

Title: Aspects of the Generalized Unitarity Method

Date: Aug 04, 2011 09:30 AM

URL: <http://pirsa.org/11080017>

Abstract: The generalized unitarity method reconstructs loop-level scattering amplitudes from tree-level scattering amplitudes. We review some of the steps of this method which are necessary for applications in any quantum field theory, discuss several points of view and emphasize some of the aspects that can be easily automated.

We mainly use  $N=4$  super-Yang-Mills theory to illustrate the discussion and comment on the necessary changes for applications to super-Yang-Mills theories with reduced supersymmetry.

## Aspects of generalized unitarity

- tree-level comments
- generalized unitarity: generalities, sums, regularization, type of cuts, etc
- Supersums; example, general props.
- numerics vs. analytics
- general strategy for supersums.
- ...

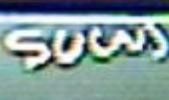
• Trees   
A<sup>(c)</sup>

• Trace  $\leftarrow$  <sup>we sum</sup>

$$A^{(t)} = N \sum_{S \neq \emptyset} Tr(T^1 \dots T^n) \cdot \tilde{A}^0(\text{L}_0, \dots \text{L}_{n-|S|})$$

$$+ \sum Tr(\quad) Tr(\quad) A^{(t)}(\quad)$$

$$+ \sum Tr(\quad) Tr(\quad) Tr(\quad) \tilde{A}^0(\quad)$$

• Traces  

$$A^{(4)} = N \sum_{\substack{s_1, s_2 \\ s_1 \neq s_2}} \text{Tr}(\tau^s \dots \tau^{s_4}) A^0(k_1, \dots, k_4)$$

$$+ \sum \text{Tr}(\quad) \text{Tr}(\quad) A^{(4)}(\quad)$$

$$+ \sum \text{Tr}(\quad) \text{Tr}(\quad) \text{Tr}(\quad) A^0(\quad)$$

$$\overline{Q} = A^0(k_1 k_2 k_3 k_4) + A^0(k_1 k_2 k_1 k_2) + A^0(k_1 k_2 k_2 k_3)$$

• Trace + sum

$$A^{(l)} = N \sum_{\sigma_1 \sigma_2} \text{Tr}(\tau^{\sigma_1} \dots \tau^{\sigma_n}) \tilde{A}^l(k_1, \dots k_n, \sigma_1 \dots \sigma_n)$$

$$+ \sum \text{Tr}(\quad) \text{Tr}(\quad) \tilde{A}^l(\quad)$$

$$+ \sum \text{Tr}(\quad) \text{Tr}(\quad) \text{Tr}(\quad) \tilde{A}^l(\quad)$$

$$\overline{G} = \tilde{A}^l(k_1 k_2 k_3 k_4) + \tilde{A}^l(k_1 k_2 k_3 k_4) + \tilde{A}^l(k_1 k_2 k_3 k_4)$$

$$\sum_{\sigma_2 \dots \sigma_n} A(l, \sigma_2 \dots \sigma_n) = 0 \quad U(l) \text{ decoupling in}$$

• less  $\neq$  more (true)

$$A^{(n)} = N \sum_{S \in \Sigma} T_r(T^1 \dots T^n) A^{\langle} (k_1, \dots k_n, \omega)$$

$$+ \sum T_r(\quad) T_r(\quad) A^{\langle} (\quad)$$

$$+ \sum T_r(\quad) T_r(\quad) T_r(\quad) A^{\langle} (\quad)$$

$$\overline{0} = A^{\langle} (k_1, k_2, k_3, k_4) + A^{\langle} (k_1, k_2, k_1, k_2) + A^{\langle} (k_1, k_2, k_2, k_3)$$

$$\sum_{\sigma_2=1} A(\_, \sigma_2 \dots \sigma_n) = 0 \quad U(1) \text{ decoupling in}$$

$O = A$  UNDE...n

$$= \underbrace{\text{Tr}_{1234} A_{1234}}_{T^4 = I} + \underbrace{\text{Tr}_{123} A_{123}}_{T^3 = 0} + \underbrace{\text{Tr}_{12} A_{12}}_{T^2 = -I}$$
$$+ \underbrace{\text{Tr}_1 A_1}_{T^1 = 0} + \underbrace{A_{0000}}_{T^0 = 1}$$



$\sigma = \lambda$

$$\therefore \underbrace{\text{Tr}_{1234} A_{1234}} + \underbrace{\text{Tr}_{1232} A_{1232}} + \underbrace{\text{Tr}_{1233} A_{1233}} \text{ is zero.}$$

$$T^3 = 1$$

$$\bullet A(1 \dots n) = (-)^n A(n \dots 1)$$

$$\begin{aligned}
 O &= A^{\text{UN} \in \dots n} \\
 &= \underbrace{\text{Tr}_{x_{234}}}_{\text{Tr}_x} A_{1234} + \underbrace{\text{Tr}_{x_{342}}}_{\text{Tr}_x} A_{1342} + \underbrace{\text{Tr}_{x_{423}}}_{\text{Tr}_x} A_{1423} \quad \text{to work.} \\
 T^* &= \Delta
 \end{aligned}$$

- $A(1\dots n) = (-)^n A(n\dots 1)$
- kreiss - kreis rel's
- BCJ relations :  $A_T = \frac{n_1 c_1}{\epsilon} + n_3$

$$O = A^{(1 \dots n)}$$

$$= \underbrace{T_{1234} A_{1234}}_{T^4 = 1} + \underbrace{Tr_{123} A_{123}}_{\text{reduzieren}} + \underbrace{Tr_{123} A_{1423}}_{\text{reduzieren}}$$

$$T^4 = 1$$

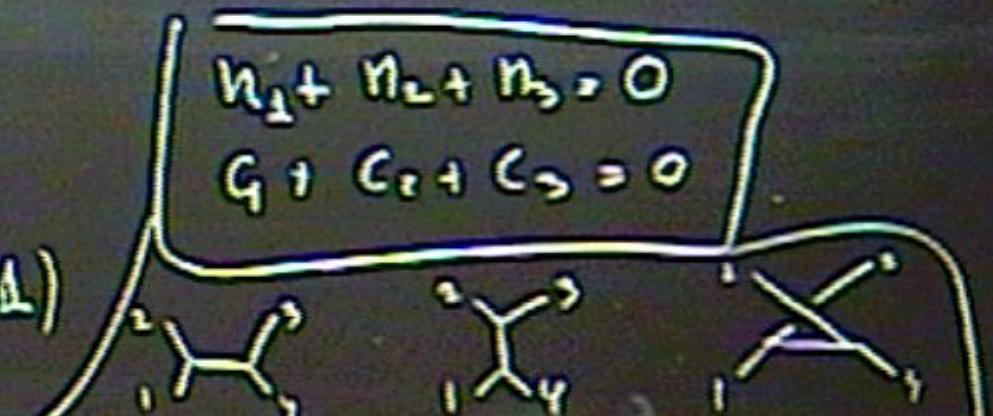
$$\bullet A^{(1 \dots n)} = (-)^n A^{(n \dots 1)}$$

