

Title: Unitarity Methods

Date: Aug 05, 2011 11:15 AM

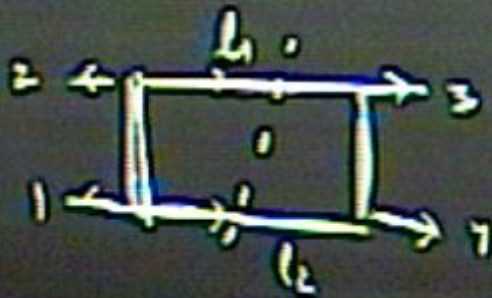
URL: <http://pirsa.org/11080015>

Abstract:

• Generalized cuts :



• Example



$$= - \frac{[34]^2}{\langle 12 \rangle^2}$$

$$= \frac{1}{(2l_1 + l_2)(2l_1 + l_2)} \\ \begin{matrix} \nearrow & \nearrow \\ (l_1 + l_2)^2 & - (l_1 + l_2)^2 \end{matrix}$$

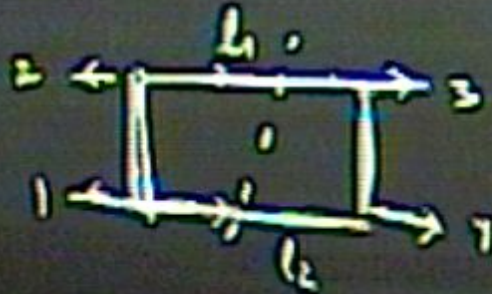
$$\frac{[31]^2}{\langle 12 \rangle^2}$$



• Generalized cuts :



• Example



$$s^2(\sum q_i \cdot \lambda_i)$$

$$= - \frac{[34]^2}{\langle 12 \rangle^4}$$

$$\frac{1}{(2l_1 \cdot l_2)(2l_1 \cdot l_3)} \cdot \frac{1}{(l_1 + l_2)^2 - (l_1 - l_2)^2}$$

$$= - \frac{[34]^2}{\langle 12 \rangle^4}$$

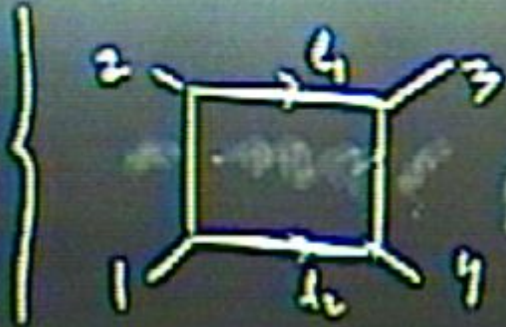
$$\frac{1}{(2l_1 + l_2)(2l_1 + l_3)} \cdot \frac{1}{(l_1 + l_2)^2 - (l_1 - l_2)^2}$$

$$\frac{[31]^2}{\langle 12 \rangle^2}$$



$$B = 0 = C = 0 \Rightarrow$$

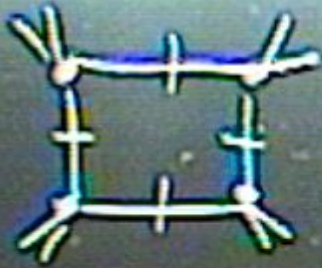
$$\frac{[31]^2}{\langle 12 \rangle^2}$$



$$B = 0 = C = 0 \Rightarrow$$

$$0 = \frac{A}{g_{11}l_1} + \frac{B}{g_{22}l_2} + C$$

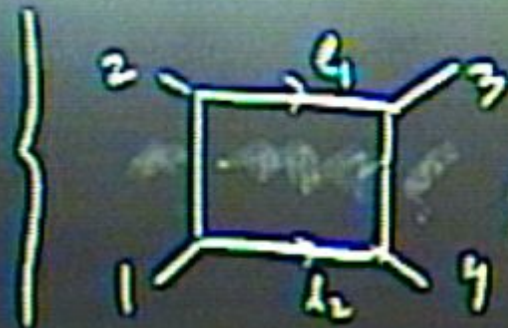
$$g_{11} + g_{22} + l_3 + l_2 = 0$$



• Analytics vs Numerics

- 1) Analytically
 - Integral reduction

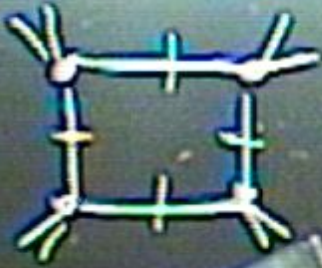
$$= \frac{[31]^2}{\langle 12 \rangle^2}$$



$$B = 0 = C = 0 \Rightarrow$$

$$0 = \frac{A}{g_1 g_2} + \frac{B}{g_3 g_4} + C$$

$$g_1 + g_2 + g_3 + g_4 = 0$$



• Analytically vs numerics

1) Analytically

• Integral reduction

- ↙ algebraic ✓
- use of \int

↘ (ignores $\int df = 0$)

