Title: Grassmanian Basics

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Abstract:

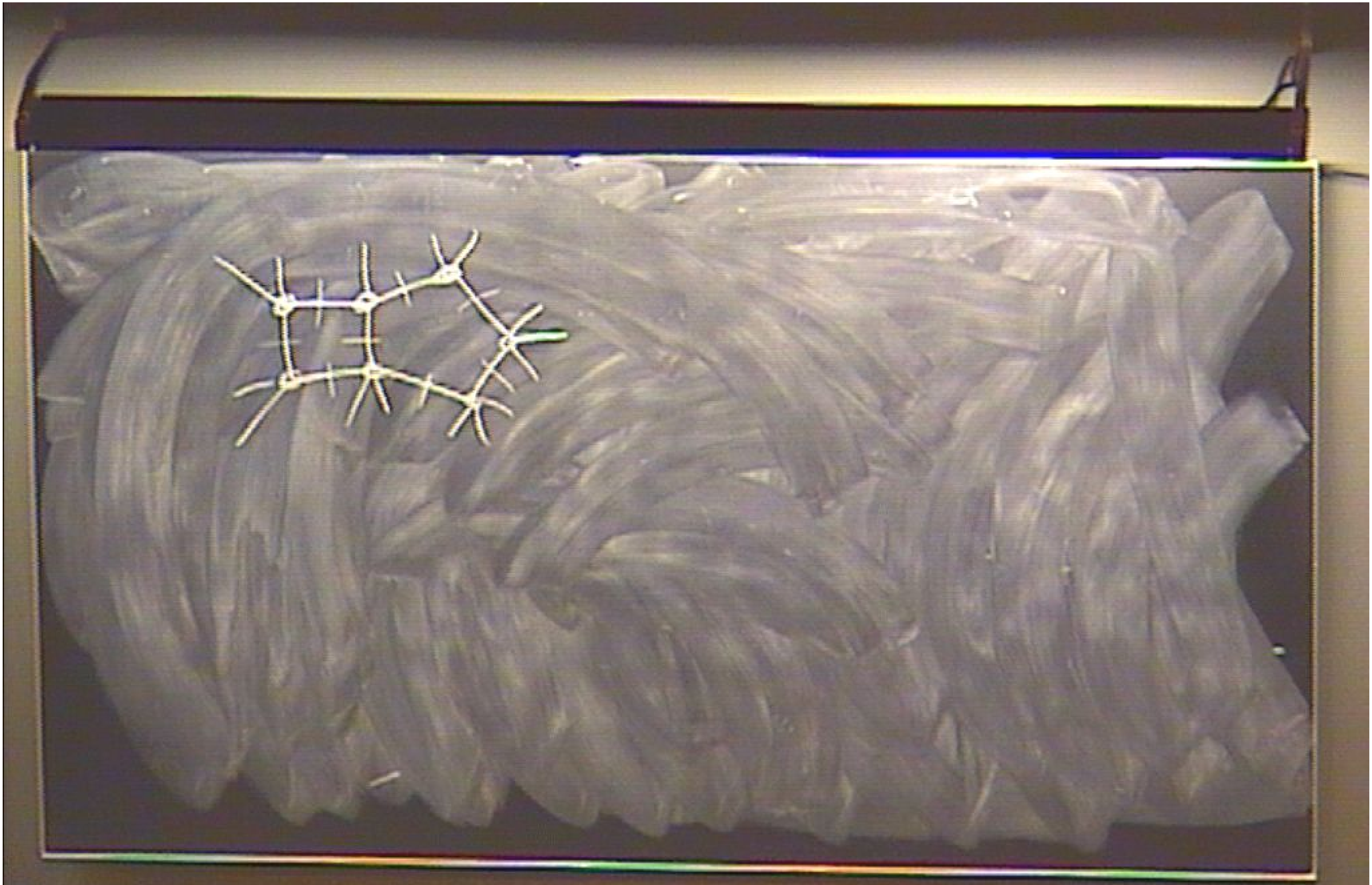
Grassmannian Basics

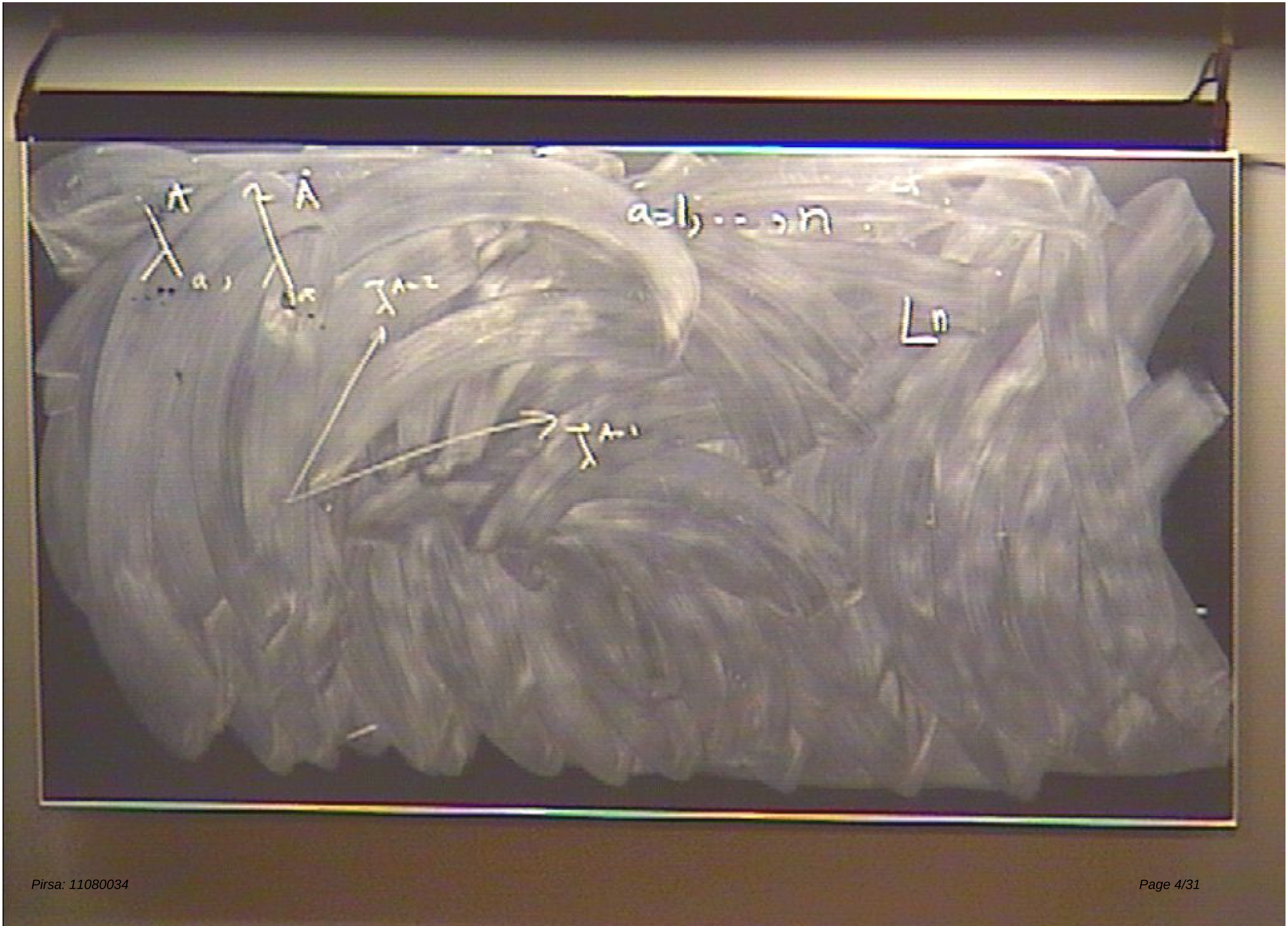
$$\mathcal{L}_{n,k} = \int \frac{d^{k\times n} C_{\alpha a}}{\mathrm{vol}\, GL(k)\, (12\cdots k)\cdots(n 1\cdots k-1)} \prod_{\alpha}^{k} \delta^{4|4}(C_{\alpha a} W_a) \longleftrightarrow \text{All loop planar integrand in Yangian invariant form}$$

