Title: Mirror TBA

Date: Aug 19, 2011 09:30 AM

URL: http://pirsa.org/11080040

Abstract: The TBA approach to the AdS/CFT spectral problem is used to compute scaling dimensions of several operators dual to two-particle states of the l.c. AdS5 x S5 string theory. The implementation of the psu(2,2|4) symmetry in the TBA framework is discussed.

Mirror TBA

Sergey Frolov

Hamilton Mathematics Institute and School of Mathematics
Trinity College Dublin

Integrability in Gauge/String Theories, Perimeter Institute, August 19, 2011

Mirror TBA

Sergey Frolov

Hamilton Mathematics Institute and School of Mathematics
Trinity College Dublin

Integrability in Gauge/String Theories, Perimeter Institute, August 19, 2011

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

TBA?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

TBA?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

TBA?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

TBA?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

TBA?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

Mirror TBA

Sergey Frolov

Hamilton Mathematics Institute and School of Mathematics
Trinity College Dublin

Integrability in Gauge/String Theories, Perimeter Institute, August 19, 2011

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

TBA?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

The 12th Claude Itzykson Meeting
Integrability in Gauge and String Theory
Saclay, 19 June 2007

Thermodynamic Bethe Ansatz?

Sergey Frolov

School of Mathematics
Trinity College Dublin

work in progress with Gleb Arutyunov and Marija Zamaklar

Outline

1. Mirror TBA
2. Excited states
3. Two-particle states
4. PSU(2, 2|4) and TBA
5. Summary

Outline

1. Mirror TBA
2. Excited states
3. Two-particle states
4. PSU(2, 2|4) and TBA
5. Summary

Outline

1. Mirror TBA
2. Excited states
3. Two-particle states
4. PSU(2, 2|4) and TBA
5. Summary

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $AdS_5 \times S^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $AdS_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times \mathrm{S}^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times \mathrm{S}^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM} $\Longleftrightarrow$ string coupling $g_s = \frac{g_{YM}^2}{4\pi}$

't Hooft coupling $\lambda = g_{YM}^2 N_c$ $\Longleftrightarrow$ String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$

SYM operators $\Longleftrightarrow$ String states

Scaling dimension $\Delta(\lambda)$ $=$ String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

"Harmonic oscillator" of AdS/CFT:

Maldacena '97

$\mathcal{N} = 4$ $SU(N_c)$ SYM $\Longleftrightarrow$ IIB strings in $\mathrm{AdS}_5 \times S^5$ geometry

YM coupling g_{YM}	$\Longleftrightarrow$	string coupling $g_s = \frac{g_{YM}^2}{4\pi}$
't Hooft coupling $\lambda = g_{YM}^2 N_c$	$\Longleftrightarrow$	String tension $\frac{R^2}{2\pi\alpha'} \equiv g = \frac{\sqrt{\lambda}}{2\pi}$
SYM operators	$\Longleftrightarrow$	String states
Scaling dimension $\Delta(\lambda)$	$=$	String energy $E(g)$

Exact spectra of $\mathcal{N} = 4$ SYM and strings on $\mathrm{AdS}_5 \times S^5$

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

- Green-Schwarz superstring — Metsaev, Tseytlin '98

$$S = -\frac{g}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN}(X) + \text{fermions}$$

- L.c. string sigma model : $E - J = \int_{-J/2}^{J/2} \mathcal{H}_{l.c.}$ — Arutyunov, Frolov '04

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

- Green-Schwarz superstring — Metsaev, Tseytlin '98

$$S = -\frac{g}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN}(X) + \text{fermions}$$

- L.c. string sigma model : $E - J = \int_{-J/2}^{J/2} \mathcal{H}_{l.c.}$ — Arutyunov, Frolov '04

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

- Green-Schwarz superstring — Metsaev, Tseytlin '98

$$S = -\frac{g}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN}(X) + \text{fermions}$$

- L.c. string sigma model : $E - J = \int_{-J/2}^{J/2} \mathcal{H}_{l.c.}$ — Arutyunov, Frolov '04

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

- Green-Schwarz superstring — Metsaev, Tseytlin '98

$$S = -\frac{g}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN}(X) + \text{fermions}$$

- L.c. string sigma model : $E - J = \int_{-J/2}^{J/2} \mathcal{H}_{l.c.}$ — Arutyunov, Frolov '04

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

- Green-Schwarz superstring — Metsaev, Tseytlin '98

$$S = -\frac{g}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN}(X) + \text{fermions}$$

- L.c. string sigma model : $E - J = \int_{-J/2}^{J/2} \mathcal{H}_{l.c.}$ — Arutyunov, Frolov '04

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

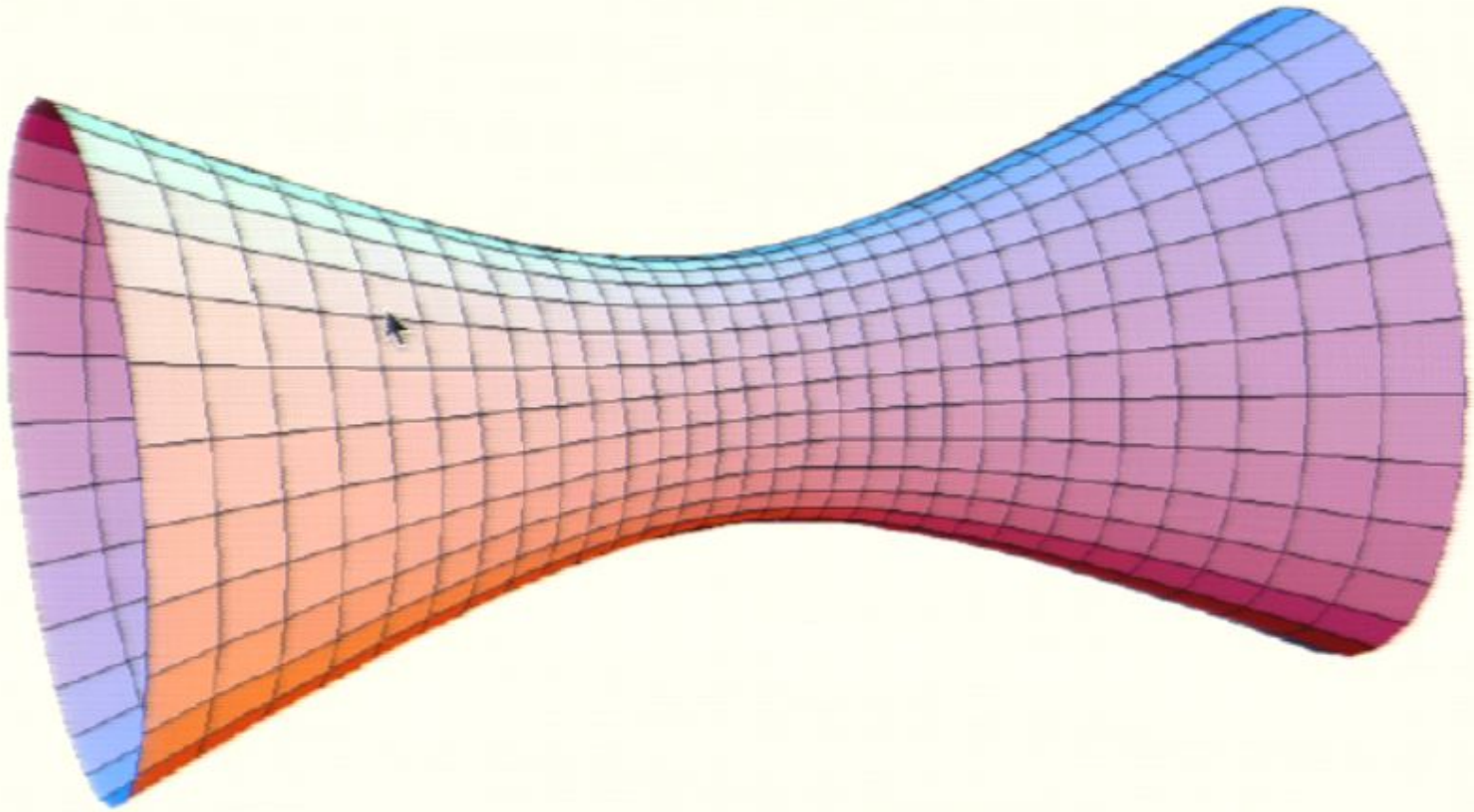
- Green-Schwarz superstring — Metsaev, Tseytlin '98

$$S = -\frac{g}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN}(X) + \text{fermions}$$

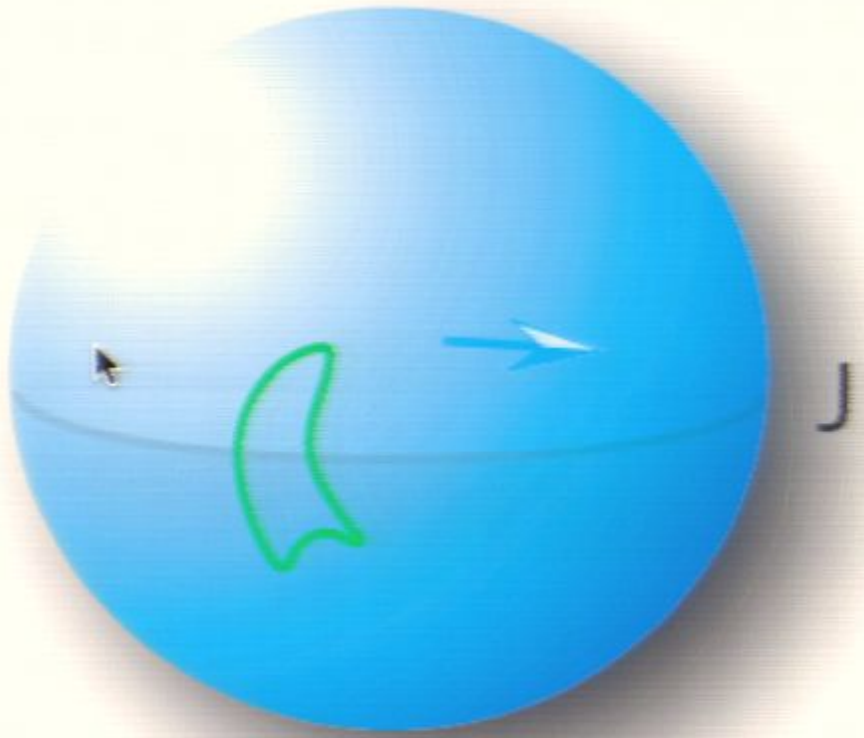
- L.c. string sigma model : $E - J = \int_{-J/2}^{J/2} \mathcal{H}_{l.c.}$ — Arutyunov, Frolov '04

Anti-de Sitter space

Space of constant negative curvature



String energy E is a conserved Noether charge corresponding to the $SO(2)$ subgroup of the conformal group $SO(4,2)$



J is a conserved Noether charge corresponding to one of the Cartan generators of $SO(6)$

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

- Green-Schwarz superstring — Metsaev, Tseytlin '98

$$S = -\frac{g}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN}(X) + \text{fermions}$$

- L.c. string sigma model : $E - J = \int_{-J/2}^{J/2} H_{l.c.}$ — Arutyunov, Frolov '04
- To compute $E(g)$ and therefore $\Delta(g)$, one needs to solve the 2-dim quantum sigma model on a cylinder!
- String integrability is the key to the solution

N=4 super Yang-Mills theory

- Maximally supersymmetric gauge theory in 4dim:

$$A_\mu, \quad \Phi^i, \quad i = 1, \ldots, 6 \quad \text{and} \quad \text{4 Weyl fermions}$$

- Introduce $X = \Phi^1 + i\Phi^2$, $Y = \Phi^3 + i\Phi^4$, $Z = \Phi^5 + i\Phi^6$, $D = D_+$
- The $\mathfrak{sl}(2)$-sector consists of linear combinations of operators

$$\mathrm{Tr}\left(\prod_{k=1}^{J} D^{n_k} Z\right), \qquad \sum_{k=1}^{J} n_k = N, \quad n_k \geq 0$$

J is the twist, and N is the spin.

N=4 super Yang-Mills theory

- Maximally supersymmetric gauge theory in 4dim:

$$A_\mu, \quad \Phi^i, \quad i = 1, \ldots, 6 \quad \text{and} \quad \text{4 Weyl fermions}$$

- Introduce $X = \Phi^1 + i\Phi^2$, $Y = \Phi^3 + i\Phi^4$, $Z = \Phi^5 + i\Phi^6$, $D = D_+$
- The $\mathfrak{sl}(2)$-sector consists of linear combinations of operators

$$\mathrm{Tr}\left(\prod_{k=1}^{J} D^{n_k} Z\right), \qquad \sum_{k=1}^{J} n_k = N, \quad n_k \geq 0$$

J is the twist, and N is the spin.

From sigma models to four-dimensional QFT

Non-linear Sigma Model in 2D → Quantum Field Theory in 4D

Planar scaling dimensions $\Delta(\lambda)$ in Yang-Mills theory should be computable by string theory! Simultaneously, this would test the conjecture.

- Green-Schwarz superstring — Metsaev, Tseytlin '98

$$S = -\frac{g}{2}\int d\tau d\sigma \sqrt{-h}h^{\alpha\beta}\partial_\alpha X^M \partial_\beta X^N G_{MN}(X) + \text{fermions}$$

- L.c. string sigma model : $E - J = \int_{-J/2}^{J/2} H_{l.c.}$ — Arutyunov, Frolov '04
- To compute $E(g)$ and therefore $\Delta(g)$, one needs to solve the 2-dim quantum sigma model on a cylinder!
- String integrability is the key to the solution

N=4 super Yang-Mills theory

- Maximally supersymmetric gauge theory in 4dim:

$$A_\mu, \quad \Phi^i, \quad i = 1, \ldots, 6 \quad \text{and} \quad \text{4 Weyl fermions}$$

- Introduce $X = \Phi^1 + i\Phi^2$, $Y = \Phi^3 + i\Phi^4$, $Z = \Phi^5 + i\Phi^6$, $D = D_+$
- The $\mathfrak{sl}(2)$-sector consists of linear combinations of operators

$$\mathrm{Tr}\left(\prod_{k=1}^{J} D^{n_k} Z\right), \qquad \sum_{k=1}^{J} n_k = N, \quad n_k \geq 0$$

J is the twist, and N is the spin.

N=4 super Yang-Mills theory

- Maximally supersymmetric gauge theory in 4dim:

$$A_\mu, \quad \Phi^i, \quad i = 1, \ldots, 6 \quad \text{and} \quad 4 \text{ Weyl fermions}$$

- Introduce $X = \Phi^1 + i\Phi^2$, $Y = \Phi^3 + i\Phi^4$, $Z = \Phi^5 + i\Phi^6$, $D = D_+$
- The $\mathfrak{sl}(2)$-sector consists of linear combinations of operators

$$\mathrm{Tr}\left(\prod_{k=1}^{J} D^{n_k} Z\right), \qquad \sum_{k=1}^{J} n_k = N, \quad n_k \geq 0$$

J is the twist, and N is the spin.

- These operators are dual to N-particle states of l.c. string theory.
- Spin-2 operators

$$\mathrm{Tr}(Z^{J-1} D^2 Z), \quad \mathrm{Tr}(Z^{k-2} DZ\, Z^{J-k} DZ)$$

- If $N = 2$ and $J = 2$ only one operator is unprotected, and it is a susy descendent of the Konishi operator

$$\mathrm{Tr}\,\Phi_i^2$$

N=4 super Yang-Mills theory

- Maximally supersymmetric gauge theory in 4dim:

$$A_\mu, \quad \Phi^i, \quad i = 1, \ldots, 6 \quad \text{and} \quad \text{4 Weyl fermions}$$

- Introduce $X = \Phi^1 + i\Phi^2$, $Y = \Phi^3 + i\Phi^4$, $Z = \Phi^5 + i\Phi^6$, $D = D_+$
- The $\mathfrak{sl}(2)$-sector consists of linear combinations of operators

$$\mathrm{Tr}\left(\prod_{k=1}^{J} D^{n_k} Z\right), \qquad \sum_{k=1}^{J} n_k = N, \quad n_k \geq 0$$

J is the twist, and N is the spin.

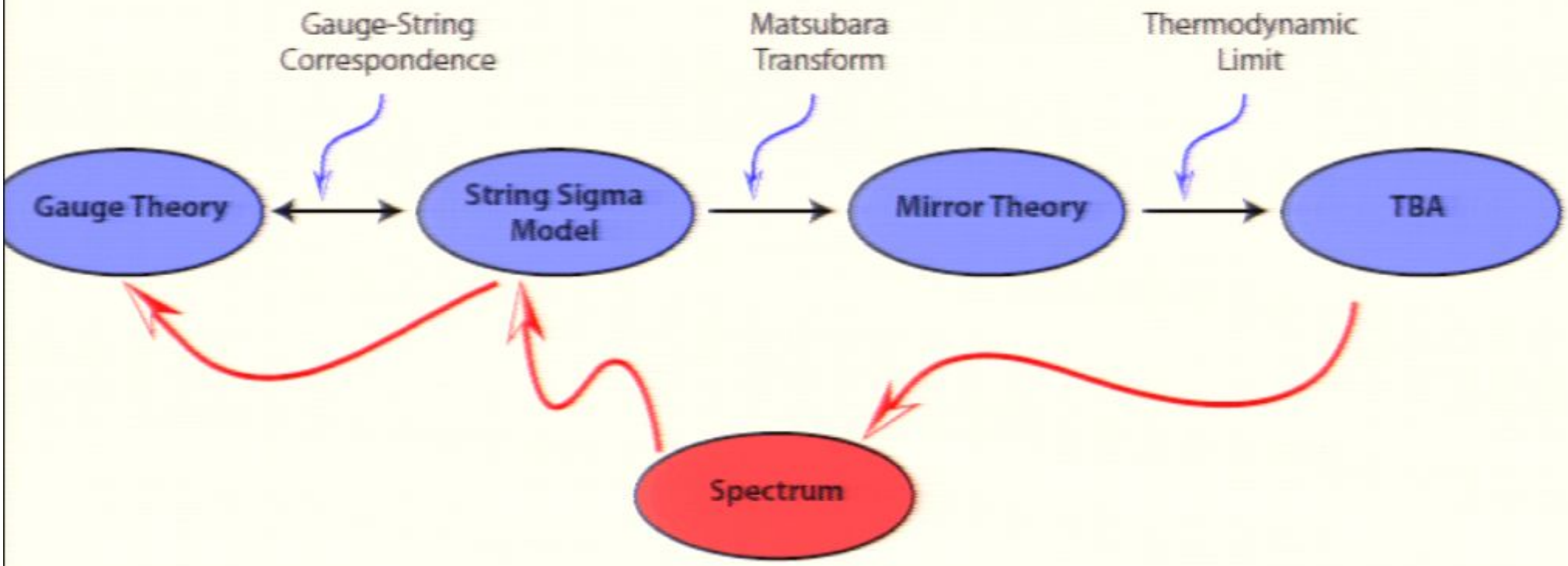
- These operators are dual to N-particle states of l.c. string theory.
- Spin-2 operators

$$\mathrm{Tr}(Z^{J-1} D^2 Z), \quad \mathrm{Tr}(Z^{k-2} DZ\, Z^{J-k} DZ)$$

- If $N = 2$ and $J = 2$ only one operator is unprotected, and it is a susy descendent of the Konishi operator

