Title: On Loops in Superspace

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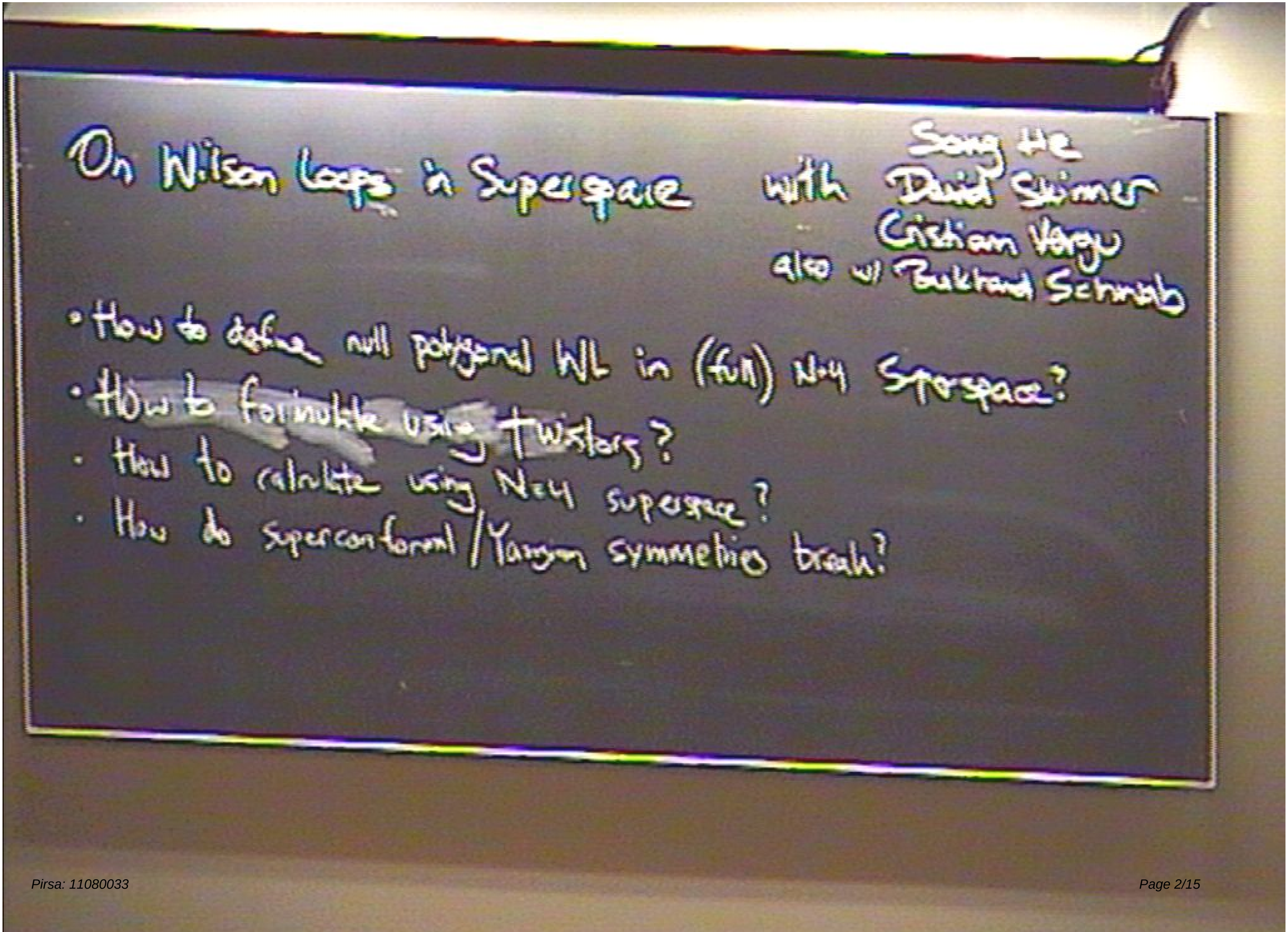
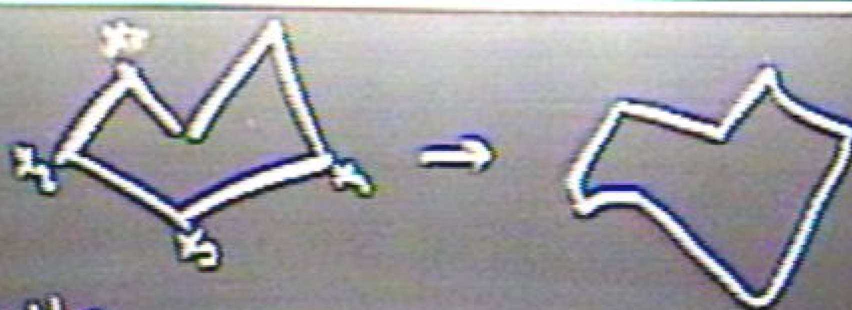
Abstract:


On Wilson loops in Superspace
with Song He
David Skinner
Cristian Vergu
also w/ Burkhard Schwab
• How to define null polygonal WL in (full) N=4 Superspace?
• How to formulate using Twistors?
• How to calculate using N=4 superspace?
• How do superconformal/Yangian symmetries break?