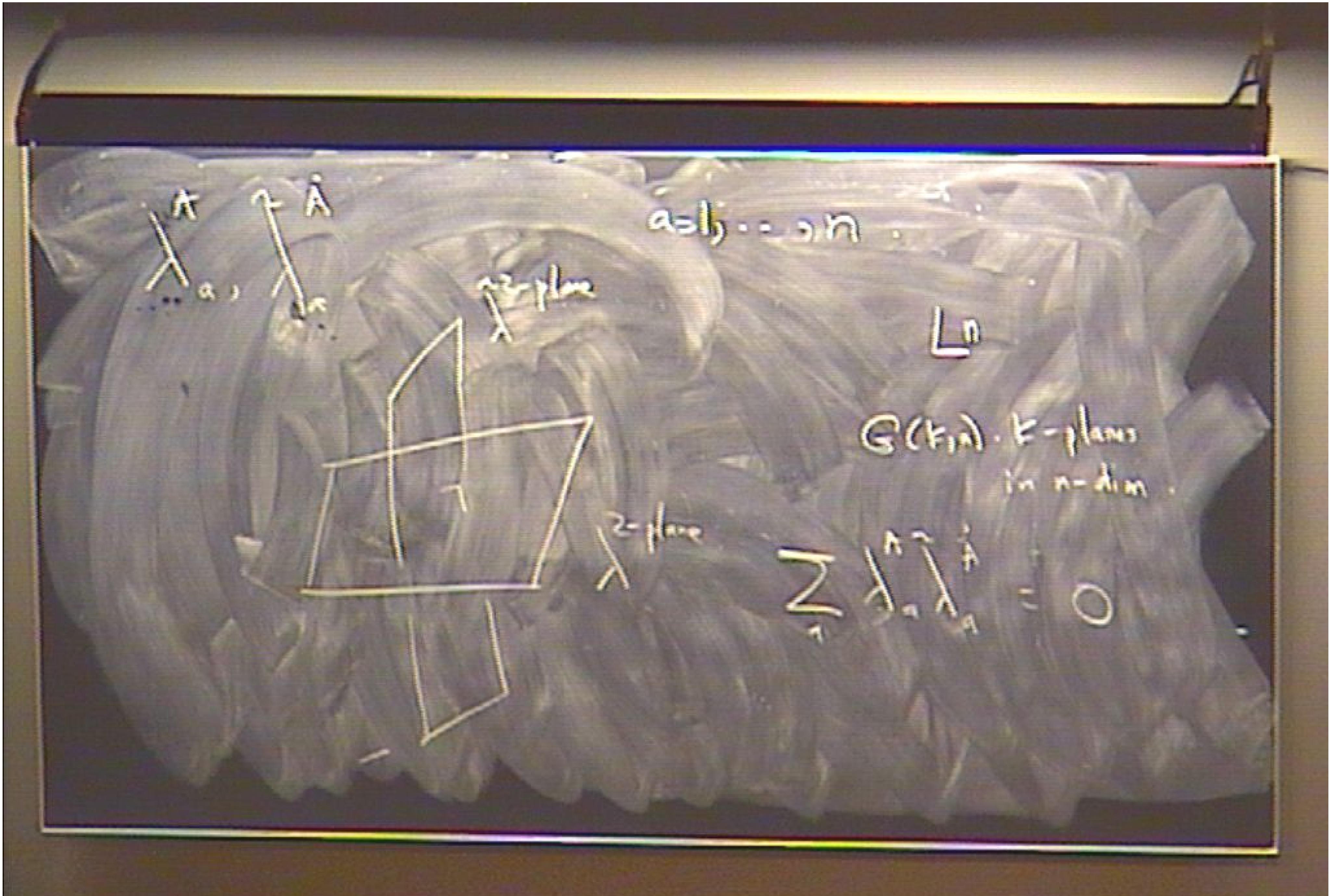
$\Big\downarrow$ Residues

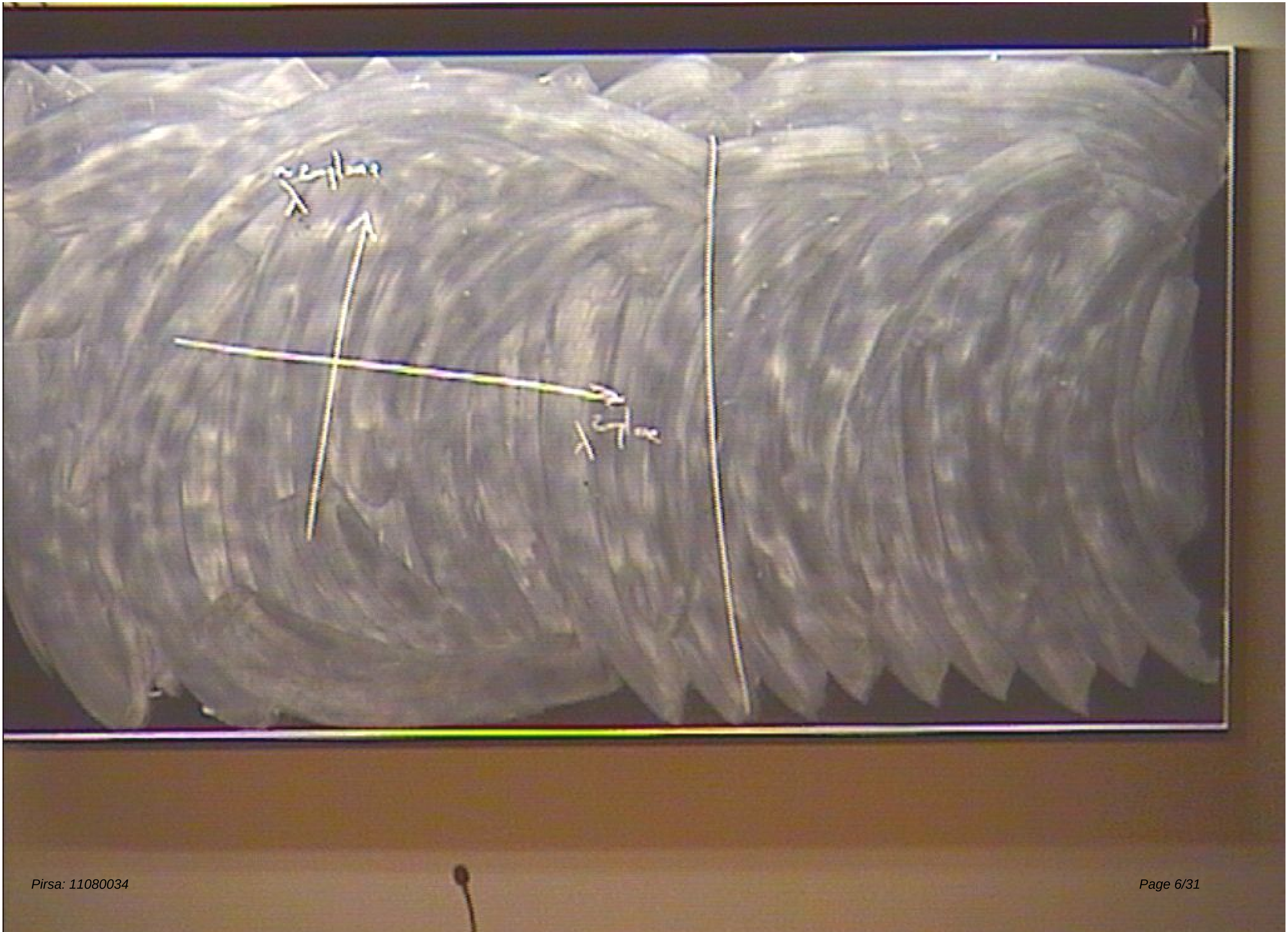
- All Yangian invariants (+ relations)