$$\mathrm{Tr}\,\Phi_i^2$$

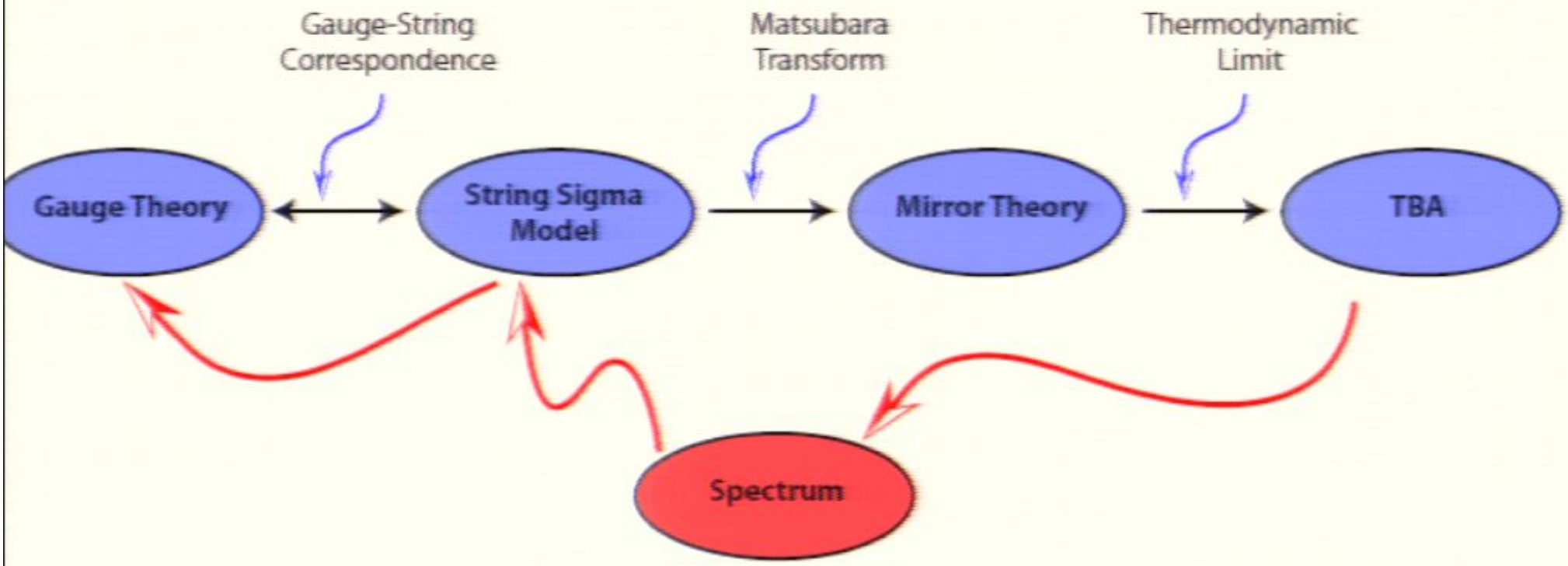
Summary of the TBA approach



String theory and N=4 SYM results

- Semi-classical strings — Berenstein, Maldacena, Nastase '02; Gubser, Klebanov, Polyakov '02; Frolov, Tseytlin '02, '03; Bena, Polchinski, Roiban '03; Kazakov, Marshakov, Minahan, Zarembo '04;
- L.c. strings in $AdS_5 \times S^5$ — Arutyunov, Frolov '04, '05; Frolov, Plefka, Zamaklar '06; Arutyunov, Frolov, Plefka, Zamaklar '06;
- Decompactification: $J \to \infty$ — Ambjorn, Janik, Kristjansen '05; Janik '06; Arutyunov, Frolov '06; Hofman, Maldacena '06;
- Symmetry algebra — Beisert '05, '06; Arutyunov, Frolov, Plefka, Zamaklar '06;
- Dispersion relations — Beisert, Dippel, Staudacher '04; Beisert '05; N.Dorey '06;
- S-matrix — Staudacher '04; Beisert '05; Arutyunov, Frolov, Zamaklar '06;
- Dressing factor and crossing eqs — Arutyunov, Frolov, Staudacher '04; Beisert, Tseytlin '05; Janik '06; Hernandez, Lopez '06; Arutyunov, Frolov '06; Beisert, Hernandez, Lopez '06; Beisert, Eden, Staudacher '06;
- Bethe ansatz — Minahan, Zarembo '02; Beisert, Dippel, Staudacher '04; Arutyunov, Frolov, Staudacher '04; Staudacher '04; Beisert, Staudacher '05;

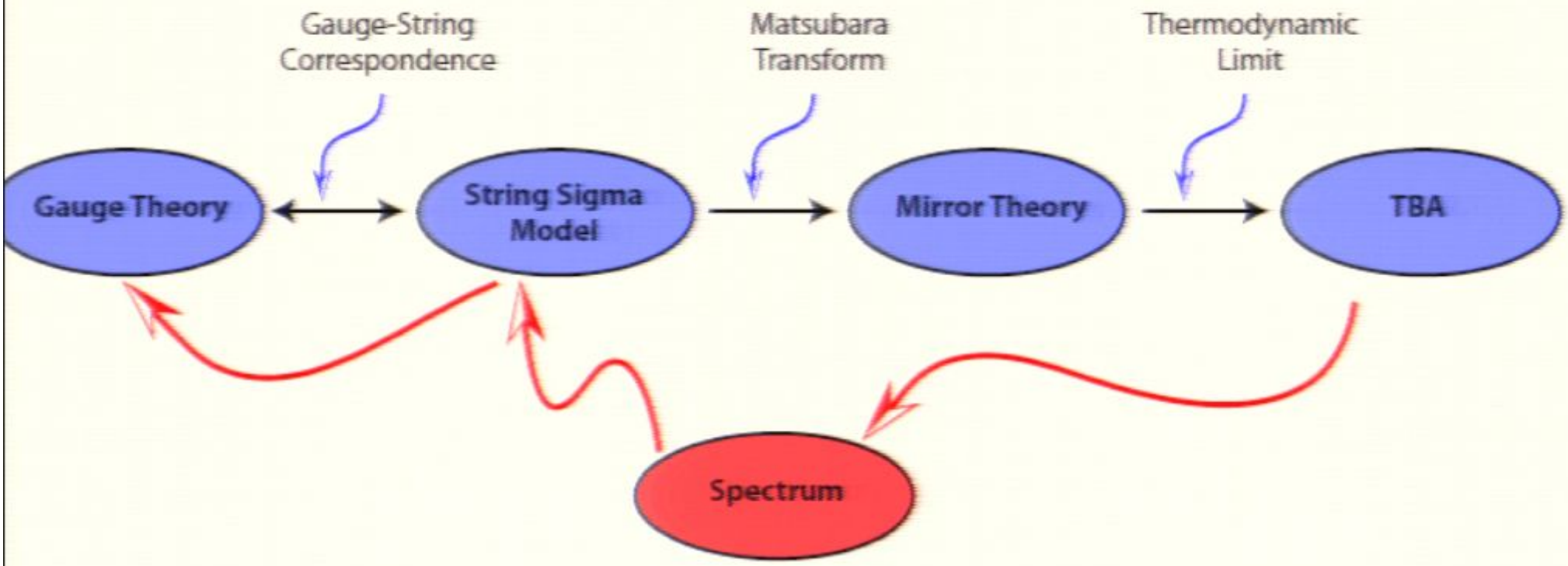
Summary of the TBA approach



String theory and N=4 SYM results

- Semi-classical strings — Berenstein, Maldacena, Nastase '02; Gubser, Klebanov, Polyakov '02; Frolov, Tseytlin '02, '03; Bena, Polchinski, Roiban '03; Kazakov, Marshakov, Minahan, Zarembo '04;
- L.c. strings in $AdS_5 \times S^5$ — Arutyunov, Frolov '04, '05; Frolov, Plefka, Zamaklar '06; Arutyunov, Frolov, Plefka, Zamaklar '06;
- Decompactification: $J \to \infty$ — Ambjorn, Janik, Kristjansen '05; Janik '06; Arutyunov, Frolov '06; Hofman, Maldacena '06;
- Symmetry algebra — Beisert '05, '06; Arutyunov, Frolov, Plefka, Zamaklar '06;
- Dispersion relations — Beisert, Dippel, Staudacher '04; Beisert '05; N.Dorey '06;
- S-matrix — Staudacher '04; Beisert '05; Arutyunov, Frolov, Zamaklar '06;
- Dressing factor and crossing eqs — Arutyunov, Frolov, Staudacher '04; Beisert, Tseytlin '05; Janik '06; Hernandez, Lopez '06; Arutyunov, Frolov '06; Beisert, Hernandez, Lopez '06; Beisert, Eden, Staudacher '06;
- Bethe ansatz — Minahan, Zarembo '02; Beisert, Dippel, Staudacher '04; Arutyunov, Frolov, Staudacher '04; Staudacher '04; Beisert, Staudacher '05;

Summary of the TBA approach



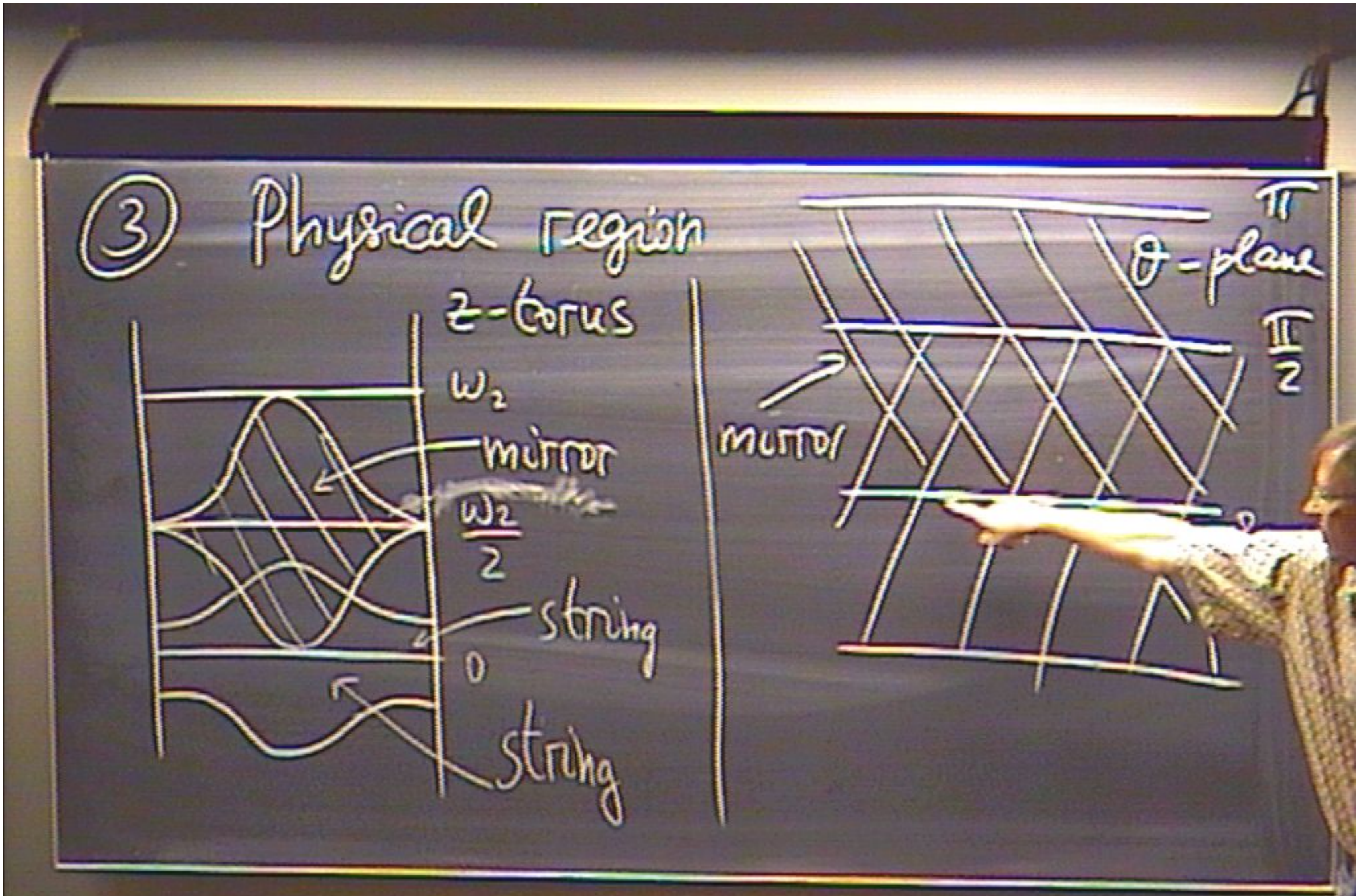
String theory and N=4 SYM results

- Semi-classical strings — Berenstein, Maldacena, Nastase '02; Gubser, Klebanov, Polyakov '02; Frolov, Tseytlin '02, '03; Bena, Polchinski, Roiban '03; Kazakov, Marshakov, Minahan, Zarembo '04;
- L.c. strings in $AdS_5 \times S^5$ — Arutyunov, Frolov '04, '05; Frolov, Plefka, Zamaklar '06; Arutyunov, Frolov, Plefka, Zamaklar '06;
- Decompactification: $J \to \infty$ — Ambjorn, Janik, Kristjansen '05; Janik '06; Arutyunov, Frolov '06; Hofman, Maldacena '06;
- Symmetry algebra — Beisert '05, '06; Arutyunov, Frolov, Plefka, Zamaklar '06;
- Dispersion relations — Beisert, Dippel, Staudacher '04; Beisert '05; N.Dorey '06;
- S-matrix — Staudacher '04; Beisert '05; Arutyunov, Frolov, Zamaklar '06;
- Dressing factor and crossing eqs — Arutyunov, Frolov, Staudacher '04; Beisert, Tseytlin '05; Janik '06; Hernandez, Lopez '06; Arutyunov, Frolov '06; Beisert, Hernandez, Lopez '06; Beisert, Eden, Staudacher '06;
- Bethe ansatz — Minahan, Zarembo '02; Beisert, Dippel, Staudacher '04; Arutyunov, Frolov, Staudacher '04; Staudacher '04; Beisert, Staudacher '05;

Comparison chart

Arutyunov, Frolov '07

	Strings	Mirrors
Dispersion relation	$\mathcal{E}_Q = \sqrt{Q^2 + 4g^2 \sin^2 \frac{p}{2}}$	$\tilde{\mathcal{E}}_Q = 2 \operatorname{arcsinh}\left(\frac{1}{2g}\sqrt{Q^2 + \tilde{p}^2}\right)$
Momentum	$-\pi \leq p < \pi$	$-\infty < \tilde{p} < \infty$
Type of theory	Lattice model	Continuum model
Giant magnon	Soliton in $R \times S^5$	Soliton in AdS_5
Bound states	Symmetric irrep su(2) sector	Antisymmetric irrep sl(2) sector
Physical region	"Fish" (?)	"Leaf" (?)
S − matrix	$S(z_1, z_2)$	$S(z_1 + \frac{\omega_2}{2}, z_2 + \frac{\omega_2}{2})$
Bethe − Yang eqs	BS; $P = 0$	extra $\sqrt{x^+/x^-}$
Dressing factor	$\sigma(1,2)^* \sigma(1,2) = 1$	$\sigma(1,2)^* \sigma(1,2) = \frac{x_1^+}{x_1^-} \frac{x_2^-}{x_2^+}$


③ Physical region
z-torus
ω_2
mirror
$\frac{\omega_2}{2}$
string
0
string
π
θ-plane
$\frac{\pi}{2}$
mirror
0

Comparison chart

Arutyunov, Frolov '07

	Strings	Mirrors
Dispersion relation	$\mathcal{E}_Q = \sqrt{Q^2 + 4g^2 \sin^2 \frac{p}{2}}$	$\tilde{\mathcal{E}}_Q = 2\,\mathrm{arcsinh}\left(\frac{1}{2g}\sqrt{Q^2 + \tilde{p}^2}\right)$
Momentum	$-\pi \leq p < \pi$	$-\infty < \tilde{p} < \infty$
Type of theory	Lattice model	Continuum model
Giant magnon	Soliton in $\mathrm{R} \times \mathrm{S}^5$	Soliton in AdS_5
Bound states	Symmetric irrep su(2) sector	Antisymmetric irrep sl(2) sector
Physical region	"Fish" (?)	"Leaf" (?)
S − matrix	$S(z_1, z_2)$	$S(z_1 + \frac{\omega_2}{2}, z_2 + \frac{\omega_2}{2})$
Bethe − Yang eqs	BS; $\mathrm{P} = 0$	extra $\sqrt{x^+/x^-}$
Dressing factor	$\sigma(1,2)^* \sigma(1,2) = 1$	$\sigma(1,2)^* \sigma(1,2) = \frac{x_1^+}{x_1^-} \frac{x_2^-}{x_2^+}$

Comparison chart

Arutyunov, Frolov '07

	Strings	Mirrors
Dispersion relation	$\mathcal{E}_Q = \sqrt{Q^2 + 4g^2 \sin^2 \frac{p}{2}}$	$\tilde{\mathcal{E}}_Q = 2 \operatorname{arcsinh}\left(\frac{1}{2g}\sqrt{Q^2 + \tilde{p}^2}\right)$
Momentum	$-\pi \leq p < \pi$	$-\infty < \tilde{p} < \infty$
Type of theory	Lattice model	Continuum model
Giant magnon	Soliton in $\mathrm{R} \times \mathrm{S}^5$	Soliton in AdS_5
Bound states	Symmetric irrep su(2) sector	Antisymmetric irrep sl(2) sector
Physical region	"Fish" (?)	"Leaf" (?)
S − matrix	$S(z_1, z_2)$	$S(z_1 + \frac{\omega_2}{2}, z_2 + \frac{\omega_2}{2})$
Bethe − Yang eqs	BS; $\mathrm{P} = 0$	extra $\sqrt{x^+/x^-}$
Dressing factor	$\sigma(1,2)^* \sigma(1,2) = 1$	$\sigma(1,2)^* \sigma(1,2) = \frac{x_1^+}{x_1^-} \frac{x_2^-}{x_2^+}$

Mirror TBA

- Ground state energy is related to the free energy of the mirror theory at temperature $T = 1/J$ Al. Zamolodchikov '90

$$E(J) = J\mathcal{F}(J)$$

- Mirror TBA for the ground state is a set of nonlinear integral equations on Y–functions. Its solution computes the free energy

Mirror TBA

- Ground state energy is related to the free energy of the mirror theory at temperature $T = 1/J$ Al. Zamolodchikov '90

$$E(J) = J\,\mathcal{F}(J)$$

- Mirror TBA for the ground state is a set of nonlinear integral equations on Y–functions. Its solution computes the free energy
- TBA eqs follow from the string hypothesis Takahashi '72
  for the mirror model Arutyunov, Frolov '09(a)
- A Bethe string leads to a Y–function ($Q = 1, 2, \ldots$)

$$Y^{(-)}_{Q|w},\ Y^{(-)}_{Q|vw},\ Y^{(-)}_{+},\ Y^{(-)}_{-},\ Y_Q,\ Y^{(+)}_{-},\ Y^{(+)}_{+},\ Y^{(+)}_{Q|vw},\ Y^{(+)}_{Q|w}$$

- Ground state energy

$$E - J = -\underbrace{\frac{1}{2\pi}\sum_{Q=1}^{\infty}\int_{-\infty}^{\infty} d\tilde{p}\,\log(1 + Y_Q)}$$

Mirror TBA

- TBA eqs can be written in various forms
  - Canonical — Arutyunov, Frolov '09(b); Bombardelli, Fioravanti, Tateo '09
  - Simplified — Arutyunov, Frolov '09(b), '09(d)
  - Hybrid — Arutyunov, Frolov, Suzuki '09
  - Quasi-local — Balog,Hegedus '11
- They lead to the Y–system conjectured by — Gromov, Kazakov, Vieira '09(a)
- and jump discontinuity conditions on Y–functions — Arutyunov, Frolov '09(b); Arutyunov, Frolov, Suzuki '09; Cavaglia, Fioravanti, Tateo '10