• Numerics

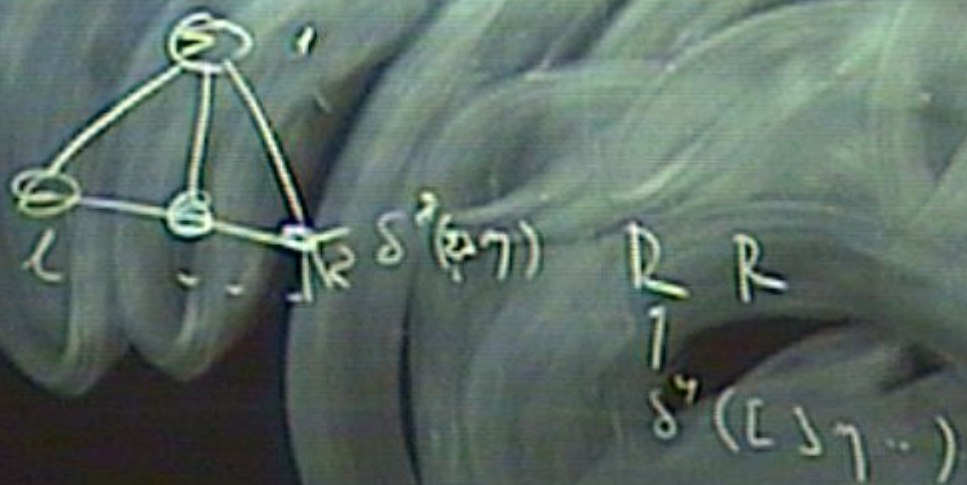
• Efficient evaluation of cuts ✓

• Ansatz { - dumb
 - smarter (later)

• No real need to assume some form of amplitude.

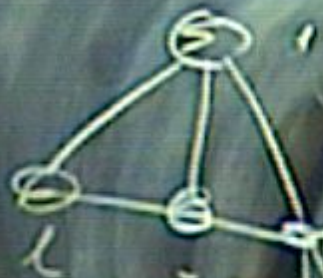
Superstrings / ~~Quantum~~

$$J = \int d\eta_1 \dots d\eta_n \prod_{i=1}^n \delta^2(\lambda \eta_i) \prod_{i=1}^n \delta^4(d s_i \eta_i)$$



• Supersums / ~~quadruple~~

$$J = \int d\eta_1 \dots d\eta_n \prod_{i=1}^n \delta^2(\eta_i) \prod_{i=1}^n \delta^2(\eta_i)$$



$$\delta^2(\eta_i) \quad R \quad R$$

$$\delta^2(L \eta \dots)$$

- Cuts of MHV
- Rest

• Supersymmetry / gauging

$$J = \int d\eta_1 \dots d\eta_n \int_{\mathcal{C}} \delta(\sum \eta_i) \int_{\mathcal{C}'} \delta(\sum \eta_i)$$



$$\delta^2(\sum \eta_i) \begin{matrix} R & R \\ | & | \\ \delta^2(L \eta \dots) \end{matrix}$$

• Cuts of MHV

• Rest

• Supersymmetry / supersymmetry

$$J = \int d^4\eta_1 \dots d^4\eta_n \prod_{i=1}^n \delta^4(\eta_i - \eta_{i+1}) \prod_{i=1}^n \delta^4(\eta_i - \eta_{i+1})$$



$\delta^4(\eta_1)$
 $\delta^4(\eta_2)$
 $\delta^4(\eta_3)$
 $\delta^4(\eta_4)$

• Cuts of MHV

• Rest

• Supersums / ~~quadrup~~

$$J = \int d^4\eta_1 \dots d^4\eta_n \prod_{i=1}^n \delta^4\left(\sum_{j=1}^n C_{ij} \eta_j\right) \prod_{i=1}^n \delta^4(\dots \eta_i)$$



$$\delta^4(\dots \eta_i) \begin{matrix} R & R \\ | & | \\ \delta^4(L \eta \dots) \end{matrix}$$

