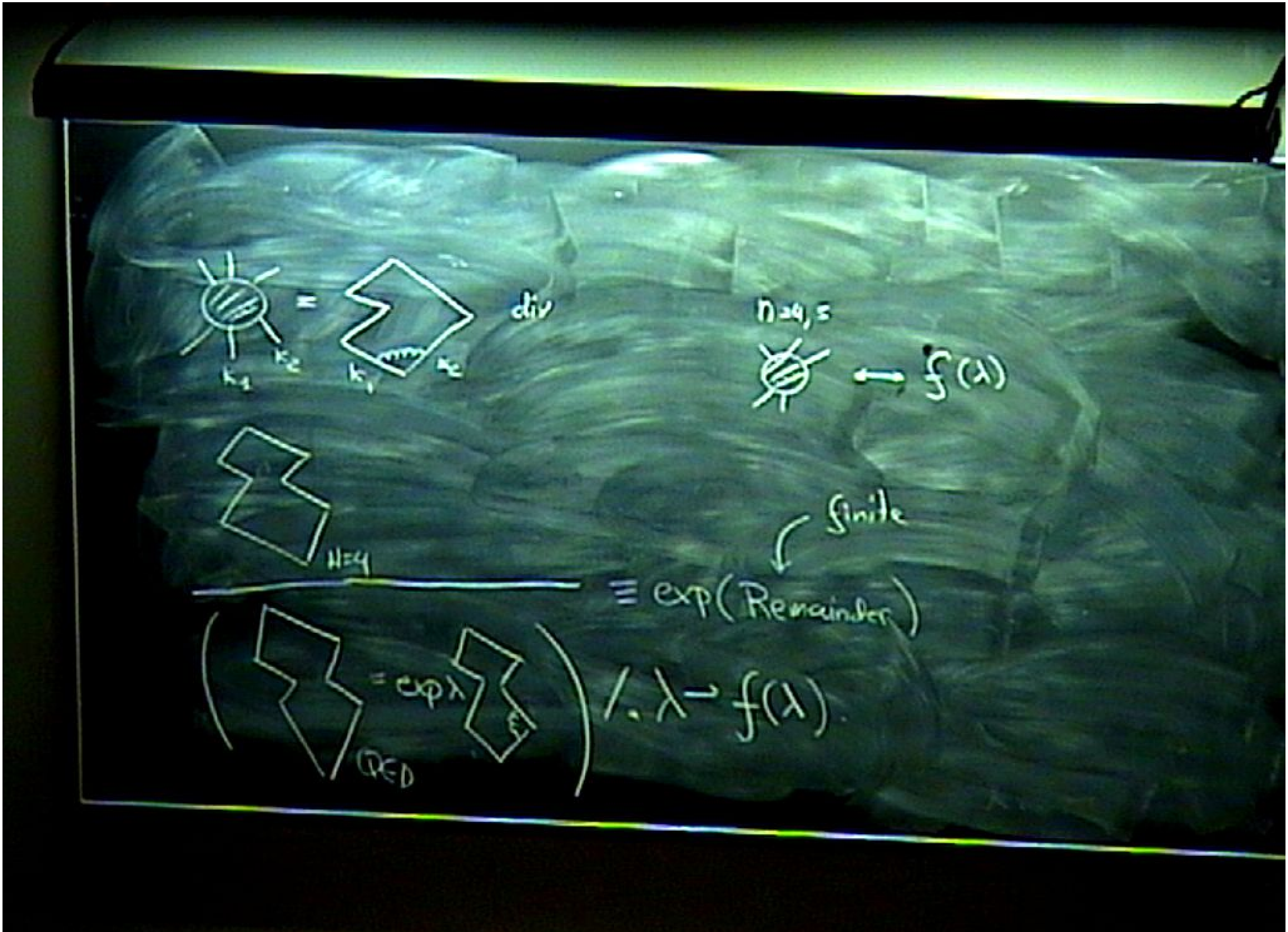
Title: Amplitudes at Strong Coupling

Date: Aug 05, 2011 12:15 PM

URL: http://pirsa.org/11080016

Abstract:


div
k1
k2
n≥4,5
f(λ)
N=4
finite
≡ exp(Remainder)
= exp λ
QED
/. λ→ f(λ)

