

Title: Mystery Talk

Date: Aug 06, 2011 09:30 AM

URL: <http://pirsa.org/11080038>

Abstract:

```

n[*6] = symbol[n_?NumberQ] := n tensor[];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

n[*8] = tensor[___, _Rational | _Integer, ___] := 0;

n[*9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];

```

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n[3] = symbol[_?NumberQ] := # tensor[];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[_Integer, x_]] :=
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n[8] = tensor[___, _Rational | _Integer, ___] := 0;

n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[i___]] := tensor[i];

shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, #, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, #, z] & /@ y;
shuffle[x___, #?NumberQ tensor[a___], y___] := # shuffle[x, tensor[a], y];
tensor[x___, #?NumberQ tensor[a___], y___] := # shuffle[x, tensor[a], y];

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n[10] = symbol[_?NumberQ] := # tensor[];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, # - 1]];

n[8] = tensor[___, _Rational | _Integer, ___] := 0;

n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, #?NumberQ tensor[a___], y___] := # shuffle[x, tensor[a], y];
tensor[x___, #?NumberQ tensor[a___], y___] := # tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : ((._tensor) ..)]

```

```

n[*8]= symbol[_n_?NumberQ] := n tensor[];
symbol[Log[_x_]] := tensor[x];
symbol[PolyLog[_n_Integer, _x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

n[*8]= tensor[____, _Rational | _Integer, ____] := 0;

n[*9]= shuffle[tensor[_a_, _b____], tensor[_c_, _d____]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[_x____, tensor[], _y____] := shuffle[x, y];
shuffle[tensor[_i____]] := tensor[i];

shuffle[_x____, _y_ : HoldPattern[Plus[(_. _tensor) ..]], _z____] :=
  shuffle[x, z, z] & /@ y;
tensor[_x____, _y_ : HoldPattern[Plus[(_. _tensor) ..]], _z____] :=
  tensor[x, z, z] & /@ y;
shuffle[_x____, _n_?NumberQ tensor[_a____], _y____] := n shuffle[x, tensor[a], y];
tensor[_x____, _n_?NumberQ tensor[_a____], _y____] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : ((_tensor) ..)]

```

```

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  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[i___]] := tensor[i];

shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
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shuffle[a_tensor, b_tensor, c : ((._tensor) ..)]

```

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  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(_ tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_ tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
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shuffle[a_tensor, b_tensor, c : ((_ tensor) ..)]

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  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, #, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, #, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : ((tensor) ..)] := shuffle[a, shuffle[b, c]]

```



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shuffle[a_tensor, b_tensor, c : ((#_tensor) ..)] := shuffle[a, shuffle[b, c]]

```

```

n[0] = symbol[n_?NumberQ] := n tensor[];
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shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
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  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
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tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : ((_tensor) ..)] := shuffle[a, shuffle[b, c]]

```

```

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symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
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n[8] = tensor[___, _Rational | _Integer, ___] := 0;

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  tensor[c, shuffle[tensor[a, b], tensor[d]]];
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shuffle[tensor[l___]] := tensor[l];

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  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a tensor, b tensor, c : (( tensor) ..)] := shuffle[a, shuffle[b, c]]

```

```

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symbol[PolyLog[_Integer, x_]] :=
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n[8] = tensor[___, _Rational | _Integer, ___] := 0;

n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(#.#_tensor) ..]], z___] :=
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tensor[x___, y : HoldPattern[Plus[(#.#_tensor) ..]], z___] :=
  tensor[x, #, z] & /@ y;
shuffle[x___, #?NumberQ tensor[a___], y___] := # shuffle[x, tensor[a], y];
tensor[x___, #?NumberQ tensor[a___], y___] := # tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : ((#_tensor) ..)] := shuffle[a, shuffle[b, c]]

```



emo.nb +

```

n[0] = symbol[_?NumberQ] := tensor[1];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

n[8] = tensor[___, _Rational | _Integer, ___] := 0;

n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(._tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(._tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : ((._tensor) ..)] := shuffle[a, shuffle[b, c]]

```

shuff

```

n[8] = symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

n[8] = tensor[___, _Rational | _Integer, ___] := 0;

n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[_, y___]] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : ((_tensor) ..)] := shuffle[a, shuffle[b, c]]

shuffle[te

```

```
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];
```

```
n[8] = tensor[___, _Rational | _Integer, ___] := 0;
```

```
n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
```

```
shuffle[x___, tensor[], y___] := shuffle[x, y];
```

```
shuffle[tensor[l___]] := tensor[l];
```

```
shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
```

```
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
```

```
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
```

```
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];
```

```
n[28] = shuffle[a_tensor, b_tensor, c_tensor] := shuffle[a, shuffle[b, c]]
```

```
▼ n[29] = shuffle[tensor[a], tensor[b], tensor[c, d]]
```

```
Out[29] = b⊗ c⊗ a⊗ d - d⊗ a - a⊗ c⊗ d -
  c⊗ (a⊗ b⊗ d - d⊗ b - b⊗ d - d⊗ b ⊗ a - a⊗ c⊗ b⊗ d - d⊗ b - a⊗ b⊗ c⊗ d
```

```
Symbol[PolyLog[__, _]] :=
```

```
-tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

```

```
n[8] = tensor[___, _Rational | _Integer, ___] := 0;

```

```
n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];

```

```
shuffle[x___, tensor[_, y___]] := shuffle[x, y];

```

```
shuffle[tensor[i___]] := tensor[i];

```

```
shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;

```

```
tensor[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;

```

```
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];

```

```
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

```

```
n[32] = shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]

```

```
▼ n[34] = shuffle[tensor[a], tensor[b], tensor[c]] // FullForm

```

```
Out[34] FullForm=

```

```
Plus[tensor[b, Plus[tensor[a, c], tensor[c, a]],
  tensor[c, Plus[tensor[a, b], tensor[b, a]], tensor[a, b, c], tensor[a, c, b]]

```

```

In[7] = tensor /: MakeBoxes[tensor[l__], fmt_ : StandardForm] := RowBox[{
  Sequence @@ Riffle[If[MemberQ[{Plus, Times}, Head[=]], RowBox[{" "}, MakeBoxes
    [=, fmt], " "]], MakeBoxes[=, fmt]] & /@ {l}, "⊗"]];

```

```

In[9] = SetAttributes[tensor, Flat];

```

```

In[11] = symbol[l_Plus] := symbol /@ l;
  symbol[n_?NumberQ x_] := n symbol[x];

```

```

In[13] = symbol[n_?NumberQ] := n tensor[];
  symbol[Log[x_]] := tensor[x];
  symbol[PolyLog[n_Integer, x_]] :=
    -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

```

```

In[18] = tensor[___, _Rational | _Integer, ___] := 0;

```

```

In[19] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
  shuffle[x___, tensor[, y___]] := shuffle[x, y];
  shuffle[tensor[l___]] := tensor[l];

  shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
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  shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
    tensor[x, =, z] & /@ y;

```

```
Symbol[PolyLog][_Integer, a_] :=
```

```
-tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];
```

```
n[8] = tensor[___, _Rational | _Integer, ___] := 0;
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n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=  
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```
shuffle[x___, tensor[_, y___]] := shuffle[x, y];
```

```
shuffle[tensor[i___]] := tensor[i];
```

```
shuffle[x___, y : HoldPattern[Plus[({_} _tensor) ..]], z___] :=  
  shuffle[x, z, z] & /@ y;
```

```
tensor[x___, y : HoldPattern[Plus[({_} _tensor) ..]], z___] :=  
  tensor[x, z, z] & /@ y;
```

```
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
```

```
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];
```

```
n[32] = shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
▼ n[34] = shuffle[tensor[a], tensor[b], tensor[c]] // FullForm
```

```
Out[34] FullForm=
```

```
Plus[tensor[b, Plus[tensor[a, c], tensor[c, a]]],  
  tensor[c, Plus[tensor[a, b], tensor[b, a]], tensor[a, b, c], tensor[a, c, b]]
```

Wolfram Mathematica 7.0 - [Demo.nb +]

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- Parallel Kernel Configuration...
- Parallel Kernel Status...
- Default Kernel
- Notebook's Kernel
- Notebook's Default Context
- Start Kernel
- Quit Kernel

```

n[8] = tensor[___,
n[9] = shuffle[tensor[a,
            tensor[c,
shuffle[x___,
shuffle[tensor[a,
            tensor[c,
shuffle[x___,
            tensor[a,
            tensor[c,
            tensor[d]]] +
            tensor[d]]];
e[x, y];
__tensor) ..]], z___] :=
tensor) ..]], z___] :=
y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ, tensor[a___], y___] := n tensor[x, tensor[a], y];

n[32] = shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
n[37] = shuffle[tensor[a], tensor[b], tensor[c]]
Out[37] = b^3 a^3 c - c^3 a^3 - c^3 a^3 b - b^3 a^3 - a^3 b^3 c - a^3 c^3 b

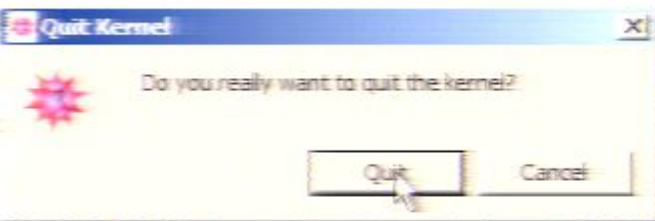
```

```
Symbol[PolyLog][a_Integer, x_] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, a - 1]];
```

```
n[8] = tensor[___, _Rational | _Integer, ___] := 0;
```

```
n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[i___]] := tensor[i];
```

```
shuffle[x___, y : HoldPattern[z___]] :=
  shuffle[x, z] & /@ z;
tensor[x___, y : HoldPattern[z___]] :=
  tensor[x, z] & /@ y;
```



```
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];
```

```
n[32] = shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
in[37] = shuffle[tensor[a], tensor[b], tensor[c]]
```

```
Out[37] = b^3 a^3 c - c^3 a - c^3 a^3 b - b^3 a - a^3 b^3 c - a^3 c^3 b
```



```
Symbol[PolyLog[__, _], _] :=
```

```
-tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];
```

```
n[8] = tensor[___, _Rational | _Integer, ___] := 0;
```

```
n[9] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=  
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +  
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
```

```
shuffle[x___, tensor[], y___] := shuffle[x, y];
```

```
shuffle[tensor[i___]] := tensor[i];
```

```
shuffle[x___, y : HoldPattern[Plus[(_, _tensor) ..]], z___] :=  
  shuffle[x, =, z] & /@ y;
```

```
tensor[x___, y : HoldPattern[Plus[(_, _tensor) ..]], z___] :=  
  tensor[x, =, z] & /@ y;
```

```
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
```

```
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];
```

```
n[32] = shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
▼ In[37] = shuffle[tensor[a], tensor[b], tensor[c]]
```

```
Out[37] = b3 a3c - c3a - c3 a3b - b3a - a3b3c - a3c3b
```

```
SetDirectory["C:\Users\cvergu\work\talks\PI2011\Demo"];
```

```
in[*]= tensor /: MakeBoxes[tensor[l__], fmt_ : StandardForm] := RowBox[{
  Sequence @@ Riffle[If[MemberQ[{Plus, Times}, Head[=]], RowBox[{"", MakeBoxes
    [=, fmt], ""}], MakeBoxes[=, fmt]] & /@ {l}, "@"]}]
```

```
n[85]= SetAttributes[tensor, {Flat, OneIdentity}];
```

```
n[*]= symbol[l_Plus] := symbol /@ l;
symbol[n_?NumberQ x_] := n symbol[x];
```

```
n[*8]= symbol[n_?NumberQ] := n tensor[];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];
```

```
n[*8]= tensor[___, _Rational | _Integer, ___] := 0;
```

```
n[*9]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];
```

```
shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
```

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```

In[4] = symbol[_Plus] := symbol /@ #;
symbol[_?NumberQ x_] := # symbol[x];

In[5] = symbol[_?NumberQ] := # tensor[];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, # - 1]];

In[6] = tensor[___, _Rational | _Integer, ___] := 0;

In[6] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, #, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, #, z] & /@ y;
shuffle[x___, _?NumberQ tensor[a___], y___] := # shuffle[x, tensor[a], y];
tensor[x___, _?NumberQ tensor[a___], y___] := # tensor[x, tensor[a], y];

In[9] = shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]

In[37] = shuffle[tensor[a], tensor[b], tensor[c]]
Pirsa: 11080038
Out[37] = b⊗ a⊗ c - c⊗ a - c⊗ a⊗ b - b⊗ a - a⊗ b⊗ c - a⊗ c⊗ b

```

```

symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

```

```

In[8]= tensor[___, _Rational | _Integer, ___] := 0;

```

```

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

```

```

shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;

```

```

shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

```

```

In[9]= shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]

```

```

▼ In[10]= shuffle[tensor[a], tensor[b], tensor[c]]

```

```

Out[10]= shuffle[a, b⊗c - c⊗b]

```

```

In[1]= SetDirectory["C:\Users\cvergu\work\talks\PI2011\Demo."];

In[2]= tensor /: MakeBoxes[tensor[l__], fmt_ : StandardForm] := RowBox[{
  Sequence @@ Riffle[If[MemberQ[{Plus, Times}, Head[=]], RowBox[{"", MakeBoxes
    [=, fmt], ""}], MakeBoxes[=, fmt]] & /@ {l}, "⊗"]]

In[3]= SetAttributes[tensor, {Flat, OneIdentity}];

In[4]= symbol[l_Plus] := symbol /@ l;
symbol[n_?NumberQ x_] := n symbol[x];

In[5]= symbol[n_?NumberQ] := n tensor[];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

In[6]= tensor[___, _Rational | _Integer, ___] := 0;

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;

```

```

In[1]= SetDirectory["C:\Users\cvergu\work\talks\PI2011\Demo"];

In[2]= tensor /: MakeBoxes[tensor[l__], fmt_ : StandardForm] := RowBox[{
  Sequence @@ Riffle[If[MemberQ[{Plus, Times}, Head[=]], RowBox[{" ", MakeBoxes
    [=, fmt], " "}], MakeBoxes[=, fmt]] & /@ {l}, "@"]]}

In[3]= SetAttributes[tensor, {Flat, OneIdentity}];

In[4]= symbol[l_Plus] := symbol /@ l;
  symbol[n_?NumberQ x_] := n symbol[x];

In[5]= symbol[n_?NumberQ] := n tensor[];
  symbol[Log[x_]] := tensor[x];
  symbol[PolyLog[n_Integer, x_]] :=
    | - tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

In[6]= tensor[___, _Rational | _Integer, ___] := 0;

In[7]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
  shuffle[x___, tensor[], y___] := shuffle[x, y];
  shuffle[tensor[l___]] := tensor[l];

  shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;

```

```

In[3]= SetAttributes[tensor, {Flat, OneIdentity}];

In[4]= symbol[1_Plus] := symbol /@ 1;
symbol[n_?NumberQ x_] := n symbol[x];

In[5]= symbol[n_?NumberQ] := n tensor[];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

In[6]= tensor[___, _Rational | _Integer, ___] := 0;

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

shuffle[x___, y : HoldPattern[Plus[(_, _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_, _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

```

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```

In[4]= symbol[_Plus] := symbol /@ #;
symbol[_?NumberQ x_] := # symbol[x];

In[5]= symbol[_?NumberQ] := # tensor[];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, # - 1]];

In[8]= tensor[___, _Rational | _Integer, ___] := 0;

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[_, y___]] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

In[7]= shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, #, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, #, z] & /@ y;
shuffle[x___, #?NumberQ tensor[a___], y___] := # shuffle[x, tensor[a], y];
tensor[x___, #?NumberQ tensor[a___], y___] := # tensor[x, tensor[a], y];

In[9]= lshuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]

In[10]= shuffle[tensor[a], tensor[b], tensor[c]]
Pirsa: 11080038
Out[10]= shuffle[a, b⊗c - c⊗b]

```



```

symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

```

```

In[8]= tensor[___, _Rational | _Integer, ___] := 0;

```

```

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[i___]] := tensor[i];

```

```

In[11]= shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;

```

```

tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;

```

```

shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

```

```

In[5]= shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]

```

```

In[6]= shuffle[tensor[a], tensor[b], tensor[c]]

```

```

Out[6]= a3b3c - a3c3b - b3a3c - b3c3a - c3a3b - c3b3a

```

```
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];
```

```
n[8] = tensor[___, _Rational | _Integer, ___] := 0;
```

```
n[6] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];
```

```
n[4] = shuffle[x___, y : HoldPattern[Plus[(_, _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_, _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];
```

```
n[5] = shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
▼ n[7] = shuffle[tensor[a], tensor[b], tensor[c, d]]
```

```
Out[7] = a⊗b⊗c⊗d - a⊗c⊗b⊗d - a⊗c⊗d⊗b - b⊗a⊗c⊗d - b⊗c⊗a⊗d - b⊗c⊗d⊗a -
  c⊗a⊗b⊗d - c⊗a⊗d⊗b - c⊗b⊗a⊗d - c⊗b⊗d⊗a - c⊗d⊗a⊗b - c⊗d⊗b⊗a
```

```

symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

n[8] = tensor[___, _Rational | _Integer, ___] := 0;

In[6] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

n[+] = shuffle[x___, y : HoldPattern[Plus[(_, _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_, _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
  (*Change attributes of shuffle to Flat*)

symbol[p_Times]

```

```

symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

symbol[Log[___]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

n[8] = tensor[___, _Rational | _Integer, ___] := 0;

n[6] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

n[5] = shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)

n[9] = symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expn]

```

```
-tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]]];
```

```
in[8] = tensor[___, _Rational | _Integer, ___] := 0;
```

```
in[6] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];
```

```
in[7] = shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes of shuffle to Flat*)
```

```
in[9] = symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
expand []
```

```

-tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

n[8] = tensor[___, _Rational | _Integer, ___] := 0;

In[6] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

n[11] = shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)

n[9] = symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := ex

```

```
-tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]]];
```

```
in[8]= tensor[___, _Rational | _Integer, ___] := 0;
```

```
in[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[_, y___]] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];
```

```
in[11]= shuffle[x___, y : HoldPattern[Plus[(_._tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_._tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
in[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
expand[expr_] := expr /. tensor[l___] => tensor @@ (Factor /@ {l})
```

```
-tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]]];
```

```
in[8] = tensor[___, _Rational | _Integer, ___] := 0;
```

```
in[6] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[_, y___]] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];
```

```
in[7] = shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
```

```
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
```

```
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
(*Change attributes of shuffle to Flat*)
```

```
in[9] = symbol[x_Times] := shuffle @@ symbol /@ List @@ x;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
expand[expr_] := expr
```



```

In[8]= tensor[___, _Rational | _Integer, ___] := 0;

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

In[11]= shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)

In[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := expr /. tensor[l___] :=> Factor[tensor[l]]

Factor

```

```

n[8] = tensor[___, _Rational | _Integer, ___] := 0;

n[6] = shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

n[7] = shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
  + "Change attributes of shuffle to Flat"

n[9] = symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := expr /. tensor[l___] :=> Factor[tensor[l]]

Factor[te

```

```

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

In[7]= shuffle[x___, y: HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y: HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c: __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)

In[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := expr /. tensor[l___] :=> Factor[tensor[l]]

In[2]= Factor[tensor[x + x y, z]]

Out[2]= (x + x y) z

```

```

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

In[7]= shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes of shuffle to Flat*)

In[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := expr /. tensor[l___] => Factor /@ tensor[l]

```

```

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

In[7]= shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)

In[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := expr /. tensor[l___] :=> Factor /@ tensor[l]

```

```

In[8]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

In[9]= shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes of shuffle to Flat*)

In[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := (expr /. tensor[l___] => Factor /@ tensor[l]) /.
  tensor[l___] => Distribute[tensor[l], Times, tensor, Plus, tensor]

```

```

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

In[7]= shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)

In[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor]

tens|

```

```

n[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

n[7]= shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
  (*Change attributes of shuffle to Flat*)

n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := (expr /. tensor[l___] => Factor /@ tensor[l]) /.
  tensor[l___] => Distribute[tensor[l], Times, tensor, Plus, tensor]

tensor[x (1 + y), z] tensor[l___] :=
  Distribute[tensor[l], Times, tensor, Plus, tensor]

```



```

    tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[_, y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

In[1]= shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
    shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
    tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
+Change attributes of shuffle to Flat+

In[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] := (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
    tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor]

In[23]= tensor[x (1+y), z] /.
    tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor]

Out[23]= x^3 z - 1 - y^3 z

```

```

    tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

n[1]= shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
    shuffle[x, =, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
    tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
    + "Change attributes to shuffle to Flat"

n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

expand[expr_] :=
    (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
    tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] // .

```

