Title: Recursion Relations

Date: Aug 04, 2011 11:15 AM

URL: http://pirsa.org/11080011

Abstract:

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Tree–Level BCFW Recursion in N=4 *Mathematica Summer School 2011*

Verifying the Recursion Relations

Outline for Today:

- **Organizing strategy for supersymmetric functions in *Mathematica***
  - ◊ (possibly also: "opt-in" human-friendly formatting for complex formulae.)
- **Evaluating tree amplitudes for particular η-components**
  - ◊ Expanding all 'hatted' momentum twistors in terms of external twistors
  - ◊ Picking out particular η-components
- ★ **Checking internal consistency of the recursed amplitudes.**
  - ◊ Generalizing our recursion algorithm for randomized `schemes.'

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e.g., for 8-point N^2MHV, we have

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Supersymmetry in Mathematica

Recall the standard "R-invariant"

$$R[a, b, c, d, e] = \frac{\delta^{0|4}(\eta_a\langle bcde\rangle+\eta_b\langle cdea\rangle+\eta_c\langle deab\rangle+\eta_d\langle eabc\rangle+\eta_e\langle abcd\rangle)}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

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We will describe all supersymmetric functions as a pair of objects:

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 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]],
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```

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Last time, we finished by defining the function **treeAmp[n_,k_]**

e.g., for 8-point N^2MHV, we have

```
In[6]:= treeAmp[8, 2]

Out[6]= {R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6], R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7],
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e.g., for 8-point N^2MHV, we have

In[6]:=

```
treeAmp[8, 2]
```

Out[6]=

```
{R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]],
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```
treeAmp[8, 2]
```

Out[6]=

```
{R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
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 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]],
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```
In[6]:= treeAmp[8, 2]

Out[6]= {R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
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 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]],
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Supersymmetry in Mathematica

Recall the standard "R-invariant"

```
{R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
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```
R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]],
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```
R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7],
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- "***dMatrix***": the matrix of coefficients for Fermionic variables appearing in the Fermionic δ-functions.

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```
R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7],
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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

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As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

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- "*residue*": a bosonic function of external data
- "*dMatrix*": the matrix of coefficients for Fermionic variables appearing in the Fermionic δ-functions.

We'll put these together as a list of the form {**residue,dMatrix**}. e.g.,

$$R[a,b,c,d,e]=\frac{\delta^{0|4}(\eta_a\langle bcde\rangle+\eta_b\langle cdea\rangle+\eta_c\langle deab\rangle+\eta_d\langle eabc\rangle+\eta_e\langle abcd\rangle)}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}\;,\;\{\langle bcde\rangle,\langle cdea\rangle,\langle deab\rangle,\langle eabc\rangle,\langle abcd\rangle\}\right\}$$

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Extracting the "residue" and "dMatrix" for each R-invariant

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$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

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$$R[a,b,c,d,e] = \frac{\delta^{0|4}(\eta_a\langle bcde\rangle+\eta_b\langle cdea\rangle+\eta_c\langle deab\rangle+\eta_d\langle eabc\rangle+\eta_e\langle abcd\rangle)}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle,\langle cdea\rangle,\langle deab\rangle,\langle eabc\rangle,\langle abcd\rangle\}\right\}$$

$$\{\text{residue},\text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

- **Extracting the "residue" and "dMatrix" for each R-invariant**

- "*residue*": a bosonic function of external data
- "*dMatrix*": the matrix of coefficients for Fermionic variables appearing in the Fermionic δ-functions.

We'll put these together as a list of the form {**residue,dMatrix**}. e.g.,

$$R[a, b, c, d, e] = \frac{\delta^{0|4}(\eta_a\langle bcde\rangle+\eta_b\langle cdea\rangle+\eta_c\langle deab\rangle+\eta_d\langle eabc\rangle+\eta_e\langle abcd\rangle)}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix"
(which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

R[a, b, c, d, e] =

$$\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle$$

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix"
(which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[7]:=
```
treeAmp[6, 1]
```
Out[7]=
```
{R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}
```

$$R[a, b, c, d, e] = \quad \langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle$$

$$\Leftrightarrow \left\{ \frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\} \right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
treeAmp[6, 1] /. R[x__] :> (Partition[{x}])
```

Out[8]=

```
{{1, 2, 3, 4, 5}, {1, 2, 3, 5, 6}, {1, 3, 4, 5, 6}}
```

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$$R[a,b,c,d,e] = \frac{\ldots}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

$$\Leftrightarrow \left\{ \frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\} \right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[9]:=

```
treeAmp[6, 1] /. R[x__] :> (Partition[{x}, 4, 1, 1])
```

Out[9]=

```
{{{1, 2, 3, 4}, {2, 3, 4, 5}, {3, 4, 5, 1}, {4, 5, 1, 2}, {5, 1, 2, 3}},
 {{1, 2, 3, 5}, {2, 3, 5, 6}, {3, 5, 6, 1}, {5, 6, 1, 2}, {6, 1, 2, 3}},
 {{1, 3, 4, 5}, {3, 4, 5, 6}, {4, 5, 6, 1}, {5, 6, 1, 3}, {6, 1, 3, 4}}}
```

$$R[a,b,c,d,e] = \frac{\ldots}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

$$\Leftrightarrow \left\{ \frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\} \right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
treeAmp[6, 1];
%[[1]] /. R[x__] :> (Partition[{x}, 4, 1, 1])
```

```
{{{1, 2, 3, 4}, {2, 3, 4, 5}, {3, 4, 5, 1}, {4, 5, 1, 2}, {5, 1, 2, 3}},
 {{1, 2, 3, 5}, {2, 3, 5, 6}, {3, 5, 6, 1}, {5, 6, 1, 2}, {6, 1, 2, 3}},
 {{1, 3, 4, 5}, {3, 4, 5, 6}, {4, 5, 6, 1}, {5, 6, 1, 3}, {6, 1, 3, 4}}}
```

R[a, b, c, d, e] = ... ⟨abcd⟩⟨bcde⟩⟨cdea⟩⟨deab⟩⟨eabc⟩

$$\Leftrightarrow \left\{ \frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle} , \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\} \right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
In[10]:= treeAmp[6, 1];
%[[1]] /. R[x__] :> (Partition[{x}, 4, 1, 1])

Out[11]= {{1, 2, 3, 4}, {2, 3, 4, 5}, {3, 4, 5, 1}, {4, 5, 1, 2}, {5, 1, 2, 3}}
```

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R[a, b, c, d, e] =

$$\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle$$

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[12]:=

```
treeAmp[6, 1]
%[[1]] /. R[x__] :> (Partition[{x}, 4, 1, 1])
```

Out[12]=

```
{R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}
```

Out[13]=

```
{{1, 2, 3, 4}, {2, 3, 4, 5}, {3, 4, 5, 1}, {4, 5, 1, 2}, {5, 1, 2, 3}}
```

R[a, b, c, d, e] = ⟨abcd⟩⟨bcde⟩⟨cdea⟩⟨deab⟩⟨eabc⟩

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

- **Extracting the "residue" and "dMatrix" for each R-invariant**

In[16]:=
```
treeAmp[6, 1][[1]]
% /. R[x__] :> (ab @@ Partition[{x}, 4, 1, 1])
```

Out[16]=
```
R[1, 2, 3, 4, 5]
```

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$$R[a, b, c, d, e] = \frac{\ldots}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

$$\Leftrightarrow \left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}, \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[19]:=
```
treeAmp[6, 1][[1]]
% /. {R[x__] :> (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

Out[19]=
```
R[1, 2, 3, 4, 5]
```

$$\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle$$

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle,\langle cdea\rangle,\langle deab\rangle,\langle eabc\rangle,\langle abcd\rangle\}\right\}$$

$$\{\text{residue},\ \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix"
(which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
{R[x__] → ((Partition[{x}, 4, 1, 1]))}
```

```
treeAmp[6, 1][[1]]
% /.
```

```
R[1, 2, 3, 4, 5]
```

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$$R[a, b, c, d, e] = \ldots \langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle$$

$$\Leftrightarrow \left\{ \frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle} , \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\} \right\}$$

$$\{\text{residue} , \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
treeAmp[6, 1][[1]]
% /.
```

```
R[1, 2, 3, 4, 5]
```

R[a, b, c, d, e] = $\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle$

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

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Extracting the "residue" and "dMatrix" for each R-invariant

```
treeAmp[6, 1][[1]]
% /. {R[x__] :>|
```

Out[21]=
```
R[1, 2, 3, 4, 5]
```

Out[22]=
```
R[1, 2, 3, 4, 5]
```

$$R[a,b,c,d,e] = \frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

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As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[25]:=
```
treeAmp[6, 1][[1]]
% /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

Out[25]=
```
R[1, 2, 3, 4, 5]
```

Out[26]=
```
                                  1
ab[1, 2, 3, 4] ab[2, 3, 4, 5] ab[3, 4, 5, 1] ab[4, 5, 1, 2] ab[5, 1, 2, 3]
```

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$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

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Extracting the "residue" and "dMatrix" for each R-invariant

In[27]:=
```
treeAmp[12, 4][[1]]
% /. {R[x__] :> 1/(Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

Out[27]=
```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

Out[28]=
```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
```

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obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[27]:=

```
treeAmp[12, 4][[1]]
% /. {R[x__] :> 1/(Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

Out[27]=

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

Out[28]=

```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
In[27]:= treeAmp[12, 4][[1]]
% /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}

Out[27]= R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]

Out[28]= 1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[27]:=
```
treeAmp[12, 4][[1]]
% /. {R[x__] :> 1/(Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

Out[27]=
```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

Out[28]=
```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

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obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[27]:=
```
treeAmp[12, 4][[1]]
% /. {R[x__] → 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

Out[27]=
```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

Out[28]=
```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

obtained by multiplying each "residue" by 4 particular entries of its "dMatrix"
(which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
treeAmp[12, 4][[1]]
% /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

obtained by multiplying each "residue" by 4 particular entries of its "dMatrix"
(which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
In[27]:=
treeAmp[12, 4][[1]]
% /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

```
Out[27]=
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
Out[28]=
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[27]= R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]

Out[28]= 1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
    ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
    ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
    ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
    ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
    ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
    ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
    ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
    ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])

In[30]:= Out[28] // nice

Out[30]= 1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]⟩
    ⟨34cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]1⟩
    ⟨456cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]⟩
    ⟨4cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]12⟩
    ⟨56cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]1⟩ ⟨678cap[{8, 9}, {11, 10, 1}]⟩
    ⟨6cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]14⟩ ⟨78cap[{8, 9}, {11, 10, 1}]1⟩
    ⟨891011⟩ ⟨8cap[{8, 9}, {11, 10, 1}]16⟩ ⟨910111⟩ ⟨101118⟩ ⟨11189⟩
    ⟨cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]123⟩
    ⟨cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]145⟩ ⟨cap[{8, 9}, {11, 10, 1}]167⟩)

nice[exprn_] := (exprn /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]⟩
   ⟨34cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]1⟩
   ⟨456cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]⟩
   ⟨4cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]12⟩
   ⟨56cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]1⟩ ⟨678cap[{8, 9}, {11, 10, 1}]⟩
   ⟨6cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]14⟩ ⟨78cap[{8, 9}, {11, 10, 1}]1⟩
   ⟨891011⟩ ⟨8cap[{8, 9}, {11, 10, 1}]16⟩ ⟨910111⟩ ⟨101118⟩ ⟨11189⟩
   ⟨cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]123⟩
   ⟨cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]145⟩ ⟨cap[{8, 9}, {11, 10, 1}]167⟩)
```

```
nice[exprn_] := (exprn /. {ab[x__] ↦ Row[{"⟨", x, "⟩"}]})
```

Out[27]=

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

Out[28]=

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[30]:=

```
Out[28] // nice
```

Out[30]=

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]⟩
   ⟨34cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]1⟩
   ⟨456cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]⟩
   ⟨4cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]12⟩
   ⟨56cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]1⟩ ⟨678cap[{8, 9}, {11, 10, 1}]⟩
   ⟨6cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]14⟩ ⟨78cap[{8, 9}, {11, 10, 1}]1⟩
   ⟨891011⟩ ⟨8cap[{8, 9}, {11, 10, 1}]16⟩ ⟨910111⟩ ⟨101118⟩ ⟨11189⟩
   ⟨cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]123⟩
   ⟨cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]145⟩ ⟨cap[{8, 9}, {11, 10, 1}]167⟩)
```

```
nice[exprn_] := (exprn /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /. {cap[x_, y_] :> Row[List["(", x, ")"]]})
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1 / (〈1234〉 〈1456〉 〈1678〉 〈18910〉 〈234cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]〉
   〈34cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]1〉
   〈456cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]〉
   〈4cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]12〉
   〈56cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]1〉 〈678cap[{8, 9}, {11, 10, 1}]〉
   〈6cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]14〉 〈78cap[{8, 9}, {11, 10, 1}]1〉
   〈891011〉 〈8cap[{8, 9}, {11, 10, 1}]16〉 〈910111〉 〈101118〉 〈11189〉
   〈cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]123〉
   〈cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]145〉 〈cap[{8, 9}, {11, 10, 1}]167〉)
```

```
nice[exprn_] := (exprn /. {ab[x__] :> Row[{"〈", x, "〉"}]})
```

Mathematica

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```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]⟩
   ⟨34cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]1⟩
   ⟨456cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]⟩
   ⟨4cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]12⟩
   ⟨56cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]1⟩ ⟨678cap[{8, 9}, {11, 10, 1}]⟩
   ⟨6cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]14⟩ ⟨78cap[{8, 9}, {11, 10, 1}]1⟩
   ⟨891011⟩ ⟨8cap[{8, 9}, {11, 10, 1}]16⟩ ⟨910111⟩ ⟨101118⟩ ⟨11189⟩
   ⟨cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]123⟩
   ⟨cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]145⟩ ⟨cap[{8, 9}, {11, 10, 1}]167⟩)
```

```
nice[exprn_] := (exprn /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

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Out[27]=

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

Out[28]=

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[32]:=

```
Out[28] // nice
```

Out[32]=

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(,{4, 5})⋂({cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1})⟩
   ⟨34(,{4, 5})⋂({cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1})1⟩
   ⟨456(,{6, 7})⋂({cap[{8, 9}, {11, 10, 1}], 8, 1})⟩
   ⟨4(,{4, 5})⋂({cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1})12⟩
   ⟨56(,{6, 7})⋂({cap[{8, 9}, {11, 10, 1}], 8, 1})1⟩ ⟨678(,{8, 9})⋂({11, 10, 1})⟩
   ⟨6(,{6, 7})⋂({cap[{8, 9}, {11, 10, 1}], 8, 1})14⟩ ⟨78(,{8, 9})⋂({11, 10, 1})1⟩
   ⟨891011⟩ ⟨8(,{8, 9})⋂({11, 10, 1})16⟩ ⟨910111⟩ ⟨101118⟩ ⟨11189⟩
   ⟨(,{4, 5})⋂({cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1})123⟩
   ⟨(,{6, 7})⋂({cap[{8, 9}, {11, 10, 1}], 8, 1})145⟩ ⟨(,{8, 9})⋂({11, 10, 1})167⟩)
```

```
nice[exprn_] := (exprn /. {cap[x_, y_] :> Row[List["(,", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

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```
R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
In[32]:= Out[28] // nice
```

```
Out[32]= 1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(,{4, 5})⋂({cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1})⟩
   ⟨34(,{4, 5})⋂({cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1})1⟩
   ⟨456(,{6, 7})⋂({cap[{8, 9}, {11, 10, 1}], 8, 1})⟩
   ⟨4(,{4, 5})⋂({cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1})12⟩
   ⟨56(,{6, 7})⋂({cap[{8, 9}, {11, 10, 1}], 8, 1})1⟩ ⟨678(,{8, 9})⋂({11, 10, 1})⟩
   ⟨6(,{6, 7})⋂({cap[{8, 9}, {11, 10, 1}], 8, 1})14⟩ ⟨78(,{8, 9})⋂({11, 10, 1})1⟩
   ⟨891011⟩ ⟨8(,{8, 9})⋂({11, 10, 1})16⟩ ⟨910111⟩ ⟨101118⟩ ⟨11189⟩
   ⟨(,{4, 5})⋂({cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1})123⟩
   ⟨(,{6, 7})⋂({cap[{8, 9}, {11, 10, 1}], 8, 1})145⟩ ⟨(,{8, 9})⋂({11, 10, 1})167⟩)
```

```
nice[exprn_] :=
  (exprn /. {cap[x_, y_] :> Row[Flatten@List["(,", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

```
R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(,45)⋂(cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]61)⟩
   ⟨34(,45)⋂(cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]61)1⟩ ⟨456(,67)⋂(cap[{8, 9}, {11, 10, 1}]81)⟩
   ⟨4(,45)⋂(cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]61)12⟩ ⟨56(,67)⋂(cap[{8, 9}, {11, 10, 1}]81)1⟩
   ⟨678(,89)⋂(11101)⟩ ⟨6(,67)⋂(cap[{8, 9}, {11, 10, 1}]81)14⟩ ⟨78(,89)⋂(11101)1⟩ ⟨891011⟩
   ⟨8(,89)⋂(11101)16⟩ ⟨910111⟩ ⟨101118⟩ ⟨11189⟩ ⟨(,45)⋂(cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]61)123⟩
   ⟨(,67)⋂(cap[{8, 9}, {11, 10, 1}]81)145⟩ ⟨(,89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn /. {cap[x_, y_] → Row[Flatten@List["(,", x, ")⋂(", y, ")"]]} /. {ab[x__] → Row[{"⟨", x, "⟩"}]})
```

```
R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂(cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]61)⟩
   ⟨34(45)⋂(cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]61)1⟩ ⟨456(67)⋂(cap[{8, 9}, {11, 10, 1}]81)⟩
   ⟨4(45)⋂(cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]61)12⟩ ⟨56(67)⋂(cap[{8, 9}, {11, 10, 1}]81)1⟩
   ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂(cap[{8, 9}, {11, 10, 1}]81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩
   ⟨8(89)⋂(11101)16⟩ ⟨910111⟩ ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂(cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]61)123⟩
   ⟨(67)⋂(cap[{8, 9}, {11, 10, 1}]81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn /. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

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```
R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
   ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
   ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
   ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] → Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] → Row[{"⟨", x, "⟩"}]})
```

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(which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
treeAmp[12, 4][[1]]
% /. {R[x__] → 1/ (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
   ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
   ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
   ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
```

(which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
treeAmp[12, 4][[1]]
% /. {R[x__] :> 1/ (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
   ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
   ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
   ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
```

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bcfw_part_deux.nb

obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

```
In[27]:= treeAmp[12, 4][[1]]
         % /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}

Out[27]= R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
          R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
          R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]

Out[28]= 1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
            ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
            ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
            ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
            ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
            ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
            ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
            ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
            ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
            ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])

In[38]:= Out[28] // nice

Out[38]= 1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
            ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
            ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
            ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(1110
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
   ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
   ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
   ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

```
R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
   ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
   ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
   ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
   ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[36]:=

```
Out[28] // nice
```

Out[36]=

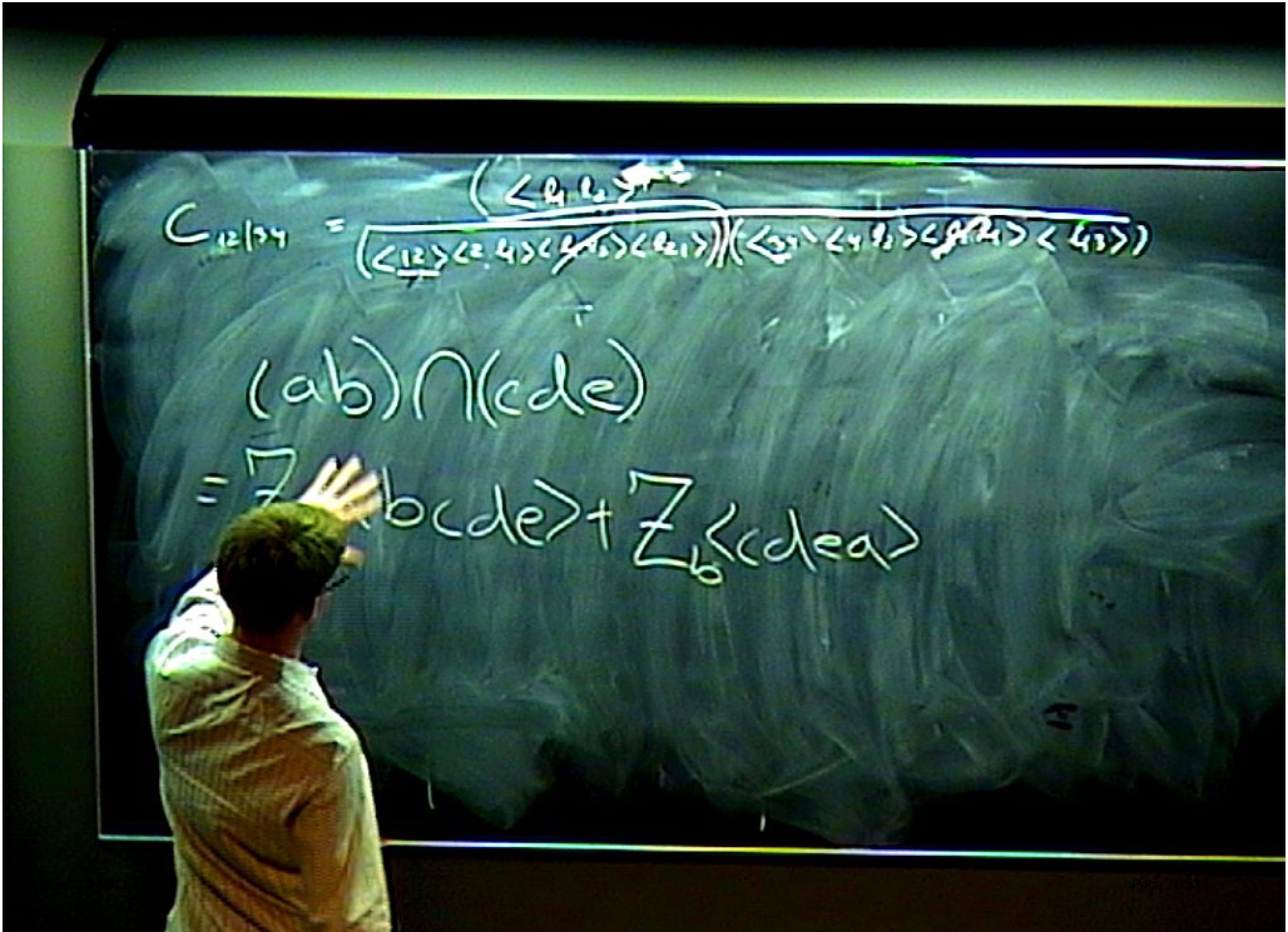
```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

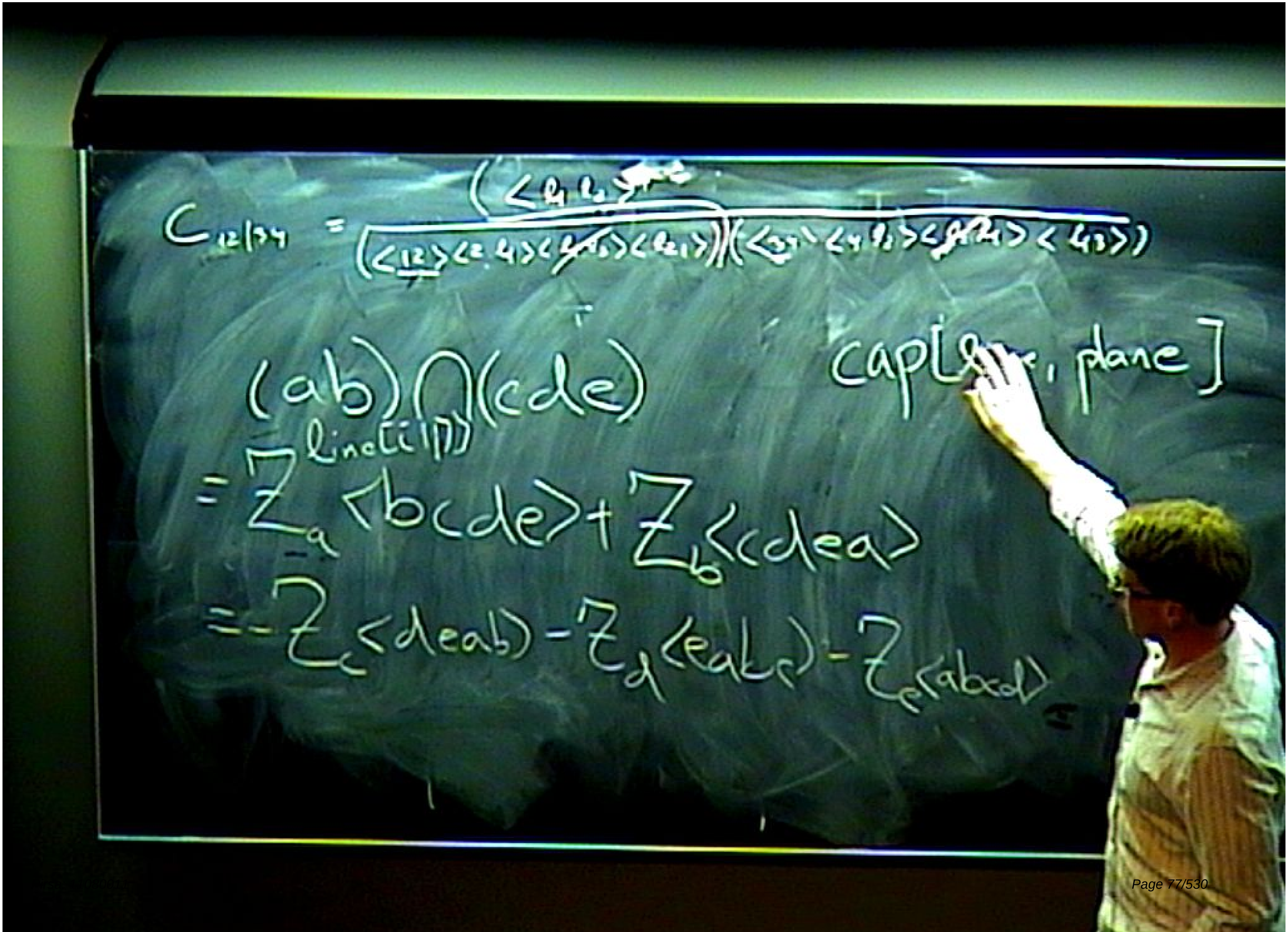
In[37]:=

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

In[39]:=

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```


(ab)∩(cde)


(ab) ∩ (cde)
line[i|j]
= Z_a <bcde> + Z_b <cdea>
= - Z_c <deab> - Z_d <eabc> - Z_e <abcd>
, plane]

```
ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[36]:=

```
Out[28] // nice
```

Out[36]=

```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

In[37]:=

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

In[39]:=

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

```
cap[x_, y
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[38]:=

```
Out[28] // nice
```

Out[38]=

```
1/(⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

In[37]:=

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

In[39]:=

```
rToResidue = {R[x__] :> 1/(Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