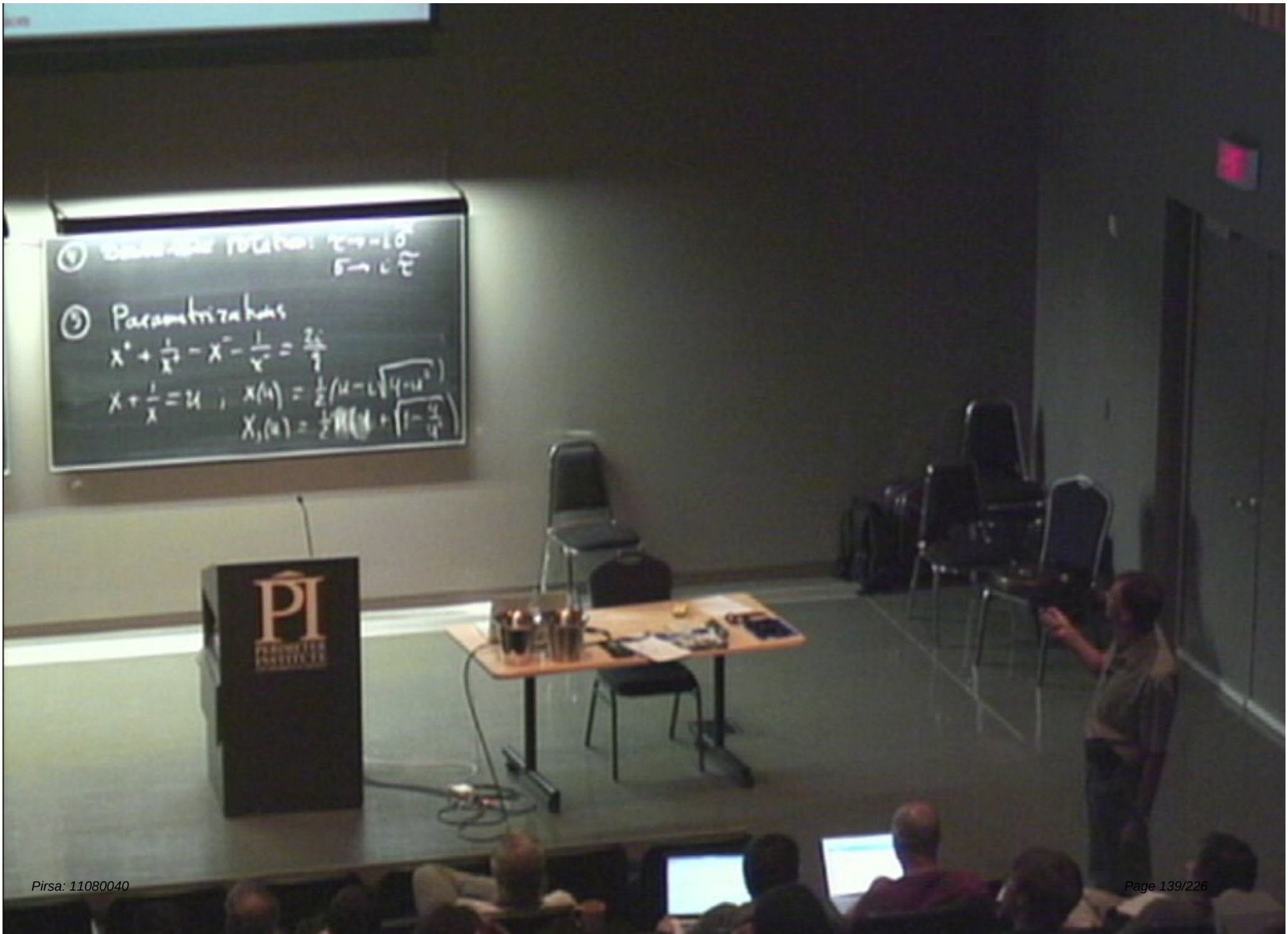
Mirror TBA

- TBA eqs can be written in various forms
  - Canonical — Arutyunov, Frolov '09(b); Bombardelli, Fioravanti, Tateo '09
  - Simplified — Arutyunov, Frolov '09(b), '09(d)
  - Hybrid — Arutyunov, Frolov, Suzuki '09
  - Quasi-local — Balog, Hegedus '11
- They lead to the Y–system conjectured by — Gromov, Kazakov, Vieira '09(a)
- and jump discontinuity conditions on Y–functions — Arutyunov, Frolov '09(b); Arutyunov, Frolov, Suzuki '09; Cavaglia, Fioravanti, Tateo '10
- TBA eqs for excited states via the contour deformation trick (inspired by P. Dorey, Tateo '96) — Gromov, Kazakov, Kozak, Vieira '09v3; Arutyunov, Frolov, Suzuki '09
- or via the Y-system and jump discontinuities (following Bazhanov, Lukyanov, Zamolodchikov '96) — Balog, Hegedus '11

Simplified TBA equations

Arutyunov, Frolov '09(b), '09(d)

- $Q = 1$-particle:
  $$\log Y_1 = -\log\left(1+\frac{1}{Y_2}\right)\star s + \log\left(1-\frac{1}{Y_-^{(-)}}\right)\left(1-\frac{1}{Y_-^{(+)}}\right)\hat{\star}\, s - \Delta \check{\star}\, s$$
- Q-particles for $Q \geq 2$: $\log Y_Q = \log \dfrac{\left(1+\frac{1}{Y^{(-)}_{Q-1|vw}}\right)\left(1+\frac{1}{Y^{(+)}_{Q-1|vw}}\right)}{(1+\frac{1}{Y_{Q-1}})(1+\frac{1}{Y_{Q+1}})} \star s$
- y-particles: $\log \dfrac{Y_+^{(\alpha)}}{Y_-^{(\alpha)}} = \log(1+Y_Q)\star K_{Qy}\,, \quad \alpha = -,+,$
  $$\log Y_+^{(\alpha)} Y_-^{(\alpha)} = 2\log\frac{1+Y^{(\alpha)}_{1|vw}}{1+Y^{(\alpha)}_{1|w}}\star s - \log(1+Y_Q)\star K_Q + 2\log(1+Y_Q)\star K_{xv}^{Q1}\star s$$
- $M|vw$-strings:
  $$\log Y^{(\alpha)}_{M|vw} = \log\frac{(1+Y^{(\alpha)}_{M-1|vw})(1+Y^{(\alpha)}_{M+1|vw})}{1+Y_{M+1}}\star s + \delta_{M1}\log\frac{1-Y_-^{(\alpha)}}{1-Y_+^{(\alpha)}}\hat{\star}\, s$$
- $M|w$-strings: $\log Y^{(\alpha)}_{M|w} = \log(1+Y^{(\alpha)}_{M-1|w})(1+Y^{(\alpha)}_{M+1|w})\star s + \delta_{M1}\log\dfrac{1-\frac{1}{Y_-^{(\alpha)}}}{1-\frac{1}{Y_+^{(\alpha)}}}\hat{\star}\, s$
- Ground state energy
  $$E = J - \underbrace{\frac{1}{2\pi}\sum_{Q=1}^{\infty}\int_{C_Q} d\tilde{p}\,\log(1+Y_Q)}$$


Parametrizations

Simplified TBA equations

Arutyunov, Frolov '09(b), '09(d)

- $Q = 1$-particle:
  $$\log Y_1 = -\log\left(1+\frac{1}{Y_2}\right)\star s + \log\left(1-\frac{1}{Y_-^{(-)}}\right)\left(1-\frac{1}{Y_-^{(+)}}\right)\hat{\star}\, s - \Delta\,\check{\star}\, s$$
- Q-particles for $Q \geq 2$: $\log Y_Q = \log \dfrac{\left(1+\frac{1}{Y^{(-)}_{Q-1|vw}}\right)\left(1+\frac{1}{Y^{(+)}_{Q-1|vw}}\right)}{(1+\frac{1}{Y_{Q-1}})(1+\frac{1}{Y_{Q+1}})} \star s$
- y-particles: $\log \dfrac{Y_+^{(\alpha)}}{Y_-^{(\alpha)}} = \log(1+Y_Q)\star K_{Qy}\,, \quad \alpha = -,+,$
  $$\log Y_+^{(\alpha)} Y_-^{(\alpha)} = 2\log\frac{1+Y^{(\alpha)}_{1|vw}}{1+Y^{(\alpha)}_{1|w}}\star s - \log(1+Y_Q)\star K_Q + 2\log(1+Y_Q)\star K_{xv}^{Q1}\star s$$
- $M|vw$-strings:
  $$\log Y^{(\alpha)}_{M|vw} = \log\frac{(1+Y^{(\alpha)}_{M-1|vw})(1+Y^{(\alpha)}_{M+1|vw})}{1+Y_{M+1}}\star s + \delta_{M1}\log\frac{1-Y_-^{(\alpha)}}{1-Y_+^{(\alpha)}}\,\hat{\star}\, s$$
- $M|w$-strings: $\log Y^{(\alpha)}_{M|w} = \log(1+Y^{(\alpha)}_{M-1|w})(1+Y^{(\alpha)}_{M+1|w})\star s + \delta_{M1}\log\dfrac{1-\frac{1}{Y_-^{(\alpha)}}}{1-\frac{1}{Y_+^{(\alpha)}}}\,\hat{\star}\, s$
- Ground state energy
  $$E = J - \underbrace{\frac{1}{2\pi}\sum_{Q=1}^{\infty}\int_{C_Q} d\tilde{p}\,\log(1+Y_Q)}$$

④ Double-Wick rotation: $\tau \to -i\tilde{\sigma}$

$$f \times g(v) = \int_C f(u)\, g(u,v)\, du \qquad \sigma \to i\tilde{\tau}$$

⑤ Parametrizations

$$x^+ + \frac{1}{x^+} - x^- - \frac{1}{x^-} = 2i$$

$$x + \frac{1}{x} = u \;;\quad x(u) = \frac{1}{2}\left(\sqrt{4-u^2}\right)$$

$$x_s(u) = \left(\sqrt{1-\frac{4}{u^2}}\right)$$

① String tension: $g = \frac{\sqrt{\lambda}}{2\pi}$

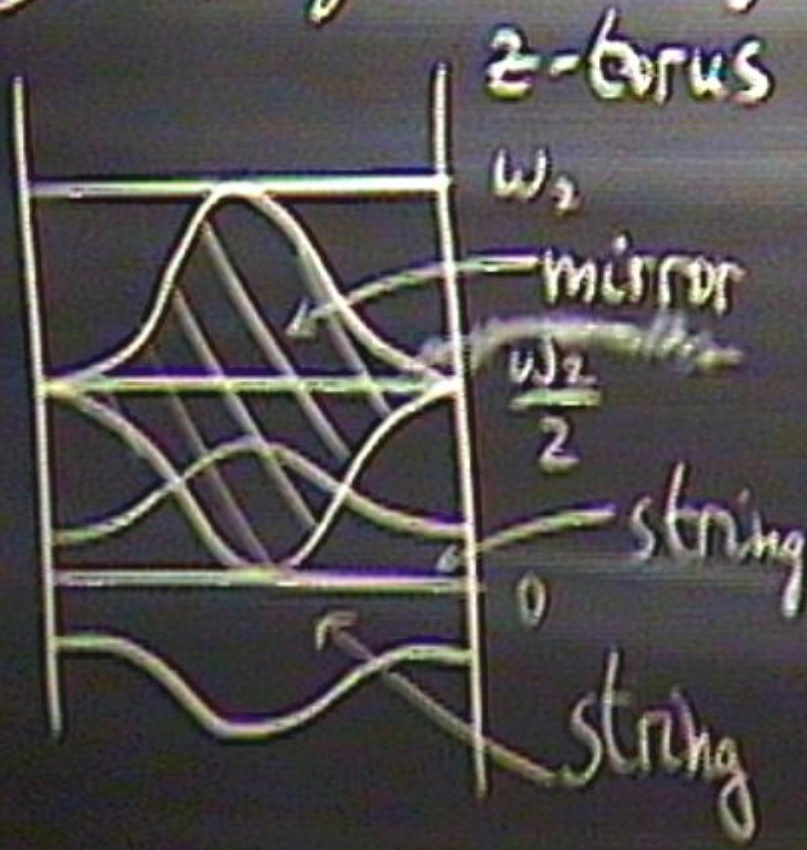
② L.C. gauge: $t = \tau$; $P_\varphi = 1$

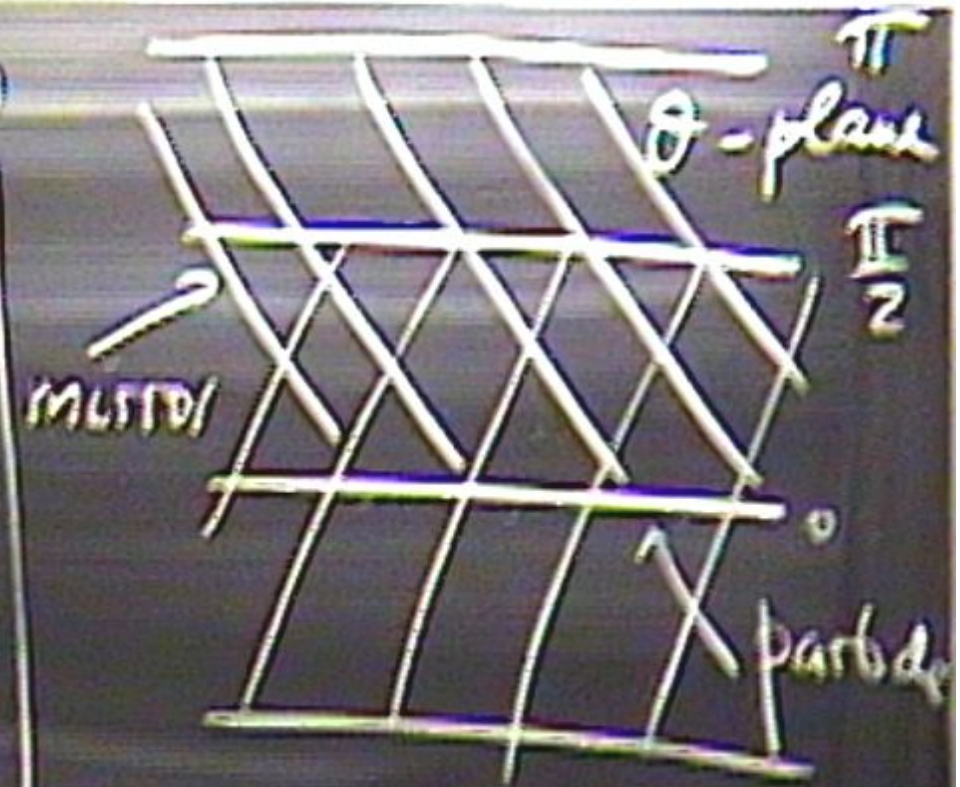
t ↑ AdS time

φ ↑ S^5 angle

$$J = \int_{-J/2}^{J/2} P_\varphi \, d\sigma \; ; \; E = -\int_{-J/2}^{J/2} P_t \, d\sigma$$

③ Physical region





Simplified TBA equations

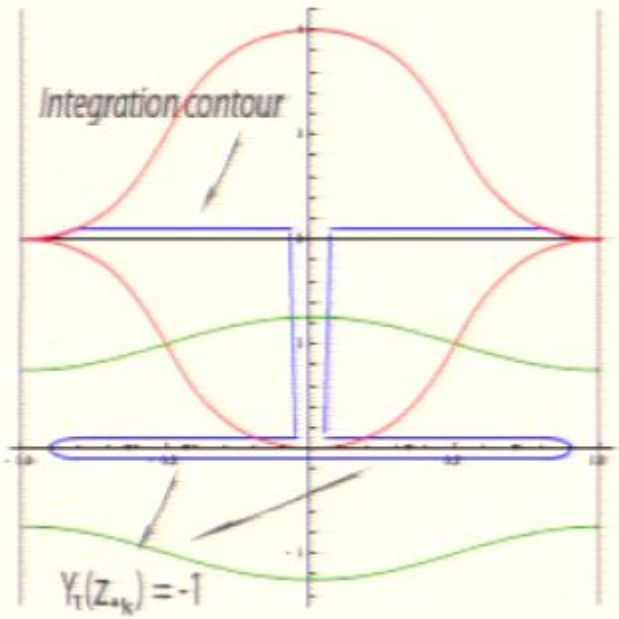
Arutyunov, Frolov '09(b), '09(d)

- $Q = 1$-particle:
  $$\log Y_1 = -\log\left(1+\frac{1}{Y_2}\right)\star s + \log\left(1-\frac{1}{Y_-^{(-)}}\right)\left(1-\frac{1}{Y_-^{(+)}}\right)\hat{\star}\, s - \Delta \check{\star}\, s$$
- Q-particles for $Q \geq 2$: $\log Y_Q = \log \dfrac{\left(1+\frac{1}{Y^{(-)}_{Q-1|vw}}\right)\left(1+\frac{1}{Y^{(+)}_{Q-1|vw}}\right)}{(1+\frac{1}{Y_{Q-1}})(1+\frac{1}{Y_{Q+1}})} \star s$
- y-particles: $\log \dfrac{Y_+^{(\alpha)}}{Y_-^{(\alpha)}} = \log(1+Y_Q)\star K_{Qy}\,, \quad \alpha = -,+,$
  $$\log Y_+^{(\alpha)} Y_-^{(\alpha)} = 2\log\frac{1+Y^{(\alpha)}_{1|vw}}{1+Y^{(\alpha)}_{1|w}}\star s - \log(1+Y_Q)\star K_Q + 2\log(1+Y_Q)\star K_{xv}^{Q1}\star s$$
- $M|vw$-strings:
  $$\log Y^{(\alpha)}_{M|vw} = \log\frac{(1+Y^{(\alpha)}_{M-1|vw})(1+Y^{(\alpha)}_{M+1|vw})}{1+Y_{M+1}}\star s + \delta_{M1}\log\frac{1-Y_-^{(\alpha)}}{1-Y_+^{(\alpha)}}\hat{\star}\, s$$
- $M|w$-strings: $\log Y^{(\alpha)}_{M|w} = \log(1+Y^{(\alpha)}_{M-1|w})(1+Y^{(\alpha)}_{M+1|w})\star s + \delta_{M1}\log\dfrac{1-\frac{1}{Y_-^{(\alpha)}}}{1-\frac{1}{Y_+^{(\alpha)}}}\hat{\star}\, s$
- Ground state energy
  $$E = J - \underbrace{\frac{1}{2\pi}\sum_{Q=1}^{\infty}\int_{C_Q} d\tilde{p}\,\log(1+Y_Q)}$$

Excited states TBA and CDT

Arutyunov, Frolov, Suzuki '09

Inspired by P. Dorey, Tateo '96



- TBA equations for excited states differ from each other only by a choice of the integration contour $\Longrightarrow$ contour deformation trick
- If $Y(z_*) = -1$ (or ∞) then taking the contour back to real mirror line produces driving term $-\log S(z_*, z)$ from $\log(1+Y) \star K$; $K(w,z) = \frac{1}{2\pi i}\frac{d}{dw}\log S(w,z)$

$$\log(1+Y)\star K \quad \Longrightarrow \quad \frac{1}{2\pi i}\oint_{z_*} \log(1+Y(w))\frac{d}{dw}\log S(w,z) =$$

$$= -\frac{1}{2\pi i}\oint_{z_*} \frac{d}{dw}\log(w-z_*)\,\log S(w,z) = \underbrace{-\log S(z_*,z)}$$

Simplified TBA equations

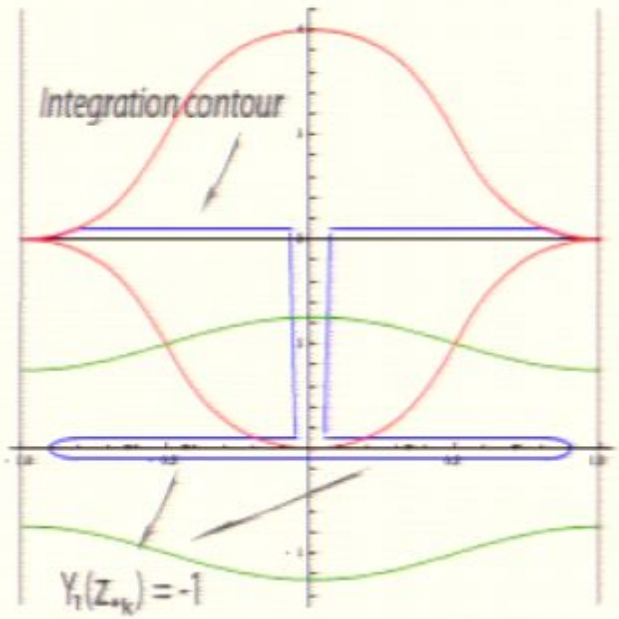
Aruryunov, Frolov '09(b), '09(d)

