Title: Baxter Q-Operators for Integrable Spin Chains

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Abstract:

Baxter Q-operators for integrable spin chains

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Exact Results in Gauge/Gravity Dualities
Perimeter Institute,

09.08.2011

arXiv:1005.3261	V. Bazhanov, TL, C. Meneghelli, M. Staudacher
arXiv:1010.3699	V. Bazhanov, R. Frassek, TL, C. Meneghelli, M. Staudacher
arXiv:1012.6021	R. Frassek, TL, C. Meneghelli, M. Staudacher
work in progress	R. Frassek, TL, C. Meneghelli, M. Staudacher, Y. Xu

Motivation

AdS/CFT correspondence

↓ ↓

Anomalous dimensions of composite operators — Energies of string excitations

Integrability

↓

Y-system, TBA equations

[D.Bombardielli, D. Fioravanti, R. Tateo '09; N.Gromov, V.Kazakov, A.Kozak, P.Vieira '09; G.Arutyunov, S.Frolov '09]

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- Still many questions and unsolved problems
- We need to find **analytic properties** of Y-functions

Motivation (II)

- Let us focus on some simpler integrable models

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- Still many questions and unsolved problems
- We need to find **analytic properties** of Y-functions
- „In integrable models it is frequently much easier to guess the exact solution than to prove and understand it!" [M. Staudacher]

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- Problem of finding one-loop anomalous dimensions is mapped to a diagonalization of the $\mathfrak{psu}(2,2|4)$-invariant spin chain [Minahan, Zarembo; Beisert, Staudacher]

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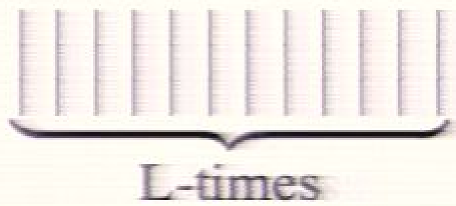
- In order to do it we will construct so called Baxter Q-operators – all Y-functions are functions of eigenvalues of Q-operators

Integrable spin chains



$= V^{\otimes L}$ + periodic boundary conditions

- Hamiltonian $H : V^L \to V^L$

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- Other approaches:
    - Derkachov, Korchemsky, Manashov, ...
    - Kazakov, Laurent, Tsuboi, Vieira

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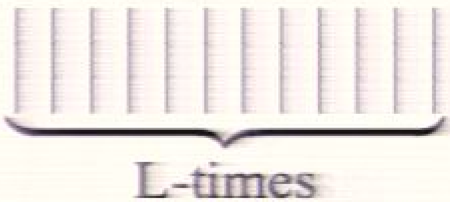

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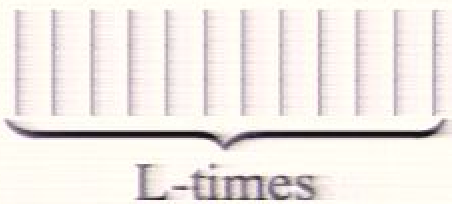

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- Hamiltonian $\quad H : V^L \to V^L$
- Nearest neighbor spin chain

$$H = \sum_{i=1}^{L} \mathcal{H}_{i,i+1}, \qquad \mathcal{H}_{i,i+1} : V^2 \to V^2$$

- We want to diagonalize it
    - directly

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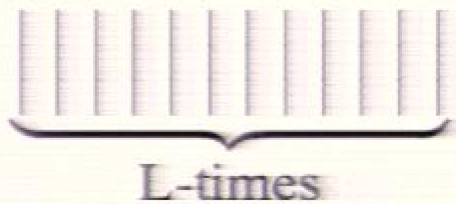

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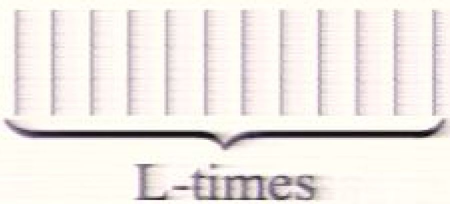

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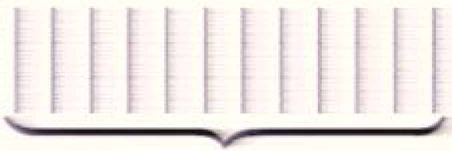

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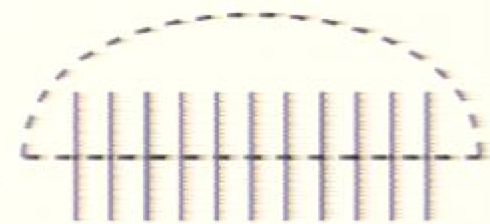
  - with use of **Bethe equations** – efficient method to find them is the Quantum Inverse Scattering Method

