Title: Wrap-up Talk

Date: Aug 06, 2011 12:15 PM

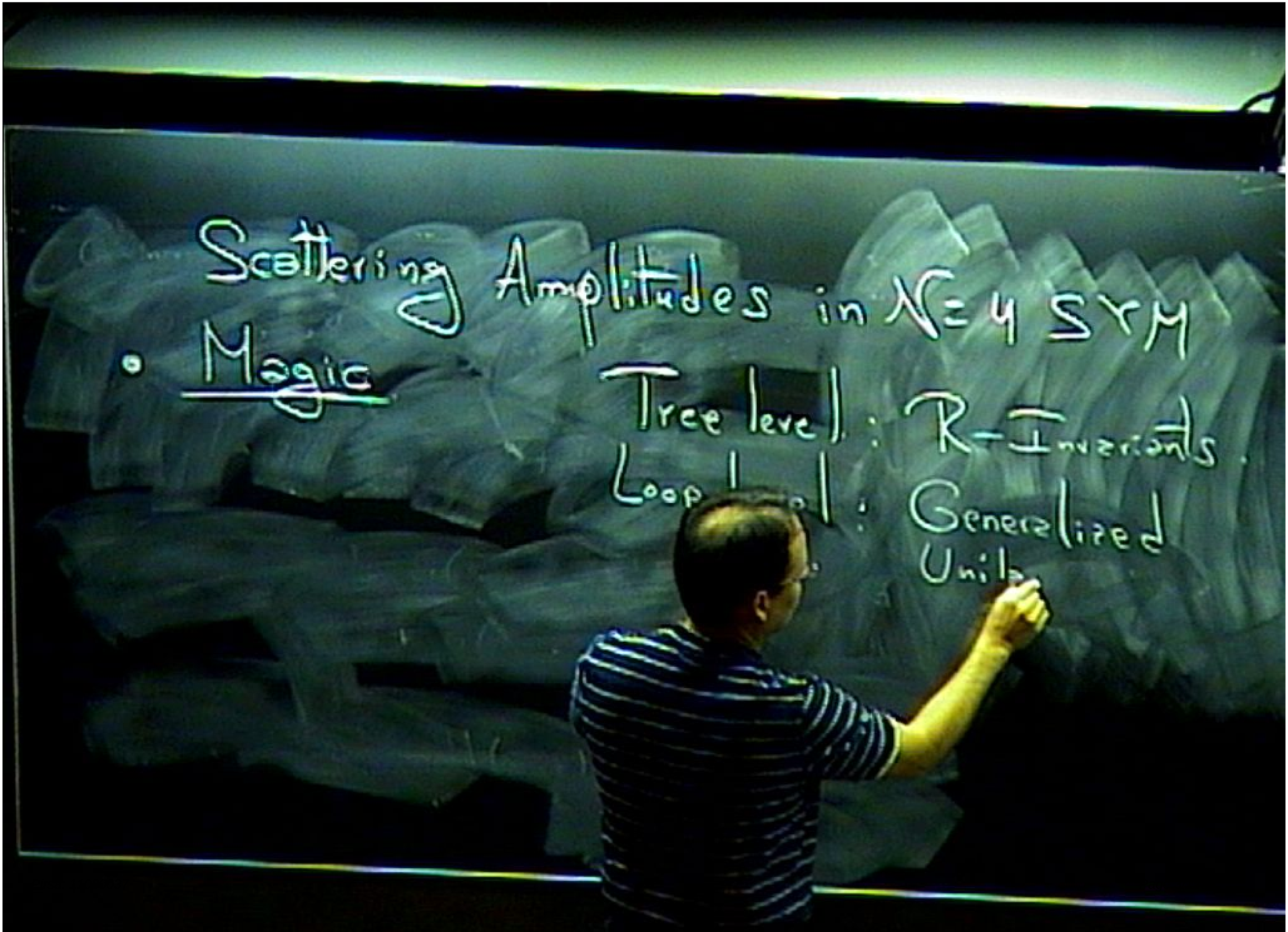
URL: http://pirsa.org/11080037

Abstract:

Scattering Amplitudes in $\mathcal{N}=4$ SYM

- <u>Magic</u>

Tree level: R-Invariants


Scattering Amplitudes in $\mathcal{N}=4$ SYM
• Magic
Tree level: R-Invariants
Loop : Generalized
Unit

Scattering Amplitudes in $\mathcal{N}=4$ SYM

- **Magic**

  Tree level : R-Invariants

  Loop level : Generalized Unitarity

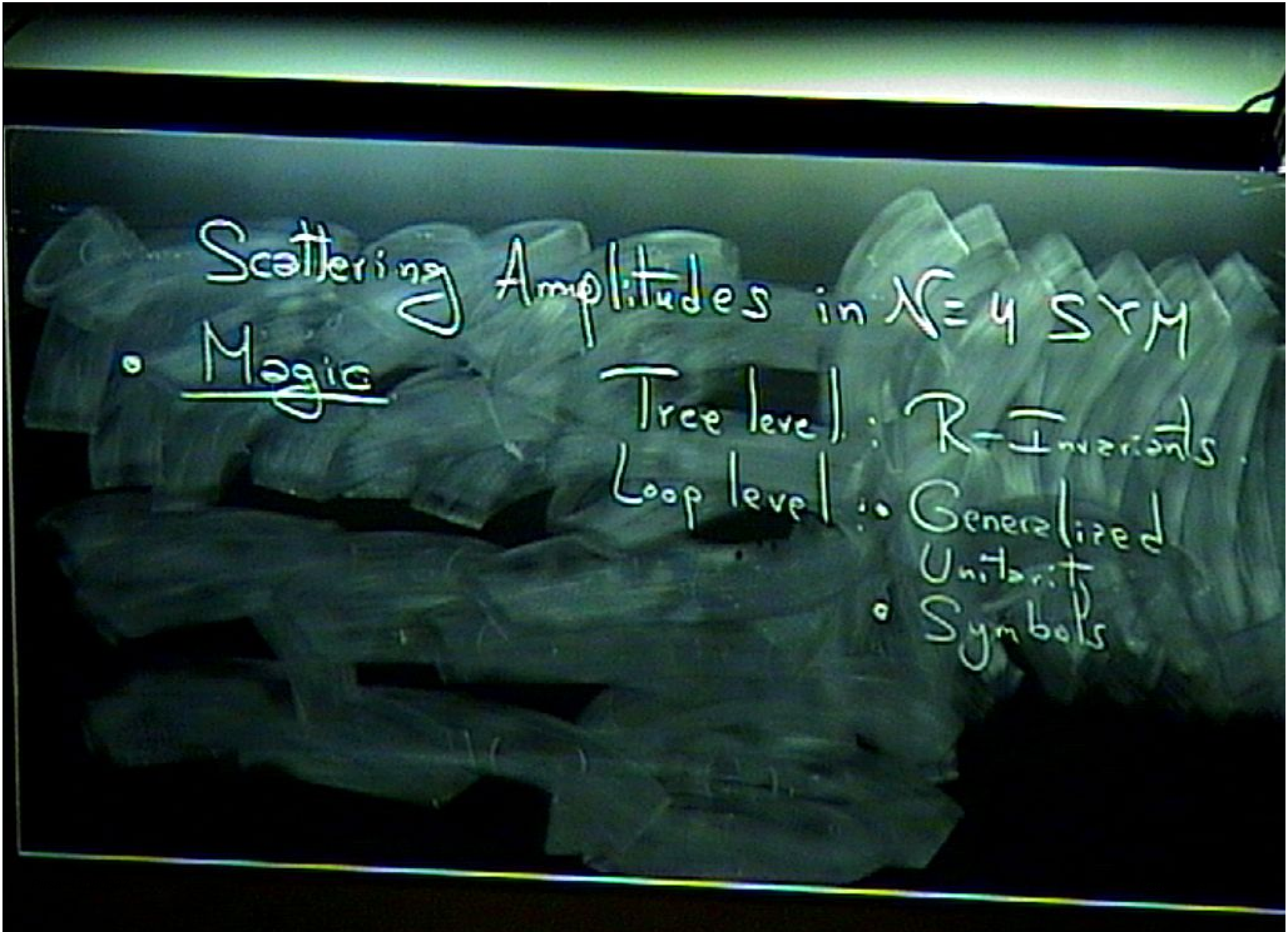
Scattering Amplitudes in $\mathcal{N}=4$ SYM

- Magic

Tree level : R-Invariants

Loop level : • Generalized Unitarity

• Symbols


Scattering Amplitudes in $\mathcal{N}=4$ SYM
• Magic
Tree level: R-Invariants
Loop level: • Generalized Unitarity
• Symbols

R-Invariants...

$$[abcde] = \frac{\int^{0|4} \left(\langle abcd \rangle \eta_e + \dots \right)}{\langle abcd \rangle \langle bcde \rangle \dots}$$

R-Invariants.

$$[abcde] = \frac{\int^{0|4} \left(\langle abcd \rangle \eta_e + \ldots \right)}{\langle abcd \rangle \langle bcde \rangle \ldots}$$

$$A_6^{NMHV} = A_6^{MHV} \left([12345] + [12356] + [13456] \right)$$

R-Invariants

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4-brackets $\rightarrow$ Dual Conformal Invariant

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4-brackets $\rightarrow$ Dual Conformal Invariant $\quad SL(4)$

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$$[abcde] = \frac{\int^{0|4} \left(\langle abcd \rangle \eta_e + \ldots \right)}{\langle abcd \rangle \langle bcde \rangle \ldots}$$

...

$$A_6^{NMHV} = A_6^{MHV} \left([12345] + [12356] + [13456] \right)$$

4-brackets $\rightarrow$ Dual Conformal Invariant $\quad SL(4)$

Dual Super Cor ″ ″

R-Invariants..

$$[abcde] = \frac{\int^{0|4} \left(\langle abcd\rangle \eta_e + \dots\right)}{\langle abcd\rangle\langle bcde\rangle \dots} = \frac{\int^{0|4}\left(t_e\eta_e + t_a\eta_a + t_b\eta_b + \dots\right)}{t_a t_b t_c t_d t_e}$$

$t_a = \langle\ \rangle$

$t_b = \langle$

$$A_6^{NMHV} = A_6^{MHV}\left([12345]+[12356]+[13456]\right)$$

4-brackets → Dual Conformal Invariant $SL(4)$

Dual Super Conf " " $SL(4|4)$

R-Invariants..

$$[abcde] = \frac{\int^{0|4}\left(\langle abcd\rangle \eta_e + \dots\right)}{\langle abcd\rangle\langle bcde\rangle \dots} = \frac{\int^{0|4}\left(t_e\eta_e + t_a\eta_a + t_b\eta_b + \dots\right)}{t_a t_b t_c t_d t_e}$$

$t_a \to \langle\ \rangle$

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$$A_6^{NMHV} = A_6^{MHV}\left([12345] + [12356] + [13456]\right)$$

4-brackets $\to$ Dual Conformal Invariant $\quad SL(4)$

Dual Super Conformal " " $\quad SL(4|4)$

$$\mathcal{Z}_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix}$$

$$\int^{4|4} \left(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \ldots + t_e \mathcal{Z}_e \right)$$

$$\mathcal{Z}_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix}$$

$$\frac{\quad}{t_a t_b \dots t_e} \delta^{4|4}\left(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \dots + t_e \mathcal{Z}_e\right)$$

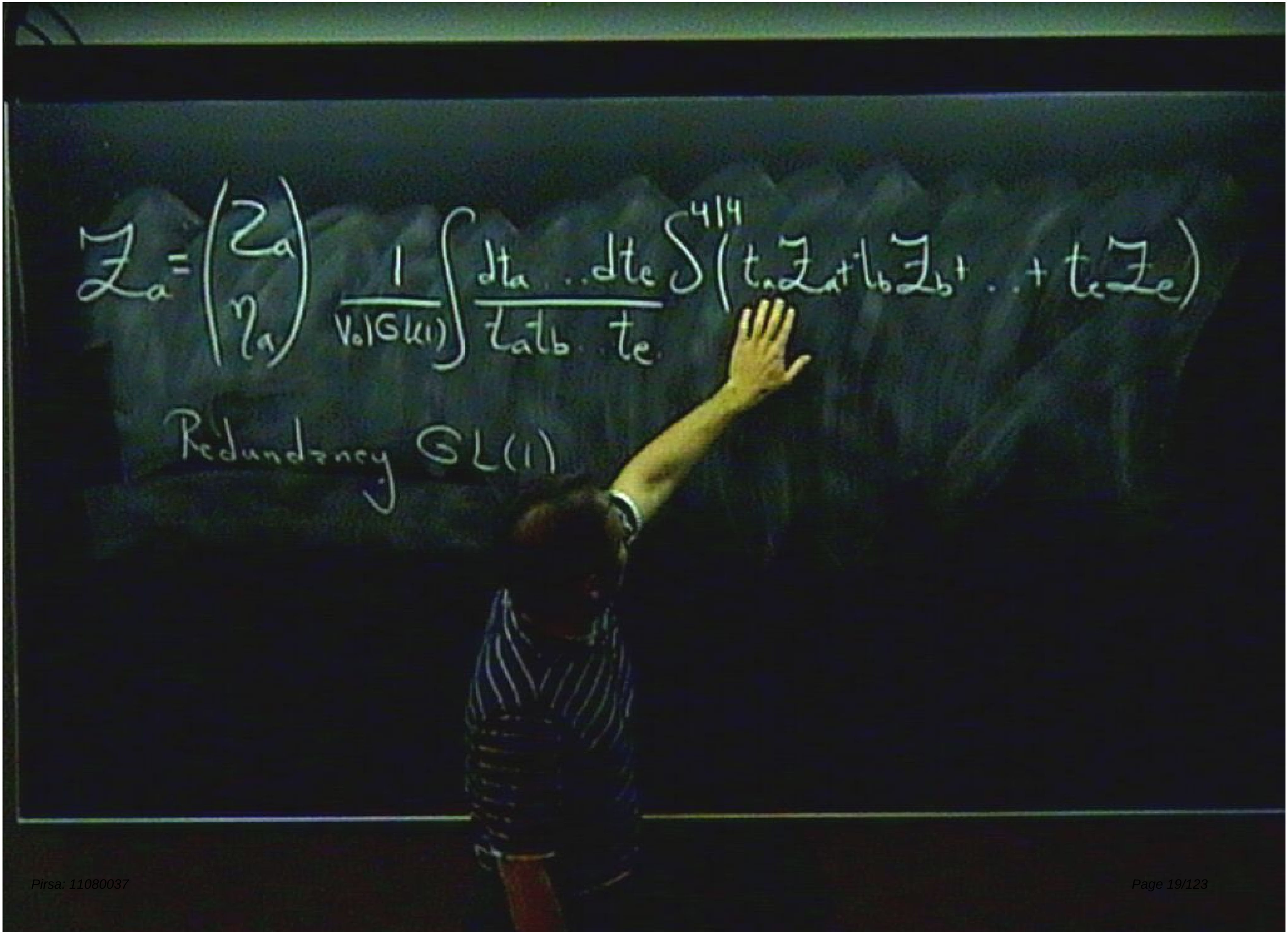
$$\mathcal{Z}_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix} \qquad \int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e} \delta^{4|4}(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \ldots + t_e \mathcal{Z}_e)$$

$$\mathcal{Z}_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix} \qquad \int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e} \, \delta^{4|4}\left(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \ldots + t_e \mathcal{Z}_e\right)$$

Redundancy GL(1)

$$\mathcal{Z}_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix} \frac{1}{\mathrm{Vol}\,G\!L(1)} \int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e} \delta^{4|4}\left(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \ldots + t_e \mathcal{Z}_e\right)$$

Redundancy $GL(1)$


$\mathcal{Z}_a = \begin{pmatrix} Z_a \\ \eta_a \end{pmatrix} \frac{1}{\mathrm{Vol}\, GL(1)} \int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e} \delta^{4|4}(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \ldots + t_e \mathcal{Z}_e)$
Redundancy GL(1)

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Redundancy $GL(1)$

$$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))} \int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e} \delta^{4|4}(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \ldots + t_e \mathcal{Z}_e)$$

Redundancy $GL(1)$

G.F.: $t_a = 1$.

$$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))} \int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e} \delta^{4|4}(t_a Z_a + t_b Z_b + \ldots + t_e Z_e)$$

Redundancy $GL(1)$ $\qquad (t_a, t_b, \ldots, t_e)$ $\in$ $\mathbb{C}^5$

G.F.: $t_a = 1$.

$$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))}\int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e}\, \delta^{4|4}(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \ldots + t_e \mathcal{Z}_e)$$

Redundancy $GL(1)$ $\qquad (t_a, t_b, \ldots, t_e) \sim \rho (t_a, t_b, \ldots, t_e)$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad \in \mathbb{C}^*$

G.F: $t_a = 1$.

$$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))}\int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e}\,\delta^{4|4}\left(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \ldots + t_e \mathcal{Z}_e\right)$$

Redundancy $GL(1)$

G.F.: $t_a = 1$.

$$\underbrace{\underset{\mathbb{C}^5}{\overset{}{(t_a, t_b, \ldots, t_e)}} \sim \rho(t_a, t_b, \ldots, t_e)}_{\mathbb{CP}^4}$$

$$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))} \int \frac{dt_a \dots dt_e}{t_a t_b \dots t_e} \delta^{4|4}(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \dots + t_e \mathcal{Z}_e)$$

Redundancy $GL(1)$

$$\underbrace{(t_a, t_b, \dots, t_e) \in \mathbb{C}^5}_{\mathbb{CP}^4} \sim \rho(t_a, t_b, \dots, t_e)$$

G.F: $t_a = 1$.

$$\int \frac{dt_b \dots dt_e}{t_b \dots t_e}$$

$$[abcde] = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_a \cdots dt_e}{t_a t_b \cdots t_e} \, \delta^{4|4}(t_a \mathcal{Z}_a + t_b \mathcal{Z}_b + \cdots + t_e \mathcal{Z}_e)$$

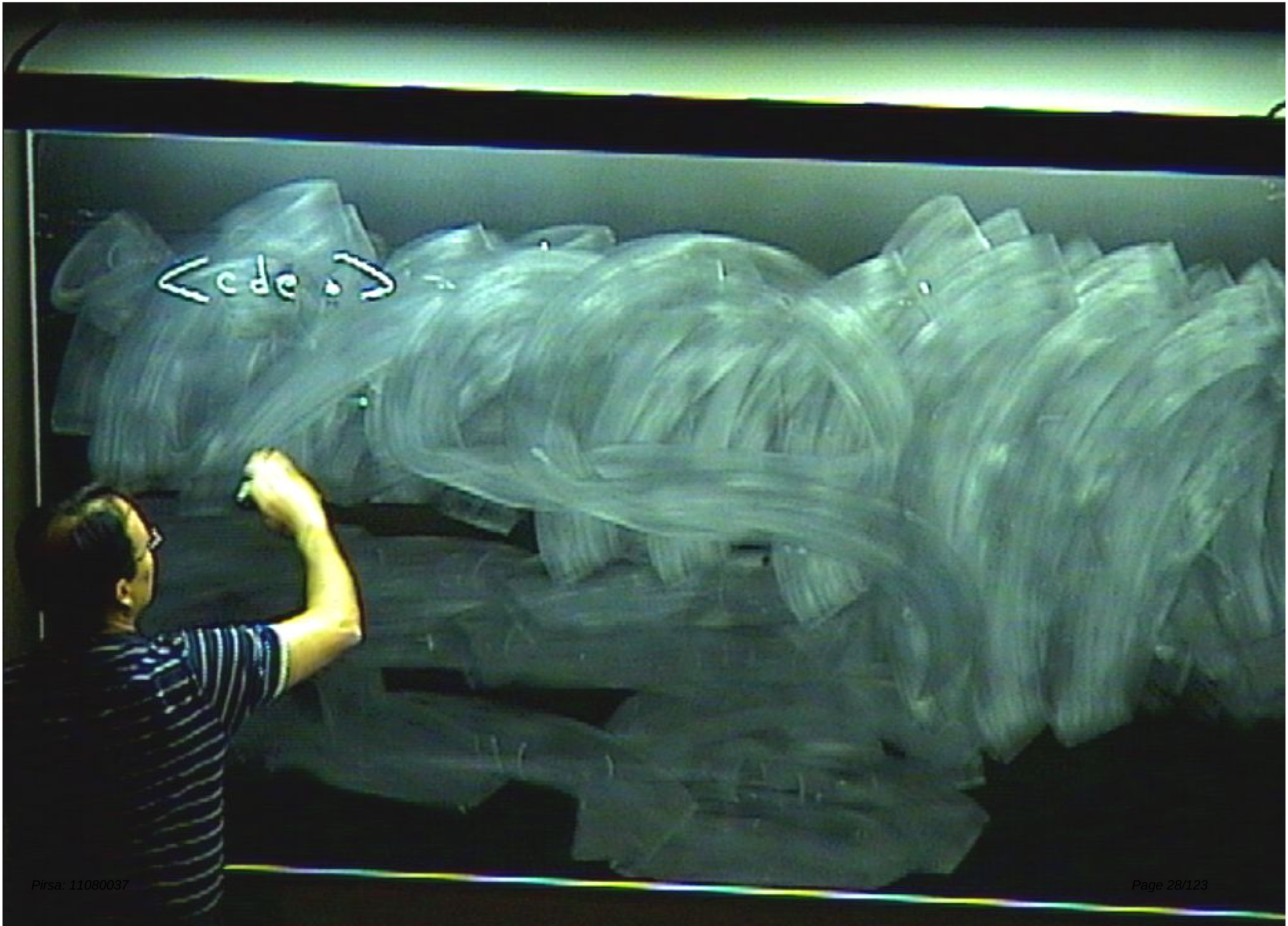
Redundancy $GL(1)$

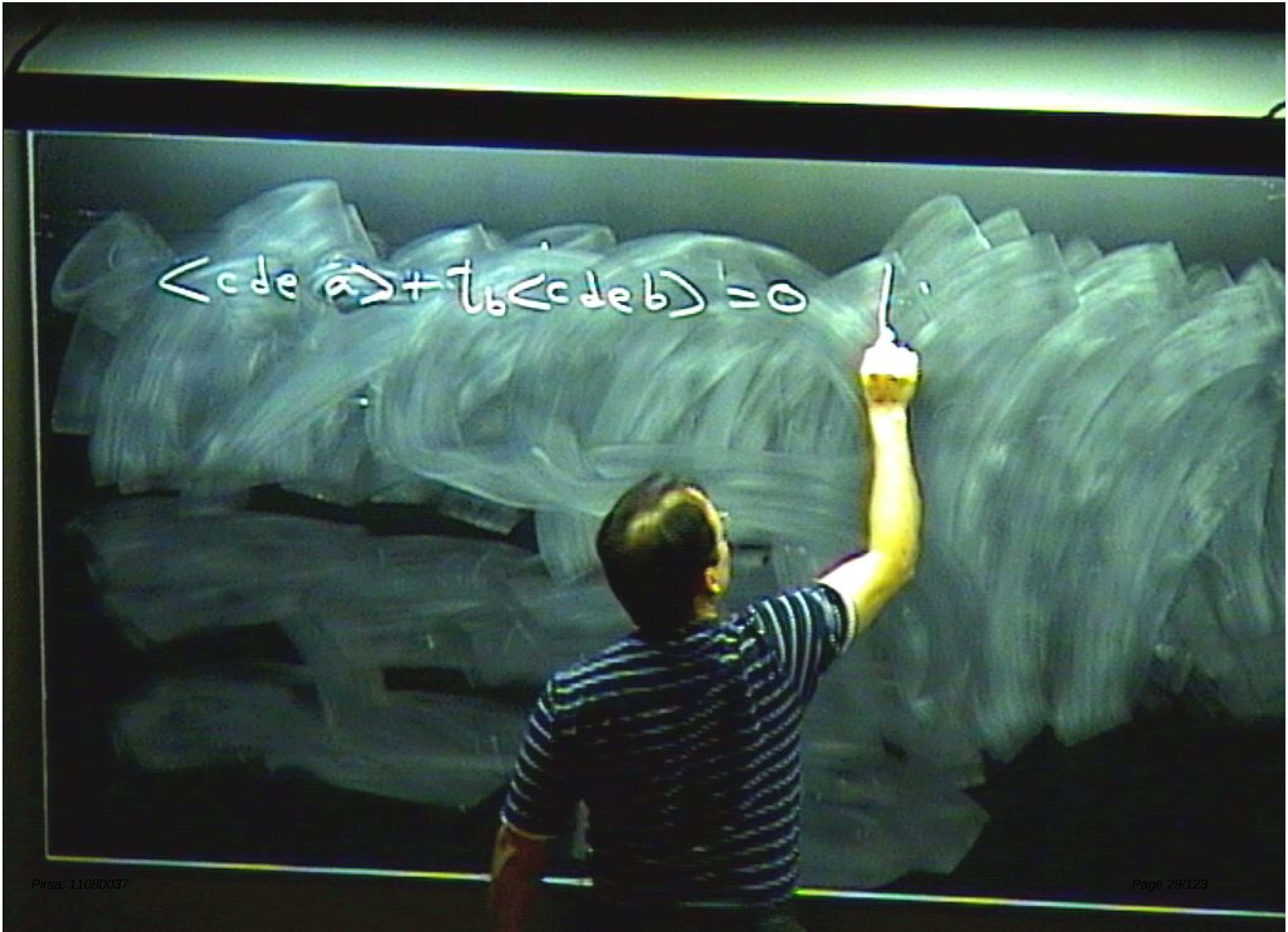
$$(t_a, t_b, \ldots, t_e) \sim \rho\,(t_a, t_b, \ldots, t_e)$$

$$\underbrace{\in \mathbb{C}^5}_{\mathbb{P}^4}$$

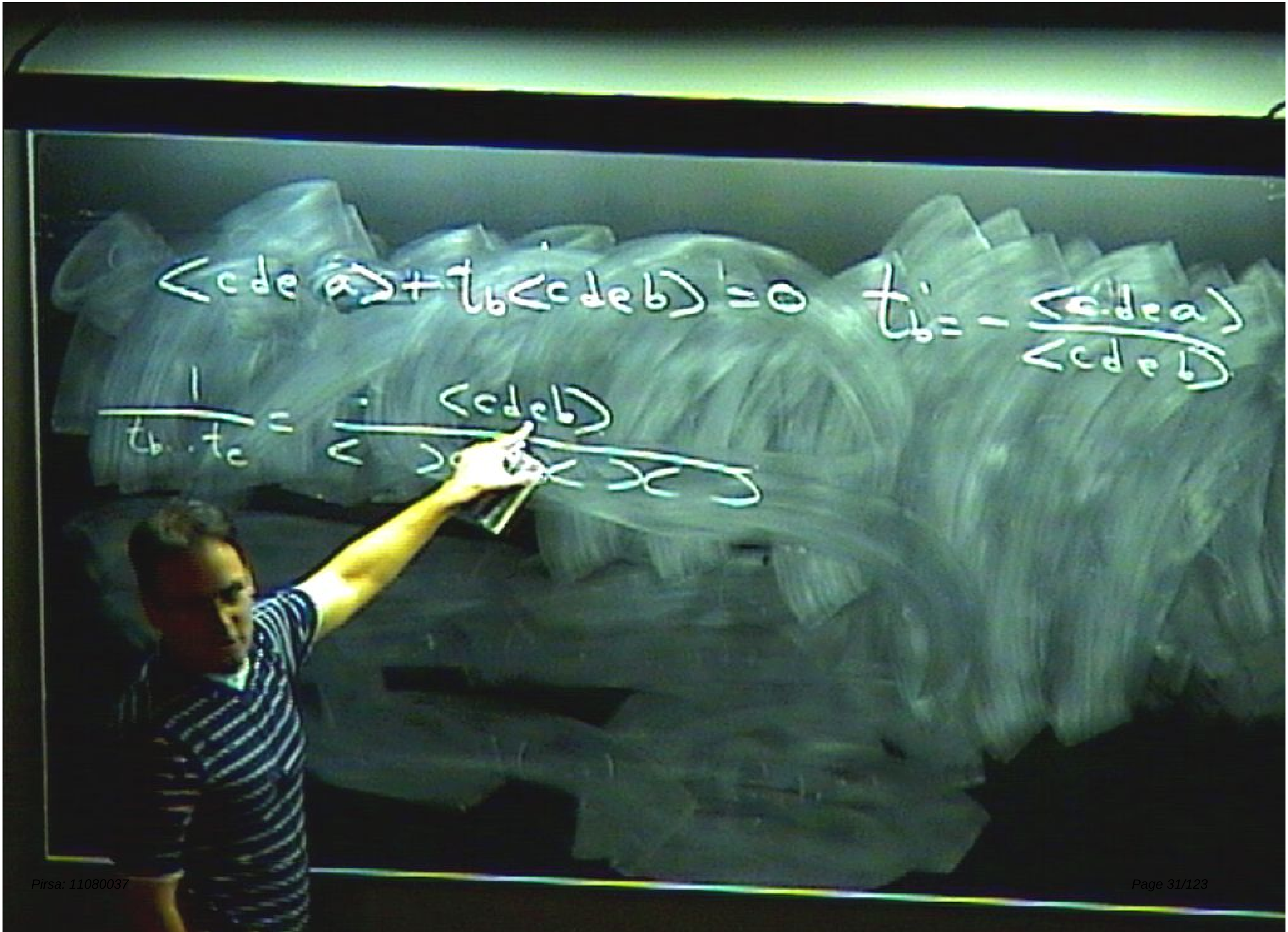
G.F.: $t_a = 1$.