$f(\lambda)$

$\longleftrightarrow \Delta_{\mathrm{Tr}(Z D^S Z)} \overset{S\to\infty}{=} \log S \;\; f(\lambda)$

scalar

lc

1) $\mathrm{tr}(\, Z \ldots \underset{\vec{p}_1}{ZDZ} \ldots \underset{\vec{p}_2}{ZDZ} \cdots Z \,)$ closed chain

$\{p_i\}$ quantized by some

alg. eq. $\forall \lambda$ integrability

2) $\# Z \gg 1$

$\# D \gg \# Z$

$\Delta = \log S \, f(\lambda) + (\ldots)$

L only enters here

$\#$ [illegible] $\sim S \to \infty$

$\Longrightarrow$ integral eqs.

Cusp Anomalous Dimension

$$\sigma(t) = \underbrace{k(t,0)}_{a} + g^2 \int_0 \underbrace{k(t,t')}_{a} \, \underbrace{\sigma(t')}_{a} \, dt'$$

$$a + bg^2 + cg^4 + \ldots$$

$\sigma(t) = a$ @ leading order

at next order

$$\sigma(t) = a + g^2 \left(b + \int a \, a \right) + \ldots$$

etc

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tp[x]

BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
Kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

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$$\hat{\sigma}(t) = \frac{t}{e^t - 1}\left(K(2gt, 0) - 4g^2 \int_0^\infty K(2gt, 2gt')\hat{\sigma}(t')\right).$$

The kernel in (2.11) is given by

$$K(t, t') = K_0(t, t') - K_1(t, t') + K_d(t, t')$$

$$K_0(t, t') = \frac{tJ_1(t)J_0(t') - t'J_0(t)J_1(t')}{t^2 - t'^2} = \frac{2}{tt'}\sum_{n=1}^{\infty}(2n-1)J_{2n-1}(t)J_{\ldots}$$

$$\frac{t'J_1(t)J_0(t') - tJ_0(t)J_1(t')}{\ldots} \quad 2\sum^{\infty}$$

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```
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```

```
tp[x]
```

BES Kernels

$$K0 = \frac{2gt\,\mathrm{BesselJ}[1, 2gt]\,\mathrm{BesselJ}[0, 2gt'] - 2gt'\,\mathrm{BesselJ}[1, 2gt']\,\mathrm{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$K1 = \frac{2gt'\,\mathrm{BesselJ}[1, 2gt]\,\mathrm{BesselJ}[0, 2gt'] - 2gt\,\mathrm{BesselJ}[1, 2gt']\,\mathrm{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$Kd = 8g^2\,(K1 /.\ t' \to t'')\,\frac{t''}{e^{t''} - 1}\,(K0 /.\ t \to t'');$$

