Title: Scattering Amplitudes and AdS/CFT Integrability

Date: Aug 17, 2011 10:50 AM

URL: http://pirsa.org/11080041

Abstract: I will give a brief review on the subject of scattering amplitudes in N=4 super Yang-Mills focussing on their infinite dimensional symmetry structure at tree-level and the fate of these symmetries at loop-level in particular employing a Highs-Regulator for the IR divergencies.

Scattering amplitudes in AdS/CFT integrability

Jan Plefka



Humboldt-Universität zu Berlin

based on work with

Fernando Alday, Lance Dixon, James Drummond,
Johannes Henn and Theodor Schuster

IGST 2011 - Perimeter Institute, 17.08.2011

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

The setting

hall focus on the maximal susy AdS_5/CFT_4 system:

> 4d max. susy Yang-Mills theory $\Leftrightarrow$ Superstring theory on $AdS_5 \times S^5$

layor progress

1. Integrability in AdS/CFT:
   - Scaling dimensions alias string spectrum from Bethe equations
     $\Rightarrow$ Close to complete solution of the spectral problem
2. Scattering amplitudes in maximally susy Yang-Mills:
   - Generalized unitarity methods and recursion relations
     $\Rightarrow$ all tree-level amplitudes and many high-loop/high-multiplicity results available
   - Relation to light-like Wilson loops/strongly coupled string description
     $\Rightarrow$ emergence of dual superconformal or Yangian symmetry

his talk: Review some of the progress and ask how to connect the two

Outline

1. Introduction
2. Trees & Symmetries: Superconformal, dual conformal and Yangian invariance

   [Drummond, Henn, JP; JHEP 0905, arXiv:0902]
3. Loops & Symmetries: Overview and novel massive regulator

   [Alday, Henn, JP, Schuster; JHEP 1001, arXiv:0908]

   [JP, Schuster; ongoing work]

Outline

1. Introduction
2. Trees & Symmetries: Superconformal, dual conformal and Yangian invariance

   [Drummond, Henn, JP; JHEP 0905, arXiv:0902]
3. Loops & Symmetries: Overview and novel massive regulator

   [Alday, Henn, JP, Schuster; JHEP 1001, arXiv:0908]

   [JP, Schuster; ongoing work]

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, \mathrm{Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

$\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

$\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, \mathrm{Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

$\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

$\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

$\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

$\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

$\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

  $\Rightarrow$ "H-atom of gauge theories"

Superconformal symmetry

- Symmetry: $\mathfrak{so}(2,4) \otimes \mathfrak{so}(6) \subset \mathfrak{psu}(2,2|4)$

  Poincaré: $p^{\alpha\dot{\alpha}} = p_\mu (\sigma^\mu)^{\dot{\alpha}\beta}$, $l_{\alpha\beta}$, $\bar{l}_{\dot{\alpha}\dot{\beta}}$

  Conformal: $k_{\alpha\dot{\alpha}}$, d (c : central charge)

  R-symmetry: r_{AB}

  Poncaré Susy: $q^{\alpha A}, \bar{q}^{\dot{\alpha}}_A$ Conformal Susy: $s_{\alpha A}, \bar{s}^A_{\dot{\alpha}}$

- $4+4$ Supermatrix notation $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^\alpha{}_\beta - \frac{1}{2}\delta^\alpha_\beta (d + \frac{1}{2}c) & k^\alpha{}_{\dot{\beta}} & s^\alpha{}_B \\ p^{\dot{\alpha}}{}_\beta & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}} (d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_B \\ q^A{}_\beta & \bar{s}^A{}_{\dot{\beta}} & -r^A{}_B - \frac{1}{4}\delta^A_B c \end{pmatrix}$$

- Algebra:

$$[J^{\bar{A}}{}_{\bar{B}}, J^{\bar{C}}{}_{\bar{D}}\} = \delta^{\bar{C}}_{\bar{B}} J^{\bar{A}}{}_{\bar{D}} - (-1)^{(|\bar{A}|+|\bar{B}|)(|\bar{C}|+|\bar{D}|)} \delta^{\bar{A}}_{\bar{D}} J^{\bar{C}}{}_{\bar{B}}$$

Superconformal symmetry

- Symmetry: $\mathfrak{so}(2,4) \otimes \mathfrak{so}(6) \subset \mathfrak{psu}(2,2|4)$

  Poincaré: $p^{\alpha\dot{\alpha}} = p_\mu (\sigma^\mu)^{\dot{\alpha}\beta}$, $l_{\alpha\beta}$, $\bar{l}_{\dot{\alpha}\dot{\beta}}$

  Conformal: $k_{\alpha\dot{\alpha}}$, d (c : central charge)

  R-symmetry: r_{AB}

  Poncaré Susy: $q^{\alpha A}, \bar{q}^{\dot{\alpha}}_A$ Conformal Susy: $s_{\alpha A}, \bar{s}^A_{\dot{\alpha}}$

- $4+4$ Supermatrix notation $\bar{A} = (\alpha, \dot{\alpha}|A)$

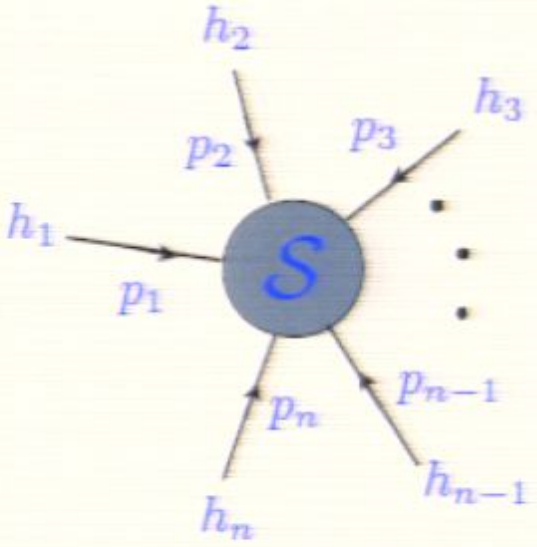
$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^\alpha{}_\beta - \frac{1}{2}\delta^\alpha_\beta (d + \frac{1}{2}c) & k^\alpha{}_{\dot{\beta}} & s^\alpha{}_B \\ p^{\dot{\alpha}}{}_\beta & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}} (d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_B \\ q^A{}_\beta & \bar{s}^A{}_{\dot{\beta}} & -r^A{}_B - \frac{1}{4}\delta^A_B c \end{pmatrix}$$

- Algebra:

$$[J^{\bar{A}}{}_{\bar{B}}, J^{\bar{C}}{}_{\bar{D}}\} = \delta^{\bar{C}}_{\bar{B}} J^{\bar{A}}{}_{\bar{D}} - (-1)^{(|\bar{A}|+|\bar{B}|)(|\bar{C}|+|\bar{D}|)} \delta^{\bar{A}}_{\bar{D}} J^{\bar{C}}{}_{\bar{B}}$$

Scattering amplitudes in $\mathcal{N} = 4$ SYM

- Consider n-particle scattering amplitude



- Planar amplitudes most conveniently expressed in color ordered formalism:

$$A_n(\{p_i, h_i, a_i\}) = \delta^{(4)}\Big(\sum_{i=1}^{n} p_i\Big) \sum_{\sigma \in S_n/Z_n} g^{n-2}\, \mathrm{tr}\big[t^{a_{\sigma_1}} \ldots t^{a_{\sigma_n}}\big] \times \mathcal{A}_n(\{p_{\sigma_1}, h_{\sigma_1}\}, \ldots, \{p_{\sigma_1}, h_{\sigma_1}\}; \lambda = g^2 N)$$

$\mathcal{A}_n$: Color ordered amplitude. Color structure is stripped off.

Helicity of ith particle: $h_i = 0$ scalar, $h_i = \pm 1$ gluon, $h_i = \pm\frac{1}{2}$ gluino

Superconformal symmetry

- Symmetry: $\mathfrak{so}(2,4) \otimes \mathfrak{so}(6) \subset \mathfrak{psu}(2,2|4)$

  Poincaré: $p^{\alpha\dot{\alpha}} = p_\mu (\sigma^\mu)^{\dot{\alpha}\beta}$, $l_{\alpha\beta}$, $\bar{l}_{\dot{\alpha}\dot{\beta}}$

  Conformal: $k_{\alpha\dot{\alpha}}$, d (c : central charge)

  R-symmetry: r_{AB}

  Poncaré Susy: $q^{\alpha A}, \bar{q}^{\dot{\alpha}}_A$ Conformal Susy: $s_{\alpha A}, \bar{s}^A_{\dot{\alpha}}$

- $4+4$ Supermatrix notation $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^\alpha{}_\beta - \frac{1}{2}\delta^\alpha_\beta (d + \frac{1}{2}c) & k^\alpha{}_{\dot{\beta}} & s^\alpha{}_B \\ p^{\dot{\alpha}}{}_\beta & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}} (d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_B \\ q^A{}_\beta & \bar{s}^A{}_{\dot{\beta}} & -r^A{}_B - \frac{1}{4}\delta^A_B c \end{pmatrix}$$

- Algebra:

$$[J^{\bar{A}}{}_{\bar{B}}, J^{\bar{C}}{}_{\bar{D}}\} = \delta^{\bar{C}}_{\bar{B}} J^{\bar{A}}{}_{\bar{D}} - (-1)^{(|\bar{A}|+|\bar{B}|)(|\bar{C}|+|\bar{D}|)} \delta^{\bar{A}}_{\bar{D}} J^{\bar{C}}{}_{\bar{B}}$$

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot\alpha} \sigma_\mu^{\dot\alpha\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot\alpha A} \sigma_I^{AB} \epsilon^{\dot\alpha\dot\beta} [\Phi^I, \bar{\Psi}_{\dot\beta B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

$\Rightarrow$ "H-atom of gauge theories"

$\mathcal{N} = 4$ super Yang Mills: The simplest interacting 4d QFT

- Field content: All fields in adjoint of $SU(N)$, $N \times N$ matrices
  - Gluons: A_μ
  - 6 real scalars: Φ_I
  - 4×4 real fermions: $\Psi_{\alpha A}$
- Action: Unique model completely fixed by SUSY

$$S = \frac{1}{{g_{\rm YM}}^2} \int d^4x \, {\rm Tr} \Big[\tfrac{1}{4} F_{\mu\nu}^2 + \tfrac{1}{2} (D_\mu \Phi_I)^2 - \tfrac{1}{4} [\Phi_I, \Phi_J][\Phi_I, \Phi_J] + \bar{\Psi}^A_{\dot{\alpha}} \sigma_\mu^{\dot{\alpha}\beta} \mathcal{D}^\mu \Psi_{\beta A} - \tfrac{i}{2} \Psi_{\alpha A} \sigma_I^{AB} \epsilon^{\alpha\beta} [\Phi^I, \Psi_{\beta B}] - \tfrac{i}{2} \bar{\Psi}_{\dot{\alpha} A} \sigma_I^{AB} \epsilon^{\dot{\alpha}\dot{\beta}} [\Phi^I, \bar{\Psi}_{\dot{\beta} B}] \Big]$$

- $\boxed{\beta_{g_{\rm YM}} = 0}$: Quantum Conformal Field Theory, 2 parameters: N & $\lambda = {g_{\rm YM}}^2 N$
- Shall consider 't Hooft planar limit: $N \to \infty$ with λ fixed.
- Is the 4d interacting QFT with highest degree of symmetry!

$\Rightarrow$ "H-atom of gauge theories"

Superconformal symmetry

- Symmetry: $\mathfrak{so}(2,4) \otimes \mathfrak{so}(6) \subset \mathfrak{psu}(2,2|4)$

  Poincaré: $p^{\alpha\dot{\alpha}} = p_\mu (\sigma^\mu)^{\dot{\alpha}\beta}$, $l_{\alpha\beta}$, $\bar{l}_{\dot{\alpha}\dot{\beta}}$

  Conformal: $k_{\alpha\dot{\alpha}}$, d (c : central charge)

  R-symmetry: r_{AB}

  Poncaré Susy: $q^{\alpha A}, \bar{q}^{\dot{\alpha}}_A$ Conformal Susy: $s_{\alpha A}, \bar{s}^A_{\dot{\alpha}}$

- $4+4$ Supermatrix notation $\bar{A} = (\alpha, \dot{\alpha}|A)$

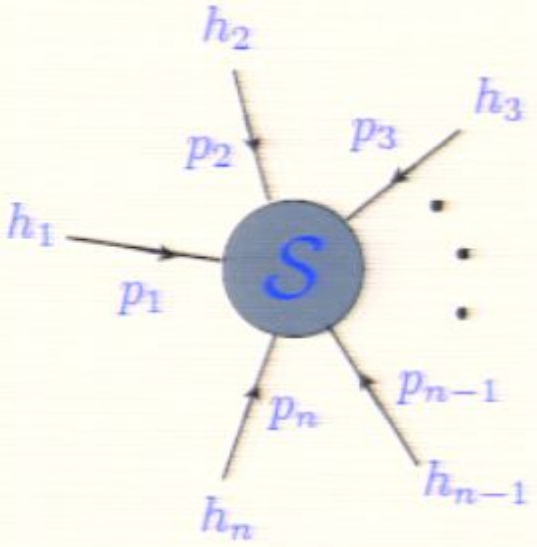
$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^\alpha{}_\beta - \frac{1}{2}\delta^\alpha_\beta (d + \frac{1}{2}c) & k^\alpha{}_{\dot{\beta}} & s^\alpha{}_B \\ p^{\dot{\alpha}}{}_\beta & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}} (d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_B \\ q^A{}_\beta & \bar{s}^A{}_{\dot{\beta}} & -r^A{}_B - \frac{1}{4}\delta^A_B c \end{pmatrix}$$

- Algebra:

$$[J^{\bar{A}}{}_{\bar{B}}, J^{\bar{C}}{}_{\bar{D}}\} = \delta^{\bar{C}}_{\bar{B}} J^{\bar{A}}{}_{\bar{D}} - (-1)^{(|\bar{A}|+|\bar{B}|)(|\bar{C}|+|\bar{D}|)} \delta^{\bar{A}}_{\bar{D}} J^{\bar{C}}{}_{\bar{B}}$$

Scattering amplitudes in $\mathcal{N} = 4$ SYM

- Consider n-particle scattering amplitude



- Planar amplitudes most conveniently expressed in color ordered formalism:

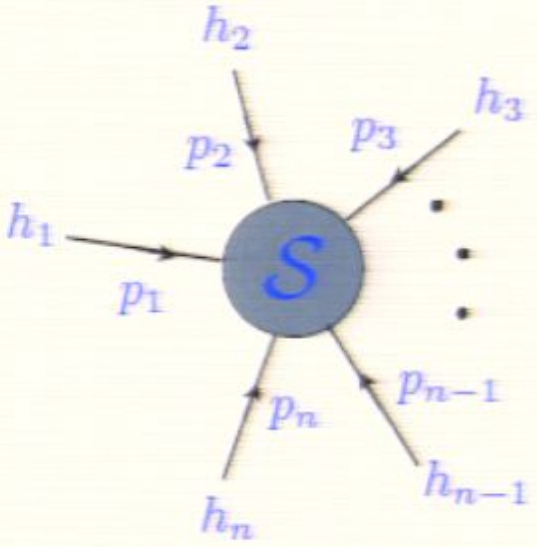
$$A_n(\{p_i, h_i, a_i\}) = \delta^{(4)}\Big(\sum_{i=1}^{n} p_i\Big) \sum_{\sigma \in S_n/Z_n} g^{n-2}\, \mathrm{tr}\big[t^{a_{\sigma_1}} \ldots t^{a_{\sigma_n}}\big] \times \mathcal{A}_n(\{p_{\sigma_1}, h_{\sigma_1}\}, \ldots, \{p_{\sigma_1}, h_{\sigma_1}\}; \lambda = g^2 N)$$

$\mathcal{A}_n$: Color ordered amplitude. Color structure is stripped off.

Helicity of ith particle: $h_i = 0$ scalar, $h_i = \pm 1$ gluon, $h_i = \pm\frac{1}{2}$ gluino

Scattering amplitudes in $\mathcal{N} = 4$ SYM

- Consider n-particle scattering amplitude



- Planar amplitudes most conveniently expressed in color ordered formalism:

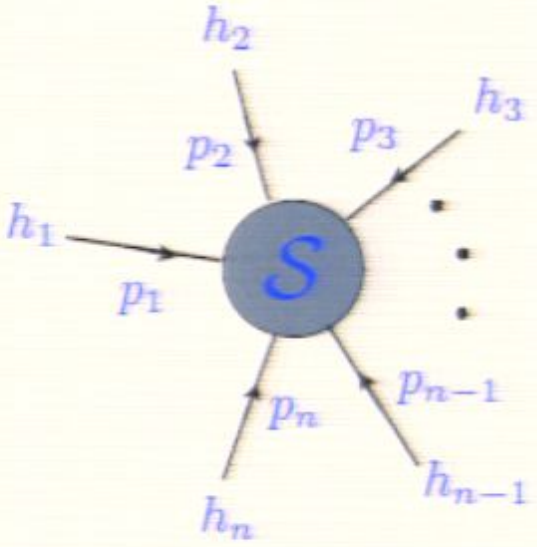
$$A_n(\{p_i, h_i, a_i\}) = \delta^{(4)}\Big(\sum_{i=1}^{n} p_i\Big) \sum_{\sigma \in S_n/Z_n} g^{n-2}\, \mathrm{tr}\big[t^{a_{\sigma_1}} \dots t^{a_{\sigma_n}}\big] \times \mathcal{A}_n(\{p_{\sigma_1}, h_{\sigma_1}\}, \dots, \{p_{\sigma_1}, h_{\sigma_1}\}; \lambda = g^2 N)$$

$\mathcal{A}_n$: Color ordered amplitude. Color structure is stripped off.

Helicity of ith particle: $h_i = 0$ scalar, $h_i = \pm 1$ gluon, $h_i = \pm\frac{1}{2}$ gluino

Scattering amplitudes in $\mathcal{N} = 4$ SYM

- Consider n-particle scattering amplitude



- Planar amplitudes most conveniently expressed in color ordered formalism:

$$A_n(\{p_i, h_i, a_i\}) = \delta^{(4)}\Big(\sum_{i=1}^{n} p_i\Big) \sum_{\sigma \in S_n/Z_n} g^{n-2}\, \mathrm{tr}\big[t^{a_{\sigma_1}} \ldots t^{a_{\sigma_n}}\big] \times \mathcal{A}_n(\{p_{\sigma_1}, h_{\sigma_1}\}, \ldots, \{p_{\sigma_1}, h_{\sigma_1}\}; \lambda = g^2 N)$$

$\mathcal{A}_n$: Color ordered amplitude. Color structure is stripped off.

Helicity of ith particle: $h_i = 0$ scalar, $h_i = \pm 1$ gluon, $h_i = \pm\frac{1}{2}$ gluino

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu \rangle} \qquad \langle \lambda\,\mu \rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu\rangle} \qquad \langle \lambda\,\mu\rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu\rangle} \qquad \langle \lambda\,\mu\rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu \rangle} \qquad \langle \lambda\,\mu \rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu\rangle} \qquad \langle \lambda\,\mu\rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu\rangle} \qquad \langle \lambda\,\mu\rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu\rangle} \qquad \langle \lambda\,\mu\rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu\rangle} \qquad \langle \lambda\,\mu\rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu\rangle} \qquad \langle \lambda\,\mu\rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu\rangle} \qquad \langle \lambda\, \mu\rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu \rangle} \qquad \langle \lambda\,\mu \rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu\rangle} \qquad \langle \lambda\,\mu\rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu \rangle} \qquad \langle \lambda\,\mu \rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu \rangle} \qquad \langle \lambda\, \mu \rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu\rangle} \qquad \langle \lambda\,\mu\rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu \rangle} \qquad \langle \lambda\,\mu \rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu\rangle} \qquad \langle \lambda\, \mu\rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu \rangle} \qquad \langle \lambda\,\mu \rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu\rangle} \qquad \langle \lambda\, \mu\rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu\rangle} \qquad \langle \lambda\, \mu\rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}}\, p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu\, p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\, \tilde{\lambda}_{\dot{\alpha}} \tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\, \mu\rangle} \qquad \langle \lambda\, \mu\rangle := \epsilon_{\alpha\beta}\, \lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\, p_1 \cdot p_2 = \langle \lambda_1, \lambda_2\rangle\, [\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2\rangle\, [2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu \rangle} \qquad \langle \lambda\,\mu \rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu \rangle} \qquad \langle \lambda\,\mu \rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Spinor helicity formalism

Asymptotic massless scattering states in D dims labeled by $|p^\mu, h\rangle$
(p^μ: Null momentum, h: $SO(D-2)$ little group label)

- Express momentum and polarizations via commuting spinors λ^α, $\tilde{\lambda}^{\dot{\alpha}}$:

$$p^{\alpha\dot{\alpha}} = (\sigma^\mu)^{\alpha\dot{\alpha}} p_\mu = \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}} \quad \Leftrightarrow \quad p_\mu p^\mu = \det p^{\alpha\dot{\alpha}} = 0$$

- Little group in 4d: $SO(2)$: Helicity ± 1. Choice of helicity determines polarization vector ε^μ of external gluon

$$h = +1 \qquad \varepsilon^{\alpha\dot{\alpha}} = \frac{\lambda^\alpha \tilde{\mu}^{\dot{\alpha}}}{[\tilde{\lambda}\,\tilde{\mu}]} \qquad [\tilde{\lambda}\,\tilde{\mu}] := \epsilon^{\dot{\alpha}\dot{\beta}}\,\tilde{\lambda}_{\dot{\alpha}}\tilde{\mu}_{\dot{\beta}}$$

$$h = -1 \qquad \tilde{\varepsilon}^{\alpha\dot{\alpha}} = \frac{\mu^\alpha \tilde{\lambda}^{\dot{\alpha}}}{\langle \lambda\,\mu \rangle} \qquad \langle \lambda\,\mu \rangle := \epsilon_{\alpha\beta}\,\lambda^\alpha \mu^\beta$$

$\mu, \bar{\mu}$ arbitrary reference spinors.

- E.g. scalar products: $2\,p_1 \cdot p_2 = \langle \lambda_1, \lambda_2 \rangle\,[\tilde{\lambda}_2, \tilde{\lambda}_1] = \langle 1, 2 \rangle\,[2, 1]$
- Helicity assignments:

$$h(\lambda^\alpha) = -1/2 \qquad h(\tilde{\lambda}^{\dot{\alpha}}) = +1/2$$

Trees

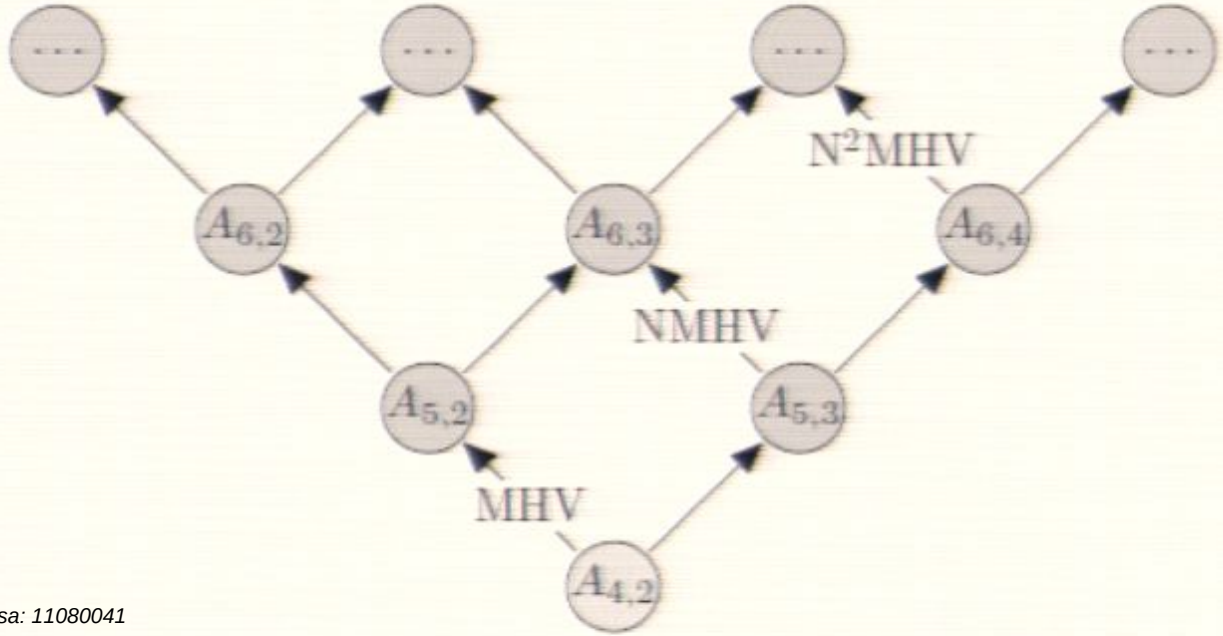
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

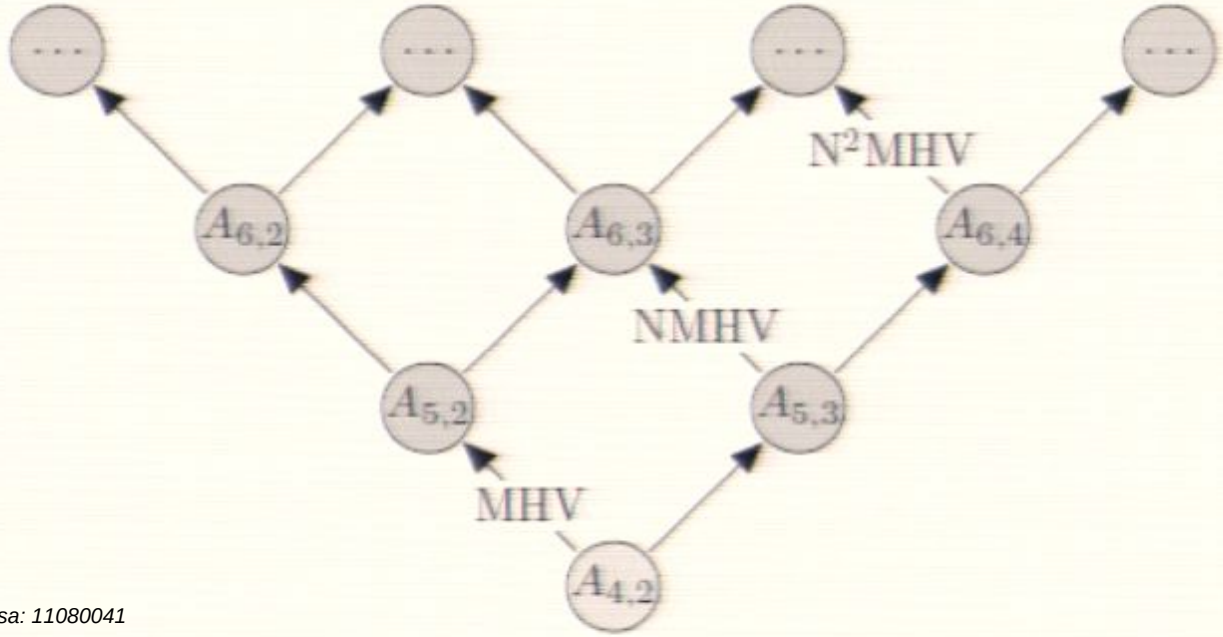
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

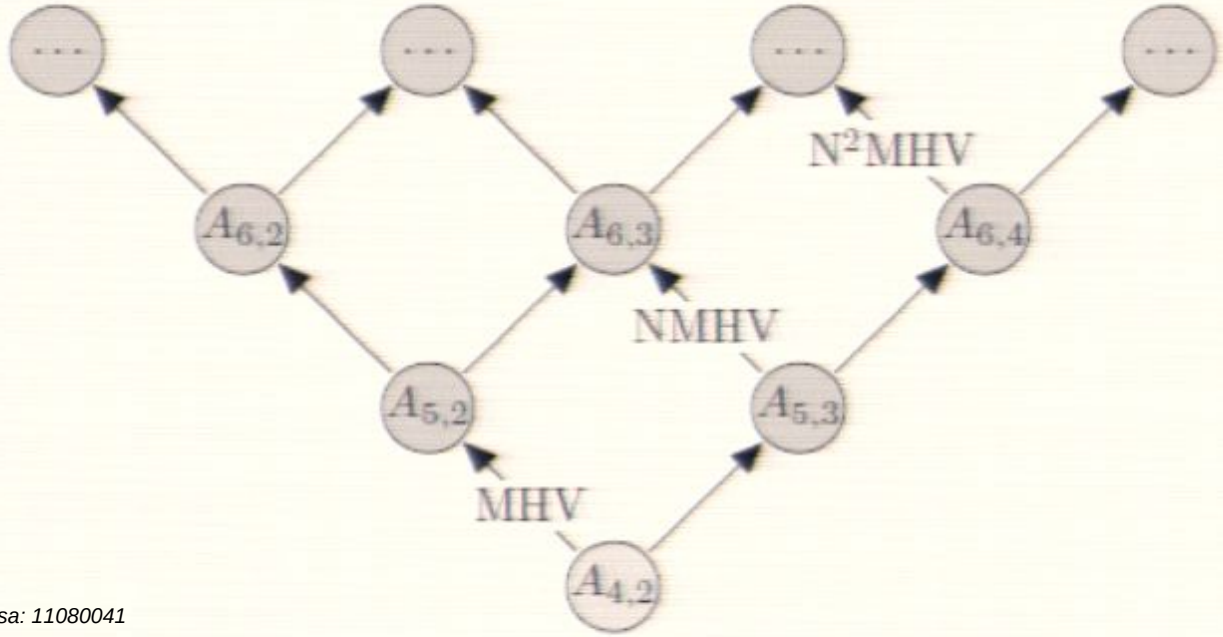
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

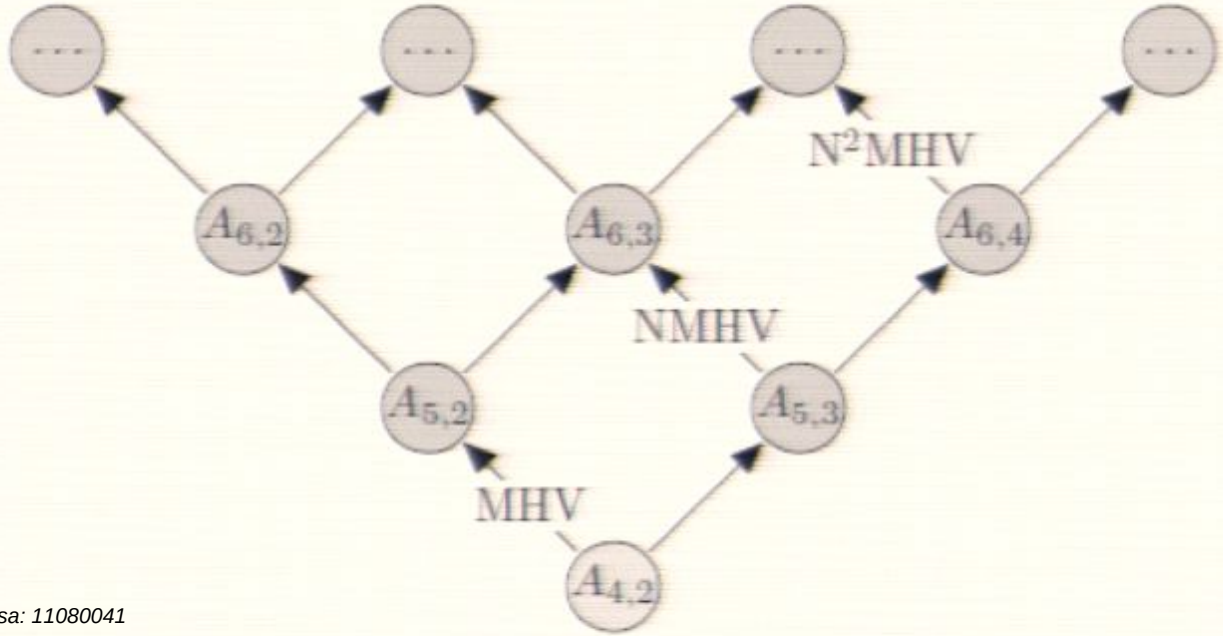
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

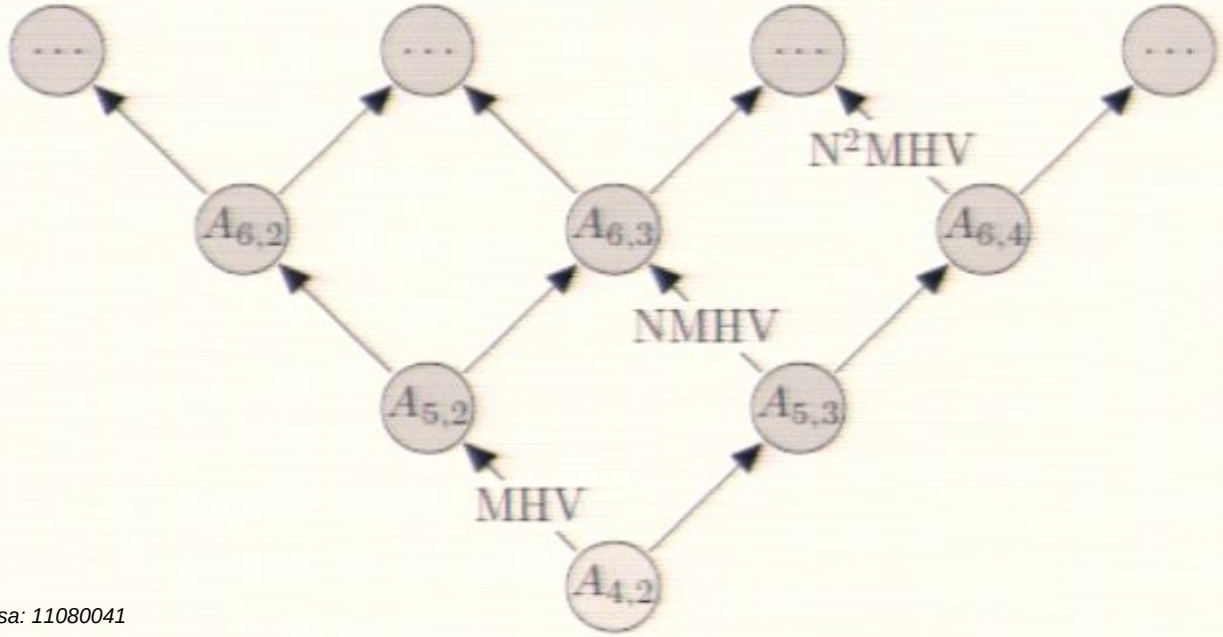
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j\rangle^4}{\langle 1, 2\rangle \langle 2, 3\rangle \ldots \langle n, 1\rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

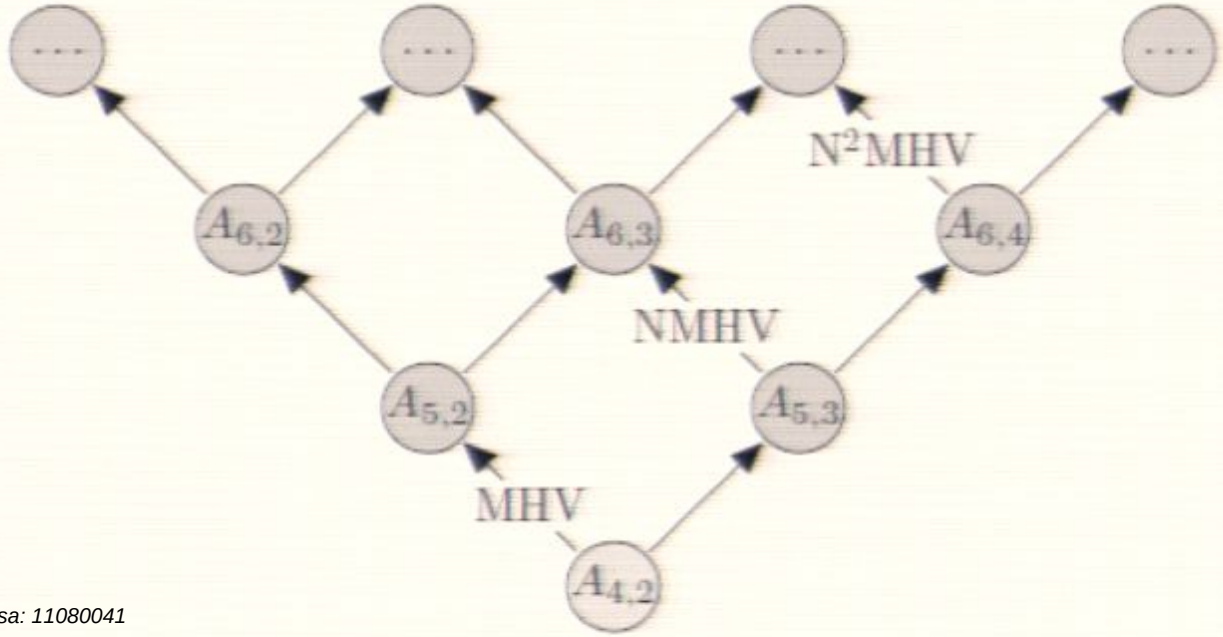
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

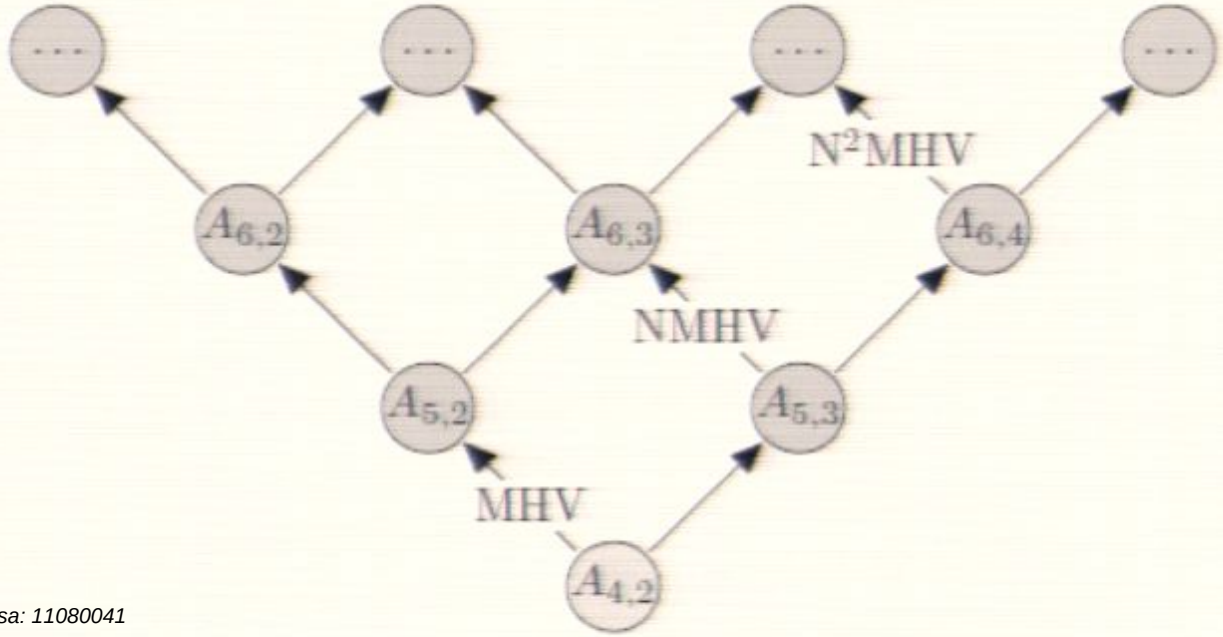
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

