Title: Integrability of Green-Schwarz Sigma-Models with Boundaries

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Abstract:

Integrability of Green-Schwarz Sigma-Models with Boundaries

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Exact Results in Gauge/Gravity Dualities

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Introduction and Motivations

- Integrable D-brane configurations in the $\mathrm{AdS}_5 \times S^5$ theory are known [Review - Zoubos]
  - *open spin chains* at weak coupling
  - *integrable sigma-model with boundaries* at strong coupling
- D3 - Giant-graviton - $\mathbb{R} \times S^3$ [Berenstein, Vazquez, Agarwal, Correa, Hofmann, Maldacena, Mann,...]
- D3-D7 System - $\mathrm{AdS}_5 \times S^3$ [Erler, Mann, Vazquez, Correa, Young, MacKay, Regelskis, McLoughlin, Swanson]
- D3-D5 - Karch-Randall - $\mathrm{AdS}_4 \times S^2$ [DeWolf, Mann, Correa, Young, Regelskis]
- Classify integrable D-brane configurations at strong coupling
- Analyze the complete sector (including fermions)

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Outline

- Integrable Super-Coset Models - Closed Strings
  - The Green-Schwarz Sigma-Model
  - The GS Action
  - Integrability
- Integrable Super-Coset Models - Open Strings
  - D-Branes in GSSM's
  - Integrability
  - Classification
- Summary and Outlook

Integrable Super-Coset Models - Closed Strings

Integrable Super-Coset Models
Closed Strings

The Green-Schwarz Sigma-Model

- GSSM for the $\mathrm{AdS}_5 \times S^5$ background was constructed a on supercoset background G/H [Metsaev, Tsyetlin]
- Using a special property of the supercoset, $\mathbb{Z}_4$-*grading*, it is possible to construct a *generating function* for ∞-set of conserved charges [Bena, Polchinski, Roiban]
- Next I will show how to construct an action for a general supercoset background with a $\mathbb{Z}_4$-automorphism and the generating function

Supercoset Backgrounds G/H

- Let $\mathfrak{g}$ be the superalgebra of the supergroup G
- $\mathbb{Z}_4$ automorphism map acts of the superalgebra $\mathfrak{g}$ as

$$\hat{\Omega}(\mathfrak{g}_k) = i^k \mathfrak{g}_k, \quad k = 0, 1, 2, 3$$

- The superalgebra decomposes

$$\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \mathfrak{g}_3$$

- Construct the background space $M = G/H$, where $\mathfrak{g}_0$ is the algebra of H
- This is our *supercoset background*, also called *semi-symmetric space*, e.g.

$$\mathrm{AdS}_5 \times S^5 \subset \frac{\mathrm{PSU}(2,2|4)}{\mathrm{SO}(4,1) \times \mathrm{SO}(5)}, \quad \mathrm{AdS}_4 \times \mathbb{C}P^3 \subset \frac{\mathrm{OSP}(6|4)}{\mathrm{U}(3) \times \mathrm{SO}(3,1)}$$

The GS Action

- Construct kappa invariant action on the supercoset background $M = G/H$
- Define the Maurer-Cartan superalgebra valued one-form

$$J = g^{-1}dg, \quad g \in G$$

- Because of $\mathbb{Z}_4$, J decomposes as $J = J^{(0)} \oplus J^{(1)} \oplus J^{(2)} \oplus J^{(3)}$
- The action takes the form

$$S = \frac{R^2}{4\pi\alpha'} \int d^2\sigma \mathrm{Str}\left(J^{(2)} \wedge *J^{(2)} + J^{(1)} \wedge J^{(3)}\right)$$

The GS Action, cont.

$$S = \frac{R^2}{4\pi\alpha'} \int d^2\sigma \mathrm{Str} \left(J^{(2)} \wedge *J^{(2)} + J^{(1)} \wedge J^{(3)} \right)$$

Symmetries:

