Title: On the Ratio of the Viscosity to Entropy Density for Quantum Gases in the Unitary Limit

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Abstract: In the so-called unitary limit of quantum gases, the scattering length diverges and the theory becomes scale invariant with dynamical exponent z=2. This point occurs precisely at the crossover between strongly coupled BEC and BCS. These systems are currently under intense experimental study using cold atoms and Feshbach resonances to tune the scattering length. We developed a new approach to the statistical mechanics of gases in higher dimensions modeled after the thermodynamic Bethe ansatz, i.e. based on the exact 2-body S-matrix. Calculations of the critical temperature Tc/T_F = 0.1 are in good agreement with experiments and Monte-Carlo studies. We also calculated the ratio of viscosity to entropy density and obtained 4.7 times the conjectured lower bound of 1/4 pi, in good agreement with very recent experiments. We also present evidence for a strongly interacting version of BEC.

# QUANTUM GASES IN THE UNITARY LIMIT

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CORNELL UNIVERSITY

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August 2011

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  BEC/BCS crossover, RG, etc.

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(unitary gases work done with Pye-ton How, 2010, JSTAT)

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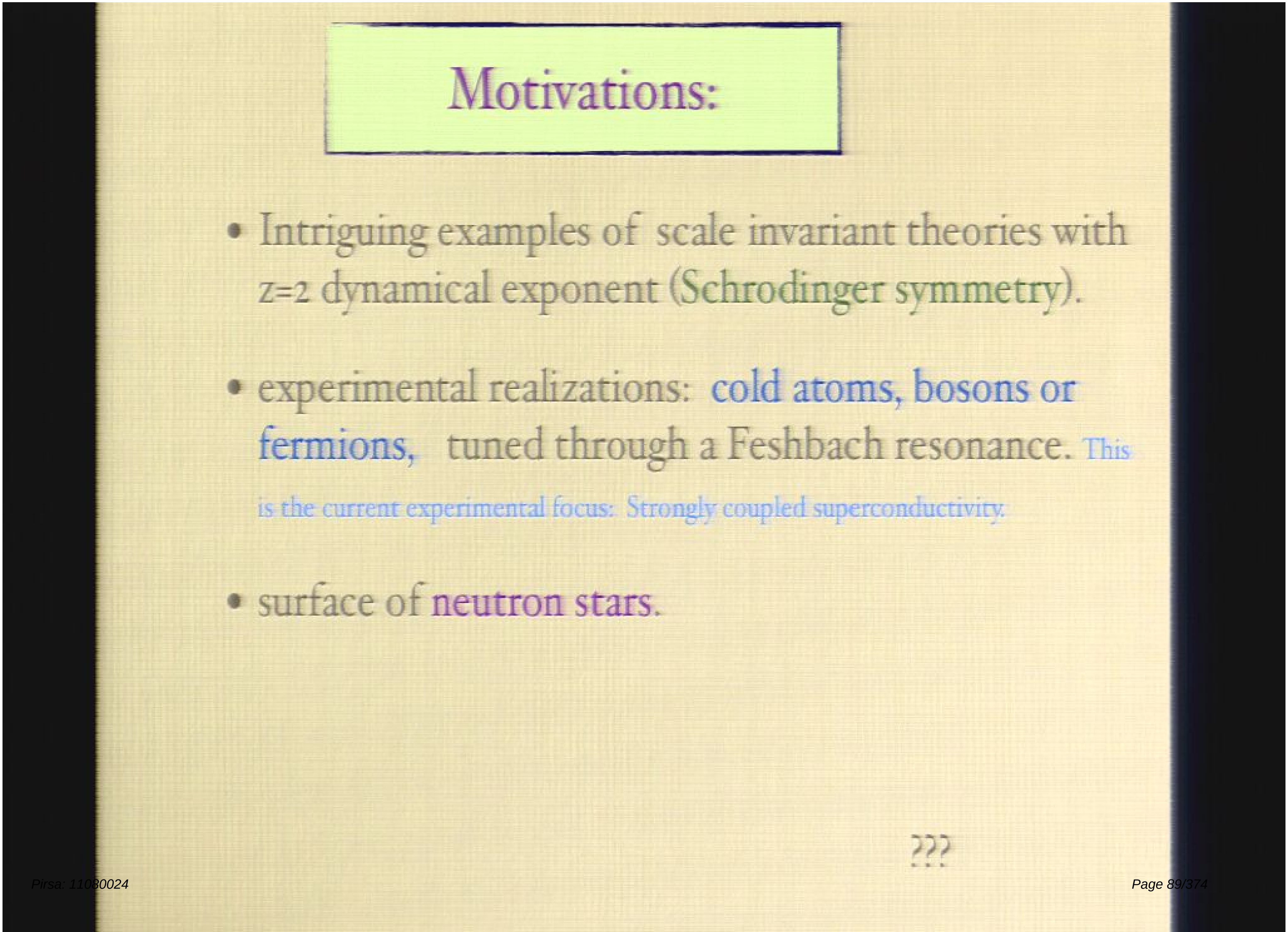
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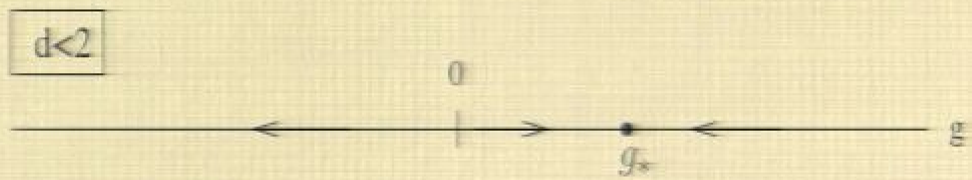


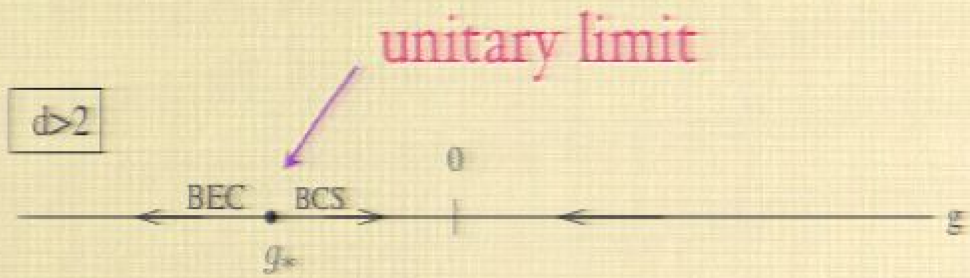


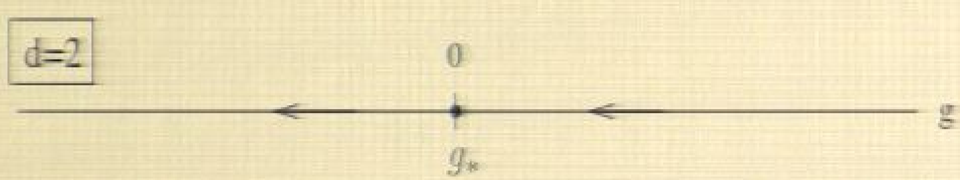

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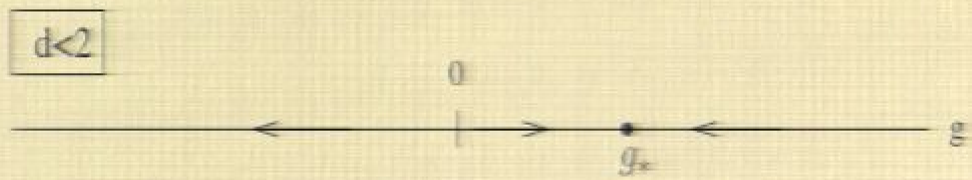


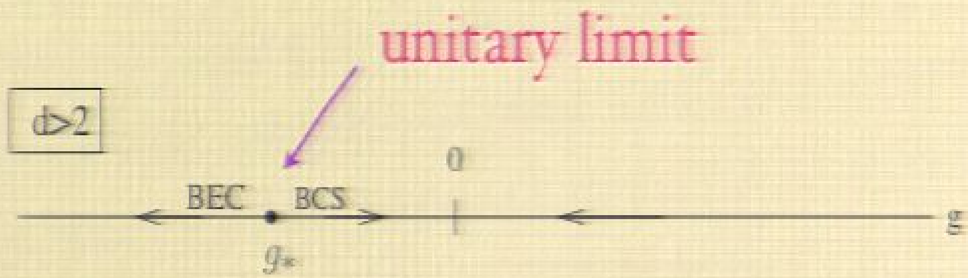


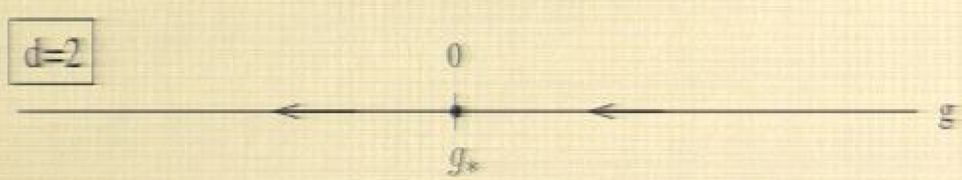

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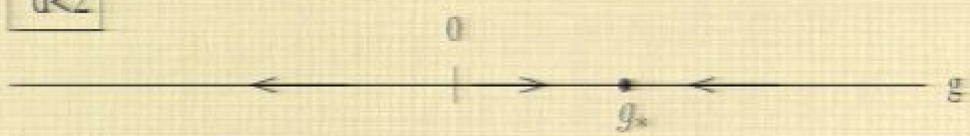
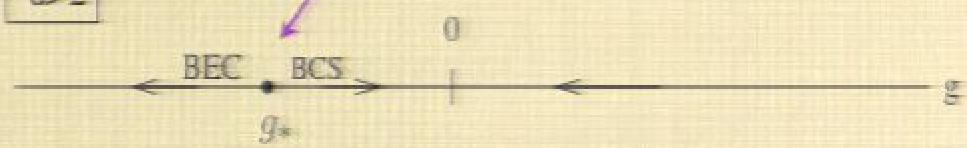
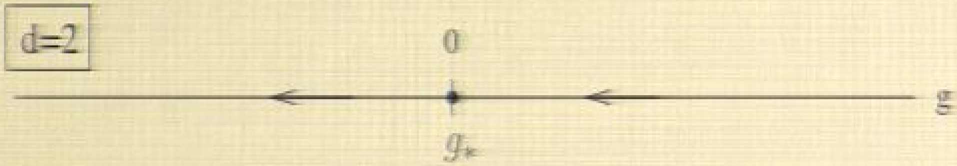



unitary limit

d>2




d=2



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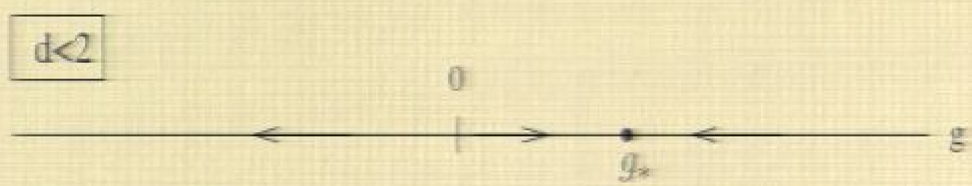


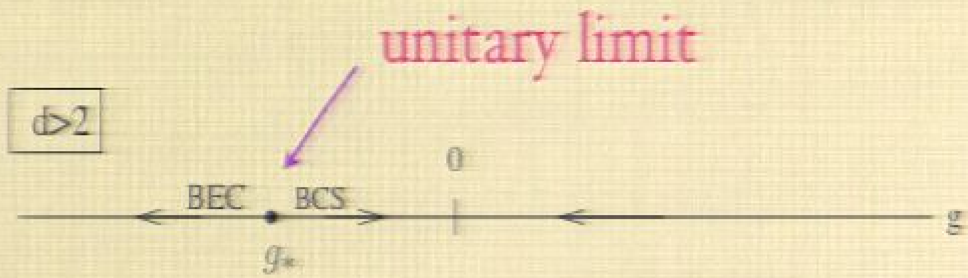


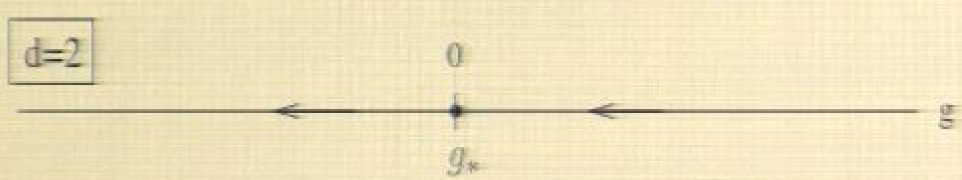

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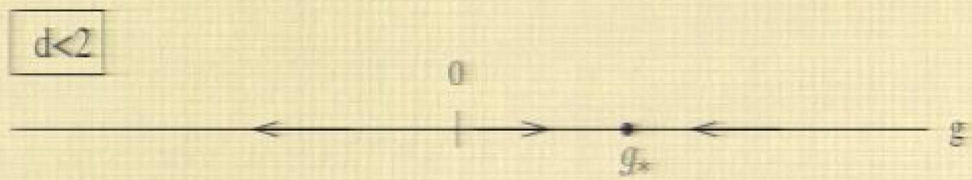


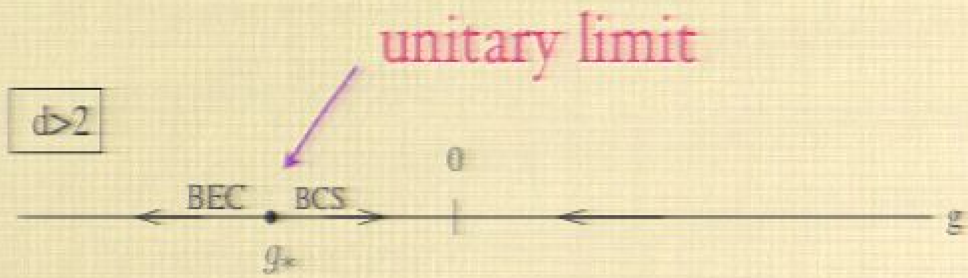


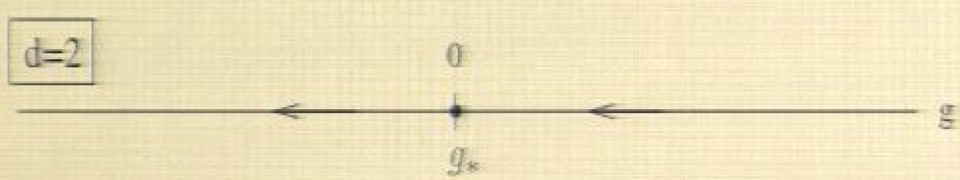

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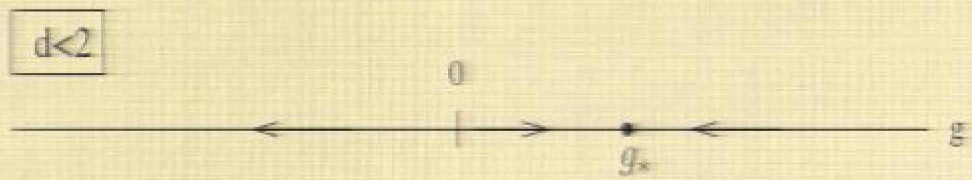


