

Title: Spin Systems as Toy Models for Emergent Gravity

Date: Aug 03, 2011 04:00 PM

URL: <http://pirsa.org/11080018>

Abstract: A number of recent proposals for a quantum theory of gravity are based on the idea that spacetime geometry and gravity are derivative concepts and only apply at an approximate level. Two fundamental challenges to any such approach are, at the conceptual level, the role of time in the emergent context and, technically, the fact that the lack of a fundamental spacetime makes difficult the straightforward application of well-known methods of statistical physics and quantum field theory to the problem. We initiate a study of such problems using spin systems as toy models for emergent geometry and gravity. These are models of quantum networks with no a priori geometric notions. In this talk we present two models. The first is a model of emergent (flat) space and matter and we show how to use methods from quantum information theory to derive features such as speed of light from a non-geometric quantum system. The second model exhibits interacting matter and geometry, with the geometry defined by the behavior of matter. This is essentially a Hubbard model on a dynamical lattice. We will see that regions of high connectivity behave like analogue black holes. Particles in their vicinity behave as if they are in a Schwarzschild geometry. Time permitting, I will show our study of the entanglement entropy of the system, which suggests particle localization near these traps.

Outline

Intro

- Quantum Gravity
- Emergent gravity & geometry
- Background independent spin systems as **models for emergent gravity & geometry**

Model 1

- **Emergent (flat) space & matter**
- **Speed of light** from a local Hamiltonian
- Spin systems on a **dynamical lattice**

Model 2

- Emergent gravity: **matter/geometry interactions**
- A **toy black hole**
- What does the matter see? **effective curved geometry**
- **Thermalization** from matter/geometry entanglement

Time, Gravity, Emergence, etc.

Summary

Emergent gravity

Gravity may be emergent:

- Thermodynamical aspects of gravity

Hawking, Unruh, Jacobson, Padmanabhan, Horava, Verlinde, ...

- AdS/CFT, matrix models

- Emergent gravity in the condensed matter sense

Volovik, Hu, Gu&Wen, Xu, ...

- Analogue models for gravity

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Emergence:

Behavior of whole system has no explanation in terms of the constituting particles, but instead comes from their collective behavior and interactions.

Distinguish between:

- complex patterns emerge from simple rules (eg game of life)



- simple structures emerge from messy and complicated building blocks (eg emergence of order)



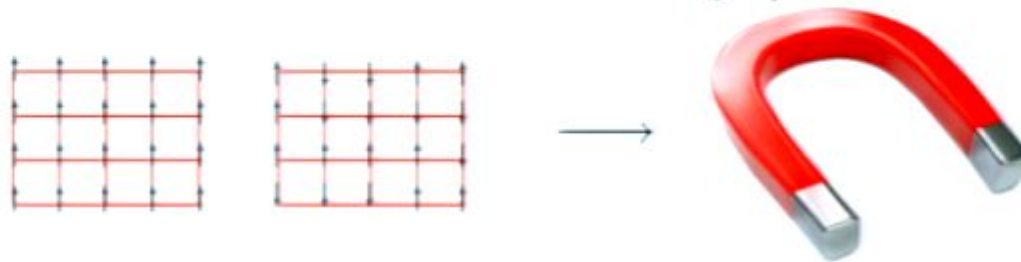
Quantum gravity=GR"+QFT

Emergent gravity: spacetime geometry and gravity are *derivative concepts*, they apply only at an approximate level.

Quantum gravity as a problem in statistical physics: we know the macrophysics (GR and QFT), we are looking for the microphysics.

Our question: How does gravity/geometry emerge from a more fundamental quantum microtheory?

- Emergence is studied in cond mat/ statistical physics. Paradigm: Ising model



What is the **Ising model for gravity**? A **universality class** for gravity?

The problem of time in quantum gravity

- Matter tells spacetime how to curve and spacetime tells matter where to go

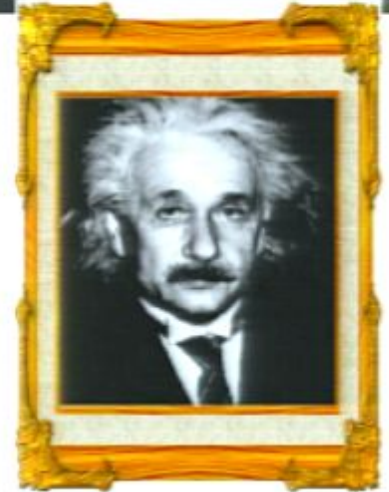
- ▶ $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = T_{\mu\nu}$

- ▶ $g_{\mu\nu}$ dynamical

- ▶ Physical quantity: $[g_{\mu\nu}]_{\text{Diff}\mathcal{M}}$

- ▶ Diffeomorphism invariance: only events and their relations are physical

Background
Independence



- In pure gravity ($T_{\mu\nu} = 0$), **time evolution is a diffeomorphism (timelessness)**.
 - ➔ If we quantize GR (LQG) we find that the Hamiltonian is a **constraint**:

$$\hat{H}|\Psi_U\rangle = 0$$

Wheeler-deWitt equation
instead of a
Schroedinger equation.
What does the RHS mean?

- ➔ Diffeos present serious problems when we try to construct local observables (already a problem in classical GR)

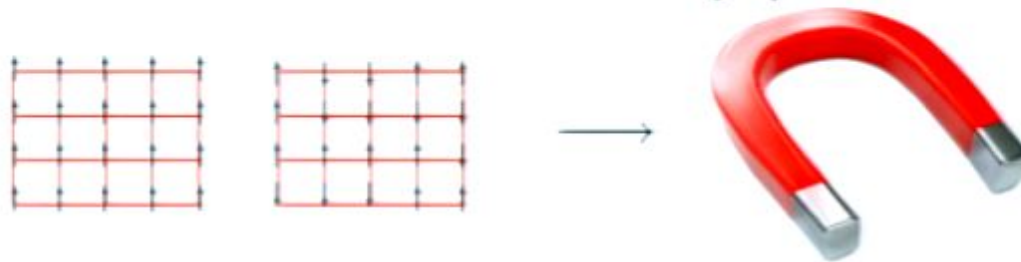
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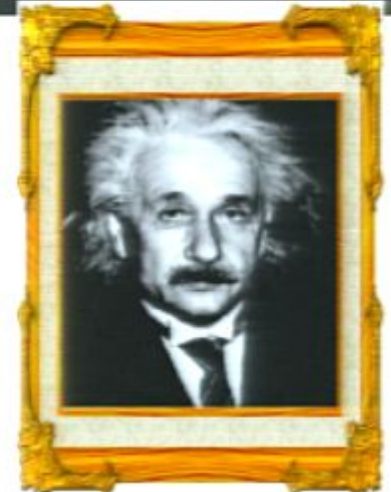
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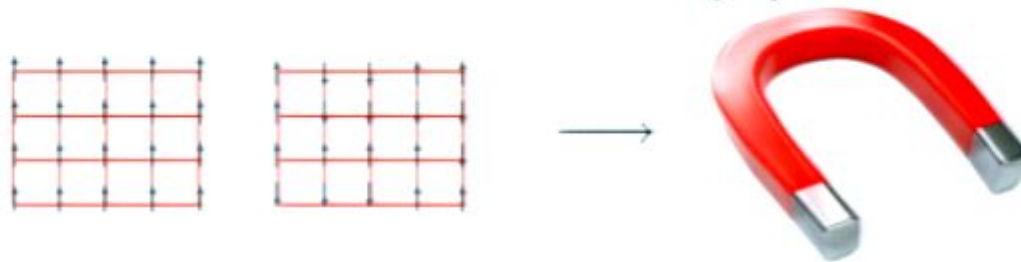
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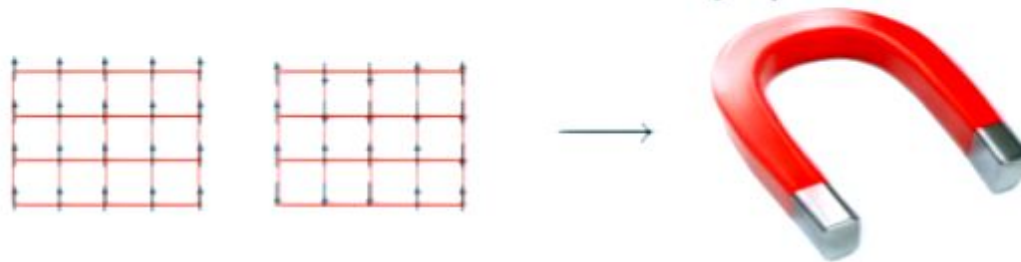
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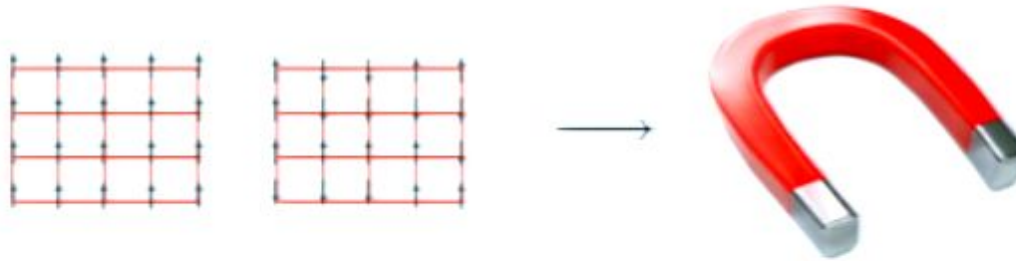


What is the **Ising model for gravity**? A **universality class** for gravity?

