

Title: Wrap-up Talk

Date: Aug 06, 2011 12:15 PM

URL: <http://pirsa.org/11080037>

Abstract:

Scattering Amplitudes in $\mathcal{N}=4$ SYM

- Magic

Tree level: \mathcal{R} -Invariants

Scattering Amplitudes in $\mathcal{N}=4$ SYM

- Magic

Tree level: R -Invariants

Loop level: Generalized
Unitarity

Scattering Amplitudes in $\mathcal{N}=4$ SYM

- Magic

Tree level: R -Invariants

Loop level: Generalized
Unitarity

Scattering Amplitudes in $\mathcal{N}=4$ SYM

- Magic

Tree level: \mathcal{R} -Invariants

Loop level: • Generalized
Unitarity
• Symbols

Scattering Amplitudes in $\mathcal{N}=4$ SYM

- Magic

Tree level: \mathcal{R} -Invariants

Loop level: • Generalized
Unitarity
• Symbols

R-Invariants.

019

$$[abcde] = \frac{\sum (\langle abcd \rangle \eta_{e+} \dots)}{\langle abcd \rangle \langle bcde \rangle \dots}$$

R-Invariants.

$$[abcde] = \frac{\sum (\langle abcd \rangle \eta_e + \dots)}{\langle abcd \rangle \langle bcde \rangle \dots}$$

$$A_6^{\text{NMHV}} = A_6^{\text{MHV}} \left([12345] + [12356] + [13456] \right)$$

R-Invariants.

$$[abcde] = \frac{\sum (\langle abcd \rangle \eta_e + \dots)}{\langle abcd \rangle \langle bcde \rangle \dots}$$

$$A_6^{\text{NMHV}} = A_6^{\text{MHV}} \left([12345] + [12356] + [13456] \right)$$

4-brackets \rightarrow Dual Conformal Invariant

R-Invariants

$$[abcde] = \frac{\sum (\langle abcd \rangle \eta_e + \dots)}{\langle abcd \rangle \langle bcde \rangle \dots}$$

$$A_6^{\text{NMHV}} = A_6^{\text{MHV}} \left([12345] + [12356] + [13456] \right)$$

4-brackets \rightarrow Dual Conformal Invariant $SL(4)$

R-Invariants.

$$[abcde] = \frac{\sum (\langle abcd \rangle \eta_e + \dots)}{\langle abcd \rangle \langle bcde \rangle \dots}$$

$$A_6^{\text{NMHV}} = A_6^{\text{MHV}} \left([12345] + [12356] + [13456] \right)$$

4-brackets \rightarrow Dual Conformal Invariant $SL(4)$
Dual Super \mathcal{E}_4 " " "

A

R-Invariants

$$[abcde] = \frac{\sum_{\text{cyclic}} \langle abcd \rangle \langle e \dots \rangle}{\langle abcd \rangle \langle bcde \rangle \dots} = \frac{\sum_{\text{cyclic}} (t_e t_f + t_a t_b + t_b t_c + \dots)}{t_a t_b t_c t_d}$$

$$A_6^{\text{MHV}} = A_6^{\text{MHV}} \left([12345] + [12356] + [13456] \right) \quad \begin{matrix} t_a = \langle 12 \rangle \\ t_b = \langle 23 \rangle \end{matrix}$$

4-brackets \rightarrow Dual Conformal Invariant $SL(4)$
 Dual Super \leftarrow " " " $SL(4|4)$

R-Invariants.

$$[abcde] = \frac{\sum_{\text{cyclic}} \langle abcde \rangle^2}{\langle abcde \rangle \langle bcdea \rangle \dots} = \frac{\sum_{\text{cyclic}} (t_e t_a + t_a t_b + t_b t_c + \dots)}{t_a t_b t_c t_d t_e}$$

$$A_6^{\text{MHV}} = A_6^{\text{MHV}} \left([12345] + [12356] + [13456] \right) \quad \begin{matrix} t_a < t_b \\ t_c < t_d \end{matrix}$$

4-brackets → Dual Conformal Invariant SL(4)
 Dual Super \mathfrak{E}_4 " " SL(4|4)

$$Z_a = \begin{pmatrix} z_a \\ \eta_a \end{pmatrix}$$

$$\int^{414} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

$$Z_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix}$$

$$\frac{1}{t_a t_b \dots t_e} \sum^{414} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

$$Z_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{414} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

$$Z_a = \begin{pmatrix} z_a \\ \eta_a \end{pmatrix} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{414} (t_a z_a + t_b z_b + \dots + t_e z_e)$$

Redundancy $GL(1)$

$$Z_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix} \frac{1}{\text{Vol}(\text{GL}(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{414} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $\text{GL}(1)$

$$Z_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix} \frac{1}{\text{Vol}(GL(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{414} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $GL(1)$

$$[\Sigma abcdo] = \frac{1}{Vol(GL(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $GL(1)$

$$[\Sigma abcdo] = \frac{1}{Vol(GL(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $GL(1)$

$$[\Sigma abcdo] = \frac{1}{Vol(GL(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $GL(1)$

G.F: $t_a = 1$.

$$[\text{abcde}] = \frac{1}{\text{Vol}(\text{GL}(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{414} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $\text{GL}(1)$ (t_a, t_b, \dots, t_e)
 \mathbb{C}^3
 G.F: $t_a = 1$.

$$[\text{abcdo}] = \frac{1}{\text{Vol}(\text{GL}(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{414} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $\text{GL}(1)$
 G.F.: $t_a = 1$.

$(t_a, t_b, \dots, t_e) \sim \int (t_a, t_b, \dots, t_e)$
 \mathbb{C}^3

$$[\text{abcdo}] = \frac{1}{\text{Vol}(\text{GL}(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $\text{GL}(1)$
 G.F: $t_a = 1$.

$(t_a, t_b, \dots, t_e) \sim \underbrace{\mathbb{C}^3}_{\mathbb{C}^2} \sim \mathbb{C}P^1$

$$[\Sigma abcde] = \frac{1}{\text{Vol}(\text{GL}(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{414} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $\text{GL}(1)$

G.F.: $t_a = 1$.

$$\int \frac{dt_b \dots dt_e}{t_b \dots t_e}$$

$(t_a, t_b, \dots, t_e) \sim f(t_a, t_b, \dots, t_e)$

\mathbb{C}^3

\mathbb{CP}^4

$$[\Sigma abcde] = \frac{1}{\text{Vol}(\mathbb{G}(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $\mathbb{G}(1)$

$(t_a, t_b, \dots, t_e) \sim f(t_a, t_b, \dots, t_e)$

G.F.: $t_a = 1$

\mathbb{C}^2

$$\int \frac{dt_b \dots dt_e}{t_b \dots t_e} \int^{0|4} (\eta_a + \dots + t_e \eta_e) \int^4 (Z_a + t_b Z_b + \dots + t_e Z_e) \mathbb{P}^4$$