```
ab[x___, cap[y_, z_], w___] :>
 (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]])
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[36]:=

```
Out[28] // nice
```

Out[36]=

```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

In[37]:=

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

In[39]:=

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

```
ab[x___, cap[y_, z_], w___] :>
 (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[2]]])
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[36]:=

```
Out[28] // nice
```

Out[36]=

```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

In[37]:=

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

In[39]:=

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

```
capRue
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[36]:=

```
Out[28] // nice
```

Out[36]=

```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

In[37]:=

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

In[39]:=

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

In[40]:=

```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[1], w] ab @@ Prepend[z, y[2]] + ab[x, y[2], w] ab @@ Append[z, y[1]]}
```

```
ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[38]:=

```
Out[28] // nice
```

Out[38]=

```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

In[37]:=

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

In[39]:=

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

In[40]:=

```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[37]:= nice[exprn_] :=
  (exprn //. {cap[x_, y_] ⧴ Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] ⧴ Row[{"⟨", x, "⟩"}]})

In[39]:= rToResidue = {R[x__] ⧴ 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};

In[40]:= capRules =
  {ab[x___, cap[y_, z_], w___] ⧴
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

Out[40]= {ab[x___, cap[y_, z_], w___] ⧴ ab[x, y⟦1⟧, w] ab @@ Prepend[z, y⟦2⟧] + ab[x, y⟦2⟧, w] ab @@ Append[z, y⟦1⟧]}

ab[]
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[37]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] ⧴ Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] ⧴ Row[{"⟨", x, "⟩"}]})
```

In[39]:=
```
rToResidue = {R[x__] ⧴ 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

In[40]:=
```
capRules =
 {ab[x___, cap[y_, z_], w___] ⧴
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=
```
{ab[x___, cap[y_, z_], w___] ⧴ ab[x, y⟦1⟧, w] ab @@ Prepend[z, y⟦2⟧] + ab[x, y⟦2⟧, w] ab @@ Append[z, y⟦1⟧]}
```

In[41]:=
```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
In[38]:= Out[28] // nice

Out[38]= 1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
In[37]:= nice[exprn_] :=
  (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

```
In[39]:= rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

```
In[40]:= capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

Out[40]= {ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

```
In[41]:= ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

Out[36]=
```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

In[37]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

In[39]:=
```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

In[40]:=
```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=
```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

In[41]:=
```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28]
```

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules

1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[8, 9, 10, 11] ab[9, 10, 11, 1]
   ab[10, 11, 1, 8] ab[11, 1, 8, 9] (ab[6, 7, 8, 8] ab[9, 11, 10, 1] + ab[6, 7, 8, 9] ab[11, 10, 1, 8])
   (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 8, 9, 1] ab[11, 10, 1, 8])
   (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[8, 9, 1, 6] ab[11, 10, 1, 8])
   (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
        ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
     ab[2, 3, 4, 5] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
        ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
   (ab[3, 4, 4, 1] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
        ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
     ab[3, 4, 5, 1] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
        ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
   (ab[4, 4, 1, 2] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
        ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
     ab[4, 5, 1, 2] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[2, 3, 4, 5] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
(ab[3, 4, 4, 1] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[3, 4, 5, 1] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
(ab[4, 4, 1, 2] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[4, 5, 1, 2] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
(ab[4, 1, 2, 3] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[5, 1, 2, 3] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))))
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
  ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[2, 3, 4, 5] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
(ab[3, 4, 4, 1] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[3, 4, 5, 1] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
(ab[4, 4, 1, 2] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[4, 5, 1, 2] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
(ab[4, 1, 2, 3] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[5, 1, 2, 3] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))))
```

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```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[8, 9, 10, 11] ab[9, 10, 11, 1]
   ab[10, 11, 1, 8] ab[11, 1, 8, 9] (ab[6, 7, 8, 8] ab[9, 11, 10, 1] + ab[6, 7, 8, 9] ab[11, 10, 1, 8])
   (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 8, 9, 1] ab[11, 10, 1, 8])
   (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[8, 9, 1, 6] ab[11, 10, 1, 8])
   (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
         ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
     ab[2, 3, 4, 5] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
         ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
   (ab[3, 4, 4, 1] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
         ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
     ab[3, 4, 5, 1] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
         ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
   (ab[4, 4, 1, 2] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
         ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
     ab[4, 5, 1, 2] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
         ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
   (ab[4, 1, 2, 3] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
         ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
     ab[5, 1, 2, 3] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[8, 9, 10, 11] ab[9, 10, 11, 1]
  ab[10, 11, 1, 8] ab[11, 1, 8, 9] (ab[6, 7, 8, 8] ab[9, 11, 10, 1] + ab[6, 7, 8, 9] ab[11, 10, 1, 8])
  (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 8, 9, 1] ab[11, 10, 1, 8])
  (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[8, 9, 1, 6] ab[11, 10, 1, 8])
  (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
  (ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
   ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
  (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
   ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
  (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
   ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
  (ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
   ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
  (ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
   ab[2, 3, 4, 5] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
  (ab[3, 4, 4, 1] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
   ab[3, 4, 5, 1] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
  (ab[4, 4, 1, 2] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
   ab[4, 5, 1, 2] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
  (ab[4, 1, 2, 3] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
   ab[5, 1, 2, 3] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules

1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[8, 9, 10, 11] ab[9, 10, 11, 1]
    ab[10, 11, 1, 8] ab[11, 1, 8, 9] (ab[6, 7, 8, 8] ab[9, 11, 10, 1] + ab[6, 7, 8, 9] ab[11, 10, 1, 8])
    (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 8, 9, 1] ab[11, 10, 1, 8])
    (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[8, 9, 1, 6] ab[11, 10, 1, 8])
    (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
    (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
    (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
    (ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
    (ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
          ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
      ab[2, 3, 4, 5] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
          ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
    (ab[3, 4, 4, 1] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
          ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
      ab[3, 4, 5, 1] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
          ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
    (ab[4, 4, 1, 2] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
          ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
```

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[37]:=
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})

In[39]:=
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};

In[40]:=
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

Out[40]=
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}

In[41]:=
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

Out[41]=
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

In[42]:=
Out[28] //. capRules

Out[42]=
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[8, 9, 10, 11] ab[9, 10, 11, 1]
   ab[10, 11, 1, 8] ab[11, 1, 8, 9] (ab[6, 7, 8, 8] ab[9, 11, 10, 1] + ab[6, 7, 8, 9] ab[11, 10, 1, 8])
   (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 8, 9, 1] ab[11, 10, 1, 8])
   (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[8, 9, 1, 6] ab[11, 10, 1, 8])
   (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

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Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
{ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
Out[40]= {ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

```
In[41]:= ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
In[42]:= Out[28] //. capRules

Out[42]= 1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[8, 9, 10, 11] ab[9, 10, 11, 1]
   ab[10, 11, 1, 8] ab[11, 1, 8, 9] (ab[6, 7, 8, 8] ab[9, 11, 10, 1] + ab[6, 7, 8, 9] ab[11, 10, 1, 8])
   (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 8, 9, 1] ab[11, 10, 1, 8])
   (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[8, 9, 1, 6] ab[11, 10, 1, 8])
   (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
        ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
     ab[2, 3, 4, 5] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
        ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
   (ab[3, 4, 4, 1] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
        ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
```

```
(ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
  ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
(ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[2, 3, 4, 5] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
(ab[3, 4, 4, 1] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[3, 4, 5, 1] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
(ab[4, 4, 1, 2] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[4, 5, 1, 2] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])))
(ab[4, 1, 2, 3] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
  ab[5, 1, 2, 3] (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
    ab[7, 6, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))))
```

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In[39]:=
```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

In[40]:=
```
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=
```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

In[41]:=
```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

In[42]:=
```
Out[28] //. capRules
```

Out[42]=
```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[8, 9, 10, 11] ab[9, 10, 11, 1]
   ab[10, 11, 1, 8] ab[11, 1, 8, 9] (ab[6, 7, 8, 8] ab[9, 11, 10, 1] + ab[6, 7, 8, 9] ab[11, 10, 1, 8])
   (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 8, 9, 1] ab[11, 10, 1, 8])
   (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[8, 9, 1, 6] ab[11, 10, 1, 8])
   (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
     ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
   (ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
       ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[39]:= rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};

In[40]:= capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

Out[40]= {ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}

In[41]:= ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

Out[28] //. capRules /. ab[x___]

Out[42]= 1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[8, 9, 10, 11] ab[9, 10, 11, 1]
    ab[10, 11, 1, 8] ab[11, 1, 8, 9] (ab[6, 7, 8, 8] ab[9, 11, 10, 1] + ab[6, 7, 8, 9] ab[11, 10, 1, 8])
    (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 8, 9, 1] ab[11, 10, 1, 8])
    (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[8, 9, 1, 6] ab[11, 10, 1, 8])
    (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[4, 5, 6, 6] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[4, 5, 6, 7] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
    (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[5, 6, 7, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
    (ab[6, 6, 1, 4] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[6, 7, 1, 4] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
    (ab[6, 1, 4, 5] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
      ab[7, 1, 4, 5] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
    (ab[2, 3, 4, 4] (ab[5, 6, 6, 1] (ab[7, 8, 8, 1] ab[9, 11, 10, 1] + ab[7, 9, 8, 1] ab[11, 10, 1, 8]) +
        ab[5, 7, 6, 1] (ab[8, 8, 1, 6] ab[9, 11, 10, 1] + ab[9, 8, 1, 6] ab[11, 10, 1, 8])) +
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[39]:=
```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

In[40]:=
```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=
```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

In[41]:=
```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

In[43]:=
```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

Out[43]=
```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

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```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};
```

```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

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Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
In[37]:= nice[exprn_] :=
          (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})

In[39]:= rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))};

In[40]:= capRules =
          {ab[x___, cap[y_, z_], w___] :>
            (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

Out[40]= {ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}

In[41]:= ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0

Out[43]= 1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
          ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
          ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
          ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
          (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
          (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
            ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[37]:= nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
```

```
In[40]:= capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

Out[40]= {ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

```
In[41]:= ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
In[42]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0

Out[42]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0;
```

In[40]:=

```
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

In[41]:=

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

In[43]:=

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

Out[43]=

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

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```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
```

In[40]:=
```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=
```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

In[41]:=
```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

In[42]:=
```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

Out[42]=
$$1/\big(ab[1,2,3,4]\, ab[1,4,5,6]\, ab[1,6,7,8]\, ab[1,8,9,10]\, ab[2,3,4,5]\, ab[3,4,5,1]\, ab[4,5,1,2]\, ab[4,5,6,7]\, ab[5,6,7,1]\, ab[6,7,1,4]\, ab[6,7,8,9]\, ab[7,6,1,4]^3\, ab[7,8,9,1]\, ab[8,9,1,6]\, ab[8,9,10,11]\, ab[9,8,1,6]^6\, ab[9,10,11,1]\, ab[10,11,1,8]\, ab[11,1,8,9]\, ab[11,10,1,8]^3\, (ab[8,1,6,7]\, ab[9,11,10,1] + ab[9,1,6,7]\, ab[11,10,1,8])\, (ab[6,1,4,5]\, ab[7,9,8,1]\, ab[11,10,1,8] + ab[7,1,4,5]\, ab[9,8,1,6]\, ab[11,10,1,8])\, (ab[4,1,2,3]\, ab[5,7,6,1]\, ab[9,8,1,6]\, ab[11,10,1,8] + ab[5,1,2,3]\, ab[7,6,1,4]\, ab[9,8,1,6]\, ab[11,10,1,8])\big)$$

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] → 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

In[44]:= treeAmp[6, 1]

Out[44]= {R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}
```

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```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] → 0
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%
```

```
{R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}
```

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```
Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

treeAmp[6, 1];
%[[1]]

Out[46]= R[1, 2, 3, 4, 5]
```

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] → 0
```

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /.
```

```
R[1, 2, 3, 5, 6]
```

Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

In[43]:=
```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] → 0
```

Out[43]=
```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

In[53]:=
```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
```

Out[54]=
```
R[1, 2, 3, 5, 6]
```

Out[55]=
$$\frac{1}{ab[1,2,3,5]\, ab[2,3,5,6]\, ab[3,5,6,1]\, ab[5,6,1,2]\, ab[6,1,2,3]}$$

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```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x__, y_, w__, y_, z__] → 0
```

```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
    ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :>
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

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```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> ab@@@Partition[{x}, 4, 1, 1]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1, 2, 3, 5]\, ab[2, 3, 5, 6]\, ab[3, 5, 6, 1]\, ab[5, 6, 1, 2]\, ab[6, 1, 2, 3]}$$

```
{ab[1, 2, 3, 5], ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], ab[6, 1, 2, 3]}
```

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] → 0
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → (ab @@@ RotateRight[Partition[{x}, 4, 1, 1]])
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

```
{ab[6, 1, 2, 3], ab[1, 2, 3, 5], ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2]}
```

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Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
In[76]:= treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]

Out[77]= R[1, 2, 3, 5, 6]
```

$$\text{Out[78]= } \frac{1}{ab[1, 2, 3, 5]\, ab[2, 3, 5, 6]\, ab[3, 5, 6, 1]\, ab[5, 6, 1, 2]\, ab[6, 1, 2, 3]}$$

```
Out[79]= {1, 2, 3, 5, 6} -> {ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x__, y_, w__, y_, z__] → 0
```

```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → TRule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1,2,3,5]\, ab[2,3,5,6]\, ab[3,5,6,1]\, ab[5,6,1,2]\, ab[6,1,2,3]}$$

```
{1, 2, 3, 5, 6} → {ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

```
Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] → 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

In[80]:= treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]

Out[81]= R[1, 2, 3, 5, 6]

Out[82]= 1 / (ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3])

Out[83]= {1 → ab[2, 3, 5, 6], 2 → ab[3, 5, 6, 1], 3 → ab[5, 6, 1, 2], 5 → ab[6, 1, 2, 3], 6 → ab[1, 2, 3, 5]}
```

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Tree−Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] → 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

In[80]:= treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]

Out[81]= R[1, 2, 3, 5, 6]

Out[82]=                              1
         ─────────────────────────────────────────────────────────────────────
         ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3]

Out[83]= {1 → ab[2, 3, 5, 6], 2 → ab[3, 5, 6, 1], 3 → ab[5, 6, 1, 2], 5 → ab[6, 1, 2, 3], 6 → ab[1, 2, 3, 5]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] → 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

In[84]:= treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]

Out[85]= R[1, 2, 3, 5, 6]

Out[86]=                               1
         ─────────────────────────────────────────────────────────────────────────
         ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3]

Out[87]= {ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

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Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

In[43]:=
```
Out[28] //. capRules /. ab[x__, y_, w__, y_, z__] → 0
```

Out[43]=
```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

In[84]:=
```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[85]=
```
R[1, 2, 3, 5, 6]
```

Out[86]=
$$\frac{1}{ab[1,2,3,5]\ ab[2,3,5,6]\ ab[3,5,6,1]\ ab[5,6,1,2]\ ab[6,1,2,3]}$$

Out[87]=
```
{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1,2,3,5]\, ab[2,3,5,6]\, ab[3,5,6,1]\, ab[5,6,1,2]\, ab[6,1,2,3]}$$

```
{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]

Out[85]= R[1, 2, 3, 5, 6]

Out[86]= 1/(ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3])

Out[87]= {ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1, 2, 3, 5]\ ab[2, 3, 5, 6]\ ab[3, 5, 6, 1]\ ab[5, 6, 1, 2]\ ab[6, 1, 2, 3]}$$

```
{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
 %[[2]]
 % /. rToResidue
 %% /]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1, 2, 3, 5]\ ab[2, 3, 5, 6]\ ab[3, 5, 6, 1]\ ab[5, 6, 1, 2]\ ab[6, 1, 2, 3]}$$

```
{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

```
In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]

Out[93]= R[1, 2, 3, 5, 6]
```

$$\text{Out[94]=}\ \frac{1}{\text{ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3]}}$$

```
Out[95]= {ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

```
In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
    ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
    ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
    ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
    (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
    (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
      ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]

Out[93]= R[1, 2, 3, 5, 6]

Out[94]=                                  1
         ---------------------------------------------------------------------------
         ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3]

{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5]}
```

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```
Out[28] //. capRules /. ab[x__, y_, w__, y_, z__] → 0

1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]

R[1, 2, 3, 5, 6]
```

$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

```
/. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

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```
In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] → 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]

Out[93]= R[1, 2, 3, 5, 6]

Out[94]= 1 / (ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3])

/. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
  ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
  ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
  ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
  (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
  (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
    ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1, 2, 3, 5]\ ab[2, 3, 5, 6]\ ab[3, 5, 6, 1]\ ab[5, 6, 1, 2]\ ab[6, 1, 2, 3]}$$

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=
```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

In[96]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[96]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

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```
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=

```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{ab[1,2,3,5]\, ab[2,3,5,6]\, ab[3,5,6,1]\, ab[5,6,1,2]\, ab[6,1,2,3]}$$

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[96]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

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```
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
 (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
 (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
   ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[5,6,1,2]\,ab[6,1,2,3]}$$

```
R[1, 3, 5, 7, 9] /.
 R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.{η /@ Range[9]}
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}}.
 {{η[1], η[2], η[3], η[4], η[5], η[6], η[7], η[8], η[9]}}
```

```
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=
```
R[1, 2, 3, 5, 6]
```

Out[94]=
$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

In[99]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
```

Out[99]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[100]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=

```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

```
R[1, 3, 5, 7, 9] /.
 R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
%%
```

Out[99]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[100]=

$$\{\text{ab}[3,5,7,9]\,\eta[1] + \text{ab}[5,7,9,1]\,\eta[3] + \text{ab}[7,9,1,3]\,\eta[5] + \text{ab}[9,1,3,5]\,\eta[7] + \text{ab}[1,3,5,7]\,\eta[9]\}$$

```
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=
```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{ab[1, 2, 3, 5]\ ab[2, 3, 5, 6]\ ab[3, 5, 6, 1]\ ab[5, 6, 1, 2]\ ab[6, 1, 2, 3]}$$

In[101]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
{%%} // nice
```

Out[101]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[102]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[103]=
```
{{⟨3579⟩, 0, ⟨5791⟩, 0, ⟨7913⟩, 0, ⟨9135⟩, 0, ⟨1357⟩}}
```

```
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1,2,3,5]\, ab[2,3,5,6]\, ab[3,5,6,1]\, ab[5,6,1,2]\, ab[6,1,2,3]}$$

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=

```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[5,6,1,2]\,ab[6,1,2,3]}$$

In[106]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle \;\; 0 \;\; \langle 5791\rangle \;\; 0 \;\; \langle 7913\rangle \;\; 0 \;\; \langle 9135\rangle \;\; 0 \;\; \langle 1357\rangle\right)$$

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```
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=
```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{ab[1, 2, 3, 5]\ ab[2, 3, 5, 6]\ ab[3, 5, 6, 1]\ ab[5, 6, 1, 2]\ ab[6, 1, 2, 3]}$$

In[105]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[105]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\begin{pmatrix} \langle 3579\rangle & 0 & \langle 5791\rangle & 0 & \langle 7913\rangle & 0 & \langle 9135\rangle & 0 & \langle 1357\rangle \end{pmatrix}$$

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Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1, 2, 3, 5]\ ab[2, 3, 5, 6]\ ab[3, 5, 6, 1]\ ab[5, 6, 1, 2]\ ab[6, 1, 2, 3]}$$

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```



```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=

```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{ab[1, 2, 3, 5]\ ab[2, 3, 5, 6]\ ab[3, 5, 6, 1]\ ab[5, 6, 1, 2]\ ab[6, 1, 2, 3]}$$

In[105]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)

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```
ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
(ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
(ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
  ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=

```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{ab[1,2,3,5]\, ab[2,3,5,6]\, ab[3,5,6,1]\, ab[5,6,1,2]\, ab[6,1,2,3]}$$

```
R[1, 3, 5, 7, 9]
```

Out[106]=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x__, y_, w__, y_, z__] → 0
```

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
  ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
  ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
  ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
  (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
  (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
    ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1,2,3,5]\, ab[2,3,5,6]\, ab[3,5,6,1]\, ab[5,6,1,2]\, ab[6,1,2,3]}$$

```
R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]]
```

```
R[1, 3, 5, 7, 9] /.
  R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
```

In[40]:=
```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=
```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

In[41]:=
```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

In[43]:=
```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

Out[43]=
```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

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```
%[[2]]
% /. rToResidue
%% /. R[x__] :→ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=

```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

```
R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]]
```

In[105]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :→ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle\ \ 0\ \ \langle 5791\rangle\ \ 0\ \ \langle 7913\rangle\ \ 0\ \ \langle 9135\rangle\ \ 0\ \ \langle 1357\rangle\right)$$

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```
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
Out[93]= R[1, 2, 3, 5, 6]
```

```
Out[94]=                                        1
         ─────────────────────────────────────────────────────────────────────────────
         ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3]
```

```
R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
 ab[x___, cap[y_, z_], w___] :>
  (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])
```

```
In[105]:= R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
Out[106]= {ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
Out[107]= {ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
Out[108]//MatrixForm=
( ⟨3579⟩  0  ⟨5791⟩  0  ⟨7913⟩  0  ⟨9135⟩  0  ⟨1357⟩ )
```

```
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=

```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[5,6,1,2]\,ab[6,1,2,3]}$$

In[109]:=

```
R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
 R[x___, cap[y_, z_], w___] :> (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])
```

Out[109]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[106]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\begin{pmatrix} \langle 3579\rangle & 0 & \langle 5791\rangle & 0 & \langle 7913\rangle & 0 & \langle 9135\rangle & 0 & \langle 1357\rangle \end{pmatrix}$$

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```
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=

```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[105]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[106]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\begin{pmatrix} \langle 3579\rangle & 0 & \langle 5791\rangle & 0 & \langle 7913\rangle & 0 & \langle 9135\rangle & 0 & \langle 1357\rangle \end{pmatrix}$$

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
  (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
    ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=
```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{ab[1, 2, 3, 5]\ ab[2, 3, 5, 6]\ ab[3, 5, 6, 1]\ ab[5, 6, 1, 2]\ ab[6, 1, 2, 3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[103]=
```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[105]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle\ \ 0\ \ \langle 5791\rangle\ \ 0\ \ \langle 7913\rangle\ \ 0\ \ \langle 9135\rangle\ \ 0\ \ \langle 1357\rangle\right)$$

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

Out[94]=

$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x__, cap[y_, z_], w__] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[103]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[105]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle\ \ 0\ \ \langle 5791\rangle\ \ 0\ \ \langle 7913\rangle\ \ 0\ \ \langle 9135\rangle\ \ 0\ \ \langle 1357\rangle\right)$$

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

Out[94]=

$$\frac{1}{\text{ab}[1,2,3,5]\,\text{ab}[2,3,5,6]\,\text{ab}[3,5,6,1]\,\text{ab}[5,6,1,2]\,\text{ab}[6,1,2,3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x__, cap[y_, z_], w__] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[109]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[105]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\begin{pmatrix} \langle 3579\rangle & 0 & \langle 5791\rangle & 0 & \langle 7913\rangle & 0 & \langle 9135\rangle & 0 & \langle 1357\rangle \end{pmatrix}$$

```
R[x__, cap[y_, z_], w__] :> (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])
```

Out[94]=
$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x__, cap[y_, z_], w__] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[103]=
```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[105]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[105]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=
$$\left(\langle 3579\rangle\ \ 0\ \ \langle 5791\rangle\ \ 0\ \ \langle 7913\rangle\ \ 0\ \ \langle 9135\rangle\ \ 0\ \ \langle 1357\rangle\right)$$

```
rToR[x__, cap[y_, z_], w__] :>
 (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])
```

Out[94]=

$$\frac{1}{ab[1,2,3,5]\ ab[2,3,5,6]\ ab[3,5,6,1]\ ab[5,6,1,2]\ ab[6,1,2,3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x__, cap[y_, z_], w__] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[103]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[106]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle \quad 0 \quad \langle 5791\rangle \quad 0 \quad \langle 7913\rangle \quad 0 \quad \langle 9135\rangle \quad 0 \quad \langle 1357\rangle\right)$$

```
rToDMatrixRow[n_] :=
 {R[x__, cap[y_, z_], w__] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])
```

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Out[94]=

$$\frac{1}{\text{ab}[1,2,3,5]\,\text{ab}[2,3,5,6]\,\text{ab}[3,5,6,1]\,\text{ab}[5,6,1,2]\,\text{ab}[6,1,2,3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x__, cap[y_, z_], w__] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[109]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[105]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\begin{pmatrix} \langle 3579\rangle & 0 & \langle 5791\rangle & 0 & \langle 7913\rangle & 0 & \langle 9135\rangle & 0 & \langle 1357\rangle \end{pmatrix}$$

```
rToDMatrixRow[n_] :=
 R|
  {R[x__, cap[y_, z_], w__] :>
    (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

Out[94]=

$$\frac{1}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[5,6,1,2]\ \text{ab}[6,1,2,3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x__, cap[y_, z_], w__] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[105]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[106]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle \quad 0 \quad \langle 5791\rangle \quad 0 \quad \langle 7913\rangle \quad 0 \quad \langle 9135\rangle \quad 0 \quad \langle 1357\rangle\right)$$

```
rToDMatrixRow[n_] :=
 {R[x__, cap[y_, z_], w__] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[94]=

$$\frac{1}{ab[1,2,3,5]\, ab[2,3,5,6]\, ab[3,5,6,1]\, ab[5,6,1,2]\, ab[6,1,2,3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[103]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[105]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle \;\; 0 \;\; \langle 5791\rangle \;\; 0 \;\; \langle 7913\rangle \;\; 0 \;\; \langle 9135\rangle \;\; 0 \;\; \langle 1357\rangle \right)$$

```
rToDMatrixRow[n_] :=
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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```
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\begin{pmatrix} \langle 3579\rangle & 0 & \langle 5791\rangle & 0 & \langle 7913\rangle & 0 & \langle 9135\rangle & 0 & \langle 1357\rangle \end{pmatrix}$$

```
rToDMatrixRow[n_] :=
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

$$\begin{pmatrix} \langle 3579\rangle & 0 & \langle 5791\rangle & 0 & \langle 7913\rangle & 0 & \langle 9135\rangle & 0 & \langle 1357\rangle \end{pmatrix}$$

```
rToDMatrixRow[n_] :=
 R[
  {R[x___, cap[y_, z_], w___] :>
    (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle \;\; 0 \;\; \langle 5791\rangle \;\; 0 \;\; \langle 7913\rangle \;\; 0 \;\; \langle 9135\rangle \;\; 0 \;\; \langle 1357\rangle\right)$$

```
rToDMatrixRow[n_] :=
 R[
  {R[x___, cap[y_, z_], w___] :>
    (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
rToDMatrixRow[n_] :=
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[103]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[105]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[105]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
rToDMatrixRow[n_][
 R :=
  {R[x___, cap[y_, z_], w___] :>
    (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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```
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[105]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[106]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
rToDMatrixRow[n_] :=
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle\ 0\ \langle 5791\rangle\ 0\ \langle 7913\rangle\ 0\ \langle 9135\rangle\ 0\ \langle 1357\rangle\right)$$

```
rToDMatrixRow[n_] :=
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

$$\left(\langle 3579\rangle \ 0 \ \langle 5791\rangle \ 0 \ \langle 7913\rangle \ 0 \ \langle 9135\rangle \ 0 \ \langle 1357\rangle\right)$$