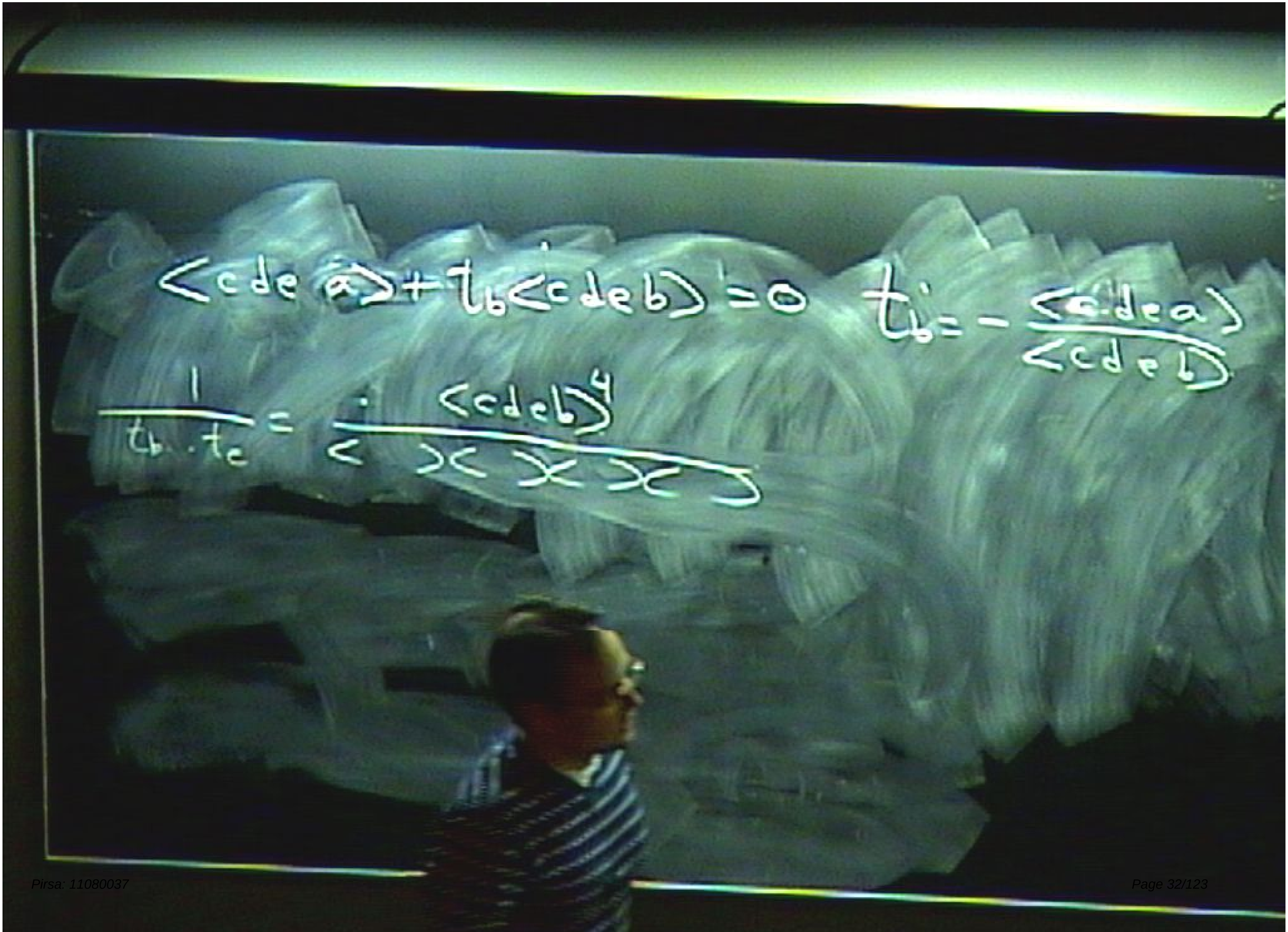
$$\int \frac{dt_b \cdots dt_e}{t_b \cdots t_e} \, \delta^{0|4}(\eta_a + \cdots + t_e \eta_e) \, \delta^{4}(Z_a + t_b Z_b + \cdots + t_e Z_e)$$

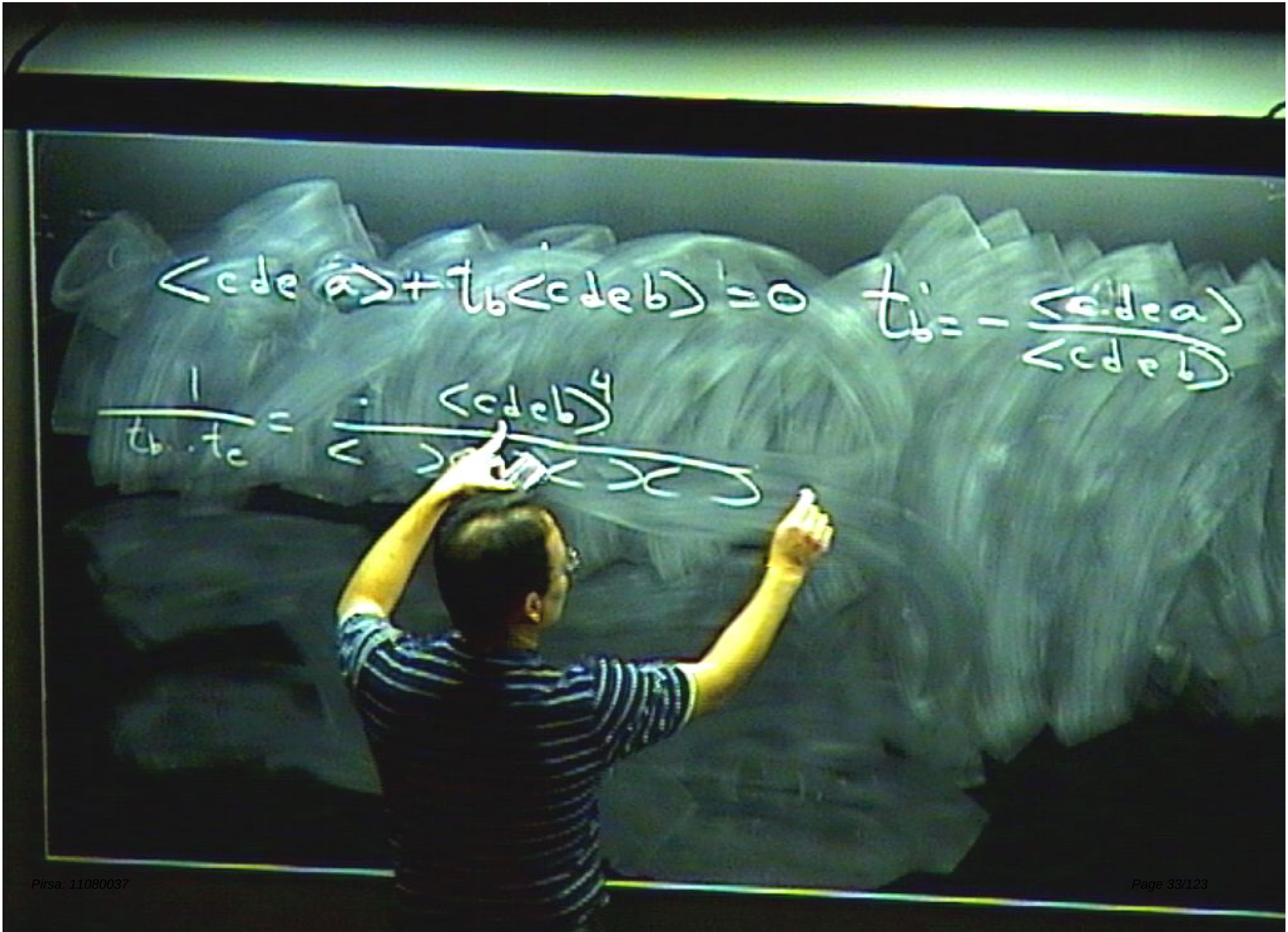


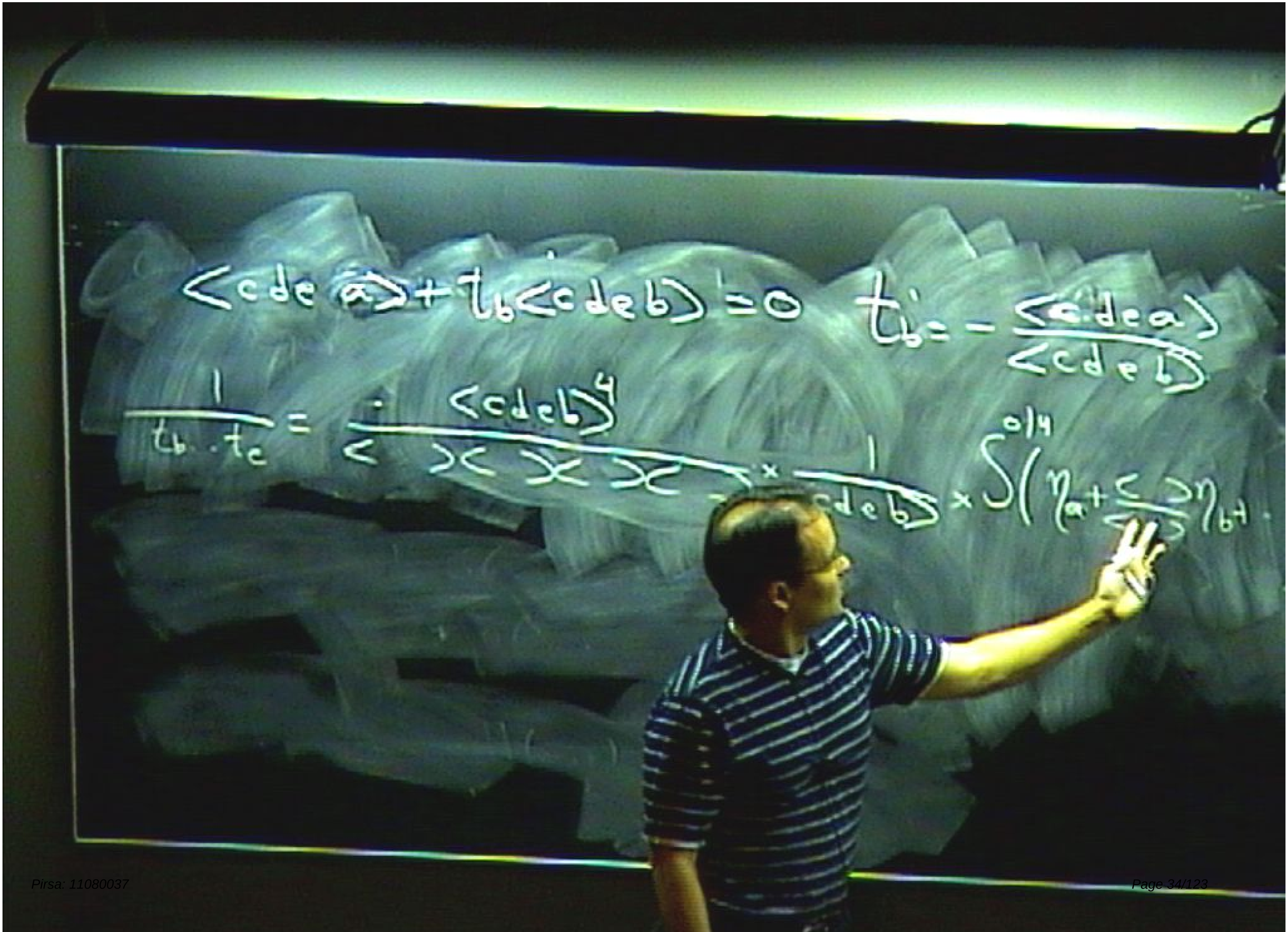

$\langle c\,d\,e\,a\rangle + \eta_b \langle c\,d\,e\,b\rangle = 0$

$$\langle c\,d\,e\,a\rangle + t_b\langle c\,d\,e\,b\rangle = 0 \qquad t_b = -\frac{\langle c\,d\,e\,a\rangle}{\langle c\,d\,e\,b\rangle}$$


$\langle c\,d\,e\,a\rangle + t_b \langle c\,d\,e\,b\rangle = 0$
$t_b = -\frac{\langle c\,d\,e\,a\rangle}{\langle c\,d\,e\,b\rangle}$
$\frac{1}{t_b \cdot t_c} = \frac{\langle c\,d\,e\,b\rangle}{\langle\ \rangle\langle\ \rangle\langle\ \rangle}$


$\langle cdea\rangle + t_b\langle cdeb\rangle = 0$
$t_b = -\frac{\langle cdea\rangle}{\langle cdeb\rangle}$
$\frac{1}{t_b \cdot t_c} = \frac{\langle cdeb\rangle^4}{\langle\;\rangle\langle\;\rangle\langle\;\rangle\langle\;\rangle}$


<cde a> + t_b <cdeb> = 0
t_b = - <cdea> / <cdeb>
1 / t_b · t_c = <cdeb>^4 / < >< >< >< >


<cdea> + t_b <cdeb> = 0
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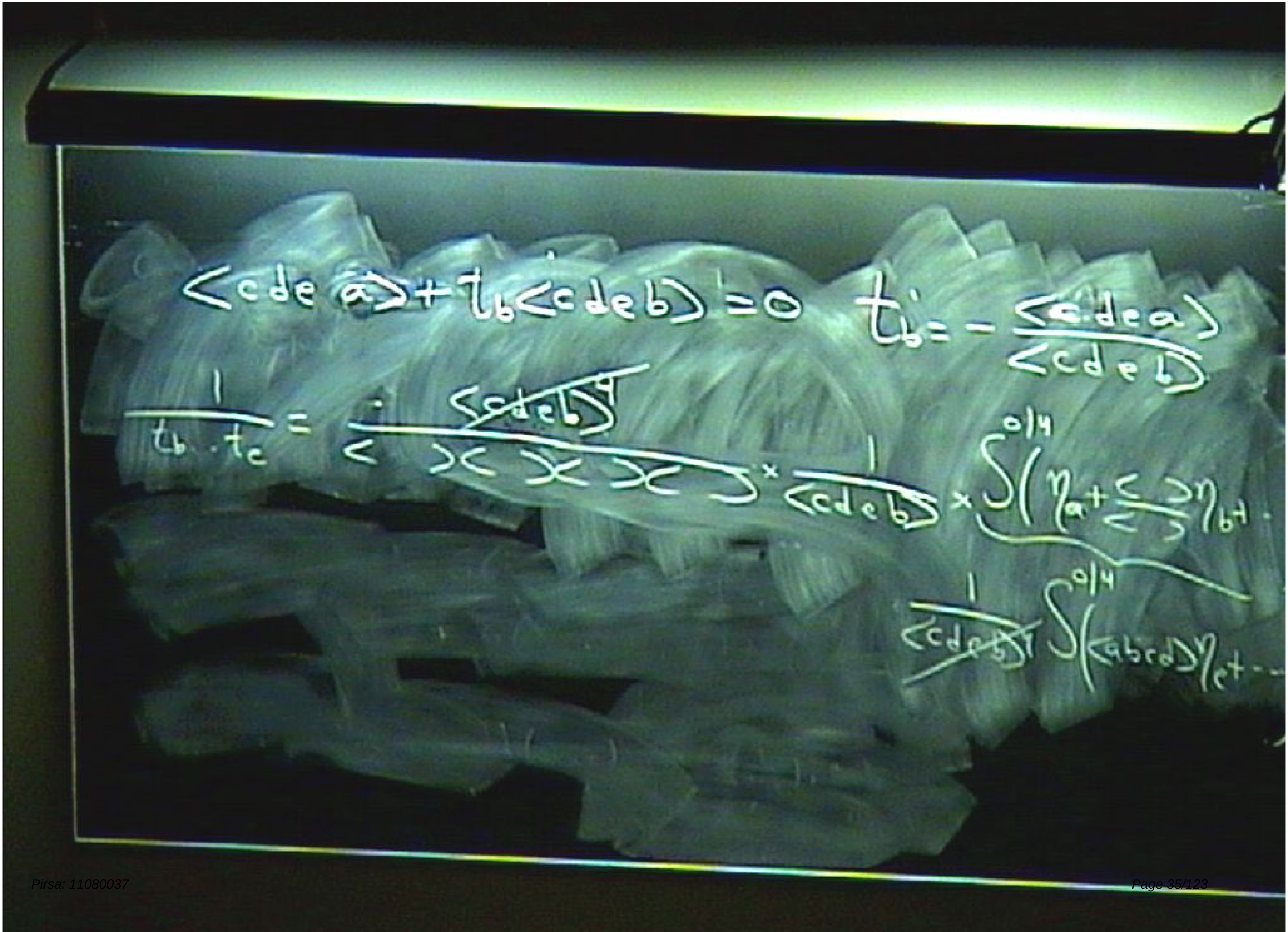
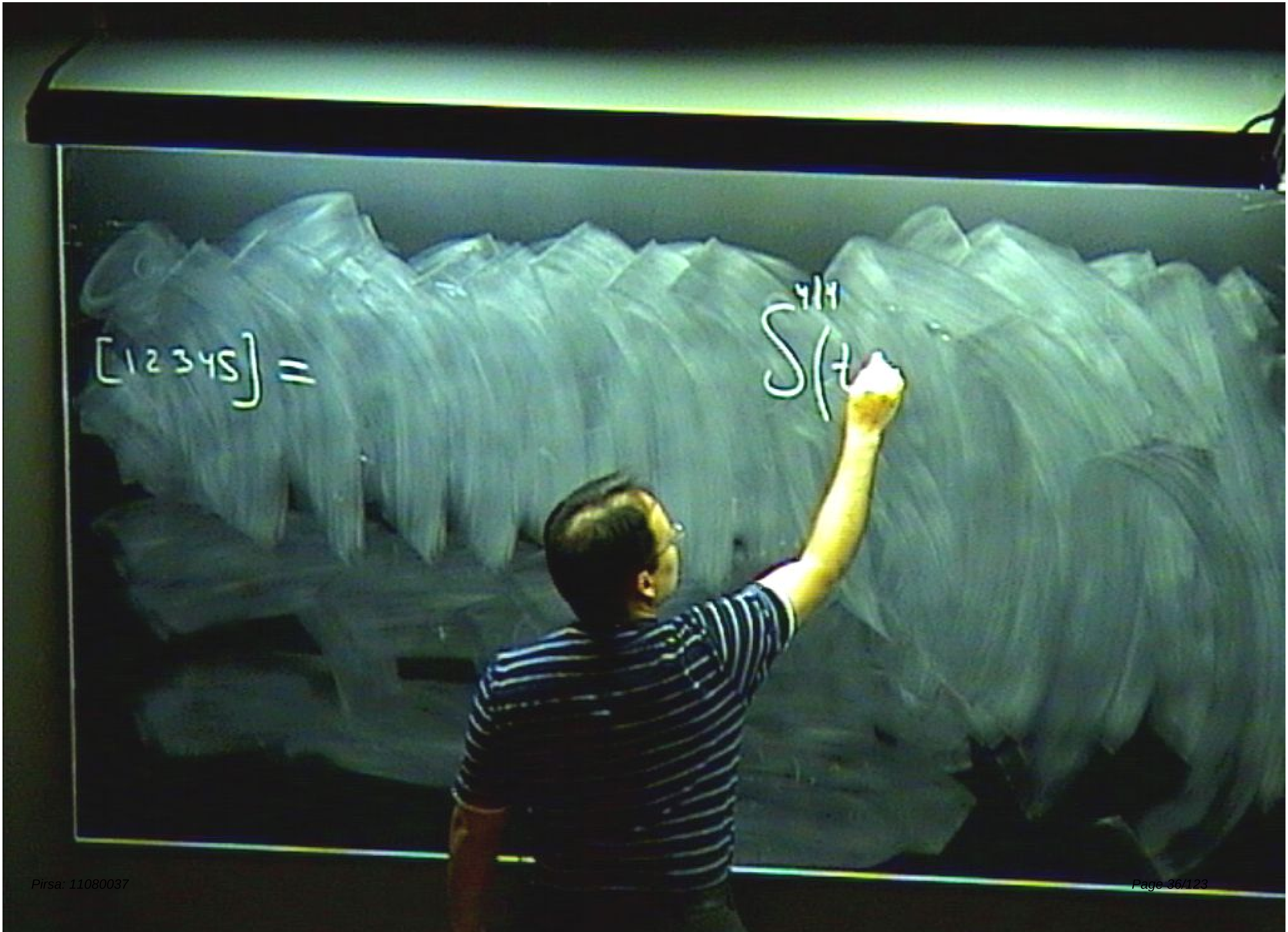

$\langle c d e a \rangle + t_b \langle c d e b \rangle = 0$
$t_b = -\frac{\langle c d e a \rangle}{\langle c d e b \rangle}$
$\frac{1}{t_b \cdot t_c} = \frac{\langle c d e b \rangle^4}{\langle \ \rangle \langle \ \rangle \langle \ \rangle \langle \ \rangle} \times \frac{1}{\langle c d e b \rangle} \times \int^{0|4} \left(\eta_a + \frac{\langle \ \rangle}{\langle \ \rangle} \eta_b + \cdots \right)$
$\frac{1}{\langle c d e b \rangle^4} \int^{0|4} \left(\langle a b c d \rangle \eta_e + \cdots \right)$


[12345] =

$$[12345] = \frac{1}{\mathrm{Vol}(GL(1))} \int_{|t_i| = \epsilon} \frac{dt_1\, dt_2 \ldots dt_6}{t_1\, t_2 \ldots t_6}\; \delta^{4|4}\left(t_1 Z_1 + t_2 Z_2 + \ldots + t_5 Z_5 + t_6 Z_6\right)$$

$=$

$$\mathcal{P}_6^{NMHV} = \frac{1}{\mathrm{vol}\,GL(1)} \oint_{|t_i|=\epsilon} \frac{dt_1\, dt_2 \cdots dt_6}{t_1\, t_2 \cdots t_6}\, \delta^{4|4}\left(t_1 \mathcal{Z}_1 + t_2 \mathcal{Z}_2 + \cdots + t_5 \mathcal{Z}_5 + t_6 \mathcal{Z}_6\right)$$

R-Invariants.

$$[abcde] = \frac{\int^{0|4} \left(\langle abcd\rangle \eta_e + \dots\right)}{\langle abcd\rangle\langle bcde\rangle \dots} = \frac{\int^{0|4} \left(t_e\eta_e + t_a\eta_a + t_b\eta_b + \dots\right)}{t_a t_b t_c t_d t_e}$$

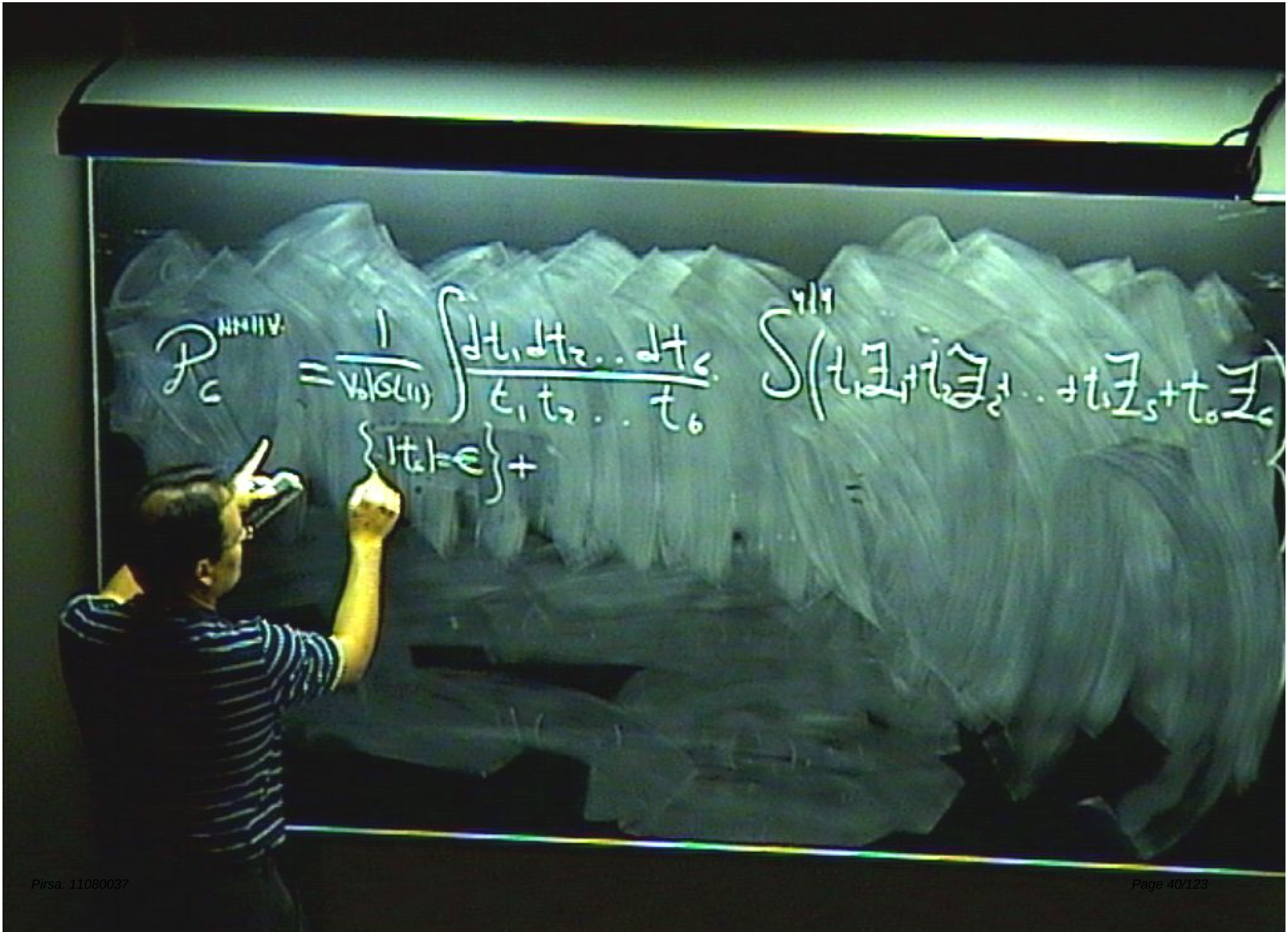
P_e

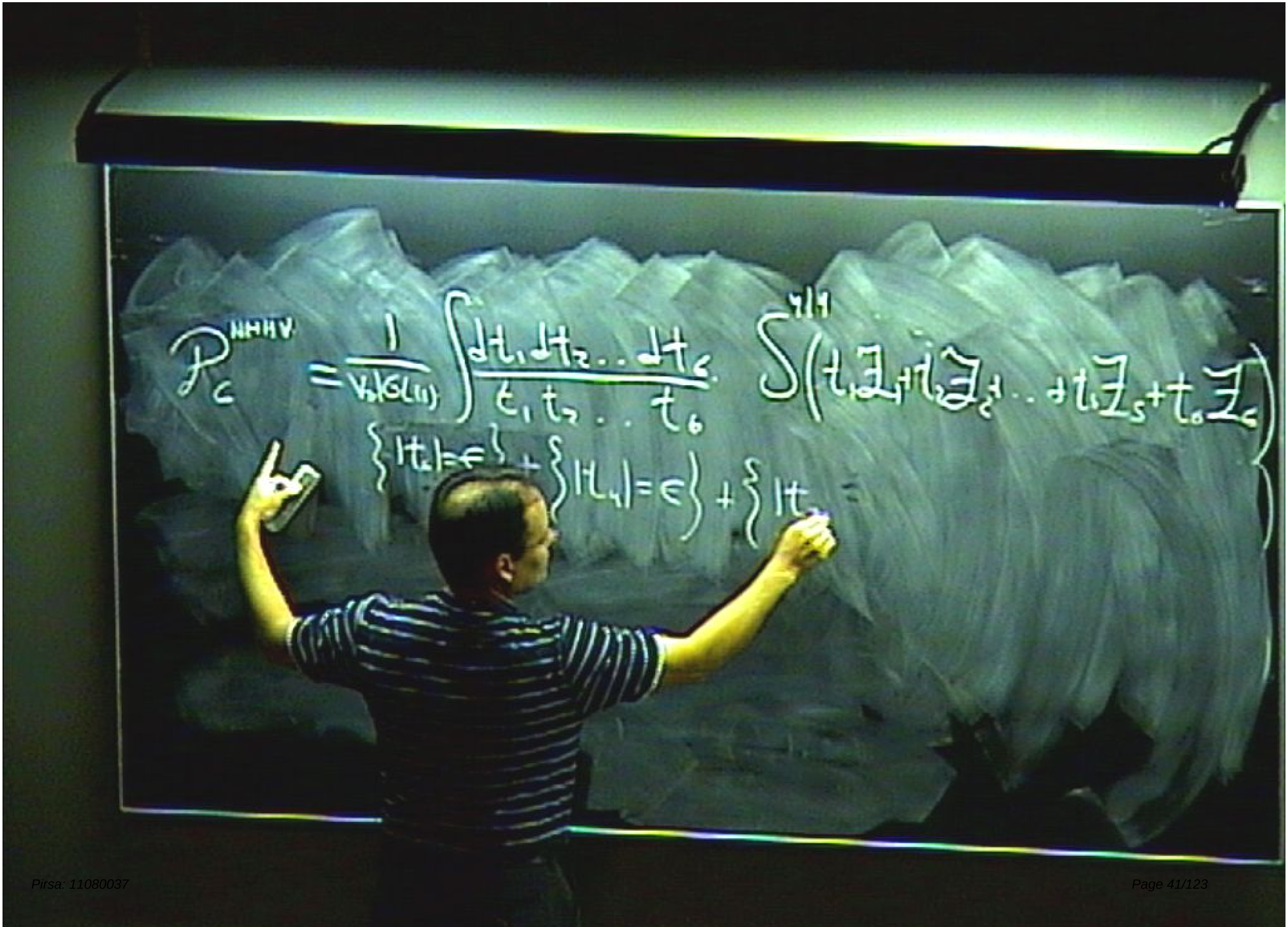
$t_a = \langle \;\rangle$
$t_b = \langle$

$$A_6^{NMHV} = A_6^{MHV}\left([12345] + [12356] + [13456]\right)$$

4-brackets → Dual Conformal Invariant SL(4)

