Title: Solving AdS/CFT Y-System

Date: Aug 10, 2011 10:00 AM

URL: http://pirsa.org/11080029

Abstract:

Solving AdS/CFT Y-system

Sébastien Leurent
LPT-ENS (Paris)

[arXiv:1108.soon] N. Gromov, V. Kazakov, SL & D. Volin

[arXiv:1007.1770] V. Kazakov & SL
[arXiv:1010.2720] N. Gromov, V.Kazakov, SL & Z.Tsuboi
[arXiv:1010.4022] V. Kazakov, SL & Z.Tsuboi

Perimeter Institute, August 10, 2011

Outline

Solving
AdS/CFT
Y-system

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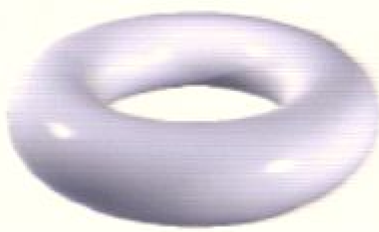
unctions
nmetries

1. Motivation : Understanding Y-system better
2. Solving Hirota → Q-functions
3. Understanding symmetries from strong coupling

Thermodynamic Bethe Ansatz

short operators
infinite time periodicity
Path integral $Z \sim e^{-RE_0(L)}$



Long operators
finite time-periodicity
$\Rightarrow$ finite temperature

"free Energy" : $f(L) = E_0(L)$

TBA equations

⇝ Equations of the form

$$Y_{a,s}(u) = \sum_{a',s'} K_{a,s}^{(a',s')} \star \log\left(1 + Y_{a',s'}(u)^{\pm 1}\right)$$
$$+\delta_{s,0}\ L\ \log \frac{x^{[-a]}}{x^{[+a]}} + (\text{Source Terms})$$

[Gromov Kazakov Kozak Vieira 09]

[Bombardelli Fioravanti Tateo 09] [Autyunov Frolov 09]

$$x^{[\pm a]} = x(u \pm a\tfrac{i}{2}) = \frac{1}{2}\frac{u \pm a\frac{i}{2}}{g} + \frac{i}{2}\sqrt{4 - \left(\frac{u \pm a\frac{i}{2}}{g}\right)^2}$$

- $Y_{a,s}(u)$ is a function of $a, s \in \mathbb{Z}$ and u in $\mathbb{R}$

- Extra assumption : Excited states obey the same equations.
  Each state correspond to a different solution of Y-system, characterized by its zeroes and poles

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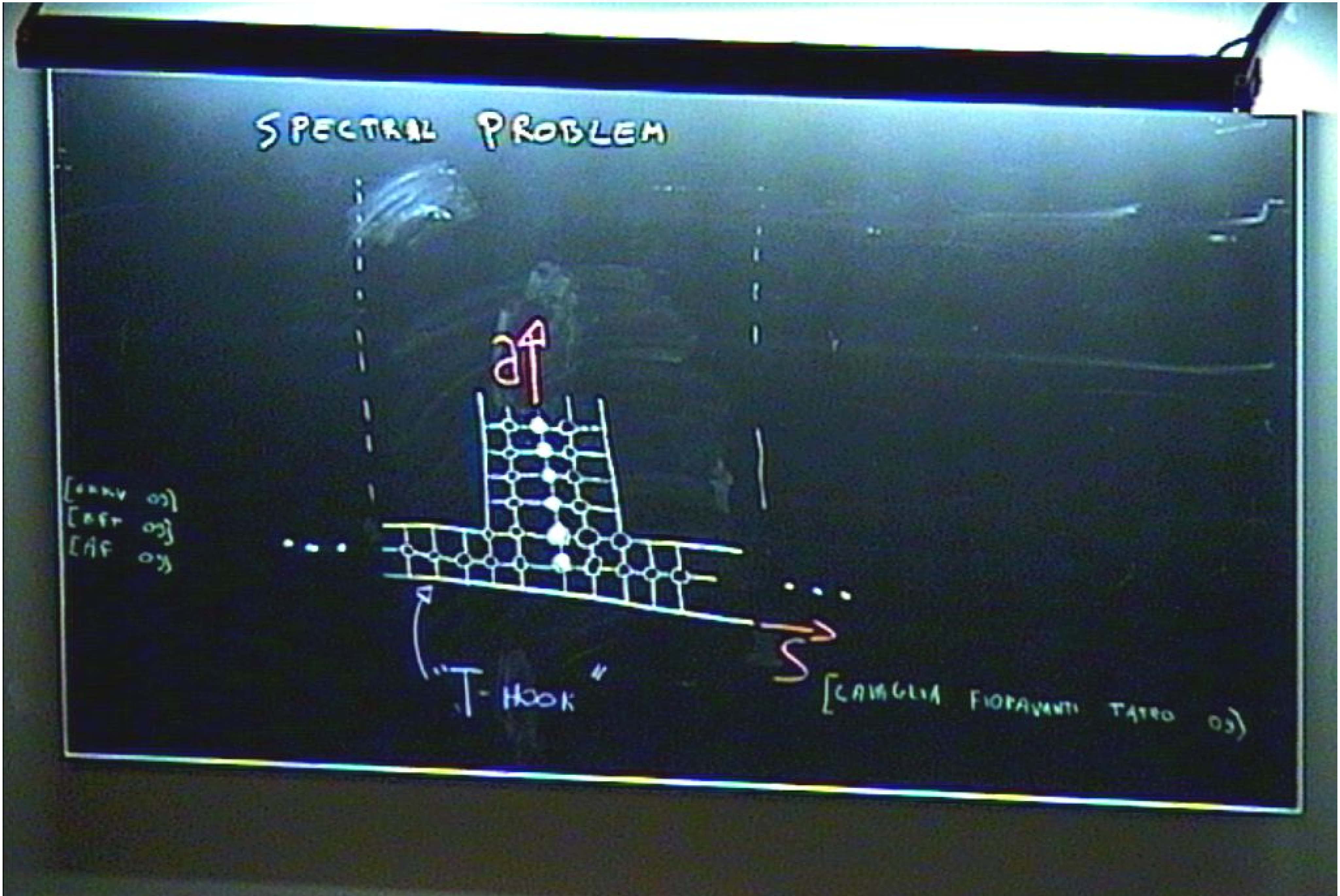
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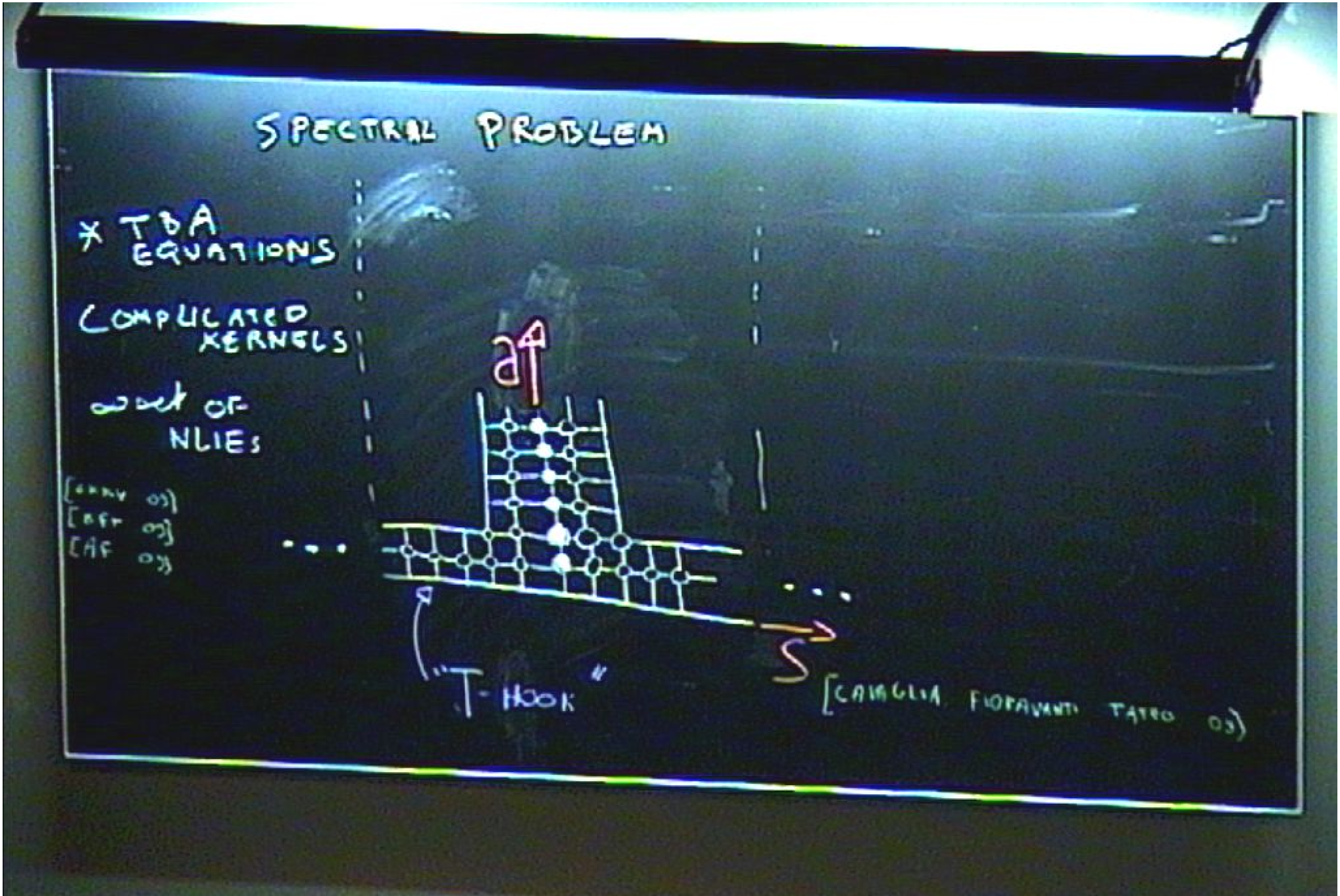
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Y-system and Hirota equation

Y-system Equation

The TBA integral equation imply the 'local' relation

$$Y^+_{a,s}\,Y^-_{a,s} = \frac{1+Y_{a,s+1}}{1+(Y_{a+1,s})^{-1}}\,\frac{1+Y_{a,s-1}}{1+(Y_{a-1,s})^{-1}}$$

[Gromov Kazakov Vieira 09] where $Y^\pm_{a,s} = Y_{a,s}(u \pm \frac{i}{2})$

- change of variable

Hirota equation

Gauge freedom

Y-functions and Hirota equation are invariant under gauge transformations

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$$T_{a,s}^{+}T_{a,s}^{-} = T_{a+1,s}T_{a-1,s} + T_{a,s+1}T_{a,s-1}$$

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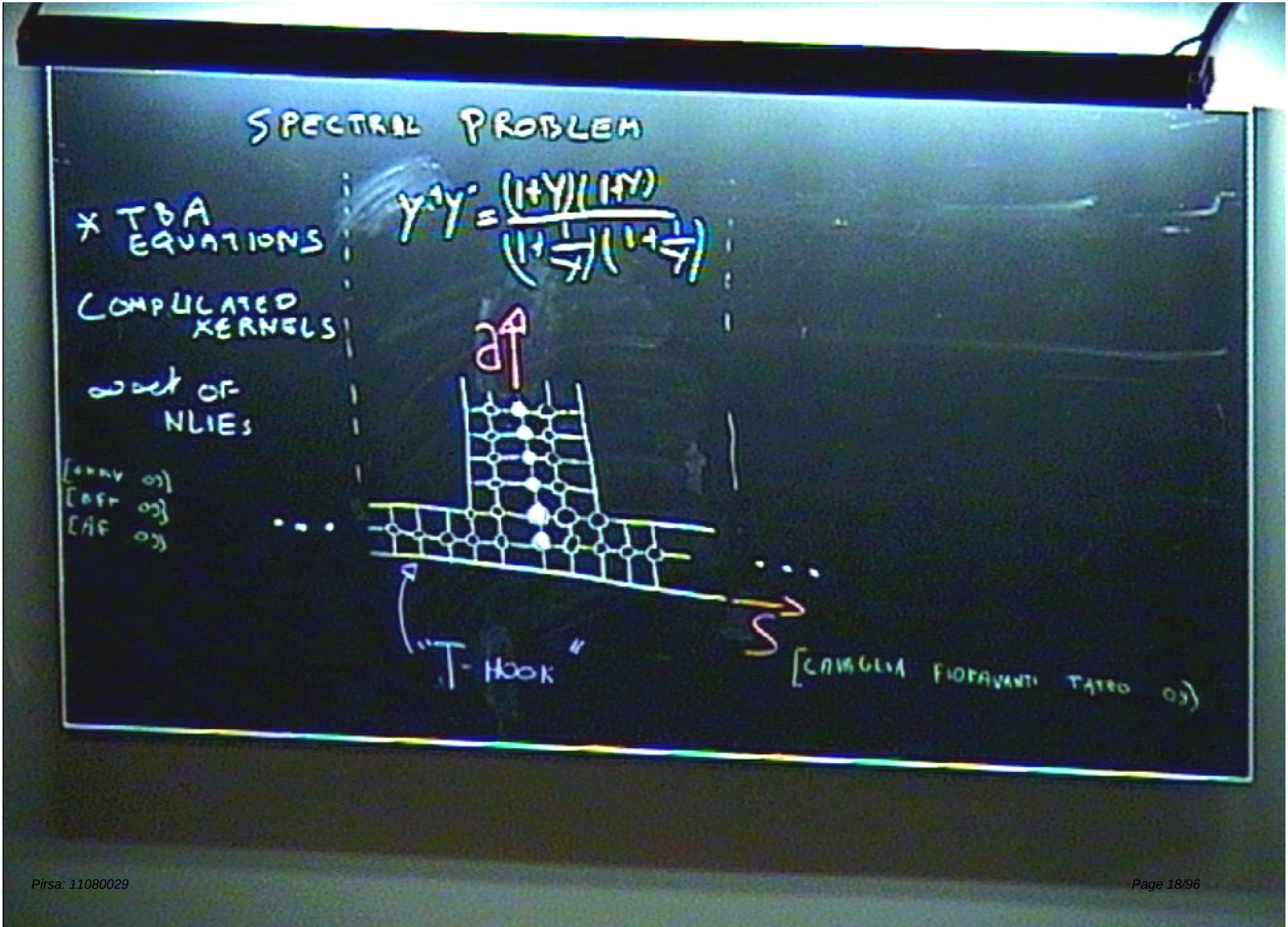
Y-functions and Hirota equation are invariant under gauge transformations $T_{a,s} \to g_1^{[a+s]}g_2^{[a-s]}g_3^{[-a+s]}g_4^{[-a-s]}T_{a,s}$

$f^{[\pm k]} = f(u \pm ik/2)$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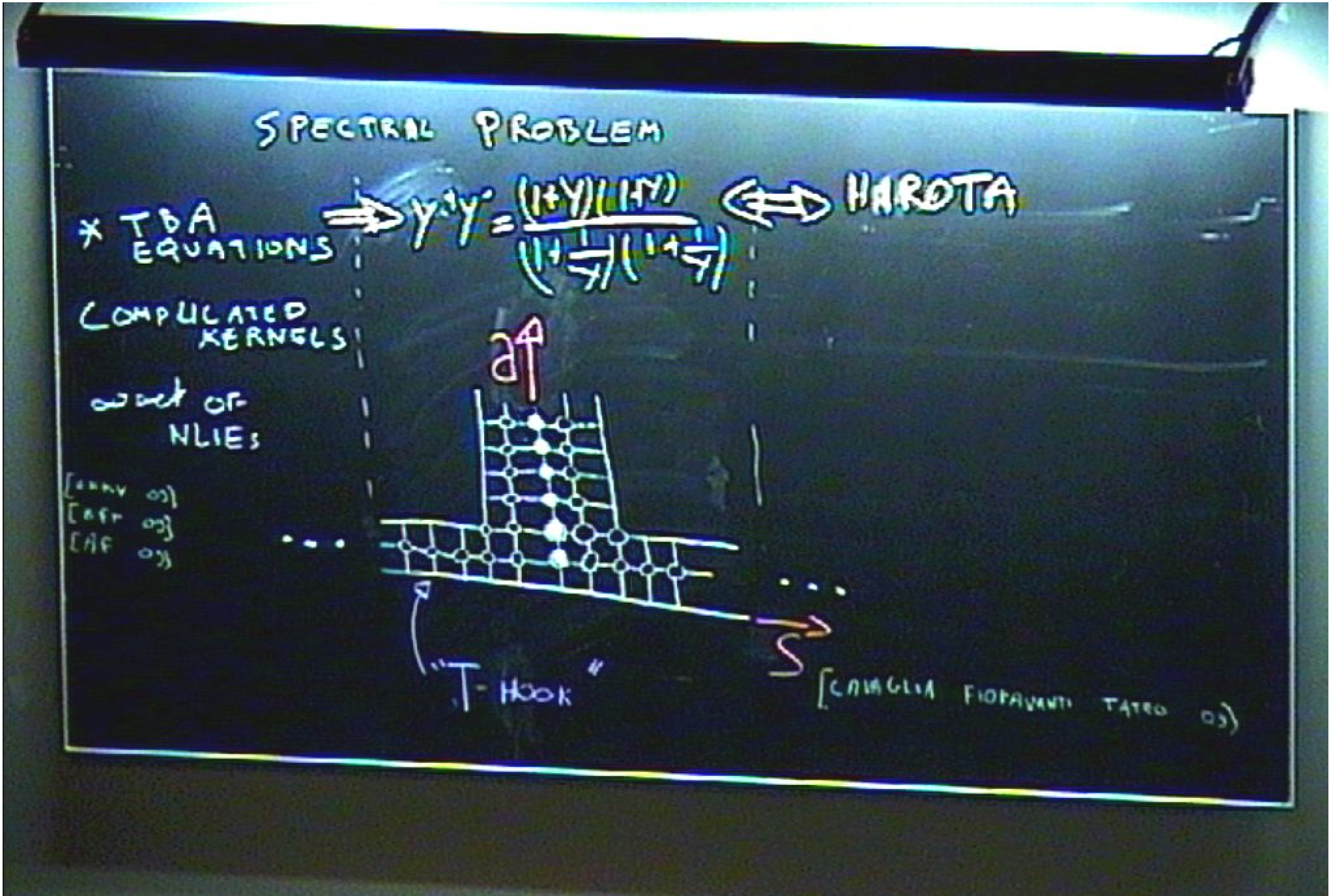