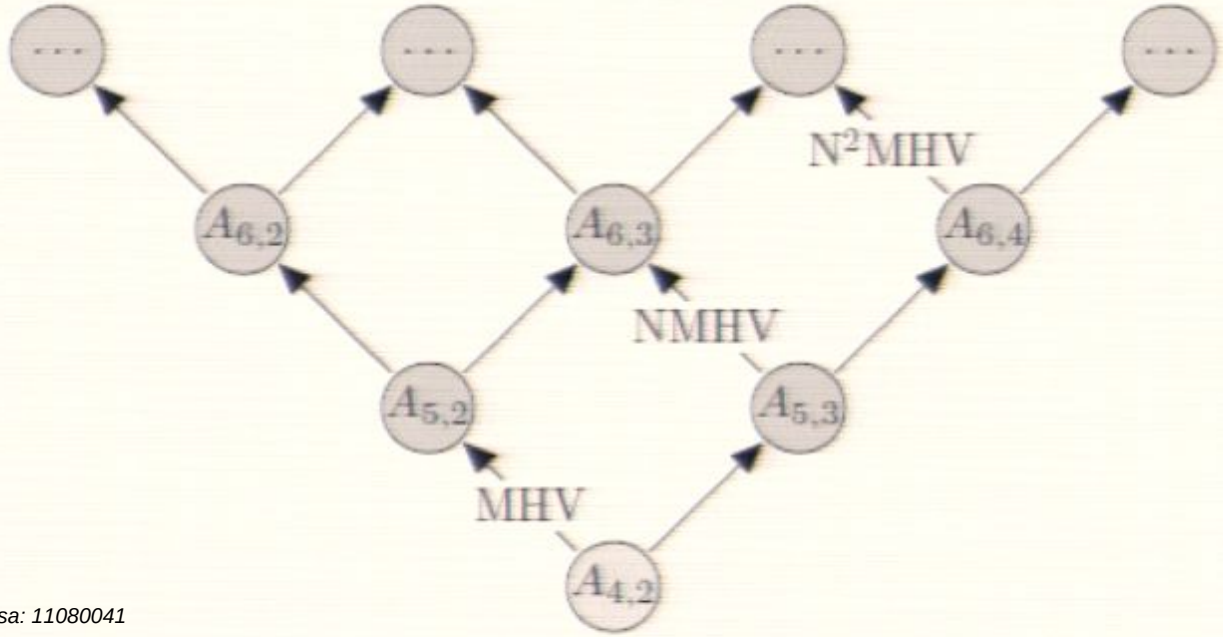
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

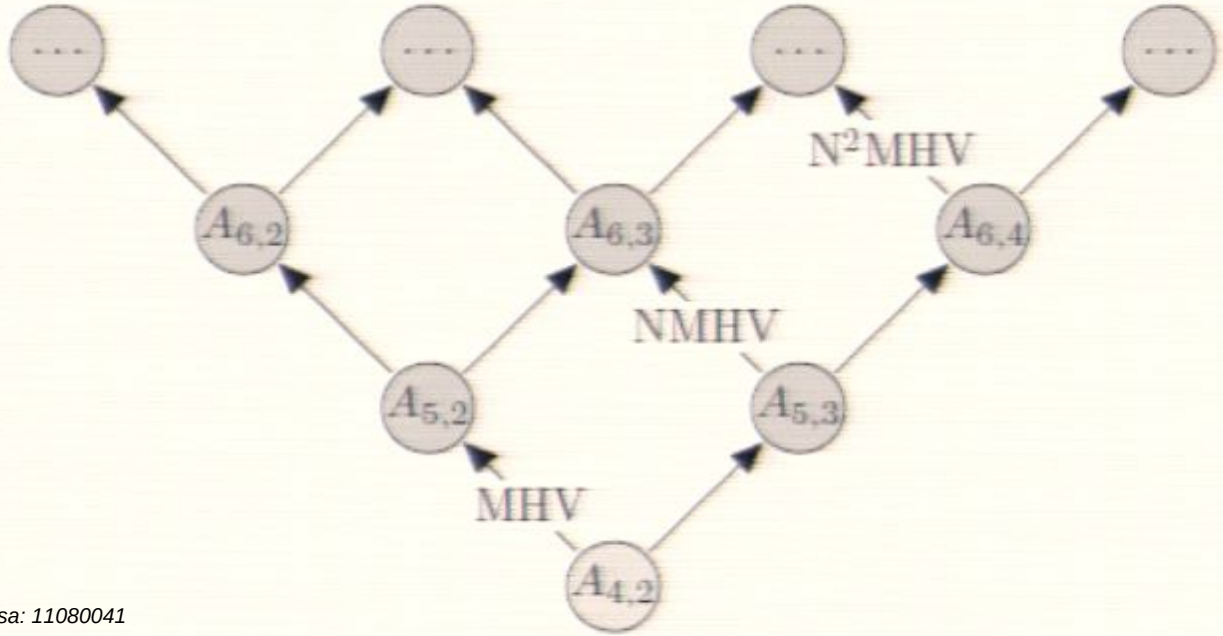
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

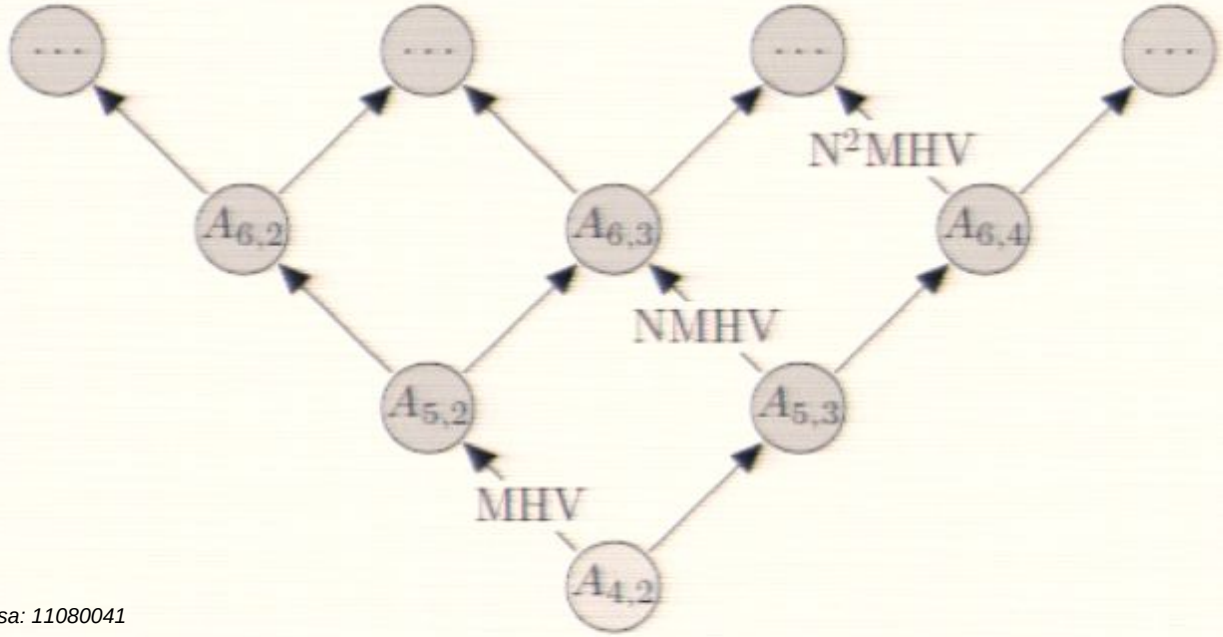
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

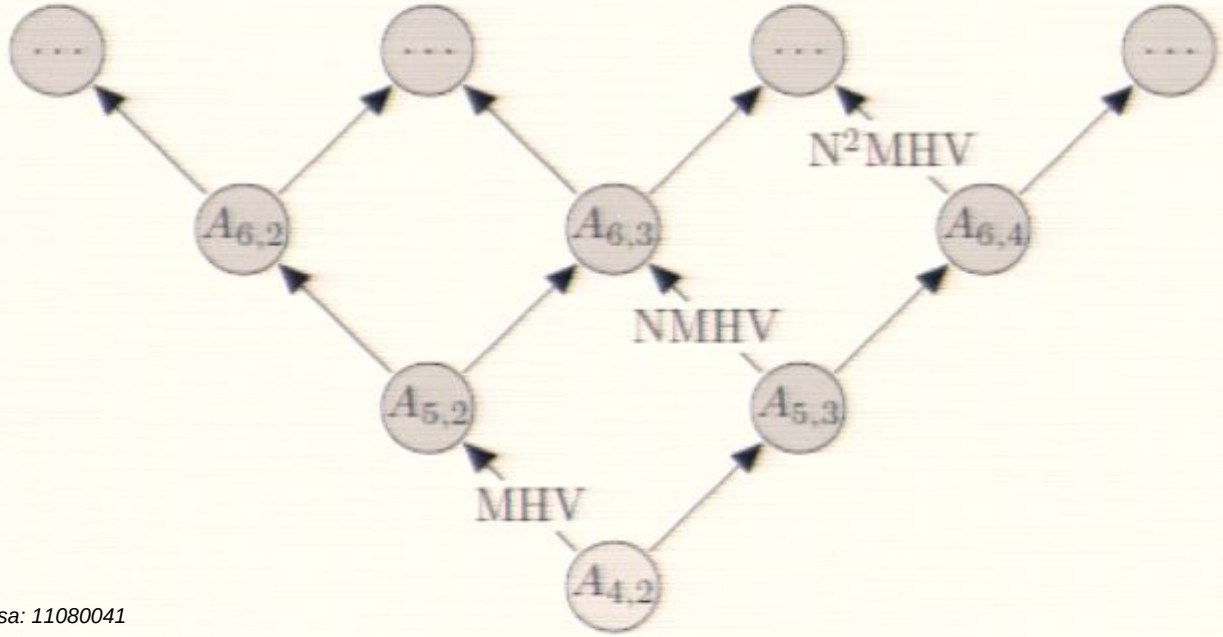
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

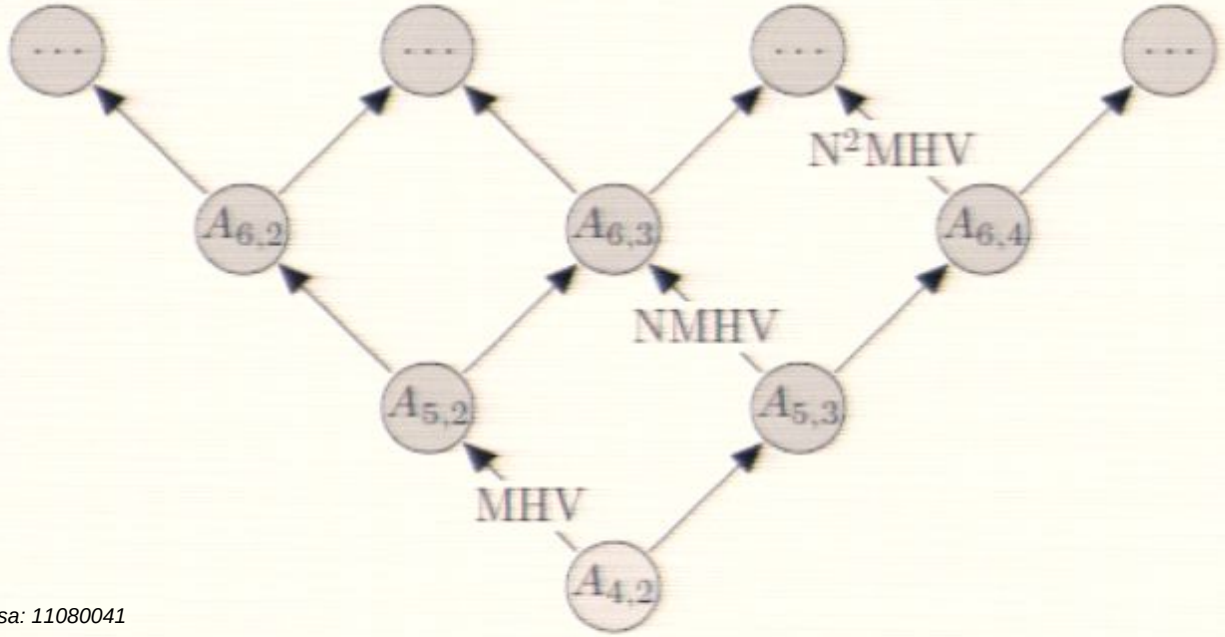
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

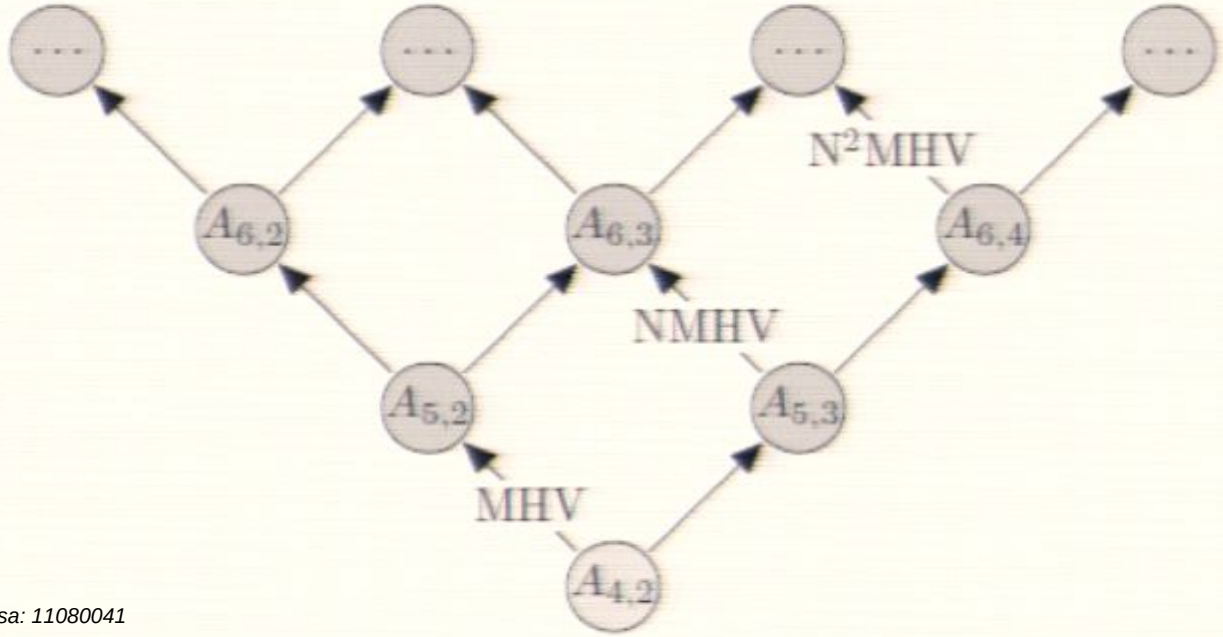
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

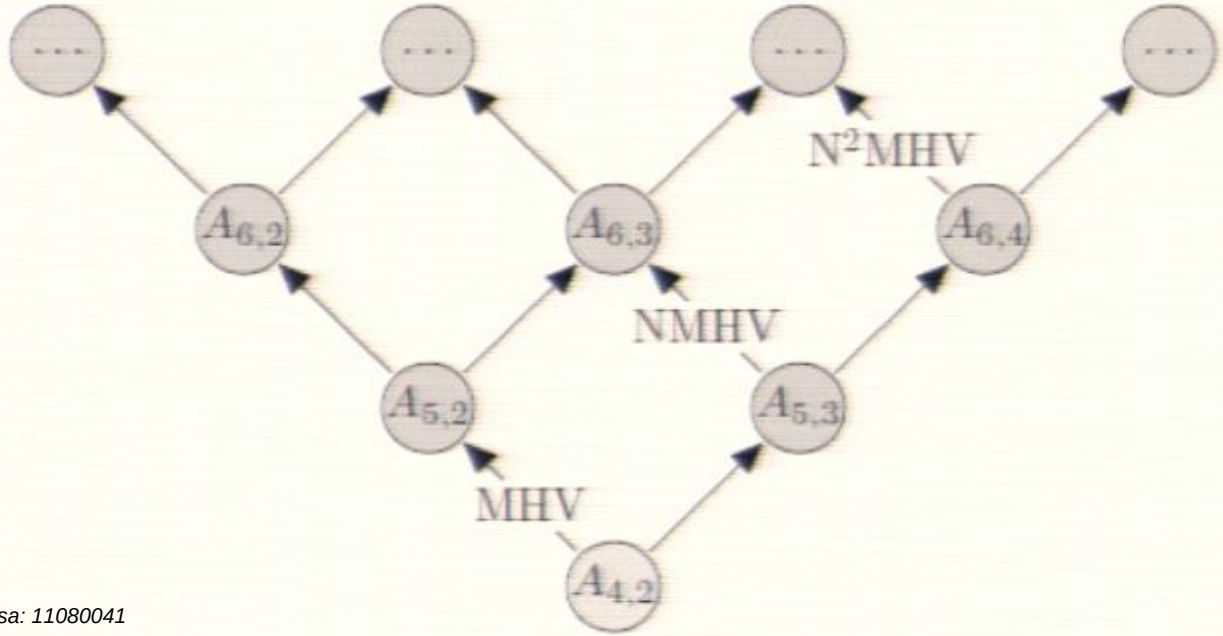
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

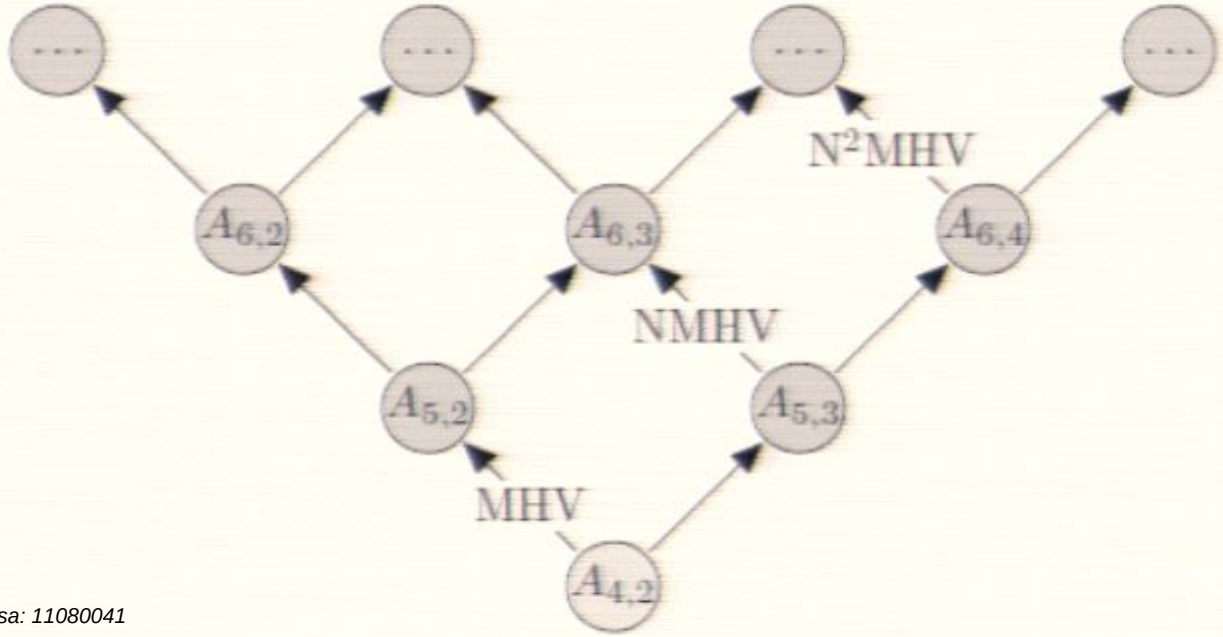
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

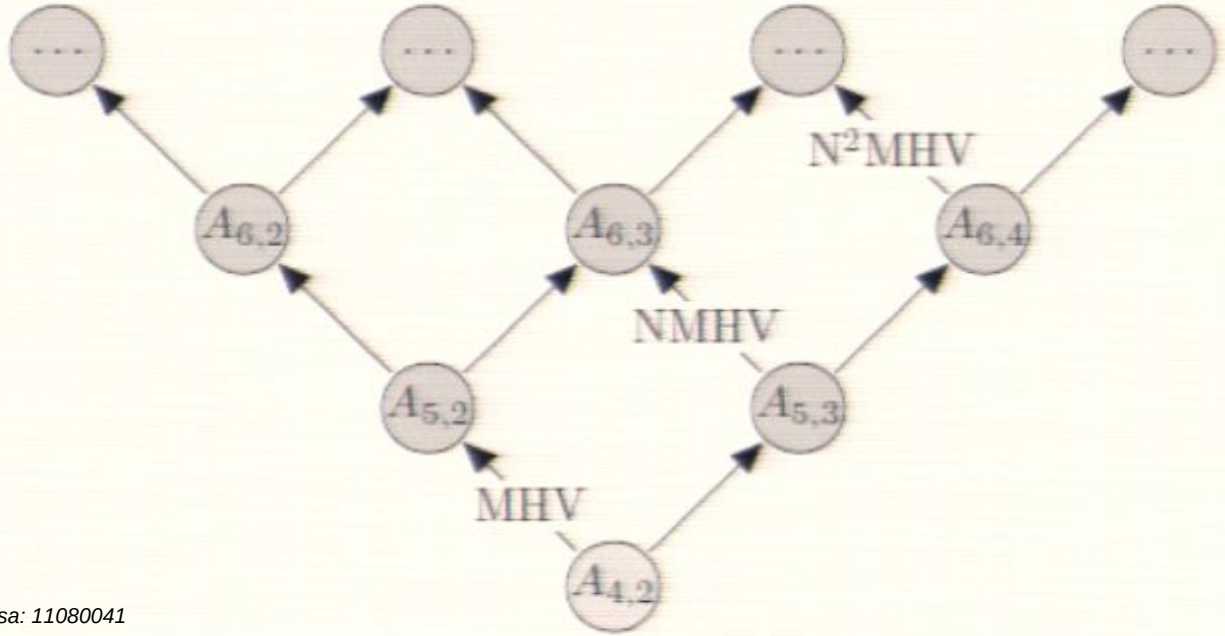
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

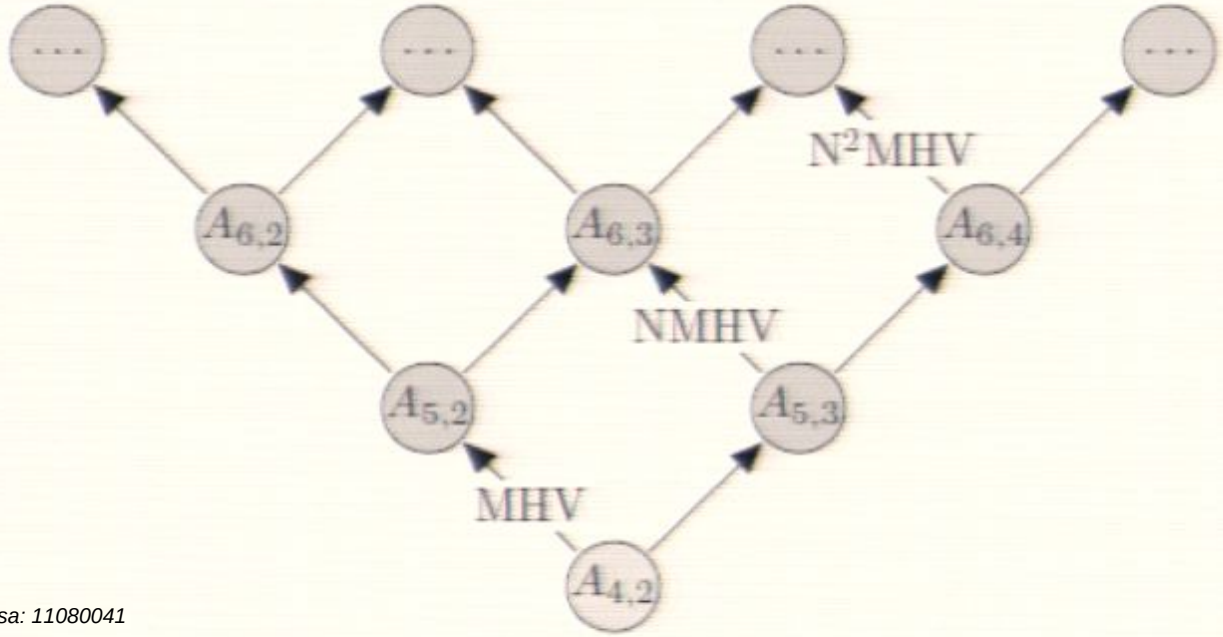
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

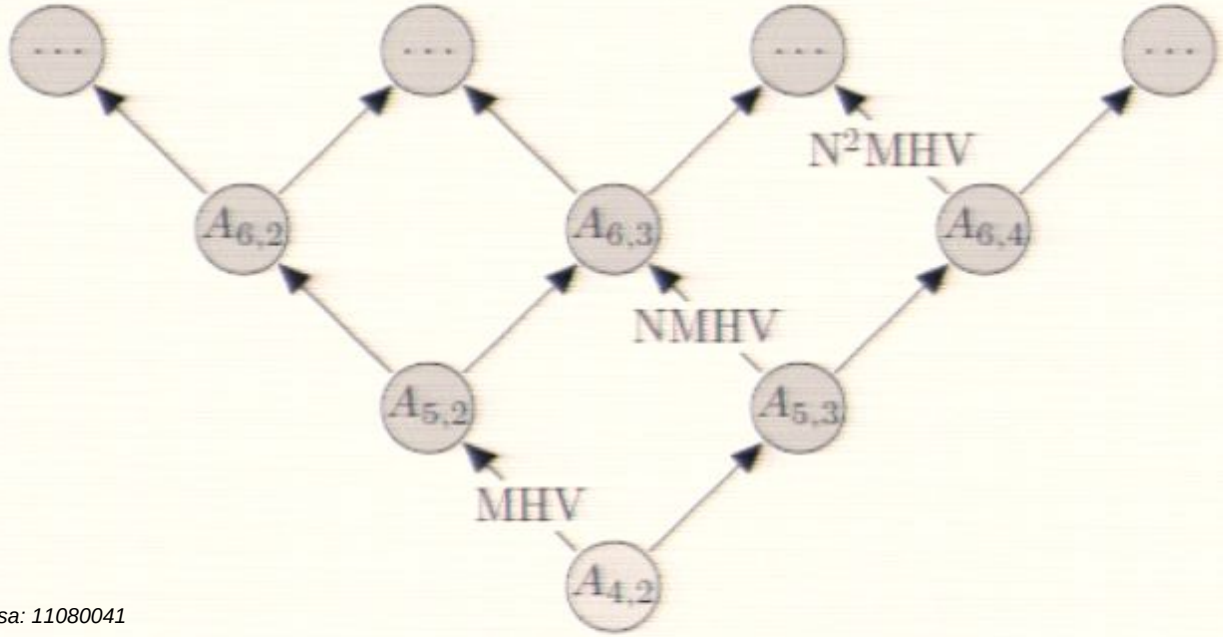
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

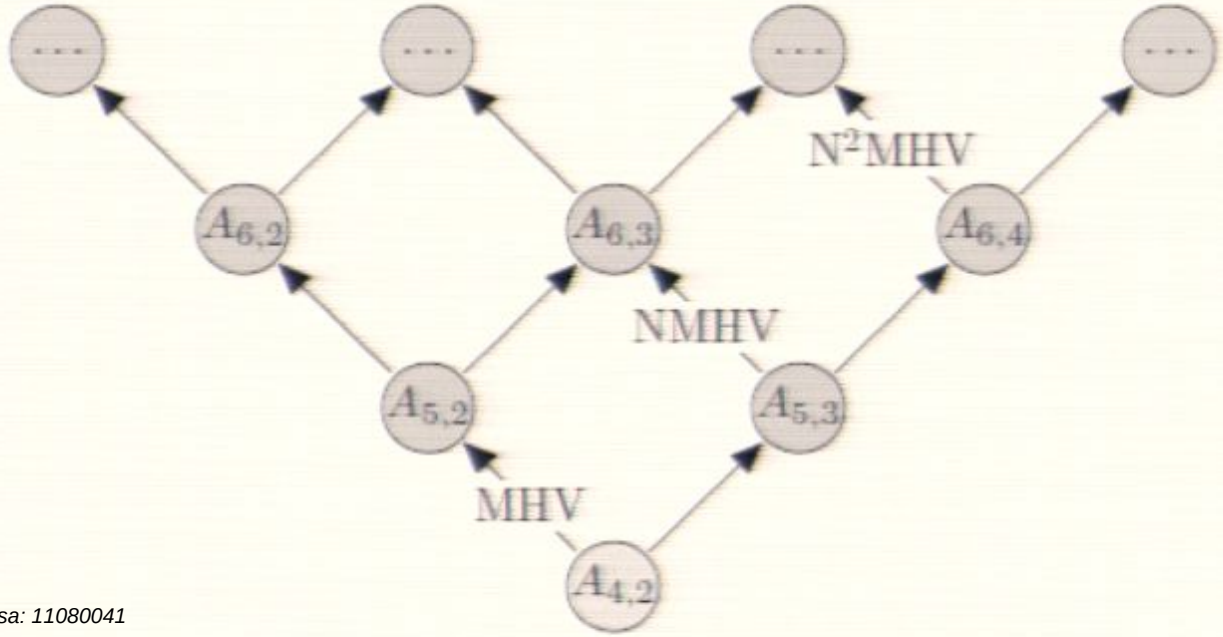
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

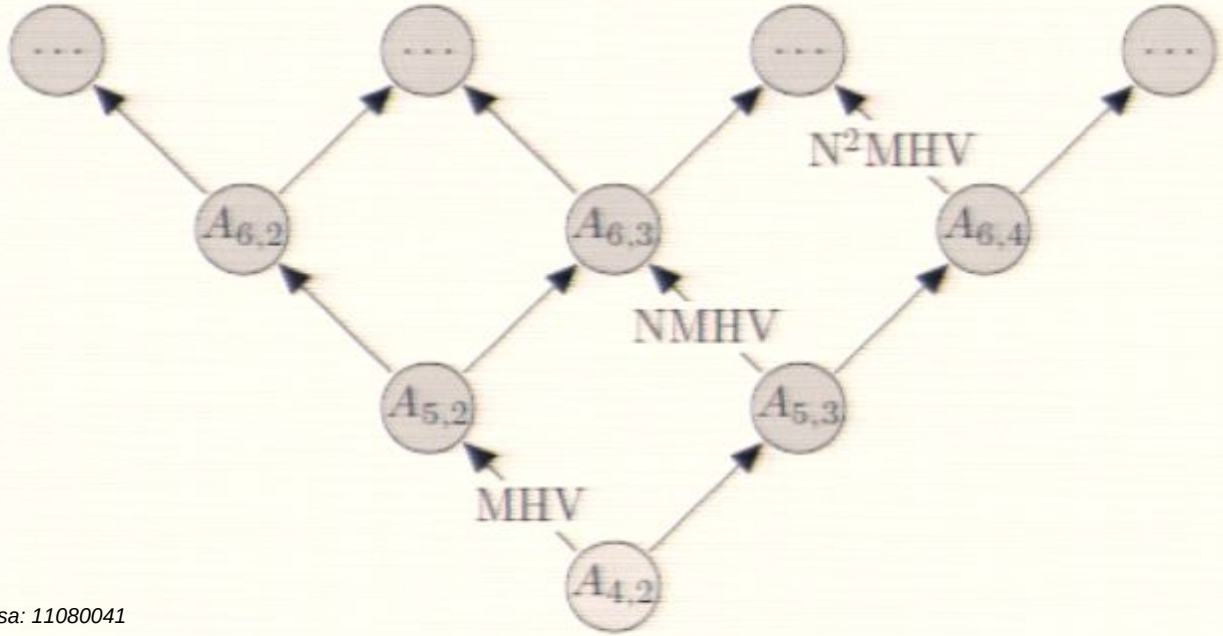
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

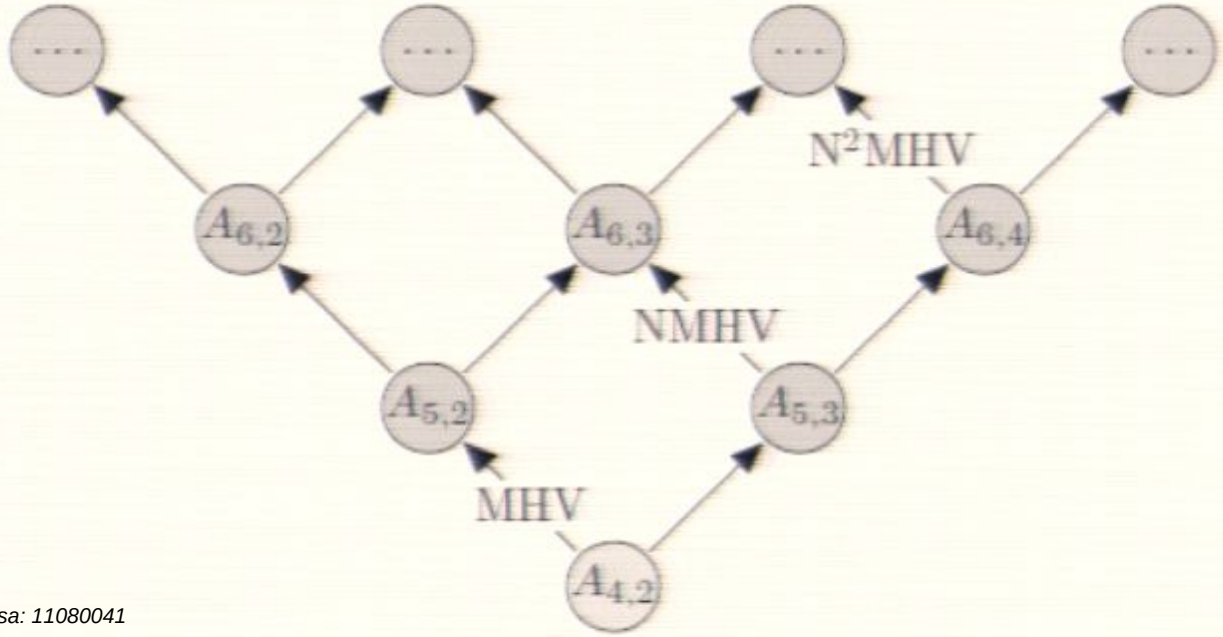
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

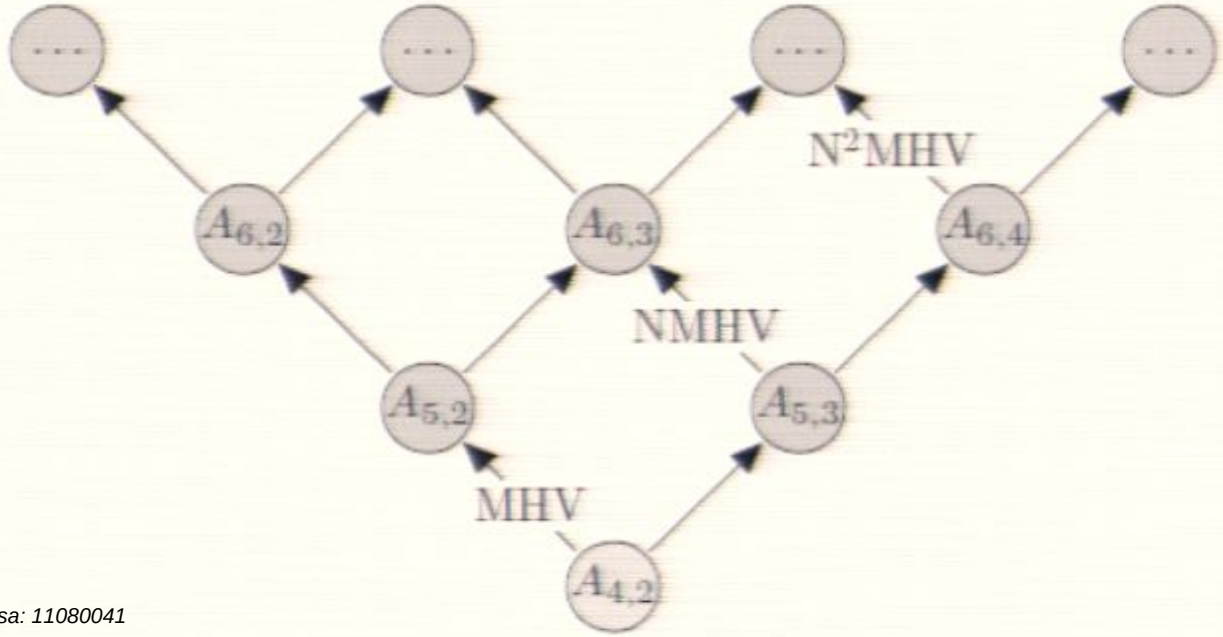
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

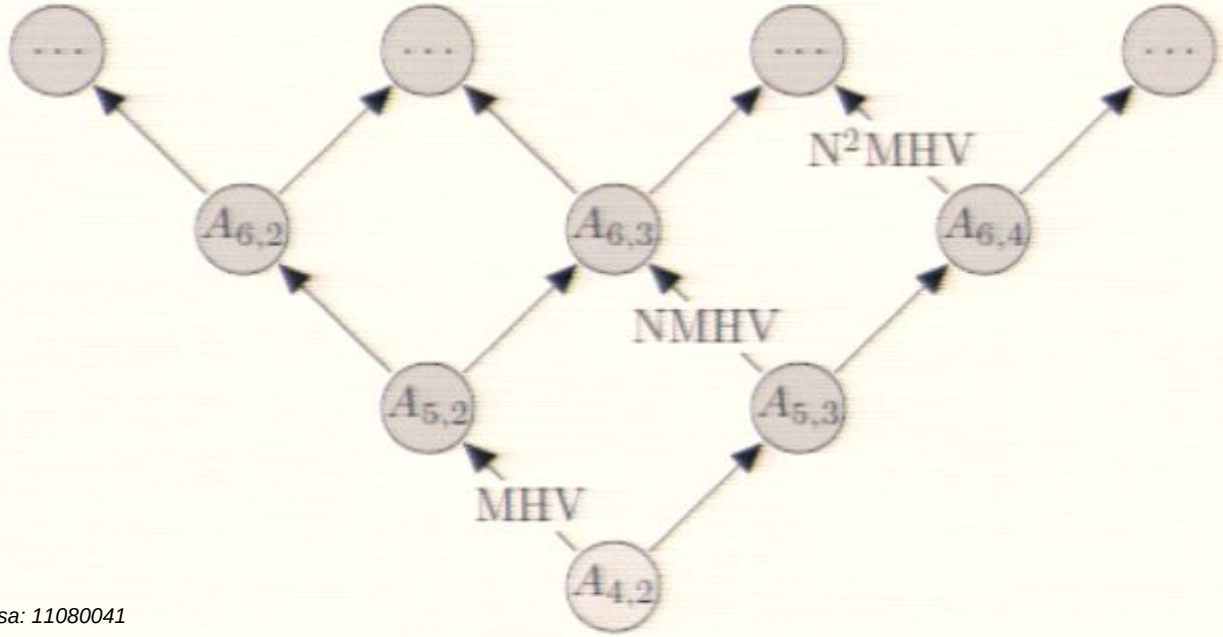
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