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```
t'[x] // FullForm

tp[x]
```

BES Kernels

$$K0 = \frac{2gt\,\mathrm{BesselJ}[1, 2gt]\,\mathrm{BesselJ}[0, 2gt'] - 2gt'\,\mathrm{BesselJ}[1, 2gt']\,\mathrm{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$K1 = \frac{2gt'\,\mathrm{BesselJ}[1, 2gt]\,\mathrm{BesselJ}[0, 2gt'] - 2gt\,\mathrm{BesselJ}[1, 2gt']\,\mathrm{BesselJ}[0, 2gt]}{(2gt)^2 - (2gt')^2};$$

$$Kd = 8g^2\,(K1\ /.\ t' \to t'')\,\frac{t''}{e^{t''} - 1}\,(K0\ /.\ t \to t'');$$

Series expanding the Kernels

$$\mathrm{Series}\left[\frac{1}{1-x}, \{x, 0, 10\}\right]$$

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + O[x]^{11}$$

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BES Kernels

$$K0 = \frac{2\,g\,t\,\mathrm{BesselJ}[1, 2\,g\,t]\,\mathrm{BesselJ}[0, 2\,g\,t'] - 2\,g\,t'\,\mathrm{BesselJ}[1, 2\,g\,t']\,\mathrm{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$K1 = \frac{2\,g\,t'\,\mathrm{BesselJ}[1, 2\,g\,t]\,\mathrm{BesselJ}[0, 2\,g\,t'] - 2\,g\,t\,\mathrm{BesselJ}[1, 2\,g\,t']\,\mathrm{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$Kd = 8\,g^2\,(K1\ /.\ t' \to t'')\,\frac{t''}{e^{t''}-1}\,(K0\ /.\ t \to t'');$$

Series expanding the Kernels

$$\text{series}\ \mathrm{X\_Collect}\left[\mathrm{Series}\left[\frac{1}{1-x}, \{x, 0, 10\}\right], x\right]$$

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$$

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BES Kernels

$$K0 = \frac{2\,g\,t\,\text{BesselJ}[1, 2\,g\,t]\,\text{BesselJ}[0, 2\,g\,t'] - 2\,g\,t'\,\text{BesselJ}[1, 2\,g\,t']\,\text{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$K1 = \frac{2\,g\,t'\,\text{BesselJ}[1, 2\,g\,t]\,\text{BesselJ}[0, 2\,g\,t'] - 2\,g\,t\,\text{BesselJ}[1, 2\,g\,t']\,\text{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$Kd = 8\,g^2\,(K1\ /.\ t' \to t'')\,\frac{t''}{e^{t''} - 1}\,(K0\ /.\ t \to t'');$$

Series expanding the Kernels

$$\text{series}[X\_, n\_] := \text{Collect}\left[\text{Series}\left[\frac{1}{1 - x}, \{x, 0, 10\}\right], x\right]$$

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$$

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BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
Kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

Series expanding the Kernels

```
series[f_, n_] := Collect[Series[f, {g, 0, n}], g]
```

$1 - x + x^2 + x^3 - x^4 + x^5 + x^6 + x^7 - x^8 + x^9 - x^{10}$

$1 + x - x^2 + x^3 + x^4 + x^5 - x^6 + x^7 - x^8 + x^9 + x^{10}$

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BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
Kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

$$K0 = \frac{2\,g\,t\,\mathrm{BesselJ}[1, 2\,g\,t]\,\mathrm{BesselJ}[0, 2\,g\,t'] - 2\,g\,t'\,\mathrm{BesselJ}[1, 2\,g\,t']\,\mathrm{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$K1 = \frac{2\,g\,t'\,\mathrm{BesselJ}[1, 2\,g\,t]\,\mathrm{BesselJ}[0, 2\,g\,t'] - 2\,g\,t\,\mathrm{BesselJ}[1, 2\,g\,t']\,\mathrm{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$Kd = 8\,g^2\,(K1 \;/.\; t' \to t'')\,\frac{t''}{e^{t''} - 1}\,(K0 \;/.\; t \to t'');$$

Series expanding the Kernels

```
series[X_, n_] := Collect[Series[X, {g, 0, n}], g]
```

```
series[Kd, max]
```

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BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

Series expanding the Kernels

```
series[X_, a_] := Collect[Series[X, {g, 0, a}], g, Factor]
```

```
series[kd, max]
```

$$\frac{g^4 t (t'')^2}{-1+e^{t''}} - \frac{g^6 t (t'')^2 \left(2t^2 - 3(t')^2 + 5(t'')^2\right)}{6\left(-1+e^{t''}\right)} + \frac{g^8 t (t'')^2 \left(t^4 + 4t^2 (t')^2 + 2(t')^4 + 7t^2 (t'')^2 + 12 (t')^2 (t'')^2 + 7(t'')^4\right)}{24\left(-1+e^{t''}\right)} - \frac{1}{720\left(-1+e^{t''}\right)} g^{10} t (t'')^2 \left(2t^6 + 15t^4 (t')^2 + 20t^2 (t')^4 + 5(t')^6 + 27t^4 (t'')^2 + 125t^2 (t')^2 (t'')^2 + 65(t')^4 (t'')^2 + 77t^2 (t'')^4 + 140(t')^2 (t'')^4 + 42(t'')^6\right)$$

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BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

Series expanding the Kernels