Dual Superconformal " " SL(4|4)


P_c^NMHV = 1/Vol(GL(1)) ∫ dt_1 dt_2 .. dt_6 / t_1 t_2 .. t_6 δ^4|4(t_1 Z_1 + t_2 Z_2 + .. + t_5 Z_5 + t_6 Z_6)
{|t_6| = ε} +


{|t₆|=ε} + {|t₄|=ε} + {|t₂

$$\mathcal{P}_6^{NMHV} = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_1\, dt_2 \ldots dt_6}{t_1\, t_2 \ldots t_6}\; \delta^{4|4}\left(t_1 Z_1 + t_2 Z_2 + \ldots + t_5 Z_5 + t_6 Z_6\right)$$

$$\{|t_6| = \epsilon\} + \{|t_4| = \epsilon\} + \{|t_2| = \epsilon\}$$

$$\mathcal{P}_c^{HHHV} = \frac{1}{\mathrm{Vol}(GL(1))} \int \frac{dt_1\, dt_2 \cdots dt_6}{t_1\, t_2 \cdots t_6}\, \delta^{4|4}\left(t_1 \mathcal{Z}_1 + t_2 \mathcal{Z}_2 + \cdots + t_5 \mathcal{Z}_5 + t_6 \mathcal{Z}_6\right)$$

$$\{|t_6| = \epsilon\} + \{|t_4| = \epsilon\} + \{|t_2| = \epsilon\}$$

$$\mathcal{P}_c^{NMHV} = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_1\, dt_2 \cdots dt_6}{t_1\, t_2 \cdots t_6}\; \delta^{4|4}\left(t_1 Z_1 + t_2 Z_2 + \cdots + t_5 Z_5 + t_6 Z_6\right)$$

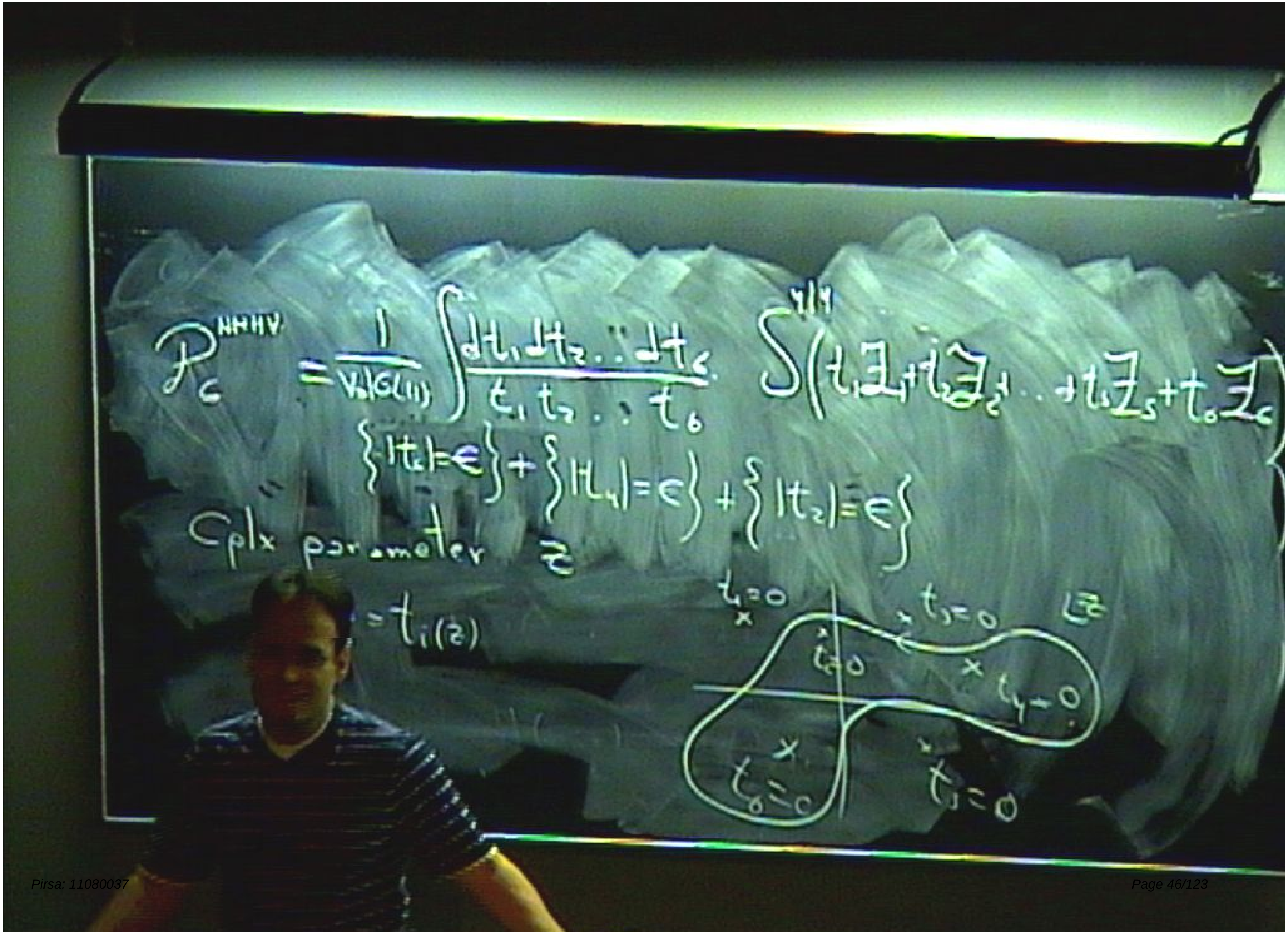
$$\{|t_6| = \epsilon\} + \{|t_4| = \epsilon\} + \{|t_2| = \epsilon\}$$

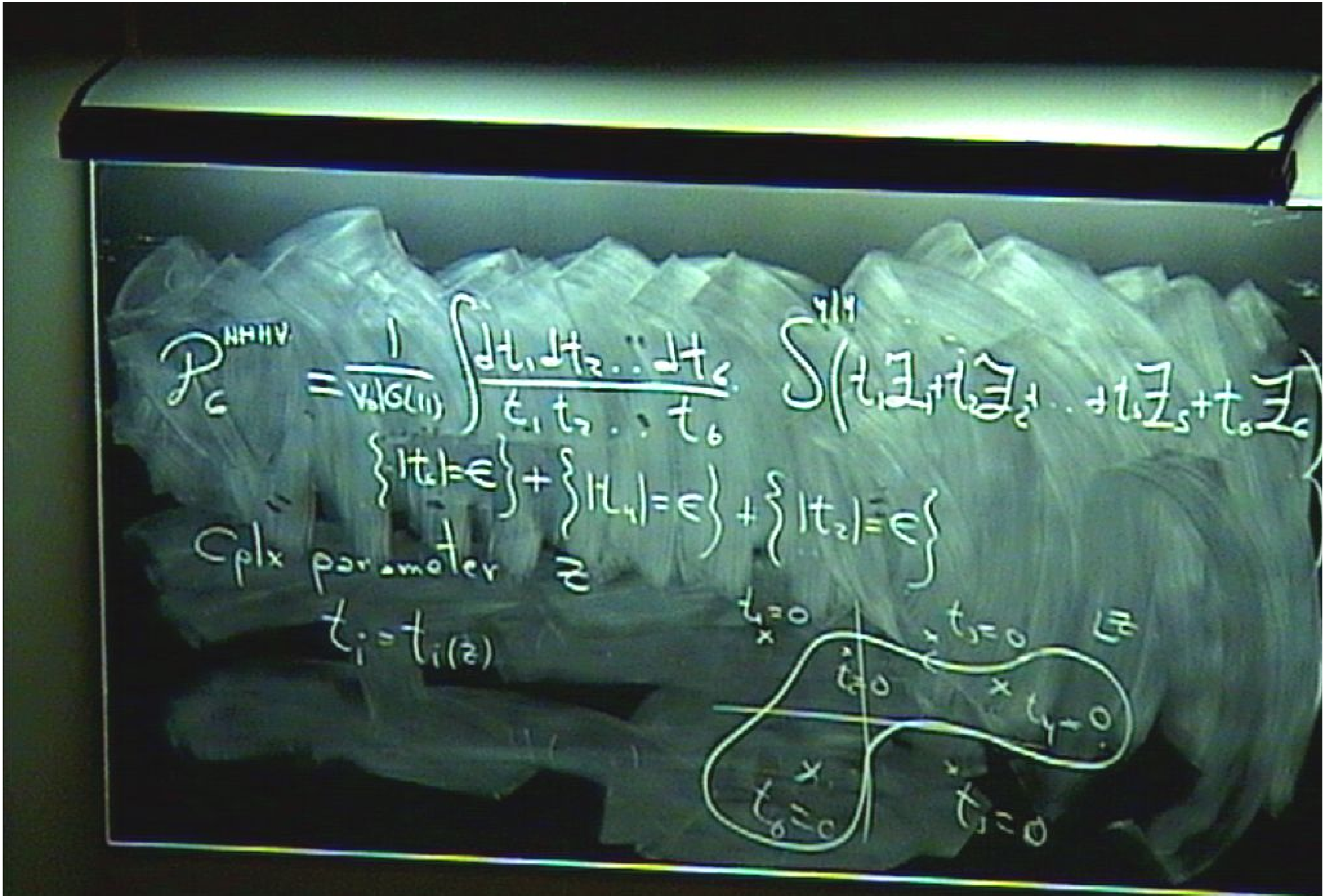
$$\mathcal{P}_c^{NMHV} = \frac{1}{\text{Vol}(GL(1))}\int \frac{dt_1\, dt_2 \cdots dt_6}{t_1\, t_2 \cdots t_6}\, \delta^{4|4}\left(t_1 \mathcal{Z}_1 + t_2 \mathcal{Z}_2 + \cdots + t_5 \mathcal{Z}_5 + t_6 \mathcal{Z}_6\right)$$

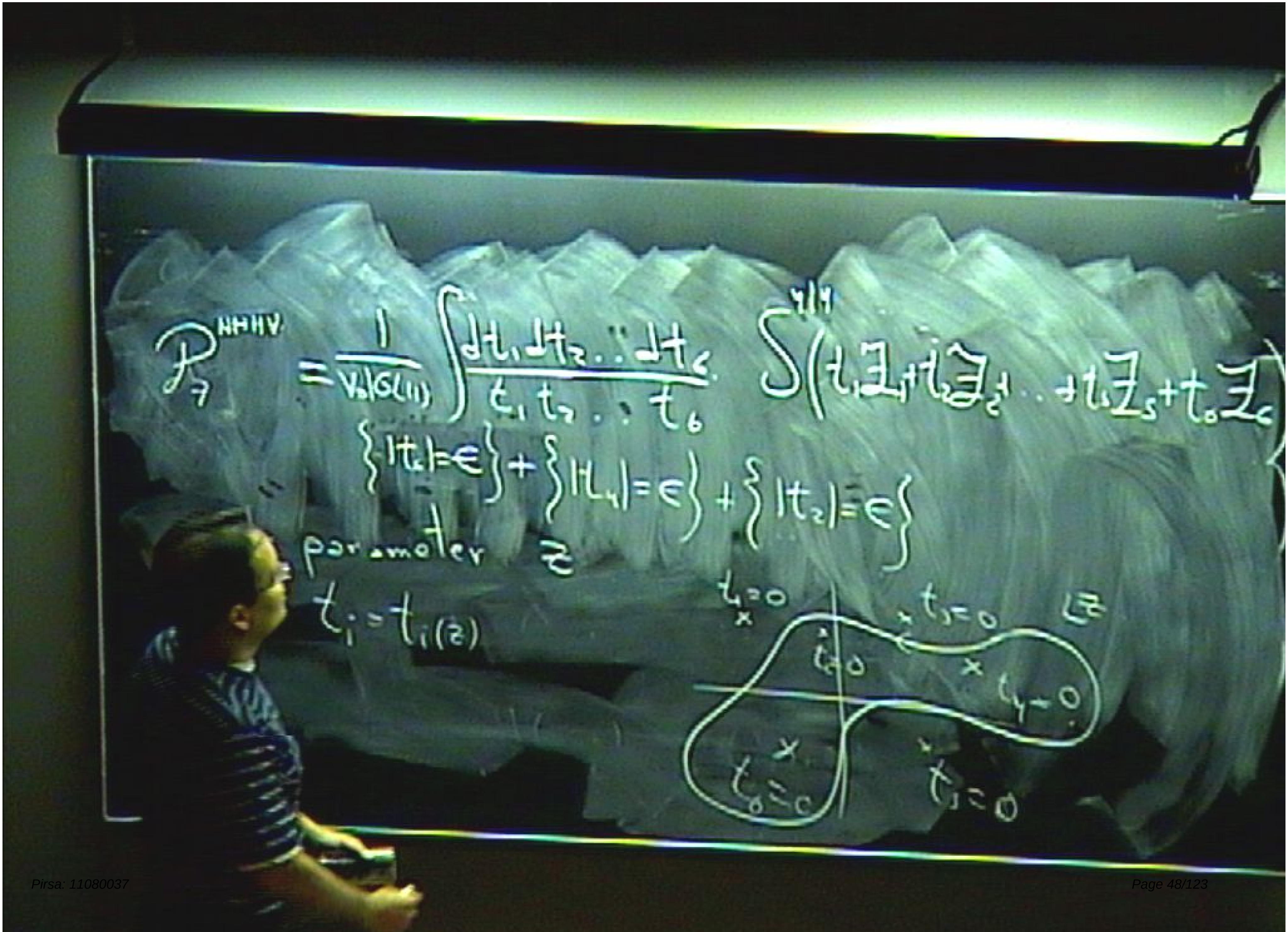
$$\{|t_6| = \epsilon\} + \{|t_4| = \epsilon\} + \{|t_2| = \epsilon\}$$

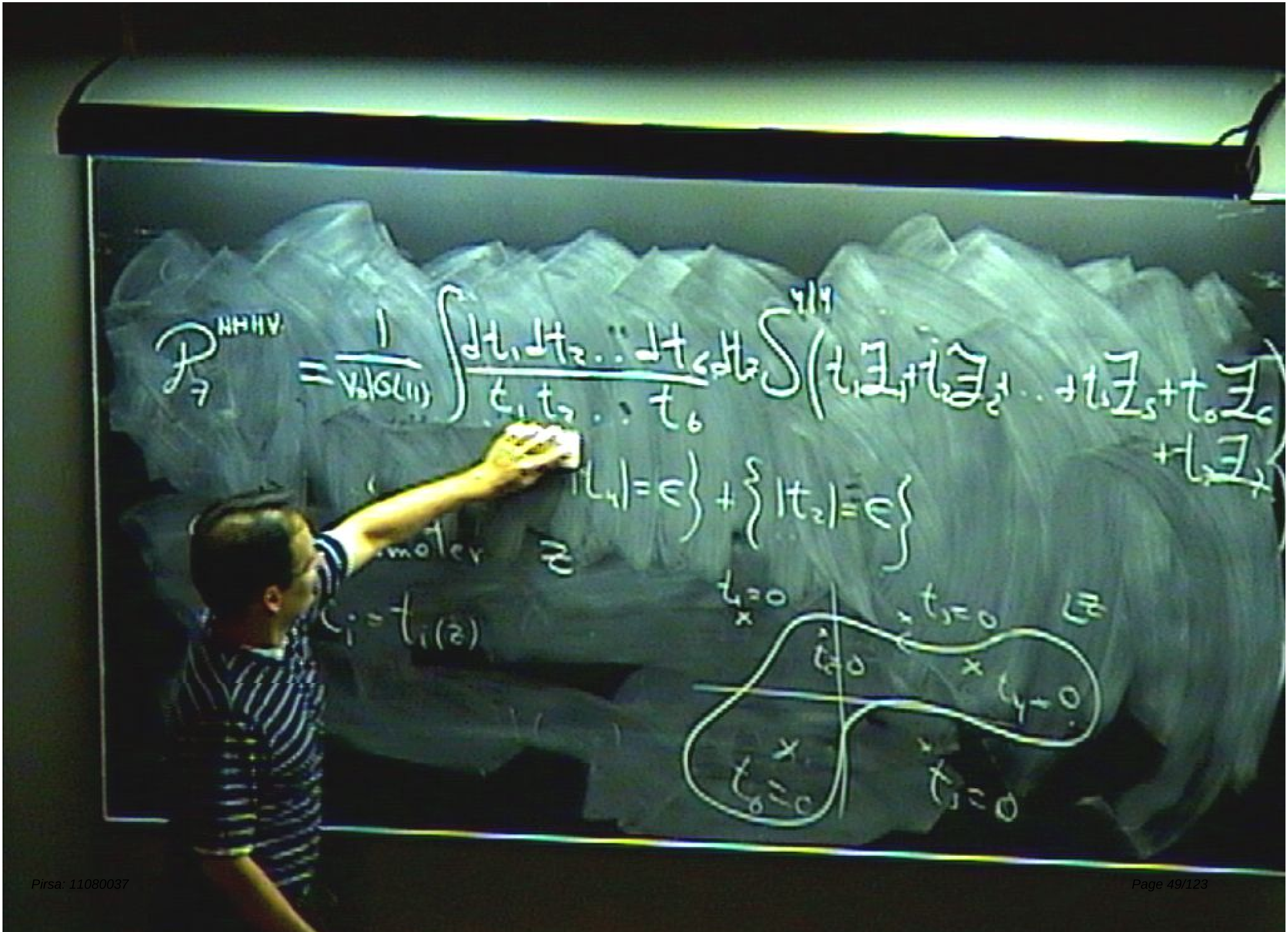
Cplx parameter z

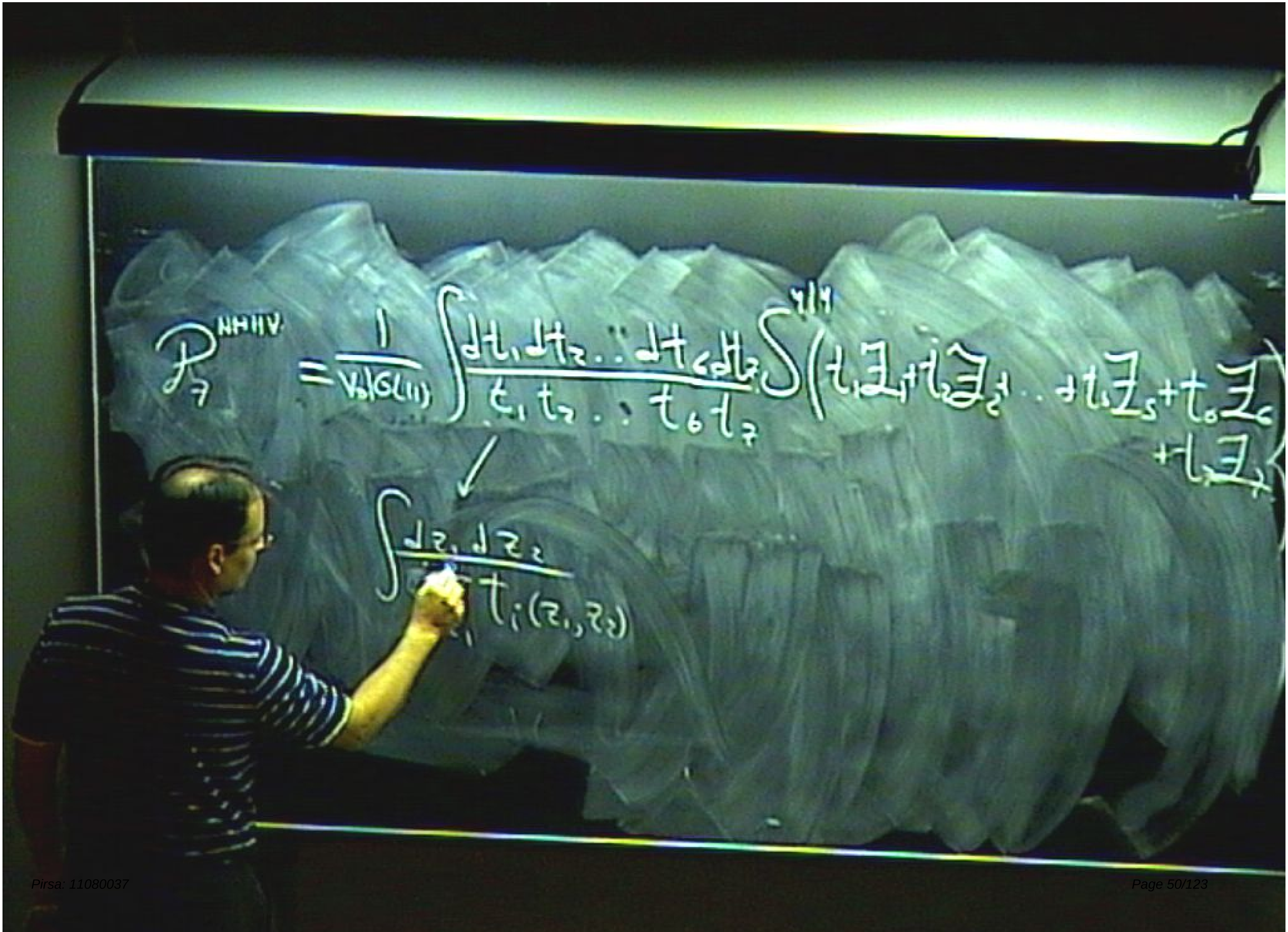
$$t_i = t_i(z)$$

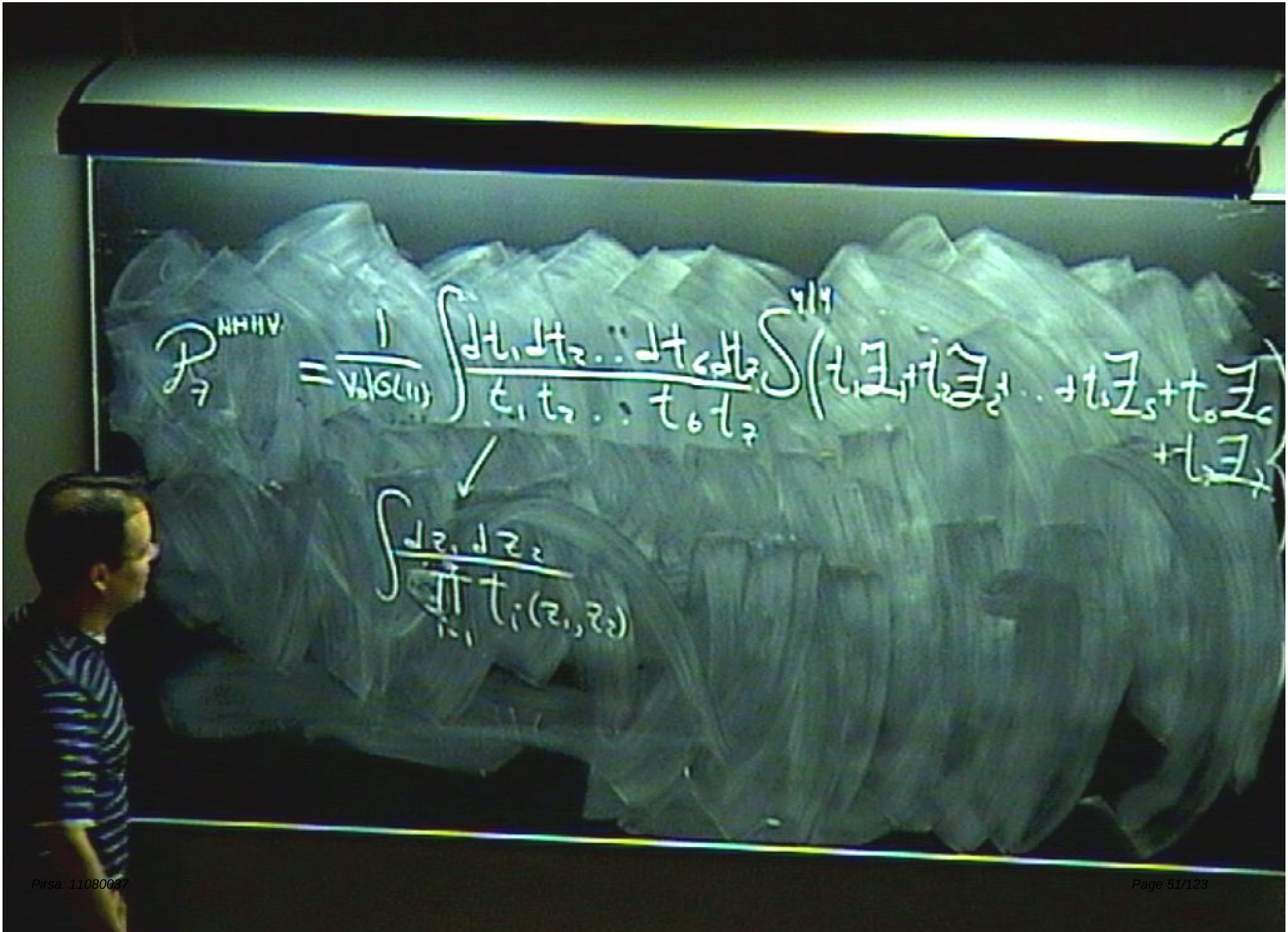

$\mathcal{P}_6^{NMHV} = \frac{1}{Vol(GL(1))} \int \frac{dt_1 dt_2 \ldots dt_6}{t_1 t_2 \ldots t_6} \delta^{4|4}(t_1 \mathcal{Z}_1 + t_2 \mathcal{Z}_2 + \ldots + t_5 \mathcal{Z}_5 + t_6 \mathcal{Z}_6)$
$\{|t_6| = \epsilon\} + \{|t_4| = \epsilon\} + \{|t_2| = \epsilon\}$
Cplx parameter z
$= t_i(z)$
$t_4=0$
$t_3=0$
$t_2=0$
$t_4=0$
$t_6=0$
$t_1=0$


$\mathcal{P}_6^{NMHV} = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_1\, dt_2 \ldots dt_6}{t_1\, t_2 \ldots t_6}\, \delta^{4|4}(t_1 Z_1 + t_2 Z_2 + \ldots + t_5 Z_5 + t_6 Z_6)$
$\{|t_6| = \epsilon\} + \{|t_4| = \epsilon\} + \{|t_2| = \epsilon\}$
Cplx parameter z
$t_i = t_i(z)$
$t_4 = 0$
$t_3 = 0$
z
$t_2 = 0$
$t_4 = 0$
$t_6 = 0$
$t_1 = 0$


