Title: Flux Tubes, Integrability and the S-matrix of N=4 SYM

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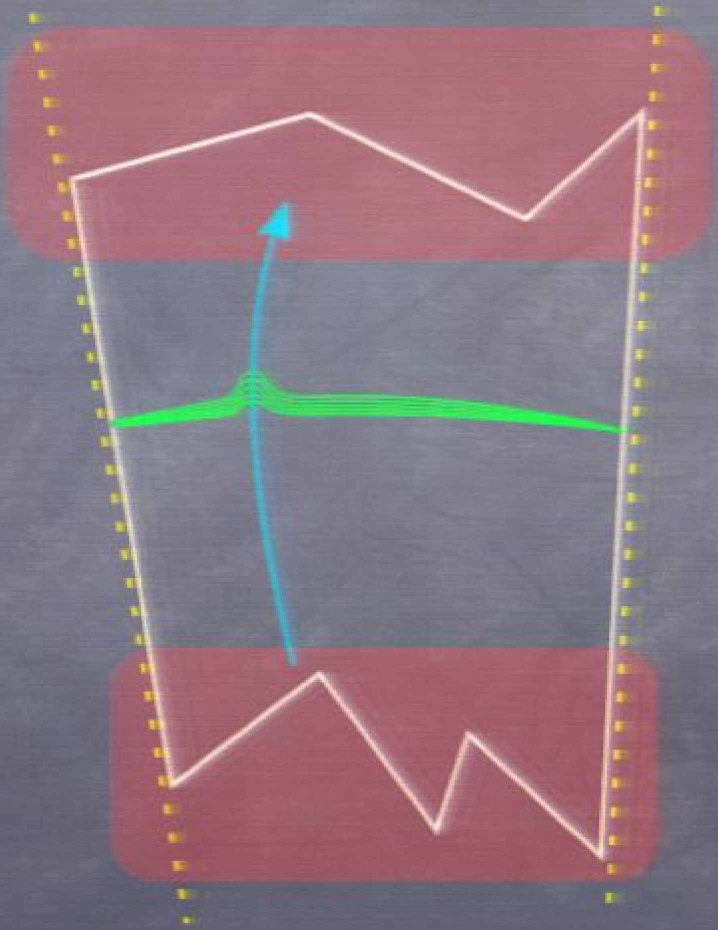
Abstract: An object which has been under attack from several fronts is the planar S-matrix of N=4 SYM. One approach towards addressing the computation of scattering amplitudes using integrability is by using an analogues of an Operator Product Expansion for these observables. It is a very general expansion that is based on the dual conformal symmetry of the amplitudes or their dual description in terms of null polygon Wilson loops. In this expansion the Wilson loop/amplitude is viewed as a transition amplitude for flux tube excitations. The flux tube in question is the color flux stretched between two fast moving quarks and the excitation are the excitations of that color flux. In the planar limit, it has an holographic description in terms of a two dimensional world sheet, known as the GKP string. For N=4 SYM, the dynamics of the flux excitation is integrable to all loops.

Flux tubes, Integrability and the S-matrix of N=4 SYM

A. Sever Perimeter Institute

F. Alday
D. Gaiotto
J. Maldacena
P. Vieira
T. Wang



Flux tubes, Integrability and the S-matrix of N=4 SYM

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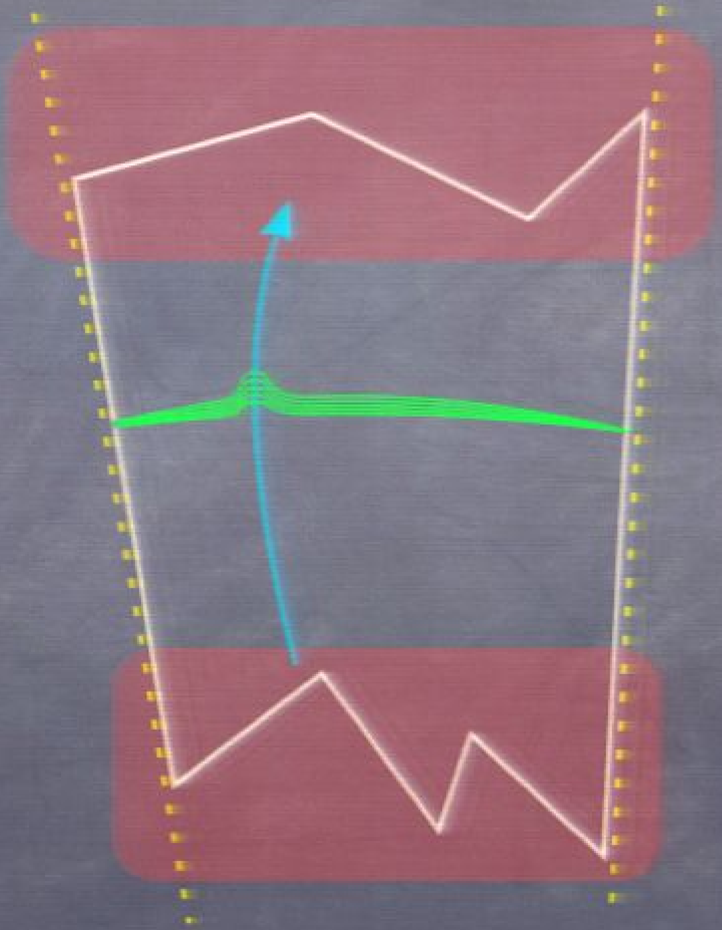
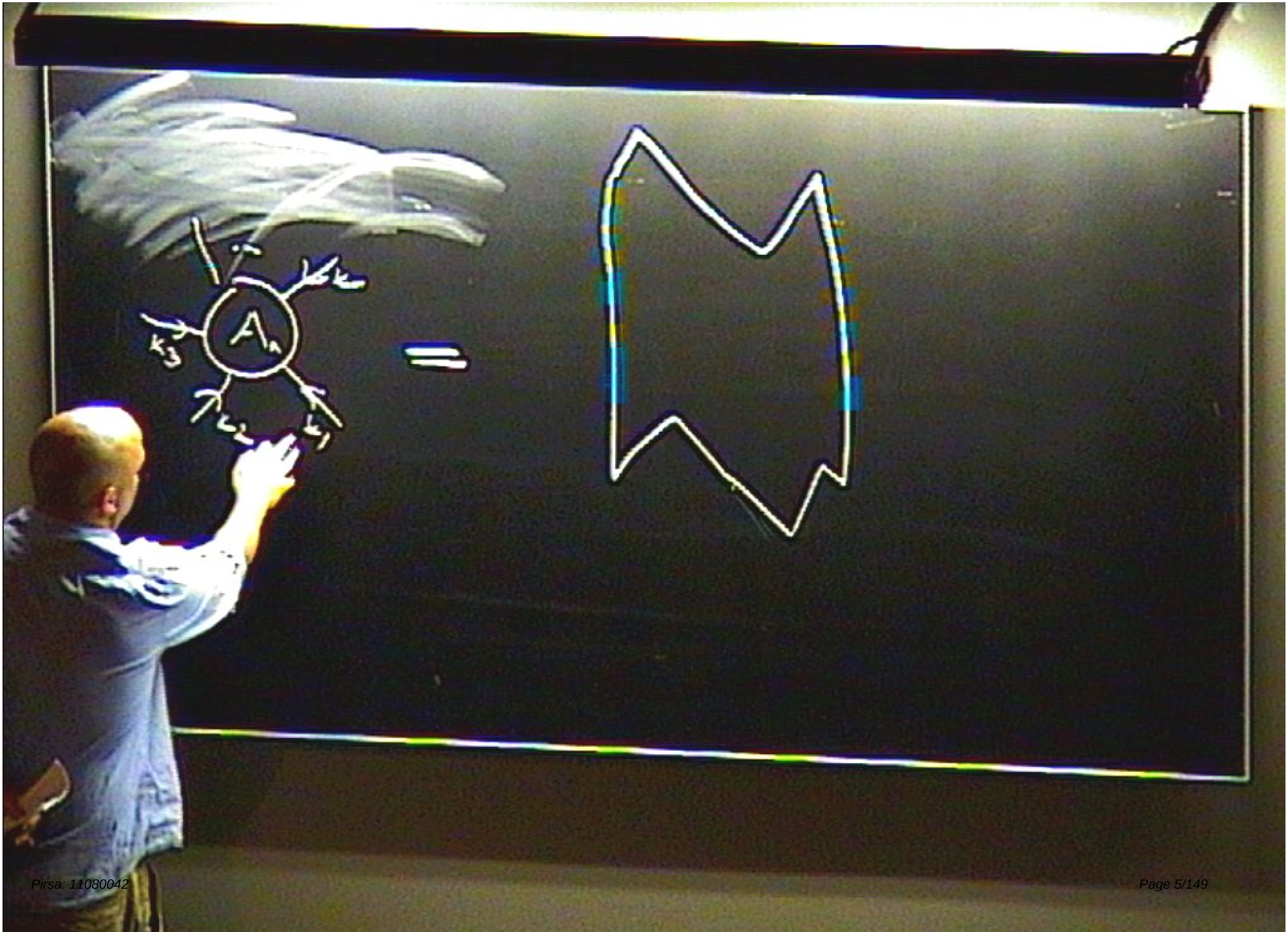
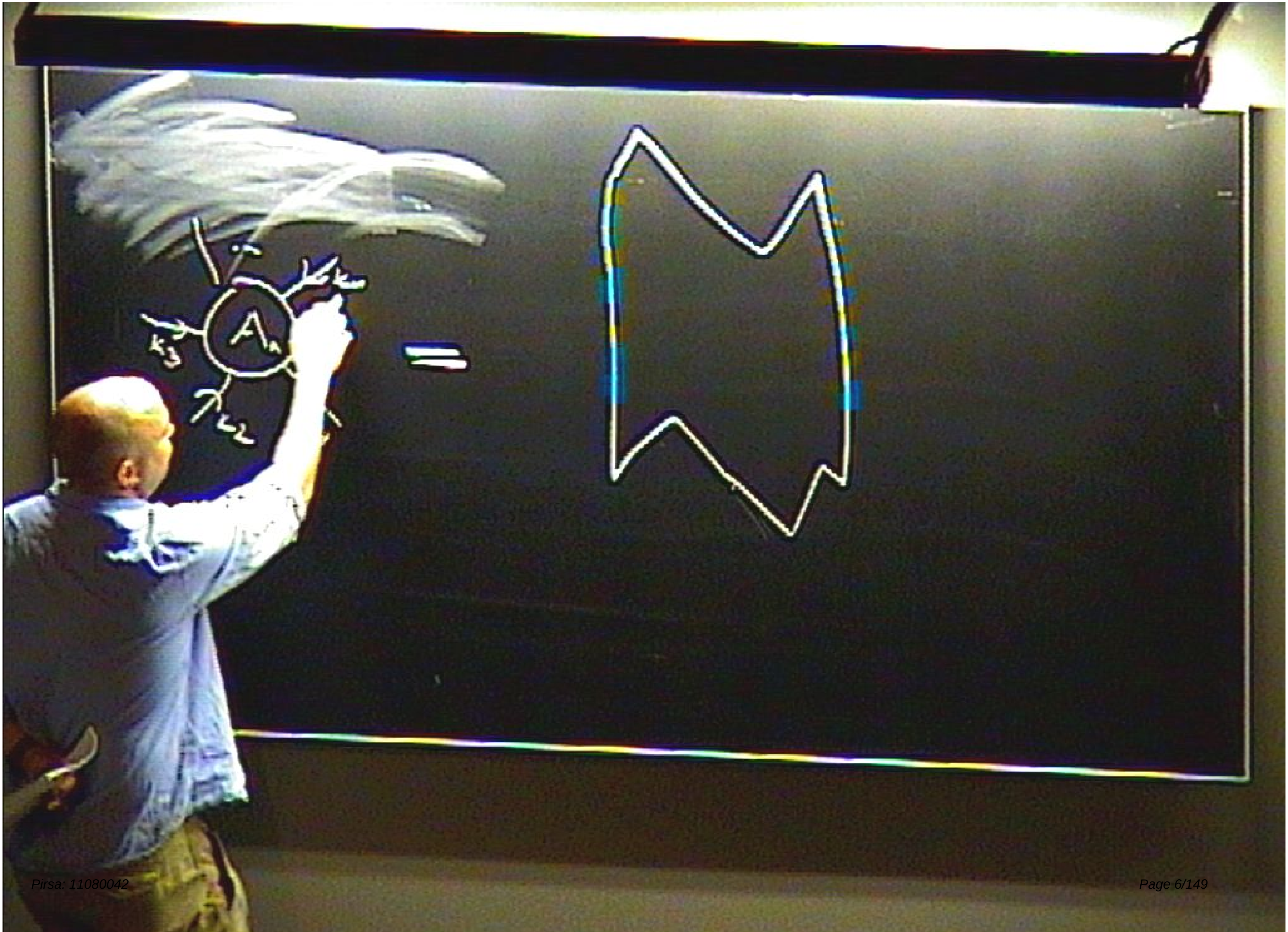
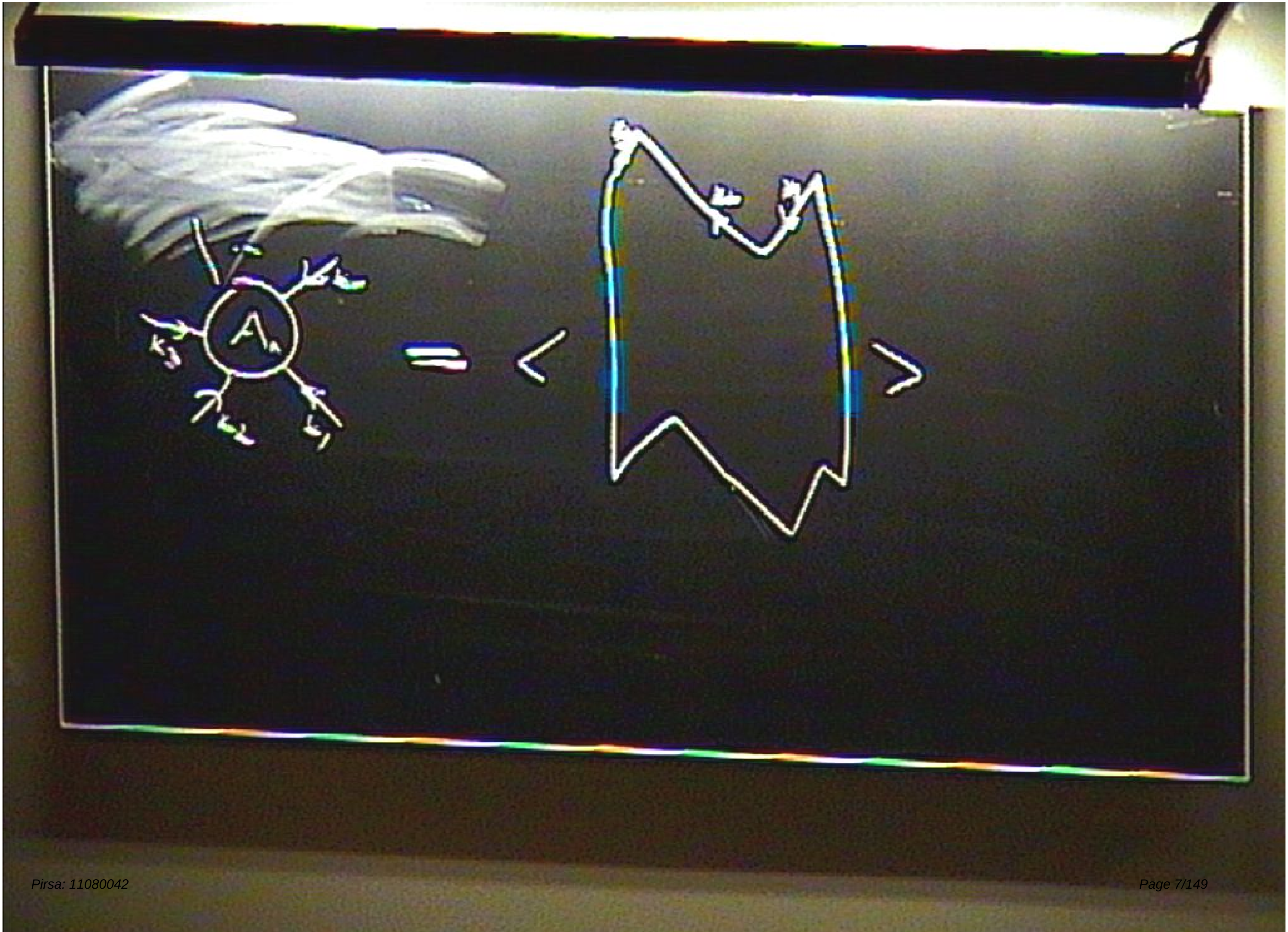



A_n
k_n
k_3
k_2
k_1
=


k_n
A_n
k_3
k_2
=


A_n
k_1
k_2
k_3
=
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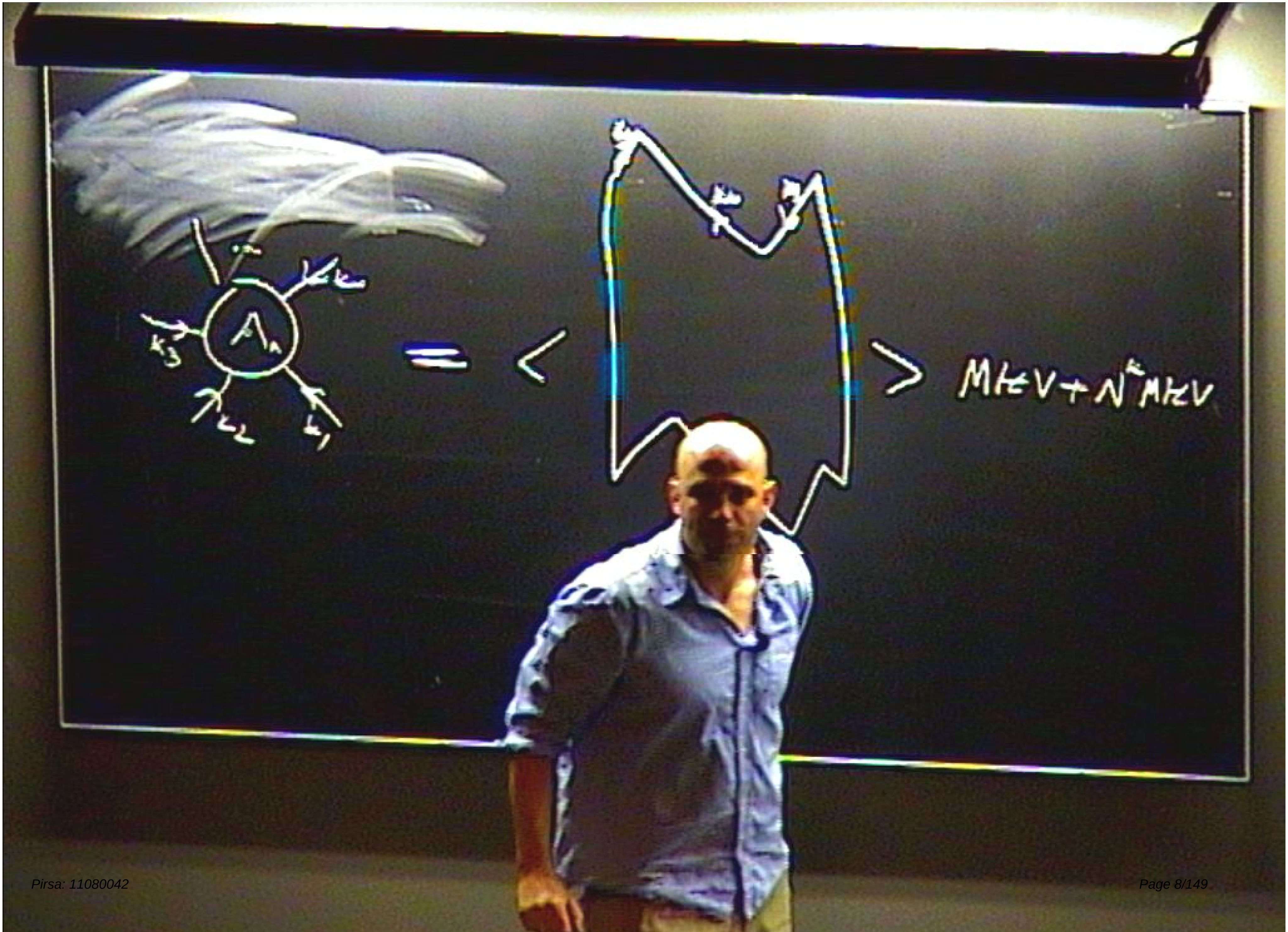
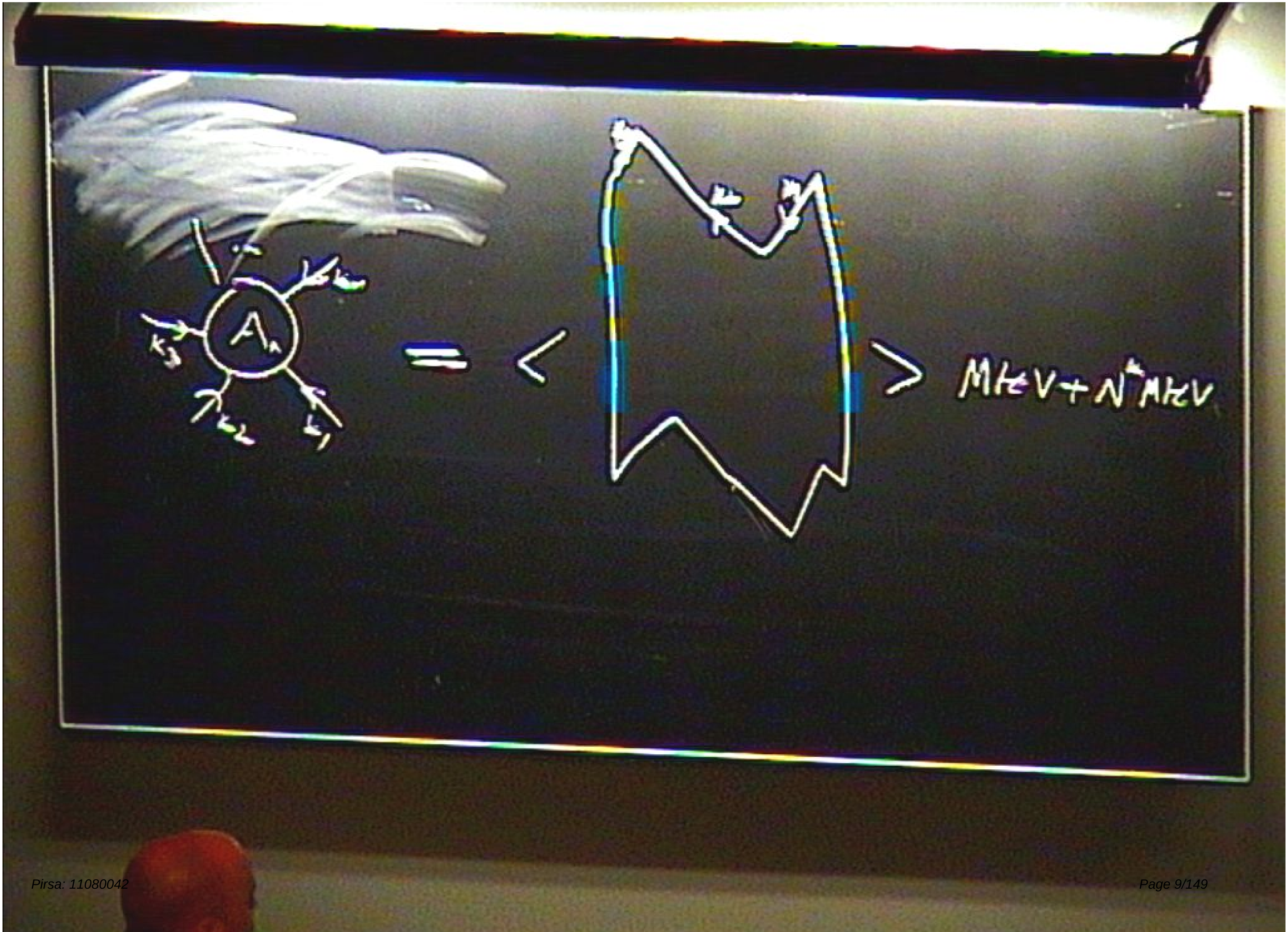
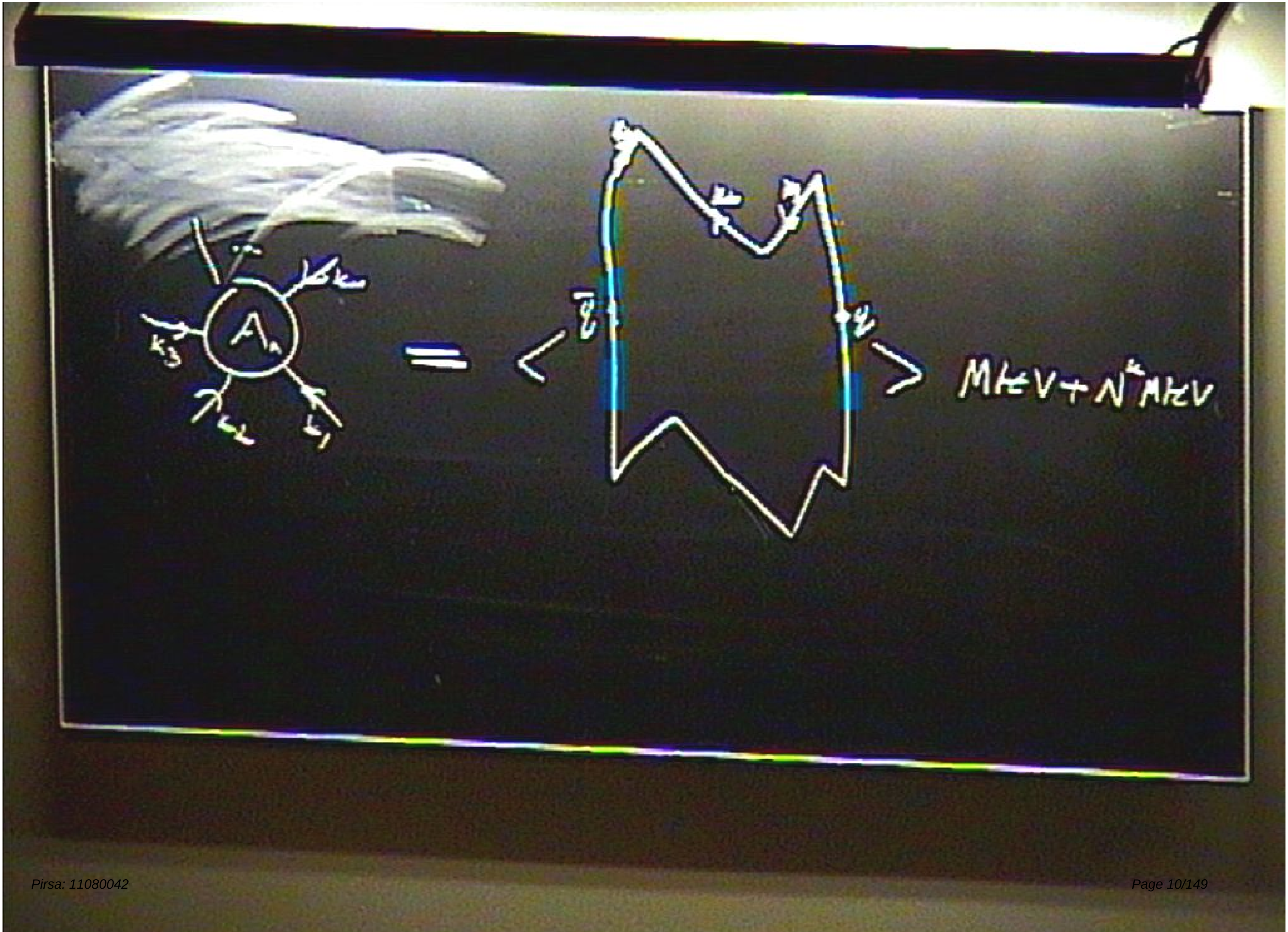
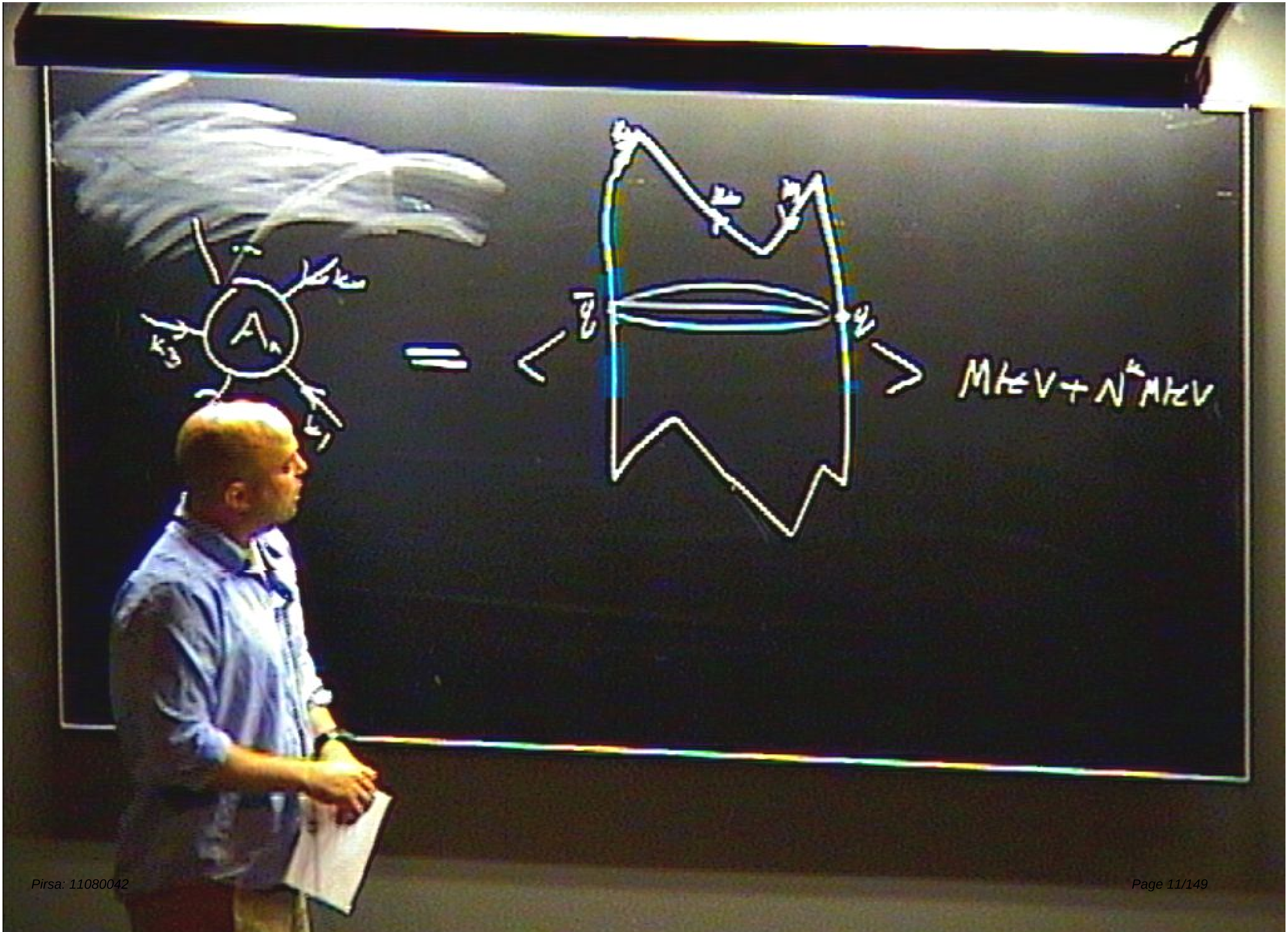
>

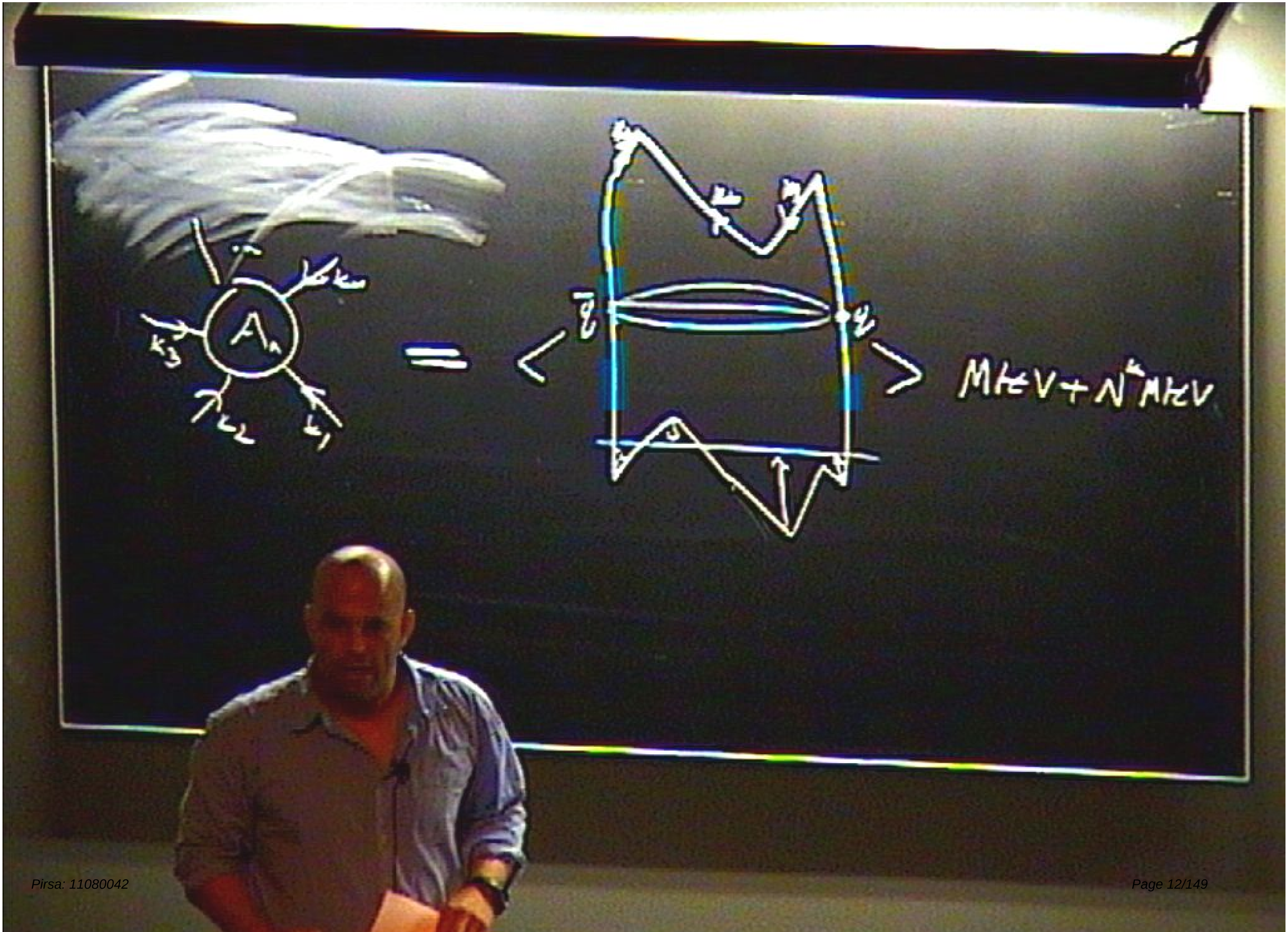

A_n
k_1
k_2
k_3
k_n
=
MHV + N^k MHV


A_n
k_1
k_2
k_3
k_n
$= \langle \quad \rangle$
MHV + N^kMHV


A_n
k_1
k_2
k_3
k_n
=
$\langle \bar{\psi}$
$\psi \rangle$
MHV + N^k MHV



A_n
k_3
= < W̄ W >
MHV + N^k MHV


A_n
k_1
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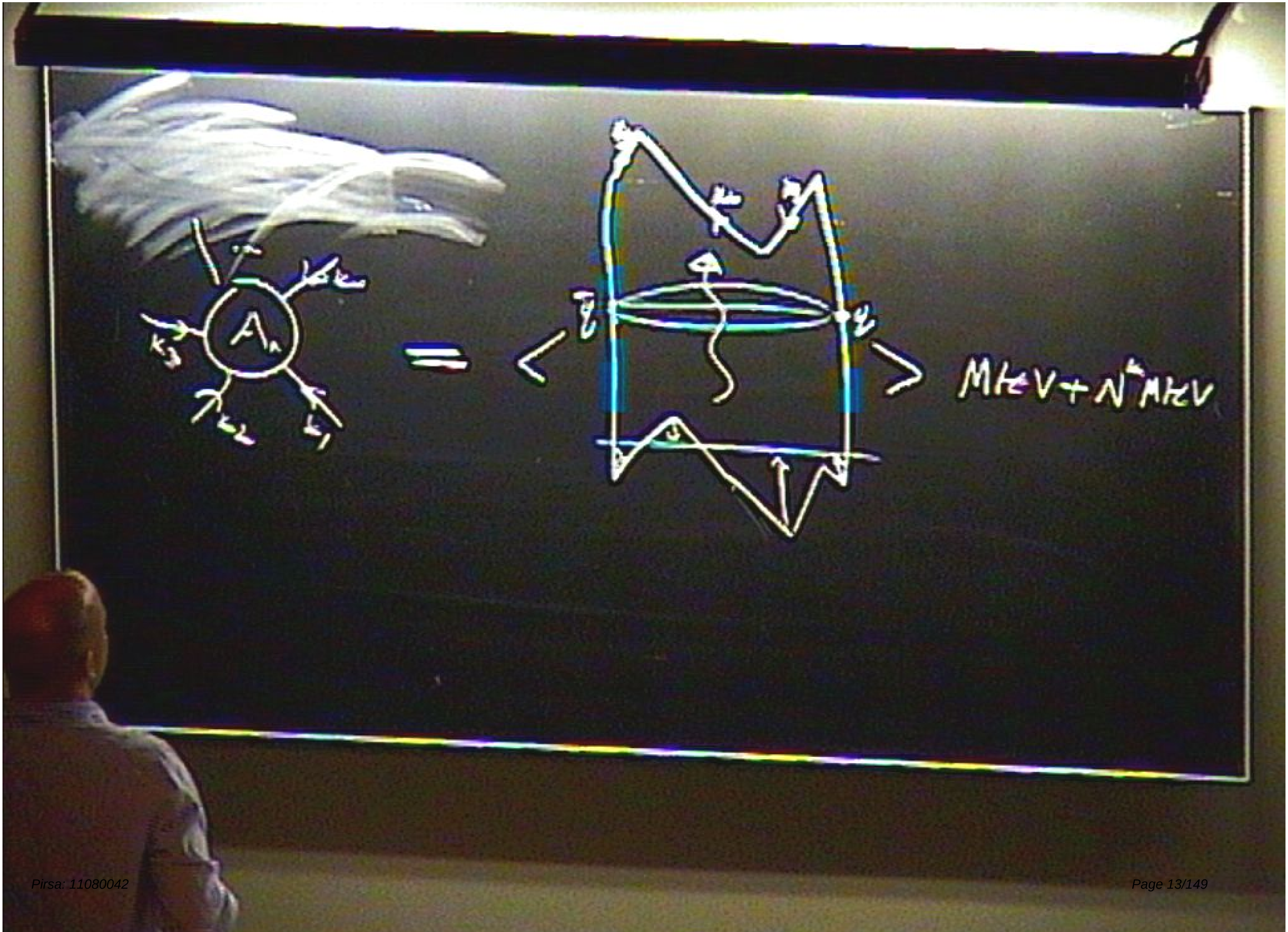
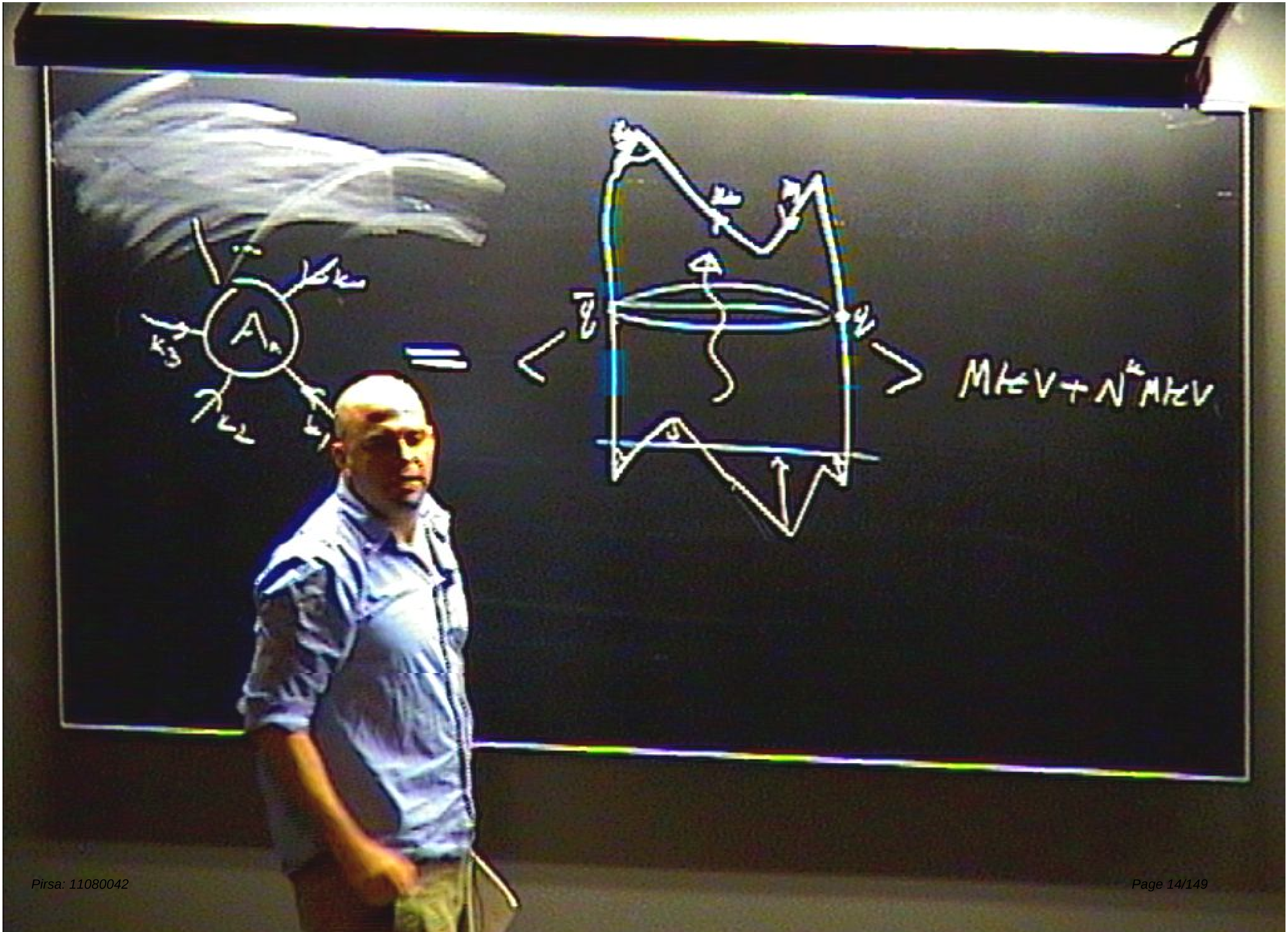
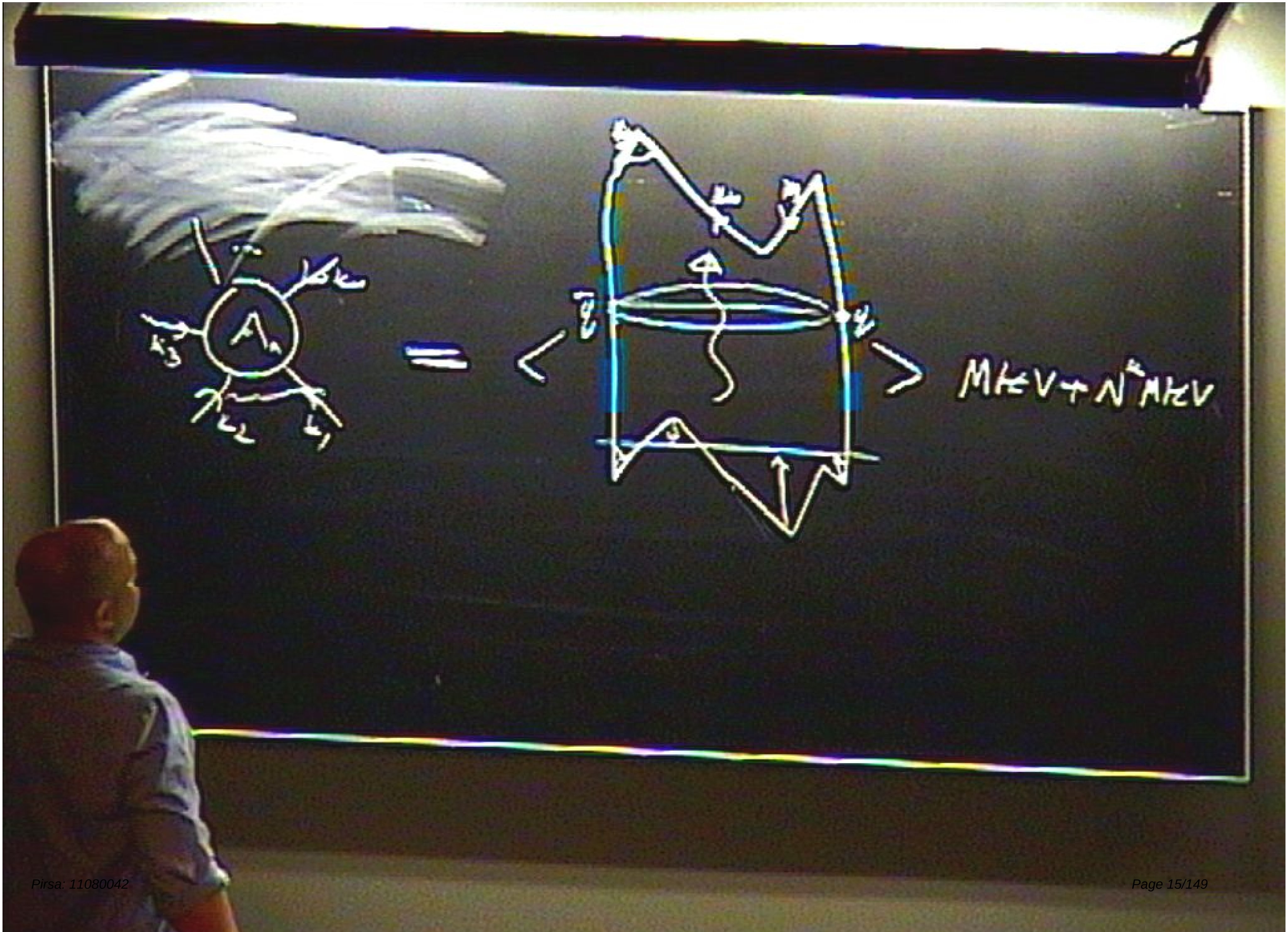
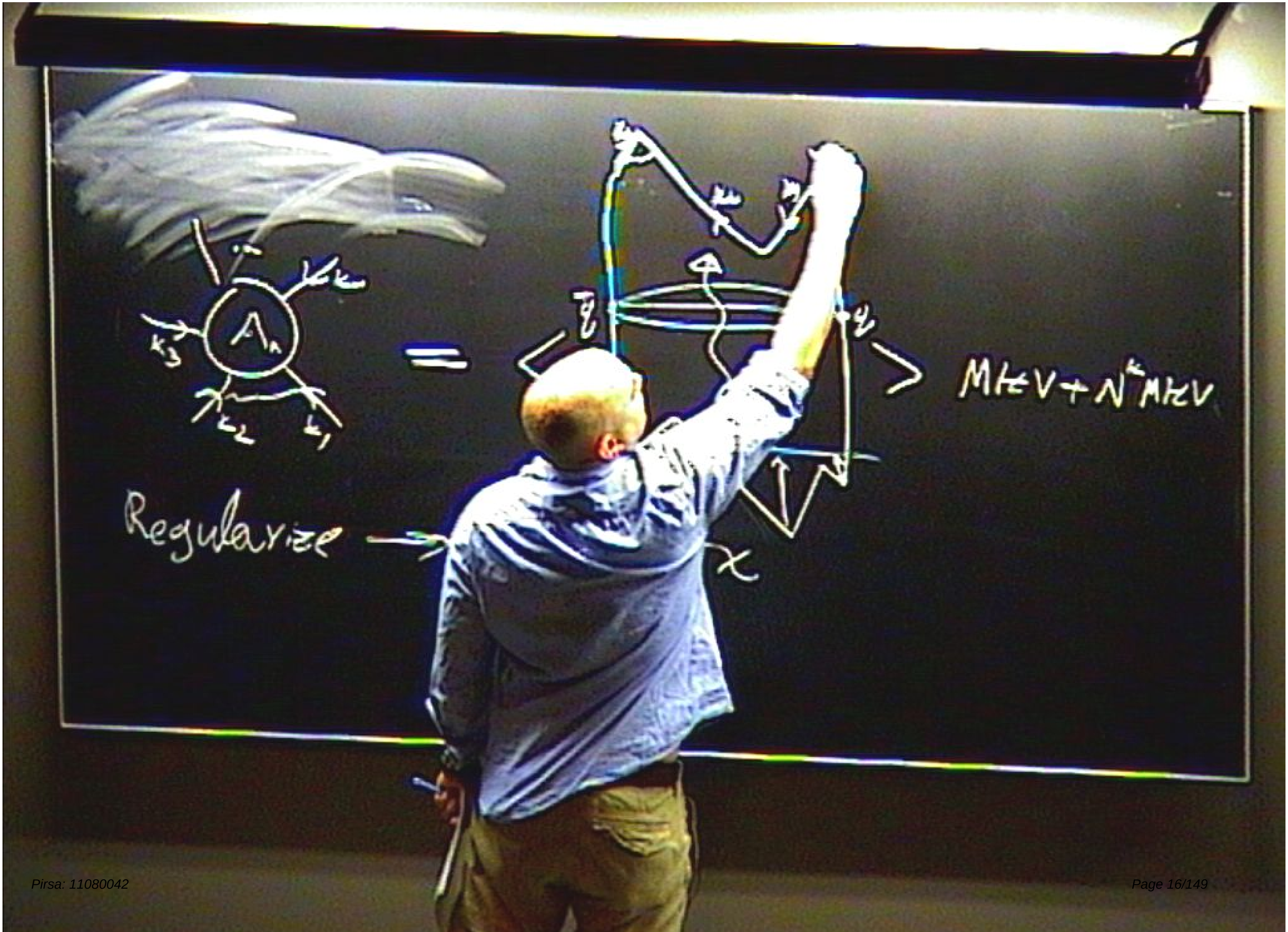
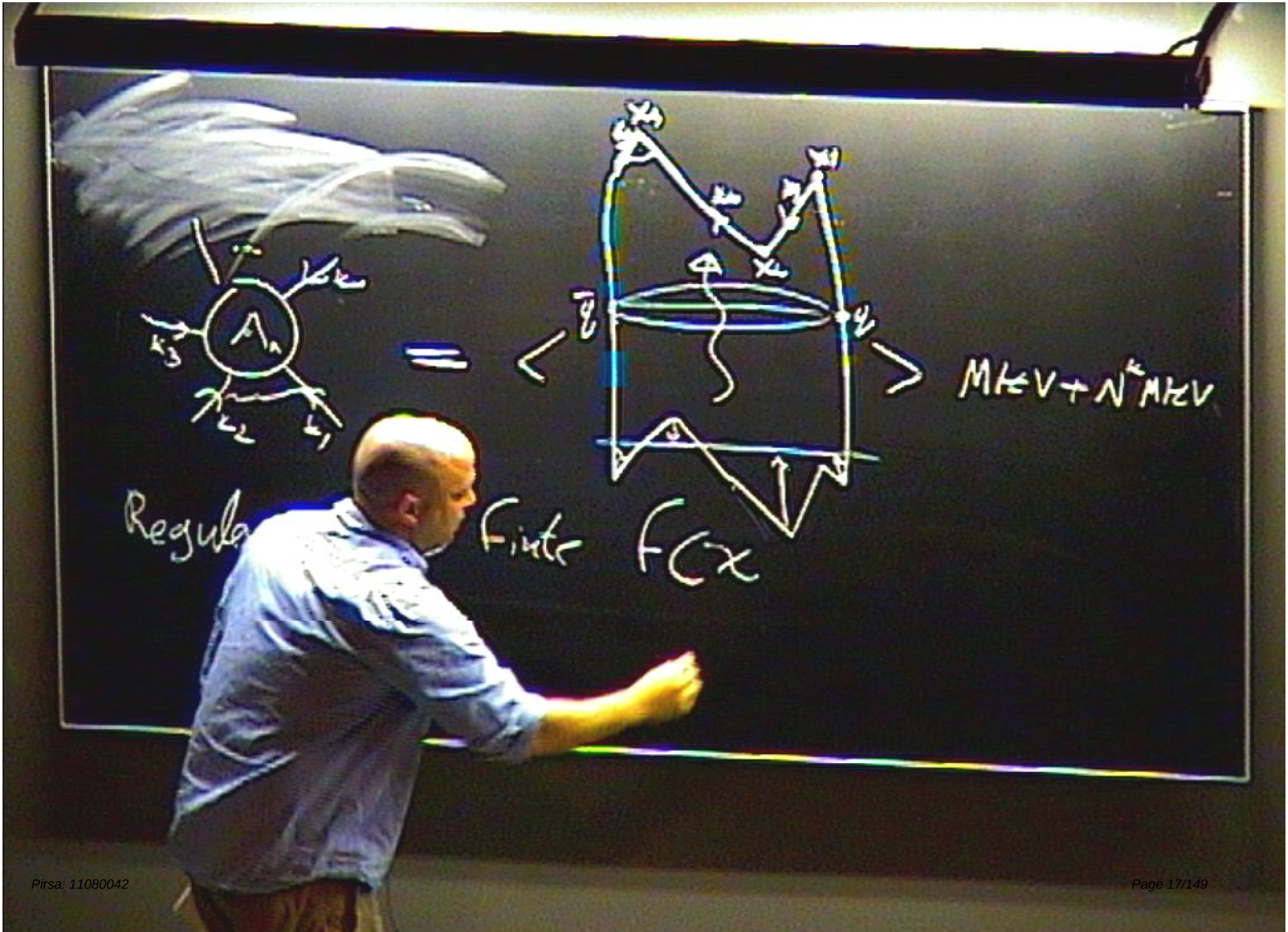
k_n
$=$
$\langle \bar{q}$
$q \rangle$
MHV + N^kMHV