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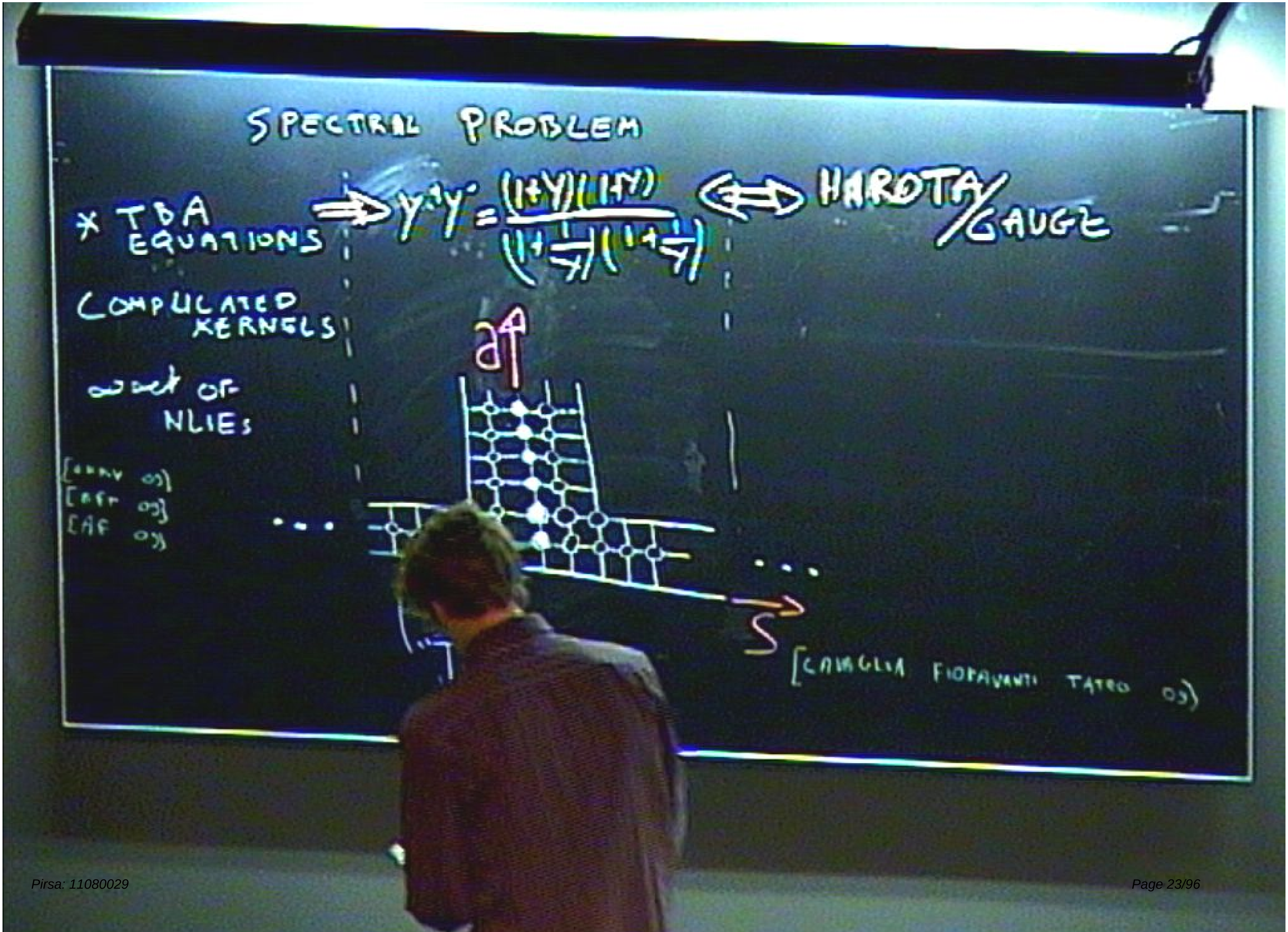
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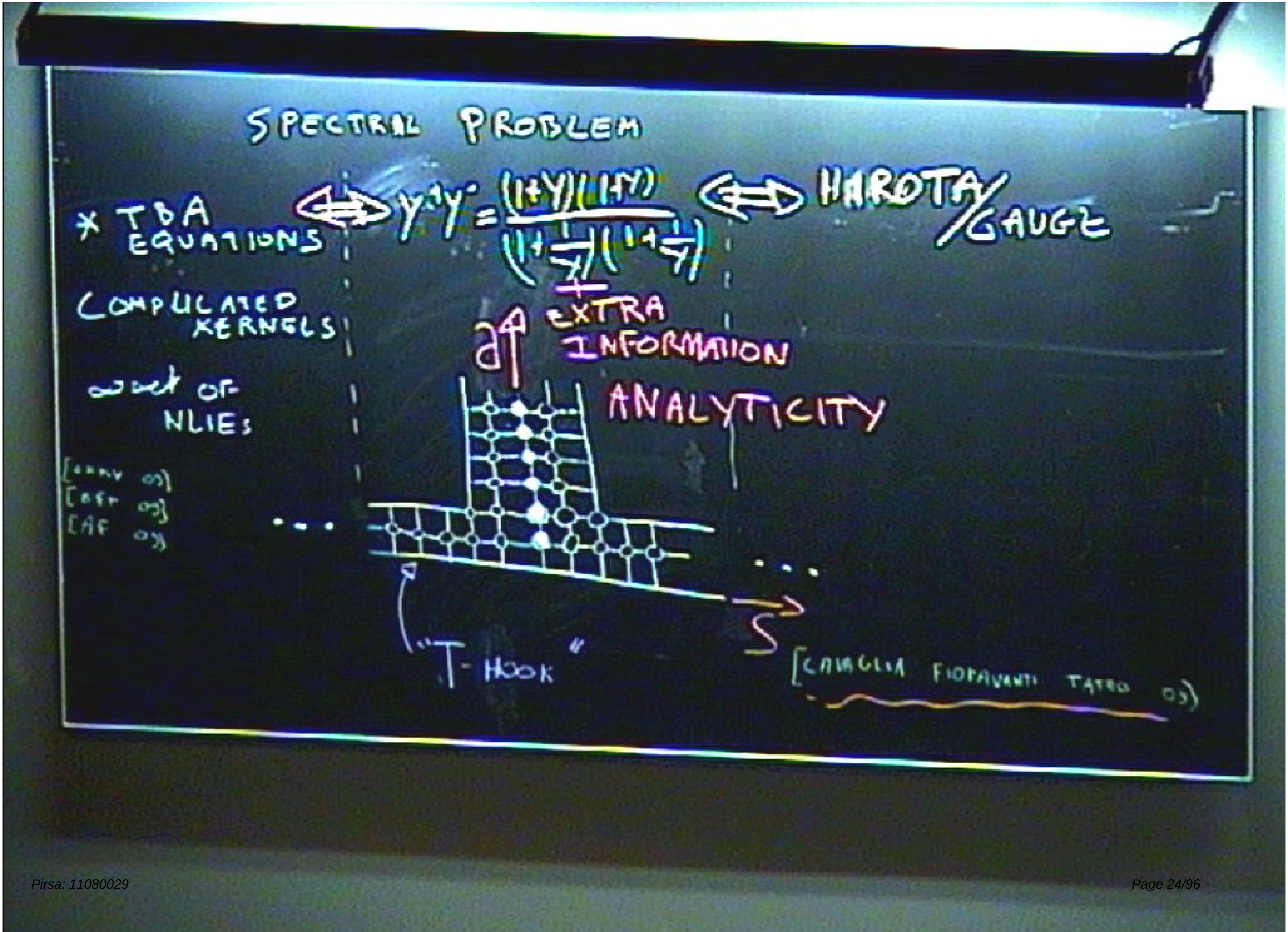
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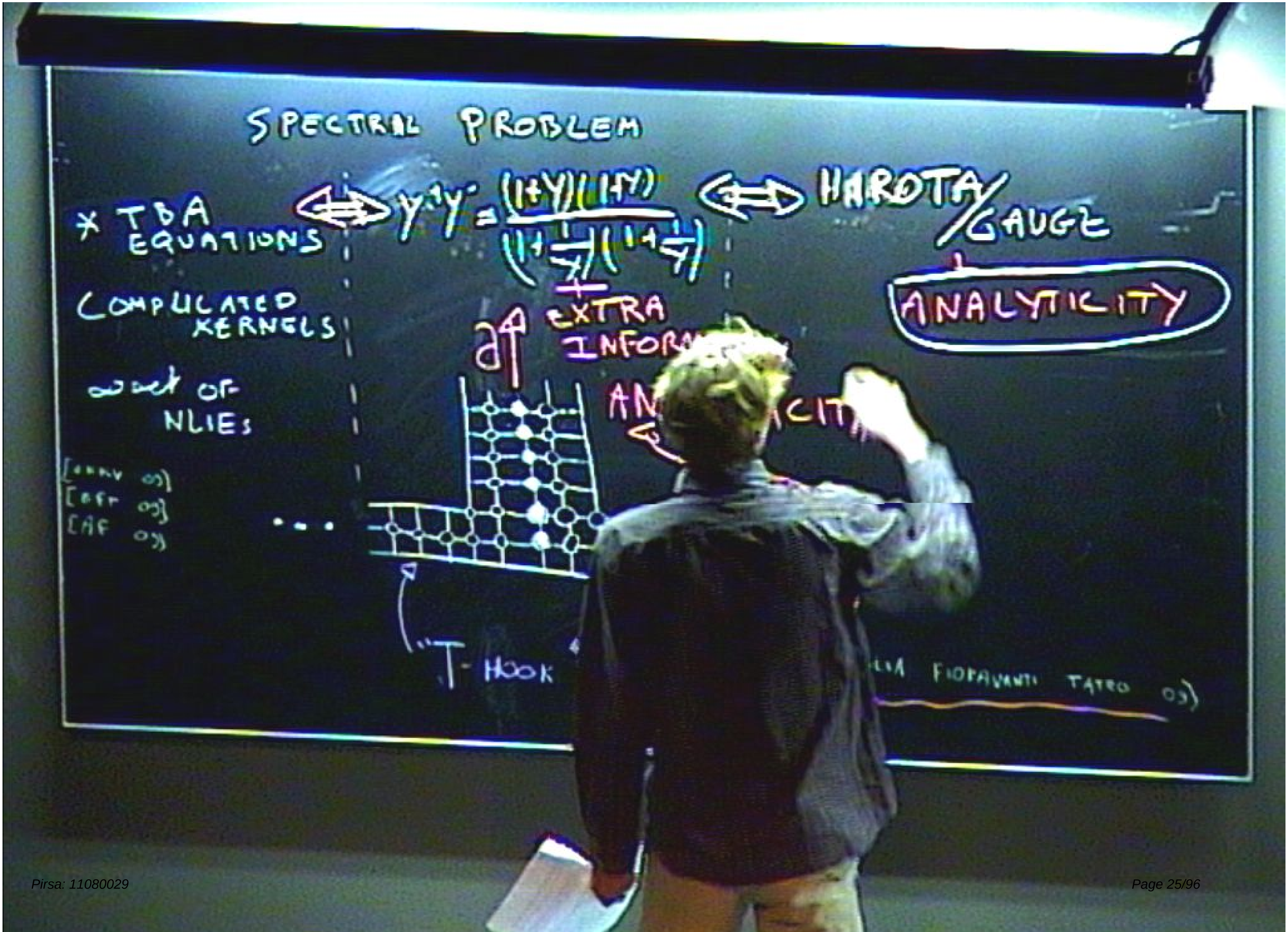
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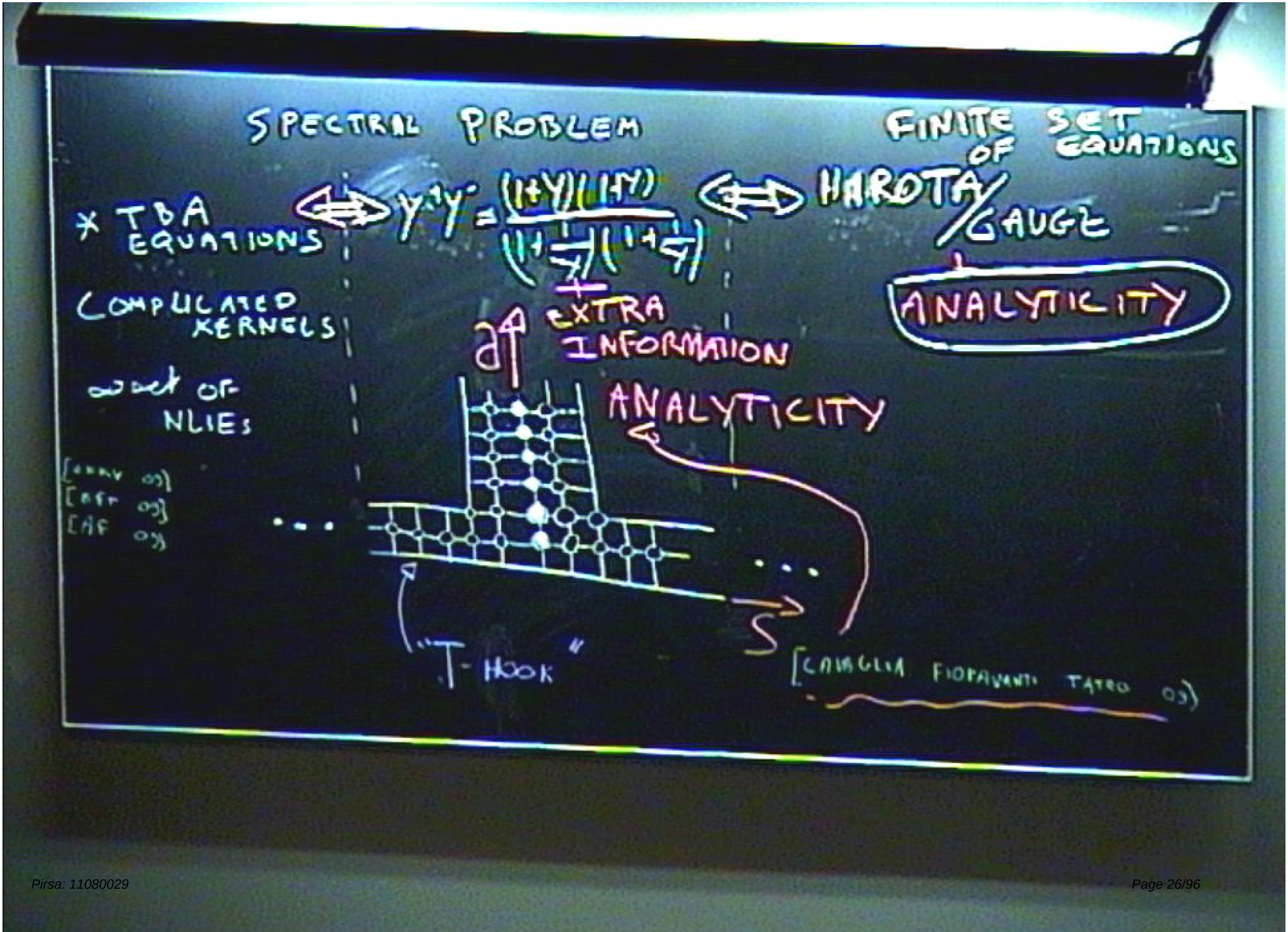
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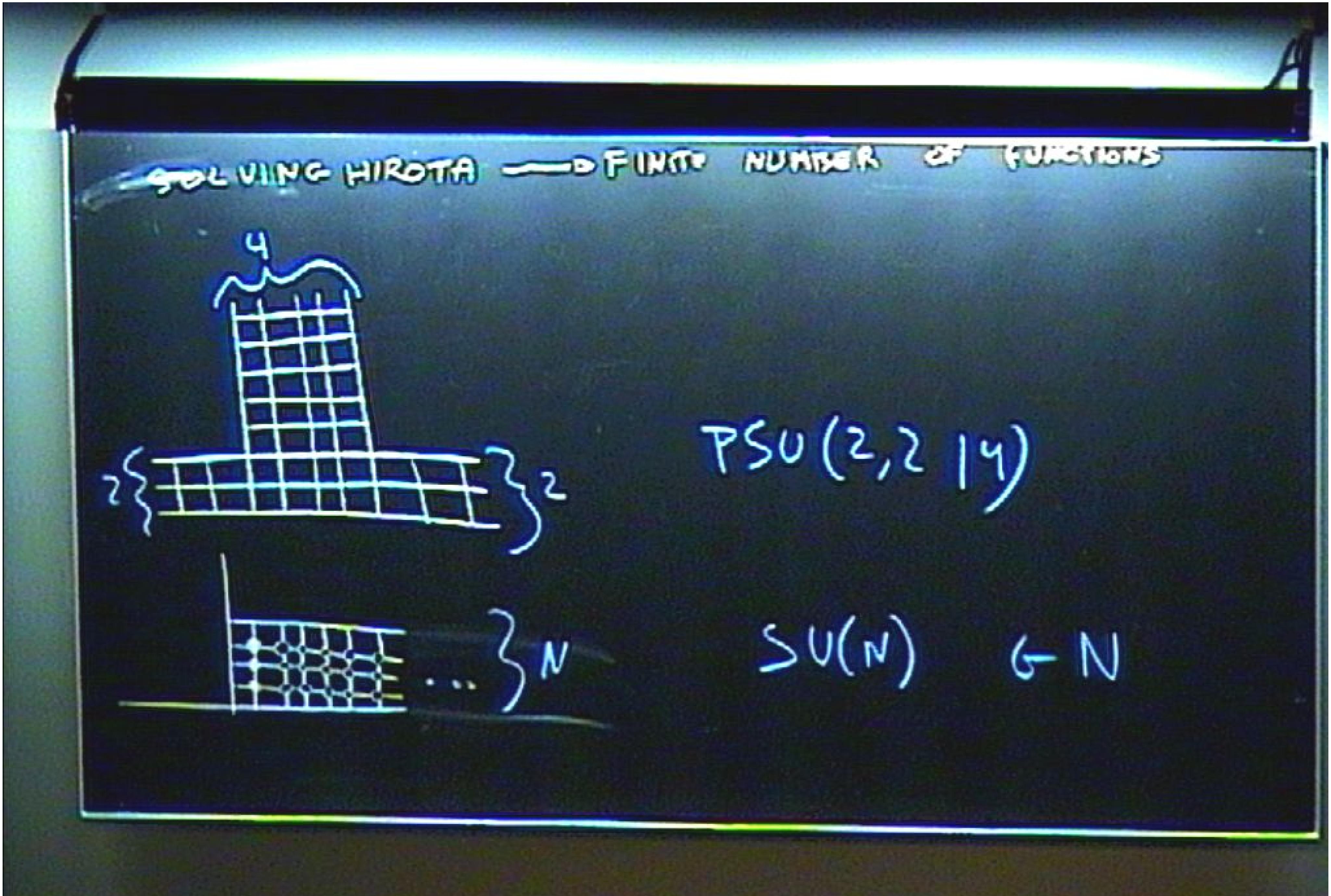
SOLVING HIROTA → FINITE NUMBER OF FUNCTIONS
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Q-functions solve Hirota equation