Quantum Inverse Scattering Method



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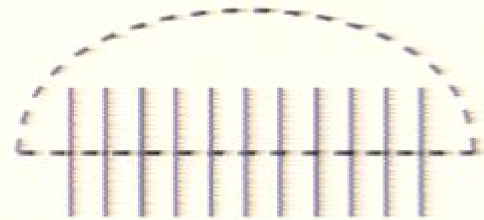

- **Transfer matrices** ($\mathbb{D}$ is for convergence)

$$T_{aux}(z) = \mathrm{Tr}_{aux}\left[\mathbb{D}\, R_{aux}(z) \otimes R_{aux}(z) \otimes \ldots \otimes R_{aux}(z)\right]$$

- From the transfer matrix with auxiliary=quantum we can extract local conserved charges

$$\begin{aligned} U &= T(z_\star) \\ H &= \frac{d}{dz} \log T(z)\Big|_{z=z\star} \end{aligned}$$

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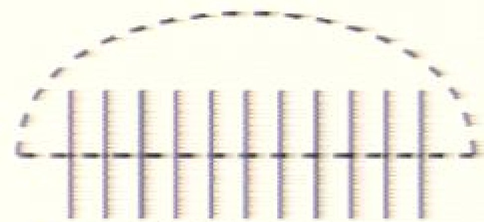

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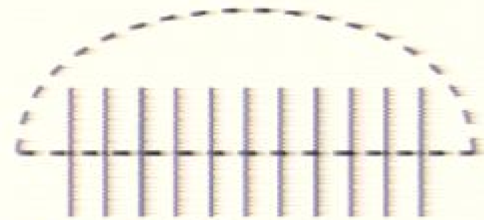

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- Transfer matrices diagonalized with use of the **Algebraic Bethe Ansatz**

Bethe equations (for the fundamental rep of $\mathfrak{sl}(n)$)

$$\left(\frac{z_{1,k}+\frac{1}{2}}{z_{1,k}-\frac{1}{2}}\right)^L = \prod_{j\neq k}\frac{z_{1,k}-z_{1,j}+1}{z_{1,k}-z_{1,j}-1}\prod_{j}\frac{z_{1,k}-z_{2,j}-\frac{1}{2}}{z_{1,k}-z_{2,j}+\frac{1}{2}}$$

$$1 = \prod_{j}\frac{z_{2,k}-z_{1,j}-\frac{1}{2}}{z_{2,k}-z_{1,j}+\frac{1}{2}}\prod_{j\neq k}\frac{z_{2,k}-z_{2,j}+1}{z_{2,k}-z_{2,j}-1}\prod_{j}\frac{z_{2,k}-z_{3,j}-\frac{1}{2}}{z_{2,k}-z_{3,j}+\frac{1}{2}}$$

$$\cdots$$

$$1 = \prod_{j}\frac{z_{n-1,k}-z_{n-2,j}-\frac{1}{2}}{z_{n-1,k}-z_{n-2,j}+\frac{1}{2}}\prod_{j\neq k}\frac{z_{n-1,k}-z_{n-1,j}+1}{z_{n-1,k}-z_{n-1,j}-1}$$

$$E = \sum_k \frac{1}{\frac{1}{4}-z_{1,k}^2}$$

Bethe equations (II)

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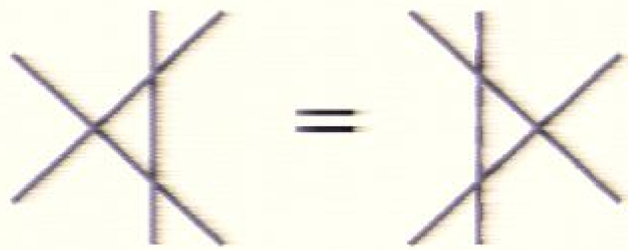
Yang-Baxter equation

- We can find R-matrices from the **Yang-Baxter equation**



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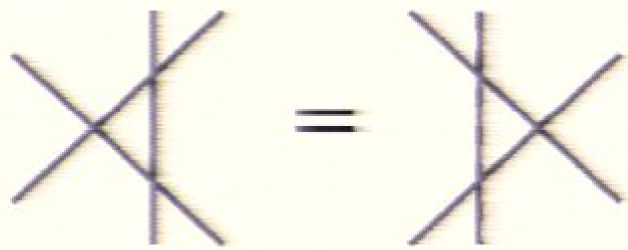