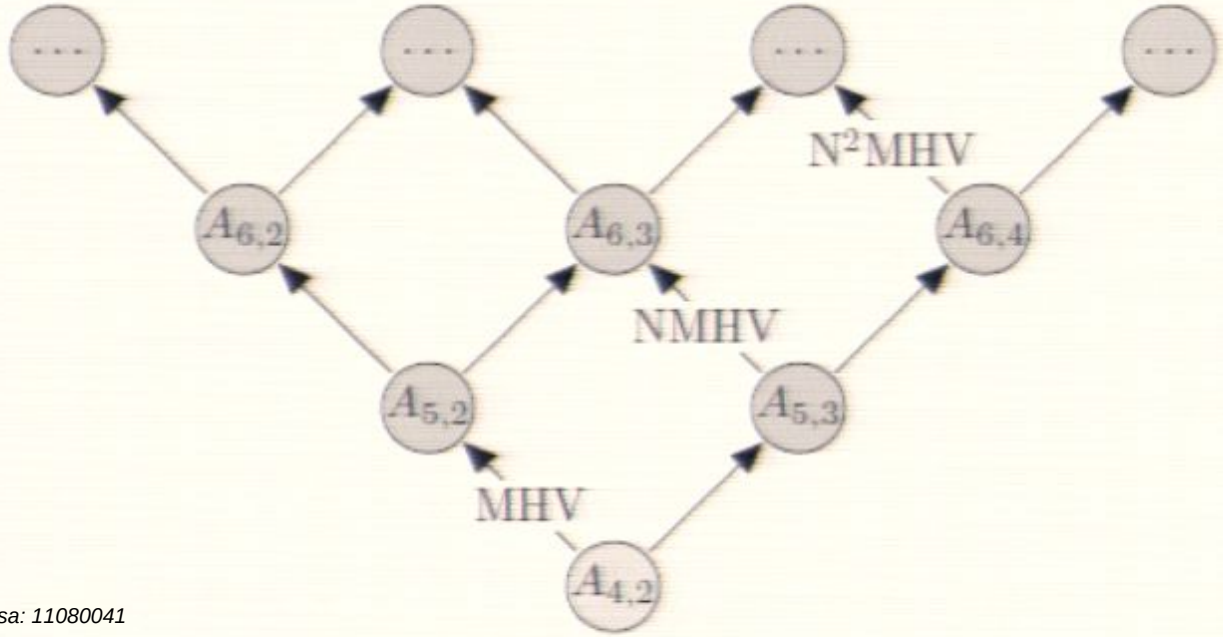
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

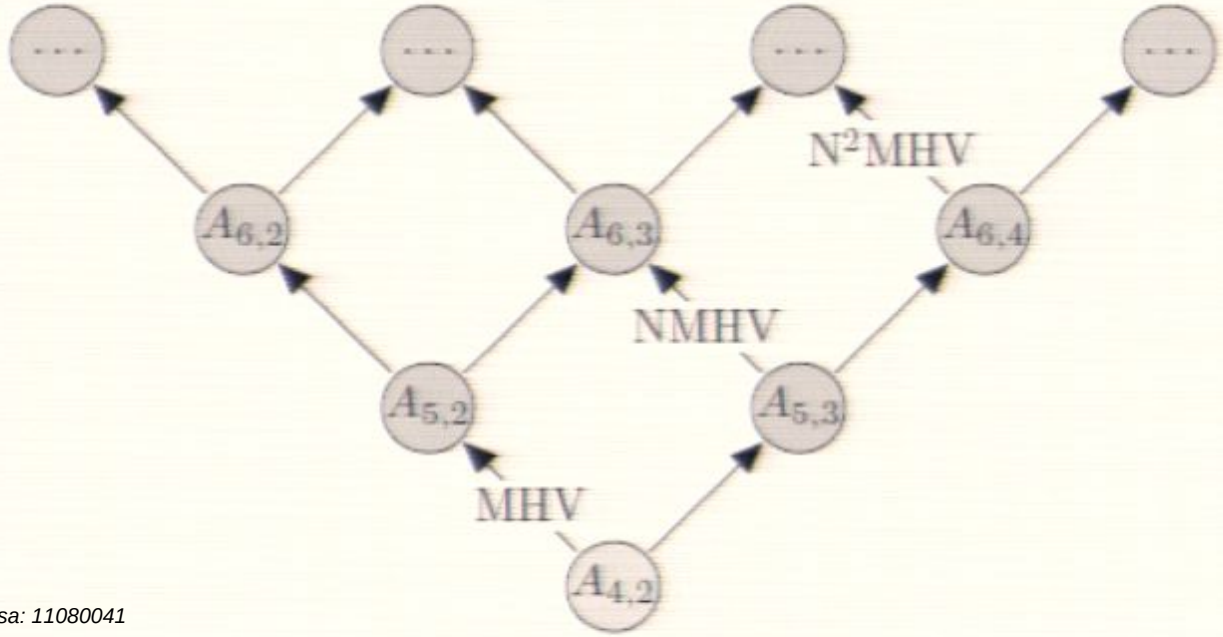
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

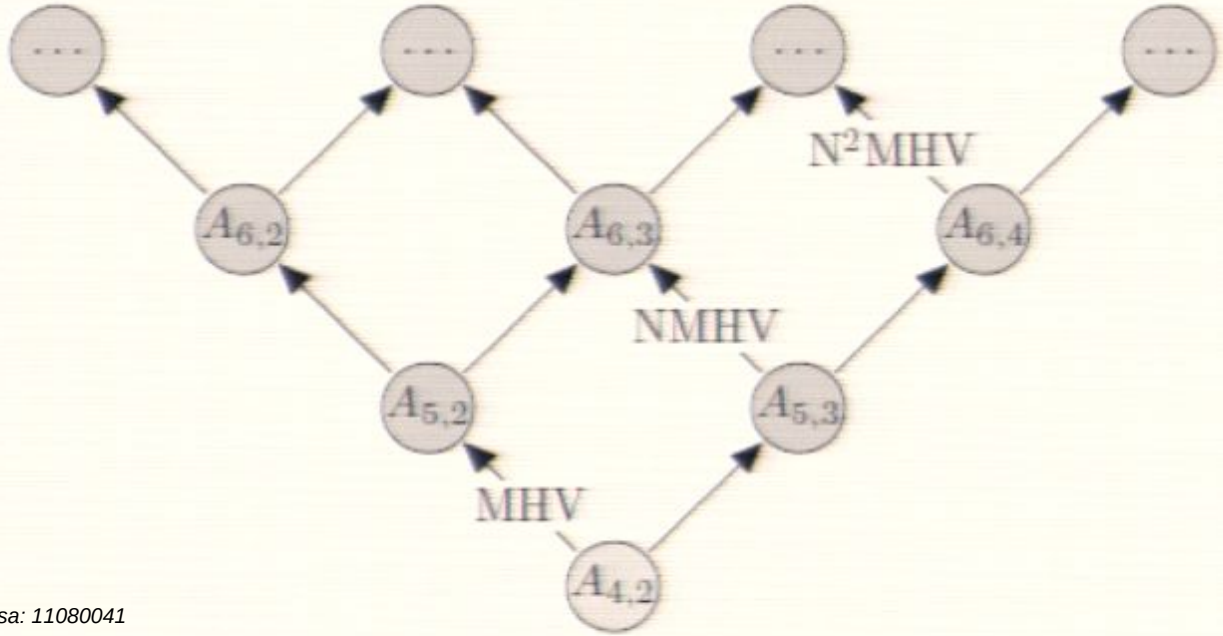
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

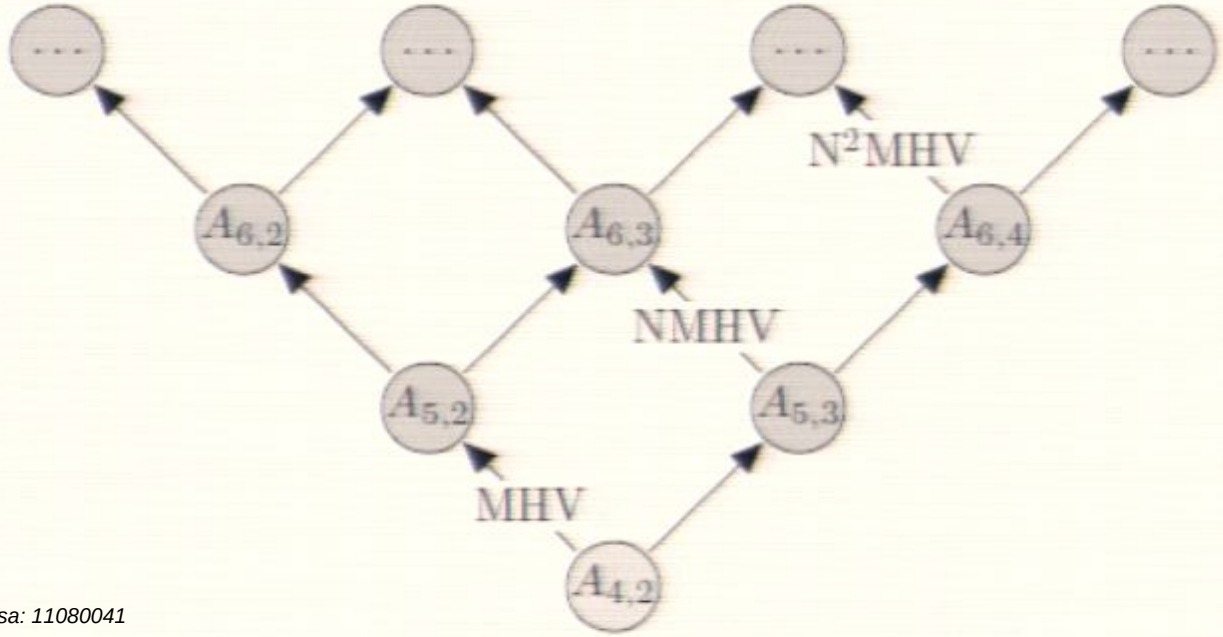
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

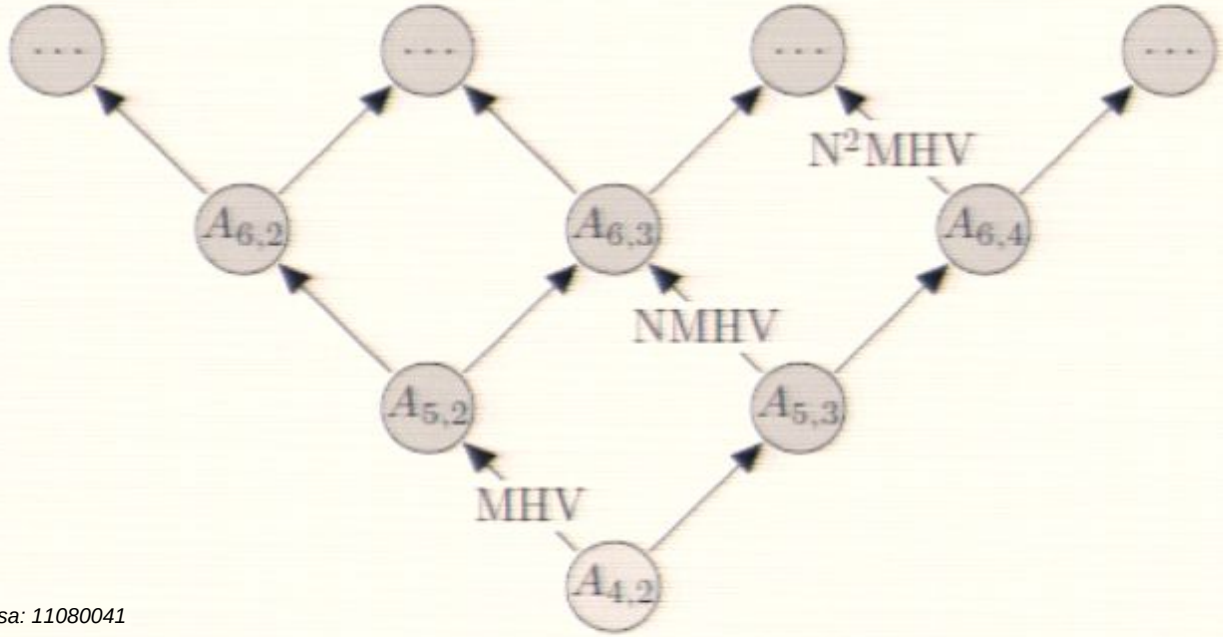
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

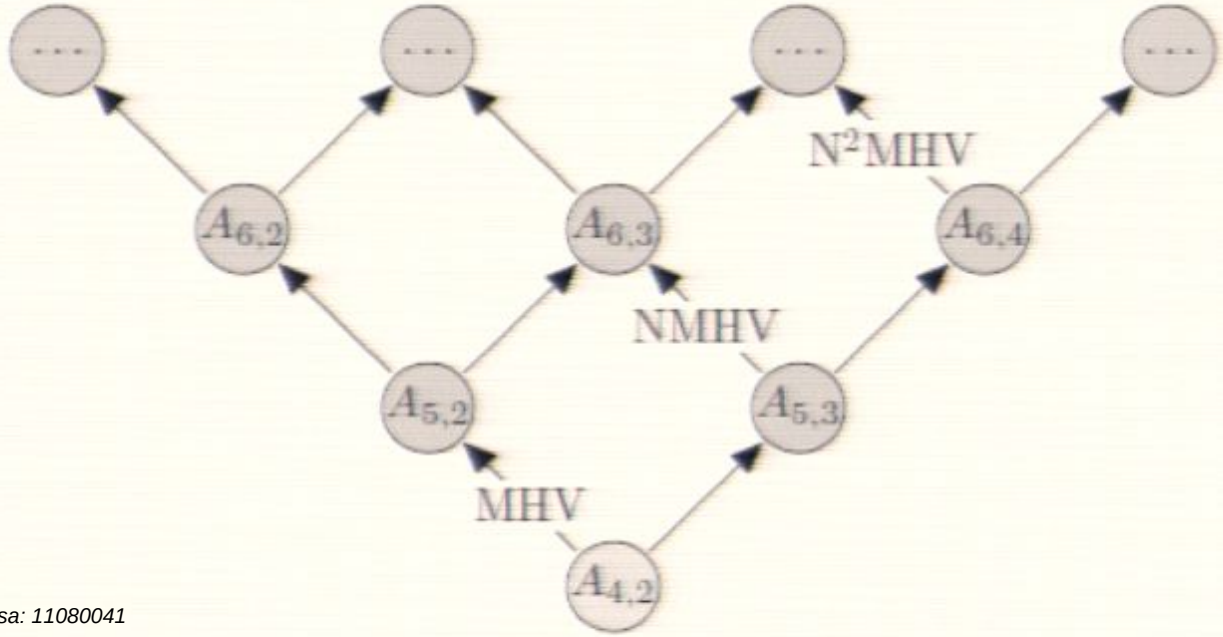
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

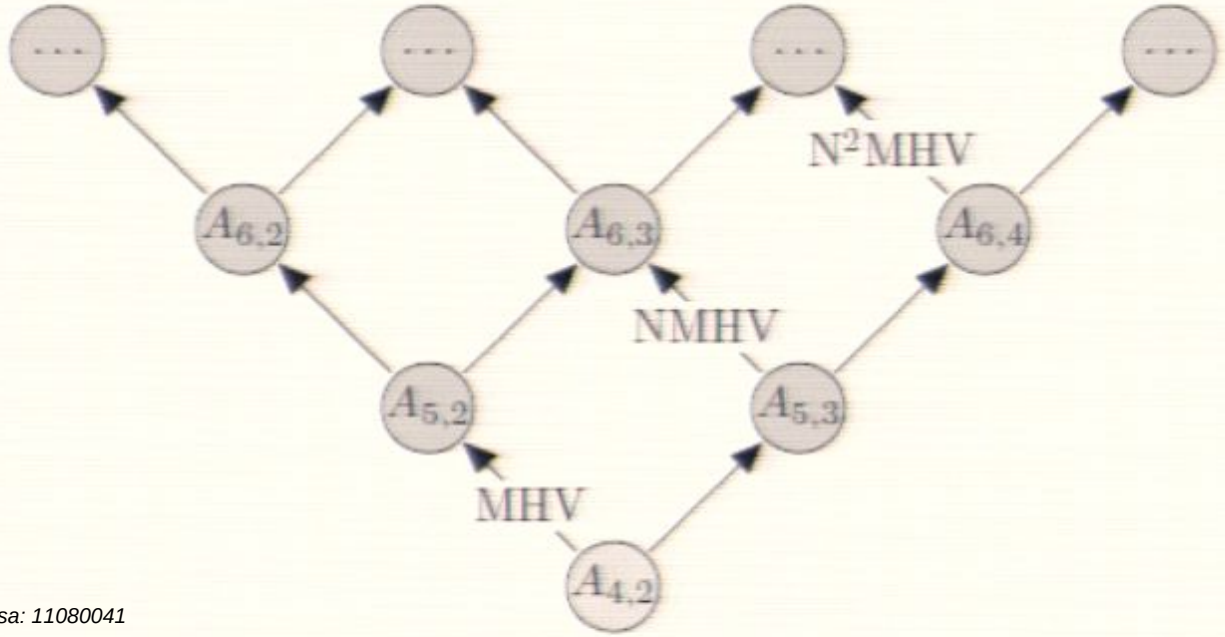
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

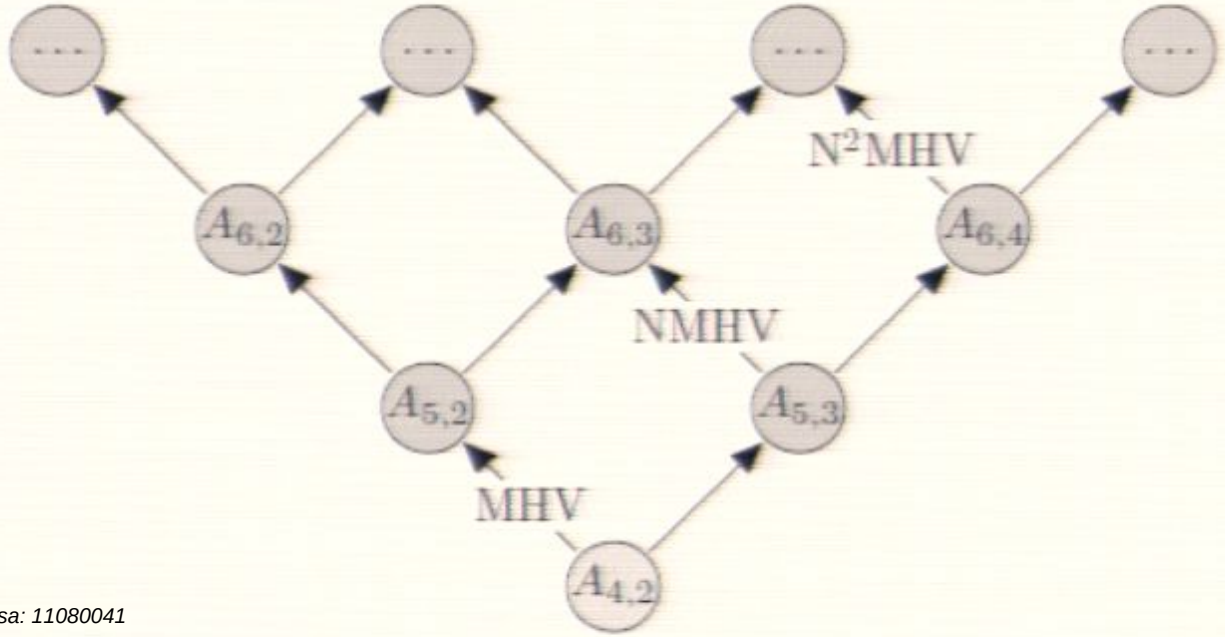
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j\rangle^4}{\langle 1, 2\rangle \langle 2, 3\rangle \ldots \langle n, 1\rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

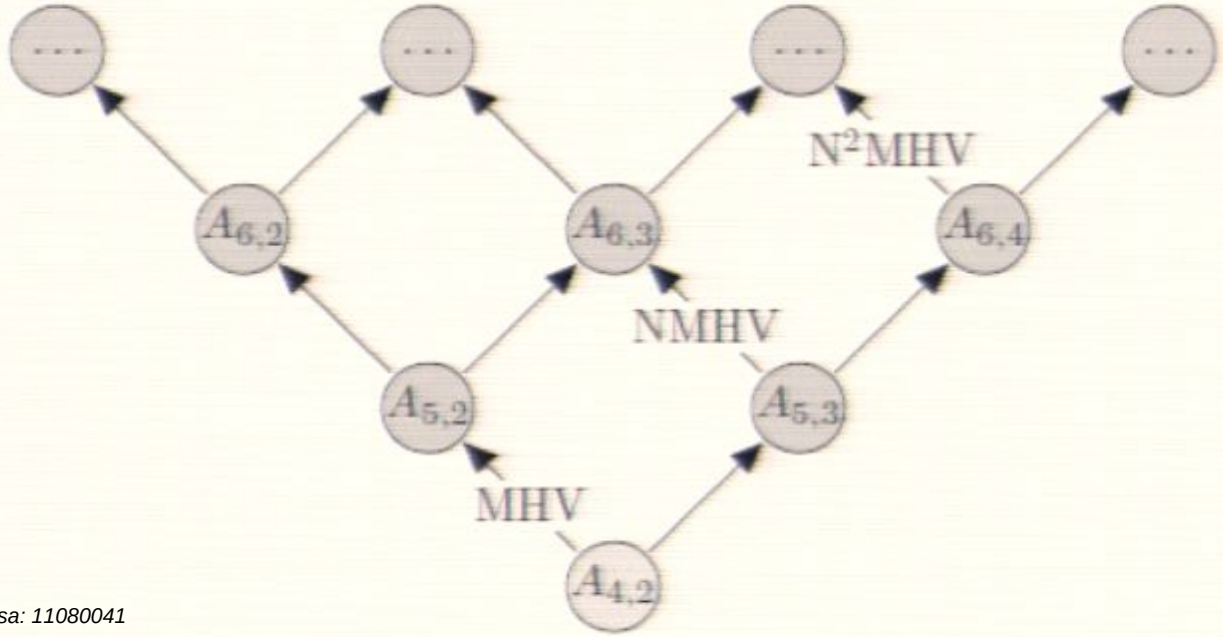
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

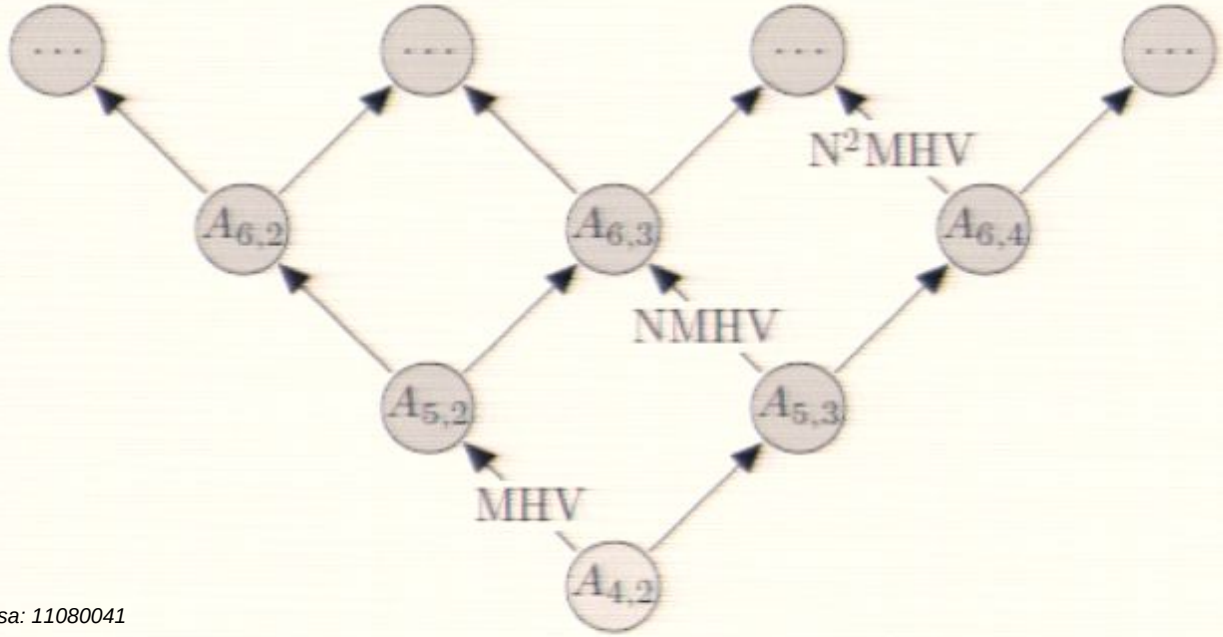
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

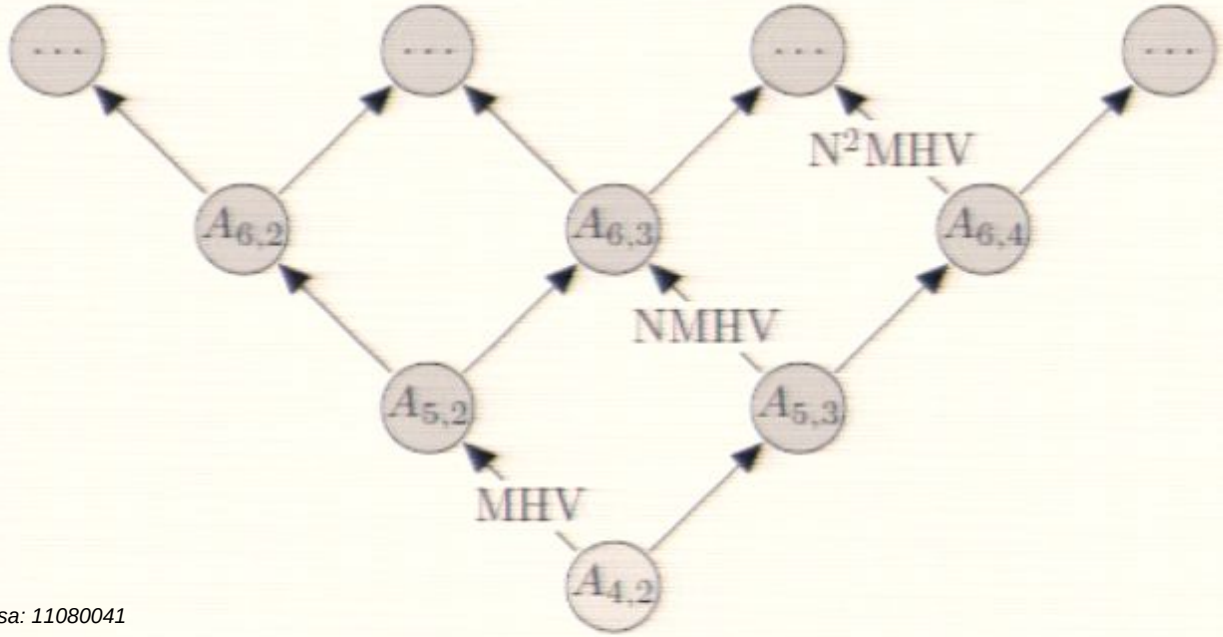
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

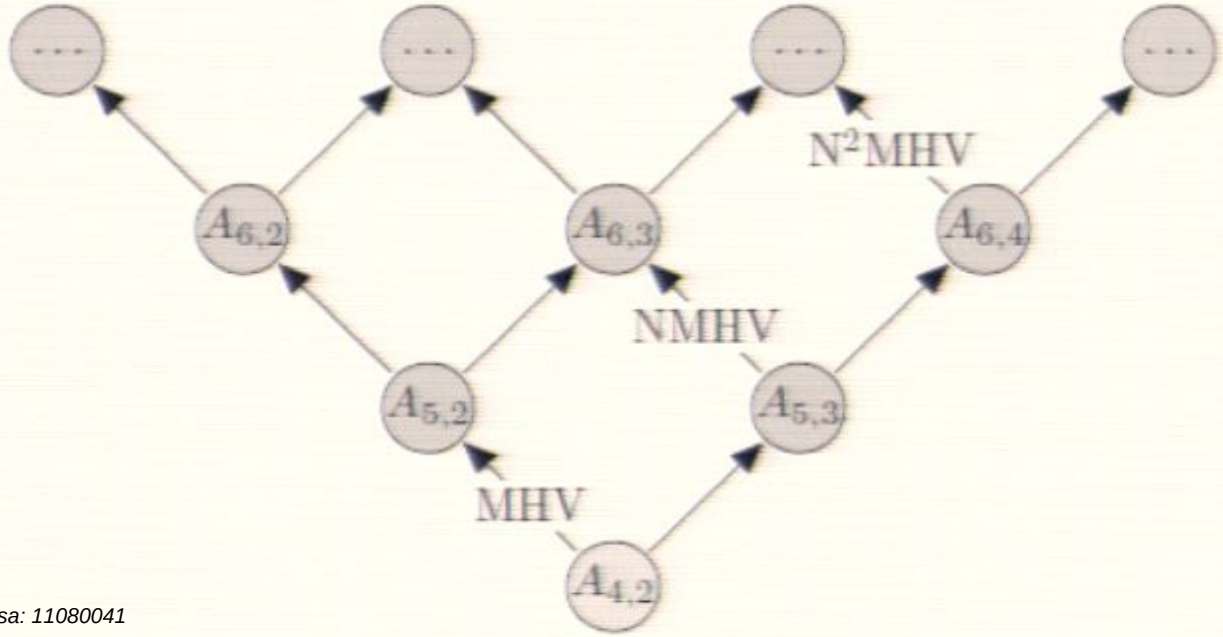
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

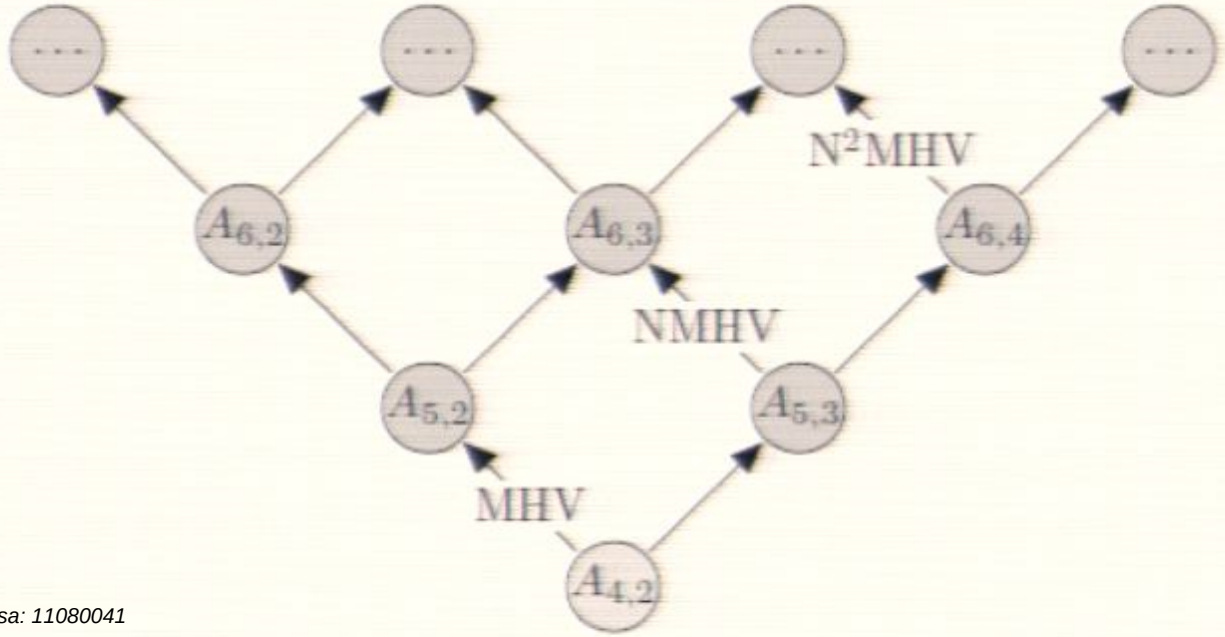
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j\rangle^4}{\langle 1, 2\rangle \langle 2, 3\rangle \ldots \langle n, 1\rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

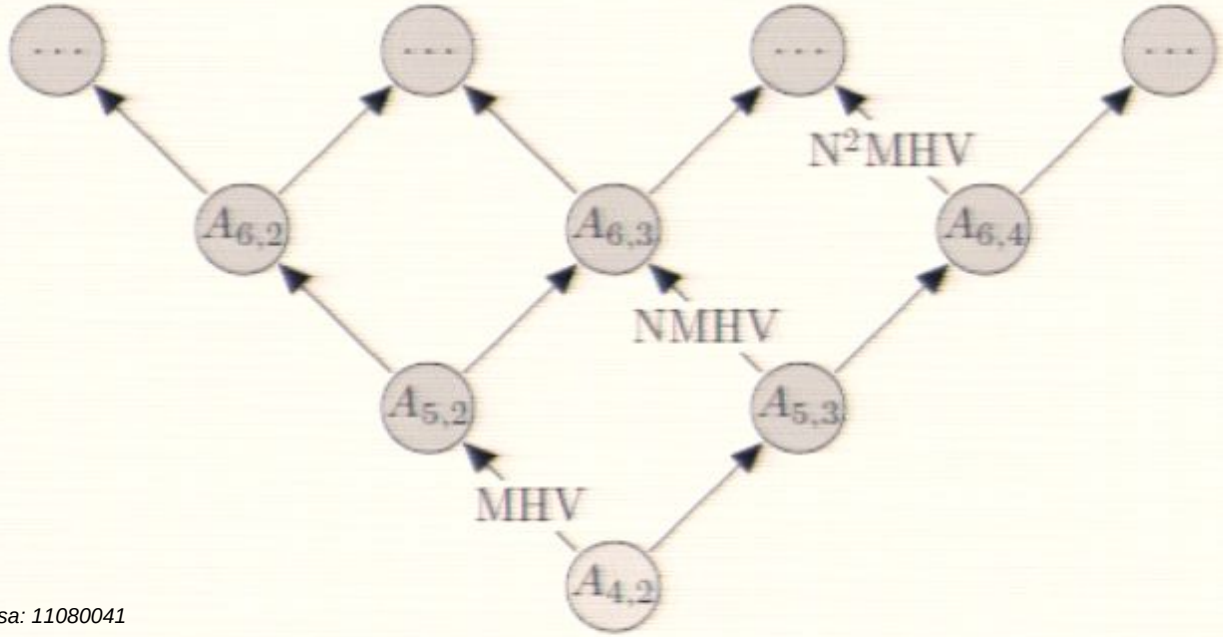
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

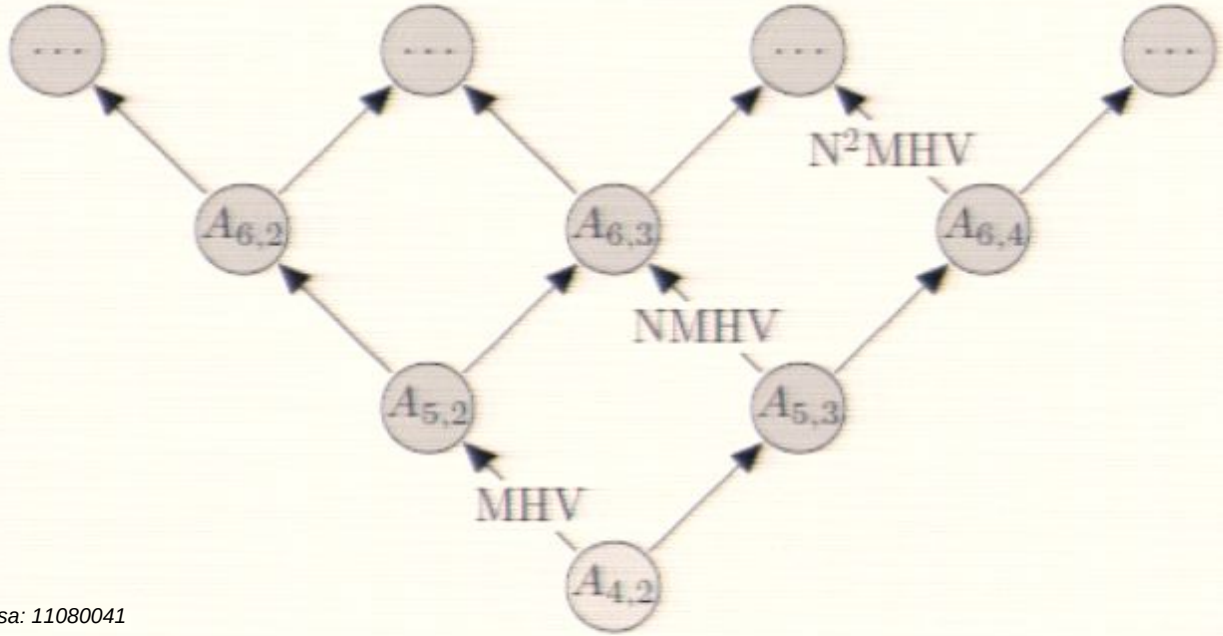
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

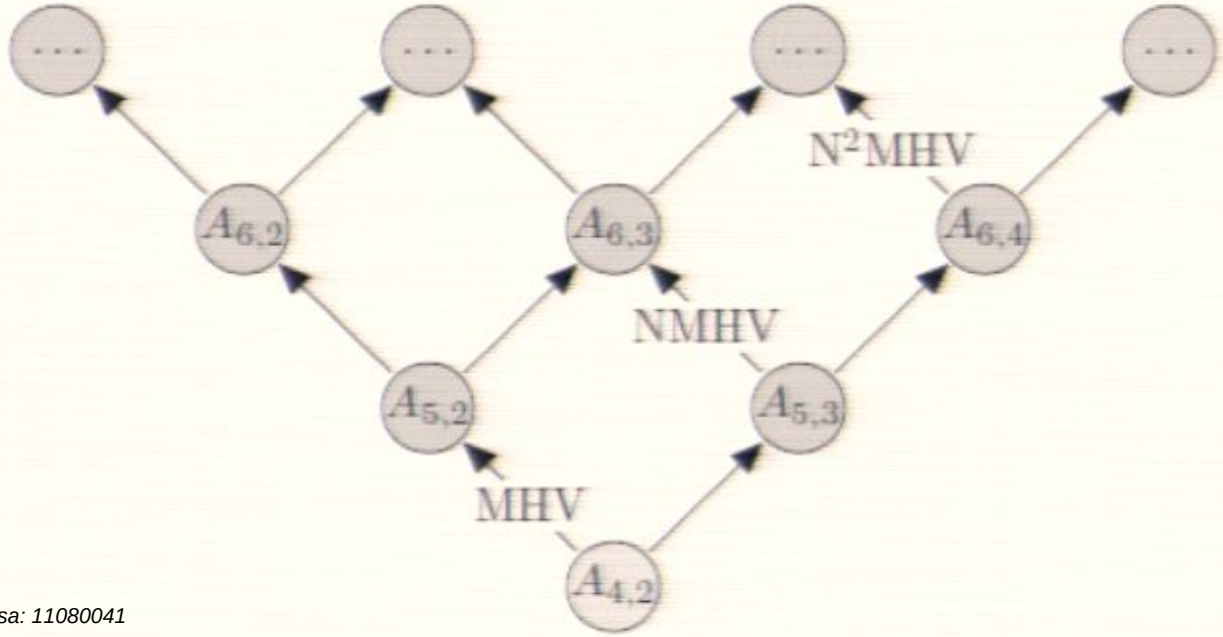
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