Solving AdS/CFT Y-system

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Symmetries

Hirota equation is solved by determinants of Q-functions : eg. for $SU(4)$,

$$T_{3,s} = \begin{vmatrix} q_1^{[+s+2]} & q_2^{[+s+2]} & q_3^{[+s+2]} & q_4^{[+s+2]} \\ q_1^{[+s]} & q_2^{[+s]} & q_3^{[+s]} & q_4^{[+s]} \\ q_1^{[+s-2]} & q_2^{[+s-2]} & q_3^{[+s-2]} & q_4^{[+s-2]} \\ \tilde{q}_1^{[-s]} & \tilde{q}_2^{[-s]} & \tilde{q}_3^{[-s]} & \tilde{q}_4^{[-s]} \end{vmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}a\\a\\a\end{matrix}}\right\} 3 \\ \}\, 4-3 \end{matrix}$$

- $q_i^{[+k]} = q_i(u + k\frac{i}{2})$

[Baxter 72], [Pasquier Gaudin 92], [Bazhanov Lykyanov Zamolodchikov 96], [Derkachov, 99], [Bytsko Teschner 06], [Bazhanov Frassek Lukowski Meneghelli Staudacher 10]

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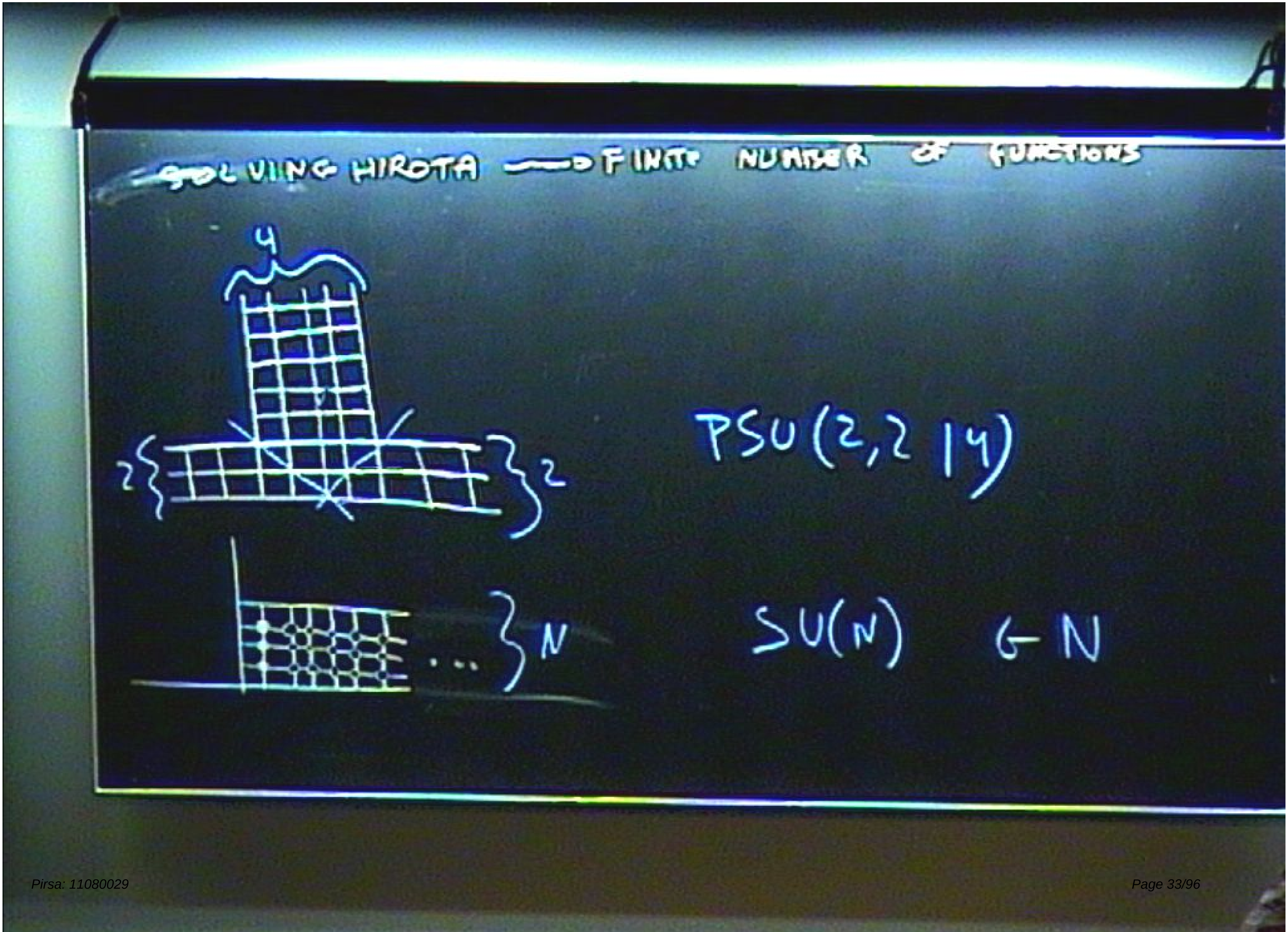
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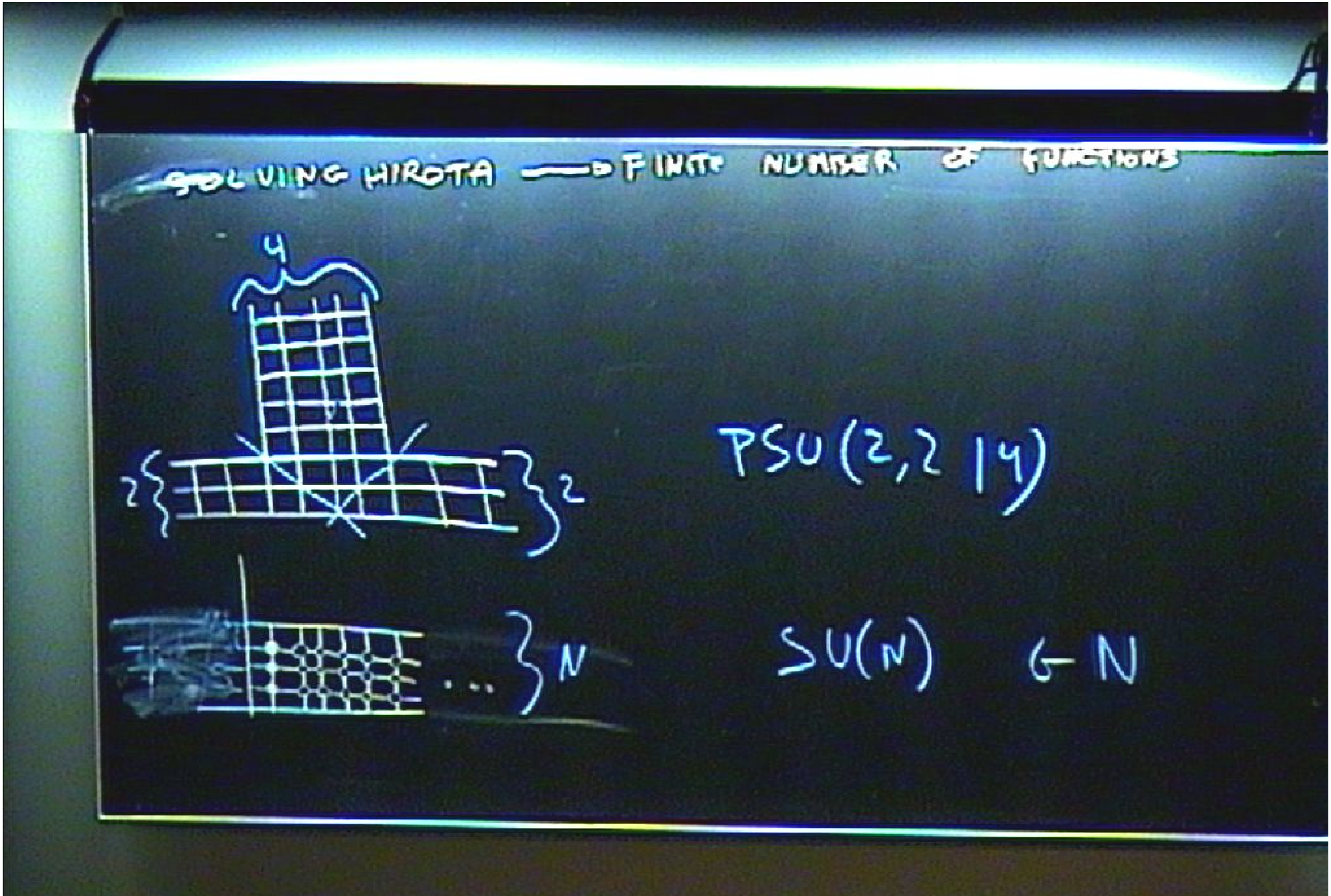
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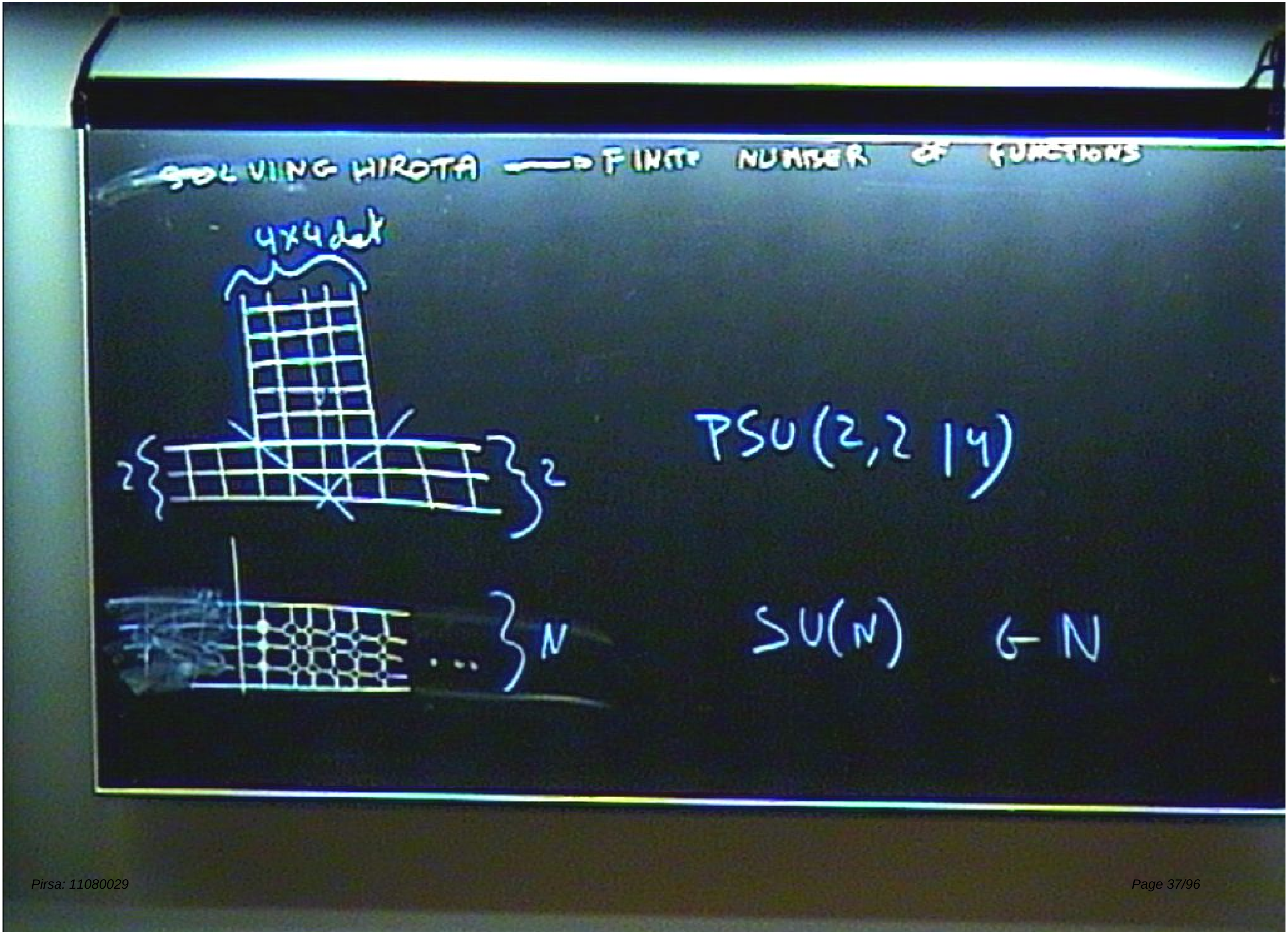
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4x4 det
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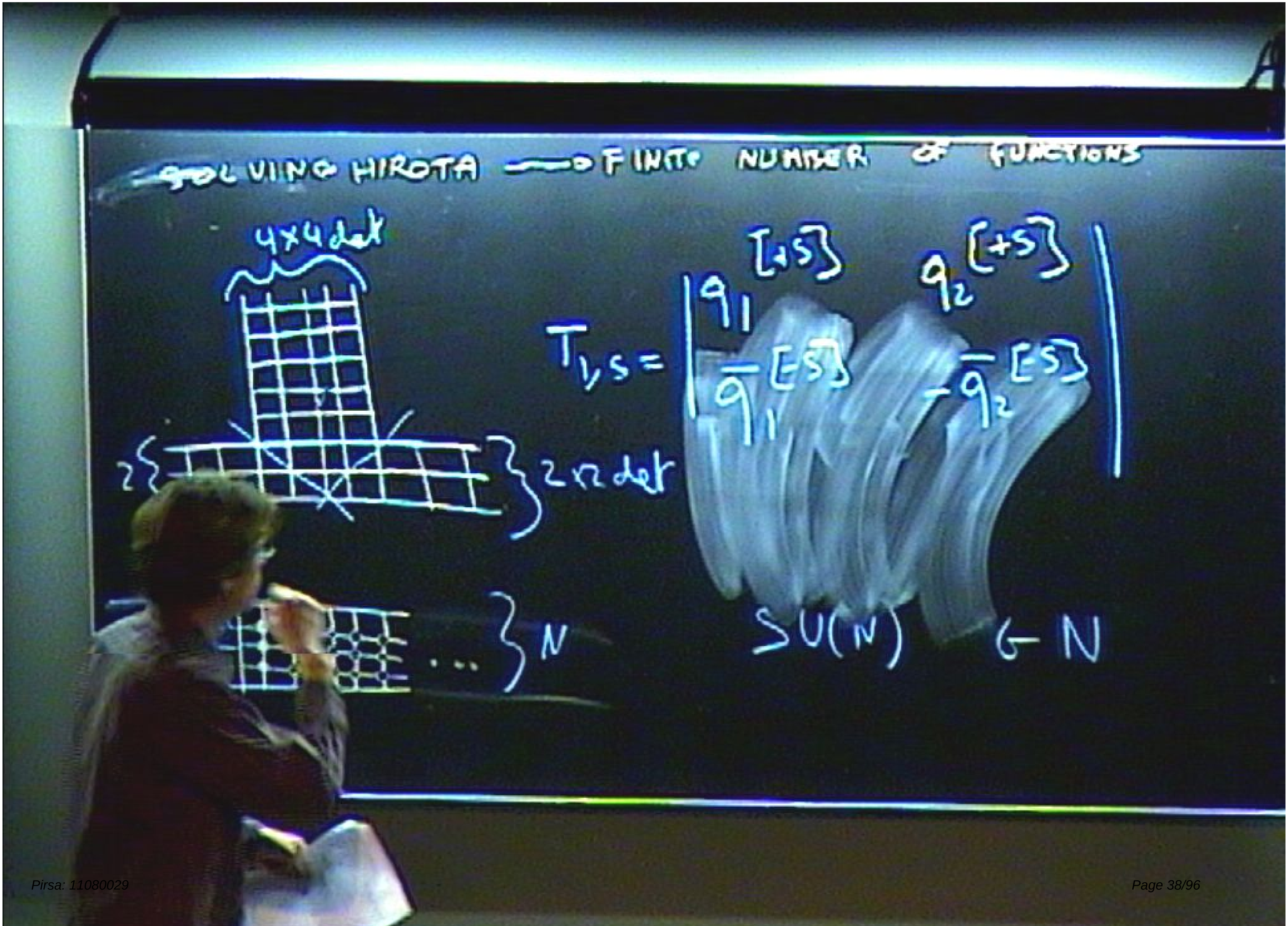
SOLVING HIROTA — FINITE NUMBER OF FUNCTIONS
4x4 det
2x2 det
2
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T1,s =
q1[+s] q2[+s]
q̄1[-s] q̄2[-s]
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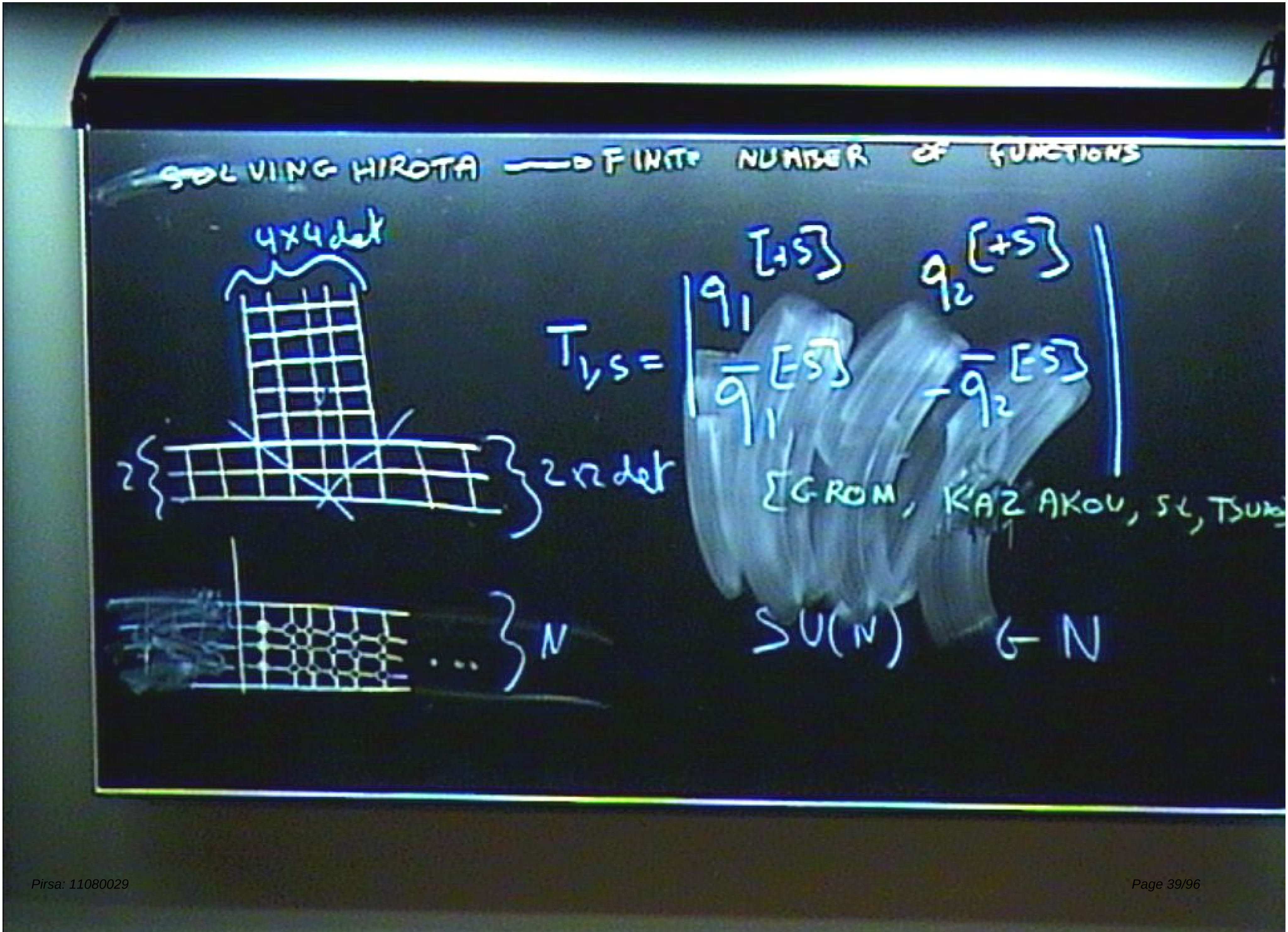