- Global: $g \to g_0 g, \quad g_0 \in G$
- Local: $g \to gh, \quad h \in H$
- Local κ-symm: $g \to g e^{\hat{\kappa}}, \quad \hat{\kappa} = [J^{(2)}_\alpha, \kappa^{(1)\alpha}_+] + [J^{(2)}_\alpha, \kappa^{(3)\alpha}_-]$

$$\kappa^\alpha_\pm = \frac{1}{2}(h^{\alpha\beta} \pm \frac{\epsilon^{\alpha\beta}}{\sqrt{h}})\kappa_\beta$$

Integrability

- The $\mathbb{Z}_4$ automorphism guarantees the existence of a one-parameter family *Lax connection* $a(z)$, built out of a combination of the one-forms $J^{(k)}$ [Bena, Polchinski, Roiban]
- The GSSM *Lax connection* is given by

$$a(z) = zj^{(1)} + \frac{1}{2}(z - z^{-1})^2 j^{(2)} + z^{-1} j^{(3)} - \frac{1}{2}(z^2 - z^{-2}) * j^{(2)}, \quad z \in \mathbb{C}$$

where $j^{(k)} = gJ^{(k)}g^{-1}$ satisfying

$$da(z) + a(z) \wedge a(z) = 0$$

by the EOM and MCE

Integrability, cont.

- Define the *transition matrix*

$$T(x,y;z) = \mathrm{P}\exp\left(\int_y^x a_\sigma(\sigma;z)d\sigma\right)$$

- One of the transition matrix properties is

$$\partial_\tau T(x,y;z) = a_\tau(x;z)T(x,y;z) - T(x,y;z)a_\tau(y;z)$$

- Define the *monodromy matrix*

$$T_L(z) = T(0,L;z)$$

and assume *periodic boundary conditions*

$$a(\sigma + L;z) = a(\sigma;z),$$

so that

$$\partial_\tau \mathrm{Str}(T_L(z)) = 0$$

Integrable Super-Coset Models - Open Strings

Integrable Super-Coset Models
Open Strings

Open Strings Integrability

- There is an alternative way to construct a *generating function* of conserved charges for (bulk) integrable systems with boundaries [Sklyanin]
- This construction was used for the *bosonic sector* of the $\mathrm{AdS}_5 \times S^5$ using the *PCM* [Mann, Vazquez]
- The MGG and D3-D7 systems were found to be integrable, while the construction for D5-branes was unsuccessful
- Note that the failure of the procedure doesn't rule out integrability of the model

Open Strings Integrability

- First we consider a class of D-branes in the GSSM
- Then we show they are integrable for the complete sector of the GSSM (including the fermionic sector)
- We classify integrable boundary conditions for $AdS_5 \times S^5$

D-Branes in GSSM's

D-Branes in GSSM's

D-Branes in GSSM's - BC's

- Take the variation of the GS action and require the boundary term to vanish

$$\delta S_B = \int d\tau \mathrm{Str}\left(\Delta\left(2J_\sigma^{(2)} + J_\tau^{(1)} - J_\tau^{(3)}\right)\right) \equiv 0,$$

$$\Delta = g^{-1}\delta g$$

- Impose the boundary conditions

$$\Omega(J_\tau^{(2)}) = J_\tau^{(2)}, \quad \Omega(J_\sigma^{(2)}) = -J_\sigma^{(2)},$$

$$\Omega(J_\tau^{(1)}) = J_\tau^{(3)}, \quad \Omega(J_\sigma^{(1)}) = -J_\sigma^{(3)}$$

- Ω is involutive: $\Omega^2 = 1$
- Ω is metric preserving: $\mathrm{Str}(\Omega(A)\Omega(B)) = \mathrm{Str}(AB)$

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D-Branes in GSSM's - Symmetries

- The bulk symmetries should satisfy $\Omega(\Delta) = \Delta$ on the boundary
- κ-symmetry variation: $\Delta = \hat{\kappa}(\tau, \sigma) = [J_\alpha^{(2)}, \kappa_-^{(1)\alpha} + \kappa_+^{(3)\alpha}]$ so

$$\Omega(\kappa_-^{(1)\tau}) = \kappa_+^{(3)\tau}, \quad \Omega(\kappa_-^{(1)\sigma}) = -\kappa_+^{(3)\sigma}$$