Kleiss-Kugff rel's

$$\bullet \text{BCJ relations: } A_4 = \frac{n_1 G_1}{s} + \frac{n_2 G_2}{t} + \frac{n_3 G_3}{u}$$

$$n_1 + n_2 + n_3 = 0$$

$$G_1 + G_2 + G_3 = 0$$



$$1 - S^+ S^- \rightarrow 2 J_\mu T = T^\mu_{\phantom{\mu}n} T^{n*}_{\phantom{n}*}$$
$$S \cdot 1 - i T$$



$$1 \cdot S^+ S^- \rightarrow 2 J_\mu T = T^\mu_{\phantom{\mu}L} T^\nu_{\phantom{\nu}R}$$

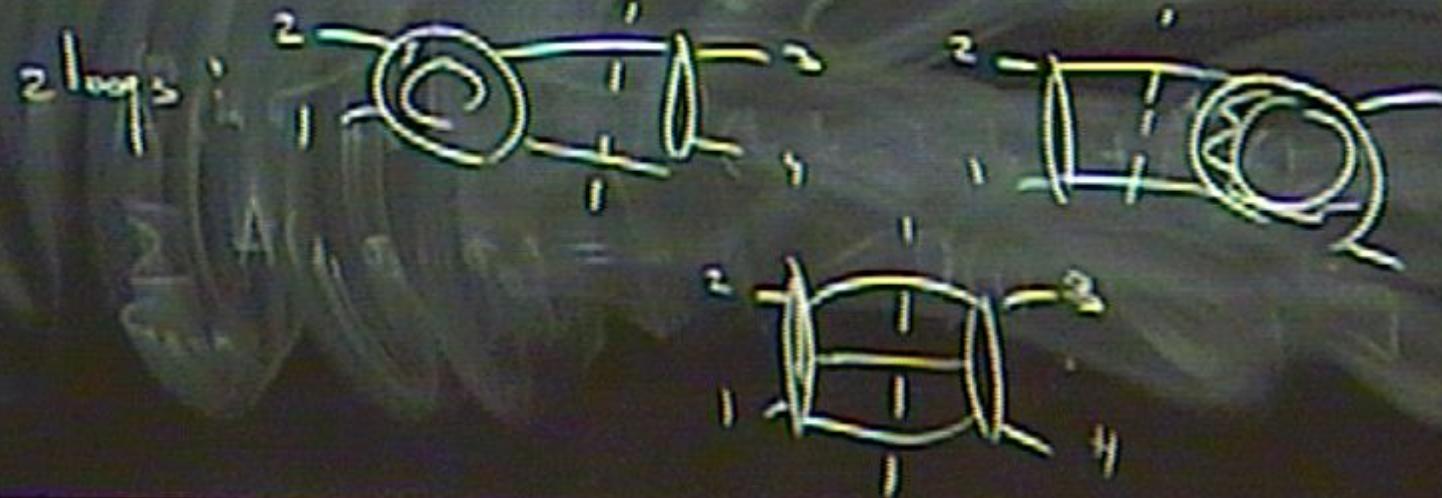


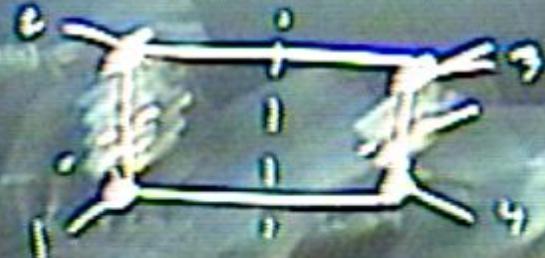
$$1 \cdot S^+ S^- \rightarrow 2 J_\mu T = T^+ T^-$$

