

Title: Reductions of the Self-Duality Equations, Twistors and Integrability

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Abstract:

Reductions of the self-duality equations, twistors and integrability

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August 10, 2011

Credits: R.S.Ward, N.M.J.Woodhouse, Hitchin,
Newman, Strachan, Tod.

Reference: 'Integrability, self-duality and twistor
theory', Mason & Woodhouse, OUP, (1996).

Twistor theory

The basic aim of Penrose's twistor theory has been to find 1-1 correspondences

$$\left\{ \begin{array}{l} \text{Solutions to Yang-} \\ \text{Mills, Einstein} \\ \text{equations} \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{l} \text{Deformed (almost)} \\ \text{complex structures} \\ \text{on twistor space} \end{array} \right\}$$

Hope: twistor space provides the correct geometric arena for the correct formulation of quantum gravity.
Works best for self-dual case where deformed complex structures are integrable

Spin off: The self-duality equations yield many integrable systems under symmetry reduction.
The correspondences for the self-duality equations provide a paradigm for 'complete integrability'.

Programme:

- (1) Classify those integrable systems that arise as reductions of the self-duality equations.
- (2) Derive the theory of these equations from that for the self-duality equations.

\mathbb{Z}^k \mathbb{CP}^3 $\alpha = 0, 1, 2$

$$\mathbb{Z}^* \cong \lambda \mathbb{Z}^*, \lambda \in \mathbb{C} - \{0\}$$

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What is integrability?

Integrable systems are differential equations that, despite nontrivial nonlinearity, are tractable.

- **Definition:** An integrable system is a $2n$ -dimensional Hamiltonian system $\{M^{2n}, \omega, H_1\}$ with n ‘constants of the motion’ H_i in involution.

Theorem: If the map $H_i : M^{2n} \rightarrow \mathbb{R}^n$ is proper and regular, $\exists \{I_i, \theta_i\}$ a coordinate system on M^{2n} of ‘action-angle’ variables, obtainable by quadratures, in which the flows are linear.

- The system is the consistency condition for an auxiliary system of overdetermined linear equations, a ‘Lax pair’.

Properties of solutions: Integrability implies that explicit formulae for solutions are readily obtainable.

- **Solitons:** Integrable equations often admit lump or ‘soliton’ solutions. These interact but maintain their integrity.

- **IST:** the scattering transform expresses the general solution as a nonlinear superposition of dispersive modes, and solitons; it transforms to action-angle variables.

Example: The SDYM equations

These are equations on a connection $D_a = \partial_a + A_a$ on a bundle E on complex Minkowski space, $\mathbb{M} = \mathbb{C}^4$. Here $\partial_a = \partial/\partial x^a$, x^a $a = 0, \dots, 3$ are coordinates on $\mathbb{M} = \mathbb{C}^4$, with metric

$$ds^2 = dx^0 dx^1 - dx^2 dx^3$$

and A_a are functions on \mathbb{M} taking values in the Lie algebra of some gauge group G .

The equations are the integrability conditions for the Lax Pair

$$L_0 = \partial_0 + A_0 - \lambda(\partial_2 + A_2), \quad L_1 = \partial_1 + A_1 - \lambda(\partial_3 + A_3)$$

where $\lambda \in \mathbb{CP}^1$.

- The equations admit real solutions on slices of \mathbb{C}^4 with ultrahyperbolic, $(2, 2)$, signature and euclidean signature only.
- They are conformally invariant.
- The role of solitons is played by instantons, finite action solutions on Euclidean \mathbb{R}^4 which extend to the conformal compactification S^4 .

\mathbb{Z}^* \mathbb{CP}^3 $\alpha = 0, 1, 2$

$$\mathbb{Z}^* \sim \lambda \mathbb{Z}^*, \lambda \in \mathbb{C}-\{0\}$$

$$\partial_{AA'} + A_{AA'}$$

$$A = I_2$$

$$A' = I_n, X \frac{\partial}{\partial x^{n+1}}$$

$$L_A = \pi^*(\epsilon_{\mu\nu} + A)$$

$$[L_A, L_B] = \epsilon$$

$$\begin{aligned} F_{AA'BD} &= \epsilon_{AB} F_{AD} \\ &\quad - \epsilon_{AC} F_{AB} \end{aligned}$$