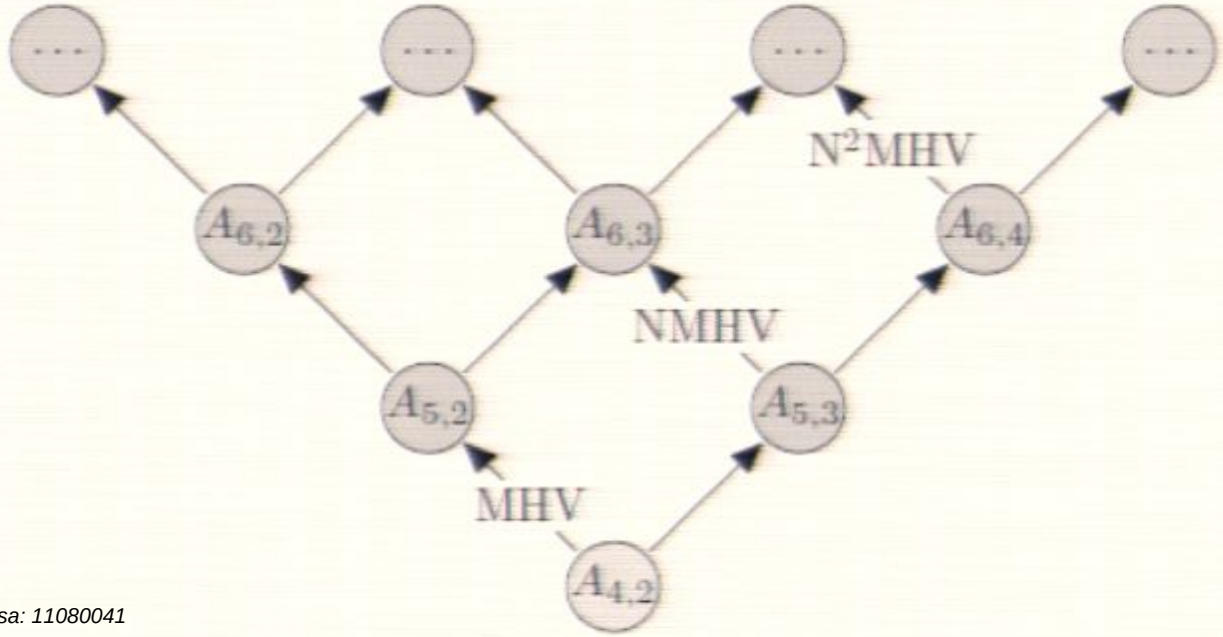
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

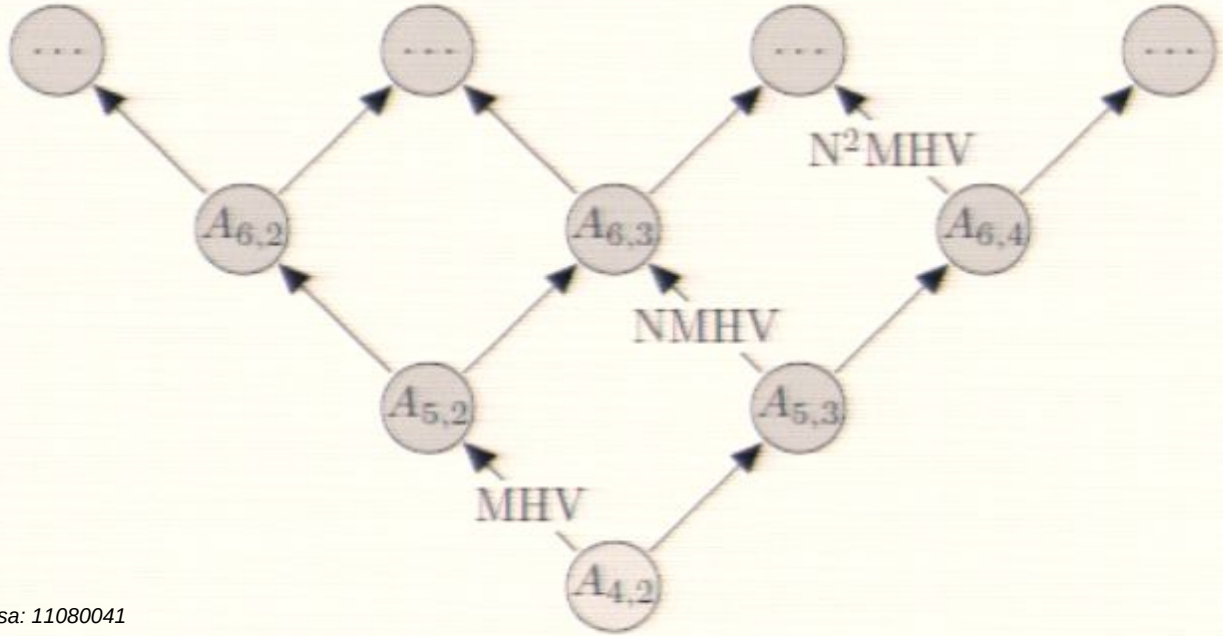
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

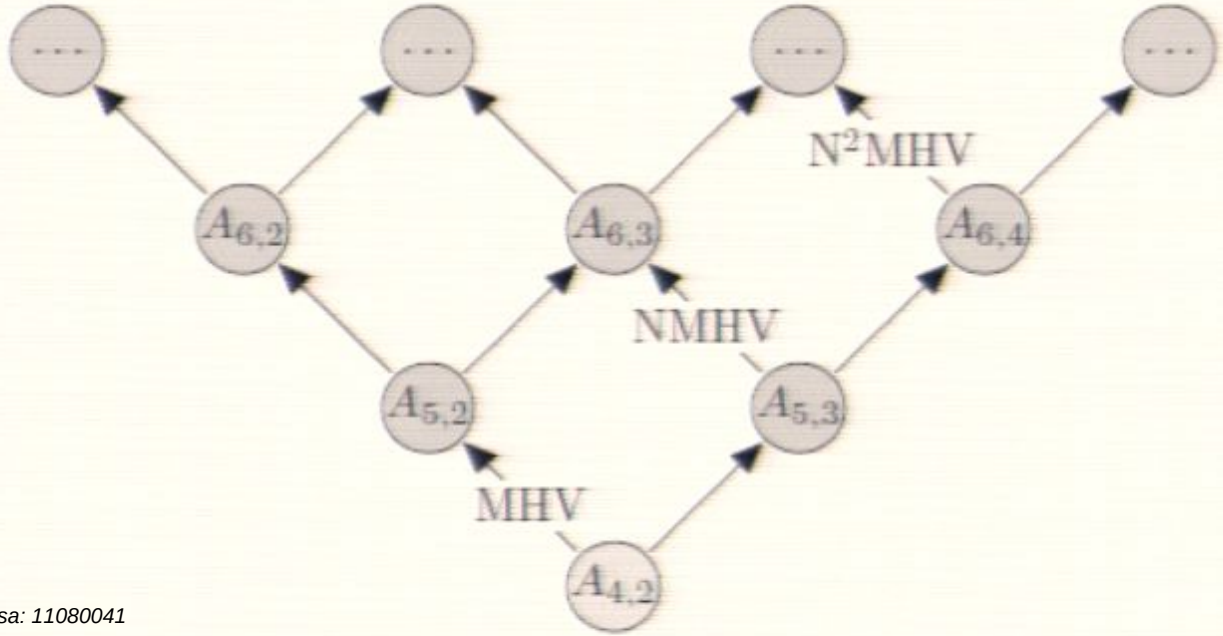
Gluon Amplitudes and Helicity Classification

lassify gluon amplitudes by # of helicity flips

- By SUSY Ward identities: $\mathcal{A}_n(1^+, 2^+, \ldots, n^+) = 0 = \mathcal{A}_n(1^-, 2^+, \ldots, n^+)$ true to all loops
- Maximally helicity violating (MHV) amplitudes

$$\mathcal{A}_n(1^+, \ldots, i^-, \ldots, j^-, \ldots n^+) = \delta^{(4)}(\sum_i p_i) \frac{\langle i, j \rangle^4}{\langle 1, 2 \rangle \langle 2, 3 \rangle \ldots \langle n, 1 \rangle}$$ [Parke, Taylor]

- Next-to-maximally helicity amplitudes (N^kMHV) have more involved structure!



$$A_{n,m} : g_+^{n-m} g_-^m$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\varphi(\lambda_2, \tilde{\lambda}_2, \eta_2)\ldots\varphi(\lambda_n, \tilde{\lambda}_n, \eta_n)\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of tree superamplitudes:

$$\mathrm{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\,\delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle\langle 2,3\rangle\ldots\langle n,1\rangle} P_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of P_n yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$P_n = P_n^{\mathrm{MHV}} + \eta^4 P_n^{\mathrm{NMHV}} + \eta^8 P_n^{\mathrm{NNMHV}} + \ldots + \eta^{4n-8} P_n^{\overline{\mathrm{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p, \eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A \eta^B S_{AB}(p) + \frac{1}{3!}\eta^A \eta^B \eta^C \epsilon_{ABCD} \bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A \eta^B \eta^C \eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of tree superamplitudes:

$$\mathrm{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1, 2\rangle \langle 2, 3\rangle \ldots \langle n, 1\rangle} P_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of P_n yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$P_n = P_n^{\mathrm{MHV}} + \eta^4 P_n^{\mathrm{NMHV}} + \eta^8 P_n^{\mathrm{NNMHV}} + \ldots + \eta^{4n-8} P_n^{\overline{\mathrm{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4\, \mathcal{P}_n^{\text{NMHV}} + \eta^8\, \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8}\, \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p, \eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A \eta^B S_{AB}(p) + \frac{1}{3!}\eta^A \eta^B \eta^C \epsilon_{ABCD} \bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A \eta^B \eta^C \eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1, 2\rangle \langle 2, 3\rangle \ldots \langle n, 1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4\, \mathcal{P}_n^{\text{NMHV}} + \eta^8\, \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8}\, \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^\pm$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^\pm$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \dots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \dots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \dots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

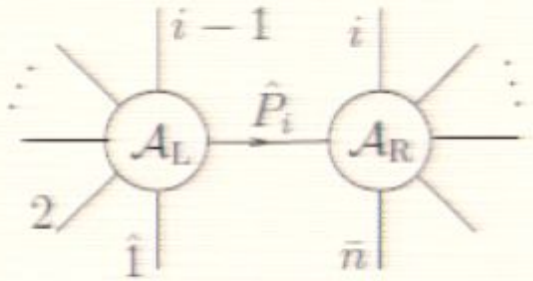
$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

On-shell superspace

- Augment λ_i^α and $\tilde{\lambda}_i^{\dot{\alpha}}$ by Grassmann variables $\eta_i^A \quad A = 1, 2, 3, 4$ [Nair]
- On-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}^{\dot{\alpha}}, \eta_i^A)$ with on-shell superfield:

$$\varphi(p,\eta) = G^+(p) + \eta^A \Gamma_A(p) + \frac{1}{2}\eta^A\eta^B S_{AB}(p) + \frac{1}{3!}\eta^A\eta^B\eta^C \epsilon_{ABCD}\bar{\Gamma}^D(p) + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D \epsilon_{ABCD} G^-(p)$$

- Superamplitudes: $\left\langle \varphi(\lambda_1, \tilde{\lambda}_1, \eta_1)\, \varphi(\lambda_2, \tilde{\lambda}_2, \eta_2) \ldots \varphi(\lambda_n, \tilde{\lambda}_n, \eta_n) \right\rangle$

  Packages all n-parton gluon$^{\pm}$-gluino$^{\pm 1/2}$-scalar amplitudes

- General form of **tree superamplitudes**:

$$\mathbb{A}_n = \frac{\delta^{(4)}(\sum_i \lambda_i \tilde{\lambda}_i)\, \delta^{(8)}(\sum_i \lambda_i \eta_i)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

  Conservation of super-momentum: $\delta^{(8)}(\sum_i \lambda^\alpha \eta_i^A) = (\sum_i \lambda^\alpha \eta_i^A)^8$

- η-expansion of $\mathcal{P}_n$ yields N^{k}MHV-classification of superamps as $h(\eta) = -1/2$

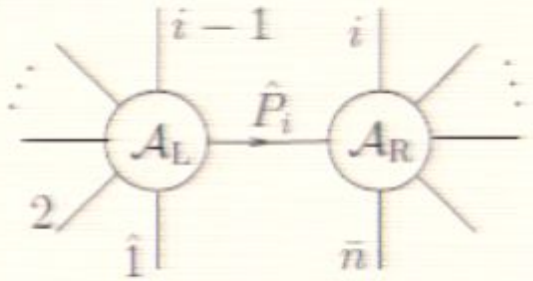
$$\mathcal{P}_n = \mathcal{P}_n^{\text{MHV}} + \eta^4 \mathcal{P}_n^{\text{NMHV}} + \eta^8 \mathcal{P}_n^{\text{NNMHV}} + \ldots + \eta^{4n-8} \mathcal{P}_n^{\overline{\text{MHV}}}$$

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

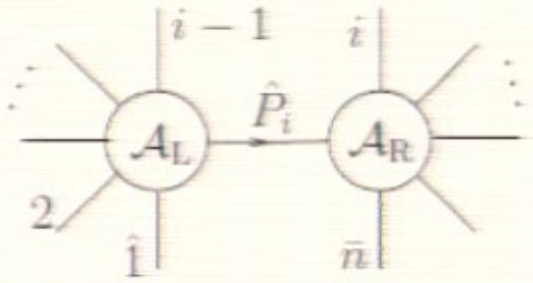
  $\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

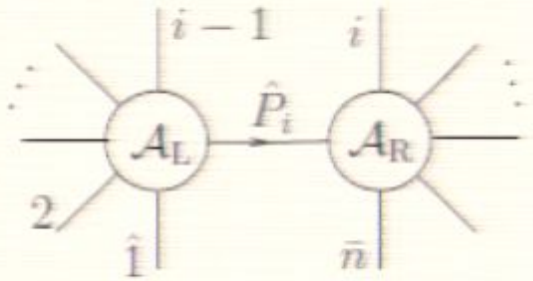
$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1}$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level

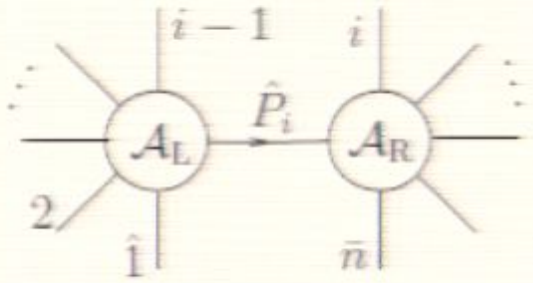
[Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

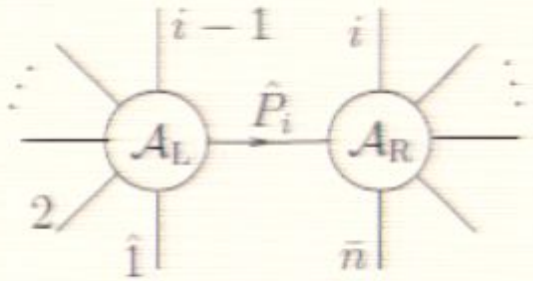
  $\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

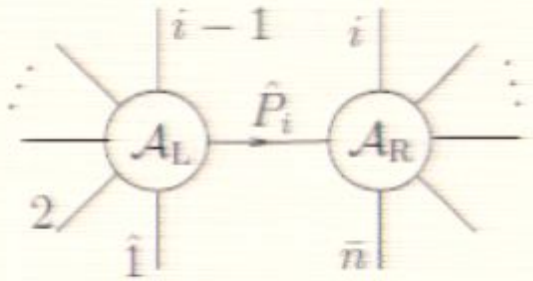
  $\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

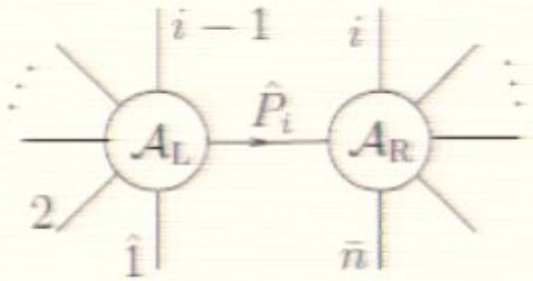
$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

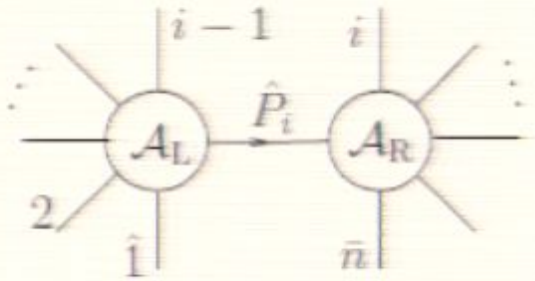
  $\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1}$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism

- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]

- Super BCFW recursion is much simpler and can be solved analytically!

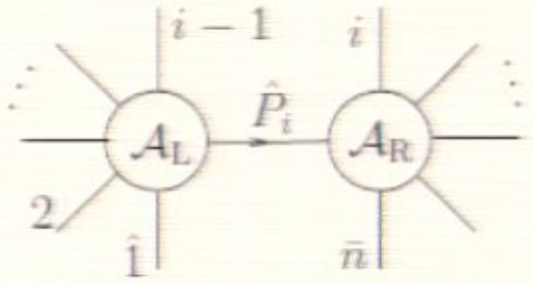
  $\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

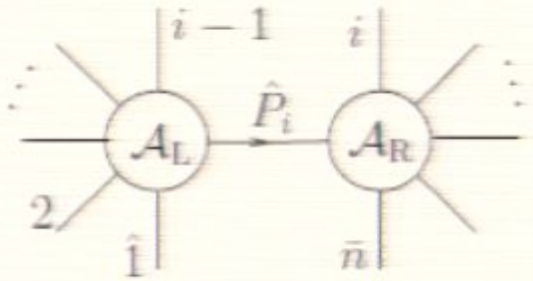
  $\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

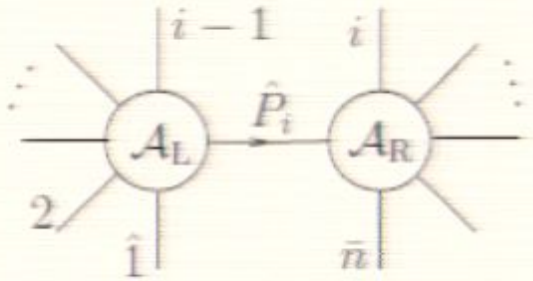
$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1}$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism

- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]

- Super BCFW recursion is much simpler and can be solved analytically!

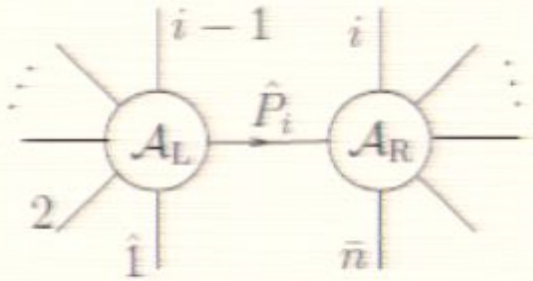
$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

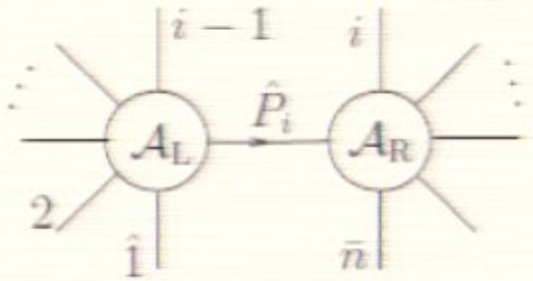
  $\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

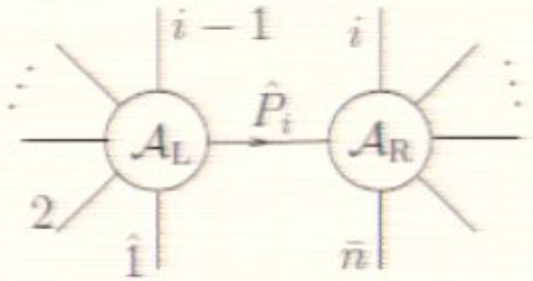
$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

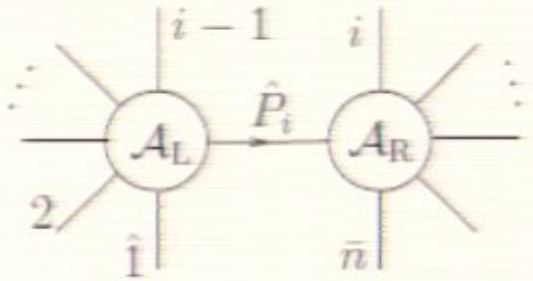
$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1}$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

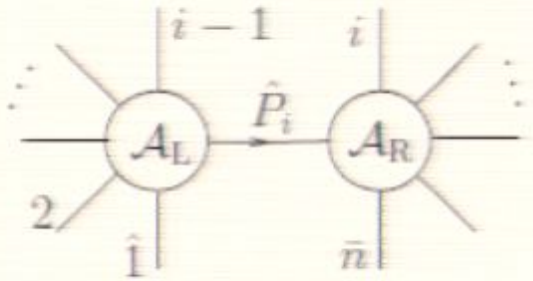
$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

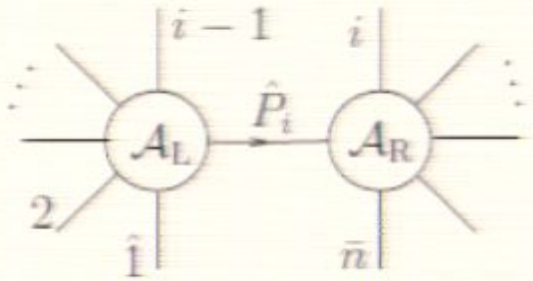
  $\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

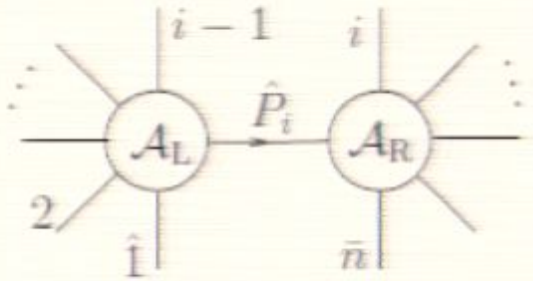
$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

Superamplitudes and BCFW recursion

- Efficient way of constructing tree-level amplitudes via BCFW recursion

  [Britto,Cachazo,Feng+Witten '04,05]

$$A_n = \sum_i A^h_{i+1} \frac{1}{P_i^2} A^{-h}_{n-i+1} \qquad \Sigma$$



  - N-point amplitudes are obtained recursively from lower-point amplitudes
  - Purely on-shell formalism
- Reformulation of recursion relations in on-shell superspace via shift in $(\lambda_i, \tilde{\lambda}_i)$ and η_i

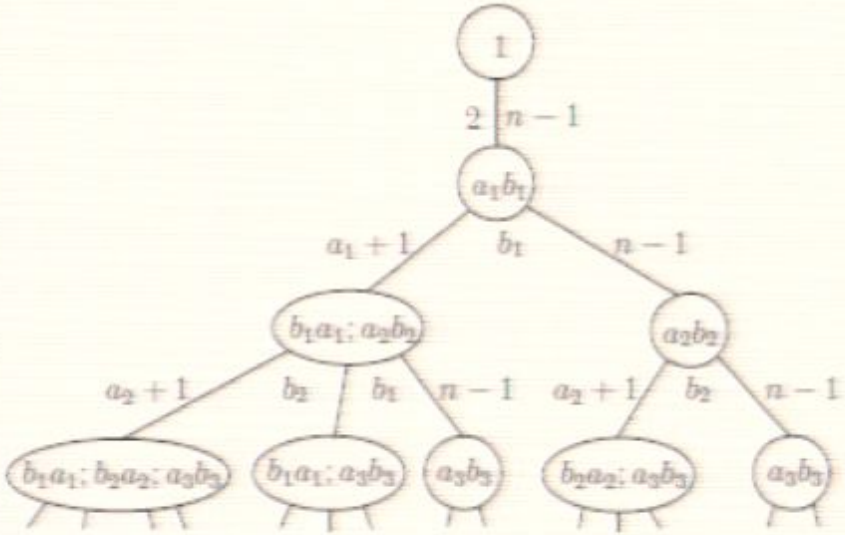
  [Elvang et al, Arkani-Hamed et al, Brandhuber et al]
- Super BCFW recursion is much simpler and can be solved analytically!

$\Rightarrow$ $\mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$ known in closed analytical form at tree-level [Drummond,Henn]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

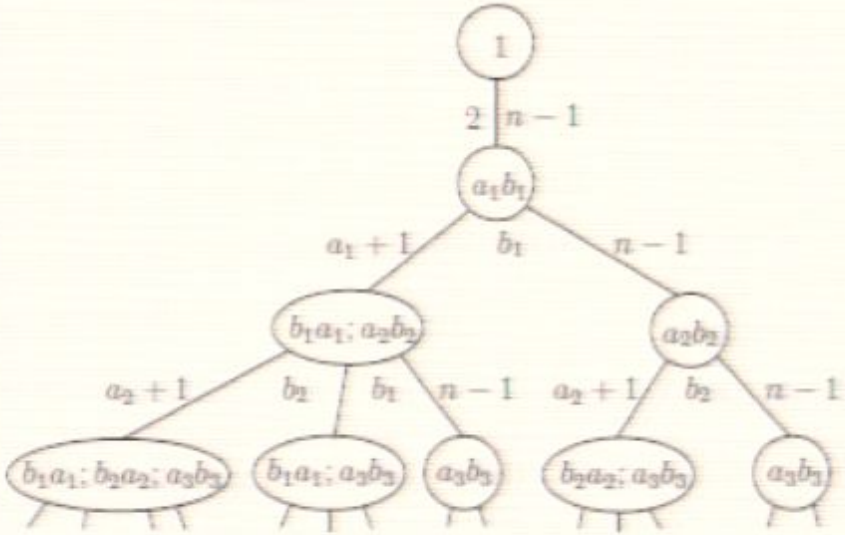
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

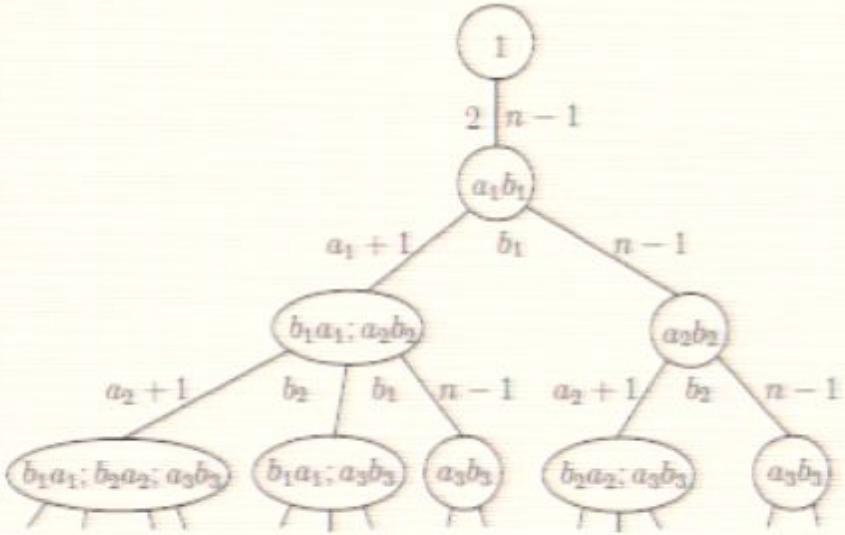
ixon, Henn, JP, Schuster]

pplication: Used in BLACKHAT for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N}^2\mathrm{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

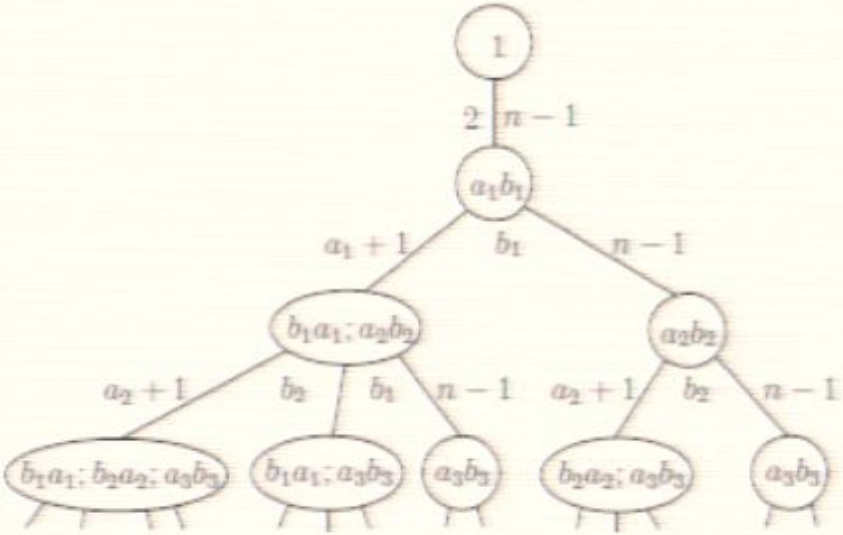
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths}\\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

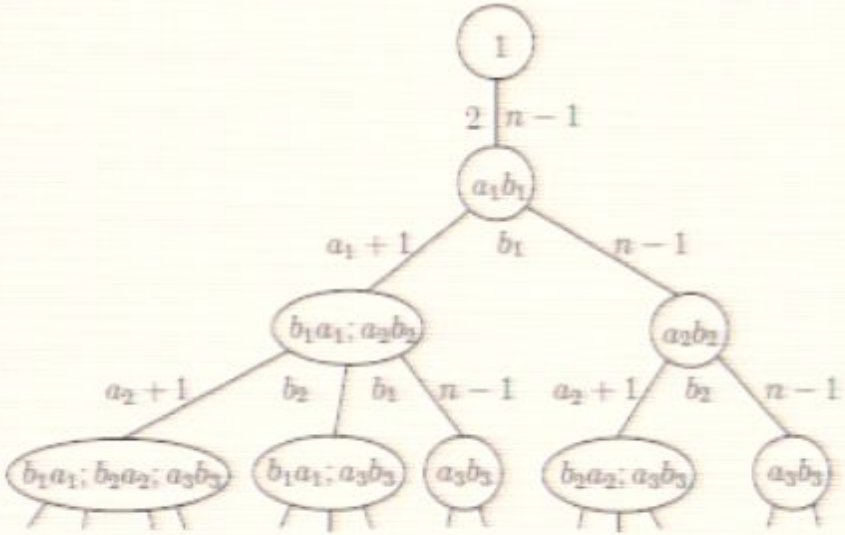
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths}\\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

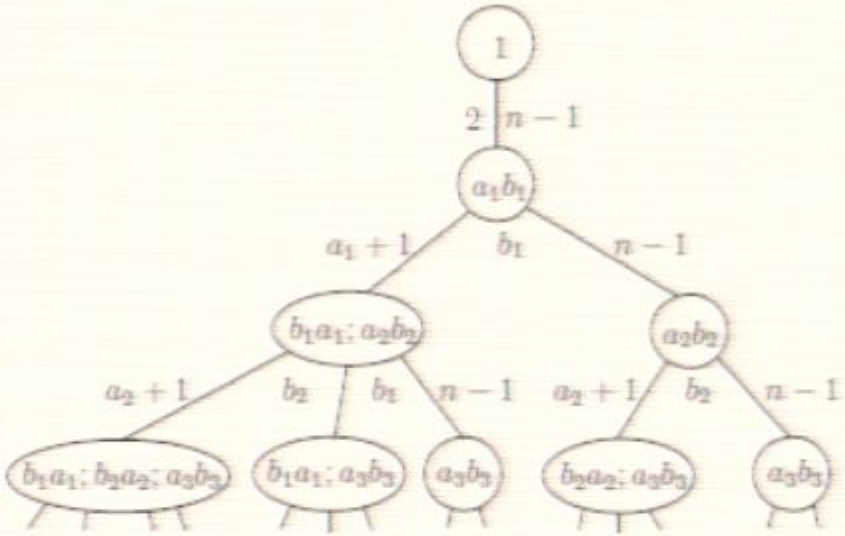
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

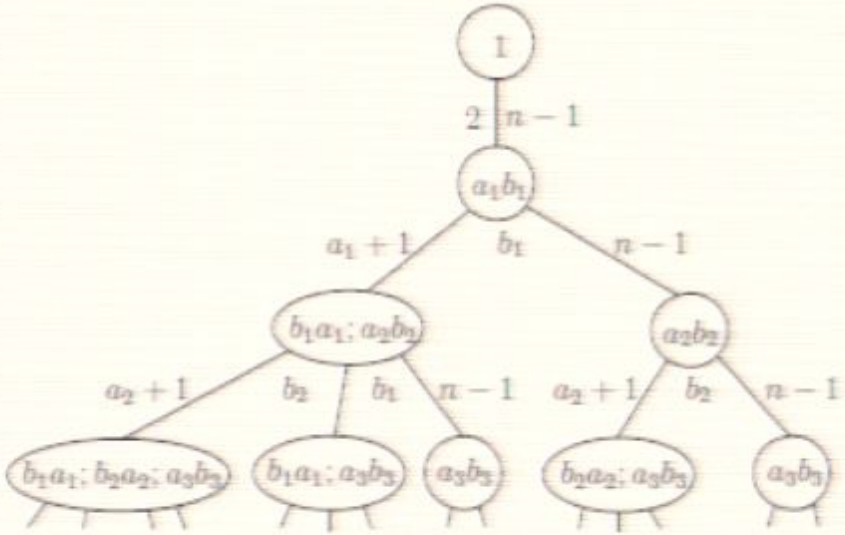
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathsf{N}^k\mathsf{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathsf{MHV}} = 1$$

$$\mathcal{P}^{\mathsf{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathsf{N}^2\mathsf{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

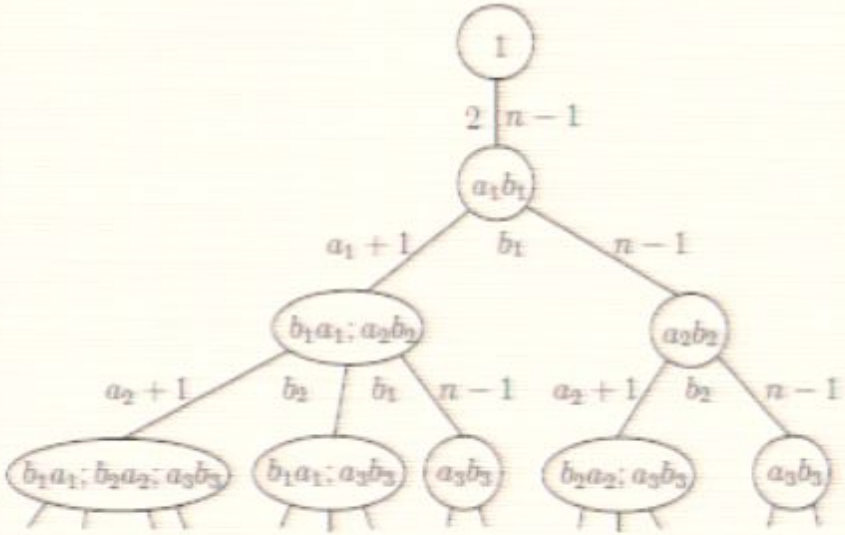
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

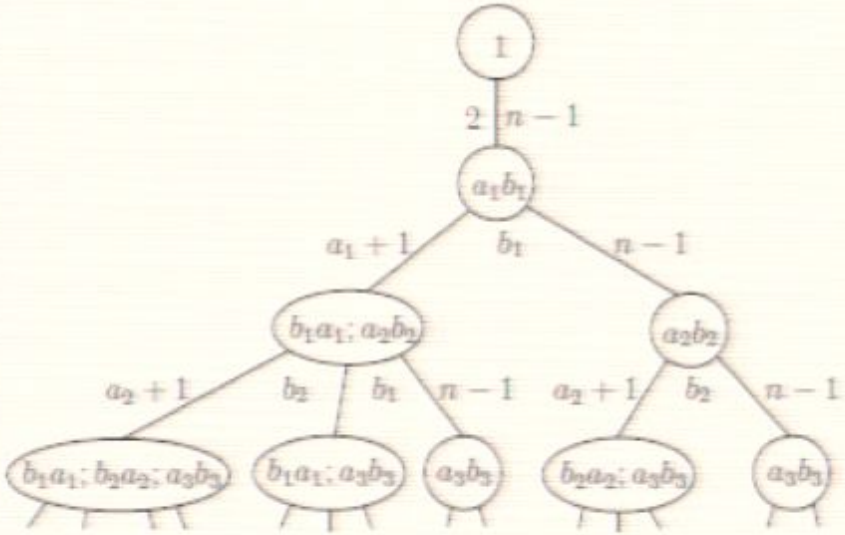
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

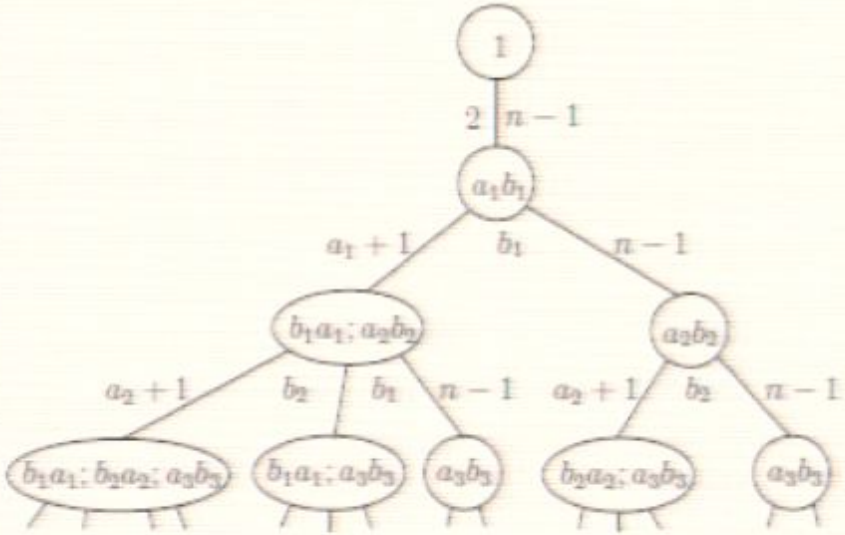
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths}\\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N}^2\mathrm{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