$S \cdot 1 \sim T$



$$1. S^+ S^- \Rightarrow 2 J_m T = T^+ T^-$$
$$S \cdot 1 \approx T$$





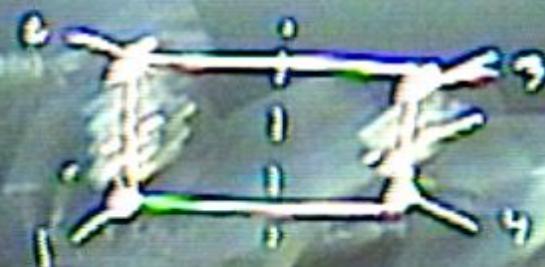
B C F

A II

N = 4

L - 100

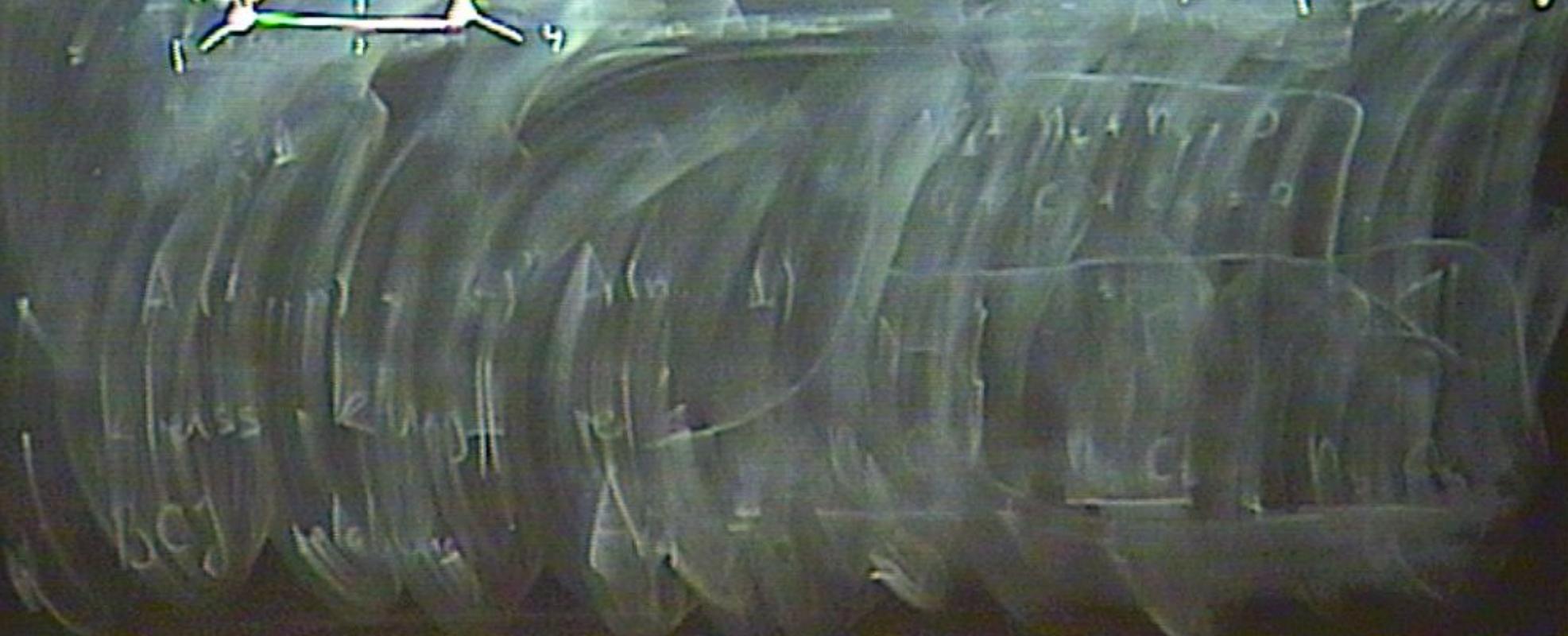


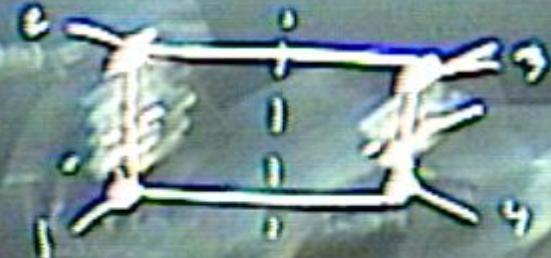


BGF

N=5

All 1-loop w/ this way





B < F

N = 8

All

1 - loop away this way



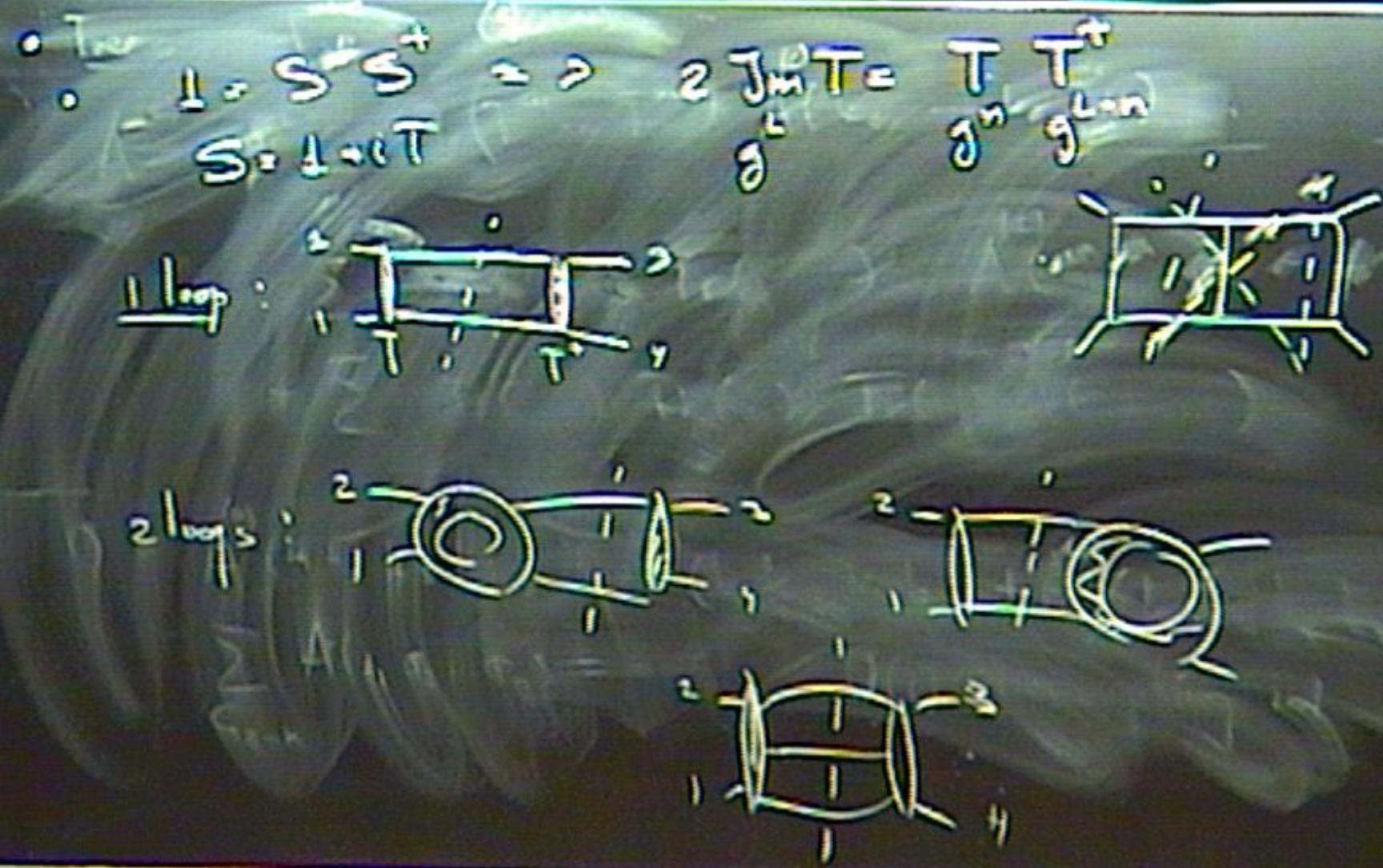
BCF

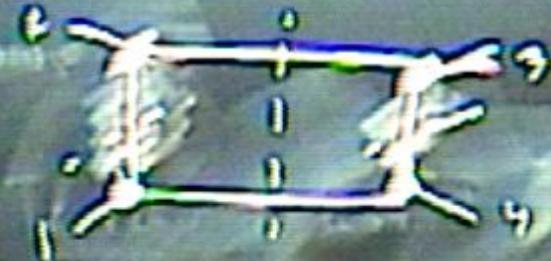
N=6

All

1-loop ways this way







BCF

N $\neq$ 5

All

1-loop ans $\rightarrow$  this way

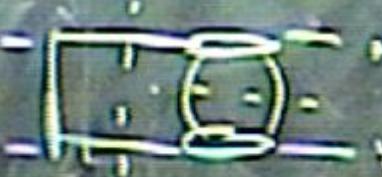




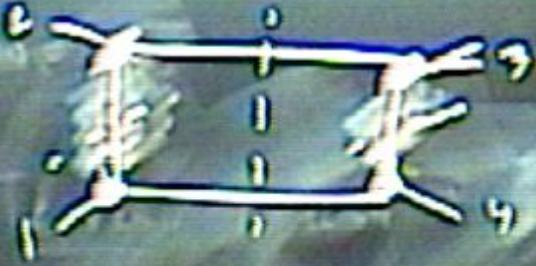
$B \subset F$

$N = 6$

All 1-loop am's this way



Cut constraints



$B \subset F$

$N=6$

All

- loop ways this way



$$+ (a l_i^+ + b l_i^-) \cdot \text{Diagram}$$

$$+ c l_i^+ l_i^-$$



Cut constraints

A person is writing on a chalkboard with a white marker. The board contains handwritten mathematical text and diagrams. At the top left, there is a small circle followed by the number '20'. To its right is the letter 'A' with three subscripts:  $a_1, a_2, a_3$ . Below this is a diagram of a rectangle divided into four quadrants by a diagonal line from the top-left corner to the bottom-right corner. The left half is labeled 'a\_1' and the right half is labeled 'a\_2'. To the right of the rectangle is the letter 'f' with three subscripts:  $f_1, f_2, f_3$ . Below the rectangle is a large bracket containing the expression  $(l_1, h_1; l_2, h_2)$ . To the right of the rectangle is another large bracket containing the expression  $(l_3, -h_3; l_4, -h_4; l_5, h_5, l_6, h_6)$ .

$$\bullet \quad 20$$
$$A(a_1, a_2, a_3)$$
$$(l_1, h_1; l_2, h_2)$$
$$f(f_1, f_2, f_3)$$
$$(l_3, -h_3; l_4, -h_4; l_5, h_5, l_6, h_6)$$

$$\begin{aligned}
 & \cdot \left( \sum_{\substack{a_1, a_2, a_3, a_4 \\ b_1, b_2, b_3, b_4}} A(b_1, b_4; b_2, b_3; h_1, h_4) \right) \times \\
 & \quad \text{Diagram: } \begin{array}{c} \text{Two rectangles} \\ \text{with vertical lines} \end{array} \quad \text{Diagram: } \begin{array}{c} \text{Two rectangles} \\ \text{with diagonal lines} \end{array} \quad A(b_1, -b_4; b_2, -b_3; h_1, h_4) \\
 & \quad \text{Tr}(\text{Tr}_{a_1} \text{Tr}_{a_4})
 \end{aligned}$$