```

tensor[_, shuffle[tensor[a_, _], tensor[_]]],
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

n[1]= shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
+Change attributes of shuffle to Flat+

n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

n[24]= expand[expr_] :=
  (expr /. tensor[l___] => Factor /@ tensor[l]) /.
  tensor[l___] => Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]

canonical =

tensor

```

```

shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

n[11]= shuffle[x___, y: HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
tensor[x___, y: HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c: __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)

n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

n[24]= expand[expr_] :=
  (expr /. tensor[l___] => Factor /@ tensor[l]) /.
  tensor[l___] => Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] => n tensor[a1, a2, a3]

canonical =

tensor[

```

```
shuffle[x___, tensor[], y___] := shuffle[x, y];
```

```
shuffle[tensor[l___]] := tensor[l];
```

```
n[11]= shuffle[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, z, z] & /@ y;
```

```
tensor[x___, y : HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, z, z] & /@ y;
```

```
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
```

```
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
*Change attributes of shuffle to Flat*
```

```
n[19]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
(expr /. tensor[l___] => Factor /@ tensor[l]) /.
```

```
tensor[l___] => Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] => n tensor[a1, a2, a3]
```

```
n[25]= canonical = tensor[l___] => Sort[{#, -#}][[1]] & /@ tensor[l]
```

```
Out[25]= l___ => Sort[{1, -1}][1]
```



Genetarov polylogs

$$G(a_1, \dots, a_n; z) = \int_0^z \frac{dt_1}{t_1 - a_1} \int_0^{t_1} \frac{dt_2}{t_2 - a_2} \dots \int_0^{t_{n-1}} \frac{dt_n}{t_n - a_n}$$

Harmonic polylogs

$$H(a, \vec{m}; z) = \int_0^z dx f_a(x) H(\vec{m}; x), \quad f_a(x) = \begin{cases} \frac{x^a}{1-x}, & a=0 \\ \frac{1}{x}, & a=1 \\ \frac{1}{1-x}, & a=-1 \end{cases}$$

$$dG(a_1, \dots, a_n; z) = G(a_1, \dots, a_n; z) d \ln \left( \frac{z - a_1}{z - a_2} \right) + G(a_1, \dots, a_{n-1}, a_n; z) d \ln \left( \frac{a_{n-1} - z}{a_n - z} \right) + \dots + G(a_1, \dots, a_{n-1}; z) d \ln \left( \frac{a_{n-1} - z}{a_n - z} \right)$$

$$dH(a, \vec{m}; z) = H(\vec{m}; z) \begin{cases} d \ln z, & a=0 \\ d \ln \left( \frac{1}{1-z} \right), & a=1 \\ d \ln(1+z), & a=-1 \end{cases}$$

Geometric polylogs

$$G(a_1, \dots, a_n; z) = \int_0^z \frac{dt_1}{t_1 - a_1} \int_0^{t_1} \frac{dt_2}{t_2 - a_2} \dots \int_0^{t_{n-1}} \frac{dt_n}{t_n - a_n}$$

Harmonic polylogs

$$H(a, \vec{m}; z) = \int_0^z dx f_a(x) H(\vec{m}; x), \quad f_a(x) = \begin{cases} \frac{1}{x}, & a=0 \\ \frac{1}{1-x}, & a=1 \\ \frac{1}{1+x}, & a=-1 \end{cases}$$

$$dG(a_1, \dots, a_n; z) = G(a_1, \dots, a_n; z) d \ln \left( \frac{z - a_1}{a_1 - a_2} \right) + G(a_1, a_3, \dots, a_n; z) d \ln \left( \frac{a_1 - a_2}{a_2 - a_3} \right) + \dots + G(a_1, \dots, a_{n-1}; z) d \ln \left( \frac{a_{n-1} - a_n}{a_n} \right)$$

$$dH(a, \vec{m}; z) = H(\vec{m}; z) \begin{cases} d \ln z, & a=0 \\ d \ln \left( \frac{1}{1-z} \right), & a=1 \\ d \ln(1+z), & a=-1 \end{cases}$$



Geometric polylogs

$$G(a_1, \dots, a_n | z) = \int_0^z \frac{dt_1}{t_1 - a_1} \int_0^{t_1} \frac{dt_2}{t_2 - a_2} \dots \int_0^{t_{n-1}} \frac{dt_n}{t_n - a_n}$$

Harmonic polylogs

$$H(a, \vec{m}; z) = \int_0^z dx f_a(x) H(\vec{m}; x), \quad f_a(x) = \begin{cases} \frac{1}{x}, & a=0 \\ \frac{1}{1-x}, & a=1 \\ \frac{1}{1+x}, & a=-1 \end{cases}$$

$$dG(a_1, \dots, a_n | z) = G(a_1, \dots, a_n | z) d \ln \left( \frac{z - a_1}{a_1 - a_2} \right) + G(a_1, a_3, \dots, a_n | z) d \ln \left( \frac{a_1 - a_2}{a_2 - a_3} \right) \\ + \dots + G(a_1, \dots, a_{n-1} | z) d \ln \left( \frac{a_{n-1} - a_n}{a_n} \right)$$

$$dH(a, \vec{m}; z) = H(\vec{m}; z) \begin{cases} d \ln z, & a=0 \\ d \ln \left( \frac{1}{1-z} \right), & a=1 \\ d \ln(1+z), & a=-1 \end{cases}$$

$$u_{jkl}^{\pm} = \frac{1 - u_j - u_k + u_l \pm \sqrt{(u_j + u_k - u_l - 1)^2 - 4(1 - u_j)(1 - u_k)u_l}}{2(1 - u_j)u_l}$$

$$v_{jkl}^{\pm} = \frac{u_k - u_l \pm \sqrt{-4u_j u_k u_l + 2u_k u_l + u_k^2 + u_l^2}}{2(1 - u_j)u_k}$$

$$G(\dots, u_{ijk}, \dots) = G(\dots, u_{ij\bar{k}}, \dots) + G(\dots, u_{i\bar{j}k}, \dots)$$

$$H\left(\dots, \frac{1}{u_{ijk}}\right) = H\left(\dots, \frac{1}{u_{ij\bar{k}}}\right) + H\left(\dots, \frac{1}{u_{i\bar{j}k}}\right)$$

and similarly for  $v_{ijk}$

$$G(\dots, u_{ijk}, \dots) = G(\dots, u_{ijk}^+, \dots) + G(\dots, u_{ijk}^-, \dots)$$

$$H\left(\dots, \frac{1}{u_{ijk}}\right) = H\left(\dots, \frac{1}{u_{ijk}^+}\right) + H\left(\dots, \frac{1}{u_{ijk}^-}\right)$$

and similarly for  $v_{ijk}$



```

In[ ]:= shuffle[x___, y : HoldPattern[Plus[(_.___tensor) ...]], z___] :=
  shuffle[x, y, z] & /@ y;
tensor[x___, y : HoldPattern[Plus[(_.___tensor) ...]], z___] :=
  tensor[x, y, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
  *Change attributes to shuffle to Flat*

```

```

In[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

```

```

In[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]

```

```

In[26]= canonical = tensor[l___] :=> Sort[{=, -=}][[1]] & /@ tensor[l];

symbol[G[a___, z_]]
  With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] ->]

```

```
Out[27]= {a, b}
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
n[19]= symbol[p_Times] := shuffle@@symbol/@List@@p;
symbol[x_^n_Integer] := shuffle@@Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor/@tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] &/@tensor[l];
```

```
symbol[G[a___, z_]] :=
  MapIndexed[
    tensor[symbol[G[Sequence@@Delete[{a}, #2], z], #1] & Most[{z, a}] - {a}]
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a, b, c}]
```

```
Out[30]= {a, b}
```

```
▼ n[29]= MapIndexed[f, {a, b, c}]
```

```
Out[29]= {f[a, {1}], f[b, {2}], f[c, {3}]}
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
  *Change attributes to shuffle to Flat*
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
  MapIndexed[
```

```
    tensor[symbol[G[Sequence @@ Delete[{a}, #2], z], #1] & Most[{z, a}] - {a}]
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a, b, c}]
```

```
Out[30]= {a, b}
```

```
▼ n[29]= MapIndexed[f, {a, b, c}]
```

```
Out[29]= {f[a, {1}], f[b, {2}], f[c, {3}]}
```



```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
n[19]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
n[31]= symbol[G[a___, z_]] :=
  MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  Most[{z, a}] - {a}]
```

```
symbol
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a, b, c}]
```

```
Out[30]= {a, b}
```

```
▼ n[32]= MapIndexed[f, {a, b, c}]
```

```
Out[32]= {f[a, 1], f[b, 2], f[c, 3]}
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
(*Change attributes to shuffle to Flat*)
```

```
n[9]= symbol[p_Times] := shuffle@@symbol/@List@@p;
```

```
symbol[x_^n_Integer] := shuffle@@Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
(expr /. tensor[l___] :=> Factor/@tensor[l]) /.
```

```
tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] &/@tensor[l];
```

```
n[31]= symbol[G[a___, z_]] :=
```

```
MapIndexed[tensor[symbol[G[Sequence@@Delete[{a}, #2], z]], #1] &,
```

```
Most[{z, a}] - {a}]
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{{} ⓓ -a2 - z } ⓓ -a1 - z , {{{} ⓓ -a1 - z } ⓓ a1 - a2 }
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a, b, c}]
```

```
Out[30]= {a, b}
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
(*Change attributes to shuffle to Flat*)
```

```
n[19]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
(expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
Most[{z, a}] - {a}]
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{{{}} ⓓ -a2 - z } ⓓ -a1 - z , {{{} ⓓ -a1 - z } ⓓ /a1 - a2 }
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a, b, c}]
```

```
Out[30]= {a, b}
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
Change attributes to shuffle to Flat:
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
(expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
```

```
Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z], #1] &,
```

```
Most[{z, a}] - {a}]]
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{{} ⊗ {-a2 - z} ⊗ -a1 - z, {{} ⊗ -a1 - z} ⊗ a1 - a2}}
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a, b, c}]
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
  *Change attributes to shuffle to Flat*
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{#, -#}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
```

```
    Most[{z, a}] - {a}] -
```

```
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z], #1] &, Most
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{{} ⊗ {-a2 - z} ⊗ -a1 - z, {{} ⊗ -a1 - z} ⊗ a1 - a2 }
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a, b, c}]
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
  + "Change attributes to shuffle to Flat"
```

```
n[19] = symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24] = expand[expr_] :=
```

```
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26] = canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
    Most[{z, a}] - {a}] -
```

```
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &, #1]
```

```
▼ n[32] = symbol[G[a1, a2, z]]
```

```
Out[32] = {{0} ⊗ (-a2 - z) ⊗ -a1 - z, {{0} ⊗ -a1 - z} ⊗ (a1 - a2)}
```

```
▼ n[28] = With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28] = {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30] = Most[{a, b, c}]
```

```
Out[30] = {a, b}
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
n[9]= symbol[p_Times] := shuffle@@symbol/@List@@p;
symbol[x_^n_Integer] := shuffle@@Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor/@tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] &/@tensor[l];
```

```
symbol[G[a___, z_]] :=
  Plus@@MapIndexed[tensor[symbol[G[Sequence@@Delete[{a}, #2], z]], #1] &,
  Most[{z, a}] - {a}] -
  Plus@@MapIndexed[tensor[symbol[G[Sequence@@Delete[{a}, #2], z]], #1] &,
  {a} - 1]
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{{} ⊗ {-a2 - z} ⊗ -a1 - z, {{} ⊗ -a1 - z} ⊗ a1 - a2}
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a, b, c}]
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  {a} - ReplacePart[RotateLeft[{a}
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{{} ⊗ (-a2 - z) ⊗ -a1 - z, {{} ⊗ -a1 - z } ⊗ a1 - a2 }
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a b, c}]
```



```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
(*Change attributes to shuffle to Flat*)
```

```
n[9] = symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24] = expand[expr_] :=
```

```
(expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26] = canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
{a} - ReplacePart[RotateLeft[{a}], #]]
```

```
▼ n[32] = symbol[G[a1, a2, z]]
```

```
Out[32] = {{{} ⊗ {-a2 - z} ⊗ -a1 - z, {{} ⊗ -a1 - z} ⊗ a1 - a2 }
```

```
▼ n[28] = With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28] = {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30] = Most[{a, b, c}]
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1]], #1] &,
    Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]]
```

```
{a}
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{( ) ⊗ (-a2 - z) ⊗ -a1 - z, ( ) ⊗ -a1 - z } ⊗ a1 - a2 }
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {a1 - z, a1 - a2, a2 - a3}
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
*Change attributes to shuffle to Flat*
```

```
In[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
In[24]= expand[expr_] :=
```

```
(expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
In[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
{a} - ReplacePart[RotateLeft[{a}], -1 -> 0]]
```

```
In[33]= {a1, a2, a3} - {b1, b2, b3}
```

```
Out[33]= {a1 - b1, a2 - b2, a3 - b3}
```

```
In[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{0}^3 (-a2 - z)^3 - a1 - z, {{0}^3 - a1 - z }^3 a1 - a2 }
```

```
In[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
→ "Change attributes to shuffle to Flat"
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
(expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] → n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1]], #1] &,
```

```
Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
```

```
{a} - ReplacePart[RotateLeft[{a}], -1 → 0]]
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{{} ⊗ {-a2 - z} ⊗ {-a1 - z}, {{} ⊗ {-a1 - z} ⊗ a1 - a2}}
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28]= {-a1 - z, a1 - a2, a2 - a3}
```

```
▼ n[30]= Most[{a b, c}]
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
Change attributes to shuffle to Flat
```

```
n[9] = symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24] = expand[expr_] :=
```

```
(expr /. tensor[l___] := Factor /@ tensor[l]) /.
```

```
tensor[l___] := Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] → n tensor[a1, a2, a3]
```

```
n[26] = canonical = tensor[l___] := Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
{a} - ReplacePart[RotateLeft[{a}], -1 → 0]]
```

Rotate

```
▼ n[32] = symbol[G[a1, a2, z]]
```

```
Out[32] = {{{} ⊗ {-a2 - z} ⊗ -a1 - z, {{}} ⊗ -a1 - z ⊗ a1 - a2 }
```

```
▼ n[28] = With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
Out[28] = {-a1 - z, a1 - a2, a2 - a3}
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
  (*Change attributes to shuffle to Flat*)
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
```

```
    Most[{z, a}] - {a}] -
```

```
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z], #1] &,
```

```
    {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]]
```

```
▼ n[34]= RotateLeft[{a, b, c}]
```

```
Out[34]= {b, c, a}
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{{z}^3 (-a2 - z)^3 -a1 - z, {{{z}^3 -a1 - z }^3 (a1 - a2)}
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1]], #1] &,
  Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]]
```

```
▼ n[34]= RotateLeft[{a, b, c}]
```

```
Out[34]= {b, c, a}
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{( )^3 (-a2 - z)^3 -a1 - z, ( )^3 -a1 - z ( )^3 (a1 - a2)}
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
    Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]]
```

```
▼ n[34]= RotateLeft[{a, b, c}]
```

```
Out[34]= {b, c, a}
```

```
▼ n[32]= symbol[G[a1, a2, z]]
```

```
Out[32]= {{0} ⊗ (-a2 - z) ⊗ -a1 - z, {{0} ⊗ -a1 - z ⊗ a1 - a2}}
```

```
▼ n[28]= With[{a = Sequence[a1, a2, a3]}, Most[{z, a}] - {a}]
```



```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
/*Change attributes to shuffle to Flat*/
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
(expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
n[36]= symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
{a} - ReplacePart[RotateLeft[{a}], -1 -> 0]]]
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
```

```
/*Change attributes to shuffle to Flat*/
```

```
n[19]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
(expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
```

```
Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
```

```
{a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
```

```
S
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{#, -#}][[1]] & /@ tensor[l];
```

```
n[36]= symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
symbol[G
```

```
shuffle[a_tensor, b_tensor, c : __tensor] := shuffle[a, shuffle[b, c]]
(*Change attributes to shuffle to Flat*)
```

```
n[19]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];
```

```
n[36]= symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];

symbol[G[]]
```

```
Change attributes to shuffle to Flat.
```

```
n[19]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
```

```
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];
```

```
n[24]= expand[expr_] :=
```

```
(expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
```

```
tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
```

```
tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] :=> Sort[{-, =}] [[1]] & /@ tensor[l];
```

```
n[36]= symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
{a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
▼ n[38]= symbol[G[a1, a2, z]]
```

```
Out[38]= - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ a1 - a2 - a2 ⊗ (-a1 - z) -
```

```
a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ a1 - a2) - (a2 - z) ⊗ (-a1 - z)
```

```
symbol
```

```
Change attributes to shuffle to Flat
```

```

n[19]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
    tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
    tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]

n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];

n[36]= symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];

n[38]= symbol[G[a1, a2, z]]

Out[38]= - a1 ⓧ (a1 - a2) - a1 ⓧ a2 - a2 ⓧ (a1 - a2) - a2 ⓧ (-a1 - z) -
  a1 - z ⓧ (a1 - a2) - a1 - z ⓧ a2 - a2 - z ⓧ (a1 - a2) - (a2 - z) ⓧ (-a1 - z)

symbol[

```

```

n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
      symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

n[24]= expand[expr_] :=
      (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
      tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
      tensor[a1___, a2_^n_, a3___] + n tensor[a1, a2, a3]

n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];

n[36]= symbol[G[a___, z_]] :=
      Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
      Most[{z, a}] - {a}] -
      Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
      {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
      symbol[G[a_, z_]] := tensor[a - z] - tensor[a];

n[38]= symbol[G[a1, a2, z]]

Out[38]= - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ (a1 - a2) - a2 ⊗ (-a1 - z) -
      a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ a1 - a2 - (a2 - z) ⊗ (-a1 - z)

      symbol[H[a_, b___, x_]] :=
      tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1,

```

```

n[9]= symbol[p_Times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

n[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor /@ tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] + n tensor[a1, a2, a3]

n[26]= canonical = tensor[l___] :=> Sort[{-, =}] [[1]] & /@ tensor[l];

n[36]= symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];

n[38]= symbol[G[a1, a2, z]]

Out[38]= - a1 ⓧ (a1 - a2) - a1 ⓧ a2 - a2 ⓧ a1 - a2 - a2 ⓧ (-a1 - z) -
  a1 - z ⓧ a1 - a2 - a1 - z ⓧ a2 - a2 - z ⓧ a1 - a2 - (a2 - z) ⓧ (-a1 - z)

symbol[H[a_, b___, x_]] :=
  tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]]

```



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```

In[19]= symbol[p_times] := shuffle @@ symbol /@ List @@ p;
symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

In[24]= expand[expr_] :=
  (expr /. tensor[l___] :=> Factor/@tensor[l]) /.
  tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] -> n tensor[a1, a2, a3]

In[26]= canonical = tensor[l___] :=> Sort[{-, -}][[1]] & /@ tensor[l];

In[36]= symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];

In[38]= symbol[G[a1, a2, z]]

Out[38]= - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ a1 - a2 ⊗ (-a1 - z) -
  a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ a1 - a2) - (a2 - z) ⊗ (-a1 - z)

symbol[H[a_, b___, x_]] :=
  tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];

```

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```

n[21]= symbol[x_>:=] := shuffle @@ symbol /> List @@ x,
      symbol[x_^n_Integer] := shuffle @@ Table[symbol[x], {n}];

n[24]= expand[expr_] :=
      (expr /. tensor[l___] :=> Factor /> tensor[l]) /.
      tensor[l___] :=> Distribute[tensor[l], Times, tensor, Plus, tensor] //.
      tensor[a1___, a2_^n_, a3___] => n tensor[a1, a2, a3]

n[26]= canonical = tensor[l___] :=> Sort[{-, -}][[l]] & /> tensor[l];

n[36]= symbol[G[a___, z_]] :=
      Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
      Most[{z, a}] - {a}] -
      Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
      {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];

n[38]= symbol[G[a1, a2, z]]

Out[38]= - a1 & (a1 - a2) - a1 & a2 - a2 & (a1 - a2) - a2 & (-a1 - z) -
      a1 - z & (a1 - a2) - a1 - z & a2 - a2 - z & (a1 - a2) - (a2 - z) & (-a1 - z)

symbol[H[a_, b___, x_]] :=
      tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];
symbol[

```

```

n[24]= expand[expr_] :=
  (expr /. tensor[l___] := Factor/@tensor[l]) /.
  tensor[l___] := Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] → n tensor[a1, a2, a3]

n[26]= canonical = tensor[l___] := Sort[{=, -=}][[1]] & /@ tensor[l];

n[36]= symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    {a} - ReplacePart[RotateLeft[{a}], -1 → 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];

n[38]= symbol[G[a1, a2, z]]
Out[38]= - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ a1 - a2 ⊗ (-a1 - z) -
  a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ a1 - a2 - (a2 - z) ⊗ (-a1 - z)

n[39]= symbol[H[a_, b___, x_]] :=
  tensor[symbol[H[b, x]], Switch[a, 1, 1/(1 - x), 0, x, -1, 1 + x]];
symbol[H[_]] := tensor[];

```

