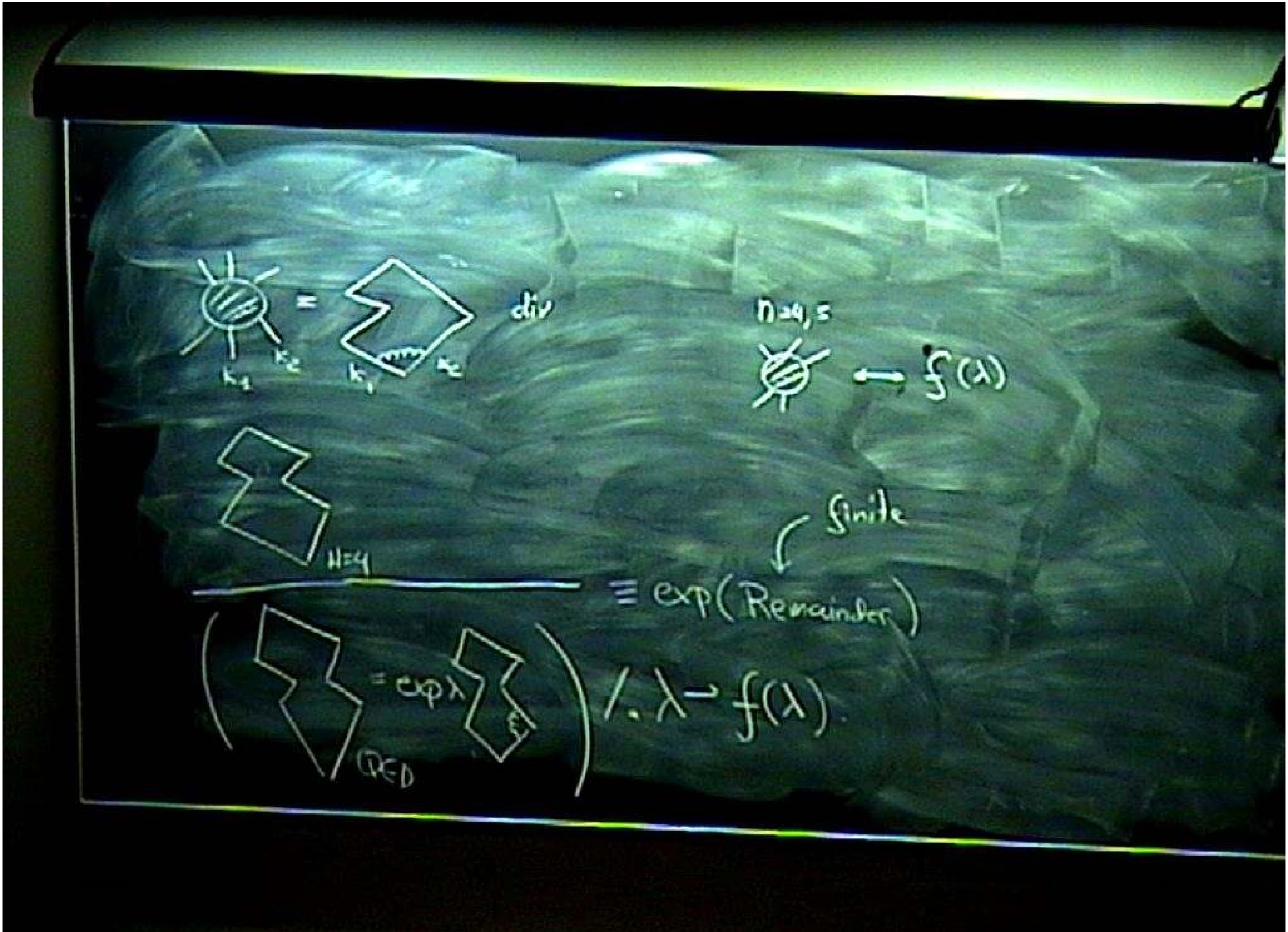
Title: Amplitudes at Strong Coupling

Date: Aug 05, 2011 12:15 PM

URL: http://pirsa.org/11080016

Abstract:


div
k_1 k_2
k_1 k_2
$f(\lambda)$
finite
$N=4$
$\equiv \exp(\text{Remainder})$
$= \exp \lambda$
QED
$/.\ \lambda \to f(\lambda)$