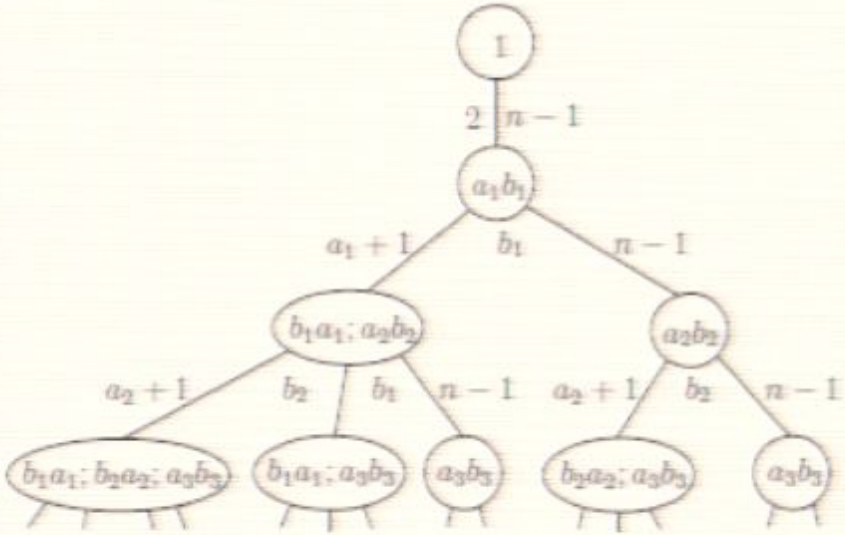
ixon, Henn, JP, Schuster]

pplication: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

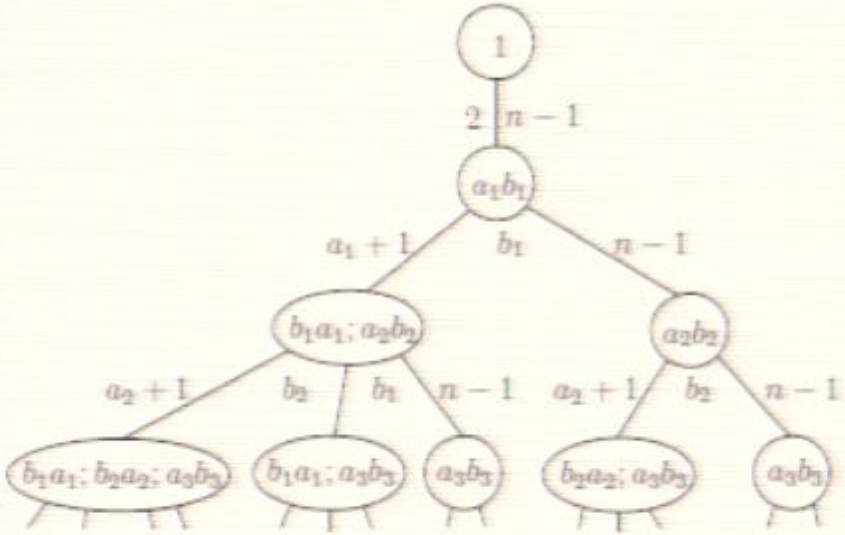
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N}^2\mathrm{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

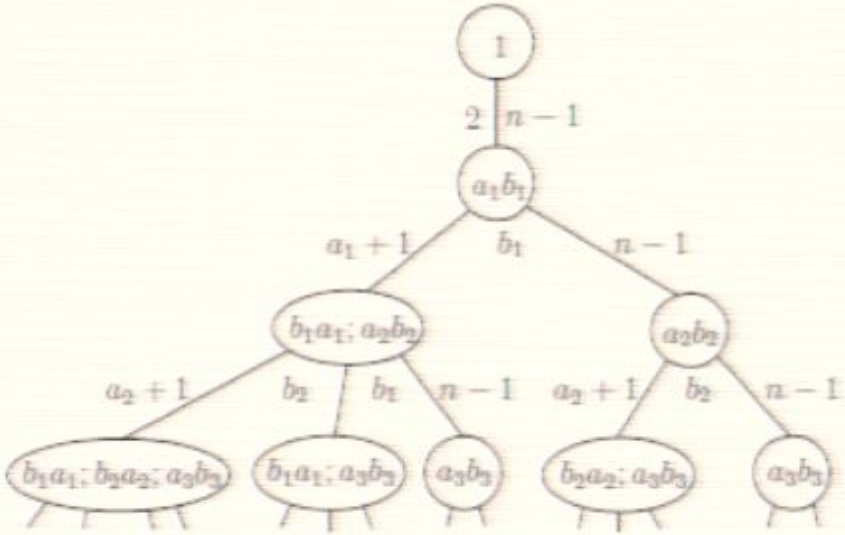
ixon, Henn, JP, Schuster]

pplication: Used in BLACKHAT for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

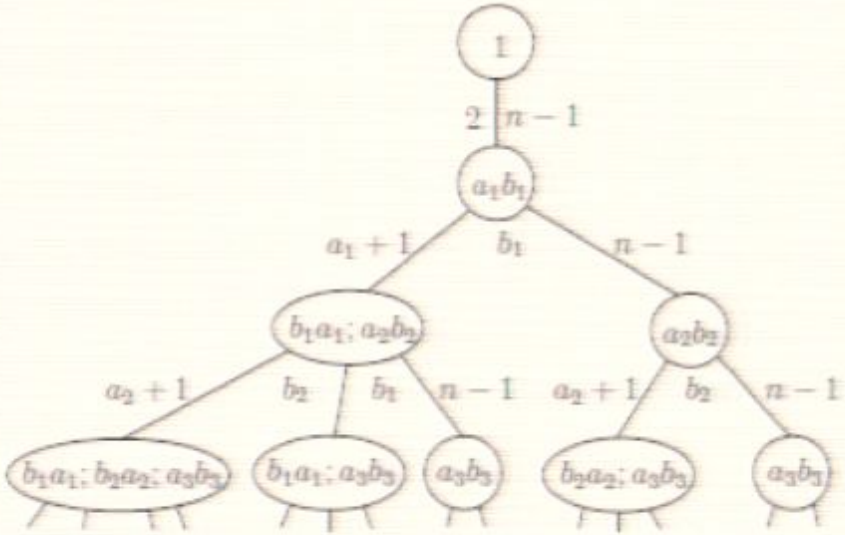
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths}\\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N}^2\mathrm{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

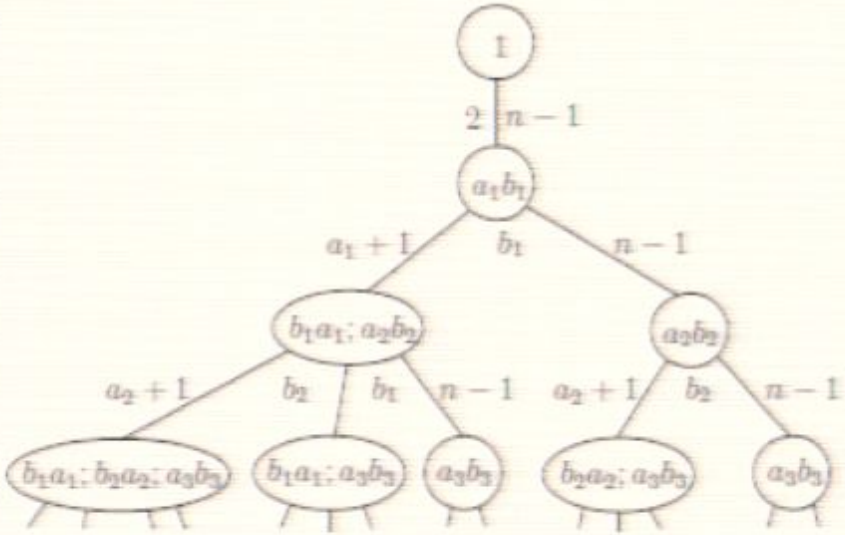
[...xon, Henn, JP, Schuster]

...pplication: Used in BlackHat for recent $Z+4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N^2MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

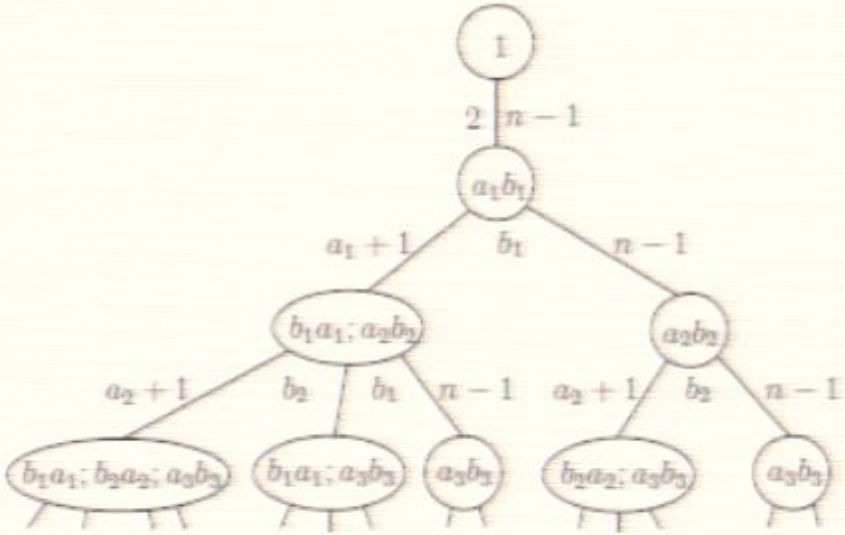
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathsf{N}^k\mathsf{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathsf{MHV}} = 1$$

$$\mathcal{P}^{\mathsf{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathsf{N}^2\mathsf{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

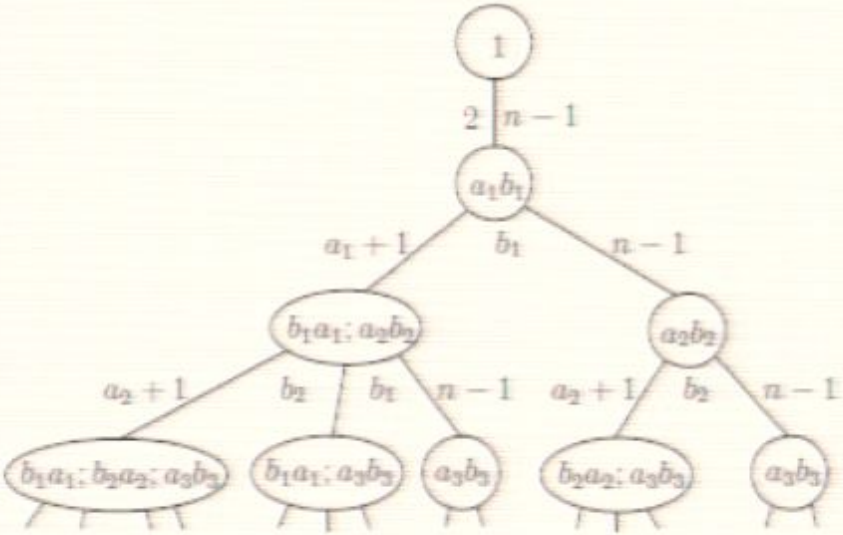
[xon, Henn, JP, Schuster]

pplication: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathsf{N}^k\mathsf{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathsf{MHV}} = 1$$

$$\mathcal{P}^{\mathsf{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathsf{N}^2\mathsf{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

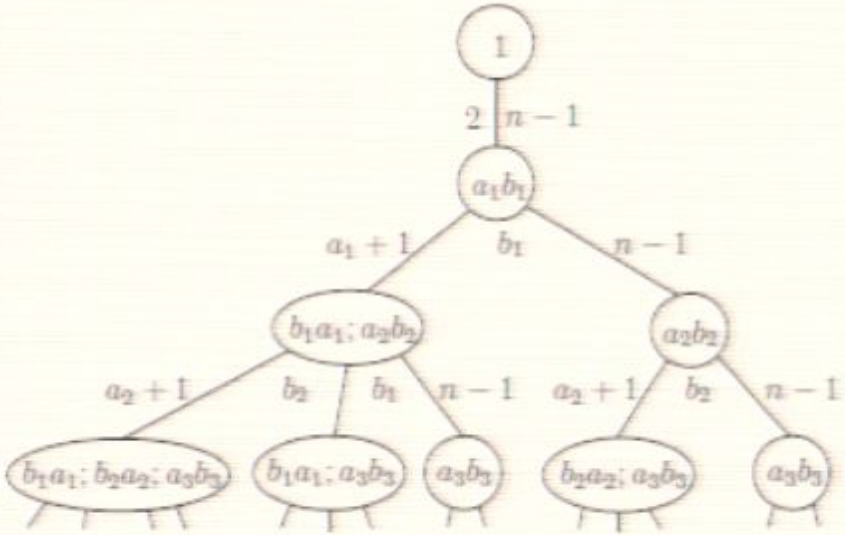
[Ixon, Henn, JP, Schuster]

Application: Used in BLACKHAT for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathsf{N}^k\mathsf{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathsf{MHV}} = 1$$

$$\mathcal{P}^{\mathsf{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathsf{N}^2\mathsf{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

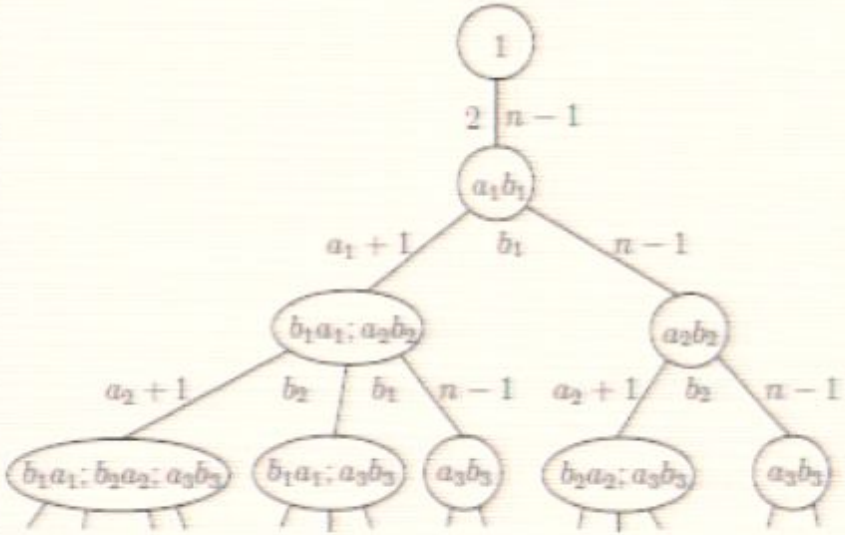
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N}^2\mathrm{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

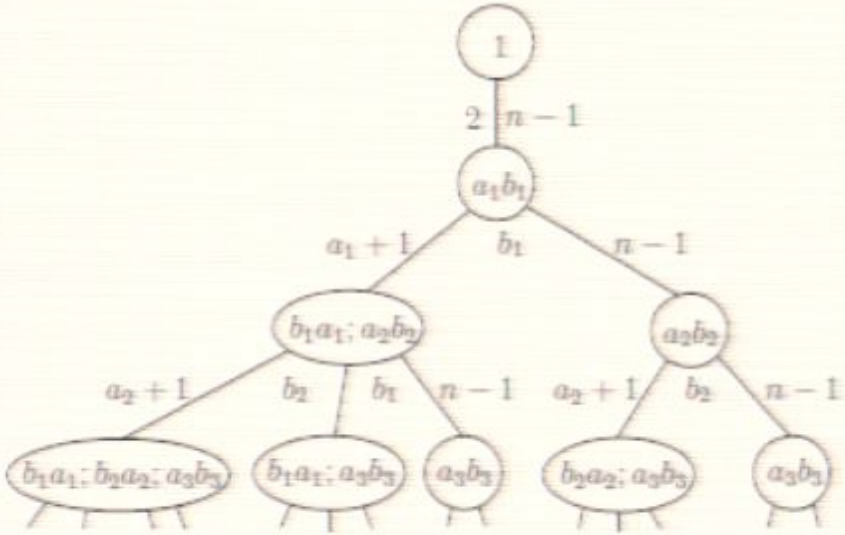
[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

The Drummond-Henn solution

$\mathcal{P}_n$ expressed as sums over R-invariants determined by paths on rooted tree

$$\mathcal{P}_n^{\mathrm{N}^k\mathrm{MHV}} = \sum_{\substack{\text{all paths} \\ \text{of length } k}} 1 \cdot R_{n,a_1b_1} \cdot R^{\{L_2\};\{U_2\}}_{n,\{I_2\},a_2b_2} \cdot \ldots \cdot R^{\{L_p\};\{U_p\}}_{n,\{I_p\},a_pb_p}$$



$$\mathcal{P}^{\mathrm{MHV}} = 1$$

$$\mathcal{P}^{\mathrm{NMHV}} = \sum_{1<a_1,b_1<n} R_{n,a_1b_1}$$

$$\mathcal{P}_n^{\mathrm{N}^2\mathrm{MHV}} = \sum_{1<a_1,b_1<n} R_{n;a_1b_1} \times \left[\sum_{a_1<a_2,b_2\leq b_1} R^{0;a_1b_1}_{n;b_1a_1;a_2b_2} + \sum_{b_1\leq a_2,b_2<n} R^{a_1b_1;0}_{n;a_2b_2} \right]$$

Projection to components yields all tree amplitudes of massless QCD

[Ixon, Henn, JP, Schuster]

Application: Used in BlackHat for recent $Z + 4$ jets cross section [Ita et. al.]

Symmetries

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle}\, \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{ obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{ less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle}\, \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha\, \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha\, A} = \sum_{i=1}^n \lambda_i^\alpha\, \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha\, A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha\, \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\, \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A\, \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle}\, \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha\, \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha\, A} = \sum_{i=1}^n \lambda_i^\alpha\, \eta_i^A \qquad \Rightarrow \text{ obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha\, A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{ less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha\, \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\, \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A\, \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle}\, \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha\, \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha\, A} = \sum_{i=1}^n \lambda_i^\alpha\, \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha\, A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha\, \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\, \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A\, \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{ obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{ less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle}\, \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{ obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{ less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{ obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{ less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{ obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{ less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle}\, \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha\, \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha\, A} = \sum_{i=1}^n \lambda_i^\alpha\, \eta_i^A \qquad \Rightarrow \text{ obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha\, A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{ less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2} \lambda_i^\alpha\, \partial_{i\,\alpha} + \frac{1}{2} \tilde{\lambda}_i^{\dot{\alpha}}\, \partial_{i\,\dot{\alpha}} + \frac{1}{2} \eta_i^A\, \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle}\, \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha\, \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha\, A} = \sum_{i=1}^n \lambda_i^\alpha\, \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha\, A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha\, \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\, \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A\, \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle} \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + \frac{1}{2}\eta_i^A \partial_{iA} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- Superamplitude: $(i = 1, \ldots, n)$

$$\mathbb{A}_n^{\text{tree}}(\{\lambda_i, \tilde{\lambda}_i, \eta_i\}) = \frac{\delta^{(4)}(\sum_i \lambda_i^\alpha \tilde{\lambda}_i^{\dot{\alpha}})\, \delta^{(8)}(\sum_i \lambda_i^\alpha \eta_i^A)}{\langle 1,2\rangle \langle 2,3\rangle \ldots \langle n,1\rangle}\, \mathcal{P}_n(\{\lambda_i, \tilde{\lambda}_i, \eta_i\})$$

- Realization of $\mathfrak{psu}(2,2|4)$ generators in **on-shell superspace**, e.g. [Witten]

$$p^{\alpha\dot{\alpha}} = \sum_{i=1}^n \lambda_i^\alpha\, \tilde{\lambda}_i^{\dot{\alpha}} \qquad q^{\alpha A} = \sum_{i=1}^n \lambda_i^\alpha\, \eta_i^A \qquad \Rightarrow \text{obvious symmetries}$$

$$k_{\alpha\dot{\alpha}} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}} \qquad s_{\alpha A} = \sum_{i=1}^n \frac{\partial}{\partial \lambda_i^\alpha} \frac{\partial}{\partial \eta_i^A} \qquad \Rightarrow \text{less obvious sym}$$

- Invariance: $\{p, k, l, \bar{l}, d, r, q, \bar{q}, s, \bar{s}, c_i\}\, \mathbb{A}_n^{\text{tree}}(\{\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A\}) = 0$
- N.B.: **Local** invariance $h_i\, \mathbb{A}_n = 1 \cdot \mathbb{A}_n$

Helicity operator: $h_i = -\frac{1}{2}\lambda_i^\alpha\, \partial_{i\,\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\, \partial_{i\,\dot{\alpha}} + \frac{1}{2}\eta_i^A\, \partial_{i\,A} = 1 - c_i$

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot\alpha}, \eta_i^A)$:

$$p^{\dot\alpha\alpha} = \sum_i \tilde{\lambda}_i^{\dot\alpha}\lambda_i^\alpha\,, \qquad k_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha}\partial_{i\dot\alpha}\,,$$

$$\overline{m}_{\dot\alpha\dot\beta} = \sum_i \tilde{\lambda}_{i(\dot\alpha}\partial_{i\dot\beta)}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)}\,,$$

$$d = \sum_i [\tfrac{1}{2}\lambda_i^\alpha\partial_{i\alpha} + \tfrac{1}{2}\tilde{\lambda}_i^{\dot\alpha}\partial_{i\dot\alpha} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A\partial_{iB} + \tfrac{1}{4}\delta^A_B\eta_i^C\partial_{iC}]\,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha\eta_i^A\,, \qquad \bar{q}^{\dot\alpha}_A = \sum_i \tilde{\lambda}_i^{\dot\alpha}\partial_{iA}\,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha}\partial_{iA}, \qquad \bar{s}^A_{\dot\alpha} = \sum_i \eta_i^A\partial_{i\dot\alpha}\,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial\lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial\eta_i^A}\,.$$

- N.B: For collinear momenta ($\lambda_2 \sim \lambda_{i-1}$) important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial\lambda^a}\frac{1}{\langle\lambda,\mu\rangle} = 2\pi\mu_a\,\delta^2(\langle\lambda,\mu\rangle)$

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A)$:

$$p^{\dot{\alpha}\alpha} = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \lambda_i^\alpha \,, \qquad k_{\alpha\dot{\alpha}} = \sum_i \partial_{i\alpha} \partial_{i\dot{\alpha}} \,,$$

$$\overline{m}_{\dot{\alpha}\dot{\beta}} = \sum_i \tilde{\lambda}_{i(\dot{\alpha}} \partial_{i\dot{\beta})}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha} \partial_{i\beta)} \,,$$

$$d = \sum_i [\tfrac{1}{2} \lambda_i^\alpha \partial_{i\alpha} + \tfrac{1}{2} \tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A \partial_{iB} + \tfrac{1}{4} \delta^A_B \eta_i^C \partial_{iC}] \,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha \eta_i^A \,, \qquad \bar{q}^{\dot{\alpha}}_A = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \partial_{iA} \,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha} \partial_{iA}, \qquad \bar{s}^A_{\dot{\alpha}} = \sum_i \eta_i^A \partial_{i\dot{\alpha}} \,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial \eta_i^A} \,.$$

- N.B: For collinear momenta $(\lambda_i \sim \lambda_{i-1})$ important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial \lambda^\alpha} \frac{1}{\langle \lambda \mu \rangle} = 2\pi \tilde{\mu}_\alpha \delta^2(\langle \lambda, \mu \rangle)$

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot\alpha}, \eta_i^A)$:

$$p^{\dot\alpha\alpha} = \sum_i \tilde{\lambda}_i^{\dot\alpha}\lambda_i^\alpha\,, \qquad k_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha}\partial_{i\dot\alpha}\,,$$

$$\overline{m}_{\dot\alpha\dot\beta} = \sum_i \tilde{\lambda}_{i(\dot\alpha}\partial_{i\dot\beta)}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)}\,,$$

$$d = \sum_i [\tfrac{1}{2}\lambda_i^\alpha\partial_{i\alpha} + \tfrac{1}{2}\tilde{\lambda}_i^{\dot\alpha}\partial_{i\dot\alpha} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A\partial_{iB} + \tfrac{1}{4}\delta^A_B\eta_i^C\partial_{iC}]\,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha\eta_i^A\,, \qquad \bar{q}^{\dot\alpha}_A = \sum_i \tilde{\lambda}_i^{\dot\alpha}\partial_{iA}\,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha}\partial_{iA}, \qquad \bar{s}^A_{\dot\alpha} = \sum_i \eta_i^A\partial_{i\dot\alpha}\,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial\lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial\eta_i^A}\,.$$

- N.B: For collinear momenta $(\lambda_i \sim \lambda_{i+1})$ important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial\tilde{\lambda}^{\dot\alpha}}\frac{1}{\langle\lambda,\mu\rangle} = 2\pi\tilde{\mu}_{\dot\alpha}\,\delta^2(\langle\lambda,\mu\rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot\alpha}, \eta_i^A)$:

$$p^{\dot\alpha\alpha} = \sum_i \tilde{\lambda}_i^{\dot\alpha}\lambda_i^\alpha\,, \qquad k_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha}\partial_{i\dot\alpha}\,,$$

$$\overline{m}_{\dot\alpha\dot\beta} = \sum_i \tilde{\lambda}_{i(\dot\alpha}\partial_{i\dot\beta)}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)}\,,$$

$$d = \sum_i [\tfrac{1}{2}\lambda_i^\alpha\partial_{i\alpha} + \tfrac{1}{2}\tilde{\lambda}_i^{\dot\alpha}\partial_{i\dot\alpha} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A\partial_{iB} + \tfrac{1}{4}\delta^A_B\eta_i^C\partial_{iC}]\,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha\eta_i^A\,, \qquad \bar{q}^{\dot\alpha}_A = \sum_i \tilde{\lambda}_i^{\dot\alpha}\partial_{iA}\,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha}\partial_{iA}, \qquad \bar{s}^A_{\dot\alpha} = \sum_i \eta_i^A\partial_{i\dot\alpha}\,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial\lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial\eta_i^A}\,.$$

- N.B: For collinear momenta $(\lambda_i \sim \lambda_{i+1})$ important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial\tilde{\lambda}^{\dot\alpha}}\frac{1}{\langle\lambda,\mu\rangle} = 2\pi\tilde{\mu}_{\dot\alpha}\,\delta^2(\langle\lambda,\mu\rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

$su(2,2|4)$ invariance

- The $su(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A)$:

$$p^{\dot{\alpha}\alpha} = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \lambda_i^\alpha \,, \qquad k_{\alpha\dot{\alpha}} = \sum_i \partial_{i\alpha} \partial_{i\dot{\alpha}} \,,$$

$$\overline{m}_{\dot{\alpha}\dot{\beta}} = \sum_i \tilde{\lambda}_{i(\dot{\alpha}} \partial_{i\dot{\beta})}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha} \partial_{i\beta)} \,,$$

$$d = \sum_i [\tfrac{1}{2} \lambda_i^\alpha \partial_{i\alpha} + \tfrac{1}{2} \tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A \partial_{iB} + \tfrac{1}{4} \delta_B^A \eta_i^C \partial_{iC}] \,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha \eta_i^A \,, \qquad \bar{q}_A^{\dot{\alpha}} = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \partial_{iA} \,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha} \partial_{iA}, \qquad \bar{s}_{\dot{\alpha}}^A = \sum_i \eta_i^A \partial_{i\dot{\alpha}} \,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial \eta_i^A} \,.$$

- N.B: For collinear momenta $(\lambda_i \sim \lambda_{i+1})$ important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial \tilde{\lambda}^{\dot{\alpha}}} \frac{1}{\langle \lambda, \mu \rangle} = 2\pi \tilde{\mu}_{\dot{\alpha}} \, \delta^2(\langle \lambda, \mu \rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot\alpha}, \eta_i^A)$:

$$p^{\dot\alpha\alpha} = \sum_i \tilde{\lambda}_i^{\dot\alpha}\lambda_i^\alpha\,, \qquad k_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha}\partial_{i\dot\alpha}\,,$$

$$\overline{m}_{\dot\alpha\dot\beta} = \sum_i \tilde{\lambda}_{i(\dot\alpha}\partial_{i\dot\beta)}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)}\,,$$

$$d = \sum_i [\tfrac{1}{2}\lambda_i^\alpha\partial_{i\alpha} + \tfrac{1}{2}\tilde{\lambda}_i^{\dot\alpha}\partial_{i\dot\alpha} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A\partial_{iB} + \tfrac{1}{4}\delta^A_B\eta_i^C\partial_{iC}]\,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha\eta_i^A\,, \qquad \bar{q}^{\dot\alpha}_A = \sum_i \tilde{\lambda}_i^{\dot\alpha}\partial_{iA}\,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha}\partial_{iA}, \qquad \bar{s}^A_{\dot\alpha} = \sum_i \eta_i^A\partial_{i\dot\alpha}\,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial\lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial\eta_i^A}\,.$$

- N.B: For collinear momenta $(\lambda_i \sim \lambda_{i+1})$ important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial\tilde{\lambda}^{\dot\alpha}}\frac{1}{\langle\lambda,\mu\rangle} = 2\pi\tilde{\mu}_{\dot\alpha}\,\delta^2(\langle\lambda,\mu\rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A)$:

$$p^{\dot{\alpha}\alpha} = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \lambda_i^\alpha \,, \qquad k_{\alpha\dot{\alpha}} = \sum_i \partial_{i\alpha} \partial_{i\dot{\alpha}} \,,$$

$$\overline{m}_{\dot{\alpha}\dot{\beta}} = \sum_i \tilde{\lambda}_{i(\dot{\alpha}} \partial_{i\dot{\beta})}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha} \partial_{i\beta)} \,,$$

$$d = \sum_i [\tfrac{1}{2} \lambda_i^\alpha \partial_{i\alpha} + \tfrac{1}{2} \tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A \partial_{iB} + \tfrac{1}{4} \delta^A_B \eta_i^C \partial_{iC}] \,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha \eta_i^A \,, \qquad \bar{q}^{\dot{\alpha}}_A = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \partial_{iA} \,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha} \partial_{iA}, \qquad \bar{s}^A_{\dot{\alpha}} = \sum_i \eta_i^A \partial_{i\dot{\alpha}} \,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial \eta_i^A} \,.$$

- N.B: For collinear momenta $(\lambda_i \sim \lambda_{i+1})$ important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial \tilde{\lambda}^{\dot{\alpha}}} \frac{1}{\langle \lambda, \mu \rangle} = 2\pi \tilde{\mu}_{\dot{\alpha}} \, \delta^2(\langle \lambda, \mu \rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A)$:

$$p^{\dot{\alpha}\alpha} = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \lambda_i^\alpha \,, \qquad k_{\alpha\dot{\alpha}} = \sum_i \partial_{i\alpha} \partial_{i\dot{\alpha}} \,,$$

$$\overline{m}_{\dot{\alpha}\dot{\beta}} = \sum_i \tilde{\lambda}_{i(\dot{\alpha}} \partial_{i\dot{\beta})}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha} \partial_{i\beta)} \,,$$

$$d = \sum_i [\tfrac{1}{2} \lambda_i^\alpha \partial_{i\alpha} + \tfrac{1}{2} \tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A \partial_{iB} + \tfrac{1}{4} \delta_B^A \eta_i^C \partial_{iC}] \,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha \eta_i^A \,, \qquad \bar{q}_A^{\dot{\alpha}} = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \partial_{iA} \,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha} \partial_{iA}, \qquad \bar{s}_{\dot{\alpha}}^A = \sum_i \eta_i^A \partial_{i\dot{\alpha}} \,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial \eta_i^A} \,.$$

- N.B: For collinear momenta $(\lambda_i \sim \lambda_{i+1})$ important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial \lambda^{\dot{\alpha}}} \frac{1}{\langle \lambda, \mu \rangle} = 2\pi \tilde{\mu}_{\dot{\alpha}} \, \delta^2(\langle \lambda, \mu \rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot\alpha}, \eta_i^A)$:

$$p^{\dot\alpha\alpha} = \sum_i \tilde{\lambda}_i^{\dot\alpha}\lambda_i^\alpha\,, \qquad k_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha}\partial_{i\dot\alpha}\,,$$

$$\overline{m}_{\dot\alpha\dot\beta} = \sum_i \tilde{\lambda}_{i(\dot\alpha}\partial_{i\dot\beta)}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)}\,,$$

$$d = \sum_i [\tfrac{1}{2}\lambda_i^\alpha\partial_{i\alpha} + \tfrac{1}{2}\tilde{\lambda}_i^{\dot\alpha}\partial_{i\dot\alpha} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A\partial_{iB} + \tfrac{1}{4}\delta^A_B\eta_i^C\partial_{iC}]\,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha\eta_i^A\,, \qquad \bar{q}^{\dot\alpha}_A = \sum_i \tilde{\lambda}_i^{\dot\alpha}\partial_{iA}\,,$$

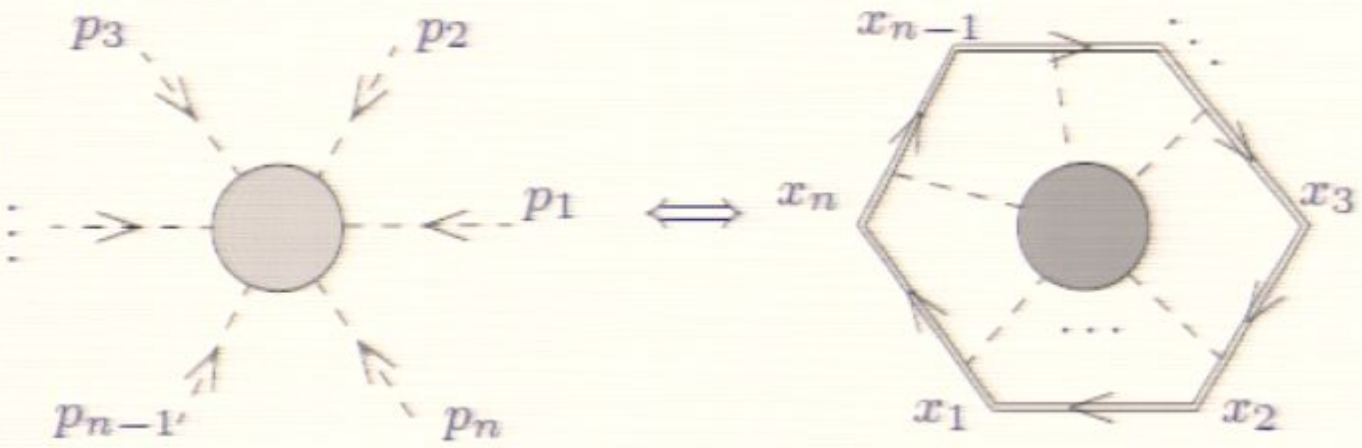
$$s_{\alpha A} = \sum_i \partial_{i\alpha}\partial_{iA}, \qquad \bar{s}^A_{\dot\alpha} = \sum_i \eta_i^A\partial_{i\dot\alpha}\,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial\lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial\eta_i^A}\,.$$

- N.B: For collinear momenta ($\lambda_i \sim \lambda_{i+1}$) important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial\tilde{\lambda}^{\dot\alpha}}\frac{1}{\langle\lambda,\mu\rangle} = 2\pi\tilde{\mu}_{\dot\alpha}\,\delta^2(\langle\lambda,\mu\rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

Wilson loop/scattering amplitude duality



$$x^\mu_{i+1} - x^\mu_i = p^\mu_i$$

- Trees are dual superconformal invariant: $(\theta_i - \theta_{i+1})^{\alpha A} = \lambda_i^\alpha \, \eta_i^A$

  [Drummond, Henn, Korchemsky, Sokatchev]

- Dual conformal generator: $K^\mu = \sum_i \left(2x_i^\mu \, x_i^\nu - x_i^2 \, \eta^{\mu\nu}\right) \frac{\partial}{\partial x_i^\nu} + \text{ferms}$

- Translate to original on-shell superspace: $x_i^{\alpha\dot\alpha} = \sum_{j=1}^{i-1} \lambda_j^\alpha \tilde\lambda_j^{\dot\alpha} + x_1^{\alpha\dot\alpha}$ etc.

$$\Rightarrow K^{\alpha\dot\alpha} = \sum_{i=1}^{n} x_i^{\alpha\dot\beta} \lambda_i^\alpha \frac{\partial}{\partial \lambda_i^\beta} + x_{i+1}^{\alpha\dot\beta} \tilde\lambda_i^{\dot\alpha} \frac{\partial}{\partial \tilde\lambda_i^{\dot\beta}} + \tilde\lambda_i^{\dot\alpha} \theta_{i+1}^{\alpha B} \frac{\partial}{\partial \eta_i^B} + x_1^{\alpha\dot\alpha}$$

Nonlocal structure!