```
(expr /. tensor[l___] := Factor /@ tensor[l]) /.
  tensor[l___] := Distribute[tensor[l], Times, tensor, Plus, tensor] //.
  tensor[a1___, a2_^n_, a3___] → n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[l___] := Sort[{=, -=}][[l]] & /@ tensor[l];
```

```
n[36]= symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, =2], z]], =1] &,
    Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, =2], z]], =1] &,
    {a} - ReplacePart[RotateLeft[{a}], -1 → 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
▼ n[38]= symbol[G[a1, a2, z]]
```

```
Out[38]= - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ (a1 - a2) - a2 ⊗ (-a1 - z) -
  a1 - z ⊗ (a1 - a2) - a1 - z ⊗ a2 - a2 - z ⊗ (a1 - a2) - a2 - z ⊗ (-a1 - z)
```

```
n[39]= symbol[H[a_, b___, x_]] :=
  tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];
symbol[H[_]] := tensor[[]];
```

```
n[4]= << "R62.dat";
```

```
rem
```

```
tensor[a1___, a2_^n_, a3___] + n tensor[a1, a2, a3]
```

```
n[26]= canonical = tensor[1___] := Sort[{=, -=}] [[1]] & /@ tensor[1];
```

```
n[36]= symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
  {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
▼ n[38]= symbol[G[a1, a2, z]]
```

```
Out[38]= - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ (a1 - a2) - a2 ⊗ (-a1 - z) -
  a1 - z ⊗ (a1 - a2) - a1 - z ⊗ a2 - a2 - z ⊗ (a1 - a2) - (a2 - z) ⊗ (-a1 - z)
```

```
n[39]= symbol[H[a_, b___, x_]] :=
```

```
tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];
symbol[H[_]] := tensor[];
```

```
n[41]= << "R62.dat";
```

```
n[42]= rem = %;
```

```
Length[%]
```

```
n[26] = canonical = tensor[1___] := Sort[{=, -=}] [[1]] & /@ tensor[1];
```

```
n[36] = symbol[G[a___, z_]] :=
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  Most[{z, a}] - {a}] -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
▼ n[38] = symbol[G[a1, a2, z]]
```

```
Out[38] = - a1 ⓧ (a1 - a2) - a1 ⓧ a2 - a2 ⓧ a1 - a2) - a2 ⓧ (-a1 - z) -
  a1 - z ⓧ a1 - a2 - a1 - z ⓧ a2 - a2 - z ⓧ a1 - a2) - a2 - z ⓧ (-a1 - z)
```

```
n[39] = symbol[H[a_, b___, x_]] :=
```

```
tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];
symbol[H[_]] := tensor[];
```

```
n[41] = << "R62.dat";
```

```
n[42] = rem = %;
```

```
▼ n[43] = Length[%]
```

```
Out[43] = 937
```

```

n[36] = symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];

```

```

▼ n[38] = symbol[G[a1, a2, z]]

```

```

Out[38] = - a1 ⓧ (a1 - a2) - a1 ⓧ a2 - a2 ⓧ (a1 - a2) - a2 ⓧ (-a1 - z) -
  a1 - z ⓧ (a1 - a2) - a1 - z ⓧ a2 - a2 - z ⓧ (a1 - a2) - (a2 - z) ⓧ (-a1 - z)

```

```

n[39] = symbol[H[a_, b___, x_]] :=
  tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];
symbol[H[_]] := tensor[];

```

```

n[41] = << "R62.dat";

```

```

n[42] = rem = %;

```

```

▼ n[43] = Length[%]

```

```

Out[43] = 937

```

```

rem [

```

```

In[36]= symbol[G[a___, z_]] :=
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
    Most[{z, a}] - {a}] -
  Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], #1], #1] &,
    {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];

```

```

▼ In[38]= symbol[G[a1, a2, z]]

```

```

Out[38]= - a1 Ⓢ (a1 - a2) - a1 Ⓢ a2 - a2 Ⓢ (a1 - a2) - a2 Ⓢ (-a1 - z) -
  a1 - z Ⓢ a1 - a2 - a1 - z Ⓢ a2 - a2 - z Ⓢ a1 - a2 - (a2 - z) Ⓢ (-a1 - z)

```

```

In[39]= symbol[H[a_, b___, x_]] :=
  tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];
symbol[H[_]] := tensor[];

```

```

In[41]= << "R62.dat";

```

```

In[42]= rem = %;

```

```

▼ In[43]= Length[%]

```

```

Out[43]= 937

```

```

rem[[-]

```



```
MOST[{z, a}] - {a} -
```

```
Plus @@ MapIndexed[tensor[symbol[G[Sequence @@ Delete[{a}, #2], z]], #1] &,
  {a} - ReplacePart[RotateLeft[{a}], -1 -> 0]];

```

```
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
▼ In[38] = symbol[G[a1, a2, z]]
```

```
Out[38] = - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ a1 - a2 ⊗ (-a1 - z) -
  a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ (a1 - a2) - a2 - z ⊗ (-a1 - z)
```

```
In[39] = symbol[H[a_, b___, x_]] :=
```

```
  tensor[symbol[H[b, x]], Switch[a, 1, 1/(1 - x), 0, x, -1, 1 + x]];

```

```
symbol[H[_]] := tensor[];
```

```
In[41] = << "R62.dat";
```

```
In[42] = rem = %;
```

```
▼ In[43] = Length[%]
```

```
Out[43] = 937
```

```
▼ In[44] = rem[[-1]]
```

```
Out[44] =  $\frac{1}{4} \text{Hca1} \left[ 1, \frac{1}{v[3, 2, 1]} \right] \cdot \text{Zeta}[3]$ 
```

```
{a} - ReplacePart[RotateLeft[{a}], -1 -> 0]]];
```

```
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
▼ In[38]= symbol[G[a1, a2, z]]
```

```
Out[38]= - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ (a1 - a2) - a2 ⊗ (-a1 - z) -  
a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ (a1 - a2) - (a2 - z) ⊗ (-a1 - z)
```

```
In[39]= symbol[H[a_, b___, x_]] :=
```

```
tensor[symbol[H[b, x]], Switch[a, 1, 1/(1 - x), 0, x, -1, 1 + x]]];
```

```
symbol[H[_]] := tensor[];
```

```
In[41]= << "R62.dat";
```

```
In[42]= rem = %;
```

```
▼ In[43]= Length[%]
```

```
Out[43]= 937
```

```
▼ In[44]= rem[[-1]]
```

```
Out[44]=  $\frac{1}{4} \text{Hca1} \left[ 1, \frac{1}{v[3, 2, 1]} \right] \cdot \text{Zeta}[3]$ 
```

```
rel
```

```
{a} - ReplacePart[RotateLeft[{a}], -1 -> 0];
```

```
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
▼ In[38]= symbol[G[a1, a2, z]]
```

```
Out[38]= - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ a1 - a2 ⊗ (-a1 - z) -  
a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ a1 - a2 - (a2 - z) ⊗ (-a1 - z)
```

```
In[39]= symbol[H[a_, b_, x_]] :=
```

```
tensor[symbol[H[b, x]], Switch[a, 1, 1/(1 - x), 0, x, -1, 1 + x]];
```

```
symbol[H[_]] := tensor[{}];
```

```
In[41]= << "R62.dat";
```

```
In[42]= rem = %;
```

```
▼ In[43]= Length[%]
```

```
Out[43]= 937
```

```
▼ In[44]= rem[[-1]]
```

```
Out[44]=  $\frac{1}{4} \text{Hca1}[1, \frac{1}{v[3, 2, 1]}] \cdot \text{Zeta}[3]$ 
```

```
rem = rem /. {Pi ->
```

```
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
▼ In[38] = symbol[G[a1, a2, z]]
```

```
Out[38] = - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ (a1 - a2) - a2 ⊗ (-a1 - z) -  
a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ a1 - a2 - a2 - z ⊗ (-a1 - z)
```

```
In[39] = symbol[H[a_, b_, x_]] :=  
tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];  
symbol[H[_]] := tensor[];
```

```
In[41] = << "R62.dat";
```

```
In[42] = rem = %;
```

```
▼ In[43] = Length[%]
```

```
Out[43] = 937
```

```
▼ In[44] = rem[[-1]]
```

```
Out[44] =  $\frac{1}{4} \text{Hca1}[1, \frac{1}{\sqrt{3, 2, 1}}] \text{Zeta}[3]$ 
```

```
In[45] = rem = rem /. {Pi → 0, Zeta[_] → 0};
```

```
Length
```

```
symbol[G[a_, z_]] := tensor[a - z] - tensor[a];
```

```
▼ In[38] = symbol[G[a1, a2, z]]
```

```
Out[38] = - a1 ⓧ (a1 - a2) - a1 ⓧ a2 - a2 ⓧ (a1 - a2) - a2 ⓧ (-a1 - z) -  
a1 - z ⓧ (a1 - a2) - a1 - z ⓧ a2 - a2 - z ⓧ (a1 - a2) - a2 - z ⓧ (-a1 - z)
```

```
In[39] = symbol[H[a_, b_, x_]] :=  
tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];  
symbol[H[_]] := tensor[];
```

```
In[41] = << "R62.dat";
```

```
In[42] = rem = %;
```

```
▼ In[43] = Length[%]
```

```
Out[43] = 937
```

```
▼ In[44] = rem[[-1]]
```

```
Out[44] =  $\frac{1}{4} \text{Hcal}\left[1, \frac{1}{\sqrt{3, 2, 1}}\right] \cdot \text{Zeta}[3]$ 
```

```
In[45] = rem = rem /. {Pi → 0, Zeta[_] → 0};
```

```
Length[rem]
```

```
▼ In[38] = Symbol[a_, b_, z_]
```

```
Out[38] = - a1 ⊗ (a1 - a2) - a1 ⊗ a2 - a2 ⊗ (a1 - a2) - a2 ⊗ (-a1 - z) -  
          a1 - z ⊗ (a1 - a2) - a1 - z ⊗ a2 - a2 - z ⊗ (a1 - a2) - a2 - z ⊗ (-a1 - z)
```

```
In[39] = symbol[H[a_, b___, x_]] :=  
        tensor[symbol[H[b, x]], Switch[a, 1, 1/(1-x), 0, x, -1, 1+x]];  
symbol[H[_]] := tensor[];
```

```
In[41] = << "R62.dat";
```

```
In[42] = rem = %;
```

```
▼ In[43] = Length[%]
```

```
Out[43] = 937
```

```
▼ In[44] = rem[[-1]]
```

```
Out[44] =  $\frac{1}{4} \text{Hca1}[1, \frac{1}{v[3, 2, 1]} \cdot \text{Zeta}[3]$ 
```

```
In[45] = rem = rem /. {Pi → 0, Zeta[_] → 0};
```

```
▼ In[46] = Length[rem]
```

```
Out[46] = 852
```

Out[39] = 
$$-a_1 \otimes (a_1 - a_2) - a_1 \otimes a_2 - a_2 \otimes (a_1 - a_2) - a_2 \otimes (-a_1 - z) -$$

$$a_1 - z \otimes (a_1 - a_2) - a_1 - z \otimes a_2 - a_2 - z \otimes (a_1 - a_2) - (a_2 - z) \otimes (-a_1 - z)$$

In[39] = `symbol[H[a_, b___, x_]] :=`  
`tensor[symbol[H[b, x]], Switch[a, 1, 1/(1-x), 0, x, -1, 1+x]];`  
`symbol[H[_]] := tensor[];`

In[41] = `<< "R62.dat";`

In[42] = `rem = %;`

▼ In[43] = `Length[%]`

Out[43] = 937

▼ In[44] = `rem[[-1]]`

Out[44] = 
$$\frac{1}{4} \text{Hca1}[1, \frac{1}{v[3, 2, 1]} \cdot \text{Zeta}[3]$$

In[45] = `rem = rem /. {Pi -> 0, Zeta[_] -> 0};`

▼ In[46] = `Length[rem]`

Out[46] = 852

tensor[/: N  
Sequence@@  
[A, fnt], "

symbol[a\_?  
symbol[a\_?]

symbol[a\_?  
symbol[Log  
symbol[Pol]

SetAttribut

tensor[\_\_\_]

shuffle[per  
shuffle[x\_

shuffle  
shuffle[per  
tensor[a, sh  
tensor[c, sh

tensor[

emo.nb +  
 In[38]= symbol[a\_, b\_, c\_]

Out[38]=  $-a^3(a-b) - a^2(a-b-c) - a(a-b-c)^2$

In[39]= symbol[H[a\_, b\_\_\_],  
 tensor[symbol[H[  
 symbol[H[\_]]] := ten

In[41]= << "R62.dat";

In[42]= rem = %;

In[43]= Length[%]

Out[43]= 937

In[44]= rem[[-1]]

Out[44]=  $\frac{1}{4} \text{Hca1}\left[1, \frac{1}{\sqrt{3, 2, 1}}\right]$

In[45]= rem = rem /. {Pi -> 0,

In[46]= Length[rem]

Out[46]= 852

Demo.m

Functions Sections Update Debug Run Package

```

tensor /: MakeBoxes[tensor[l_], fmt_:StandardForm] := RowBox[
Sequence@@Riffle[If[MemberQ[{Plus, Times}, Head[l]], RowBox[{"(", MakeBoxes
[l, fmt], ")"}], MakeBoxes[l, fmt]]@{1}, "@"]]]

(* The symbol is a local identifier *)
symbol[_Flap] := symbol @ 1;
symbol[_Number] a_ := a symbol[a];

(* List of symbols of some local identifiers *)
symbol[_Number] := a tensor[];
symbol[log[x_]] := tensor[x];
symbol[PolyLog[_Integer, x_]] := -tensor[1 - x, Sequence @@ Table[x, {0,
-1}]];

SetAttributes[tensor, {Flap, OneIdentity}];

tensor[___, _Rational _Integer, ___] := 0;

(* Shuffle symbols *)
shuffle[tensor[l_]] := tensor[l];
shuffle[a___, tensor[b], c___] := shuffle[a, b, c];

(* Shuffle symbols and the arguments *)
shuffle[tensor[a_, b___], tensor[c_, d___]] :=
tensor[a, shuffle[tensor[b], tensor[c, d]]] -
tensor[c, shuffle[tensor[a, b], tensor[d]]];

(* Local identifier *)
tensor[a] := a;
    
```



emo.nb +

▼ In[38] = symbol[a1, a2, a3]

Out[38] = - a1 & a1 - a2 - a1  
a1 - z & a1 - a2 -

In[39] = symbol[H[a\_, b\_\_\_],  
tensor[symbol[H[  
symbol[H[\_]]] := ten

In[41] = << "R62.dat";

In[42] = rem = %;

▼ In[43] = Length[%]

Out[43] = 937

▼ In[44] = rem[[-1]]

Out[44] =  $\frac{1}{4} \text{Hca1} \left[ 1, \frac{1}{v[3, 2, 1]} \right]$

In[45] = rem = rem /. {Pi -> 0,

▼ In[46] = Length[rem]

Out[46] = 852

Demo.m

Functions Sections Update Debug Run Package

```

symbol[B[a_, b_] :=
  Plus ## MapIndexed[
    tensor[symbol[B[Sequence ## Delete[{a}, #], #], #] &],
    Most[{a, #} - {a}], -
  Plus ## MapIndexed[
    tensor[symbol[
      B[Sequence ## Delete[{a}, #], #], #] &], {a} -
    ReplacePart[RotateLeft[{a}], -1 -> 0]]

symbol[R[a_, b___, c_] :=
  tensor[symbol[R[a, c]], Switch[a, 1, 1 - x, 2, x, -1, 1 - x]];

symbol[R[a_] := tensor[];

<< "R62.dat";

rem = %;

Length[rem]

rem = rem /. {Pi -> 0, Deba[_] -> 0};

Length[rem]

rem = rem /. {
  Pos[a___, b[a_, c_, d_] :=
  B[x, up[a, c, d], y] - B[x, um[a, c, d], y];

```

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emo.nb \*  
 In[38]= Symbol[a1, a2, a3]

Out[38]=  $-a_1 \otimes (a_1 - a_2) - a_1$   
 $a_1 - z \otimes (a_1 - a_2) -$

In[39]= symbol[H[a\_, b\_],  
 tensor[symbol[H[  
 symbol[H[\_]] := ten

In[41]= << "R62.dat";

In[42]= rem = %;

In[43]= Length[%]

Out[43]= 937

In[44]= rem[[-1]]

Out[44]=  $\frac{1}{4} \text{Hcal}[1, \frac{1}{v[3, 2, 1]}$

In[45]= rem = rem /. {Pi -> 0,

In[46]= Length[rem]

Out[46]= 852

Demo.m

Functions Sections Update Debug Run Package

```

symbol[H[x_]] := tensor[];
<< "R62.dat";
rem = %;
Length[rem];
rem = rem /. {Pi -> 0, Data[_] -> 0};
Length[rem];
rem = rem /. {
Goal[x_, u[i_, j_, k]^p_., y_] ->
G[x, up[i, j, k]^p, y] + G[x, um[i, j, k]^p, y];
Goal[x_, v[i_, j_, k]^p_., y_] ->
G[x, vp[i, j, k]^p, y] + G[x, vm[i, j, k]^p, y];
Hcal[x_, u[i_, j_, k]^p_., y_] ->
H[x, up[i, j, k]^p, y] + H[x, um[i, j, k]^p, y];
Hcal[x_, v[i_, j_, k]^p_., y_] ->
H[x, vp[i, j, k]^p, y] + H[x, vm[i, j, k]^p, y];
};
export = {}

```

```
▼ In[38] = Symbol[a1, a2, z]
```

```
Out[38] = - a1 ⊗ a1 - a2 - a1 ⊗ a2 - a2 ⊗ a1 - a2 ⊗ -a1 - z -  
a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ a1 - a2 - (a2 - z) ⊗ (-a1 - z)
```

```
In[39] = symbol[H[a_, b___, x_]] :=  
tensor[symbol[H[b, x]], Switch[a, 1, 1 / (1 - x), 0, x, -1, 1 + x]];  
symbol[H[_]] := tensor[];
```

```
In[41] = << "R62.dat";
```

```
In[42] = rem = %;
```

```
▼ In[43] = Length[%]
```

```
Out[43] = 937
```

```
▼ In[44] = rem[[-1]]
```

```
Out[44] =  $\frac{1}{4} \text{Hca1}[1, \frac{1}{v[3, 2, 1]}, \text{Zeta}[3]$ 
```

```
In[45] = rem = rem /. {Pi → 0, Zeta[_] → 0};
```

```
▼ In[46] = Length[rem]
```

```
Out[46] = 852
```

emo.nb \*

In[38]= Symbol[a1, a2, a3]

Out[38]= - a1 ⓧ (a1 - a2) - a1  
a1 - z ⓧ a1 - a2 -

In[39]= symbol[H[a\_, b\_\_\_,  
tensor[symbol[H[  
symbol[H[\_]] := ten

In[41]= << "R62.dat";

In[42]= rem = %;

In[43]= Length[%]

Out[43]= 937

In[44]= rem[[-1]]

Out[44]=  $\frac{1}{4} \text{Hcal}[1, \frac{1}{v[3, 2, 1]}$

In[45]= rem = rem /. {Pi -> 0,

In[46]= Length[rem]

Out[46]= 852

Demo.m

Functions Sections Update Debug Run Package

```

symbol[H[x_]] := tensor[];

<< "R62.dat";

rem = %;

Length[rem];

rem = rem /. {Pi -> 0, Data[_] -> 0};

Length[rem];

rem = rem //. {
  Goal[x___, u[i_, j_, k]^p_., y___] ->
    G[x, up[i, j, k]^p, y] + G[x, um[i, j, k]^p, y],
  Goal[x___, v[i_, j_, k]^p_., y___] ->
    G[x, vp[i, j, k]^p, y] + G[x, vm[i, j, k]^p, y],
  Hcal[x___, u[i_, j_, k]^p_., y___] ->
    H[x, up[i, j, k]^p, y] + H[x, um[i, j, k]^p, y],
  Hcal[x___, v[i_, j_, k]^p_., y___] ->
    H[x, vp[i, j, k]^p, y] + H[x, vm[i, j, k]^p, y]};

export = {
  a1 - 1 - sqrt[2], a2, a3] ->
  1 - sqrt[2, 0, k] - a1[1 - sqrt[2, 0, k]^2 -
  a1[k] + a1[1 - sqrt[2, 0, k] - a1[1] a1[1 - a1[k]^2],
  a1 - 1 - sqrt[2], a2, a3] ->
  1 - sqrt[2, 0, k] - a1[1 - sqrt[2, 0, k]^2 -

```

emo.nb +

