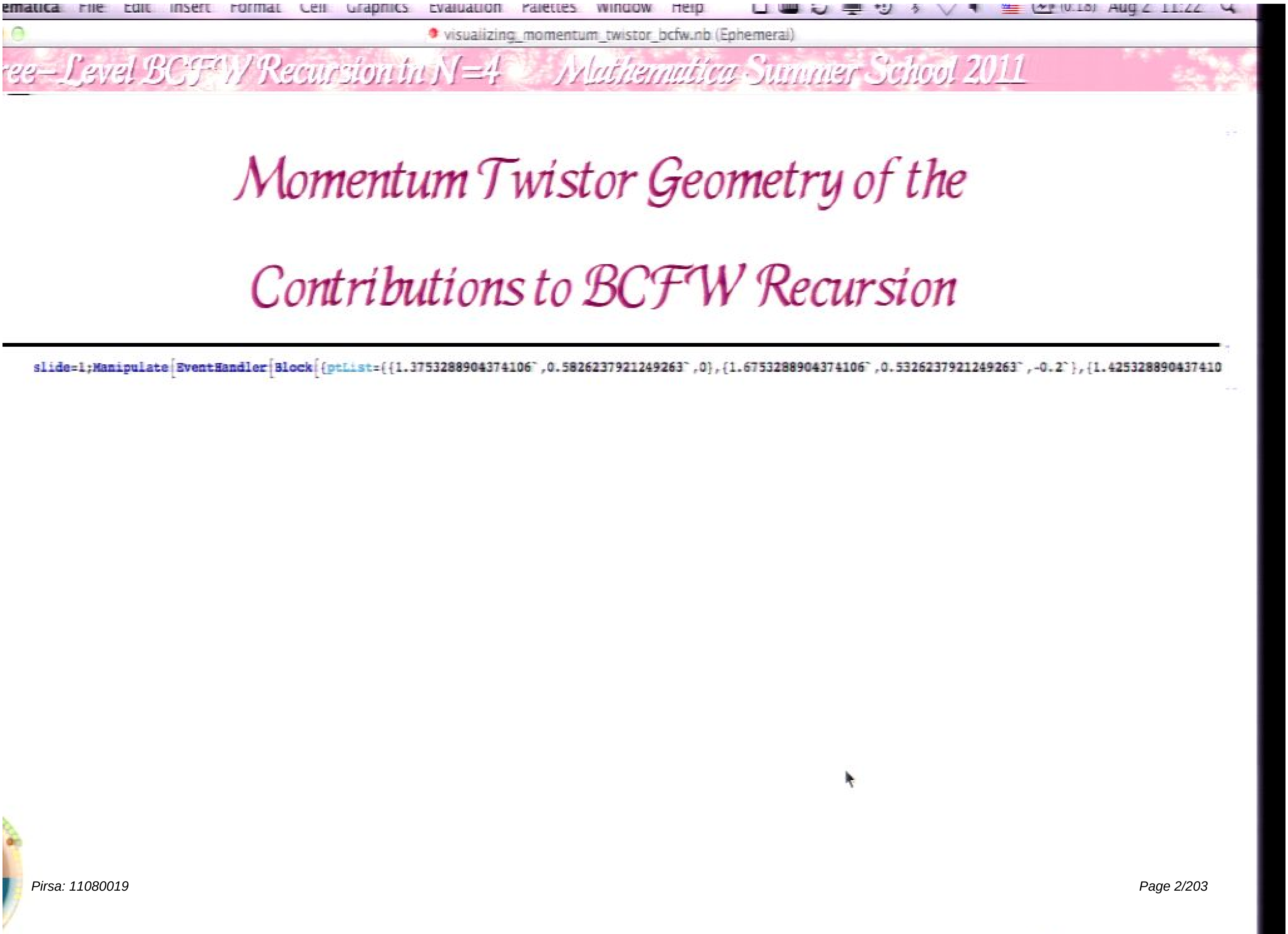
Title: Recursion Relations

Date: Aug 02, 2011 11:15 AM

URL: http://pirsa.org/11080019

Abstract: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visualizing_momentum_twistor_bcfw.nb (Ephemeral)
ree–Level BCFW Recursion in N=4
Mathematica Summer School 2011

# Momentum Twistor Geometry of the Contributions to BCFW Recursion

```
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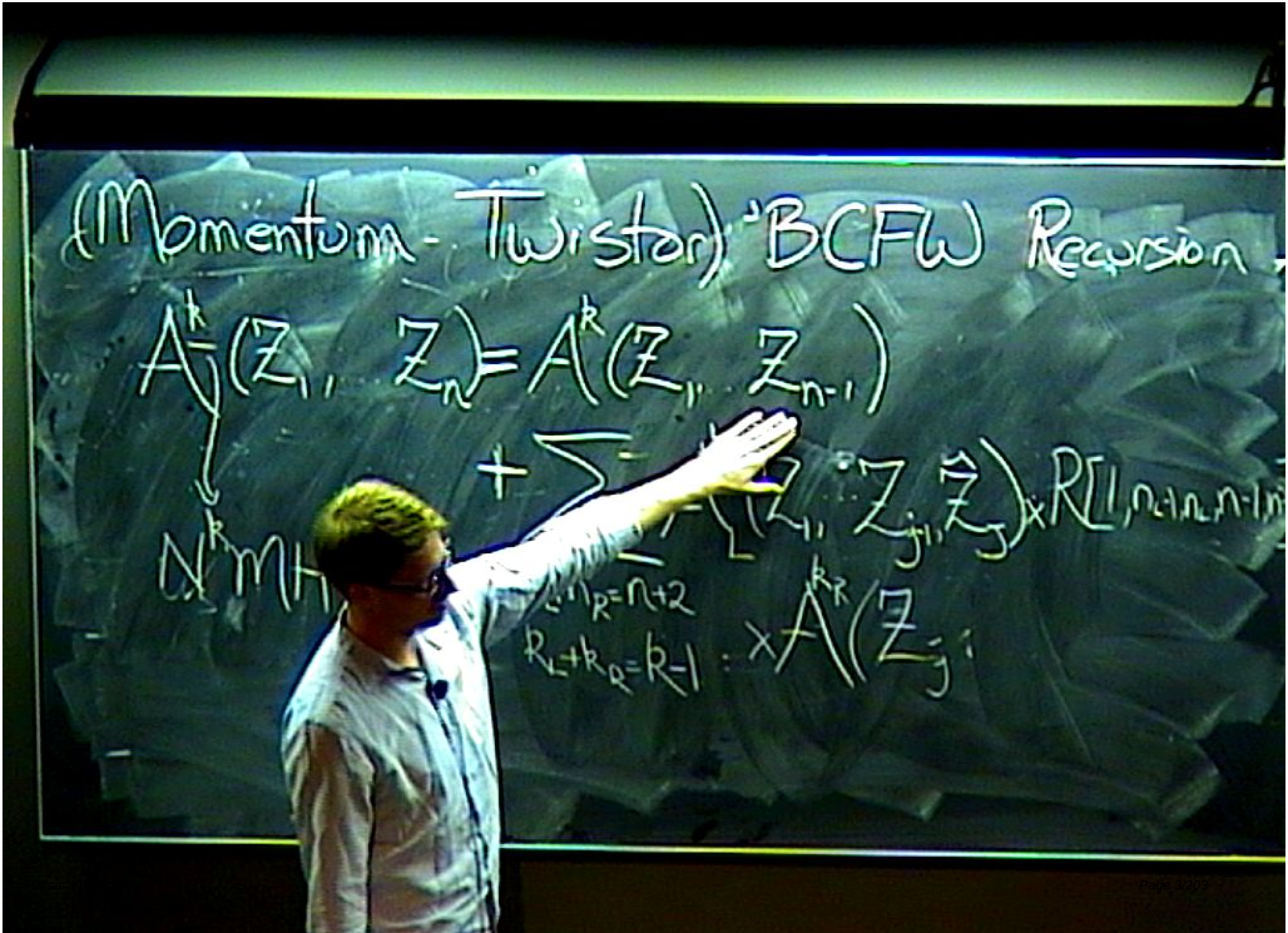
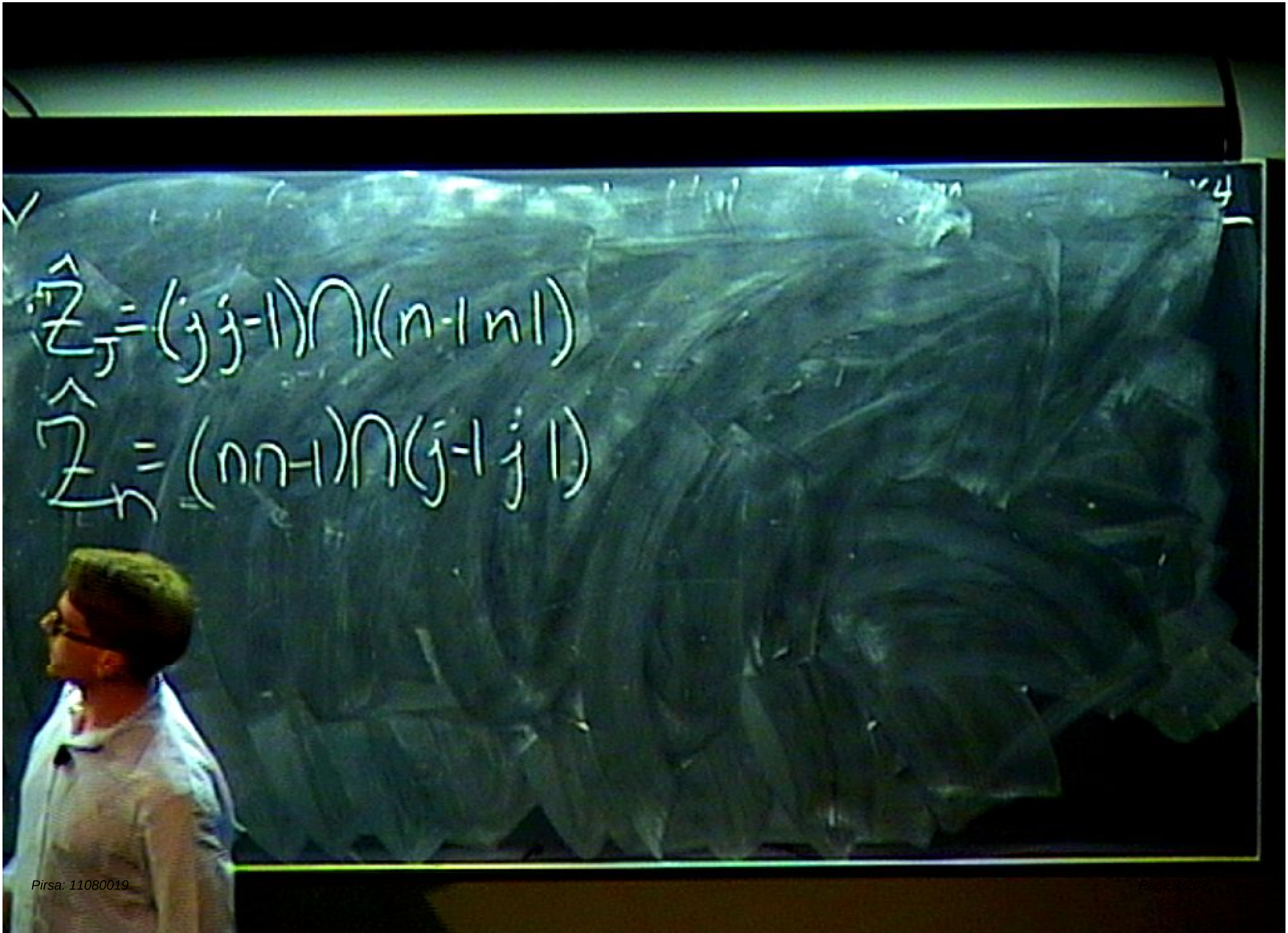


(Momentum-Twistor) 'BCFW Recursion

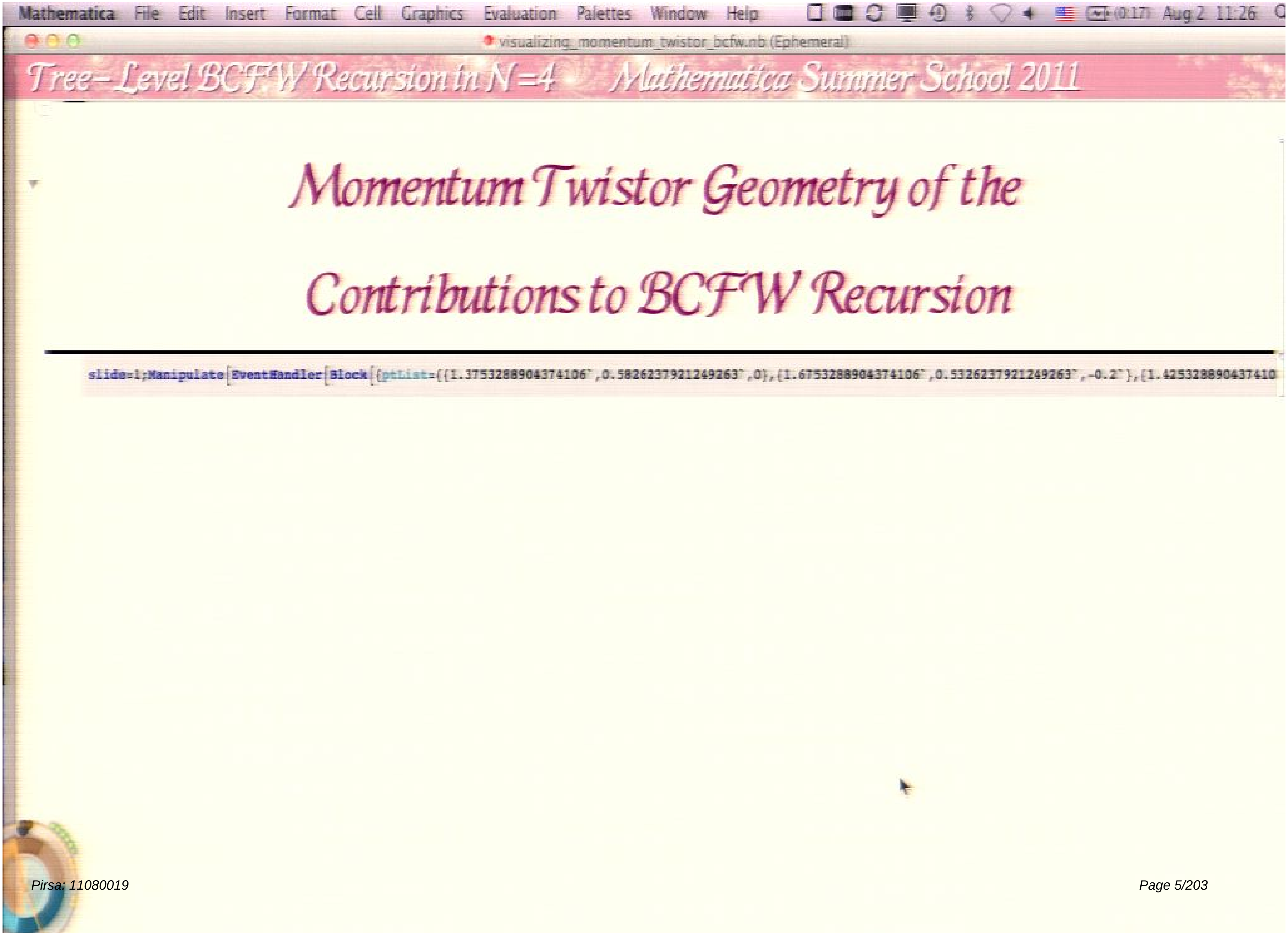
$$A^k_n(Z_1, \dots Z_n) = A^k(Z_1, \dots Z_{n-1}) + \sum_{\substack{n_L+n_R=n+2\\ k_L+k_R=k-1}} A^{k_L}_L(Z_1, \dots Z_{j-1}, \hat{Z}_j) \times R[1, n-1, n, n-1, n \times A^{k_R}_R(\hat{Z}_j$$

$N^k MHV$



$$\hat{Z}_J = (j\ j-1)\cap(n-1\ n\ 1)$$

$$\hat{Z}_n = (n\ n-1)\cap(j-1\ j\ 1)$$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visualizing_momentum_twistor_bcfw.nb (Ephemeral)

Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

# Momentum Twistor Geometry of the Contributions to BCFW Recursion

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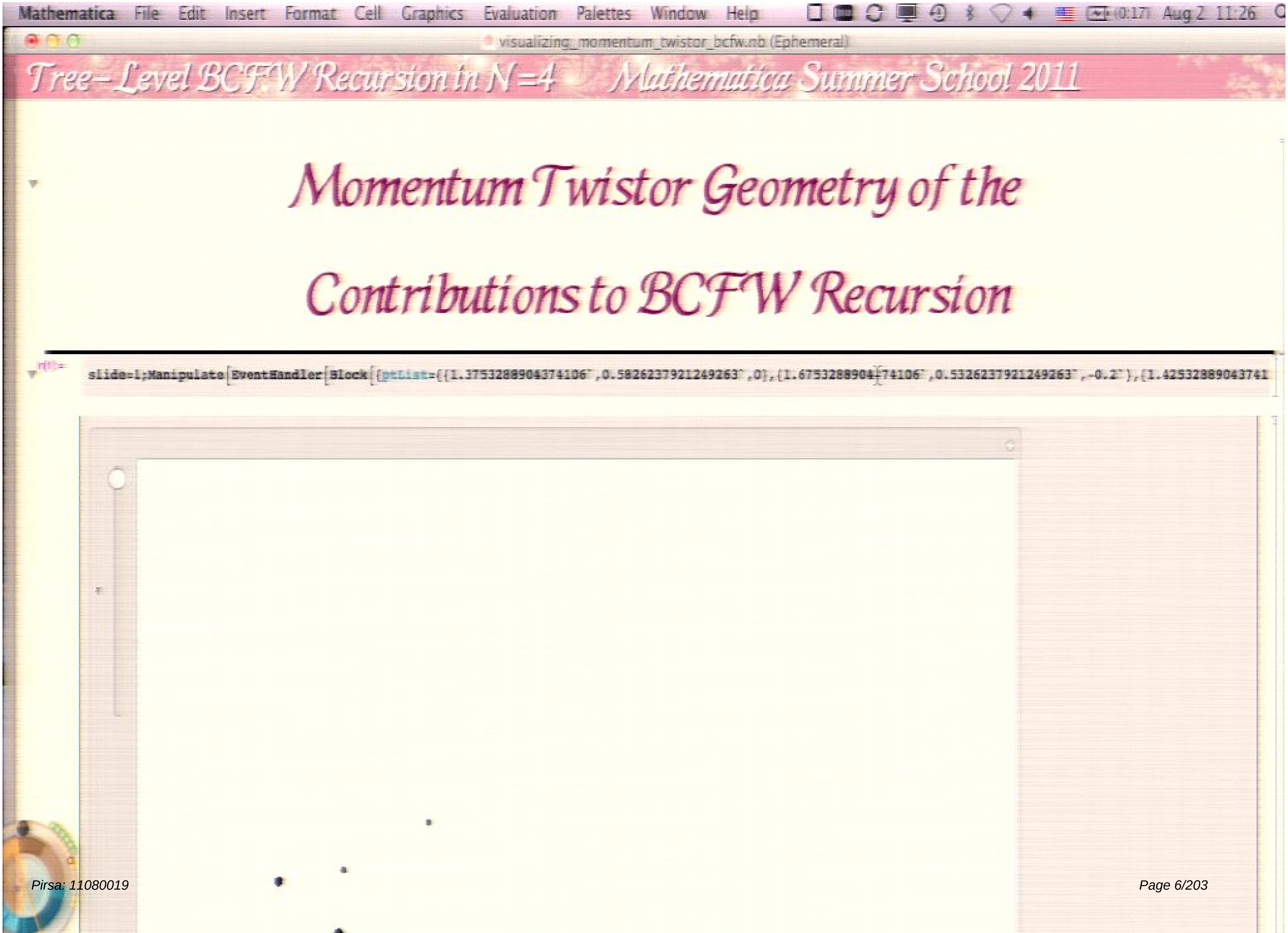
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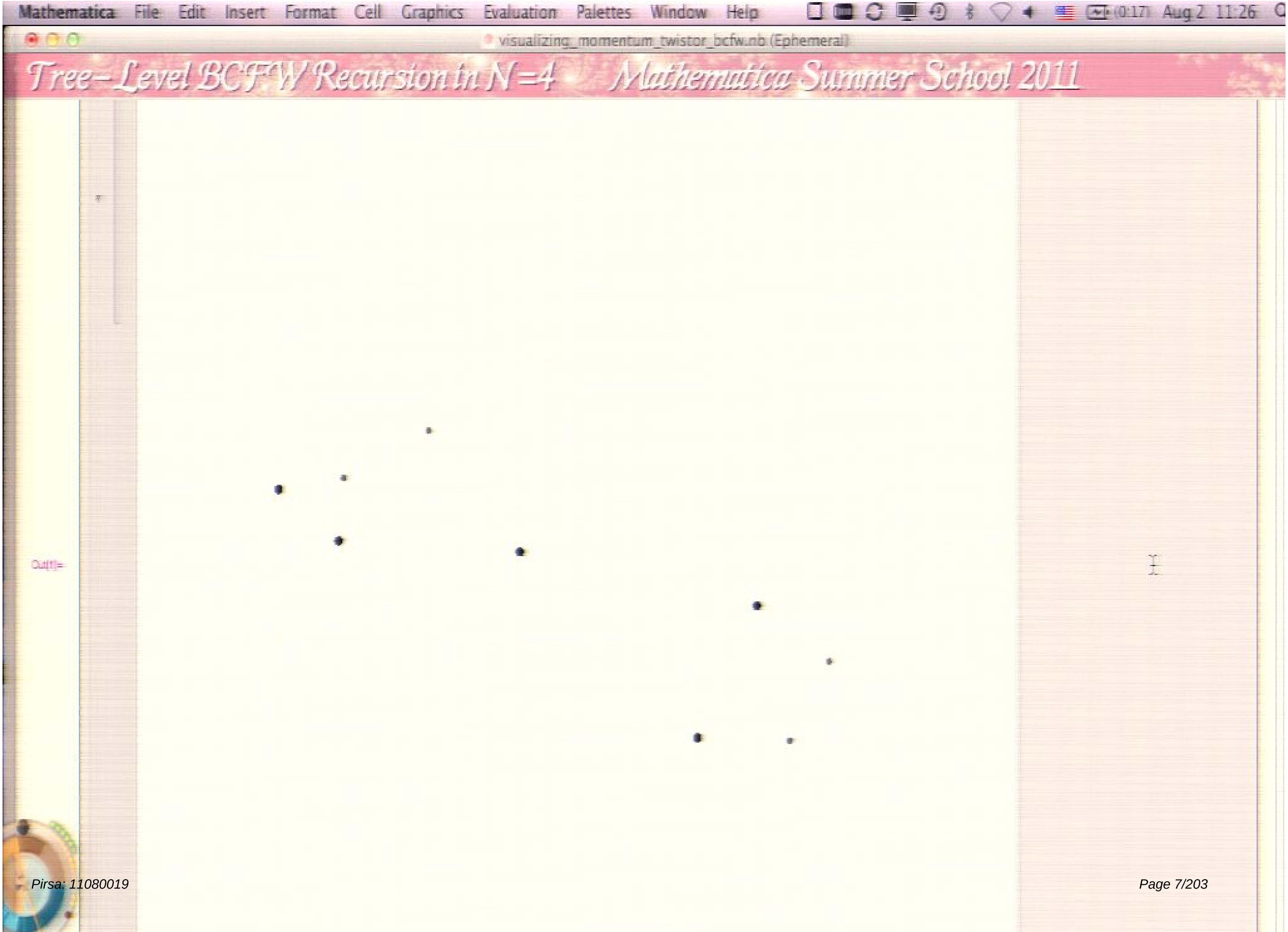

visualizing_momentum_twistor_bcfw.nb (Ephemeral)

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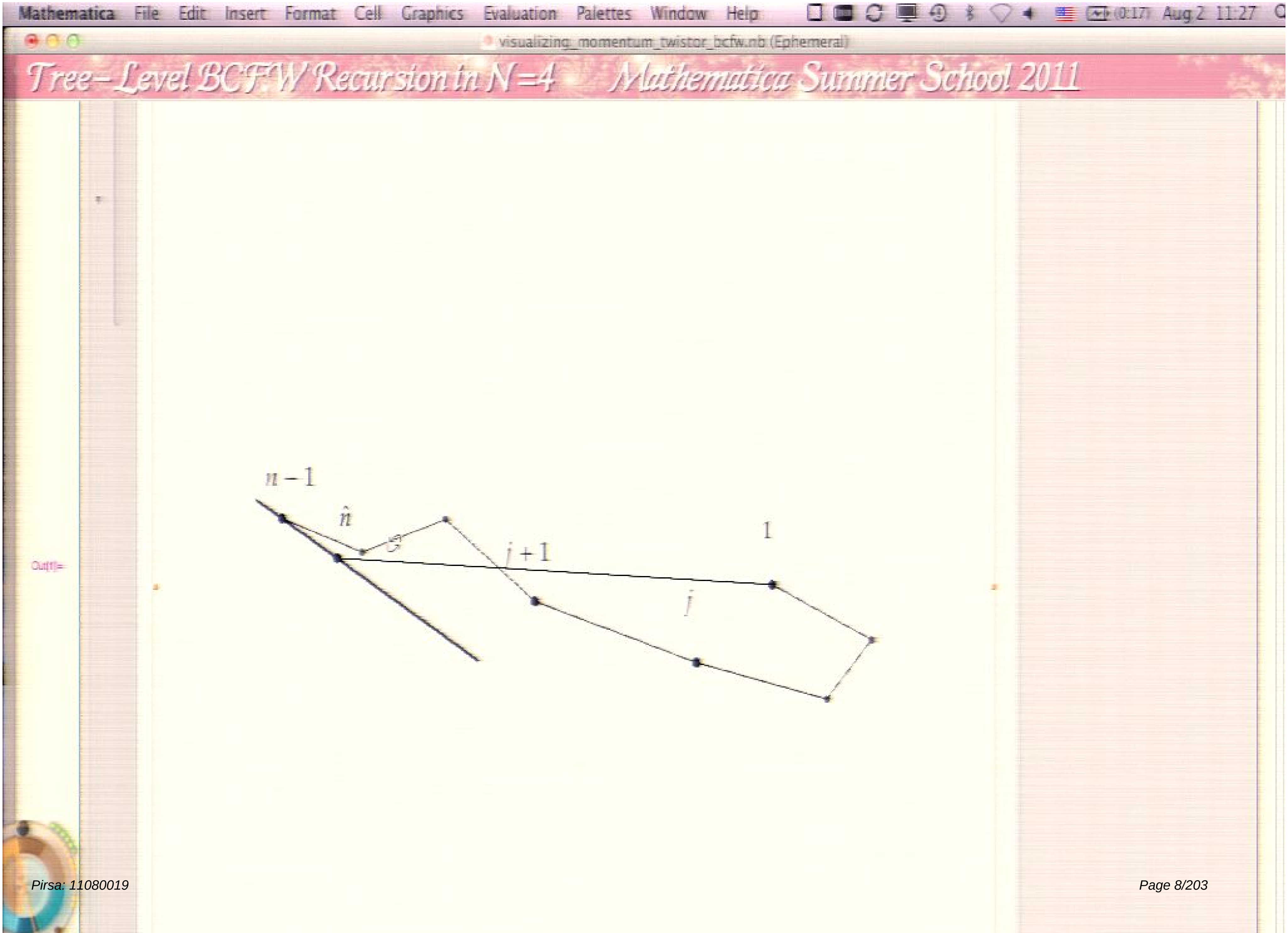
Out[1]=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$n-1$
$\hat{n}$
$j+1$
$1$
$j$

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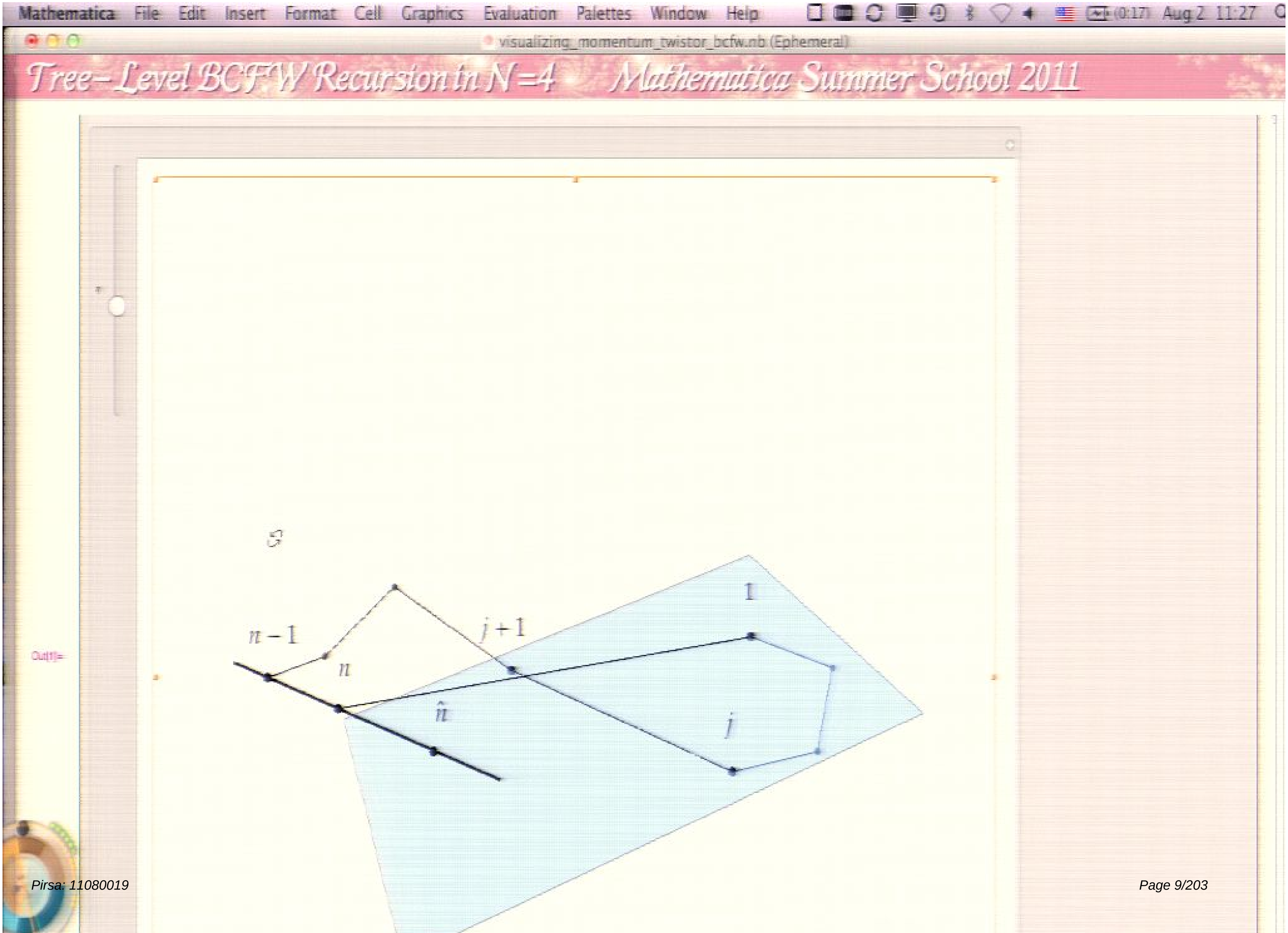
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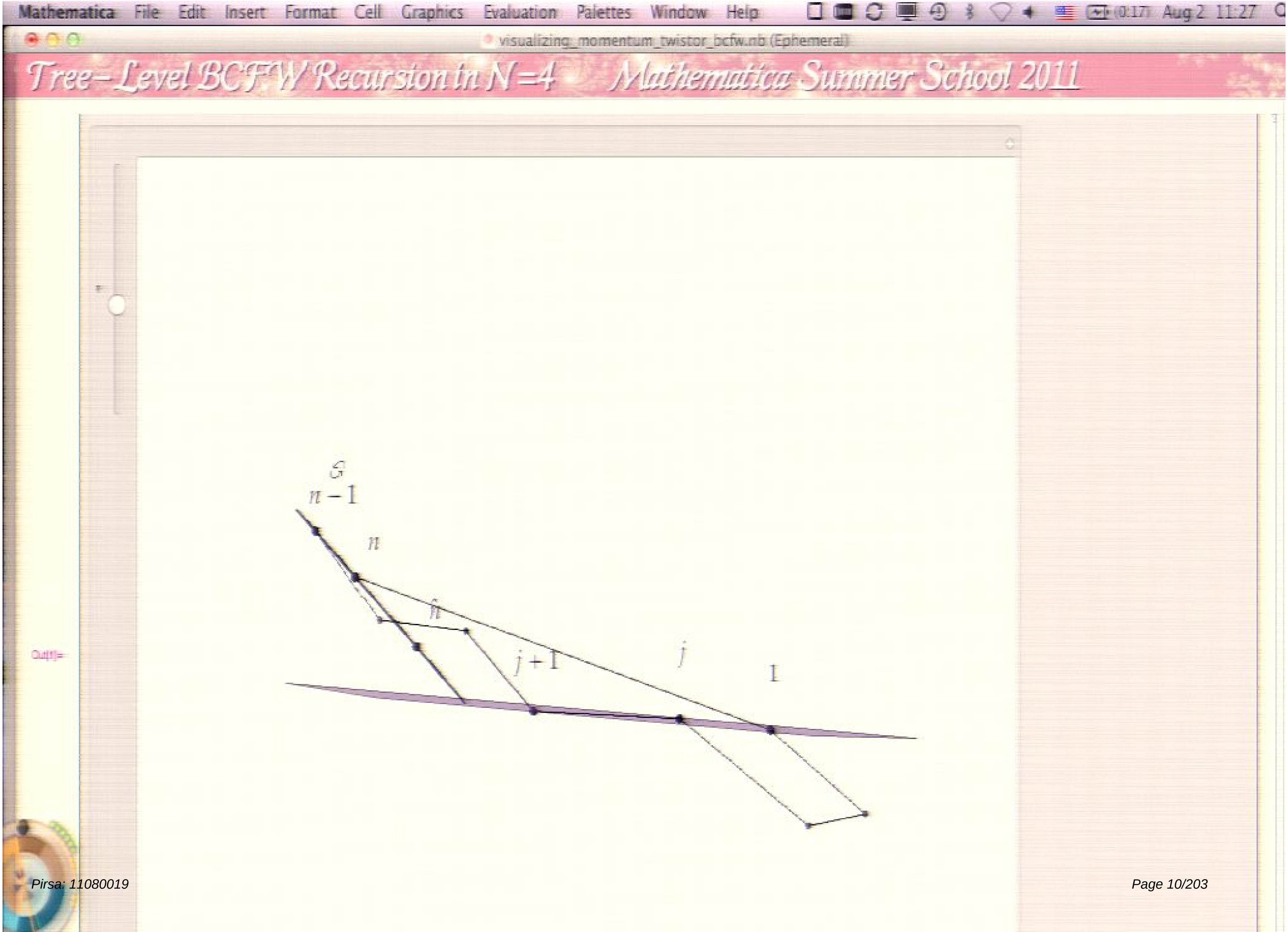
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011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$\mathcal{G}$
$n-1$
$n$
$\hat{n}$
$j+1$
$j$
$1$

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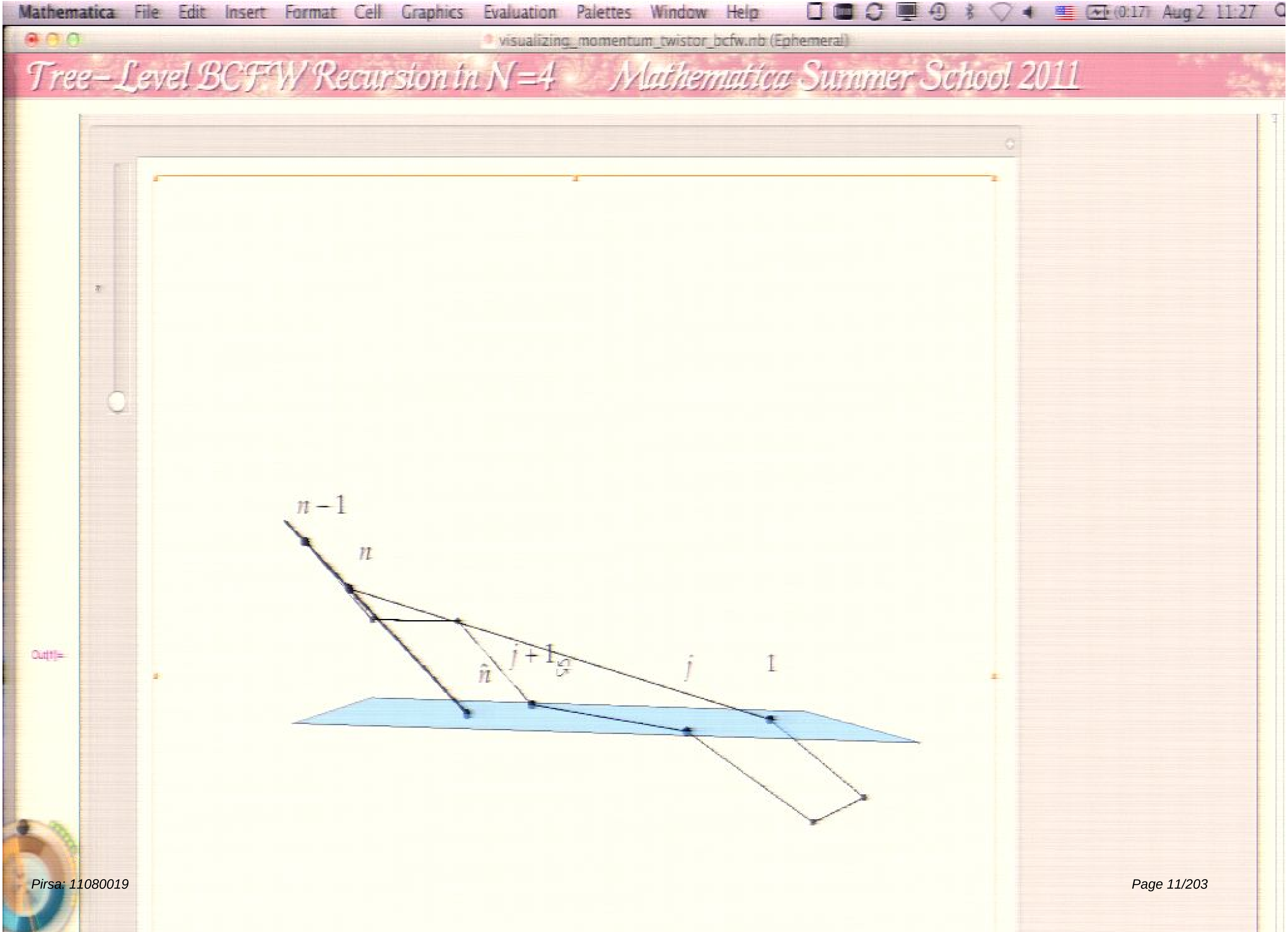

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$j$
$1$

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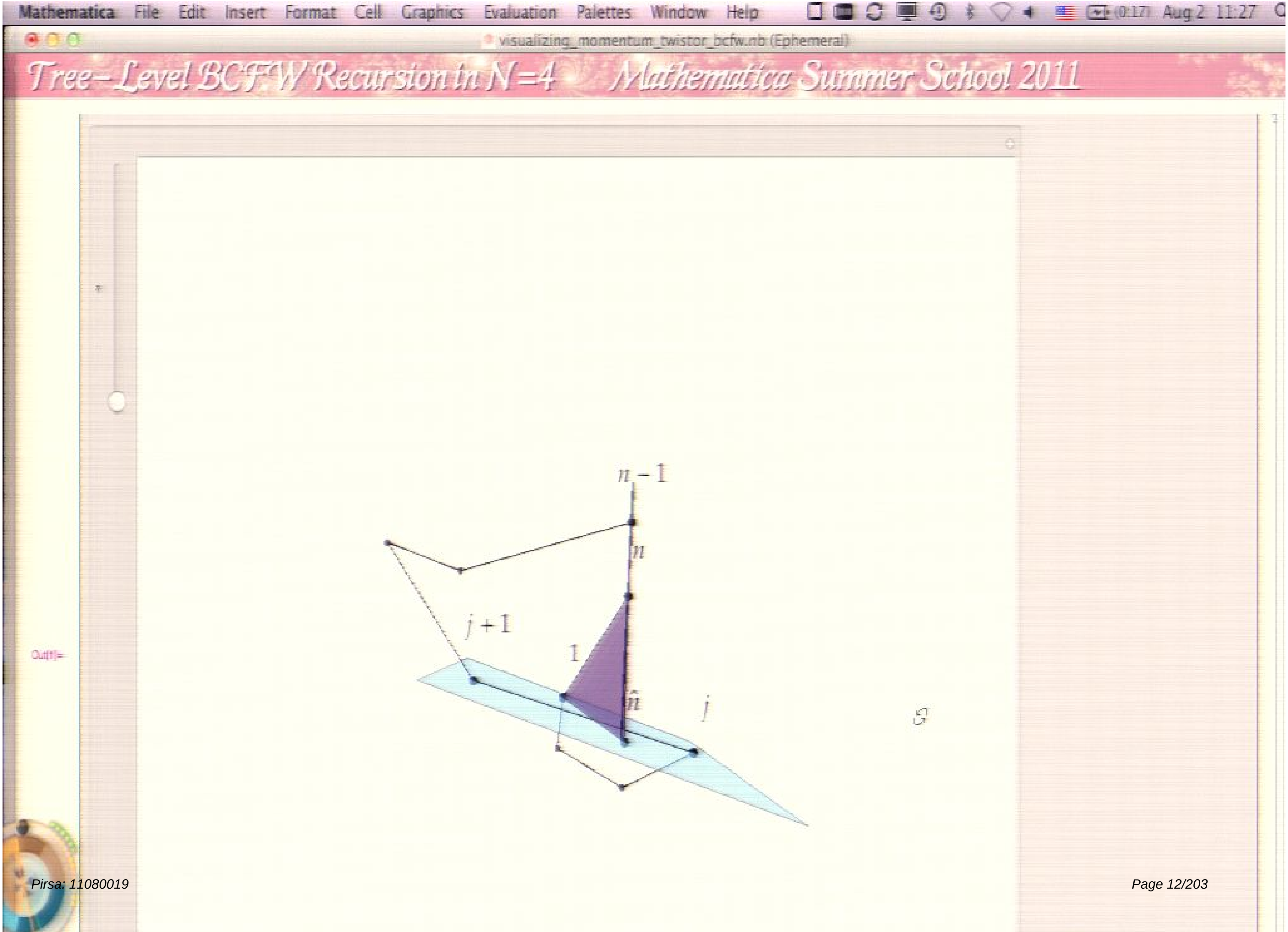
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$j+1$
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$\hat{n}$
$j$

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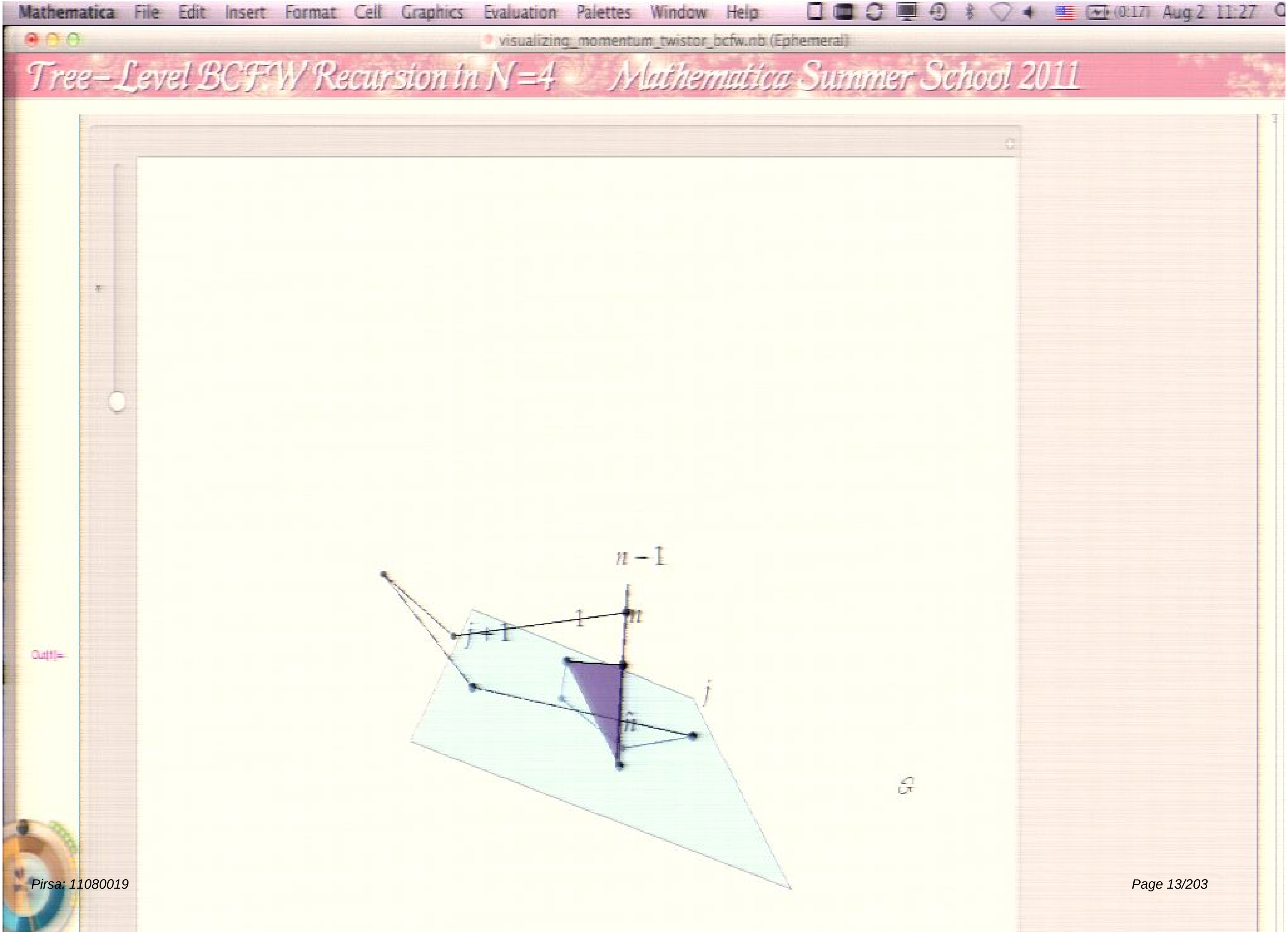
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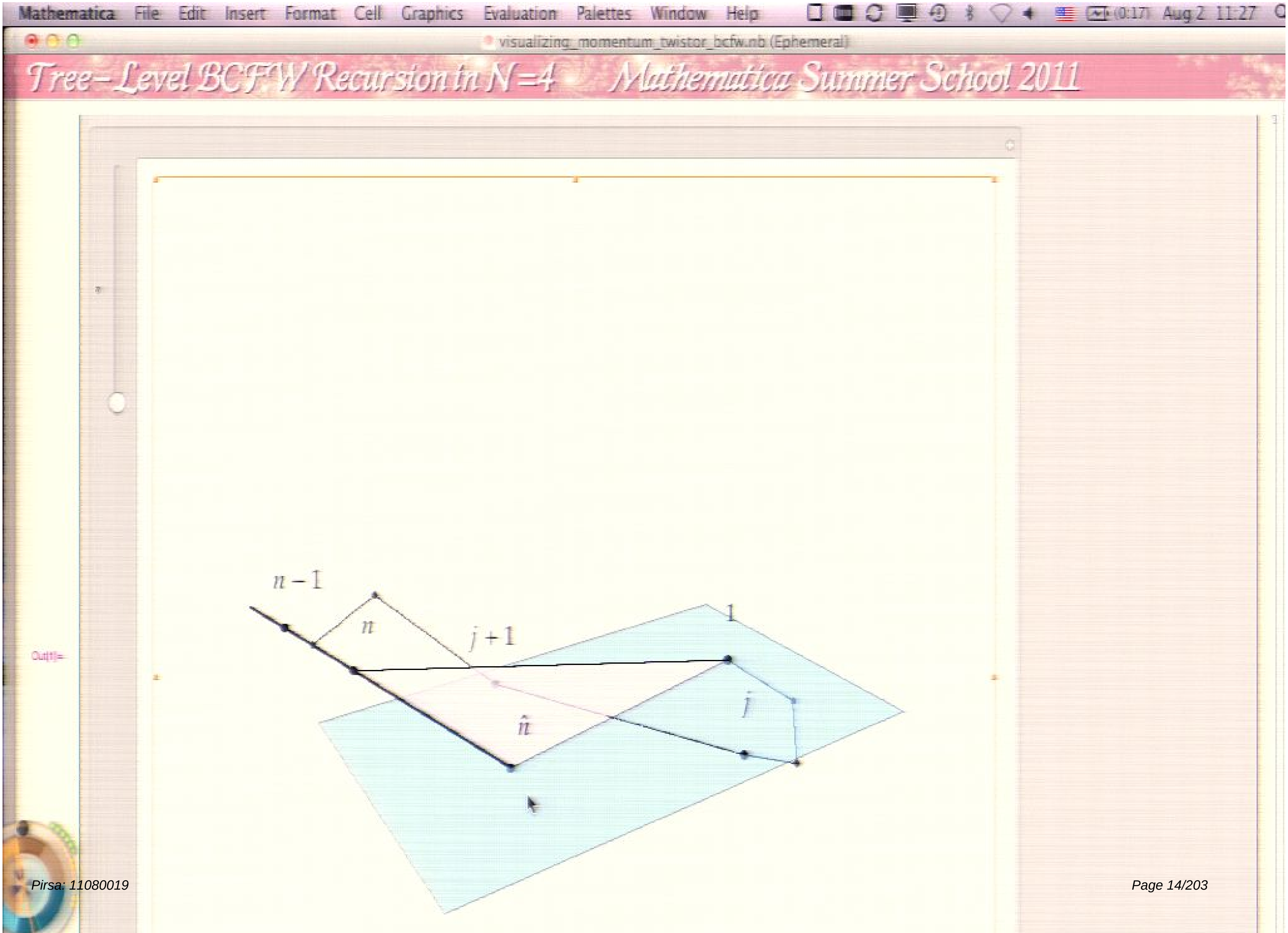
$1$
$j+1$
$j$
$\hat{n}$

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011



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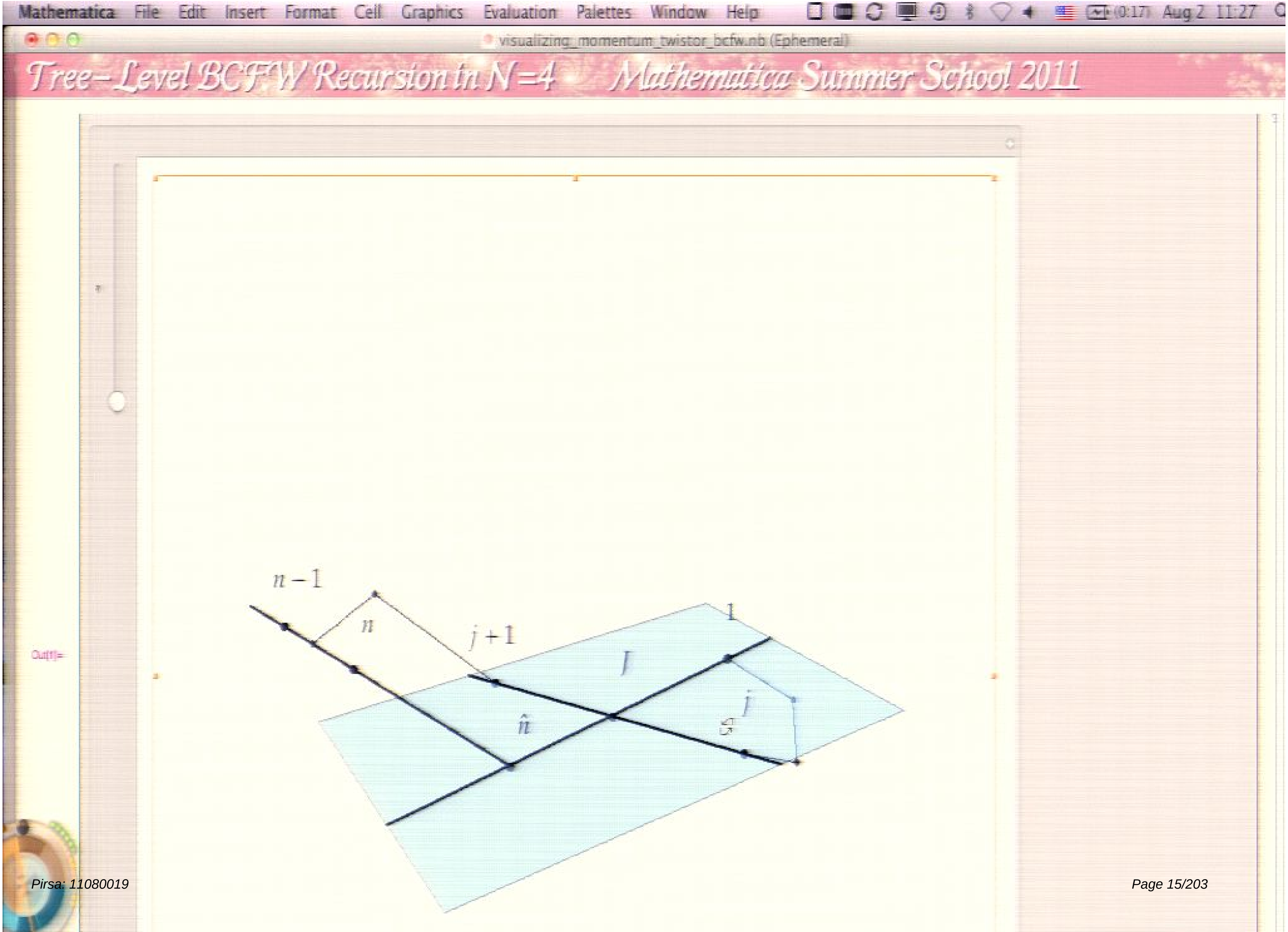
$j+1$
$1$
$j$
$\hat{n}$

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011



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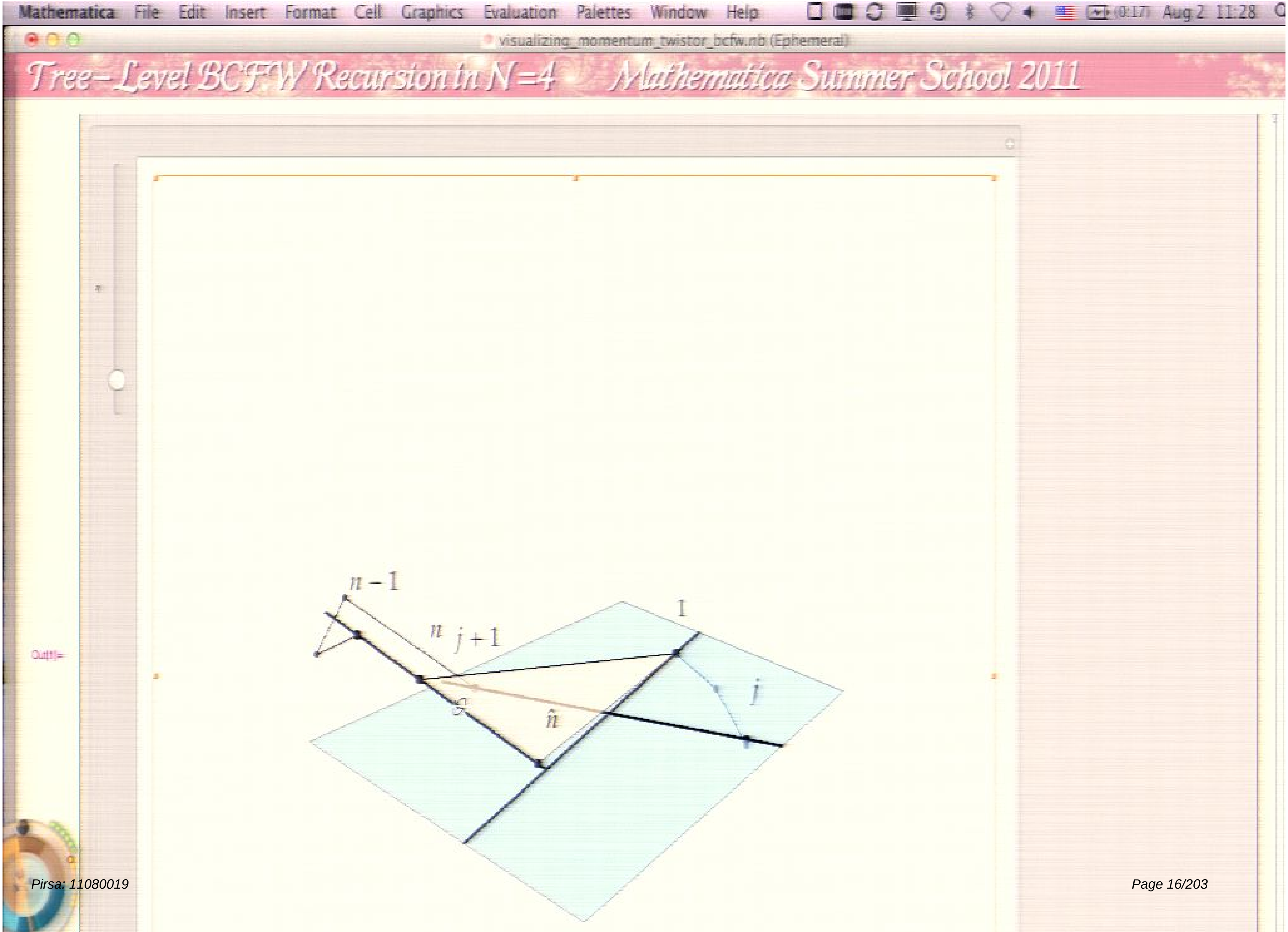
Out[1]=
$n-1$
$n$
$j+1$
$1$
$J$
$j$
$\hat{n}$

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011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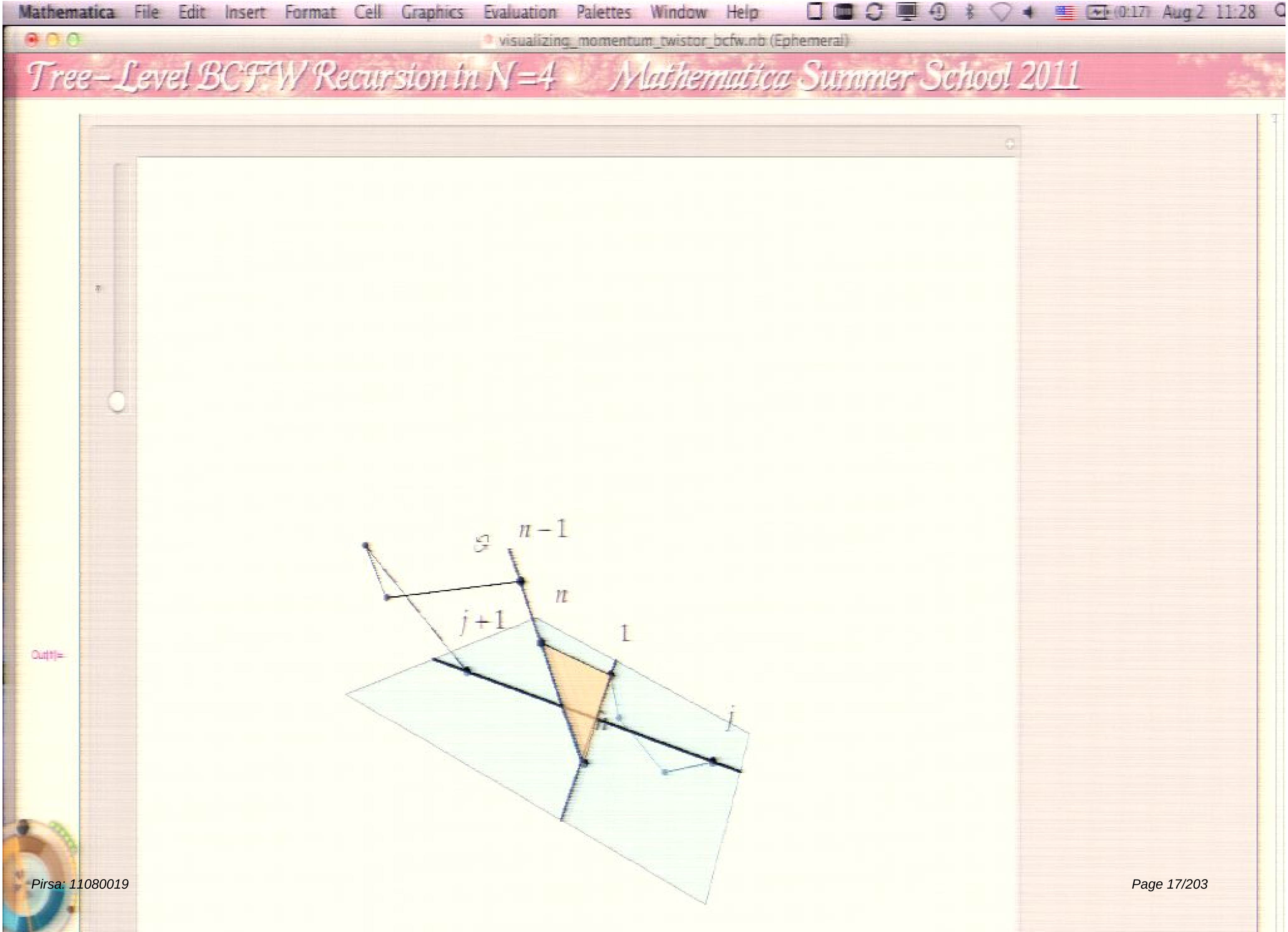
visualizing_momentum_twistor_bcfw.nb (Ephemeral)
Out[1]=
$n-1$
$n$
$j+1$
$I$
$j$
$\hat{n}$
$\mathcal{J}$

# Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011



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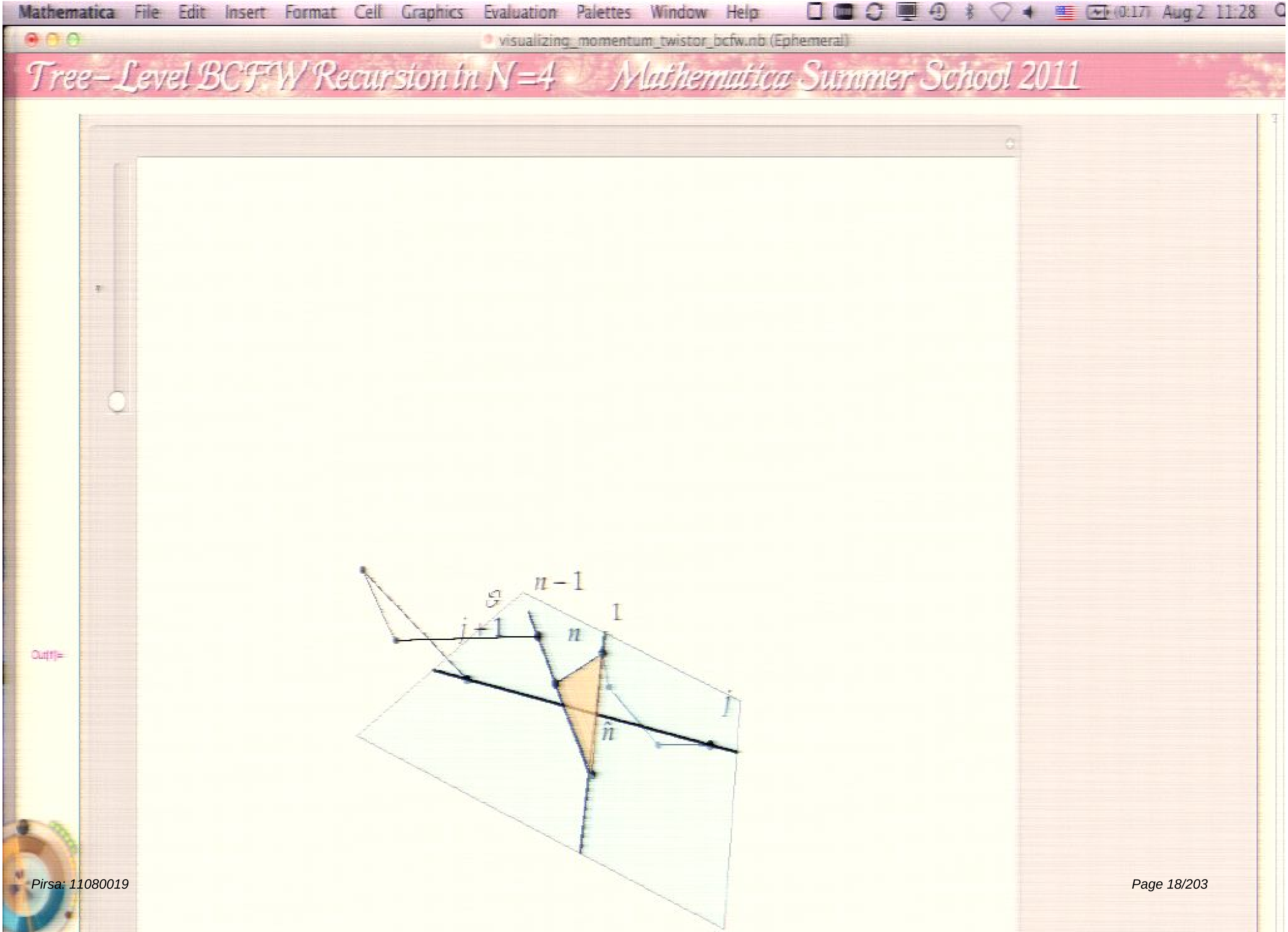
visualizing_momentum_twistor_bcfw.nb (Ephemeral)

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

Out[1]=

$\mathcal{S}$
$n-1$
$n$
$j+1$
$1$
$j$



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visualizing_momentum_twistor_bcfw.nb (Ephemeral)

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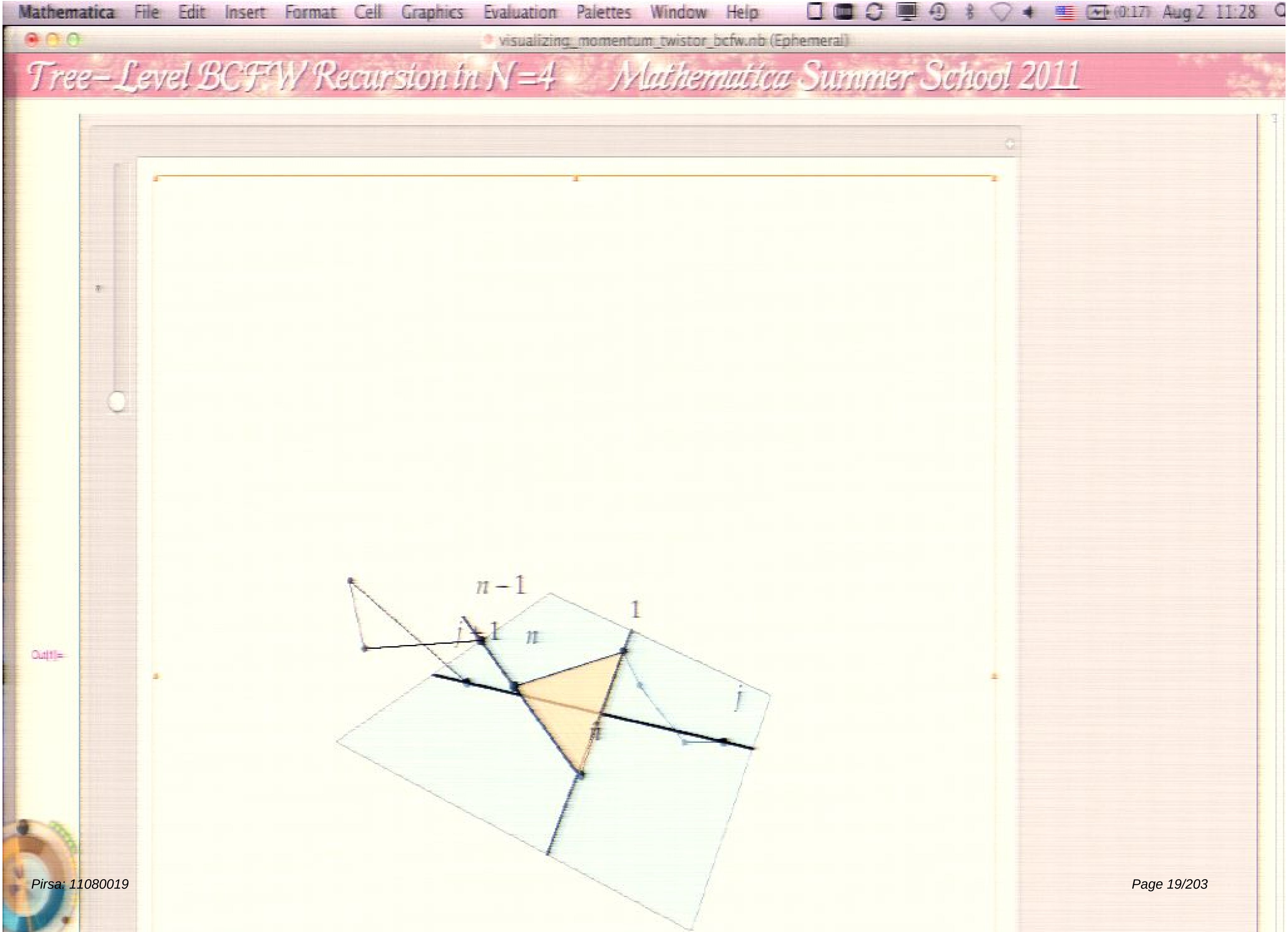
Out[]=
$n-1$
$1$
$j+1$
$n$
$j$
$\hat{n}$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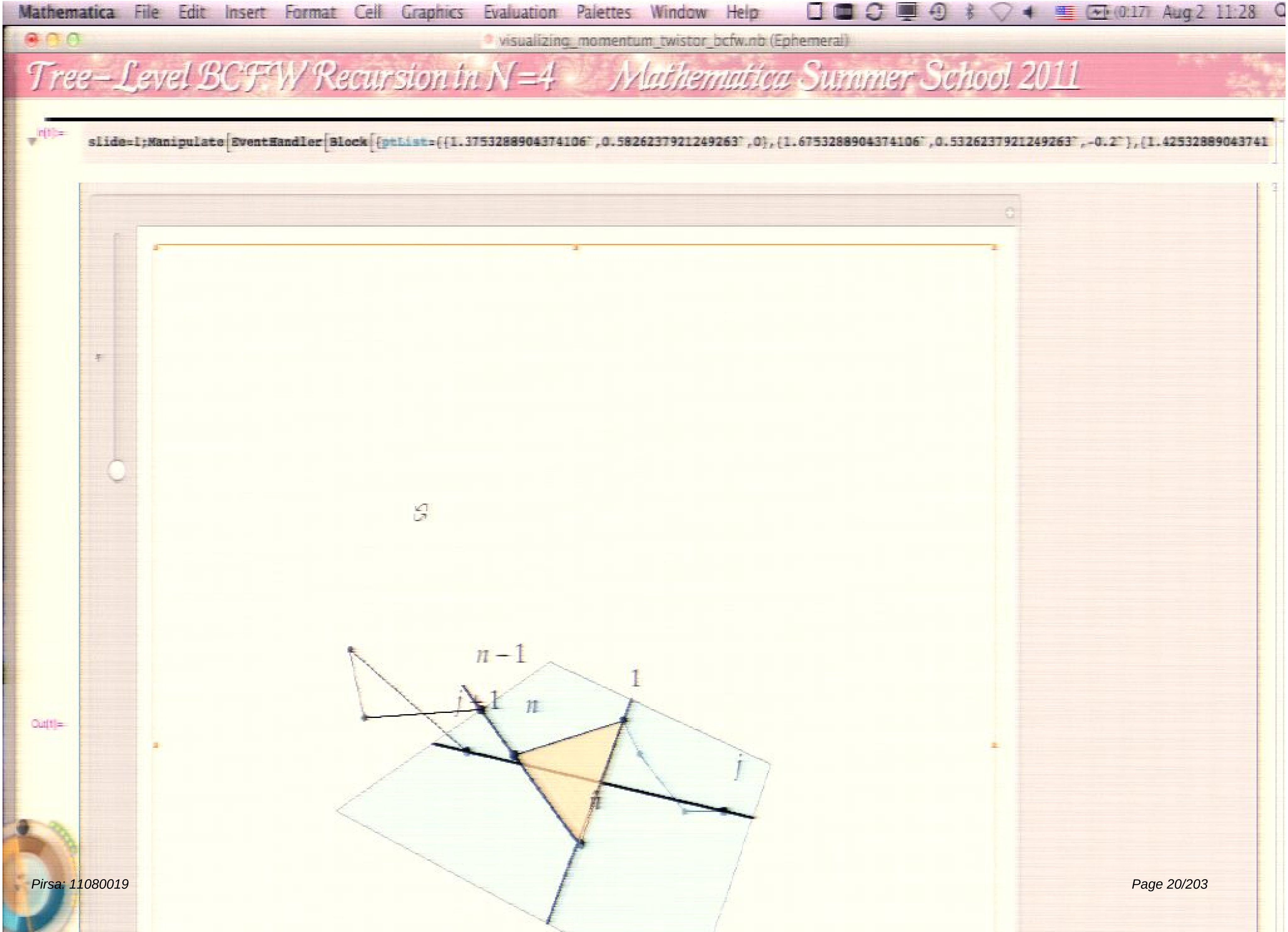
Out[1]=
$n-1$
$1$
$j+1$
$n$
$j$
$\hat{n}$

Tree–Level BCFW Recursion in N =4 Mathematica Summer School 2011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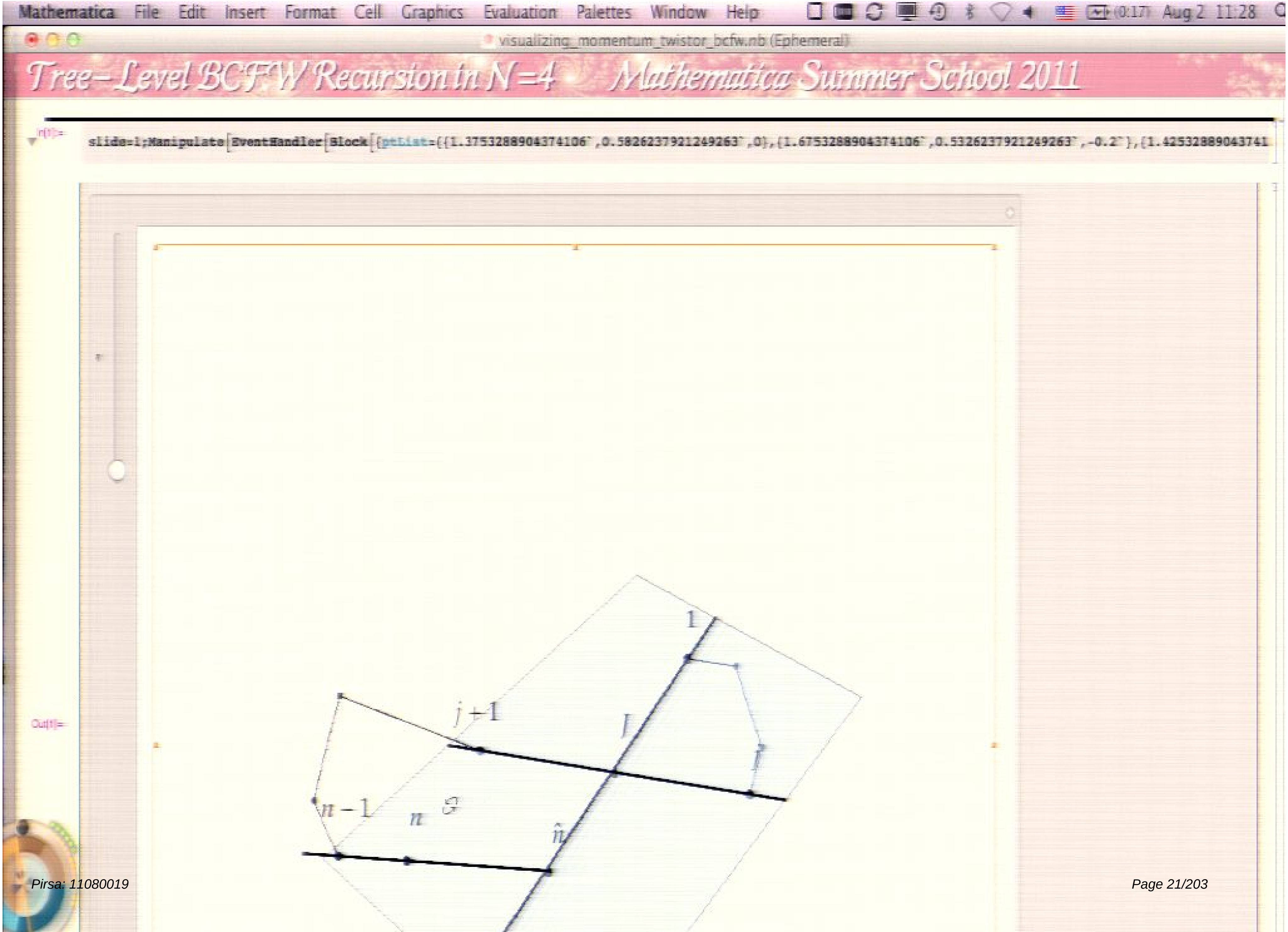
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011
In[1]:=
Out[1]=
n − 1
1
j+1
n
j
n̂

```
slide=1;Manipulate[EventHandler[Block[{ptList={{1.3753288904374106`,0.5826237921249263`,0},{1.6753288904374106`,0.5326237921249263`,-0.2`},{1.42532889043741
```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# Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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slide=1;Manipulate[EventHandler[Block[{ptList={{1.3753288904374106`,0.5826237921249263`,0},{1.6753288904374106`,0.5326237921249263`,-0.2`},{1.42532889043741
```
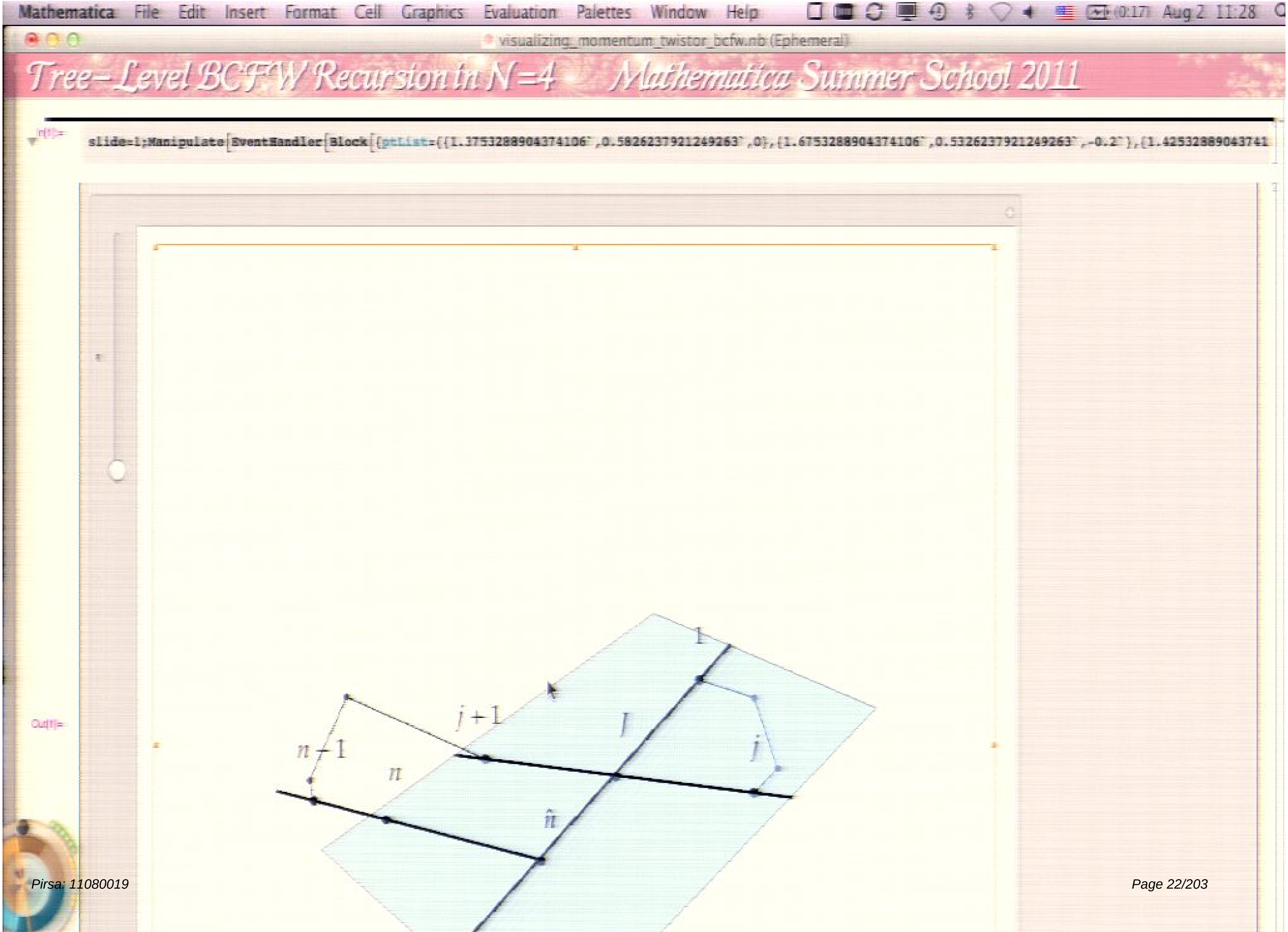

Out[1]=
1
$j+1$
$J$
$I$
$n-1$
$n$
$\mathcal{I}$
$\hat{n}$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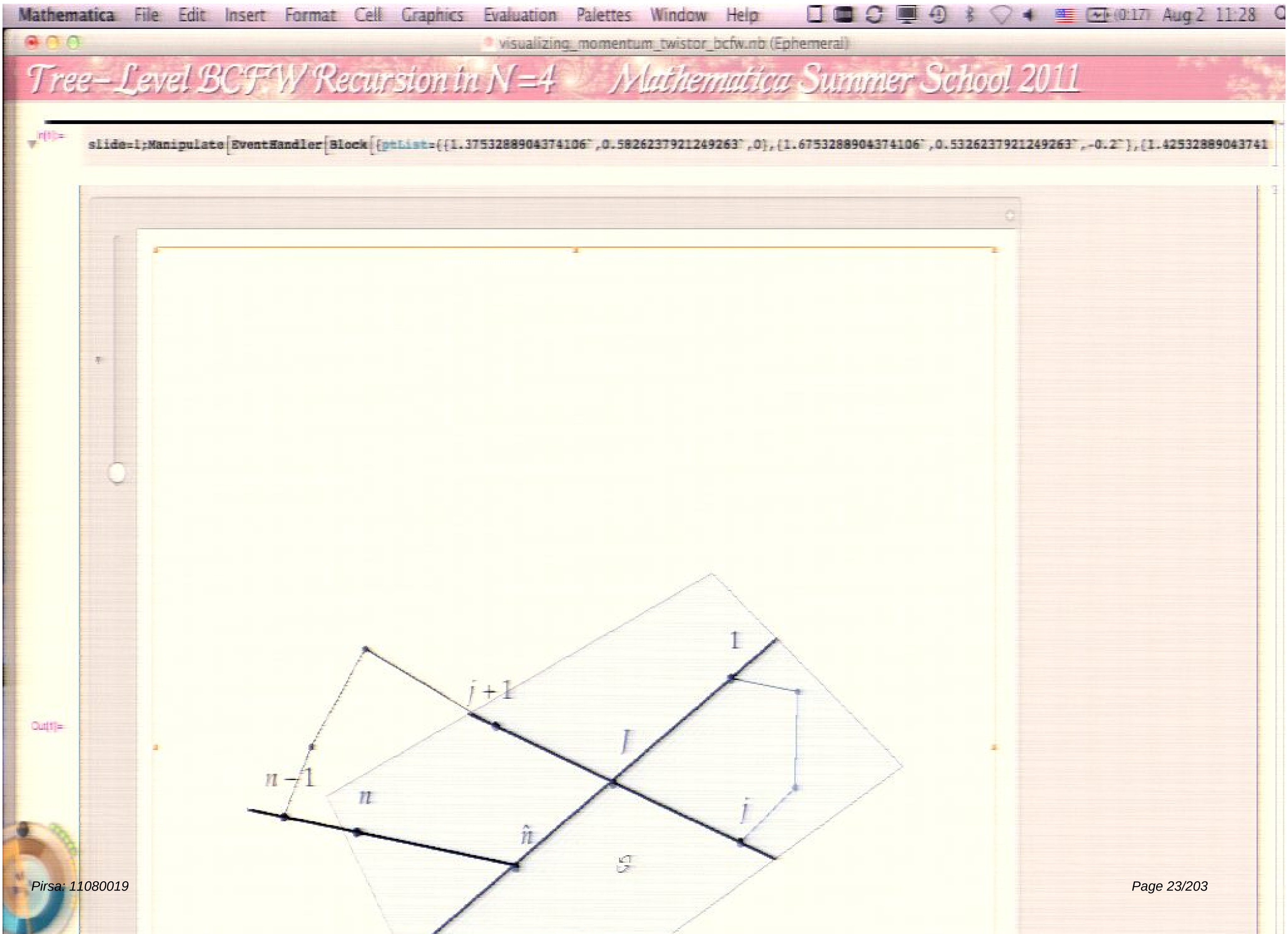
visualizing_momentum_twistor_bcfw.nb (Ephemeral)
In[1]:=
Out[1]=
1
$j+1$
$I$
$j$
$n-1$
$n$
$\hat{n}$

# Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
slide=1;Manipulate[EventHandler[Block[{ptList={{1.3753288904374106`,0.5826237921249263`,0},{1.6753288904374106`,0.5326237921249263`,-0.2`},{1.42532889043741
```



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visualizing_momentum_twistor_bcfw.nb (Ephemeral)
In[1]:=
Out[1]=
1
j + 1
J
n − 1
n
j
n̂

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
slide=1;Manipulate[EventHandler[Block[{ptList={{1.3753288904374106`,0.5826237921249263`,0},{1.6753288904374106`,0.5326237921249263`,-0.2`},{1.42532889043741
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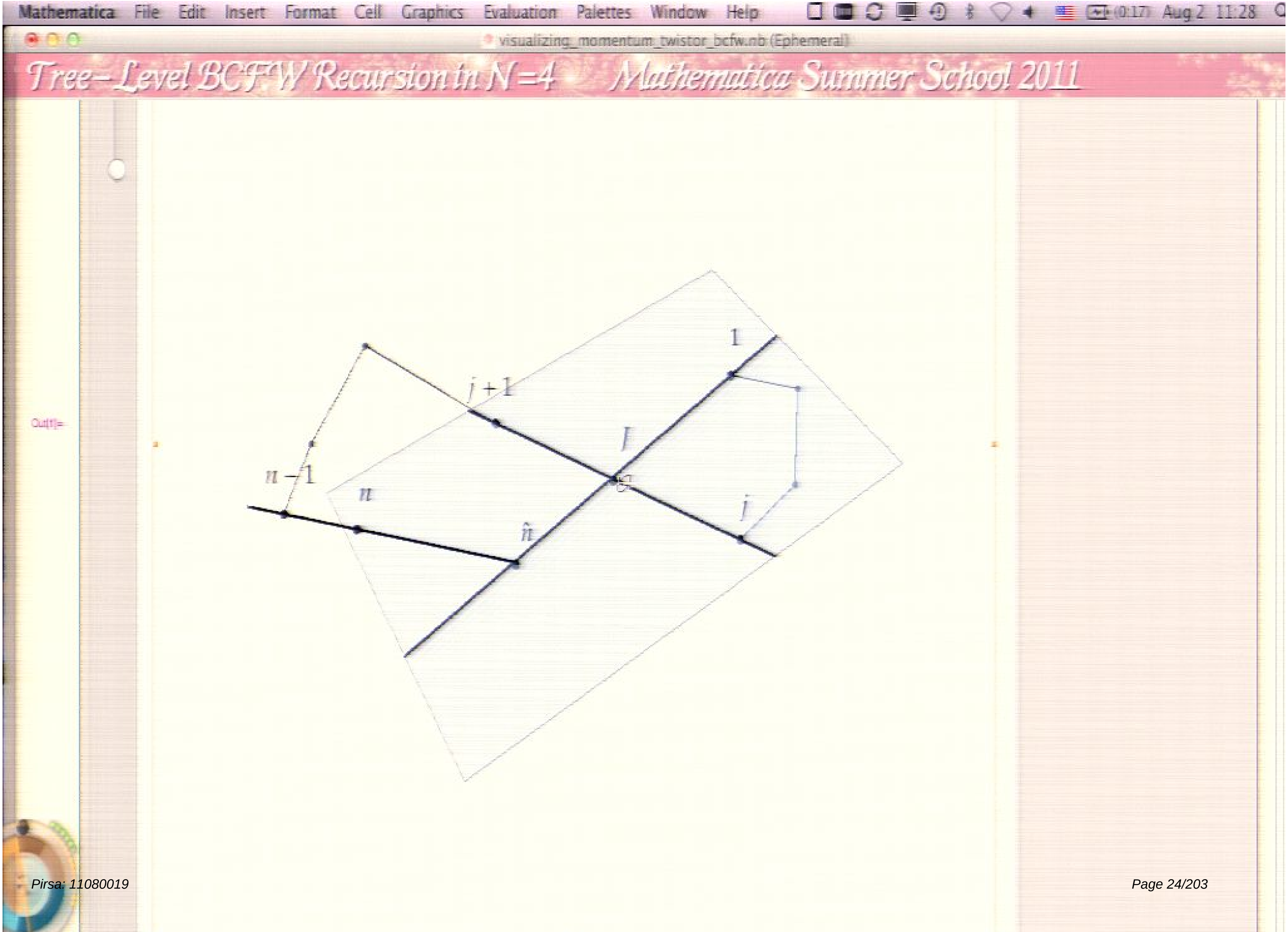

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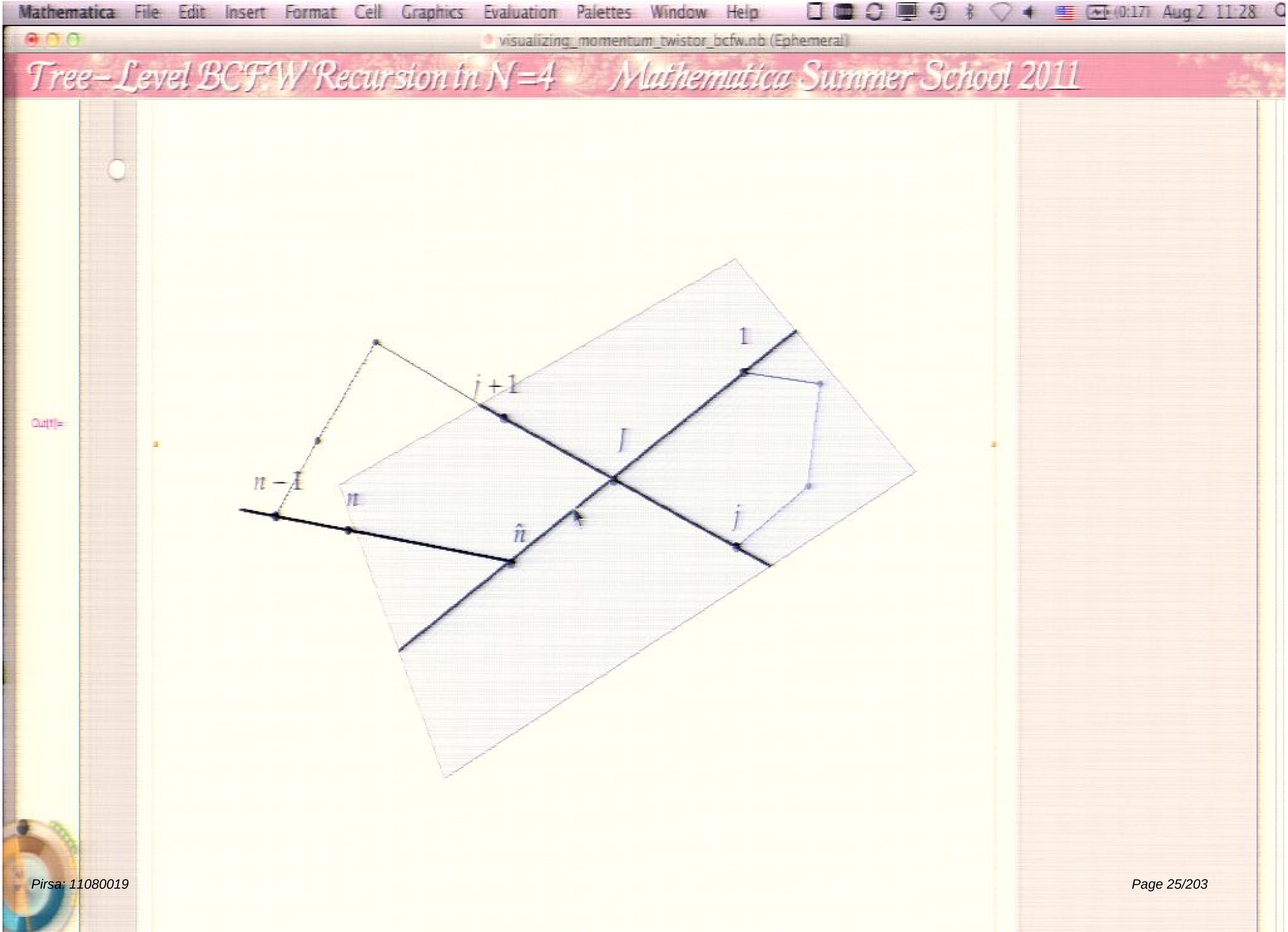
visualizing_momentum_twistor_bcfw.nb (Ephemeral)

# Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

Out[1]=

1
j + 1
J
n − 1
n
$\hat{n}$
j



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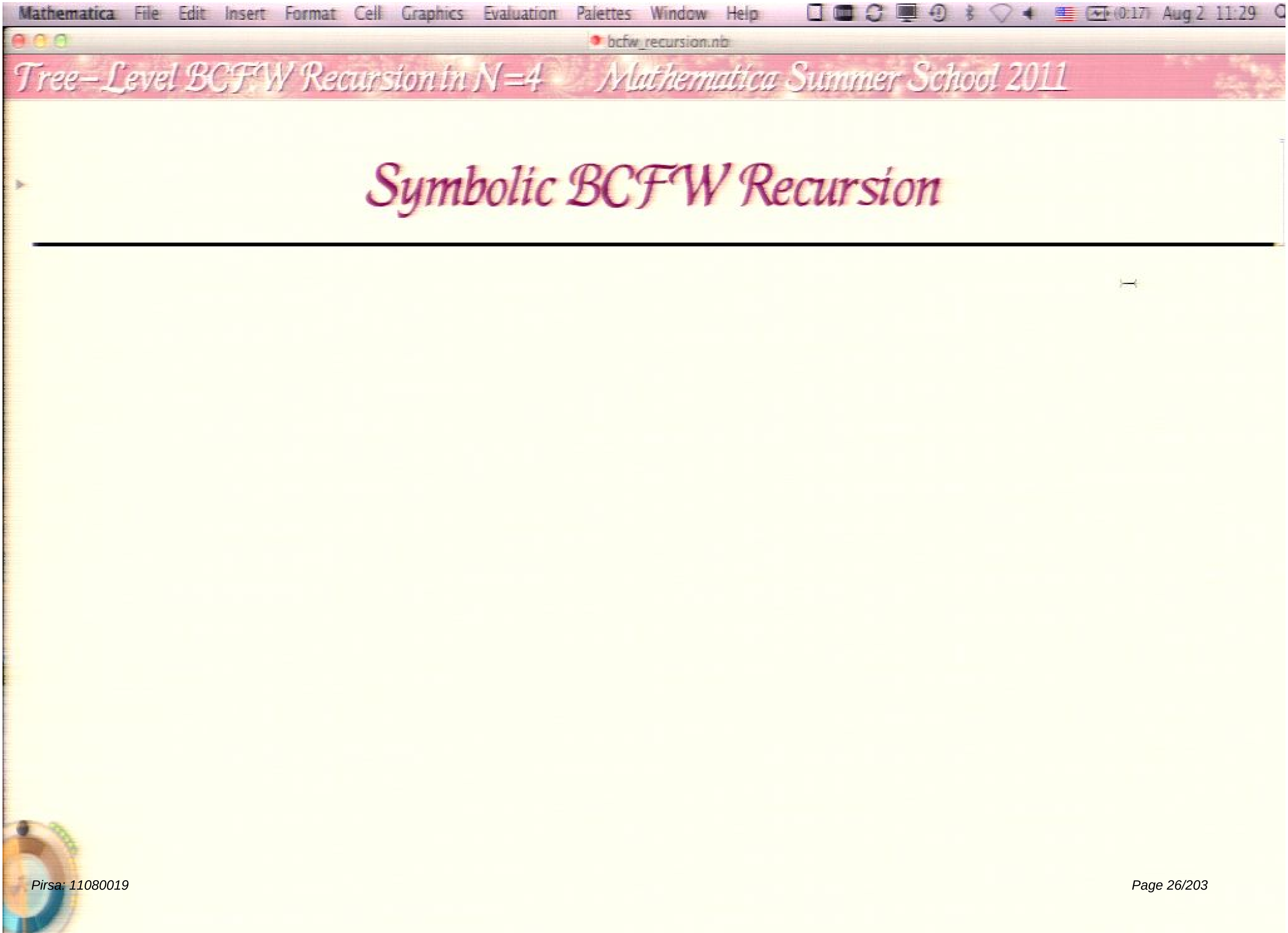
$j+1$
$I$
$n-1$
$n$
$\hat{n}$
$j$

Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011



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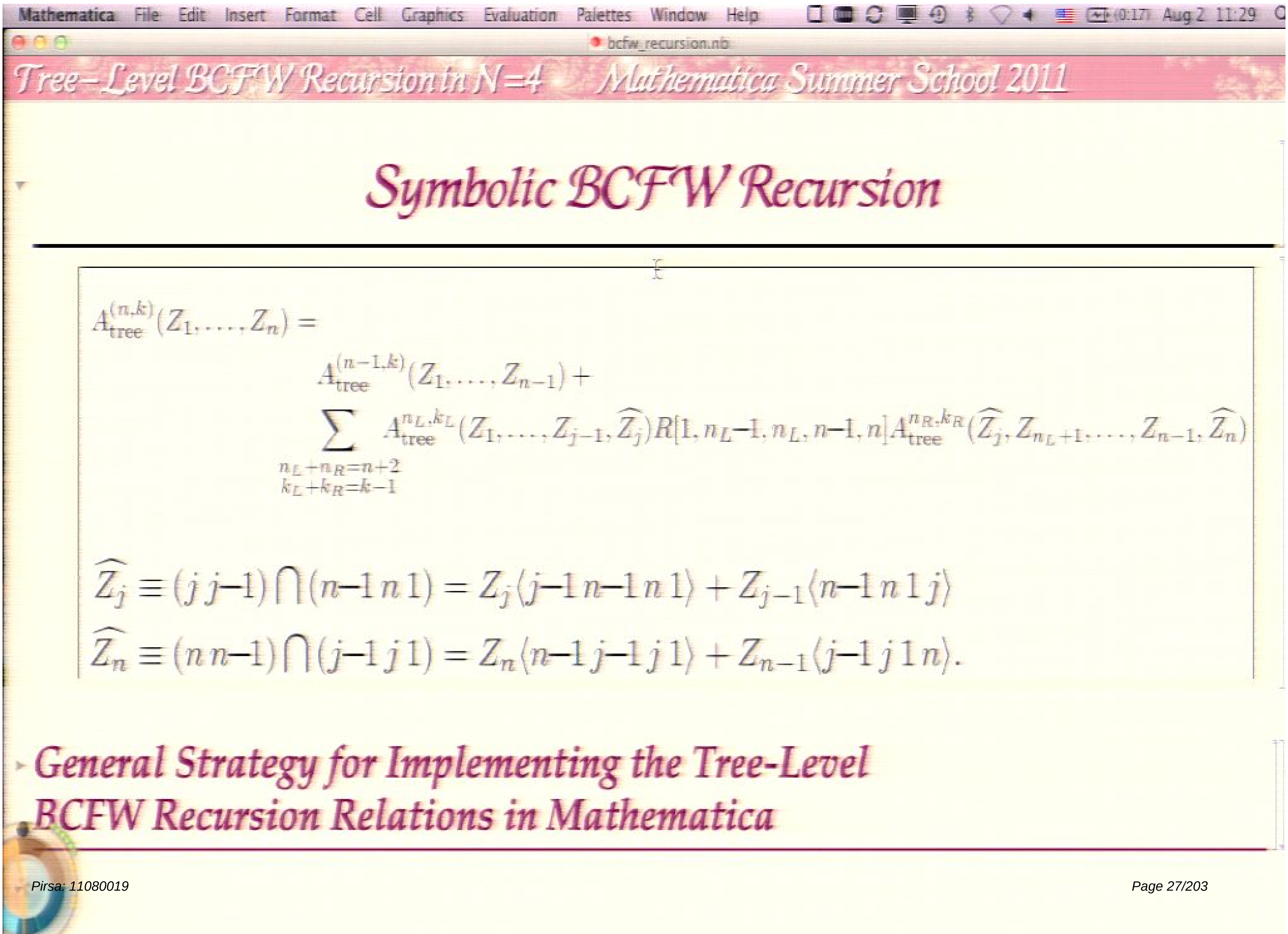
bcfw_recursion.nb

*Tree–Level BCFW Recursion in N=4*   *Mathematica Summer School 2011*

# Symbolic BCFW Recursion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bcfw_recursion.nb

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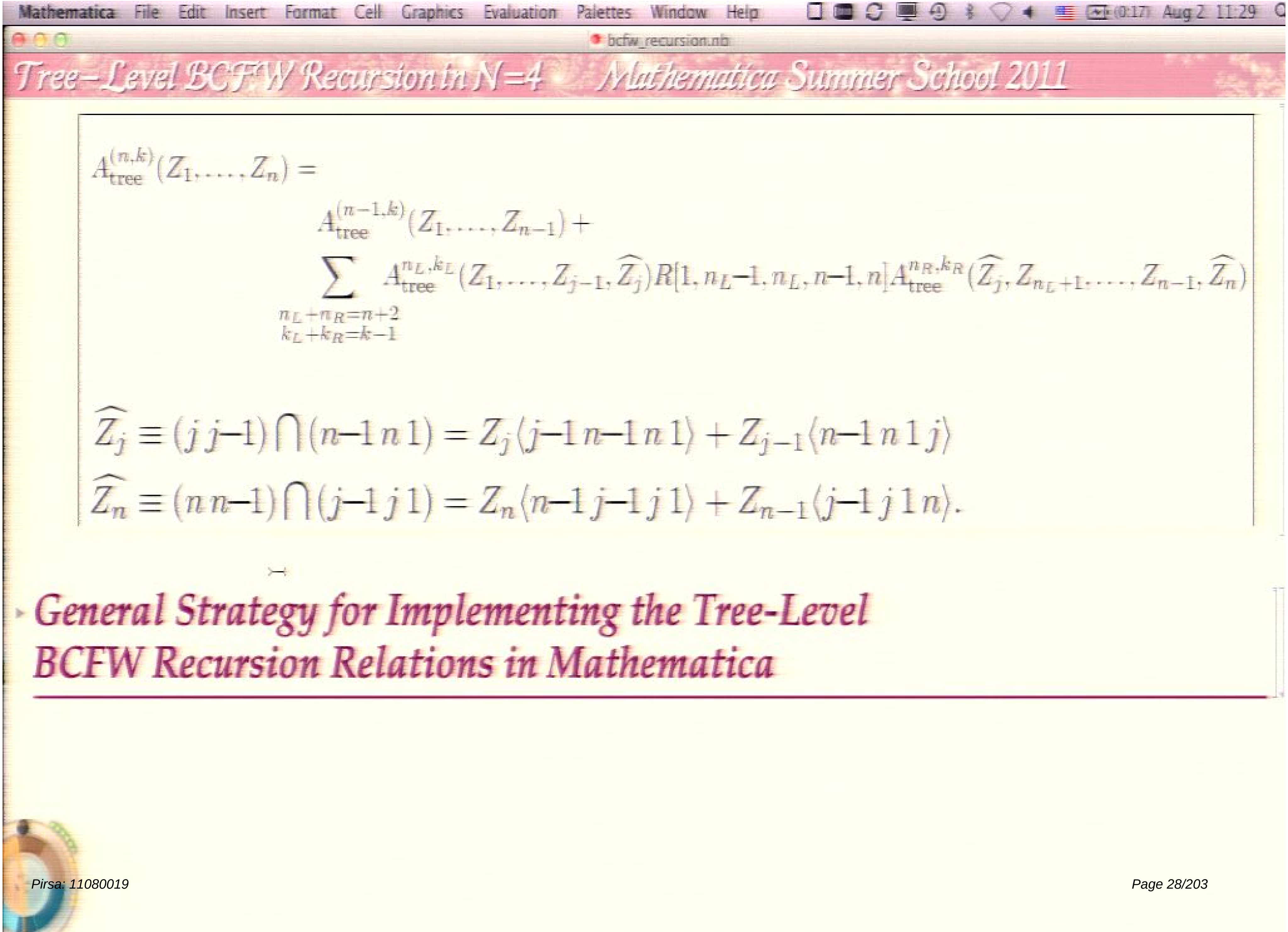
## Symbolic BCFW Recursion

$$A^{(n,k)}_{\text{tree}}(Z_1,\ldots,Z_n) = \\ A^{(n-1,k)}_{\text{tree}}(Z_1,\ldots,Z_{n-1}) + \\ \sum_{\substack{n_L+n_R=n+2\\ k_L+k_R=k-1}} A^{n_L,k_L}_{\text{tree}}(Z_1,\ldots,Z_{j-1},\widehat{Z_j})R[1,n_L-1,n_L,n-1,n]A^{n_R,k_R}_{\text{tree}}(\widehat{Z_j},Z_{n_L+1},\ldots,Z_{n-1},\widehat{Z_n})$$

$$\widehat{Z_j} \equiv (j\, j{-}1)\bigcap(n{-}1\, n\, 1) = Z_j\langle j{-}1\, n{-}1\, n\, 1\rangle + Z_{j-1}\langle n{-}1\, n\, 1\, j\rangle$$

$$\widehat{Z_n} \equiv (n\, n{-}1)\bigcap(j{-}1\, j\, 1) = Z_n\langle n{-}1\, j{-}1\, j\, 1\rangle + Z_{n-1}\langle j{-}1\, j\, 1\, n\rangle.$$

## General Strategy for Implementing the Tree-Level BCFW Recursion Relations in Mathematica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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

$$A_{\text{tree}}^{(n,k)}(Z_1,\ldots,Z_n) =$$

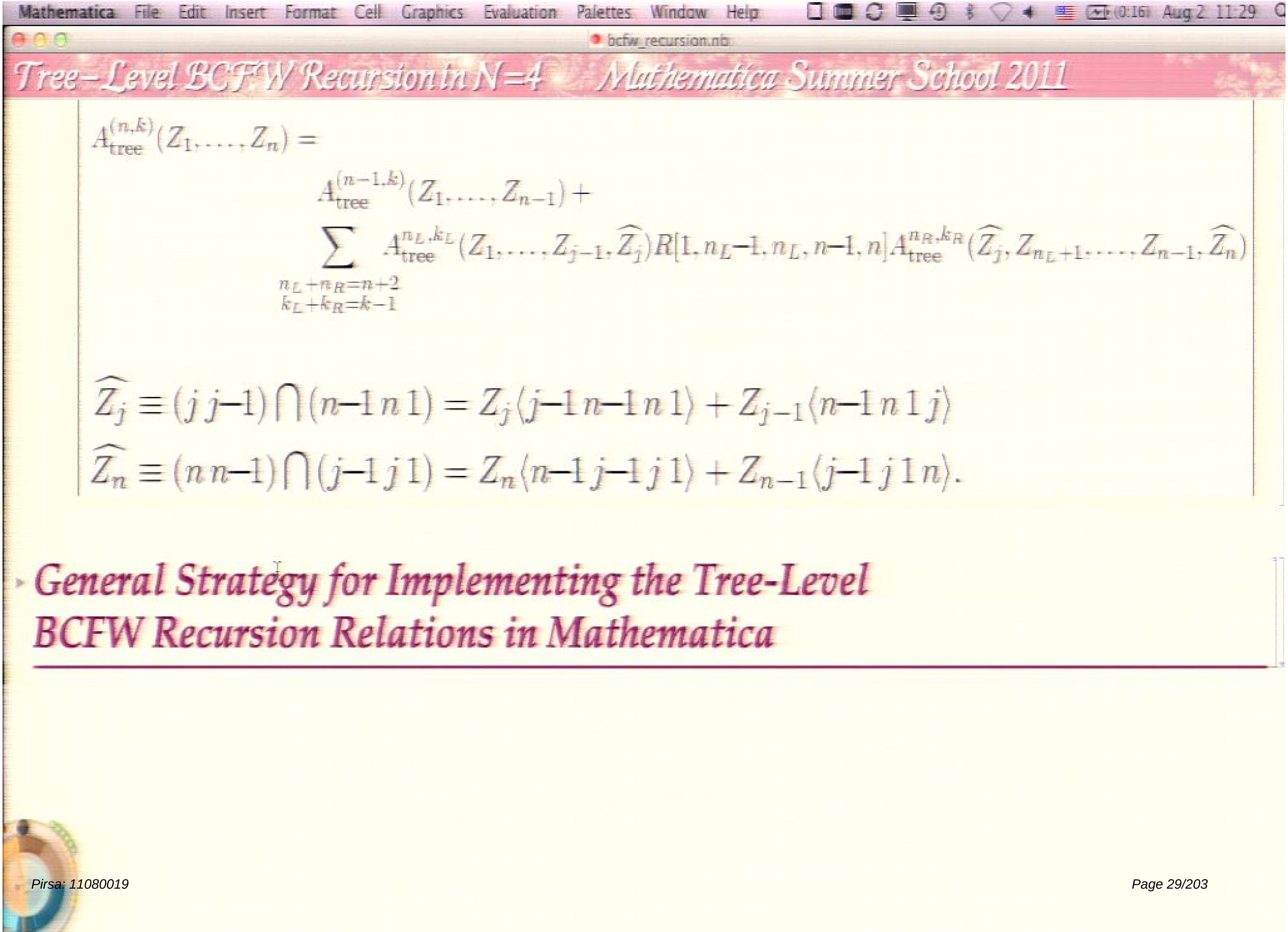
$$A_{\text{tree}}^{(n-1,k)}(Z_1,\ldots,Z_{n-1}) +$$

$$\sum_{\substack{n_L+n_R=n+2\\ k_L+k_R=k-1}} A_{\text{tree}}^{n_L,k_L}(Z_1,\ldots,Z_{j-1},\widehat{Z_j})R[1,n_L{-}1,n_L,n{-}1,n]A_{\text{tree}}^{n_R,k_R}(\widehat{Z_j},Z_{n_L+1},\ldots,Z_{n-1},\widehat{Z_n})$$

$$\widehat{Z_j} \equiv (j\,j{-}1)\bigcap(n{-}1\,n\,1) = Z_j\langle j{-}1\,n{-}1\,n\,1\rangle + Z_{j-1}\langle n{-}1\,n\,1\,j\rangle$$

$$\widehat{Z_n} \equiv (n\,n{-}1)\bigcap(j{-}1\,j\,1) = Z_n\langle n{-}1\,j{-}1\,j\,1\rangle + Z_{n-1}\langle j{-}1\,j\,1\,n\rangle.$$

## General Strategy for Implementing the Tree-Level BCFW Recursion Relations in Mathematica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bcfw_recursion.nb

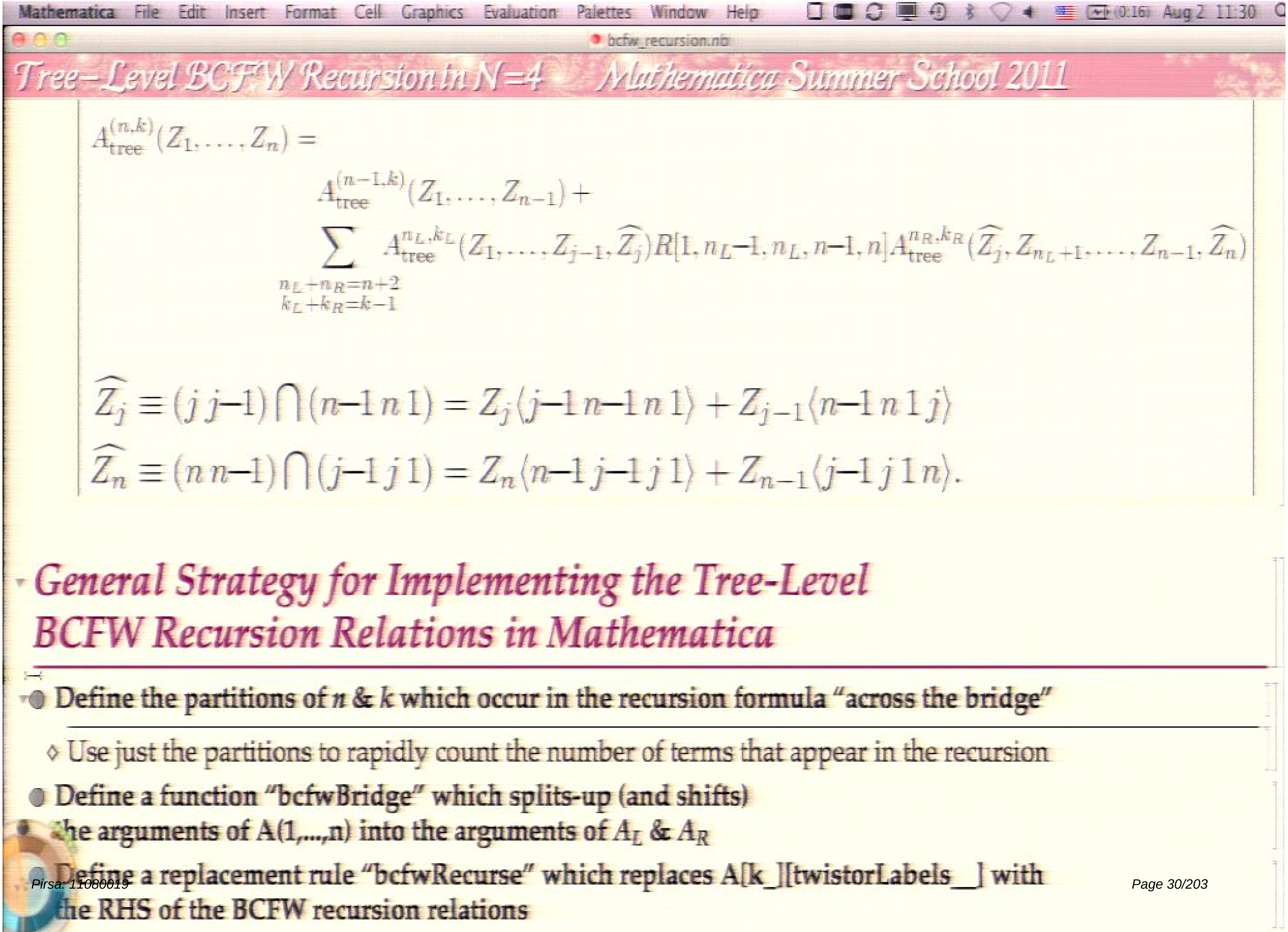
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

$$A_{\text{tree}}^{(n,k)}(Z_1,\ldots,Z_n) = A_{\text{tree}}^{(n-1,k)}(Z_1,\ldots,Z_{n-1}) + \sum_{\substack{n_L+n_R=n+2\\k_L+k_R=k-1}} A_{\text{tree}}^{n_L,k_L}(Z_1,\ldots,Z_{j-1},\widehat{Z_j})R[1,n_L-1,n_L,n-1,n]A_{\text{tree}}^{n_R,k_R}(\widehat{Z_j},Z_{n_L+1},\ldots,Z_{n-1},\widehat{Z_n})$$

$$\widehat{Z_j} \equiv (j\,j-1)\bigcap(n-1\,n\,1) = Z_j\langle j-1\,n-1\,n\,1\rangle + Z_{j-1}\langle n-1\,n\,1\,j\rangle$$

$$\widehat{Z_n} \equiv (n\,n-1)\bigcap(j-1\,j\,1) = Z_n\langle n-1\,j-1\,j\,1\rangle + Z_{n-1}\langle j-1\,j\,1\,n\rangle.$$

## General Strategy for Implementing the Tree-Level BCFW Recursion Relations in Mathematica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bcfw_recursion.nb

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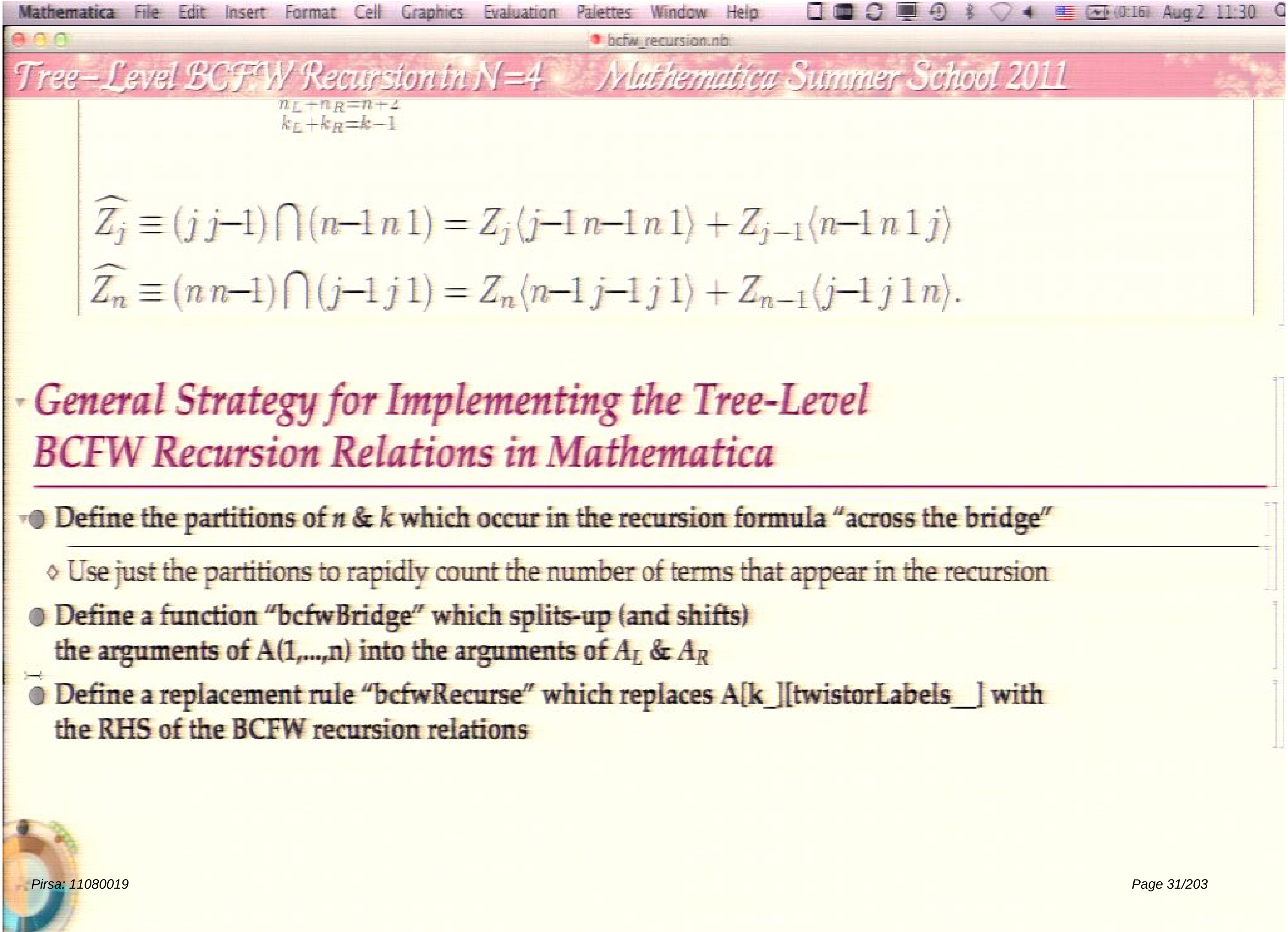
$$A_{\text{tree}}^{(n,k)}(Z_1,\ldots,Z_n) = A_{\text{tree}}^{(n-1,k)}(Z_1,\ldots,Z_{n-1}) + \sum_{\substack{n_L+n_R=n+2\\ k_L+k_R=k-1}} A_{\text{tree}}^{n_L,k_L}(Z_1,\ldots,Z_{j-1},\widehat{Z_j})R[1,n_L{-}1,n_L,n{-}1,n]A_{\text{tree}}^{n_R,k_R}(\widehat{Z_j},Z_{n_L+1},\ldots,Z_{n-1},\widehat{Z_n})$$

$$\widehat{Z_j} \equiv (j\,j{-}1)\bigcap(n{-}1\,n\,1) = Z_j\langle j{-}1\,n{-}1\,n\,1\rangle + Z_{j-1}\langle n{-}1\,n\,1\,j\rangle$$
$$\widehat{Z_n} \equiv (n\,n{-}1)\bigcap(j{-}1\,j\,1) = Z_n\langle n{-}1\,j{-}1\,j\,1\rangle + Z_{n-1}\langle j{-}1\,j\,1\,n\rangle.$$

## General Strategy for Implementing the Tree-Level BCFW Recursion Relations in Mathematica

- Define the partitions of *n* & *k* which occur in the recursion formula “across the bridge”
  - ◇ Use just the partitions to rapidly count the number of terms that appear in the recursion
- Define a function “bcfwBridge” which splits-up (and shifts)
- the arguments of A(1,...,n) into the arguments of $A_L$ & $A_R$
- Define a replacement rule “bcfwRecurse” which replaces A[k_][twistorLabels__] with the RHS of the BCFW recursion relations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bcfw_recursion.nb

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$$n_L+n_R=n+2$$
$$k_L+k_R=k-1$$

$$\widehat{Z}_j \equiv (j\, j{-}1)\bigcap(n{-}1\, n\, 1) = Z_j\langle j{-}1\, n{-}1\, n\, 1\rangle + Z_{j-1}\langle n{-}1\, n\, 1\, j\rangle$$
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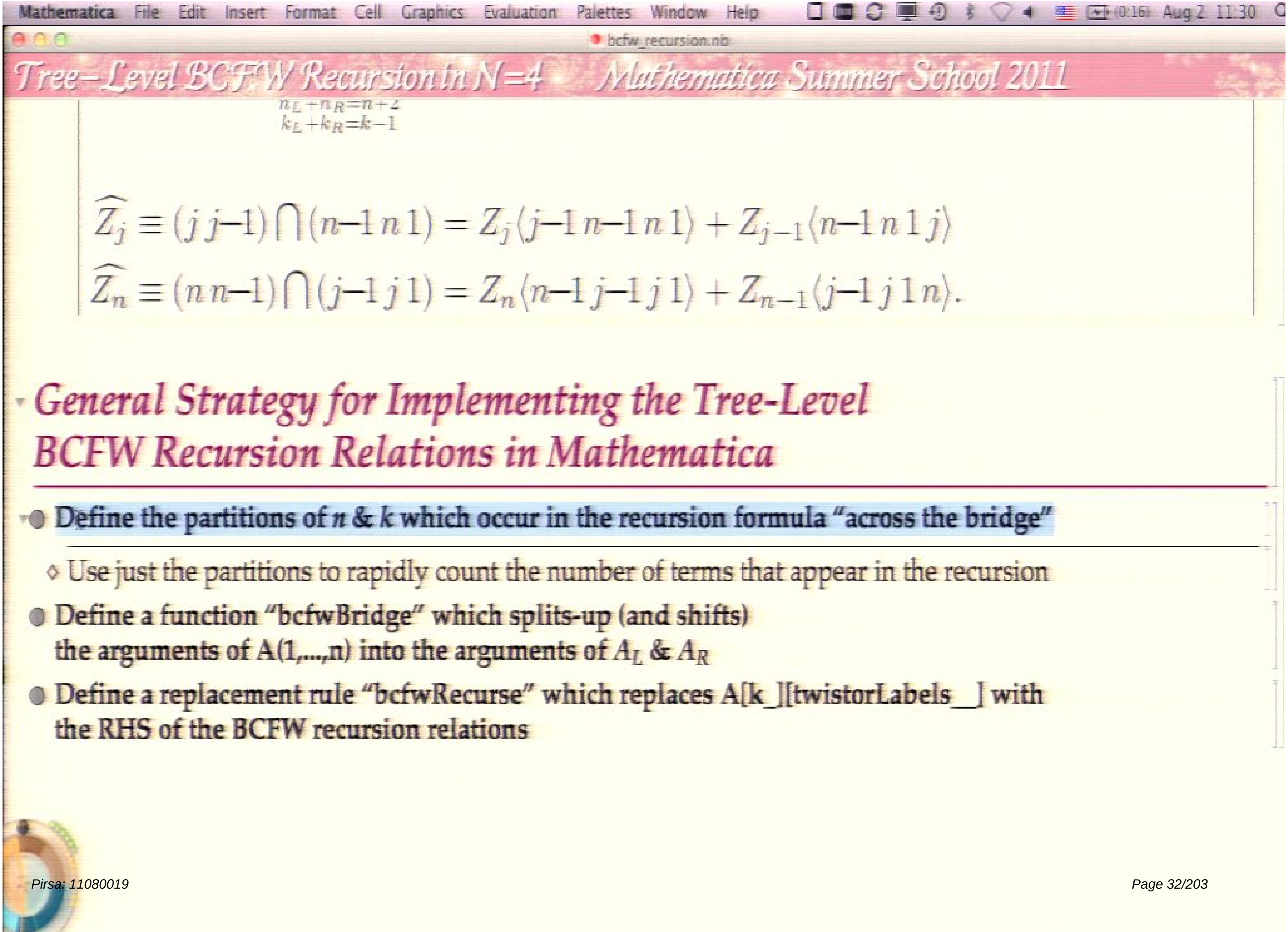
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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

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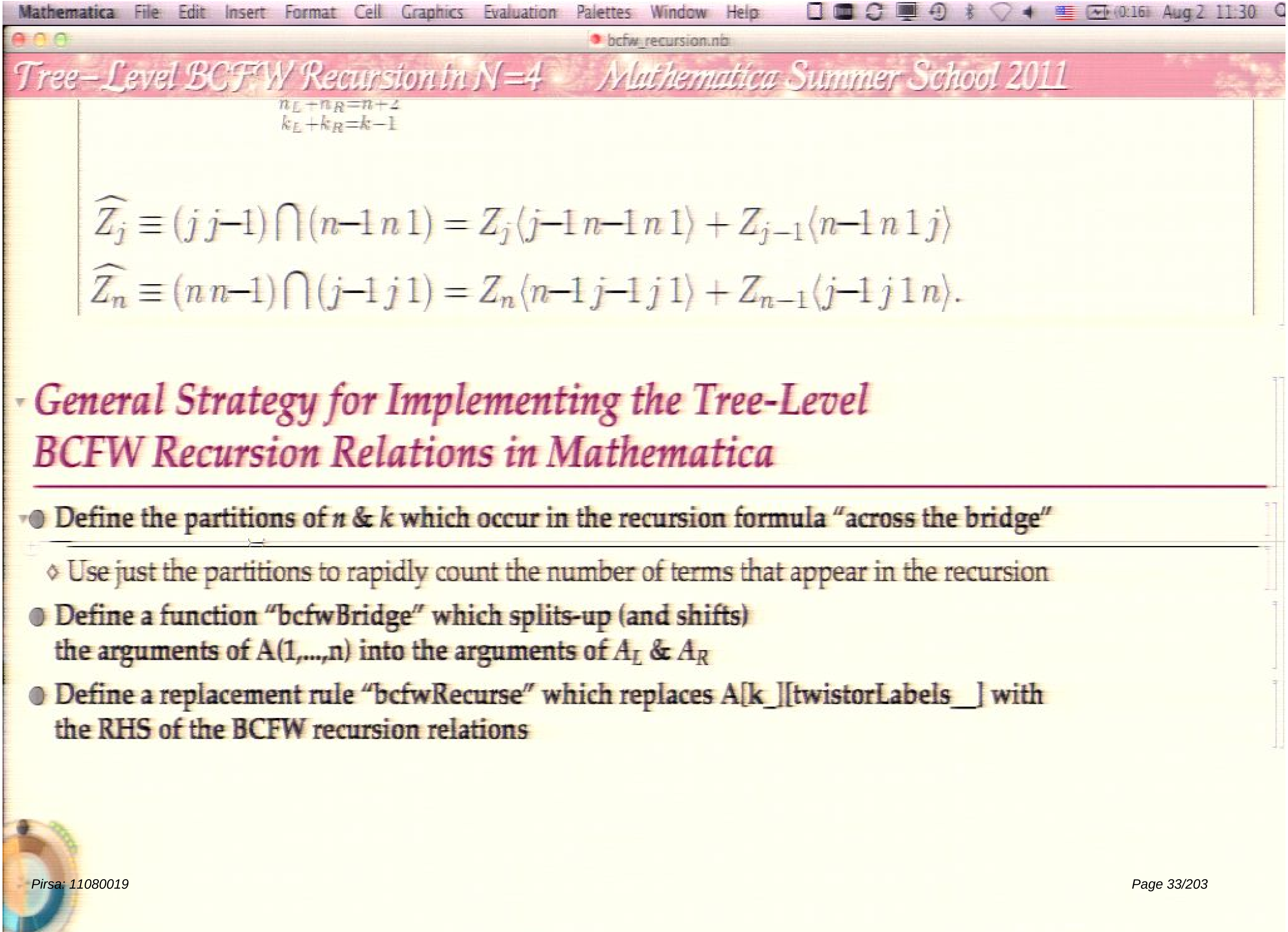
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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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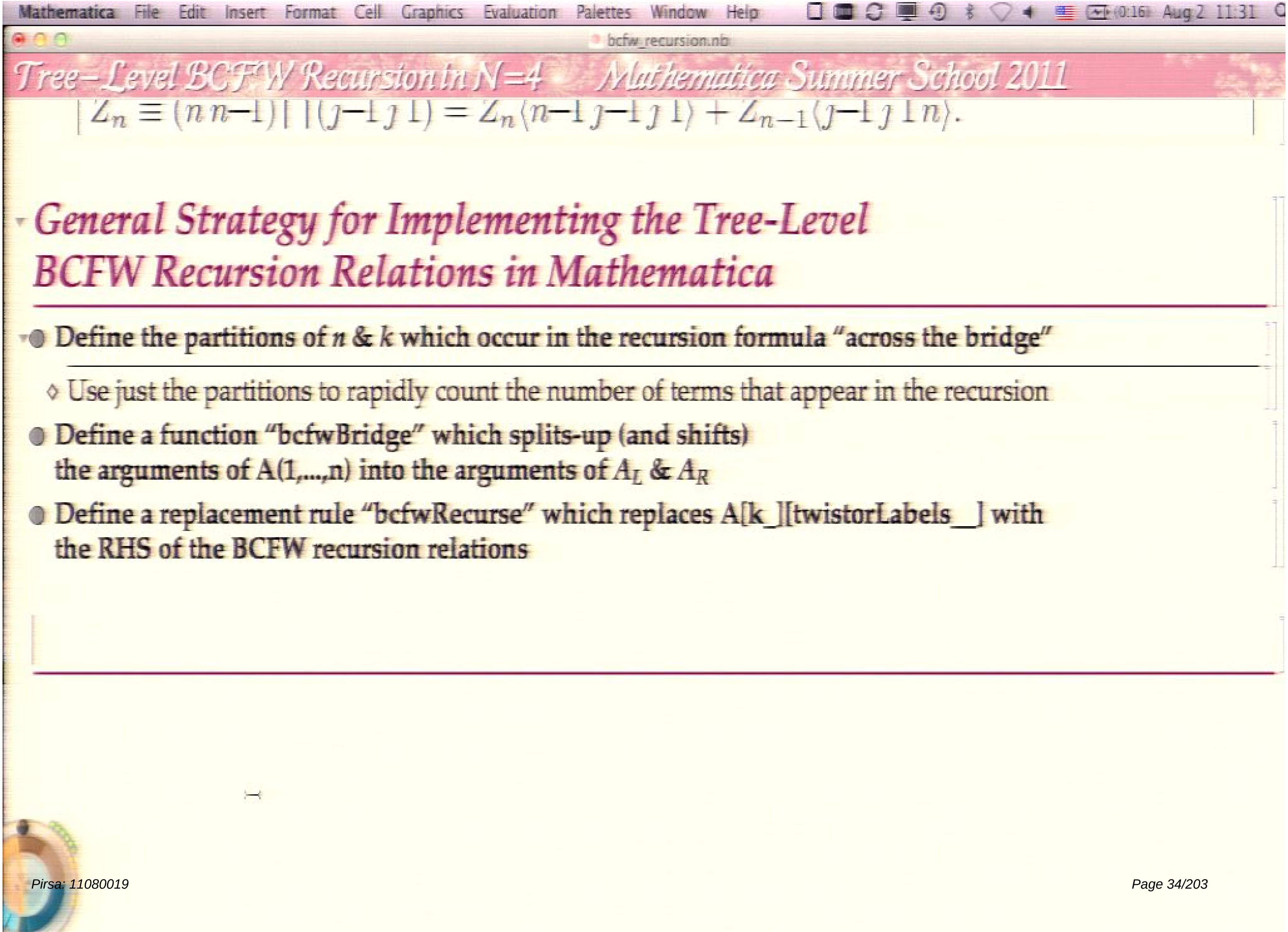

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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

$$Z_n \equiv (n\, n{-}1) \bigcap (j{-}1\, j\, 1) = Z_n \langle n{-}1\, j{-}1\, j\, 1\rangle + Z_{n-1}\langle j{-}1\, j\, 1\, n\rangle.$$

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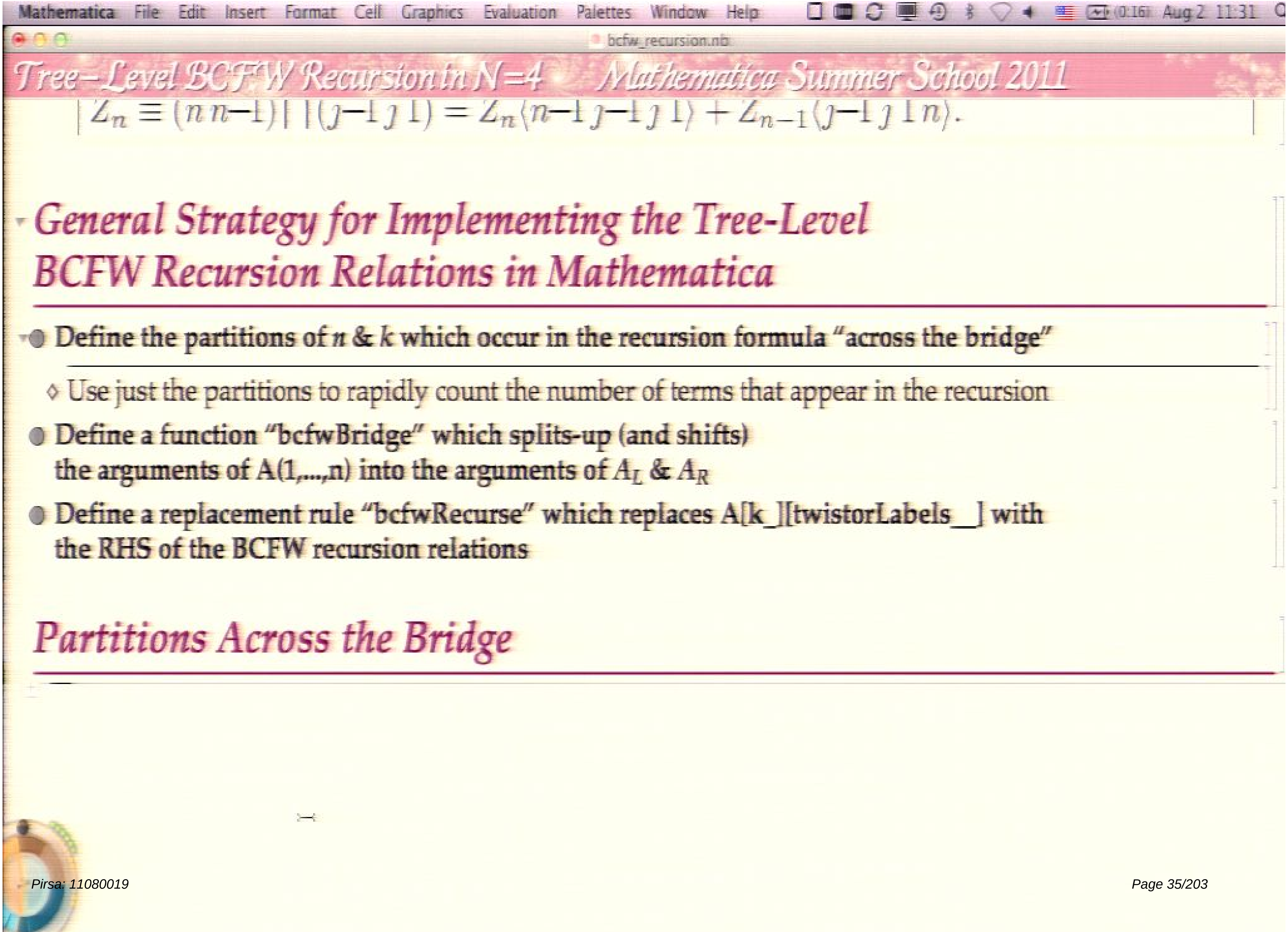
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Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

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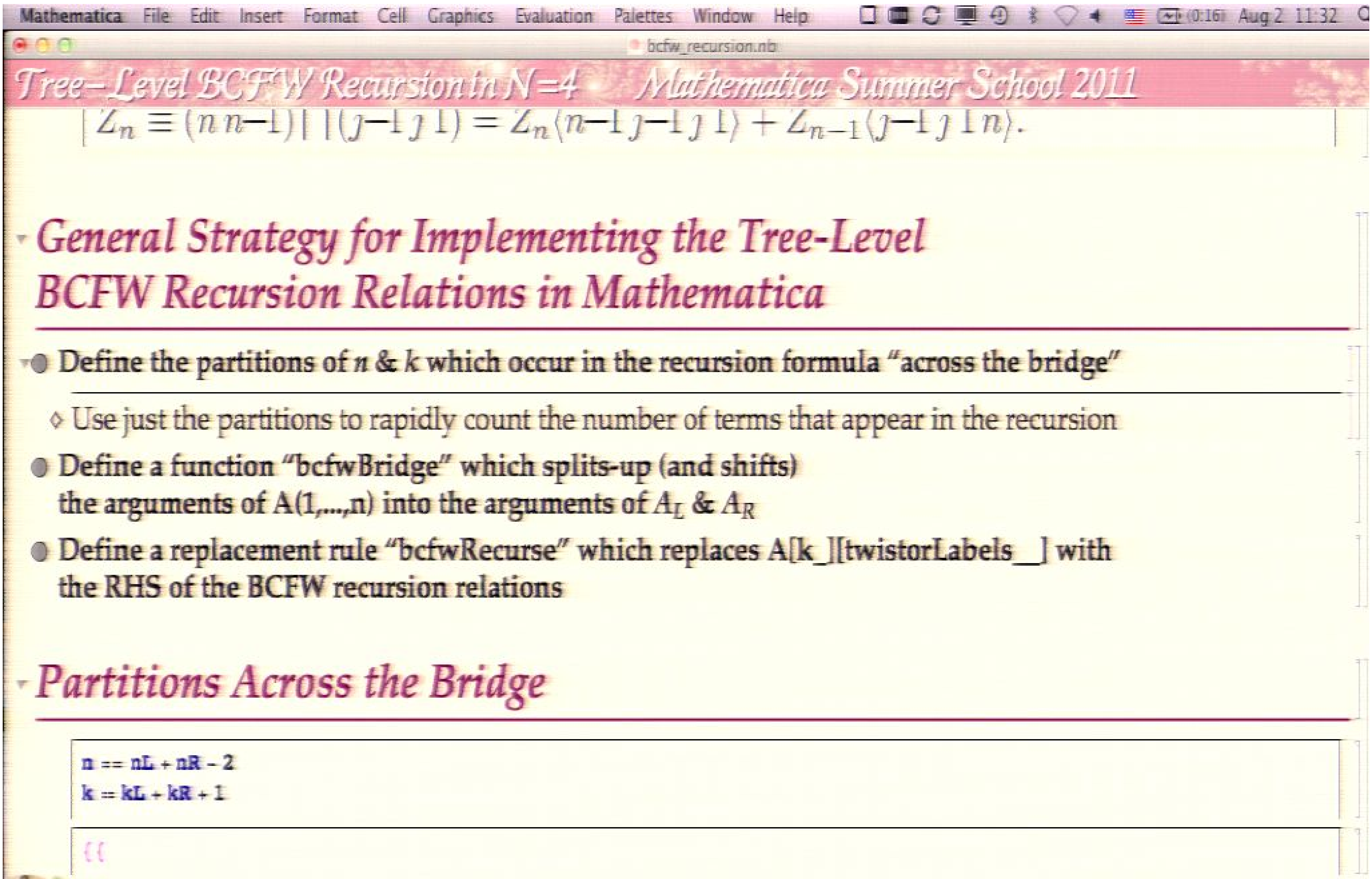
## Partitions Across the Bridge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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

$$Z_n \equiv (n\,n{-}1) \cap (j{-}1\,j\,1) = Z_n\langle n{-}1\,j{-}1\,j\,1\rangle + Z_{n-1}\langle j{-}1\,j\,1\,n\rangle.$$

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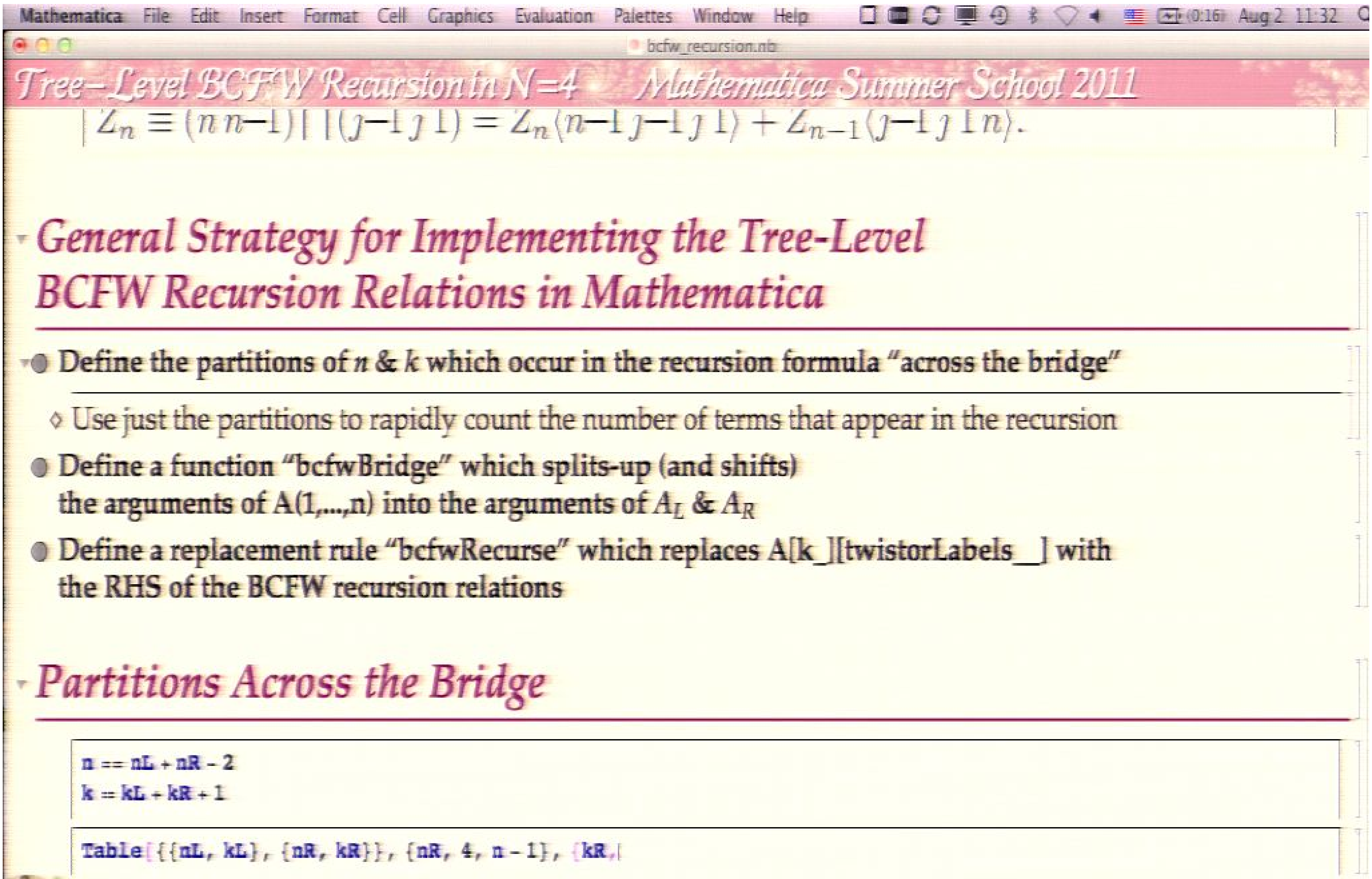
## Partitions Across the Bridge

```
n == nL + nR - 2
k == kL + kR + 1
```

```
{{
```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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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```

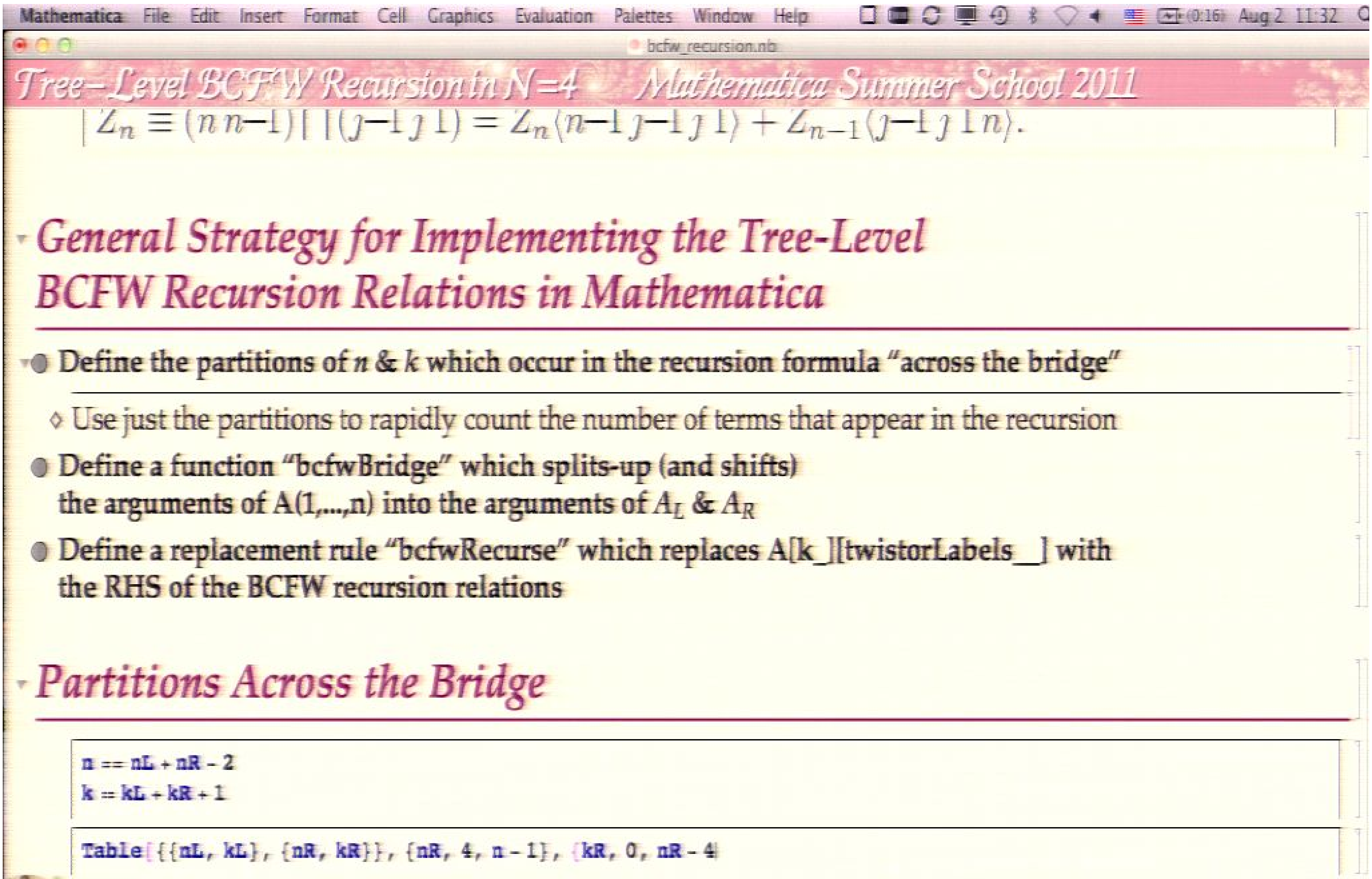
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```
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Table[{{nL, kL}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4
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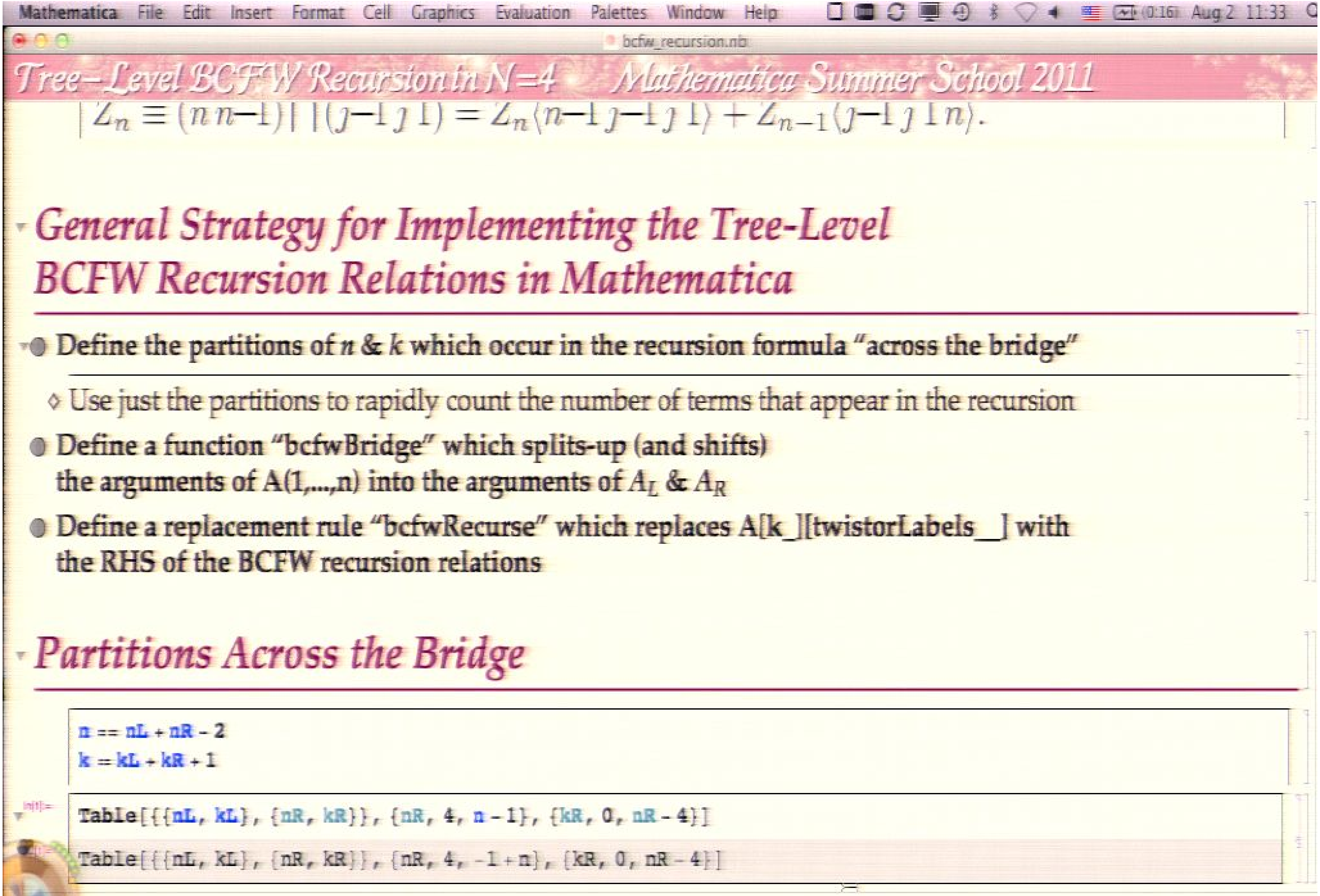
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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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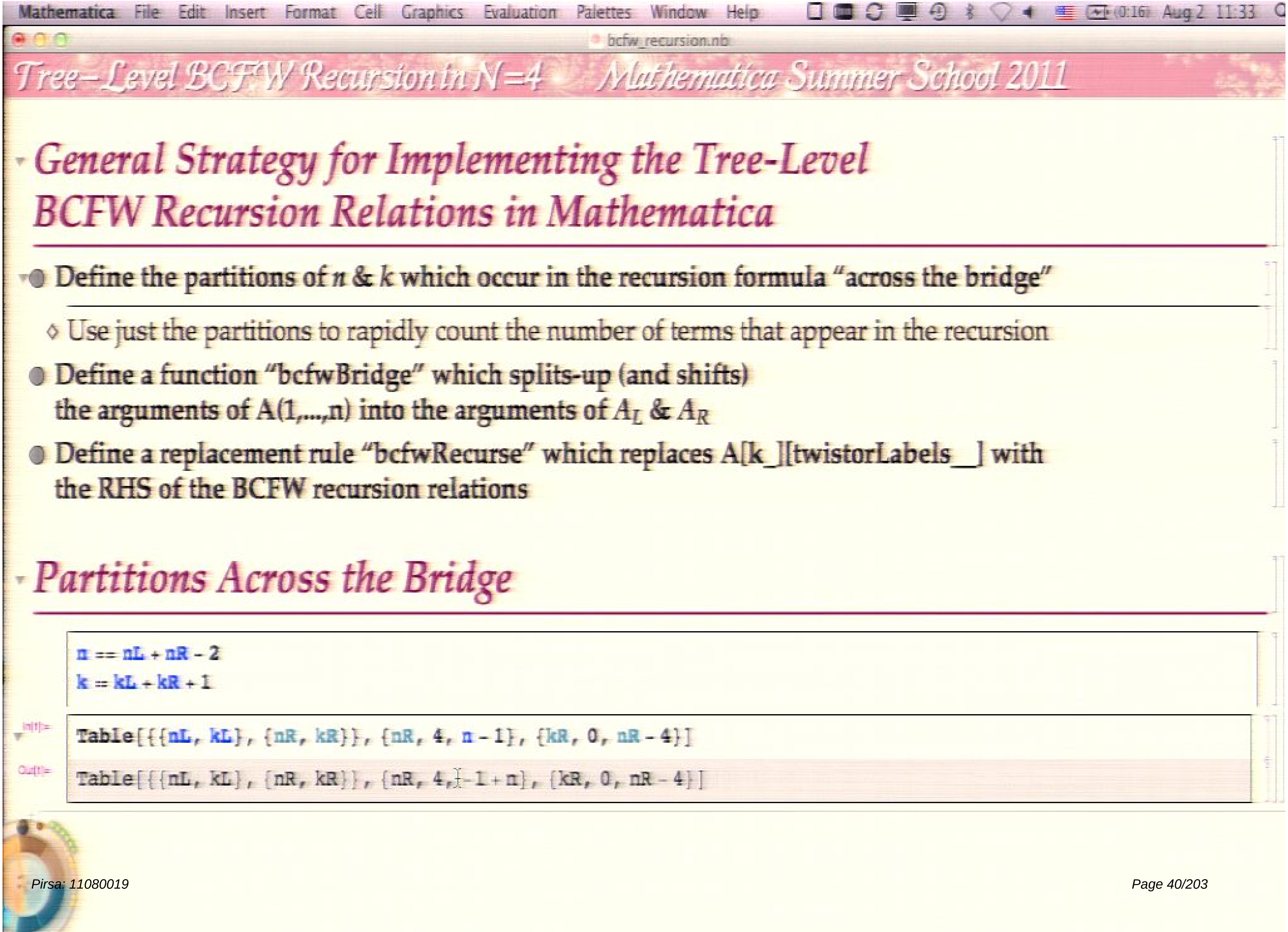
## Partitions Across the Bridge

```
n == nL + nR - 2
k = kL + kR + 1
```

```
In[1]:= Table[{{nL, kL}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}]
```

```
Table[{{nL, kL}, {nR, kR}}, {nR, 4, -1 + n}, {kR, 0, nR - 4}]
```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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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```
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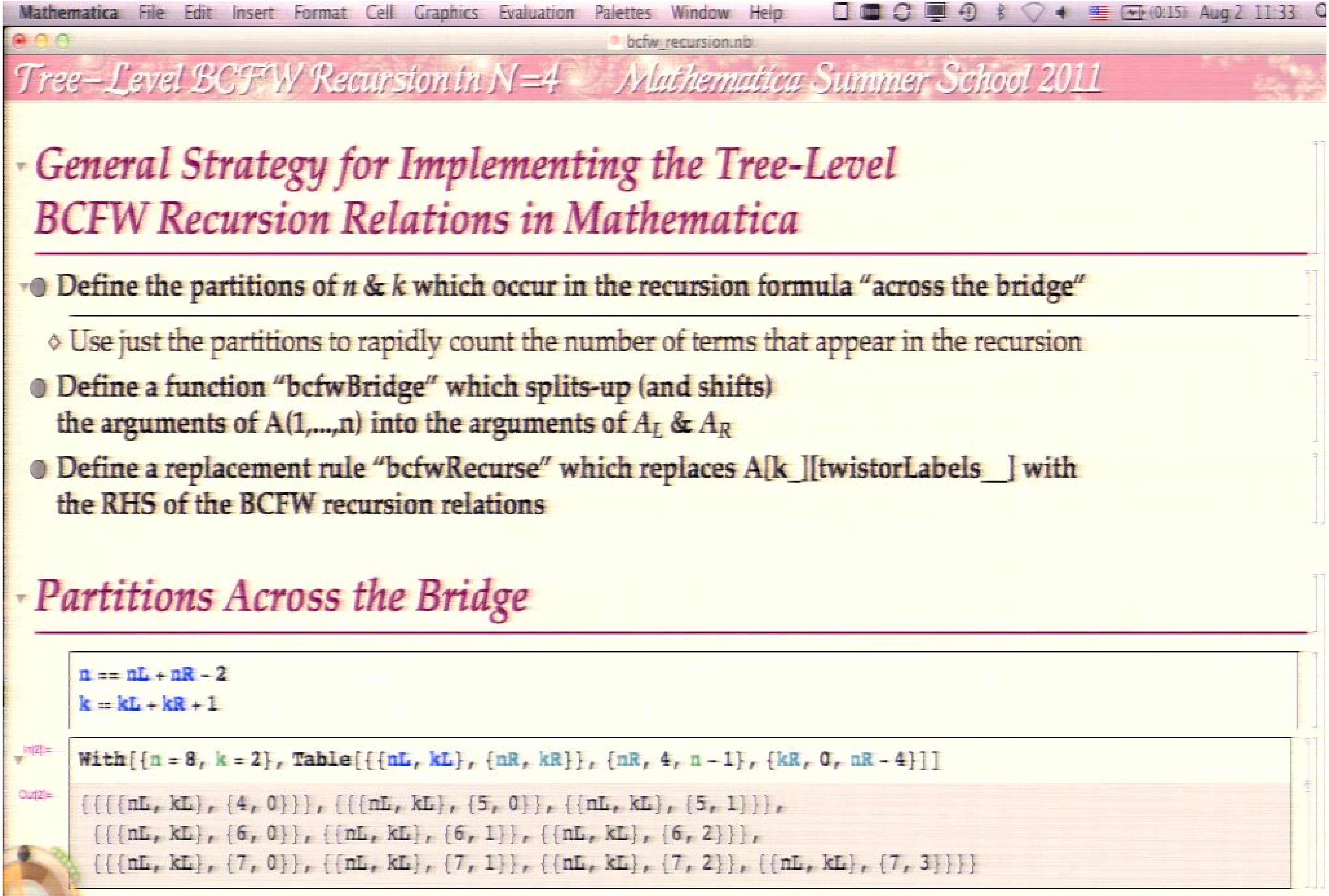
```
In[1]:= Table[{{nL, kL}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}]

Out[1]= Table[{{nL, kL}, {nR, kR}}, {nR, 4, -1 + n}, {kR, 0, nR - 4}]
```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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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## Partitions Across the Bridge

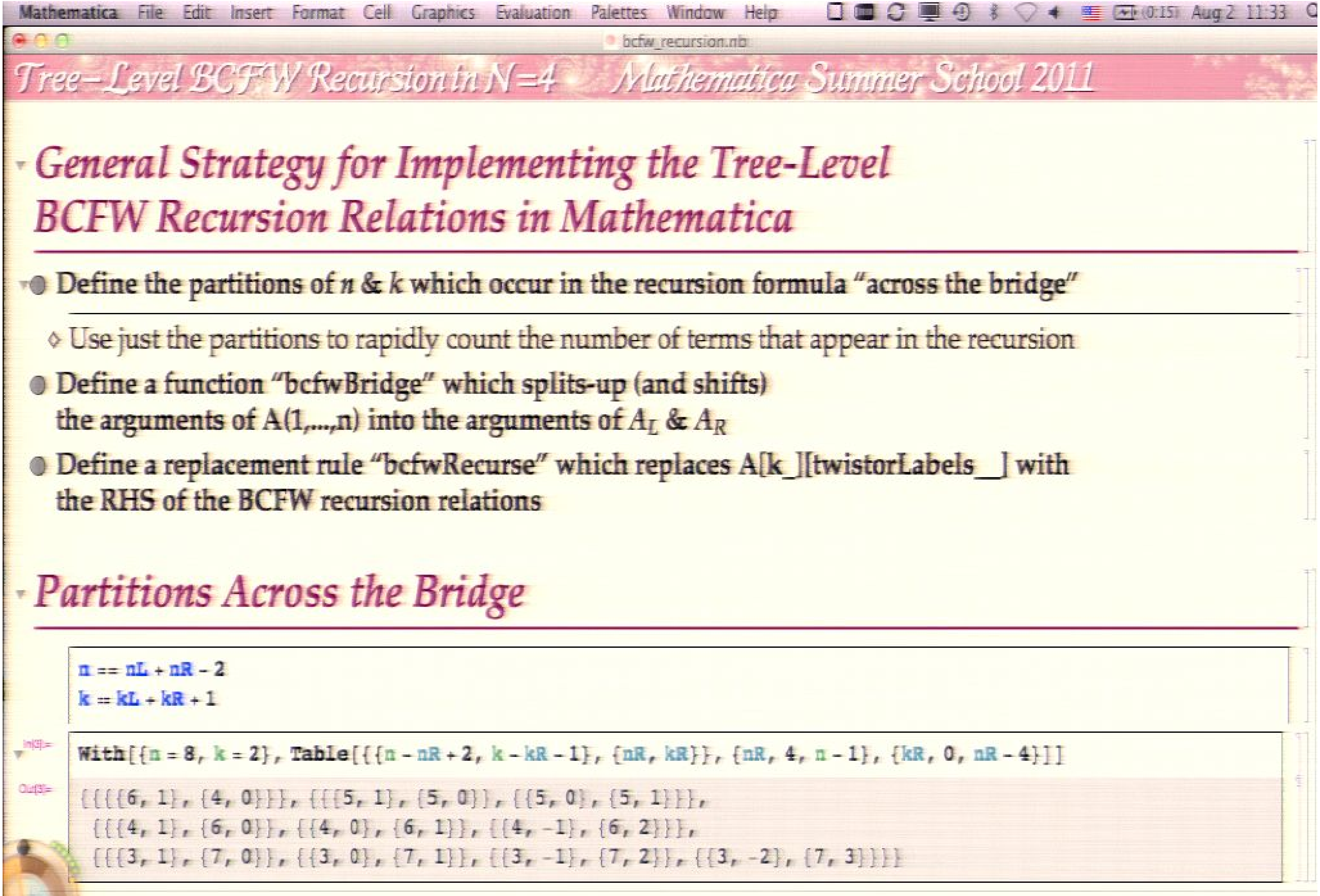
```
n == nL + nR - 2
k == kL + kR + 1
```

In[2]:=

```
With[{n = 8, k = 2}, Table[{{nL, kL}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}]]
```

Out[2]=

```
{{{{nL, kL}, {4, 0}}}, {{{nL, kL}, {5, 0}}, {{nL, kL}, {5, 1}}},
 {{{nL, kL}, {6, 0}}, {{nL, kL}, {6, 1}}, {{nL, kL}, {6, 2}}},
 {{{nL, kL}, {7, 0}}, {{nL, kL}, {7, 1}}, {{nL, kL}, {7, 2}}, {{nL, kL}, {7, 3}}}}
```



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Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

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# Partitions Across the Bridge

```
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```

```
In[3]:= With[{n = 8, k = 2}, Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}]]

Out[3]= {{{{6, 1}, {4, 0}}}, {{{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}},
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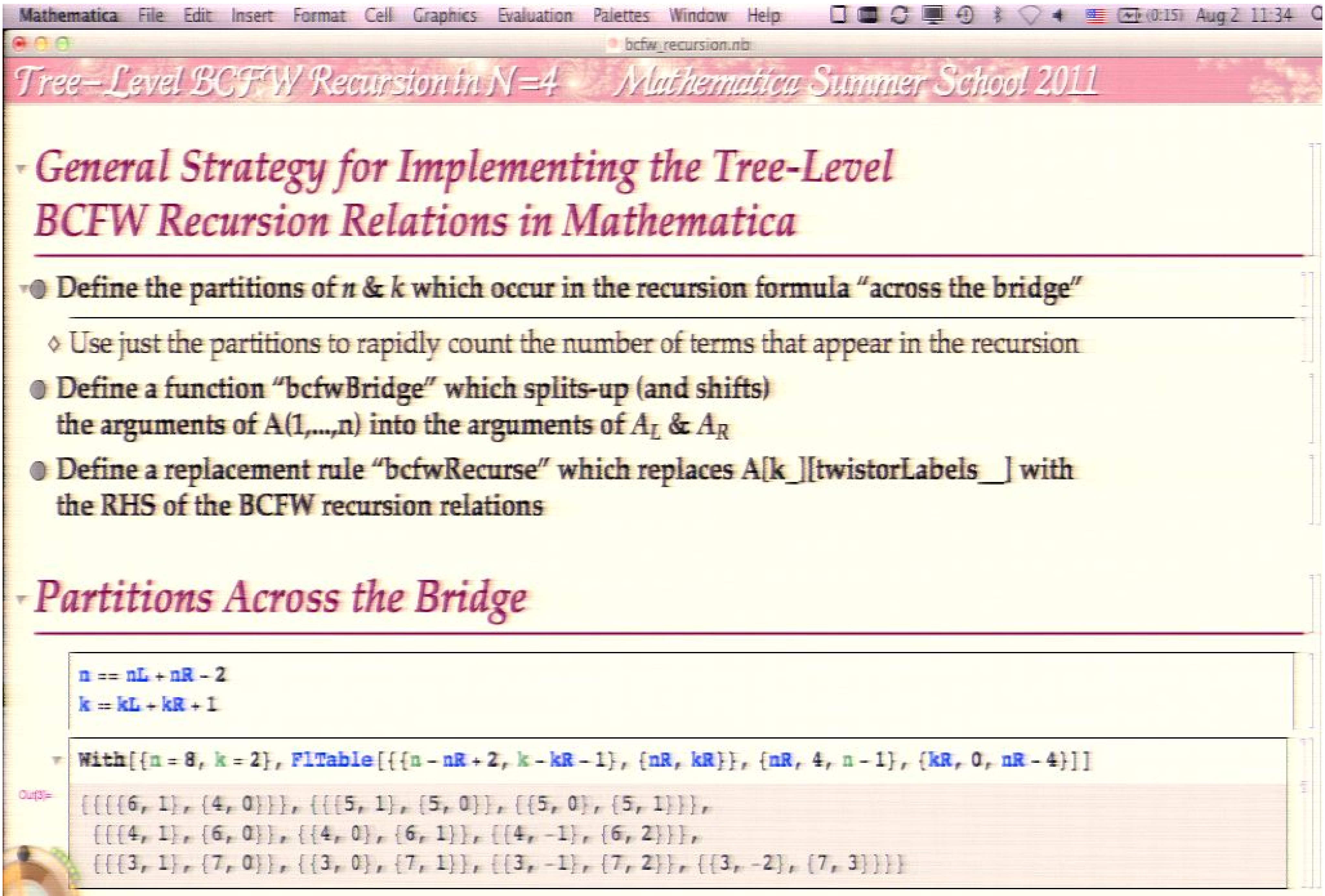
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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

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```
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With[{n = 8, k = 2}, FlTable[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}]]
```

Out[3]=

```
{{{{6, 1}, {4, 0}}}, {{{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}},
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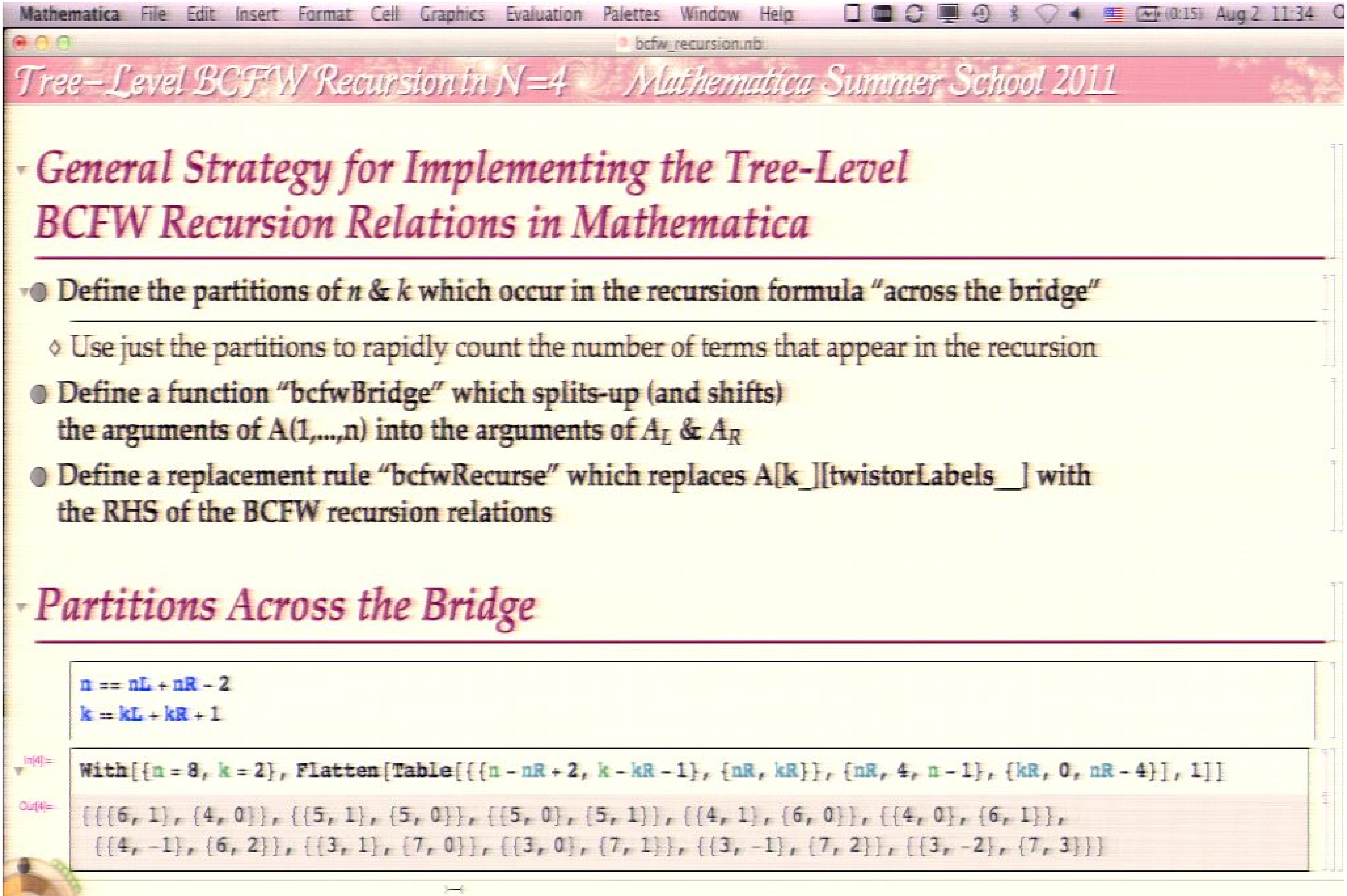
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Tree–Level BCFW Recursion in N=4  Mathematica Summer School 2011

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# Partitions Across the Bridge

```
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```
In[4]:= With[{n = 8, k = 2}, Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1]]

Out[4]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 1}, {6, 0}}, {{4, 0}, {6, 1}},
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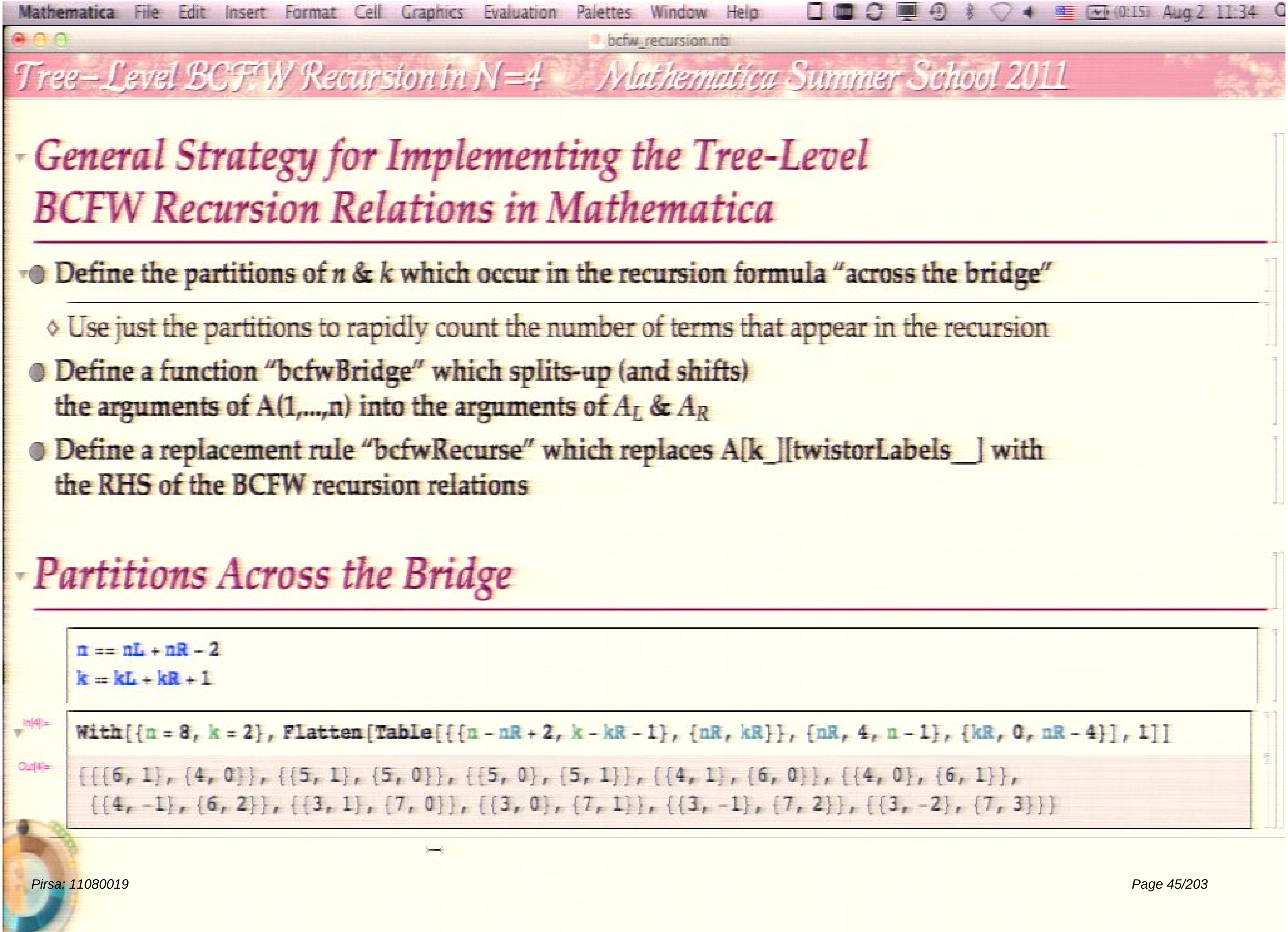
```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bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011
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Out[4]=

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```
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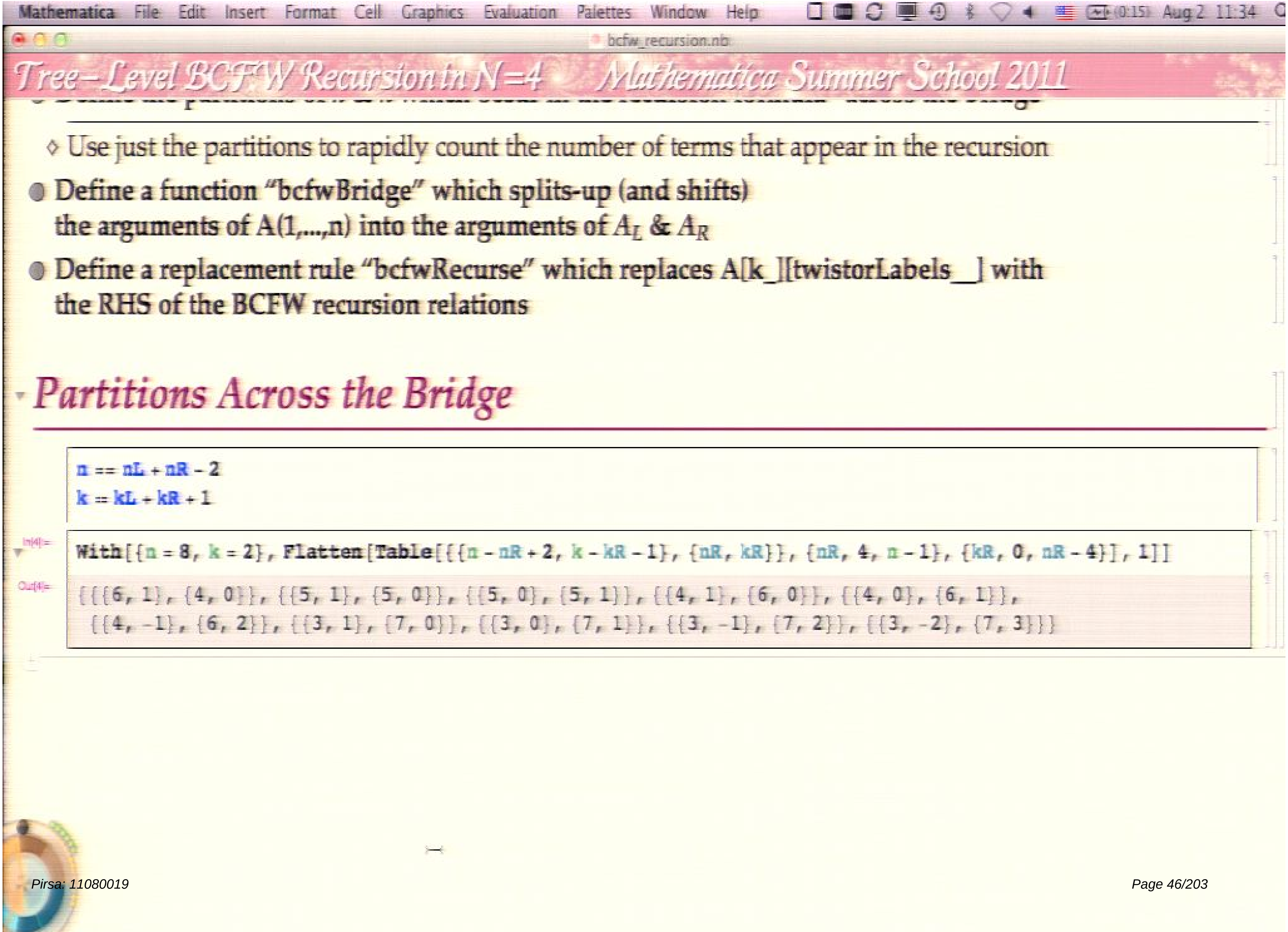
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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

◇ Use just the partitions to rapidly count the number of terms that appear in the recursion

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## *Partitions Across the Bridge*

```
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```
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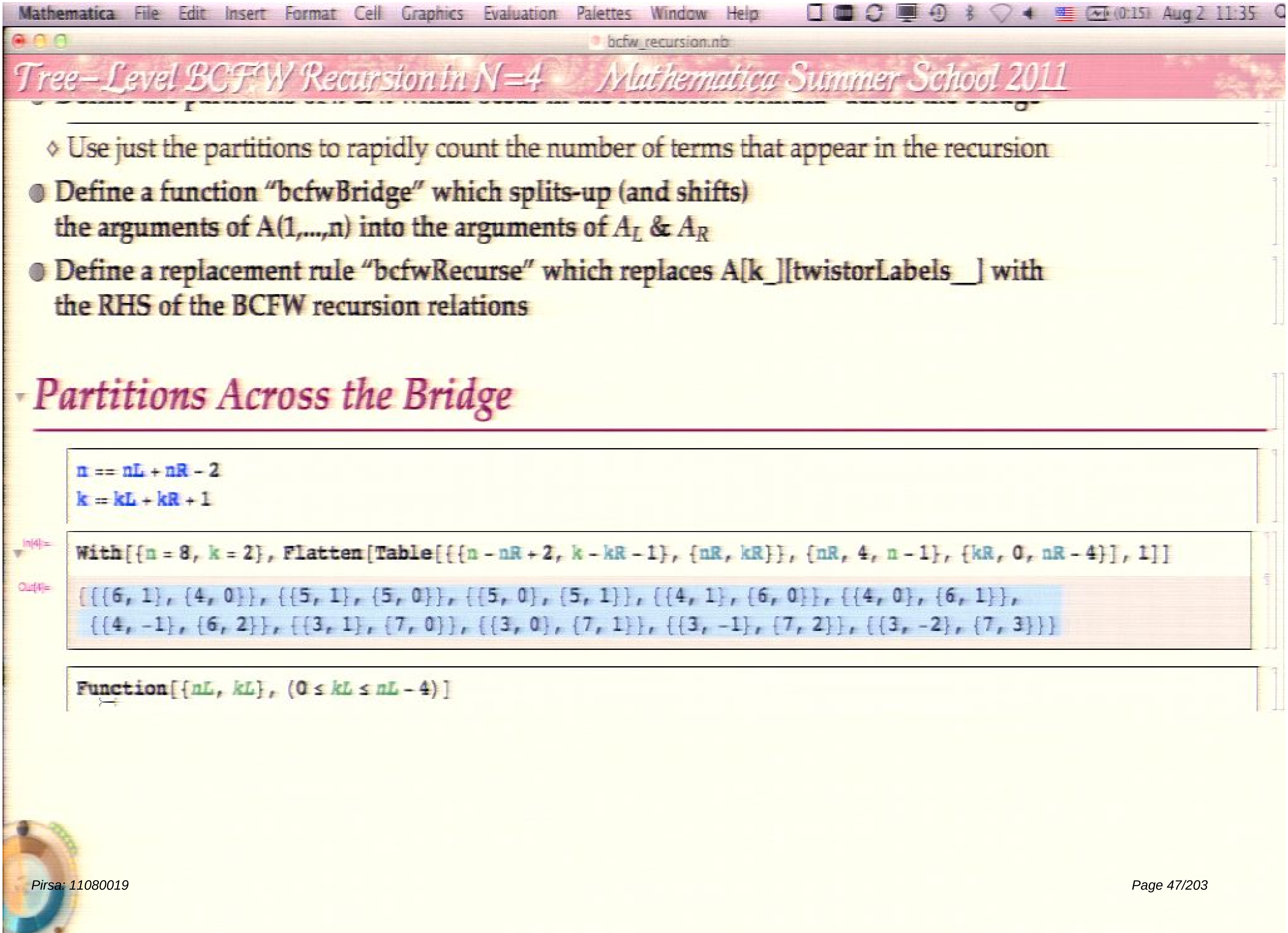
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bcfw_recursion.nb
In[4]:=
Out[4]=

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

- ◇ Use just the partitions to rapidly count the number of terms that appear in the recursion
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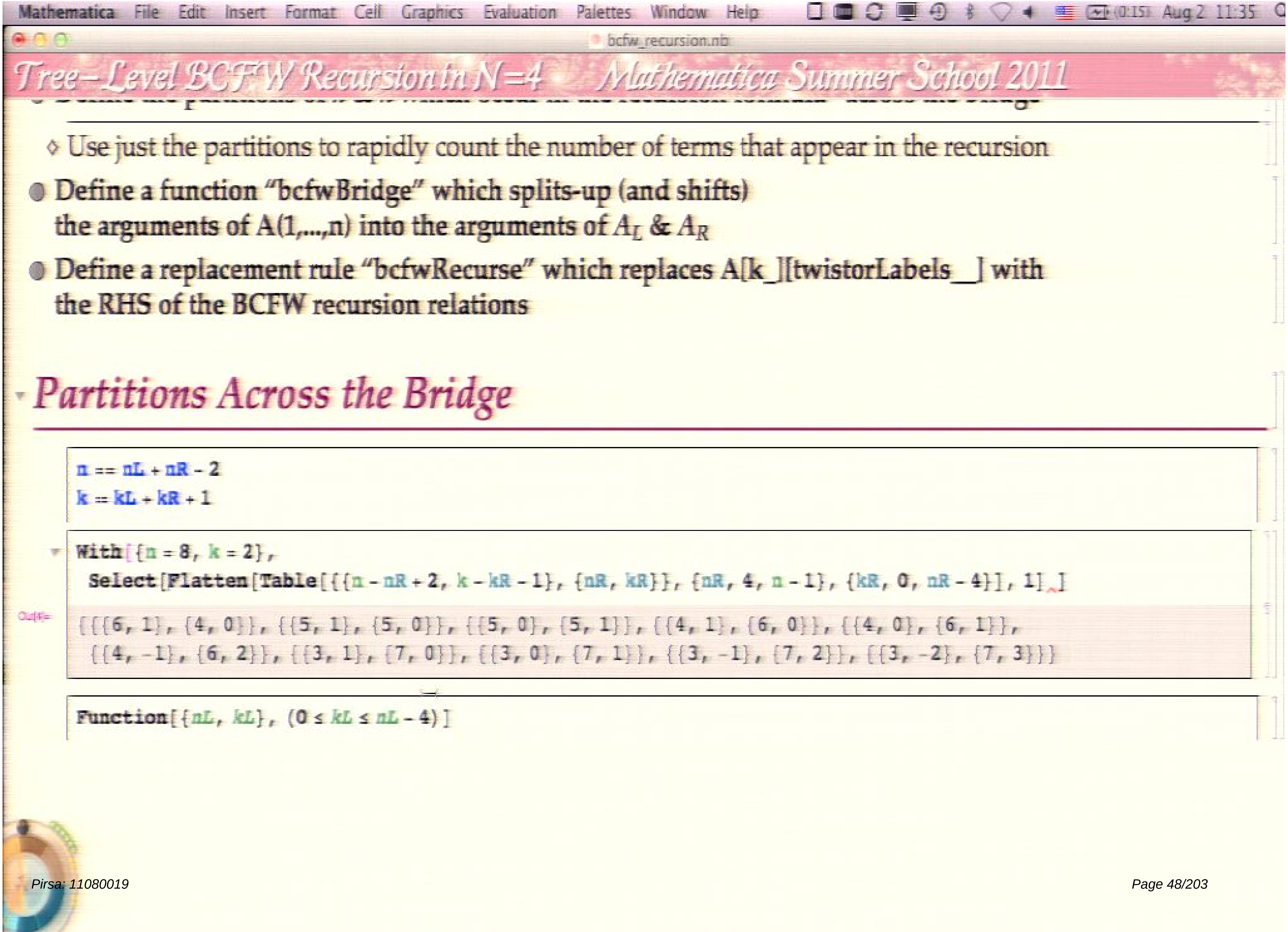
 {{4, -1}, {6, 2}}, {{3, 1}, {7, 0}}, {{3, 0}, {7, 1}}, {{3, -1}, {7, 2}}, {{3, -2}, {7, 3}}}
```

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
```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bcfw_recursion.nb

*Tree–Level BCFW Recursion in N=4* *Mathematica Summer School 2011*

- ◇ Use just the partitions to rapidly count the number of terms that appear in the recursion
- **Define a function “bcfwBridge” which splits-up (and shifts) the arguments of A(1,...,n) into the arguments of $A_L$ & $A_R$**
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## *Partitions Across the Bridge*

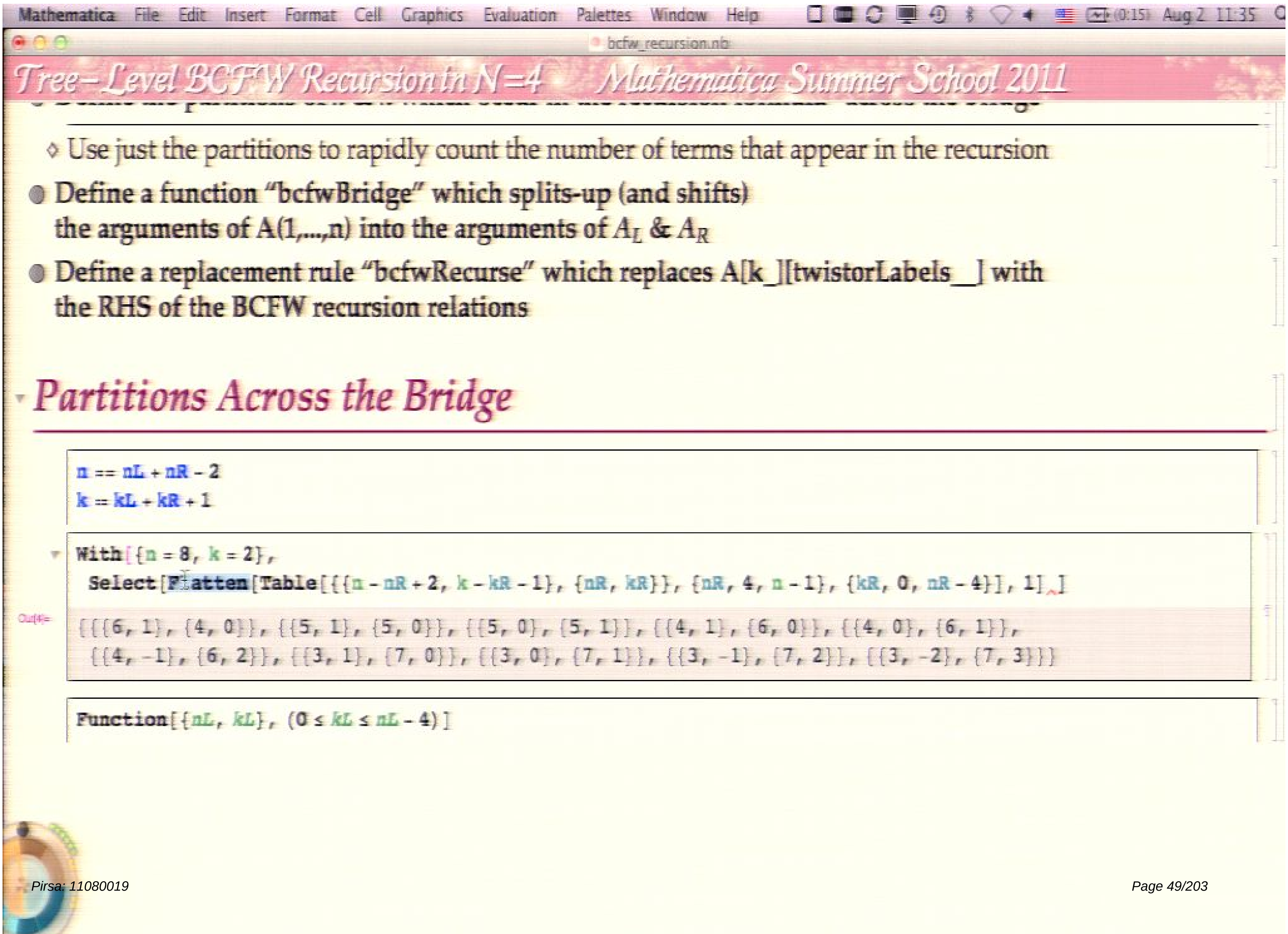
```
n == nL + nR - 2
k == kL + kR + 1
```

```
With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1]]
```

Out[4]=

```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 1}, {6, 0}}, {{4, 0}, {6, 1}},
 {{4, -1}, {6, 2}}, {{3, 1}, {7, 0}}, {{3, 0}, {7, 1}}, {{3, -1}, {7, 2}}, {{3, -2}, {7, 3}}}
```

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
```



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With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1] ]
```
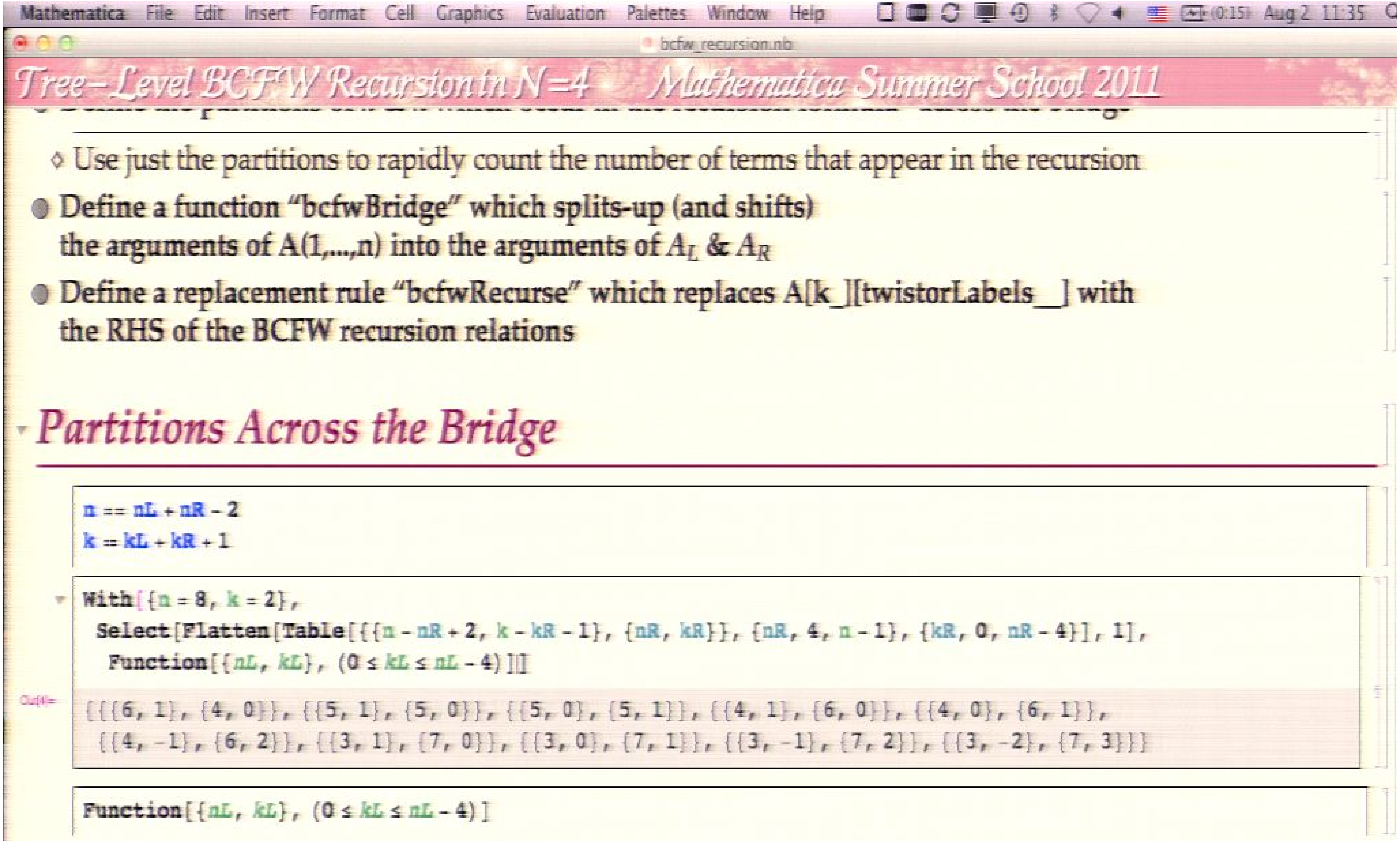

```
Out[4]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 1}, {6, 0}}, {{4, 0}, {6, 1}},
 {{4, -1}, {6, 2}}, {{3, 1}, {7, 0}}, {{3, 0}, {7, 1}}, {{3, -1}, {7, 2}}, {{3, -2}, {7, 3}}}
```

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
```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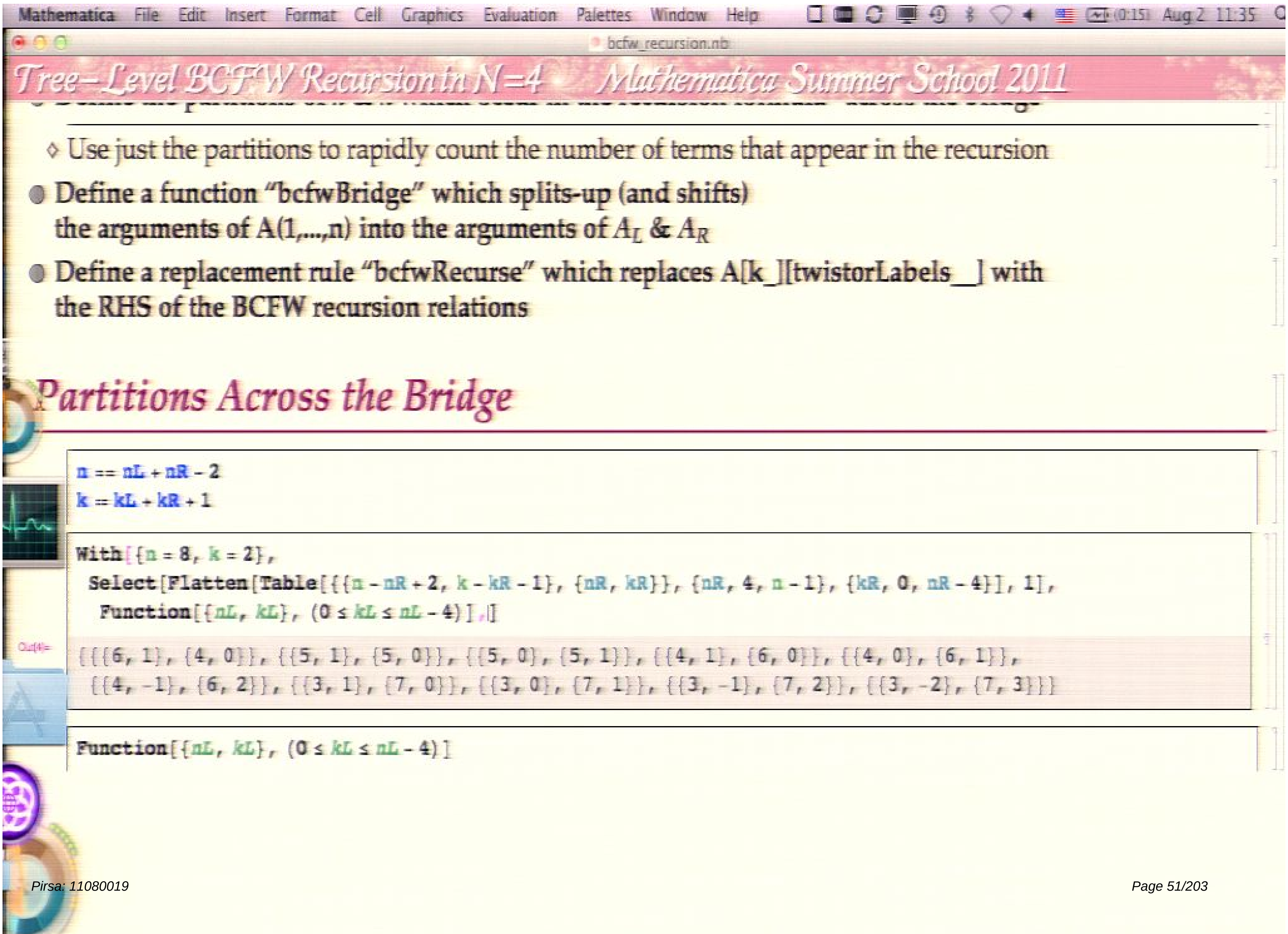
  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]]]
```

Out[4]=

```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 1}, {6, 0}}, {{4, 0}, {6, 1}},
 {{4, -1}, {6, 2}}, {{3, 1}, {7, 0}}, {{3, 0}, {7, 1}}, {{3, -1}, {7, 2}}, {{3, -2}, {7, 3}}}
```

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
```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```
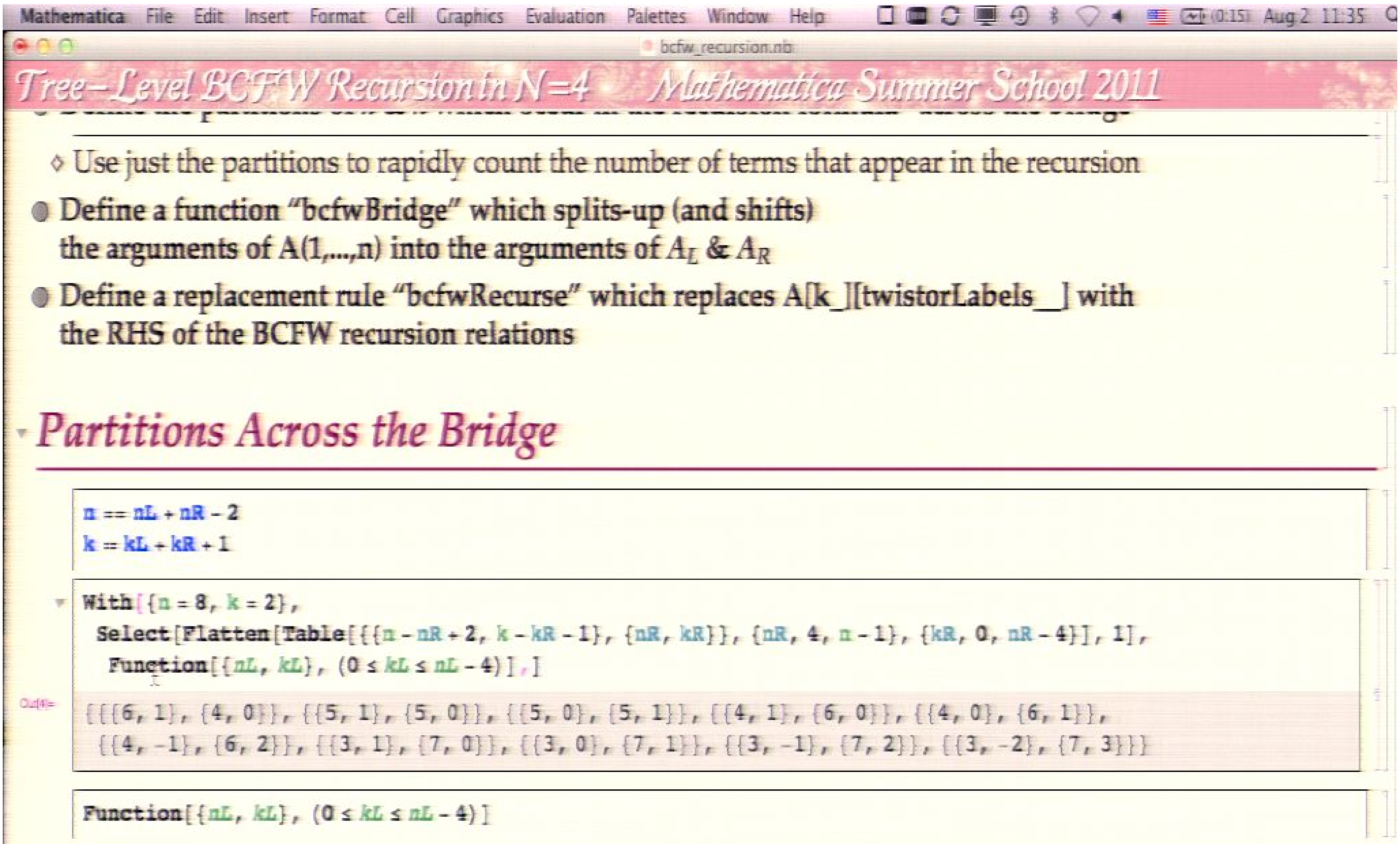