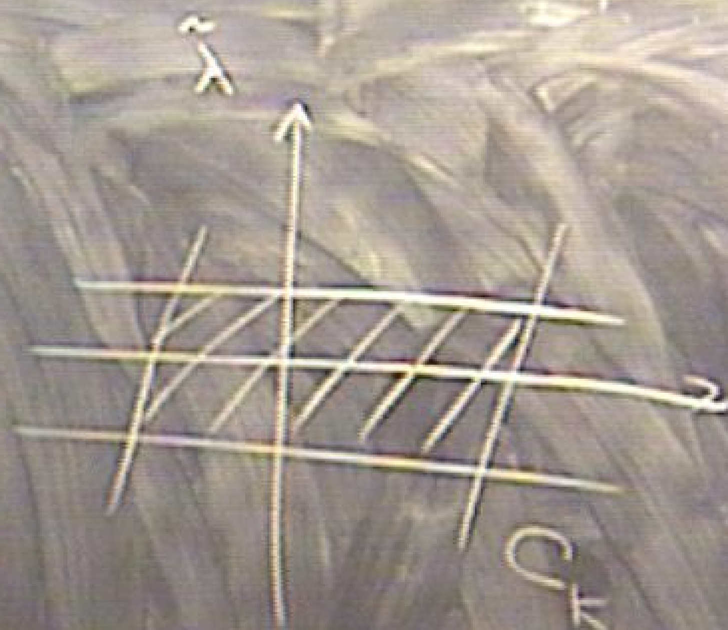
- Leading singularities




a=1, . . . , n
Ln


a=1, . . . , n
Ln
n-2-plane
2-plane
G(k,n) . k-planes
in n-dim .
$\sum_a \lambda_a^{\alpha} \tilde{\lambda}_a^{\dot{\alpha}} = 0$





C_k contains $\lambda^{\geq -p'}$

$\perp$ $\tilde{\lambda}^{\geq -p'}$

(Impossible if $k=0,1$)

$\tilde{C}_{n-k}$ $\perp$ $\lambda^{\geq -p'}$

contains $\tilde{\lambda}^{\geq -p'}$.

$$\begin{pmatrix} 1 & a_1 & & 1 \\ 1 & a_2 & & 1 \\ \vdots & \vdots & & \vdots \\ 1 & a_k & & 1 \end{pmatrix} = C_{\alpha a}$$

$\alpha = 1, \dots, k \qquad a = 1, \dots, n$

$$\begin{pmatrix} 1 & 0 & 0 & \boxed{\begin{matrix} k(n-k) \\ C^{\alpha}_{s} \end{matrix}} \\ 0 & 1 & 0 & \\ 0 & 0 & \vdots & \\ \vdots & \vdots & 1 & \end{pmatrix}$$

"Invariant" "gauge redundancy" $\quad C_{\alpha a} \to L_{\alpha}^{\ \beta} C_{\beta a} \qquad GL(k)$

$$\dim G(k,n) = k\,n - k^2 = k(n-k) = \dim G(n-k,n)$$

Grassmannian Basics

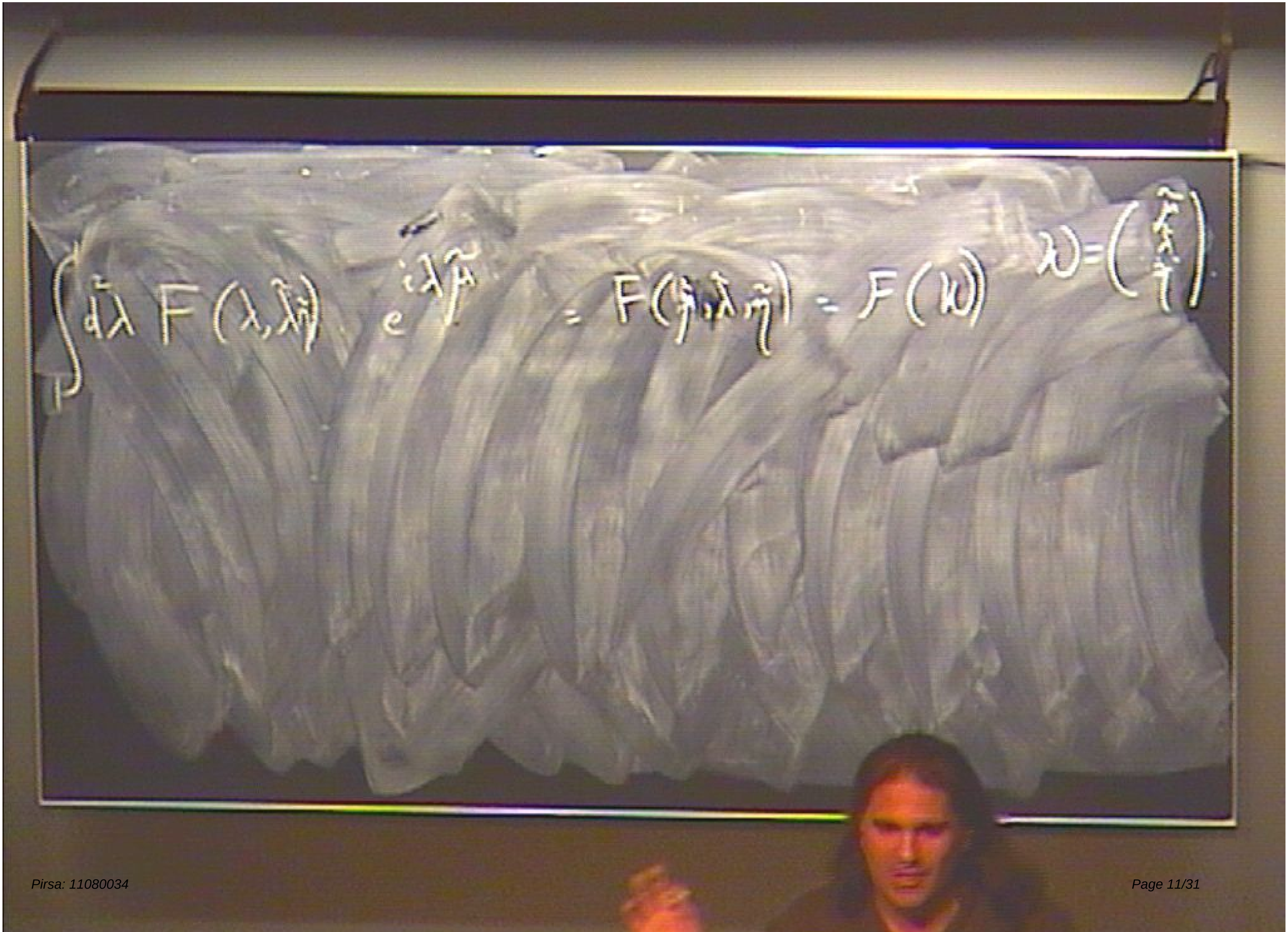
$$\prod_{\alpha=1}^{k} \delta^2[C_{\alpha a}\tilde{\lambda}_a] \int$$

Grassmannian Basics

$$\prod_{\alpha=1}^{k} \delta^2[C_{\alpha a}\tilde{\lambda}_a] \int d^{2\times k}\rho^{\alpha} \prod_{a=1}^{n} \delta^2[\rho^{\alpha} C_{\alpha a} - \lambda_a]$$

$$C \to tC$$

$$t^{-2k}\, t^{-2k}\, t^{+4k} = 1 \qquad \prod_{\alpha=1}^{4} \delta^{4|4}(C_{\alpha a}\tilde{\eta}_a)$$