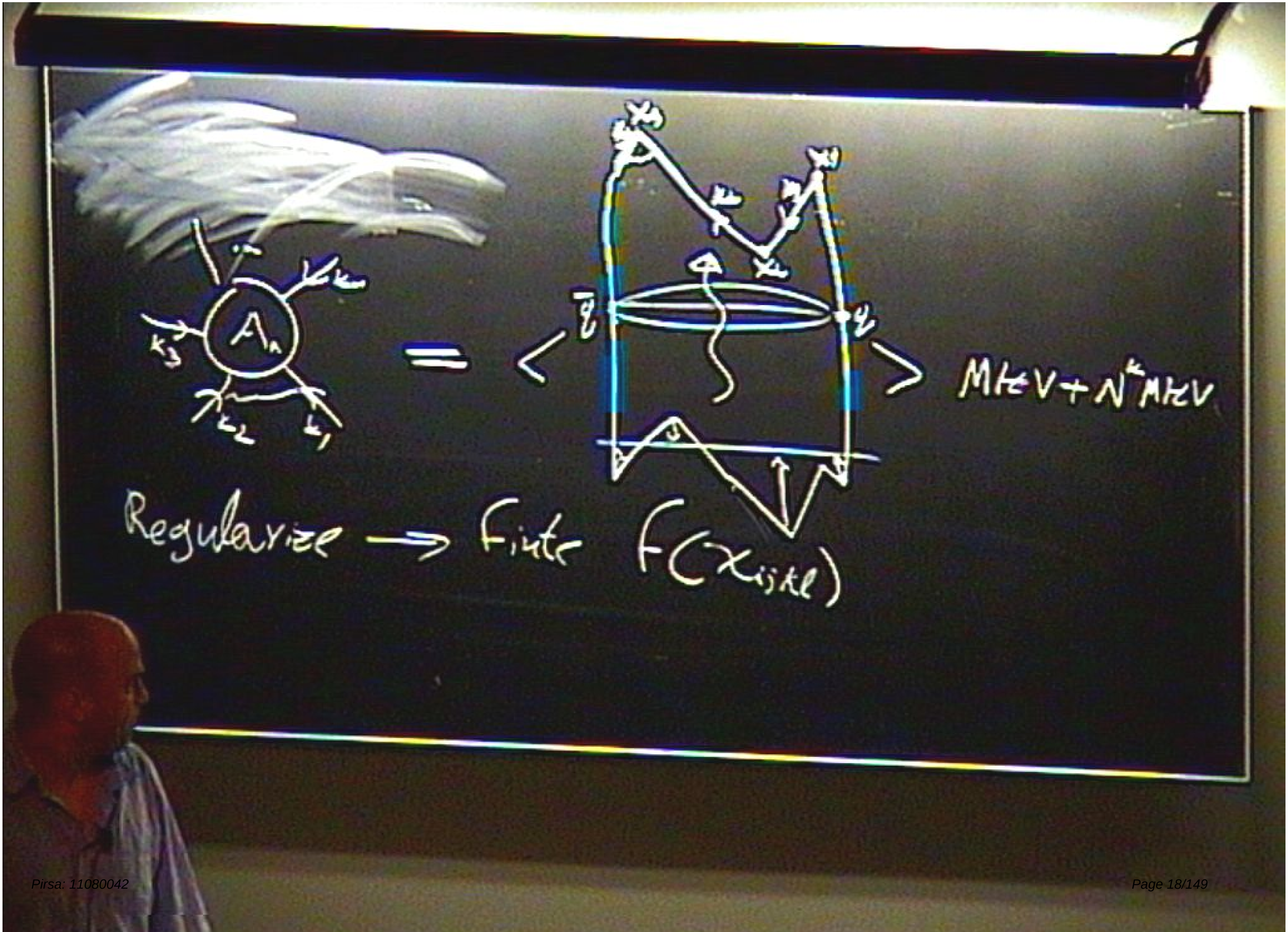

A_n
k_1
k_2
k_3
k_n
=
< >
MHV + N^k MHV

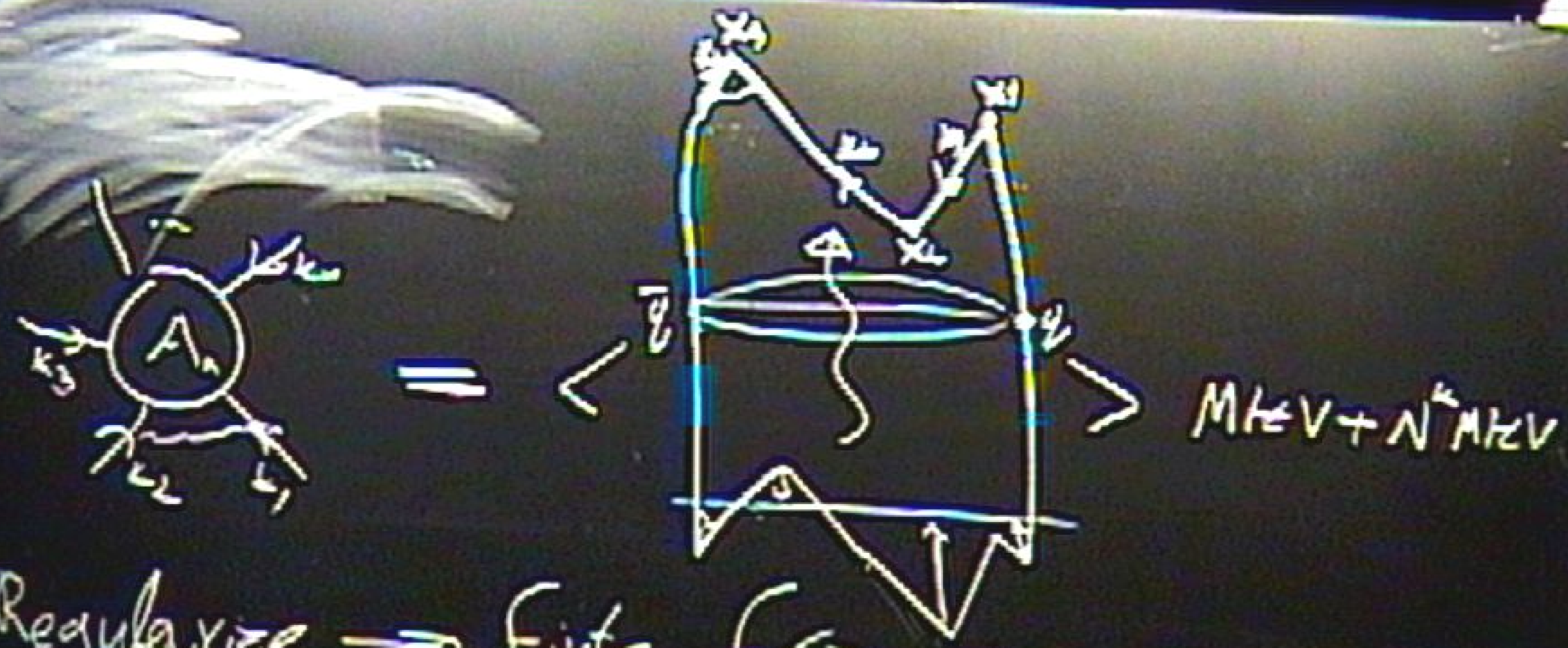

A_n
k_1
k_2
k_3
k_n
$= \langle \bar{\psi} \quad \psi \rangle$
MHV + N^kMHV


k_3
k_n
k_2
k_1
=
MkV + N^k MkV


A_n
k_n
k_3
k_2
k_1
=
MHV + N^k MHV
Regularize
x

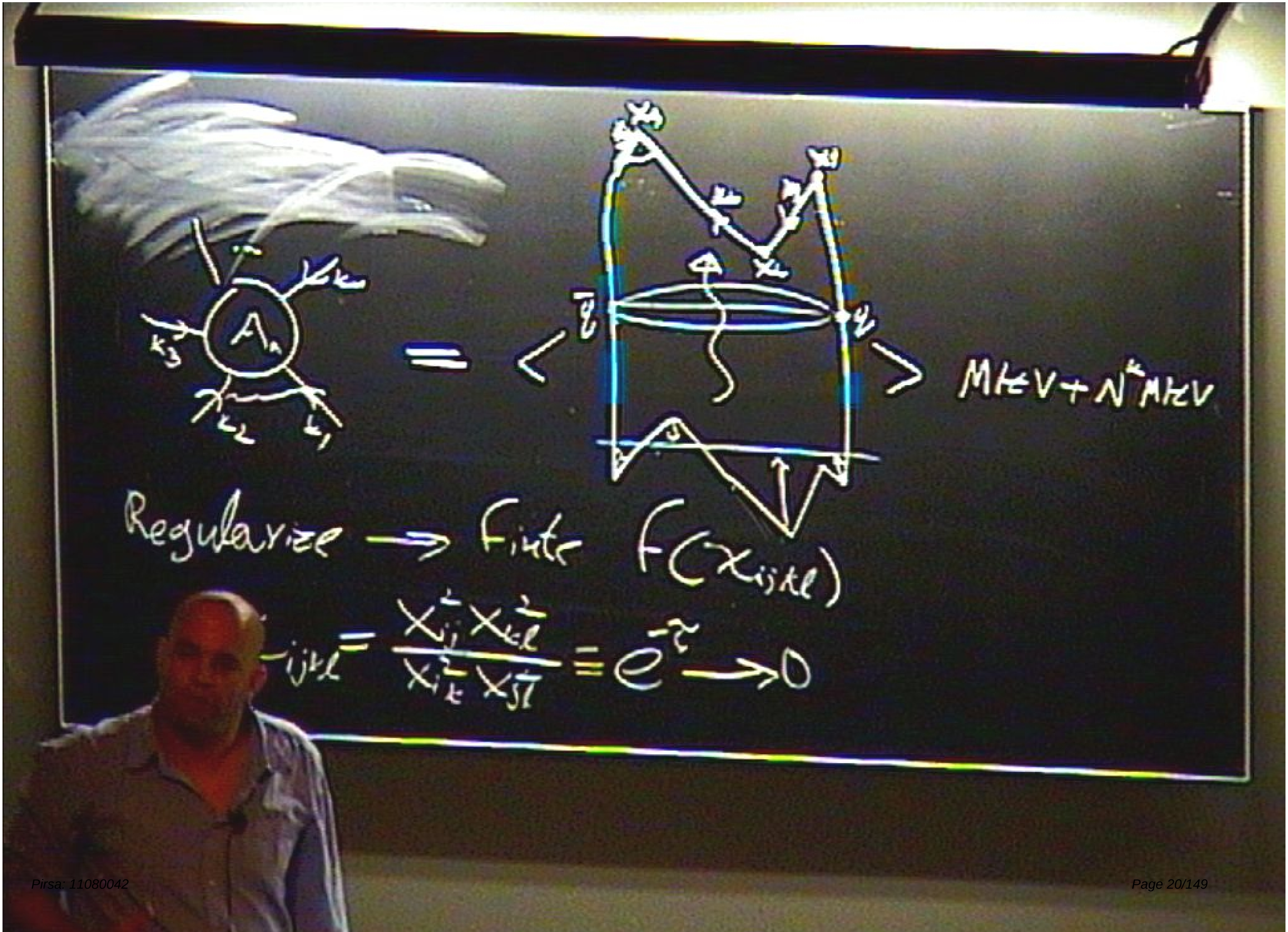

A_n
k_n
k_3
k_2
k_1
=
<
>
MHV + N^k MHV
Regula
Finite
FCx


A_n
k_n
k_3
k_2
k_1
$= \langle \bar{\psi}$
$\psi \rangle$
$MHV + N^k MHV$
Regularize → Finite $f(x_{ijkl})$



Regularize → Finite $F(\chi_{ijkl})$

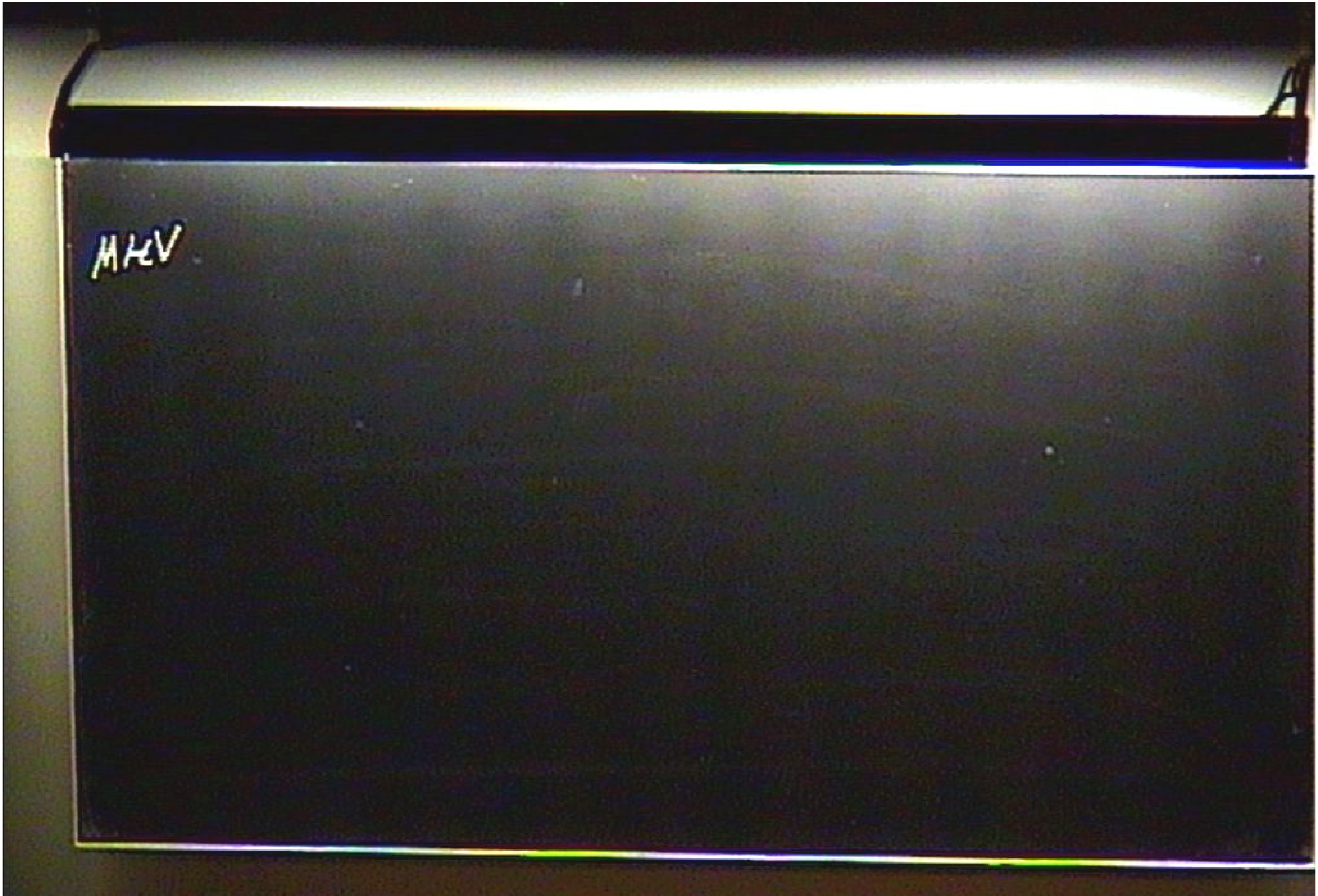
$$\chi_{ijkl} = \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2}$$



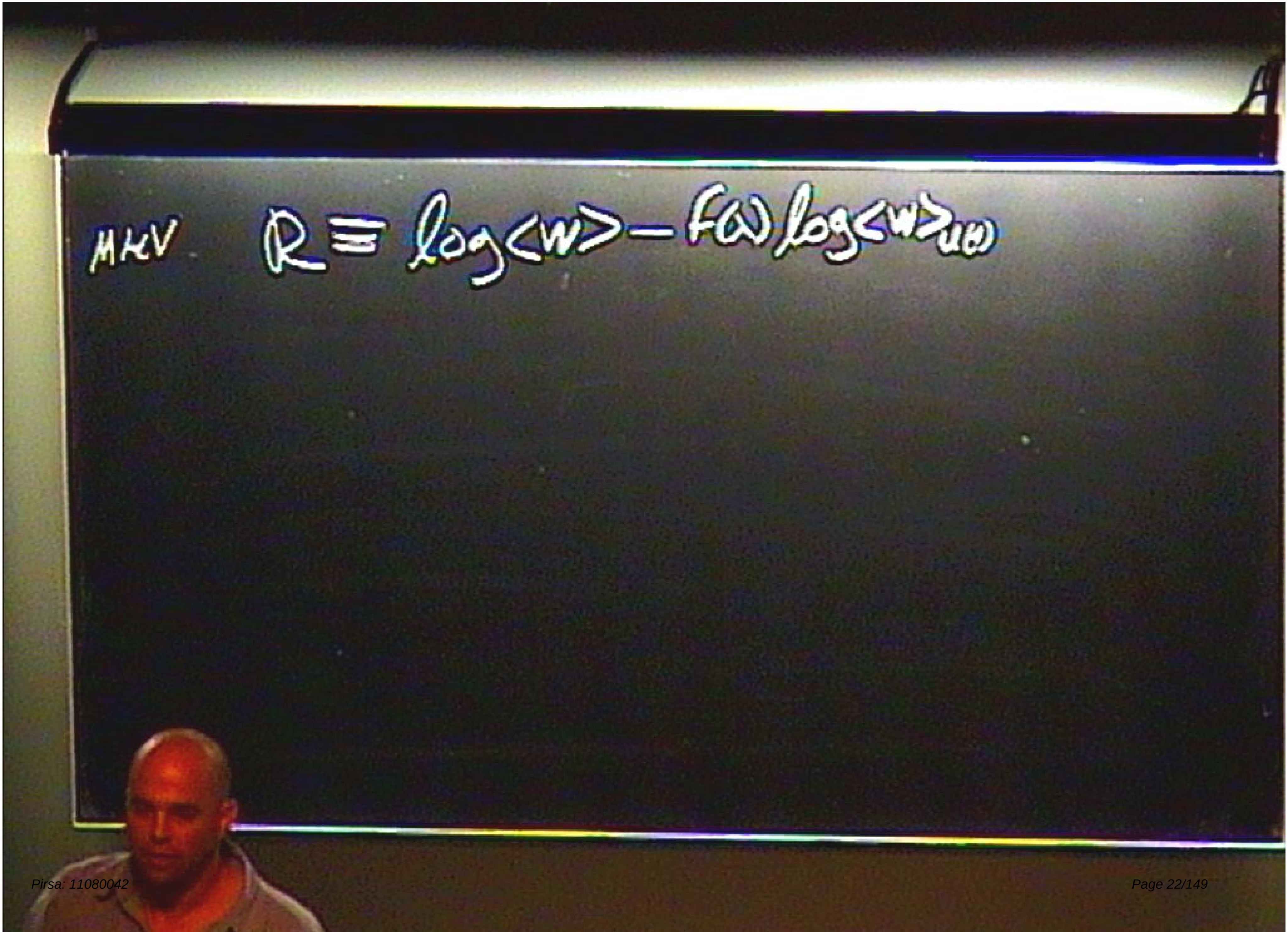
MHV + N^kMHV

Regularize ⟶ Finite $f(\chi_{ijkl})$

$$_{ijkl} = \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2} = e^{-\tau} \longrightarrow 0$$



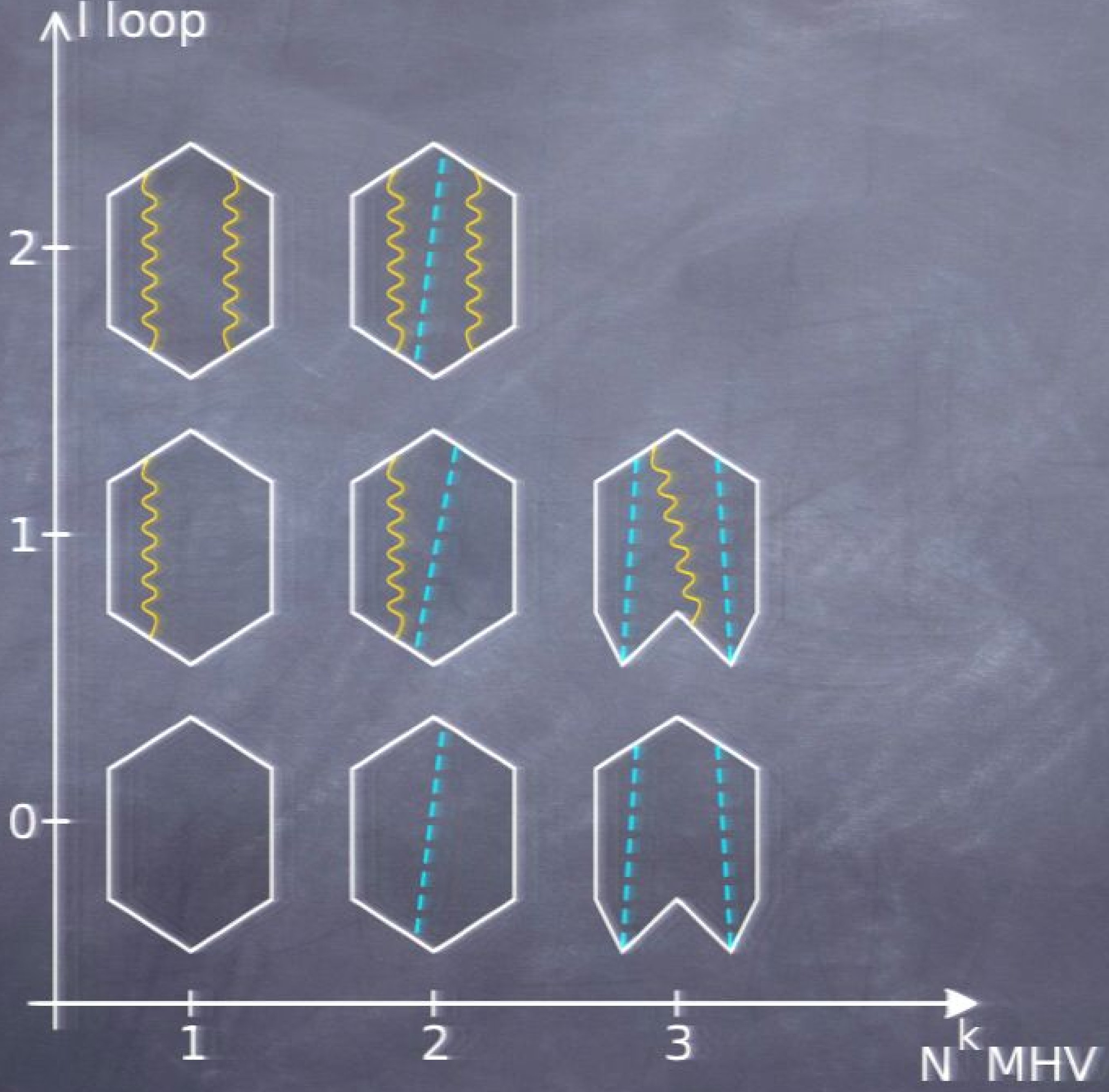
MHV

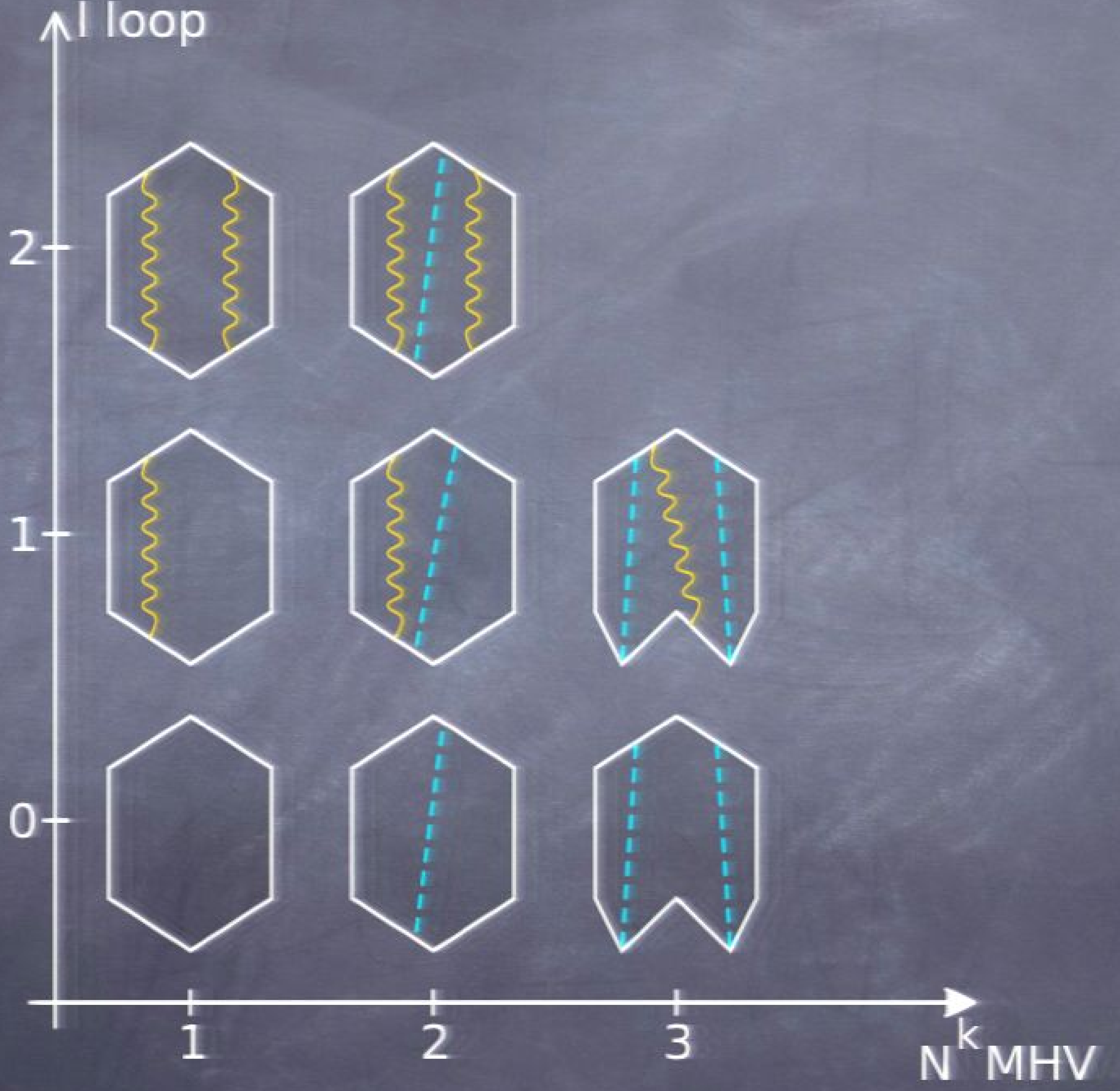


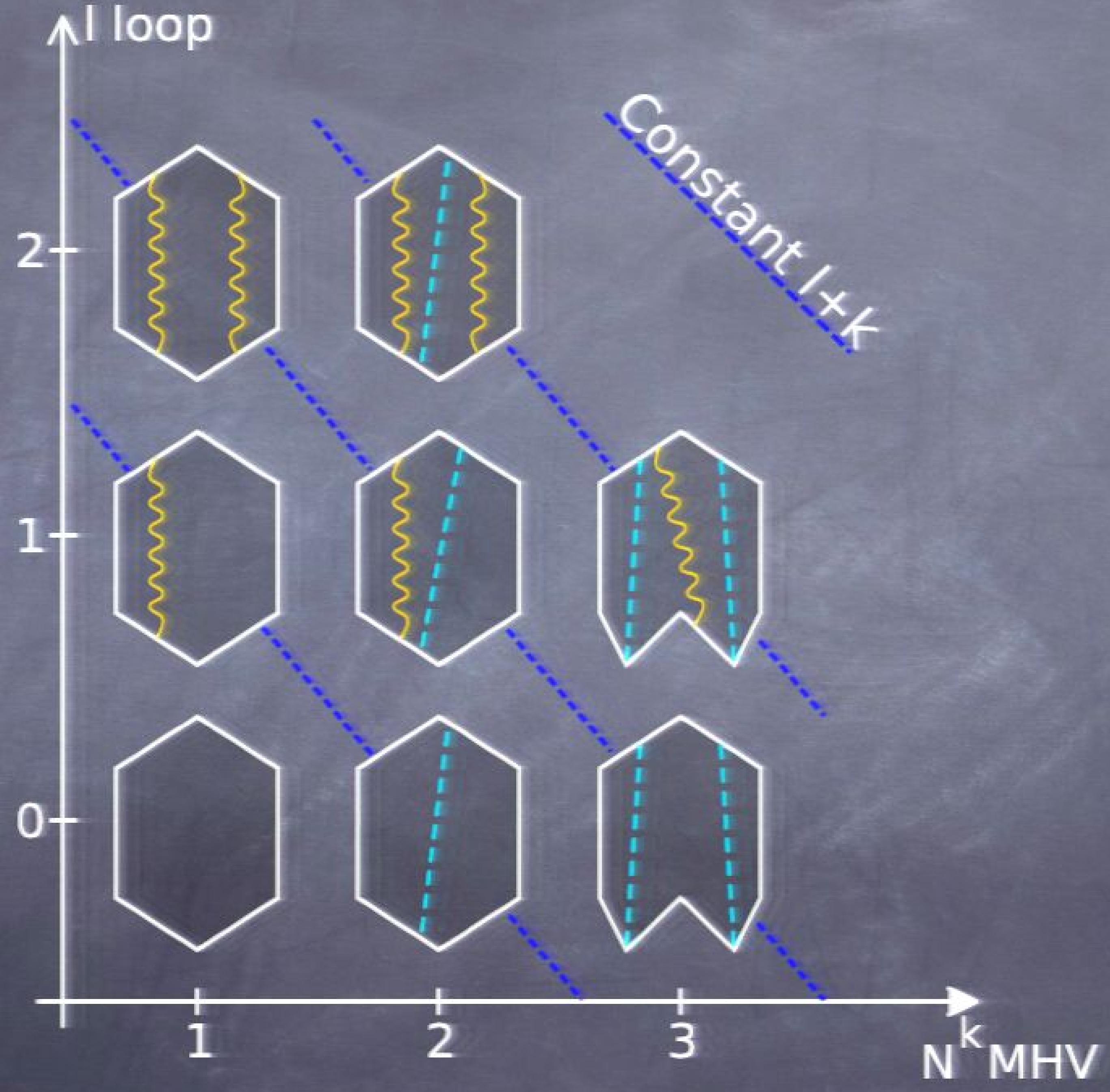
MKV $\mathbb{R} \equiv \log\langle w\rangle - f(\omega)\log\langle w\rangle_{u(\omega)}$

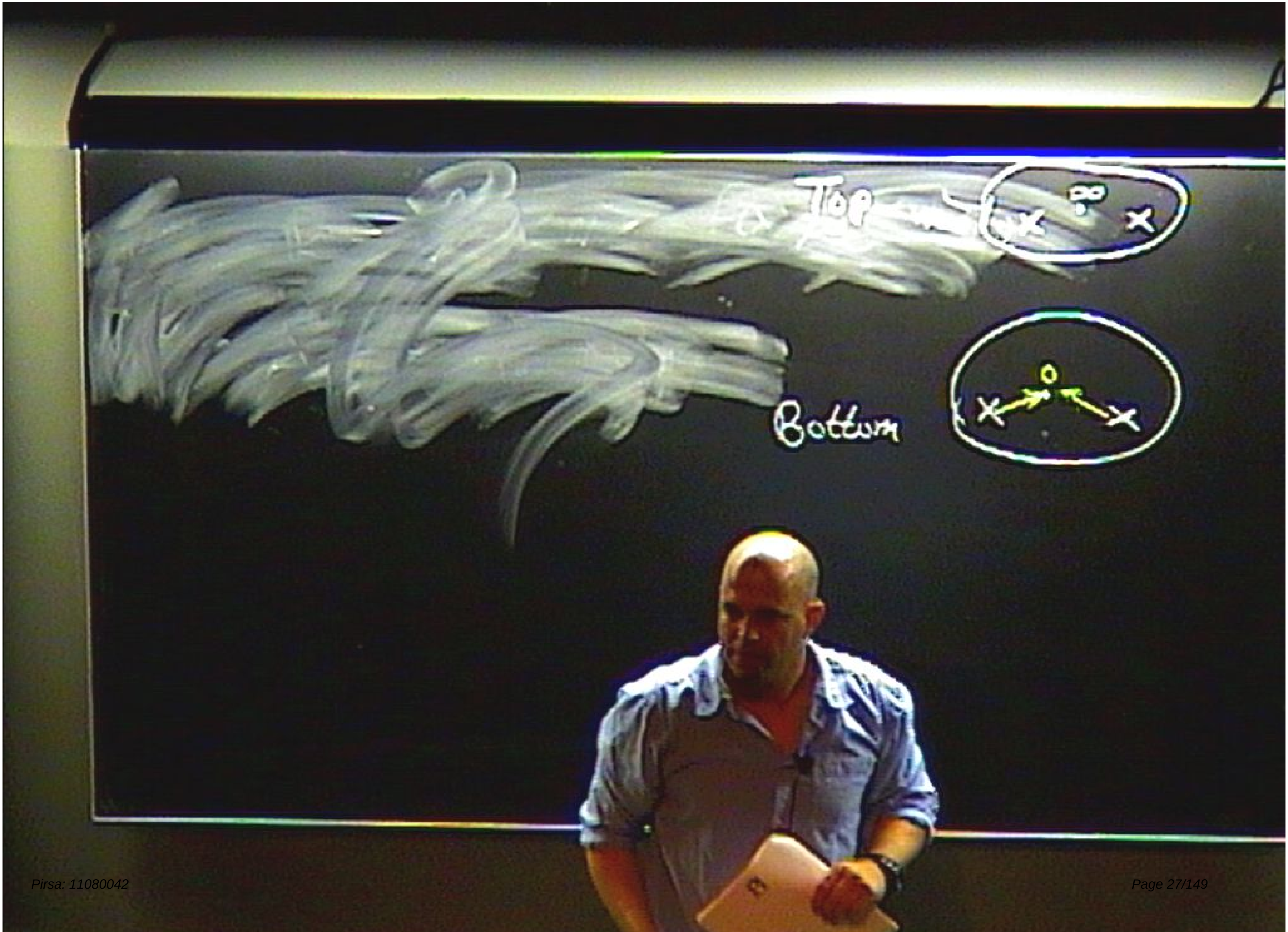
MHV $\mathcal{R} \equiv \log\langle W\rangle - f(\omega)\log\langle W\rangle_{U(1)}$