```
With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)] ,]]
```

Out[4]=

```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 1}, {6, 0}}, {{4, 0}, {6, 1}},
 {{4, -1}, {6, 2}}, {{3, 1}, {7, 0}}, {{3, 0}, {7, 1}}, {{3, -1}, {7, 2}}, {{3, -2}, {7, 3}}}
```

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
```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```
n == nL + nR - 2
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```
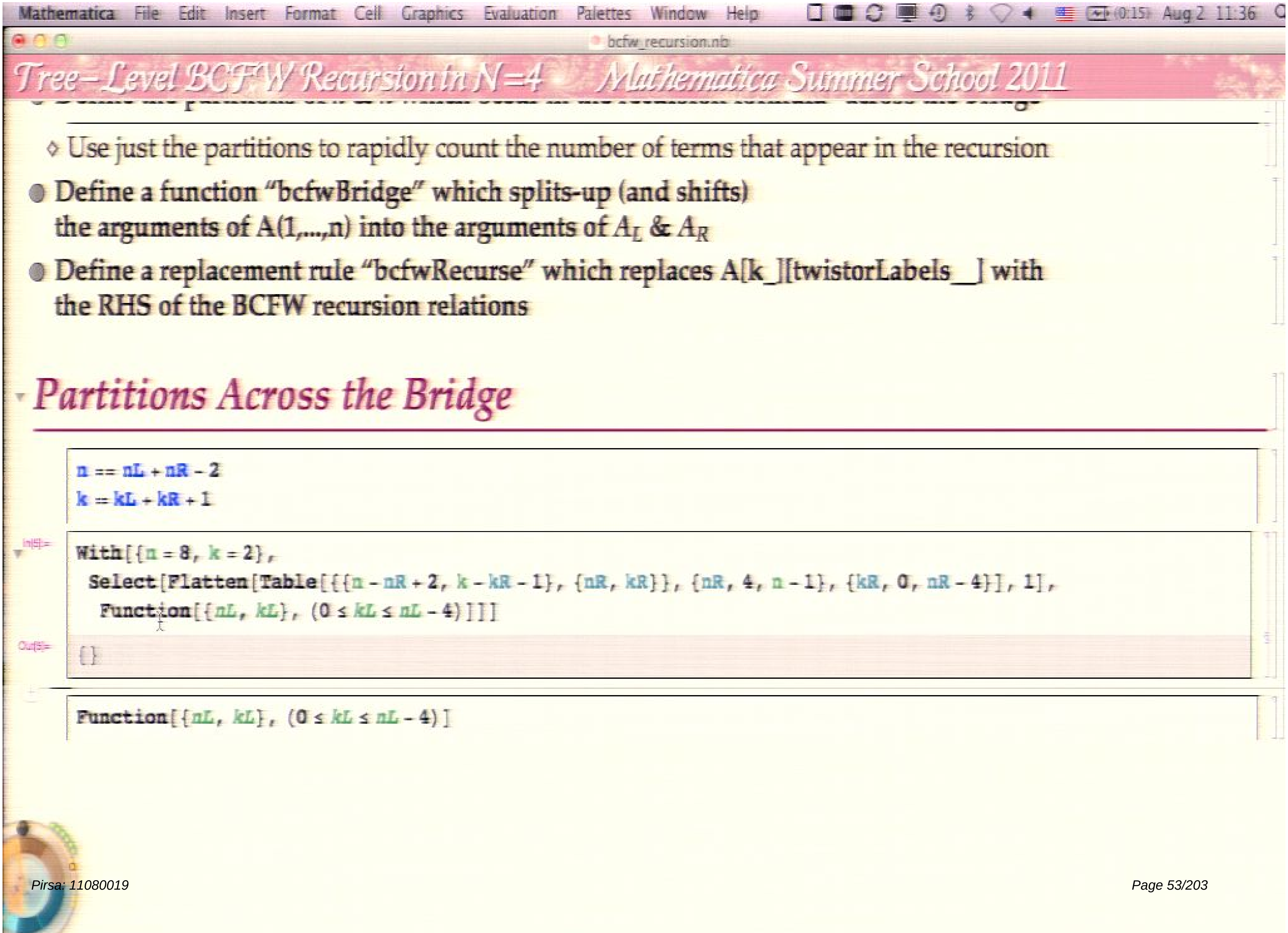

```
With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)],]
```

Out[4]=
```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 1}, {6, 0}}, {{4, 0}, {6, 1}},
 {{4, -1}, {6, 2}}, {{3, 1}, {7, 0}}, {{3, 0}, {7, 1}}, {{3, -1}, {7, 2}}, {{3, -2}, {7, 3}}}
```

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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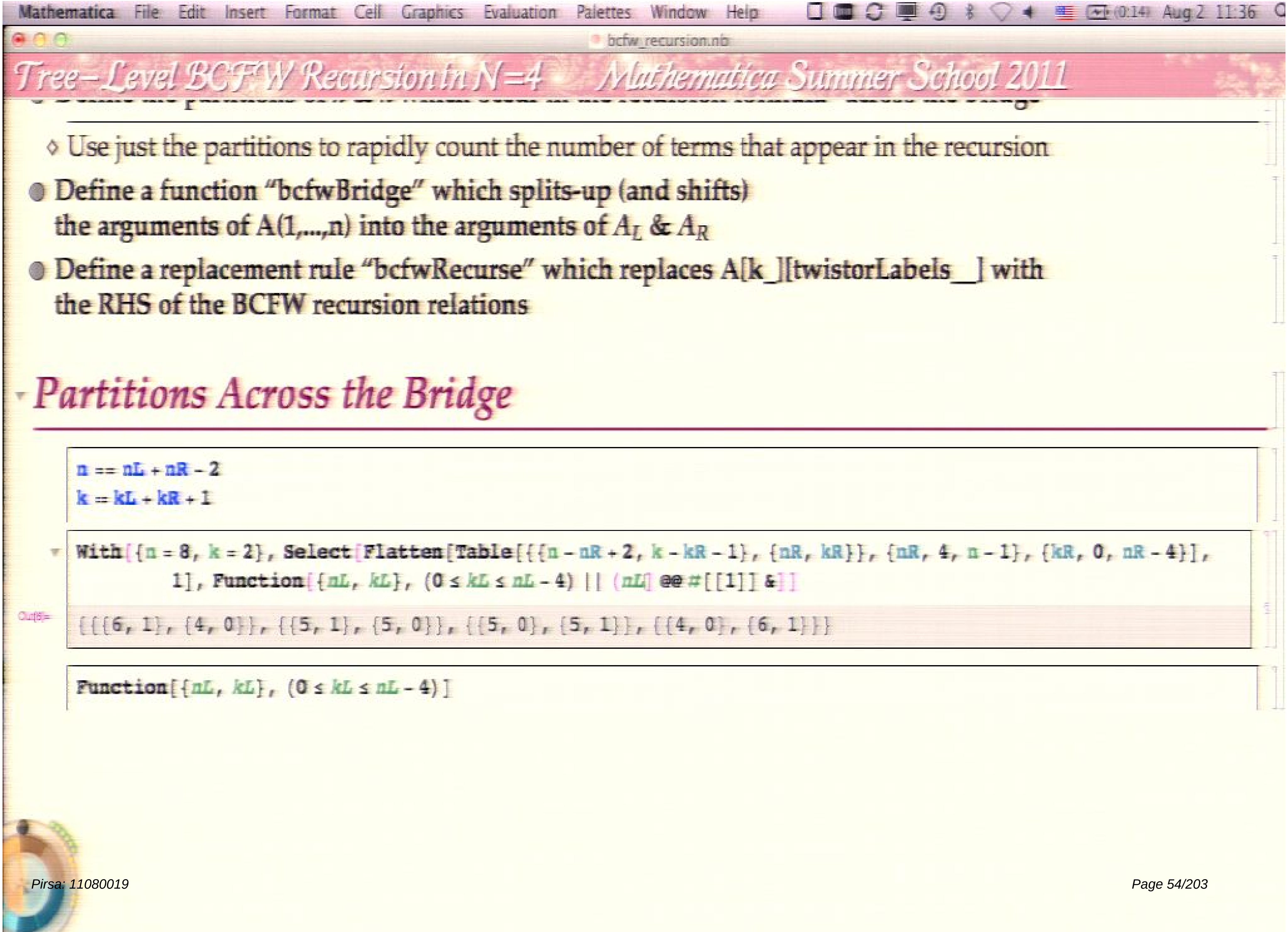
## *Partitions Across the Bridge*

```
n == nL + nR - 2
k = kL + kR + 1
```

```
In[5]:= With[{n = 8, k = 2},
         Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
          Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]]]

Out[5]= {}
```

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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## Partitions Across the Bridge

```
n == nL + nR - 2
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```
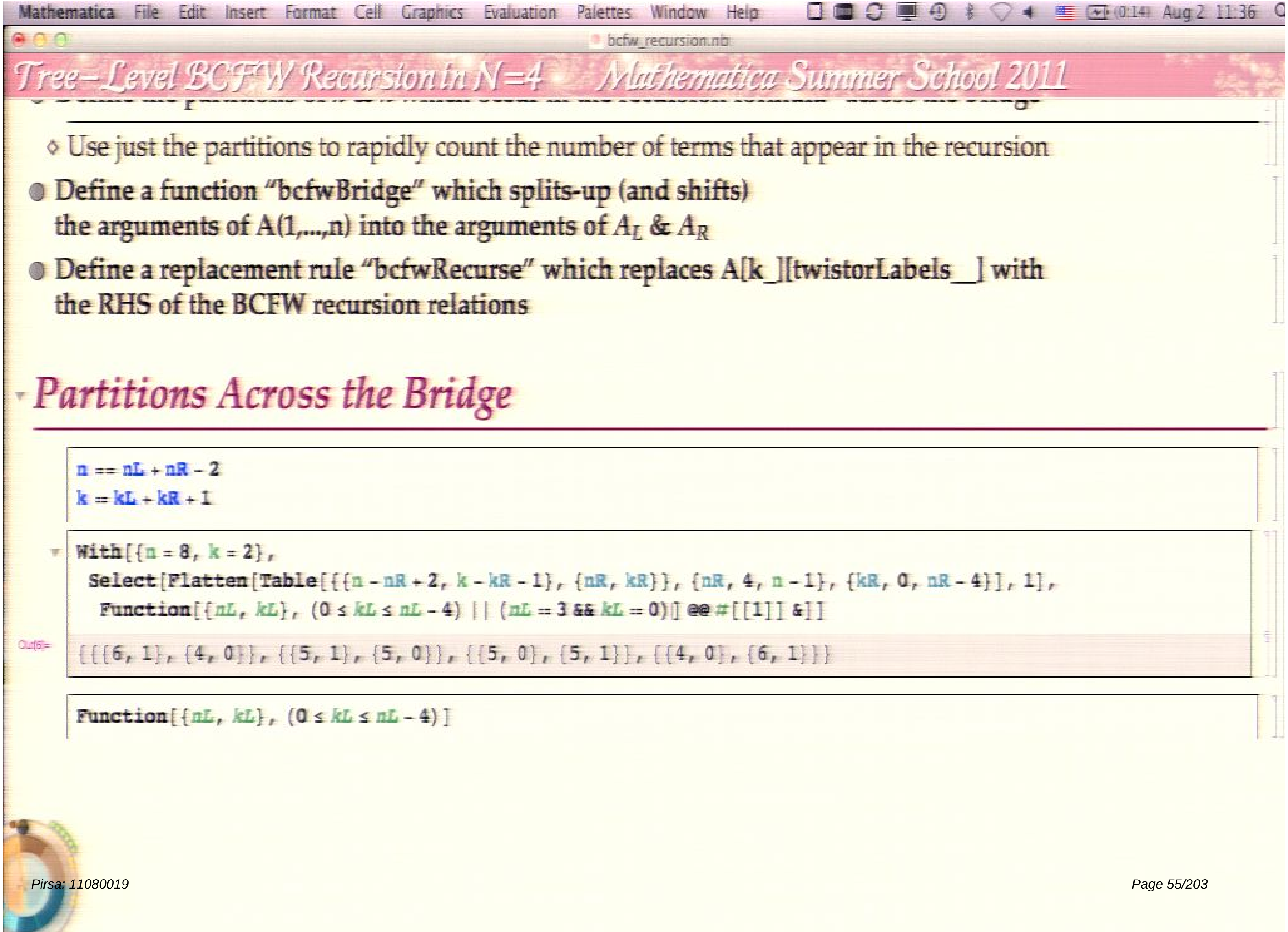

```
With[{n = 8, k = 2}, Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}],
        1], Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL] @@ #[[1]] &]]
```

Out[6]=
```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}}
```

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
```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*Tree–Level BCFW Recursion in N=4*   *Mathematica Summer School 2011*

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  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[6]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}}
```

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
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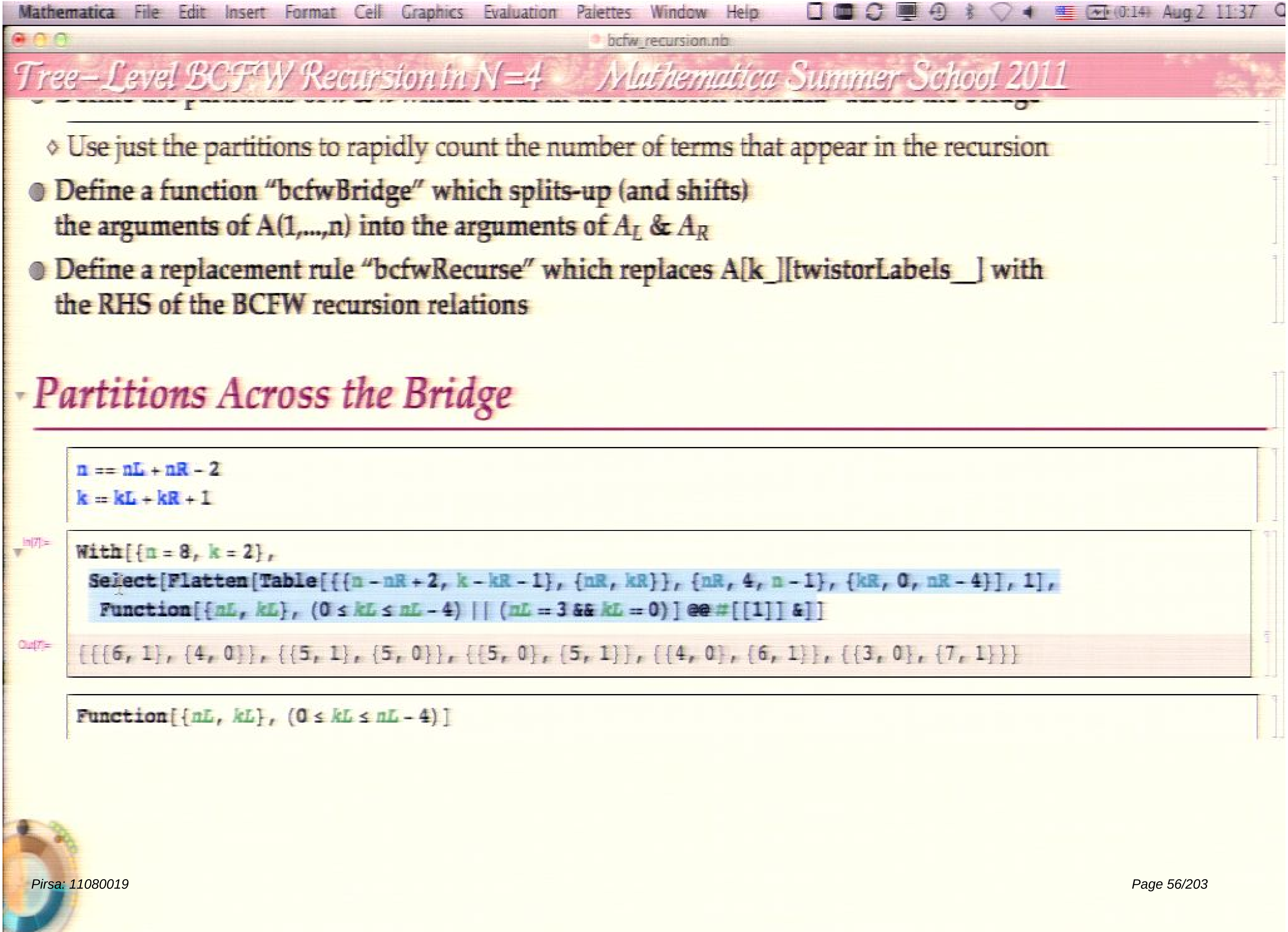
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bcfw_recursion.nb
In[7]:=
Out[7]=

*Tree–Level BCFW Recursion in N=4*  *Mathematica Summer School 2011*

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```

```
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```

```
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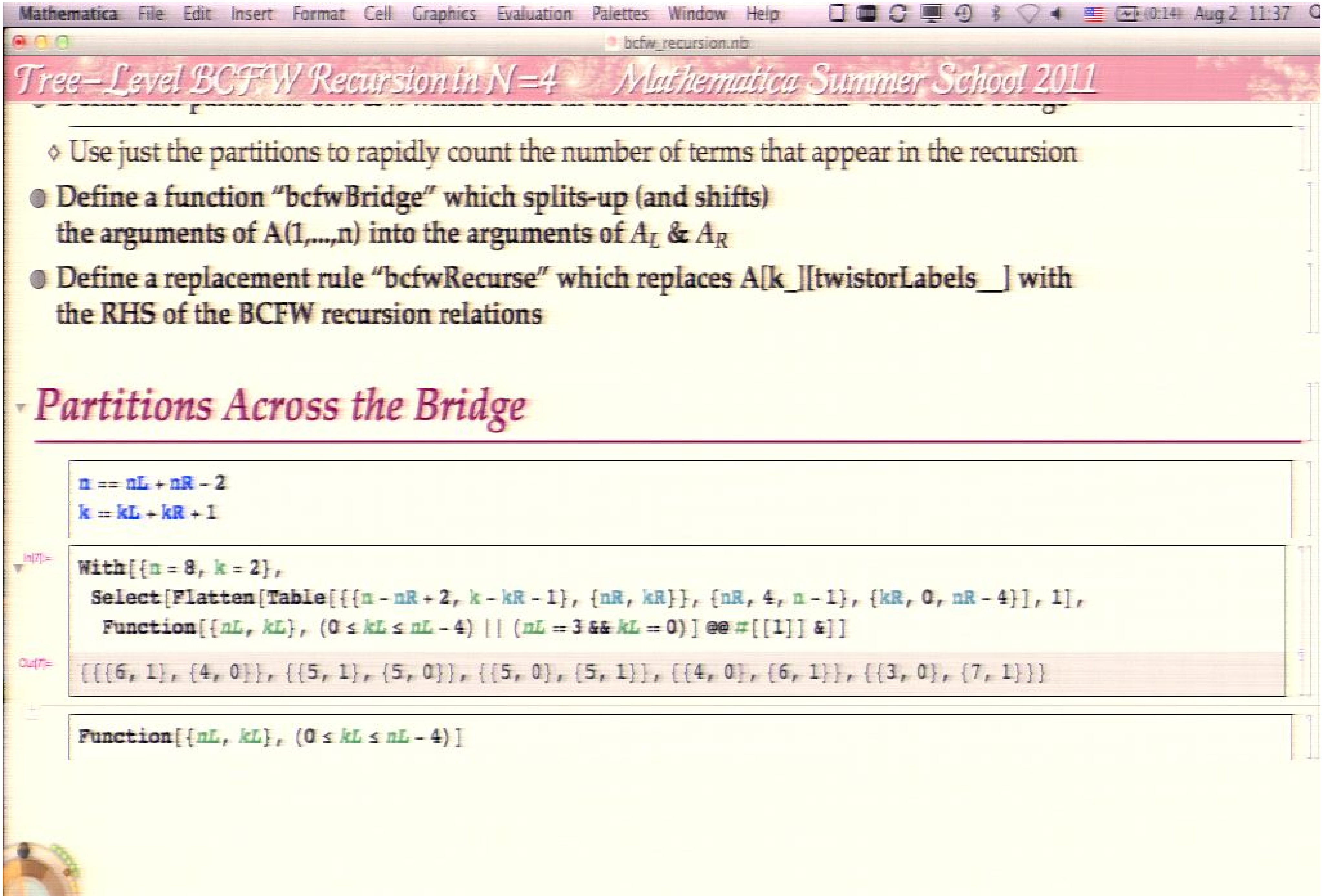

*Tree–Level BCFW Recursion in N=4* *Mathematica Summer School 2011*

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```

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```

```
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```
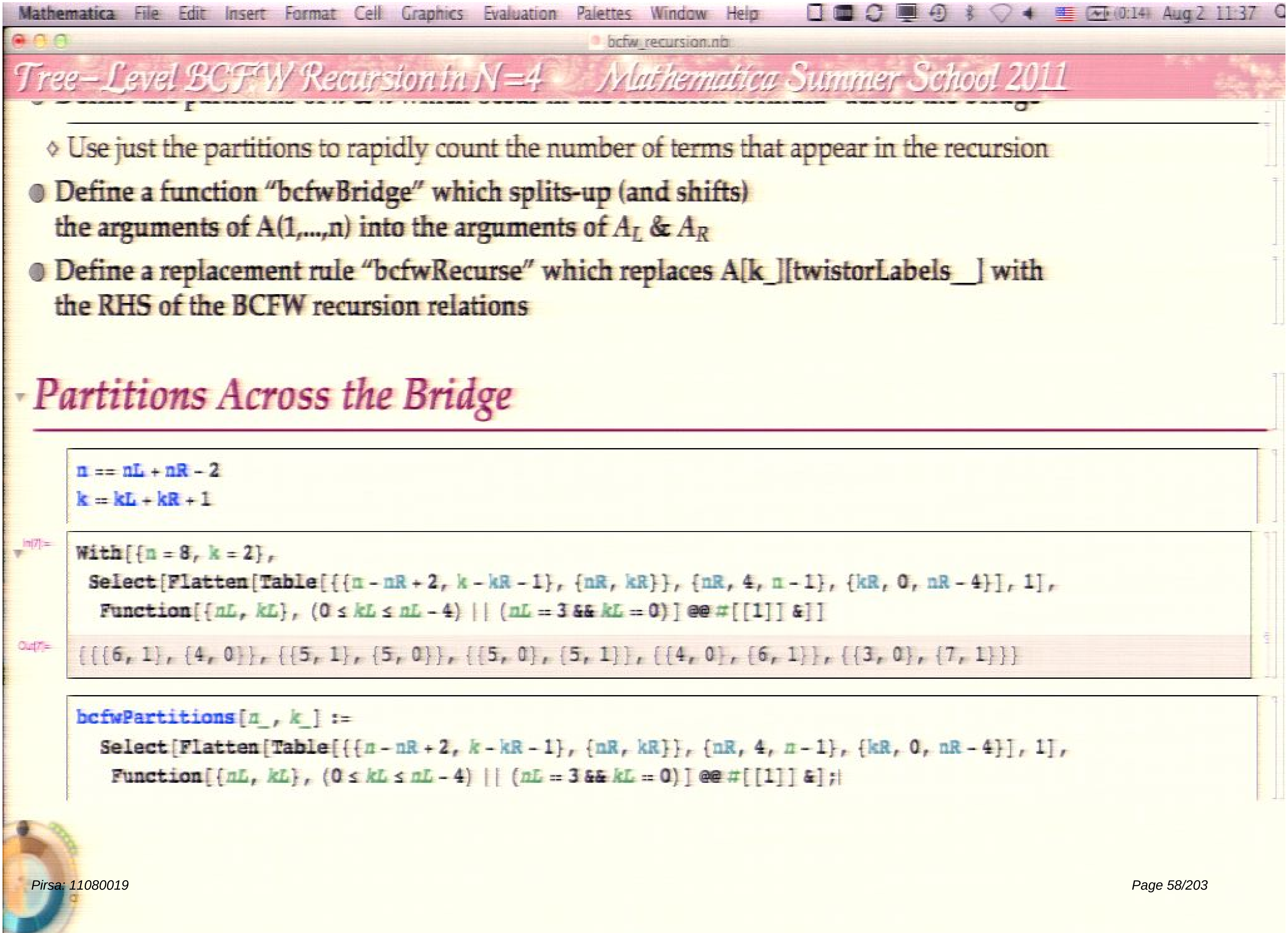

```
Function[{nL, kL}, (0 ≤ kL ≤ nL - 4)]
```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  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]
```

Out[7]=
```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
bcfwPartitions[n_, k_] :=
  Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
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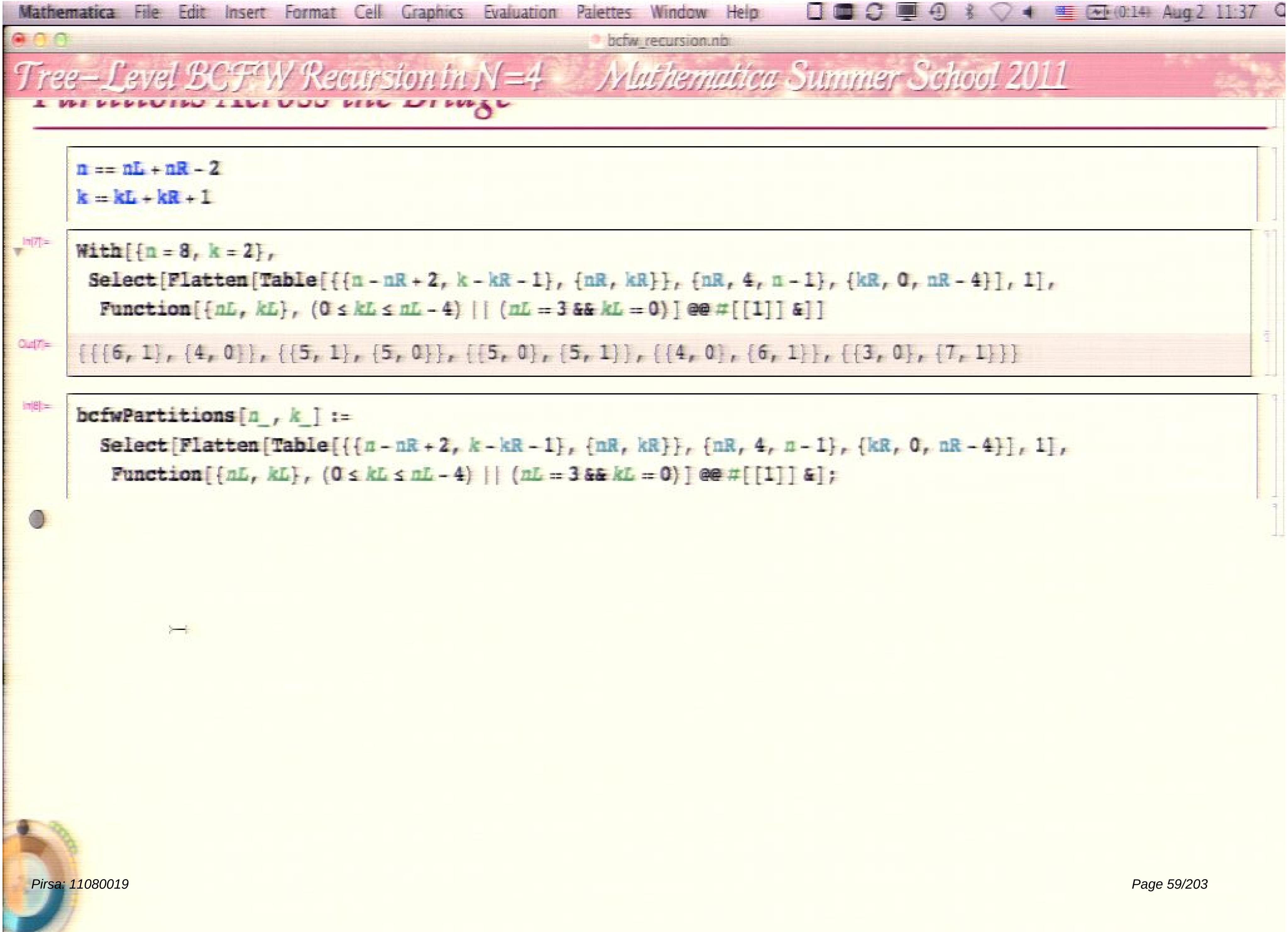

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

## Partitions Across the Bridge

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```

```
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          Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[8]:= bcfwPartitions[n_, k_] :=
          Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
           Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
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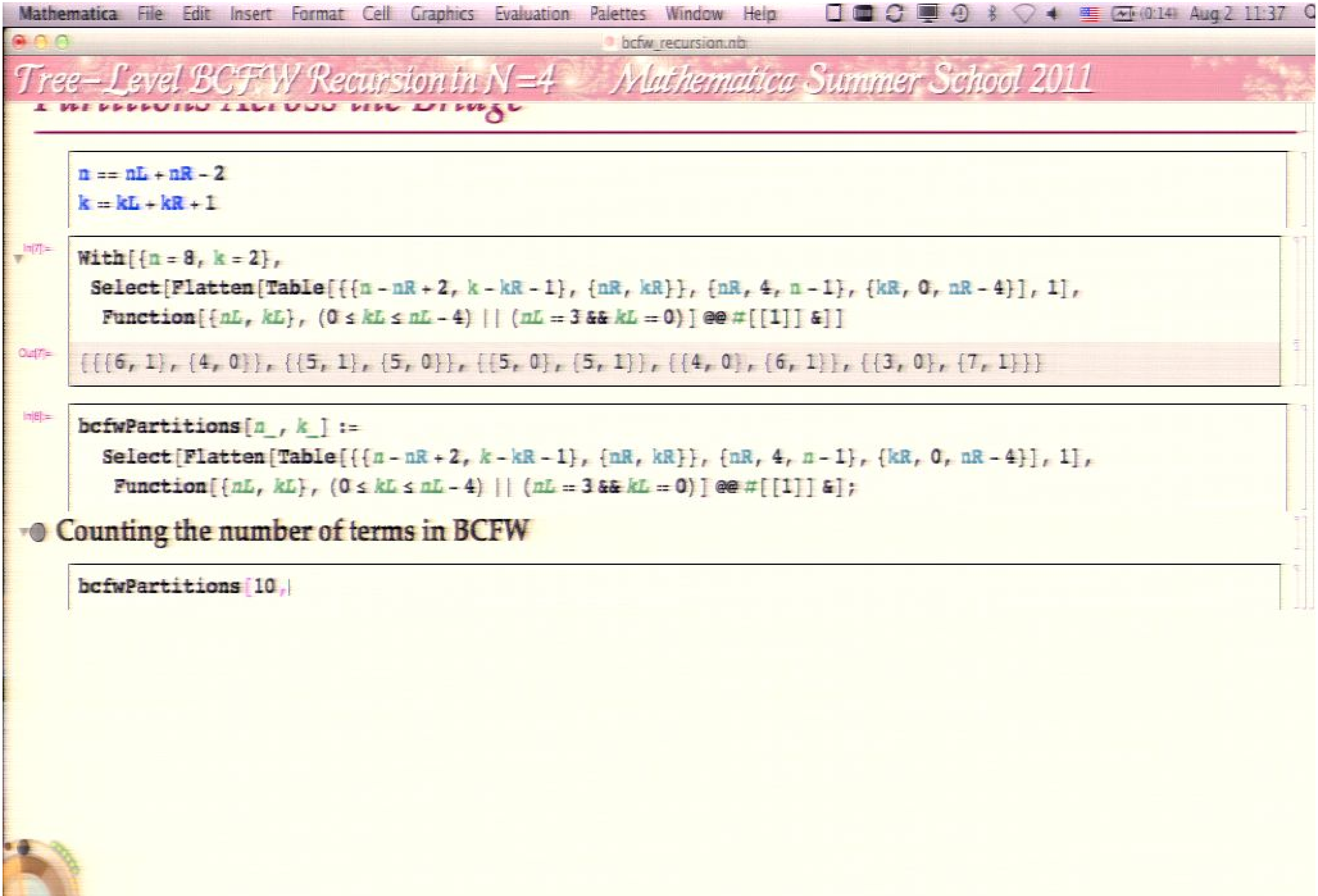
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bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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```
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          Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[8]:= bcfwPartitions[n_, k_] :=
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           Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```
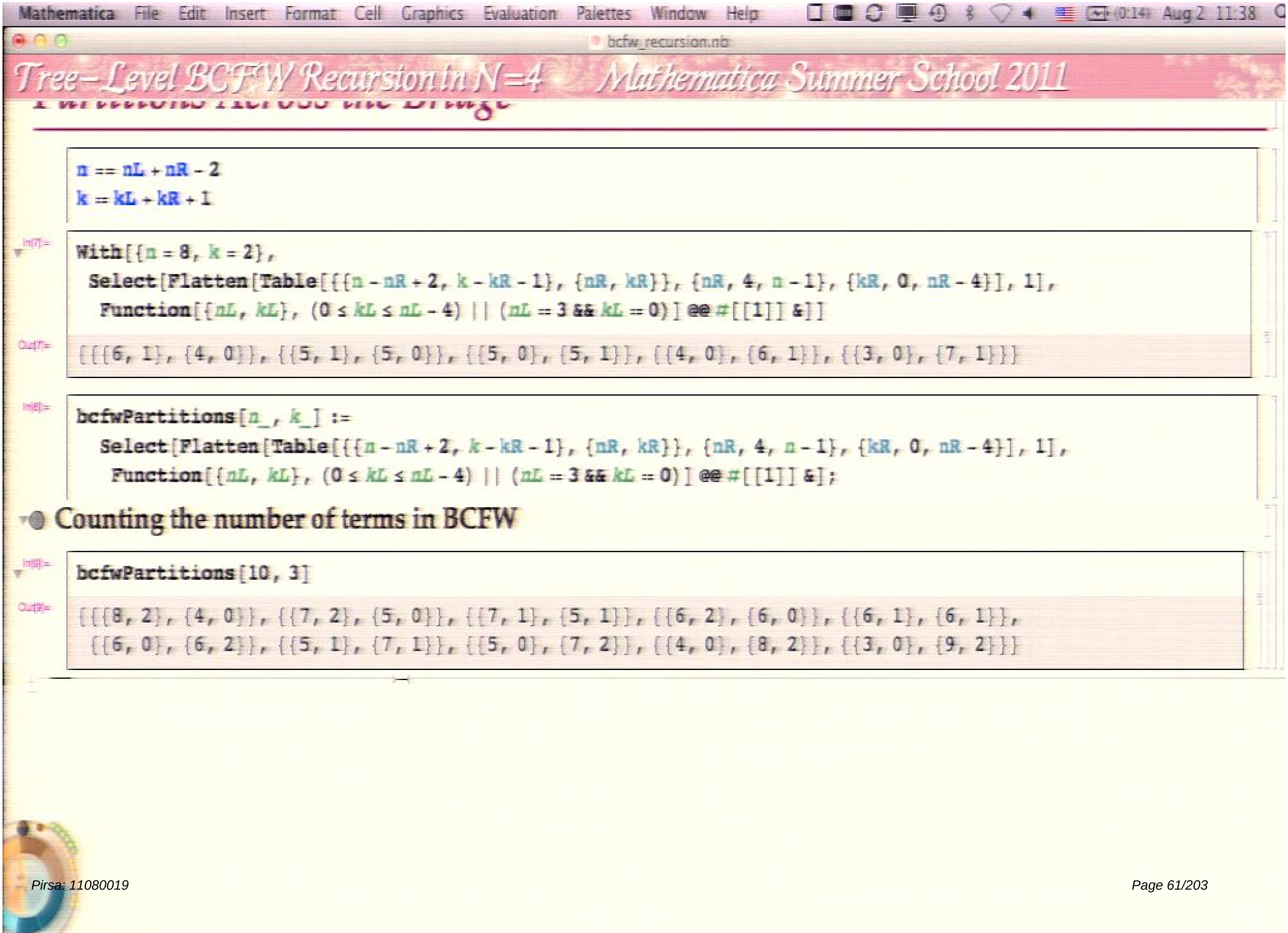

## Counting the number of terms in BCFW

```
bcfwPartitions[10,
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

## Partitions Across the Bridge

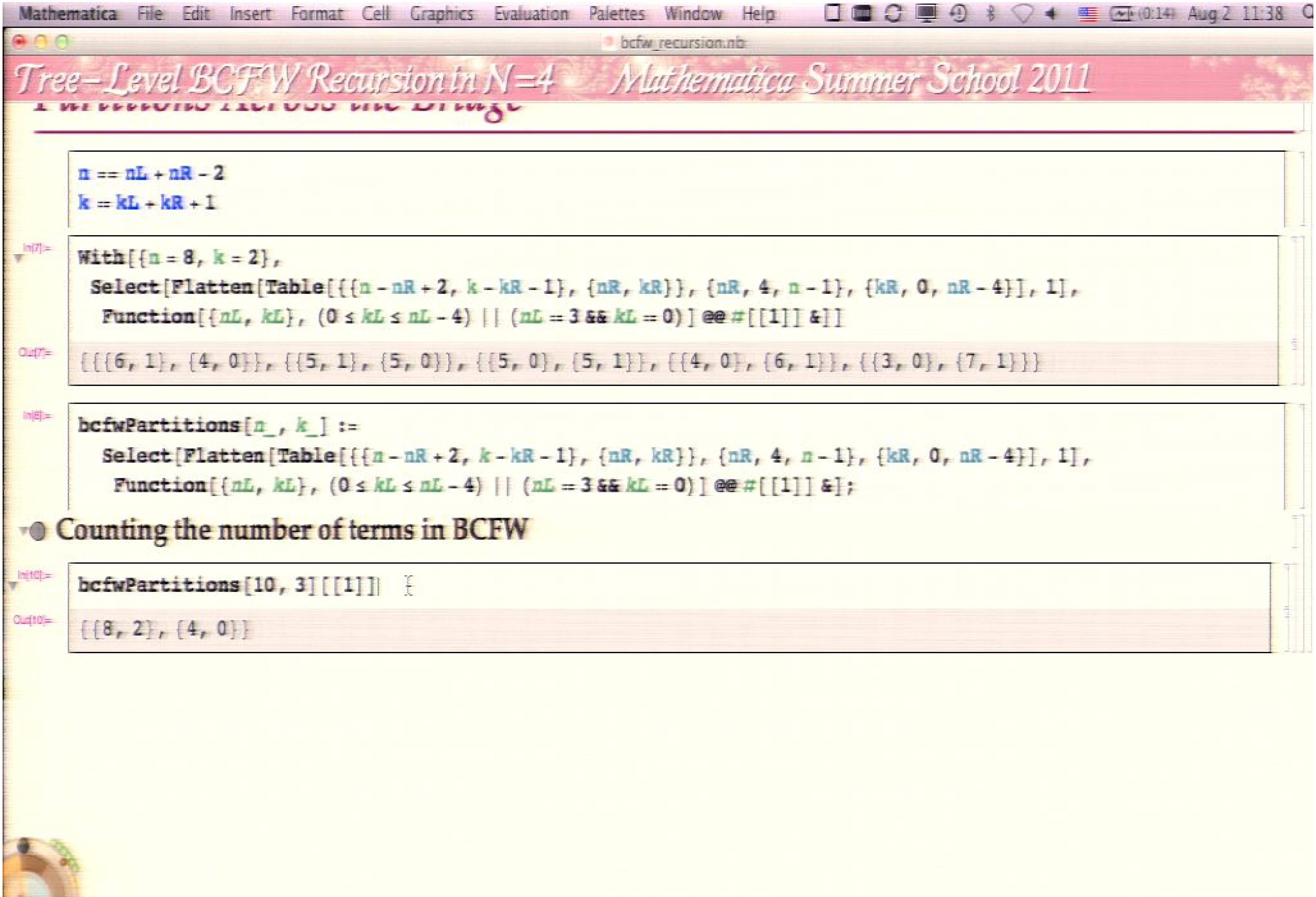
```
n == nL + nR - 2
k == kL + kR + 1
```

```
In[7]:= With[{n = 8, k = 2},
          Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
           Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[8]:= bcfwPartitions[n_, k_] :=
          Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
           Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

## Counting the number of terms in BCFW

```
In[9]:= bcfwPartitions[10, 3]

Out[9]= {{{8, 2}, {4, 0}}, {{7, 2}, {5, 0}}, {{7, 1}, {5, 1}}, {{6, 2}, {6, 0}}, {{6, 1}, {6, 1}},
         {{6, 0}, {6, 2}}, {{5, 1}, {7, 1}}, {{5, 0}, {7, 2}}, {{4, 0}, {8, 2}}, {{3, 0}, {9, 2}}}
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

Partitions Across the Bridge

```
n == nL + nR - 2
k = kL + kR + 1
```

```
In[7]:= With[{n = 8, k = 2},
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          Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```
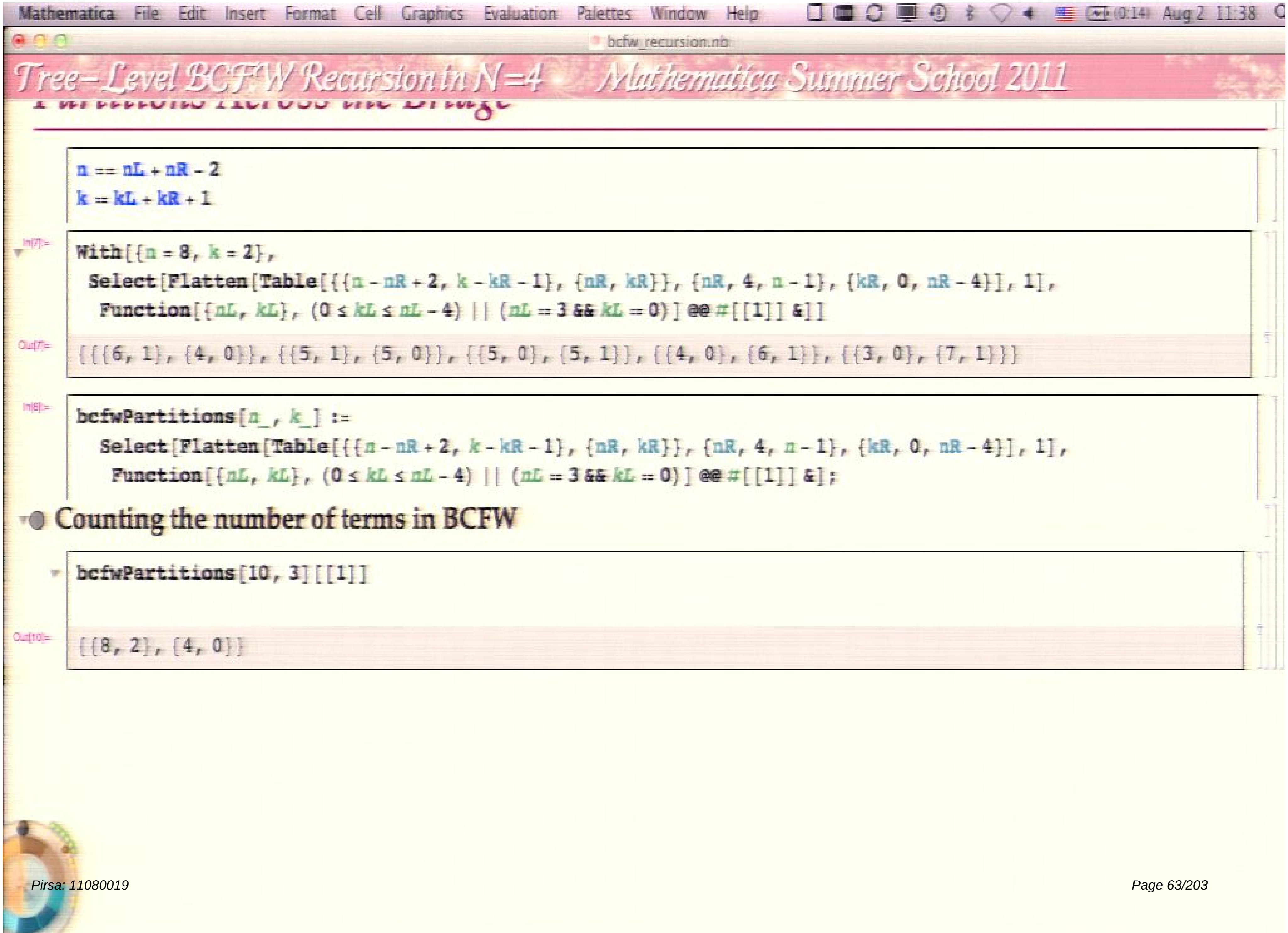

```
In[8]:= bcfwPartitions[n_, k_] :=
          Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
           Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

## Counting the number of terms in BCFW

```
In[10]:= bcfwPartitions[10, 3][[1]]

Out[10]= {{8, 2}, {4, 0}}
```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Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

## Partitions Across the Bridge

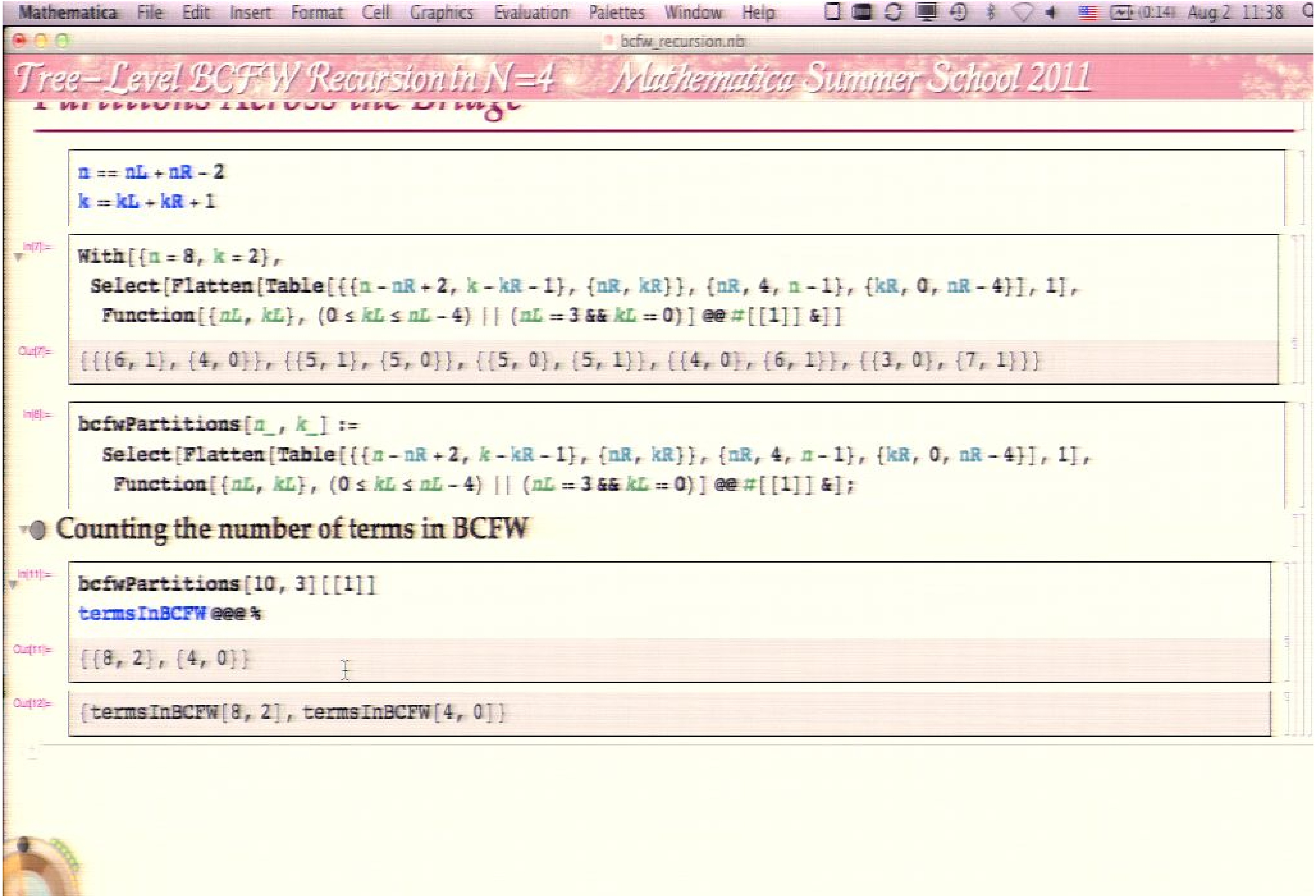
```
n == nL + nR - 2
k == kL + kR + 1
```

```
In[7]:= With[{n = 8, k = 2},
         Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
          Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[8]:= bcfwPartitions[n_, k_] :=
          Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
           Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

### Counting the number of terms in BCFW

```
bcfwPartitions[10, 3][[1]]

Out[10]= {{8, 2}, {4, 0}}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:14) Aug 2 11:38
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

## Partitions Across the Bridge

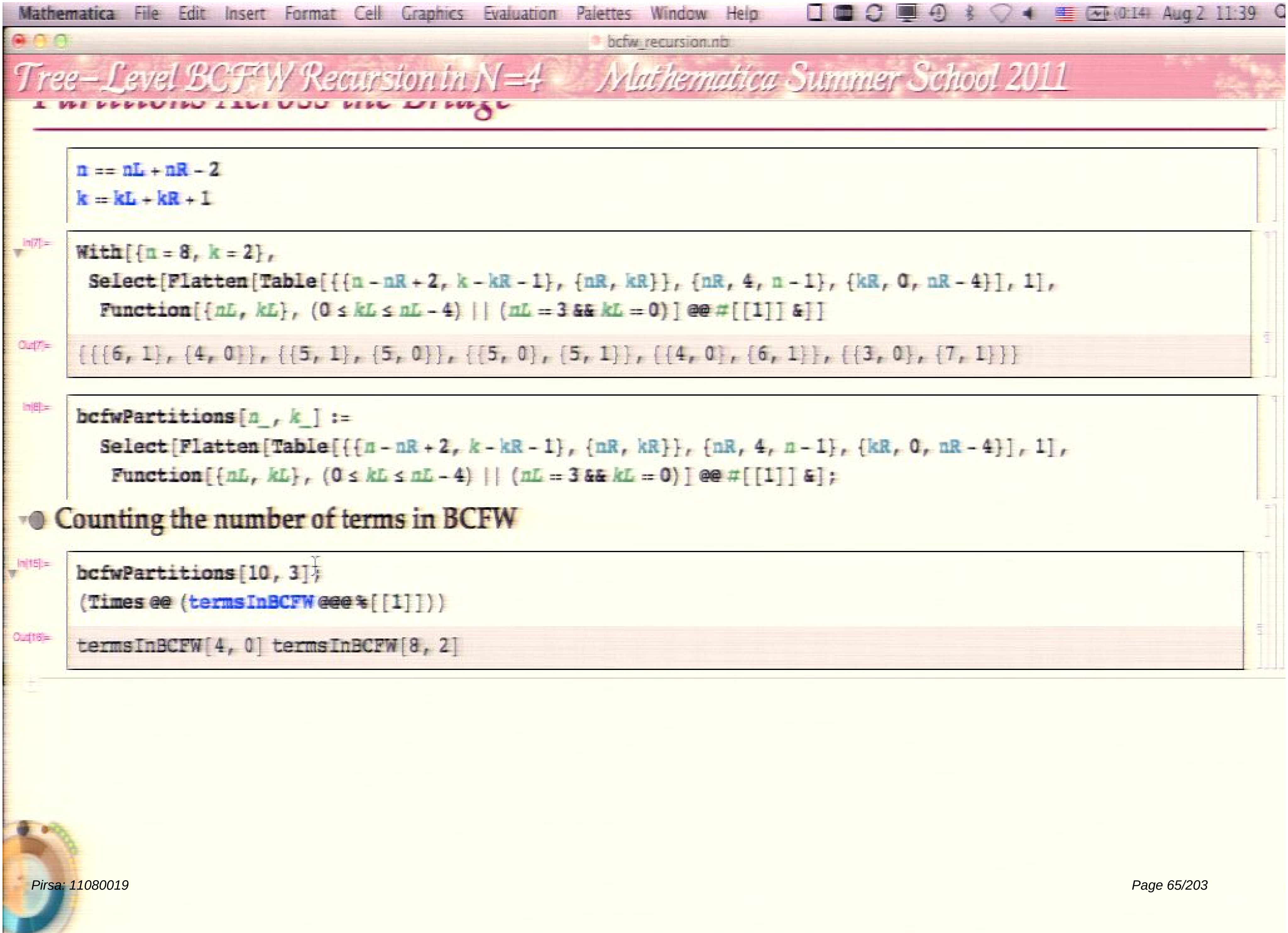
```
n == nL + nR - 2
k = kL + kR + 1
```

```
In[7]:= With[{n = 8, k = 2},
         Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
          Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[8]:= bcfwPartitions[n_, k_] :=
          Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
           Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

### Counting the number of terms in BCFW

```
In[11]:= bcfwPartitions[10, 3][[1]]
         termsInBCFW @@@ %

Out[11]= {{8, 2}, {4, 0}}

Out[12]= {termsInBCFW[8, 2], termsInBCFW[4, 0]}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:14) Aug 2 11:39
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

## Partitions Across the Bridge

```
n == nL + nR - 2
k == kL + kR + 1
```

In[7]:=

```
With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]
```

Out[7]=

```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

In[8]:=

```
bcfwPartitions[n_, k_] :=
  Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
   Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

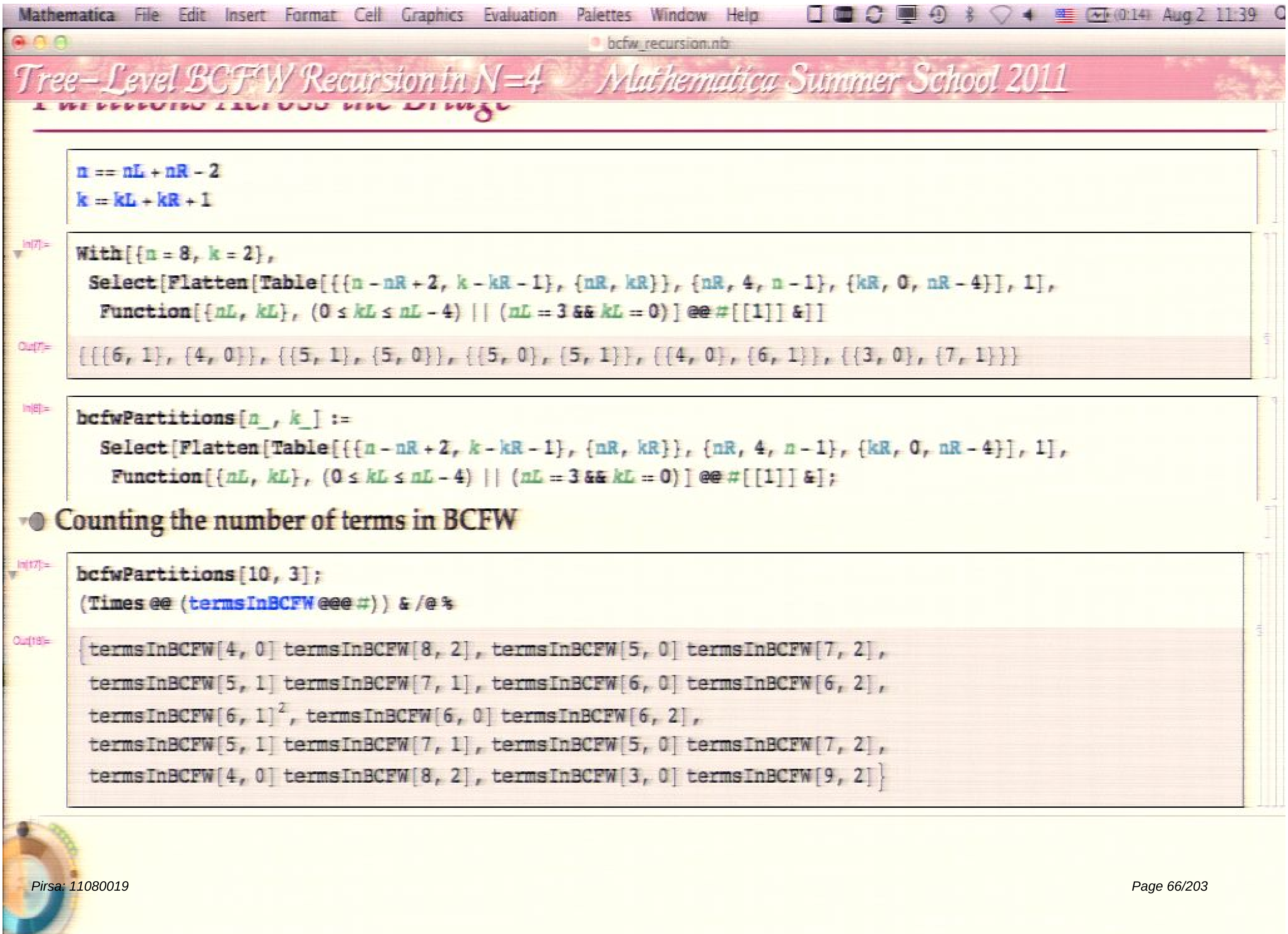
### Counting the number of terms in BCFW

In[15]:=

```
bcfwPartitions[10, 3];
(Times @@ (termsInBCFW @@@ %[[1]]))
```

Out[16]=

```
termsInBCFW[4, 0] termsInBCFW[8, 2]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:14) Aug 2 11:39
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

## Partitions Across the Bridge

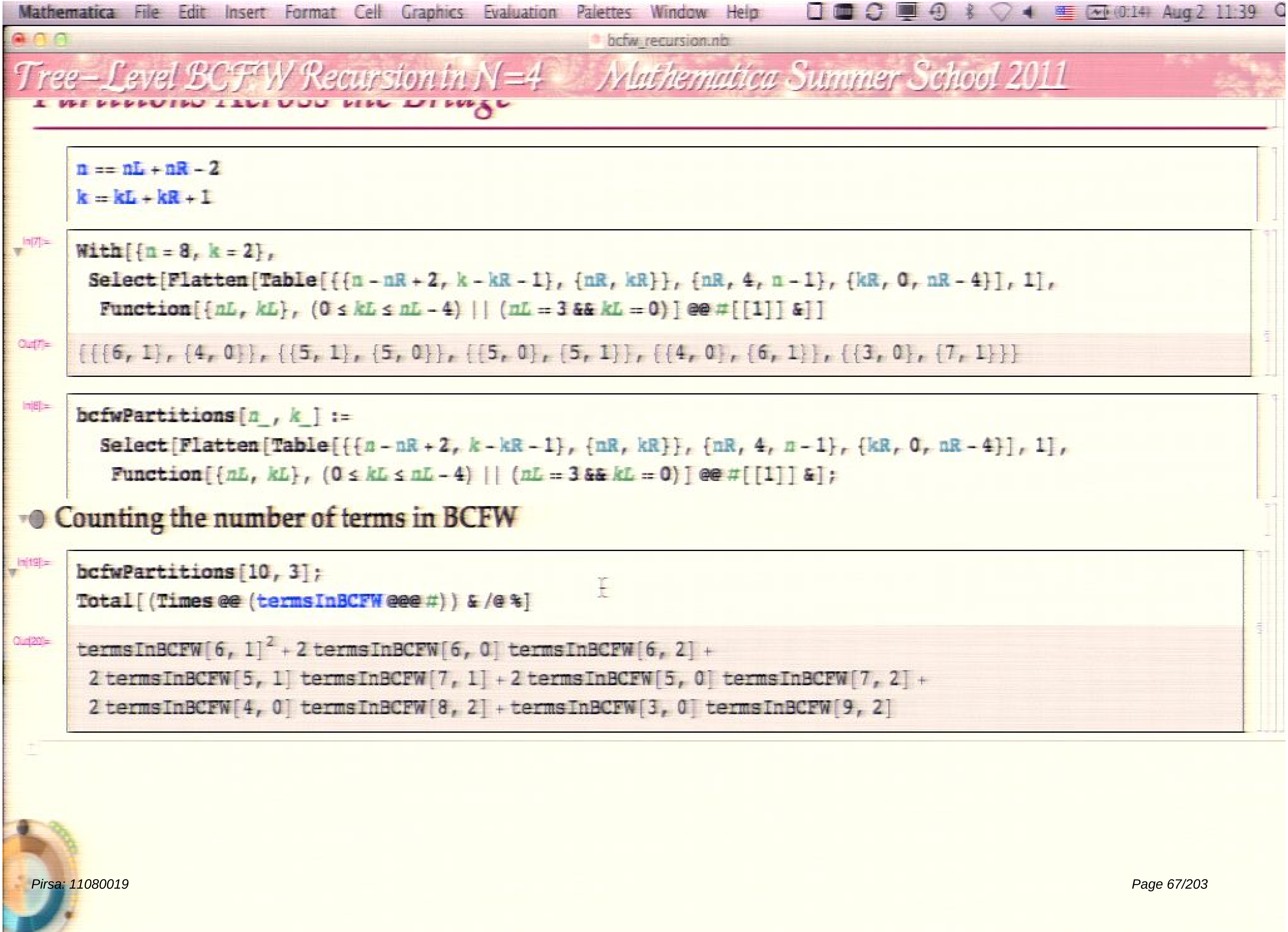
```
n == nL + nR - 2
k == kL + kR + 1
```

```
In[7]:=
With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]=
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[8]:=
bcfwPartitions[n_, k_] :=
  Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
   Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

## Counting the number of terms in BCFW

```
In[17]:=
bcfwPartitions[10, 3];
(Times @@ (termsInBCFW @@@ #)) & /@ %

Out[18]=
{termsInBCFW[4, 0] termsInBCFW[8, 2], termsInBCFW[5, 0] termsInBCFW[7, 2],
 termsInBCFW[5, 1] termsInBCFW[7, 1], termsInBCFW[6, 0] termsInBCFW[6, 2],
 termsInBCFW[6, 1]^2, termsInBCFW[6, 0] termsInBCFW[6, 2],
 termsInBCFW[5, 1] termsInBCFW[7, 1], termsInBCFW[5, 0] termsInBCFW[7, 2],
 termsInBCFW[4, 0] termsInBCFW[8, 2], termsInBCFW[3, 0] termsInBCFW[9, 2]}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:14) Aug 2 11:39
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4  Mathematica Summer School 2011

## Partitions Across the Bridge

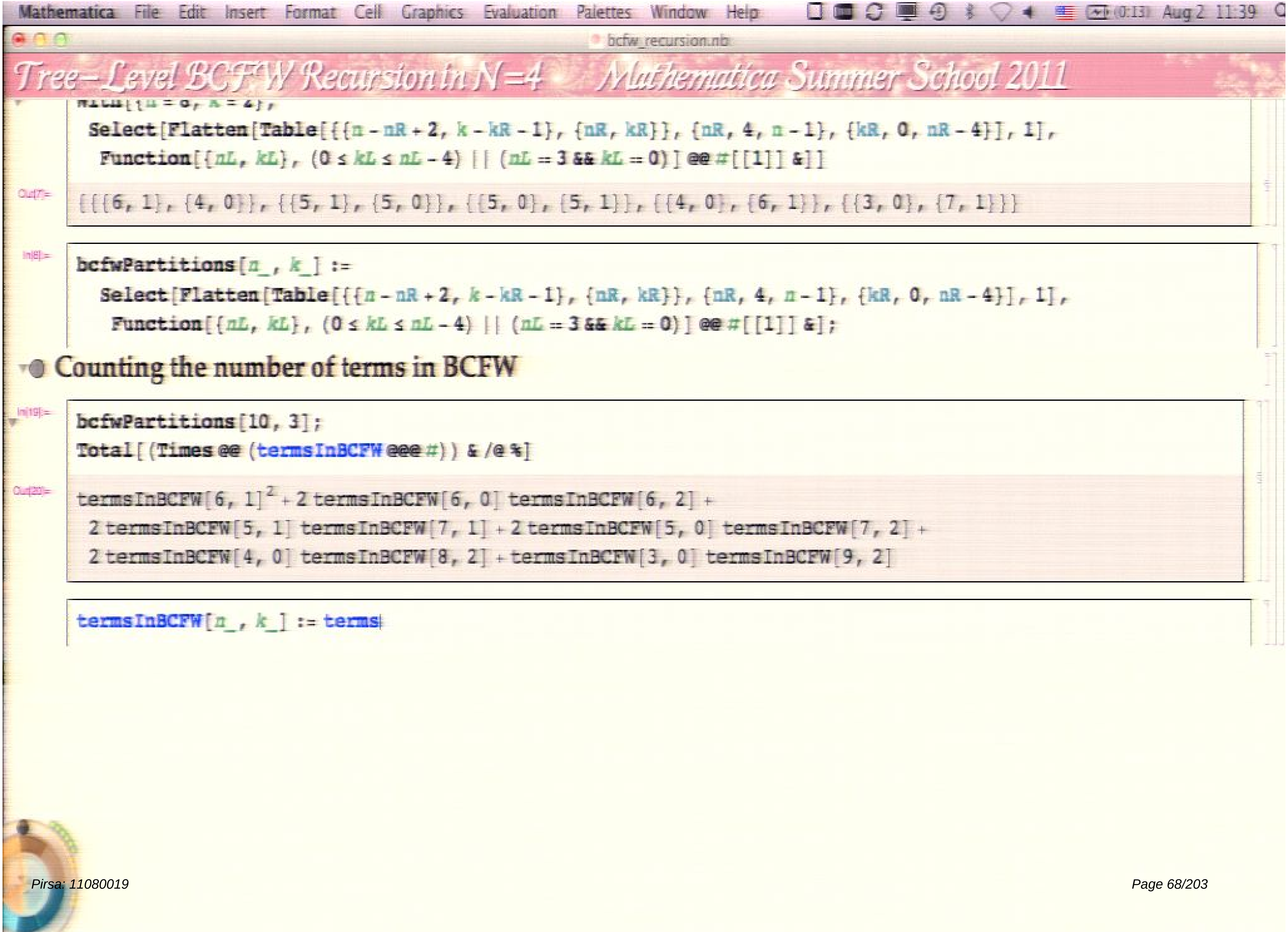
```
n == nL + nR - 2
k == kL + kR + 1
```

```
In[7]:= With[{n = 8, k = 2},
         Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
          Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[8]:= bcfwPartitions[n_, k_] :=
          Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
           Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

### Counting the number of terms in BCFW

```
In[19]:= bcfwPartitions[10, 3];
         Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]= termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
          2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
          2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]
```

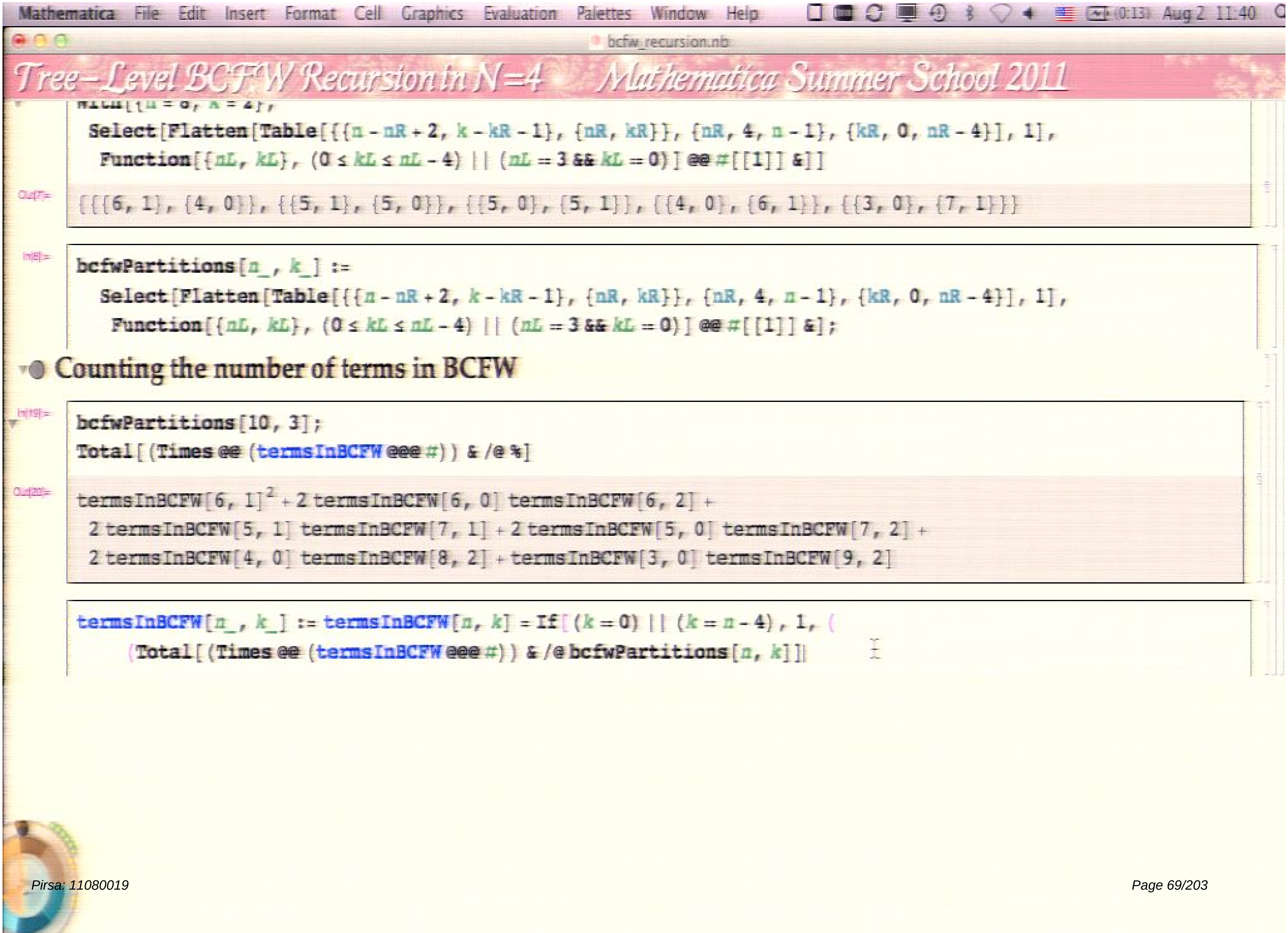

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:13) Aug 2 11:39
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[8]:= bcfwPartitions[n_, k_] :=
  Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
   Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

## Counting the number of terms in BCFW

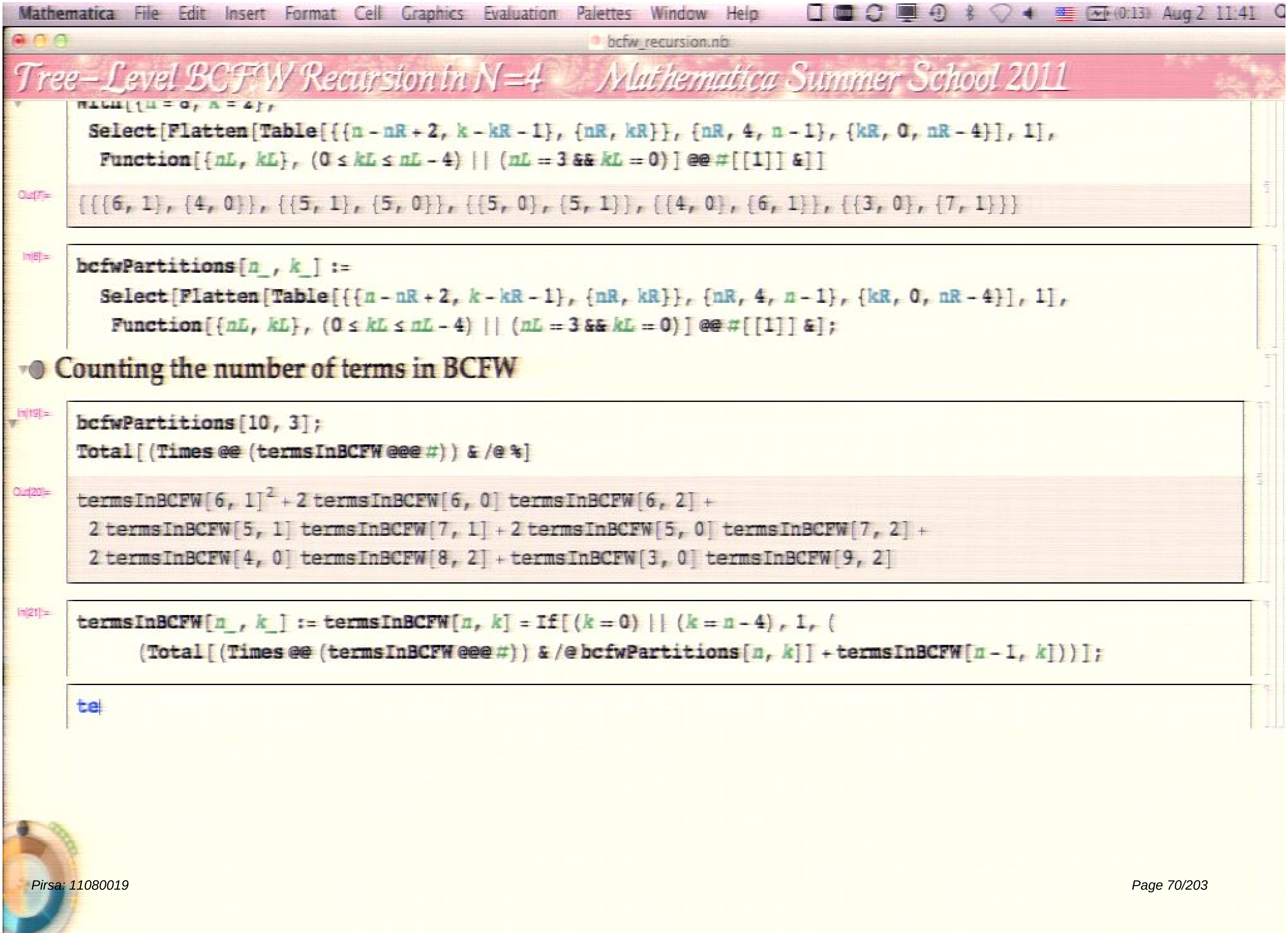
```
In[19]:= bcfwPartitions[10, 3];
Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]= termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
 2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
 2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]

termsInBCFW[n_, k_] := terms
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:13) Aug 2 11:40
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[8]:= bcfwPartitions[n_, k_] :=
  Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
   Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

## Counting the number of terms in BCFW

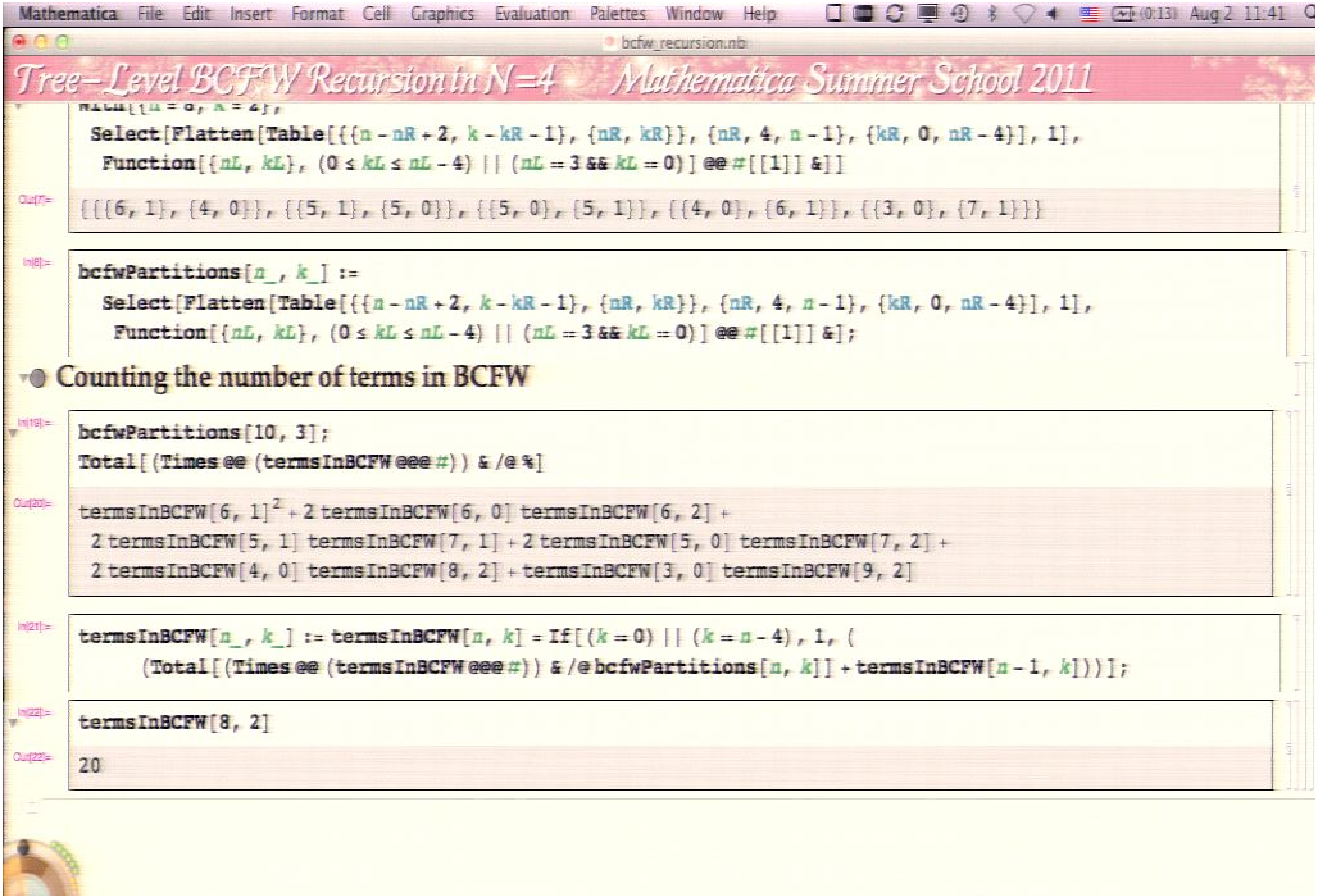
```
In[19]:= bcfwPartitions[10, 3];
Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]= termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
 2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
 2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]

termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
    (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:13) Aug 2 11:41
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4
Mathematica Summer School 2011

```
With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[8]:= bcfwPartitions[n_, k_] :=
  Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
   Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

## Counting the number of terms in BCFW

```
In[19]:= bcfwPartitions[10, 3];
Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]= termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
 2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
 2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]

In[21]:= termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
     (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

te
```

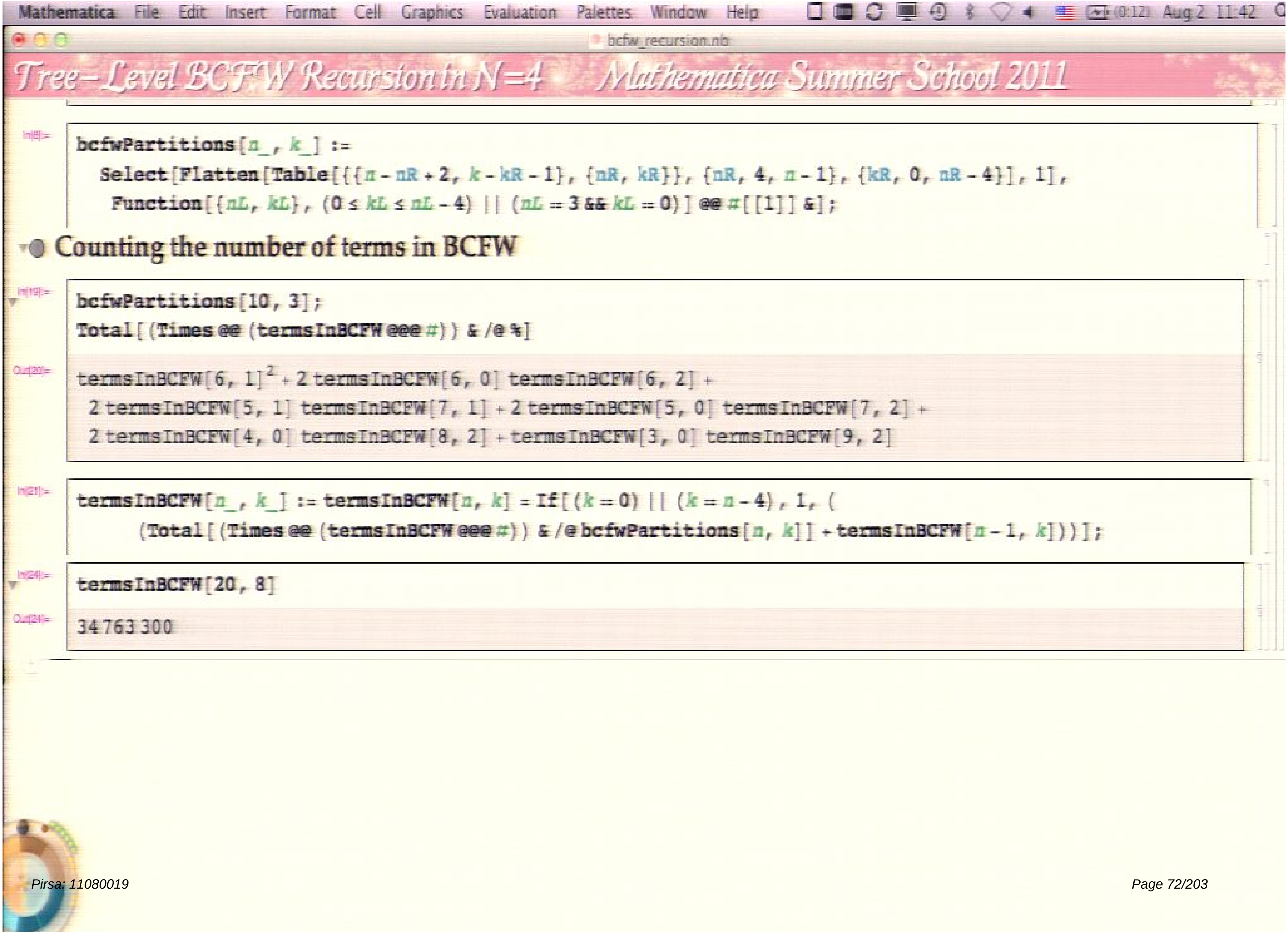

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:13) Aug 2 11:41
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
With[{n = 8, k = 2},
 Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
  Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &]]

Out[7]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[8]:= bcfwPartitions[n_, k_] :=
  Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
   Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

## Counting the number of terms in BCFW

```
In[19]:= bcfwPartitions[10, 3];
Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]= termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
 2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
 2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]

In[21]:= termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
      (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[22]:= termsInBCFW[8, 2]

Out[22]= 20
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:12) Aug 2 11:42
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

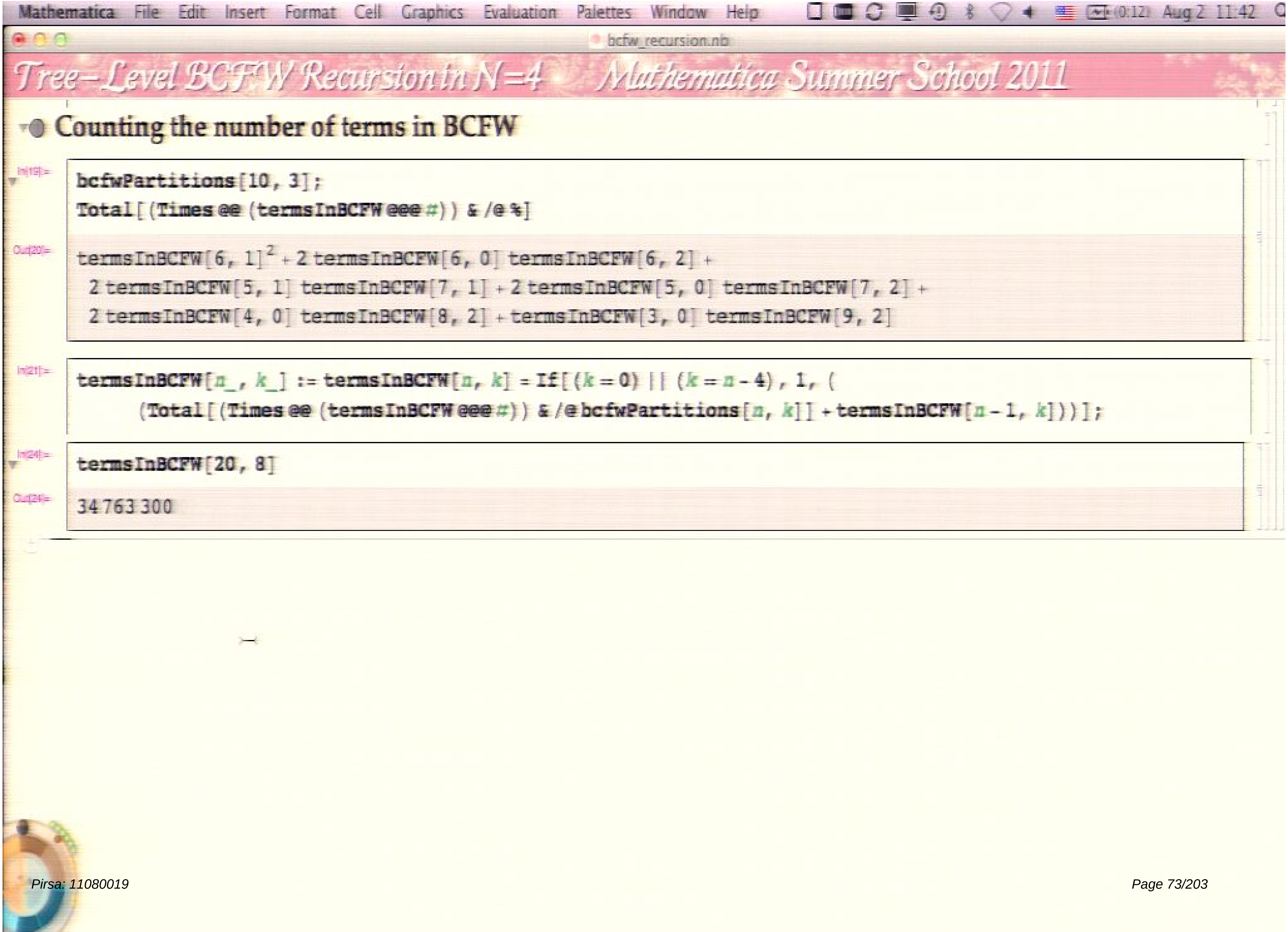
```
In[8]:=
bcfwPartitions[n_, k_] :=
  Select[Flatten[Table[{{n - nR + 2, k - kR - 1}, {nR, kR}}, {nR, 4, n - 1}, {kR, 0, nR - 4}], 1],
   Function[{nL, kL}, (0 ≤ kL ≤ nL - 4) || (nL == 3 && kL == 0)] @@ #[[1]] &];
```

## Counting the number of terms in BCFW

```
In[19]:=
bcfwPartitions[10, 3];
Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]=
termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
 2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
 2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]
```

```
In[21]:=
termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
     (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
```

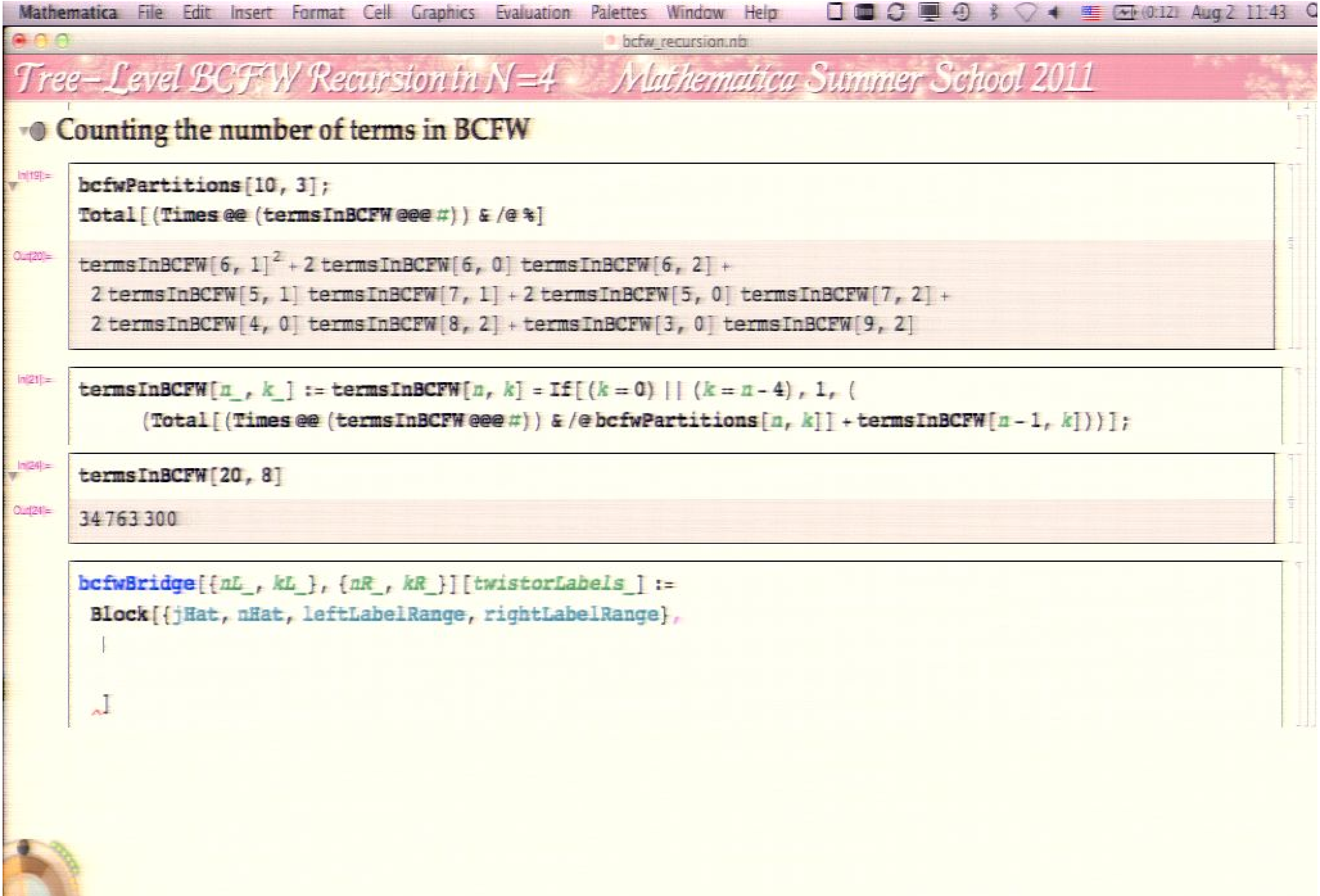
```
In[24]:=
termsInBCFW[20, 8]

Out[24]=
34 763 300
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:12) Aug 2 11:42
bcfw_recursion.nb

*Tree–Level BCFW Recursion in N=4*    *Mathematica Summer School 2011*

## Counting the number of terms in BCFW

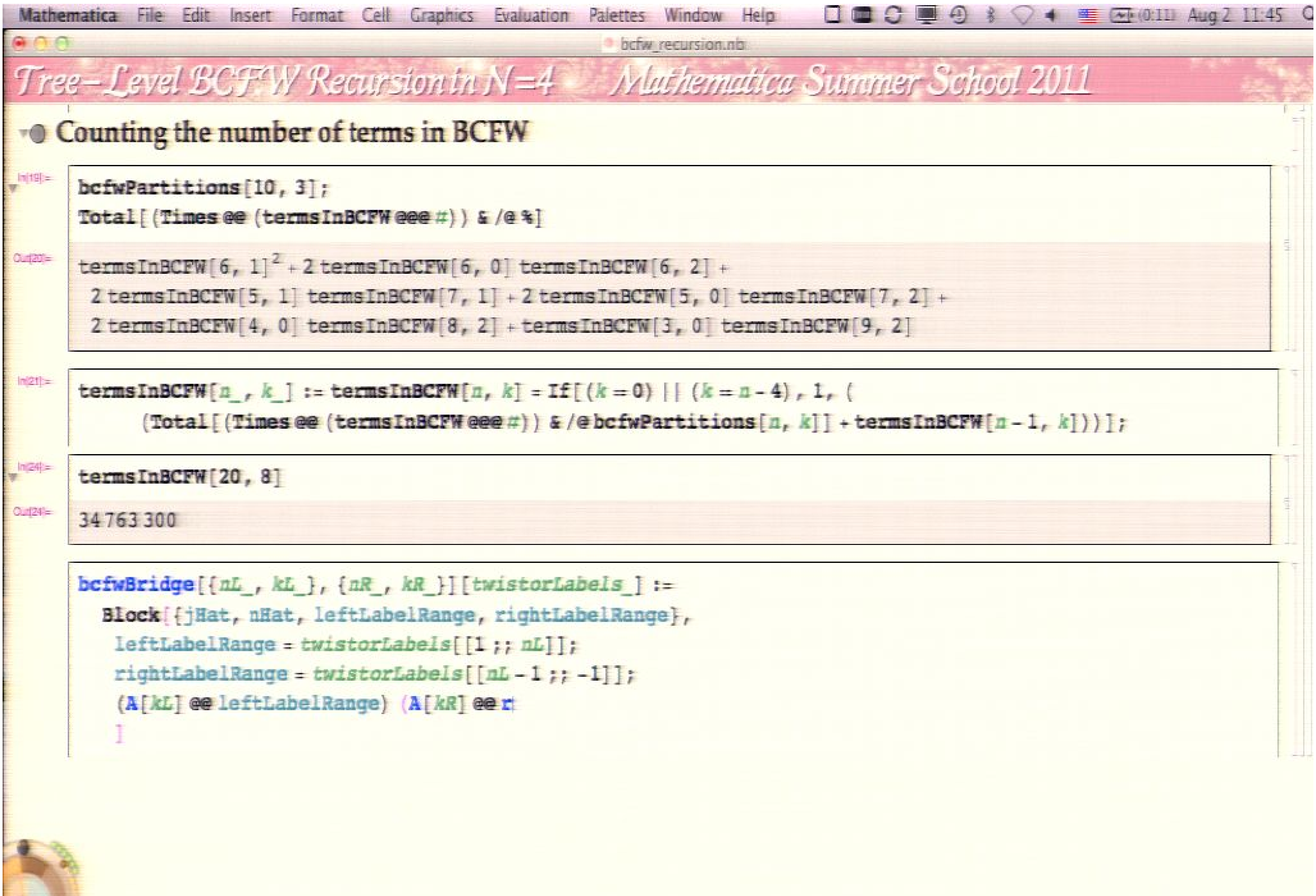
```
In[19]:= bcfwPartitions[10, 3];
         Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]= termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
          2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
          2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]

In[21]:= termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
              (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[24]:= termsInBCFW[20, 8]

Out[24]= 34 763 300
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:12) Aug 2 11:43
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

## Counting the number of terms in BCFW

```
In[19]:=
bcfwPartitions[10, 3];
Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]=
termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
 2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
 2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]

In[21]:=
termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
     (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[24]:=
termsInBCFW[20, 8]

Out[24]=
34 763 300

bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange},
  |


 ]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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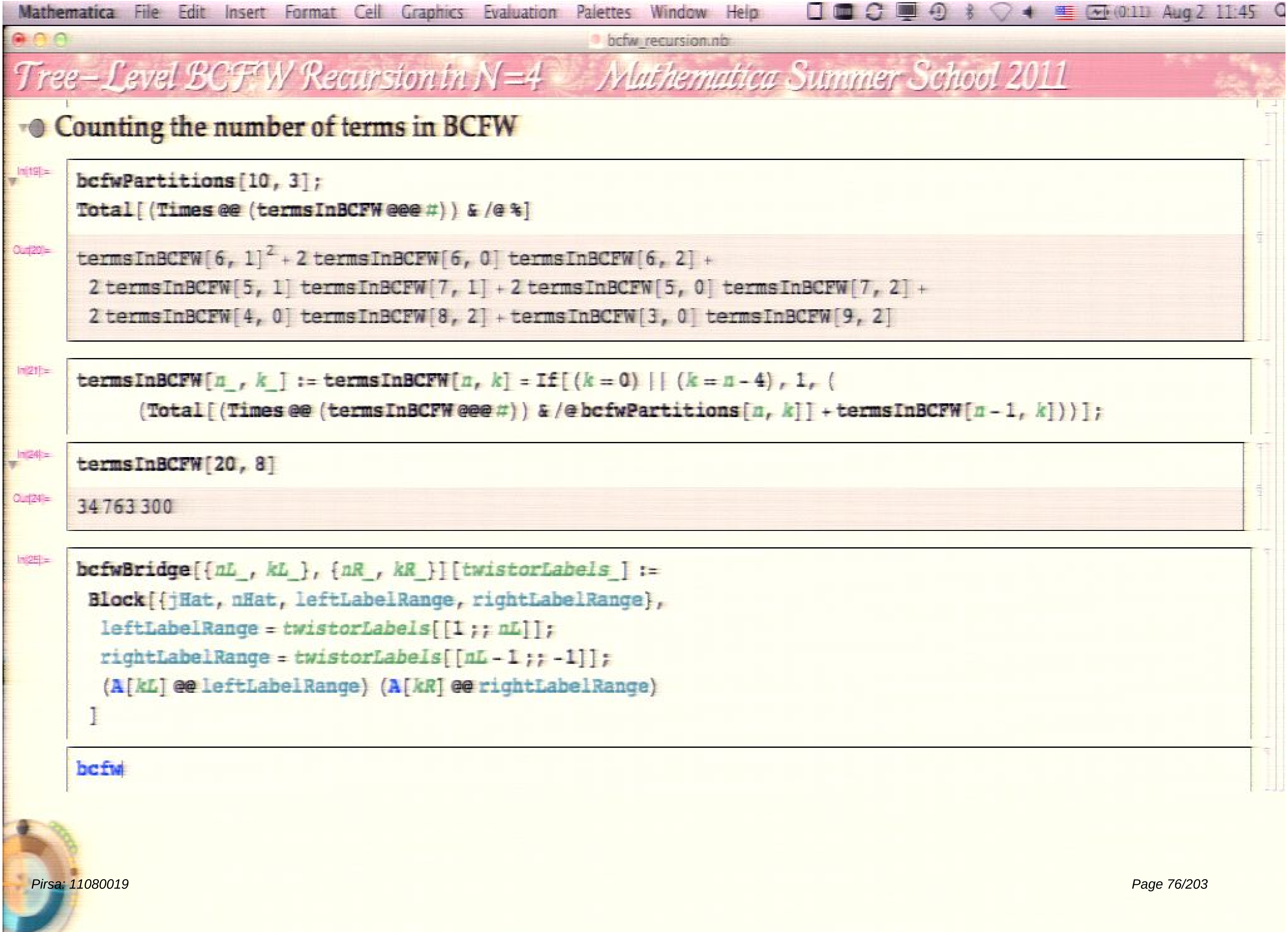
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

## Counting the number of terms in BCFW

```
In[19]:= bcfwPartitions[10, 3];
         Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]= termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
          2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
          2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]

In[21]:= termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
              (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[24]:= termsInBCFW[20, 8]

Out[24]= 34763300

bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange},
   leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   (A[kL] @@ leftLabelRange) (A[kR] @@ r
   ]
```



Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

## Counting the number of terms in BCFW

In[19]:=
```
bcfwPartitions[10, 3];
Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]
```

Out[20]=
```
termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
 2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
 2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]
```

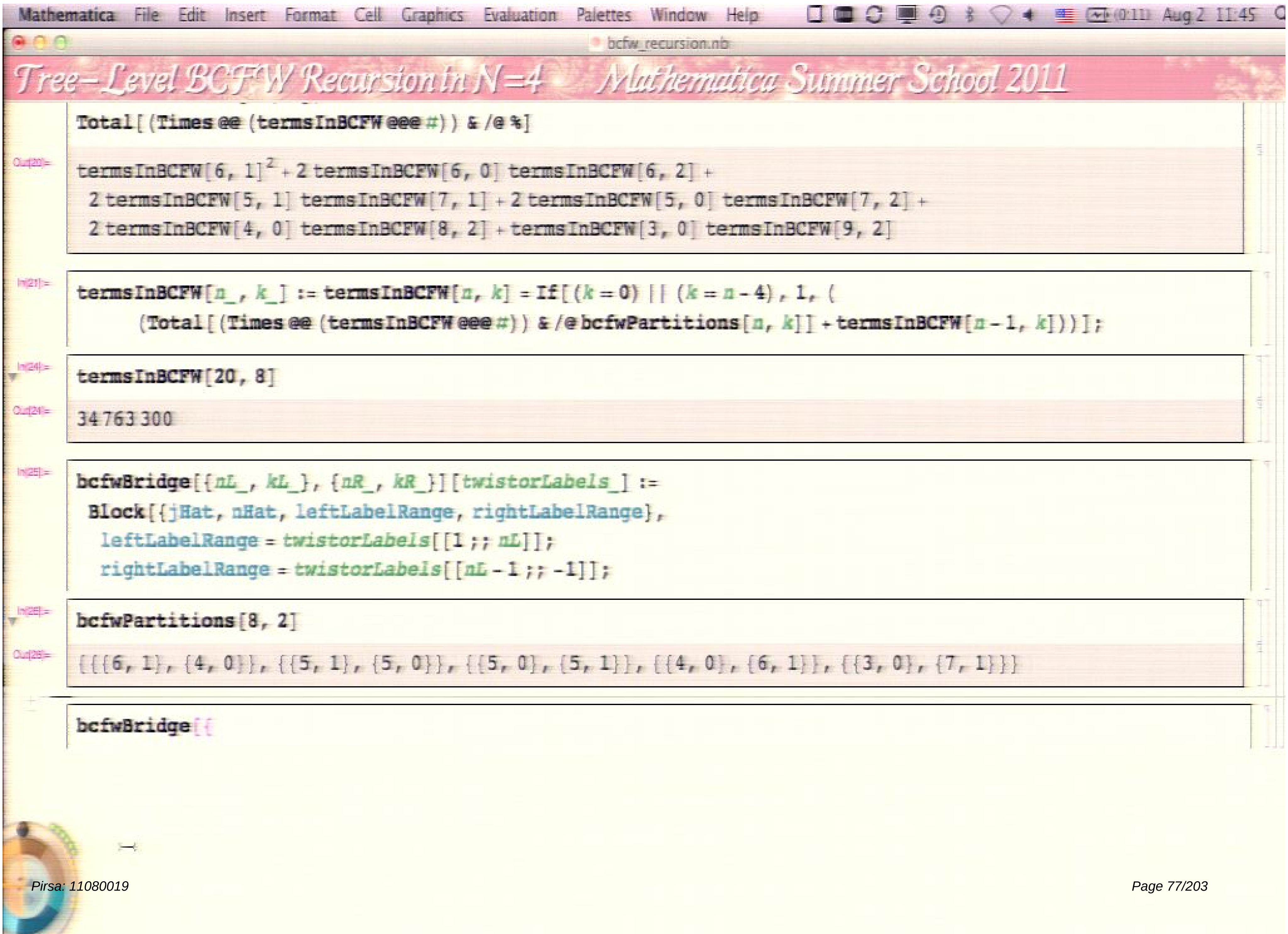
In[21]:=
```
termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
     (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
```

In[24]:=
```
termsInBCFW[20, 8]
```

Out[24]=
```
34 763 300
```

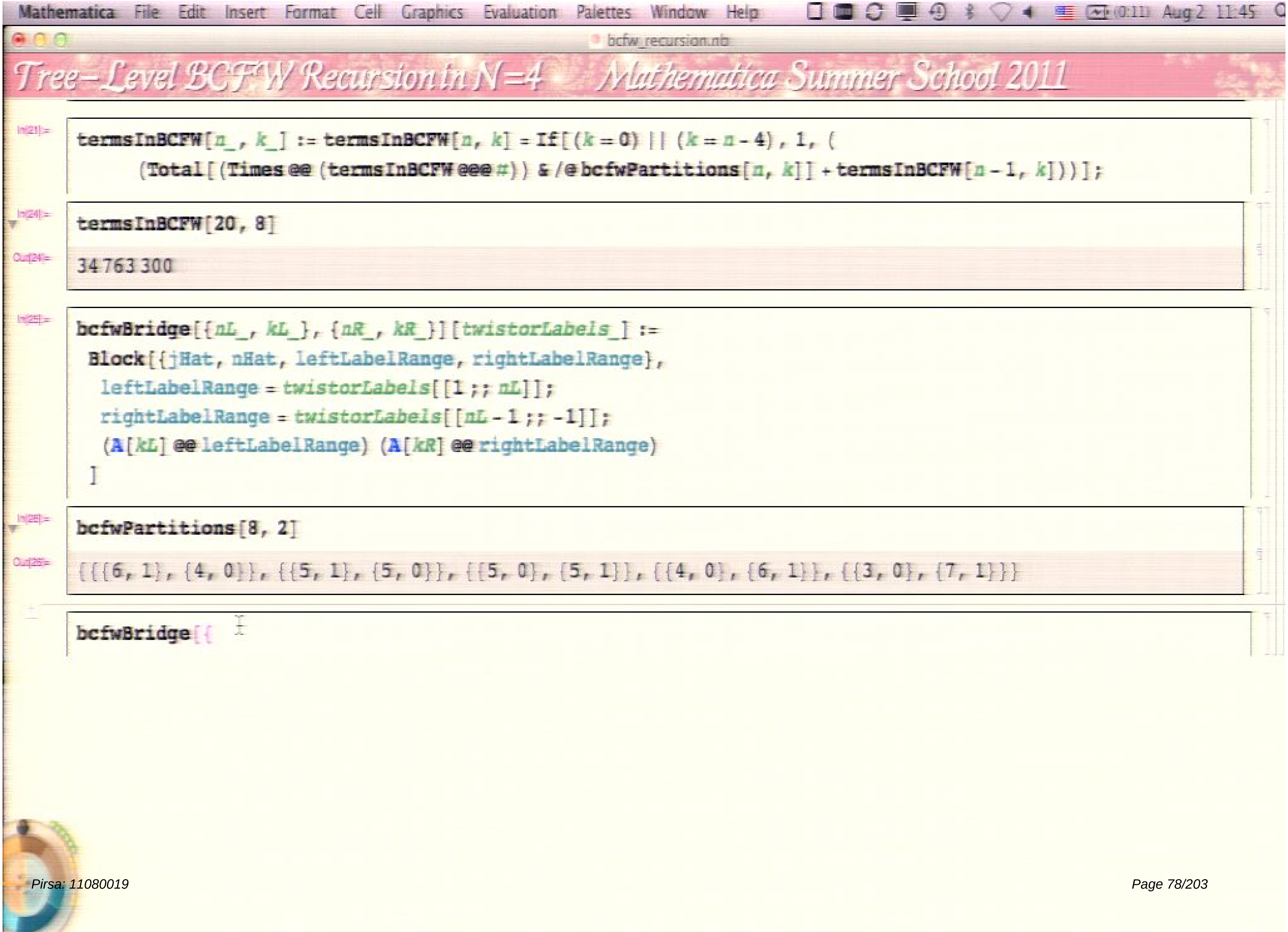
In[25]:=
```
bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange},
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  (A[kL] @@ leftLabelRange) (A[kR] @@ rightLabelRange)
 ]
```