- Supersymmetry variation: $\Delta = g^{-1}\epsilon g = g^{-1}(\epsilon^{(1)} + \epsilon^{(3)})g$. We encounter two kinds of automorphisms:
  - $\Omega(AB) = \Omega(A)\Omega(B)$, so that $\Omega(g) = g \Rightarrow \Omega(\epsilon^{(1)}) = \epsilon^{(3)}$
  - $\Omega(AB) = -\Omega(B)\Omega(A)$, so that $\Omega(g) = -g^{-1} \Rightarrow \Omega(\epsilon^{(1)}) = \epsilon^{(3)}$
- Exactly half of the supersymmetry is broken, since Ω is linear

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D-Branes in GSSM's, cont.

- Boundary conditions for these half-BPS configurations can be summarize as

$$\Omega(a_\tau(z)) = a_\tau(z^{-1}), \quad \Omega(a_\sigma(z)) = -a_\sigma(z^{-1})$$

or using conformal coordinates $a = a_\tau + a_\sigma, \quad \bar{a} = a_\tau - a_\sigma$

$$\Omega(a(z)) = \bar{a}(z^{-1})$$

- Geometric interpretation: note that $J^{(2)} \in span\{P_a\}$, so Ω decomposes $P = P^+ \oplus P^-$

$$P^+ - \text{Neumann}, \quad P^- - \text{Dirichlet}$$

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Integrability of Open Strings

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Construction of The Generating Function

- The *generating function* for *boundary integrable systems* adapted for the GSSM is

$$T(z) = U_0 T^{-1}(\pi, 0; z^{-1}) U_\pi T(\pi, 0; z)$$

$T(\pi, 0; z)$ - *transition matrix*, $\sigma \in [0, \pi]$ - *string range*

- Requiring $\partial_z \mathrm{Str}(T(z)) = 0$ we find

$$U_0 a_z(0; z^{-1}) - a_z(0; z) U_0 = 0, \quad U_\pi a_z(\pi; z) - a_z(\pi; z^{-1}) U_\pi = 0$$

- Define the automorphism map $\Omega_U(x) = U x U^{-1}$ then

$$\Omega_U(a_z(z)) = a_z(z^{-1})$$

- This is the half-BPS boundary condition for a *special class* of automorphisms

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Construction of The Generating Function

- What about other automorphisms?
- Define

$$T_{\tilde{\Omega}}(x,y;z) = \mathrm{P}\exp\left(\int_y^x \tilde{\Omega}(a(\sigma;z)_\sigma)d\sigma\right)$$

and

$$T(z) = U_0 T_{\tilde{\Omega}}^{-1}(\pi,0;z^{-1}) U_\pi T(\pi,0;z)$$

- Require $\partial_\tau \mathrm{Str}(T(z)) = 0$
- Define the automorphism composition $\Omega(x) = \Omega_U(\tilde{\Omega}(x))$ we find again

$$\Omega(a_\tau(z)) = a_\tau(z^{-1})$$

- This is again the half-BPS D-brane boundary condition we found

Classification

Examples

MGG - $\mathbb{R}^1 \times S^3 \subset AdS_5 \times S^5$

- The $AdS_5 \times S^5$ metric in global coordinates

$$ds^2_{AdS_5} = d\rho^2 - \cosh^2\rho dt^2 + \sinh^2\rho(d\alpha^2 + \sin^2\alpha d\beta^2 + \cos^2\alpha d\gamma^2),$$

$$ds^2_{S^5} = d\tilde{\theta}^2 + \sin\tilde{\theta}^2 d\phi^2 + \cos^2\tilde{\theta}(d\psi^2 + \sin^2\psi d\eta^2 + \cos^2\psi d\varphi^2)$$

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- B.C's: $\rho = 0, \tilde{\theta} = 0$ gives for the *bosonic sector*

$$J_\tau^{(2)} = P_0\partial_\tau t + P_5 \sin\psi\partial_\tau\eta + P_6\partial_\tau\psi + P_8\cos\psi\partial_\tau\varphi$$

$$J_\sigma^{(2)} = P_1\partial_\sigma\rho + P_7\partial_\sigma\tilde{\theta}$$

- Using *conformal algebra* and *Clifford algebra*

$$U = aP_0 + bP_5P_6P_8, \quad a, b \in \mathbb{C}$$

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MGG - $\mathbb{R}^1 \times S^3 \subset AdS_5 \times S^5$, cont.

