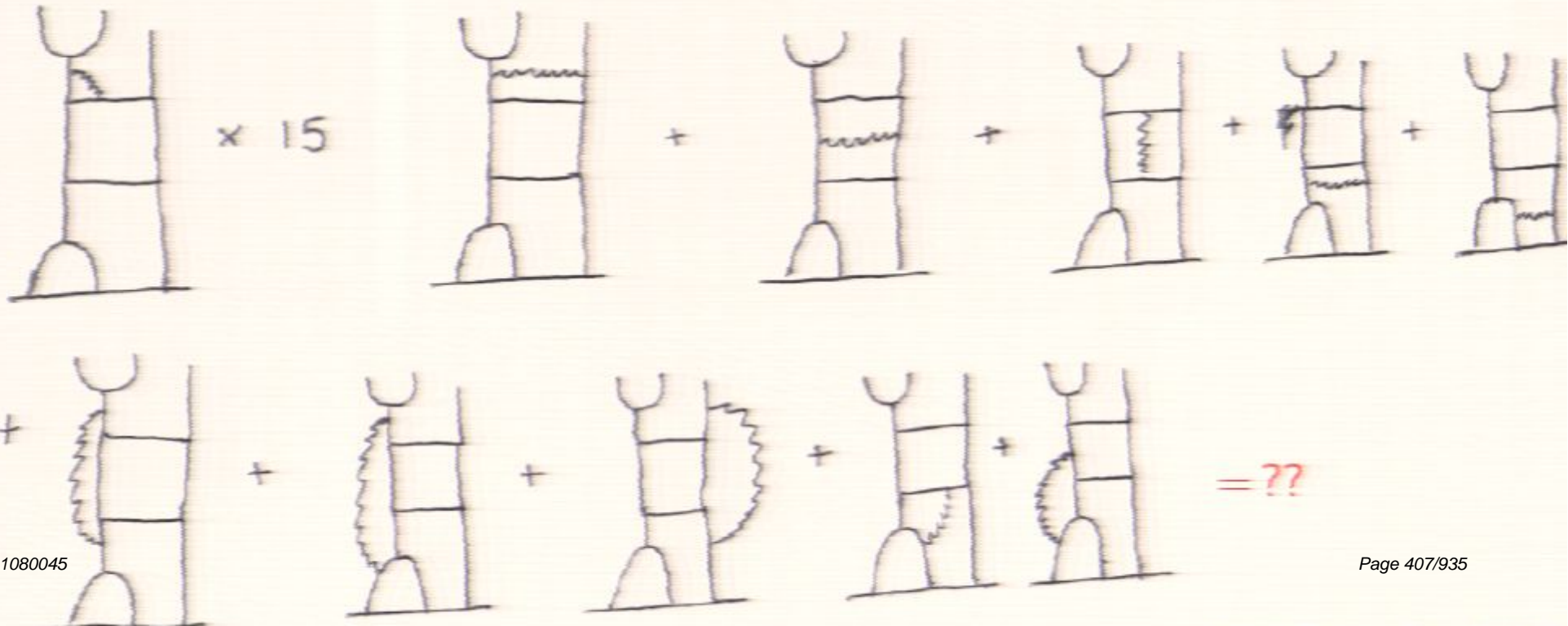
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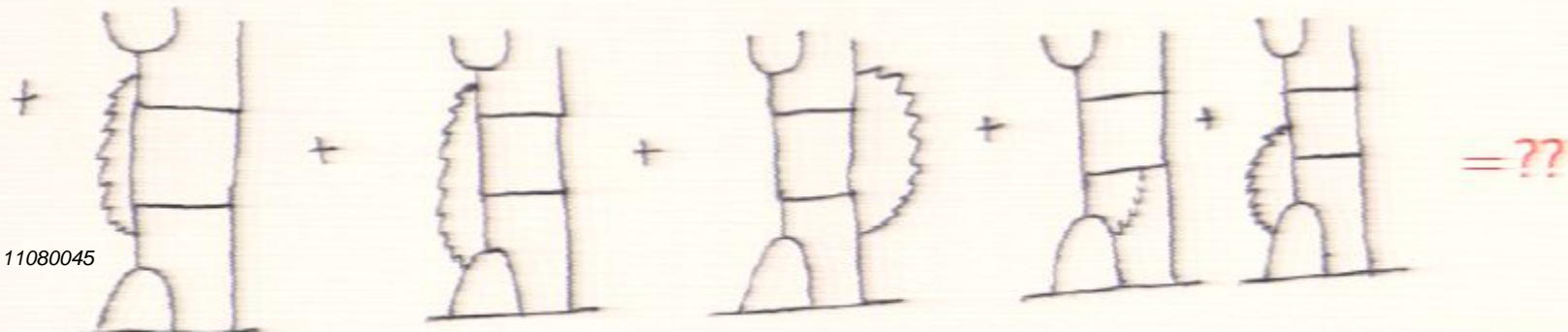
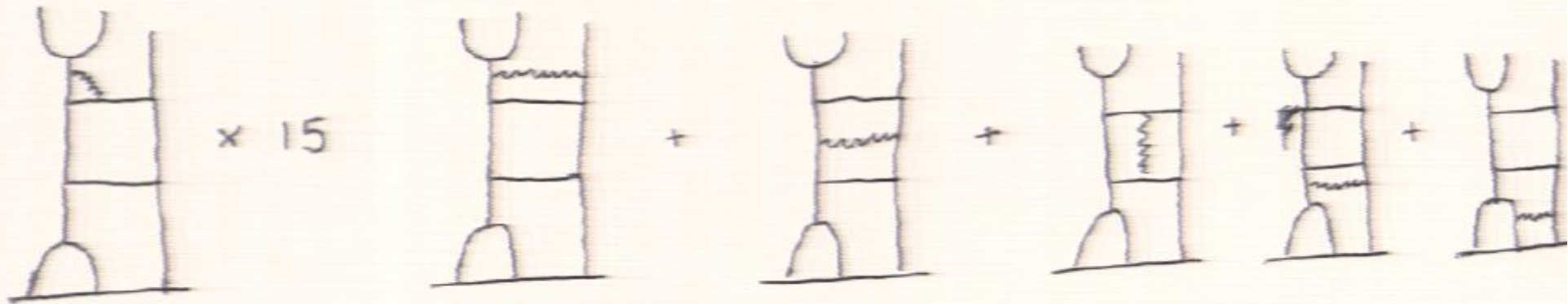
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$$W = \kappa \left(\text{tr}(XYZ - qXZY) + \frac{h}{3}(X^3 + Y^3 + Z^3) \right) \quad q \text{ \& } h \text{ complex.}$$

Conformal condition : $\gamma_i = 0$

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Leigh-Strassler deformations

$$W = \kappa \left(\text{tr}(XYZ - qXZY) + \frac{h}{3}(X^3 + Y^3 + Z^3) \right) \quad q \text{ \& } h \text{ complex.}$$

Conformal condition : $\gamma_i = 0$

$$2g_{\text{YM}}^2 = \kappa \bar{\kappa} (1 + q\bar{q} + h\bar{h}) \quad (\text{planar})$$

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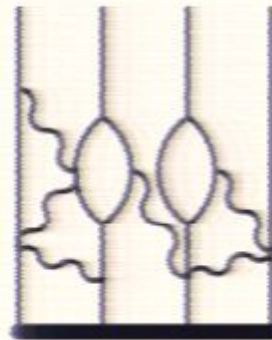
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Planar protected states in the cubic model:



Graphs where neighboring legs are only connected by vectors are finite
(after removing subdivergences)

e.g.

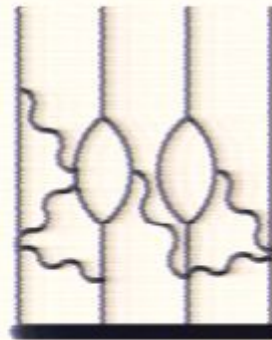


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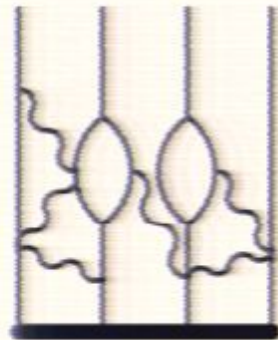


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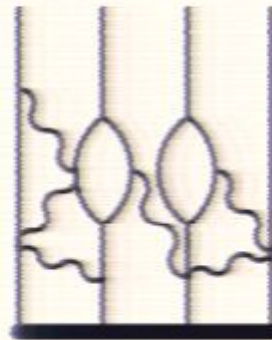


Planar protected states in the cubic model:

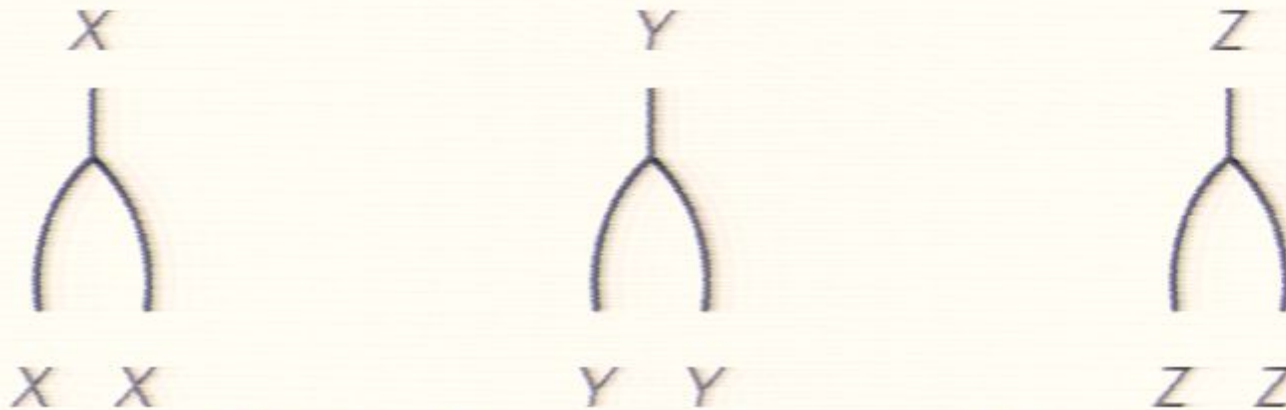


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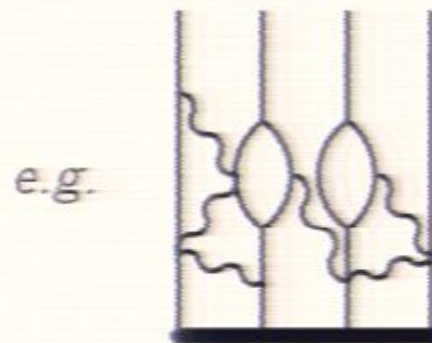
e.g.



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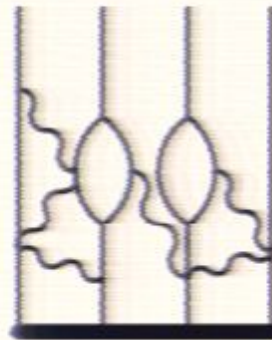


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e.g.

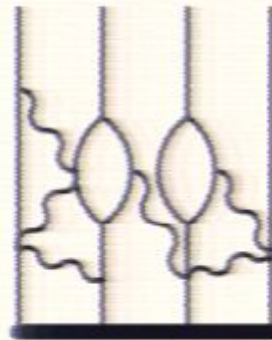


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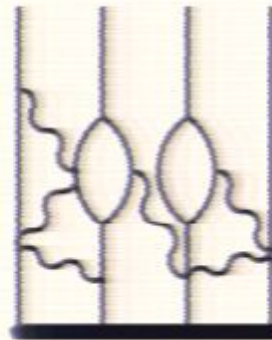


Planar protected states in the cubic model:



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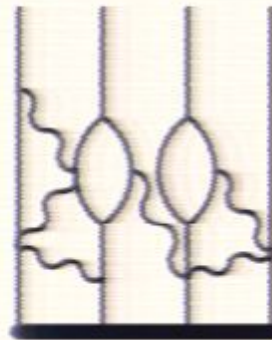


Planar protected states in the cubic model:



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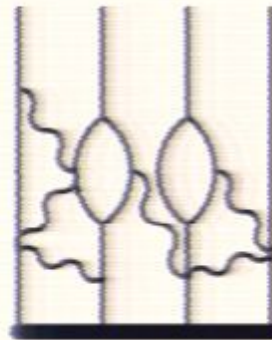


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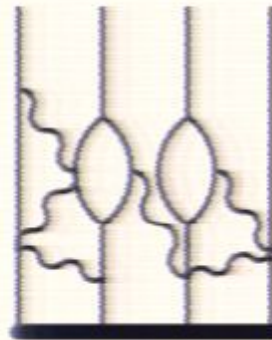
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Planar protected states in the cubic model:

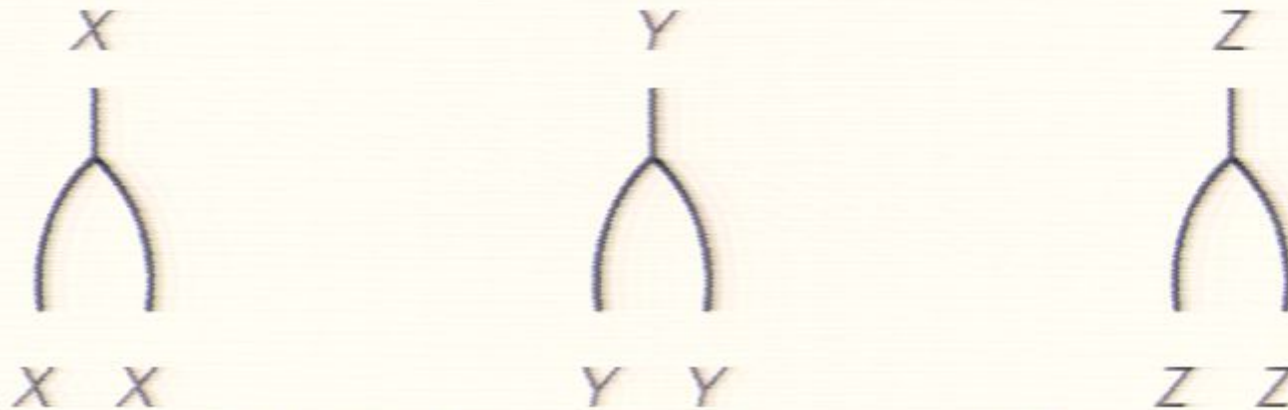


Graphs where neighboring legs are only connected by vectors are finite
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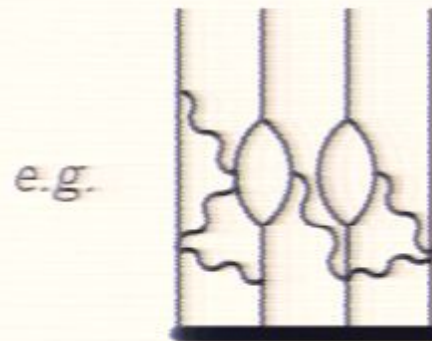
e.g.



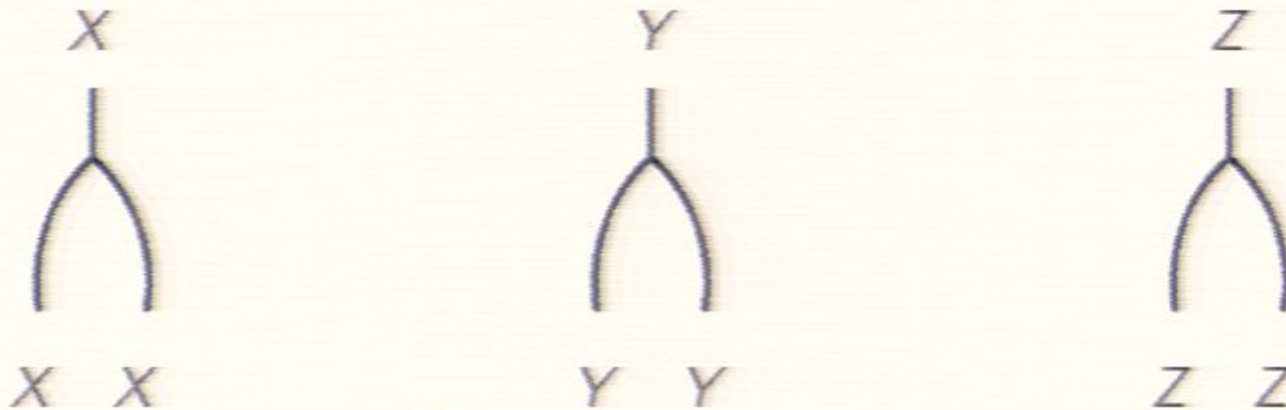
Planar protected states in the cubic model:



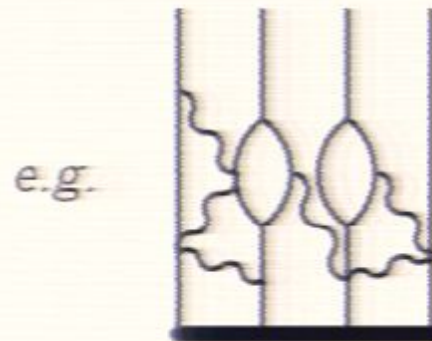
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Planar protected states in the cubic model:



Graphs where neighboring legs are only connected by vectors are finite
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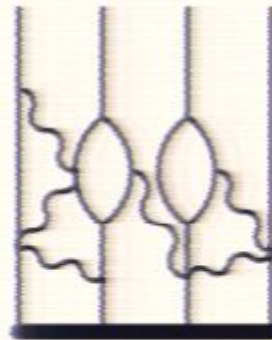


Planar protected states in the cubic model:

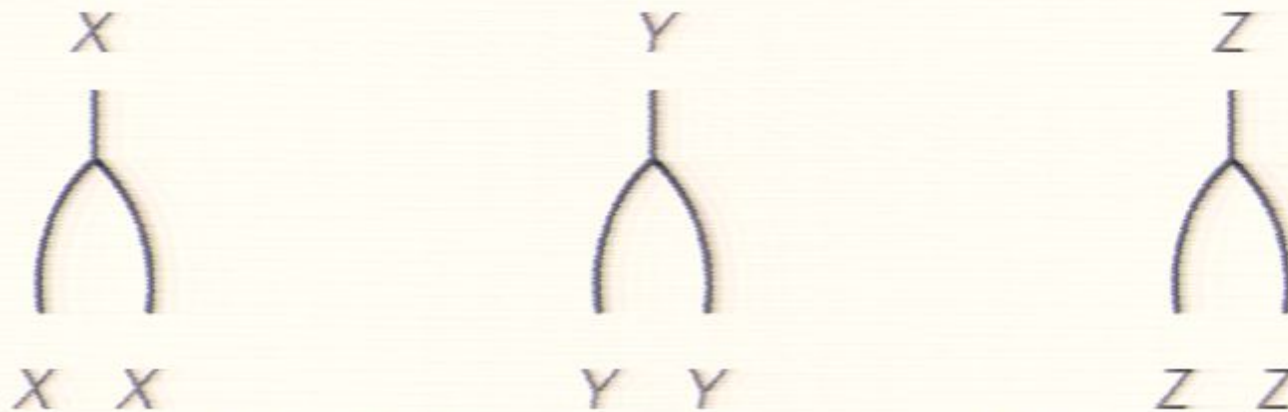


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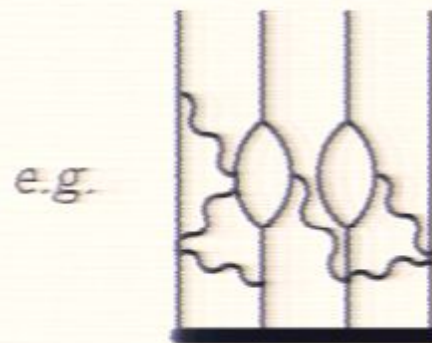
e.g.



Planar protected states in the cubic model:



Graphs where neighboring legs are only connected by vectors are finite
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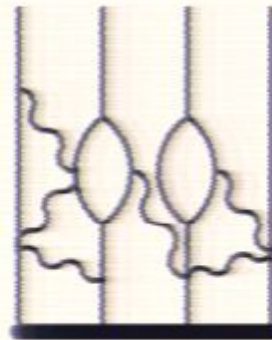


Planar protected states in the cubic model:



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e.g.

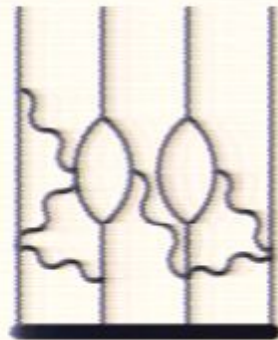


Planar protected states in the cubic model:



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e.g.

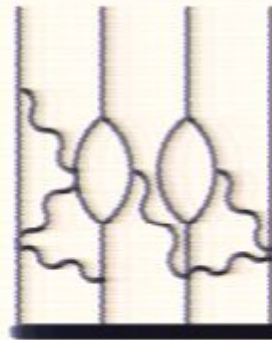


Planar protected states in the cubic model:



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e.g.

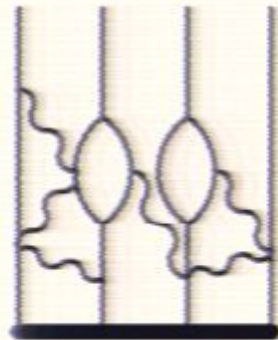


Planar protected states in the cubic model:

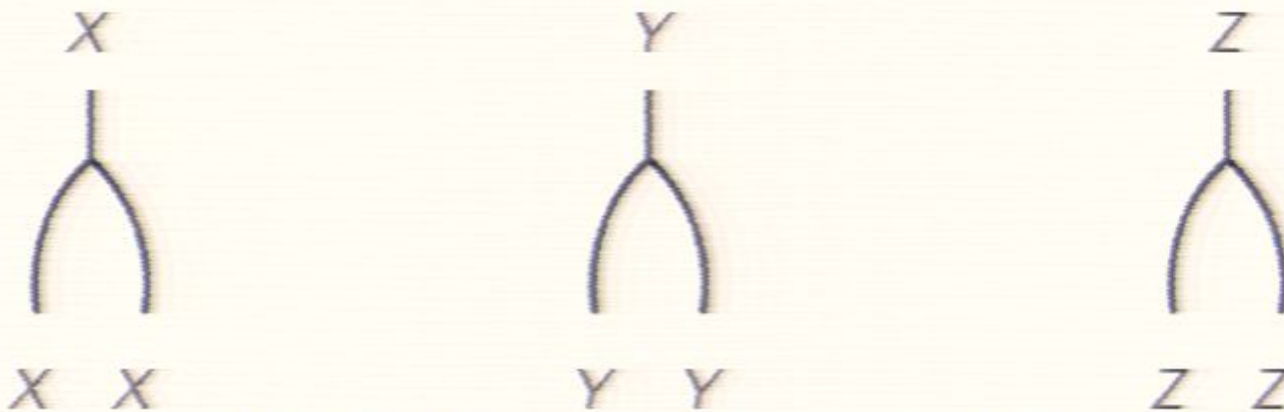


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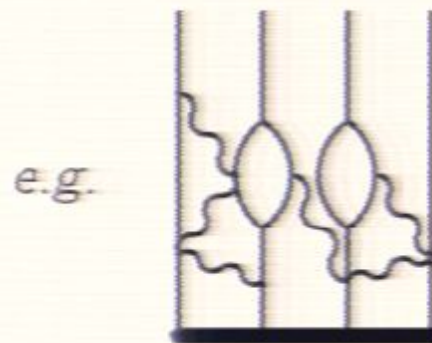
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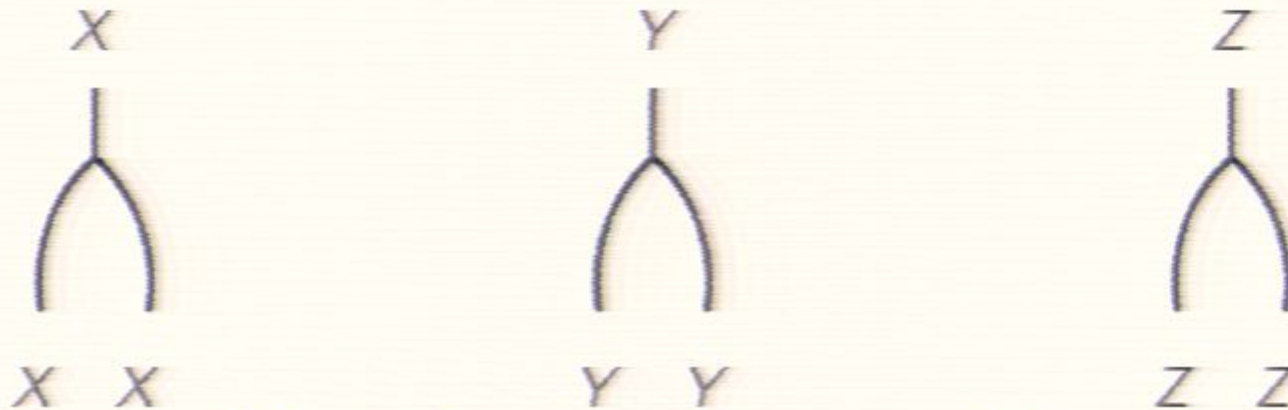
Planar protected states in the cubic model:



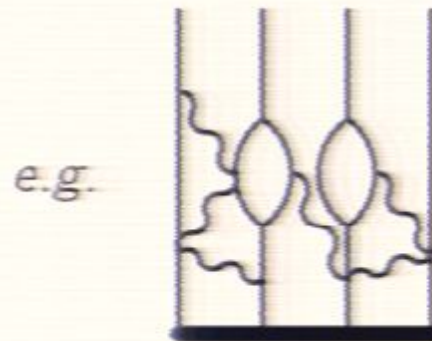
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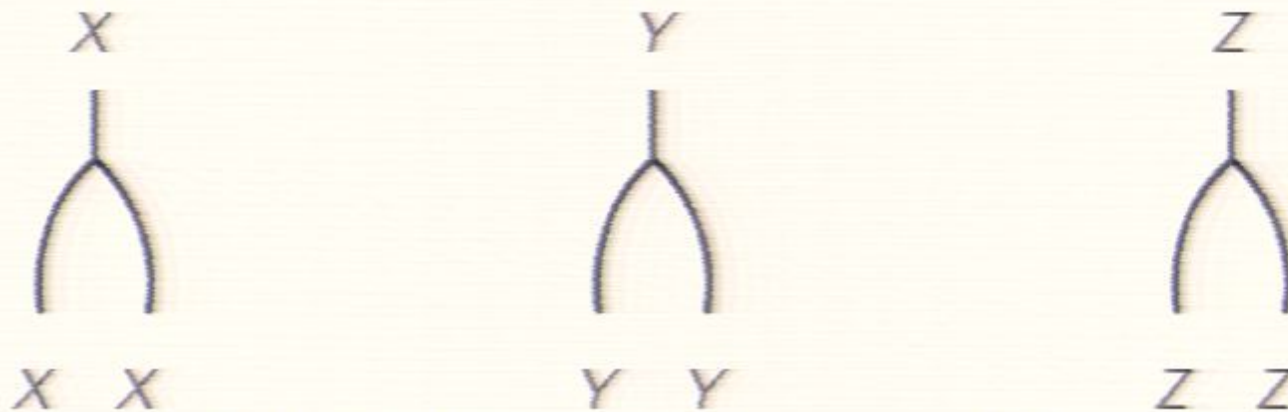
Planar protected states in the cubic model:



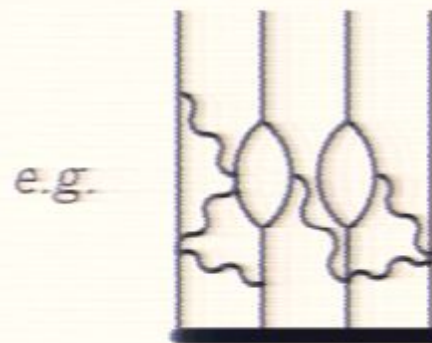
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Planar protected states in the cubic model:



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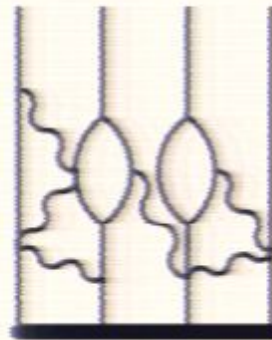


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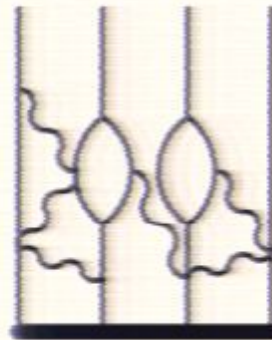


Planar protected states in the cubic model:



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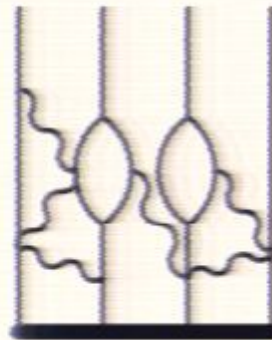


Planar protected states in the cubic model:

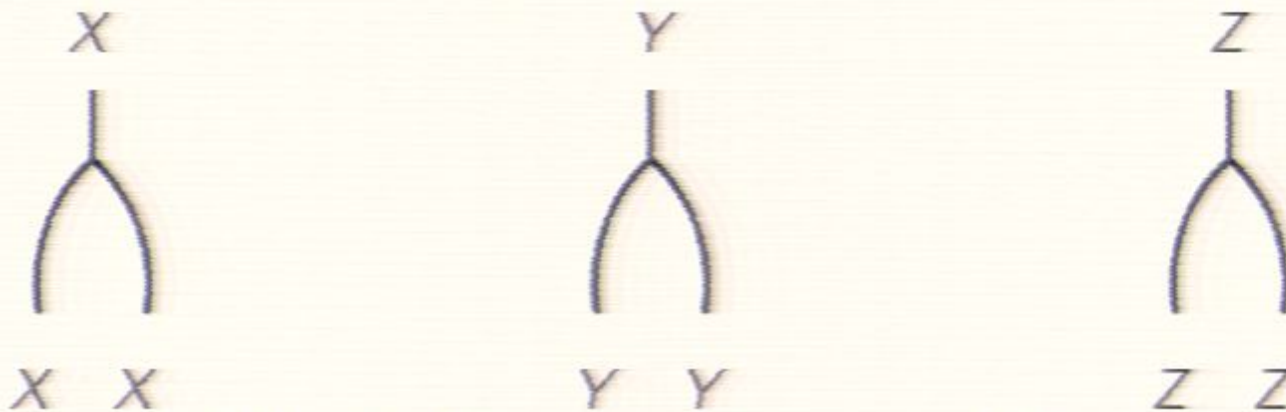


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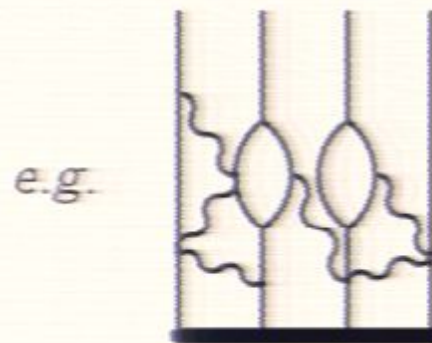
e.g.



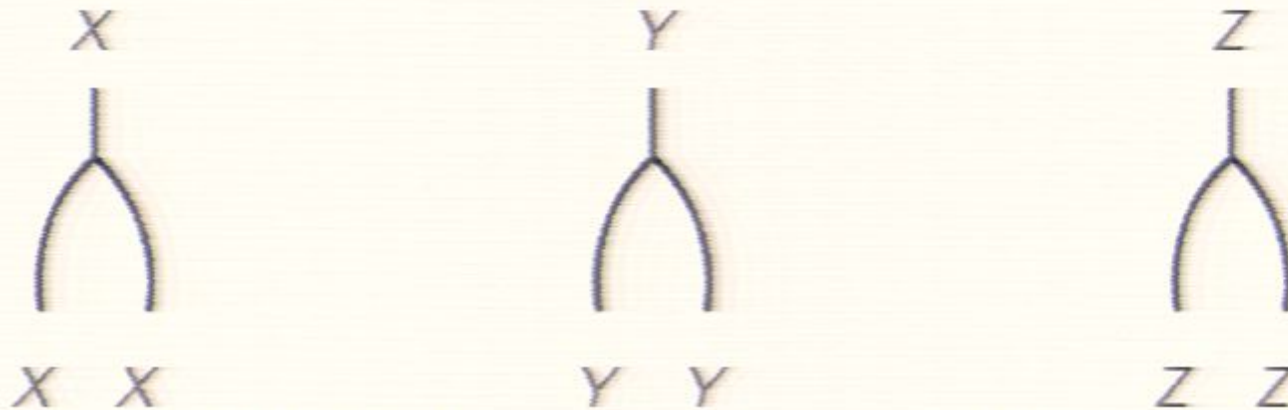
Planar protected states in the cubic model:



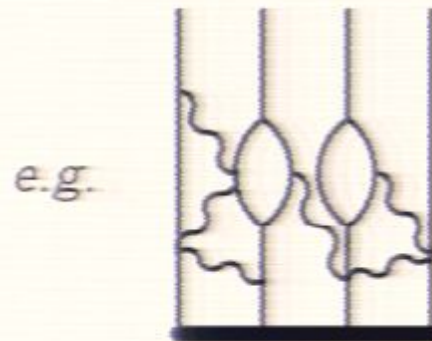
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Planar protected states in the cubic model:



Graphs where neighboring legs are only connected by vectors are finite
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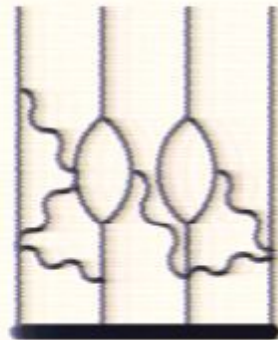


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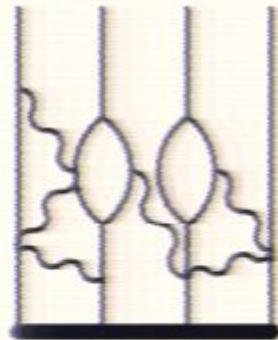


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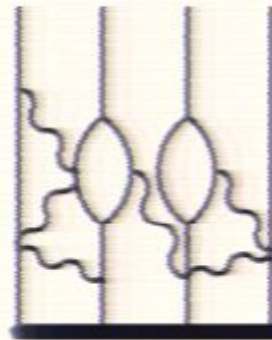


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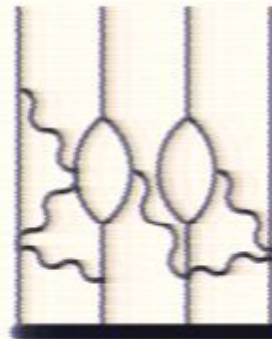


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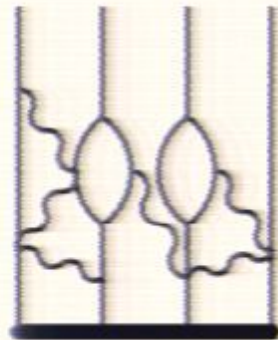


Planar protected states in the cubic model:

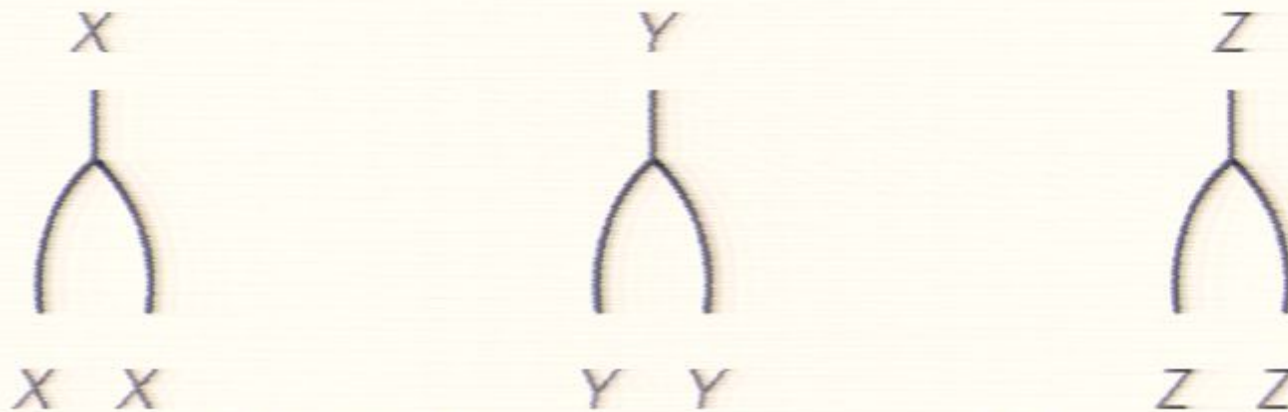


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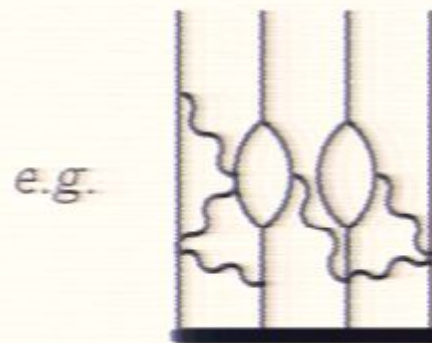
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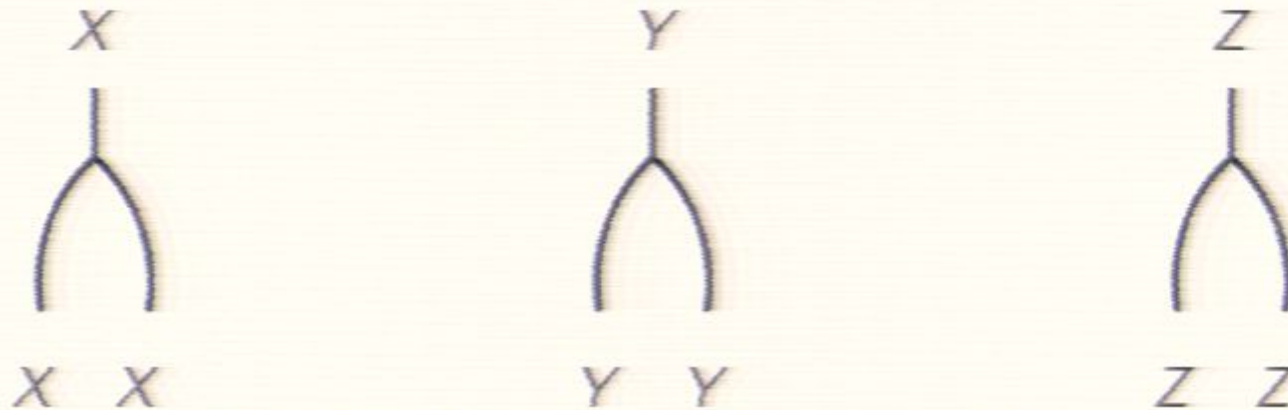
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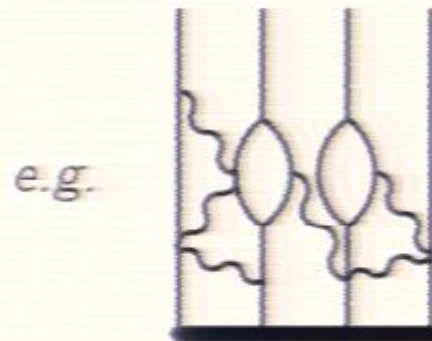
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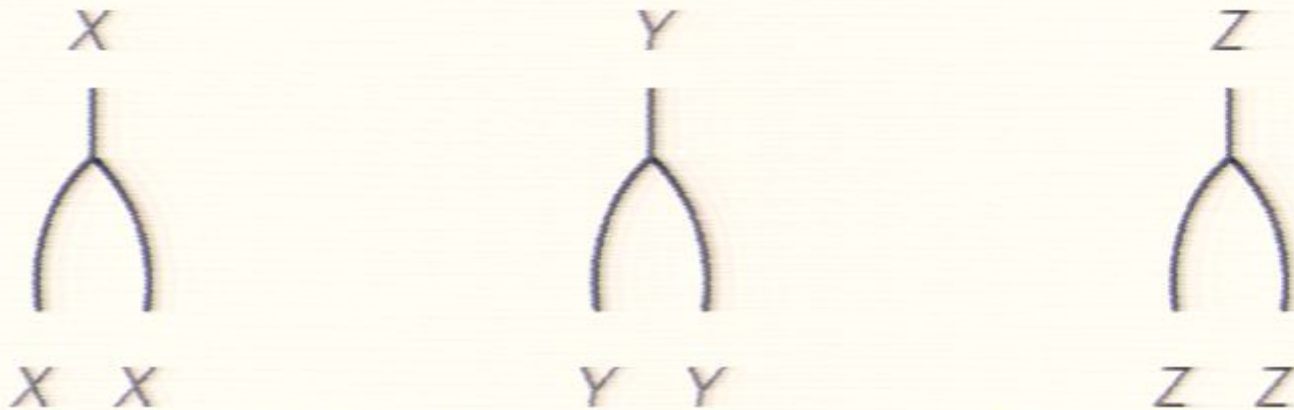
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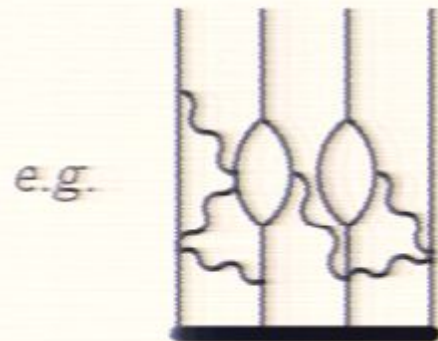
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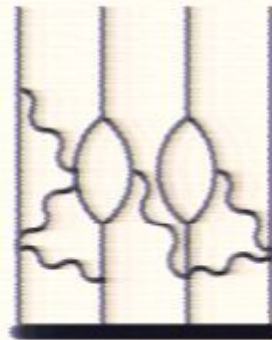