```
bcfw
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:11) Aug 2 11:45
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
Total[(Times @@ (termsInBCFW @@@ #)) & /@ %]

Out[20]= termsInBCFW[6, 1]^2 + 2 termsInBCFW[6, 0] termsInBCFW[6, 2] +
          2 termsInBCFW[5, 1] termsInBCFW[7, 1] + 2 termsInBCFW[5, 0] termsInBCFW[7, 2] +
          2 termsInBCFW[4, 0] termsInBCFW[8, 2] + termsInBCFW[3, 0] termsInBCFW[9, 2]

In[21]:= termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
              (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[24]:= termsInBCFW[20, 8]

Out[24]= 34 763 300

In[25]:= bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
          Block[{jHat, nHat, leftLabelRange, rightLabelRange},
           leftLabelRange = twistorLabels[[1 ;; nL]];
           rightLabelRange = twistorLabels[[nL - 1 ;; -1]];

In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

bcfwBridge[{
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

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             (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[24]:= termsInBCFW[20, 8]

Out[24]= 34 763 300

In[25]:= bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
          Block[{jHat, nHat, leftLabelRange, rightLabelRange},
           leftLabelRange = twistorLabels[[1 ;; nL]];
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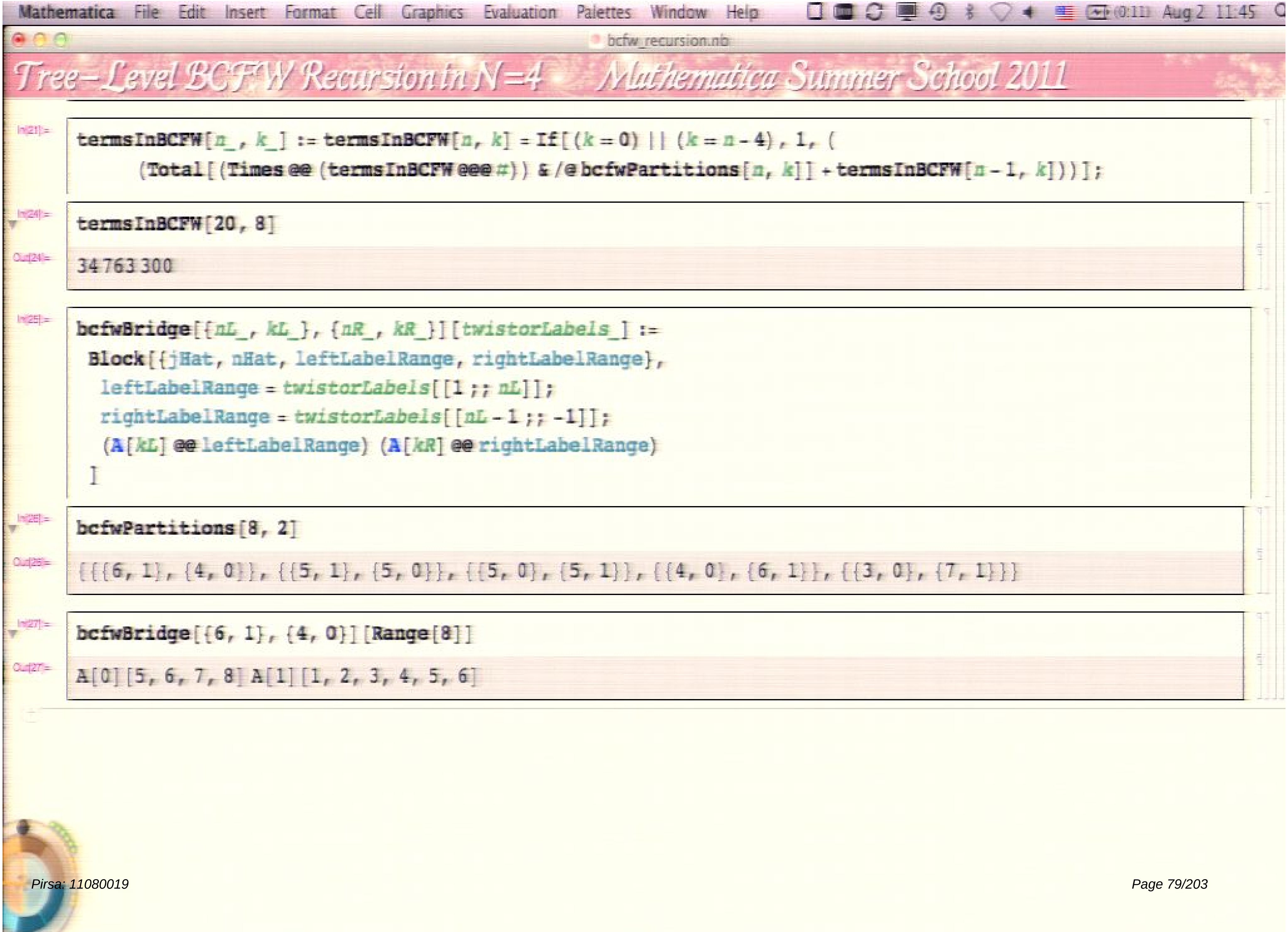
           (A[kL] @@ leftLabelRange) (A[kR] @@ rightLabelRange)
          ]

In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

bcfwBridge[{
```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Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[21]:= termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
              (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[24]:= termsInBCFW[20, 8]

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In[25]:= bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
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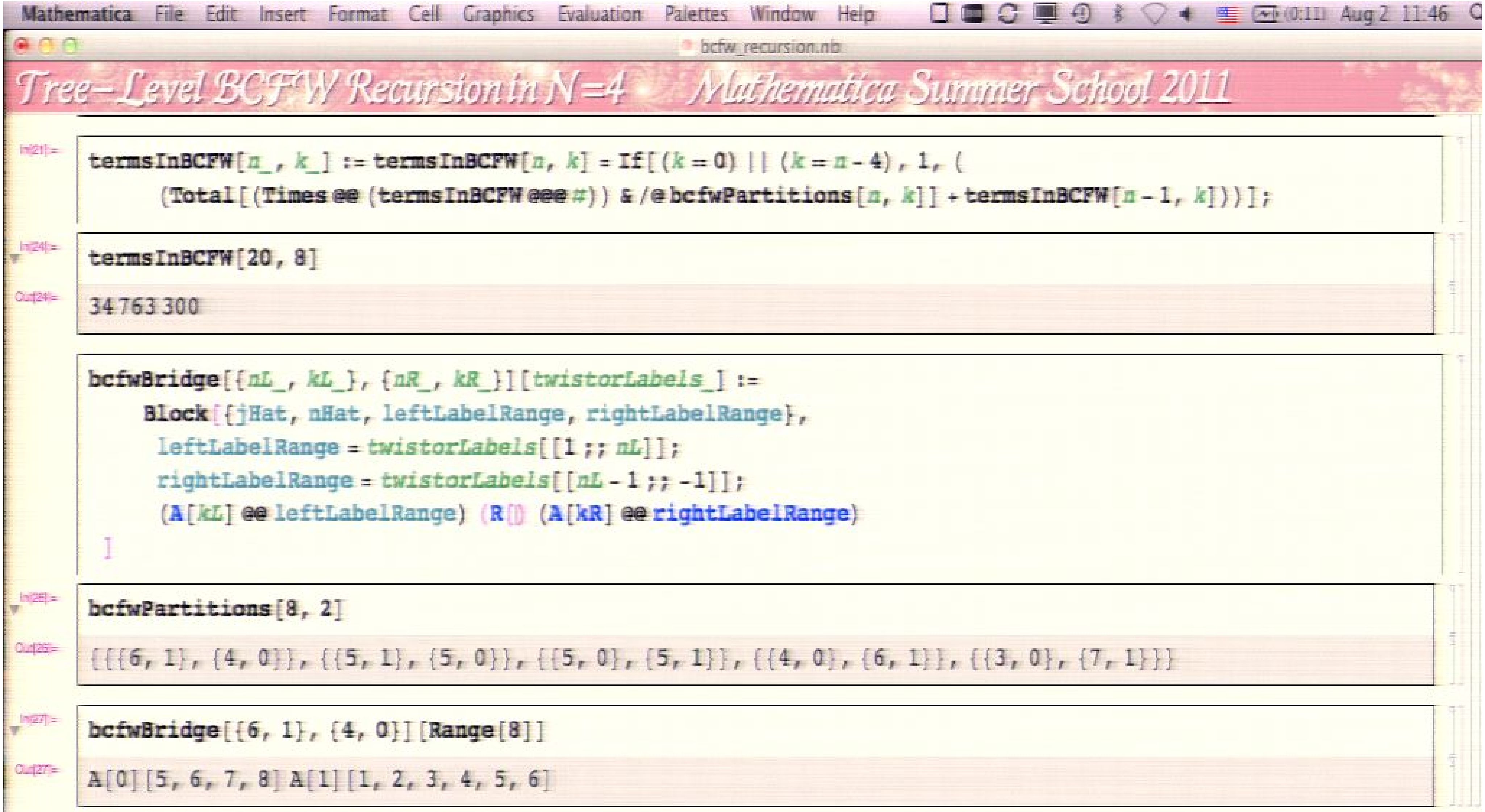
           leftLabelRange = twistorLabels[[1 ;; nL]];
           rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
           (A[kL] @@ leftLabelRange) (A[kR] @@ rightLabelRange)
          ]

In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[27]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[27]= A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:11) Aug 2 11:46
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

```
In[21]:= termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
             (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[24]:= termsInBCFW[20, 8]

Out[24]= 34 763 300

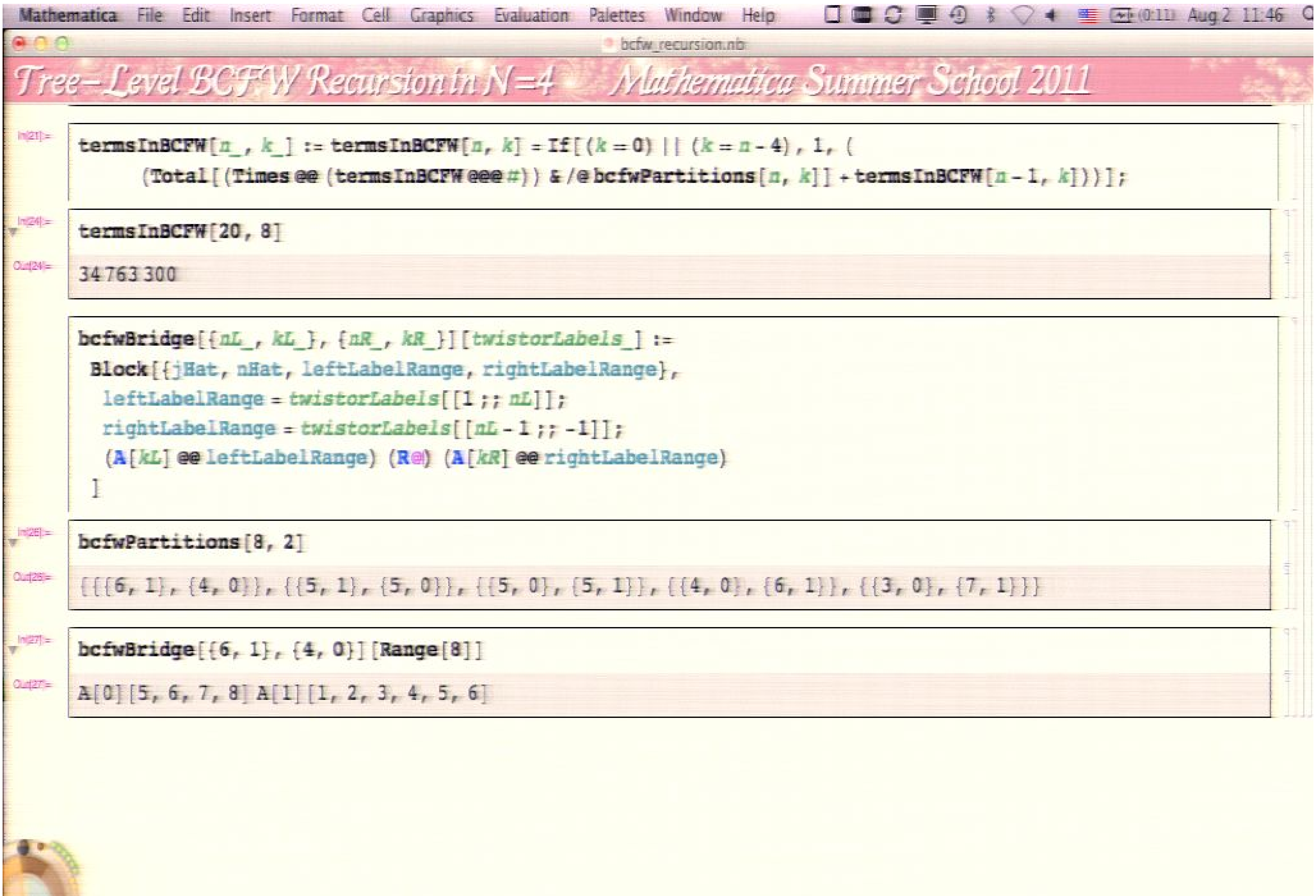
         bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
           Block[{jHat, nHat, leftLabelRange, rightLabelRange},
            leftLabelRange = twistorLabels[[1 ;; nL]];
            rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
            (A[kL] @@ leftLabelRange) (R[]) (A[kR] @@ rightLabelRange)
           ]

In[25]:= bcfwPartitions[8, 2]

Out[25]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[27]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[27]= A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:11) Aug 2 11:46
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011
In[21]:=
In[24]:=
Out[24]=
In[26]:=
Out[26]=
In[27]:=
Out[27]=

```
termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
     (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
```

```
termsInBCFW[20, 8]
```

```
34 763 300
```

```
bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
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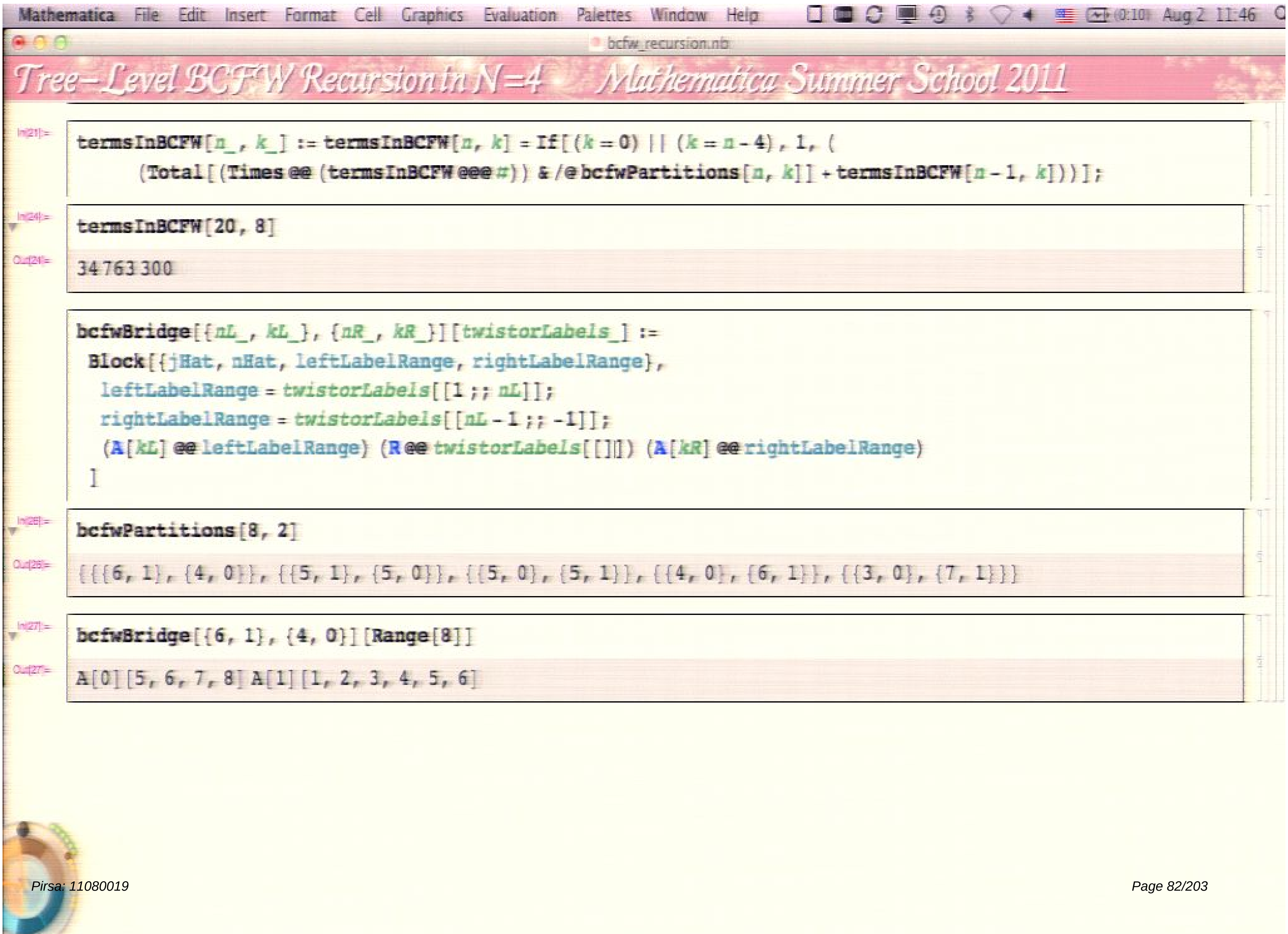
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  (A[kL] @@ leftLabelRange) (R@) (A[kR] @@ rightLabelRange)
 ]
```

```
bcfwPartitions[8, 2]
```

```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
```

```
A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
0:10 Aug 2 11:46
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[21]:= termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1, (
              (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[24]:= termsInBCFW[20, 8]

Out[24]= 34 763 300

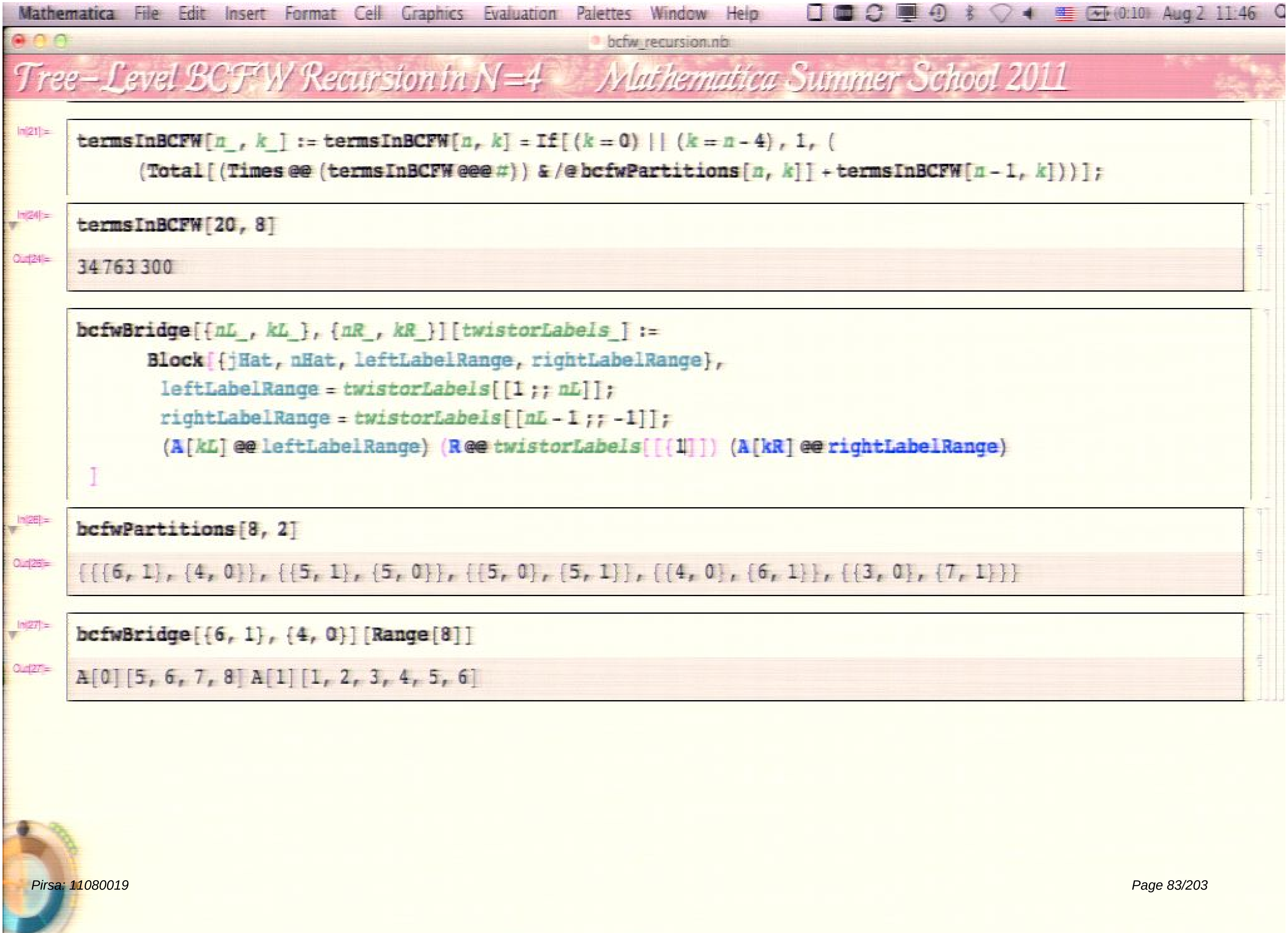
bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange},
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[]]) (A[kR] @@ rightLabelRange)
 ]

In[25]:= bcfwPartitions[8, 2]

Out[25]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[27]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[27]= A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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       leftLabelRange = twistorLabels[[1 ;; nL]];
       rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
       (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1]]) (A[kR] @@ rightLabelRange)
  ]

In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[27]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[27]= A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```

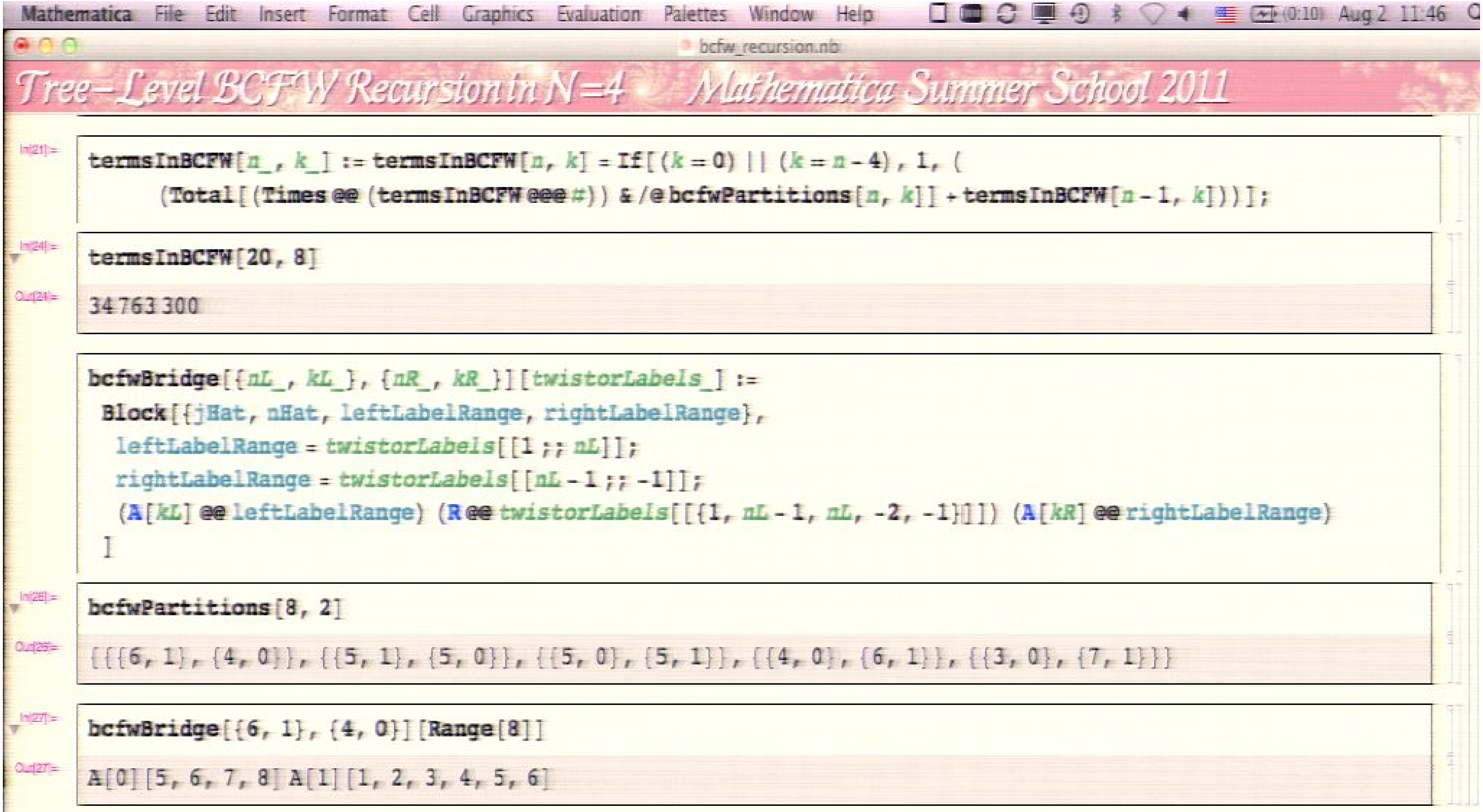
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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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Out[24]= 34 763 300

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  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)
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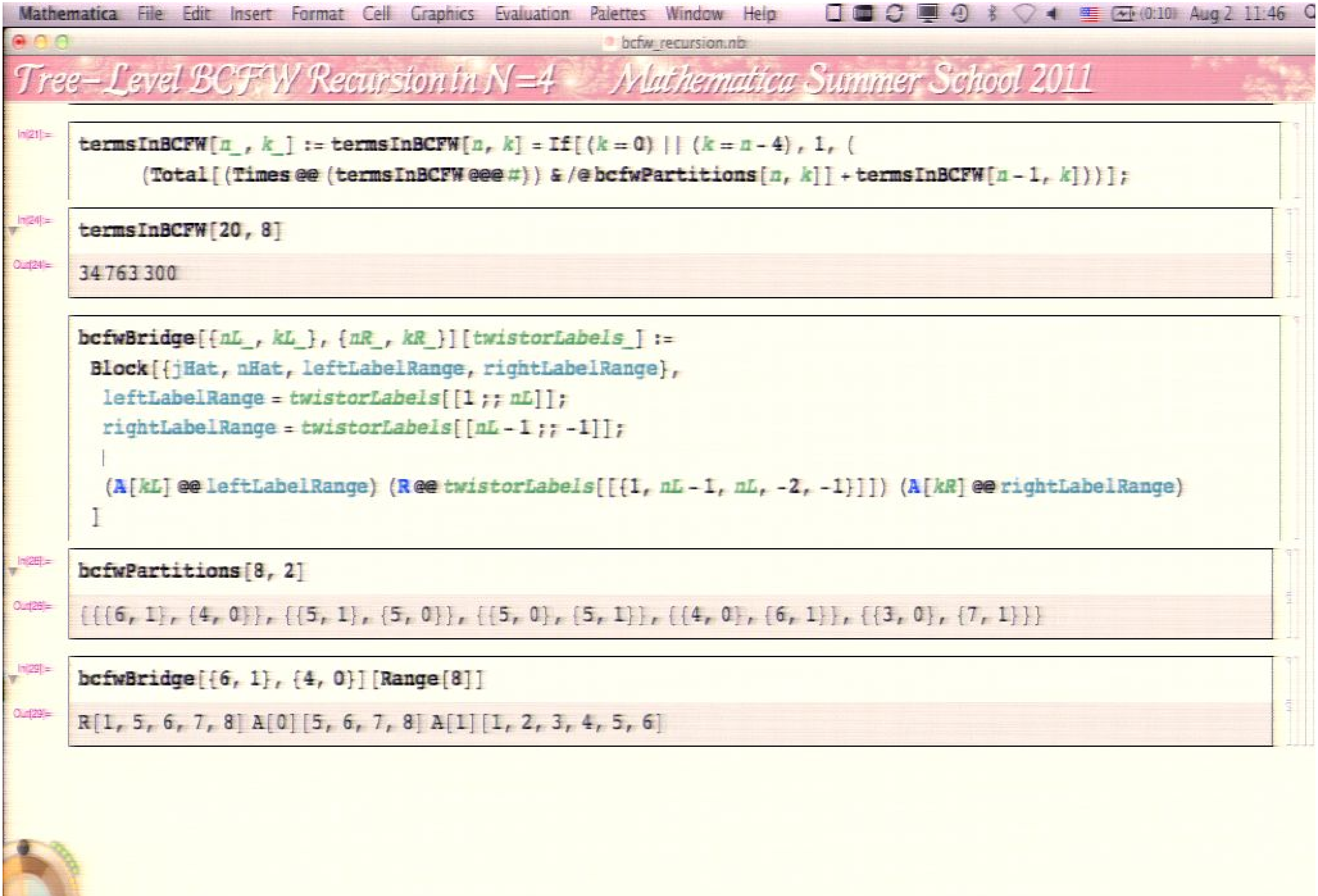
In[25]:= bcfwPartitions[8, 2]

Out[25]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[27]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[27]= A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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Out[24]= 34 763 300

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  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  |
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In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[29]= R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```

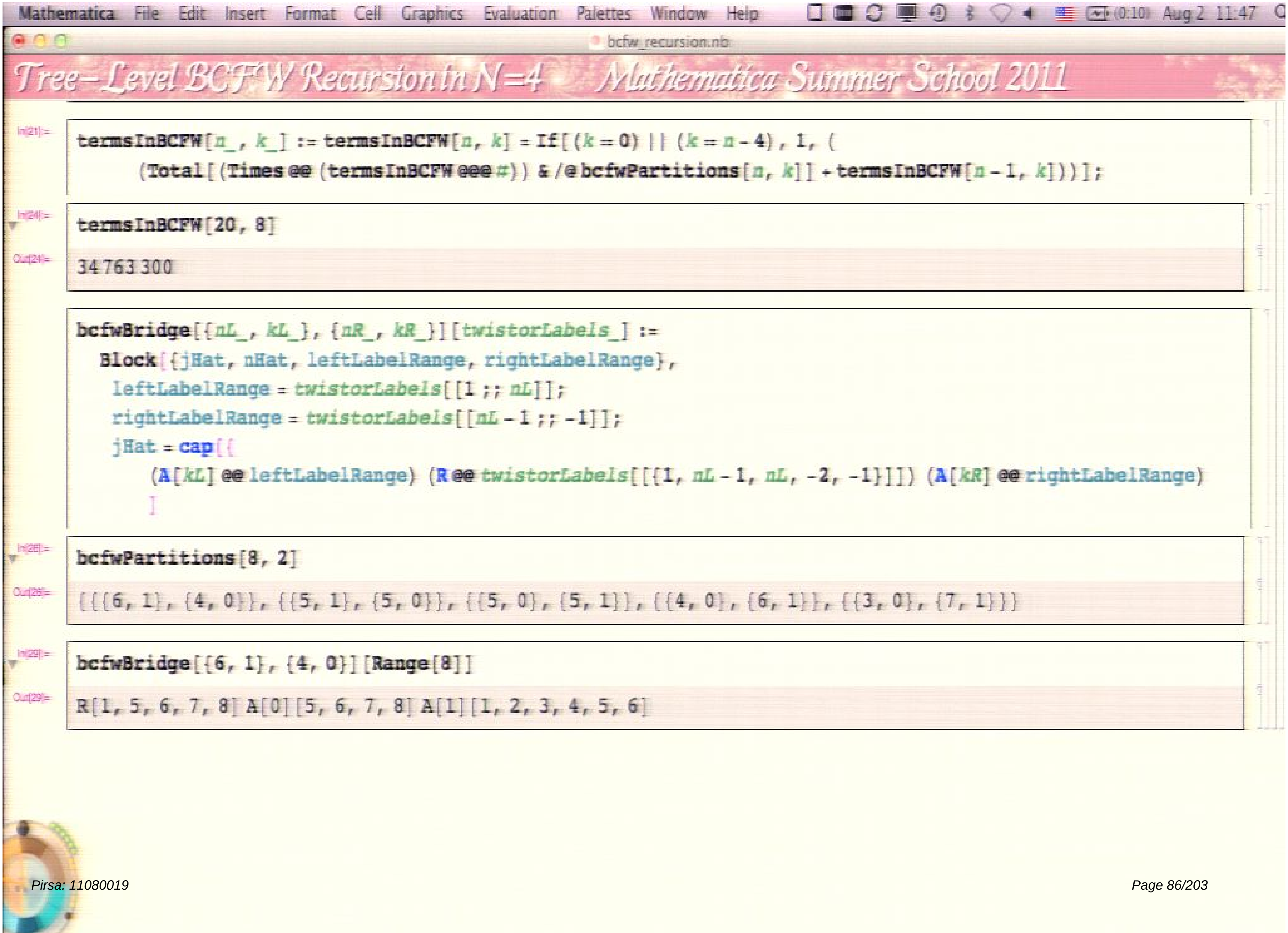
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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

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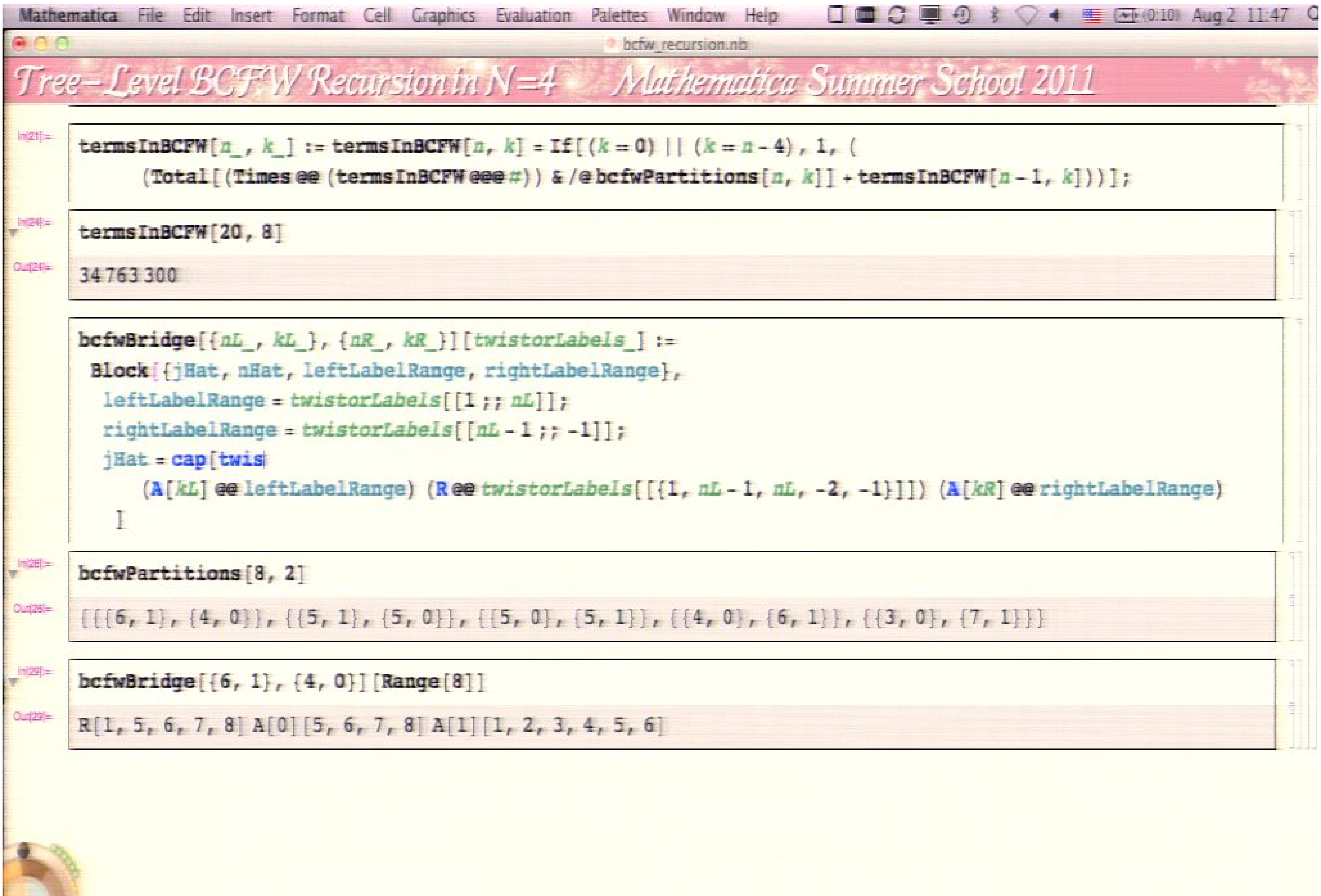
   jHat = cap[{
       (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)
       ]

In[28]:= bcfwPartitions[8, 2]

Out[28]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[29]= R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

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In[28]:= bcfwPartitions[8, 2]

Out[28]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

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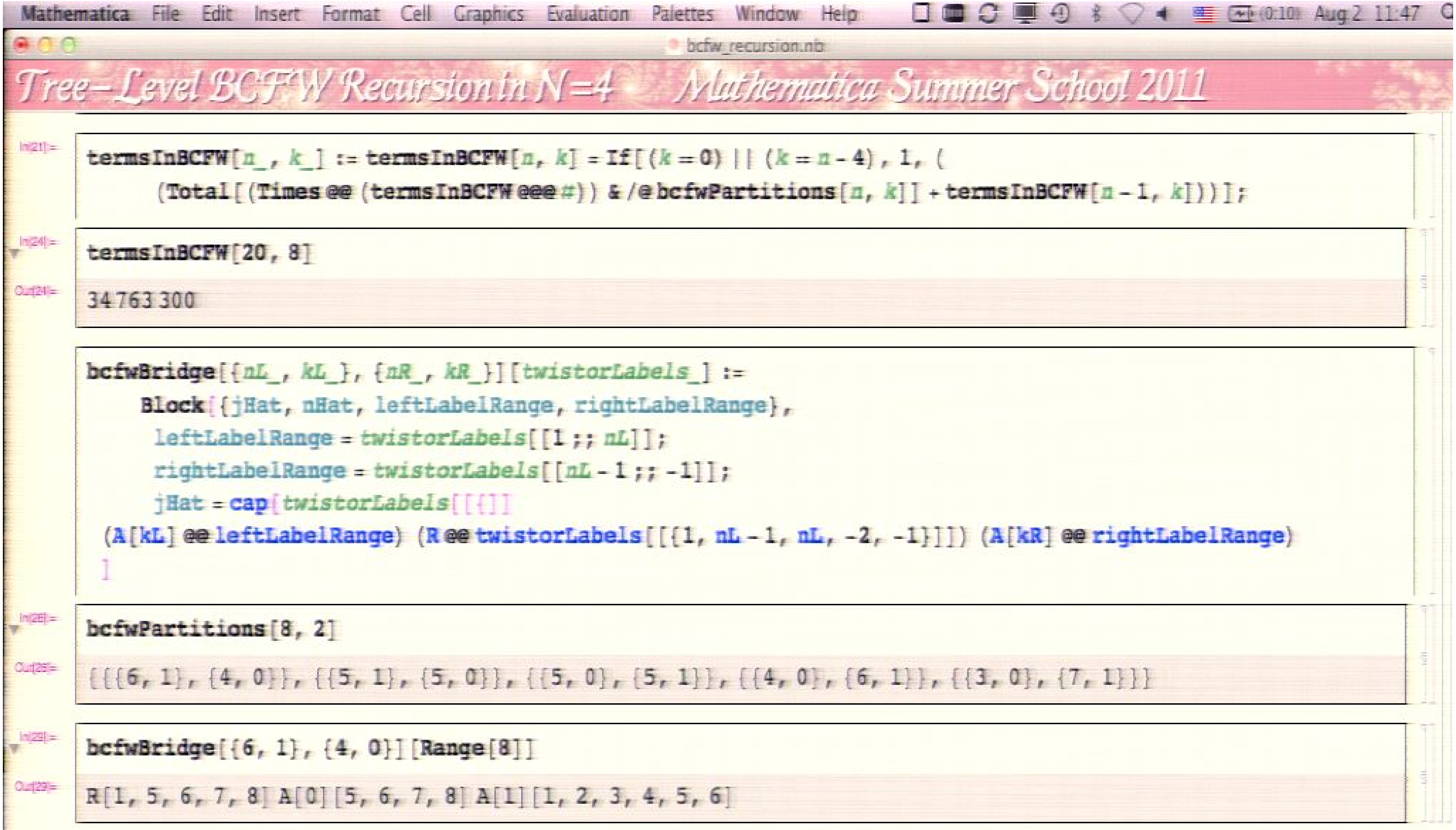
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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bcfw_recursion.nb

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     rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
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In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

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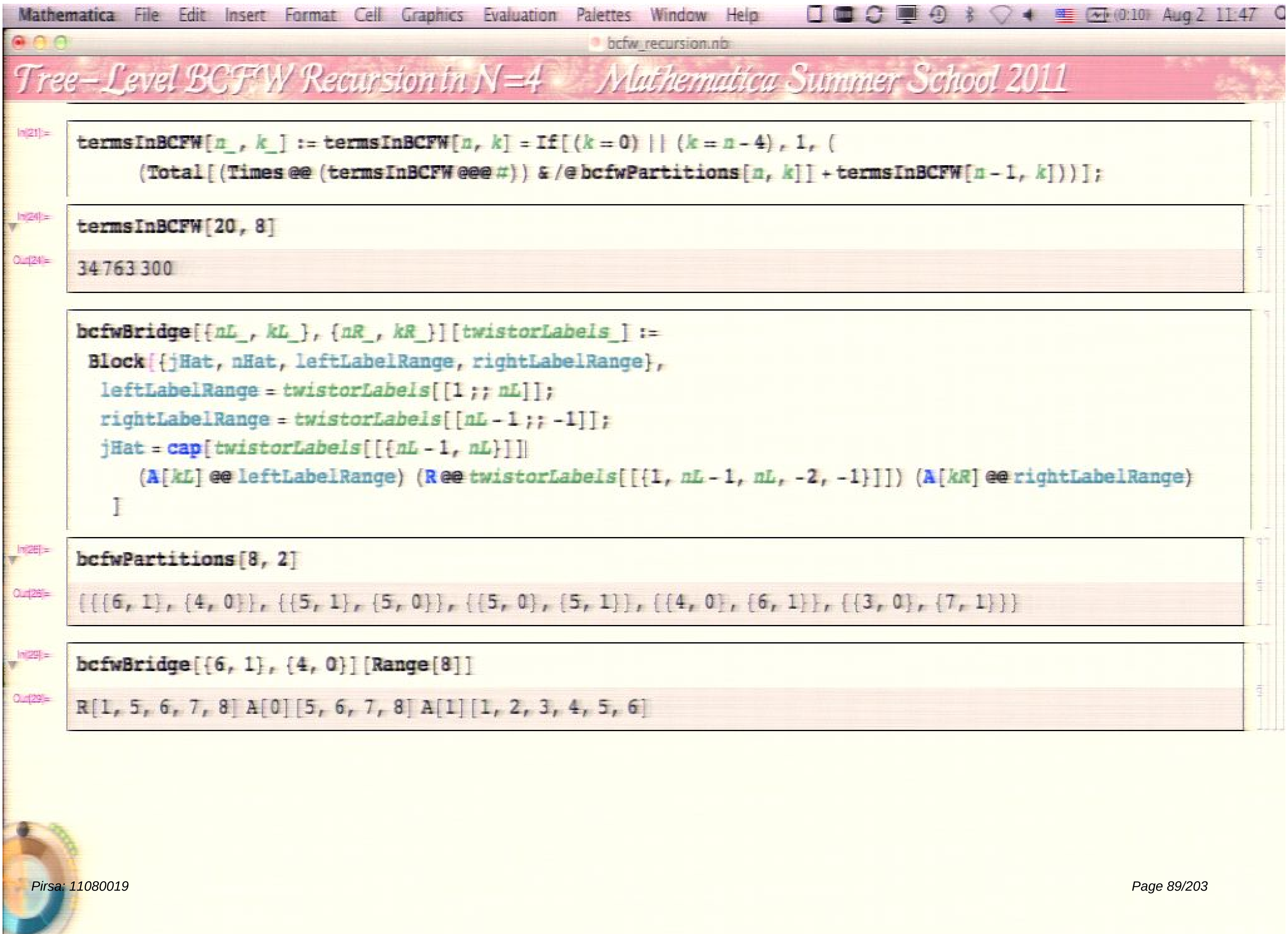

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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

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  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  jHat = cap[twistorLabels[[{nL - 1, nL}]]]
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   ]

In[28]:=
bcfwPartitions[8, 2]

Out[28]=
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:=
bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[29]=
R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```

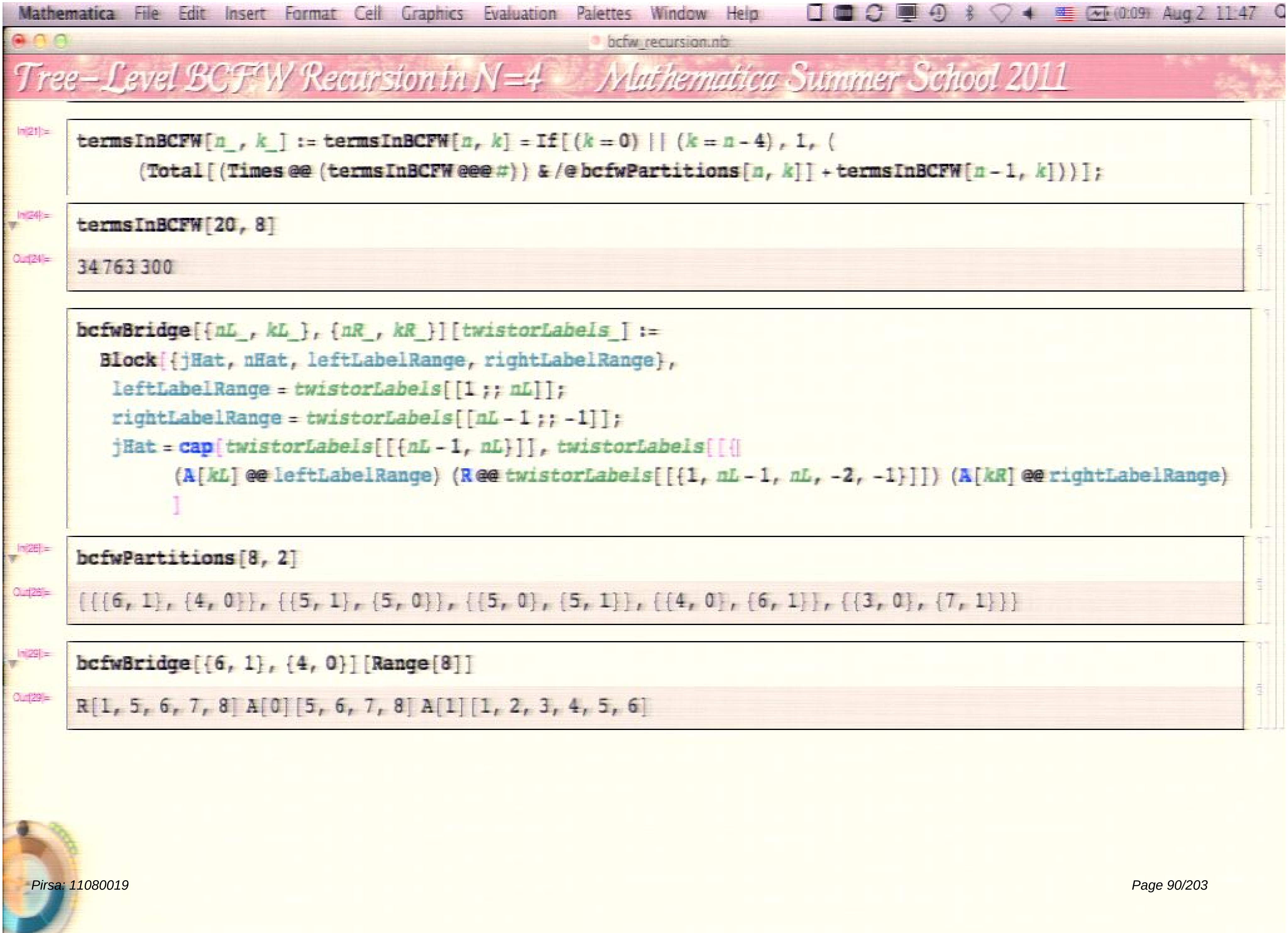
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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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   leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{
        (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)
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In[28]:= bcfwPartitions[8, 2]

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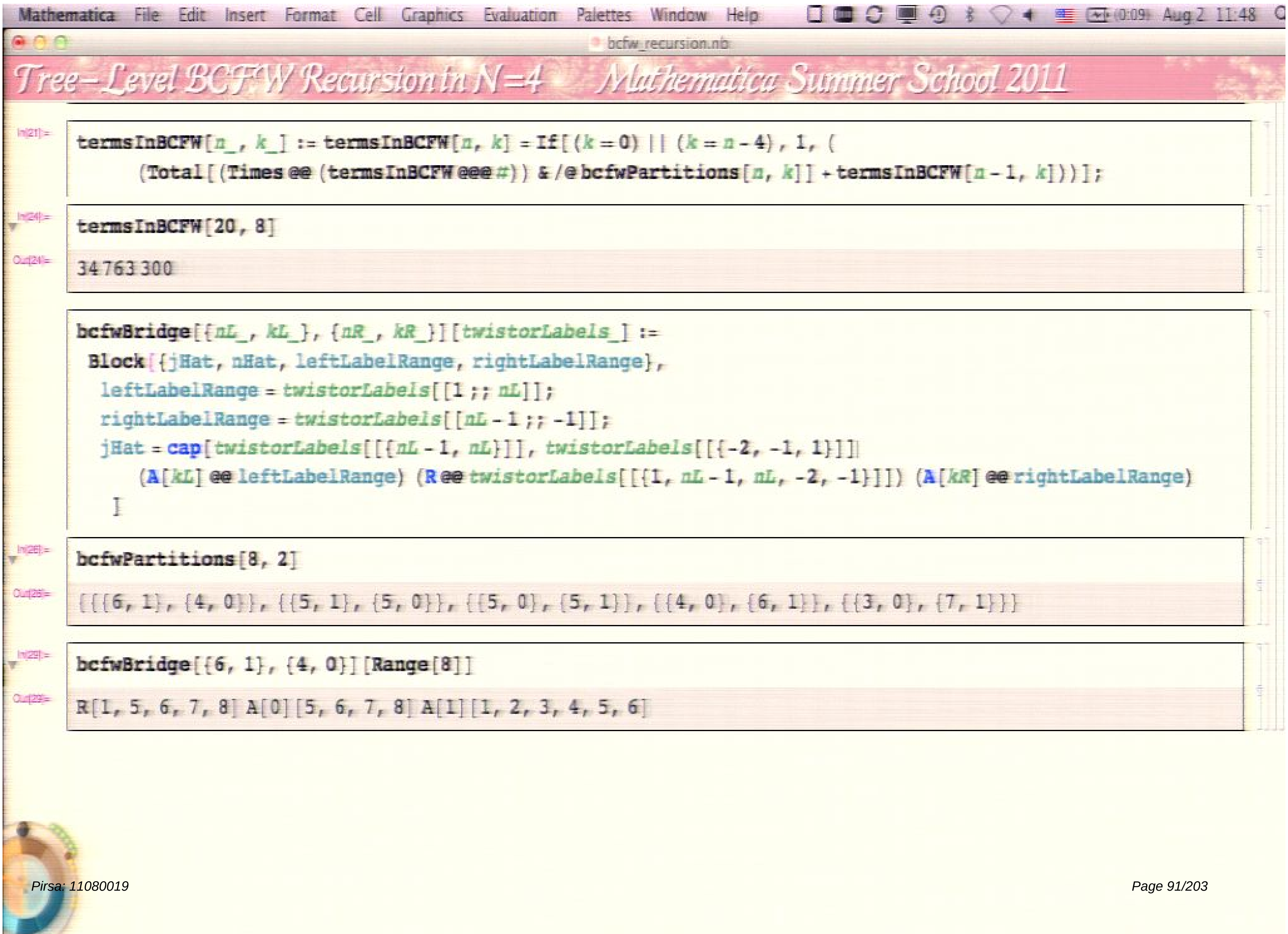
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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

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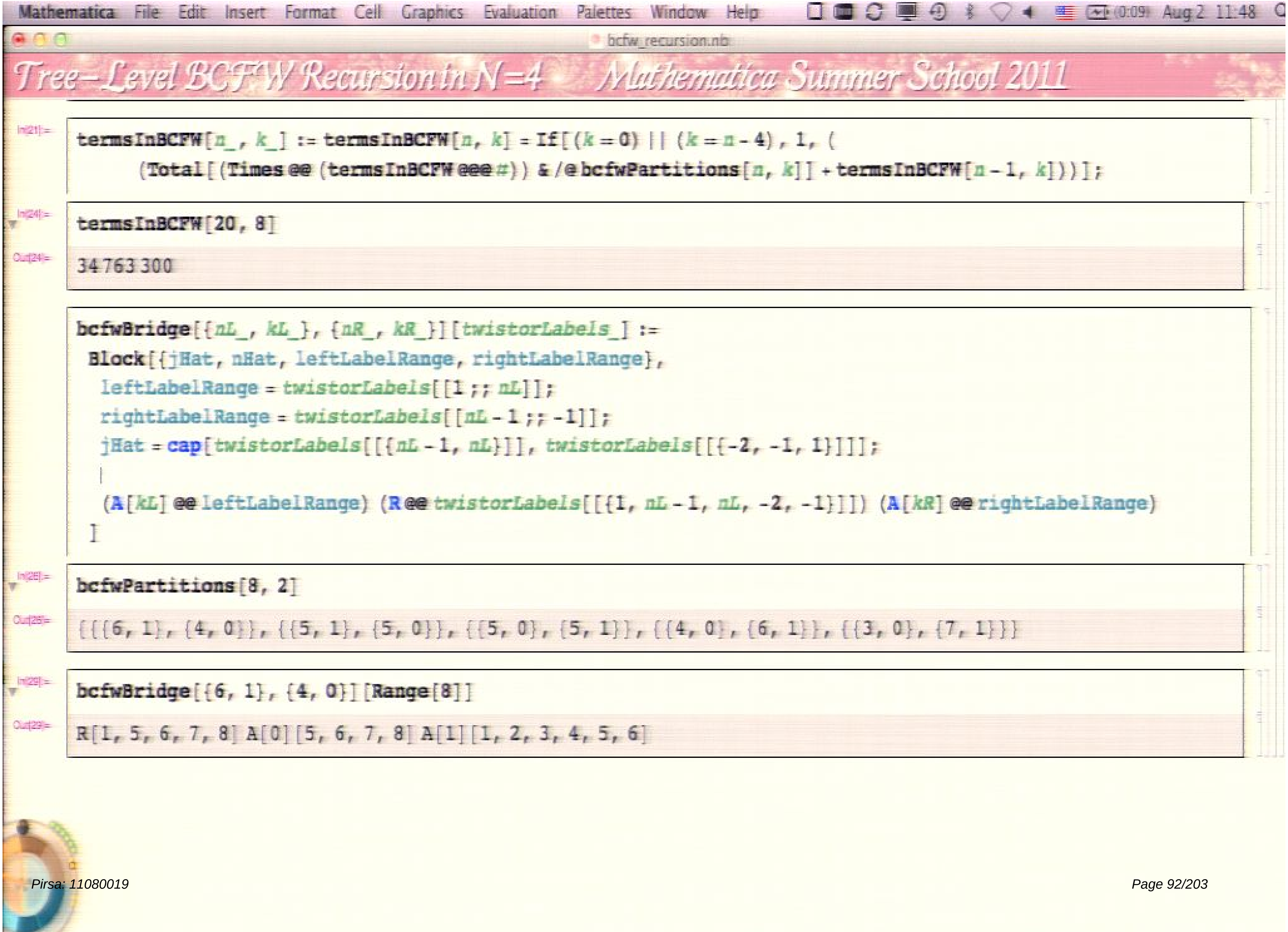
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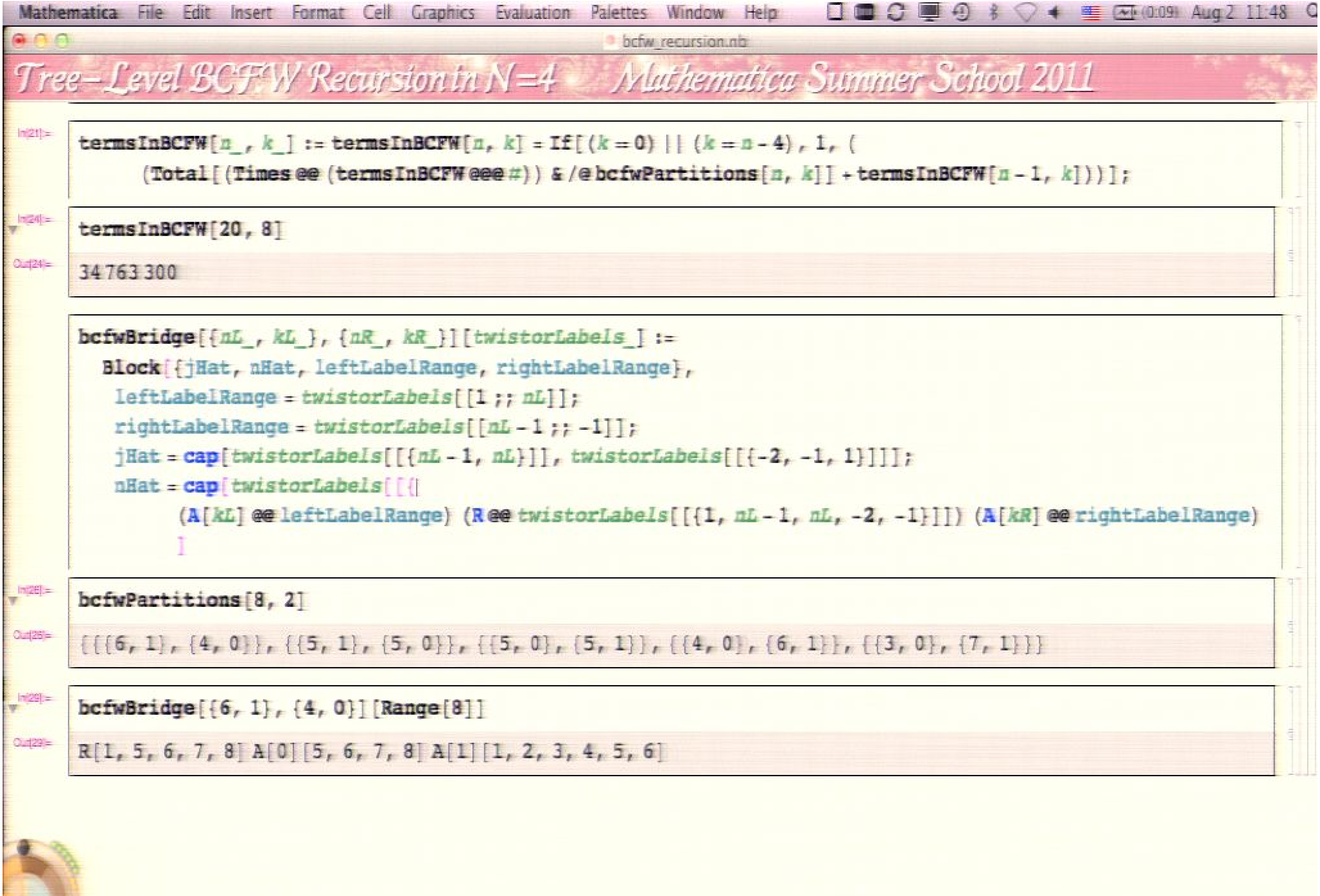
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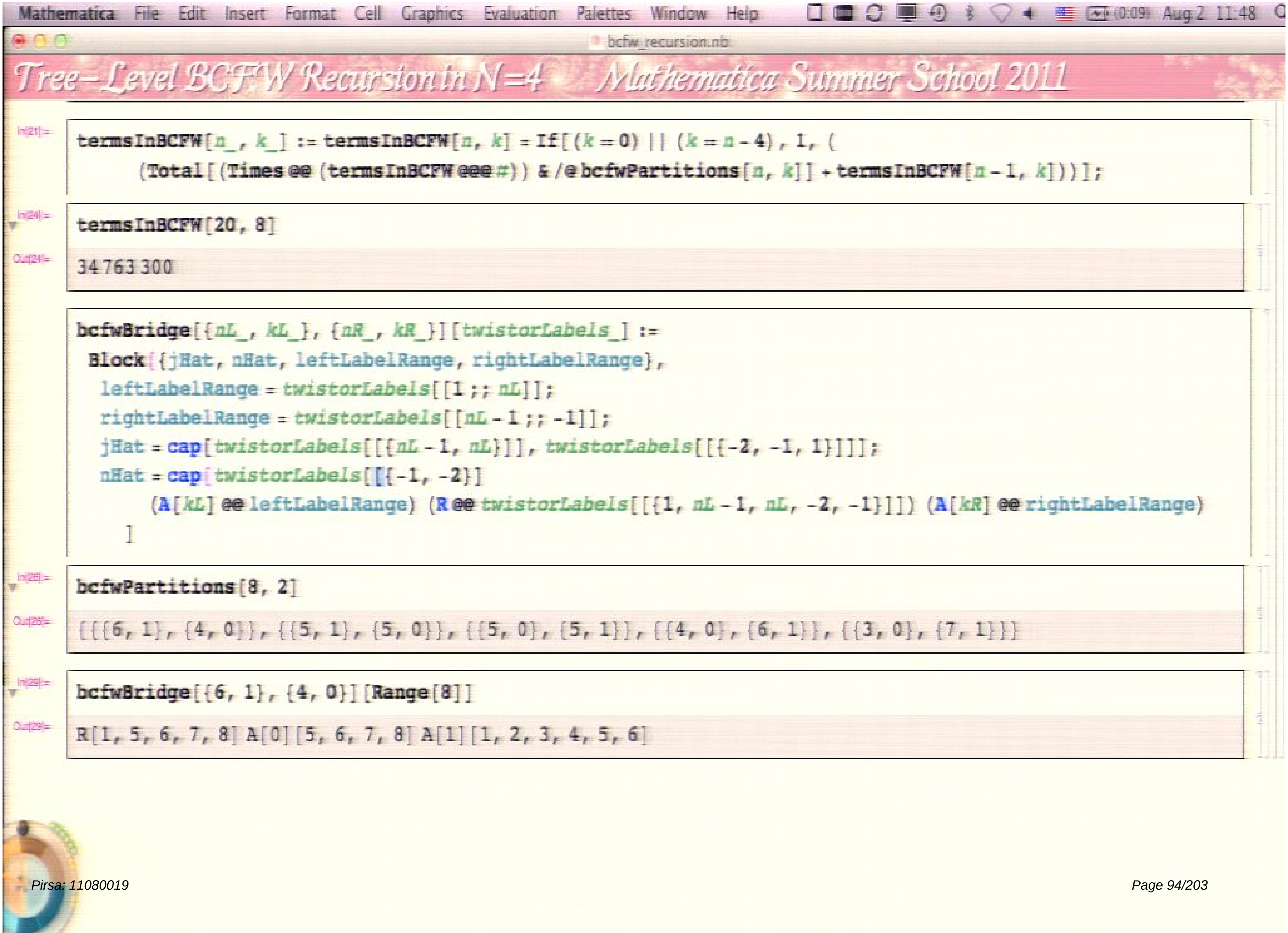
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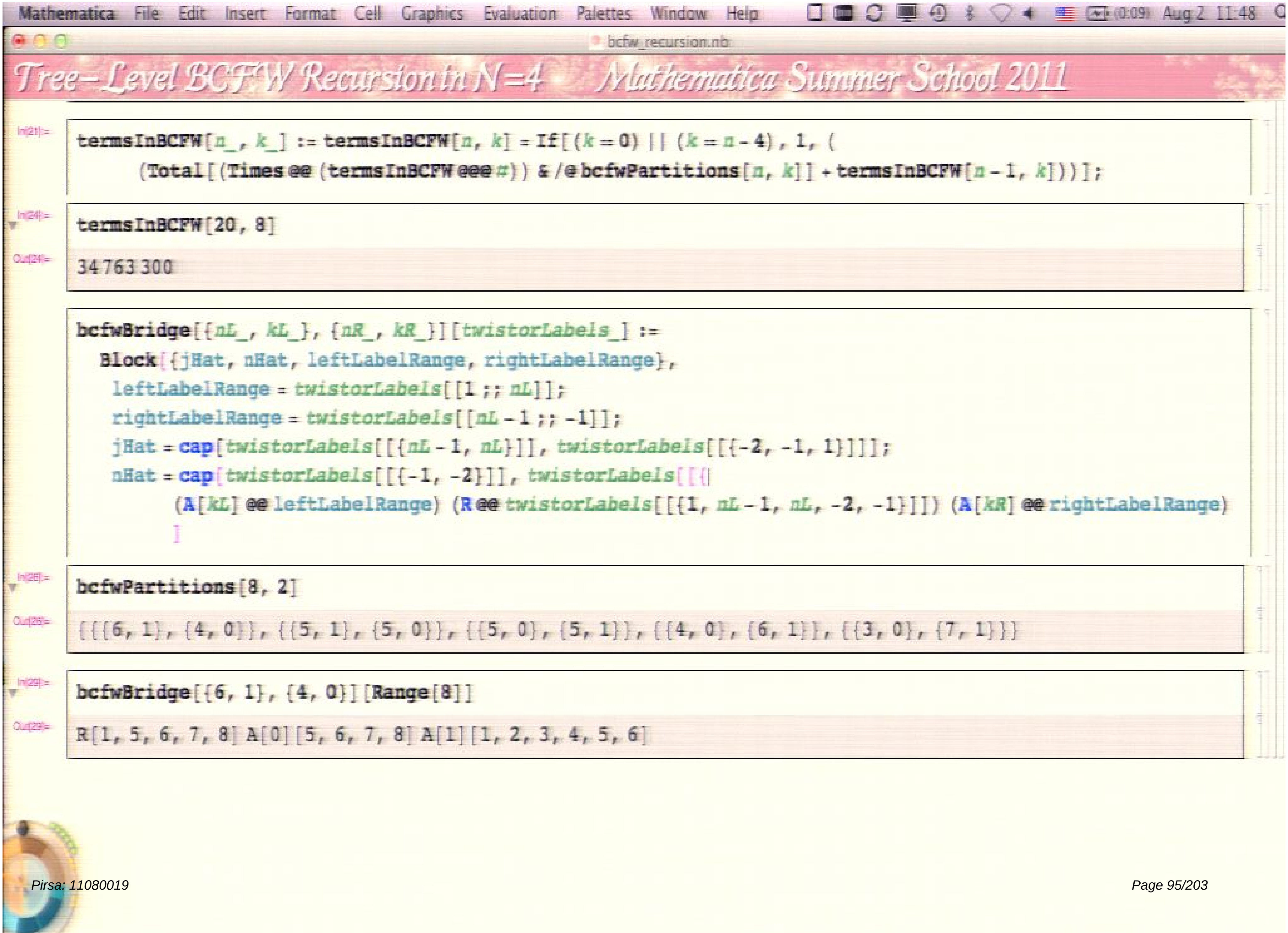
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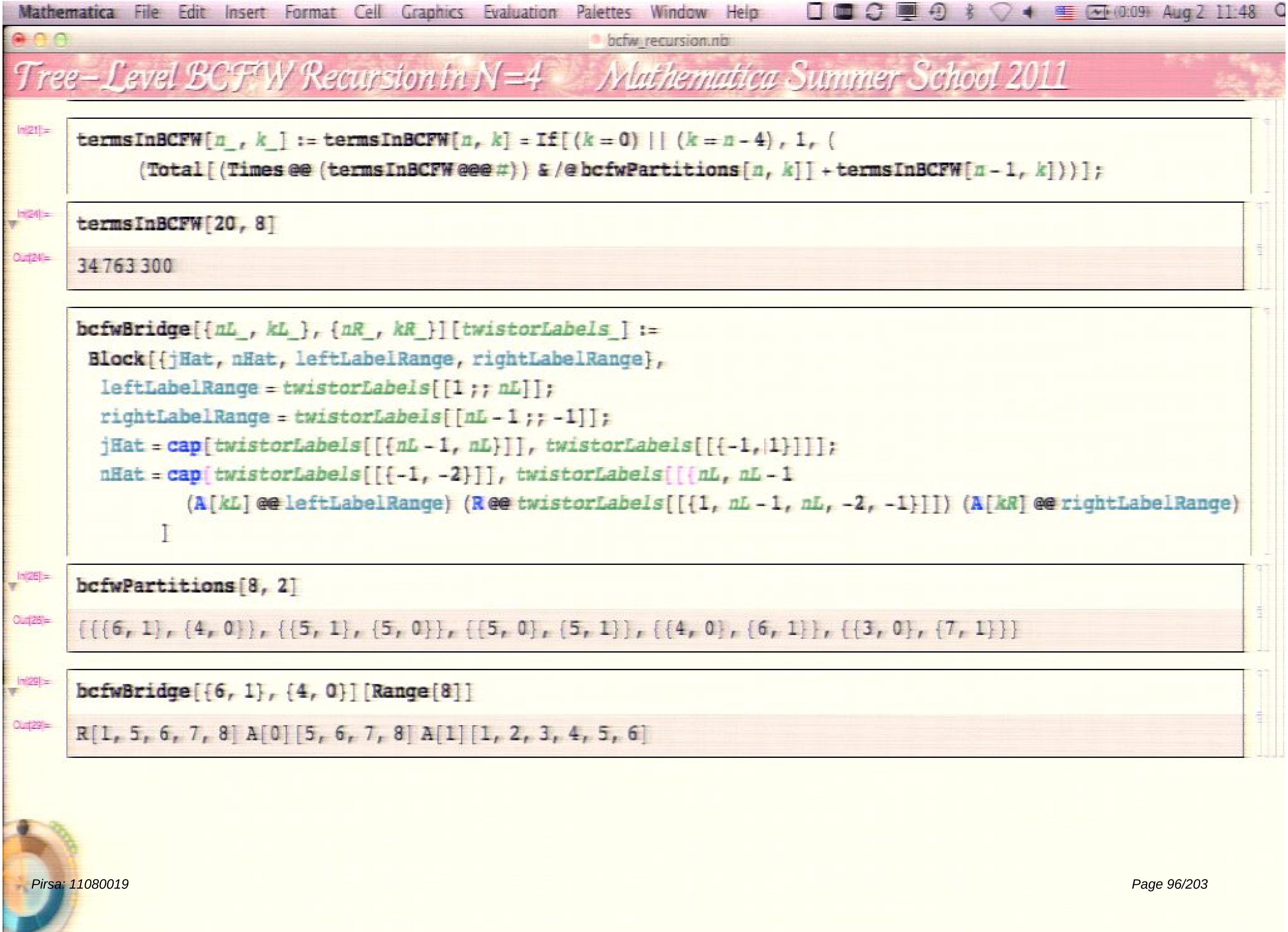
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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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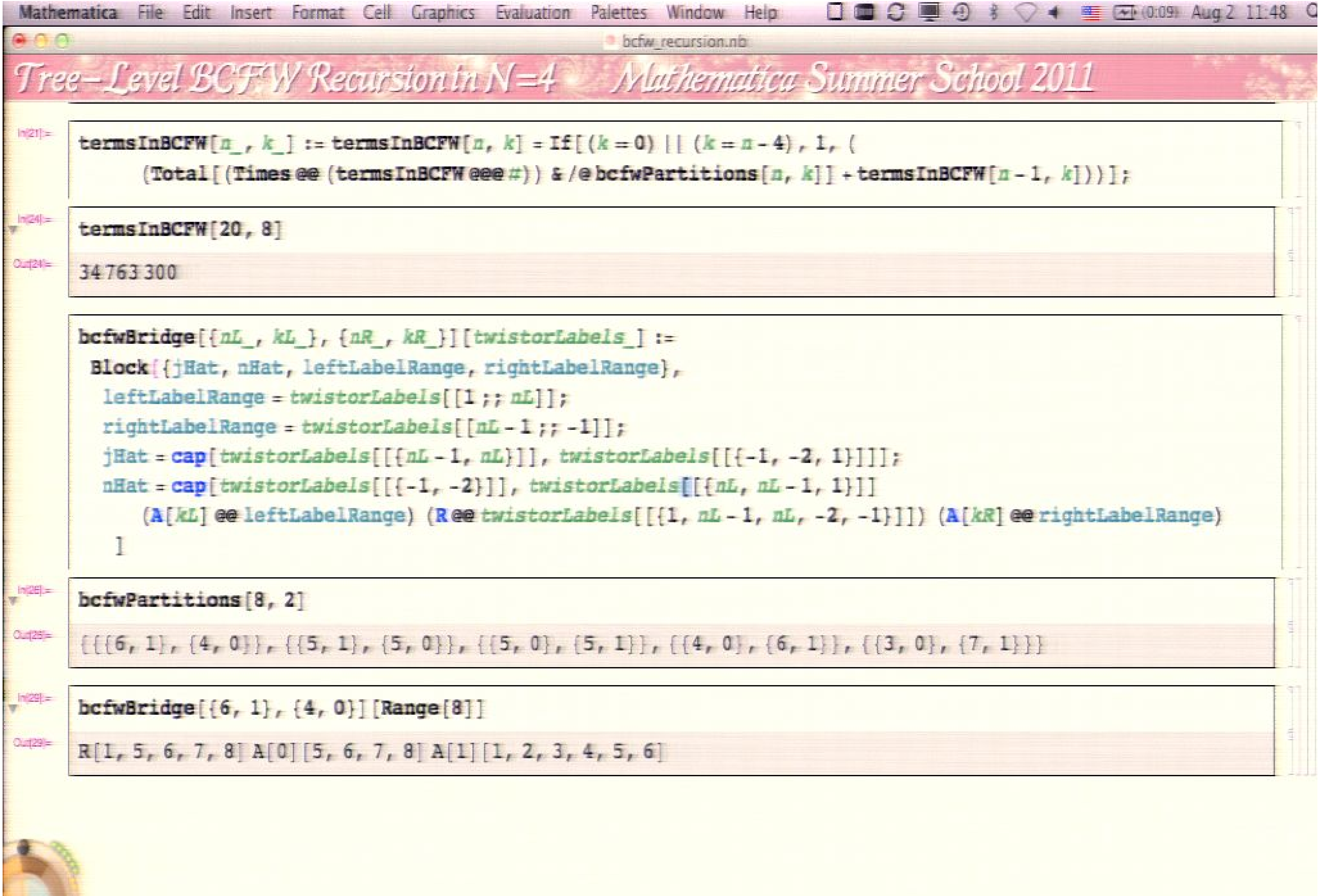
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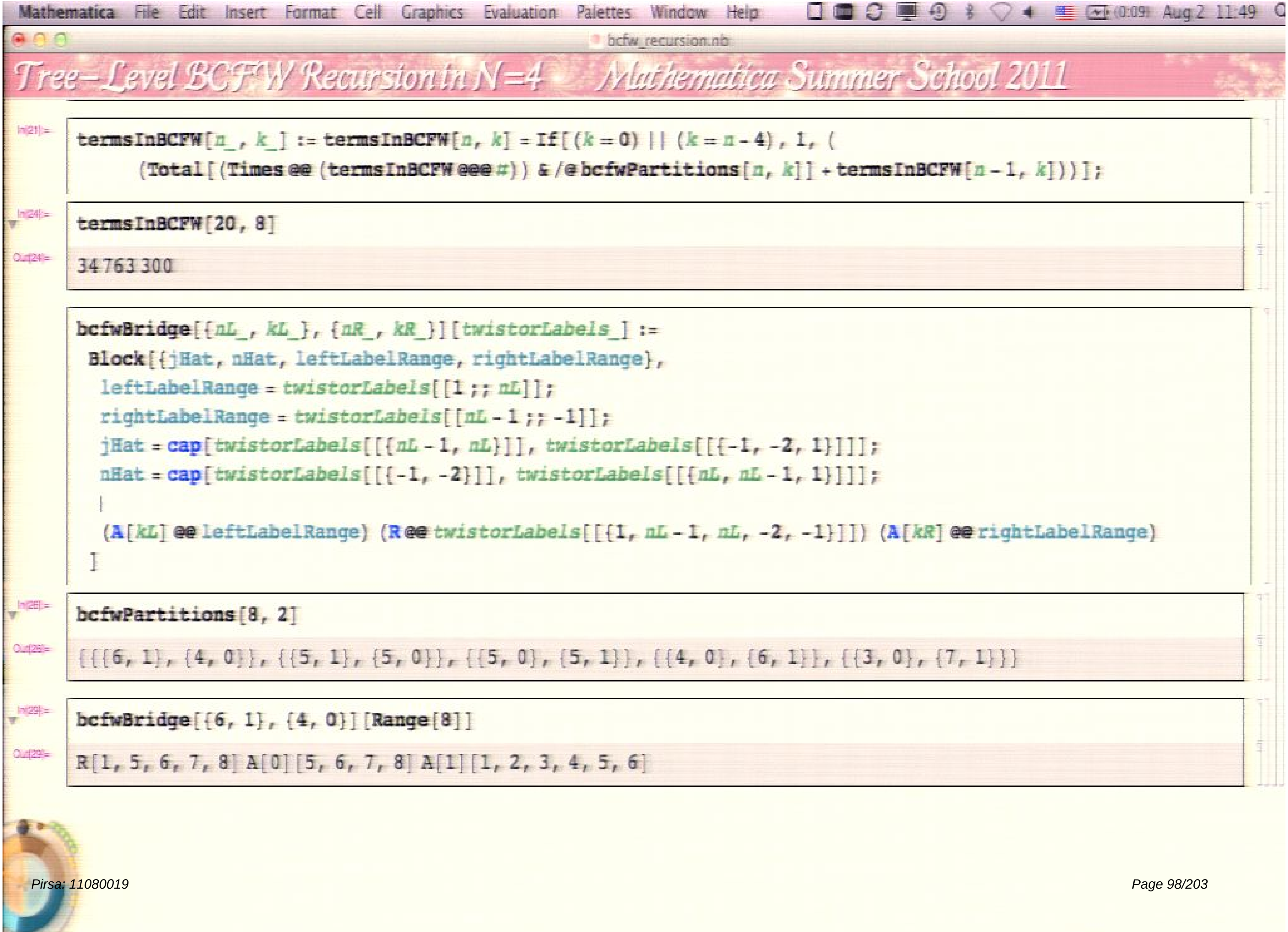
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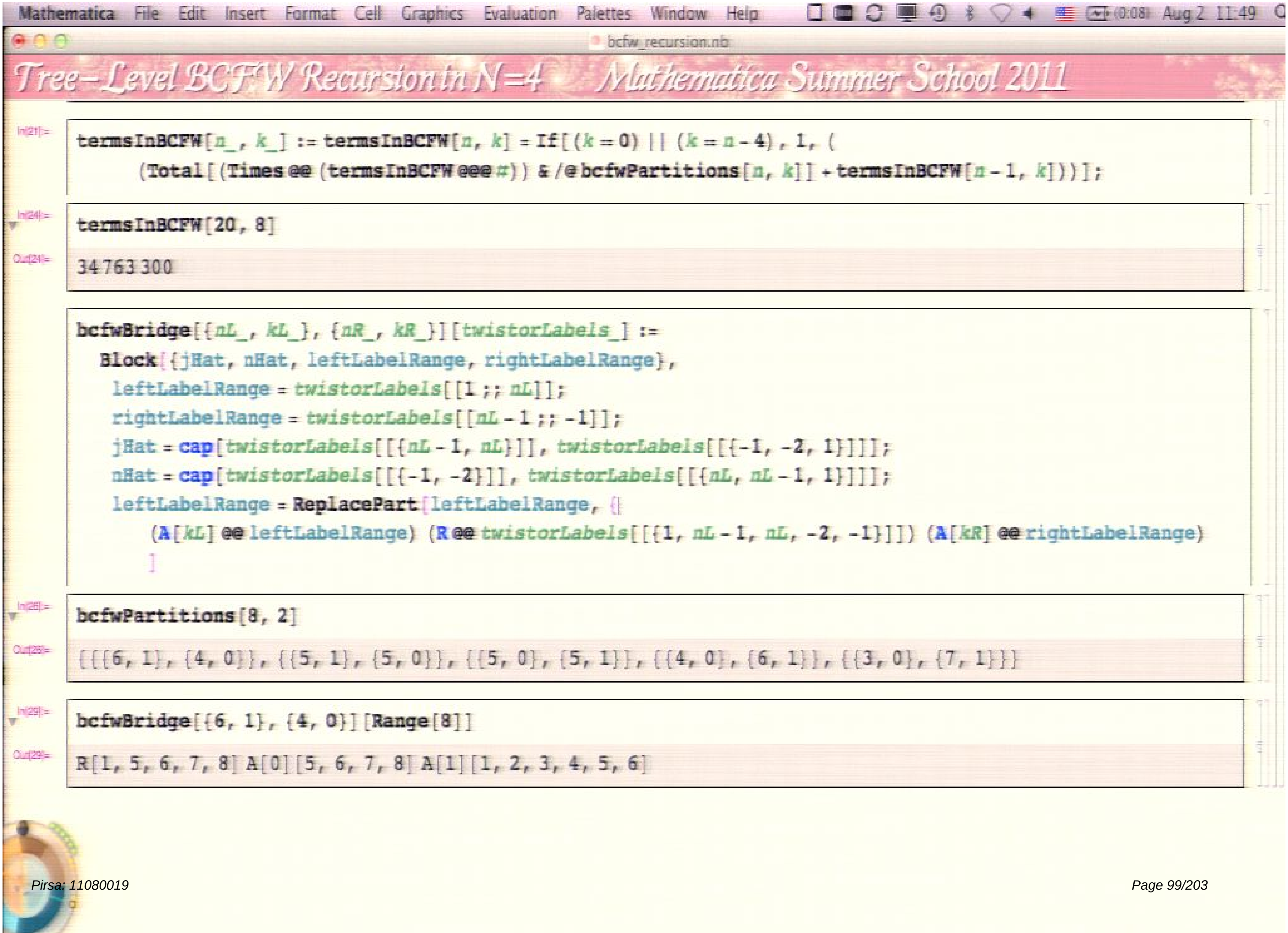
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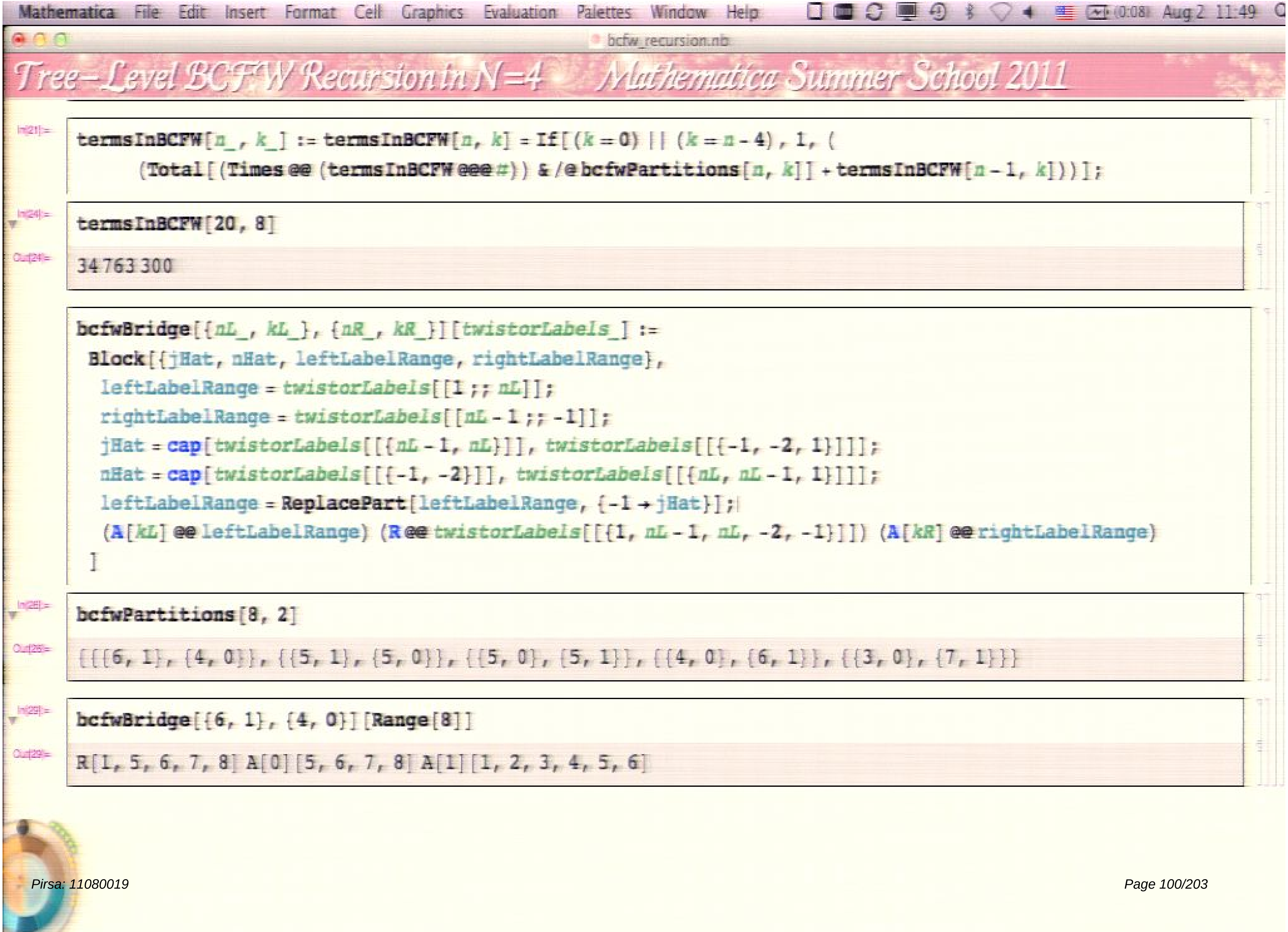
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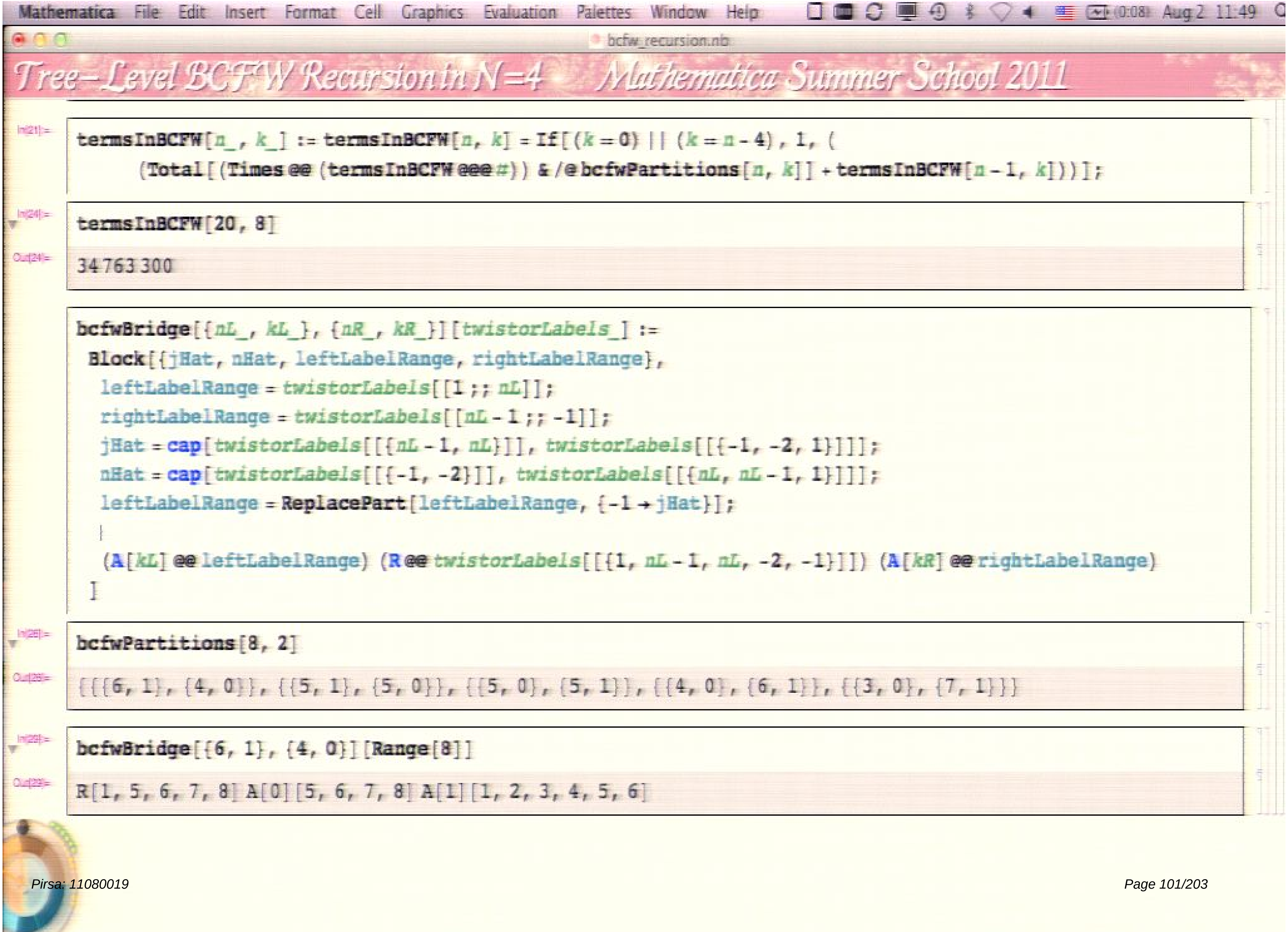
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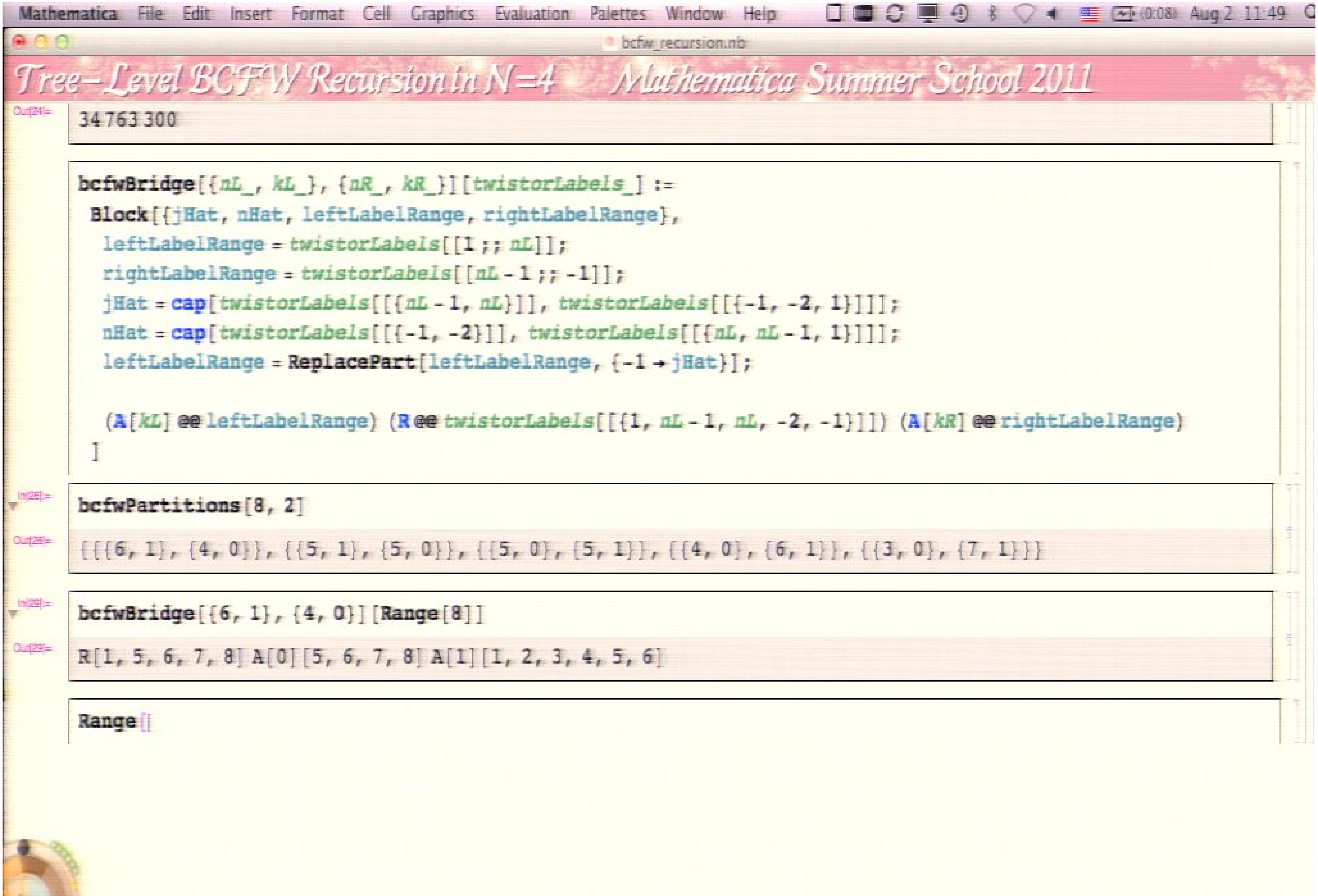
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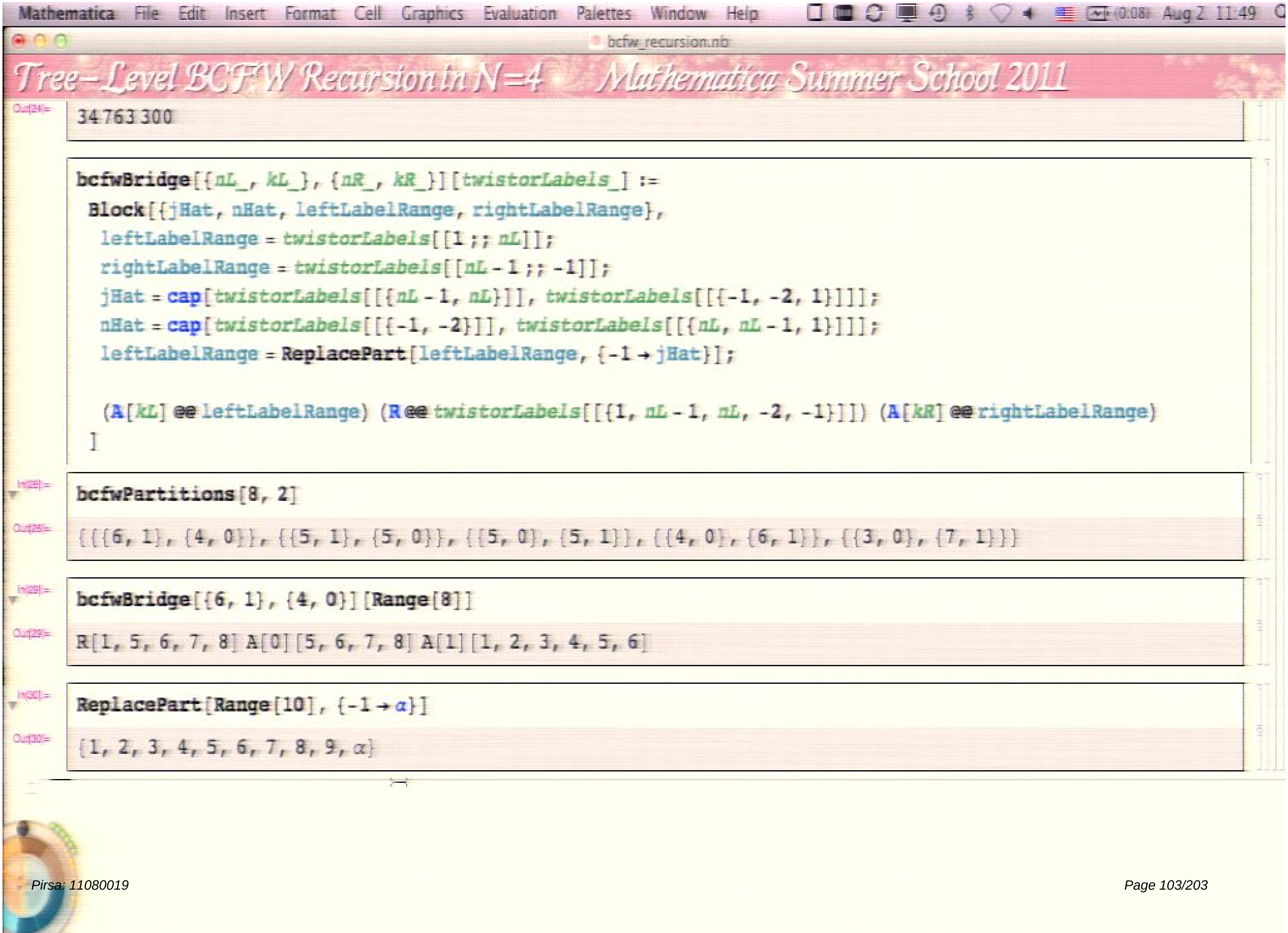
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  nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];

  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)
 ]
```

```
In[28]:= bcfwPartitions[8, 2]

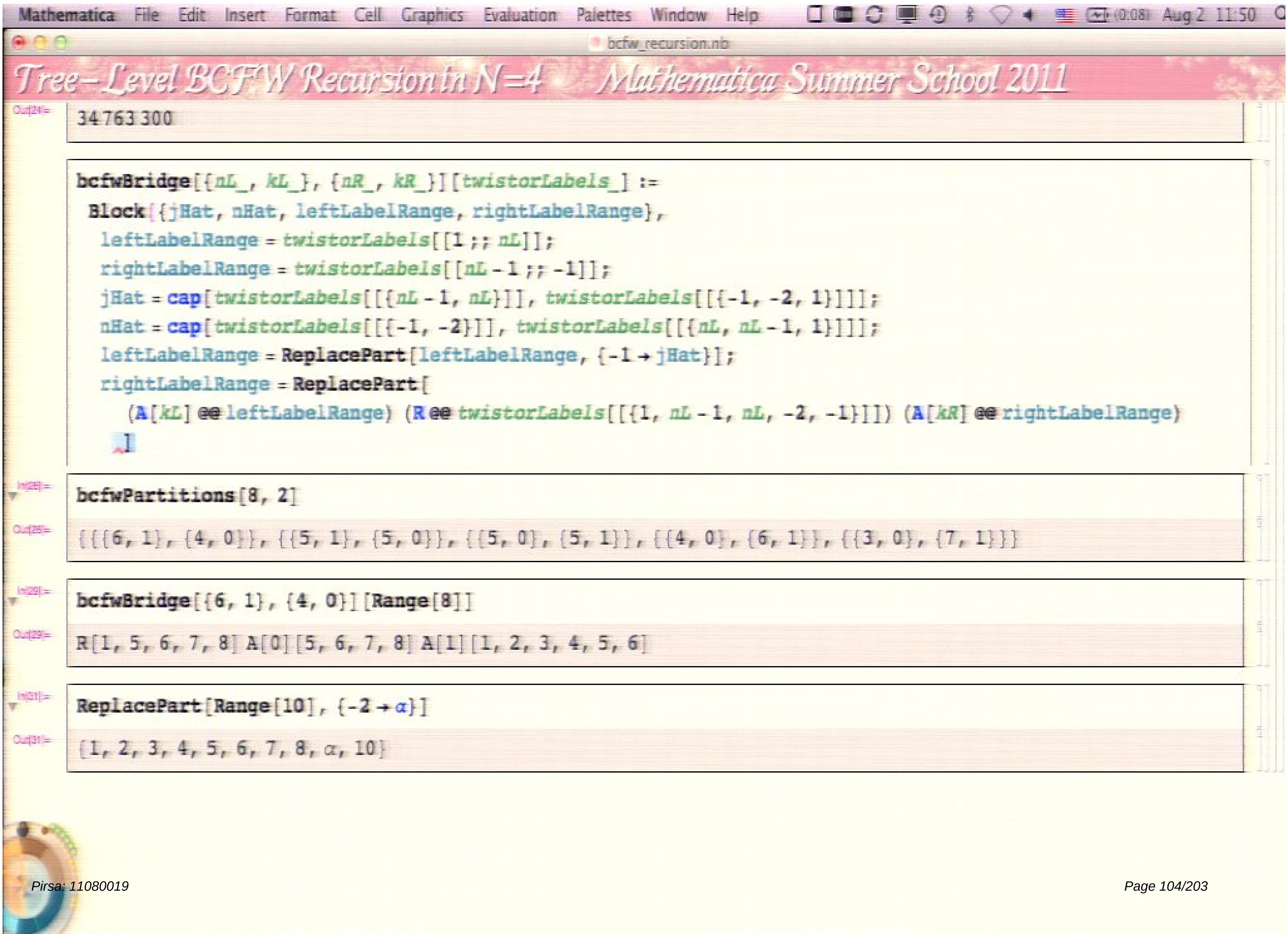
Out[28]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[29]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[29]= R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]
```

```
In[30]:= ReplacePart[Range[10], {-1 → α}]

Out[30]= {1, 2, 3, 4, 5, 6, 7, 8, 9, α}
```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(0:08) Aug 2 11:50
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
Out[24]= 34 763 300

bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange},
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
  nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[
    (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)
   ]

In[26]:= bcfwPartitions[8, 2]