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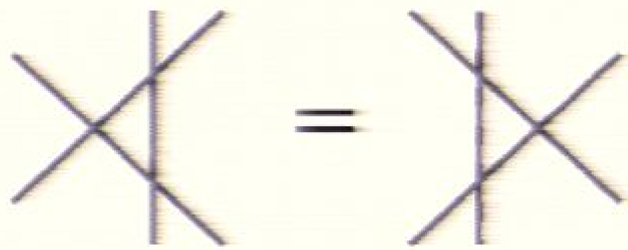

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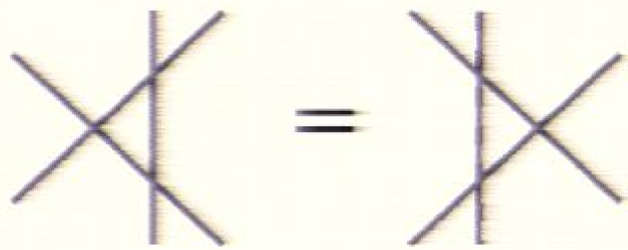

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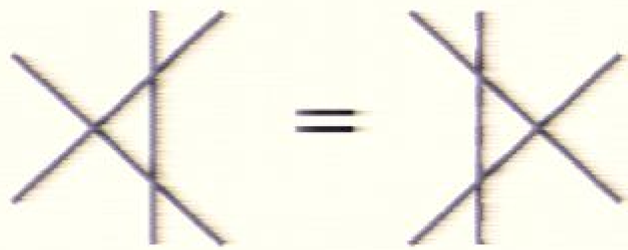

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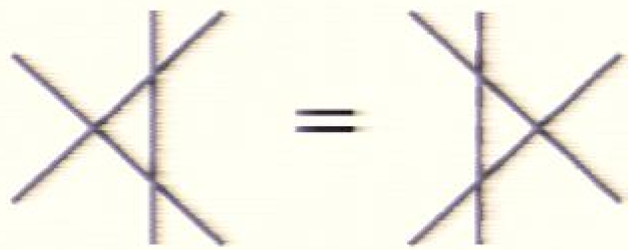

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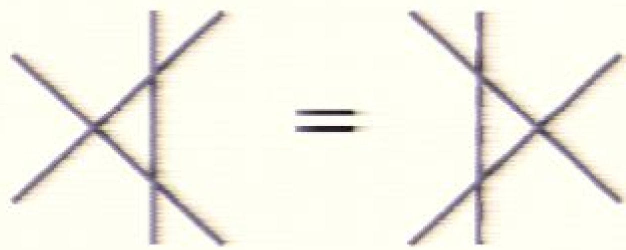

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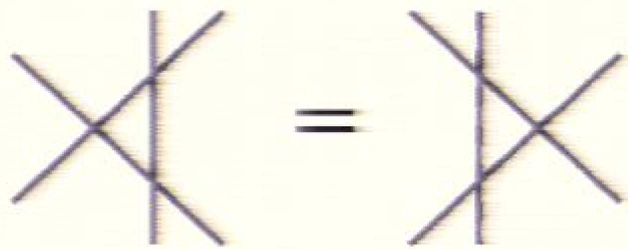

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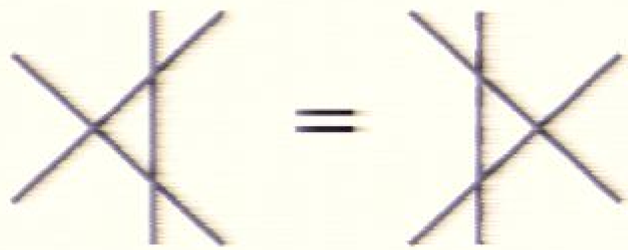

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Spin chain in the fundamental representation

- Take $\Box \otimes \Box \otimes W$ (W – any space) and try to find **all linear solutions** of

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- W can be the representation space of the tensor product of any $\mathfrak{gl}(p)$ ($p \leq n$) representation and an **oscillator algebra** representation
- For $p = n$ we have previous solutions – transfer matrices
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$$Q_p(z) = \mathrm{Tr}_{aux}\left[\mathbb{D}\, R_p(z) \otimes R_p(z) \otimes \ldots \otimes R_p(z)\right]$$

- Belong to the same family of commuting operators: $[Q_p(z), T(z')] = 0$
- Even more Q-operators – $\binom{n}{p}$ different ways of fixing $\mathfrak{gl}(p)$ in $\mathfrak{gl}(n)$

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Spin chain in the fundamental representation

- Take $\square \otimes \square \otimes W$ (W – any space) and try to find **all linear solutions** of

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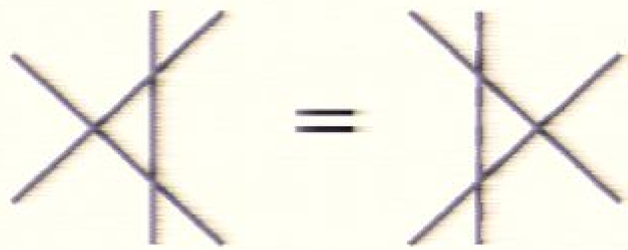
Yang-Baxter equation