N^kMHV — $A^{(n)} = A^{(n)}_{MHV} \times \mathcal{R}_{\|}$


l loop
2
1
0
1
2
3
N^k MHV


l loop
2
1
0
1
2
3
N^k MHV


l loop
Constant l+k
2
1
0
1
2
3
N^k MHV


Top
x
x
Bottom
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o
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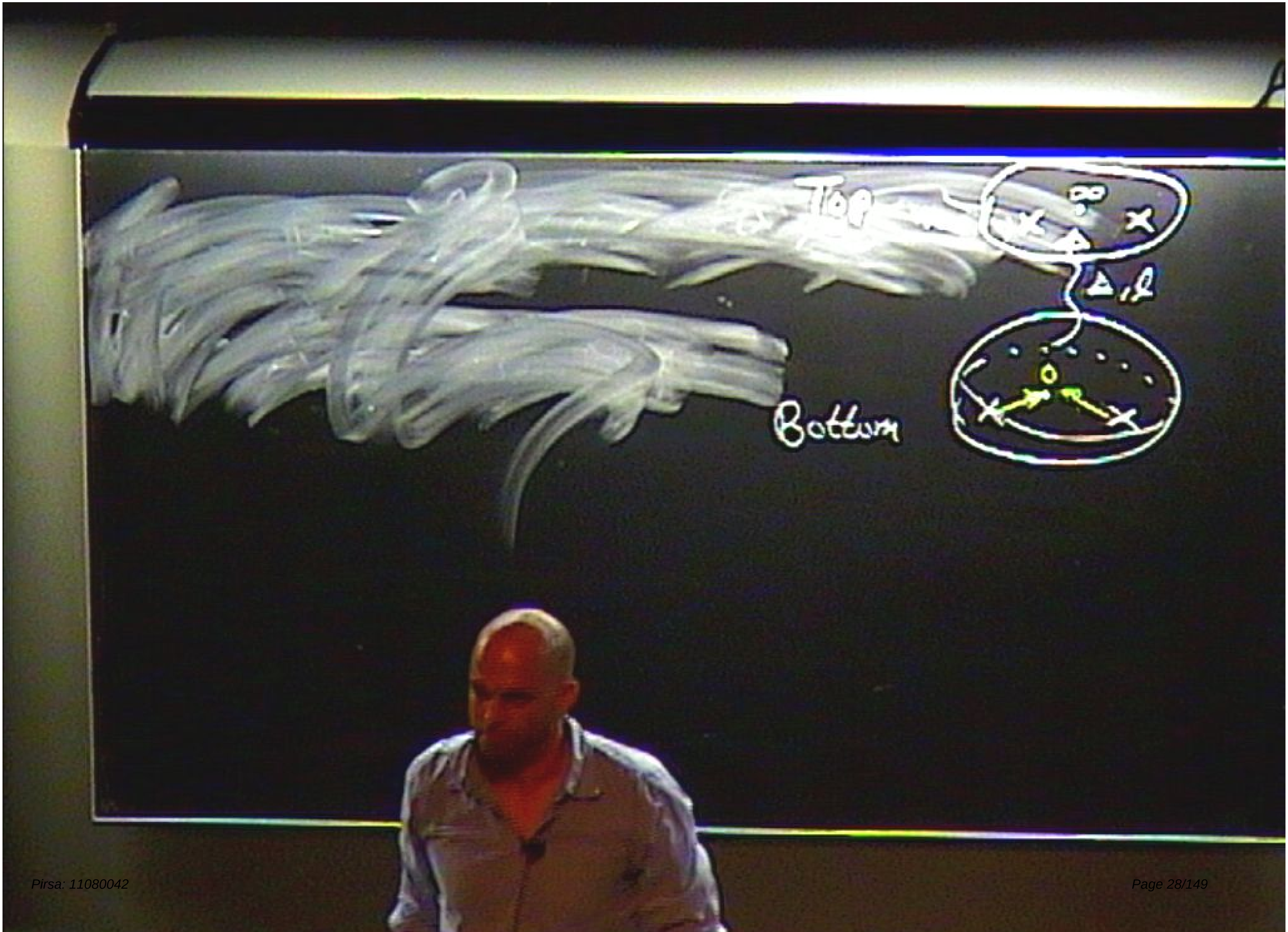
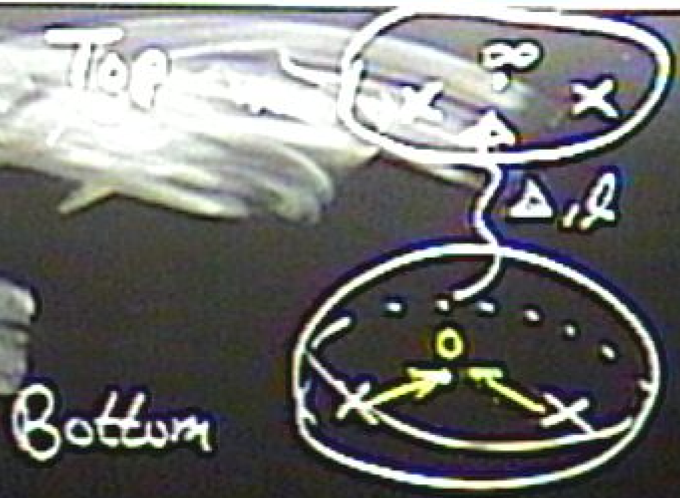

Top
∞
Δ, ℓ
0
Bottom

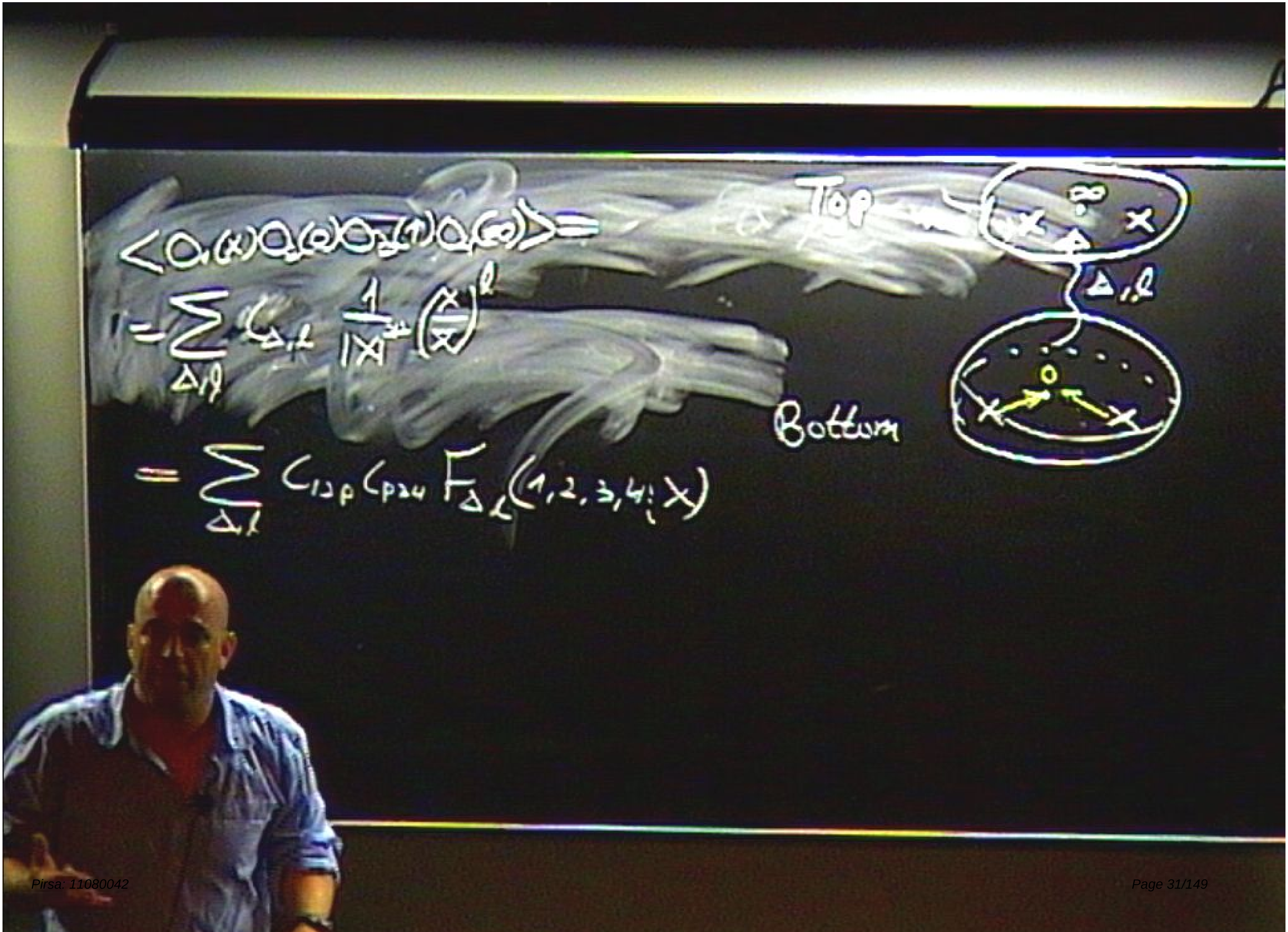
$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle = \sum_{\Delta, l} C_{\Delta, l} \frac{1}{|x|^{[illegible]}} \left(\frac{x}{|x|}\right)^{l}$$



$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle =$$

$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{\bar{x}}\right)^{\ell}$$

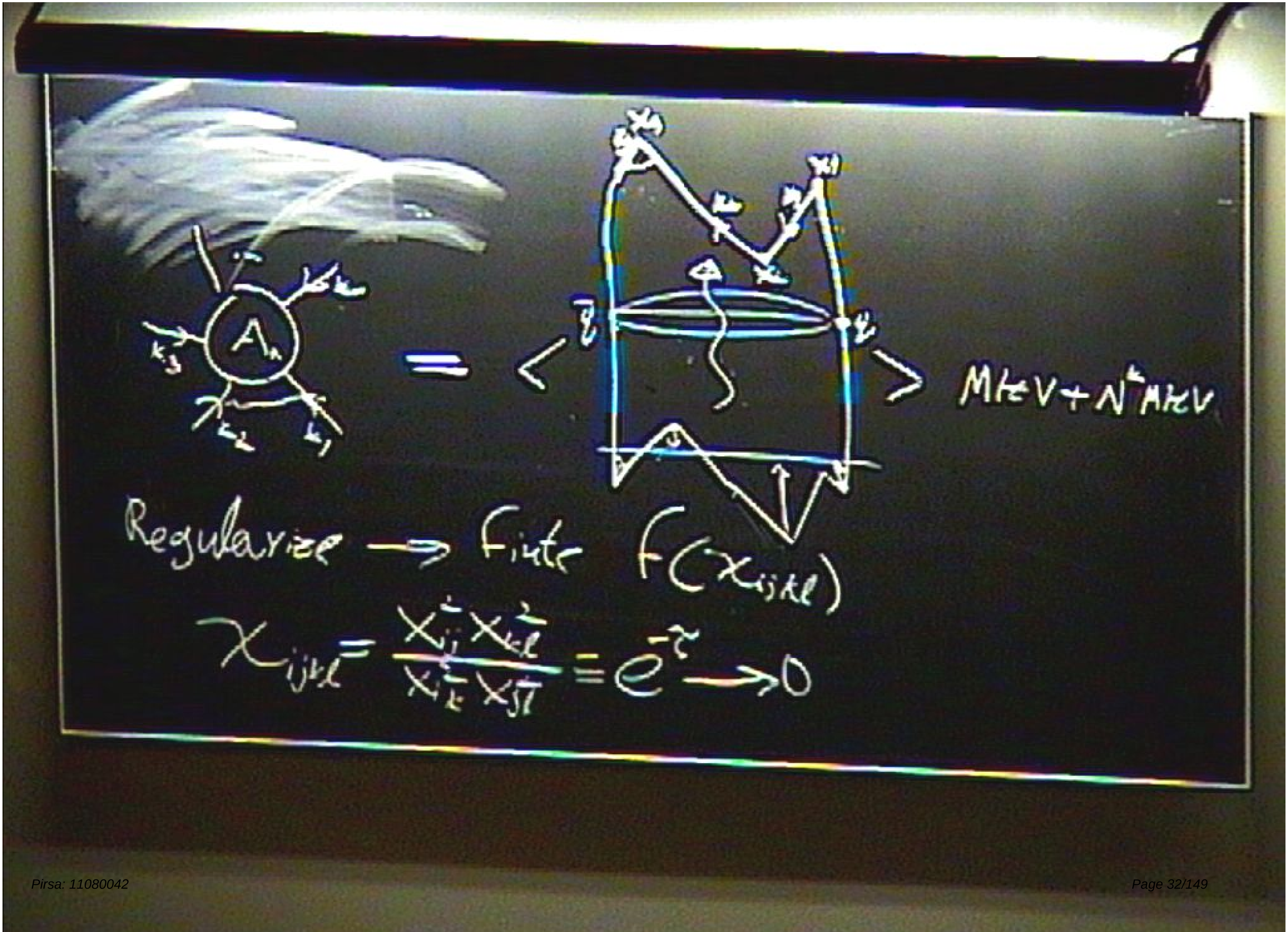
$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F$$



$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle =$$

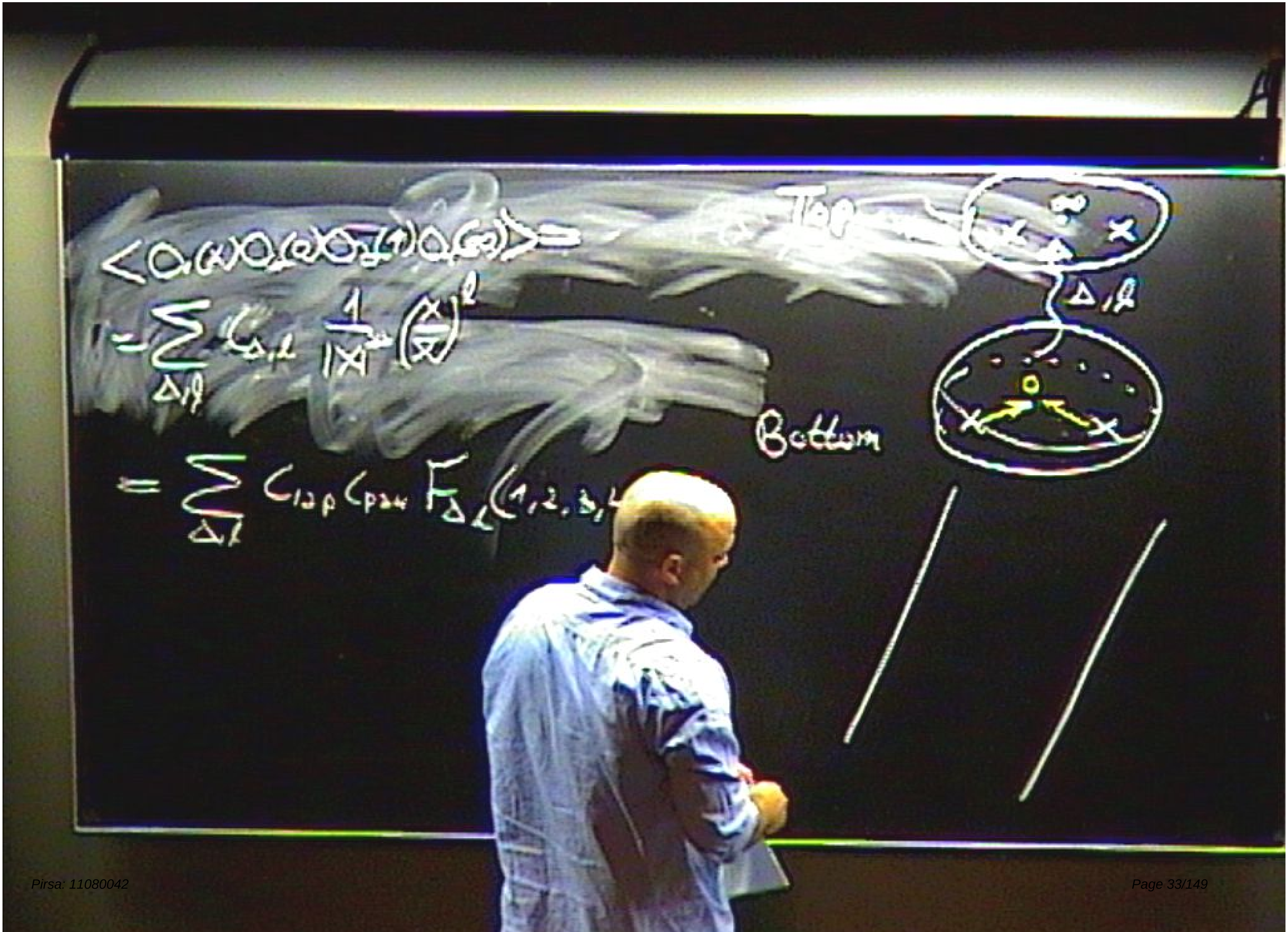
$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{\bar{x}}\right)^{\ell}$$

$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4; x)$$



Regularize → Finite $f(\chi_{ijkl})$

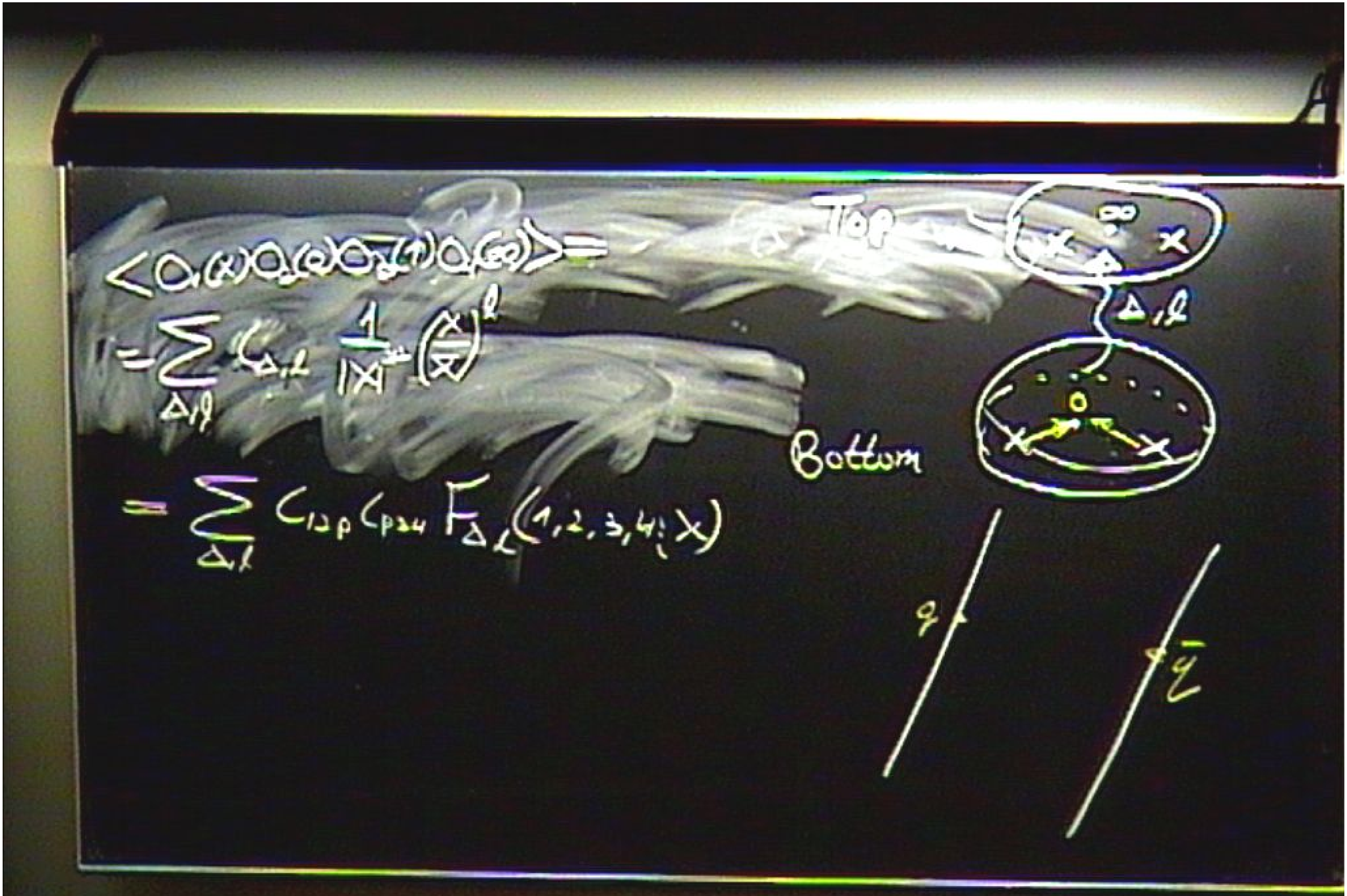
$$\chi_{ijkl} = \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2} = e^{-\tau} \to 0$$



$\langle O_1(x)O_2(0)O_3(1)O_4(\infty)\rangle =$

$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{|x|}\right)^{\ell}$

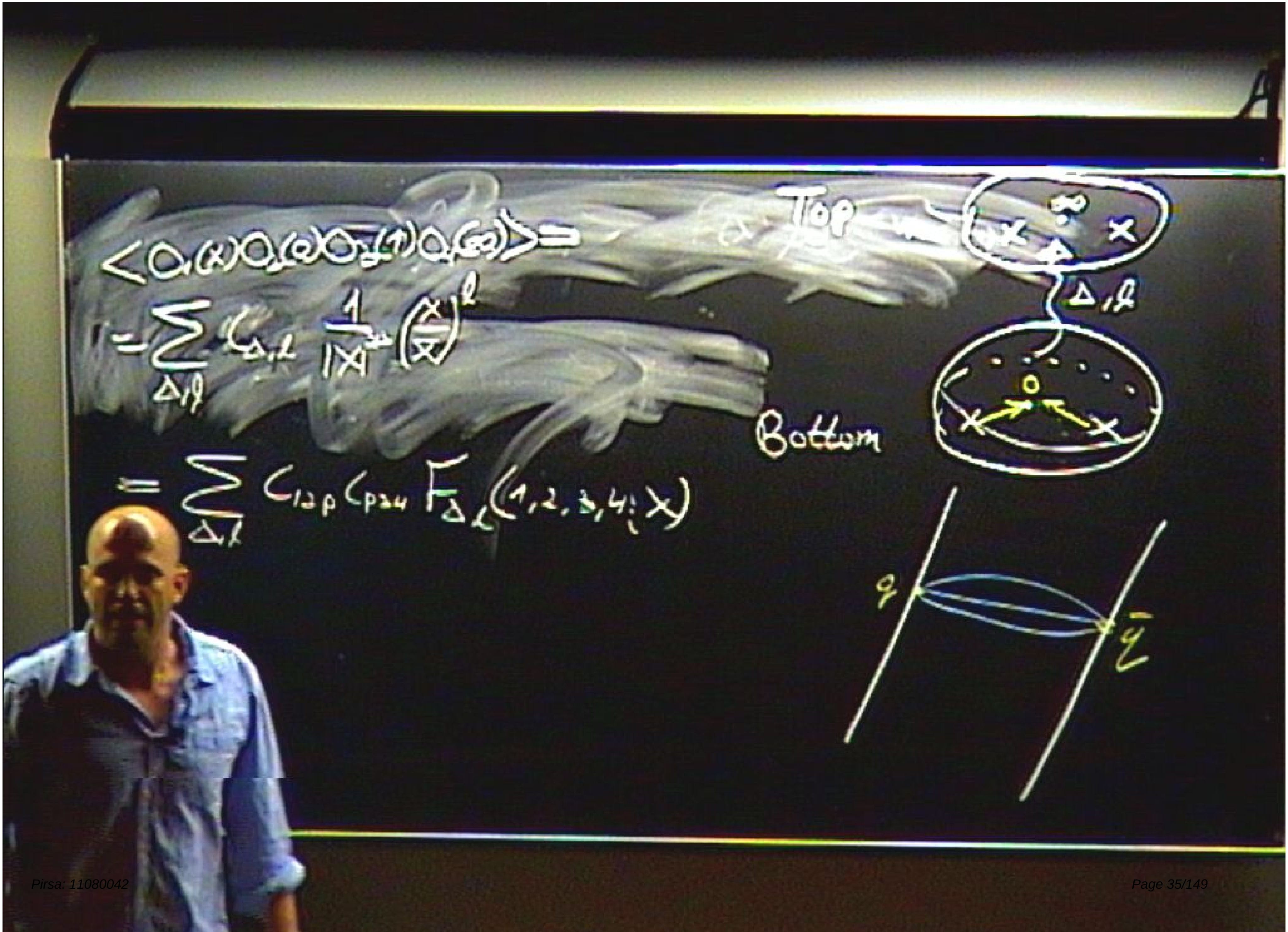
$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4$



$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle =$$

$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{\bar{x}}\right)^{\ell}$$

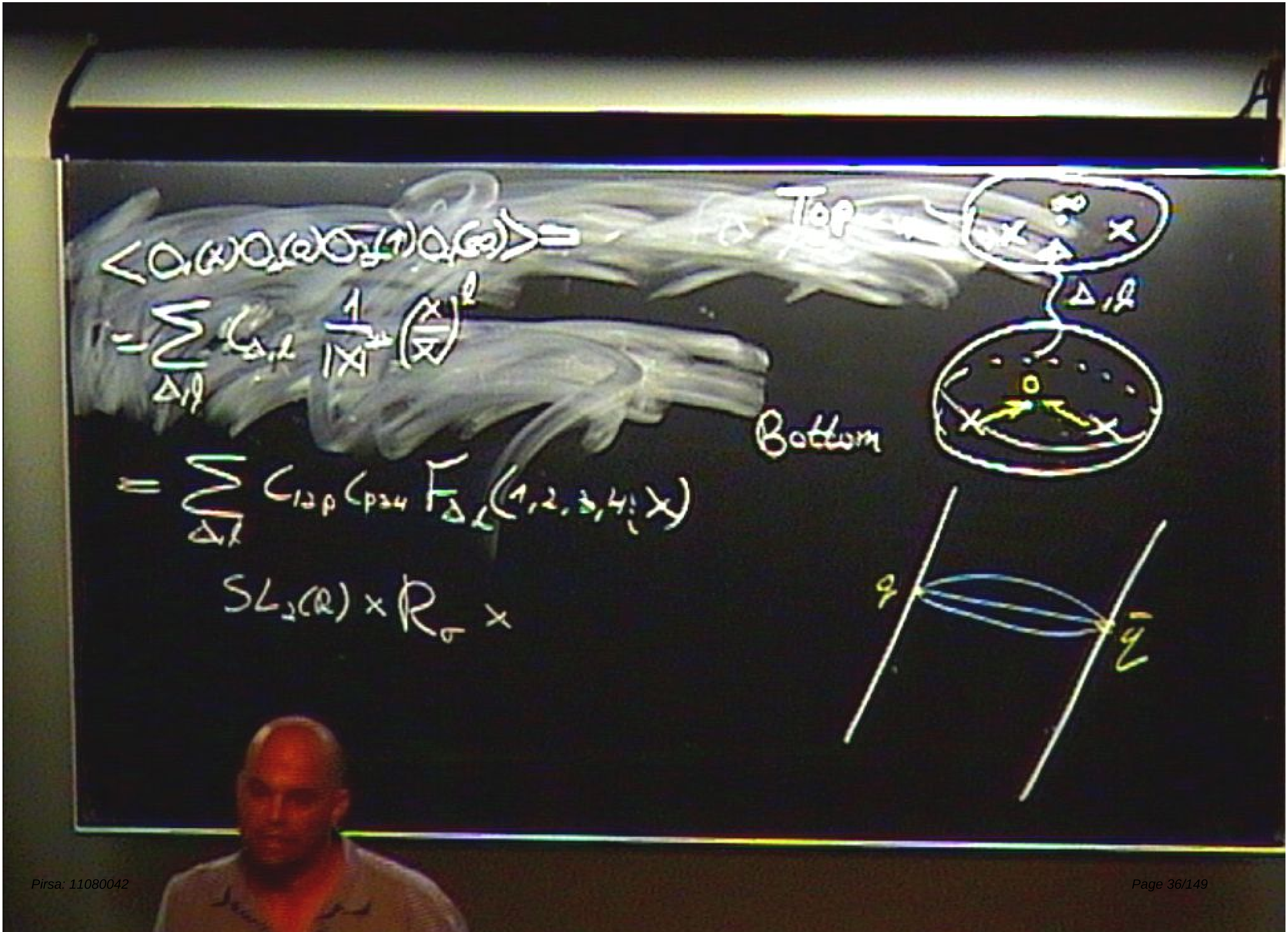
$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4;x)$$



$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle =$$

$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{\bar{x}}\right)^{\ell}$$

$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4; x)$$

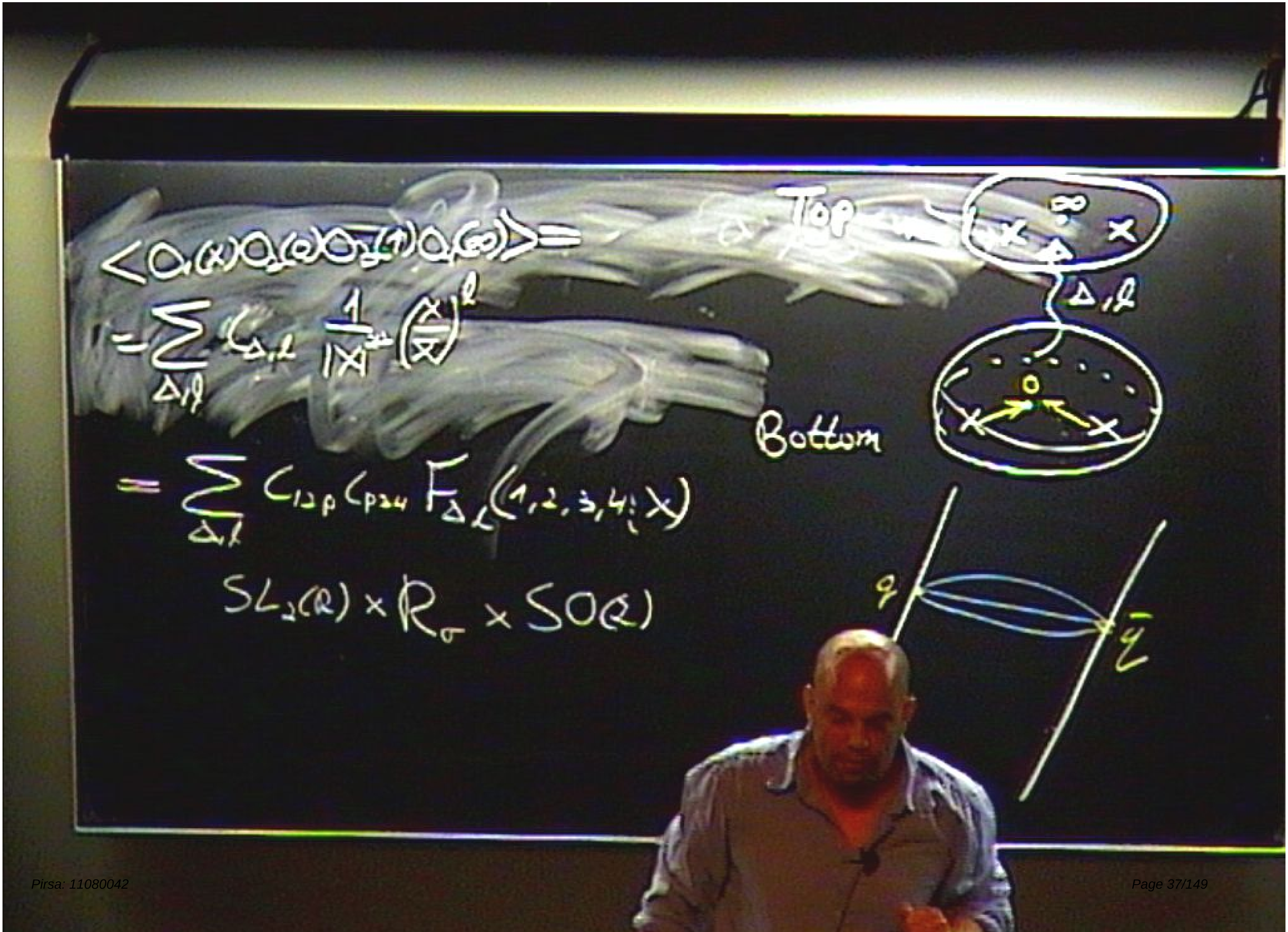


$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty)\rangle =$$

$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{|x|}\right)^{\ell}$$

$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4;x)$$

$$SL_2(\mathbb{R}) \times \mathbb{R}_{\sigma} \times$$

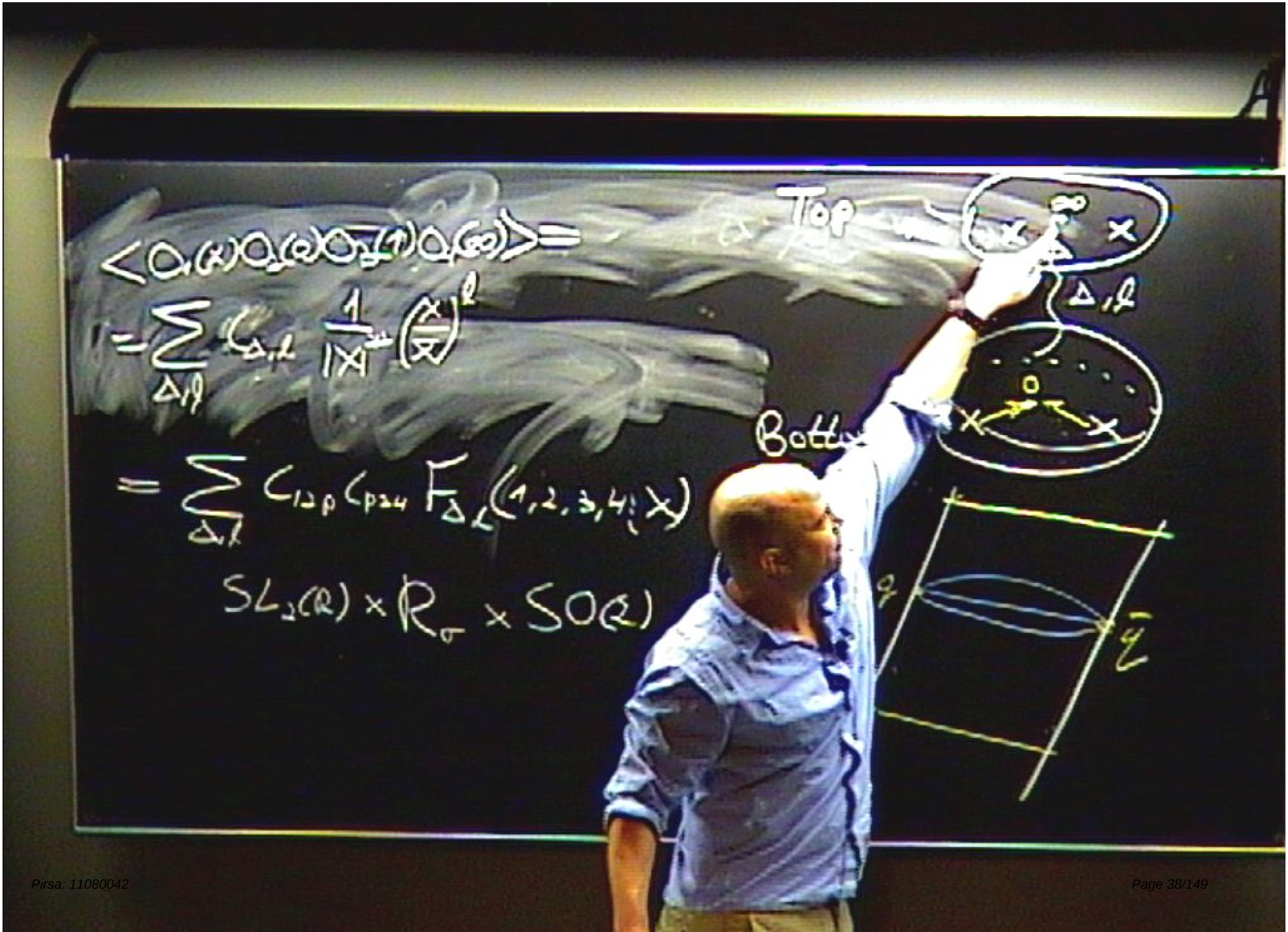


$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle =$$

$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{|x|}\right)^{\ell}$$

$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4; x)$$

$$SL_2(\mathbb{R}) \times \mathbb{R}_{\sigma} \times SO(2)$$



$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle =$$

$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|} \left(\frac{x}{x}\right)^{\ell}$$

$$= \sum_{\Delta,\ell} C_{12p} C_{p24} F_{\Delta,\ell}(1,2,3,4; x)$$

$$SL_2(\mathbb{R}) \times \mathbb{R}_\sigma \times SO(2)$$

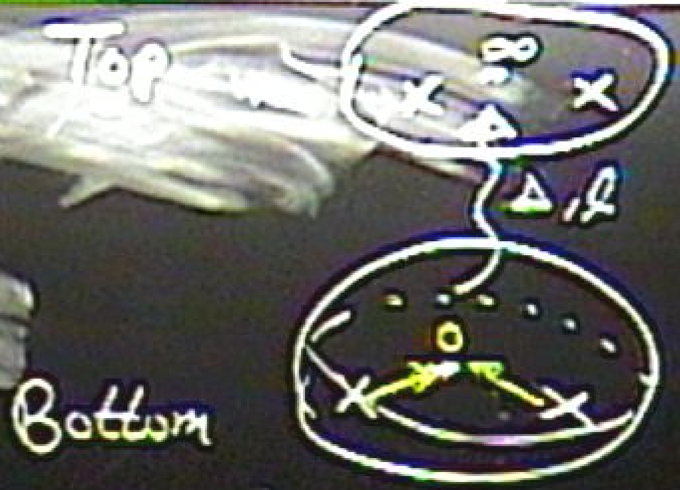
$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle =$$

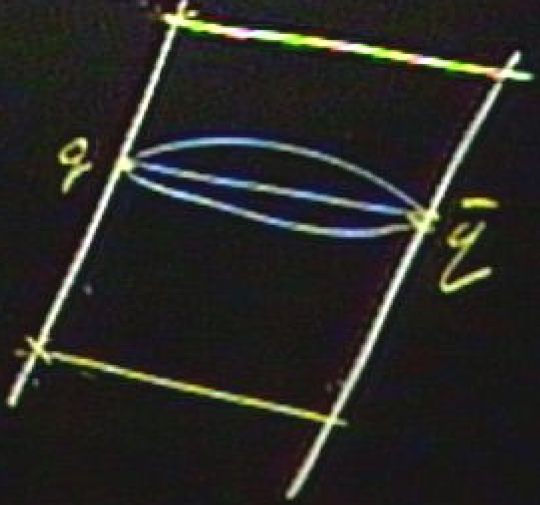
$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{|x|}\right)^{\ell}$$

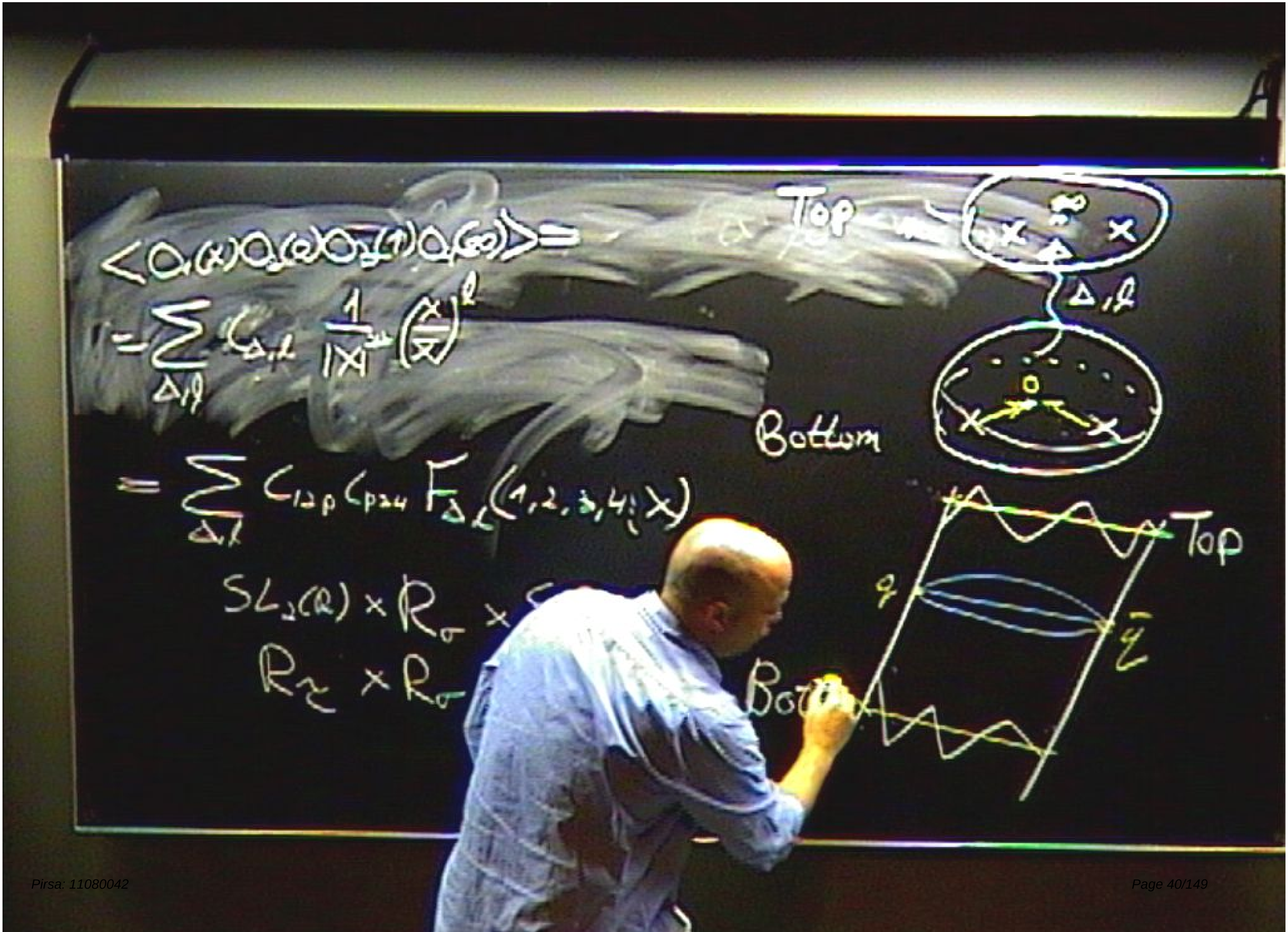
$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4;x)$$

$$SL_2(\mathbb{R}) \times R_\sigma \times SO(2)$$

$$R_\tau \times R_\sigma \times SO(2)_\rho$$





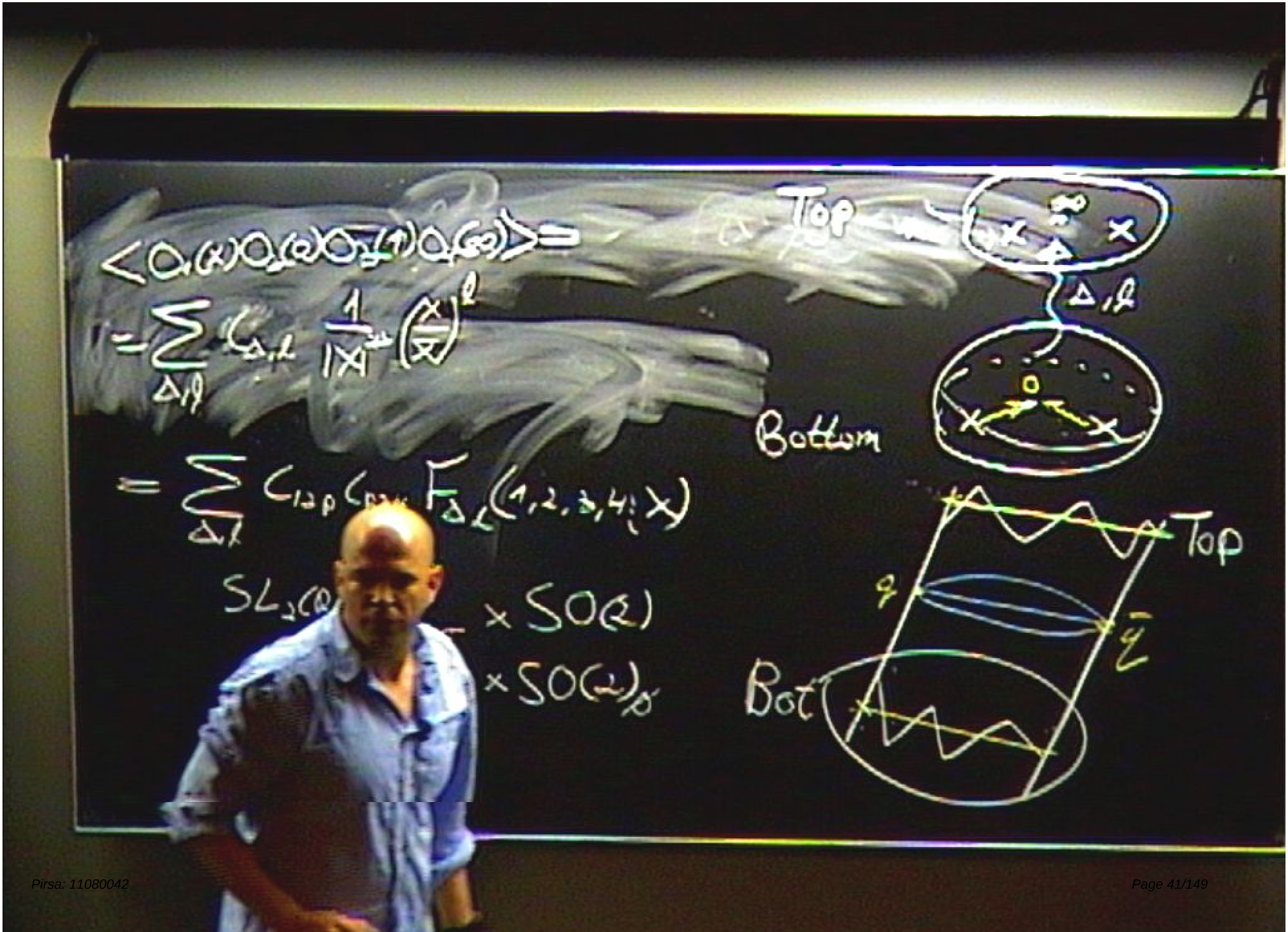


$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty)\rangle = \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{2\Delta}} \left(\frac{x}{\bar{x}}\right)^{\ell}$$