<code>

$$\langle cde | a \rangle + \tau_b \langle cdeb \rangle = 0$$

$$\langle cde | \hat{a} \rangle + t_b \langle cdeb \rangle = 0 \quad t_b = - \frac{\langle cde | \hat{a} \rangle}{\langle cdeb \rangle}$$

$$\langle cde a \rangle + t_b \langle cdeb \rangle = 0 \quad t_b = - \frac{\langle cdea \rangle}{\langle cdeb \rangle}$$

$$\frac{1}{t_b \cdot t_c} = \frac{\langle cdeb \rangle}{\langle \dots \rangle}$$

$$\langle cde | \hat{a} \rangle + t_b \langle cdeb \rangle = 0 \quad t_b = - \frac{\langle cdea \rangle}{\langle cdeb \rangle}$$

$$\frac{1}{t_b \cdot t_c} = \frac{\langle cdeb \rangle^4}{\langle \rangle \langle \rangle \langle \rangle \langle \rangle}$$

$$\langle cde a \rangle + t_b \langle cdeb \rangle = 0 \quad t_b = - \frac{\langle cdea \rangle}{\langle cdeb \rangle}$$

$$\frac{1}{t_b \cdot t_c} = \frac{\langle cdeb \rangle^2}{\langle \quad \rangle \langle \quad \rangle}$$



$$\langle cde | a \rangle + t_b \langle cde | b \rangle = 0 \quad t_b = - \frac{\langle cde | a \rangle}{\langle cde | b \rangle}$$

$$\frac{1}{t_b \cdot t_c} = \frac{\langle cde | b \rangle^4}{\langle cde | b \rangle \times \langle cde | b \rangle \times \langle cde | b \rangle \times \langle cde | b \rangle} \times \int (\eta_a + \frac{c}{s} \eta_b)$$

$$\langle cde a \rangle + t_b \langle cdeb \rangle = 0 \quad t_b = - \frac{\langle cdea \rangle}{\langle cdeb \rangle}$$

$$\frac{1}{t_b \cdot t_c} = \frac{\langle cdeb \rangle^4}{\langle \dots \rangle \langle \dots \rangle \langle \dots \rangle} \times \frac{1}{\langle cdeb \rangle} \times \int^{0|4} \left(\eta_a + \frac{c}{\langle \dots \rangle} \eta_b \right)$$

$$\frac{1}{\langle cdeb \rangle} \int^{0|4} \langle abcd \rangle \eta_c$$

$$[12345] =$$

$$S(4) =$$

$$[12345] = \frac{1}{\sqrt{5!}} \int_{|t_i|=1} \frac{dt_1 dt_2 \dots dt_5}{t_1 t_2 \dots t_5} \sum_{\sigma \in S_5} (t_1 z_{\sigma(1)} + t_2 z_{\sigma(2)} + \dots + t_5 z_{\sigma(5)})$$

$|t_i|=1$

$$P_c^{\text{NHV}} = \frac{1}{V_0 \Omega_{\text{LH}}} \int_{|t_i| = \epsilon} dt_1 dt_2 \dots dt_n \int (t_1 \mathcal{Z}_1 + t_2 \mathcal{Z}_2 + \dots + t_n \mathcal{Z}_n + t_0 \mathcal{Z}_0)$$

R-Invariants

$$[abcde] = \frac{\int (\langle abcd \rangle \eta_c + \dots)}{\langle abcd \rangle \langle bcde \rangle \dots} = \frac{\int (t_c \eta_c + t_a \eta_a + t_b \eta_b + \dots)}{t_a t_b t_c t_d}$$

$$A_6^{\text{MHVV}} = A_6^{\text{MHV}} \left([12345] + [12356] + [13456] \right)$$

$t_a = \langle 12 \rangle$
 $t_b = \langle 23 \rangle$

4-brackets \rightarrow Dual Conformal Invariant $SL(4)$
 Dual Super \leftarrow " " " $SL(4|4)$

$$P_c^{\text{HHIV}} = \frac{1}{V_0 |G_{\text{HHIV}}|} \int_{\{ |t_i| = \epsilon \}^+} \frac{dt_1 dt_2 \dots dt_6}{t_1 t_2 \dots t_6} \int_{\text{HHIV}} (t_1 Z_1 + t_2 Z_2 + \dots + t_5 Z_5 + t_6 Z_6)$$

$$P_c^{HHV} = \frac{1}{V_0 K(L_1)} \int_{\epsilon_1, t_2 \dots t_6} dt_1 dt_2 \dots dt_6 \int_{\{t_1 Z_1 + t_2 Z_2 + \dots + t_5 Z_5 + t_6 Z_6\}} dt$$

$$\{ |t_1| = \epsilon \} + \{ |t_4| = \epsilon \} + \{ |t_6| = \epsilon \}$$

$$\begin{aligned}
 P_c^{\text{NHIV}} &= \frac{1}{V_0 K(\mu)} \int \frac{dt_1 dt_2 \dots dt_6}{t_1 t_2 \dots t_6} \int_{\mathcal{H}} (t_1 \mathcal{Z}_1 + t_2 \mathcal{Z}_2 + \dots + t_5 \mathcal{Z}_5 + t_6 \mathcal{Z}_6) \\
 &\quad \left\{ |t_1| = \epsilon \right\} + \left\{ |t_4| = \epsilon \right\} + \left\{ |t_2| = \epsilon \right\}
 \end{aligned}$$

$$\begin{aligned}
 P_c^{HHHV} &= \frac{1}{V_1 V_2 \dots V_n} \int_{t_1, t_2, \dots, t_n} dt_1 dt_2 \dots dt_n \int_{\{ |t_i| = \epsilon \}} (t_1 Z_1 + t_2 Z_2 + \dots + t_n Z_n) \\
 &\quad \{ |t_1| = \epsilon \} + \{ |t_2| = \epsilon \} + \dots
 \end{aligned}$$

$$\begin{aligned}
 P_c^{HHV} &= \frac{1}{V_0 K(\mu)} \int_{t_1, t_2, \dots, t_6} dt_1 dt_2 \dots dt_6 \int_{\{t_1 Z_1 + t_2 Z_2 + \dots + t_5 Z_5 + t_6 Z_6\}} \\
 &\quad \{ |t_1| = \epsilon \} + \{ |t_4| = \epsilon \} + \{ |t_2| = \epsilon \}
 \end{aligned}$$

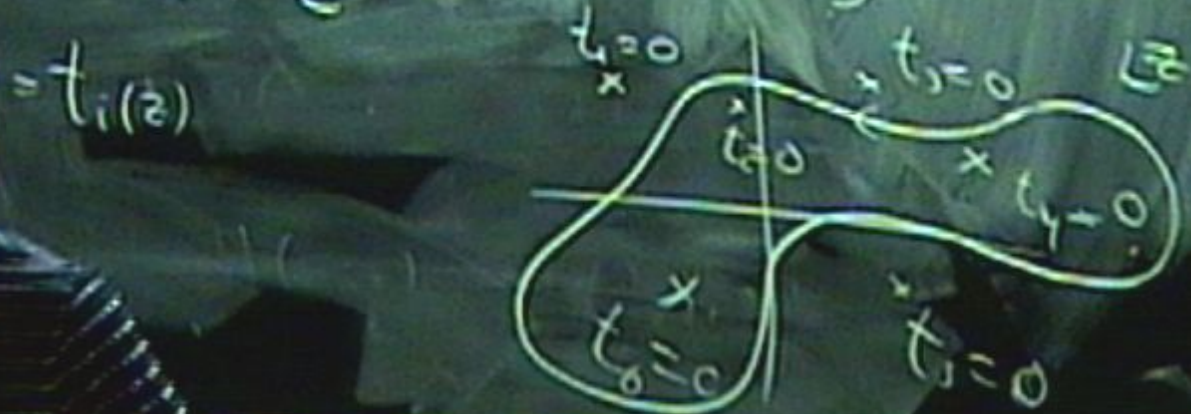
$$P_c^{\text{HHV}} = \frac{1}{V(\Omega)} \int_{\substack{t_1, t_2, \dots, t_6 \\ \{|t_i|=\epsilon\}}} dt_1 dt_2 \dots dt_6 \int_{\substack{z_1, z_2, \dots, z_6 \\ \{|z_i|=\epsilon\}}} (t_1 z_1 + t_2 z_2 + \dots + t_5 z_5 + t_6 z_6)$$