- $Q = 1$-particle:
  $$\log Y_1 = -\log\left(1+\frac{1}{Y_2}\right)\star s + \log\left(1-\frac{1}{Y_-^{(-)}}\right)\left(1-\frac{1}{Y_-^{(+)}}\right)\hat{\star}\, s - \Delta \check{\star}\, s$$
- Q-particles for $Q \geq 2$: $\log Y_Q = \log \dfrac{\left(1+\frac{1}{Y^{(-)}_{Q-1|vw}}\right)\left(1+\frac{1}{Y^{(+)}_{Q-1|vw}}\right)}{(1+\frac{1}{Y_{Q-1}})(1+\frac{1}{Y_{Q+1}})} \star s$
- y-particles: $\log \dfrac{Y_+^{(\alpha)}}{Y_-^{(\alpha)}} = \log(1+Y_Q)\star K_{Qy}\,, \quad \alpha = -,+,$
  $$\log Y_+^{(\alpha)} Y_-^{(\alpha)} = 2\log\frac{1+Y^{(\alpha)}_{1|vw}}{1+Y^{(\alpha)}_{1|w}}\star s - \log(1+Y_Q)\star K_Q + 2\log(1+Y_Q)\star K_{xv}^{Q1}\star s$$
- $M|vw$-strings:
  $$\log Y^{(\alpha)}_{M|vw} = \log\frac{(1+Y^{(\alpha)}_{M-1|vw})(1+Y^{(\alpha)}_{M+1|vw})}{1+Y_{M+1}}\star s + \delta_{M1}\log\frac{1-Y_-^{(\alpha)}}{1-Y_+^{(\alpha)}}\hat{\star}\, s$$
- $M|w$-strings: $\log Y^{(\alpha)}_{M|w} = \log(1+Y^{(\alpha)}_{M-1|w})(1+Y^{(\alpha)}_{M+1|w})\star s + \delta_{M1}\log\dfrac{1-\frac{1}{Y_-^{(\alpha)}}}{1-\frac{1}{Y_+^{(\alpha)}}}\hat{\star}\, s$
- Ground state energy
  $$E = J - \underbrace{\frac{1}{2\pi}\sum_{Q=1}^{\infty}\int_{C_Q} d\tilde{p}\,\log(1+Y_Q)}$$

Excited states TBA and CDT

Arutyunov, Frolov, Suzuki '09

Inspired by P. Dorey, Tateo '96



- TBA equations for excited states differ from each other only by a choice of the integration contour $\Longrightarrow$ contour deformation trick
- If $Y(z_*) = -1$ (or ∞) then taking the contour back to real mirror line produces driving term $-\log S(z_*, z)$ from $\log(1+Y) \star K$; $K(w,z) = \frac{1}{2\pi i}\frac{d}{dw}\log S(w,z)$

$$\log(1+Y)\star K \quad \Longrightarrow \quad \frac{1}{2\pi i}\oint_{z_*} \log(1+Y(w))\frac{d}{dw}\log S(w,z) =$$

$$= -\frac{1}{2\pi i}\oint_{z_*} \frac{d}{dw}\log(w-z_*)\log S(w,z) = \underbrace{-\log S(z_*,z)}$$

Simplified TBA equations

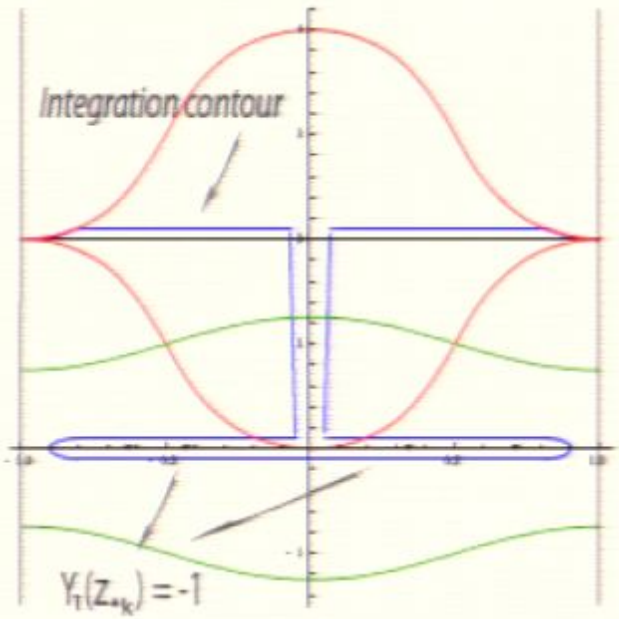
Arutyunov, Frolov '09(b), '09(d)

- $Q = 1$-particle:
  $$\log Y_1 = -\log\left(1+\frac{1}{Y_2}\right)\star s + \log\left(1-\frac{1}{Y_-^{(-)}}\right)\left(1-\frac{1}{Y_-^{(+)}}\right)\hat{\star}\, s - \Delta \check{\star}\, s$$
- Q-particles for $Q \geq 2$: $\log Y_Q = \log \dfrac{\left(1+\frac{1}{Y^{(-)}_{Q-1|vw}}\right)\left(1+\frac{1}{Y^{(+)}_{Q-1|vw}}\right)}{(1+\frac{1}{Y_{Q-1}})(1+\frac{1}{Y_{Q+1}})} \star s$
- y-particles: $\log \dfrac{Y_+^{(\alpha)}}{Y_-^{(\alpha)}} = \log(1+Y_Q)\star K_{Qy}\,, \quad \alpha = -,+,$
  $$\log Y_+^{(\alpha)} Y_-^{(\alpha)} = 2\log\frac{1+Y^{(\alpha)}_{1|vw}}{1+Y^{(\alpha)}_{1|w}}\star s - \log(1+Y_Q)\star K_Q + 2\log(1+Y_Q)\star K_{xv}^{Q1}\star s$$
- $M|vw$-strings:
  $$\log Y^{(\alpha)}_{M|vw} = \log\frac{(1+Y^{(\alpha)}_{M-1|vw})(1+Y^{(\alpha)}_{M+1|vw})}{1+Y_{M+1}}\star s + \delta_{M1}\log\frac{1-Y_-^{(\alpha)}}{1-Y_+^{(\alpha)}}\hat{\star}\, s$$
- $M|w$-strings: $\log Y^{(\alpha)}_{M|w} = \log(1+Y^{(\alpha)}_{M-1|w})(1+Y^{(\alpha)}_{M+1|w})\star s + \delta_{M1}\log\dfrac{1-\frac{1}{Y_-^{(\alpha)}}}{1-\frac{1}{Y_+^{(\alpha)}}}\hat{\star}\, s$
- Ground state energy
  $$E = J - \underbrace{\frac{1}{2\pi}\sum_{Q=1}^{\infty}\int_{C_Q} d\tilde{p}\,\log(1+Y_Q)}$$

Excited states TBA and CDT

Arutyunov, Frolov, Suzuki '09

Inspired by P. Dorey, Tateo '96



- TBA equations for excited states differ from each other only by a choice of the integration contour $\Longrightarrow$ contour deformation trick
- If $Y(z_*) = -1$ (or ∞) then taking the contour back to real mirror line produces driving term $-\log S(z_*, z)$ from $\log(1+Y) \star K$; $K(w,z) = \frac{1}{2\pi i}\frac{d}{dw}\log S(w,z)$

$$\log(1+Y)\star K \quad \Longrightarrow \quad \frac{1}{2\pi i}\oint_{z_*} \log(1+Y(w))\frac{d}{dw}\log S(w,z) =$$

$$= -\frac{1}{2\pi i}\oint_{z_*} \frac{d}{dw}\log(w-z_*)\,\log S(w,z) = \underbrace{-\log S(z_*,z)}$$

Exact Bethe equations

The spectrum of excited states

$$E = J + \underbrace{\sum_{k=1}^{N} \mathcal{E}(p_k)}_{\text{Bethe}-\text{Yang}} - \underbrace{\frac{1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} d\tilde{p} \, \log(1 + Y_Q)}_{\text{finite}-\text{size contribution}}$$

Momenta p_k (or rapidities u_k) are found from the *exact Bethe equations* (quantization cond.)

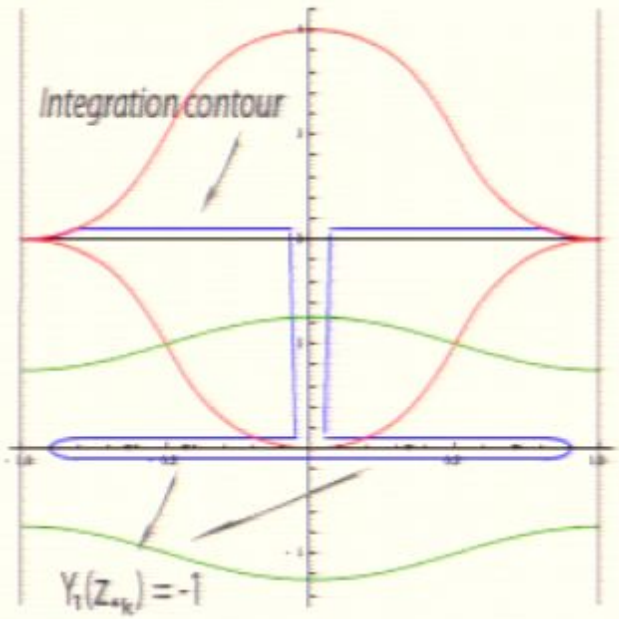
Bazhanov, Lukyanov, Zamolodchikov '96; P. Dorey, Tateo '96

$$Y_{1_*}(p_k) = -1$$

Excited states TBA and CDT

Arutyunov, Frolov, Suzuki '09

Inspired by P. Dorey, Tateo '96



- TBA equations for excited states differ from each other only by a choice of the integration contour $\Longrightarrow$ contour deformation trick
- If $Y(z_*) = -1$ (or ∞) then taking the contour back to real mirror line produces driving term $-\log S(z_*, z)$ from $\log(1+Y) \star K$; $K(w,z) = \frac{1}{2\pi i}\frac{d}{dw}\log S(w,z)$

$$\log(1+Y)\star K \quad \Longrightarrow \quad \frac{1}{2\pi i}\oint_{z_*} \log(1+Y(w))\frac{d}{dw}\log S(w,z) =$$

$$= -\frac{1}{2\pi i}\oint_{z_*} \frac{d}{dw}\log(w-z_*)\,\log S(w,z) = \underbrace{-\log S(z_*,z)}$$

Exact Bethe equations

The spectrum of excited states

$$E = J + \underbrace{\sum_{k=1}^{N} \mathcal{E}(p_k)}_{\text{Bethe}-\text{Yang}} - \underbrace{\frac{1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} d\tilde{p} \, \log(1 + Y_Q)}_{\text{finite}-\text{size contribution}}$$

Momenta p_k (or rapidities u_k) are found from the *exact Bethe equations* (quantization cond.)

Bazhanov, Lukyanov, Zamolodchikov '96; P. Dorey, Tateo '96

$$Y_{1_*}(p_k) = -1$$

Exact Bethe equations

The spectrum of excited states

$$E = J + \underbrace{\sum_{k=1}^{N} \mathcal{E}(p_k)}_{\text{Bethe}-\text{Yang}} - \underbrace{\frac{1}{2\pi} \sum_{Q=1}^{\infty} \int_{-\infty}^{\infty} d\tilde{p} \log(1 + Y_Q)}_{\text{finite}-\text{size contribution}}$$

Momenta p_k (or rapidities u_k) are found from the *exact Bethe equations* (quantization cond.)

Bazhanov, Lukyanov, Zamolodchikov '96; P. Dorey, Tateo '96

$$Y_{1_*}(p_k) = -1$$

The EBE work fine for *real* p_k.
Do they need a modification for *complex* ones?

Bajnok-Janik asymptotic solution (large J or small g and finite J)

- Generalized Lüscher formulae give the large J asymptotic solution Bajnok, Janik '08

$$Y_Q^o(v) = \Upsilon_Q(v)\, T_{Q,-1}(v|\bar{u})\, T_{Q,1}(v|\bar{u})$$

$$\Upsilon_Q(v) = e^{-J\tilde{\mathcal{E}}_Q(v)} \prod_{i=1}^{N} S^{Q1}_{*\,\mathfrak{sl}(2)}(v, u_i) \quad \leftarrow \quad \text{exp suppressed}$$

- $T_{Q,1}$ is an eigenvalue of a properly normalized $\mathfrak{su}(2|2)_C$ transfer matrix

$$\mathcal{T}_{Q,1}(v|\bar{u}) = \mathrm{str}_A\, S^{Q,1}_{A1}(v, u_1) S^{Q,1}_{A2}(v, u_2) \cdots S^{Q,1}_{AN}(v, u_N)\,,$$

conjectured by Beisert '06
and derived by Arutyunov, de Leeuw, Suzuki, Torrieli '09

Bajnok-Janik asymptotic solution (large J or small g and finite J)

- Generalized Lüscher formulae give the large J asymptotic solution Bajnok, Janik '08

$$Y_Q^o(v) = \Upsilon_Q(v)\, T_{Q,-1}(v|\bar{u})\, T_{Q,1}(v|\bar{u})$$

$$\Upsilon_Q(v) = e^{-J\tilde{\mathcal{E}}_Q(v)} \prod_{i=1}^{N} S_{\mathfrak{sl}(2)}^{Q1_*}(v, u_i) \quad \leftarrow \quad \text{exp suppressed}$$

- $T_{Q,1}$ is an eigenvalue of a properly normalized $\mathfrak{su}(2|2)_C$ transfer matrix

$$\mathcal{T}_{Q,1}(v|\bar{u}) = \mathrm{str}_A\, S_{A1}^{Q,1}(v, u_1) S_{A2}^{Q,1}(v, u_2) \cdots S_{AN}^{Q,1}(v, u_N)\,,$$

conjectured by Beisert '06
and derived by Arutyunov, de Leeuw, Suzuki, Torrieli '09

- The BY equations (in the $\mathfrak{sl}(2)$ sector) follow from $\tilde{\mathcal{E}}_{1_*}(u_k) = -i\, p_k$ and

$$T_{1_*,\pm 1}(u_k|\bar{u}) = 1 \implies -1 = e^{i\,J p_k} \prod_{j=1}^{N} S_{\mathfrak{sl}(2)}^{1_* 1_*}(u_k, u_j)$$

- All auxiliary Y^o-functions are fixed by Y_Q^o Arutyunov, Frolov '10
and (almost) agree with Gromov, Kazakov, Vieira '09(a)

BY equations

Beisert, Staudacher '05

The main Bethe equations have the form

$$1 = e^{iJp_k} \prod_{l\neq k}^{K^{\rm I}} S_{su(2)}(u_k, u_l) \prod_{l=1}^{K_-^{\rm II}} \frac{x_k^- - y_l^{(-)}}{x_k^+ - y_l^{(-)}} \sqrt{\frac{x_k^+}{x_k^-}} \prod_{l=1}^{K_+^{\rm II}} \frac{x_k^- - y_l^{(+)}}{x_k^+ - y_l^{(+)}} \sqrt{\frac{x_k^+}{x_k^-}}$$

They are supplied with auxiliary Bethe equations for the roots $y^{(\alpha)}$ and $w^{(\alpha)}$, $\alpha = \pm$

$$\prod_{i=1}^{K^{\rm I}} \frac{y_k^{(\alpha)} - x_i^-}{y_k^{(\alpha)} - x_i^+} \sqrt{\frac{x_i^+}{x_i^-}} = \prod_{i=1}^{K_\alpha^{\rm III}} \frac{w_i^{(\alpha)} - \nu_k^{(\alpha)} - \frac{i}{g}}{w_i^{(\alpha)} - \nu_k^{(\alpha)} + \frac{i}{g}}, \qquad \nu_k^{(\alpha)} \equiv y_k^{(\alpha)} + \frac{1}{y_k^{(\alpha)}}$$

$$\prod_{i=1}^{K_\alpha^{\rm II}} \frac{w_k^{(\alpha)} - \nu_i^{(\alpha)} + \frac{i}{g}}{w_k^{(\alpha)} - \nu_i^{(\alpha)} - \frac{i}{g}} = -\prod_{i=1}^{K_\alpha^{\rm III}} \frac{w_k^{(\alpha)} - w_i^{(\alpha)} + \frac{2i}{g}}{w_k^{(\alpha)} - w_i^{(\alpha)} - \frac{2i}{g}}.$$

Here $K^{\rm I}$ is # of particles, $K_\alpha^{\rm II}$, $K_\alpha^{\rm III}$ are weights of four $SU(2)$'s of manifest symmetry of the l.c. string sigma model. $SU(4)$ weights $[q_1, p, q_2]$ and the spins $[s_1, s_2]$ are

$$q_1 = K_-^{\rm II} - 2K_-^{\rm III}, \quad q_2 = K_+^{\rm II} - 2K_+^{\rm III} \qquad s_1 = K^{\rm I} - K_-^{\rm II}$$
$$p = J - \tfrac{1}{2}(K_-^{\rm II} + K_+^{\rm II}) + K_-^{\rm III} + K_+^{\rm III} \qquad s_2 = K^{\rm I} - K_+^{\rm II}$$

Instead of weights of $su(4)$ one can use the weights (J_1, J_2, J_3) of $SO(6)$

$$J \equiv J_1 = \frac{1}{2}(q_1 + 2p + q_2), \qquad J_2 = \frac{1}{2}(q_1 + q_2), \qquad J_3 = \frac{1}{2}(q_2 - q_1).$$

CDT: General strategy

1. Solve the BY equations (in any grading) for a fixed set of integers

$$J, \quad N = K^{\mathrm{I}}, \quad (K_-^{\mathrm{III}}, K_-^{\mathrm{II}}, K_+^{\mathrm{II}}, K_+^{\mathrm{III}})$$

The energy at $g = 0$ (canonical dimension) is

$$E_0 = \Delta_0 = J + N$$

Choose a solution. It is characterized by a definite set of g-dependent momenta.

Auxiliary roots are completely fixed by the momenta p_k and play no independent role in the description of the state

2. Compute asymptotic Y-functions and find zeroes and poles of Y_{I} and Y_{II}

3. Choose contours and engineer TBA equations for the state so that the asymptotic TBA equations obtained by dropping terms $\propto \ln(1 - Y_k)$ are solved by the asymptotic Y-functions

4. Exact momenta α_k are found from the exact Baxter equations (cuts $\mathcal{C}_k$ are $\mathbb{R}$)

$$Y_k(\alpha_k) = -1$$

Cannot be analytically continued the excited state TBA equations for Y_k

CDT: General strategy

1. Solve the BY equations (in any grading) for a fixed set of integers

$$J, \quad N = K^{\mathrm{I}}, \quad (K_-^{\mathrm{III}}, K_-^{\mathrm{II}}, K_+^{\mathrm{II}}, K_+^{\mathrm{III}})$$

The energy at $g = 0$ (canonical dimension) is

$$E_0 = \Delta_0 = J + N$$

Choose a solution. It is characterized by a definite set of g-dependent momenta.

Auxiliary roots are completely fixed by the momenta p_k and play no independent role in the description of the state

2. Compute asymptotic Y-functions and find zeroes and poles of $1 + Y$ and Y

3. Choose contours and engineer TBA equations for the state so that the asymptotic TBA equations obtained by dropping terms with $\log(1 + Y_Q)$ are solved by the asymptotic Y-functions

BY equations

Beisert, Staudacher '05

The main Bethe equations have the form

$$1 = e^{iJp_k} \prod_{l\neq k}^{K^{\rm I}} S_{su(2)}(u_k, u_l) \prod_{l=1}^{K_-^{\rm II}} \frac{x_k^- - y_l^{(-)}}{x_k^+ - y_l^{(-)}} \sqrt{\frac{x_k^+}{x_k^-}} \prod_{l=1}^{K_+^{\rm II}} \frac{x_k^- - y_l^{(+)}}{x_k^+ - y_l^{(+)}} \sqrt{\frac{x_k^+}{x_k^-}}$$

They are supplied with auxiliary Bethe equations for the roots $y^{(\alpha)}$ and $w^{(\alpha)}$, $\alpha = \pm$

$$\prod_{i=1}^{K^{\rm I}} \frac{y_k^{(\alpha)} - x_i^-}{y_k^{(\alpha)} - x_i^+} \sqrt{\frac{x_i^+}{x_i^-}} = \prod_{i=1}^{K_\alpha^{\rm III}} \frac{w_i^{(\alpha)} - \nu_k^{(\alpha)} - \frac{i}{g}}{w_i^{(\alpha)} - \nu_k^{(\alpha)} + \frac{i}{g}}, \qquad \nu_k^{(\alpha)} \equiv y_k^{(\alpha)} + \frac{1}{y_k^{(\alpha)}}$$

$$\prod_{i=1}^{K_\alpha^{\rm II}} \frac{w_k^{(\alpha)} - \nu_i^{(\alpha)} + \frac{i}{g}}{w_k^{(\alpha)} - \nu_i^{(\alpha)} - \frac{i}{g}} = -\prod_{i=1}^{K_\alpha^{\rm III}} \frac{w_k^{(\alpha)} - w_i^{(\alpha)} + \frac{2i}{g}}{w_k^{(\alpha)} - w_i^{(\alpha)} - \frac{2i}{g}}.$$

Here $K^{\rm I}$ is # of particles, $K_\alpha^{\rm II}$, $K_\alpha^{\rm III}$ are weights of four $SU(2)$'s of manifest symmetry of the l.c. string sigma model. $SU(4)$ weights $[q_1, p, q_2]$ and the spins $[s_1, s_2]$ are

$$q_1 = K_-^{\rm II} - 2K_-^{\rm III}, \quad q_2 = K_+^{\rm II} - 2K_+^{\rm III} \qquad s_1 = K^{\rm I} - K_-^{\rm II}$$
$$p = J - \tfrac{1}{2}(K_-^{\rm II} + K_+^{\rm II}) + K_-^{\rm III} + K_+^{\rm III} \qquad s_2 = K^{\rm I} - K_+^{\rm II}$$

Instead of weights of $su(4)$ one can use the weights (J_1, J_2, J_3) of $SO(6)$

$$J \equiv J_1 = \frac{1}{2}(q_1 + 2p + q_2), \qquad J_2 = \frac{1}{2}(q_1 + q_2), \qquad J_3 = \frac{1}{2}(q_2 - q_1).$$

CDT: General strategy

1. Solve the BY equations (in any grading) for a fixed set of integers

$$J, \quad N = K^{\mathrm{I}}, \quad (K_-^{\mathrm{III}}, K_-^{\mathrm{II}}, K_+^{\mathrm{II}}, K_+^{\mathrm{III}})$$

The energy at $g = 0$ (canonical dimension) is

$$E_0 = \Delta_0 = J + N$$

Choose a solution. It is characterized by a definite set of g-dependent momenta.