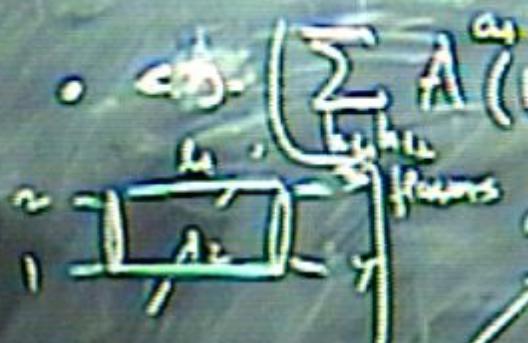
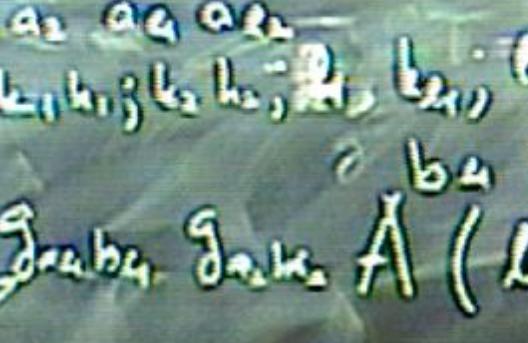
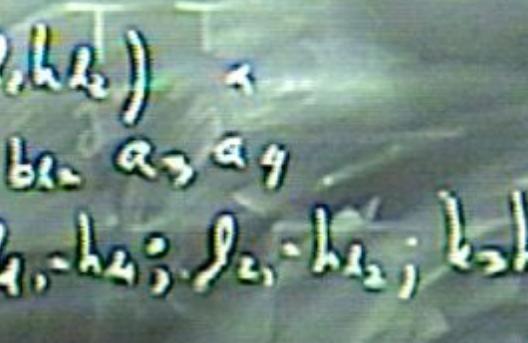
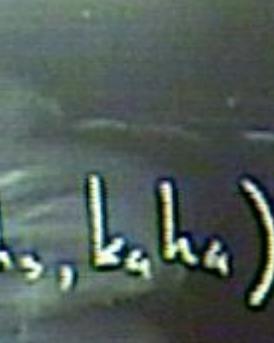
- $\text{Tr} \left( \sum_{\text{all } h_1, h_2} A(h_1, h_2; b_1, b_2; a_1, a_2) \right)$   


Diagram: A rectangular loop with four ports labeled  $b_1, b_2$  on the top and  $a_1, a_2$  on the bottom. The left edge is labeled  $b$  and the right edge is labeled  $a$ .

$$A(h_1, h_2; b_1, b_2; a_1, a_2)$$
- $\text{Tr}(T)$
- $\text{Tr}(T^q)$   
planar cut

- $\sum A(a_1, a_2; b_1, b_2)$   

- $A(a_1, a_2; b_1, b_2)$   

- $T_r(\dots)$   

- $T_r(a_1, a_2, b_1, b_2)$   


- $\text{cd} \cdot \left( \sum_{\substack{\text{bonds} \\ \text{in } T}} A(a_1 a_2; a_3 a_4) \right)$   

Diagram of a planar graph  $T$  with four vertices labeled  $a_1, a_2, a_3, a_4$ . The edges are  $(a_1, a_2), (a_1, a_3), (a_2, a_4), (a_3, a_4)$ .
- $\text{Tr}(T^{a_1} T^{a_2})$   

Diagram of a knot diagram with two components, each labeled  $a_1$  or  $a_2$ .
- $\text{Tr}(T^{a_1} T^{a_2} \dots T^{a_n})$   

Diagram of a knot diagram with  $n$  components, each labeled  $a_i$ .
- $\text{Tr}(T^{a_1} T^{a_2} T^{a_3} T^{a_4})$  : planar cut  

Diagram of a knot diagram with four components, labeled  $a_1, a_2, a_3, a_4$ , with a vertical line passing through it.

- $\sum_{\text{links}} A(l_1, b_1; l_2, b_2; l_3, b_3; l_4, b_4)$   

  
 $A(l_1, b_1; l_2, b_2; l_3, b_3; l_4, b_4)$   
 $Tr(T^{l_1} T^{b_1})$
- $Tr(\ ) \bar{Tr}(\ ) \dots \bar{Tr}(\ )$   

 $Tr(T^a T^b) \bar{Tr}(T^c T^d)$
- $Tr(T^a T^b T^c T^d)$  : planar cut  


### Super Sums

- add them 1 by 1
- say, sum over multiples



### Super Sums

- add them 1 by 1
- say . . sum over multiplets
  - Word identifications.
- $N=4$  - later

### SuperSums

- add them 1 by 1
- say... sum over multiples
  - Ward identities.
- $N=4$  - later
- Regularization: needed |
  - 1) Stick to d = 4 reg: -
  - 2) compare w/
- ✓  $N=4$  : massive regulator

- add them "by hand"
- say... sum over multiplets
- Ward identities.
- $N=4$  - (for)
- Regularization: needed
  - 1) Stick to  $d=4$
  - 2) compare w/  $d$
- add reg. -
- $N=4$  : massive regulator

Cuts

$\overset{2}{\text{P}}$   $\overset{1}{\text{D}}$   $\overset{3}{\text{C}}$

$\overset{2}{\text{J}}$ ,  $\overset{3}{\text{J}}$ ,  $\overset{3}{\text{J}}$ ,  $\overset{1}{\text{J}}$ )

a) ss (clapses to 1 form)

Cuts

• 2<sup>+</sup> - DDC 3<sup>-0</sup>

• 3<sup>+</sup> 3<sup>-0</sup> 3<sup>-0</sup>

a) ss collapses to 1 form

- Cuts



a) ss collapses to 1 form - singlet cuts

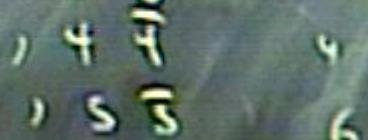
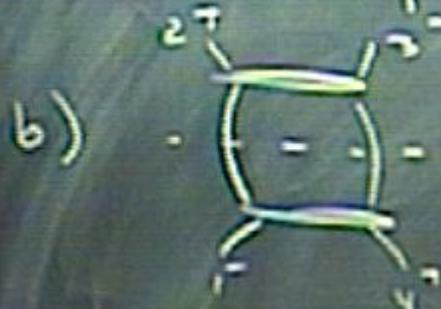


1  
4  
6

- Cuts



a) ss collapses to 1 form - singlet cuts



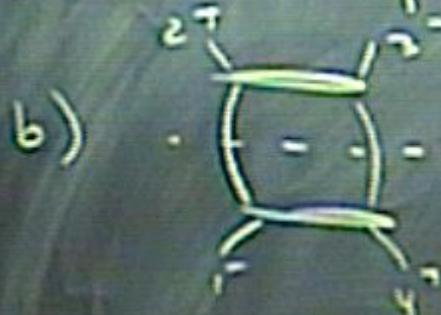
- non-singlet cuts

c) Super am's.