$\{|t_i|=\epsilon\} + \{|z_i|=\epsilon\}$
 Cplx parameter z



$$P_c^{\text{NHIV}} = \frac{1}{V(\Omega)} \int_{\substack{t_1, t_2, \dots, t_6 \\ \{|t_k|=\epsilon\}}} dt_1 dt_2 \dots dt_6 \int_{\substack{z_1, z_2, \dots, z_6 \\ \{|z_k|=1\}}} (t_1 z_1 + t_2 z_2 + \dots + t_5 z_5 + t_6 z_6)$$

Cplx parameter z

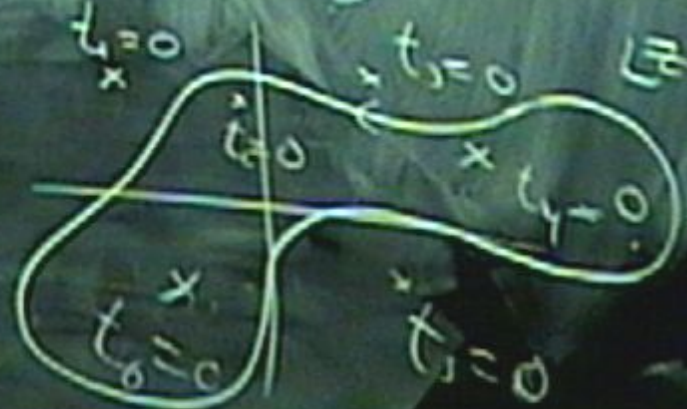


$$P_c^{NHIV} = \frac{1}{V_0 |G_{111}|} \int \frac{dt_1 dt_2 \dots dt_c}{t_1 t_2 \dots t_c} \int_{\mathcal{C}} (t_1 Z_1 + t_2 Z_2 + \dots + t_s Z_s + t_c Z_c)$$

$$\{ |t_1| = \epsilon \} + \{ |t_2| = \epsilon \} + \{ |t_3| = \epsilon \}$$

Cplx parameter z

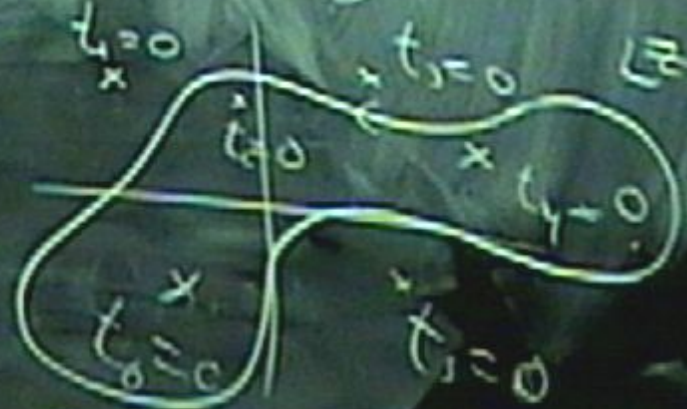
$$t_i = t_i(z)$$



$$P_7^{\text{HHIV}} = \frac{1}{\sqrt{|G(\mathbf{t})|}} \int_{\{ |t_i| = \epsilon \}} \frac{dt_1 dt_2 \dots dt_6}{t_1 t_2 \dots t_6} \int_{\mathbb{R}^6} (t_1 Z_1 + t_2 Z_2 + \dots + t_5 Z_5 + t_6 Z_6)$$

$\{ |t_i| = \epsilon \} + \{ |t_4| = \epsilon \} + \{ |t_2| = \epsilon \}$
 parameter \mathbb{R}^6

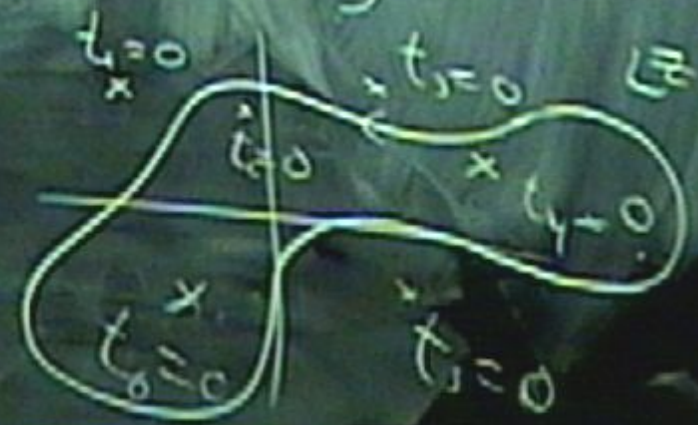
$$t_i = t_i(z)$$



$$P_7^{\text{HHIV}} = \frac{1}{V_0 |G_{11}|} \int dt_1 dt_2 \dots dt_6 dt_7 \int \left(t_1 Z_1 + t_2 Z_2 \dots + t_5 Z_5 + t_6 Z_6 + t_7 Z_7 \right)$$

$$\left(|t_1| = \epsilon \right) + \left(|t_2| = \epsilon \right)$$

molecul
 $c_i = t_i(z)$



$$P_7^{NHIV} = \frac{1}{V_0 |G_{II}|} \int \frac{dt_1 dt_2 \dots dt_n dt_1 dt_2}{t_1 t_2 \dots t_n t_1 t_2} \int \left(t_1 Z_1 + t_2 Z_2 \dots + t_1 Z_s + t_n Z_n + t_1 Z_1 \right)$$

$$\int \frac{dz_1 dz_2}{T_i(z_1, z_2)}$$

$$P_7^{\text{NHIV}} = \frac{1}{V_0 |G_{111}|} \int dt_1 dt_2 \dots dt_6 dt_7 \int \frac{d^3z_1 d^3z_2 \dots d^3z_6 d^3z_7}{t_1 t_2 \dots t_6 t_7} \mathcal{S}(t_1 z_1 + t_2 z_2 \dots + t_6 z_6 + t_7 z_7)$$

$$\int \frac{d^3z_1 d^3z_2}{\prod_{i=1}^2 t_i(z_1, z_2)}$$

$$P_7^{NHIV} = \frac{1}{\sqrt{|G|}} \int dt_1 dt_2 \dots dt_n \int_{t_1, t_2, \dots, t_n} \left(t_1 Z_1 + t_2 Z_2 + \dots + t_5 Z_5 + t_6 Z_6 + t_7 Z_7 \right)$$

$$\int \frac{dz_1 dz_2}{\prod_{i=1}^n t_i(z_1, z_2)}$$

Global Residue Thm.

$$P_7^{\text{HHIV}} = \frac{1}{\text{Vol}(G_{11})} \int dt_1 dt_2 \dots dt_6 dt_7 \int_{\mathcal{H}} (t_1 Z_1 + t_2 Z_2 \dots + t_5 Z_5 + t_6 Z_6 + t_7 Z_7)$$

$$\int \frac{dz_1 dz_2}{\prod_{i=1}^7 t_i(z_1, z_2)}$$

Global Residue Thm.