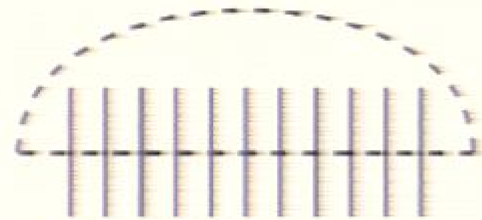
- We can find R-matrices from the **Yang-Baxter equation**



$$R_{12}(x-y)R_{13}(x)R_{23}(y) = R_{23}(y)R_{13}(x)R_{12}(x-y)$$

- YBE is a defining relation for **Yangian** – every solution of the YBE gives a representation of Yangian

Quantum Inverse Scattering Method



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- From the transfer matrix with auxiliary=quantum we can extract local conserved charges

$$\begin{aligned} U &= T(z_\star) \\ H &= \frac{d}{dz} \log T(z)\Big|_{z=z\star} \end{aligned}$$

- Transfer matrices form a family of commuting operators

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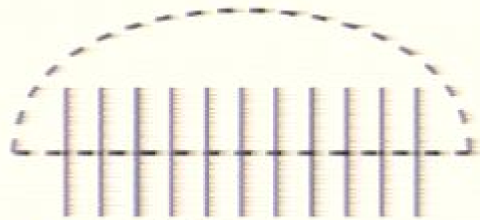
- Transfer matrices diagonalized with use of the **Algebraic Bethe Ansatz**

Quantum Inverse Scattering Method



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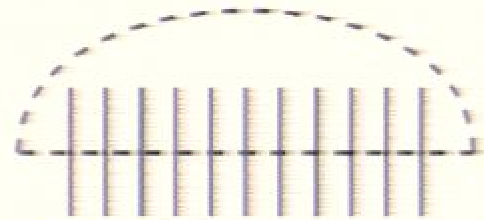

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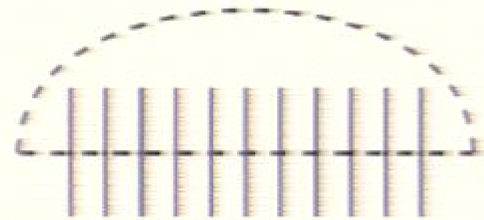

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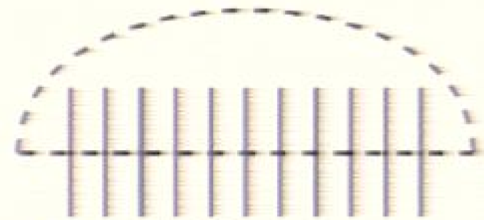

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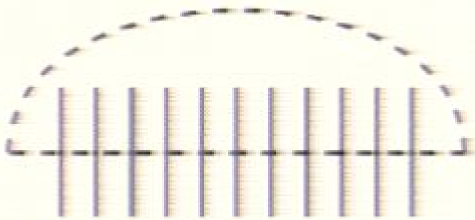

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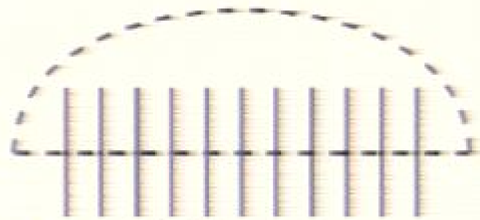

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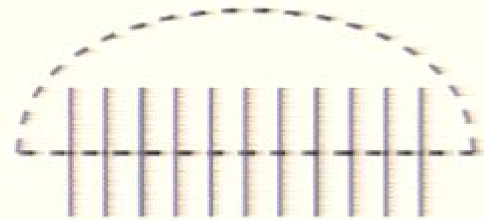

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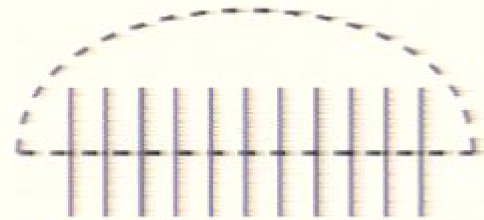

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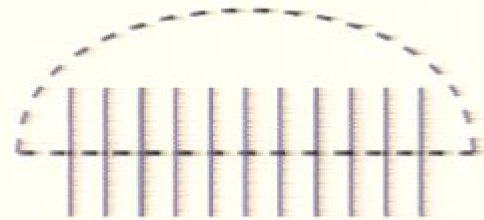