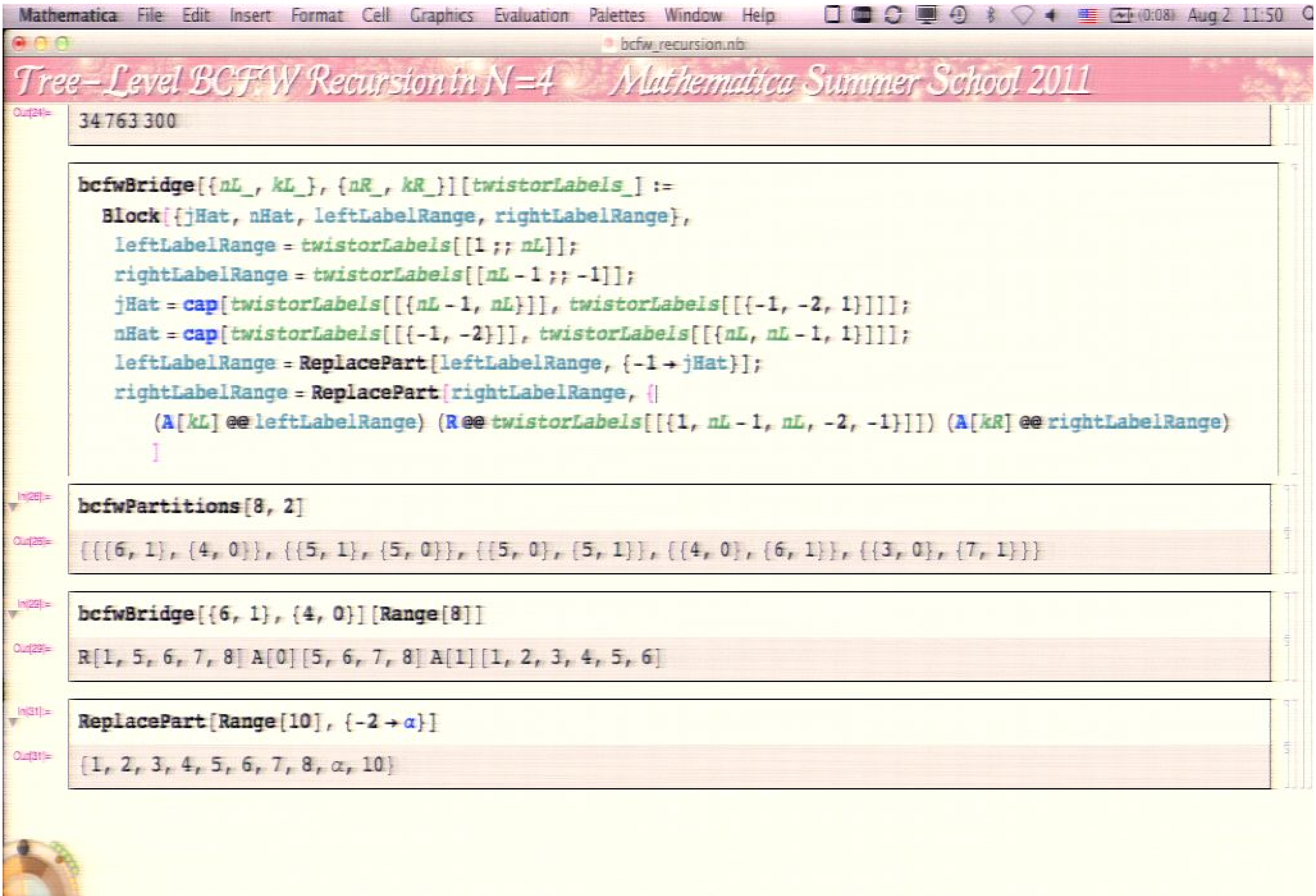
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[29]= R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]

In[31]:= ReplacePart[Range[10], {-2 → α}]

Out[31]= {1, 2, 3, 4, 5, 6, 7, 8, α, 10}
```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(0:08) Aug 2 11:50
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

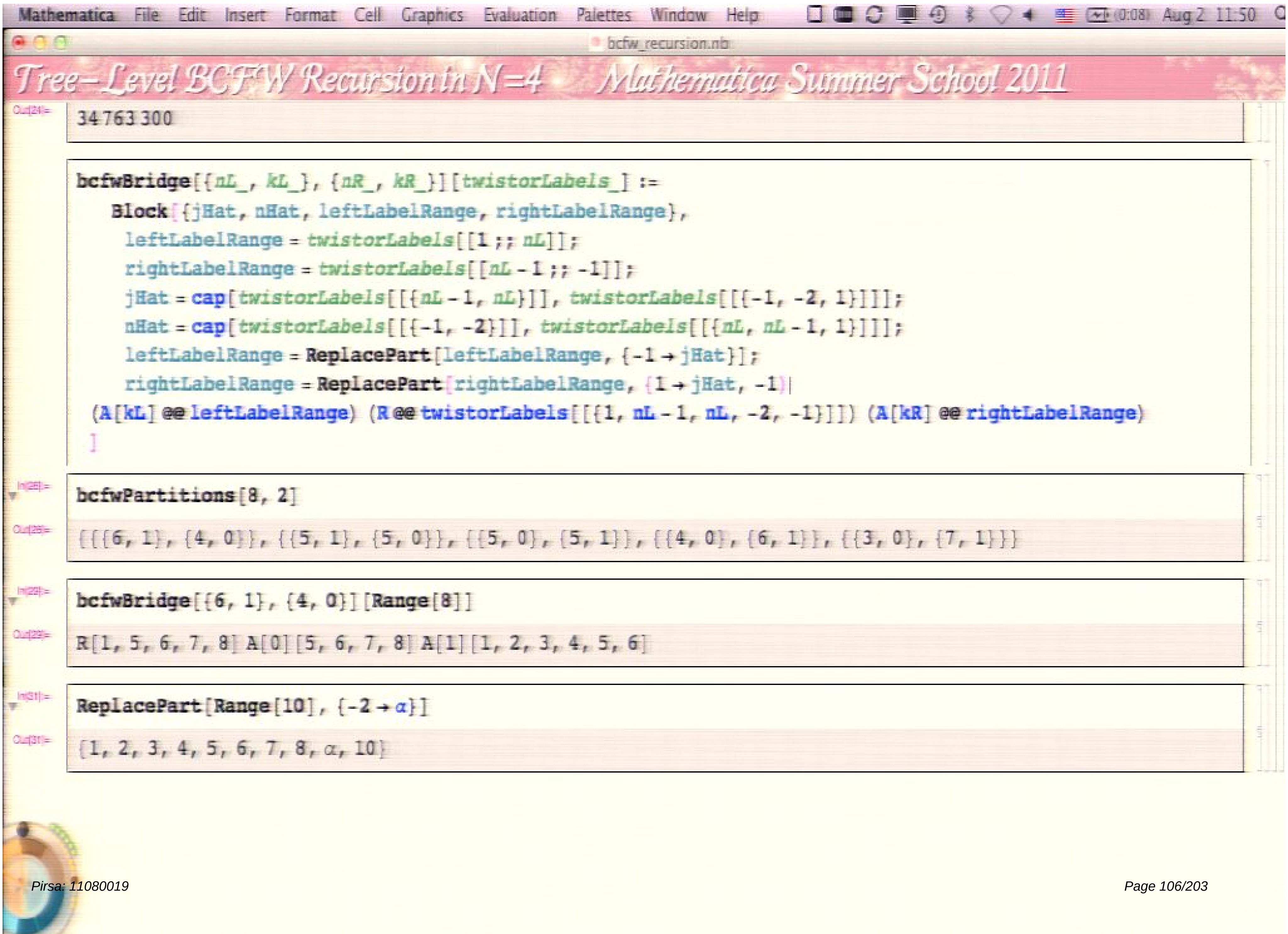
```
Out[24]=  34 763 300

bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange},
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
  nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[rightLabelRange, {|
     (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)
     ]

In[26]:=  bcfwPartitions[8, 2]
Out[26]=  {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:=  bcfwBridge[{6, 1}, {4, 0}][Range[8]]
Out[29]=  R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]

In[31]:=  ReplacePart[Range[10], {-2 → α}]
Out[31]=  {1, 2, 3, 4, 5, 6, 7, 8, α, 10}
```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(0:08) Aug 2 11:50
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
Out[24]= 34 763 300

bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
   Block[{jHat, nHat, leftLabelRange, rightLabelRange},
    leftLabelRange = twistorLabels[[1 ;; nL]];
    rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
    jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
    nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
    leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
    rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1}|
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)
  ]

In[26]:= bcfwPartitions[8, 2]

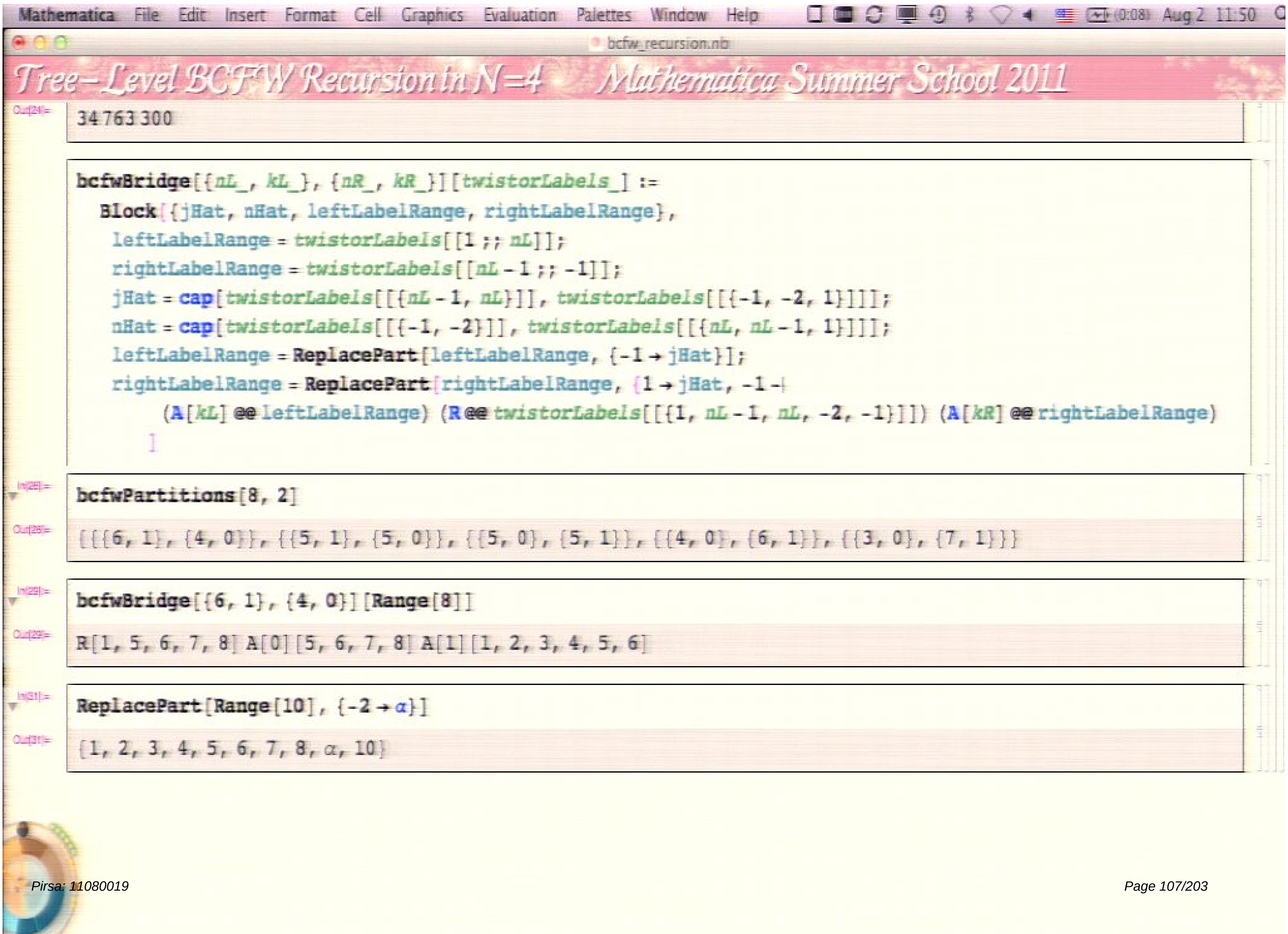
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[29]= R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]

In[31]:= ReplacePart[Range[10], {-2 → α}]

Out[31]= {1, 2, 3, 4, 5, 6, 7, 8, α, 10}
```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(0:08) Aug 2 11:50
bcfw_recursion.nb

# Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[24]= 34763300

bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange},
   leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 -
       (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)
     ]

In[28]:= bcfwPartitions[8, 2]

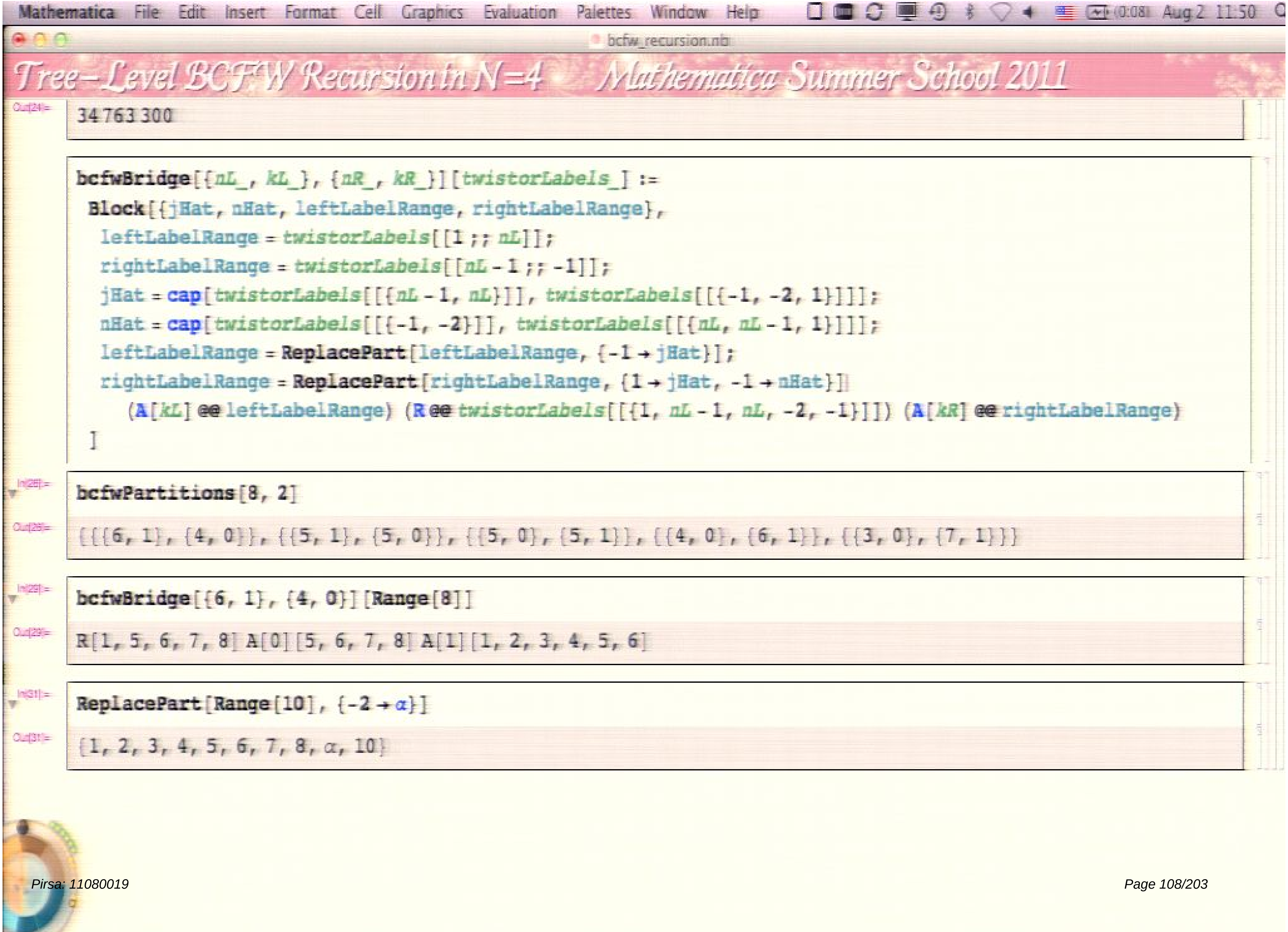
Out[28]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[29]= R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]

In[31]:= ReplacePart[Range[10], {-2 → α}]

Out[31]= {1, 2, 3, 4, 5, 6, 7, 8, α, 10}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:08) Aug 2 11:50
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[24]=  34 763 300

bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange},
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
  nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}]
    (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)
 ]

In[28]:=  bcfwPartitions[8, 2]

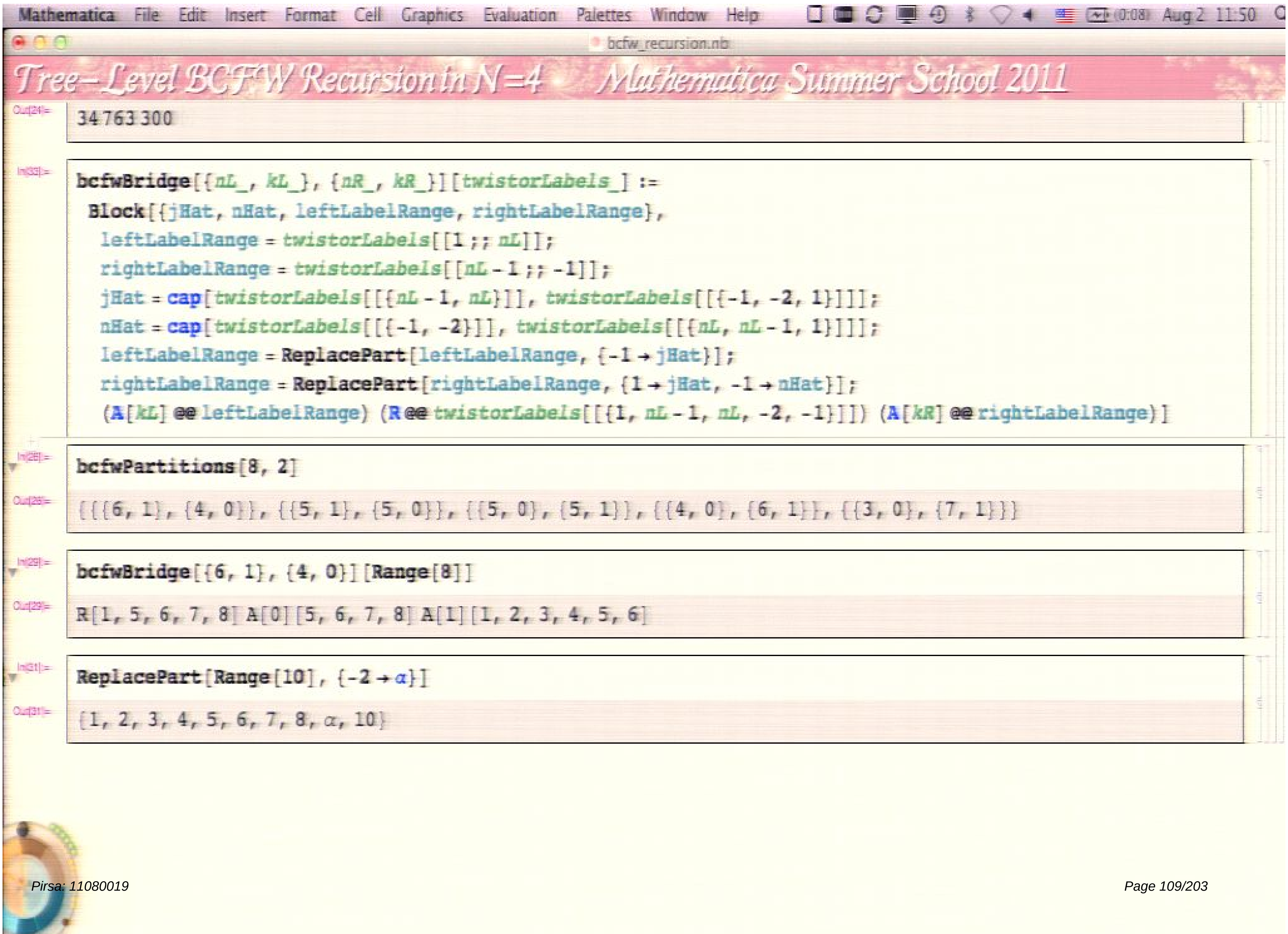
Out[28]=  {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:=  bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[29]=  R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]

In[31]:=  ReplacePart[Range[10], {-2 → α}]

Out[31]=  {1, 2, 3, 4, 5, 6, 7, 8, α, 10}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:08) Aug 2 11:50
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
Out[24]= 34763300

In[33]:= bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
          Block[{jHat, nHat, leftLabelRange, rightLabelRange},
           leftLabelRange = twistorLabels[[1 ;; nL]];
           rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
           jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
           nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
           leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
           rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
           (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]

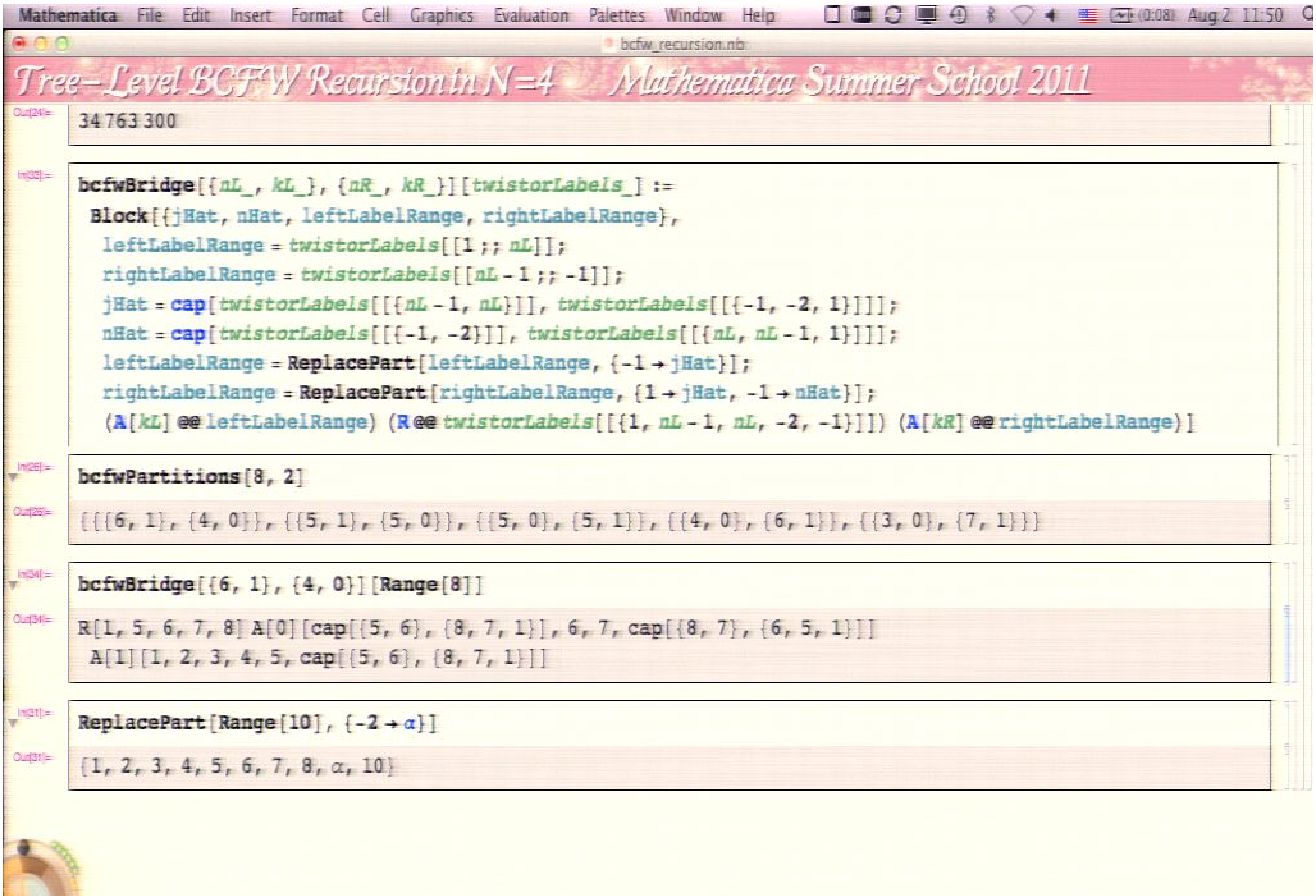
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[29]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[29]= R[1, 5, 6, 7, 8] A[0][5, 6, 7, 8] A[1][1, 2, 3, 4, 5, 6]

In[31]:= ReplacePart[Range[10], {-2 → α}]

Out[31]= {1, 2, 3, 4, 5, 6, 7, 8, α, 10}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:08) Aug 2 11:50
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4  Mathematica Summer School 2011

```
Out[24]=  34 763 300

In[33]:=  bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
           Block[{jHat, nHat, leftLabelRange, rightLabelRange},
            leftLabelRange = twistorLabels[[1 ;; nL]];
            rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
            jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
            nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
            leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
            rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
            (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:=  bcfwPartitions[8, 2]

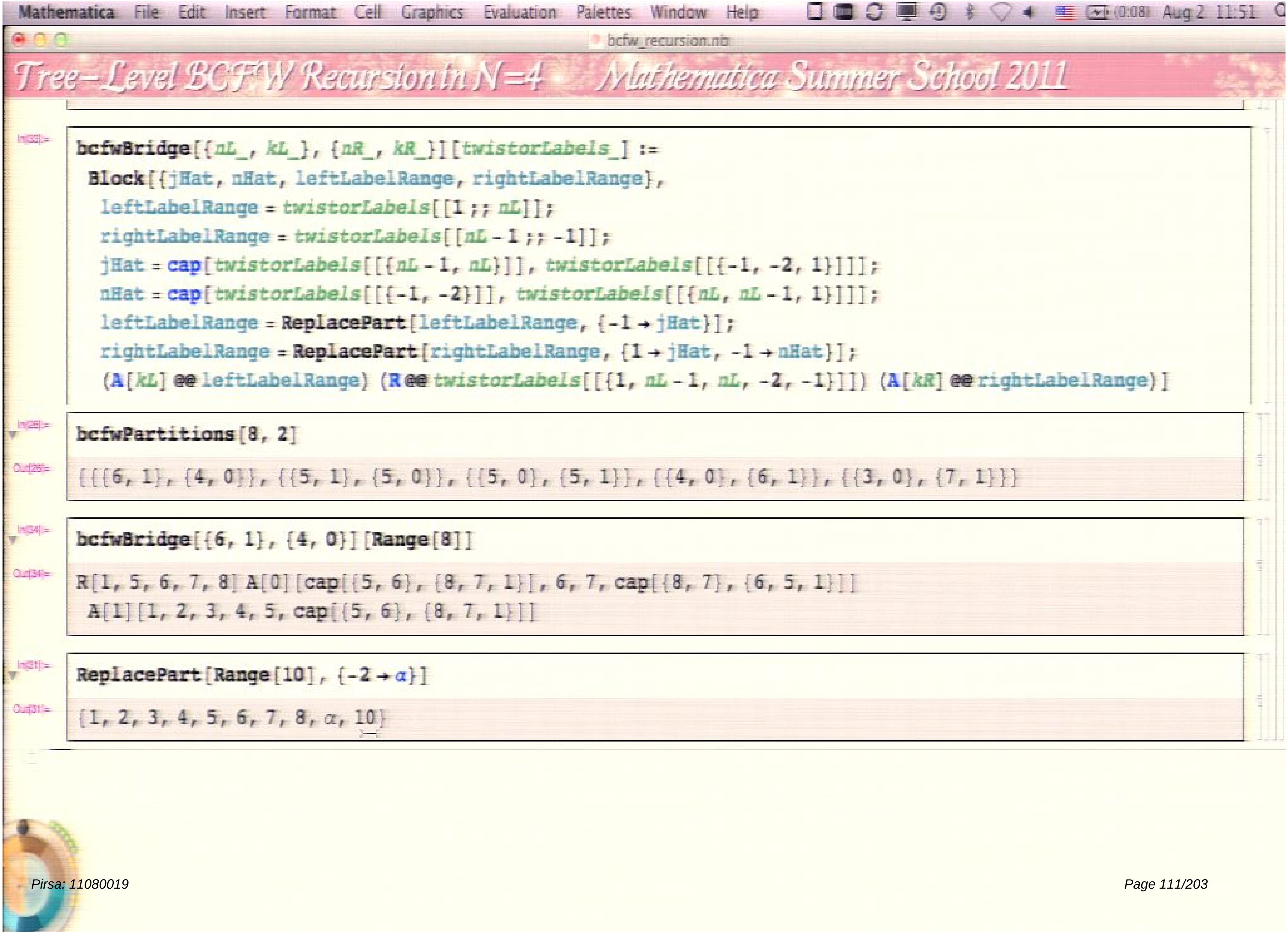
Out[26]=  {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:=  bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]=  R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
           A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]

In[31]:=  ReplacePart[Range[10], {-2 → α}]

Out[31]=  {1, 2, 3, 4, 5, 6, 7, 8, α, 10}
```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(0:08) Aug 2 11:51
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[33]:= bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
          Block[{jHat, nHat, leftLabelRange, rightLabelRange},
           leftLabelRange = twistorLabels[[1 ;; nL]];
           rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
           jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
           nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
           leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
           rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
           (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]

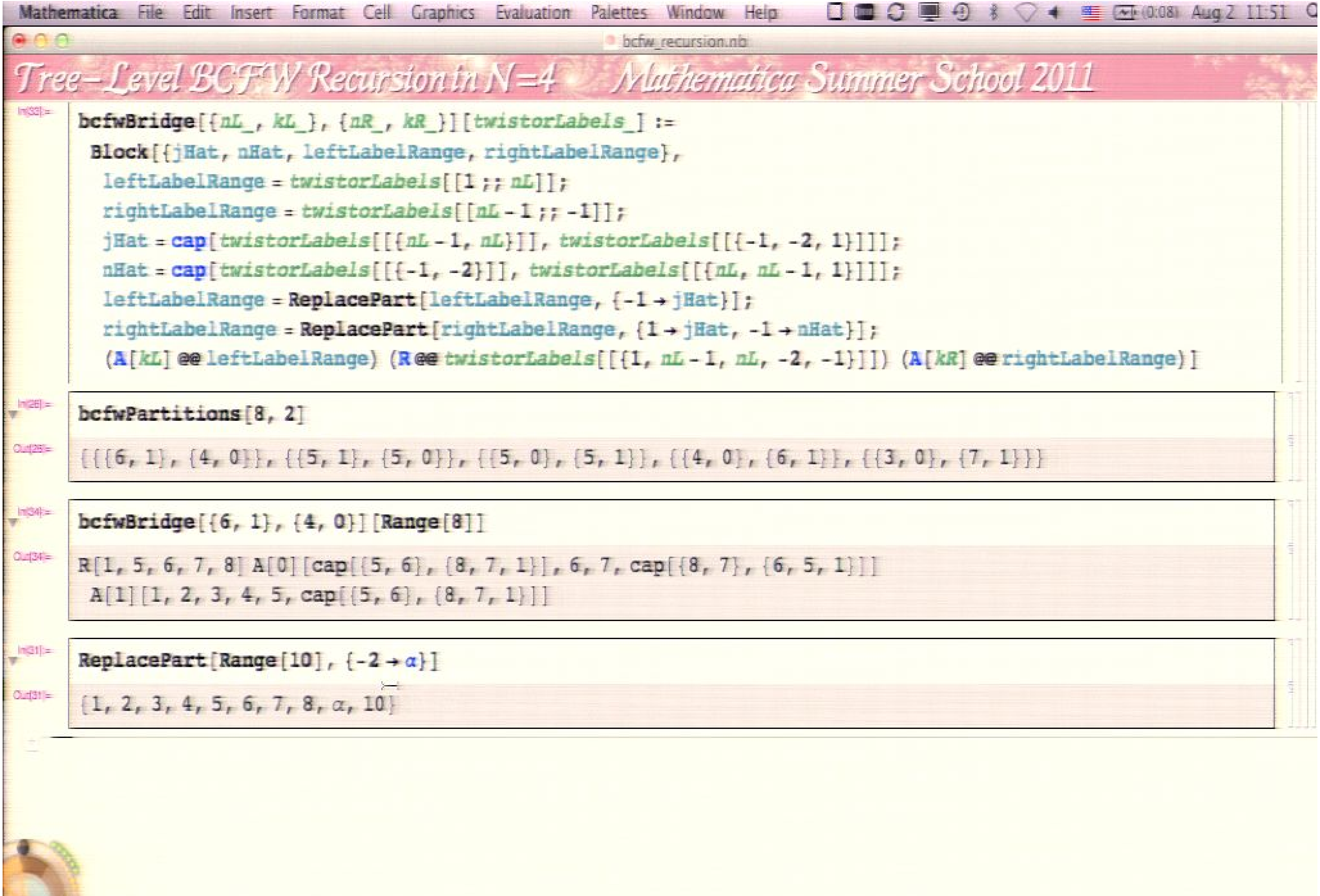
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]

In[31]:= ReplacePart[Range[10], {-2 → α}]

Out[31]= {1, 2, 3, 4, 5, 6, 7, 8, α, 10}
```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(0:08) Aug 2 11:51
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[33]:= bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
          Block[{jHat, nHat, leftLabelRange, rightLabelRange},
           leftLabelRange = twistorLabels[[1 ;; nL]];
           rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
           jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
           nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
           leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
           rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
           (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]

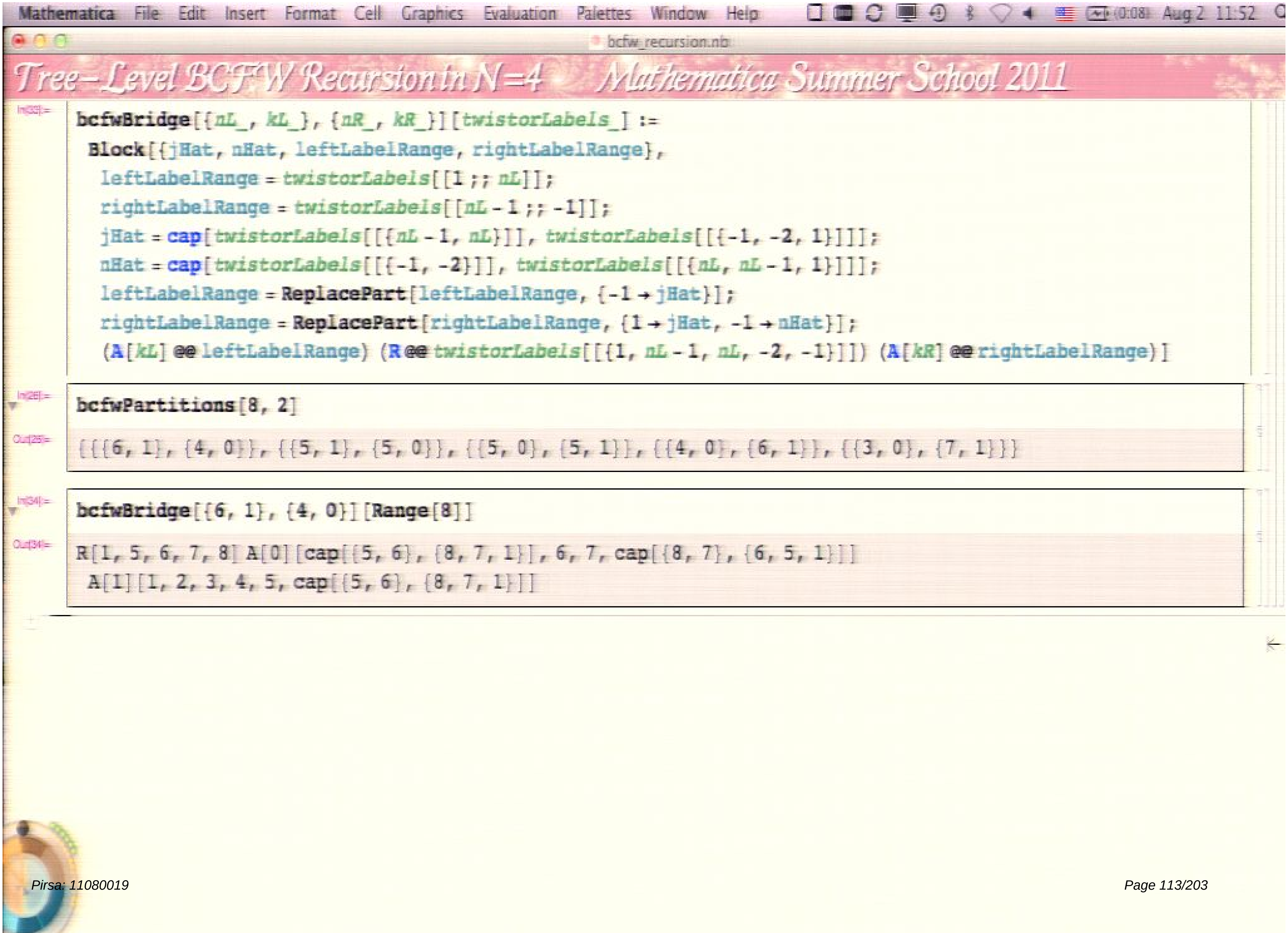
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]

In[31]:= ReplacePart[Range[10], {-2 → α}]

Out[31]= {1, 2, 3, 4, 5, 6, 7, 8, α, 10}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:08) Aug 2 11:52
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

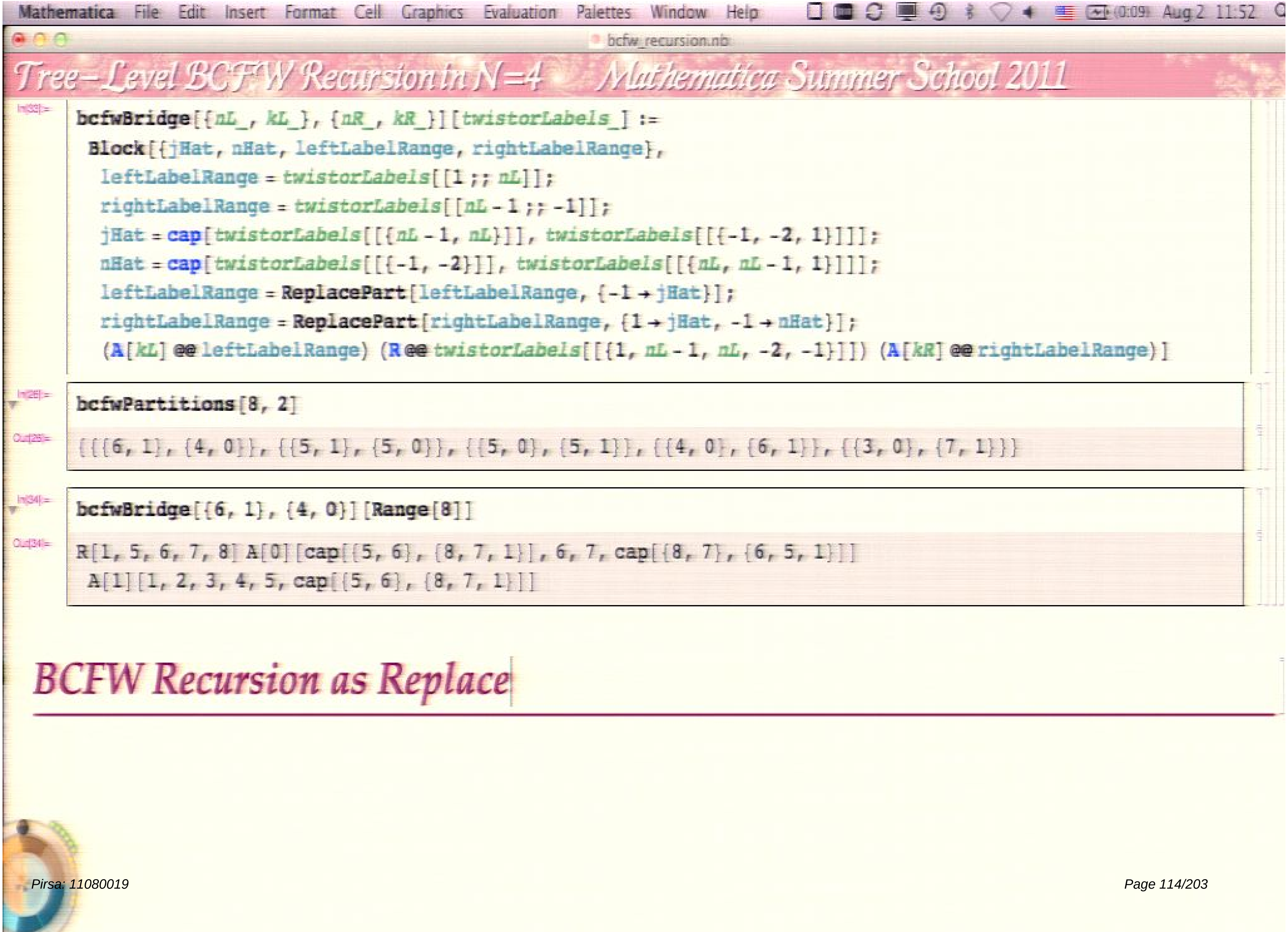
```
In[33]:=
bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange},
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
  nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:09) Aug 2 11:52
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[33]:=

```
bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange},
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
  nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

In[26]:=

```
bcfwPartitions[8, 2]
```

Out[26]=

```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

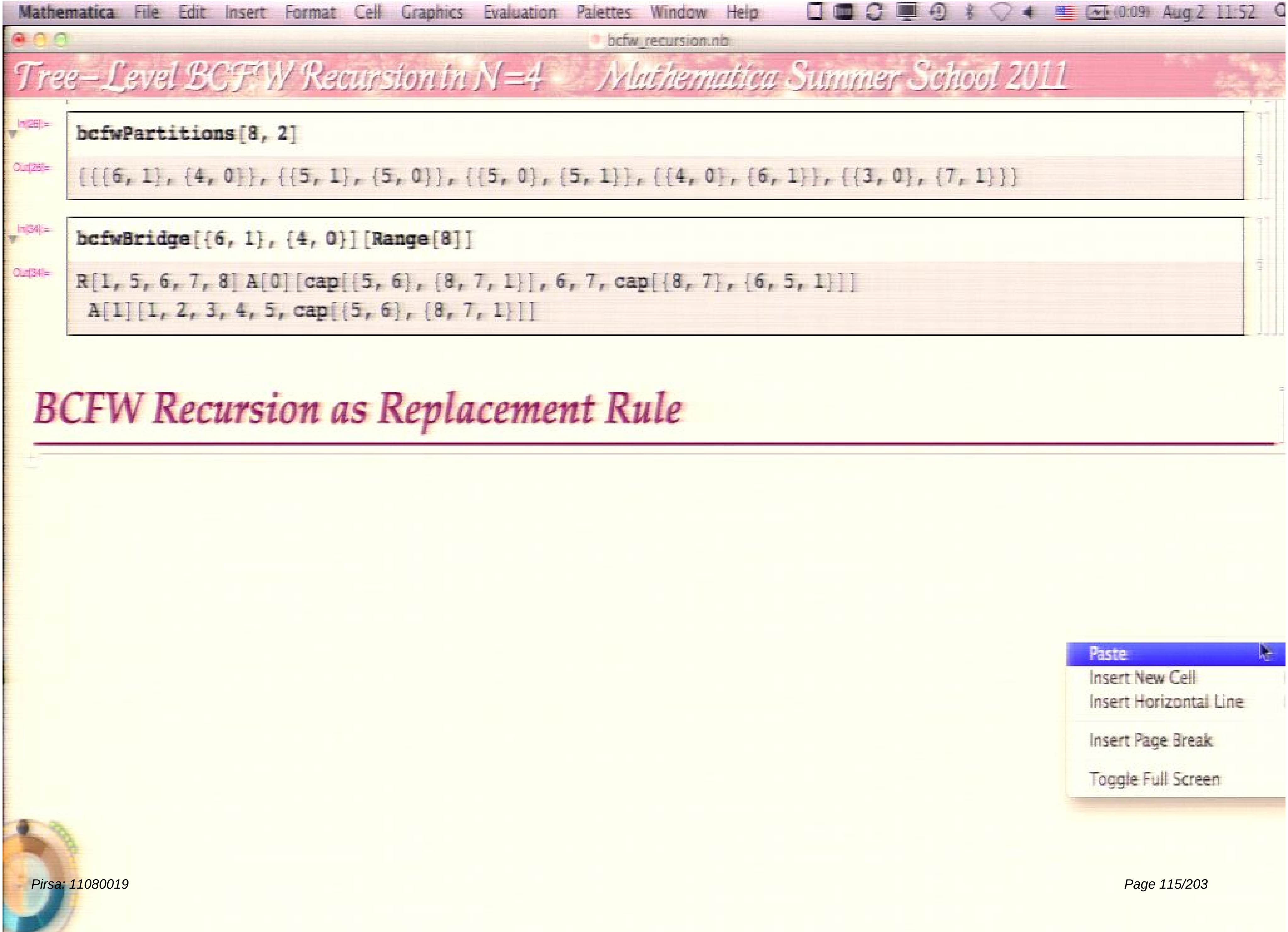
In[34]:=

```
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
```

Out[34]=

```
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

# BCFW Recursion as Replace


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:09) Aug 2 11:52
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[26]:= bcfwPartitions[8, 2]

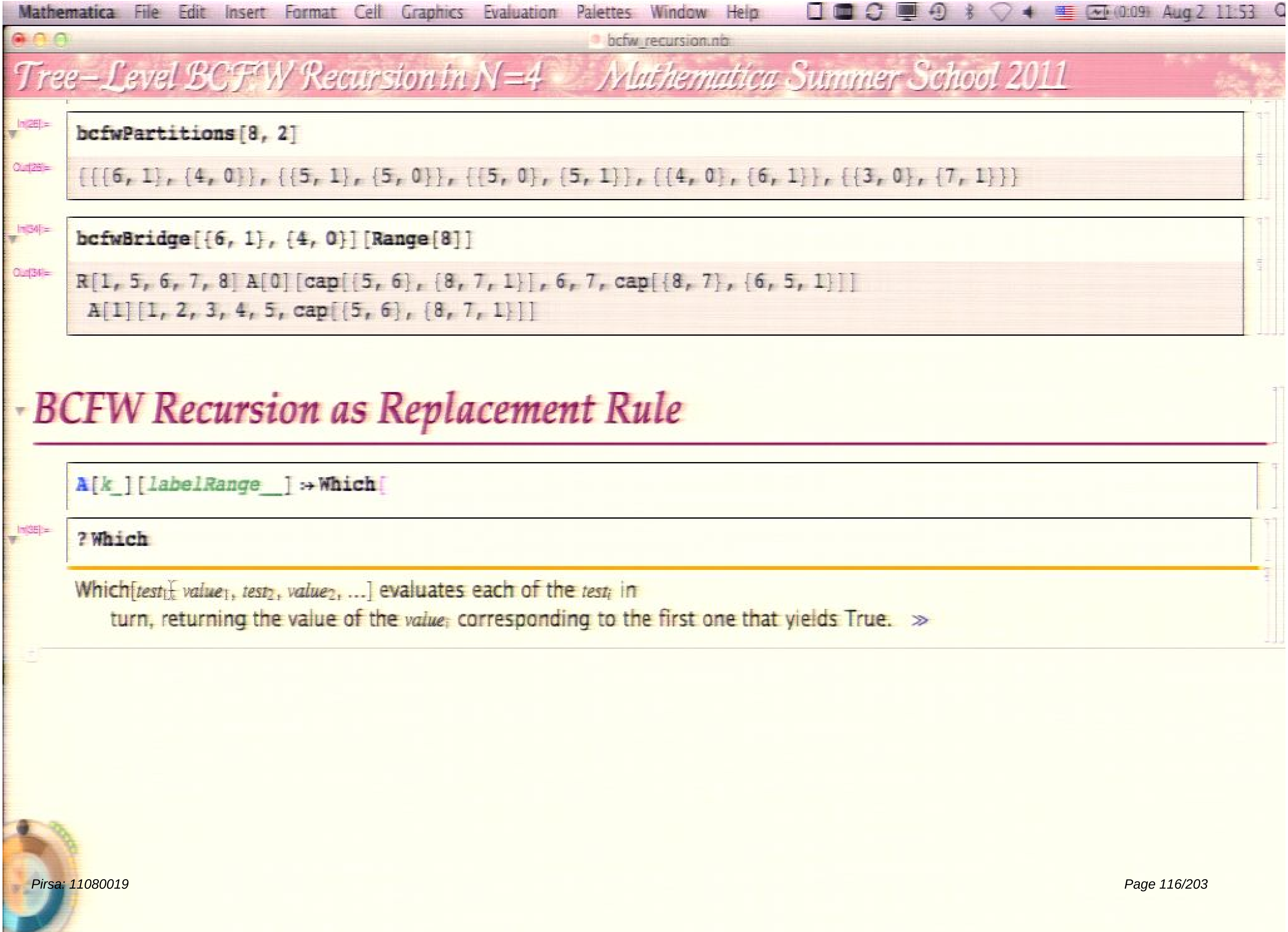
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

# BCFW Recursion as Replacement Rule

Paste
Insert New Cell
Insert Horizontal Line
Insert Page Break
Toggle Full Screen


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:09) Aug 2 11:53
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011
In[26]:=
Out[26]=
In[34]:=
Out[34]=
In[35]:=

```
bcfwPartitions[8, 2]
```

```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
```

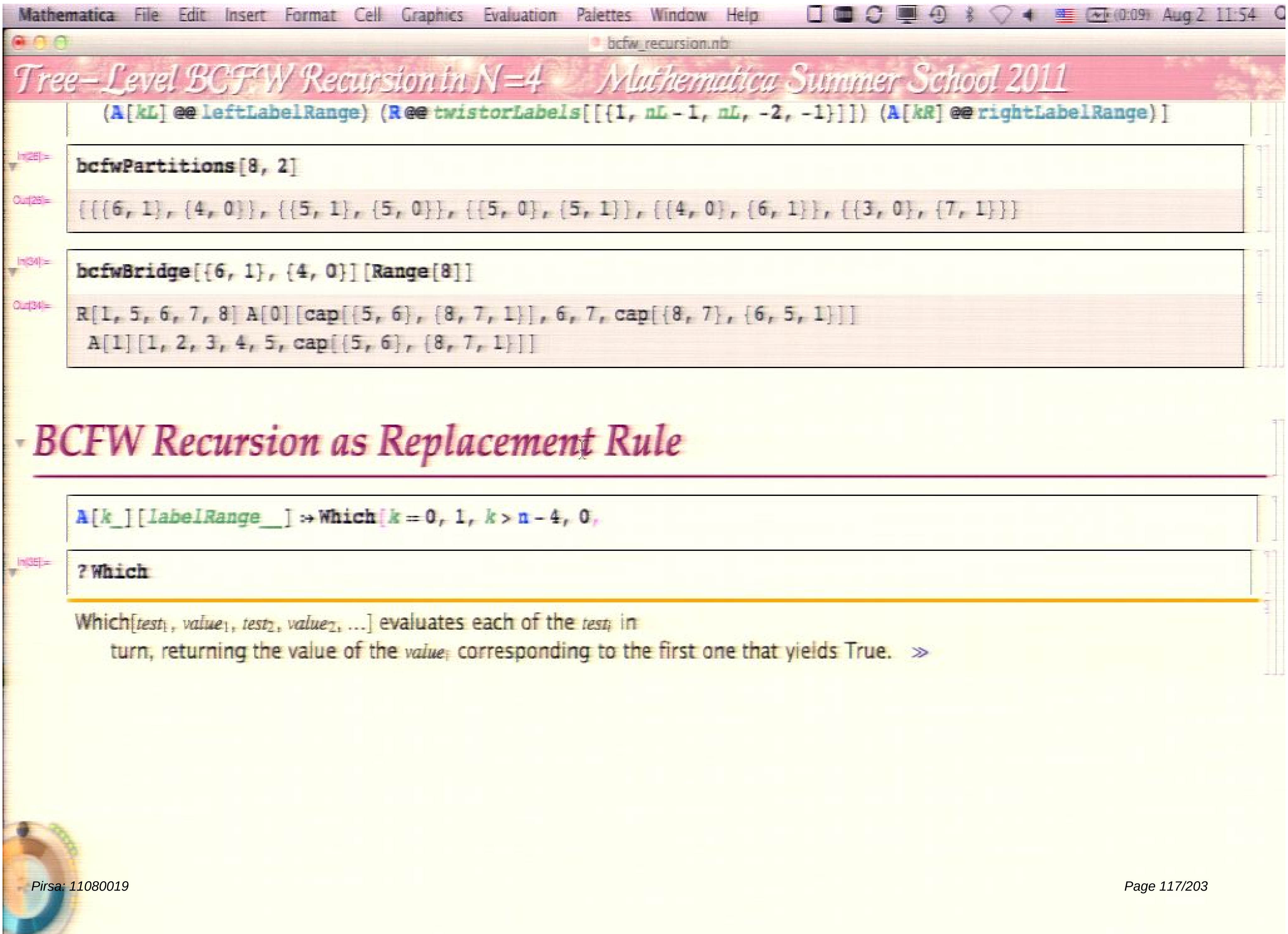
```
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :> Which[
```

```
?Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:09) Aug 2 11:54
bcfw_recursion.nb

Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

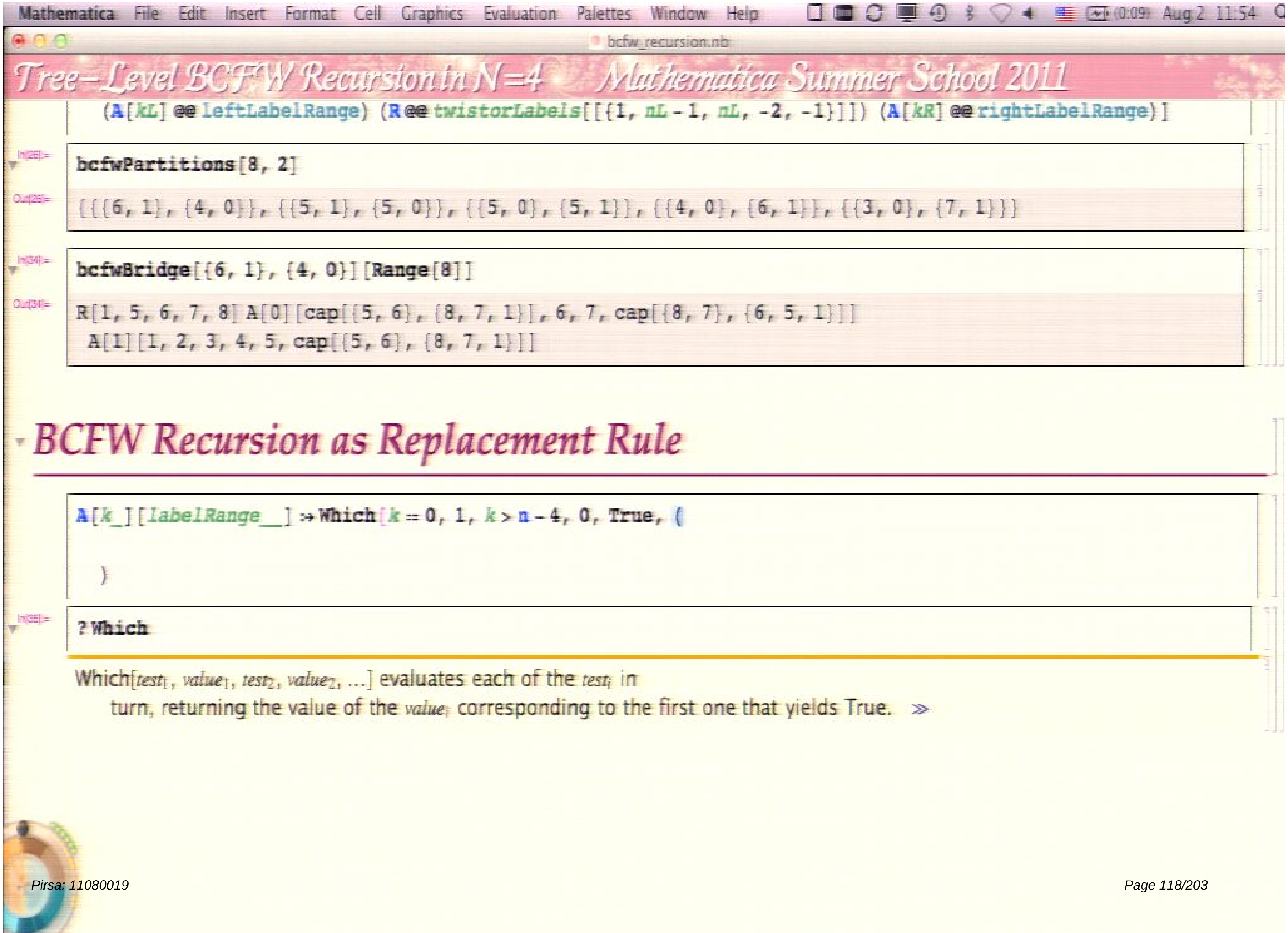
Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :→ Which[k == 0, 1, k > n - 4, 0,
```

```
In[35]:= ?Which
```

Which[*test*₁, *value*₁, *test*₂, *value*₂, ...] evaluates each of the *test*ᵢ in
    turn, returning the value of the *value*ᵢ corresponding to the first one that yields True. »


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:09) Aug 2 11:54
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

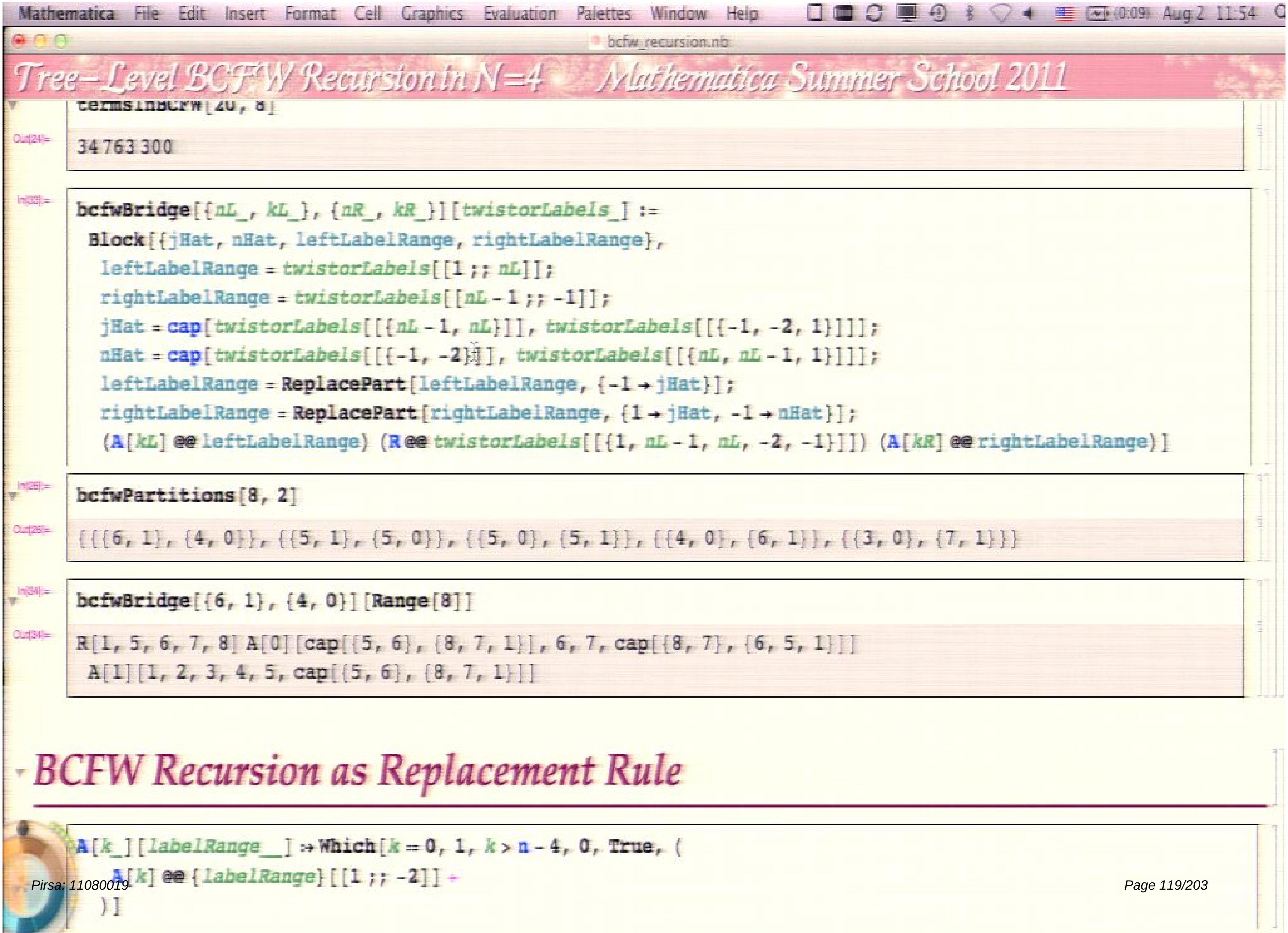
## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :→ Which[k == 0, 1, k > n - 4, 0, True, (


  )
```

```
In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in
   turn, returning the value of the $value_i$ corresponding to the first one that yields True. »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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
termsInBCFW[20, 8]
Out[24]= 34 763 300

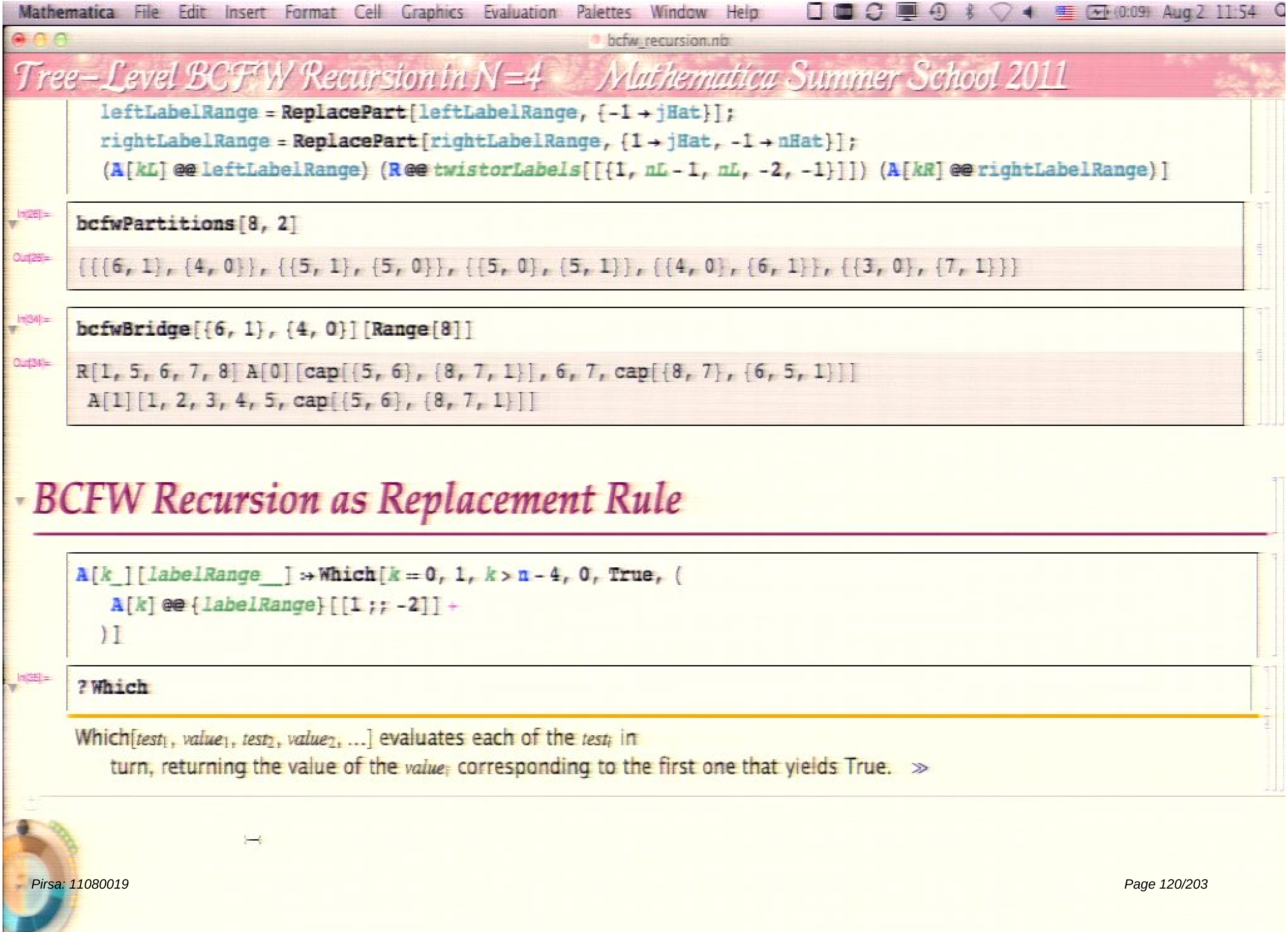
In[33]:= bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
          Block[{jHat, nHat, leftLabelRange, rightLabelRange},
           leftLabelRange = twistorLabels[[1 ;; nL]];
           rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
           jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
           nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
           leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
           rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
           (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]
Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :→ Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:09) Aug 2 11:54
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

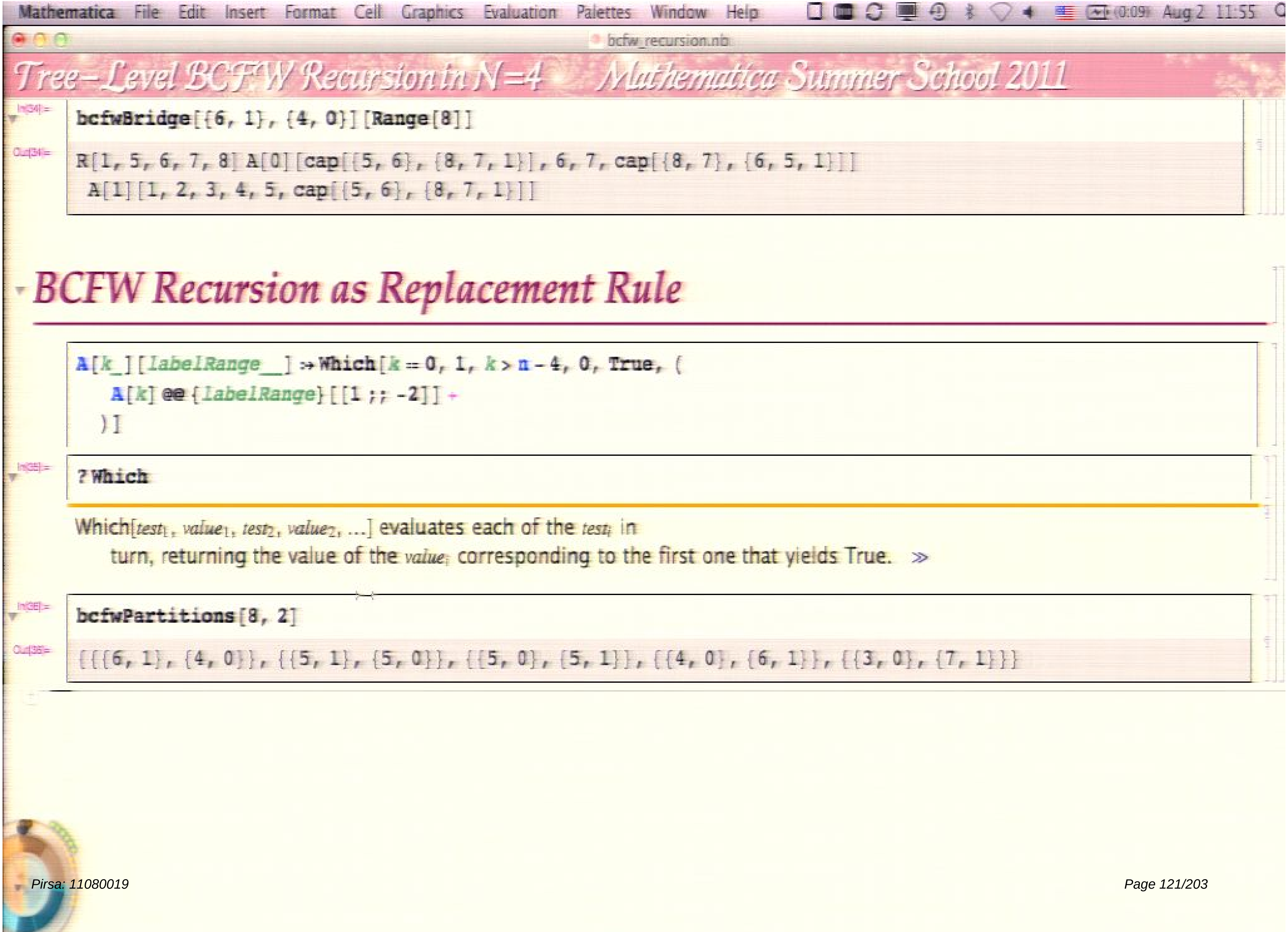
Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ?Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

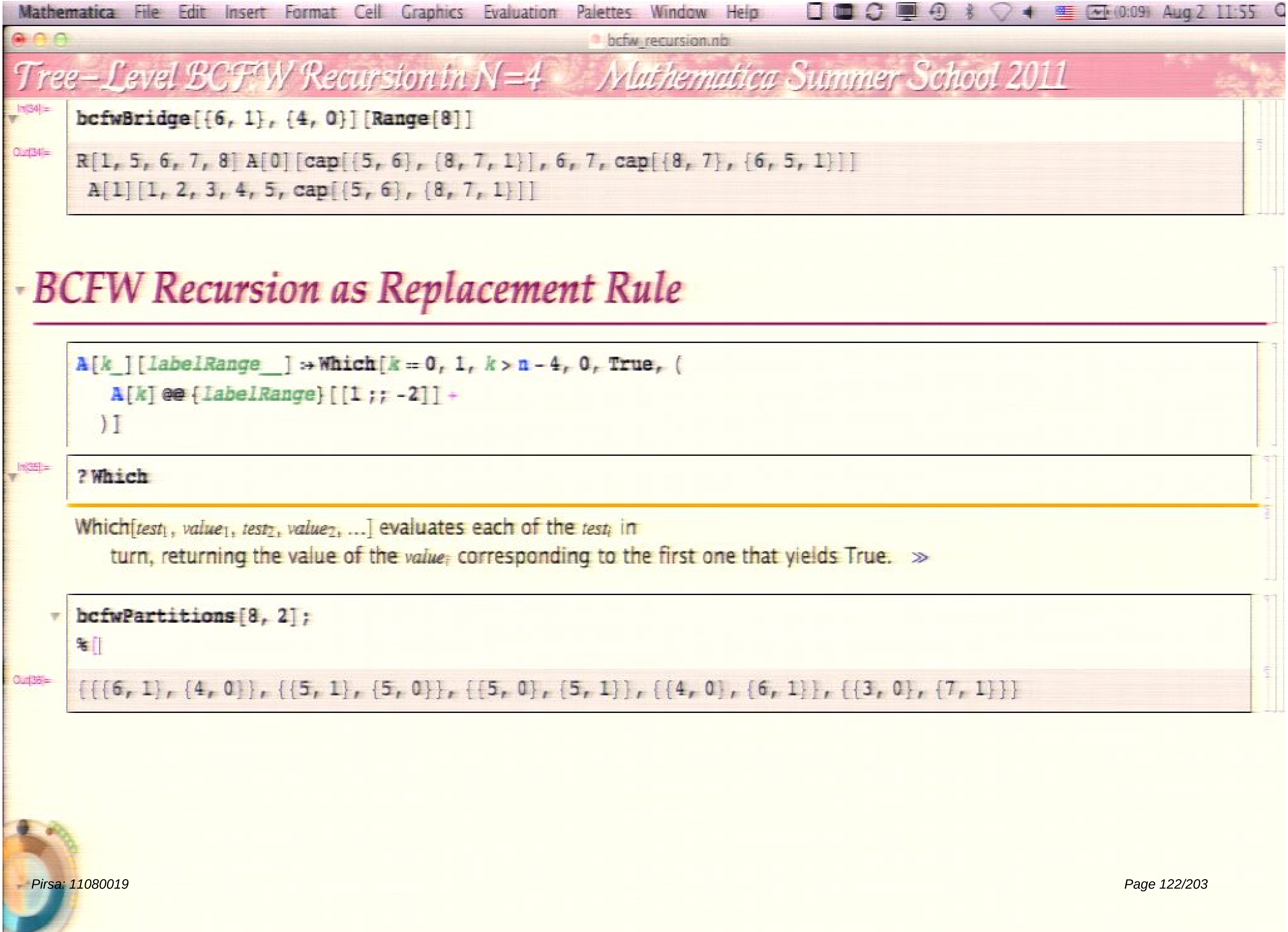
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ?Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[36]:= bcfwPartitions[8, 2]

Out[36]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```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(0:09) Aug 2 11:55
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

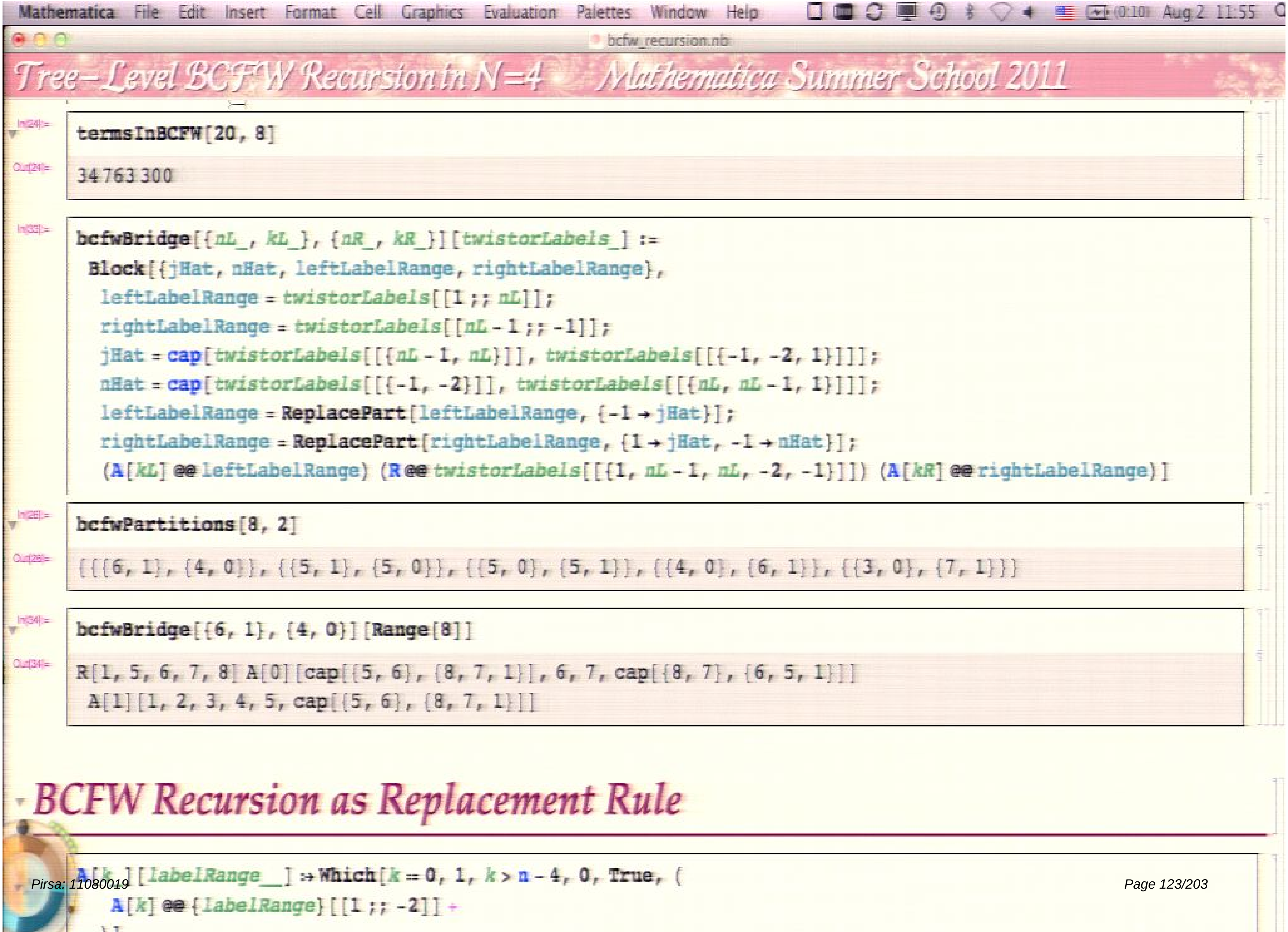
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]]
  )]
```

```
In[35]:= ?Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
bcfwPartitions[8, 2];
%[[

Out[36]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:10) Aug 2 11:55
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
In[24]:= termsInBCFW[20, 8]

Out[24]= 34 763 300

In[33]:= bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
          Block[{jHat, nHat, leftLabelRange, rightLabelRange},
           leftLabelRange = twistorLabels[[1 ;; nL]];
           rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
           jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
           nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
           leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
           rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
           (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]

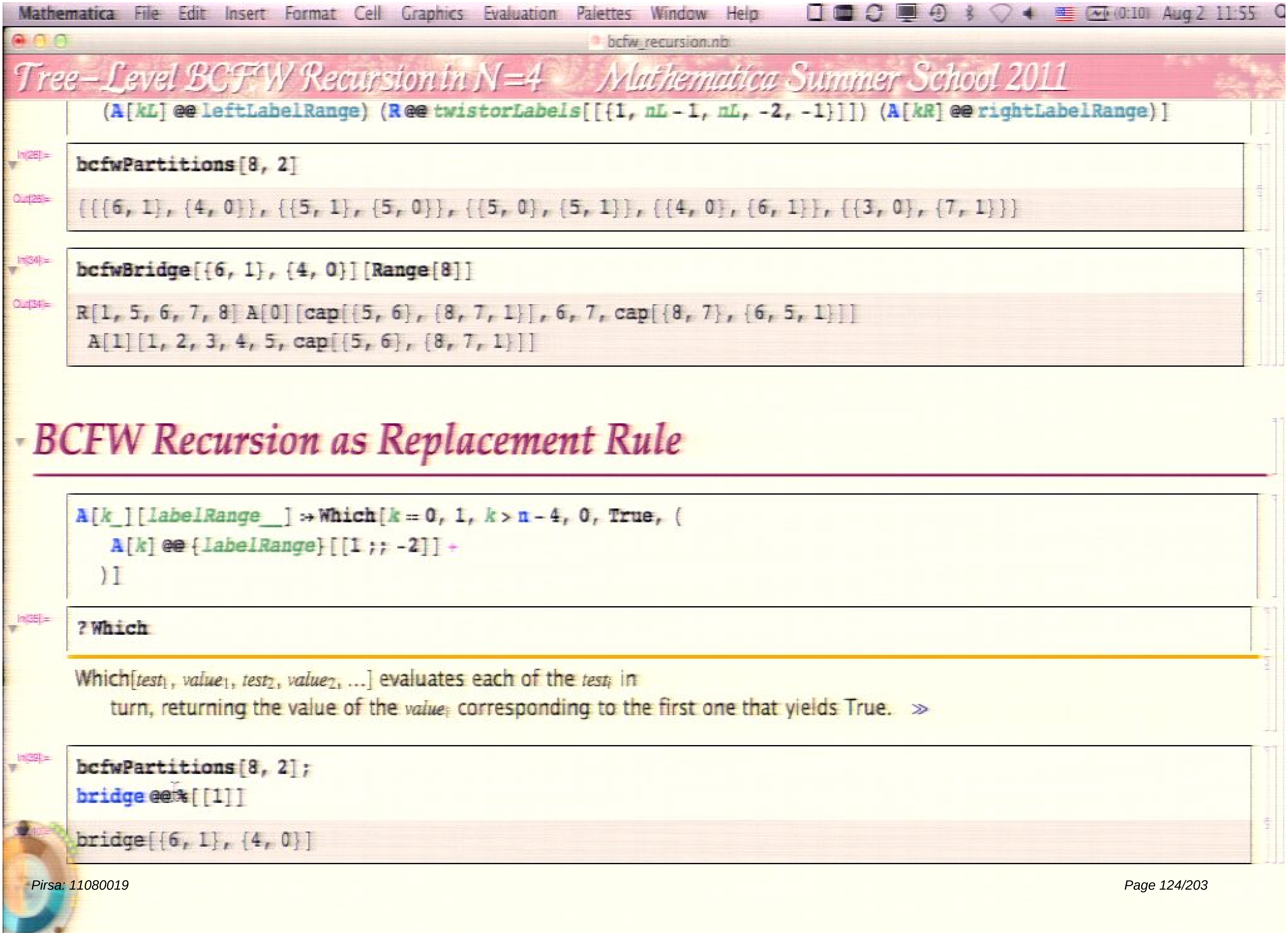
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
```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Aug 2 11:55
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

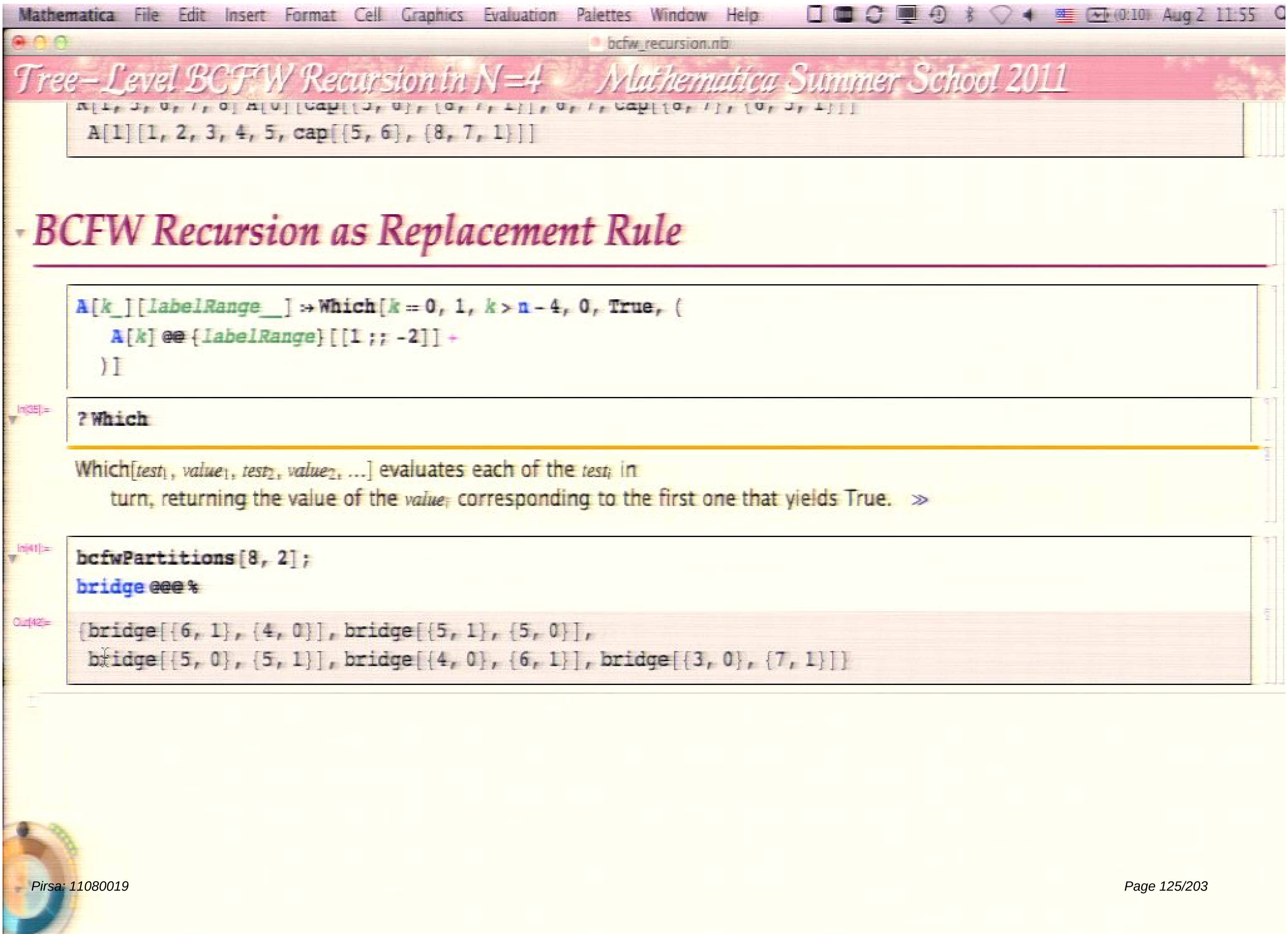
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. ≫

```
In[39]:= bcfwPartitions[8, 2];
         bridge @@ %[[1]]

         bridge[{6, 1}, {4, 0}]
```



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

# BCFW Recursion as Replacement Rule

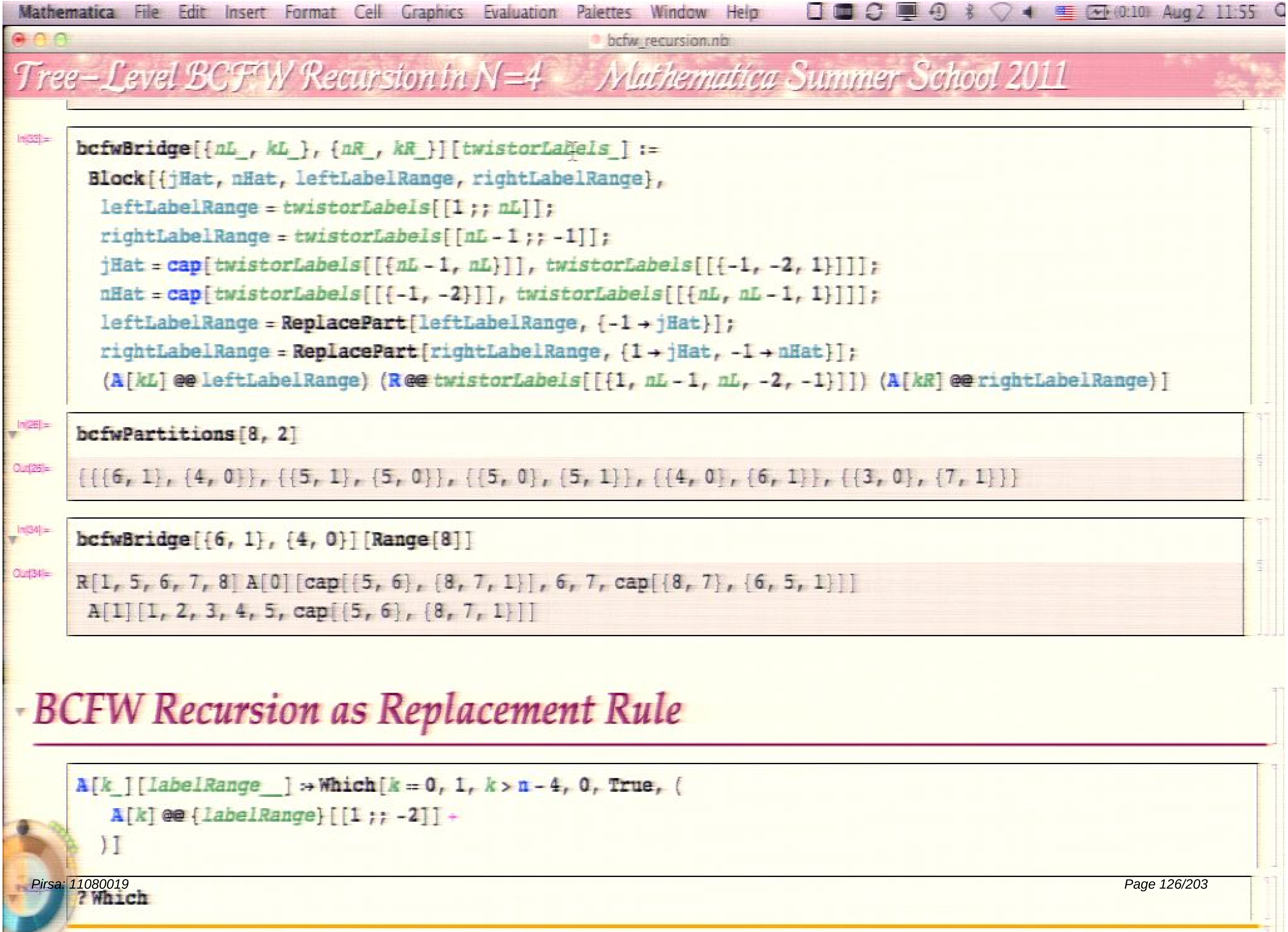
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ?Which
```

Which[*test*$_1$, *value*$_1$, *test*$_2$, *value*$_2$, …] evaluates each of the *test*$_i$ in turn, returning the value of the *value*$_i$ corresponding to the first one that yields True. »

```
In[41]:= bcfwPartitions[8, 2];
         bridge @@@ %

Out[42]= {bridge[{6, 1}, {4, 0}], bridge[{5, 1}, {5, 0}],
          bridge[{5, 0}, {5, 1}], bridge[{4, 0}, {6, 1}], bridge[{3, 0}, {7, 1}]}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:10) Aug 2 11:55
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[33]:=
bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange},
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
  nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

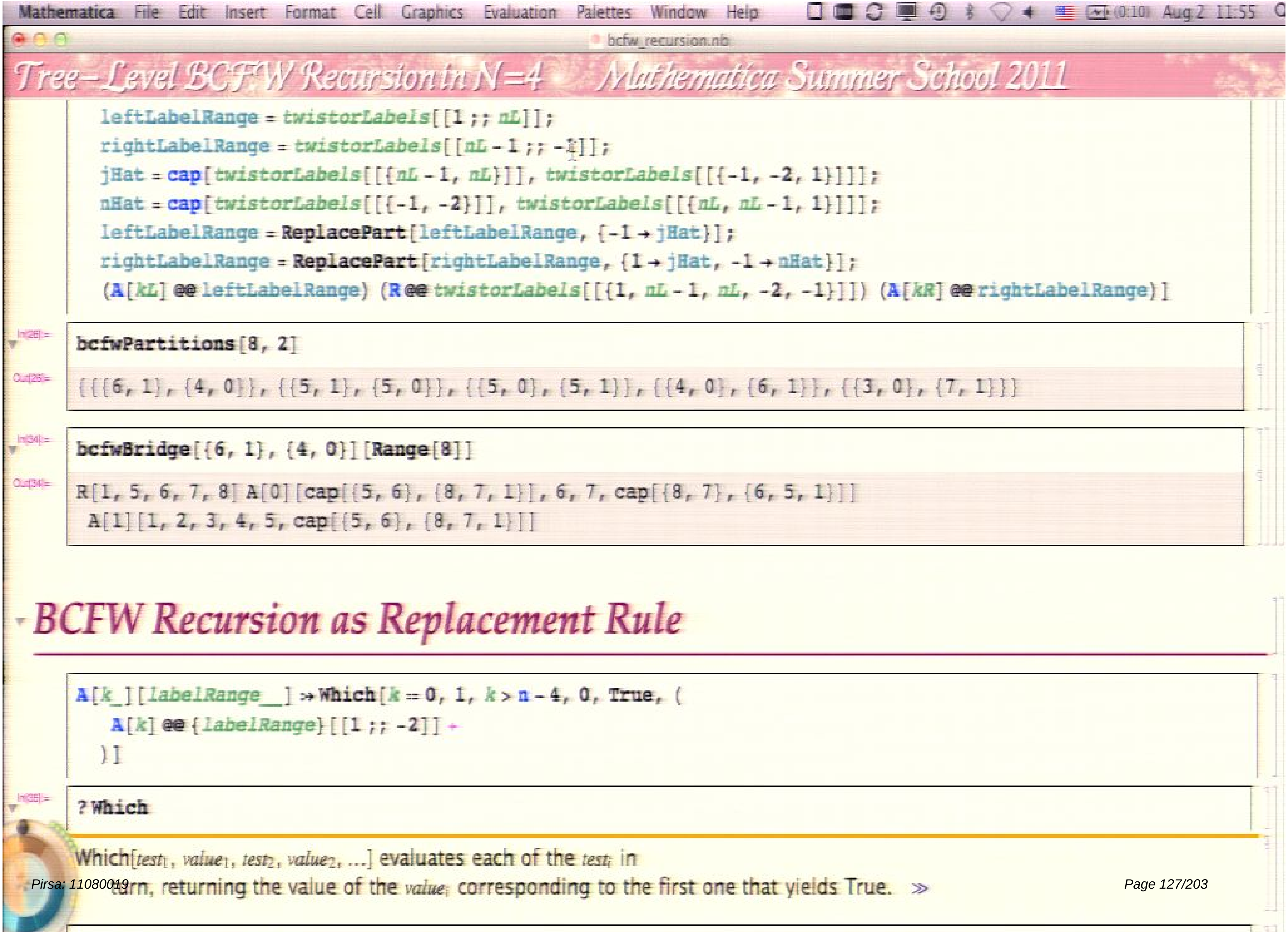
In[26]:=
bcfwPartitions[8, 2]
Out[26]=
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:=
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
Out[34]=
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
? Which
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:10) Aug 2 11:55
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
  leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
  nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

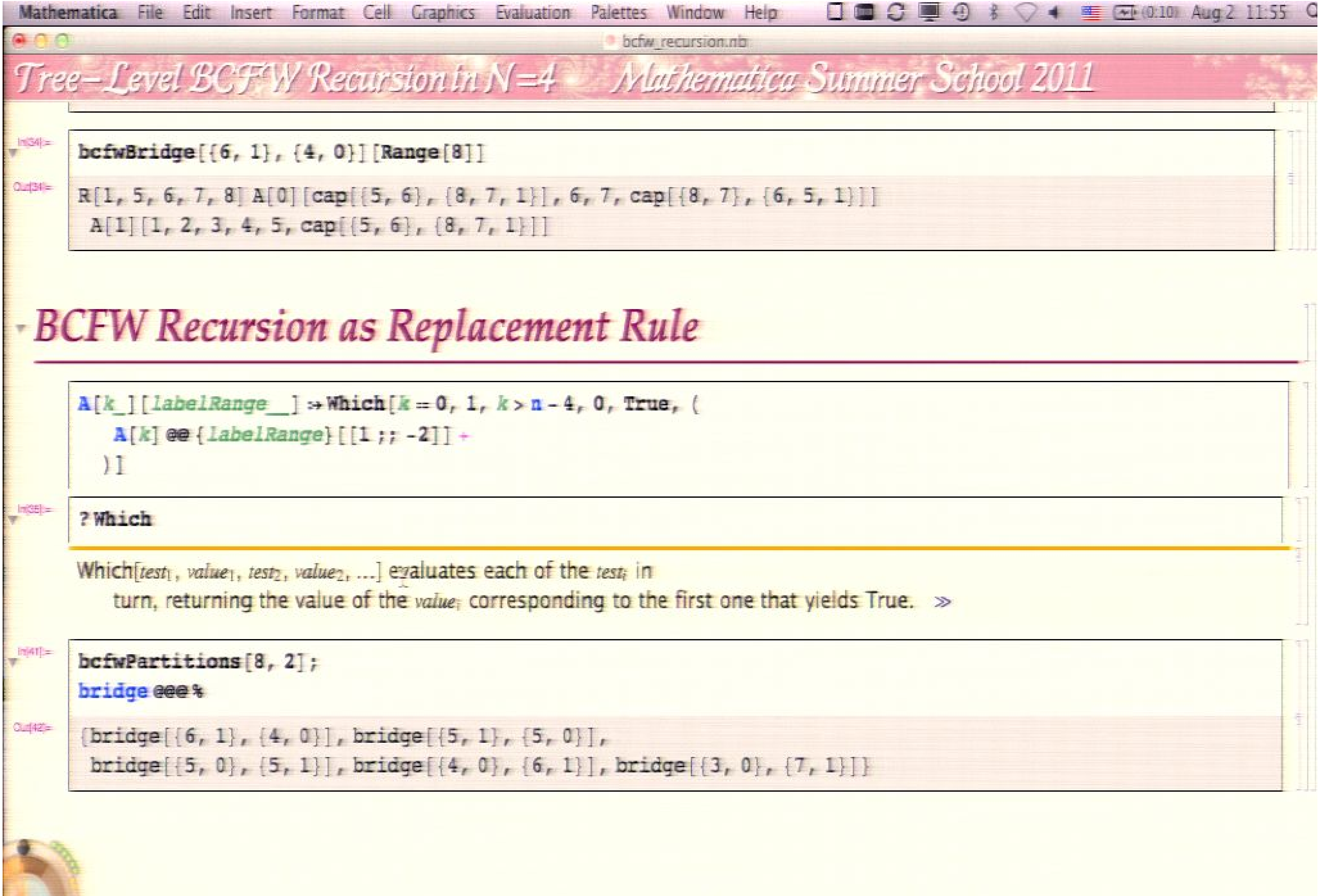
Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ? Which
```

Which[*test*<sub></sub>1, *value*1, *test*2, *value*2, …] evaluates each of the *test*i in turn, returning the value of the *value*i corresponding to the first one that yields True. »


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:10) Aug 2 11:55
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

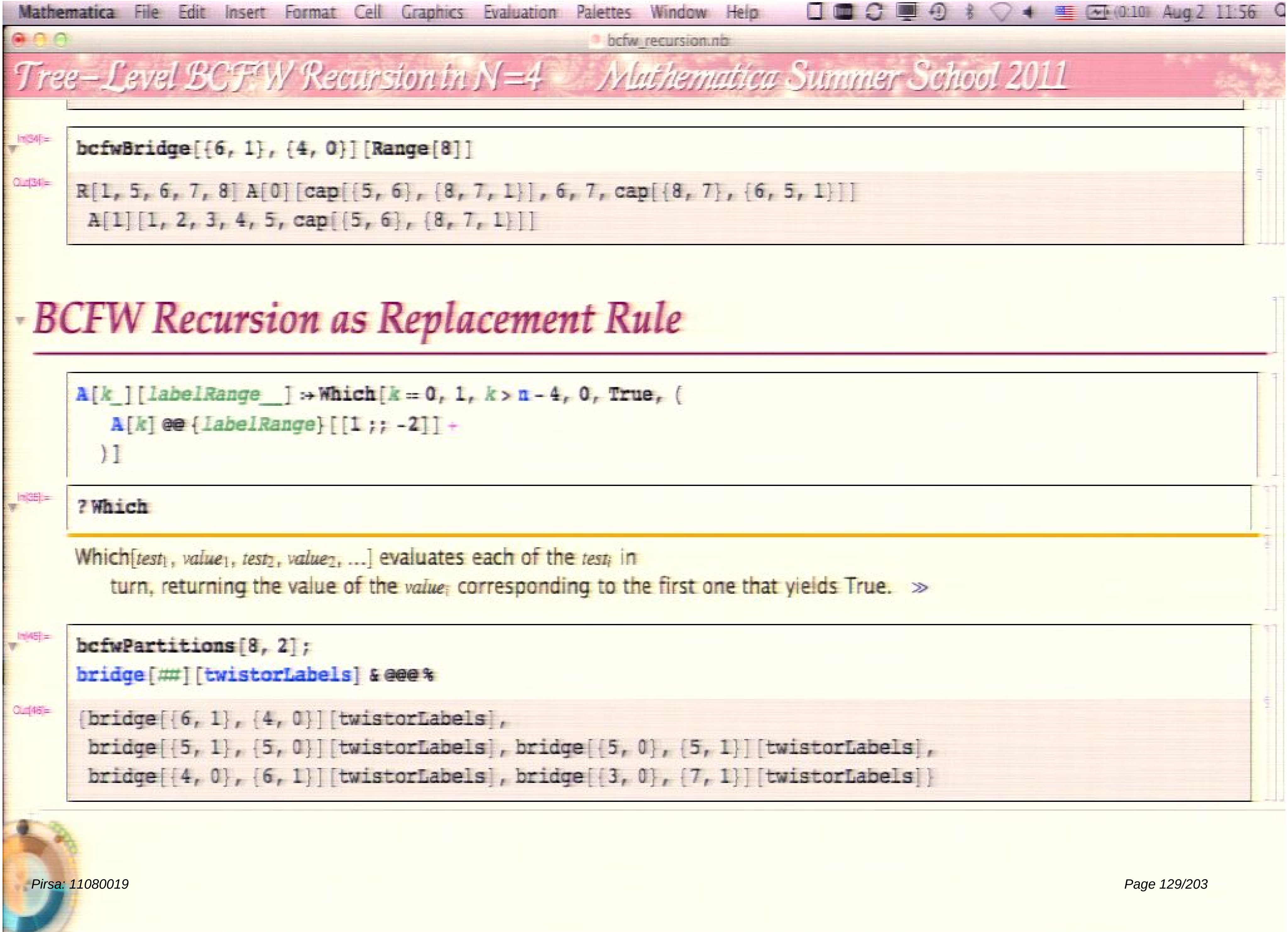
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[41]:= bcfwPartitions[8, 2];
         bridge @@@ %

Out[42]= {bridge[{6, 1}, {4, 0}], bridge[{5, 1}, {5, 0}],
          bridge[{5, 0}, {5, 1}], bridge[{4, 0}, {6, 1}], bridge[{3, 0}, {7, 1}]}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:10) Aug 2 11:56
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

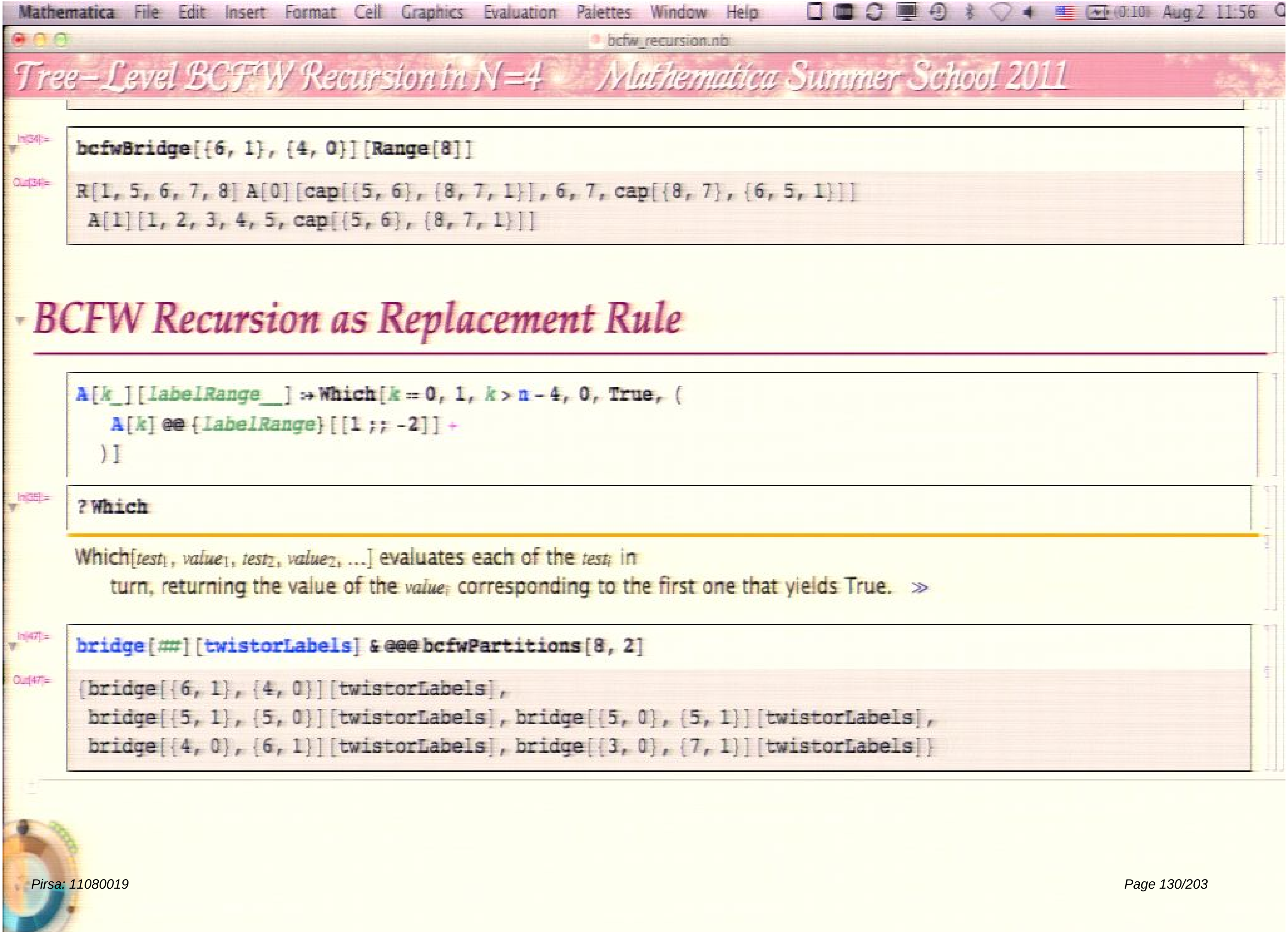
```
A[k_][labelRange__] ⧴ Which[k ⩵ 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in
turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[45]:= bcfwPartitions[8, 2];
         bridge[##][twistorLabels] & @@@ %