$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4;x)$$

$SL_2(\mathbb{R}) \times \mathbb{R}_\sigma \times$

$\mathbb{R}_\tau \times \mathbb{R}_\sigma$



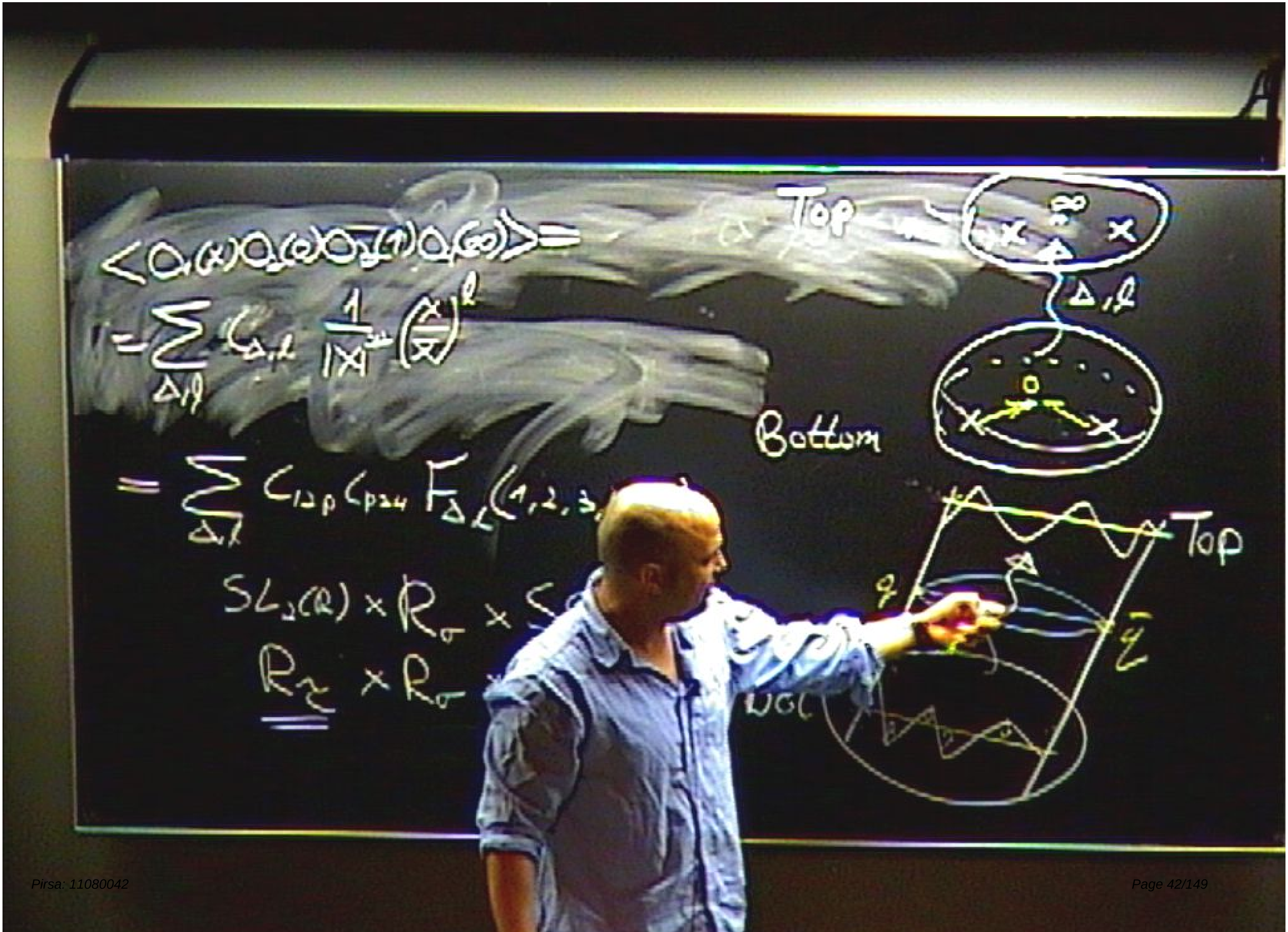
$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle =$

$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{\bar{x}}\right)^{\ell}$

$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4; x)$

$SL_2(\mathbb{C})$ $\times SO(2)$

$\times SO(2)_p$


$\langle O_1(x) O_2(0) O_3(1) O_4(\infty) \rangle =$
$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{\bar{x}}\right)^{\ell}$
$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,$
$SL_2(\mathbb{R}) \times R_\sigma \times S$
$R_\tau \times R_\sigma$
Top
∞
Δ, ℓ
Bottom
0
Top
q
$\bar{q}$

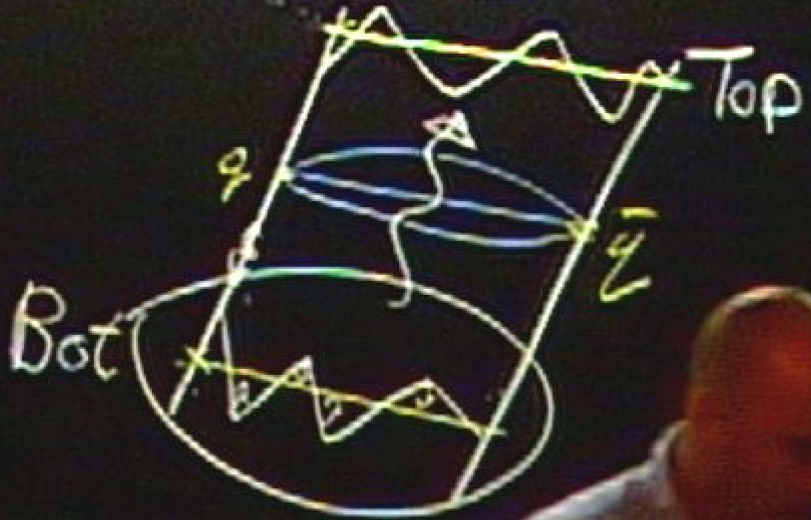
$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty)\rangle =$$

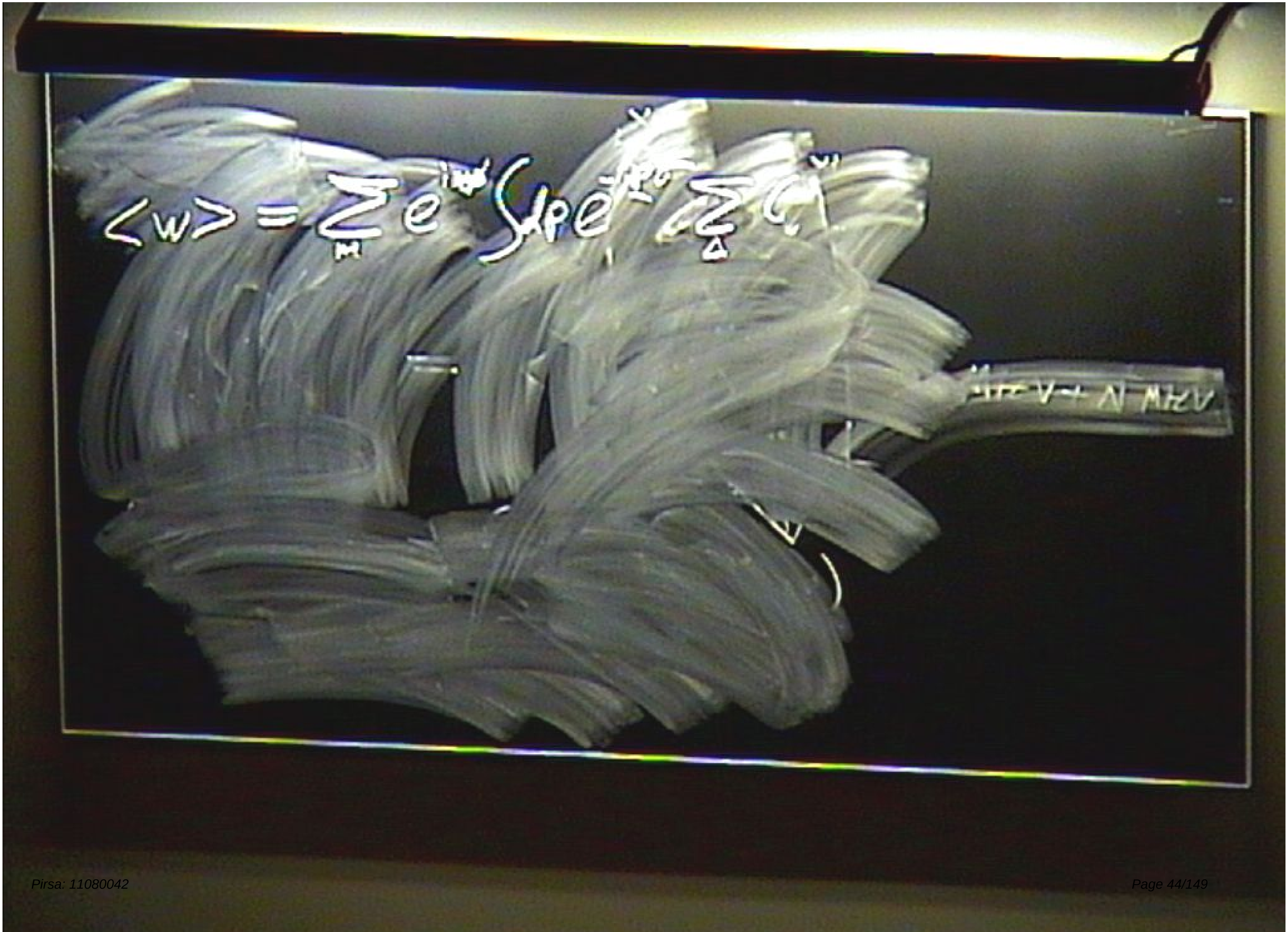
$$= \sum_{\Delta,l} C_{\Delta,l} \frac{1}{|x|^{\Delta}} \left(\frac{x}{\bar{x}}\right)^{l}$$

$$= \sum_{\Delta,l} C_{12p} C_{p34} F_{\Delta,l}(1,2,3,4;x)$$

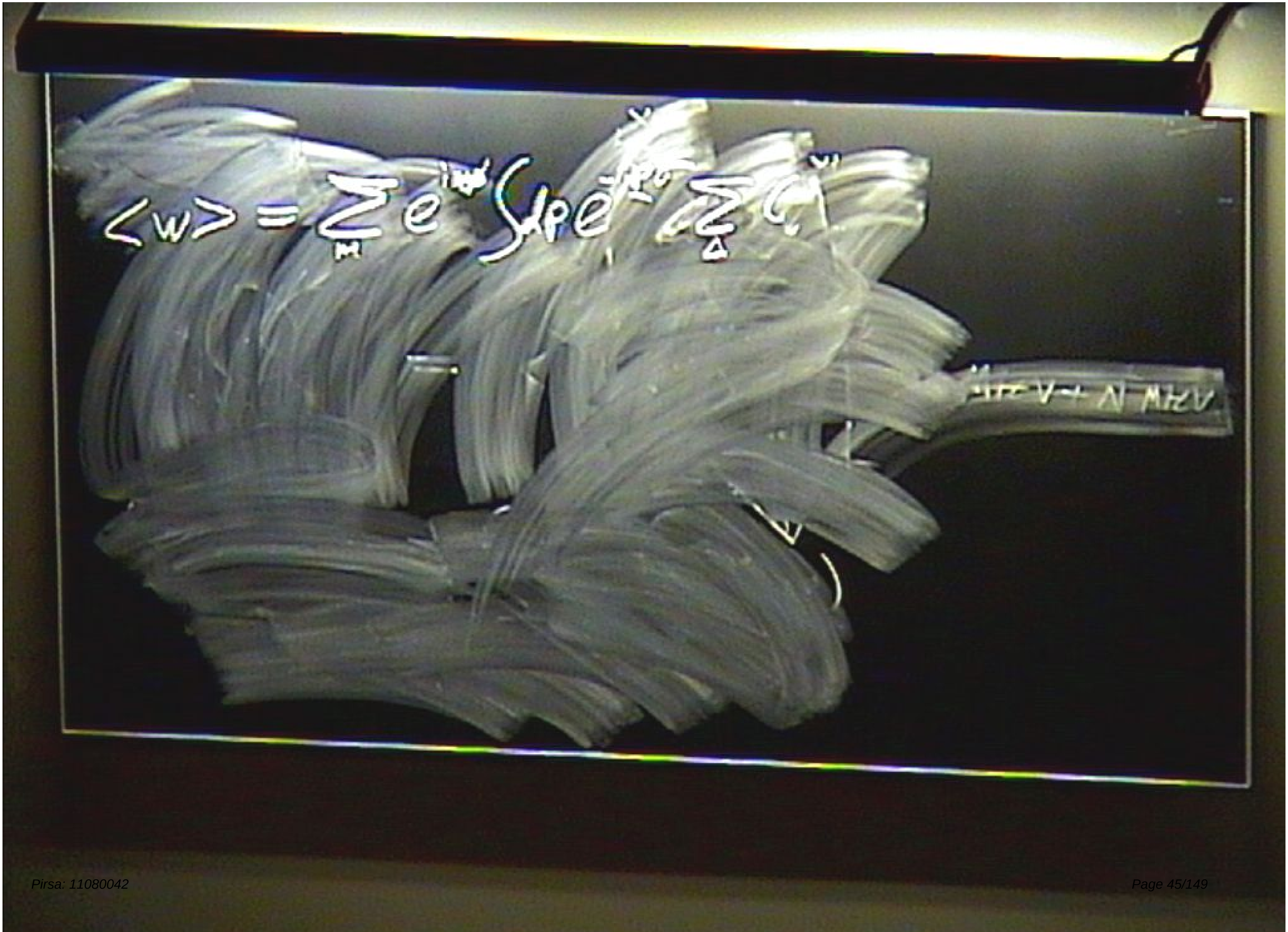
$$SL_2(\mathbb{R}) \times R_\sigma \times SO(2)$$

$$\underline{\underline{R_\tau}} \times R_\sigma \times SO(2)_\rho$$

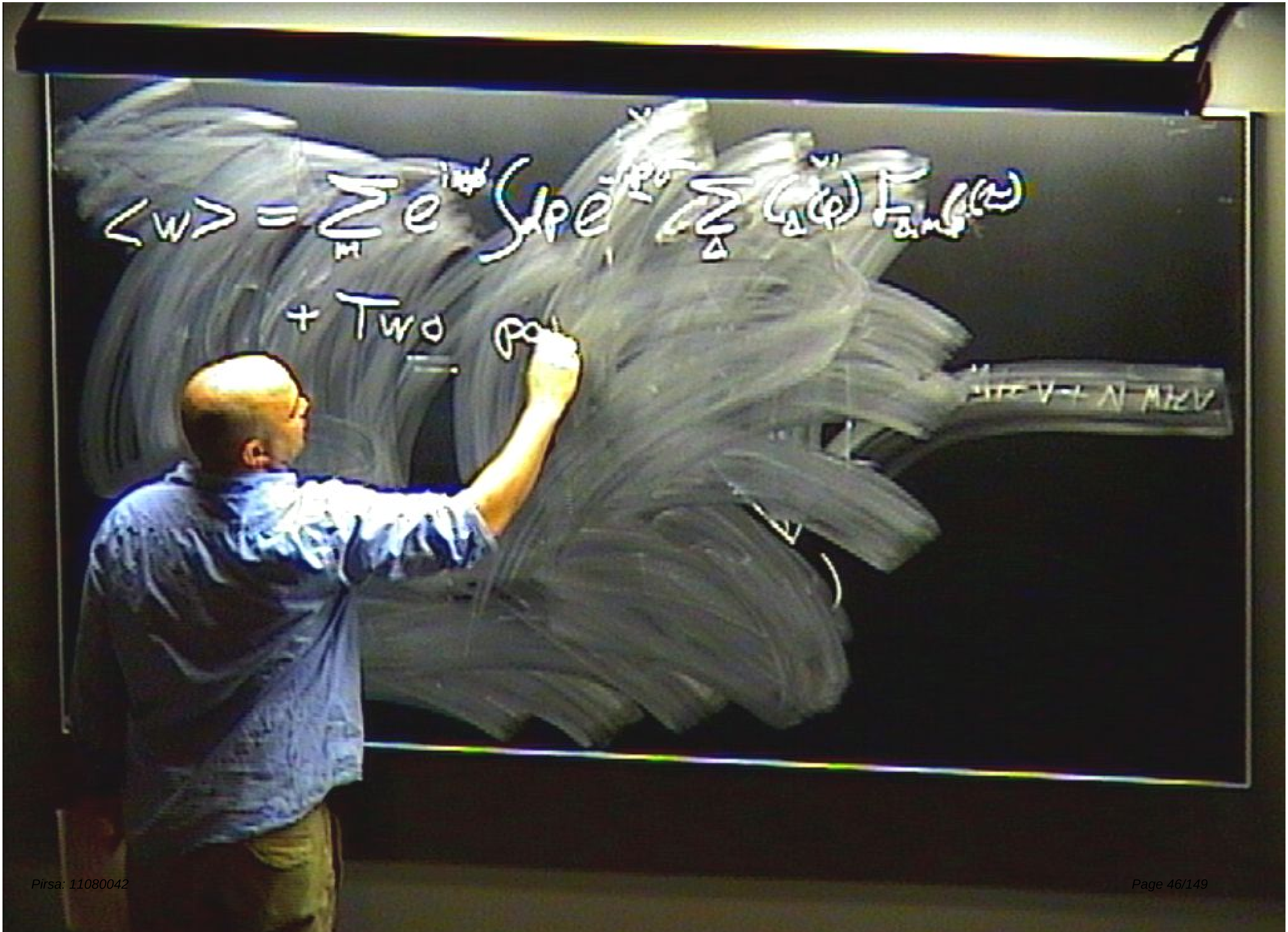




$$\langle W \rangle = \sum_{M} e^{iM\theta} \int dp\, e^{-ip\theta} \sum_{\Delta} C_{\Delta}$$

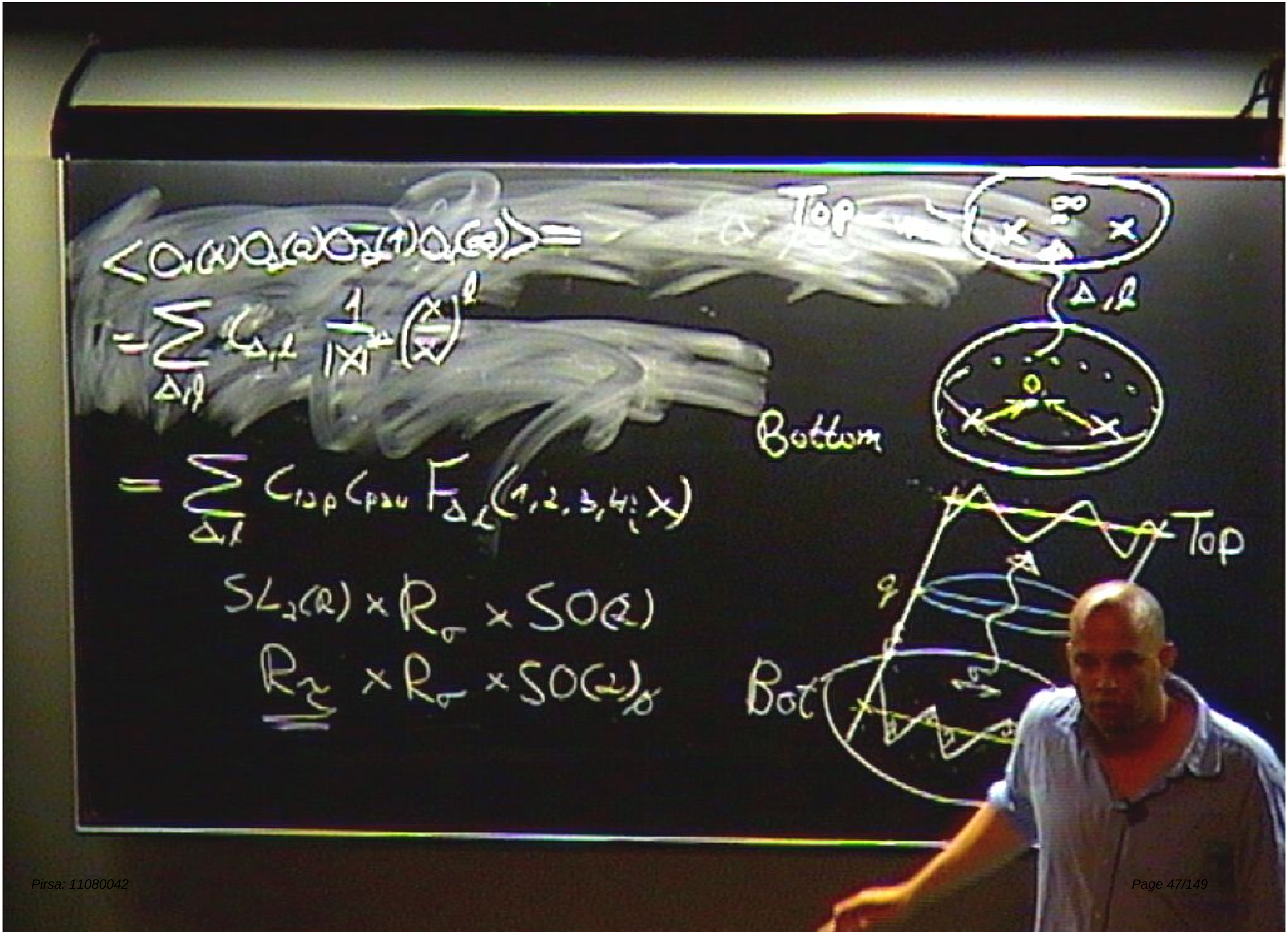


$$\langle W \rangle = \sum_{M} e^{[illegible]} \int dp\, e^{[illegible]} \sum_{\Delta} C$$



$$\langle W \rangle = \sum_{m} e^{im\phi} \int dp\, e^{-ip\sigma} \sum_{\Delta} C_{\Delta}(p)\, F_{\Delta,m,G}(u)$$

$+$ Two po



$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty)\rangle =$$

$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{\bar{x}}{x}\right)^{\ell}$$

$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4;x)$$

$$SL_2(\mathbb{R}) \times R_\sigma \times SO(2)$$

$$\underline{R_\tau} \times R_\sigma \times SO(2)_\rho$$

$$R^{1\text{-loop}}_{2356} = \tau^{\ell} D_0 + \tau^{\ell-1} D_1 + \dots + D_\ell$$

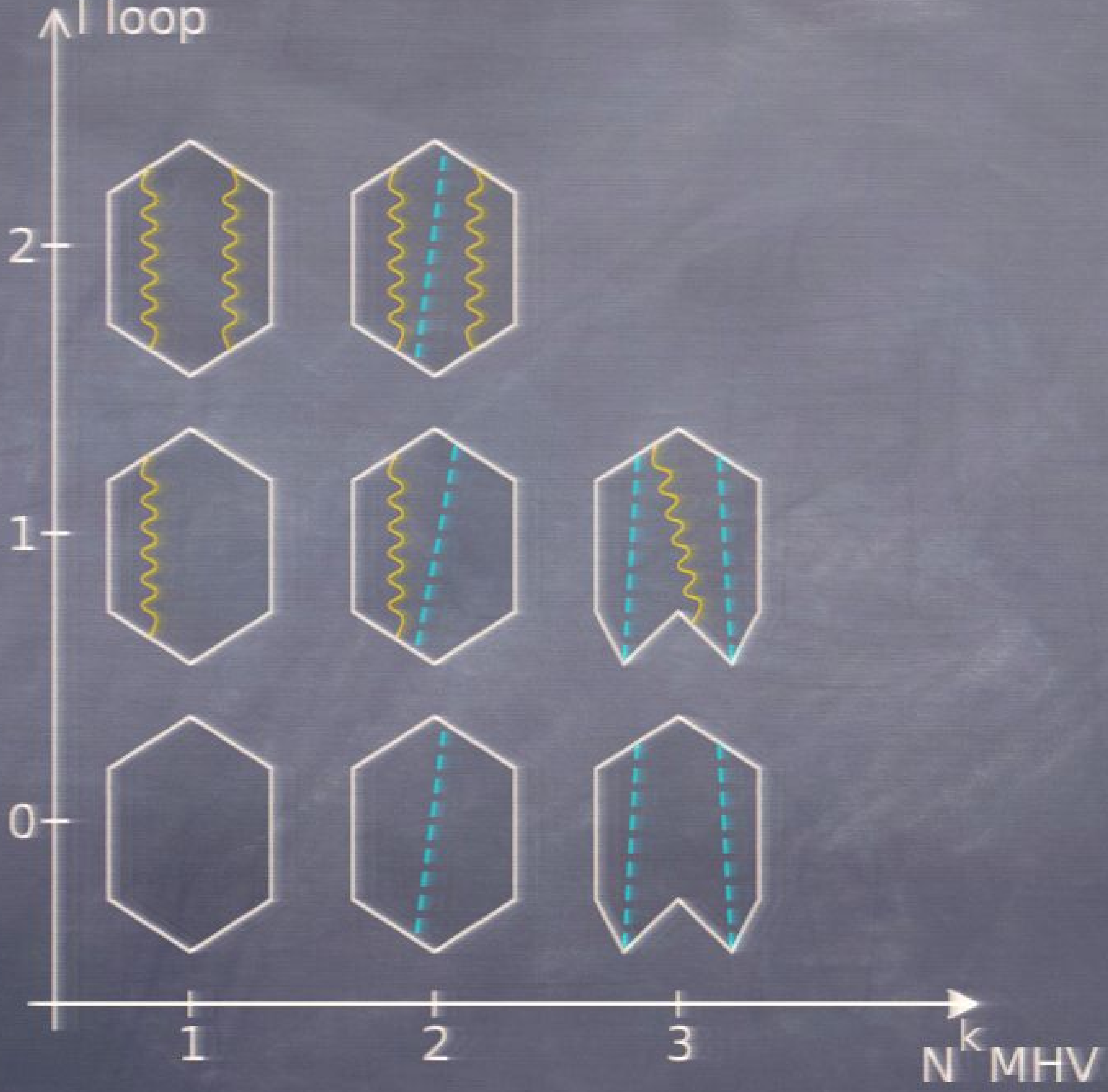
$$\gamma_{\pm}(p) = 2g^2\left[\psi\left(\tfrac{1}{2}+\tfrac{ip}{2}\right) + \psi\left(\tfrac{1}{2}-\tfrac{ip}{2}\right) - 2\psi(1)\right]$$

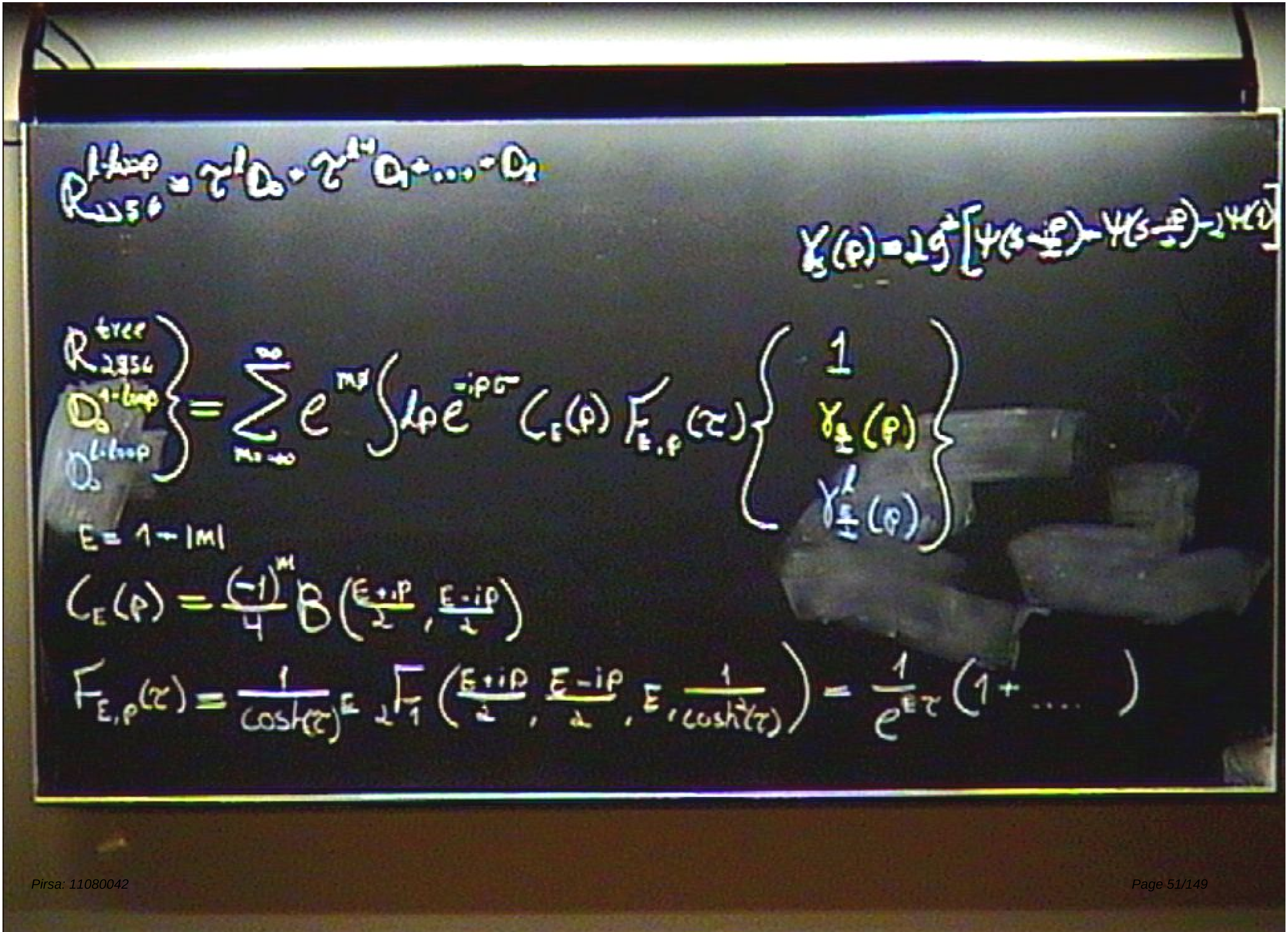
$$\left.\begin{matrix} R^{\text{tree}}_{2356} \\ D_0^{1\text{-loop}} \\ D_0^{\ell\text{-loop}} \end{matrix}\right\} = \sum_{m=-\infty}^{\infty} e^{m\phi} \int dp\, e^{-ip\sigma}\, C_E(p)\, F_{E,p}(\tau) \left\{\begin{matrix} 1 \\ \gamma_{\pm}(p) \\ \gamma^{\ell}_{\pm}(p) \end{matrix}\right\}$$

$$E = 1 - |m|$$

$$C_E(p) = \frac{(-1)^m}{4} B\left(\frac{E+ip}{2}, \frac{E-ip}{2}\right)$$

$$F_{E,p}(\tau) = \frac{1}{\cosh(\tau)^E}\, {}_2F_1\left(\frac{E+ip}{2}, \frac{E-ip}{2}, E, \frac{1}{\cosh^2(\tau)}\right) = \frac{1}{e^{E\tau}}\left(1 + \dots\right)$$


l loop
2
1
0
1
2
3
N^{k}MHV



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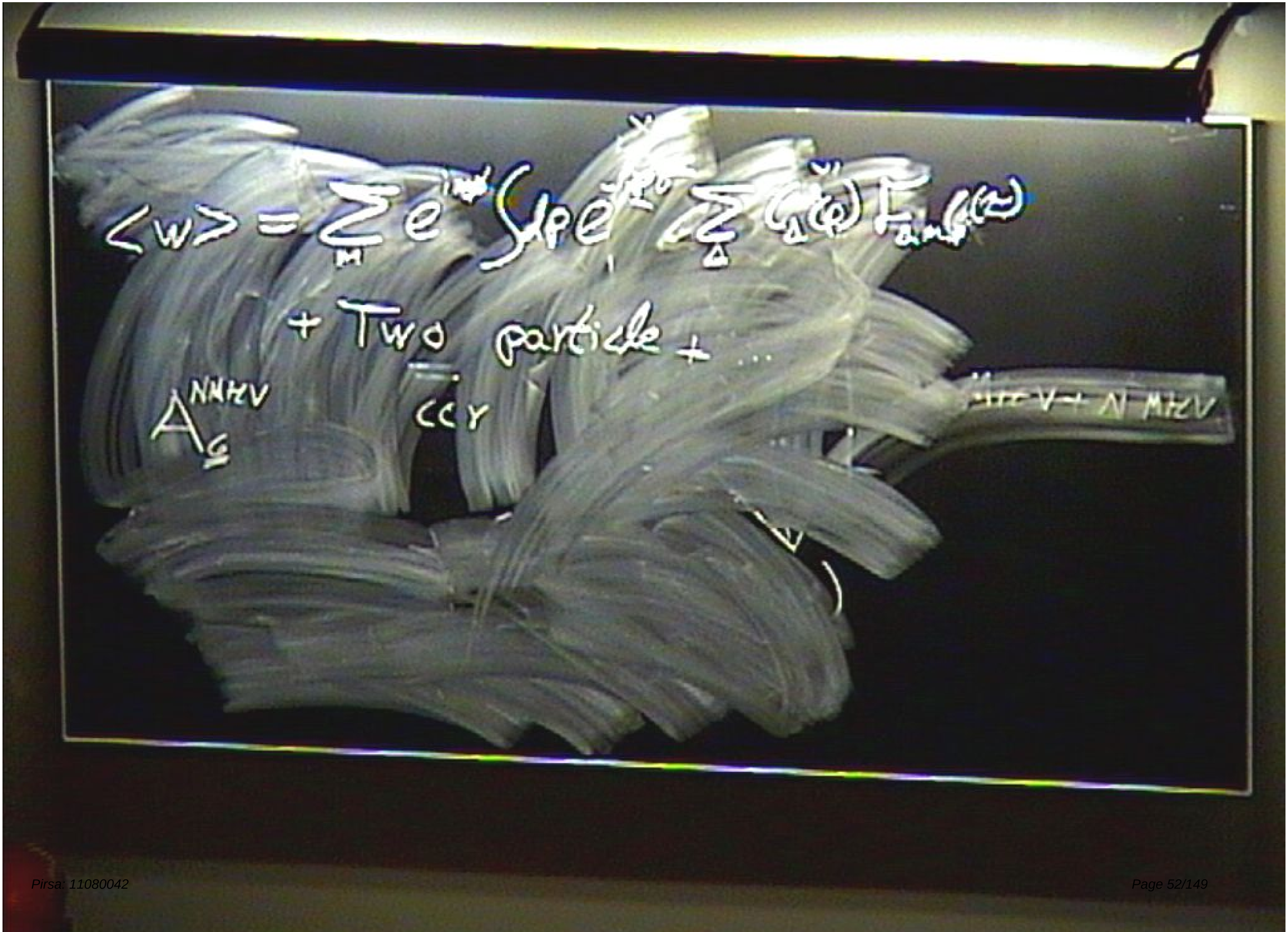


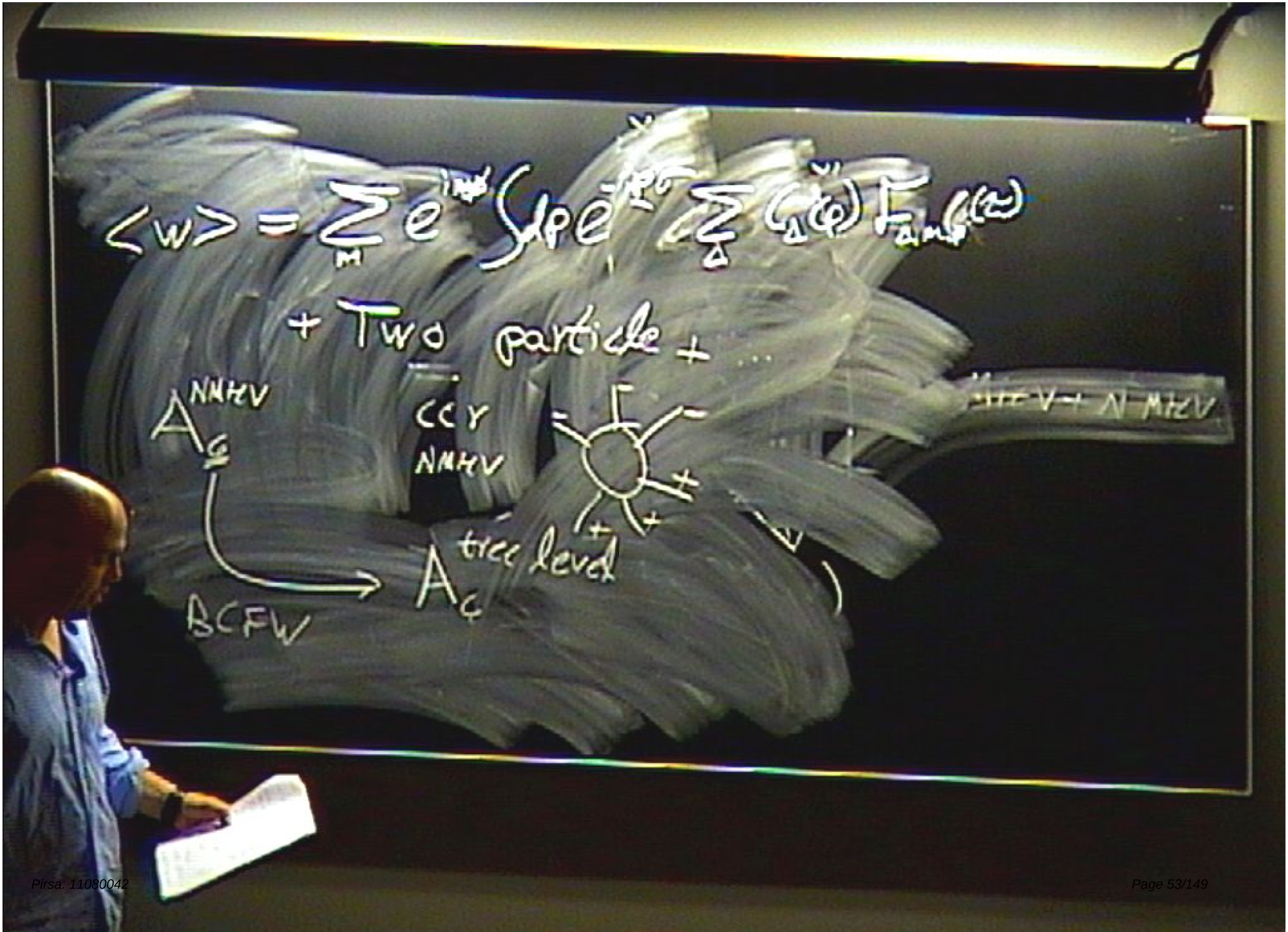
$$\langle W \rangle = \sum_{M} e^{iM\phi} \int dp\, e^{-E\tau} \sum_{\Delta} C_{\Delta}(\phi) F_{\Delta}(\tau)$$

+ Two particle + ...

A_6^{NMHV}

$\overline{CCY}$

MHV + N MHV



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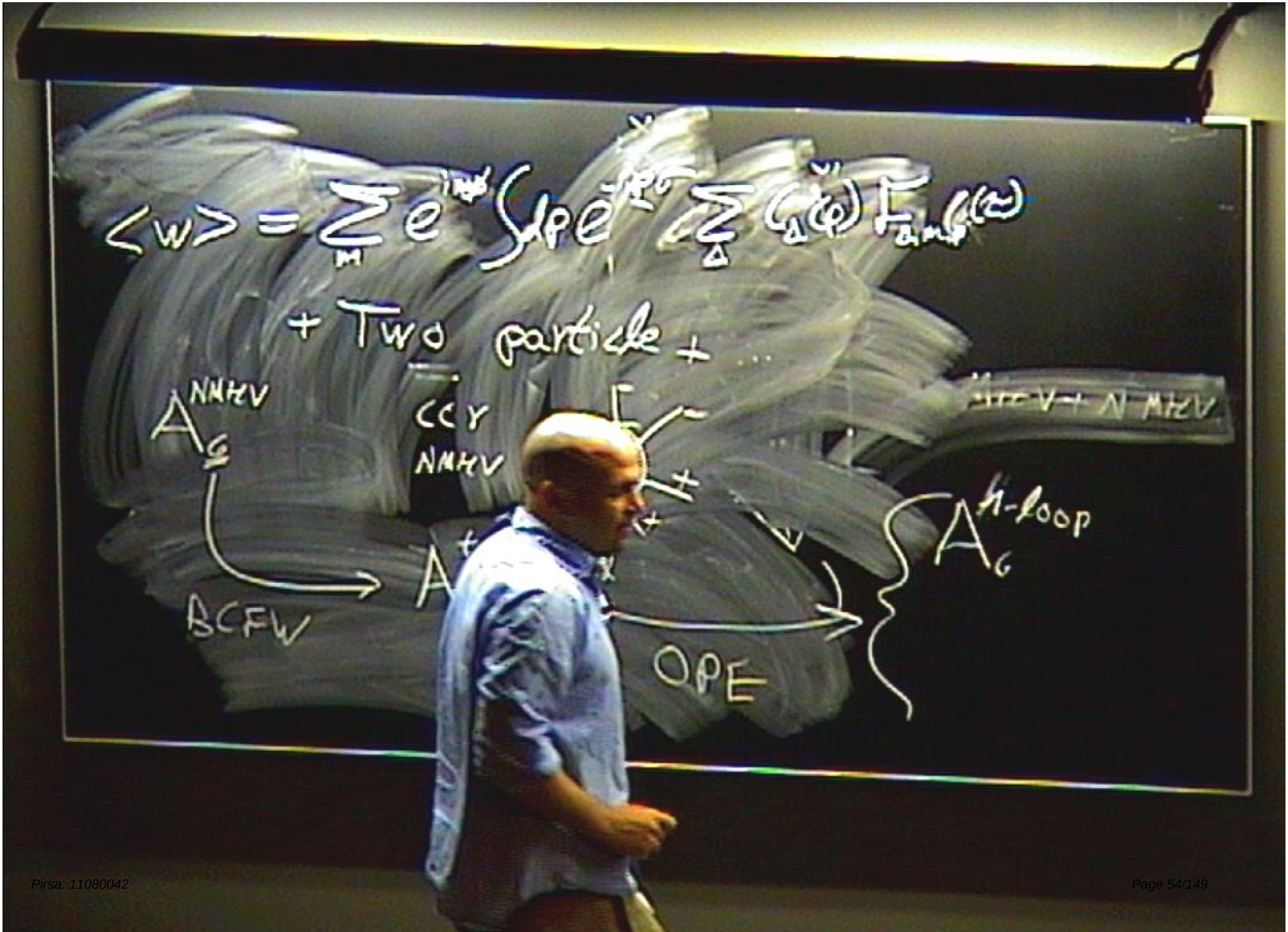
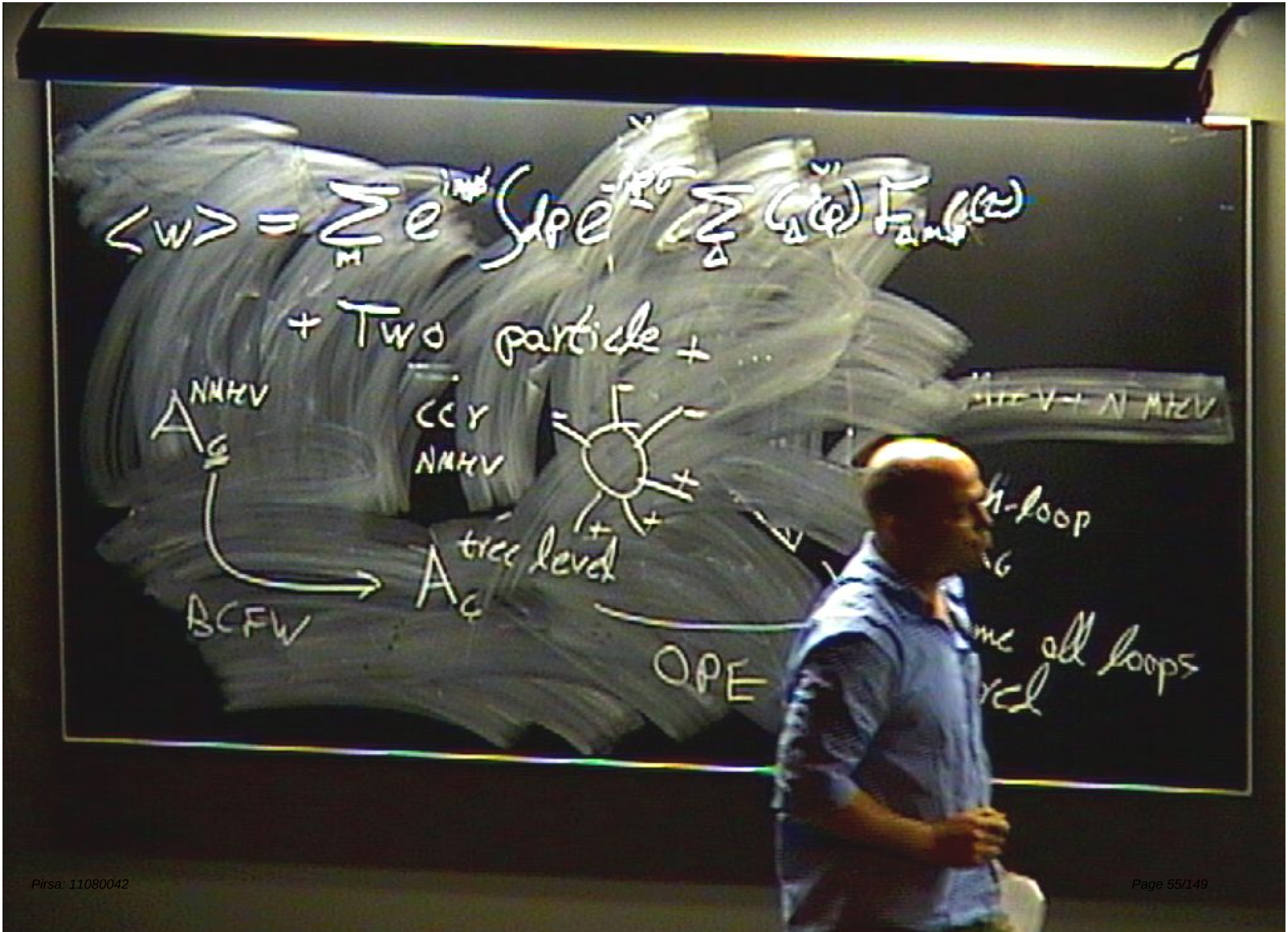