$P_7^{NHHV} = \frac{1}{Vol(GL(1))} \int \frac{dt_1 dt_2 \cdots dt_6}{t_1 t_2 \cdots t_6} \delta^{4|4}(t_1 Z_1 + t_2 Z_2 + \cdots + t_5 Z_5 + t_6 Z_6)$
$\{|t_6| = \epsilon\} + \{|t_4| = \epsilon\} + \{|t_2| = \epsilon\}$
parameter z
$t_i = t_i(z)$
$t_4=0$
$t_3=0$
$\mathbb{C}z$
$t_2=0$
$t_4=0$
$t_6=0$
$t_1=0$


$\{|t_1|=\epsilon\} + \{|t_2|=\epsilon\}$
$t_4=0$
$t_5=0$
$t_3=0$
$t_6=0$
$t_7=0$

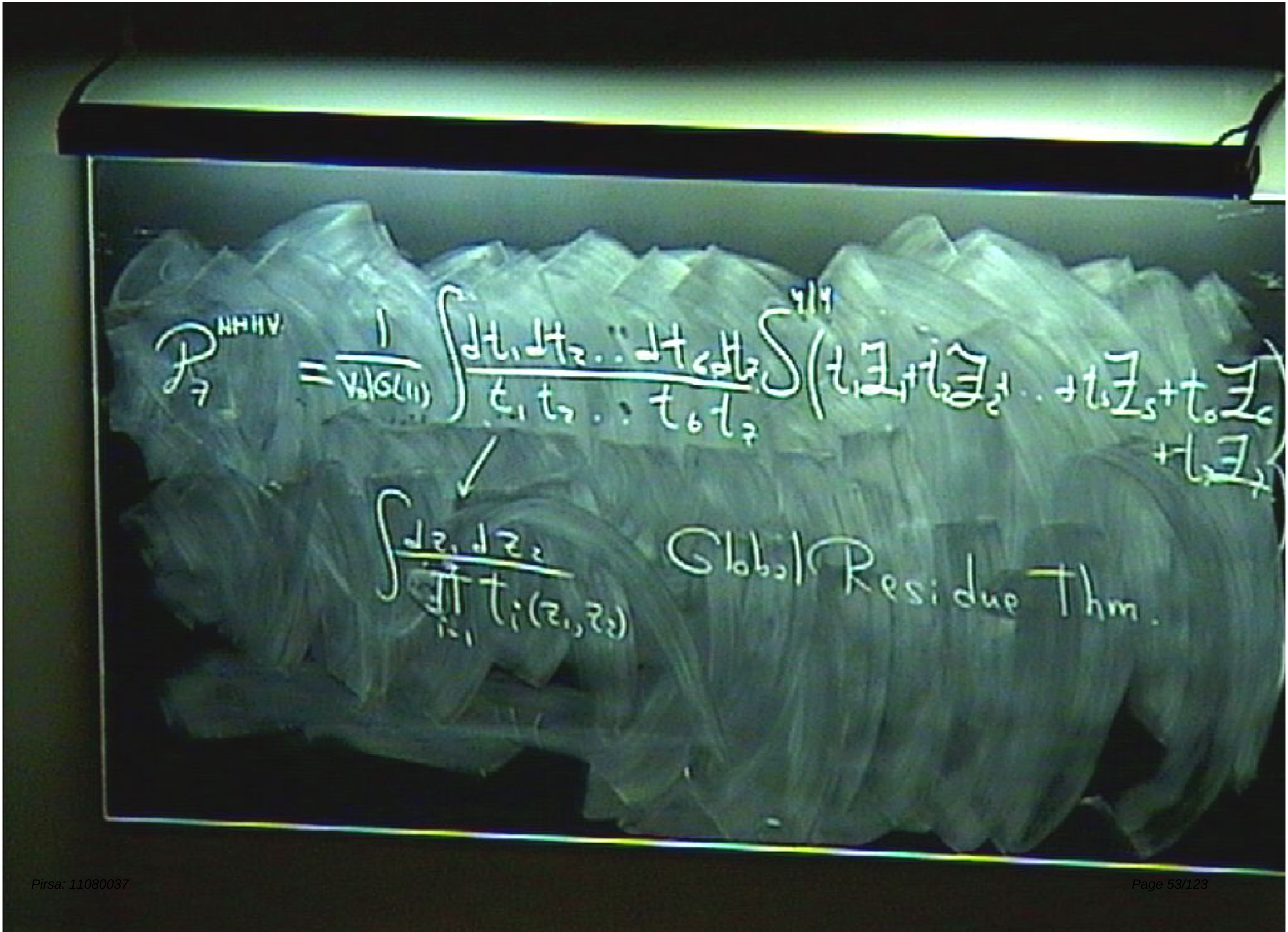

$\mathcal{P}_7^{\text{NMHV}} = \frac{1}{\text{Vol}(G(1))} \int \frac{dt_1\, dt_2 \cdots dt_6\, dt_7}{t_1\, t_2 \cdots t_6\, t_7}\, \delta^{4|4}(t_1 \mathcal{Z}_1 + t_2 \mathcal{Z}_2 + \cdots + t_5 \mathcal{Z}_5 + t_6 \mathcal{Z}_6 + t_7 \mathcal{Z}_7)$
$\int \frac{dz_1\, dz_2}{\cdots\, t_i(z_1, z_2)}$

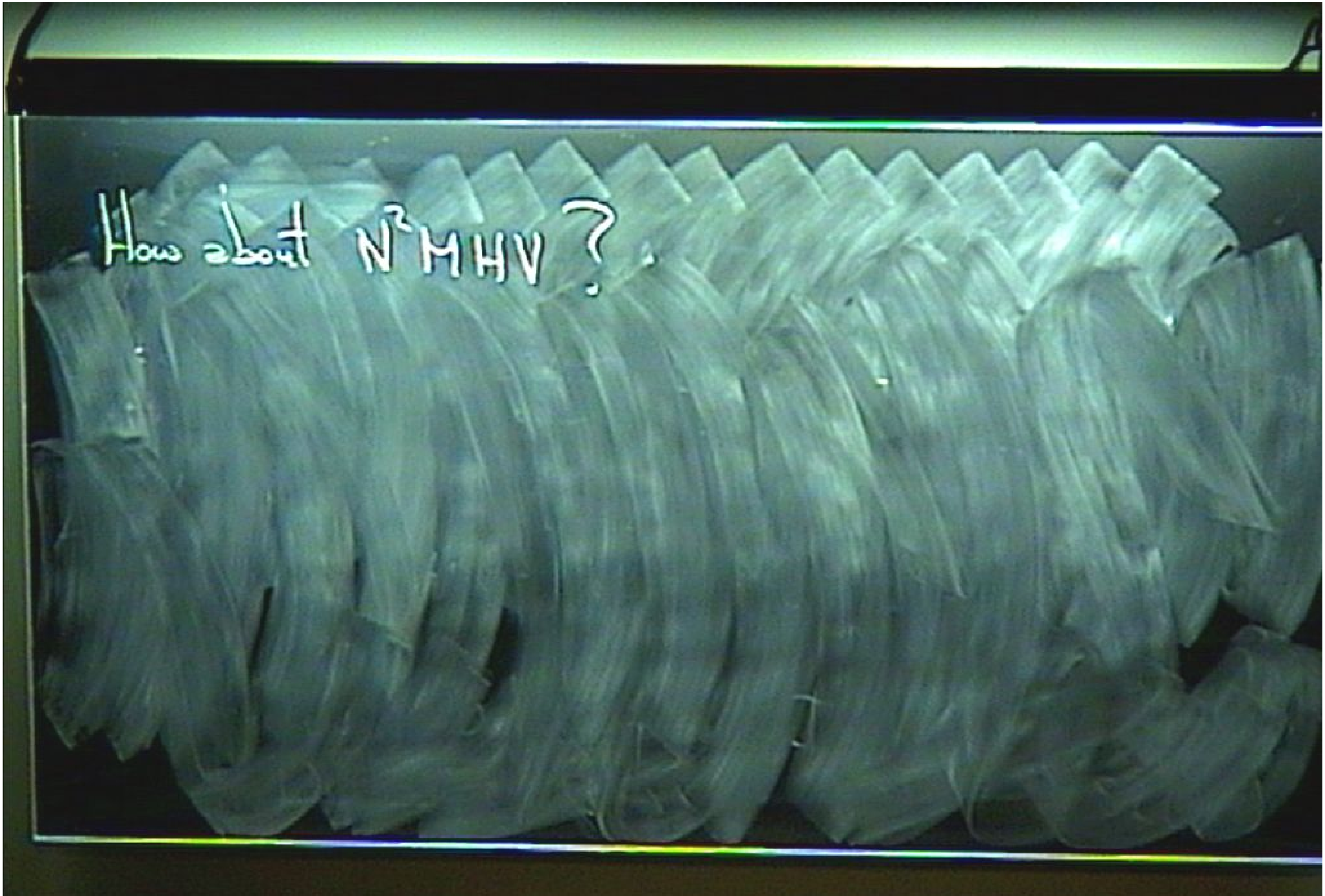

$\mathcal{P}_7^{NMHV} = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_1\, dt_2 \ldots dt_6\, dt_7}{t_1 t_2 \ldots t_6 t_7} \delta^{4|4}(t_1 Z_1 + t_2 Z_2 + \ldots + t_5 Z_5 + t_6 Z_6 + t_7 Z_7)$
$\int \frac{dz_1\, dz_2}{\prod_{i=1}^{7} t_i(z_1, z_2)}$

$$\mathcal{P}_7^{NMHV} = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_1\, dt_2 \cdots dt_6\, dt_7}{t_1\, t_2 \cdots t_6\, t_7}\, \delta^{4|4}\left(t_1 \mathcal{Z}_1 + t_2 \mathcal{Z}_2 + \cdots + t_5 \mathcal{Z}_5 + t_6 \mathcal{Z}_6 + t_7 \mathcal{Z}_7\right)$$

$$\int \frac{dz_1\, dz_2}{\prod_{i=1}^{7} t_i(z_1, z_2)}$$

Global Residue Thm.


$\mathcal{P}_7^{\text{NHHV}} = \frac{1}{\text{Vol}(GL(1))}\int \frac{dt_1\, dt_2 \cdots dt_6\, dt_7}{t_1\, t_2 \cdots t_6\, t_7}\, \delta^{4|4}(t_1\mathcal{Z}_1 + t_2\mathcal{Z}_2 + \cdots + t_5\mathcal{Z}_5 + t_6\mathcal{Z}_6 + t_7\mathcal{Z}_7)$
$\int \frac{dz_1\, dz_2}{\prod_{i=1}^{7} t_i(z_1, z_2)}$
Global Residue Thm.


How about N^2MHV?

$$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))}\int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e}\,\delta^{4|4}(t_a Z_a + t_b Z_b + \ldots + t_e Z_e)$$

Redundancy $GL(1)$ $\quad (t_a, t_b, \ldots, t_e) \sim \underbrace{\rho}_{\in\, \mathbb{C}^*}(t_a, t_b, \ldots t_e)$

G.F: $t_a = 1$.

$$\int \frac{dt_b \ldots dt_e}{t_b \ldots t_e}\,\delta^{0|4}(\eta_a + \ldots + t_e \eta_e)\,\delta^4(\underbrace{Z_a + t_b Z_b + \ldots + t_e Z_e}_{=0}) \in \mathbb{P}^4$$

$$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))}\int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e}\, \delta^{4|4}(t_a Z_a + t_b Z_b + \ldots + t_e Z_e)$$

Redundancy $GL(1)$ $\quad (t_a, t_b, \ldots, t_e) \sim \rho(t_a, t_b, \ldots, t_e)$

$(t_a, t_b, \ldots, t_e) \in \mathbb{C}^5 \qquad \mathbb{CP}^4$

G.F: $t_a = 1$.

$$\int \frac{dt_b \ldots dt_e}{t_b \ldots t_e}\, \delta^{0|4}(\eta_a + \ldots + t_e \eta_e)\, \delta^4(\underbrace{Z_a + t_b Z_b + \ldots + t_e Z_e}_{=0})$$

$$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))}\int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e}\, \delta^{4|4}(t_a Z_a + t_b Z_b + \ldots + t_e Z_e)$$

Redundancy $GL(1)$ $\quad (t_a, t_b, \ldots, t_e) \sim \rho(t_a, t_b, \ldots, t_e)$

$$\underset{\mathbb{C}^5}{\cap} \qquad \mathbb{CP}^4$$

G.F: $t_a = 1$.

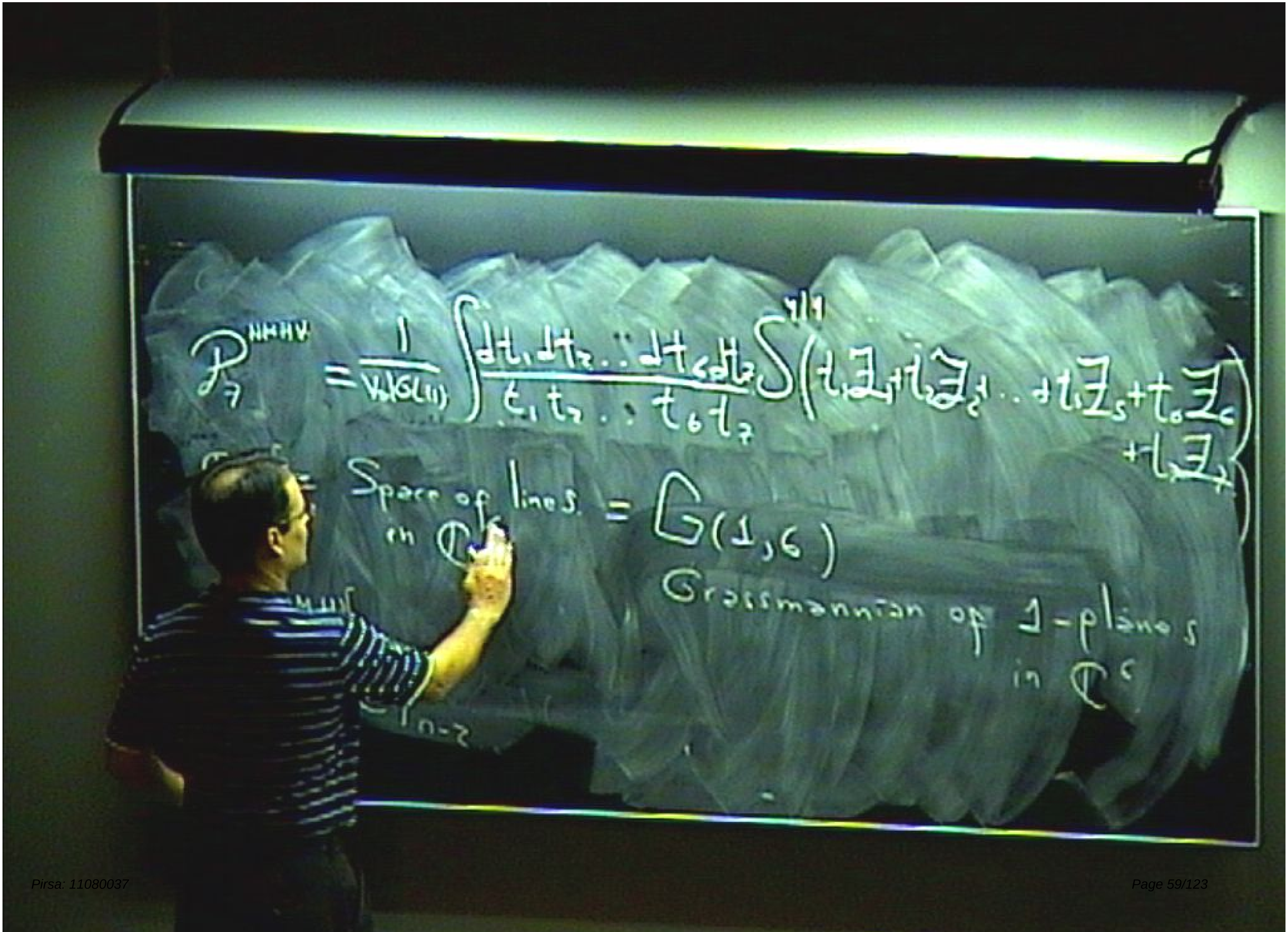
$$\int \frac{dt_b \ldots dt_e}{t_b \ldots t_e}\, \delta^{0|4}(\eta_a + \ldots + t_e \eta_e)\, \delta^4\underbrace{(Z_a + t_b Z_b + \ldots + t_e Z_e)}_{= 0}$$

$$\mathcal{P}_7^{NMHV} = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_1\, dt_2 \cdots dt_6\, dt_7}{t_1\, t_2 \cdots t_6\, t_7}\, \delta^{4|4}\left(t_1 Z_1 + t_2 Z_2 + \cdots + t_5 Z_5 + t_6 Z_6 + t_7 Z_7\right)$$

$\mathbb{CP}^5 =$ Space of lines in $\mathbb{C}^6$ $= G(1,6)$

Grassmannian of 1-planes in $\mathbb{C}^6$

$\mathcal{P}_n^{NMHV}$ $\quad \mathbb{CP}_{n-2}$


$\mathcal{P}_7^{NMHV} = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_1\, dt_2 \cdots dt_6\, dt_7}{t_1 t_2 \cdots t_6 t_7} \delta^{4|4}\left(t_1 \mathcal{Z}_1 + t_2 \mathcal{Z}_2 + \cdots + t_5 \mathcal{Z}_5 + t_6 \mathcal{Z}_6 + t_7 \mathcal{Z}_7\right)$
Space of lines in $\mathbb{C}^6$ = $G(1,6)$
Grassmannian of 1-planes in $\mathbb{C}^6$

$$\mathcal{P}_7^{NMHV} = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_1\, dt_2 \cdots dt_6\, dt_7}{t_1\, t_2 \cdots t_6\, t_7}\, \delta^{4|4}\left(t_1 Z_1 + t_2 Z_2 + \cdots + t_5 Z_5 + t_6 Z_6 + t_7 Z_7\right)$$

$\mathbb{CP}^5 =$ Space of lines in $\mathbb{C}^7$ $= G(1,6)$

Grassmannian of 1-planes in $\mathbb{C}^6$

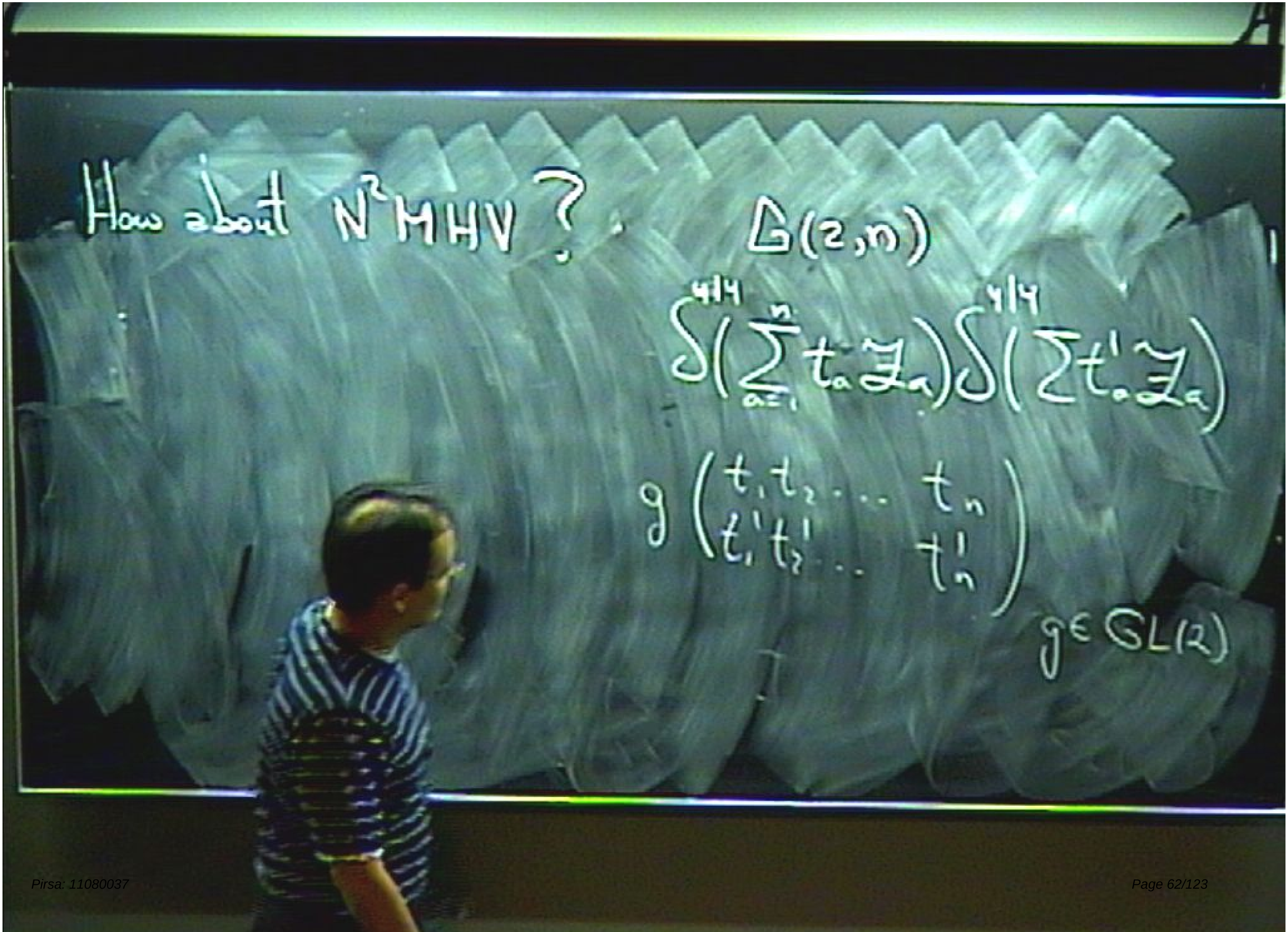
$\mathcal{P}_n^{NMHV}$ $\quad \mathbb{CP}_{n-2}$

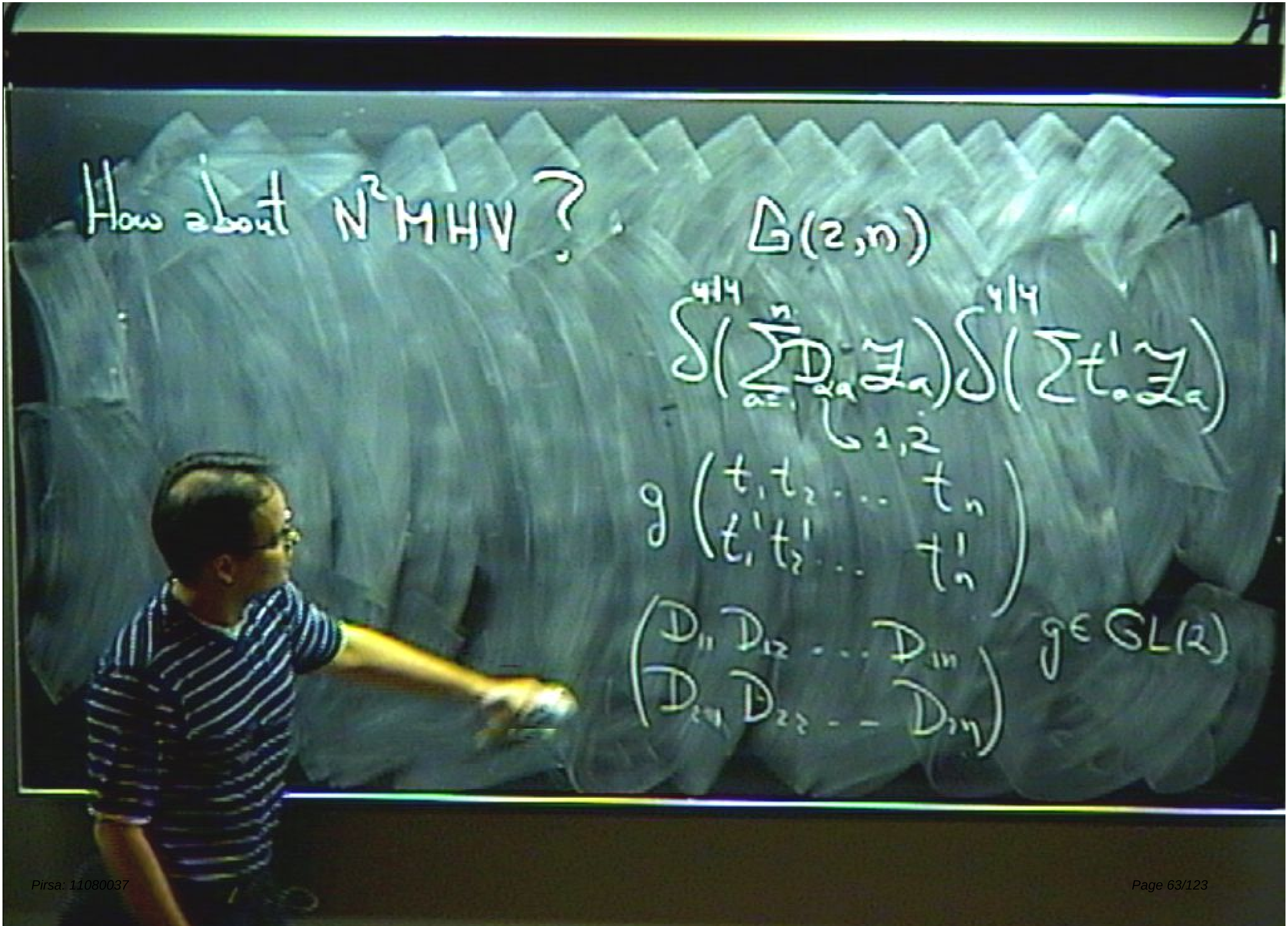
$$\mathcal{P}_7^{NMHV} = \frac{1}{\mathrm{Vol}\,GL(1)} \int \frac{dt_1\, dt_2 \cdots dt_6\, dt_7}{t_1 t_2 \cdots t_6 t_7}\, \delta^{4|4}\left(t_1 Z_1 + t_2 Z_2 + \cdots + t_5 Z_5 + t_6 Z_6 + t_7 Z_7\right)$$

$\mathbb{CP}^5$ = Space of lines in $\mathbb{C}^7$ = $G(1,6)$

Grassmannian of 1-planes in $\mathbb{C}^6$

$\mathcal{P}_n^{NMHV}$ $\quad \mathbb{CP}_{n-2}$


How about N^2MHV ?
$G(2,n)$
$\delta^{4|4}\left(\sum_{a=1}^{n} t_a \mathcal{Z}_a\right)\delta^{4|4}\left(\sum t'_a \mathcal{Z}_a\right)$
$g\begin{pmatrix} t_1 & t_2 & \dots & t_n \\ t'_1 & t'_2 & \dots & t'_n \end{pmatrix}$
$g \in GL(2)$


How about N² MHV ?
G(2,n)
δ^{4|4}(Σ_{a=1}^{n} D_{1,2 a} Z_a) δ^{4|4}(Σ t'_a Z_a)
g (t_1 t_2 … t_n / t'_1 t'_2 … t'_n)
(D_11 D_12 … D_1n / D_21 D_22 … D_2n)
g ∈ GL(2)

How about N^2MHV ?

$G(2,n)$

$$\int d^{2n} D_{\alpha a} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a} \mathcal{Z}_a\right)$$

$$\begin{pmatrix} D_{11} & D_{12} & \cdots & D_{1n} \\ D_{21} & D_{22} & \cdots & D_{2n} \end{pmatrix}$$

How about N^2MHV ?