- Method: A relativist's viewpoint (the microscopic analogue of spacetime should be dynamical) with a cond matter toolbox (quantum many body physics, quantum information theory).

- The scale of quantum gravity is *not* beyond observable physics.

Quantum gravity=GR"+"QFT



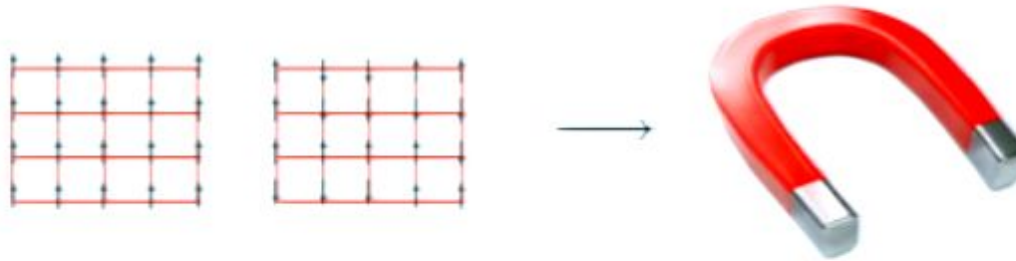
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Can we get gravity as an emergent phenomenon from a quantum many body system?

- Can **emergence** coexist with the **timelessness** of General Relativity?
 - Is there a **unified picture** underlying the different emergence hints?
 - Emergent gravity should be **testable** (e.g. approximate symmetries).
- Look for robust signatures of emergent gravity.

We propose to study the questions raised by emergence of gravity in the explicit context of a **spin system**.

Background independent spin models



What is the **Ising model for gravity**? A **universality class** for gravity?

Model 1: A spin system on a **dynamical lattice**. **The links are the spins.**
A role for quantum information theory: Information before spacetime geometry.

Model 2: “**Geometry tells matter where to go and matter tells geometry how to curve**”, in a quantum Hamiltonian.

The dynamical lattice: lattice links as spins

T.Konopka, FM & L.Smolín, hep-th/0611197

Basic idea:

Lattice = adjacency = locality (geometry) = interactions in a Hamiltonian

$$s_i \bullet \xrightarrow{J_{ij}} s_j \quad H = \sum_{ij} J_{ij} s_i s_j \quad \begin{cases} J_{ij} \neq 0 & s_i, s_j \text{ neighbors} \\ J_{ij} = 0 & s_i, s_j \text{ not adjacent} \end{cases}$$

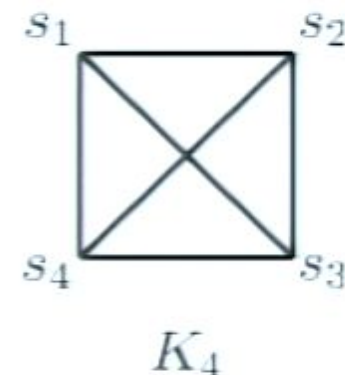
Promote link to a quantum degree of freedom $\{|1\rangle, |0\rangle\}$

qubits of adjacency

For $s_i = 1, \dots, N$, there are $\frac{N(N-1)}{2}$ possible links.

quantum geometry:
superposition of
adjacent/not adjacent

State space of models: $\mathcal{H} = \bigotimes_{e \in K_N} \mathcal{H}_e$



Spin models on a dynamical lattice

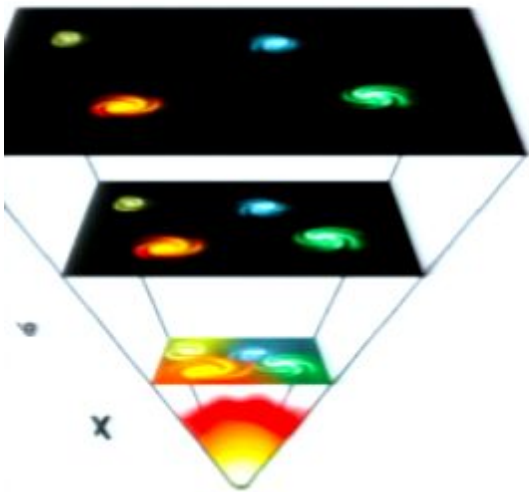
- Task: Find a Hamiltonian on a spin system that:
 - ▶ shows how a **regular geometry** emerges from “disordered” or *no* geometry
 - ▶ exhibits **primitive notions of gravity**: attraction, horizons, $c < 0$, gravitons
 - ▶ is **quantum**: matter/geometry entanglement, interference of quantum geometries, ...
 - ▶ Investigate emergent gravity and develop **new tools** in an explicit context
- So far: Questions we have looked at:
 - ▶ emergence of (flat) space
 - ▶ emergence of matter (Wen)
 - ▶ speed of light from first principles
 - ▶ matter/geometry interactions & entanglement
 - ▶ combinatorial analogue of attraction
 - ▶ quantum cosmology: quantum \rightarrow statistical?
 - ▶ Lorentz & diffeomorphism invariance vs The Lattice

Model 1

Model 1: Emergent space and matter

We currently assume an FRW geometry all the way to the Big Bang.

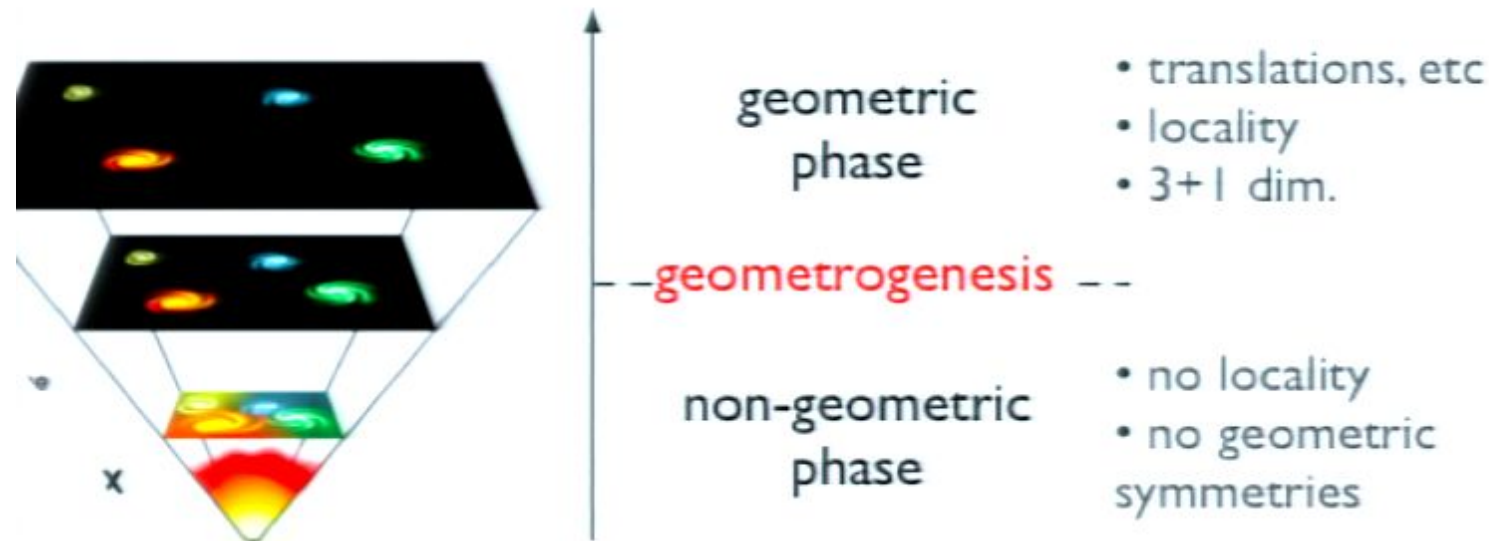
Alternative scenario: What if geometry is not fundamental?



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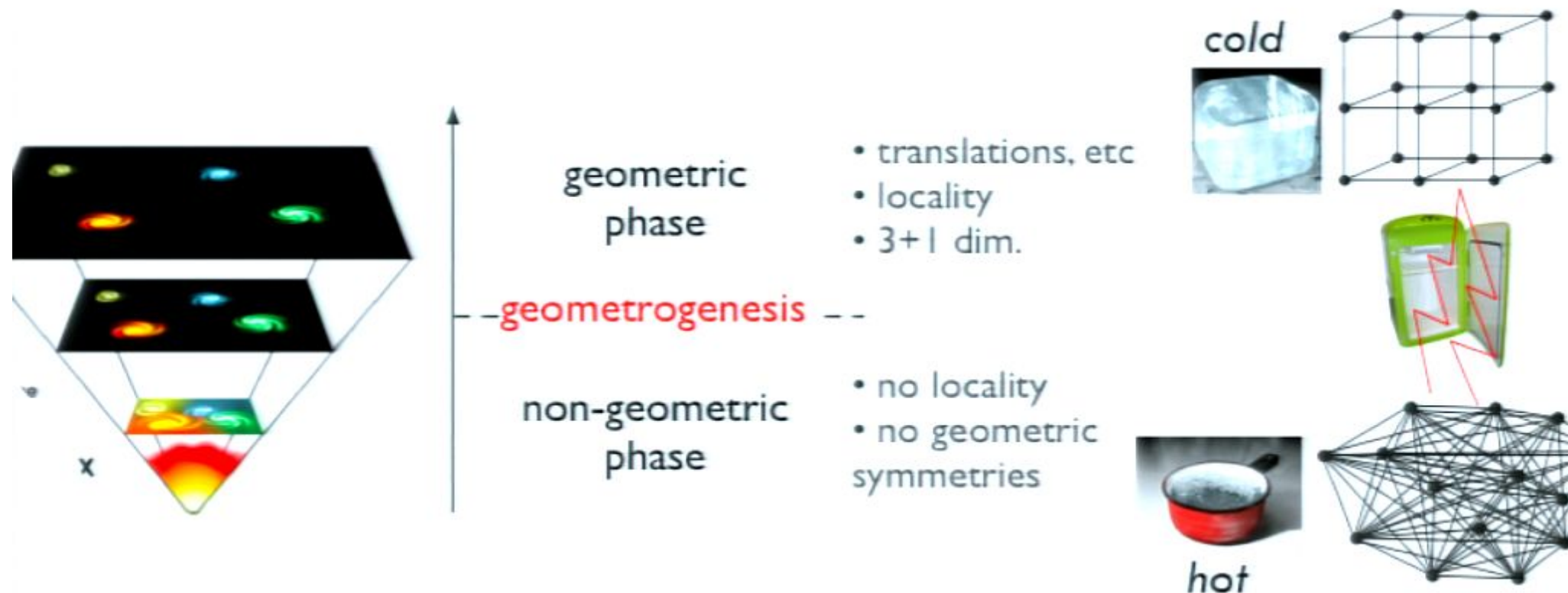
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T. Konopka, FM & S. Severini, PRD 08