⟨W⟩ = Σ e^{imφ} ∫dp e^{-iσp} Σ C_Δ(φ) F_{Δ,m,p}(u)
m
Δ
+ Two particle + …
A_6^{NMHV}
CCY
NMHV
BCFW
A
MHV + NMHV
A_6^{h-loop}
OPE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A_C^{NMHV}
CCY
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BCFW
A_C tree level
OPE
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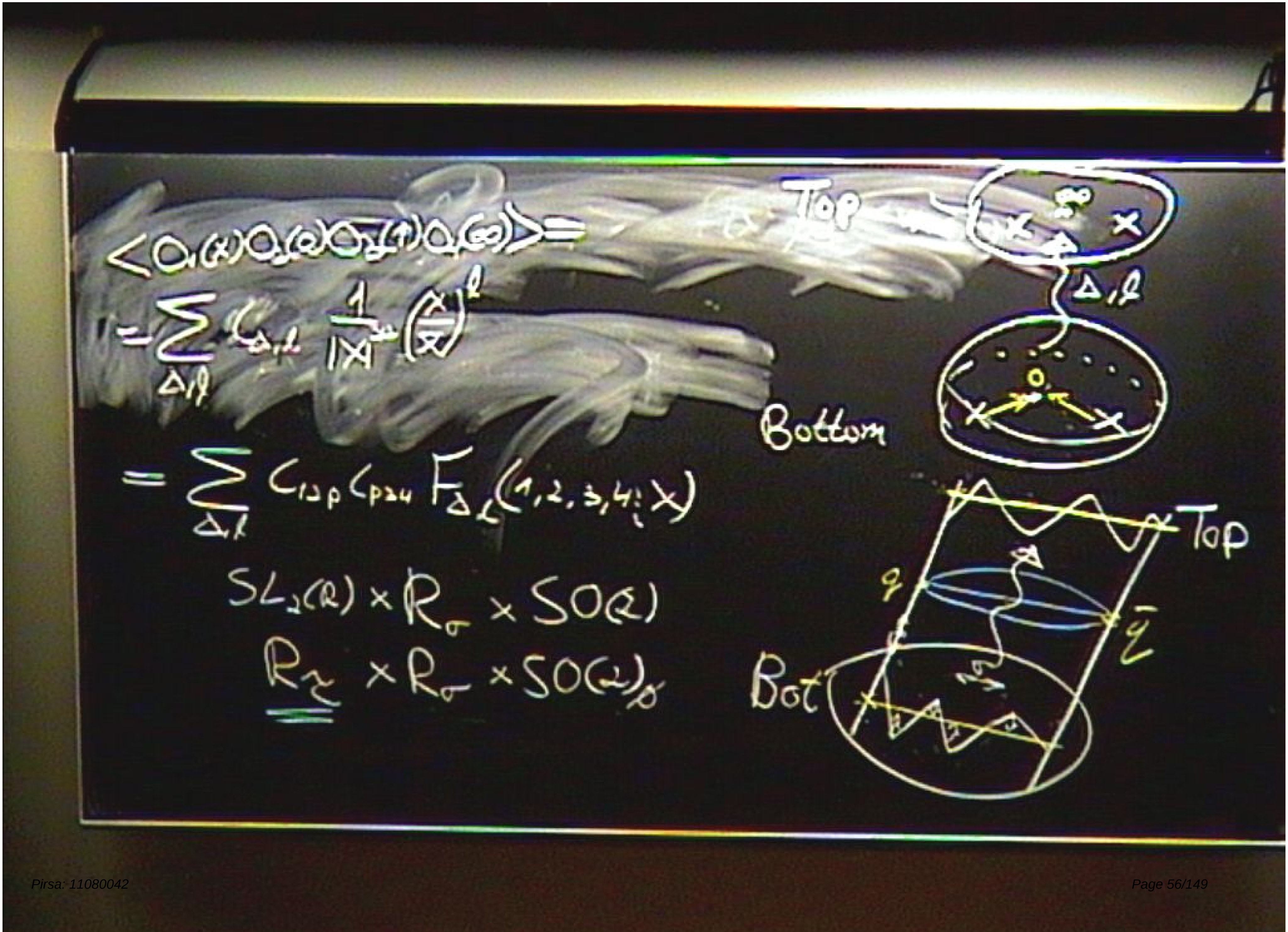
-loop
all loops
rcl



$$\langle O_1(x) O_2(0) O_3(1) O_4(\infty)\rangle =$$

$$= \sum_{\Delta,\ell} C_{\Delta,\ell} \frac{1}{|x|^{\Delta}} \left(\frac{x}{\bar{x}}\right)^{\ell}$$

$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4;x)$$

$$SL_2(\mathbb{R}) \times R_\sigma \times SO(2)$$

$$\underline{\underline{R_\tau}} \times R_\sigma \times SO(2)_\rho$$

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- Trivialize the momentum conservation and null constraints.
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1, 2, 3, 4

Momentum twistor

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$SL(4)$

Null polygon

$$Y = Y^+ \left(1, x^2, x^\mu\right)$$

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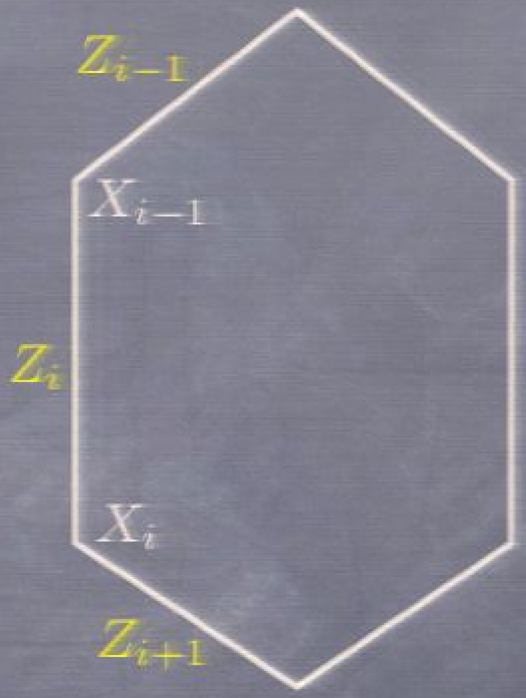

$$(X_i - X_{i-1})^2 = \left\{ \begin{array}{l} X_i^+ X_{i-1}^+ (x_i - x_{i-1})^2 \\ \\ -2 Z_{i-1} \wedge Z_i \wedge Z_i \wedge Z_{i+1} \end{array} \right. = 0$$

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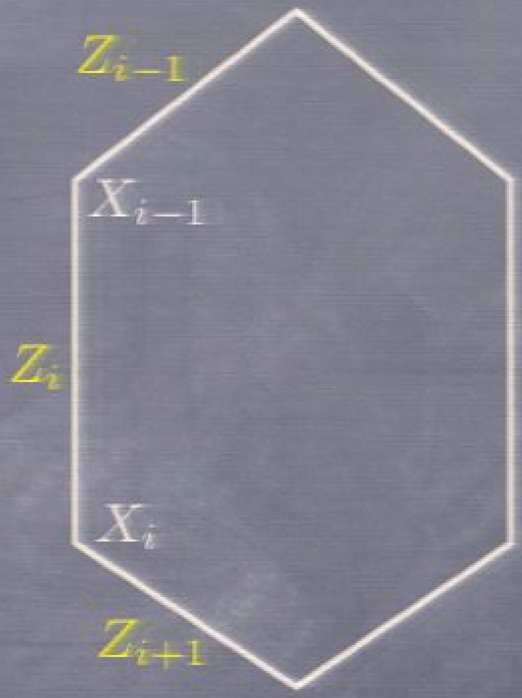

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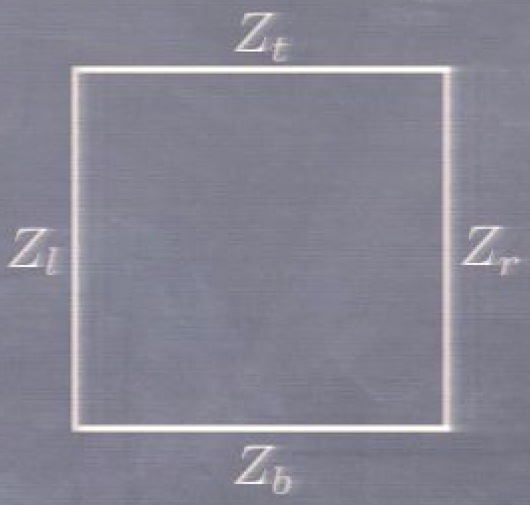
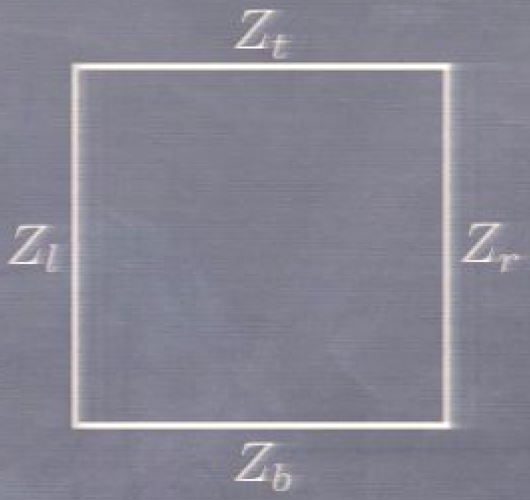
$$Z_i \wedge Z_j \wedge Z_k \wedge Z_l \equiv \langle i, j, k, l \rangle$$

The reference square



The reference square

$$\begin{pmatrix} Z_{\text{left}} \\ Z_{\text{top}} \\ Z_{\text{right}} \\ Z_{\text{bottom}} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$



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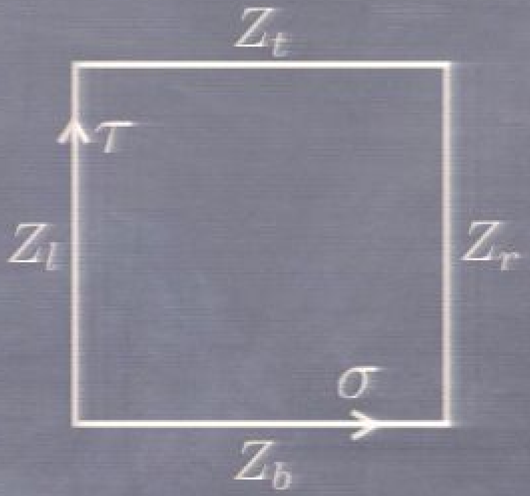

Symmetries $R_\tau \times R_\sigma \times SO(2)$

$$M_{SL(4)}(\tau, \sigma, \phi) = \begin{pmatrix} e^{\sigma+\phi/2} & & & \\ & e^{-\sigma+\phi/2} & & \\ & & e^{\tau-\phi/2} & \\ & & & e^{-\tau-\phi/2} \end{pmatrix}.$$

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$$X_k = a Z_a \wedge Z_b + b Z_d \wedge Z_c$$

$$= \sum_{\Delta,\ell} C_{12p} C_{p34} F_{\Delta,\ell}(1,2,3,4;X)$$

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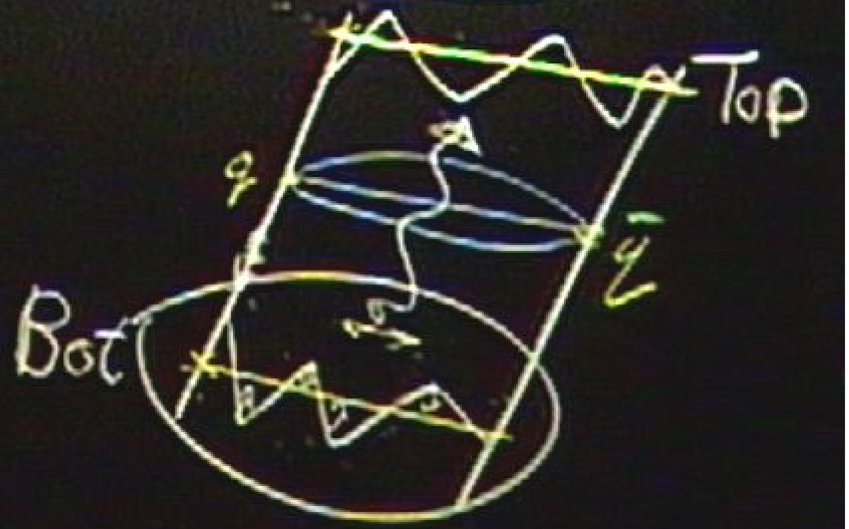
$$\underline{\underline{\mathbb{R}_\tau}} \times \mathbb{R}_\sigma \times SO(2)_\phi$$



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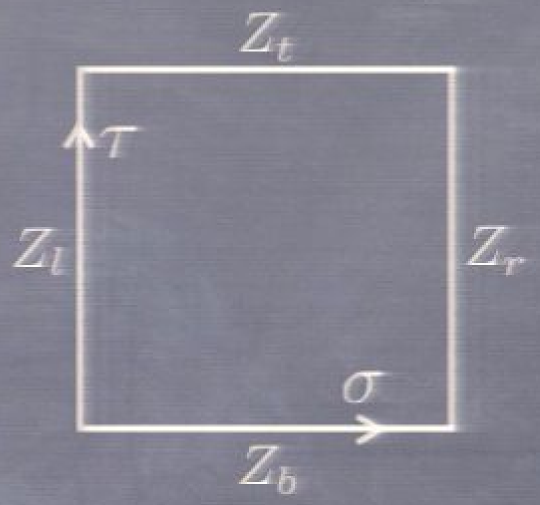

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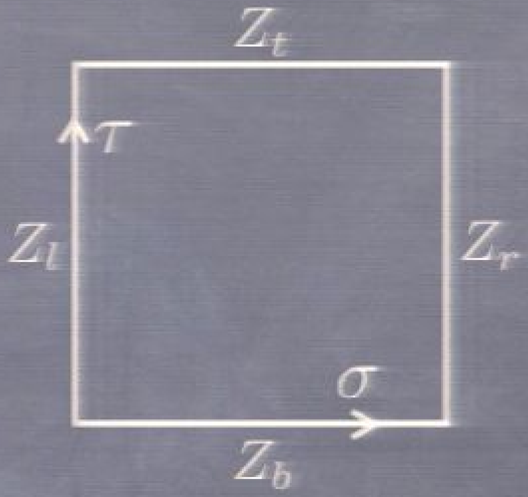

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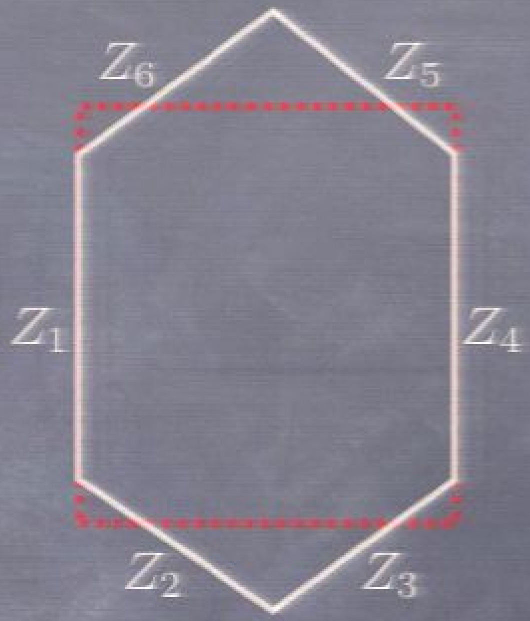
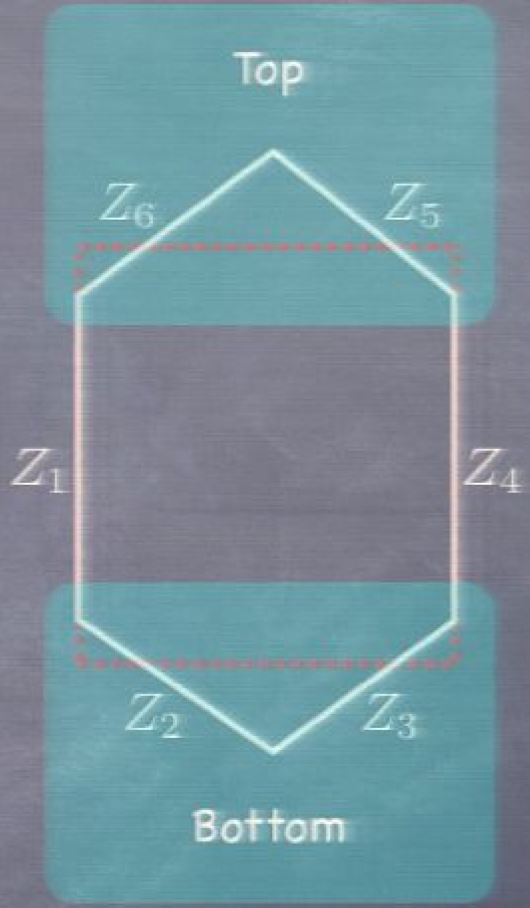
$$SL(2)_\tau$$

The hexagon



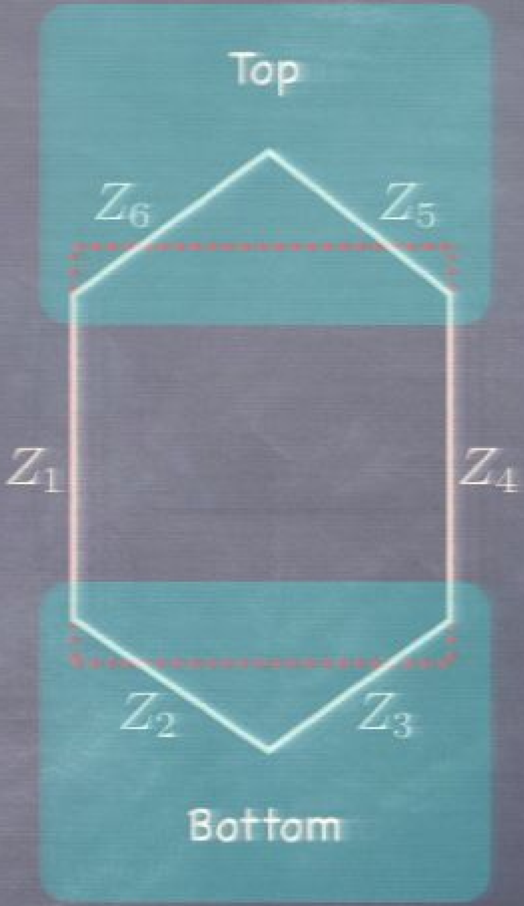
The hexagon



The hexagon

Three parameters family of hexagons

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} Z_1 \\ M \cdot \widetilde{Z}_2 \\ M \cdot \widetilde{Z}_3 \\ \widetilde{Z}_4 \\ \widetilde{Z}_5 \\ \widetilde{Z}_6 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & e^{\frac{\phi}{2}-\sigma} & -e^{\tau-\frac{\phi}{2}} & e^{-\tau-\frac{\phi}{2}} \\ e^{\sigma+\frac{\phi}{2}} & 0 & -e^{\tau-\frac{\phi}{2}} & -e^{-\tau-\frac{\phi}{2}} \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$



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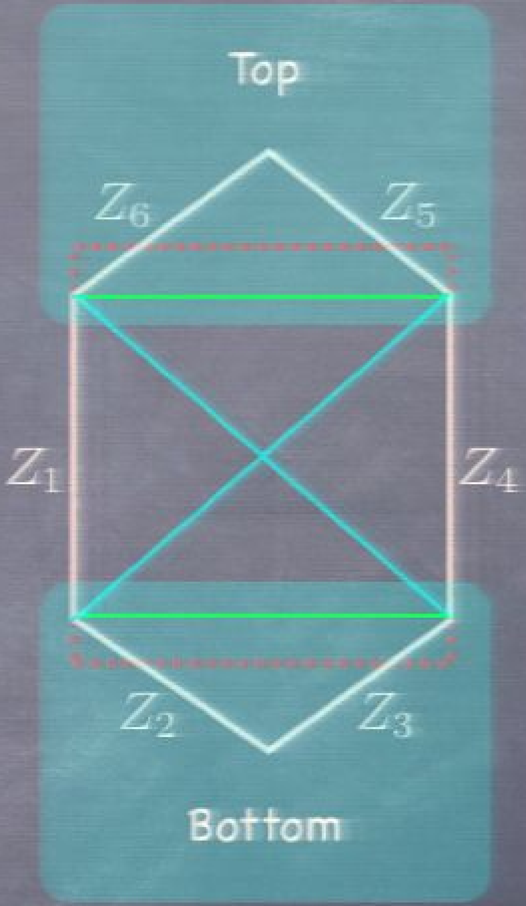

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Three conformal cross ratios

$$u_2 = \frac{X_{1,3}^2 X_{4,6}^2}{X_{1,4}^2 X_{3,6}^2} = \frac{\langle 1234 \rangle \langle 4561 \rangle}{\langle 1245 \rangle \langle 3461 \rangle} = \frac{1}{\cosh \tau}$$

The tree level Ratio function

$$\mathcal{R}_6^{\text{NMHV tree}} = \frac{\mathcal{A}_6^{\text{NMHV tree}}}{\mathcal{A}_n^{\text{MHV tree}}} = \mathcal{R}_{2356}\,\eta_2\eta_3\eta_5\eta_6 \;+\; \ldots \;\overset{}{\underset{\text{BCFW}}{=}}\; \sum_{1<i<j<6-2} [1, i, i+1, j, j+2]$$

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$$[a,b,c,d,e] \equiv \frac{\delta^{0|4}\left(\eta_a \langle bcde\rangle + \eta_b \langle cdea\rangle + \eta_c \langle deab\rangle + \eta_d \langle eabc\rangle + \eta_e \langle abcd\rangle\right)}{\langle abcd\rangle \langle bcde\rangle \langle cdea\rangle \langle deab\rangle \langle eabc\rangle}$$

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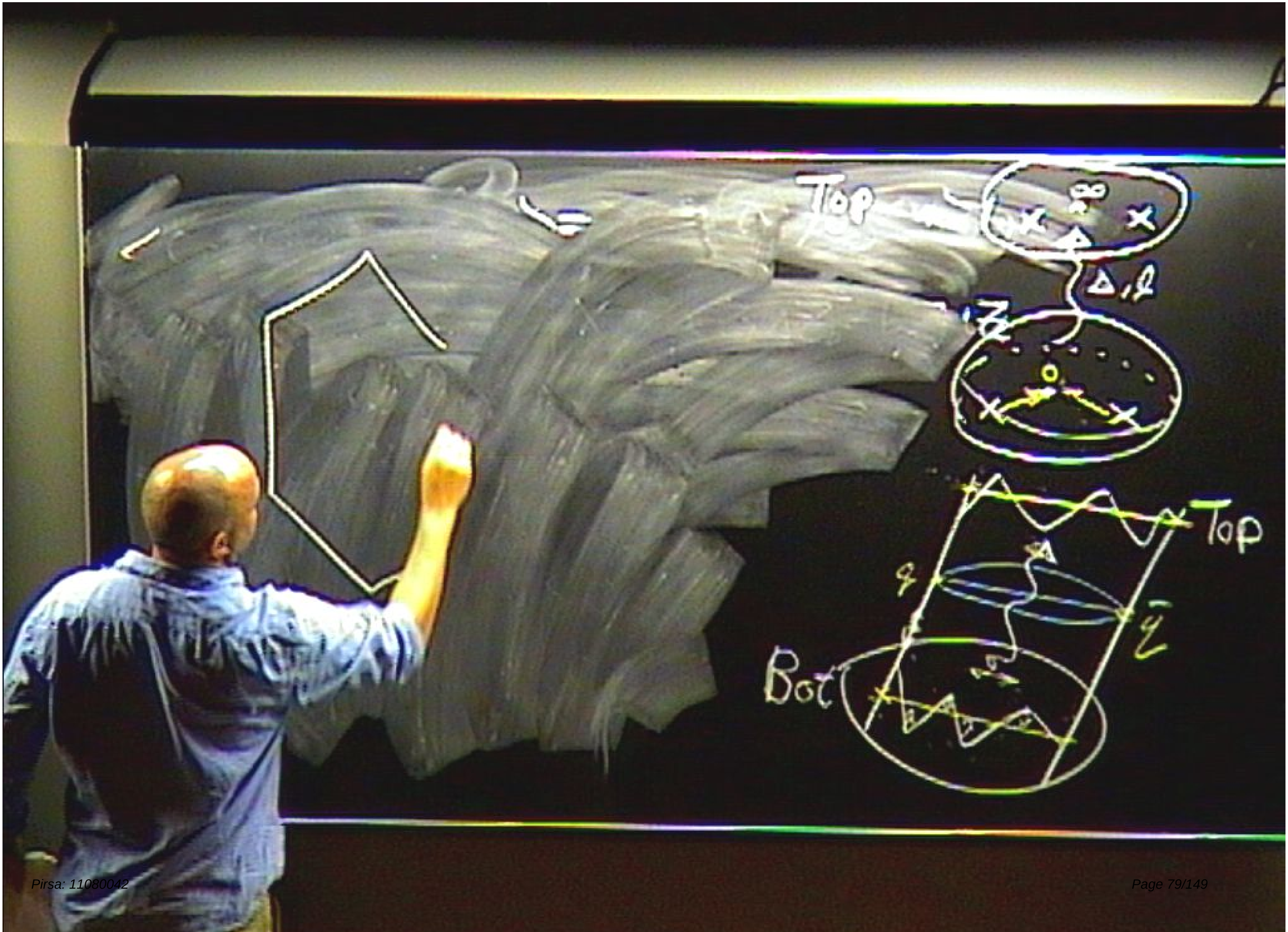
$$\mathcal{R}_{2356} = \frac{1}{\langle 2356\rangle} = \frac{1}{4[\cosh(\tau)\cosh(\sigma) + \cosh(\phi)]}$$


Top
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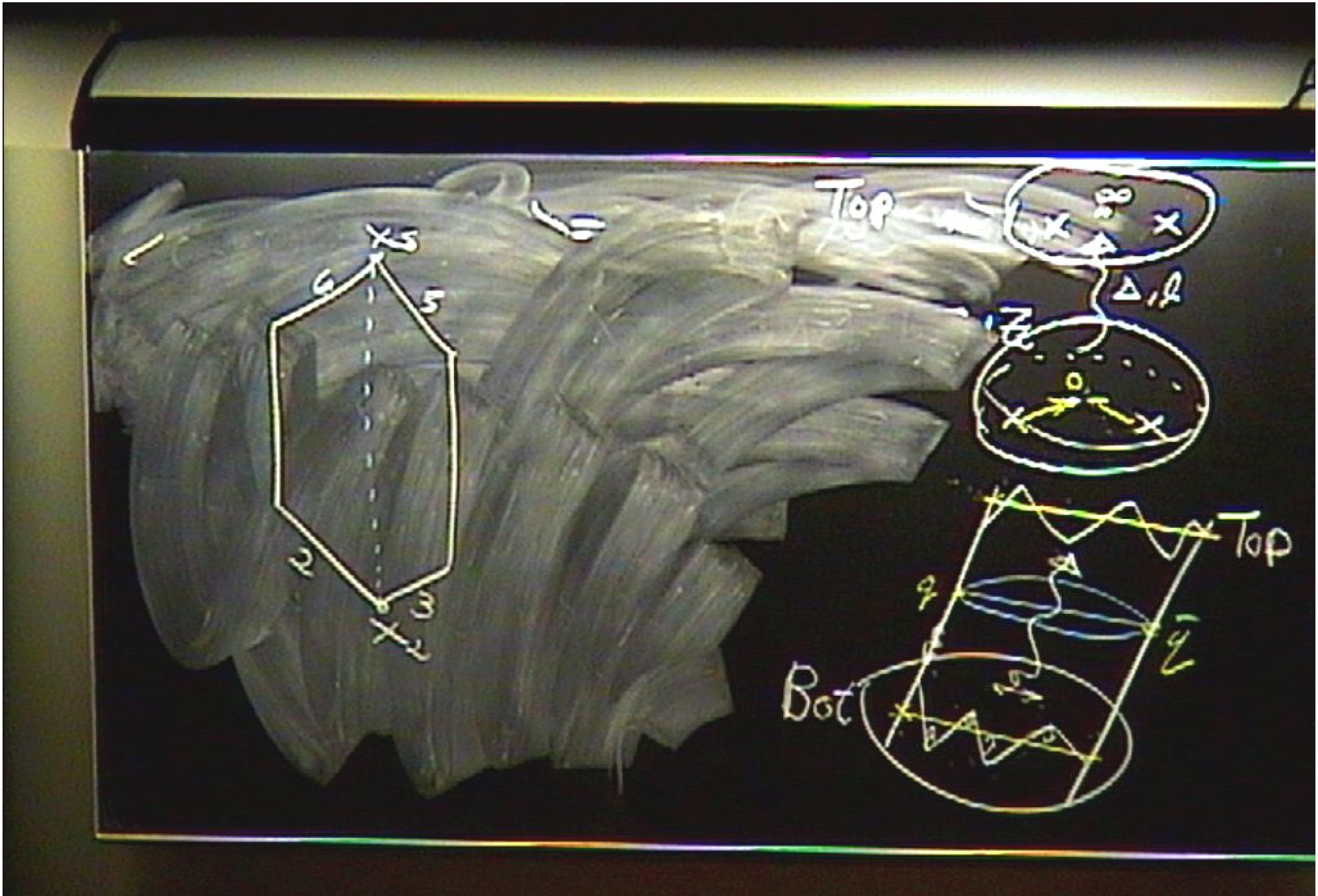
1
×5
6
5
2
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×
Δ, ℓ
Z
0
Top
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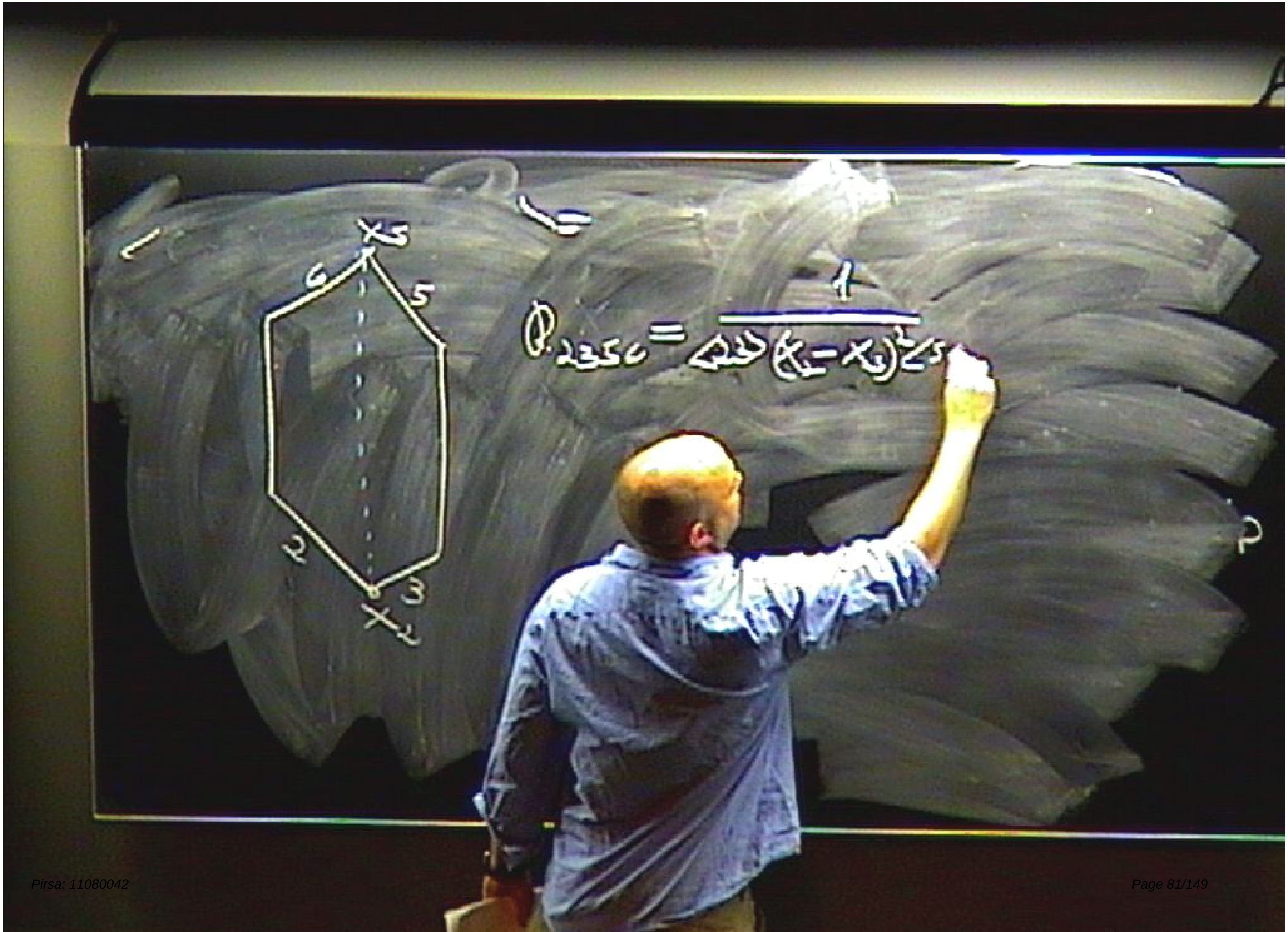
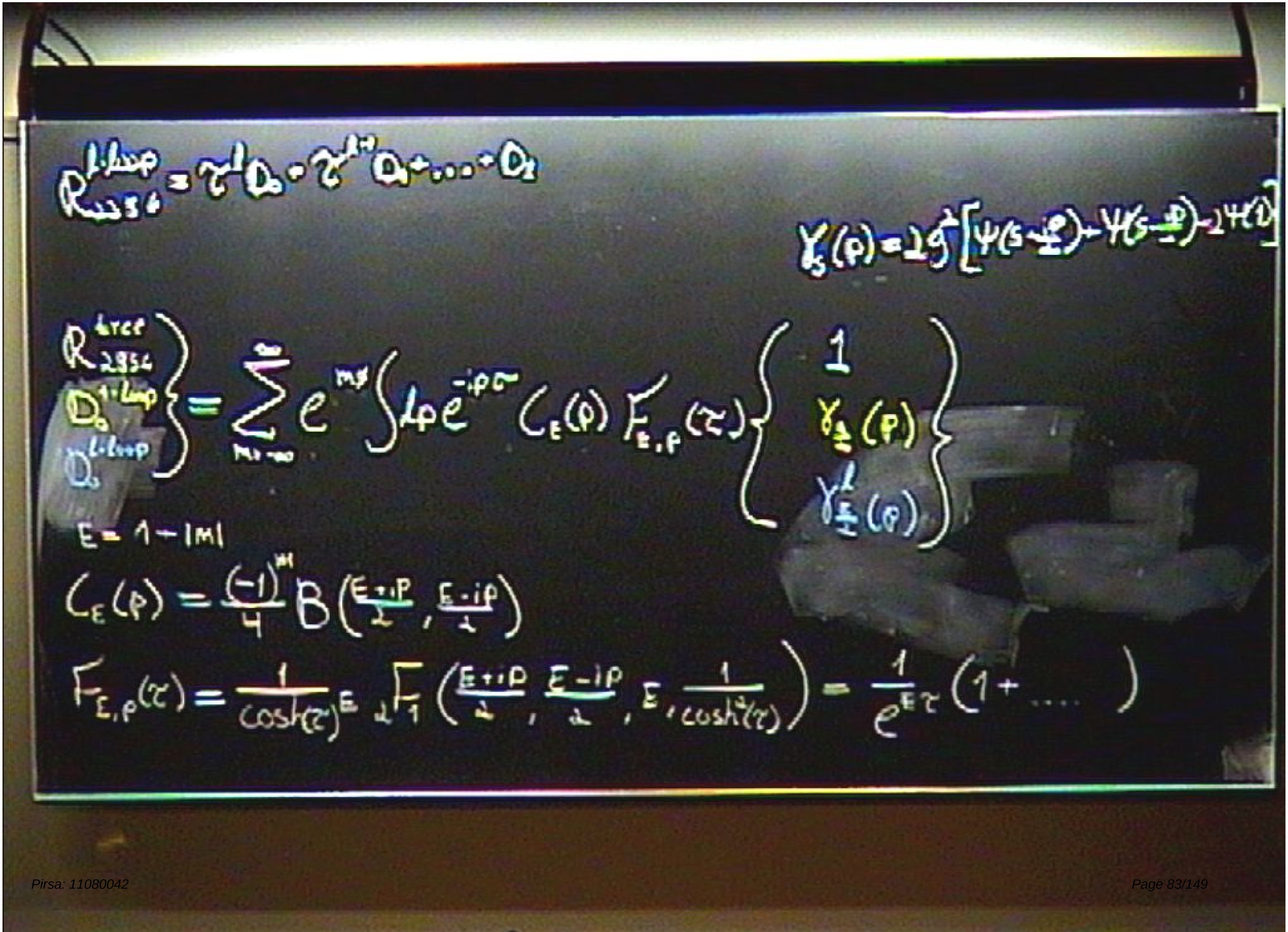
q̄
Bot



$$\Phi_{2356} = \frac{1}{4\pi^2 (x_2 - x_5)^2 \epsilon_5}$$

$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_2-x_3)^2\langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$



$$R^{l\text{-loop}}_{2356} = \tau^{l} D_0 + \tau^{l-1} D_1 + \dots + D_l$$

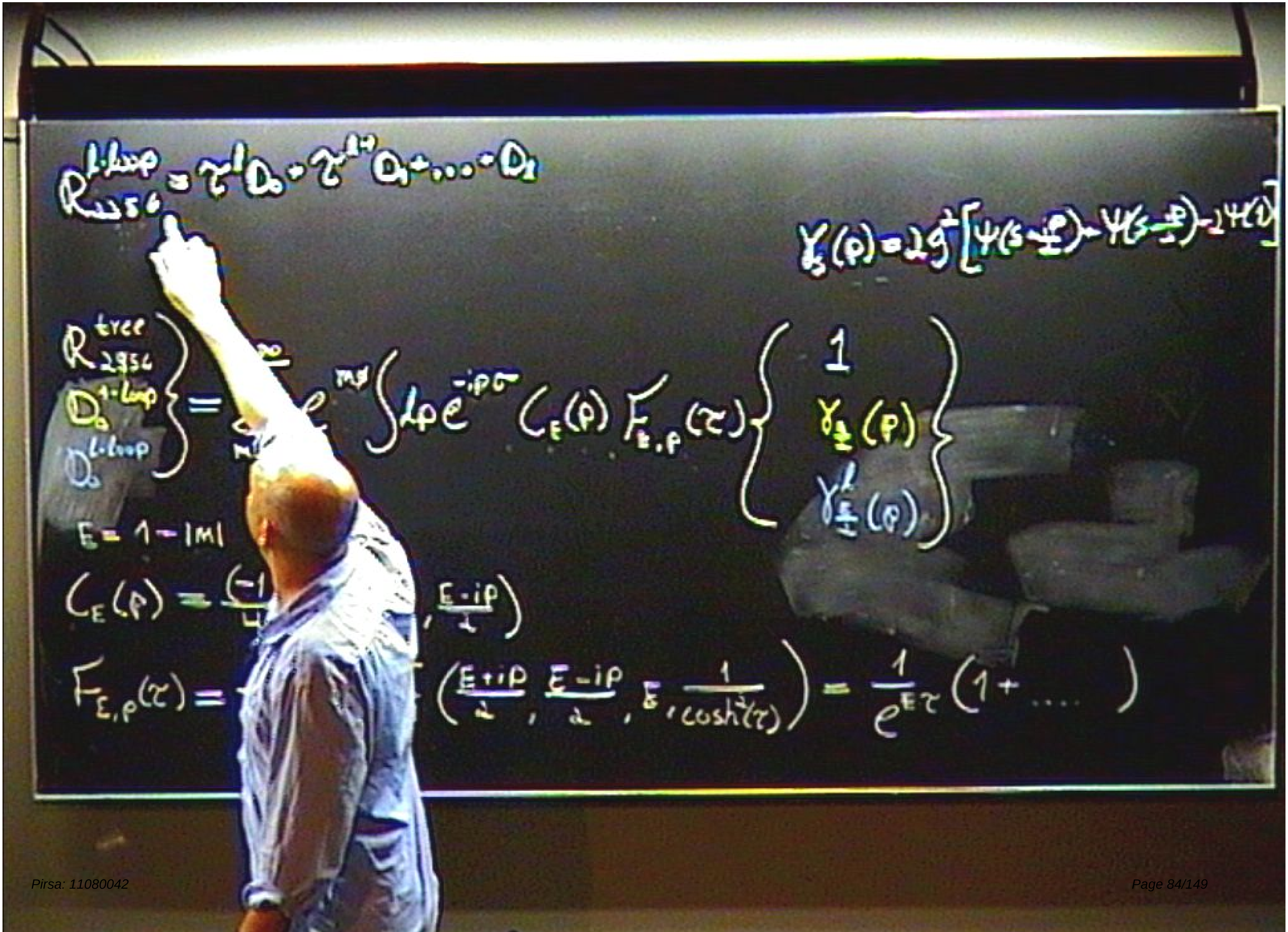
$$\gamma_S(p) = 2g^2\left[\psi\left(S+\frac{ip}{2}\right) + \psi\left(S-\frac{ip}{2}\right) - 2\psi(1)\right]$$

$$\left.\begin{matrix} R^{\text{tree}}_{2356} \\ D_0^{1\text{-loop}} \\ D_0^{l\text{-loop}} \end{matrix}\right\} = \sum_{m=-\infty}^{\infty} e^{im\phi} \int dp\, e^{-ip\sigma}\, C_E(p)\, F_{E,p}(\tau) \left\{\begin{matrix} 1 \\ \gamma_{\frac{E}{2}}(p) \\ \gamma^{l}_{\frac{E}{2}}(p) \end{matrix}\right\}$$

$$E = 1 - |m|$$

$$C_E(p) = \frac{(-1)^m}{4} B\left(\frac{E+ip}{2}, \frac{E-ip}{2}\right)$$

$$F_{E,p}(\tau) = \frac{1}{\cosh(\tau)^E}\, {}_2F_1\left(\frac{E+ip}{2}, \frac{E-ip}{2}, E, \frac{1}{\cosh^2(\tau)}\right) = \frac{1}{e^{E\tau}}\left(1 + \dots\right)$$



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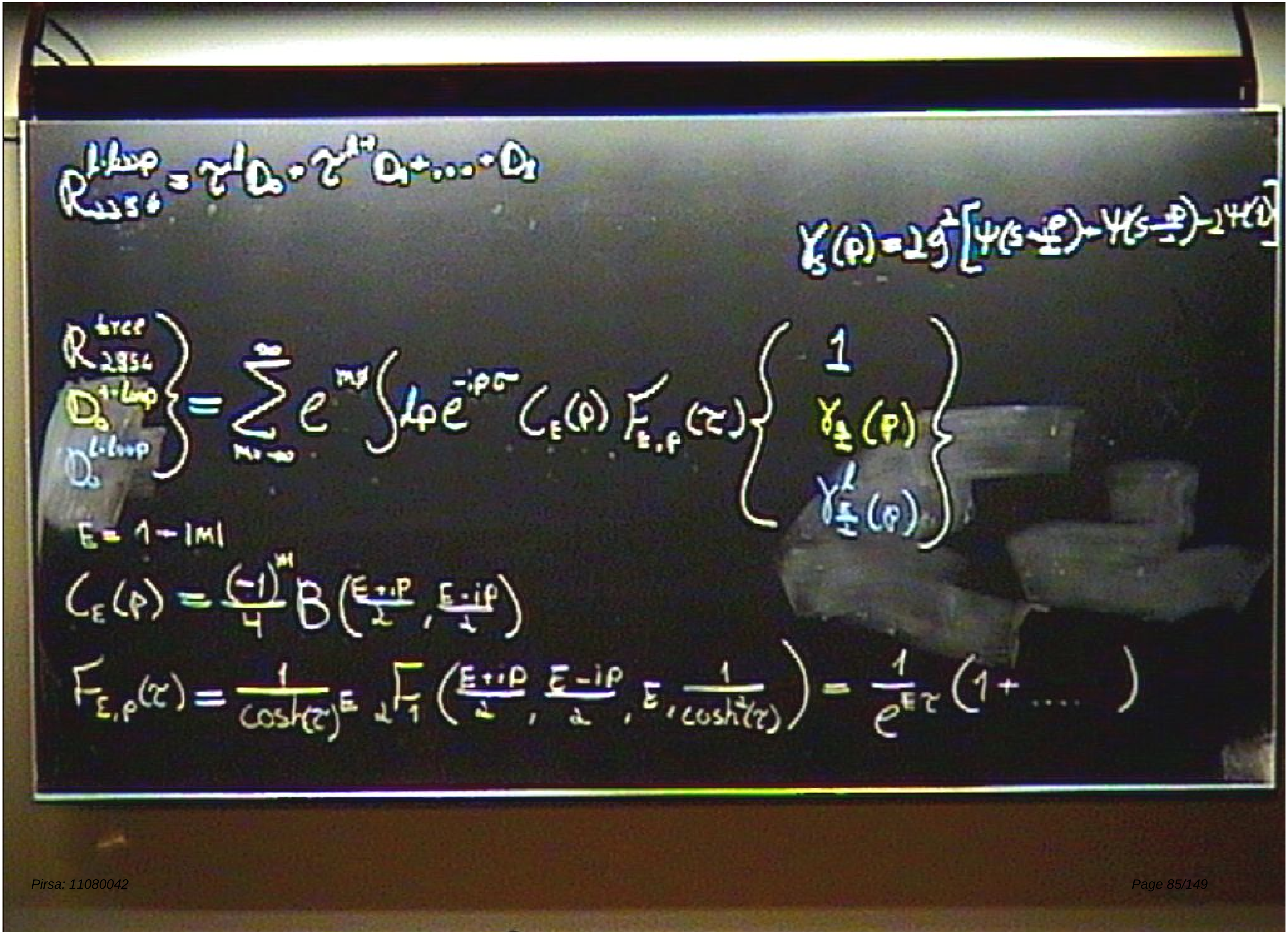

$$R^{l\text{-loop}}_{2356} = \tau^l D_0 + \tau^{l-1} D_1 + \dots + D_l$$

$$\gamma_s(p) = 2g^2\left[\psi\left(s+\frac{ip}{2}\right) + \psi\left(s-\frac{ip}{2}\right) - 2\psi(1)\right]$$

$$\left.\begin{matrix} R^{tree}_{2356} \\ D_0^{1\text{-loop}} \\ D_0^{l\text{-loop}} \end{matrix}\right\} = \sum_{m=-\infty}^{\infty} e^{m\phi} \int dp\, e^{-ip\sigma}\, C_E(p)\, F_{E,p}(\tau) \left\{\begin{matrix} 1 \\ \gamma_{\frac{E}{2}}(p) \\ \gamma^l_{\frac{E}{2}}(p) \end{matrix}\right\}$$

$$E = 1 - |m|$$

$$C_E(p) = \frac{(-1)^m}{4} B\left(\frac{E+ip}{2}, \frac{E-ip}{2}\right)$$

$$F_{E,p}(\tau) = \frac{1}{\cosh(\tau)^E}\, {}_2F_1\left(\frac{E+ip}{2}, \frac{E-ip}{2}, E, \frac{1}{\cosh^2(\tau)}\right) = \frac{1}{e^{E\tau}}\left(1 + \dots\right)$$

The tree level Ratio function

$$\mathcal{R}_6^{\text{NMHV tree}} = \frac{\mathcal{A}_6^{\text{NMHV tree}}}{\mathcal{A}_n^{\text{MHV tree}}} = \mathcal{R}_{2356}\,\eta_2\eta_3\eta_5\eta_6 + \ldots = \sum_{1<i<j<6-2} [1, i, i+1, j, j+2]$$

BCFW

$$[a,b,c,d,e] \equiv \frac{\delta^{0|4}\left(\eta_a \langle bcde\rangle + \eta_b \langle cdea\rangle + \eta_c \langle deab\rangle + \eta_d \langle eabc\rangle + \eta_e \langle abcd\rangle\right)}{\langle abcd\rangle \langle bcde\rangle \langle cdea\rangle \langle deab\rangle \langle eabc\rangle}$$

$$\mathcal{R}_{2356} = \frac{1}{\langle 2356\rangle} = \frac{1}{4[\cosh(\tau)\cosh(\sigma) + \cosh(\phi)]}$$

$$= \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \sum_{\beta_m} \mathcal{C}_{\beta_m}(p) \mathcal{F}_{\beta_m,p}(\tau)$$

Conformal blocks

Form factors

Primaries

Conformal blocks

$\left[\mathcal{C} - \left(\partial_\phi^2 - 1\right)\right] \mathcal{R}_{2356} = 0$ $\Rightarrow$ Decomposition in scalars conformal blocks exist!