```
rToDMatrixRow[n_] :=
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
rToDMatrixRow[n_] :=
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

$$\left(\langle 3579\rangle \;\; 0 \;\; \langle 5791\rangle \;\; 0 \;\; \langle 7913\rangle \;\; 0 \;\; \langle 9135\rangle \;\; 0 \;\; \langle 1357\rangle \right)$$

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[109]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[106]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle \;\; 0 \;\; \langle 5791\rangle \;\; 0 \;\; \langle 7913\rangle \;\; 0 \;\; \langle 9135\rangle \;\; 0 \;\; \langle 1357\rangle\right)$$

```
tree
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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```
    (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[105]=

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[106]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=

$$\left(\langle 3579\rangle \; 0 \; \langle 5791\rangle \; 0 \; \langle 7913\rangle \; 0 \; \langle 9135\rangle \; 0 \; \langle 1357\rangle\right)$$

In[110]:=

```
treeAmp[8, 2]
```

Out[110]=

```
{R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6], R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]],
```

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```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[105]=
```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /. R[x__] :> SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[106]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[107]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[108]//MatrixForm=
```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

In[110]:=
```
treeAmp[8, 2]
```

Out[110]=
```
{R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6], R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]],
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7],
```

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```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))

Out[109]= ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]

In[111]:= R[1, 3, 5, 7, 9] /. R[x__] :> SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]

Out[111]= SparseArray[<5>, {9}]

Out[112]= {ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}

Out[113]//MatrixForm=
( ab[3, 5, 7, 9]  0  ab[5, 7, 9, 1]  0  ab[7, 9, 1, 3]  0  ab[9, 1, 3, 5]  0  ab[1, 3, 5, 7] )

In[110]:= treeAmp[8, 2]

Out[110]= {R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6], R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]],
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7],
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))

Out[109]= ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]

R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
  {%}.η /@ Range[9]
  MatrixForm[{%%} // nice]

Out[111]= SparseArray[<5>, {9}]

Out[112]= {ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}

Out[113]//MatrixForm=
( ab[3, 5, 7, 9]  0  ab[5, 7, 9, 1]  0  ab[7, 9, 1, 3]  0  ab[9, 1, 3, 5]  0  ab[1, 3, 5, 7] )

In[110]:= treeAmp[8, 2]

Out[110]= {R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6], R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]],
```

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bcfw_part_deux.nb

Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[114]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

In[110]:=
```
treeAmp[8, 2]
```

Out[110]=
```
{R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7],
 R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 5, 6],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 4, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 5, 6], R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]],
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 6, 7],
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {4, 3, 1}]],
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {4, 3, 1}]],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[114]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[114]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

In[117]:=

```
treeAmp[8, 2][[13]]
```

Out[117]=

```
R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7]
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[114]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[114]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=

$$\left(\langle 3579\rangle \;\; 0 \;\; \langle 5791\rangle \;\; 0 \;\; \langle 7913\rangle \;\; 0 \;\; \langle 9135\rangle \;\; 0 \;\; \langle 1357\rangle\right)$$

```
(List @@ treeAmp[8, 2][[13]])
```

Out[118]=

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7]}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

In[114]:=

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[114]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=

(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)

```
(List @@ treeAmp[8, 2][[13]]) //.
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[118]=

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 6, 7]}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[114]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
( ⟨3579⟩  0  ⟨5791⟩  0  ⟨7913⟩  0  ⟨9135⟩  0  ⟨1357⟩ )
```

In[120]:=
```
(List @@ treeAmp[8, 2][[14]]) //.
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[120]=
```
{R[1, 2, 3, 7, 8], ab[3, 8, 7, 1] (ab[3, 2, 1, 8] R[2, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[2, 4, 5, 7, 8]) +
  ab[8, 7, 1, 2] (ab[3, 2, 1, 8] R[3, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[3, 4, 5, 7, 8])}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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In[114]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
( ⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩ )
```

```
(List @@ treeAmp[8, 2][[14]]
 ) //.
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[120]=
```
{R[1, 2, 3, 7, 8], ab[3, 8, 7, 1] (ab[3, 2, 1, 8] R[2, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[2, 4, 5, 7, 8]) +
  ab[8, 7, 1, 2] (ab[3, 2, 1, 8] R[3, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[3, 4, 5, 7, 8])}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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In[114]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

In[123]:=
```
List @@ treeAmp[8, 2][[14]]
(%) //.
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[123]=
```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[124]=
```
{R[1, 2, 3, 7, 8], ab[3, 8, 7, 1] (ab[3, 2, 1, 8] R[2, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[2, 4, 5, 7, 8]) +
  ab[8, 7, 1, 2] (ab[3, 2, 1, 8] R[3, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[3, 4, 5, 7, 8])}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

```
%[[2]]
% /. rToResidue
%% /. R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[5,6,1,2]\,ab[6,1,2,3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x___, cap[y_, z_], w___] ⧴
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

$$\begin{pmatrix} \langle 3579\rangle & 0 & \langle 5791\rangle & 0 & \langle 7913\rangle & 0 & \langle 9135\rangle & 0 & \langle 1357\rangle \end{pmatrix}$$

```
List @@ treeAmp[8, 2][[14]]
(%) //.
 {R[x___, cap[y_, z_], w___] ⧴
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]),
```

Out[38]=

```
1/(⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

In[37]:=

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
```

In[40]:=

```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y⟦1⟧, w] ab @@ Prepend[z, y⟦2⟧] + ab[x, y⟦2⟧, w] ab @@ Append[z, y⟦1⟧]}
```

In[41]:=

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

In[43]:=

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

Out[43]=

```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
  ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
  ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
  ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
  (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[38]:= Out[28] // nice

Out[38]= 1/(⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
   ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
   ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
   ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)

In[37]:= nice[exprn_] :=
  (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})

rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0;

In[40]:= capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

Out[40]= {ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}

In[41]:= ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

```
R[1, 2, 3, 5, 6]
```

$$\frac{1}{ab[1, 2, 3, 5]\, ab[2, 3, 5, 6]\, ab[3, 5, 6, 1]\, ab[5, 6, 1, 2]\, ab[6, 1, 2, 3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
%[[1]] /@ Range[9]
MatrixForm[{%%} // nice]
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[40]:= capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

Out[40]= {ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}

In[41]:= ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules

Out[41]= ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]

In[43]:= Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0

Out[43]= 1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
   ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
   ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
   (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
   (ab[4, 1, 2, 3] ab[5, 7, 6, 1] ab[9, 8, 1, 6] ab[11, 10, 1, 8] +
     ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))

treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]

R[1, 2, 3, 5, 6]

1
ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3]
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
 {R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{R[1, 2, 3, 7, 8], ab[3, 8, 7, 1] (ab[3, 2, 1, 8] R[2, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[2, 4, 5, 7, 8]) +
  ab[8, 7, 1, 2] (ab[3, 2, 1, 8] R[3, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[3, 4, 5, 7, 8])}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree–Level BCFW Recursion in N=4 *Mathematica Summer School 2011*

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
  {R[x__, cap[y_, z_], w__] :>
    (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
 ab[x___, y_, w___, y_, z___] :> 0
```

Out[123]=
```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[124]=
```
{R[1, 2, 3, 7, 8], ab[3, 8, 7, 1] (ab[3, 2, 1, 8] R[2, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[2, 4, 5, 7, 8]) +
  ab[8, 7, 1, 2] (ab[3, 2, 1, 8] R[3, 4, 5, 7, 7] + ab[7, 3, 2, 1] R[3, 4, 5, 7, 8])}
```

```
{R[x__, cap[y_, z_], w__] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
((3579) 0 (5791) 0 (7913) 0 (9135) 0 (1357))
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
  {R[x__, cap[y_, z_], w__] :>
    (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
 R[x__, y_, w__, y_, z__] :> 0
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{R[1, 2, 3, 7, 8], ab[3, 8, 7, 1] ab[7, 3, 2, 1] R[2, 4, 5, 7, 8] + ab[7, 3, 2, 1] ab[8, 7, 1, 2] R[3, 4, 5, 7, 8]}
```

```
{R[x__, cap[y_, z_], w__] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x__, cap[y_, z_], w__] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{R[1, 2, 3, 7, 8], ab[3, 8, 7, 1] ab[7, 3, 2, 1] R[2, 4, 5, 7, 8] + ab[7, 3, 2, 1] ab[8, 7, 1, 2] R[3, 4, 5, 7, 8]}
```

```
{R[x__, cap[y_, z_], w__] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

In[129]:=
```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x__, cap[y_, z_], w__] ⇸
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] ⇸ 0 /.
 {R[x__] ⇸ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
% // nice
```

Out[129]=
```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[130]=
```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

Out[131]=
```
{{⟨2378⟩, ⟨3781⟩, ⟨7812⟩, 0, 0, 0, ⟨8123⟩, ⟨1237⟩}, {0, ⟨3871⟩ ⟨4578⟩ ⟨7321⟩, ⟨4578⟩ ⟨7321⟩ ⟨8712⟩,
  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩, ⟨3871⟩ ⟨7321⟩ ⟨7824⟩ + ⟨7321⟩ ⟨7834⟩ ⟨8712⟩, 0,
  ⟨3871⟩ ⟨7321⟩ ⟨8245⟩ + ⟨7321⟩ ⟨8345⟩ ⟨8712⟩, ⟨2457⟩ ⟨3871⟩ ⟨7321⟩ + ⟨3457⟩ ⟨7321⟩ ⟨8712⟩}}
```

```
{R[x__, cap[y_, z_], w__] ⇸
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[114]= {ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}

Out[115]= {ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}

Out[116]//MatrixForm=
( ⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩ )

In[132]:= List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x__, cap[y_, z_], w__] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
MatrixForm[% // nice]

Out[132]= {R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}

Out[133]= {{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}

Out[134]//MatrixForm=
( ⟨2378⟩        ⟨3781⟩              ⟨7812⟩                    0
  0     ⟨3871⟩ ⟨4578⟩ ⟨7321⟩  ⟨4578⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨7321⟩ ⟨7

{R[x__, cap[y_, z_], w__] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
MatrixForm[% // nice]
```

⟨8123⟩ ⟨1237⟩

1⟩ ⟨7321⟩ ⟨8245⟩ + ⟨7321⟩ ⟨8345⟩ ⟨8712⟩ ⟨2457⟩ ⟨3871⟩ ⟨7321⟩ + ⟨3457⟩ ⟨7321⟩ ⟨8712⟩

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
MatrixForm[% // nice]
```

⟨8123⟩ ⟨1237⟩

1⟩ ⟨7321⟩ ⟨8245⟩ + ⟨7321⟩ ⟨8345⟩ ⟨8712⟩ ⟨2457⟩ ⟨3871⟩ ⟨7321⟩ + ⟨3457⟩ ⟨7321⟩ ⟨8712⟩

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
MatrixForm[% // nice]
```

⟨8123⟩ ⟨1237⟩

45⟩ + ⟨7321⟩ ⟨8345⟩ ⟨8712⟩ ⟨2457⟩ ⟨3871⟩ ⟨7321⟩ + ⟨3457⟩ ⟨7321⟩ ⟨8712⟩

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
((⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩))
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
  {R[x__, cap[y_, z_], w__] :>
    (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
 R[x__, y_, w__, y_, z__] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
MatrixForm[% // nice]
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

```
(⟨2378⟩         ⟨3781⟩              ⟨7812⟩                              0
   0    ⟨3871⟩ ⟨4578⟩ ⟨7321⟩  ⟨4578⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨7321⟩ ⟨7
```

```
{R[x__, cap[y_, z_], w__] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x__, cap[y_, z_], w__] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
MatrixForm[% // nice]
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

```
( ⟨2378⟩            ⟨3781⟩                    ⟨7812⟩                          0
   0      ⟨3871⟩ ⟨4578⟩ ⟨7321⟩  ⟨4578⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨7321⟩ ⟨7
```

```
{R[x__, cap[y_, z_], w__] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
```

```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
  ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
  ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
  ab[11, 1, 8, 9] ab[11, 10, 1, 8]^9 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
  (ab[6, 1, 4, 5] ab[7, 9, 8, 1] ab[11, 10, 1, 8] + ab[7, 1, 4, 5] ab[9, 8, 1, 6] ab[11, 10, 1, 8])
```

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```
Out[28] // nice
```

```
1/(⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]})
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
```

```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[1], w] ab @@ Prepend[z, y[2]] + ab[x, y[2], w] ab @@ Append[z, y[1]]}
```

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1/(ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
  ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
  ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
  ab[11, 1, 8, 9] ab[11, 10, 1, 8]^3 (ab[8, 1, 6, 7] ab[9, 11, 10, 1] + ab[9, 1, 6, 7] ab[11, 10, 1, 8])
```

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e–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[38]:=
```
Out[28] // nice
```

Out[38]=
```
1/ (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?Matrix)
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
```

In[40]:=
```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=
```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

In[41]:=
```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

In[43]:=
```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1/ (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
  ab[4, 5, 1, 2] ab[4, 5, 6, 7] ab[5, 6, 7, 1] ab[6, 7, 1, 4] ab[6, 7, 8, 9] ab[7, 6, 1, 4]^3
  ab[7, 8, 9, 1] ab[8, 9, 1, 6] ab[8, 9, 10, 11] ab[9, 8, 1, 6]^6 ab[9, 10, 11, 1] ab[10, 11, 1, 8]
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
R[1, 3, 5, 7, 9] /.
 R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

```
Out[114]= {ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
Out[115]= {ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
Out[116]//MatrixForm=
( ⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩ )
```

```
In[132]:= List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] ⇸
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] ⇸ 0 /.
 {R[x__] ⇸ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
MatrixForm[% // nice]
```

```
Out[132]= {R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
Out[133]= {{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

```
Out[134]//MatrixForm=
( ⟨2378⟩   ⟨3781⟩   ⟨7812⟩   0
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[136]:=

```
List @@ treeAmp[8, 2][[14]]
(%) //.
    {R[x__, cap[y_, z_], w__] :>
      (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[136]=

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[137]=

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

Out[138]//MatrixForm=

```
( ⟨2378⟩        ⟨3781⟩                ⟨7812⟩                        0
    0     ⟨3871⟩ ⟨4578⟩ ⟨7321⟩  ⟨4578⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨7321⟩ ⟨7
```

```
{R[x__, cap[y_, z_], w__] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x__, cap[y_, z_], w__] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

```
(⟨2378⟩                    ⟨7812⟩              0
   0     ⟨387  ⟨4578⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨7321⟩ ⟨7
```

Cut
Copy
Paste
Copy As
Generate Palette
Convert To
Get Help
Speak Selection
Toggle Full Screen

```
{R[x__, cap[y
  (R[x, y[[1]]  d[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Norm  Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x__, cap[y_, z_], w__] ⇸
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] ⇸ 0 /.
 {R[x__] ⇸ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

Out[138]//MatrixForm=

```
( ⟨2378⟩        ⟨3781⟩                  ⟨7812⟩                                0
     0     ⟨3871⟩ ⟨4578⟩ ⟨7321⟩  ⟨4578⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨7321⟩ ⟨7
```

```
{R[x__, cap[y_, z_], w__] ⇸
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] ⇸ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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bcfw_part_deux.nb

Tree−Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

//MatrixForm=

$$\begin{pmatrix} \langle 2378\rangle & \langle 3781\rangle & \langle 7812\rangle & 0 \\ 0 & \langle 3871\rangle\langle 4578\rangle\langle 7321\rangle & \langle 4578\rangle\langle 7321\rangle\langle 8712\rangle & \langle 3871\rangle\langle 5782\rangle\langle 7321\rangle + \langle 5783\rangle\langle 7321\rangle\langle 8712\rangle & \langle 3871\rangle\langle 7321\rangle\langle 7 \end{pmatrix}$$

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

$$\left(\langle 3579\rangle \;\; 0 \;\; \langle 5791\rangle \;\; 0 \;\; \langle 7913\rangle \;\; 0 \;\; \langle 9135\rangle \;\; 0 \;\; \langle 1357\rangle\right)$$

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x__, cap[y_, z_], w__] ⧴
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] ⧴ 0 /.
 {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

$$\begin{pmatrix} \langle 2378\rangle & \langle 3781\rangle & \langle 7812\rangle & 0 \\ 0 & \langle 3871\rangle\langle 4578\rangle\langle 7321\rangle & \langle 4578\rangle\langle 7321\rangle\langle 8712\rangle & \langle 3871\rangle\langle 5782\rangle\langle 7321\rangle + \langle 5783\rangle\langle 7321\rangle\langle 8712\rangle \;\; \langle 3871\rangle\langle 7321\rangle\langle 7 \end{pmatrix}$$

```
{R[x__, cap[y_, z_], w__] ⧴
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
   ab[5, 1, 2, 3] ab[7, 6, 1, 4] ab[9, 8, 1, 6] ab[11, 10, 1, 8]))
```

```
treeAmp[6, 1];
%[[2]]
% /. rToResidue
%% /. R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
```

Out[93]=
```
R[1, 2, 3, 5, 6]
```

Out[94]=

$$\frac{1}{ab[1, 2, 3, 5]\ ab[2, 3, 5, 6]\ ab[3, 5, 6, 1]\ ab[5, 6, 1, 2]\ ab[6, 1, 2, 3]}$$

```
(R[1, 3, 5, 7, cap[{10, 11}, {12, 13, 19}]] /.
  R[x___, cap[y_, z_], w___] :>
   (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]]))
```

Out[109]=
```
ab[11, 12, 13, 19] R[1, 3, 5, 7, 10] + ab[12, 13, 19, 10] R[1, 3, 5, 7, 11]
```

In[114]:=
```
R[1, 3, 5, 7, 9] /.
 R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]
{%}.η /@ Range[9]
MatrixForm[{%%} // nice]
```

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x__, cap[y_, z_], w__] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

Out[138]//MatrixForm=

```
( ⟨2378⟩        ⟨3781⟩                ⟨7812⟩                           0
   0    ⟨3871⟩ ⟨4578⟩ ⟨7321⟩  ⟨4578⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨7321⟩ ⟨7
```

```
{R[x__, cap[y_, z_], w__] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
MatrixForm[{%%} // nice]

Out[114]= {ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}

Out[115]= {ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}

Out[116]//MatrixForm=
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)

In[136]:=
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]

Out[136]= {R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}

Out[137]= {{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}

Out[138]//MatrixForm=
(⟨2378⟩        ⟨3781⟩                 ⟨7812⟩                                  0
   0    ⟨3871⟩ ⟨4578⟩ ⟨7321⟩  ⟨4578⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨7321⟩ ⟨7

{R[x___, cap[y_, z_], w___] :>
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
MatrixForm[{%%} // nice]
```

Out[114]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=

(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)

```
termT
```

In[136]:=

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x__, cap[y_, z_], w__] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x__, y_, w__, y_, z__] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[136]=

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[137]=

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

Out[138]//MatrixForm=

(⟨2378⟩ ⟨3781⟩ ⟨7812⟩ 0
0 ⟨3871⟩ ⟨4578⟩ ⟨7321⟩ ⟨4578⟩ ⟨7321⟩ ⟨8712⟩ ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩ ⟨3871⟩ ⟨7321⟩ ⟨7

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bcfw_part_deux.nb

Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
MatrixForm[{%%} // nice]
```

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
(⟨3579⟩  0  ⟨5791⟩  0  ⟨7913⟩  0  ⟨9135⟩  0  ⟨1357⟩)
```

```
termToDMatrix[term_]
```

In[136]:=
```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] ⧴
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] ⧴ 0 /.
 {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[136]=
```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[137]=
```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

Out[138]//MatrixForm=
```
( ⟨2378⟩   ⟨3781⟩                 ⟨7812⟩                 0
  0        ⟨3871⟩ ⟨4578⟩ ⟨7321⟩   ⟨4578⟩ ⟨7321⟩ ⟨8712⟩   ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩   ⟨3871⟩ ⟨7321⟩ ⟨7
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
MatrixForm[{%%}] // nice]

Out[114]= {ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}

Out[115]= {ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}

Out[116]//MatrixForm=
( ⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩ )

termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},

In[136]:= List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]

Out[136]= {R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}

Out[137]= {{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}

Out[137]//MatrixForm=
( ⟨2378⟩        ⟨3781⟩              ⟨7812⟩                       0
  0     ⟨3871⟩ ⟨4578⟩ ⟨7321⟩  ⟨4578⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩  ⟨3871⟩ ⟨7321⟩ ⟨7
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
MatrixForm[{%%} // nice]
```

Out[114]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=

$$(\langle 3579 \rangle \;\; 0 \;\; \langle 5791 \rangle \;\; 0 \;\; \langle 7913 \rangle \;\; 0 \;\; \langle 9135 \rangle \;\; 0 \;\; \langle 1357 \rangle)$$

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
```

In[136]:=

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[136]=

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[137]=

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

Out[137]//MatrixForm=

⟨2378⟩ ⟨3781⟩ ⟨7812⟩ 0

0 ⟨3871⟩⟨4578⟩⟨7321⟩ ⟨4578⟩⟨7321⟩⟨8712⟩ ⟨3871⟩⟨5782⟩⟨7321⟩ + ⟨5783⟩⟨7321⟩⟨8712⟩ ⟨3871⟩⟨7321⟩⟨7

```
MatrixForm[{%%} // nice]
```

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
( ⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩ )
```

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
```

In[136]:=
```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] ⧴
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] ⧴ 0 /.
 {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[136]=
```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[137]=
```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

Out[138]//MatrixForm=
```
( ⟨2378⟩   ⟨3781⟩   ⟨7812⟩   0
  0   ⟨3871⟩ ⟨4578⟩ ⟨7321⟩   ⟨4578⟩ ⟨7321⟩ ⟨8712⟩   ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩   ⟨3871⟩ ⟨7321⟩ ⟨7
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
MatrixForm[{%%} // nice]
```

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=

$$\left(\langle 3579\rangle \;\; 0 \;\; \langle 5791\rangle \;\; 0 \;\; \langle 7913\rangle \;\; 0 \;\; \langle 9135\rangle \;\; 0 \;\; \langle 1357\rangle\right)$$

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
```

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

//MatrixForm=

⟨2378⟩ ⟨3781⟩ ⟨7812⟩ 0

0 ⟨3871⟩⟨4578⟩⟨7321⟩ ⟨4578⟩⟨7321⟩⟨8712⟩ ⟨3871⟩⟨5782⟩⟨7321⟩ + ⟨5783⟩⟨7321⟩⟨8712⟩ ⟨3871⟩⟨7321⟩⟨7

```
MatrixForm[{%%} // nice]
```

Out[114]=

```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  rList /. //.
     {R[x___, cap[y_, z_], w___] :>
       (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] :> 0 /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

In[136]:=

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[136]=

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[137]=

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
MatrixForm[{%%} // nice]
```

Out[114]=
```
{ab[3, 5, 7, 9], 0, ab[5, 7, 9, 1], 0, ab[7, 9, 1, 3], 0, ab[9, 1, 3, 5], 0, ab[1, 3, 5, 7]}
```

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
(⟨3579⟩  0  ⟨5791⟩  0  ⟨7913⟩  0  ⟨9135⟩  0  ⟨1357⟩)
```

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] :> 0 /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

In[136]:=
```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[136]=
```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[137]=
```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
```

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Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]})
```

In[136]:=

```
List @@ treeAmp[8, 2][[14]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[136]=

```
{R[1, 2, 3, 7, 8], R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]]}
```

Out[137]=

```
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7]},
 {0, ab[3, 8, 7, 1] ab[4, 5, 7, 8] ab[7, 3, 2, 1], ab[4, 5, 7, 8] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[5, 7, 8, 2] ab[7, 3, 2, 1] + ab[5, 7, 8, 3] ab[7, 3, 2, 1] ab[8, 7, 1, 2],
  ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[7, 8, 2, 4] + ab[7, 3, 2, 1] ab[7, 8, 3, 4] ab[8, 7, 1, 2],
  0, ab[3, 8, 7, 1] ab[7, 3, 2, 1] ab[8, 2, 4, 5] + ab[7, 3, 2, 1] ab[8, 3, 4, 5] ab[8, 7, 1, 2],
  ab[2, 4, 5, 7] ab[3, 8, 7, 1] ab[7, 3, 2, 1] + ab[3, 4, 5, 7] ab[7, 3, 2, 1] ab[8, 7, 1, 2]}}
```

Out[138]//MatrixForm=

```
⟨2378⟩   ⟨3781⟩   ⟨7812⟩   0
0   ⟨3871⟩ ⟨4578⟩ ⟨7321⟩   ⟨4578⟩ ⟨7321⟩ ⟨8712⟩   ⟨3871⟩ ⟨5782⟩ ⟨7321⟩ + ⟨5783⟩ ⟨7321⟩ ⟨8712⟩   ⟨3871⟩ ⟨7321⟩ ⟨7
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩}
```

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]})
```

In[139]:=

```
List @@ treeAmp[8, 2][[2]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[139]=

```
{R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]], R[1, 4, 5, 7, 8]}
```

Out[140]=

```
{{ab[2, 3, 4, 5] ab[8, 7, 1, 4], ab[3, 4, 5, 1] ab[8, 7, 1, 4],
  ab[4, 5, 1, 2] ab[8, 7, 1, 4], ab[5, 1, 2, 3] ab[8, 7, 1, 4], ab[1, 2, 3, 4] ab[8, 7, 1, 4]},
 {ab[4, 5, 7, 8], 0, 0, ab[5, 7, 8, 1], ab[7, 8, 1, 4], 0, ab[8, 1, 4, 5], ab[1, 4, 5, 7]}}
```

Out[141]=

```
{{⟨2345⟩ ⟨8714⟩, ⟨3451⟩ ⟨8714⟩, ⟨4512⟩ ⟨8714⟩, ⟨5123⟩ ⟨8714⟩, ⟨1234⟩ ⟨8714⟩},
 {⟨4578⟩, 0, 0, ⟨5781⟩, ⟨7814⟩, 0, ⟨8145⟩, ⟨1457⟩}}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(rList //.
    {R[x___, cap[y_, z_], w___] :>
      (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
   R[x___, y_, w___, y_, z___] :> 0 /.
  {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]})
```

```
In[139]:= List @@ treeAmp[8, 2][[2]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]

Out[139]= {R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]], R[1, 4, 5, 7, 8]}

Out[140]= {{ab[2, 3, 4, 5] ab[8, 7, 1, 4], ab[3, 4, 5, 1] ab[8, 7, 1, 4],
  ab[4, 5, 1, 2] ab[8, 7, 1, 4], ab[5, 1, 2, 3] ab[8, 7, 1, 4], ab[1, 2, 3, 4] ab[8, 7, 1, 4]},
 {ab[4, 5, 7, 8], 0, 0, ab[5, 7, 8, 1], ab[7, 8, 1, 4], 0, ab[8, 1, 4, 5], ab[1, 4, 5, 7]}}