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$$\alpha = 0, 1, 2, 3$$

$$\mathbb{Z}^* \sim \lambda \mathbb{Z}^*, \lambda \in \mathbb{C} - \{0\}$$

$$\partial_{AA'} + A_{AA'}$$

$$A = i, x \quad \frac{\partial}{\partial x^i}$$

$$L_A = \pi^*(\epsilon_{AB} + \delta_{AB})$$

$$[L_A, L_B] = \epsilon_{ABC} \pi^* F_{BC} = \epsilon_{ABC} F_{BC}$$

$= 0$ if A.S.D. condition
connection is S.D.

Z^* \mathbb{CP}^3

$$\alpha = 0, 1, 2$$

$$Z^* \sim \lambda Z^*, \lambda \in \mathbb{C}-\{0\}$$

$$\partial_{AA'} + A_{AA'}$$

$$A = I_2$$

$$A = I_2, x \frac{\partial}{\partial x^M}$$

$$L_A = \pi^*(e_M - h_M)$$

$$F_{AA'AB}$$

$$= E_{AB} F_{AA'}$$

$$[L_A, L_B] = \epsilon_{AB} \pi^* p^{MK} F_{AK} = -\epsilon_{BC} F_{AB}$$

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The Korteweg de Vries & KP equations

KdV: $u = u(x, t)$ = height of shallow water waves in a channel

$$4u_t - u_{xxx} + 6uu_x = 0.$$

\Leftrightarrow integrability, $[L_0, L_1] = 0$, of the Lax pair

$$L_0 = \partial_x + \begin{pmatrix} q & -1 \\ p & -q \end{pmatrix} - \lambda \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad L_1 = \partial_t + B - \lambda \partial_x$$

(here $u = -2q_x$ and p and B are determined by q).

Solitons: The one soliton solution is

$$u = 2c \cosh^{-2}(c(x - ct)), \quad c = \text{velocity}.$$

The KP equations control 2d shallow water waves

$$u := u(x, y, t), \quad \partial_x(4u_t - u_{xxx} + 6uu_x) = u_{yy},$$

with Lax pair

$$L_0 = \partial_y - \partial_x^2 - 2u, \quad L_1 = \partial_t - \partial_x^3 + 3u\partial_x + v.$$

Reductions of integrable systems

Reduction means the imposition of a symmetry or the specialization of a parameter in a system of equations. E.g. ∂_y symmetry gives KP \rightarrow KdV.

Reduction gives a partial ordering on the set of integrable equations.

Question: Is there a universal integrable system?

Probably not usefully, but self-dual Yang-Mills (SDYM) is a good start for classifying reductions.

Twistor theory then reduces to give self contained theory for each reduction.

Programme: Classification of reductions of SDYM.

Reductions are classified by choice of

- reality structure,
- gauge group,
- symmetry subgroup of the conformal group of 4 dim. Minkowski space,
- constants of integration that arise in the reduction.