Auxiliary roots are completely fixed by the momenta p_k and play no independent role in the description of the state

CDT: General strategy

1. Solve the BY equations (in any grading) for a fixed set of integers

$$J, \quad N = K^{\mathrm{I}}, \quad (K_-^{\mathrm{III}}, K_-^{\mathrm{II}}, K_+^{\mathrm{II}}, K_+^{\mathrm{III}})$$

The energy at $g = 0$ (canonical dimension) is

$$E_0 = \Delta_0 = J + N$$

Choose a solution. It is characterized by a definite set of g-dependent momenta.

Auxiliary roots are completely fixed by the momenta p_k and play no independent role in the description of the state

2. Compute asymptotic Y-functions and find zeroes and poles of $1 + Y$ and Y

3. Choose contours and engineer TBA equations for the state so that the asymptotic TBA equations obtained by dropping terms with $\log(1 + Y_Q)$ are solved by the asymptotic Y-functions

BY equations

Beisert, Staudacher '05

The main Bethe equations have the form

$$1 = e^{iJp_k} \prod_{l\neq k}^{K^{\rm I}} S_{su(2)}(u_k,u_l) \prod_{l=1}^{K_-^{\rm II}} \frac{x_k^- - y_l^{(-)}}{x_k^+ - y_l^{(-)}} \sqrt{\frac{x_k^+}{x_k^-}} \prod_{l=1}^{K_+^{\rm II}} \frac{x_k^- - y_l^{(+)}}{x_k^+ - y_l^{(+)}} \sqrt{\frac{x_k^+}{x_k^-}}$$

They are supplied with auxiliary Bethe equations for the roots $y^{(\alpha)}$ and $w^{(\alpha)}$, $\alpha = \pm$

$$\prod_{i=1}^{K^{\rm I}} \frac{y_k^{(\alpha)} - x_i^-}{y_k^{(\alpha)} - x_i^+} \sqrt{\frac{x_i^+}{x_i^-}} = \prod_{i=1}^{K_\alpha^{\rm III}} \frac{w_i^{(\alpha)} - \nu_k^{(\alpha)} - \frac{i}{g}}{w_i^{(\alpha)} - \nu_k^{(\alpha)} + \frac{i}{g}}, \qquad \nu_k^{(\alpha)} \equiv y_k^{(\alpha)} + \frac{1}{y_k^{(\alpha)}}$$

$$\prod_{i=1}^{K_\alpha^{\rm II}} \frac{w_k^{(\alpha)} - \nu_i^{(\alpha)} + \frac{i}{g}}{w_k^{(\alpha)} - \nu_i^{(\alpha)} - \frac{i}{g}} = -\prod_{i=1}^{K_\alpha^{\rm III}} \frac{w_k^{(\alpha)} - w_i^{(\alpha)} + \frac{2i}{g}}{w_k^{(\alpha)} - w_i^{(\alpha)} - \frac{2i}{g}}.$$

Here $K^{\rm I}$ is # of particles, $K_\alpha^{\rm II}$, $K_\alpha^{\rm III}$ are weights of four $SU(2)$'s of manifest symmetry of the l.c. string sigma model. $SU(4)$ weights $[q_1, p, q_2]$ and the spins $[s_1, s_2]$ are

$$q_1 = K_-^{\rm II} - 2K_-^{\rm III}, \quad q_2 = K_+^{\rm II} - 2K_+^{\rm III} \qquad s_1 = K^{\rm I} - K_-^{\rm II}$$
$$p = J - \tfrac{1}{2}(K_-^{\rm II} + K_+^{\rm II}) + K_-^{\rm III} + K_+^{\rm III} \qquad s_2 = K^{\rm I} - K_+^{\rm II}$$

Instead of weights of $su(4)$ one can use the weights (J_1, J_2, J_3) of $SO(6)$

$$J \equiv J_1 = \frac{1}{2}(q_1 + 2p + q_2), \qquad J_2 = \frac{1}{2}(q_1 + q_2), \qquad J_3 = \frac{1}{2}(q_2 - q_1).$$

CDT: General strategy

1. Solve the BY equations (in any grading) for a fixed set of integers

$$J, \quad N = K^{\mathrm{I}}, \quad (K_-^{\mathrm{III}}, K_-^{\mathrm{II}}, K_+^{\mathrm{II}}, K_+^{\mathrm{III}})$$

The energy at $g = 0$ (canonical dimension) is

$$E_0 = \Delta_0 = J + N$$

Choose a solution. It is characterized by a definite set of g-dependent momenta.

Auxiliary roots are completely fixed by the momenta p_k and play no independent role in the description of the state

CDT: General strategy

1. Solve the BY equations (in any grading) for a fixed set of integers

$$J, \quad N = K^{\mathrm{I}}, \quad (K_-^{\mathrm{III}}, K_-^{\mathrm{II}}, K_+^{\mathrm{II}}, K_+^{\mathrm{III}})$$

The energy at $g = 0$ (canonical dimension) is

$$E_0 = \Delta_0 = J + N$$

Choose a solution. It is characterized by a definite set of g-dependent momenta.

Auxiliary roots are completely fixed by the momenta p_k and play no independent role in the description of the state

2. Compute asymptotic Y-functions and find zeroes and poles of $1 + Y$ and Y

3. Choose contours and engineer TBA equations for the state so that the asymptotic TBA equations obtained by dropping terms with $\log(1 + Y_Q)$ are solved by the asymptotic Y-functions

CDT: General strategy

1. Solve the BY equations (in any grading) for a fixed set of integers

$$J, \quad N = K^{\mathrm{I}}, \quad (K_-^{\mathrm{III}}, K_-^{\mathrm{II}}, K_+^{\mathrm{II}}, K_+^{\mathrm{III}})$$

The energy at $g = 0$ (canonical dimension) is

$$E_0 = \Delta_0 = J + N$$

Choose a solution. It is characterized by a definite set of g-dependent momenta.

Auxiliary roots are completely fixed by the momenta p_k and play no independent role in the description of the state

2. Compute asymptotic Y-functions and find zeroes and poles of $1 + Y$ and Y

3. Choose contours and engineer TBA equations for the state so that the asymptotic TBA equations obtained by dropping terms with $\log(1 + Y_Q)$ are solved by the asymptotic Y-functions

4. Exact momenta p_k are found from the *exact Bethe equations* (quantization cond.)

$$Y_{1*}(p_k) = -1$$

derived by analytically continuing the excited state TBA equation for Y_1

Comments I

- There are no equations to determine the exact location of the auxiliary roots.

  Y-functions are regular at the locations of auxiliary roots, *i.e.* do not have neither zeroes, poles or -1's there.

- Asymptotic Y-functions have zeroes, poles or -1's at other locations, and exact Y-functions have similar properties too.

  Exact locations of these -1's are determined by analytically continuing the TBA equations to these points, and setting the Y-functions there to -1.

- Some zeroes, poles or -1's of asymptotic Y-functions might be spurious and be absent in exact Y-functions.

  These spurious zeroes, poles or -1's are outside the analyticity strip.

Comments I

- There are no equations to determine the exact location of the auxiliary roots.

  Y-functions are regular at the locations of auxiliary roots, *i.e.* do not have neither zeroes, poles or -1's there.

- Asymptotic Y-functions have zeroes, poles or -1's at other locations, and exact Y-functions have similar properties too.

  Exact locations of these -1's are determined by analytically continuing the TBA equations to these points, and setting the Y-functions there to -1.

  These are the generalized quantization conditions

- Some zeroes, poles or -1's of asymptotic Y-functions might be spurious and be absent in exact Y-functions.

  These spurious zeroes, poles or -1's are outside the analyticity strip.

Comments I

- There are no equations to determine the exact location of the auxiliary roots.

  Y-functions are regular at the locations of auxiliary roots, *i.e.* do not have neither zeroes, poles or -1's there.

- Asymptotic Y-functions have zeroes, poles or -1's at other locations, and exact Y-functions have similar properties too.

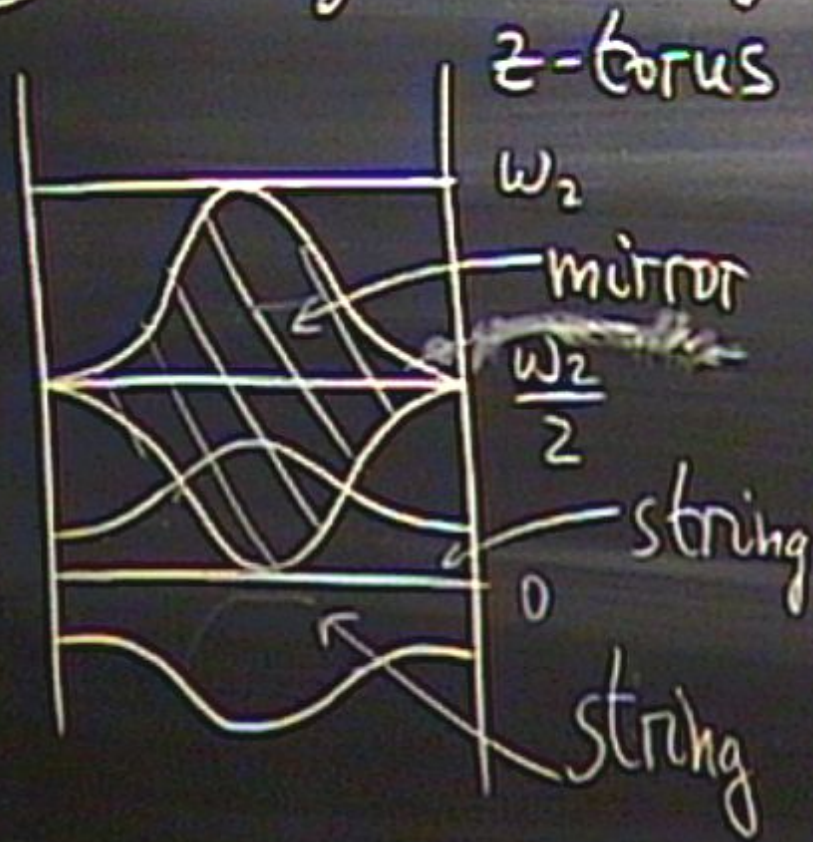
  Exact locations of these -1's are determined by analytically continuing the TBA equations to these points, and setting the Y-functions there to -1.

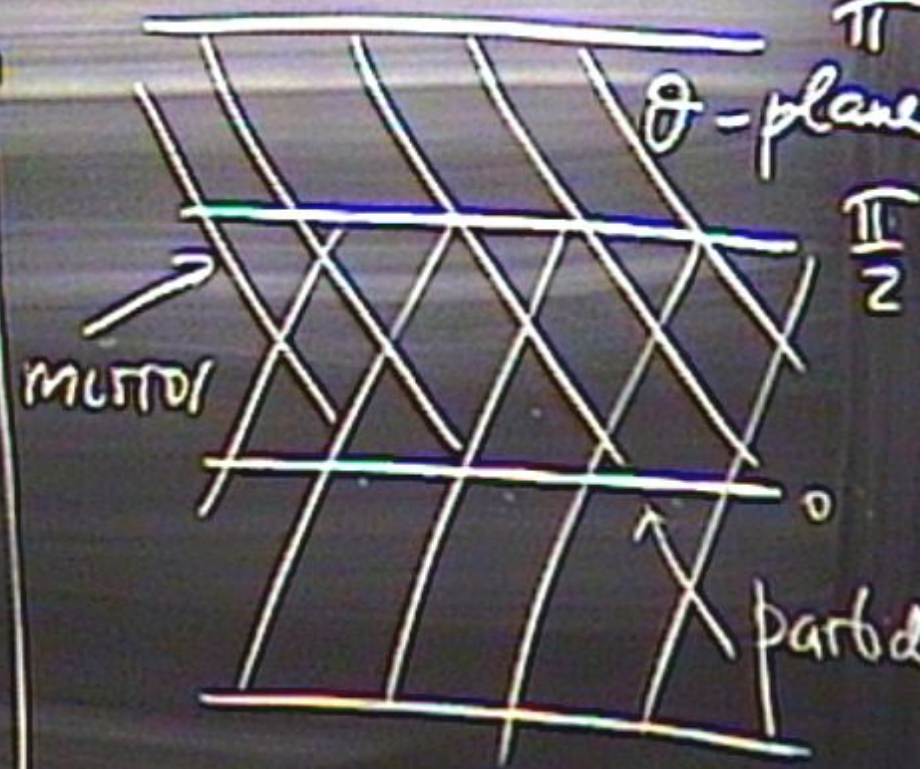
  These are the generalized quantization conditions

- Some zeroes, poles or -1's of asymptotic Y-functions might be spurious and be absent in exact Y-functions.

  These spurious zeroes, poles or -1's are outside the analyticity strip.

③ Physical region





Comments II

- The energy spectrum computed by using the asymptotic Bethe equations is *very degenerate*. Most of this degeneracy is *lifted* as soon as the leading exponential corrections are taken into account.

Consider the states

$$K^{\mathrm{I}} = 4\,, \qquad K^{\mathrm{II}}_{-} + K^{\mathrm{II}}_{+} = 2\,, \qquad K^{\mathrm{III}}_{\alpha} = 0$$

Equations for "left" and "right" y-roots is the same

$$\prod_{i=1}^{K^{\mathrm{I}}} \frac{y_k^{(\alpha)} - x_i^-}{y_k^{(\alpha)} - x_i^+} \sqrt{\frac{x_i^+}{x_i^-}} = 1 \quad \Longrightarrow \quad \prod_{i=1}^{K^{\mathrm{I}}} \frac{y_k - x_i^-}{y_k - x_i^+} \sqrt{\frac{x_i^+}{x_i^-}} = 1$$

Three y-root distributions are possible:

- $K^{\mathrm{II}}_{-} = 0, K^{\mathrm{II}}_{+} = 2$
- $K^{\mathrm{II}}_{-} = 2, K^{\mathrm{II}}_{+} = 0$
- $K^{\mathrm{II}}_{-} = 1, K^{\mathrm{II}}_{+} = 1$

All three states have the same asymptotic energy!

Comments II

- The energy spectrum computed by using the asymptotic Bethe equations is *very degenerate*. Most of this degeneracy is *lifted* as soon as the leading exponential corrections are taken into account.

  Consider the states

$$K^{\mathrm{I}} = 4\,, \qquad K_-^{\mathrm{II}} + K_+^{\mathrm{II}} = 2\,, \qquad K_\alpha^{\mathrm{III}} = 0$$

  Equations for "left" and "right" y-roots is the same

$$\prod_{i=1}^{K^{\mathrm{I}}} \frac{y_k^{(\alpha)} - x_i^-}{y_k^{(\alpha)} - x_i^+} \sqrt{\frac{x_i^+}{x_i^-}} = 1 \quad \Longrightarrow \quad \prod_{i=1}^{K^{\mathrm{I}}} \frac{y_k - x_i^-}{y_k - x_i^+} \sqrt{\frac{x_i^+}{x_i^-}} = 1$$

  Three y-root distributions are possible:

  - $K_-^{\mathrm{II}} = 0, K_+^{\mathrm{II}} = 2$
  - $K_-^{\mathrm{II}} = 2, K_+^{\mathrm{II}} = 0$
  - $K_-^{\mathrm{II}} = 1, K_+^{\mathrm{II}} = 1$

  All three states have the same asymptotic energy!

- All these should be valid for states with *real* momenta. States with *complex* momenta require special treatment!

Arutyunov, Frolov, van Tongeren, work in progress

Known excited state TBA equations

Almost all states are from the $sl(2)$ sector

$$\mathrm{Tr}\left(\prod_{k=1}^{J} D^{n_k} Z\right), \qquad \sum_{k=1}^{J} n_k = N, \quad n_k \geq 0$$

J is the twist, and N is the spin or the number of particles.

- Canonical TBA eqs for two-particle Konishi-like states, $\lambda < \lambda_{\mathrm{cr}}$

  Gromov, Kazakov, Kozak, Vieira '09v3;

- Hybrid, simplified and canonical TBA eqs for arbitrary two-particle states

  Arutyunov, Frolov, Suzuki '09;

- Hybrid TBA eqs for twist-2 N-particle lightest state, $\lambda < \lambda_{\mathrm{cr}}$

  Balog, Hegedus '10;

- TBA eqs for a subsector of the $sl(2)$ sector, $\lambda < \lambda_{\mathrm{cr}}$

  Balog, Hegedus '11;

- TBA eqs for the two states not from the $sl(2)$ sector which are degenerate asymptotically, $\lambda < \lambda_{\mathrm{cr}}$

  Sfondrini, van Tongeren. '11;

Two-particle states in perturbation theory

- dual to spin-2 operators

$$\mathrm{Tr}(Z^{J-1}D^2Z)\,, \quad \mathrm{Tr}(Z^{k-2}DZ\,Z^{J-k}DZ)$$

- Log of the BY equation for two-particle states with $p_1 + p_2 = 0$

$$ip(J+1) - \log\frac{1+\frac{1}{x^{+2}}}{1+\frac{1}{x^{-2}}} - 2i\,\underbrace{\theta(p,-p)}_{\text{dressing phase}} = 2\pi i\,n\,, \quad n > 0$$

- At $g \to 0$ the momentum is

$$p^o_{J,n} = \frac{2\pi n}{J+1}\,, \quad n = 1\,,\ldots,\left[\frac{J+1}{2}\right]\,,$$

Known excited state TBA equations

Almost all states are from the $sl(2)$ sector

$$\mathrm{Tr}\left(\prod_{k=1}^{J} D^{n_k} Z\right), \qquad \sum_{k=1}^{J} n_k = N, \quad n_k \geq 0$$

J is the twist, and N is the spin or the number of particles.