Bosonic Null Pol. WL

Null polygons & conf. transf.



light-like $\dot{x}^2=0$ $\xrightarrow{\text{conf.}}$ light-like

Straight $\ddot{x}\sim\dot{x}$ $\xrightarrow{\text{conf.}}$ ~~Straight~~

null straight $\longrightarrow$ null straight

mom-twistors

$$x_{k+1}-x_k=\lambda_k\bar{\lambda}_k \qquad SU(2,2)$$

$$\mu_k = x_k\cdot\lambda_k \qquad W_i=(\lambda_i,\mu_i)$$

one-loop $\langle W\rangle = \ldots \ Li_2,\ \log^2$ $\quad [W_a,W_b,W_c,W_d]\sim\epsilon^{\cdots}\,WWWW$

towards superspace $\left(x, \underset{8}{\theta}, \underset{8}{\bar\theta}\right)$ 8 complex θ / 16 real θ in $(3,1)$ sign.

$\{Q, \bar Q\} = 2P$ → torsion

$\underline{\text{light-like}}$ $\dot{x}$ is not stable under S-pieces

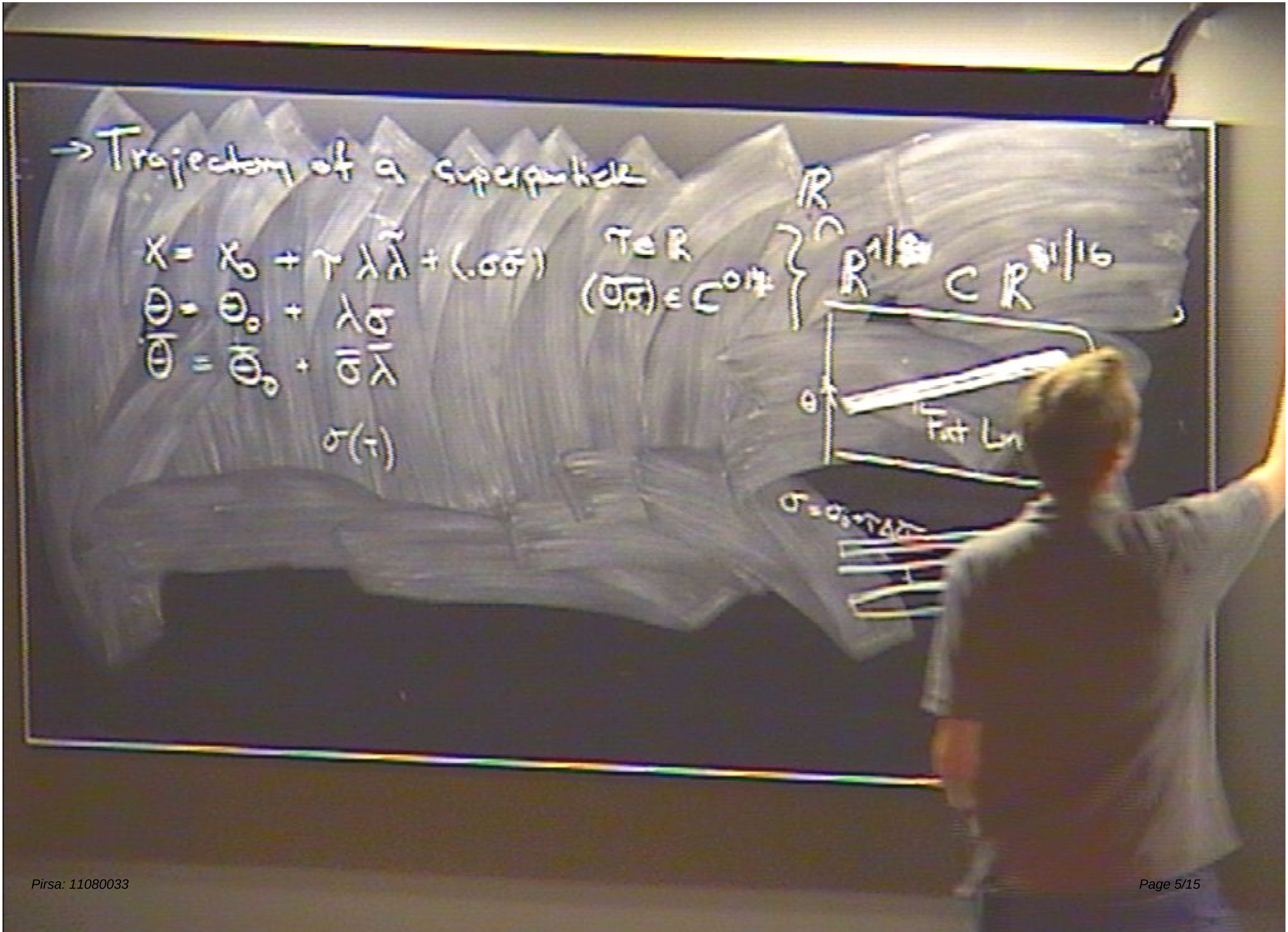
$$e = \dot x + \dot\theta\bar\theta - \theta\dot{\bar\theta}$$