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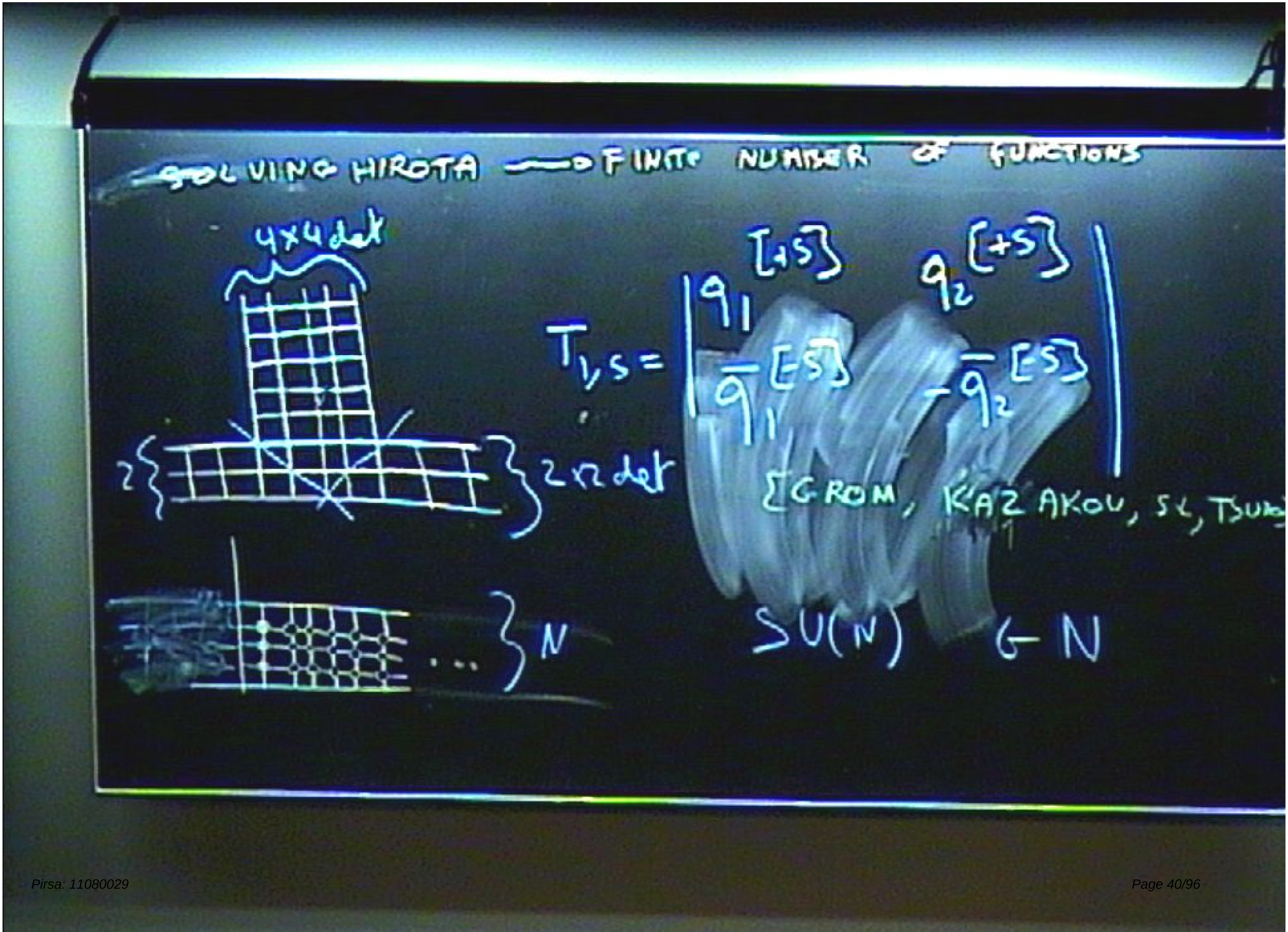
GROM, KAZAKOU, SL,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2x2 det
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T1,s =
GROM, KAZAKOV, SK, TSUB
SU(N)
G N

Q-functions solve Hirota equation

Solving
AdS/CFT
Y-system

S. Leurent

tivation
unctions
nmetries

Hirota equation is solved by determinants of Q-functions : eg. for $SU(4)$,

$$T_{3,s} = \begin{vmatrix} q_1^{[+s+2]} & q_2^{[+s+2]} & q_3^{[+s+2]} & q_4^{[+s+2]} \\ q_1^{[+s]} & q_2^{[+s]} & q_3^{[+s]} & q_4^{[+s]} \\ q_1^{[+s-2]} & q_2^{[+s-2]} & q_3^{[+s-2]} & q_4^{[+s-2]} \\ \tilde{q}_1^{[-s]} & \tilde{q}_2^{[-s]} & \tilde{q}_3^{[-s]} & \tilde{q}_4^{[-s]} \end{vmatrix} \begin{matrix} \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} 3 \\ \left.\begin{matrix} \\ \end{matrix}\right\} 4-3 \end{matrix}$$

- $q_i^{[+k]} = q_i(u + k\frac{i}{2})$

[Baxter 72], [Pasquier Gaudin 92], [Bazhanov Lykyanov Zamolodchikov 96], [Derkachov, 99], [Bytsko Teschner 06],
[Bazhanov Frassek Lukowski Meneghelli Staudacher 10]

[Kazakov S.L. Tsuboi 10]

Classical limit

In the classical limit, $g \to \infty$, and $T_{a,s} \to T_{a,s}(u/g)$.
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[Gromov Kazakov Tsuboi]

- Actually, the $PSU(2,2|4)$ symmetry imposes more constraints:
  - $\det = 1$
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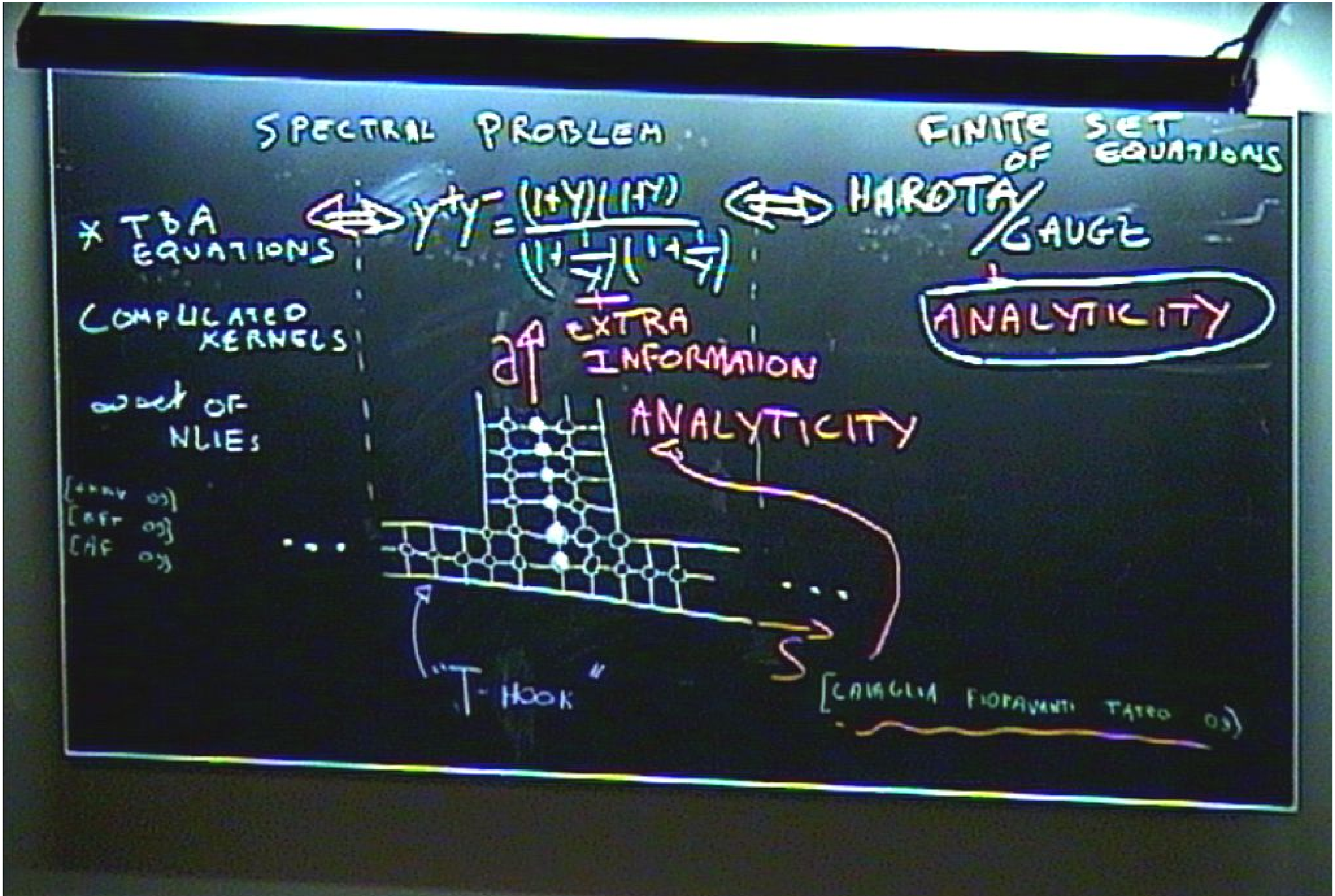
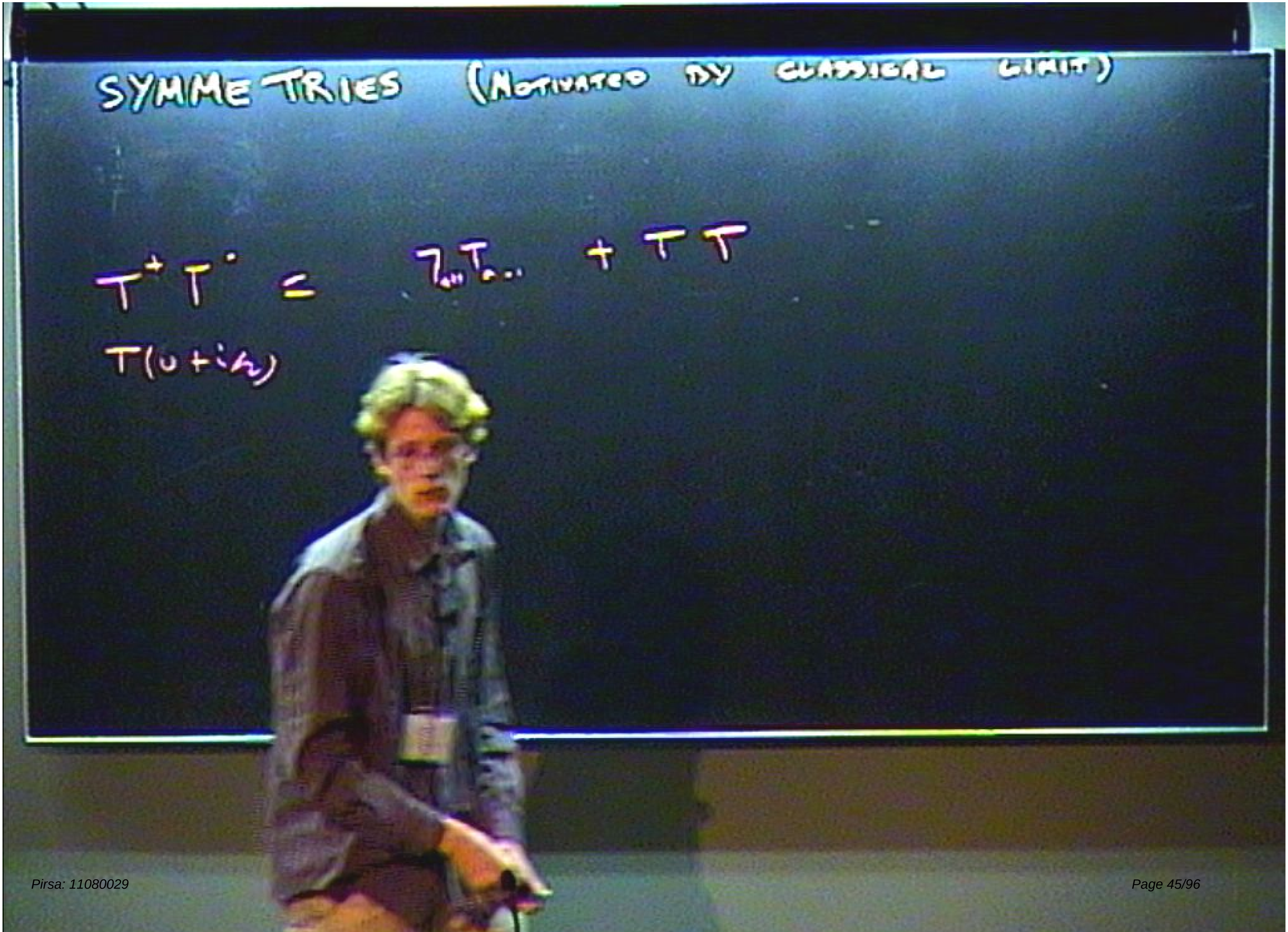
That gives extra symmetries of the characters (generalizing to symmetries of T-functions at finite size).