- Cuts of MHV
- Rest

• Supersums / ~~generalized~~

$$J = \int d\eta_1 \dots d\eta_n \prod_{i=1}^n \delta\left(\sum_{j=1}^n C_{ij} \eta_j\right) \prod_{i=1}^n \delta^2(\sum_{j=1}^n \eta_j) \leftarrow \delta^2(\sum \eta_j)$$

(22+47-1)



$$\delta^2(\sum \eta_j) \quad R \quad R$$

$$\delta^2(L \cup \eta \dots)$$

- Cuts of MHV
- No more $\delta(\eta)$
- Rest

• Supersums / ~~generalized~~

$$J = \int d\eta_{L1} \dots d\eta_{L2} \prod_{i=1}^n \delta\left(\sum_{j=1}^n C_{ij} \eta_j\right) \prod_{a=1}^r \delta^4\left(\sum_{i=1}^n c_{ai} \eta_i\right) \propto \delta^4(\eta)$$

(2L+4r-1)



$$\delta^4(\eta) \quad R \quad R$$

$$\delta^4(L \eta \dots)$$

- Cuts of MHV
- No more $\delta(\eta)$
- 4th power

$$J = \left(\det \begin{pmatrix} C_{11} & \dots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{r1} & \dots & C_{rn} \\ d_{1+} \end{pmatrix} \right)^4$$

$$\int \delta^2(q_{12} - \lambda_1 \eta_{12} - \lambda_2 \eta_{12}) \int \delta^2(q_{12} + \eta_{12} \lambda_1 + \eta_{12} \lambda_2)$$

$$\int \delta^2(q_{12} + q_{12}) \prod_{a=1}^4 \int \delta^2(q_{12} + \eta_{12} \lambda_1 + \lambda_2 \eta_{12})$$

$$\int \delta(\eta - \text{small})$$

$$\begin{cases} \eta_{12}^1 \lambda_1^1 + \eta_{12}^2 \lambda_2^1 = -q_{12}^1 \\ \eta_{12}^1 \lambda_1^2 + \eta_{12}^2 \lambda_2^2 = -q_{12}^2 \end{cases}$$

$$\begin{pmatrix} \lambda_1^1 & \lambda_2^1 \\ \lambda_1^2 & \lambda_2^2 \end{pmatrix}$$

$$\begin{pmatrix} \delta(\eta^1 - \text{small}) \\ \delta(\eta^2 - \text{small}) \end{pmatrix}$$

$$\times \int = \int (M \cdot \eta + X)$$

$$\begin{pmatrix} \lambda_1^1 \lambda_2^2 - \lambda_2^1 \lambda_1^2 \end{pmatrix}$$

$$\langle \eta \rangle$$

$$= \int \delta(\eta + M^{-1}X) \det M$$

$\delta^2 \left(\eta_{12}^2 + \eta_{14}^2 + \eta_{23}^2 \right)$
 $\delta^2 \left(\eta_{12}^2 + \eta_{13}^2 + \eta_{14}^2 + \eta_{23}^2 + \eta_{24}^2 + \eta_{34}^2 \right)$
 $\delta^2 \left(\eta_{13}^2 + \eta_{14}^2 + \eta_{23}^2 + \eta_{24}^2 \right)$

$$\frac{16}{24} = 16$$

$$= \langle l_1 l_2 \rangle^4 \langle l_3 l_4 \rangle^4$$

$$\begin{array}{l}
 \text{1.} \\
 \text{2.} \\
 \text{3.} \\
 \text{4.}
 \end{array}
 \left. \begin{array}{l}
 \text{---} \\
 \text{---} \\
 \text{---} \\
 \text{---}
 \end{array} \right\} \begin{array}{l}
 \int \delta^2 (q_{12}^a + \eta_{14}^a \lambda_{24}^a + \eta_{13}^a \lambda_{23}^a) \\
 \int \delta^2 (q_{13}^a + \eta_{12}^a \lambda_{23}^a - \eta_{14}^a \lambda_{24}^a) \\
 \int \delta^2 (q_{14}^a + \eta_{13}^a \lambda_{23}^a - \eta_{12}^a \lambda_{24}^a) \\
 \int \delta^2 (q_{23}^a + \eta_{13}^a \lambda_{23}^a + \eta_{14}^a \lambda_{24}^a)
 \end{array}$$