Cuts



a) ss (collapses to 1 form) - singlet cuts



- non-singlet  
cuts

c) Super ann's.

$$A((k_1 q_1, k_2 q_2, k_3 q_3), L, \eta_L) A(\eta_1, q_1, r(\eta_1, q_1, k_1 q_1, k_2 q_2, k_3 q_3))$$

$$\int d\eta_1 d\eta_2 A(k_1 q_1, k_2 q_2, k_3 \eta_{12}, k_4 \eta_{12}) A(k_1 q_1, k_2 q_2, k_3 \eta_1, k_4 \eta_2)$$

$$\int d^4 \eta F$$

$$\int d\eta_1 d\eta_2 A(\eta_1, \eta_2, \eta_{12}, \eta_{12}) A(\eta_1, \eta_2, \eta_{12}, \eta_1, \eta_2)$$

$$\int d\eta F(\eta) G(\eta) =$$

$$= \overbrace{\partial^4 F(\eta) G(\eta)} + \underbrace{(\partial^2 \eta)_a F(\partial^2 \eta)^a G}_{G} \\ + \underbrace{(\partial^2 \eta)_a F((\partial^2 \eta)^a G)} + (\partial \eta)^a F(\partial^2 \eta)_a G$$

$$\int d\eta_1 d\eta_2 A(k_1 \eta_1, k_2 \eta_2, k_3 \eta_3) L(\eta_1) L(\eta_2) |A(k_1, k_2, k_3, k_4 \eta_1, k_5 \eta_2, k_6 \eta_3)|$$

$$\begin{aligned} & \gamma^4 F(\eta) G(\eta) + (\partial_\eta^2)^2 F(\eta) G \\ & + \underbrace{(\partial_\eta^2)^2 F((\partial_\eta^2 \eta)^{ab}) G}_{G} + (\partial_\eta^2)^2 F(\eta) G \end{aligned}$$

$$\int d\eta_1 d\eta_2 A(k_1 q_1, k_2 q_2, \ell_1 \eta_{1L}, \ell_2 \eta_{2L}) A(k_1 q_1, k_2 q_2, \ell_1 \eta_{1R}, \ell_2 \eta_{2R})$$

$$\int d\eta_1 d\eta_2 A(k_1 q_1, k_2 q_2, 4\eta_L) L(\eta_L) |A(k_1 q_1, k_2 q_2, k_3 q_3, k_4 q_4)|$$

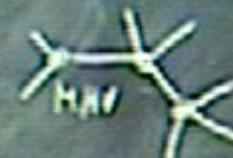
Cuts w/  
 $N^L$  HHV

$$\int d\eta_1 d\eta_2 \bar{A}(k_1 q_1, k_2 q_2, k_3 q_3) \bar{A}(k_4 q_4, k_5 q_5, k_6 q_6, k_7 q_7)$$

- Cuts w/  $N^k$  MHV

- BCFW

- OCSW



- MHV cuts (only  $\pm$ MHV or  $\pm \overline{\text{MHV}}$  factors)

A<sup>HHV</sup> - ଶ୍ରୀ ପାତ୍ର ମହାନ୍ତିର  
ପାତ୍ରମାନ

A<sup>HHV</sup>

$$A^{H\bar{H}V} = \delta(\eta) \frac{\epsilon(\vec{k}, \vec{n}, \vec{r})}{\int d\omega \int_{-\infty}^{\infty} d\omega' \delta(\eta - \omega' - \omega)}$$

$$A^{H\bar{H}V} = \frac{i(-)^k}{\int d\omega \int_{-\infty}^{\infty} d\omega' \delta(\eta - \omega' - \omega)}$$



$$\int d\eta_1 d\eta_2 A(k_1 q_1, k_2 q_2, \ell_1 \eta_{1L}, \ell_2 \eta_{2L}) A(k_1 q_1, k_2 q_2, \ell_1 \eta_{1R}, \ell_2 \eta_{2R})$$



$$\int d\eta_1 d\eta_2 A(k_1 q_1, k_2 q_2, k_1 \eta_{1L}, k_2 \eta_{2L}) / A(k_1 q_1, k_2 q_2, k_1 \eta_{1L}, k_2 \eta_{2L})$$

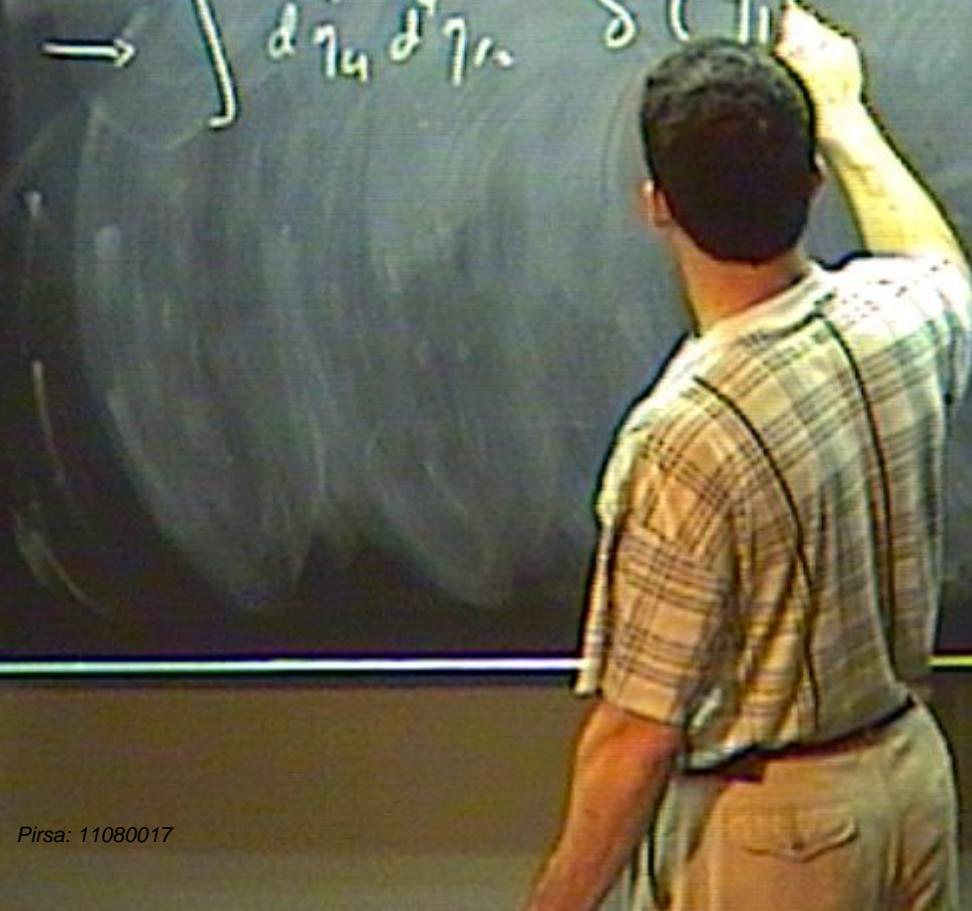
$\approx$

$$1 - \frac{1}{k_1^2 k_2^2}$$

$$\int d\eta_4 d\eta_5 \tilde{A}(k_1 q_1, k_2 q_2, k_3 q_3) \tilde{C}_1 \eta_4 \tilde{C}_2 \eta_5 / \tilde{A}(k_1 q_1, k_2 q_2, k_3 q_3, k_4 q_4)$$

