Title: Unitarity Methods

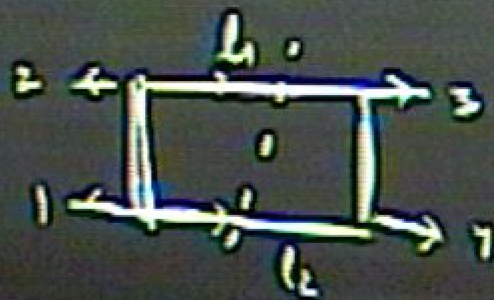
Date: Aug 05, 2011 11:15 AM

URL: http://pirsa.org/11080015

Abstract:

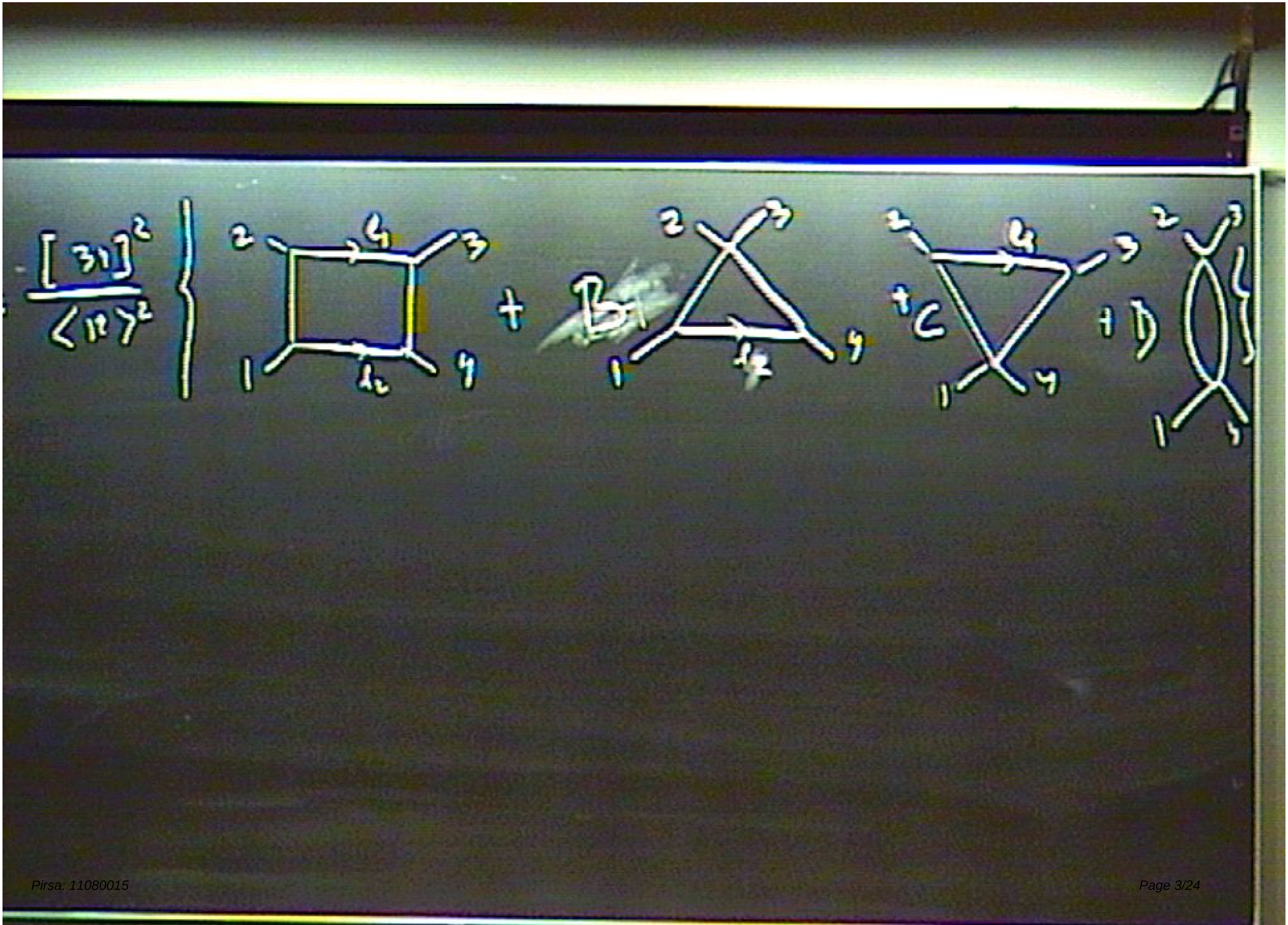
- Generalized cuts:

- Example



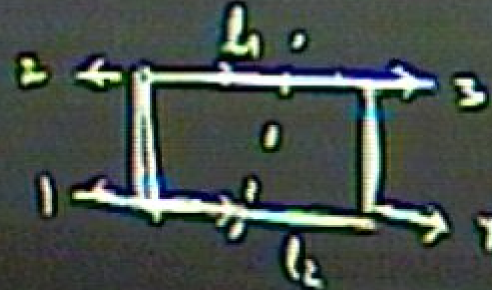
$$= -\frac{[34]^2}{\langle 12\rangle^2}\;\frac{1}{(2 l_2\cdot k_1)(2 l_1\cdot k_3)}$$

$(2 l_2\cdot k_1) \leftarrow (l_2+k_1)^2$, $\quad (2 l_1\cdot k_3) \leftarrow -(l_1-k_3)^2$


[31]^2
<12>^2
+ B
+ C
+ D

- Generalized cuts :

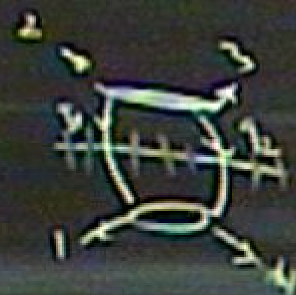
- Example



$$\delta^8\left(\sum \eta_i \lambda_i\right)$$

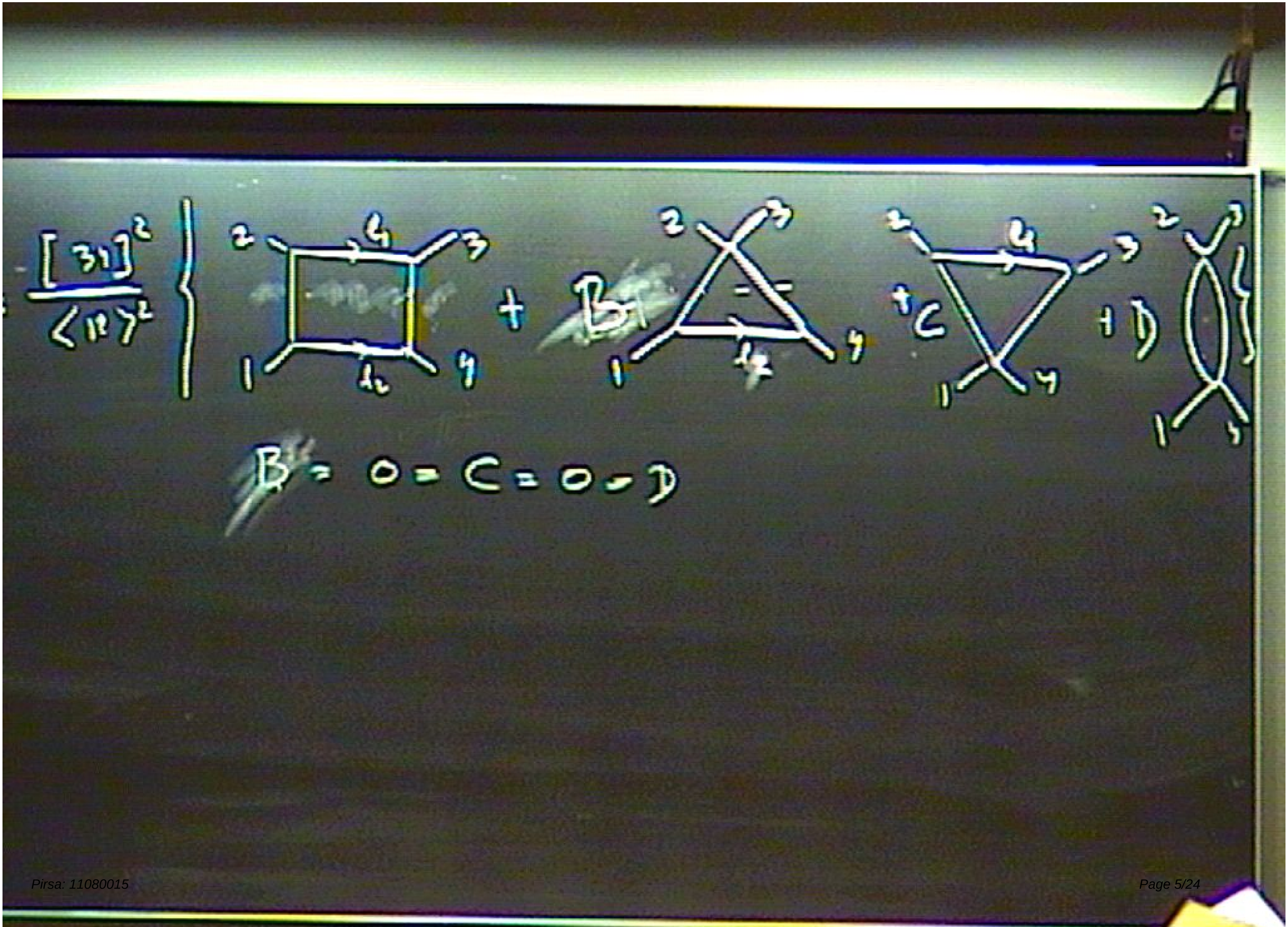
$$\int \;=\; -\frac{[34]^2}{\langle 12\rangle^2}\,\frac{1}{(2\ell_2\cdot k_4)(2\ell_1\cdot k_3)}$$