Out[141]= {{⟨2345⟩ ⟨8714⟩, ⟨3451⟩ ⟨8714⟩, ⟨4512⟩ ⟨8714⟩, ⟨5123⟩ ⟨8714⟩, ⟨1234⟩ ⟨8714⟩},
 {⟨4578⟩, 0, 0, ⟨5781⟩, ⟨7814⟩, 0, ⟨8145⟩, ⟨1457⟩}}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]})
```

In[139]:=
```
List @@ treeAmp[8, 2][[2]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[139]=
```
{R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]], R[1, 4, 5, 7, 8]}
```

Out[140]=
```
{{ab[2, 3, 4, 5] ab[8, 7, 1, 4], ab[3, 4, 5, 1] ab[8, 7, 1, 4],
  ab[4, 5, 1, 2] ab[8, 7, 1, 4], ab[5, 1, 2, 3] ab[8, 7, 1, 4], ab[1, 2, 3, 4] ab[8, 7, 1, 4]},
 {ab[4, 5, 7, 8], 0, 0, ab[5, 7, 8, 1], ab[7, 8, 1, 4], 0, ab[8, 1, 4, 5], ab[1, 4, 5, 7]}}
```

Out[141]=
```
{{⟨2345⟩ ⟨8714⟩, ⟨3451⟩ ⟨8714⟩, ⟨4512⟩ ⟨8714⟩, ⟨5123⟩ ⟨8714⟩, ⟨1234⟩ ⟨8714⟩},
 {⟨4578⟩, 0, 0, ⟨5781⟩, ⟨7814⟩, 0, ⟨8145⟩, ⟨1457⟩}}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
     {R[x___, cap[y_, z_], w___] :>
       (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] :> 0 /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]})
```

```
In[139]:= List @@ treeAmp[8, 2][[2]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]

Out[139]= {R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]], R[1, 4, 5, 7, 8]}

Out[140]= {{ab[2, 3, 4, 5] ab[8, 7, 1, 4], ab[3, 4, 5, 1] ab[8, 7, 1, 4],
  ab[4, 5, 1, 2] ab[8, 7, 1, 4], ab[5, 1, 2, 3] ab[8, 7, 1, 4], ab[1, 2, 3, 4] ab[8, 7, 1, 4]},
 {ab[4, 5, 7, 8], 0, 0, ab[5, 7, 8, 1], ab[7, 8, 1, 4], 0, ab[8, 1, 4, 5], ab[1, 4, 5, 7]}}

Out[141]= {{⟨2345⟩ ⟨8714⟩, ⟨3451⟩ ⟨8714⟩, ⟨4512⟩ ⟨8714⟩, ⟨5123⟩ ⟨8714⟩, ⟨1234⟩ ⟨8714⟩},
 {⟨4578⟩, 0, 0, ⟨5781⟩, ⟨7814⟩, 0, ⟨8145⟩, ⟨1457⟩}}
```

```
{R[x___, cap[y_, z_], w___] :>
  (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

{R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]})
```

```
List @@ treeAmp[8, 2][[2]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

```
{R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]], R[1, 4, 5, 7, 8]}
```

```
{{ab[2, 3, 4, 5] ab[8, 7, 1, 4], ab[3, 4, 5, 1] ab[8, 7, 1, 4],
  ab[4, 5, 1, 2] ab[8, 7, 1, 4], ab[5, 1, 2, 3] ab[8, 7, 1, 4], ab[1, 2, 3, 4] ab[8, 7, 1, 4]},
 {ab[4, 5, 7, 8], 0, 0, ab[5, 7, 8, 1], ab[7, 8, 1, 4], 0, ab[8, 1, 4, 5], ab[1, 4, 5, 7]}}
```

```
{{⟨2345⟩ ⟨8714⟩, ⟨3451⟩ ⟨8714⟩, ⟨4512⟩ ⟨8714⟩, ⟨5123⟩ ⟨8714⟩, ⟨1234⟩ ⟨8714⟩},
 {⟨4578⟩, 0, 0, ⟨5781⟩, ⟨7814⟩, 0, ⟨8145⟩, ⟨1457⟩}}
```

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Tree-Level BCFW Recursion in N=4 *Mathematica Summer School 2011*

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]});
```

```
List @@ treeAmp[8, 2][[2]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

```
{R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]], R[1, 4, 5, 7, 8]}
```

```
{{ab[2, 3, 4, 5] ab[8, 7, 1, 4], ab[3, 4, 5, 1] ab[8, 7, 1, 4],
  ab[4, 5, 1, 2] ab[8, 7, 1, 4], ab[5, 1, 2, 3] ab[8, 7, 1, 4], ab[1, 2, 3, 4] ab[8, 7, 1, 4]},
 {ab[4, 5, 7, 8], 0, 0, ab[5, 7, 8, 1], ab[7, 8, 1, 4], 0, ab[8, 1, 4, 5], ab[1, 4, 5, 7]}}
```

```
{{⟨2345⟩ ⟨8714⟩, ⟨3451⟩ ⟨8714⟩, ⟨4512⟩ ⟨8714⟩, ⟨5123⟩ ⟨8714⟩, ⟨1234⟩ ⟨8714⟩},
 {⟨4578⟩, 0, 0, ⟨5781⟩, ⟨7814⟩, 0, ⟨8145⟩, ⟨1457⟩}}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

Out[115]=
```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=
```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
```

In[139]:=
```
List @@ treeAmp[8, 2][[2]]
(%) //.
   {R[x___, cap[y_, z_], w___] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] :> 0 /.
 {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[139]=
```
{R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]], R[1, 4, 5, 7, 8]}
```

Out[140]=
```
{{ab[2, 3, 4, 5] ab[8, 7, 1, 4], ab[3, 4, 5, 1] ab[8, 7, 1, 4],
  ab[4, 5, 1, 2] ab[8, 7, 1, 4], ab[5, 1, 2, 3] ab[8, 7, 1, 4], ab[1, 2, 3, 4] ab[8, 7, 1, 4]},
 {ab[4, 5, 7, 8], 0, 0, ab[5, 7, 8, 1], ab[7, 8, 1, 4], 0, ab[8, 1, 4, 5], ab[1, 4, 5, 7]}}
```

```
{{⟨2345⟩ ⟨8714⟩, ⟨3451⟩ ⟨8714⟩, ⟨4512⟩ ⟨8714⟩, ⟨5123⟩ ⟨8714⟩, ⟨1234⟩ ⟨8714⟩},
 {⟨4578⟩, 0, 0, ⟨5781⟩, ⟨7814⟩, 0, ⟨8145⟩, ⟨1457⟩}}
```

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Out[115]=

```
{ab[3, 5, 7, 9] η[1] + ab[5, 7, 9, 1] η[3] + ab[7, 9, 1, 3] η[5] + ab[9, 1, 3, 5] η[7] + ab[1, 3, 5, 7] η[9]}
```

Out[116]//MatrixForm=

```
(⟨3579⟩ 0 ⟨5791⟩ 0 ⟨7913⟩ 0 ⟨9135⟩ 0 ⟨1357⟩)
```

In[142]:=

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] ⧴
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] ⧴ 0 /.
    {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
```

In[139]:=

```
List @@ treeAmp[8, 2][[2]]
(%) //.
   {R[x___, cap[y_, z_], w___] ⧴
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
  R[x___, y_, w___, y_, z___] ⧴ 0 /.
 {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]]]]}
nice[%]
```

Out[139]=

```
{R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]], R[1, 4, 5, 7, 8]}
```

Out[140]=

```
{{ab[2, 3, 4, 5] ab[8, 7, 1, 4], ab[3, 4, 5, 1] ab[8, 7, 1, 4],
  ab[4, 5, 1, 2] ab[8, 7, 1, 4], ab[5, 1, 2, 3] ab[8, 7, 1, 4], ab[1, 2, 3, 4] ab[8, 7, 1, 4]},
 {ab[4, 5, 7, 8], 0, 0, ab[5, 7, 8, 1], ab[7, 8, 1, 4], 0, ab[8, 1, 4, 5], ab[1, 4, 5, 7]}}
```

```
{{⟨2345⟩ ⟨8714⟩, ⟨3451⟩ ⟨8714⟩, ⟨4512⟩ ⟨8714⟩, ⟨5123⟩ ⟨8714⟩, ⟨1234⟩ ⟨8714⟩},
 {⟨4578⟩, 0, 0, ⟨5781⟩, ⟨7814⟩, 0, ⟨8145⟩, ⟨1457⟩}}
```

$$\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle$$

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[27]:=

```
treeAmp[12, 4][[1]]
% /. {R[x__] → 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

Out[27]=

```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

Out[28]=

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
```

```
ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

In[38]:=
```
Out[28] // nice
```

Out[38]=
```
1/(⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
 ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
 ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
 ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

In[35]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue = {R[x__] :> 1/(Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
```

In[40]:=
```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

Out[40]=
```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

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```
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
```

```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
```

```
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
```

```
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```



```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
In[38]:= Out[28] // nice

Out[38]= 1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
   ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
   ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
   ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
In[35]:= nice[exprn_] :=
  (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
    m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
```

```
In[40]:= capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}

Out[40]= {ab[x___, cap[y_, z_], w___] :> ab[x, y⟦1⟧, w] ab @@ Prepend[z, y⟦2⟧] + ab[x, y⟦2⟧, w] ab @@ Append[z, y⟦1⟧]}
```

```
In[41]:= ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1/(⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue = {R[x__] :> 1/(Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
```

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[1], w] ab @@ Prepend[z, y[2]] + ab[x, y[2], w] ab @@ Append[z, y[1]]}
```

```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
ab[cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4, 5] ab[cap[{8, 9}, {11, 10, 1}], 1, 6, 7])
```

```
Out[28] // nice
```

```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])}
```

```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

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In[35]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

In[43]:=
```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

Out[40]=
```
{ab[x___, cap[y_, z_], w___] :> ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]]}
```

In[41]:=
```
ab[1, 2, 3, cap[{4, 5}, {6, 7, 8}]] /. capRules
```

Out[41]=
```
ab[1, 2, 3, 4] ab[5, 6, 7, 8] + ab[1, 2, 3, 5] ab[6, 7, 8, 4]
```

```
Out[28] //. capRules /. ab[x___, y_, w___, y_, z___] :> 0
```

```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10] ab[2, 3, 4, 5] ab[3, 4, 5, 1]
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    m_?MatrixQ → MatrixForm[m])
```

In[143]:=

```
rToResidue = {R[x__] → 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
    ab[x___, y_, w___, y_, z___] → 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] →
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] → 0 /.
    {R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] →
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    m_?MatrixQ ⇒ MatrixForm[m])
```

In[43]:=

```
rToResidue = {R[x__] ⇒ 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] ⇒ 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
     {R[x___, cap[y_, z_], w___] ⇒
       (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] ⇒ 0 /.
   {R[x__] ⇒ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] ⇒
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[46]:=

```
treeAmp[10, 3][[50]]
```

Out[46]=

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

$$\Leftrightarrow\left\{\frac{1}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle},\ \{\langle bcde\rangle, \langle cdea\rangle, \langle deab\rangle, \langle eabc\rangle, \langle abcd\rangle\}\right\}$$

$$\{\text{residue}, \text{dMatrix}\}$$

As reviewed during Pedro's lecture on Sunday, each component-amplitude of an NMHV amplitude is obtained by multiplying each "residue" by 4 particular entries of its "dMatrix" (which we can view as four 1x1 determinants).

Extracting the "residue" and "dMatrix" for each R-invariant

In[27]:=
```
treeAmp[12, 4][[1]]
% /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))}
```

Out[27]=
```
R[1, 2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
 R[1, 4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
 R[1, 6, 7, 8, cap[{8, 9}, {11, 10, 1}]] R[1, 8, 9, 10, 11]
```

Out[28]=
```
1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 6] ab[1, 6, 7, 8] ab[1, 8, 9, 10]
   ab[2, 3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}]]
   ab[3, 4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1]
   ab[4, 5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}]]
   ab[4, cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2]
   ab[5, 6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1] ab[6, 7, 8, cap[{8, 9}, {11, 10, 1}]]
   ab[6, cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 1, 4] ab[7, 8, cap[{8, 9}, {11, 10, 1}], 1]
   ab[8, 9, 10, 11] ab[8, cap[{8, 9}, {11, 10, 1}], 1, 6] ab[9, 10, 11, 1] ab[10, 11, 1, 8]
   ab[11, 1, 8, 9] ab[cap[{4, 5}, {cap[{6, 7}, {cap[{8, 9}, {11, 10, 1}], 8, 1}], 6, 1}], 1, 2, 3]
```

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```
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{|
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

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```
⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :→ Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :→ Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :→ MatrixForm[m])
```

```
rToResidue = {R[x__] :→ 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :→ 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :→
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :→ 0 /.
    {R[x__] :→ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :→
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{residue, dMatrix}
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
〈101118〉〈11189〉〈(45)⋂((67)⋂((89)⋂(11101)81)61)123〉〈(67)⋂((89)⋂(11101)81)145〉〈(89)⋂(11101)167〉
```

In[135]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"〈", x, "〉"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

In[143]:=
```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[147]:=
```
treeAmp[10, 3][[50]]
{residue, termToDMatrix[%, 10]}
```

Out[147]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[148]=
```
{residue, {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

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⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)

In[135]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

In[143]:=
```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{residue, termToDMatrix[%, 10]};
```

Out[147]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[148]=
```
{residue, {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
  ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩⟩

In[135]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

In[143]:=
```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{residue, termToDMatrix[%, 10]};
nice@
```

Out[147]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[148]=
```
{residue, {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

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Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

In[135]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] ⧴ Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] ⧴ Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ ⧴ MatrixForm[m])
```

In[143]:=
```
rToResidue = {R[x__] ⧴ 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] ⧴ 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] ⧴
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] ⧴ 0 /.
    {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] ⧴
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[149]:=
```
treeAmp[10, 3][[50]]
{residue, termToDMatrix[%, 10]};
nice@%
```

Out[149]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[151]=
```
{residue, ( ⟨3467⟩   0       ⟨4671⟩                 ⟨6713⟩                  0                    ⟨7134⟩
            ⟨78910⟩  0         0                      0                     0                      0
              0      0  ⟨4567⟩ ⟨4761⟩ ⟨6431⟩  ⟨4761⟩ ⟨5673⟩ ⟨6431⟩  ⟨4761⟩ ⟨6431⟩ ⟨6734⟩  ⟨4761⟩ ⟨6431⟩ ⟨7345⟩  ⟨34
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(exprn //. {cap[x_, y_] → Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] → Row[{"⟨", x, "⟩"}]} /.
  m_?MatrixQ → MatrixForm[m])
```

In[143]:=
```
rToResidue = {R[x__] → 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] → 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
     {R[x___, cap[y_, z_], w___] →
       (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] → 0 /.
   {R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] →
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[149]:=
```
treeAmp[10, 3][[50]]
{residue, termToDMatrix[%, 10]};
nice@%
```

Out[149]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[151]=
```
{residue, ( ⟨3467⟩  0  ⟨4671⟩                      ⟨6713⟩                      0                           ⟨7134⟩
            ⟨78910⟩ 0  0                           0                           0                           0
            0       0  ⟨4567⟩ ⟨4761⟩ ⟨6431⟩  ⟨4761⟩ ⟨5673⟩ ⟨6431⟩  ⟨4761⟩ ⟨6431⟩ ⟨6734⟩  ⟨4761⟩ ⟨6431⟩ ⟨7345⟩  ⟨34
```

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
   (rList //.
       {R[x___, cap[y_, z_], w___] :>
          (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
      R[x___, y_, w___, y_, z___] :> 0 /.
     {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{residue, termToDMatrix[%, 10]};
nice@%
```

```
1}]]
```

		⟨7134⟩		⟨1346⟩		0	0	0	
		0		⟨89101⟩		⟨91017⟩	⟨10178⟩	⟨1789⟩	
4⟩	⟨4761⟩	⟨6431⟩	⟨7345⟩	⟨3456⟩	⟨4761⟩	⟨6431⟩	0	0	0

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```
    ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[149]:= treeAmp[10, 3][[50]]
{residue, termToDMatrix[%, 10]};
nice@%

Out[149]= R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]

Out[151]= {residue, ( ⟨3467⟩  0  ⟨4671⟩  ⟨6713⟩  0  ⟨7134⟩
                      ⟨78910⟩ 0  0  0  0  0
                      0  0  ⟨4567⟩ ⟨4761⟩ ⟨6431⟩  ⟨4761⟩ ⟨5673⟩ ⟨6431⟩  ⟨4761⟩ ⟨6431⟩ ⟨6734⟩  ⟨4761⟩ ⟨6431⟩ ⟨7345⟩  ⟨34
```

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```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] ⧴ Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] ⧴ Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ ⧴ MatrixForm[m])
```

```
rToResidue = {R[x__] ⧴ 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] ⧴ 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
     {R[x___, cap[y_, z_], w___] ⧴
       (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] ⧴ 0 /.
   {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] ⧴
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{residue, termToDMatrix[%, 10]};
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{residue, (⟨3467⟩   0  ⟨4671⟩                      ⟨6713⟩                      0                           ⟨7134⟩
           ⟨78910⟩  0  0                           0                           0                           0
           0        0  ⟨4567⟩ ⟨4761⟩ ⟨6431⟩  ⟨4761⟩ ⟨5673⟩ ⟨6431⟩  ⟨4761⟩ ⟨6431⟩ ⟨6734⟩  ⟨4761⟩ ⟨6431⟩ ⟨7345⟩  ⟨34
```

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```
1 / (⟨1234⟩ ⟨1456⟩ ⟨1678⟩ ⟨18910⟩ ⟨234(45)⋂((67)⋂((89)⋂(11101)81)61)⟩ ⟨34(45)⋂((67)⋂((89)⋂(11101)81)61)1⟩
  ⟨456(67)⋂((89)⋂(11101)81)⟩ ⟨4(45)⋂((67)⋂((89)⋂(11101)81)61)12⟩ ⟨56(67)⋂((89)⋂(11101)81)1⟩
  ⟨678(89)⋂(11101)⟩ ⟨6(67)⋂((89)⋂(11101)81)14⟩ ⟨78(89)⋂(11101)1⟩ ⟨891011⟩ ⟨8(89)⋂(11101)16⟩ ⟨910111⟩
  ⟨101118⟩ ⟨11189⟩ ⟨(45)⋂((67)⋂((89)⋂(11101)81)61)123⟩ ⟨(67)⋂((89)⋂(11101)81)145⟩ ⟨(89)⋂(11101)167⟩)
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{%, termToDMatrix[%, 10]};
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

	⟨3467⟩	0	⟨4671⟩	⟨6713⟩	0	⟨7134⟩
{residue,	⟨78910⟩	0	0	0	0	0

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[152]:= treeAmp[10, 3][[50]]
{% /. rToResidue, termToDMatrix[%, 10]};
nice@%

Out[152]= R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]

Out[154]= {1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨456(76)⋂(431)⟩ ⟨4671⟩ ⟨56(76)⋂(431) (34)⋂(761)⟩ ⟨6713⟩ ⟨6(76)⋂(431) (34)⋂(761) 4⟩
   ⟨7134⟩ ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩ ⟨(34)⋂(761) 456⟩ ⟨(76)⋂(431) (34)⋂(761) 45⟩),
   ( ⟨3467⟩  0  ⟨46
     ⟨78910⟩ 0  0
     0  0  ⟨4567⟩ ⟨47
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[158]:= treeAmp[10, 3][[50]]
{(% /. rToResidue //. capRules), termToDMatrix[%, 10]};
nice@%
```

```
Out[158]= R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
Out[160]= {1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨4671⟩ (⟨4317⟩ ⟨4566⟩ + ⟨4567⟩ ⟨6431⟩) ⟨6713⟩ ⟨7134⟩ (⟨3456⟩ ⟨4761⟩ + ⟨4456⟩ ⟨7613⟩)
    (⟨4317⟩ (⟨4761⟩ ⟨5663⟩ + ⟨5664⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩))
    (⟨4317⟩ (⟨4761⟩ ⟨6634⟩ + ⟨6644⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨6734⟩ + ⟨6744⟩ ⟨7613⟩))
    (⟨4317⟩ (⟨4761⟩ ⟨6345⟩ + ⟨6445⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨7345⟩ + ⟨7445⟩ ⟨7613⟩))
    ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),
      ⟨3467⟩   0   ⟨4671⟩                      ⟨6713⟩                      0
      ⟨78910⟩  0   0                           0                           0
      0        0   ⟨4567⟩ ⟨4761⟩ ⟨6431⟩   ⟨4761⟩ ⟨5673⟩ ⟨6431⟩   ⟨4761⟩ ⟨6431⟩ ⟨67:
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[143]:=
```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
     {R[x___, cap[y_, z_], w___] :>
       (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] :> 0 /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[158]:=
```
treeAmp[10, 3][[50]]
{(% /. rToResidue //. capRules), termToDMatrix[%, 10]};
nice@%
```

Out[158]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[160]=
```
{1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨4671⟩ (⟨4317⟩ ⟨4566⟩ + ⟨4567⟩ ⟨6431⟩) ⟨6713⟩ ⟨7134⟩ (⟨3456⟩ ⟨4761⟩ + ⟨4456⟩ ⟨7613⟩)
    (⟨4317⟩ (⟨4761⟩ ⟨5663⟩ + ⟨5664⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩))
    (⟨4317⟩ (⟨4761⟩ ⟨6634⟩ + ⟨6644⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨6734⟩ + ⟨6744⟩ ⟨7613⟩))
    (⟨4317⟩ (⟨4761⟩ ⟨6345⟩ + ⟨6445⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨7345⟩ + ⟨7445⟩ ⟨7613⟩))
                                              ⟨3467⟩   0        ⟨4671⟩                  ⟨6713⟩                  0
    ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),        ⟨78910⟩   0          0                       0                    0
                                                0      0   ⟨4567⟩ ⟨4761⟩ ⟨6431⟩   ⟨4761⟩ ⟨5673⟩ ⟨6431⟩   ⟨4761⟩ ⟨6431⟩ ⟨67…
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] :> 0 /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{(% /. rToResidue //. capRules), termToDMatrix[%, 10]};
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

{1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨4671⟩ (⟨4317⟩ ⟨4566⟩ + ⟨4567⟩ ⟨6431⟩) ⟨6713⟩ ⟨7134⟩ (⟨3456⟩ ⟨4761⟩ + ⟨4456⟩ ⟨7613⟩)
 (⟨4317⟩ (⟨4761⟩ ⟨5663⟩ + ⟨5664⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩))
 (⟨4317⟩ (⟨4761⟩ ⟨6634⟩ + ⟨6644⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨6734⟩ + ⟨6744⟩ ⟨7613⟩))
 (⟨4317⟩ (⟨4761⟩ ⟨6345⟩ + ⟨6445⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨7345⟩ + ⟨7445⟩ ⟨7613⟩))
 ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),

⟨3467⟩	0	⟨4671⟩		⟨6713⟩	0
⟨78910⟩	0	0		0	0
0	0	⟨4567⟩ ⟨4761⟩ ⟨6431⟩	⟨4761⟩ ⟨5673⟩ ⟨6431⟩	⟨4761⟩ ⟨6431⟩ ⟨67…	