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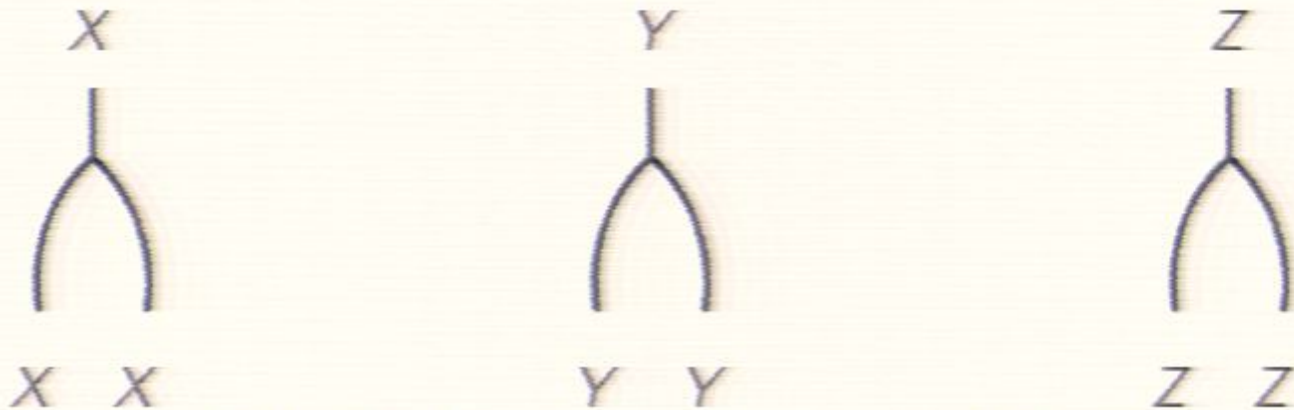


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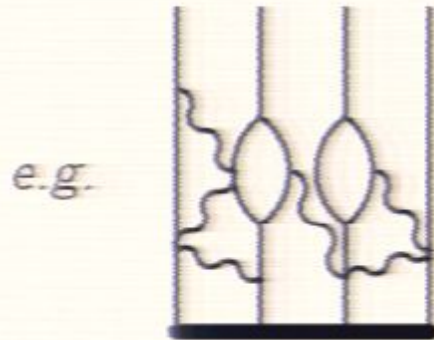
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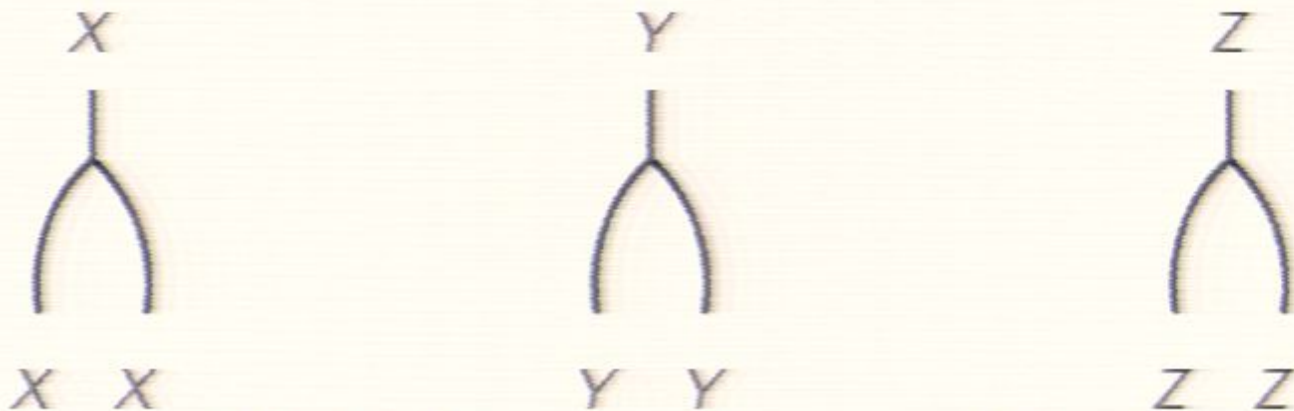
Graphs where neighboring legs are only connected by vectors are finite
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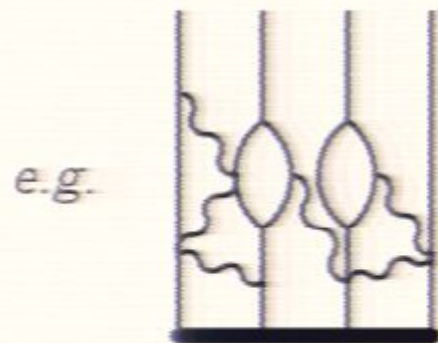
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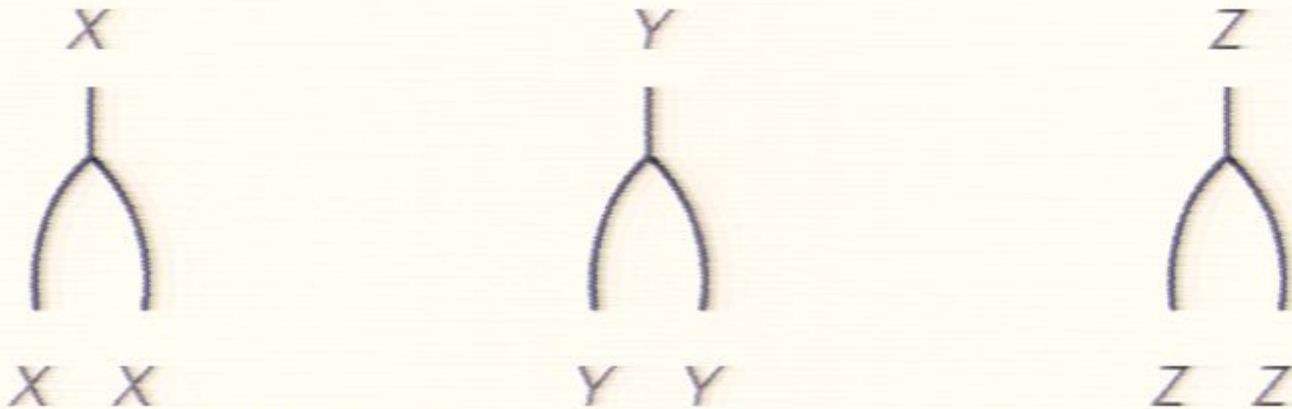
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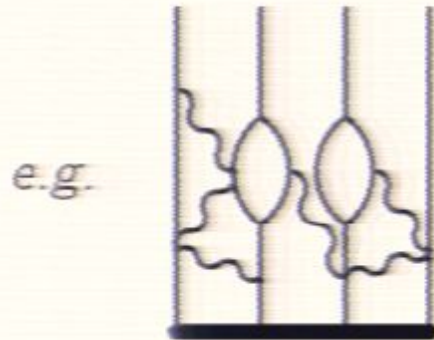
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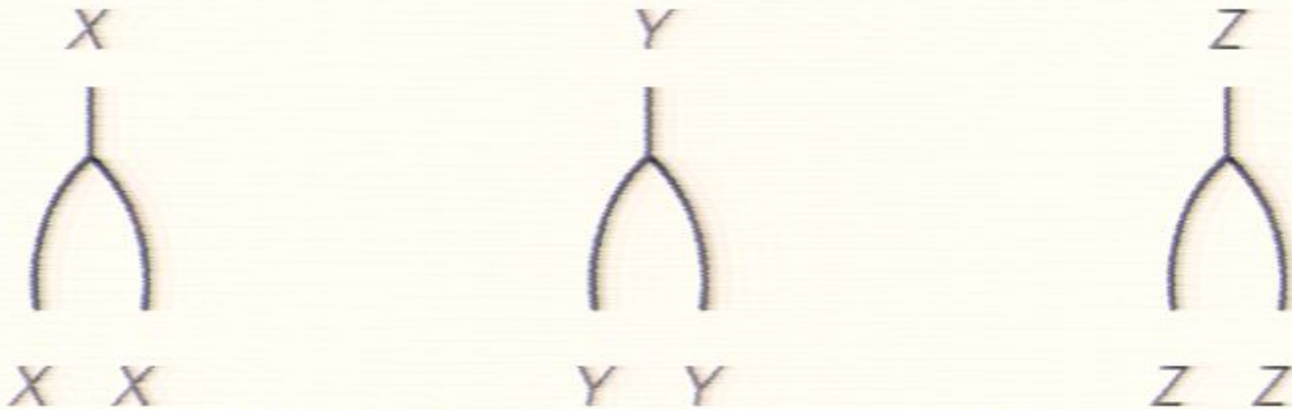
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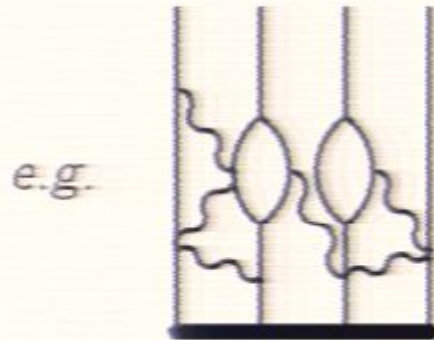
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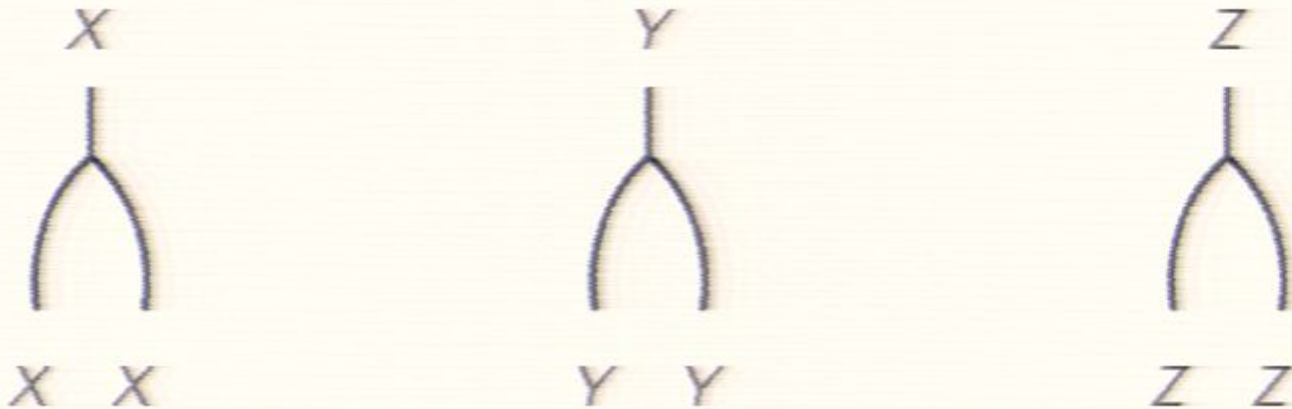
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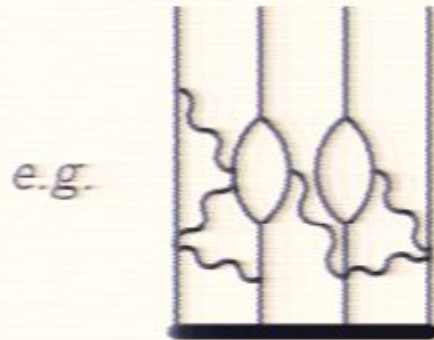
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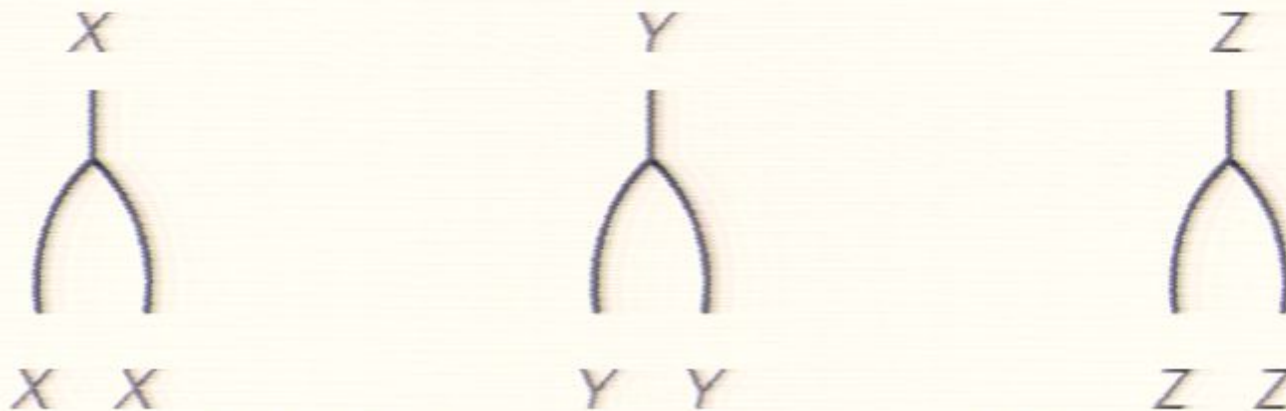
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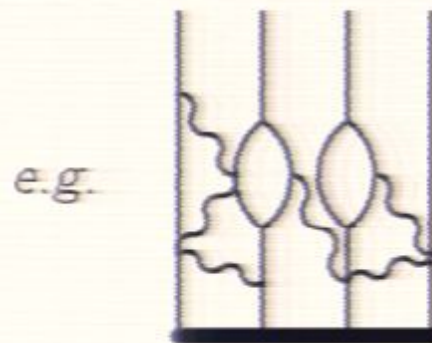
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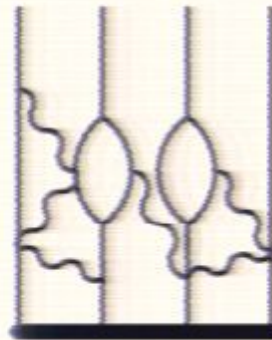
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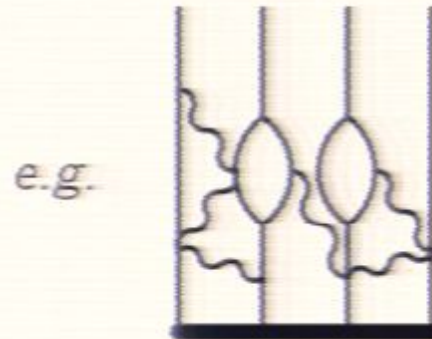
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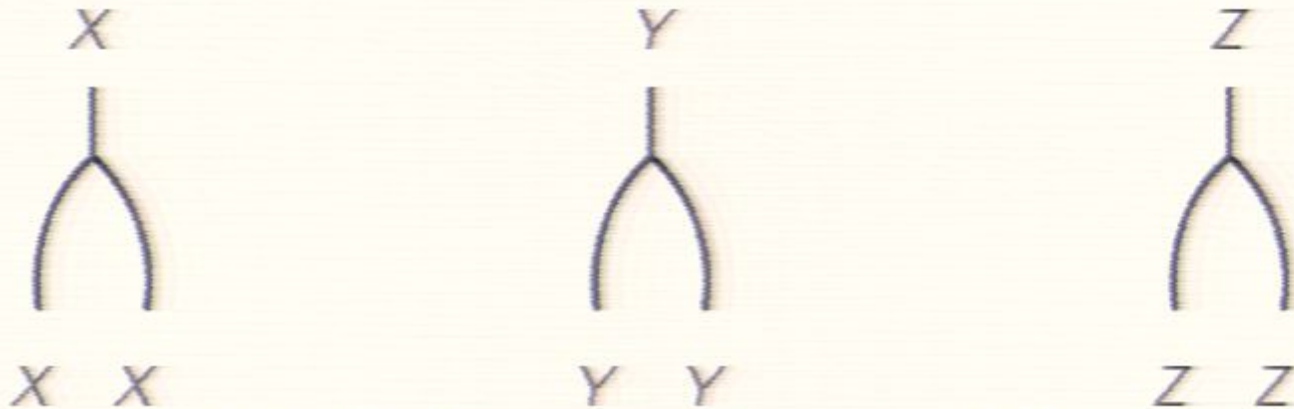
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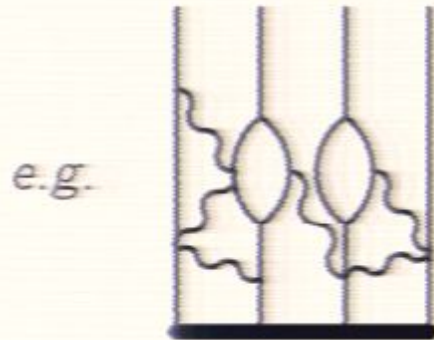
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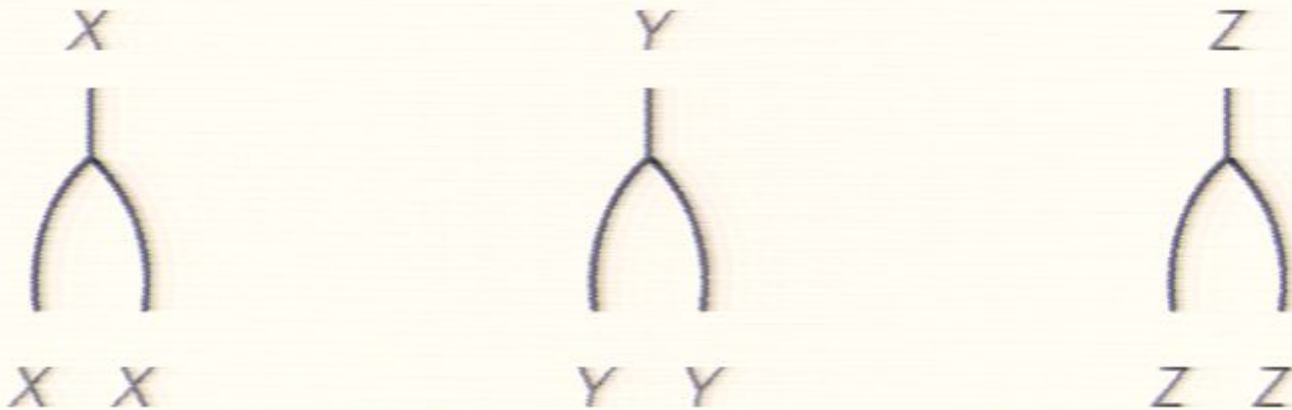
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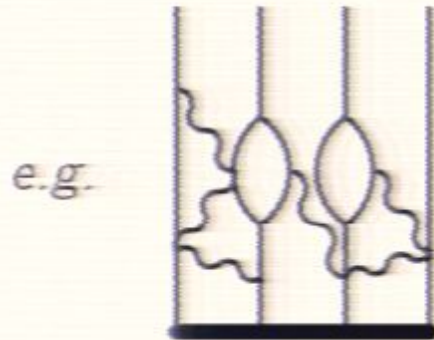
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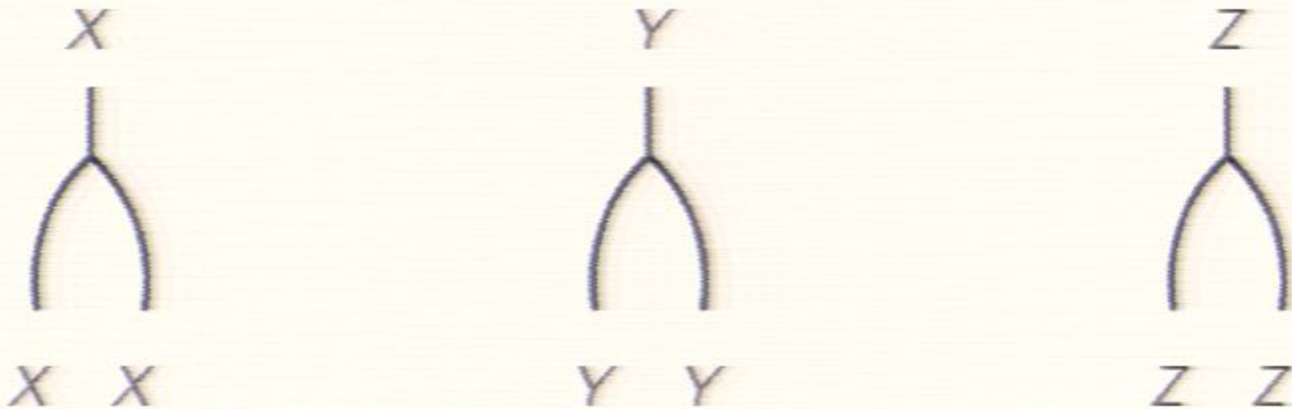
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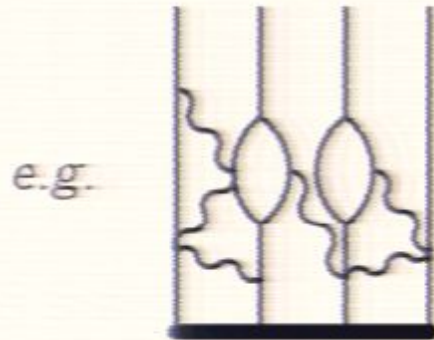
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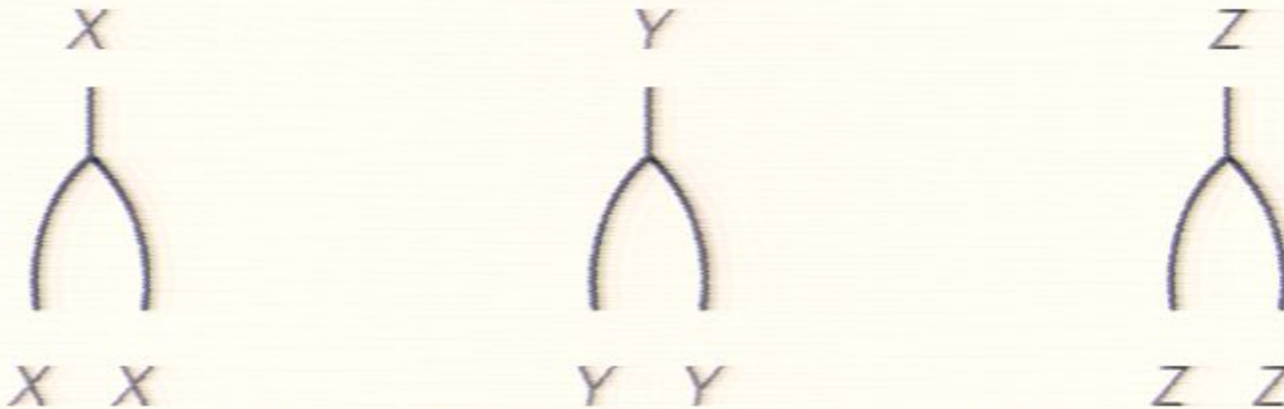
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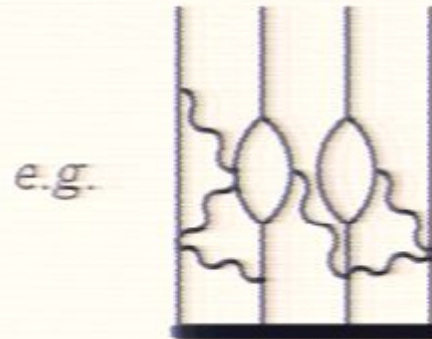
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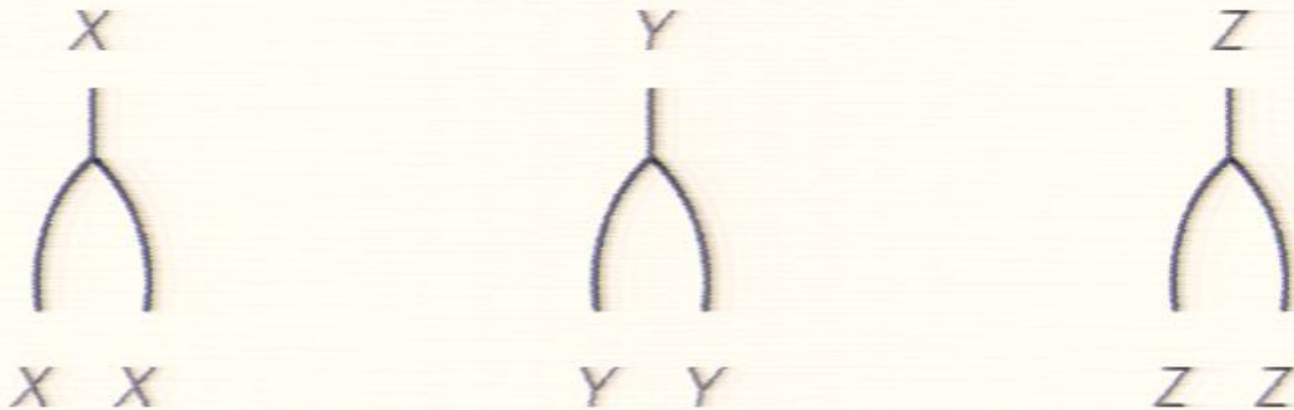
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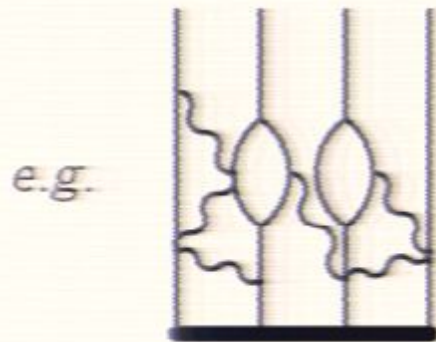
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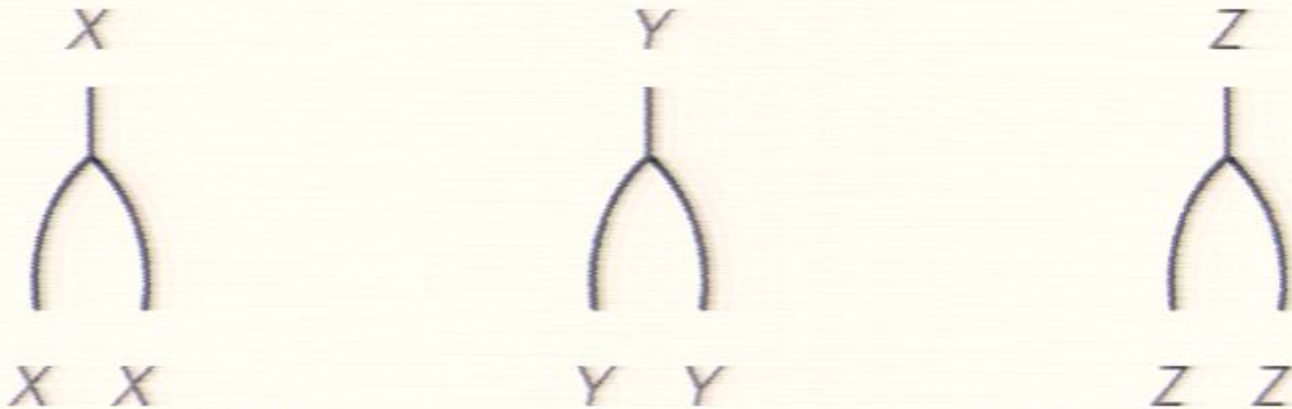
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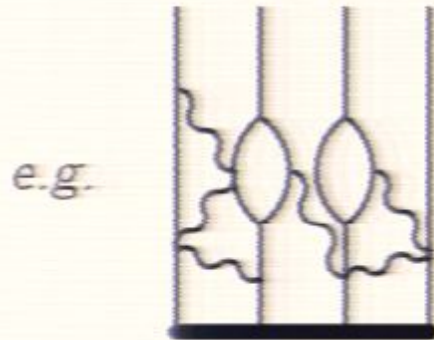
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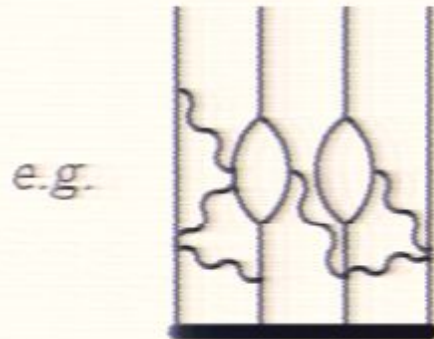
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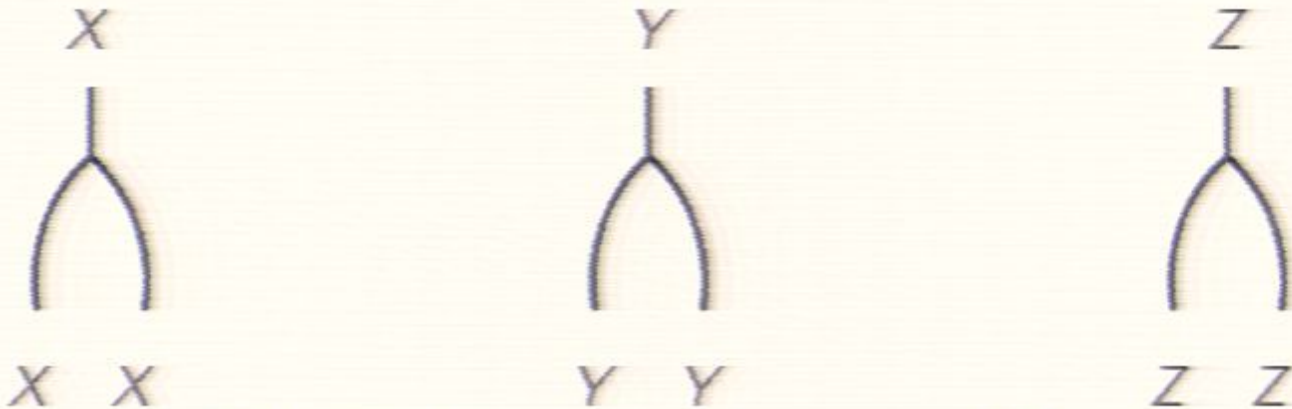
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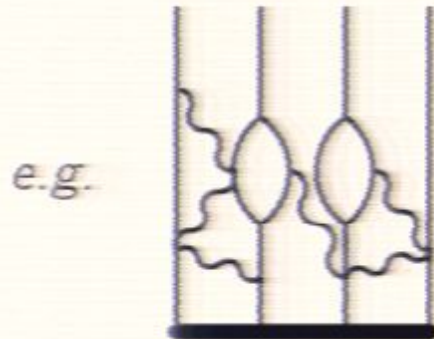
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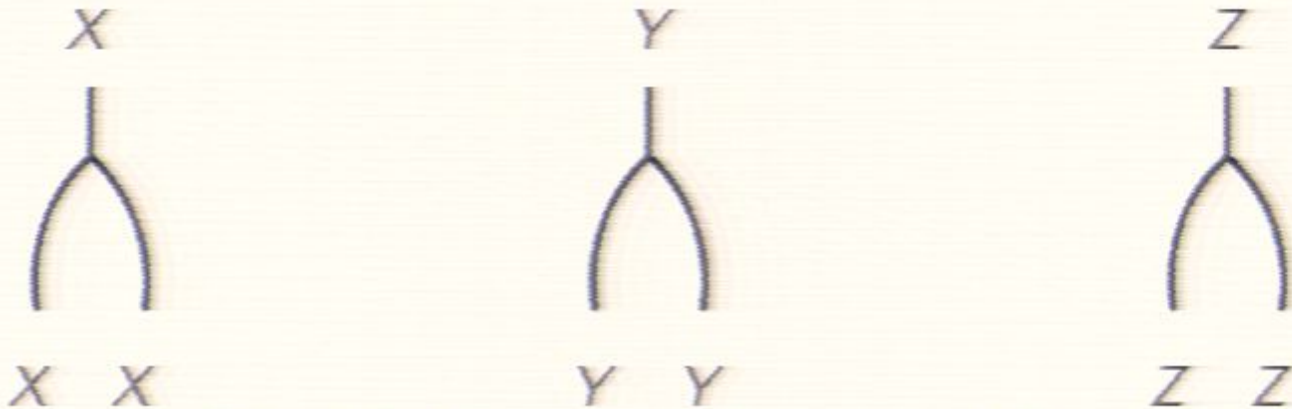
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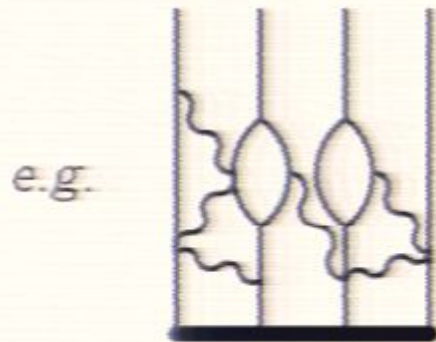
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Planar protected states in the cubic model:



Graphs where neighboring legs are only connected by vectors are finite
 (after removing subdivergences)



Let $\mathcal{O} = \text{tr}[XYXYZX \dots XYZYZY]$ All neighbors different $\Rightarrow \delta = 0$

All graphs in the plane are finite since legs can't connect by chiral verts.

Leigh-Strassler deformations

$$W = \kappa \left(\text{tr}(XYZ - qXZY) + \frac{h}{3}(X^3 + Y^3 + Z^3) \right) \quad q \text{ \& } h \text{ complex.}$$

Conformal condition : $\gamma_i = 0$

$$2g_{\text{YM}}^2 = \kappa \bar{\kappa} (1 + q\bar{q} + h\bar{h}) \quad (\text{planar})$$

All loop exact if $h = 0$, $q = e^{-2\pi i\beta}$, β real. 4-loop corrections otherwise.

"Cubic Model" : $\kappa \rightarrow 0$, $h \rightarrow \infty$, $\hat{h} = \kappa h$ finite.

$$\hat{h} = \sqrt{2} g_{\text{YM}} + 4\text{-loop corrections}$$

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Conjecture

The all-loop sum of supergraphs with the connected chiral function $\chi(a_1, a_2 \dots a_j, \dots a_n)$ is given by

$$F_n(g^2)\chi(a_1, a_2 \dots a_j, \dots a_n) \\ + \text{(maybe transcendental terms)}$$

- ▶ This is a statement about supergraphs and can be applied outside the $SU(2)$ sector where

$$\chi(a_1 \dots a_j, a_j \pm 1, a_j \dots a_n) \neq \chi(a_1 \dots a_j \dots a_n)$$

- ▶ Does not directly require $\mathcal{N} = 4$ susy.
- ▶ Can be applied it to $\mathcal{N} = 1$ superconformal theories with different chiral functions.

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Single trace operators made with ϕ^i or W_α

Scalar operators: $\mathcal{O} = \text{tr}[XYYXZZZYXXXX]$

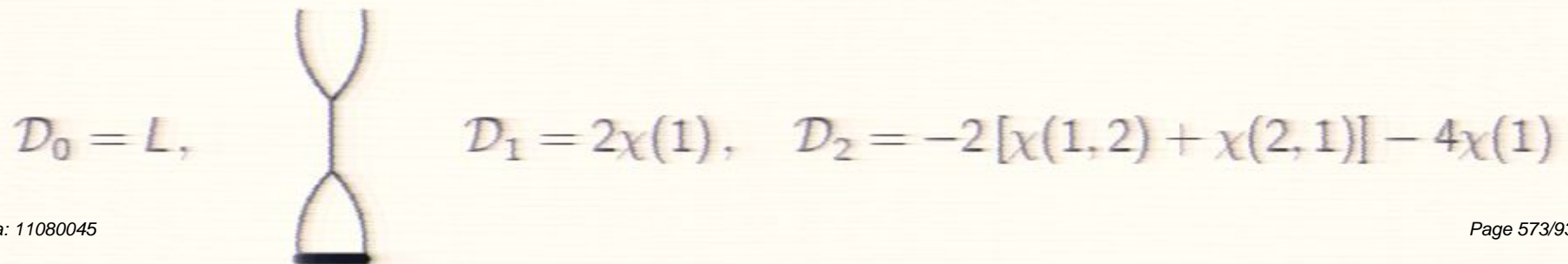
Scalar operators as chiral vertices:



Putting everything together:

Compute:

$$\mathcal{D} = \sum_{\ell=0}^{\infty} g^{2\ell} \mathcal{D}_\ell$$



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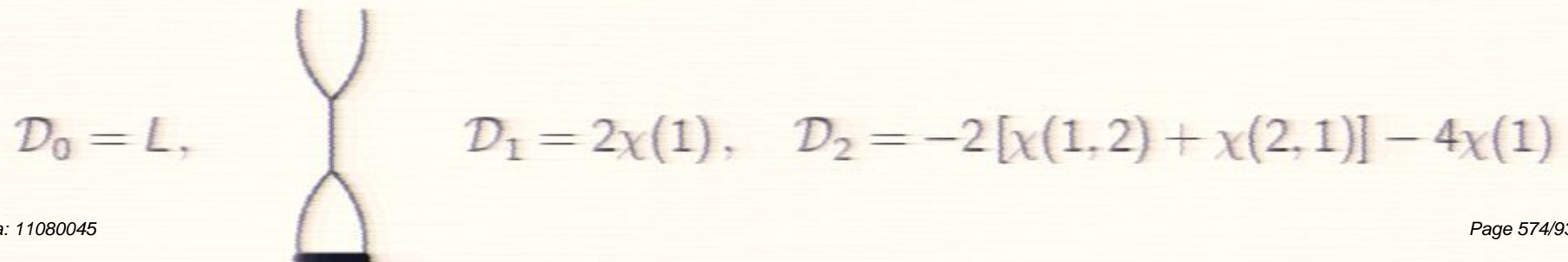
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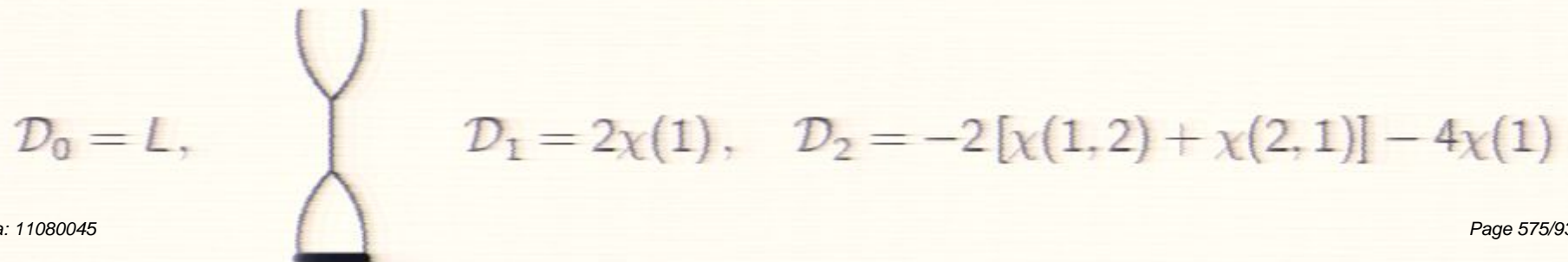
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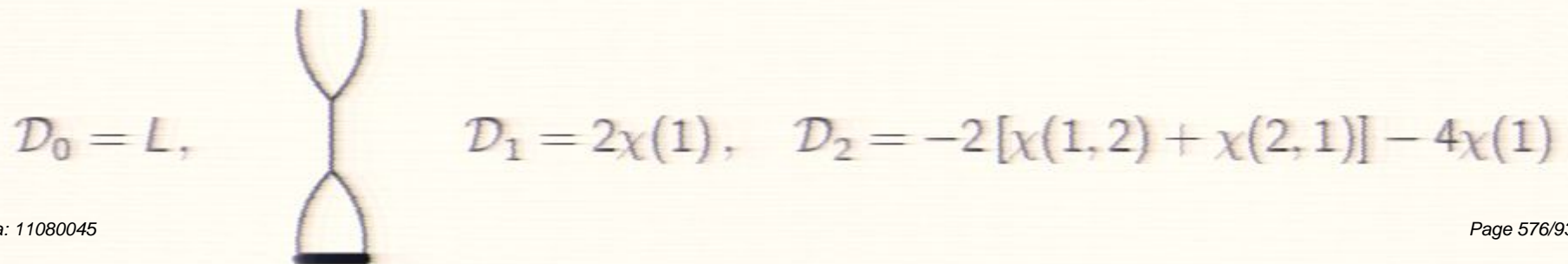
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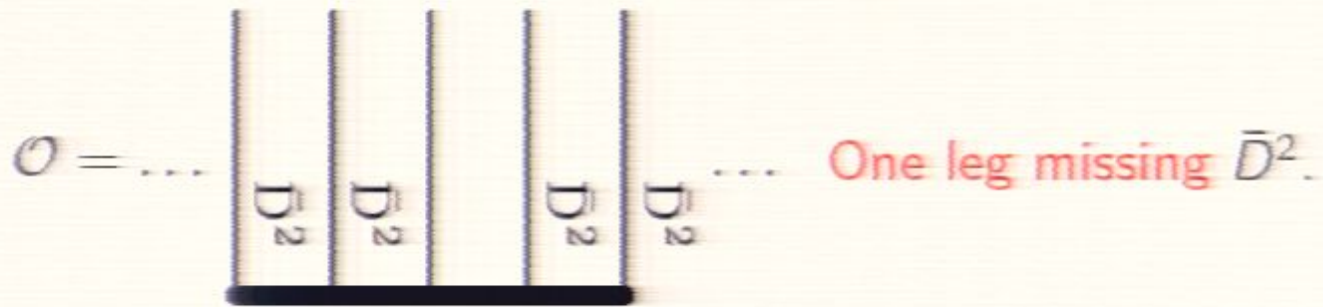


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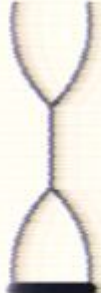
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$\mathcal{D}_0 = L,$  $\mathcal{D}_1 = 2\chi(1), \quad \mathcal{D}_2 = -2[\chi(1,2) + \chi(2,1)] - 4\chi(1)$

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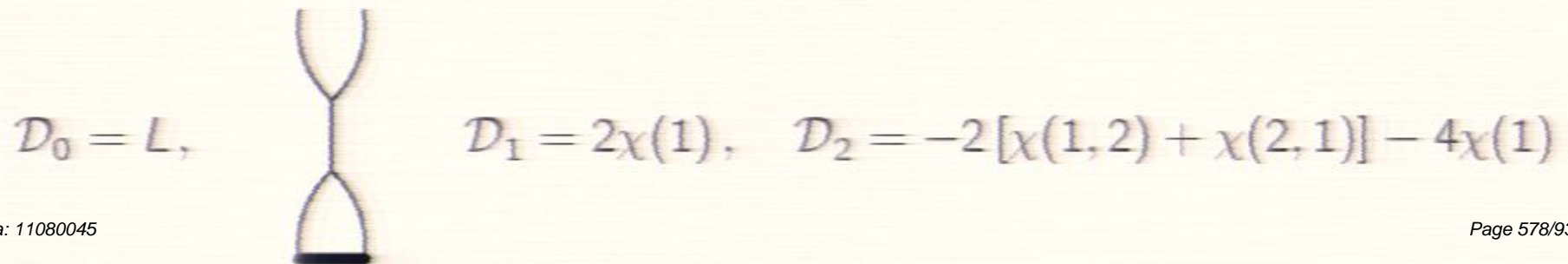
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n.b. No matter what the outcome of the conjecture, it is still true that the connected chiral components sum to the dispersion relation.

e.g. If $\chi(1, 2, 1)$ is known at four loops, then so is $\chi(1)$.