$\rightarrow$

$$\int d\eta_4 d\eta_5 \delta^4(q_1 + q_2 + q_3 + q_4)$$



$$\int d\gamma_4 d\gamma_5 A(\gamma_1, \gamma_2, \gamma_3, \gamma_4) A(\gamma_1, \gamma_2, \gamma_3, \gamma_4)$$

$$P = |P\rangle \langle P|$$

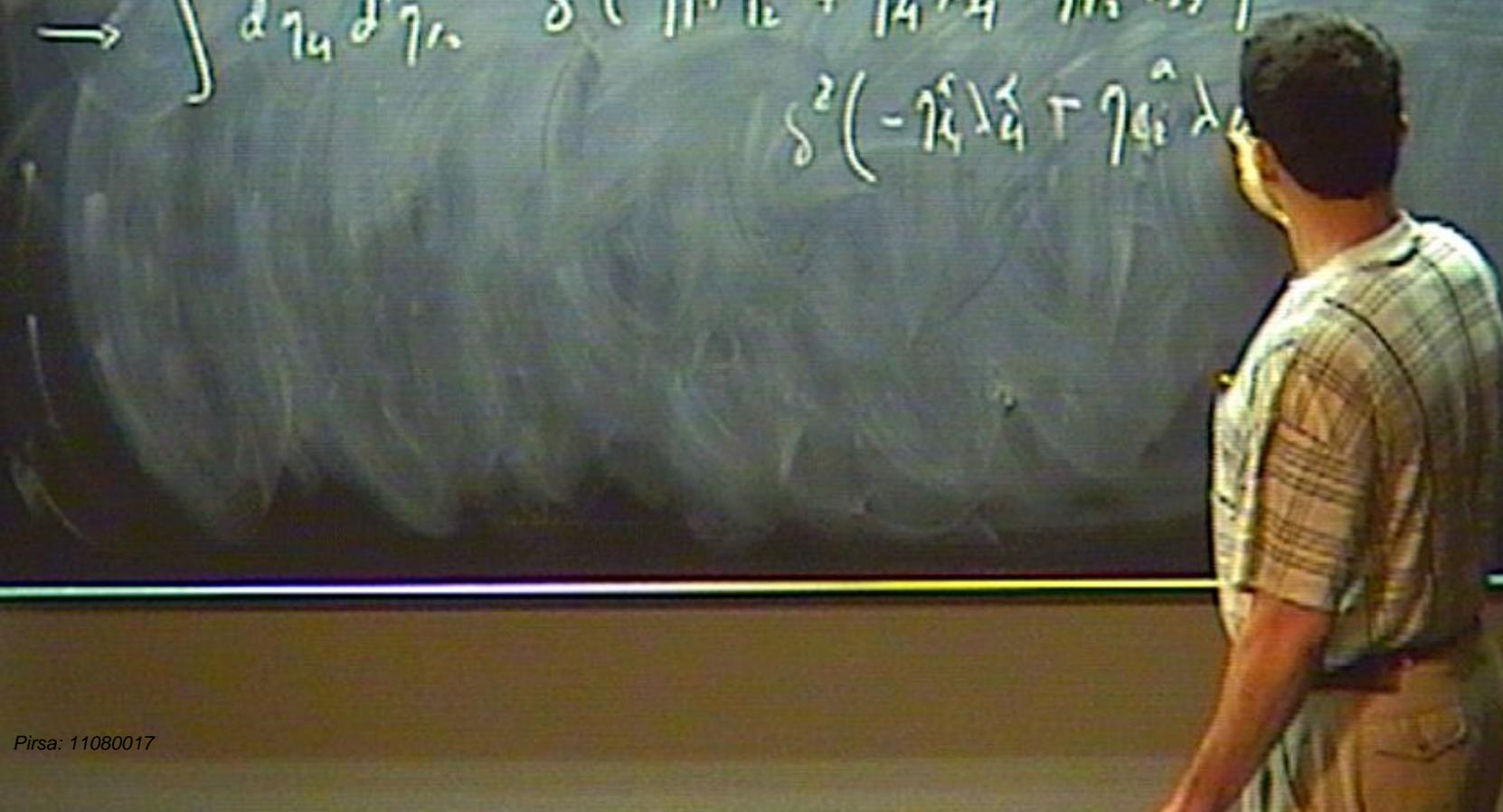
Integrate

$$\rightarrow \int d\gamma_4 d\gamma_5 \delta^4(\gamma_1, \gamma_2 + \gamma_4 \lambda_4 - \gamma_5 \lambda_5)$$

$$\delta^2($$

$$\int d\eta_1 d\eta_2 A(k_1 q_1, k_2 q_2, k_3 q_3, k_4 q_4) A(k_1 p_1, k_2 p_2, k_3 p_3, k_4 p_4)$$

$$\stackrel{!}{=} \int d\eta_1 d\eta_2 \delta(\eta_1 + \eta_4 \lambda_4 - \eta_3 \lambda_3) \delta^2(-\eta_4 \lambda_4 + \eta_3 \lambda_3)$$



$$\int d\eta_1 d\eta_2 A(k_1 q_1, k_2 q_2, \gamma_{1L} \gamma_{2L}, \gamma_{1R} \gamma_{2R}) A(k_1 q_1, \gamma_{1L}^*, \gamma_{1R}^*, k_2 q_2, k_2 q_2)$$

$$= \overline{P} = \overline{P}^\dagger$$

$$P = |P\rangle \langle P|$$

$$-P = (-|P\rangle) \langle P|$$

$$\rightarrow \int d\eta_1 d\eta_2 \delta(\eta_1 + \eta_2 \lambda_4^* - \eta_1 \lambda_4) \delta^2(-\eta_1 \lambda_4^* + \eta_2 \lambda_4^* + \eta_2 \gamma_4)$$

=

$$\begin{aligned}
 & \int d\eta_4 d\eta_5 A(k_1 q_1, k_2 q_2, k_3 q_3) A(k_4 q_4, k_5 q_5, k_6 q_6) \\
 & \stackrel{\text{def}}{=} \int d\eta_4 d\eta_5 \delta^6(q_1 + q_4 \lambda_4^\alpha + q_5 \lambda_5^\alpha) \\
 & \quad \delta^2(-q_4^\alpha \lambda_4^\beta + q_5^\alpha \lambda_5^\beta - q_6) \\
 & = \delta(\sum q_i) \int d\eta_4 d\eta_5 \delta^6(q_1 + q_4 \lambda_4^\alpha + q_5 \lambda_5^\alpha)
 \end{aligned}$$

$$\begin{aligned}
 & \int d\eta_4 d\eta_6 A(k_0 \eta_0, k_2 \eta_2, k_4 \eta_4) A(\bar{k}_0 \bar{\eta}_0, \bar{k}_2 \bar{\eta}_2, \bar{k}_4 \bar{\eta}_4) \\
 & \stackrel{\text{Def}}{=} \int d\eta_4 d\eta_6 \delta^4(\eta_0 + \eta_4 \lambda_4 + \eta_6 \lambda_6) \\
 & \quad \delta^2(-\eta_4 \lambda_4 + \eta_6 \lambda_6 + \eta_0 + \eta_4) \\
 & = \delta(\sum \eta_i) \int d\eta_4 d\eta_6 \delta^4(\eta_0 + \eta_4 \lambda_4 + \eta_6 \lambda_6) \\
 & = \delta(\sum \eta_i) \langle \lambda_4, \lambda_6 \rangle^4
 \end{aligned}$$

