

Title: Leaving Behind N=4 SYM: Why are Three Loops Interesting?

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Abstract:

Leaving behind N=4 SYM: why are three loops interesting?

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08.08.11, Exact Results in Gauge/Gravity Dualities, PI

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- ① motivation
- ② short review of the formalism in No. 4, 5, 6
- ③ the interpolating theory : three loop results

Why are three Loop interesting?

$N=1$ Superspace : $N=4$ Syc, β -deformation

Why are three loop interesting?

$N=1$ superspace : $N=4$ Syc, β -deformation, ...

composite operators : to $(2, \bar{2}, \phi, \psi, \dots)$ \leftrightarrow closed
superfields

one- and two-loop.

Why are three loop interesting?

$N=1$ superspace ; $N=4$ Syc, β -deformation, ...

composite operators : to $(\mathbb{Z} \cdot \mathbb{Z} \cdot \mathbb{Z} \cdot \mathbb{Z} \dots) \Leftrightarrow$ closed
superfields

one- and two-loop.

- fermion manipulations (closed functions)

associated with mapon dispersion relation

$$2 \text{ step: } \mathbb{R}^2 = \Delta \rightarrow \frac{1}{(4\pi)^2} \left(-\frac{1}{2\epsilon} + \frac{1}{2\epsilon} \right)$$



Over^a

$$2 \text{ loop: } \mathbb{R}^2 \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$$



$$\text{Oven}^a = Z^a_b O^b_{bme}$$

$$Z^a_b = \mathbb{1} - \sum U V p_{ab}$$

didaktik operator:

$$2 \text{ loop: } \mathbb{R}^2 = \Delta \rightarrow \frac{1}{(4\pi)^2} \left(-\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \right)$$



$$\text{Oven}^a = \mathbb{Z}^a_b \mathcal{O}^b_{bme}$$

$$\mathbb{Z}^a_b = \mathbb{1} - \sum UV \text{ paths}$$

$$\text{dilatation operator: } \mathcal{D} = \mu \frac{\partial}{\partial \mu} \ln \mathbb{Z}(g, \mu^c)$$

$$2 \text{ loop: } \Gamma_2 = \Delta \rightarrow \frac{1}{(4\pi)^4} \left(-\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \right)$$



$$\text{Oven}^a = Z^a_b O^b_{\text{one}}$$

$$Z^a_b = \mathbb{1} - \sum \text{UV poles}$$

$$\text{dilatation operator: } \mathcal{D} = \mu \frac{\partial}{\partial \mu} \ln Z(g, \mu^{\epsilon})$$

$\ln Z$ is free of higher order poles: $\nexists \frac{1}{\epsilon^n}, n \geq 2$

$$2 \text{ loop: } \Gamma_2 = \Delta \rightarrow \frac{1}{(4\pi)^2} \left(-\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \right)$$



$$\text{Over}^a = Z^a_b O^b_{bme}$$

$$Z^a_b = \Lambda - \sum UV \text{ poles}$$

$$\text{dilatation operator: } \mathcal{D} = \mu \frac{\partial}{\partial \mu} \ln Z(g\mu^e)$$

$\ln Z$ is free of higher order poles: $\exists \frac{1}{\epsilon^n}, n \geq 2$

$$\Downarrow$$

$$D_n \Rightarrow D_2$$

$$\ln D = \underbrace{\int^2 D_1}_{\text{"ln"}} + D_2 + \dots$$

$$2 \text{ loop: } \mathbb{R}^2 = \Delta \rightarrow \frac{1}{(4\pi)^2} \left(-\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \right)$$



$$\text{Oven}^a = \mathbb{Z}^a_b O^b_{bme}$$

$$\mathbb{Z}^a_b = \Lambda - \sum UV \text{ poles}$$

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$\ln \mathbb{Z}$ is free of higher order poles: $\nexists \frac{1}{\epsilon^n}, n \geq 2$

$$\Downarrow$$

$$D_n \Rightarrow D_1 \quad \text{in } \mathcal{D} = \underbrace{\dot{g}^1 D_1 + \dot{g}^2 D_2 + \dots}_{\text{"minimal"}}$$

$$2 \text{ loop: } \Gamma_2 = \Delta \rightarrow \frac{1}{(4\pi)^4} \left(-\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \right)$$



$$\text{Oven}^a = Z^a_b O^b_{\text{one}}$$

$$Z^a_b = \mathbb{1} - \sum \text{UV poles}$$


$$\text{dilatation operator: } \mathcal{D} = \mu \frac{\partial}{\partial \mu} \ln Z(g, \mu^2)$$

$\ln Z$ is free of higher order poles: $\nabla \frac{1}{\epsilon^n}, n \geq 2$



$$\Rightarrow \mathcal{D}_2 \quad \text{in } \mathcal{D} = \underbrace{\tilde{g}^2 \mathcal{D}_1 + \tilde{g}^4 \mathcal{D}_2 + \dots}_{\text{"minimal"}}$$

Three loops:

- flavor manipulations that exclusively contribute to scattering of impurities
- extended integral basis: 

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Three loops:

- Flavor manipulations that exclusively contribute to scattering of impurities

- extended integral basis:



② Short summary of $N=4$ case

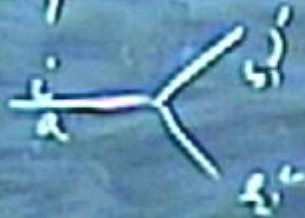
Superpotential



$$d_i = (z, d_{i+1})$$

② Short summary of $N=4$ case

Supersymmetric



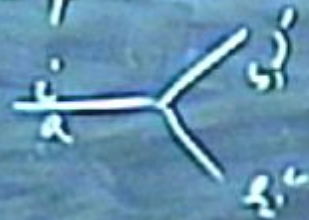
\rightarrow i sym \mathcal{E} is in $\mathfrak{so}(1,3)$

$$d_i = (\mathbb{Z}, d, \mathbb{Z})$$



② Short summary of $N=4$ case

Superpotential

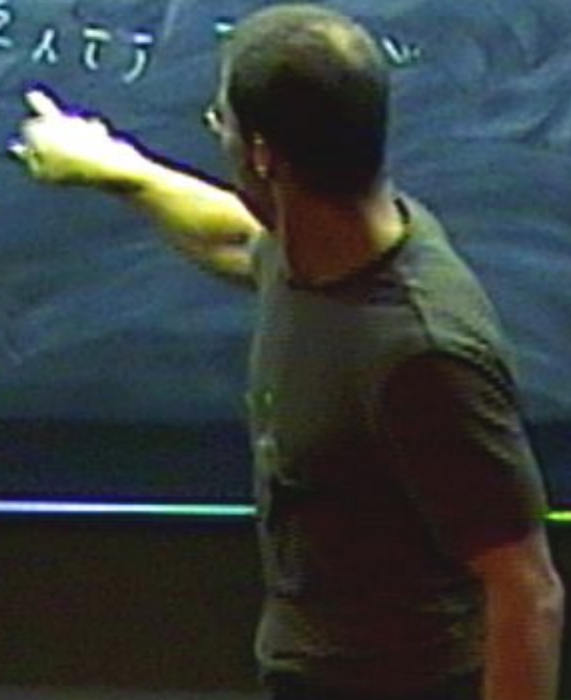


$$\Rightarrow i g_{ij} \epsilon_{ijk} \ln(\tau^2 [1^2 2^2])$$

$$d_i = (2, d_i, 4)$$

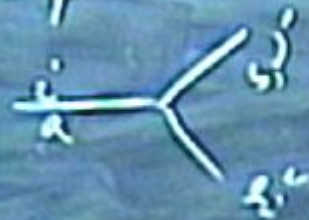


$$\sim \epsilon_{ijk} \epsilon_{lmn}$$



② Short summary of $N=4$ case

Supersymmetric



$$\Rightarrow i g_{\text{YM}}^2 \epsilon_{ijk} \ln(\tau^2 [1^2 2^2])$$

$$d_i = (2, 0, 4)$$



$$\sim \epsilon^{ijkl} \epsilon_{\alpha\beta\gamma\delta} = \delta_{\alpha\beta}^{\gamma\delta} - \delta_{\alpha\gamma}^{\beta\delta} + \delta_{\alpha\delta}^{\beta\gamma}$$



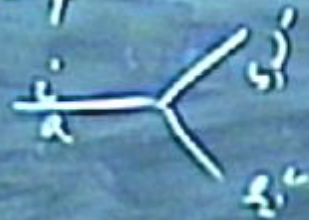
P_1



$\mathbb{1}$

② Short summary of $N=4$ case

Supersymmetric



$$\Rightarrow i g_{\mu\nu} \epsilon_{ijk} \tau_a (\tau^a)^{\mu\nu}$$

$$d_i = (2, 0, 4)$$



$$\sim \epsilon^{ijkl} \epsilon_{lmnp} = \delta_{ij}^{\dot{m}\dot{n}} \delta_{kl}^{\dot{p}\dot{q}} - \delta_{ij}^{\dot{p}\dot{q}} \delta_{kl}^{\dot{m}\dot{n}}$$