How about N^2 MHV?

$$[\Sigma abcdo] = \frac{1}{V_0(GU)} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $GL(1)$

$(t_a, t_b, \dots, t_e) \sim f(t_a, t_b, \dots, t_e)$

G.F: $t_a = 1$

\mathbb{C}^3

$$\int \frac{dt_b \dots dt_e}{t_b \dots t_e} \int^{0|4} (t_a \eta_a + \dots + t_e \eta_e) \int^4 (Z_a + t_b Z_b + \dots + t_e Z_e) \mathbb{P}^4$$

\mathbb{O}^4

$$[\Sigma abc d_0] = \frac{1}{V(SU(4))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $SU(4)$

$$(t_a, t_b, \dots, t_e) \sim f(t_a, t_b, \dots, t_e)$$

G.F: $t_a = 1$

$$S^3$$

$$CP^4$$

$$\int \frac{dt_b \dots dt_e}{t_b \dots t_e} \int^{0|4} (t_a \eta_a + \dots + t_e \eta_e) \int^4 (Z_a + t_b Z_b + \dots + t_e Z_e)$$

$$[\Sigma abc d_0] = \frac{1}{V_0(SU_4)} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $SU(4)$

$$(t_a, t_b, \dots, t_e) \sim f(t_a, t_b, \dots, t_e)$$

G.F: $t_a = 1$

$$\mathbb{C}^3 \quad \mathbb{CP}^4$$

$$\int \frac{dt_b \dots dt_e}{t_b \dots t_e} \int^{0|4} (t_a \eta_a + \dots + t_e \eta_e) \int^4 (Z_a + t_b Z_b + \dots + t_e Z_e)$$

$$P_7^{NHV} = \frac{1}{\sqrt{16!}} \int \frac{dt_1 dt_2 \dots dt_6 dt_7}{t_1 t_2 \dots t_6 t_7} \int \left(t_1 Z_1 + t_2 Z_2 + \dots + t_5 Z_5 + t_6 Z_6 + t_7 Z_7 \right)$$

$$\mathbb{CP}^5 = \text{Space of lines in } \mathbb{C}P^6 = G(1,6)$$

Grassmannian of 1-planes in $\mathbb{C}P^6$

$$P_n^{NMHV} = \mathbb{CP}^{n-2}$$

$$\mathcal{P}_7^{\text{HHHV}} = \frac{1}{\sqrt{16!}} \int dt_1 dt_2 \dots dt_6 dt_7 \int \frac{d^6 z}{t_1 t_2 \dots t_6 t_7} \left(t_1 Z_1 + t_2 Z_2 + \dots + t_5 Z_5 + t_6 Z_6 + t_7 Z_7 \right)$$

Space of lines in $\mathbb{P}^6 = G(1,6)$

Großmannian of 1-planes in \mathbb{P}^6

$$P_7^{NMHV} = \frac{1}{\text{Vol}(SL(4))} \int dt_1 dt_2 \dots dt_6 dt_7 \int \left(t_1 Z_1 + t_2 Z_2 + \dots + t_6 Z_6 + t_7 Z_7 \right)$$

$$\mathbb{CP}^5 = \text{Space of lines in } \mathbb{CP}^7 = G(1, 6)$$

Grosmannian of 1-planes in \mathbb{CP}^6

$$P_n^{NMHV} = \mathbb{CP}^{n-2}$$

$$P_7^{NHV} = \frac{1}{\text{Vol}(S^1)} \int dt_1 dt_2 \dots dt_6 dt_7 \int_{(t_1, \mathbb{Z}_1 + t_2 \mathbb{Z}_2 + \dots + t_5 \mathbb{Z}_5 + t_6 \mathbb{Z}_6 + t_7 \mathbb{Z}_7)}$$

$$\mathbb{CP}^5 = \text{Space of lines in } \mathbb{P}^7 = G(1, 6)$$

Grosmannian of 1-planes in \mathbb{P}^6

$$P_n^{NMHV} \quad \mathbb{CP}_{n-2}$$

How about N^2 MHV?

$\Delta(2, n)$

$$\int^{\text{y/y}} \left(\sum_{a=1}^n t_a \mathcal{Z}_a \right) \int^{\text{y/y}} \left(\sum t'_a \mathcal{Z}_a \right)$$

$$g \begin{pmatrix} t_1, t_2, \dots, t_n \\ t'_1, t'_2, \dots, t'_n \end{pmatrix}$$

$$g \in GL(2)$$

How about N^2 MHV?

$\Delta(2, n)$

$$\int_{\mathcal{D}} \left(\sum_{a=1}^n D_{qa} \mathcal{Z}_a \right) \int_{\mathcal{D}'} \left(\sum_{a=1}^n t'_a \mathcal{Z}_a \right)$$

$$g \begin{pmatrix} t_1 & t_2 & \dots & t_n \\ t'_1 & t'_2 & \dots & t'_n \end{pmatrix}$$

$$\begin{pmatrix} D_{11} & D_{12} & \dots & D_{1n} \\ D_{21} & D_{22} & \dots & D_{2n} \end{pmatrix}$$

$g \in GL(2)$

How about N^2 MHV?

$$\int \frac{d^{2n}D_{2n}}{d^2D_{2n}}$$

$\Gamma(2, n)$

$$\prod_{\alpha=1}^2 \int \left(\sum_{\alpha=1}^n \frac{d^2D_{\alpha}}{d^2D_{\alpha}} \mathcal{F}_{\alpha} \right)$$

$$\begin{pmatrix} D_{11} & D_{12} & \dots & D_{1n} \\ D_{21} & D_{22} & \dots & D_{2n} \end{pmatrix}$$

How about N^2 MHV?

$$\int \frac{d^{2n}D_{\alpha a}}$$

$\Gamma(2, n)$

$$\prod_{\alpha=1}^2 \int \left(\sum_{a=1}^n D_{\alpha a} \mathcal{F}_a \right)$$

$$\begin{pmatrix} D_{11} & D_{12} & \dots & D_{1n} \\ D_{21} & D_{22} & \dots & D_{2n} \end{pmatrix}$$

How about N^2 MHV?

$$\int \frac{d^{2n}D_{\alpha a}}{d^{2n}D_{\alpha a}}$$

$\Delta(2, n)$

$$\prod_{\alpha=1}^2 \int \left(\sum_{\alpha a} d^{2n}D_{\alpha a} \mathcal{F}_a \right)$$

$$\begin{pmatrix} D_{11} & D_{12} & \dots & D_{1n} \\ D_{21} & D_{22} & \dots & D_{2n} \end{pmatrix}$$

$$[abcde] = \frac{1}{\text{Vol}(G/U)} \int dt_a \dots dt_e \int_{\mathbb{C}P^4} (t_a Z_a + t_b Z_b + \dots + t_e Z_e)$$

Redundancy $\mathbb{C}P^4$

$(t_a, t_b, \dots, t_e) \sim f(t_a, t_b, \dots, t_e)$

$\mathbb{C}P^4$

$\mathbb{C}P^4$

G.F.:

$$\int dt_a \dots dt_e \int_{\mathbb{C}P^4} (t_a \eta_a + \dots + t_e \eta_e) \int_{\mathbb{C}P^4} (Z_a + t_b Z_b + \dots + t_e Z_e)$$

$$[abcde] = \frac{1}{\text{Vol}(GL(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a z_a + t_b z_b + \dots + t_e z_e)$$

Redundancy $GL(1)$

$$(t_a, t_b, \dots, t_e) \sim f(t_a, t_b, \dots, t_e)$$

G.F.: $t_a = 1$

$$\mathbb{C}^3$$

$$\mathbb{CP}^4$$

$$\int \frac{dt_b \dots dt_e}{t_b \dots t_e} \int^{0|4} (\eta_a + \dots + t_e \eta_e) \int^4 (z_a + t_b z_b + \dots + t_e z_e)$$

=

How about N^2 MHV?