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Planar protected states in the cubic model:

- ▶ For scalar ops. of length L there are $\sim 2^L/L$ planar protected ops.

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Flavor choices

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Same as 4-loop prop

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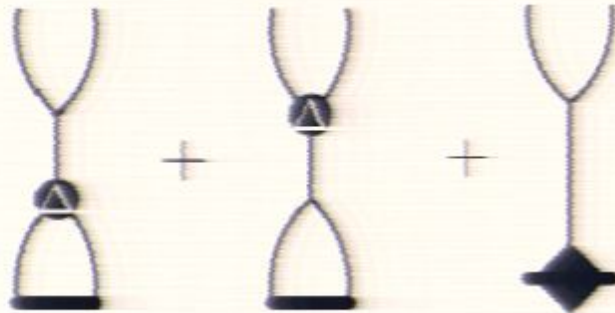


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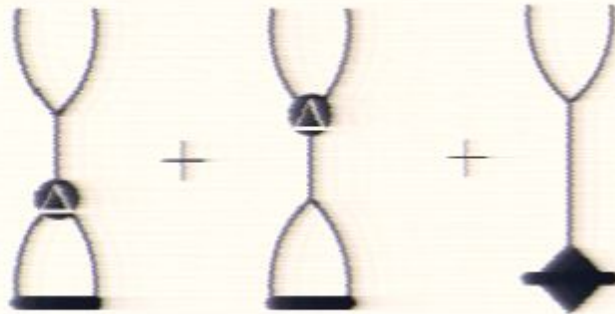


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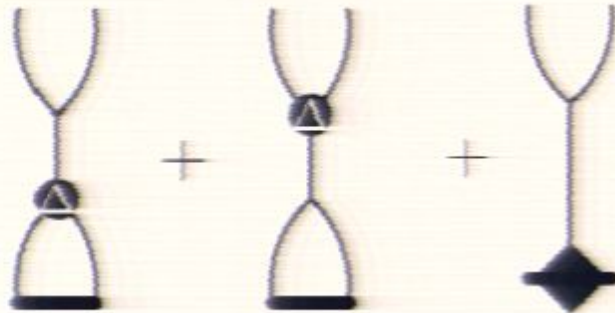


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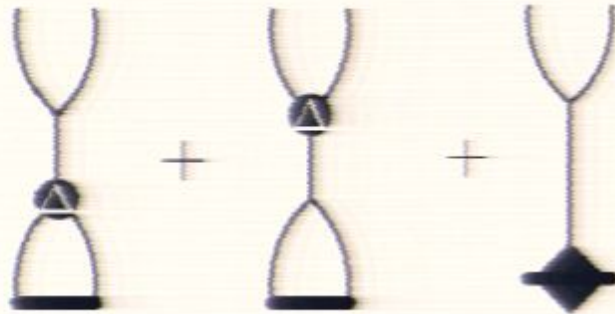


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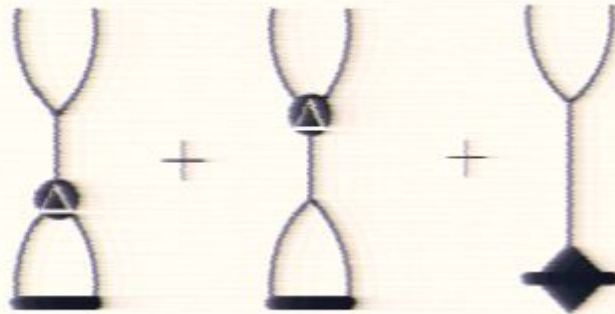


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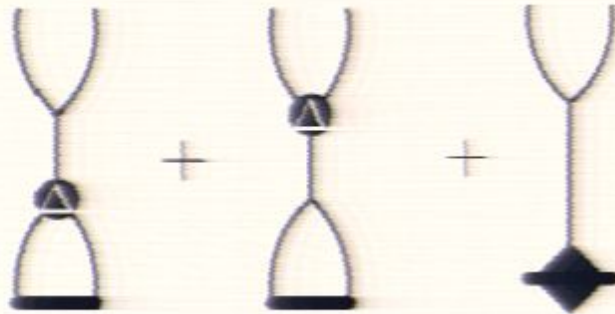


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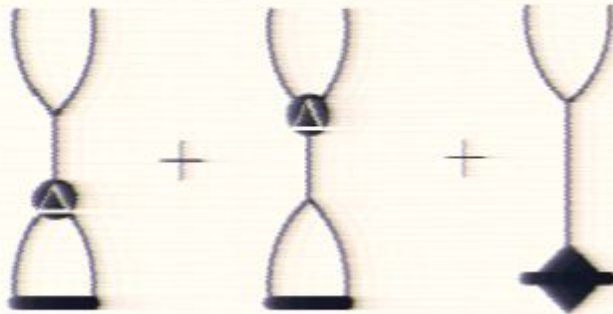
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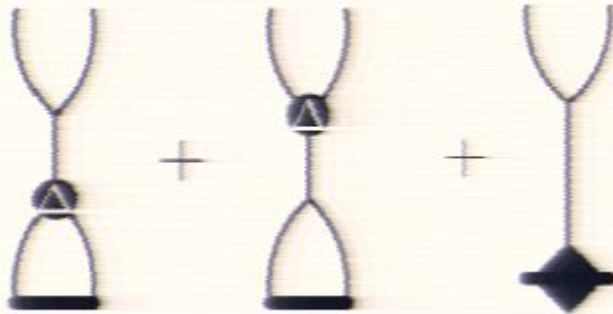


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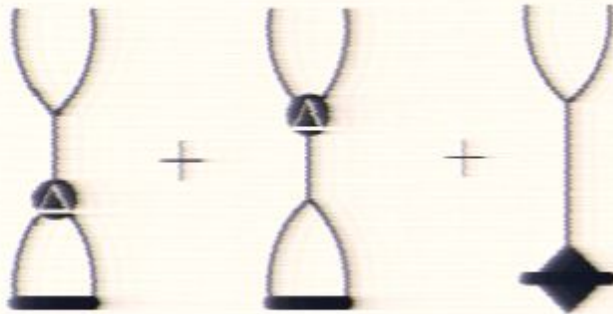
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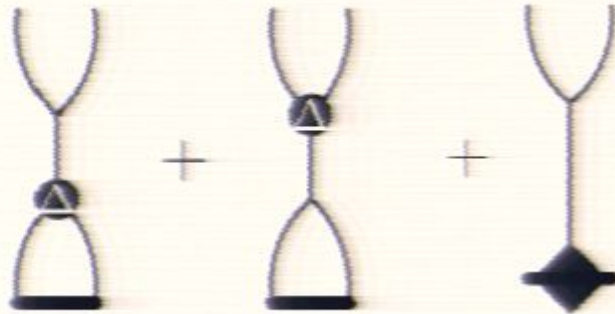


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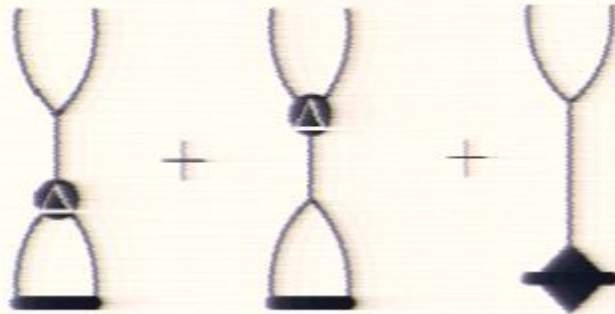
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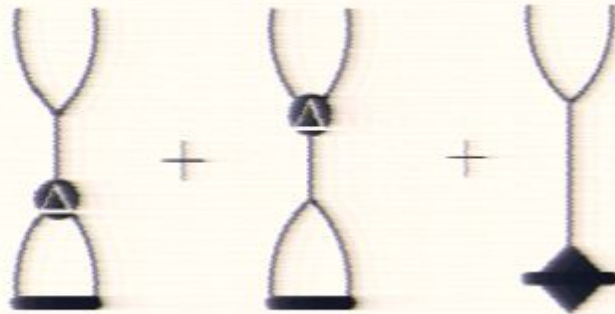


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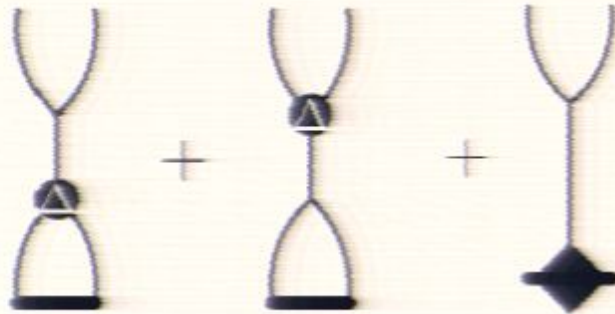
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Same as 4-loop prop

Corrections:



Same as 4-loop prop

Same topology as chiral prop. corr. \Rightarrow Corrections cancel.

4-loop anom. dim. is $\chi(1)$ from $\mathcal{N} = 4$.

$$2F_1(g^2) = \sqrt{1 + 8g^2} - 1$$

$$\delta_4 = -160 g^8 N_p + ?$$

Higher loops:

4-loop correction to anom. dimension for a one-pair state
(or multipairs separated by 3 or more)



Flavor choices

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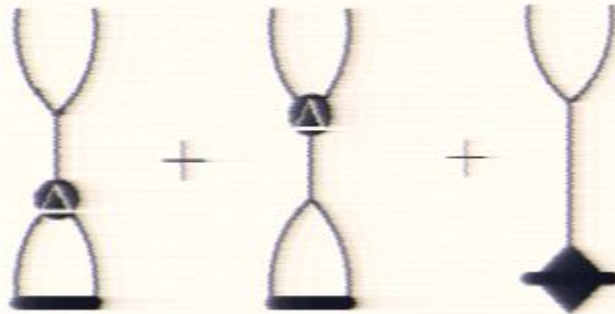
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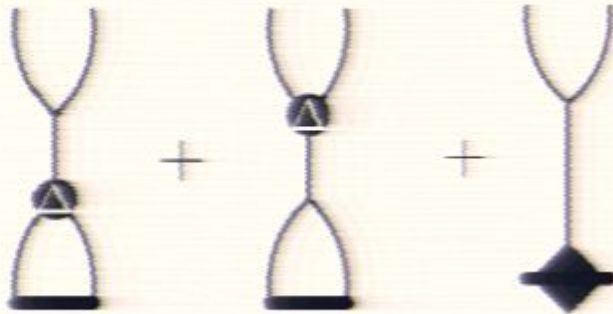


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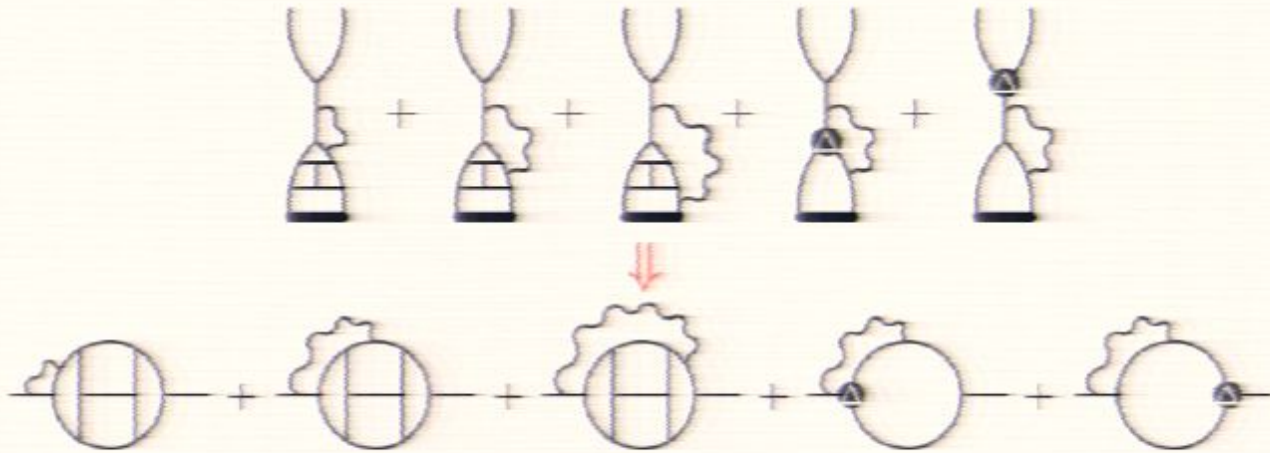
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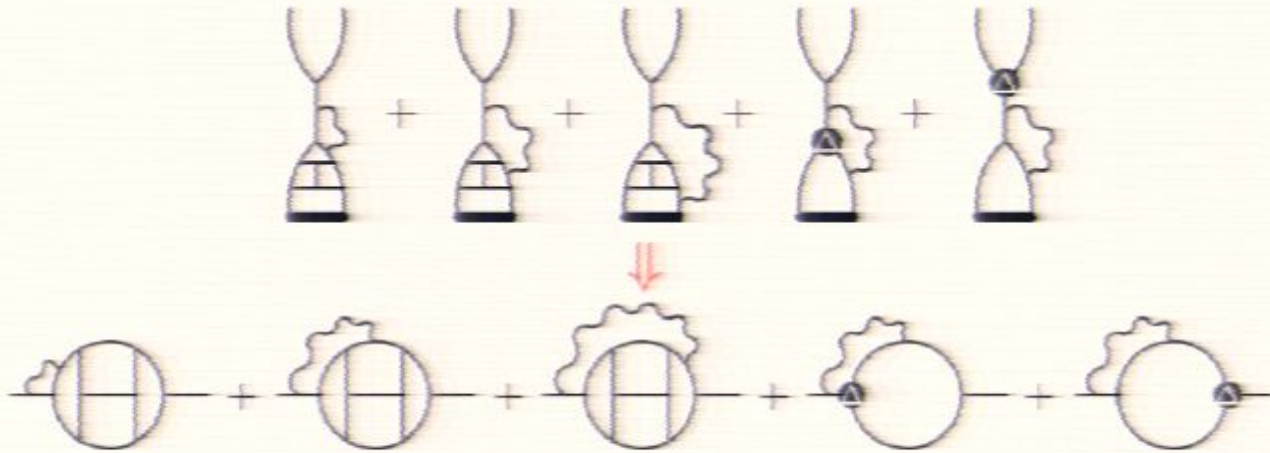
Higher loops:

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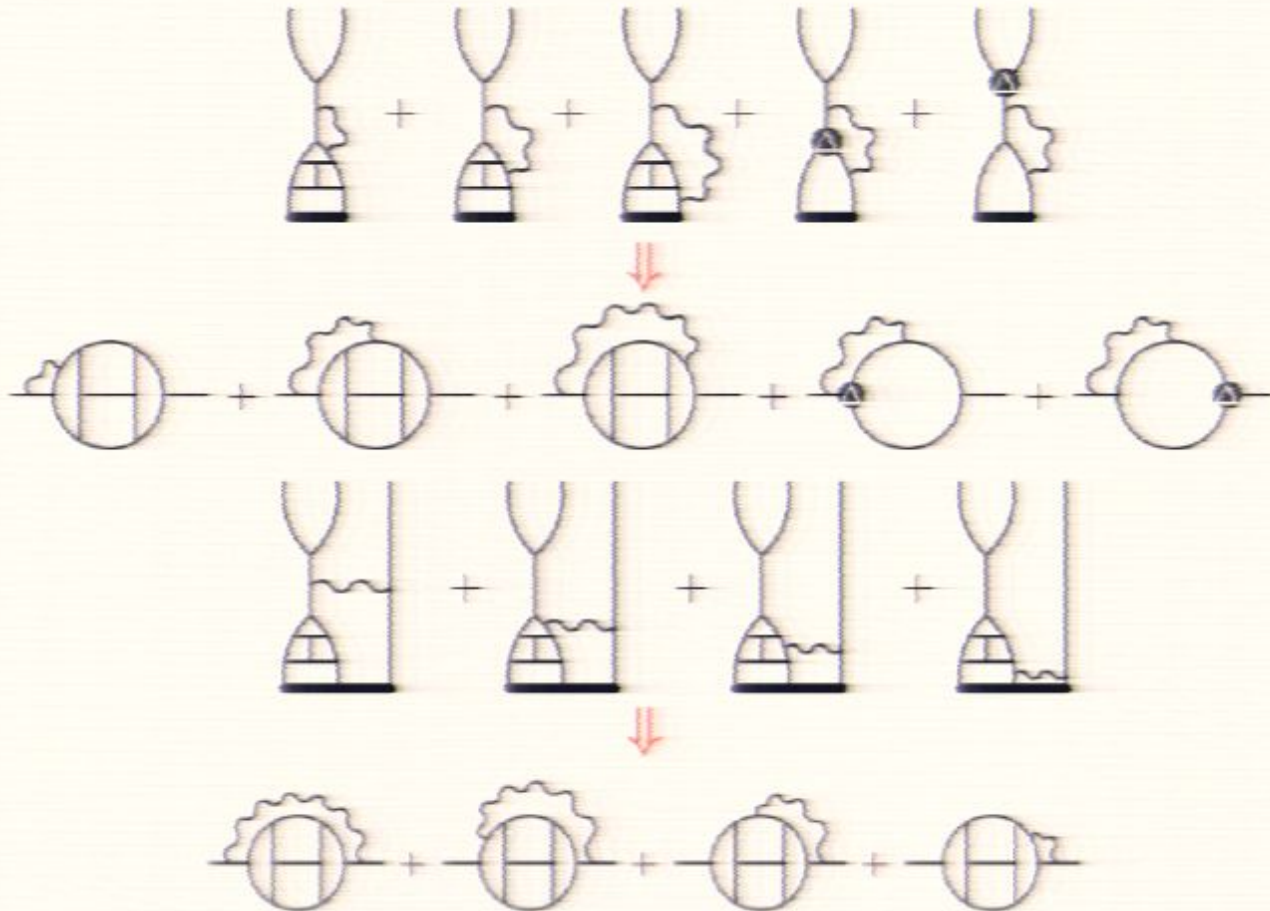
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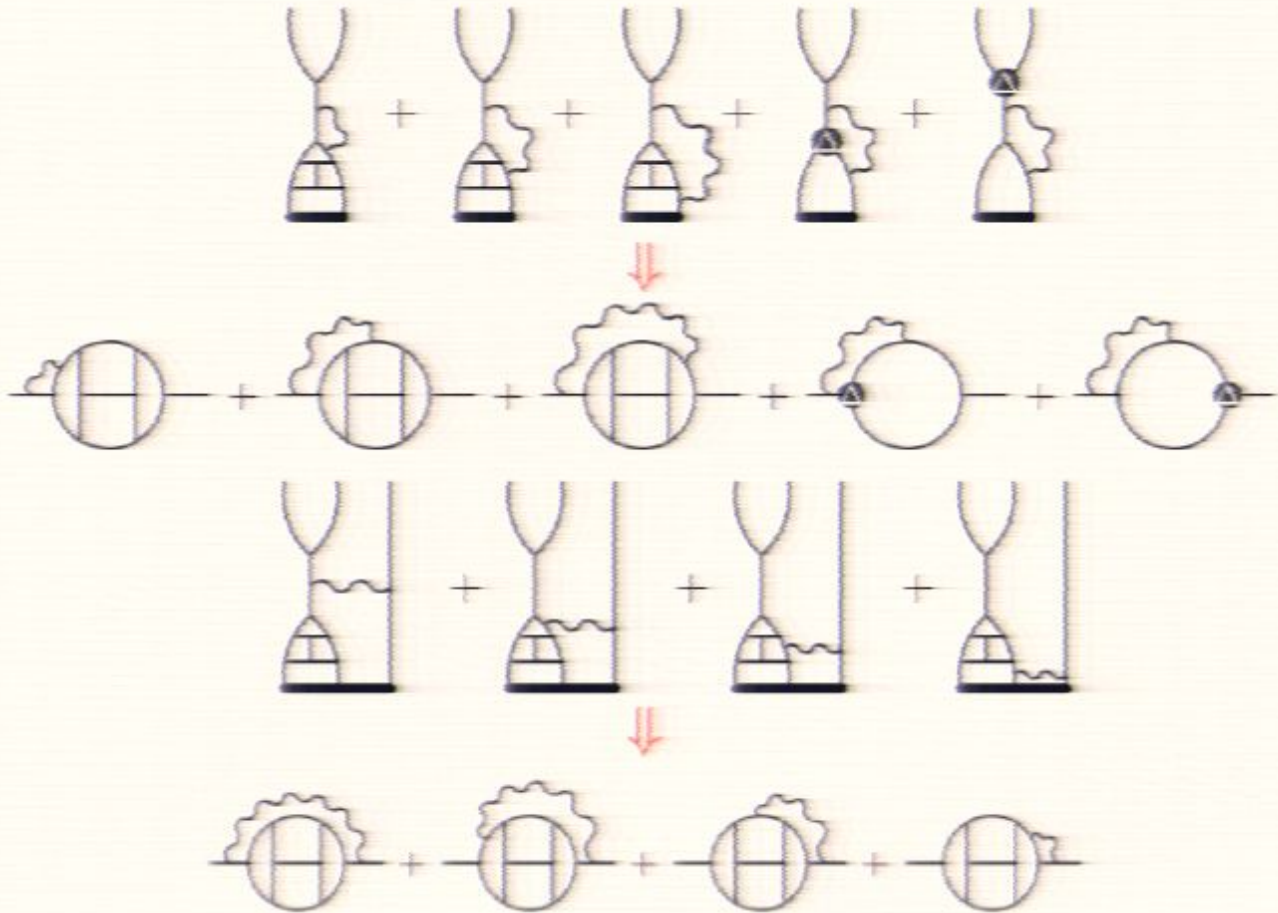


Canceling 5-loop divs. in props \Rightarrow Cancels 5-loop div. in ops.

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Higher loops:

5-loop correction to anom. dimension for a one-pair state

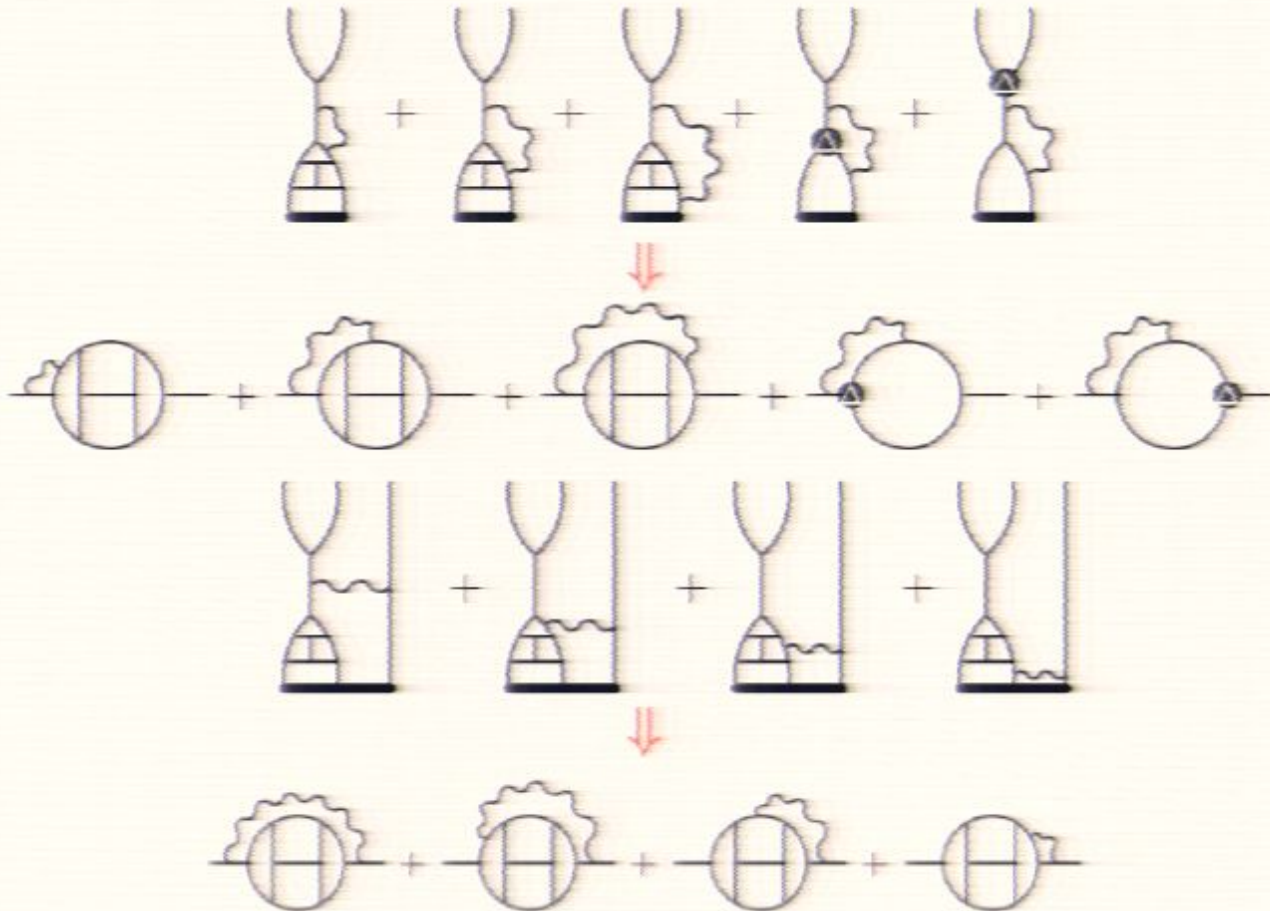


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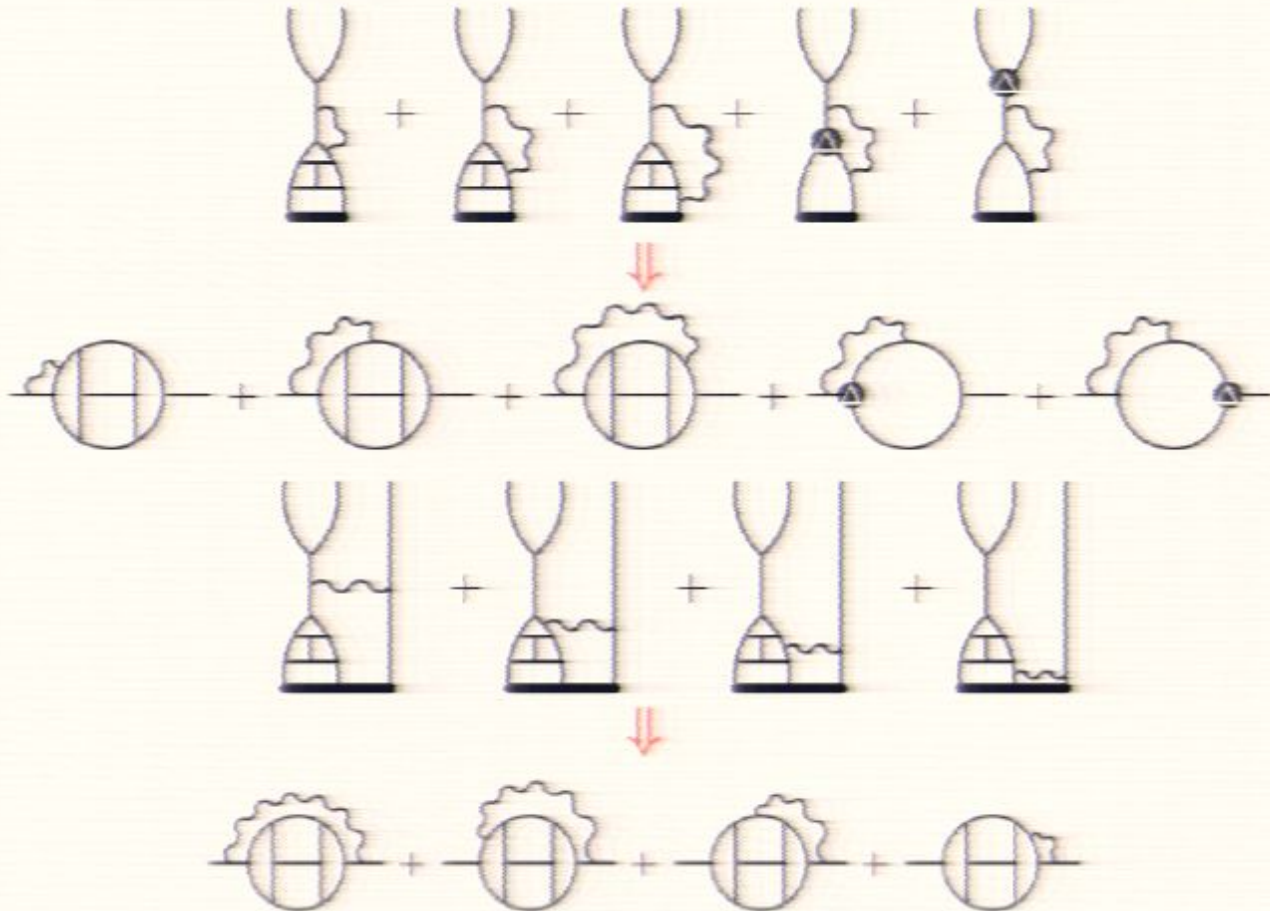


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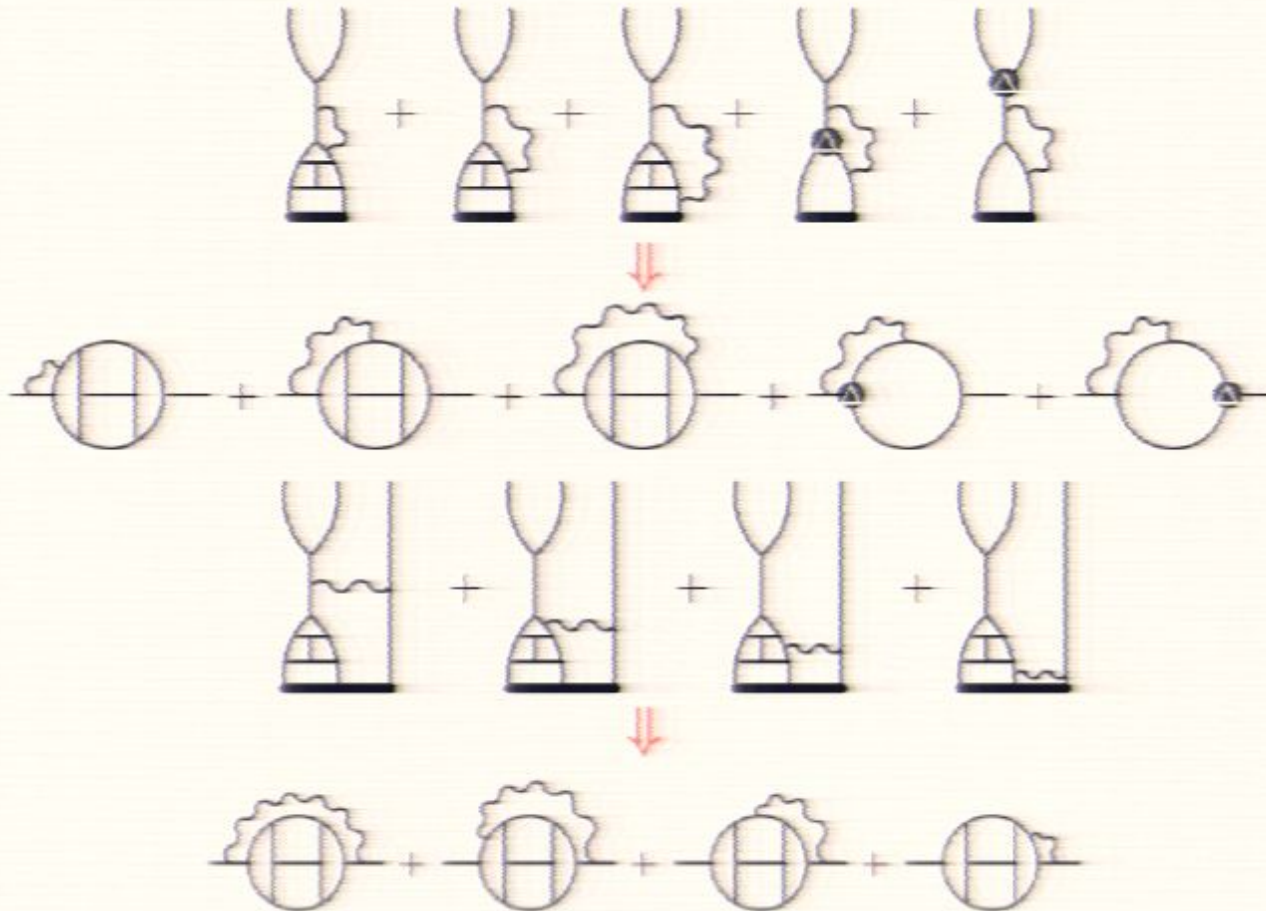


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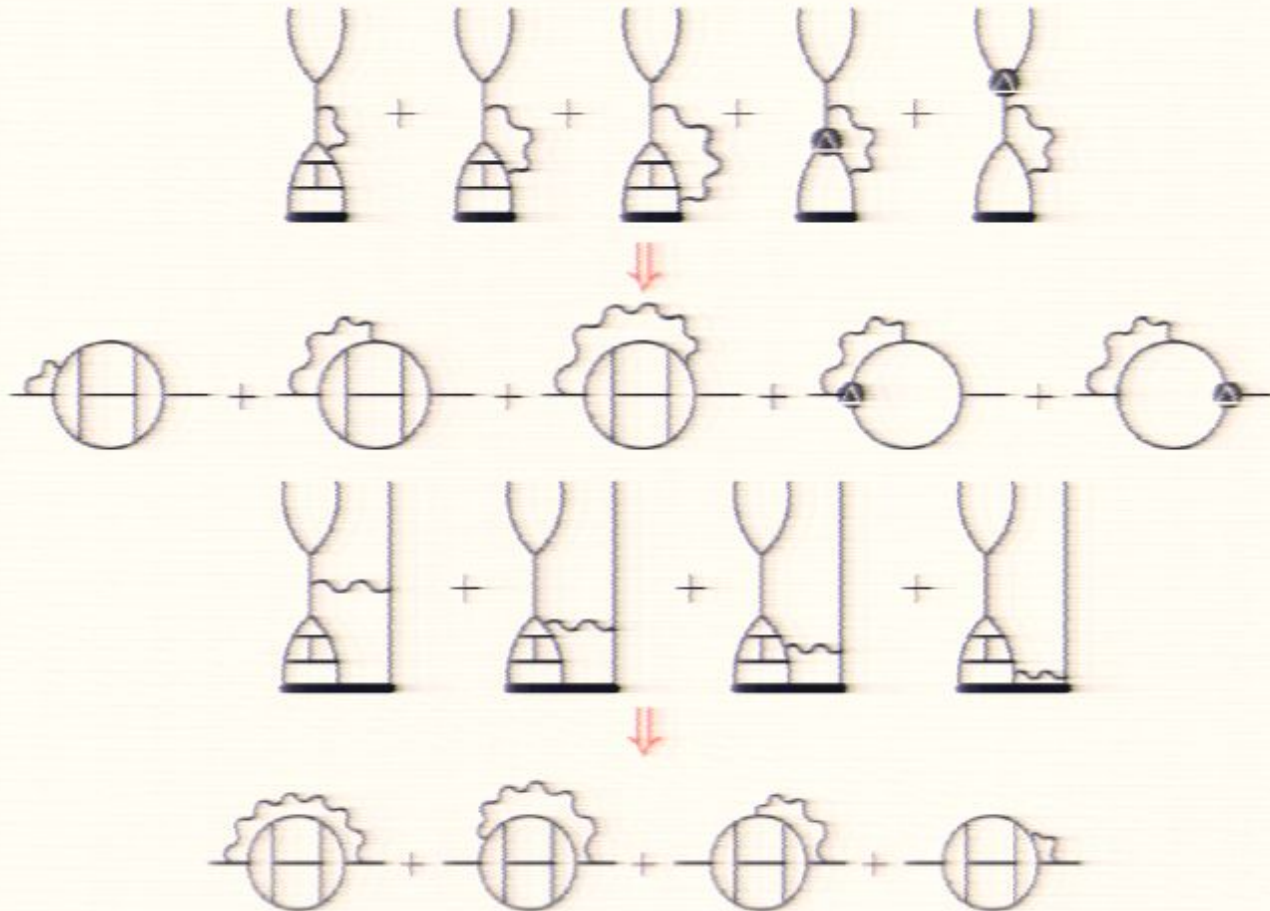


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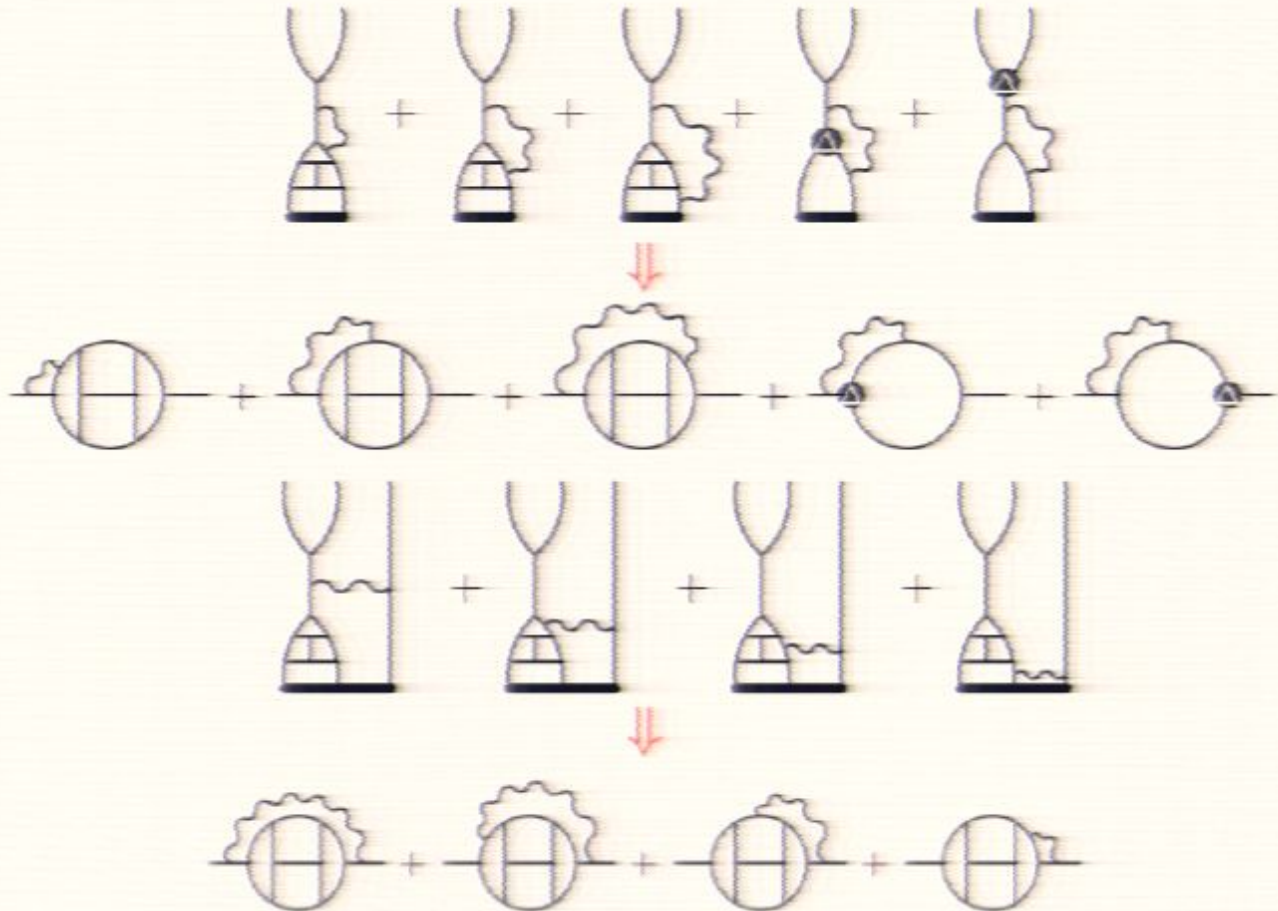


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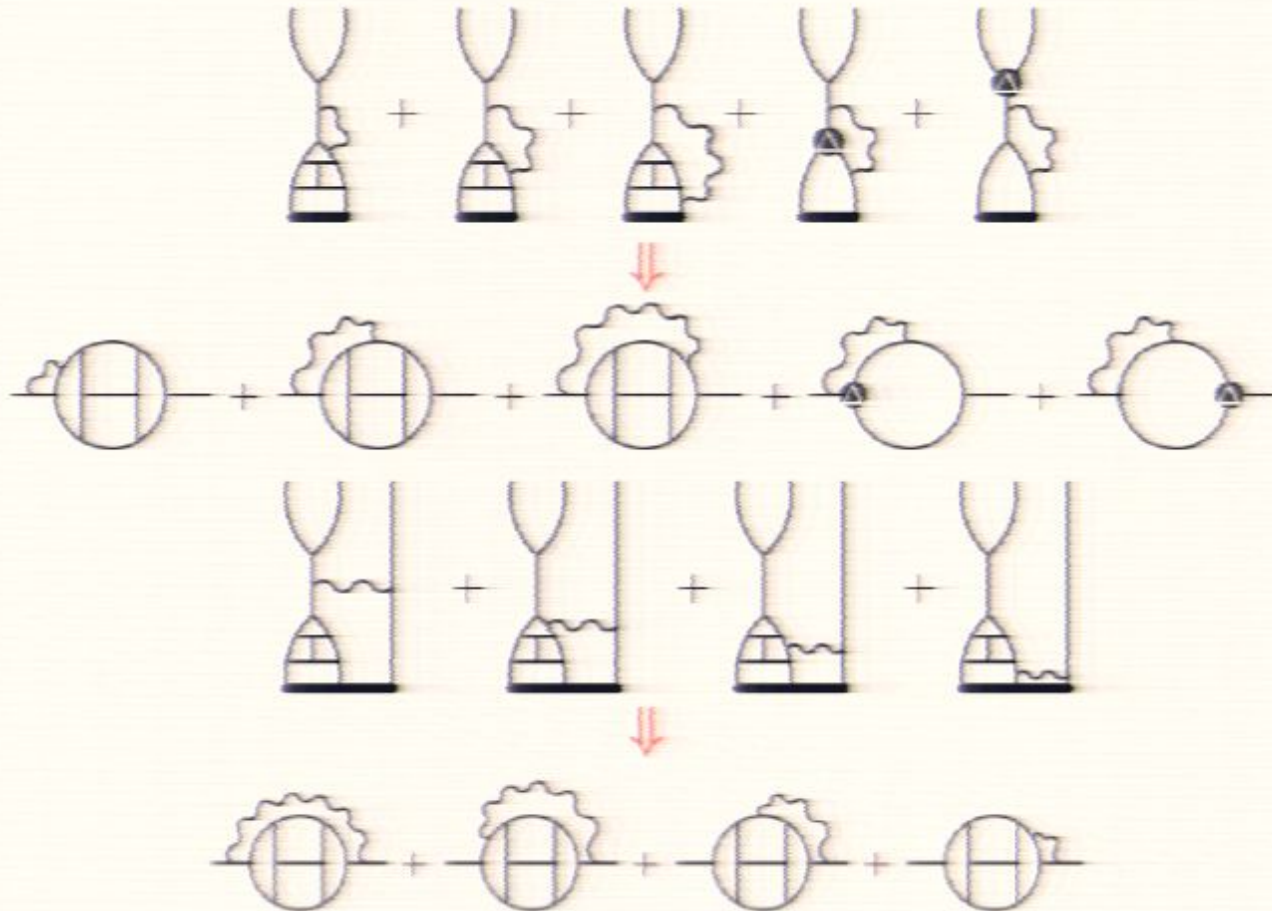