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot{\alpha}}, \eta_i^A)$:

$$p^{\dot{\alpha}\alpha} = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \lambda_i^\alpha \,, \qquad k_{\alpha\dot{\alpha}} = \sum_i \partial_{i\alpha} \partial_{i\dot{\alpha}} \,,$$

$$\overline{m}_{\dot{\alpha}\dot{\beta}} = \sum_i \tilde{\lambda}_{i(\dot{\alpha}} \partial_{i\dot{\beta})}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha} \partial_{i\beta)} \,,$$

$$d = \sum_i [\tfrac{1}{2} \lambda_i^\alpha \partial_{i\alpha} + \tfrac{1}{2} \tilde{\lambda}_i^{\dot{\alpha}} \partial_{i\dot{\alpha}} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A \partial_{iB} + \tfrac{1}{4} \delta^A_B \eta_i^C \partial_{iC}] \,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha \eta_i^A \,, \qquad \bar{q}^{\dot{\alpha}}_A = \sum_i \tilde{\lambda}_i^{\dot{\alpha}} \partial_{iA} \,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha} \partial_{iA}, \qquad \bar{s}^A_{\dot{\alpha}} = \sum_i \eta_i^A \partial_{i\dot{\alpha}} \,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial \eta_i^A} \,.$$

- N.B: For collinear momenta $(\lambda_i \sim \lambda_{i+1})$ important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial \tilde{\lambda}^{\dot{\alpha}}} \frac{1}{\langle \lambda, \mu \rangle} = 2\pi \tilde{\mu}_{\dot{\alpha}} \, \delta^2(\langle \lambda, \mu \rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot\alpha}, \eta_i^A)$:

$$p^{\dot\alpha\alpha} = \sum_i \tilde{\lambda}_i^{\dot\alpha}\lambda_i^\alpha\,, \qquad k_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha}\partial_{i\dot\alpha}\,,$$

$$\overline{m}_{\dot\alpha\dot\beta} = \sum_i \tilde{\lambda}_{i(\dot\alpha}\partial_{i\dot\beta)}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)}\,,$$

$$d = \sum_i [\tfrac{1}{2}\lambda_i^\alpha\partial_{i\alpha} + \tfrac{1}{2}\tilde{\lambda}_i^{\dot\alpha}\partial_{i\dot\alpha} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A\partial_{iB} + \tfrac{1}{4}\delta^A_B\eta_i^C\partial_{iC}]\,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha\eta_i^A\,, \qquad \bar{q}^{\dot\alpha}_A = \sum_i \tilde{\lambda}_i^{\dot\alpha}\partial_{iA}\,,$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha}\partial_{iA}, \qquad \bar{s}^A_{\dot\alpha} = \sum_i \eta_i^A\partial_{i\dot\alpha}\,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial\lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial\eta_i^A}\,.$$

- N.B: For collinear momenta $(\lambda_i \sim \lambda_{i+1})$ important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial\tilde{\lambda}^{\dot\alpha}}\frac{1}{\langle\lambda,\mu\rangle} = 2\pi\tilde{\mu}_{\dot\alpha}\,\delta^2(\langle\lambda,\mu\rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

$\mathfrak{su}(2,2|4)$ invariance

- The $\mathfrak{su}(2,2|4)$ generators acting in on-shell superspace $(\lambda_i^\alpha, \tilde{\lambda}_i^{\dot\alpha}, \eta_i^A)$:

$$p^{\dot\alpha\alpha} = \sum_i \tilde{\lambda}_i^{\dot\alpha}\lambda_i^\alpha\,, \qquad k_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha}\partial_{i\dot\alpha}\,,$$

$$\overline{m}_{\dot\alpha\dot\beta} = \sum_i \tilde{\lambda}_{i(\dot\alpha}\partial_{i\dot\beta)}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)}\,,$$

$$d = \sum_i [\tfrac{1}{2}\lambda_i^\alpha\partial_{i\alpha} + \tfrac{1}{2}\tilde{\lambda}_i^{\dot\alpha}\partial_{i\dot\alpha} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A\partial_{iB} + \tfrac{1}{4}\delta^A_B\eta_i^C\partial_{iC}]\,,$$

$$q^{\alpha A} = \sum_i \lambda_i^\alpha\eta_i^A\,, \qquad \bar{q}^{\dot\alpha}_A = \sum_i \tilde{\lambda}_i^{\dot\alpha}\partial_{iA}\,,$$

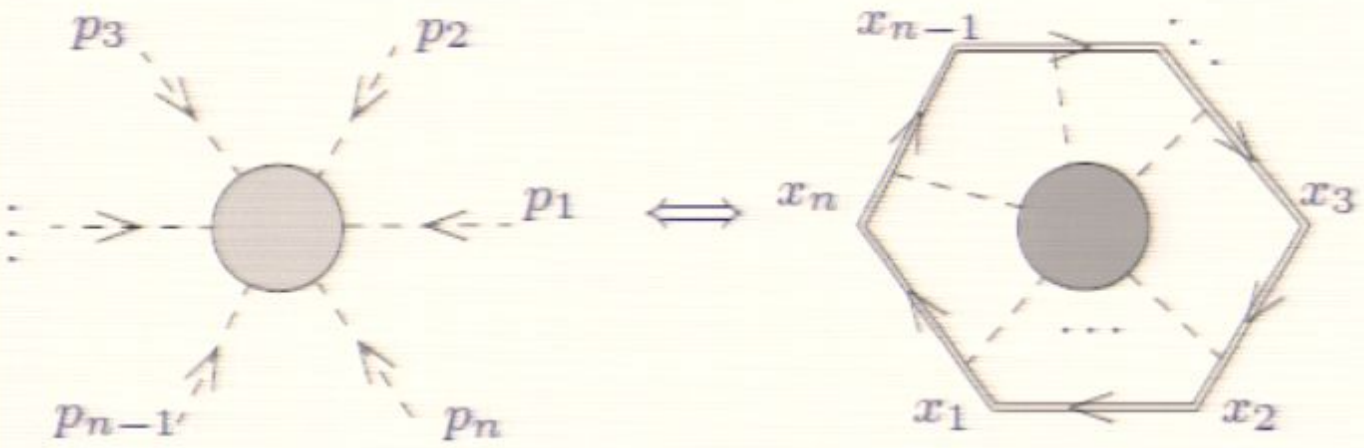
$$s_{\alpha A} = \sum_i \partial_{i\alpha}\partial_{iA}, \qquad \bar{s}^A_{\dot\alpha} = \sum_i \eta_i^A\partial_{i\dot\alpha}\,,$$

$$\partial_{i\alpha} := \frac{\partial}{\partial\lambda_i^\alpha} \qquad \partial_{iA} := \frac{\partial}{\partial\eta_i^A}\,.$$

- N.B: For collinear momenta ($\lambda_i \sim \lambda_{i+1}$) important additional length changing terms, due to holomorphic anomaly $\frac{\partial}{\partial\tilde{\lambda}^{\dot\alpha}}\frac{1}{\langle\lambda,\mu\rangle} = 2\pi\tilde{\mu}_{\dot\alpha}\,\delta^2(\langle\lambda,\mu\rangle)$

[Bargheer, Beisert, Galleas, Loebbert,McLoughlin] [Korchemsky, Sokatchev] [Skinner,Mason][Arkani-Hamed, Cachazo, Kaplan]

Wilson loop/scattering amplitude duality



$$x^{\mu}_{i+1} - x^{\mu}_{i} = p^{\mu}_{i}$$

- Trees are dual superconformal invariant: $(\theta_i - \theta_{i+1})^{\alpha A} = \lambda_i^{\alpha}\,\eta_i^{A}$

  [Drummond, Henn, Korchemsky, Sokatchev]

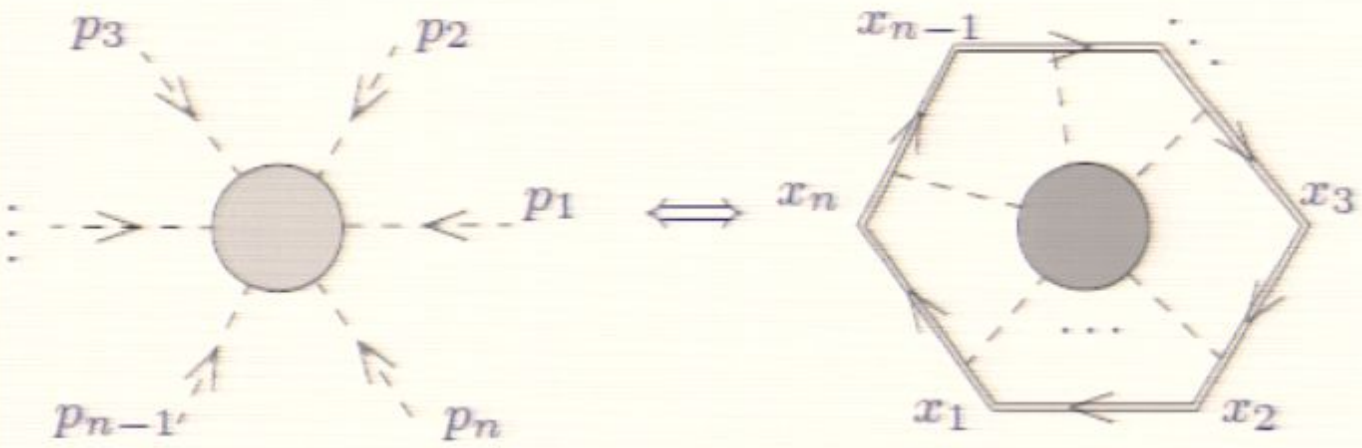
- Dual conformal generator: $K^{\mu} = \sum_i \left(2x_i^{\mu}\, x_i^{\nu} - x_i^2\,\eta^{\mu\nu}\right)\frac{\partial}{\partial x_i^{\nu}} + \text{ferms}$

- Translate to original on-shell superspace: $x_i^{\alpha\dot{\alpha}} = \sum_{j=1}^{i-1} \lambda_j^{\alpha}\tilde{\lambda}_j^{\dot{\alpha}} + x_1^{\alpha\dot{\alpha}}$ etc.

$$\Rightarrow\quad K^{\alpha\dot{\alpha}} = \sum_{i=1}^{n} x_i^{\alpha\dot{\beta}}\,\lambda_i^{\alpha}\frac{\partial}{\partial\lambda_i^{\beta}} + x_{i+1}^{\alpha\dot{\beta}}\tilde{\lambda}_i^{\dot{\alpha}}\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\beta}}} + \tilde{\lambda}_i^{\dot{\alpha}}\theta_{i+1}^{\alpha B}\frac{\partial}{\partial\eta_i^{B}} + x_1^{\alpha\dot{\alpha}}$$

Nonlocal structure!

Wilson loop/scattering amplitude duality



$$x^{\mu}_{i+1} - x^{\mu}_{i} = p^{\mu}_{i}$$

- Trees are dual superconformal invariant: $(\theta_i - \theta_{i+1})^{\alpha A} = \lambda_i^{\alpha}\,\eta_i^{A}$

  [Drummond, Henn, Korchemsky, Sokatchev]

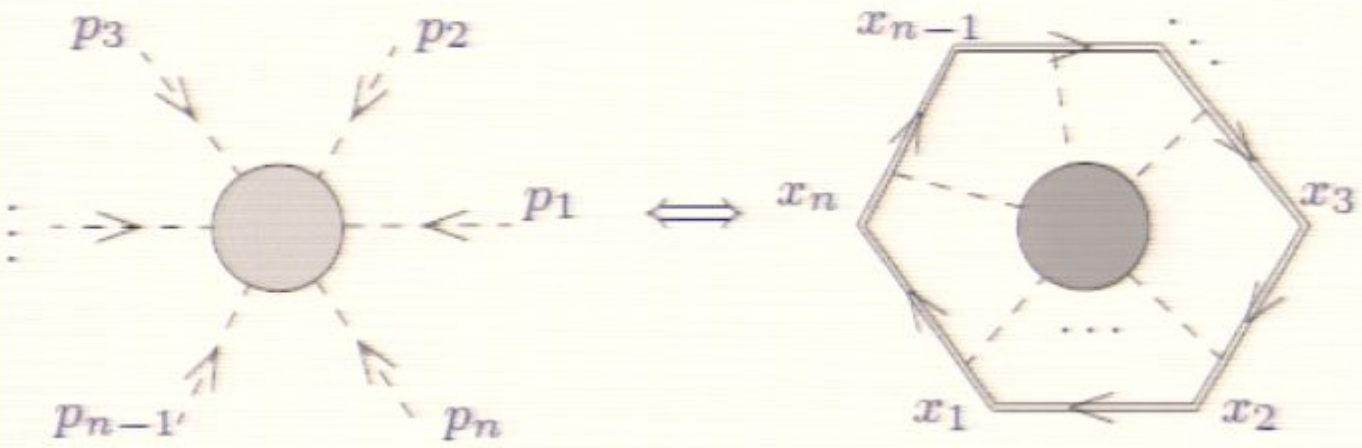
- Dual conformal generator: $K^{\mu} = \sum_i \left(2x_i^{\mu}\, x_i^{\nu} - x_i^2\,\eta^{\mu\nu}\right)\frac{\partial}{\partial x_i^{\nu}} + \text{ferms}$

- Translate to original on-shell superspace: $x_i^{\alpha\dot{\alpha}} = \sum_{j=1}^{i-1} \lambda_j^{\alpha}\tilde{\lambda}_j^{\dot{\alpha}} + x_1^{\alpha\dot{\alpha}}$ etc.

$$\Rightarrow K^{\alpha\dot{\alpha}} = \sum_{i=1}^{n} x_i^{\alpha\dot{\beta}}\,\lambda_i^{\alpha}\frac{\partial}{\partial\lambda_i^{\beta}} + x_{i+1}^{\alpha\dot{\beta}}\,\tilde{\lambda}_i^{\dot{\alpha}}\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\beta}}} + \tilde{\lambda}_i^{\dot{\alpha}}\,\theta_{i+1}^{\alpha B}\frac{\partial}{\partial\eta_i^{B}} + x_1^{\alpha\dot{\alpha}}$$

Nonlocal structure!

Wilson loop/scattering amplitude duality



$$x^\mu_{i+1} - x^\mu_i = p^\mu_i$$

- Trees are dual superconformal invariant: $(\theta_i - \theta_{i+1})^{\alpha A} = \lambda_i^\alpha \eta_i^A$

  [Drummond, Henn, Korchemsky, Sokatchev]

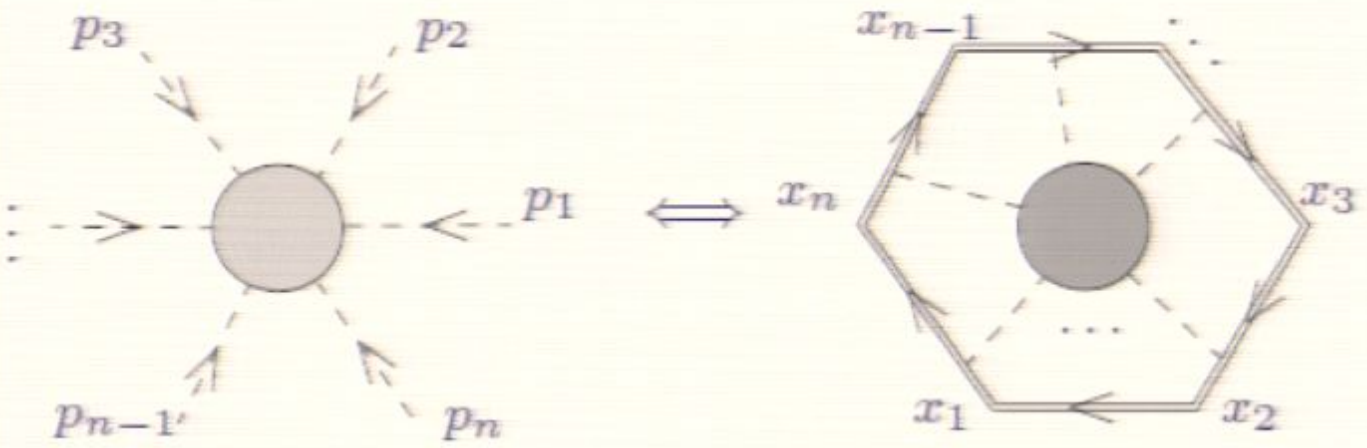
- Dual conformal generator: $K^\mu = \sum_i (2x_i^\mu x_i^\nu - x_i^2 \eta^{\mu\nu}) \frac{\partial}{\partial x_i^\nu} + \text{ferms}$

- Translate to original on-shell superspace: $x_i^{\alpha\dot\alpha} = \sum_{j=1}^{i-1} \lambda_j^\alpha \tilde\lambda_j^{\dot\alpha} + x_1^{\alpha\dot\alpha}$ etc.

$$\Rightarrow \quad K^{\alpha\dot\alpha} = \sum_{i=1}^{n} x_i^{\dot\alpha\beta} \lambda_i^\alpha \frac{\partial}{\partial \lambda_i^\beta} + x_{i+1}^{\alpha\dot\beta} \tilde\lambda_i^{\dot\alpha} \frac{\partial}{\partial \tilde\lambda_i^{\dot\beta}} + \tilde\lambda_i^{\dot\alpha} \theta_{i+1}^{\alpha B} \frac{\partial}{\partial \eta_i^B} + x_i^{\alpha\dot\alpha}$$

Nonlocal structure!

Wilson loop/scattering amplitude duality



$$x^{\mu}_{i+1} - x^{\mu}_{i} = p^{\mu}_{i}$$

- Trees are dual superconformal invariant: $(\theta_i - \theta_{i+1})^{\alpha A} = \lambda_i^{\alpha}\, \eta_i^{A}$

  [Drummond, Henn, Korchemsky, Sokatchev]

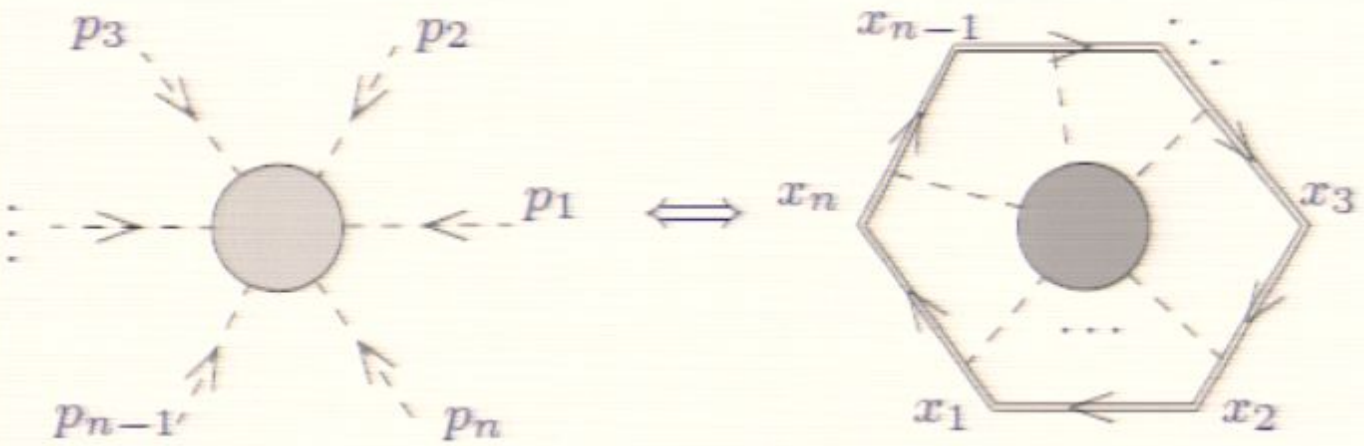
- Dual conformal generator: $K^{\mu} = \sum_i \left(2x_i^{\mu}\, x_i^{\nu} - x_i^2\, \eta^{\mu\nu}\right) \frac{\partial}{\partial x_i^{\nu}} + \text{ferms}$

- Translate to original on-shell superspace: $x_i^{\alpha\dot{\alpha}} = \sum_{j=1}^{i-1} \lambda_j^{\alpha}\, \tilde{\lambda}_j^{\dot{\alpha}} + x_1^{\alpha\dot{\alpha}}$ etc.

$$\Rightarrow \quad K^{\alpha\dot{\alpha}} = \sum_{i=1}^{n} x_i^{\dot{\alpha}\beta}\, \lambda_i^{\alpha} \frac{\partial}{\partial \lambda_i^{\beta}} + x_{i+1}^{\alpha\dot{\beta}}\, \tilde{\lambda}_i^{\dot{\alpha}} \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\beta}}} + \tilde{\lambda}_i^{\dot{\alpha}}\, \theta_{i+1}^{\alpha B} \frac{\partial}{\partial \eta_i^{B}} + x_i^{\alpha\dot{\alpha}}$$

Nonlocal structure!

Wilson loop/scattering amplitude duality



$$x_{i+1}^{\mu} - x_i^{\mu} = p_i^{\mu}$$

- Trees are dual superconformal invariant: $(\theta_i - \theta_{i+1})^{\alpha A} = \lambda_i^{\alpha}\,\eta_i^{A}$

  [Drummond, Henn, Korchemsky, Sokatchev]

- Dual conformal generator: $K^{\mu} = \sum_i \left(2x_i^{\mu}\, x_i^{\nu} - x_i^2\,\eta^{\mu\nu}\right)\frac{\partial}{\partial x_i^{\nu}} + \text{ferms}$

- Translate to original on-shell superspace: $x_i^{\alpha\dot{\alpha}} = \sum_{j=1}^{i-1} \lambda_j^{\alpha}\,\tilde{\lambda}_j^{\dot{\alpha}} + x_1^{\alpha\dot{\alpha}}$ etc.

$$\Rightarrow \quad K^{\alpha\dot{\alpha}} = \sum_{i=1}^{n} x_i^{\dot{\alpha}\beta}\,\lambda_i^{\alpha}\frac{\partial}{\partial\lambda_i^{\beta}} + x_{i+1}^{\alpha\dot{\beta}}\,\tilde{\lambda}_i^{\dot{\alpha}}\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\beta}}} + \tilde{\lambda}_i^{\dot{\alpha}}\,\theta_{i+1}^{\alpha B}\frac{\partial}{\partial\eta_i^{B}} + x_i^{\alpha\dot{\alpha}}$$

Nonlocal structure!

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators

$$\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar{l}_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar{q}_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators

$$\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i\, \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators

$$\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2} \sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators

$$\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)}\, J_{j,c}^{(0)}}$$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)}\, \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\, \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha} C\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i\, \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar{l}_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar{q}_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar{l}_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar{q}_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha} C\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

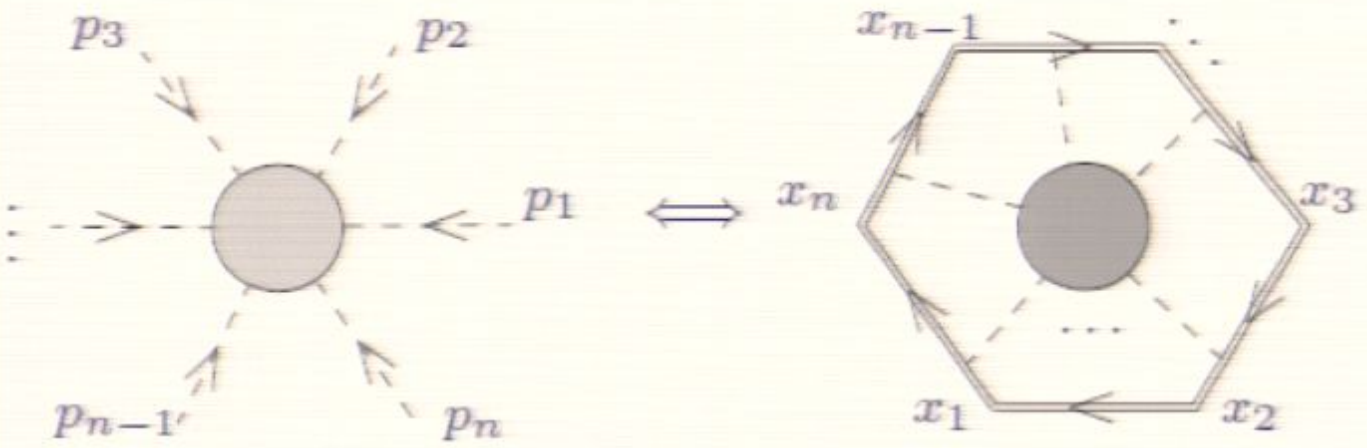
and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Wilson loop/scattering amplitude duality



$$x^\mu_{i+1} - x^\mu_i = p^\mu_i$$

- Trees are dual superconformal invariant: $(\theta_i - \theta_{i+1})^{\alpha A} = \lambda_i^\alpha \eta_i^A$

  [Drummond, Henn, Korchemsky, Sokatchev]

- Dual conformal generator: $K^\mu = \sum_i (2x_i^\mu x_i^\nu - x_i^2 \eta^{\mu\nu}) \frac{\partial}{\partial x_i^\nu} + \text{ferms}$

- Translate to original on-shell superspace: $x_i^{\alpha\dot\alpha} = \sum_{j=1}^{i-1} \lambda_j^\alpha \tilde\lambda_j^{\dot\alpha} + x_1^{\alpha\dot\alpha}$ etc.

$$\Rightarrow \quad K^{\alpha\dot\alpha} = \sum_{i=1}^{n} x_i^{\dot\alpha\beta} \lambda_i^\alpha \frac{\partial}{\partial \lambda_i^\beta} + x_{i+1}^{\alpha\dot\beta} \tilde\lambda_i^{\dot\alpha} \frac{\partial}{\partial \tilde\lambda_i^{\dot\beta}} + \tilde\lambda_i^{\dot\alpha} \theta_{i+1}^{\alpha B} \frac{\partial}{\partial \eta_i^B} + x_i^{\alpha\dot\alpha}$$

Nonlocal structure!

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i\, \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar{l}_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar{q}_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar{l}_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar{q}_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar{l}_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar{q}_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators

$$\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators

$$\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i\, \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i\, \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators

$$\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar{l}_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar{q}_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar{l}_{i,\dot\alpha}{}^{\dot\gamma}\delta_\alpha^\gamma - d_i\,\delta_\alpha^\gamma\delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar{q}_{i,\dot\alpha} C\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i\, \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- Superconformal + Dual superconformal algebra = Yangian $Y[\mathfrak{psu}(2,2|4)]$

[Drummond, Henn, JP]

$$[J_a^{(0)}, J_b^{(0)}\} = f_{ab}{}^c J_c^{(0)} \qquad \text{conventional superconformal symmetry}$$

$$[J_a^{(0)}, J_b^{(1)}\} = f_{ab}{}^c J_c^{(1)} \qquad \text{from dual conformal symmetry}$$

with nonlocal generators $\boxed{J_a^{(1)} = f^{cb}{}_a \sum_{1<j<i<n} J_{i,b}^{(0)} J_{j,c}^{(0)}}$

and super Serre relations (representation dependent). [Dolan,Nappi,Witten]

- In particular: Bosonic invariance $\boxed{p_{\alpha\dot\alpha}^{(1)} \mathbb{A}_n = 0}$ with

$$p_{\alpha\dot\alpha}^{(1)} = K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha}$$
$$= \frac{1}{2}\sum_{i<j} (l_{i,\alpha}{}^\gamma \delta_{\dot\alpha}^{\dot\gamma} + \bar l_{i,\dot\alpha}{}^{\dot\gamma} \delta_\alpha^\gamma - d_i \delta_\alpha^\gamma \delta_{\dot\alpha}^{\dot\gamma})\, p_{j,\gamma\dot\gamma} + \bar q_{i,\dot\alpha C}\, q_{j,\alpha}^C - (i \leftrightarrow j)$$

- In fact $J_a^{(0)}$ and $p_{\alpha\dot\alpha}^{(1)}$ alone generate all of $Y[\mathfrak{psu}(2,2|4)]$

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\,(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\,(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\,(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\,(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\,(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drummond, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A}=(\alpha,\dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}}=\begin{pmatrix} l^{\alpha}{}_{\beta}-\frac{1}{2}\delta^{\alpha}_{\beta}\,(d+\frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}}+\frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d-\frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B}-\frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}}:=\sum_{i<j}(-1)^{|\bar{C}|}(J_i^{\bar{A}}{}_{\bar{C}}\,J_j^{\bar{C}}{}_{\bar{B}}-J_j^{\bar{A}}{}_{\bar{C}}\,J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0 \qquad J^{(1)\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A}=(\alpha,\dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}}=\begin{pmatrix} l^{\alpha}{}_{\beta}-\frac{1}{2}\delta^{\alpha}_{\beta}\left(d+\frac{1}{2}c\right) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}}+\frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\left(d-\frac{1}{2}c\right) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B}-\frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}}:=\sum_{i<j}(-1)^{|\bar{C}|}\left(J_i^{\bar{A}}{}_{\bar{C}}\,J_j^{\bar{C}}{}_{\bar{B}}-J_j^{\bar{A}}{}_{\bar{C}}\,J_i^{\bar{C}}{}_{\bar{B}}\right)}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0 \qquad J^{(1)\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drummond, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}} J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}} J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for tree-level $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\left(d + \frac{1}{2}c\right) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\left(d - \frac{1}{2}c\right) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} \left(J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}}\right)}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}} J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}} J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\left(d + \frac{1}{2}c\right) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\left(d - \frac{1}{2}c\right) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} \left(J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}}\right)}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\,(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for tree-level $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drummond, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\left(d + \frac{1}{2}c\right) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\left(d - \frac{1}{2}c\right) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} \left(J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}}\right)}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\,(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Loops

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\,(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}\,(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drummond, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A}=(\alpha,\dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}}=\begin{pmatrix} l^{\alpha}{}_{\beta}-\frac{1}{2}\delta^{\alpha}_{\beta}\,(d+\frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}}+\frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d-\frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B}-\frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}}:=\sum_{i<j}(-1)^{|\bar{C}|}(J_i^{\bar{A}}{}_{\bar{C}}\,J_j^{\bar{C}}{}_{\bar{B}}-J_j^{\bar{A}}{}_{\bar{C}}\,J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for tree-level $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0 \qquad J^{(1)\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A}=(\alpha,\dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}}=\begin{pmatrix} l^{\alpha}{}_{\beta}-\frac{1}{2}\delta^{\alpha}_{\beta}\,(d+\frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}}+\frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\,(d-\frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B}-\frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}}:=\sum_{i<j}(-1)^{|\bar{C}|}(J_i^{\bar{A}}{}_{\bar{C}}\,J_j^{\bar{C}}{}_{\bar{B}}-J_j^{\bar{A}}{}_{\bar{C}}\,J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0 \qquad J^{(1)\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drummond, Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A}=(\alpha,\dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}}=\begin{pmatrix} l^{\alpha}{}_{\beta}-\frac{1}{2}\delta^{\alpha}_{\beta}\left(d+\frac{1}{2}c\right) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}}+\frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}\left(d-\frac{1}{2}c\right) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B}-\frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}}:=\sum_{i<j}(-1)^{|\bar{C}|}\left(J_i^{\bar{A}}{}_{\bar{C}}\,J_j^{\bar{C}}{}_{\bar{B}}-J_j^{\bar{A}}{}_{\bar{C}}\,J_i^{\bar{C}}{}_{\bar{B}}\right)}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0 \qquad J^{(1)\bar{A}}{}_{\bar{B}}\circ\mathbb{A}_n=0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin, Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld; Beisert, Galleas, Loebbert, McLoughlin; Korchemsky, Sokatchev]

Yangian symmetry of scattering amplitudes in $\mathcal{N}=4$ SYM

- In supermatrix notation: $\bar{A} = (\alpha, \dot{\alpha}|A)$

$$J^{\bar{A}}{}_{\bar{B}} = \begin{pmatrix} l^{\alpha}{}_{\beta} - \frac{1}{2}\delta^{\alpha}_{\beta}(d + \frac{1}{2}c) & k^{\alpha}{}_{\dot{\beta}} & s^{\alpha}{}_{B} \\ p^{\dot{\alpha}}{}_{\beta} & \bar{l}^{\dot{\alpha}}{}_{\dot{\beta}} + \frac{1}{2}\delta^{\dot{\alpha}}_{\dot{\beta}}(d - \frac{1}{2}c) & \bar{q}^{\dot{\alpha}}{}_{B} \\ q^{A}{}_{\beta} & \bar{s}^{A}{}_{\dot{\beta}} & -r^{A}{}_{B} - \frac{1}{4}\delta^{A}_{B}\,c \end{pmatrix}$$

$$\Rightarrow \boxed{J^{(1)\,\bar{A}}{}_{\bar{B}} := \sum_{i<j} (-1)^{|\bar{C}|} (J_i^{\bar{A}}{}_{\bar{C}}\, J_j^{\bar{C}}{}_{\bar{B}} - J_j^{\bar{A}}{}_{\bar{C}}\, J_i^{\bar{C}}{}_{\bar{B}})}$$

- Implies an infinite-dimensional symmetry algebra for **tree-level** $\mathcal{N}=4$ SYM scattering amplitudes! $\Leftrightarrow$ spin chain picture

$$\boxed{J^{\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0 \qquad J^{(1)\bar{A}}{}_{\bar{B}} \circ \mathbb{A}_n = 0}$$

- Including correction terms arising from collinear momenta this symmetry is constructive: Unambiguously fixes tree-level amplitudes.

[Drinfeld, Beisert, Galleas, Loebbert, McLoughlin, Korchemsky, Sokatchev]

Loops

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]
  Best way to understand dual conformal symmetry in the field theory:
  ⇒ Inspired by AdS/CFT [Alday, Maldacena, Schabinger, 2008; Sever, McGreevy]
  ⇒ IR divergences regulated by masses, at least for large N
  ⇒ Conjecture: Existence of an extended dual conformal symmetry
  [Alday, Henn, JP, Schuster]
  ⇒ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin, Boels, Bern, Dennen, Huang]
  ⇒ Now essentially proven through 6D SYM [Caron-Huot, O'Connell, Dennen, Huang, 2010]
  ⇒ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry
  [Arkani-Hamed et al., 2010]
  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]
  Best way to understand dual conformal symmetry in the field theory:
  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena, Schubinger, 2008; Sever, McGreevy]
  $\Rightarrow$ IR divergences regulated by masses, at least for large N
  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry
  [Alday, Henn, JP, Schuster]
  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin, Roels, Bern, Dennen, Huang]
  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, O'Connell, Dennen, Huang, 2010]
  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry
  [Arkani-Hamed et al, 2010]
  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al. 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higher loops and Higgs regulator

- Beyond tree-level: Need of regularization (IR divergences) a priori breaks conformal and dual conformal symmetry
- Standard regularization method Dim reduction $10 \to 4 - \epsilon$
- Alternative method: Massive or Higgs regulator [Alday, Henn, JP, Schuster]

  Best way to understand dual conformal symmetry in the field theory:

  $\Rightarrow$ Inspired by AdS/CFT [Alday, Maldacena; Schabinger, 2008; Sever, McGreevy]

  $\Rightarrow$ IR divergences regulated by masses, at least for large N

  $\Rightarrow$ Conjecture: Existence of an extended dual conformal symmetry

  [Alday, Henn, JP, Schuster]

  $\Rightarrow$ Lots of supporting evidence [Naculich, Henn, Schnitzer, Spradlin; Boels, Bern, Dennen, Huang]

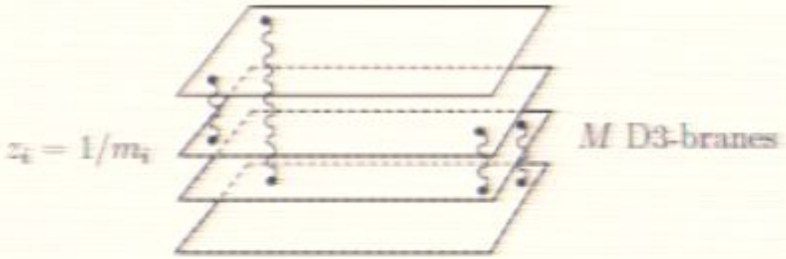
  $\Rightarrow$ Now essentially proven through 6D SYM [Caron-Huot, OConnel; Dennen, Huang, 2010]

  $\Rightarrow$ Restricts the loop integrand/integrals!
- Related development: (Unregulated) planar integrand has Yangian symmetry

  [Arkani-Hamed et al, 2010]

  Higgs regulator and its extended dual conformal symmetry is used to justify transition to regulated integrand

Higgs regularization [Alday, Henn, JP, Schuster]



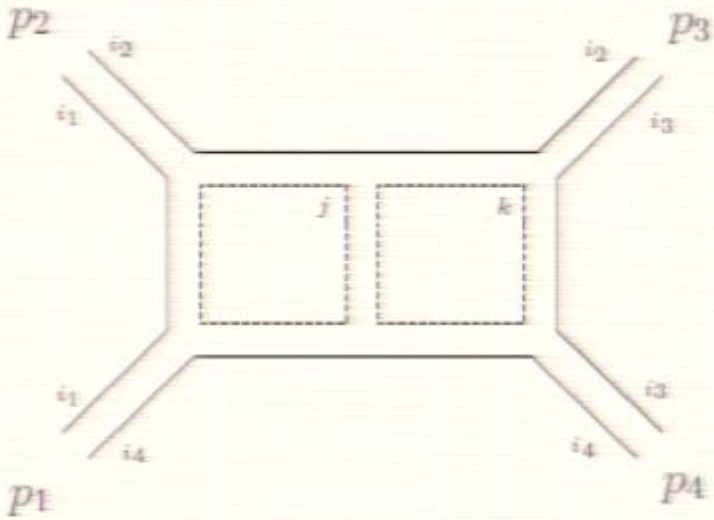
- Take string picture serious:



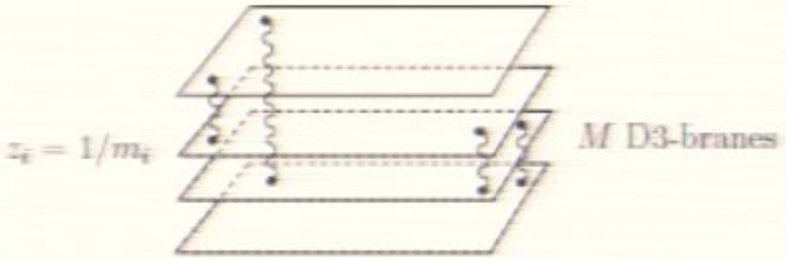
(a)

- Field Theory: Higgsing $U(N+M) \to U(N) \times U(1)^M$. One brane for every scattered particle, $N \gg M$.

Renders amplitudes IR finite. Have light $(m_i - m_j)$ and heavy m_i fields



Higgs regularization [Alday, Henn, JP, Schuster]



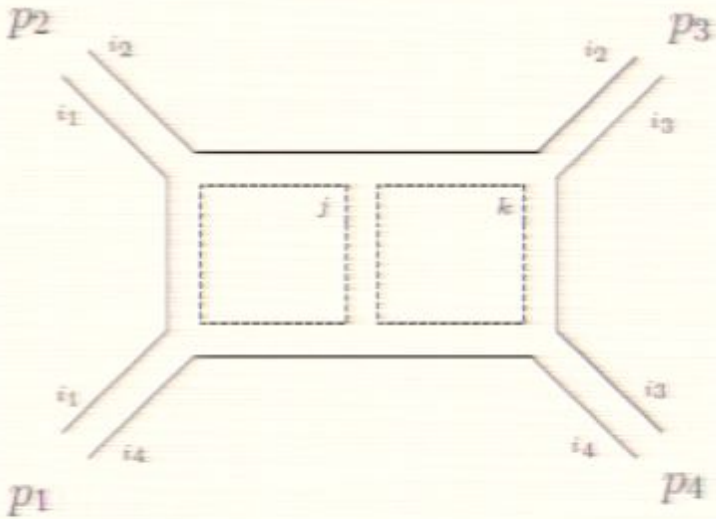
- Take string picture serious:



(a)

- Field Theory: Higgsing $U(N+M) \to U(N) \times U(1)^M$. One brane for every scattered particle, $N \gg M$.

Renders amplitudes IR finite. Have light $(m_i - m_j)$ and heavy m_i fields



Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

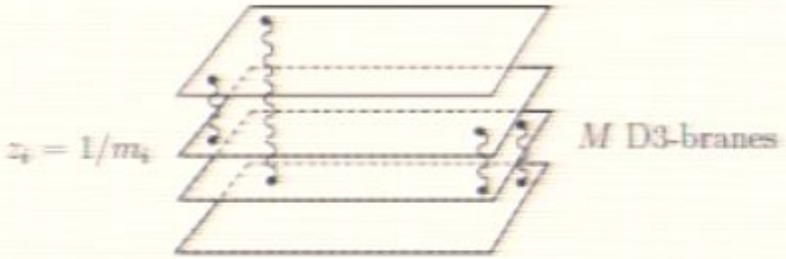
$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

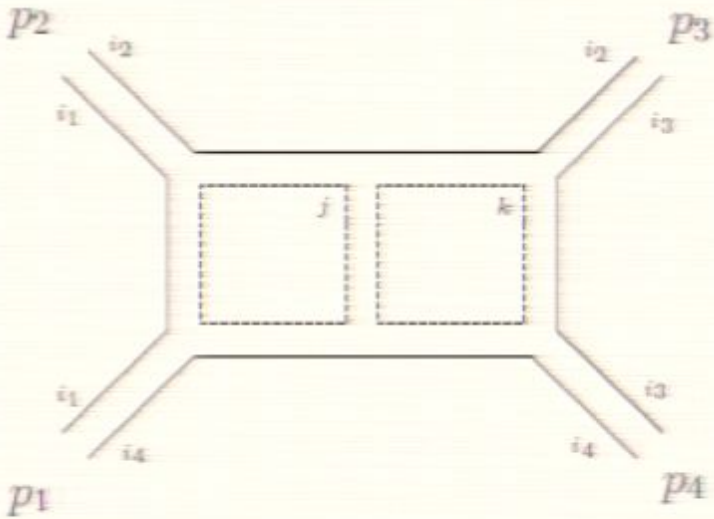
Higgs regularization [Alday, Henn, JP, Schuster]



(a)

- Take string picture serious:
- Field Theory: Higgsing $U(N+M) \to U(N) \times U(1)^M$. One brane for every scattered particle, $N \gg M$.