The tree level Ratio function

$$\mathcal{R}_6^{\text{NMHV tree}} = \frac{\mathcal{A}_6^{\text{NMHV tree}}}{\mathcal{A}_n^{\text{MHV tree}}} = \mathcal{R}_{2356}\,\eta_2\eta_3\eta_5\eta_6 + \ \ldots \ = \sum_{1<i<j<6-2} [1, i, i+1, j, j+2]$$

BCFW

$$[a,b,c,d,e] \equiv \frac{\delta^{0|4}\left(\eta_a \langle bcde\rangle + \eta_b \langle cdea\rangle + \eta_c \langle deab\rangle + \eta_d \langle eabc\rangle + \eta_e \langle abcd\rangle\right)}{\langle abcd\rangle \langle bcde\rangle \langle cdea\rangle \langle deab\rangle \langle eabc\rangle}$$

$$\mathcal{R}_{2356} = \frac{1}{\langle 2356\rangle} = \frac{1}{4[\cosh(\tau)\cosh(\sigma) + \cosh(\phi)]}$$

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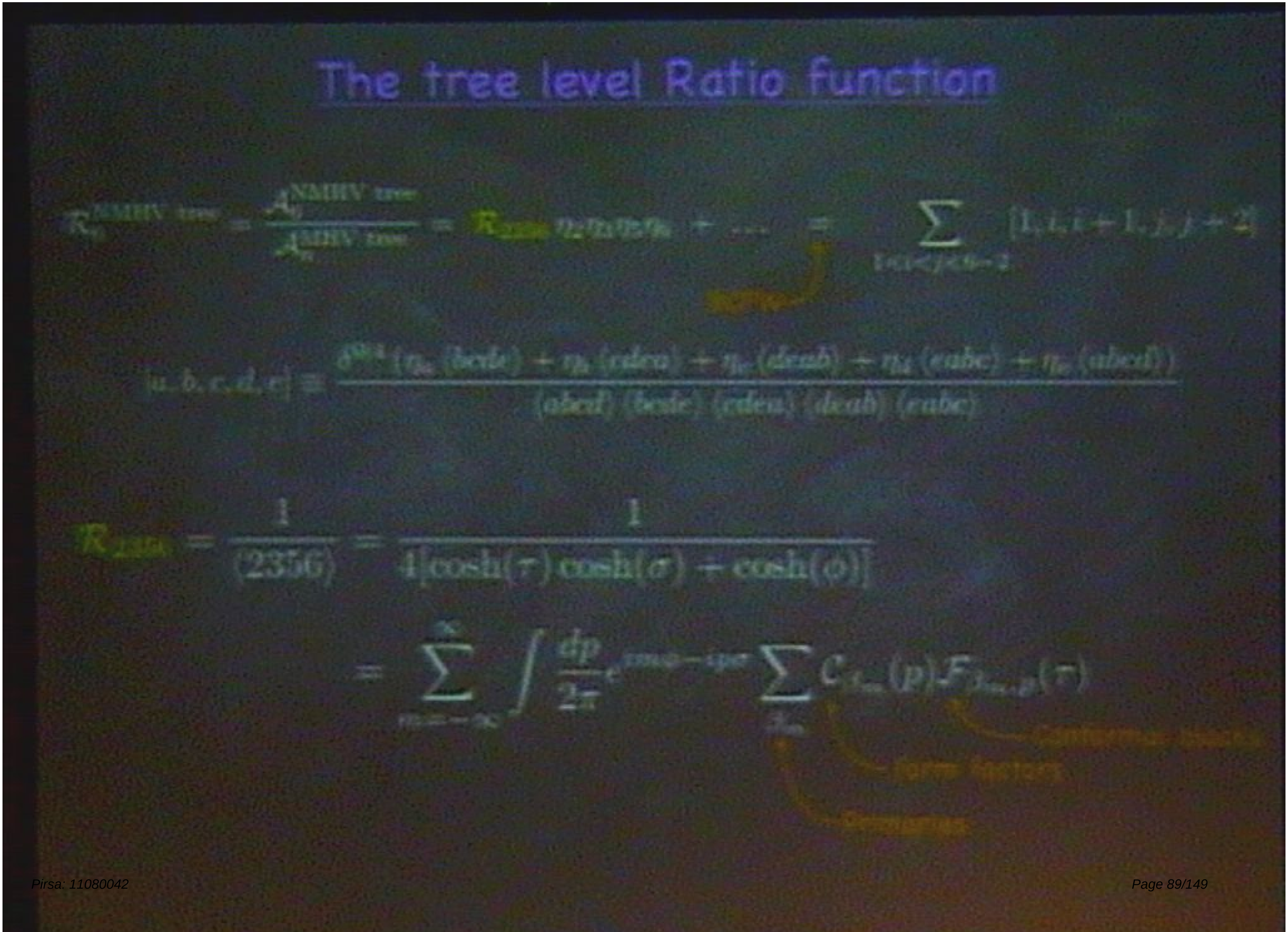
Conformal blocks

Form factors

Primaries



The tree level Ratio function

$$\mathcal{R}_n^{\text{NMHV tree}} = \frac{\mathcal{A}_n^{\text{NMHV tree}}}{\mathcal{A}_n^{\text{MHV tree}}} = R_{2356}\,\eta_2\eta_3\eta_5\eta_6 + \ldots = \sum_{1<i<j<n-2} [1, i, i+1, j, j+2]$$

$$[a, b, c, d, e] \equiv \frac{\delta^{0|4}\left(\eta_a\,(bcde) + \eta_b\,(cdea) + \eta_c\,(deab) + \eta_d\,(eabc) + \eta_e\,(abcd)\right)}{(abcd)\,(bcde)\,(cdea)\,(deab)\,(eabc)}$$

$$R_{2356} = \frac{1}{(2356)} = \frac{1}{4[\cosh(\tau)\cosh(\sigma) + \cosh(\phi)]}$$

$$= \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \sum_{j_m} C_{j_m}(p) \mathcal{F}_{j_m,p}(\tau)$$

The tree level Ratio function

$$\mathcal{R}_n^{\text{NMHV tree}} = \frac{\mathcal{A}_n^{\text{NMHV tree}}}{\mathcal{A}_n^{\text{MHV tree}}} = \mathcal{R}_{2356}\,\eta_2\eta_3\eta_5\eta_6 + \dots = \sum_{1<i<j<n-2} [1,i,i+1,j,j+2]$$

[illegible]

$$[a,b,c,d,e] \equiv \frac{\delta^{0|4}\left(\eta_a\langle bcde\rangle + \eta_b\langle cdea\rangle + \eta_c\langle deab\rangle + \eta_d\langle eabc\rangle + \eta_e\langle abcd\rangle\right)}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

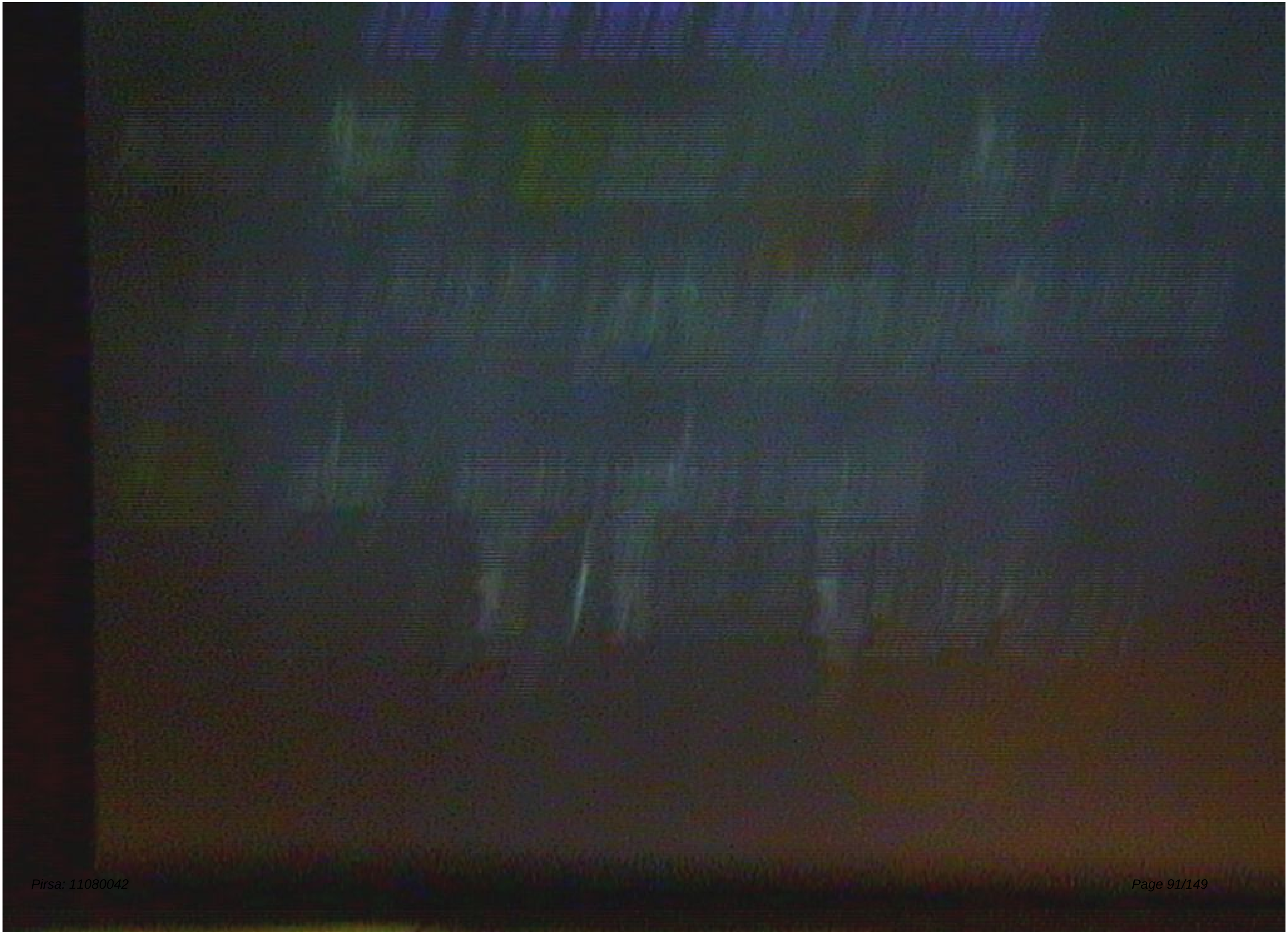
$$\mathcal{R}_{2356} = \frac{1}{\langle 2356\rangle} = \frac{1}{4[\cosh(\tau)\cosh(\sigma) + \cosh(\phi)]}$$

$$= \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \sum_{\psi_m} C_{\psi_m}(p)\mathcal{F}_{\psi_m,p}(\tau)$$

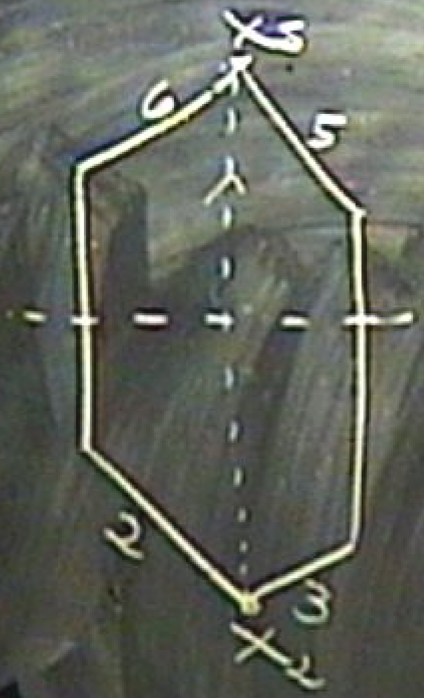
conformal blocks

form factors

[illegible]



$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_2 - x_5)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$



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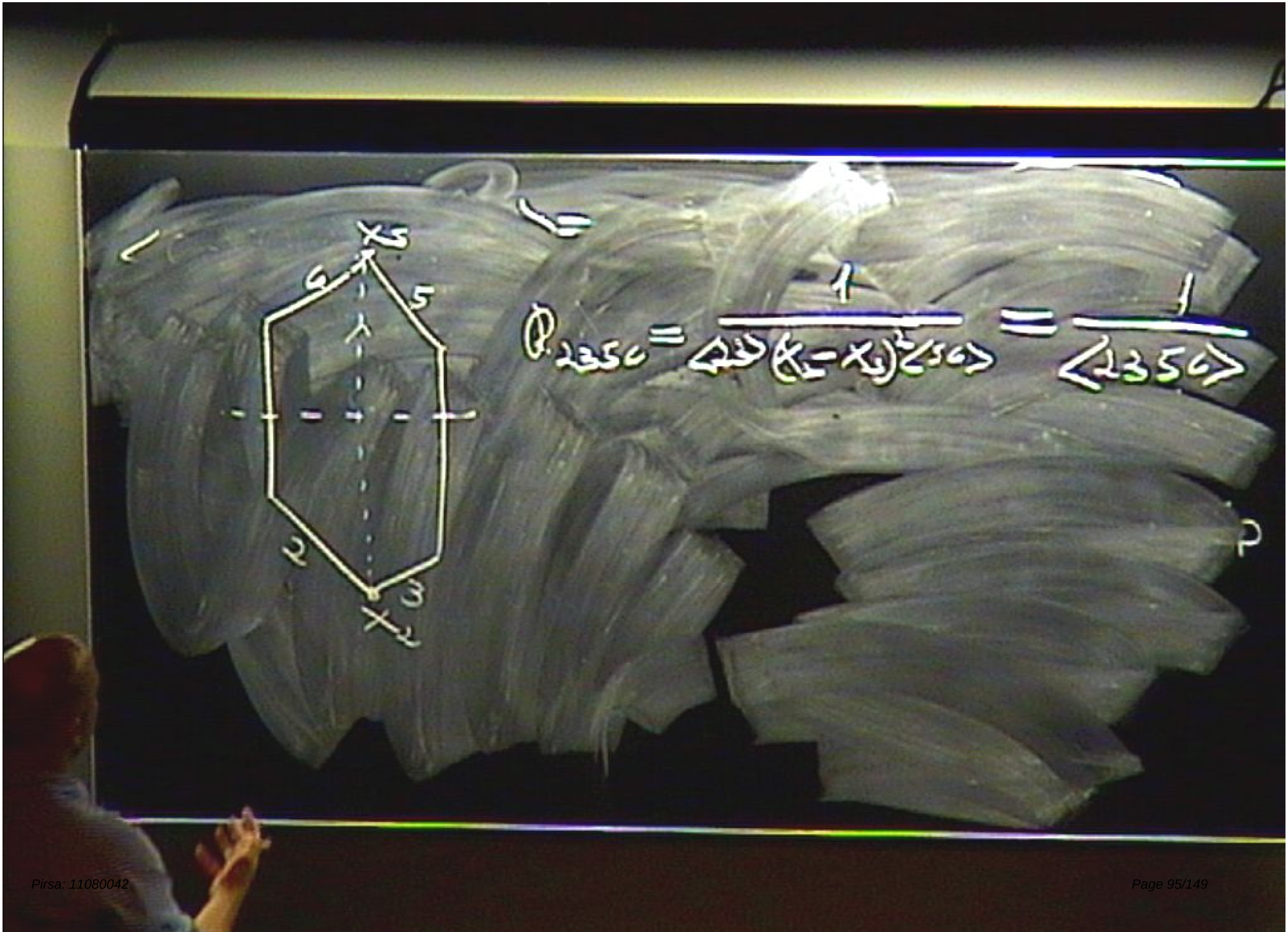

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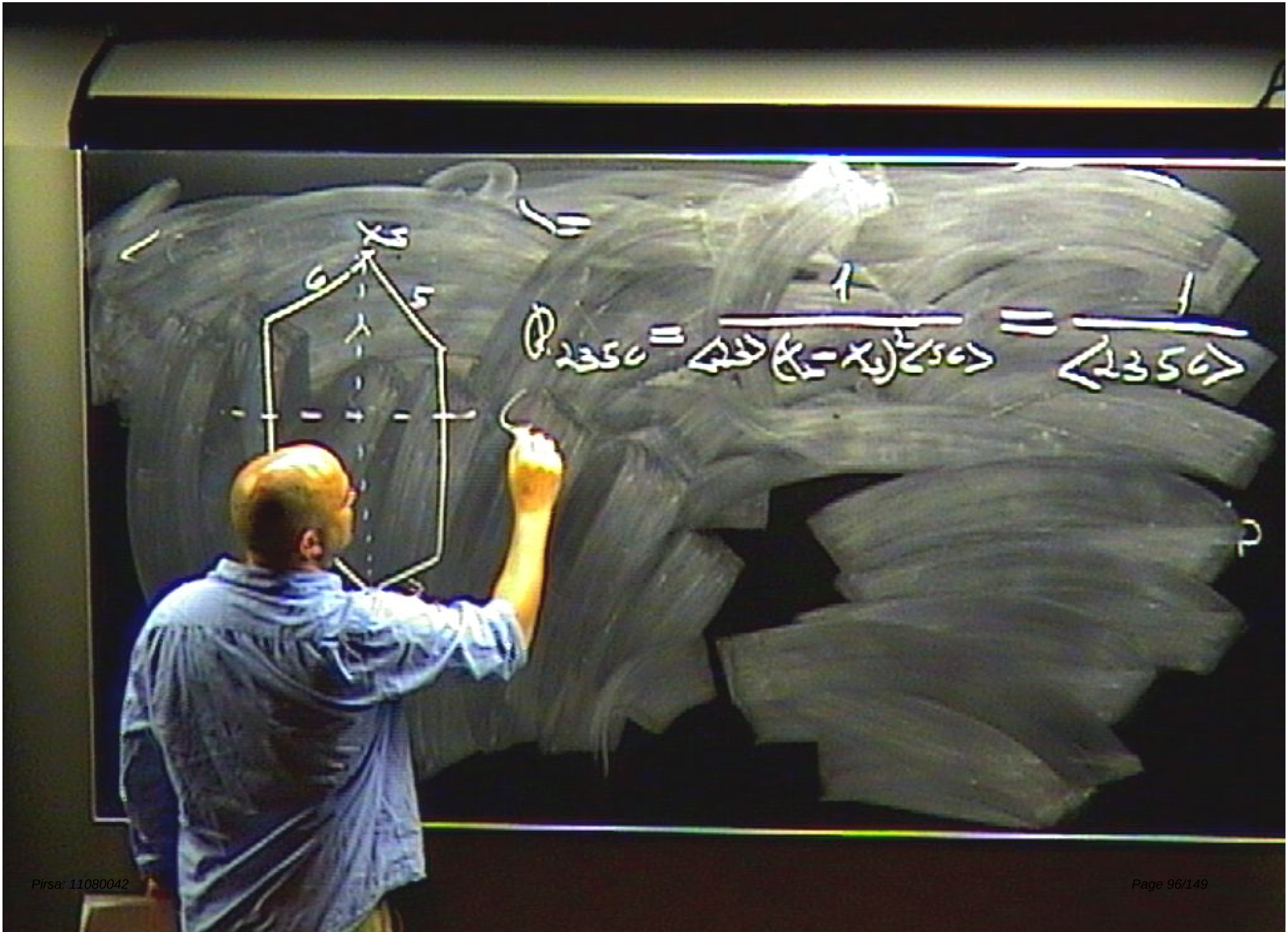
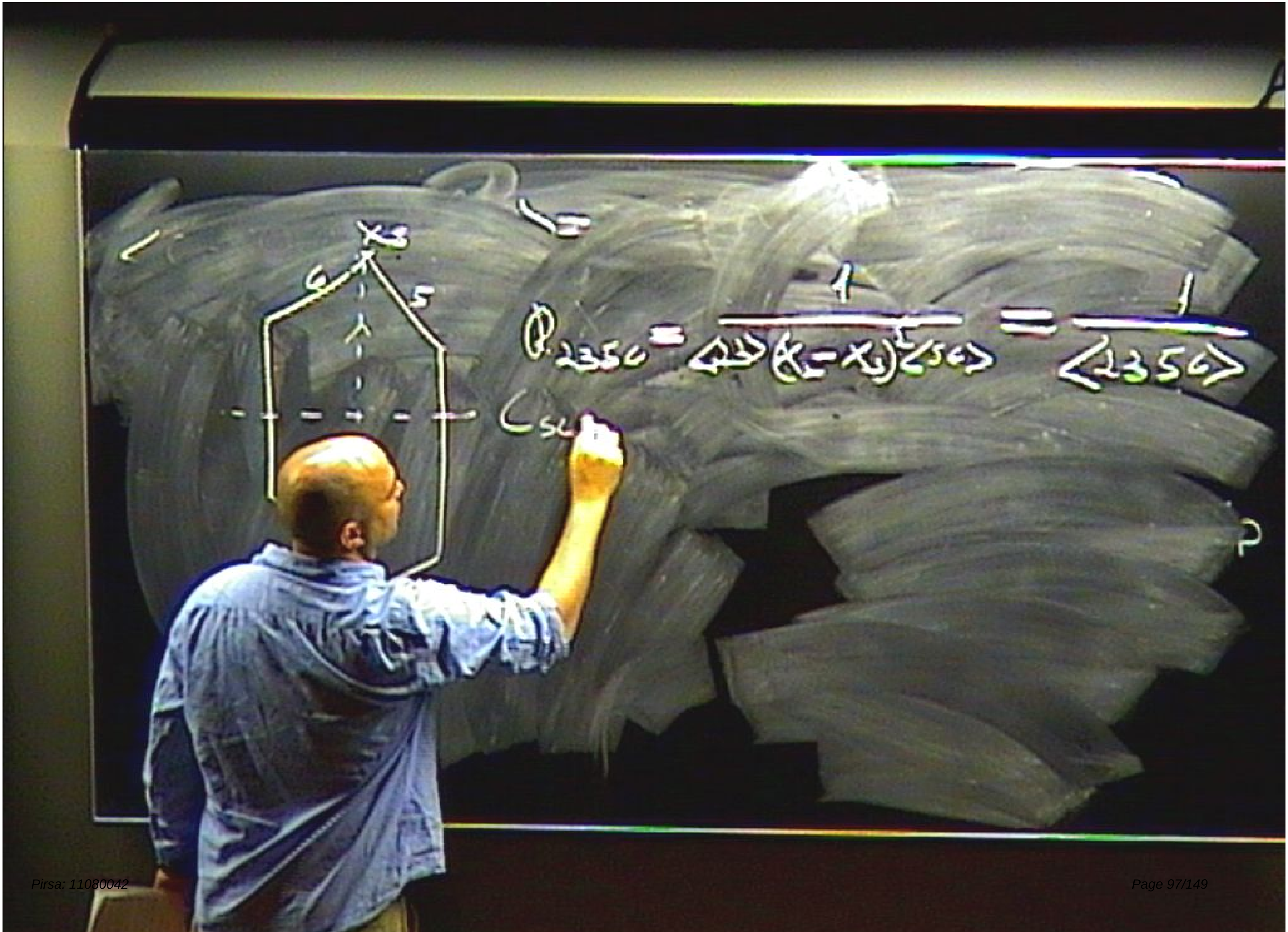
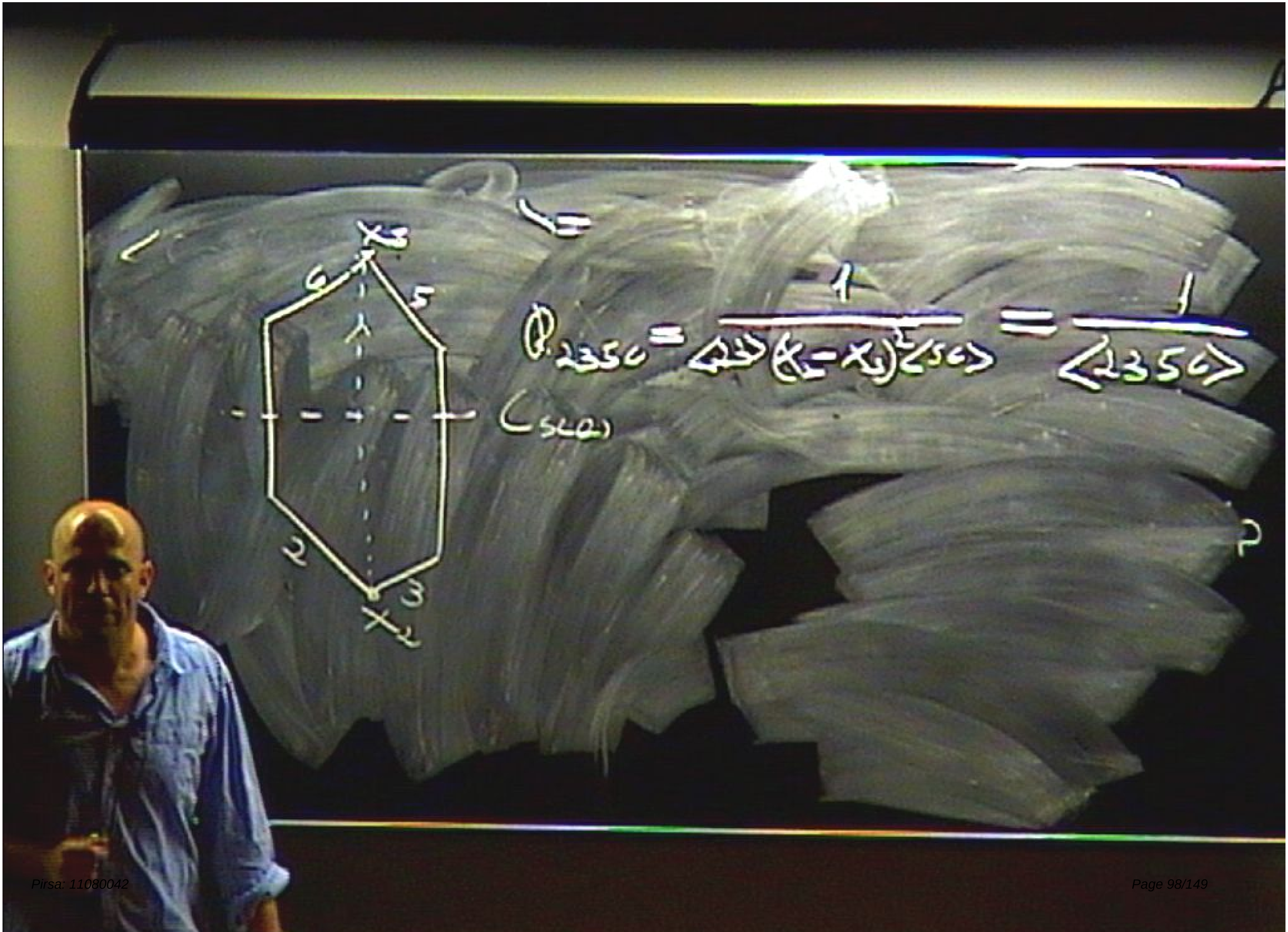
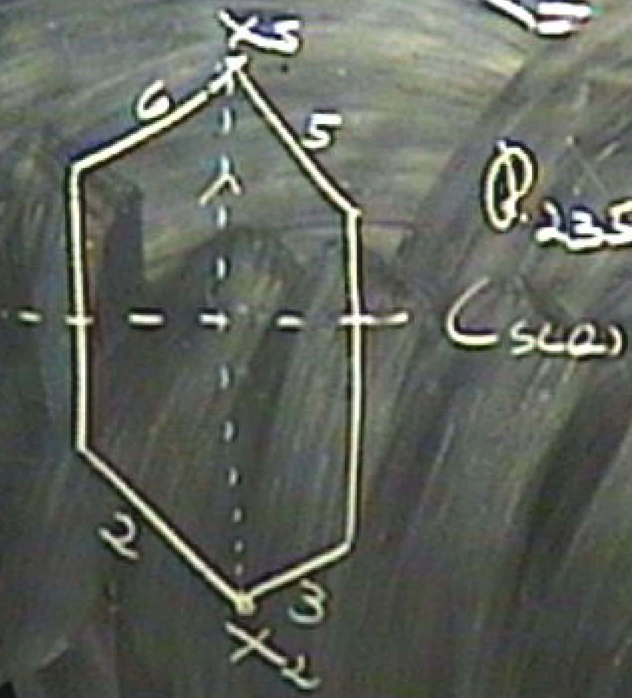


$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_2 - x_5)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$

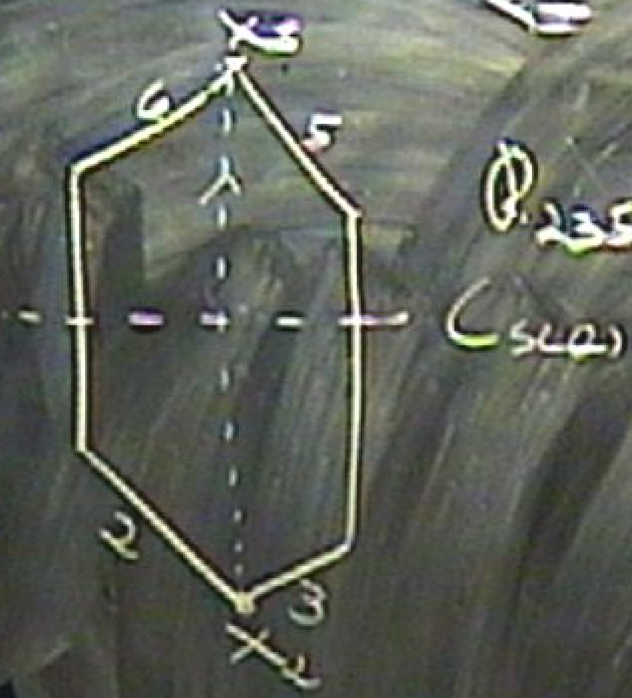

x_3
6
5
λ
$\Phi_{2350} = \frac{1}{\langle 23\rangle (x_2 - x_3)^2 \langle 50\rangle} = \frac{1}{\langle 2350\rangle}$
L_{s}



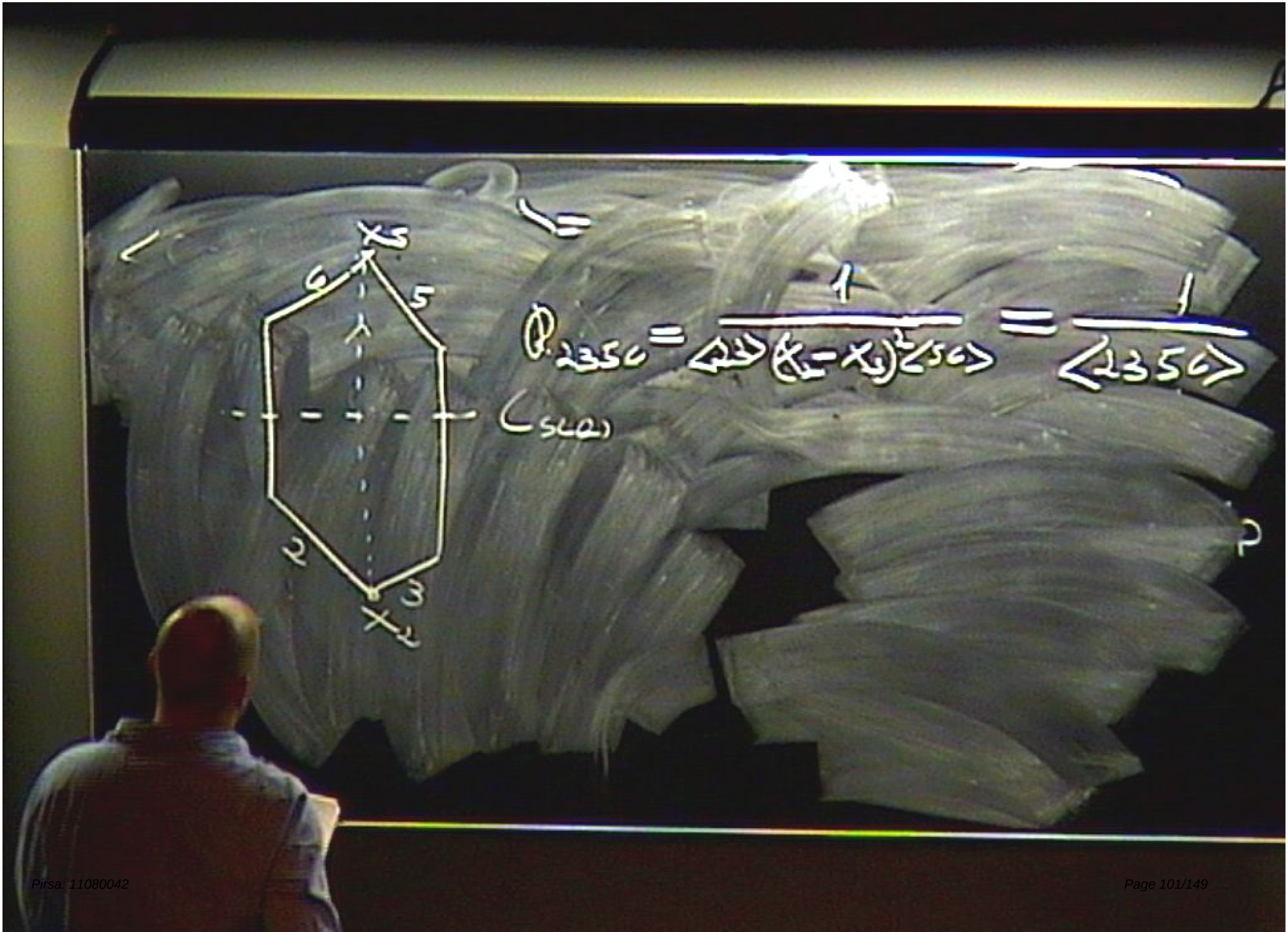
$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_1 - x_3)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$



$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_2 - x_5)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$



$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_2 - x_5)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$



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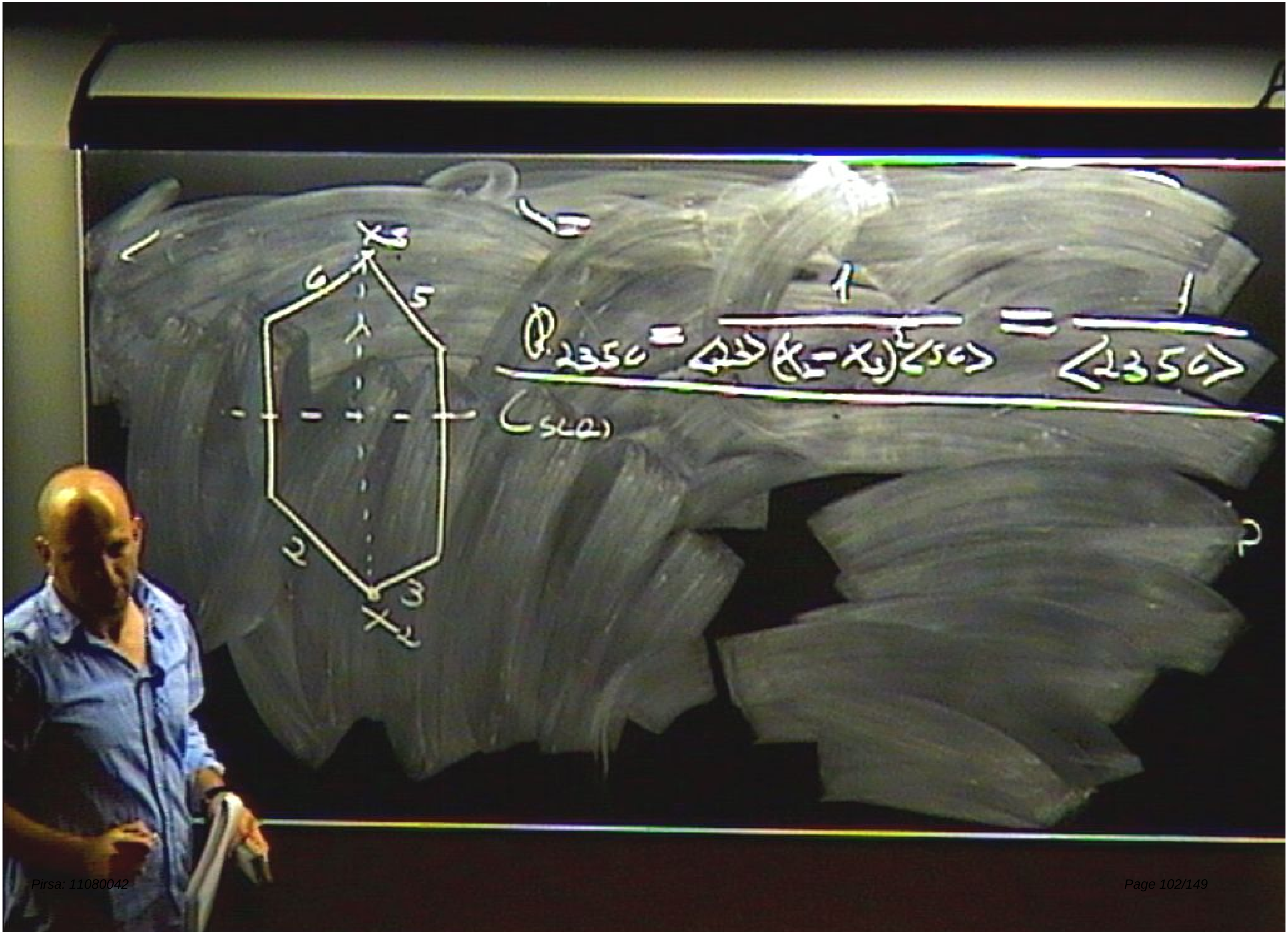
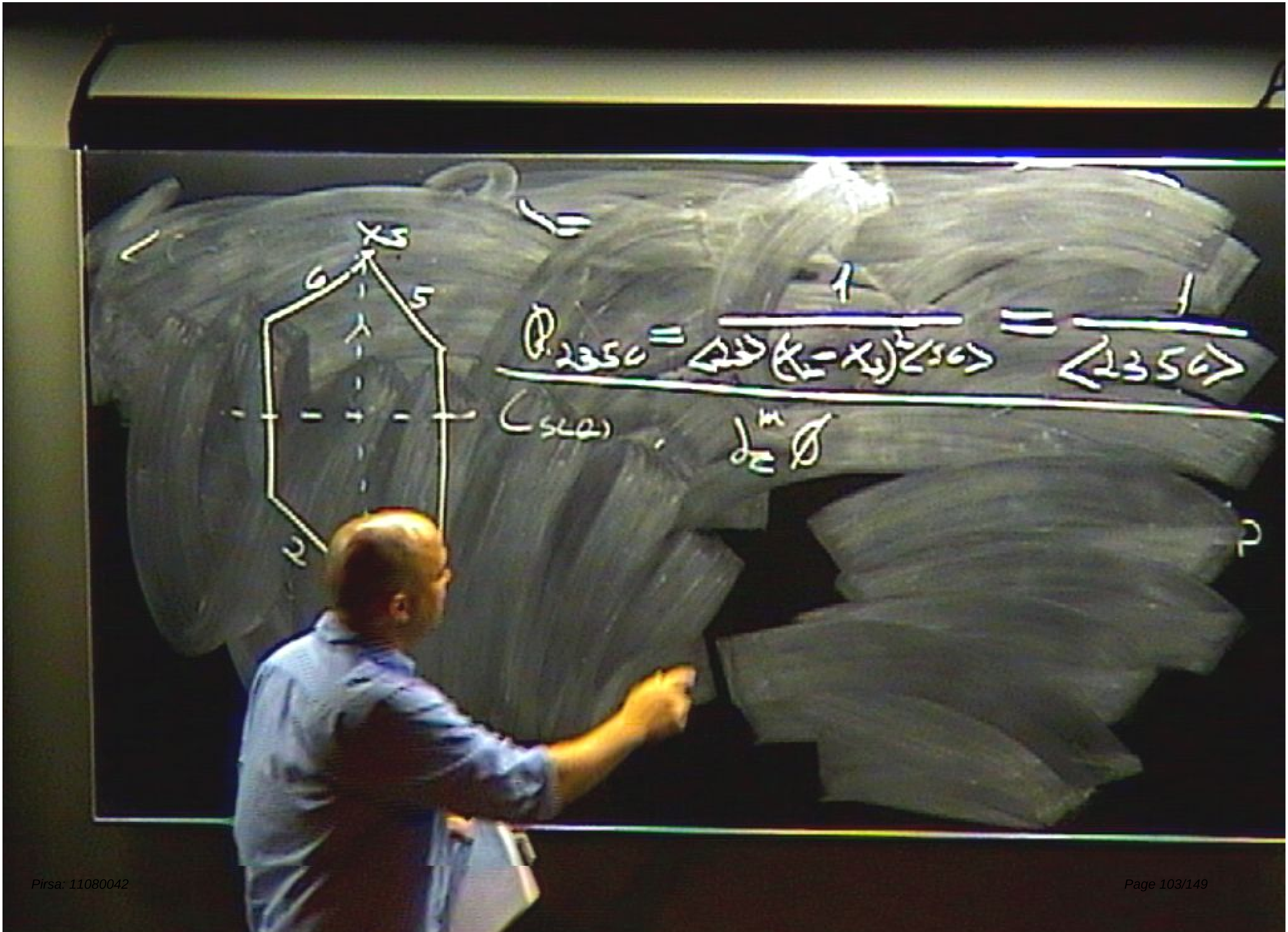
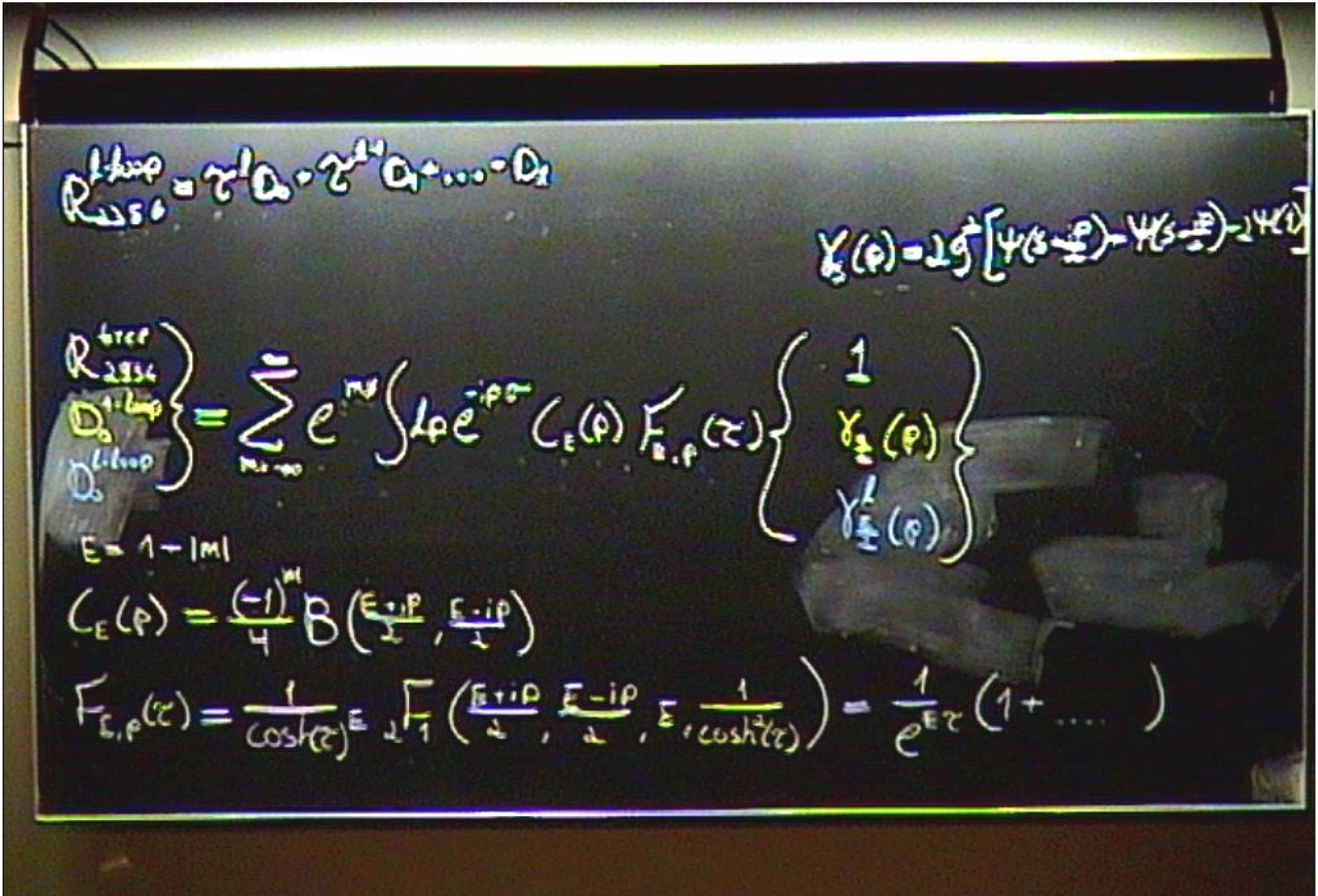