$f(\lambda)$

$$\longleftrightarrow \Delta_{\mathrm{Tr}(Z D^S Z)} \overset{S\to\infty}{=} \log S \; f(\lambda)$$

scalar (→ Z)   lc (→ D)

1) $\mathrm{tr}(Z \ldots \underset{\vec{p}_1}{ZDZ} \ldots \underset{\vec{p}_2}{ZDZ} \cdots Z)$ closed chain

$\{p_i\}$ quantized by some dif. eq. $\forall \lambda$ integrability

↑ $\# u_k \sim S \to \infty$

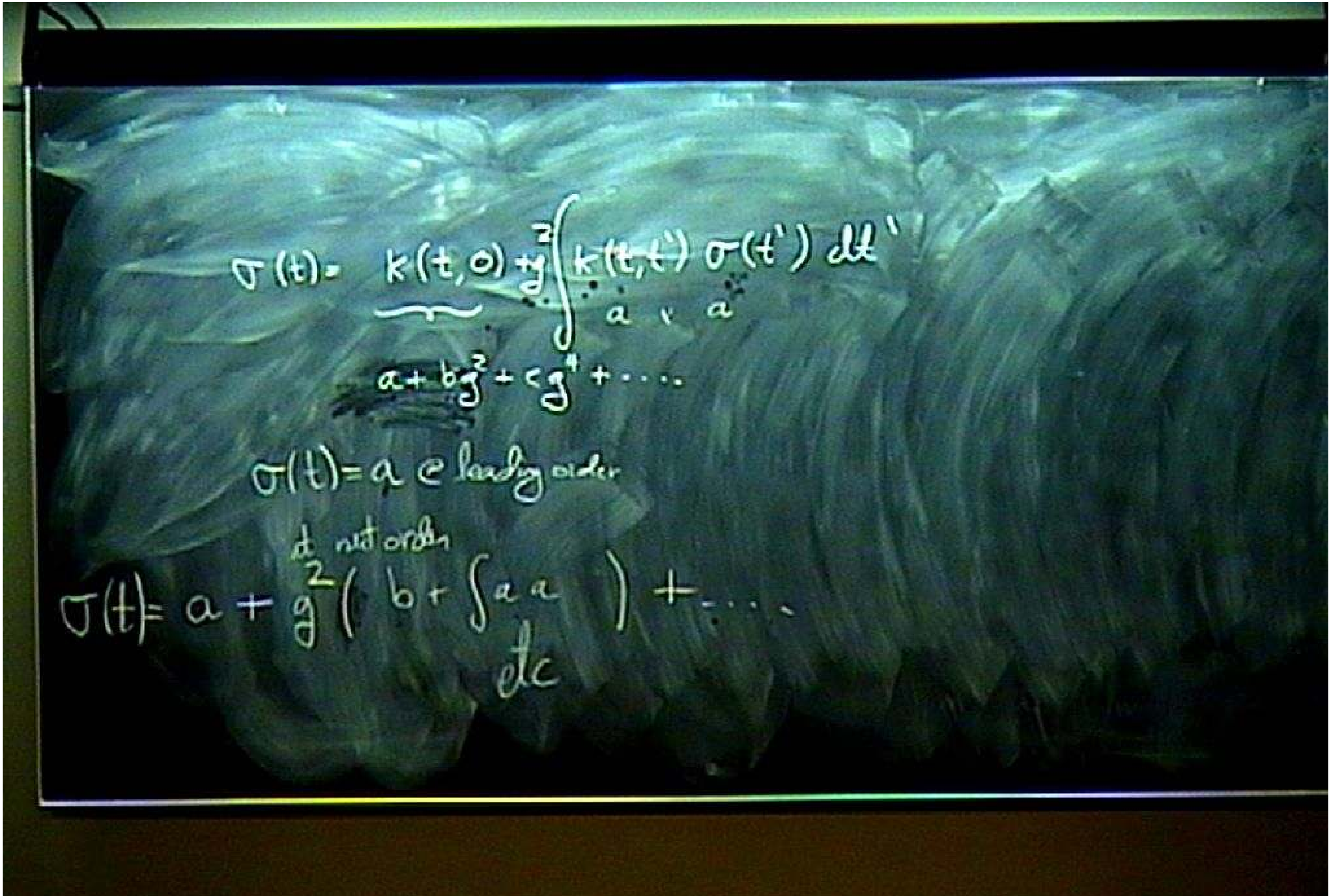
$\Longrightarrow$ integral eq.