SPECTRAL PROBLEM
FINITE SET OF EQUATIONS
* TBA EQUATIONS
HIROTA / GAUGE
COMPLICATED KERNELS
ANALYTICITY
EXTRA INFORMATION
OF NLIEs
ANALYTICITY
"T-HOOK"
[CAVAGLIA FIORAVANTI TATEO 09]


SYMMETRIES (MOTIVATED BY CLASSICAL LIMIT)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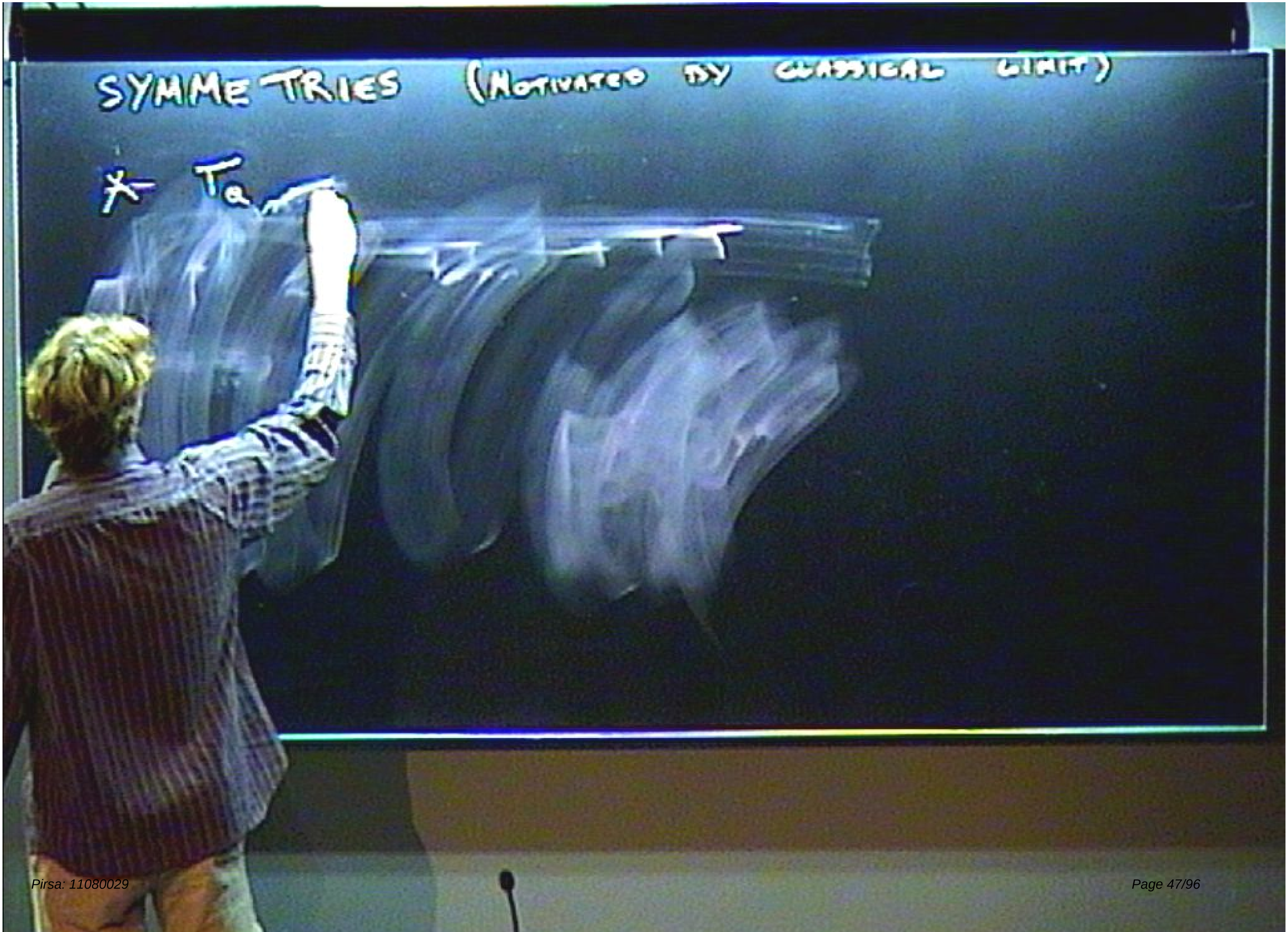
SYMMETRIES (MOTIVATED BY CLASSICAL LIMIT)
* Ta


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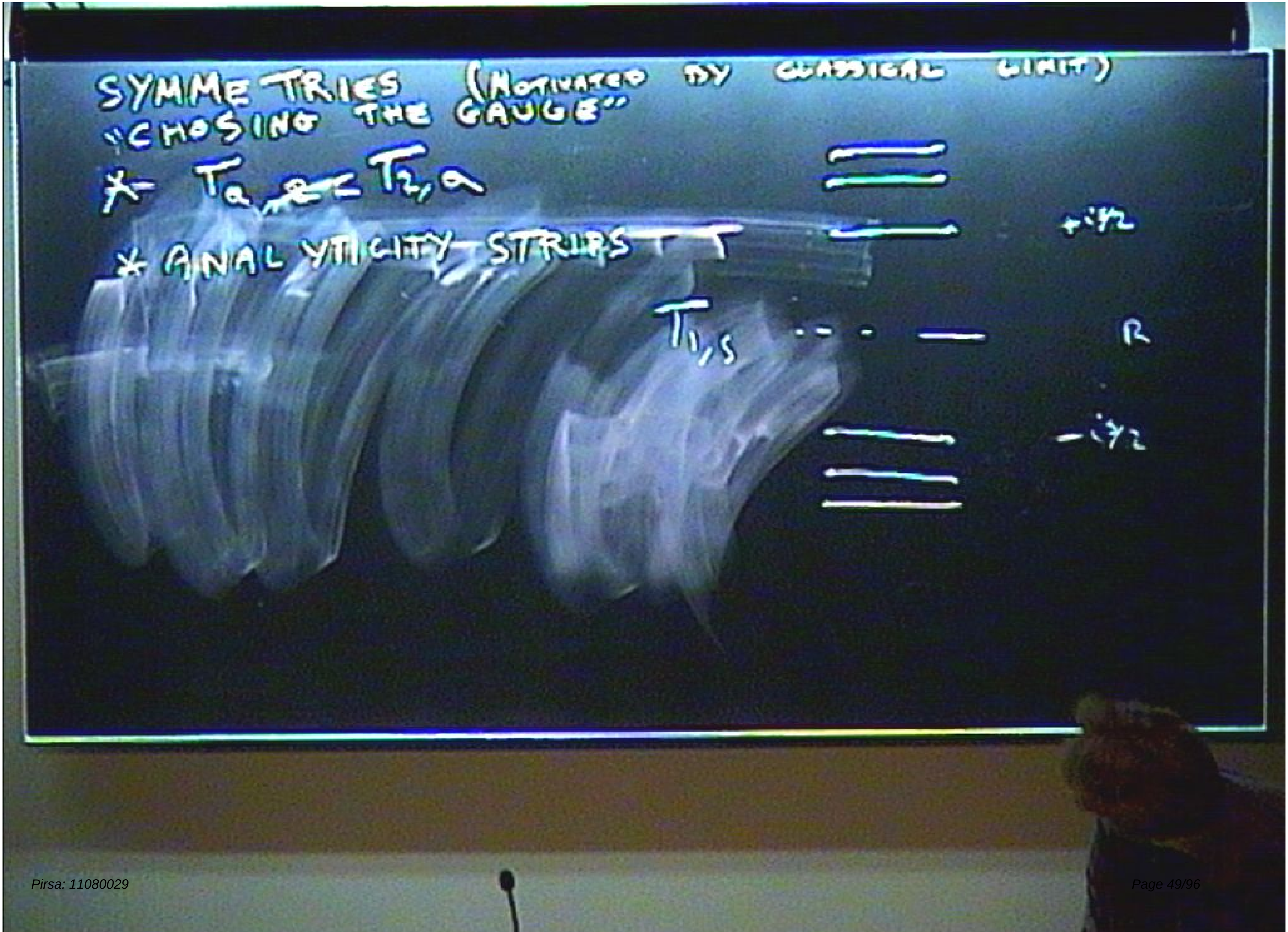
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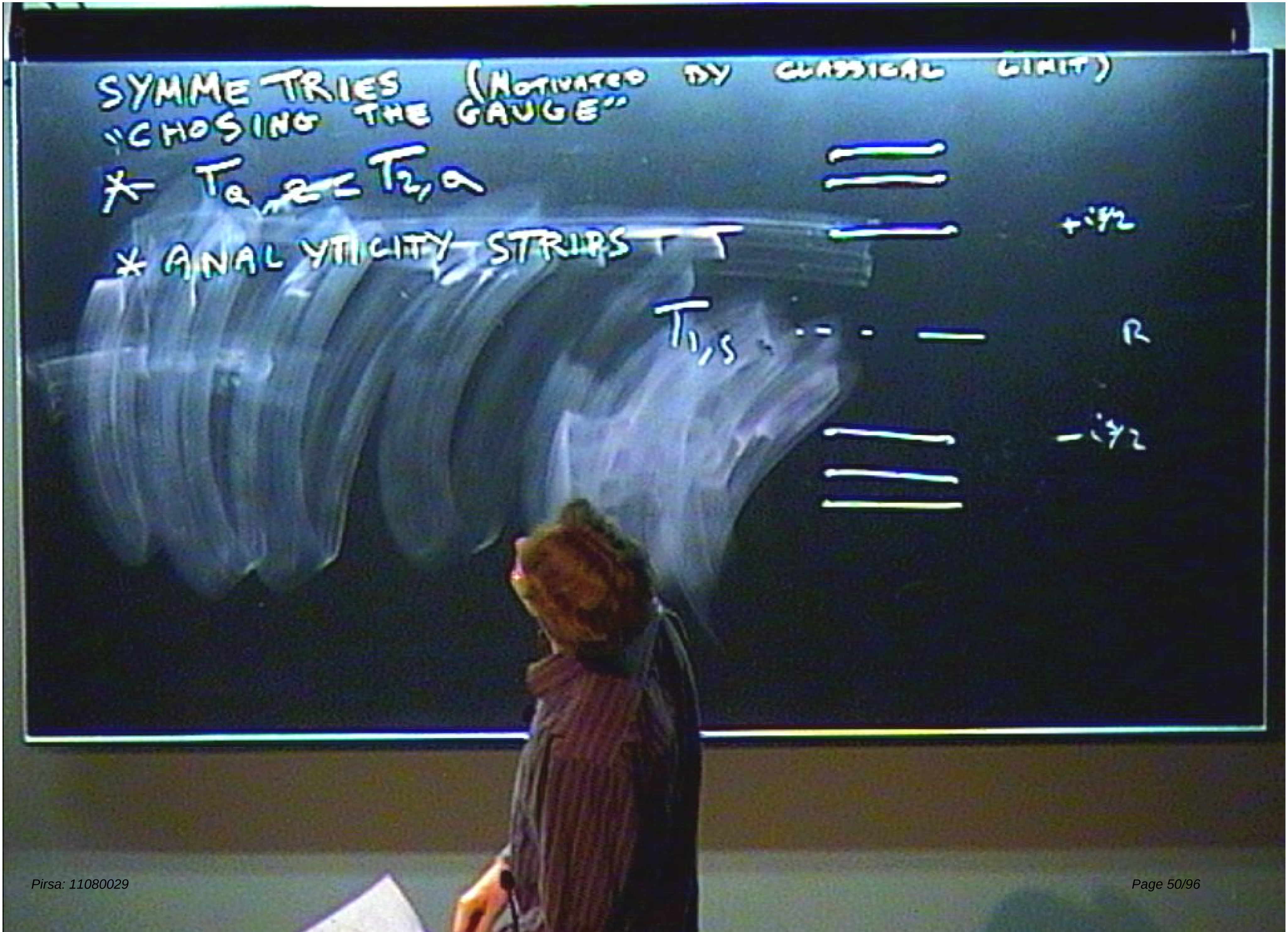

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Solving AdS/CFT Y-system

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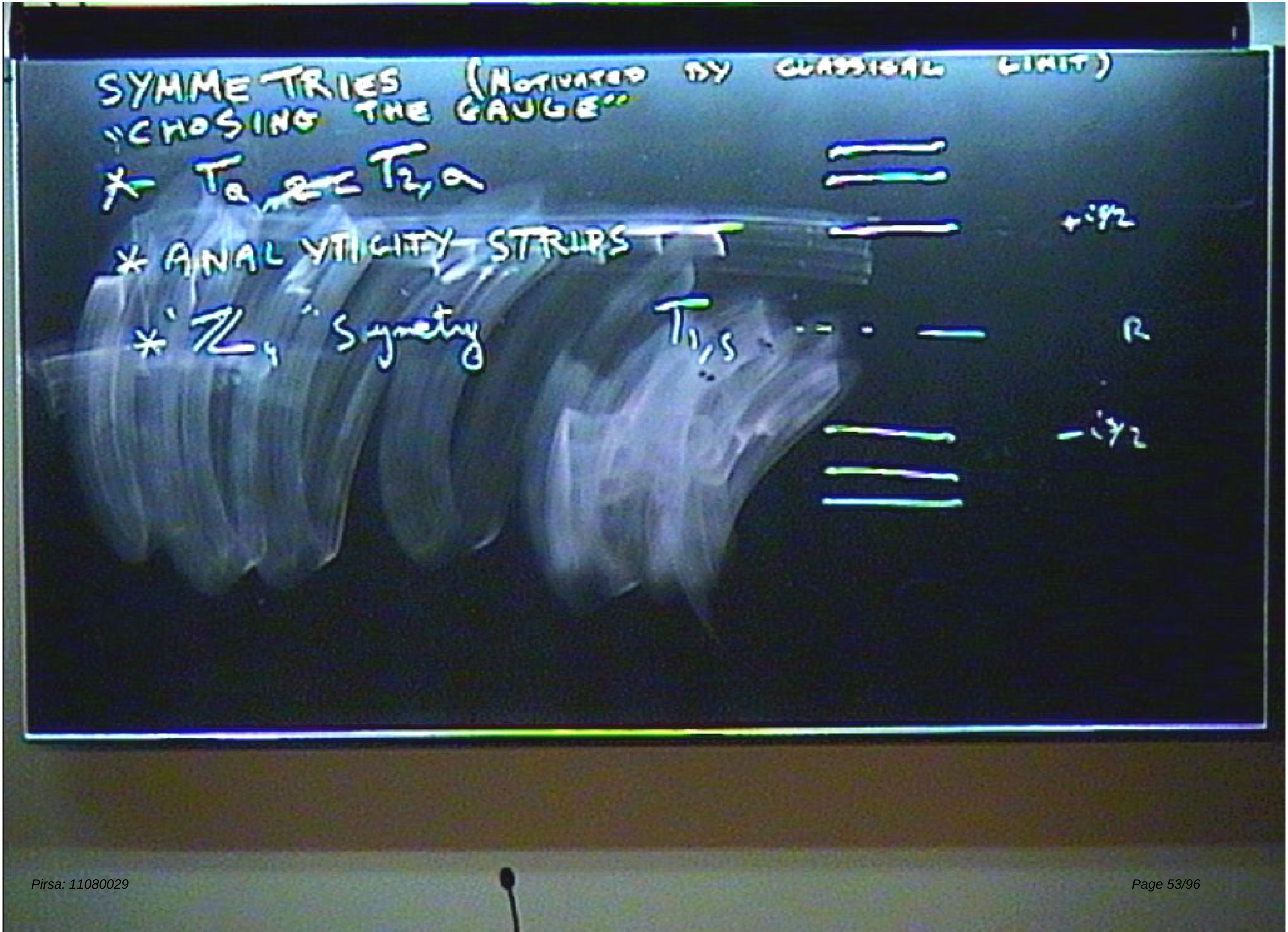
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$\mathbb{Z}_4$ symmetry