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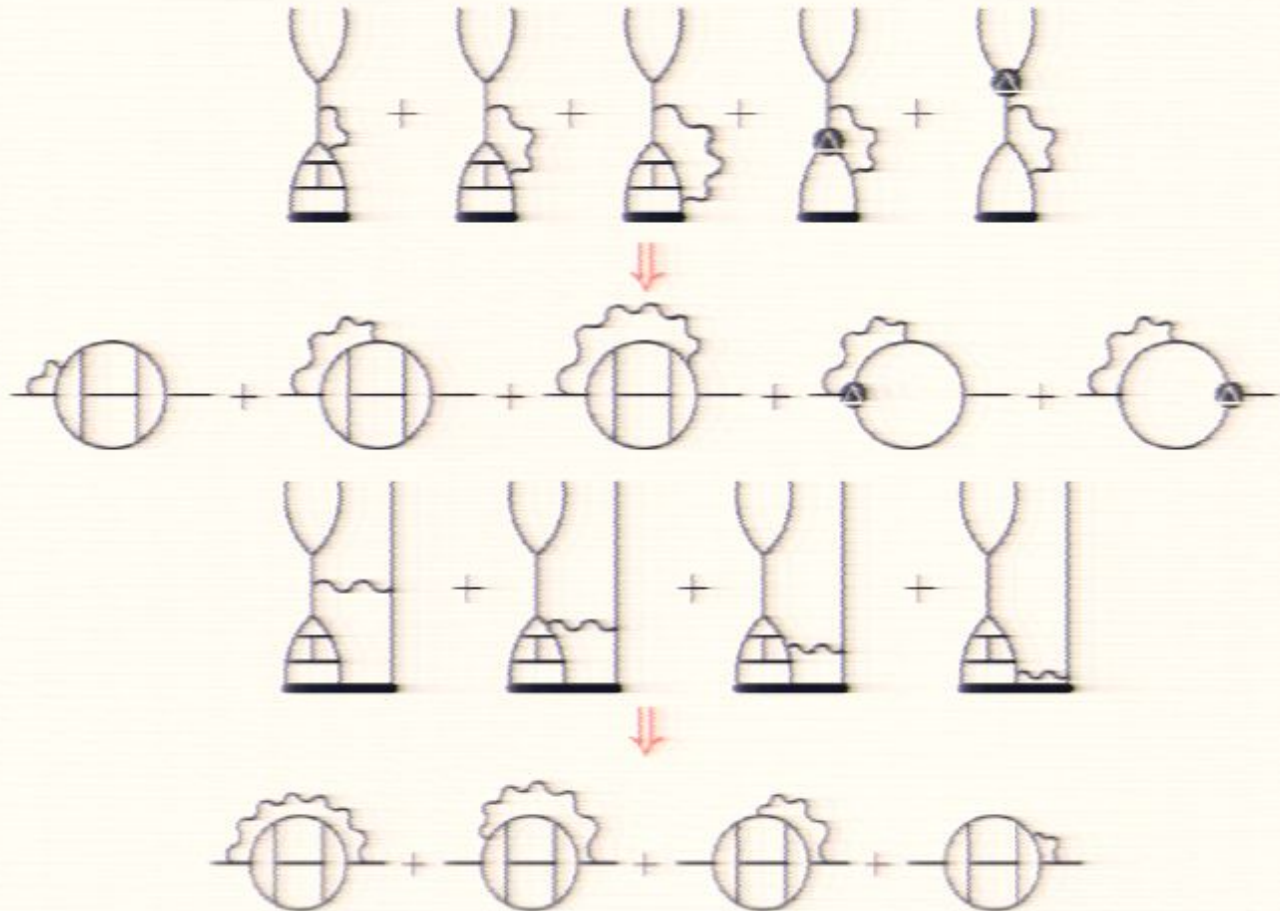


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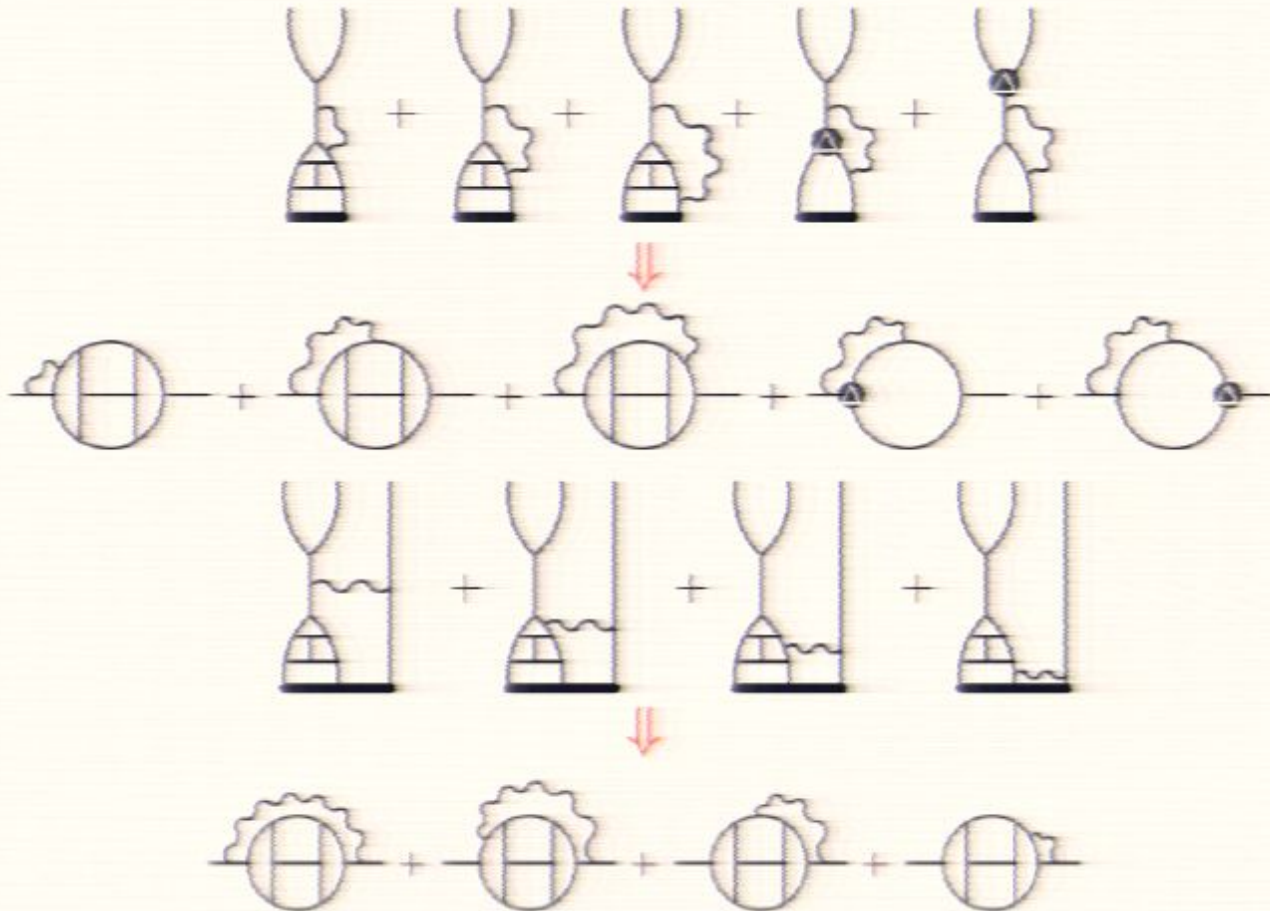


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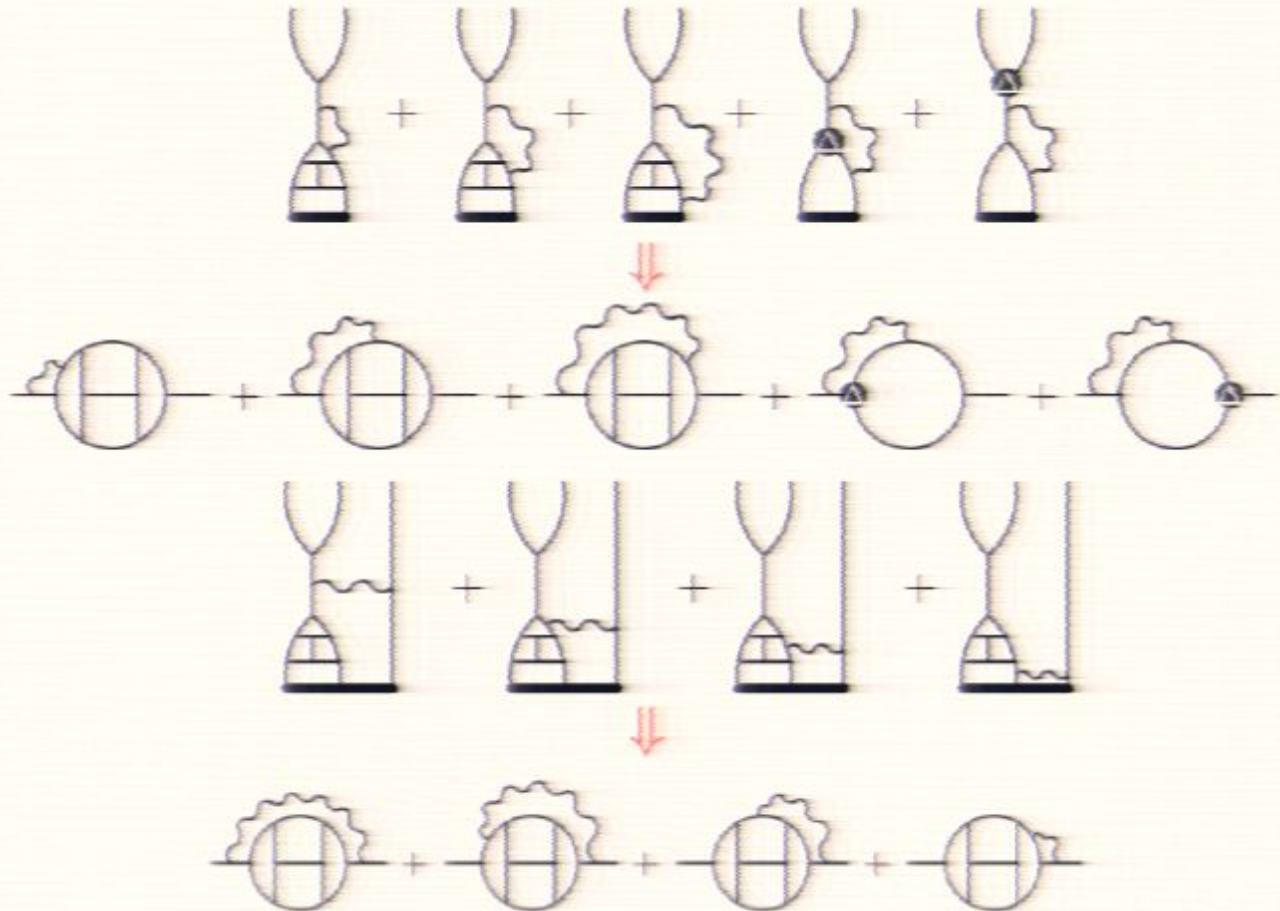


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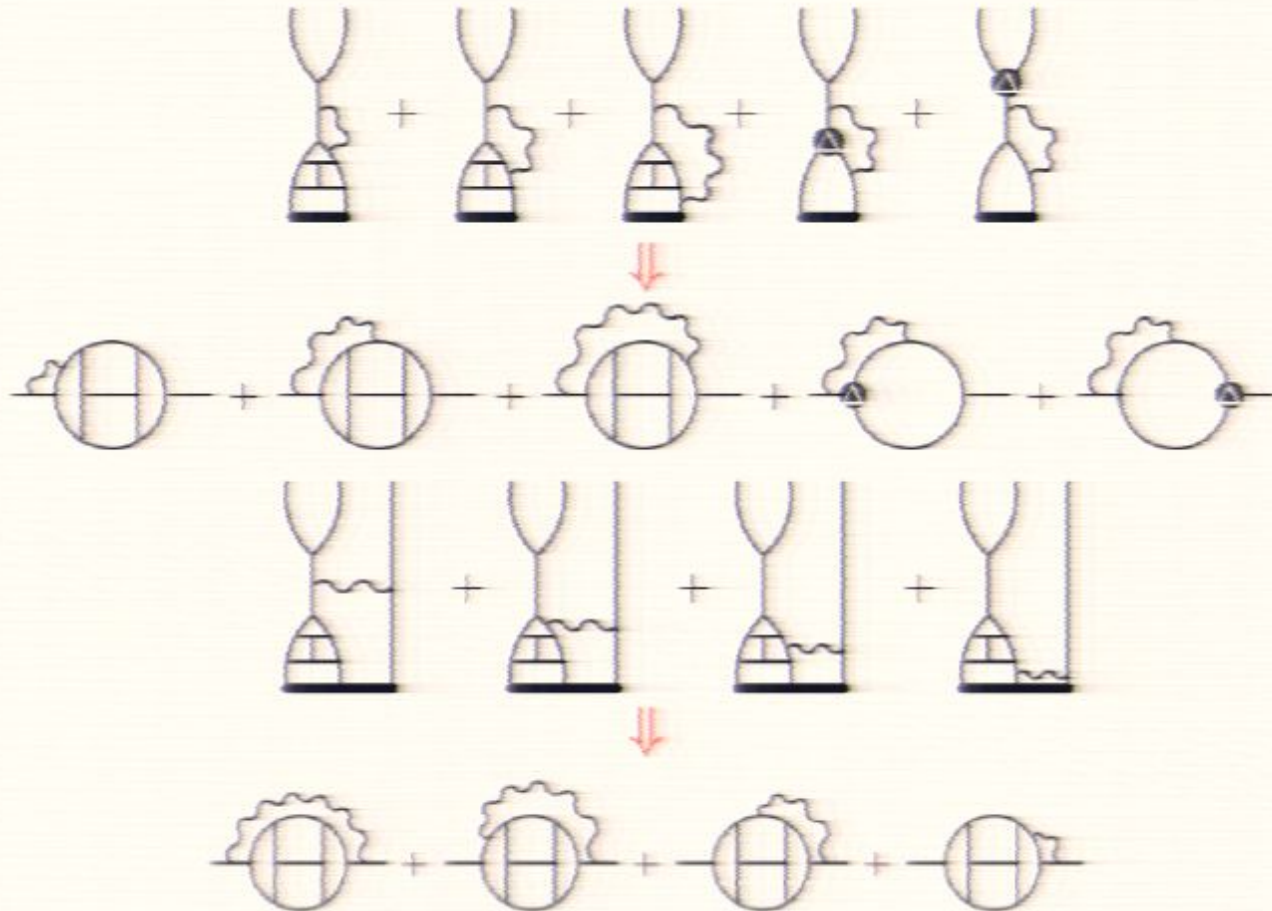


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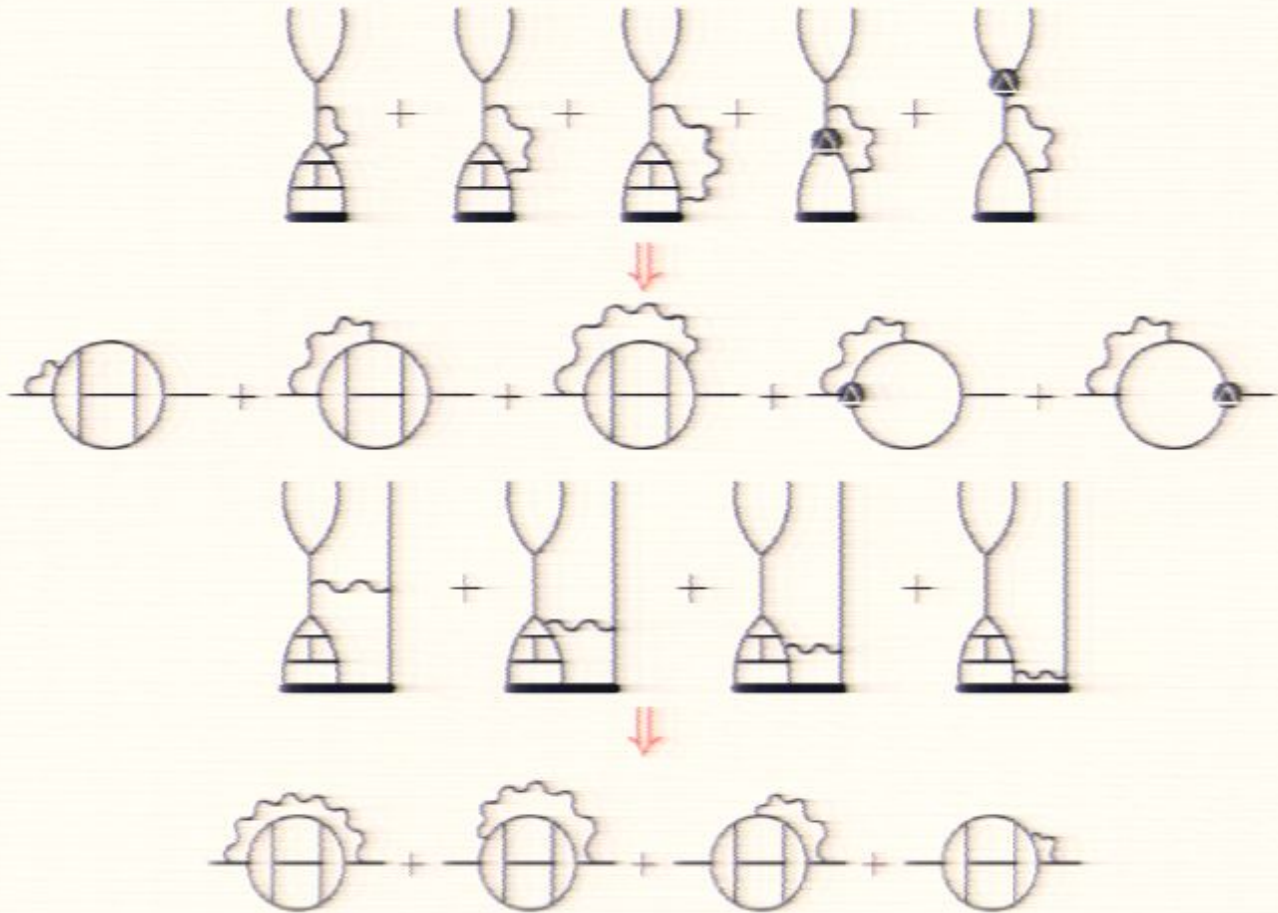


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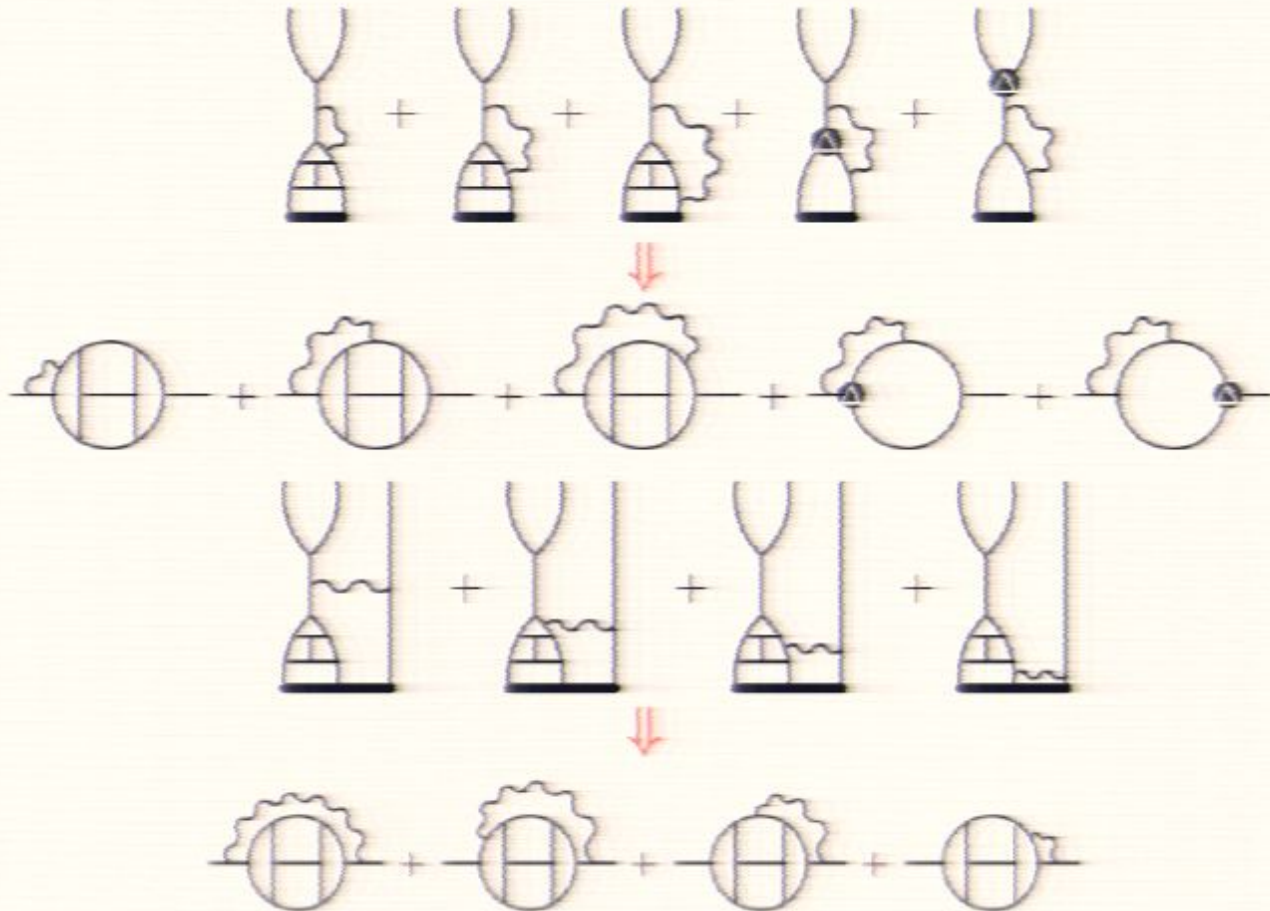


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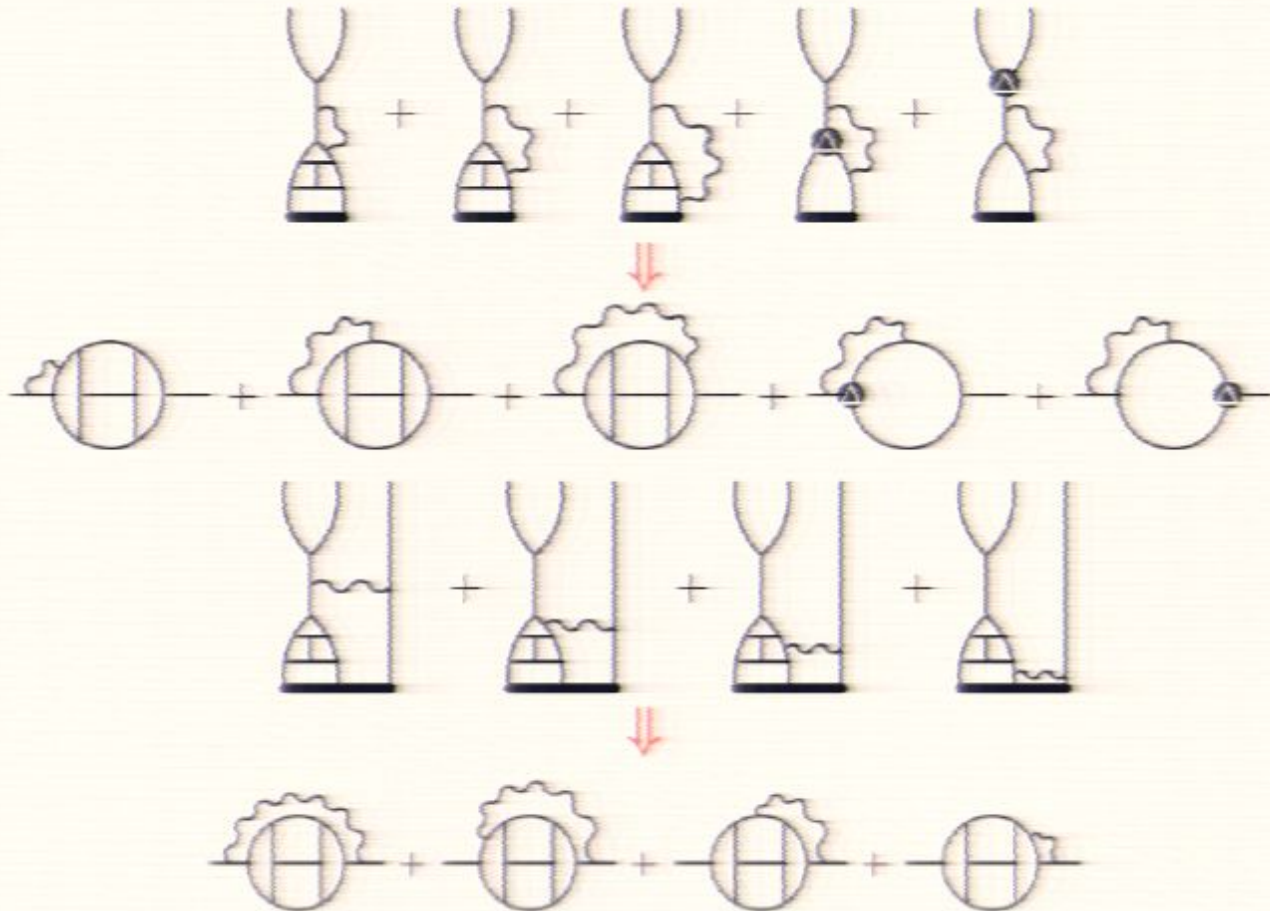


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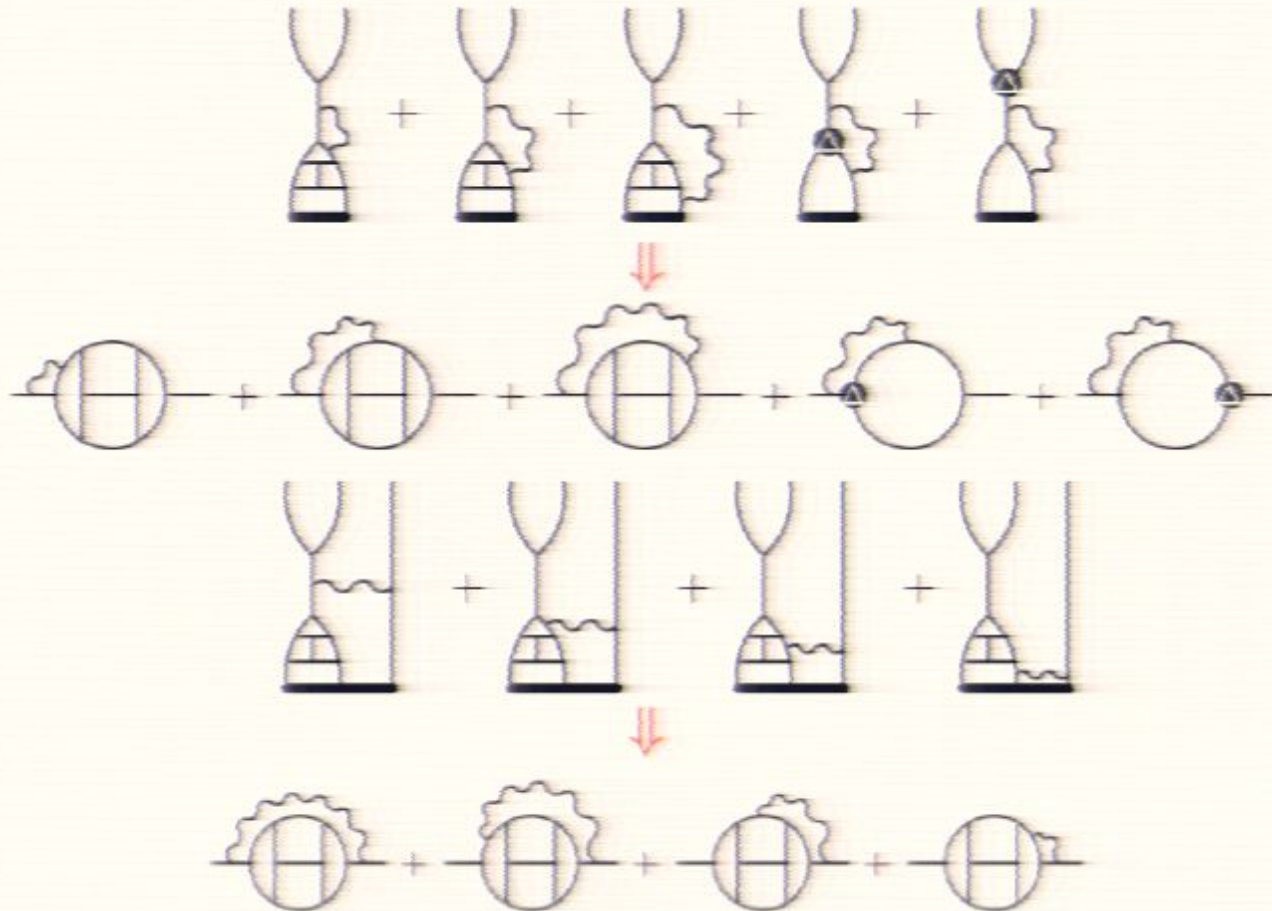


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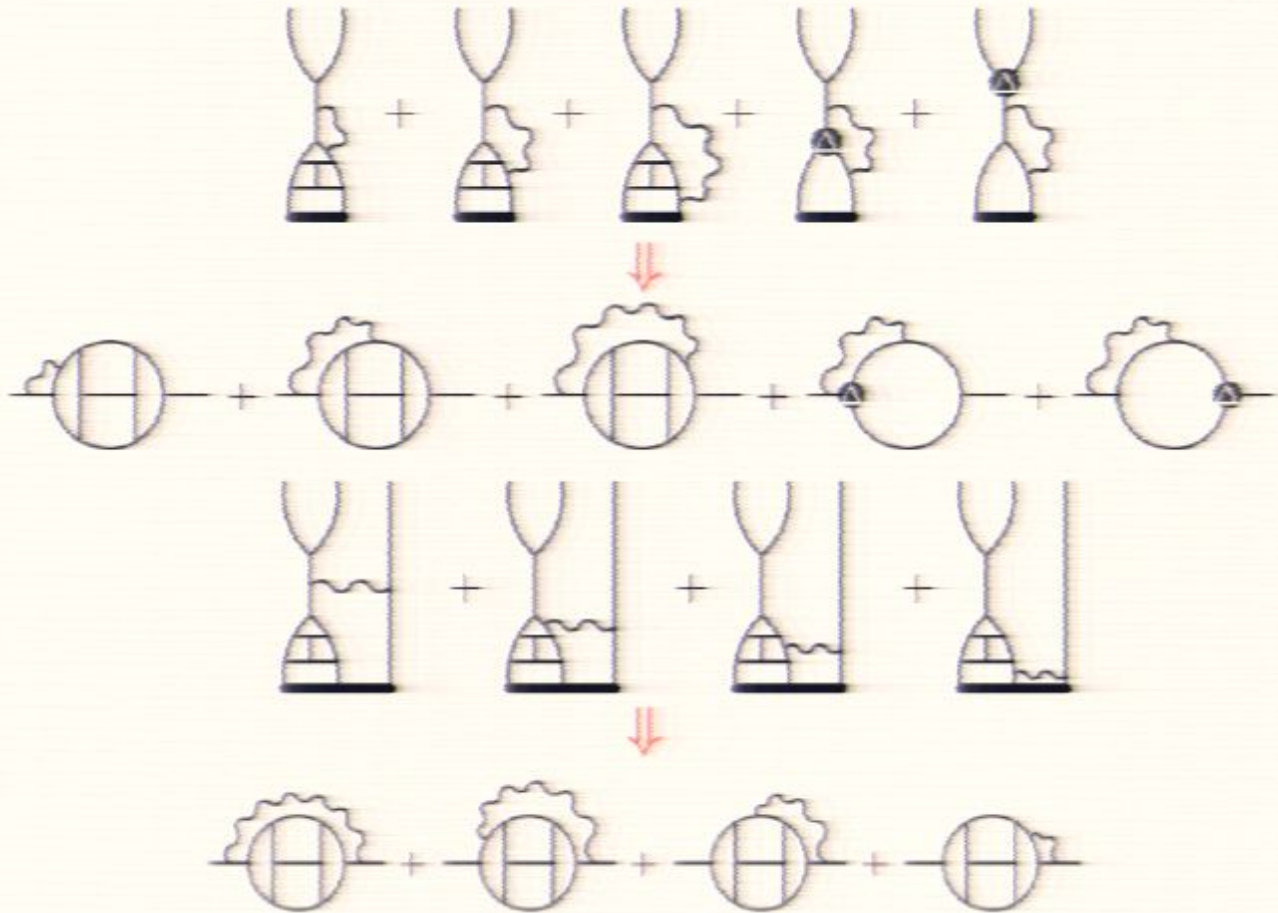


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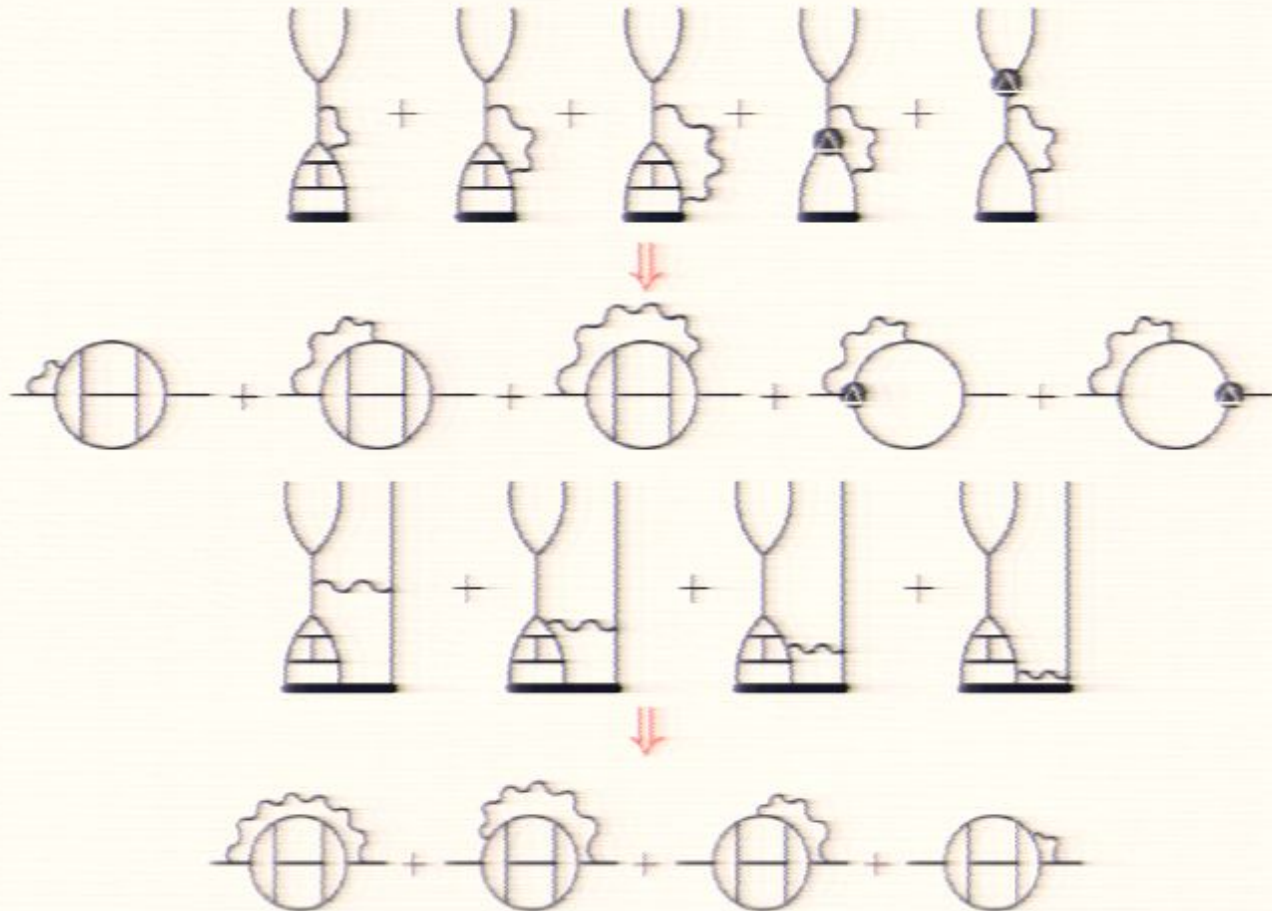


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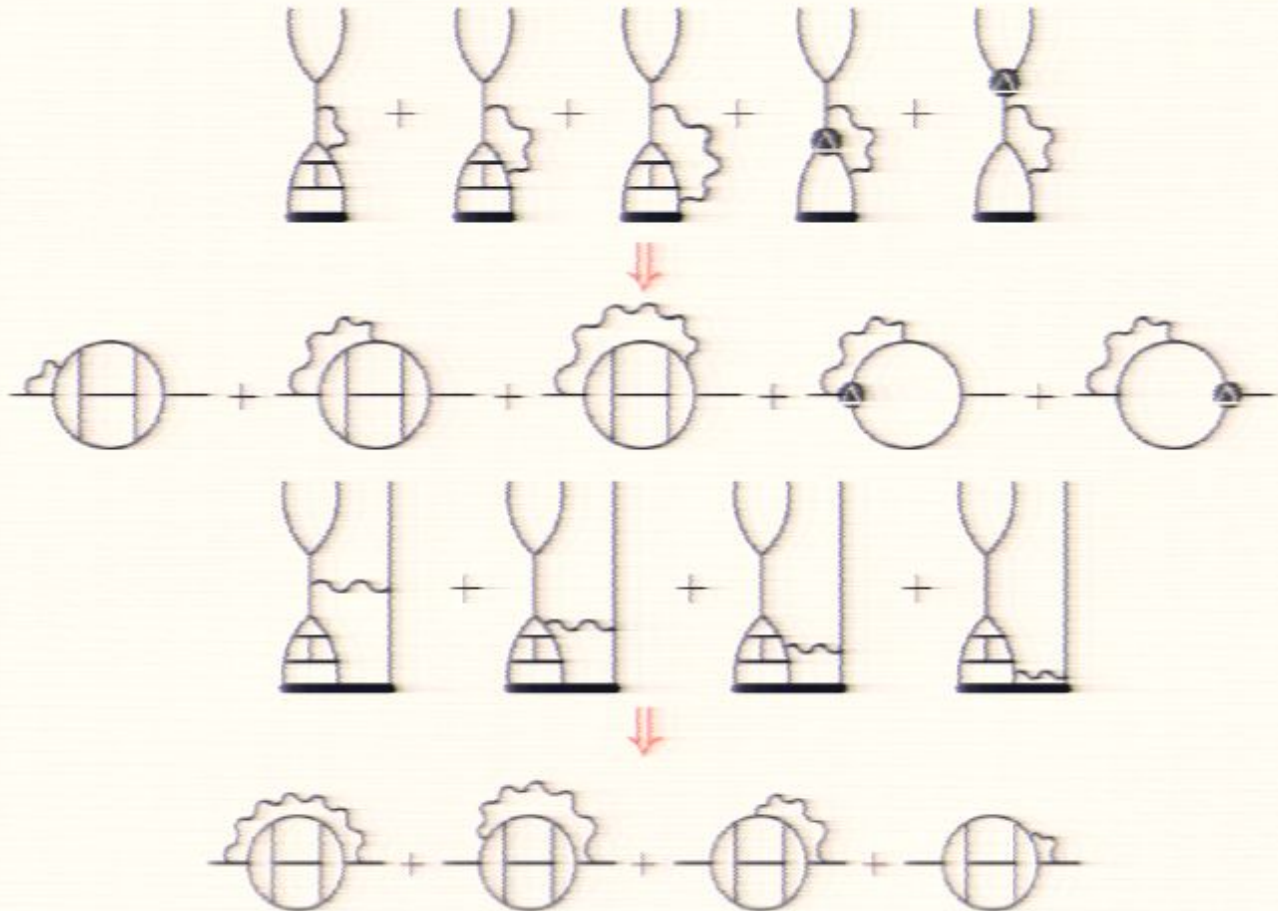