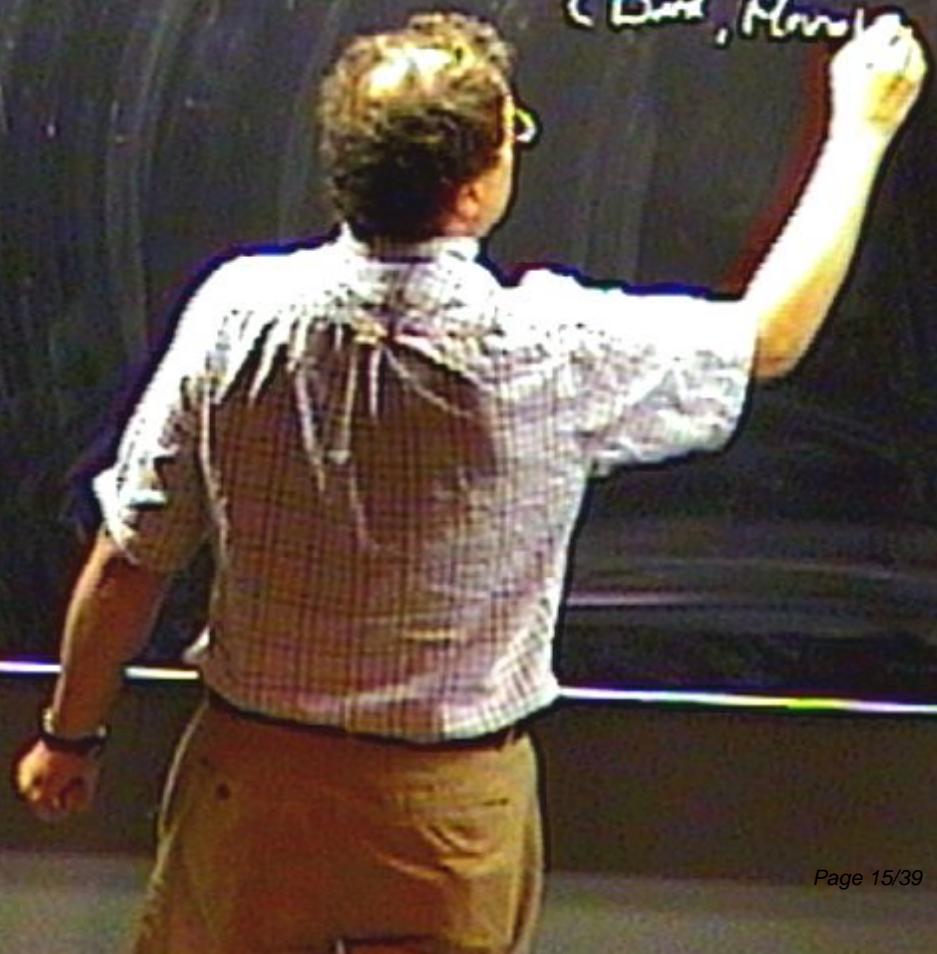
Poset

$Z+1/3$

SDYM

Bogomolny; eq

Chiral models with torsion
(Lind, Henneke)



SDYM

ZH/3

Bogomolny; PT

Chiral Models with fermions
(Lund, Manton &?)

Sugakov, Chiral Model, PTod., Henneaux, Ernst eqn., KdV, NLS

SDYM

$Z+1/3$

Bogomolny; PT

Chiral Models with terms
(Lax, Morozov?)

$2/1,$ Smirnov, Chiral Model, MToda, Hirota eqns, E_{inst} eqn, KdV, NLS
I
Painlevé, Tops ...

Z^*

$\alpha = 0, 1, 2, \dots$

 $\mathbb{C}\mathbb{P}^3$ $\partial_1 \theta \partial_1$ $\mathbb{C}\mathbb{P}^1$

$Z^* \sim \lambda Z^*, \lambda \in \mathbb{C} - \{0\}$

 T_{A^*}

$\partial_{AA'} + A_{AA'}$

$A = I_2 \quad \begin{matrix} 1 \\ 1 \\ 1 = r, c \end{matrix} \quad \frac{\partial}{\partial x^m}$

$L_A = T^*(\partial_{Ax} + A_{Ax})$

$F_{AA'BD}$

$= E_{AB} F_{AC}$

$= E_{BC} F_{AB}$

$[L_A, L_B] = \epsilon_{ABC} T^* \pi^B F_{AC} \quad \underbrace{=}_{= 0} \quad \text{if A.S.D. Connection}$

$\text{Connection is S.D.}$

\mathbb{Z}^k

$$\alpha = 0, 1, 2, \dots$$

 $\mathbb{C}\mathbb{P}^3$ $\partial_1 \theta \partial_1$ $\mathbb{C}\mathbb{P}^1$

$$z^* \sim \lambda z^*, \quad \lambda \in \mathbb{C} - \{0\}$$

 T_{A^*}

$$\partial_{AA'} + A_{AA'}$$

$$\begin{aligned} A &= i\omega \frac{\partial}{\partial x^A} \\ A' &= i\omega \frac{\partial}{\partial x^{A'}} \end{aligned}$$

$$L_A = \pi^*(\partial_{A^*} + A_{A^*})$$

$$F_{AA'BB'}$$

$$= E_{AB} F_{AC} \\ = E_{BC} F_{AB}$$

$$[L_A, L_B] = \epsilon_{ABC} \pi^* \tilde{F}^C F_{AB} \rightarrow \epsilon_{ABC} F_{AB}$$

$$= 0 \quad \text{if } \tilde{F} \text{ is S.D.} \\ \text{Connection is S.D.}$$

The Painlevé property

The Painlevé Property: Solutions to complex nonlinear equations generally have essential and branching singularities even where the equation itself is regular.