$G(2,n)$

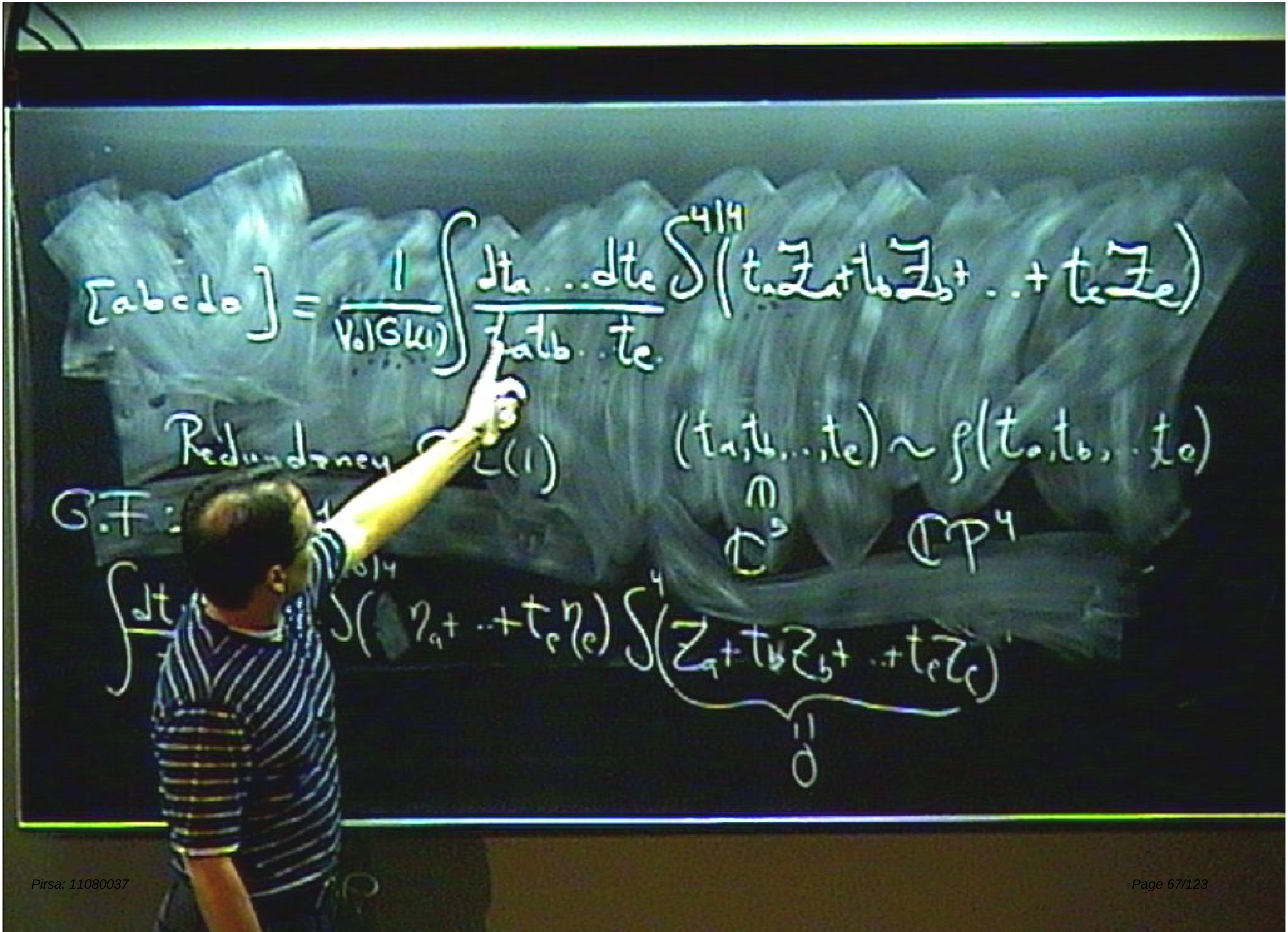
$$\int d^{2n} D_{\alpha a} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a} \mathcal{Z}_a\right)$$

$$\begin{pmatrix} D_{11} & D_{12} & \cdots & D_{1n} \\ D_{21} & D_{22} & \cdots & D_{2n} \end{pmatrix}$$

How about N^2MHV ? $G(2,n)$

$$\int \overset{\boxed{2n}}{d D_{\alpha a}} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a} \mathcal{Z}_a\right)$$

$$\begin{pmatrix} D_{11} & D_{12} & \cdots & D_{1n} \\ D_{21} & D_{22} & \cdots & D_{2n} \end{pmatrix}$$


$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))} \int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e} \delta^{4|4}(t_a Z_a + t_b Z_b + \ldots + t_e Z_e)$
Redundancy $GL(1)$
$(t_a, t_b, \ldots, t_e) \sim \rho(t_a, t_b, \ldots, t_e)$
$\cap$
$\mathbb{C}^5$
$\mathbb{CP}^4$
G.F.
$\delta(\ldots \eta_a + \ldots + t_e \eta_e)\, \delta^4(Z_a + t_b Z_b + \ldots + t_e Z_e)$
0

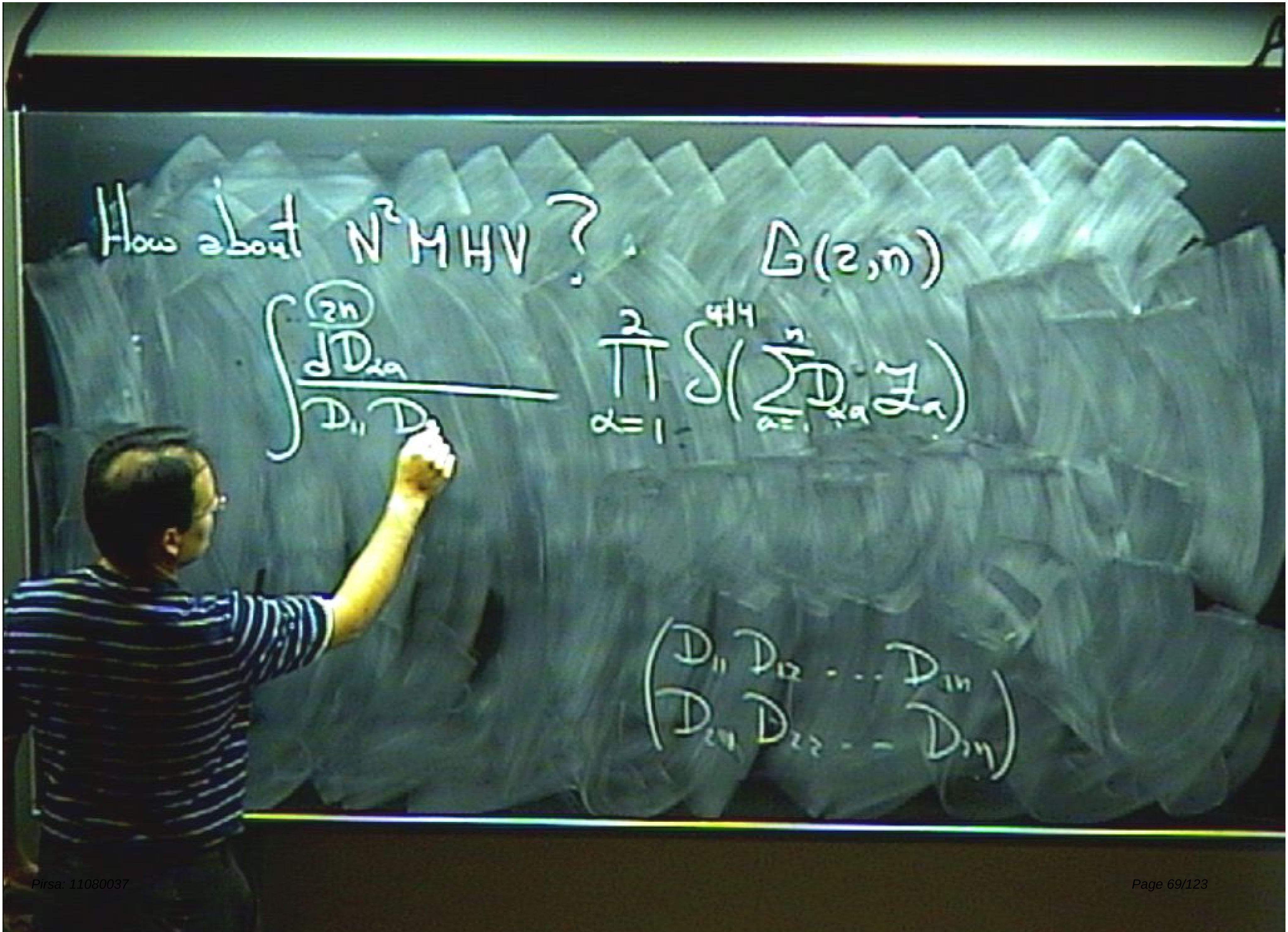
$$[abcde] = \frac{1}{\mathrm{Vol}(GL(1))} \int \frac{dt_a \ldots dt_e}{t_a t_b \ldots t_e} \delta^{4|4}(t_a Z_a + t_b Z_b + \ldots + t_e Z_e)$$

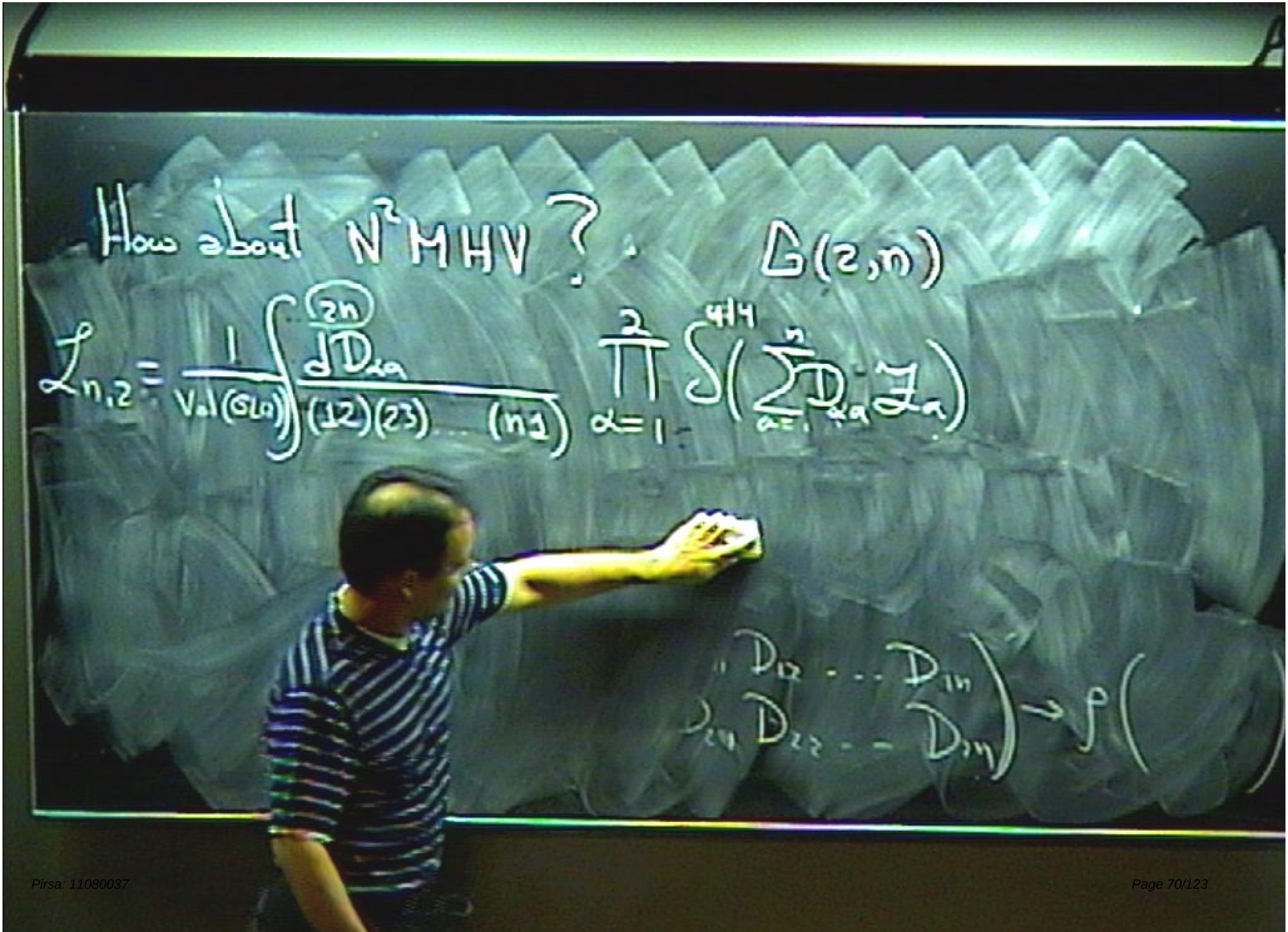
Redundancy $GL(1)$ $\quad (t_a, t_b, \ldots, t_e) \sim \rho(t_a, t_b, \ldots t_e)$

$$(t_a, t_b, \ldots, t_e) \in \mathbb{C}^5 \qquad \mathbb{CP}^4$$

G.F: $t_a = 1$.

$$\int \frac{dt_b \ldots dt_e}{t_b \ldots t_e} \delta^{0|4}(\eta_a + \ldots + t_e \eta_e)\, \delta^4 \underbrace{(Z_a + t_b Z_b + \ldots + t_e Z_e)}_{!! \atop 0}$$


How about N^2MHV ?
$G(2,n)$
$\int \frac{d^{2n} D_{\alpha a}}{D_{11} D_{\dots}} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a} \mathcal{Z}_a\right)$
$\begin{pmatrix} D_{11} & D_{12} & \dots & D_{1n} \\ D_{21} & D_{22} & \dots & D_{2n} \end{pmatrix}$

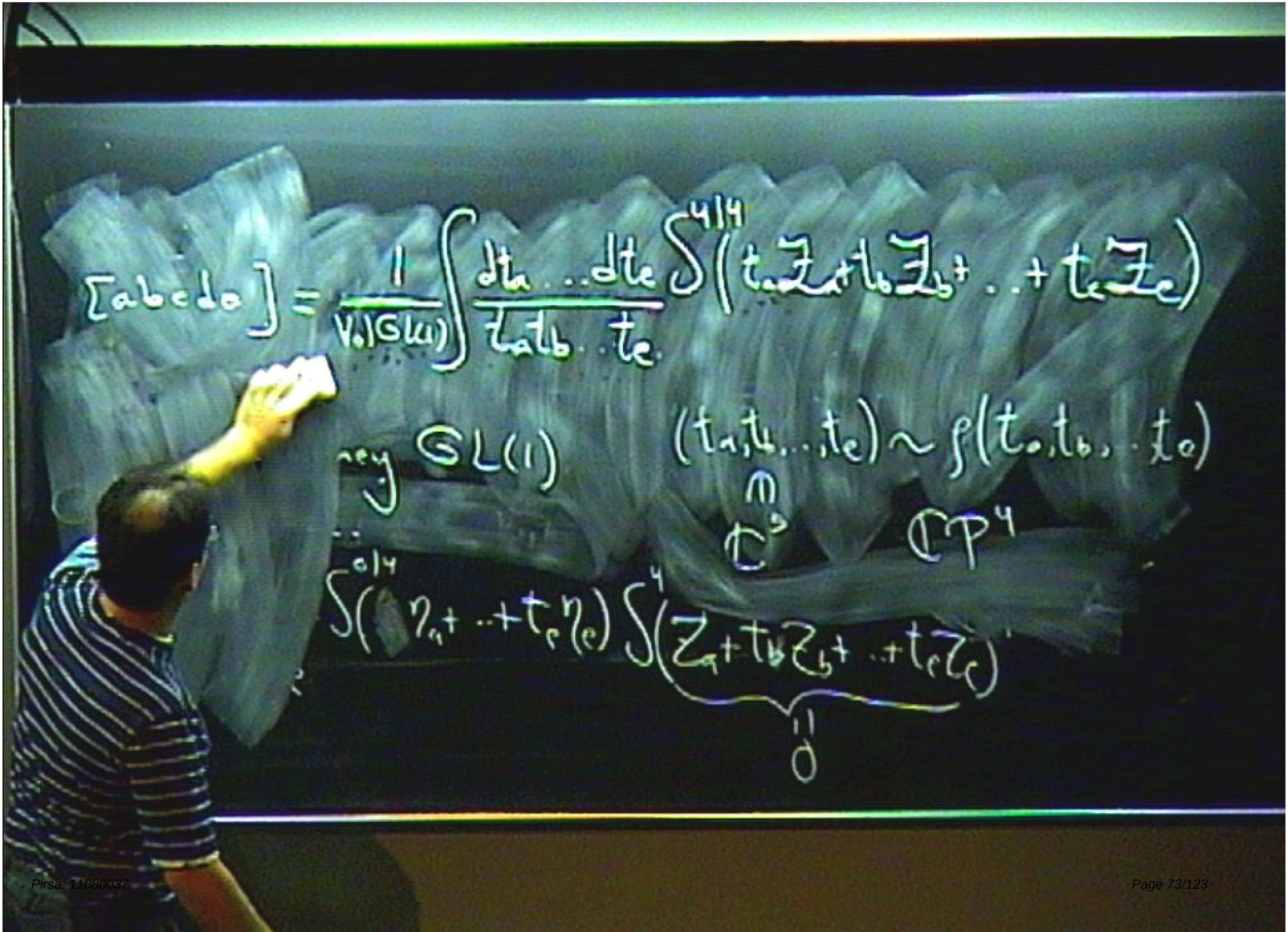

How about N^2MHV ?
$G(2,n)$
$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))} \int \frac{d^{2n} D_{\alpha a}}{(12)(23)\ldots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a} \mathcal{Z}_a\right)$
$D_{12} \ldots D_{1n}$
$D_{22} \ldots D_{2n}$

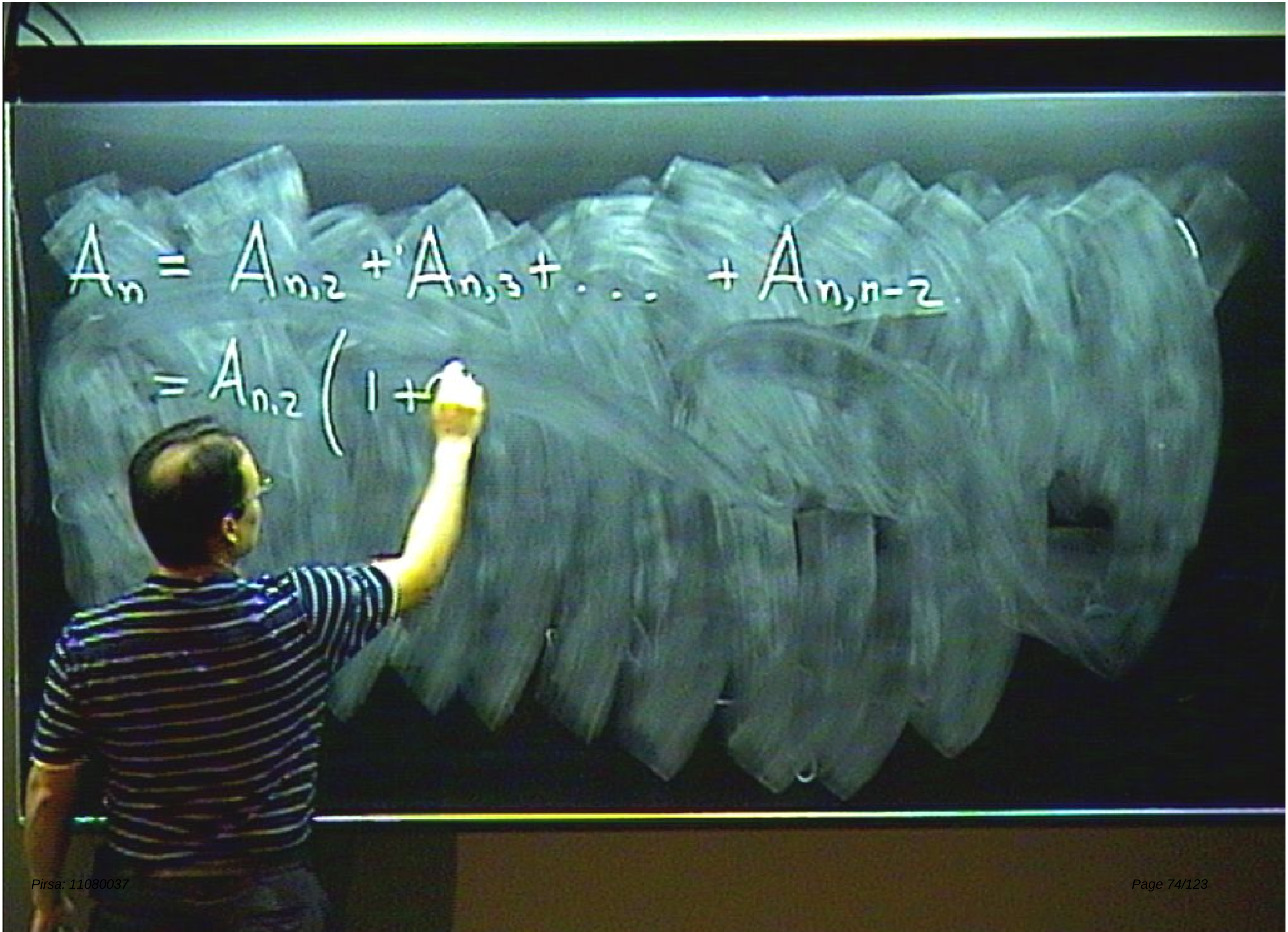
How about N^2MHV? $G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))} \int \frac{d^{2n} D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a} \mathcal{Z}_a\right)$$

How about N^2MHV ? $G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n}D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\right)$$


[abcde] = 1/Vol(GL(1)) ∫ dt_a ... dt_e / t_a t_b ... t_e δ^{4|4}(t_a Z_a + t_b Z_b + ... + t_e Z_e)
GL(1)
(t_a, t_b, ..., t_e) ~ ρ(t_a, t_b, ..., t_e)
∩
C^5
CP^4
δ^{0|4}(η_a + ... + t_e η_e) δ^4(Z_a + t_b Z_b + ... + t_e Z_e)
0


$A_n = A_{n,2} + A_{n,3} + \ldots + A_{n,n-2}$
$= A_{n,2} \left(1 + \right.$

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2}$$

$$= A_{n,2}\left(1 + P_{n,1} + \dots + P_{n,n-4}\right)$$

How about N^2MHV ?

$G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n} D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\right)$$

$$J^A{}_B = \sum_{a=1}^{n} \mathcal{Z}^A_a \frac{\partial}{\partial \mathcal{Z}^B_a}$$

Dual Superconf

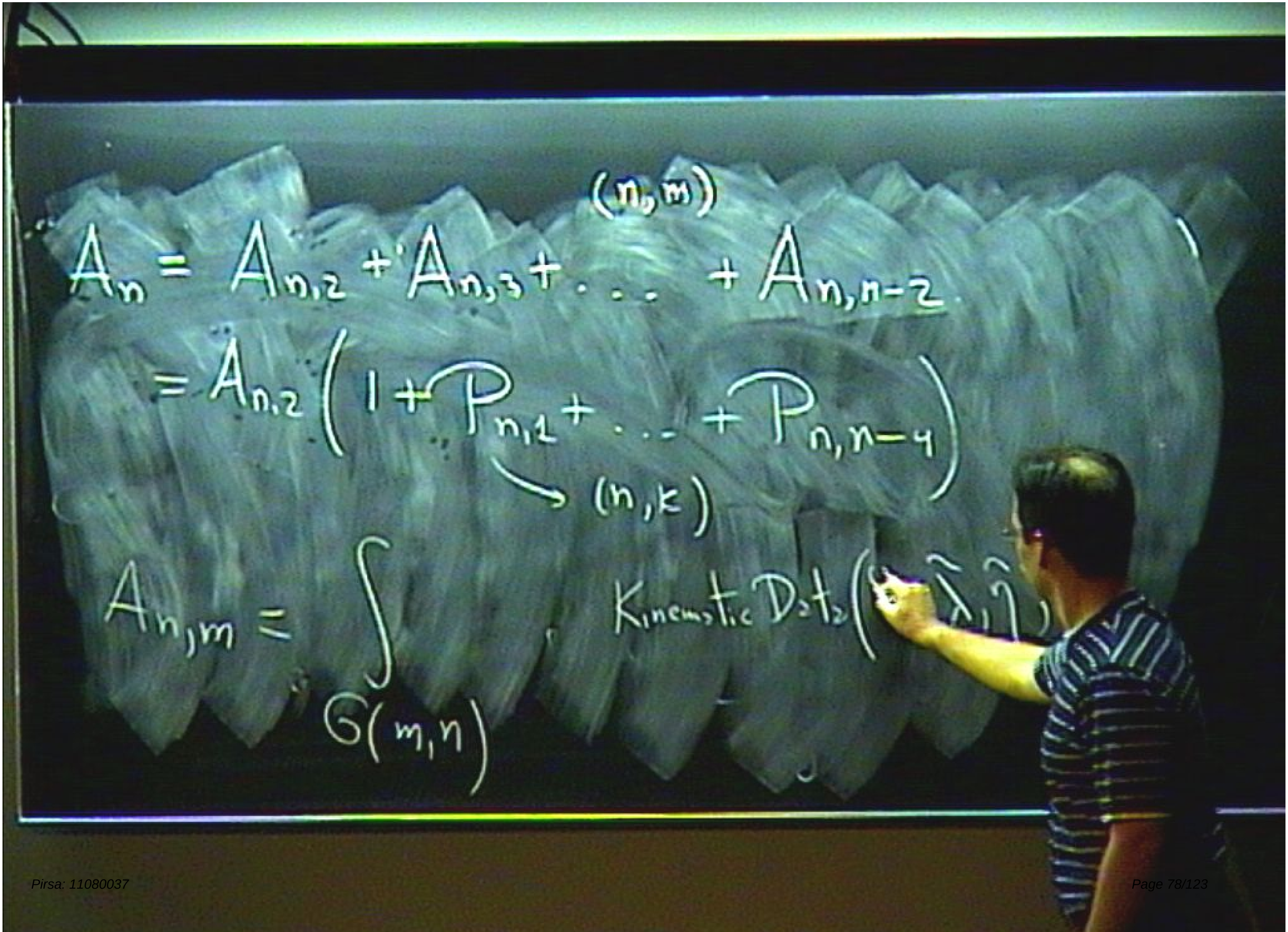
$SL(4|4)$

$$A_n = A_{n,2} + A_{n,3} + \ldots + \overset{(n,m)}{A_{n,n-2}}$$

$$= A_{n,2}\left(1 + P_{n,1} + \ldots + P_{n,n-4}\right)$$

(n,k)

A


(n,m)
A_n = A_{n,2} + A_{n,3} + . . . + A_{n,n-2}
= A_{n,2} (1 + P_{n,1} + . . . + P_{n,n-4})
(n,k)
A_{n,m} = ∫_{G(m,n)} Kinematic Data

$$(n,m)$$

$$A_n = A_{n,2} + A_{n,3} + \ldots + A_{n,n-2}$$

$$= A_{n,2}\left(1 + P_{n,1} + \ldots + P_{n,n-4}\right)$$

$$(n,k)$$

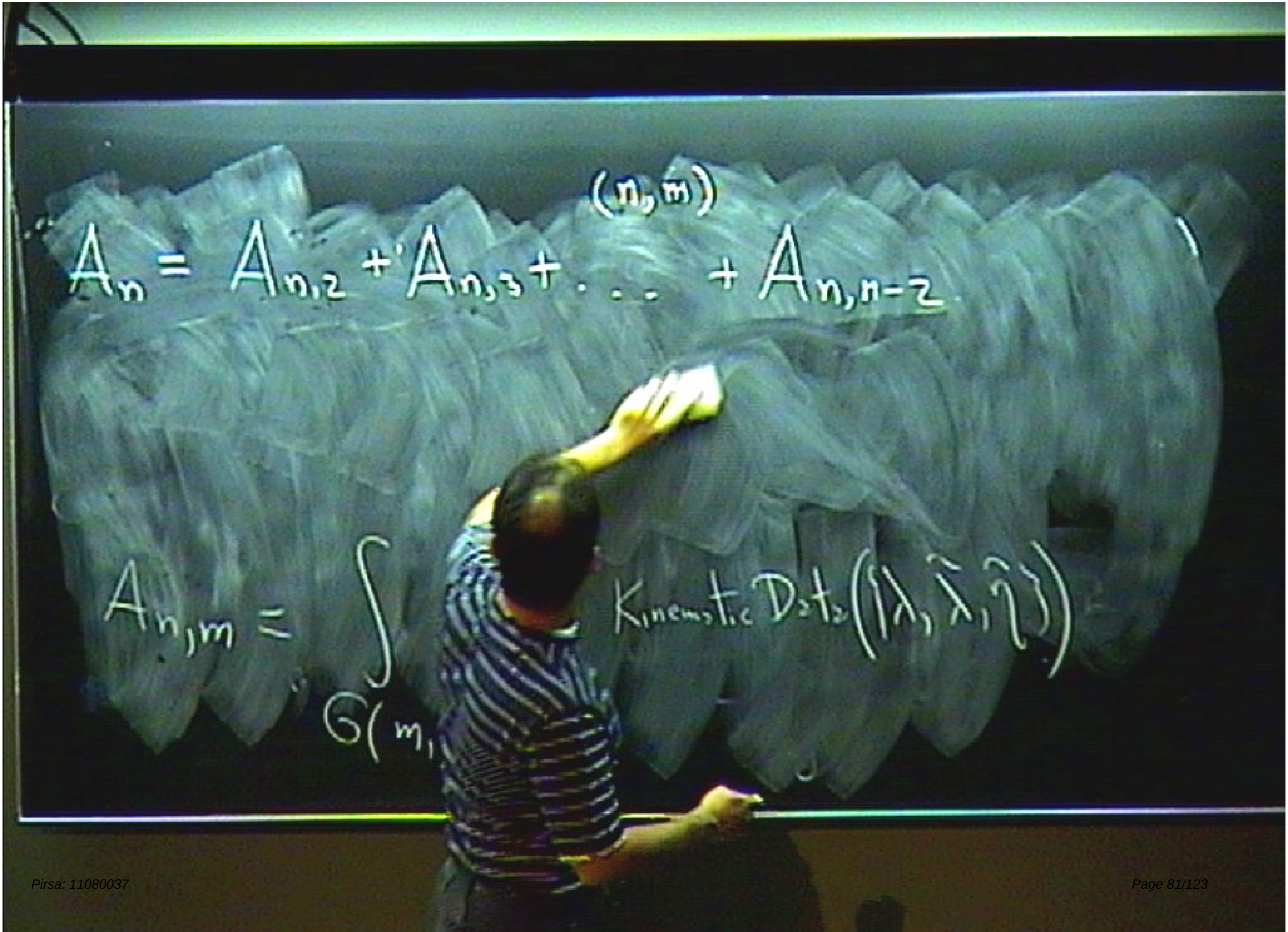
$$A_{n,m} = \int_{G(m,n)} \text{Kinematic Data}\left(\{\lambda, \tilde{\lambda}, \eta\}\right)$$