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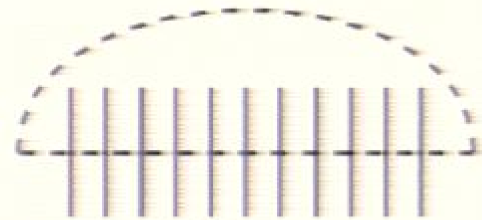

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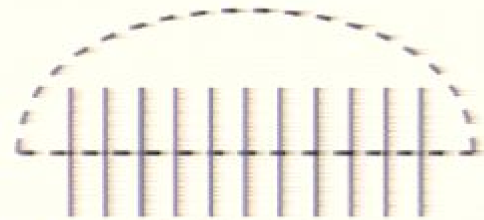

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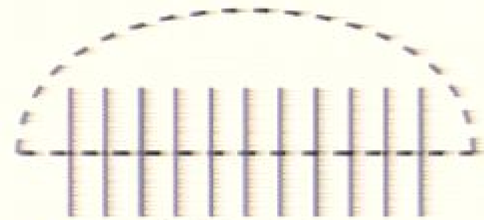

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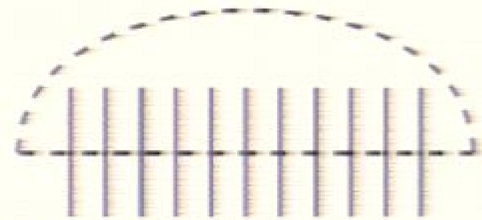

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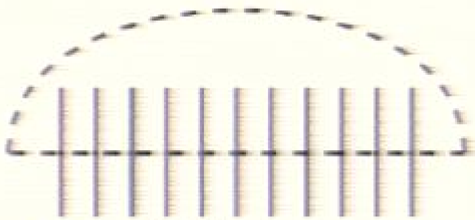
Quantum Inverse Scattering Method



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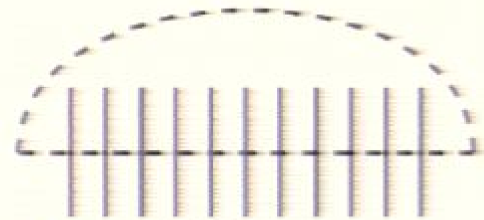

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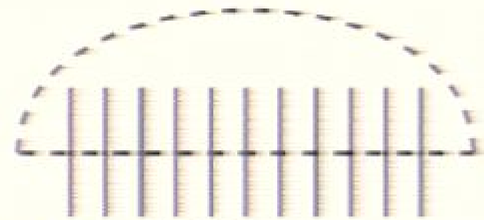

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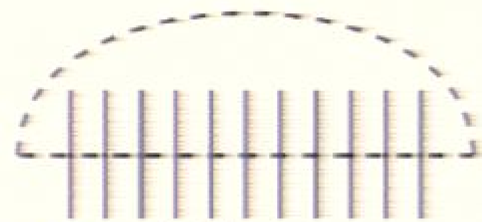

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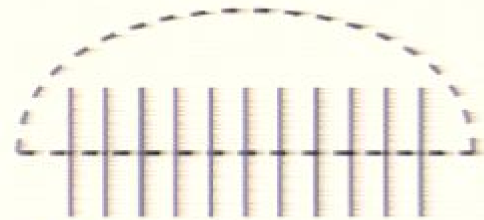

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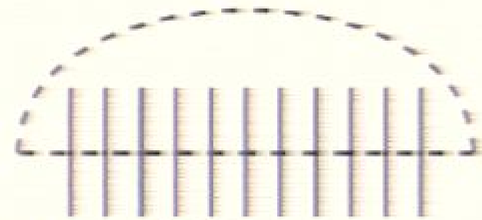

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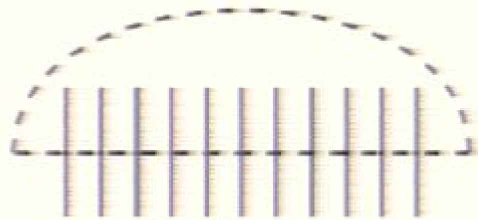

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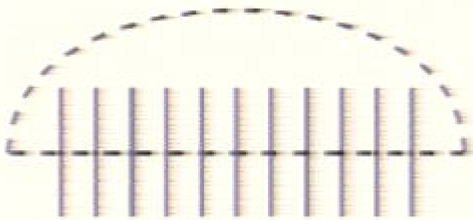

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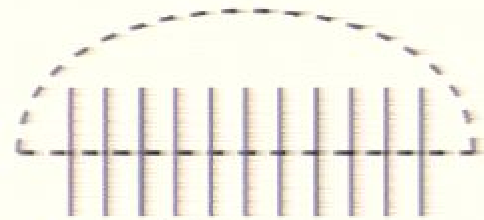