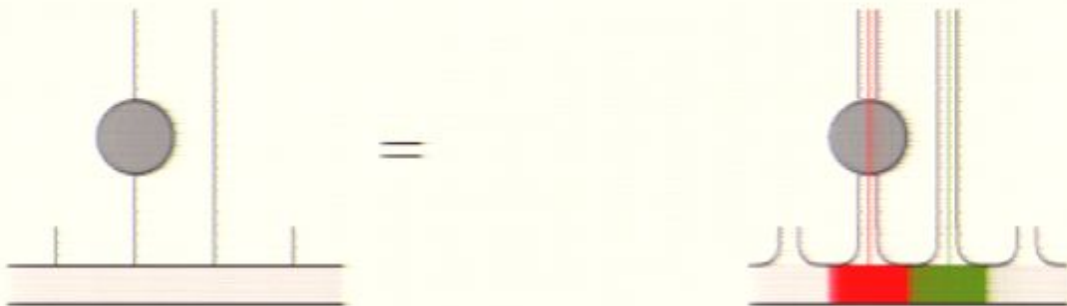
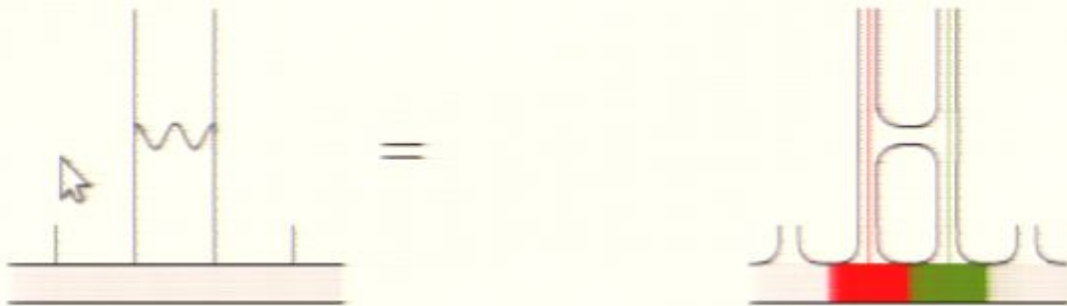
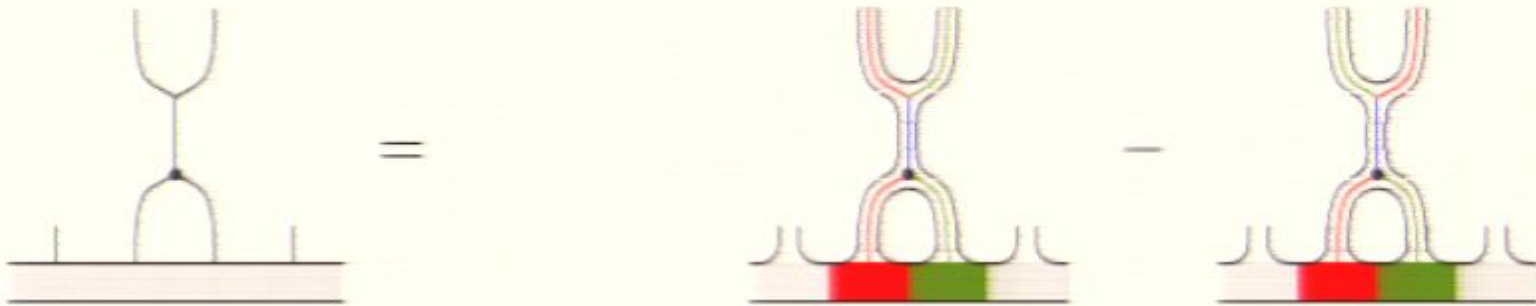
P_1



P_2

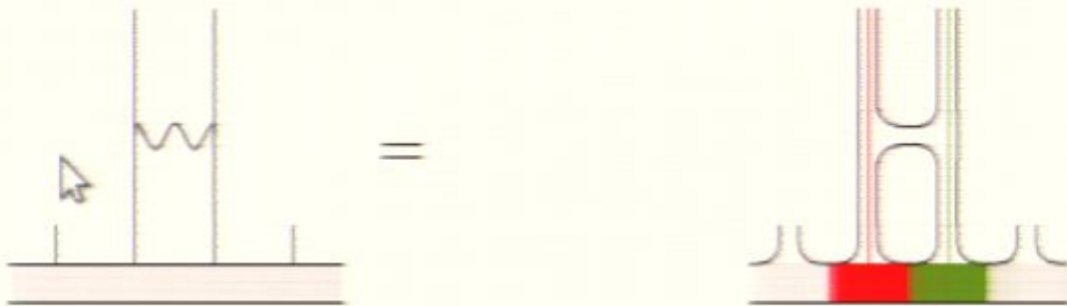
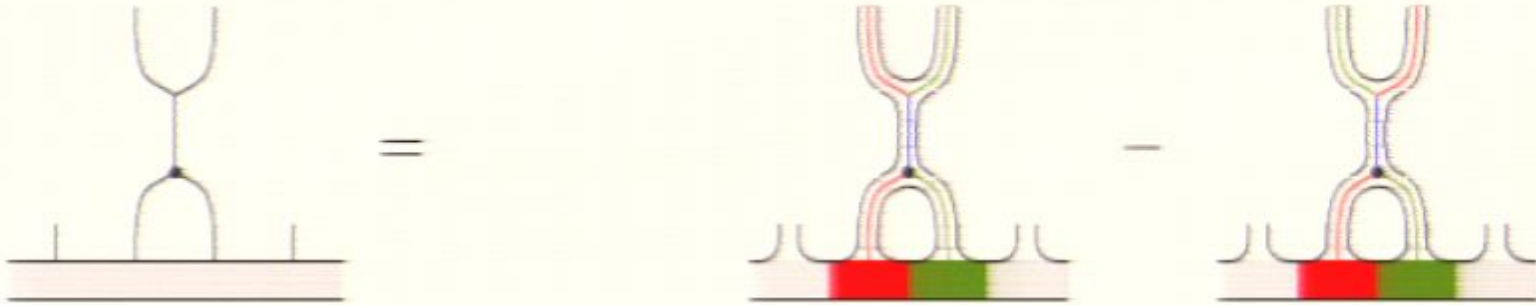
One loop

$$\text{tr}(\psi[Z, \phi]) = i \left(\text{triple junction} - \text{triple junction} \right), \quad -i \text{tr}(\bar{\psi}[\bar{\phi}, \bar{Z}]) = -i \left(\text{triple junction} - \text{triple junction} \right)$$



One loop

$$\text{tr}(\psi[Z, \phi]) = i \left(\text{Y}_1 - \text{Y}_2 \right), \quad -i \text{tr}(\bar{\psi}[\bar{\phi}, \bar{Z}]) = -i \left(\text{Y}_3 - \text{Y}_4 \right)$$



Chiral functions

$$\chi(1) = - \{ \} + \{ 1 \}$$

$$\chi(1,2) = \{ \} - \{ 1 \} - \{ 1 \} + \{ 1,2 \}$$

$$\{ a_1, \dots, a_n \} = \sum_{i=1}^L P_{i+a_1, i+a_1+1} \dots P_{i+a_n, i+a_n+1}$$

$$\chi(1,2,3) = - \{ \} + 3 \{ 1 \} - 2 \{ 1,2 \} - \{ 1,3 \} + \{ 1,2,3 \}$$

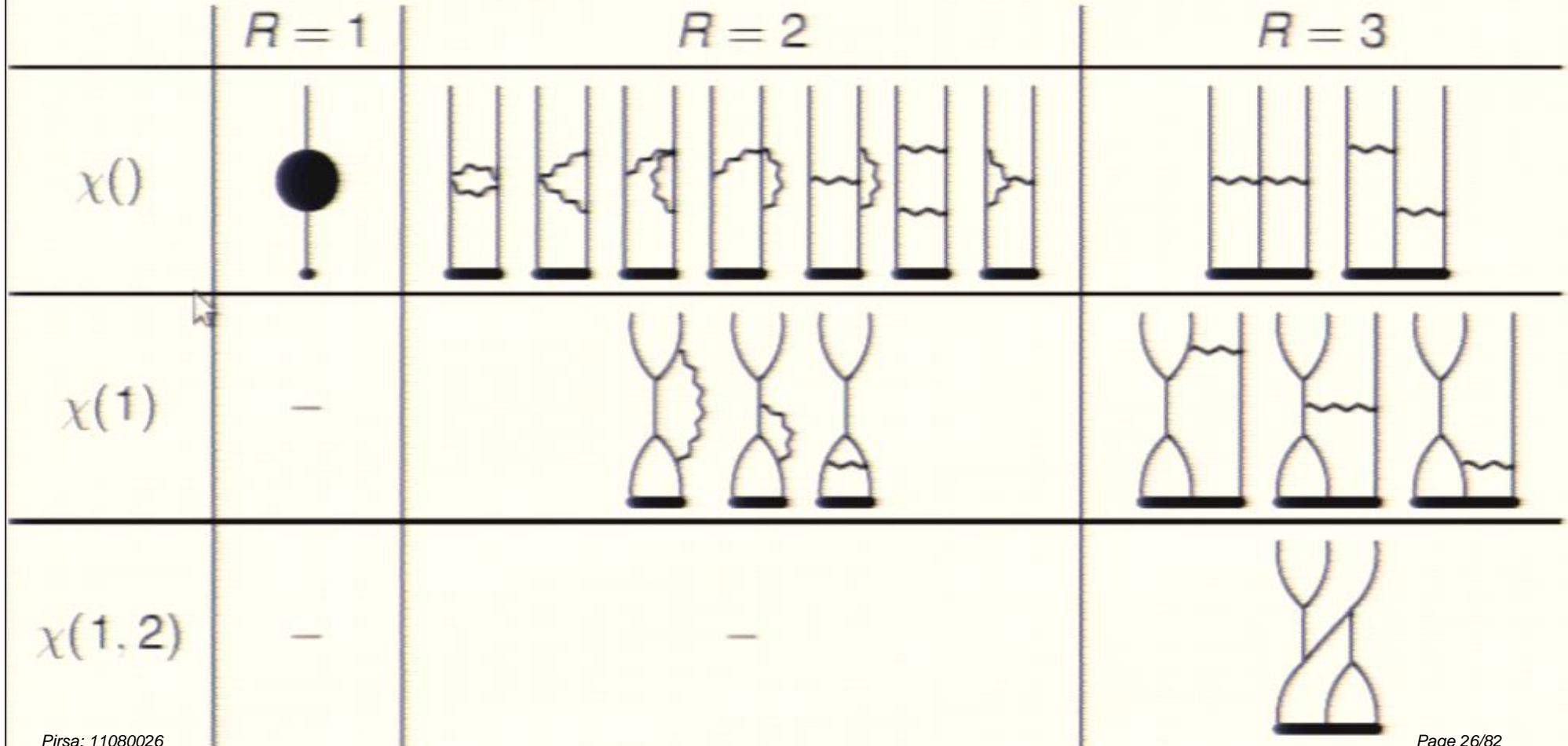
Two loops

▶ all diagrams (apart from reflections, one-loop wave function ren.)