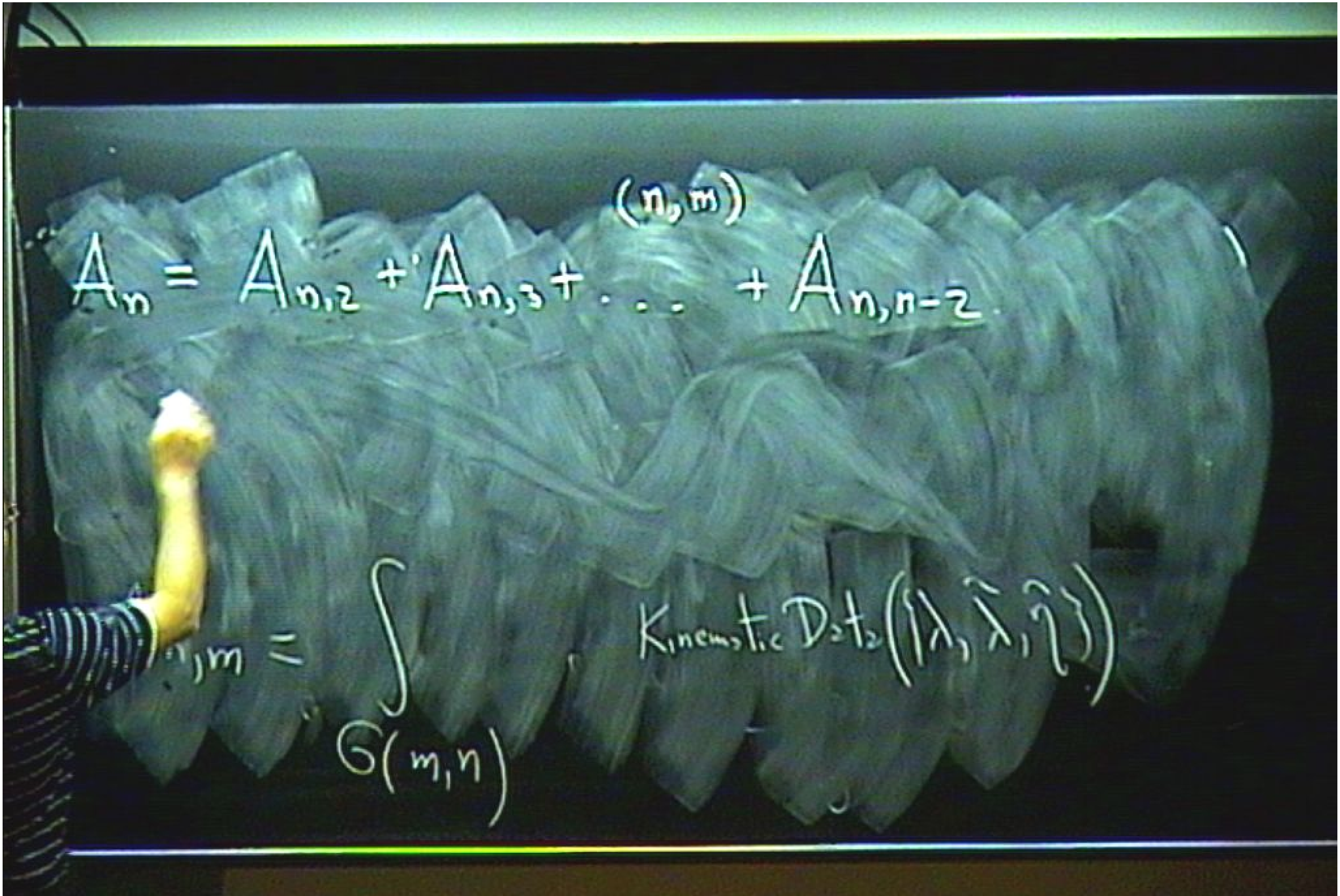
$$A_n = A_{n,2} + A_{n,3} + \ldots + \overset{(n,m)}{} + A_{n,n-2}$$

$$= A_{n,2}\left(1 + P_{n,1} + \ldots + P_{n,n-4}\right)$$

(n,k)

$$A_{n,m} = \int_{G(m,n)} \text{Kinematic Data}\left(\{\lambda, \tilde{\lambda}, \tilde{\eta}\}\right)$$


(n,m)
$A_n = A_{n,2} + A_{n,3} + \ldots + A_{n,n-2}$
$A_{n,m} = \int_{G(m,} \text{Kinematic Data}(\{\lambda, \tilde{\lambda}, \tilde{\eta}\})$



(n,m)
$A_n = A_{n,2} + A_{n,3} + \ldots + A_{n,n-2}$
$A_{n,m} = \int_{G(m,n)}$ Kinematic Data$(\{\lambda, \tilde\lambda, \tilde\eta\})$

$$A_n = A_{n,2} + A_{n,3} + \ldots + A_{n,n-2}$$

(n,m)

$$A_{n,m} = \int_{G(m,n)} \text{Kinematic Data}(\{\lambda, \tilde{\lambda}, \tilde{\eta}\})$$

$$A_n = A_{n,2} + A_{n,3} + \ldots + \underbrace{A_{n,n-2}}_{}$$

$$(n,m)$$

$$\int d^2\lambda_a\, e^{i\langle \lambda_a \mu_a \rangle} A_n(\{\lambda\, \tilde{\lambda}\, \tilde{\eta}\}) = \tilde{A}_n(\{\mu, \tilde{\lambda}\, \tilde{\eta}\})$$

$$\int_{G(m,n)} \text{Kinematic Data}(\{\lambda, \tilde{\lambda}, \tilde{\eta}\})$$

$$(n,m)$$

$$A_n = A_{n,2} + A_{n,3} + \dots + A_{n,n-2}$$

$$\int d^2\lambda_a \, e^{i\langle \lambda_a \mu_a \rangle} A_n(\{\lambda \tilde{\lambda} \tilde{\eta}\}) = \tilde{A}_n(\{\underbrace{\mu_a \tilde{\lambda}_a \tilde{\eta}_a}_{\mathcal{W}_a}\})$$

$$\mathcal{W}_a = \begin{pmatrix} W_a \\ \tilde{\eta}_a \end{pmatrix}$$

How about N^2MHV? $G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n}D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\right)$$

$$J^A{}_B = \sum_{a=1}^{n} \mathcal{Z}^A_a \frac{\partial}{\partial \mathcal{Z}^B_a}$$

Dual Superconf

$SL(4|4)$

How about N^2MHV ?

$G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n} D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\Big(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\Big)$$

$$J^{A}{}_{B} = \sum_{a=1}^{n} \mathcal{Z}^{A}_{a}\frac{\partial}{\partial \mathcal{Z}^{B}_{a}}$$

Dual Superconf

$SL(4|4)$

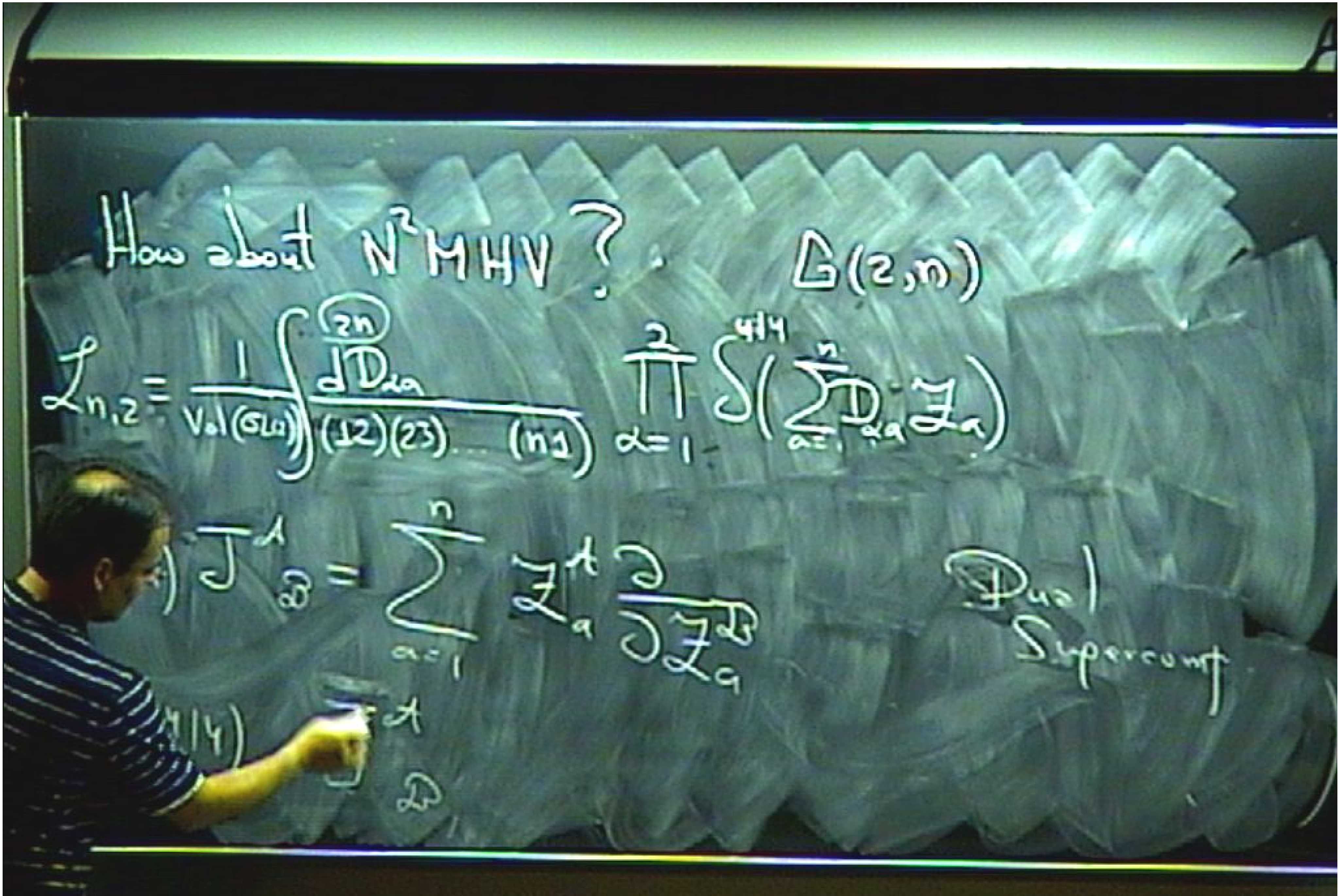
How about N^2MHV? $\quad G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n}D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a}\,\mathcal{Z}_a\right)$$

$$J^{A}{}_{B} = \sum_{a=1}^{n} \mathcal{Z}^{A}_{a} \frac{\partial}{\partial \mathcal{Z}^{B}_{a}}$$

Dual Superconf

$SL(4|4)$ $\quad J$

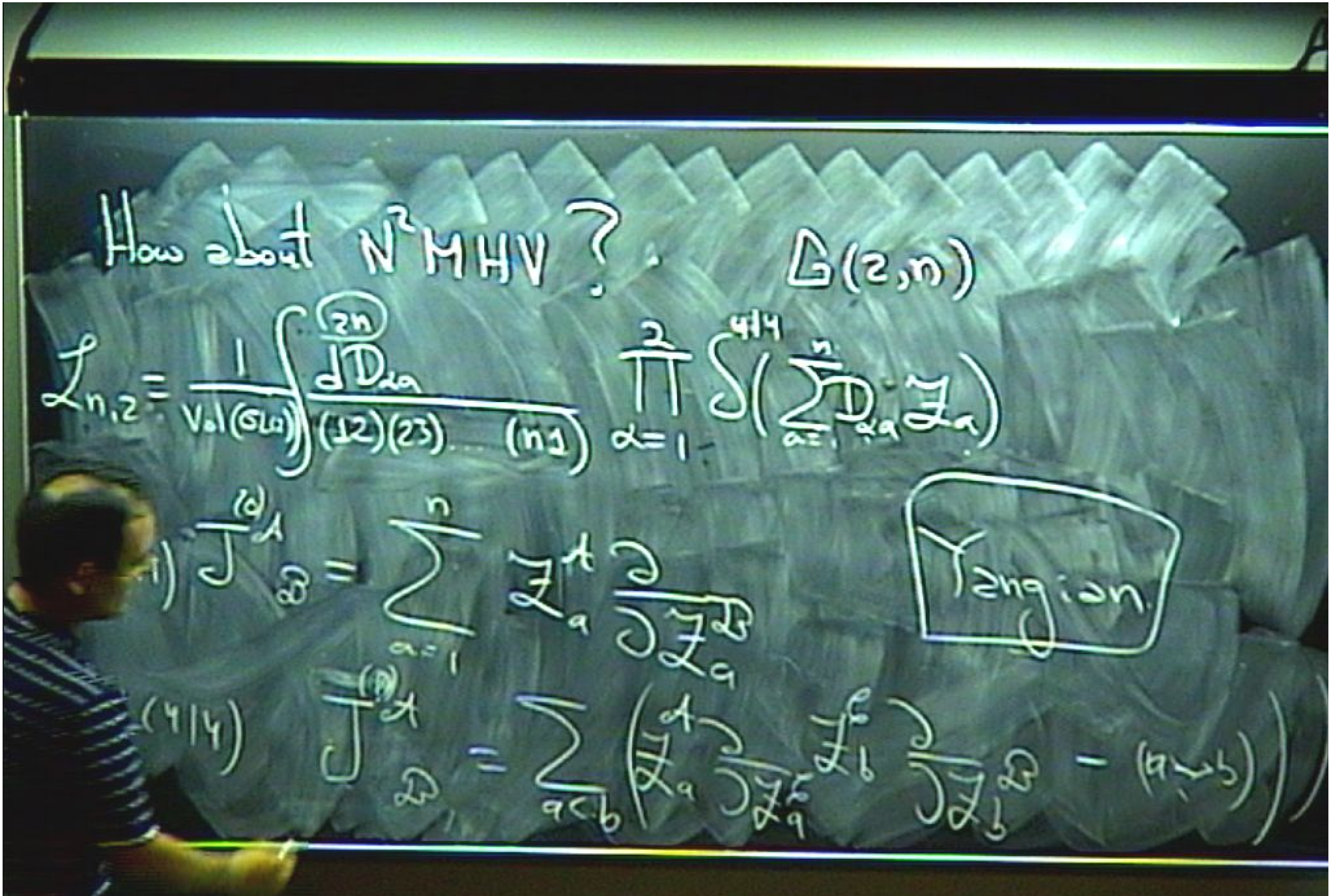

How about N²MHV?
G(2,n)
$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))} \int \frac{d^{2n} D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a} \mathcal{Z}_a\right)$
$J^{A}{}_{B} = \sum_{a=1}^{n} \mathcal{Z}_a^{A} \frac{\partial}{\partial \mathcal{Z}_a^{B}}$
Dual Superconf
(4|4)

How about N^2MHV? $\quad G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n}D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\right)$$

SL(4|4) $\quad J^{A}{}_{B} = \sum_{a=1}^{n} \mathcal{Z}_a^{A} \frac{\partial}{\partial \mathcal{Z}_a^{B}}$ $\quad$ Dual Superconf

SL(4|4) $\quad J'^{A}{}_{B} = \sum_{a<b}\left(\mathcal{Z}_a^{A} \frac{\partial}{\partial \mathcal{Z}_a^{C}}\mathcal{Z}_b^{C}\frac{\partial}{\partial \mathcal{Z}_b^{B}} - (a\leftrightarrow b)\right)$


How about N^2MHV ?
$G(2,n)$
$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n}D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\right)$
$J^{(0)\mathcal{A}}{}_{\mathcal{B}} = \sum_{a=1}^{n} \mathcal{Z}_a^{\mathcal{A}} \frac{\partial}{\partial \mathcal{Z}_a^{\mathcal{B}}}$
Yangian
$(4|4)$
$J^{(1)\mathcal{A}}{}_{\mathcal{B}} = \sum_{a<b}\left(\mathcal{Z}_a^{\mathcal{A}}\frac{\partial}{\partial\mathcal{Z}_a^{\mathcal{C}}}\mathcal{Z}_b^{\mathcal{C}}\frac{\partial}{\partial \mathcal{Z}_b^{\mathcal{B}}} - (a\leftrightarrow b)\right)$

How about N^2MHV? $G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n}D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\Big(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\Big)$$

$SL(4|4)$
$$J^{(0)\mathcal{A}}{}_{\mathcal{B}} = \sum_{a=1}^{n} \mathcal{Z}_a^{\mathcal{A}} \frac{\partial}{\partial \mathcal{Z}_a^{\mathcal{B}}}$$

$\boxed{\text{Yangian}}$

$SL(4|4)$
$$J^{(1)\mathcal{A}}{}_{\mathcal{B}} = \sum_{a<b}\Big(\mathcal{Z}_a^{\mathcal{A}}\frac{\partial}{\partial \mathcal{Z}_a^{\mathcal{C}}} \mathcal{Z}_b^{\mathcal{C}}\frac{\partial}{\partial \mathcal{Z}_b^{\mathcal{B}}} - (a\leftrightarrow b)\Big)$$

How about N^2MHV? $G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n}D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\right)$$

$$SL(4|4) \quad J^{(0)\mathcal{A}}{}_{\mathcal{B}} = \sum_{a=1}^{n} \mathcal{Z}_a^{\mathcal{A}} \frac{\partial}{\partial \mathcal{Z}_a^{\mathcal{B}}}$$

$\boxed{\text{Yangian}}$

$$J^{(1)\mathcal{A}}{}_{\mathcal{B}} = \sum_{a<b}\left(\mathcal{Z}_a^{\mathcal{A}}\frac{\partial}{\partial \mathcal{Z}_a^{\mathcal{C}}}\mathcal{Z}_b^{\mathcal{C}}\frac{\partial}{\partial \mathcal{Z}_b^{\mathcal{B}}} - (a\leftrightarrow b)\right)$$

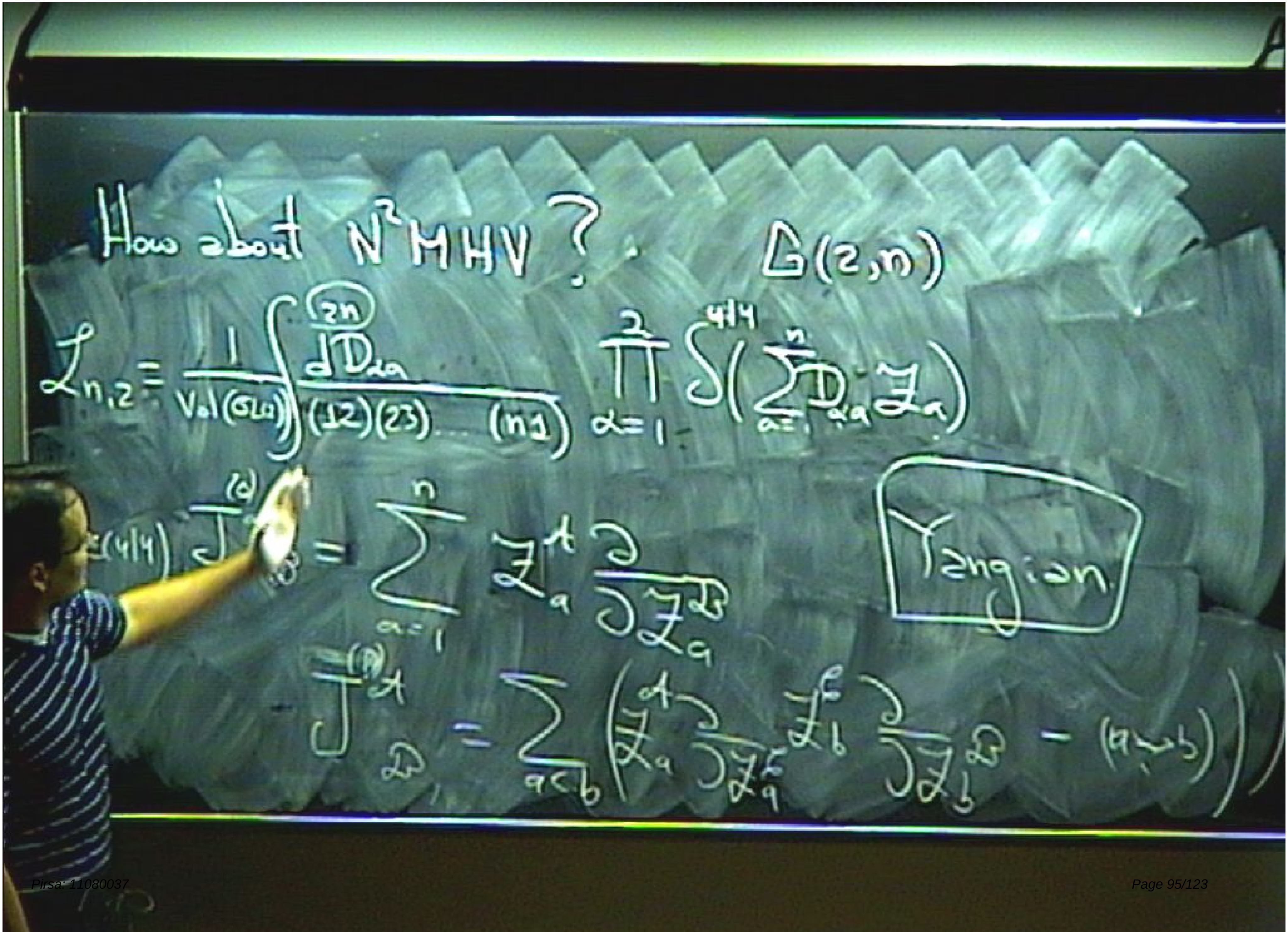
How about N^2MHV ? $G(2,n)$

$$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n}D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2}\delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\right)$$

$\boxed{\text{Yangian}}$

$$SL(4|4) \quad J^{(0)\mathcal{A}}{}_{\mathcal{B}} = \sum_{a=1}^{n} \mathcal{Z}_a^{\mathcal{A}}\frac{\partial}{\partial \mathcal{Z}_a^{\mathcal{B}}}$$

$$J^{(1)\mathcal{A}}{}_{\mathcal{B}} = \sum_{a<b}\left(\mathcal{Z}_a^{\mathcal{A}}\frac{\partial}{\partial \mathcal{Z}_a^{\mathcal{C}}}\mathcal{Z}_b^{\mathcal{C}}\frac{\partial}{\partial \mathcal{Z}_b^{\mathcal{B}}} - (a\leftrightarrow b)\right)$$


How about N²MHV ?
G(2,n)
$\mathcal{L}_{n,2} = \frac{1}{\mathrm{Vol}(GL(2))}\int \frac{d^{2n}D_{\alpha a}}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}\left(\sum_{a=1}^{n} D_{\alpha a}\mathcal{Z}_a\right)$
$(4|4)\ J^{(0)\mathcal{A}}_{\mathcal{B}} = \sum_{a=1}^{n} \mathcal{Z}_a^{\mathcal{A}} \frac{\partial}{\partial \mathcal{Z}_a^{\mathcal{B}}}$
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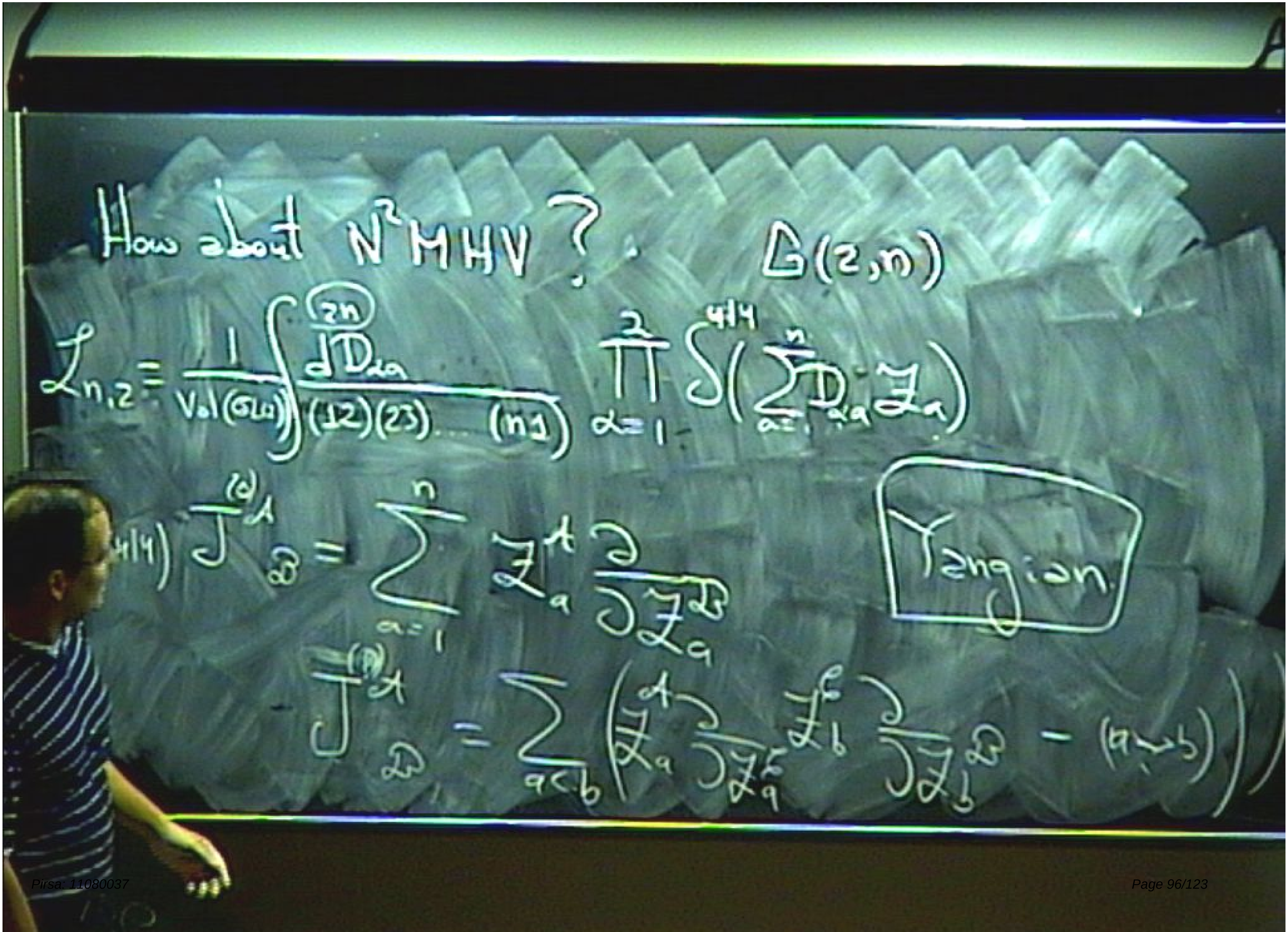
Yangian