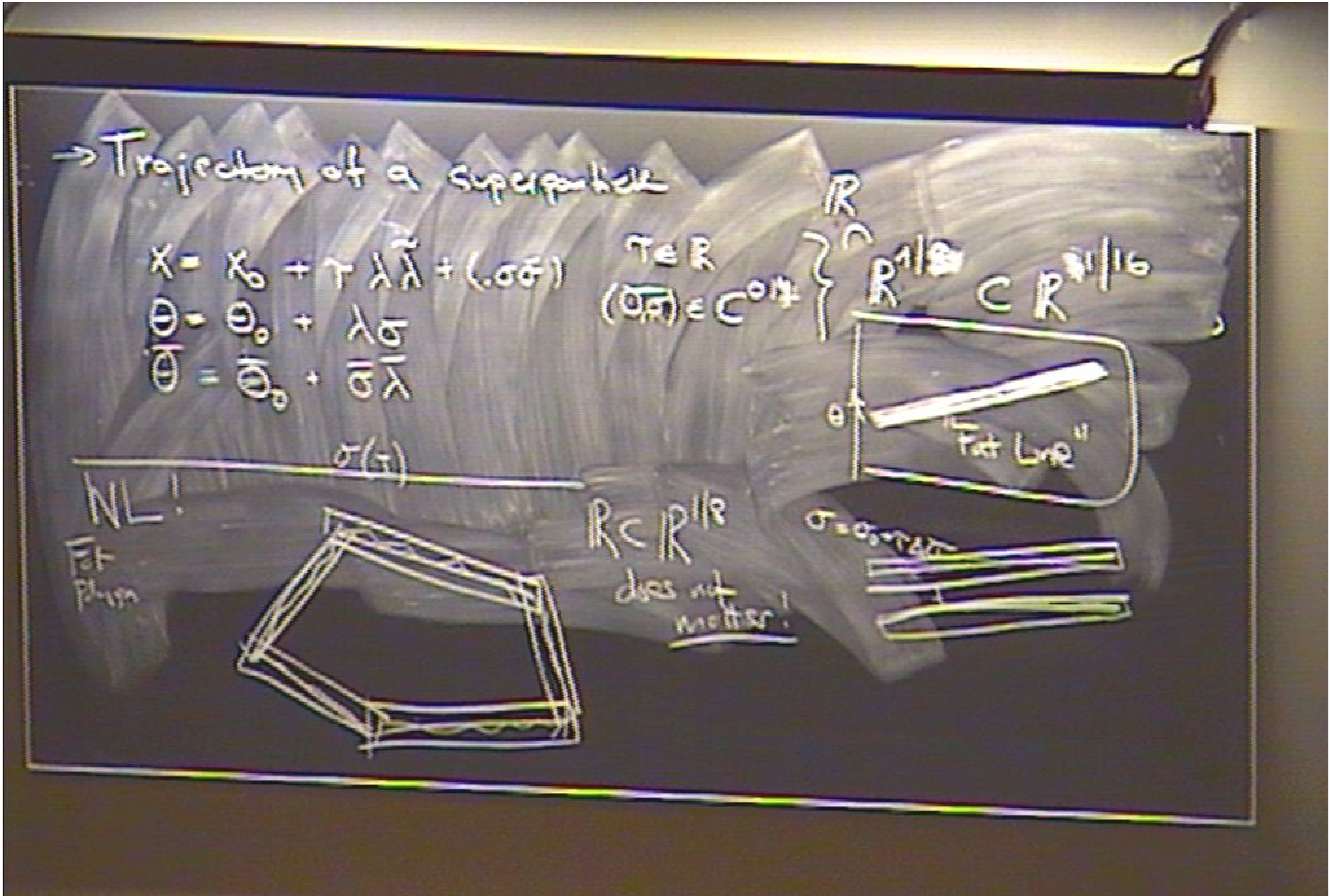
$$e^2 = 0 \qquad e\cdot\dot\theta = 0 \qquad e\,\dot{\bar\theta} = 0 \Longleftarrow \begin{array}{l} \dot\theta^2 = 0 \\ \dot{\bar\theta}^2 = 0 \\ \dot x \; \dot\theta\dot{\bar\theta}\;\theta\bar\theta \end{array}$$