$$\phi_{2350} = \frac{1}{\langle 23\rangle (x_2 - x_3)^2 \langle 50\rangle} = \frac{1}{\langle 2350\rangle}$$


x5
6
5
2
⟨2350⟩
$d_c^m \phi$



$$R^{1\text{-loop}}_{2356} = \tau^{1} D_0 + \tau^{2+4} D_1 + \dots + D_2$$

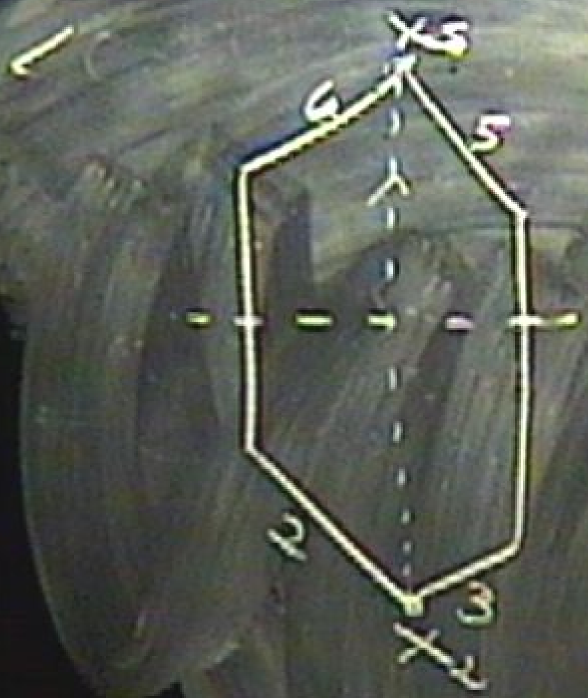
$$\gamma_{\pm}(p) = 2g^2\left[\psi\left(\tfrac{3}{2}+\tfrac{ip}{2}\right) + \psi\left(\tfrac{3}{2}-\tfrac{ip}{2}\right) - 2\psi(1)\right]$$

$$\left.\begin{matrix} R^{\text{tree}}_{2356} \\ D_0^{1\text{-loop}} \\ D_0^{L\text{-loop}} \end{matrix}\right\} = \sum_{m=-\infty}^{\infty} e^{im\phi} \int dp\, e^{-ip\sigma}\, C_E(p)\, F_{E,p}(\tau) \left\{\begin{matrix} 1 \\ \gamma_{\pm}(p) \\ \gamma^{L}_{\pm}(p) \end{matrix}\right\}$$

$$E = 1 - |m|$$

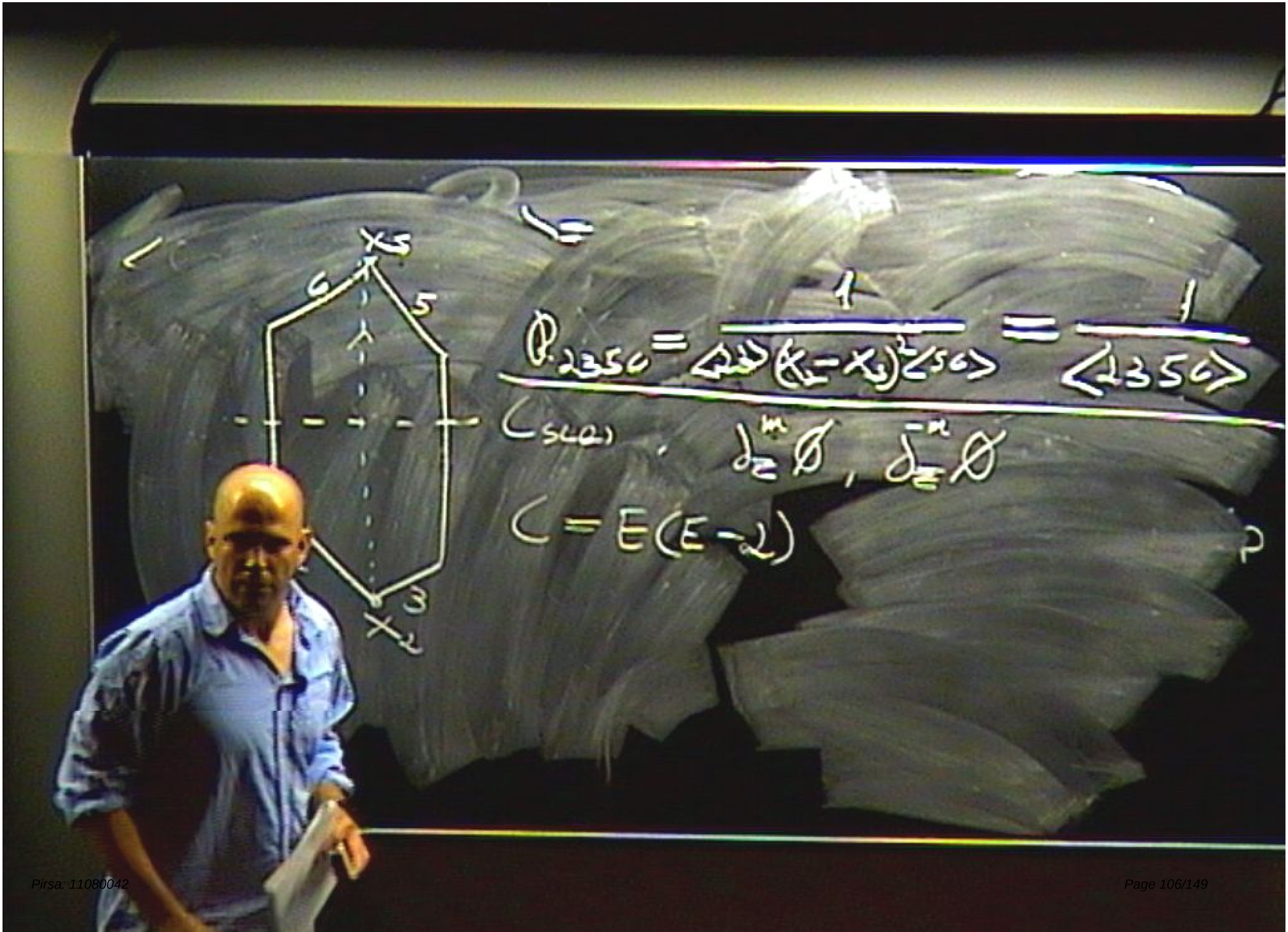
$$C_E(p) = \frac{(-1)^m}{4} B\left(\frac{E+ip}{2}, \frac{E-ip}{2}\right)$$

$$F_{E,p}(\tau) = \frac{1}{\cosh(\tau)^E}\, {}_2F_1\left(\frac{E+ip}{2}, \frac{E-ip}{2}, E, \frac{1}{\cosh^2(\tau)}\right) = \frac{1}{e^{E\tau}}\left(1 + \dots\right)$$



$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_2 - x_3)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$

$$\partial_z^{m} \phi, \quad \partial_z^{-m} \phi$$



$$\Phi_{2356} = \frac{1}{\langle 23 \rangle (x_2 - x_5)^2 \langle 56 \rangle} = \frac{1}{\langle 2356 \rangle}$$

$C_{SL(2)}$: $\partial_z^{m} \phi , \partial_{\bar{z}}^{-m} \phi$

$C = E(E-2)$

$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_2-x_3)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$

$C_{SU(2)}$ $\qquad \partial_z^{m}\phi \, , \; \partial_z^{-m}\phi$

$C = E(E-2)$

$C \circ R = (J_1^2 - J_2^2 + J_3^2) \circ R$

$$R_{2356} = \frac{1}{\langle 23\rangle (x_2 - x_3)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$

$$L_{SL(2)} \quad , \quad \partial_z^{m}\phi \, , \, \partial_z^{-n}\phi$$

$$C = E(E-2) \qquad p$$

$$C \circ R = (J_1^2 - J_2^2 + J_3^2) \circ R$$

$$= \sum_{a=1}^{3} (-1)^a \partial_a^2 R[z_{top}, z_2, z_1, e^{\sigma_a} z_{top}]$$

$R_{2356}^{1\text{-loop}}$

R_{2356}^{tree}

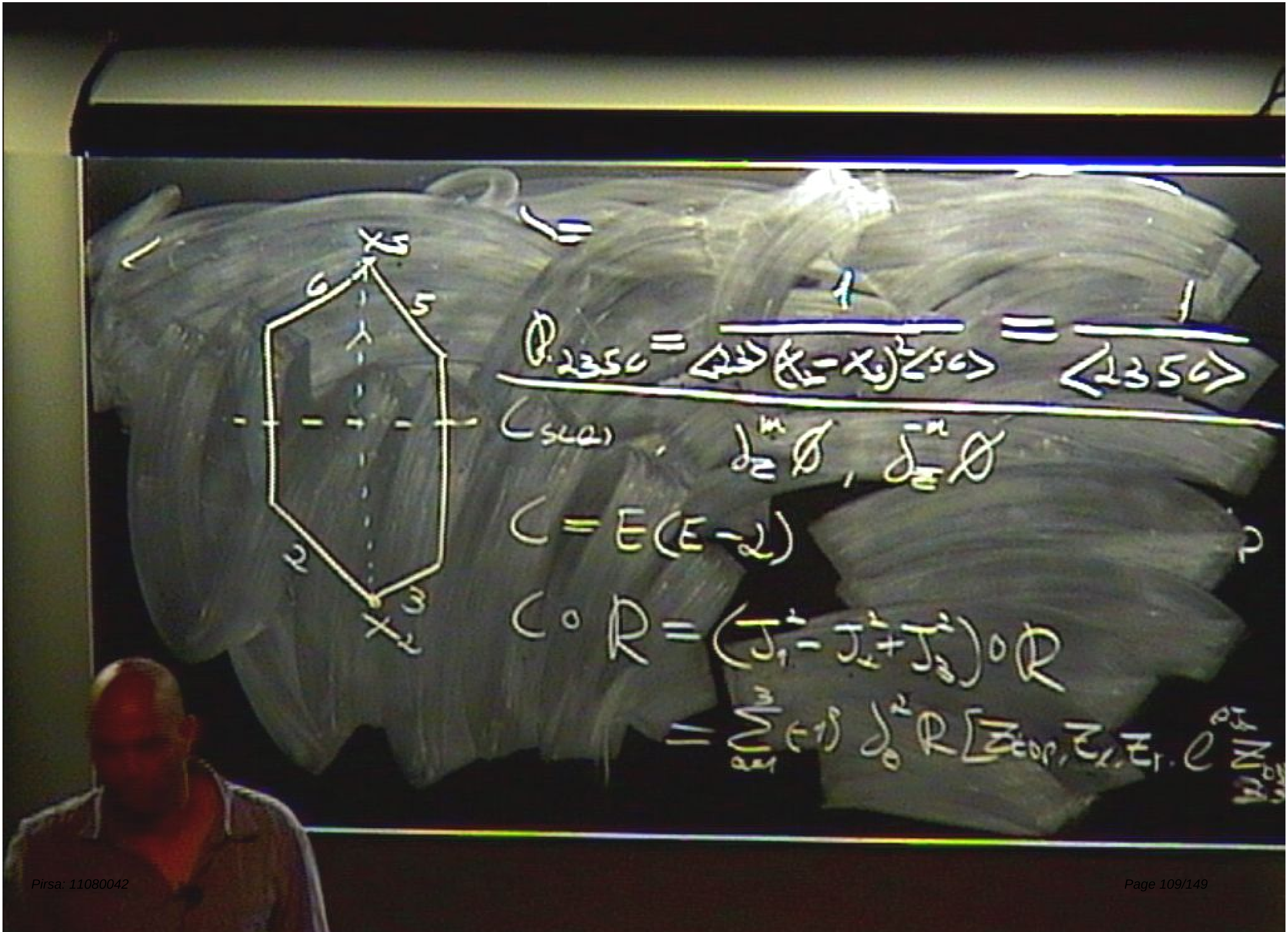
$D_0^{1\text{-loop}}$

$D_0^{1\text{-loop}}$

$E = 1 -$

$C_E(p)$

$F_{E,p}(z$



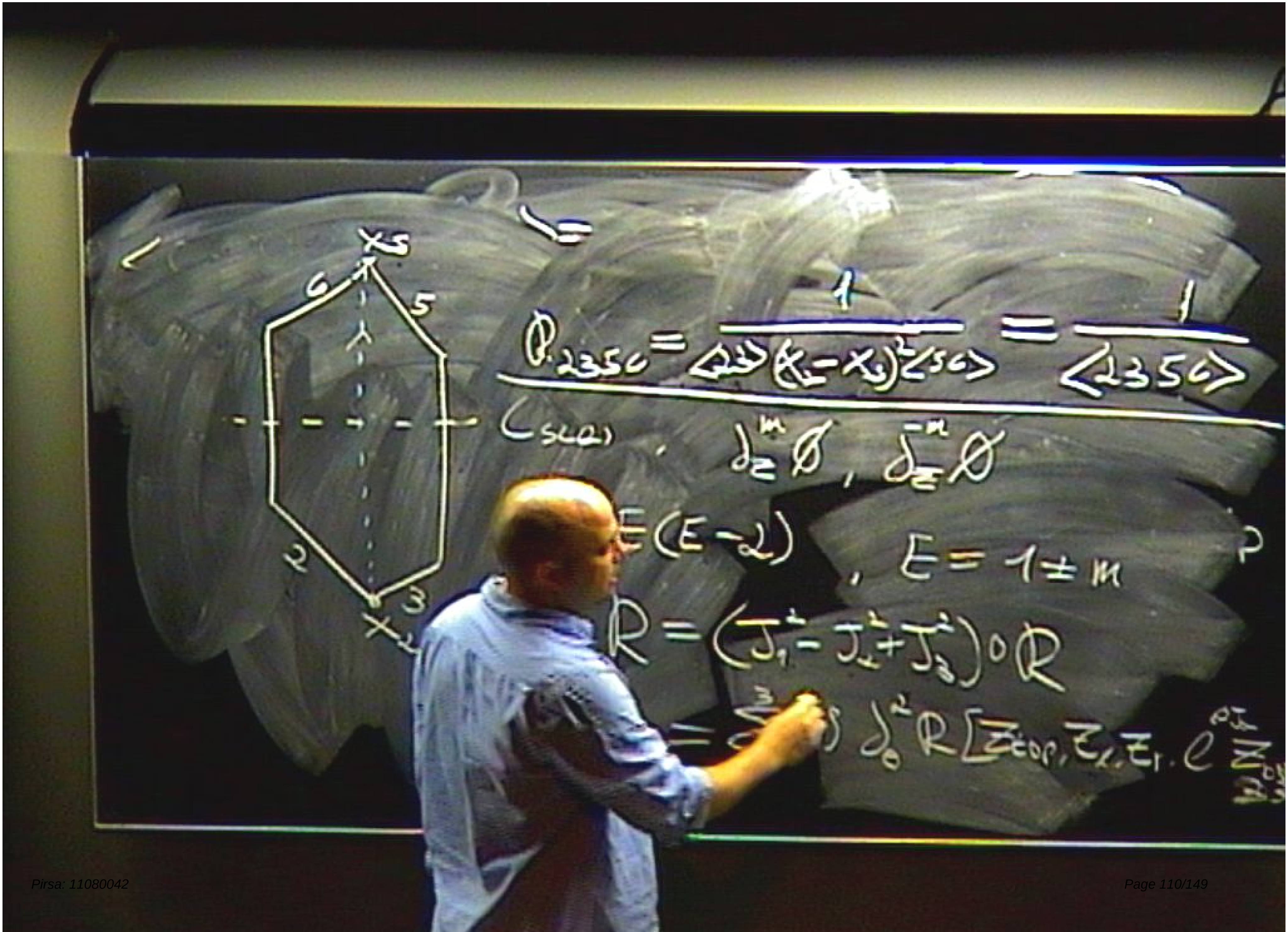
$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_2 - x_5)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$

$C_{SL(2)}$: $\partial_z^{m}\phi$, $\partial_{\bar{z}}^{-m}\phi$

$$C = E(E-2)$$

$$C \circ R = (J_1^2 - J_2^2 + J_3^2) \circ R$$

$$= \sum_{a=1}^{3} (-1)^a \, \partial_\alpha^2 R[Z_{top}, Z_L, Z_r \cdot e^{\alpha J_a} Z_{bo}$$



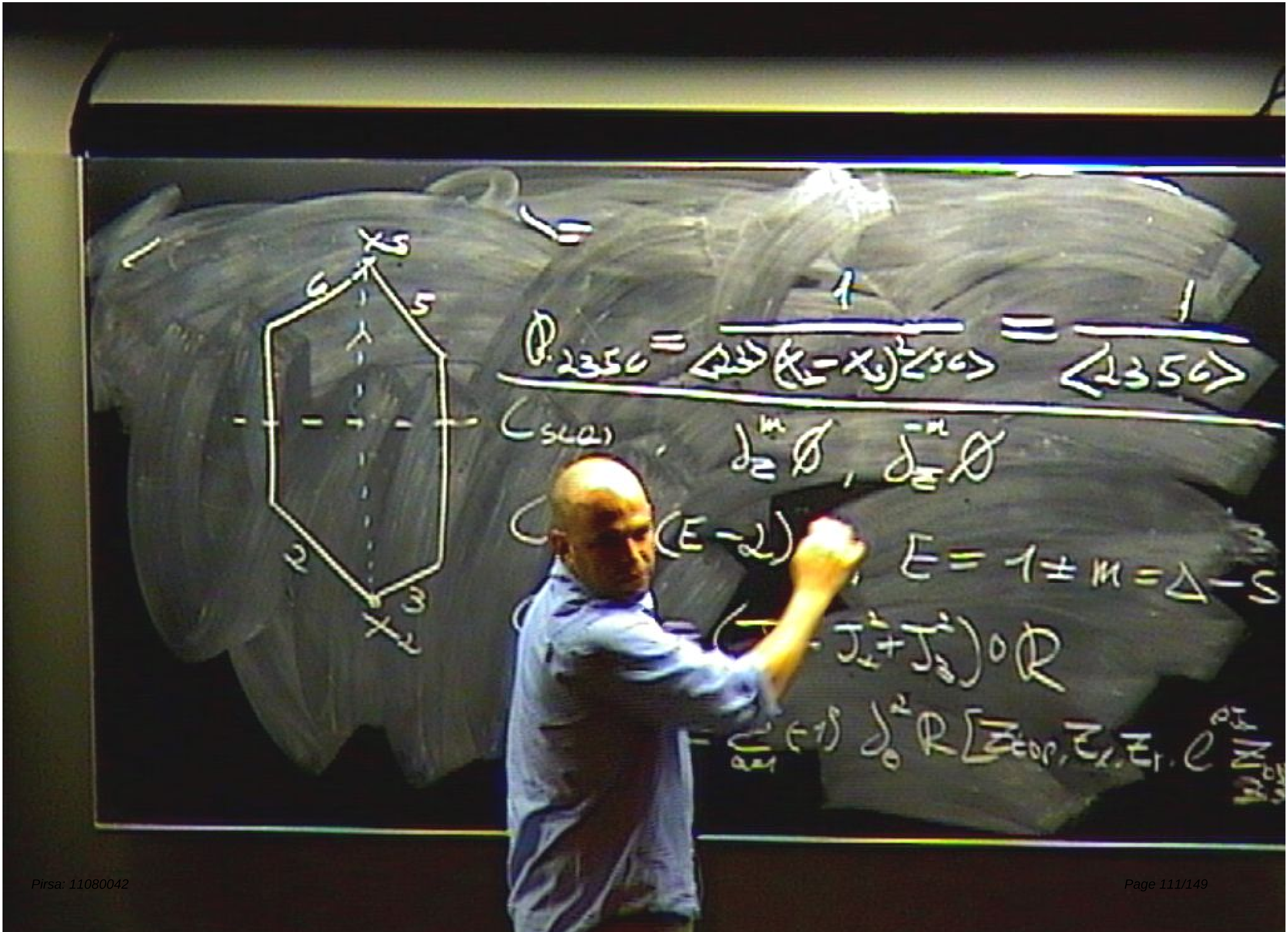
$$\Phi_{2356} = \frac{1}{\langle 23\rangle (x_2 - x_5)^2 \langle 56\rangle} = \frac{1}{\langle 2356\rangle}$$

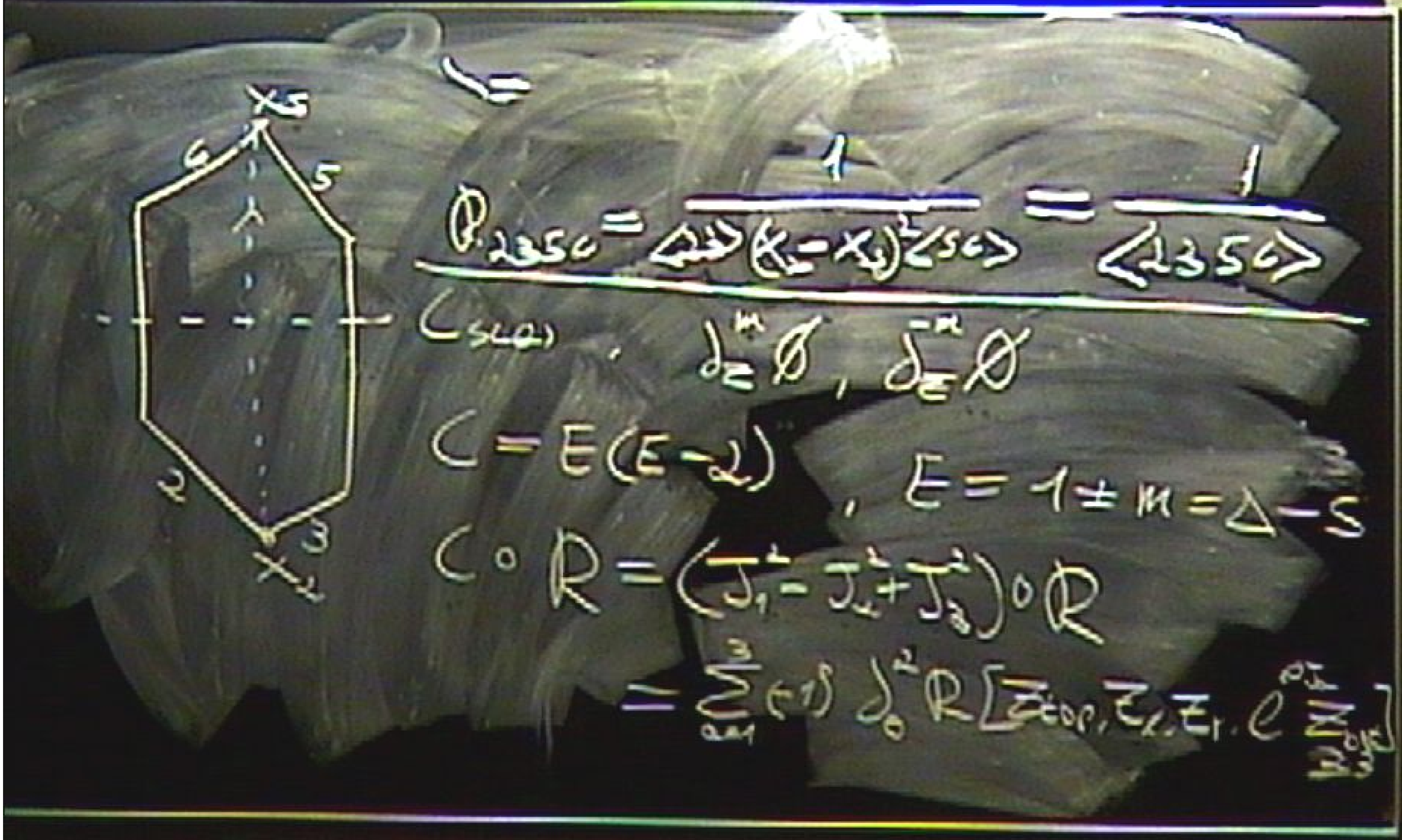
$\partial_z^{m}\phi$, $\partial_z^{-m}\phi$

$E(E-2)$, $E = 1 \pm m$?

$R = (J_1^2 - J_2^2 + J_3^2) \circ R$

$= \int^{3} d\sigma\, \partial_\sigma^2 R[Z_{top}, Z_1, Z_1 \cdot e^{\sigma J_2} Z_{bot}$


x5
6
5
2
3
x2
L_{SL(2)}
Φ_{2356} = 1 / (⟨23⟩(x_2 − x_3)^2⟨56⟩) = 1 / ⟨2356⟩
∂_z^m Φ, ∂_{\bar z}^{−m} Φ
(E − 2)
E = 1 ± m = Δ − S
J_2^2 + J_3^2) ∘ R
(−1) ∂_σ^2 R [Z_{top}, Z_1, Z_r, e



$C =$

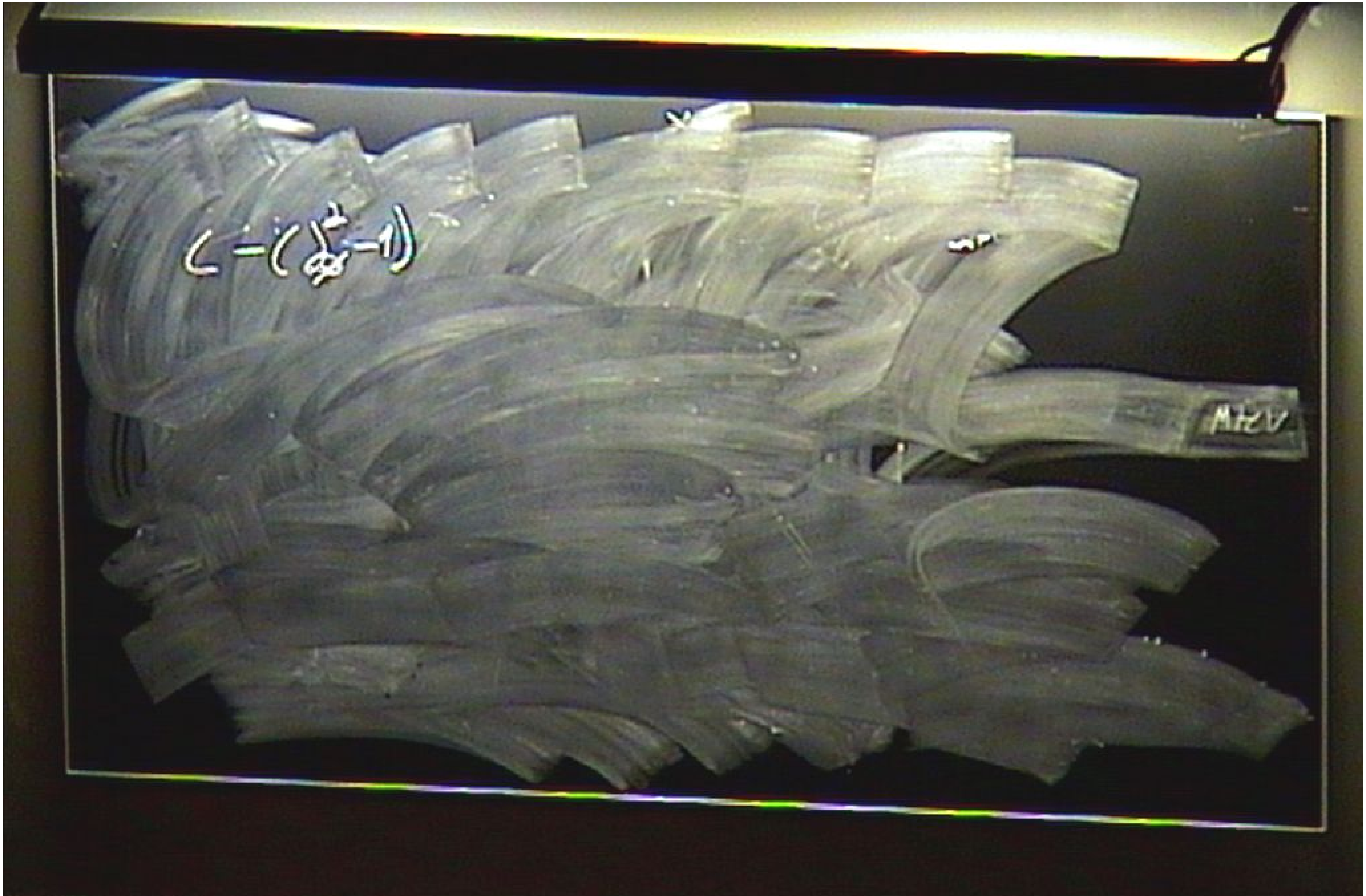
$$R_{2350} = \frac{1}{\langle 23\rangle (x_2 - x_3)^2 \langle 50\rangle} = \frac{1}{\langle 2350\rangle}$$

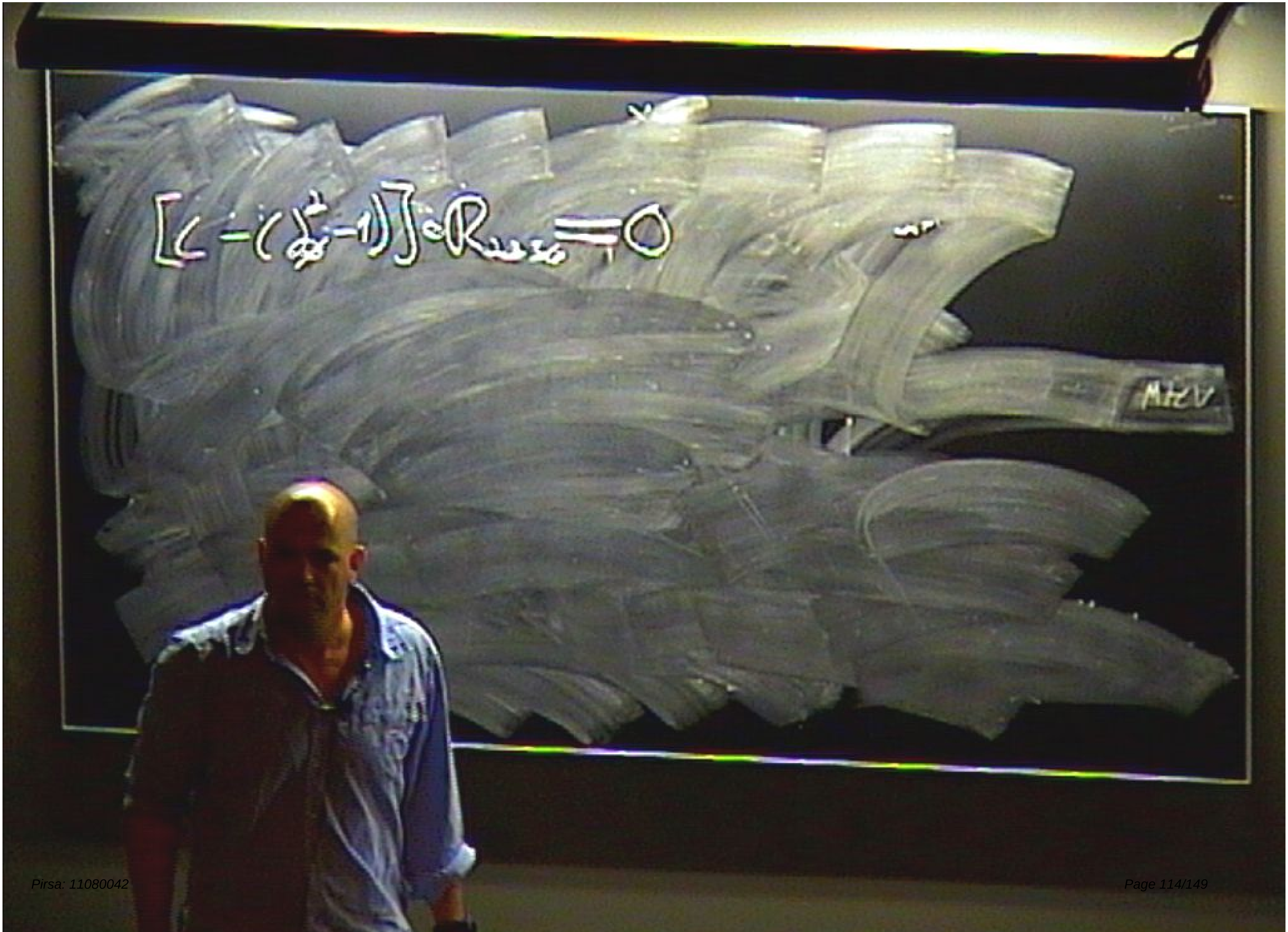
$C_{SL(2)}$, $\partial_z^{m}\emptyset , \partial_z^{-m}\emptyset$

$C = E(E-2)$, $E = 1 \pm m = \Delta - S$

$C \circ R = (J_1^2 - J_2^2 + J_3^2) \circ R$

$$= \sum_{a=1}^{3} (-1)^{a} \partial_0^2 R[Z_{top}, Z_L, Z_R, e^{\rho J_a} Z_{bot}]$$


$L -\left(\frac{t^2}{x^2}-1\right)$


$\left[L - \left(\frac{\omega^2}{\omega_0^2} - 1 \right) \right] \delta R_{uuz_0} = 0$
MHV

The tree level Ratio function

$$\mathcal{R}_6^{\text{NMHV tree}} = \frac{\mathcal{A}_6^{\text{NMHV tree}}}{\mathcal{A}_n^{\text{MHV tree}}} = \mathcal{R}_{2356}\,\eta_2\eta_3\eta_5\eta_6 + \ldots \underset{\substack{\uparrow \\ \text{BCFW}}}{=} \sum_{1<i<j<6-2} [1, i, i+1, j, j+2]$$

$$[a, b, c, d, e] \equiv \frac{\delta^{0|4}\left(\eta_a \langle bcde\rangle + \eta_b \langle cdea\rangle + \eta_c \langle deab\rangle + \eta_d \langle eabc\rangle + \eta_e \langle abcd\rangle\right)}{\langle abcd\rangle \langle bcde\rangle \langle cdea\rangle \langle deab\rangle \langle eabc\rangle}$$

$$\mathcal{R}_{2356} = \frac{1}{\langle 2356\rangle} = \frac{1}{4[\cosh(\tau)\cosh(\sigma) + \cosh(\phi)]}$$

$$= \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \sum_{\beta_m} \mathcal{C}_{\beta_m}(p) \mathcal{F}_{\beta_m,p}(\tau)$$

Conformal blocks

Form factors

Primaries

Conformal blocks

$\left[\mathcal{C} - \left(\partial_\phi^2 - 1\right)\right] \mathcal{R}_{2356} = 0 \quad \Rightarrow$ Decomposition in scalars conformal blocks exist!

Conformal blocks

$\left[\mathcal{C} - (\partial_\phi^2 - 1)\right] \mathcal{R}_{2356} = 0$ ⇒ Decomposition in scalars conformal blocks exist!

Conformal blocks

$[\mathcal{C} - (\partial_\phi^2 - 1)]\, \mathcal{R}_{2356} = 0 \qquad \Rightarrow \qquad$ Decomposition in scalars conformal blocks exist!

$$\left[\left(\partial_\tau^2 + 2\coth(2\tau)\partial_\tau - \frac{p^2}{\cosh^2(\tau)}\right) - E(E-2)\right]\mathcal{F}^{2356}_{E,p}(\tau) = 0$$

$$\Rightarrow \qquad \mathcal{F}^{2356}_{E,p}(\tau) = \frac{1}{\cosh^E(\tau)}\,{}_2F_1\left(\frac{E+ip}{2}, \frac{E-ip}{2}, E, \frac{1}{\cosh^2(\tau)}\right) = \frac{1}{e^{E\tau}}(1+\ldots)$$

Conformal blocks

$[\mathcal{C} - (\partial_\phi^2 - 1)]\,\mathcal{R}_{2356} = 0 \quad \Rightarrow$ Decomposition in scalars conformal blocks exist!

$$\left[\left(\partial_\tau^2 + 2\coth(2\tau)\partial_\tau - \frac{p^2}{\cosh^2(\tau)}\right) - E(E-2)\right]\mathcal{F}^{2356}_{E,p}(\tau) = 0$$

$$\Rightarrow \qquad \mathcal{F}^{2356}_{E,p}(\tau) = \frac{1}{\cosh^E(\tau)}\,{}_2F_1\left(\frac{E+ip}{2}, \frac{E-ip}{2}, E, \frac{1}{\cosh^2(\tau)}\right) = \frac{1}{e^{E\tau}}(1+\dots)$$

Note that $\mathcal{F}^{2356}_{E,p}$ depends on (2356).

$\mathcal{R}_{2356}$ is an analog of $\langle \mathcal{O}^{\mu\nu}\mathcal{O}^{\sigma}\mathcal{O}^{\rho}\mathcal{O}^{\gamma\delta}\rangle$ carrying spin

Decomposition

$$\mathcal{R}_{2356} = \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \mathcal{C}_E(p) \left(\frac{1}{e^{E\tau}} + \dots \right) = \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \mathcal{C}_E(p) \mathcal{F}_{E,p}(\tau)$$

$$E(m) = |m| + 1$$

Decomposition

$$\mathcal{R}_{2356} = \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \mathcal{C}_E(p) \left(\frac{1}{e^{E\tau}} + \ldots \right) = \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \mathcal{C}_E(p) \mathcal{F}_{E,p}(\tau)$$

$$E(m) = |m| + 1$$

$$\mathcal{C}_E(p) = \mathcal{C}_E^{2356}(p) = \frac{(-1)^m}{4} B\left(\frac{E+ip}{2}, \frac{E-ip}{2} \right) \simeq \frac{\text{Rational}(p)}{\sinh(p)}$$

Decomposition

$$\mathcal{R}_{2356} = \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \mathcal{C}_E(p) \left(\frac{1}{e^{E\tau}} + \ldots \right) = \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi} e^{im\phi - ip\sigma} \mathcal{C}_E(p) \mathcal{F}_{E,p}(\tau)$$

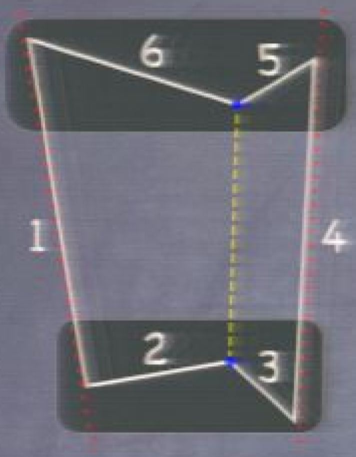
$$E(m) = |m| + 1$$

$$\mathcal{C}_E(p) = \mathcal{C}_E^{2356}(p) = \frac{(-1)^m}{4} B\left(\frac{E+ip}{2}, \frac{E-ip}{2} \right) \simeq \frac{\text{Rational}(p)}{\sinh(p)}$$

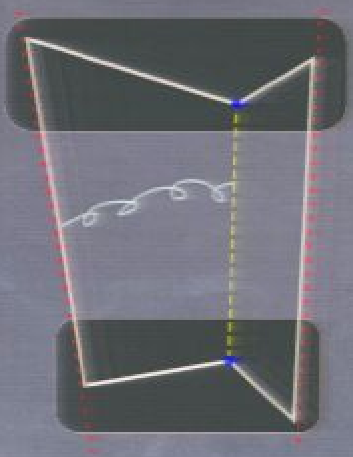
ery similar to other components (and $\Box r_{U(1)}$ of 1-loop MHV).

$$\begin{aligned} \mathcal{F}_{E,p}^{(2256)}(\tau) &= \tanh(\tau)\, \text{sech}^E(\tau)\ {}_2F_1\left[\frac{E-ip}{2}, \frac{E+ip+2}{2}, E, \text{sech}^2(\tau) \right]. \\ \mathcal{C}_m^{(2256)}(p) &= \mathcal{C}_m^{(2356)}(p - 2i\,\delta_{m\geq 0}) \end{aligned}$$