Out[46]= {bridge[{6, 1}, {4, 0}][twistorLabels],
          bridge[{5, 1}, {5, 0}][twistorLabels], bridge[{5, 0}, {5, 1}][twistorLabels],
          bridge[{4, 0}, {6, 1}][twistorLabels], bridge[{3, 0}, {7, 1}][twistorLabels]}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:10) Aug 2 11:56
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011
In[34]:=
Out[34]=
In[35]:=
In[47]:=
Out[47]=

```
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
```

```
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

# BCFW Recursion as Replacement Rule

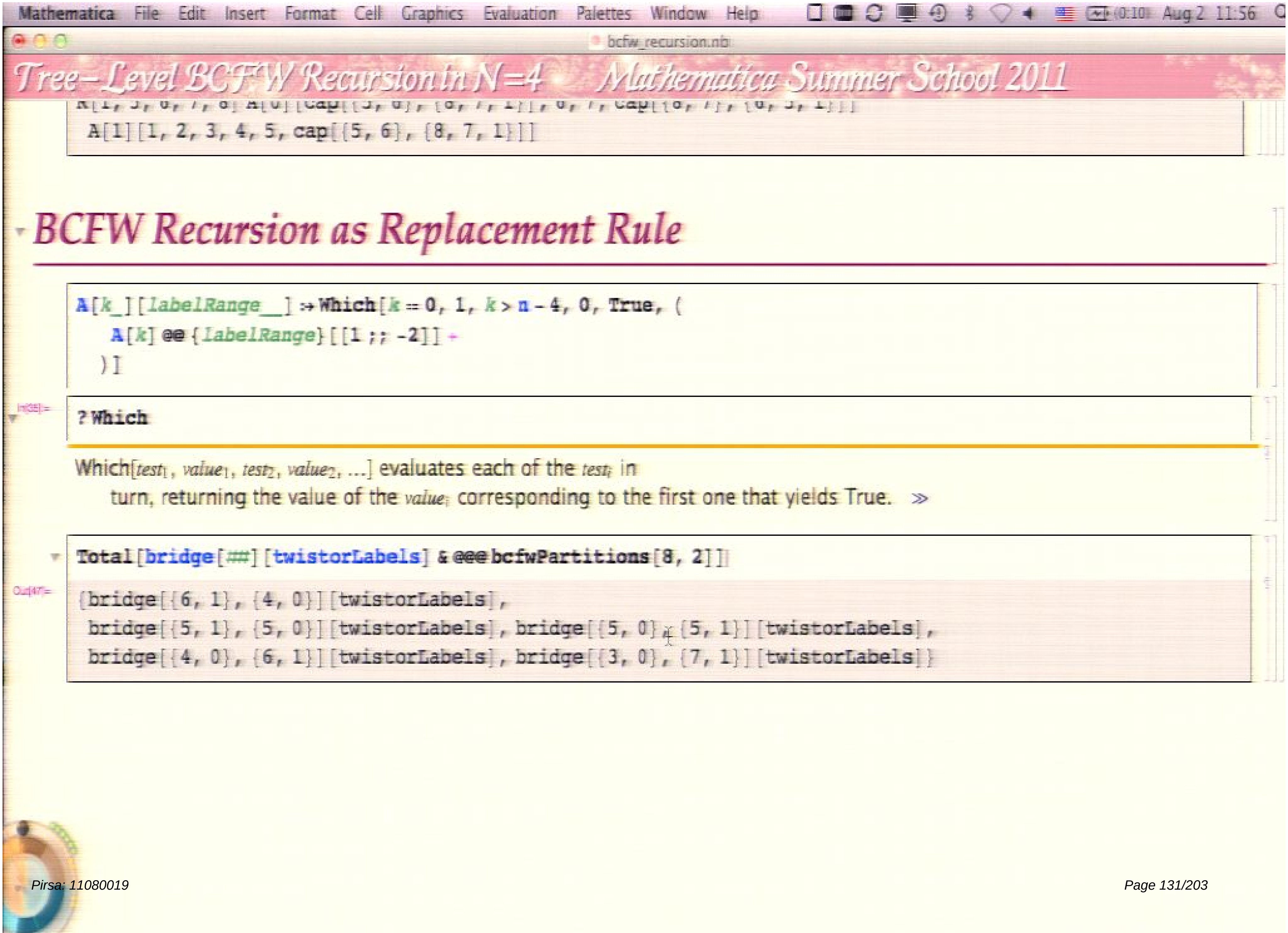
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
?Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]
```

```
{bridge[{6, 1}, {4, 0}][twistorLabels],
 bridge[{5, 1}, {5, 0}][twistorLabels], bridge[{5, 0}, {5, 1}][twistorLabels],
 bridge[{4, 0}, {6, 1}][twistorLabels], bridge[{3, 0}, {7, 1}][twistorLabels]}
```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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
A[1, 3, 6, 7, 8] A[0][cap[{3, 6}, {6, 7, 1}], 6, 7, cap[{6, 7}, {6, 3, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

# BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :→ Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

In[35]:=

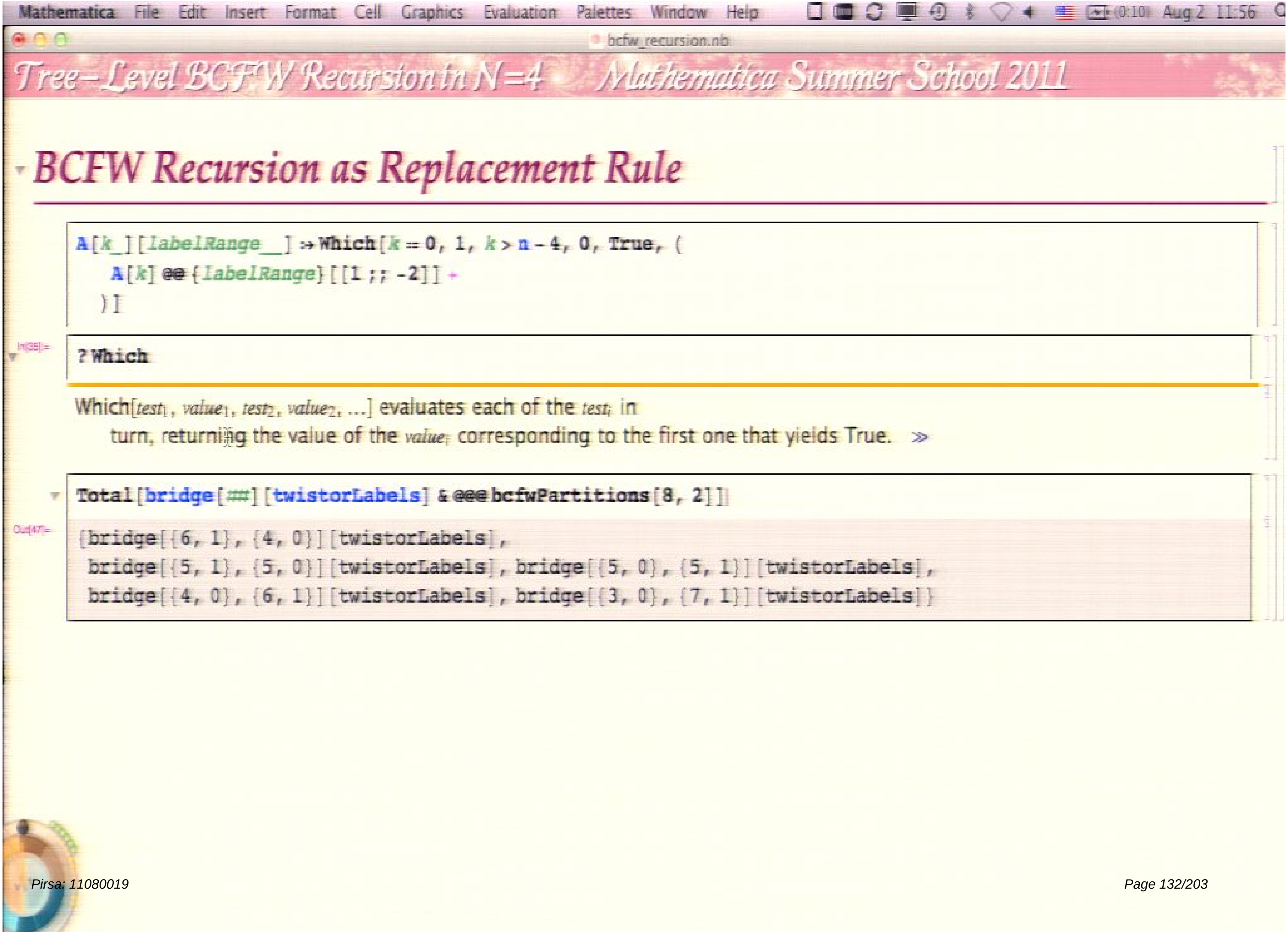
```
?Which
```

Which[*test*$_1$, *value*$_1$, *test*$_2$, *value*$_2$, ...] evaluates each of the *test*$_i$ in turn, returning the value of the *value*$_i$ corresponding to the first one that yields True. »

```
Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]
```

Out[47]=

```
{bridge[{6, 1}, {4, 0}][twistorLabels],
 bridge[{5, 1}, {5, 0}][twistorLabels], bridge[{5, 0}, {5, 1}][twistorLabels],
 bridge[{4, 0}, {6, 1}][twistorLabels], bridge[{3, 0}, {7, 1}][twistorLabels]}
```



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# BCFW Recursion as Replacement Rule

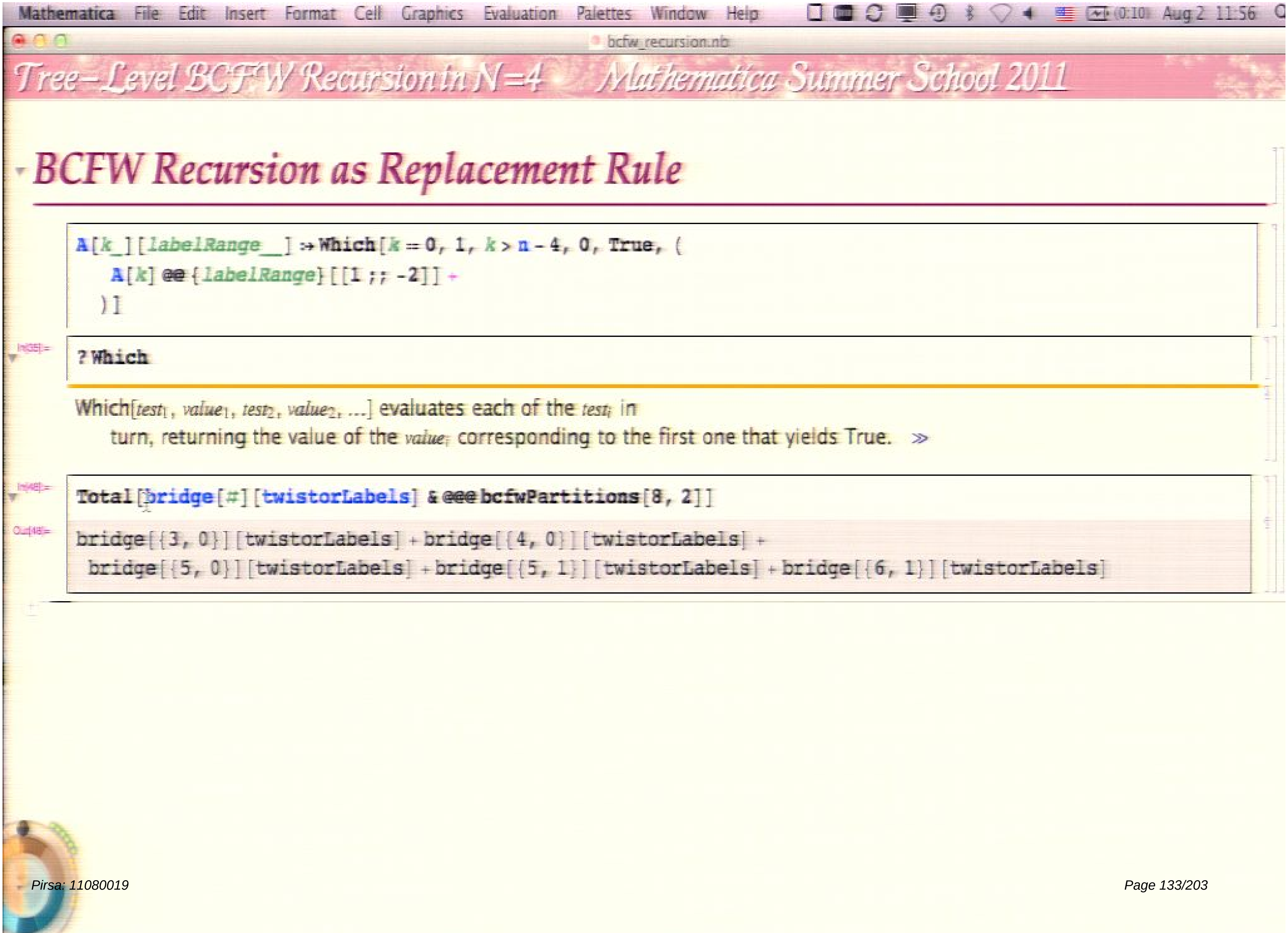
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ?Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[47]= {bridge[{6, 1}, {4, 0}][twistorLabels],
 bridge[{5, 1}, {5, 0}][twistorLabels], bridge[{5, 0}, {5, 1}][twistorLabels],
 bridge[{4, 0}, {6, 1}][twistorLabels], bridge[{3, 0}, {7, 1}][twistorLabels]}
```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# BCFW Recursion as Replacement Rule

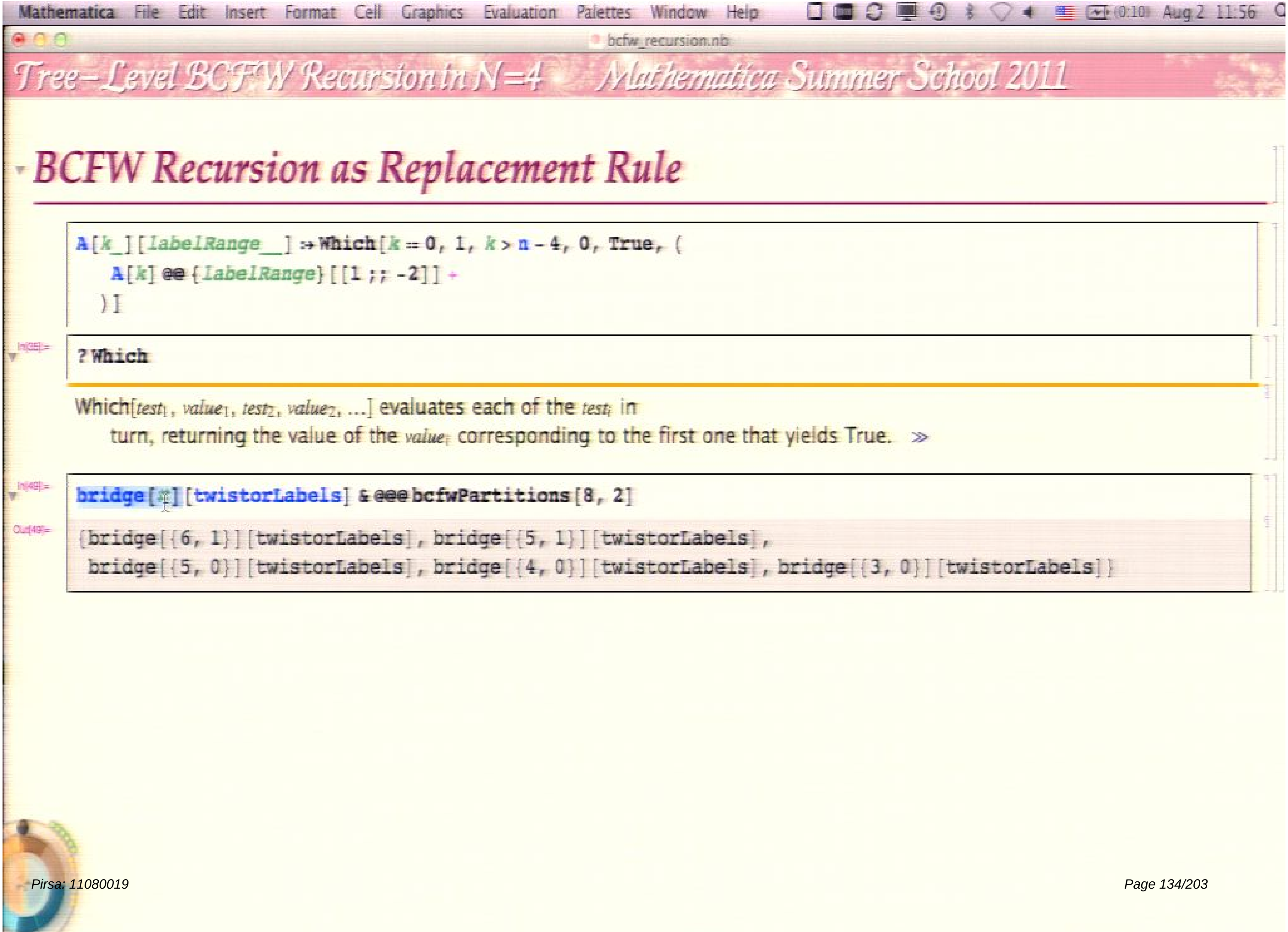
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[48]:= Total[bridge[#][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[48]= bridge[{3, 0}][twistorLabels] + bridge[{4, 0}][twistorLabels] +
          bridge[{5, 0}][twistorLabels] + bridge[{5, 1}][twistorLabels] + bridge[{6, 1}][twistorLabels]
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

# BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

In[35]:=

```
? Which
```

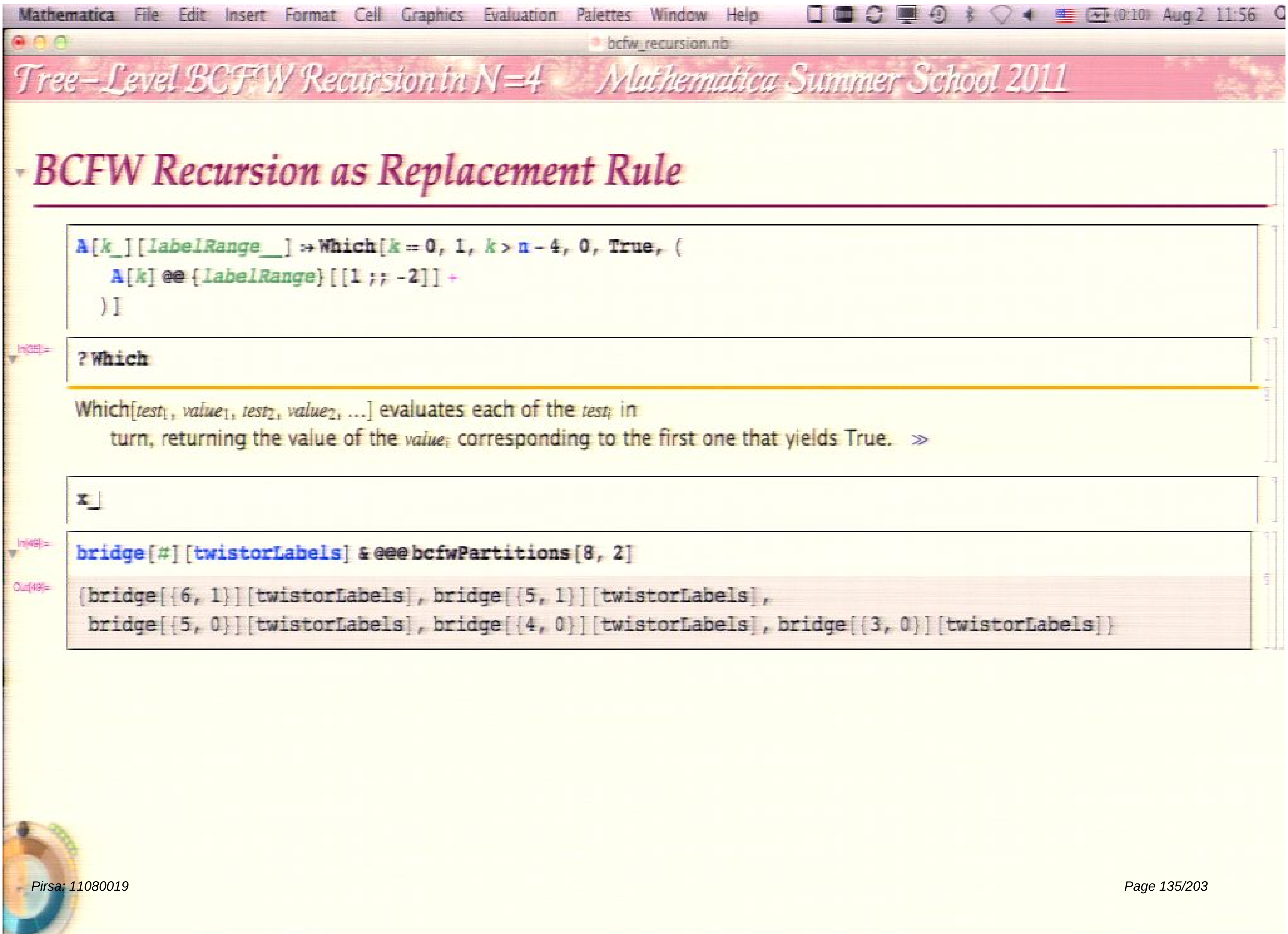
Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

In[49]:=

```
bridge[#][twistorLabels] & @@@ bcfwPartitions[8, 2]
```

Out[49]=

```
{bridge[{6, 1}][twistorLabels], bridge[{5, 1}][twistorLabels],
 bridge[{5, 0}][twistorLabels], bridge[{4, 0}][twistorLabels], bridge[{3, 0}][twistorLabels]}
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

# BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :→ Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

In[35]:=

```
? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

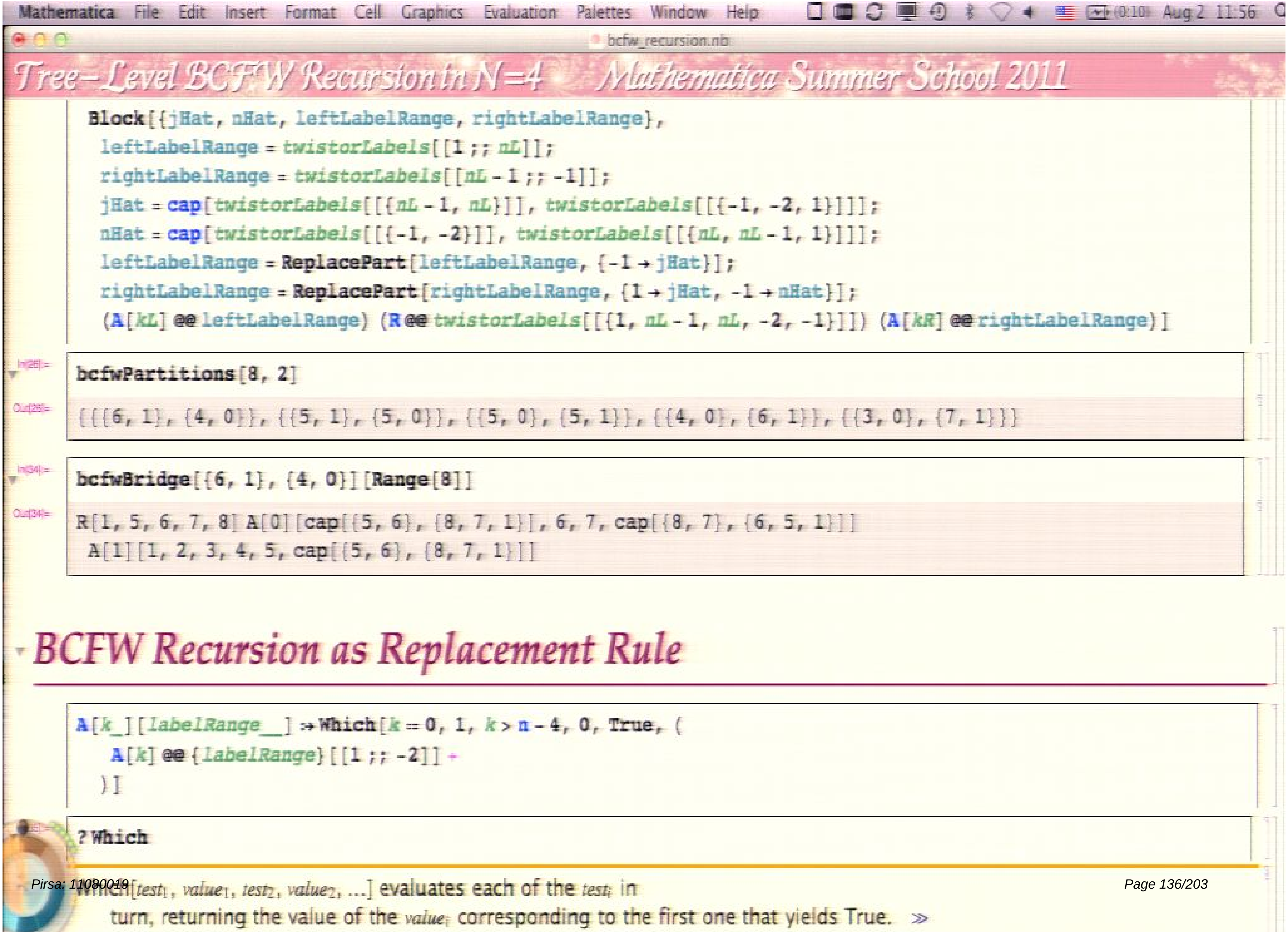
```
x_
```

In[49]:=

```
bridge[#][twistorLabels] & @@@ bcfwPartitions[8, 2]
```

Out[49]=

```
{bridge[{6, 1}][twistorLabels], bridge[{5, 1}][twistorLabels],
 bridge[{5, 0}][twistorLabels], bridge[{4, 0}][twistorLabels], bridge[{3, 0}][twistorLabels]}
```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Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

```
Block[{jHat, nHat, leftLabelRange, rightLabelRange},
 leftLabelRange = twistorLabels[[1 ;; nL]];
 rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
 jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
 nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
 leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
 rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
 (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

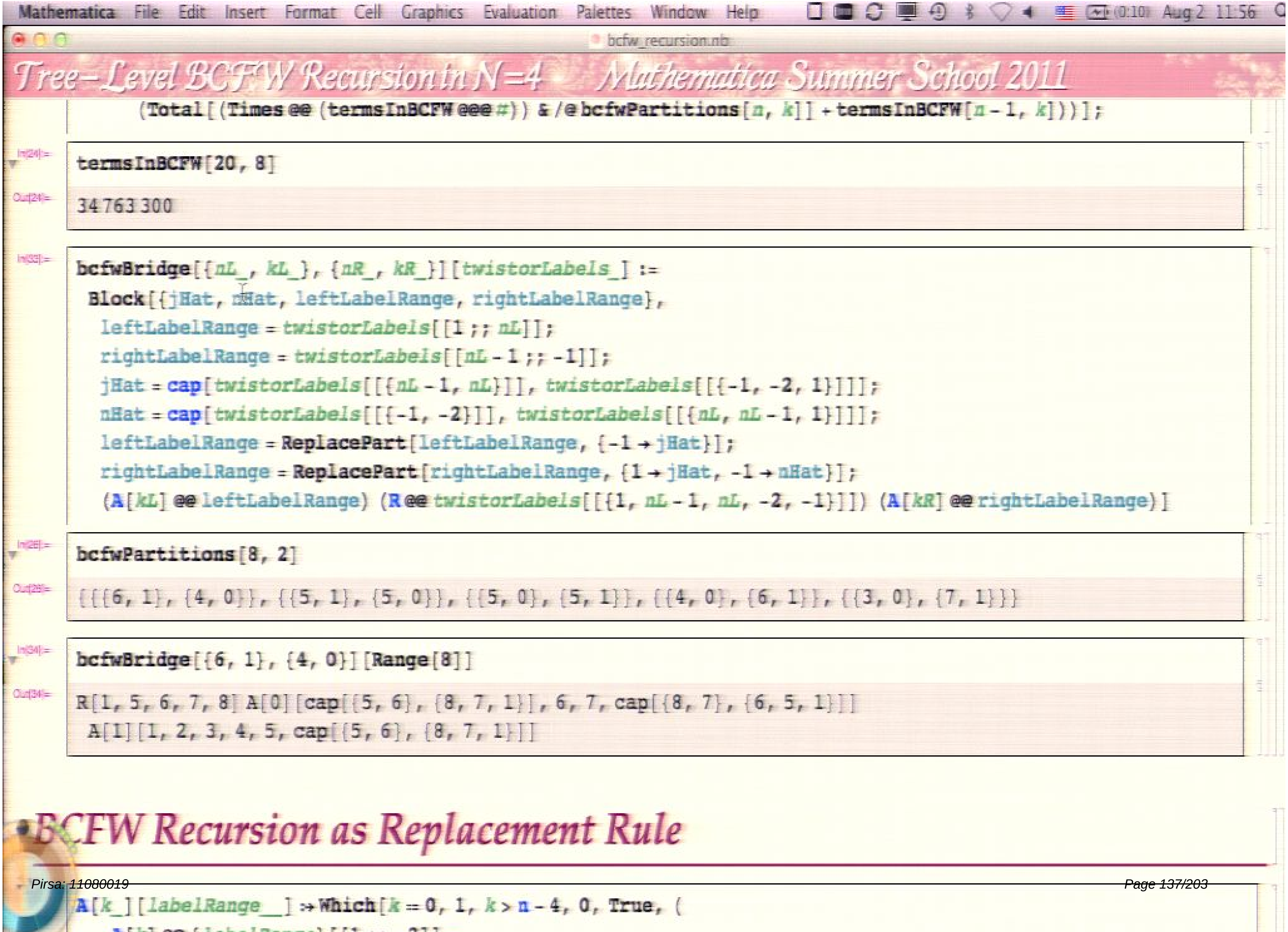
Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
        (Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];

In[24]:= termsInBCFW[20, 8]

Out[24]= 34 763 300

In[33]:= bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
          Block[{jHat, nHat, leftLabelRange, rightLabelRange},
           leftLabelRange = twistorLabels[[1 ;; nL]];
           rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
           jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
           nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
           leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
           rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
           (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]

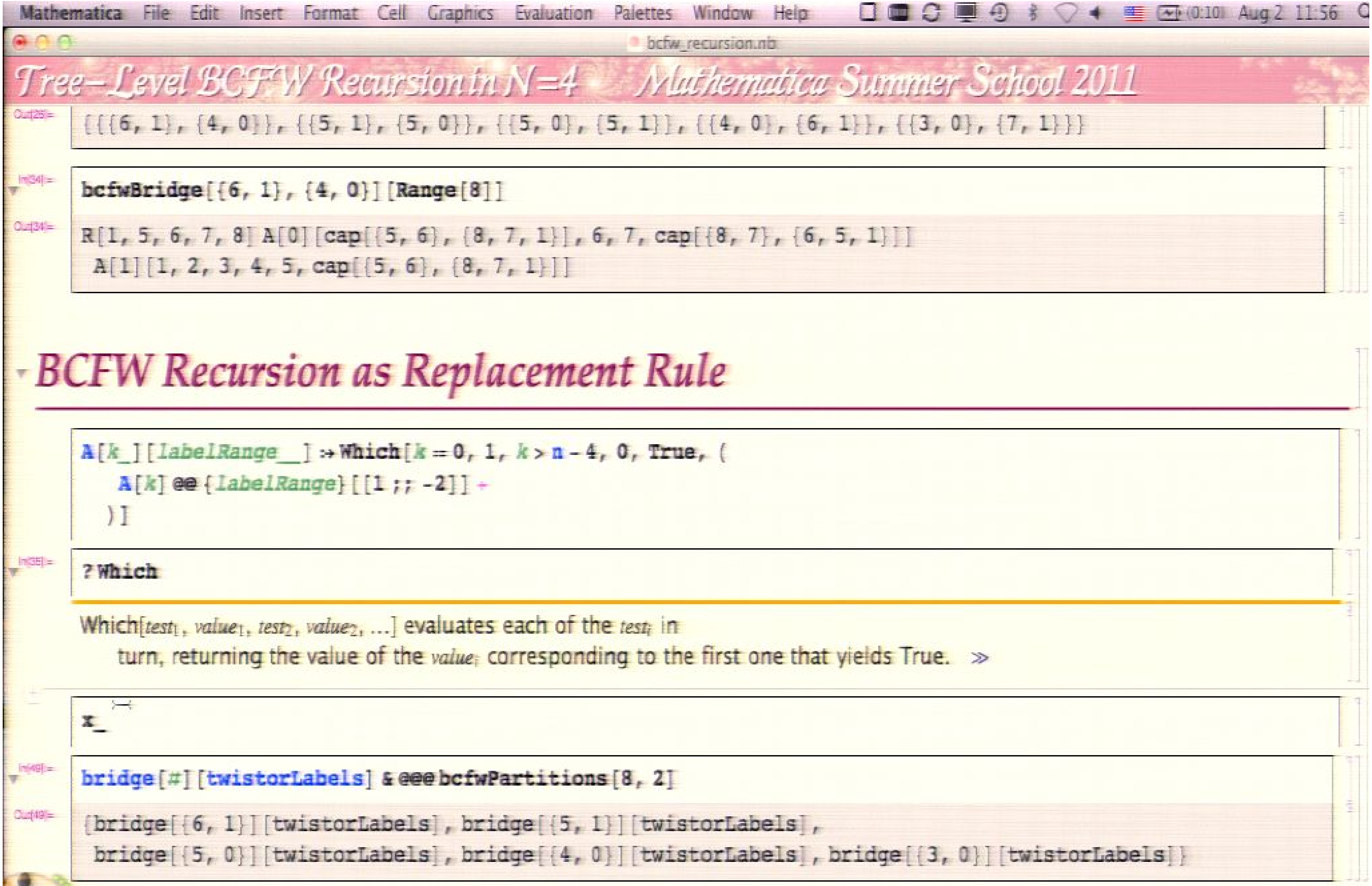
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] ⧴ Which[k == 0, 1, k > n - 4, 0, True, (
```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Out[26]=
```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

In[34]:=
```
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
```

Out[34]=
```
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] ⧴ Which[k ⩵ 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

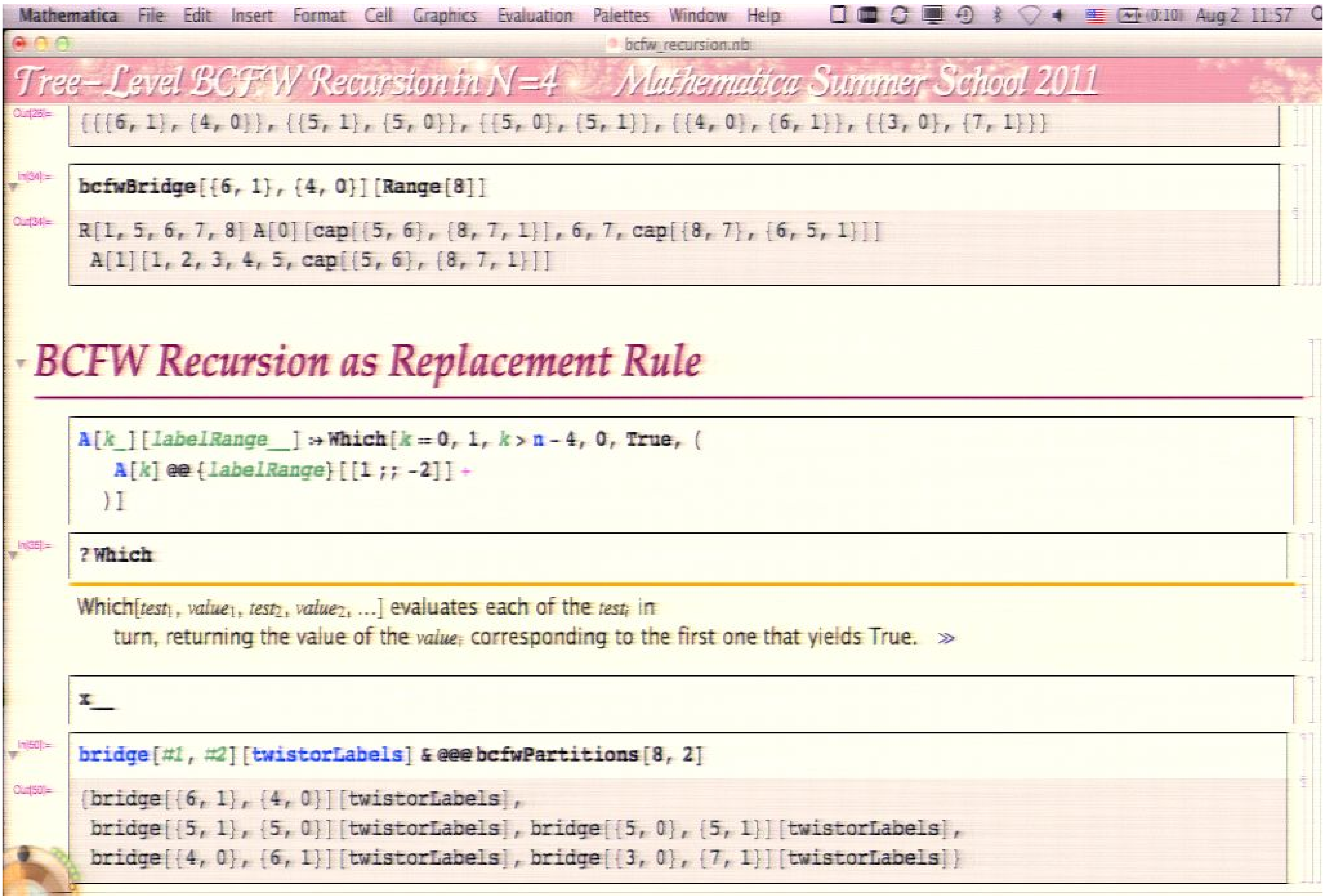
In[35]:=
```
? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
x_
```

In[49]:=
```
bridge[#][twistorLabels] & @@@ bcfwPartitions[8, 2]
```

Out[49]=
```
{bridge[{6, 1}][twistorLabels], bridge[{5, 1}][twistorLabels],
 bridge[{5, 0}][twistorLabels], bridge[{4, 0}][twistorLabels], bridge[{3, 0}][twistorLabels]}
```



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Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

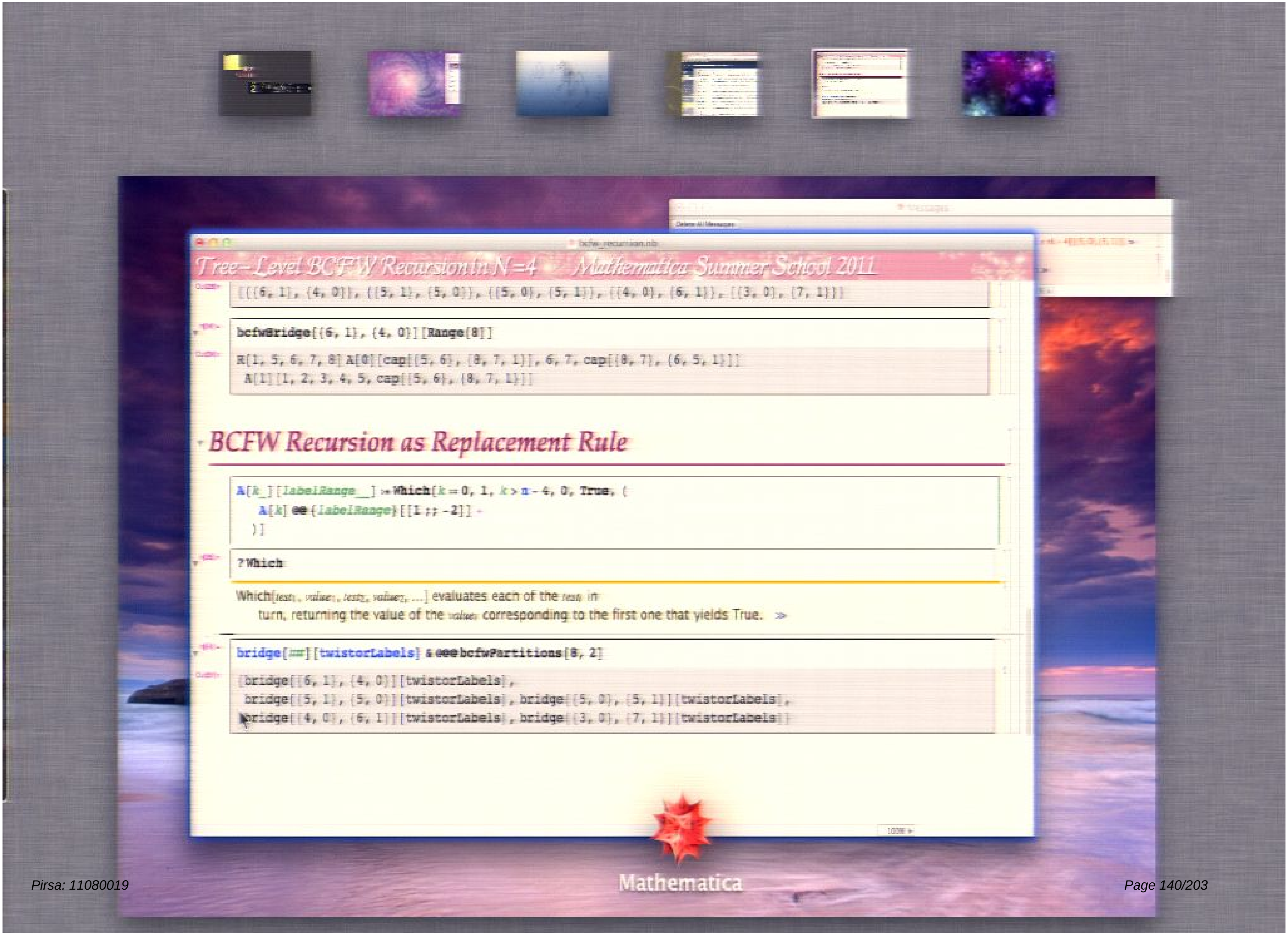
```
In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
x_
```

```
In[50]:= bridge[#1, #2][twistorLabels] & @@@ bcfwPartitions[8, 2]

Out[50]= {bridge[{6, 1}, {4, 0}][twistorLabels],
          bridge[{5, 1}, {5, 0}][twistorLabels], bridge[{5, 0}, {5, 1}][twistorLabels],
          bridge[{4, 0}, {6, 1}][twistorLabels], bridge[{3, 0}, {7, 1}][twistorLabels]}
```



Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
```

```
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

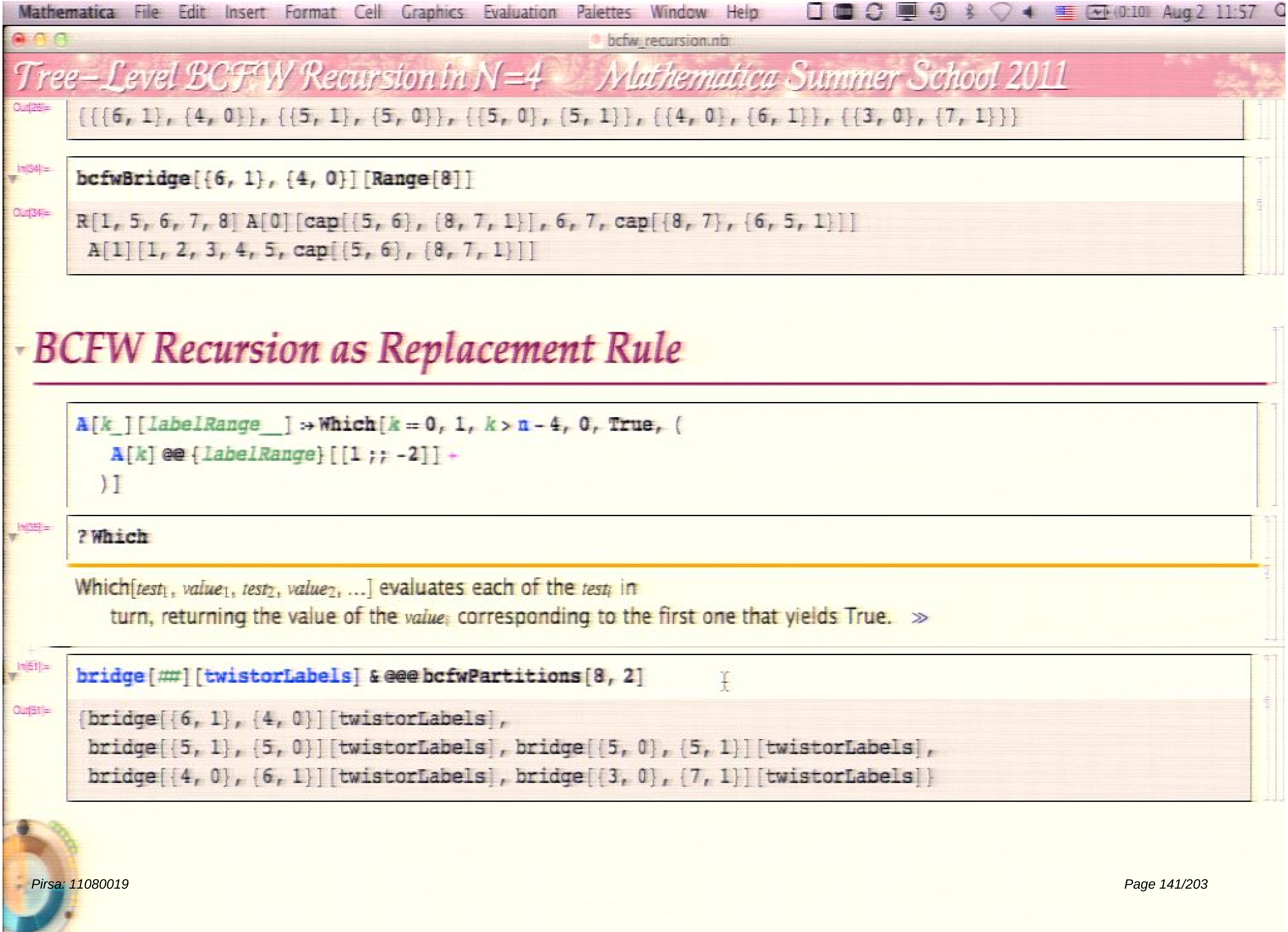
```
A[k_][labelRange__] := Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]]
  )]
```

```
?Which
```

Which[*test*₁, *value*₁, *test*₂, *value*₂, …] evaluates each of the *test*ᵢ in turn, returning the value of the *value*ᵢ corresponding to the first one that yields True. >>

```
bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]
```

```
{bridge[{6, 1}, {4, 0}][twistorLabels],
 bridge[{5, 1}, {5, 0}][twistorLabels], bridge[{5, 0}, {5, 1}][twistorLabels],
 bridge[{4, 0}, {6, 1}][twistorLabels], bridge[{3, 0}, {7, 1}][twistorLabels]}
```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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[26]=
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:=
bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]=
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

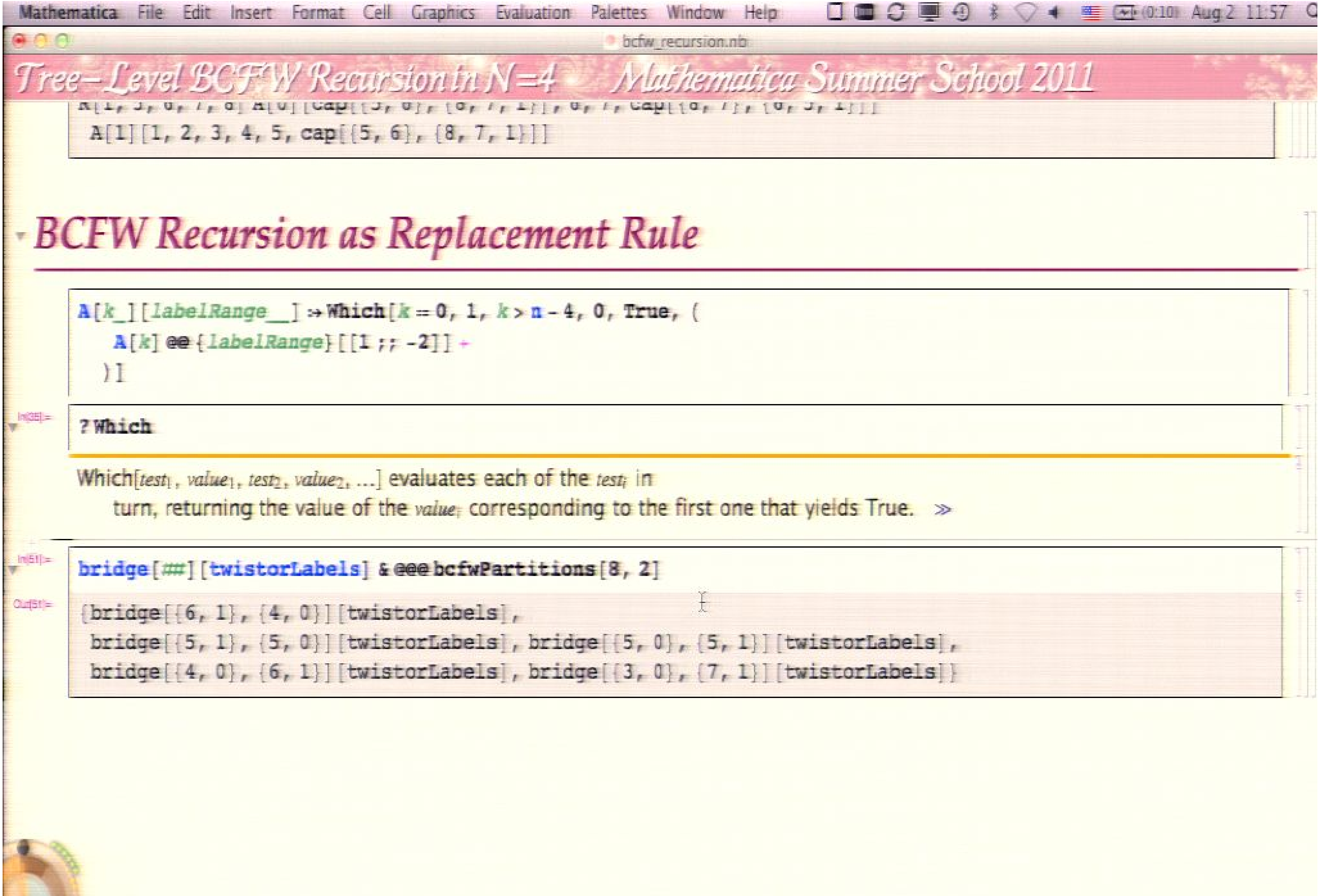
```
A[k_][labelRange__] ⧴ Which[k ⩵ 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:=
? Which
```

Which[*test*$_1$, *value*$_1$, *test*$_2$, *value*$_2$, …] evaluates each of the *test*$_i$ in turn, returning the value of the *value*$_i$ corresponding to the first one that yields True. »

```
In[51]:=
bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]

Out[51]=
{bridge[{6, 1}, {4, 0}][twistorLabels],
 bridge[{5, 1}, {5, 0}][twistorLabels], bridge[{5, 0}, {5, 1}][twistorLabels],
 bridge[{4, 0}, {6, 1}][twistorLabels], bridge[{3, 0}, {7, 1}][twistorLabels]}
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

# BCFW Recursion as Replacement Rule

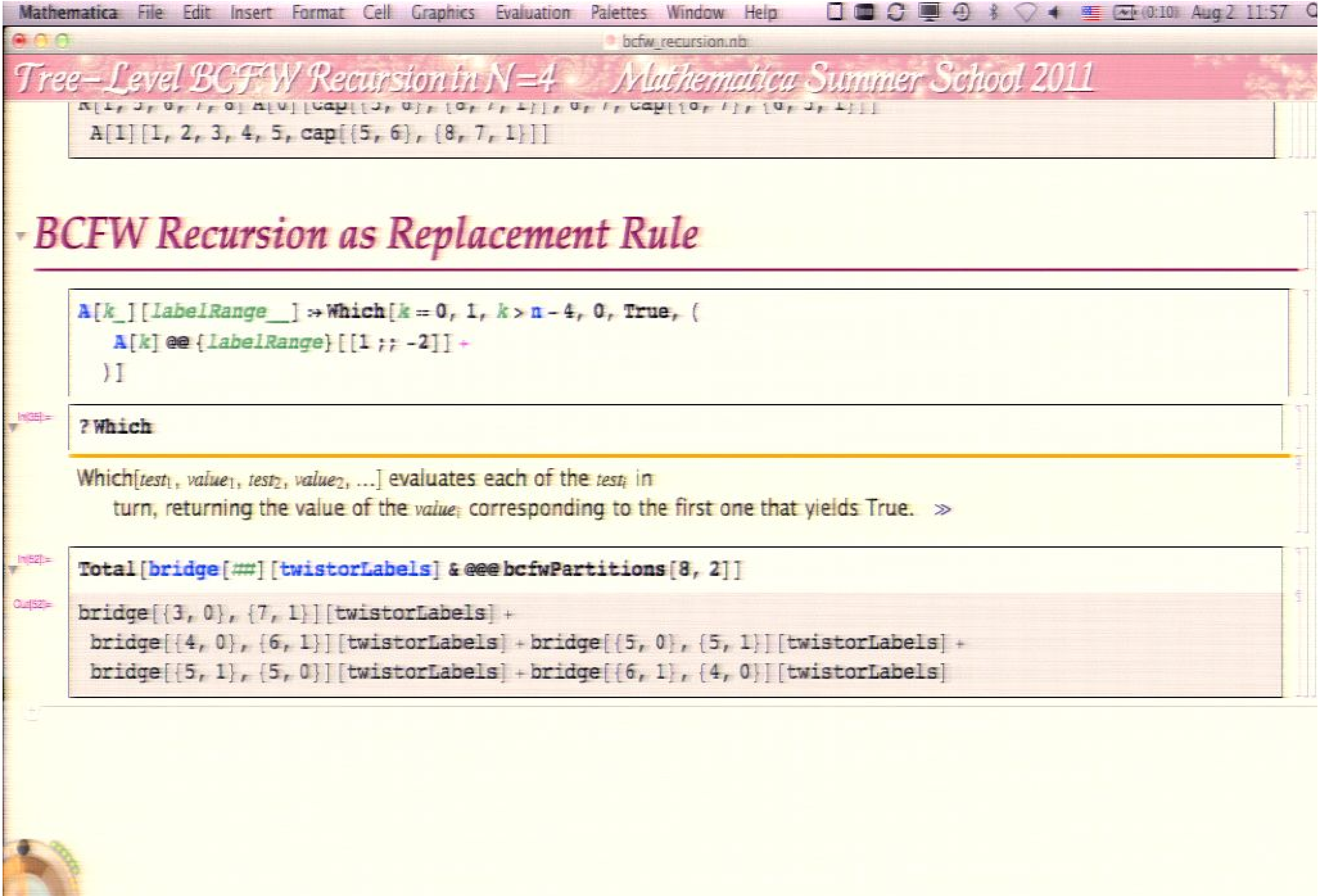
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
In[35]:= ? Which
```

Which[*test*₁, *value*₁, *test*₂, *value*₂, ...] evaluates each of the *test*ᵢ in turn, returning the value of the *value*ᵢ corresponding to the first one that yields True. »

```
In[51]:= bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]

Out[51]= {bridge[{6, 1}, {4, 0}][twistorLabels],
 bridge[{5, 1}, {5, 0}][twistorLabels], bridge[{5, 0}, {5, 1}][twistorLabels],
 bridge[{4, 0}, {6, 1}][twistorLabels], bridge[{3, 0}, {7, 1}][twistorLabels]}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

# BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :→ Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

In[35]:=

```
?Which
```

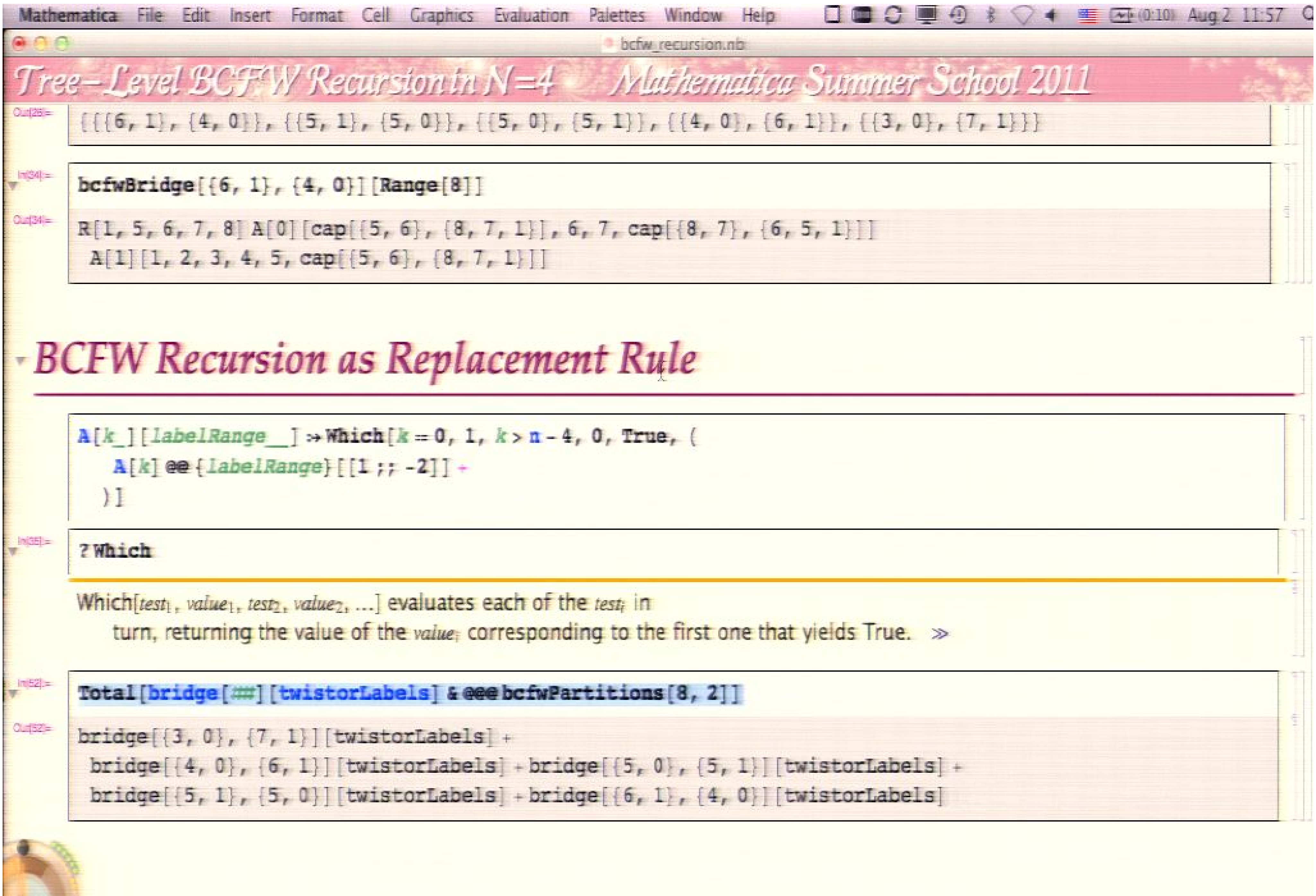
Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

In[52]:=

```
Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]
```

Out[52]=

```
bridge[{3, 0}, {7, 1}][twistorLabels] +
 bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
 bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```



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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

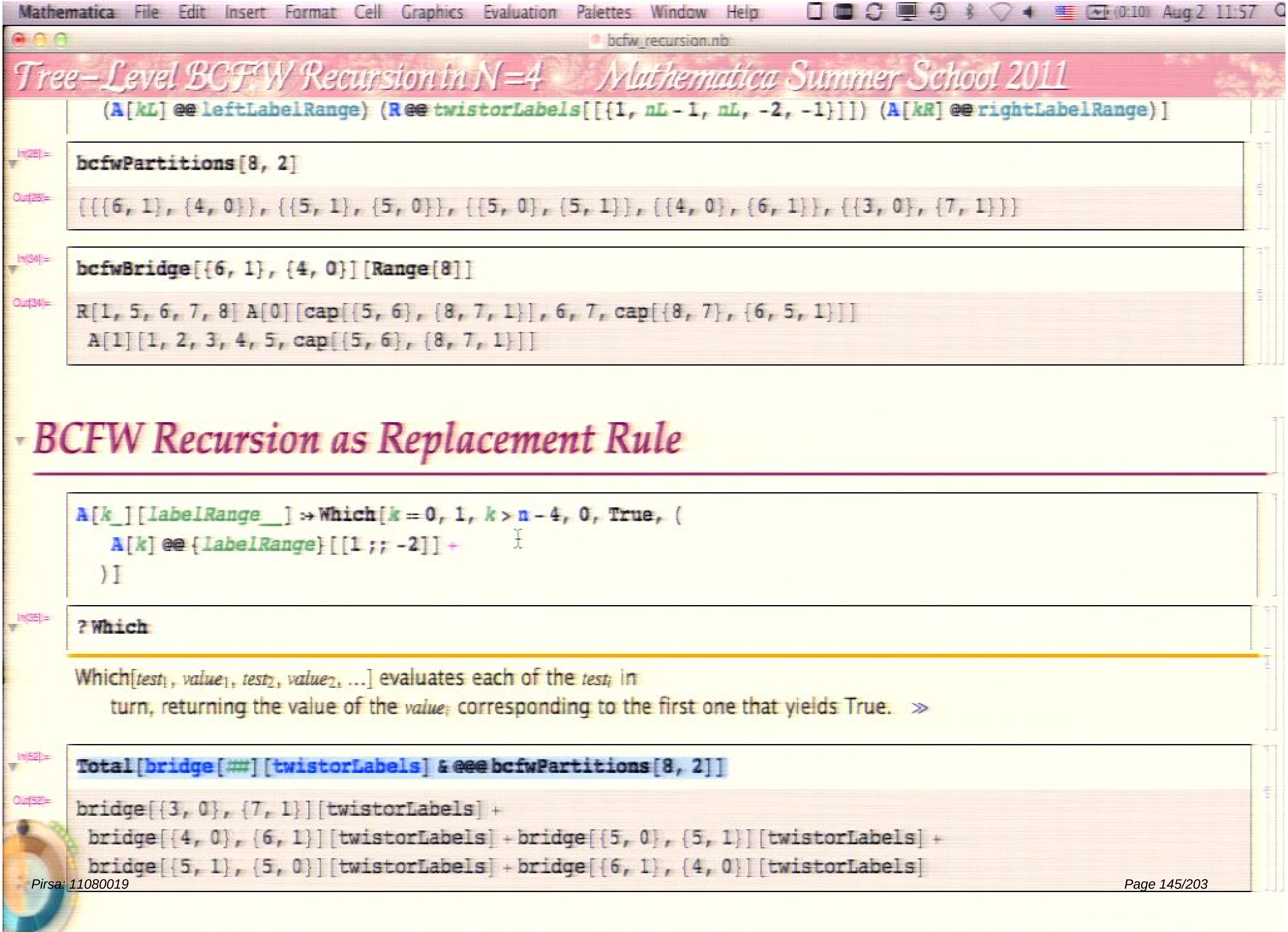
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]

In[35]:= ?Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[52]:= Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[52]= bridge[{3, 0}, {7, 1}][twistorLabels] +
          bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
          bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```



*Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011*

```
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
bcfwPartitions[8, 2]
```

```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
```

```
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

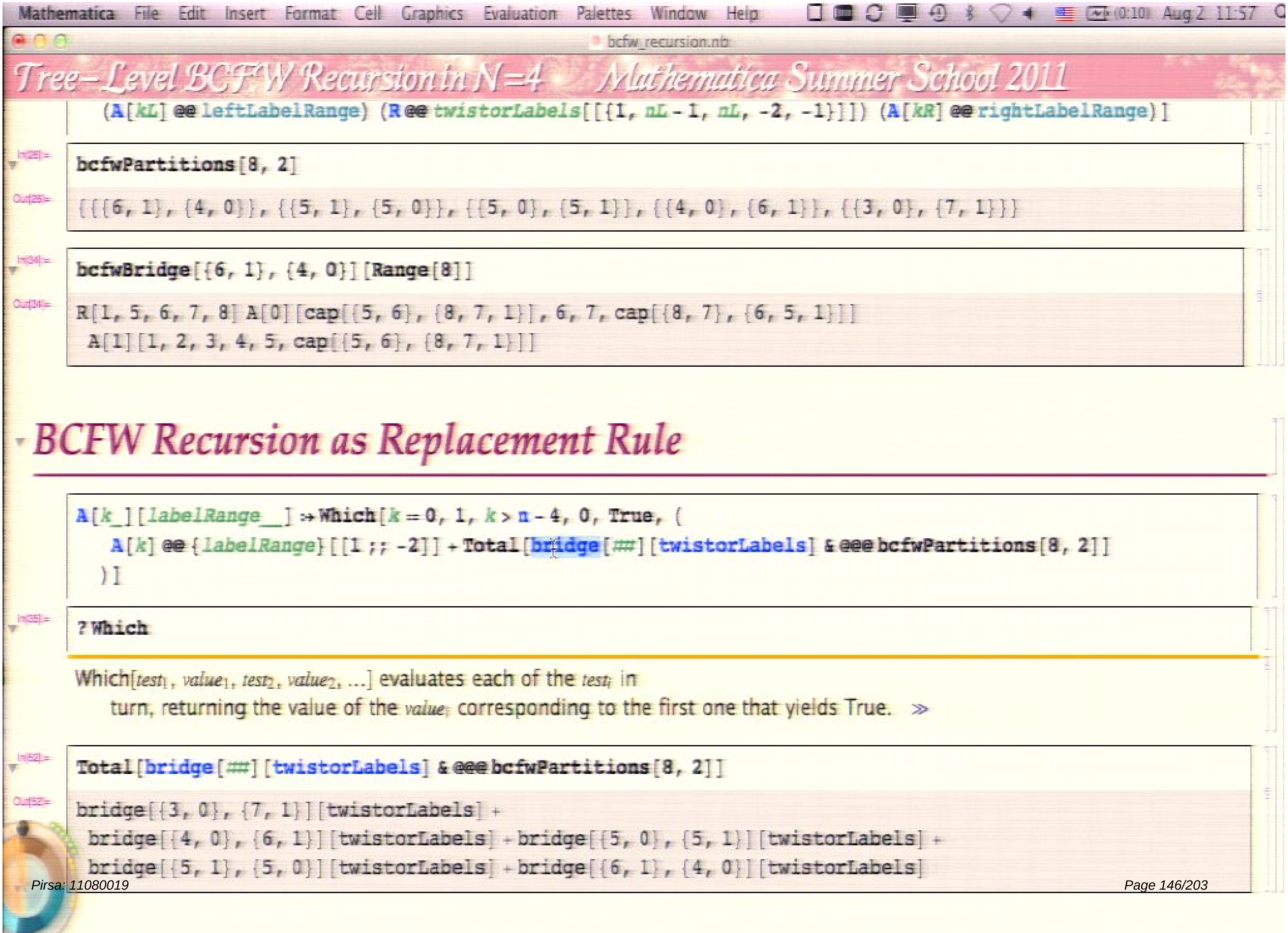
```
A[k_][labelRange__] ⧴ Which[k ⩵ 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
  )]
```

```
? Which
```

Which[*test*$_1$, *value*$_1$, *test*$_2$, *value*$_2$, …] evaluates each of the *test*$_i$ in
 turn, returning the value of the *value*$_i$ corresponding to the first one that yields True. »

```
Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]
```

```
bridge[{3, 0}, {7, 1}][twistorLabels] +
 bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
 bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

# BCFW Recursion as Replacement Rule

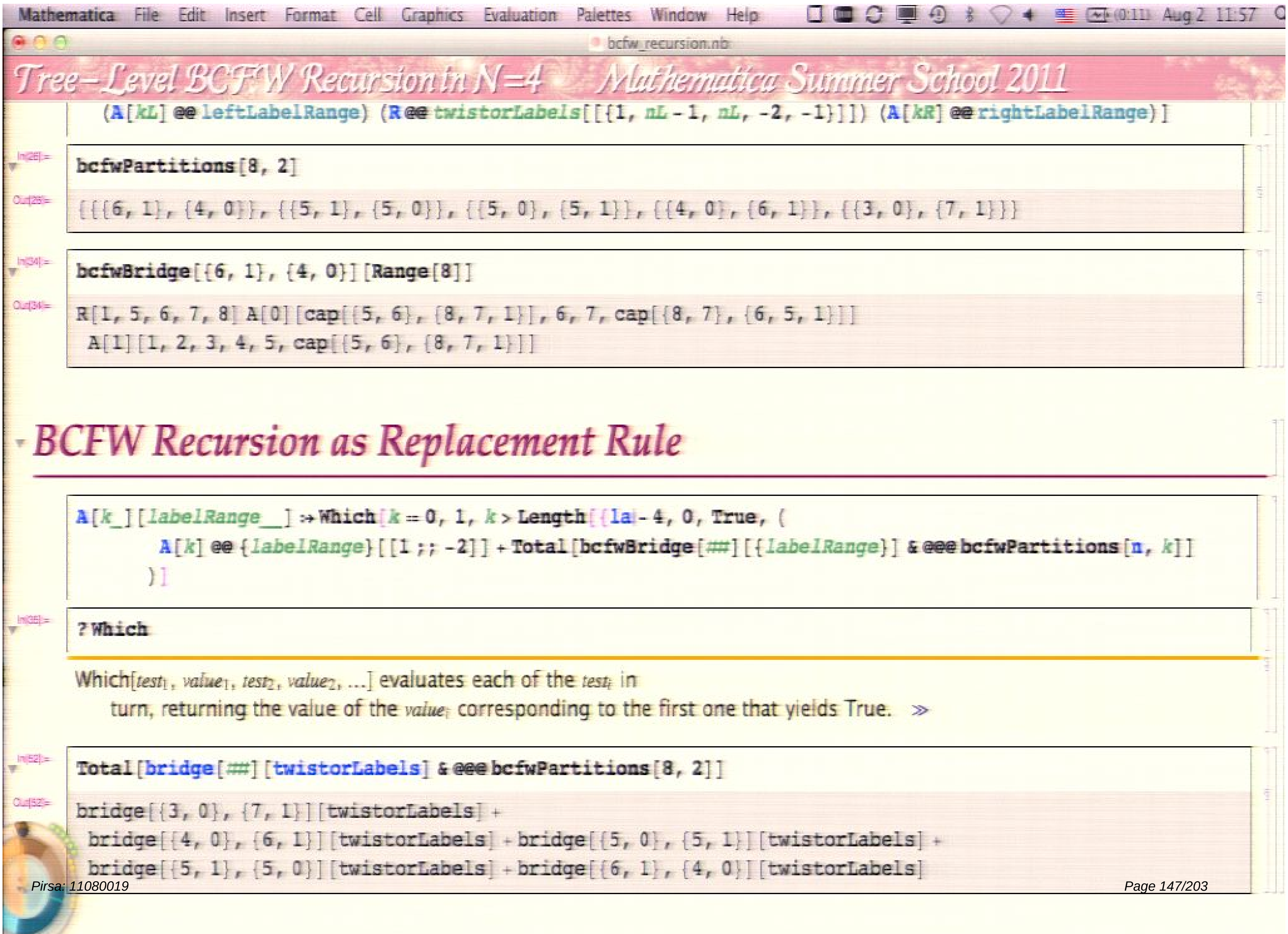
```
A[k_][labelRange__] :> Which[k == 0, 1, k > n - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] + Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]
  )]
```

```
In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. ≫

```
In[52]:= Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[52]= bridge[{3, 0}, {7, 1}][twistorLabels] +
          bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
          bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:11) Aug 2 11:57
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

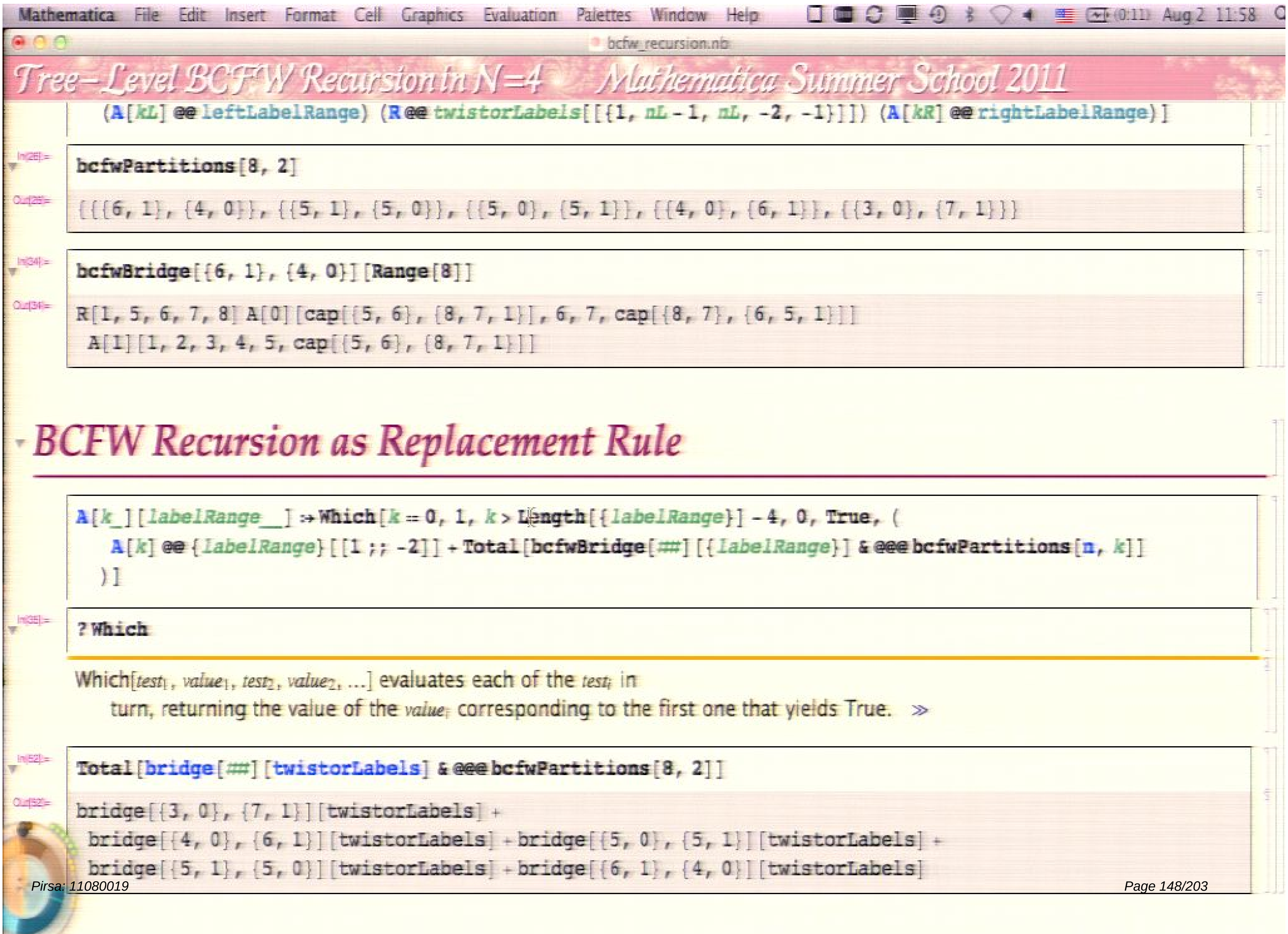
```
A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{la} - 4, 0, True, (
       A[k] @@ {labelRange}[[1 ;; -2]] + Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[n, k]]
     )]
```

```
In[35]:= ? Which
```

Which[*test*₁, *value*₁, *test*₂, *value*₂, ...] evaluates each of the *test*ᵢ in
  turn, returning the value of the *value*ᵢ corresponding to the first one that yields True. »

```
In[52]:= Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[52]= bridge[{3, 0}, {7, 1}][twistorLabels] +
          bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
          bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```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(0:11) Aug 2 11:58
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

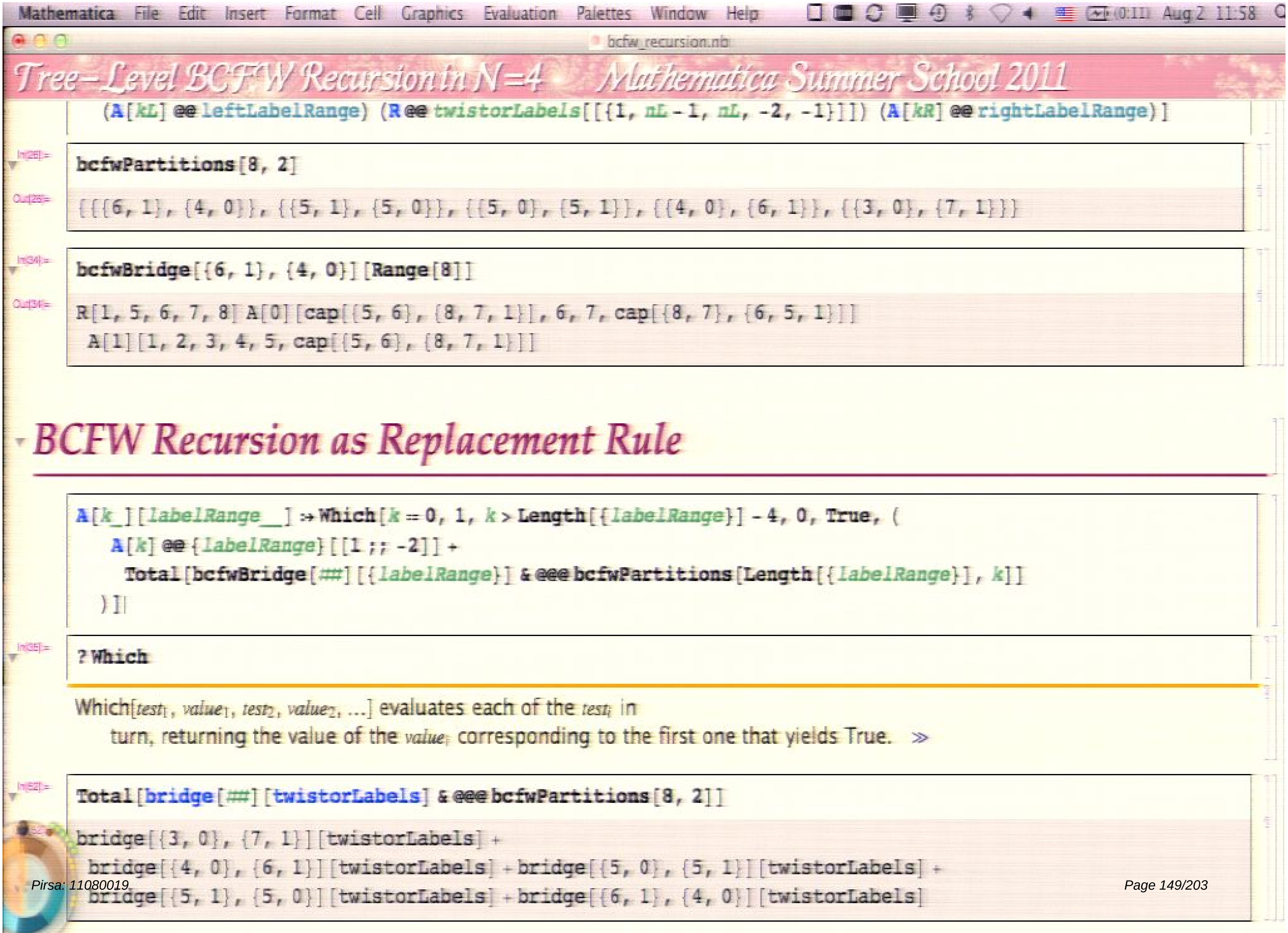
```
A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] + Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[n, k]]
  )]

In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in
turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[52]:= Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[52]= bridge[{3, 0}, {7, 1}][twistorLabels] +
          bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
          bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```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Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
  (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

# BCFW Recursion as Replacement Rule

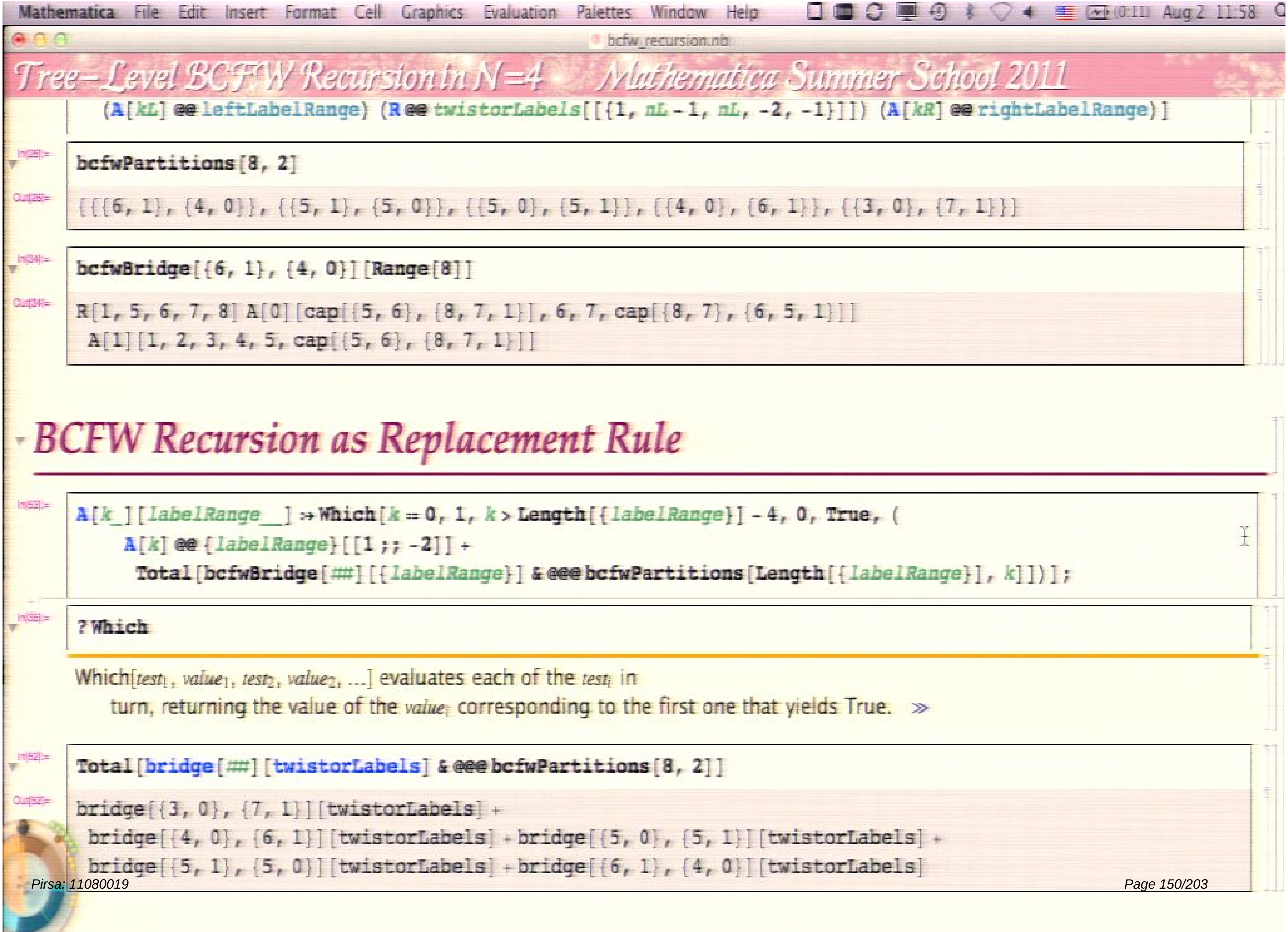
```
A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
   A[k] @@ {labelRange}[[1 ;; -2]] +
    Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]]
  )]
```

```
In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[52]:= Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

bridge[{3, 0}, {7, 1}][twistorLabels] +
 bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
 bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```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(0:11) Aug 2 11:58
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
    (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

In[26]:=
```
bcfwPartitions[8, 2]
```
Out[26]=
```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

In[34]:=
```
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
```
Out[34]=
```
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

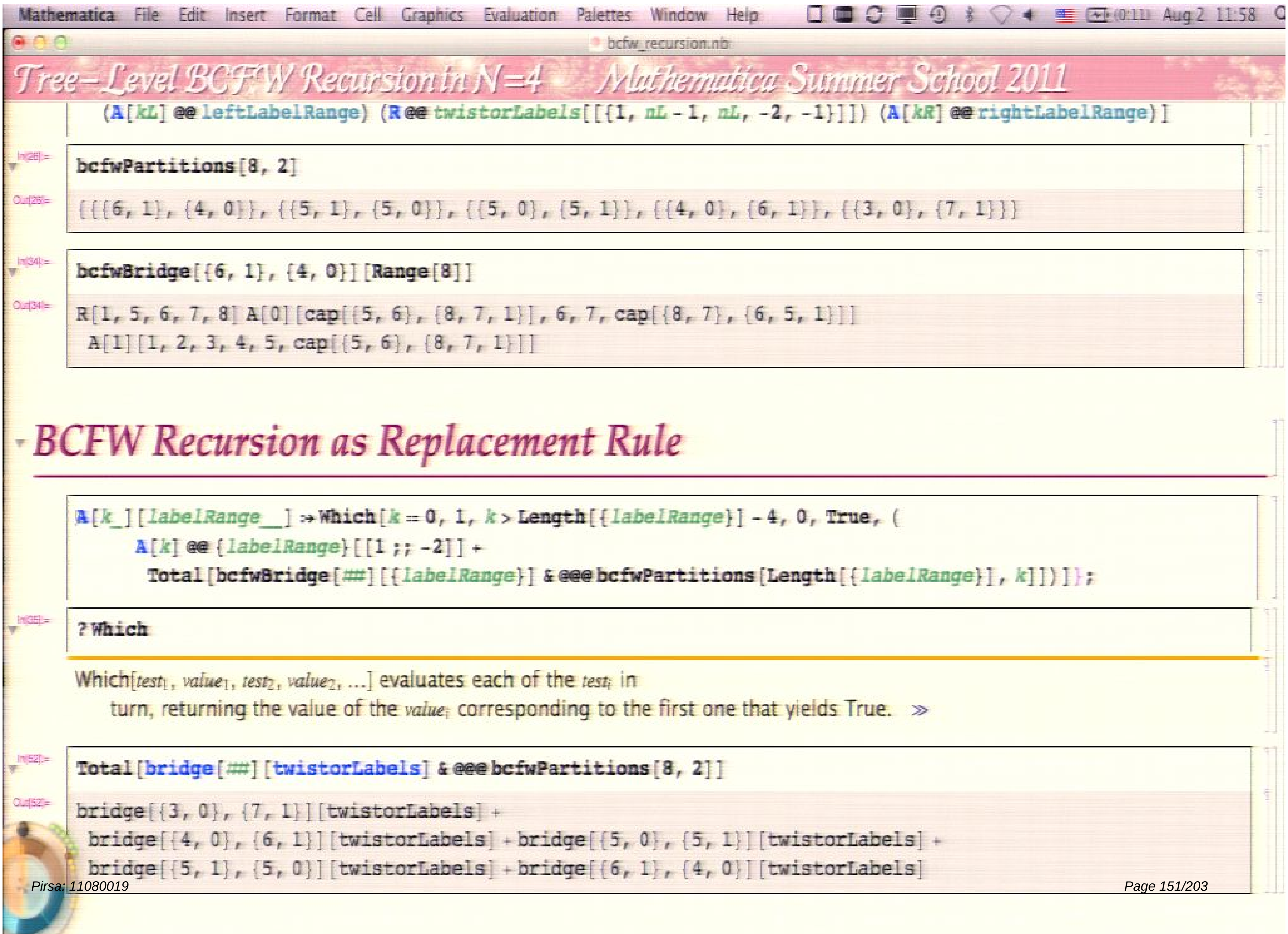
## BCFW Recursion as Replacement Rule

In[53]:=
```
A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
    A[k] @@ {labelRange}[[1 ;; -2]] +
     Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])];
```

In[35]:=
```
? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in
turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

In[52]:=
```
Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]
```
Out[52]=
```
bridge[{3, 0}, {7, 1}][twistorLabels] +
 bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
 bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```



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(0:11) Aug 2 11:58
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
     A[k] @@ {labelRange}[[1 ;; -2]] +
      Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])];
```

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In[35]:= ? Which
```
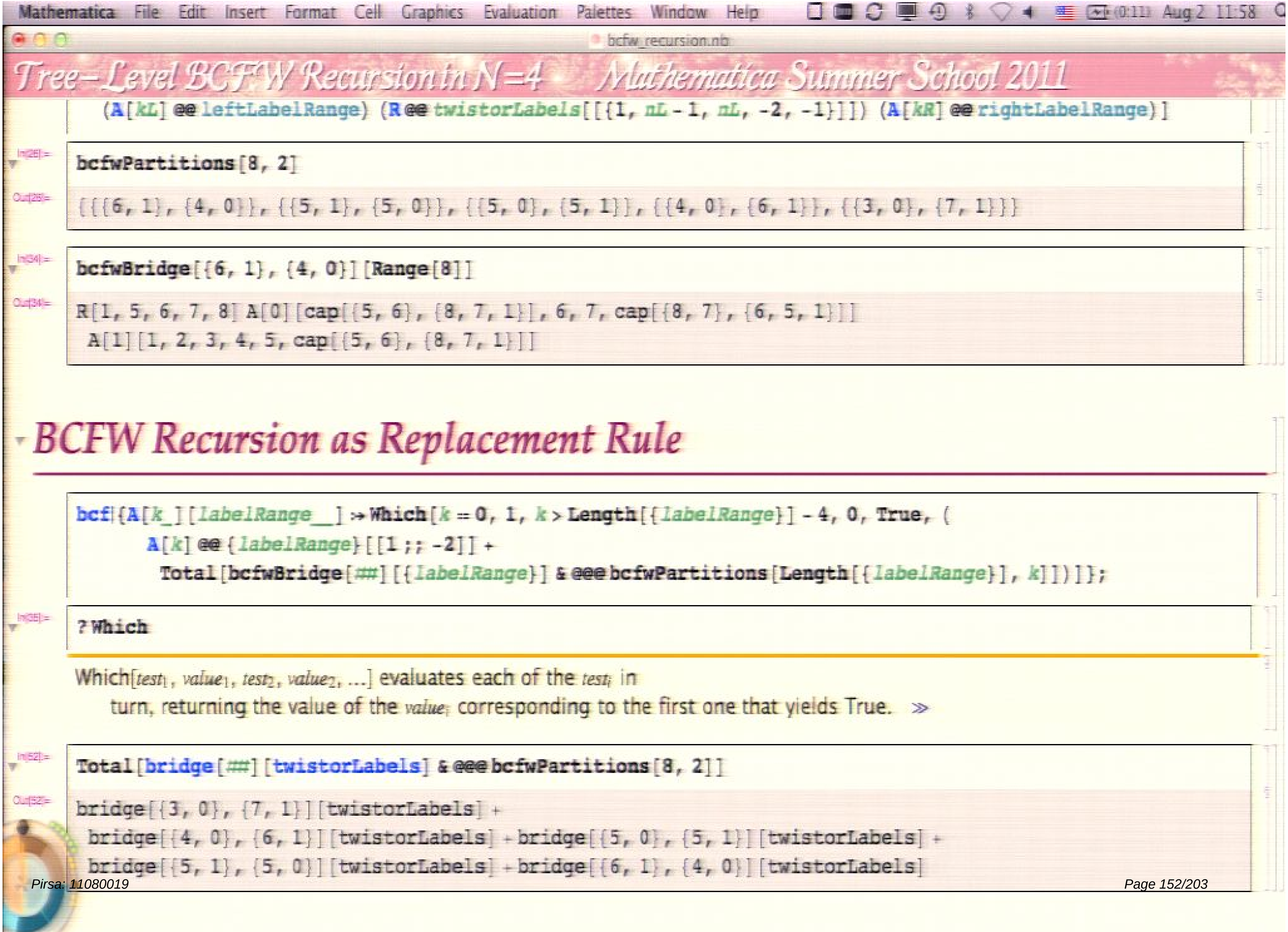

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[52]:= Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[52]= bridge[{3, 0}, {7, 1}][twistorLabels] +
          bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
          bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

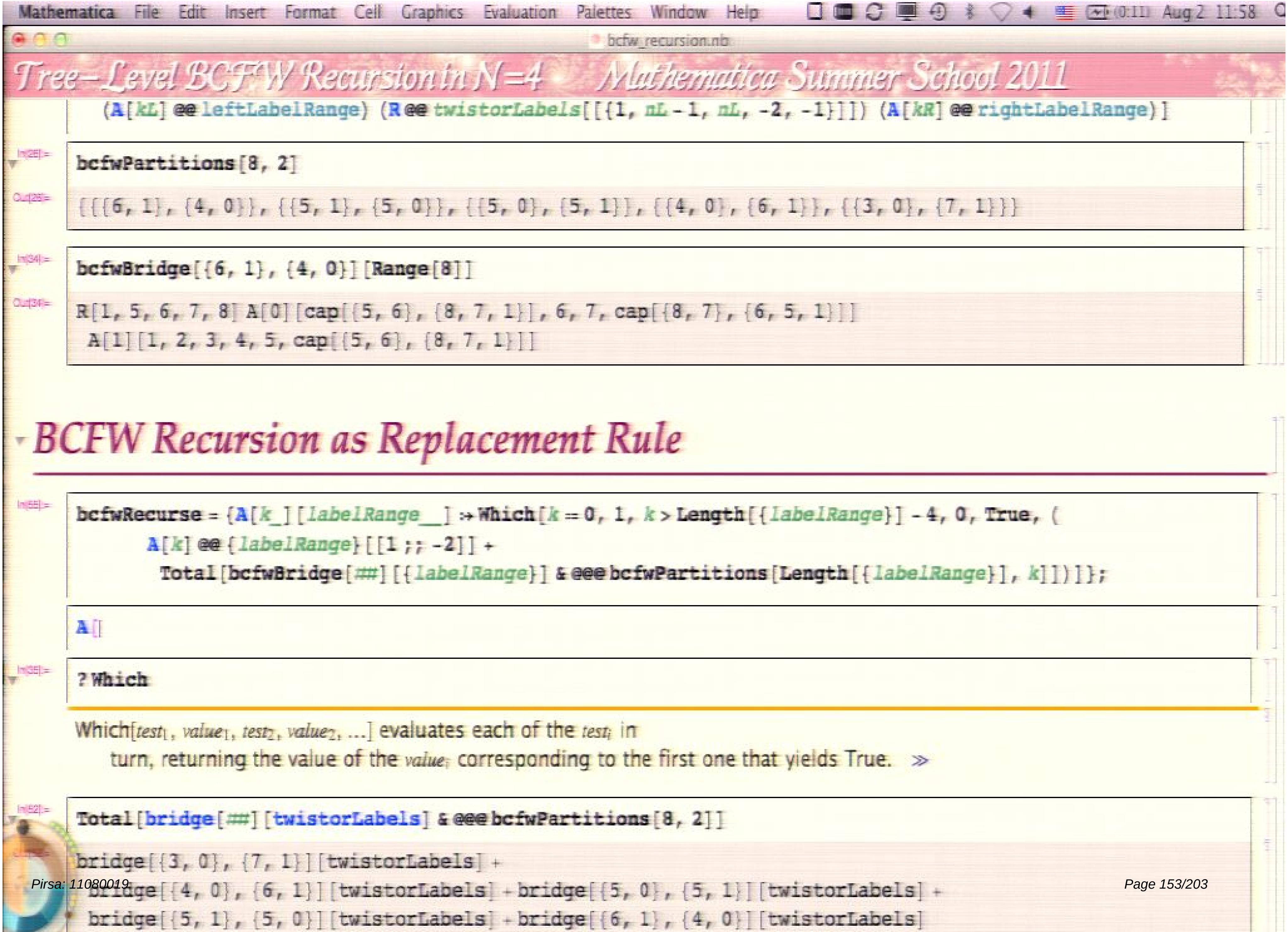
```
bcf|{A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
      A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
```

```
In[35]:= ?Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in
  turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[52]:= Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[52]= bridge[{3, 0}, {7, 1}][twistorLabels] +
          bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
          bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

In[26]:=
```
bcfwPartitions[8, 2]
```

Out[26]=
```
{{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

In[34]:=
```
bcfwBridge[{6, 1}, {4, 0}][Range[8]]
```

Out[34]=
```
R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
 A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

In[55]:=
```
bcfwRecurse = {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
      A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
```

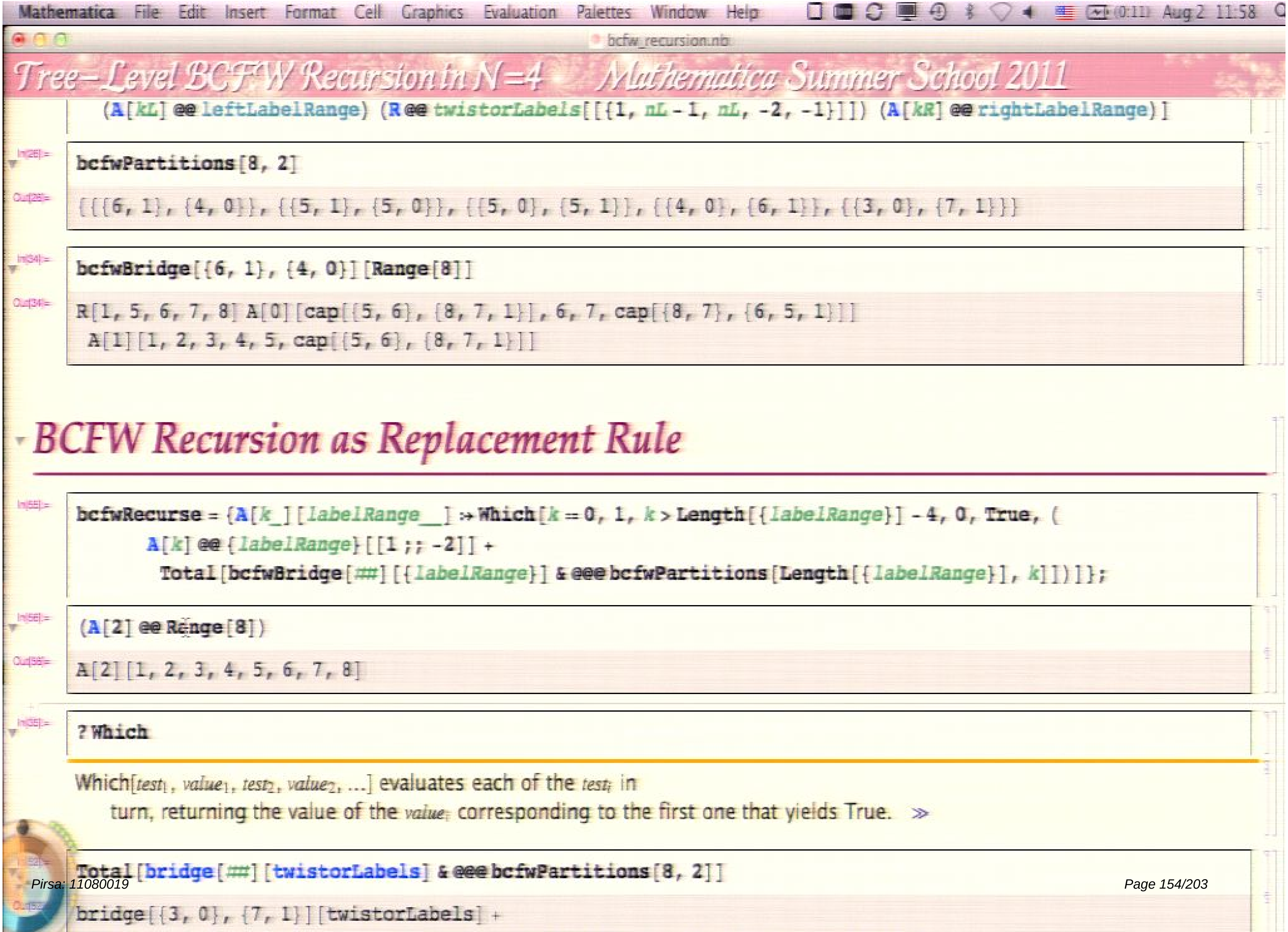
```
A[]
```

In[35]:=
```
? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. ≫

In[52]:=
```
Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]
```

```
bridge[{3, 0}, {7, 1}][twistorLabels] +
 bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
 bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

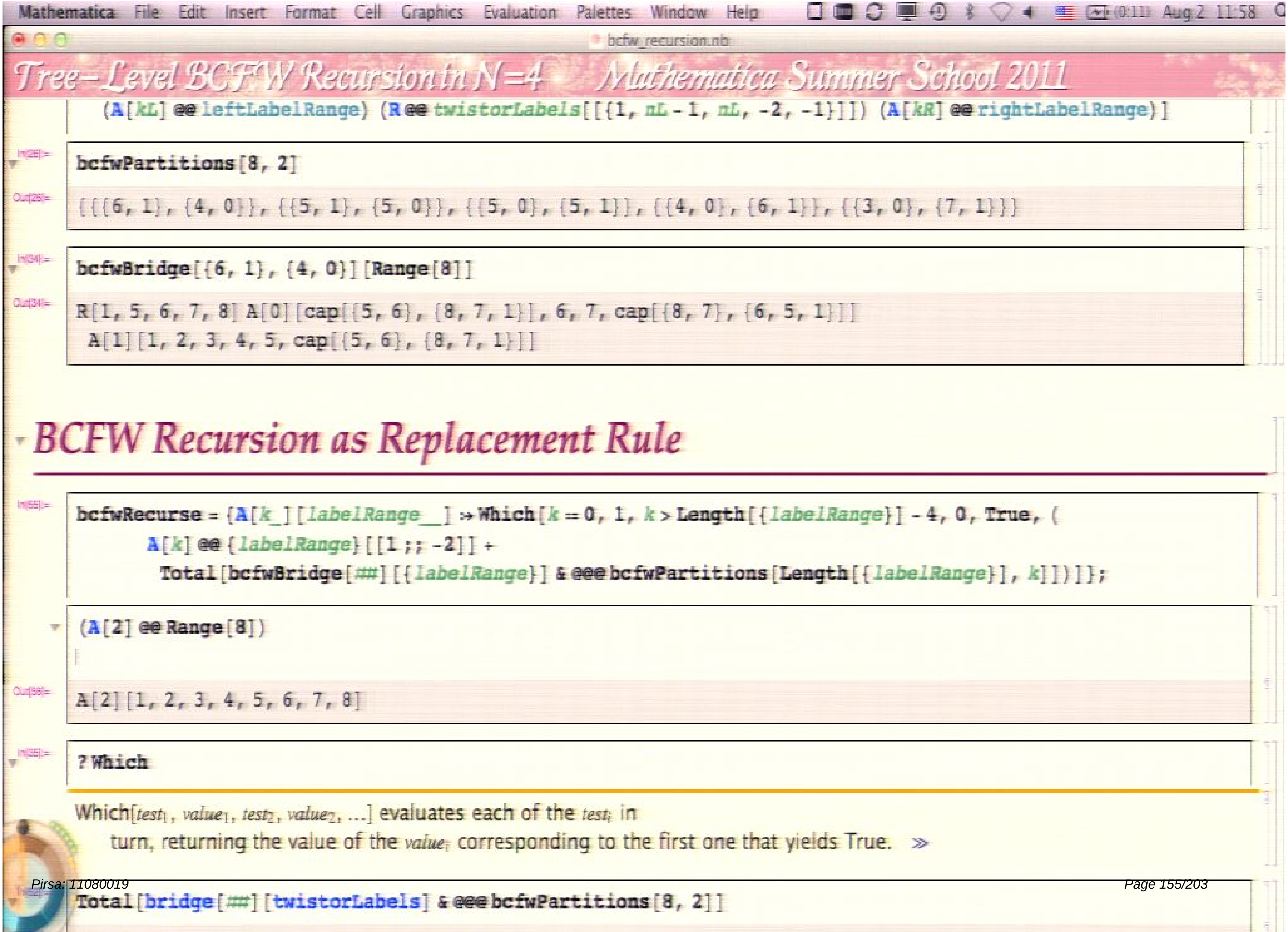
```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
                A[k] @@ {labelRange}[[1 ;; -2]] +
                 Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[56]:= (A[2] @@ Range[8])