```

In[38]= Symbol[a_, b_, c_]

```

```

Out[38]= - a1 ⊗ a1 - a2 - a1 ⊗ a2 - a2 ⊗ a1 - a2 - a2 ⊗ (-a1 - z) -
          a1 - z ⊗ a1 - a2 - a1 - z ⊗ a2 - a2 - z ⊗ a1 - a2) - a2 - z) ⊗ (-a1 - z)

```

```

In[39]= symbol[H[a_, b___, x_]] :=
        tensor[symbol[H[b, x]], Switch[a, 1, 1/(1-x), 0, x, -1, 1+x]];
symbol[H[_]] := tensor[];

```

```

In[41]= << "R62.dat";

```

```

In[42]= rem = %;

```

```

In[43]= Length[%]

```

```

Out[43]= 937

```

```

In[44]= rem[[-1]]

```

```

Out[44]=  $\frac{1}{4} \text{Hca1}\left[1, \frac{1}{v[3, 2, 1]}\right] \text{Zeta}[3]$ 

```

```

In[45]= rem = rem /. {Pi -> 0, Zeta[_] -> 0};

```

```

In[46]= Length[rem]

```

```

Out[46]= 852

```

emo.nb \*

```
▼ In[43]= Length[%]
```

```
Out[43]= 937
```

```
▼ In[44]= rem[[-1]]
```

```
Out[44]=  $\frac{1}{4} \text{Hcal}\left[1, \frac{1}{v[3, 2, 1]}\right] \text{Zeta}[3]$ 
```

```
In[45]= rem = rem /. {Pi -> 0, Zeta[_] -> 0};
```

```
▼ In[46]= Length[rem]
```

```
Out[46]= 852
```

```
rem = rem //. {
  Gcal[x___, u[i_, j_, k_]^p_, y___] ->
    G[x, up[i, j, k]^p, y] + G[x, um[i, j, k]^p, y],
  Gcal[x___, v[i_, j_, k_]^p_, y___] ->
    G[x, vp[i, j, k]^p, y] + G[x, vm[i, j, k]^p, y],
  Hcal[x___, u[i_, j_, k_]^p_, y___] ->
    H[x, up[i, j, k]^p, y] + H[x, um[i, j, k]^p, y],
  Hcal[x___, v[i_, j_, k_]^p_, y___] ->
    H[x, vp[i, j, k]^p, y] + H[x, vm[i, j, k]^p, y]};
```

In[43] = Length[%]

Out[43] = 937

In[44] = rem[[-1]]

Out[44] =  $\frac{1}{4} \text{Hcal}\left[1, \frac{1}{v[3, 2, 1]}\right]$

In[45] = rem = rem /. {Pi -> 0,

In[46] = Length[rem]

Out[46] = 852

In[47] = rem = rem //. {

Gcal[x\_\_\_, u[i\_, j, k]

G[x, up[i, j, k]^

Gcal[x\_\_\_, v[i\_, j, k]

G[x, vp[i, j, k]^

Hcal[x\_\_\_, u[i\_, j, k]

H[x, up[i, j, k]^

Hcal[x\_\_\_, v[i\_, j, k]

H[x, vp[i, j, k]^

**Demo.m**

Functions Sections Update Debug Run Package

```

symbol[E[x_]] := censor[];

<< "EPI.def";

rem = {};

Length[rem];

rem = rem /. {Pi -> 0, Data[_] -> 0};

Length[rem];

rem = rem //. {
  Gcal[x___, u[i_, j, k]^p_., y___] ->
    G[x, up[i, j, k]^p, y] + G[x, um[i, j, k]^p, y],
  Gcal[x___, v[i_, j, k]^p_., y___] ->
    G[x, vp[i, j, k]^p, y] + G[x, vm[i, j, k]^p, y],
  Hcal[x___, u[i_, j, k]^p_., y___] ->
    H[x, up[i, j, k]^p, y] + H[x, um[i, j, k]^p, y],
  Hcal[x___, v[i_, j, k]^p_., y___] ->
    H[x, vp[i, j, k]^p, y] + H[x, vm[i, j, k]^p, y]};

extrem = {}
  
```

▼ In[43] = Length[%]

Out[43] = 937

▼ In[44] = rem[[-1]]

Out[44] =  $\frac{1}{4} \text{Hcal}\left[1, \frac{1}{\sqrt{3, 2, 1}}\right]$

In[45] = rem = rem /. {Pi -> 0,

▼ In[46] = Length[rem]

Out[46] = 852

In[47] = rem = rem //. {

```
Gcal[x___, u[i_, j, k]]
G[x, up[i, j, k]^
Gcal[x___, v[i_, j, k]]
G[x, vp[i, j, k]^
Hcal[x___, u[i_, j, k]]
H[x, up[i, j, k]^
Hcal[x___, v[i_, j, k]]
H[x, vp[i, j, k]^
```

Demo.m

Functions Sections Update

Debug Run Package

```
Goal[x___, u[i_, j, k]^p_, y___] ->
  G[x, up[i, j, k]^p, y] - G[x, um[i, j, k]^p, y];
Goal[x___, v[i_, j, k]^p_, y___] ->
  G[x, vp[i, j, k]^p, y] - G[x, vm[i, j, k]^p, y];
Goal[x___, u[i_, j, k]^p_, y___] ->
  G[x, up[i, j, k]^p, y] - G[x, um[i, j, k]^p, y];
Goal[x___, v[i_, j, k]^p_, y___] ->
  G[x, vp[i, j, k]^p, y] - G[x, vm[i, j, k]^p, y];
```

```
extraw = {
  n_ (-1 + vp[i_, j_, k_]) ->
    ((1 - vp[i, j, k] + u[i]+vp[i, j, k])^2+
    (u[k] + u[j]+vp[i, j, k])) u[j]/(u[i]+u[k]^2),
  n_ (1 - vp[i_, j_, k_]) ->
    ((1 - vp[i, j, k] + u[i]+vp[i, j, k])^2+
    (u[k] + u[j]+vp[i, j, k])) u[j]/(u[i]+u[k]^2),
  n_ (-u[k_] - u[j_]+vp[i_, j_, k_] + u[i_]+u[j_]+vp[i_, j_, k_]) ->
    -((n u[i]+u[k])/((1 - vp[i, j, k] + u[i]+vp[i, j, k]))),
  tensor[xx___, n_ - vp[i_, j_, k_], yy___] ->
    tensor[xx, n ((1 - vp[i, j, k] + u[i]+vp[i, j, k]) +
    (u[k] + u[j]+vp[i, j, k]))/(u[i]+u[k]), yy],
  {++}
  n_ (-1 + vm[i_, j_, k_]) ->
    ((1 - vm[i, j, k] + u[i]+vm[i, j, k])^2+
    (u[k] + u[j]+vm[i, j, k])) u[j]/(u[i]+u[k]^2),
  n_ (1 - vm[i_, j_, k_]) ->
    ((1 - vm[i, j, k] + u[i]+vm[i, j, k])^2+
    (u[k] + u[j]+vm[i, j, k])) u[j]/(u[i]+u[k]^2)
```



emo.nb \*

▼ In[43] = Length[%]

Out[43] = 937

▼ In[44] = rem[[-1]]

Out[44] =  $\frac{1}{4} \text{Hcal}\left[1, \frac{1}{v[3, 2, 1]}\right]$

In[45] = rem = rem /. {Pi -> 0,

▼ In[46] = Length[rem]

Out[46] = 852

In[47] = rem = rem //. {

Gcal[x\_\_\_, u[i\_, :]

G[x, up[i, j, k]^

Gcal[x\_\_\_, v[i\_, :]

G[x, vp[i, j, k]^

Hcal[x\_\_\_, u[i\_, :]

H[x, up[i, j, k]^

Hcal[x\_\_\_, v[i\_, :]

H[x, vp[i, j, k]^

Demo.m

Functions Sections Update

Debug Run Package

```
extraw = {
  n_ (-1 + vp[i_, j_, k_]) ->
  ((1 - vp[i, j, k] + u[i]*vp[i, j, k])^2 +
  (u[k] + u[j]*vp[i, j, k])) u[j]/(u[i]+u[k]^2),
  n_ (1 - vp[i_, j_, k_]) ->
  ((1 - vp[i, j, k] + u[i]*vp[i, j, k])^2 +
  (u[k] + u[j]*vp[i, j, k])) u[j]/(u[i]+u[k]^2),
  n_ (-u[k_] - u[j]*vp[i_, j_, k_] + u[i_] + u[j]*vp[i_, j_, k_]) ->
  -((n u[i]+u[k])/(1 - vp[i, j, k] + u[i]*vp[i, j, k])),
  tensor[xx___, n_ - vp[i_, j_, k_], yy___] ->
  tensor[xx, n ((1 - vp[i, j, k] + u[i]*vp[i, j, k]) +
  (u[k] + u[j]*vp[i, j, k]))/(u[i]+u[k]), yy],
  (++)
  n_ (-1 + vm[i_, j_, k_]) ->
  ((1 - vm[i, j, k] + u[i]*vm[i, j, k])^2 +
  (u[k] + u[j]*vm[i, j, k])) u[j]/(u[i]+u[k]^2),
  n_ (1 - vm[i_, j_, k_]) ->
  ((1 - vm[i, j, k] + u[i]*vm[i, j, k])^2 +
  (u[k] + u[j]*vm[i, j, k])) u[j]/(u[i]+u[k]^2),
  n_ (-u[k_] - u[j]*vm[i_, j_, k_] + u[i_] + u[j]*vm[i_, j_, k_]) ->
  -((n u[i]+u[k])/(1 - vm[i, j, k] + u[i]*vm[i, j, k])),
  tensor[xx___, n_ - vm[i_, j_, k_], yy___] ->
  tensor[xx, n ((1 - vm[i, j, k] + u[i]*vm[i, j, k]) +
  (u[k] + u[j]*vm[i, j, k]))/(u[i]+u[k]), yy],
};
```

eliminatePlus =

```
▼ In[43]= Length[%]
```

```
Out[43]= 937
```

```
▼ In[44]= rem[[-1]]
```

```
Out[44]=  $\frac{1}{4} \text{Hcal}\left[1, \frac{1}{v[3, 2, 1]}\right] \text{Zeta}[3]$ 
```

```
In[45]= rem = rem /. {Pi -> 0, Zeta[_] -> 0};
```

```
▼ In[46]= Length[rem]
```

```
Out[46]= 852
```

```
In[47]= rem = rem //. {
```

```
  Gcal[x___, u[i_, j_, k_]^p_, y___] ->
```

```
  G[x, up[i, j, k]^p, y] + G[x, um[i, j, k]^p, y],
```

```
  Gcal[x___, v[i_, j_, k_]^p_, y___] ->
```

```
  G[x, vp[i, j, k]^p, y] + G[x, vm[i, j, k]^p, y],
```

```
  Hcal[x___, u[i_, j_, k_]^p_, y___] ->
```

```
  H[x, up[i, j, k]^p, y] + H[x, um[i, j, k]^p, y],
```

```
  Hcal[x___, v[i_, j_, k_]^p_, y___] ->
```

```
  H[x, vp[i, j, k]^p, y] + H[x, vm[i, j, k]^p, y]};
```

```

n_. (1 - vp[i_, j_, k_]) ->
  ((1 - vp[i, j, k] + u[i]*vp[i, j, k])^2 *
   (u[k] + u[j]*vp[i, j, k])) u[j] / (u[i]*u[k]^2),
n_. (-u[k_] - u[j_]*vp[i_, j_, k_] + u[i_]*u[j_]*vp[i_, j_, k_]) ->
  -((n u[i]*u[k]) / (1 - vp[i, j, k] + u[i]*vp[i, j, k])),
tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
  tensor[xx, n ((1 - vp[i, j, k] + u[i]*vp[i, j, k]) *
   (u[k] + u[j]*vp[i, j, k])) / (u[i]*u[k]), yy],
--
n_. (-1 + vm[i_, j_, k_]) ->
  ((1 - vm[i, j, k] + u[i]*vm[i, j, k])^2 *
   (u[k] + u[j]*vm[i, j, k])) u[j] / (u[i]*u[k]^2),
n_. (1 - vm[i_, j_, k_]) ->
  ((1 - vm[i, j, k] + u[i]*vm[i, j, k])^2 *
   (u[k] + u[j]*vm[i, j, k])) u[j] / (u[i]*u[k]^2),
n_. (-u[k_] - u[j_]*vm[i_, j_, k_] + u[i_]*u[j_]*vm[i_, j_, k_]) ->
  -((n u[i]*u[k]) / (1 - vm[i, j, k] + u[i]*vm[i, j, k])),
tensor[xx___, n_. vm[i_, j_, k_], yy___] ->
  tensor[xx, n ((1 - vm[i, j, k] + u[i]*vm[i, j, k]) *
   (u[k] + u[j]*vm[i, j, k])) / (u[i]*u[k]), yy]
};

```

- Basic Math Assistant
- Classroom Assistant
- Writing Assistant
- Slide Show
- Chart Element Schemes
- Color Schemes
- Special Characters
- Other
- Generate Palette from Selection
- Generate Notebook from Palette
- Install Palette...

```

n_. (1 - vp[i_, j_, k_]) ^ 2 *
((1 - vp[i, j, k] + u[i] * vm[i, j, k]) ^ 2 *
(u[k] + u[j] * vp[i, j, k])) / (u[i] * u[k]), yy],
n_. (-u[k_] - u[j_] * vm[i_, j_, k_] + u[i_] * u[j_] * vm[i_, j_, k_]) ->
-((n u[i] * u[k]) / (1 - vm[i, j, k] + u[i] * vm[i, j, k])),
tensor[xx___, n_. vm[i_, j_, k_], yy___] ->
tensor[xx, n ((1 - vm[i, j, k] + u[i] * vm[i, j, k]) *
(u[k] + u[j] * vm[i, j, k])) / (u[i] * u[k]), yy]
};

```

- Magnification
- Show Ruler
- Show Toolbar
- Show Handwriting Input
- Stack Windows
- Tile Windows Wide
- Tile Windows Tall
- Full Screen: F12
- Messages**
- Demo.m
- Demo.nb +

\_ | □ | ×

▲

$$^2 * u[k]^2),$$

$$[i_] * u[j_] * vp[i_, j_, k_]) ->$$

$$i] * vp[i, j, k]),$$

$$_]) ->$$

$$* vp[i, j, k]) * (u[k] + u[j] * vp[i, j, k])) / (u[i] * u[k]), yy],$$

```

n_. (1 - vp[i_, j_, k_]
  ((1 - vp[i, j, k] + u
  (u[k] + u[j] * vp[i, j
n_. (-u[k_] - u[j_] * v
  - ((n u[i] * u[k]) / (1 -
tensor[xx___, n_. vp[
  tensor[xx, n ((1 - vp
  (u[k] + u[j] * vp[i, j, k])) / (u[i] * u[k]), yy],
...
n_. (-1 + vm[i_, j_, k_]) ->
  ((1 - vm[i, j, k] + u[i] * vm[i, j, k]) ^2 *
  (u[k] + u[j] * vm[i, j, k])) u[j] / (u[i] * u[k]^2),
n_. (1 - vm[i_, j_, k_]) ->
  ((1 - vm[i, j, k] + u[i] * vm[i, j, k]) ^2 *
  (u[k] + u[j] * vm[i, j, k])) u[j] / (u[i] * u[k]^2),
n_. (-u[k_] - u[j_] * vm[i_, j_, k_] + u[i_] * u[j_] * vm[i_, j_, k_]) ->
  - ((n u[i] * u[k]) / (1 - vm[i, j, k] + u[i] * vm[i, j, k])),
tensor[xx___, n_. vm[i_, j_, k_], yy___] ->
  tensor[xx, n ((1 - vm[i, j, k] + u[i] * vm[i, j, k]) *
  (u[k] + u[j] * vm[i, j, k])) / (u[i] * u[k]), yy]
};

```

```

n_. (1 - vp[i_, j_, k_]) ->
((1 - vp[i, j, k]
(u[k] + u[j]*vp[i
n_. (-u[k_] - u[j_]
-((n u[i]*u[k])/
tensor[xx___, n_.
tensor[xx, n ((1
(u[k] + u[j]*vp[
--
n_. (-1 + vm[i_, j_
((1 - vm[i, j, k]
(u[k] + u[j]*vm[i
n_. (1 - vm[i_, j_
((1 - vm[i, j, k]
(u[k] + u[j]*vm[i
n_. (-u[k_] - u[j_]
-((n u[i]*u[k])/
tensor[xx___, n_.
tensor[xx, n ((1
(u[k] + u[j]*vm[
];

```

```

Demo.m
Functions Sections Update
Debug Run Package

extraw = [
n_. (-1 + vp[i_, j_, k_]) ->
((1 - vp[i, j, k] + u[i]*vp[i, j, k])^2 +
(u[k] + u[j]*vp[i, j, k]) u[j]/(u[i]+u[k]^2),
n_. (1 - vp[i_, j_, k_]) ->
((1 - vp[i, j, k] + u[i]*vp[i, j, k])^2 +
(u[k] + u[j]*vp[i, j, k]) u[j]/(u[i]+u[k]^2),
n_. (-u[k_] - u[j_] + vp[i_, j_, k_] + u[i_] + u[j_] + vp[i_, j_, k_]) ->
-((n u[i]*u[k])/(1 - vp[i, j, k] + u[i]*vp[i, j, k])),
tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
tensor[xx, n ((1 - vp[i, j, k] + u[i]*vp[i, j, k]) +
(u[k] + u[j]*vp[i, j, k]))/(u[i]+u[k]), yy],
{++}
n_. (-1 + vm[i_, j_, k_]) ->
((1 - vm[i, j, k] + u[i]*vm[i, j, k])^2 +
(u[k] + u[j]*vm[i, j, k]) u[j]/(u[i]+u[k]^2),
n_. (1 - vm[i_, j_, k_]) ->
((1 - vm[i, j, k] + u[i]*vm[i, j, k])^2 +
(u[k] + u[j]*vm[i, j, k]) u[j]/(u[i]+u[k]^2),
n_. (-u[k_] - u[j_] + vm[i_, j_, k_] + u[i_] + u[j_] + vm[i_, j_, k_]) ->
-((n u[i]*u[k])/(1 - vm[i, j, k] + u[i]*vm[i, j, k])),
tensor[xx___, n_. vm[i_, j_, k_], yy___] ->
tensor[xx, n ((1 - vm[i, j, k] + u[i]*vm[i, j, k]) +
(u[k] + u[j]*vm[i, j, k]))/(u[i]+u[k]), yy],
];
eliminatePlus =

```

```

u_. (1 - vp[i_, j_, k_]) ->
((1 - vp[i, j, k]
(u[k] + u[j]*vp[i
u_. (-u[k_] - u[j_]
- ((n u[i]*u[k]) /
tensor[xx___, u_.
tensor[xx, n ((1
(u[k] + u[j]*vp[.
--
u_. (-1 + vm[i_, j_
((1 - vm[i, j, k]
(u[k] + u[j]*vm[i
u_. (1 - vm[i_, j_
((1 - vm[i, j, k]
(u[k] + u[j]*vm[i
u_. (-u[k_] - u[j_]
- ((n u[i]*u[k]) /
tensor[xx___, u_.
tensor[xx, n ((1
(u[k] + u[j]*vm[
];

```

Demo.m

Functions Sections Update

Debug Run Package

```

((1 - vm[i, j, k] + u[i]*vm[i, j, k])^2 +
(u[k] + u[j]*vm[i, j, k]) u[j]/(u[i]+u[k]^2),
u_. (-u[k_] - u[j_] + vm[i_, j_, k_] + u[i_] + u[j_] + vm[i_, j_, k_] ->
-((n u[i]*u[k])/(1 - vm[i, j, k] + u[i]*vm[i, j, k])),
tensor[xx___, u_. vm[i_, j_, k_] , yy___] ->
tensor[xx, n ((1 - vm[i, j, k] + u[i]*vm[i, j, k]) +
(u[k] + u[j]*vm[i, j, k])/(u[i]+u[k]), yy]
];

```

```

{ eliminatePlus = {
u_. (-1 + vp[i_, j_, k_]) -> n u[i] (-1 + u[i] (-1 + vm[i, j, k] ,
tensor[xx___, u_. vp[i_, j_, k_], yy___] ->
tensor[xx, n u[k] (-1 - u[i] + u[i] vm[i, j, k], yy],
u_. (1 - vp[i_, j_, k_]) + u[i_] vp[i_, j_, k_] ->
n u[i] (-u[k] + u[i] (1 - vm[i, j, k] - u[i] vm[i, j, k] ,
u_. u[k_] - u[i_] vp[i_, j_, k_] ->
n u[i] + u[k]^2 (-1 - u[i] + u[k] - u[i] vm[i, j, k]),
u_. -u[k_] - u[i_] vp[i_, j_, k_] - u[i_] u[i_] vp[i_, j_, k_] ->
n u[i] u[i] u[k] (-u[k] - u[i] vm[i, j, k] - u[i] u[i] vm[i, j, k]
};

symb = symbol[rem];
symb = symb . {
vm[i_Integer, u_Integer, j_Integer] :> u[i] -> -u[i] u[k] vp[i, j, k],
vp[i_Integer, u_Integer, j_Integer] :> u[i] -> -u[i] u[k] vm[i, j, k]
};

```

