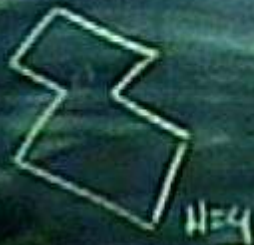


Title: Amplitudes at Strong Coupling

Date: Aug 05, 2011 12:15 PM

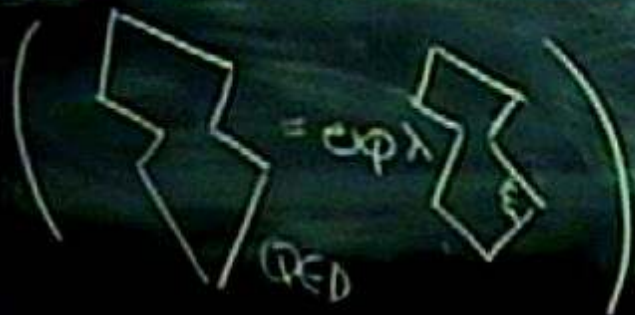
URL: <http://pirsa.org/11080016>

Abstract:

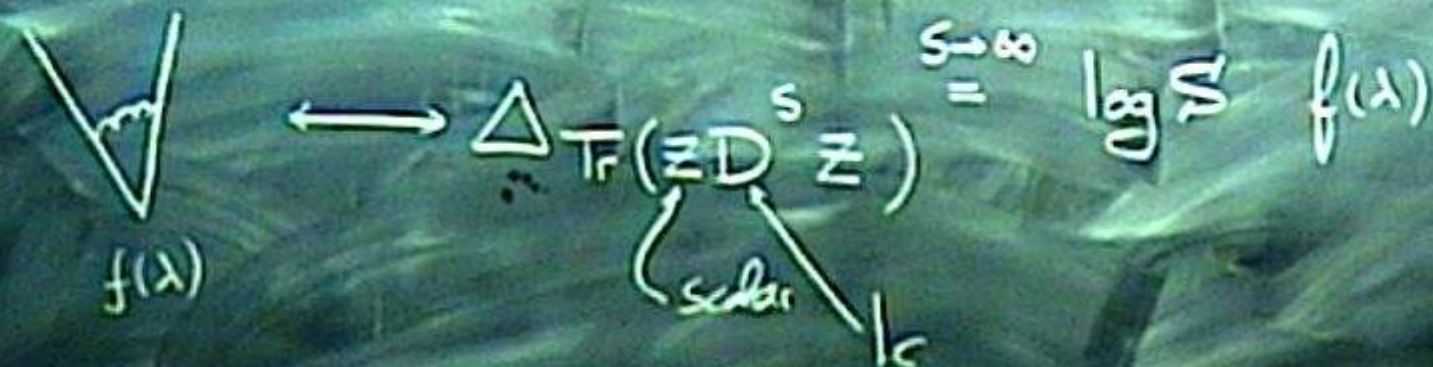


$\equiv \exp(\text{Remainder})$

finite



$\lambda \rightarrow f(\lambda)$



$1) \left(\underbrace{Z \dots Z}_N D \underbrace{Z \dots Z}_N \dots \right)$

closed chain

$\{p_i\}$ quantized by some
 dy. eq. $\forall \lambda$ integrability

$2) \begin{cases} \#Z \gg 1 \\ \#D \gg \#Z \end{cases}$

$\Delta = \log S f(\lambda) + (\dots)$

L only enters here

$\# \mu \rightarrow S \rightarrow \infty$

\Rightarrow integral eqs.

Cusp Anomalous Dimension

$$\sigma(t) = \underbrace{k(t,0)}_{a + bg^2 + cg^4 + \dots} + g^2 \int_a^a k(t,t') \sigma(t') dt'$$

$$\sigma(t) = a \text{ @ leading order}$$

$$\sigma(t) = a + g^2 \left(\begin{array}{c} \text{next order} \\ b + \int a a \\ \text{etc} \end{array} \right) + \dots$$

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BES Kernels

$$K_0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$K_1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$K_d = 2g^2 (K_1 /. t' \rightarrow t'') \frac{e^{t''}}{e^{t''} - 1} (K_0 /. t \rightarrow t'');$$

$$\hat{\sigma}(t) = \frac{t}{e^t - 1} \left(K(2gt, 0) - 4g^2 \int_0^\infty K(2gt, 2gt') \hat{\sigma}(t') \right).$$