2) $\# Z \gg 1$

$\# D \gg \# Z$

$\Delta = \log S \, f(\lambda) + (\ldots)$ ← L only enters here

Cusp Anomalous Dimension


σ(t) = k(t,0) + g² ∫ k(t,t') σ(t') dt'
a a
a + bg² + cg⁴ + ...
σ(t) = a @ leading order
@ next order
σ(t) = a + g²(b + ∫ a a) + ...
etc

BES lecture.nb

BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
Kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

Notation

Notation

Notation

Symbolize

InfixNotation

AddInputAlias

Full Notation Palette

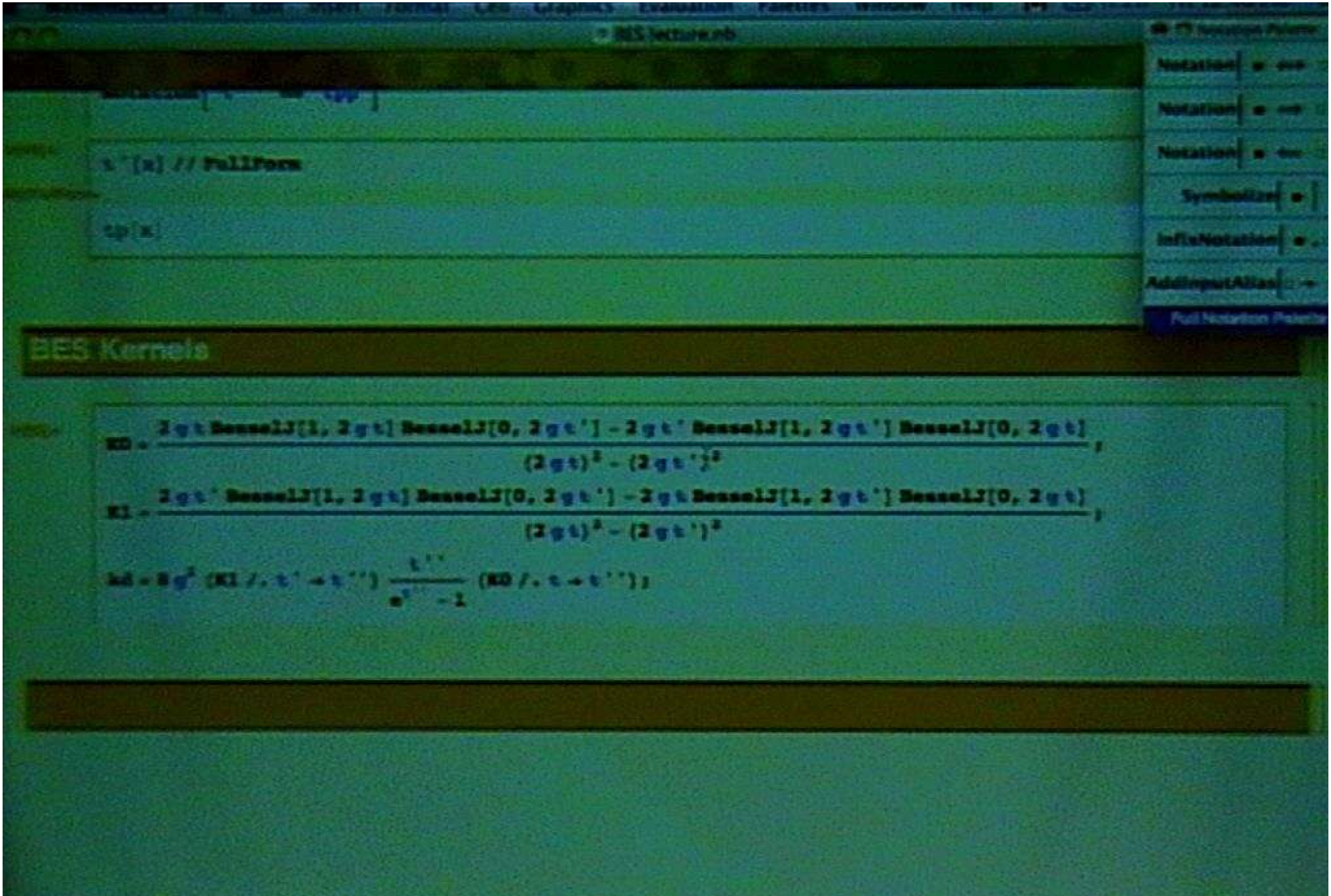
$$\hat{\sigma}(t) = \frac{t}{e^t - 1}\left(K(2gt, 0) - 4g^2 \int_0^\infty K(2gt, 2gt')\hat{\sigma}(t')\right).$$