$(2\ell_2\cdot k_4) \leftarrow (\ell_2+k_4)^2$, $\quad (2\ell_1\cdot k_3) \leftarrow -(\ell_1-k_3)^2$



$$= -\frac{[34]^2}{\langle 12\rangle^2}\,\frac{1}{(2q_1\cdot k_2)(2q_2\cdot k_4)}$$

$(2q_1\cdot k_2) \leftarrow (q_1+k_2)^2$, $\quad (2q_2\cdot k_4) \leftarrow -(q_2-k_4)^2$

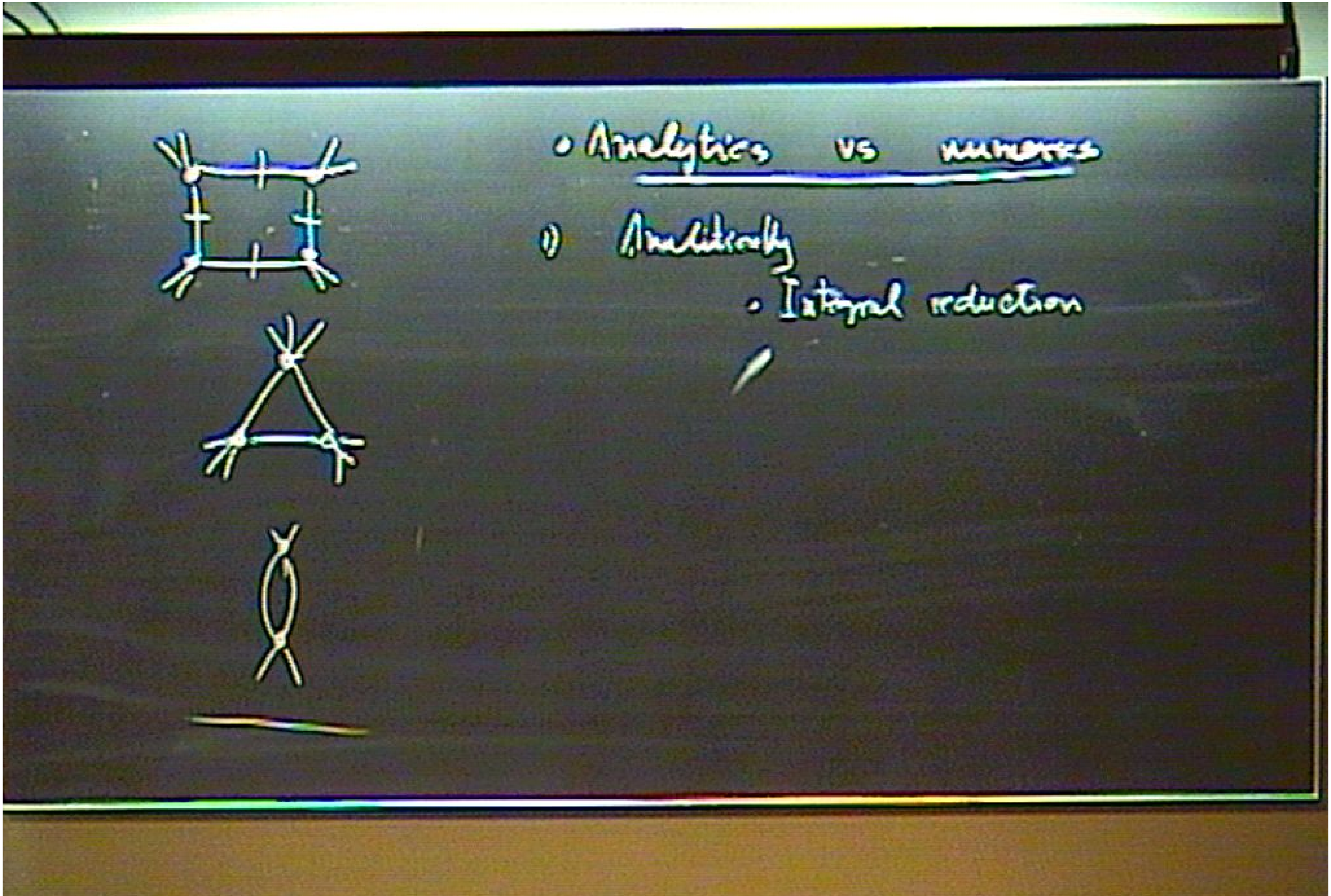

[31]^2 / ⟨12⟩^2
+ B
+ C
+ D
B = 0 = C = 0 = D

$$= \frac{[31]^2}{\langle 12\rangle^2} \Bigg|$$ 

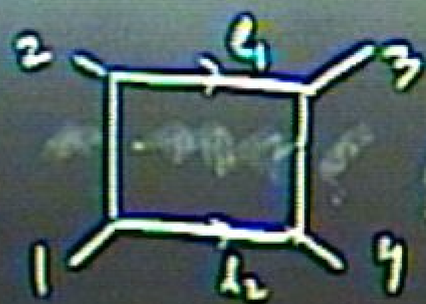
$$B = 0 = C = 0 = D$$

$$0 = \frac{A}{q_1 \cdot k_1} + \frac{B}{q_2 \cdot k_4} + C$$

$$q_1 + q_2 + k_3 + k_2 = 0$$


• Analytics vs numerics
1) Analytically
• Integral reduction

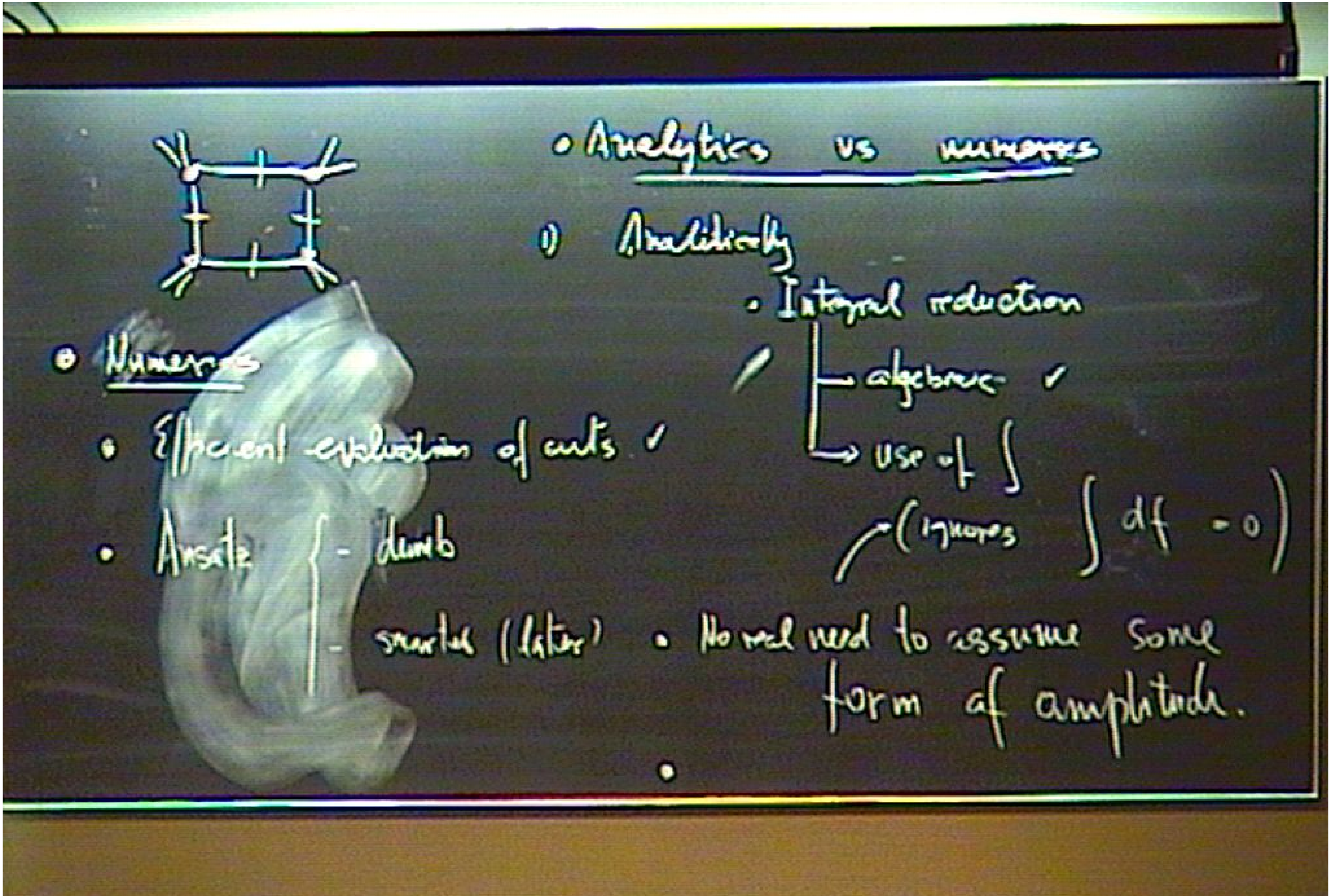
$$= \frac{[34]^2}{\langle 12\rangle^2}$$