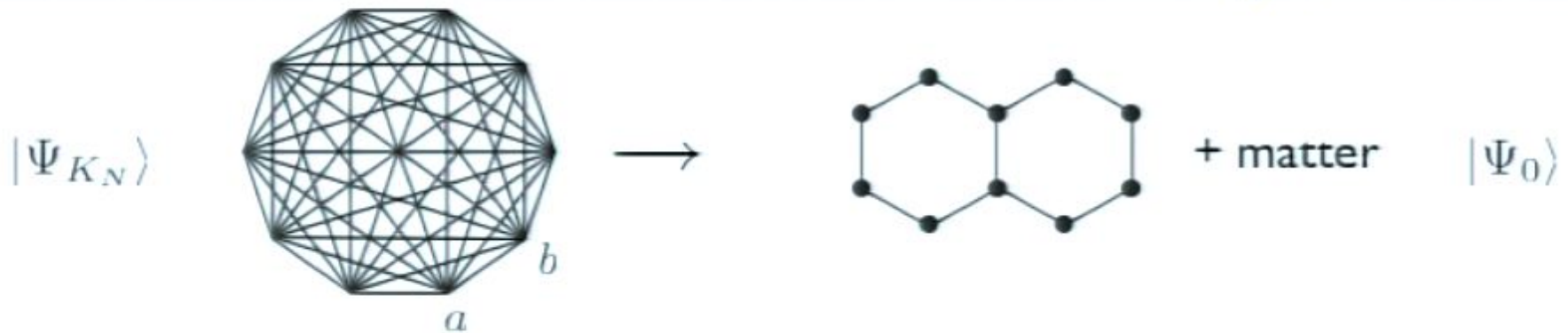


permutation symmetry
no subsystems
➡ no locality
➡ no space

translation symmetry
(FRW)
➡ meaningful distances
➡ space







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j_{ab}	m_{ab}
1  edge "on"	+1  -1  0 
0  edge "off"	0 

K_N : complete graph on N vertices.

$N(N-1)/2$ edges.

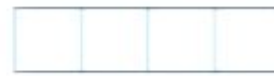
• $\mathcal{H}_{\text{edge } ab} = |j_{ab}, m_{ab}\rangle$

• $\mathcal{H} = \bigotimes_{ab} \mathcal{H}_{\text{edge } ab}$

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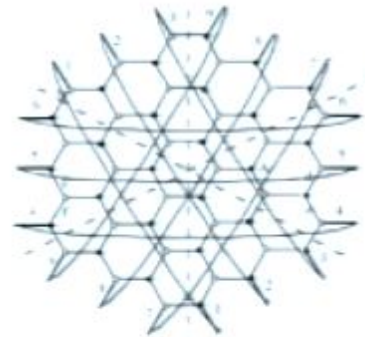
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- $$\text{on } \mathcal{H}|1,0\rangle$$

$$H_{\text{loops}} = \sum_a \left(- \sum_b g_B \delta_{ab} \sum_{L=0}^{\infty} \frac{r^L}{L!} N_{ab}^{(L)} \right)$$

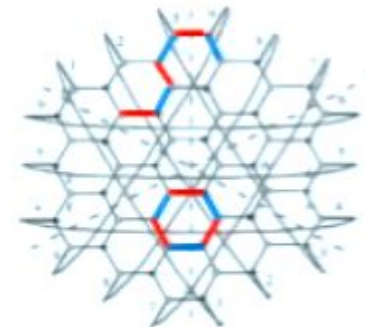


flat space is stable
local minimum

- $$\text{on full } \mathcal{H}$$

$$H_{\text{loops}} = - \sum_a \sum_{L=0}^{\infty} \frac{r^L}{L!} g_B \prod_{i=1}^L M_i^{\pm}$$

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emergent photons
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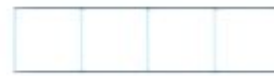
X.G.Wen, Quantum Field Theory of Many Body Systems

➡ Emergent matter when space emerges.

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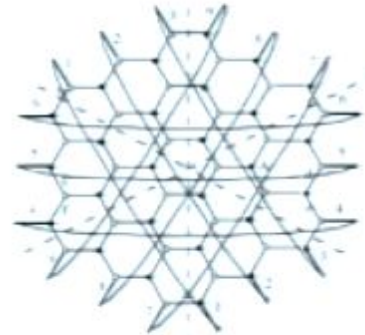
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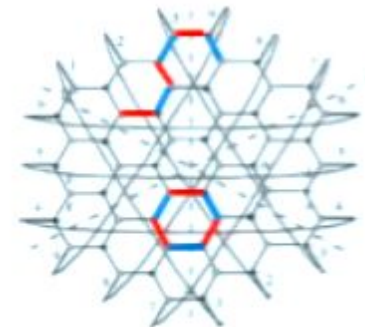


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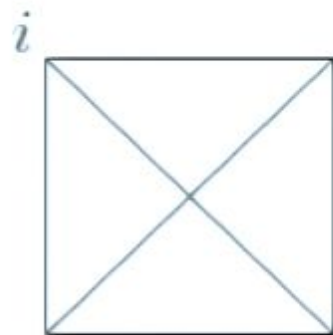
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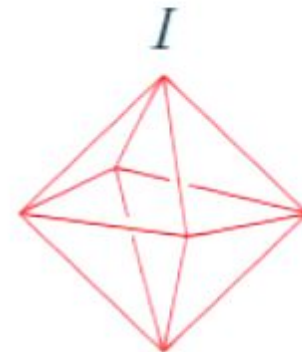
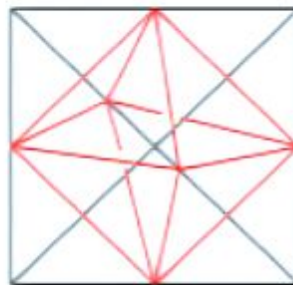
New tool: from a varying to a fixed lattice

F. Caravelli & FM, PRD 11

$K_N \longrightarrow$ Line graph of K_N



$$i = 1, \dots, N$$



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\hat{H} on dynamical lattice \longrightarrow Ising on fixed lattice

Vertex degree is a good order parameter.

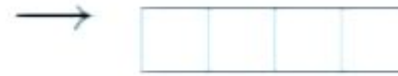
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Is a zero temperature transition good enough?

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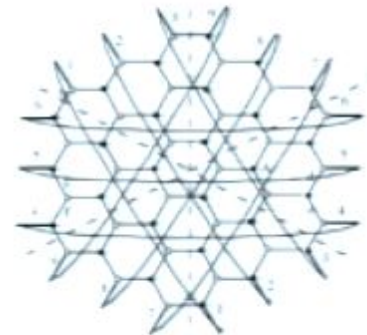
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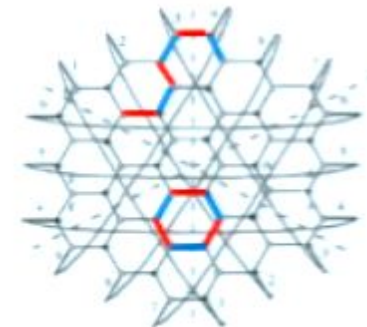


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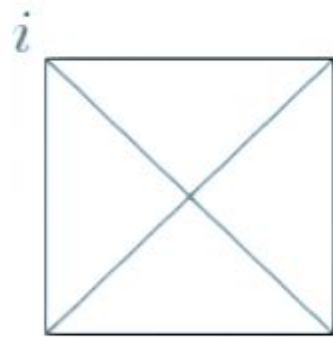
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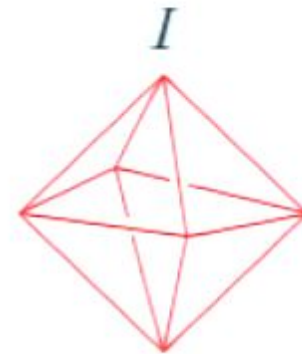
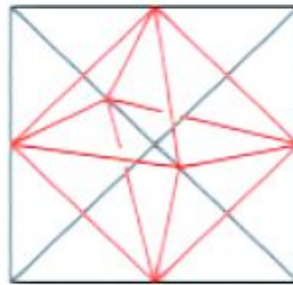
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F. Caravelli & FM, PRD 11