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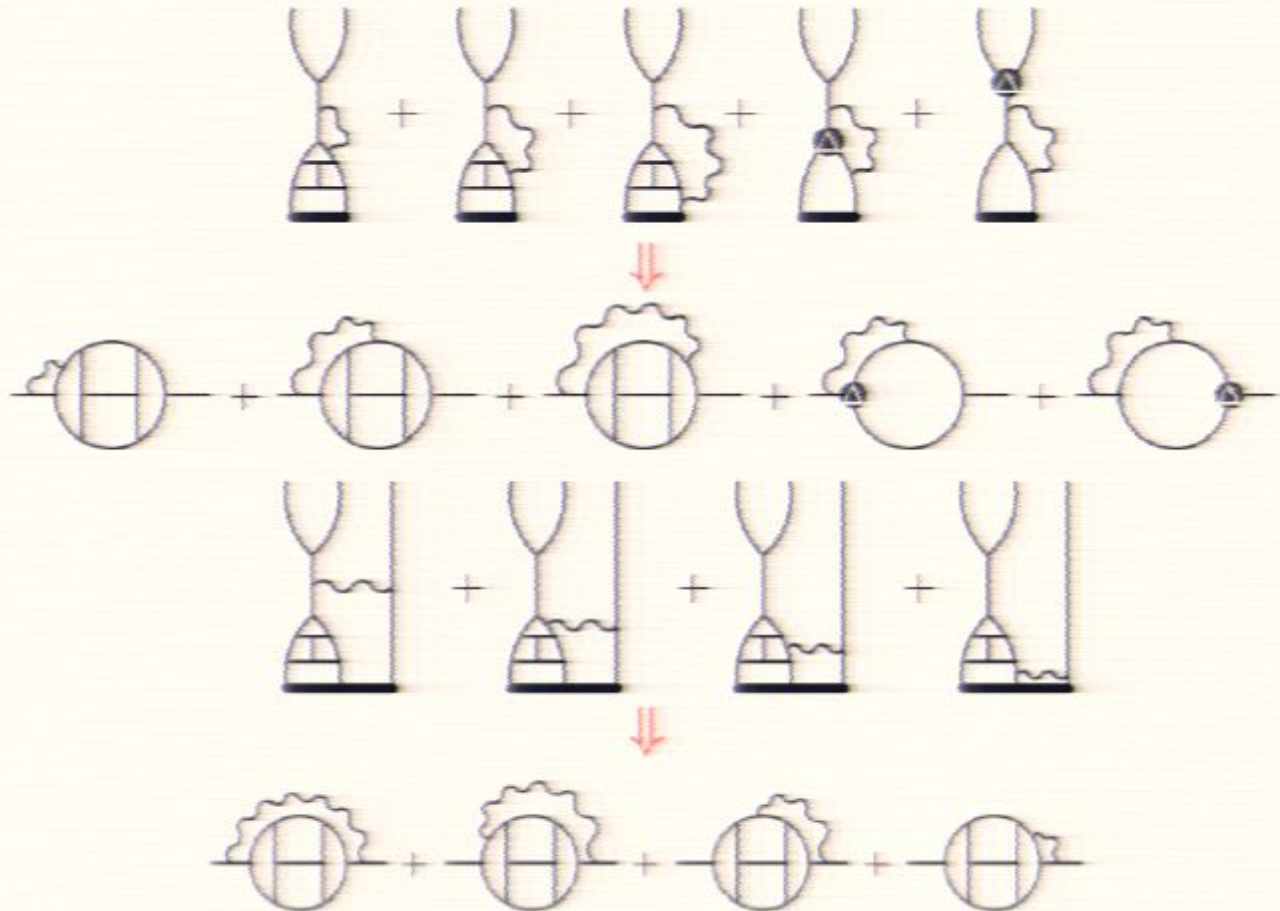


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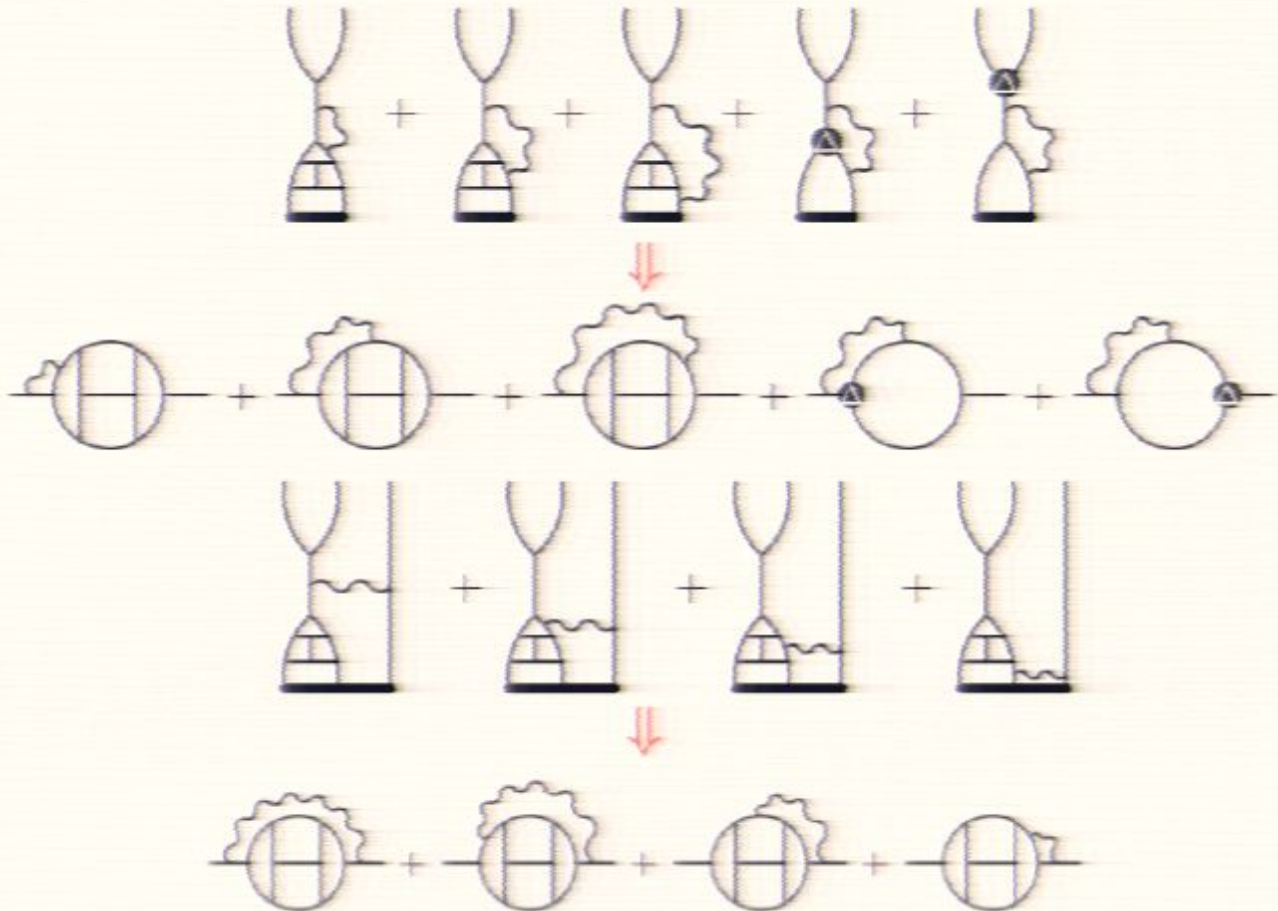


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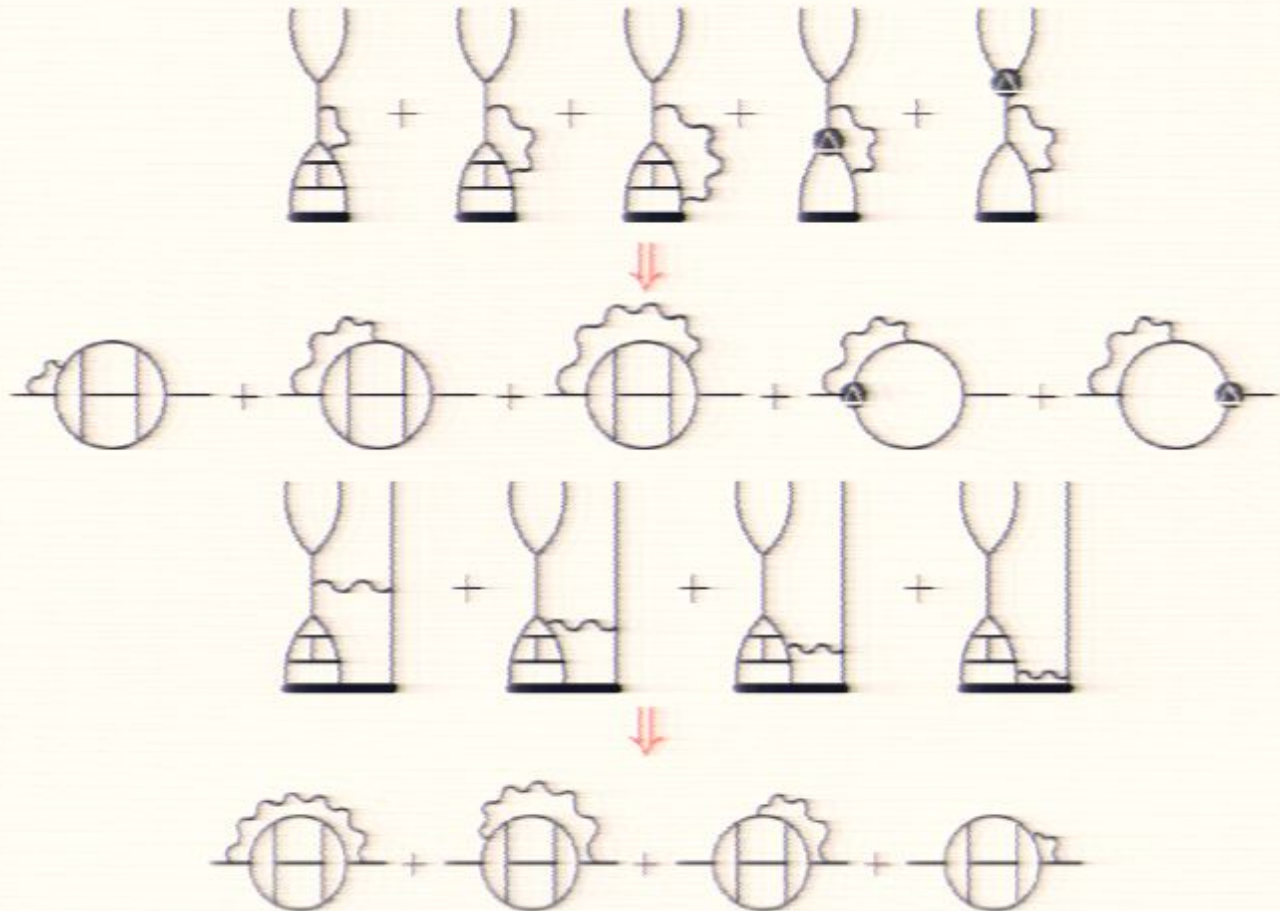


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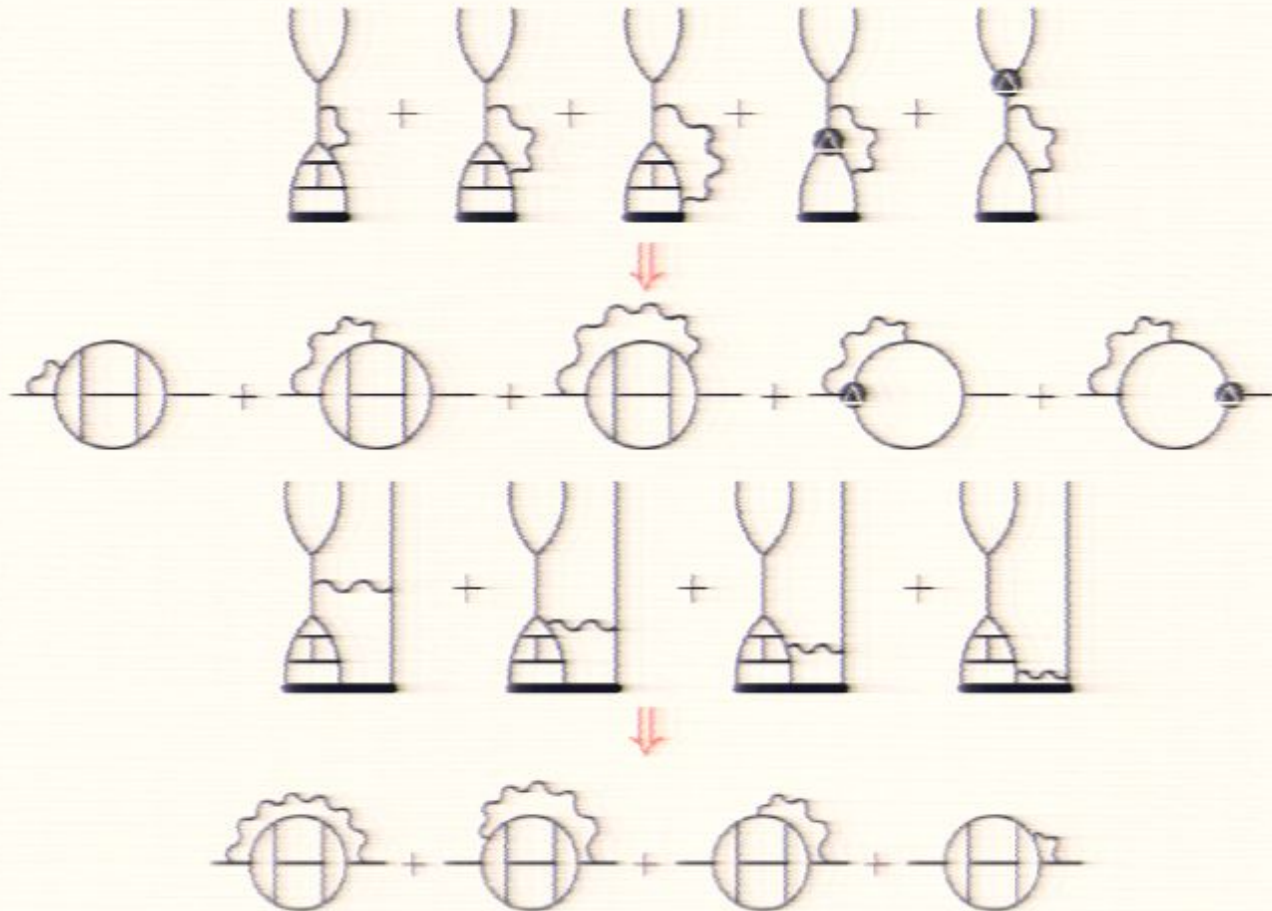


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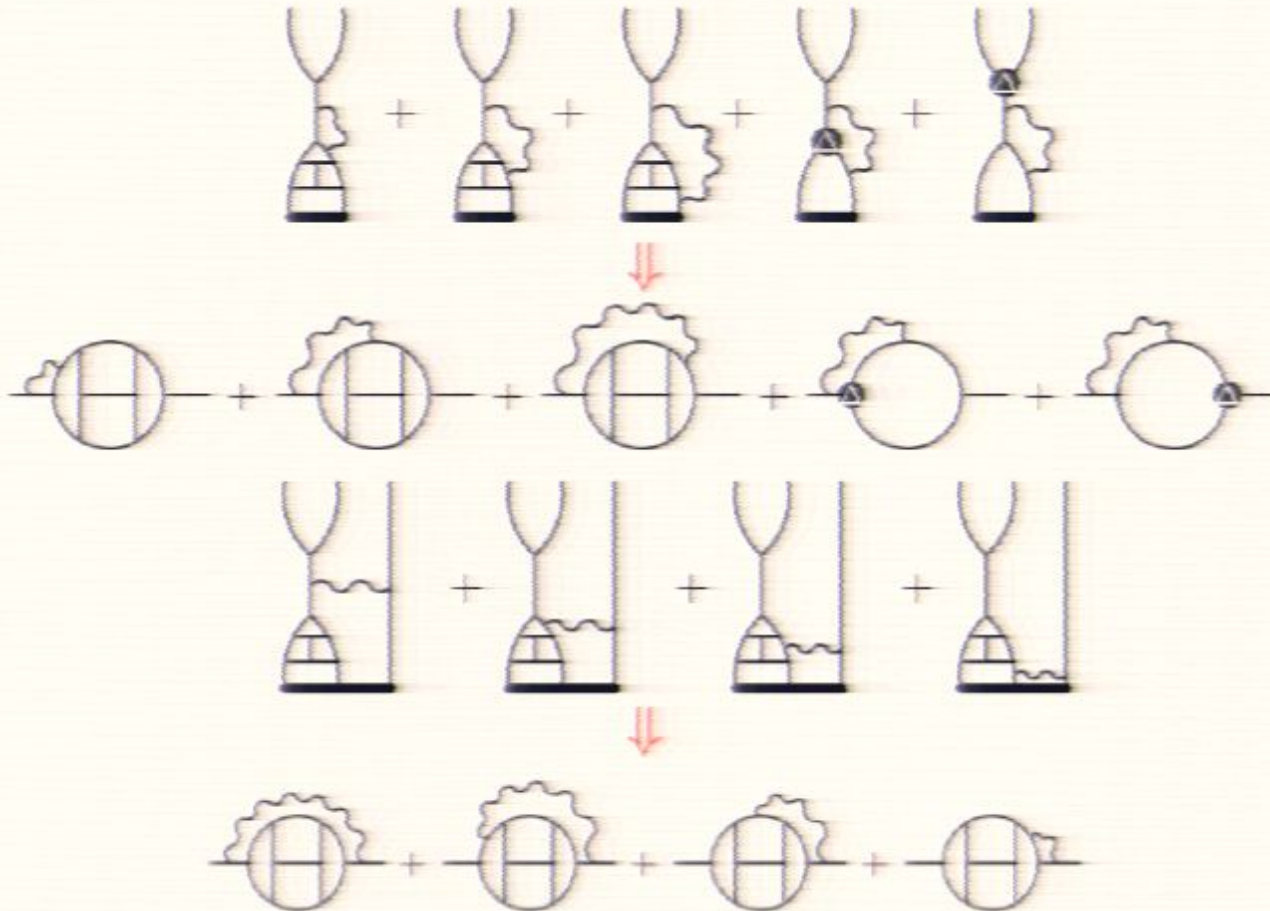


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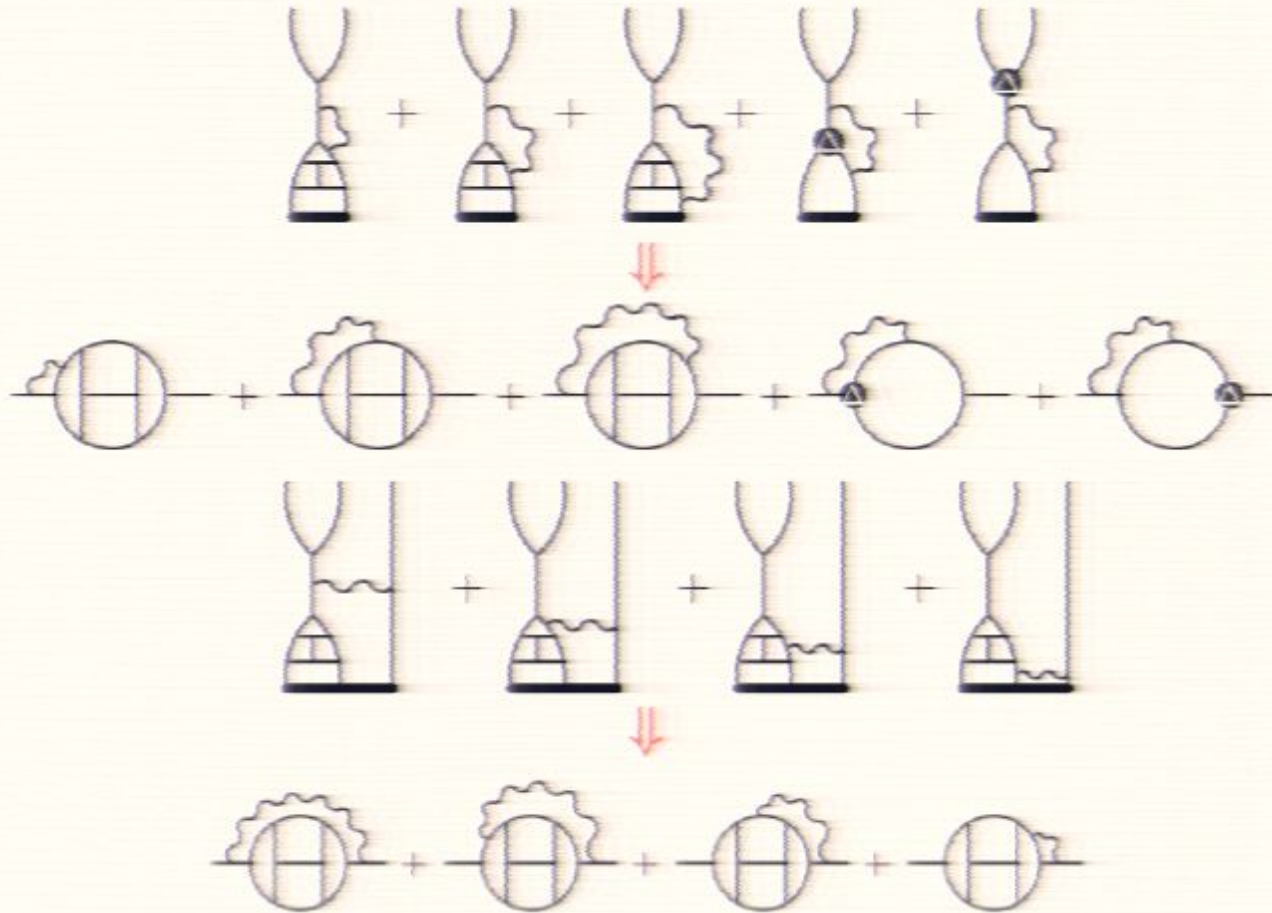


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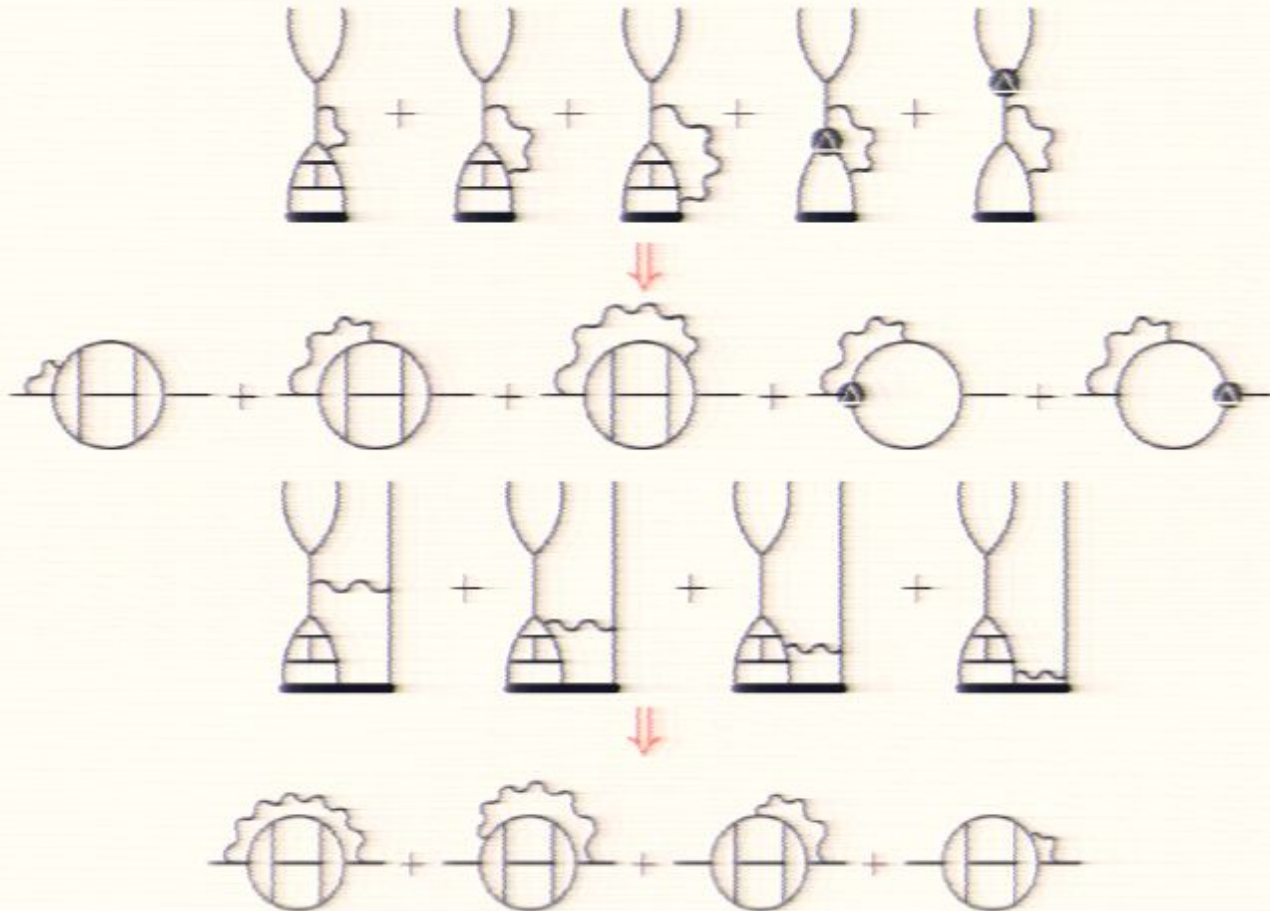


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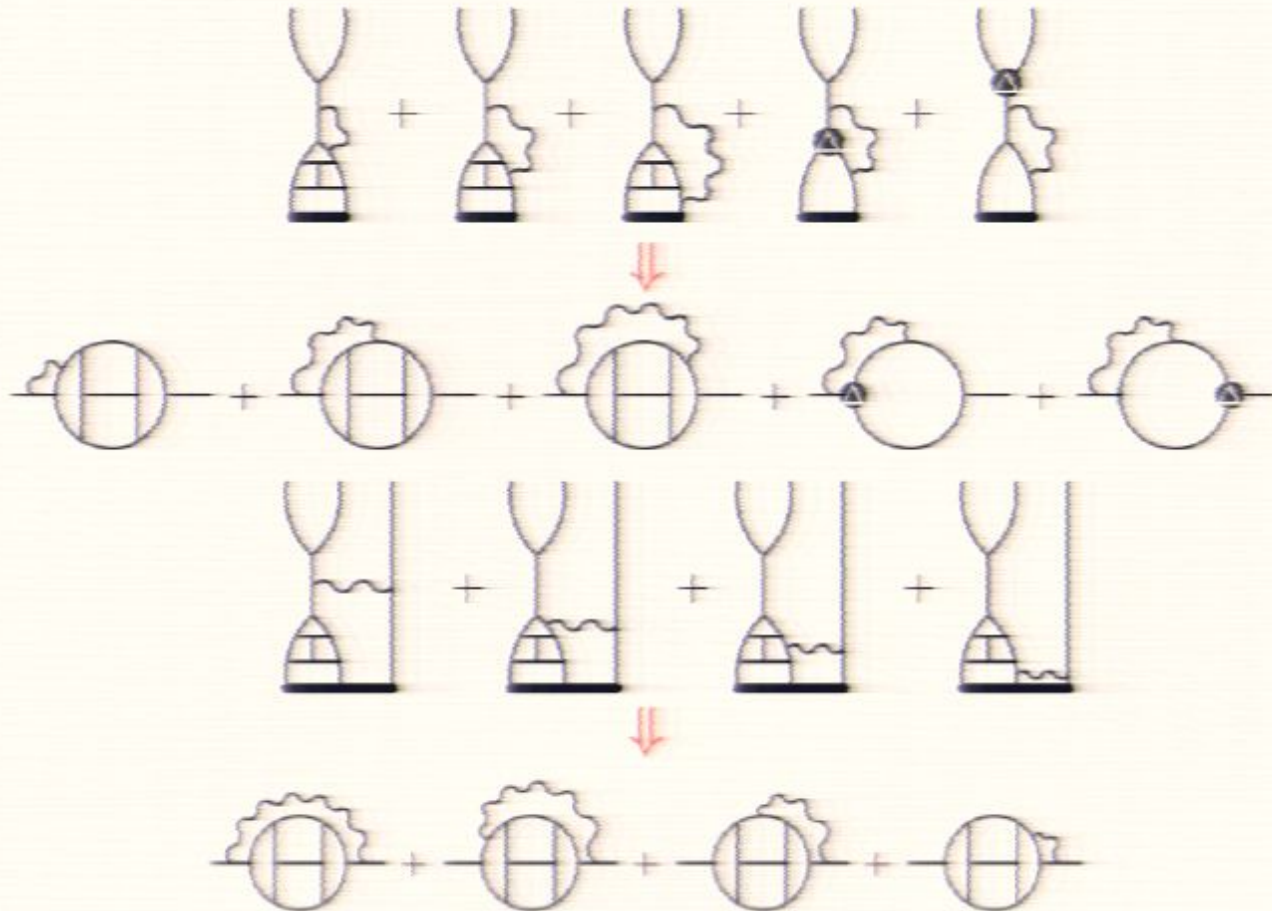


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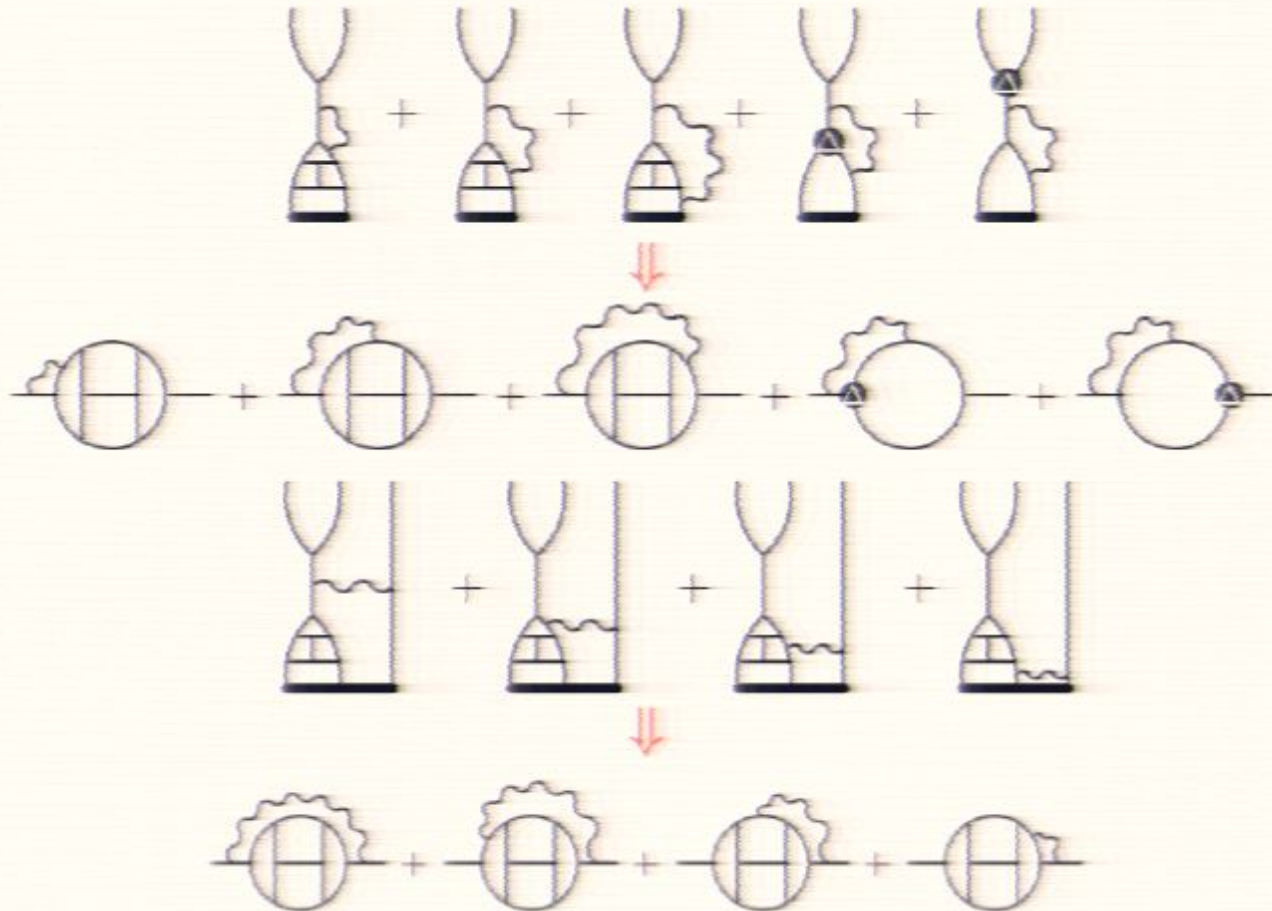


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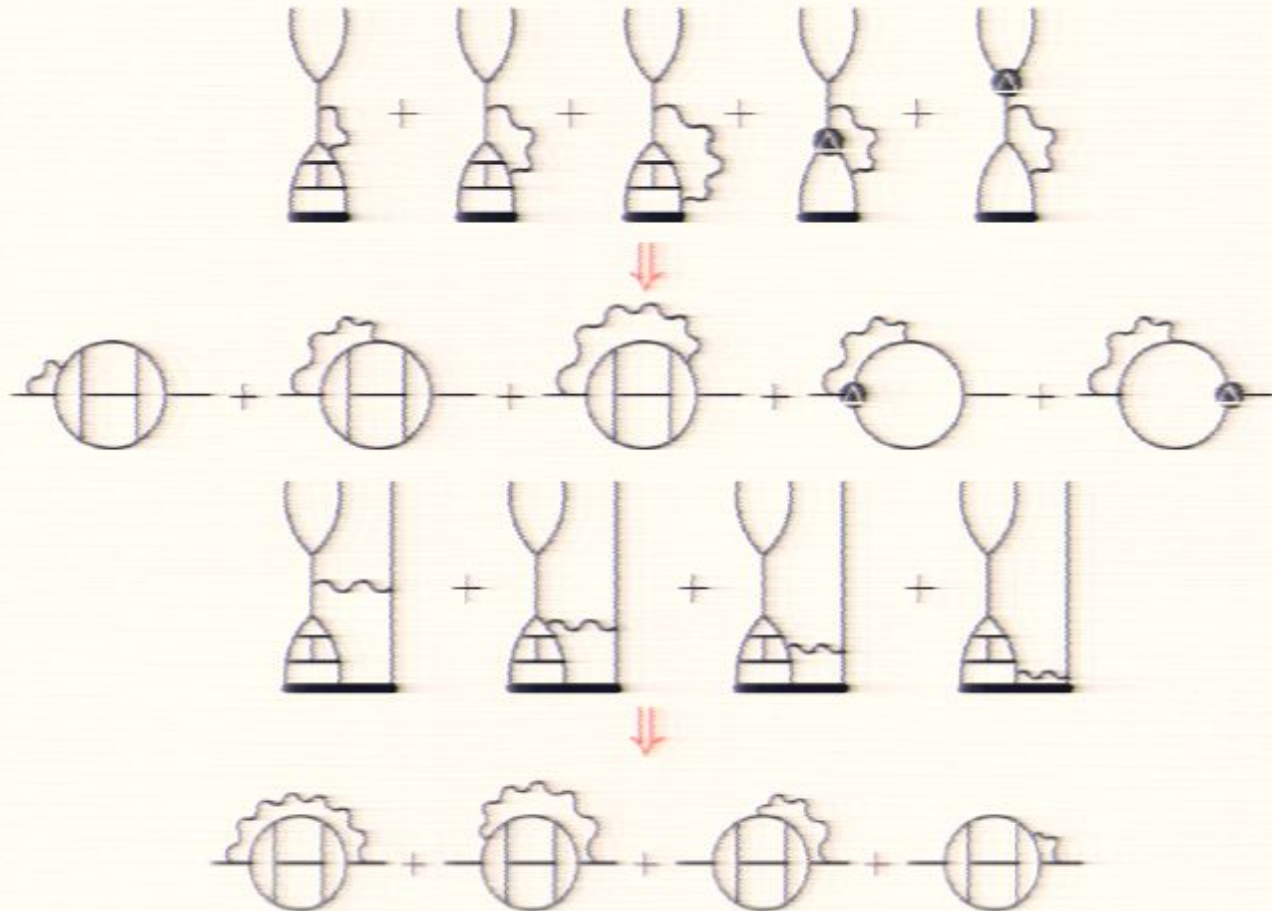


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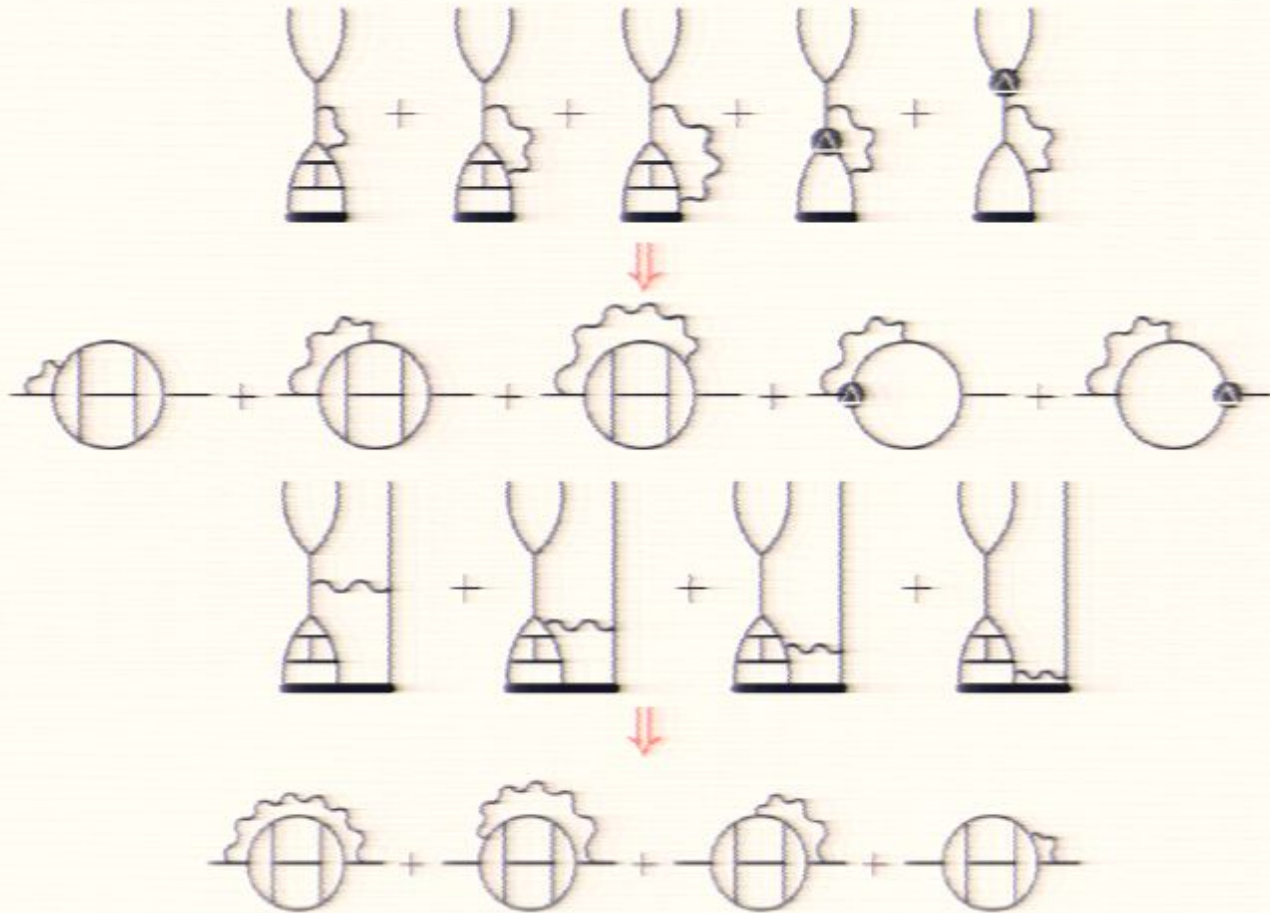


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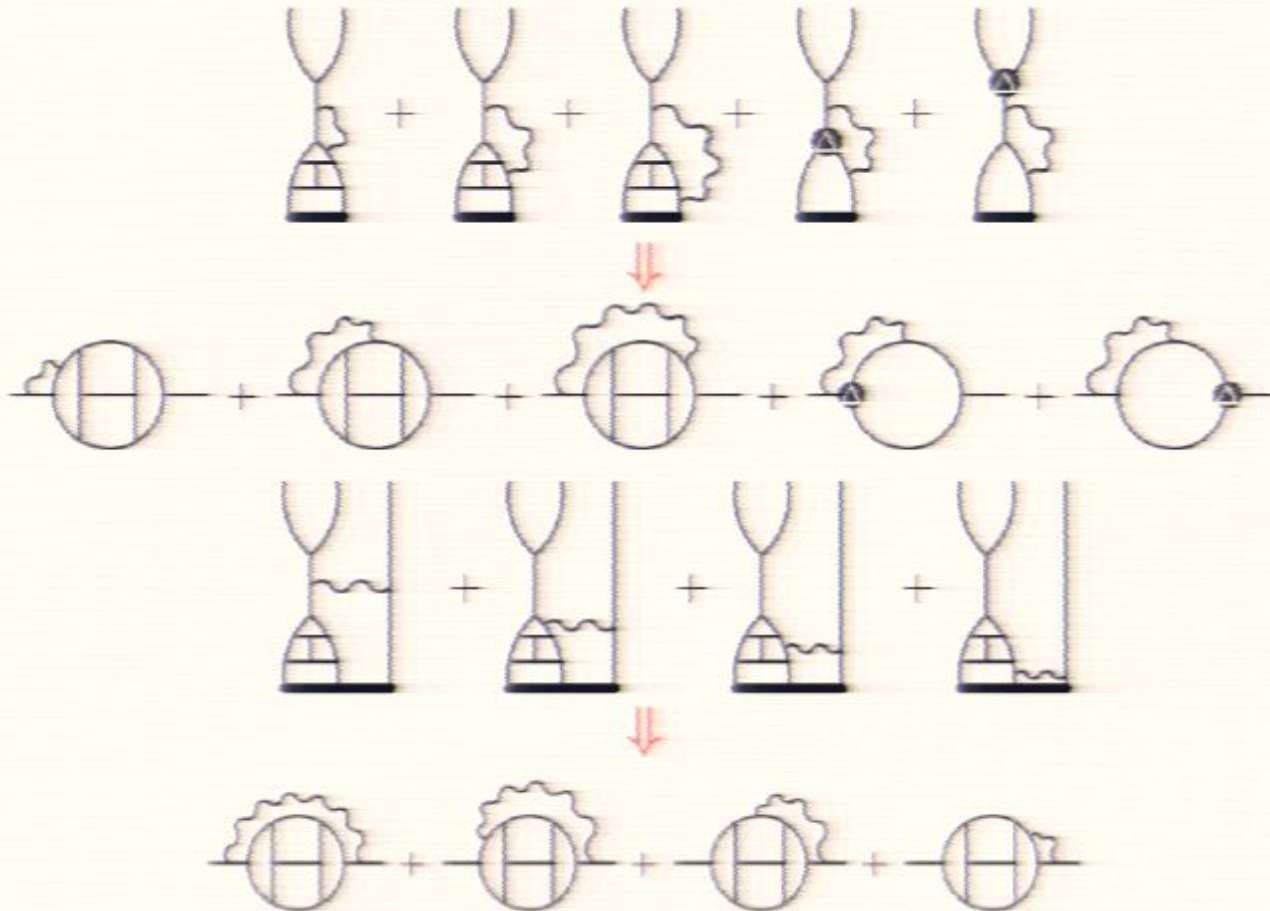