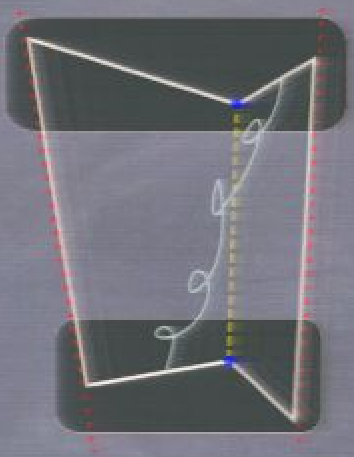
From tree level to one loop



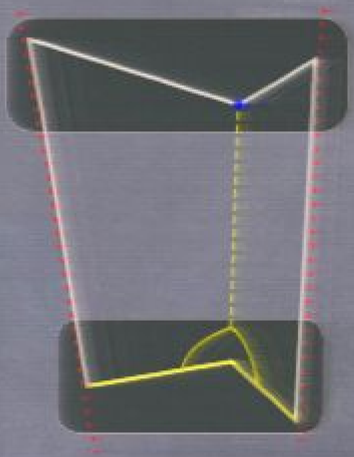
Tree level



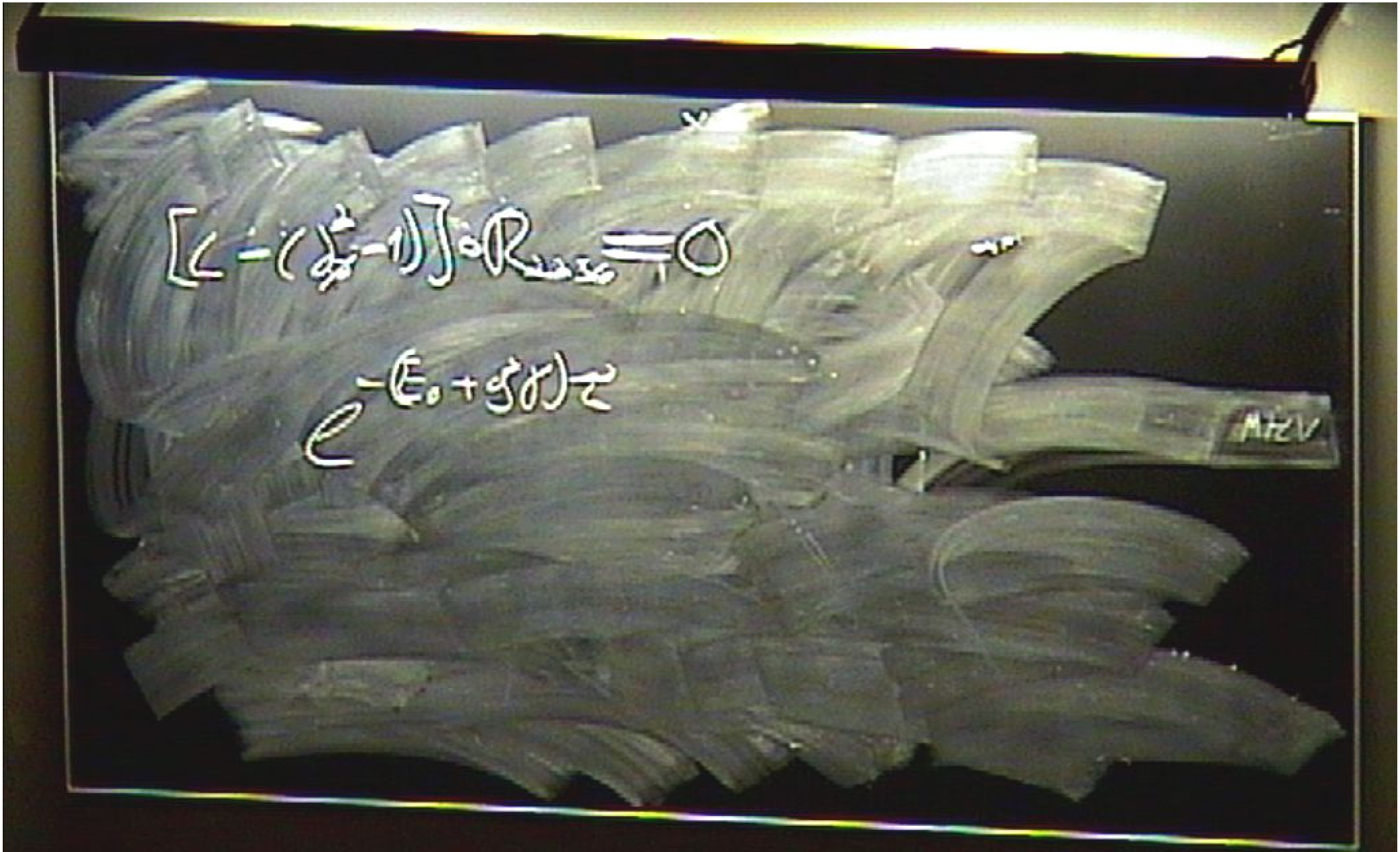
Flux tube interaction



Two particles



Form factor correction

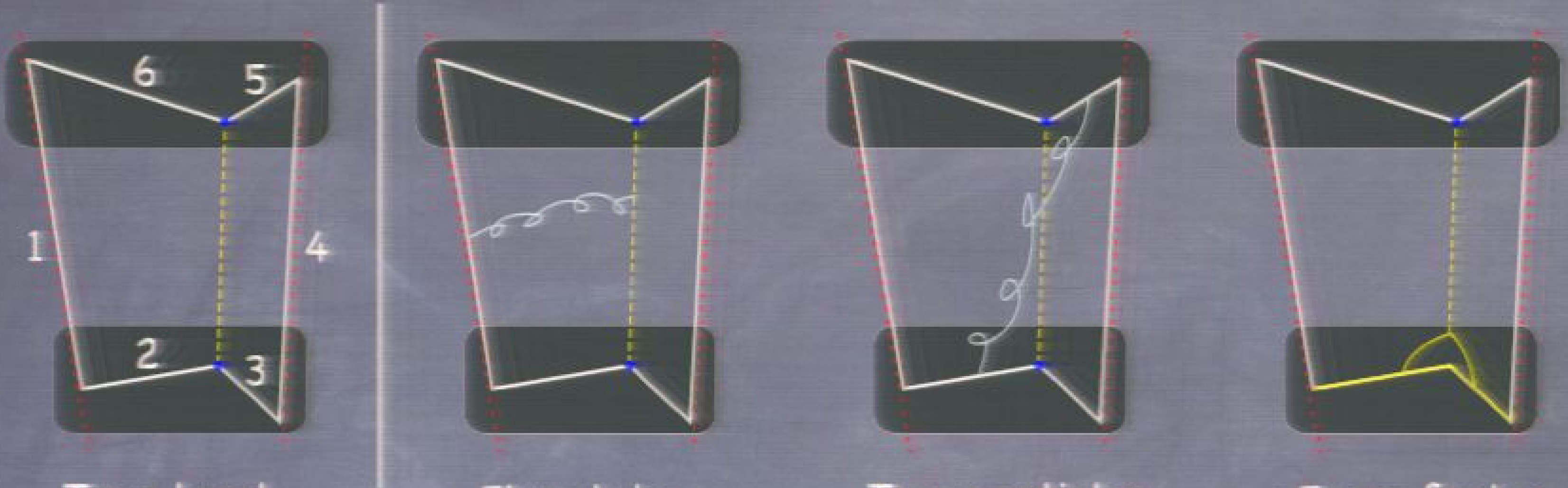


$$\left[L-\left(g^2-1\right)\right]\circ R_{\omega\omega z}=0$$

$$e^{-(E_0+g\gamma)\tau}$$

MHV

From tree level to one loop



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$$\mathcal{R}^{\text{1-loop}} = \tau D + \widetilde{D}$$

$$D_{2356} = \sum_{m=-\infty}^{\infty} \int \frac{dp}{2\pi}\, e^{im\phi - ip\sigma}\, \mathcal{C}_E^{(2356)}(p)\, \mathcal{F}_{E,p}^{(2356)}(\tau)\, \gamma_{E/2}(p)$$

$$\gamma_s(p) = 2g^2 \left[\psi(s + ip/2) + \psi(s - ip/2) - 2\psi(1)\right]$$

See B. Basso talk

Conformal spin $=(\Delta + S)/2$

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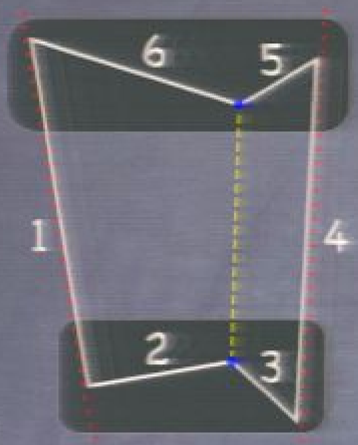

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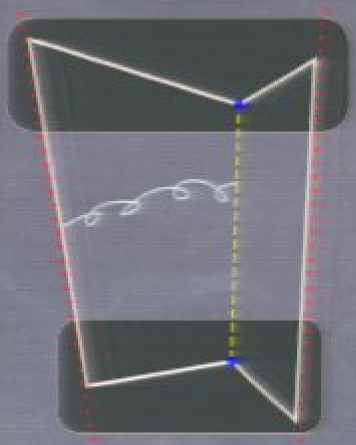

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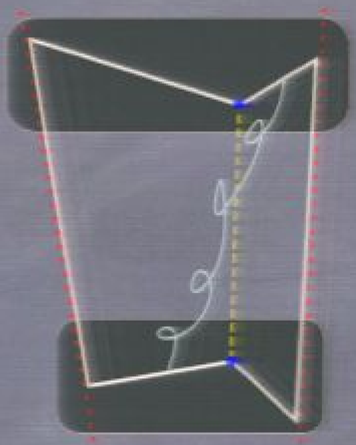

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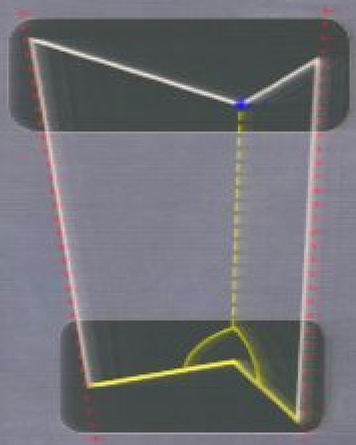

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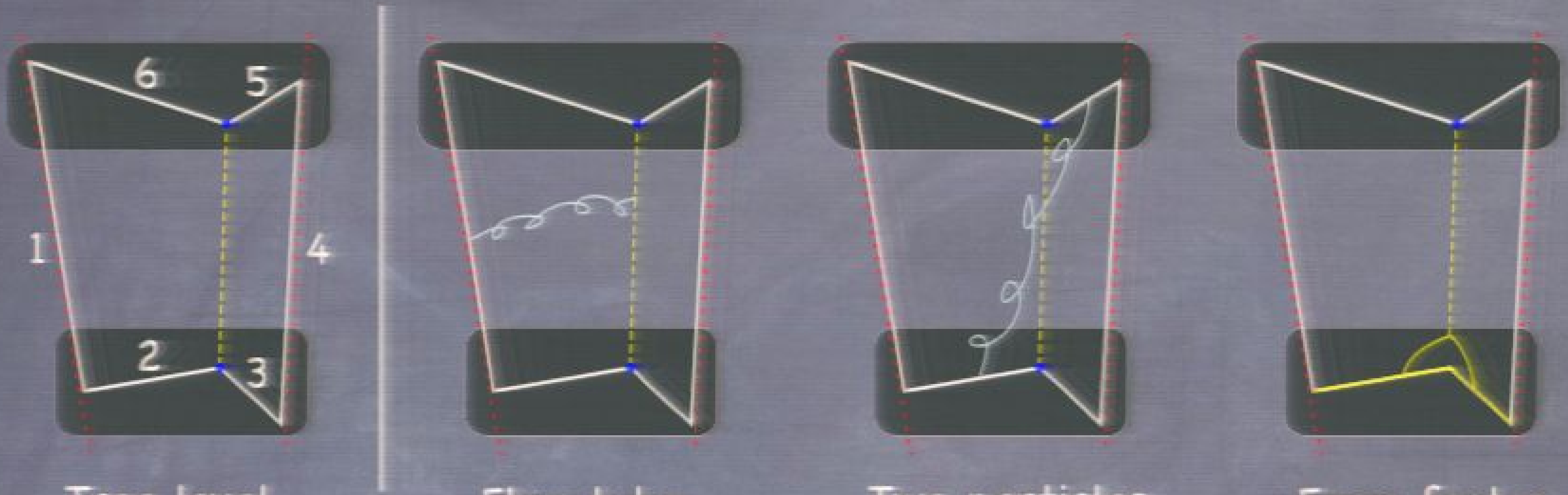

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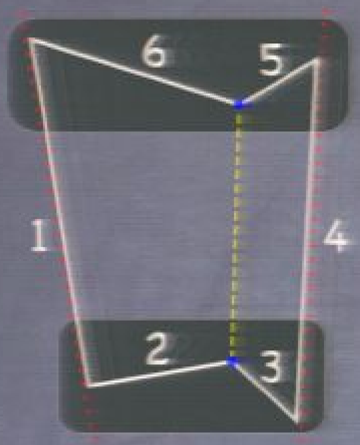

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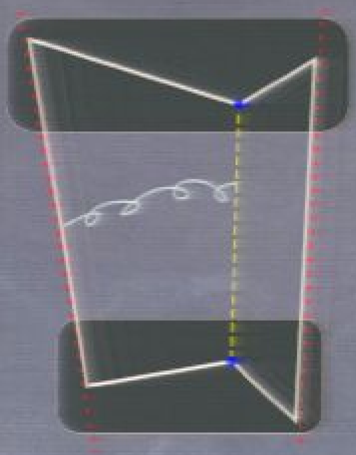

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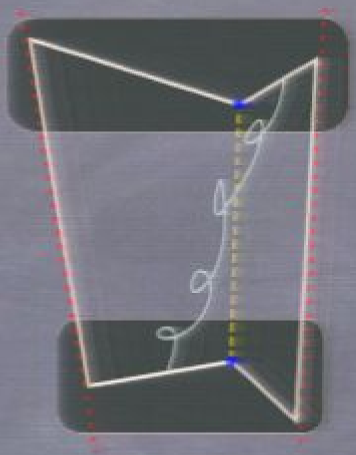

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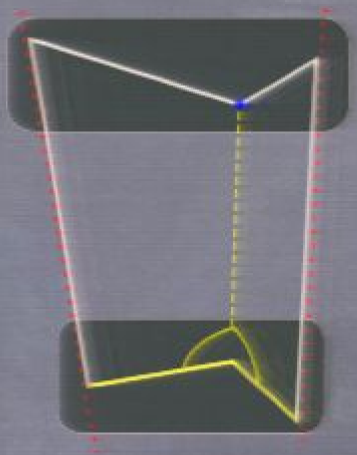

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$$\boxed{D_{2356} = \mathcal{R}_{2356}^{\text{tree}} \times 2\log\left(\frac{u_1 u_3}{1 - u_2}\right)}$$

SUSY

- No cyclicity
- $\mathcal{R}^{\text{1-loop}}_{2356}$ have discontinuities in $u_1,\ u_3$ as well
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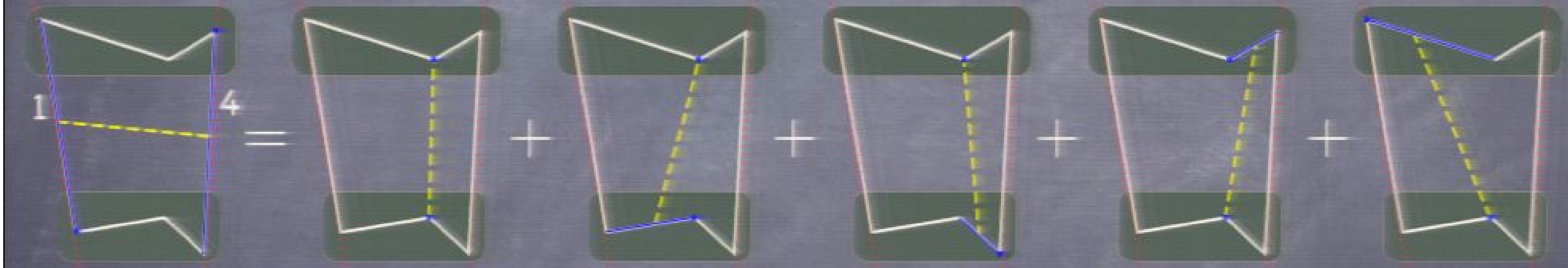

$\eta_1\eta_1\eta_4\eta_4$ = $\eta_2\eta_3\eta_5\eta_6$ + $\eta_2\eta_2\eta_5\eta_6$ + $\eta_3\eta_3\eta_5\eta_6$ ($\sigma \to -\sigma$) + $\eta_2\eta_3\eta_5\eta_5$ ($\phi \to -\phi$) + $\eta_2\eta_3\eta_6\eta_6$ ($\sigma \to -\sigma$, $\phi \to -\phi$)

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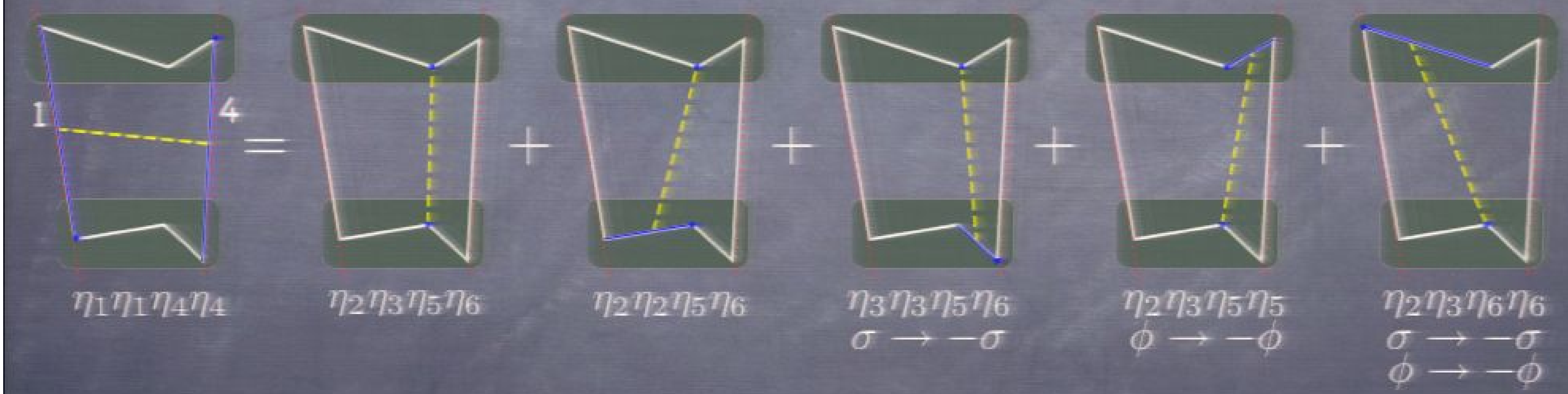

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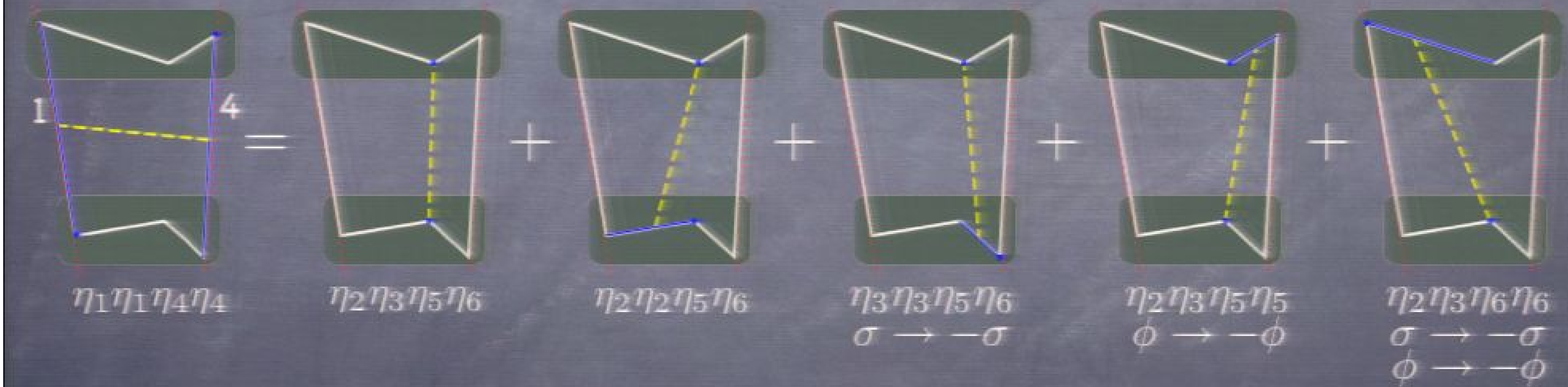

$\Rightarrow$ Everything in the u_2 channel $\Rightarrow$ Everything in the $u_{1,3}$ channel

$$\frac{D_1^{(2356)}}{\mathcal{R}^{(2356)}} = \left(\frac{D_2^{(1245)}}{\mathcal{R}^{(1245)}}\right)_{u_i \to u_{i-1}}$$

Bootstrapping the full 1-loop amplitude

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$$\begin{aligned} D_{u_2}^{(2356)} &= 2\mathcal{R}_{\text{tree}}^{(2356)}\left[\log u_1 + \log u_3 - \log(1-u_2)\right] \\ D_{u_{1/3}}^{(2356)} &= 2\mathcal{R}_{\text{tree}}^{(2356)}\left[\log u_2 - \log u_{3/1} - \log(1-u_{1/3})\right]. \end{aligned}$$

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\end{aligned}
$$

$$
\Rightarrow \quad \mathcal{R}^{(2356)}_{1-loop} = 2\mathcal{R}^{(2356)}_{\text{tree}}\left[\log(u_2)\log(u_1u_3) - \left\{\begin{array}{c}\log(u_2)\log(1-u_2)\\ \text{or}\\ \text{Li}_2(1-u_2)\end{array}\right\}\right] + \ldots
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$$
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No cut at $u_2 = 1$ on the Euclidian sheet

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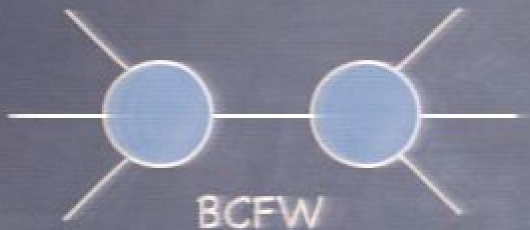
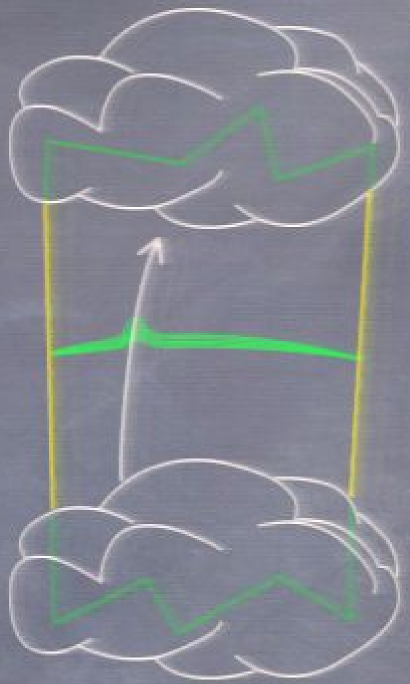
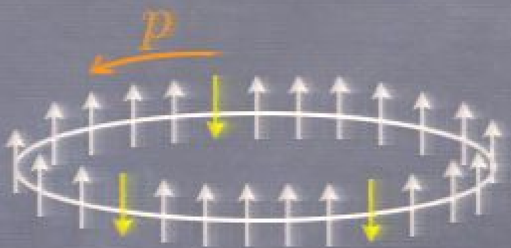
Logic


BCFW

All tree level amplitudes





Dynamics from spin chain
(see B .Basso talk)

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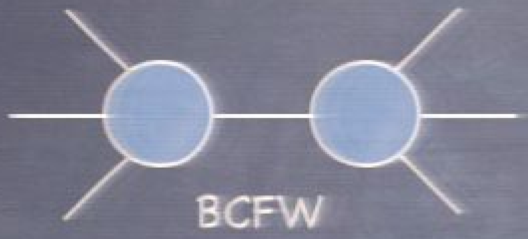

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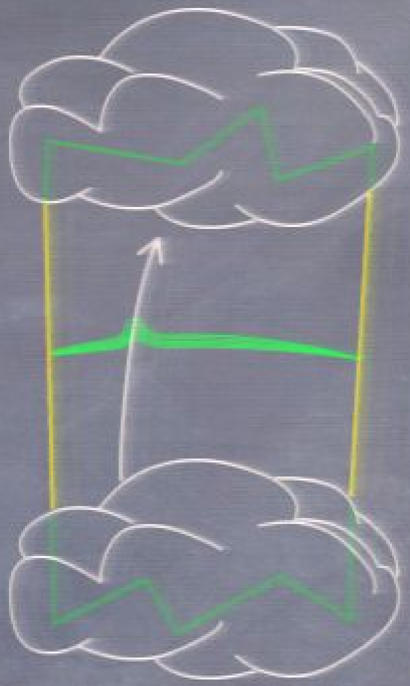

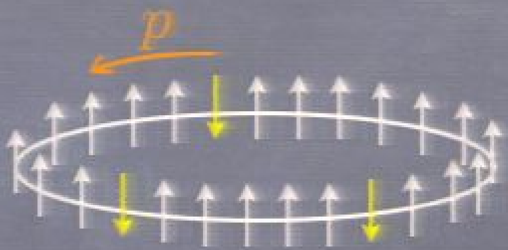

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- No Feynmann diagrams! No 4d. Only tree level data from a 2d integrable approach
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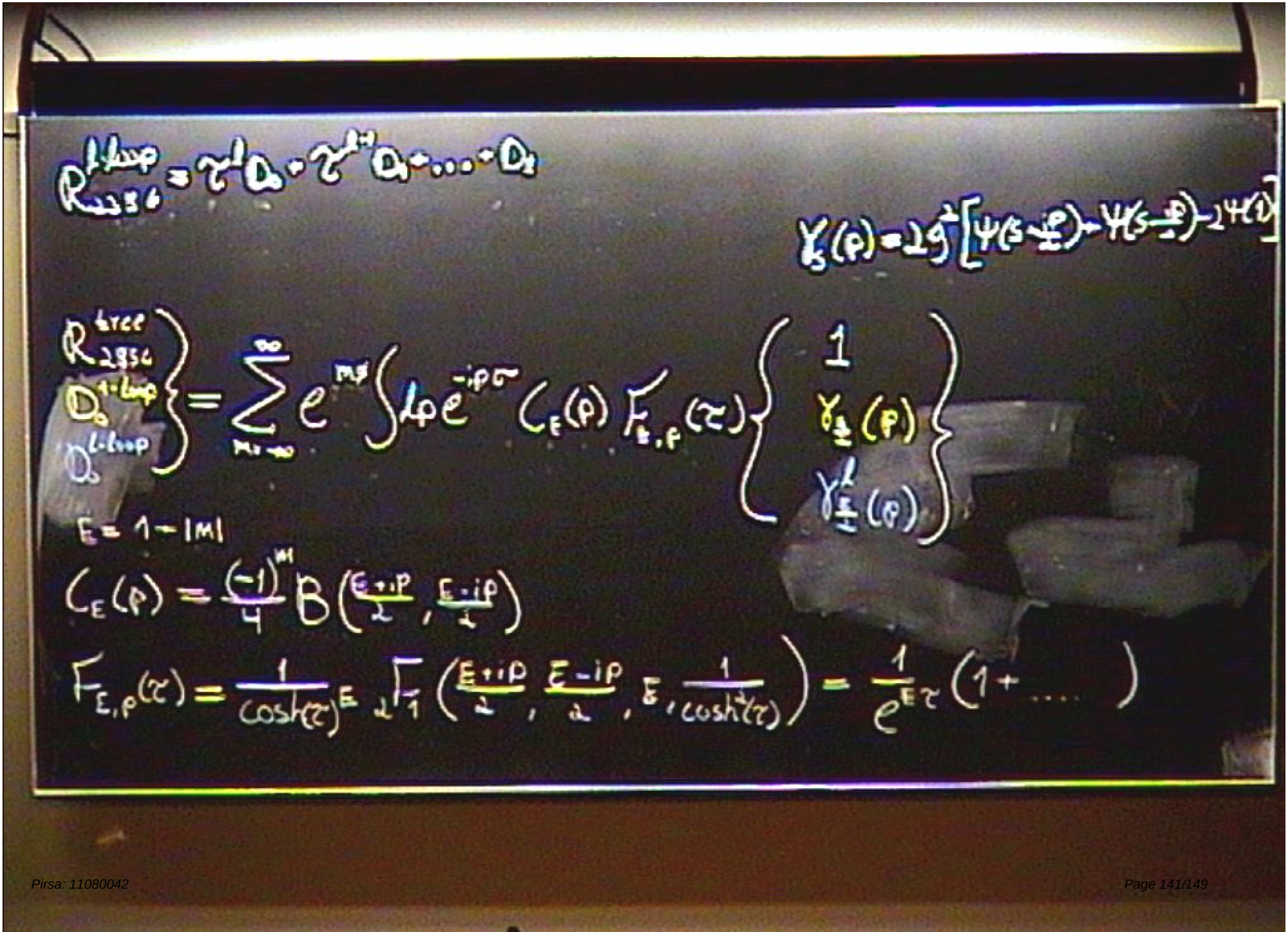


$$R^{l\text{-loop}}_{2356} = \tau^{l} D_0 + \tau^{l-1} D_1 + \dots + D_l$$

$$\gamma_{s}(p) = 2g^2\left[\psi\left(s+\frac{ip}{2}\right)+\psi\left(s-\frac{ip}{2}\right)-2\psi(1)\right]$$

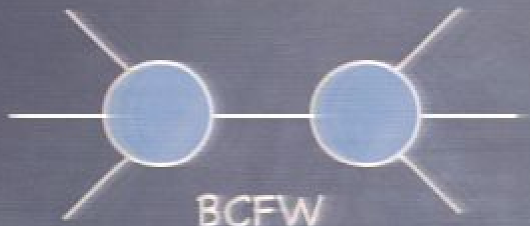
$$\left.\begin{matrix}R^{tree}_{2356}\\ D_0^{1\text{-loop}}\\ D_0^{l\text{-loop}}\end{matrix}\right\} = \sum_{m=-\infty}^{\infty} e^{ms}\int dp\, e^{-ip\sigma}\, C_E(p)\, F_{E,p}(\tau)\left\{\begin{matrix}1\\ \gamma_{\frac{E}{2}}(p)\\ \gamma^{l}_{\frac{E}{2}}(p)\end{matrix}\right\}$$

$$E = 1-|m|$$

$$C_E(p) = \frac{(-1)^m}{4} B\left(\frac{E+ip}{2}, \frac{E-ip}{2}\right)$$

$$F_{E,p}(\tau) = \frac{1}{\cosh(\tau)^E}\, {}_2F_1\left(\frac{E+ip}{2}, \frac{E-ip}{2}, E, \frac{1}{\cosh^2(\tau)}\right) = \frac{1}{e^{E\tau}}\left(1+\dots\right)$$

Logic



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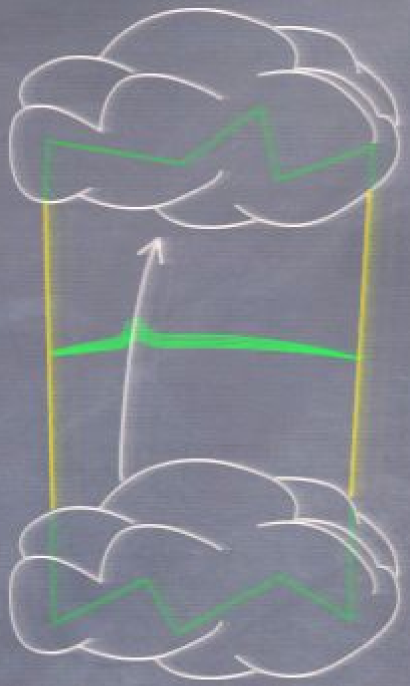

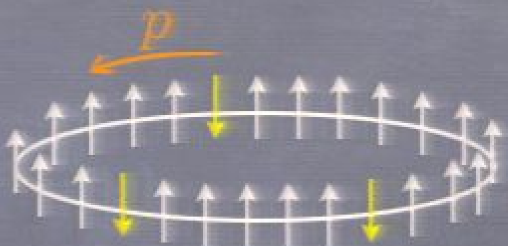

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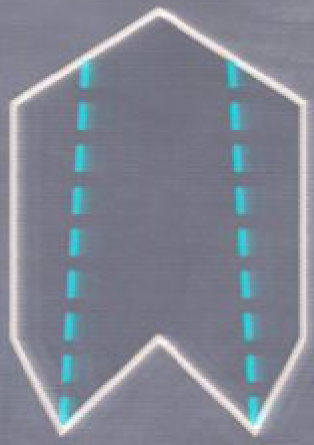

N^2MHV

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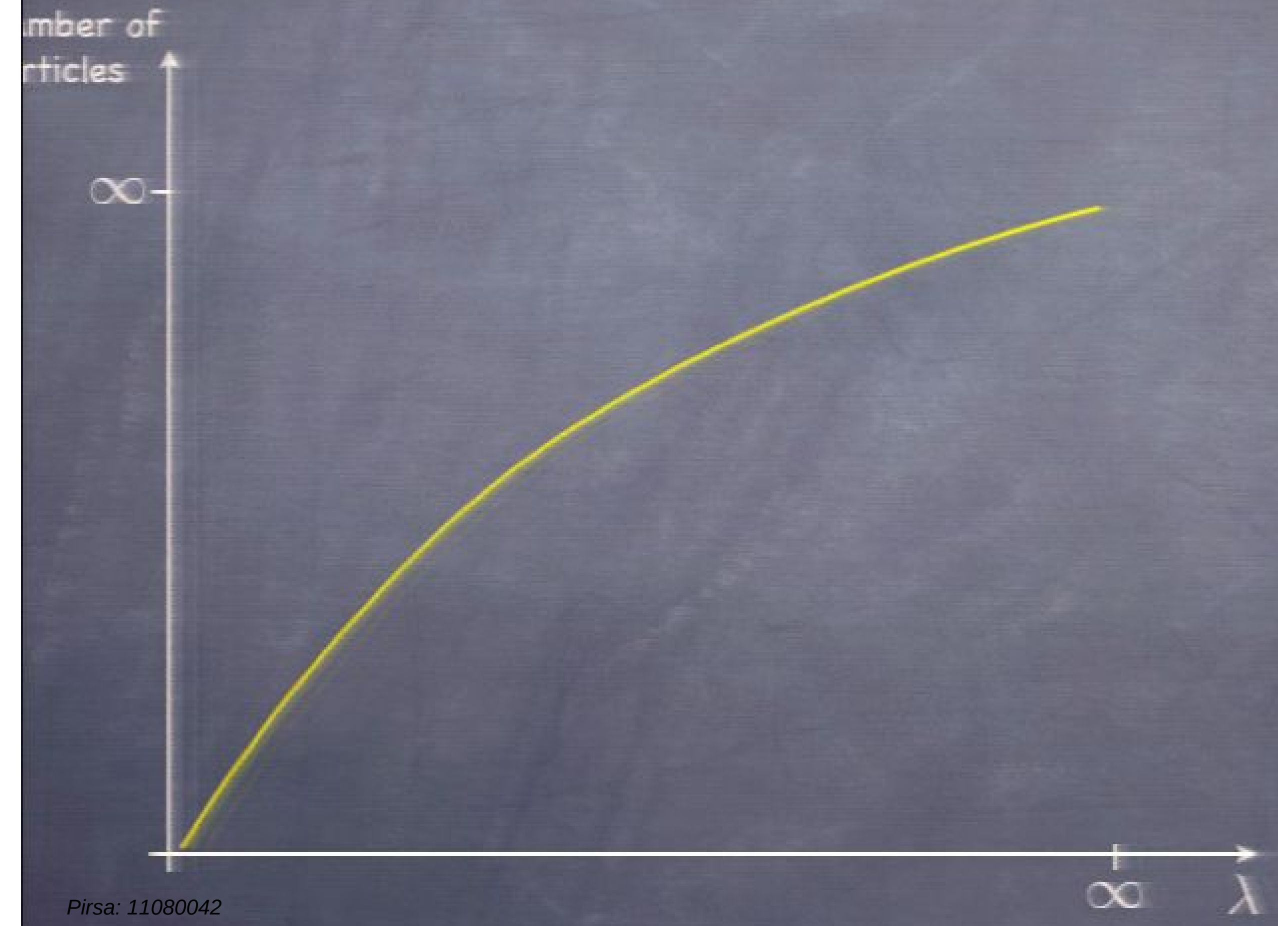

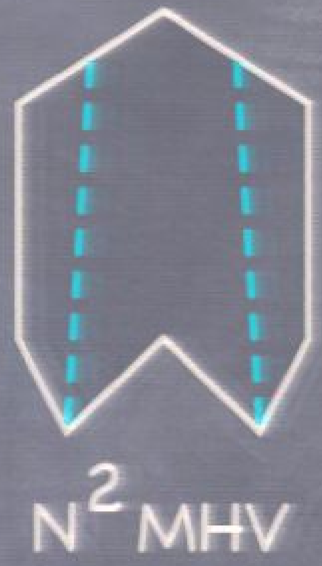

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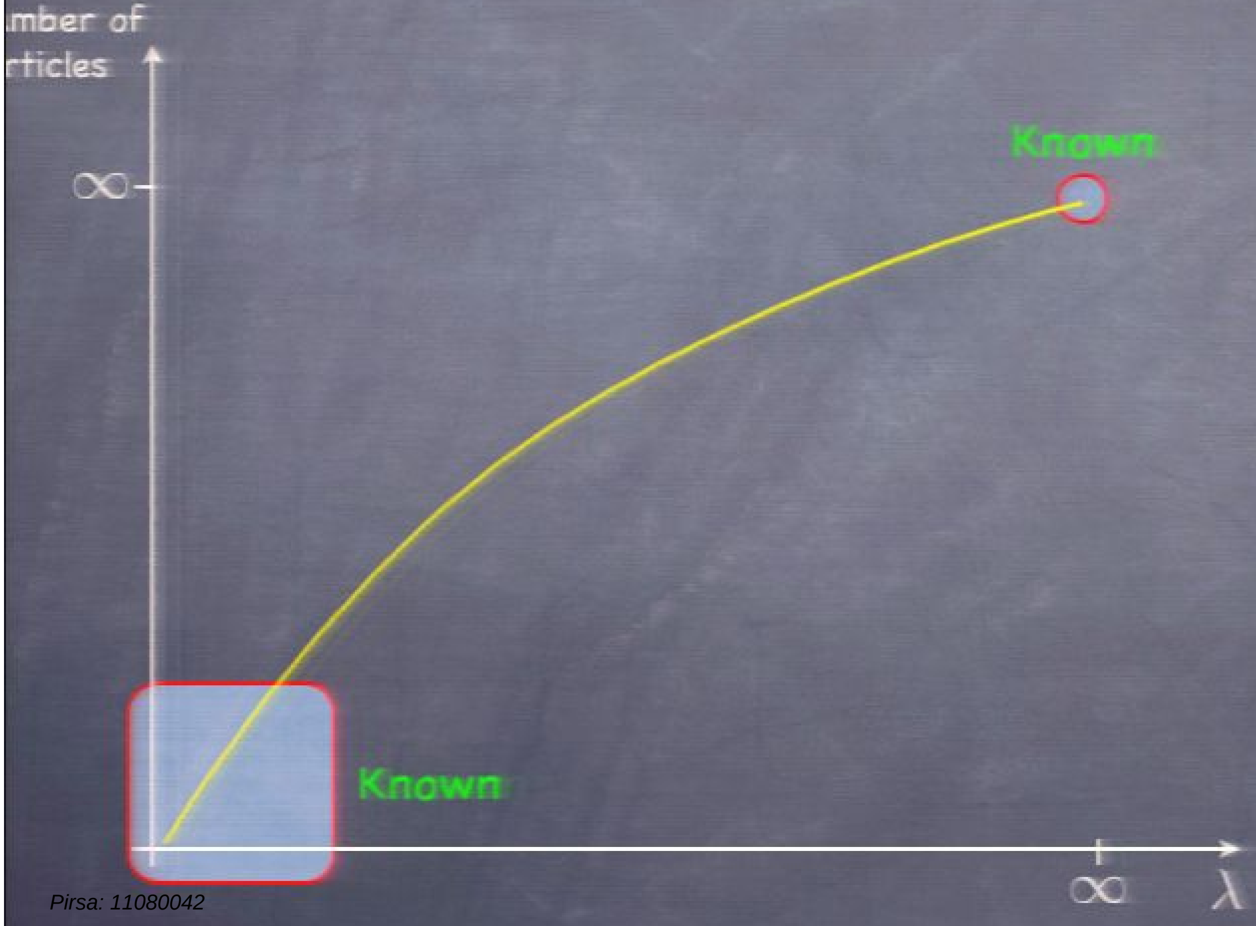

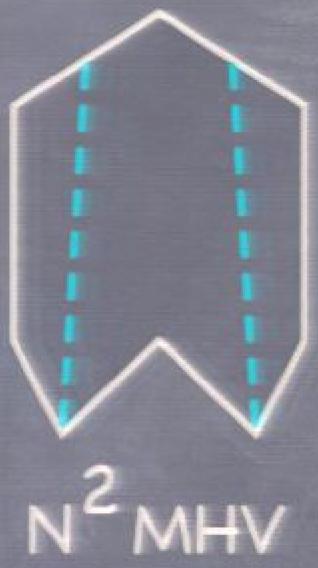

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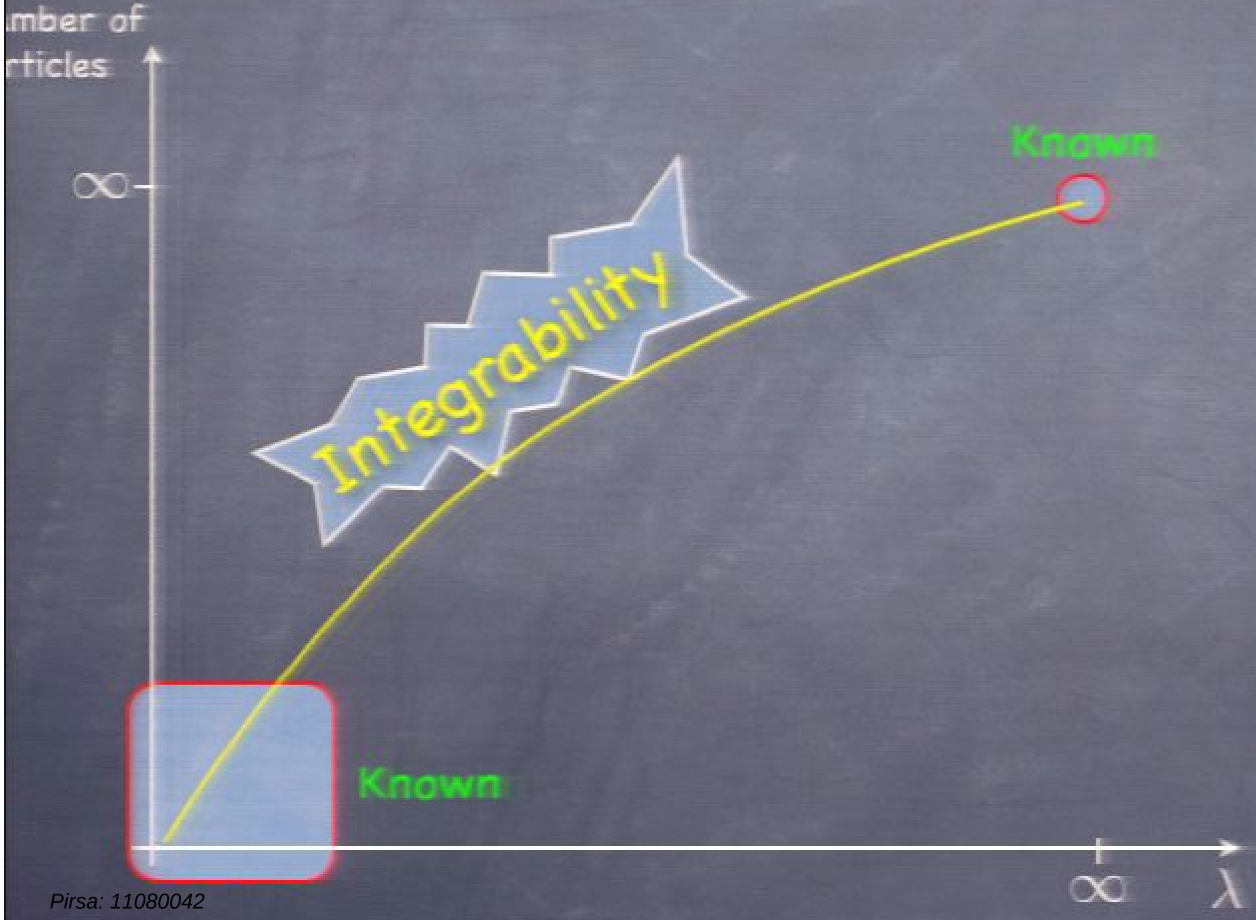

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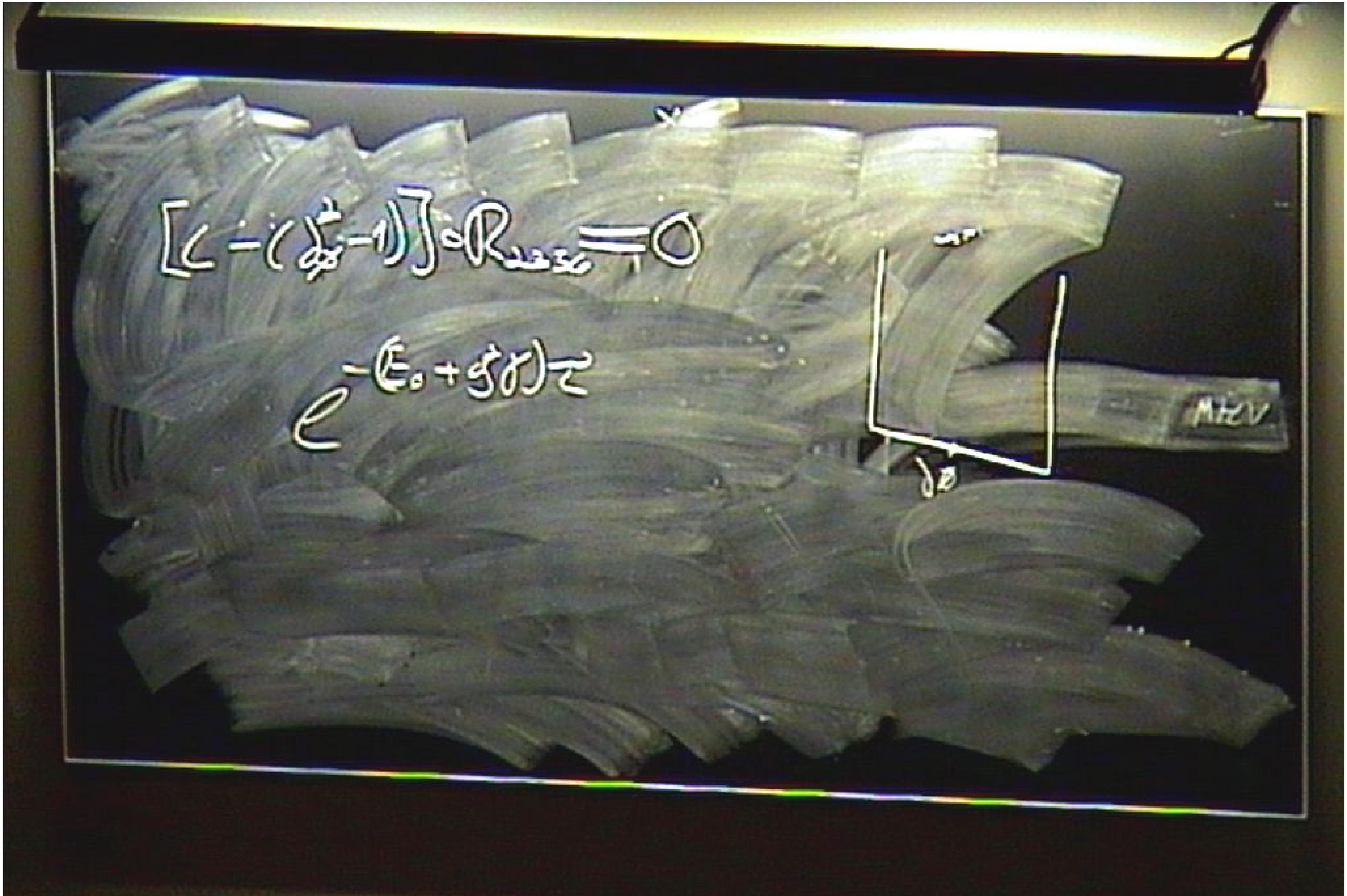

$$\left[L-\left(\frac{g^2}{g_0^2}-1\right)\right]\circ R_{aa3c}=0$$

$$e^{-(E_0+g^2\gamma)\tau}$$

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