$\int d^2\tilde{\lambda}\, F(\lambda, \tilde{\lambda}\eta)\, e^{i\tilde{\lambda}\tilde{\mu}} = F(\tilde{\mu}, \tilde{\lambda}\eta) = F(W)$
$W = \begin{pmatrix} \tilde{\mu} \\ \tilde{\lambda} \\ \eta \end{pmatrix}$

$$\int d^2\lambda \, F(\lambda,\tilde{\lambda},\tilde{\eta}) \; e^{i\lambda\tilde{\mu}} = F(\tilde{\mu},\tilde{\lambda},\tilde{\eta}) = F(\mathcal{W}) \qquad \mathcal{W} = \begin{pmatrix} \tilde{\mu} \\ \tilde{\lambda} \\ \tilde{\eta} \end{pmatrix}$$

$$\downarrow$$

$$\prod_{\alpha=1}^{k} \delta^2(C_{\alpha a}\tilde{\lambda}_a)\; \delta^4(C_{\alpha a}\tilde{\eta}_a)\; \delta^2(C_{\alpha a}\tilde{\mu}_a)$$

$$\int d^2\lambda \, F(\lambda, \tilde\lambda, \tilde\eta) \, e^{i\lambda\tilde\mu} = F(\tilde\mu, \tilde\lambda, \tilde\eta) = F(W) \qquad W = \begin{pmatrix} \tilde\mu \\ \tilde\lambda \\ \tilde\eta \end{pmatrix}$$

$$\downarrow$$

$$\prod_{\alpha=1}^{k} \delta^2(C_{\alpha a}\tilde\lambda_a)\, \delta^4(C_{\alpha a}\tilde\eta_a)\, \delta^2(C_{\alpha a}\tilde\mu_a)$$

$$= \prod_{\alpha=1}^{k} \delta^{4|4}(C_{\alpha a} W_a)$$

$$\begin{pmatrix} 1 & 0 & 0 & \\ 0 & 1 & 0 & \boxed{} \\ 0 & 0 & \vdots & \\ \vdots & \vdots & 1 & \end{pmatrix}$$

$$\int \frac{d^{k\times n} C}{\mathrm{vol}\,GL(k)} \prod_{\alpha=1}^{k} \delta^{4|4}\left(C_{\alpha a} W_a\right)$$