$\mathbb{Z}_4$ symmetry of the classical limit

$$\Omega = \hat{C}^{-1}(\Omega^{-1})^T \hat{C}$$

or

$$\{\lambda_i\} = \{1/\lambda_i\}$$

[Bena Polchinksi Roiban]

Quantum case

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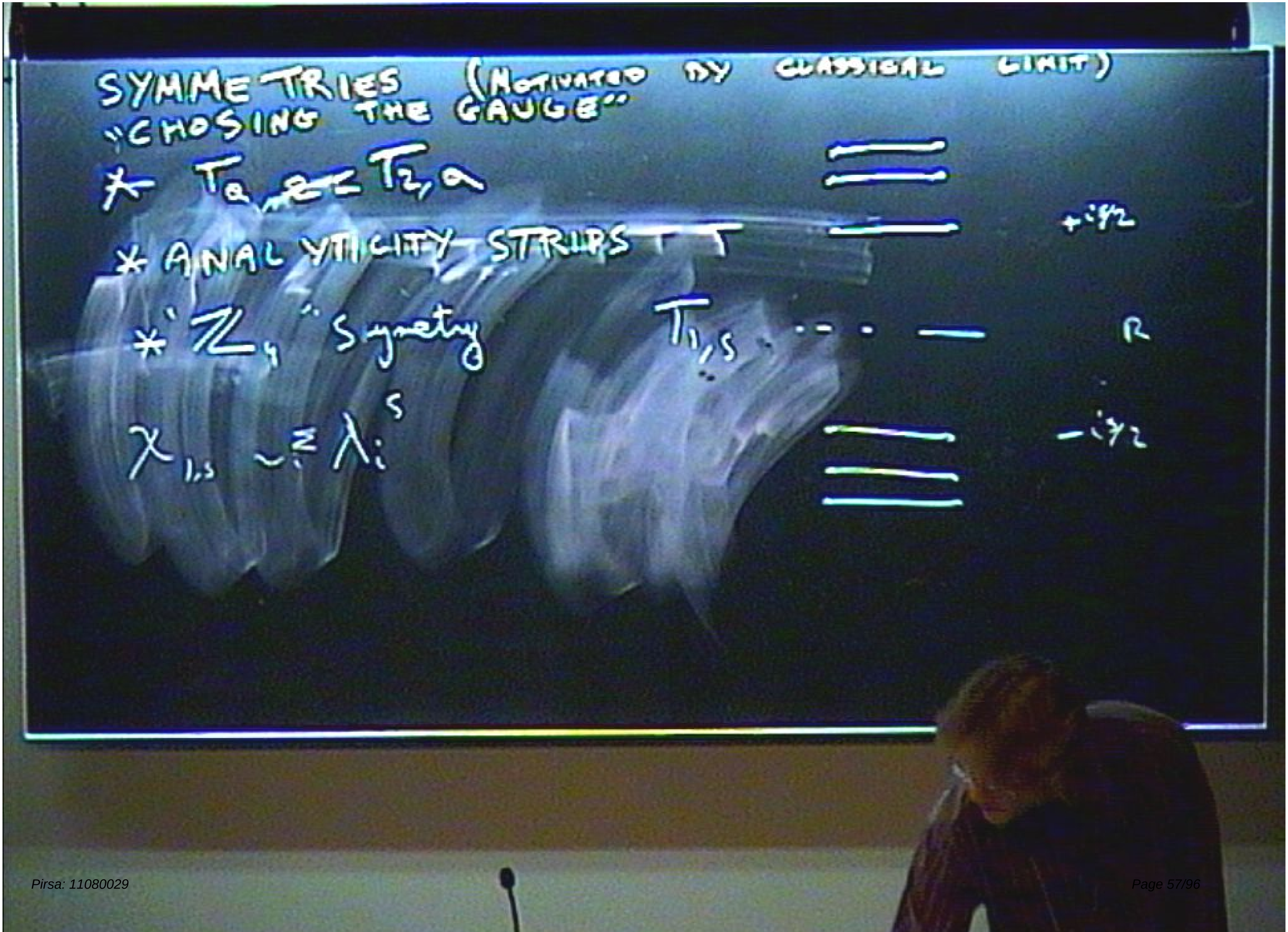
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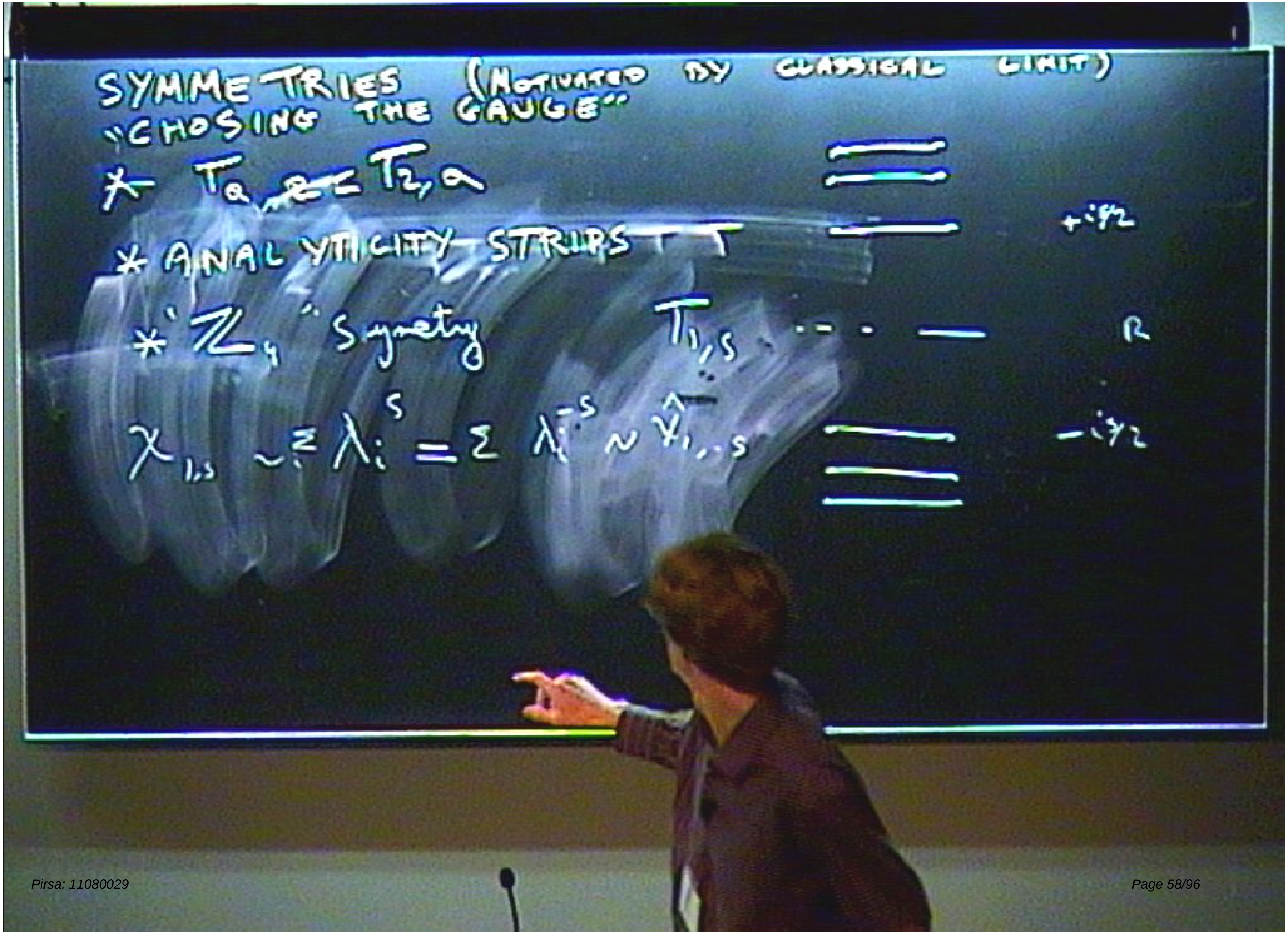
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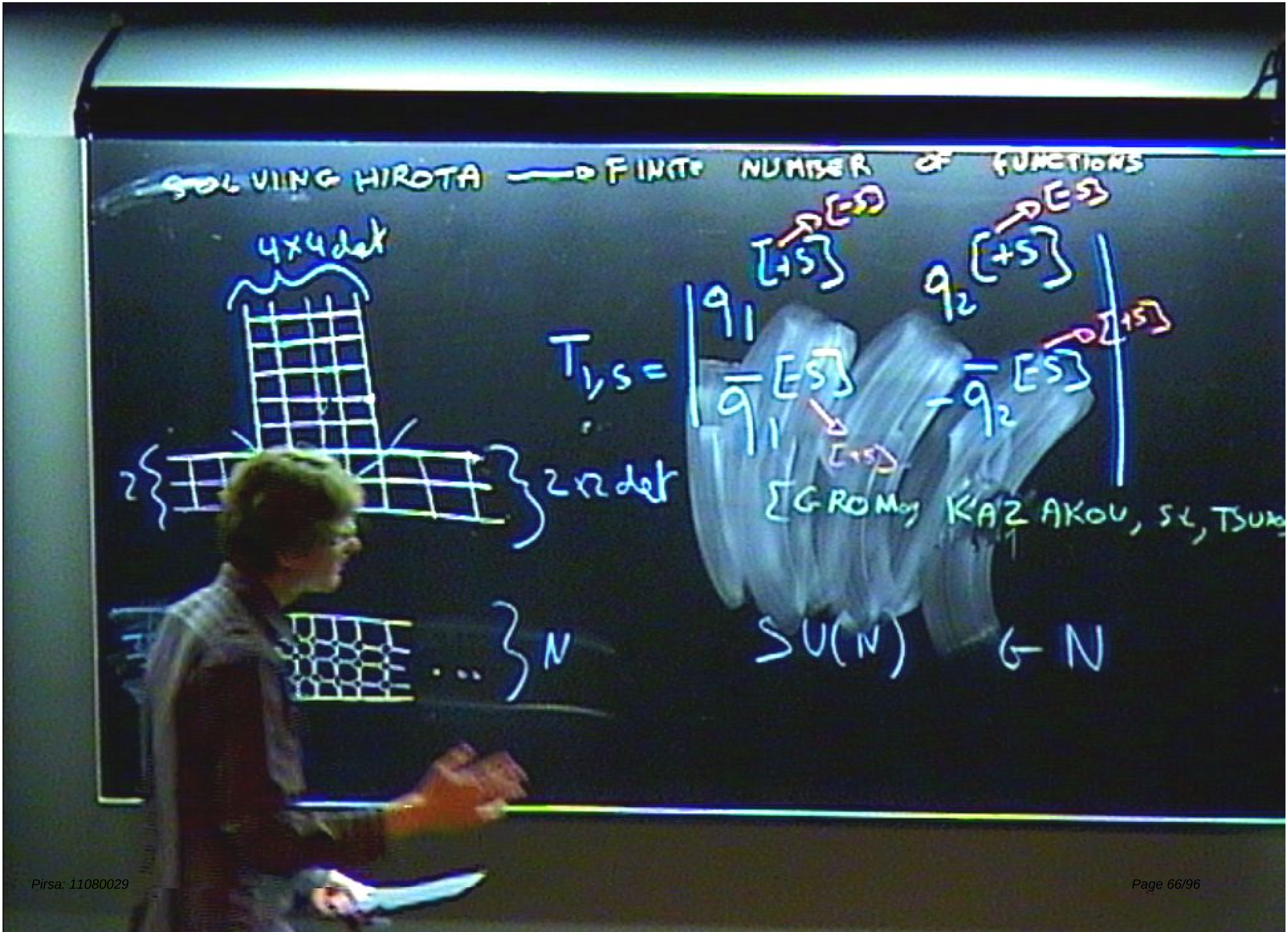
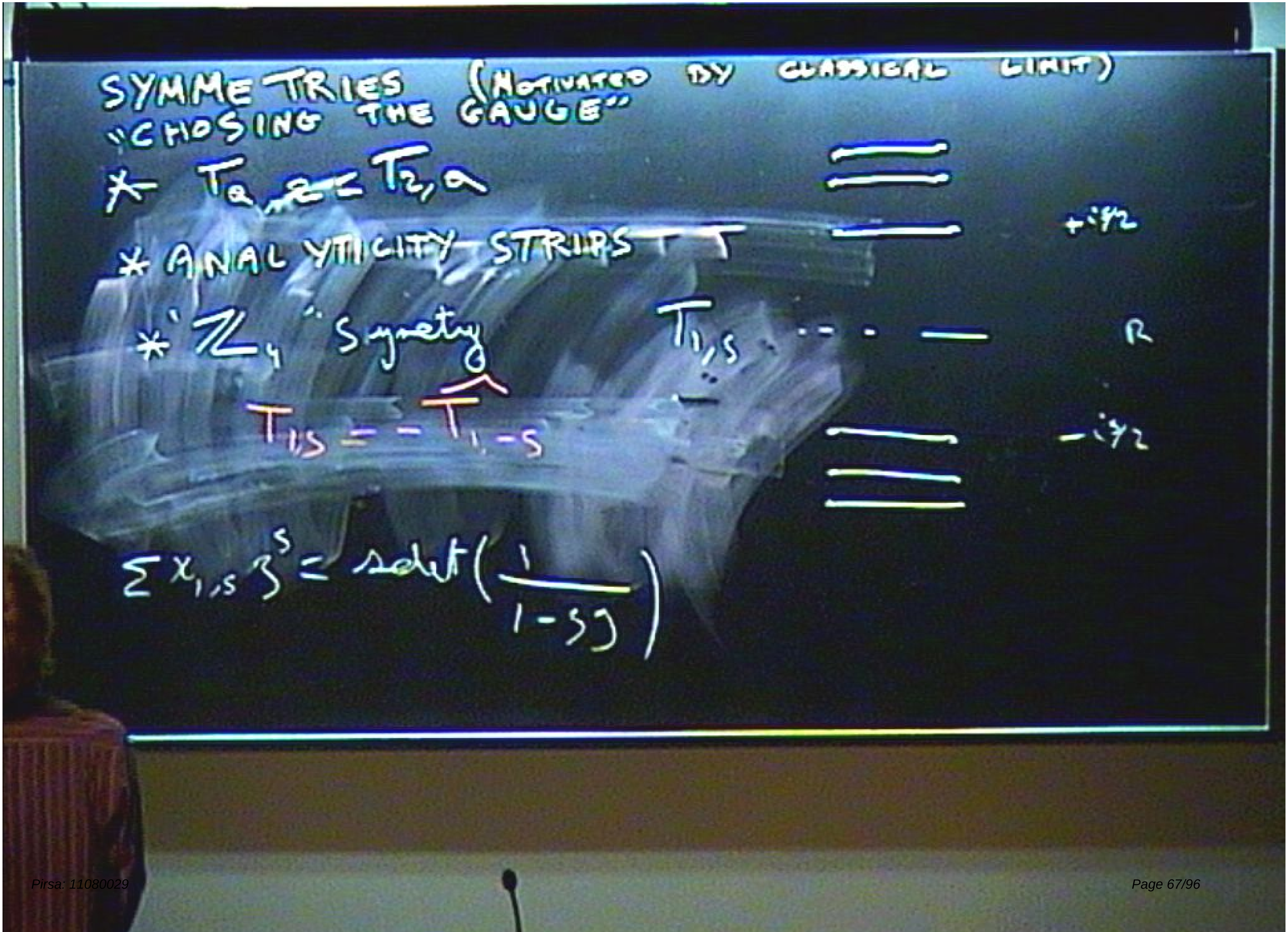