$\Delta(2, m)$

$$\int \frac{\mathcal{D}^{2n} \mathcal{D}_{x_a}}{\mathcal{D}_{11} \mathcal{D}_{12} \dots \mathcal{D}_{1n}}$$

$$\prod_{\alpha=1}^2 \int \mathcal{D}^{4H} \left(\prod_{\alpha=1}^m \mathcal{D}_{x_a} \mathcal{F}_a \right)$$

$$\begin{pmatrix} \mathcal{D}_{11} & \mathcal{D}_{12} & \dots & \mathcal{D}_{1n} \\ \mathcal{D}_{21} & \mathcal{D}_{22} & \dots & \mathcal{D}_{2n} \end{pmatrix}$$

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SU_n)} \int \frac{\mathcal{D}^{2n} \lambda}{(12)(23)\dots(n1)} \prod_{\alpha=1}^2 \int \left(\sum_{a=1}^{n-1} \mathcal{D}_{\alpha a} \mathcal{F}_a \right)$$

$$\left(\begin{matrix} D_{12} & \dots & D_{1n} \\ D_{21} & & D_{2n} \\ & & \dots \\ & & D_{n1} \end{matrix} \right) \rightarrow \mathcal{F} \left(\dots \right)$$

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SU_n)} \int \frac{\mathcal{D}^{2n} D_{\alpha a}}{(12)(23)\dots(n1)} \prod_{\alpha=1}^2 \int \left(\sum_{a=1}^n D_{\alpha a} \mathcal{F}_a \right)$$

How about N^2 MHV?

$G(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(GL_n)} \int \prod_{i=1}^{2n} dD_{i\alpha} \frac{\prod_{\alpha=1}^2 \int \left(\sum_{a=1}^{4|1} D_{i\alpha} \mathcal{F}_a \right)}$$

$$[\text{abcdo}] = \frac{1}{V_1(G_k)} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \int^{4|4} (t_a z_a + t_b z_b + \dots + t_e z_e)$$

neg $GL(1)$

$(t_a, t_b, \dots, t_e) \sim f(t_a, t_b, \dots, t_e)$

\mathbb{C}^3

$\mathbb{C}P^4$

$$\int^{0|4} (t_a \eta_a + \dots + t_e \eta_e) \int^4 (z_a + t_b z_b + \dots + t_e z_e)$$

0=

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2}$$

$$= A_{n,2} (1 + \dots)$$

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2}$$

$$= A_{n,2} (1 + P_{n,2} + \dots + P_{n,n-2})$$

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(GL_n)} \int \prod_{a=1}^{2n} D_{\alpha a} \frac{\prod_{\alpha=1}^2 \int \left(\prod_{a=1}^n D_{\alpha a} \mathcal{L}_a \right)}$$

$$J_{\omega}^1 = \sum_{a=1}^n \mathcal{L}_a^1 \frac{\partial}{\partial \mathcal{L}_a^{\omega}}$$

$SL(4|4)$

Dual
Superconformal

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2} \quad (n, m)$$

$$= A_{n,2} \left(1 + P_{n,2} + \dots + P_{n,n-4} \right)$$

\searrow (n, k)

A

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2} \quad (n, m)$$

$$= A_{n,2} \left(1 + P_{n,2} + \dots + P_{n,n-4} \right)$$

\searrow (n, k)

$$A_{n,m} = \int_{G(m,n)} \text{Kinematic Dots}_2 \left(\hat{\lambda}_1, \hat{\lambda}_2 \right)$$

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2} \quad (n, m)$$

$$= A_{n,2} \left(1 + P_{n,2} + \dots + P_{n,n-2} \right)$$

\searrow (n, k)

$$A_{n,m} = \int_{G(m,n)} \text{Kinematic Dst}_2(\lambda, \tilde{\lambda}, \tilde{\eta})$$

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2} \quad (n, m)$$

$$= A_{n,2} \left(1 + P_{n,2} + \dots + P_{n,n-2} \right)$$

\searrow (n, k)

$$A_{n,m} = \int_{G(m,n)} \text{Kinematic Dots}_2(\lambda, \hat{\lambda}, \hat{\eta})$$

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2} \quad (n, m)$$

$$A_{n,m} = \int_{G(m)} \text{Kinematic Data}(\lambda, \tilde{\lambda}, \tilde{\eta})$$

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2}^{(n,m)}$$

$$A_{n,m} = \int_{G(m,n)} \text{Kinematic Dst}_2(\lambda, \tilde{\lambda}, \tilde{\eta})$$

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2} \quad (n, m)$$

$$A_{n,m} = \int_{G(m,n)} \text{Kinematic Data}(\lambda, \tilde{\lambda}, \tilde{\eta})$$

(n, m)

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2}$$

$$\int d\lambda_a e^{i(\lambda_a \mu_a)} A_n(\{\lambda, \tilde{\lambda}, \tilde{\eta}\}) = \tilde{A}_n(\{\mu, \tilde{\lambda}, \tilde{\eta}\})$$

$$\int_{G(m,n)} \text{Kinematic Det}_2(\{\lambda, \tilde{\lambda}, \tilde{\eta}\})$$

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2} \quad (n, m)$$

$$\int d\lambda_a e^{i(\lambda_a \mu_a)} A_n(\{\lambda, \tilde{\lambda}, \tilde{\eta}\}) = \tilde{A}_n(\underbrace{\{\mu, \tilde{\lambda}, \tilde{\eta}\}}_{\tilde{W}_a})$$

$$\tilde{W}_a = \begin{pmatrix} W_a \\ \tilde{\eta}_a \\ a \end{pmatrix}$$

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SL_n)} \int \frac{d^{2n} D_{\alpha a}}{(12)(23) \dots (n1)} \prod_{\alpha=1}^2 \int_{\mathbb{P}^{4|4}} \left(\prod_{a=1}^n D_{\alpha a} \mathcal{F}_a \right)$$

$$\mathcal{J}_{\omega}^1 = \sum_{\alpha=1}^n \frac{\mathcal{F}_a^1}{\mathcal{F}_a^{\omega}}$$

Dual
Superconformal

$SL(4|4)$

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SL)} \frac{\int \prod_{a=1}^{2n} D_{\alpha a}}{(12)(23)\dots(n1)} \prod_{\alpha=1}^2 \int \left(\prod_{a=1}^{4|4} D_{\alpha a} \mathcal{F}_a \right)$$

$$\mathcal{J}_{\omega}^1 = \sum_{a=1}^n \frac{\mathcal{F}_a^1}{\mathcal{F}_a^{\omega}}$$

Dual
Superconformal

$SL(4|4)$

How about N^2 MHV?