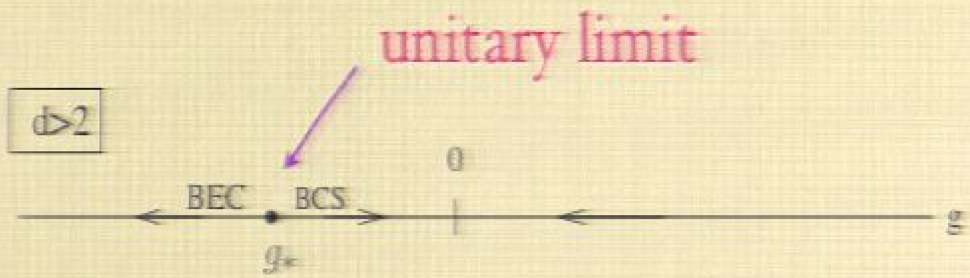


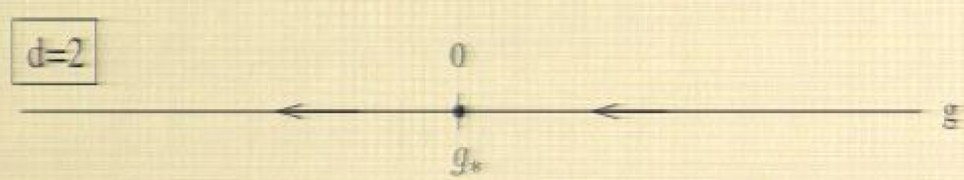

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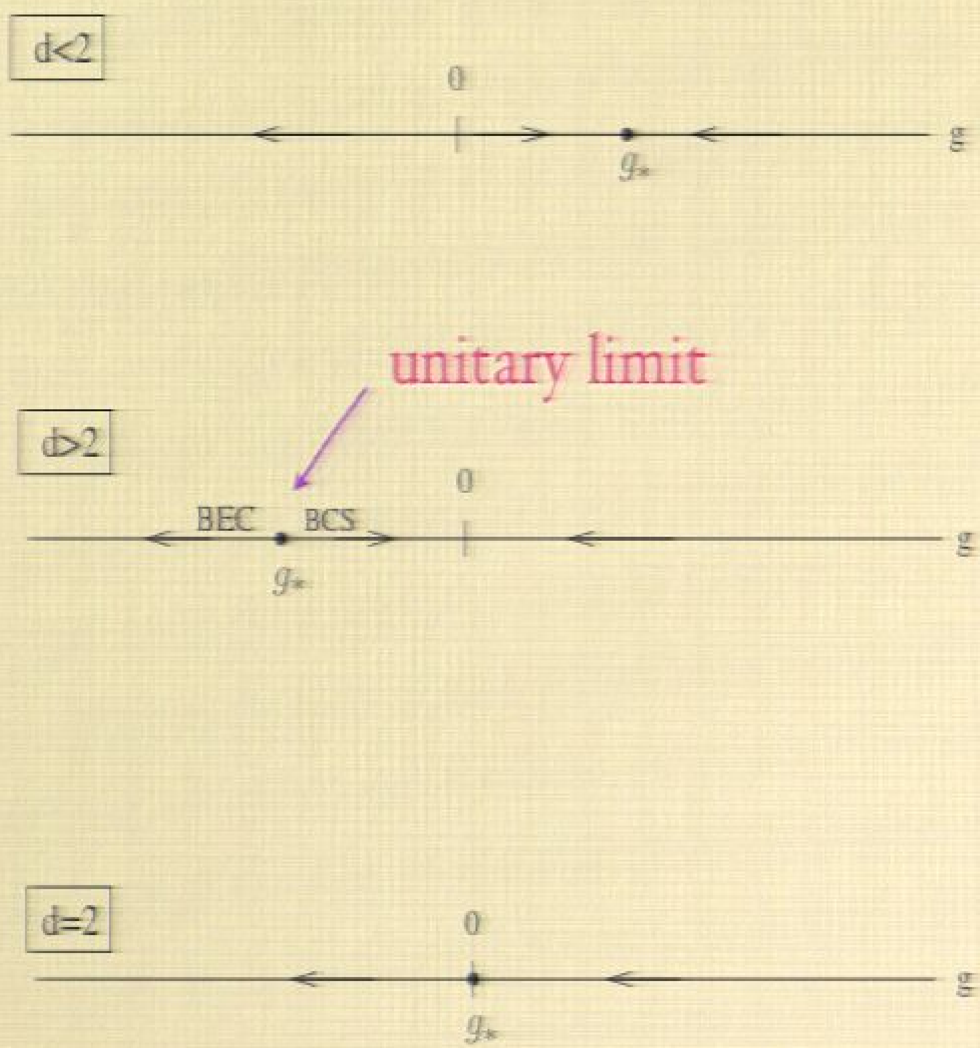

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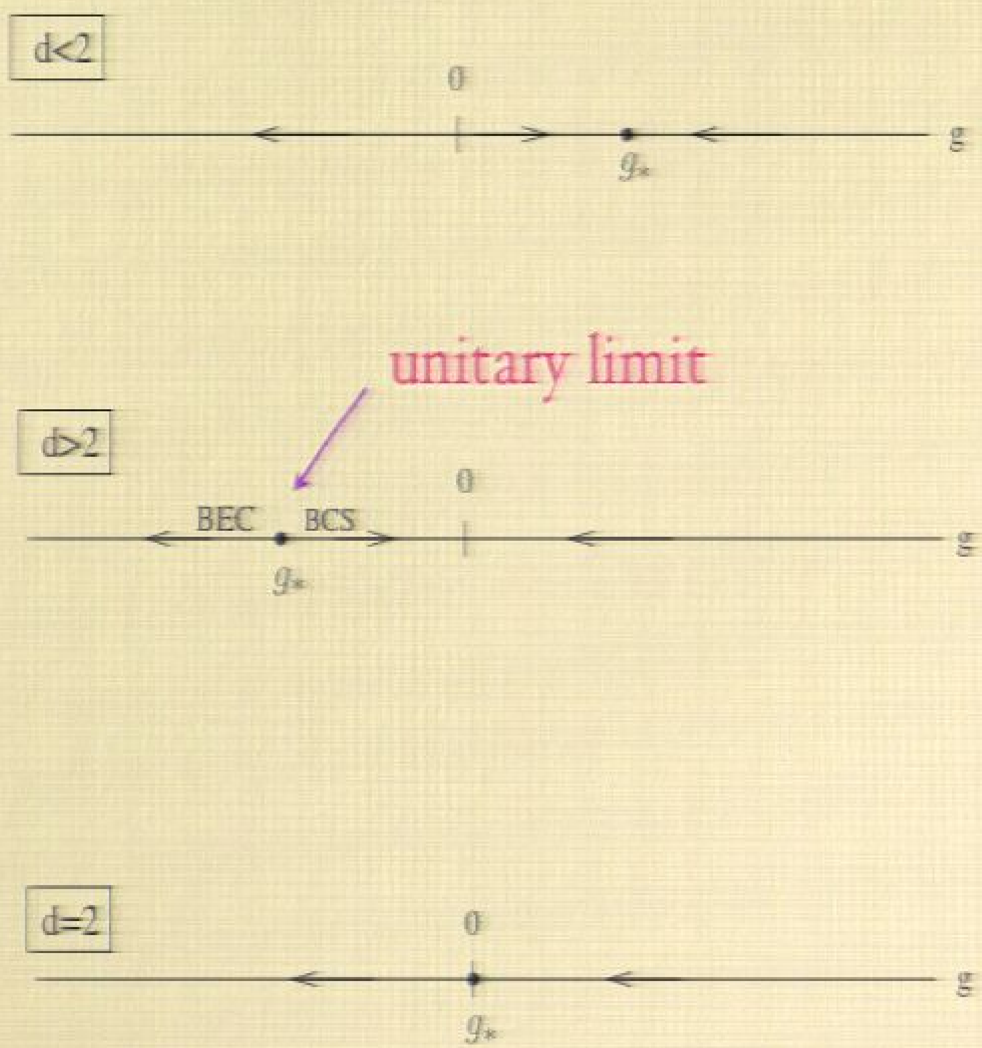

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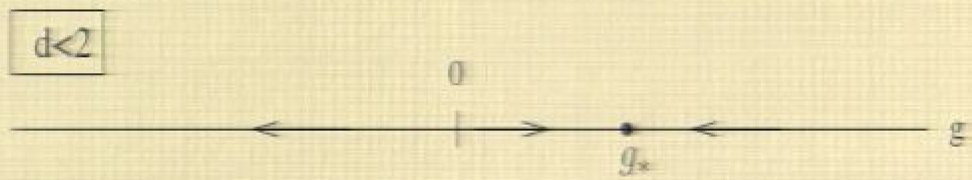


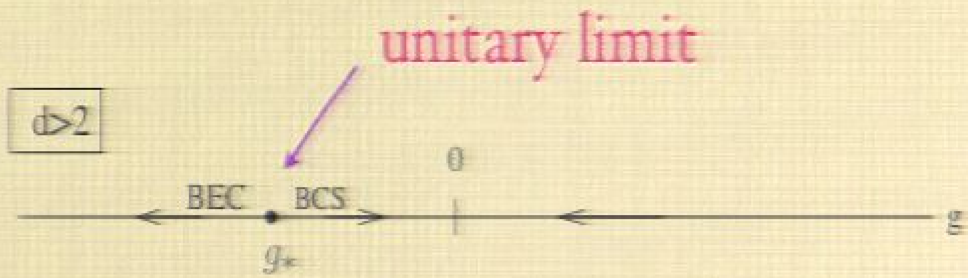


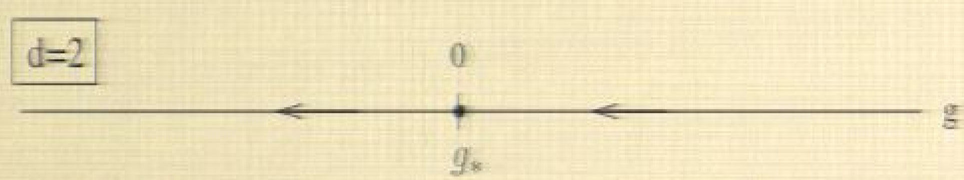

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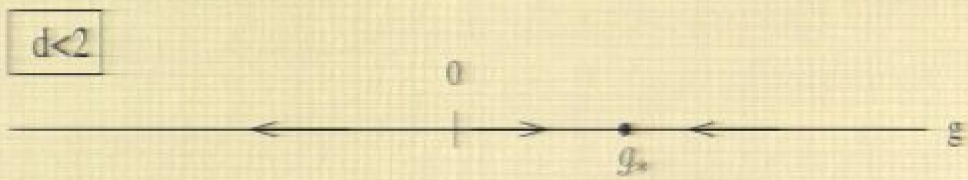


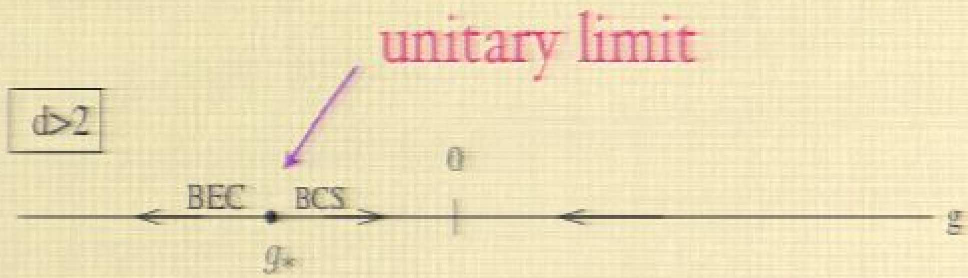


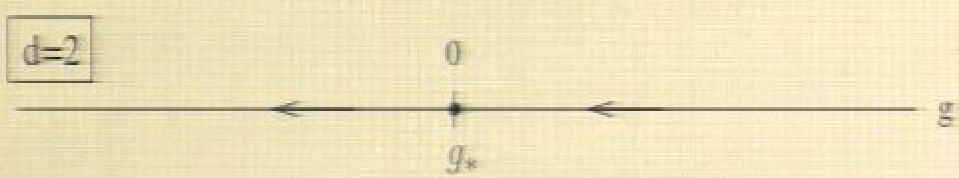

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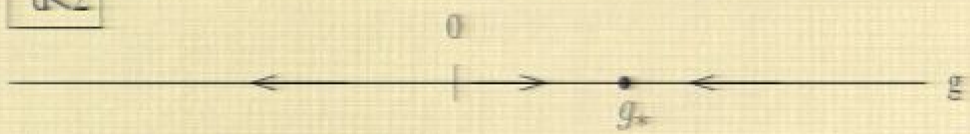
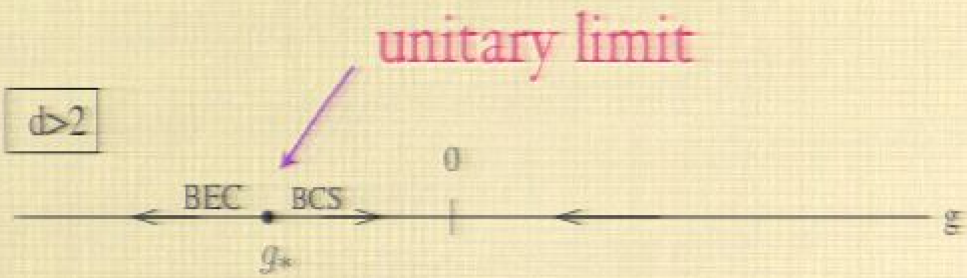
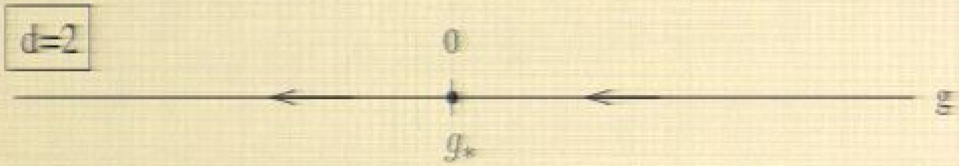



d>2




d=2



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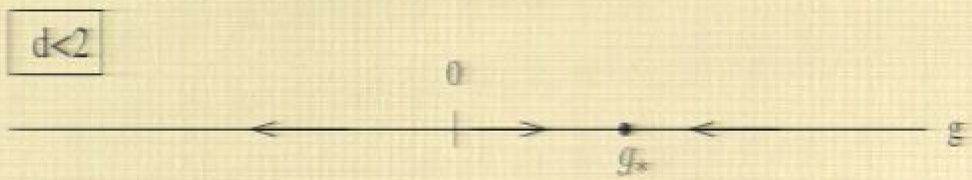