```

n_. (1 - vp[i_, j_, k_]) ->
((1 - vp[i, j, k]
(u[k] + u[j]*vp[i
n_. (-u[k_] - u[j_]
-((n u[i]*u[k])/
tensor[xx___, n_.
tensor[xx, n ((1
(u[k] + u[j]*vp[
**
n_. (-1 + vm[i_, j_
((1 - vm[i, j, k]
(u[k] + u[j]*vm[i
n_. (1 - vm[i_, j_]
((1 - vm[i, j, k]
(u[k] + u[j]*vm[i
n_. (-u[k_] - u[j_]
-((n u[i]*u[k])/
tensor[xx___, n_.
tensor[xx, n ((1
(u[k] + u[j]*vm[
];

```

```

Demo.m
Functions Sections Update
Debug Run Package

1 - vm[i, j, k] - u[i] - vm[i, j, k]^2 +
u[k] - u[i] - vm[i, j, k] - u[i] - u[i]^2 +
u[j] - u[i] - u[j] - vm[i, j, k] - u[i] - u[i]^2 + vm[i, j, k] ->
- u[i] - u[k] - 1 - vm[i, j, k] - u[i] - vm[i, j, k]
tensor[xx___, n_. - vm[i_, j_, k_] / yy___] ->
tensor[xx, n_. 1 - vm[i, j, k] - u[i] - vm[i, j, k] +
u[k] - u[i] - vm[i, j, k] - u[i] - u[k] / yy]

eliminatePlus = [
n_. (-1 + vp[i_, j_, k_]) -> n u[i]/(-1 + u[i])/(-1 + vm[i, j, k]),
tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
tensor[xx, n u[k]/((-1 + u[i]) + u[j])/vm[i, j, k], yy],
n_. (1 - vp[i_, j_, k_] + u[i_] vp[i_, j_, k_]) ->
n (u[i] + u[k])/u[j]/(1 - vm[i, j, k] + u[i] vm[i, j, k]),
n_. (u[k_] + u[j_] vp[i_, j_, k_]) ->
n (u[i] + u[k]^2)/(-1 + u[i])/u[k] + u[j] vm[i, j, k],
n_. (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
n u[i] u[j] u[k]/(-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
];

symb = symbol[rem];
symb = symb /. [
vm[i_Integer, j_Integer, k_Integer] ; &] -> -u[j] u[k] vp[i, j, k],
vp[i_Integer, j_Integer, k_Integer] ; &] -> -u[j] u[k] vm[i, j, k]
];

```



```

n_. (1 - vp[i_, j_, k_]) ->
((1 - vp[i, j, k]
(u[k] + u[j] * vp[i
n_. (-u[k_] - u[j_]
- ((n u[i] * u[k]) /
tensor[xx___, n_.
tensor[xx, n ((1
(u[k] + u[j] * vp[.
**
n_. (-1 + vm[i_, j_
((1 - vm[i, j, k]
(u[k] + u[j] * vm[i
n_. (1 - vm[i_, j_]
((1 - vm[i, j, k]
(u[k] + u[j] * vm[i
n_. (-u[k_] - u[j_]
- ((n u[i] * u[k]) /
tensor[xx___, n_.
tensor[xx, n ((1
(u[k] + u[j] * vm[
};

```

```

Demo.m
Functions Sections Update
Debug Run Package

1 - vm[i, j, k] - u[i] - vm[i, j, k]^2 +
u[k] - u[j] - vm[i, j, k] - u[i] - u[i]^2 +
u[j] - u[k] - u[j] - vm[i, j, k] - u[i] - u[i]^2 + vm[i, j, k] ->
- n u[i] + u[k] - 1 - vm[i, j, k] - u[i] - vm[i, j, k]
tensor[xx___, n - vm[i, j, k] - u[i] - u[i]^2 ->
tensor[xx, n - 1 - vm[i, j, k] - u[i] - vm[i, j, k] +
u[k] - u[j] - vm[i, j, k] - u[i] - u[i]^2 + u[j]
];

eliminatePlus = [
n_. (-1 + vp[i_, j_, k_]) -> n u[i] / (-1 + u[i]) / (-1 + vm[i, j, k]),
tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
tensor[xx, n u[k] / ((-1 + u[i]) + u[j]) / vm[i, j, k], yy],
n_. (1 - vp[i_, j_, k_]) + u[i_] vp[i_, j_, k_] ->
n (u[i] + u[k]) / u[j] / (1 - vm[i, j, k]) + u[i] vm[i, j, k],
n_. (u[k_] + u[j_] vp[i_, j_, k_]) ->
n (u[i] + u[k]^2) / (-1 + u[i]) / (u[k] + u[j] vm[i, j, k]),
n_. (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
];

symb = symbol[rem];
symb = symb /. {
vm[i_Integer, k_Integer, j_Integer] /: k > j -> -u[j] / u[k] vp[i, j, k],
vp[i_Integer, k_Integer, j_Integer] /: k > j -> -u[j] / u[k] vm[i, j, k]
};
];

```

```

n_. (1 - vp[i_, j_, k_]) ->
  ((1 - vp[i, j, k] + u[i]*vp[i, j, k])^2 *
   (u[k] + u[j]*vp[i, j, k])) u[j] / (u[i]*u[k]^2),
n_. (-u[k_] - u[j_]*vp[i_, j_, k_] + u[i_]*u[j_]*vp[i_, j_, k_]) ->
  -((n u[i]*u[k]) / (1 - vp[i, j, k] + u[i]*vp[i, j, k])),
tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
  tensor[xx, n ((1 - vp[i, j, k] + u[i]*vp[i, j, k]) *
   (u[k] + u[j]*vp[i, j, k])) / (u[i]*u[k]), yy],
--
n_. (-1 + vm[i_, j_, k_]) ->
  ((1 - vm[i, j, k] + u[i]*vm[i, j, k])^2 *
   (u[k] + u[j]*vm[i, j, k])) u[j] / (u[i]*u[k]^2),
n_. (1 - vm[i_, j_, k_]) ->
  ((1 - vm[i, j, k] + u[i]*vm[i, j, k])^2 *
   (u[k] + u[j]*vm[i, j, k])) u[j] / (u[i]*u[k]^2),
n_. (-u[k_] - u[j_]*vm[i_, j_, k_] + u[i_]*u[j_]*vm[i_, j_, k_]) ->
  -((n u[i]*u[k]) / (1 - vm[i, j, k] + u[i]*vm[i, j, k])),
tensor[xx___, n_. vm[i_, j_, k_], yy___] ->
  tensor[xx, n ((1 - vm[i, j, k] + u[i]*vm[i, j, k]) *
   (u[k] + u[j]*vm[i, j, k])) / (u[i]*u[k]), yy]
};
eliminatePlus = {
  n_. (-1 + vp[i_, j_, k_]) -> n u[i] / (-1 + u[i]) / (-1 + vm[i, j, k]),
  tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
  tensor[xx, n u[k] / ((-1 + u[i]) * u[j]) / vm[i, j, k], yy],

```

```

-((n u[i]*u[k]) / (1 - vm[i, j, k] + u[i]*vm[i, j, k])),
tensor[xx___, n_. vm[i_, j_, k_], yy___] ->
tensor[xx, n ((1 - vm[i, j, k] + u[i]*vm[i, j, k]) *
(u[k] + u[j]*vm[i, j, k])) / (u[i]*u[k]), yy]
};

eliminatePlus = {
n_. (-1 + vp[i_, j_, k_]) -> n u[i] / (-1 + u[i]) / (-1 + vm[i, j, k]),
tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
tensor[xx, n u[k] / ((-1 + u[i])*u[j]) / vm[i, j, k], yy],
n_. (1 - vp[i_, j_, k_] + u[i_] vp[i_, j_, k_]) ->
n (u[i]*u[k]) / u[j] / (1 - vm[i, j, k] + u[i] vm[i, j, k]),
n_. (u[k_] + u[j_] vp[i_, j_, k_]) ->
n (u[i]*u[k]^2) / (-1 + u[i]) / (u[k] + u[j] vm[i, j, k]),
n_. (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
};

symb = symbol[rem];
symb = symb /. {
vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vp[i, j, k],
vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vm[i, j, k]
};

```

```

Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Running_Demo.nb *
- ((n u[i] * u[k]) / (1 -
tensor[xx___, n_., vm[
tensor[xx, n ((1 - vm
(u[k] + u[j] * vm[i, j
});
eliminatePlus = {
n_. (-1 + vp[i_, j_, k_
tensor[xx___, n_. vp[i_, j_, k_] ->
tensor[xx, n u[k] / ((-1 + u[i]) * u[j]) / vm[i, j, k], yy],
n_. (1 - vp[i_, j_, k_] + u[i_] vp[i_, j_, k_] ->
n (u[i] * u[k]) / u[j] / (1 - vm[i, j, k] + u[i] vm[i, j, k]),
n_. (u[k_] + u[j_] vp[i_, j_, k_] ->
n (u[i] * u[k]^2) / (-1 + u[i]) / (u[k] + u[j] vm[i, j, k]),
n_. (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_] ->
n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
});

symb = symbol[rem];
symb = symb /. {
vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vp[i, j, k],
vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vm[i, j, k]
};

```

- Magnification
- Show Ruler
- Show Toolbar
- Show Handwriting Input
- Stack Windows
- Tile Windows Wide
- Tile Windows Tall
- Full Screen F12
- Messages
- Detvoim**
- Running...Demo.nb \*

```

[i] * vm[i, j, k]),
_ ] ->
* vm[i, j, k]) *
k]), yy]
+ u[i]) / (-1 + vm[i, j, k]),
->
yy],
->
k]),
->
k])

```

```

-((n u[i] * u[k]) / (1 - vm[i, j, k] + u[i] + u[j] + u[k]))
tensor[xx___, n_] :=
  tensor[xx, n ((1
  (u[k] + u[j] * vm[
  ]);
eliminatePlus = {
  n_ (-1 + vp[i_, j_, k_])
  tensor[xx___, n_]
  tensor[xx, n u[k]
  n_ (1 - vp[i_, j_, k_])
  n (u[i] * u[k]) / u[
  n_ (u[k_] + u[j_]
  n (u[i] * u[k]^2) /
  n_ (-u[k_] - u[j_]
  n u[i] u[j] u[k] /
  };

  symb = symbol[rem]
  symb = symb /. {
    vm[i_Integer, k_Integer, j_Integer]
    vp[i_Integer, k_Integer, j_Integer]
  };

```

Demo.m

Functions Sections Update

Debug Run Package

```

1 - vm[i, j, k] - u[i] - vm[i, j, k]^2 +
u[k] - u[i] - vm[i, j, k] - u[j] - u[i] + u[k]^2,
u_ -> u[k] - u[i] - vm[i, j, k] - u[j] + u[i] + u[k]^2 ->
- u u[i] + u[k] - 1 - vm[i, j, k] - u[i] - vm[i, j, k]
tensor[xx___, n_] -> vm[i_, j_, k_] ->
tensor[xx, n (1 - vm[i, j, k] - u[i] - vm[i, j, k] +
u[k] - u[i] - vm[i, j, k] - u[i] + u[k]^2), yy]
};

```

```

eliminatePlus = {
  n_ (-1 + vp[i_, j_, k_]) -> n u[i] / (-1 + u[i]) / (-1 + vm[i, j, k]),
  tensor[xx___, n_ - vp[i_, j_, k_], yy___] ->
  tensor[xx, n u[k] / ((-1 + u[i]) + u[j]) / vm[i, j, k], yy],
  n_ (1 - vp[i_, j_, k_]) + u[i_] vp[i_, j_, k_] ->
  n (u[i] + u[k]) / u[j] / (1 - vm[i, j, k] + u[i] vm[i, j, k]),
  n_ (u[k_] + u[j_] vp[i_, j_, k_]) ->
  n (u[i] + u[k]^2) / (-1 + u[i]) / (u[k] + u[j] vm[i, j, k]),
  n_ (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
  n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
};

symb = symbol[rem];
symb = symb /. {
  vm[i_Integer, k_Integer, j_Integer] /: k > j -> -u[j] / u[k] vp[i, j, k],
  vp[i_Integer, k_Integer, j_Integer] /: k > j -> -u[j] / u[k] vm[i, j, k]
};

```

```

-((n u[i] * u[k]) / (1 - vm[i + k] + u[i] + vm[i + k]))
tensor[xx___, n_] :=
tensor[xx, n ((1
(u[k] + u[j] * vm[
]);
eliminatePlus = {
n_ (-1 + vp[i_, j_
tensor[xx___, n_]
tensor[xx, n u[k]
n_ (1 - vp[i_, j_
n (u[i] * u[k]) / u[
n_ (u[k_] + u[j_]
n (u[i] * u[k]^2) /
n_ (-u[k_] - u[j_]
n u[i] u[j] u[k] /
});

symb = symbol[rem]
symb = symb /. {
vm[i_Integer, k_Integer
vp[i_Integer, k_Integer
};

```

Demo.m

Functions Sections Update

Debug Run Package

```

n_ (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
n u[i] u[j] u[k] / (-u[k_] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
];
symb = symbol[rem];
symb = symb /. {
vm[i_Integer, k_Integer, j_Integer] /: k > j -> -u[j] / u[k] vp[i, j, k],
vp[i_Integer, k_Integer, j_Integer] /: k > j -> -u[j] / u[k] vm[i, j, k]
};

```

```

symbe = Expand[expand[symb]];
symbe1 = Expand[expand[symbe, expand]];
symbe2 = Expand[expand[symbe1, eliminatePlus]];

length = {rem, symb, symbe, symbe1, symbe2};

Select[symbe2, (FreeQ[#, vm | vp | m | {1, 2, 3}] &)]

```

```

uprules = {
vp[1, 2, 3] -> -1 - delta - u[1] - u[2] - u[3], 2 - -1 - u[1] + u[3],
vp[2, 3, 1] -> -1 - delta - u[1] - u[2] - u[3], 2 - -1 - u[2] + u[1],
vp[3, 1, 2] -> -1 - delta - u[1] - u[2] - u[3], 2 - u[2] - -1 - u[3],
vm[1, 2, 3] -> -1 - delta - u[1] - u[2] - u[3], 2 - -1 - u[1] + u[3],
vm[2, 3, 1] -> -1 - delta - u[1] - u[2] - u[3], 2 - -1 - u[2] + u[1],
vm[3, 1, 2] -> -1 - delta - u[1] - u[2] - u[3], 2 - u[2] - -1 - u[3]
};

```

```

-((n u[i]*u[k]) / (1 - vm[i, j, k] + u[i]*vm[i, j, k])),
tensor[xx___, n_. vm[i_, j_, k_], yy___] ->
tensor[xx, n ((1 - vm[i, j, k] + u[i]*vm[i, j, k]) *
(u[k] + u[j]*vm[i, j, k])) / (u[i]*u[k]), yy]
];
eliminatePlus = {
n_. (-1 + vp[i_, j_, k_]) -> n u[i] / (-1 + u[i]) / (-1 + vm[i, j, k]),
tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
tensor[xx, n u[k] / ((-1 + u[i]) * u[j]) / vm[i, j, k], yy],
n_. (1 - vp[i_, j_, k_] + u[i_] vp[i_, j_, k_]) ->
n (u[i]*u[k]) / u[j] / (1 - vm[i, j, k] + u[i] vm[i, j, k]),
n_. (u[k_] + u[j_] vp[i_, j_, k_]) ->
n (u[i]*u[k]^2) / (-1 + u[i]) / (u[k] + u[j] vm[i, j, k]),
n_. (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
];

symb = symbol[rem];
symb = symb /. {
vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vp[i, j, k],
vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vm[i, j, k]
};

```

```

tensor[xx___, n_. vm[i_, j_, k_], yy___] ->
  tensor[xx, n ((1 - vm[i, j, k] + u[i] + vm[i, j, k]) *
    (u[k] + u[j] + vm[i, j, k])) / (u[i] * u[k]), yy]
];

eliminatePlus = {
  n_. (-1 + vp[i_, j_, k_]) -> n u[i] / (-1 + u[i]) / (-1 + vm[i, j, k]),
  tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
    tensor[xx, n u[k] / ((-1 + u[i]) * u[j]) / vm[i, j, k], yy],
  n_. (1 - vp[i_, j_, k_] + u[i_] vp[i_, j_, k_]) ->
    n (u[i] * u[k]) / u[j] / (1 - vm[i, j, k] + u[i] vm[i, j, k]),
  n_. (u[k_] + u[j_] vp[i_, j_, k_]) ->
    n (u[i] * u[k]^2) / (-1 + u[i]) / (u[k] + u[j] vm[i, j, k]),
  n_. (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
    n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
};

symb = symbol[rem];
symb = symb /. {
  vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vp[i, j, k],
  vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vm[i, j, k]
};

sym

```



```

(u[k] + u[j] * vm[i, j, k]) / (u[i] * u[k]), yy]
};
eliminatePlus = {
  u_ . (-1 + vp[i_, j_, k_]) -> n u[i] / (-1 + u[i]) / (-1 + vm[i, j, k]),
  tensor[xx___, u_ . vp[i_, j_, k_], yy___] ->
  tensor[xx, n u[k] / ((-1 + u[i]) * u[j]) / vm[i, j, k], yy],
  u_ . (1 - vp[i_, j_, k_] + u[i_] vp[i_, j_, k_]) ->
  n (u[i] * u[k]) / u[j] / (1 - vm[i, j, k] + u[i] vm[i, j, k]),
  u_ . (u[k_] + u[j_] vp[i_, j_, k_]) ->
  n (u[i] * u[k]^2) / (-1 + u[i]) / (u[k] + u[j] vm[i, j, k]),
  u_ . (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
  n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
};

symb = symbol[rem];
symb = symb /. {
  vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vp[i, j, k],
  vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vm[i, j, k]
};

```

```

In[52]= symb // Length

```

```

Out[52]= 4809

```

```

/ ,
eliminatePlus = {
  n_ . (-1 + vp[i_, j_, k_]) -> n u[i] / (-1 + u[i]) / (-1 + vm[i, j, k]),
  tensor[xx___, n_ . vp[i_, j_, k_], yy___] ->
  tensor[xx, n u[k] / ((-1 + u[i]) * u[j]) / vm[i, j, k], yy],
  n_ . (1 - vp[i_, j_, k_] + u[i_] vp[i_, j_, k_]) ->
  n (u[i] * u[k]) / u[j] / (1 - vm[i, j, k] + u[i] vm[i, j, k]),
  n_ . (u[k_] + u[j_] vp[i_, j_, k_]) ->
  n (u[i] * u[k]^2) / (-1 + u[i]) / (u[k] + u[j] vm[i, j, k]),
  n_ . (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
  n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
};

symb = symbol[rem];
symb = symb /. {
  vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vp[i, j, k],
  vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vm[i, j, k]
};

```

▼ In[52] = `symb // Length`

Out[52] = 4809

`symb`

```

eliminatePlus = {
  n_. (-1 + vp[i_, j_, k_]) -> nu[i]/(-1 + u[i])/(-1 + vm[i, j, k]),
  tensor[xx___, n_. vp[i_, j_, k_], yy___] ->
  tensor[xx, nu[k]/((-1 + u[i])*u[j])/vm[i, j, k], yy],
  n_. (1 - vp[i_, j_, k_] + u[i_] vp[i_, j_, k_]) ->
  n (u[i]*u[k])/u[j]/(1 - vm[i, j, k] + u[i] vm[i, j, k]),
  n_. (u[k_] + u[j_] vp[i_, j_, k_]) ->
  n (u[i]*u[k]^2)/(-1 + u[i])/(u[k] + u[j] vm[i, j, k]),
  n_. (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
  nu[i] u[j] u[k]/(-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
};

symb = symbol[rem];
symb = symb /. {
  vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j]/u[k] vp[i, j, k],
  vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j]/u[k] vm[i, j, k]
};

```

▼ n[52]= symb // Length

Out[52]= 4809

Take[sy

```

n_. (1 - vp[i_, j_, k_] + u[i_] vp[i_, j_, k_]) ->
  n (u[i] * u[k]) / u[j] / (1 - vm[i, j, k] + u[i] vm[i, j, k]),
n_. (u[k_] + u[j_] vp[i_, j_, k_]) ->
  n (u[i] * u[k]^2) / (-1 + u[i]) / (u[k] + u[j] vm[i, j, k]),
n_. (-u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_]) ->
  n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
};

symb = symbol[rem];
symb = symb /. {
  vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vp[i, j, k],
  vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vm[i, j, k]
};

```