The Painlevé property: all *moveable* non-characteristic singularities are rational.

Examples: $dy/dt = y^n$ has solutions

$$y = Ae^t, \quad n = 1, \quad \frac{1}{(n-1)}(t - t_0)^{-\frac{1}{(n-1)}}, \quad n \neq 1.$$

These are Painlevé only for $n = 1, 2$.

Conjecture: A system satisfying the Painlevé property is integrable.

Theorem 1. [Ward 1984] \exists a gauge in which the SDYM equation satisfy the Painlevé property.

Painlevé's Classification

The 2nd order Painlevé equations that are rational in y are

$$P_I \quad y'' = 6y^2 + t$$

$$P_{II} \quad y'' = 2y^3 + ty + \alpha$$

$$P_{III} \quad y'' = \frac{y'^2}{y} - \frac{y'}{t} + \frac{\alpha y^2 + \beta}{t} + \gamma y^3 + \frac{\delta}{y}$$

$$P_{IV} \quad y'' = \frac{y'^2}{2y} + \frac{3y^3}{2} + 4ty^2 + 2(t^2 - \alpha)y + \frac{\beta}{y}$$

$$P_V \quad y'' = y'^2 \left(\frac{1}{2y} + \frac{1}{y-1} \right) - \frac{y'}{t} + \frac{(y-1)^2}{t^2} \left(\alpha y + \frac{\beta}{y} \right) \\ + \frac{\gamma y}{t} + \frac{\delta y(y+1)}{y-1}$$

$$P_{VI} \quad y'' = \frac{1}{2} \left(\frac{1}{y} + \frac{1}{y-1} + \frac{1}{y-t} \right) y'^2 \\ - \left(\frac{1}{t} + \frac{1}{t-1} + \frac{1}{y-t} \right) y' \\ + \frac{y(y-1)(y-t)}{t^2(t-1)^2} \left(\alpha + \frac{\beta t}{y^2} + \frac{\gamma(t-1)}{(y-1)^2} + \frac{\delta t(t-1)}{(y-t)^2} \right)$$

Reductions of SDYM to Painlevé

Proposition 1. [M. & Woodhouse] SDYM with gauge group $\text{SL}(2, \mathbb{C})$ and 'nondegenerate' abelian 3-dim. groups of conformal symmetries yields the Painlevé equations precisely.

The Painlevé equation obtained corresponds to the choice of symmetry. Using conformal group $= \text{SL}(4, \mathbb{C})$, the correspondence becomes:

$$\begin{aligned} P_{I,II} &\leftrightarrow \begin{pmatrix} d & c & b & a \\ 0 & d & c & b \\ 0 & 0 & d & c \\ 0 & 0 & 0 & d \end{pmatrix} & P_{III} &\leftrightarrow \begin{pmatrix} d & b & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & c & a \\ 0 & 0 & 0 & c \end{pmatrix} \\ P_{IV} &\leftrightarrow \begin{pmatrix} c & b & a & 0 \\ 0 & c & b & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix} & P_V &\leftrightarrow \begin{pmatrix} b & a & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d \end{pmatrix} \\ P_{VI} &\leftrightarrow \begin{pmatrix} d & 0 & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & a \end{pmatrix} \end{aligned}$$

Twistor theory

Twistor correspondences arise from *double fibrations*

$$\begin{array}{ccc} M \times \mathbb{CP}^1 & & \\ p \swarrow & & \searrow q \\ M & & \mathbb{PT} \end{array}$$

Here p is projection onto the 1st factor and q is projection along the vector fields

$$V_0 = \partial_0 - \lambda \partial_2, \quad V_1 = \partial_1 - \lambda \partial_3$$

underlying the Lax pair. Twistor space $\mathbb{PT} = \mathbb{CP}^3 - \mathbb{CP}^1$ is the quotient of $M \times \mathbb{CP}^1$ by $\{V_0, V_1\}$.