The kernel in (2.11) is given by

$$K(t, t') = K_0(t, t') - K_1(t, t') + K_d(t, t')$$

$$K_0(t, t') = \frac{t J_1(t) J_0(t') - t' J_0(t) J_1(t')}{t^2 - t'^2} = \frac{2}{tt'} \sum_{n=1}^{\infty} (2n-1) J_{2n-1}(t) J_{2n-1}(t')$$

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$\lambda^2(x)$ // FullForm

cp[x]

BES Kernels

$$K0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2}$$

$$K1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2}$$

$$k0 = 2g^2 (K1 /. t' \rightarrow t'') \frac{t''}{e^{2t''} - 1} (K0 /. t \rightarrow t'')$$

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 RefreshCellGroup

$t'[x]$ // FullForm

$tp[x]$

BES Kernels

$$K0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$K1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$kd = 8g^2 (K1 /. t' \rightarrow t'') \frac{t''}{e^{2t''} - 1} (K0 /. t \rightarrow t'');$$

Series expanding the Kernels

Series $\left[\frac{1}{1-x}, \{x, 0, 10\} \right]$

$$1 - x - x^2 - x^3 - x^4 - x^5 - x^6 - x^7 - x^8 - x^9 - x^{10} - O[x]^{11}$$

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BES Kernels

$$K0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$K1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$Kd = 2g^2 (K1 /. t' \rightarrow t^{**}) \frac{e^{t^{**}}}{e^{2t^{**}} - 1} (K0 /. t \rightarrow t^{**});$$

Series expanding the Kernels

```
series & Collect[Series[1/(1-x), {x, 0, 10}], x]
```

$$1 - x - x^2 - x^3 - x^4 - x^5 - x^6 - x^7 - x^8 - x^9 - x^{10}$$

$$1 - x - x^2 - x^3 - x^4 - x^5 - x^6 - x^7 - x^8 - x^9 - x^{10}$$

BES Kernels

$$K0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$K1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$k0 = 2g^2 (K1 /. t' -> t'') \frac{e^{t''}}{e^{t''} - 1} (K0 /. t -> t'');$$

Series expanding the Kernels

```
series K0, n0, Collect[Series[1/(1-x), {x, 0, 10}], x]
```

```
1 - x - x^2 - x^3 - x^4 - x^5 - x^6 - x^7 - x^8 - x^9 - x^10
```

```
1 - x - x^2 - x^3 - x^4 - x^5 - x^6 - x^7 - x^8 - x^9 - x^10
```

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BES Kernels

$$K0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$K1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$kE = 8g^2 (K1 /. t' \rightarrow t'') \frac{e^{t''}}{e^{t''} - 1} (K0 /. t \rightarrow t'');$$

Series expanding the Kernels

```
series[x, n] := Collect[Series[x, {y, 0, n}], y]
1 - x - x^2 - x^3 - x^4 - x^5 - x^6 - x^7 - x^8 - x^9 - x^10
1 - x - x^2 - x^3 - x^4 - x^5 - x^6 - x^7 - x^8 - x^9 - x^10
```

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AllNotations | a

BES Kernels

$$K0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2}$$

$$K1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2}$$

$$k2 = 5g^2 (K1 /. t' -> t'') \frac{t''}{e^{t''} - 1} (K0 /. t -> t'')$$

Series expanding the Kernels

```
series[x_, n_] := Collect[Series[x, {g, 0, n}], g]

series[k2, max]
```

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Substitution | ω

BES Kernels

$$K0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2}$$

$$K1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2}$$

$$kd = 2g^2 (K1 /. t' \rightarrow t'') \frac{e^{t''}}{e^{t''} - 1} (K0 /. t \rightarrow t'')$$