$$\overline{16} \quad \overline{24 - 8} = 16$$

$$= \langle l_3 l_4 \rangle^4 \langle l_3 l_4 \rangle^4$$

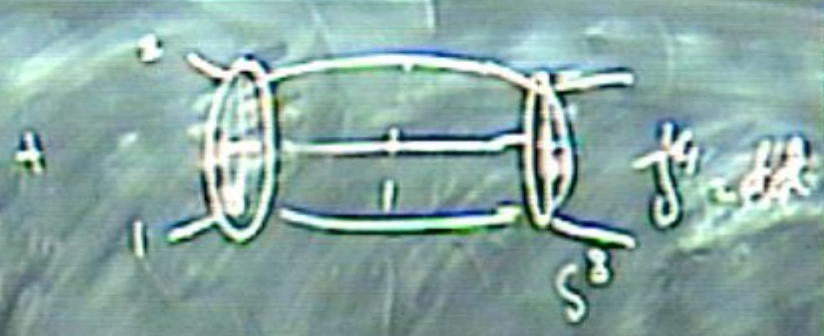
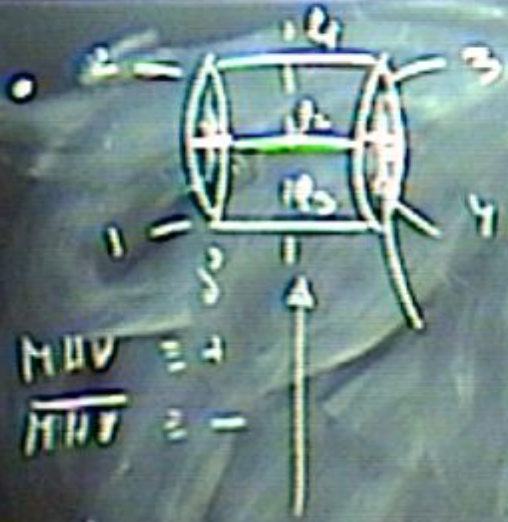
$$\det \left[\begin{array}{cc|cc}
 \lambda_{14}^a & \lambda_{13}^a & \lambda_{23}^a & \lambda_{24}^a \\
 0^a & 0^a & \lambda_{13}^a & \lambda_{14}^a
 \end{array} \right]$$

• Supersums / gauge

$$J = \int d^4\eta_1 \dots d^4\eta_n \prod_{i=1}^n \delta\left(\sum_{j=1}^n C_{ij} \eta_j\right) \prod_{i=1}^n \delta(\eta_i) \dots$$

$$\Delta(n) = \frac{i(-1)^n}{\prod_{i=1}^n C_{ii}} \int d^4\omega \prod_{i=1}^n \delta(\eta_i - \omega C_{ii})$$

$$J = \left(\det(\text{matrix of } \omega\omega\text{'s}) \right)^4$$



$$\begin{bmatrix} 1 & 0 & 0 & -\tilde{\lambda}_4^1 - \tilde{\lambda}_4^2 \\ 0 & 1 & 0 & \end{bmatrix}$$

$$\int d\eta_{11} d\eta_{12} d\eta_{13} \int_0^1 \delta \left(\eta_{11}^a + \eta_{14}^a \tilde{\lambda}_4^x + \eta_{12}^a \tilde{\lambda}_2^x + \eta_{13}^a \tilde{\lambda}_3^x \right)$$

$$\int \left(\prod_{j=3}^4 \delta \left(\eta_j^a - \omega_2^a \tilde{\lambda}_j^x \right) \right) \delta \left(\eta_{14}^a - \omega_2^a \tilde{\lambda}_4^x \right) \delta \left(\eta_{12}^a - \dots \right) \delta \left(\eta_{13}^a - \dots \right)$$

• Supersums / general

$$J = \int d^4\eta_1 \dots d^4\eta_n \prod_{i=1}^n \delta\left(\sum_{j=1}^n C_{ij} \eta_j\right) \prod_{i=1}^n \delta^4(\eta_i) \propto \delta^4(\sum_{i=1}^n \eta_i)$$