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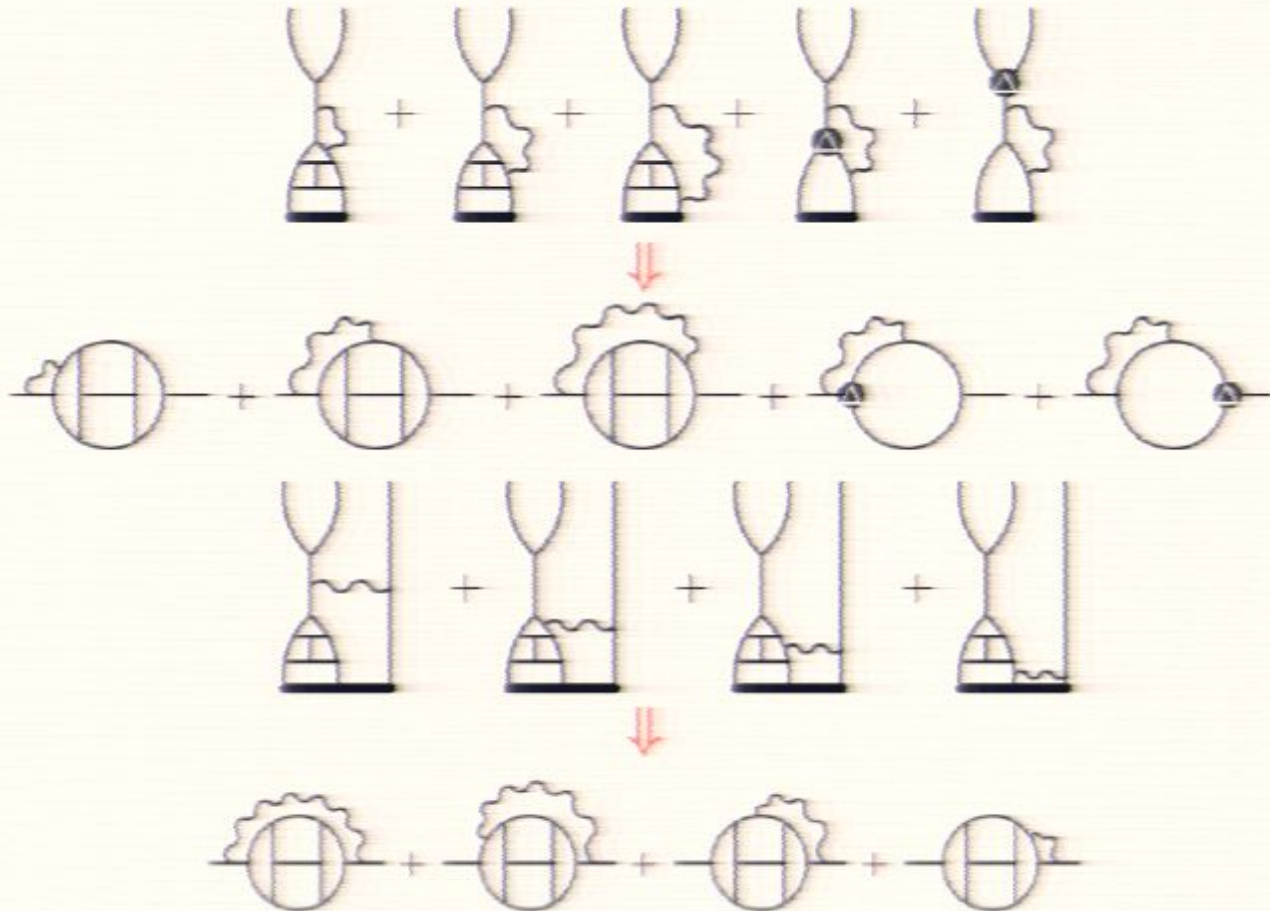


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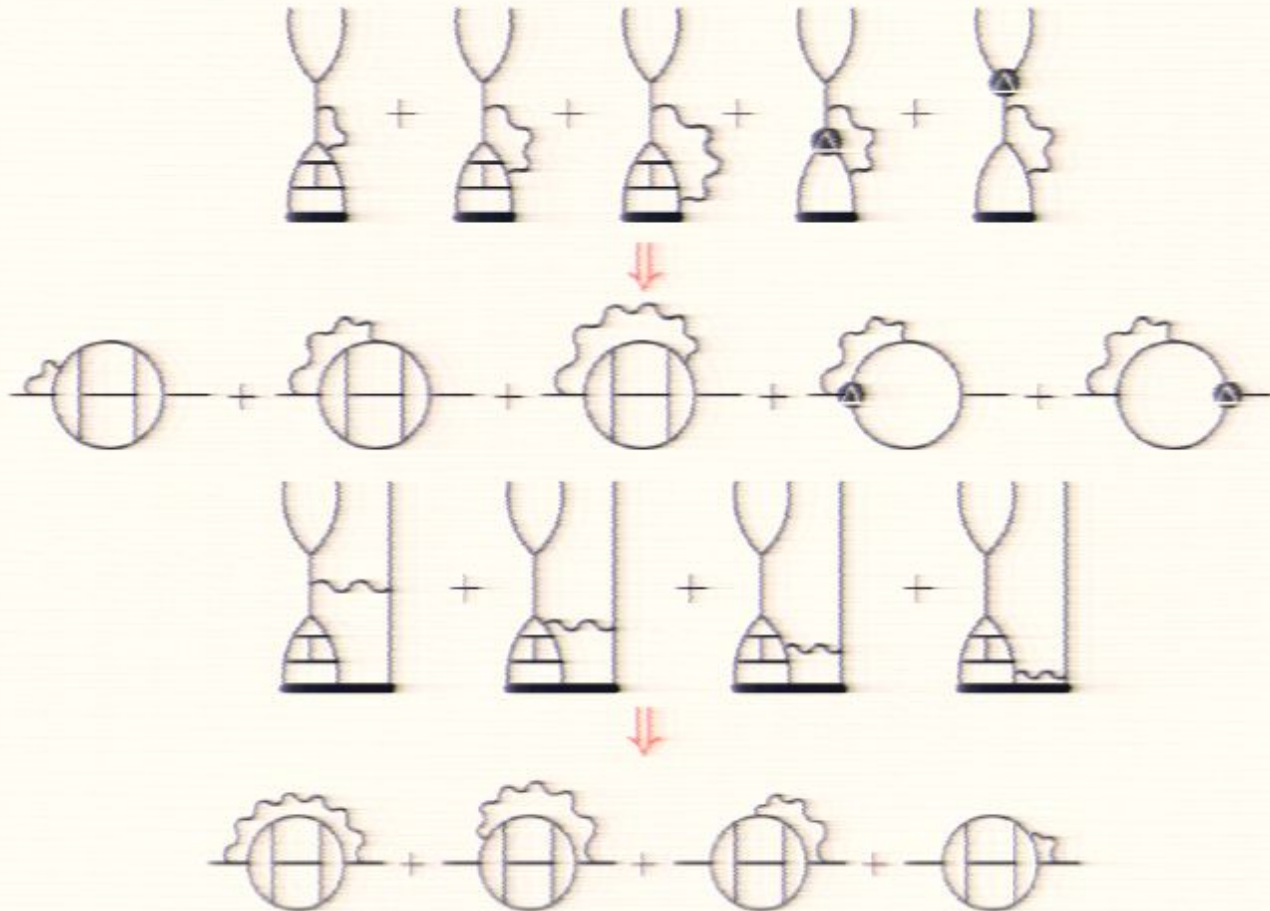


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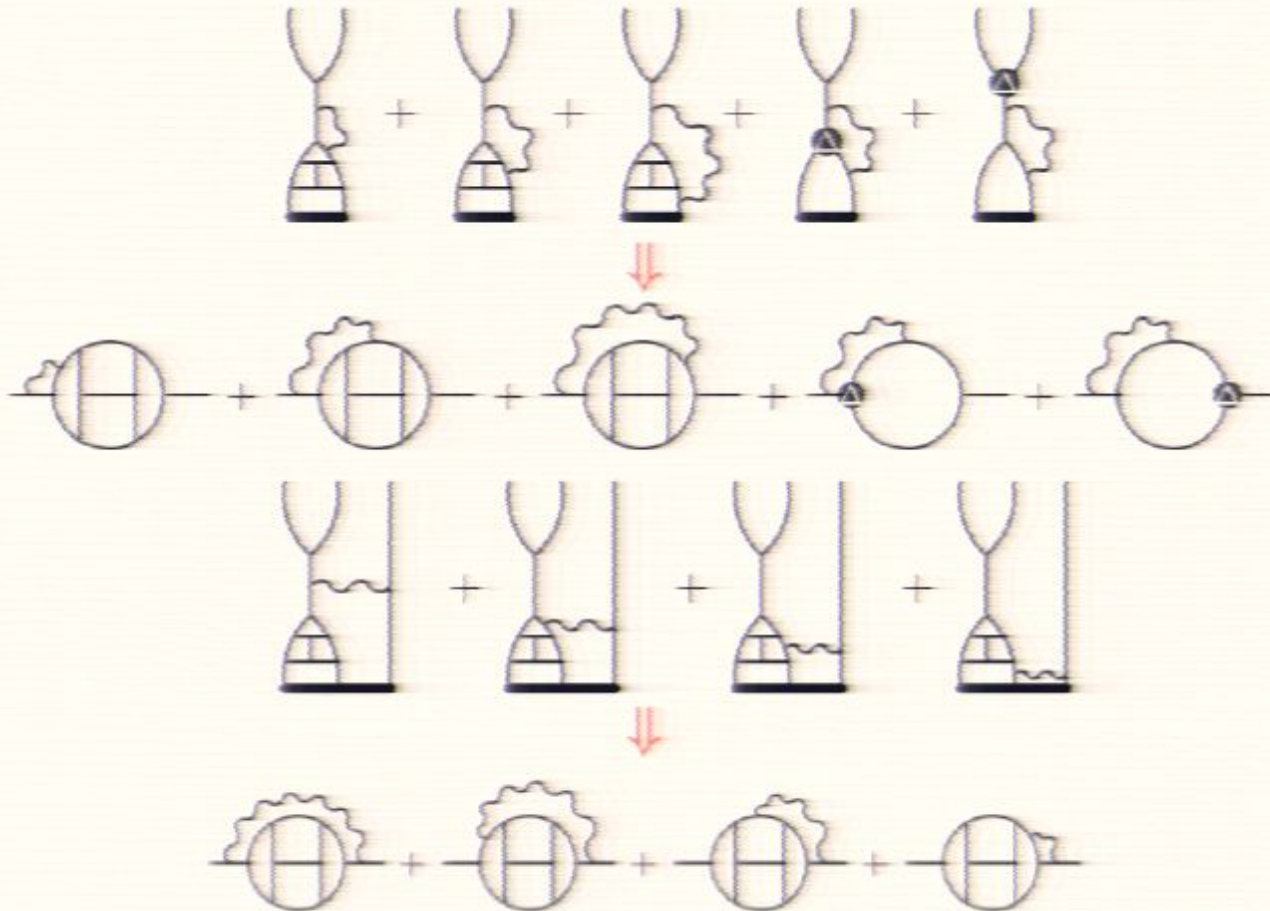


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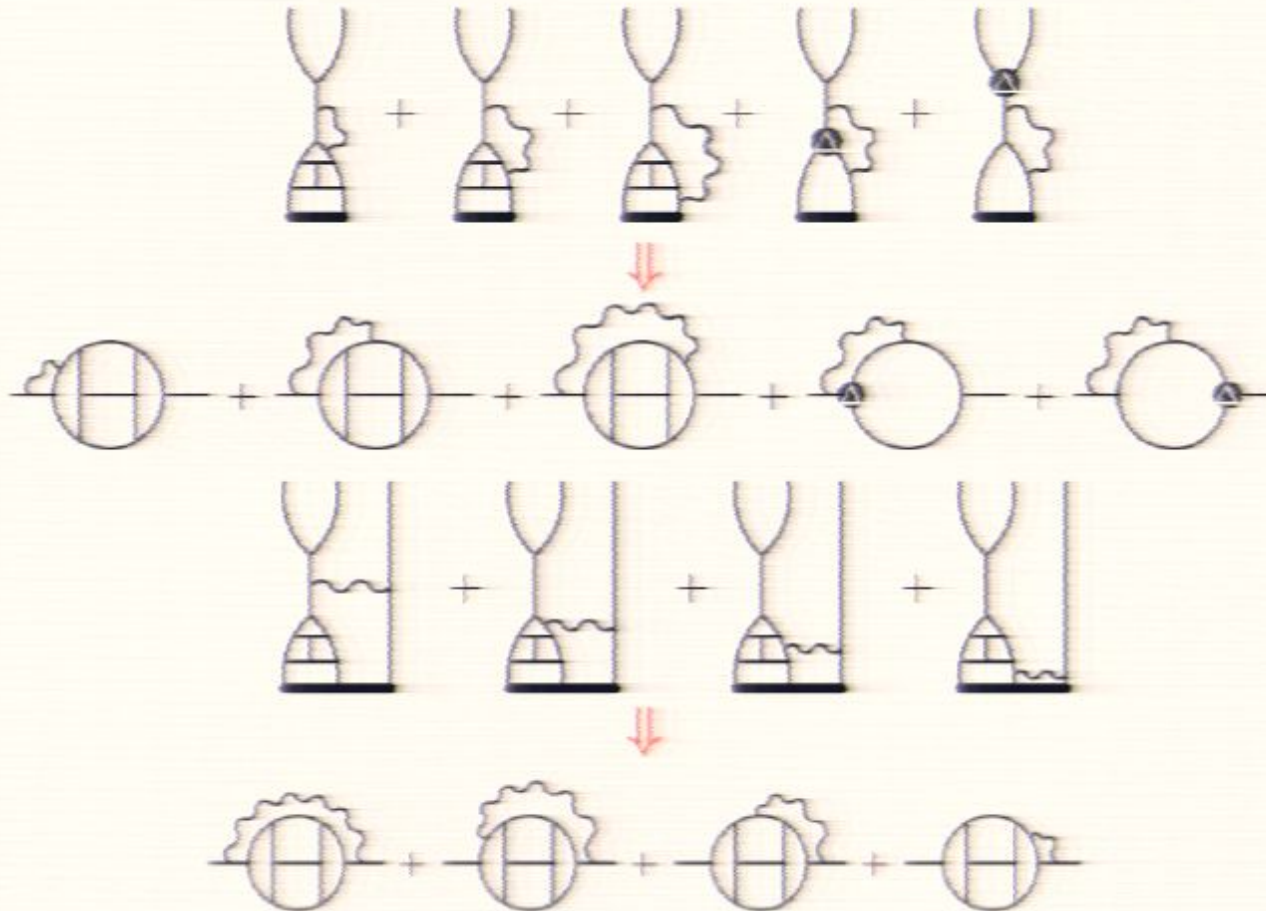


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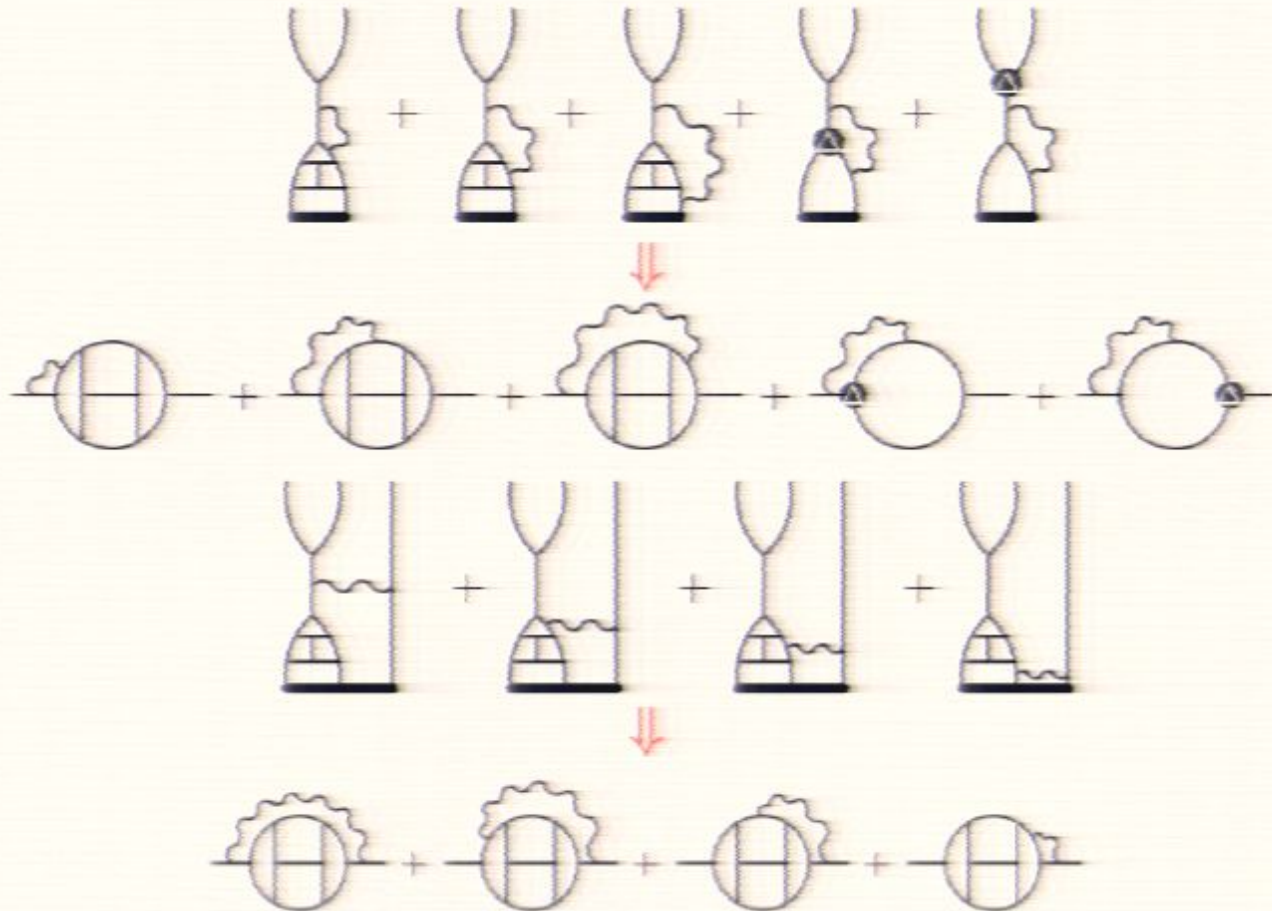


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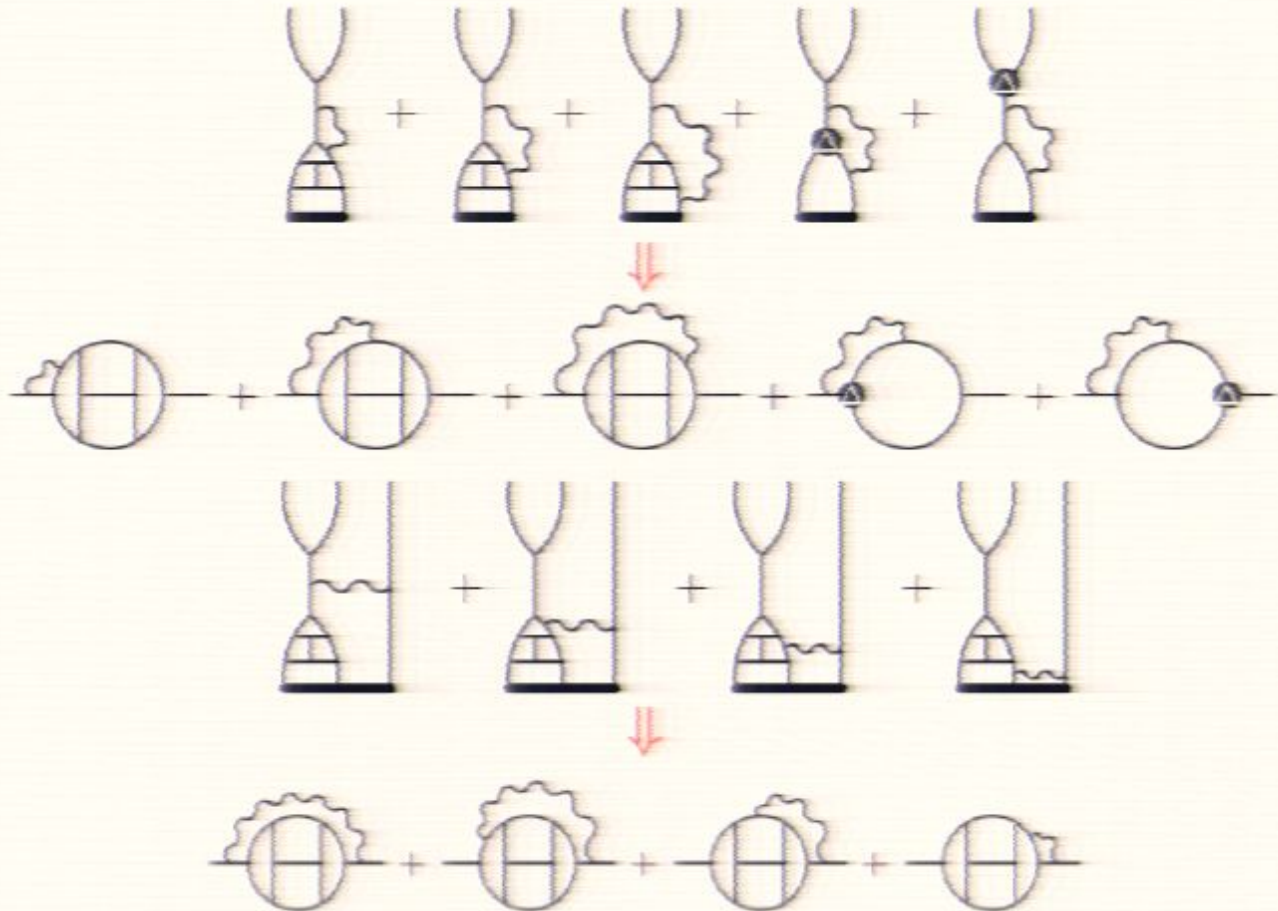


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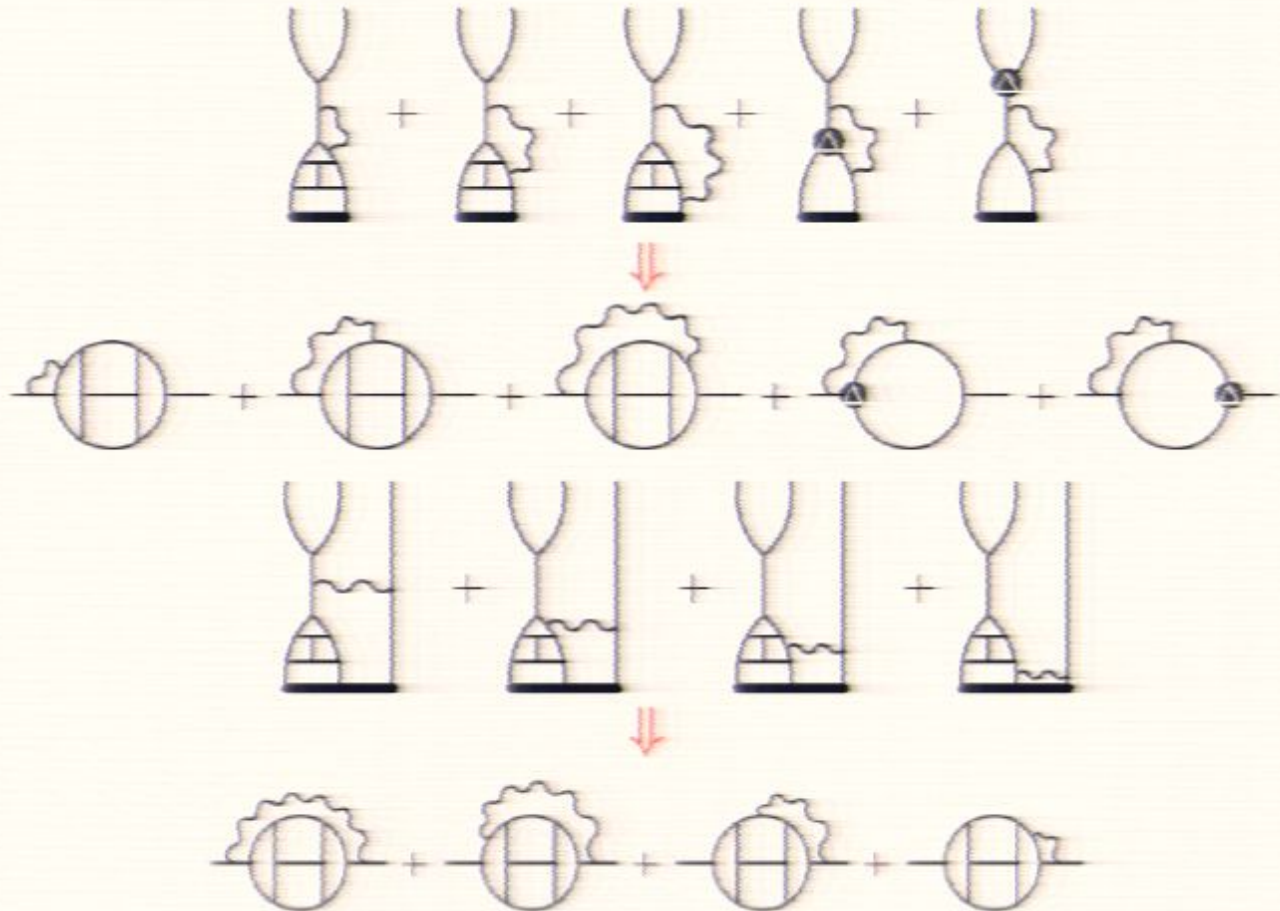


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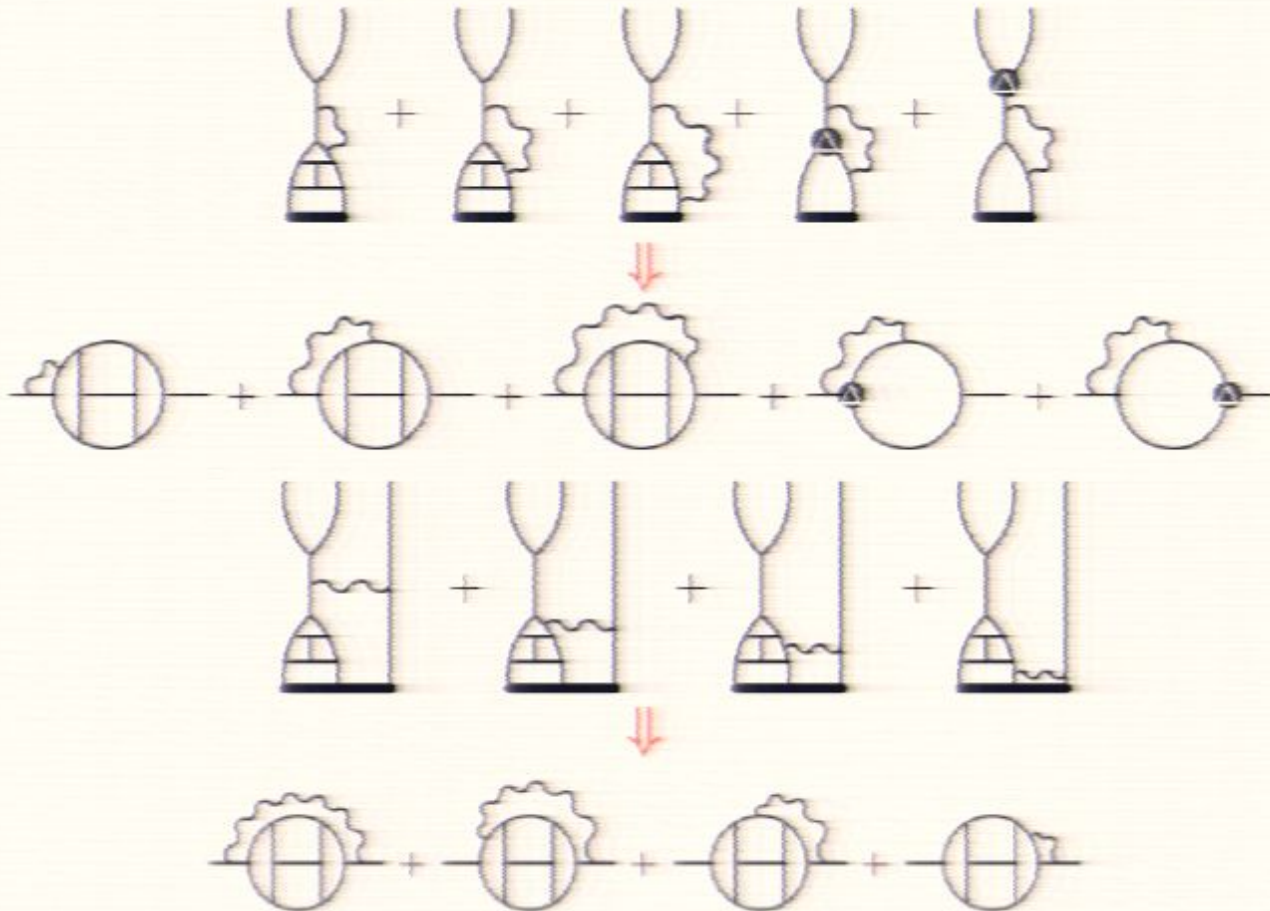


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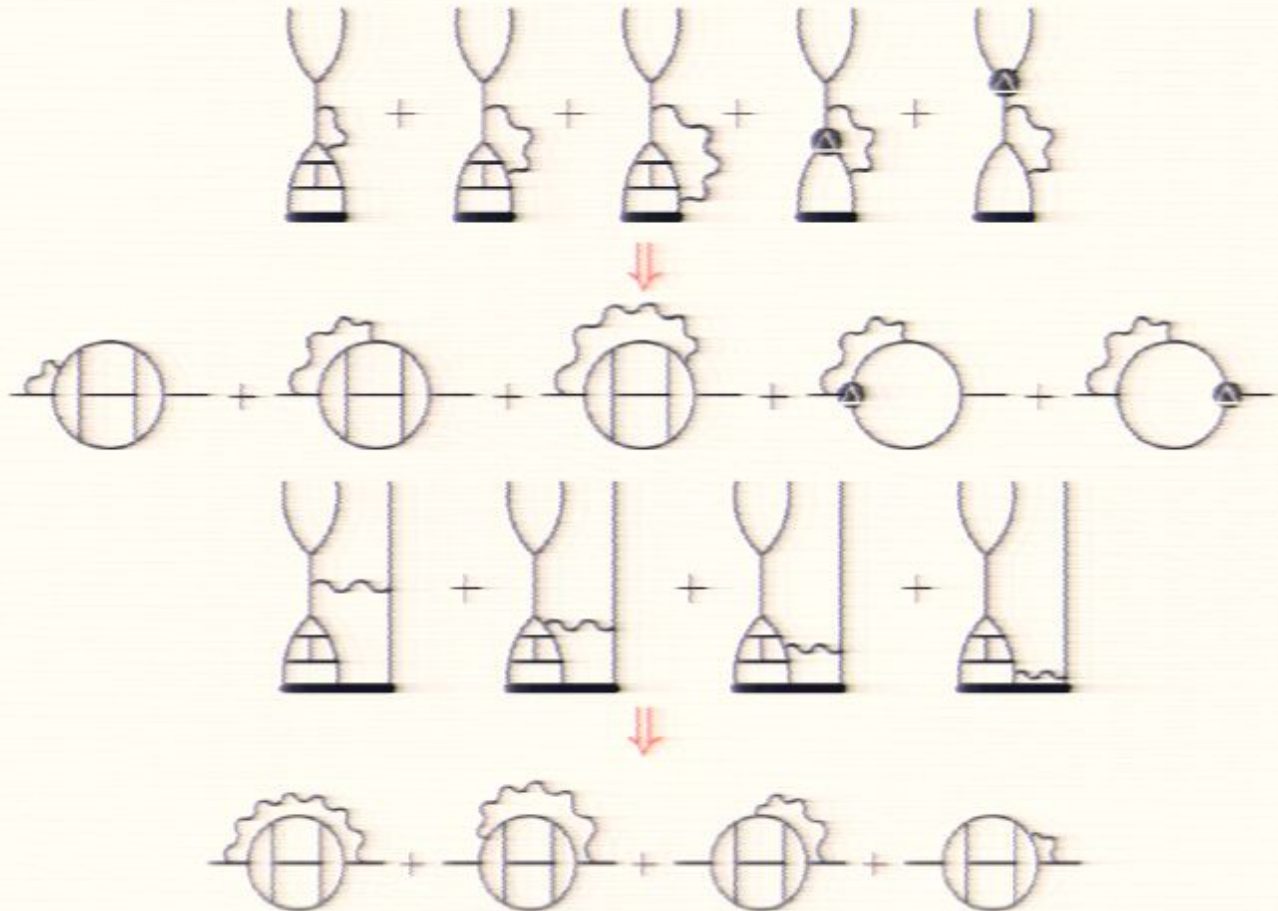


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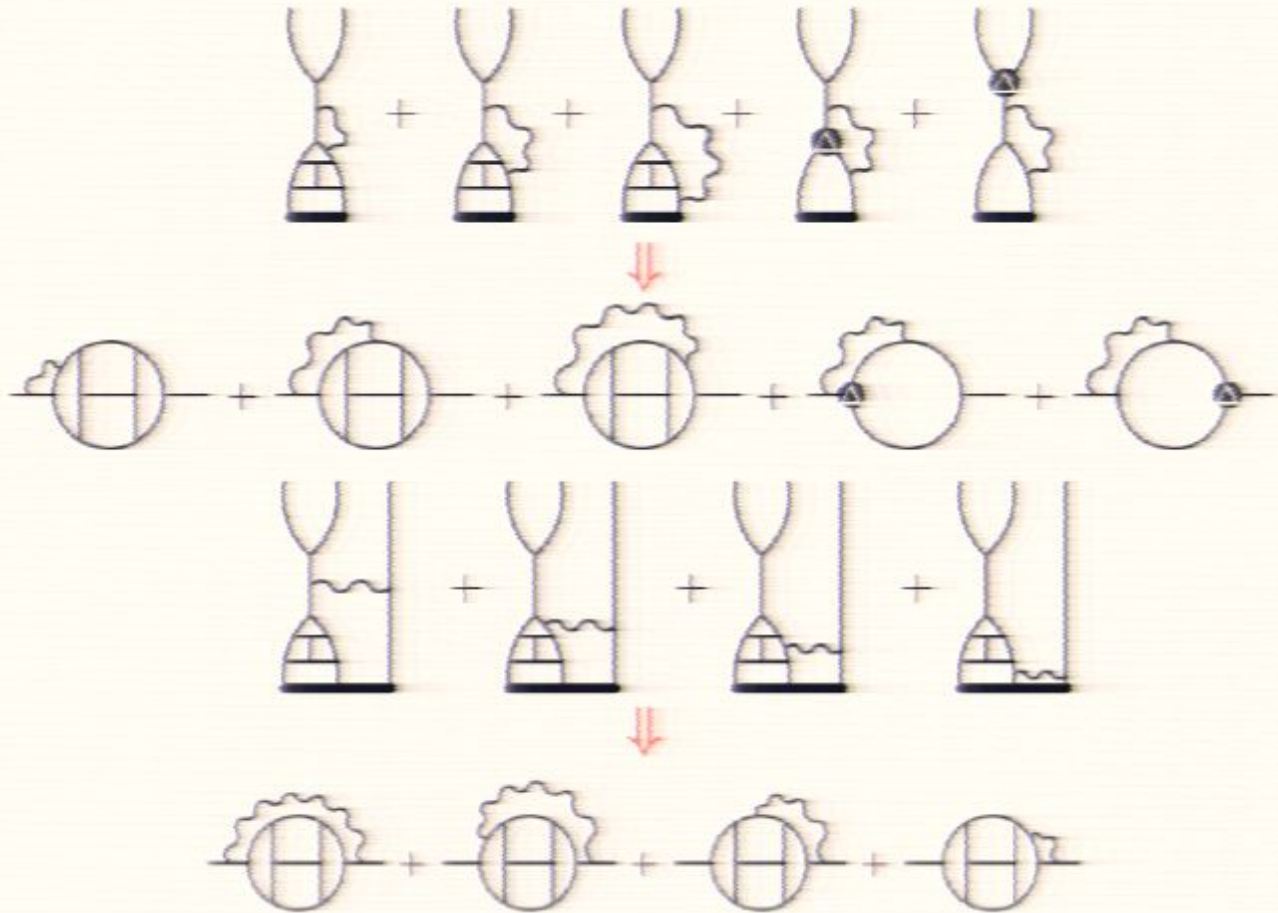


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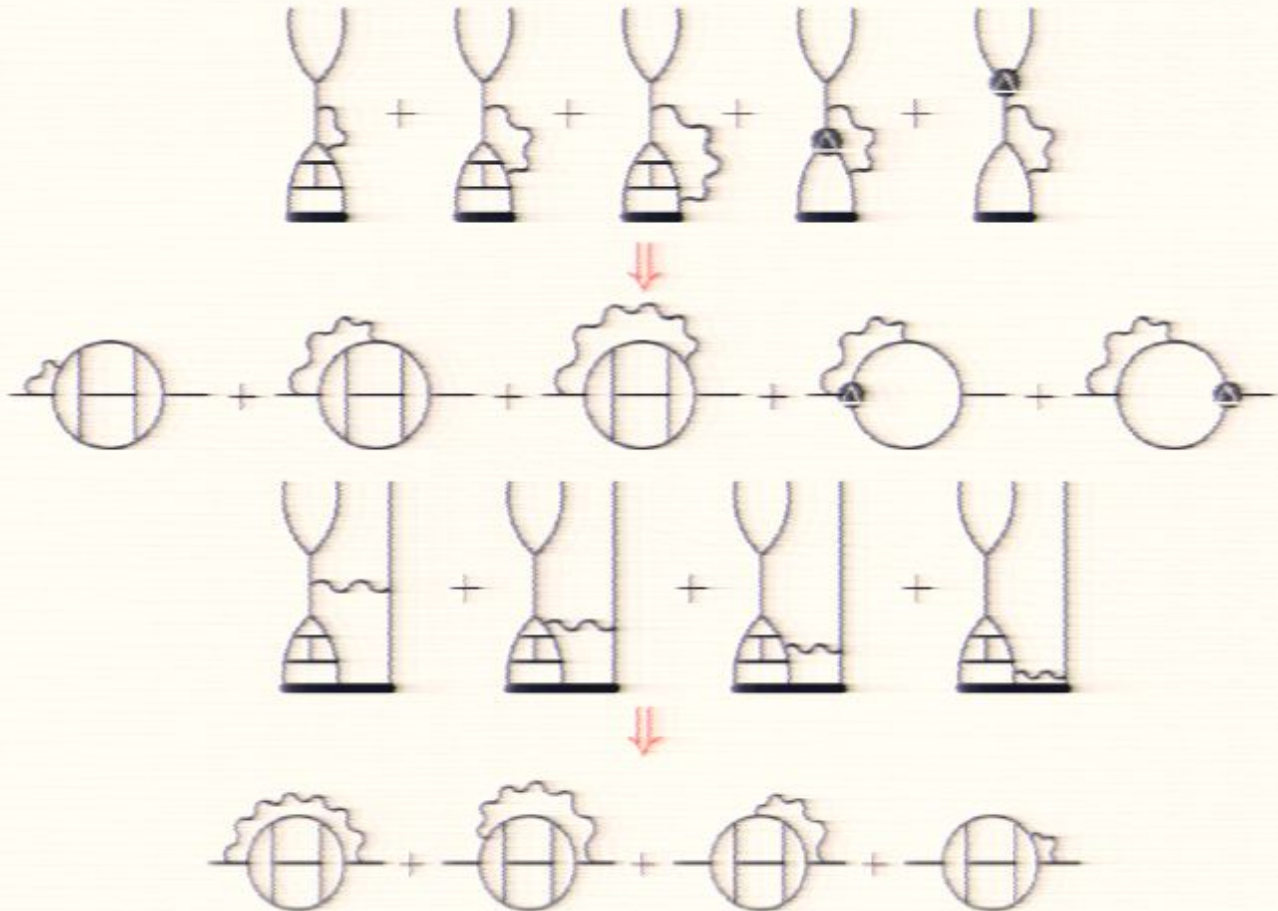