$$\begin{aligned}
 & \int d\eta_1 d\eta_2 A(k_1 q_1, k_2 q_2, k_3 q_3) A(k_4 q_4, k_5 q_5, k_6 q_6) \\
 & \stackrel{\text{Feynman}}{\rightarrow} \int d\eta_1 d\eta_2 \delta(\eta_1 + \eta_4 \lambda_4^* - \eta_5 \lambda_5^*) \\
 & = \delta(\sum \eta_i) \left\langle \lambda_4^* \lambda_5^* \right\rangle = \delta(\pi) \sim 0
 \end{aligned}$$

$$\begin{aligned}
 & \int d\eta_1 d\eta_2 A(k_1 q_1, k_2 q_2, k_3 q_3) C(\eta_1) C(\eta_2) |A(k_1 q_1, k_2 q_2, k_3 q_3) \\
 & \stackrel{\text{def}}{=} \int d\eta_1 d\eta_2 \delta^3(q_1 + q_2 + q_3) \delta^3(\eta_1 + \eta_2) \\
 & \rightarrow \int d\eta_4 d\eta_5 \delta^3(q_1 + q_2 + q_3 - \eta_4 - \eta_5) \\
 & \quad \delta^2(-\eta_4^i \lambda_4^i + \eta_5^i \lambda_5^i + q_3^i q_4^i) \\
 & = \delta(\sum q_i) \left[ \int d\eta_4 d\eta_5 \delta^3(q_1 + q_2 + q_3 - \eta_4 - \eta_5) \right] \\
 & = \delta(\sum q_i) \langle \lambda_4 \lambda_5 \rangle^3
 \end{aligned}$$

$$\begin{aligned}
 & \int d\eta_4 d\eta_6 A(k_1 q_1, k_2 q_2, k_3 q_3, k_4 q_4) / A(k_1 q_1, k_2 q_2, k_3 q_3, k_4 q_4) \\
 & \stackrel{\text{def}}{=} \frac{P}{-P} = \frac{|P|}{|P|} = \sin \theta \\
 & \rightarrow \int d\eta_4 d\eta_6 \delta^2(q_1 + q_2 + q_3 + q_4) \\
 & \quad \delta^2(-q_1 - q_2 - q_3 - q_4) \\
 & = \delta(\sum q_i) \left[ \int d\eta_4^1 d\eta_6^1 \delta^2(q_1 + q_2 + q_3 + q_4) \right]^{(n)} \\
 & = \delta^2(\sum q_i) \langle \lambda_4, \lambda_6 \rangle^n
 \end{aligned}$$

## Apects of generalized unitarity

- ✓ • tree-level comments
- generalized unitarity: generalities', sunis' regularization', type of cuts, etc
- Superamps; example, general props.
- Parameters vs. analytics
- general strategy for superamps.
- ...

$$C_{12|34} = \frac{\langle 4_1 4_2 \rangle^*}{(\langle 1_2 \rangle \langle 2_4 \rangle \langle 4_3 \rangle \langle 4_1 \rangle)(\langle 3_1 \rangle \langle 4_1 \rangle \langle 4_2 \rangle \langle 4_3 \rangle)}$$

$$C_{12|34} = \frac{\langle 1_2 | 3_4 \rangle^*}{\langle 1_2 \rangle \langle 2_4 \rangle \langle 3_4 \rangle \langle 4_1 \rangle} (\langle 3_1 \rangle \langle 4_1 \rangle \langle 3_4 \rangle \langle 4_1 \rangle)$$

=



$$C_{12|34} = \frac{\langle k_1 k_2 \rangle}{\langle k_1 \rangle \langle k_2 \rangle \langle k_3 \rangle \langle k_4 \rangle} \frac{\langle k_3 k_4 \rangle}{\langle k_3 \rangle \langle k_4 \rangle \langle k_1 \rangle \langle k_2 \rangle}$$

$$= \frac{[34]^2}{\Delta_{12} \langle k_{12} \rangle} \frac{\overline{T}_{r+}(k_1 k_2 k_3 k_4)}{(2k_2 + k_1)(2k_3 + k_2)(2k_4 + k_3)(2k_1 + k_4)}$$

$$\begin{aligned}
 C_{12|34} &= \frac{\langle k_1 k_2 \rangle}{\langle k_1 \rangle \langle k_2 \rangle \langle k_3 \rangle \langle k_4 \rangle (\langle k_1 \rangle \langle k_2 \rangle \langle k_3 \rangle \langle k_4 \rangle)} \\
 &= \frac{\frac{[34]}{2}}{\delta_{12} \langle k_{12} \rangle} \frac{T_{F+}(k_1 k_2 k_3 k_4)}{(2k_2 \cdot k_1) (2k_4 \cdot k_2) (2k_3 \cdot k_4) (2k_1 \cdot k_3)} \\
 &\quad \times \frac{1}{\langle k_{ij} \rangle} \rightarrow \frac{\frac{[34]}{2}}{2k_i \cdot k_j}
 \end{aligned}$$

$$\begin{aligned}
 C_{12|34} &= \frac{\langle k_1 k_2 \rangle}{\langle k_1 \rangle \langle k_2 \rangle \langle k_3 \rangle \langle k_4 \rangle (\langle k_3 \rangle \langle k_1 \rangle \langle k_2 \rangle \langle k_4 \rangle \langle k_3 \rangle)} \\
 &= \frac{\frac{[34]^2}{\Delta_{12} \langle k_{12} \rangle}}{(2k_2 \cdot k_1) (2k_1 \cdot k_2) (2k_3 \cdot k_4) (2k_4 \cdot k_3)} \\
 &\quad \frac{1}{\langle k_{ij} \rangle} \rightarrow \frac{[ij]}{2k_i \cdot k_j} \\
 \langle i \rangle [jk] \langle kl \rangle \dots &\rightarrow \bar{T}_{r+}(\times \times \dots)
 \end{aligned}$$

$$C_{12|34} = \frac{\langle b_1 b_4 \rangle}{\langle 1_2 \rangle \langle 2_4 \rangle \langle 3_1 \rangle \langle 4_2 \rangle} (\langle 3_2 \rangle \langle 4_1 \rangle \langle 3_4 \rangle \langle 4_3 \rangle)$$