How about N^2MHV ? $G(2,n)$

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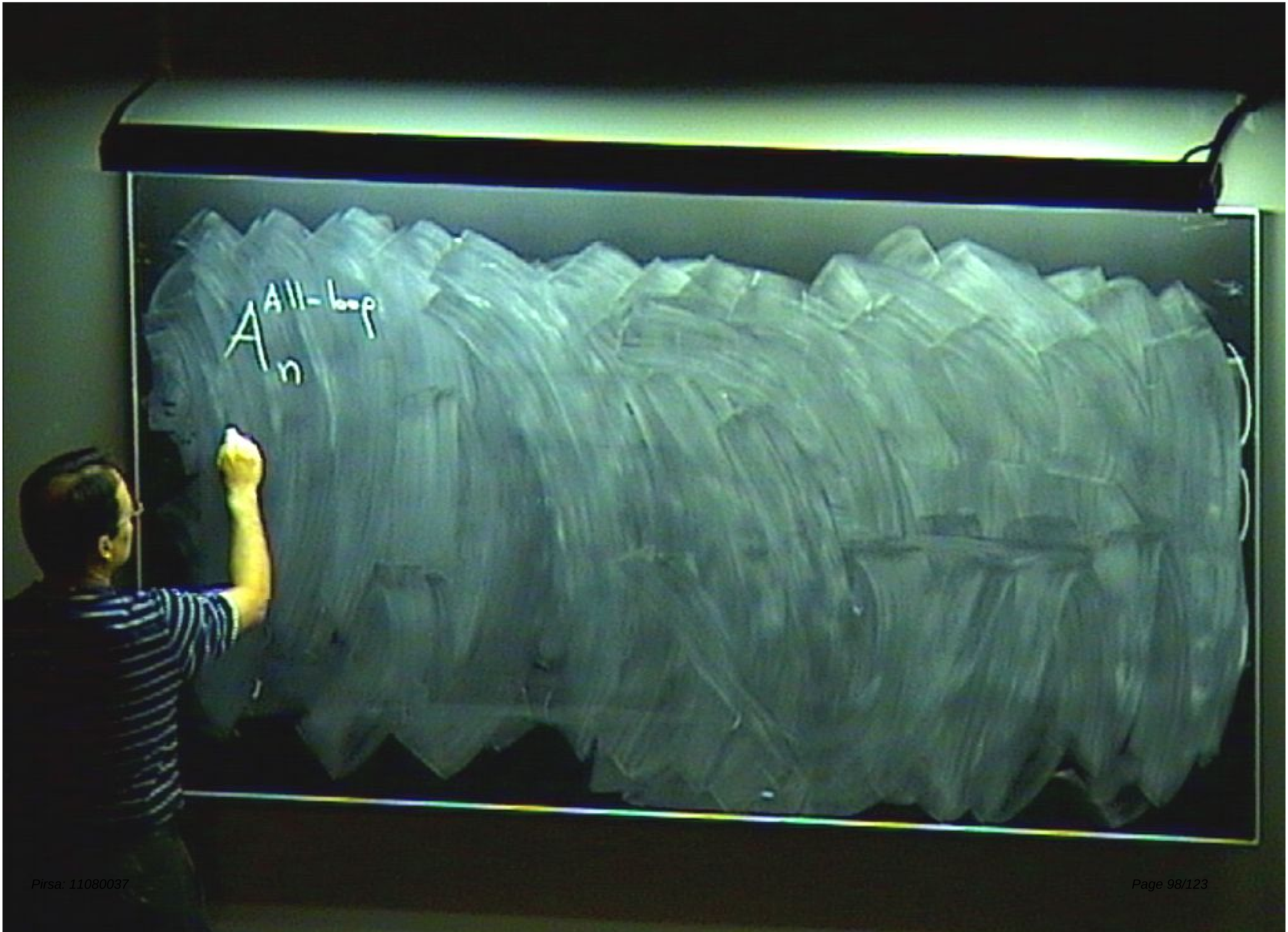
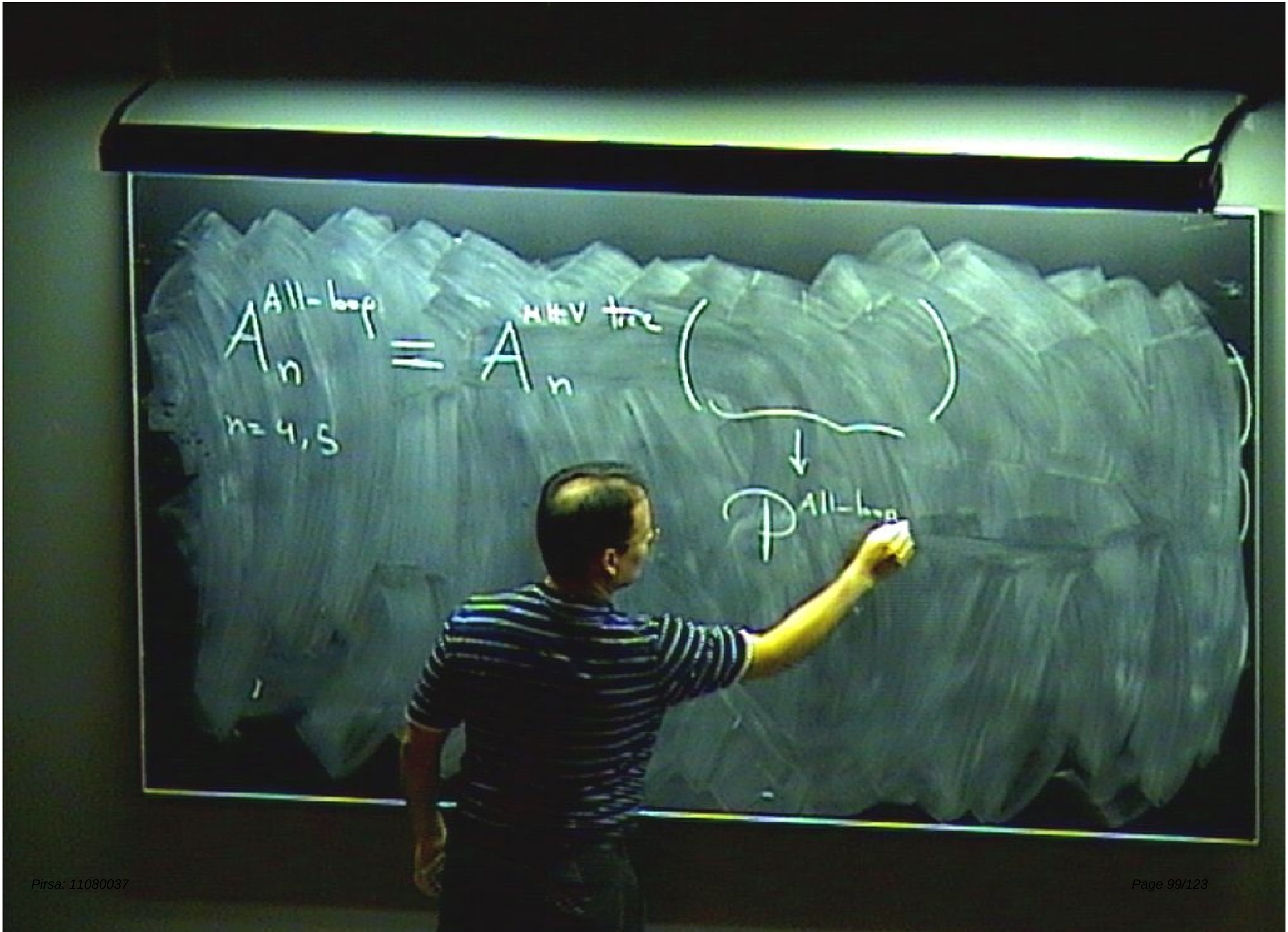
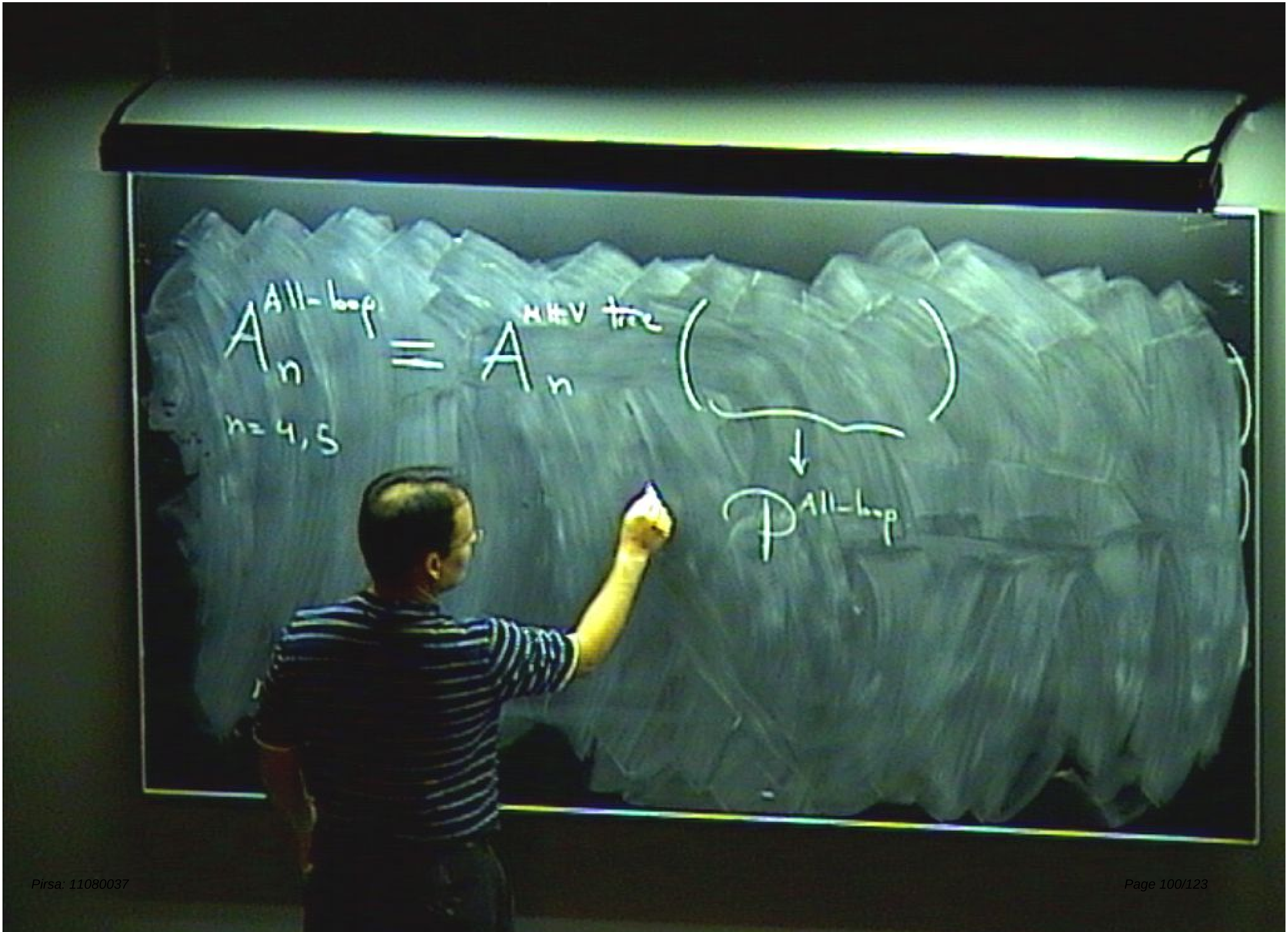

$A_n^{\text{All-loop}}$


A_n^All-loop ≡ A_n^MHV tree ( )
n=4,5
P^All-loop


$A_n^{\text{All-loop}} = A_n^{\text{MHV tree}} \left(\quad \right)$
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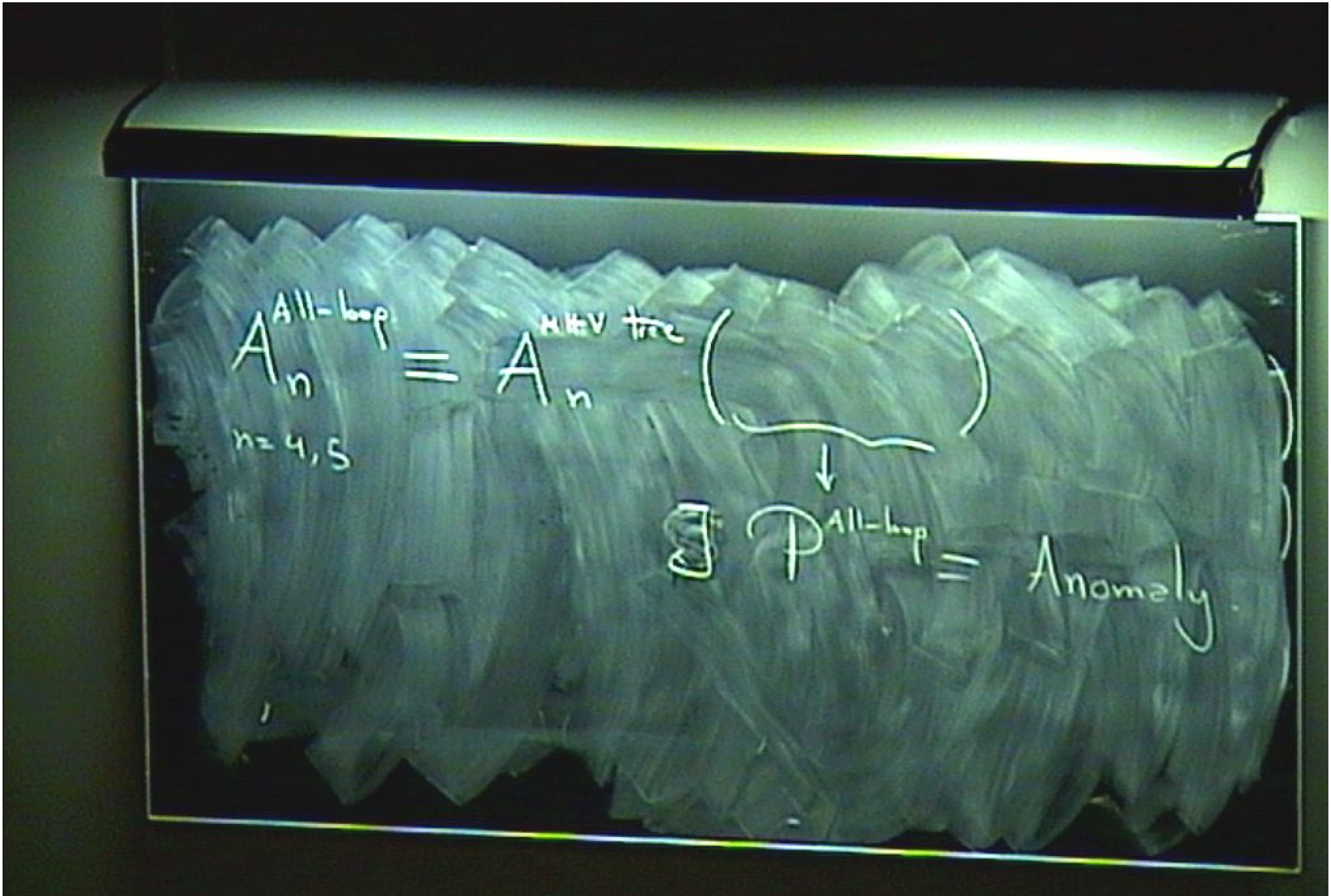
$$\exists \; \mathcal{D}^{\text{All-loop}} = \text{Anomaly}.$$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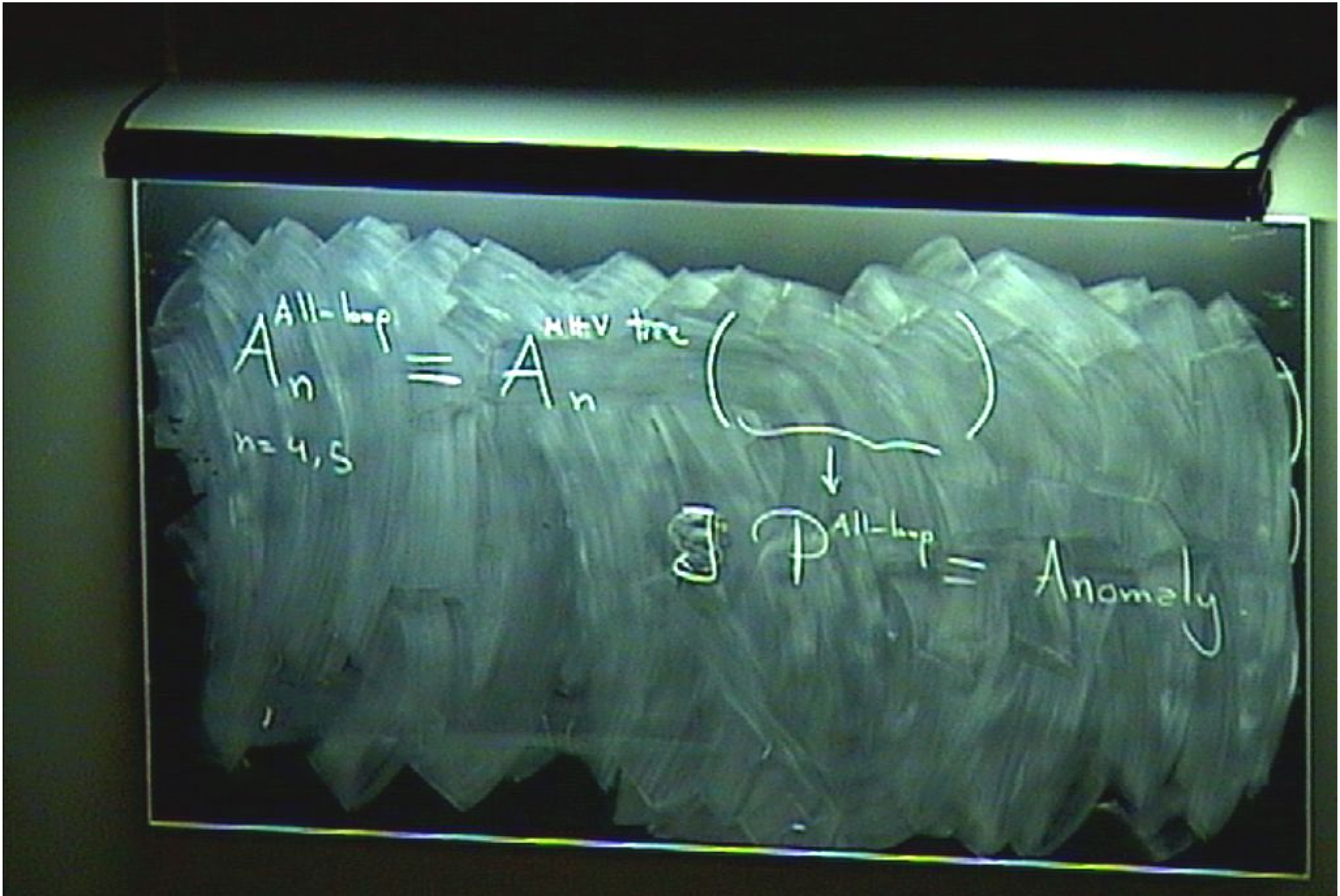
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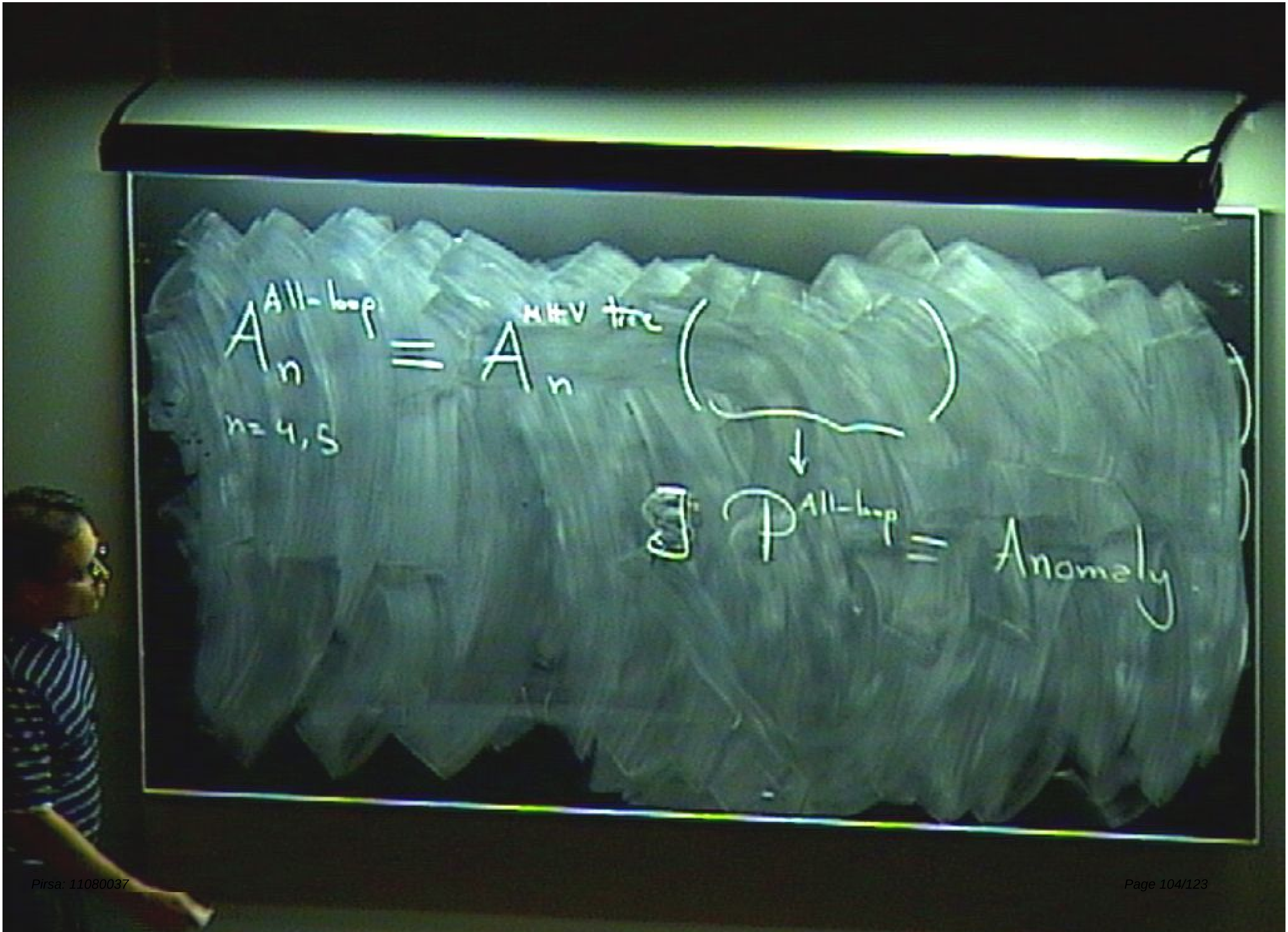
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$$\downarrow$$

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1-loop:

$$A_4^{\text{1-loop}} = A_4^{\text{tree}} \int \frac{d^4\ell}{\ell^2(\ell+p_1)^2(\ell+p_1+p_2)^2(\ell+p_1+p_2+p_3)^2}$$

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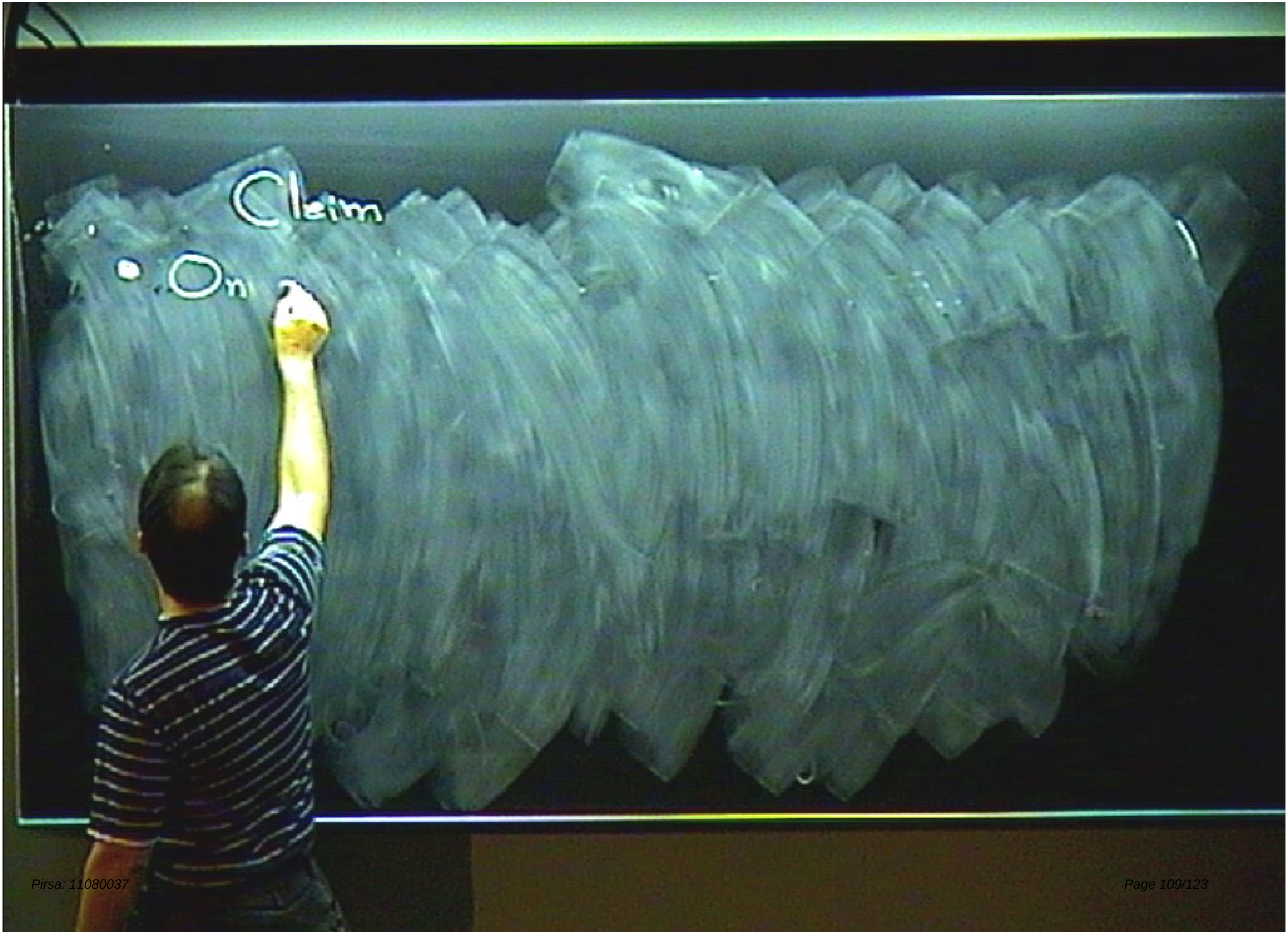
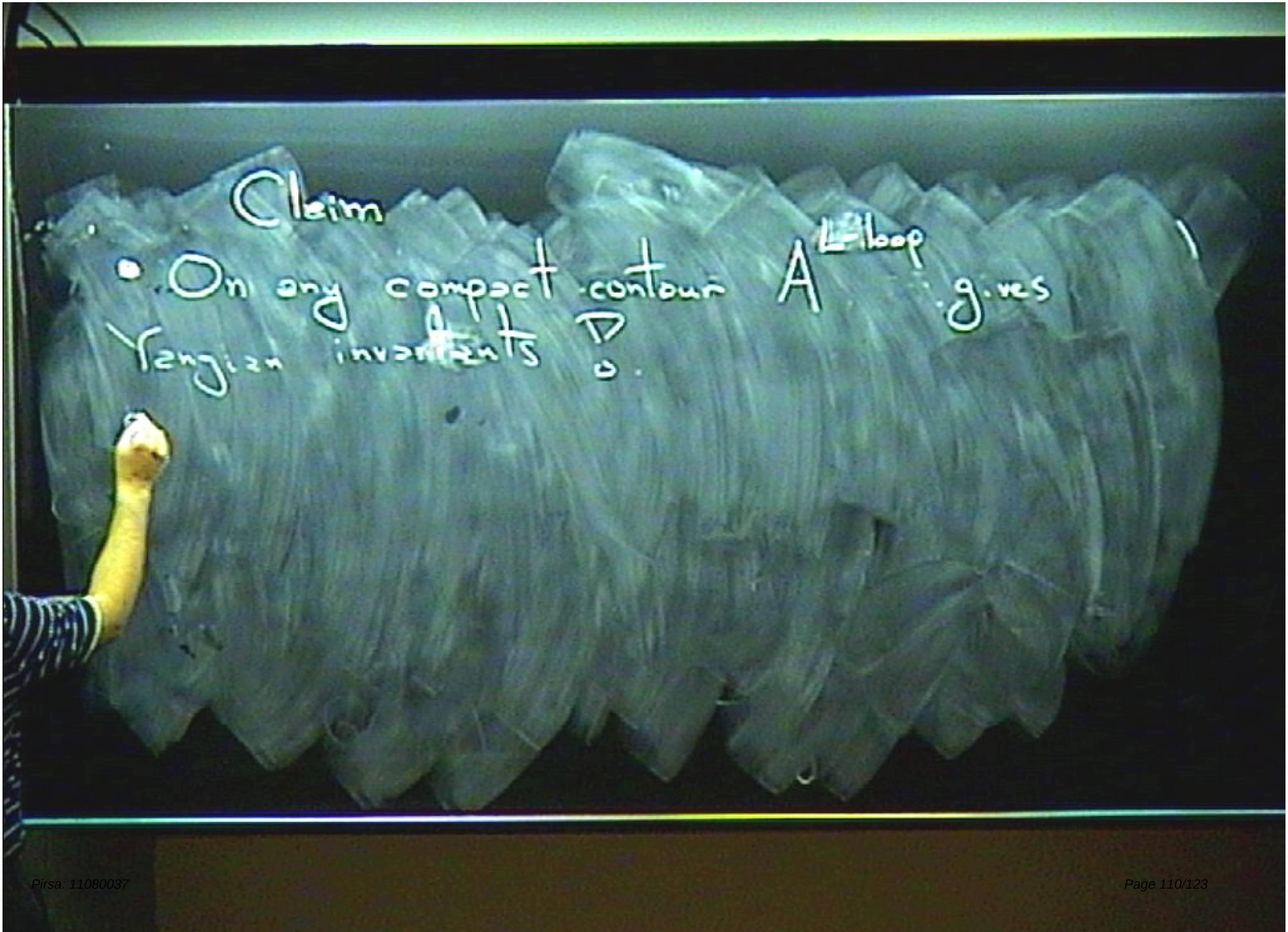
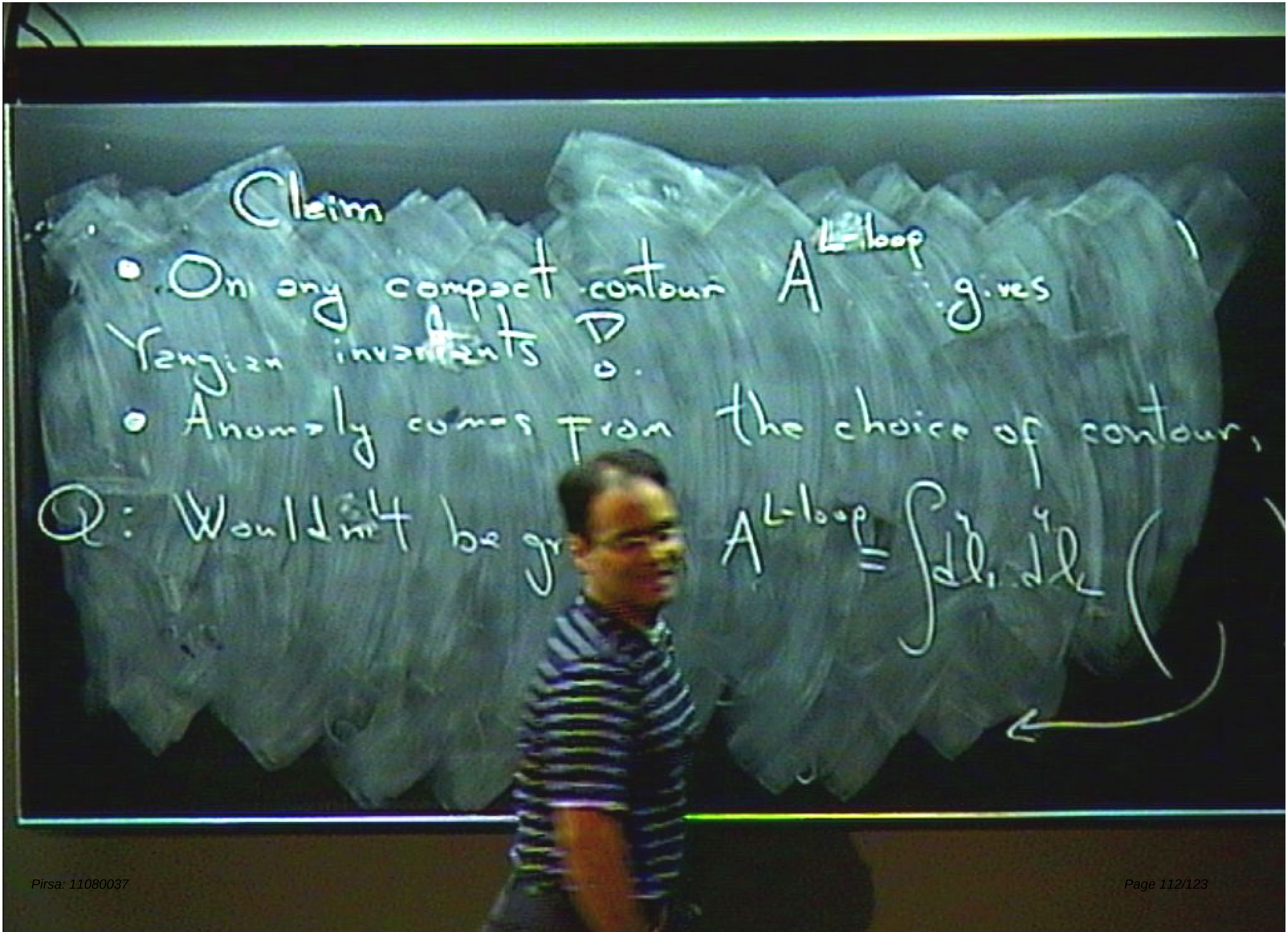