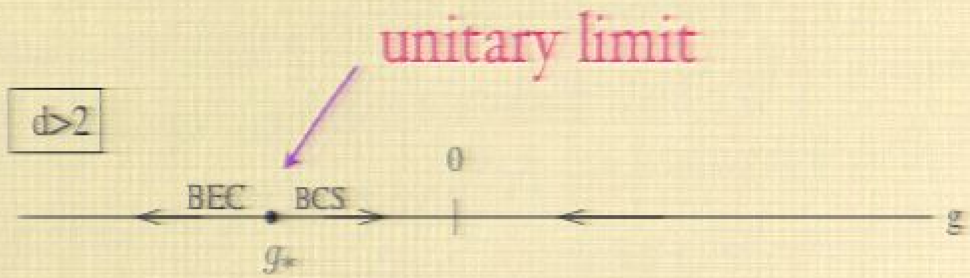


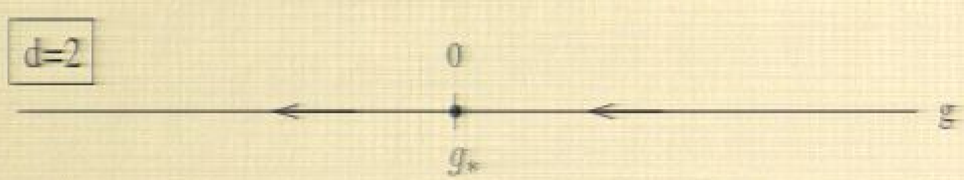

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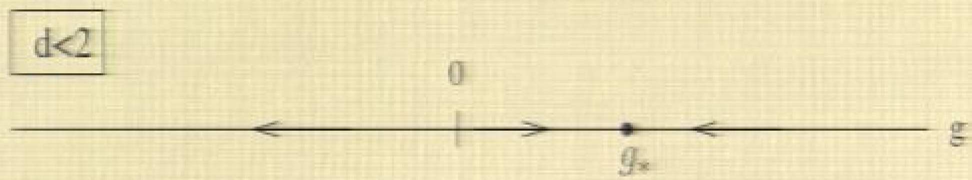


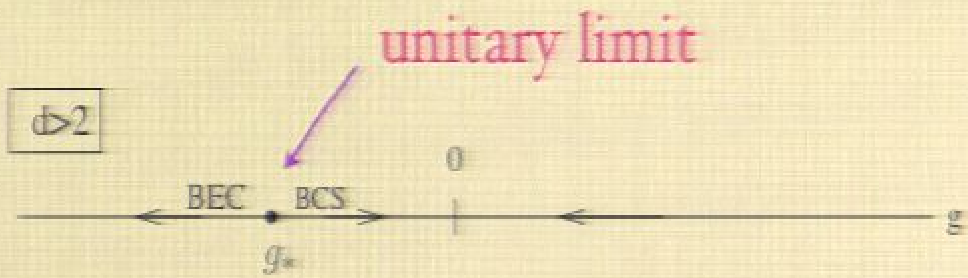


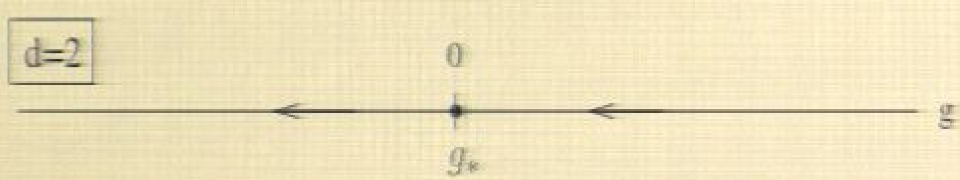

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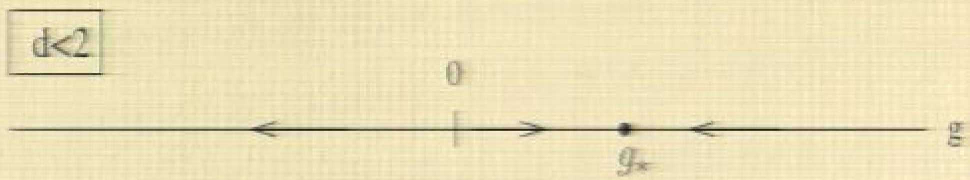


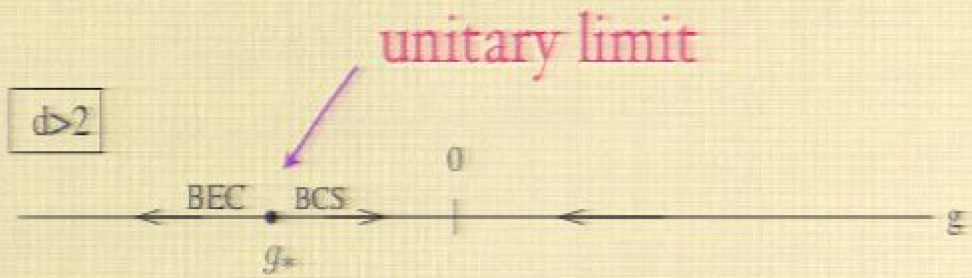


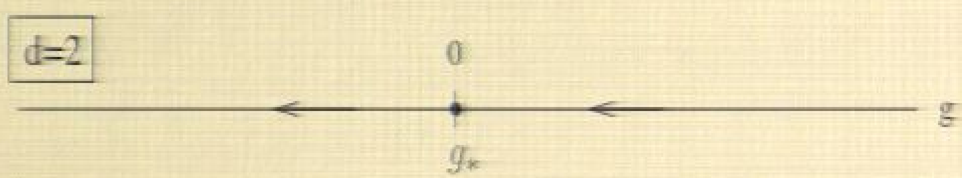

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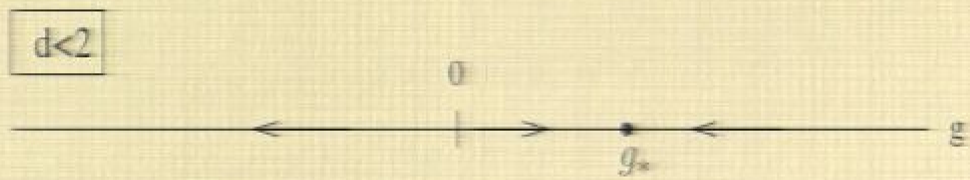


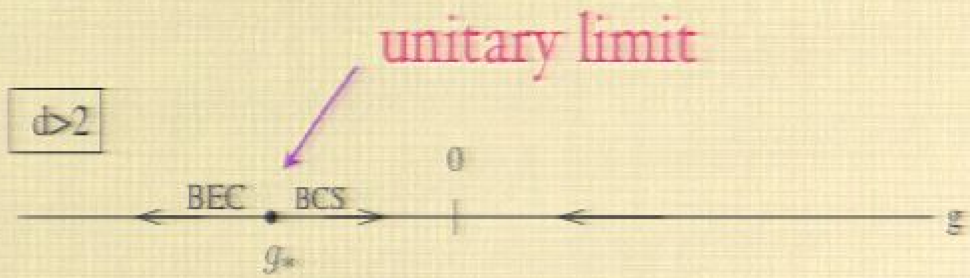


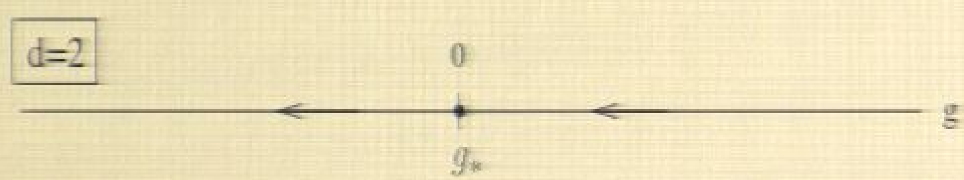

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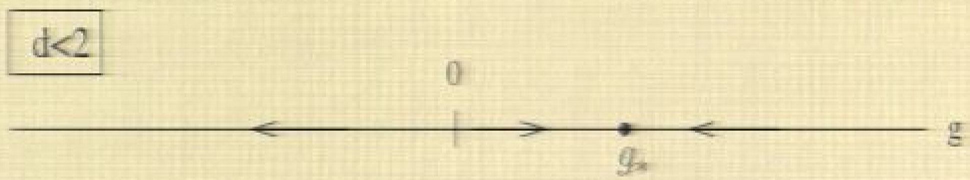


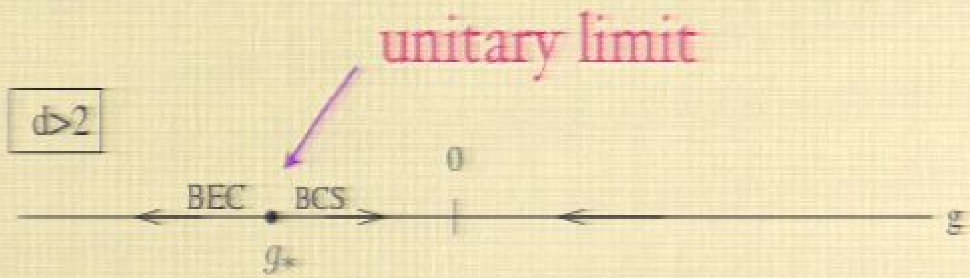


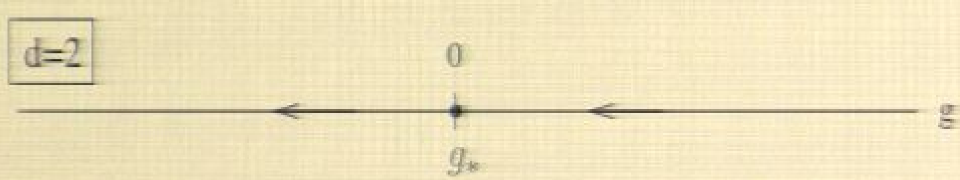

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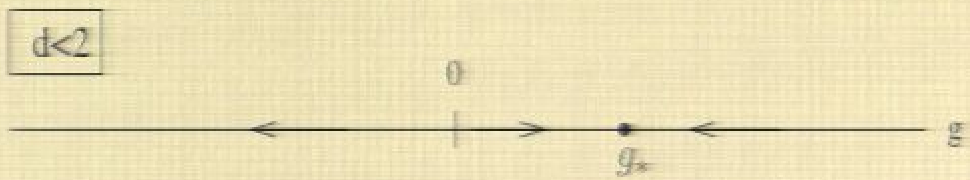


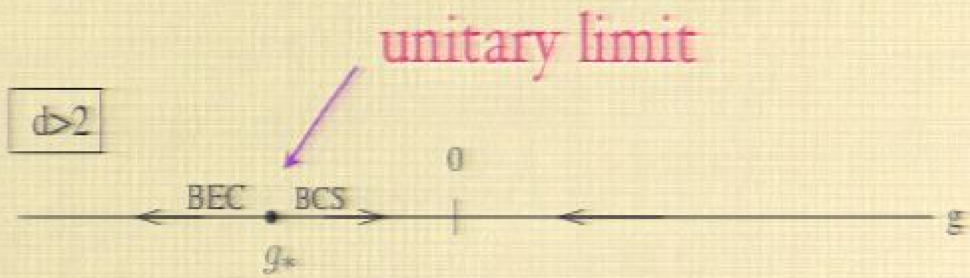


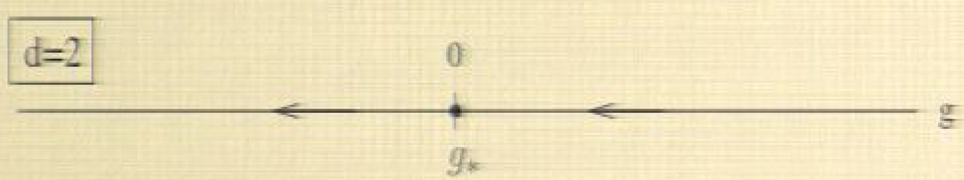

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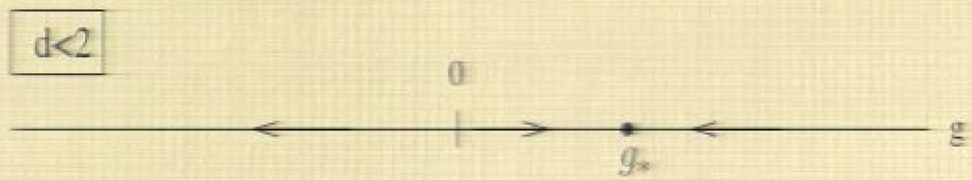