```

In[52] = symb // Length

```

```

Out[52] = 4809

```

```

In[53] = Take[symb, 5]

```

```

Out[53] = - -180808 -  $\frac{u[2] \text{vm}[1, 2, 3]}{u[3]}$  - -180808  $\frac{u[2] \text{vm}[1, 2, 3]}{u[3]}$  -
-180808 -vm[2, 1, 3] - -180808vm[2, 1, 3] - -180808  $-\frac{u[1] \text{vm}[3, 1, 2]}{u[2]}$ 

```

```

n_ ( -u[k_] - u[j_] vp[i_, j_, k_] + u[i_] u[j_] vp[i_, j_, k_] ) ->
  n u[i] u[j] u[k] / ( -u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k] )
};

```

```

symb = symbol[rem];

```

```

symb = symb /. {

```

```

  vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vp[i, j, k],

```

```

  vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vm[i, j, k]

```

```

};

```

```

In[52]= symb // Length

```

```

Out[52]= 4809

```

```

In[53]= Take[symb, 5]

```

```

Out[53]= - -180808 -  $\frac{u[2] vm[1, 2, 3]}{u[3]}$  - -180808  $\frac{u[2] vm[1, 2, 3]}{u[3]}$  -
-180808 -vm[2, 1, 3] - -180808 vm[2, 1, 3] - -180808  $\left( -\frac{u[1] vm[3, 1, 2]}{u[2]} \right)$ 

```

```

n u[i] u[j] u[k] / (-u[k] - u[j] vm[i, j, k] + u[i] u[j] vm[i, j, k])
};

```

```

symb = symbol[rem];
symb = symb /. {
  vm[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vp[i, j, k],
  vp[i_Integer, k_Integer, j_Integer] /; k > j -> -u[j] / u[k] vm[i, j, k]
};

```

```

In[52]= symb // Length

```

```

Out[52]= 4809

```

```

In[53]= Take[symb, 5]

```

```

Out[53]= -180303 -  $\frac{u[2] \text{vm}[1, 2, 3]}{u[3]}$  -180303  $\frac{u[2] \text{vm}[1, 2, 3]}{u[3]}$  -
-180303 -vm[2, 1, 3] -180303vm[2, 1, 3] -180303  $\left(-\frac{u[1] \text{vm}[3, 1, 2]}{u[2]}\right)$ 

```

emo.nb \*

Out[52]= 4809

In[53]= **Take[symb, 5]**

$$\text{Out[53]} = \left( -1 \otimes 0 \otimes 0 \otimes \frac{u[2] \text{vm}[1, 2, 3]}{u[3]} - -1 \otimes 0 \otimes 0 \otimes \frac{u[2] \text{vm}[1, 2, 3]}{u[3]} \right) -$$

$$-1 \otimes 0 \otimes 0 \otimes -\text{vm}[2, 1, 3] - -1 \otimes 0 \otimes 0 \otimes \text{vm}[2, 1, 3] - -1 \otimes 0 \otimes 0 \otimes \left( -\frac{u[1] \text{vm}[3, 1, 2]}{u[2]} \right)$$
In[54]= **tensor[a, 1, b]**Out[54]=  $a \otimes 1 \otimes b$ In[55]= **?? tensor**

Global`tensor

Attributes[tensor] = {Flat, OneIdentity}

x\_\_\_⊗y:HoldPattern[- \_\_. \_tensor.. ]⊗z\_\_\_ := (x⊗=1⊗z &amp; /@y

x\_\_\_⊗n\_?NumberQ a\_\_\_⊗y\_\_\_ := nx⊗a⊗y

MakeBoxes[l\_\_\_, fmt\_:StandardForm]^:=

RowBox[{Sequence@@Riffle[If[MemberQ[{Plus, Times}, Head[=1]],

RowBox[[], MakeBoxes[=1, fmt], []], MakeBoxes[=1, fmt]] &amp;] /@ {l}, ⊗]]

```

In[1]= SetDirectory["C:\Users\cvergu\work\talks\PI2011\Demo/"];

In[2]= tensor /: MakeBoxes[tensor[l__], fmt_ : StandardForm] := RowBox[{
  Sequence @@ Riffle[If[MemberQ[{Plus, Times}, Head[=]], RowBox[{" "}], MakeBoxes
    [=, fmt], " "], MakeBoxes[=, fmt]] & /@ {l}, "@"]];

In[3]= SetAttributes[tensor, {Flat, OneIdentity}];

In[4]= symbol[l_Plus] := symbol /@ l;
  symbol[n_?NumberQ x_] := n symbol[x];

In[5]= symbol[n_?NumberQ] := n tensor[];
  symbol[Log[x_]] := tensor[x];
  symbol[PolyLog[n_Integer, x_]] :=
    -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

In[6]= tensor[___, _Rational | _Integer, ___] := 0;

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[l___]] := tensor[l];

In[7]= shuffle[x___, y : HoldPattern[Plus[(._. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;

```



```

In[3]= SetAttributes[tensor, {Flat, OneIdentity}];

In[4]= symbol[1_Plus] := symbol /@ 1;
symbol[n_?NumberQ x_] := n symbol[x];

In[5]= symbol[n_?NumberQ] := n tensor[];
symbol[Log[x_]] := tensor[x];
symbol[PolyLog[n_Integer, x_]] :=
  -tensor[1 - x, Sequence @@ ConstantArray[x, n - 1]];

In[8]= tensor[___, _Rational | _Integer, ___] := 0;

In[6]= shuffle[tensor[a_, b___], tensor[c_, d___]] :=
  tensor[a, shuffle[tensor[b], tensor[c, d]]] +
  tensor[c, shuffle[tensor[a, b], tensor[d]]];
shuffle[x___, tensor[], y___] := shuffle[x, y];
shuffle[tensor[1___]] := tensor[1];

In[7]= shuffle[x___, y: HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  shuffle[x, =, z] & /@ y;
tensor[x___, y: HoldPattern[Plus[(_. _tensor) ..]], z___] :=
  tensor[x, =, z] & /@ y;
shuffle[x___, n_?NumberQ tensor[a___], y___] := n shuffle[x, tensor[a], y];
tensor[x___, n_?NumberQ tensor[a___], y___] := n tensor[x, tensor[a], y];

shuffle[a_tensor, b_tensor, c: ___tensor] := shuffle[a, shuffle[b, c]]

```

```
MakeBoxes[l___, fmt_ : StandardForm] ^:=  
  RowBox[Sequence @@ Riffle[ If[MemberQ[{Plus, Times}, Head[l]],  
    RowBox[l], MakeBoxes[l, fmt], - ]], MakeBoxes[l, fmt]] & /@ {l}, 3]]
```

Out[52]= 4809

▼ In[53]= **Take[symb, 5]**

Out[53]= 
$$\begin{aligned} & -1 \otimes 0 \otimes 0 \otimes \frac{u[2] \text{vm}[1, 2, 3]}{u[3]} - 1 \otimes 0 \otimes 0 \otimes \frac{u[2] \text{vm}[1, 2, 3]}{u[3]} - \\ & -1 \otimes 0 \otimes 0 \otimes \text{vm}[2, 1, 3] - 1 \otimes 0 \otimes 0 \otimes \text{vm}[2, 1, 3] - 1 \otimes 0 \otimes 0 \otimes \left( -\frac{u[1] \text{vm}[3, 1, 2]}{u[2]} \right) \end{aligned}$$

▼ In[54]= **tensor[a, 1, b]**

Out[54]= a  $\otimes$  1  $\otimes$  b

▼ In[55]= **?? tensor**

▼ Global`tensor

Attributes[tensor] = {Flat, OneIdentity}

x\_\_\_  $\otimes$  y: HoldPattern[- \_\_. \_tensor.. ]  $\otimes$  z\_\_\_ := (x  $\otimes$  1  $\otimes$  z & /@ y

x\_\_\_  $\otimes$  n\_?NumberQ a\_\_\_  $\otimes$  y\_\_\_ := n x  $\otimes$  a  $\otimes$  y

MakeBoxes[l\_\_\_, fmt\_: StandardForm] ^:=

RowBox[{Sequence @@ Riffle[ If[MemberQ[{Plus, Times}, Head[=1]],

RowBox[[] , MakeBoxes[=1, fmt[], []], MakeBoxes[=1, fmt[]] &] /@ {l},  $\otimes$ ]]]

▼ In[59]= `symp // Length`

Out[59]= 1677

▼ In[58]= `Take[symp, 5]`

Out[58]= 
$$\left[ \begin{array}{l} -1 - \frac{1}{u[1]} \otimes -\frac{1}{u[1]} \otimes -\frac{1}{u[1]} \otimes 1 - \frac{1}{u[2]} \\ -1 - \frac{1}{u[1]} \otimes -\frac{1}{u[1]} \otimes -\frac{1}{u[1]} \otimes \frac{1}{u[2]} \\ -1 - \frac{1}{u[1]} \otimes -\frac{1}{u[1]} \otimes -\frac{1}{u[1]} \otimes 1 - \frac{1}{u[3]} \\ -1 - \frac{1}{u[1]} \otimes -\frac{1}{u[1]} \otimes -\frac{1}{u[1]} \otimes \frac{1}{u[3]} \\ -1 - \frac{1}{u[1]} \otimes -\frac{1}{u[1]} \otimes \frac{1}{u[1]} \otimes -\frac{1}{u[1] - u[2]} \end{array} \right]$$

▼ In[60]= `syambe = Expand[expand[symp]];  
syambe2 = Expand[expand[syambe // . extrav]];  
syambe3 = Expand[expand[syambe2 // . eliminatePlus]];`

`Length /@ {rem, symp, syambe, syambe2, syambe3}`

Out[60]= {855, 1677, 8912, 4329, 3729}

```
Out[63]= {855, 1677, 8912, 4329, 3729}
```

```
▼ In[64]= upmrules = {
  up[1, 2, 3] -> -(1 + delta - u[1] - u[2] + u[3]) / (2 * (-1 + u[1]) * u[3]),
  up[2, 3, 1] -> -(1 + delta + u[1] - u[2] - u[3]) / (2 * (-1 + u[2]) * u[1]),
  up[3, 1, 2] -> -(1 + delta - u[1] + u[2] - u[3]) / (2 * u[2] * (-1 + u[3])),
  um[1, 2, 3] -> -(1 - delta - u[1] - u[2] + u[3]) / (2 * (-1 + u[1]) * u[3]),
  um[2, 3, 1] -> -(1 - delta + u[1] - u[2] - u[3]) / (2 * (-1 + u[2]) * u[1]),
  um[3, 1, 2] -> -(1 - delta - u[1] + u[2] - u[3]) / (2 * u[2] * (-1 + u[3]))
}
```

```
Out[64]= {up[1, 2, 3] -> 
$$\frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 (-1 - u[1]) u[3]}$$
,
  up[2, 3, 1] -> 
$$\frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] (-1 - u[2])}$$
,
  up[3, 1, 2] -> 
$$\frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] (-1 - u[3])}$$
,
  um[1, 2, 3] -> 
$$\frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 (-1 - u[1]) u[3]}$$
,
  um[2, 3, 1] -> 
$$\frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] (-1 - u[2])}$$
,
  um[3, 1, 2] -> 
$$\frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] (-1 - u[3])}$$

}
```

- Magnification: ▶
- Show Ruler
- Show Toolbar
- Show Handwriting Input
- Stack Windows
- Tile Windows Wide
- Tile Windows Tall
- Full Screen F12
- Messages
- Demo**
- Demo.nb \*

Out[63]= {855, 1677, 3912, 4329,

```

In[64]= upmrules = {
  up[1, 2, 3] -> -(1 + d
  up[2, 3, 1] -> -(1 + d
  up[3, 1, 2] -> -(1 + d
  um[1, 2, 3] -> -(1 - d
  um[2, 3, 1] -> -(1 - delta + u[1] - u[2]
  um[3, 1, 2] -> -(1 - delta - u[1] + u[2]
}

```

$$\begin{aligned}
 &+ u[3]) / (2 * (-1 + u[1]) * u[3]), \\
 &- u[3]) / (2 * (-1 + u[2]) * u[1]), \\
 &- u[3]) / (2 * u[2] * (-1 + u[3])), \\
 &+ u[3]) / (2 * (-1 + u[1]) * u[3]), \\
 &- u[3]) / (2 * (-1 + u[2]) * u[1]), \\
 &- u[3]) / (2 * u[2] * (-1 + u[3]))
 \end{aligned}$$

$$\begin{aligned}
 \text{Out[64]} = \{ & \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 - 1 - u[1]} u[3], \\
 & \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] - 1 - u[2]}, \\
 & \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] - 1 - u[3]}, \\
 & \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 - 1 - u[1]} u[3], \\
 & \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] - 1 - u[2]}, \\
 & \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] - 1 - u[3]} \}
 \end{aligned}$$

Out[83]= {1855, 1677, 8912, 4329, 3729}

Demo.m

Functions Sections Update

Debug Run Package

```
select_symbes, {FreeQ[#, vm | vp | v | 1, 2, 3]] & /;
```

```
upmrules = {
  up[1,2,3] -> -(1 + delta - u[1] - u[2] + u[3]) / (2*(-1 + u[1])*u[3]),
  up[2,3,1] -> -(1 + delta + u[1] - u[2] - u[3]) / (2*(-1 + u[2])*u[1]),
  up[3,1,2] -> -(1 + delta - u[1] + u[2] - u[3]) / (2*u[2]*(-1 + u[3])),
  um[1,2,3] -> -(1 - delta - u[1] - u[2] + u[3]) / (2*(-1 + u[1])*u[3]),
  um[2,3,1] -> -(1 - delta + u[1] - u[2] - u[3]) / (2*(-1 + u[2])*u[1]),
  um[3,1,2] -> -(1 - delta - u[1] + u[2] - u[3]) / (2*u[2]*(-1 + u[3]))
}
```

```
cod = {
```

```
delta ->
```

```
(z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] - z[2]*z[3]*z[5] -
 z[1]*z[4]*z[5] - z[2]*z[4]*z[5] - z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
 z[1]*z[4]*z[6] - z[3]*z[4]*z[6] - z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
(z[1] - z[4] + z[2] - z[5] + z[3] - z[6]),
```

```
u[1] -> (z[2]-z[3] - z[5]-z[6]) / (z[2]-z[5] - z[3]-z[6]),
```

```
u[2] -> (z[3]-z[4] - z[6]-z[1]) / (z[3]-z[6] - z[4]-z[1]),
```

```
u[3] -> (z[4]-z[5] - z[1]-z[2]) / (z[4]-z[1] - z[5]-z[2]);
```

Out[91]= {955, 1677, 3912, 4329, 3729}

Demo.m

Functions Sections Update

Debug Run Package

```

up[1,2,3] -> -(1 + delta - u[1] - u[2] + u[3]) / (2*(-1 + u[1])*u[3]),
up[2,3,1] -> -(1 + delta + u[1] - u[2] - u[3]) / (2*(-1 + u[2])*u[1]),
up[3,1,2] -> -(1 + delta - u[1] + u[2] - u[3]) / (2*u[2]*(-1 + u[3])),
um[1,2,3] -> -(1 - delta - u[1] - u[2] + u[3]) / (2*(-1 + u[1])*u[3]),
um[2,3,1] -> -(1 - delta + u[1] - u[2] - u[3]) / (2*(-1 + u[2])*u[1]),
um[3,1,2] -> -(1 - delta - u[1] + u[2] - u[3]) / (2*u[2]*(-1 + u[3]))

```

tot = {}

delta ->

$$\frac{z[1]z[2]z[4] - z[1]z[3]z[4] - z[1]z[2]z[5] - z[2]z[3]z[5] - z[1]z[4]z[5] - z[2]z[4]z[5] - z[1]z[3]z[6] - z[2]z[3]z[6] - z[1]z[4]z[6] - z[3]z[4]z[6] - z[2]z[5]z[6] - z[3]z[5]z[6]}{(z[1] - z[4])z[2] - z[5]z[3] - z[6]}$$

$$u[1] \rightarrow \frac{z[2] - z[3]}{z[5]z[6]} \frac{z[2] - z[5]}{z[3] - z[6]}$$

$$u[2] \rightarrow \frac{z[3] - z[4]}{z[6] - z[1]} \frac{z[3] - z[6]}{z[4] - z[1]}$$

$$u[3] \rightarrow \frac{z[4] - z[5]}{z[1] - z[2]} \frac{z[4] - z[1]}{z[5] - z[2]}$$

```

tot2 = Join[uprules, tot, {x_ -> y_ :> (x -> Factor[y])},
Drop[tot, 1]]

```



Out[81]= {855, 1677, 8912, 4329, 3729}

Demo.m

Functions Sections Update

Debug Run Package

```

up[1,4,3] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[1]) * u[3]),
up[2,3,1] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[2]) * u[1]),
up[3,1,2] -> -1 - delta - u[1] - u[2] - u[3] / (2*u[2] * (-1 - u[3])),
um[1,2,3] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[1]) * u[3]),
um[2,3,1] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[2]) * u[1]),
um[3,1,2] -> -1 - delta - u[1] - u[2] - u[3] / (2*u[2] * (-1 - u[3]))

```

tot = {}

delta ->

```

(z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] - z[2]*z[3]*z[5] -
z[1]*z[4]*z[5] - z[2]*z[4]*z[5] - z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
z[1]*z[4]*z[6] - z[3]*z[4]*z[6] - z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
(z[1] - z[4] * z[2] - z[5] * z[3] - z[6] ,

```

```

u[1] -> ((z[2]-z[3]) (z[5]-z[6]))/((z[2]-z[5]) (z[3]-z[6])),
u[2] -> ((z[3]-z[4]) (z[6]-z[1]))/((z[3]-z[6]) (z[4]-z[1])),
u[3] -> ((z[4]-z[5]) (z[1]-z[2]))/((z[4]-z[1]) (z[5]-z[2]));

```

```

tot2 = Join[ uprules , tot , {x_ -> y_ :> (x -> Factor[y])},
Drop[tot, 1]]

```

Out[31]= {955, 1677, 9912, 4329, 3729}

Demo.m

Functions Sections Update Debug Run Package

```

up[1, 4, 3] -> - 1 - delta - u[1] - u[4] - u[3] / (2*(-1 - u[1]) * u[3]),
up[2, 3, 1] -> - 1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[2]) * u[1]),
up[3, 1, 2] -> - 1 - delta - u[1] - u[2] - u[3] / (2*u[2] * -1 - u[3]),
um[1, 2, 3] -> - 1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[1]) * u[3]),
um[2, 3, 1] -> - 1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[2]) * u[1]),
um[3, 1, 2] -> - 1 - delta - u[1] - u[2] - u[3] / (2*u[2] * -1 - u[3])

```

tot = {}

delta ->

```

(z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] - z[2]*z[3]*z[5] -
z[1]*z[4]*z[5] - z[2]*z[4]*z[5] - z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
z[1]*z[4]*z[6] - z[3]*z[4]*z[6] - z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
(z[1] - z[4] * z[2] - z[5] * z[3] - z[6]),
u[1] -> (z[2]-z[3] z[5]-z[6] z[2]-z[5] z[3]-z[6]),
u[2] -> (z[3]-z[4] z[6]-z[1] z[3]-z[6] z[4]-z[1]),
u[3] -> (z[4]-z[5] z[1]-z[2] z[4]-z[1] z[5]-z[2])

```

```

tot2 = Join[ uprules , tot , {x_ -> y_ :> {x -> Factor[y]}}
Drop[tot, 1]

```

Out[89]= {955, 1677, 8912, 4329, 3729}

Demo.m

Functions Sections Update Debug Run Package

```

up[1, 4, 5] -> - 1 - delta - u[1] - u[4] - u[5] / (2 * (-1 - u[1]) * u[5]),
up[2, 3, 1] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * (-1 - u[2]) * u[1]),
up[3, 1, 2] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * u[2] * (-1 - u[3])),
um[1, 2, 3] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * (-1 - u[1]) * u[3]),
um[2, 3, 1] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * (-1 - u[2]) * u[1]),
um[3, 1, 2] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * u[2] * (-1 - u[3]))

```

toz = {

```

delta ->
(z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] + z[2]*z[3]*z[5] +
z[1]*z[4]*z[5] - z[2]*z[4]*z[5] + z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
z[1]*z[4]*z[6] + z[3]*z[4]*z[6] + z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
((z[1] - z[4])*(z[2] - z[5])*(z[3] - z[6])),

```