$$C \to t\,C$$

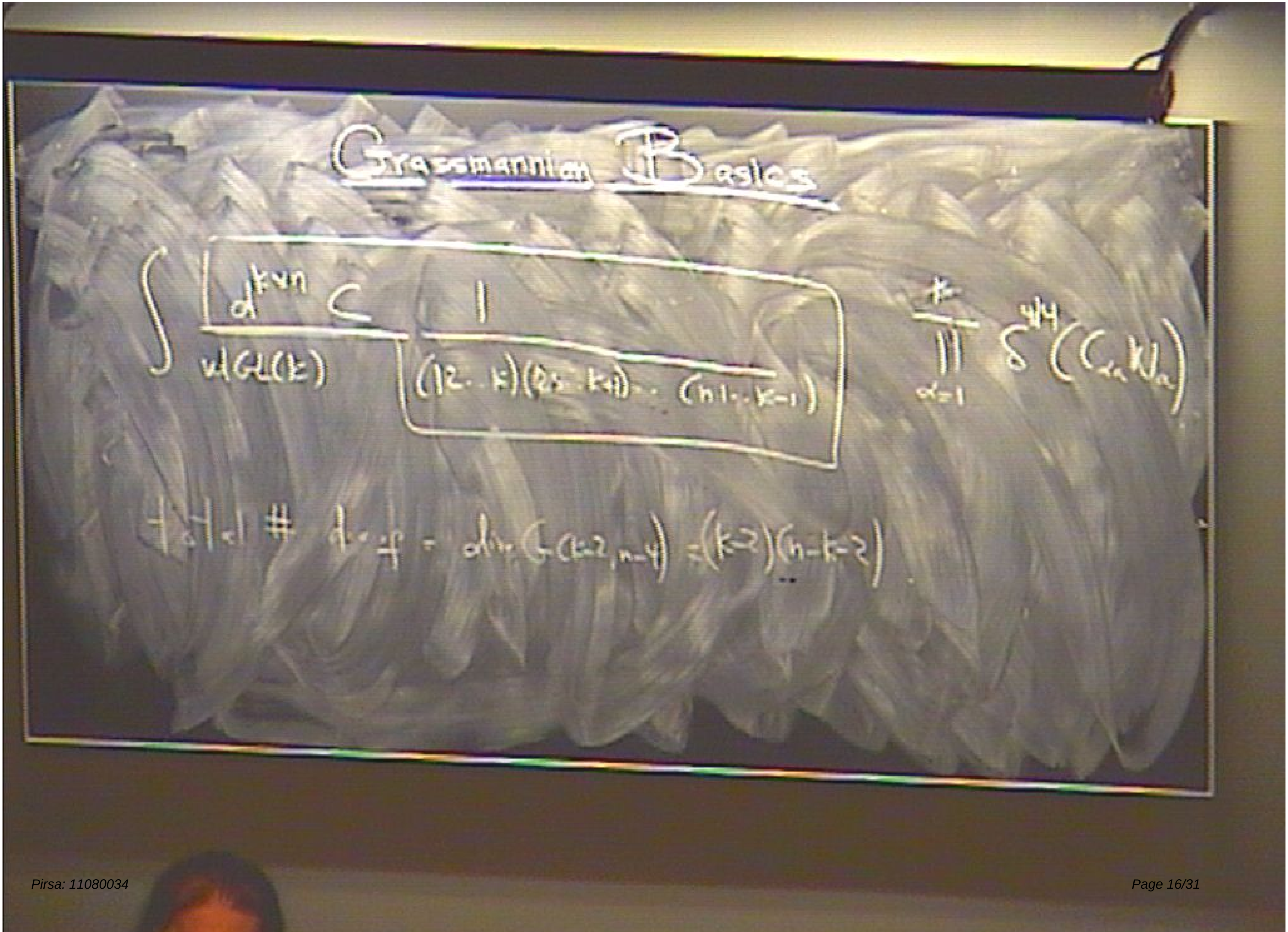
$$t^{k\times n}$$

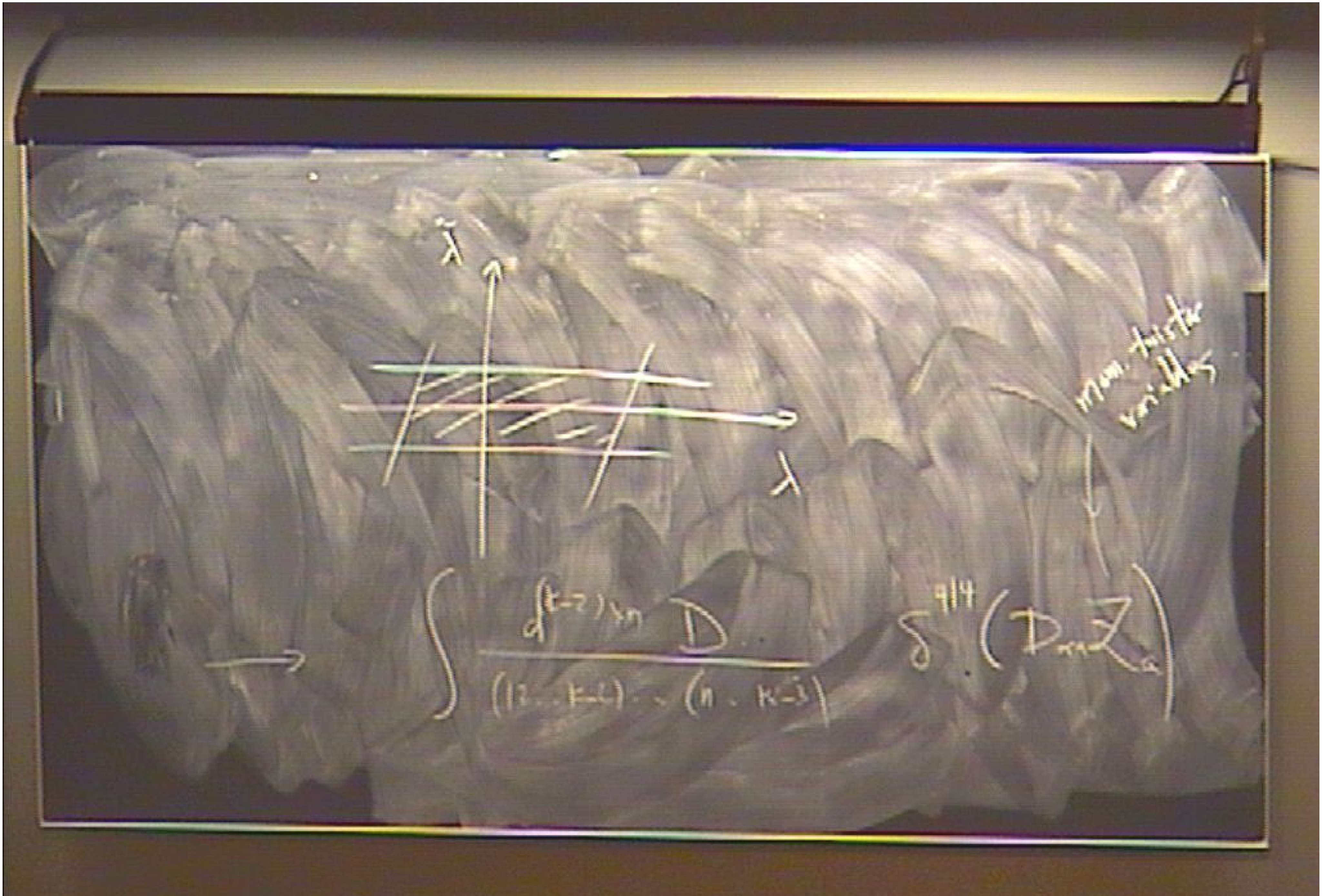
$$\left(\qquad \right)$$

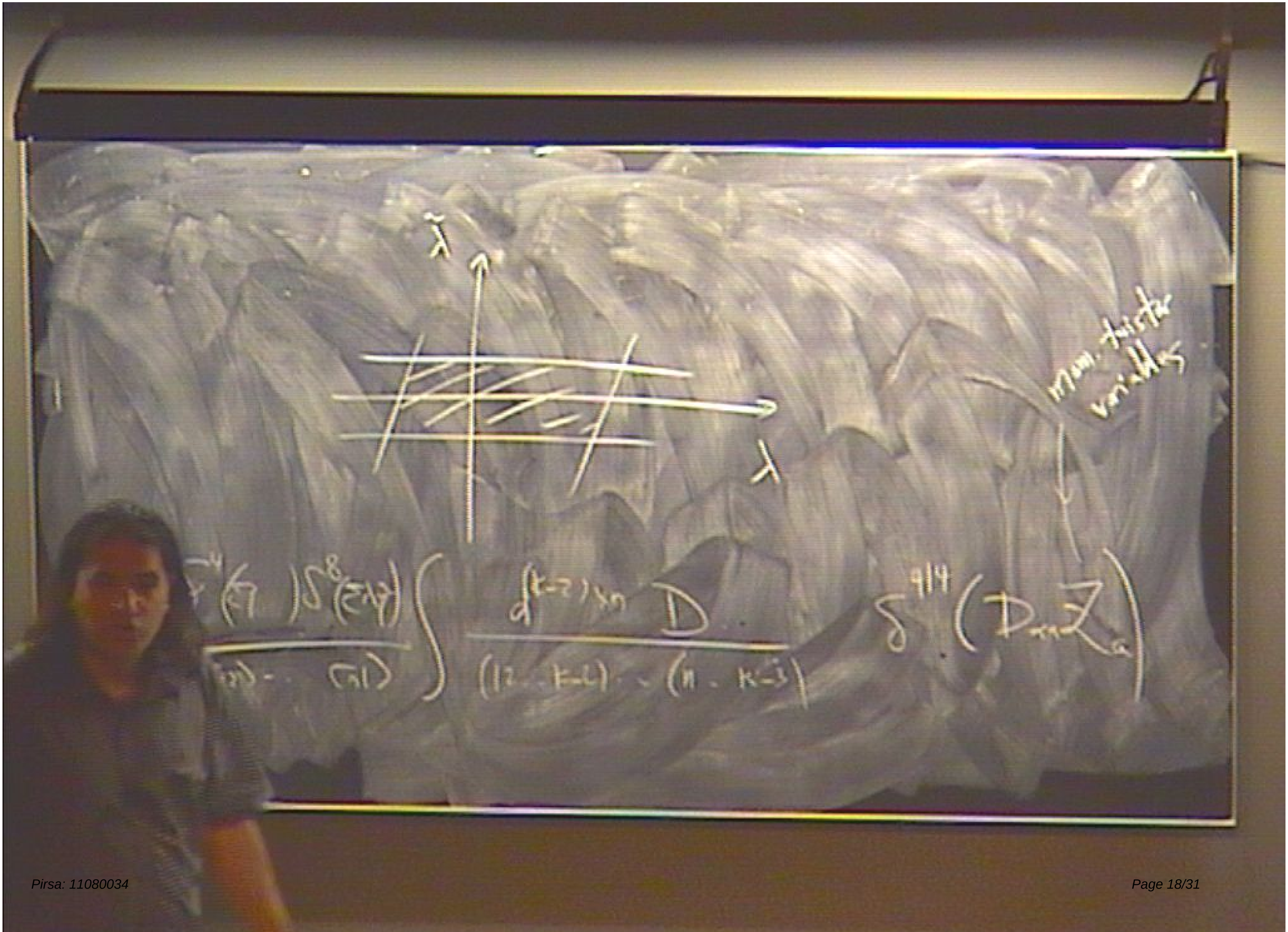
$$(m_1 \ldots m_k) = \epsilon^{\alpha_1 \ldots \alpha_k} C_{\alpha_1 m_1} \ldots C_{\alpha_k m_k}$$