constraints on D-algebra

flatness conditions and finite 2-loop self energy

absence of



Two loops

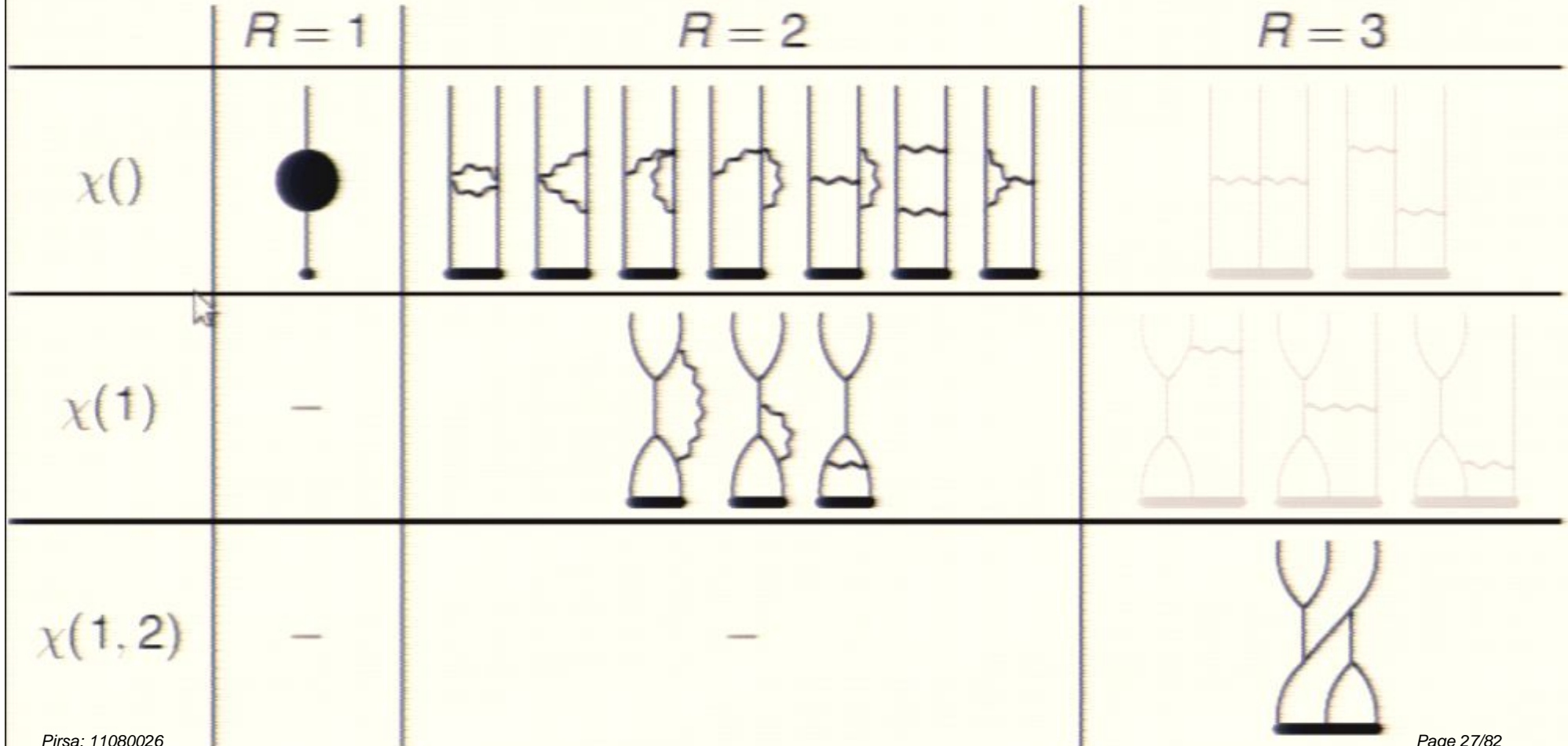
Two diagrams: each with two vertices and a ϕ^2 wave function:

constraints on D-algebra

[Fiamberti, Santambrogio, C.S., Zanoni]

interesting conditions are those \mathbb{Z}_2 -odd set energy

classifies



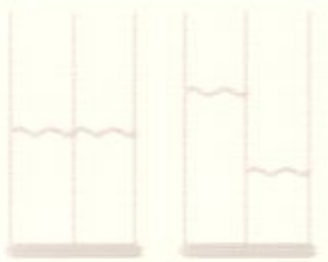
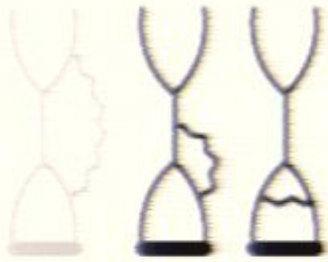




Two loops

2-loop diagrams with 2 external legs and 2 external fermion lines
 containing 2 fermions

- ▶ finiteness conditions and finite 2-loop self energy
 \Rightarrow absence of $\chi()$

[C.S.]

	$R = 1$	$R = 2$	$R = 3$
$\chi()$			
$\chi(1)$	—		
$\chi(1, 2)$	—	—	

$$D_2 = 4\chi(1) - 2[\chi(1, 2) + \chi(2, 1)]$$

$$\bar{z} = 1 - \lambda \mathbb{I}_1(x_6) - \lambda^2 \mathbb{I}_2[x(1,2) + x(2,1) - 2x_6]$$



$$\bar{E} = 1 - \lambda T_1 x(t) - \lambda^2 T_2 [x(1,2) + x(2,1) - 2x(t)]$$

$x(t)^2 =$ disconnected.

$$\ln \bar{E} = -\lambda T_1 x(t) - (T_2 + \frac{1}{2} T_1^2) x(t)^2 - \text{disconn.}$$

$\swarrow \quad \searrow$
 $-\frac{1}{2} T_1^2 \quad (\frac{1}{2})^2$



$$\bar{z} = 1 - \lambda T_1 x(t) - \lambda^2 T_2 [x(1,2) + x(2,1) - 2x(t)]$$

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$$\ln \bar{z} = -\lambda T_1 x(t) - (T_2 + \frac{1}{2} T_1^2) (x(t))^2 - \text{disconnected}$$

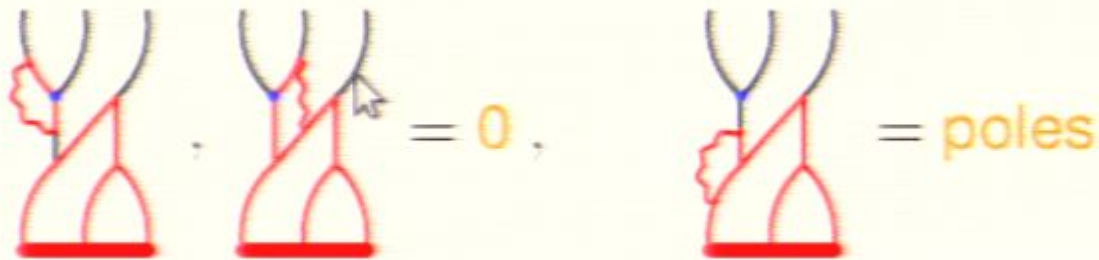
$\swarrow \quad \searrow$
 $-\frac{1}{2\epsilon^2} \quad (\frac{1}{2})^2$

Three loop, feasible transfer to feedback condition

Finiteness conditions

For all $R \geq 2$ diagrams with \mathcal{O}_L of the $SU(2)$ sector (also beyond?),
 no **overall UV divergence** if at least one of the conditions holds:

- 1. all chiral vertices take part in loops, e.g.:



- 2. one of its spinor derivatives D_α is brought outside loops
- 3. for v_0 chiral vertices outside loops:
 more than $2(v_0 - 1)$ spinor derivatives $\bar{D}_{\dot{\alpha}}$ are brought outside loops

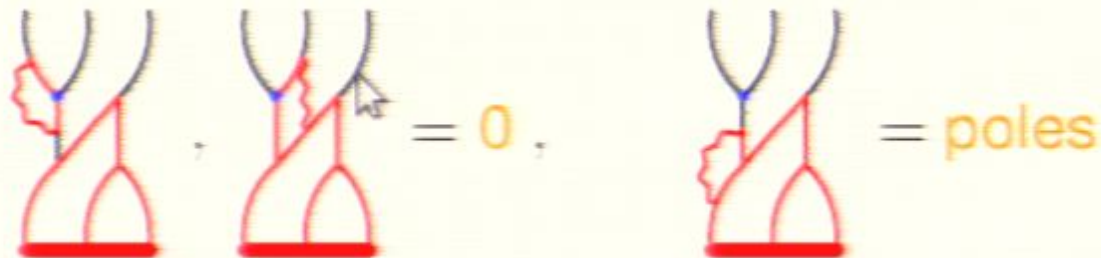
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> **cancellations** between certain diagrams involving vector fields

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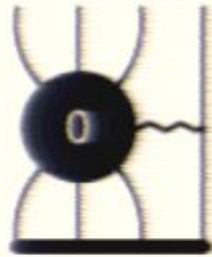
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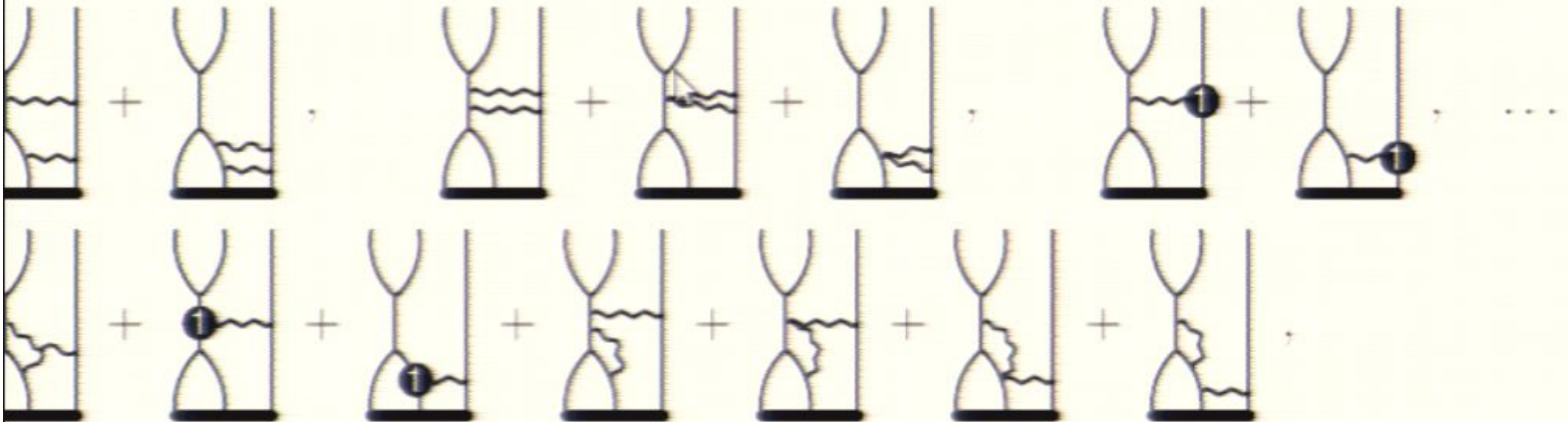
> **cancellations** between certain diagrams involving vector fields