$G(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SL_4)} \int \frac{\mathcal{D}^{2n} \mathcal{D}x_a}{(12)(23) \dots (n1)} \prod_{\alpha=1}^2 \int \mathcal{D}^{4|4} \left(\sum_{a=1}^n \mathcal{D}_{\alpha a} \mathcal{Z}_a \right)$$

$$\mathcal{J}_{\omega}^1 = \sum_{\alpha=1}^n \mathcal{Z}_a^{\alpha} \frac{\partial}{\partial \mathcal{Z}_a^{\alpha}}$$

Dual
Superconformal

$SL(4|4)$

\mathcal{J}

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(GL_n)} \int \frac{\prod_{a=1}^{2n} D_{\alpha a}}{(12)(23)\dots(n1)} \prod_{\alpha=1}^2 \int \left(\prod_{a=1}^{4|4} D_{\alpha a} \mathcal{F}_a \right)$$

$$\mathcal{J}_\omega^1 = \sum_{a=1}^n \mathcal{F}_a^1 \frac{\partial}{\partial \mathcal{F}_a^2}$$

Dual
Superconformal

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SL_n)} \int \frac{d^{2n} D_{\alpha a}}{(12)(23) \dots (n1)} \prod_{\alpha=1}^2 \int_{S^{4|4}} \left(\prod_{a=1}^n D_{\alpha a} \mathcal{F}_a \right)$$

$$SL(4|4) \mathcal{J}_{\mathcal{Q}}^1 = \sum_{a=1}^n \mathcal{F}_a^{\dagger} \frac{\partial}{\partial \mathcal{F}_a}$$

Dual
Superconformal

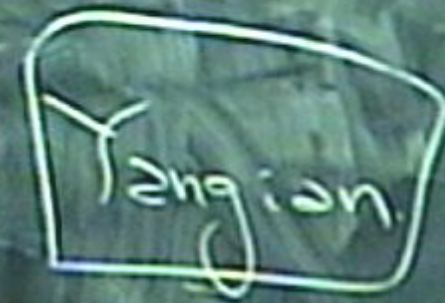
$$SL(4|4) \mathcal{J}_{\mathcal{Q}}^{\dagger} = \sum_{a < b} \left(\mathcal{F}_a^{\dagger} \frac{\partial}{\partial \mathcal{F}_a} \frac{\partial}{\partial \mathcal{F}_b} \mathcal{F}_b^{\dagger} - (a \leftrightarrow b) \right)$$

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(GL_n)} \int \frac{\mathcal{D}^{(2n)} \mathcal{D}x_a}{(12)(23)\dots(n1)} \prod_{\alpha=1}^2 \int \mathcal{D}^{(4|4)} \left(\prod_{a=1}^n \mathcal{D}x_a \mathcal{Z}_a \right)$$

$$1) \int \mathcal{D}^{(4)} \mathcal{Z}_a = \sum_{a=1}^n \mathcal{Z}_a^{\alpha} \frac{\partial}{\partial \mathcal{Z}_a^{\beta}}$$



$$(4|4) \int \mathcal{D}^{\alpha} \mathcal{Z}_a = \sum_{a < b} \left(\mathcal{Z}_a^{\alpha} \frac{\partial}{\partial \mathcal{Z}_a^{\beta}} \frac{\partial}{\partial \mathcal{Z}_b^{\gamma}} - (a \leftrightarrow b) \right)$$

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(GL_n)} \int \prod_{a=1}^{2n} D_{x_a} \frac{\prod_{\alpha=1}^2 \int \prod_{a=1}^{4|4} \mathcal{Z}_a}{(12)(23) \dots (n1)}$$

$$SL(4|4) \int_{\mathcal{D}} \mathcal{Z}_a = \sum_{a=1}^n \mathcal{Z}_a \frac{\partial}{\partial \mathcal{Z}_a}$$

Yangian

$$SL(4|4) \int_{\mathcal{D}} \mathcal{Z}_a = \sum_{a < b} \left(\mathcal{Z}_a \frac{\partial}{\partial \mathcal{Z}_a} \mathcal{Z}_b \frac{\partial}{\partial \mathcal{Z}_b} - (\mathcal{Z}_b \frac{\partial}{\partial \mathcal{Z}_b} \mathcal{Z}_a \frac{\partial}{\partial \mathcal{Z}_a}) \right)$$

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SL_n)} \int \frac{D^{2n} \alpha}{(12)(23) \dots (n1)} \prod_{\alpha=1}^2 \int \left(\prod_{a=1}^n D_{\alpha a} \chi_a \right)$$

$$SL(4|4) \int_{\mathcal{D}} \chi_a = \sum_{a=1}^n \int \chi_a \frac{\partial}{\partial \chi_a}$$

Yangian

$$\int_{\mathcal{D}} \chi_a = \sum_{a < b} \left(\chi_a \frac{\partial}{\partial \chi_a} \chi_b \frac{\partial}{\partial \chi_b} - \chi_b \frac{\partial}{\partial \chi_b} \chi_a \frac{\partial}{\partial \chi_a} \right)$$

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SL_n)} \frac{\int \prod_{i=1}^{2n} D_{i,a}}{(12)(23)\dots(n1)} \prod_{\alpha=1}^2 \int \left(\prod_{a=1}^{4|4} D_{i,a} \mathcal{F}_a \right)$$

$$SL(4|4) \int_{\mathcal{D}} \mathcal{J}_1^{(4)} = \sum_{a=1}^n \mathcal{F}_a^{\mathcal{A}} \frac{\partial}{\partial \mathcal{F}_a^{\mathcal{B}}}$$

$$\int_{\mathcal{D}} \mathcal{J}_2^{(4)} = \sum_{a \subset b} \left(\mathcal{F}_a^{\mathcal{A}} \frac{\partial}{\partial \mathcal{F}_a^{\mathcal{B}}} \frac{\partial}{\partial \mathcal{F}_b^{\mathcal{C}}} \right) - (a \rightarrow b)$$

Yangian

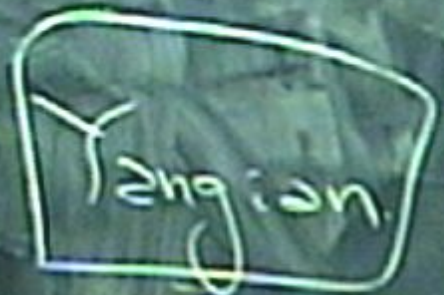
How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(GL_n)} \int \frac{\prod_{i=1}^{2n} dD_{ia}}{(12)(23)\dots(n1)} \prod_{\alpha=1}^2 \int \left(\prod_{a=1}^n D_{\alpha a} \mathcal{F}_a \right)$$

$$\int_{(4|4)} \mathcal{J}^{(4)} = \sum_{a=1}^n \mathcal{F}_a^T \frac{\partial}{\partial \mathcal{F}_a}$$

$$\int_{\mathcal{D}} \mathcal{J}^{(4)} = \sum_{a < b} \left(\mathcal{F}_a^T \frac{\partial}{\partial \mathcal{F}_a} \frac{\partial}{\partial \mathcal{F}_b} \frac{\partial}{\partial \mathcal{F}_b} - (a \leftrightarrow b) \right)$$



How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(GL_n)} \int \frac{\delta^{(2n)}(\sum_{a=1}^n D_{aa})}{(12)(23)\dots(n1)} \prod_{a=1}^n \int \delta(\sum_{a=1}^n D_{aa} \lambda_a)$$

$$\int \delta^{(4)}(\omega) \int \lambda_a = \sum_{a=1}^n \int \lambda_a \frac{\partial}{\partial \lambda_a}$$

$$\int \lambda_a = \sum_{a \subset b} \left(\int \lambda_a \frac{\partial}{\partial \lambda_a} \right) \left(\int \lambda_b \frac{\partial}{\partial \lambda_b} - (a,b) \right)$$

Yangian

How about N^2 MHV?