```

u[1] -> { z[2]-z[3] z[5]-z[6] z[2]-z[5] z[3]-z[6] },
u[2] -> { z[3]-z[4] z[6]-z[1] z[3]-z[6] z[4]-z[1] },
u[3] -> { z[4]-z[5] z[1]-z[2] z[4]-z[1] z[5]-z[2] };

```

```

toz2 = Join[ uprules , toz , { x_ -> y_ :> {x -> Factor[y]}},
Drop[toz, 1]]

```

Out[31]= {955, 1677, 9912, 4329, 3729}

Demo.m

Functions Sections Update Debug Run Package

```

up[1, 4, 3] -> - 1 - delta - u[1] - u[4] - u[3] / (2 * (-1 - u[1]) * u[3]),
up[2, 3, 1] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * (-1 - u[2]) * u[1]),
up[3, 1, 2] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * u[2] * (-1 - u[3])),
um[1, 2, 3] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * (-1 - u[1]) * u[3]),
um[2, 3, 1] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * (-1 - u[2]) * u[1]),
um[3, 1, 2] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * u[2] * (-1 - u[3]))

```

tot = {}

delta ->

```

z[1] * z[2] * z[4] - z[1] * z[3] * z[4] - z[1] * z[2] * z[5] - z[2] * z[3] * z[5] -
z[1] * z[4] * z[5] - z[2] * z[4] * z[5] - z[1] * z[3] * z[6] - z[2] * z[3] * z[6] -
z[1] * z[4] * z[6] - z[3] * z[4] * z[6] - z[2] * z[5] * z[6] - z[3] * z[5] * z[6]) /
(z[1] - z[4] * z[2] - z[5] * z[3] - z[6]) ,

```

```

u[1] -> (z[2] - z[3] z[5] - z[6] z[2] - z[5] z[3] - z[6] z[3]) ,
u[2] -> (z[3] - z[4] z[6] - z[1] z[3] - z[6] z[4] - z[1] z[4]) ,
u[3] -> (z[4] - z[5] z[1] - z[2] z[4] - z[1] z[5] - z[2] z[5]) ;

```

```

tot2 = Join[ uprules , tot , { x_ -> y_ :> (x -> Factor[y])},
Drop[tot, 1]]

```

Out[31]= {955, 1677, 9912, 4329, 3729}

Demo.m

Functions Sections Update Debug Run Package

```

up[1,4,3] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[1]) * u[3]),
up[2,3,1] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[2]) * u[1]),
up[3,1,2] -> -1 - delta - u[1] - u[2] - u[3] / (2*u[2] * (-1 - u[3])),
um[1,2,3] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[1]) * u[3]),
um[2,3,1] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[2]) * u[1]),
um[3,1,2] -> -1 - delta - u[1] - u[2] - u[3] / (2*u[2] * (-1 - u[3]))

```

tot = {}

delta ->

```

(z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] + z[2]*z[3]*z[5] +
z[1]*z[4]*z[5] - z[2]*z[4]*z[5] + z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
z[1]*z[4]*z[6] + z[3]*z[4]*z[6] + z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
((z[1] - z[4])*(z[2] - z[5])*(z[3] - z[6])),

```

```

u[1] -> (z[2]-z[3] z[5]-z[6] z[2]-z[5] z[3]-z[6]),
u[2] -> (z[3]-z[4] z[6]-z[1] z[3]-z[6] z[4]-z[1]),
u[3] -> (z[4]-z[5] z[1]-z[2] z[4]-z[1] z[5]-z[2]);

```

```

tot2 = Join[uprules, tot, {x_ -> y_ :> (x -> Factor[y])},
Drop[tot, 1]]

```

Out[31]= {955, 1677, 8912, 4329, 3729}

Demo.m

```

Functions ▾ Sections ▾ Update Debug Run Package
up[1, 4, 2] -> - 1 - delta - u[1] - u[4] - u[2] / (2 * (-1 - u[1]) * u[2]),
up[2, 3, 1] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * (-1 - u[2]) * u[1]),
up[3, 1, 2] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * u[2] * (-1 - u[3])),
um[1, 2, 3] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * (-1 - u[1]) * u[3]),
um[2, 3, 1] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * (-1 - u[2]) * u[1]),
um[3, 1, 2] -> - 1 - delta - u[1] - u[2] - u[3] / (2 * u[2] * (-1 - u[3]))

```

```

toz = {
  delta ->
  (z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] + z[2]*z[3]*z[5] +
  z[1]*z[4]*z[5] - z[2]*z[4]*z[5] + z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
  z[1]*z[4]*z[6] + z[3]*z[4]*z[6] + z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
  ((z[1] - z[4]) * (z[2] - z[5]) * (z[3] - z[6])),
  u[1] -> ((z[2]-z[3]) (z[5]-z[6])) / ((z[2]-z[5]) (z[3]-z[6])),
  u[2] -> ((z[3]-z[4]) (z[6]-z[1])) / ((z[3]-z[6]) (z[4]-z[1])),
  u[3] -> ((z[4]-z[5]) (z[1]-z[2])) / ((z[4]-z[1]) (z[5]-z[2]))};

```

```

toz2 = Join[uprules, toz, {x_ -> y_ :> (x -> Factor[y])},
  Drop[toz, 1]]

```

Out[89]= {855, 1677, 8912, 4329, 3729}

Demo.m

Functions Sections Update

Debug Run Package

```

up[2, 3, 1] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[2])*u[1]),
up[3, 1, 2] -> -1 - delta - u[1] - u[2] - u[3] / (2*u[2]*(-1 - u[3])),
um[1, 2, 3] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[1])*u[3]),
um[2, 3, 1] -> -1 - delta - u[1] - u[2] - u[3] / (2*(-1 - u[2])*u[1]),
um[3, 1, 2] -> -1 - delta - u[1] - u[2] - u[3] / (2*u[2]*(-1 - u[3]))

```

```

toz = {
delta ->
(z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] + z[2]*z[3]*z[5] +
z[1]*z[4]*z[5] - z[2]*z[4]*z[5] + z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
z[1]*z[4]*z[6] + z[3]*z[4]*z[6] + z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
((z[1] - z[4])*(z[2] - z[5])*(z[3] - z[6])),
u[1] -> ((z[2]-z[3]) (z[5]-z[6]))/((z[2]-z[5]) (z[3]-z[6])),
u[2] -> ((z[3]-z[4]) (z[6]-z[1]))/((z[3]-z[6]) (z[4]-z[1])),
u[3] -> ((z[4]-z[5]) (z[1]-z[2]))/((z[4]-z[1]) (z[5]-z[2]));

```

```

toz2 = Join[uprules, toz, {x_ -> y_ :> {x -> Factor[y]}},
Drop[toz, 1]]

```

Out[63]= {855, 1677, 8912, 4329, 3729}

▼ In[64]= upmrules = {  

$$\text{up}[1, 2, 3] \rightarrow -(1 + \text{delta} - u[1] - u[2] + u[3]) / (2 * (-1 + u[1]) * u[3]),$$

$$\text{up}[2, 3, 1] \rightarrow -(1 + \text{delta} + u[1] - u[2] - u[3]) / (2 * (-1 + u[2]) * u[1]),$$

$$\text{up}[3, 1, 2] \rightarrow -(1 + \text{delta} - u[1] + u[2] - u[3]) / (2 * u[2] * (-1 + u[3])),$$

$$\text{um}[1, 2, 3] \rightarrow -(1 - \text{delta} - u[1] - u[2] + u[3]) / (2 * (-1 + u[1]) * u[3]),$$

$$\text{um}[2, 3, 1] \rightarrow -(1 - \text{delta} + u[1] - u[2] - u[3]) / (2 * (-1 + u[2]) * u[1]),$$

$$\text{um}[3, 1, 2] \rightarrow -(1 - \text{delta} - u[1] + u[2] - u[3]) / (2 * u[2] * (-1 + u[3]))$$
}

Out[64]= {
$$\text{up}[1, 2, 3] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 (-1 - u[1]) u[3]},$$
  

$$\text{up}[2, 3, 1] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] (-1 - u[2])},$$
  

$$\text{up}[3, 1, 2] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] (-1 - u[3])},$$
  

$$\text{um}[1, 2, 3] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 (-1 - u[1]) u[3]},$$
  

$$\text{um}[2, 3, 1] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] (-1 - u[2])},$$
  

$$\text{um}[3, 1, 2] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] (-1 - u[3])}$$
}



}

$$\text{Out}[84]= \left\{ \begin{array}{l} \text{up}[1, 2, 3] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 - 1 - u[1]} u[3], \\ \text{up}[2, 3, 1] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] - 1 - u[2]}, \\ \text{up}[3, 1, 2] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] - 1 - u[3]}, \\ \text{um}[1, 2, 3] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 - 1 - u[1]} u[3], \\ \text{um}[2, 3, 1] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] - 1 - u[2]}, \\ \text{um}[3, 1, 2] \rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] - 1 - u[3]} \end{array} \right\}$$

toz = {

delta -&gt;

$$\begin{aligned} & (z[1] * z[2] * z[4] - z[1] * z[3] * z[4] - z[1] * z[2] * z[5] + z[2] * z[3] * z[5] + \\ & z[1] * z[4] * z[5] - z[2] * z[4] * z[5] + z[1] * z[3] * z[6] - z[2] * z[3] * z[6] - \\ & z[1] * z[4] * z[6] + z[3] * z[4] * z[6] + z[2] * z[5] * z[6] - z[3] * z[5] * z[6]) / \\ & ((z[1] - z[4]) * (z[2] - z[5]) * (z[3] - z[6])), \end{aligned}$$

$$u[1] \rightarrow ((z[2] - z[3]) (z[5] - z[6])) / ((z[2] - z[5]) (z[3] - z[6])),$$

$$u[2] \rightarrow ((z[3] - z[4]) (z[6] - z[1])) / ((z[3] - z[6]) (z[4] - z[1])),$$

$$u[3] \rightarrow ((z[4] - z[5]) (z[1] - z[2])) / ((z[4] - z[1]) (z[5] - z[2]));$$

}

Demo.m

Functions Sections Update

Debug Run Package

```

up[2,3,1] -> -1 - delta - u[1] - u[2] - u[3]/(2*(-1 - u[2])*u[1]),
up[3,1,2] -> -1 - delta - u[1] - u[2] - u[3]/(2*u[2]*(-1 - u[3])),
um[1,2,3] -> -1 - delta - u[1] - u[2] - u[3]/(2*(-1 - u[1])*u[3]),
um[2,3,1] -> -1 - delta - u[1] - u[2] - u[3]/(2*(-1 - u[2])*u[1]),
um[3,1,2] -> -1 - delta - u[1] - u[2] - u[3]/(2*u[2]*(-1 - u[3]))

```

```

toz = {
  delta ->
  (z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] + z[2]*z[3]*z[5] +
  z[1]*z[4]*z[5] - z[2]*z[4]*z[5] + z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
  z[1]*z[4]*z[6] + z[3]*z[4]*z[6] + z[2]*z[5]*z[6] - z[3]*z[5]*z[6])/
  ((z[1] - z[4])*(z[2] - z[5])*(z[3] - z[6])),
  u[1] -> ((z[2]-z[3]) (z[5]-z[6]))/((z[2]-z[5]) (z[3]-z[6])),
  u[2] -> ((z[3]-z[4]) (z[6]-z[1]))/((z[3]-z[6]) (z[4]-z[1])),
  u[3] -> ((z[4]-z[5]) (z[1]-z[2]))/((z[4]-z[1]) (z[5]-z[2]))};

```

```

toz2 = Join[uprules, toz, {x_ -> y_ :> {x -> Factor[y]}],
  Drop[toz, 1]]

```

}

Demo.m

Functions Sections Update

Debug Run Package

```

up[3,1,2] -> -1 - delta - u[1] - u[2] - u[3] (2*u[2]*-1 - u[3]),
um[1,2,3] -> -1 - delta - u[1] - u[2] - u[3] (2*-1 - u[1])*u[3],
um[2,3,1] -> -1 - delta - u[1] - u[2] - u[3] (2*-1 - u[2])*u[1],
um[3,1,2] -> -1 - delta - u[1] - u[2] - u[3] (2*u[2]*-1 - u[3])

```

toz = {}

delta ->

```

(z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] - z[2]*z[3]*z[5] -
z[1]*z[4]*z[5] - z[2]*z[4]*z[5] - z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
z[1]*z[4]*z[6] - z[3]*z[4]*z[6] - z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
(z[1] - z[4] + z[2] - z[5] + z[3] - z[6]),
u[1] -> (z[2]-z[3] z[5]-z[6] z[2]-z[5] z[3]-z[6]),
u[2] -> (z[3]-z[4] z[6]-z[1] z[3]-z[6] z[4]-z[1]),
u[3] -> (z[4]-z[5] z[1]-z[2] z[4]-z[1] z[5]-z[2]);

```

```

toz2 = Join[(uprules /. toz) /. (x_ -> y_) :=> (x -> Factor[y]),
Drop[toz,1]]

```

```

symbe4 = Expand[expand[symbe3 /. toz2]];

```

}

$$\text{Out}[64] = \left\{ \begin{aligned} \text{up}[1, 2, 3] &\rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 - 1 - u[1] - u[3]}, \\ \text{up}[2, 3, 1] &\rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] - 1 - u[2]}, \\ \text{up}[3, 1, 2] &\rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] - 1 - u[3]}, \\ \text{um}[1, 2, 3] &\rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 - 1 - u[1] - u[3]}, \\ \text{um}[2, 3, 1] &\rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[1] - 1 - u[2]}, \\ \text{um}[3, 1, 2] &\rightarrow \frac{-1 - \text{delta} - u[1] - u[2] - u[3]}{2 u[2] - 1 - u[3]} \end{aligned} \right\}$$

n[65] = toz = {

delta -&gt;

$$\begin{aligned} & (z[1] * z[2] * z[4] - z[1] * z[3] * z[4] - z[1] * z[2] * z[5] + z[2] * z[3] * z[5] + \\ & z[1] * z[4] * z[5] - z[2] * z[4] * z[5] + z[1] * z[3] * z[6] - z[2] * z[3] * z[6] - \\ & z[1] * z[4] * z[6] + z[3] * z[4] * z[6] + z[2] * z[5] * z[6] - z[3] * z[5] * z[6]) / \\ & ((z[1] - z[4]) * (z[2] - z[5]) * (z[3] - z[6])), \end{aligned}$$

$$u[1] \rightarrow ((z[2] - z[3]) (z[5] - z[6])) / ((z[2] - z[5]) (z[3] - z[6])),$$

$$u[2] \rightarrow ((z[3] - z[4]) (z[6] - z[1])) / ((z[3] - z[6]) (z[4] - z[1])),$$

$$u[3] \rightarrow ((z[4] - z[5]) (z[1] - z[2])) / ((z[4] - z[1]) (z[5] - z[2]));$$

emo.nb \*

```

z[1] * z[4] * z[6] + z[3] * z[4] * z[6] + z[2] * z[5] * z[6] - z[3] * z[5] * z[6]) /
((z[1] - z[4]) * (z[2] - z[5]) * (z[3] - z[6])),
u[1] -> ((z[2] - z[3]) (z[5] - z[6])) / ((z[2] - z[5]) (z[3] - z[6])),
u[2] -> ((z[3] - z[4]) (z[6] - z[1])) / ((z[3] - z[6]) (z[4] - z[1])),
u[3] -> ((z[4] - z[5]) (z[1] - z[2])) / ((z[4] - z[1]) (z[5] - z[2]));

```

```

In[66]= toz2 = Join[(upmrules /. toz) /. (x_ -> y_) :> (x -> Factor[y]),
Drop[toz, 1]]

```

$$\text{Out}[66] = \left\{ \begin{aligned}
 & \text{up}[1, 2, 3] \rightarrow -\frac{z[1] - z[3]}{z[1] - z[2]} \frac{z[2] - z[5]}{-z[3] - z[5]}, \\
 & \text{up}[2, 3, 1] \rightarrow \frac{-z[1] - z[5]}{-z[1] - z[3]} \frac{z[3] - z[6]}{z[5] - z[6]}, \\
 & \text{up}[3, 1, 2] \rightarrow -\frac{-z[1] - z[4]}{z[3] - z[4]} \frac{z[3] - z[5]}{z[1] - z[5]}, \\
 & \text{um}[1, 2, 3] \rightarrow \frac{z[2] - z[5]}{z[4] - z[5]} \frac{z[4] - z[6]}{z[2] - z[6]}, \\
 & \text{um}[2, 3, 1] \rightarrow -\frac{z[2] - z[4]}{z[2] - z[3]} \frac{z[3] - z[6]}{-z[4] - z[6]}, \\
 & \text{um}[3, 1, 2] \rightarrow \frac{-z[1] - z[4]}{-z[2] - z[4]} \frac{z[2] - z[6]}{z[1] - z[6]}, \quad u[1] \rightarrow \frac{z[2] - z[3]}{z[2] - z[5]} \frac{(z[5] - z[6])}{(z[3] - z[6])}, \\
 & u[2] \rightarrow \frac{z[3] - z[4]}{-z[1] - z[4]} \frac{-z[1] - z[6]}{z[3] - z[6]}, \quad u[3] \rightarrow \frac{(z[1] - z[2]) (z[4] - z[5])}{-z[1] - z[4]} \frac{1}{(-z[2] - z[5])} \end{aligned} \right\}$$

$$\frac{z[1] * z[4] * z[6] + z[3] * z[4] * z[6] + z[2] * z[5] * z[6] - z[3] * z[5] * z[6]}{}$$

```
up[3,1,2] -> -1 - delta - u[1] - u[2] - u[3] (2*u[2]*-1 - u[3]),
um[1,2,3] -> -1 - delta - u[1] - u[2] - u[3] (2*-1 - u[1])*u[3],
um[2,3,1] -> -1 - delta - u[1] - u[2] - u[3] (2*-1 - u[2])*u[1],
um[3,1,2] -> -1 - delta - u[1] - u[2] - u[3] (2*u[2]*-1 - u[3])
```

toz = {

delta ->

```
{z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] - z[2]*z[3]*z[5] -
z[1]*z[4]*z[5] - z[2]*z[4]*z[5] - z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
z[1]*z[4]*z[6] - z[3]*z[4]*z[6] - z[2]*z[5]*z[6] - z[3]*z[5]*z[6] /
(z[1] - z[4] + z[2] - z[5] * z[3] - z[6]) ,
```

```
u[1] -> {z[2]-z[3] z[5]-z[6] z[2]-z[5] z[3]-z[6]},
u[2] -> {z[3]-z[4] z[6]-z[1] z[3]-z[6] z[4]-z[1]},
u[3] -> {z[4]-z[5] z[1]-z[2] z[4]-z[1] z[5]-z[2]}];
```

```
toz2 = Join[(uprules /. toz) /. (x_ -> y_) :=> (x -> Factor[y]),
Drop[toz,1]]
```

```
symbe4 = Expand[expand[symbe3 /. toz2]];
```

$$\frac{z[1] * z[4] * z[6] + z[3] * z[4] * z[6] + z[2] * z[5] * z[6] - z[3] * z[5] * z[6]}{}$$

```

toz = {
  delta ->
  (z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] - z[2]*z[3]*z[5] -
  z[1]*z[4]*z[5] - z[2]*z[4]*z[5] - z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
  z[1]*z[4]*z[6] - z[3]*z[4]*z[6] - z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
  (z[1] - z[4] * z[2] - z[5] * z[3] - z[6] ),
  u[1] -> { z[2]-z[3] z[5]-z[6] z[2]-z[5] z[3]-z[6] },
  u[2] -> { z[3]-z[4] z[6]-z[1] z[3]-z[6] z[4]-z[1] },
  u[3] -> { z[4]-z[5] z[1]-z[2] z[4]-z[1] z[5]-z[2] };

```

```

toz2 = Join[(upmrules /. toz) /. (x_ -> y_) :=> (x -> Factor[y]),
  Drop[toz,1]]

```

```

symbe4 = Expand[expand[symbe3 /. toz2]];

```

```

Length /@ {rem, symb, symbe, symbe2, symbe3, symbe4}

```

$$z[1] * z[4] * z[6] + z[3] * z[4] * z[6] + z[2] * z[5] * z[6] - z[3] * z[5] * z[6]) /$$