Cancellations

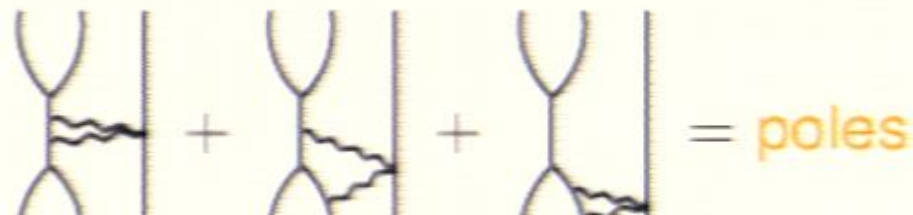
maximum range $R = 4$ diagrams if χ is not of maximum range **irrelevant**



cancellations for next-to-maximum range $R = 3$ diagrams



but cancellations not complete



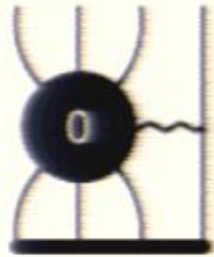
The interpolating theory: field content

[Gadde, Pomoni, Rastelli]

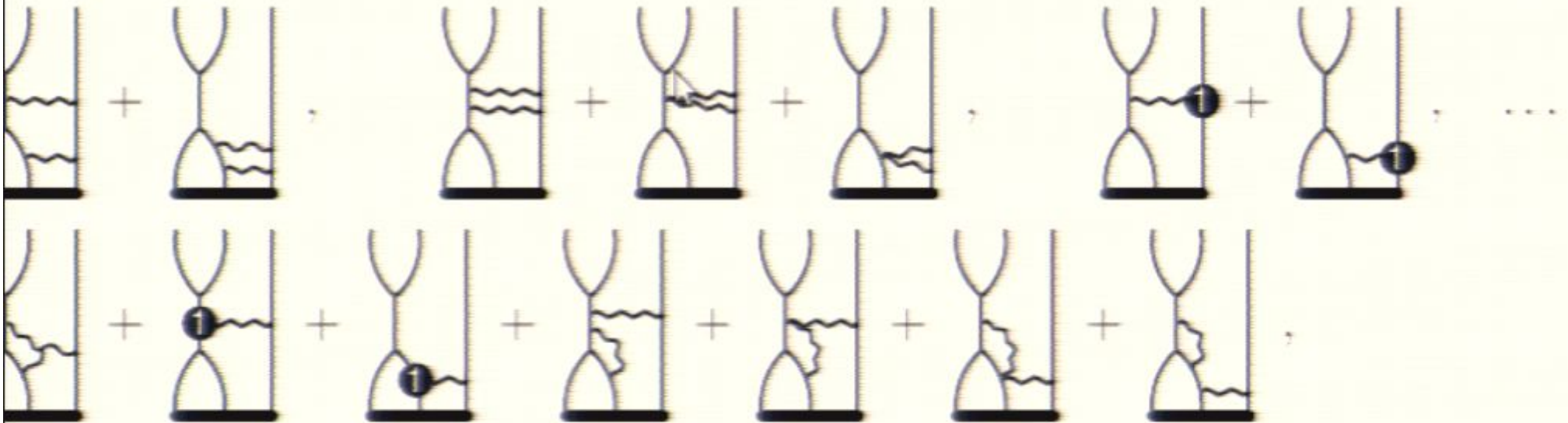
field	$SU(N) \times SU(N)$	$SU(2)_L$	$SU(2)_R$	$U(1)$
V	(adj., 1)	1	1	0
Φ	(adj., 1)	1	1	1
$\bar{\Phi}$	(adj., 1)	1	1	-1
\hat{V}	(1, adj.)	1	1	0
$\hat{\Phi}$	(1, adj.)	1	1	1
$\hat{\bar{\Phi}}$	(1, adj.)	1	1	-1
Q_i	(\square , $\bar{\square}$)	\square	\square	0
$Q_i^{\prime\prime}$	(\square , $\bar{\square}$)	\square		0
Q_i^{\prime}	($\bar{\square}$, \square)	$\bar{\square}$	$\bar{\square}$	0
$Q_i^{\prime\prime}$	($\bar{\square}$, \square)	$\bar{\square}$		0

Cancellations

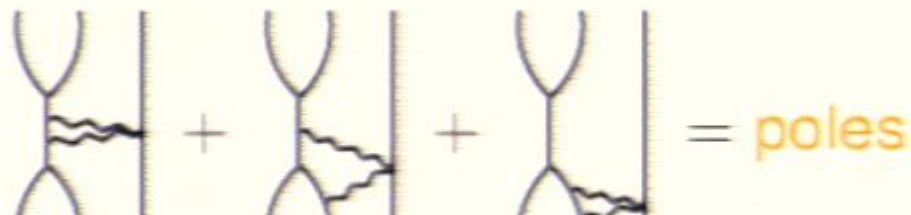
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cancellations for next-to-maximum range $R = 3$ diagrams



but cancellations not complete



③ Interpolation's Heavy

③ Interpolation, Theory: Gadda, Prouni, Rastelli

$N=2$ q_{11}

③ Interpolating theory: Gaiotto-Panouri, Rastelli

$N=2$ quiver Yang theory: $SU(N) \times SU(N)$



$\mathbb{Z}_2 \times SU(N) \times U(1)$
 \uparrow
 R -Symmetry

③ Interpolating theory: Gaiotto-Panouri, Rastelli

$N=2$ quiver gauge theory: $SU(N) \times SU(N)$



$SU(N)_L \times SU(N)_R \times U(1)$
 \uparrow
 $\leftrightarrow \mathbb{Z}$

$N = \mathbb{Z}$

R-symmetry

$\mathcal{J} = \mathcal{S}$

$\mathcal{J} = 0$

\mathbb{Z}_2 orbifold of $N=4$ Sym

$N=2$ SCQCD

closed class subject:

closed class subgroups:

protected states: $W(\phi, \delta)$, $L(\phi, \delta)$, $L(\phi, \delta, \dots)$

by orders
symmetric
state in dia

closed class subsector:

protected states: $L(\phi, \delta)$, $L(\phi, \delta^T)$, $L(\underbrace{\phi, \delta^i}_{\substack{\text{by vectors} \\ \text{symmetric} \\ \text{state vector}}})$

$L(\phi, \phi, \phi, \phi, \dots)$

perp

closed chiral subsector:

protected states: $L_0(\phi, \bar{\phi})$, $L_0(\phi, \bar{\phi}^\dagger)$, $L_0(\phi, \bar{\phi}^{\dagger n})$

by no less
symmetric
state in dia

$L_0(\phi, \bar{\phi} \otimes_2 \bar{\phi} \dots \bar{\phi} \otimes^n \dots)$



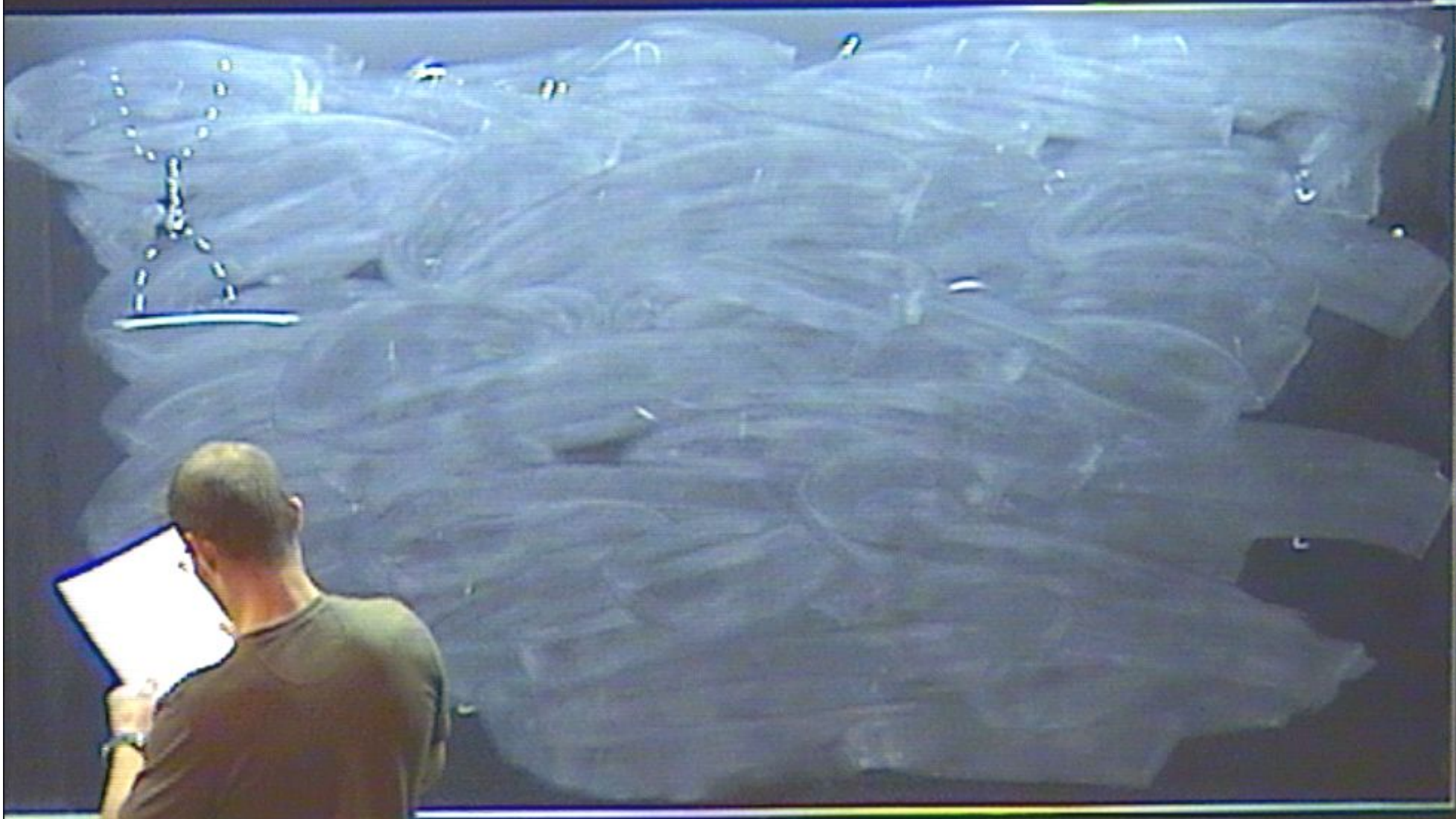
closed class subject:

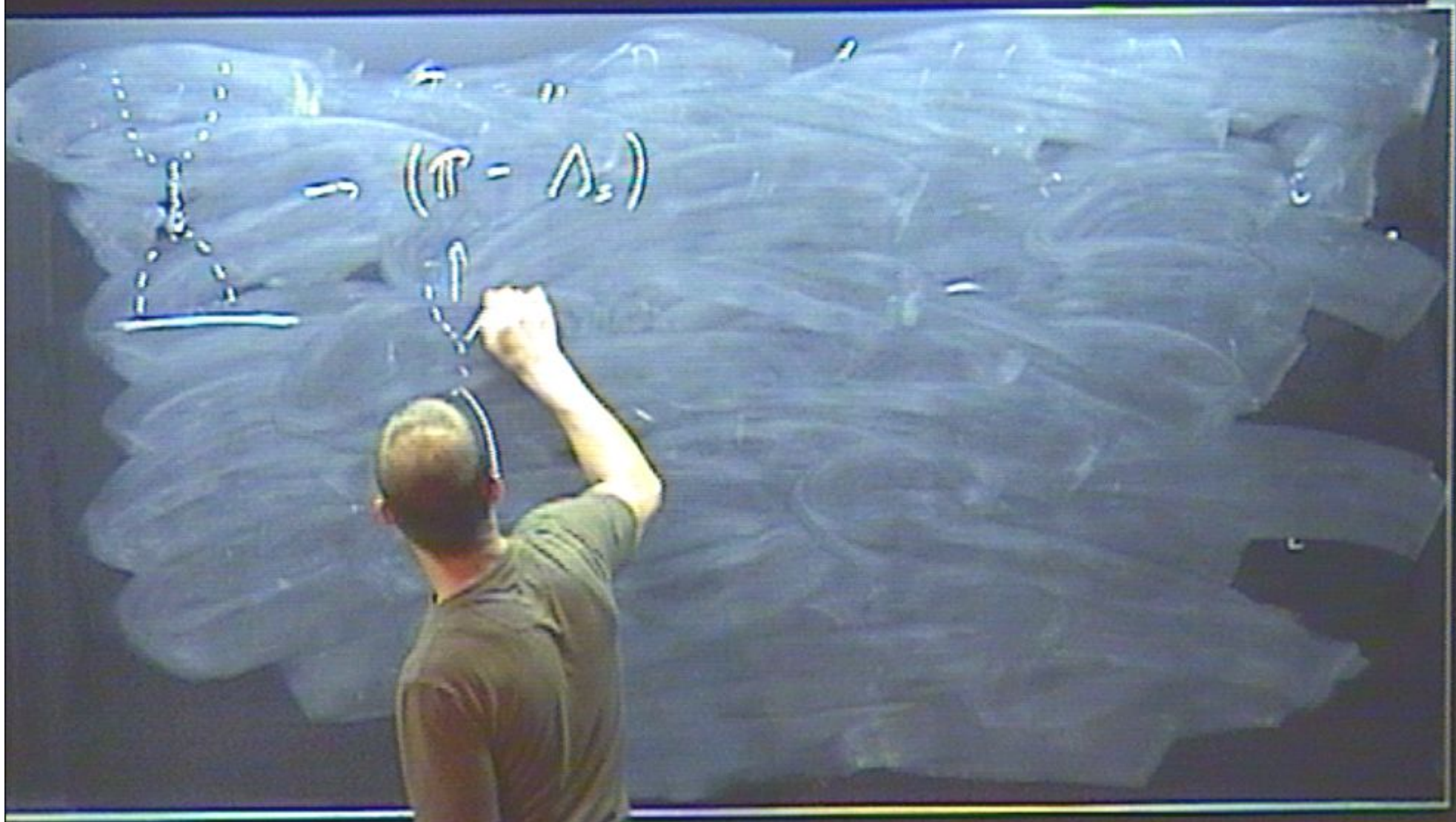
protected state: $W(\phi, \delta)$, $L(\phi, \delta^T)$, $L(\phi, \delta^T, \dots)$