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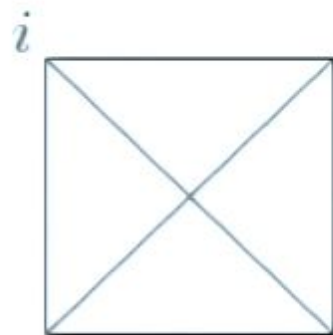
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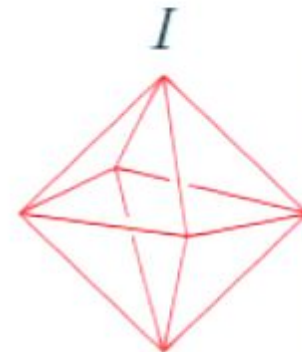
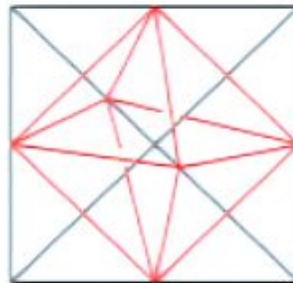
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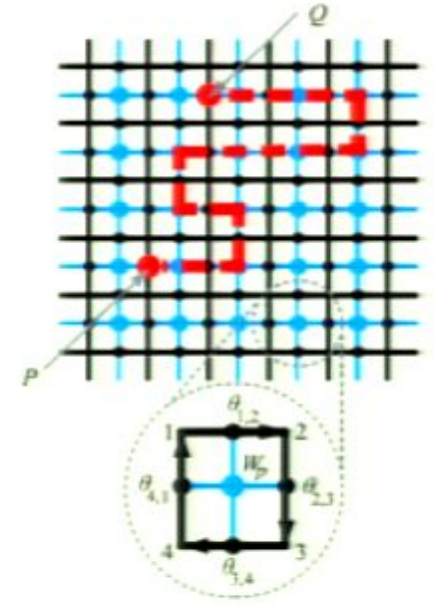
A. Hamma, FM, I. Premont-Schwarz, S. Severini, PRL 2009

Given local Hamiltonian $H = \sum_{\langle ij \rangle} h_{ij}$,

Lieb-Robinson speed of information propagation:

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$$\|[O_P(t), O_Q(0)]\| \leq 2\|O_P\|\|O_Q\| C \exp[-a(d_{PQ} - vt)]$$



Find: $v_{LR} = \sqrt{2g_B g_C}$

- $v_{LR} = c$ in the emergent Maxwell equations
- Effective finite light cones consistent with non-relativistic quantum mechanics
- $v_{LR} \sim d$

Approximating the model with a hypercube whose dimension varies with time $v_{LR}(t) \sim D(t)$

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Does an emergent observer see Minkowski space?

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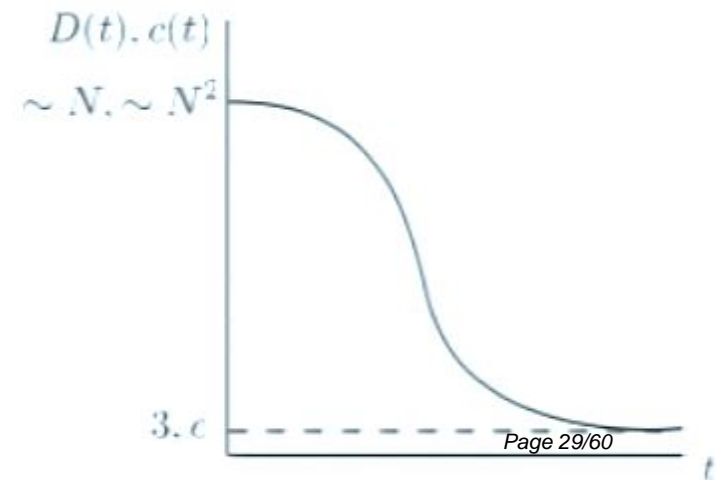
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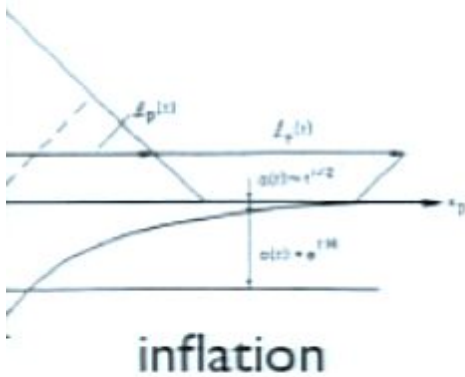
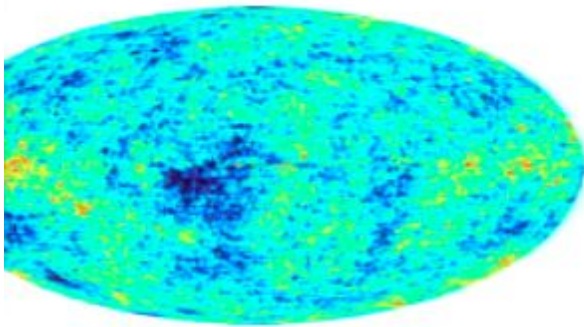
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Mismatched locality

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Evolving speed of light and the horizon problem:



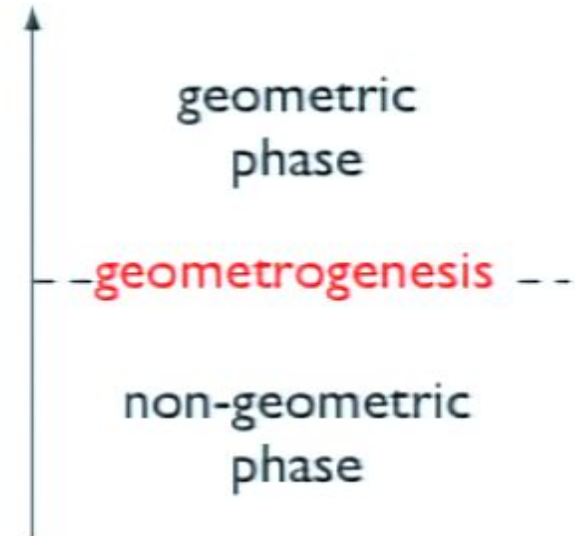
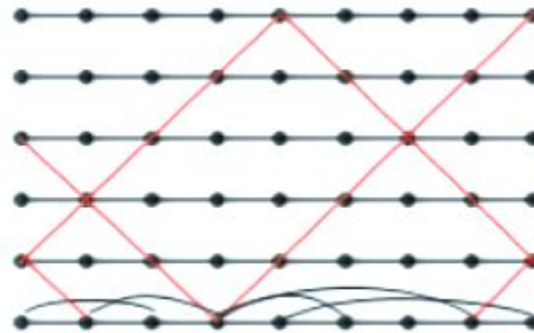
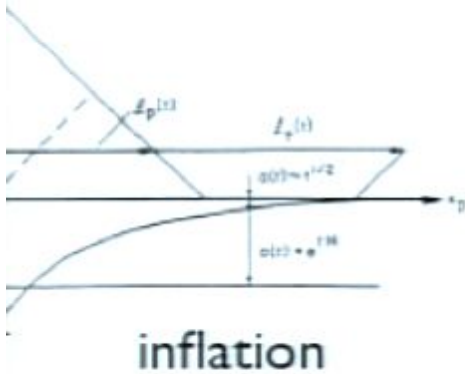
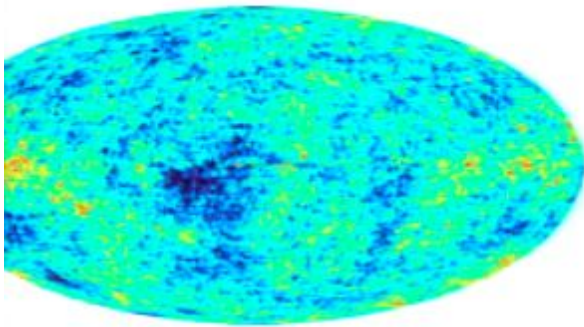
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A.Hamma, FM, I. Premont-Schwarz, S.Severini, PRL 2009

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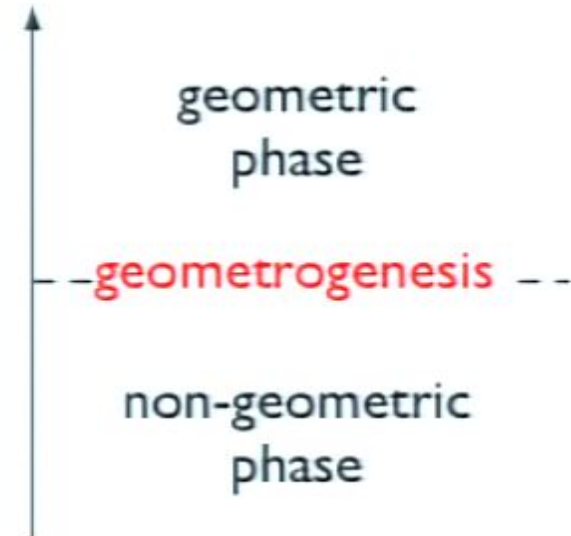
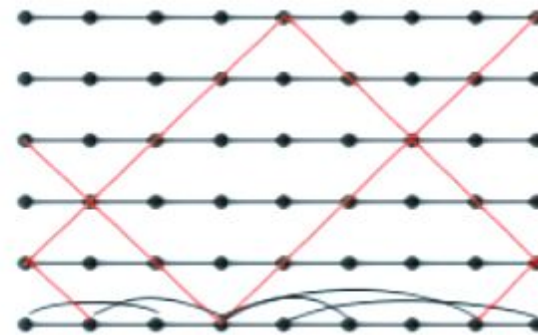
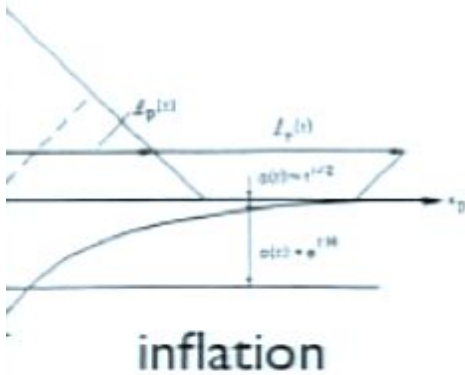
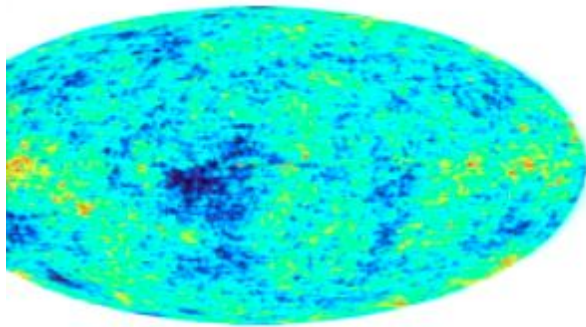