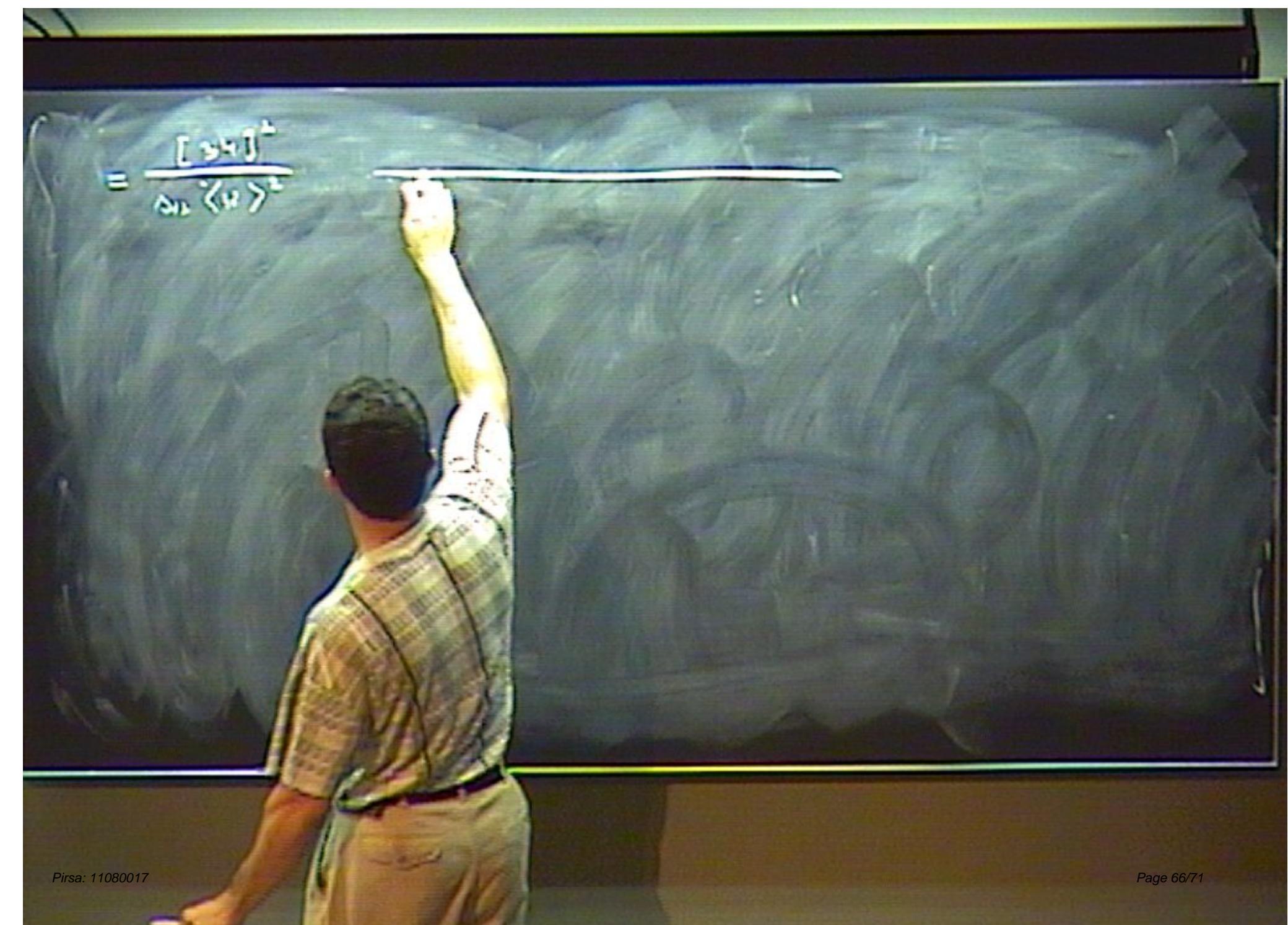
$$= \frac{[341^2]}{S_{12} \langle 12 \rangle} \frac{\bar{T}_{r+}(k_1 k_2 k_3 k_4 k_5)}{(2k_2 \cdot k_1) (2k_4 \cdot k_2) (2k_3 \cdot k_1) (2k_5 \cdot k_2)}$$

$$\frac{1}{\langle i,j \rangle} \rightarrow \frac{[ij]}{2k_i \cdot k_j}$$

$$\langle i,j \rangle [jk] \langle kl \rangle \dots \rightarrow \bar{T}_{r+}(\star \star k \dots)$$

$$\langle b_1 b_4 \rangle = \langle b_3 b_2 \rangle [b_2 b_4] \langle b_3 b_1 \rangle [b_3 b_4]$$

$$= \frac{[34]}{\Delta x \langle u \rangle^2}$$



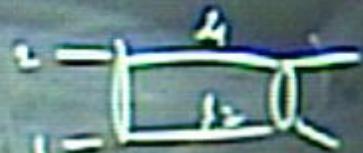
$$= \frac{[34]}{\Delta \nu \langle u \rangle^2}$$

$$\frac{(2\lambda_{45}) (2\lambda_1 \cdot \lambda_{41}) (2\lambda_2 \cdot \lambda_2)}{(\ ) (\ ) (\ ) (\ )}$$



$$= \frac{[34]^2}{\langle u \rangle^2}$$

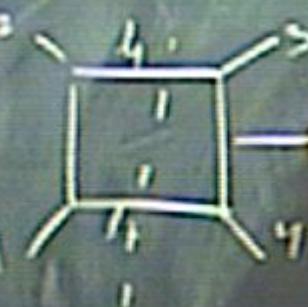
$$= \frac{(2k_1 l_1)(2k_2 l_2)(2k_3 l_3)}{( ) ( ) ( ) ( )}$$



$$= \frac{[34]^2}{\langle u \rangle^2}$$

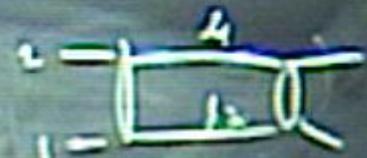
$$= \frac{1}{(2k_1 l_1)(2k_2 l_2)}$$

$$= \left( \frac{[34]}{\langle u \rangle} \right)^2$$



$$= \frac{[34]^2}{\langle u \rangle^2}$$

$$= \frac{(2k_1, l_1)(2k_2, l_2)(2k_3, l_3)}{( ) ( ) ( ) ( )}$$



$$\frac{[34]^2}{\langle u \rangle^2}$$

$$= \frac{1}{(2k_1, l_1)(2k_2, l_2)}$$

$$= \left( \frac{[34]}{\langle u \rangle} \right)^2$$

$$= \frac{1}{(k_2 + l_1)^2}$$

$$= \frac{[34]}{\langle u \rangle^2}$$

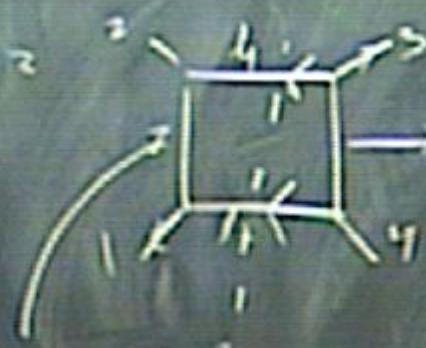
$$= \frac{(2k_1) (2k_2) (2k_3)}{( ) ( ) ( ) ( )}$$



$$= \frac{[34]}{\langle u \rangle^2}$$

$$= \frac{1}{(2k_1 \cdot l_1)(2k_2 \cdot l_1)}$$

$$= \left( \frac{[34]}{\langle u \rangle} \right)^2 \frac{1}{(l_2 - l_1)^2} \rightarrow (l_2 - l_1)^2$$



Color Symms

$$\begin{aligned}
 C_{12|34} &= \frac{\langle l_1 l_2 |}{\langle l_1 l_2 \rangle \langle l_3 l_4 \rangle \langle l_1 l_3 \rangle \langle l_2 l_4 \rangle} (\langle l_3 \rangle \langle l_4 | \langle l_1 l_2 | \langle l_3 l_4 \rangle \langle l_1 l_3 \rangle \langle l_2 l_4 \rangle) \\
 &= \frac{[34]}{\delta_{12} \langle l_1 l_2 \rangle} \frac{\text{Tr}_+ (l_1 l_2 l_3 l_4)}{(2k_1 + k_1)(2k_1 + k_2)(2k_2 + k_1)(2k_2 + k_2)} \\
 &\quad - \frac{1}{\langle l_1 l_2 \rangle} = \frac{[34]}{2k_1 + k_2}
 \end{aligned}$$