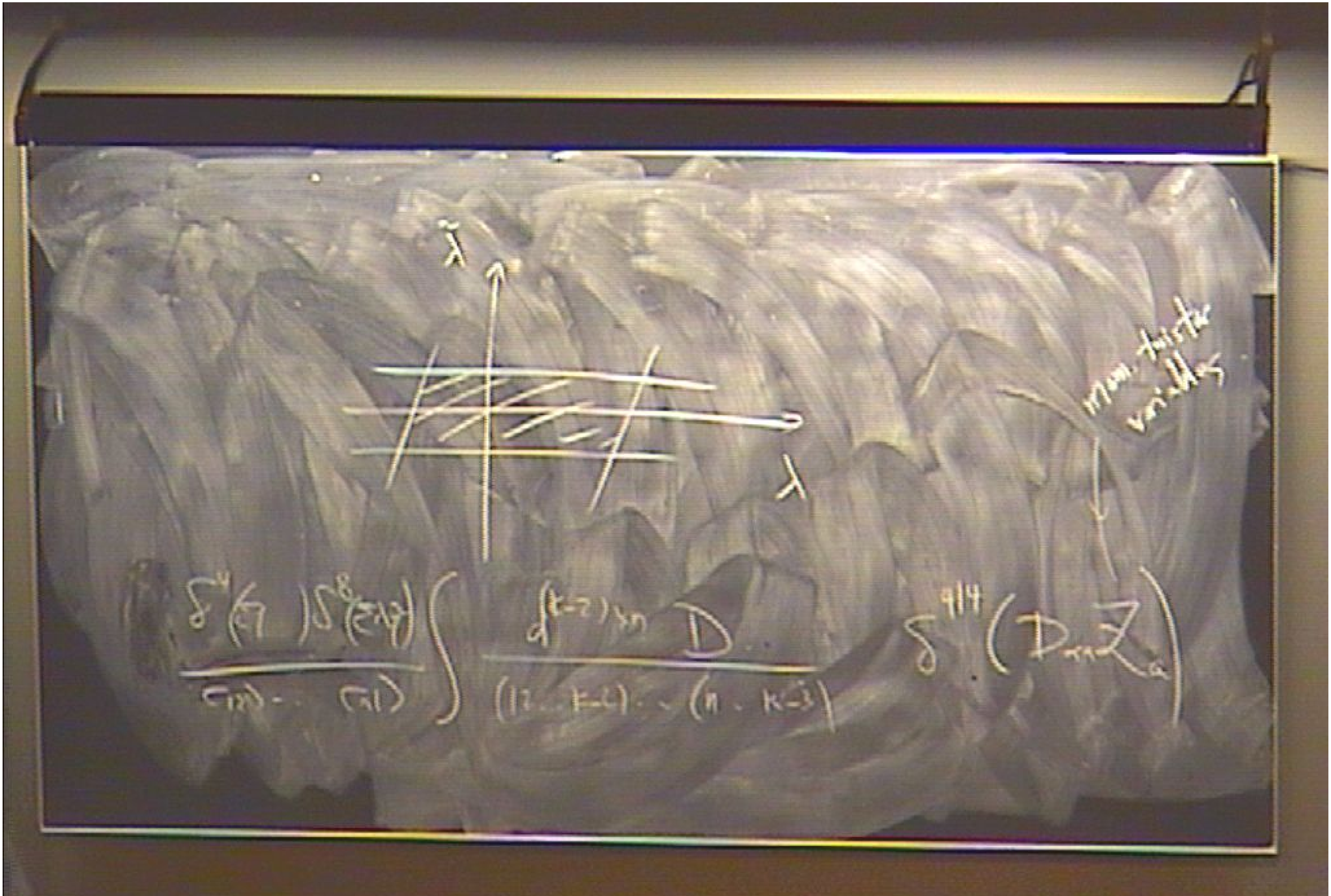
Grassmannian Basics

$$\int \frac{d^{k\times n} C}{\mathrm{vol}(GL(k))} \, \frac{1}{(12\cdots k)(23\cdots k{+}1)\cdots(n1\cdots k{-}1)} \, \prod_{\alpha=1}^{k} \delta^{4|4}\left(C_{\alpha a} \mathcal{W}_a\right)$$


Grassmannian Basics
(12…k)(23…k+1)…(n1…k−1)
dim G(k−2, n−4) = (k−2)(n−k−2)


λ̃
λ
mom. twistor
variables


mom. twistor variables


$\tilde{\lambda}$
λ
mom. twistor variables
$\frac{\delta^4(\ )\,\delta^8(\ )}{\langle 12\rangle \cdots \langle n1\rangle} \int \frac{d^{(k-2)\times n} D}{(12 \ldots k-2) \cdots (n \ldots k-3)} \; \delta^{4|4}(D \cdot Z)$

Simplest residue: mom. twistor space $k \cdot z = \underline{1}$

$$\int \frac{1}{\mathrm{vol}} \frac{dD_1}{D_1} \cdots \frac{dD_n}{D_n} \quad \delta^{4|4}(D \cdot Z)$$

Simplest residue: mom. twistor space $k - 2 = 1$.

$$\int \frac{1}{GL(1)} \frac{dD_1}{D_1} \cdots \frac{dD_5}{D_5} \quad \delta^{4|4}(D_a Z_a)$$

$$Z_5 + D_1 Z_1 + \cdots + D_4 Z_4 = 0$$

Simplest residue: mom. twistor space $\chi \cdot z = \Delta$

$$\int \frac{1}{GL(1)} \frac{dD_1}{D_1} \cdots \frac{dD_5}{D_5} \; \delta^{4|4}(D_a Z_a)$$

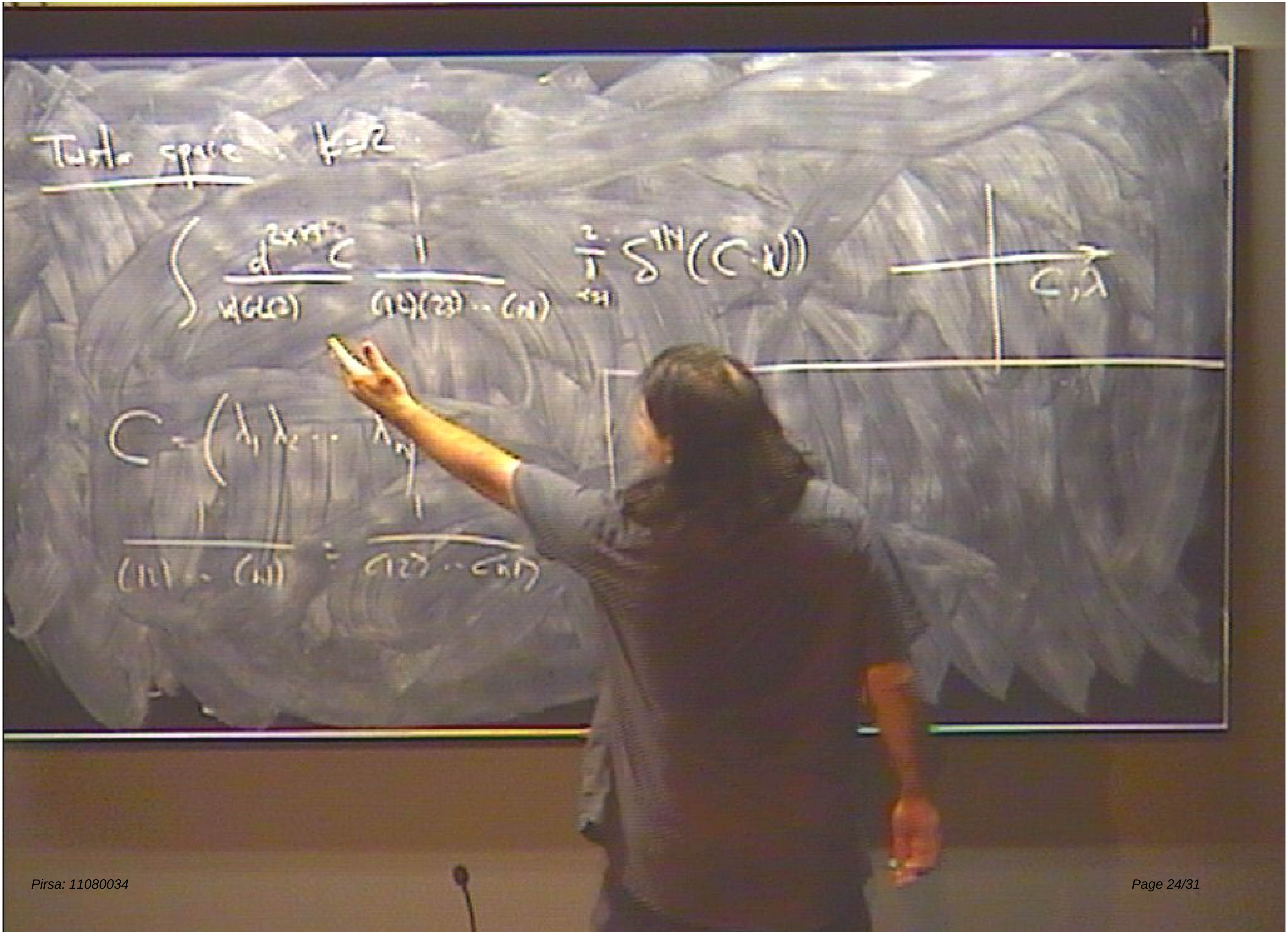
$$Z_5 + D_1 Z_1 + \cdots + D_4 Z_4 = 0$$

$$= [12345]$$

$$= \frac{\delta^4(\eta_1 \langle 2345 \rangle + \text{cyclic})}{\langle 1234 \rangle \cdots \langle 5123 \rangle}$$