- Let us add the *fermionic sector*. Take the coset representative $g = g_B g_F$, $g_F = e^F$, $F = \theta \cdot Q$
- Remember we need $\Omega_U(g_F) = g_F$ so we require $\Omega_U(F) = F$, then

$$\Omega_U(F) = \theta \cdot \Omega_U(Q) = \theta \cdot (U^{(s)} Q)$$

$$\Rightarrow \theta = \theta U^{(s)} \text{ on the boundary}$$

- $U^{(s)}$ is a product of $\Gamma_{32\times 32}$ times ϵ acting on internal indices so Ω_U takes $J^{(1)}$ to $J^{(3)}$
- Together with $\Omega_U(J^{(1)}_{\tau/\sigma}) = \pm J^{(3)}_{\tau/\sigma}$ the fermions boundary conditions are

$$\partial_\tau \theta = \partial_\tau \theta \cdot U^{(s)}, \quad \partial_\sigma \theta = -\partial_\sigma \theta \cdot U^{(s)}$$

MGG - $\mathbb{R}^1 \times S^3 \subset AdS_5 \times S^5$, cont.

- For consistency *Weyl* and *Majorana* conditions must be preserved, as well as $\Omega^2 = 1$
- Finally we find

$$U = 2P_0 + i2^3 P_5 P_6 P_8, \quad \text{and} \quad U^{(s)} = \Gamma^0\Gamma^5\Gamma^6\Gamma^8 \otimes \epsilon$$

Karch-Randall - $AdS_4 \times S^2 \subset AdS_5 \times S^5$

- The $AdS_5 \times S^5$ metric in Poincare coordinates

$$ds^2_{AdS_5} = \frac{dx^\mu dx_\mu + dy^2}{y^2}$$

$$ds^2_{S^5} = d\theta_9^2 + \cos^2\theta_9 (d\theta_8^2 + \cos^2\theta_8 (d\theta_7^2 + \cos^2\theta_7 (d\theta_6^2 + \cos^2\theta_6 d\theta_5^2)))$$

- B.C's: $y^2 = 0$, $\theta_7 = \theta_8 = \theta_9 = 0$ gives for the *bosonic sector*

- But, $\exists\, \Omega$ such that

Karch-Randall - $AdS_4 \times S^2 \subset AdS_5 \times S^5$

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- B.C's: $x^2 = 0, \theta_7 = \theta_8 = \theta_9 = 0$ gives for the *bosonic sector*

$$J_\tau^{(2)} = \frac{1}{y}(P_0\partial_\tau t + P_1\partial_\tau x_1 + P_3\partial_\tau x_3 - P_4\partial_\tau y) + P_8\partial_\tau\theta_8 + \cos\theta_8 P_5\partial_\tau\theta_5$$

$$J_\sigma^{(2)} = P_2\frac{\partial_\sigma x_2}{y} + P_9\partial_\sigma\theta_9 + P_6\partial_\sigma\theta_6 + P_7\partial_\sigma\theta_7$$

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$$J_\tau^{(2)} = \frac{1}{y}(P_0\partial_\tau t + P_1\partial_\tau x_1 + P_3\partial_\tau x_3 - P_4\partial_\tau y) + P_8\partial_\tau\theta_8 + \cos\theta_8 P_5\partial_\tau\theta_5$$

$$J_\sigma^{(2)} = P_2\frac{\partial_\sigma x_2}{y} + P_9\partial_\sigma\theta_9 + P_6\partial_\sigma\theta_6 + P_7\partial_\sigma\theta_7$$

- But, $\nexists\, U$ such that $\Omega_U(J^{(2)}_{\tau/\sigma}) = \pm J^{(2)}_{\tau/\sigma}$

Karch-Randall - $AdS_4 \times S^2 \subset AdS_5 \times S^5$, cont.