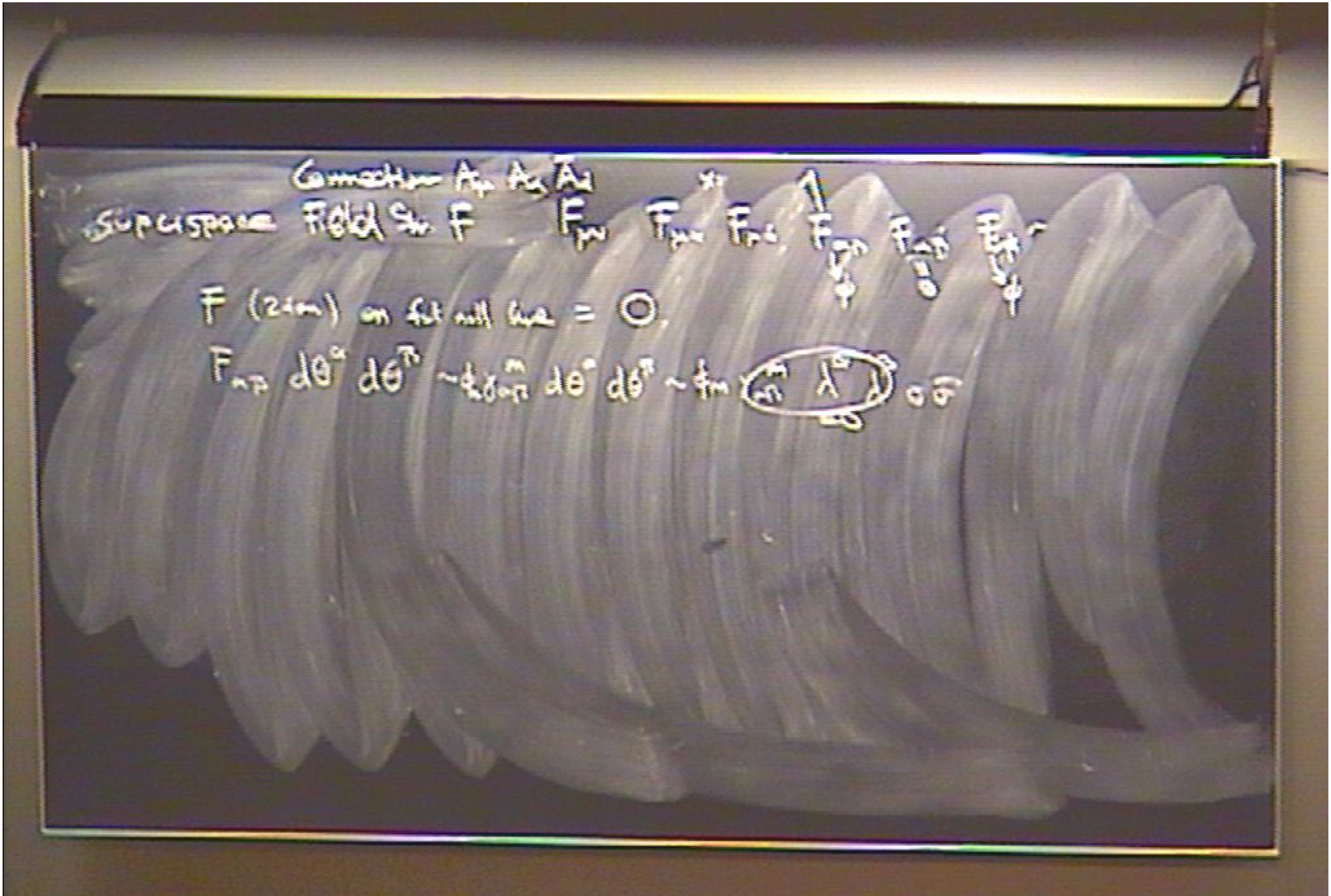
$$e = \lambda\bar\lambda \qquad \dot\theta \sim \lambda\eta \qquad \dot{\bar\theta} \sim \bar\eta\bar\lambda$$