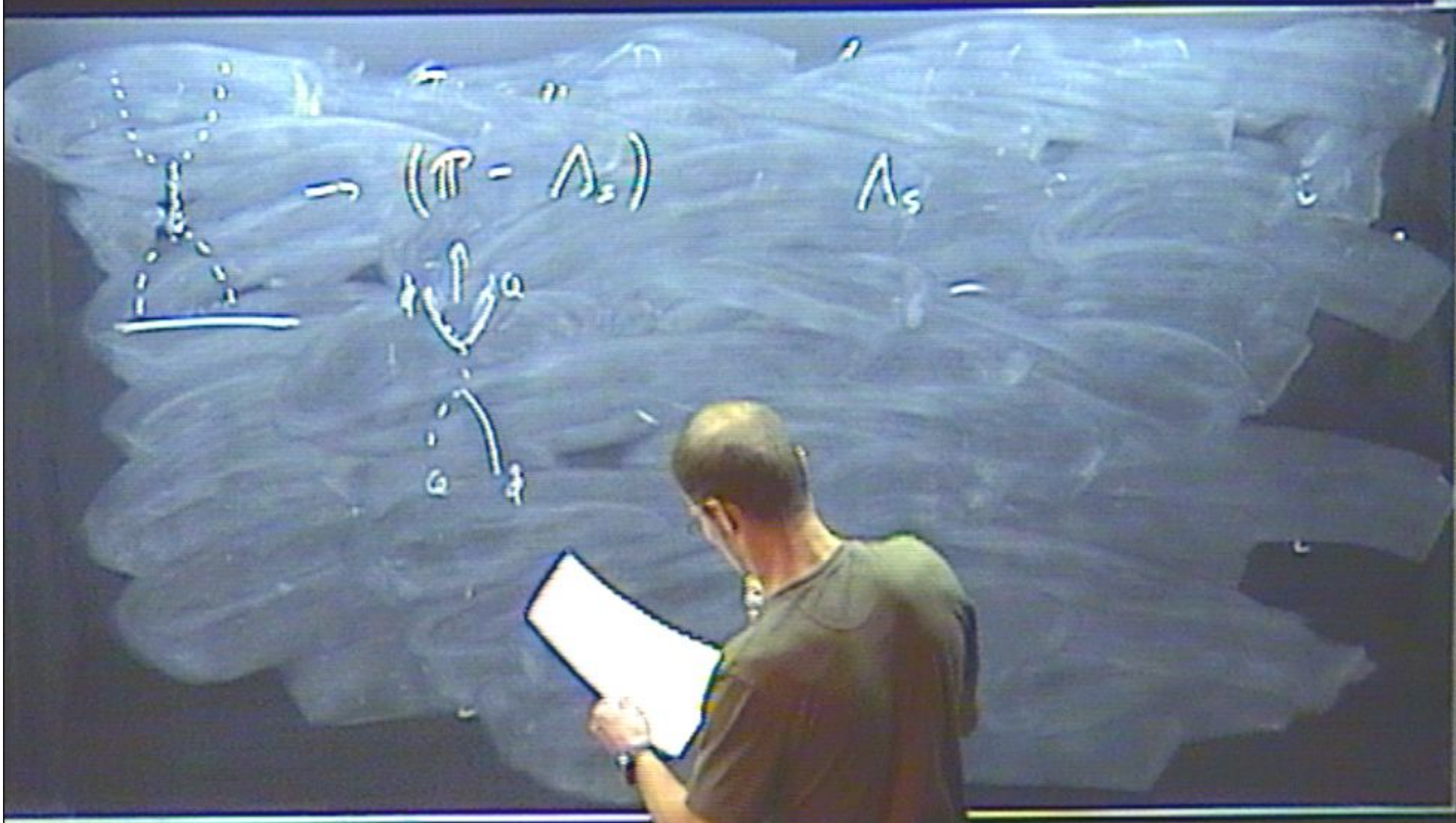
$L(\phi, \delta, \phi, \delta, \dots)$

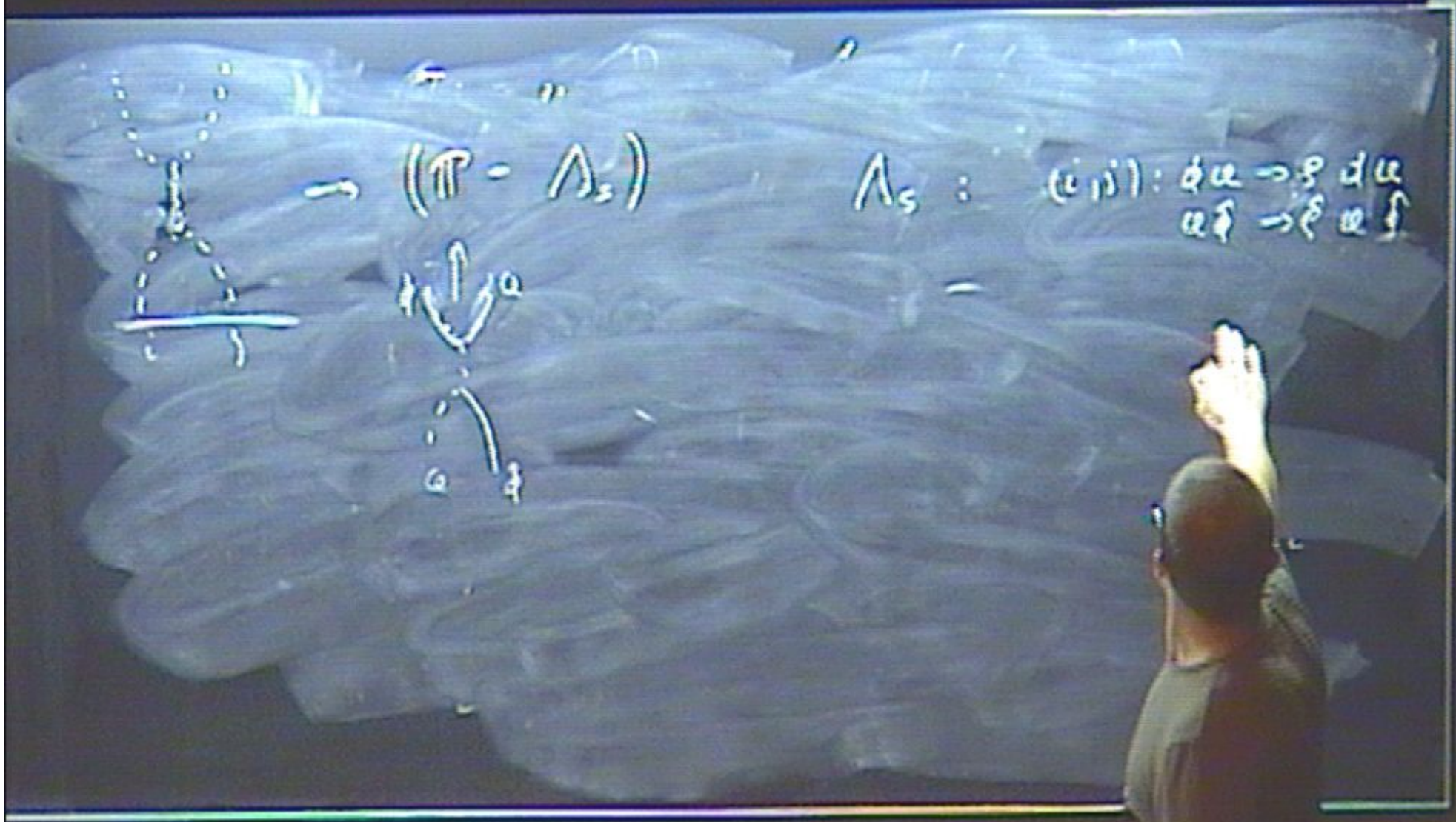
by orders
symmetric
state vector

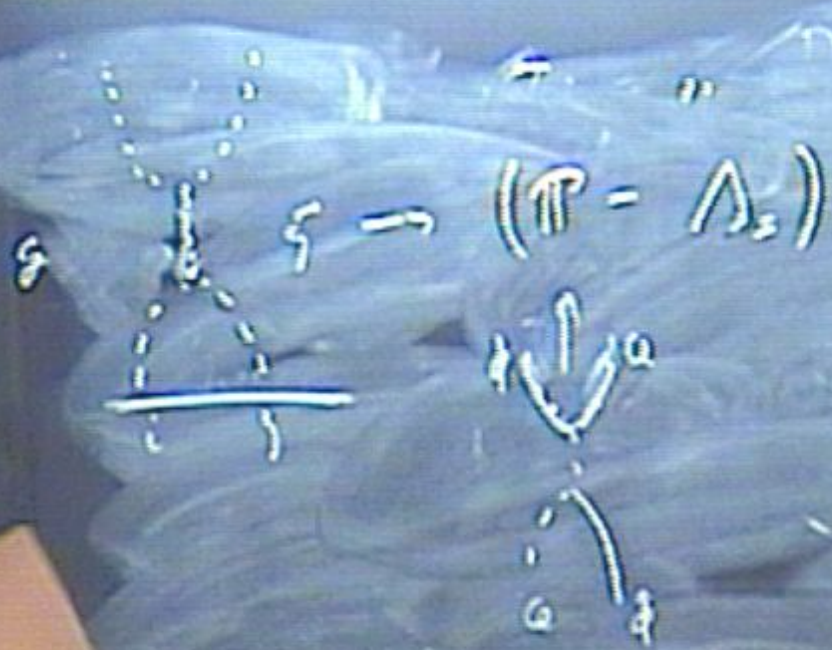






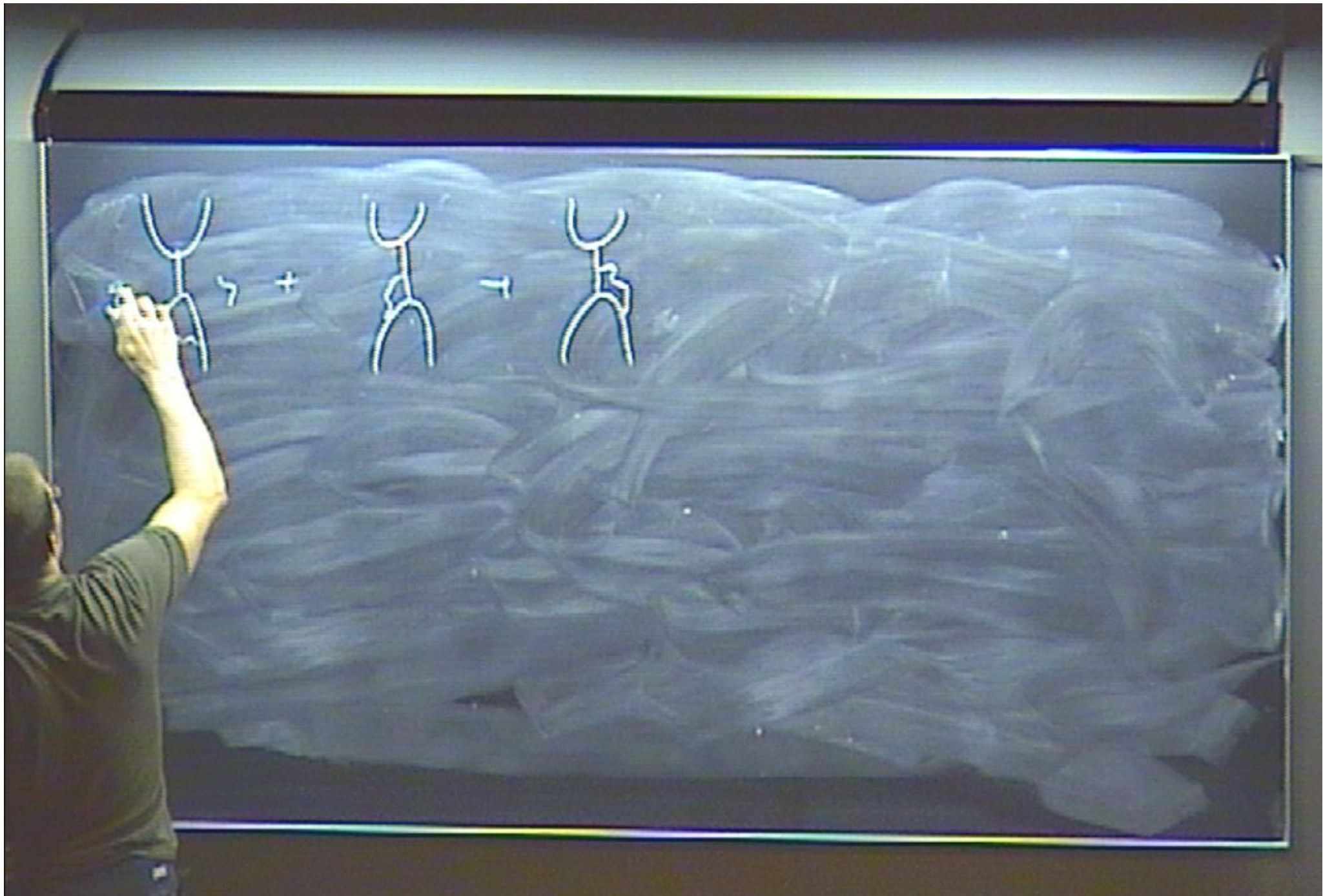




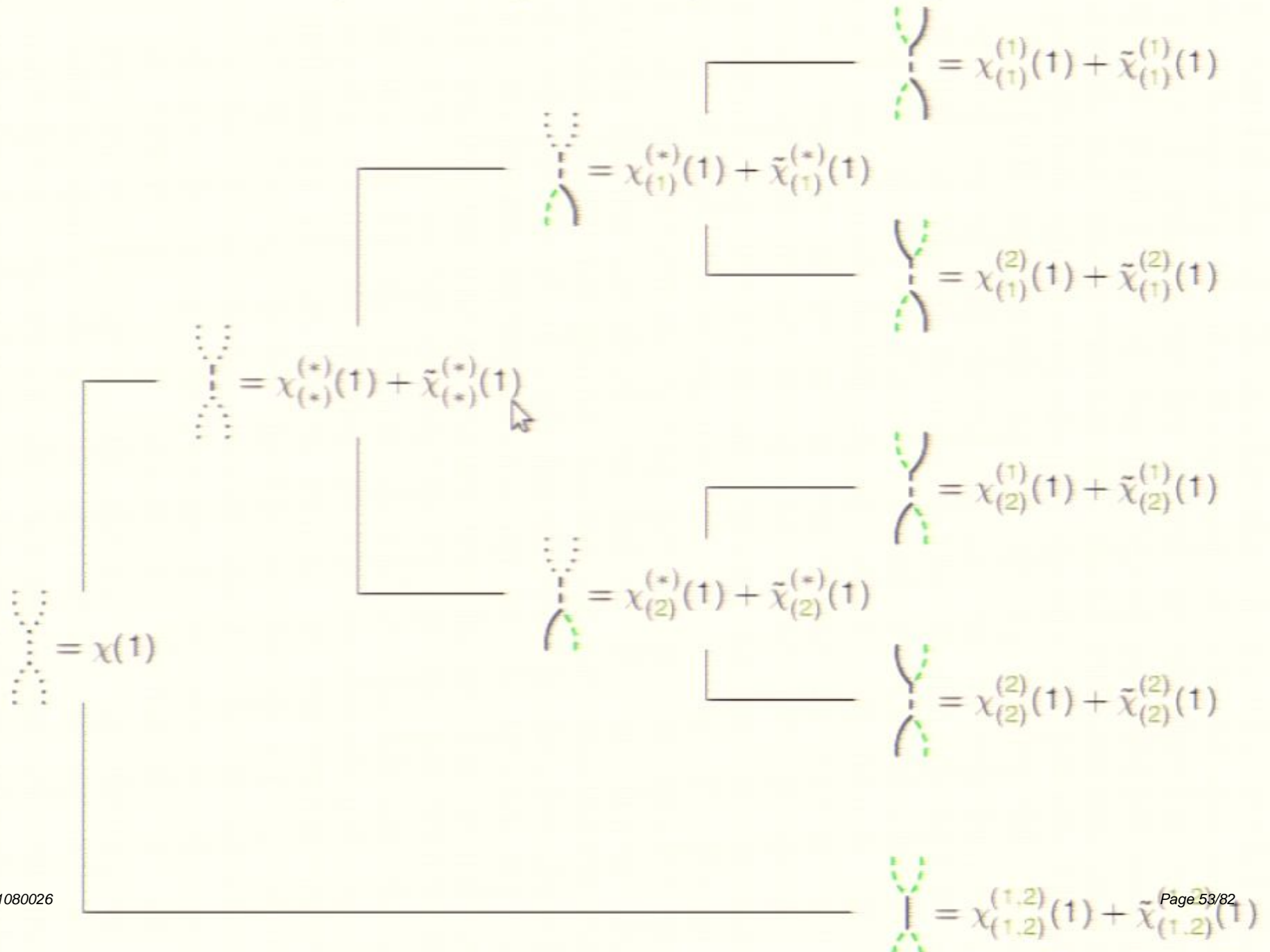


$$g = \frac{g}{g}, \quad \dot{g} = \frac{g}{g}$$

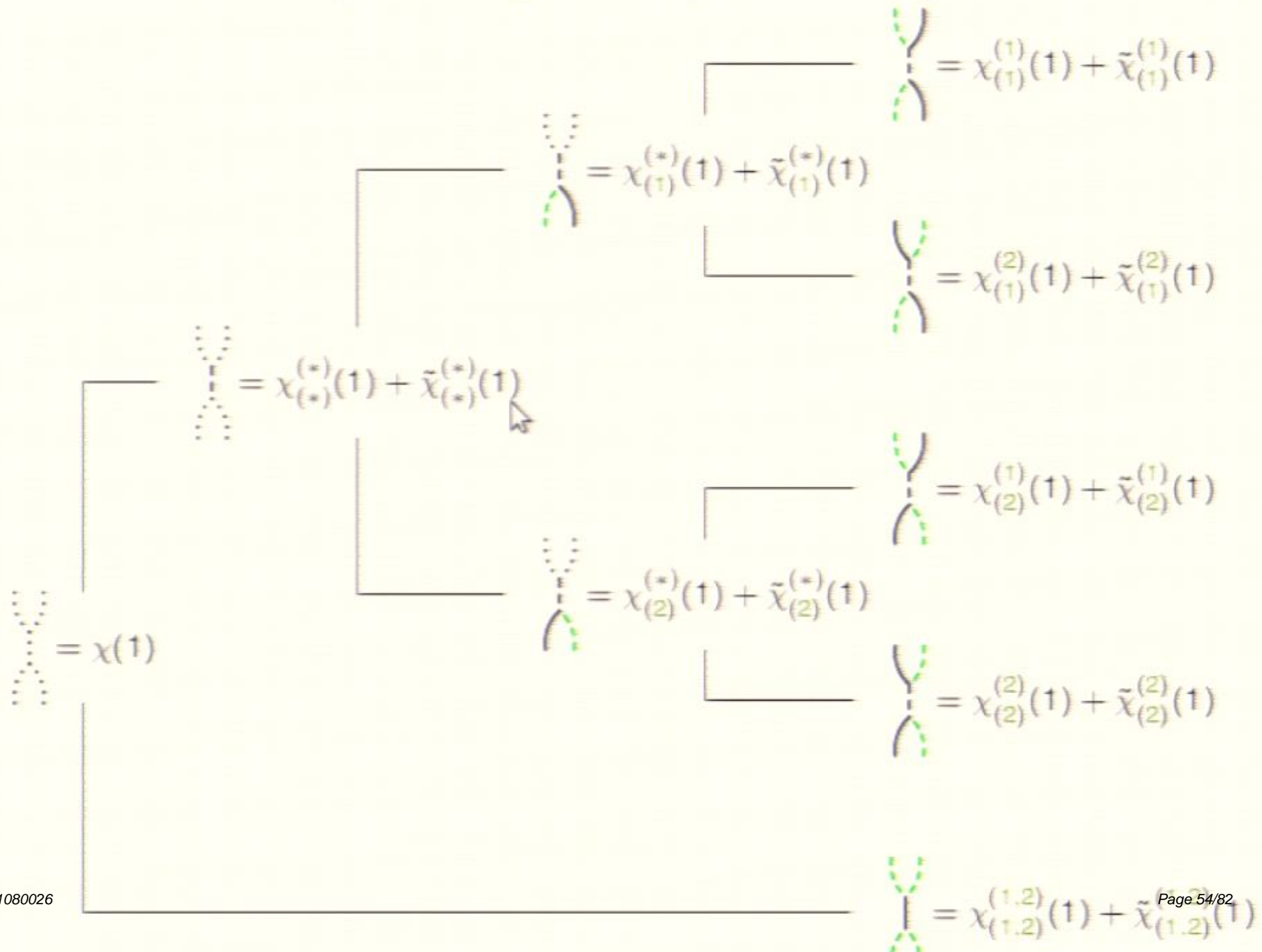
$$\Lambda_s : (i, j) : \begin{matrix} d\alpha \rightarrow g d\alpha \\ \alpha \dot{g} \rightarrow \dot{g} \alpha \dot{g} \end{matrix}$$



The interpolating theory: building block

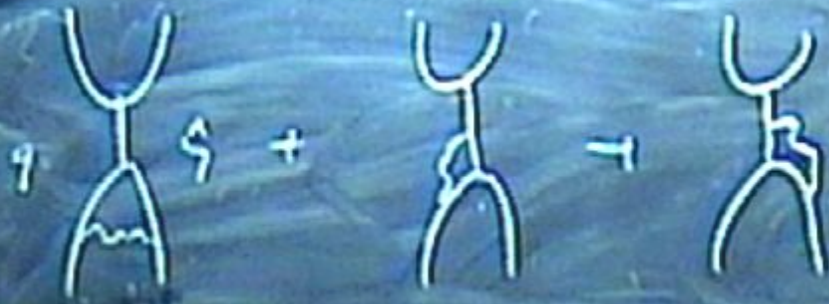


The interpolating theory: building block









$$\sim (S \cdot \epsilon) S(\tau) X(\mu, z)$$





$$\sim (S \cdot \epsilon) S(3) \times (1, 2)$$



$$\rightarrow h(S, S) \sim P(3)$$



The interpolating theory: dilatation operator

$$\mathcal{D}_1 = -2\chi(1),$$

The interpolating theory: dilatation operator

$$D_1 = -2\chi(\mathbf{1}),$$

The interpolating theory: dilatation operator

- ▶ deformed pure scattering term
- ▶ anti-hermitean contributions: removed by non-unitary similarity trafo

$$\mathcal{D}_1 = -2 \chi(\mathbf{1}),$$

$$\mathcal{D}_2 = -2(\chi(1,2) + \chi(2,1)) + 2(\rho + \hat{\rho}) \chi(\mathbf{1})$$

$$\mathcal{D}_3 = -4(\chi(1,2,3) + \chi(3,2,1)) - 2(\rho + \hat{\rho})(\chi(1,2) + \chi(2,1)) \\ + \chi(1,2,1) + \chi(2,1,2) + (\rho + \hat{\rho})^2 \chi(\mathbf{1})$$

$$- 4(\hat{\rho} \chi_{(*)}^{(*)}(\mathbf{1}, \mathbf{3}) + \rho \tilde{\chi}_{(*)}^{(*)}(\mathbf{1}, \mathbf{3})) + 2(\chi(2,1,3) - \chi(1,3,2))$$

$$+ 2\zeta(\mathbf{3})(\rho - \hat{\rho})((\rho - \hat{\rho}) \chi(\mathbf{1}))$$

$$- \hat{\rho}(\chi_{(1)}^{(*)}(\mathbf{1}) + \tilde{\chi}_{(2)}^{(*)}(\mathbf{1})) + \rho(\chi_{(2)}^{(*)}(\mathbf{1}) + \tilde{\chi}_{(1)}^{(*)}(\mathbf{1}))$$

$$- \chi_{(2)}^{(*)}(\mathbf{1}, \mathbf{2}) + \chi_{(1,2)}^{(*,3)}(\mathbf{1}, \mathbf{2}) + \chi_{(2)}^{(*)}(\mathbf{2}, \mathbf{1}) + \chi_{(2,3)}^{(1,*)}(\mathbf{2}, \mathbf{1})$$

$$+ \tilde{\chi}_{(2)}^{(*)}(\mathbf{1}, \mathbf{2}) - \tilde{\chi}_{(1,2)}^{(*,3)}(\mathbf{1}, \mathbf{2}) - \tilde{\chi}_{(2)}^{(*)}(\mathbf{2}, \mathbf{1}) - \tilde{\chi}_{(2,3)}^{(1,*)}(\mathbf{2}, \mathbf{1})$$