$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SL_n)} \int \frac{\prod_{a=1}^{2n} D_{\alpha a}}{(12)(23)\dots(n1)} \prod_{\alpha=1}^2 \int_{\mathbb{P}^{4|4}} \left(\prod_{a=1}^n D_{\alpha a} \mathcal{F}_a \right)$$

$$SL(4|4) \int_{\mathbb{P}^{4|4}} \mathcal{F}_1 = \sum_{a=1}^n \int_{\mathbb{P}^{4|4}} \mathcal{F}_a \frac{\partial}{\partial \mathcal{F}_a}$$

$$\int_{\mathbb{P}^{4|4}} \mathcal{F}_1 = \sum_{a < b} \left(\int_{\mathbb{P}^{4|4}} \mathcal{F}_a \frac{\partial}{\partial \mathcal{F}_a} \int_{\mathbb{P}^{4|4}} \mathcal{F}_b \frac{\partial}{\partial \mathcal{F}_b} - (a,b) \right)$$

Yangian

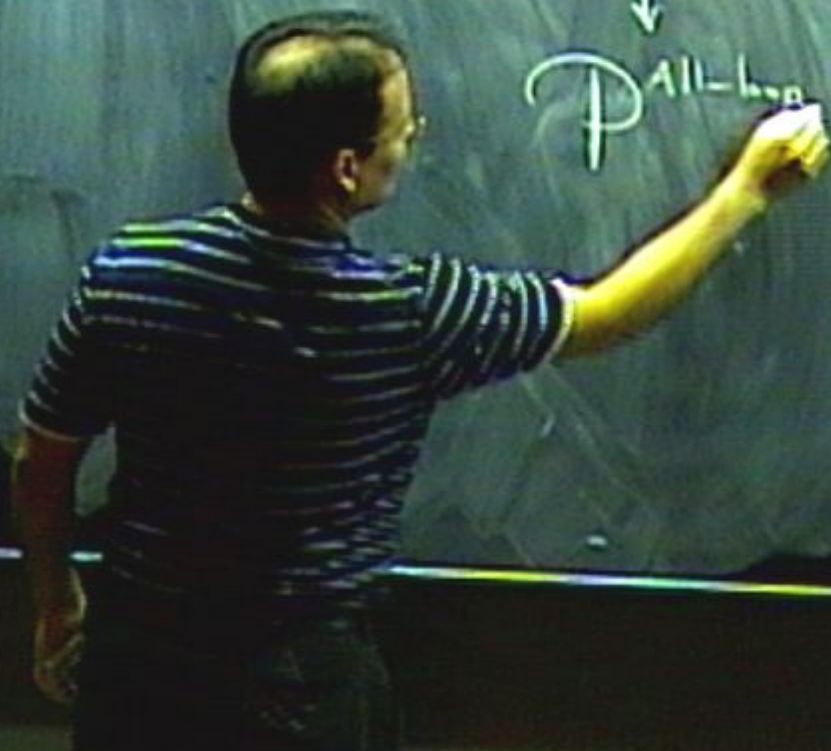
$A_{n \text{ All-loop}}$

$$A_n^{\text{All-loop}} \equiv A_n^{\text{MHV tree}} \left(\right)$$

$n=4,5$

\downarrow

$$\mathcal{P}^{\text{All-loop}}$$



$$A_n^{\text{All-loop}} = A_n^{\text{MHV tree}} \left(\right)$$

$n=4,5$

\downarrow

$\mathcal{P}^{\text{All-loop}}$

$$A_n^{\text{All-loop}}$$

$n=4,5$

$=$

$$A_n^{\text{MHV tree}}$$



\mathcal{P}

$$P^{\text{All-loop}}$$

$=$

Anomaly

$$A_n^{\text{All-loop}} = A_n^{\text{MHV tree}} \left(\right)$$

↓

$$\mathcal{B} \mathcal{P}^{\text{All-loop}} = \text{Anomaly}$$

$$A_n^{\text{All-loop}}$$

$n=4,5$

$=$

$$A_n^{\text{MHV tree}}$$

()



\mathcal{P}

$$P^{\text{All-loop}}$$

$=$

Anomaly

$$A_n^{\text{All-loop}} \stackrel{=}{=} A_n^{\text{MHV tree}} \quad \left(\right)$$

↓

$$\int \mathcal{P}^{\text{All-loop}} = \text{Anomaly}$$

$$A_n^{\text{All-loop}} \stackrel{n=4,5}{=} A_n^{\text{MHV tree}}$$

$$\int \mathcal{P}^{\text{All-loop}} = \text{Anomaly}$$

1-loop

$$A_4^{\text{1-loop}} = A_4^{\text{tree}}$$

$$\int \frac{d^4 \ell}{\ell^2 (\ell+p)^2 (\ell+p_1+p_2)^2 (\ell+p_1+p)^2}$$

$$A_n^{\text{All-loop}} = A_n^{\text{MHV tree}} \quad (n=4,5)$$

$$\int \mathcal{P}^{\text{All-loop}} = \text{Anomaly}$$

1-loop

$$A_4^{\text{1-loop}} = A_4^{\text{tree}}$$

$$\int \frac{d^4 \ell}{\ell^2 (\ell+p)^2 (\ell+p_1+p_2)^2 (\ell+p_1+p_2+p)^2}$$

$$A_n^{\text{All-loop}} = A_n^{\text{MHV tree}} \left(\right)$$

$n=4,5$

↓

$$\mathcal{P}^{\text{All-loop}} = \text{Anomaly}$$

1-loop

$$A_4^{\text{1-loop}} = A_4^{\text{tree}}$$

$$\int \frac{d^4 l}{l^2 (l+p)^2 (l+p_1+p_2)^2 (l+p_1+p_2+p)^2}$$

How about N^2 MHV?