SOLVING HIROTA → FINITE NUMBER OF FUNCTIONS
4x4 det
2x2 det
2
N
T1,s =
SU(N)
← N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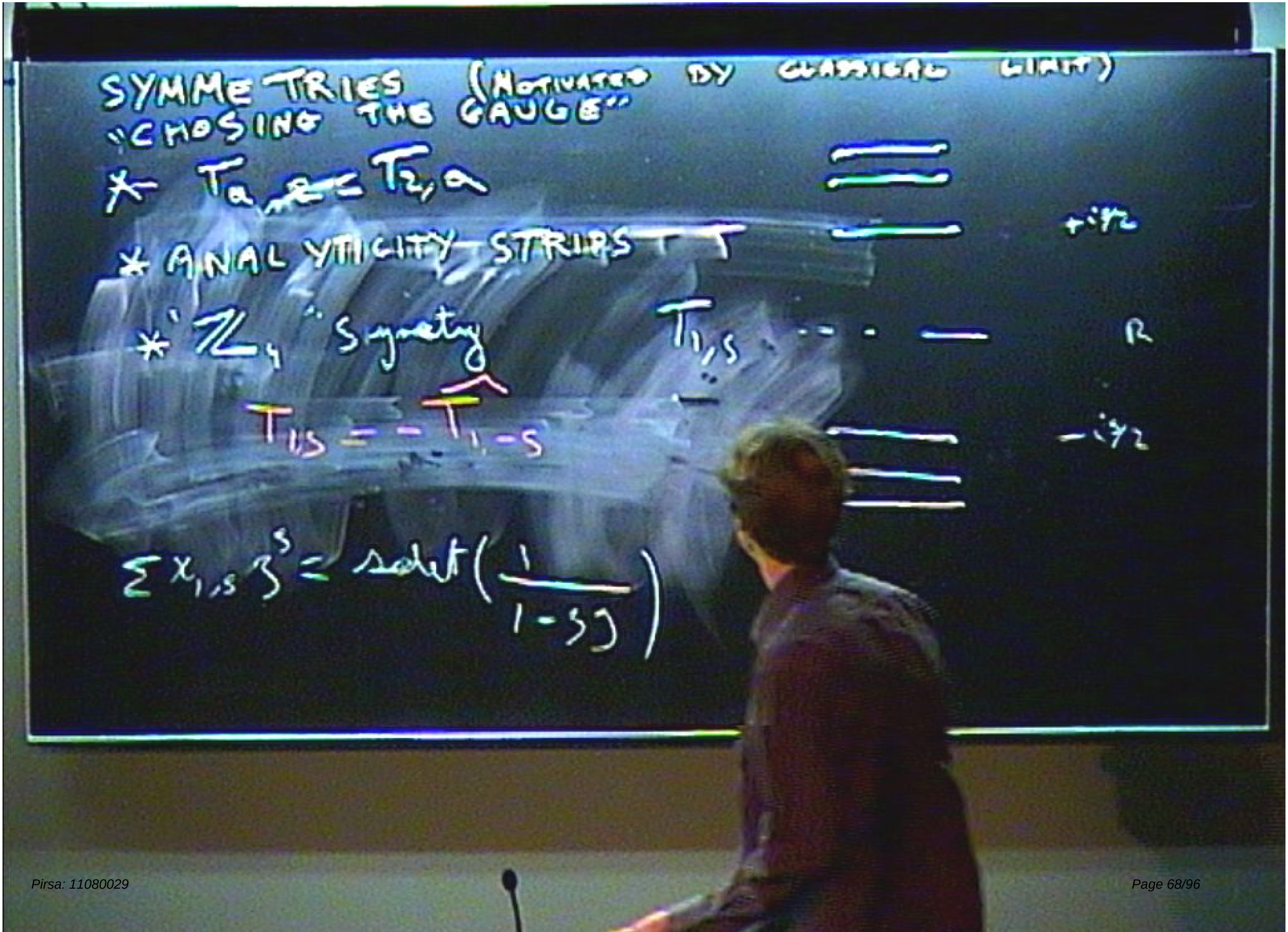
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T1,s = − T1,−s
T1,0 = − T1,0
T1,s
+iγ/2
R
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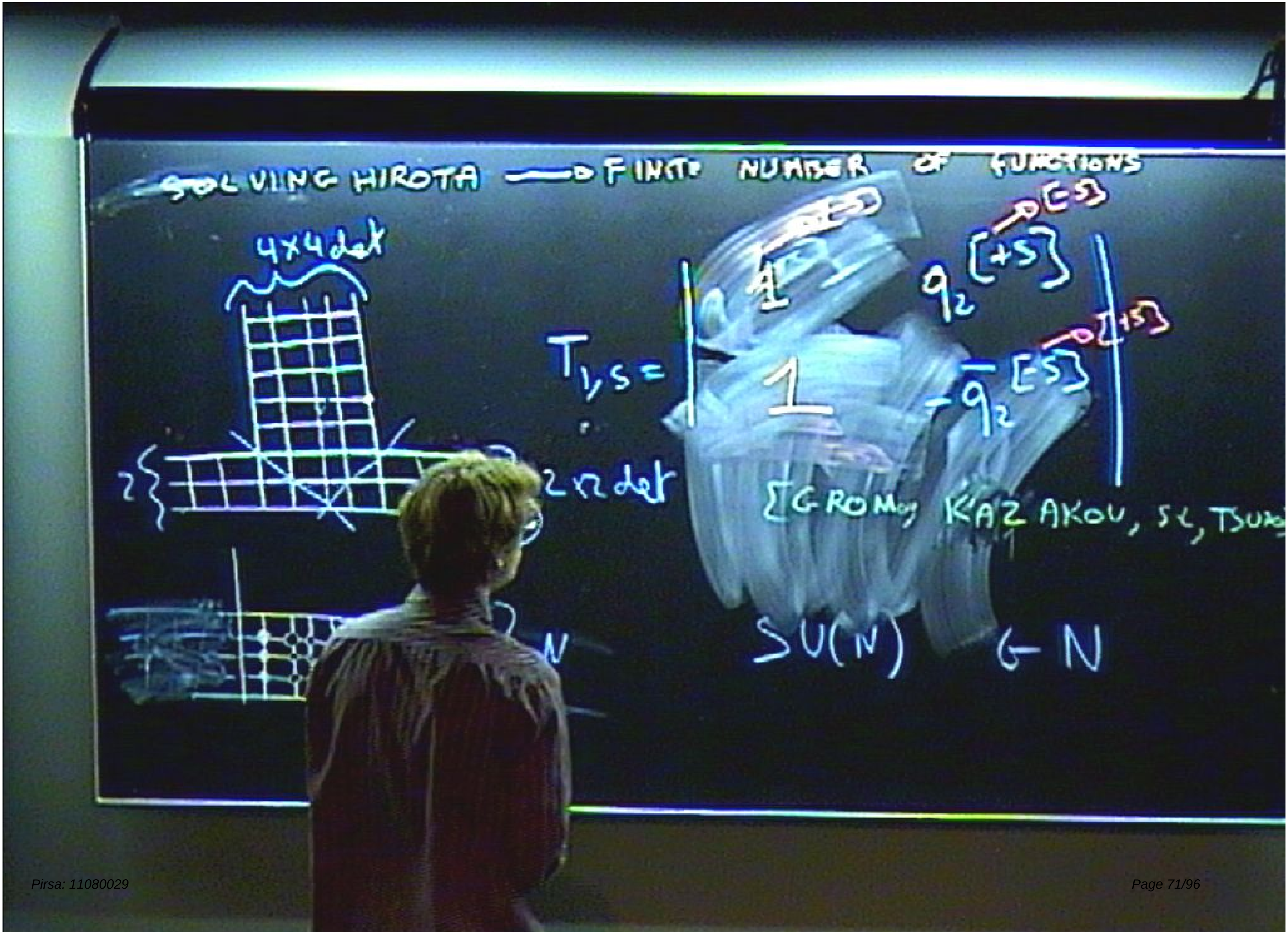
Example of the right Strip

For instance, in a real gauge where $q_1 = 1$,

$$\hat{T}_{1,0} = -\bar{q}_2(u) - q_2(u)$$

- Hence $q_2(u) =$ [illegible]
- generalizable to all T-functions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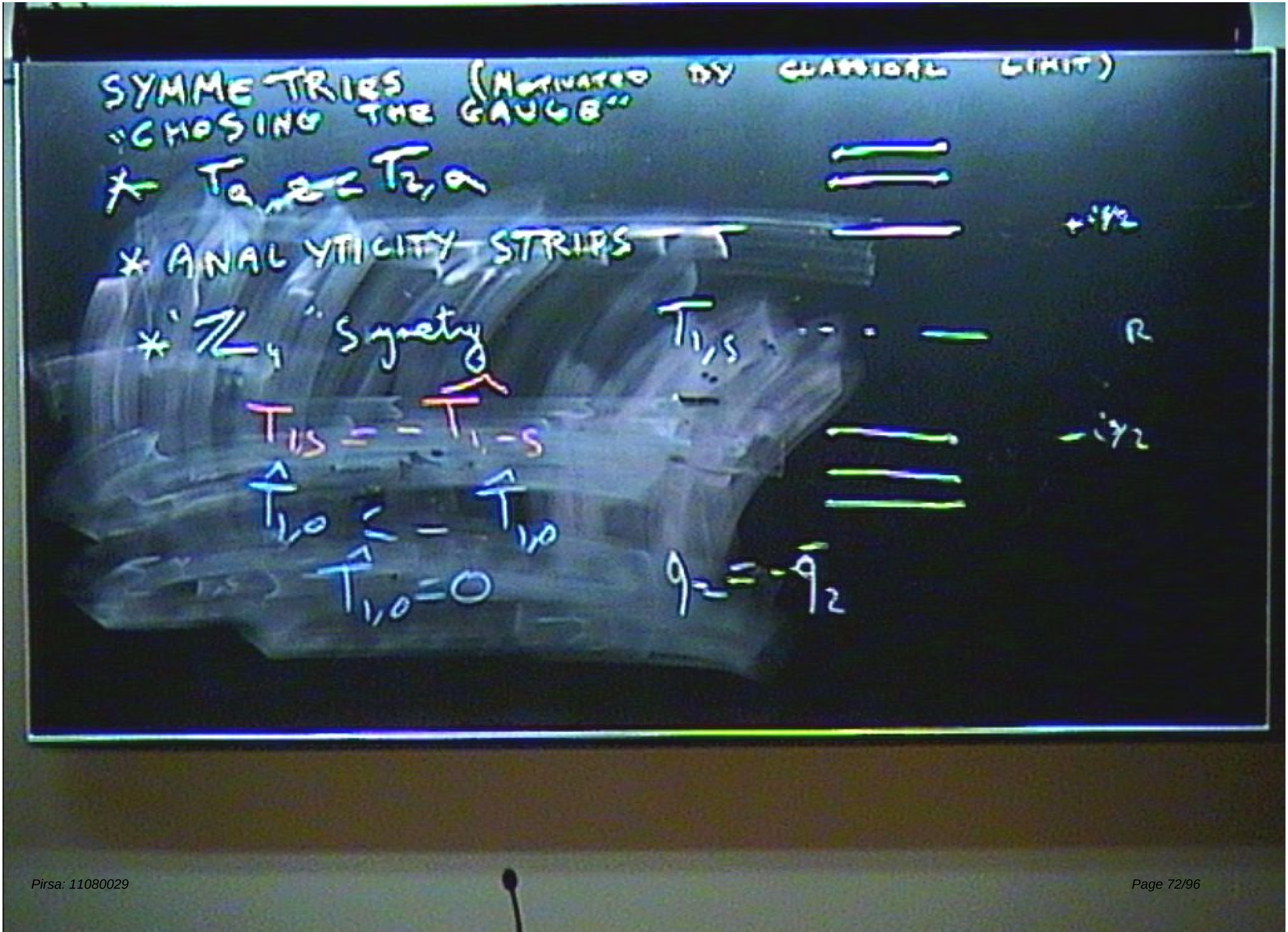
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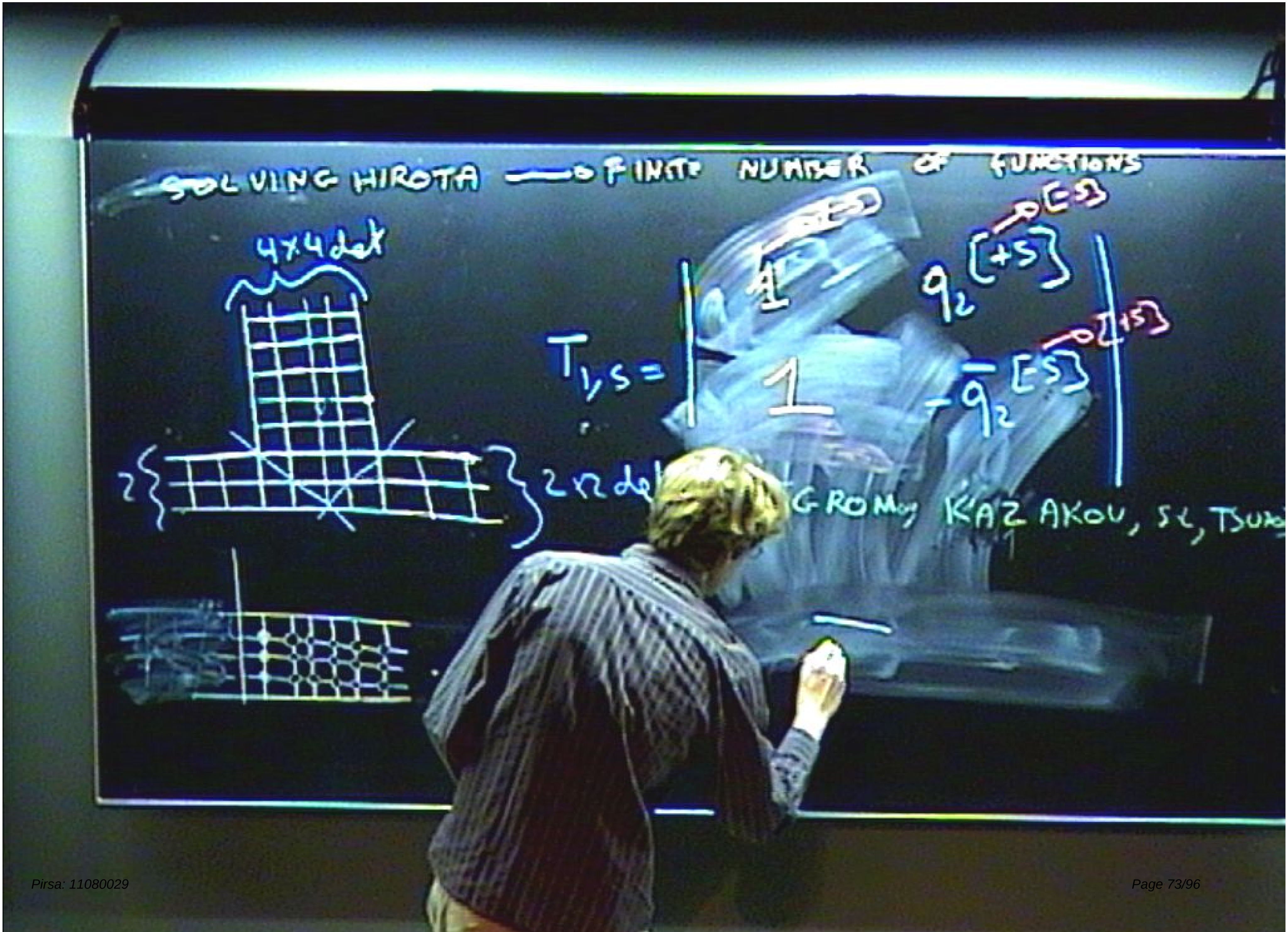
SOLVING HIROTA — FINITE NUMBER OF FUNCTIONS
4x4 det
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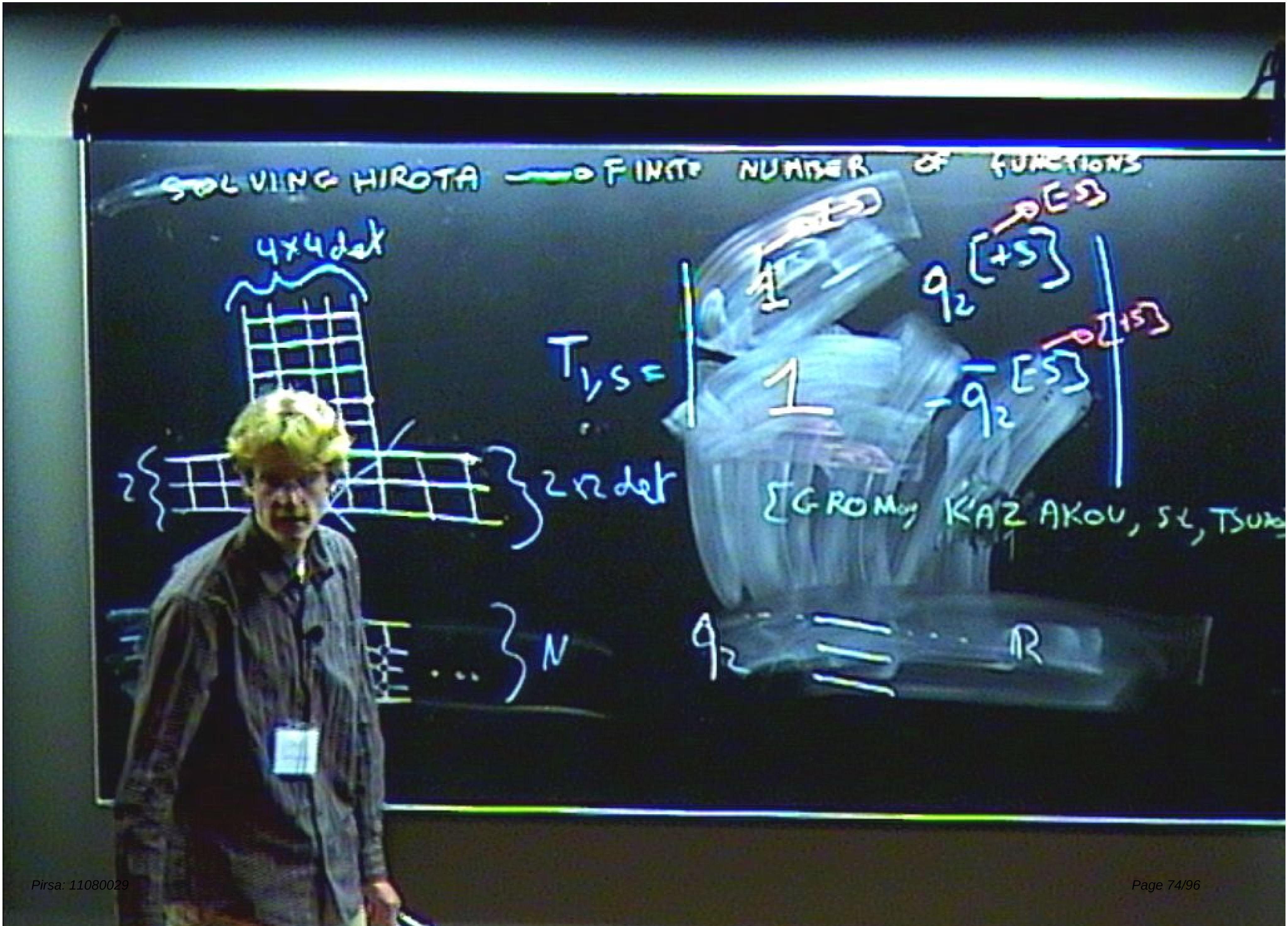
T1,s =
q2 [+s]
[-s]
[+s]
KAZAKOV, SL,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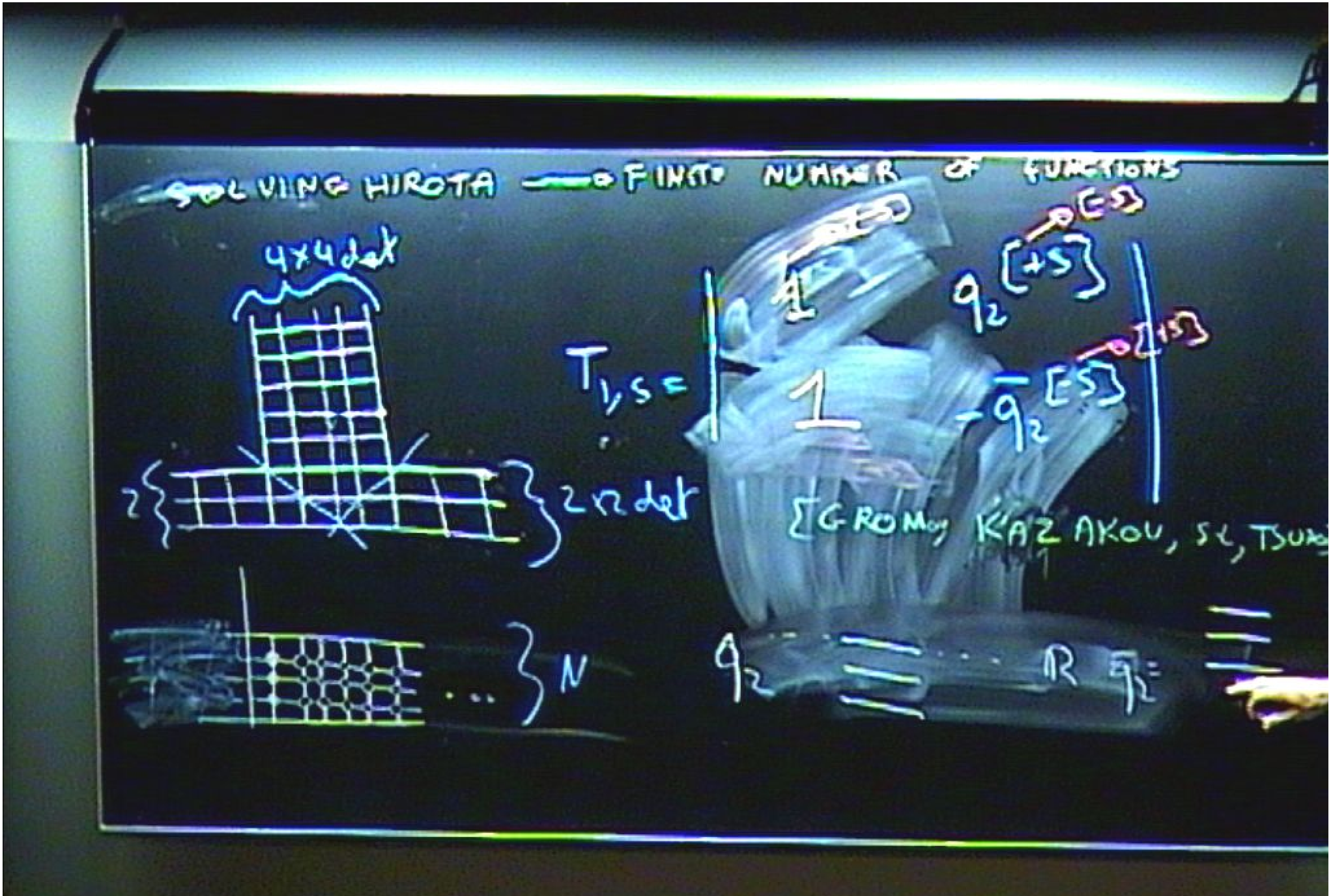