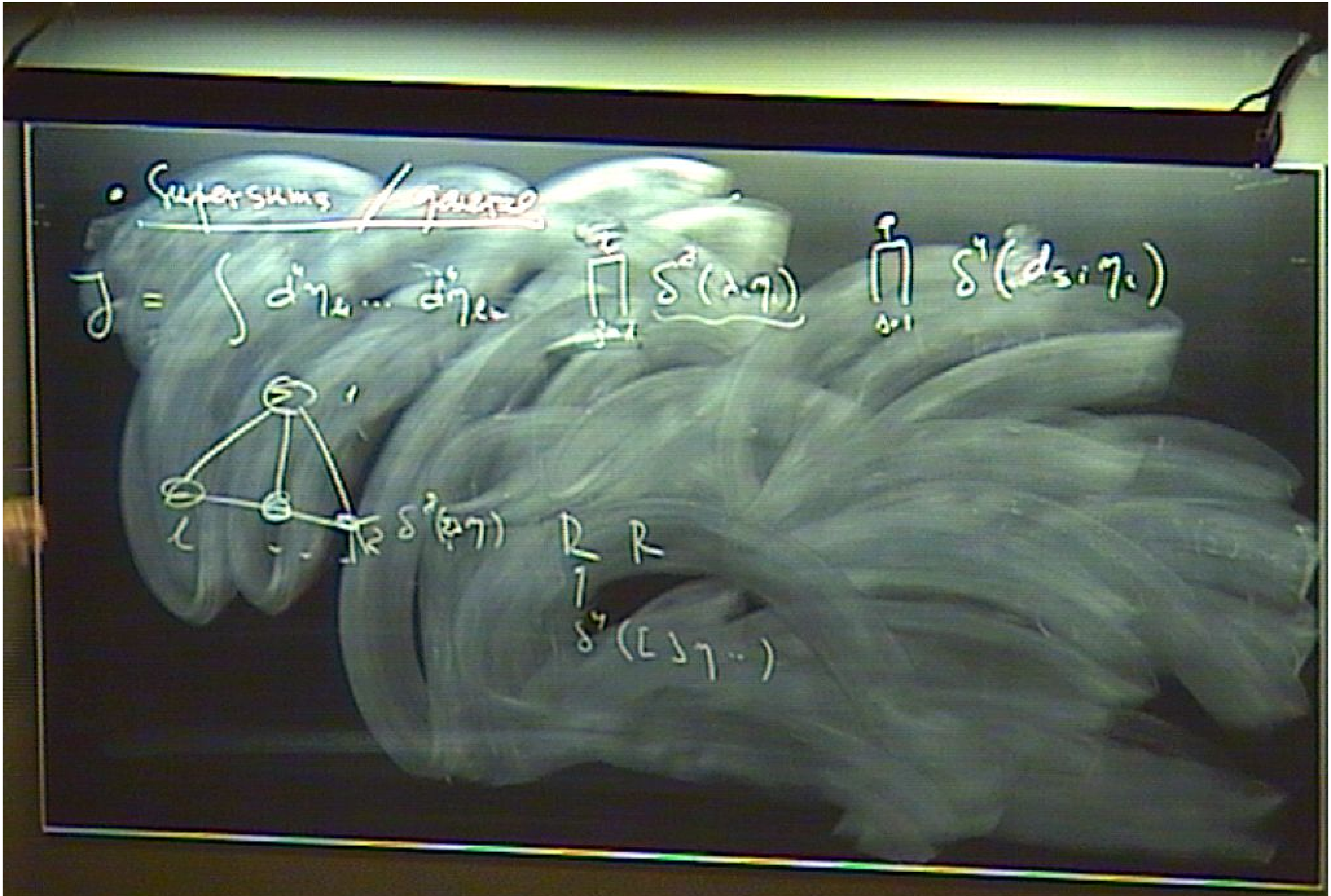


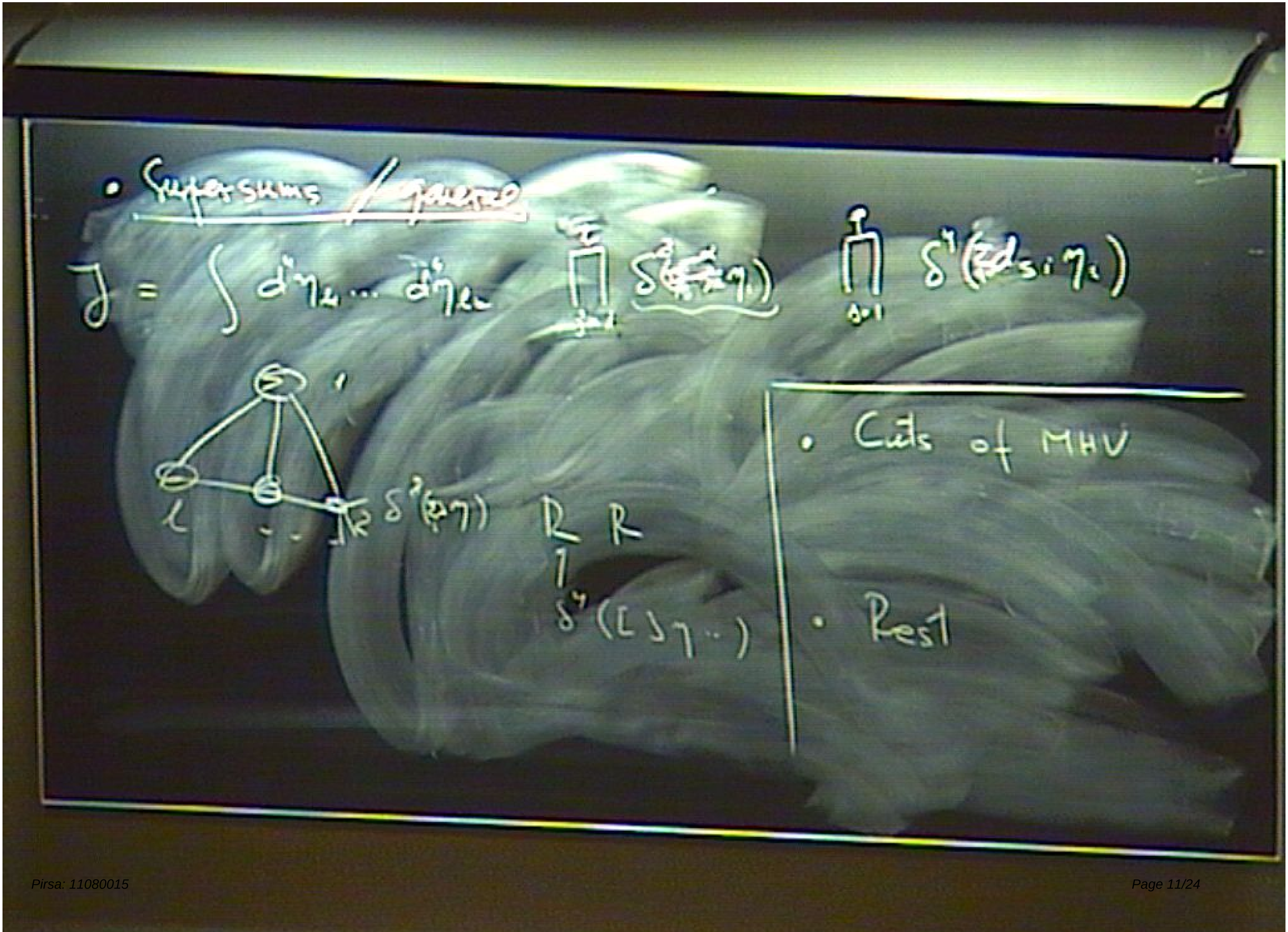
$$B = 0 = C = 0 = D$$

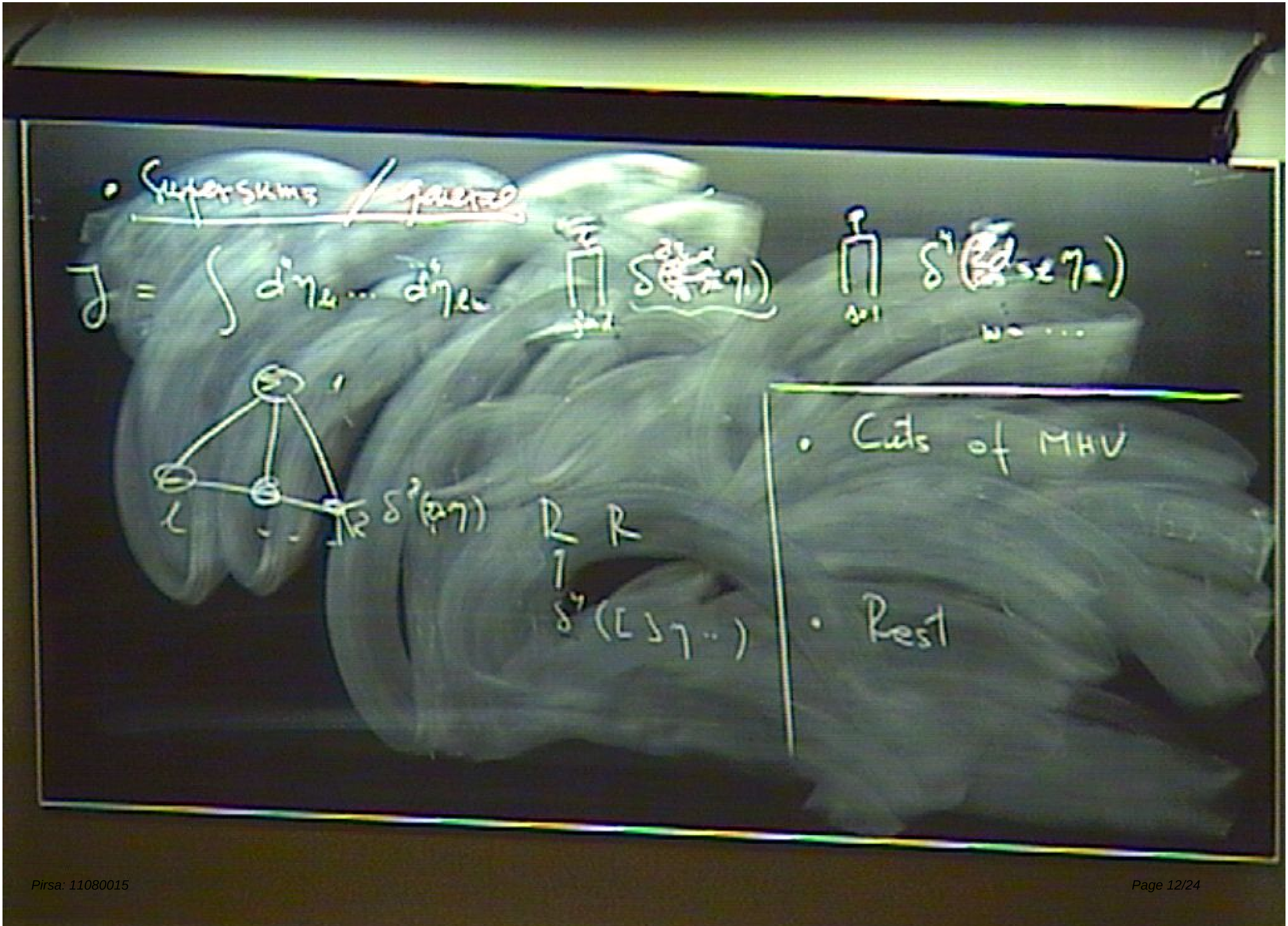
$$0 = \frac{A}{q_1 \cdot k_2} + \frac{B}{q_2 \cdot k_4} + C$$

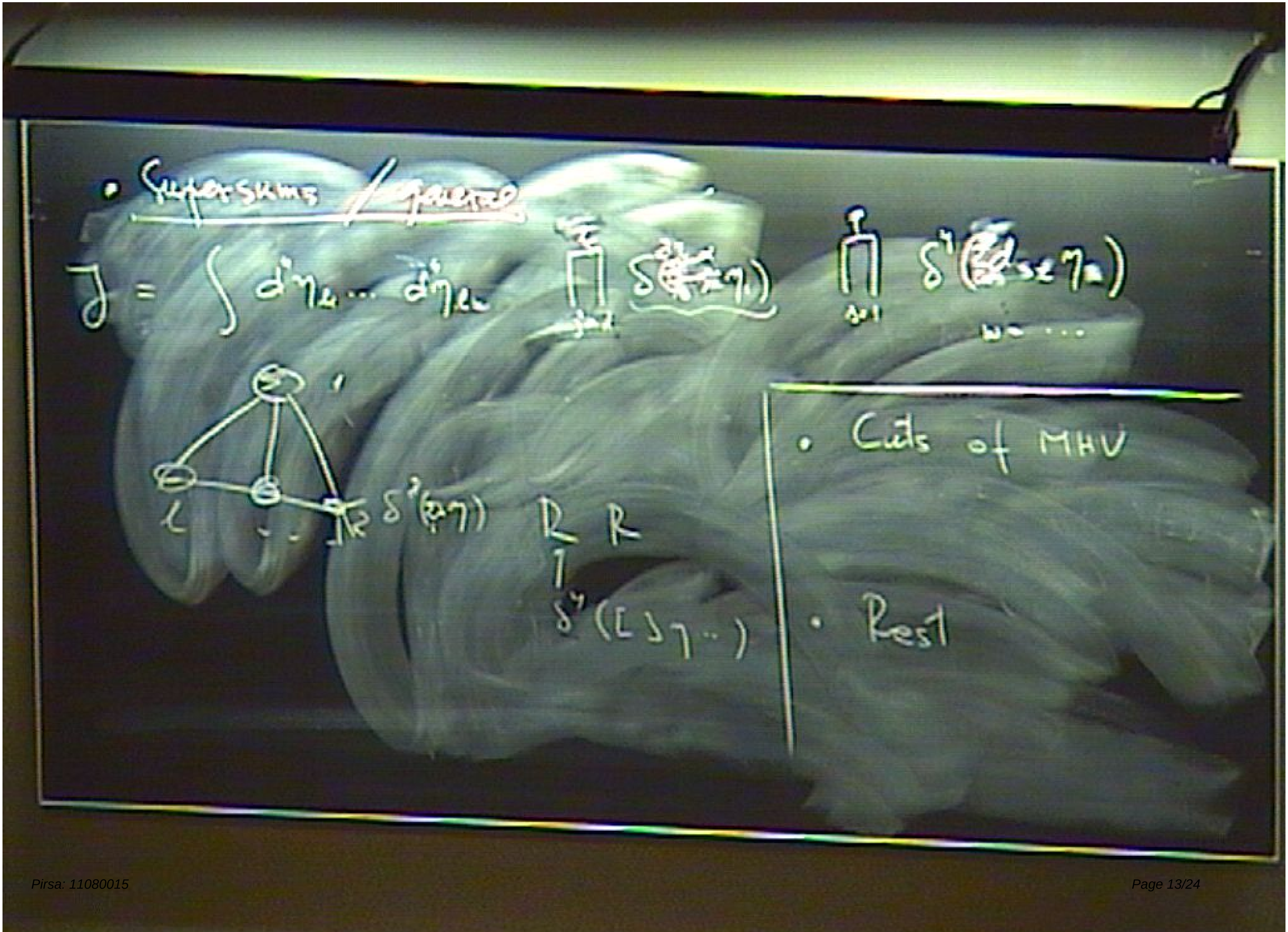
$$q_1 + q_2 + k_3 + k_2 = 0$$


• Analytics vs numerics
1) Analitically
• Integral reduction
algebraic ✓
use of ∫
(ignores ∫ df = 0)
• Numerics
• Efficient evaluation of cuts ✓
• Ansatz
- dumb
- smarter (later)
• No real need to assume some form of amplitude.