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue[] = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0;
 termToDMatrix[term_, n_] :=
  Block[{rList = If[Head[term] === R, {term}, List @@ term]},
    (rList //. {R[x___, cap[y_, z_], w___] :>
          (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
       R[x___, y_, w___, y_, z___] :> 0 /.
      {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
  capRules =
   {ab[x___, cap[y_, z_], w___] :>
     (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{(% /. rToResidue //. capRules), termToDMatrix[%, 10]};
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨4671⟩ (⟨4317⟩ ⟨4566⟩ + ⟨4567⟩ ⟨6431⟩) ⟨6713⟩ ⟨7134⟩ (⟨3456⟩ ⟨4761⟩ + ⟨4456⟩ ⟨7613⟩)
   (⟨4317⟩ (⟨4761⟩ ⟨5663⟩ + ⟨5664⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩))
   (⟨4317⟩ (⟨4761⟩ ⟨6634⟩ + ⟨6644⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨6734⟩ + ⟨6744⟩ ⟨7613⟩))
   (⟨4317⟩ (⟨4761⟩ ⟨6345⟩ + ⟨6445⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨7345⟩ + ⟨7445⟩ ⟨7613⟩))
                                              ⟨3467⟩  0       ⟨4671⟩             ⟨6713⟩             0
   ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),         ⟨78910⟩ 0       0                  0                  0
                                              0       0 ⟨4567⟩ ⟨4761⟩ ⟨6431⟩ ⟨4761⟩ ⟨5673⟩ ⟨6431⟩ ⟨4761⟩ ⟨6431⟩ ⟨67
```

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue =
 exp = {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /. ab[x___, y_, w___, y_, z___] :> 0;
 termToDMatrix[term_, n_] :=
  Block[{rList = If[Head[term] === R, {term}, List @@ term]},
    (rList //. {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
       R[x___, y_, w___, y_, z___] :> 0 /.
      {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
  capRules =
   {ab[x___, cap[y_, z_], w___] :>
     (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{(% /. rToResidue //. capRules), termToDMatrix[%, 10]};
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨4671⟩ (⟨4317⟩ ⟨4566⟩ + ⟨4567⟩ ⟨6431⟩) ⟨6713⟩ ⟨7134⟩ (⟨3456⟩ ⟨4761⟩ + ⟨4456⟩ ⟨7613⟩)
   (⟨4317⟩ (⟨4761⟩ ⟨5663⟩ + ⟨5664⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩))
   (⟨4317⟩ (⟨4761⟩ ⟨6634⟩ + ⟨6644⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨6734⟩ + ⟨6744⟩ ⟨7613⟩))
   (⟨4317⟩ (⟨4761⟩ ⟨6345⟩ + ⟨6445⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨7345⟩ + ⟨7445⟩ ⟨7613⟩))
                                                 ⟨3467⟩   0        ⟨4671⟩                   ⟨6713⟩               0
   ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),           ⟨78910⟩  0         0                        0                    0
                                                   0      0  ⟨4567⟩ ⟨4761⟩ ⟨6431⟩  ⟨4761⟩ ⟨5673⟩ ⟨6431⟩  ⟨4761⟩ ⟨6431⟩ ⟨67
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] ⧴ Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] ⧴ Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ ⧴ MatrixForm[m])

rToResidue[exprn_] := {R[x__] ⧴ 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] ⧴ 0;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] ⧴
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] ⧴ 0 /.
    {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] ⧴
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};

treeAmp[10, 3][[50]]
{(% /. rToResidue //. capRules), termToDMatrix[%, 10]};
nice@%

R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]

{1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨4671⟩ (⟨4317⟩ ⟨4566⟩ + ⟨4567⟩ ⟨6431⟩) ⟨6713⟩ ⟨7134⟩ (⟨3456⟩ ⟨4761⟩ + ⟨4456⟩ ⟨7613⟩)
   (⟨4317⟩ (⟨4761⟩ ⟨5663⟩ + ⟨5664⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩))
   (⟨4317⟩ (⟨4761⟩ ⟨6634⟩ + ⟨6644⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨6734⟩ + ⟨6744⟩ ⟨7613⟩))
   (⟨4317⟩ (⟨4761⟩ ⟨6345⟩ + ⟨6445⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨7345⟩ + ⟨7445⟩ ⟨7613⟩))
                                              ⟨3467⟩  0       ⟨4671⟩              ⟨6713⟩              0
   ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),          ⟨78910⟩ 0         0                   0                 0
                                                0     0 ⟨4567⟩ ⟨4761⟩ ⟨6431⟩  ⟨4761⟩ ⟨5673⟩ ⟨6431⟩  ⟨4761⟩ ⟨6431⟩ ⟨67
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue[exprn_] :=
 (exprn /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
   ab[x___, y_, w___, y_, z___] :> 0);
 termToDMatrix[term_, n_] :=
  Block[{rList = If[Head[term] === R, {term}, List @@ term]},
    (rList //. {R[x___, cap[y_, z_], w___] :>
          (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
       R[x___, y_, w___, y_, z___] :> 0 /.
      {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
   capRules =
   {ab[x___, cap[y_, z_], w___] :>
     (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{(% /. rToResidue //. capRules), termToDMatrix[%, 10]};
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨4671⟩ (⟨4317⟩ ⟨4566⟩ + ⟨4567⟩ ⟨6431⟩) ⟨6713⟩ ⟨7134⟩ (⟨3456⟩ ⟨4761⟩ + ⟨4456⟩ ⟨7613⟩)
   (⟨4317⟩ (⟨4761⟩ ⟨5663⟩ + ⟨5664⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩))
   (⟨4317⟩ (⟨4761⟩ ⟨6634⟩ + ⟨6644⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨6734⟩ + ⟨6744⟩ ⟨7613⟩))
   (⟨4317⟩ (⟨4761⟩ ⟨6345⟩ + ⟨6445⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨7345⟩ + ⟨7445⟩ ⟨7613⟩))
                                   ⟨3467⟩  0       ⟨4671⟩        ⟨6713⟩        0
   ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩)  ⟨78910⟩ 0       0             0             0
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{(% /. rToResidue //. capRules), termToDMatrix[%, 10]};
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨4671⟩ (⟨4317⟩ ⟨4566⟩ + ⟨4567⟩ ⟨6431⟩) ⟨6713⟩ ⟨7134⟩ (⟨3456⟩ ⟨4761⟩ + ⟨4456⟩ ⟨7613⟩)
    (⟨4317⟩ (⟨4761⟩ ⟨5663⟩ + ⟨5664⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩))
    (⟨4317⟩ (⟨4761⟩ ⟨6634⟩ + ⟨6644⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨6734⟩ + ⟨6744⟩ ⟨7613⟩))
    (⟨4317⟩ (⟨4761⟩ ⟨6345⟩ + ⟨6445⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨7345⟩ + ⟨7445⟩ ⟨7613⟩))
                                   ⟨3467⟩  0      ⟨4671⟩      ⟨6713⟩      0
    ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩) . ⟨78910⟩  0   0   0   0
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] ⧴ Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] ⧴ Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ ⧴ MatrixForm[m])
```

```
Clear
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] ⧴ 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
     ab[x___, y_, w___, y_, z___] ⧴ 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] ⧴
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] ⧴ 0 /.
    {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] ⧴
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{(% /. rToResidue //. capRules), termToDMatrix[%, 10]};
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{1 / (⟨1346⟩ ⟨1789⟩ ⟨3467⟩ ⟨4671⟩ (⟨4317⟩ ⟨4566⟩ + ⟨4567⟩ ⟨6431⟩) ⟨6713⟩ ⟨7134⟩ (⟨3456⟩ ⟨4761⟩ + ⟨4456⟩ ⟨7613⟩)
   (⟨4317⟩ (⟨4761⟩ ⟨5663⟩ + ⟨5664⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩))
   (⟨4317⟩ (⟨4761⟩ ⟨6634⟩ + ⟨6644⟩ ⟨7613⟩) + ⟨6431⟩ (⟨4761⟩ ⟨6734⟩ + ⟨6744⟩ ⟨7613⟩))
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
     R[x___, y_, w___, y_, z___] → 0 /.
    {R[x__] → Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] →
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[165]:= treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]};
nice@%
```

```
Out[165]= R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[167]= $\Big\{\_ / \big(\langle 1346\rangle \langle 1789\rangle \langle 3456\rangle \langle 3467\rangle \langle 4567\rangle \langle 4671\rangle \langle 4761\rangle^2$

$\langle 6431\rangle^3 \langle 6713\rangle \langle 6734\rangle \langle 7134\rangle (\langle 4317\rangle \langle 4761\rangle \langle 6345\rangle + \langle 4761\rangle \langle 6431\rangle \langle 7345\rangle)$

$(\langle 4761\rangle \langle 5673\rangle + \langle 5674\rangle \langle 7613\rangle) \langle 78910\rangle \langle 89101\rangle \langle 91017\rangle \langle 10178\rangle\big), \begin{pmatrix} \langle 3467\rangle & 0 & & \langle 4671\rangle & & \langle \\ \langle 78910\rangle & 0 & & 0 & & \\ 0 & 0 & \langle 4567\rangle & \langle 4761\rangle & \langle 6431\rangle & \langle 4761\rangle \langle \end{pmatrix}$

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bcfw_part_deux.nb


```
m_?MatrixQ :> MatrixForm[m]}
```

In[161]:=
```
ClearAll[rToResidue]
```

In[162]:=
```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[165]:=
```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]};
nice@%
```

Out[165]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[167]=
```
{_/ (⟨1346⟩ ⟨1789⟩ ⟨3456⟩ ⟨3467⟩ ⟨4567⟩ ⟨4671⟩ ⟨4761⟩^2
   ⟨6431⟩^3 ⟨6713⟩ ⟨6734⟩ ⟨7134⟩ (⟨4317⟩ ⟨4761⟩ ⟨6345⟩ + ⟨4761⟩ ⟨6431⟩ ⟨7345⟩)
                                                                    ⟨3467⟩  0          ⟨4671⟩              ⟨
   (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩) ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),  ⟨78910⟩ 0          0
                                                                    0       0  ⟨4567⟩ ⟨4761⟩ ⟨6431⟩ ⟨4761⟩ ⟨
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[161]:=
```
ClearAll[rToResidue]
```

In[162]:=
```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[165]:=
```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]};
nice@%
```

Out[165]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[167]=

$$\Big\{\_ / \big(\langle 1346\rangle \langle 1789\rangle \langle 3456\rangle \langle 3467\rangle \langle 4567\rangle \langle 4671\rangle \langle 4761\rangle^2$$
$$\langle 6431\rangle^3 \langle 6713\rangle \langle 6734\rangle \langle 7134\rangle (\langle 4317\rangle \langle 4761\rangle \langle 6345\rangle + \langle 4761\rangle \langle 6431\rangle \langle 7345\rangle)$$
$$(\langle 4761\rangle \langle 5673\rangle + \langle 5674\rangle \langle 7613\rangle) \langle 78910\rangle \langle 89101\rangle \langle 91017\rangle \langle 10178\rangle\big), \begin{pmatrix} \langle 3467\rangle & 0 & \langle 4671\rangle & \langle \\ \langle 78910\rangle & 0 & 0 & \\ 0 & 0 \; \langle 4567\rangle & \langle 4761\rangle \langle 6431\rangle & \langle 4761\rangle \langle \end{pmatrix}$$

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[161]:= ClearAll[rToResidue]

In[162]:= rToResidue[exprn_] :=
  (exprn /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};

In[165]:= treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]};
nice@%

Out[165]= R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]

Out[167]= {1 / (⟨1346⟩ ⟨1789⟩ ⟨3456⟩ ⟨3467⟩ ⟨4567⟩ ⟨4671⟩ ⟨4761⟩^2
   ⟨6431⟩^3 ⟨6713⟩ ⟨6734⟩ ⟨7134⟩ (⟨4317⟩ ⟨4761⟩ ⟨6345⟩ + ⟨4761⟩ ⟨6431⟩ ⟨7345⟩)
   (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩) ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),  ( ⟨3467⟩   0          ⟨4671⟩             ⟨
                                                                     ⟨78910⟩  0          0
                                                                     0        0 ⟨4567⟩ ⟨4761⟩ ⟨6431⟩ ⟨4761⟩ ⟨
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[161]:=
```
ClearAll[rToResidue]
```

In[162]:=
```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[165]:=
```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]};
nice@%
```

Out[165]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[167]=

$\{1/(\langle 1346\rangle\langle 1789\rangle\langle 3456\rangle\langle 3467\rangle\langle 4567\rangle\langle 4671\rangle\langle 4761\rangle^2$
$\langle 6431\rangle^3\langle 6713\rangle\langle 6734\rangle\langle 7134\rangle(\langle 4317\rangle\langle 4761\rangle\langle 6345\rangle+\langle 4761\rangle\langle 6431\rangle\langle 7345\rangle)$
$(\langle 4761\rangle\langle 5673\rangle+\langle 5674\rangle\langle 7613\rangle)\langle 78910\rangle\langle 89101\rangle\langle 91017\rangle\langle 10178\rangle),\ \begin{pmatrix} \langle 3467\rangle & 0 & \langle 4671\rangle & \langle \\ \langle 78910\rangle & 0 & 0 & \\ 0 & 0 & \langle 4567\rangle\langle 4761\rangle\langle 6431\rangle & \langle 4761\rangle\langle \end{pmatrix}$

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[168]:= treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%

Out[168]= R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]

Out[169]= {1/(ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
     ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
     (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
     (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
     ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
  {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
   {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
   {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
    ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
    ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

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Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> 1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{_ / (ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
  ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
  (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[135]:=
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")⋂(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])

In[161]:=
ClearAll[rToResidue]

rToResidue[exprn_] := (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1]))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};

In[168]:=
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%

Out[168]=
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]

Out[169]=
{_/ (ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
    ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
    (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
    (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{_/ (ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
  ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
  (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
```

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Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1/ (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[174]:=
```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
```

Out[174]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[175]=
```
{_/ (ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
    ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
    (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
    (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[61]:=
```
ClearAll[rToResidue]
```

In[71]:=
```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1/ (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[74]:=
```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
```

Out[74]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[75]=
```
{_/ (ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
    ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
    (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
    (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0) _;
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{_/ (ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
   ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
   (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
   (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[180]:=
```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
```

Out[180]=
```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[181]=
```
{1/(ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
   ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
   (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
   (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

```
    {R[x__, cap[y_, z_], w__] :>
      (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
   R[x__, y_, w__, y_, z__] :> 0 /.
  {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
 {ab[x__, cap[y_, z_], w__] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{1/(ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
   ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
   (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
   (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

$$1/(\langle 1346\rangle\langle 1789\rangle\langle 3456\rangle\langle 3467\rangle\langle 4567\rangle\langle 4671\rangle\langle 4761\rangle^2$$
$$\langle 6431\rangle^3\langle 6713\rangle\langle 6734\rangle\langle 7134\rangle(\langle 4317\rangle\langle 4761\rangle\langle 6345\rangle+\langle 4761\rangle\langle 6431\rangle\langle 7345\rangle)$$

```
  {R[x__, cap[y_, z_], w__] ⧴
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
   R[x__, y_, w__, y_, z__] ⧴ 0 /.
  {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
 {ab[x__, cap[y_, z_], w__] ⧴
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{1/(ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
   ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
   (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
   (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

$$\{1/(\langle 1346\rangle\langle 1789\rangle\langle 3456\rangle\langle 3467\rangle\langle 4567\rangle\langle 4671\rangle\langle 4761\rangle^2$$
$$\langle 6431\rangle^3\langle 6713\rangle\langle 6734\rangle\langle 7134\rangle(\langle 4317\rangle\langle 4761\rangle\langle 6345\rangle+\langle 4761\rangle\langle 6431\rangle\langle 7345\rangle)$$

```
   {R[x__, cap[y_, z_], w__] :>
     (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
   R[x__, y_, w__, y_, z__] :> 0 /.
  {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x__, cap[y_, z_], w__] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{1/(ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
   ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
   (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
   (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

```
{1/(⟨1346⟩ ⟨1789⟩ ⟨3456⟩ ⟨3467⟩ ⟨4567⟩ ⟨4671⟩ ⟨4761⟩^2
```

```
    {R[x___, cap[y_, z_], w___] :>
      (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
   R[x___, y_, w___, y_, z___] :> 0 /.
  {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
%%
```

Out[180]=

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

Out[181]=

```
{1/(ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
    ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
    (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
    (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

Out[182]=

$1/(\langle 1346\rangle\langle 1789\rangle\langle 3456\rangle\langle 3467\rangle\langle 4567\rangle\langle 4671\rangle\langle 4761\rangle^2$

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[183]:=
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) &@%%

Out[183]=
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]

Out[184]=
{1/(ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
   ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
   (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
   (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}

Out[185]=
{1/(⟨1346⟩ ⟨1789⟩ ⟨3456⟩ ⟨3467⟩ ⟨4567⟩ ⟨4671⟩ ⟨4761⟩^2
   ⟨6431⟩^3 ⟨6713⟩ ⟨6734⟩ ⟨7134⟩ (⟨4317⟩ ⟨4761⟩ ⟨6345⟩ + ⟨4761⟩ ⟨6431⟩ ⟨7345⟩)
   (⟨4761⟩ ⟨5673⟩ + ⟨5674⟩ ⟨7613⟩) ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),
   ( ⟨3467⟩   0   ⟨4671⟩                   ⟨
     ⟨78910⟩  0   0
     0        0   ⟨4567⟩ ⟨4761⟩ ⟨6431⟩   ⟨4761⟩ ⟨

{Det[#2[[All, {1, 2, 3}]]]^4/(ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
   ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
   (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
treeAmp[10, 3][[50]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %%
```

```
R[1, 3, 4, 6, 7] R[1, 7, 8, 9, 10] R[cap[{3, 4}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {4, 3, 1}]]
```

```
{1/(ab[1, 3, 4, 6] ab[1, 7, 8, 9] ab[3, 4, 5, 6] ab[3, 4, 6, 7] ab[4, 5, 6, 7]
   ab[4, 6, 7, 1] ab[4, 7, 6, 1]^2 ab[6, 4, 3, 1]^3 ab[6, 7, 1, 3] ab[6, 7, 3, 4] ab[7, 1, 3, 4]
   (ab[4, 3, 1, 7] ab[4, 7, 6, 1] ab[6, 3, 4, 5] + ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5])
   (ab[4, 7, 6, 1] ab[5, 6, 7, 3] + ab[5, 6, 7, 4] ab[7, 6, 1, 3])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[3, 4, 6, 7], 0, ab[4, 6, 7, 1], ab[6, 7, 1, 3], 0, ab[7, 1, 3, 4], ab[1, 3, 4, 6], 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, 0, ab[4, 5, 6, 7] ab[4, 7, 6, 1] ab[6, 4, 3, 1], ab[4, 7, 6, 1] ab[5, 6, 7, 3] ab[6, 4, 3, 1],
   ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[6, 7, 3, 4], ab[4, 7, 6, 1] ab[6, 4, 3, 1] ab[7, 3, 4, 5],
   ab[3, 4, 5, 6] ab[4, 7, 6, 1] ab[6, 4, 3, 1], 0, 0, 0}}}
```

$$\Big\{1/\big(\langle 1346\rangle\langle 1789\rangle\langle 3456\rangle\langle 3467\rangle\langle 4567\rangle\langle 4671\rangle\langle 4761\rangle^2 \langle 6431\rangle^3\langle 6713\rangle\langle 6734\rangle\langle 7134\rangle(\langle 4317\rangle\langle 4761\rangle\langle 6345\rangle+\langle 4761\rangle\langle 6431\rangle\langle 7345\rangle) ((\langle 4761\rangle\langle 5673\rangle+\langle 5674\rangle\langle 7613\rangle)\langle 78910\rangle\langle 89101\rangle\langle 91017\rangle\langle 10178\rangle\big), \begin{pmatrix} \langle 3467\rangle & 0 & \langle 4671\rangle & \langle \\ \langle 78910\rangle & 0 & 0 & \\ 0 & 0 & \langle 4567\rangle\langle 4761\rangle\langle 6431\rangle & \langle 4761\rangle\langle \end{pmatrix}$$

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```
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[191]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %%
```

Out[191]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[192]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
   ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[193]=
```
{1/(⟨1235⟩ ⟨1789⟩ ⟨2345⟩ ⟨2356⟩ ⟨3456⟩ ⟨3561⟩ ⟨3651⟩^2
   ⟨5321⟩^3 ⟨5612⟩ ⟨5623⟩ ⟨6123⟩ (⟨3216⟩ ⟨3651⟩ ⟨5234⟩ + ⟨3651⟩ ⟨5321⟩ ⟨6234⟩)
   (⟨3651⟩ ⟨4562⟩ + ⟨4563⟩ ⟨6512⟩) ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),
                                                  ( ⟨2356⟩     ⟨3561⟩                       ⟨56
                                                    ⟨78910⟩    0                            0
                                                    0          ⟨3456⟩ ⟨3651⟩ ⟨5321⟩ ⟨3651⟩ ⟨45
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
{{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
{ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
{0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[193]=

$$\Big\{1 \Big/ \Big(\langle 1235\rangle \langle 1789\rangle \langle 2345\rangle \langle 2356\rangle \langle 3456\rangle \langle 3561\rangle \langle 3651\rangle^2$$
$$\langle 5321\rangle^3 \langle 5612\rangle \langle 5623\rangle \langle 6123\rangle (\langle 3216\rangle \langle 3651\rangle \langle 5234\rangle + \langle 3651\rangle \langle 5321\rangle \langle 6234\rangle)$$
$$(\langle 3651\rangle \langle 4562\rangle + \langle 4563\rangle \langle 6512\rangle) \langle 78910\rangle \langle 89101\rangle \langle 91017\rangle \langle 10178\rangle\Big),\ \left(\begin{array}{ccc} \langle 2356\rangle & \langle 3561\rangle & \langle 56 \\ \langle 78910\rangle & 0 & 0 \\ 0 & \langle 3456\rangle \langle 3651\rangle \langle 5321\rangle & \langle 3651\rangle \langle 45 \end{array}\right.$$

Out[194]=

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
  ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
 ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
 ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
{ab[x__, cap[y_, z_], w__] :>
  (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %%
```

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
   ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

$$\Big\{1\Big/\big(\langle 1235\rangle\langle 1789\rangle\langle 2345\rangle\langle 2356\rangle\langle 3456\rangle\langle 3561\rangle\langle 3651\rangle^2 \langle 5321\rangle^3\langle 5612\rangle\langle 5623\rangle\langle 6123\rangle(\langle 3216\rangle\langle 3651\rangle\langle 5234\rangle+\langle 3651\rangle\langle 5321\rangle\langle 6234\rangle) (\langle 3651\rangle\langle 4562\rangle+\langle 4563\rangle\langle 6512\rangle)\langle 78910\rangle\langle 89101\rangle\langle 91017\rangle\langle 10178\rangle\big),\ \begin{pmatrix} \langle 2356\rangle & \langle 3561\rangle & \langle 56 \\ \langle 78910\rangle & 0 & 0 \\ 0 & \langle 3456\rangle\langle 3651\rangle\langle 5321\rangle & \langle 3651\rangle\langle 45 \end{pmatrix}$$


```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
```

```
  {ab[x__, cap[y_, z_], w__] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[191]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %%
```

```
Out[191]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
Out[192]= {1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
     ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
     (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
     (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
     ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
  {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
   {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
   {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
    ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
    ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[193]= $\{1/(\langle 1235\rangle \langle 1789\rangle \langle 2345\rangle \langle 2356\rangle \langle 3456\rangle \langle 3561\rangle \langle 3651\rangle^2$
$\langle 5321\rangle^3 \langle 5612\rangle \langle 5623\rangle \langle 6123\rangle (\langle 3216\rangle \langle 3651\rangle \langle 5234\rangle + \langle 3651\rangle \langle 5321\rangle \langle 6234\rangle)$
$(\langle 3651\rangle \langle 4562\rangle + \langle 4563\rangle \langle 6512\rangle) \langle 78910\rangle \langle 89101\rangle \langle 91017\rangle \langle 10178\rangle),$
$\begin{pmatrix} \langle 2356\rangle & \langle 3561\rangle & \langle 56 \\ \langle 78910\rangle & 0 & 0 \\ 0 & \langle 3456\rangle \langle 3651\rangle \langle 5321\rangle & \langle 3651\rangle \langle 45 \end{pmatrix}$


```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
```

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Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
{ab[x___, cap[y_, z_], w___] :>
  (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[191]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %%
```

Out[191]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[192]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[193]=

$$\Big\{1\Big/\Big(\langle 1235\rangle\langle 1789\rangle\langle 2345\rangle\langle 2356\rangle\langle 3456\rangle\langle 3561\rangle\langle 3651\rangle^2 \langle 5321\rangle^3\langle 5612\rangle\langle 5623\rangle\langle 6123\rangle(\langle 3216\rangle\langle 3651\rangle\langle 5234\rangle+\langle 3651\rangle\langle 5321\rangle\langle 6234\rangle) (\langle 3651\rangle\langle 4562\rangle+\langle 4563\rangle\langle 6512\rangle)\langle 78910\rangle\langle 89101\rangle\langle 91017\rangle\langle 10178\rangle\Big), \begin{pmatrix} \langle 2356\rangle & \langle 3561\rangle & \langle 56 \\ \langle 78910\rangle & 0 & 0 \\ 0 & \langle 3456\rangle\langle 3651\rangle\langle 5321\rangle & \langle 3651\rangle\langle 45 \end{pmatrix}$$

In[194]:=
```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
```

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```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %%
```

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
   ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

$$\Big\{1\Big/\Big(\langle 1235\rangle\langle 1789\rangle\langle 2345\rangle\langle 2356\rangle\langle 3456\rangle\langle 3561\rangle\langle 3651\rangle^2 \langle 5321\rangle^3\langle 5612\rangle\langle 5623\rangle\langle 6123\rangle(\langle 3216\rangle\langle 3651\rangle\langle 5234\rangle+\langle 3651\rangle\langle 5321\rangle\langle 6234\rangle) (\langle 3651\rangle\langle 4562\rangle+\langle 4563\rangle\langle 6512\rangle)\langle 78910\rangle\langle 89101\rangle\langle 91017\rangle\langle 10178\rangle\Big), \begin{pmatrix} \langle 2356\rangle & \langle 3561\rangle & \langle 56 \\ \langle 78910\rangle & 0 & 0 \\ 0 & \langle 3456\rangle\langle 3651\rangle\langle 5321\rangle & \langle 3651\rangle\langle 45 \end{pmatrix}$$

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
  ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4/
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[191]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
nice@%
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %%
```

Out[191]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[192]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[193]=
```
{1/(⟨1235⟩ ⟨1789⟩ ⟨2345⟩ ⟨2356⟩ ⟨3456⟩ ⟨3561⟩ ⟨3651⟩^2
    ⟨5321⟩^3 ⟨5612⟩ ⟨5623⟩ ⟨6123⟩ (⟨3216⟩ ⟨3651⟩ ⟨5234⟩ + ⟨3651⟩ ⟨5321⟩ ⟨6234⟩)
                                                                            ⟨2356⟩        ⟨3561⟩                  ⟨56
    (⟨3651⟩ ⟨4562⟩ + ⟨4563⟩ ⟨6512⟩) ⟨78910⟩ ⟨89101⟩ ⟨91017⟩ ⟨10178⟩),     ⟨78910⟩         0                       0
                                                                              0    ⟨3456⟩ ⟨3651⟩ ⟨5321⟩  ⟨3651⟩ ⟨45
```

```
(ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
```

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Tree−Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[195]:=
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %%
```

```
Out[195]=
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
Out[196]=
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
Out[197]=
Det[R[1, 7, 8, 9, 10][[All, {1, 2, 3}]]]^4 R[1, 2, 3, 5, 6]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[200]=
```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
{FToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) &@@ %
```

Out[198]=

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[200]=

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
{{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
 {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
 {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
  ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
  ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
[200]= (-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
   ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
  ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
  (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
  (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
  ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
  ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
  (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
  (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
  ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList},
```

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Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList},
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    R[x___, y_, w___, y_, z___] :> 0 /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
   ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[200]=

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
   ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[198]:=

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) &@@%
```

Out[198]=

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[200]=

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidutreeAmp[n, k]
```

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```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[200]=

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4/
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  residueList = rToResidue /@ treeAmp[n, k];
```

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```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix /@ treeAmp[n, k];
```

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```
In[161]:= ClearAll[rToResidue]
```

```
In[177]:= rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[198]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
Out[199]= {1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
  {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
   {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
```

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```
{{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
 {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
 {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
  ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
  ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[200]=

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
  ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
  (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
  (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
  ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[n] /@ treeAmp[n, k]; Transpose[ ]
```

```
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[200]=
```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

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```
{0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
 ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
 ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}
```

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
   ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
  ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
  (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
  (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
  ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[ ]
```

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```
{0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
 ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
 ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}
```

Out[200]=

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
   ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
  ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
  (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
  (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
  ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[202]:=

```
componentAmp[6, 1]
```

Out[202]=

```
{{1 / (ab[1, 2, 3, 4] ab[2, 3, 4, 5] ab[3, 4, 5, 1] ab[4, 5, 1, 2] ab[5, 1, 2, 3]),
  {{ab[2, 3, 4, 5], ab[3, 4, 5, 1], ab[4, 5, 1, 2], ab[5, 1, 2, 3], ab[1, 2, 3, 4], 0}}},
 {1 / (ab[1, 2, 3, 5] ab[2, 3, 5, 6] ab[3, 5, 6, 1] ab[5, 6, 1, 2] ab[6, 1, 2, 3]),
  {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5]}}},
 {1 / (ab[1, 3, 4, 5] ab[3, 4, 5, 6] ab[4, 5, 6, 1] ab[5, 6, 1, 3] ab[6, 1, 3, 4]),
  {{ab[3, 4, 5, 6], 0, ab[4, 5, 6, 1], ab[5, 6, 1, 3], ab[6, 1, 3, 4], ab[1, 3, 4, 5]}}}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
  ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
  (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
  (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
  ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[201]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[202]:= componentAmp[6, 1]
```

Out[202]=

$$\left\{\left\{\frac{1}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[4,5,1,2]\,ab[5,1,2,3]},\right.\right.$$
$$\left.\{\{ab[2,3,4,5],ab[3,4,5,1],ab[4,5,1,2],ab[5,1,2,3],ab[1,2,3,4],0\}\}\right\},$$
$$\left\{\frac{1}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[5,6,1,2]\,ab[6,1,2,3]},\right.$$
$$\left.\{\{ab[2,3,5,6],ab[3,5,6,1],ab[5,6,1,2],0,ab[6,1,2,3],ab[1,2,3,5]\}\}\right\},$$
$$\left\{\frac{1}{ab[1,3,4,5]\,ab[3,4,5,6]\,ab[4,5,6,1]\,ab[5,6,1,3]\,ab[6,1,3,4]},\right.$$
$$\left.\left.\{\{ab[3,4,5,6],0,ab[4,5,6,1],ab[5,6,1,3],ab[6,1,3,4],ab[1,3,4,5]\}\}\right\}\right\}$$

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```
ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[203]:=