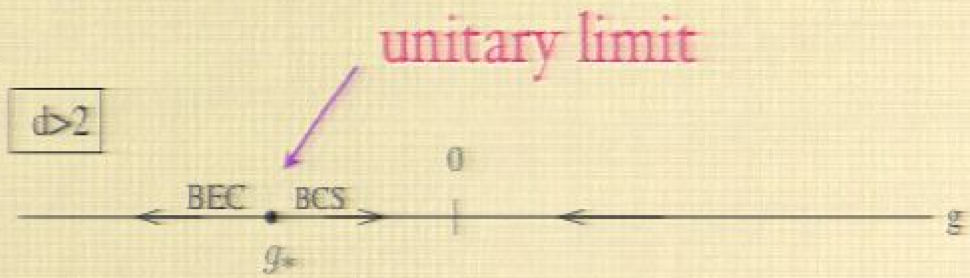


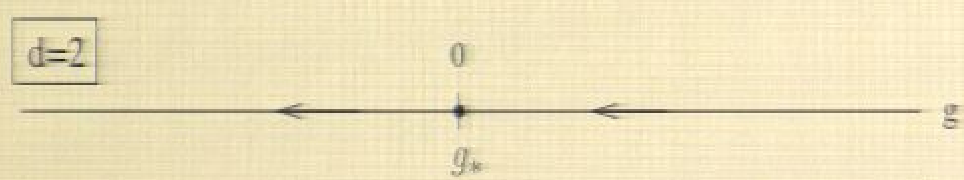

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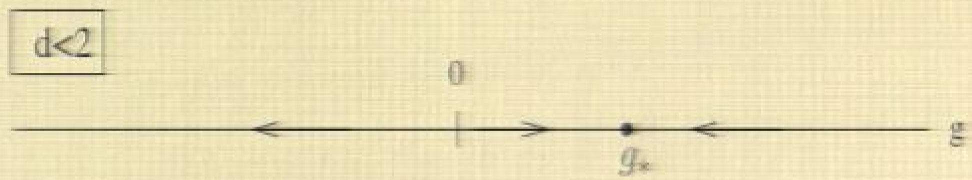


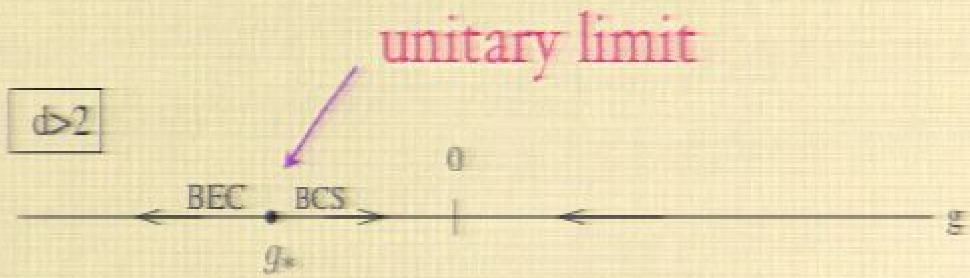


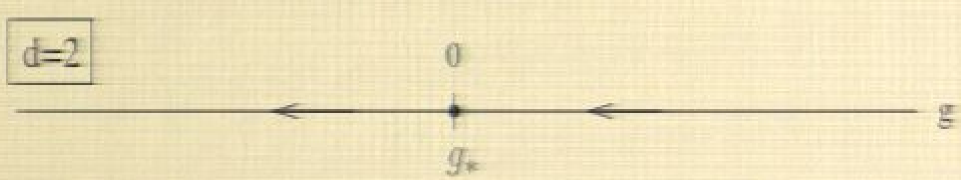

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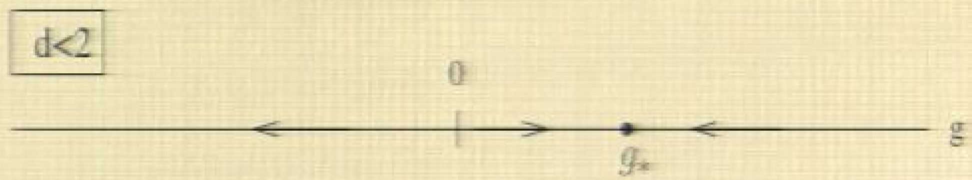


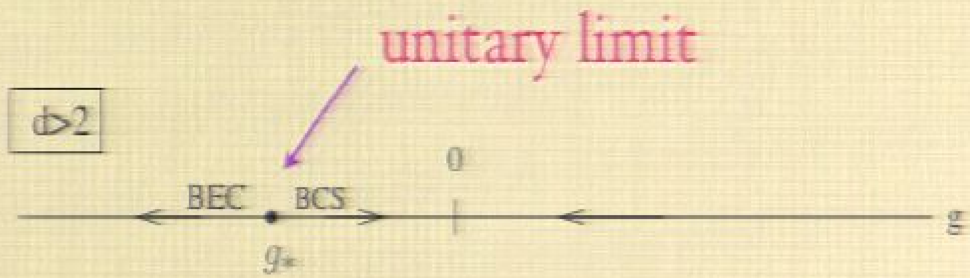


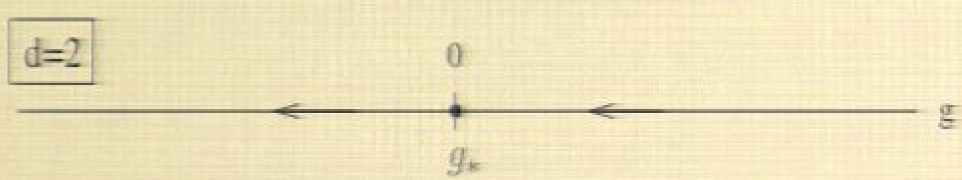

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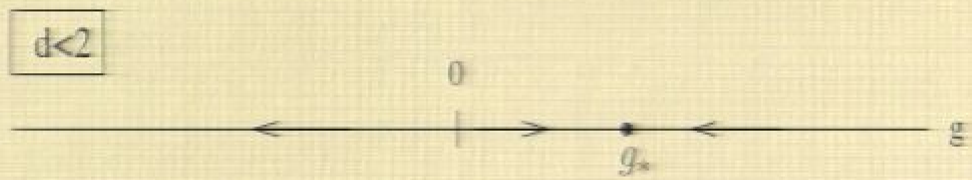


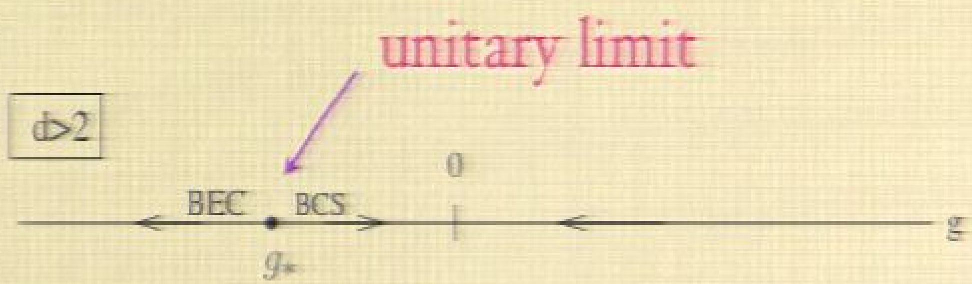


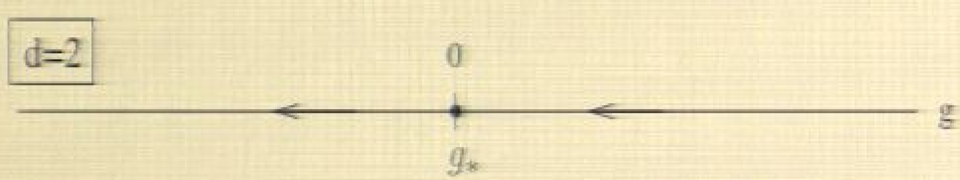

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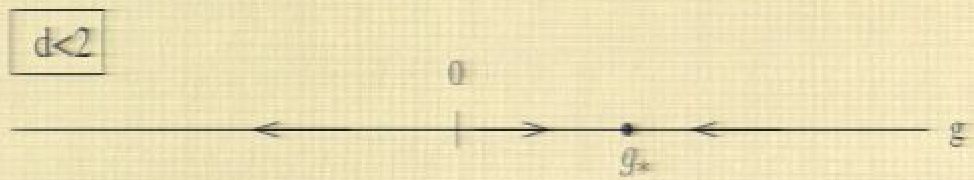


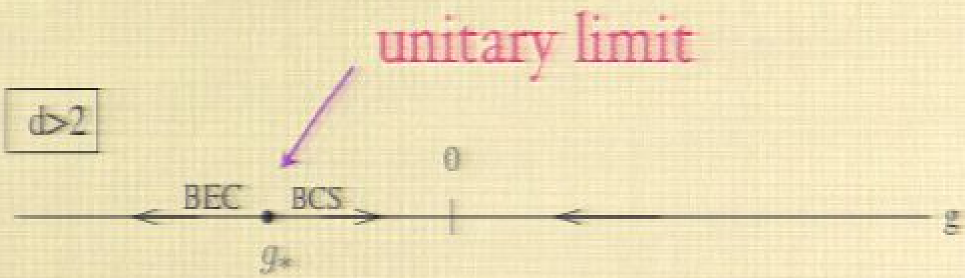


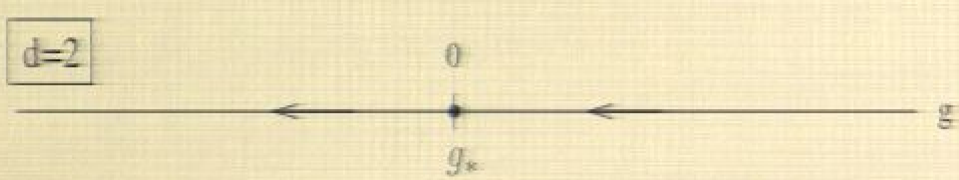

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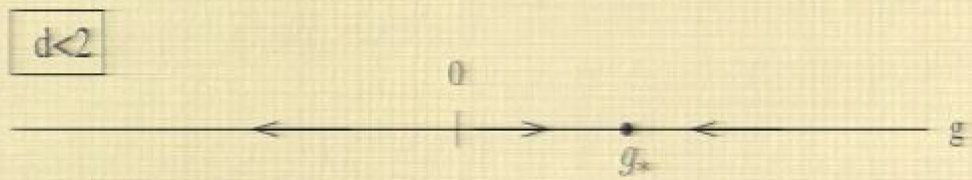


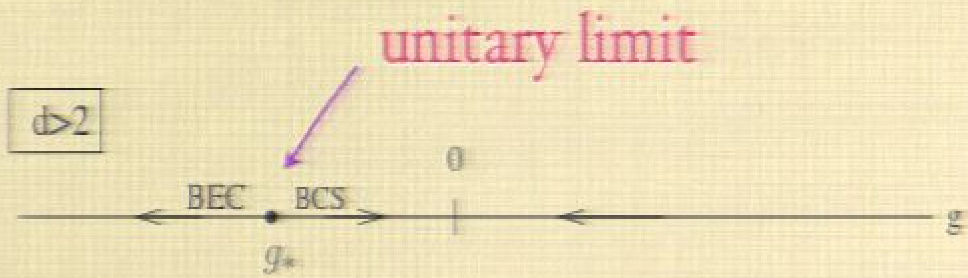


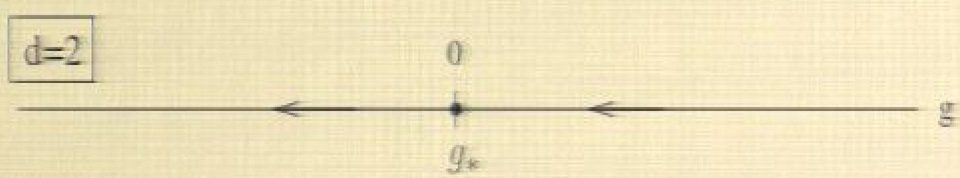

# Renormalization group:

## flows to low energy:

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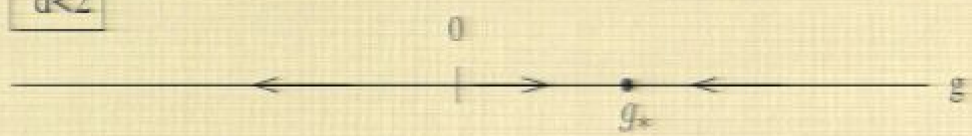
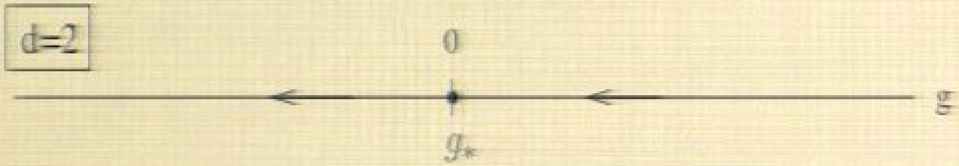



unitary limit

d>2


d=2



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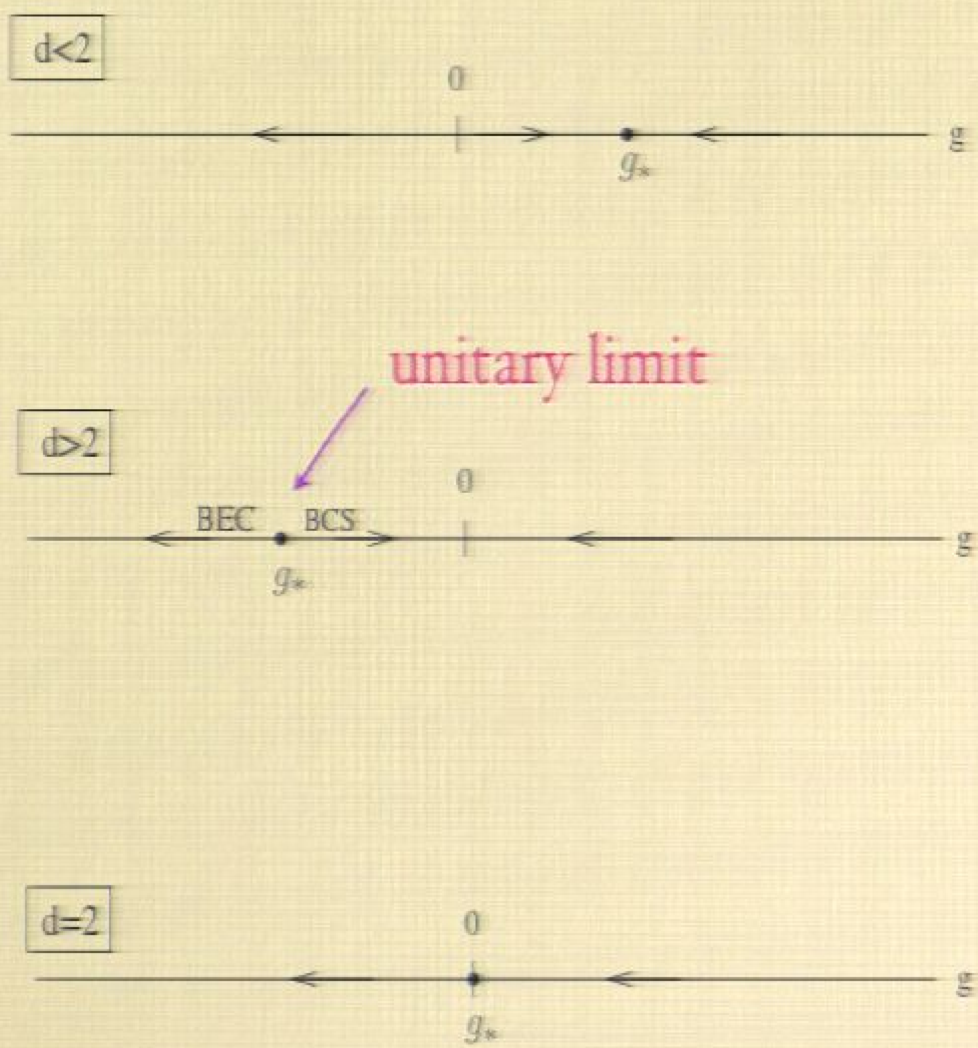