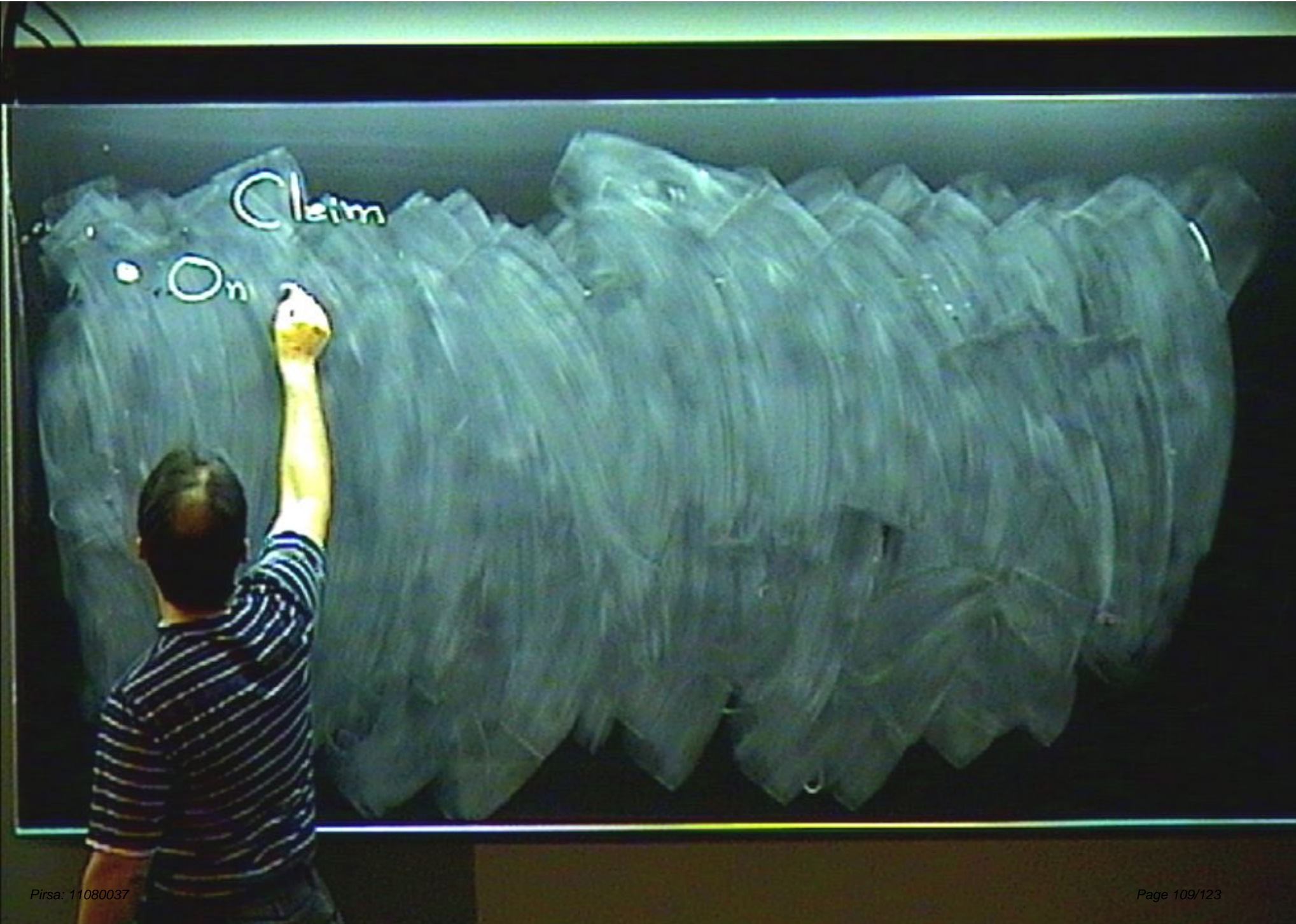
$\Delta(2, n)$

$$\mathcal{L}_{n,2} = \frac{1}{\text{Vol}(SU(4))} \int \frac{d^{2n}D_{\alpha a}}{(12)(23)\dots(n1)} \prod_{\alpha=1}^2 \int \left(\prod_{a=1}^n D_{\alpha a} \mathcal{Z}_a \right)$$

$$SU(4) \int_{\mathcal{E}_2} \mathcal{Z}_a = \sum_{a=1}^n \mathcal{Z}_a \frac{\partial}{\partial \mathcal{Z}_a}$$

$$\int_{\mathcal{E}_2} \mathcal{Z}_a = \sum_{a \subset b} \left(\mathcal{Z}_a \frac{\partial}{\partial \mathcal{Z}_a} \mathcal{Z}_b \frac{\partial}{\partial \mathcal{Z}_b} \dots \right) - (a \rightarrow b)$$

Yangian



Claim

- On any compact contour A $\int_{A} dz$ gives Yangian invariants \mathcal{D} .

Claim

- On any compact contour A ^{loop} gives Yangian invariants \mathcal{D} .
- Anomaly comes from the choice of contour,

Claim

- On any compact contour $A^{\text{L-loop}}$ gives Yangian invariants \mathcal{D} .

- Anomaly comes from the choice of contour,

Q: Wouldn't be gr $A^{\text{L-loop}} = \int d\ell_1 d\ell_2$ (

Claim

- On any compact contour $A^{\text{L-loop}}$ gives Yangian invariants \mathcal{D} .

- Anomaly comes from the choice of contour,

Q: Wouldn't be great $A^{\text{L-loop}} = \int d\ell_1 d\ell_2$ (

Claim

- On any compact contour $A^{\text{L-loop}}$ gives Yangian invariants \mathcal{D} .

- Anomaly comes from the choice of contour,

Q: Wouldn't be great $A^{\text{L-loop}} = \int d\ell_1 d\ell_2$ (Total derivative)

Claim

• On any compact contour $A^{\text{L-loop}}$ gives Yangian invariants \mathcal{D} .

• Anomaly comes from the choice of contour,

Q: Wouldn't be great $A^{\text{L-loop}} = \int d\ell_1 d\ell_2$ (

Total derivative

All-loop BCFW Recursion Relation

- \exists Integrand \rightarrow Rational function

All-loop BCFW Recursion Relation

• \exists Integrand \rightarrow Rational function

$$I_n \rightarrow I_n + z I_{n-1}$$

"

All-loop BCFW Recursion Relation

• \exists Integrand \rightarrow Rational Function

$$I_n \rightarrow I_n + z I_{n-1} \quad \left(\frac{dz}{z} M(z) = M(z) \right)$$



=

All-loop BCFW Recursion Relation

• \exists Integrand \rightarrow Rational function

$$\mathcal{I}_n \rightarrow \mathcal{I}_n + z \mathcal{I}_{n-1}$$

$$\int \frac{dz}{z} M(z) = M(0)$$

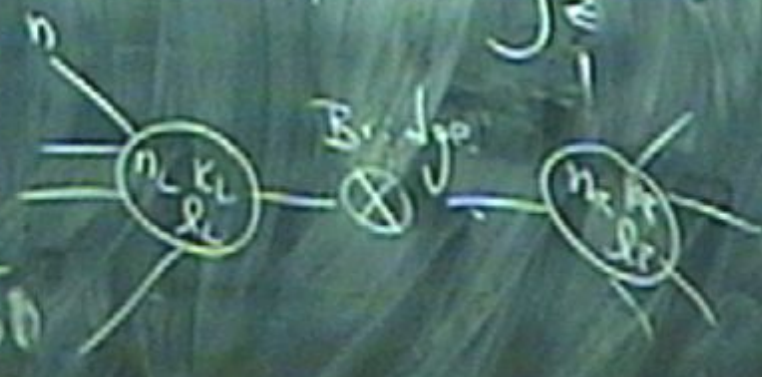


All-loop BCFW Recursion Relation

• \exists Integrand \rightarrow Rational function

$$I_n \rightarrow I_n + z I_{n-1}$$

$$\oint \frac{dz}{z} M(z) = M(0)$$



+ Forward Limit

$$\text{Forward Limit} = \int [A \otimes \dots \otimes A] M$$



Forward Limit = $\int_{GL(2)}$ ^{Loop variable} $M_{(1, \dots, \hat{n}_{AB}, \hat{A}, \hat{B})}$

$$\hat{n}_{AB} = (n-1, n) \cap (A, B)$$

$$\hat{A} = (A, B) \cap (n-1, n)$$

Forward Limit = $\int_{GL(2)} [AB_{n-1, n, 1}] \mathcal{M}_{n+2, k+1, l-1}^{(1, \dots, \hat{n}_{AB}, \hat{A}, \hat{B})}$ Loop variable

$$\hat{n}_{AB} = (n-1, n) \cap (A, B, \downarrow)$$

$$\hat{A} = (A, B) \cap (n-1, n, \uparrow)$$