- Canonical TBA eqs for two-particle Konishi-like states, $\lambda < \lambda_{\mathrm{cr}}$

  Gromov, Kazakov, Kozak, Vieira '09v3;

- Hybrid, simplified and canonical TBA eqs for arbitrary two-particle states

  Arutyunov, Frolov, Suzuki '09;

- Hybrid TBA eqs for twist-2 N-particle lightest state, $\lambda < \lambda_{\mathrm{cr}}$

  Balog, Hegedus '10;

- TBA eqs for a subsector of the $sl(2)$ sector, $\lambda < \lambda_{\mathrm{cr}}$

  Balog, Hegedus '11;

- TBA eqs for the two states not from the $sl(2)$ sector which are degenerate asymptotically, $\lambda < \lambda_{\mathrm{cr}}$

  Sfondrini, van Tongeren. '11;

Two-particle states in perturbation theory

- dual to spin-2 operators

$$\mathrm{Tr}(Z^{J-1}D^2Z)\,, \quad \mathrm{Tr}(Z^{k-2}DZ\,Z^{J-k}DZ)$$

- Log of the BY equation for two-particle states with $p_1 + p_2 = 0$

$$ip(J+1) - \log\frac{1+\frac{1}{x^{+2}}}{1+\frac{1}{x^{-2}}} - 2i\,\underbrace{\theta(p,-p)}_{\text{dressing phase}} = 2\pi i\,n\,, \quad n > 0$$

- At $g \to 0$ the momentum is

$$p^o_{J,n} = \frac{2\pi n}{J+1}\,, \quad n = 1\,,\ldots,\left[\frac{J+1}{2}\right]\,,$$

Two-particle states in perturbation theory

- dual to spin-2 operators

$$\mathrm{Tr}(Z^{J-1}D^2Z)\,, \quad \mathrm{Tr}(Z^{k-2}DZ\,Z^{J-k}DZ)$$

- Log of the BY equation for two-particle states with $p_1 + p_2 = 0$

$$ip(J+1) - \log\frac{1+\frac{1}{x^{+2}}}{1+\frac{1}{x^{-2}}} - 2i\,\underbrace{\theta(p,-p)}_{\text{dressing phase}} = 2\pi i\,n\,, \quad n > 0$$

- At $g \to 0$ the momentum is

$$p^o_{J,n} = \frac{2\pi n}{J+1}\,, \quad n = 1\,,\ldots,\left[\frac{J+1}{2}\right]\,,$$

- The corresponding rapidity

$$u_{J,n} \to \frac{1}{g}u^o_{J,n}\,, \quad u^o_{J,n} = \cot\frac{\pi n}{J+1}$$

- At large g the integer n coincides with the string level
- For Konishi $J = 2$ and $n = 1$

Types of states

At $g \sim 0$ the following classification of two-particle states in the $sl(2)$-sector takes place

Arutyunov, Frolov, Suzuki '09

Type of a state	Y-functions	Number of zeros
I	$Y_{1\|vw}$	2
II	$Y_{1\|vw}$, $Y_{2\|vw}$	2+2
III	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$	4+2+2
IV	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$, $Y_{4\|vw}$	4+4+2+2
$\vdots$	$\vdots$	$\vdots$
$k \to \infty$	$Y_{1\|vw}$, $Y_{2\|vw}$, $\ldots$	4+4+ $\ldots$

Type of a state depends on how many Y_{vw}-functions have zeroes in the rescaled analyticity strip $|\mathrm{Im}(u)| < 1$

Evolution of asymptotic Y-functions

Arutyunov, Frolov, Suzuki '09

Initial cond. $\rightarrow$	$Y_{1\|vw}$, $Y_{2\|vw}$	2+2
	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$	4+2+2
$g \downarrow$	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$, $Y_{4\|vw}$	4+4+2+2
	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$, $Y_{4\|vw}$, $Y_{5\|vw}$	4+4+4+2+2
$\vdots$	$\vdots$	$\vdots$
	$Y_{1\|vw}$, $Y_{2\|vw}$, $\ldots$	4+4+ $\ldots$

The change of analytic properties of Y's in the analyticity strip changes the TBA equations. This leads to the issue of critical values of the coupling.

Types of states

At $g \sim 0$ the following classification of two-particle states in the $\mathfrak{sl}(2)$-sector takes place

Arutyunov, Frolov, Suzuki '09

Type of a state	Y-functions	Number of zeros
I	$Y_{1\|vw}$	2
II	$Y_{1\|vw}$, $Y_{2\|vw}$	2+2
III	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$	4+2+2
IV	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$, $Y_{4\|vw}$	4+4+2+2
$\vdots$	$\vdots$	$\vdots$
$k \to \infty$	$Y_{1\|vw}$, $Y_{2\|vw}$, $\ldots$	4+4+ $\ldots$

Type of a state depends on how many Y_{vw}-functions have zeroes in the rescaled analyticity strip $|\mathrm{Im}(u)| < 1$

Evolution of asymptotic Y-functions

Arutyunov, Frolov, Suzuki '09

Initial cond. $\rightarrow$	$Y_{1\|vw}$, $Y_{2\|vw}$	2+2
	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$	4+2+2
$g \downarrow$	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$, $Y_{4\|vw}$	4+4+2+2
	$Y_{1\|vw}$, $Y_{2\|vw}$, $Y_{3\|vw}$, $Y_{4\|vw}$, $Y_{5\|vw}$	4+4+4+2+2
$\vdots$	$\vdots$	$\vdots$
	$Y_{1\|vw}$, $Y_{2\|vw}$, $\ldots$	4+4+ $\ldots$

The change of analytic properties of Y's in the analyticity strip changes the TBA equations. This leads to the issue of critical values of the coupling.

Strong coupling expansion

$$E_{(J,n)}(\lambda) = c_{-1}\sqrt[4]{n^2\lambda} + c_0 + \frac{c_1}{\sqrt[4]{n^2\lambda}} + \frac{c_2}{\sqrt{n^2\lambda}} + \frac{c_3}{(n^2\lambda)^{3/4}} + \frac{c_4}{n^2\lambda} + \frac{c_5}{(n^2\lambda)^{5/4}} + \cdots$$

- Expansion is in $1/\sqrt[4]{n^2\lambda}$, n is the string level of a 2-particle state

Strong coupling expansion

$$E_{(J,n)}(\lambda) = c_{-1}\sqrt[4]{n^2\lambda} + c_0 + \frac{c_1}{\sqrt[4]{n^2\lambda}} + \frac{c_2}{\sqrt{n^2\lambda}} + \frac{c_3}{(n^2\lambda)^{3/4}} + \frac{c_4}{n^2\lambda} + \frac{c_5}{(n^2\lambda)^{5/4}} + \cdots$$

- Expansion is in $1/\sqrt[4]{n^2\lambda}$, n is the string level of a 2-particle state
- $c_{-1} = 2$ from the flat space string spectrum and BYE — Gubser, Klebanov, Polyakov '98; Arutyunov, Frolov, Staudacher '04

Strong coupling expansion

$$E_{(J,n)}(\lambda) = c_{-1}\sqrt[4]{n^2\lambda} + c_0 + \frac{c_1}{\sqrt[4]{n^2\lambda}} + \frac{c_2}{\sqrt{n^2\lambda}} + \frac{c_3}{(n^2\lambda)^{3/4}} + \frac{c_4}{n^2\lambda} + \frac{c_5}{(n^2\lambda)^{5/4}} + \cdots$$

- Expansion is in $1/\sqrt[4]{n^2\lambda}$, n is the string level of a 2-particle state
- $c_{-1} = 2$ from the flat space string spectrum and BYE — Gubser, Klebanov, Polyakov '98; Arutyunov, Frolov, Staudacher '04
- $c_0 = 0$ from BYE and free fermion model — Arutyunov, Frolov '05

Strong coupling expansion

$$E_{(J,n)}(\lambda) = c_{-1}\sqrt[4]{n^2\lambda} + c_0 + \frac{c_1}{\sqrt[4]{n^2\lambda}} + \frac{c_2}{\sqrt{n^2\lambda}} + \frac{c_3}{(n^2\lambda)^{3/4}} + \frac{c_4}{n^2\lambda} + \frac{c_5}{(n^2\lambda)^{5/4}} + \cdots$$

- Expansion is in $1/\sqrt[4]{n^2\lambda}$, n is the string level of a 2-particle state
- $c_{-1} = 2$ from the flat space string spectrum and BYE — Gubser, Klebanov, Polyakov '98; Arutyunov, Frolov, Staudacher '04
- $c_0 = 0$ from BYE and free fermion model — Arutyunov, Frolov '05
- Other coefficients are functions of J and n

Strong coupling expansion

$$E_{(J,n)}(\lambda) = c_{-1}\sqrt[4]{n^2\lambda} + c_0 + \frac{c_1}{\sqrt[4]{n^2\lambda}} + \frac{c_2}{\sqrt{n^2\lambda}} + \frac{c_3}{(n^2\lambda)^{3/4}} + \frac{c_4}{n^2\lambda} + \frac{c_5}{(n^2\lambda)^{5/4}} + \cdots$$

- Expansion is in $1/\sqrt[4]{n^2\lambda}$, n is the string level of a 2-particle state
- $c_{-1} = 2$ from the flat space string spectrum and BYE — Gubser, Klebanov, Polyakov '98; Arutyunov, Frolov, Staudacher '04
- $c_0 = 0$ from BYE and free fermion model — Arutyunov, Frolov '05
- Other coefficients are functions of J and n
- $c_2 = 0$??? due to supersymmetry — Roiban, Tseytlin '09

Strong coupling expansion

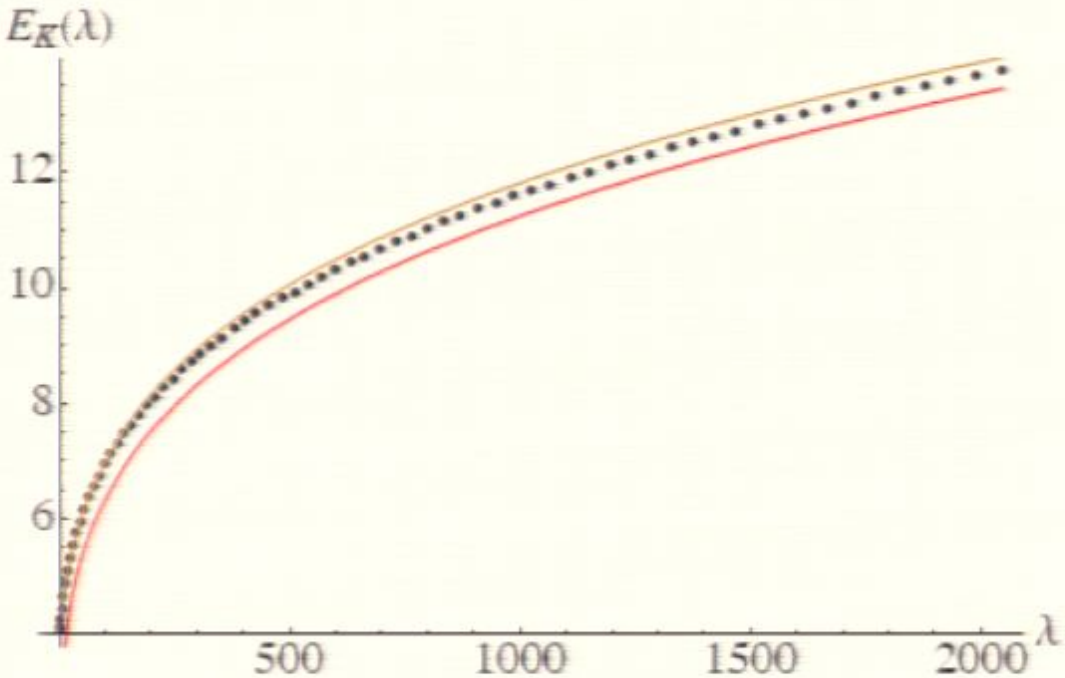
$$E_{(J,n)}(\lambda) = c_{-1}\sqrt[4]{n^2\lambda} + c_0 + \frac{c_1}{\sqrt[4]{n^2\lambda}} + \frac{c_2}{\sqrt{n^2\lambda}} + \frac{c_3}{(n^2\lambda)^{3/4}} + \frac{c_4}{n^2\lambda} + \frac{c_5}{(n^2\lambda)^{5/4}} + \cdots$$

- Expansion is in $1/\sqrt[4]{n^2\lambda}$, n is the string level of a 2-particle state
- $c_{-1} = 2$ from the flat space string spectrum and BYE — Gubser, Klebanov, Polyakov '98; Arutyunov, Frolov, Staudacher '04
- $c_0 = 0$ from BYE and free fermion model — Arutyunov, Frolov '05
- Other coefficients are functions of J and n
- $c_2 = 0$??? due to supersymmetry — Roiban, Tseytlin '09
- $c_{2k} = 0$!?!?!? — Gromov, Kazakov, Vieira '09(b)

$$E_{(J,n)}(\lambda) = \sqrt[4]{n^2\lambda}\left(2 + \frac{c_1}{\sqrt{n^2\lambda}} + \frac{c_3}{n^2\lambda} + \frac{c_5}{(n^2\lambda)^{3/2}} + \cdots\right)$$

Konishi dimension from TBA

TBA eqs were solved numerically up to $g = 7.2$ ($\lambda \approx 2047$) Frolov '10



dots – numerics,
red – $2\sqrt[4]{\lambda}$,
brown –
$E_{\rm asym} = 2 + 2\sqrt{1 + 4g^2 \sin^2 \frac{p_{\rm asym}}{2}}$

- Up to $g = 4.1$ ($\lambda \approx 664$) agreement with Gromov, Kazakov, Vieira '09(b)
- Setting $c_0 = c_2 = c_4 = 0$, one gets $c_1 = 2$ from GKV numerics

$$\overline{E}_K^{GKV}(\lambda) = \sqrt[4]{\lambda}\left(2.0004 + \frac{1.988}{\sqrt{\lambda}} - \frac{2.60}{\lambda} + \frac{6.2}{\lambda^{3/2}}\right)$$

Large λ expansion from numerics

- In general an asymptotic series cannot be found reliably from numerical data

$$2\frac{1-e^{100-\sqrt{\lambda+1}}}{1+e^{100-\sqrt{\lambda+1}}}\sqrt[4]{\lambda+1} \to 2\sqrt[4]{\lambda}$$

From numerics one would conclude that it asymptotes to $-2\sqrt[4]{\lambda}$

Large λ expansion from numerics

- In general an asymptotic series cannot be found reliably from numerical data

$$2\frac{1-e^{100-\sqrt{\lambda+1}}}{1+e^{100-\sqrt{\lambda+1}}}\sqrt[4]{\lambda+1} \to 2\sqrt[4]{\lambda}$$

From numerics one would conclude that it asymptotes to $-2\sqrt[4]{\lambda}$

- We have to assume that exponentially suppressed terms become very small already at the values of λ we are dealing with.

Large λ expansion from numerics

- In general an asymptotic series cannot be found reliably from numerical data

$$2\frac{1-e^{100-\sqrt{\lambda+1}}}{1+e^{100-\sqrt{\lambda+1}}}\sqrt[4]{\lambda+1} \to 2\sqrt[4]{\lambda}$$

From numerics one would conclude that it asymptotes to $-2\sqrt[4]{\lambda}$

- We have to assume that exponentially suppressed terms become very small already at the values of λ we are dealing with.
- If λ is not large enough then one needs to make an assumption about the structure of the large λ expansion, for example to decide if the series contains all possible terms or some of them vanish.

Large λ expansion from numerics

- In general an asymptotic series cannot be found reliably from numerical data

$$2\frac{1-e^{100-\sqrt{\lambda+1}}}{1+e^{100-\sqrt{\lambda+1}}}\sqrt[4]{\lambda+1} \to 2\sqrt[4]{\lambda}$$

From numerics one would conclude that it asymptotes to $-2\sqrt[4]{\lambda}$

- We have to assume that exponentially suppressed terms become very small already at the values of λ we are dealing with.
- If λ is not large enough then one needs to make an assumption about the structure of the large λ expansion, for example to decide if the series contains all possible terms or some of them vanish.
- Fitting numerical data one should decide how many terms to keep in an asymptotic series, and what fitting interval to use.

Large λ expansion from numerics

- In general an asymptotic series cannot be found reliably from numerical data

$$2\frac{1-e^{100-\sqrt{\lambda+1}}}{1+e^{100-\sqrt{\lambda+1}}}\sqrt[4]{\lambda+1} \to 2\sqrt[4]{\lambda}$$

From numerics one would conclude that it asymptotes to $-2\sqrt[4]{\lambda}$

- We have to assume that exponentially suppressed terms become very small already at the values of λ we are dealing with.
- If λ is not large enough then one needs to make an assumption about the structure of the large λ expansion, for example to decide if the series contains all possible terms or some of them vanish.
- Fitting numerical data one should decide how many terms to keep in an asymptotic series, and what fitting interval to use.
- A function can approach its asymptotic series monotonically or in oscillations, and it does not seem possible to single out one from numerics. In fact, using the standard least-square fitting procedure would always lead to an oscillating behavior of numerical data about a fitting function.

Large λ expansion from numerics

Fitting the data for $g \in [1.4, 7.2]$, one gets

Frolov '10

- No condition on c_i

$$\overline{E}_K(\lambda) = 1.99\sqrt[4]{\lambda} + 0.21 - \frac{0.06}{\sqrt[4]{\lambda}} + \frac{10.43}{\sqrt{\lambda}} - \frac{31.78}{\lambda^{3/4}} + \frac{42.20}{\lambda} - \frac{17.56}{\lambda^{5/4}}$$

Large λ expansion from numerics

Fitting the data for $g \in [1.4, 7.2]$, one gets

Frolov '10

- No condition on c_i

$$\overline{E}_K(\lambda) = 1.99\sqrt[4]{\lambda} + 0.21 - \frac{0.06}{\sqrt[4]{\lambda}} + \frac{10.43}{\sqrt{\lambda}} - \frac{31.78}{\lambda^{3/4}} + \frac{42.20}{\lambda} - \frac{17.56}{\lambda^{5/4}}$$

- $c_{-1} = 2$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} - 0.027 + \frac{2.59}{\sqrt[4]{\lambda}} - \frac{5.16}{\sqrt{\lambda}} + \frac{18.86}{\lambda^{3/4}} - \frac{44.17}{\lambda} + \frac{42.89}{\lambda^{5/4}}$$

- $c_{-1} = 2, c_0 = 0$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{1.99}{\sqrt[4]{\lambda}} + \frac{0.15}{\sqrt{\lambda}} - \frac{4.01}{\lambda^{3/4}} + \frac{4.15}{\lambda} + \frac{2.79}{\lambda^{5/4}}$$

- $c_{-1} = 2, c_0 = 0, c_1 = 2$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{2}{\sqrt[4]{\lambda}} - \frac{0.034}{\sqrt{\lambda}} - \frac{2.85}{\lambda^{3/4}} + \frac{0.92}{\lambda} + \frac{6.08}{\lambda^{5/4}},$$

Large λ expansion from numerics

Fitting the data for $g \in [1.4, 7.2]$, one gets

Frolov '10

- No condition on c_i

$$\overline{E}_K(\lambda) = 1.99\sqrt[4]{\lambda} + 0.21 - \frac{0.06}{\sqrt[4]{\lambda}} + \frac{10.43}{\sqrt{\lambda}} - \frac{31.78}{\lambda^{3/4}} + \frac{42.20}{\lambda} - \frac{17.56}{\lambda^{5/4}}$$

- $c_{-1} = 2$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} - 0.027 + \frac{2.59}{\sqrt[4]{\lambda}} - \frac{5.16}{\sqrt{\lambda}} + \frac{18.86}{\lambda^{3/4}} - \frac{44.17}{\lambda} + \frac{42.89}{\lambda^{5/4}}$$

- $c_{-1} = 2,\ c_0 = 0$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{1.99}{\sqrt[4]{\lambda}} + \frac{0.15}{\sqrt{\lambda}} - \frac{4.01}{\lambda^{3/4}} + \frac{4.15}{\lambda} + \frac{2.79}{\lambda^{5/4}}$$

- $c_{-1} = 2,\ c_0 = 0,\ c_1 = 2$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{2}{\sqrt[4]{\lambda}} - \frac{0.034}{\sqrt{\lambda}} - \frac{2.85}{\lambda^{3/4}} + \frac{0.92}{\lambda} + \frac{6.08}{\lambda^{5/4}},$$

- $c_{-1} = 2,\ c_0 = 0,\ c_1 = 2,\ c_2 = 0$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{2}{\sqrt[4]{\lambda}} - \frac{3.28}{\lambda^{3/4}} + \frac{2.68}{\lambda} + \frac{3.76}{\lambda^{5/4}}$$

- $c_0 = 0,\ c_2 = 0,\ c_4 = 0$

$$\overline{E}_K(\lambda) = 2.00005\sqrt[4]{\lambda} + \frac{1.99237}{\sqrt[4]{\lambda}} - \frac{2.72847}{\lambda^{3/4}} + \frac{7.45145}{\lambda^{5/4}}$$

$c_1 = 2$ was confirmed ??? by

Vallilo, Mazzucato '11

Large λ expansion from numerics

Fitting the data for $g \in [1.4, 7.2]$, one gets

Frolov '10

- No condition on c_i

$$\overline{E}_K(\lambda) = 1.99\sqrt[4]{\lambda} + 0.21 - \frac{0.06}{\sqrt[4]{\lambda}} + \frac{10.43}{\sqrt{\lambda}} - \frac{31.78}{\lambda^{3/4}} + \frac{42.20}{\lambda} - \frac{17.56}{\lambda^{5/4}}$$