```

}

toz = {
  delta ->
  (z[1]*z[2]*z[4] - z[1]*z[3]*z[4] - z[1]*z[2]*z[5] - z[2]*z[3]*z[5] -
  z[1]*z[4]*z[5] - z[2]*z[4]*z[5] - z[1]*z[3]*z[6] - z[2]*z[3]*z[6] -
  z[1]*z[4]*z[6] - z[3]*z[4]*z[6] - z[2]*z[5]*z[6] - z[3]*z[5]*z[6]) /
  (z[1] - z[4] * z[2] - z[5] * z[3] - z[6] ),
  u[1] -> { z[2]-z[3]    z[5]-z[6]    z[2]-z[5]    (z[3]-z[6]) },
  u[2] -> { z[3]-z[4]    z[6]-z[1]    z[3]-z[6]    z[4]-z[1] },
  u[3] -> { z[4]-z[5]    z[1]-z[2]    z[4]-z[1]    z[5]-z[2] };

toz2 = Join[ uprules , toz , { x_ -> y_ :> {x -> Factor[y]}},
  Drop[toz,1]]

symbe4 = Expand[expand[symbe3 /. toz2]];
}

Length /@ {rem, symb, symbe, symbe2, symbe3, symbe4}

```



$$\frac{z[1] * z[4] * z[0] + z[3] * z[4] * z[0] + z[2] * z[5] * z[0] - z[3] * z[5] * z[0]}{}$$

Functions Sections Update

Debug Run Package

```
u[4] -> z[4]-z[5]
u[3] -> z[4]-z[5] z[1]-z[2] z[4]-z[1] z[5]-z[2]};
```

```
toz2 = Join[upmrules, toz, {x_ -> y_ :> (x -> Factor[y])},
Drop[toz, 1]]
```

```
symbe4 = Expand[expand[symbe3 /. toz2]];
```

```
Length[{rem, symb, symbe, symbe2, symbe3, symbe4}]
```

```
final /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[b, a, c, d];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[a, b, d, c];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[c, d, a, b];
Expand[%]
```

emo.nb \*

```

z[1] * z[4] * z[6] + z[3] * z[4] * z[6] + z[2] * z[5] * z[6] - z[3] * z[5] * z[6]) /
((z[1] - z[4]) * (z[2] - z[5]) * (z[3] - z[6])),
u[1] -> ((z[2] - z[3]) (z[5] - z[6])) / ((z[2] - z[5]) (z[3] - z[6])),
u[2] -> ((z[3] - z[4]) (z[6] - z[1])) / ((z[3] - z[6]) (z[4] - z[1])),
u[3] -> ((z[4] - z[5]) (z[1] - z[2])) / ((z[4] - z[1]) (z[5] - z[2]));

```

```

In[66]= toz2 = Join[(uprules /. toz) /. (x_ -> y_) :> (x -> Factor[y]),
Drop[toz, 1]]

```

$$\text{Out}[66]= \left\{ \begin{aligned}
 &\text{up}[1, 2, 3] \rightarrow -\frac{z[1] - z[3]}{z[1] - z[2]} \frac{z[2] - z[5]}{-z[3] - z[5]}, \\
 &\text{up}[2, 3, 1] \rightarrow \frac{-z[1] - z[5]}{-z[1] - z[3]} \frac{z[3] - z[6]}{z[5] - z[6]}, \\
 &\text{up}[3, 1, 2] \rightarrow -\frac{-z[1] - z[4]}{z[3] - z[4]} \frac{z[3] - z[5]}{z[1] - z[5]}, \\
 &\text{um}[1, 2, 3] \rightarrow \frac{z[2] - z[5]}{z[4] - z[5]} \frac{z[4] - z[6]}{z[2] - z[6]}, \\
 &\text{um}[2, 3, 1] \rightarrow -\frac{z[2] - z[4]}{z[2] - z[3]} \frac{z[3] - z[6]}{-z[4] - z[6]}, \\
 &\text{um}[3, 1, 2] \rightarrow \frac{-z[1] - z[4]}{-z[2] - z[4]} \frac{z[2] - z[6]}{z[1] - z[6]}, \quad u[1] \rightarrow \frac{z[2] - z[3]}{z[2] - z[5]} \frac{(z[5] - z[6])}{(z[3] - z[6])}, \\
 &u[2] \rightarrow \frac{z[3] - z[4]}{-z[1] - z[4]} \frac{-z[1] - z[6]}{z[3] - z[6]}, \quad u[3] \rightarrow \frac{(z[1] - z[2]) (z[4] - z[5])}{-z[1] - z[4]} \frac{1}{(-z[2] - z[5])} \end{aligned} \right\}$$

```

((z[1] - z[2]) (z[4] - z[5]) (z[3] - z[6])) / ((z[2] - z[5]) (z[3] - z[6])),
u[1] -> ((z[2] - z[3]) (z[5] - z[6])) / ((z[2] - z[5]) (z[3] - z[6])),
u[2] -> ((z[3] - z[4]) (z[6] - z[1])) / ((z[3] - z[6]) (z[4] - z[1])),
u[3] -> ((z[4] - z[5]) (z[1] - z[2])) / ((z[4] - z[1]) (z[5] - z[2]));

```

```

In[66]:= toz2 = Join[(uprules /. toz) /. (x_ -> y_) :> (x -> Factor[y]),
Drop[toz, 1]]

```

$$\text{Out[66]} = \left\{ \begin{aligned}
 & \text{up}[1, 2, 3] \rightarrow -\frac{z[1] - z[3]}{z[1] - z[2]} \frac{z[2] - z[5]}{-z[3] - z[5]}, \\
 & \text{up}[2, 3, 1] \rightarrow \frac{-z[1] - z[5]}{-z[1] - z[3]} \frac{z[3] - z[6]}{z[5] - z[6]}, \\
 & \text{up}[3, 1, 2] \rightarrow -\frac{-z[1] - z[4]}{z[3] - z[4]} \frac{z[3] - z[5]}{z[1] - z[5]}, \\
 & \text{um}[1, 2, 3] \rightarrow \frac{z[2] - z[5]}{z[4] - z[5]} \frac{z[4] - z[6]}{z[2] - z[6]}, \\
 & \text{um}[2, 3, 1] \rightarrow -\frac{z[2] - z[4]}{z[2] - z[3]} \frac{z[3] - z[6]}{-z[4] - z[6]}, \\
 & \text{um}[3, 1, 2] \rightarrow \frac{-z[1] - z[4]}{-z[2] - z[4]} \frac{z[2] - z[6]}{z[1] - z[6]}, \quad u[1] \rightarrow \frac{z[2] - z[3]}{z[2] - z[5]} \frac{(z[5] - z[6])}{(z[3] - z[6])}, \\
 & u[2] \rightarrow \frac{z[3] - z[4]}{-z[1] - z[4]} \frac{-z[1] - z[6]}{z[3] - z[6]}, \quad u[3] \rightarrow \frac{(z[1] - z[2]) (z[4] - z[5])}{-z[1] - z[4]} \frac{1}{(-z[2] - z[5])} \end{aligned} \right\}$$

<<

emo.nb \*

```

((z[1] - z[2]) * (z[2] - z[3]) * (z[3] - z[4])),
u[1] -> ((z[2] - z[3]) (z[5] - z[6])) / ((z[2] - z[5]) (z[3] - z[6])),
u[2] -> ((z[3] - z[4]) (z[6] - z[1])) / ((z[3] - z[6]) (z[4] - z[1])),
u[3] -> ((z[4] - z[5]) (z[1] - z[2])) / ((z[4] - z[1]) (z[5] - z[2]));

```

```

In[66]= toz2 = Join[(uprules /. toz) /. (x_ -> y_) :> (x -> Factor[y]),
Drop[toz, 1]]

```

$$\text{Out}[66] = \left\{ \begin{aligned}
 & \text{up}[1, 2, 3] \rightarrow -\frac{z[1] - z[3]}{z[1] - z[2]} \frac{z[2] - z[5]}{-z[3] - z[5]}, \\
 & \text{up}[2, 3, 1] \rightarrow \frac{-z[1] - z[5]}{-z[1] - z[3]} \frac{z[3] - z[6]}{z[5] - z[6]}, \\
 & \text{up}[3, 1, 2] \rightarrow -\frac{-z[1] - z[4]}{z[3] - z[4]} \frac{z[3] - z[5]}{z[1] - z[5]}, \\
 & \text{um}[1, 2, 3] \rightarrow \frac{z[2] - z[5]}{z[4] - z[5]} \frac{z[4] - z[6]}{z[2] - z[6]}, \\
 & \text{um}[2, 3, 1] \rightarrow -\frac{z[2] - z[4]}{z[2] - z[3]} \frac{z[3] - z[6]}{-z[4] - z[6]}, \\
 & \text{um}[3, 1, 2] \rightarrow \frac{-z[1] - z[4]}{-z[2] - z[4]} \frac{z[2] - z[6]}{z[1] - z[6]}, \quad u[1] \rightarrow \frac{z[2] - z[3]}{z[2] - z[5]} \frac{(z[5] - z[6])}{(z[3] - z[6])}, \\
 & u[2] \rightarrow \frac{z[3] - z[4]}{-z[1] - z[4]} \frac{-z[1] - z[6]}{z[3] - z[6]}, \quad u[3] \rightarrow \frac{(z[1] - z[2]) (z[4] - z[5])}{-z[1] - z[4]} \frac{1}{(-z[2] - z[5])} \end{aligned} \right\}$$

All Messages

(line 7284 of "emo.nb")

```

((z[1] - z[2]) * (z[2] - z[3]) * (z[3] - z[4])) /
u[1] -> ((z[2] - z[3]) (z[5] - z[6])) / ((z[2] - z[5]) (z[3] - z[6])),
u[2] -> ((z[3] - z[4]) (z[6] - z[1])) / ((z[3] - z[6]) (z[4] - z[1])),
u[3] -> ((z[4] - z[5]) (z[1] - z[2])) / ((z[4] - z[1]) (z[5] - z[2]));

```

```

n[66] = toz2 = Join[(uprules /. toz) /. (x_ -> y_) :> (x -> Factor[y]),
Drop[toz, 1]]

```

$$\text{Out}[66] = \left\{ \begin{aligned}
& \text{up}[1, 2, 3] \rightarrow -\frac{z[1] - z[3]}{z[1] - z[2]} \frac{z[2] - z[5]}{-z[3] - z[5]}, \\
& \text{up}[2, 3, 1] \rightarrow \frac{-z[1] - z[5]}{-z[1] - z[3]} \frac{z[3] - z[6]}{z[5] - z[6]}, \\
& \text{up}[3, 1, 2] \rightarrow -\frac{-z[1] - z[4]}{z[3] - z[4]} \frac{z[3] - z[5]}{z[1] - z[3]}, \\
& \text{um}[1, 2, 3] \rightarrow \frac{z[2] - z[5]}{z[4] - z[5]} \frac{z[4] - z[6]}{z[2] - z[6]}, \\
& \text{um}[2, 3, 1] \rightarrow -\frac{z[2] - z[4]}{z[2] - z[3]} \frac{-z[3] - z[6]}{-z[4] - z[6]}, \\
& \text{um}[3, 1, 2] \rightarrow \frac{-z[1] - z[4]}{-z[2] - z[4]} \frac{z[2] - z[6]}{z[1] - z[6]}, \quad u[1] \rightarrow \frac{z[2] - z[3]}{z[2] - z[5]} \frac{z[5] - z[6]}{z[3] - z[6]}, \\
& u[2] \rightarrow \frac{z[3] - z[4]}{-z[1] - z[4]} \frac{-z[1] - z[6]}{z[3] - z[6]}, \quad u[3] \rightarrow \frac{(z[1] - z[2]) (z[4] - z[5])}{-z[1] - z[4]} \frac{1}{(-z[2] - z[5])} \end{aligned} \right\}$$

```
n[67] = << final.m;
```

emo.nb \*

```
u[2] -> ((z[3] - z[4]) (z[6] - z[1])) / ((z[3] - z[6]) (z[4] - z[1])),
u[3] -> ((z[4] - z[5]) (z[1] - z[2])) / ((z[4] - z[1]) (z[5] - z[2]));
```

```
In[66]= toz2 = Join[(upmrules /. toz) /. (x_ -> y_) :> (x -> Factor[y]),
Drop[toz, 1]]
```

```
Out[66]= {up[1, 2, 3] -> -frac(z[1] - z[3] z[2] - z[5])}{z[1] - z[2] -z[3] - z[5]},
up[2, 3, 1] -> -frac(-z[1] - z[5] z[3] - z[6])}{-z[1] - z[3] z[5] - z[6]},
up[3, 1, 2] -> -frac(-z[1] - z[4] z[3] - z[5])}{z[3] - z[4] z[1] - z[5]},
um[1, 2, 3] -> frac(z[2] - z[5] z[4] - z[6])}{z[4] - z[5] z[2] - z[6]},
um[2, 3, 1] -> -frac(z[2] - z[4] z[3] - z[6])}{z[2] - z[3] -z[4] - z[6]},
um[3, 1, 2] -> -frac(-z[1] - z[4] z[2] - z[6])}{-z[2] - z[4] z[1] - z[6]}, u[1] -> frac(z[2] - z[3] z[5] - z[6])}{z[2] - z[5] z[3] - z[6]},
u[2] -> frac(z[3] - z[4] -z[1] - z[6])}{-z[1] - z[4] z[3] - z[6]}, u[3] -> frac((z[1] - z[2]) (z[4] - z[5]))}{-z[1] - z[4] (-z[2] - z[5])}
```

```
In[67]= << final.m;
```

**Length**

emo.nb \*

```
u[2] -> ((z[3] - z[4]) (z[6] - z[1])) / ((z[3] - z[6]) (z[4] - z[1])),
u[3] -> ((z[4] - z[5]) (z[1] - z[2])) / ((z[4] - z[1]) (z[5] - z[2]));
```

```
▼ n[66] = toz2 = Join[(upmrules /. toz) /. (x_ -> y_) :> (x -> Factor[y]),
  Drop[toz, 1]]
```

```
Out[66] = {up[1, 2, 3] -> -frac(z[1] - z[3] z[2] - z[5])
  frac(z[1] - z[2] - z[3] - z[5]),
  up[2, 3, 1] -> frac(-z[1] - z[5] z[3] - z[6])
  frac(-z[1] - z[3] z[5] - z[6]),
  up[3, 1, 2] -> -frac(-z[1] - z[4] z[3] - z[5])
  frac(z[3] - z[4] z[1] - z[5]),
  um[1, 2, 3] -> frac(z[2] - z[5] z[4] - z[6])
  frac(z[4] - z[5] z[2] - z[6]),
  um[2, 3, 1] -> -frac(z[2] - z[4] z[3] - z[6])
  frac(z[2] - z[3] - z[4] - z[6]),
  um[3, 1, 2] -> frac(-z[1] - z[4] z[2] - z[6])
  frac(-z[2] - z[4] z[1] - z[6]), u[1] -> frac(z[2] - z[3] z[5] - z[6])
  frac(z[2] - z[5] z[3] - z[6]),
  u[2] -> frac(z[3] - z[4] - z[1] - z[6])
  frac(-z[1] - z[4] z[3] - z[6]), u[3] -> frac(z[1] - z[2] z[4] - z[5])
  frac(-z[1] - z[4] (-z[2] - z[5]))}
```

```
n[67] = << final.m;
```

```
Length[final]
```

```

In[66] = toz2 = Join[(uprules /. toz) /. (x_ -> y_) :> (x -> Factor[y]),
  Drop[toz, 1]]

```

```

Out[66] = {up[1, 2, 3] -> -\frac{z[1] - z[3]}{z[1] - z[2]} \frac{z[2] - z[5]}{-z[3] - z[5]},
  up[2, 3, 1] -> \frac{-z[1] - z[5]}{-z[1] - z[3]} \frac{z[3] - z[6]}{z[5] - z[6]},
  up[3, 1, 2] -> -\frac{-z[1] - z[4]}{z[3] - z[4]} \frac{z[3] - z[5]}{z[1] - z[5]},
  um[1, 2, 3] -> \frac{z[2] - z[5]}{z[4] - z[5]} \frac{z[4] - z[6]}{z[2] - z[6]},
  um[2, 3, 1] -> -\frac{z[2] - z[4]}{z[2] - z[3]} \frac{z[3] - z[6]}{-z[4] - z[6]},
  um[3, 1, 2] -> \frac{-z[1] - z[4]}{-z[2] - z[4]} \frac{z[2] - z[6]}{z[1] - z[6]}, u[1] -> \frac{z[2] - z[3]}{z[2] - z[5]} \frac{(z[5] - z[6])}{(z[3] - z[6])},
  u[2] -> \frac{z[3] - z[4]}{-z[1] - z[4]} \frac{-z[1] - z[6]}{z[3] - z[6]}, u[3] -> \frac{(z[1] - z[2]) (z[4] - z[5])}{-z[1] - z[4]} \frac{(-z[2] - z[5])}{(-z[2] - z[5])}}

```

```

In[67] = << final.m;

```

```

In[68] = Length[final]

```

```

Out[68] = 7272

```



```

u[4] -> {z[4]-z[5] z[1]-z[2] z[4]-z[1] z[5]-z[2]};

```

```

toz2 = Join[upmrules, toz] /. {x_ -> y_} -> {x -> Factor[y]},
Drop[toz2, 1]

```

```

symbe4 = Expand[expand[symbe3 /. toz2]];

```

```

Length /@ {rem, symb, symbe, symbe2, symbe3, symbe4}

```

```

final /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[b, a, c, d];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[a, b, d, c];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[c, d, a, b];
Expand[%]

```

Demo.m

Functions Sections Update

Debug Run Package

```
u[3] -> {z[4]-z[5] z[1]-z[2] z[4]-z[1] z[5]-z[2]};
```

```
toz2 = Join[upmrules, toz, {x_ -> y_ :> (x -> Factor[y])},
  Drop[toz, 1]]
```

```
symbe4 = Expand[expand[symbe3, toz2]];
```

```
Length /@ {rem, symb, symbe, symbe2, symbe3, symbe4}
```

```
final /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[b, a, c, d];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[a, b, d, c];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[c, d, a, b];
Expand[%]
```

$$\begin{aligned}
 \text{up}[2, 3, 1] &\rightarrow \frac{-z[1] - z[5]}{-z[1] - z[3]} \frac{z[3] - z[6]}{z[5] - z[6]}, \\
 \text{up}[3, 1, 2] &\rightarrow -\frac{-z[1] - z[4]}{z[3] - z[4]} \frac{z[3] - z[5]}{z[1] - z[5]}, \\
 \text{um}[1, 2, 3] &\rightarrow \frac{z[2] - z[5]}{z[4] - z[5]} \frac{z[4] - z[6]}{z[2] - z[6]}, \\
 \text{um}[2, 3, 1] &\rightarrow -\frac{z[2] - z[4]}{z[2] - z[3]} \frac{z[3] - z[6]}{-z[4] - z[6]}, \\
 \text{um}[3, 1, 2] &\rightarrow \frac{-z[1] - z[4]}{-z[2] - z[4]} \frac{z[2] - z[6]}{z[1] - z[6]}, \quad u[1] \rightarrow \frac{z[2] - z[3]}{z[2] - z[5]} \frac{z[5] - z[6]}{z[3] - z[6]}, \\
 u[2] &\rightarrow \frac{z[3] - z[4]}{-z[1] - z[4]} \frac{-z[1] - z[6]}{z[3] - z[6]}, \quad u[3] \rightarrow \frac{(z[1] - z[2])(z[4] - z[5])}{-z[1] - z[4]} \frac{1}{(-z[2] - z[5])}
 \end{aligned}$$

```
n[67] = << final.m;
```

```
▼ n[68] = Length[final]
```

```
Out[68]= 7272
```

```

final /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[b, a, c, d];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[a, b, d, c];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[c, d, a, b];
Expand[%]

```

$$\begin{aligned}
 \text{up}[3, 1, 2] &\rightarrow -\frac{-z[1] - z[4]}{z[3] - z[4]} \frac{z[3] - z[5]}{z[1] - z[5]}, \\
 \text{um}[1, 2, 3] &\rightarrow \frac{z[2] - z[5]}{z[4] - z[5]} \frac{z[4] - z[6]}{z[2] - z[6]}, \\
 \text{um}[2, 3, 1] &\rightarrow -\frac{z[2] - z[4]}{z[2] - z[3]} \frac{z[3] - z[6]}{-z[4] - z[6]}, \\
 \text{um}[3, 1, 2] &\rightarrow \frac{-z[1] - z[4]}{-z[2] - z[4]} \frac{z[2] - z[6]}{z[1] - z[6]}, \quad u[1] \rightarrow \frac{z[2] - z[3]}{z[2] - z[5]} \frac{z[5] - z[6]}{z[3] - z[6]}, \\
 u[2] &\rightarrow \frac{z[3] - z[4]}{-z[1] - z[4]} \frac{-z[1] - z[6]}{z[3] - z[6]}, \quad u[3] \rightarrow \frac{(z[1] - z[2])(z[4] - z[5])}{-z[1] - z[4]} \frac{1}{(-z[2] - z[5])}
 \end{aligned}$$

```
In[67]= << final.m;
```

```
▼ In[68]= Length[final]
```

```
Cut[68]= 7272
```

```
▼ In[69]= final /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[b, a, c, d];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[a, b, d, c];
% /. tensor[a_, b_, c_, d_] -> tensor[a, b, c, d] - tensor[c, d, a, b];
Expand[%]
```

```
Cut[72]= 0
```