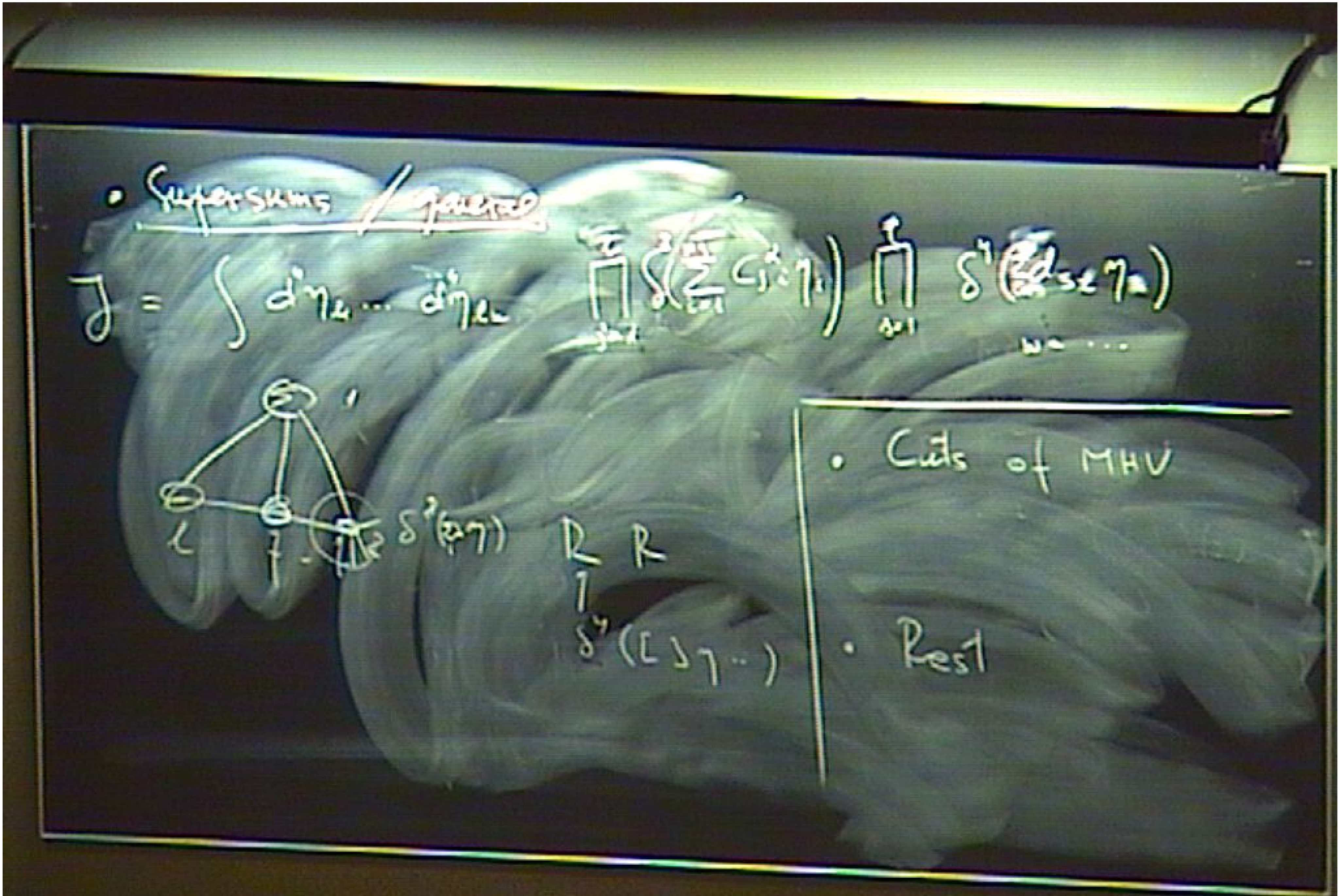


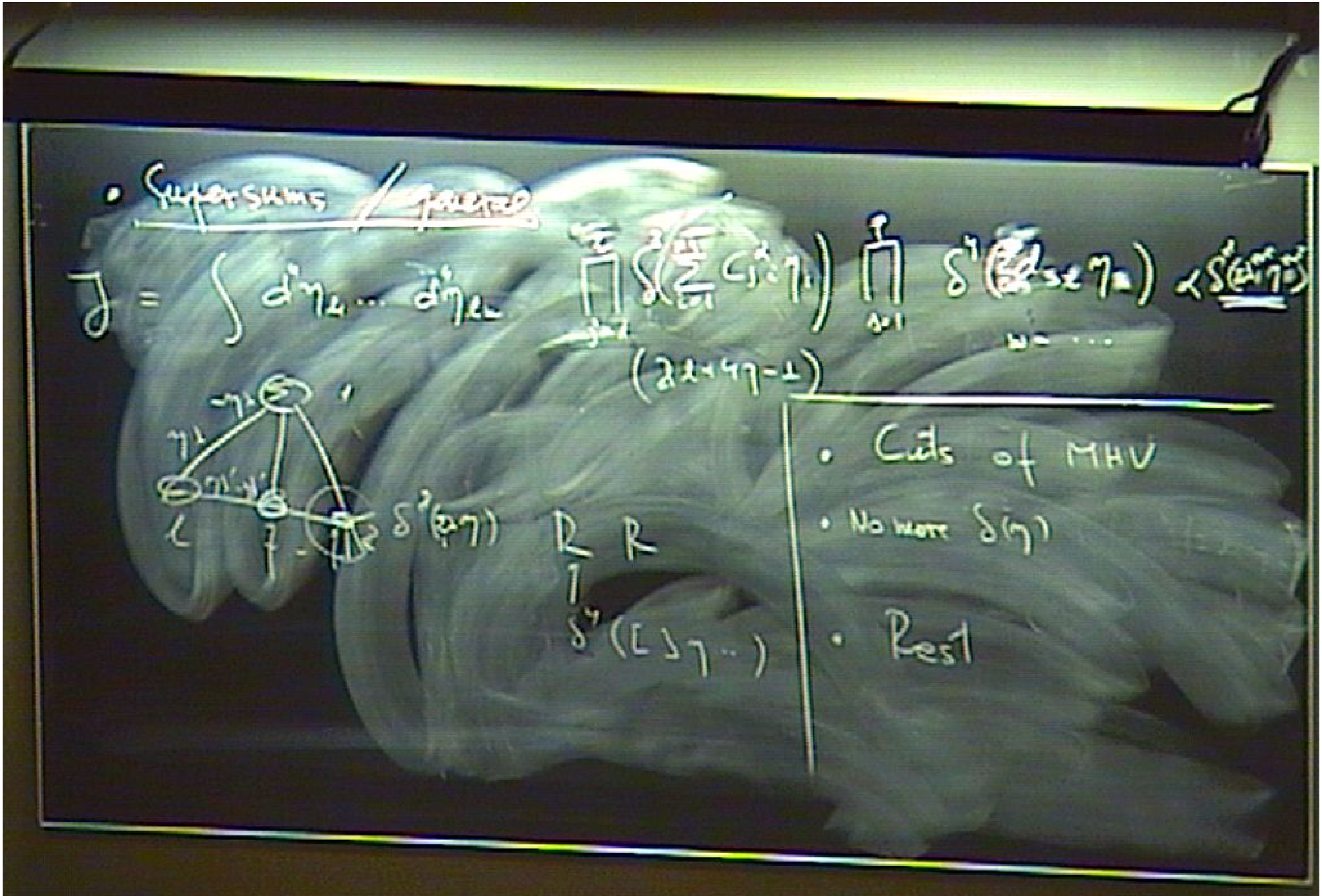

• Supersums
• Cuts of MHV
• Rest


• Supersums
• Cuts of MHV
• Rest



• Cuts of MHV
• Rest

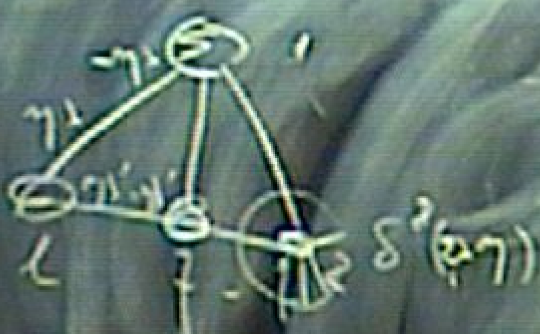

• Cuts of MHV
• Rest


Cuts of MHV
No more δ(η)
Rest

• Supersums / general

$$J = \int d^4\eta_{L_1} \cdots d^4\eta_{L_m} \prod_{j=1} \delta\Big(\sum_{i=1} c_{ji}\, \eta_i\Big) \prod_{s=1} \delta^4\Big(\sum d_{st}\, \eta_t\Big) \times \delta^8(\ldots)$$