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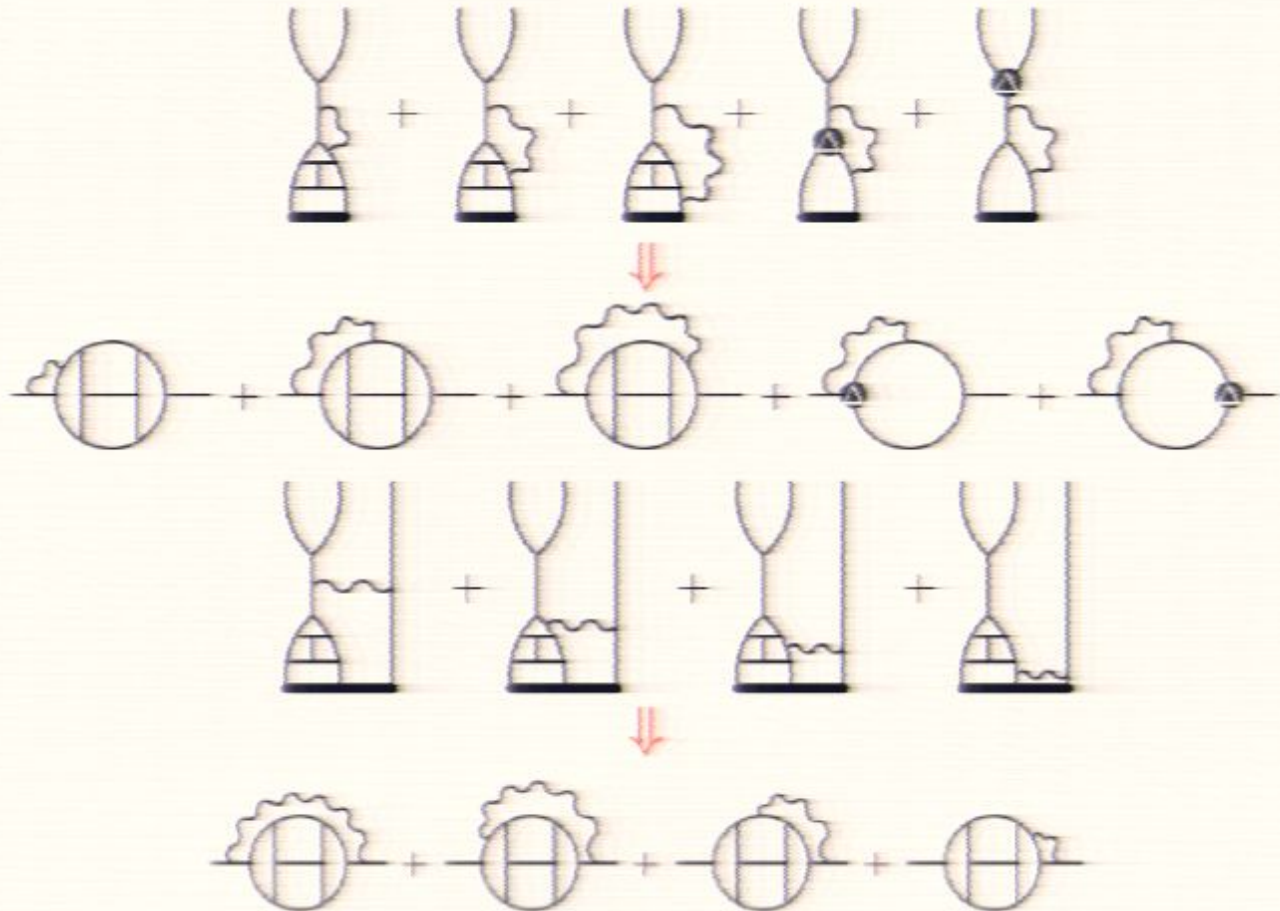


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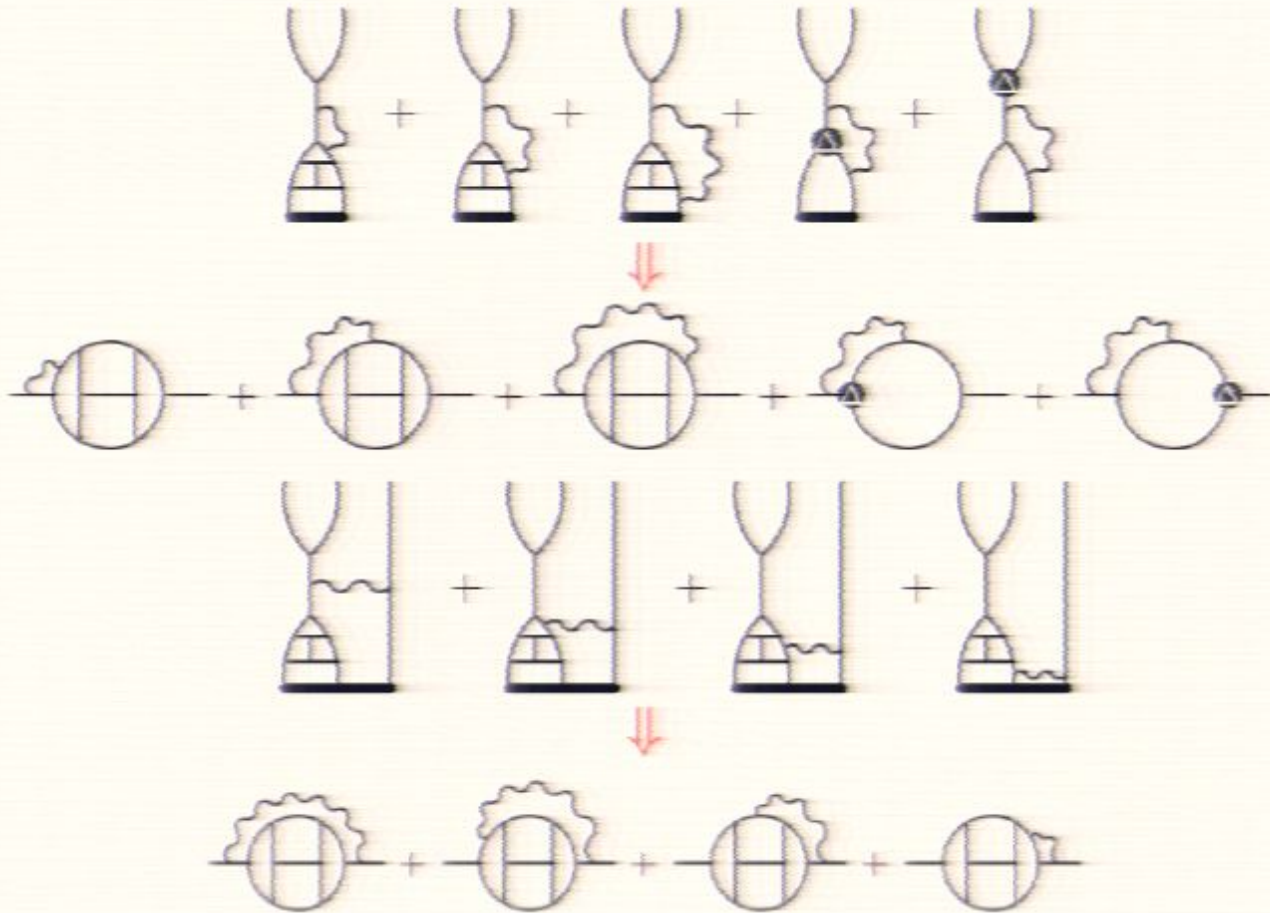


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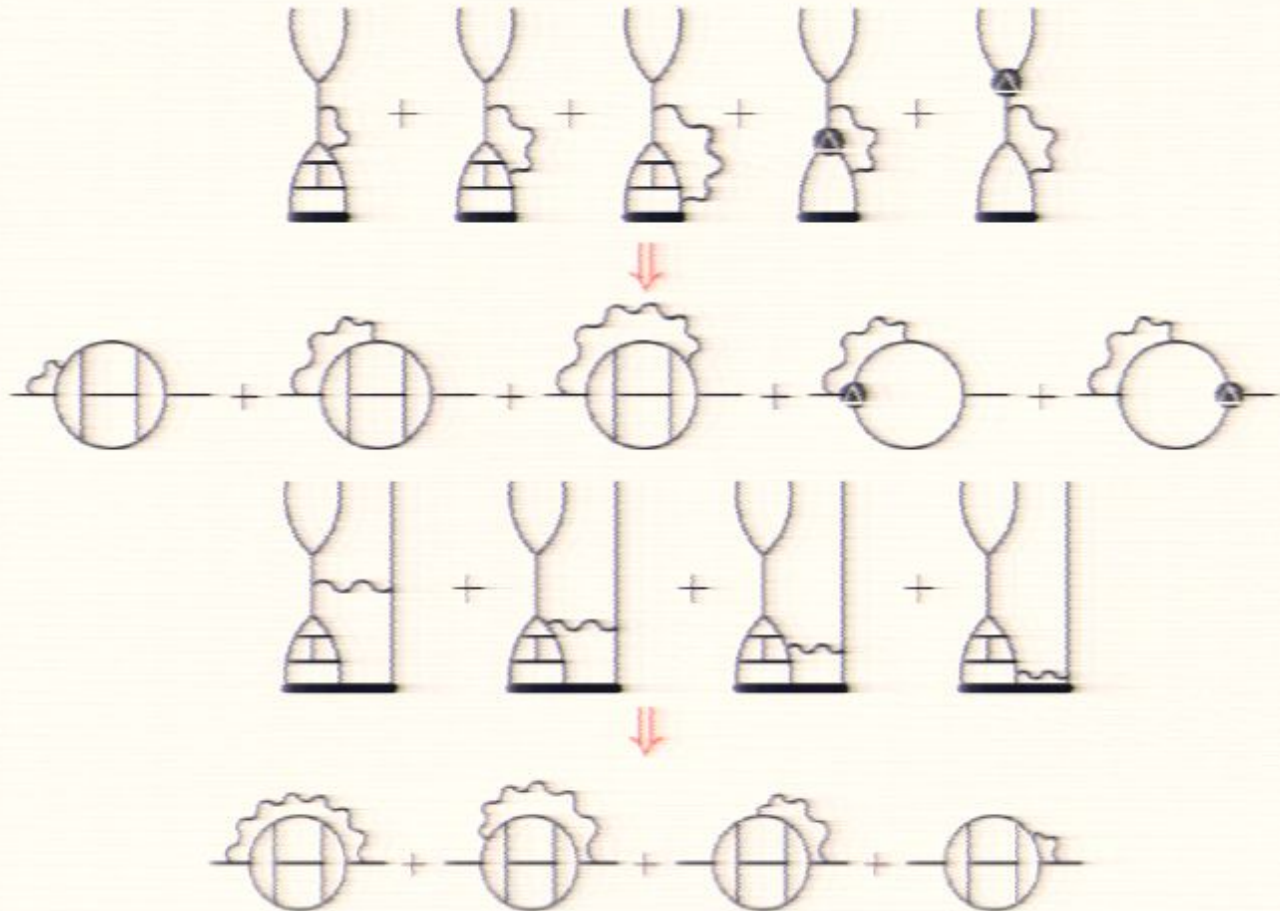


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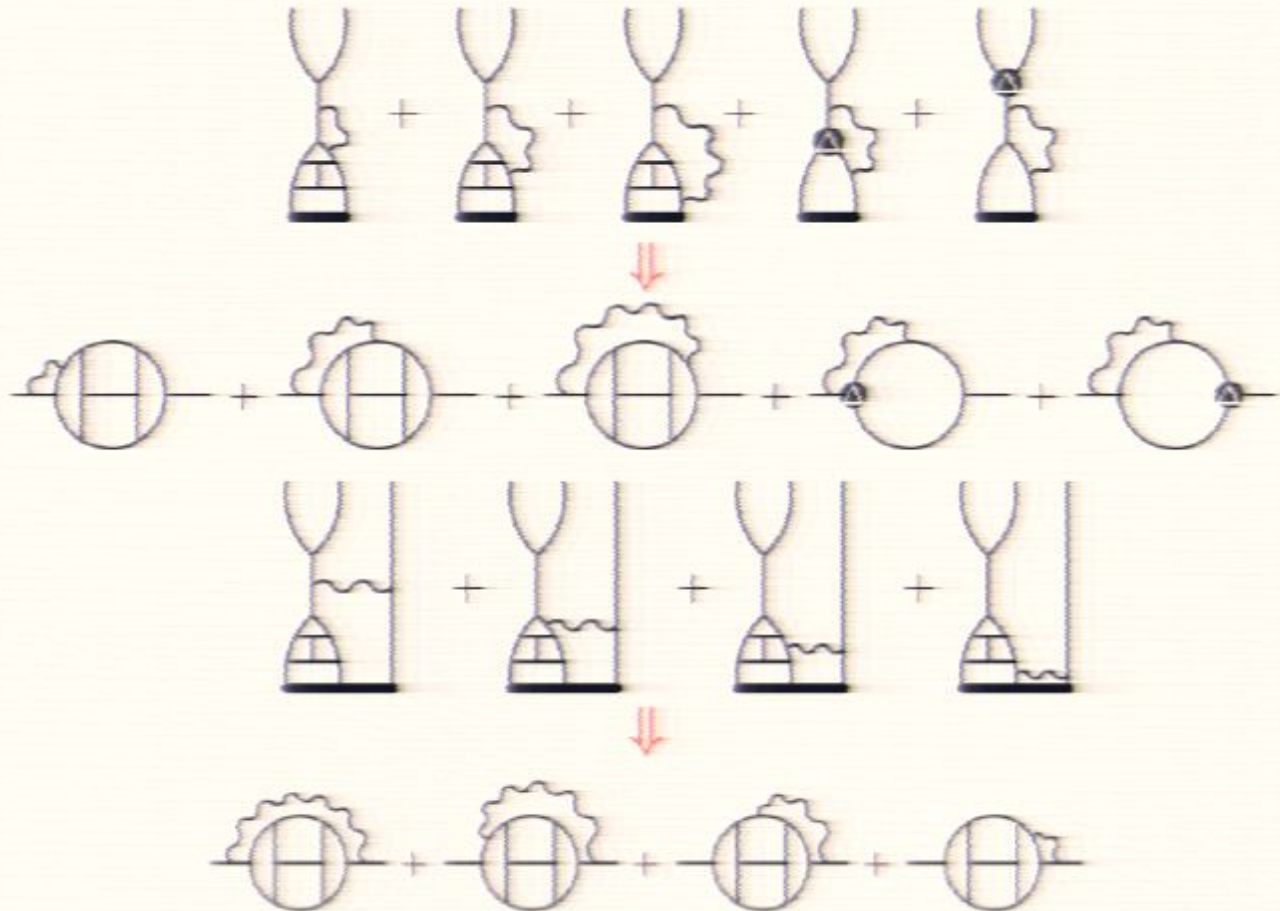


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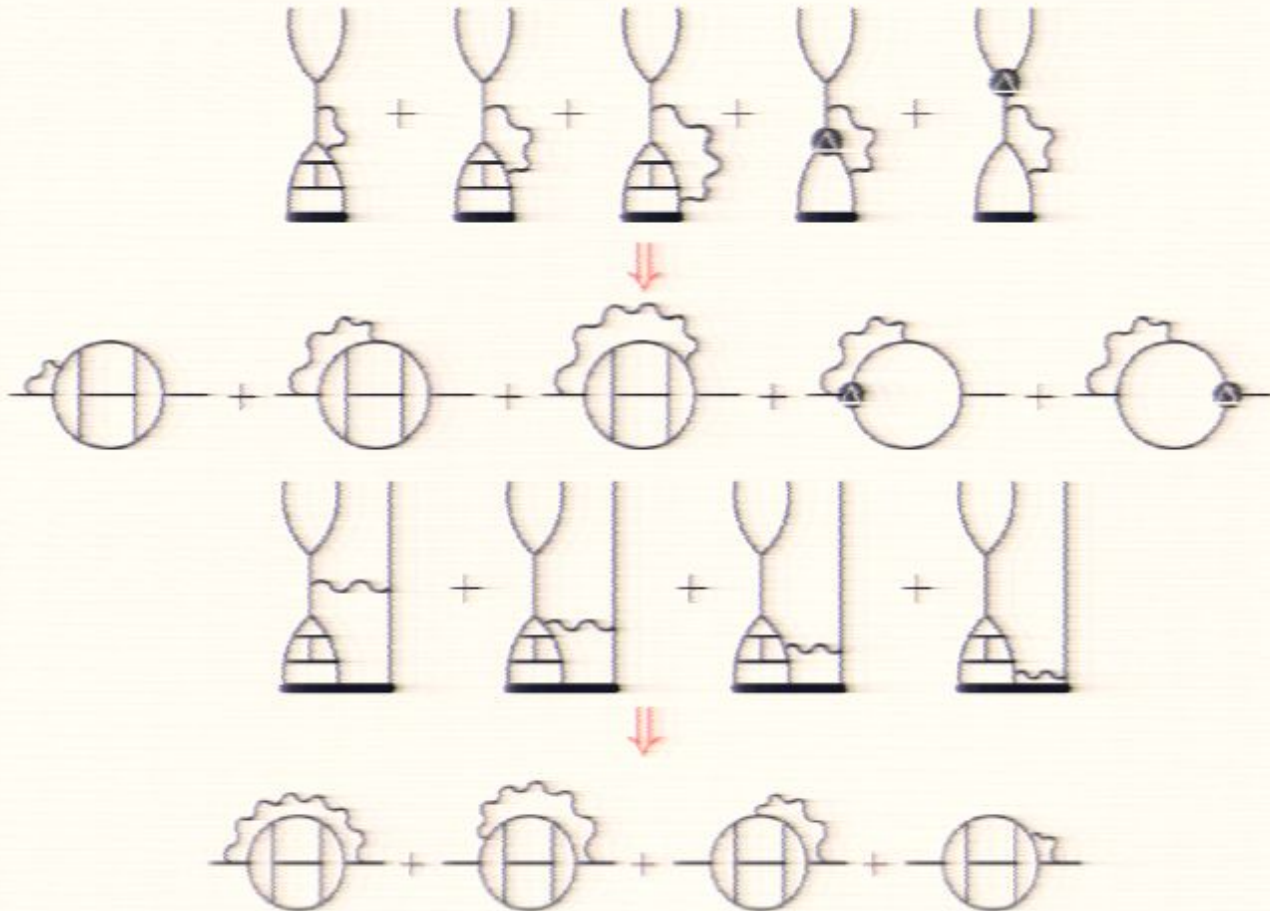


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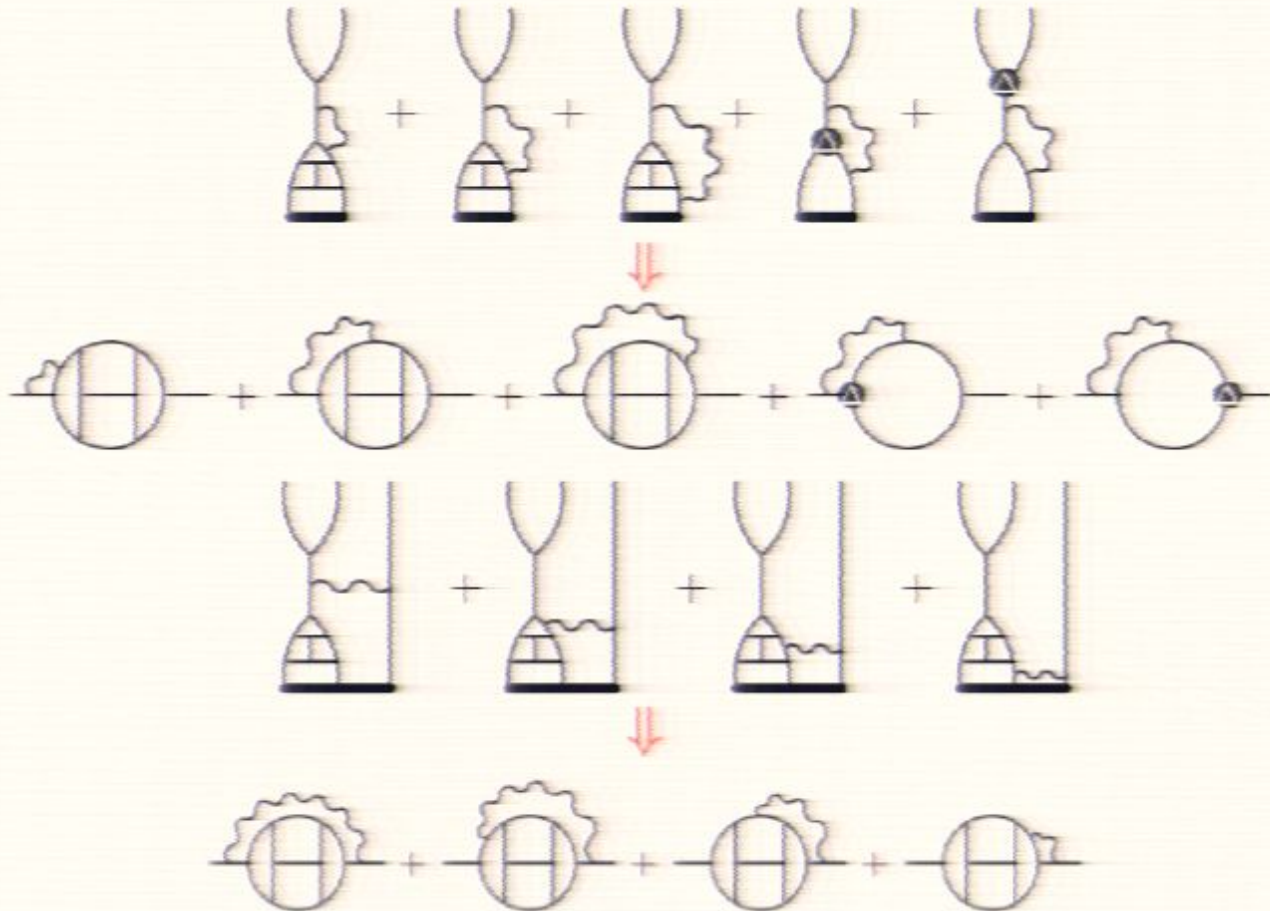


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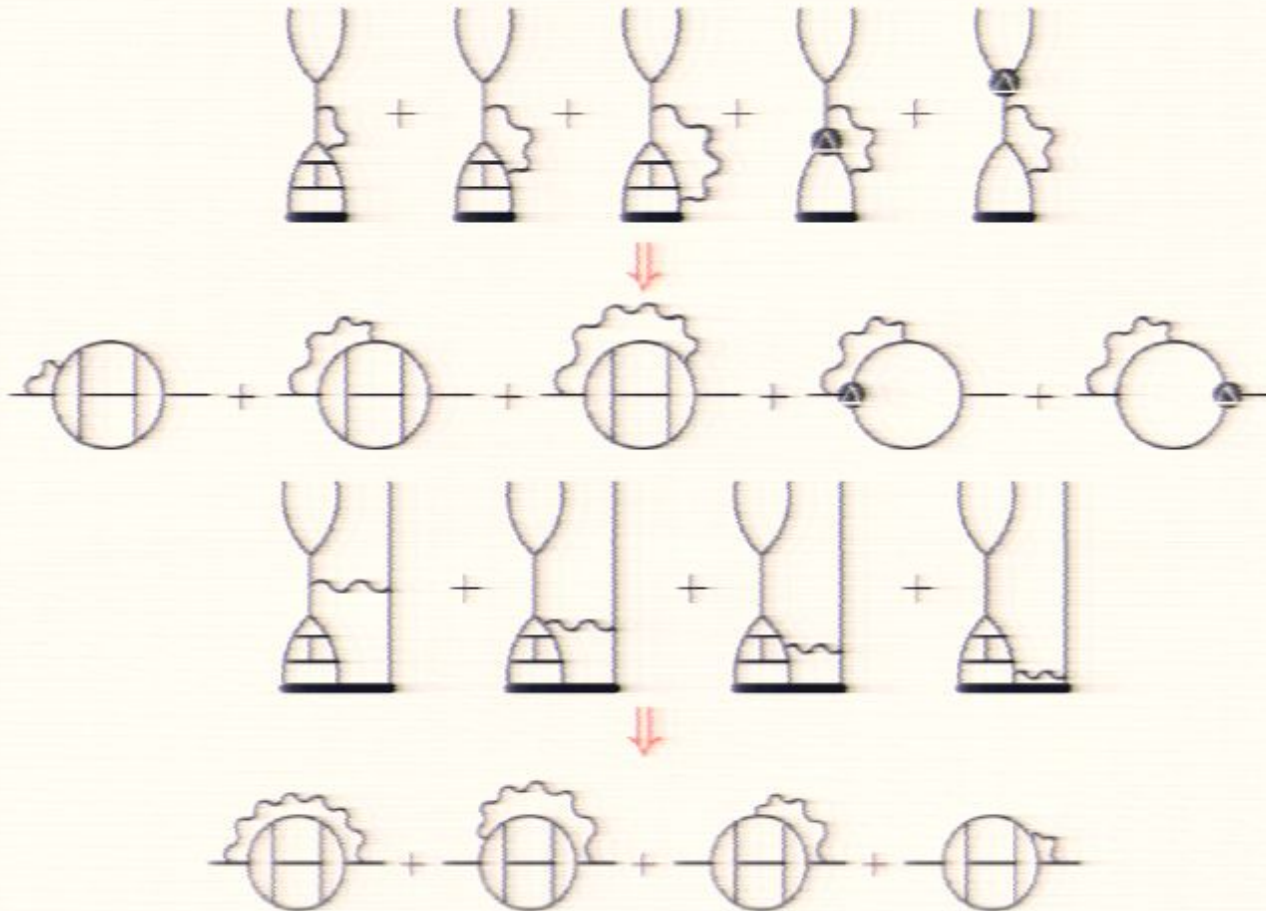


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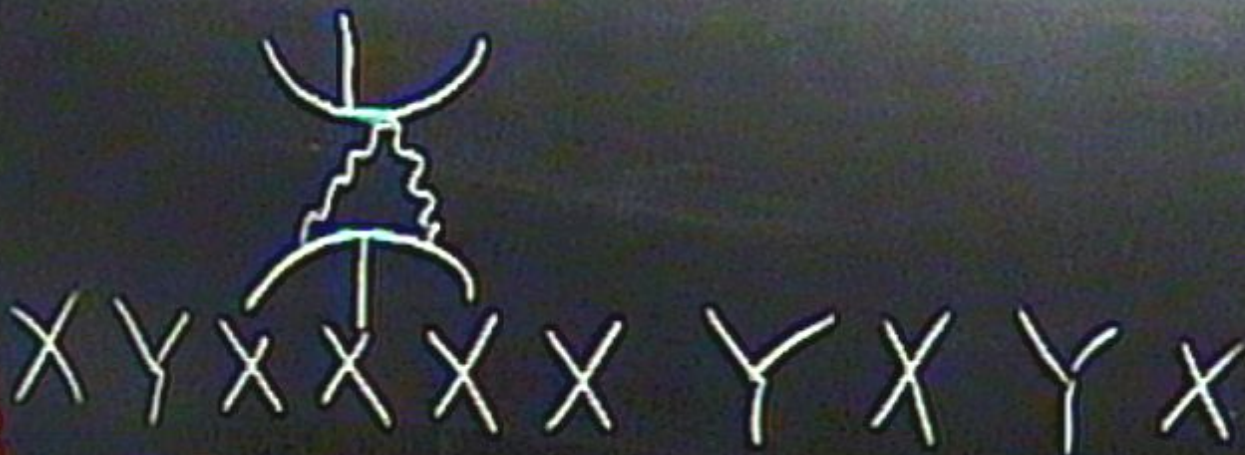
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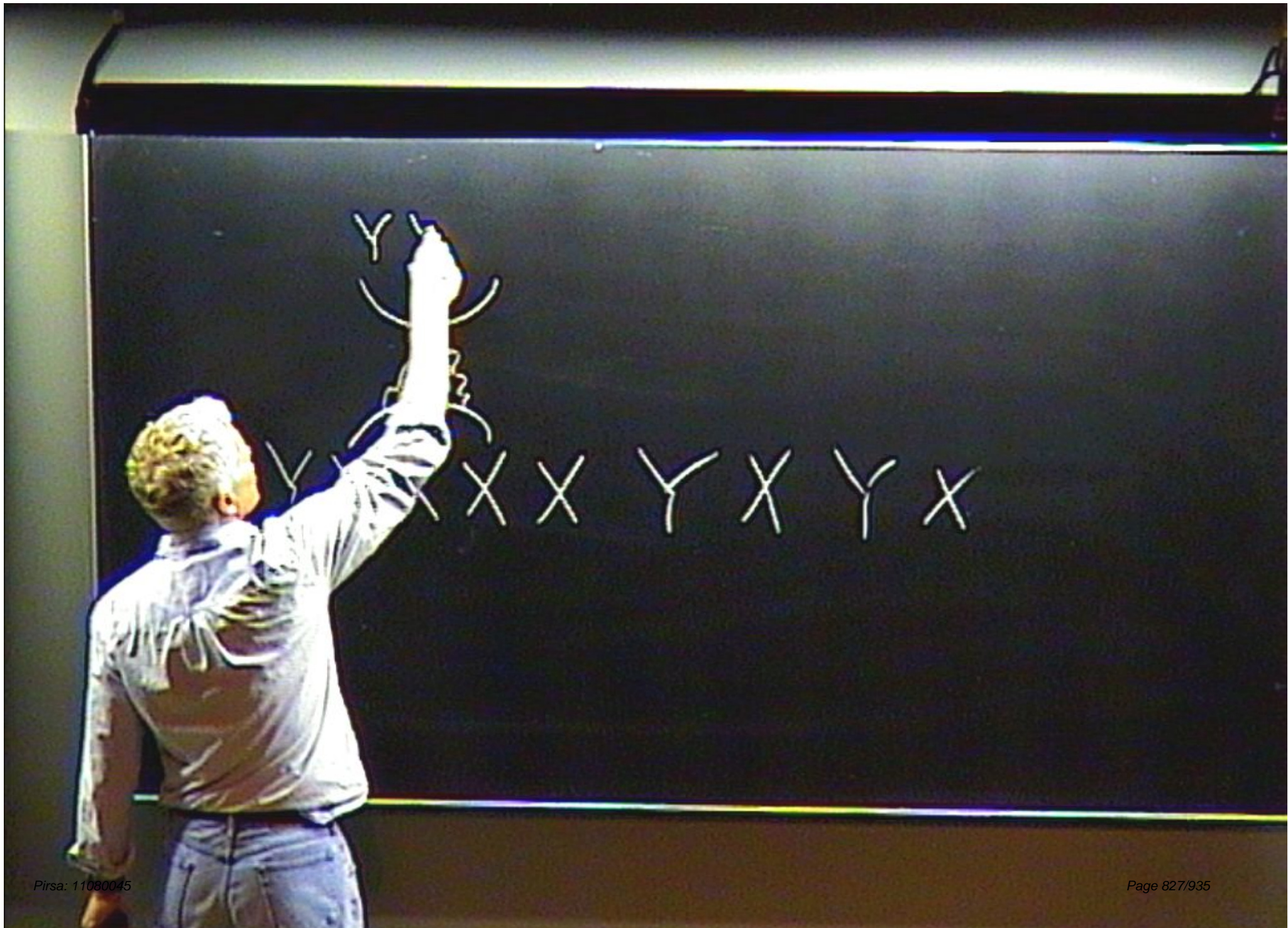