The interpolating theory: dilatation operator

- ▶ deformed pure scattering term
- ▶ maximal transcendental contributions $\rightarrow h^2(\rho, \hat{\rho}), f(\rho, \hat{\rho})$

$$\mathcal{D}_1 = -2 \chi(\mathbf{1}),$$

$$\mathcal{D}_2 = -2(\chi(\mathbf{1}, 2) + \chi(\mathbf{2}, 1)) + 2(\rho + \hat{\rho}) \chi(\mathbf{1})$$

$$\mathcal{D}_3 = -4(\chi(\mathbf{1}, 2, 3) + \chi(\mathbf{3}, 2, 1)) - 2(\rho + \hat{\rho})(\chi(\mathbf{1}, 2) + \chi(\mathbf{2}, 1)) \\ + \chi(\mathbf{1}, 2, 1) + \chi(\mathbf{2}, 1, 2) + (\rho + \hat{\rho})^2 \chi(\mathbf{1})$$

$$- 4(\hat{\rho} \chi_{(*)}^{(*)}(\mathbf{1}, 3) + \rho \tilde{\chi}_{(*)}^{(*)}(\mathbf{1}, 3))$$

$$+ 2\zeta(3)(\rho - \hat{\rho})((\rho - \hat{\rho}) \chi(\mathbf{1}))$$

$$- (\rho + \hat{\rho})(\chi_{(1)}^{(1)}(\mathbf{1}) - \chi_{(2)}^{(2)}(\mathbf{1}) - \tilde{\chi}_{(1)}^{(1)}(\mathbf{1}) + \tilde{\chi}_{(2)}^{(2)}(\mathbf{1}))$$

$$\psi(\mathbf{r}) = \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \phi(\mathbf{k})$$

$$\psi(\rho) = \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\rho} \phi(\mathbf{k}) \delta(\mathbf{k} - \mathbf{k}')$$

$$\tilde{\psi}(\rho) = \psi(\rho) \quad \left| \begin{array}{l} \phi \leftrightarrow \tilde{\phi} \\ \mathbf{k} \leftrightarrow \tilde{\mathbf{k}} \end{array} \right.$$

$$E(\rho) = \sqrt{1 + \hbar^2(\mathbf{g}, \tilde{\mathbf{g}}) \times \hbar}$$

$$\psi(\rho) = \sum_m e^{i\mathbf{k}_m \cdot \rho} \phi(\rho) \alpha'_m \delta$$

$$\tilde{\psi}(\rho) = \psi(\rho) \left| \begin{array}{l} \phi \leftrightarrow \hat{\phi} \\ \alpha \leftrightarrow \hat{\alpha} \end{array} \right.$$

$$E(\rho) = \sqrt{\gamma + \hbar^2(\mathbf{g}, \hat{\mathbf{g}}) \times \hbar} - 1 + 4(\mathbf{g}, \hat{\mathbf{g}}) (\hat{\mathbf{g}} - \hat{\mathbf{g}}) \frac{\sin}{\tau}$$

$$\psi(\rho) = \sum_m e^{i\mathbf{k} \cdot \mathbf{r}} \phi(\rho) \alpha \hat{\mathbf{e}} \cdot \hat{\mathbf{e}}$$

$$\tilde{\psi}(\rho) = \psi(\rho) \left| \begin{array}{l} \phi \leftrightarrow \hat{\phi} \\ \alpha \leftrightarrow \hat{\alpha} \end{array} \right.$$