- $c_{-1} = 2$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} - 0.027 + \frac{2.59}{\sqrt[4]{\lambda}} - \frac{5.16}{\sqrt{\lambda}} + \frac{18.86}{\lambda^{3/4}} - \frac{44.17}{\lambda} + \frac{42.89}{\lambda^{5/4}}$$

- $c_{-1} = 2,\ c_0 = 0$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{1.99}{\sqrt[4]{\lambda}} + \frac{0.15}{\sqrt{\lambda}} - \frac{4.01}{\lambda^{3/4}} + \frac{4.15}{\lambda} + \frac{2.79}{\lambda^{5/4}}$$

- $c_{-1} = 2,\ c_0 = 0,\ c_1 = 2$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{2}{\sqrt[4]{\lambda}} - \frac{0.034}{\sqrt{\lambda}} - \frac{2.85}{\lambda^{3/4}} + \frac{0.92}{\lambda} + \frac{6.08}{\lambda^{5/4}},$$

- $c_{-1} = 2,\ c_0 = 0,\ c_1 = 2,\ c_2 = 0$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{2}{\sqrt[4]{\lambda}} - \frac{3.28}{\lambda^{3/4}} + \frac{2.68}{\lambda} + \frac{3.76}{\lambda^{5/4}}$$

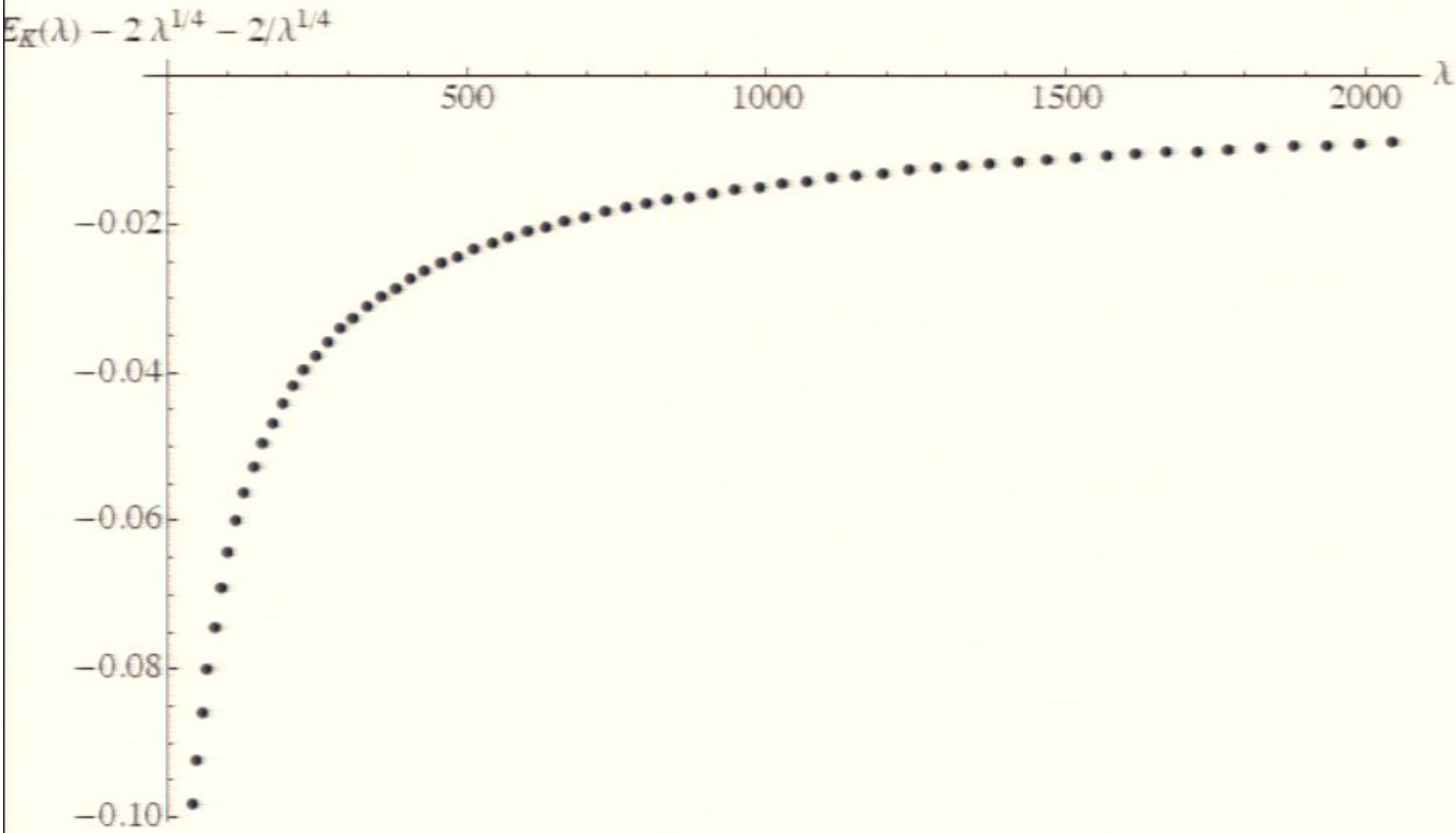
- $c_0 = 0,\ c_2 = 0,\ c_4 = 0$

$$\overline{E}_K(\lambda) = 2.00005\sqrt[4]{\lambda} + \frac{1.99237}{\sqrt[4]{\lambda}} - \frac{2.72847}{\lambda^{3/4}} + \frac{7.45145}{\lambda^{5/4}}$$

$c_1 = 2$ was confirmed ??? by

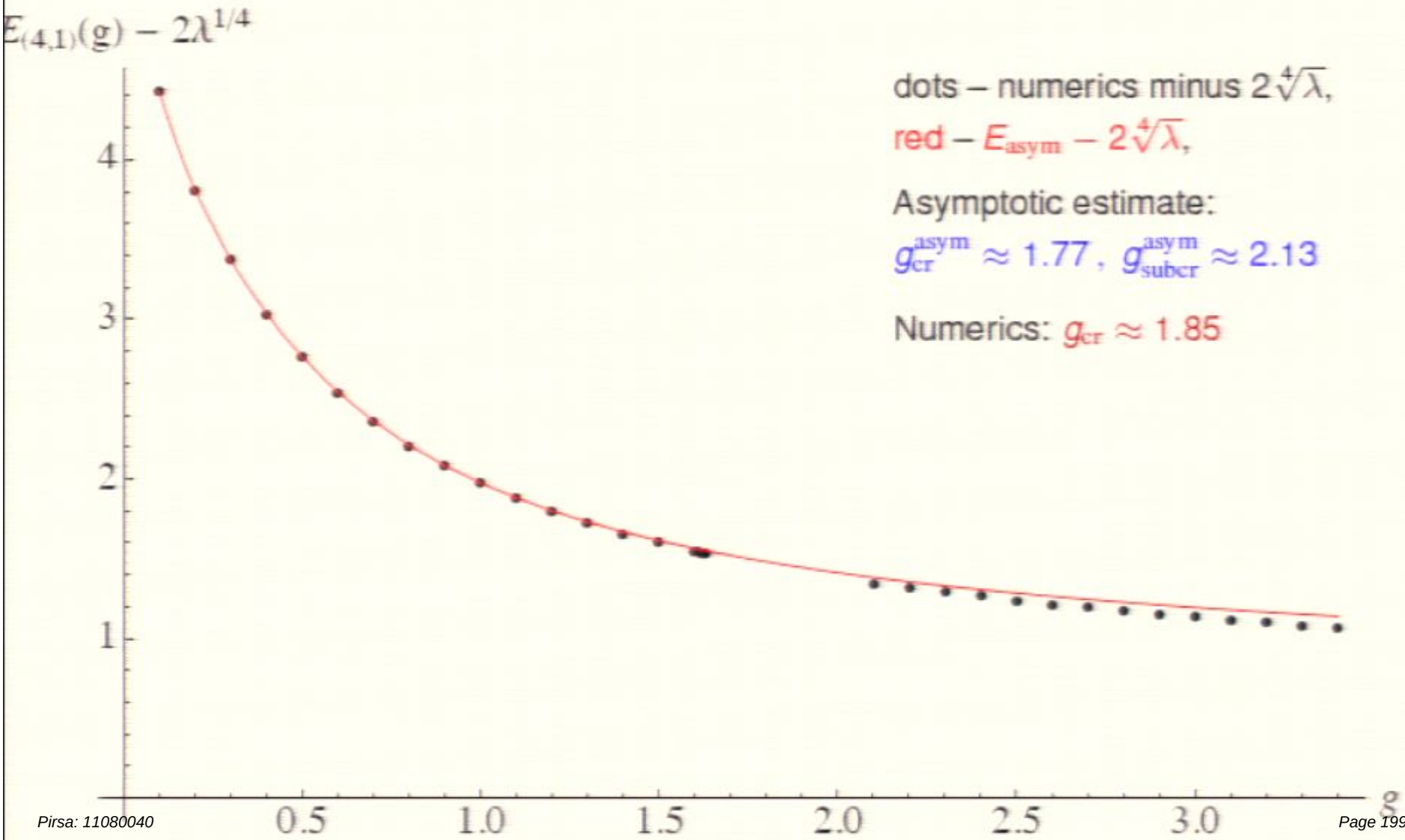
Vallilo, Mazzucato '11

Konishi dimension from TBA



J=4, n=1 state

Frolov '??



dots – numerics minus $2\sqrt[4]{\lambda}$,

red – $E_{\rm asym} - 2\sqrt[4]{\lambda}$,

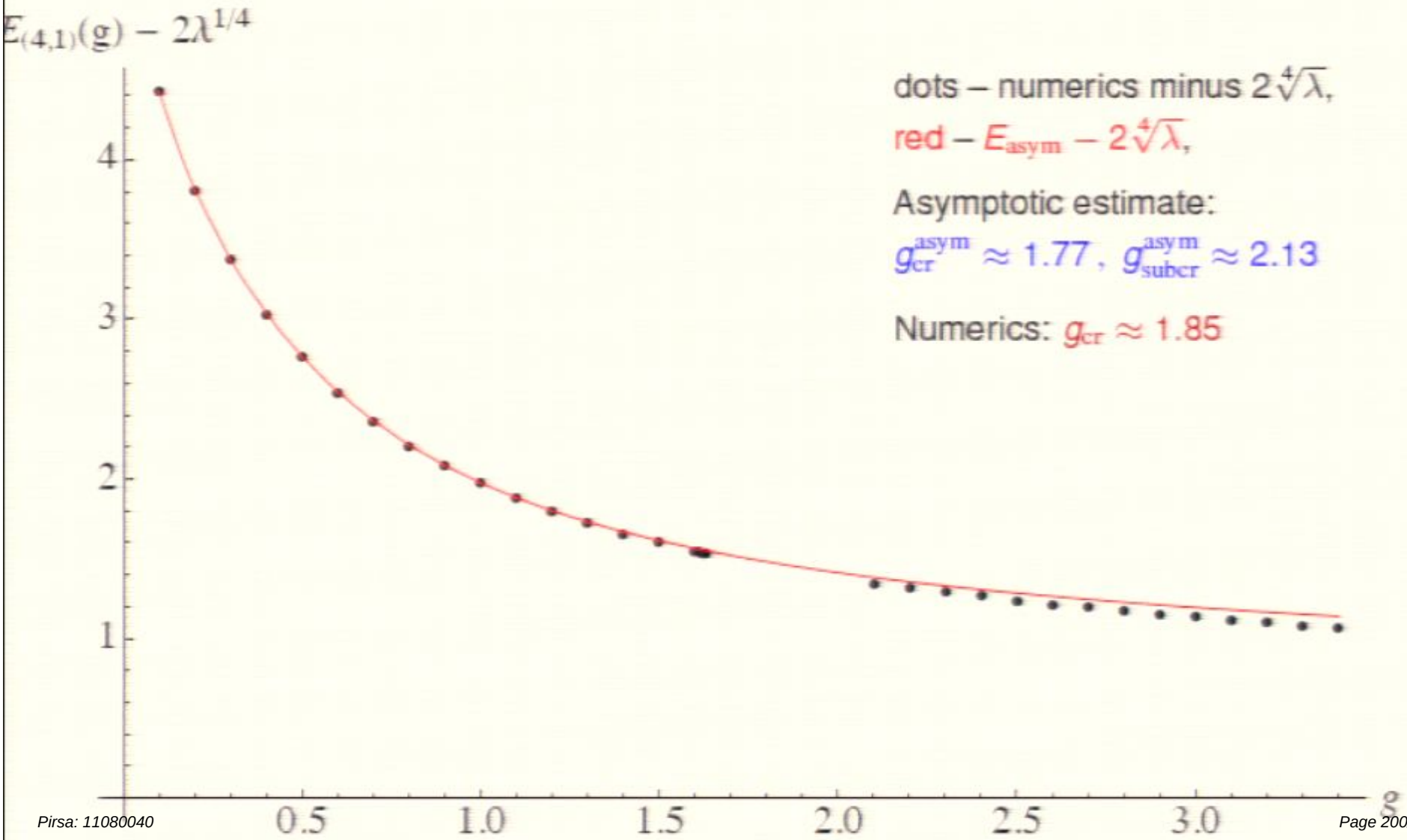
Asymptotic estimate:

$g_{\rm cr}^{\rm asym} \approx 1.77$, $g_{\rm subcr}^{\rm asym} \approx 2.13$

Numerics: $g_{\rm cr} \approx 1.85$

J=4, n=1 state

Frolov '??



dots – numerics minus $2\sqrt[4]{\lambda}$,

red – $E_{\rm asym} - 2\sqrt[4]{\lambda}$,

Asymptotic estimate:

$g_{\rm cr}^{\rm asym} \approx 1.77$, $g_{\rm subcr}^{\rm asym} \approx 2.13$

Numerics: $g_{\rm cr} \approx 1.85$

J=4, n=1 state

Frolov '??

- Fitting the data for $g \in [1.4, 3.4]$, one gets
  - $c_0 = 0$, $c_2 = 0$, $c_4 = 0$

$$\overline{E}_{(4,1)}(\lambda) = 2.0022\sqrt[4]{\lambda} + \frac{4.91748}{\sqrt[4]{\lambda}} - \frac{1.24309}{\lambda^{3/4}} + \frac{9.64361}{\lambda^{5/4}}$$

J=4, n=1 state

Frolov '??

- Fitting the data for $g \in [1.4, 3.4]$, one gets
  - $c_0 = 0,\ c_2 = 0,\ c_4 = 0$

$$\overline{E}_{(4,1)}(\lambda) = 2.0022\sqrt[4]{\lambda} + \frac{4.91748}{\sqrt[4]{\lambda}} - \frac{1.24309}{\lambda^{3/4}} + \frac{9.64361}{\lambda^{5/4}}$$

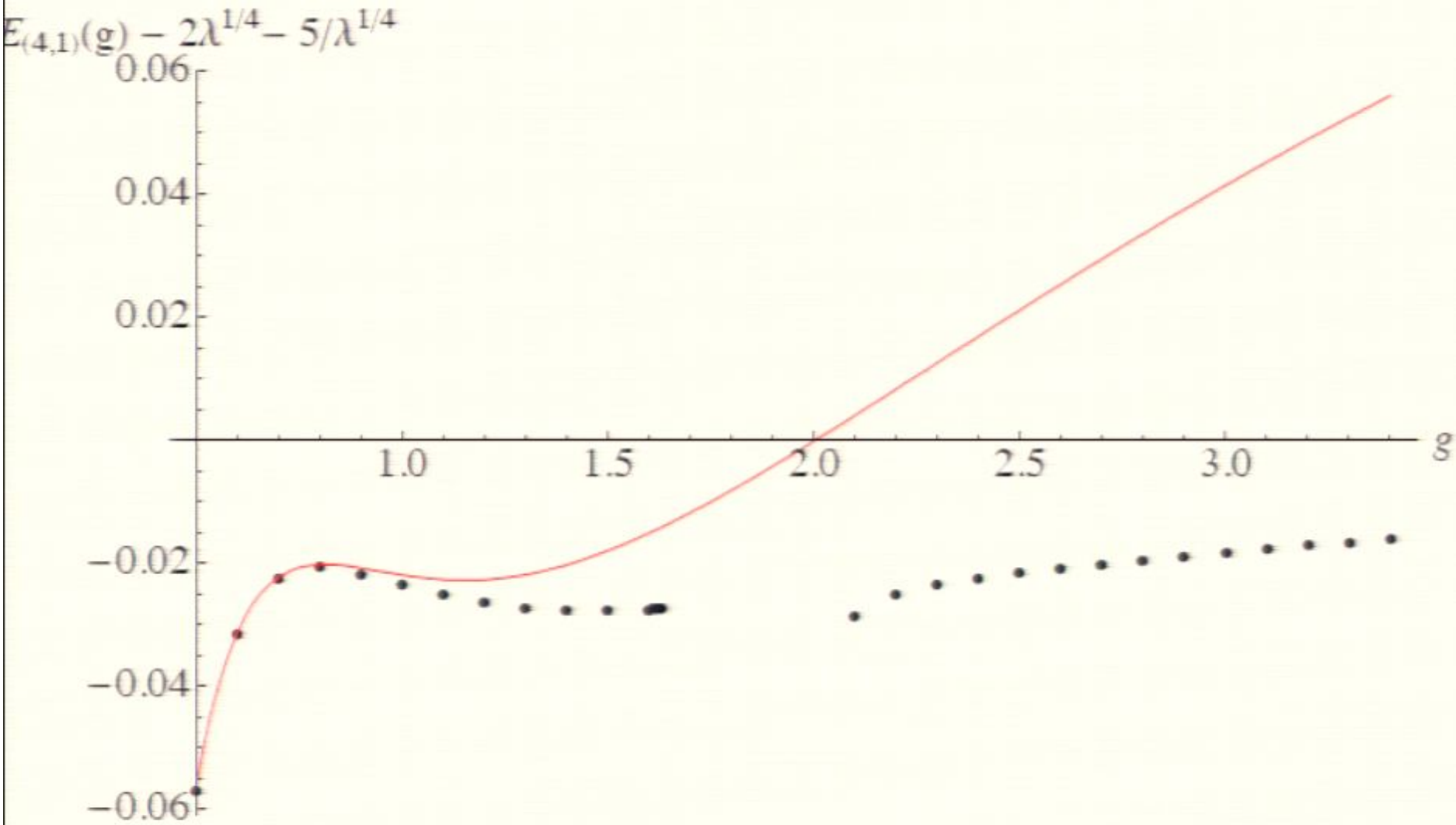
  - $c_{-1} = 2,\ c_0 = 0,\ c_2 = 0,\ c_4 = 0$

$$\overline{E}_{(4,1)}(\lambda) = 2\sqrt[4]{\lambda} + \frac{5.00948}{\sqrt[4]{\lambda}} - \frac{2.47288}{\lambda^{3/4}} + \frac{14.9027}{\lambda^{5/4}}$$

  - Is $c_1 = 5$???

J=4, n=1 state

Frolov '??



$c_1(J,n) = ?,\ c_2(J,n) = 0\ ?$

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

$c_1(J,n) = ?,\ c_2(J,n) = 0\ ?$

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

$c_1(J,n) = ?,\ c_2(J,n) = 0\ ?$

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

- $c_1^{\rm asym}(J,n) = \frac{J^2}{4} + \frac{1}{2}$ from BYE and free fermions — Arutyunov, Frolov '05
- From TBA — Frolov '??
  - $c_2(2,1) \approx 0$, $c_1(2,1) \approx 2$, $\Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - If $c_2(3,1) = c_2(4,1) = 0$, then
    $c_1(3,1) \approx \frac{13}{4}$, $c_1(4,1) \approx 5 \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$

$c_1(J,n) = ?,\ c_2(J,n) = 0\ ?$

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

- $c_1^{\rm asym}(J,n) = \frac{J^2}{4} + \frac{1}{2}$ from BYE and free fermions — Arutyunov, Frolov '05
- From TBA — Frolov '??
  - $c_2(2,1) \approx 0,\ c_1(2,1) \approx 2, \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - If $c_2(3,1) = c_2(4,1) = 0$, then
    $c_1(3,1) \approx \frac{13}{4},\ c_1(4,1) \approx 5 \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - $c_2 \approx 0,\ c_1(4,2) \approx 5,\ c_1(5,2) \approx \frac{29}{4} \Rightarrow c_1(J,2) = \frac{J^2}{4} + 1$
  - If $c_2 = 0$ then

$$c_1(6,3) \approx 9\,, \quad c_1(7,3) \approx \frac{49}{4} \quad \Rightarrow \quad c_1(J,3) = \frac{J^2}{4}$$

$c_1(J,n) = ?,\ c_2(J,n) = 0$?