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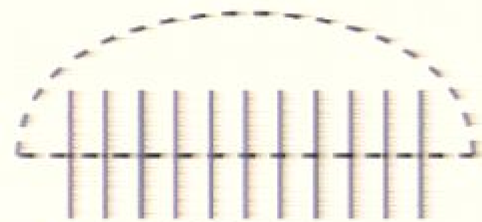

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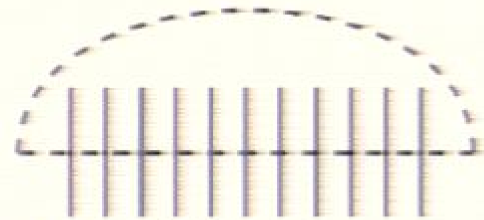

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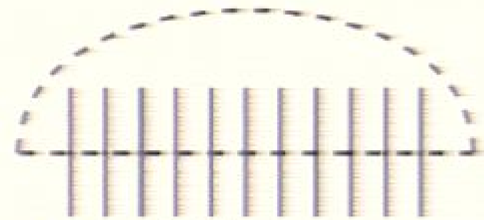

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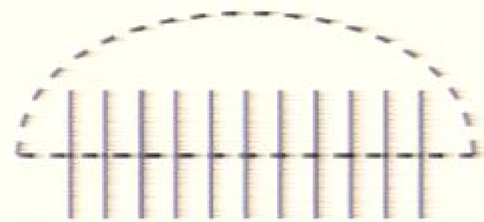

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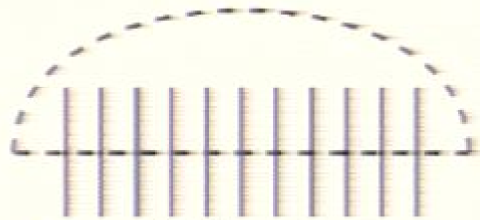

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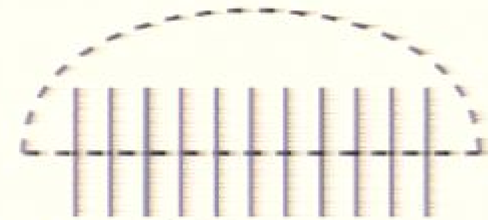

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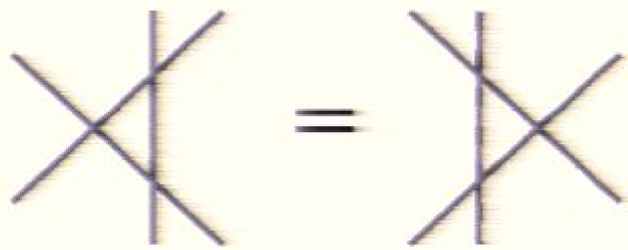
Yang-Baxter equation

- We can find R-matrices from the **Yang-Baxter equation**



$$R_{12}(x-y)R_{13}(x)R_{23}(y) = R_{23}(y)R_{13}(x)R_{12}(x-y)$$

- YBE is a defining relation for **Yangian** – every solution of the YBE gives a representation of Yangian
- Example $\square \otimes \square \otimes \square$

$$R_{12}(z) = zI + P_{12}$$

Spin chain in the fundamental representation

- Take $\square \otimes \square \otimes W$ (W – any space) and try to find **all linear solutions** of

$$R_{\square,\square}(x-y)R_{\square,W}(x)R_{\square,W}(y) = R_{\square,W}(y)R_{\square,W}(x)R_{\square,\square}(x-y)$$

- W can be the representation space of the tensor product of any $\mathfrak{gl}(p)$ ($p \leq n$) representation and an **oscillator algebra** representation
- For $p = n$ we have previous solutions – transfer matrices
- For the singlet in $\mathfrak{gl}(p)$ we define ($R_{\square,W_p} \equiv R_p$)

$$Q_p(z) = \mathrm{Tr}_{aux}\left[\mathbb{D}\, R_p(z) \otimes R_p(z) \otimes \ldots \otimes R_p(z)\right]$$

- Belong to the same family of commuting operators: $[Q_p(z), T(z')] = 0$
- Even more Q-operators – $\binom{n}{p}$ different ways of fixing $\mathfrak{gl}(p)$ in $\mathfrak{gl}(n)$