- Here is where we have to use a different kind of automorphism
- Define
$$\Omega(x) = -Ux^{st}U^{-1}$$
- Taking
$$U = 2P_4 - 4P_6P_7$$
we are done! (Repeat the MGG analysis...)
- Note that
$$U^{(s)} = \gamma_5\gamma_6 \otimes \gamma_0\gamma_1\gamma_3\gamma_4 \otimes 1 \otimes \epsilon$$

General Solution

General Solution

$AdS_5 \times S^5$ - General Solution

Using our consistency conditions:

- The automorphism should be *involutive*: $\Omega^2 = 1$
- The automorphism should map $\mathcal{H}_1 \leftrightarrow \mathcal{H}_3$:

$$\hat{\Omega}(\Omega(\mathcal{H}_{1/3})) = \mp i\Omega(\mathcal{H}_{1/3})$$

- The automorphism should *preserve the chirality*: even number of gamma matrices
- The automorphism should *preserve Majorana condition*: if $Q^\dagger = CQ$ then

$$\Omega(Q^\dagger) = \Omega(CQ) = C\Omega(Q) = CUQ = CUC^{-1}Q^\dagger$$

$AdS_5 \times S^5$ - General Solution

- Using the traditional reflection matrices, that is

$$\Omega(x) = UxU^{-1}$$

we find

$$(1,3),(3,1),(3,5) \text{ and } (5,3) \text{ D}-\text{branes}$$

$AdS_5 \times S^5$ - General Solution

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- Examples

$\mathbb{R}^{1,0} \times S^3$	$U = P_0 + iP_{5,6,9}$	$\rho = 0; \theta = 0$
$\mathbb{R}_+^{0,1} \times S^3$	$U = P_1 + P_{5,6,9}$	$t = 0, \alpha = 0; \gamma = 0, \theta = 0$
$AdS_3 \times S^1$	$U = P_{0,1,4} + iP_8$	$\gamma = \beta = 0; \theta = \psi = 0$
$\mathbb{H}^3 \times S^1$	$U = P_{1,2,3} + P_8$	$t = 0, \beta = 0; \theta = \psi = 0$
$AdS_3 \times S^5$	$U = P_{0,1,3} + iP_{5,6,7,8,9}$	$\alpha = 0$
$\mathbb{H}^3 \times S^5$	$U = P_{1,2,3} + P_{5,6,7,8,9}$	$t = 0, \beta = 0$
$AdS_5 \times S^3$	$U = P_{0,1,2,3,4} + iP_{5,6,8}$	$\theta = 0$

$AdS_5 \times S^5$ - General Solution

- Using $\Omega(x) = -U^{-1} x^{st} U$ we find

$$(0,2), (2,0), (2,4) \text{ and } (4,2) \text{ D}-\text{branes}$$

- Examples

AdS_2	$U = P_1 - P_{[illegible]}$	[illegible] $= 0$
$S^2 \subset S^5$	$U = P_{2,4} - iP_{[illegible]}$	$\rho = 0$, [illegible] $= 0$
[illegible]	$U = P_{[illegible]} - iP_{[illegible]}$	[illegible] $= 0$
$AdS_2 \times S^4$	$U = P_{[illegible]} - P_{[illegible]}$	[illegible] $= 0$
[illegible] $\times S^4$	$U = P_{[illegible]} - iP_{[illegible]}$	[illegible] $= 0$
$AdS_2 \times S^2$	$U = P_{[illegible]} - P_{[illegible]}$	[illegible] $= 0$
[illegible] $\times S^2$	$U = P_{[illegible]} - iP_{[illegible]}$	[illegible] $= 0$