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Model illustrates the effect of a **transition in the local structure**

Note: a phase transition in the speed of light can also reproduce the scale invariant CMB spectrum (Variable Speed of Light cosmology)

Model 2

Motivation for Model 2:

- The regular geometry appears at low energy. How does the system lower its energy? What we did is equivalent to an external heat bath.

We want a **unitary model** of the universe.

- **Matter and geometry** come from the same microscopic degrees of freedom. This is a very interesting feature of this model but we also want to study a model with state space $\mathcal{H} = \mathcal{H}_{\text{geometry}} \otimes \mathcal{H}_{\text{matter}}$

- Can we write an Ising model-type system that realizes “geometry tells matter where to go and matter tells geometry how to curve”? If so, investigate:

Gravity \rightleftharpoons **geometry tells matter where to go and matter tells geometry how to curve.**

Model 2: Interacting matter-geometry

Hamma, Lloyd, FM, Caravelli, Severini, Markstrom, PRD2010

Gravity:

Geometry tells matter where to go and matter tells geometry how to curve.



$$\mathcal{H} = \bigotimes_e \mathcal{H}_e \bigotimes_v \mathcal{H}_v$$

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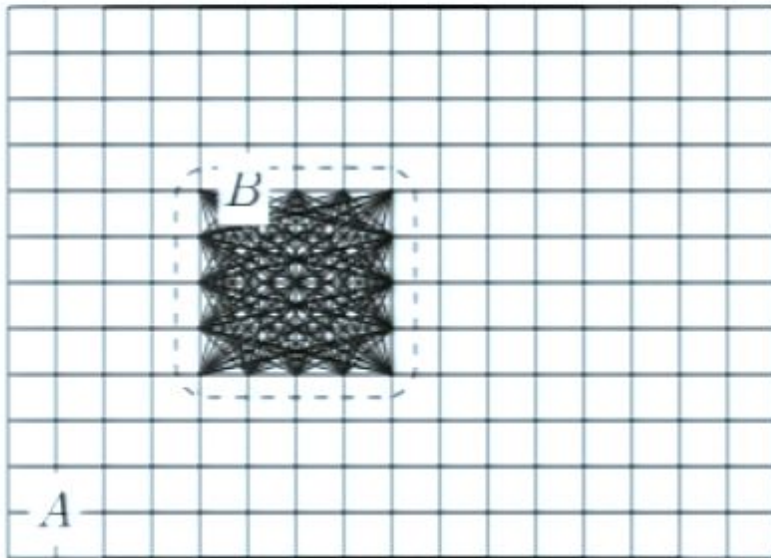
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Model 2: A toy black hole

Hamma, Lloyd, FM, Caravelli, Severini, Markstrom, PRD2010



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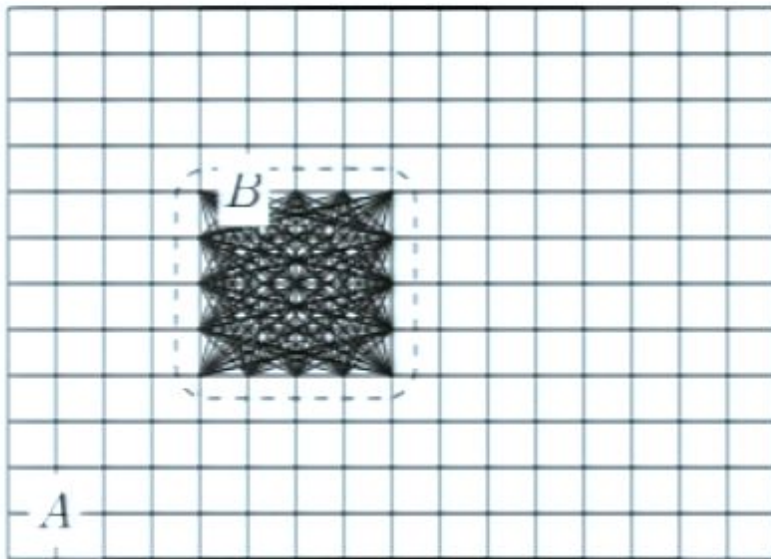
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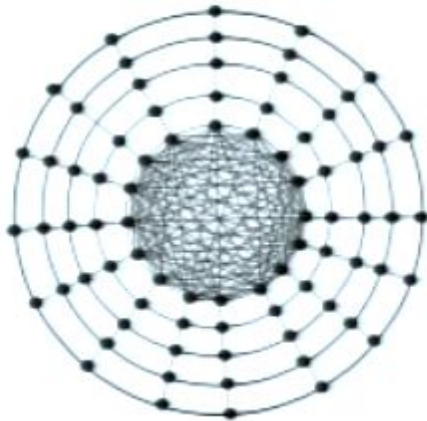
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Caravelli, Hamma, FM, Riera

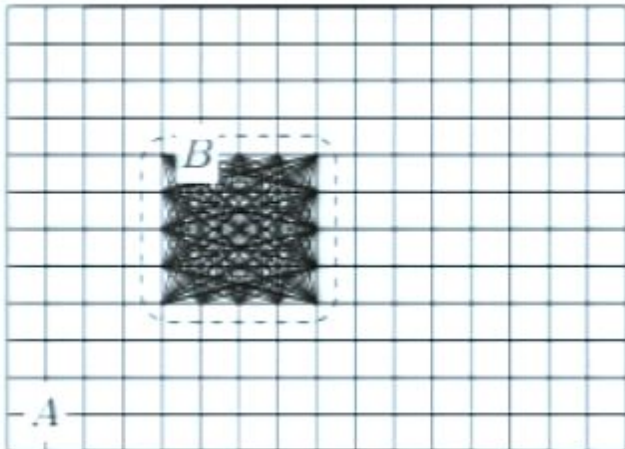


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Hamma, Lloyd, FM, Caravelli, Severini, Markstrom, PRD2010, Hamma, FM NJP2011

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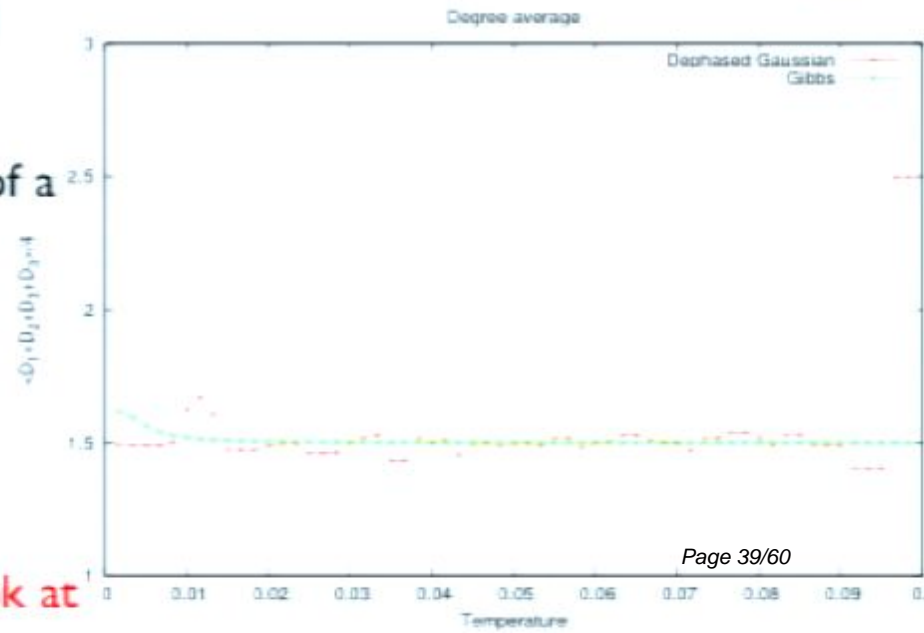
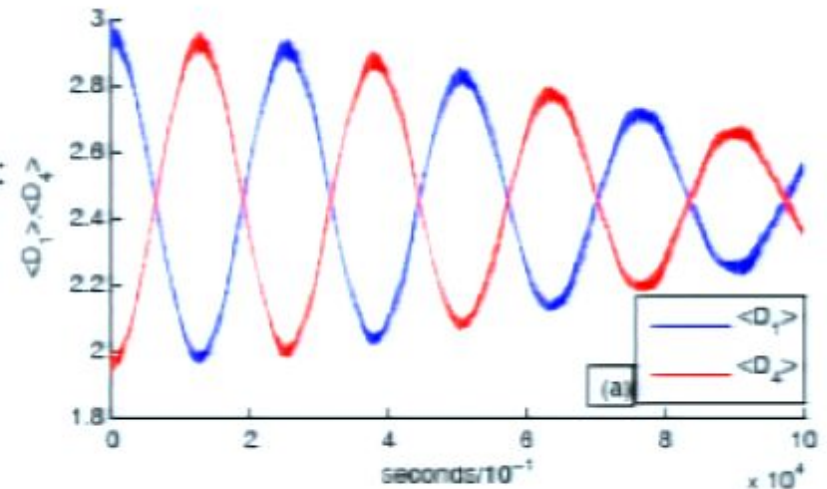
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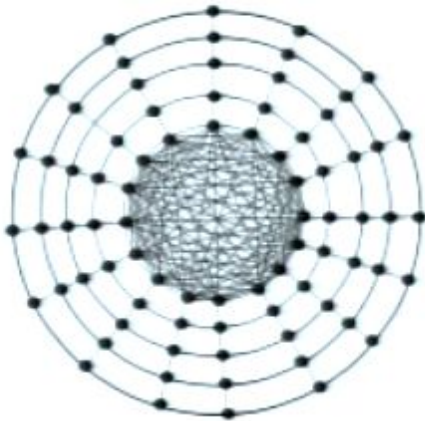
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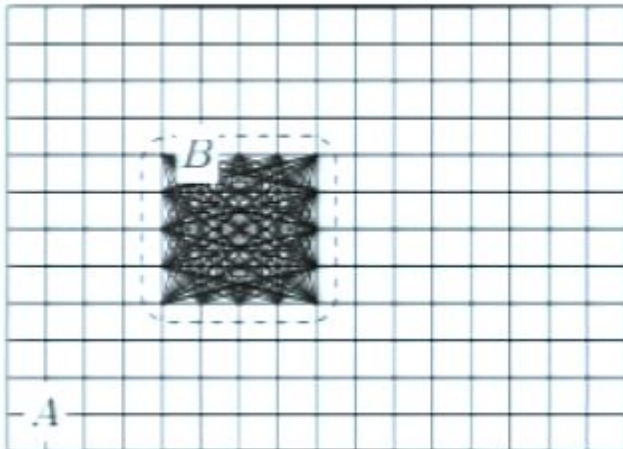