```
nice@componentAmp[6, 1]
```

Out[203]=

$$\left\{\left\{\frac{1}{\langle 1234\rangle\langle 2345\rangle\langle 3451\rangle\langle 4512\rangle\langle 5123\rangle},\ (\langle 2345\rangle\ \langle 3451\rangle\ \langle 4512\rangle\ \langle 5123\rangle\ \langle 1234\rangle\ 0)\right\},\right.$$

$$\left\{\frac{1}{\langle 1235\rangle\langle 2356\rangle\langle 3561\rangle\langle 5612\rangle\langle 6123\rangle},\ (\langle 2356\rangle\ \langle 3561\rangle\ \langle 5612\rangle\ 0\ \langle 6123\rangle\ \langle 1235\rangle)\right\},$$

$$\left.\left\{\frac{1}{\langle 1345\rangle\langle 3456\rangle\langle 4561\rangle\langle 5613\rangle\langle 6134\rangle},\ (\langle 3456\rangle\ 0\ \langle 4561\rangle\ \langle 5613\rangle\ \langle 6134\rangle\ \langle 1345\rangle)\right\}\right\}$$

```
ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[204]:=

```
nice@componentAmp[7, 1]
```

Out[204]=

$$\left\{\left\{\frac{1}{\langle 1234\rangle\langle 2345\rangle\langle 3451\rangle\langle 4512\rangle\langle 5123\rangle}, \left(\langle 2345\rangle\ \langle 3451\rangle\ \langle 4512\rangle\ \langle 5123\rangle\ \langle 1234\rangle\ 0\ 0\right)\right\},\right.$$
$$\left\{\frac{1}{\langle 1235\rangle\langle 2356\rangle\langle 3561\rangle\langle 5612\rangle\langle 6123\rangle}, \left(\langle 2356\rangle\ \langle 3561\rangle\ \langle 5612\rangle\ 0\ \langle 6123\rangle\ \langle 1235\rangle\ 0\right)\right\},$$
$$\left\{\frac{1}{\langle 1236\rangle\langle 2367\rangle\langle 3671\rangle\langle 6712\rangle\langle 7123\rangle}, \left(\langle 2367\rangle\ \langle 3671\rangle\ \langle 6712\rangle\ 0\ 0\ \langle 7123\rangle\ \langle 1236\rangle\right)\right\},$$
$$\left\{\frac{1}{\langle 1345\rangle\langle 3456\rangle\langle 4561\rangle\langle 5613\rangle\langle 6134\rangle}, \left(\langle 3456\rangle\ 0\ \langle 4561\rangle\ \langle 5613\rangle\ \langle 6134\rangle\ \langle 1345\rangle\ 0\right)\right\},$$
$$\left\{\frac{1}{\langle 1346\rangle\langle 3467\rangle\langle 4671\rangle\langle 6713\rangle\langle 7134\rangle}, \left(\langle 3467\rangle\ 0\ \langle 4671\rangle\ \langle 6713\rangle\ 0\ \langle 7134\rangle\ \langle 1346\rangle\right)\right\},$$
$$\left.\left\{\frac{1}{\langle 1456\rangle\langle 4567\rangle\langle 5671\rangle\langle 6714\rangle\langle 7145\rangle}, \left(\langle 4567\rangle\ 0\ 0\ \langle 5671\rangle\ \langle 6714\rangle\ \langle 7145\rangle\ \langle 1456\rangle\right)\right\}\right\}$$

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
nice@componentAmp[10, 1]
```

$\left\{\left\{\frac{1}{\langle 1234\rangle\langle 2345\rangle\langle 3451\rangle\langle 4512\rangle\langle 5123\rangle}, (\langle 2345\rangle\ \langle 3451\rangle\ \langle 4512\rangle\ \langle 5123\rangle\ \langle 1234\rangle\ 0\ 0\ 0\ 0\ 0)\right\},\right.$

$\left\{\frac{1}{\langle 1235\rangle\langle 2356\rangle\langle 3561\rangle\langle 5612\rangle\langle 6123\rangle}, (\langle 2356\rangle\ \langle 3561\rangle\ \langle 5612\rangle\ 0\ \langle 6123\rangle\ \langle 1235\rangle\ 0\ 0\ 0\ 0)\right\},$

$\left\{\frac{1}{\langle 1236\rangle\langle 2367\rangle\langle 3671\rangle\langle 6712\rangle\langle 7123\rangle}, (\langle 2367\rangle\ \langle 3671\rangle\ \langle 6712\rangle\ 0\ 0\ \langle 7123\rangle\ \langle 1236\rangle\ 0\ 0\ 0)\right\},$

$\left\{\frac{1}{\langle 1237\rangle\langle 2378\rangle\langle 3781\rangle\langle 7812\rangle\langle 8123\rangle}, (\langle 2378\rangle\ \langle 3781\rangle\ \langle 7812\rangle\ 0\ 0\ 0\ \langle 8123\rangle\ \langle 1237\rangle\ 0\ 0)\right\},$

$\left\{\frac{1}{\langle 1238\rangle\langle 2389\rangle\langle 3891\rangle\langle 8912\rangle\langle 9123\rangle}, (\langle 2389\rangle\ \langle 3891\rangle\ \langle 8912\rangle\ 0\ 0\ 0\ 0\ \langle 9123\rangle\ \langle 1238\rangle\ 0)\right\},$

$\left\{\frac{1}{\langle 1239\rangle\langle 23910\rangle\langle 39101\rangle\langle 91012\rangle\langle 10123\rangle}, (\langle 23910\rangle\ \langle 39101\rangle\ \langle 91012\rangle\ 0\ 0\ 0\ 0\ 0\ \langle 10123\rangle\ \langle 1239\rangle)\right\},$

$\left\{\frac{1}{\langle 1345\rangle\langle 3456\rangle\langle 4561\rangle\langle 5613\rangle\langle 6134\rangle}, (\langle 3456\rangle\ 0\ \langle 4561\rangle\ \langle 5613\rangle\ \langle 6134\rangle\ \langle 1345\rangle\ 0\ 0\ 0\ 0)\right\},$

$\left\{\frac{1}{\langle 1346\rangle\langle 3467\rangle\langle 4671\rangle\langle 6713\rangle\langle 7134\rangle}, (\langle 3467\rangle\ 0\ \langle 4671\rangle\ \langle 6713\rangle\ 0\ \langle 7134\rangle\ \langle 1346\rangle\ 0\ 0\ 0)\right\},$

$\left\{\frac{1}{\langle 1347\rangle\langle 3478\rangle\langle 4781\rangle\langle 7813\rangle\langle 8134\rangle}, (\langle 3478\rangle\ 0\ \langle 4781\rangle\ \langle 7813\rangle\ 0\ 0\ \langle 8134\rangle\ \langle 1347\rangle\ 0\ 0)\right\},$

$\left\{\frac{1}{\langle 1348\rangle\langle 3489\rangle\langle 4891\rangle\langle 8913\rangle\langle 9134\rangle}, (\langle 3489\rangle\ 0\ \langle 4891\rangle\ \langle 8913\rangle\ 0\ 0\ 0\ \langle 9134\rangle\ \langle 1348\rangle\ 0)\right\},$

$\left\{\frac{1}{\langle 1349\rangle\langle 34910\rangle\langle 49101\rangle\langle 91013\rangle\langle 10134\rangle}, (\langle 34910\rangle\ 0\ \langle 49101\rangle\ \langle 91013\rangle\ 0\ 0\ 0\ 0\ \langle 10134\rangle\ \langle 1349\rangle)\right\},$

$\left\{\frac{1}{\langle 1456\rangle\langle 4567\rangle\langle 5671\rangle\langle 6714\rangle\langle 7145\rangle}, (\langle 4567\rangle\ 0\ 0\ \langle 5671\rangle\ \langle 6714\rangle\ \langle 7145\rangle\ \langle 1456\rangle\ 0\ 0\ 0)\right\},$

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
componentAmp[10, 1]
```

In[205]:=
```
nice@componentAmp[10, 1]
```

Out[205]=

$$\left\{\left\{\frac{1}{\langle 1234\rangle\langle 2345\rangle\langle 3451\rangle\langle 4512\rangle\langle 5123\rangle},\ (\langle 2345\rangle\ \langle 3451\rangle\ \langle 4512\rangle\ \langle 5123\rangle\ \langle 1234\rangle\ 0\ 0\ 0\ 0\ 0)\right\},\right.$$

$$\left\{\frac{1}{\langle 1235\rangle\langle 2356\rangle\langle 3561\rangle\langle 5612\rangle\langle 6123\rangle},\ (\langle 2356\rangle\ \langle 3561\rangle\ \langle 5612\rangle\ 0\ \langle 6123\rangle\ \langle 1235\rangle\ 0\ 0\ 0\ 0)\right\},$$

$$\left\{\frac{1}{\langle 1236\rangle\langle 2367\rangle\langle 3671\rangle\langle 6712\rangle\langle 7123\rangle},\ (\langle 2367\rangle\ \langle 3671\rangle\ \langle 6712\rangle\ 0\ 0\ \langle 7123\rangle\ \langle 1236\rangle\ 0\ 0\ 0)\right\},$$

$$\left\{\frac{1}{\langle 1237\rangle\langle 2378\rangle\langle 3781\rangle\langle 7812\rangle\langle 8123\rangle},\ (\langle 2378\rangle\ \langle 3781\rangle\ \langle 7812\rangle\ 0\ 0\ 0\ \langle 8123\rangle\ \langle 1237\rangle\ 0\ 0)\right\},$$

$$\left\{\frac{1}{\langle 1238\rangle\langle 2389\rangle\langle 3891\rangle\langle 8912\rangle\langle 9123\rangle},\ (\langle 2389\rangle\ \langle 3891\rangle\ \langle 8912\rangle\ 0\ 0\ 0\ 0\ \langle 9123\rangle\ \langle 1238\rangle\ 0)\right\},$$

$$\left\{\frac{1}{\langle 1239\rangle\langle 23910\rangle\langle 39101\rangle\langle 91012\rangle\langle 10123\rangle},\ (\langle 23910\rangle\ \langle 39101\rangle\ \langle 91012\rangle\ 0\ 0\ 0\ 0\ 0\ \langle 10123\rangle\ \langle 1239\rangle)\right\},$$

$$\left\{\frac{1}{\langle 1345\rangle\langle 3456\rangle\langle 4561\rangle\langle 5613\rangle\langle 6134\rangle},\ (\langle 3456\rangle\ 0\ \langle 4561\rangle\ \langle 5613\rangle\ \langle 6134\rangle\ \langle 1345\rangle\ 0\ 0\ 0\ 0)\right\},$$

$$\left\{\frac{1}{\langle 1346\rangle\langle 3467\rangle\langle 4671\rangle\langle 6713\rangle\langle 7134\rangle},\ (\langle 3467\rangle\ 0\ \langle 4671\rangle\ \langle 6713\rangle\ 0\ \langle 7134\rangle\ \langle 1346\rangle\ 0\ 0\ 0)\right\},$$

$$\left\{\frac{1}{\langle 1347\rangle\langle 3478\rangle\langle 4781\rangle\langle 7813\rangle\langle 8134\rangle},\ (\langle 3478\rangle\ 0\ \langle 4781\rangle\ \langle 7813\rangle\ 0\ 0\ \langle 8134\rangle\ \langle 1347\rangle\ 0\ 0)\right\},$$

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
componentAmp[10, 1]
(#1 * Power[Det[#2[[All, {1}]]], 4]) & @@@ %
```

$$\left\{\left\{\frac{1}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[4,5,1,2]\,ab[5,1,2,3]},\right.\right.$$
{{ab[2, 3, 4, 5], ab[3, 4, 5, 1], ab[4, 5, 1, 2], ab[5, 1, 2, 3], ab[1, 2, 3, 4], 0, 0, 0, 0, 0}}},

$$\left\{\frac{1}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[5,6,1,2]\,ab[6,1,2,3]},\right.$$
{{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0}}},

$$\left\{\frac{1}{ab[1,2,3,6]\,ab[2,3,6,7]\,ab[3,6,7,1]\,ab[6,7,1,2]\,ab[7,1,2,3]},\right.$$
{{ab[2, 3, 6, 7], ab[3, 6, 7, 1], ab[6, 7, 1, 2], 0, 0, ab[7, 1, 2, 3], ab[1, 2, 3, 6], 0, 0, 0}}},

$$\left\{\frac{1}{ab[1,2,3,7]\,ab[2,3,7,8]\,ab[3,7,8,1]\,ab[7,8,1,2]\,ab[8,1,2,3]},\right.$$
{{ab[2, 3, 7, 8], ab[3, 7, 8, 1], ab[7, 8, 1, 2], 0, 0, 0, ab[8, 1, 2, 3], ab[1, 2, 3, 7], 0, 0}}},

$$\left\{\frac{1}{ab[1,2,3,8]\,ab[2,3,8,9]\,ab[3,8,9,1]\,ab[8,9,1,2]\,ab[9,1,2,3]},\right.$$
{{ab[2, 3, 8, 9], ab[3, 8, 9, 1], ab[8, 9, 1, 2], 0, 0, 0, 0, ab[9, 1, 2, 3], ab[1, 2, 3, 8], 0}}},

$$\left\{\frac{1}{ab[1,2,3,9]\,ab[2,3,9,10]\,ab[3,9,10,1]\,ab[9,10,1,2]\,ab[10,1,2,3]},\right.$$

$$\text{ab}[1,3,4,7]\,\text{ab}[4,7,8,1]\,\text{ab}[7,8,1,3]\,\text{ab}[8,1,3,4]$$

$$+\frac{\text{ab}[4,5,7,8]^3}{\text{ab}[1,4,5,7]\,\text{ab}[5,7,8,1]\,\text{ab}[7,8,1,4]\,\text{ab}[8,1,4,5]}+$$

$$\frac{\text{ab}[5,6,7,8]^3}{\text{ab}[1,5,6,7]\,\text{ab}[6,7,8,1]\,\text{ab}[7,8,1,5]\,\text{ab}[8,1,5,6]}+$$

$$\frac{\text{ab}[2,3,8,9]^3}{\text{ab}[1,2,3,8]\,\text{ab}[3,8,9,1]\,\text{ab}[8,9,1,2]\,\text{ab}[9,1,2,3]}+$$

$$\frac{\text{ab}[3,4,8,9]^3}{\text{ab}[1,3,4,8]\,\text{ab}[4,8,9,1]\,\text{ab}[8,9,1,3]\,\text{ab}[9,1,3,4]}+$$

$$\frac{\text{ab}[4,5,8,9]^3}{\text{ab}[1,4,5,8]\,\text{ab}[5,8,9,1]\,\text{ab}[8,9,1,4]\,\text{ab}[9,1,4,5]}+$$

$$\frac{\text{ab}[5,6,8,9]^3}{\text{ab}[1,5,6,8]\,\text{ab}[6,8,9,1]\,\text{ab}[8,9,1,5]\,\text{ab}[9,1,5,6]}+$$

$$\frac{\text{ab}[6,7,8,9]^3}{\text{ab}[1,6,7,8]\,\text{ab}[7,8,9,1]\,\text{ab}[8,9,1,6]\,\text{ab}[9,1,6,7]}+$$

$$\frac{\text{ab}[2,3,9,10]^3}{\text{ab}[1,2,3,9]\,\text{ab}[3,9,10,1]\,\text{ab}[9,10,1,2]\,\text{ab}[10,1,2,3]}+$$

$$\frac{\text{ab}[3,4,9,10]^3}{\text{ab}[1,3,4,9]\,\text{ab}[4,9,10,1]\,\text{ab}[9,10,1,3]\,\text{ab}[10,1,3,4]}+$$

$$\frac{\text{ab}[4,5,9,10]^3}{\text{ab}[1,4,5,9]\,\text{ab}[5,9,10,1]\,\text{ab}[9,10,1,4]\,\text{ab}[10,1,4,5]}+$$

$$\frac{\text{ab}[5,6,9,10]^3}{\text{ab}[1,5,6,9]\,\text{ab}[6,9,10,1]\,\text{ab}[9,10,1,5]\,\text{ab}[10,1,5,6]}+$$

$$\text{ab}[6,7,9,10]^3 \; +$$

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Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {1}]]], 4]) & @@@ %]
```

$$\frac{\text{ab}[2,3,4,5]^3}{\text{ab}[1,2,3,4]\,\text{ab}[3,4,5,1]\,\text{ab}[4,5,1,2]\,\text{ab}[5,1,2,3]}+$$

$$\frac{\text{ab}[2,3,5,6]^3}{\text{ab}[1,2,3,5]\,\text{ab}[3,5,6,1]\,\text{ab}[5,6,1,2]\,\text{ab}[6,1,2,3]}+$$

$$\frac{\text{ab}[3,4,5,6]^3}{\text{ab}[1,3,4,5]\,\text{ab}[4,5,6,1]\,\text{ab}[5,6,1,3]\,\text{ab}[6,1,3,4]}+$$

$$\frac{\text{ab}[2,3,6,7]^3}{\text{ab}[1,2,3,6]\,\text{ab}[3,6,7,1]\,\text{ab}[6,7,1,2]\,\text{ab}[7,1,2,3]}+$$

$$\frac{\text{ab}[3,4,6,7]^3}{\text{ab}[1,3,4,6]\,\text{ab}[4,6,7,1]\,\text{ab}[6,7,1,3]\,\text{ab}[7,1,3,4]}+$$

$$\frac{\text{ab}[4,5,6,7]^3}{\text{ab}[1,4,5,6]\,\text{ab}[5,6,7,1]\,\text{ab}[6,7,1,4]\,\text{ab}[7,1,4,5]}+$$

$$\frac{\text{ab}[2,3,7,8]^3}{\text{ab}[1,2,3,7]\,\text{ab}[3,7,8,1]\,\text{ab}[7,8,1,2]\,\text{ab}[8,1,2,3]}+$$

$$\frac{\text{ab}[3,4,7,8]^3}{\text{ab}[1,3,4,7]\,\text{ab}[4,7,8,1]\,\text{ab}[7,8,1,3]\,\text{ab}[8,1,3,4]}+$$

$$\frac{\text{ab}[4,5,7,8]^3}{\text{ab}[1,4,5,7]\,\text{ab}[5,7,8,1]\,\text{ab}[7,8,1,4]\,\text{ab}[8,1,4,5]}+$$

$$\text{ab}[5,6,7,8]^3$$

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

$$\frac{\text{ab}[4,5,8,9]^3}{\text{ab}[1,4,5,8]\,\text{ab}[5,8,9,1]\,\text{ab}[8,9,1,4]\,\text{ab}[9,1,4,5]}+$$
$$\frac{\text{ab}[5,6,8,9]^3}{\text{ab}[1,5,6,8]\,\text{ab}[6,8,9,1]\,\text{ab}[8,9,1,5]\,\text{ab}[9,1,5,6]}+$$
$$\frac{\text{ab}[6,7,8,9]^3}{\text{ab}[1,6,7,8]\,\text{ab}[7,8,9,1]\,\text{ab}[8,9,1,6]\,\text{ab}[9,1,6,7]}+$$
$$\frac{\text{ab}[2,3,9,10]^3}{\text{ab}[1,2,3,9]\,\text{ab}[3,9,10,1]\,\text{ab}[9,10,1,2]\,\text{ab}[10,1,2,3]}+$$
$$\frac{\text{ab}[3,4,9,10]^3}{\text{ab}[1,3,4,9]\,\text{ab}[4,9,10,1]\,\text{ab}[9,10,1,3]\,\text{ab}[10,1,3,4]}+$$
$$\frac{\text{ab}[4,5,9,10]^3}{\text{ab}[1,4,5,9]\,\text{ab}[5,9,10,1]\,\text{ab}[9,10,1,4]\,\text{ab}[10,1,4,5]}+$$
$$\frac{\text{ab}[5,6,9,10]^3}{\text{ab}[1,5,6,9]\,\text{ab}[6,9,10,1]\,\text{ab}[9,10,1,5]\,\text{ab}[10,1,5,6]}+$$
$$\frac{\text{ab}[6,7,9,10]^3}{\text{ab}[1,6,7,9]\,\text{ab}[7,9,10,1]\,\text{ab}[9,10,1,6]\,\text{ab}[10,1,6,7]}+$$
$$\frac{\text{ab}[7,8,9,10]^3}{\text{ab}[1,7,8,9]\,\text{ab}[8,9,10,1]\,\text{ab}[9,10,1,7]\,\text{ab}[10,1,7,8]}$$

In[205]:=
```
nice@componentAmp[10, 1]
```

Out[205]=

$$\left\{\left\{\frac{1}{\langle 1234\rangle\langle 2345\rangle\langle 3451\rangle\langle 4512\rangle\langle 5123\rangle},\ (\langle 2345\rangle\ \langle 3451\rangle\ \langle 4512\rangle\ \langle 5123\rangle\ \langle 1234\rangle\ 0\ 0\ 0\ 0\ 0)\right\},\right.$$
$$\left\{\frac{1}{\langle 1235\rangle\langle 2356\rangle\langle 3561\rangle\langle 5612\rangle\langle 6123\rangle},\ (\langle 2356\rangle\ \langle 3561\rangle\ \langle 5612\rangle\ 0\ \langle 6123\rangle\ \langle 1235\rangle\ 0\ 0\ 0\ 0)\right\},$$

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
   ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
  ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
  (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
  (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
  ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {1}]]], 4]) & @@@ %]
```

$$\frac{ab[2,3,4,5]^3}{ab[1,2,3,4]\ ab[3,4,5,1]\ ab[4,5,1,2]\ ab[5,1,2,3]}+$$
$$\frac{ab[2,3,5,6]^3}{ab[1,2,3,5]\ ab[3,5,6,1]\ ab[5,6,1,2]\ ab[6,1,2,3]}+$$
$$\frac{ab[3,4,5,6]^3}{ab[1,3,4,5]\ ab[4,5,6,1]\ ab[5,6,1,3]\ ab[6,1,3,4]}+$$
$$\frac{ab[2,3,6,7]^3}{ab[1,2,3,6]\ ab[3,6,7,1]\ ab[6,7,1,2]\ ab[7,1,2,3]}+$$
$$\frac{ab[3,4,6,7]^3}{ab[1,3,4,6]\ ab[4,6,7,1]\ ab[6,7,1,3]\ ab[7,1,3,4]}+$$
$$ab[4,5,6,7]^3$$

$$\frac{\ldots}{\mathrm{ab}[1,2,3,4]\,\mathrm{ab}[2,3,4,5]\,\mathrm{ab}[3,4,5,1]\,\mathrm{ab}[5,1,2,3]}+$$

$$\frac{\mathrm{ab}[5,6,1,2]^3}{\mathrm{ab}[1,2,3,5]\,\mathrm{ab}[2,3,5,6]\,\mathrm{ab}[3,5,6,1]\,\mathrm{ab}[6,1,2,3]}+$$

$$\frac{\mathrm{ab}[4,5,6,1]^3}{\mathrm{ab}[1,3,4,5]\,\mathrm{ab}[3,4,5,6]\,\mathrm{ab}[5,6,1,3]\,\mathrm{ab}[6,1,3,4]}+$$

$$\frac{\mathrm{ab}[6,7,1,2]^3}{\mathrm{ab}[1,2,3,6]\,\mathrm{ab}[2,3,6,7]\,\mathrm{ab}[3,6,7,1]\,\mathrm{ab}[7,1,2,3]}+$$

$$\frac{\mathrm{ab}[4,6,7,1]^3}{\mathrm{ab}[1,3,4,6]\,\mathrm{ab}[3,4,6,7]\,\mathrm{ab}[6,7,1,3]\,\mathrm{ab}[7,1,3,4]}+$$

$$\frac{\mathrm{ab}[7,8,1,2]^3}{\mathrm{ab}[1,2,3,7]\,\mathrm{ab}[2,3,7,8]\,\mathrm{ab}[3,7,8,1]\,\mathrm{ab}[8,1,2,3]}+$$

$$\frac{\mathrm{ab}[4,7,8,1]^3}{\mathrm{ab}[1,3,4,7]\,\mathrm{ab}[3,4,7,8]\,\mathrm{ab}[7,8,1,3]\,\mathrm{ab}[8,1,3,4]}+$$

$$\frac{\mathrm{ab}[8,9,1,2]^3}{\mathrm{ab}[1,2,3,8]\,\mathrm{ab}[2,3,8,9]\,\mathrm{ab}[3,8,9,1]\,\mathrm{ab}[9,1,2,3]}+$$

$$\frac{\mathrm{ab}[4,8,9,1]^3}{\mathrm{ab}[1,3,4,8]\,\mathrm{ab}[3,4,8,9]\,\mathrm{ab}[8,9,1,3]\,\mathrm{ab}[9,1,3,4]}+$$

$$\frac{\mathrm{ab}[9,10,1,2]^3}{\mathrm{ab}[1,2,3,9]\,\mathrm{ab}[2,3,9,10]\,\mathrm{ab}[3,9,10,1]\,\mathrm{ab}[10,1,2,3]}+$$

$$\frac{\mathrm{ab}[4,9,10,1]^3}{\mathrm{ab}[1,3,4,9]\,\mathrm{ab}[3,4,9,10]\,\mathrm{ab}[9,10,1,3]\,\mathrm{ab}[10,1,3,4]}$$

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[200]=

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
  ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
 ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
 ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[212]:=

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[5,1,2,3]} + \frac{ab[5,6,1,2]^3}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[6,1,2,3]} + \frac{ab[4,5,6,1]^3}{ab[1,3,4,5]\,ab[3,4,5,6]\,ab[5,6,1,3]\,ab[6,1,3,4]} + \frac{ab[6,7,1,2]^3}{ab[1,2,3,6]\,ab[2,3,6,7]\,ab[3,6,7,1]\,ab[7,1,2,3]} + \frac{ab[4,6,7,1]^3}{\ldots} + \ldots$$

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[5,1,2,3]} + \frac{ab[5,6,1,2]^3}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[6,1,2,3]} + \frac{ab[4,5,6,1]^3}{ab[1,3,4,5]\,ab[3,4,5,6]\,ab[5,6,1,3]\,ab[6,1,3,4]} + \frac{ab[6,7,1,2]^3}{ab[1,2,3,6]\,ab[2,3,6,7]\,ab[3,6,7,1]\,ab[7,1,2,3]} + \frac{ab[4,6,7,1]^3}{ab[1,3,4,6]\,ab[3,4,6,7]\,ab[6,7,1,3]\,ab[7,1,3,4]} + \frac{ab[7,8,1,2]^3}{ab[1,2,3,7]\,ab[2,3,7,8]\,ab[3,7,8,1]\,ab[8,1,2,3]} + \frac{ab[4,7,8,1]^3}{ab[1,3,4,7]\,ab[3,4,7,8]\,ab[7,8,1,3]\,ab[8,1,3,4]} + \frac{ab[8,9,1,2]^3}{\cdots} +$$

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```
ab[3, 6, 5, 1]^4 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
splitA
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[5,1,2,3]}+$$
$$\frac{ab[5,6,1,2]^3}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[6,1,2,3]}+$$
$$\frac{ab[4,5,6,1]^3}{ab[1,3,4,5]\,ab[3,4,5,6]\,ab[5,6,1,3]\,ab[6,1,3,4]}+$$
$$\frac{ab[6,7,1,2]^3}{ab[1,2,3,6]\,ab[2,3,6,7]\,ab[3,6,7,1]\,ab[7,1,2,3]}+$$
$$\frac{ab[4,6,7,1]^3}{ab[1,3,4,6]\,ab[3,4,6,7]\,ab[6,7,1,3]\,ab[7,1,3,4]}+$$
$$\frac{ab[7,8,1,2]^3}{ab[1,2,3,7]\,ab[2,3,7,8]\,ab[3,7,8,1]\,ab[8,1,2,3]}+$$
$$\frac{ab[4,7,8,1]^3}{ab[1,3,4,7]\,ab[3,4,7,8]\,ab[7,8,1,3]\,ab[8,1,3,4]}+$$

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 6, 5, 1]^4 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=
```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
splitHelicityAmplitude[n_, k
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[5,1,2,3]} + \frac{ab[5,6,1,2]^3}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[6,1,2,3]} + \frac{ab[4,5,6,1]^3}{ab[1,3,4,5]\,ab[3,4,5,6]\,ab[5,6,1,3]\,ab[6,1,3,4]} + \frac{ab[6,7,1,2]^3}{ab[1,2,3,6]\,ab[2,3,6,7]\,ab[3,6,7,1]\,ab[7,1,2,3]} + \frac{ab[4,6,7,1]^3}{ab[1,3,4,6]\,ab[3,4,6,7]\,ab[6,7,1,3]\,ab[7,1,3,4]} + \frac{ab[7,8,1,2]^3}{ab[1,2,3,7]\,ab[2,3,7,8]\,ab[3,7,8,1]\,ab[8,1,2,3]} + \frac{ab[4,7,8,1]^3}{ab[1,3,4,7]\,ab[3,4,7,8]\,ab[7,8,1,3]\,ab[8,1,3,4]} +$$

```
ab[3, 6, 5, 1]^4 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[201]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[5,1,2,3]} +$$

$$\frac{ab[5,6,1,2]^3}{ab[1,2,3,5]\,ab[2,3,5,6]\,ab[3,5,6,1]\,ab[6,1,2,3]} +$$

$$\frac{ab[4,5,6,1]^3}{ab[1,3,4,5]\,ab[3,4,5,6]\,ab[5,6,1,3]\,ab[6,1,3,4]} +$$

$$\frac{ab[6,7,1,2]^3}{ab[1,2,3,6]\,ab[2,3,6,7]\,ab[3,6,7,1]\,ab[7,1,2,3]} +$$

$$\frac{ab[4,6,7,1]^3}{ab[1,3,4,6]\,ab[3,4,6,7]\,ab[6,7,1,3]\,ab[7,1,3,4]} +$$

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 6, 5, 1]^4 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{\text{ab}[4,5,1,2]^3}{\text{ab}[1,2,3,4]\ \text{ab}[2,3,4,5]\ \text{ab}[3,4,5,1]\ \text{ab}[5,1,2,3]} +$$

$$\frac{\text{ab}[5,6,1,2]^3}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[6,1,2,3]} +$$

$$\frac{\text{ab}[4,5,6,1]^3}{\text{ab}[1,3,4,5]\ \text{ab}[3,4,5,6]\ \text{ab}[5,6,1,3]\ \text{ab}[6,1,3,4]} +$$

$$\frac{\text{ab}[6,7,1,2]^3}{\text{ab}[1,2,3,6]\ \text{ab}[2,3,6,7]\ \text{ab}[3,6,7,1]\ \text{ab}[7,1,2,3]} +$$