$\underline{\text{Straightness}}$ $\ln$

$$\begin{aligned} x &= x_0 + \tau\Delta x \\ \theta &= \theta_0 + \tau\Delta\theta \\ \bar\theta &= \bar\theta_0 + \tau\Delta\bar\theta \end{aligned} \xrightarrow{\text{grav!}} \cancel{\text{straight}}$$


→ Trajectory of a superparticle
$X = X_0 + \tau \lambda \tilde{\lambda} + (, \sigma \bar{\sigma})$
$\tau \in \mathbb{R}$
$\Theta = \Theta_0 + \lambda \sigma$
$\bar{\Theta} = \bar{\Theta}_0 + \bar{\sigma} \bar{\lambda}$
$\sigma(\tau)$
$\mathbb{R}$
$\subset \mathbb{R}^{11|16}$
θ


→ Trajectory of a superparticle
x = x0 + τ λλ̃ + (…σσ̄)
τ ∈ R
θ = θ0 + λσ
θ̄ = θ̄0 + σ̄λ̄
R
R^{1|8} ⊂ R^{11|16}
σ(τ)
NL!
"Fat Line"
R ⊂ R^{1|8}
does not matter!


Connection
Superspace Field Str. F
F (2-form) on flat null line = 0

Connection A_μ A_α $\bar{A}_{\dot\alpha}$

Superspace Field Str. F $\quad F_{\mu\nu}$ $\quad F_{\mu\alpha}$ $\quad F_{\mu\dot\alpha}$ $\quad F_{\alpha\beta}$ $\quad F_{\alpha\dot\beta}$ $\quad F_{\dot\alpha\dot\beta}$

$\phi \quad 0 \quad \phi$

F (2-form) on flat null line = 0.

$F_{\alpha\beta}\, d\theta^\alpha\, d\theta^\beta \sim \epsilon\,\delta^m_{\alpha\beta}\, d\theta^\alpha\, d\theta^\beta \sim \mathrm{tr}\,(\ldots\ \lambda^\alpha\ \lambda)\ \sigma\bar\sigma$

$F_{\mu\beta}\, e^\mu\, d\theta^\beta$

• two segments $\mathbb{R}^{1|8}$

meet in point $\mathbb{R}^{0|0}$

$16-8-8=0$

• there's a unique $\mathbb{R}^{1|8}$ joining two null-sep. pts.

Null • $e = x_1 - x_2 + \theta_1\bar\theta_2 - \theta_2\bar\theta_1$ $\quad e^2=0$ $\quad$ null

but $\Delta x = x_1 - x_2$ $\quad$ close

Scattering ampl.

$r = e$?

$r = \Delta x$

Twistors for Sst. null Polygons.

• Polyg. is determ. by seq. of null-sep. pts $(x_k, \theta_k, \bar\theta_k)$ in superspace

Step 1 - Spin. hel.

$$x_{k+1} - x_k - \theta_k \bar\theta_{k+1} + \theta_{k+1} \bar\theta_k = \lambda_k \bar\lambda_k$$

$$\theta_{k+1} - \theta_k = \lambda_k \eta_k \qquad \bar\theta_{k+1} - \bar\theta_k = \bar\eta_k \bar\lambda_k$$

Step 2 twistors

$$\boxed{\begin{aligned} \mu_k &= \lambda_k \cdot x_k^+ \\ \chi_k &= \lambda_k \cdot \theta_k \end{aligned}} \qquad \begin{aligned} \bar\mu_k &= x_k^- \bar\lambda_k \\ \bar\chi_k &= \bar\theta_k \cdot \bar\lambda_k \end{aligned} \qquad x^\pm = x \pm \theta\bar\theta$$

mom. twistor $\rightarrow W_k = (\lambda_k, \mu_k, \chi_k)$ PSU(2,2|4)

$\bar W_k = (\bar\lambda_k, \bar\mu_k, \bar\chi_k) \leftarrow$ conjugate $(3,1)$ psu(2,2|4)

W_k's $\bar{W}_k'$ are constrained can show: $W_k \cdot \bar{W}_k = Q_k$ $\left\{ \begin{array}{l} x^{\pm} = x \pm \theta\bar{\theta} \\ x^{+} - x^{-} = 2\theta\bar{\theta} = \end{array} \right.$