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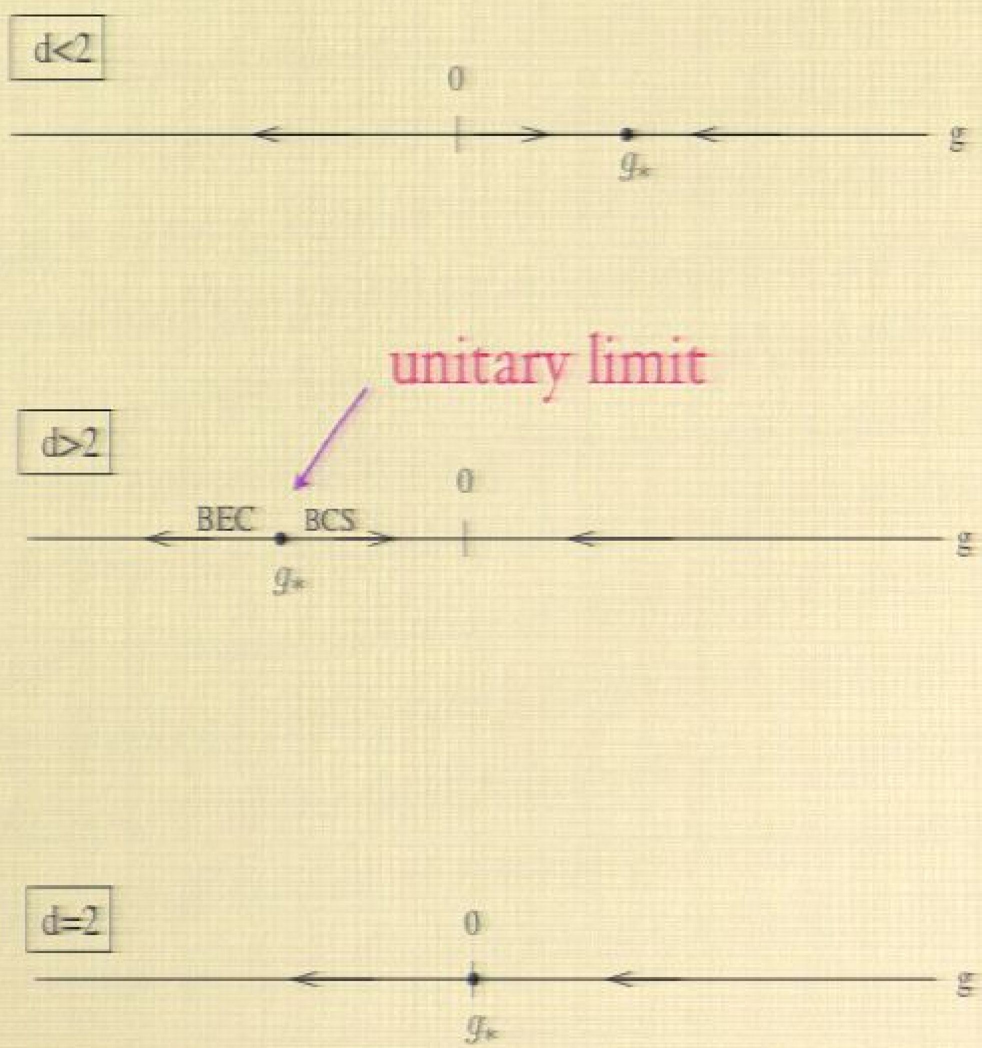

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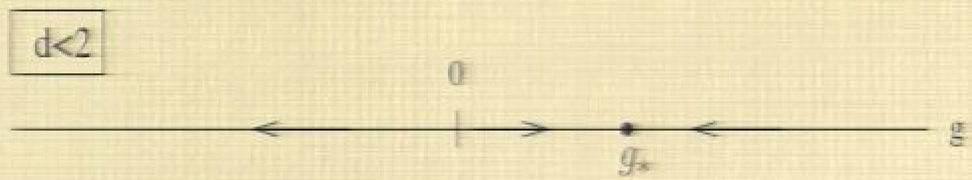


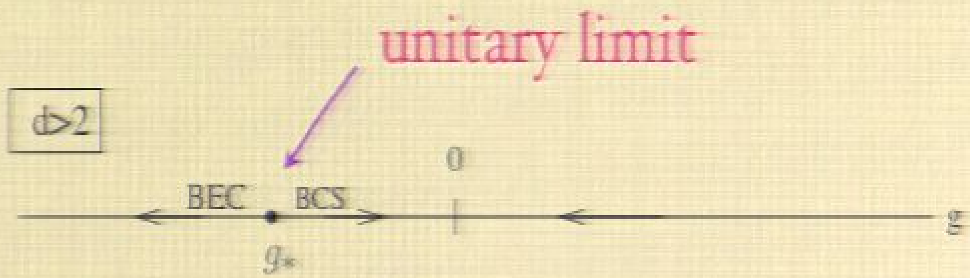


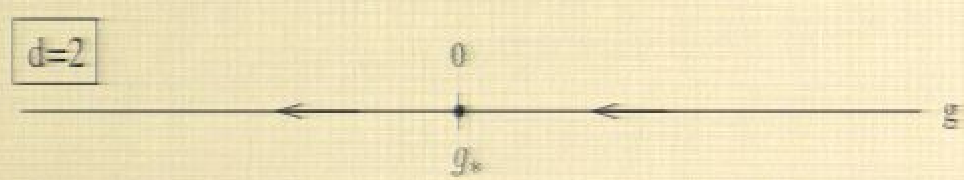

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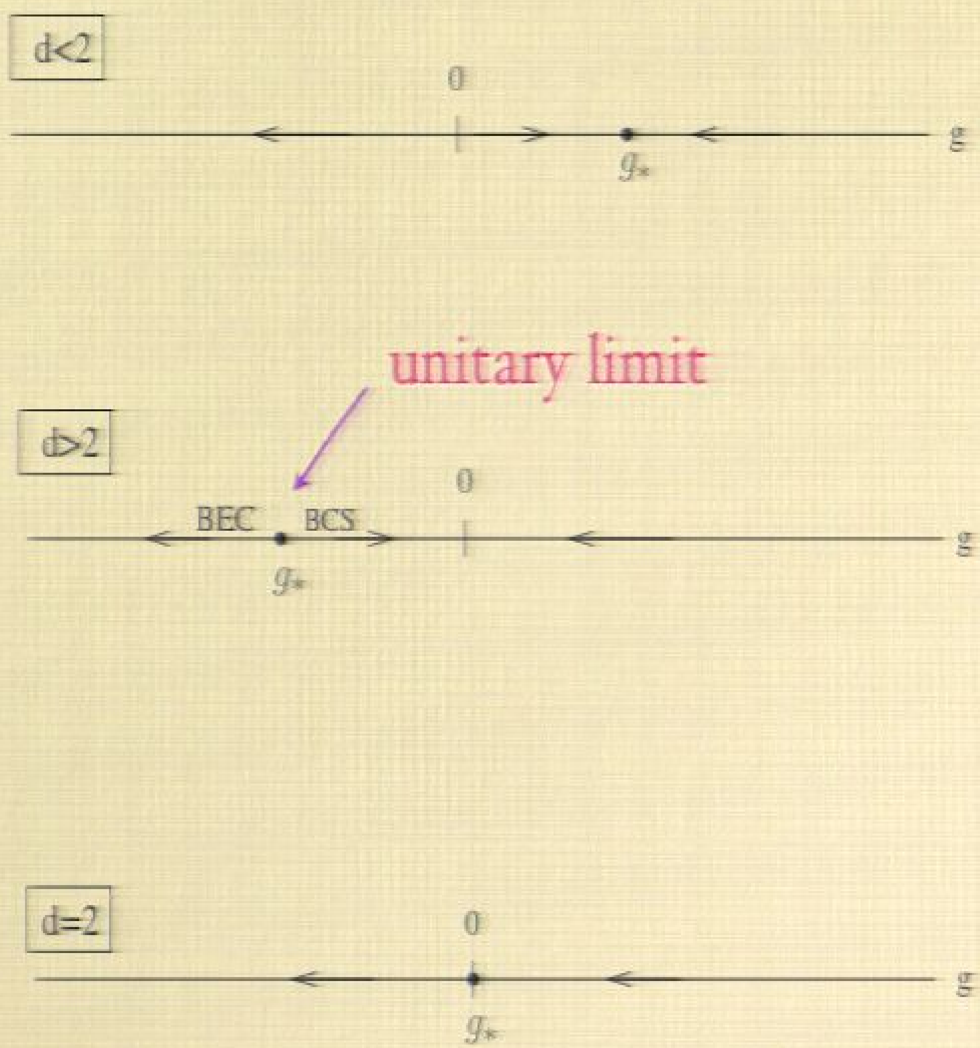

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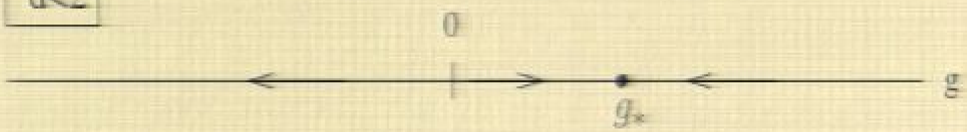
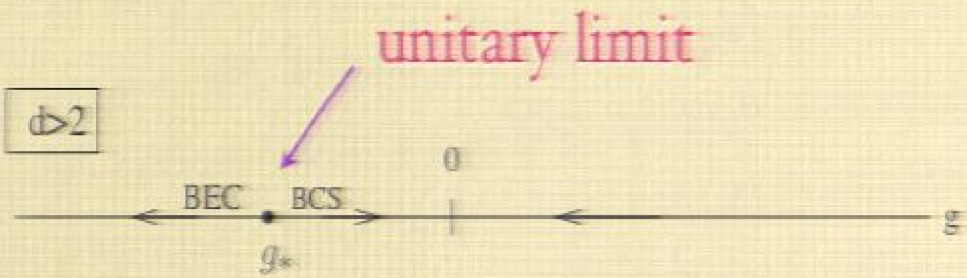
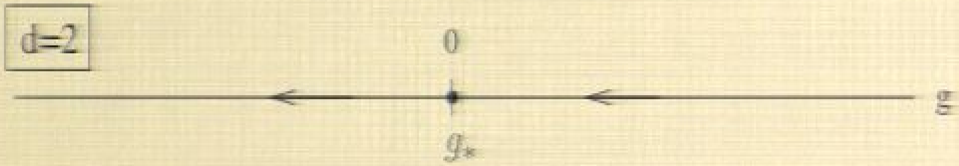



d>2




d=2



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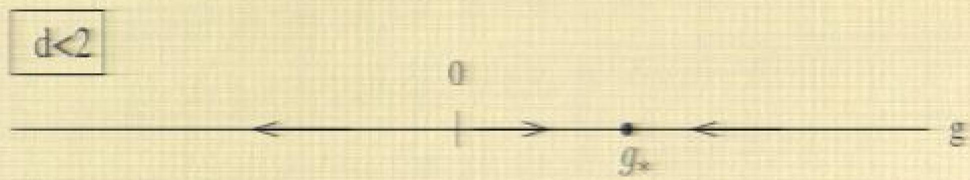


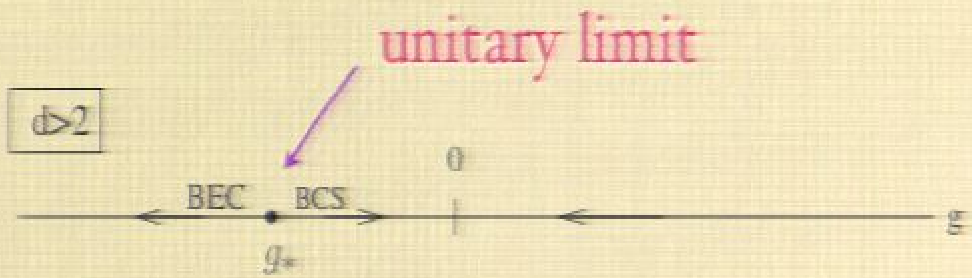


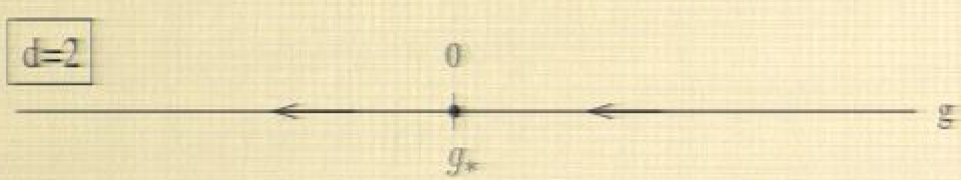

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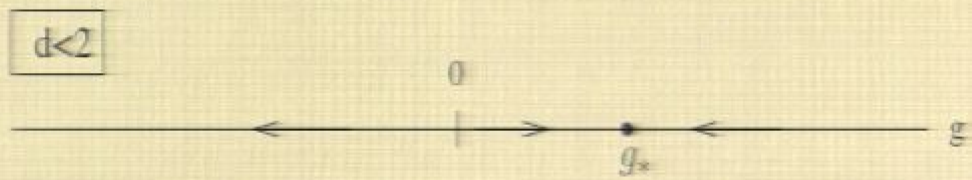