$(2k + 4\eta - 1)$



$R \quad R$

$\delta^4(L\, J\, \eta \ldots)$

• Cuts of MHV

• No more $\delta(\eta)$

• 4th power

$$J = \left(\det \begin{pmatrix} c^1_{j,i} \\ c^2_{j,i} \\ d_{st} \end{pmatrix} \right)^4$$

$$\delta^8(q_{12} - \lambda_{l_1}\eta_{l_1} - \lambda_{l_2}\eta_{l_2})\ \delta^8(\eta_{34} + \eta_{l_1}\lambda_{l_1} + \eta_{l_2}\lambda_{l_2})$$

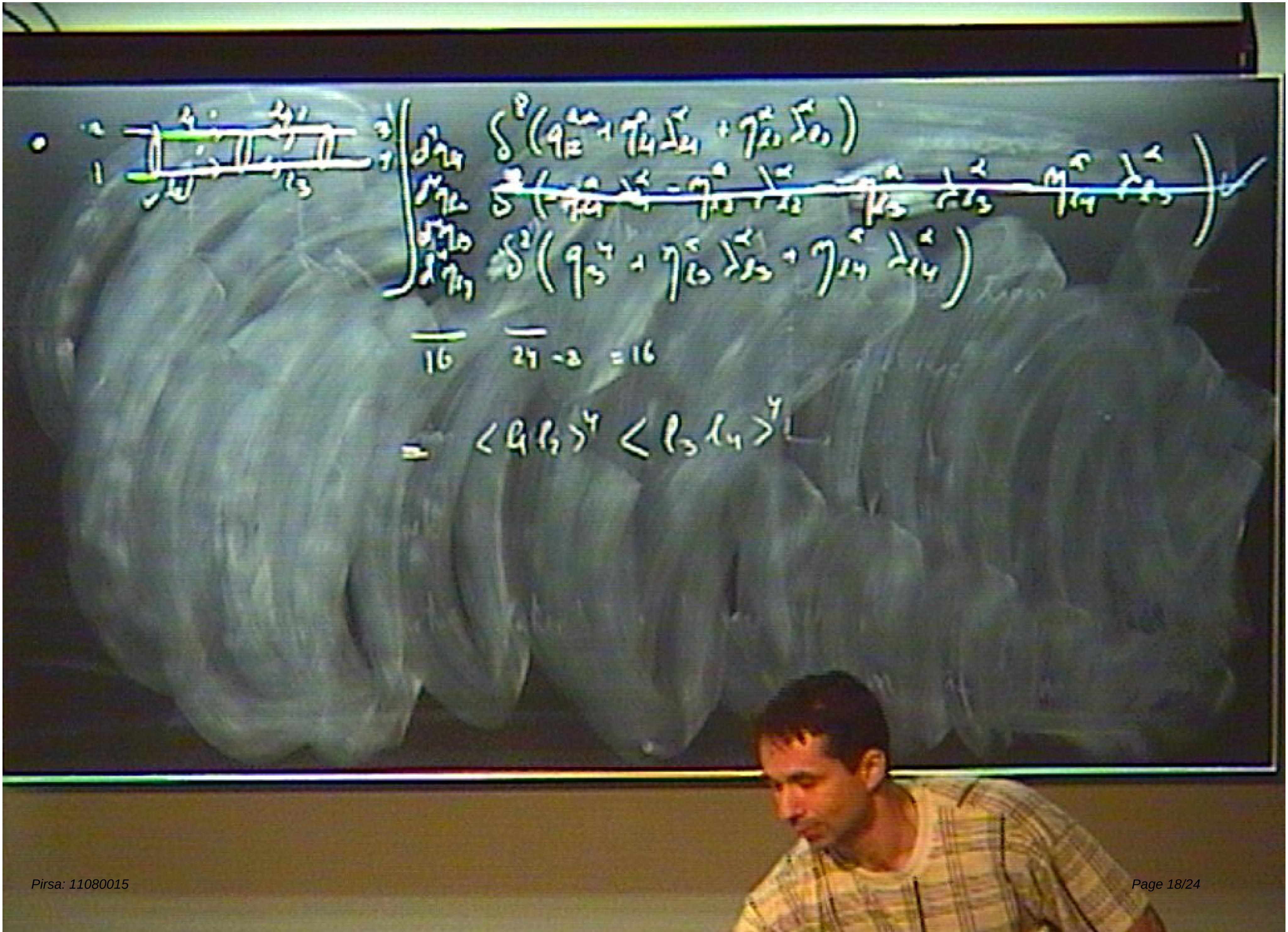
$$\delta^2(q_{12}+q_{34}) \prod_{a=1}^{8} \delta(\eta^{a}_{34} + \eta^a_{l_1}\lambda^{\alpha}_{l_1} + \lambda^{\alpha}_{l_2}\eta^a_{l_2})$$