```
ab[3, 6, 5, 1]^4 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[201]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]

In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];

In[215]:= Zs = RandomInteger[{-500, 500}, {10, 4}]

Out[215]= {{-84, 135, 491, -256}, {366, 215, -145, -137}, {-207, -176, -35, 121},
 {-212, 141, -463, 190}, {-273, -98, 3, 111}, {189, -329, -50, 82},
 {71, 444, -400, -434}, {84, -206, 164, 22}, {-199, 328, 117, 175}, {216, -91, 198, -438}}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\ ab[2,3,4,5]\ ab[3,4,5,1]\ ab[5,1,2,3]} + \frac{ab[5,6,1,2]^3}{ab[1,2,3,5]\ ab[2,3,5,6]\ ab[3,5,6,1]\ ab[6,1,2,3]} +$$

$$ab[4,5,6,1]^3$$

```
ab[3, 6, 5, 1]^4 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[201]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]

Out[215]= {{-84, 135, 491, -256}, {366, 215, -145, -137}, {-207, -176, -35, 221},
 {-212, 141, -463, 190}, {-273, -98, 3, 111}, {189, -329, -50, 82},
 {71, 444, -400, -434}, {84, -206, 164, 22}, {-199, 328, 117, 175}, {216, -91, 198, -438}}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{\text{ab}[4,5,1,2]^3}{\text{ab}[1,2,3,4]\,\text{ab}[2,3,4,5]\,\text{ab}[3,4,5,1]\,\text{ab}[5,1,2,3]} +$$

$$\frac{\text{ab}[5,6,1,2]^3}{\text{ab}[1,2,3,5]\,\text{ab}[2,3,5,6]\,\text{ab}[3,5,6,1]\,\text{ab}[6,1,2,3]} +$$

```
ab[3, 6, 5, 1]^4 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[201]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
splitHelicityAmplitude[10, 1] /. ab[x__] :>
```

```
Out[215]= {{-84, 135, 491, -256}, {366, 215, -145, -137}, {-207, -176, -35, 221},
 {-212, 141, -463, 190}, {-273, -98, 3, 111}, {189, -329, -50, 82},
 {71, 444, -400, -434}, {84, -206, 164, 22}, {-199, 328, 117, 175}, {216, -91, 198, -438}}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\, ab[2,3,4,5]\, ab[3,4,5,1]\, ab[5,1,2,3]} +$$

$$\frac{ab[5,6,1,2]^3}{ab[1,2,3,5]\, ab[2,3,5,6]\, ab[3,5,6,1]\, ab[6,1,2,3]} +$$

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 6, 5, 1]^4 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[201]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
In[216]:= Zs = RandomInteger[{-500, 500}, {10, 4}]
splitHelicityAmplitude[10, 1] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
Out[216]= {{288, -366, 260, 89}, {95, -498, -218, 179}, {-369, 161, -479, 134},
 {-198, 259, -108, 362}, {-135, -183, 243, -178}, {-385, -348, 43, 218},
 {386, -132, 190, -3}, {127, -187, -488, -307}, {-76, -389, 141, 52}, {158, 401, 257, -46}}
```

```
Out[217]= -37 579 989 774 635 412 671 939 543 074 861 118 170 712 200 084 826 715 353 910 043 742 136 294 874 845 961 640 812 899 134 \
   127 169 391 796 585 719 535 973 818 338 647 /
  6 977 493 765 614 536 433 868 123 669 068 067 031 746 302 267 687 321 755 202 212 754 430 963 711 296 327 304 853 149 129 \
   476 684 633 736 182 080 887 681 623 058 980 833 835 160
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

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```
ab[3, 6, 5, 1]^4 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[201]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
In[218]:= Zs = RandomInteger[{-500, 500}, {10, 4}]
splitHelicityAmplitude[10, 2] /. ab[x__] :> Det[Zs[[{x}]]]

Out[218]= {{-189, -200, -392, -15}, {278, -72, 331, 461}, {56, -89, 148, -153},
 {-57, 399, -340, 301}, {-286, -140, 258, 445}, {470, 156, -66, -169},
 {360, -141, -142, 431}, {-81, 385, -475, -389}, {-55, -316, 479, 448}, {31, 415, -42, 436}}

Out[219]= -296 470 780 127 423 233 233 777 804 140 673 102 132 460 913 753 029 881 780 801 135 496 508 899 890 424 684 346 234 185 560 \
   007 366 825 993 431 457 081 270 198 106 222 735 882 744 316 802 651 237 /
  1 670 853 741 254 481 575 953 947 267 335 130 783 697 174 807 837 681 003 356 240 599 504 167 541 489 308 614 433 942 315 \
   331 768 661 892 582 780 748 999 766 818 752 951 825 467 751 614 057 206 803 782 535 437 674 097 408
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

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```
ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

In[220]:=

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
splitHelicityAmplitude[10, 3] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[220]=

```
{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451},
 {448, -62, -249, -63}, {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184},
 {417, 128, -22, 64}, {227, -281, 259, -448}, {-450, 333, -324, -97}}
```

Out[221]=

```
847 940 525 964 420 245 040 834 410 595 447 801 796 629 647 062 021 857 044 263 754 709 197 875 187 079 565 660 067 266 738
   886 150 617 039 884 861 842 535 703 081 993 444 017 553 553 367 770 602 377 430 085 071 810 608 378 893 203 808 485 791 393
   750 690 262 200 238 919 424 226 793 738 172 855 834 282 670 901 427 651 858 627 850 076 607 236 062 620 307 277 076 409 045
   145 126 952 367 867 697 852 131 967 951 383 /
  25 860 330 741 292 466 517 830 093 082 565 850 903 817 543 916 950 567 129 129 914 748 319 592 064 208 177 616 122 028 384
   329 367 380 395 637 583 774 073 165 574 559 636 746 137 201 844 684 891 749 388 464 084 526 384 654 533 116 152 919 960
   804 482 005 353 359 431 456 072 569 494 057 133 594 928 399 650 603 196 537 449 265 437 133 288 812 573 835 740 202 037
   565 893 256 457 639 724 927 581 749 140 026 649 639 123 338 196 288 620 915 112 802 400 000 +
 (834 070 882 275 548 170 154 545 367 229 811 981 529
   Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
      {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
      {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{5, 6}, {8, 7, 1}], 5, 1}]]]^4) /
```

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In[201]:=
```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=
```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

In[220]:=
```
Zs = RandomInteger[{-500, 500}, {10, 4}]
splitHelicityAmplitude[10, 3] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[220]=
```
{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451},
 {448, -62, -249, -63}, {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184},
 {417, 128, -22, 64}, {227, -281, 259, -448}, {-450, 333, -324, -97}}
```

Out[221]=
```
847 940 525 964 420 245 040 834 410 595 447 801 796 629 647 062 021 857 044 263 754 709 197 875 187 079 565 660 067 266 738 :
   886 150 617 039 884 861 842 535 703 081 993 444 017 553 553 367 770 602 377 430 085 071 810 608 378 893 203 808 485 791 393 :
   750 690 262 200 238 919 424 226 793 738 172 855 834 282 670 901 427 651 858 627 850 076 607 236 062 620 307 277 076 409 045 :
   145 126 952 367 867 697 852 131 967 951 383 /
  25 860 330 741 292 466 517 830 093 082 565 850 903 817 543 916 950 567 129 129 914 748 319 592 064 208 177 616 122 028 384 :
   329 367 380 395 637 583 774 073 165 574 559 636 746 137 201 844 684 891 749 388 464 084 526 384 654 533 116 152 919 960 :
   804 482 005 353 359 431 456 072 569 494 057 133 594 928 399 650 603 196 537 449 265 437 133 288 812 573 835 740 202 037 :
   565 893 256 457 639 724 927 581 749 140 026 649 639 123 338 196 288 620 915 112 802 400 000 +
 (834 070 882 275 548 170 154 545 367 229 811 981 529
   Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
       {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
       {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{5, 6}, {8, 7, 1}], 5, 1}]]^4) /
  233 686 332 958 137 464 415 469 682 333 715 577 508 530 293 486 336 838 987 695 732 653 345 925 285 927 812 130 047 950 713 :
   087 897 505 155 474 067 574 659 595 156 438 358 412 861 862 005 096 089 600 000 000 +
 (202 802 682 163 863 456 054 193 399 122 984 363 794 086 853
   Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
       {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
```

```
{227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{6, 7}, {10, 9, 1}], 6, 1}]]^4) /
3 851 381 621 752 712 347 333 426 467 181 486 044 595 390 487 916 962 428 691 384 721 578 317 158 759 517 349 534 779 090
 909 826 862 541 899 012 959 444 451 213 898 556 432 824 272 377 378 788 096 858 175 830 520 406 047 792 148 953 158 517
 404 361 952 846 466 252 800 000 000 000 -
(10 095 489 960 702 723 322 747 566 008 576 743 908 547 289 204 193 223 065 311 658 831 423 668 612 394 547 046 537 678 453
  794 971 439 227 564 027 799
 Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
    {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
    {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{7, 8}, {10, 9, 1}], 7, 1}]]^4) /
8 683 941 456 827 537 061 881 218 377 507 607 278 283 095 355 478 288 513 933 367 363 891 350 242 171 248 602 594 559 419
 600 051 805 666 350 184 102 297 759 360 418 628 020 239 535 070 624 816 105 237 695 541 118 033 764 409 879 945 764 740
 848 285 711 176 701 841 827 474 637 551 359 643 847 400 000 +
(62 003 773 483 156 690 176 616 437 647 857 614 493 524 446 097 898 702 610 381 073 960 601 052 551 111
 Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
    {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
    {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{4, 8, 7, cap[{2, 3}, {9, 8, 1}]}]]^4) /
27 180 937 403 104 658 277 863 582 766 822 432 115 156 188 067 936 578 283 433 729 635 502 343 299 803 164 000 510 275 726
 570 281 481 082 846 747 342 696 647 588 593 127 005 340 952 783 788 397 553 764 800 697 495 114 979 640 436 605 980 632
 000 000 + (202 969 578 366 622 873 924 319 973 662 110 683 489 107 753 839 966 281 783 409 307 194 831 459 009
 Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
    {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
    {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{4, 8, 7, cap[{2, 3}, {10, 9, 1}]}]]^4) /
15 187 170 956 634 316 638 800 011 664 681 852 964 107 230 537 037 858 738 453 787 496 505 360 623 634 111 952 575 927 255
 662 216 915 259 291 872 368 827 425 322 646 662 436 609 636 206 870 700 985 153 604 694 672 668 841 975 969 330 000 000
 000 000 -
(46 498 182 373 187 900 383 215 170 703 120 202 887 130 805 108 918 038 668 114 555 689 332 397 750 598 431 778 004 647 969
```

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```
ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
 ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
 ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[201]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

In[220]:=

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
splitHelicityAmplitude[10, 3] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[220]=

```
{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451},
 {448, -62, -249, -63}, {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184},
 {417, 128, -22, 64}, {227, -281, 259, -448}, {-450, 333, -324, -97}}
```

Out[221]=

```
847 940 525 964 420 245 040 834 410 595 447 801 796 629 647 062 021 857 044 263 754 709 197 875 187 079 565 660 067 266 738
  886 150 617 039 884 861 842 535 703 081 993 444 017 553 553 367 770 602 377 430 085 071 810 608 378 893 203 808 485 791 393
  750 690 262 200 238 919 424 226 793 738 172 855 834 282 670 901 427 651 858 627 850 076 607 236 062 620 307 277 076 409 045
  145 126 952 367 867 697 852 131 967 951 383 /
 25 860 330 741 292 466 517 830 093 082 565 850 903 817 543 916 950 567 129 129 914 748 319 592 064 208 177 616 122 028 384
  329 367 380 395 637 583 774 073 165 574 559 636 746 137 201 844 684 891 749 388 464 084 526 384 654 533 116 152 919 960
  804 482 005 353 359 431 456 072 569 494 057 133 594 928 399 650 603 196 537 449 265 437 133 288 812 573 835 740 202 037
  565 893 256 457 639 724 927 581 749 140 026 649 639 123 338 196 288 620 915 112 802 400 000 +
 (834 070 882 275 548 170 154 545 367 229 811 981 529
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
  m_?MatrixQ :> MatrixForm[m])

In[161]:= ClearAll[rToResidue]

In[177]:= rToResidue[exprn_] :=
           (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
             ab[x___, y_, w___, y_, z___] :> 0);
          termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
           (rList //.
               {R[x___, cap[y_, z_], w___] :>
                 (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
              R[x___, y_, w___, y_, z___] :> 0 /.
             {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
          capRules =
           {ab[x___, cap[y_, z_], w___] :>
             (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};

In[196]:= treeAmp[10, 3][[10]]
          {rToResidue[%], termToDMatrix[%, 10]}
          (#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %

Out[196]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]

Out[199]= {1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
             ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
             (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
             (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
             ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
          {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
m_?MatrixQ :> MatrixForm[m]}
```

In[161]:=
```
ClearAll[rToResidue]
```

In[177]:=
```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
     {R[x___, cap[y_, z_], w___] :>
       (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] :> 0 /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
m_?MatrixQ :> MatrixForm[m]}
```

In[161]:=

```
ClearAll[rToResidue]
```

In[177]:=

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]}]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=

```
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
```

```
m_?MatrixQ ⧴ MatrixForm[m]}
```

```
In[161]:= ClearAll[rToResidue]
```

```
In[177]:= rToResidue[exprn_] :=
  (exprn /. {R[x__] ⧴ (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] ⧴ 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] ⧴
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] ⧴ 0 /.
    {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] ⧴
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[198]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
Out[199]= {1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
m_?MatrixQ :> MatrixForm[m]}
```

In[161]:=
```
ClearAll[rToResidue]
```

In[177]:=
```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  (rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
    R[x___, y_, w___, y_, z___] :> 0 /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[161]:= ClearAll[rToResidue]

rToResidue[exprn_] :=
    (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
      ab[x___, y_, w___, y_, z___] :> 0);
  termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
    (rList //. {R[x___, cap[y_, z_], w___] :>
          (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
       R[x___, y_, w___, y_, z___] :> 0) /.
     {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]}]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};

In[198]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %

Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]

Out[199]= {1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
     ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
     (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
     (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
     ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
  {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
   {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
```

```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
      R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
m_?MatrixQ :> MatrixForm[m]]

In[161]:= ClearAll[rToResidue]

rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1/ (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};

In[198]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %

Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]

Out[199]= {1/ (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
  {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
{227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{6, 7}, {9, 8, 1}], 6, 1}]]^4) /
511 396 263 902 898 330 115 447 519 463 652 138 572 723 800 314 505 966 626 401 734 489 633 759 269 769 563 439 023 650 809
964 298 967 697 645 930 153 379 838 724 395 271 198 570 117 159 757 872 896 715 914 146 511 548 192 198 872 623 181 022
976 643 249 509 236 736 -
(1 417 095 598 006 906 410 297 932 868 446 768 329 564 475 556 886 123 613 538 906 261 955 406 328 020 079 422 682 688 275
702 557 Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
{-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
{227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{6, 7}, {10, 9, 1}], 6, 1}]]^4) /
3 851 381 621 752 712 347 333 426 467 181 486 044 595 390 487 916 962 428 691 384 721 578 317 158 759 517 349 534 779 090
909 826 862 541 899 012 959 444 451 213 898 556 432 824 272 377 378 788 096 858 175 830 520 406 047 792 148 953 158 517
404 361 952 846 466 252 800 000 000 000 -
(10 095 489 960 702 723 322 747 566 008 576 743 908 547 289 204 193 223 065 311 658 831 423 668 612 394 547 046 537 678 453
794 971 439 227 564 027 799
Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
{-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
{227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{7, 8}, {10, 9, 1}], 7, 1}]]^4) /
8 683 941 456 827 537 061 881 218 377 507 607 278 283 095 355 478 288 513 933 367 363 891 350 242 171 248 602 594 559 419
600 051 805 666 350 184 102 297 759 360 418 628 020 239 535 070 624 816 105 237 695 541 118 033 764 409 879 945 764 740
848 285 711 176 701 841 827 474 637 551 359 643 847 400 000 +
(62 003 773 483 156 690 176 616 437 647 857 614 493 524 446 097 898 702 610 381 073 960 601 052 551 111
Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
{-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
{227, -281, 259, -448}, {-450, 333, -324, -97}}[[{4, 8, 7, cap[{2, 3}, {9, 8, 1}]}]]^4) /
27 180 937 403 104 658 277 863 582 766 822 432 115 156 188 067 936 578 283 433 729 635 502 343 299 803 164 000 510 275 726
370 281 481 082 846 747 342 696 647 588 593 127 005 340 952 783 788 397 553 764 800 697 495 114 979 640 436 605 980 032
000 000 + (202 969 578 366 622 873 924 319 973 662 110 683 489 107 753 839 966 281 783 409 307 194 831 459 009
```

```
108 977 335 736 667 261 907 804 188 520 672 691 266 426 260 466 642 035 819 976 303 957 915 773 437 566 590 957 468 474 967
 440 909 095 903 364 558 052 115 755 307 152 871 843 353 105 874 714 062 500 000 000 000 +
(1 229 843 938 302 683 757 537 443 585 725 697 570 642 611 124 061 034 816 244 357 645 758 851 273 653 548 632 125 625
  Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
      {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
      {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{6, 7}, {9, 8, 1}], 6, 1}]]]^4) /
511 396 263 902 898 330 115 447 519 463 652 138 572 723 800 314 505 966 626 401 734 489 633 759 269 769 563 439 023 650 809
 964 298 967 697 645 930 153 379 838 724 395 271 198 570 117 159 757 872 896 715 914 146 511 548 192 198 872 623 181 022
 976 643 249 509 236 736 -
(1 417 095 598 006 906 410 297 932 868 446 768 329 564 475 556 886 123 613 538 906 261 955 406 328 020 079 422 682 688 275
   702 557 Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
      {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
      {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{6, 7}, {10, 9, 1}], 6, 1}]]]^4) /
3 851 381 621 752 712 347 333 426 467 181 486 044 595 390 487 916 962 428 691 384 721 578 317 158 759 517 349 534 779 090
 909 826 862 541 899 012 959 444 451 213 898 556 432 824 272 377 378 788 096 858 175 830 520 406 047 792 148 953 158 517
 404 361 952 846 466 252 800 000 000 000 -
(10 095 489 960 702 723 322 747 566 008 576 743 908 547 289 204 193 223 065 311 658 831 423 668 612 394 547 046 537 678 453
   794 971 439 227 564 027 799
  Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
      {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
      {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{7, 8}, {10, 9, 1}], 7, 1}]]]^4) /
8 683 941 456 827 537 061 881 218 377 507 607 278 283 095 355 478 288 513 933 367 363 891 350 242 171 248 602 594 559 419
 600 051 805 666 350 184 102 297 759 360 418 628 020 239 535 070 624 816 105 237 695 541 118 033 764 409 879 945 764 740
 848 285 711 176 701 841 827 474 637 551 359 643 847 400 000 +
(62 003 773 483 156 690 176 616 437 647 857 614 493 524 446 097 898 702 610 381 073 960 601 052 551 111
  Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(1 788 782 927 536 251 297 091 172 056 869 992 041 418 198 980 711
   Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
      {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
      {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{5, 6}, {10, 9, 1}], 5, 1}]]]^4) /
 108 977 335 736 667 261 907 804 188 520 672 691 266 426 260 466 642 035 819 976 303 957 915 773 437 566 590 957 468 474 967
  440 909 095 903 364 558 052 115 755 307 152 871 843 353 105 874 714 062 500 000 000 000 +
(1 229 843 938 302 683 757 537 443 585 725 697 570 642 611 124 061 034 816 244 357 645 758 851 273 653 548 632 125 625
   Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
      {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
      {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{6, 7}, {9, 8, 1}], 6, 1}]]]^4) /
 511 396 263 902 898 330 115 447 519 463 652 138 572 723 800 314 505 966 626 401 734 489 633 759 269 769 563 439 023 650 809
  964 298 967 697 645 930 153 379 838 724 395 271 198 570 117 159 757 872 896 715 914 146 511 548 192 198 872 623 181 022
  976 643 249 509 236 736 -
(1 417 095 598 006 906 410 297 932 868 446 768 329 564 475 556 886 123 613 538 906 261 955 406 328 020 079 422 682 688 275
    702 557 Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
      {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
      {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{6, 7}, {10, 9, 1}], 6, 1}]]]^4) /
 3 851 381 621 752 712 347 333 426 467 181 486 044 595 390 487 916 962 428 691 384 721 578 317 158 759 517 349 534 779 090
  909 826 862 541 899 012 959 444 451 213 898 556 432 824 272 377 378 788 096 858 175 830 520 406 047 792 148 953 158 517
  404 361 952 846 466 252 800 000 000 000 -
(10 095 489 960 702 723 322 747 566 008 576 743 908 547 289 204 193 223 065 311 658 831 423 668 612 394 547 046 537 678 453
    794 971 439 227 564 027 799
   Det[{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451}, {448, -62, -249, -63},
      {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184}, {417, 128, -22, 64},
      {227, -281, 259, -448}, {-450, 333, -324, -97}}[[{3, cap[{7, 8}, {10, 9, 1}], 7, 1}]]]^4) /
 8 683 941 456 827 537 061 881 218 377 507 607 278 283 095 355 478 288 513 933 367 363 891 350 242 171 248 602 594 559 419
```

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

In[161]:=
```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

In[161]:=

```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1/ (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=

```
{1/ (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
   ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] :=
  (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
   ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[135]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

In[161]:=
```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] := (exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))}) //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
      R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[135]:=
```
nice[exprn_] :=
 (exprn //. {cap[x_, y_] :> Row[Flatten@List["(", x, ")∩(", y, ")"]]} /. {ab[x__] :> Row[{"⟨", x, "⟩"}]} /.
   m_?MatrixQ :> MatrixForm[m])
```

In[161]:=
```
ClearAll[rToResidue]
```

In[222]:=
```
rToResidue[exprn_] :=
  ((exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))}) //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
```

```
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %

Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]

Out[199]= {1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
  {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
   {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
   {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
    ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
    ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}

Out[200]= (-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
  (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])

In[201]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]

splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
 ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
 ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[225]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

In[220]:=

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
splitHelicityAmplitude[10, 3] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[220]=

```
{{-415, -36, -309, -88}, {68, -339, -304, 173}, {39, -227, 85, 451},
 {448, -62, -249, -63}, {-4, -39, -23, 252}, {469, 18, -115, 246}, {305, -362, -400, -184},
 {417, 128, -22, 64}, {227, -281, 259, -448}, {-450, 333, -324, -97}}
```

Out[221]=

```
847 940 525 964 420 245 040 834 410 595 447 801 796 629 647 062 021 857 044 263 754 709 197 875 187 079 565 660 067 266 738 \
  886 150 617 039 884 861 842 535 703 081 993 444 017 553 553 367 770 602 377 430 085 071 810 608 378 893 203 808 485 791 393 \
  750 690 262 200 238 919 424 226 793 738 172 855 834 282 670 901 427 651 858 627 850 076 607 236 062 620 307 277 076 409 045 \
  145 126 952 367 867 697 852 131 967 951 383 /
 25 860 330 741 292 466 517 830 093 082 565 850 903 817 543 916 950 567 129 129 914 748 319 592 064 208 177 616 122 028 384 \
  329 367 380 395 637 583 774 073 165 574 559 636 746 137 201 844 684 891 749 388 464 084 526 384 654 533 116 152 919 960 \
  804 482 005 353 359 431 456 072 569 494 057 133 594 928 399 650 603 196 537 449 265 437 133 288 812 573 835 740 202 037 \
  565 893 256 457 639 724 927 581 749 140 026 649 639 123 338 196 288 620 915 112 802 400 000 +
 (834 070 882 275 548 170 154 545 367 229 811 981 529
```

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Running...bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
 ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
 ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
In[226]:= Zs = RandomInteger[{-500, 500}, {10, 4}]
splitHelicityAmplitude[10, 3] /. ab[x__] :> Det[Zs[[{x}]]]