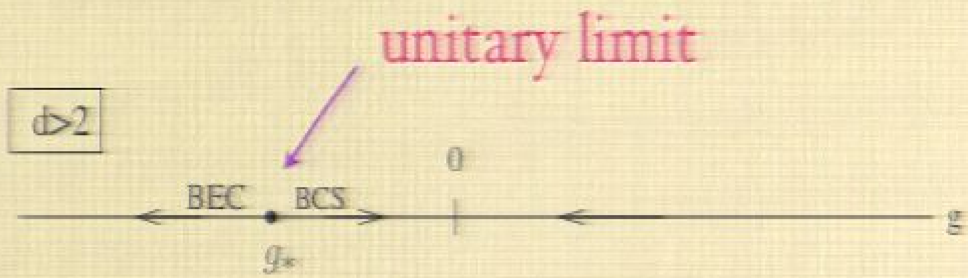


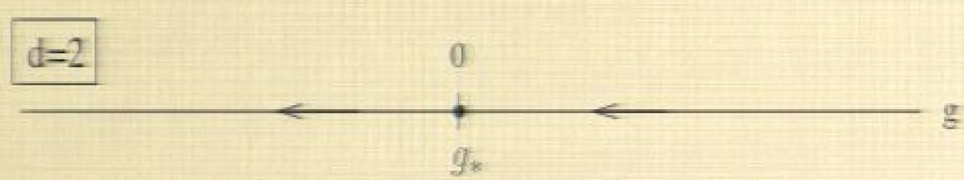

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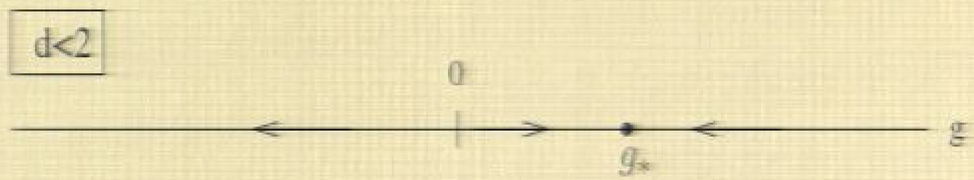


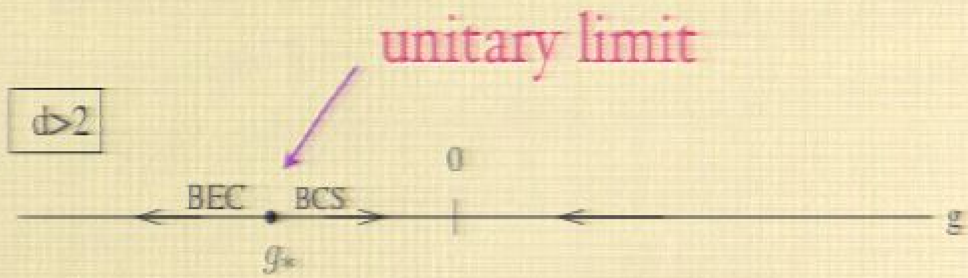


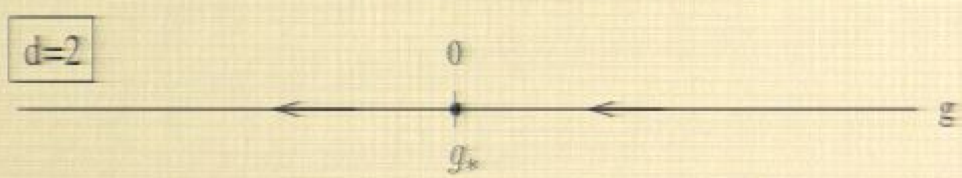

## d=3 case:

- at the fixed point, a=scattering length diverges.
- z=2 scale invariant theory. Only energy scales are the chemical potential and temperature.
- On BEC side, the 2-fermion bound state can condense.
- BCS side well described by BCS theory at small coupling. (no bound state on this side.)
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$$g \to \infty$$

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Turns out to be a free fermion. Difficult to see perturbatively, but clear from the TBA.

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density: $$n = -\partial_\mu \mathcal{F} = \int \frac{d^d\mathbf{k}}{(2\pi)^d} f(\mathbf{k})$$

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$$Z = Z_0 + \frac{1}{2\pi}\int dE\, \mathrm{e}^{-\beta E}\, \mathrm{Tr}\, \mathrm{Im}\, \partial_{\mathrm{E}} \log \widehat{\mathrm{S}}(\mathrm{E})$$

(Dashen, Ma, Bernstein, 1969)

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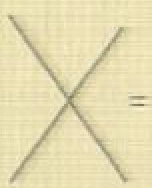
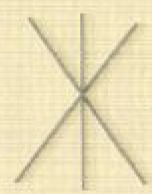


= <2 particle| log S | 2 particle>



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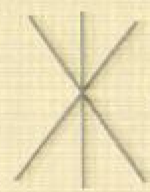

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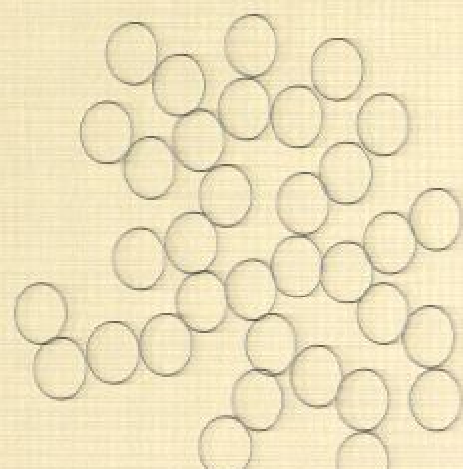
## binary approximation:

re-summation of “foam diagrams”



Final result (after much work).

Variational principle gives:

$$f(\mathbf{k}) = \frac{1}{e^{\beta\varepsilon(\mathbf{k})} + 1}$$

$$\varepsilon(\mathbf{k}) = \omega_{\mathbf{k}} - \mu - \int \frac{d^d\mathbf{k}}{(2\pi)^d} G(\mathbf{k}, \mathbf{k}') \frac{1}{e^{\beta\varepsilon(\mathbf{k}')} + 1}$$

pseudo-energy integral eqn

$$\mathcal{F} = -T \int \frac{d^2\mathbf{k}}{(2\pi)^2} \left[ \log(1 + e^{-\beta\varepsilon}) + \frac{\beta}{2} \frac{1}{e^{\beta\varepsilon} + 1} (\varepsilon - \omega + \mu) \right]$$

(different signs for bosons)

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$$2\pi\delta(E-\omega_{\mathbf{k}}-\omega_{\mathbf{k}'})\, V\, G(\mathbf{k},\mathbf{k}') = -i < \mathbf{k},\mathbf{k}'|\log\widehat{S}(E)|\mathbf{k},\mathbf{k}' >$$

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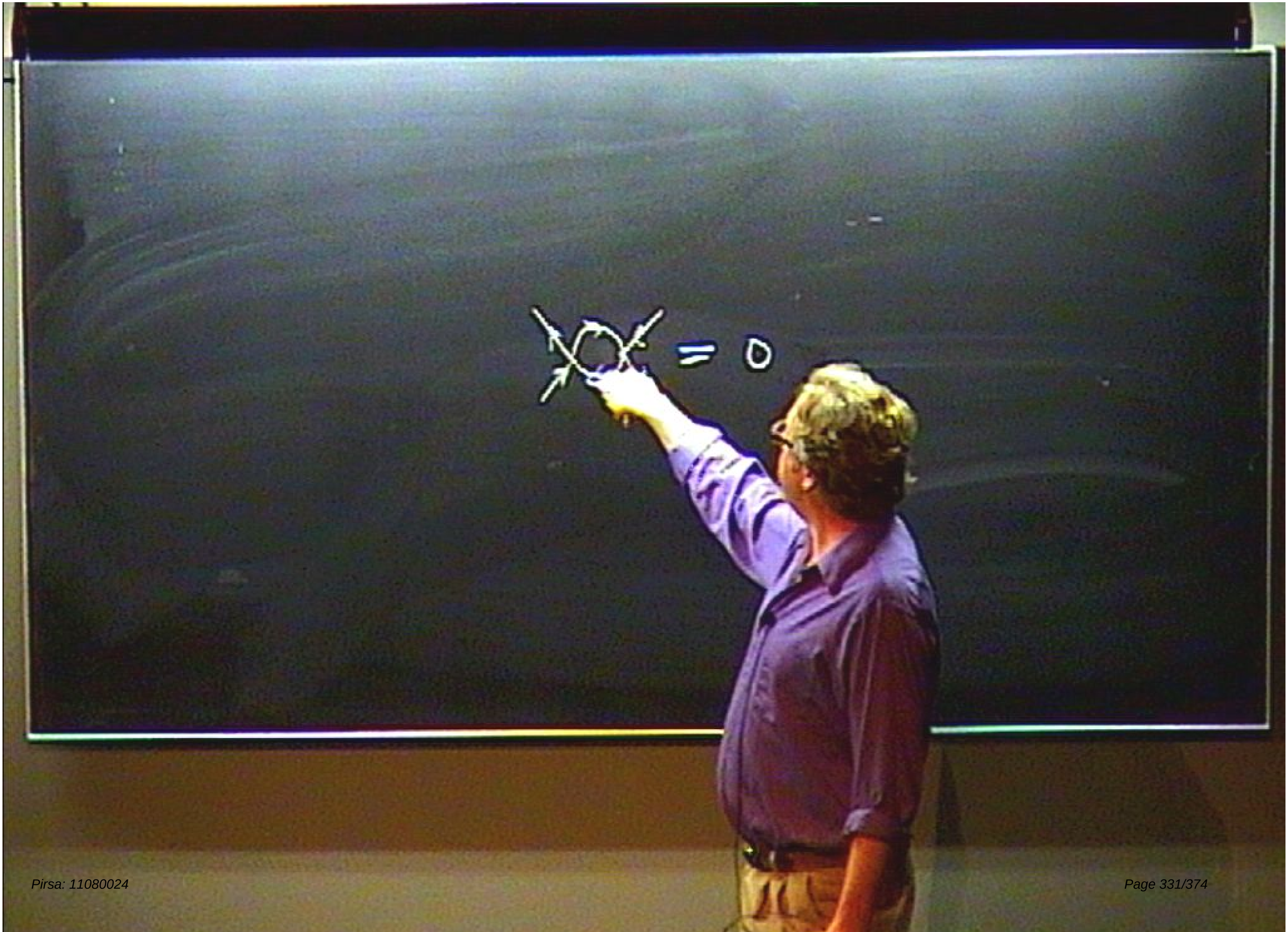
$$G(\mathbf{k},\mathbf{k}') = -i\frac{8\pi}{m|\mathbf{k}-\mathbf{k}'|}\log\left(\frac{1/g_R - im|\mathbf{k}-\mathbf{k}'|/16\pi}{1/g_R + im|\mathbf{k}-\mathbf{k}'|/16\pi}\right)$$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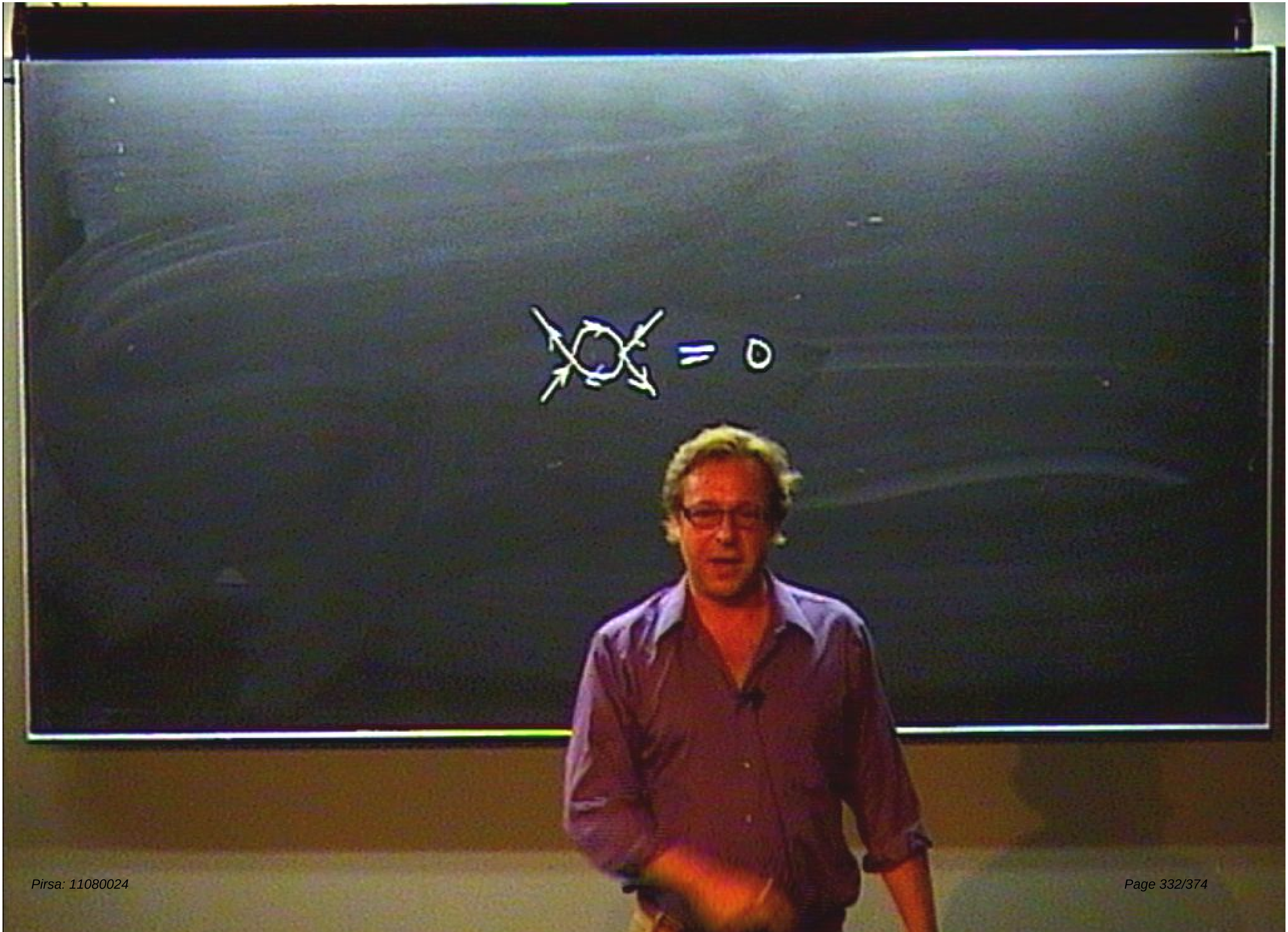
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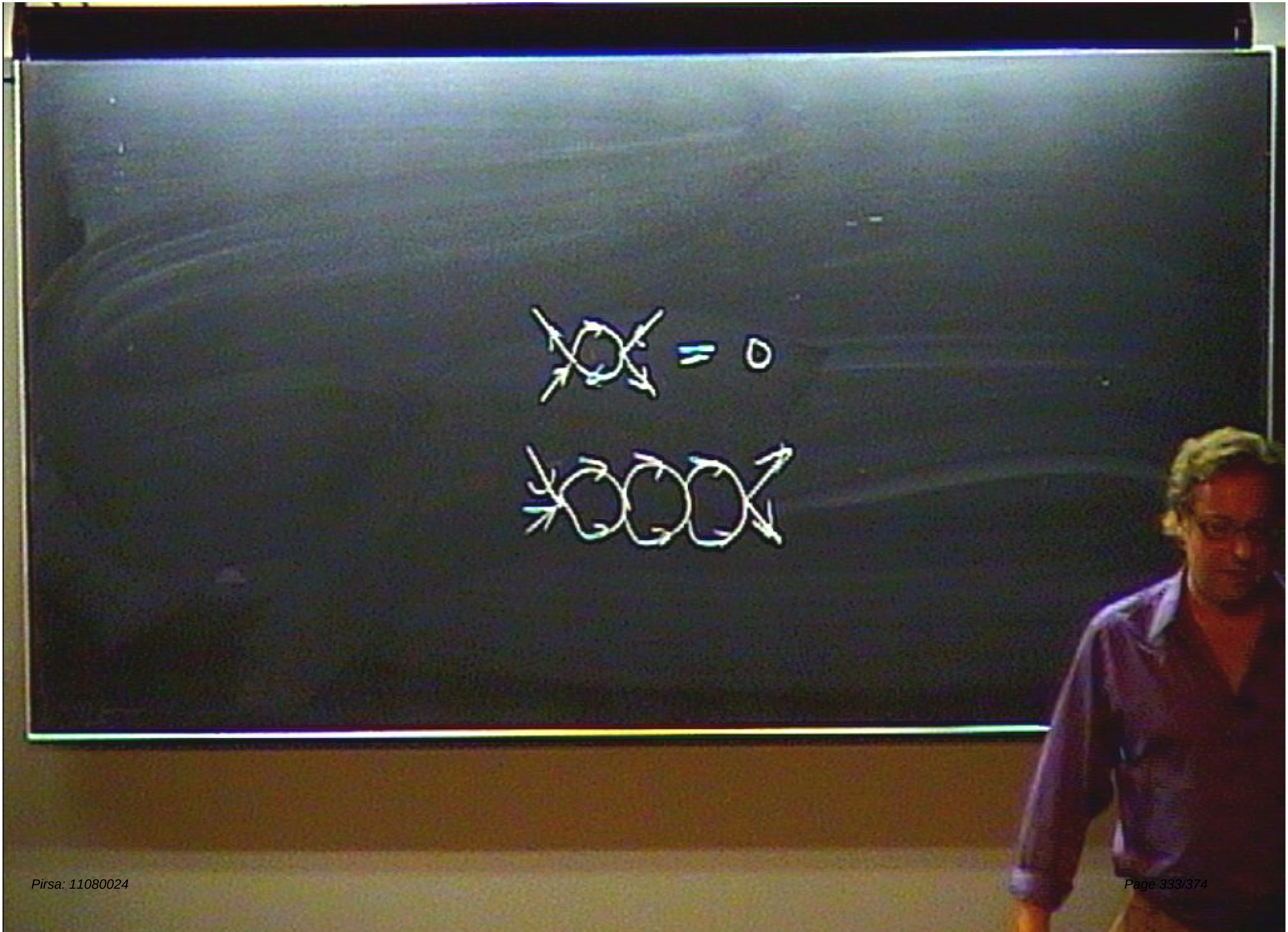
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# Application to 3d unitary gas