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Our toy BHs are an explicit example of the above (infinite-dim spin systems).

Analogue models for gravity, or more?

Hindsight-aided motivation: analogue gravity



World of observer inside fluid (water, BEC, ...):

- is Lorentzian
- has horizons
- has Hawking radiation
- ➔ Gravity analogues in the lab.

Unruh. Cirac. Visser.
Weinfurtner. Liberati...

Our spin systems can be viewed as a new kind of analogue gravity models: no background fluid, fully dynamical

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{ Quantum many-body system \rightarrow Gravity (FAPP) : quantum gravity!
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Find fundamental obstructions to the analogy: learn why gravity is *not* emergent

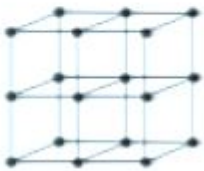
Summary

A. Hamma, FM NJP(2011), 1011.5754

Underlying spatial geometry is the notion of adjacency.



metric: neighbours or not?



E.g. a 3d euclidean geometry is a particular *order* of adjacencies that exhibits certain symmetries. Our geometric world is a *phase* (geometrogenesis), we froze to that phase.

By promoting adjacency to a qubit and considering any network as a subgraph of K_N we have a convenient way to deal with superpositions of quantum geometries.

When the adjacency qubits are dynamical dofs, we have an important ingredient of GR (dynamical geometry).

Local interactions mean a finite speed of information propagation. Given local dynamics on network of adjacencies, we can define a spacetime with finite lightcone structure. (Is it also universal?)

If we find microscopic dynamics that simulate GR, we will have reconciled quantum with gravity. With emergent gravity, we need to *explain*, not quantize the Einstein Equations!

Summary

Spin systems on a dynamical lattice as models for emergent gravity.

Questions we have looked at:

- ▶ emergence of (flat) space
- ▶ emergence of matter (Wen)
- ▶ speed of light from first principles
- ▶ matter/geometry interactions & entanglement
- ▶ combinatorial analogue of attraction
- ▶ quantum cosmology: quantum vs statistical?
- ▶ Lorentz & diffeomorphism invariance vs The Lattice

What we'd like to understand:

- ▶ Emergence, time and background independence
- ▶ Fundamental time in Hamiltonian vs geometric time: can emergence allow us to have our cake and eat it?
- ▶ Can symmetries such as Lorentz invariance, diffeomorphisms, be emergent?
- ▶ Quantization of GR: fundamental vs "phonons". Observational signature?
- ▶ Gravity from first principles: emergent

gravity requires explaining gravity.

Some of the things to do next:

- Thermalization and equilibration to a regular lattice
- Timescale of thermalization (scrambling)
- Lieb-Robinson speed of light in the continuum limit and for infinite-dimensional systems
- Tensor network renormalization for the low energy physics (Gu&Wen, Vidal, Vedral, ...)
- Recast the partition function of the spin system in the stabilizer formalism (Briegel et al)
- A new type of analogue models? Can a designer fluid (instead of BEC or water) show dynamical aspects of GR?

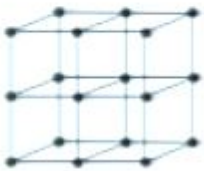
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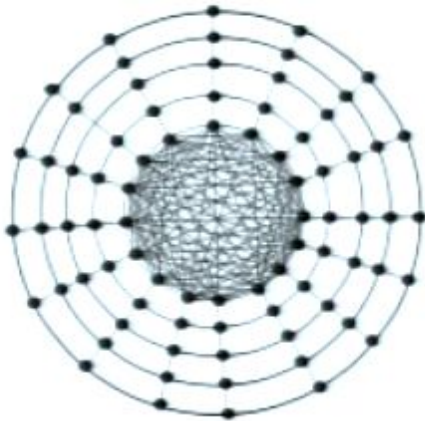
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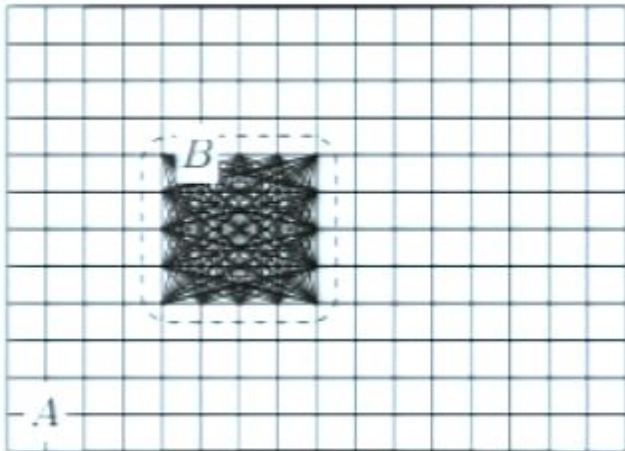
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Caravelli, Hamma, FM, Riera



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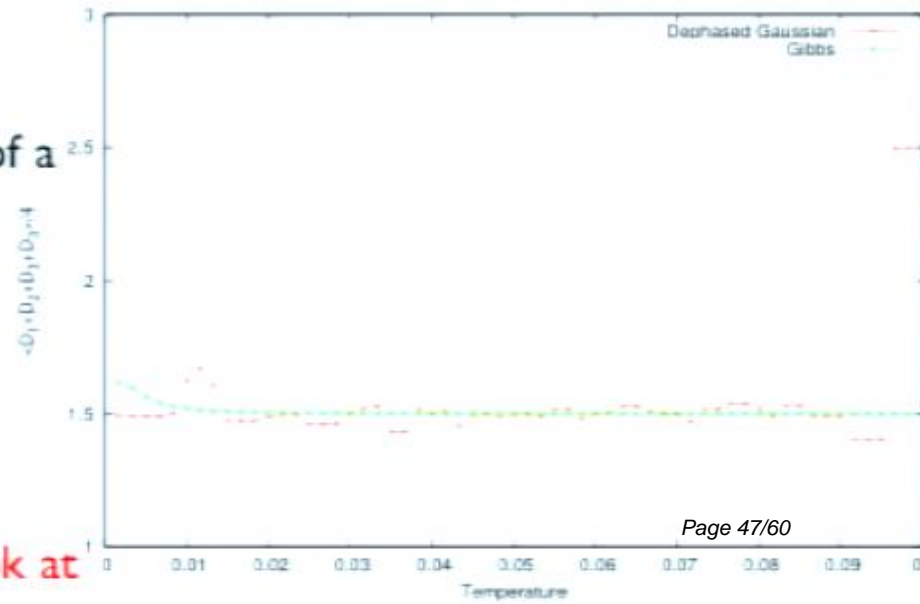
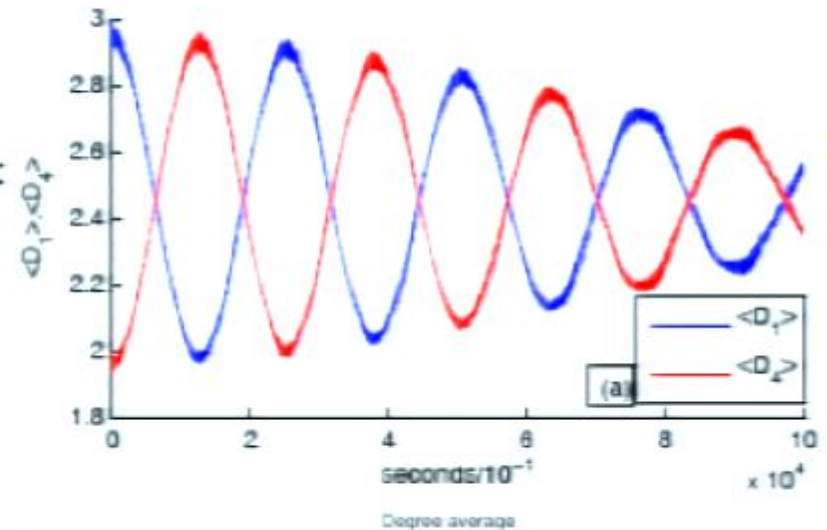
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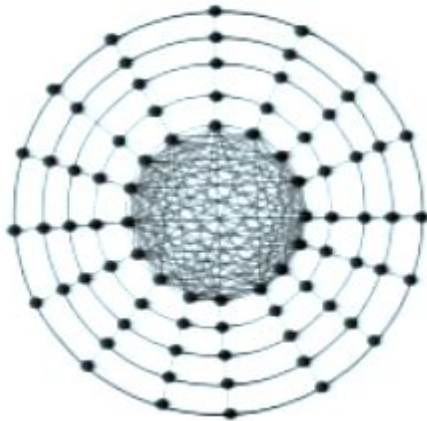
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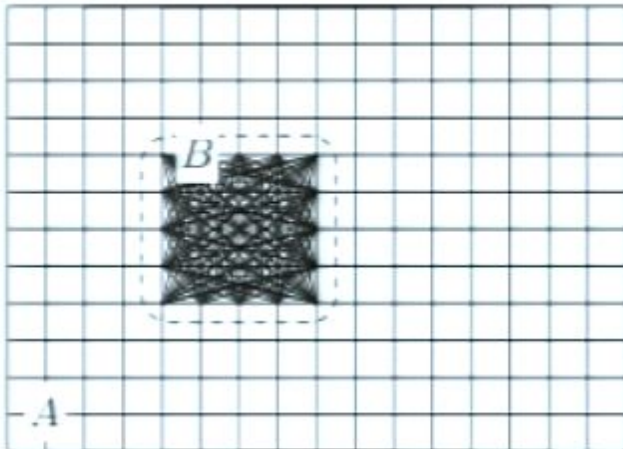