$W_k \cdot \bar{W}_{k+1} = 0$

$(x_k, \theta_k, \bar{\theta}_k)$ n vertices ambi-twistor(s) mom. ambi-twistors

$4n \quad 16n$

LL. $n \quad 3n$

DOF $3n|8n$

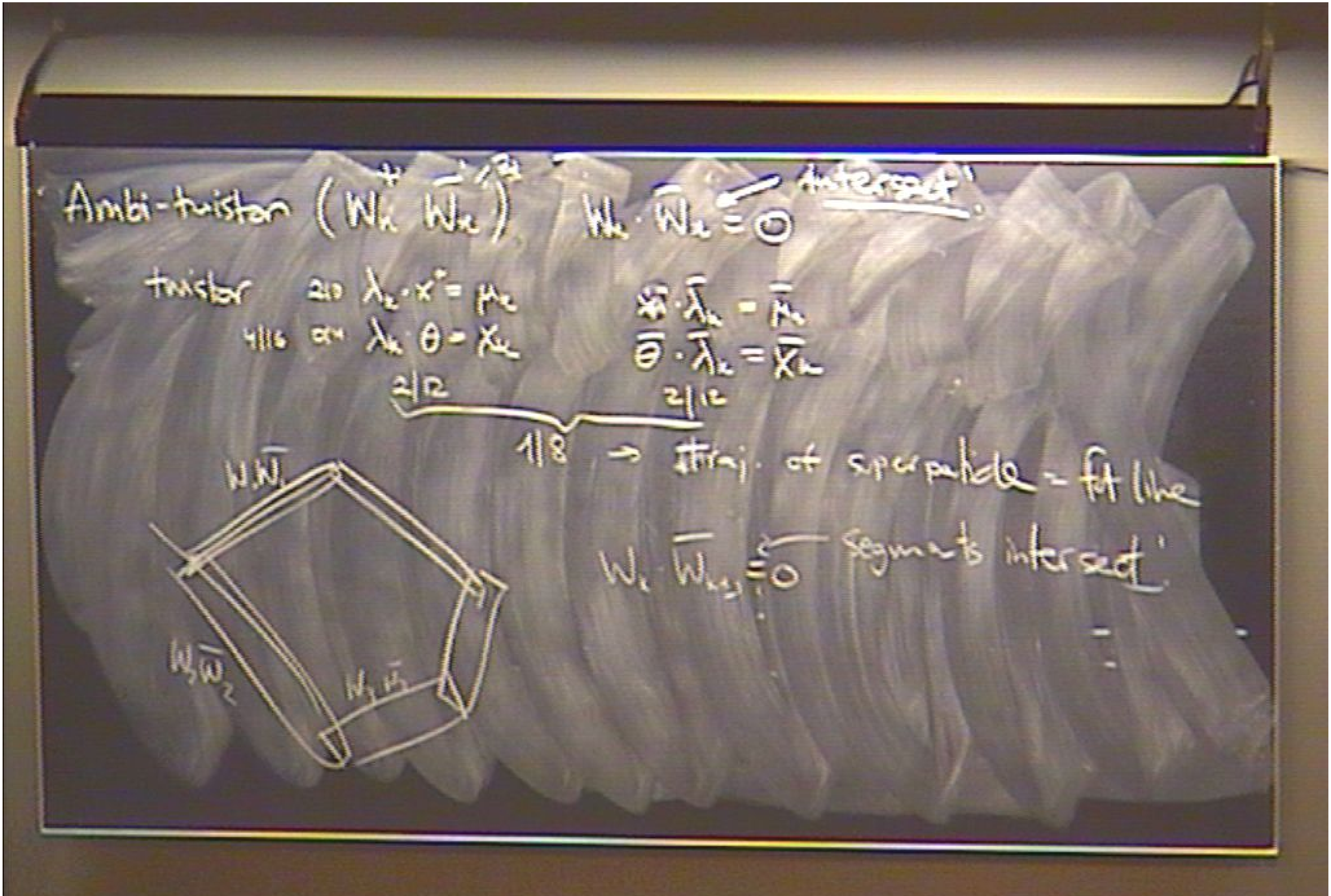
$W_k \quad 4n|4n \qquad \bar{W}_k \quad 4n|4n$

proj. $-n|0 \qquad -n|0$

const. $6n|8n$

$-3n|0$

$3n|8n$


Ambi-twistor (W_k W̄_k)
W_k · W̄_k = 0
intersect!
twistor 2|0 λ_k · x^+ = μ_k
x̄ · λ̄_k = μ̄_k
4|16 or λ_k θ = χ_k
θ̄ · λ̄_k = χ̄_k
2|2
2|2
1|8 → traj. of superparticle = fat line
W_k W̄_{k+1} = 0 segments intersect!
W_1 W̄_1
W_2 W̄_2
W_3 W̄_3

How to compute?