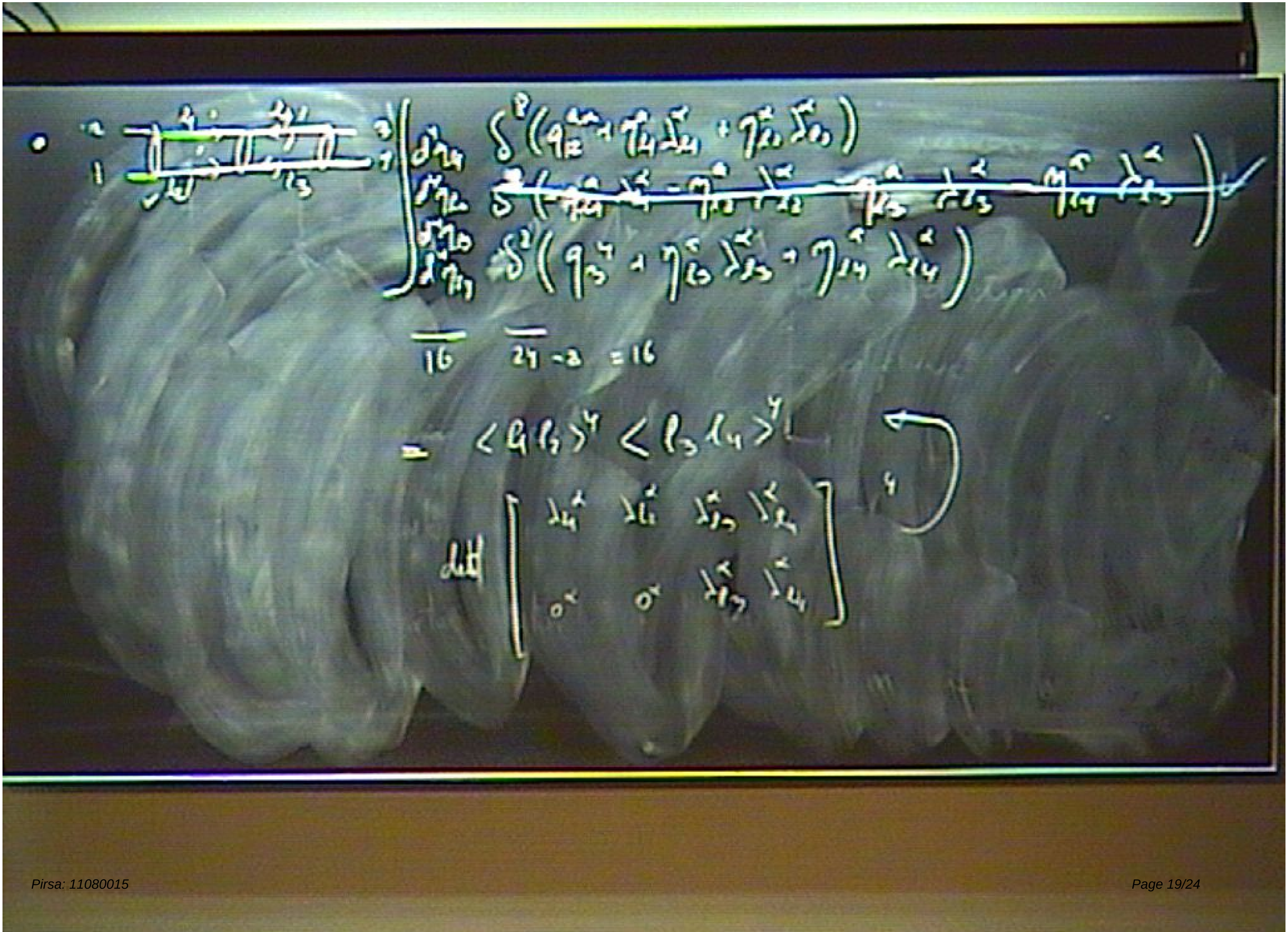
$$\int \delta(\eta - \text{soln})$$

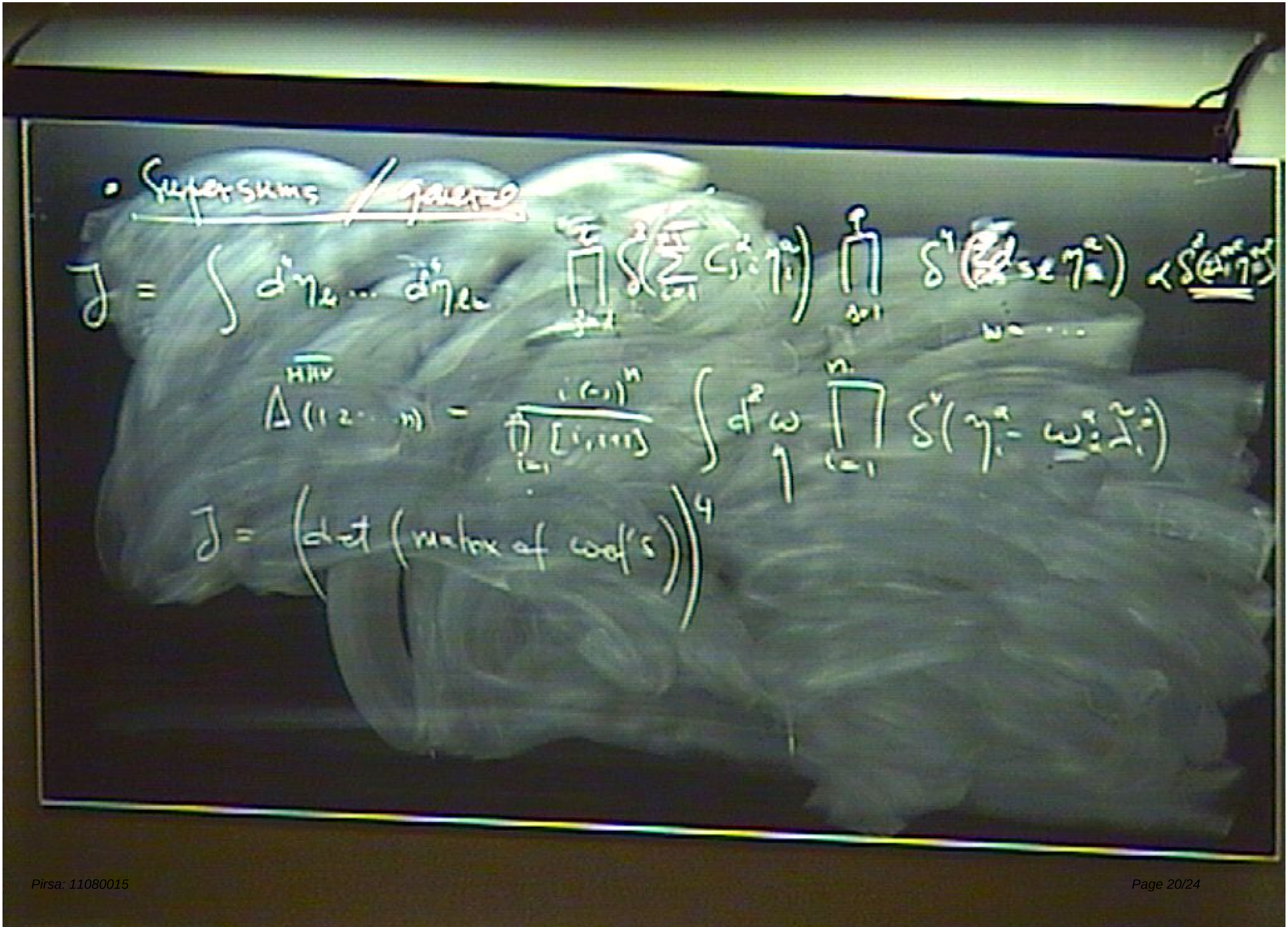
$$\begin{cases} \eta^1_{l_1}\lambda^1_{l_1} + \eta^1_{l_2}\lambda^1_{l_2} = -q^{11} \\ \eta^1_{l_1}\lambda^2_{l_1} + \eta^1_{l_2}\lambda^2_{l_2} = -q^{12} \end{cases}$$

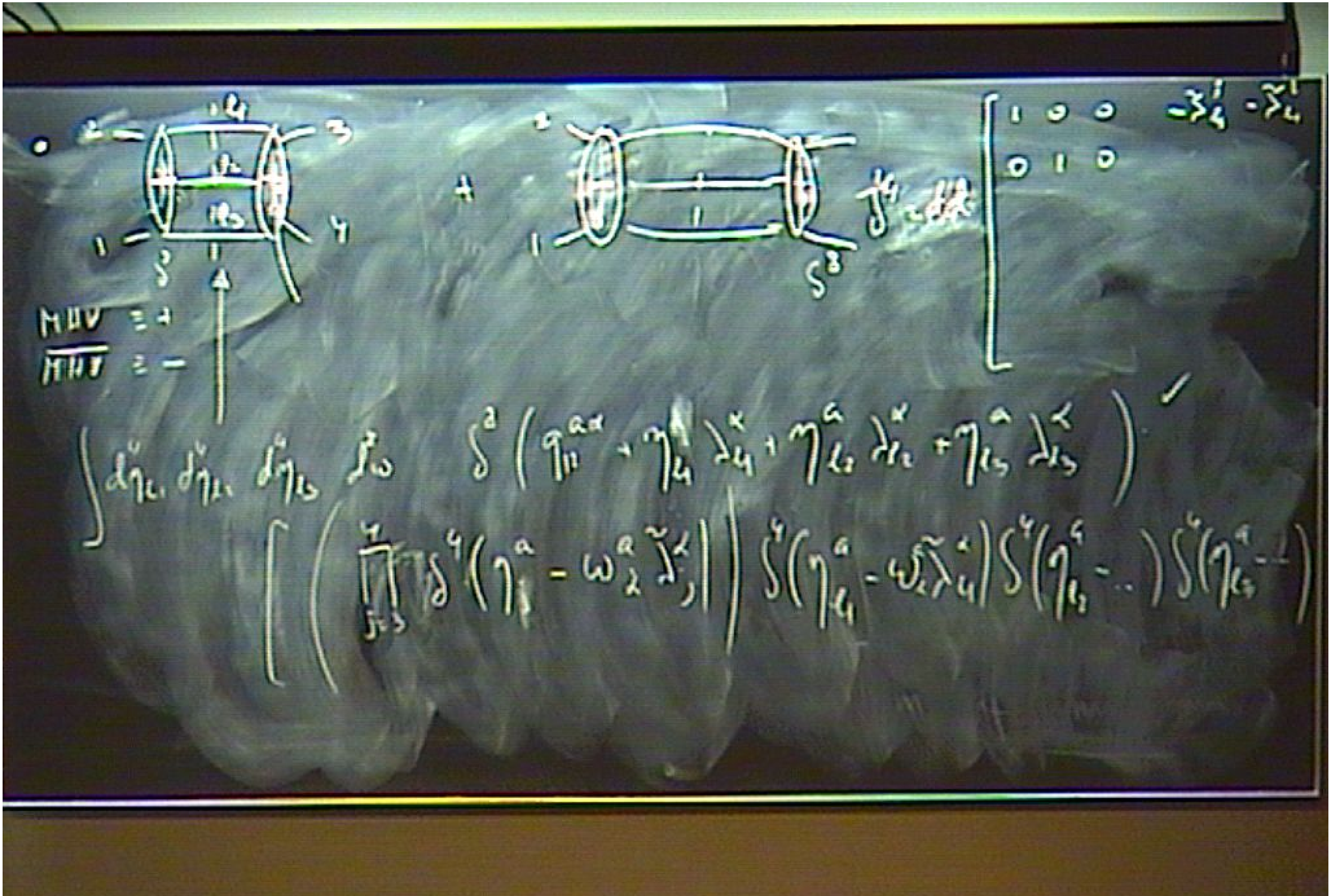
$$\left(\det\begin{pmatrix}\lambda^1_{l_1} & \lambda^1_{l_2}\\ \lambda^2_{l_1} & \lambda^2_{l_2}\end{pmatrix}\right)^4 \qquad \begin{pmatrix}\delta(\eta^1 - \text{soln})\\ \delta(\eta^2 - \text{soln})\end{pmatrix} \times J = \delta^2(M\cdot\eta + X)$$

$$(\lambda^1_{l_1}\lambda^2_{l_2} - \lambda^1_{l_2}\lambda^2_{l_1})^4 \sim \langle l_1 l_2\rangle^4 \qquad = \delta(\eta + M^{-1}X)\det M$$