The kernel in (2.11) is given by

$$K(t,t') = K_0(t,t') - K_1(t,t') + K_d(t,t')$$

$$K_0(t,t') = \frac{tJ_1(t)J_0(t') - t'J_0(t)J_1(t')}{t^2 - t'^2} = \frac{2}{tt'}\sum_{n=1}^{\infty}(2n-1)J_{2n-1}(t)J$$

$$\frac{t'J_1(t)J_0(t') - tJ_0(t)J_1(t')}{} \quad 2\sum^{\infty}$$


BES lecture.nb
t'[x] // FullForm
tp[x]
Notation
Notation
Notation
Symbolize
InfixNotation
AddInputAlias
Full Notation Palette
BES Kernels
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
Kd = 8 g^2 (K1 /. t' → t'') t''/(e^t'' - 1) (K0 /. t → t'');

BES lecture.nb

```
t'[x] // FullForm
```

```
tp[x]
```

Notation
Notation
Notation
Symbolize
InfixNotation
AddInputAlias
Full Notation Palette

BES Kernels

$$K0 = \frac{2\,g\,t\,\text{BesselJ}[1, 2\,g\,t]\,\text{BesselJ}[0, 2\,g\,t'] - 2\,g\,t'\,\text{BesselJ}[1, 2\,g\,t']\,\text{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$K1 = \frac{2\,g\,t'\,\text{BesselJ}[1, 2\,g\,t]\,\text{BesselJ}[0, 2\,g\,t'] - 2\,g\,t\,\text{BesselJ}[1, 2\,g\,t']\,\text{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$Kd = 8\,g^2\,(K1\ /.\ t' \to t'')\,\frac{t''}{e^{t''} - 1}\,(K0\ /.\ t \to t'');$$

Series expanding the Kernels

$$\text{Series}\left[\frac{1}{1 - x}, \{x, 0, 10\}\right]$$

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + O[x]^{11}$$

BES lecture.nb

BES Kernels

$$K0 = \frac{2\,g\,t\,\mathrm{BesselJ}[1, 2\,g\,t]\,\mathrm{BesselJ}[0, 2\,g\,t'] - 2\,g\,t'\,\mathrm{BesselJ}[1, 2\,g\,t']\,\mathrm{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$K1 = \frac{2\,g\,t'\,\mathrm{BesselJ}[1, 2\,g\,t]\,\mathrm{BesselJ}[0, 2\,g\,t'] - 2\,g\,t\,\mathrm{BesselJ}[1, 2\,g\,t']\,\mathrm{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$Kd = 8\,g^2\,(K1\,/.\,t' \to t'')\,\frac{t''}{e^{t''} - 1}\,(K0\,/.\,t \to t'');$$

Series expanding the Kernels

```
series X_Collect[Series[1/(1 - x), {x, 0, 10}], x]
```

$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
kd = 8 g^2 (K1 /. t' → t'') t''/(e^t'' - 1) (K0 /. t → t'');
```

Series expanding the Kernels

```
series[f_, n_] := Collect[Series[1/(1 - x), {x, 0, 10}], x]
```

$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

BES lecture.nb

Notation | Notation | Notation | Symbolize | InfixNotation | AddInputAlias

BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
Kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

Series expanding the Kernels

```
series[f_, n_] := Collect[Series[f, {g, 0, n}], g]
```

$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10}$

BES lecture.nb

BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);

K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);

Kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

Notation

Notation

Notation

Symbolize

InfixNotation

AddInputAlias

Full Notation Palette

Series expanding the Kernels