Renders amplitudes IR finite. Have light $(m_i - m_j)$ and heavy m_i fields



Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Extended dual conformal symmetry: The string picture

- Consider the string description of the IR-regulated amplitude in the T-dual theory: The radial coordinates are related by

$$\boxed{1/z = r = m}$$

- The $SO(2,4)$ isometry of AdS_5 in T-dual theory is generated by J_{MN} with embedding coordinates $M = -1, 0, 1, 2, 3, 4$.
  In Poincaré coordinates (r, x^μ) we have

$$J_{-1,4} = r\partial_r + x^\mu \partial_\mu = \hat{D}$$
$$J_{4,\mu} - J_{-1,\mu} = \partial_\mu = \hat{P}_\mu$$
$$J_{4,\mu} + J_{-1,\mu} = 2x_\mu(x_\nu \partial^\nu + r\partial_r) - (x^2 + r^2)\partial_\mu = \hat{K}_\mu$$

- Expectation: Amplitudes regulated by Higgsing should be invariant exactly under extended dual conformal symmetry $\hat{K}_\mu$ and $\hat{D}$ with $r \to m$!

Higgsing $\mathcal{N} = 4$ Super Yang-Mills

- Action

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x \, \mathrm{Tr}\Big(-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}(D_\mu \hat{\Phi}_I)^2 + \frac{g^2}{4}[\hat{\Phi}_I, \hat{\Phi}_J]^2 + \text{ferms} \Big) ,$$

- Decompose into $N + M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix} , \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\frac{m_i}{g}\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

$$a, b = 1, \ldots, N , \quad i, j = N+1, \ldots, N+M ,$$

- Quadratic terms ($A_M := (A_\mu, \Phi_I)$)

$$S_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i - m_j)^2 (A_M)_{ij}(A^M)_{ji} - m_i^2 (A_M)_{ia}(A^M)_{ai} + \text{ferms} \Big\}$$

- Plus novel bosonic 3-point interactions proportional to m_i

Higgsing $\mathcal{N} = 4$ Super Yang-Mills

- Action

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x \, \mathrm{Tr}\Big(-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}(D_\mu \hat{\Phi}_I)^2 + \frac{g^2}{4}[\hat{\Phi}_I, \hat{\Phi}_J]^2 + \text{ferms} \Big) ,$$

- Decompose into $N + M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix} , \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\frac{m_i}{g}\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

$$a, b = 1, \ldots, N , \quad i, j = N+1, \ldots, N+M ,$$

- Quadratic terms ($A_M := (A_\mu, \Phi_I)$)

$$S_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i - m_j)^2 (A_M)_{ij}(A^M)_{ji} - m_i^2 (A_M)_{ia}(A^M)_{ai} + \text{ferms} \Big\}$$

- Plus novel bosonic 3-point interactions proportional to m_i

Higgsing $\mathcal{N} = 4$ Super Yang-Mills

- Action

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x \, \mathrm{Tr}\Big(-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}(D_\mu \hat{\Phi}_I)^2 + \frac{g^2}{4}[\hat{\Phi}_I, \hat{\Phi}_J]^2 + \text{ferms} \Big) ,$$

- Decompose into $N + M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix} , \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\frac{m_i}{g}\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

$$a, b = 1, \ldots, N , \quad i, j = N+1, \ldots, N+M ,$$

- Quadratic terms ($A_M := (A_\mu, \Phi_I)$)

$$\hat{S}_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i - m_j)^2 (A_M)_{ij} (A^M)_{ji} - m_i^2 (A_M)_{ia} (A^M)_{ai} + \text{ferms} \Big\}$$

- Plus novel bosonic 3-point interactions proportional to m_i

Higgsing $\mathcal{N} = 4$ Super Yang-Mills

- Action

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x \,\mathrm{Tr}\Big(-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}(D_\mu\hat{\Phi}_I)^2 + \frac{g^2}{4}[\hat{\Phi}_I,\hat{\Phi}_J]^2 + \text{ferms}\Big)\,,$$

- Decompose into $N+M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix}\,, \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\,\frac{m_i}{g}\,\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

$$a,b = 1,\ldots,N\,, \quad i,j = N+1,\ldots,N+M\,,$$

- Quadratic terms $(A_M := (A_\mu, \Phi_I))$

$$\hat{S}_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i - m_j)^2\,(A_M)_{ij}\,(A^M)_{ji} - m_i^2\,(A_M)_{ia}\,(A^M)_{ai} + \text{ferms}\Big\}$$

- Plus novel bosonic 3-point interactions proportional to m_i

Higgsing $\mathcal{N}=4$ Super Yang-Mills

- Action

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x \,\mathrm{Tr}\Big(-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}(D_\mu\hat{\Phi}_I)^2 + \frac{g^2}{4}[\hat{\Phi}_I,\hat{\Phi}_J]^2 + \text{ferms}\Big)\,,$$

- Decompose into $N+M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix}\,, \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\,\frac{m_i}{g}\,\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

$$a,b = 1,\ldots,N\,, \quad i,j = N+1,\ldots,N+M\,,$$

- Quadratic terms $(A_M := (A_\mu,\Phi_I))$

$$\hat{S}_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i-m_j)^2\,(A_M)_{ij}\,(A^M)_{ji} - m_i^2\,(A_M)_{ia}\,(A^M)_{ai} + \text{ferms}\Big\}$$

- Plus novel bosonic 3-point interactions proportional to m_i

Higgsing $\mathcal{N}=4$ Super Yang-Mills

- Action

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x \,\mathrm{Tr}\Big(-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}(D_\mu\hat{\Phi}_I)^2 + \frac{g^2}{4}[\hat{\Phi}_I,\hat{\Phi}_J]^2 + \text{ferms}\Big)\,,$$

- Decompose into $N+M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix}\,, \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\,\frac{m_i}{g}\,\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

$$a,b = 1,\ldots,N\,, \quad i,j = N+1,\ldots,N+M\,,$$

- Quadratic terms $(A_M := (A_\mu,\Phi_I))$

$$\hat{S}_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i-m_j)^2\,(A_M)_{ij}\,(A^M)_{ji} - m_i^2\,(A_M)_{ia}\,(A^M)_{ai} + \text{ferms}\Big\}$$

- Plus novel bosonic 3-point interactions proportional to m_i

Higgsing $\mathcal{N}=4$ Super Yang-Mills

- Action

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x \,\mathrm{Tr}\Big(-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}(D_\mu\hat{\Phi}_I)^2 + \frac{g^2}{4}[\hat{\Phi}_I,\hat{\Phi}_J]^2 + \text{ferms}\Big)\,,$$

- Decompose into $N+M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix}\,, \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\,\frac{m_i}{g}\,\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

$$a,b = 1,\ldots,N\,, \quad i,j = N+1,\ldots,N+M\,,$$

- Quadratic terms $(A_M := (A_\mu,\Phi_I))$

$$\hat{S}_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i-m_j)^2\,(A_M)_{ij}\,(A^M)_{ji} - m_i^2\,(A_M)_{ia}\,(A^M)_{ai} + \text{ferms}\Big\}$$

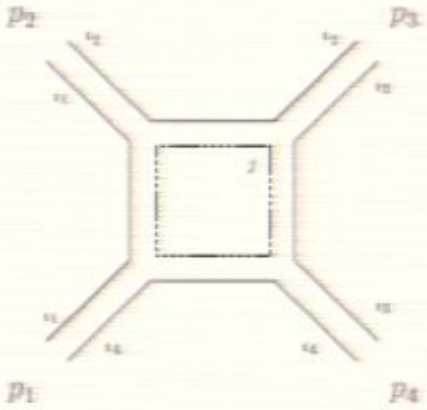
- Plus novel bosonic 3-point interactions proportional to m_i

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

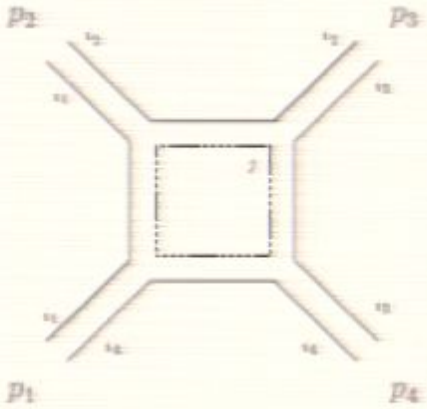
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

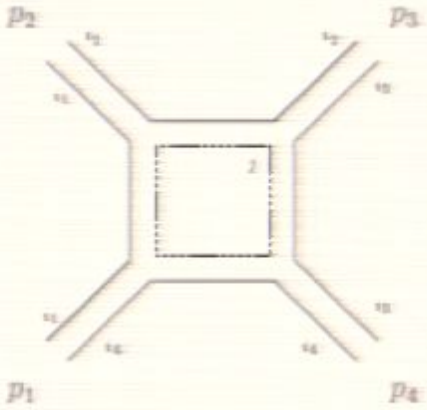
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

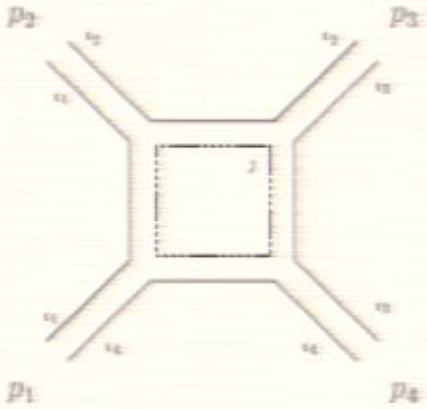
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

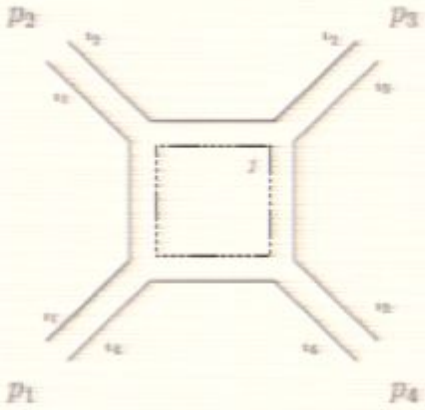
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

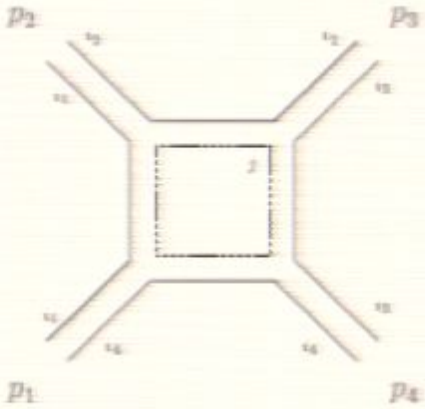
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

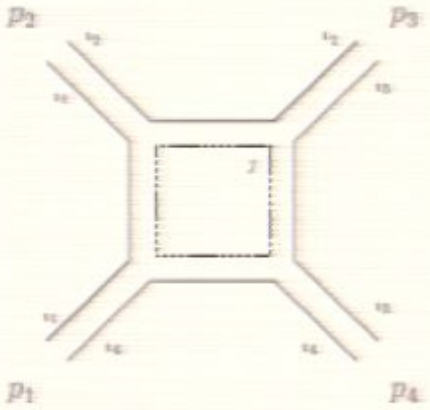
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

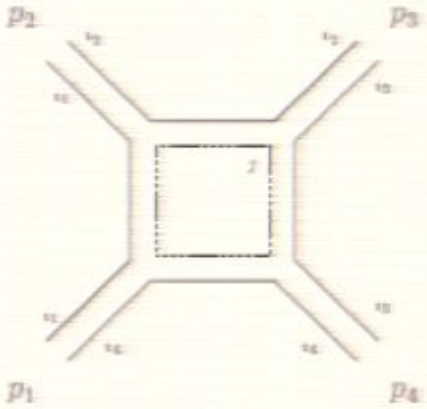
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

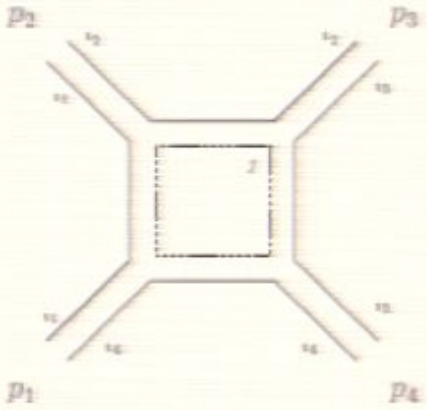
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

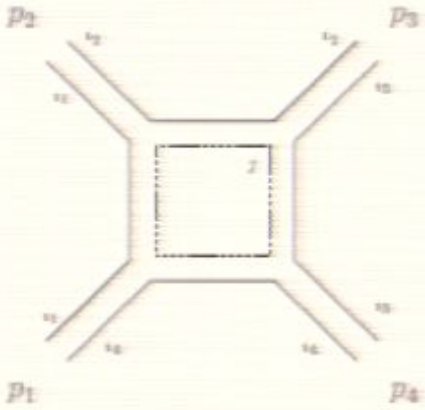
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

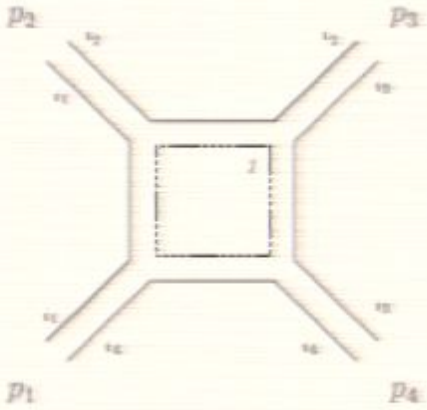
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g^2_{\rm YM}\left(1+\lambda\, I^{(1)}(s,t,m_i)+O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2+(m_1-m_3)^2)(x_{24}^2+(m_2-m_4)^2)}{(x_{15}^2+m_1^2)(x_{25}^2+m_2^2)(x_{35}^2+m_3^2)(x_{45}^2+m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1\ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2\hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2\hat{x}_{25}^2\hat{x}_{35}^2\hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

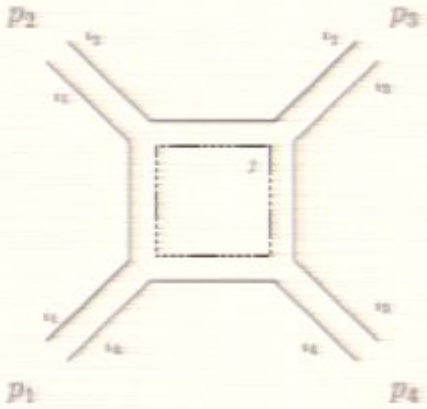
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

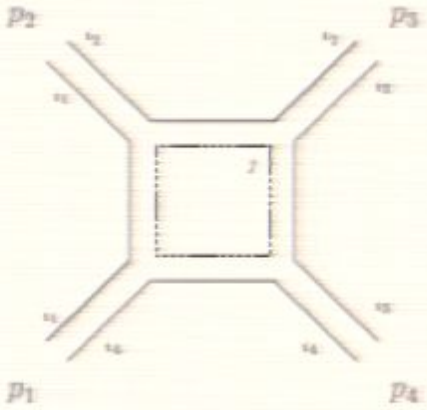
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

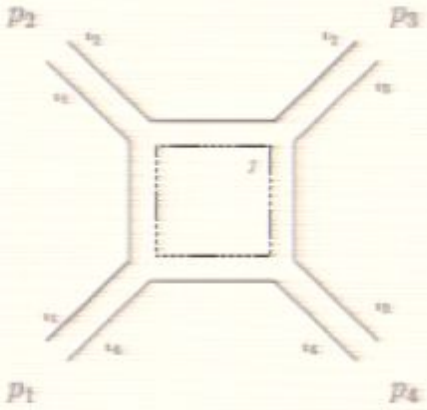
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

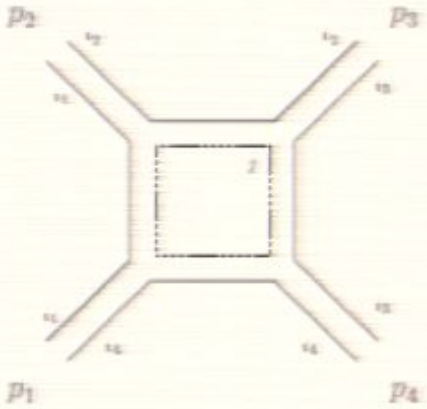
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

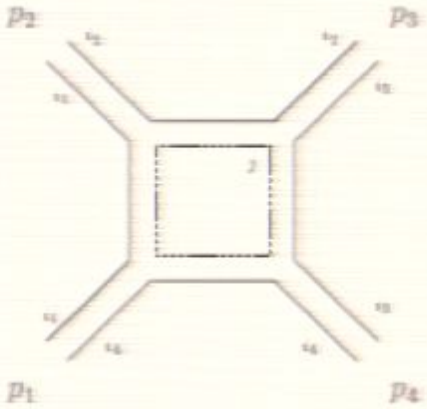
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

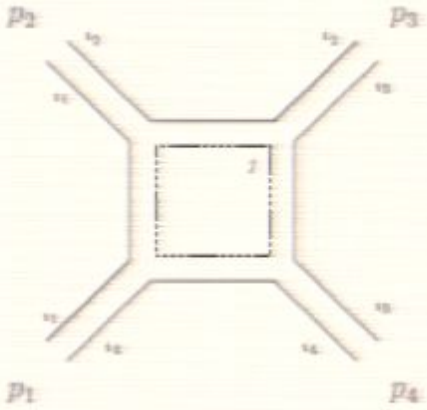
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

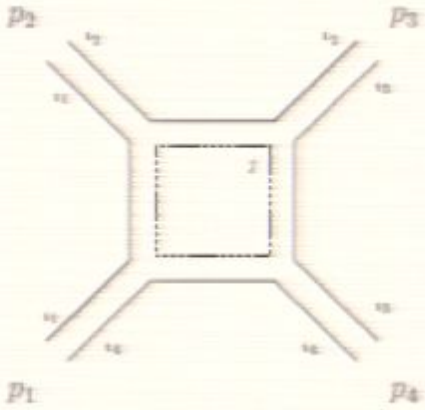
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g^2_{\rm YM}\left(1+\lambda\, I^{(1)}(s,t,m_i)+O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2+(m_1-m_3)^2)(x_{24}^2+(m_2-m_4)^2)}{(x_{15}^2+m_1^2)(x_{25}^2+m_2^2)(x_{35}^2+m_3^2)(x_{45}^2+m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1\ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2\hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2\hat{x}_{25}^2\hat{x}_{35}^2\hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

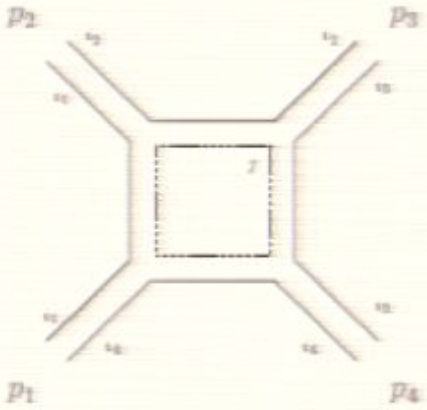
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

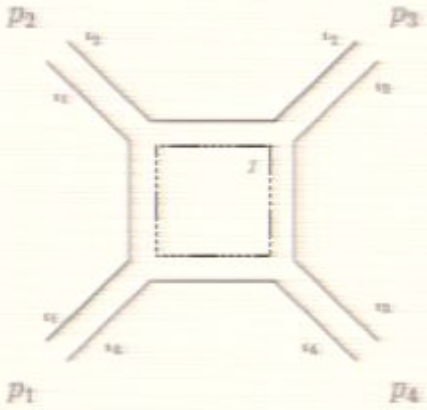
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

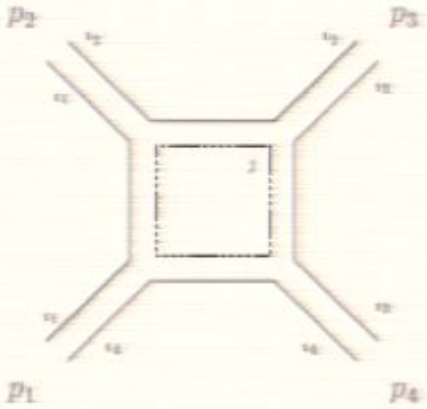
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

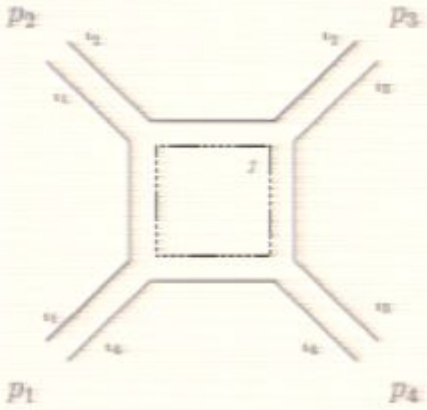
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

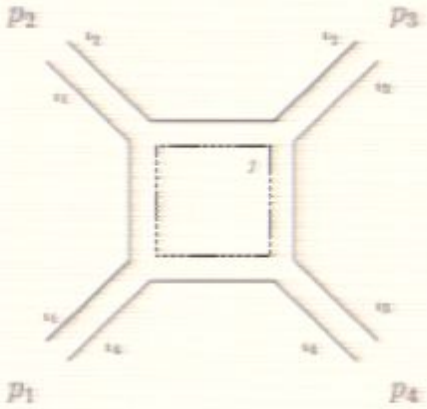
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

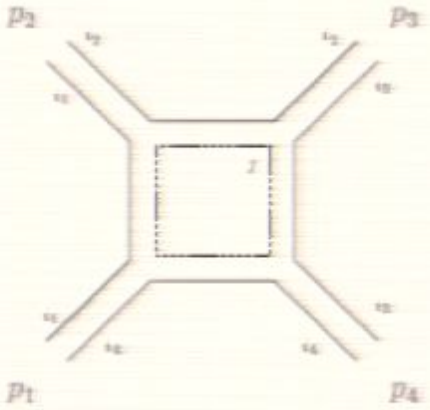
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

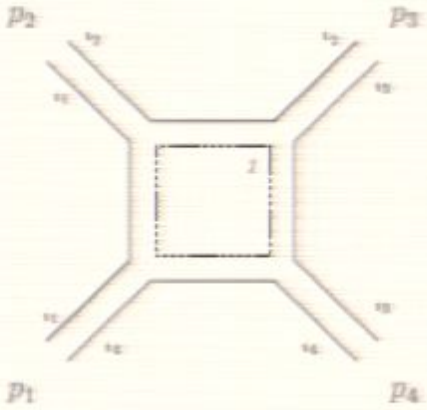
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g^2_{\rm YM}\left(1+\lambda\, I^{(1)}(s,t,m_i)+O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2+(m_1-m_3)^2)(x_{24}^2+(m_2-m_4)^2)}{(x_{15}^2+m_1^2)(x_{25}^2+m_2^2)(x_{35}^2+m_3^2)(x_{45}^2+m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1\ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2\hat{x}_{24}^2\int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2\hat{x}_{25}^2\hat{x}_{35}^2\hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

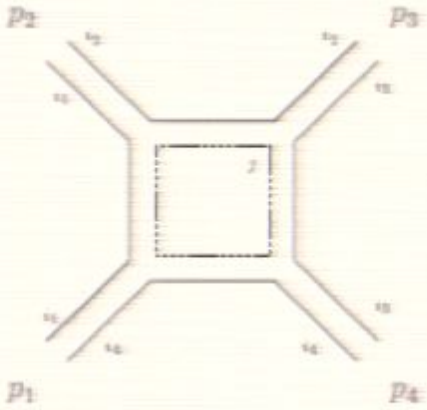
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

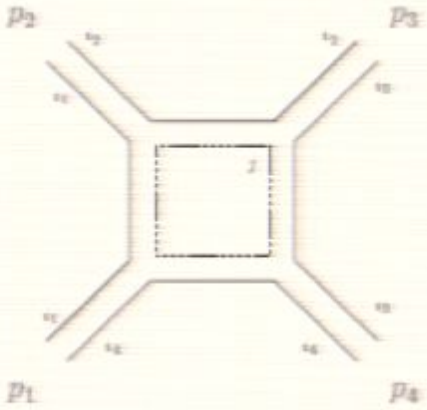
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

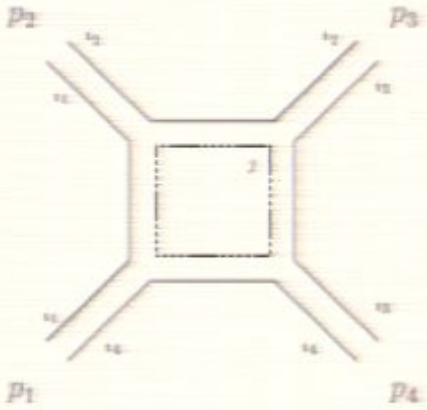
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g^2_{\rm YM}\left(1+\lambda\, I^{(1)}(s,t,m_i)+O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2+(m_1-m_3)^2)(x_{24}^2+(m_2-m_4)^2)}{(x_{15}^2+m_1^2)(x_{25}^2+m_2^2)(x_{35}^2+m_3^2)(x_{45}^2+m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1\ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2\hat{x}_{24}^2\int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2\hat{x}_{25}^2\hat{x}_{35}^2\hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

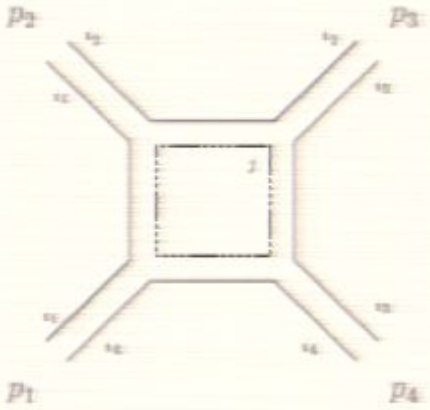
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

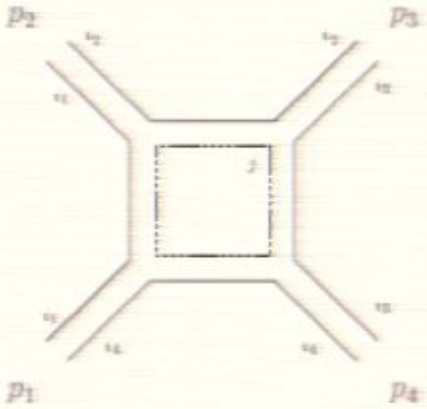
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

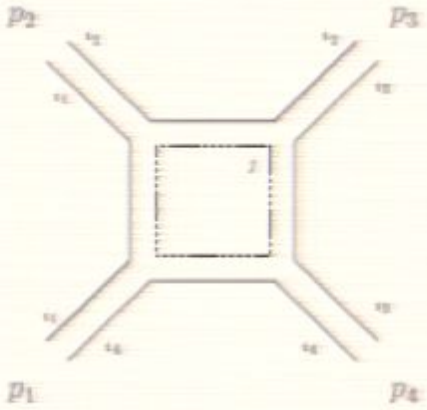
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

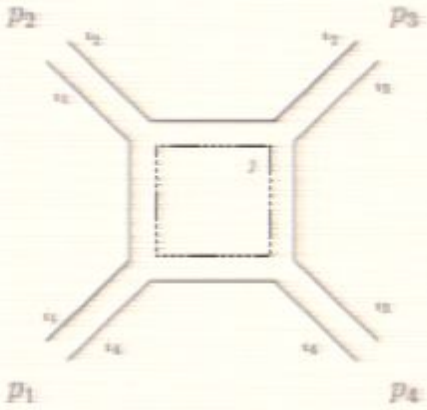
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

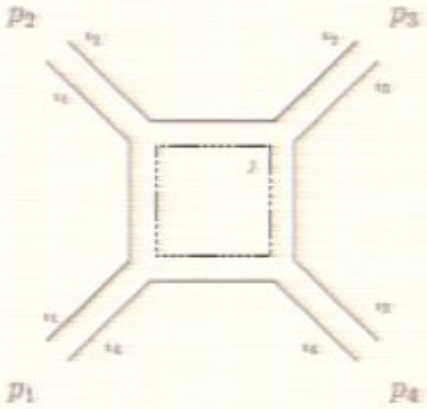
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

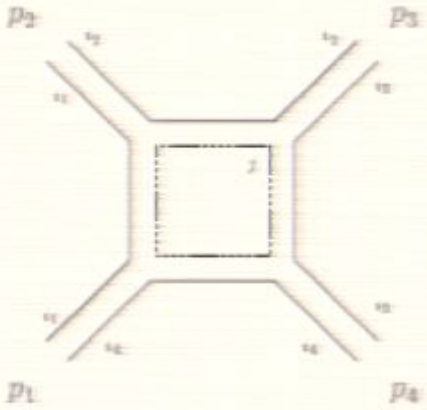
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

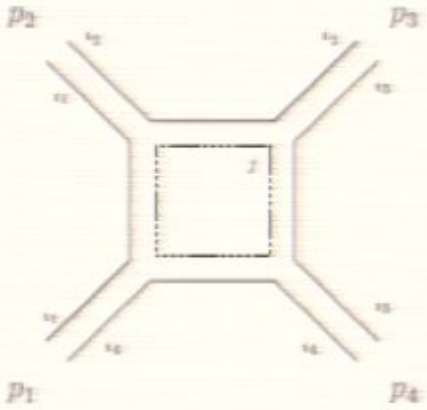
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g^2_{\rm YM}\left(1+\lambda\, I^{(1)}(s,t,m_i)+O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x^2_{13}+(m_1-m_3)^2)(x^2_{24}+(m_2-m_4)^2)}{(x^2_{15}+m_1^2)(x^2_{25}+m_2^2)(x^2_{35}+m_3^2)(x^2_{45}+m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}^\mu_i := x^\mu_i\,, \quad \hat{x}^4_i := m_i\,,$ $\quad i = 1\ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}^2_{13}\hat{x}^2_{24}\int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}^2_{15}\hat{x}^2_{25}\hat{x}^2_{35}\hat{x}^2_{45}}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

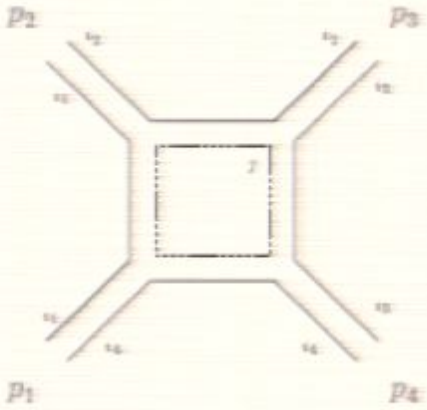
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

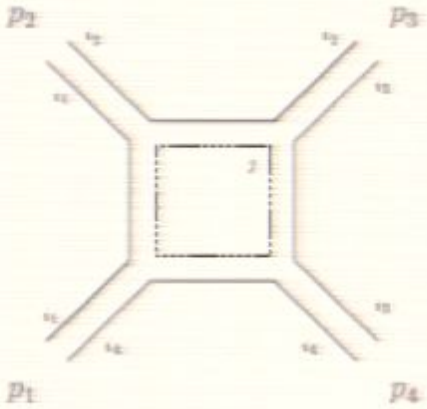
Pirsa: 11080041

One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\,\Phi_5(p_2)\,\Phi_4(p_3)\,\Phi_5(p_4)\rangle = i g^2_{\rm YM}\left(1+\lambda\, I^{(1)}(s,t,m_i)+O(\lambda^2)\right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2+(m_1-m_3)^2)(x_{24}^2+(m_2-m_4)^2)}{(x_{15}^2+m_1^2)(x_{25}^2+m_2^2)(x_{35}^2+m_3^2)(x_{45}^2+m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i=1\ldots4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2\hat{x}_{24}^2\int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2\hat{x}_{25}^2\hat{x}_{35}^2\hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

Pirsa: 11080041

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) :=$$
$$\sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right]I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) :=$$
$$\sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right]I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) :=$$
$$\sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right]I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) :=$$
$$\sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right]I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Symmetries of massively regulated loop amplitudes

[ongoing work]

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

Symmetries of massively regulated loop amplitudes

[ongoing work]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi'''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4$ $a = 1,2$ $\dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab}$ $p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a$ & $\tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi'''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4$ $a = 1,2$ $\dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab}$ $p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a$ & $\tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g_a^{\dot{a}}\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi'''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi'''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g_a^{\dot{a}}\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \ \& \ \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \ \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \ \& \ \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi'''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4$ $a = 1,2$ $\dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab}$ $p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a$ & $\tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g_a^{\dot{a}}\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g_a^{\dot{a}}\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g_a^{\dot{a}}\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g_a^{\dot{a}}\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i (2x_i^\mu x_i^\nu - x_i^2 \eta^{\mu\nu}) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB} x_i^{BC} \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu) \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi'''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

Higgsing $\mathcal{N}=4$ Super Yang-Mills

- Action

$$\hat{S}^{U(N+M)}_{\mathcal{N}=4} = \int d^4x \,\mathrm{Tr}\Big(-\frac{1}{4}\hat{F}^2_{\mu\nu} - \frac{1}{2}(D_\mu\hat{\Phi}_I)^2 + \frac{g^2}{4}[\hat{\Phi}_I,\hat{\Phi}_J]^2 + \text{ferms}\Big)\,,$$

- Decompose into $N+M$ blocks

$$\hat{A}_\mu = \begin{pmatrix} (A_\mu)_{ab} & (A_\mu)_{aj} \\ (A_\mu)_{ia} & (A_\mu)_{ij} \end{pmatrix}\,, \qquad \hat{\Phi}_I = \begin{pmatrix} (\Phi_I)_{ab} & (\Phi_I)_{aj} \\ (\Phi_I)_{ia} & \delta_{I9}\,\frac{m_i}{g}\,\delta_{ij} + (\Phi_I)_{ij} \end{pmatrix}$$

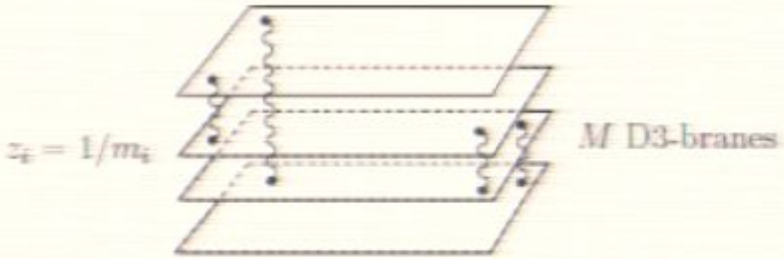
$$a,b = 1,\ldots,N\,, \quad i,j = N+1,\ldots,N+M\,,$$

- Quadratic terms $(A_M := (A_\mu,\Phi_I))$

$$\hat{S}_{\mathcal{N}=4}\Big|_{\text{quad}} = \int d^4x \Big\{ -\tfrac{1}{2}\mathrm{Tr}(\partial_\mu A_M)^2 - \tfrac{1}{2}(m_i-m_j)^2\,(A_M)_{ij}\,(A^M)_{ji} - m_i^2\,(A_M)_{ia}\,(A^M)_{ai} + \text{ferms}\Big\}$$

- Plus novel bosonic 3-point interactions proportional to m_i

Higgs regularization [Alday, Henn, JP, Schuster]



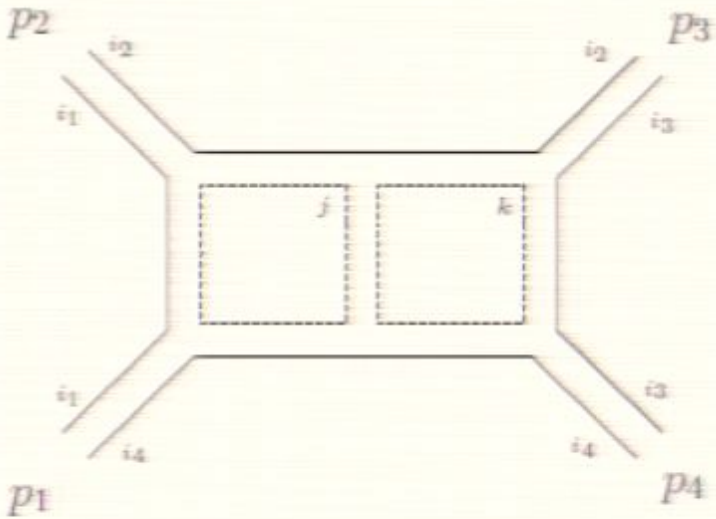
- Take string picture serious:



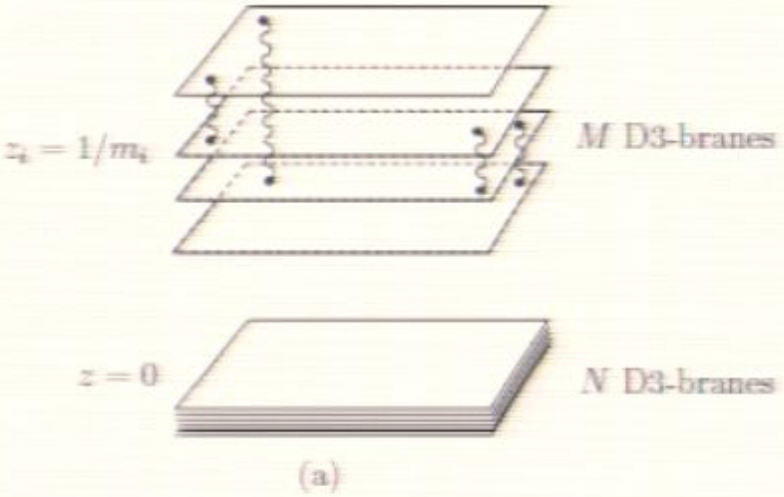
(a)

- Field Theory: Higgsing $U(N+M) \to U(N) \times U(1)^M$. One brane for every scattered particle, $N \gg M$.

Renders amplitudes IR finite. Have light $(m_i - m_j)$ and heavy m_i fields

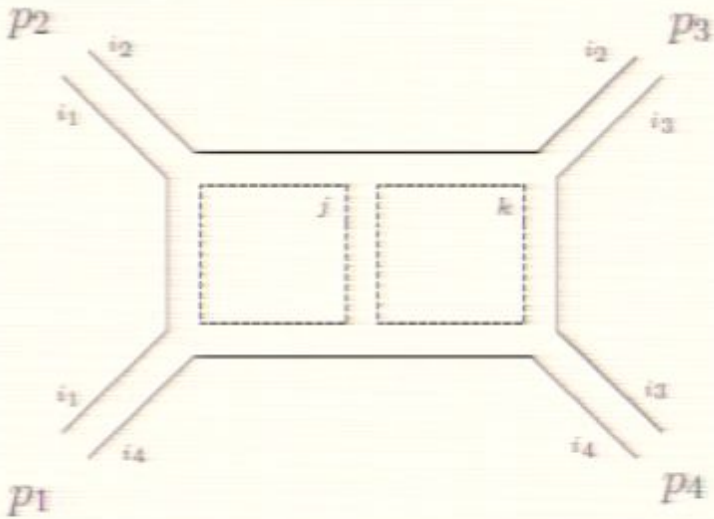


Higgs regularization [Alday, Henn, JP, Schuster]



- Take string picture serious:
- Field Theory: Higgsing $U(N+M) \to U(N) \times U(1)^M$. One brane for every scattered particle, $N \gg M$.