The SDYM correspondence:

SDYM fields on $M \leftrightarrow$ holomorphic bundles on \mathbb{PT} .

\rightarrow : Given SD D_a on $E \rightarrow M$, define $E' \hookrightarrow \mathbb{PT}$

$$E'|_Z = \{s \in \Gamma(p^* E, q^{-1}(Z)); L_0 s = L_1 s = 0\}$$

The remarkable feature is that it is reversible:

Theorem 2. [Ward] *The self-dual $\{E, D_a\}$ can be reconstructed from the hol. vector bundle E' .*

Symmetries on $M \leftrightarrow$ hol symmetries on \mathbb{PT} so

Corollary 1. *Reductions of SDYM \leftrightarrow hol vector bundles on \mathbb{PT} with symmetries.*

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$$Z^* = (\omega^A, \tau_A)$$

$$\omega^A = i X^M \tau_A$$

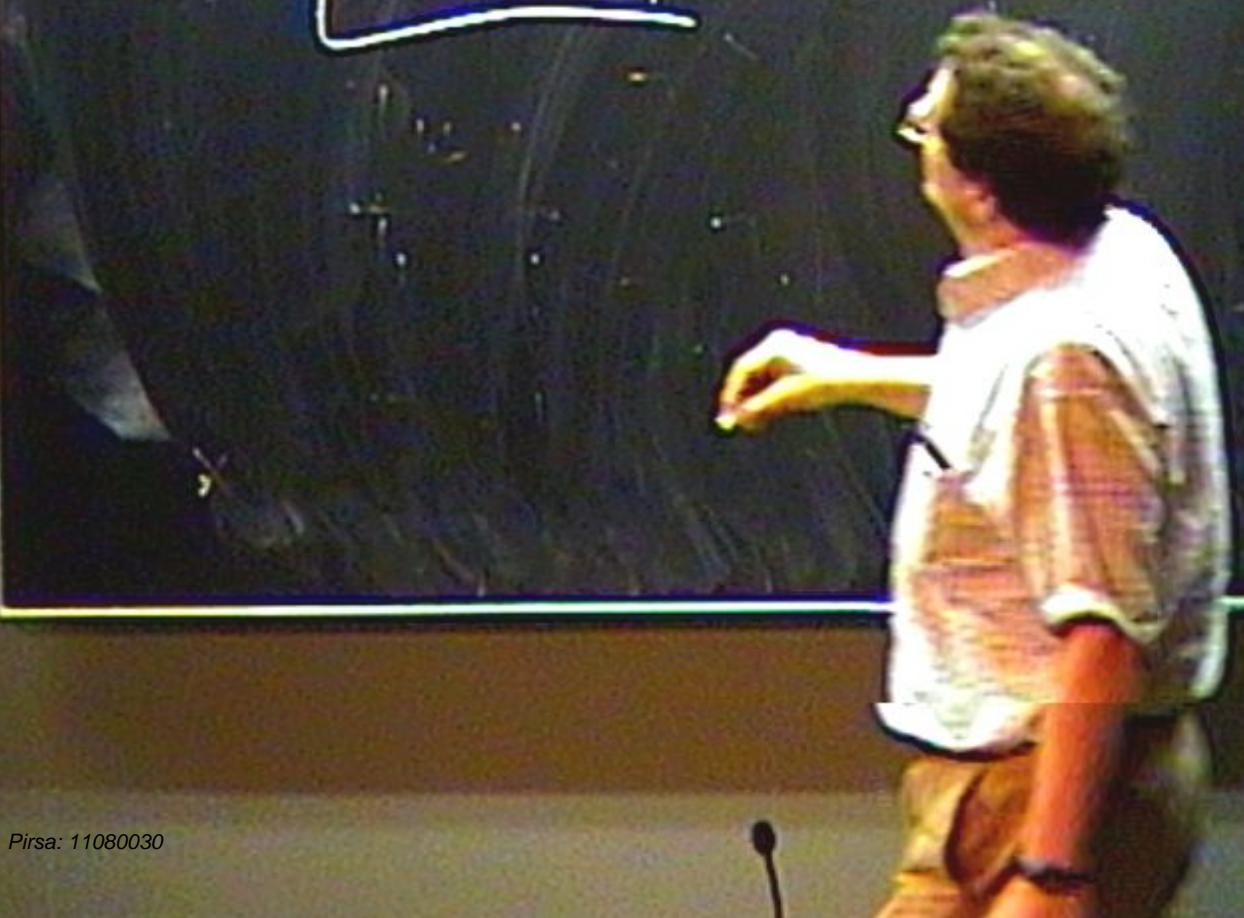


$$\mathcal{Z}^* = (\omega^*, \pi_{\mathcal{R}})$$

$$\omega^* = i X^M T \pi_M$$



$\mathbb{C}\mathbb{P}$