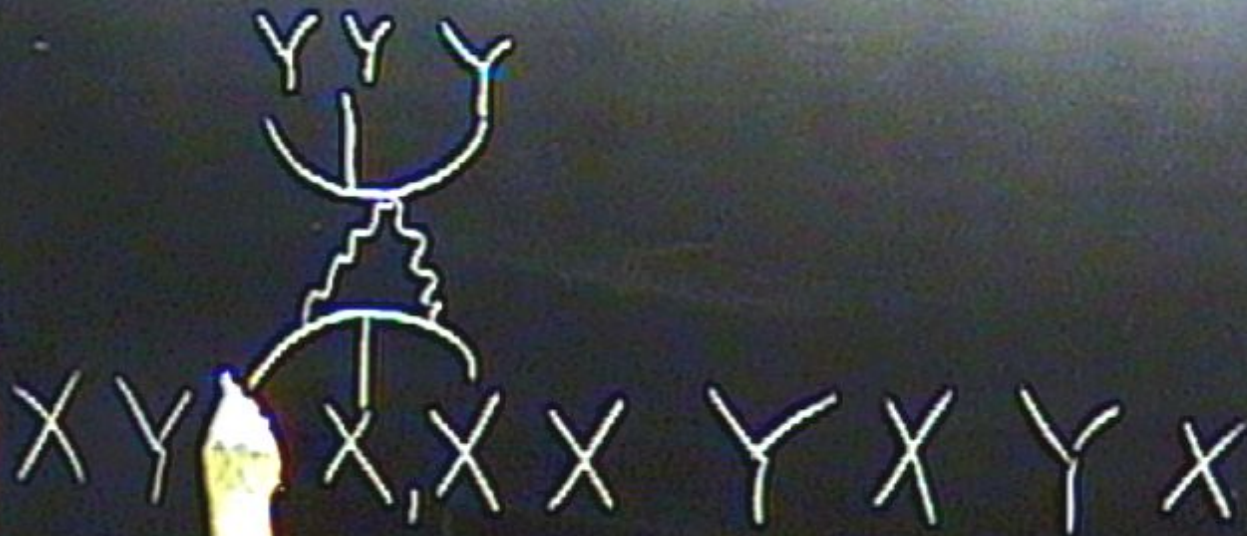


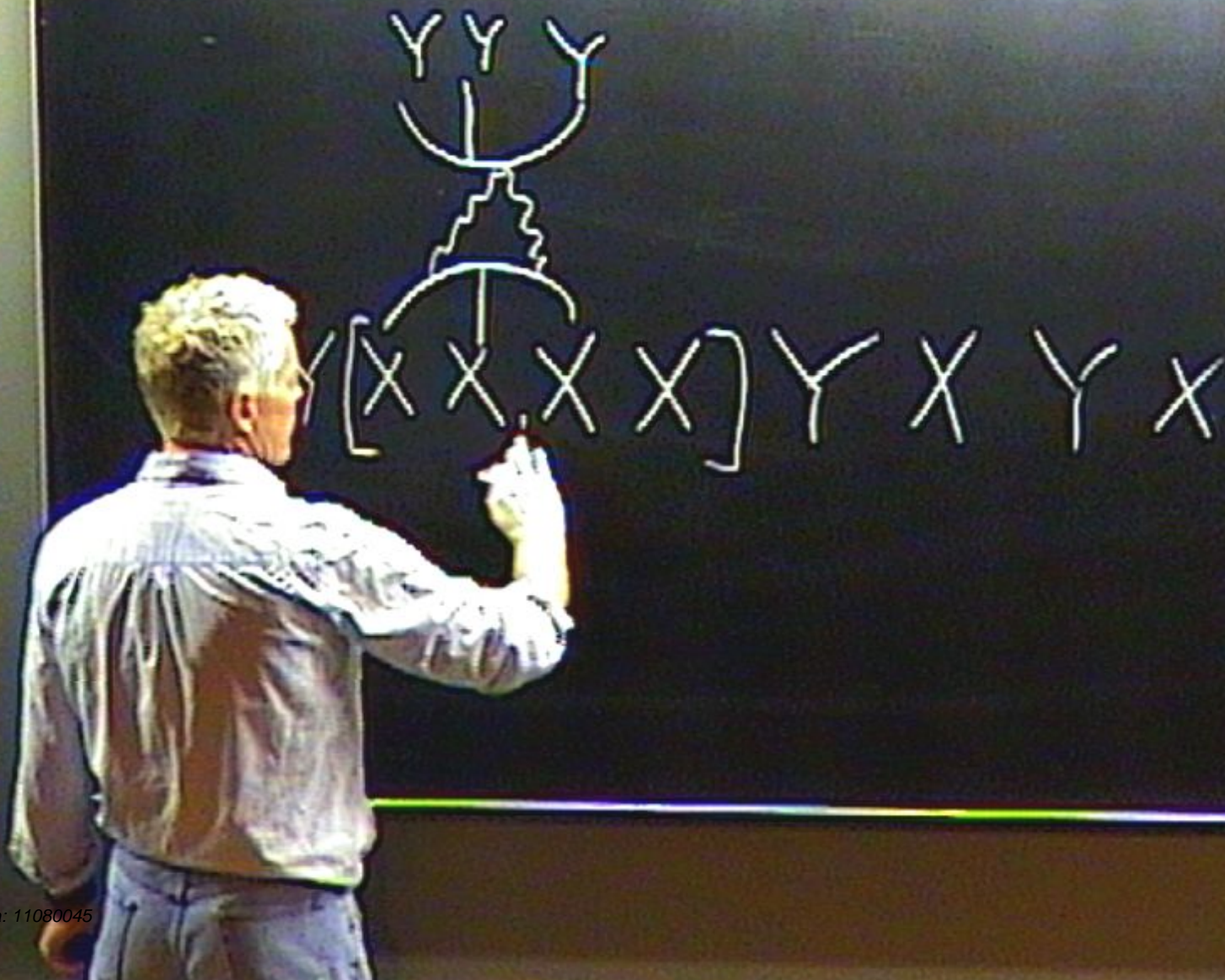


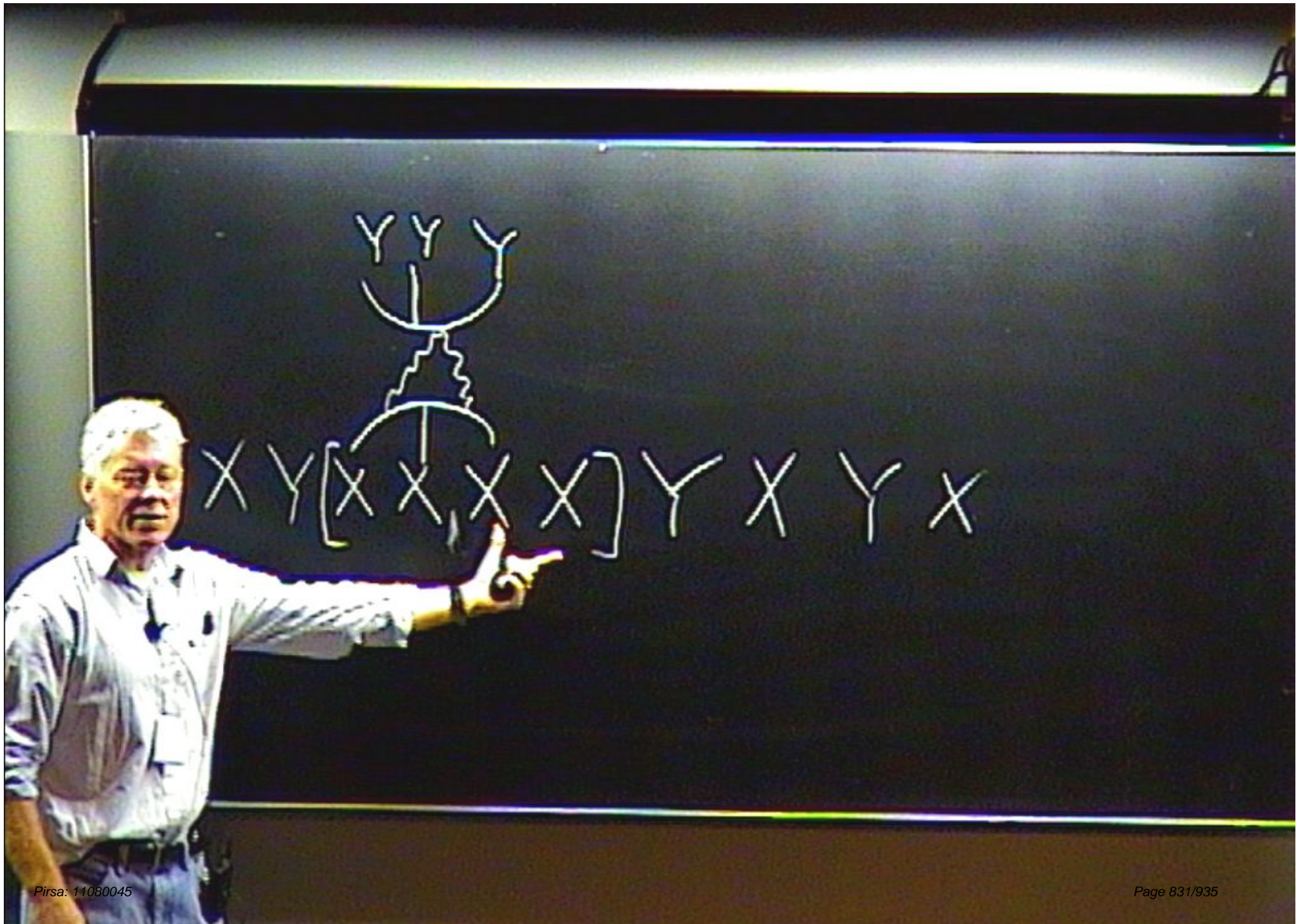




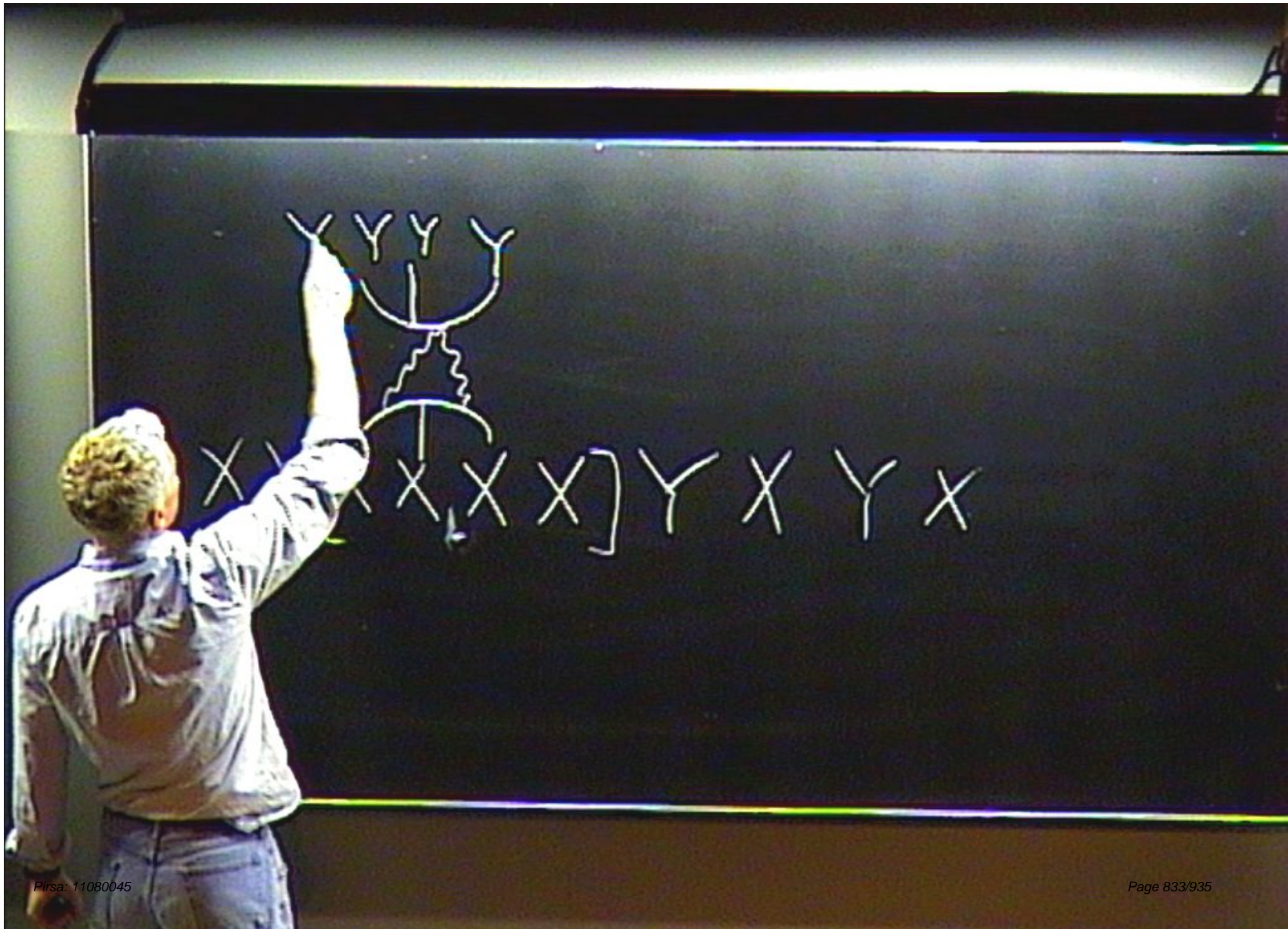


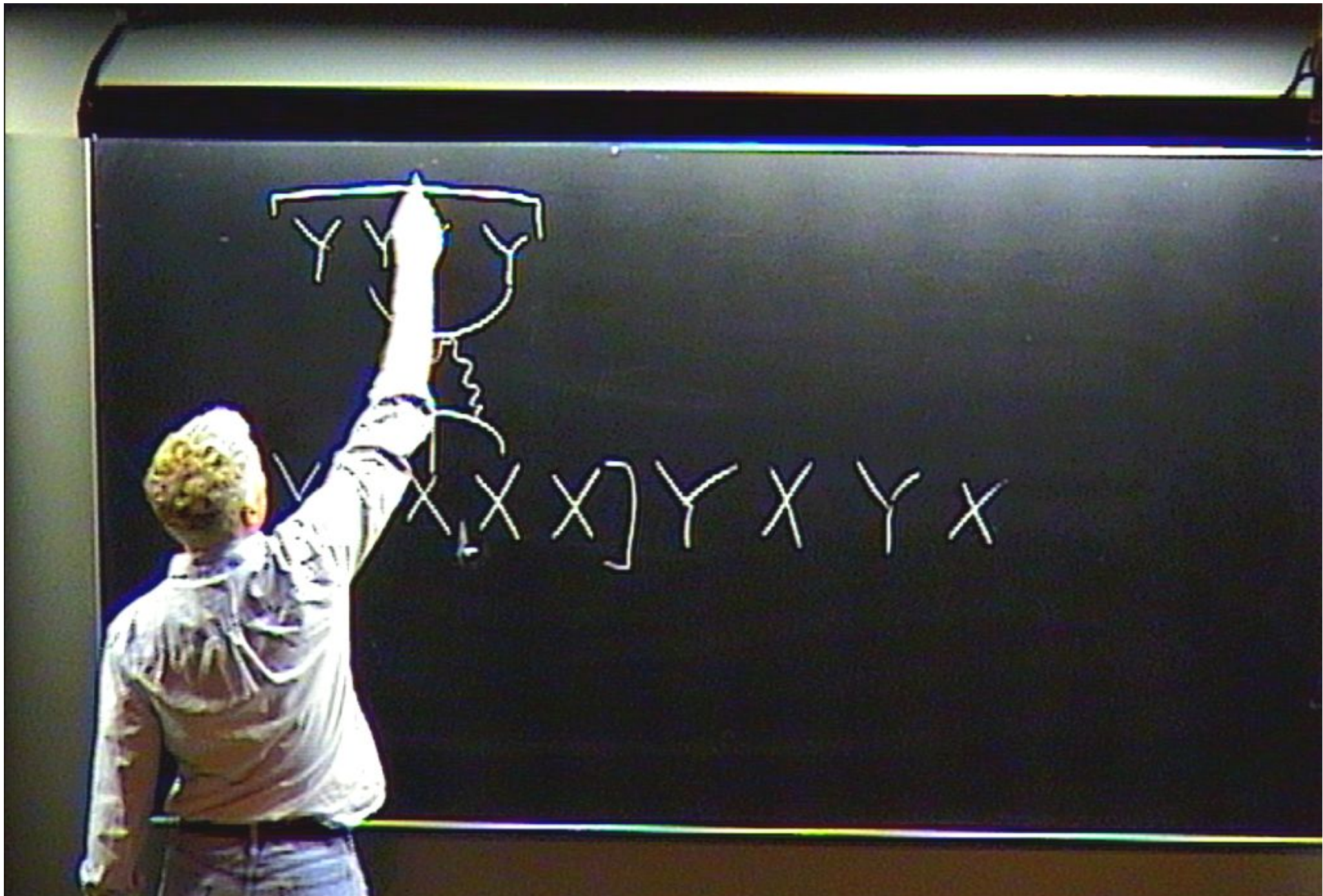


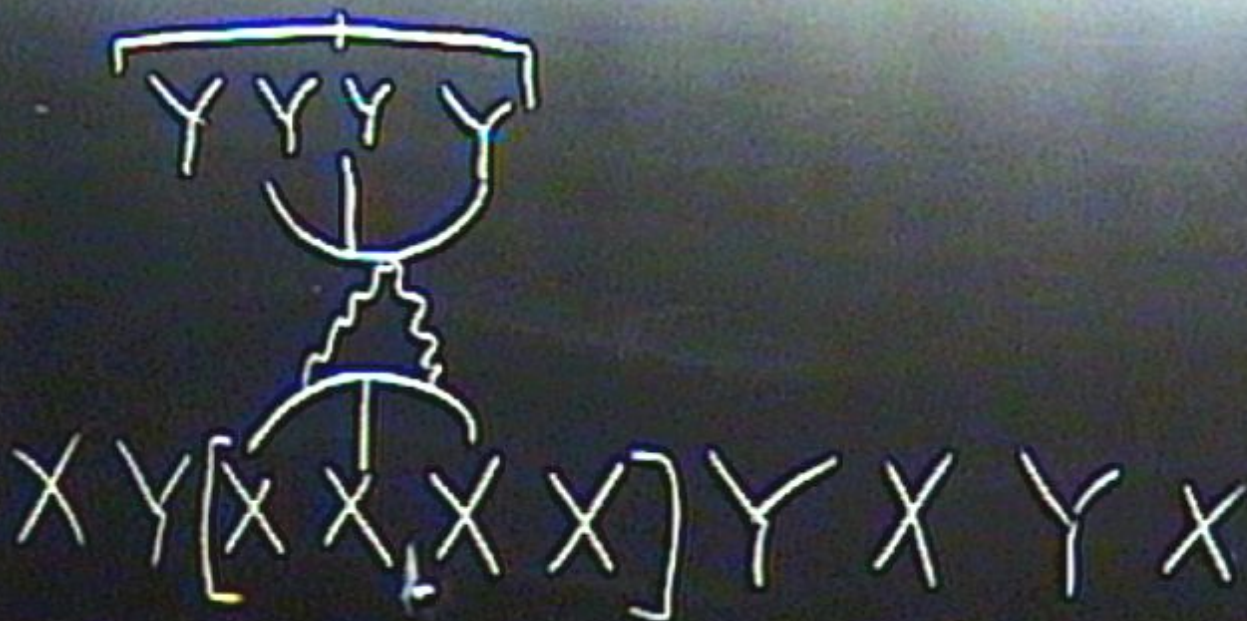


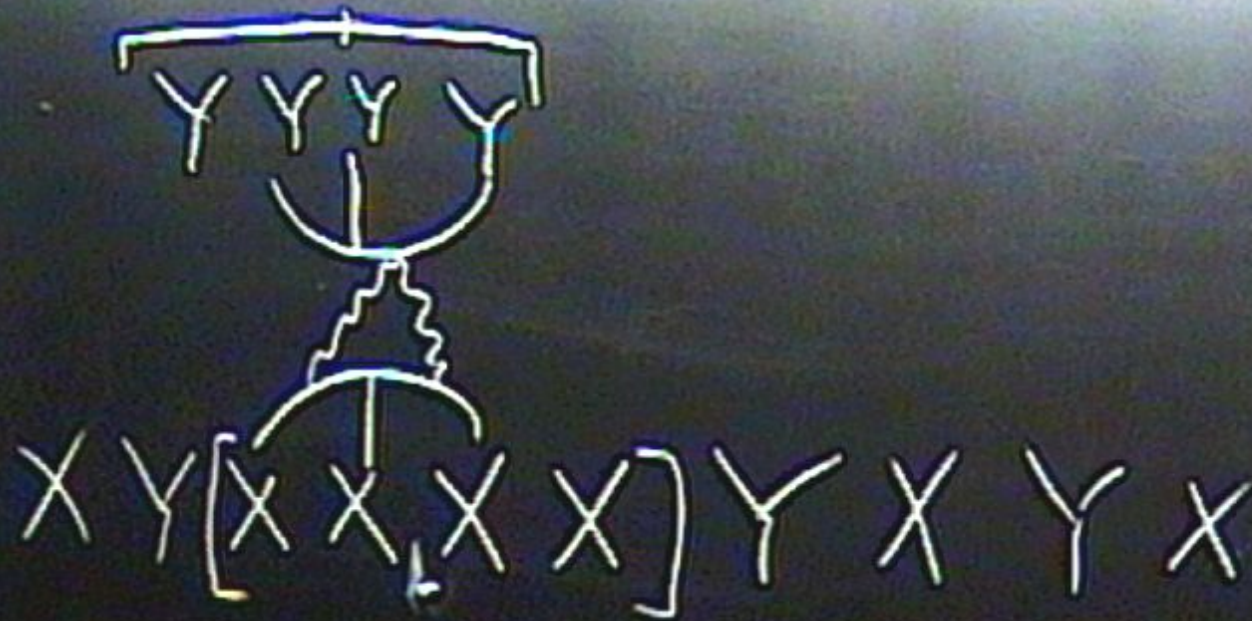


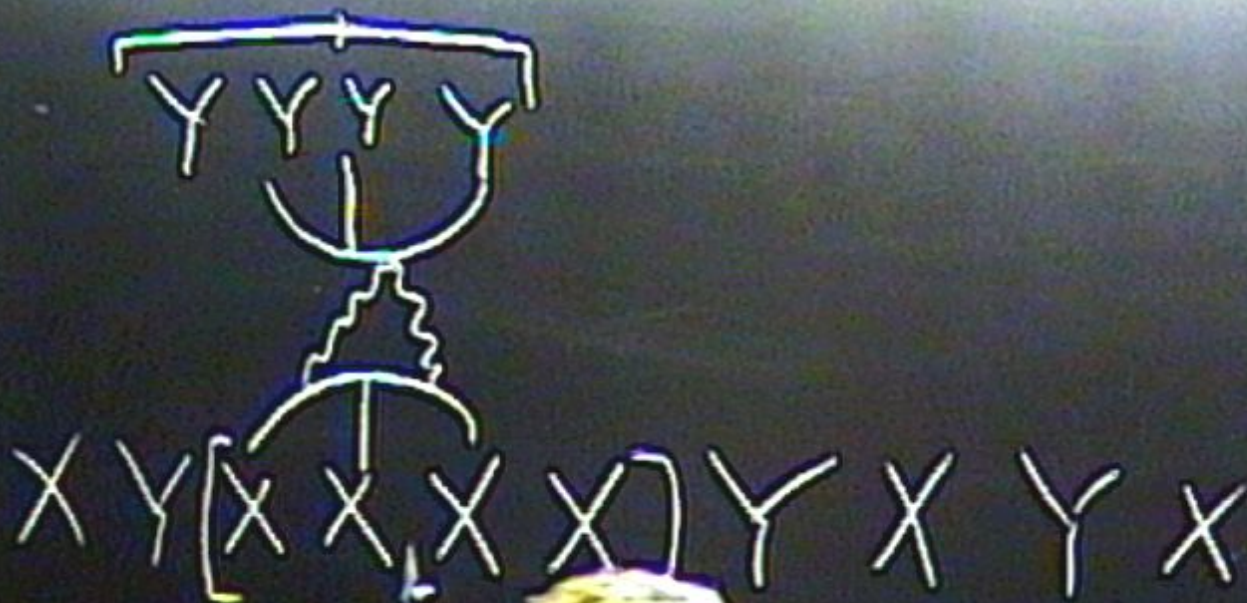


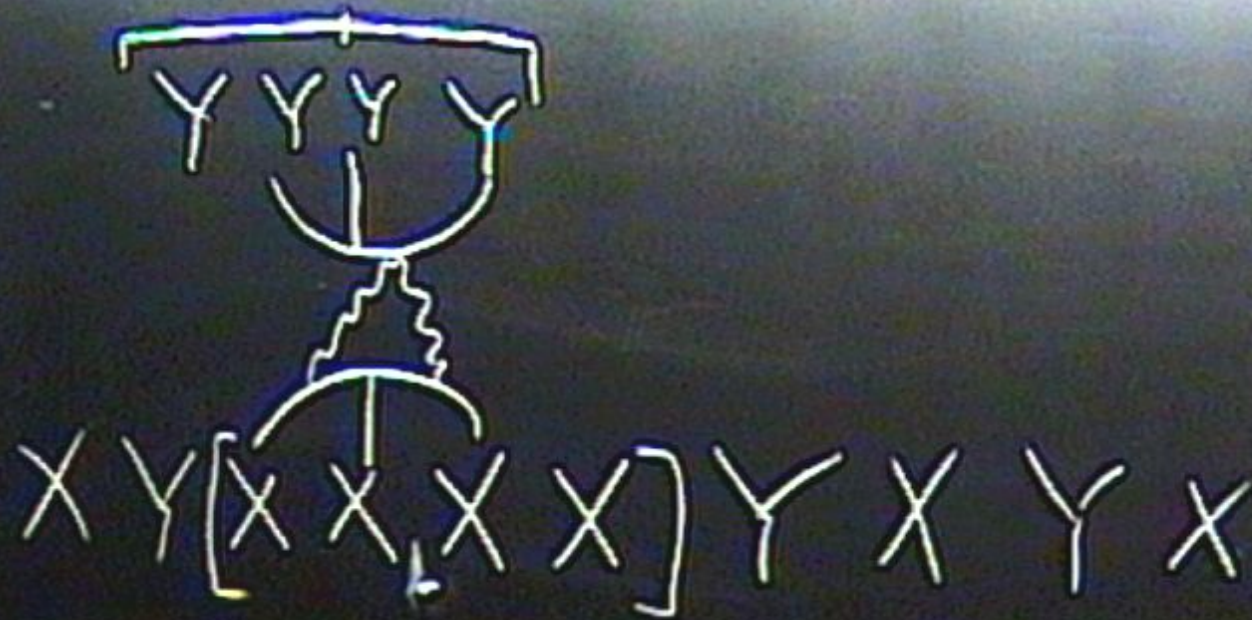


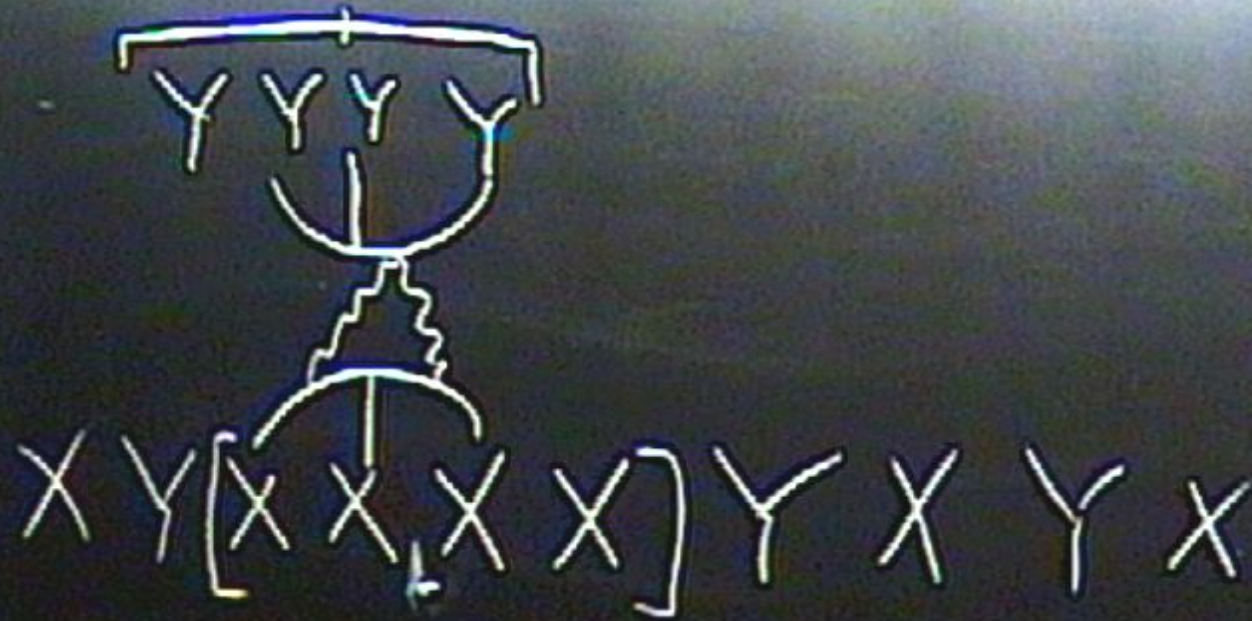












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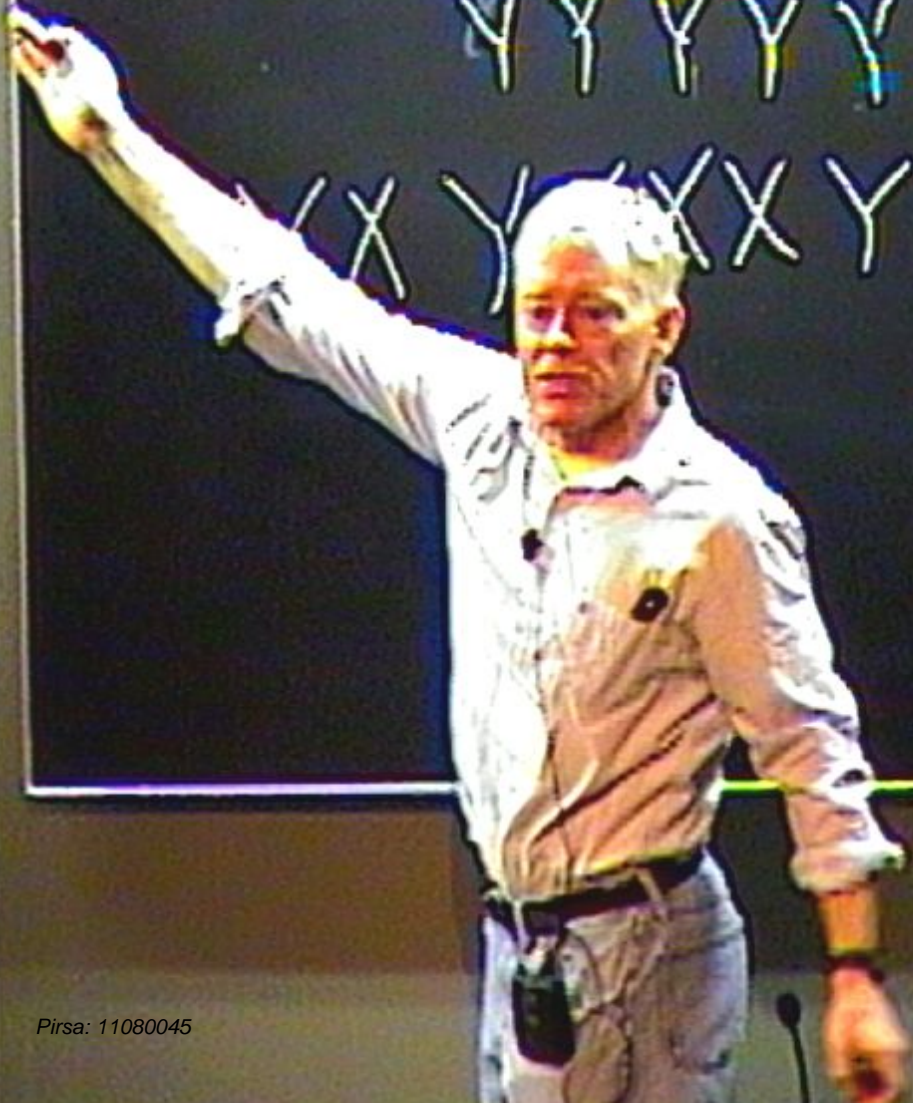
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Y Y Y Y Y
Y X Y X X X Y X

X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

XXXYXY
YXYXY
YXXXYX



XXXYYY
YYYY
YXXXYX

X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

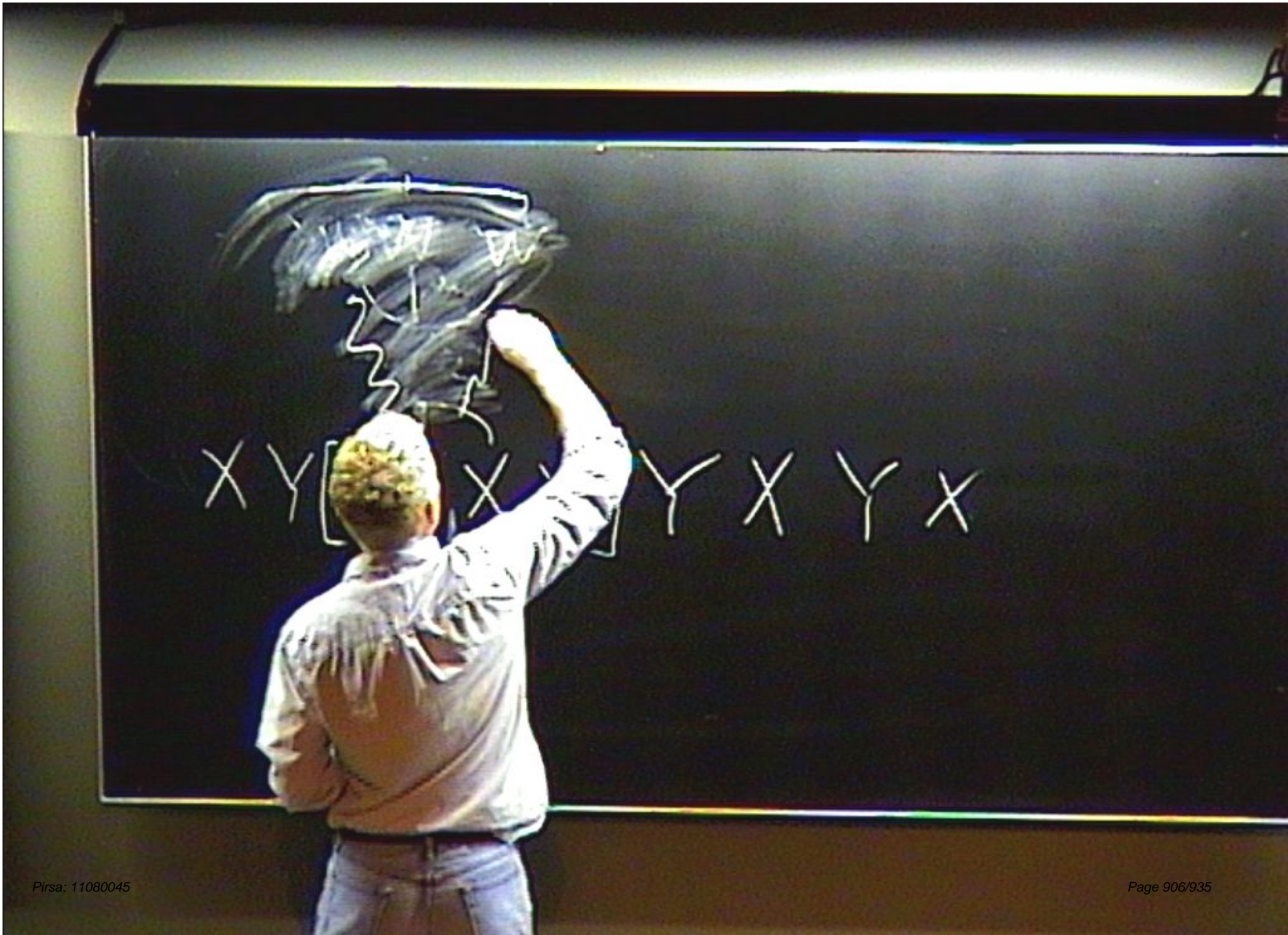
X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

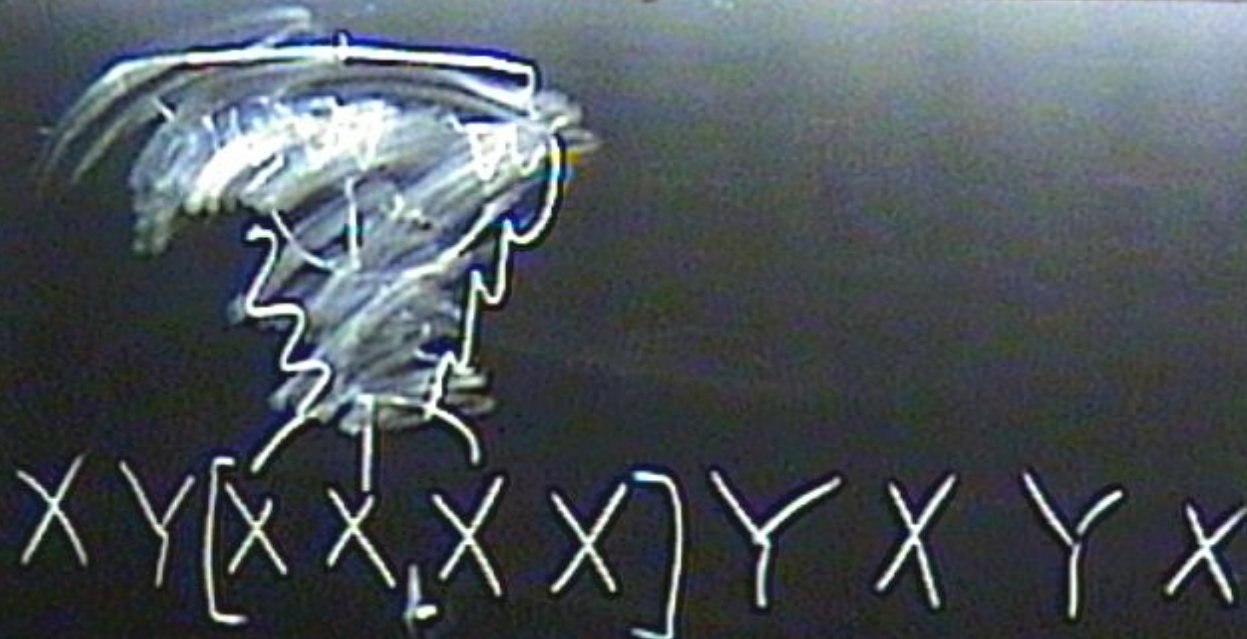
X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X

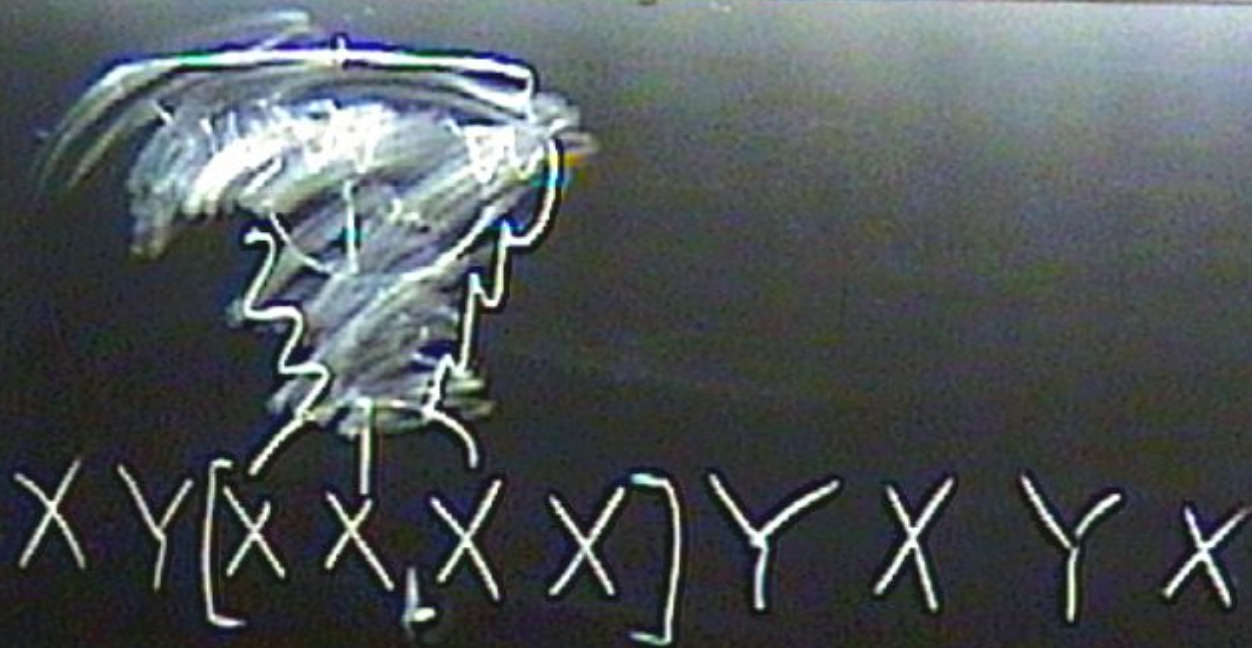
X X X X Y Y
Y Y Y Y Y
Y X Y X X X Y X








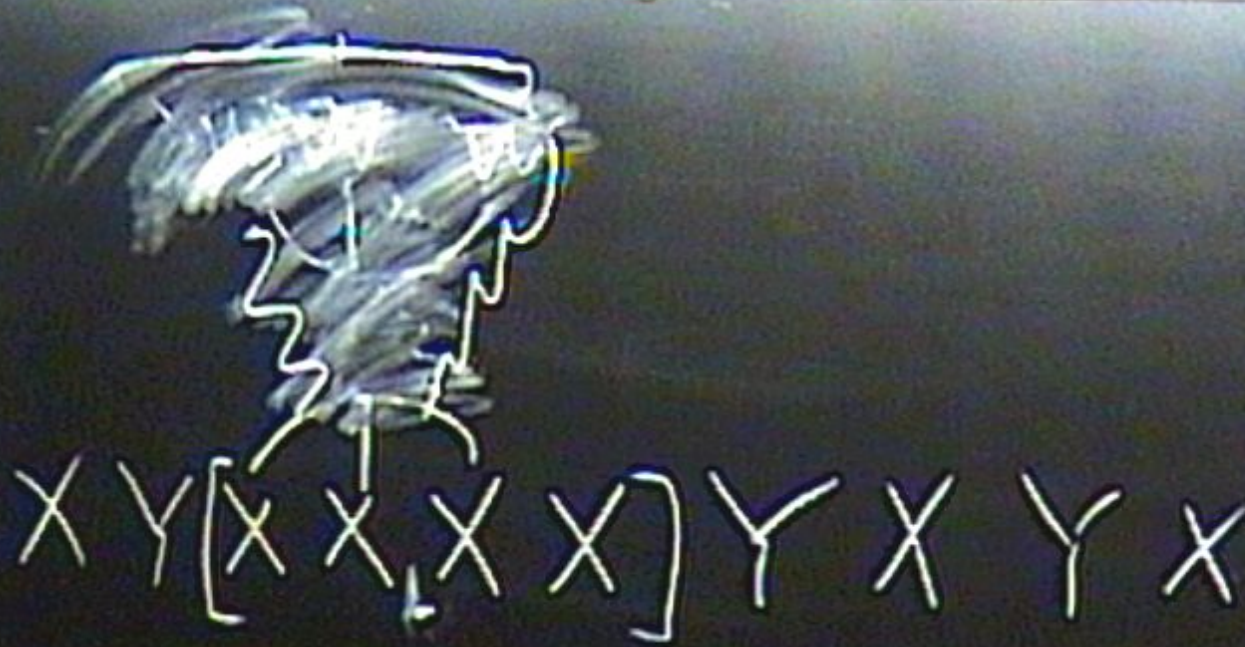





X Y [X X, X X] Y X Y X




X Y [X X X X] Y X Y X






X Y [X X X X] Y X Y X

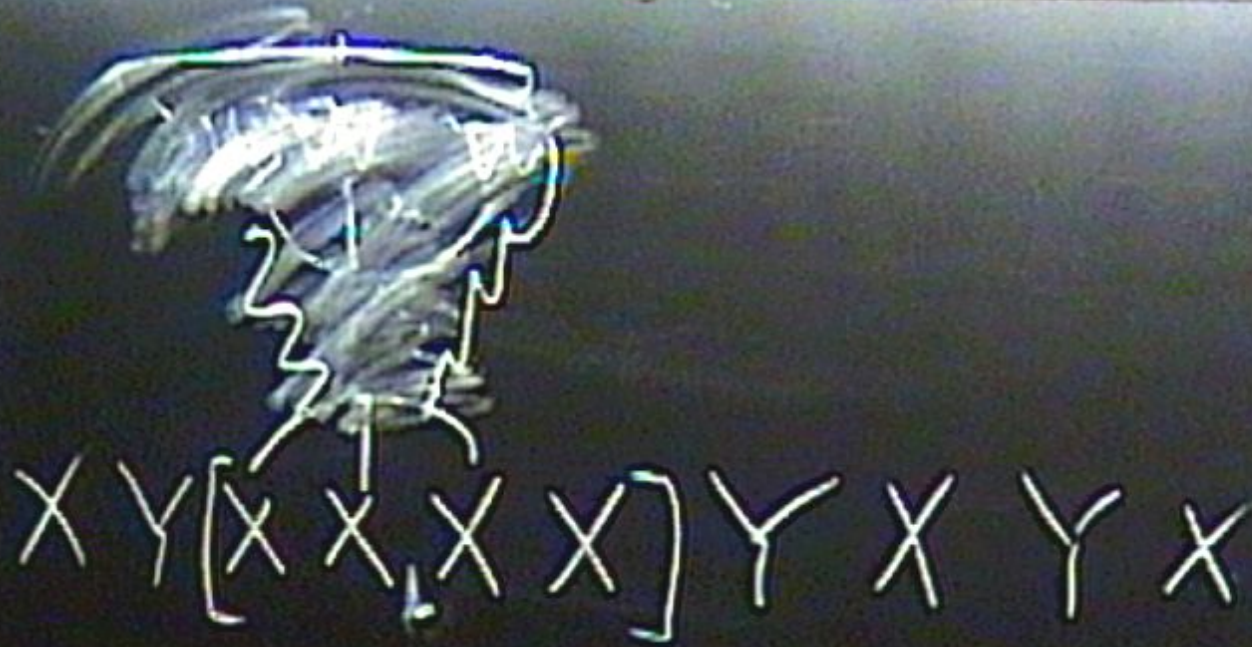


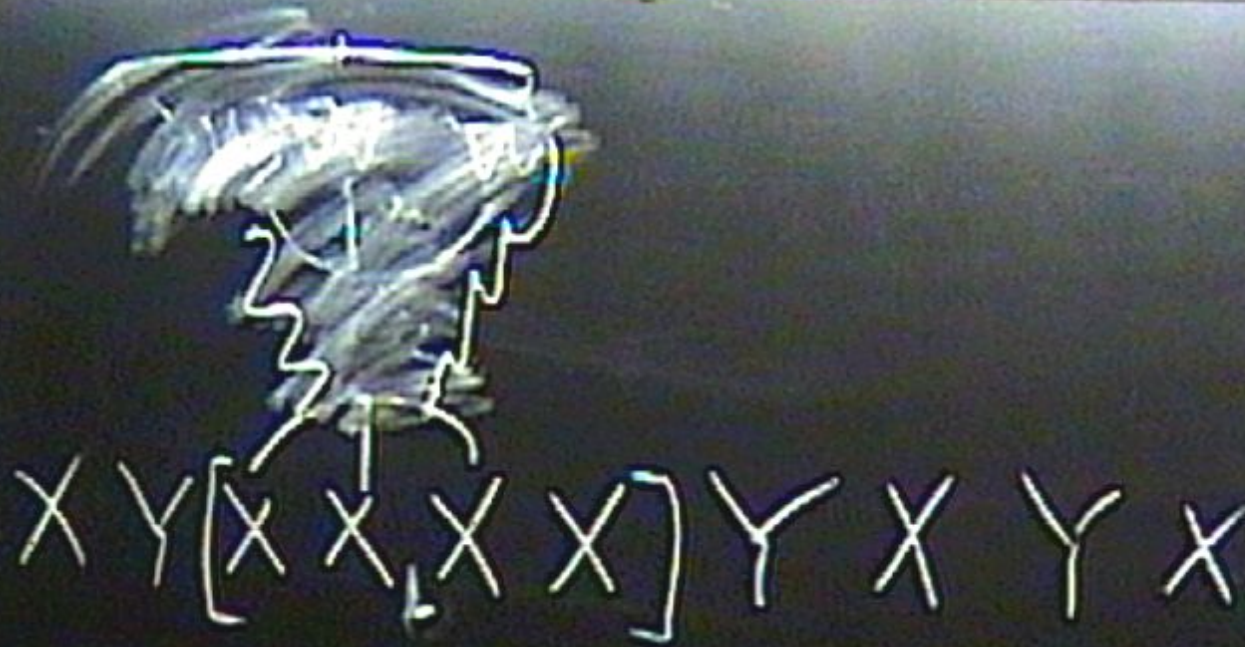
X Y [X X X X] Y X Y X






X Y [X X X X] Y X Y X

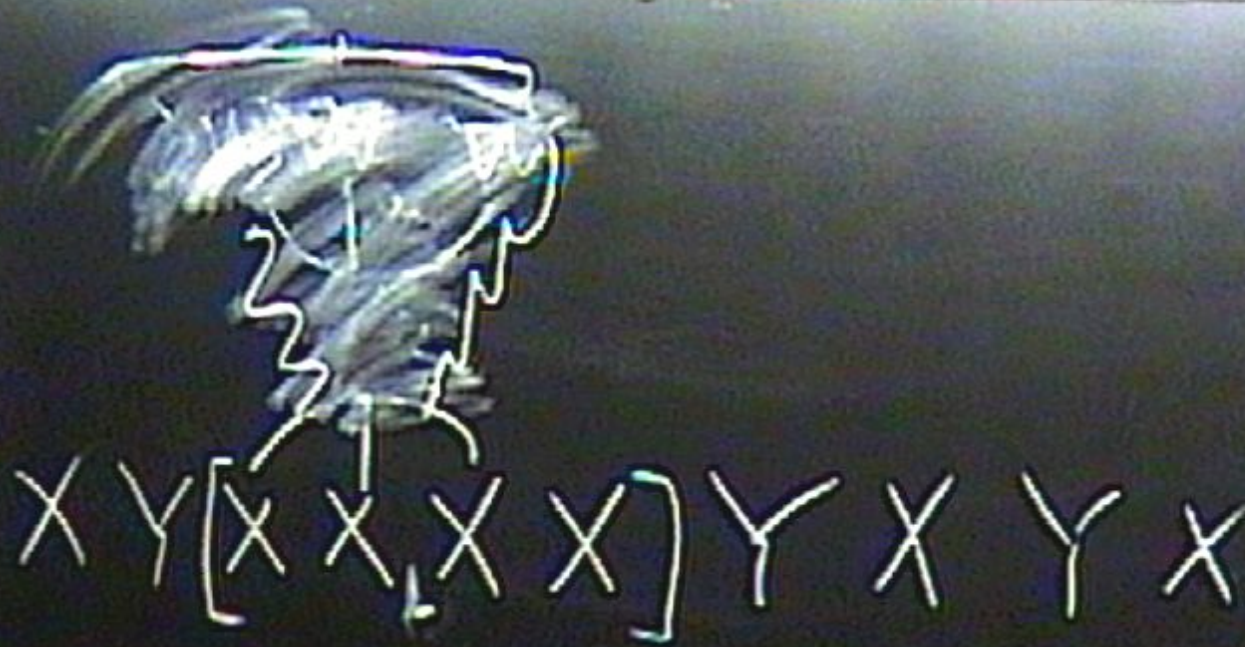





X Y [X X, X X] Y X Y X


X Y [X X X X] Y X Y X


X Y [X X, X X] Y X Y X



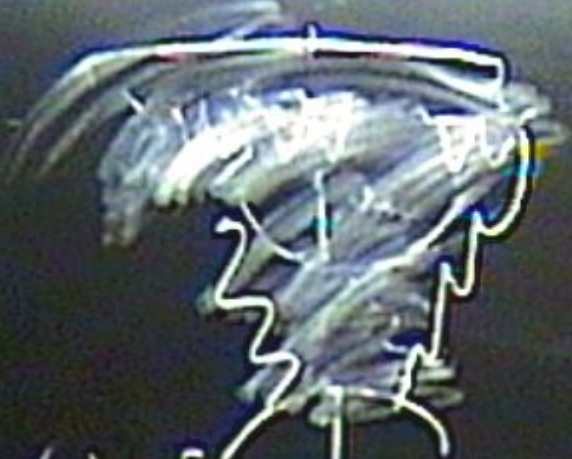
X Y [X X, X X] Y X Y X




$XY[X, X, X, X]Y X Y X$





$XY[X, X, X, X]Y X Y X$




X Y [X X X X] Y X Y X



X Y [X X X X] Y X Y X


X Y [X X X X] Y X Y X





X Y [X X X X] Y X Y X



X Y [X X X X] Y X Y X




X Y [X X X X] Y X Y X



X Y [X X X X] Y X Y X


X Y [X X X X] Y X Y X



$XY[X, X, X, X]YX Y X$


X Y [X X X X] Y X Y X



X Y [X X X X] Y X Y X