Out[56]= A[2][1, 2, 3, 4, 5, 6, 7, 8]

In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, ...] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

bridge[{3, 0}, {7, 1}][twistorLabels] +
```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(0:11) Aug 2 11:58
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

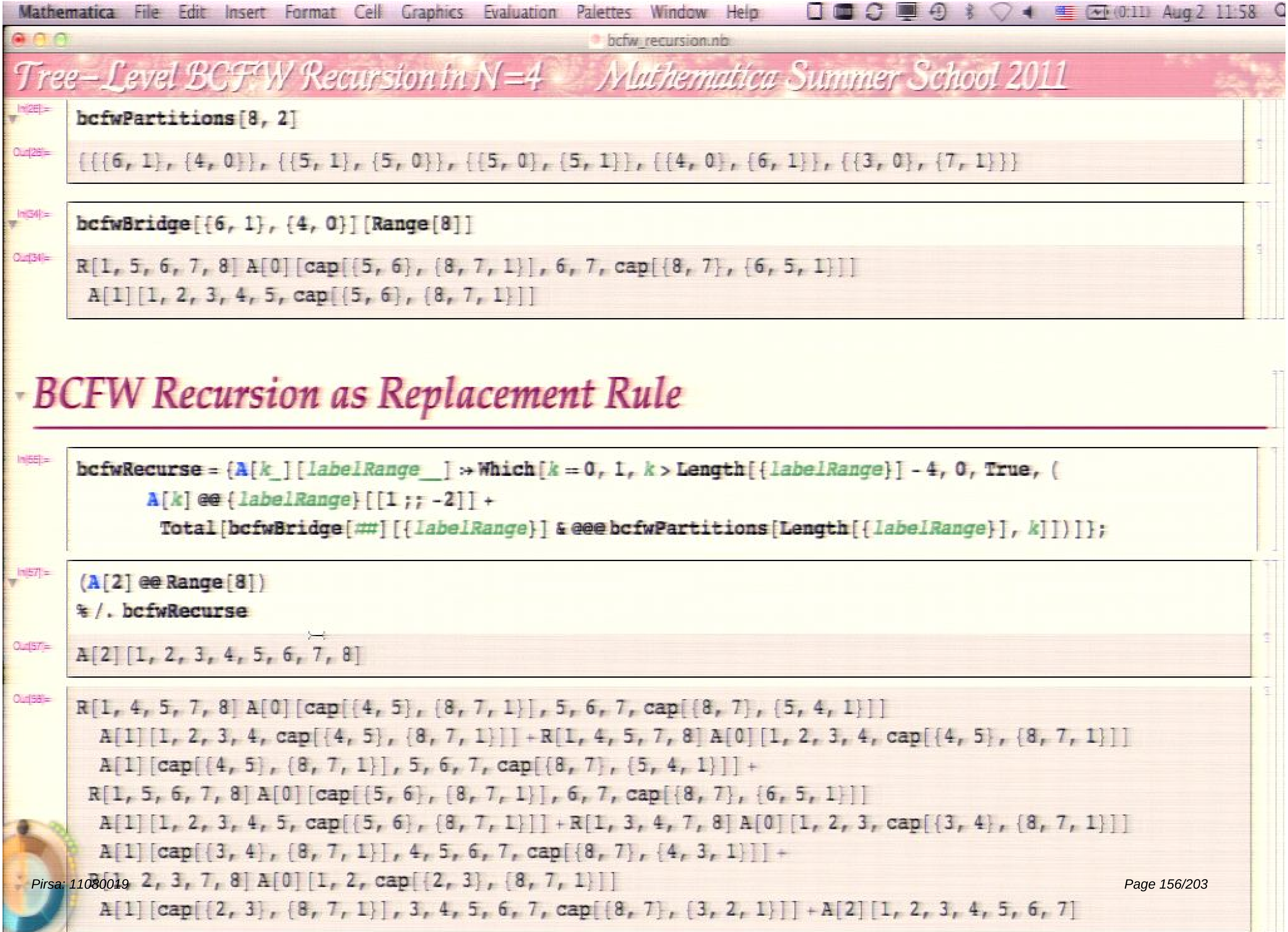
```
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]

In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] → Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
              A[k] @@ {labelRange}[[1 ;; -2]] +
               Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

         (A[2] @@ Range[8])

Out[56]= A[2][1, 2, 3, 4, 5, 6, 7, 8]

In[35]:= ? Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]
```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(0:11) Aug 2 11:58
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

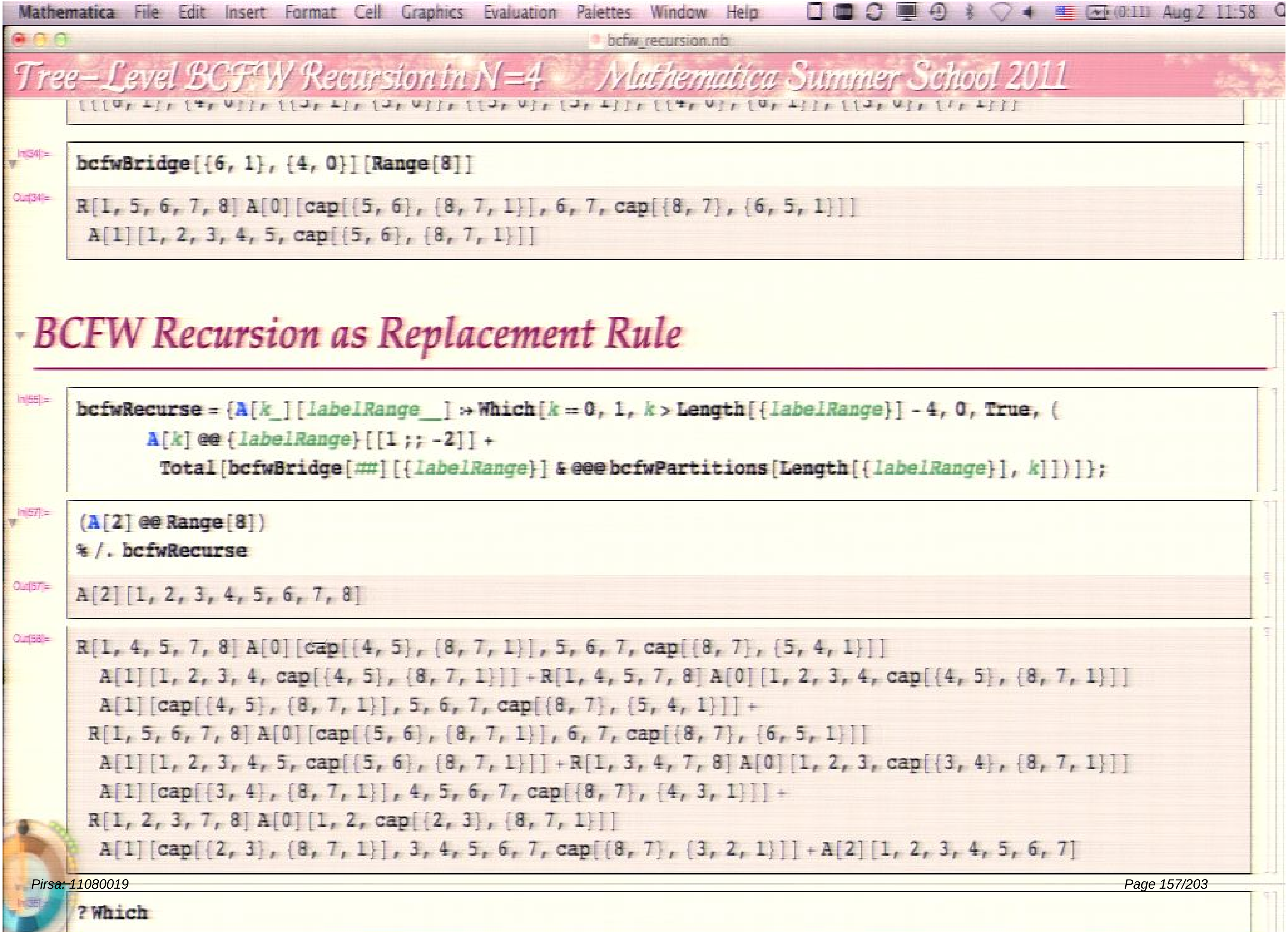
## BCFW Recursion as Replacement Rule

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
               A[k] @@ {labelRange}[[1 ;; -2]] +
                Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
```

```
In[57]:= (A[2] @@ Range[8])
         % /. bcfwRecurse

Out[57]= A[2][1, 2, 3, 4, 5, 6, 7, 8]

Out[58]= R[1, 4, 5, 7, 8] A[0][cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]]
           A[1][1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] + R[1, 4, 5, 7, 8] A[0][1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]]
           A[1][cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]] +
          R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
           A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] + R[1, 3, 4, 7, 8] A[0][1, 2, 3, cap[{3, 4}, {8, 7, 1}]]
           A[1][cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7, cap[{8, 7}, {4, 3, 1}]] +
          R[1, 2, 3, 7, 8] A[0][1, 2, cap[{2, 3}, {8, 7, 1}]]
           A[1][cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6, 7, cap[{8, 7}, {3, 2, 1}]] + A[2][1, 2, 3, 4, 5, 6, 7]
```



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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[54]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[54]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

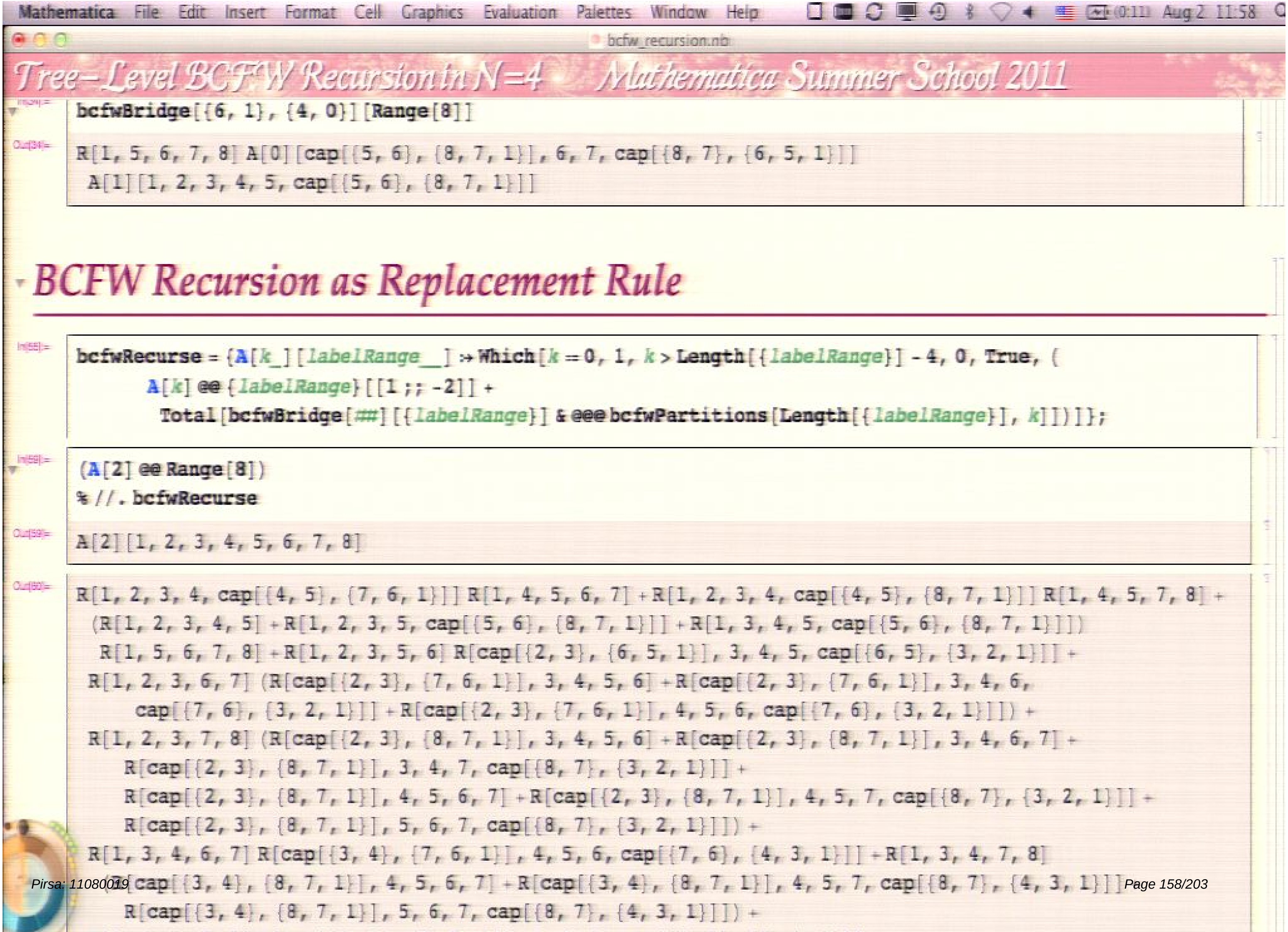
```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
               A[k] @@ {labelRange}[[1 ;; -2]] +
                Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[57]:= (A[2] @@ Range[8])
         % /. bcfwRecurse

Out[57]= A[2][1, 2, 3, 4, 5, 6, 7, 8]

Out[58]= R[1, 4, 5, 7, 8] A[0][cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]]
           A[1][1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] + R[1, 4, 5, 7, 8] A[0][1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]]
           A[1][cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]] +
          R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
           A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] + R[1, 3, 4, 7, 8] A[0][1, 2, 3, cap[{3, 4}, {8, 7, 1}]]
           A[1][cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7, cap[{8, 7}, {4, 3, 1}]] +
          R[1, 2, 3, 7, 8] A[0][1, 2, cap[{2, 3}, {8, 7, 1}]]
           A[1][cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6, 7, cap[{8, 7}, {3, 2, 1}]] + A[2][1, 2, 3, 4, 5, 6, 7]
```

```
?Which
```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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

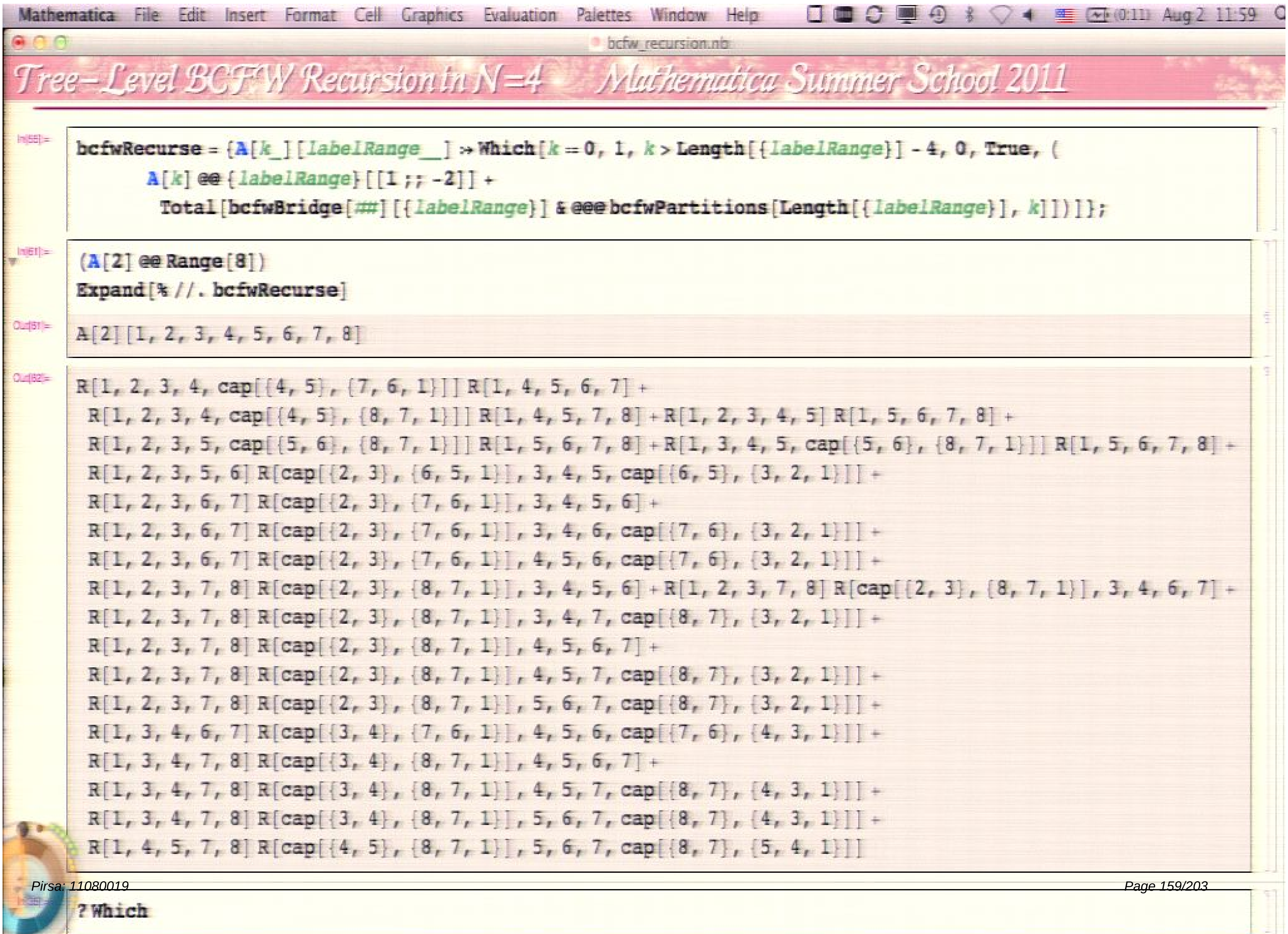
## BCFW Recursion as Replacement Rule

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] ⧴ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
               A[k] @@ {labelRange}[[1 ;; -2]] +
                Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[58]:= (A[2] @@ Range[8])
         % //. bcfwRecurse

Out[59]= A[2][1, 2, 3, 4, 5, 6, 7, 8]

Out[60]= R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7] + R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8] +
          (R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] + R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]])
           R[1, 5, 6, 7, 8] + R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]] +
          R[1, 2, 3, 6, 7] (R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6] + R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6,
              cap[{7, 6}, {3, 2, 1}]] + R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]]) +
          R[1, 2, 3, 7, 8] (R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6] + R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7] +
             R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]] +
             R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7] + R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]] +
             R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]]) +
          R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]] + R[1, 3, 4, 7, 8]
           (R[cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7] + R[cap[{3, 4}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {4, 3, 1}]]
             R[cap[{3, 4}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {4, 3, 1}]]) +
```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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

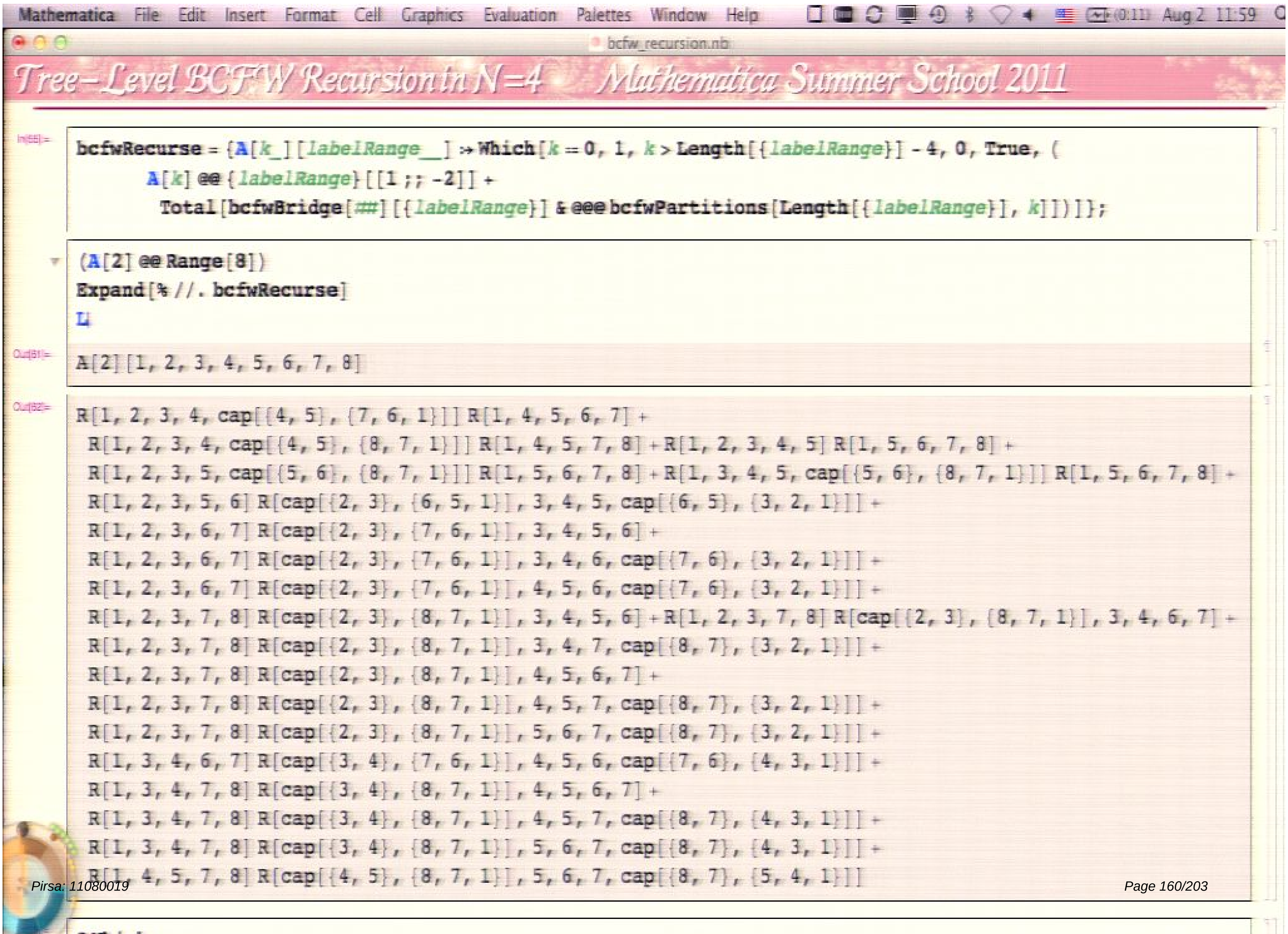
```
In[55]:= bcfwRecurse = {A[k_][labelRange__] ⧴ Which[k ⩵ 0, 1, k > Length[{labelRange}] - 4, 0, True, (
               A[k] @@ {labelRange}[[1 ;; -2]] +
                Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[61]:= (A[2] @@ Range[8])
         Expand[% //. bcfwRecurse]

Out[61]= A[2][1, 2, 3, 4, 5, 6, 7, 8]

Out[62]= R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7] +
          R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8] + R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8] +
          R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8] + R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8] +
          R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]] +
          R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6] +
          R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]] +
          R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6] + R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]] +
          R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]] +
          R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7] +
          R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {4, 3, 1}]] +
          R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {4, 3, 1}]] +
          R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]]

?Which
```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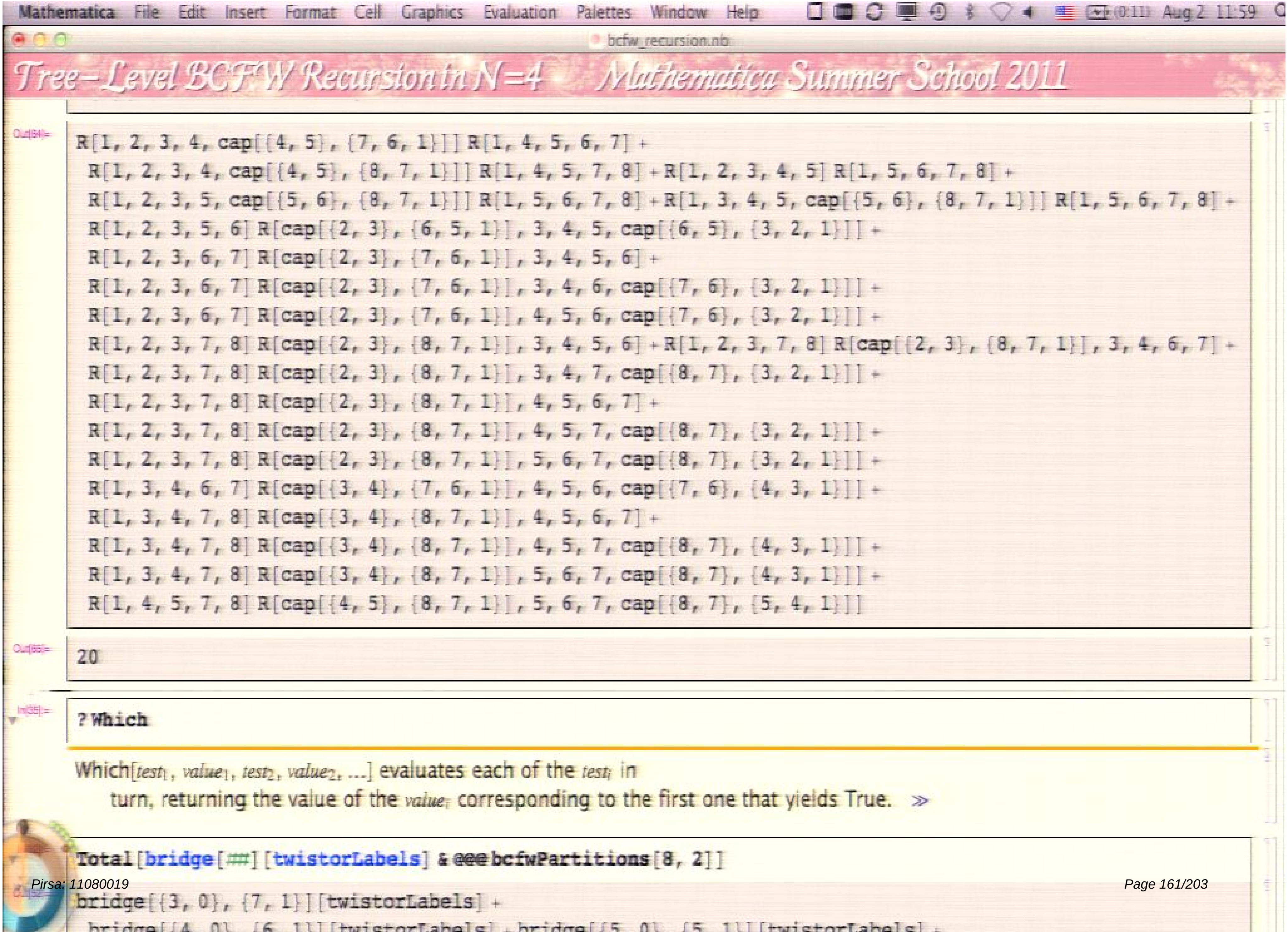
Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[55]:=
bcfwRecurse = {A[k_][labelRange__] ⧴ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
      A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
```

```
(A[2] @@ Range[8])
Expand[% //. bcfwRecurse]
```

```
Out[61]=
A[2][1, 2, 3, 4, 5, 6, 7, 8]
```

```
Out[62]=
R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7] +
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8] + R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8] +
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8] + R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8] +
 R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]] +
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6] +
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]] +
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6] + R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]] +
 R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]] +
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7] +
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {4, 3, 1}]] +
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {4, 3, 1}]] +
 R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]]
```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bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011
Out[64]=
Out[65]=
In[35]=

```
R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7] +
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8] + R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8] +
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8] + R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8] +
 R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]] +
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6] +
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]] +
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6] + R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]] +
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]] +
 R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]] +
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7] +
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {4, 3, 1}]] +
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {4, 3, 1}]] +
 R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]]
```

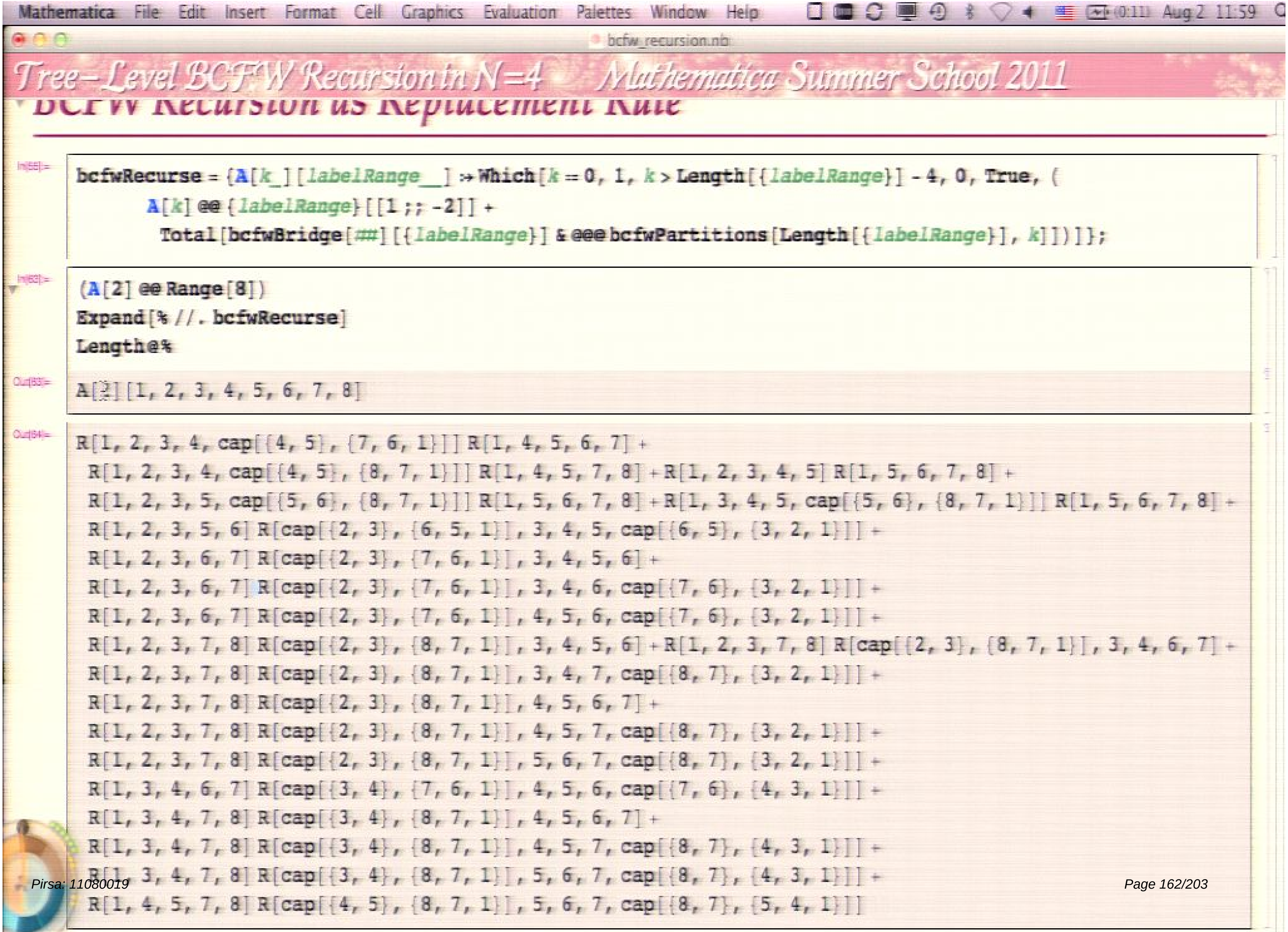
```
20
```

```
? Which
```

Which[*test*$_1$, *value*$_1$, *test*$_2$, *value*$_2$, ...] evaluates each of the *test*$_i$ in turn, returning the value of the *value*$_i$ corresponding to the first one that yields True. »

```
Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]
```

```
bridge[{3, 0}, {7, 1}][twistorLabels] +
 bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
```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(0:11) Aug 2 11:59
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

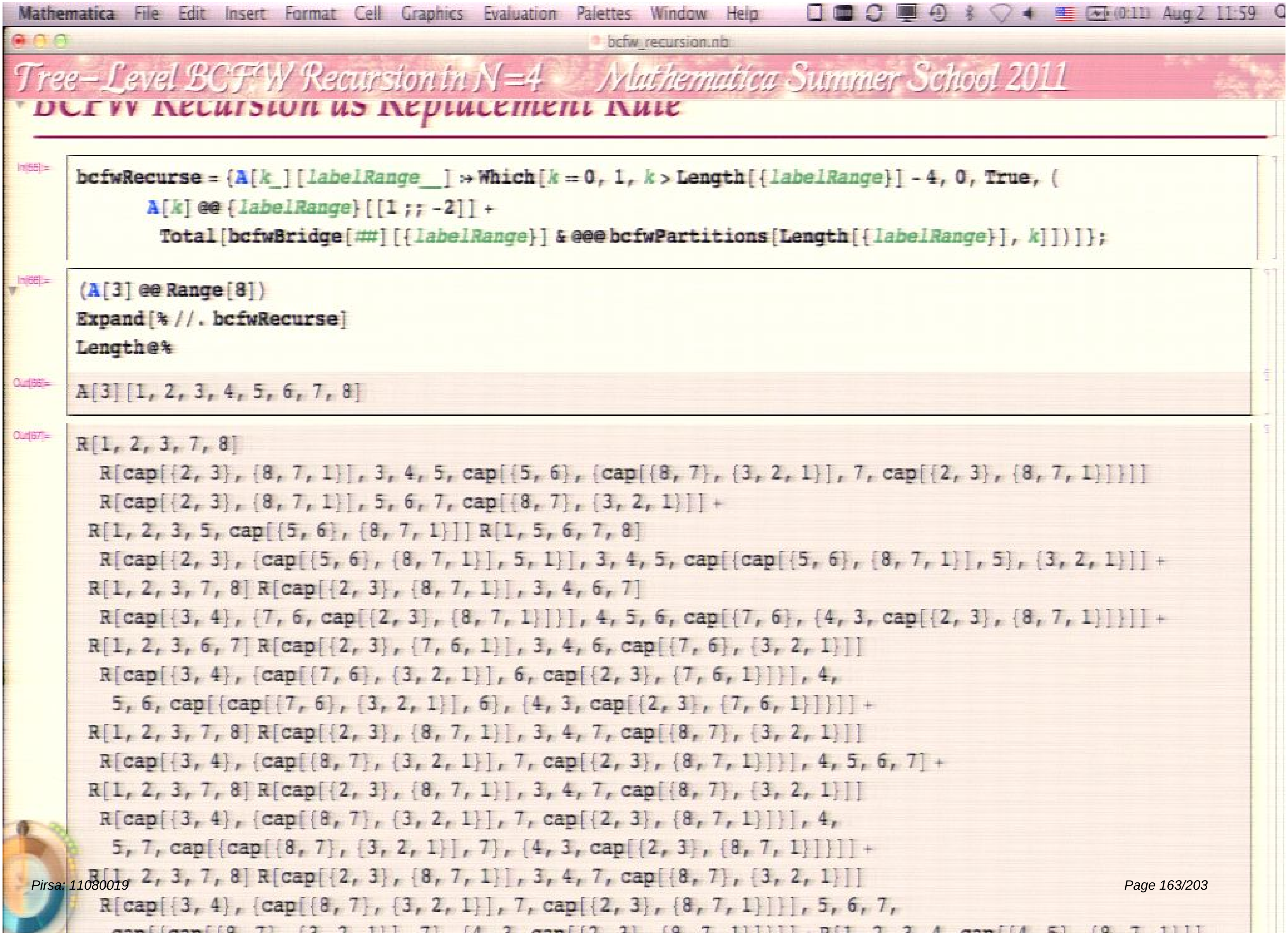
## BCFW Recursion as Replacement Rule

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
              A[k] @@ {labelRange}[[1 ;; -2]] +
               Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[63]:= (A[2] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[63]= A[2][1, 2, 3, 4, 5, 6, 7, 8]

Out[64]= R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7] +
          R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8] + R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8] +
          R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8] + R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8] +
          R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]] +
          R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6] +
          R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]] +
          R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6] + R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]] +
          R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]] +
          R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7] +
          R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {4, 3, 1}]] +
          R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {4, 3, 1}]] +
          R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]]
```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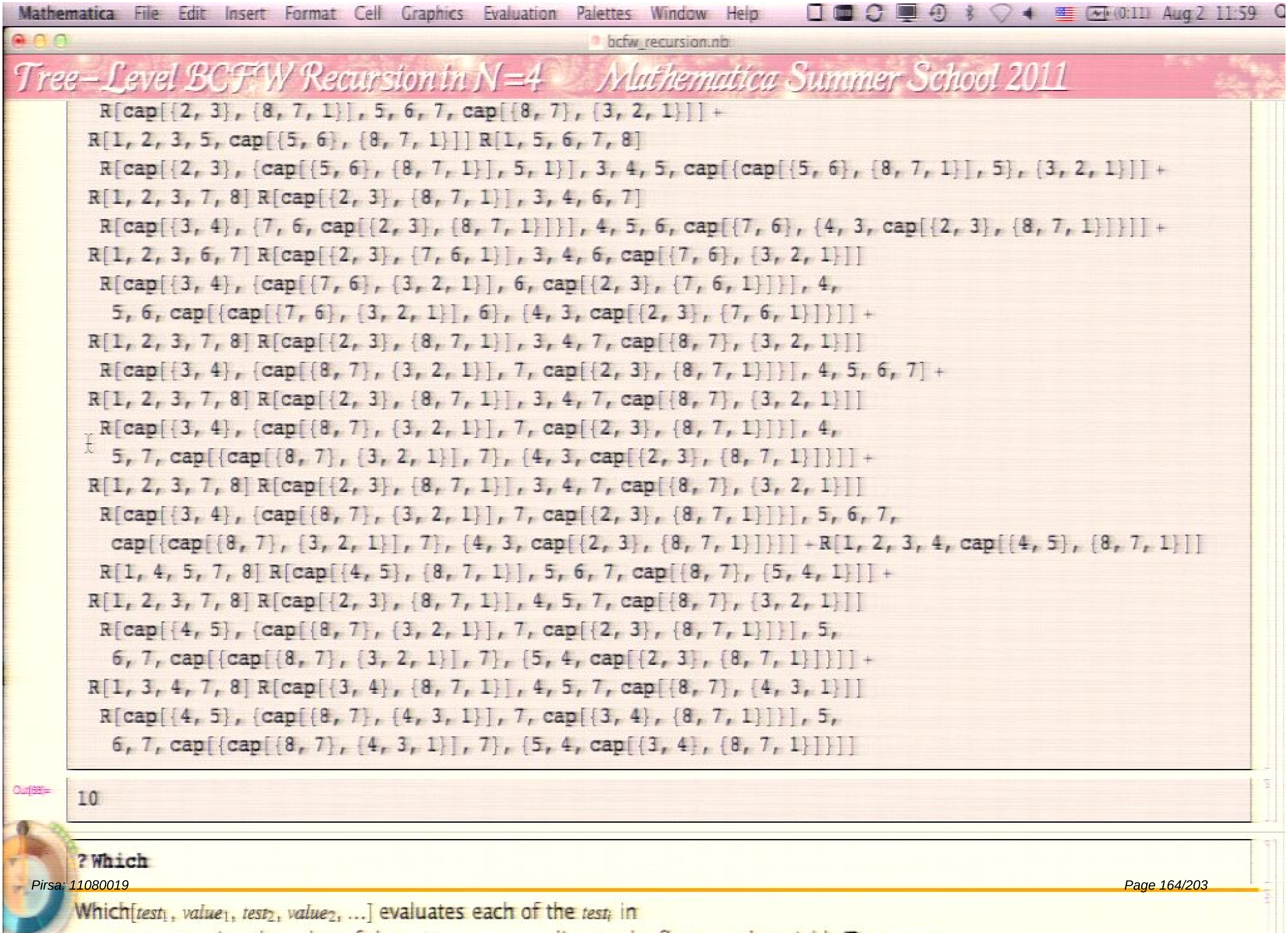
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

## BCFW Recursion as Replacement Rule

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] ⧴ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
             A[k] @@ {labelRange}[[1 ;; -2]] +
              Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[66]:= (A[3] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[66]= A[3][1, 2, 3, 4, 5, 6, 7, 8]

Out[67]= R[1, 2, 3, 7, 8]
           R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, cap[{5, 6}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}]]
           R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]] +
          R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8]
           R[cap[{2, 3}, {cap[{5, 6}, {8, 7, 1}], 5, 1}], 3, 4, 5, cap[{cap[{5, 6}, {8, 7, 1}], 5}, {3, 2, 1}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7]
           R[cap[{3, 4}, {7, 6, cap[{2, 3}, {8, 7, 1}]}], 4, 5, 6, cap[{7, 6}, {4, 3, cap[{2, 3}, {8, 7, 1}]}]] +
          R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]]
           R[cap[{3, 4}, {cap[{7, 6}, {3, 2, 1}], 6, cap[{2, 3}, {7, 6, 1}]}], 4,
            5, 6, cap[{cap[{7, 6}, {3, 2, 1}], 6}, {4, 3, cap[{2, 3}, {7, 6, 1}]}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
           R[cap[{3, 4}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 4, 5, 6, 7] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
           R[cap[{3, 4}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 4,
            5, 7, cap[{cap[{8, 7}, {3, 2, 1}], 7}, {4, 3, cap[{2, 3}, {8, 7, 1}]}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
           R[cap[{3, 4}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 5, 6, 7,
            cap[{cap[{8, 7}, {3, 2, 1}], 7}, {4, 3, cap[{2, 3}, {8, 7, 1}]}]] + R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:11) Aug 2 11:59
bcfw_recursion.nb

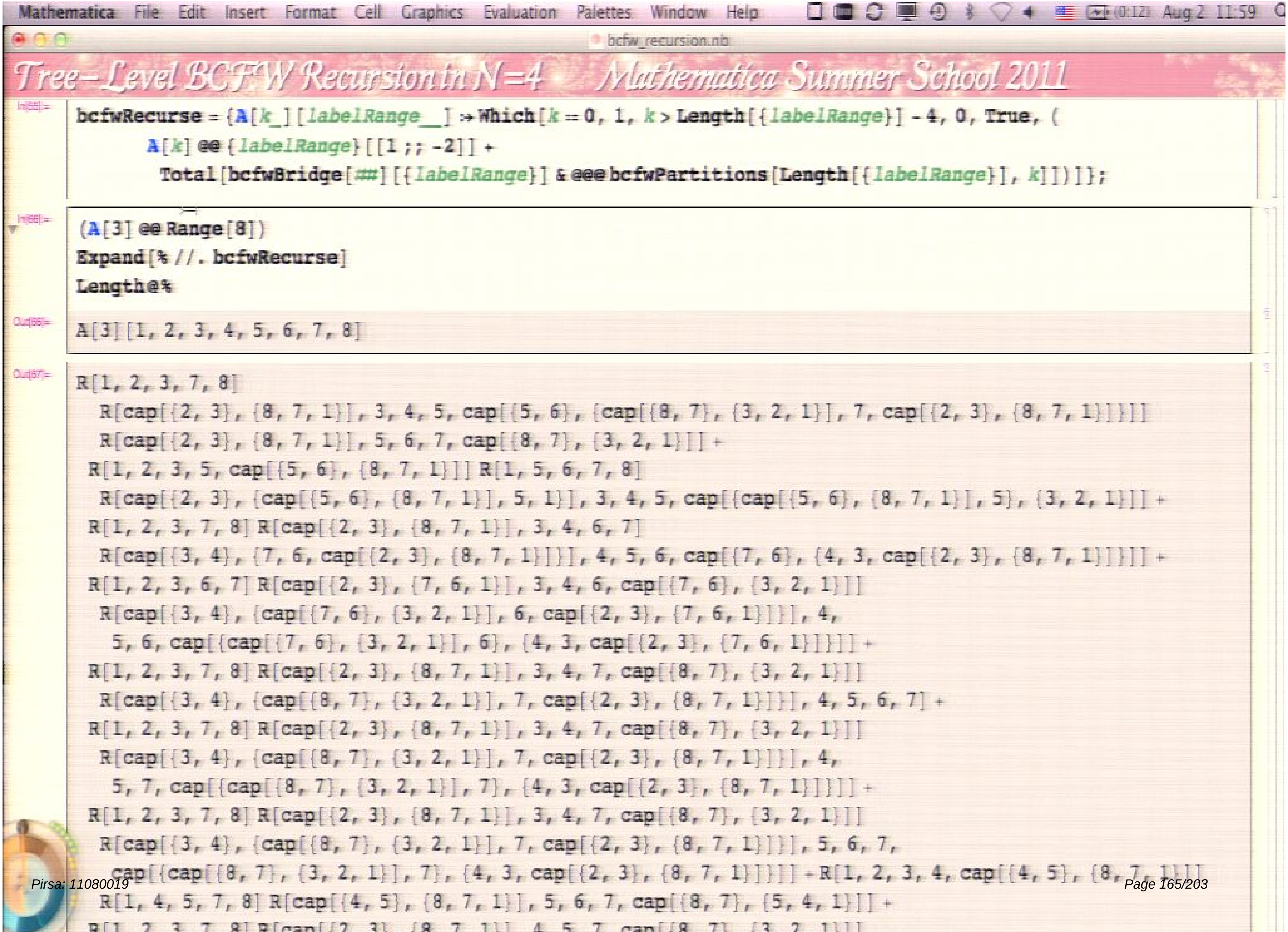
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
 R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]] +
R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8]
 R[cap[{2, 3}, {cap[{5, 6}, {8, 7, 1}], 5, 1}], 3, 4, 5, cap[{cap[{5, 6}, {8, 7, 1}], 5}, {3, 2, 1}]] +
R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7]
 R[cap[{3, 4}, {7, 6, cap[{2, 3}, {8, 7, 1}]}], 4, 5, 6, cap[{7, 6}, {4, 3, cap[{2, 3}, {8, 7, 1}]}]] +
R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]]
 R[cap[{3, 4}, {cap[{7, 6}, {3, 2, 1}], 6, cap[{2, 3}, {7, 6, 1}]}], 4,
  5, 6, cap[{cap[{7, 6}, {3, 2, 1}], 6}, {4, 3, cap[{2, 3}, {7, 6, 1}]}]] +
R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
 R[cap[{3, 4}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 4, 5, 6, 7] +
R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
 R[cap[{3, 4}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 4,
  5, 7, cap[{cap[{8, 7}, {3, 2, 1}], 7}, {4, 3, cap[{2, 3}, {8, 7, 1}]}]] +
R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
 R[cap[{3, 4}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 5, 6, 7,
  cap[{cap[{8, 7}, {3, 2, 1}], 7}, {4, 3, cap[{2, 3}, {8, 7, 1}]}]] + R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]]
 R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]] +
R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]
 R[cap[{4, 5}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 5,
  6, 7, cap[{cap[{8, 7}, {3, 2, 1}], 7}, {5, 4, cap[{2, 3}, {8, 7, 1}]}]] +
R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {4, 3, 1}]]
 R[cap[{4, 5}, {cap[{8, 7}, {4, 3, 1}], 7, cap[{3, 4}, {8, 7, 1}]}], 5,
  6, 7, cap[{cap[{8, 7}, {4, 3, 1}], 7}, {5, 4, cap[{3, 4}, {8, 7, 1}]}]]
```

```
Out[88]= 10
```

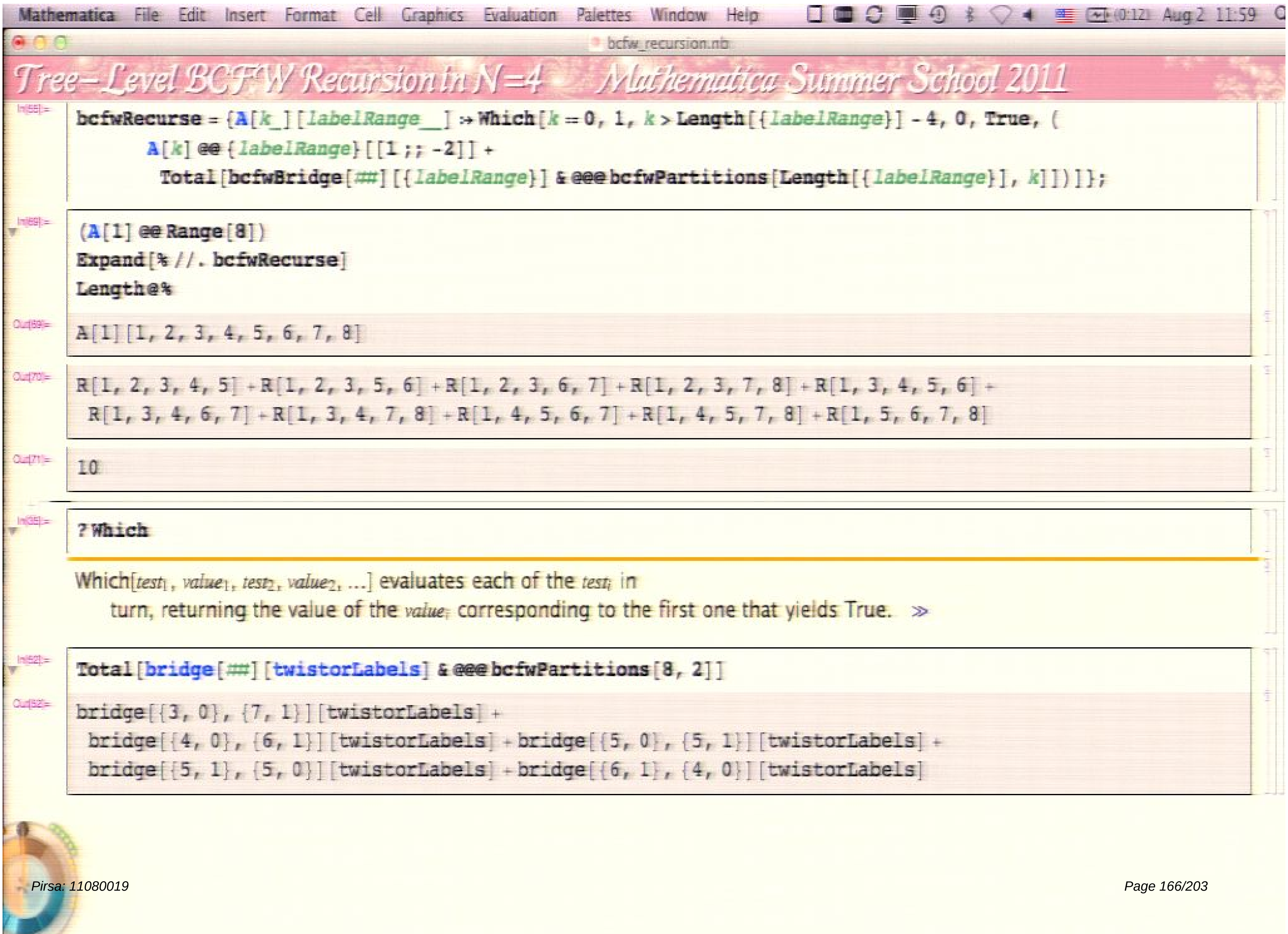
```
?Which
```

Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:12) Aug 2 11:59
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[65]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
             A[k] @@ {labelRange}[[1 ;; -2]] +
              Total[bcfwBridge[##][[{labelRange}]] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[66]:= (A[3] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[66]= A[3][1, 2, 3, 4, 5, 6, 7, 8]

Out[67]= R[1, 2, 3, 7, 8]
           R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, cap[{5, 6}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}]]
           R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]] +
          R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8]
           R[cap[{2, 3}, {cap[{5, 6}, {8, 7, 1}], 5, 1}], 3, 4, 5, cap[{cap[{5, 6}, {8, 7, 1}], 5}, {3, 2, 1}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7]
           R[cap[{3, 4}, {7, 6, cap[{2, 3}, {8, 7, 1}]}], 4, 5, 6, cap[{7, 6}, {4, 3, cap[{2, 3}, {8, 7, 1}]}]] +
          R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]]
           R[cap[{3, 4}, {cap[{7, 6}, {3, 2, 1}], 6, cap[{2, 3}, {7, 6, 1}]}], 4,
            5, 6, cap[{cap[{7, 6}, {3, 2, 1}], 6}, {4, 3, cap[{2, 3}, {7, 6, 1}]}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
           R[cap[{3, 4}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 4, 5, 6, 7] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
           R[cap[{3, 4}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 4,
            5, 7, cap[{cap[{8, 7}, {3, 2, 1}], 7}, {4, 3, cap[{2, 3}, {8, 7, 1}]}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
           R[cap[{3, 4}, {cap[{8, 7}, {3, 2, 1}], 7, cap[{2, 3}, {8, 7, 1}]}], 5, 6, 7,
            cap[{cap[{8, 7}, {3, 2, 1}], 7}, {4, 3, cap[{2, 3}, {8, 7, 1}]}]] + R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]]
           R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]] +
          R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]
```

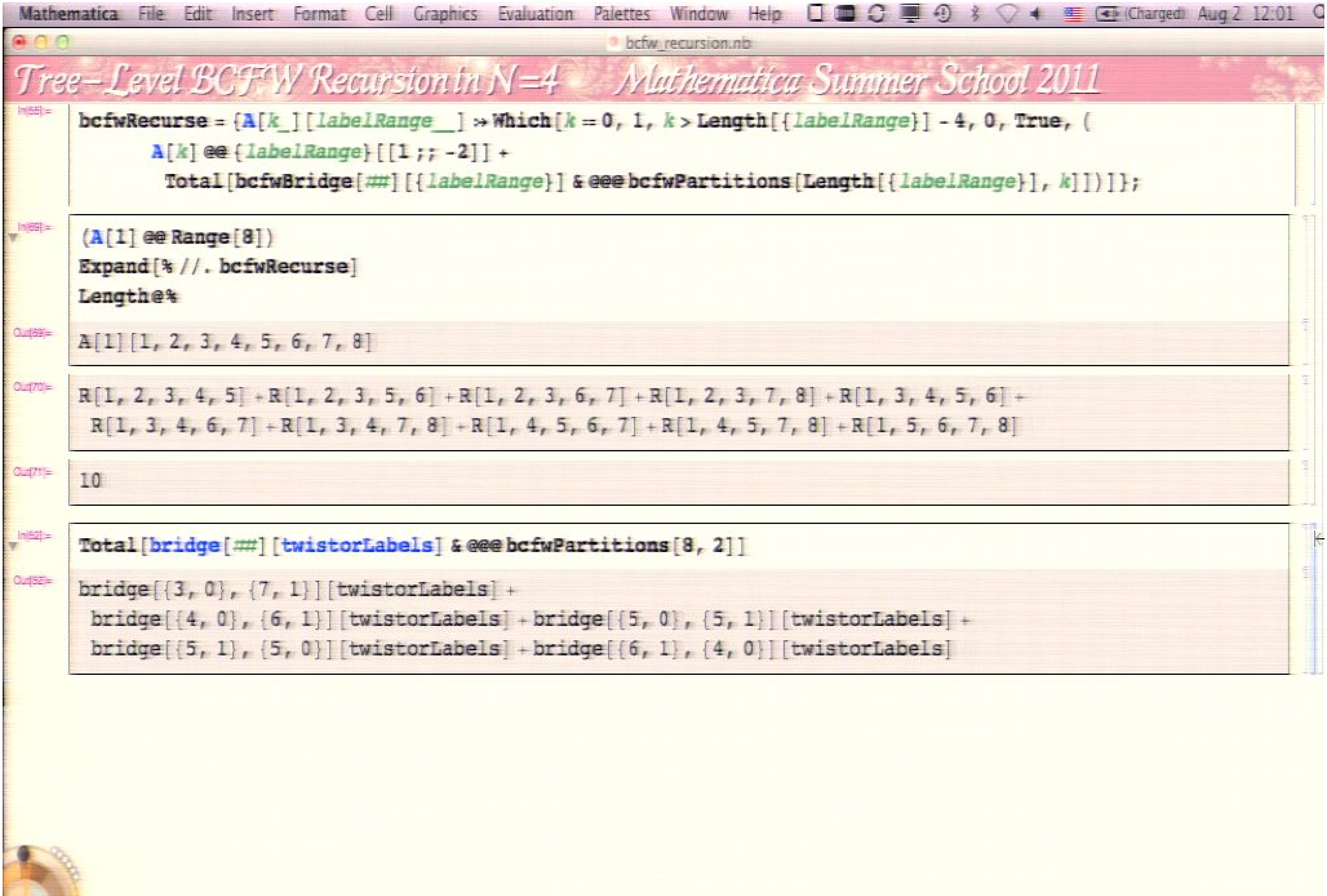

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(0:12) Aug 2 11:59
bcfw_recursion.nb

# Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] ⧴ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
             A[k] @@ {labelRange}[[1 ;; -2]] +
              Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[35]:= ? Which
```

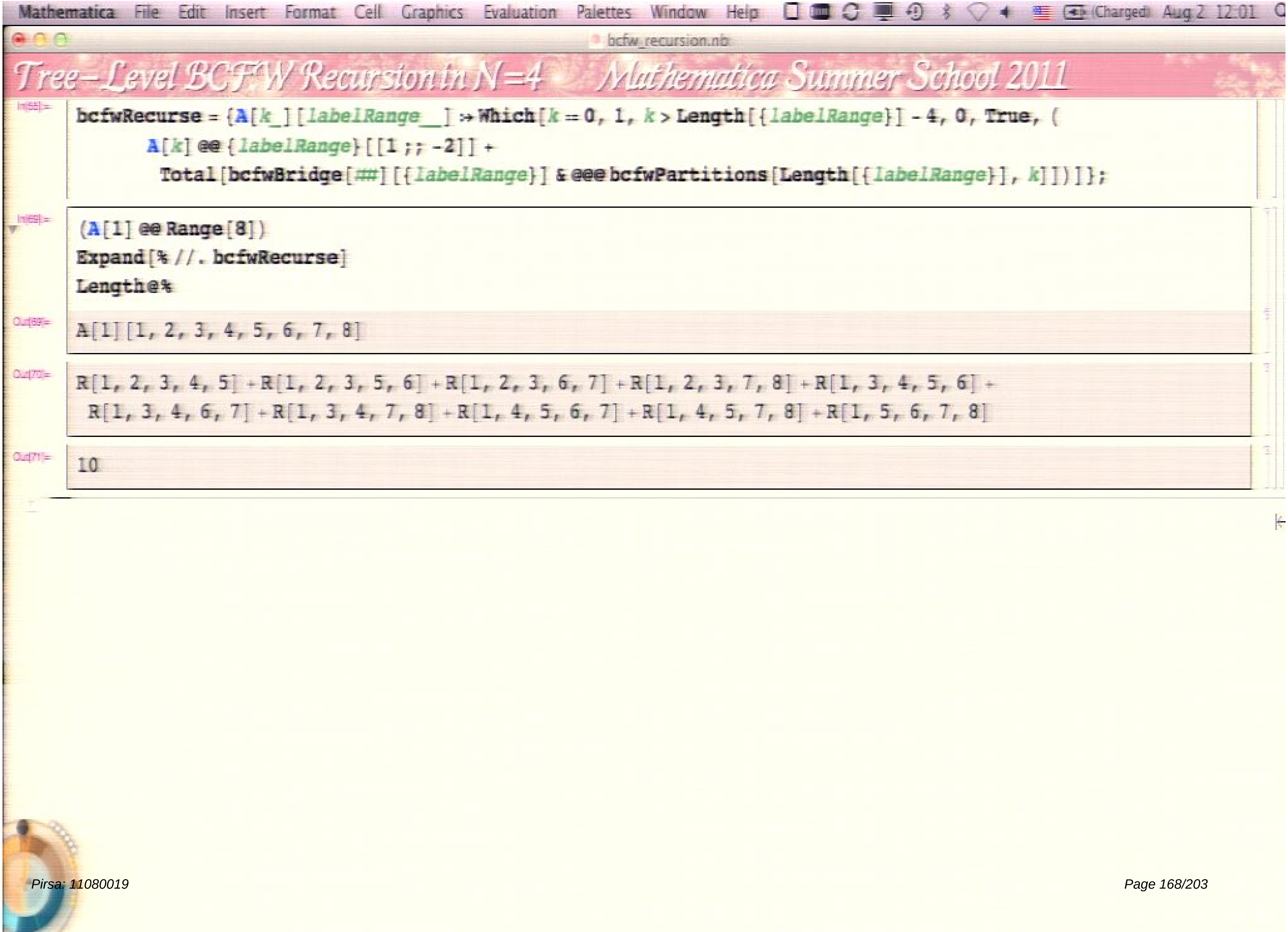
Which[$test_1$, $value_1$, $test_2$, $value_2$, …] evaluates each of the $test_i$ in turn, returning the value of the $value_i$ corresponding to the first one that yields True. »

```
In[52]:= Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[52]= bridge[{3, 0}, {7, 1}][twistorLabels] +
          bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
          bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(Charged) Aug 2 12:01
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
       A[k] @@ {labelRange}[[1 ;; -2]] +
        Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[52]:= Total[bridge[##][twistorLabels] & @@@ bcfwPartitions[8, 2]]

Out[52]= bridge[{3, 0}, {7, 1}][twistorLabels] +
          bridge[{4, 0}, {6, 1}][twistorLabels] + bridge[{5, 0}, {5, 1}][twistorLabels] +
          bridge[{5, 1}, {5, 0}][twistorLabels] + bridge[{6, 1}, {4, 0}][twistorLabels]
```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(Charged) Aug 2 12:01
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

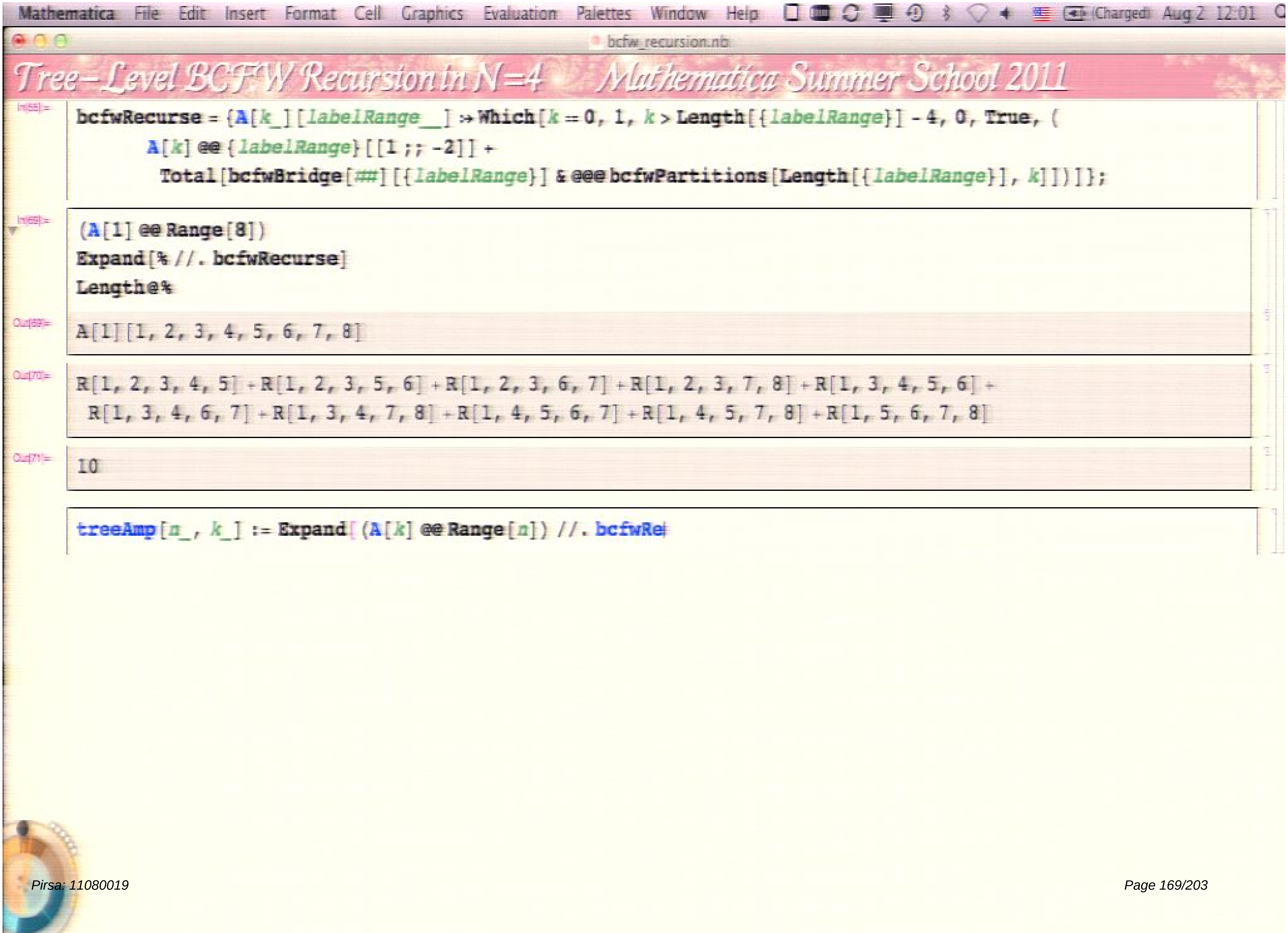
```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
              A[k] @@ {labelRange}[[1 ;; -2]] +
               Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10
```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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

In[55]:=

```
bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
      A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
```

In[69]:=

```
(A[1] @@ Range[8])
Expand[% //. bcfwRecurse]
Length@%
```

Out[69]=

```
A[1][1, 2, 3, 4, 5, 6, 7, 8]
```

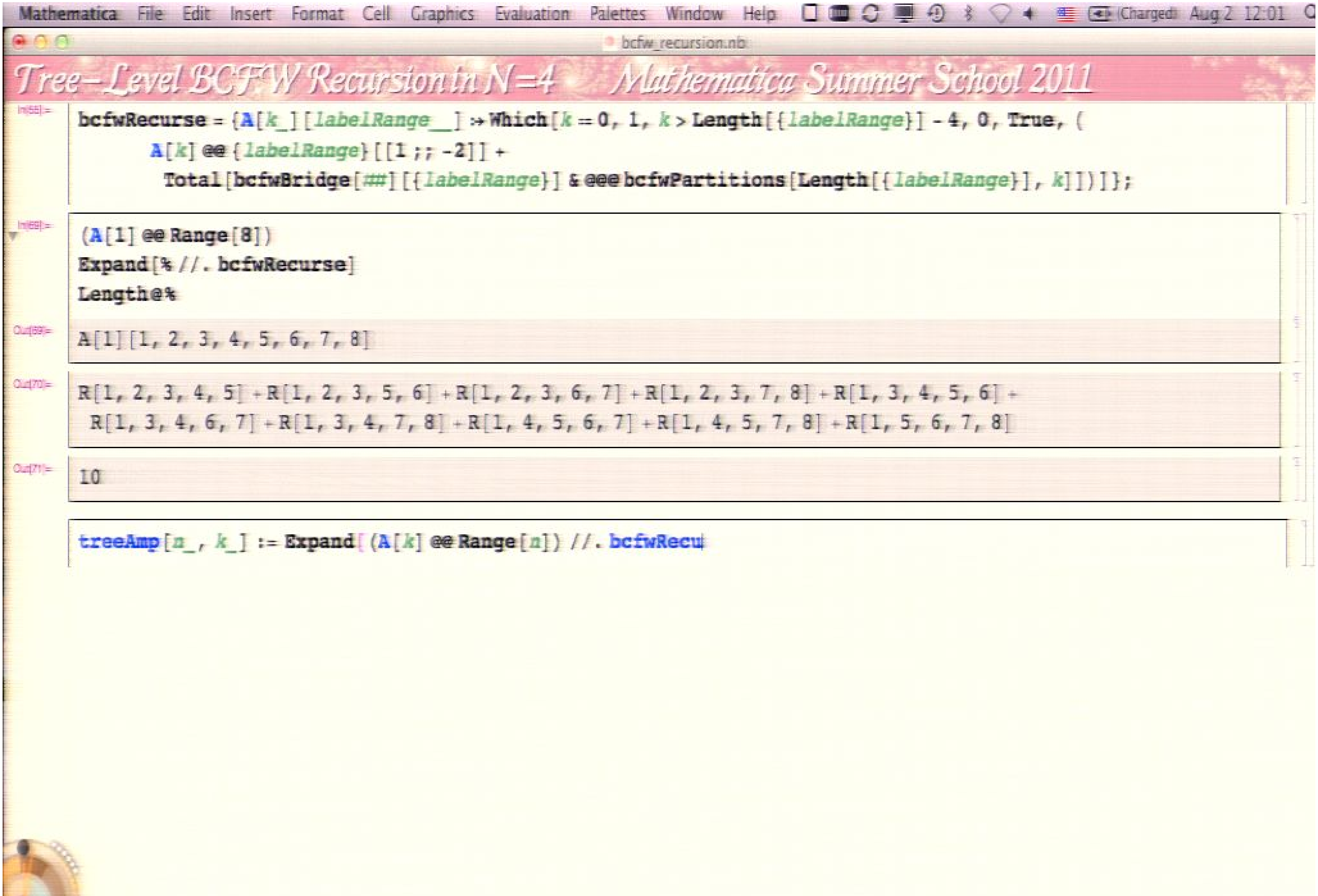
Out[70]=

```
R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
 R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]
```

Out[71]=

```
10
```

```
treeAmp[n_, k_] := Expand[(A[k] @@ Range[n]) //. bcfwRe
```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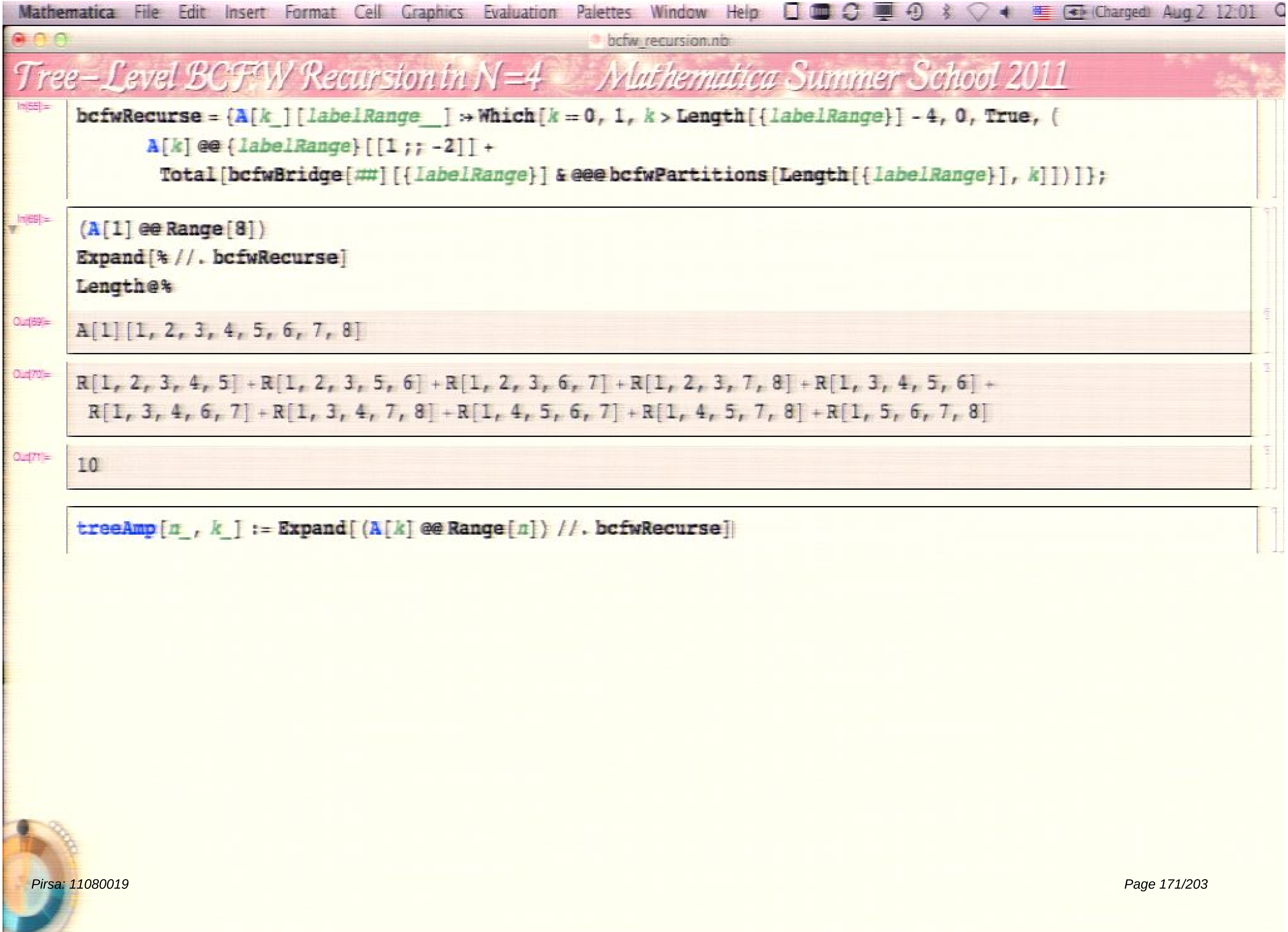
(Charged) Aug 2 12:01
bcfw_recursion.nb

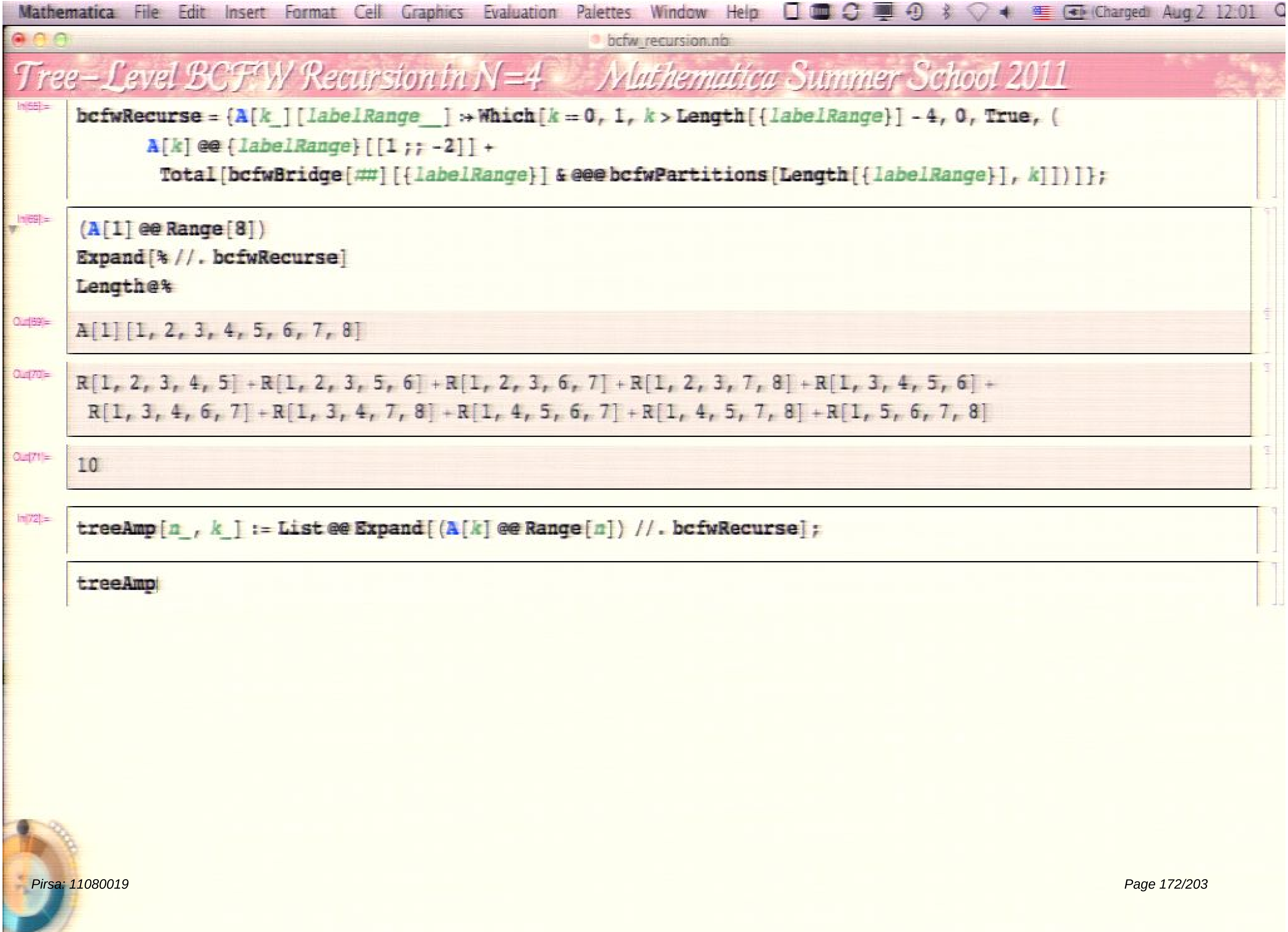
Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
              A[k] @@ {labelRange}[[1 ;; -2]] +
               Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

treeAmp[n_, k_] := Expand[(A[k] @@ Range[n]) //. bcfwRecu
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(Charged) Aug 2 12:01
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] ⧴ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
              A[k] @@ {labelRange}[[1 ;; -2]] +
               Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

         treeAmp[n_, k_] := Expand[(A[k] @@ Range[n]) //. bcfwRecurse]
```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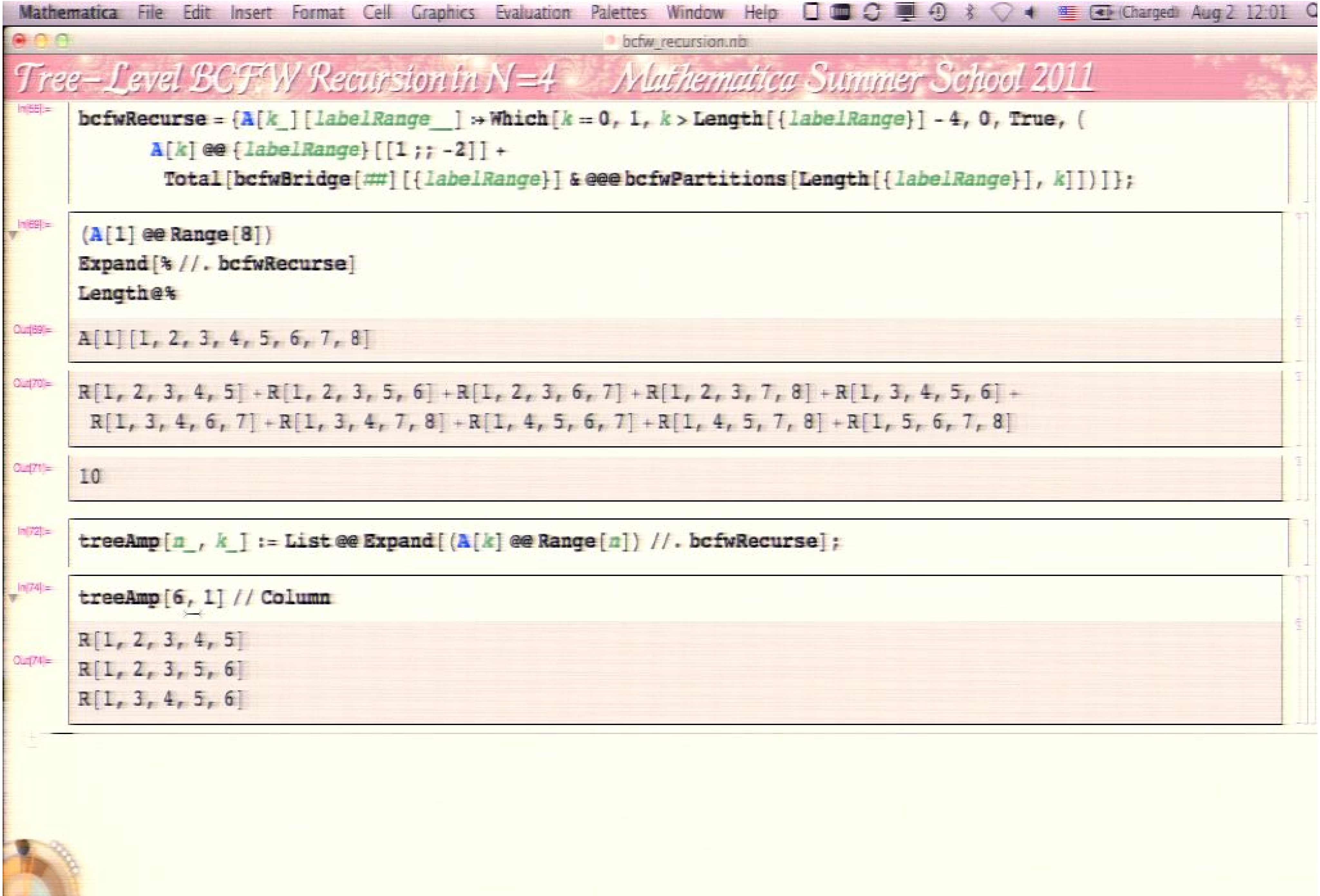
(Charged) Aug 2 12:01
bcfw_recursion.nb

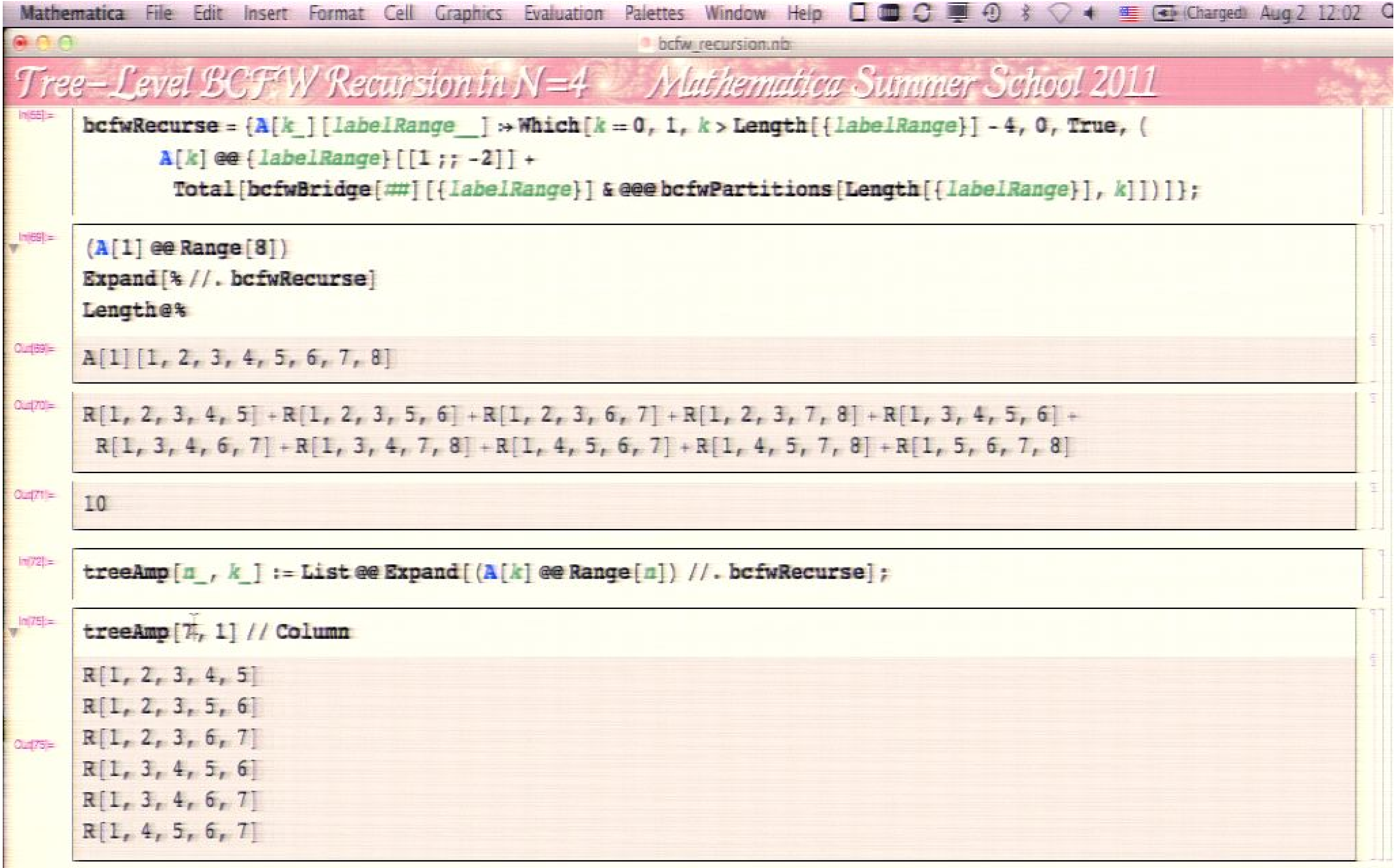
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[65]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
               A[k] @@ {labelRange}[[1 ;; -2]] +
                Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[72]:= treeAmp[n_, k_] := List @@ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

         treeAmp
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(Charged) Aug 2 12:01
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
              A[k] @@ {labelRange}[[1 ;; -2]] +
               Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[72]:= treeAmp[n_, k_] := List @@ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[74]:= treeAmp[6, 1] // Column

Out[74]= R[1, 2, 3, 4, 5]
         R[1, 2, 3, 5, 6]
         R[1, 3, 4, 5, 6]
```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(Charged) Aug 2 12:02
bcfw_recursion.nb

# Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[65]:=
```
bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
      A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
```

In[69]:=
```
(A[1] @@ Range[8])
Expand[% //. bcfwRecurse]
Length@%
```

Out[69]=
```
A[1][1, 2, 3, 4, 5, 6, 7, 8]
```

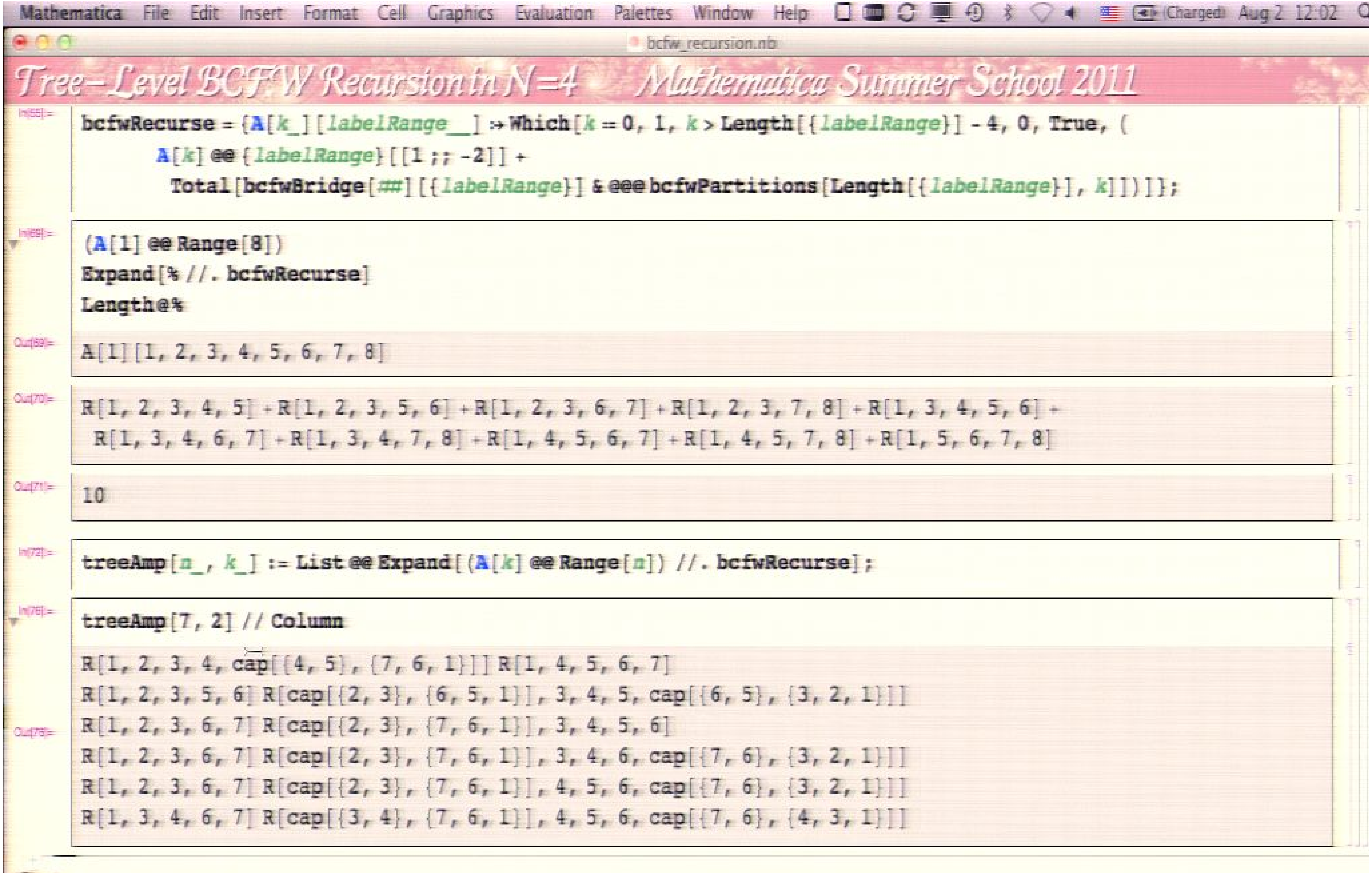
Out[70]=
```
R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
 R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]
```

Out[71]=
```
10
```

In[72]:=
```
treeAmp[n_, k_] := List @@ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
```

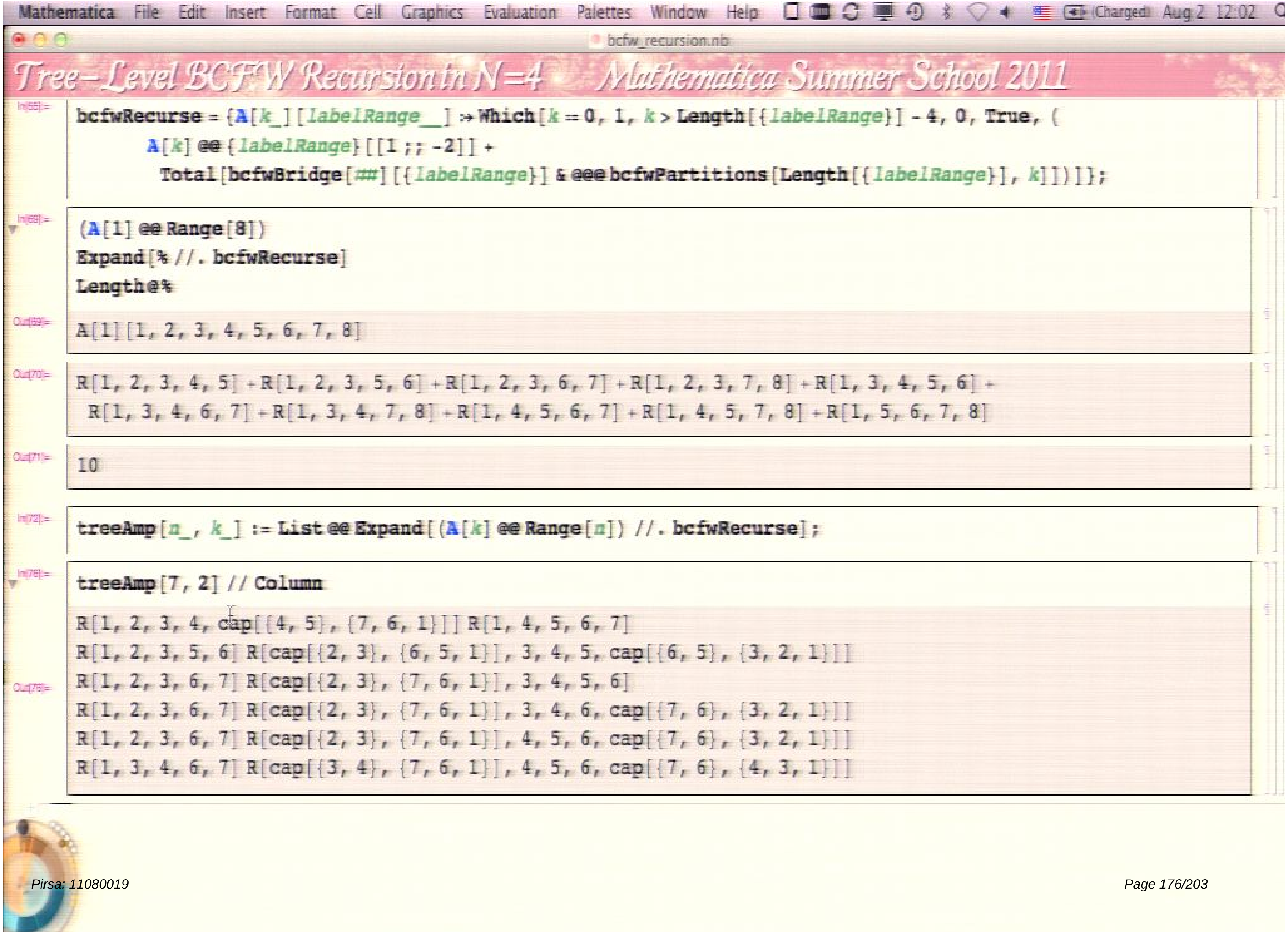
In[75]:=
```
treeAmp[7, 1] // Column
```

Out[75]=
```
R[1, 2, 3, 4, 5]
R[1, 2, 3, 5, 6]
R[1, 2, 3, 6, 7]
R[1, 3, 4, 5, 6]
R[1, 3, 4, 6, 7]
R[1, 4, 5, 6, 7]
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(Charged) Aug 2 12:02
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
              A[k] @@ {labelRange}[[1 ;; -2]] +
               Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[72]:= treeAmp[n_, k_] := List @@ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[76]:= treeAmp[7, 2] // Column

         R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7]
         R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
Out[76]= R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6]
         R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]]
         R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]]
         R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```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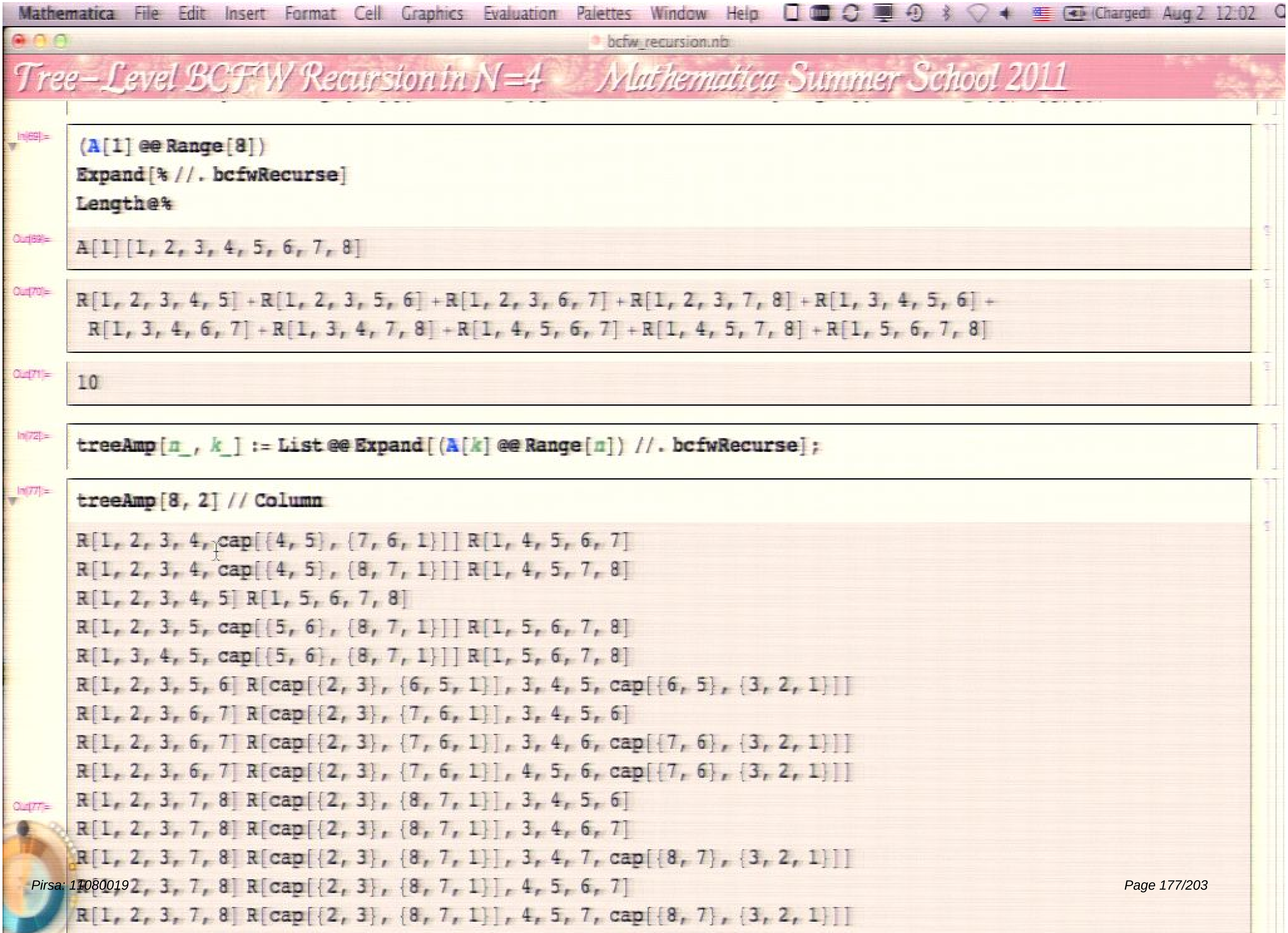
(Charged) Aug 2 12:02
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[55]:=
bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
      A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:=
(A[1] @@ Range[8])
Expand[% //. bcfwRecurse]
Length@%

Out[69]=
A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]=
R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
 R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]=
10

In[72]:=
treeAmp[n_, k_] := List @@ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[76]:=
treeAmp[7, 2] // Column

Out[76]=
R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7]
R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6]
R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]]
R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]]
R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```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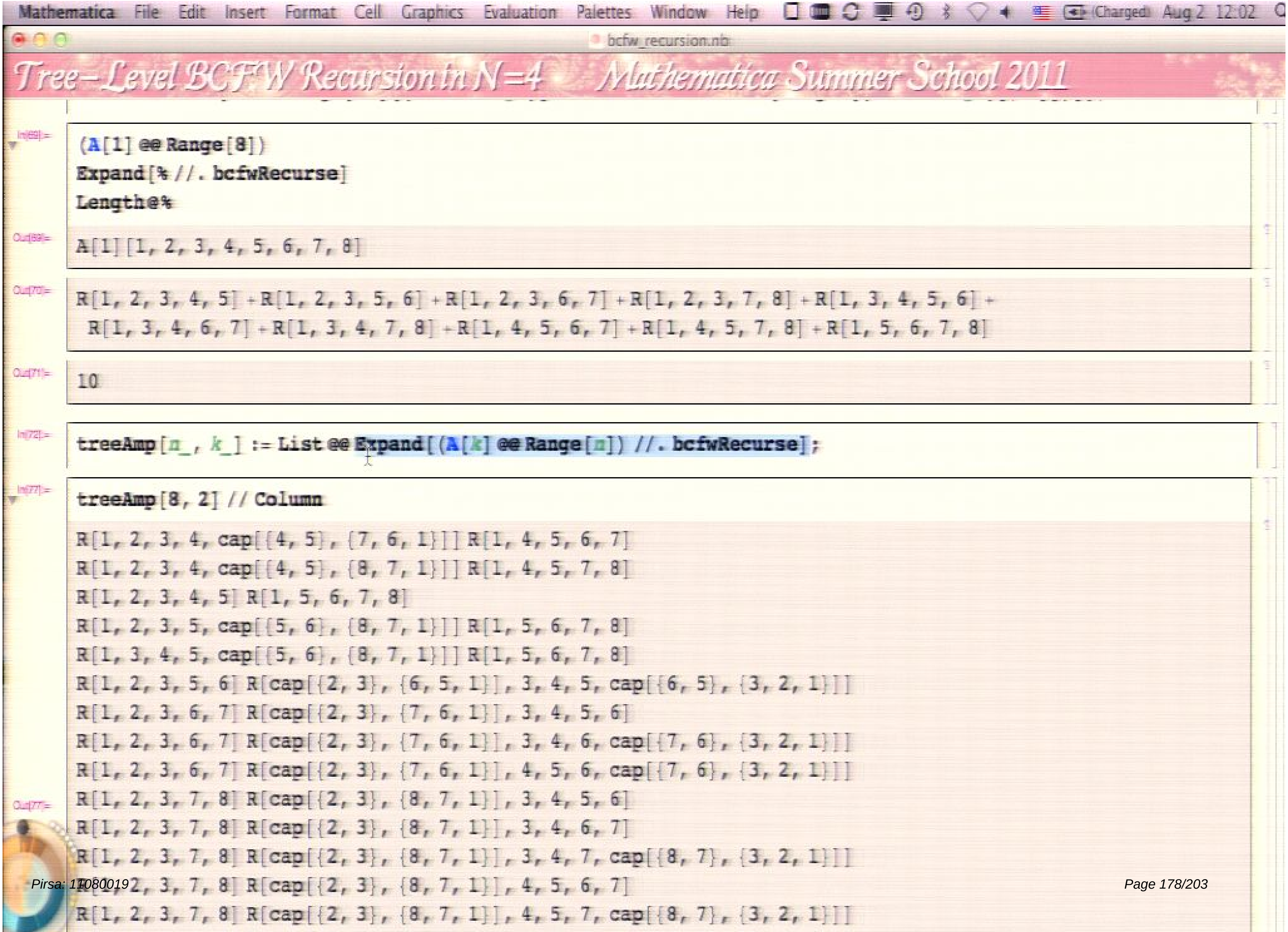
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[72]:= treeAmp[n_, k_] := List @@ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[77]:= treeAmp[8, 2] // Column

Out[77]= R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7]
         R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8]
         R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8]
         R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8]
         R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8]
         R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
         R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6]
         R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]]
         R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]
```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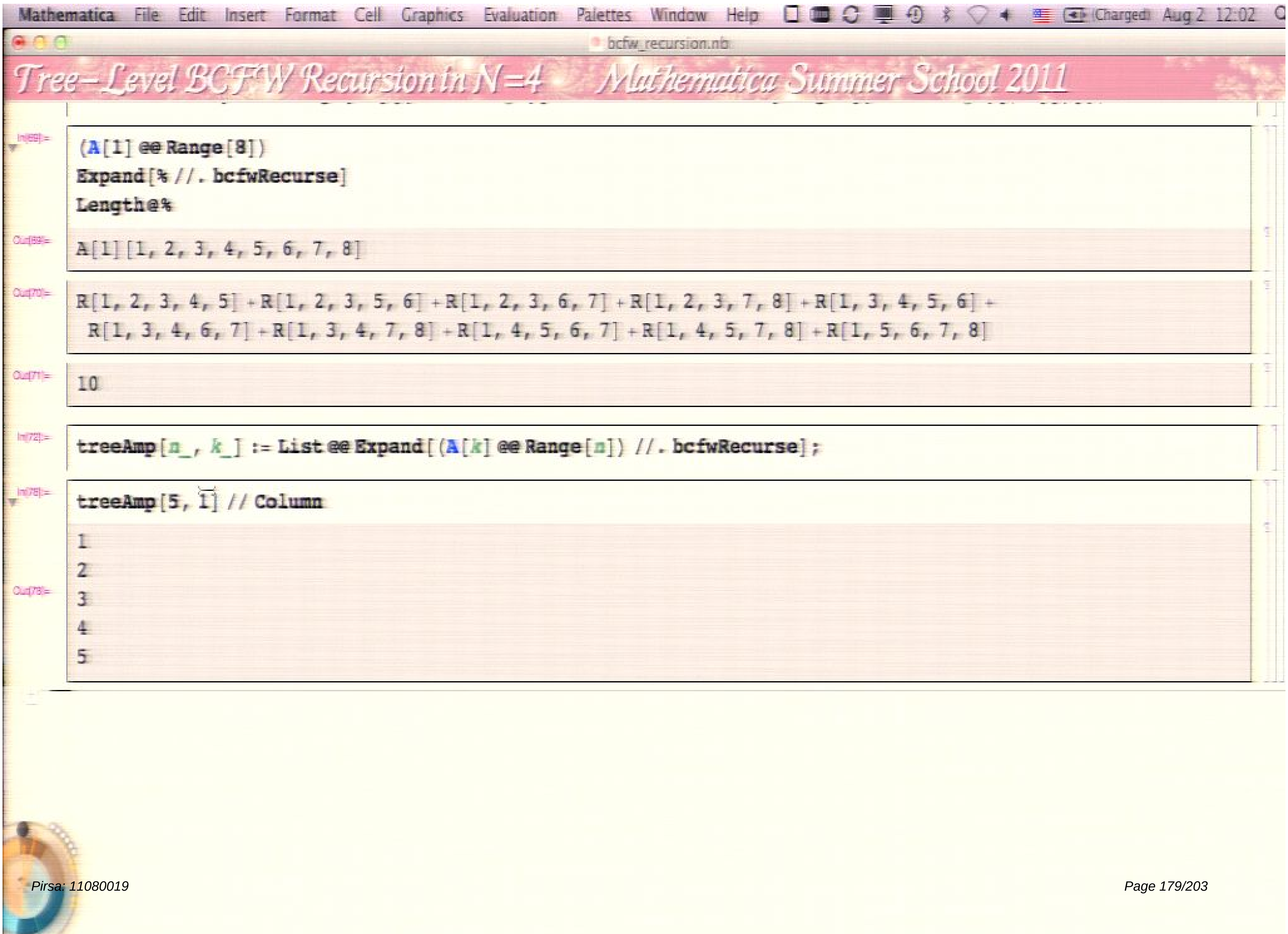
bcfw_recursion.nb
Tree–Level BCFW Recursion in N=4
Mathematica Summer School 2011

```
In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[72]:= treeAmp[n_, k_] := List @@ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[77]:= treeAmp[8, 2] // Column

Out[77]= R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7]
         R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8]
         R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8]
         R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8]
         R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8]
         R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
         R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6]
         R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]]
         R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7]
         R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]
```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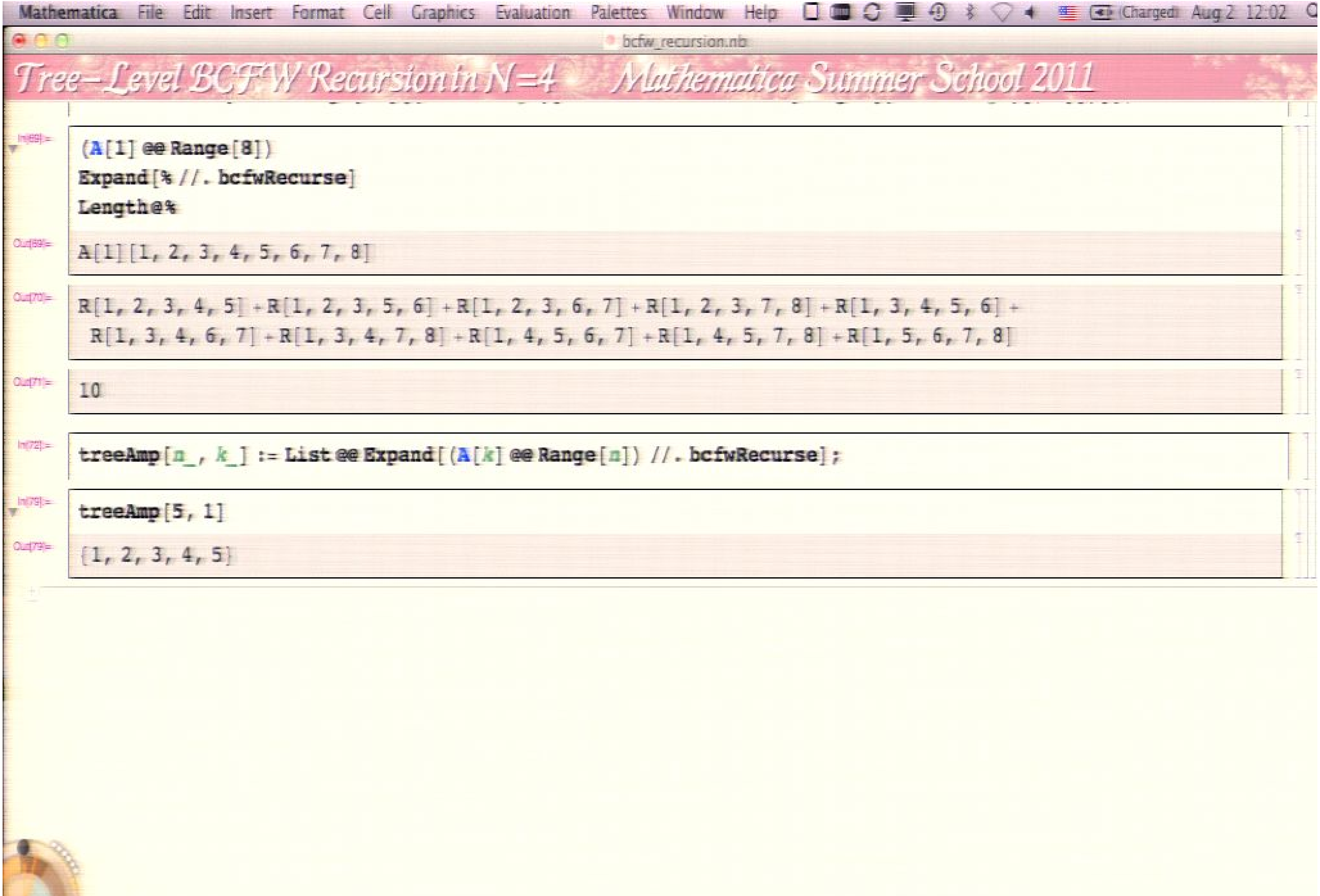
(Charged) Aug 2 12:02
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[69]:=
(A[1] @@ Range[8])
Expand[% //. bcfwRecurse]
Length@%