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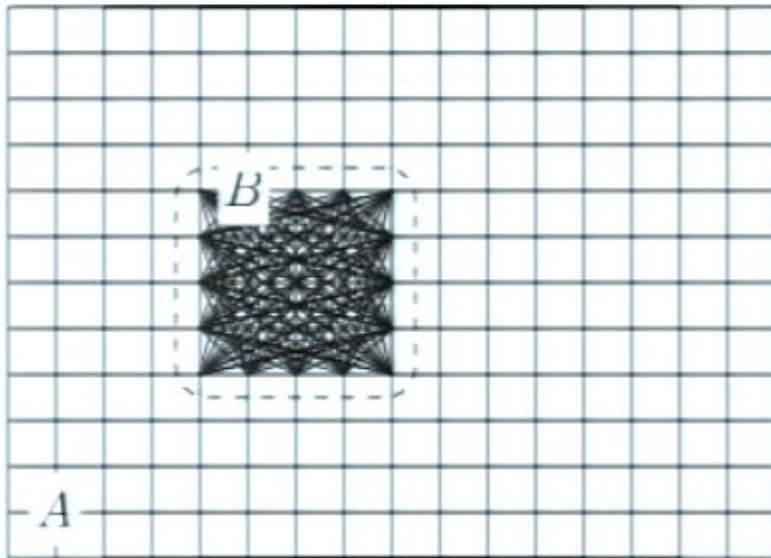
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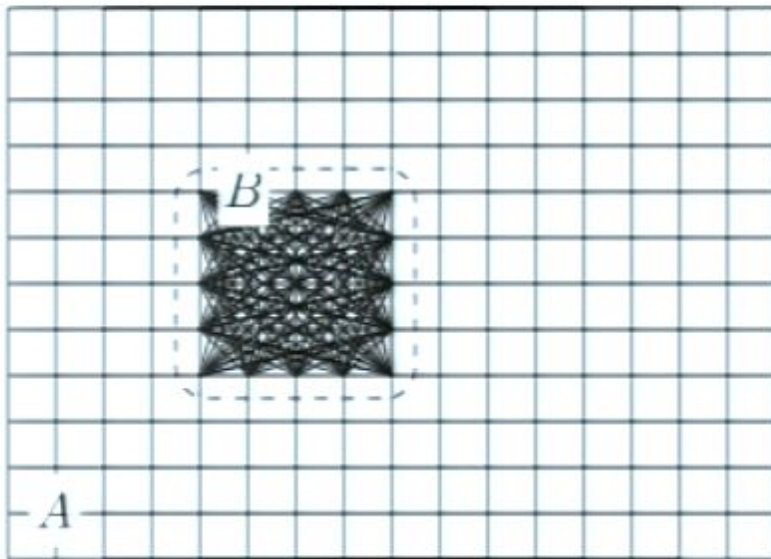
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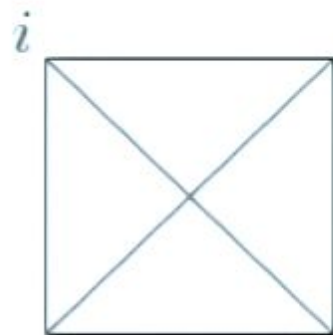
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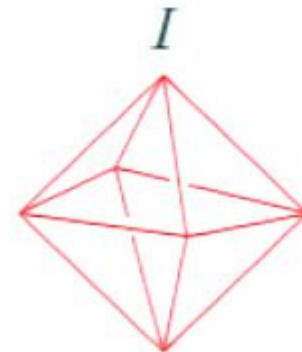
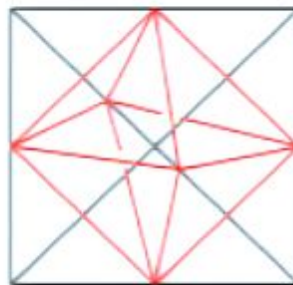
New tool: from a varying to a fixed lattice

F. Caravelli & FM, PRD 11

$K_N \longrightarrow$ Line graph of K_N



$i = 1, \dots, N$



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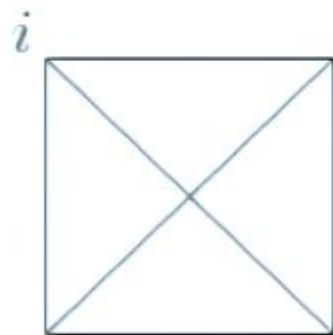
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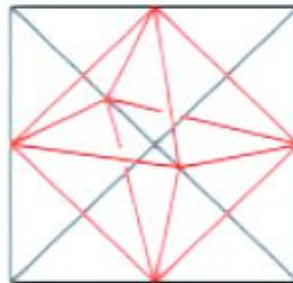
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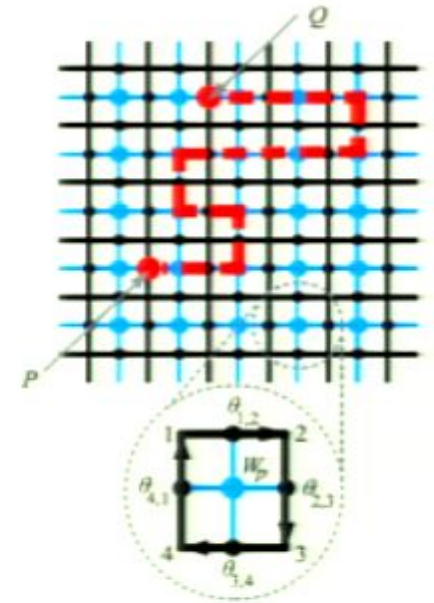
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$$\|[O_P(t), O_Q(0)]\| \leq 2\|O_P\|\|O_Q\| \sum \frac{(2|t|h_{max})^n}{n!} N_{PQ}(n)$$

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Find: $v_{LR} = \sqrt{2g_B g_C}$

- $v_{LR} = c$ in the emergent Maxwell equations
- Effective finite light cones consistent with non-relativistic quantum mechanics
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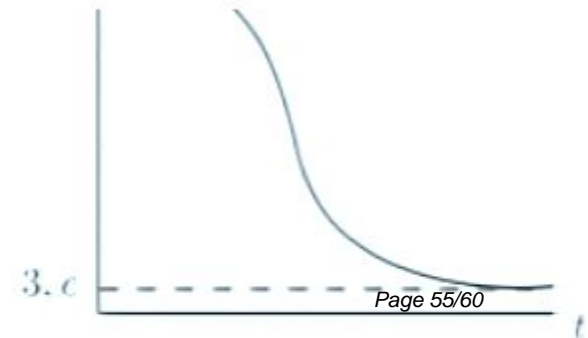
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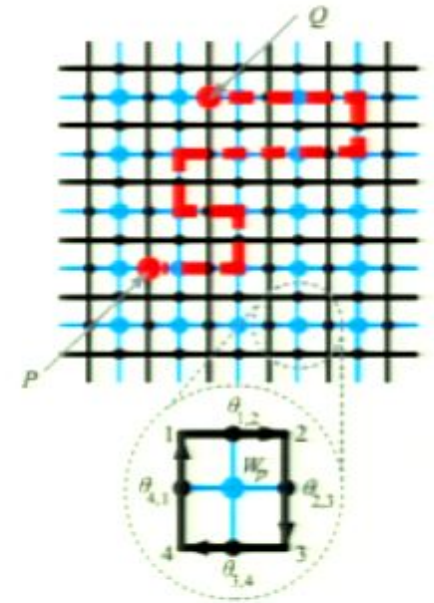
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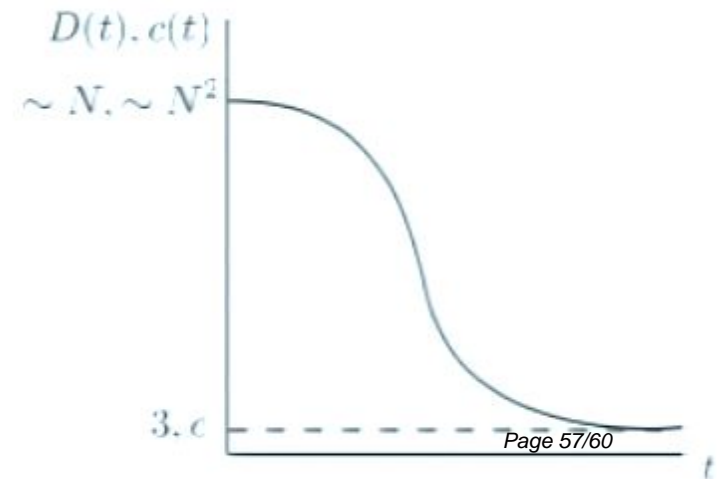
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- $v_{LR} \sim d$