$AdS_5 \times S^5$ - General Solution

- Using $\Omega(x) = -U^{-1}x^{st}U$ we find

$$(0,2), (2,0), (2,4) \text{ and } (4,2) \text{ D-branes}$$

- Examples

AdS_2	$U = P_3 + P_{7,9}$	$\alpha = \beta = \gamma = 0; \theta = \psi = \varphi = 0$
$S^2 \subset S^5$	$U = P_{2,4} + iP_6$	$\rho = 0, t = 0; \theta_6 = \theta_7 = \theta_9 = 0$
$\mathbb{H}^2$	$U = P_0 + iP_{7,9}$	$t = 0, \alpha = 0; \theta = \psi = \varphi = 0$
$AdS_2 \times S^4$	$U = P_{0,2} + P_7$	$x^i = 0; \theta_9 = 0$
$\mathbb{H}^2 \times S^4$	$U = P_0 + iP_{6,8}$	$t = 0, \alpha = 0; \theta_5 = 0$
$AdS_4 \times S^2$	$U = P_4 - P_{6,7}$	$x^2 = 0; \theta_5 = \theta_7 = \theta_8 = 0$
$\mathbb{H}^4 \times S^2$	$U = P_{1,3} + iP_6$	$t = 0; \theta_6 = \theta_7 = \theta_9 = 0$

$\mathrm{AdS}_4 \times \mathbb{CP}^3$

- We also analyzed the $\mathrm{AdS}_4 \times \mathbb{CP}^3$

$$ds^2_{\mathrm{AdS}_4} = d\rho^2 - \cosh^2\rho dt^2 + \sinh^2\rho(d\alpha^2 + \sin^2\alpha d\beta^2)$$

$$ds^2_{\mathbb{CP}^3} = d\mu^2 + \cos^2\mu\sin^2\mu(d\psi - \frac{1}{2}\cos\theta_1 d\varphi_1 + \frac{1}{2}\cos\theta_2 d\varphi_2)^2$$

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- For example

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$\mathbb{R}^{1,0}$	$U = P_0 + (M_{14} + M_{25} + M_{36})$	$\rho = 0; \mu = 0, \theta_2 = -\frac{\pi}{2}, \varphi_2 = 0$
$\mathbb{R}^{0,1}_+$	$U = P_3 + i(M_{14} + M_{25} + M_{36})$	$t = 0, \alpha = 0; \mu = 0, \theta_2 = -\frac{\pi}{2}, \varphi_2 = 0$
$S^3 \subset \mathrm{CP}^3$	$U = P_{0,1,2,3} + 2i(T_1R_1 + T_3R_3 + T_6R_6)$	$\rho = 0, t = 0; \varphi_1 = \varphi_2 = \psi = 0$
$\mathrm{AdS}_2 \times S^3$	$U = P_{0,3} + 2i(T_1R_1 + T_3R_3 + T_6R_6)$	$\alpha = 0; \varphi_1 = \varphi_2 = \psi = 0$
$\mathrm{AdS}_3 \times S^2$	$U = P_{0,1,3} + (M^2_{14} - M^2_{25} + M^2_{36})$	$\beta = 0, \mu = 0$
$\mathbb{H}^3 \times S^2$	$U = P_{1,2,3} + i(M^2_{14} - M^2_{25} + M^2_{36})$	$t = 0, \mu = 0$
$\mathrm{AdS}_4 \times S^3$	$U = P_{1,1} + 2(T_1R_1 + T_3R_3 + T_6R_6)$	$\varphi_1 = \varphi_2 = \psi = 0$
$\mathrm{AdS}_3 \times \mathrm{CP}^3$	$U = P_{0,1,3} + (M^2_{14} + M^2_{25} + M^2_{36})$	$\beta = 0$
$\mathbb{H}^3 \times \mathrm{CP}^3$	$U = P_{1,2,3} + i(M^2_{14} + M^2_{25} + M^2_{36})$	$t = 0$

Summary and Outlook

- We gave an automorphism gluing boundary conditions for half-BPS D-branes
- These D-brane configuration are integrable as explained
- Confirmed integrability of $AdS_4 \times S^2$ at strong coupling
- Find less supersymmetric integrable configurations
- Use pure-spinor formalism for to prove the existence of the charges at the quantum level
- Complete the classification for other interesting backgrounds
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Thank You!