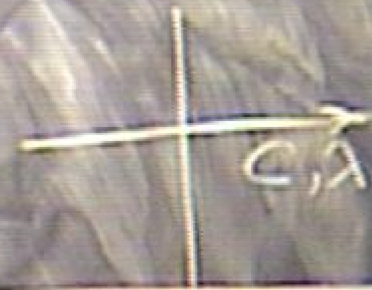
Twistor space : $k=2$

$$\int \frac{d^{2\times n} C}{\mathrm{vol}(GL(2))} \frac{1}{(12)(23)\cdots(n1)} \prod_{\alpha=1}^{2} \delta^{4|4}(C\cdot W)$$


Twistor space : k=2

Twistor space : $k=2$

$$\int \frac{d^{2\times n} C}{\mathrm{vol}(GL(2))} \frac{1}{(12)(23)\cdots(n1)} \prod_{a=1}^{2} \delta^{4|4}(C\cdot W)$$



$$C = \begin{pmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_n \end{pmatrix}$$

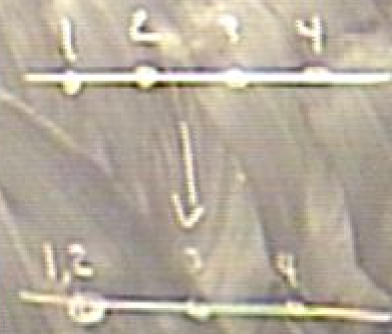
$$\frac{1}{(12)\cdots(n1)} = \frac{1}{\langle 12\rangle \cdots \langle n1\rangle}$$

$G(2,4)$

$$C = \begin{pmatrix} 1 & x_2 & 0 & x_4 \\ 0 & y_2 & 1 & y_4 \end{pmatrix}$$

$$\int \frac{dx_2\, dy_2\, dx_4\, dy_4}{x_2\, y_2\, x_4\, y_4} \qquad \int \delta^{4|4}(C\,W)$$

$$\int \frac{dx_2}{x_2} \frac{dz_2}{z_2} \frac{dz_4}{z_4}$$

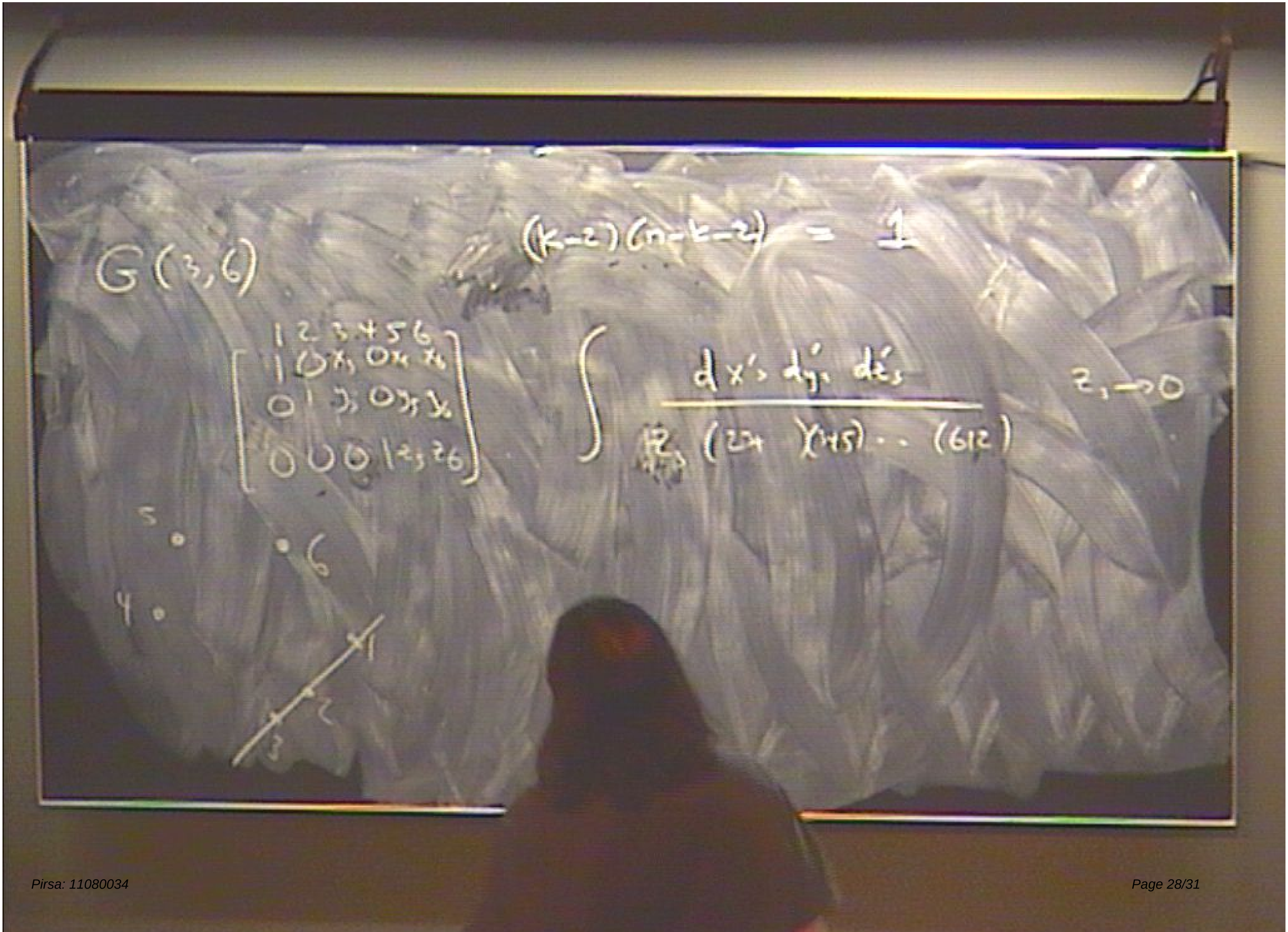


$G(3,6)$

$$(k-2)(n-k-2) = 1$$

$$\begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 1 & 0 & x_3 & 0 & x_5 & x_6 \\ 0 & 1 & y_3 & 0 & y_5 & y_6 \\ 0 & 0 & 0 & 1 & z_5 & z_6 \end{bmatrix} \end{array}$$

$$\int \frac{dx'_3 \, dy'_3 \, dz'_3}{z_3 \, (234)(345) \cdots (612)} \qquad z_1 \to 0$$