Spin chain in any representation

- Solve YBE for $\square \otimes \square \otimes \Lambda$ – leads to $R_{\square,\Lambda}$ (quadratic)

$$R_{\square,\Lambda}(z) = z\, e_{ii} + e_{ij} \otimes J_{ji}$$

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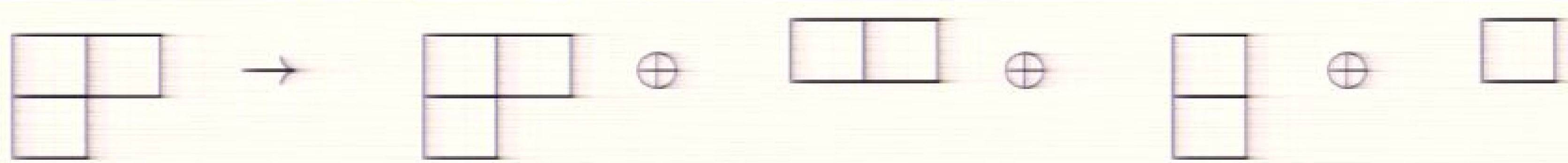

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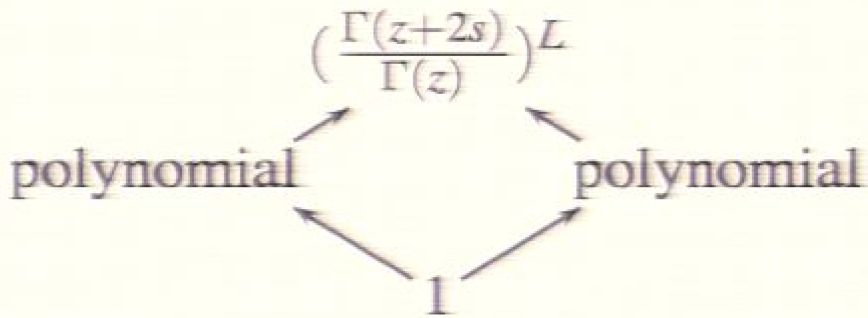
Analytic properties of Q-functions

- Hasse diagram for $s > 0$ – compact case



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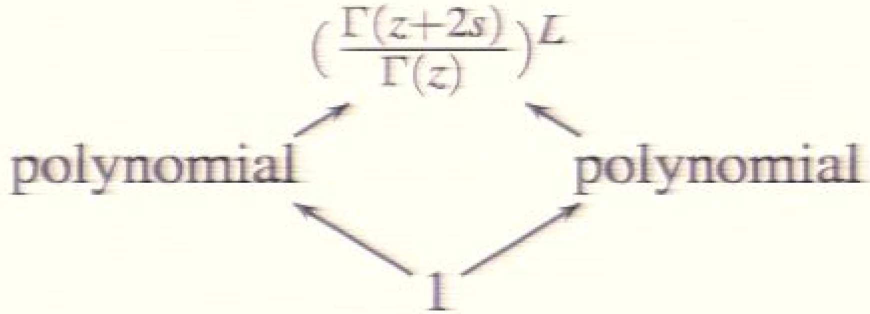

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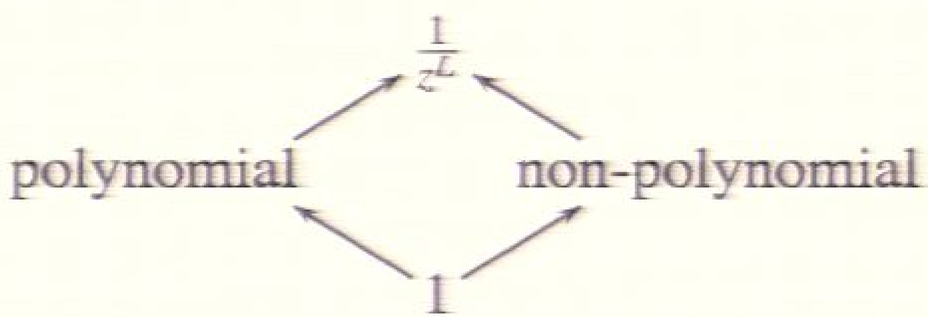

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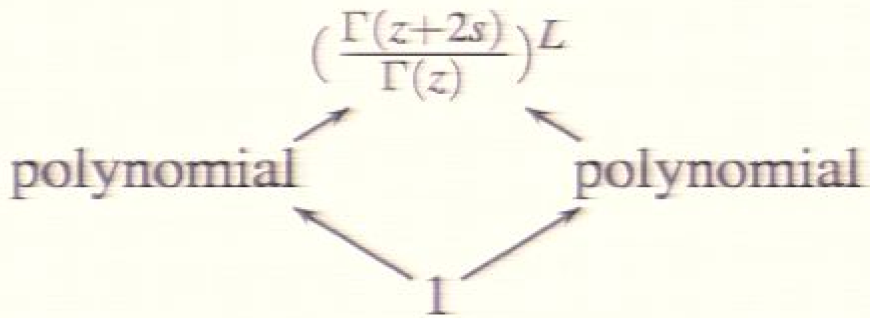