```
series[X_, n_, f_:Factor] := Collect[Series[X, {g, 0, n}], g, f]
series[kd, max, Identity]
```

$$\frac{g^4 t (t'')^2}{-1+e^{t''}} + g^6\left(-\frac{t^3 (t'')^2}{3(-1+e^{t''})} - \frac{t (t')^2 (t'')^2}{2(-1+e^{t''})} - \frac{5 t (t'')^4}{6(-1+e^{t''})}\right) + g^8\left(\frac{t^5 (t'')^2}{24(-1+e^{t''})} + \frac{t^3 (t')^2 (t'')^2}{6(-1+e^{t''})} + \frac{t (t')^4 (t'')^2}{12(-1+e^{t''})} + \frac{7 t^3 (t'')^4}{24(-1+e^{t''})} + \frac{t (t')^2 (t'')^4}{2(-1+e^{t''})} + \frac{7 t (t'')^6}{24(-1+e^{t''})}\right) +$$

$$g^{10}\left(-\frac{t^7 (t'')^2}{360(-1+e^{t''})} - \frac{t^5 (t')^2 (t'')^2}{48(-1+e^{t''})} - \frac{t^3 (t')^4 (t'')^2}{36(-1+e^{t''})} - \frac{t (t')^6 (t'')^2}{144(-1+e^{t''})} - \frac{3 t^5 (t'')^4}{80(-1+e^{t''})} - \frac{25 t^3 (t')^2 (t'')^4}{144(-1+e^{t''})} - \frac{13 t (t')^4 (t'')^4}{144(-1+e^{t''})} - \frac{77 t^3 (t'')^6}{720(-1+e^{t''})} - \frac{7 t (t')^2 (t'')^6}{36(-1+e^{t''})} - \frac{7 t (t'')^8}{120(-1+e^{t''})}\right)$$

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BES Kernels

$$K0 = \frac{2\,g\,t\,\mathrm{BesselJ}[1, 2\,g\,t]\,\mathrm{BesselJ}[0, 2\,g\,t'] - 2\,g\,t'\,\mathrm{BesselJ}[1, 2\,g\,t']\,\mathrm{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$K1 = \frac{2\,g\,t'\,\mathrm{BesselJ}[1, 2\,g\,t]\,\mathrm{BesselJ}[0, 2\,g\,t'] - 2\,g\,t\,\mathrm{BesselJ}[1, 2\,g\,t']\,\mathrm{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$kd = 8\,g^2\,(K1\,/.\,t' \to t'')\,\frac{t''}{e^{t''} - 1}\,(K0\,/.\,t \to t'');$$

Series expanding the Kernels

```
series[X_, n_, f_:Factor] := Collect[Series[X, {g, 0, n}], g, f]
```

```
series[kd, max]
```

$$\frac{g^4\,t\,(t'')^2}{-1+e^{t''}} - \frac{g^6\,t\,(t'')^2\,(2t^2 + 3\,(t')^2 + 5\,(t'')^2)}{6\,(-1+e^{t''})} +$$

$$\frac{g^8\,t\,(t'')^2\,(t^4 + 4t^2\,(t')^2 + 2\,(t')^4 + 7\,t^2\,(t'')^2 + 12\,(t')^2\,(t'')^2 + 7\,(t'')^4)}{24\,(-1+e^{t''})} -$$

$$\frac{1}{720\,(-1+e^{t''})}\,g^{10}\,t\,(t'')^2\,(2\,t^6 + 15\,t^4\,(t')^2 + 20\,t^2\,(t')^4 + 5\,(t')^6 + 27\,t^4\,(t'')^2 +$$

$$125\,t^2\,(t')^2\,(t'')^2 + 65\,(t')^4\,(t'')^2 + 77\,t^2\,(t'')^4 + 140\,(t')^2\,(t'')^4 + 42\,(t'')^6)$$