$$\mathcal{Z}^* = (\omega^*, \pi_\alpha)$$

M



Lx
totally null
SD

$$\omega^* = i X^M T \pi_\alpha$$



$\in \mathbb{P}$

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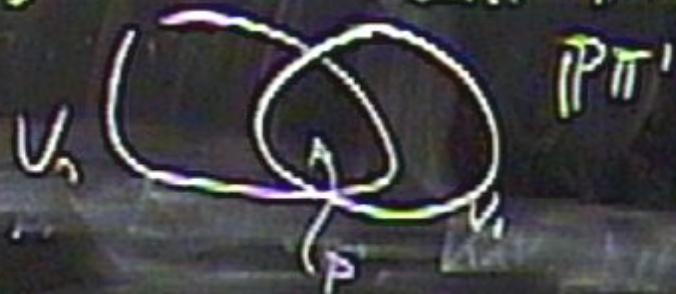
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SDG II
Holomorphic V.b $E \rightarrow P\pi'$

Loc.

Patching function (over $P\pi' - ?$)



SDT
Holomorphic V.b. $E \rightarrow P\pi'$

I. Patching function (over $P\pi' \cap U_i$)



? $\bar{\partial}$ -operator $\bar{\partial}_U f_U$

$$(\bar{\partial} f_U)^* = 0$$

$f_U = \phi(\omega, \tau, \bar{\tau}) \tilde{f} d\bar{\tau}$ Hol function
of \mathbb{R}^n on $U_i \cap U_j$,
values + g(k).

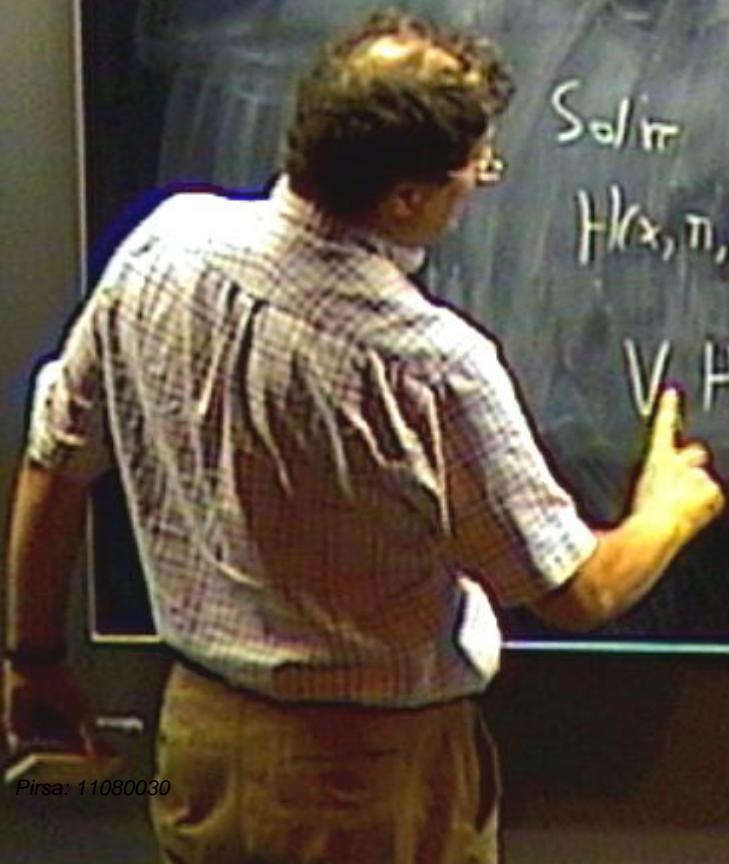
Cox 1. for each L_x must have $H_0(x, \pi) = H(x, \pi)$

St. $P H_0 = H$.

hol or $D_0 \cap L_x$
 $D_1 \cap L_x$
resp.