WL in $N=4$ better manifest $N=4$ FT

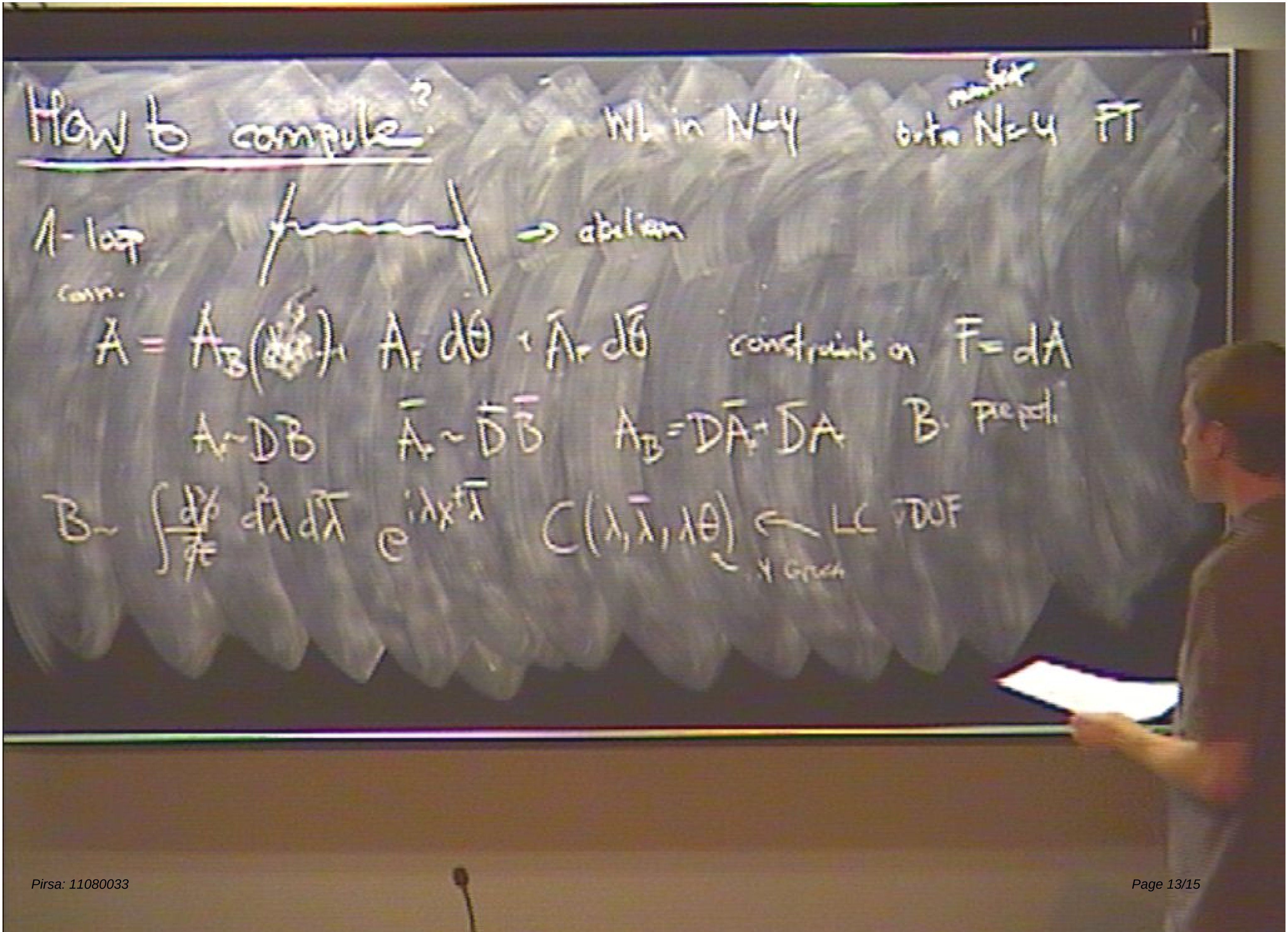
1-loop  $\rightarrow$ abelian

conn.

$$A = A_B(x) + A_F\, d\theta + \bar{A}_F\, d\bar{\theta} \qquad \text{constraints on } F = dA$$

$$A_F \sim DB \qquad \bar{A}_F \sim \bar{D}\bar{B} \qquad A_B = D\bar{A} + \bar{D}A \qquad B \text{ prepot.}$$

$B \sim$


How to compute?
WL in N=4
constraints on F=dA
B. prepot.
LC DOF

How to compute?

WL in $N=4$ twistor $N=4$ FT

1-loop  $\Rightarrow$ abelian

conn.