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

- $c_1^{\rm asym}(J,n) = \frac{J^2}{4} + \frac{1}{2}$ from BYE and free fermions — Arutyunov, Frolov '05
- From TBA — Frolov '??
  - $c_2(2,1) \approx 0$, $c_1(2,1) \approx 2$, $\Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - If $c_2(3,1) = c_2(4,1) = 0$, then
    $c_1(3,1) \approx \frac{13}{4}$, $c_1(4,1) \approx 5 \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - $c_2 \approx 0$, $c_1(4,2) \approx 5$, $c_1(5,2) \approx \frac{29}{4} \Rightarrow c_1(J,2) = \frac{J^2}{4} + 1$
  - If $c_2 = 0$ then

$$c_1(6,3) \approx 9\,, \quad c_1(7,3) \approx \frac{49}{4} \quad \Rightarrow \quad c_1(J,3) = \frac{J^2}{4}$$

$c_1(J,n) = ?,\ c_2(J,n) = 0\ ?$

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

- $c_1^{\rm asym}(J,n) = \frac{J^2}{4} + \frac{1}{2}$ from BYE and free fermions Arutyunov, Frolov '05
- From TBA Frolov '??
  - $c_2(2,1) \approx 0$, $c_1(2,1) \approx 2$, $\Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - If $c_2(3,1) = c_2(4,1) = 0$, then
    $c_1(3,1) \approx \frac{13}{4}$, $c_1(4,1) \approx 5 \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - $c_2 \approx 0$, $c_1(4,2) \approx 5$, $c_1(5,2) \approx \frac{29}{4} \Rightarrow c_1(J,2) = \frac{J^2}{4} + 1$
  - If $c_2 = 0$ then
    $$c_1(6,3) \approx 9\,, \quad c_1(7,3) \approx \frac{49}{4} \quad \Rightarrow \quad c_1(J,3) = \frac{J^2}{4}$$
- $c_1^{\rm exact}(J,n) = \frac{J^2}{4} + c(n)$???, e.g. $c(n) = n(3-n)/2$

$c_1(J,n) = ?,\ c_2(J,n) = 0$?

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

- $c_1^{\rm asym}(J,n) = \frac{J^2}{4} + \frac{1}{2}$ from BYE and free fermions — Arutyunov, Frolov '05
- From TBA — Frolov '??
  - $c_2(2,1) \approx 0$, $c_1(2,1) \approx 2$, $\Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - If $c_2(3,1) = c_2(4,1) = 0$, then
    $c_1(3,1) \approx \frac{13}{4}$, $c_1(4,1) \approx 5 \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - $c_2 \approx 0$, $c_1(4,2) \approx 5$, $c_1(5,2) \approx \frac{29}{4} \Rightarrow c_1(J,2) = \frac{J^2}{4} + 1$
  - If $c_2 = 0$ then
    $$c_1(6,3) \approx 9\,, \quad c_1(7,3) \approx \frac{49}{4} \quad \Rightarrow \quad c_1(J,3) = \frac{J^2}{4}$$
- $c_1^{\rm exact}(J,n) = \frac{J^2}{4} + c(n)$???, e.g. $c(n) = n(3-n)/2$

PSU(2, 2|4): General facts

- Dynkin labels of a state are related to excitation numbers

$$q_1 = K_-^{\mathrm{II}} - 2K_-^{\mathrm{III}} \qquad s_1 = K^{\mathrm{I}} - K_-^{\mathrm{II}}$$
$$p = J - \tfrac{1}{2}(K_-^{\mathrm{II}} + K_+^{\mathrm{II}}) + K_-^{\mathrm{III}} + K_+^{\mathrm{III}} \qquad s_2 = K^{\mathrm{I}} - K_+^{\mathrm{II}}$$
$$q_2 = K_+^{\mathrm{II}} - 2K_+^{\mathrm{III}}$$

- PSU(2, 2|4) acts on asymptotic solutions provided the level-matching is satisfied

PSU(2, 2|4): General facts

- Dynkin labels of a state are related to excitation numbers

$$q_1 = K_-^{\mathrm{II}} - 2K_-^{\mathrm{III}} \qquad s_1 = K^{\mathrm{I}} - K_-^{\mathrm{II}}$$
$$p = J - \tfrac{1}{2}(K_-^{\mathrm{II}} + K_+^{\mathrm{II}}) + K_-^{\mathrm{III}} + K_+^{\mathrm{III}} \qquad s_2 = K^{\mathrm{I}} - K_+^{\mathrm{II}}$$
$$q_2 = K_+^{\mathrm{II}} - 2K_+^{\mathrm{III}}$$

- PSU(2, 2|4) acts on asymptotic solutions provided the level-matching is satisfied
- States in the same multiplet have the same anomalous dimension and canonical ones which may differ by a (half-)integer

$$E = J + \sum_{k=1}^{K^{\mathrm{I}}} \sqrt{1 + 4g^2 \sin^2 \frac{p_k}{2}}\,.$$

Putting $g = 0$ gives the canonical dimension $E = J + K^{\mathrm{I}}$. Adding a particle with zero momentum $p = 0$ ($u = \infty$) increases canonical dimension by 1.

- Superconformal primary has the lowest canonical dimension in the multiplet!
- States can have roots $\nu^{(\alpha)} = y^{(\alpha)} + \frac{1}{y^{(\alpha)}}$ and $w^{(\alpha)}$ located at infinity

A state in a multiplet

Typical multiplet of 2^{16} states

$$|\mathcal{O}\rangle = \prod \underbrace{(Q^d_{-\infty})^{n^d_\infty}(Q^d_{+0})^{n^d_0}}_{J\to+1/2}\underbrace{(Q^k_{+\infty})^{n^k_\infty}(Q^k_{+2\infty,+0})^{n^k_{\infty,0}}}_{J\to-1/2,\ K^1\to+1}|\mathrm{hws}\rangle$$

- The superconformal primary (hws) has four y-roots and two w-roots at ∞

$$E_{hws} = E_{reg} - 2\,, \qquad J_{hws} = J_{reg} - 2\,.$$

PSU(2, 2|4)-symmetry is built-in to TBA!

$$J = J_{reg} + \tfrac{1}{2}(\mathcal{K}_0^{\mathrm{II}} - \mathcal{K}_\infty^{\mathrm{II}})$$

- Bajnok-Janik formula for the asymptotic Y_Q works for any state

$$Y_Q^o(v) = \Upsilon_Q(v)\, T_{Q,-1}(v|\vec{u})\, T_{Q,1}(v|\vec{u}), \quad \Upsilon_Q(v) = e^{-J\bar{\mathcal{E}}_Q(v)} \prod_{i=1}^{N} S_{sl(2)}^{Q1\,*}(v, u_i)$$

- Since $S_{sl(2)}^{Q1\,*}(v, \infty) = 1$, one gets

$$\Upsilon_Q = \Upsilon_Q^{reg}\, e^{-\frac{1}{2}(\mathcal{K}_0^{\mathrm{II}} - \mathcal{K}_\infty^{\mathrm{II}})\bar{\mathcal{E}}_Q}$$

- On the other hand

Thus, L_{TBA} coincides with the maximum J-charge in a susy multiplet!

Only for the vacuum $L_{TBA} = J_{reg}$, as the corresponding susy multiplet is atypical (short)

The mirror TBA: from string hypothesis to the exact spectrum

1. Ground state TBA eqs follow from the string hypothesis
2. TBA eqs for excited states are determined by the ground state ones via the CDT
3. TBA eqs are written for the whole superconformal multiplet. PSU(2, 2|4) invariance is built-in Arutyunov, Frolov '11
4. TBA eqs for states composed of particles with *real momenta* can be obtained
5. States containing particles with *complex momenta* (e.g. bound states) still represent a challenge Arutyunov, Frolov, van Tongeren (work in progress)
6. The form of the equations depends on λ. For infinitely many states *there are critical values of* λ, crossing which the TBA eqs must be modified.
7. For a given operator, is the number of critical values *infinite* or *finite* (or even 0)?
8. We found *no evidence* that up to the overall factor $2\sqrt[4]{n^2\lambda}$ the large λ expansion is in powers of $1/\sqrt{\lambda}$. Is the expansion in powers of $1/\sqrt[4]{\lambda}$?
9. The numerics we performed is not sufficient to give definite answers to (m)any questions. Analytical methods are necessary. NLIE ???

Assumptions

1. Quantum integrability of l.c. string theory and its mirror
2. Symmetry algebra of l.c. string theory and its mirror in the decompactification limit — Beisert '05; Arutyunov, Frolov, Plefka, Zamaklar '06
3. BES dressing factor — Beisert, Eden, Staudacher '06
4. Bajnok-Janik asymptotic solution — Bajnok, Janik '08
5. String hypothesis for the mirror model — Arutyunov, Frolov '09
6. Universality of the contour deformation trick
7. Universality of the exact Bethe equations $Y_{1*}(p_k) = -1$

The mirror TBA: from string hypothesis to the exact spectrum

1. Ground state TBA eqs follow from the string hypothesis
2. TBA eqs for excited states are determined by the ground state ones via the CDT
3. TBA eqs are written for the whole superconformal multiplet. PSU(2, 2|4) invariance is built-in Arutyunov, Frolov '11
4. TBA eqs for states composed of particles with *real momenta* can be obtained
5. States containing particles with *complex momenta* (e.g. bound states) still represent a challenge Arutyunov, Frolov, van Tongeren (work in progress)
6. The form of the equations depends on λ. For infinitely many states *there are critical values of* λ, crossing which the TBA eqs must be modified.
7. For a given operator, is the number of critical values *infinite* or *finite* (or even 0)?
8. We found *no evidence* that up to the overall factor $2\sqrt[4]{n^2\lambda}$ the large λ expansion is in powers of $1/\sqrt{\lambda}$. Is the expansion in powers of $1/\sqrt[4]{\lambda}$?
9. The numerics we performed is not sufficient to give definite answers to (m)any questions. Analytical methods are necessary. NLIE ???

The mirror TBA: from string hypothesis to the exact spectrum

1. Ground state TBA eqs follow from the string hypothesis
2. TBA eqs for excited states are determined by the ground state ones via the CDT
3. TBA eqs are written for the whole superconformal multiplet. PSU(2, 2|4) invariance is built-in Arutyunov, Frolov '11
4. TBA eqs for states composed of particles with *real momenta* can be obtained
5. States containing particles with *complex momenta* (e.g. bound states) still represent a challenge Arutyunov, Frolov, van Tongeren (work in progress)
6. The form of the equations depends on λ. For infinitely many states *there are critical values of* λ, crossing which the TBA eqs must be modified.
7. For a given operator, is the number of critical values *infinite* or *finite* (or even 0)?
8. We found *no evidence* that up to the overall factor $2\sqrt[4]{n^2\lambda}$ the large λ expansion is in powers of $1/\sqrt{\lambda}$. Is the expansion in powers of $1/\sqrt[4]{\lambda}$?
9. The numerics we performed is not sufficient to give definite answers to (m)any questions. Analytical methods are necessary. NLIE ???

Assumptions

1. Quantum integrability of l.c. string theory and its mirror
2. Symmetry algebra of l.c. string theory and its mirror in the decompactification limit — Beisert '05; Arutyunov, Frolov, Plefka, Zamaklar '06
3. BES dressing factor — Beisert, Eden, Staudacher '06
4. Bajnok-Janik asymptotic solution — Bajnok, Janik '08
5. String hypothesis for the mirror model — Arutyunov, Frolov '09
6. Universality of the contour deformation trick
7. Universality of the exact Bethe equations $Y_{1*}(p_k) = -1$

Assumptions

1. Quantum integrability of l.c. string theory and its mirror
2. Symmetry algebra of l.c. string theory and its mirror in the decompactification limit — Beisert '05; Arutyunov, Frolov, Plefka, Zamaklar '06
3. BES dressing factor — Beisert, Eden, Staudacher '06
4. Bajnok-Janik asymptotic solution — Bajnok, Janik '08
5. String hypothesis for the mirror model — Arutyunov, Frolov '09
6. Universality of the contour deformation trick
7. Universality of the exact Bethe equations $Y_{1*}(p_k) = -1$

$c_1(J,n) = ?,\ c_2(J,n) = 0\ ?$

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

- $c_1^{\rm asym}(J,n) = \frac{J^2}{4} + \frac{1}{2}$ from BYE and free fermions — Arutyunov, Frolov '05
- From TBA — Frolov '??
  - $c_2(2,1) \approx 0,\ c_1(2,1) \approx 2, \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - If $c_2(3,1) = c_2(4,1) = 0$, then
    $c_1(3,1) \approx \frac{13}{4},\ c_1(4,1) \approx 5 \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - $c_2 \approx 0,\ c_1(4,2) \approx 5,\ c_1(5,2) \approx \frac{29}{4} \Rightarrow c_1(J,2) = \frac{J^2}{4} + 1$
  - If $c_2 = 0$ then

$$c_1(6,3) \approx 9\,, \quad c_1(7,3) \approx \frac{49}{4} \quad \Rightarrow \quad c_1(J,3) = \frac{J^2}{4}$$

$c_1(J,n) = ?,\ c_2(J,n) = 0$?

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

- $c_1^{\rm asym}(J,n) = \frac{J^2}{4} + \frac{1}{2}$ from BYE and free fermions — Arutyunov, Frolov '05
- From TBA — Frolov '??
  - $c_2(2,1) \approx 0,\ c_1(2,1) \approx 2, \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - If $c_2(3,1) = c_2(4,1) = 0$, then
    $c_1(3,1) \approx \frac{13}{4},\ c_1(4,1) \approx 5 \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - $c_2 \approx 0,\ c_1(4,2) \approx 5,\ c_1(5,2) \approx \frac{29}{4} \Rightarrow c_1(J,2) = \frac{J^2}{4} + 1$
  - If $c_2 = 0$ then
    $$c_1(6,3) \approx 9\,, \quad c_1(7,3) \approx \frac{49}{4} \quad \Rightarrow \quad c_1(J,3) = \frac{J^2}{4}$$
- $c_1^{\rm exact}(J,n) = \frac{J^2}{4} + c(n)$???, e.g. $c(n) = n(3-n)/2$

Large λ expansion from numerics

Fitting the data for $g \in [1.4, 7.2]$, one gets

Frolov '10

- No condition on c_i

$$\overline{E}_K(\lambda) = 1.99\sqrt[4]{\lambda} + 0.21 - \frac{0.06}{\sqrt[4]{\lambda}} + \frac{10.43}{\sqrt{\lambda}} - \frac{31.78}{\lambda^{3/4}} + \frac{42.20}{\lambda} - \frac{17.56}{\lambda^{5/4}}$$

- $c_{-1} = 2$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} - 0.027 + \frac{2.59}{\sqrt[4]{\lambda}} - \frac{5.16}{\sqrt{\lambda}} + \frac{18.86}{\lambda^{3/4}} - \frac{44.17}{\lambda} + \frac{42.89}{\lambda^{5/4}}$$

- $c_{-1} = 2, c_0 = 0$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{1.99}{\sqrt[4]{\lambda}} + \frac{0.15}{\sqrt{\lambda}} - \frac{4.01}{\lambda^{3/4}} + \frac{4.15}{\lambda} + \frac{2.79}{\lambda^{5/4}}$$

- $c_{-1} = 2, c_0 = 0, c_1 = 2$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{2}{\sqrt[4]{\lambda}} - \frac{0.034}{\sqrt{\lambda}} - \frac{2.85}{\lambda^{3/4}} + \frac{0.92}{\lambda} + \frac{6.08}{\lambda^{5/4}},$$

- $c_{-1} = 2, c_0 = 0, c_1 = 2, c_2 = 0$

$$\overline{E}_K(\lambda) = 2\sqrt[4]{\lambda} + \frac{2}{\sqrt[4]{\lambda}} - \frac{3.28}{\lambda^{3/4}} + \frac{2.68}{\lambda} + \frac{3.76}{\lambda^{5/4}}$$

- $c_0 = 0, c_2 = 0, c_4 = 0$

$$\overline{E}_K(\lambda) = 2.00005\sqrt[4]{\lambda} + \frac{1.99237}{\sqrt[4]{\lambda}} - \frac{2.72847}{\lambda^{3/4}} + \frac{7.45145}{\lambda^{5/4}}$$

$c_1 = 2$ was confirmed ??? by

Vallilo, Mazzucato '11

Strong coupling expansion

$$E_{(J,n)}(\lambda) = c_{-1}\sqrt[4]{n^2\lambda} + c_0 + \frac{c_1}{\sqrt[4]{n^2\lambda}} + \frac{c_2}{\sqrt{n^2\lambda}} + \frac{c_3}{(n^2\lambda)^{3/4}} + \frac{c_4}{n^2\lambda} + \frac{c_5}{(n^2\lambda)^{5/4}} + \cdots$$

- Expansion is in $1/\sqrt[4]{n^2\lambda}$, n is the string level of a 2-particle state
- $c_{-1} = 2$ from the flat space string spectrum and BYE — Gubser, Klebanov, Polyakov '98; Arutyunov, Frolov, Staudacher '04
- $c_0 = 0$ from BYE and free fermion model — Arutyunov, Frolov '05
- Other coefficients are functions of J and n
- $c_2 = 0$??? due to supersymmetry — Roiban, Tseytlin '09
- $c_{2k} = 0$!?!?!? — Gromov, Kazakov, Vieira '09(b)

$$E_{(J,n)}(\lambda) = \sqrt[4]{n^2\lambda}\left(2 + \frac{c_1}{\sqrt{n^2\lambda}} + \frac{c_3}{n^2\lambda} + \frac{c_5}{(n^2\lambda)^{3/2}} + \cdots\right)$$

$c_1(J,n) = ?,\ c_2(J,n) = 0\ ?$

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

- $c_1^{\rm asym}(J,n) = \frac{J^2}{4} + \frac{1}{2}$ from BYE and free fermions Arutyunov, Frolov '05
- From TBA Frolov '??
  - $c_2(2,1) \approx 0$, $c_1(2,1) \approx 2$, $\Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - If $c_2(3,1) = c_2(4,1) = 0$, then
    $c_1(3,1) \approx \frac{13}{4}$, $c_1(4,1) \approx 5 \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - $c_2 \approx 0$, $c_1(4,2) \approx 5$, $c_1(5,2) \approx \frac{29}{4} \Rightarrow c_1(J,2) = \frac{J^2}{4} + 1$
  - If $c_2 = 0$ then
    $$c_1(6,3) \approx 9\,, \quad c_1(7,3) \approx \frac{49}{4} \quad \Rightarrow \quad c_1(J,3) = \frac{J^2}{4}$$
- $c_1^{\rm exact}(J,n) = \frac{J^2}{4} + c(n)$???, e.g. $c(n) = n(3-n)/2$

$c_1(J,n) = ?,\ c_2(J,n) = 0\ ?$

$$E_{(J,n)}(\lambda) = 2\sqrt[4]{n^2\lambda} + \frac{c_1(J,n)}{\sqrt[4]{n^2\lambda}} + \frac{c_2(J,n)}{\sqrt{n^2\lambda}} + \frac{c_3(J,n)}{(n^2\lambda)^{3/4}} + \cdots$$

- $c_1^{\rm asym}(J,n) = \frac{J^2}{4} + \frac{1}{2}$ from BYE and free fermions — Arutyunov, Frolov '05
- From TBA — Frolov '??
  - $c_2(2,1) \approx 0,\ c_1(2,1) \approx 2, \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - If $c_2(3,1) = c_2(4,1) = 0$, then
    $c_1(3,1) \approx \frac{13}{4},\ c_1(4,1) \approx 5 \Rightarrow c_1(J,1) = \frac{J^2}{4} + 1$
  - $c_2 \approx 0,\ c_1(4,2) \approx 5,\ c_1(5,2) \approx \frac{29}{4} \Rightarrow c_1(J,2) = \frac{J^2}{4} + 1$
  - If $c_2 = 0$ then
    $$c_1(6,3) \approx 9\,, \quad c_1(7,3) \approx \frac{49}{4} \quad \Rightarrow \quad c_1(J,3) = \frac{J^2}{4}$$
- $c_1^{\rm exact}(J,n) = \frac{J^2}{4} + c(n)$???, e.g. $c(n) = n(3-n)/2$