Solve $(\bar{\partial} + b) H(x, \pi, \bar{\pi}) \Big|_{L_x} = 0$
 $H(x, \pi, \bar{\pi})$

$$V H = H \pi^* A(x)$$



Cor 1. for each L_x must have $H_0(x, \pi) = H(x, \pi)$

St. $\int_{L_x} P H_0 \in H,$

hol on $D_0 \cap L_x$
 $D_1 \cap L_x$
resp.

Soln $(\partial + h) H(x, \pi, \bar{\pi}) \Big|_{L_x} = 0$
 $H(x, \pi, \bar{\pi})$

$\sim V_A H = H \pi^* A(x)$

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SDF

Holomorphy $V_b : E \rightarrow PT^*$

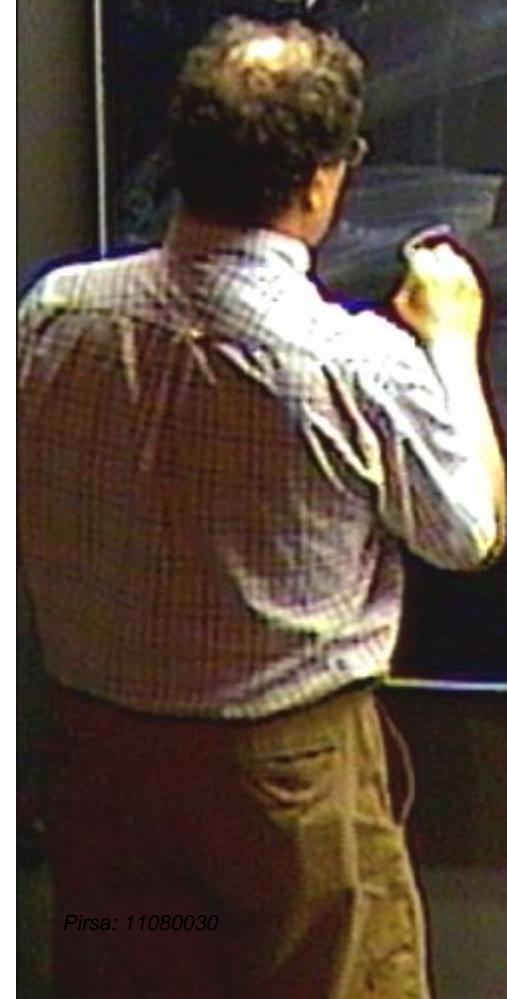
$(A_{ab}, \zeta - \epsilon \in \text{ZASD}(2-\text{nm}))$

Value in Lie \mathfrak{g}

function?

$$F = 0 \quad d_A \zeta = 0$$

$$\int G_A F$$



SDS

Hilbert space V^* , $E \rightarrow PT$

$(A_{ASD}, G - E)$ (ASD 12-ha)

Value in Lie G function?

$F = 0$ $d_A G = 0$

$$S_{ASD} = \int G_A F + S_{\text{SUSY}}^{\text{N=6}}$$

+ SUSY completion \sim Hol Chan-Sen
on $PT^{1/4}$

$$S_{\text{full}} = \int G_A F + \frac{1}{2} \lambda \int G^2$$

$\text{SDG} \rightarrow \text{Hilb. pl. } V_{\lambda} : E \rightarrow PT$

$(A_{\alpha\beta}, G - \in \text{CASD 12-hm})$ with
Value in Lie G function (λ)

$F = 0$ $d_A G = 0$
 $\int G_A F + \text{SUSY completion} \sim \text{Hol Chan-Sen}$
 $= \int G_F + \frac{1}{2} \lambda \int G^2$ $\text{on } PT^{1/4}$

S_{SD}

Holonomy $\rightarrow V_L \rightarrow PT$

$(A_{ab}, G - C)$ ASD 12-form

Value in Lie G

function ψ ?