16 24 - 8 = 16


16
24 - 8 = 16
det


• Supersums / generic
J =
J = (det (matrix of ω's))^4



• Supersums / general

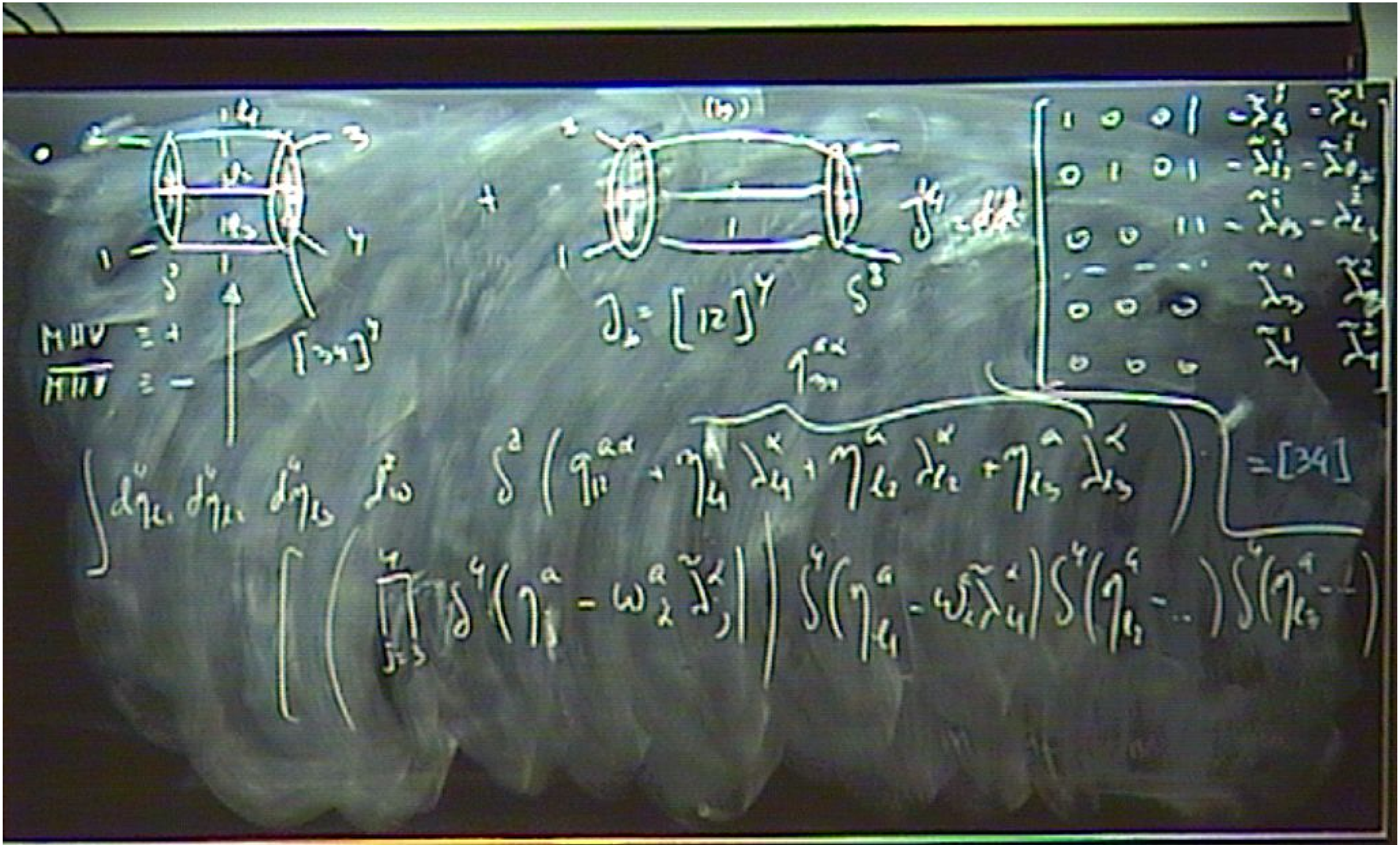
$$J = \int d^4\eta_{k_1} \cdots d^4\eta_{k_n} \prod_{j=1} \delta\left(\sum_{i=1}^{n} c_{ji}\,\eta_i\right) \prod_{j=1} \delta^4\left(\sum \eta_i^a\right) \propto \delta^8\left(\sum \eta_i\right)$$

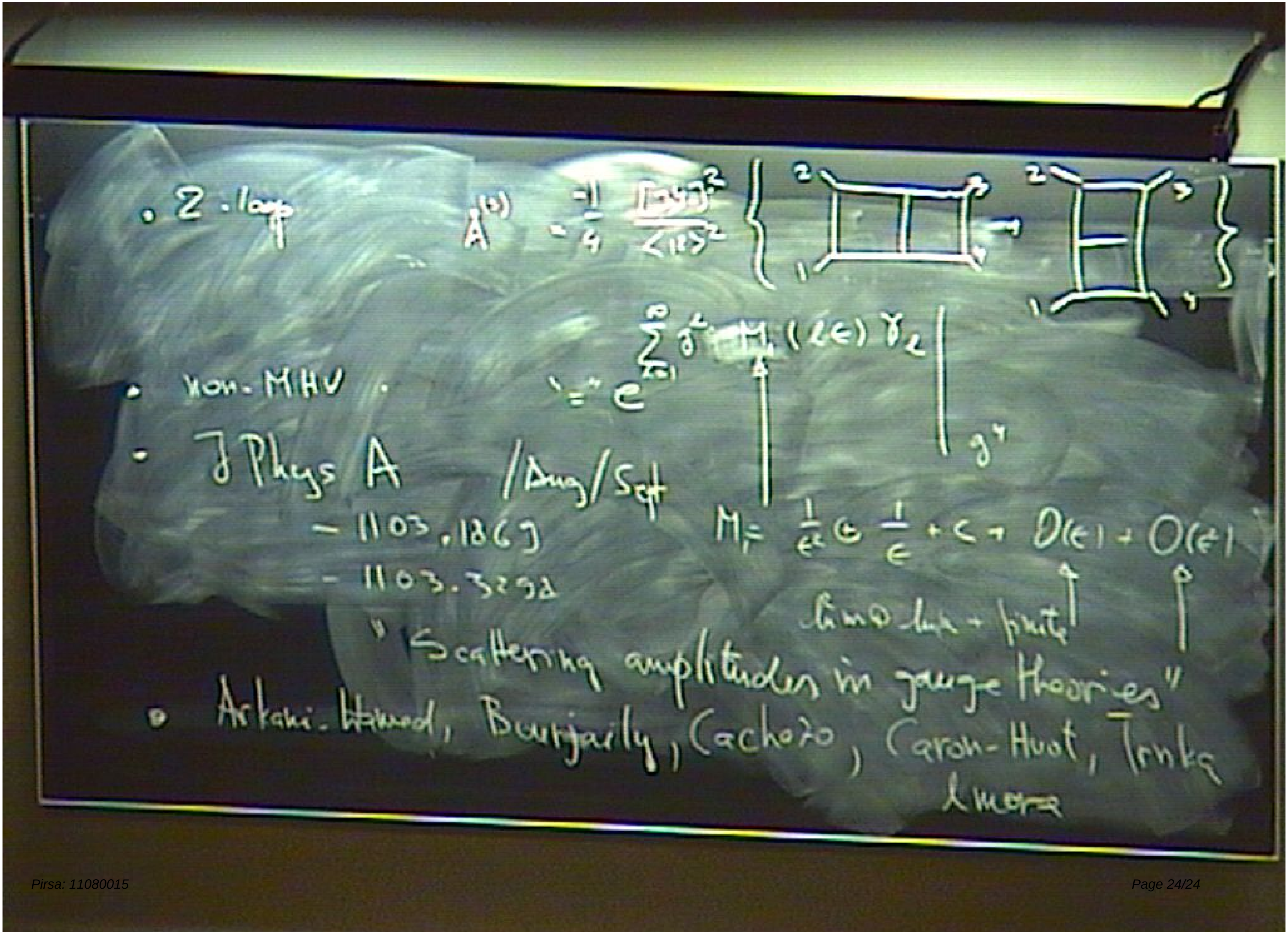
$$\Delta^{\overline{MHV}}(1\,2 \cdots n) = \frac{i(-1)^n}{\prod_{i=1}^{n} [i, i+1]} \int d^2\omega \prod_{i=1}^{n} \delta^4\left(\eta_i^a - \omega^a_{\dot a}\tilde\lambda_i^{\dot a}\right)$$

$$J = \left(\det\left(\text{matrix of coef's}\right)\right)^4$$

$$Q = \sum_{i=1}^{n} \lambda_i \left(\eta_i^a - \omega^a_{\dot a}\tilde\lambda_i^{\dot a}\right)$$

$$= \sum \lambda \eta_i^a + \omega \underbrace{\sum \lambda \tilde\lambda}_{0}$$


MHV = +
MHV = −
[34]^4
[12]^4
= [34]


• 2-loop
non-MHV
• J Phys A /Aug/Sept
– 1103.1869
– 1103.3298
"Scattering amplitudes in gauge theories"
• Arkani-Hamed, Bourjaily, Cachazo, Caron-Huot, Trnka
& more