Out[69]=
A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]=
R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
 R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]=
10

In[72]:=
treeAmp[n_, k_] := List @@ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[78]:=
treeAmp[5, 1] // Column

Out[78]=
1
2
3
4
5
```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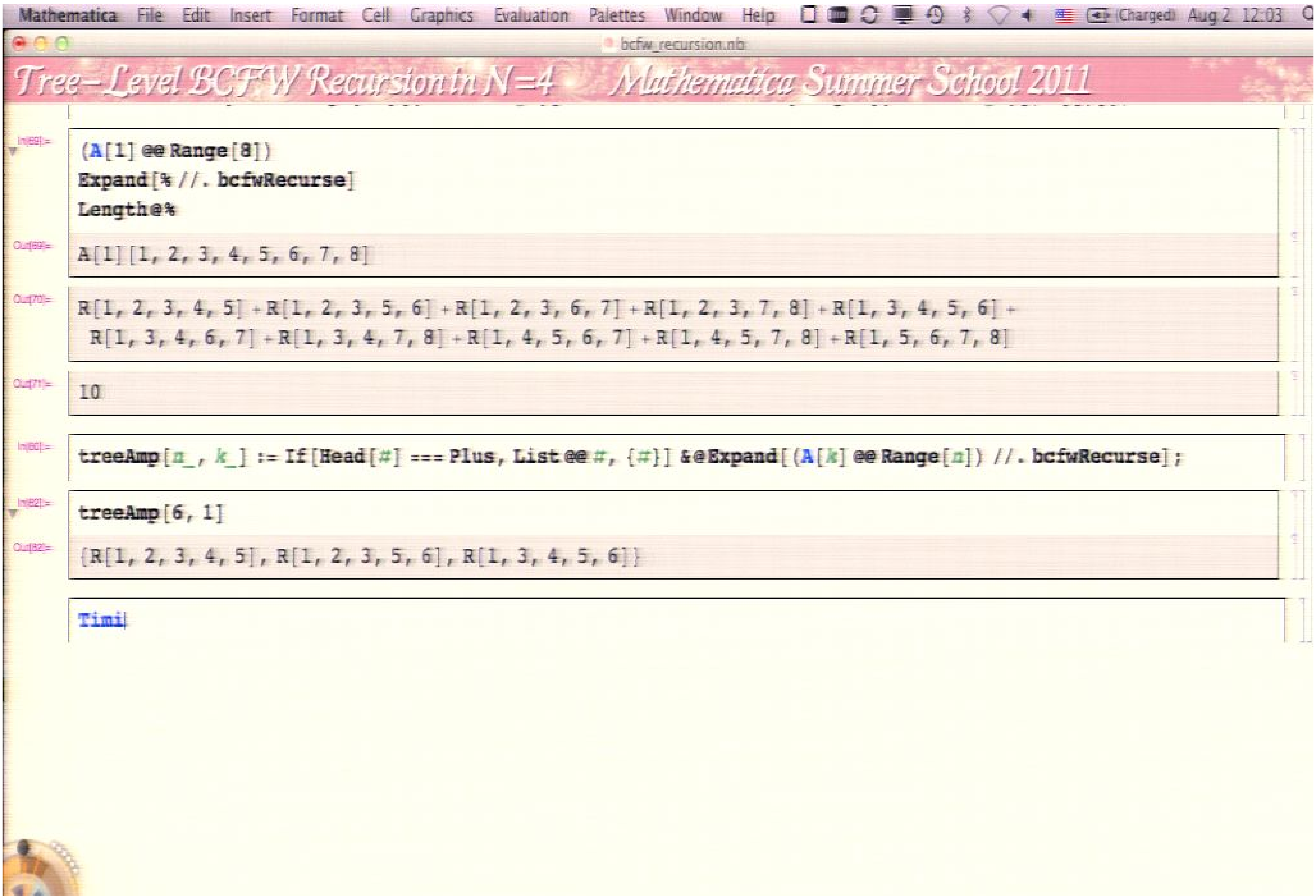
(Charged) Aug 2 12:02
bcfw_recursion.nb

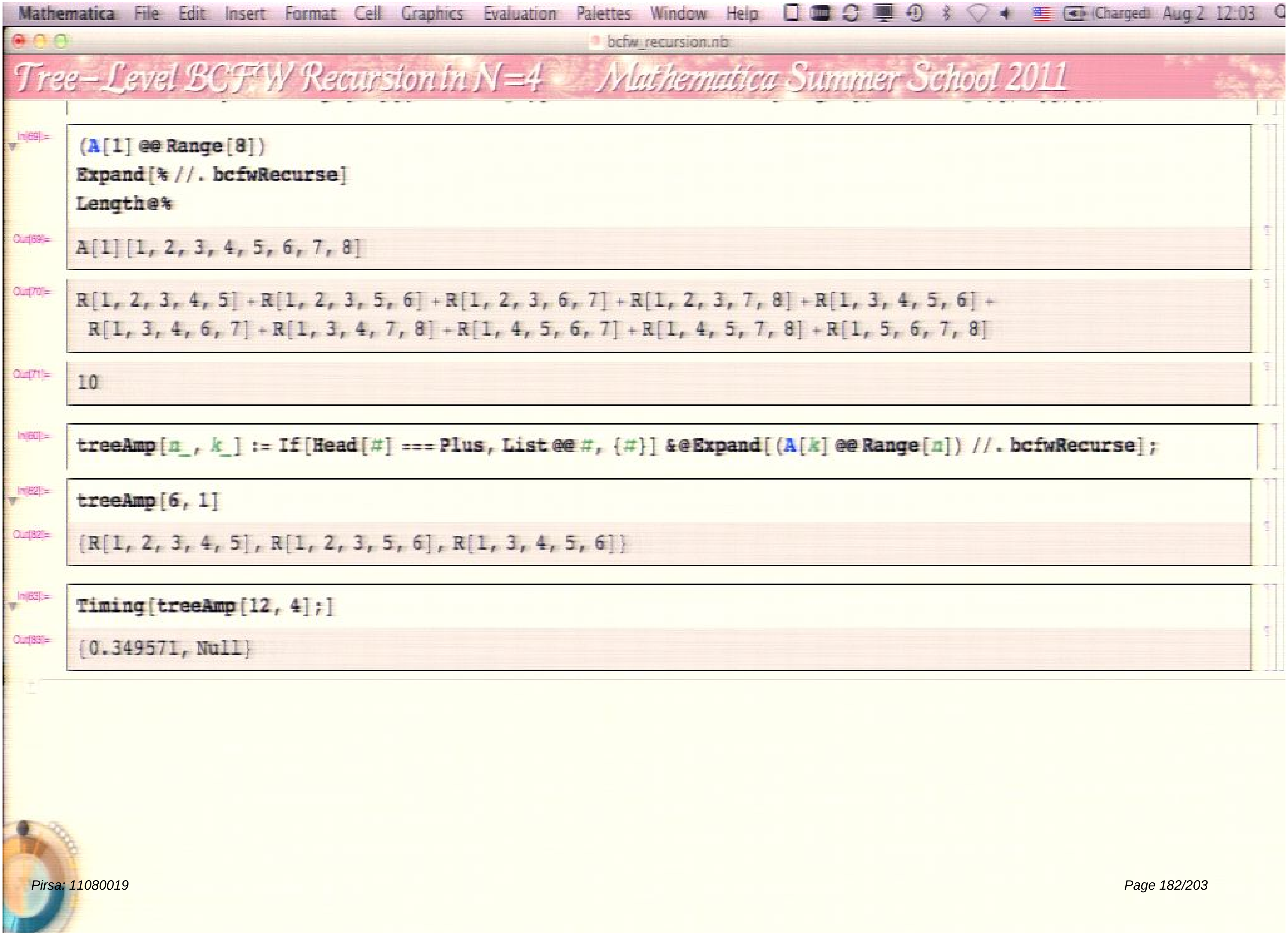
Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
In[69]:=
(A[1] @@ Range[8])
Expand[% //. bcfwRecurse]
Length@%

Out[69]=
A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]=
R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
 R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]=
10

In[72]:=
treeAmp[n_, k_] := List @@ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[79]:=
treeAmp[5, 1]

Out[79]=
{1, 2, 3, 4, 5}
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(Charged) Aug 2 12:03
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[60]:= treeAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[62]:= treeAmp[6, 1]

Out[62]= {R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}

Timi
```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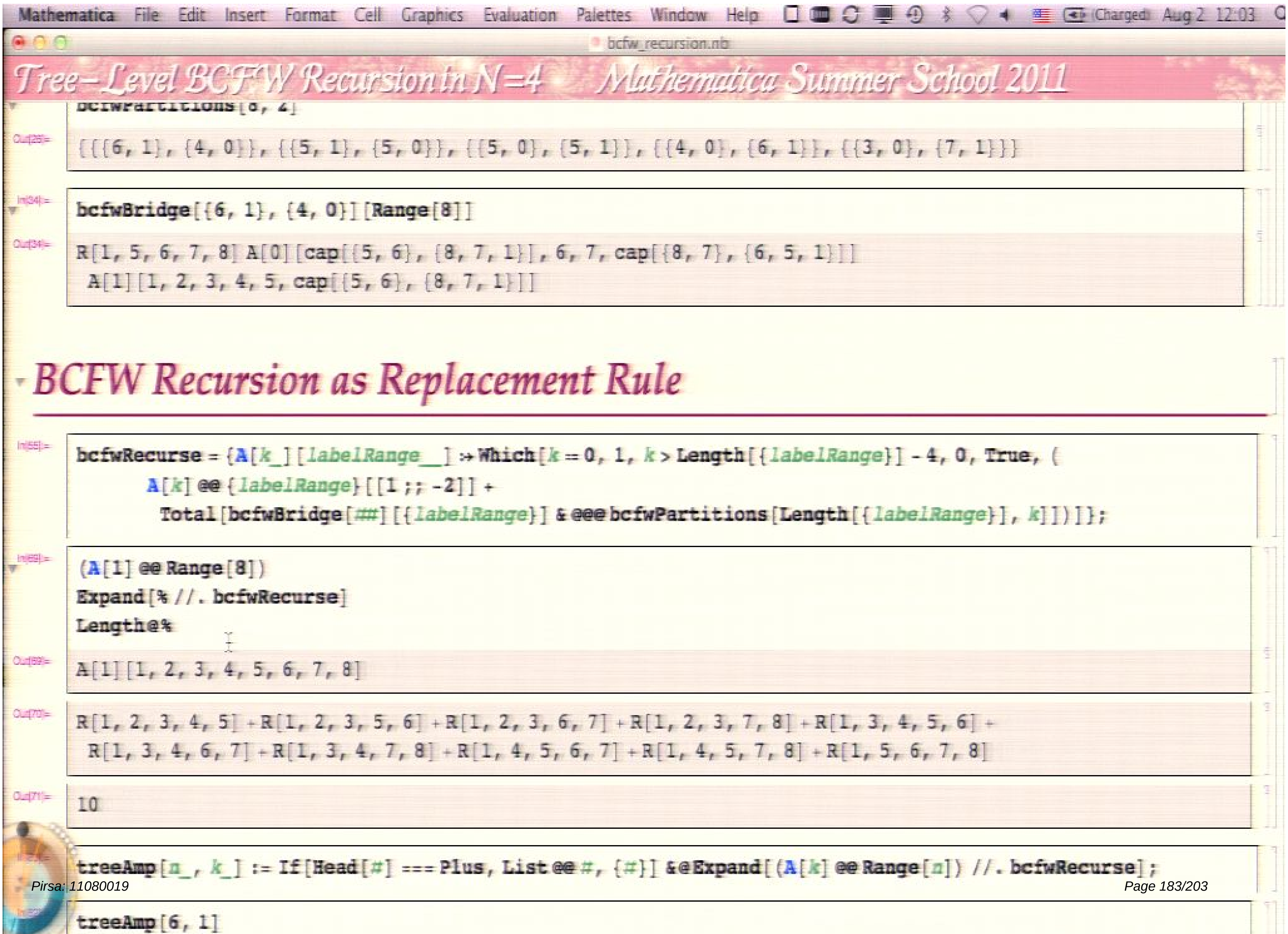
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

```
In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[80]:= treeAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[82]:= treeAmp[6, 1]

Out[82]= {R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}

In[83]:= Timing[treeAmp[12, 4];]

Out[83]= {0.349571, Null}
```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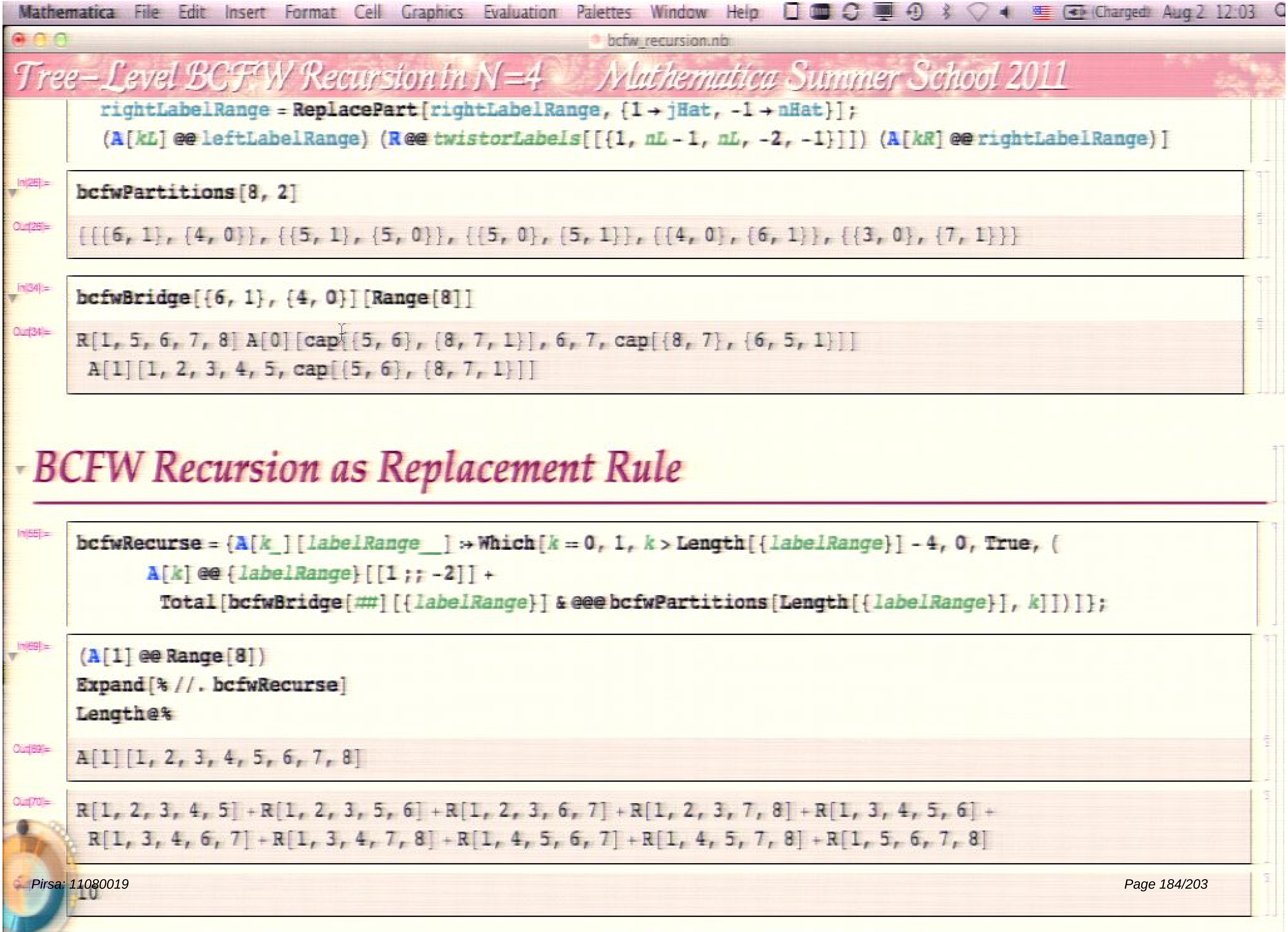
Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
bcfwPartitions[8, 2]
Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}

In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] ⧴ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
                A[k] @@ {labelRange}[[1 ;; -2]] +
                 Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

treeAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

treeAmp[6, 1]
```



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bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4    Mathematica Summer School 2011

```
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)]
```

```
In[26]:= bcfwPartitions[8, 2]

Out[26]= {{{6, 1}, {4, 0}}, {{5, 1}, {5, 0}}, {{5, 0}, {5, 1}}, {{4, 0}, {6, 1}}, {{3, 0}, {7, 1}}}
```

```
In[34]:= bcfwBridge[{6, 1}, {4, 0}][Range[8]]

Out[34]= R[1, 5, 6, 7, 8] A[0][cap[{5, 6}, {8, 7, 1}], 6, 7, cap[{8, 7}, {6, 5, 1}]]
          A[1][1, 2, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]]
```

## BCFW Recursion as Replacement Rule

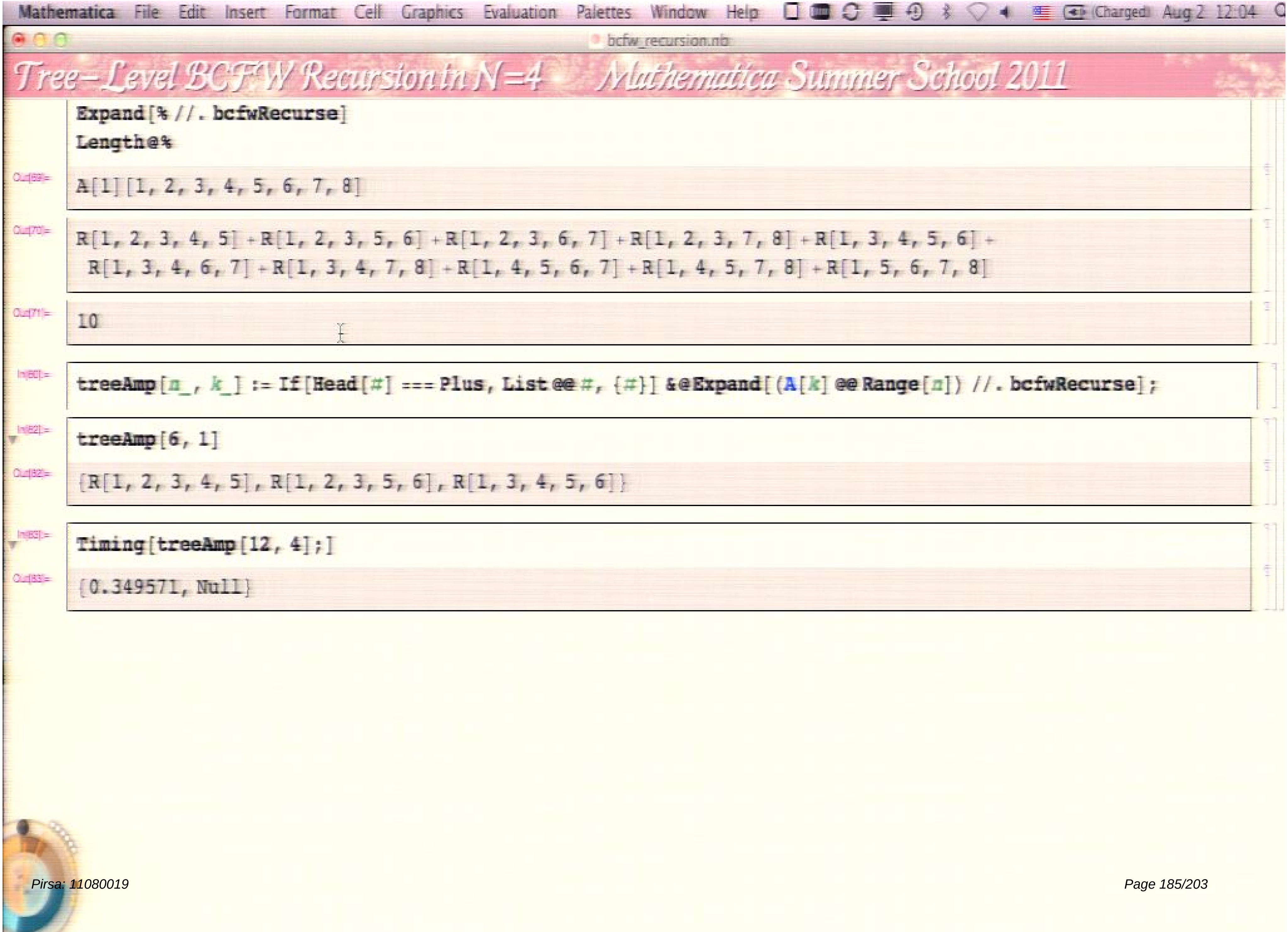
```
In[55]:= bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
              A[k] @@ {labelRange}[[1 ;; -2]] +
               Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
```

```
In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

         10
```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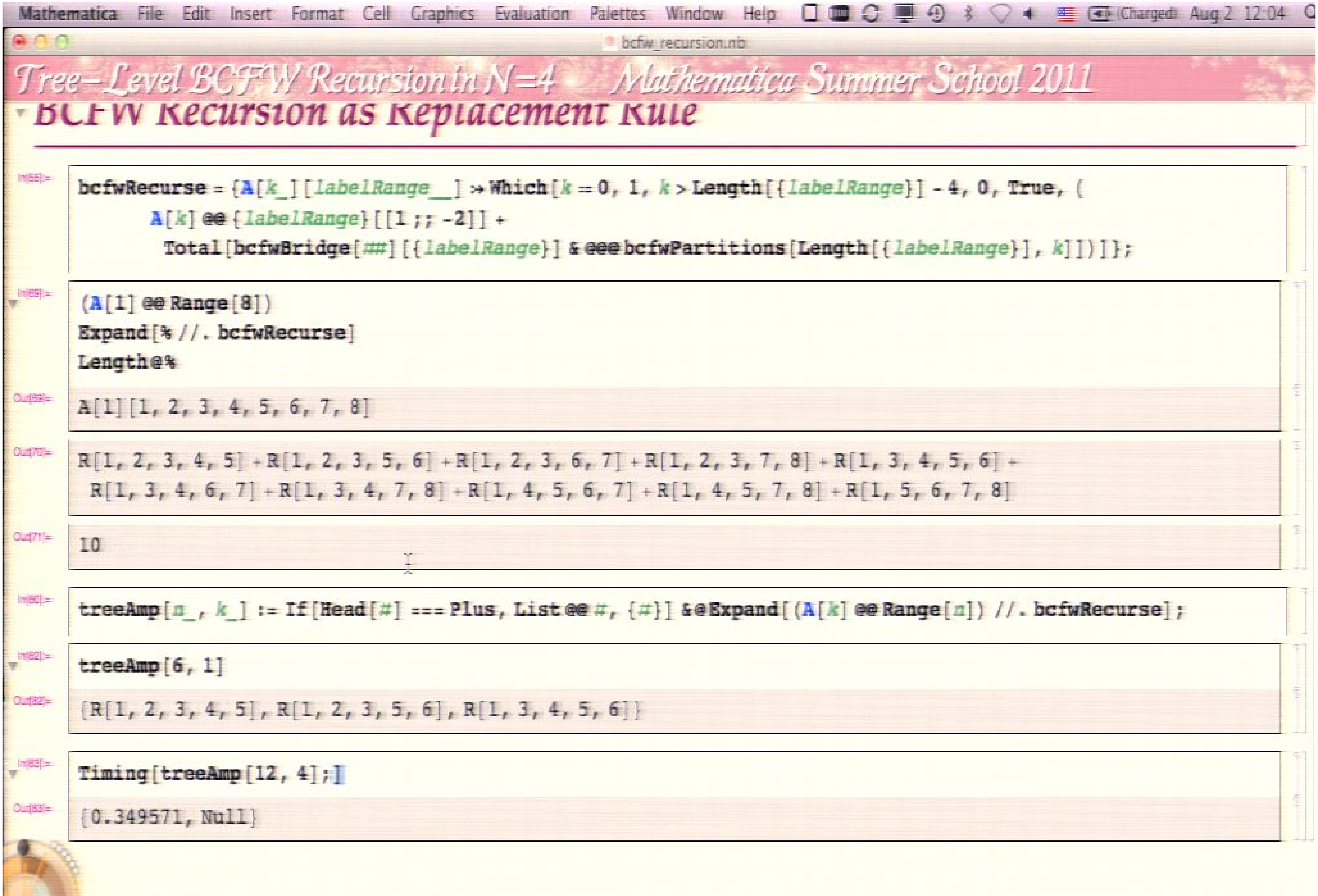
bcfw_recursion.nb

Tree–Level BCFW Recursion In N=4 Mathematica Summer School 2011

```
Expand[% //. bcfwRecurse]
Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[80]:= treeAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[82]:= treeAmp[6, 1]

Out[82]= {R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}

In[83]:= Timing[treeAmp[12, 4];]

Out[83]= {0.349571, Null}
```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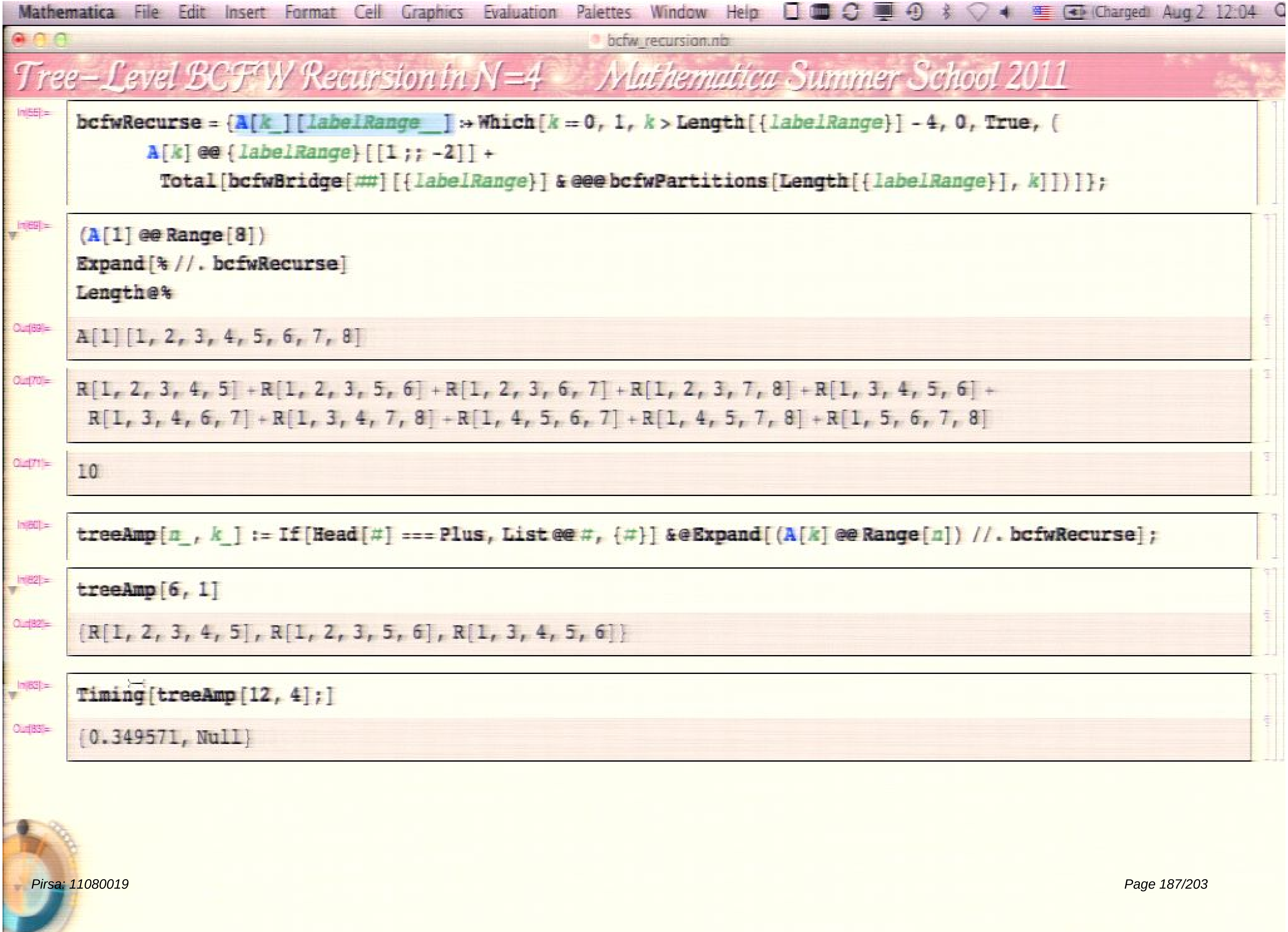
bcfw_recursion.nb

Tree–Level BCFW Recursion in N=4     Mathematica Summer School 2011

## BCFW Recursion as Replacement Rule

```
In[55]:= bcfwRecurse = {A[k_][labelRange__] ⧴ Which[k ⩵ 0, 1, k > Length[{labelRange}] - 4, 0, True, (
                A[k] @@ {labelRange}[[1 ;; -2]] +
                 Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};

In[69]:= (A[1] @@ Range[8])
         Expand[% //. bcfwRecurse]
         Length@%

Out[69]= A[1][1, 2, 3, 4, 5, 6, 7, 8]

Out[70]= R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
          R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]

Out[71]= 10

In[80]:= treeAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];

In[82]:= treeAmp[6, 1]

Out[82]= {R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}

In[83]:= Timing[treeAmp[12, 4];]

Out[83]= {0.349571, Null}
```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bcfw_recursion.nb

Tree-Level BCFW Recursion in N=4    Mathematica Summer School 2011

In[55]:=
```
bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True, (
      A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
```

In[69]:=
```
(A[1] @@ Range[8])
Expand[% //. bcfwRecurse]
Length@%
```

Out[69]=
```
A[1][1, 2, 3, 4, 5, 6, 7, 8]
```

Out[70]=
```
R[1, 2, 3, 4, 5] + R[1, 2, 3, 5, 6] + R[1, 2, 3, 6, 7] + R[1, 2, 3, 7, 8] + R[1, 3, 4, 5, 6] +
 R[1, 3, 4, 6, 7] + R[1, 3, 4, 7, 8] + R[1, 4, 5, 6, 7] + R[1, 4, 5, 7, 8] + R[1, 5, 6, 7, 8]
```

Out[71]=
```
10
```

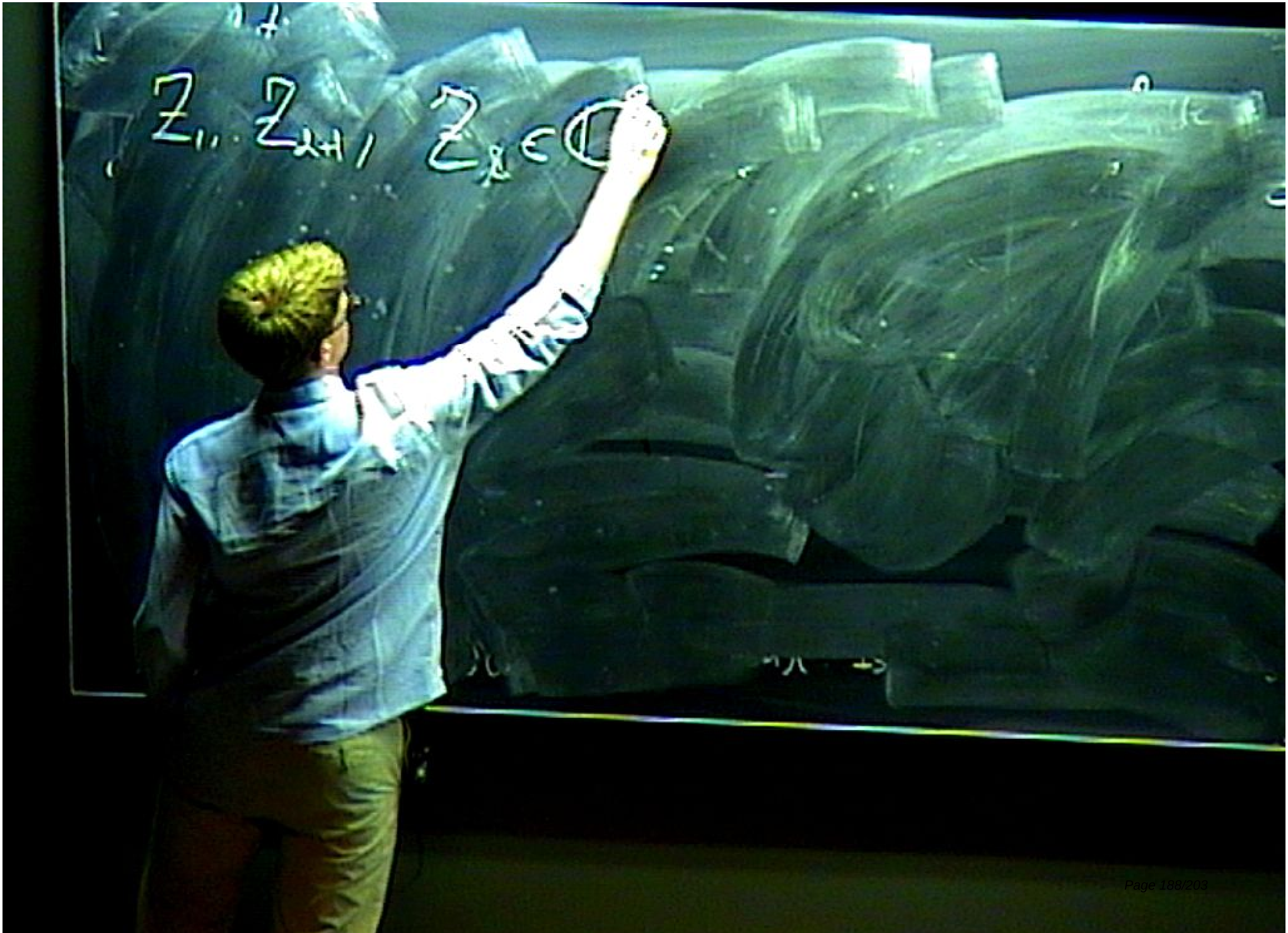
In[80]:=
```
treeAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
```

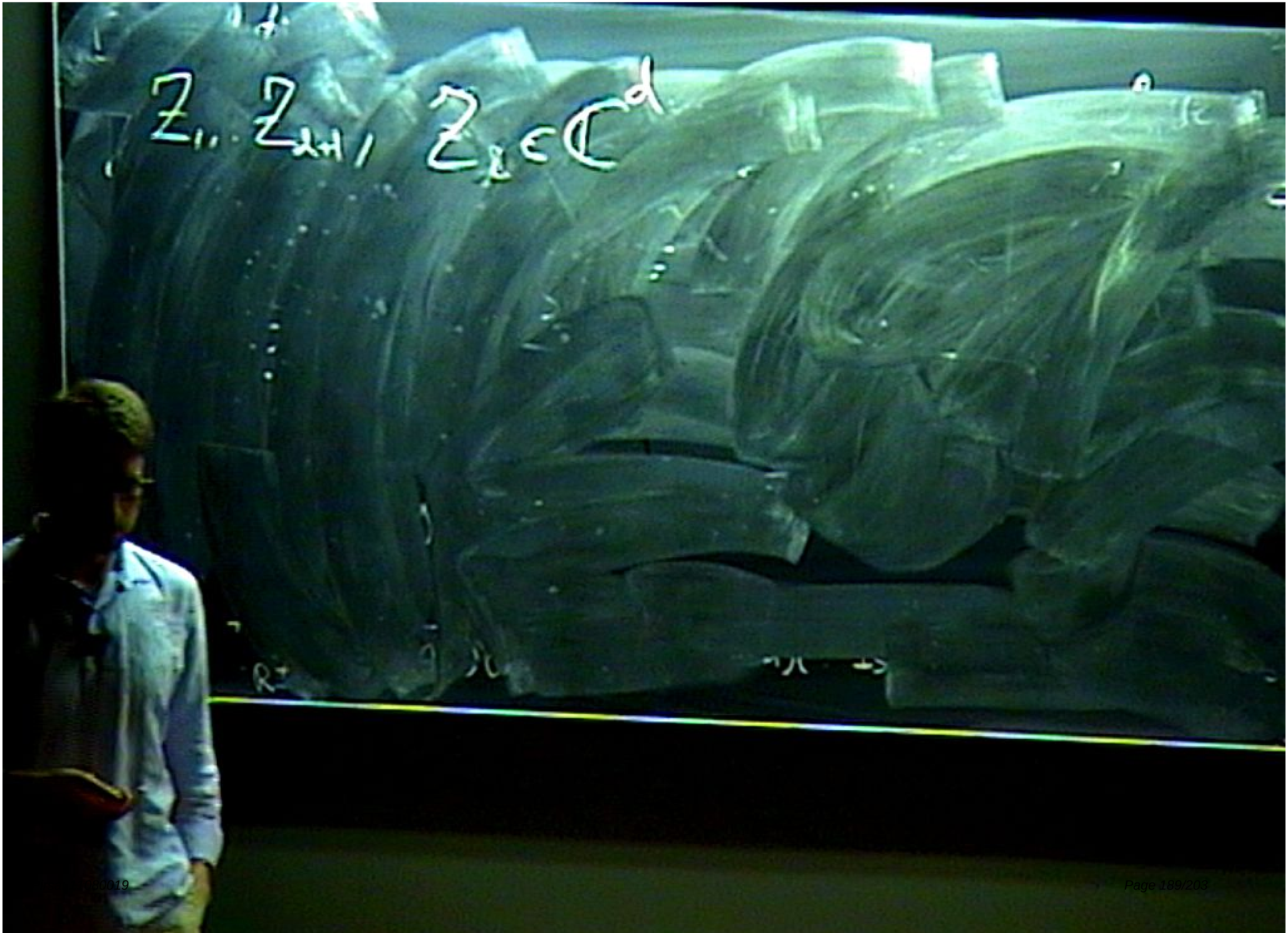
In[82]:=
```
treeAmp[6, 1]
```

Out[82]=
```
{R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}
```

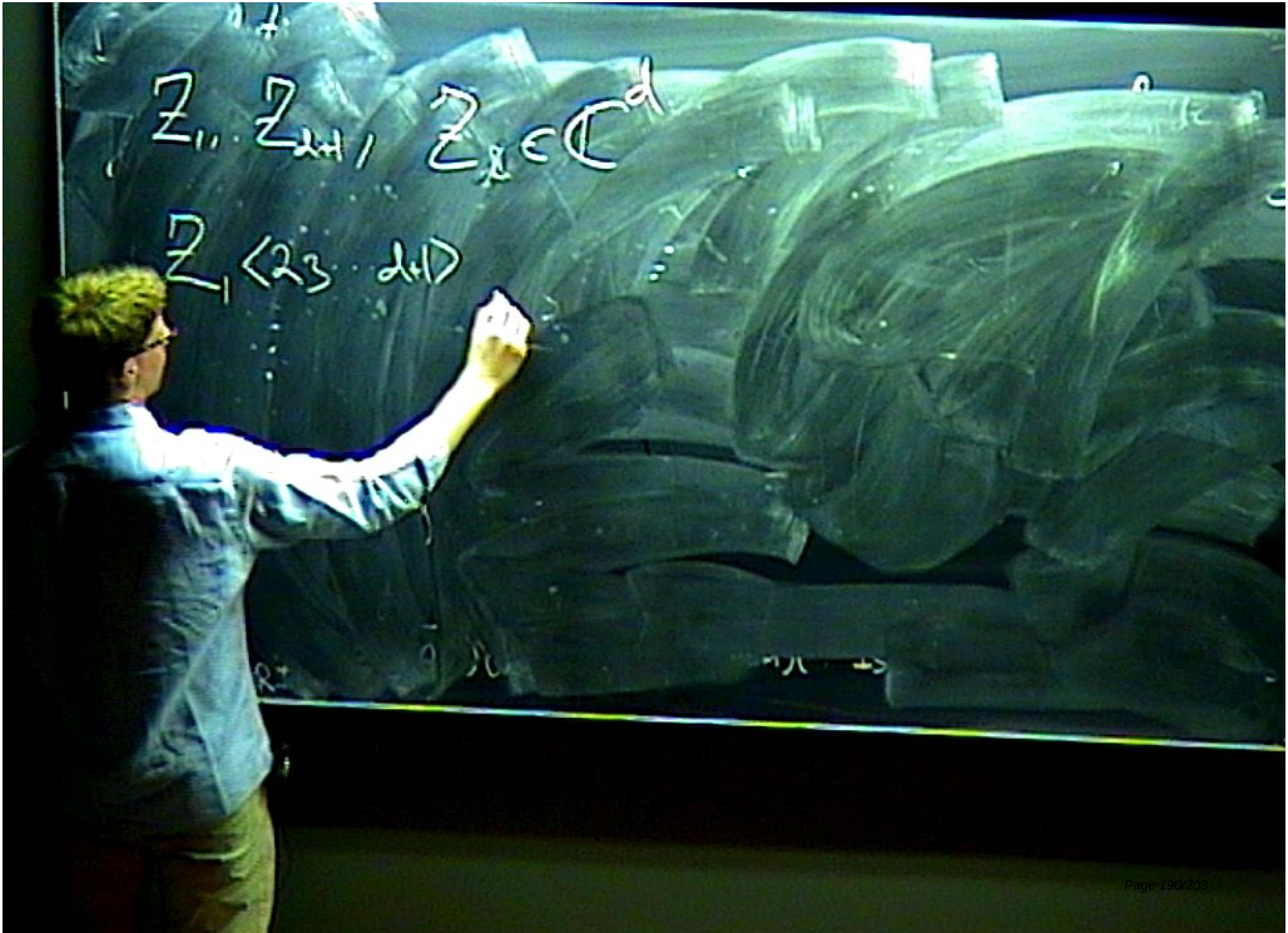
In[83]:=
```
Timing[treeAmp[12, 4];]
```

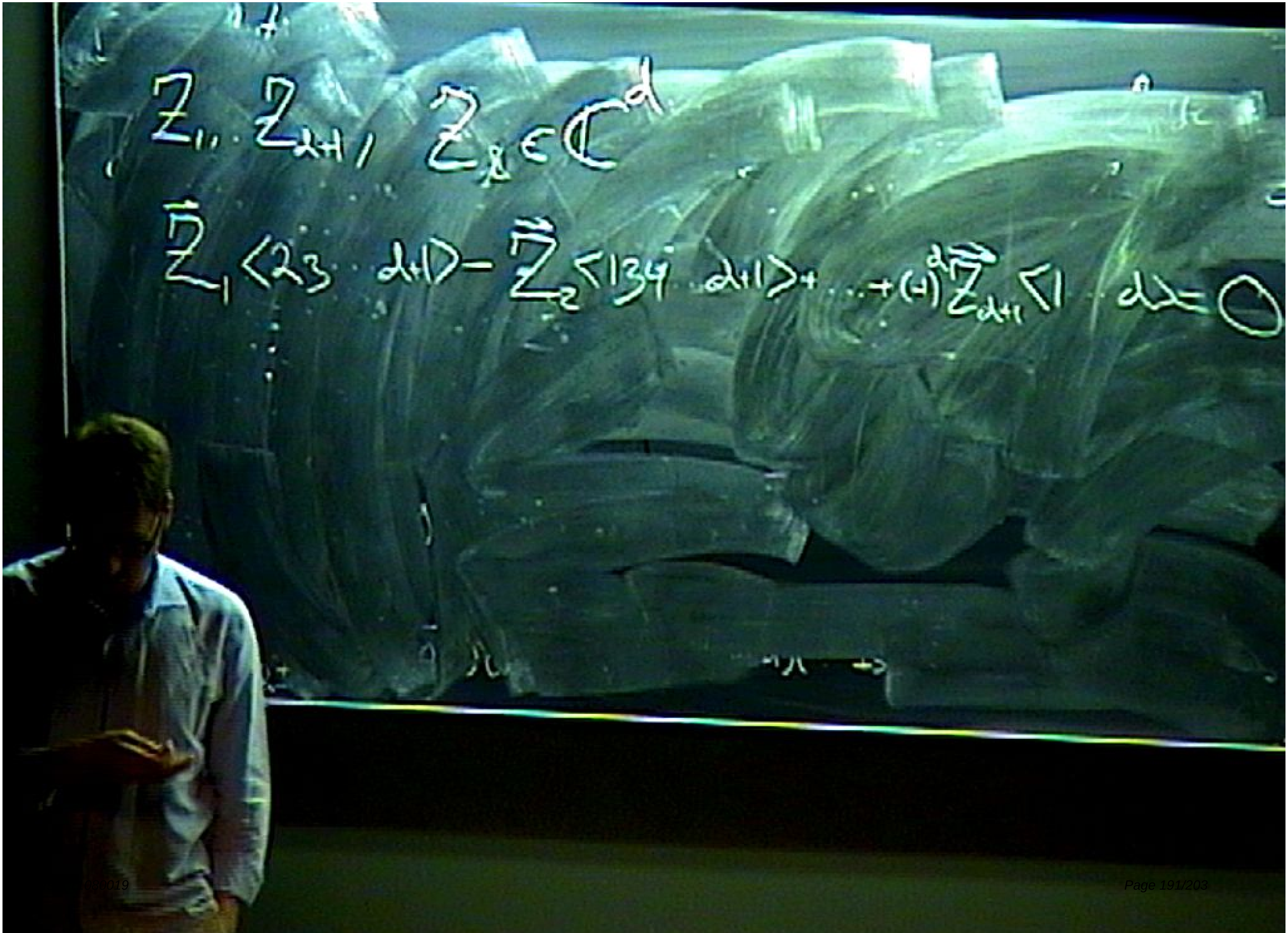
Out[83]=
```
{0.349571, Null}
```


$Z_1 \ldots Z_{2l+1}, Z_l \in \mathbb{C}$



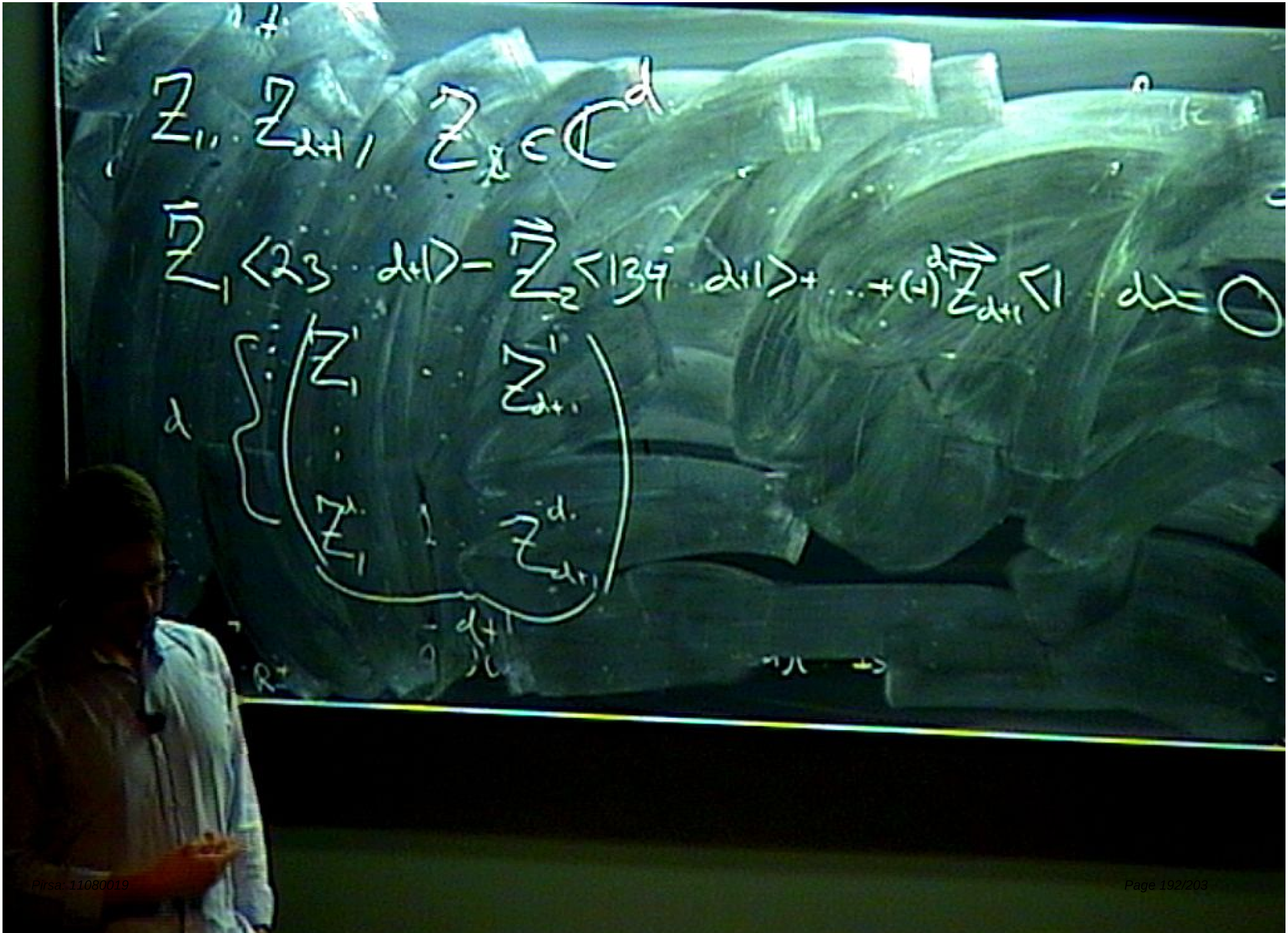
$Z_1 \ldots Z_{2H},\ Z_k \in \mathbb{C}^d$


$Z_1 \dots Z_{d+1}, \; Z_\ell \in \mathbb{C}^d$
$Z_1 \langle 2 3 \dots d+1 \rangle$



$$Z_1 \ldots Z_{d+1},\ Z_* \in \mathbb{C}^d$$

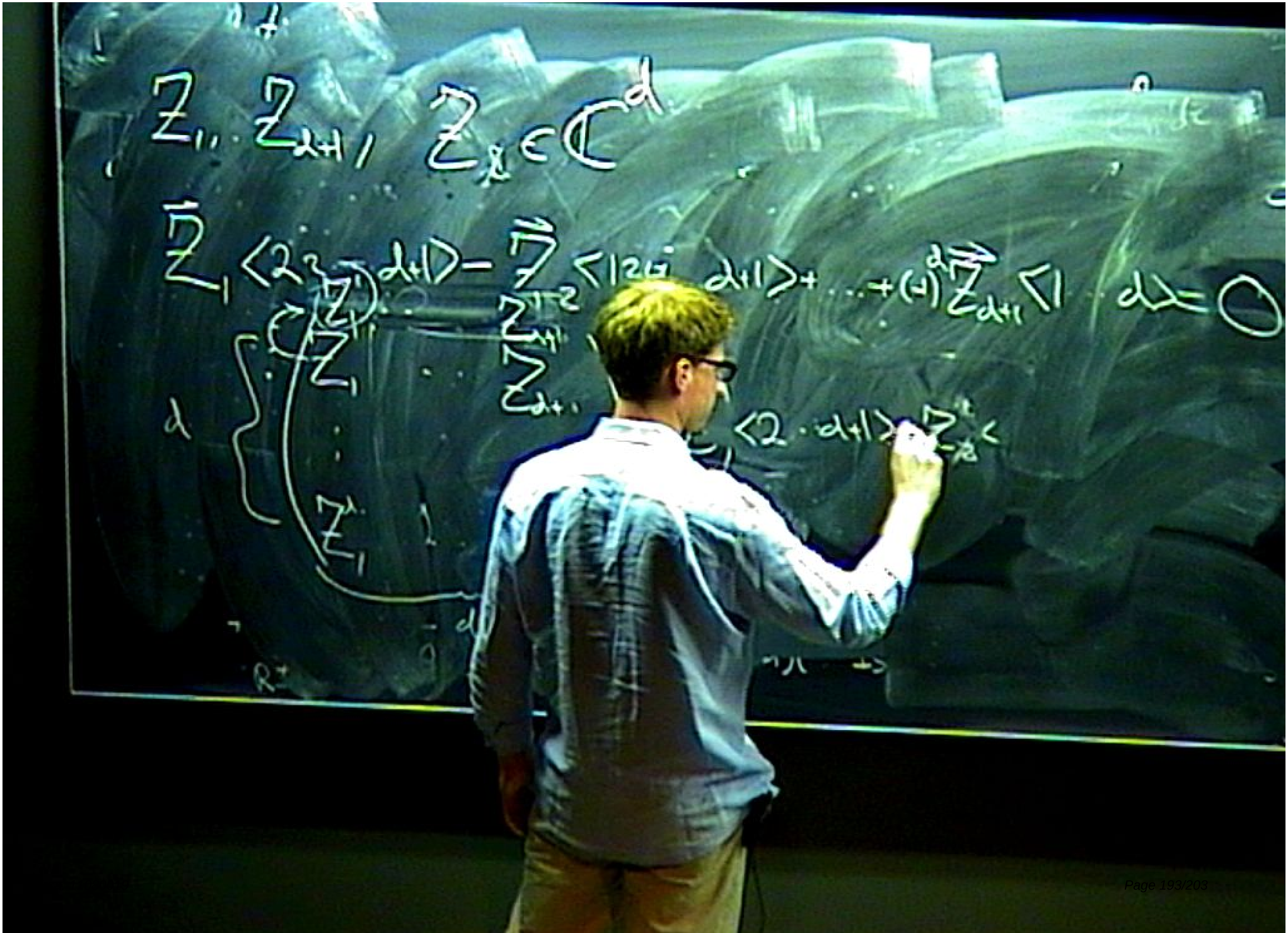
$$\vec{Z}_1 \langle 23 \cdots d+1\rangle - \vec{Z}_2 \langle 134 \cdots d+1\rangle + \cdots + (-1)^d \vec{Z}_{d+1} \langle 1 \cdots d\rangle = 0$$



$$Z_1 \ldots Z_{d+1}, \quad Z_i \in \mathbb{C}^d$$

$$\vec{Z}_1 \langle 2 3 \ldots d{+}1 \rangle - \vec{Z}_2 \langle 1 3 4 \ldots d{+}1 \rangle + \ldots + (-1)^d \vec{Z}_{d+1} \langle 1 \ldots d \rangle = 0$$

$$d\left\{\underbrace{\begin{pmatrix} Z_1^1 & \cdots & Z_{d+1}^1 \\ \vdots & & \\ Z_1^d & \cdots & Z_{d+1}^d \end{pmatrix}}_{d+1}\right.$$

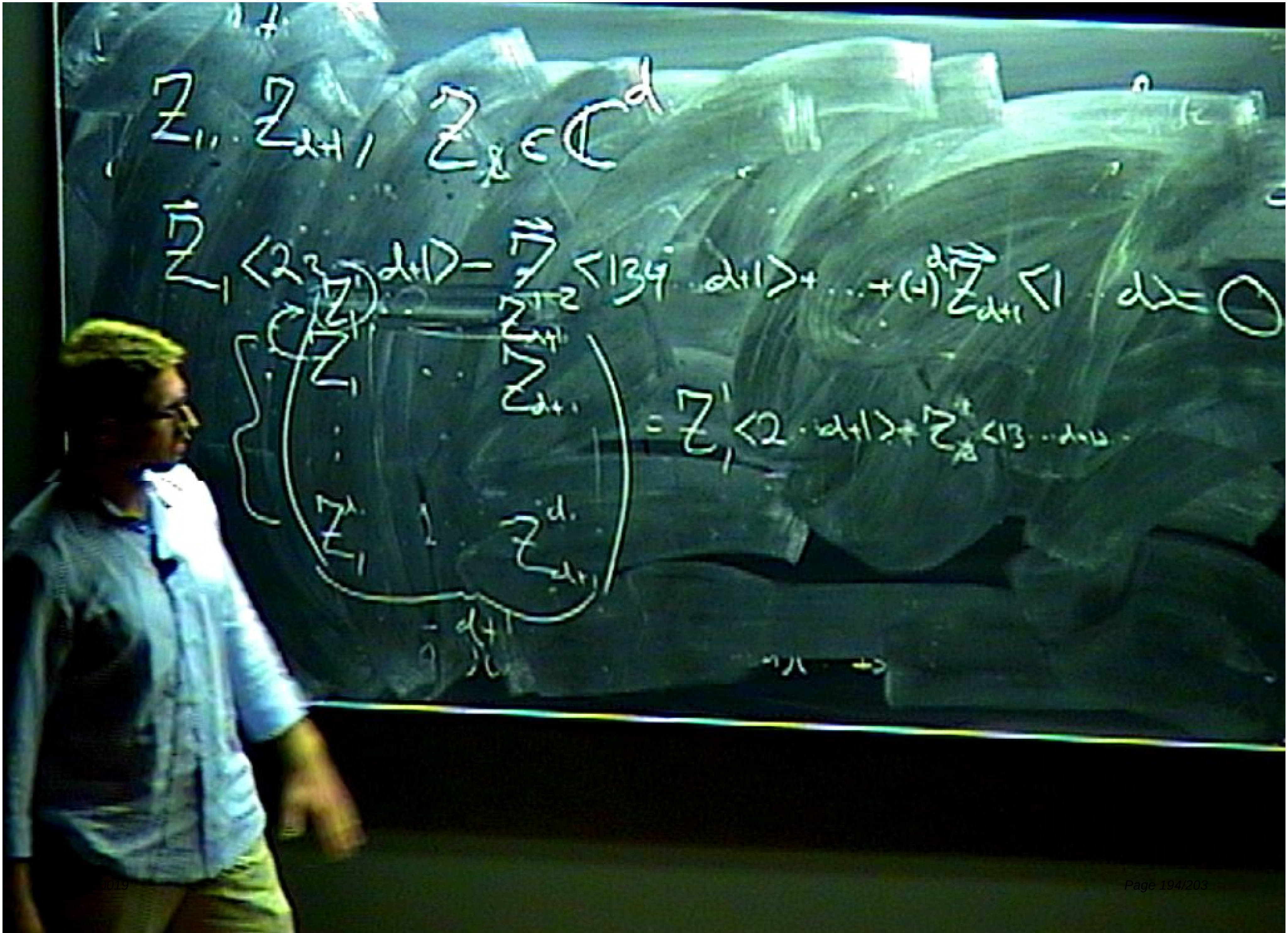


$Z_1, \ldots Z_{d+1}, \; Z_k \in \mathbb{C}^d$

$\vec{Z}_1 \langle 23 \cdots d+1 \rangle - \vec{Z}_2 \langle 13 \cdots d+1 \rangle + \cdots + (-1)^d \vec{Z}_{d+1} \langle 1 \cdots d \rangle = 0$

$$d \left\{ \begin{pmatrix} Z_1^1 & \cdots & Z_{d+1}^1 \\ Z_1 & \cdots & Z_{d+1} \\ \vdots & & \\ Z_1 & & \end{pmatrix} \right.$$

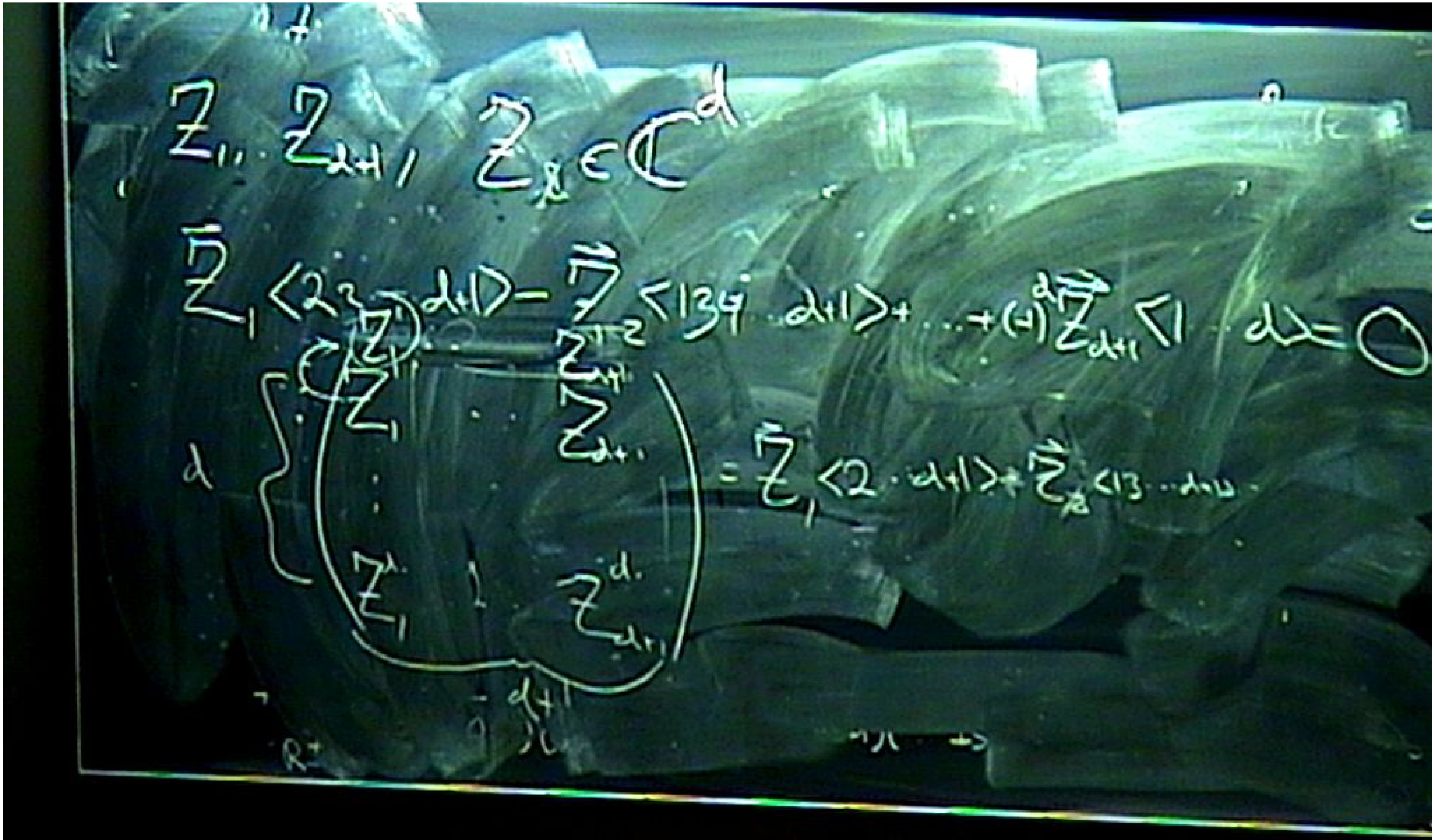
$\langle 2 \cdots d+1 \rangle \, Z_k^{I} <$



$Z_1, \ldots Z_{d+1},\ Z_* \in \mathbb{C}^d$

$\vec{Z}_1 \langle 2 3 \ldots d+1 \rangle - \vec{Z}_2 \langle 1 3 4 \ldots d+1 \rangle + \ldots + (-1)^d \vec{Z}_{d+1} \langle 1 \ldots d \rangle = 0$

$$\begin{pmatrix} Z_1^1 & \cdots & Z_{d+1}^1 \\ \vdots & & \\ Z_1^d & 1 & Z_{d+1}^d \end{pmatrix} = Z_1^1 \langle 2 \ldots d+1 \rangle + Z_*^1 \langle 1 3 \ldots d+1 \rangle \ldots$$



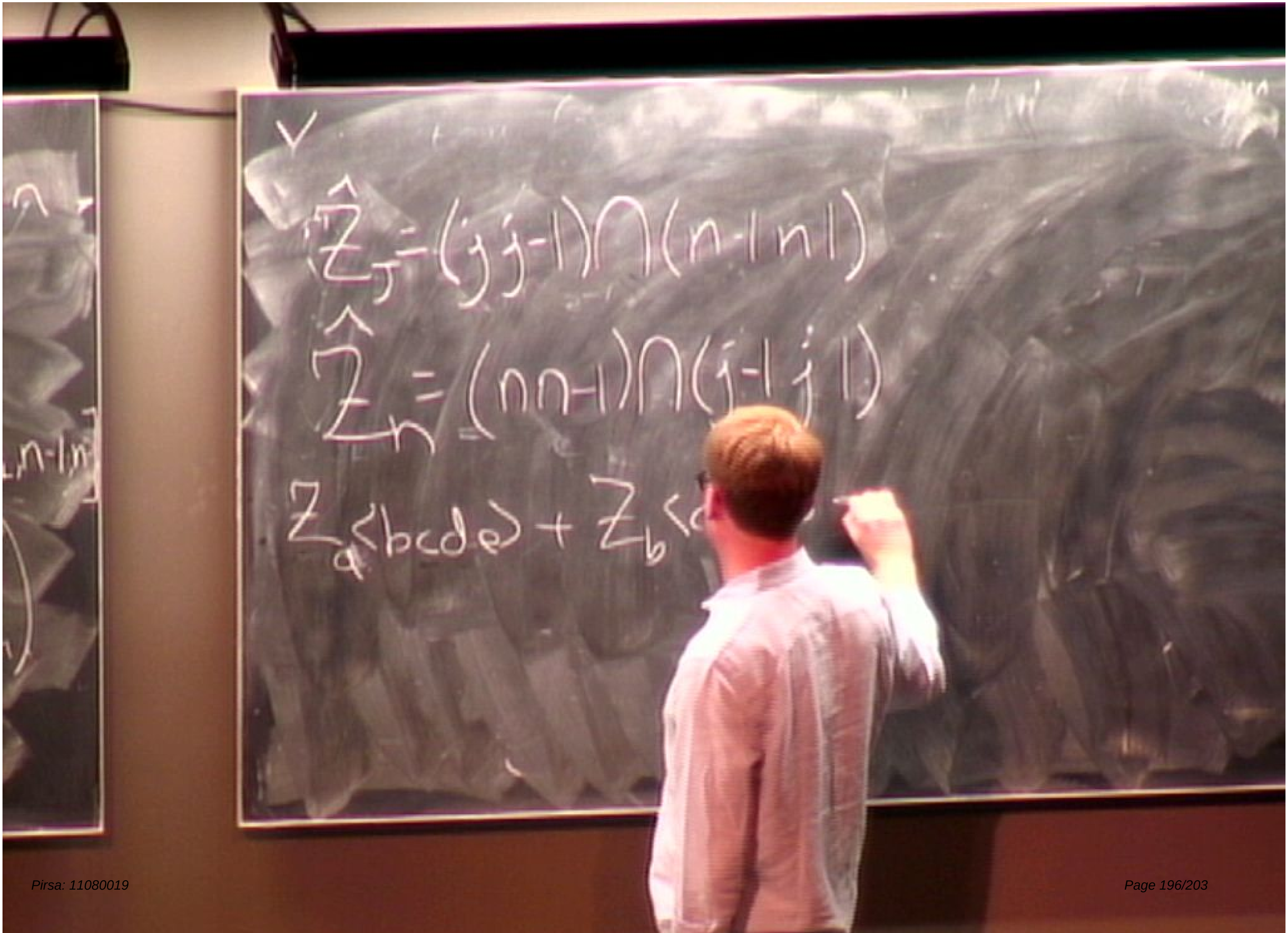
$Z_1 \dots Z_{d+1}, \; Z_k \in \mathbb{C}^d$

$\vec{Z}_1 \langle 2 3 \dots d+1 \rangle - \vec{Z}_2 \langle 1 3 4 \dots d+1 \rangle + \dots + (-1)^d \vec{Z}_{d+1} \langle 1 \dots d \rangle = 0$

$$d \left\{ \begin{pmatrix} Z_1^1 & \dots & Z_{d+1}^1 \\ \vdots & & \\ Z_1^d & \dots & Z_{d+1}^d \end{pmatrix} \right. = \vec{Z}_1 \langle 2 \dots d+1 \rangle + \vec{Z}_2 \langle 1 3 \dots d+1 \rangle \dots$$

$d+1$

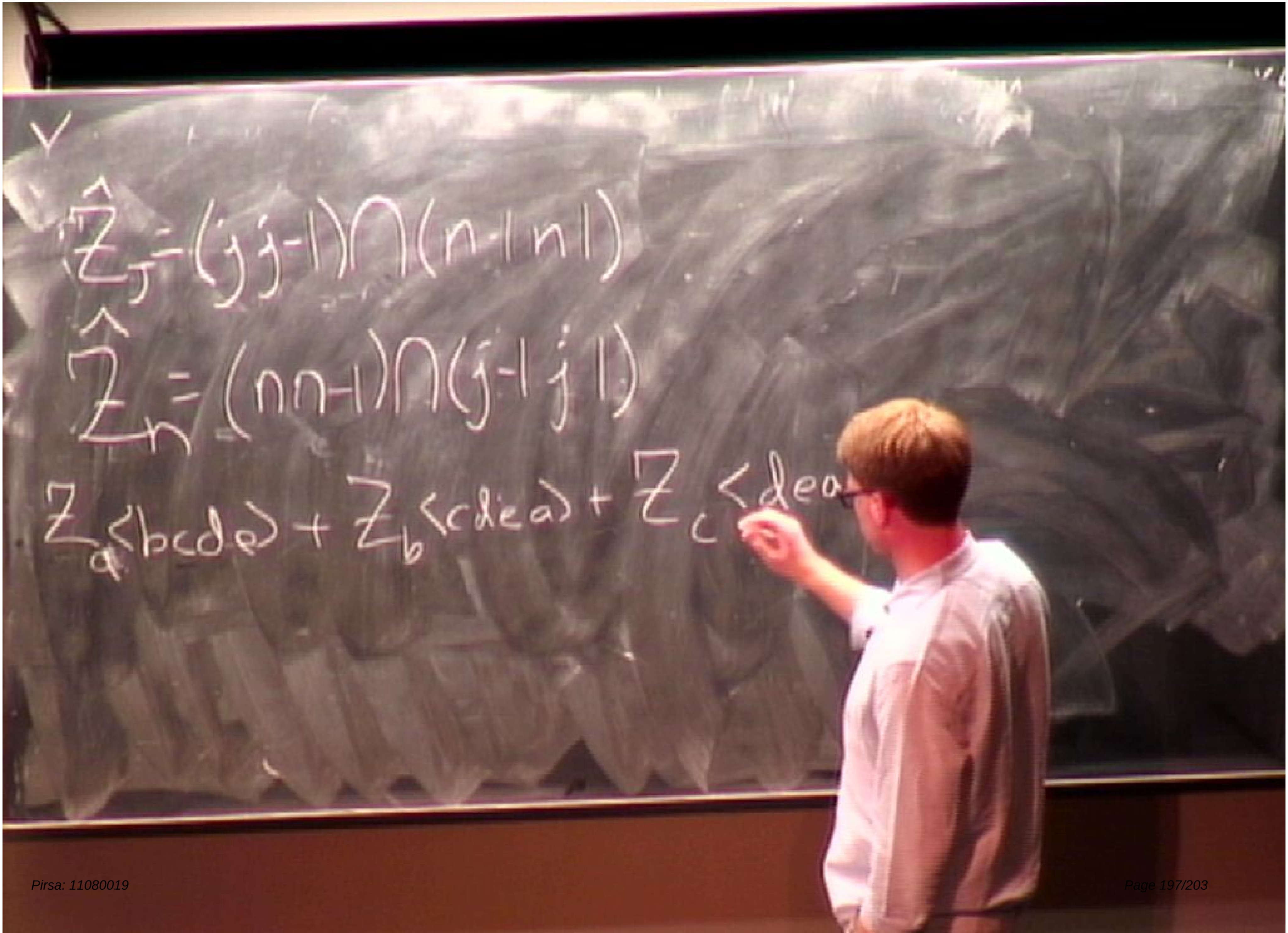
$R^+$



$\hat{Z}_j = (j\, j{-}1) \cap (n{-}1\, n\, 1)$

$\hat{Z}_n = (n\, n{-}1) \cap (j{-}1\, j\, 1)$

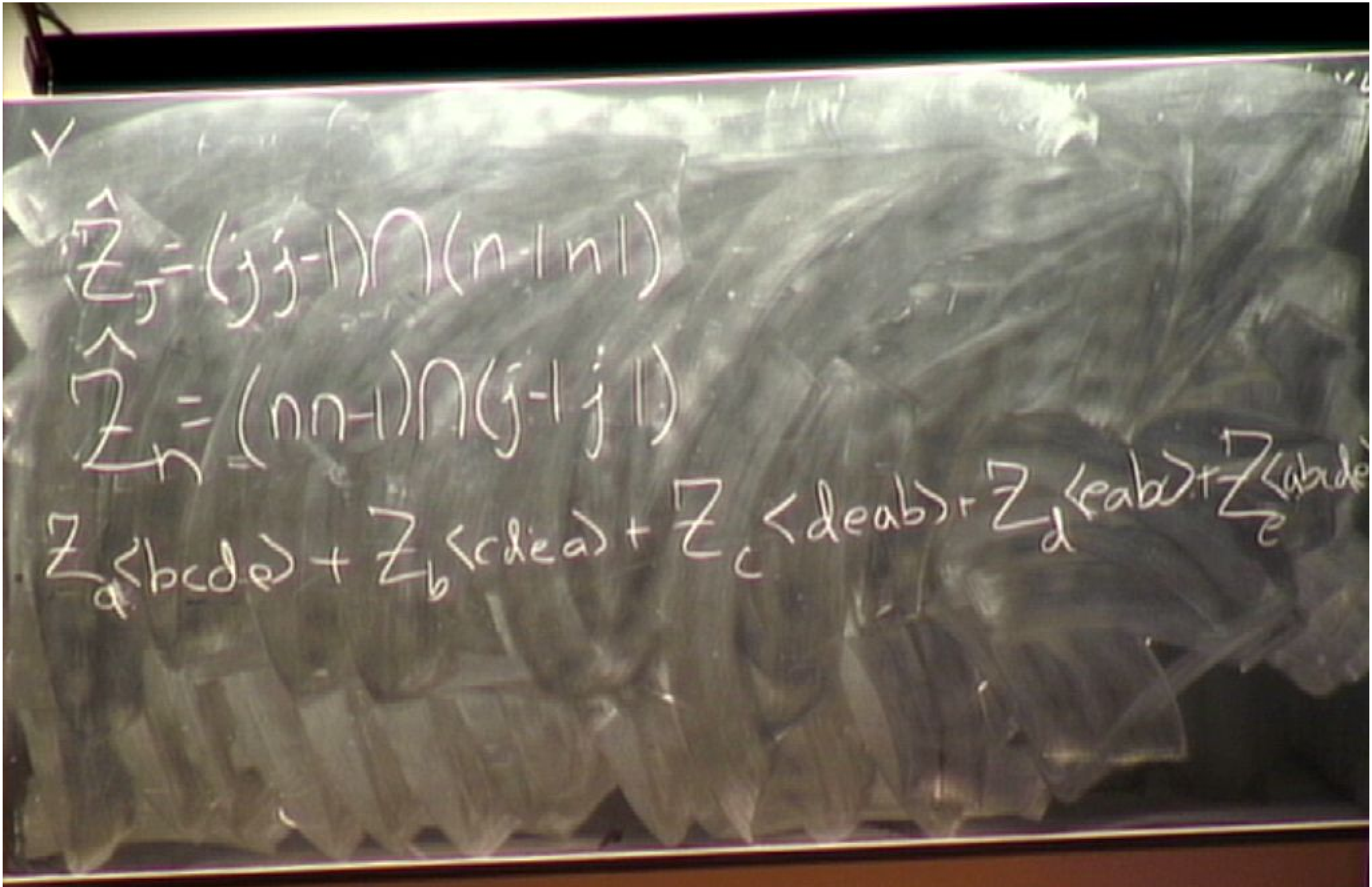
$Z_a \langle bcde \rangle + Z_b \langle c$



$$\hat{Z}_j = (j\; j-1) \cap (n-1\; n\; 1)$$

$$\hat{Z}_n = (n\; n-1) \cap (j-1\; j\; 1)$$

$$Z_a \langle bcde \rangle + Z_b \langle cdea \rangle + Z_c \langle dea$$

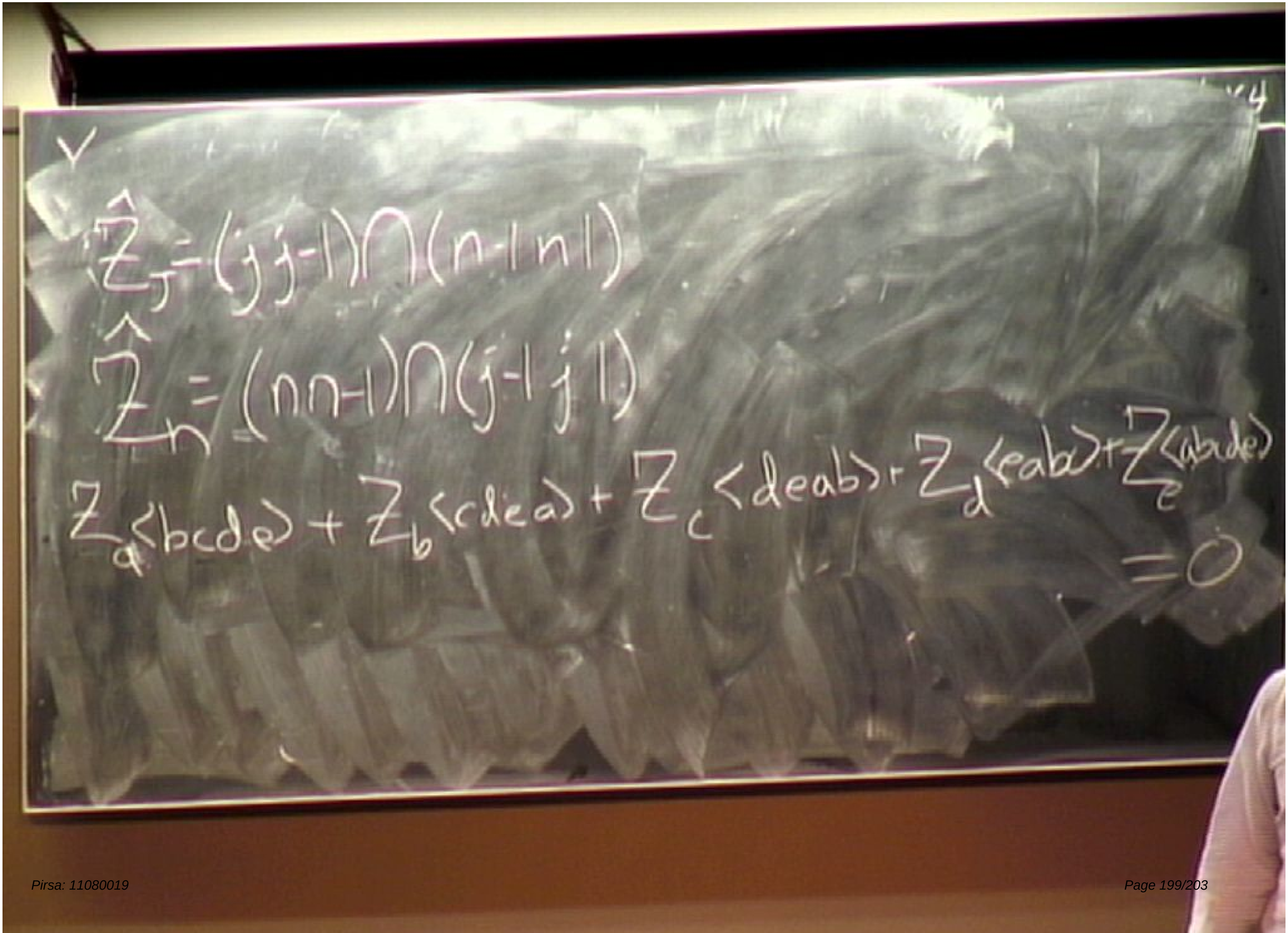


V

$$\hat{Z}_j = (j\, j-1) \cap (n-1\, n\, 1)$$

$$\hat{Z}_n = (n\, n-1) \cap (j-1\, j\, 1)$$

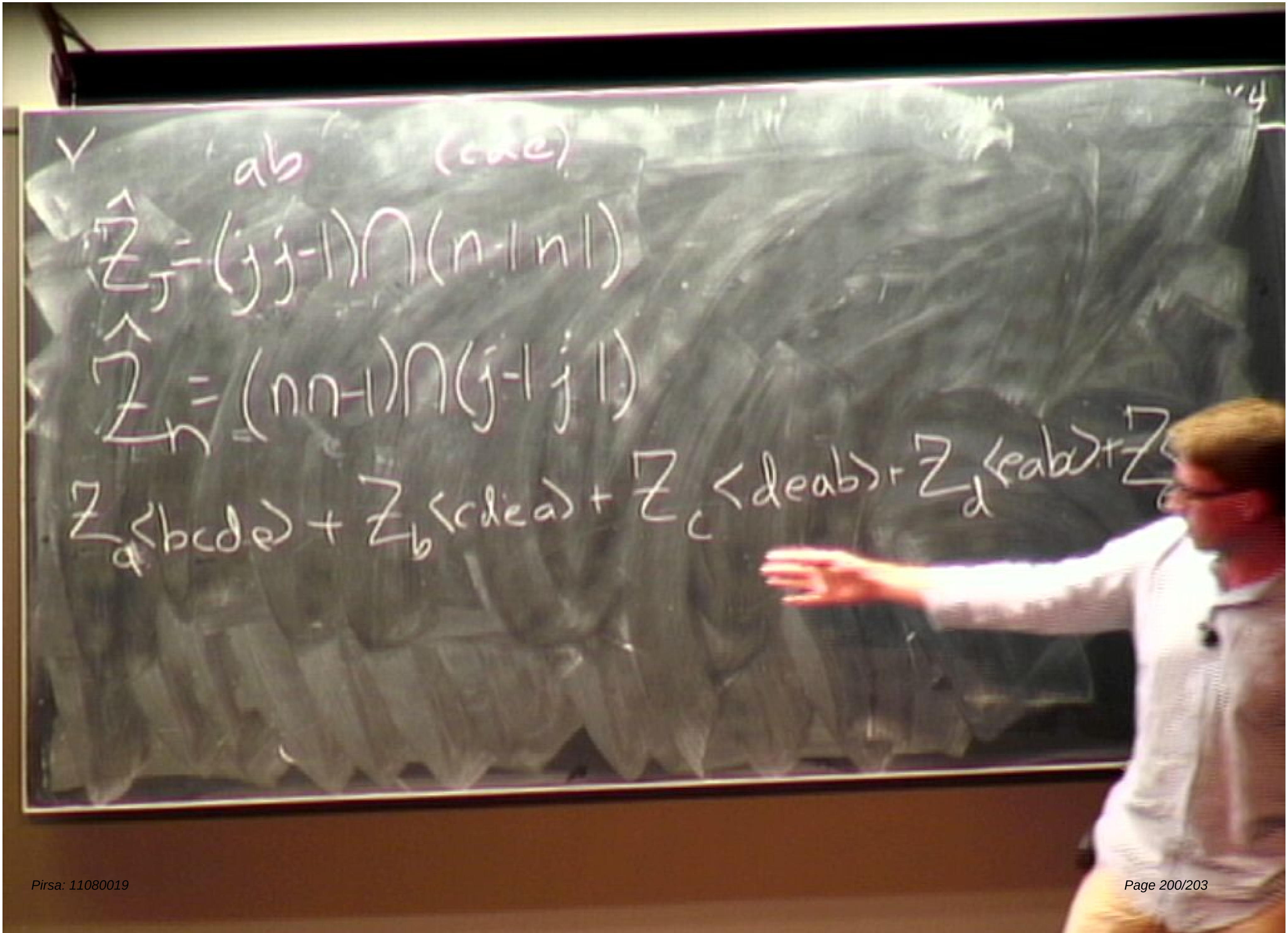
$$Z_a \langle bcde \rangle + Z_b \langle cdea \rangle + Z_c \langle deab \rangle + Z_d \langle eabc \rangle + Z_e \langle abcd$$



$$\hat{Z}_j = (j\ j{-}1) \cap (n{-}1\ n\ 1)$$

$$\hat{Z}_n = (n\ n{-}1) \cap (j{-}1\ j\ 1)$$

$$Z_a\langle bcde\rangle + Z_b\langle cdea\rangle + Z_c\langle deab\rangle + Z_d\langle eabc\rangle + Z_e\langle abcde\rangle = 0$$



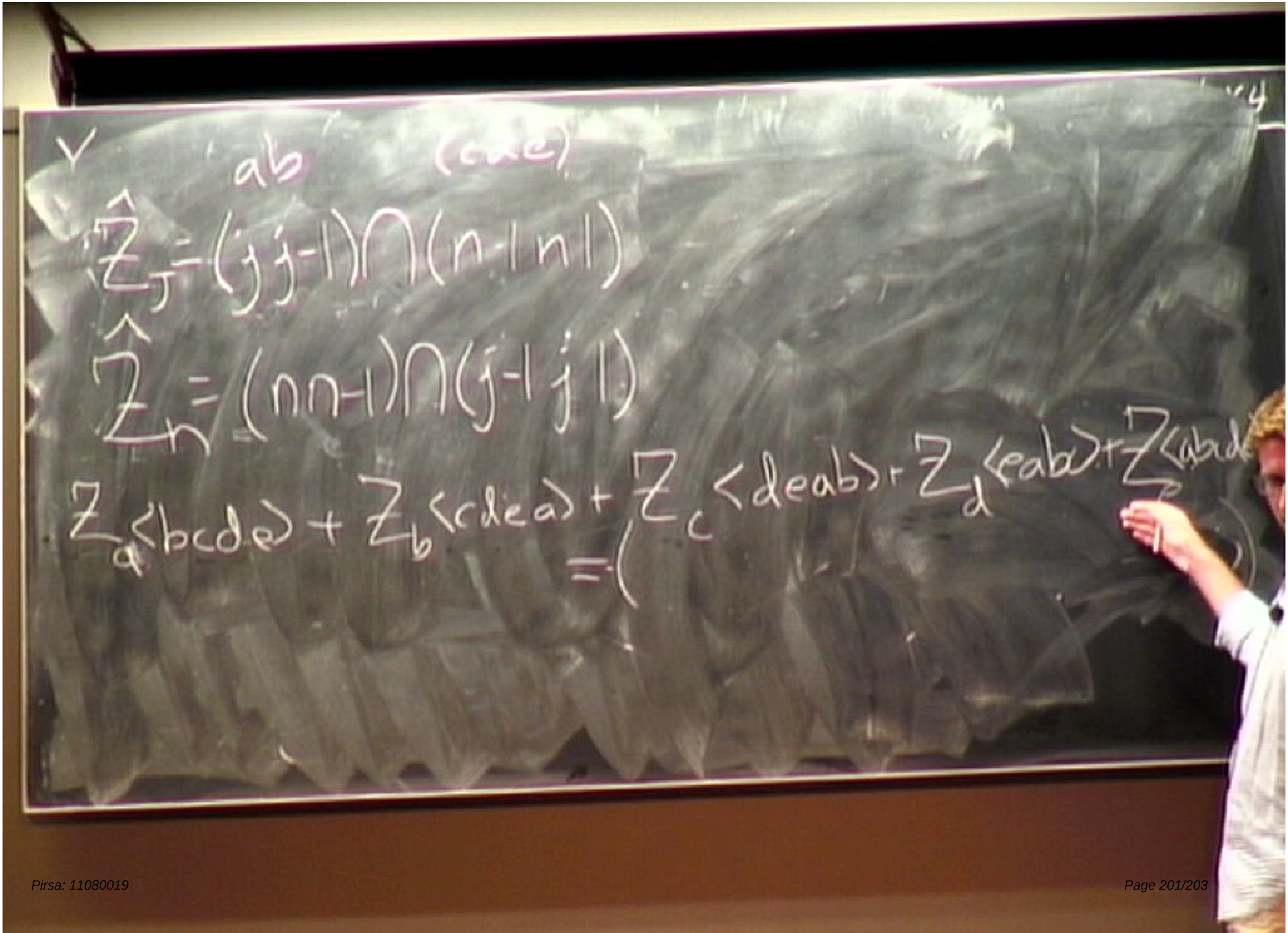
V

ab (cde)

$$\hat{Z}_j = (j\; j-1) \cap (n-1\; n\; 1)$$

$$\hat{Z}_n = (n\; n-1) \cap (j-1\; j\; 1)$$

$$Z_a \langle bcde\rangle + Z_b \langle cdea\rangle + Z_c \langle deab\rangle + Z_d \langle eab\rangle + Z_e$$



V

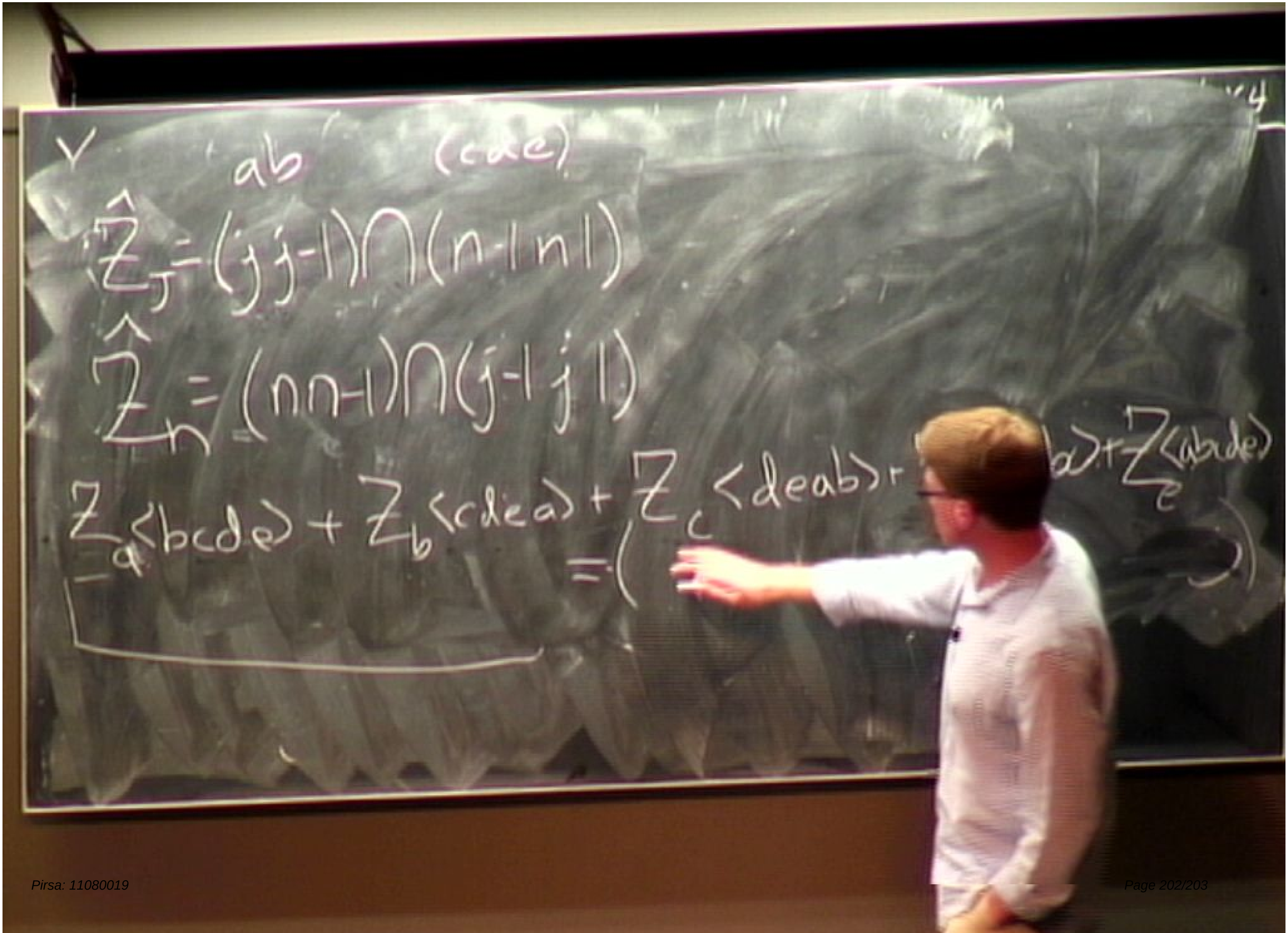
ab (cde)

$$\hat{Z}_j = (j\, j\text{-}1) \cap (n\text{-}1\, n\, 1)$$

$$\hat{Z}_n = (n\, n\text{-}1) \cap (j\text{-}1\, j\, 1)$$

$$Z_a\langle bcde\rangle + Z_b\langle cdea\rangle + Z_c\langle deab\rangle + Z_d\langle eabc\rangle + Z\langle abcd$$

$$= ($$



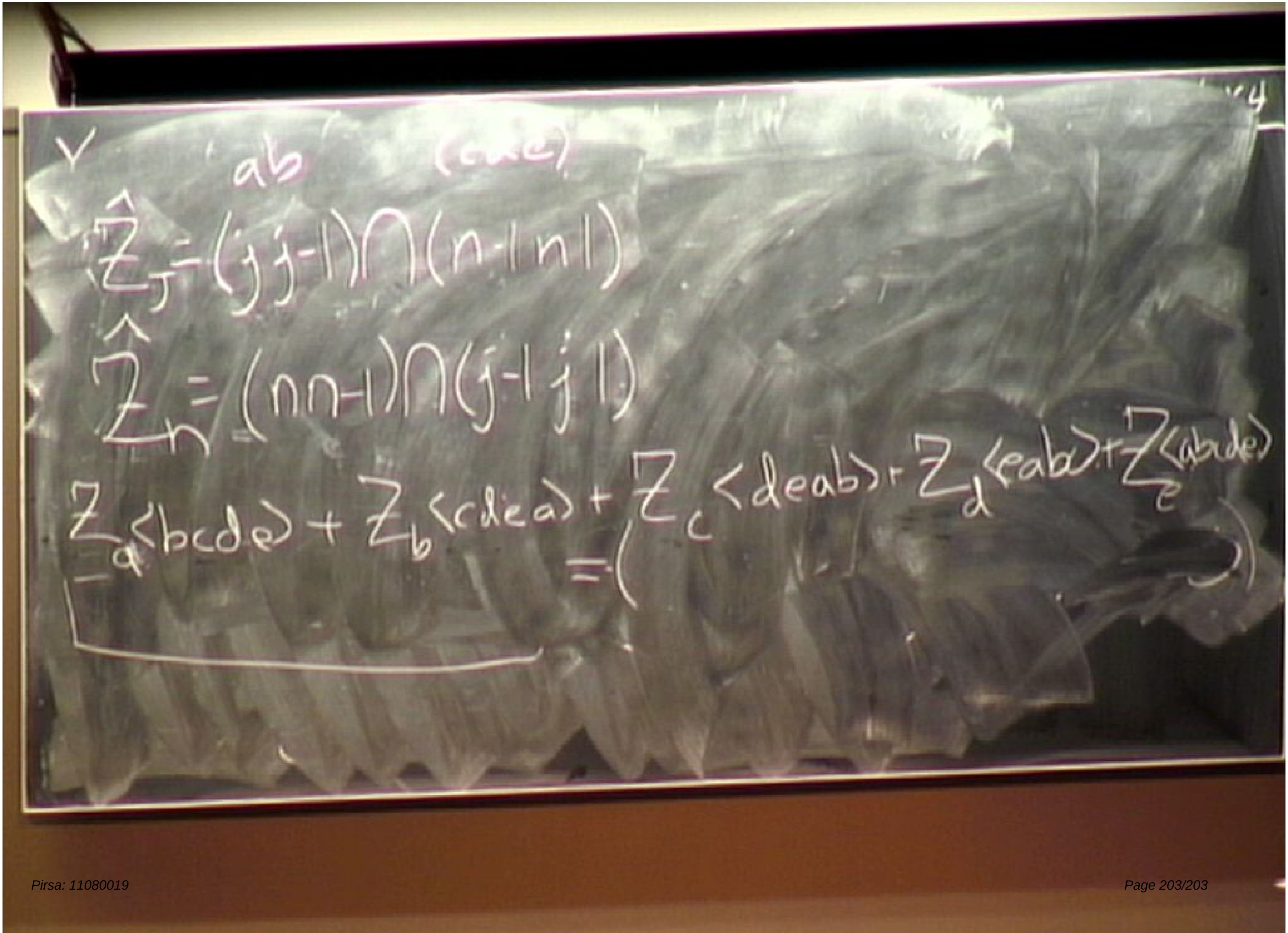
$V \quad ab \quad (cde)$

$$\hat{Z}_j = (j\, j{-}1) \cap (n{-}1\, n\, 1)$$

$$\hat{Z}_n = (n\, n{-}1) \cap (j{-}1\, j\, 1)$$

$$Z_a\langle bcde\rangle + Z_b\langle cdea\rangle + Z_c\langle deab\rangle + \quad a\rangle + Z_e\langle abcd\rangle$$

$$= ( \qquad )$$



ab (cde)

$$\hat{Z}_j = (j\, j-1) \cap (n-1\, n\, 1)$$

$$\hat{Z}_n = (n\, n-1) \cap (j-1\, j\, 1)$$

$$Z_a \langle bcde \rangle + Z_b \langle cdea \rangle + Z_c \langle deab \rangle + Z_d \langle eab \rangle + Z_e \langle abcde \rangle$$

$$= ( \quad )$$