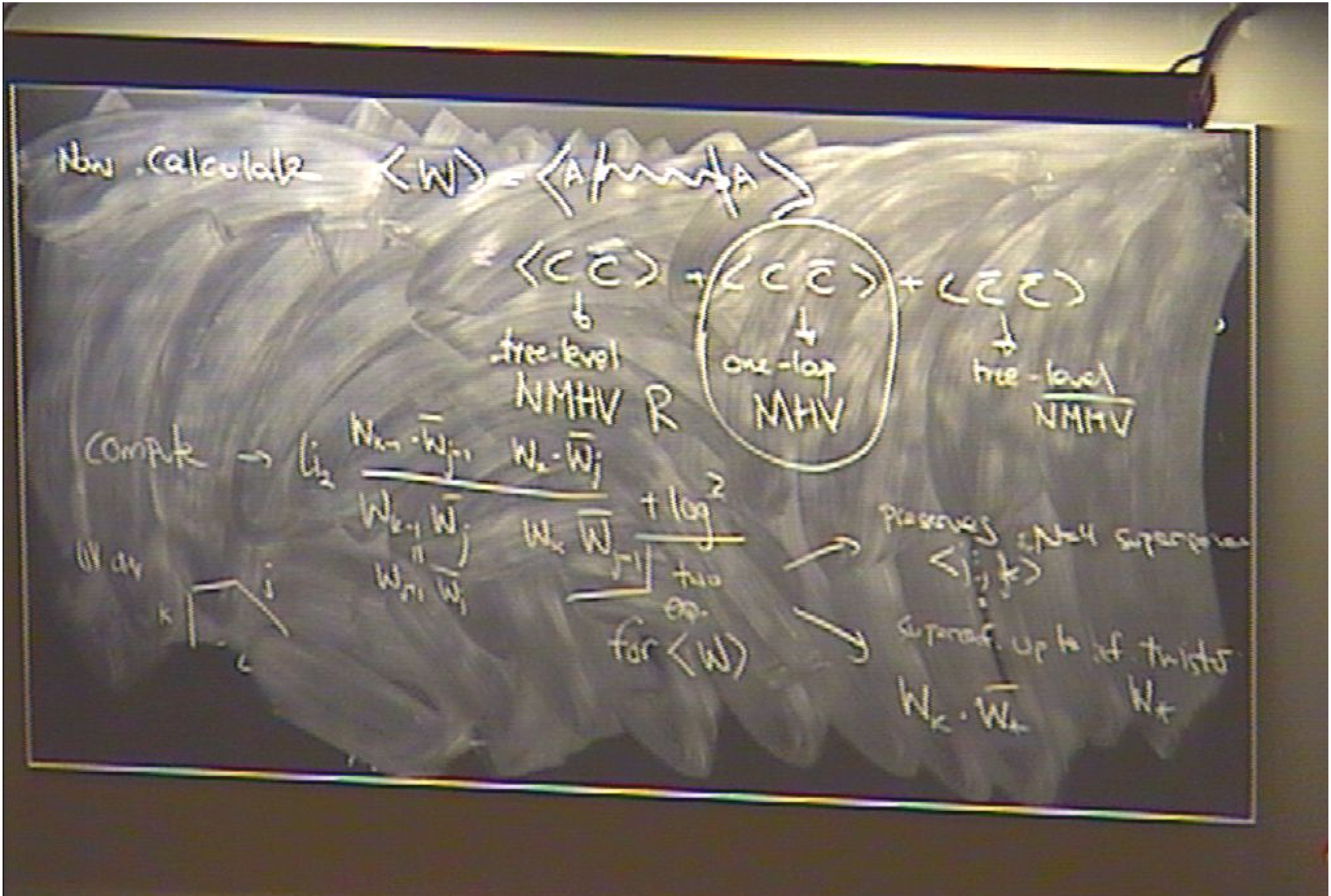
$$A = A_B(\ldots) + A_F\, d\theta + \bar{A}_F\, d\bar{\theta} \qquad \text{constraints on } F = dA$$

$$A_F \sim DB \qquad \bar{A}_F \sim \bar{D}\bar{B} \qquad A_B = D\bar{A}_F + \bar{D}A_F \qquad B \text{ prepot.}$$

$$B \sim \int \frac{d\lambda}{\lambda}\, d^2\lambda\, d^2\bar{\lambda}\; e^{i\lambda x \bar{\lambda}}\; C(\lambda, \bar{\lambda}, \lambda\theta) \leftarrow \text{LC DOF}$$

4 Grass.

$$\bar{B} \sim \qquad \langle C(\lambda, \bar{\lambda}, X)\, C(\lambda', \bar{\lambda}', X') \rangle \sim \delta^2(\lambda + \lambda')\, \delta^2(\bar{\lambda} - \bar{\lambda}')\, \delta^4(X - X')$$


Now calculate
tree-level
NMHV R
one-loop
MHV
tree-level
NMHV
compute
for ⟨W⟩