$$E(\rho) = \sqrt{\gamma + \hbar^2(\mathbf{g}, \hat{\mathbf{g}}) \times \hbar} - 1 + 4(\mathbf{g}, \hat{\mathbf{g}}) (\mathbf{g} - \hat{\mathbf{g}}) \sin \rho$$

$$\hbar^2(\mathbf{g}, \hat{\mathbf{g}}) = 4\hat{g}^2 - 4\hat{g}'(\mathbf{g} \cdot \hat{\mathbf{g}}) + \dots$$

$$\psi(\rho) = \sum_m e^{i\mathbf{k} \cdot \mathbf{r}} \phi(\rho) \alpha' \hat{\mathbf{r}} \cdot \hat{\mathbf{r}}$$

$$\tilde{\psi}(\rho) = \psi(\rho) \left| \begin{array}{l} \phi \leftrightarrow \hat{\phi} \\ \alpha \leftrightarrow \hat{\alpha} \end{array} \right.$$

$$E(\rho) = \sqrt{1 + \hbar^2 (\mathbf{g}, \hat{\mathbf{r}}) \times \hbar} - 1 + 4(\mathbf{g}, \hat{\mathbf{r}}) (\mathbf{g} - \hat{\mathbf{r}}) \sin \rho$$

$$\hbar^2 (\mathbf{g}, \hat{\mathbf{r}}) = \hbar^2 \mathbf{g}^2 - 4 \mathbf{g}^2 (\mathbf{g} - \hat{\mathbf{r}})^2 \mathbf{g}(\mathbf{r}) + \dots$$

$$\psi(\rho) = \sum_m e^{i\mu r} \phi - \rho \omega \hat{q} \dots \hat{q}$$

$$\tilde{\psi}(\rho) = \psi(\rho) \left| \begin{matrix} \phi \rightarrow \hat{q} \\ \omega \rightarrow \alpha \end{matrix} \right. \text{Gadde Rastelli}$$

$$E(\rho) = \sqrt{1 + h^2(\eta, \tilde{\eta}) \times h} - 1 + 4(\eta, \tilde{\eta}) (\eta - \tilde{\eta} \sin \rho)$$

$$h^2(\eta, \tilde{\eta}) = 4\tilde{\eta}^2 - 4\tilde{\eta}^4 (\eta - \tilde{\eta})^2 \rho(3) + \dots$$

$$4(\eta, \tilde{\eta}) = -2\tilde{\eta}^4 (\eta^2 - \tilde{\eta}^2) \rho(3)$$

$$\psi(\rho) = \sum_n e^{i n \rho} \phi_{-n} \alpha'_{\tilde{\alpha}} \cdot \tilde{\alpha}$$

$$\tilde{\psi}(\rho) = \psi(\rho) \left| \begin{array}{l} \phi \leftrightarrow \tilde{\phi} \\ \alpha \leftrightarrow \tilde{\alpha} \end{array} \right. \text{Gadde Rastelli}$$

$$E(\rho) = \sqrt{1 + h^2(\eta, \tilde{\eta}) \times h} - 1 + 4(\eta, \tilde{\eta}) (\eta - \tilde{\eta}) \sin \rho$$

$$h^2(\eta, \tilde{\eta}) = 4 \eta^2 - 4 \eta^4 (\eta - \tilde{\eta})^2 \rho(3) + \dots$$

$$4(\eta, \tilde{\eta}) = -2 \eta^4 (\eta^2 - \tilde{\eta}^2) \rho(3)$$

$$\mathcal{D}' = e^{-x} \mathcal{D} e^x$$

$x =$ expansion in Feynman diagrams

$$E \rightarrow E(p')$$

$$\mathcal{D}' = e^{-x} \mathcal{D} e^x$$

$x =$ expression in Feynman diagrams

$$E \rightarrow E(p') = \sqrt{1 + 4^2 \lambda u} = 1 + 4(s, \vec{\beta})(\beta \cdot \vec{\beta})$$

$$\mathcal{D}' = e^{-x} \mathcal{D} e^x$$

$x =$ expression in Feynman diagrams

$$E \rightarrow E(p') = \sqrt{1 + 4^2 \lambda u} = 1 + 4(s, \bar{s})(\bar{s}, s')$$

$$P' = p + i \bar{s}' (s' - \bar{s}') s(z)$$

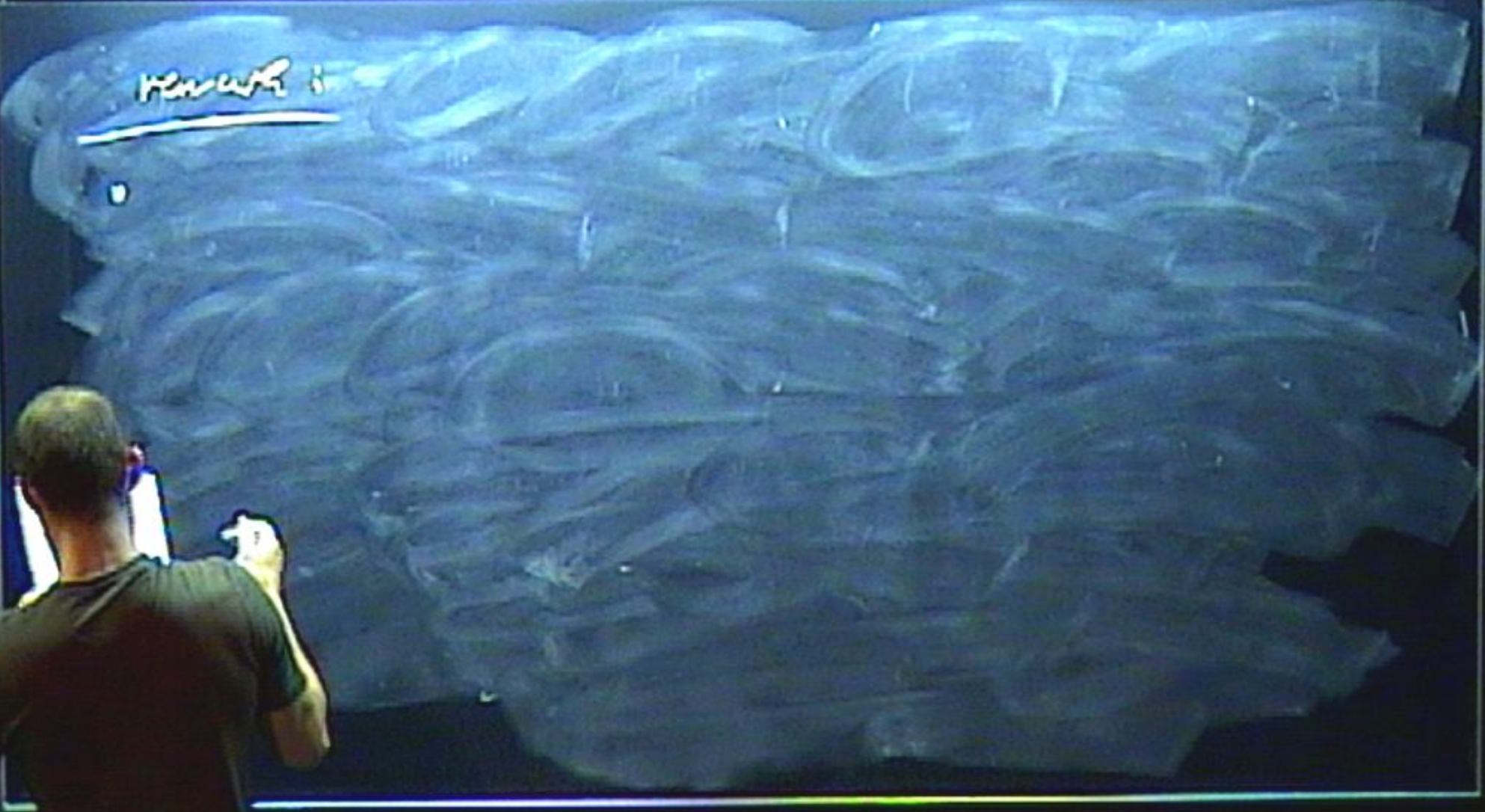
$$\mathcal{D}' = e^{-x} \mathcal{D} e^x$$

$x = \text{expression in Feynman diagrams}$

$$E \rightarrow E(p') = \sqrt{1 + 4^2 \lambda u} - 1 + 4(s, \vec{s}) (s \cdot \vec{s}')$$

$$P' = p \pm i \vec{s}' (s^2 - s'^2) \vec{s}(z)$$

Research 1



Remark 1

• $h(s, \beta)$ is not transcendental as in AB44)

Remark 1

- $h(\xi, \eta)$ maximal transcendental as in ABY(4)
Cases
also in the ABY case

Remark:

• $h(\xi, \eta)$ maximal transcendental as in ABY(1)

Case

also in the ABY case: deformation of h_{res}

ξ -ambrosi: at 6-loop



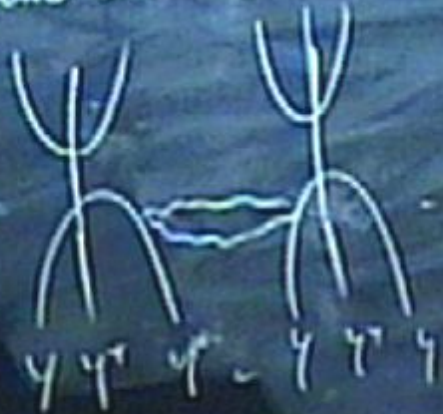
Remark 1

• $h(\xi, \eta)$ maximal transcendent as in ABY(4)

Case

also in the ABY case: deformation of the

\mathcal{S} -structure: at 6-loop



• in the interpolating theory:
Sector \rightarrow S_{eff} , $S_{\text{eff}}(T)$ of $N=4$

• in the interpolating theory:
 sections \leftrightarrow $S_2(\sigma)$, $S_4(\sigma)$ of $N=4$

$$\begin{array}{c} \text{---} \circlearrowleft \text{---} \end{array} \rightarrow \Sigma_2 = -2\mathbb{I} \int p^2 (n \cdot \xi) \begin{array}{c} \text{---} \triangle \text{---} \\ \sim S(3) \end{array}$$

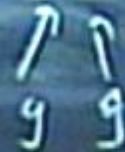
• in the interpolating theory:
 Section \leftrightarrow $S_2(a)$, $S_4(a)$ of $N=4$



\rightarrow

$\Sigma_2 =$

$-2 \overline{\Sigma}$



$p^2(a-s)$



$\sim S(3)$