Claim
• On


Claim
• On any compact contour A^{L-loop} gives Yangian invariants ?

Claim

- On any compact contour $A^{L\text{-loop}}$ gives Yangian invariants ?
- Anomaly comes from the choice of contour,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Total derivative

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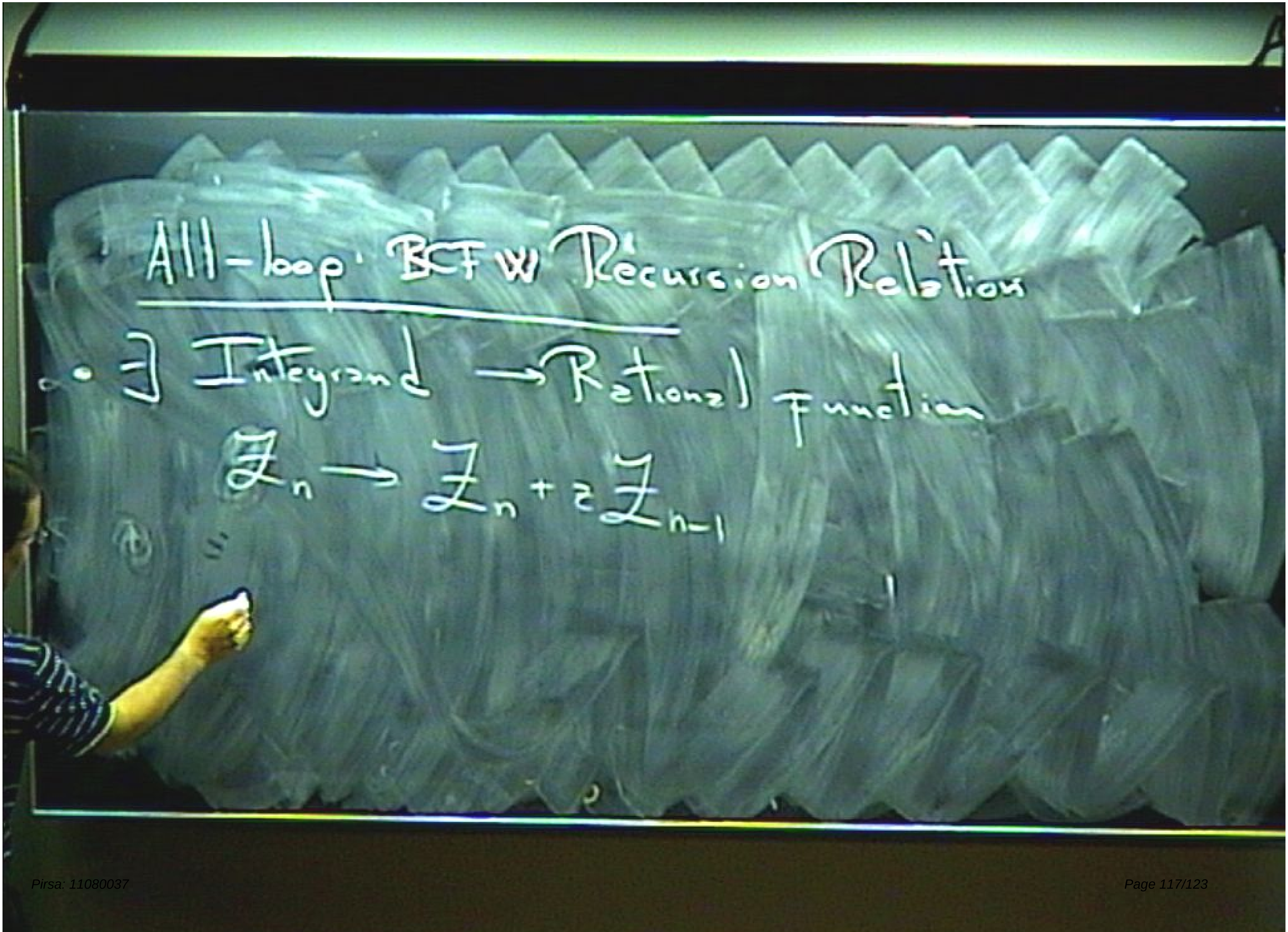
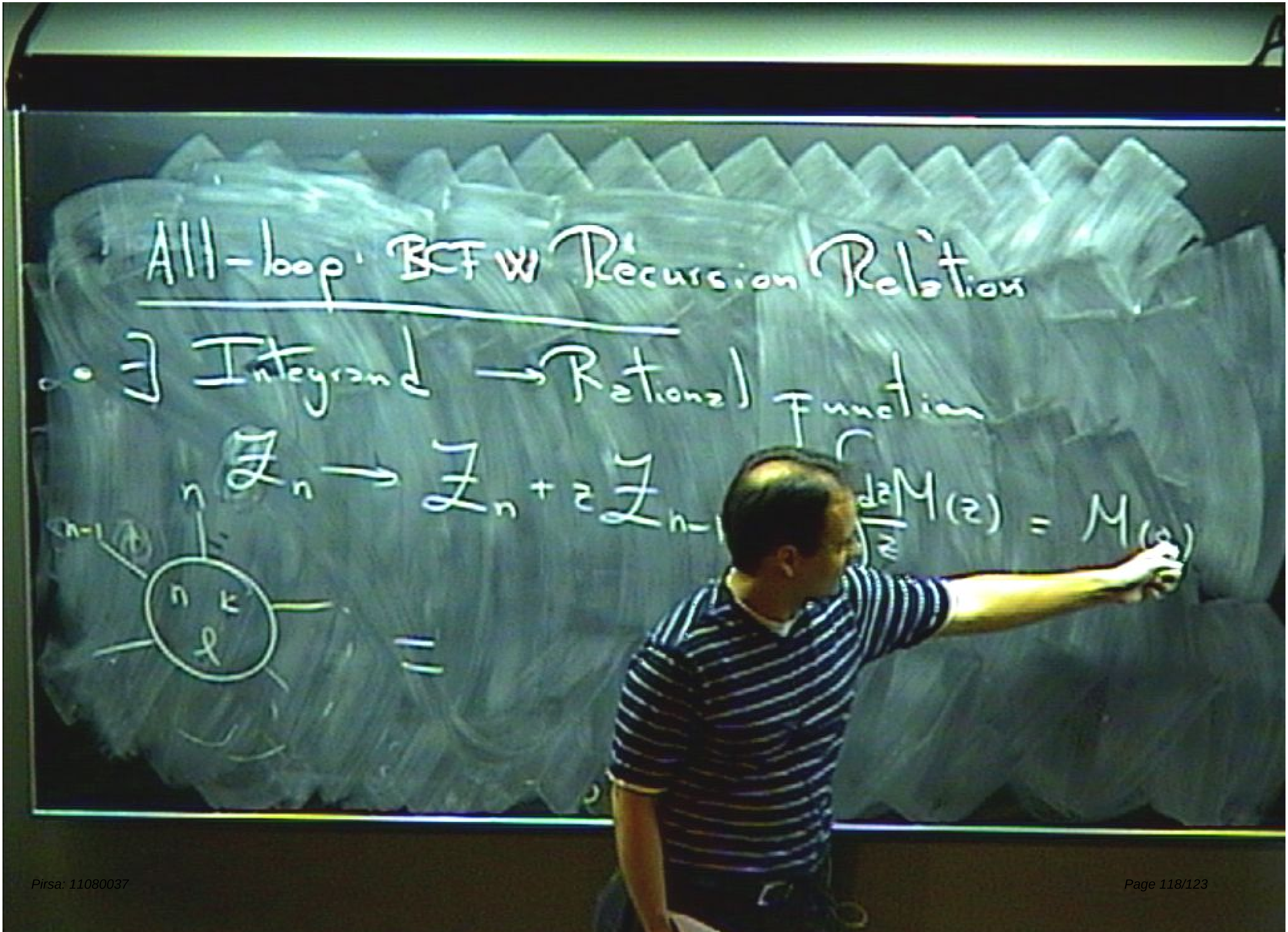
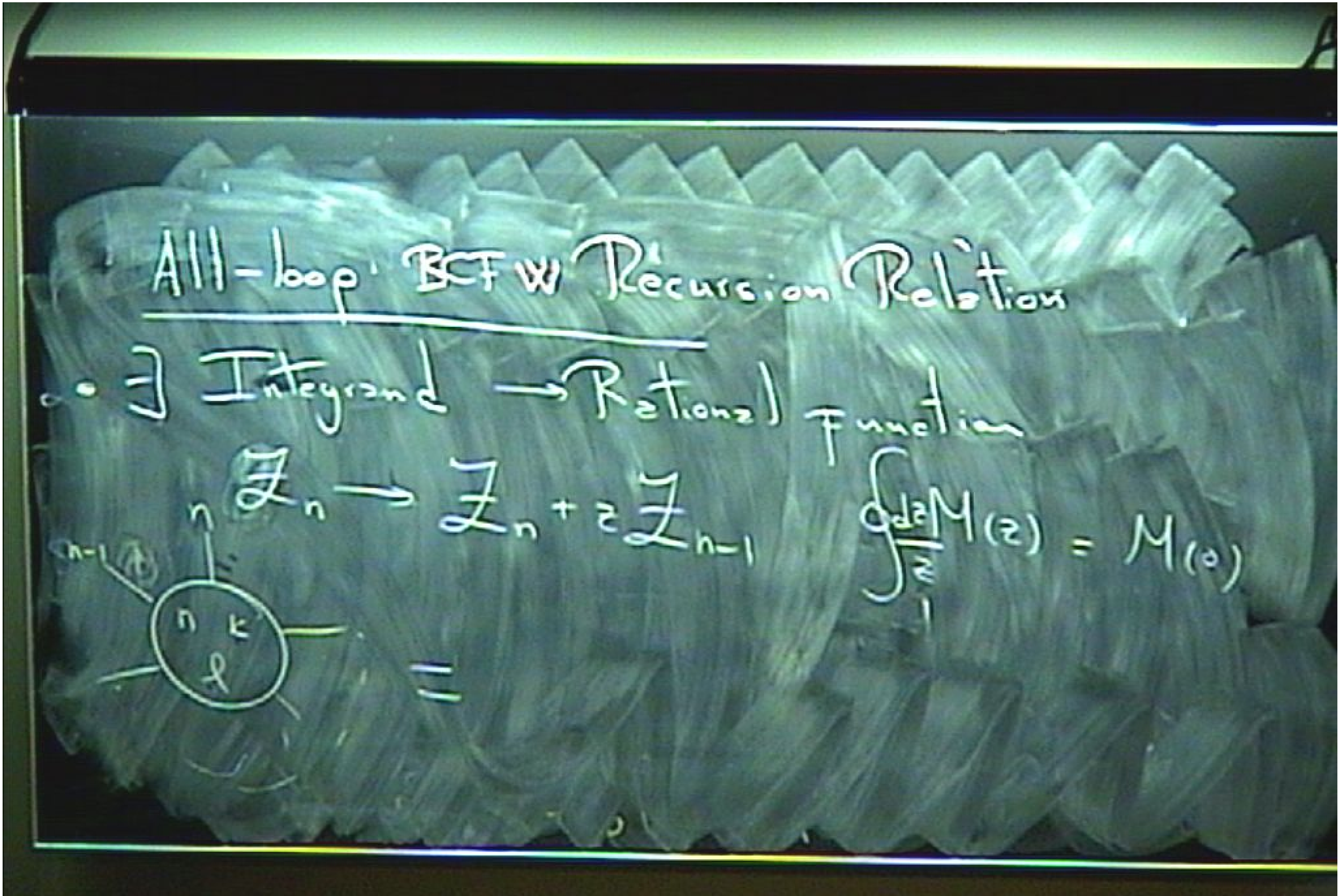
Total derivative

All-loop BCFW Recursion Relation

- $\exists$ Integrand $\rightarrow$ Rational Function


All-loop BCFW Recursion Relation
• ∃ Integrand → Rational Function
$\mathcal{Z}_n \to \mathcal{Z}_n + z\,\mathcal{Z}_{n-1}$


All-loop BCFW Recursion Relation
∃ Integrand → Rational Function
Zn → Zn + z Zn-1
dz/z M(z) = M(0)
n
n-1
n k
ℓ
=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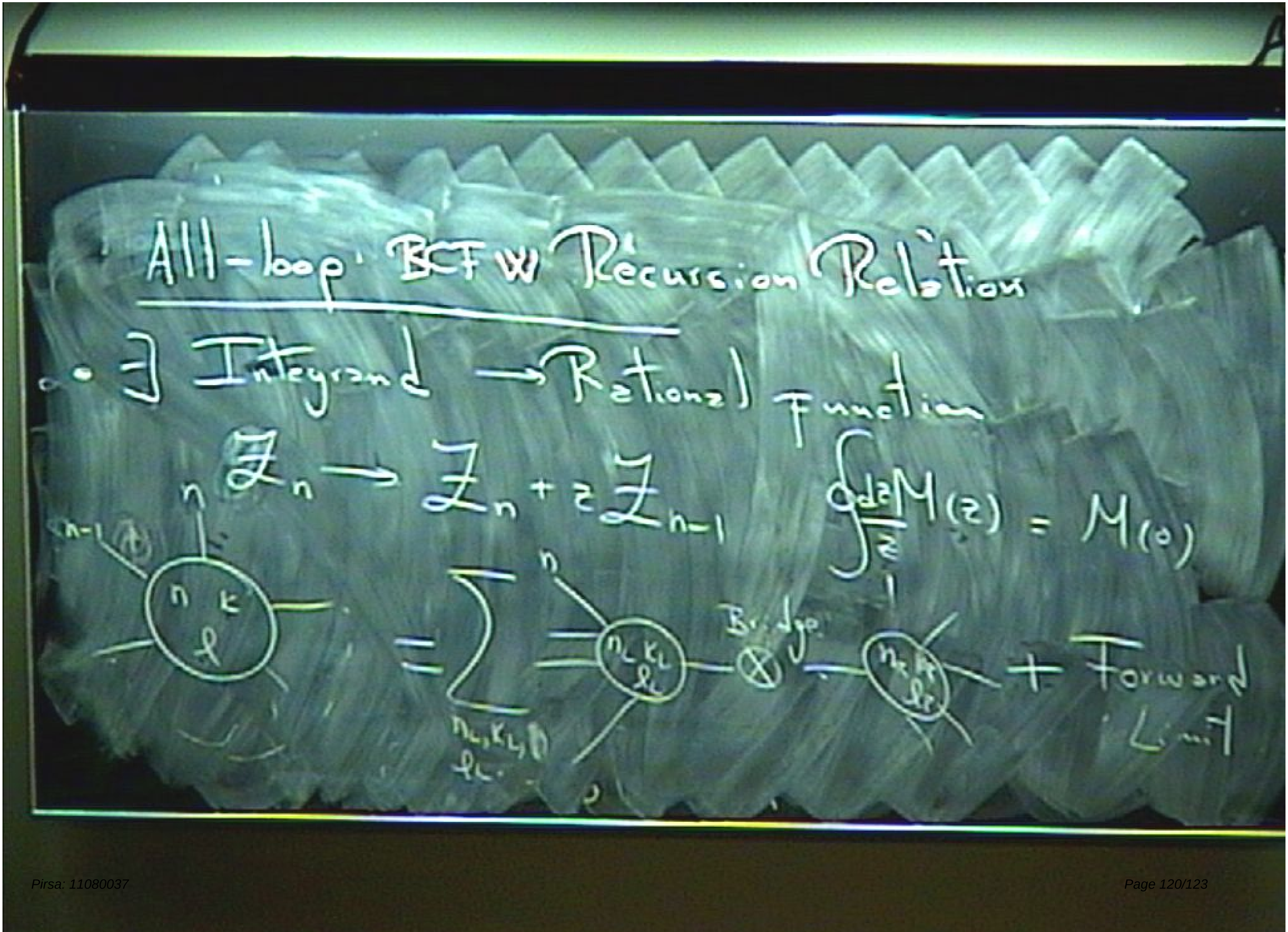
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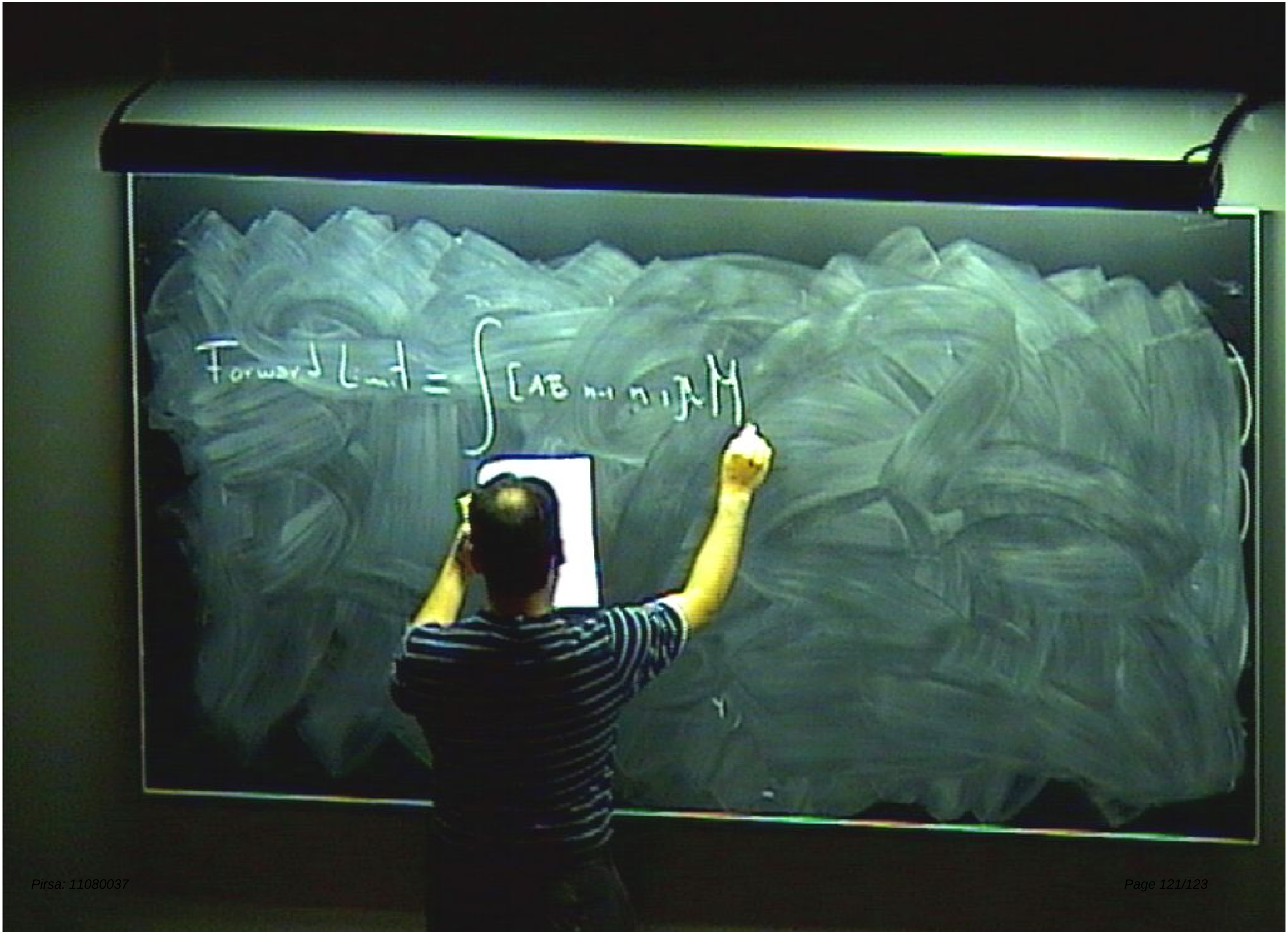
Bridge
+ Forward Limit

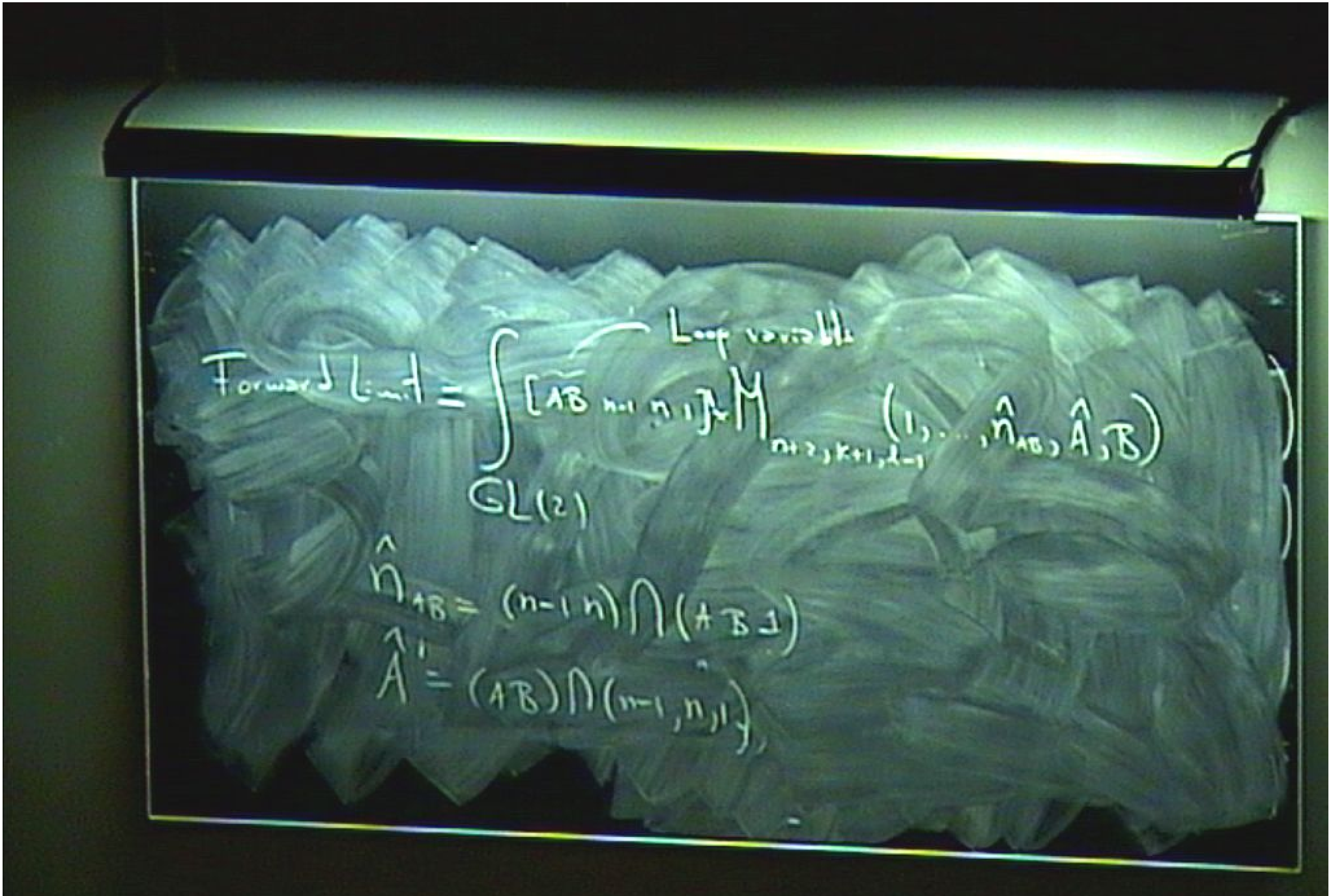

Forward Limit =

Loop variables

$$\text{Forward Limit} = \int_{GL(2)} [AB\; n{-}1\; n\; 1]\, M_{n+2,\,k+1,\,l-1}\left(1, \ldots, \hat{n}_{AB}, \hat{A}, \hat{B}\right)$$

$$\hat{n}_{AB} = (n{-}1\; n) \cap (A\, B\, 1)$$

$$\hat{A}' = (AB) \cap (n{-}1, n, 1)$$


Loop variable
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