Renders amplitudes IR finite. Have light $(m_i - m_j)$ and heavy m_i fields

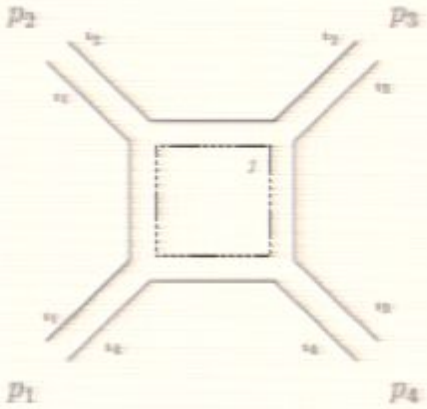


One loop test of extended dual conformal symmetry 1

- Consider the (special) purely scalar amplitude:

$$A_4 = \langle \Phi_4(p_1)\, \Phi_5(p_2)\, \Phi_4(p_3)\, \Phi_5(p_4) \rangle = i g_{\rm YM}^2 \left(1 + \lambda\, I^{(1)}(s,t,m_i) + O(\lambda^2) \right)$$

$I^{(1)}(s,t,m_i)$: Massive box integral in dual variables $(p_i = x_i - x_{i+1})$



$$= \int d^4x_5 \frac{(x_{13}^2 + (m_1 - m_3)^2)(x_{24}^2 + (m_2 - m_4)^2)}{(x_{15}^2 + m_1^2)(x_{25}^2 + m_2^2)(x_{35}^2 + m_3^2)(x_{45}^2 + m_4^2)}$$

- Reexpressed in 5d variables $\hat{x}^M$: $\hat{x}_i^\mu := x_i^\mu\,, \quad \hat{x}_i^4 := m_i\,,$ $\quad i = 1 \ldots 4$

$$I^{(1)}(s,t,m_i) = \hat{x}_{13}^2 \hat{x}_{24}^2 \int d^5\hat{x}_5 \frac{\delta(\hat{x}_5^{M=4})}{\hat{x}_{15}^2 \hat{x}_{25}^2 \hat{x}_{35}^2 \hat{x}_{45}^2}$$

Indeed $I^{(1)}(s,t,m_i)$ is extended dual conformal invariant: $\hat{K}_\mu I^{(1)}(s,t,m_i) = 0$

Pirsa: 11080041

Extended dual conformal invariance

- Extended dual conformal invariance

$$\hat{K}_\mu I^{(1)}(s,t,m_i) := \sum_{i=1}^{4}\left[2x_{i\mu}\left(x_i^\nu\frac{\partial}{\partial x_i^\nu}+m_i\frac{\partial}{\partial m_i}\right)-(x_i^2+m_i^2)\frac{\partial}{\partial x_i^\mu}\right] I^{(1)}(s,t,m_i)=0$$

- m_i is the fifth coordinate $x^M=(x^\mu,m)$.
- Triangle and bubble graphs are forbidden by extended conformal symmetry!
- Indeed an explicit one-loop calculation shows the cancelation of triangles.
- Has been extended to higher loops & higher multiplicities as well as Regge limit

[Henn, Naculich, Schnitzer, Drummond]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi'''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi'''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi'''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i (2x_i^\mu\, x_i^\nu - x_i^2\, \eta^{\mu\nu})\, \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC}\, \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu)\, \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\, \delta^{(4)}(q)\, \delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i \left(2x_i^\mu\, x_i^\nu - x_i^2\, \eta^{\mu\nu}\right) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC}\, \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu)\, \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\, \delta^{(4)}(q)\, \delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i \left(2x_i^\mu\, x_i^\nu - x_i^2\, \eta^{\mu\nu}\right) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC}\, \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu)\, \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\, \delta^{(4)}(q)\, \delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i (2x_i^\mu\, x_i^\nu - x_i^2\, \eta^{\mu\nu}) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC} \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu) \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

On shell superspace for 6d $\mathcal{N} = (1,1)$ sYM

4d massively regulated $\mathcal{N} = 4$ SYM amplitudes	=	6d massless $\mathcal{N} = (1,1)$ SYM amps with $p^5 + ip^6 = m_{4d}$ & internal loop momenta in 4d

On shell 6d superspace:

- Lorentz group $SO(1,5) \simeq SU(4)^*$ Little group $SO(4) \simeq SU(2) \times SU(2)$
- Helicity spinors: $\boxed{\lambda^{Aa} \;\&\; \tilde{\lambda}_{A\dot{a}}}$ $A = 1,2,3,4 \quad a = 1,2 \;\; \dot{a} = 1,2$
- 6d momentum: $p^{AB} = \lambda^{Aa}\,\lambda^{Bb}\,\epsilon_{ab} \quad p_{AB} = \tilde{\lambda}_{A\dot{a}}\,\lambda_{B\dot{b}}\,\epsilon^{\dot{a}\dot{b}} = \frac{1}{2}\,\epsilon_{ABCD}p^{AB}$
- SUSY partners: $\boxed{\eta_a \;\&\; \tilde{\eta}^{\dot{a}}}$
- Supertranslations: $q^A = \lambda^{Aa}\,\eta_a \quad \& \quad \tilde{q}_A = \tilde{\lambda}_{A\dot{a}}\,\tilde{\eta}^{\dot{a}}$
- On-shell superfield as η expansion:

$$\Omega(p,\eta,\tilde{\eta}) = \phi + \psi^a\eta_a + \tilde{\psi}_a\tilde{\eta}^a + \phi'\,\eta^2 + \phi''\,\tilde{\eta}^2 + g^{\dot{a}}_a\,\eta^a\,\tilde{\eta}_{\dot{a}} \ldots + \phi''\,\eta^2\,\bar{\eta}^2$$

- Perturbative results:
  - 3,4,5 point amplitudes @ tree-level [Cheung,O'Connell]
  - Super BCFW recursion [Dennen,Huang,Siegel]
  - 4pt amplitude @ 1 & 2 loops [Bern,Carrasco,Dennen,Huang,Ita]

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i \left(2x_i^\mu\, x_i^\nu - x_i^2\, \eta^{\mu\nu}\right) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC}\, \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu)\, \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\, \delta^{(4)}(q)\, \delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i \left(2x_i^\mu\, x_i^\nu - x_i^2\, \eta^{\mu\nu}\right) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC}\, \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu)\, \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\, \delta^{(4)}(q)\, \delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i \left(2x_i^\mu\, x_i^\nu - x_i^2\,\eta^{\mu\nu}\right) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC}\, \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu)\, \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i \left(2x_i^\mu\, x_i^\nu - x_i^2\, \eta^{\mu\nu}\right) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC}\, \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu)\, \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\, \delta^{(4)}(q)\, \delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

Sketch of proof

- Tree level:

  Write $K^\mu = I\, P^\mu\, I$ with inversion op. I. For $\mathbb{A}_n^{\text{tree}} = \delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})\,f_n$ one shows inductively via super BCFW recursion: $I[f_n] = (\prod_i x_i^2)\, f_n$

  $\Rightarrow \quad (K^\mu + 2\sum_i x_i^\mu)\, f_n$

- Loop level: $\mathbb{A}_n^{L\text{-loop}} = \delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q}) \int (\prod_{r=1}^{L} (d^6 x_{l_r})\, I_n^{L\text{-loop}}$ L-loop

  integrand $I_n^{L\text{-loop}}$ is well defined for color ordered amplitudes in planar theory

  Inverts as $I[I_n^{L\text{-loop}}] = (\prod_i x_i^2)\,(\prod_{r=1}^{L} (x_{l_r}^2)^4)\, I_n^{L\text{-loop}}$ (from generalized unitarity)

  Hence restricting the loop integration by hand one has

$$I[\int (\prod_{r=1}^{L} (d^4 x_{l_r})\, I_n^{L\text{-loop}}] = (\prod_i x_i^2) \int (\prod_{r=1}^{L} (d^4 x_{l_r})\, I_n^{L\text{-loop}}$$

  and thus $(K^\mu + 2\sum_i x_i^\mu)\, \dfrac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0$

Sketch of proof

- Tree level:

  Write $K^\mu = I\, P^\mu\, I$ with inversion op. I. For $\mathbb{A}_n^{\text{tree}} = \delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})\,f_n$ one shows inductively via super BCFW recursion: $I[f_n] = (\prod_i x_i^2)\, f_n$

  $\Rightarrow \quad (K^\mu + 2\sum_i x_i^\mu)\, f_n$

- Loop level: $\mathbb{A}_n^{L\text{-loop}} = \delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q}) \int (\prod_{r=1}^{L} (d^6 x_{l_r})\, I_n^{L\text{-loop}}$ L-loop

  integrand $I_n^{L\text{-loop}}$ is well defined for color ordered amplitudes in planar theory

  Inverts as $I[I_n^{L\text{-loop}}] = (\prod_i x_i^2)\,(\prod_{r=1}^{L} (x_{l_r}^2)^4)\, I_n^{L\text{-loop}}$ (from generalized unitarity)

  Hence restricting the loop integration by hand one has

$$I[\int (\prod_{r=1}^{L} (d^4 x_{l_r})\, I_n^{L\text{-loop}}] = (\prod_i x_i^2) \int (\prod_{r=1}^{L} (d^4 x_{l_r})\, I_n^{L\text{-loop}}$$

  and thus $(K^\mu + 2\sum_i x_i^\mu)\, \dfrac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\,\beta)} + \mu_{i(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i \left(2x_i^\mu\, x_i^\nu - x_i^2\, \eta^{\mu\nu}\right) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC}\, \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu)\, \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\, \delta^{(4)}(q)\, \delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

Dual conformal symmetry for 6d $\mathcal{N} = (1,1)$ SYM

- Introduce dual on-shell superspace

$$x_i^{AB} - x_{i+1}^{AB} = p_i^{AB} \quad \theta_i^A - \theta_{i+1}^A = q_i^A \quad \tilde{\theta}_{i\,A} - \tilde{\theta}_{i+1\,A} = \tilde{q}_{iA}$$

- Dual conformal generator:

$$K^\mu = \sum_i (2x_i^\mu\, x_i^\nu - x_i^2\, \eta^{\mu\nu}) \frac{\partial}{\partial x_i^\nu} + \theta_i^A (\sigma^\mu)_{AB}\, x_i^{BC} \frac{\partial}{\partial \theta_i^C} + \dots$$

- Statement: K^μ is symmetry of 6d stripped super amplitudes [Dennen,Huang]

$$\boxed{(K^\mu + 2\sum_i x_i^\mu) \frac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0}$$

For $L > 0$ iff loop integration is in 4d

$\Rightarrow$ Proof of exact dual conformal symmetry of massively regulated $\mathcal{N} = 4$ SYM!

Sketch of proof

- Tree level:

  Write $K^\mu = I\, P^\mu\, I$ with inversion op. I. For $\mathbb{A}_n^{\text{tree}} = \delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})\,f_n$ one shows inductively via super BCFW recursion: $I[f_n] = (\prod_i x_i^2)\, f_n$

  $\Rightarrow \quad (K^\mu + 2\sum_i x_i^\mu)\, f_n$

- Loop level: $\mathbb{A}_n^{L\text{-loop}} = \delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q}) \int (\prod_{r=1}^{L} (d^6 x_{l_r})\, I_n^{L\text{-loop}}$ L-loop

  integrand $I_n^{L\text{-loop}}$ is well defined for color ordered amplitudes in planar theory

  Inverts as $I[I_n^{L\text{-loop}}] = (\prod_i x_i^2)\,(\prod_{r=1}^{L} (x_{l_r}^2)^4)\, I_n^{L\text{-loop}}$ (from generalized unitarity)

  Hence restricting the loop integration by hand one has

$$I[\int (\prod_{r=1}^{L} (d^4 x_{l_r})\, I_n^{L\text{-loop}}] = (\prod_i x_i^2) \int (\prod_{r=1}^{L} (d^4 x_{l_r})\, I_n^{L\text{-loop}}$$

  and thus $(K^\mu + 2\sum_i x_i^\mu)\, \dfrac{\mathbb{A}_n^{L\text{-loop}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\,\beta)} + \mu_{i(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\,\beta)} + \mu_{i(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\,\beta)} + \mu_{i(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\,\beta)} + \mu_{i(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\,\beta)} + \mu_{i(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\,\beta)} + \mu_{i(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\,\beta)} + \mu_{i(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\,\beta)} + \mu_{i(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)} + \mu_{i(\alpha}\delta_{i\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\dot\alpha}\partial_{i\alpha} - \tilde\lambda_{i\dot\alpha}\delta_{i\alpha} + \mu_{i\alpha}\tilde\partial_{i\dot\alpha} - \lambda_{i\alpha}\tilde\delta_{i\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\dot\alpha} + \mu_i^\alpha\delta_{i\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}}$$ (two sets of spinors)

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:
Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

Question:

Can we again interpret dual conformal K_μ as the level one Yangian generator $p_\mu^{(1)}$ of massively regularised $\mathcal{N}=4$ SYM upon dim reduction to 4d?

- Needs to compactify to 4d: $p^{AB} = \begin{pmatrix} m\,\epsilon_{\alpha\beta} & -p_\alpha{}^{\dot\beta} \\ p^{\dot\alpha}{}_\beta & -\bar m\,\epsilon^{\dot\alpha\dot\beta} \end{pmatrix}$
- Helicity spinors for massive 4d particles:

$$\boxed{p^{\alpha\dot\alpha} = \lambda^\alpha \tilde\lambda^{\dot\alpha} + \mu^\alpha \tilde\mu^{\dot\alpha}} \qquad \text{(two sets of spinors)}$$

with constraints $\langle \lambda\, \mu\rangle = m$ and $[\tilde\mu\, \tilde\lambda] = \bar m$. Reality constraint: $m = \bar m$.

- Inherited Lorentz symmetries of massively regulated $\mathcal{N}=4$ amplitudes

$$l_{\alpha\beta} = \sum_i \lambda_{i\,(\alpha}\partial_{i\,\beta)} + \mu_{i\,(\alpha}\delta_{i\,\beta)} \qquad \bar l_{\dot\alpha\dot\beta} = \ldots \qquad \Leftrightarrow \quad l_{\mu\nu}$$

$$h_{\alpha\dot\alpha} = \sum_i \tilde\mu_{i\,\dot\alpha}\partial_{i\,\alpha} - \tilde\lambda_{i\,\dot\alpha}\delta_{i\,\alpha} + \mu_{i\,\alpha}\tilde\partial_{i\,\dot\alpha} - \lambda_{i\,\alpha}\tilde\delta_{i\,\dot\alpha} \qquad \Leftrightarrow \quad l_{\mu 5} + i l_{\mu 6}$$

$$d = \sum_i \tfrac{1}{2}(\lambda_i^\alpha\partial_{i\,\alpha} + \tilde\lambda_i^{\dot\alpha}\tilde\partial_{i\,\dot\alpha} + \mu_i^\alpha\delta_{i\,\alpha} + \tilde\mu_i^{\dot\alpha}\tilde\delta_{i\,\dot\alpha}) + 1 \qquad \Leftrightarrow \quad l_{56}$$

with $\delta_{i\alpha} := \frac{\partial}{\partial \mu_i^\alpha}$ $\qquad \partial_{i\alpha} := \frac{\partial}{\partial \lambda_i^\alpha}$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix}$ $\bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}\,b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha\,b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta\,b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}\,b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathrm{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\,\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a} Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\,\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a} Q_j^{a\gamma} - \bar{Q}_{i\,a}^{\dot\gamma} Q^a_{j\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\gamma a} Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a} Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}_{i\,a}^{\dot\gamma}\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\,h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\,\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a} Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\,\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathrm{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\,\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a} Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\,\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\,h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}^{\dot\gamma}_{i\,a}\, Q^a_{j\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathrm{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}^{\dot\gamma}_{i\,a}\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}^{\dot\gamma}_{i\,a}\, Q^a_{j\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\gamma a}\, Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}_{i\,a}^{\dot\gamma}\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}^{\dot\gamma}_{i\,a}\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\,\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\,\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\,h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\,\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a} Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\,\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\gamma a}\, Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j)\Big]$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j} \Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\, \bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j) \Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i} \Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a} Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j) \Big]$

Nonlocal symmetries of massively regulated $\mathcal{N} = 4$ 4d SYM

- However, no ∞-dim symmetry structure emerges as $p^{(1)}_{\alpha\dot{\alpha}}$ and $m^{(1)}$ form an ideal of the algebra:

$$\mathfrak{i} = \{p, m\} \subset \mathfrak{a} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(0)}] = \mathfrak{i}^{(0)} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(1)}] = \mathfrak{i}^{(1)}$$

- Conclusion:

$$\{p, m, l, \bar{l}, h, d; q, \tilde{q}; p^{(1)}, m^{(1)}\} \frac{\mathbb{A}_n^{L\text{-loop, } \mathcal{N}=4 \text{ SYM}}}{\delta^{(6)}(p)\, \delta^{(4)}(q)\, \delta^{(4)}(\tilde{q})} = 0$$

- Have not yet found level 1 SUSY generators
- Seems like ∞-dim symmetry structure hinges upon conformal symmetry at level 0 as there

$$[k, p] \sim l + \bar{l} + d$$

Nonlocal symmetries of massively regulated $\mathcal{N} = 4$ 4d SYM

- However, no ∞-dim symmetry structure emerges as $p^{(1)}_{\alpha\dot{\alpha}}$ and $m^{(1)}$ form an ideal of the algebra:

$$\mathfrak{i} = \{p, m\} \subset \mathfrak{a} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(0)}] = \mathfrak{i}^{(0)} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(1)}] = \mathfrak{i}^{(1)}$$

- Conclusion:

$$\{p, m, l, \bar{l}, h, d; q, \tilde{q}\,;\, p^{(1)}, m^{(1)}\} \frac{\mathbb{A}_n^{L\text{-loop, } \mathcal{N}=4 \text{ SYM}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0$$

- Have not yet found level 1 SUSY generators
- Seems like ∞-dim symmetry structure hinges upon conformal symmetry at level 0 as there

$$[k, p] \sim l + \bar{l} + d$$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\,\alpha} = \lambda_{i\,\alpha}\bar{\xi}^a_i - \mu_{i\,\alpha}\xi^a_i \qquad Q^a_{i\,\dot{\alpha}} = \tilde{\lambda}_{i\,\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\,\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\,\dot{\alpha}\,a} = \tilde{\lambda}_{i\,\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\,\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\,\alpha\,a} = \lambda_{i\,\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\,\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}\,b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha\,b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta\,b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}\,b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a}\, Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\, h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a} Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulated $\mathcal{N} = 4$ 4d SYM

- However, no ∞-dim symmetry structure emerges as $p^{(1)}_{\alpha\dot{\alpha}}$ and $m^{(1)}$ form an ideal of the algebra:

$$\mathfrak{i} = \{p, m\} \subset \mathfrak{a} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(0)}] = \mathfrak{i}^{(0)} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(1)}] = \mathfrak{i}^{(1)}$$

- Conclusion:

$$\{p, m, l, \bar{l}, h, d; q, \tilde{q}\,;\, p^{(1)}, m^{(1)}\} \frac{\mathbb{A}_n^{L\text{-loop, } \mathcal{N}=4 \text{ SYM}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0$$

- Have not yet found level 1 SUSY generators
- Seems like ∞-dim symmetry structure hinges upon conformal symmetry at level 0 as there

$$[k, p] \sim l + \bar{l} + d$$

From 6 to 4

- Inherited supersymmetries of massively regulated $\mathcal{N} = 4$ amplitudes

$$Q^a_{i\alpha} = \lambda_{i\alpha}\bar{\xi}^a_i - \mu_{i\alpha}\xi^a_i \qquad Q^a_{i\dot{\alpha}} = \tilde{\lambda}_{i\dot{\alpha}}\xi^a_i + \tilde{\mu}_{i\dot{\alpha}}\bar{\xi}^a_i$$

$$\bar{Q}_{i\dot{\alpha}a} = \tilde{\lambda}_{i\dot{\alpha}}\frac{\partial}{\partial\bar{\xi}^a_i} - \tilde{\mu}_{i\dot{\alpha}}\frac{\partial}{\partial\xi^a_i} \qquad \bar{Q}_{i\alpha a} = \lambda_{i\alpha}\frac{\partial}{\partial\xi^a_i} + \mu_{i\alpha}\frac{\partial}{\partial\bar{\xi}^a_i}$$

with $\xi^a = \begin{pmatrix} \eta_1 \\ -\tilde{\eta}^{\dot{1}} \end{pmatrix} \quad \bar{\xi}^a = \begin{pmatrix} \eta_2 \\ \tilde{\eta}^{\dot{2}} \end{pmatrix}$

- (Bosonic) algebra

$$[h_{\alpha\dot{\alpha}}, p_{\beta\dot{\beta}}] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot{\alpha}\dot{\beta}}\,m\,, \qquad [h_{\alpha\dot{\alpha}}, m] = p_{\alpha\dot{\alpha}}\,,$$

$$[h_{\alpha\dot{\alpha}}, h_{\beta\dot{\beta}}] = 2\epsilon_{\alpha\beta}\,\bar{l}_{\dot{\alpha}\dot{\beta}} + 2\epsilon_{\dot{\alpha}\dot{\beta}}\,l_{\alpha\beta}\,, \qquad [l_{\beta\gamma}, h_{\alpha\dot{\alpha}}] = \epsilon_{\alpha(\beta}\,h_{\gamma)\dot{\alpha}}\,,$$

- SUSY algebra

$$\{Q^a_\alpha, \bar{Q}_{\dot{\alpha}b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\alpha b}\} = p_{\alpha\dot{\alpha}}\,\delta^a_b$$

$$\{Q^a_\alpha, \bar{Q}_{\beta b}\} = m\,\epsilon_{\alpha\beta}\,\delta^a_b \qquad \{Q^a_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}b}\} = -m\,\epsilon_{\dot{\alpha}\dot{\beta}}\,\delta^a_b$$

Nonlocal symmetries of massively regulates $\mathcal{N}=4$ SYM

- Rewriting the 6d dual conformal K^μ $(\mu = 0,1,2,3)$ in the massive on-shell 4d superspace $\{\lambda_\alpha, \tilde{\lambda}_{\dot\alpha}; \mu_\alpha, \tilde{\mu}_{\dot\alpha}; \xi^a, \bar{\xi}^a\}$ yields

$$K_{\alpha\dot\alpha} + \Delta K_{\alpha\dot\alpha} + 2\sum_i x_{i\,\alpha\dot\alpha} - x_1\text{-terms}$$

$$= \sum_{i<j}\Big[(\epsilon_{\dot\alpha\dot\beta}\, l_{i\,\alpha\beta} + \epsilon_{\alpha\beta}\,\bar{l}_{i\,\dot\alpha\dot\beta} + \epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, d_i\,)\, p_j^{\beta\dot\beta} + h_{i\,\alpha\dot\alpha}\, m_j$$

$$- \bar{Q}_{i\,\alpha a} Q^a_{j\dot\alpha} - \bar{Q}_{i\,\dot\alpha a} Q^a_{j\alpha} - (i \leftrightarrow j)\Big] = \mathbf{p}^{(1)}_{\alpha\dot\alpha}$$

Is level-one Yangian like extension of translational part of Poincaré algebra

$$[\,h_{\alpha\dot\alpha}, p^{(1)}_{\beta\dot\beta}\,] = 2\,\epsilon_{\alpha\beta}\,\epsilon_{\dot\alpha\dot\beta}\, m^{(1)}$$

with $m^{(1)} = \frac{1}{2}\sum_{j<i}\Big[h_{i\gamma\dot\gamma}\, p_j^{\gamma\dot\gamma} + 2 d_i\, m_j - \bar{Q}_{i\,\gamma a} Q_j^{a\gamma} - \bar{Q}_i^{\dot\gamma}{}_a\, Q^a_{j\dot\gamma} - (i \leftrightarrow j)\Big]$

Nonlocal symmetries of massively regulated $\mathcal{N} = 4$ 4d SYM

- However, no ∞-dim symmetry structure emerges as $p^{(1)}_{\alpha\dot{\alpha}}$ and $m^{(1)}$ form an ideal of the algebra:

$$\mathfrak{i} = \{p, m\} \subset \mathfrak{a} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(0)}] = \mathfrak{i}^{(0)} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(1)}] = \mathfrak{i}^{(1)}$$

- Conclusion:

$$\{p, m, l, \bar{l}, h, d; q, \tilde{q}; p^{(1)}, m^{(1)}\} \frac{\mathbb{A}_n^{L\text{-loop, } \mathcal{N}=4 \text{ SYM}}}{\delta^{(6)}(p)\, \delta^{(4)}(q)\, \delta^{(4)}(\tilde{q})} = 0$$

- Have not yet found level 1 SUSY generators
- Seems like ∞-dim symmetry structure hinges upon conformal symmetry at level 0 as there

$$[k, p] \sim l + \bar{l} + d$$

Nonlocal symmetries of massively regulated $\mathcal{N} = 4$ 4d SYM

- However, no ∞-dim symmetry structure emerges as $p^{(1)}_{\alpha\dot{\alpha}}$ and $m^{(1)}$ form an ideal of the algebra:

$$\mathfrak{i} = \{p, m\} \subset \mathfrak{a} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(0)}] = \mathfrak{i}^{(0)} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(1)}] = \mathfrak{i}^{(1)}$$

- Conclusion:

$$\{p, m, l, \bar{l}, h, d; q, \tilde{q}; p^{(1)}, m^{(1)}\} \frac{\mathbb{A}_n^{L\text{-loop, } \mathcal{N}=4 \text{ SYM}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0$$

- Have not yet found level 1 SUSY generators
- Seems like ∞-dim symmetry structure hinges upon conformal symmetry at level 0 as there

$$[k, p] \sim l + \bar{l} + d$$

Nonlocal symmetries of massively regulated $\mathcal{N} = 4$ 4d SYM

- However, no ∞-dim symmetry structure emerges as $p^{(1)}_{\alpha\dot{\alpha}}$ and $m^{(1)}$ form an ideal of the algebra:

$$\mathfrak{i} = \{p, m\} \subset \mathfrak{a} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(0)}] = \mathfrak{i}^{(0)} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(1)}] = \mathfrak{i}^{(1)}$$

- Conclusion:

$$\{p, m, l, \bar{l}, h, d; q, \tilde{q}; p^{(1)}, m^{(1)}\} \frac{\mathbb{A}_n^{L\text{-loop, } \mathcal{N}=4 \text{ SYM}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0$$

- Have not yet found level 1 SUSY generators
- Seems like ∞-dim symmetry structure hinges upon conformal symmetry at level 0 as there

$$[k, p] \sim l + \bar{l} + d$$

Nonlocal symmetries of massively regulated $\mathcal{N} = 4$ 4d SYM

- However, no ∞-dim symmetry structure emerges as $p^{(1)}_{\alpha\dot{\alpha}}$ and $m^{(1)}$ form an ideal of the algebra:

$$\mathfrak{i} = \{p, m\} \subset \mathfrak{a} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(0)}] = \mathfrak{i}^{(0)} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(1)}] = \mathfrak{i}^{(1)}$$

- Conclusion:

$$\{p, m, l, \bar{l}, h, d; q, \tilde{q}; p^{(1)}, m^{(1)}\} \frac{\mathbb{A}_n^{L\text{-loop, } \mathcal{N}=4 \text{ SYM}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0$$

- Have not yet found level 1 SUSY generators
- Seems like ∞-dim symmetry structure hinges upon conformal symmetry at level 0 as there

$$[k, p] \sim l + \bar{l} + d$$

Nonlocal symmetries of massively regulated $\mathcal{N} = 4$ 4d SYM

- However, no ∞-dim symmetry structure emerges as $p^{(1)}_{\alpha\dot{\alpha}}$ and $m^{(1)}$ form an ideal of the algebra:

$$\mathfrak{i} = \{p, m\} \subset \mathfrak{a} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(0)}] = \mathfrak{i}^{(0)} \qquad [\mathfrak{a}^{(0)}, \mathfrak{i}^{(1)}] = \mathfrak{i}^{(1)}$$

- Conclusion:

$$\{p, m, l, \bar{l}, h, d; q, \tilde{q}\,;\, p^{(1)}, m^{(1)}\} \frac{\mathbb{A}_n^{L\text{-loop, } \mathcal{N}=4 \text{ SYM}}}{\delta^{(6)}(p)\,\delta^{(4)}(q)\,\delta^{(4)}(\tilde{q})} = 0$$

- Have not yet found level 1 SUSY generators
- Seems like ∞-dim symmetry structure hinges upon conformal symmetry at level 0 as there

$$[k, p] \sim l + \bar{l} + d$$

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

$$\boxed{\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin,JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert,Vieira][Alday,Henn,Maldacena,JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

$$\boxed{\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever Vieira][Beisert Henn McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert,Henn,McLoughlin,JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert, Ricci, Tseytlin, Wolf]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert,Henn,McLoughlin,JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

$$\boxed{\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira,Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert, Henn, McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert,Henn,McLoughlin,JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert,Henn,McLoughlin,JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert,Ricci,Tseytlin,Wolf]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert, Henn, McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert, Henn, McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert,Henn,McLoughlin,JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

$$\boxed{\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever Vieira!Beisert Henn McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

$$\boxed{\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira,Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

$$\boxed{\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert Henn McLoughlin JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert, Henn, McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert, Henn, McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

$$\boxed{\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled?
  Yes! Perturbative construction in dim. regularization [Beisert Henn McLoughlin JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

$$\boxed{\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N} = 4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N} = 4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  $$\text{Superpoincaré} + h_\mu + d + p_\mu^{(1)} + m^{(1)}$$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

Summary and Outlook

- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?

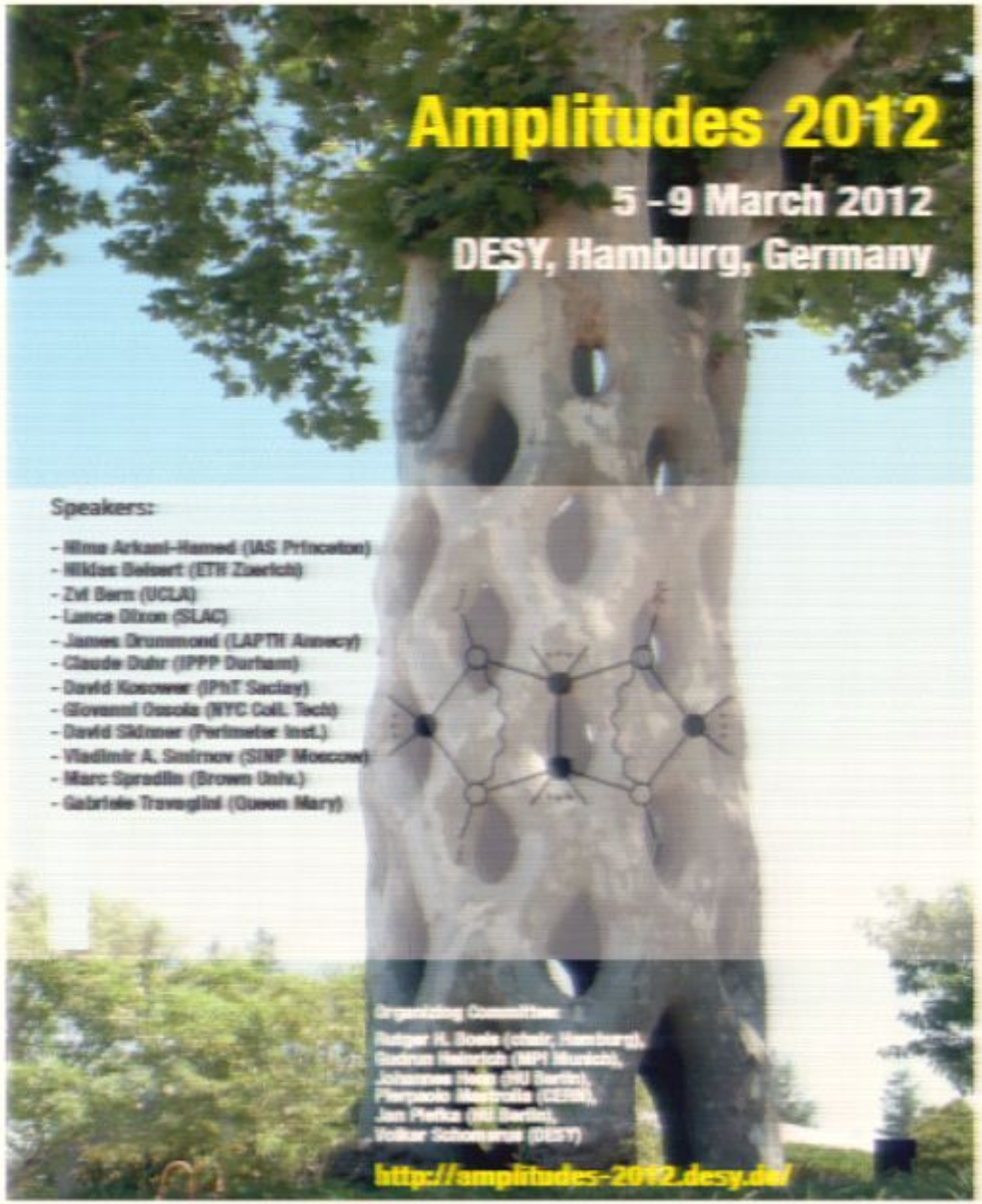
Summary and Outlook

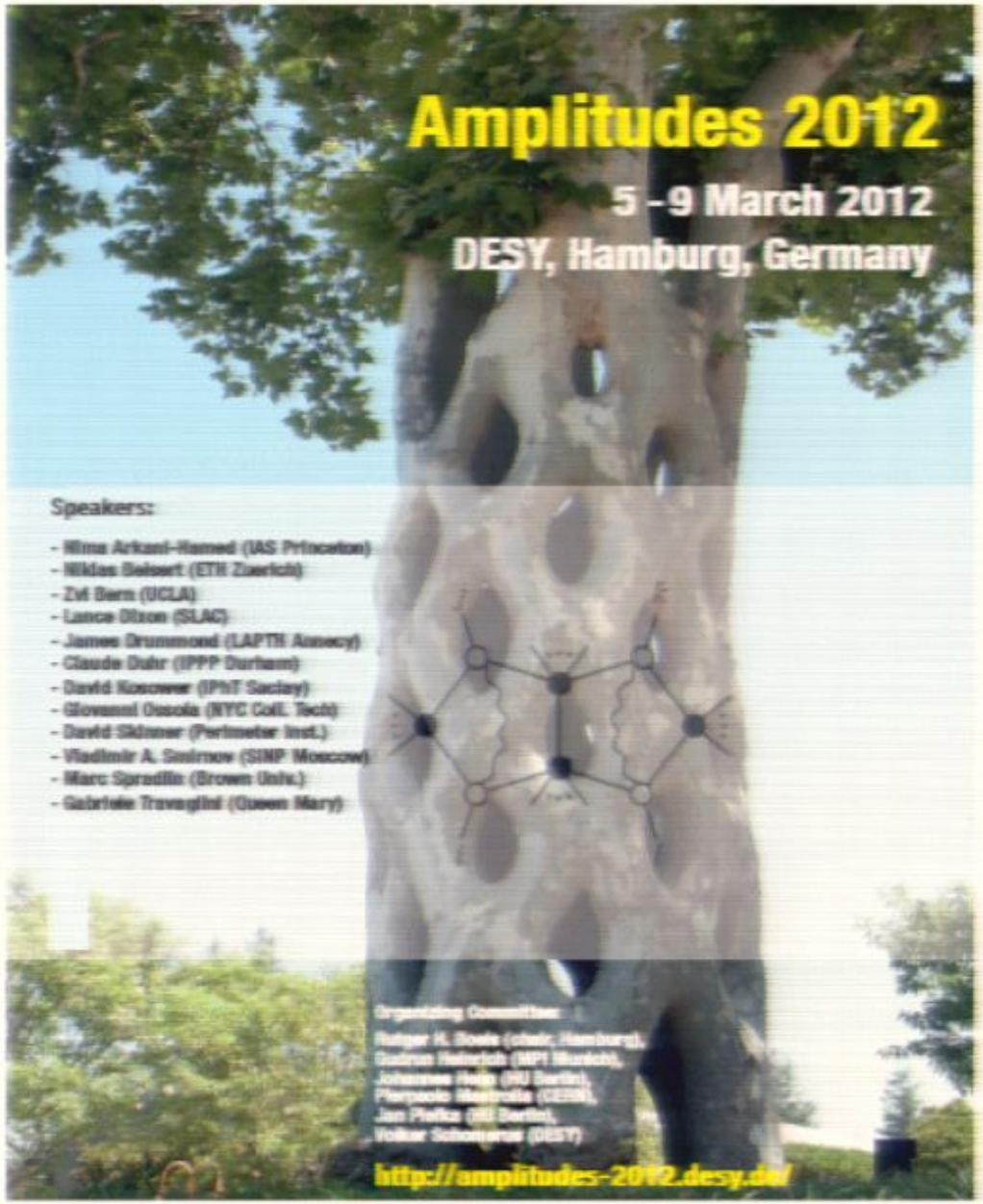
- All tree-level amplitudes in $\mathcal{N}=4$ SYM known analytically
- Tree level amplitudes are invariant under an infinite dimensional Yangian symmetry
- Challenge at weak coupling: Does Yangian symmetry extend to the loop level?
- Breaking of dual conformal invariance at loop level under control: Best seen in massive (Higgs) regulator
- Restriction of possible integrals at higher loops.
- Established symmetry structure of massively regulated $\mathcal{N}=4$ SYM:

  Superpoincaré $+ h_\mu + d + p_\mu^{(1)} + m^{(1)}$

- Can breaking of standard conformal invariance at loop level be controlled? Yes! Perturbative construction in dim. regularization [Sever,Vieira][Beisert,Henn,McLoughlin, JP]
- Relation to massive regularization?

Does integrability determine the all loop planar scattering amplitudes?


Amplitudes 2012
5 -9 March 2012
DESY, Hamburg, Germany
Speakers:
- Nima Arkani-Hamed (IAS Princeton)
- Niklas Beisert (ETH Zuerich)
- Zvi Bern (UCLA)
- Lance Dixon (SLAC)
- James Drummond (LAPTH Annecy)
- Claude Duhr (IPPP Durham)
- David Kosower (IPhT Saclay)
- Giovanni Ossola (NYC Coll. Tech)
- David Skinner (Perimeter Inst.)
- Vladimir A. Smirnov (SINP Moscow)
- Marc Spradlin (Brown Univ.)
- Gabriele Travaglini (Queen Mary)
http://amplitudes-2012.desy.de/


Amplitudes 2012
5 -9 March 2012
DESY, Hamburg, Germany
Speakers:
- Nima Arkani-Hamed (IAS Princeton)
- Niklas Beisert (ETH Zuerich)
- Zvi Bern (UCLA)
- Lance Dixon (SLAC)
- James Drummond (LAPTH Annecy)
- Claude Duhr (IPPP Durham)
- David Kosower (IPhT Saclay)
- Giovanni Ossola (NYC Coll. Tech)
- David Skinner (Perimeter Inst.)
- Vladimir A. Smirnov (SINP Moscow)
- Marc Spradlin (Brown Univ.)
- Gabriele Travaglini (Queen Mary)
http://amplitudes-2012.desy.de/