$$G(\mathbf{k}, \mathbf{k}') = -i \frac{8\pi}{m|\mathbf{k} - \mathbf{k}'|} \log \left( \frac{1/g_R - im|\mathbf{k} - \mathbf{k}'|/16\pi}{1/g_R + im|\mathbf{k} - \mathbf{k}'|/16\pi} \right)$$

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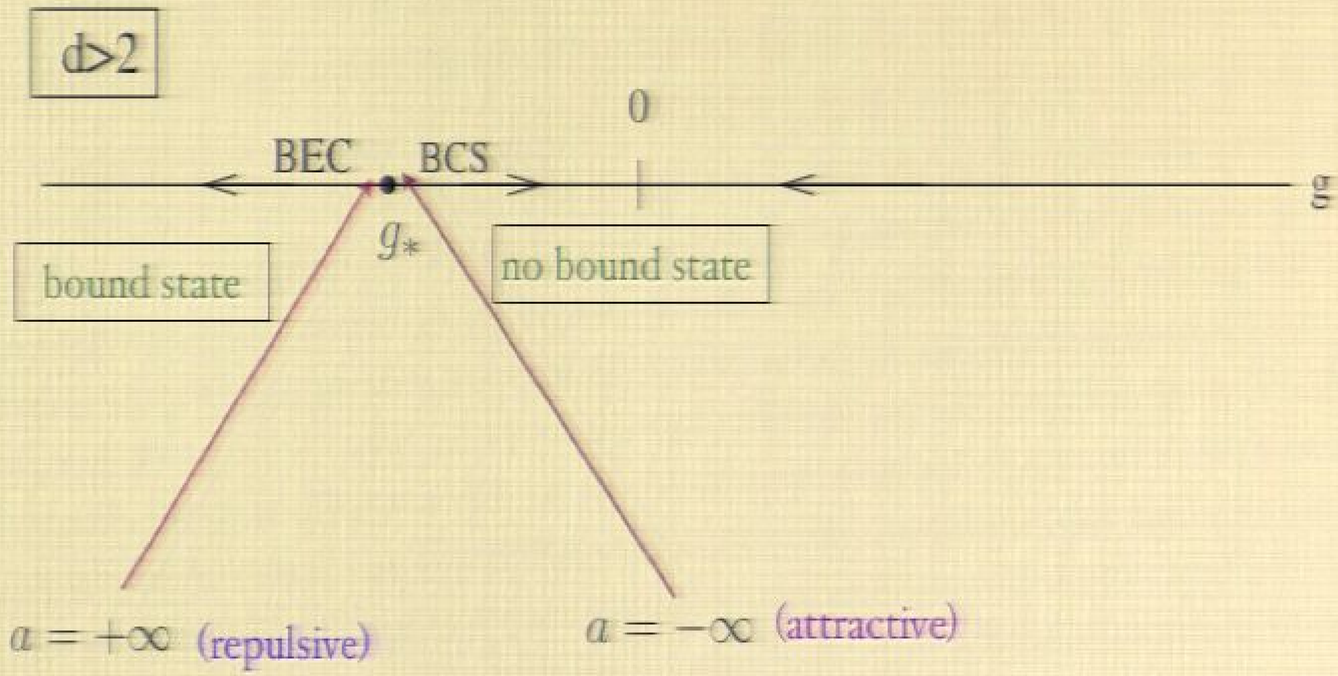
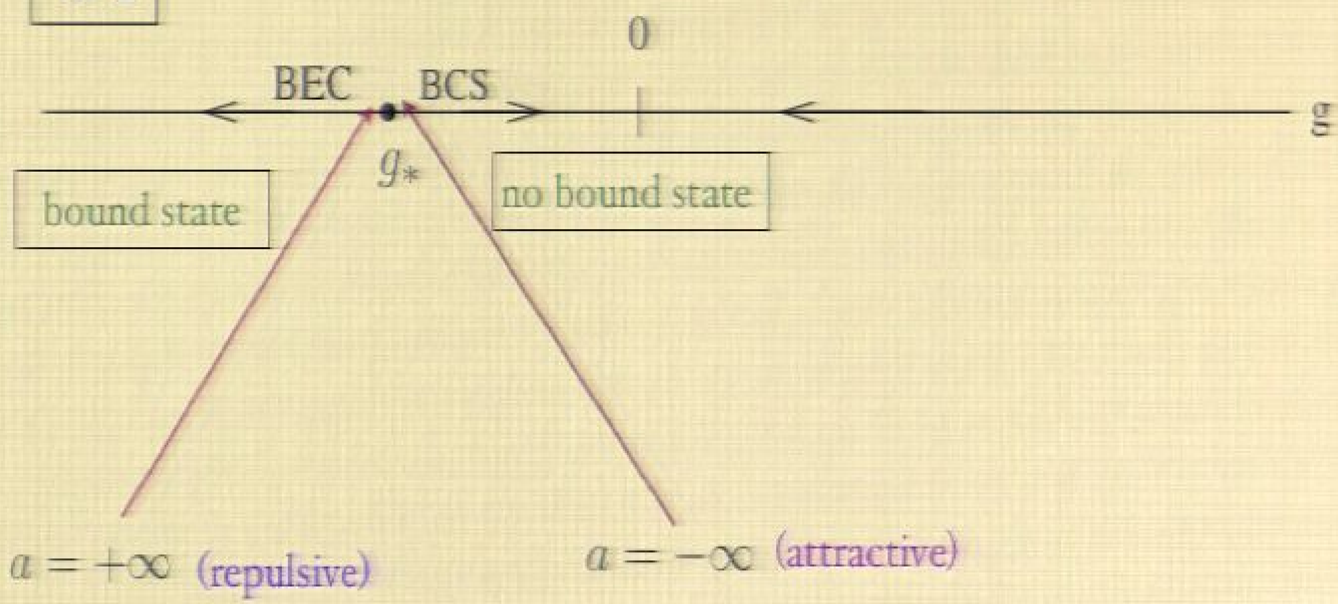

d>2
0
BEC
BCS
g
$g_*$
bound state
no bound state
$a = +\infty$ (repulsive)
$a = -\infty$ (attractive)

d>2




$$G \to \mp \frac{8\pi^2}{m|\mathbf{k} - \mathbf{k}'|}$$

-/+ corresponds to repulsive/attractive

(for small g , G = -g )

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Critical points must occur at fixed values of $\mu/T$.
These points can be expressed as:

$$n\lambda_{T_c}^d = \text{constant} \qquad \text{(bosons)} \qquad \lambda_T = \sqrt{2\pi/mT}.$$

$$T_c/T_F = \text{constant} \qquad \text{(fermions)}$$

# Fermions

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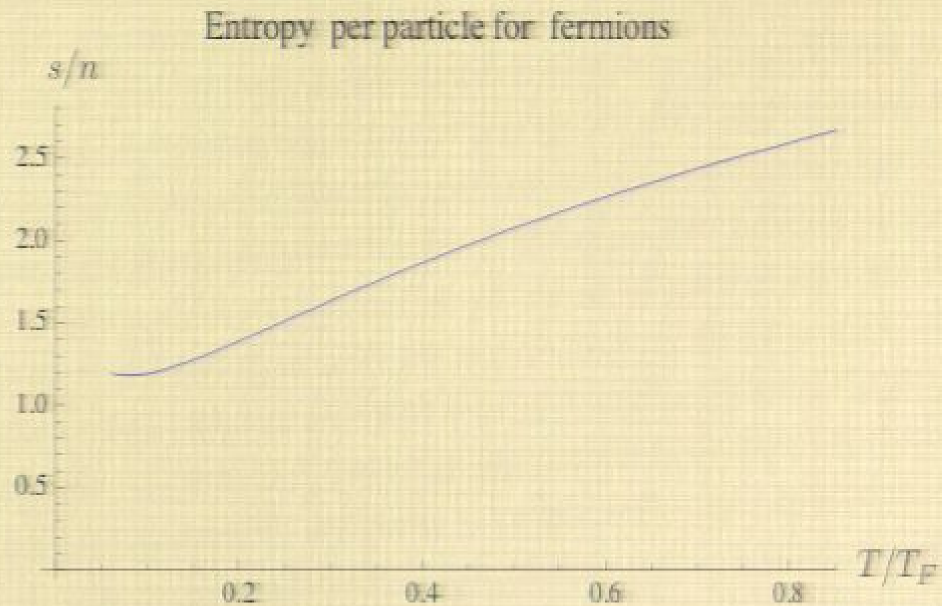
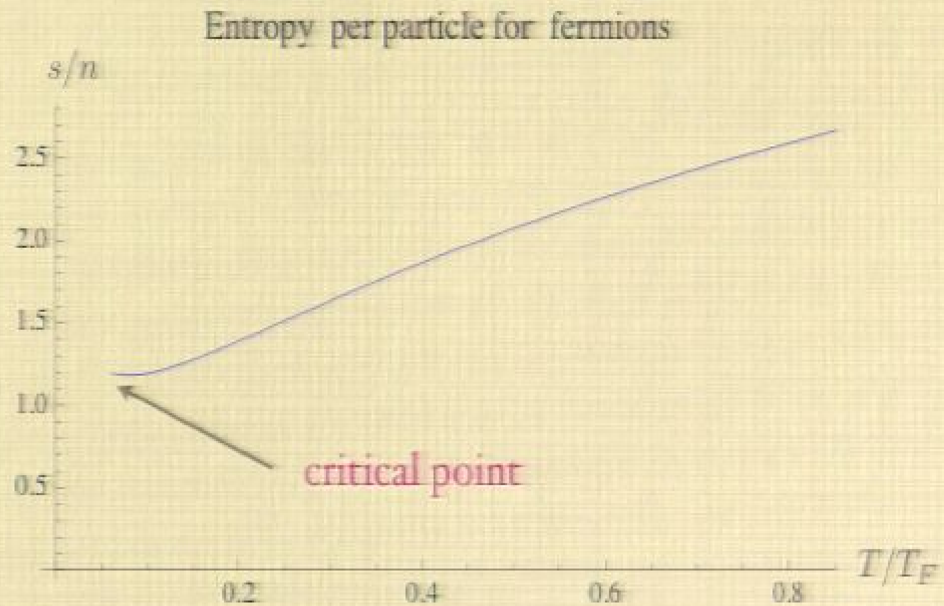