```
series[X_, n_] := Collect[Series[X, {g, 0, n}], g]

series[Kd, max]
```

BES lecture.nb

Notation | Notation | Notation | Symbolize | InfixNotation | AddInputAlias

Full Notation Palette

BES Kernels

$$K0 = \frac{2\,g\,t\,\text{BesselJ}[1, 2\,g\,t]\,\text{BesselJ}[0, 2\,g\,t'] - 2\,g\,t'\,\text{BesselJ}[1, 2\,g\,t']\,\text{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$K1 = \frac{2\,g\,t'\,\text{BesselJ}[1, 2\,g\,t]\,\text{BesselJ}[0, 2\,g\,t'] - 2\,g\,t\,\text{BesselJ}[1, 2\,g\,t']\,\text{BesselJ}[0, 2\,g\,t]}{(2\,g\,t)^2 - (2\,g\,t')^2};$$

$$kd = 8\,g^2\,(K1\,/.\,t' \to t'')\,\frac{t''}{e^{t''} - 1}\,(K0\,/.\,t \to t'');$$

Series expanding the Kernels

```
series[X_, n_] := Collect[Series[X, {g, 0, n}], g, Factor]
```

```
series[kd, max]
```

$$\frac{g^4\,t\,(t'')^2}{-1 + e^{t''}} - \frac{g^6\,t\,(t'')^2\,(2\,t^2 + 3\,(t')^2 + 5\,(t'')^2)}{6\,(-1 + e^{t''})} +$$

$$\frac{g^8\,t\,(t'')^2\,(t^4 + 4\,t^2\,(t')^2 + 2\,(t')^4 + 7\,t^2\,(t'')^2 + 12\,(t')^2\,(t'')^2 + 7\,(t'')^4)}{24\,(-1 + e^{t''})} -$$

$$\frac{1}{720\,(-1 + e^{t''})}\,g^{10}\,t\,(t'')^2\,(2\,t^6 + 15\,t^4\,(t')^2 + 20\,t^2\,(t')^4 + 5\,(t')^6 + 27\,t^4\,(t'')^2 +$$

$$125\,t^2\,(t')^2\,(t'')^2 + 65\,(t')^4\,(t'')^2 + 77\,t^2\,(t'')^4 + 140\,(t')^2\,(t'')^4 + 42\,(t'')^6$$

BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
Kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

Series expanding the Kernels

```
series[f_, n_, f_:Factor] := Collect[Series[f, {g, 0, n}], g, f]
```

```
series[Kd, max, Identity]
```

BES lecture.nb

Notation
Notation
Notation
Symbolize
InfixNotation
AddInputAlias
Full Notation Palette

BES Kernels

```
K0 = (2 g t BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t' BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
K1 = (2 g t' BesselJ[1, 2 g t] BesselJ[0, 2 g t'] - 2 g t BesselJ[1, 2 g t'] BesselJ[0, 2 g t]) / ((2 g t)^2 - (2 g t')^2);
kd = 8 g^2 (K1 /. t' -> t'') t''/(e^t'' - 1) (K0 /. t -> t'');
```

Series expanding the Kernels

```
series[X_, n_, f_:Factor] := Collect[Series[X, {g, 0, n}], g, f]
```

```
series[kd, max]
```

$$\frac{g^4 t (t'')^2}{-1+e^{t''}} - \frac{g^6 t (t'')^2 (2t^2 + 3(t')^2 + 5(t'')^2)}{6(-1+e^{t''})} + \frac{g^8 t (t'')^2 (t^4 + 4t^2 (t')^2 + 2(t')^4 + 7t^2 (t'')^2 + 12(t')^2 (t'')^2 + 7(t'')^4)}{24(-1+e^{t''})} - \frac{2}{720(-1+e^{t''})} g^{10} t (t'')^2 (2t^6 + 15t^4 (t')^2 + 20t^2 (t')^4 + 5(t')^6 + 27t^4 (t'')^2 + 125t^2 (t')^2 (t'')^2 + 65(t')^4 (t'')^2 + 77t^2 (t'')^4 + 140(t')^2 (t'')^4 + 42(t'')^6)$$