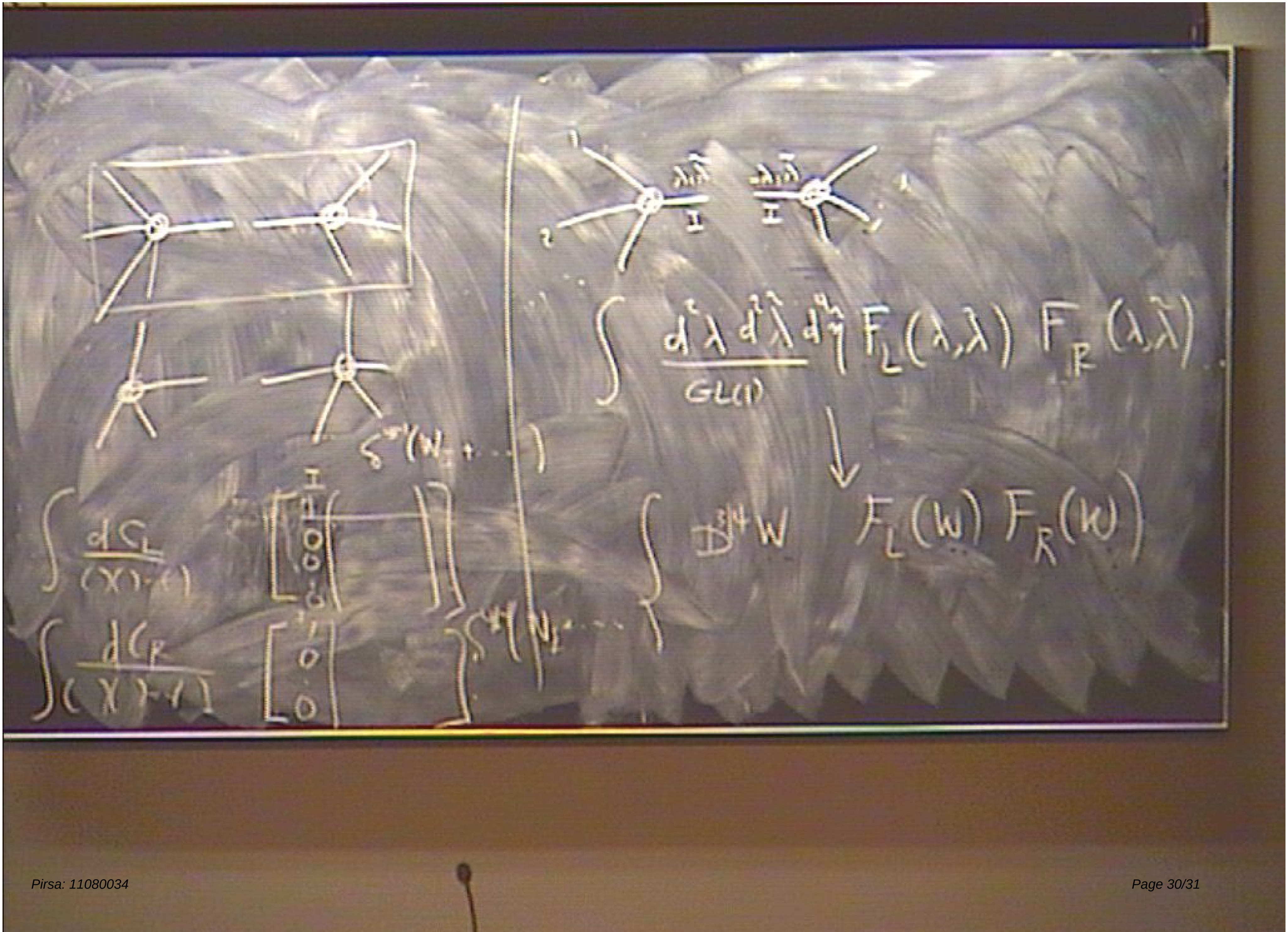

G(3,6)
(k-2)(n-k-2) = 1
1 2 3 4 5 6
z3 → 0

$G(3,6)$

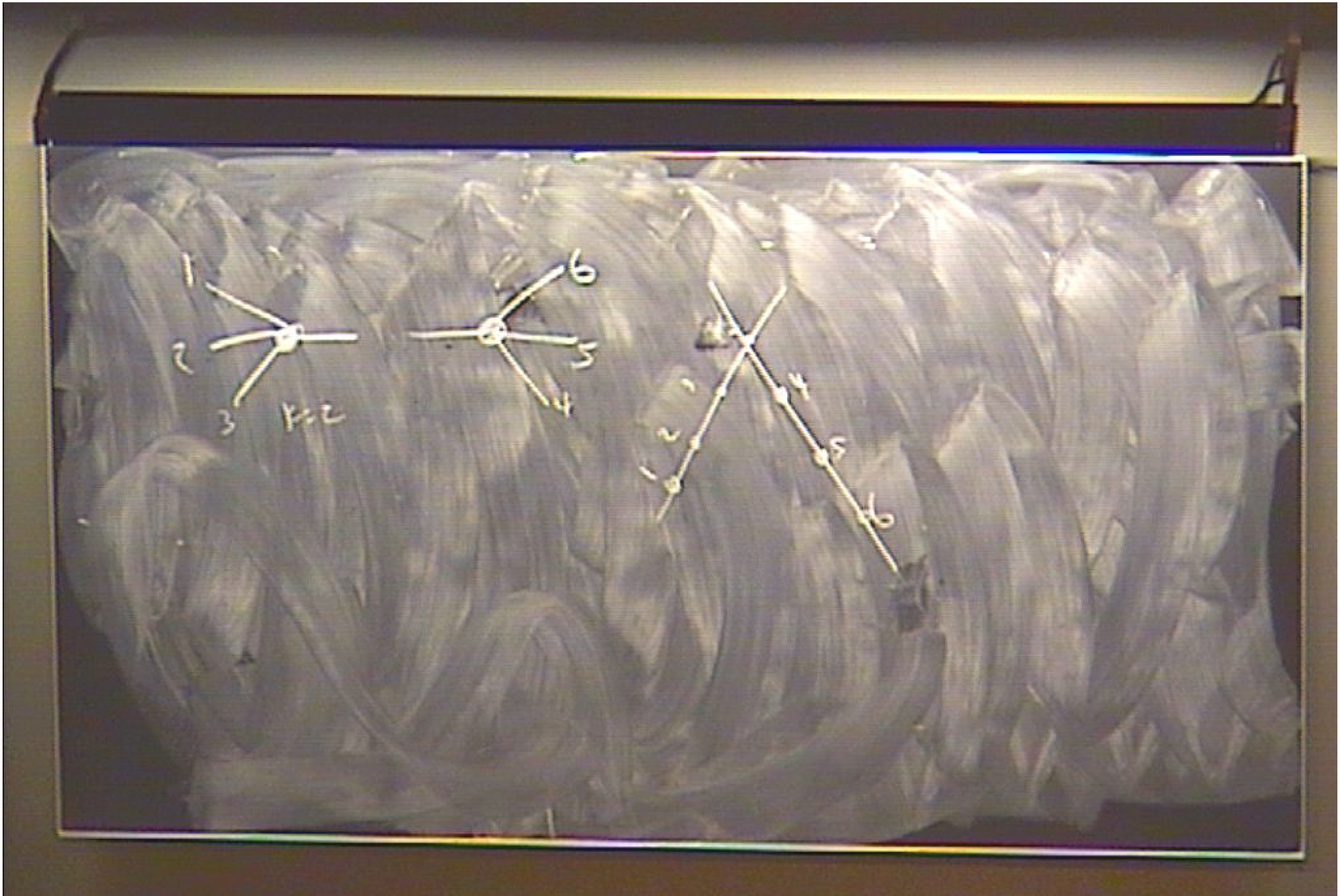
$$(k-2)(n-k-2) = \underline{1}$$

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$
$$\begin{bmatrix} 1 & 0 & x_3 & 0 & x_5 & x_6 \\ 0 & 1 & y_3 & 0 & y_5 & y_6 \\ 0 & 0 & 0 & 1 & z_5 & z_6 \end{bmatrix}$$

$$\int \frac{dx'_3\, dy'_3\, dz'_5}{z_5\,(234)(345)\cdots(612)} \qquad z_5 \to 0$$



$\int \frac{d^2\lambda\, d^2\tilde\lambda\, d^4\eta}{GL(1)} F_L(\lambda,\lambda)\, F_R(\lambda,\tilde\lambda)$
$\int D^{3|4}W \;\; F_L(W)\, F_R(W)$



1
2
3
k=2
6
5
4
1
2
4
5
6