$$F = 0$$

$$d_A \varphi = 0$$

$$+ S_{\text{SUSY}}^{\text{N=6}}$$

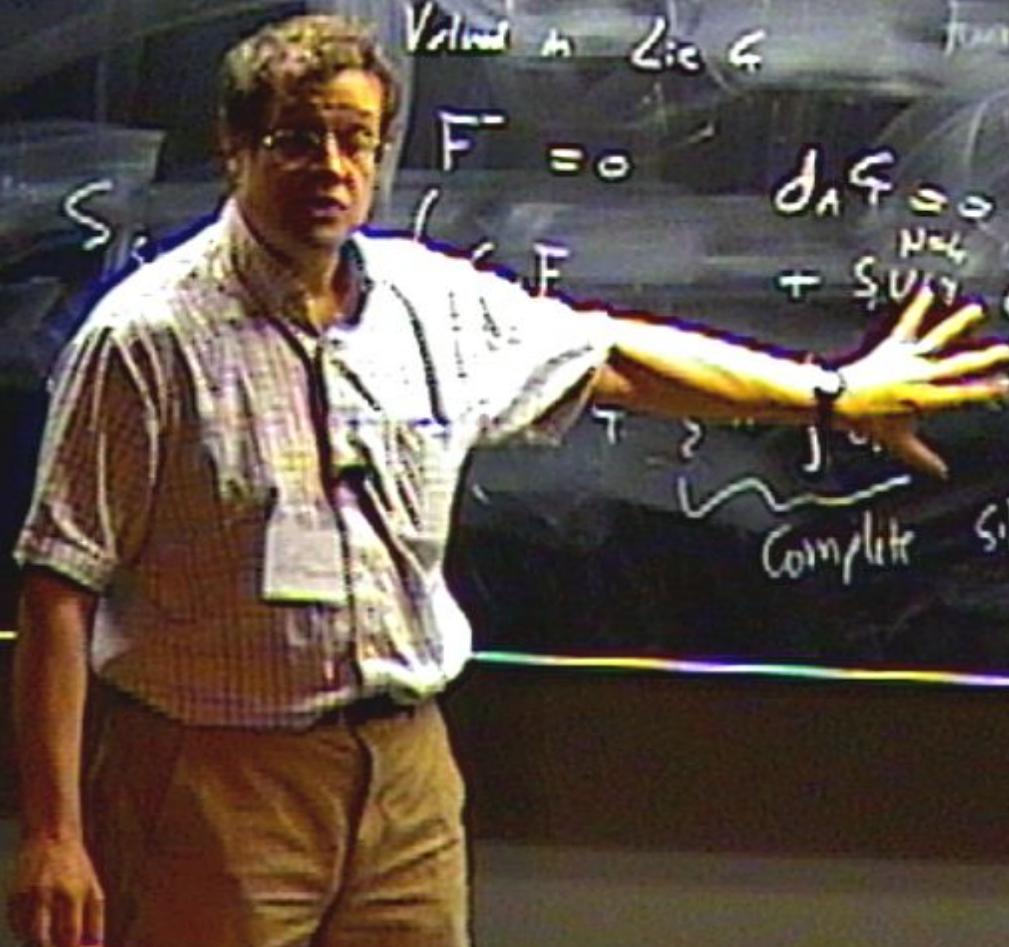
+ SUSY completion \sim Hol Chan-Sen

on $PT^{1/4}$

$$S_{SD} = \int G_A F$$

$$S_{\text{full}} = \int G_A F + \frac{1}{2} \lambda \int G^2$$

Complete SUGRA full theory



SDT
Holonomy $V_L : E \rightarrow PT$
 $(A_{\alpha}, \varphi \in \text{ASD 12-hm})$
Values in Lie \mathfrak{g} functions
 $F = 0$ $d_A \varphi = 0$
 $+ SU(N)$ complete \sim Hol Chan-Sen
on $PT^{1/4}$
Complete SdS sector full theory

SDG

Holonomy $\tilde{V}_A \cdot E \rightarrow PT$

$(A_{\mu\nu}, G - \in ZASD 12-\text{form})$

Value in Lie G

function \tilde{V}_A

$$F = 0$$

$$d_A G = 0$$

+ SUSY completion \sim Hol Chan-Sen
on $PT^{1/4}$

$$S_{SD} =$$

$$\int G_A F$$

$$S_{full} =$$

$$\int G_A F + \frac{1}{2} \lambda \int G^2$$

Complete SD Sector full theory