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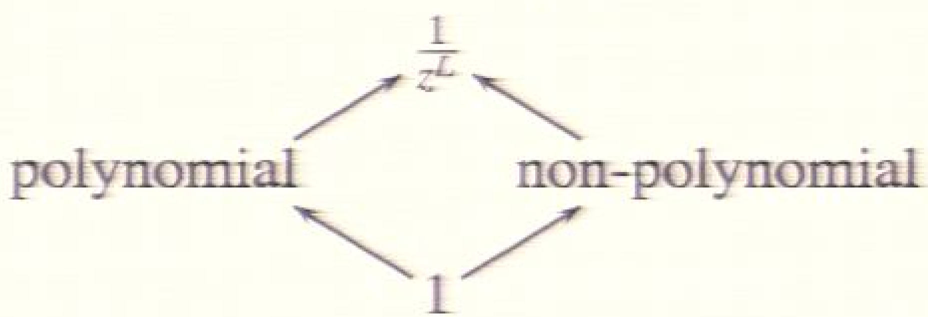

- For $\mathfrak{su}(2,2|4)$ we have 2^8 different Q-operators with various analytic properties - some of them are polynomials and some are very complicated meromorphic functions

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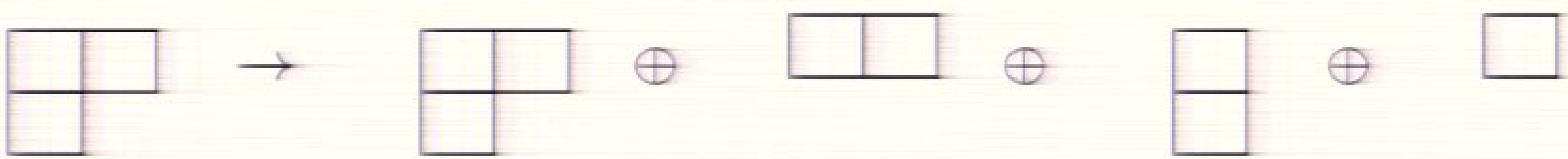

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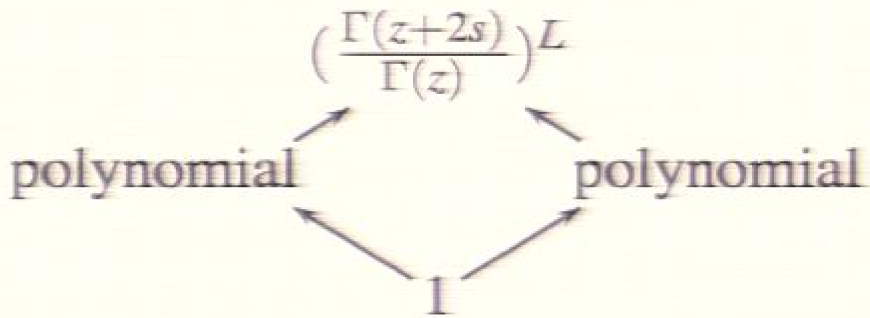

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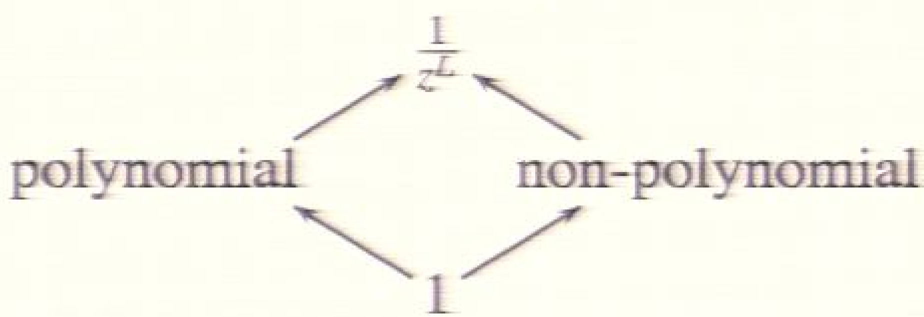

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- Similar situation in all loop $\mathcal{N} = 4$ SYM – it was conjectured that the solution is given by the Y-system/TBA equations....

  However, what is the Hamiltonian?

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- This stems from the fact that for non-rectangular representations we have multiplicities in the decomposition of the tensor product
- The resulting Hamiltonian takes the form

$$\mathcal{H} = \sum_{k=1}^{n} \psi(z_\star + A_k) + \mathrm{const}(n, z_\star)$$

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Thank you!

No Signal

VGA-1

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