Series expanding the Kernels

```
series[X_, a_] := Collect[Series[X, {g, 0, a}], g, Factor]
```

```
series[kd, max]
```

$$\frac{g^4 e^{t''} (t'')^2}{-1 - e^{t''}} - \frac{g^4 t e^{t''} (t'')^2 (2t^2 - 3(t'')^2 - 3(t'')^4)}{6(-1 - e^{t''})}$$

$$\frac{g^4 t e^{t''} (t'')^2 (t^4 + 4t^2 (t'')^2 + 2(t'')^4 - 7t^2 (t'')^2 - 12(t'')^2 (t'')^2 - 7(t'')^4)}{24(-1 - e^{t''})}$$

$$\frac{g^4}{720(-1 - e^{t''})} (g^{2t} e^{t''} (t'')^2 (2t^5 - 15t^3 (t'')^2 - 20t^2 (t'')^4 - 5(t'')^6 - 27t^4 (t'')^2 - 125t^2 (t'')^2 (t'')^2 - 65(t'')^2 (t'')^2 - 77t^2 (t'')^4 + 140(t'')^2 (t'')^4 - 42(t'')^6)$$

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BES Kernels

$$K0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$K1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$kd = Bg^2 (K1 /. t \rightarrow t') \frac{e^{t'}}{e^{t'} - 1} (K0 /. t \rightarrow t');$$

Series expanding the Kernels

```
series[X_, n_, f_?Factor] := Collect[Series[X, {g, 0, n}], g, f]
```

```
series[kd, max, Identity]
```

$$g^2 \frac{t^2 (t')^2}{-1 - e^{t'}} - g^2 \left\{ \frac{t^2 (t')^2}{3(-1 - e^{t'})} - \frac{t (t')^2 (t')^2}{2(-1 - e^{t'})} - \frac{5t (t')^4}{6(-1 - e^{t'})} \right\} - g^3$$

$$\frac{t^2 (t')^2}{24(-1 - e^{t'})} - \frac{t^2 (t')^2 (t')^2}{8(-1 - e^{t'})} - \frac{t (t')^4 (t')^2}{12(-1 - e^{t'})} - \frac{7t^3 (t')^4}{24(-1 - e^{t'})} - \frac{t (t')^2 (t')^4}{2(-1 - e^{t'})} - \frac{7t (t')^4}{24(-1 - e^{t'})}$$

$$g^4 \left\{ \frac{t^2 (t')^2}{160(-1 - e^{t'})} - \frac{t^2 (t')^2 (t')^2}{48(-1 - e^{t'})} - \frac{t^2 (t')^4 (t')^2}{36(-1 - e^{t'})} - \frac{t (t')^4 (t')^2}{144(-1 - e^{t'})} - \frac{3t^3 (t')^4}{80(-1 - e^{t'})} \right.$$

$$\left. - \frac{20t^2 (t')^2 (t')^2}{144(-1 - e^{t'})} - \frac{13t (t')^4 (t')^4}{144(-1 - e^{t'})} - \frac{77t^2 (t')^4}{720(-1 - e^{t'})} - \frac{7t (t')^2 (t')^4}{36(-1 - e^{t'})} - \frac{7t (t')^4}{120(-1 - e^{t'})} \right\}$$

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BES Kernels

$$K0 = \frac{2gt \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt' \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2}$$

$$K1 = \frac{2gt' \text{BesselJ}[1, 2gt] \text{BesselJ}[0, 2gt'] - 2gt \text{BesselJ}[1, 2gt'] \text{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2}$$

$$kd = 8g^2 (K1 /. t' \rightarrow t'') \frac{t''}{e^{t''} - 1} (K0 /. t \rightarrow t'')$$

Series expanding the Kernels

```
series[x_, n_, f_ : Factor] := Collect[Series[x, {g, 0, n}], g, f]
```

```
series[kd, max]
```

$$\frac{g^2 t (t'')^2}{2 - e^2} - \frac{g^2 t (t'')^2 (2t^2 - 3(t'')^2 - 5(t'')^4)}{6(-1 - e^2)}$$

$$\frac{g^2 t (t'')^2 (t^2 - 4t^2 (t'')^2 - 2(t'')^4 - 7t^2 (t'')^2 - 12(t'')^2 (t'')^2 - 7(t'')^4)}{24(-1 - e^2)}$$

$$\frac{g^2}{720(-1 - e^2)} \frac{g^{22} t (t'')^2 (2t^2 - 15t^2 (t'')^2 - 20t^2 (t'')^4 - 5(t'')^6 - 27t^2 (t'')^2)}{125t^2 (t'')^2 (t'')^2 - 65(t'')^4 (t'')^2 - 77t^2 (t'')^6 - 140(t'')^2 (t'')^4 - 42(t'')^6}$$