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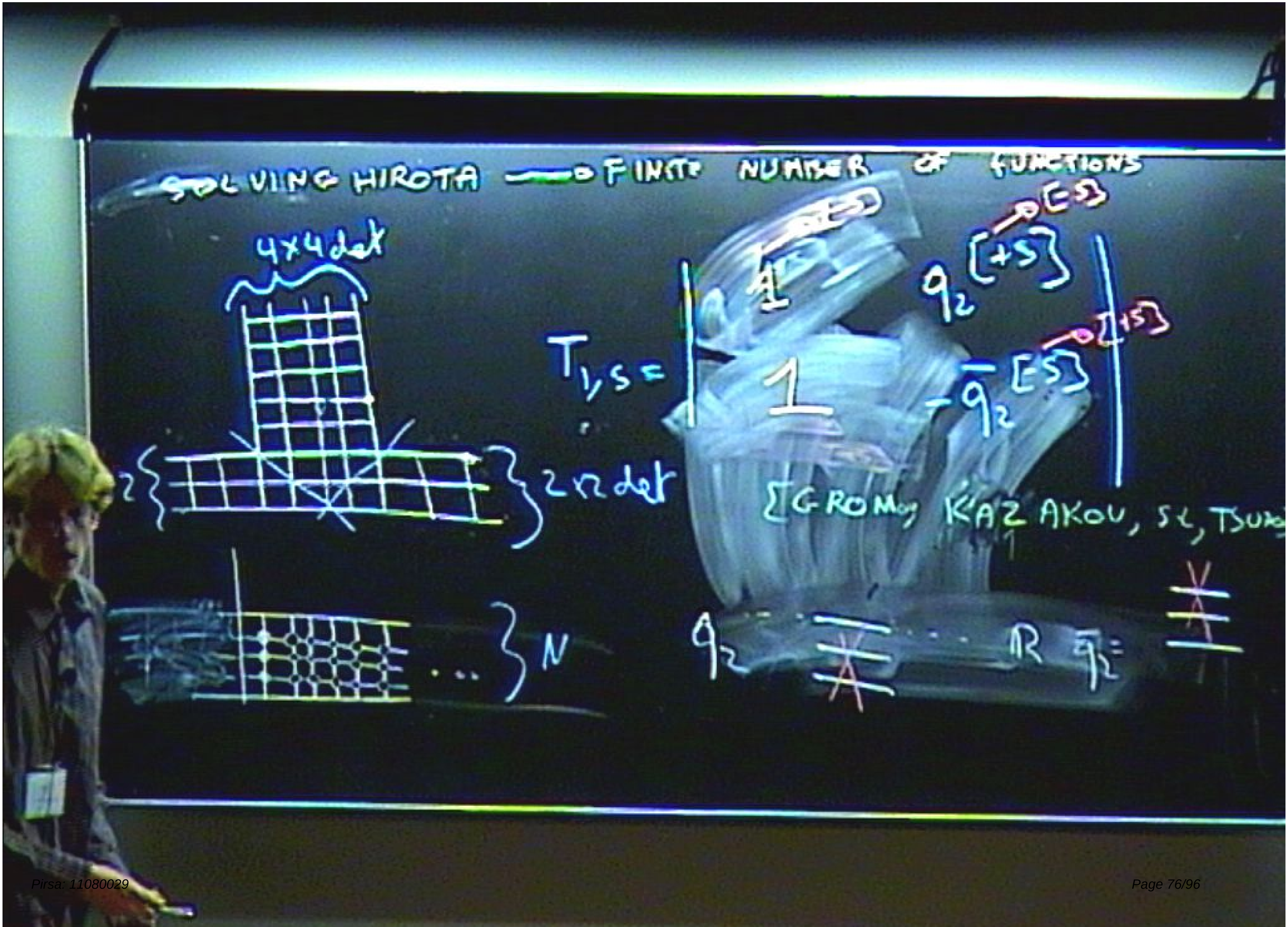
GROMOV, KAZAKOV, SL, TSUBOI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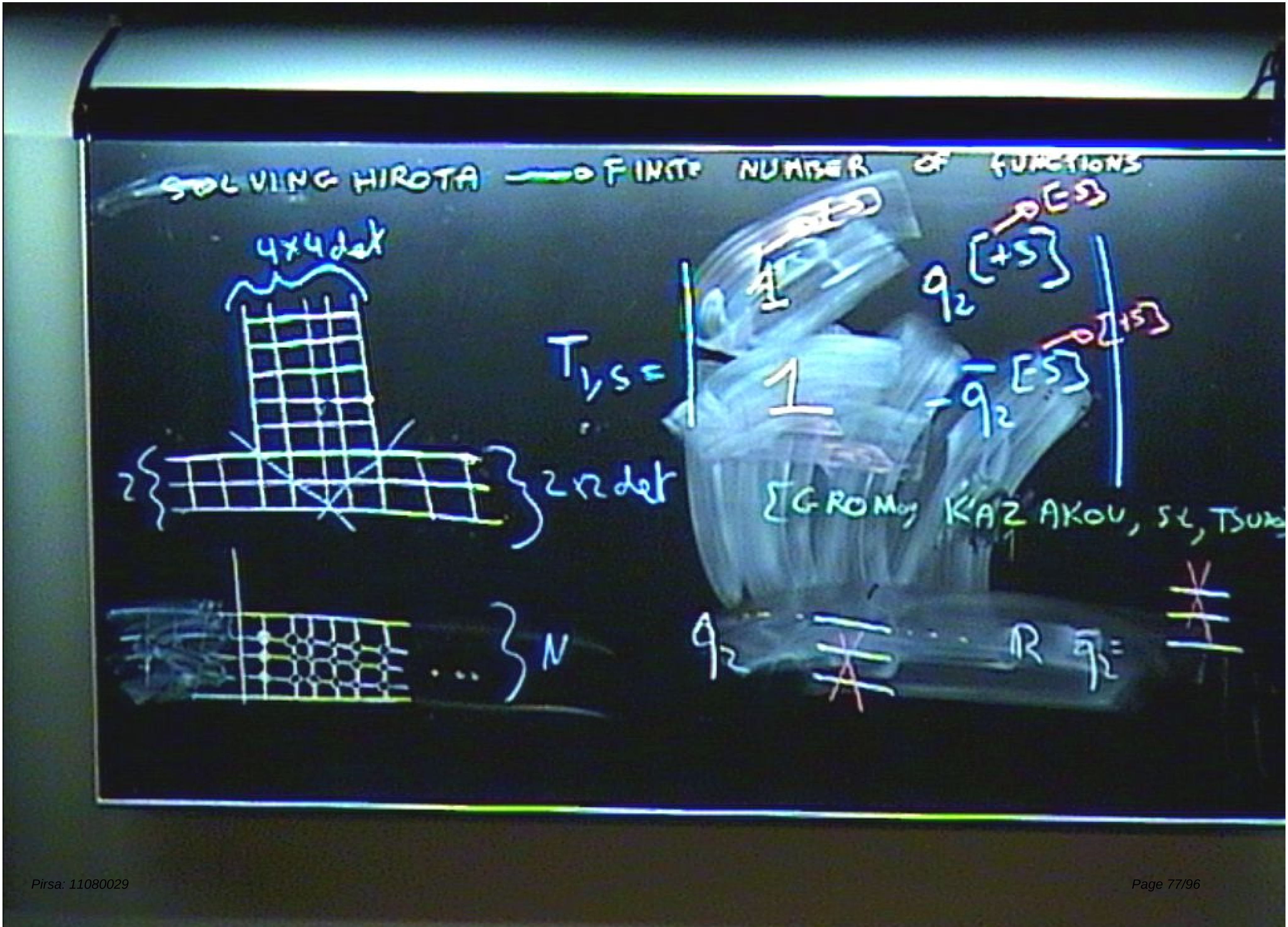
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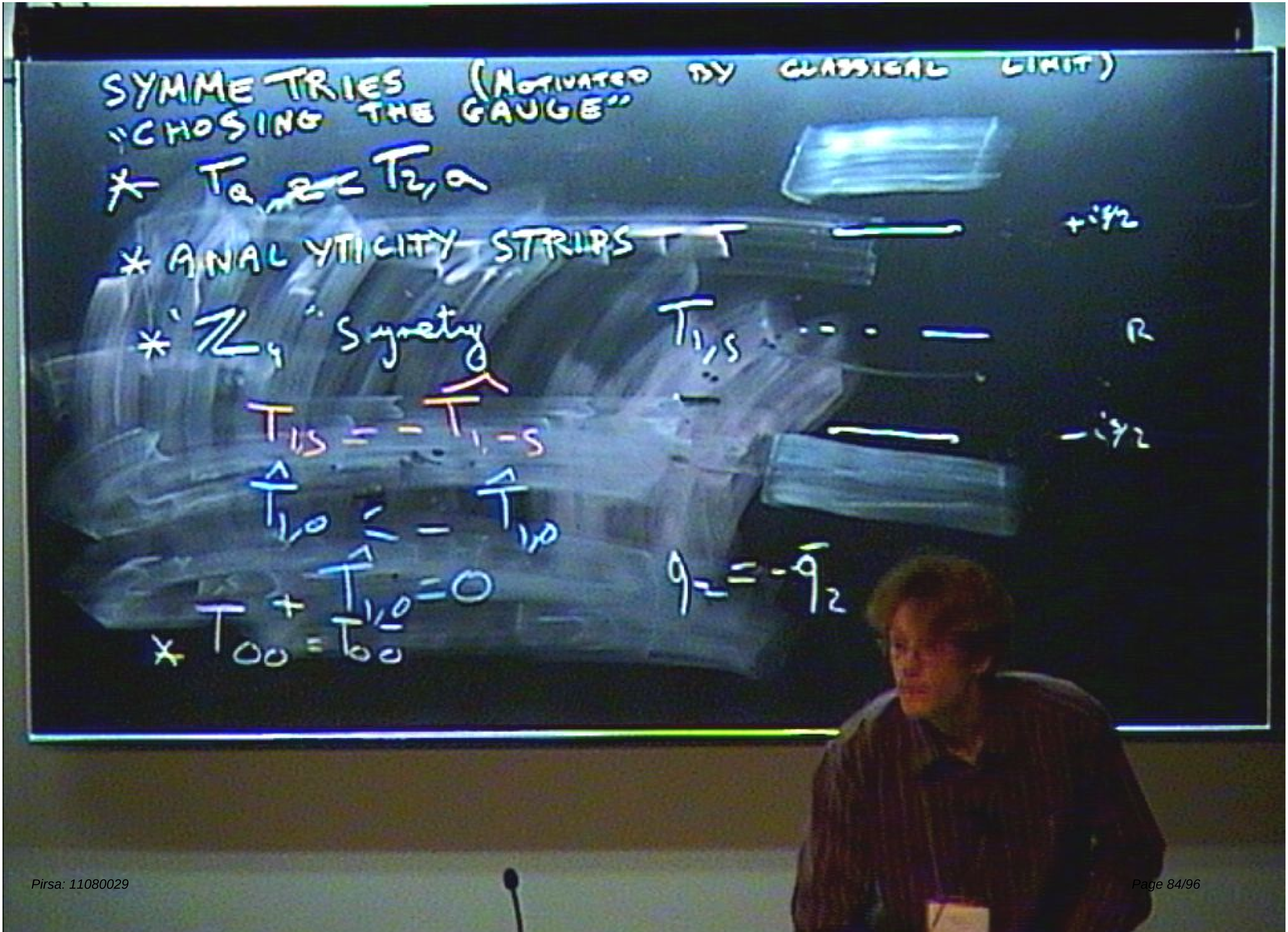
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* T00 = T00

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  - strong coupling construction of T
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Y-system

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Really

Thank you !

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  - $\left\{ \begin{array}{l} \text{numeric efficiency} \\ \text{best FiNLIE formulation} \end{array} \right.$ are to be studied
  - application to other Y-systems ?
  - BFKL
  - strong coupling construction of T (? $T = \langle \mathrm{trace}\Omega \rangle$)
  - weak coupling interpretation of T

Conclusion

- A better understanding of Y-system
  - analytic properties
  - new symmetries
  - Finite set of NLIEs
  - $\partial \log T_{0,0} \xrightarrow{u\to\infty} \frac{2E}{\cdots}$

Really

Thank you !

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Example of the right Strip

For instance, in a real gauge where $q_1 = 1$,

$$\hat{T}_{1,0} = -\bar{q}_2(u) - q_2(u)$$

- Hence $q_2(u) = -iu + \frac{1}{2i\pi}\int_{-2g}^{2g}\frac{\rho(v)}{v-u}$
- generalizable to all T-functions

Example of the right Strip

Solving AdS/CFT Y-system

S. Leurent

tivation

unctions

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