Approximating the model with a hypercube whose dimension varies with time $v_{LR}(t) \sim D(t)$

Does an emergent observer see Minkowski space?

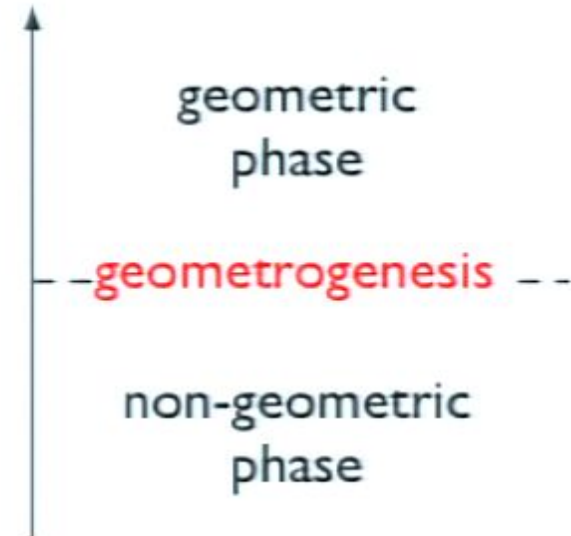
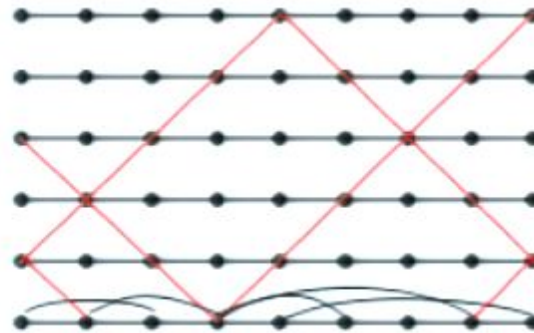
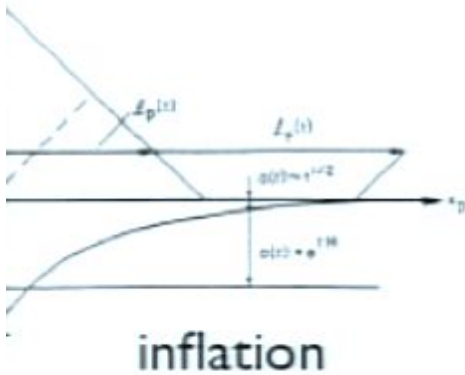
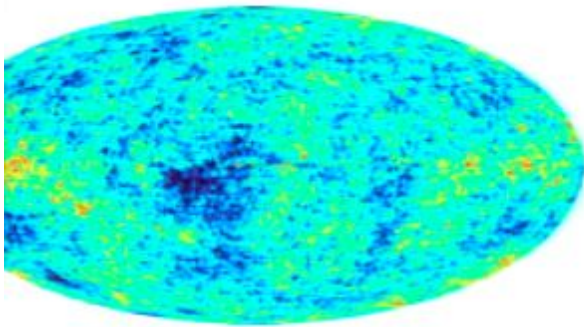


Mismatched locality

A. Hamma, FM, I. Premont-Schwarz, S. Severini, PRL 2009

Evolving speed of light and the horizon problem:

F. Markopoulou and L. Smolin, "Disordered locality in loop quantum gravity states",
Classical and Quantum Gravity
 24: 3813-3824 (2007) (gr-qc/0702044).



Why don't we see quantum spacetimes?

Hamma, Lloyd, FM, Caravelli, Severini, Markstrom, PRD2010, Hamma, FM NJP2011

Matter as the heat bath for geometry:

Thermalization in quantum cosmology: While the whole system evolves unitarily, locally it can look thermal (Page 13). How to prove?

New results in thermalization in closed quantum systems:

- Consider part of a closed system:

$$H = H_{\text{graph}} + H_{\text{matter}} + H^I$$

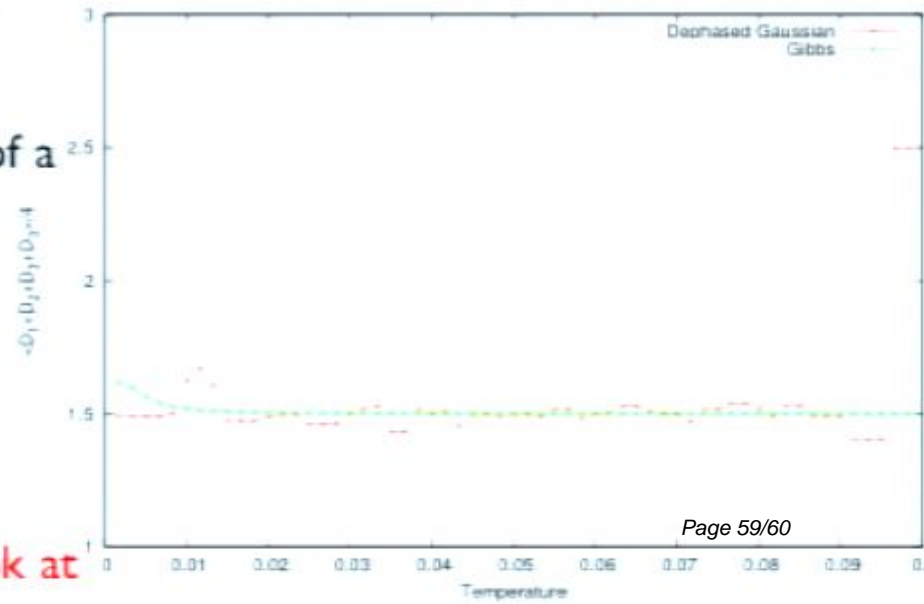
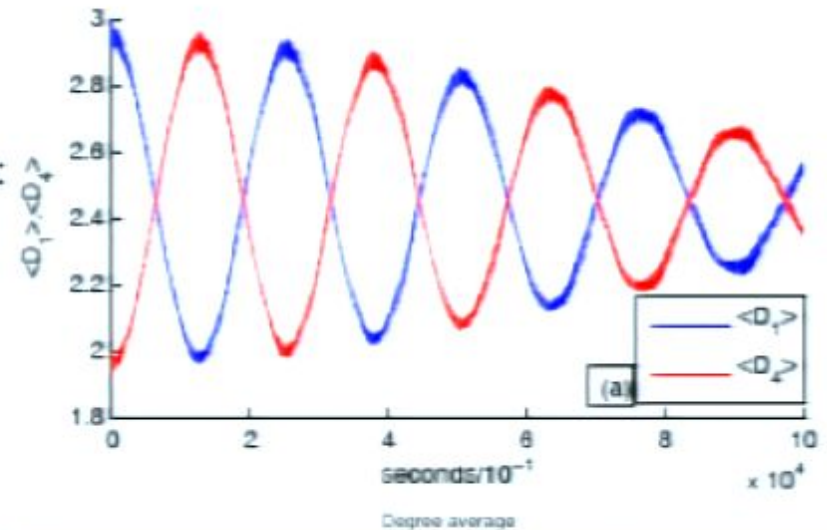
$$\rho_{\text{matter}}(t) = \text{Tr}_{\text{graph}} \rho(t)$$

- Insulator/superfluid regions: thermalization of initial state of no particles on K_N

- Typical values of observables correspond to those of a canonical ensemble:

$$\langle \Gamma \rangle(t) = \text{Tr} \{ \rho(t) \Gamma \otimes \mathbf{1}_{\text{matter}} \}$$

$$\lim_{t \rightarrow \infty} \langle \Gamma \rangle(t) \simeq \frac{\text{Tr} \{ \Gamma e^{-\beta(E) H_{\text{graph}}} \}}{\text{Tr} \{ e^{-\beta(E) H_{\text{graph}}} \}}$$



Analogue models for gravity, or more?

Hindsight-aided motivation: analogue gravity



World of observer inside fluid (water, BEC, ...):

- is Lorentzian
- has horizons
- has Hawking radiation
- ➔ Gravity analogues in the lab.

Unruh. Cirac. Visser.
Weinfurtner. Liberati...

Our spin systems can be viewed as a new kind of
analogue gravity models: no background fluid, fully dynamical

ush analogy:

{ Quantum many-body system \rightarrow Gravity (FAPP) : quantum gravity!
(no geometric dofs)

Find fundamental obstructions to the analogy: learn why gravity is *not* emergent