Entropy per particle for fermions
s/n
2.5
2.0
1.5
1.0
0.5
0.2
0.4
0.6
0.8
T/T_F

# Fermions

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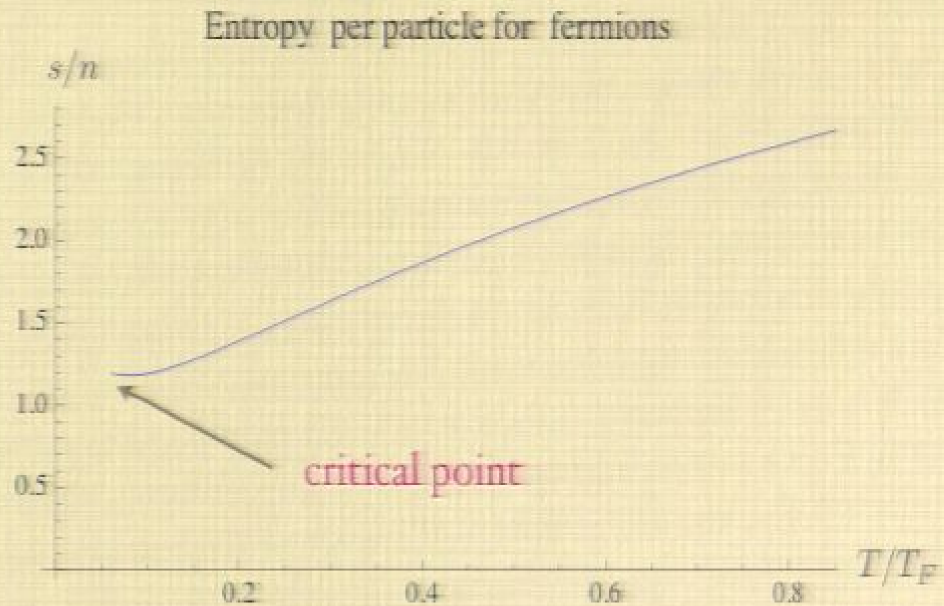


$$T_c/T_F \approx 0.1.$$

consistent with lattice Monte Carlo, where $T_c/T_F = 0.15$

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$$\boxed{\eta_{\text{boson}} = \frac{m^3 \bar{v}^3}{48\sqrt{2}\pi}}$$

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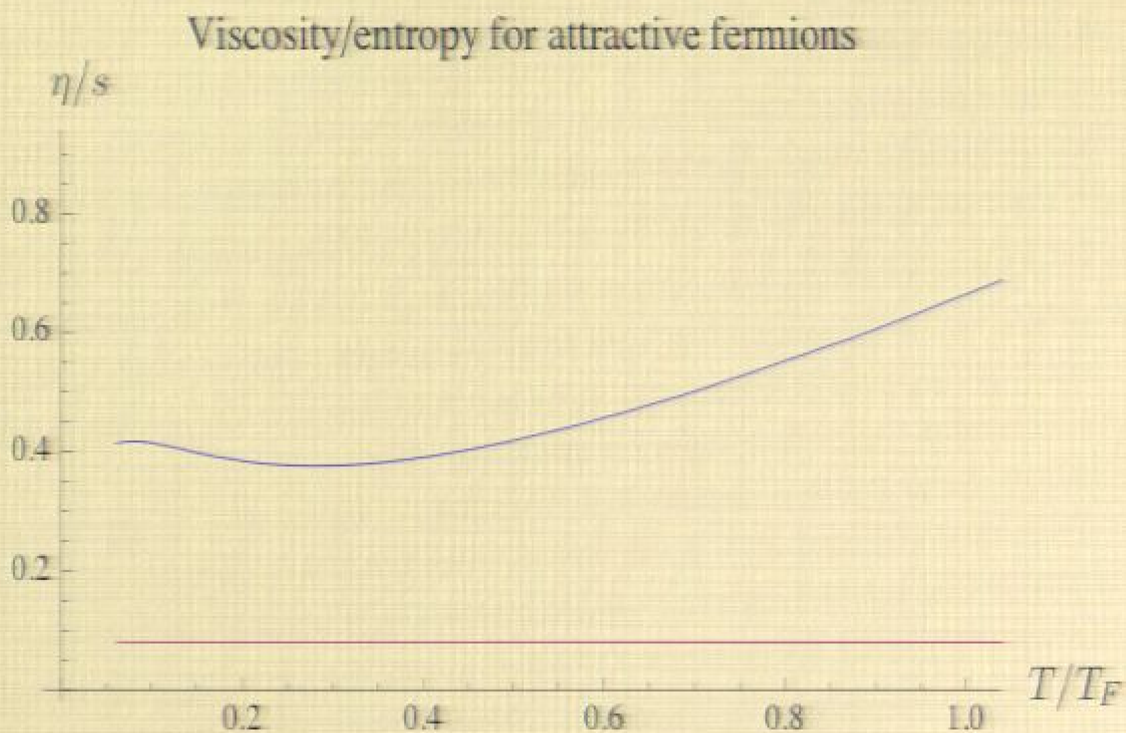

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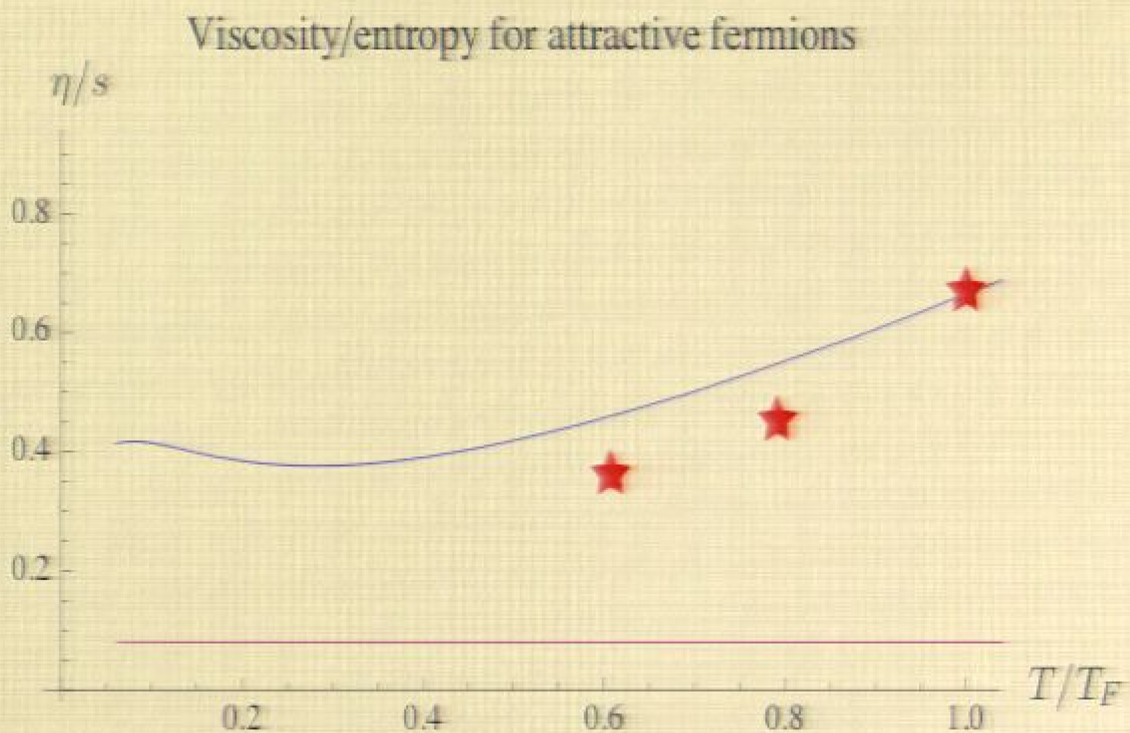


★ = data (Duke group (Thomas) Science 2011)

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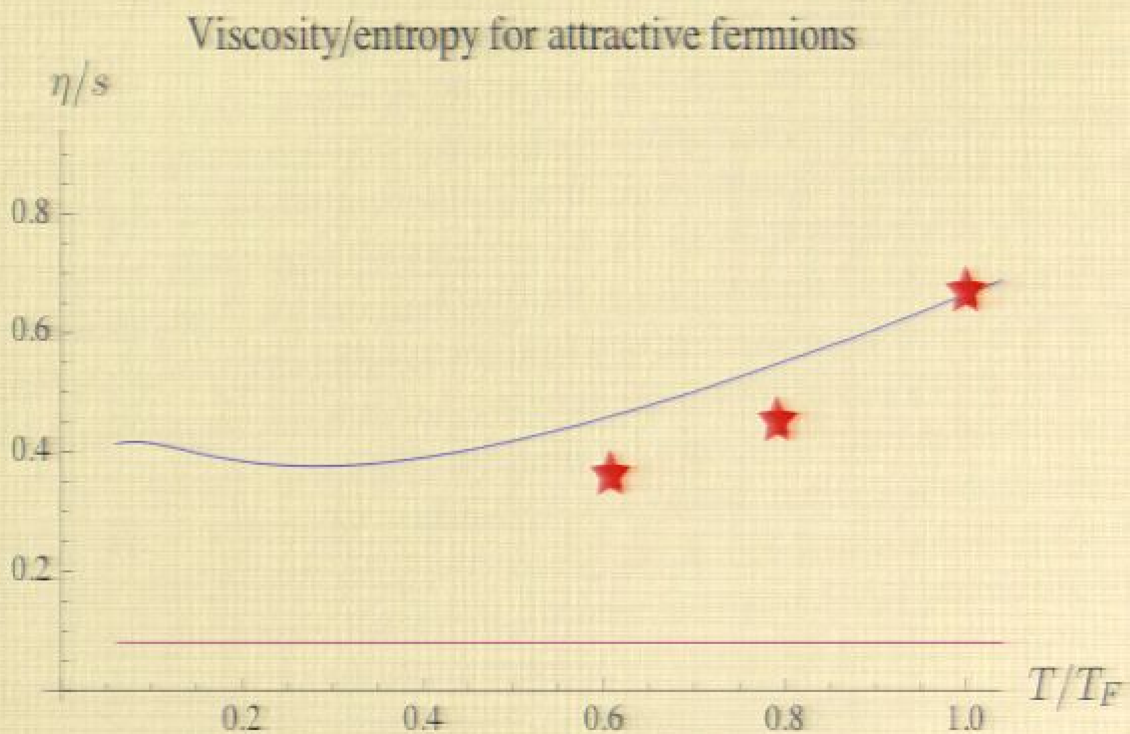


★ = data (Duke group (Thomas) Science 2011)

$$\frac{\eta}{s} > 4.72\frac{\hbar}{4\pi k_B}$$

(data: 4-5 times bound)

Bosons

## Bosons

Evidence for an interacting version of BEC (new)

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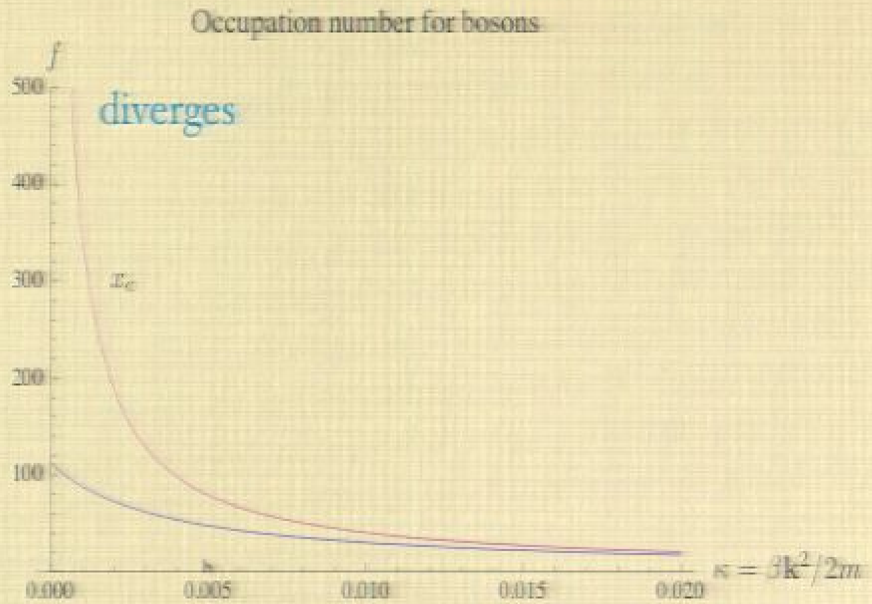
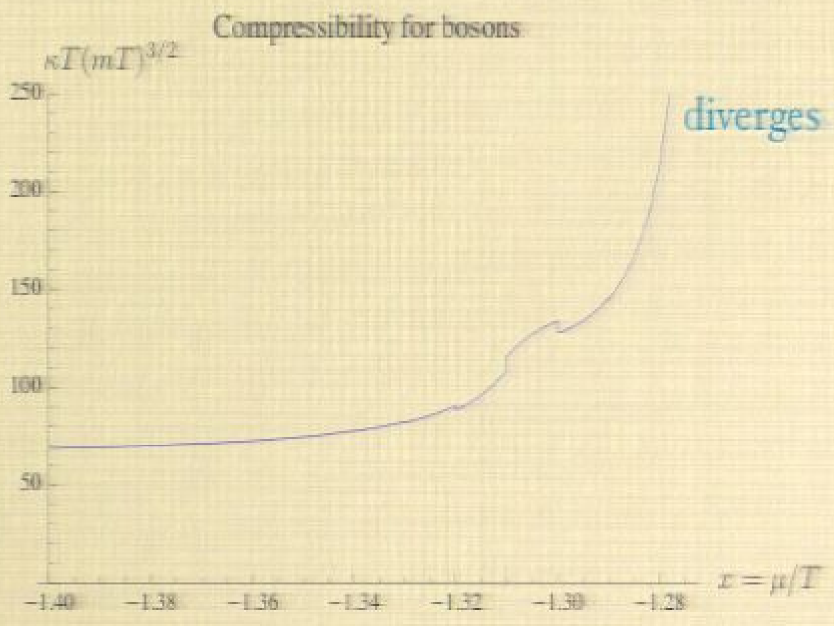
$$n_c \lambda_T^3 = 1.325, \qquad (\mu/T = x_c = -1.2741)$$

compare with non-interacting BEC:

$$x_c = 0 \text{ and } n_c \lambda_T^3 = \zeta(3/2) = 2.61.$$


Occupation number for bosons
f
500
400
300
200
100
diverges
$x_c$
0.000
0.005
0.010
0.015
0.020
$\kappa = \beta \mathbf{k}^2/2m$


Compressibility for bosons
$\kappa T(mT)^{3/2}$
250
200
150
100
50
diverges
−1.40
−1.38
−1.36
−1.34
−1.32
−1.30
−1.28
$x = \mu/T$

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- Although an approximation, a 'TBA -like' formalism exists for non-integrable models in any dimension that can give reliable results.
- Unitary gases: an accurate AdS/CFT description appears unlikely based on the data. Fortunately other methods (like ours) are working reasonably well.
- Is there a non-relativistic quantum gas that satisfies the bound (supersymmetric?).

the End