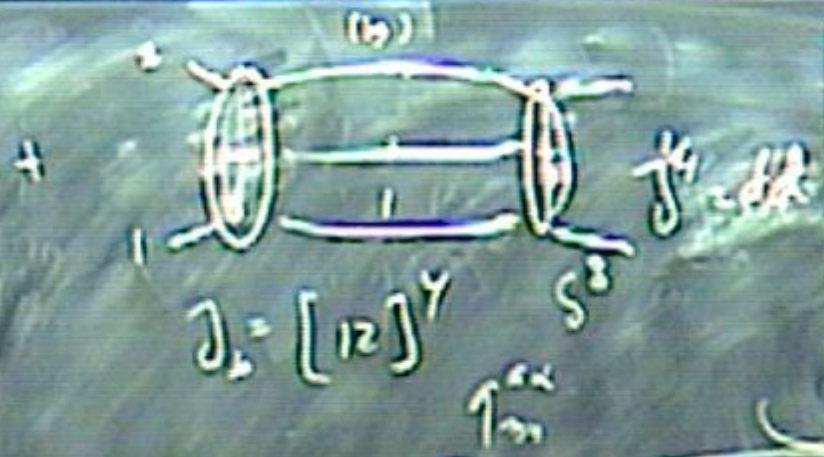
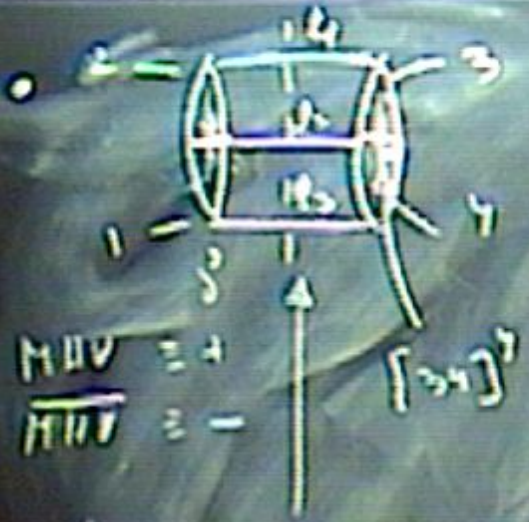
HBV

$$\Delta((1, 2, \dots, n)) = \frac{i^{(n-1)^2}}{\prod_{i=1}^{n-1} (i+1)} \int d^2\omega \prod_{i=1}^n \delta^4(\eta_i - \omega_i \lambda_i)$$

$$J = \left(\det(\text{matrix of coef's}) \right)^4$$

$$0 = \sum_{i=1}^n \lambda_i (\eta_i^\alpha - \omega_i^\alpha \lambda_i)$$

$$= \sum \lambda_i \eta_i^\alpha + \omega \sum \lambda_i^2$$



1	0	0	1	λ_1^2	λ_1
0	1	0	1	λ_2^2	λ_2
0	0	1	1	λ_3^2	λ_3
0	0	0	1	λ_4^2	λ_4
0	0	0	0	λ_5^2	λ_5

$$\int d\eta_{11} d\eta_{12} d\eta_{13} \int_0^{\infty} \delta \left(\eta_{11}^{ax} + \eta_{14}^x \lambda_4^x + \eta_{12}^x \lambda_2^x + \eta_{13}^x \lambda_3^x \right) = [34]$$

$$\int \left(\prod_{j=3}^4 \delta \left(\eta_j^a - \omega_2^a \tilde{\lambda}_j^x \right) \right) \delta \left(\eta_{14}^a - \omega_{14}^a \tilde{\lambda}_4^x \right) \delta \left(\eta_{12}^a - \dots \right) \delta \left(\eta_{13}^a - \dots \right)$$

• 2-loop

$$A^{(2)} = \frac{-1}{4} \frac{[123]^2}{\langle 12 \rangle^2} \left\{ \begin{array}{l} \text{Diagram 1} \\ \text{Diagram 2} \end{array} \right\}$$

• non-MHV

• J Phys A / Aug / Sept
 - 1103.1869
 - 1103.3292

$$i = e \sum_{L=1}^{\infty} \delta^{L-1} M_L(\epsilon) \gamma_L$$

$$M_L = \frac{1}{\epsilon^L} \oplus \frac{1}{\epsilon} + c + O(\epsilon) + O(\epsilon^2)$$

"Scattering amplitudes in gauge theories"

• Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka
 & more