Out[226]= {{315, -470, 403, -142}, {-290, 422, -270, -218}, {321, -251, -120, 409},
 {-470, 432, -253, -453}, {298, -298, -278, 87}, {-166, -262, 76, -204},
 {356, 461, 196, -398}, {282, -356, 170, 53}, {-67, -35, 390, -78}, {23, 295, 186, 366}}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]

ab[4, 5, 1, 2]^3
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
In[226]:= Zs = RandomInteger[{-500, 500}, {10, 4}]
splitHelicityAmplitude[10, 3] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
Out[226]= {{315, -470, 403, -142}, {-290, 422, -270, -218}, {321, -251, -120, 409},
 {-470, 432, -253, -453}, {298, -298, -278, 87}, {-166, -262, 76, -204},
 {356, 461, 196, -398}, {282, -356, 170, 53}, {-67, -35, 390, -78}, {23, 295, 186, 366}}
```

```
Out[227]= -472761013918836367287086732703200787661190749562172891115634477737437452403009273158201705129
   333034959887561442200090946331518868372368352322738947116783949408668608136832906783029740
   184602754519304940552403769954190804799114850493753433867006472220630346894005600030479116
   678417581635740239061331343 /
  206158076898970207316550802589049881637287632507648843617003873266143224247327503159951008
   479453414287574638854904284803762468249558239613349575408816029644508249530816146149071072
   264739189711460910956757461145199107337965933970996014250560861990620205286633603783746102
   228536150986270087477165851195335983402708775307758464000 +
 (555008321660942976029762762593310269875199784508132051101
   Det[{{315, -470, 403, -142}, {-290, 422, -270, -218}, {321, -251, -120, 409}, {-470, 432, -253, -453},
      {298, -298, -278, 87}, {-166, -262, 76, -204}, {356, 461, 196, -398}, {282, -356, 170, 53},
      {-67, -35, 390, -78}, {23, 295, 186, 366}}[[{3, cap[{5, 6}, {8, 7, 1}], 5, 1}]]^4) /
  1068517630708147881343509311605507554070352215855939059385891939601732205496616276483201126
   666453274995624275383089592726169539235051896934988305561825870307502587904000 -
 (45524339214719448905989476946318645091939030414375
   Det[{{315, -470, 403, -142}, {-290, 422, -270, -218}, {321, -251, -120, 409}, {-470, 432, -253, -453},
      {298, -298, -278, 87}, {-166, -262, 76, -204}, {356, 461, 196, -398}, {282, -356, 170, 53},
```

```
dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
In[228]:= Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k] /. ab[x__] :> Det[Zs[[{x}]]]

Out[228]= {{-86, -62, 332, 396}, {83, 308, -456, 159}, {99, 333, 271, 190},
 {-395, -298, 271, -104}, {21, -451, 415, 375}, {76, -182, -41, 193},
 {123, -366, 414, -208}, {194, -348, 312, 411}, {-397, 374, -180, -387}, {-319, 283, 6, -60}}
```

```
Out[229]= {Det[{k == 0, 1, k > -3, 0, True, A[k][]}[[All, Range[k]]]]^4 Which[k == 0, 1, k > Length[{n}] - 4, 0, True,
   A[k] @@ {n}[[1 ;; -2]] + Total[Apply[bcfwBridge[##1][{n}] &, bcfwPartitions[Length[{n}], k], {1}]]]}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{\text{ab}[4,5,1,2]^3}{\text{ab}[1,2,3,4]\ \text{ab}[2,3,4,5]\ \text{ab}[3,4,5,1]\ \text{ab}[5,1,2,3]} +$$

$$\frac{\text{ab}[5,6,1,2]^3}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[6,1,2,3]} +$$

$$\frac{\text{ab}[4,5,6,1]^3}{\text{ab}[1,3,4,5]\ \text{ab}[3,4,5,6]\ \text{ab}[5,6,1,3]\ \text{ab}[6,1,3,4]} +$$

```
dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=
```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

In[230]:=
```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=
```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\ ab[2,3,4,5]\ ab[3,4,5,1]\ ab[5,1,2,3]} +$$

$$\frac{ab[5,6,1,2]^3}{ab[1,2,3,5]\ ab[2,3,5,6]\ ab[3,5,6,1]\ ab[6,1,2,3]} +$$

$$\frac{ab[4,5,6,1]^3}{ab[1,3,4,5]\ ab[3,4,5,6]\ ab[5,6,1,3]\ ab[6,1,3,4]} +$$

$$\frac{ab[6,7,1,2]^3}{ab[1,2,3,6]\ ab[2,3,6,7]\ ab[3,6,7,1]\ ab[7,1,2,3]} +$$

$$ab[4,6,7,1]^3$$

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
In[230]:= Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
Out[231]= {0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\ ab[2,3,4,5]\ ab[3,4,5,1]\ ab[5,1,2,3]} +$$

$$\frac{ab[5,6,1,2]^3}{ab[1,2,3,5]\ ab[2,3,5,6]\ ab[3,5,6,1]\ ab[6,1,2,3]} +$$

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=
```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=
```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

Out[231]=

A very large output was generated. Here is a sample of it:

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
componentAmp[10, 4]
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\ ab[2,3,4,5]\ ab[3,4,5,1]\ ab[5,1,2,3]} +$$

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
{0, 0, <<101>>, 0, 0}
```

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```
Position[componentAmp[10, 4], _cap, {
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[5,1,2,3]} +$$

```
dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
Position[componentAmp[10, 4], _cap, {0, ∞}]
```

A very large output was generated. Here is a sample of it:

```
{{1, 2, 2, 2, 3, 1, 2, 1}, {1, 2, 2, 2, 3, 1, 2, 3}, {1, 2, 2, 2, 3, 1},
 {1, 2, 2, 2, 3, 3}, {1, 2, 2, 3, 3, 1, 2, 1}, {1, 2, 2, 3, 3, 1, 2, 3}, {1, 2, 2, 3, 3, 1},
 {1, 2, 2, 3, 3, 3}, {1, 2, 2, 4, 3, 1, 2, 1}, {1, 2, 2, 4, 3, 1, 2, 3}, {1, 2, 2, 4, 3, 1},
 <<4668>>, {105, 2, 4, 9, 4, 4}, {105, 2, 4, 10, 2, 2, 1, 1}, {105, 2, 4, 10, 2, 2, 2, 3},
 {105, 2, 4, 10, 2, 2}, {105, 2, 4, 10, 2, 4, 2, 1}, {105, 2, 4, 10, 2, 4, 2, 3}, {105, 2, 4, 10, 2, 4},
 {105, 2, 4, 10, 4, 4}, {105, 2, 4, 10, 5, 4, 2, 1}, {105, 2, 4, 10, 5, 4, 2, 3}, {105, 2, 4, 10, 5, 4}}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=
```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=
```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

Out[231]=
```
{0, 0, <<101>>, 0, 0}
```

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In[233]:=
```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

Out[233]=
```
{1, 2, 2, 2, 3, 1, 2, 1}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```



```
ab[4, 5, 1, 2]^3
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]

In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];

Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] -> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}

A very large output was generated. Here is a sample of it:

Out[231]= {0, 0, <<101>>, 0, 0}

Show Less  Show More  Show Full Output  Set Size Limit...

In[232]:= Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]

Out[232]= {1, 2, 2, 2, 3, 1, 2, 1}

componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

```
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]

In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];

Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
Out[231]= {0, 0, <<101>>, 0, 0}
```

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```
In[233]:= Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]

Out[233]= {1, 2, 2, 2, 3, 1, 2, 1}
```

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```
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[225]:=
```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=
```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] → Det[Zs[[{x}]]]
```

Out[230]=
```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

Out[231]=
```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

In[233]:=
```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

Out[233]=
```
{1, 2, 2, 2, 3, 1, 2, 1}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[225]:=
```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=
```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=
```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

Out[231]=
```
{0, 0, <<101>>, 0, 0}
```

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In[233]:=
```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

Out[233]=
```
{1, 2, 2, 2, 3, 1, 2, 1}
```

```
  m_?MatrixQ :> MatrixForm[m])
```

In[161]:=
```
ClearAll[rToResidue]
```

In[222]:=
```
rToResidue[exprn_] :=
  ((exprn /. {R[x__] :> (1/(Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))}) //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
      R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
```

```
m_?MatrixQ :> MatrixForm[m]}
```

In[161]:=
```
ClearAll[rToResidue]
```

In[222]:=
```
rToResidue[exprn_] :=
  ((exprn /. {R[x__] :> (1/(Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))}) //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
  {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
```

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```
ClearAll[rToResidue]
```

```
rToResidue[exprn_] :=
  ((exprn /. {R[x__] :> (1/ (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))}) //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
      R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
{1/ (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
```

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```
In[161]:= ClearAll[rToResidue]

In[222]:= rToResidue[exprn_] :=
  ((exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))}) //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};

In[198]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %

Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]

Out[199]= {1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
```

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```
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
   ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

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```
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
{{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
{ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
{0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
Out[200]= (-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
   ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
  ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
  (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
  (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
  ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] -> Det[Zs[[{x}]]]
```

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Out[200]= (-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
  (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])

In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]

In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];

Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}

A very large output was generated. Here is a sample of it:

Out[231]= {0, 0, <<101>>, 0, 0}

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Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

```
ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
 ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
 ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
Out[231]= {0, 0, <<101>>, 0, 0}
```

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```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]

{1, 2, 2, 2, 3, 1, 2, 1}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^2 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
 ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
Out[231]= {0, 0, <<101>>, 0, 0}
```

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```
In[233]:= Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]

Out[233]= {1, 2, 2, 2, 3, 1, 2, 1}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
      R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=

```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Out[200]=

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
  (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
```

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```
  ((rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[198]:= treeAmp[10, 3][[10]]
          {rToResidue[%], termToDMatrix[%, 10]}
          (#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %

Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]

Out[199]= {1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
      ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
      (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
      (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
      ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
   {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
    {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
    {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
     ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
     ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}

   (ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
      ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
```

```
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
   {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[198]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
Out[199]= {1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```



```
Out[200]= (-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

Out[231]=

```
{0, 0, <<101>>, 0, 0}
```

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In[232]:=

```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

Out[232]=

```
{1, 2, 2, 2, 3, 1, 2, 1}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

Out[213]=

$$\frac{\text{ab}[4,5,1,2]^3}{\text{ab}[1,2,3,4]\ \text{ab}[2,3,4,5]\ \text{ab}[3,4,5,1]\ \text{ab}[5,1,2,3]} + \frac{\text{ab}[5,6,1,2]^3}{\text{ab}[1,2,3,5]\ \text{ab}[2,3,5,6]\ \text{ab}[3,5,6,1]\ \text{ab}[6,1,2,3]} +$$

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=
```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

Out[231]=
```
{0, 0, <<101>>, 0, 0}
```

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```
componentAmp[10, 4][[1]
```

In[233]:=
```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

Out[233]=
```
{1, 2, 2, 2, 3, 1, 2, 1}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[5,1,2,3]} +$$

$$ab[5,6,1,2]^3$$

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```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
{0, 0, <<101>>, 0, 0}
```

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```
componentAmp[10, 4][[1]]
```

```
{1/(ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
    ab[3, 10, 9, 1]^2 ab[5, 6, 2, 3] ab[5, 6, 7, 8] ab[7, 8, 9, 10] ab[9, 3, 2, 1]^9 ab[9, 10, 1, 2]
    ab[10, 1, 2, 3] (ab[2, 5, 6, 7] ab[3, 10, 9, 1] + ab[3, 5, 6, 7] ab[10, 9, 1, 2])
    (ab[2, 7, 8, 9] ab[3, 10, 9, 1] + ab[3, 7, 8, 9] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[6, 7, 8, 2] + ab[6, 7, 8, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[7, 8, 2, 5] + ab[7, 8, 3, 5] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])^3
    (ab[3, 10, 9, 1] ab[8, 9, 10, 2] + ab[8, 9, 10, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[9, 10, 2, 7] + ab[9, 10, 3, 7] ab[10, 9, 1, 2])
    (ab[3, 2, 1, 10] (ab[3, 10, 9, 1] ab[9, 2, 7, 8] + ab[9, 3, 7, 8] ab[10, 9, 1, 2]) +
      ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[10, 2, 7, 8] + ab[10, 3, 7, 8] ab[10, 9, 1, 2]))
    (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])^6
    (ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[7, 2, 5, 6] + ab[7, 3, 5, 6] ab[10, 9, 1, 2])
```

```
(ab[3, 10, 9, 1] ab[7, 8, 2, 5] + ab[7, 8, 3, 5] ab[10, 9, 1, 2])
(ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])^3
(ab[3, 10, 9, 1] ab[8, 9, 10, 2] + ab[8, 9, 10, 3] ab[10, 9, 1, 2])
(ab[3, 10, 9, 1] ab[9, 10, 2, 7] + ab[9, 10, 3, 7] ab[10, 9, 1, 2])
(ab[3, 2, 1, 10] (ab[3, 10, 9, 1] ab[9, 2, 7, 8] + ab[9, 3, 7, 8] ab[10, 9, 1, 2]) +
  ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[10, 2, 7, 8] + ab[10, 3, 7, 8] ab[10, 9, 1, 2]))
(ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])^6
(ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[7, 2, 5, 6] + ab[7, 3, 5, 6] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[8, 10, 9, 2] + ab[8, 10, 9, 3] ab[10, 9, 1, 2]) +
  ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 2, 5, 6] + ab[8, 3, 5, 6] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))
(ab[3, 10, 9, 1] ab[5, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[6, 8, 7, 2] + ab[6, 8, 7, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]) + ab[3, 10, 9, 1]
   ab[6, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))),
{{ab[2, 3, 9, 10], ab[3, 9, 10, 1], ab[9, 10, 1, 2], 0, 0, 0, 0, 0, ab[10, 1, 2, 3], ab[1, 2, 3, 9]},
 {0, ab[3, 4, 5, 6] ab[3, 10, 9, 1]
   ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}], 7, cap[{2, 3}, {10, 9, 1}], 5],
  ab[3, 10, 9, 1] ab[4, 5, 6, 2] ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}],
    7, cap[{2, 3}, {10, 9, 1}], 5], ab[3, 10, 9, 1] ab[5, 6, 2, 3]
   ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}], 7, cap[{2, 3}, {10, 9, 1}], 5],
  ab[3, 10, 9, 1] ab[6, 2, 3, 4] ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}],
    7, cap[{2, 3}, {10, 9, 1}], 5], ab[2, 3, 4, 5] ab[3, 10, 9, 1]
   ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}], 7, cap[{2, 3}, {10, 9, 1}], 5],
  0, 0, 0, 0}, {0, ab[3, 10, 9, 1] ab[5, 6, 7, 8] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[5, 6, 7, 8] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7], 0,
  ab[3, 10, 9, 1] ab[6, 7, 8, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
   ab[6, 7, 8, 3] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[3, 10, 9, 1] ab[7, 8, 2, 5] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
```

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A very large output was generated. Here is a sample of it:

Out[231]= {0, 0, <<101>>, 0, 0}

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In[234]:= componentAmp[10, 4][[1]]

```
Out[234]= {1/(ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
    ab[3, 10, 9, 1]^2 ab[5, 6, 2, 3] ab[5, 6, 7, 8] ab[7, 8, 9, 10] ab[9, 3, 2, 1]^9 ab[9, 10, 1, 2]
    ab[10, 1, 2, 3] (ab[2, 5, 6, 7] ab[3, 10, 9, 1] + ab[3, 5, 6, 7] ab[10, 9, 1, 2])
    (ab[2, 7, 8, 9] ab[3, 10, 9, 1] + ab[3, 7, 8, 9] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[6, 7, 8, 2] + ab[6, 7, 8, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[7, 8, 2, 5] + ab[7, 8, 3, 5] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])^3
    (ab[3, 10, 9, 1] ab[8, 9, 10, 2] + ab[8, 9, 10, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[9, 10, 2, 7] + ab[9, 10, 3, 7] ab[10, 9, 1, 2])
    (ab[3, 2, 1, 10] (ab[3, 10, 9, 1] ab[9, 2, 7, 8] + ab[9, 3, 7, 8] ab[10, 9, 1, 2]) +
      ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[10, 2, 7, 8] + ab[10, 3, 7, 8] ab[10, 9, 1, 2]))
    (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])^6
    (ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[7, 2, 5, 6] + ab[7, 3, 5, 6] ab[10, 9, 1, 2])
       (ab[3, 10, 9, 1] ab[8, 10, 9, 2] + ab[8, 10, 9, 3] ab[10, 9, 1, 2]) +
      ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 2, 5, 6] + ab[8, 3, 5, 6] ab[10, 9, 1, 2])
       (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))
    (ab[3, 10, 9, 1] ab[5, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[6, 8, 7, 2] + ab[6, 8, 7, 3] ab[10, 9, 1, 2])
       (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]) + ab[3, 10, 9, 1]
```

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bcfw_part_deux.nb

Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

A very large output was generated. Here is a sample of it:

Out[231]= {0, 0, <<101>>, 0, 0}

Show Less | Show More | Show Full Output | Set Size Limit...

```
%[[1
```

In[234]:=

```
componentAmp[10, 4][[1]]
```

Out[234]=

```
{1/(ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
    ab[3, 10, 9, 1]^2 ab[5, 6, 2, 3] ab[5, 6, 7, 8] ab[7, 8, 9, 10] ab[9, 3, 2, 1]^9 ab[9, 10, 1, 2]
    ab[10, 1, 2, 3] (ab[2, 5, 6, 7] ab[3, 10, 9, 1] + ab[3, 5, 6, 7] ab[10, 9, 1, 2])
    (ab[2, 7, 8, 9] ab[3, 10, 9, 1] + ab[3, 7, 8, 9] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[6, 7, 8, 2] + ab[6, 7, 8, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[7, 8, 2, 5] + ab[7, 8, 3, 5] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])^3
    (ab[3, 10, 9, 1] ab[8, 9, 10, 2] + ab[8, 9, 10, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[9, 10, 2, 7] + ab[9, 10, 3, 7] ab[10, 9, 1, 2])
    (ab[3, 2, 1, 10] (ab[3, 10, 9, 1] ab[9, 2, 7, 8] + ab[9, 3, 7, 8] ab[10, 9, 1, 2]) +
      ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[10, 2, 7, 8] + ab[10, 3, 7, 8] ab[10, 9, 1, 2]))
    (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])^6
    (ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[7, 2, 5, 6] + ab[7, 3, 5, 6] ab[10, 9, 1, 2])
       (ab[3, 10, 9, 1] ab[8, 10, 9, 2] + ab[8, 10, 9, 3] ab[10, 9, 1, 2]) +
      ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 2, 5, 6] + ab[8, 3, 5, 6] ab[10, 9, 1, 2])
       (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

A very large output was generated. Here is a sample of it:

{0, 0, <<101>>, 0, 0}

Show Less | Show More | Show Full Output | Set Size Limit...

```
%[[1]]
```

```
1/(ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
   ab[3, 10, 9, 1]^2 ab[5, 6, 2, 3] ab[5, 6, 7, 8] ab[7, 8, 9, 10] ab[9, 3, 2, 1]^9 ab[9, 10, 1, 2]
   ab[10, 1, 2, 3] (ab[2, 5, 6, 7] ab[3, 10, 9, 1] + ab[3, 5, 6, 7] ab[10, 9, 1, 2])
   (ab[2, 7, 8, 9] ab[3, 10, 9, 1] + ab[3, 7, 8, 9] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[6, 7, 8, 2] + ab[6, 7, 8, 3] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[7, 8, 2, 5] + ab[7, 8, 3, 5] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])^3
   (ab[3, 10, 9, 1] ab[8, 9, 10, 2] + ab[8, 9, 10, 3] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[9, 10, 2, 7] + ab[9, 10, 3, 7] ab[10, 9, 1, 2])
   (ab[3, 2, 1, 10] (ab[3, 10, 9, 1] ab[9, 2, 7, 8] + ab[9, 3, 7, 8] ab[10, 9, 1, 2]) +
     ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[10, 2, 7, 8] + ab[10, 3, 7, 8] ab[10, 9, 1, 2]))
   (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])^6
   (ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[7, 2, 5, 6] + ab[7, 3, 5, 6] ab[10, 9, 1, 2])
      (ab[3, 10, 9, 1] ab[8, 10, 9, 2] + ab[8, 10, 9, 3] ab[10, 9, 1, 2]) +
     ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 2, 5, 6] + ab[8, 3, 5, 6] ab[10, 9, 1, 2])
      (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))
   (ab[3, 10, 9, 1] ab[5, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[6, 8, 7, 2] + ab[6, 8, 7, 3] ab[10, 9, 1, 2])
      (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]) +
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[10, 1, 2, 3] (ab[2, 5, 6, 7] ab[3, 10, 9, 1] + ab[3, 5, 6, 7] ab[10, 9, 1, 2])
 (ab[2, 7, 8, 9] ab[3, 10, 9, 1] + ab[3, 7, 8, 9] ab[10, 9, 1, 2])
 (ab[3, 10, 9, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[10, 9, 1, 2])
 (ab[3, 10, 9, 1] ab[6, 7, 8, 2] + ab[6, 7, 8, 3] ab[10, 9, 1, 2])
 (ab[3, 10, 9, 1] ab[7, 8, 2, 5] + ab[7, 8, 3, 5] ab[10, 9, 1, 2])
 (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])^3
 (ab[3, 10, 9, 1] ab[8, 9, 10, 2] + ab[8, 9, 10, 3] ab[10, 9, 1, 2])
 (ab[3, 10, 9, 1] ab[9, 10, 2, 7] + ab[9, 10, 3, 7] ab[10, 9, 1, 2])
 (ab[3, 2, 1, 10] (ab[3, 10, 9, 1] ab[9, 2, 7, 8] + ab[9, 3, 7, 8] ab[10, 9, 1, 2]) +
   ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[10, 2, 7, 8] + ab[10, 3, 7, 8] ab[10, 9, 1, 2]))
 (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])^6
 (ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[7, 2, 5, 6] + ab[7, 3, 5, 6] ab[10, 9, 1, 2])
     (ab[3, 10, 9, 1] ab[8, 10, 9, 2] + ab[8, 10, 9, 3] ab[10, 9, 1, 2]) +
   ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 2, 5, 6] + ab[8, 3, 5, 6] ab[10, 9, 1, 2])
     (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))
 (ab[3, 10, 9, 1] ab[5, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[6, 8, 7, 2] + ab[6, 8, 7, 3] ab[10, 9, 1, 2])
     (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]) +
   ab[3, 10, 9, 1] ab[6, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])
     (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])))
```

```
In[234]:= componentAmp[10, 4][[1]]

Out[234]= {1/(ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
    ab[3, 10, 9, 1]^2 ab[5, 6, 2, 3] ab[5, 6, 7, 8] ab[7, 8, 9, 10] ab[9, 3, 2, 1]^9 ab[9, 10, 1, 2]
    ab[10, 1, 2, 3] (ab[2, 5, 6, 7] ab[3, 10, 9, 1] + ab[3, 5, 6, 7] ab[10, 9, 1, 2])
     (ab[2, 7, 8, 9] ab[3, 10, 9, 1] + ab[3, 7, 8, 9] ab[10, 9, 1, 2])
     (ab[3, 10, 9, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[10, 9, 1, 2])
     (ab[3, 10, 9, 1] ab[6, 7, 8, 2] + ab[6, 7, 8, 3] ab[10, 9, 1, 2])
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[235]:=
```
%[[1]]
```

Out[235]=
```
1/(ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
   ab[3, 10, 9, 1]^2 ab[5, 6, 2, 3] ab[5, 6, 7, 8] ab[7, 8, 9, 10] ab[9, 3, 2, 1]^9 ab[9, 10, 1, 2]
   ab[10, 1, 2, 3] (ab[2, 5, 6, 7] ab[3, 10, 9, 1] + ab[3, 5, 6, 7] ab[10, 9, 1, 2])
   (ab[2, 7, 8, 9] ab[3, 10, 9, 1] + ab[3, 7, 8, 9] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[6, 7, 8, 2] + ab[6, 7, 8, 3] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[7, 8, 2, 5] + ab[7, 8, 3, 5] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])^3
   (ab[3, 10, 9, 1] ab[8, 9, 10, 2] + ab[8, 9, 10, 3] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[9, 10, 2, 7] + ab[9, 10, 3, 7] ab[10, 9, 1, 2])
   (ab[3, 2, 1, 10] (ab[3, 10, 9, 1] ab[9, 2, 7, 8] + ab[9, 3, 7, 8] ab[10, 9, 1, 2]) +
     ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[10, 2, 7, 8] + ab[10, 3, 7, 8] ab[10, 9, 1, 2]))
   (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])^6
   (ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[7, 2, 5, 6] + ab[7, 3, 5, 6] ab[10, 9, 1, 2])
       (ab[3, 10, 9, 1] ab[8, 10, 9, 2] + ab[8, 10, 9, 3] ab[10, 9, 1, 2]) +
     ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 2, 5, 6] + ab[8, 3, 5, 6] ab[10, 9, 1, 2])
       (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))
   (ab[3, 10, 9, 1] ab[5, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[6, 8, 7, 2] + ab[6, 8, 7, 3] ab[10, 9, 1, 2])
       (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]) +
     ab[3, 10, 9, 1] ab[6, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])
       (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])))
```

```
componentAmp[10, 4][[1]]
```

```
1/(ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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In[235]:=
```
%[[1]]
```

In[234]:=
```
componentAmp[10, 4][[1]]
```

Out[234]=
```
{1/(ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
   ab[3, 10, 9, 1]^2 ab[5, 6, 2, 3] ab[5, 6, 7, 8] ab[7, 8, 9, 10] ab[9, 3, 2, 1]^9 ab[9, 10, 1, 2]
   ab[10, 1, 2, 3] (ab[2, 5, 6, 7] ab[3, 10, 9, 1] + ab[3, 5, 6, 7] ab[10, 9, 1, 2])
   (ab[2, 7, 8, 9] ab[3, 10, 9, 1] + ab[3, 7, 8, 9] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[6, 7, 8, 2] + ab[6, 7, 8, 3] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[7, 8, 2, 5] + ab[7, 8, 3, 5] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])^3
   (ab[3, 10, 9, 1] ab[8, 9, 10, 2] + ab[8, 9, 10, 3] ab[10, 9, 1, 2])
   (ab[3, 10, 9, 1] ab[9, 10, 2, 7] + ab[9, 10, 3, 7] ab[10, 9, 1, 2])
   (ab[3, 2, 1, 10] (ab[3, 10, 9, 1] ab[9, 2, 7, 8] + ab[9, 3, 7, 8] ab[10, 9, 1, 2]) +
     ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[10, 2, 7, 8] + ab[10, 3, 7, 8] ab[10, 9, 1, 2]))
   (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7])^6
   (ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[7, 2, 5, 6] + ab[7, 3, 5, 6] ab[10, 9, 1, 2])
      (ab[3, 10, 9, 1] ab[8, 10, 9, 2] + ab[8, 10, 9, 3] ab[10, 9, 1, 2]) +
     ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 2, 5, 6] + ab[8, 3, 5, 6] ab[10, 9, 1, 2])
      (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))
   (ab[3, 10, 9, 1] ab[5, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[6, 8, 7, 2] + ab[6, 8, 7, 3] ab[10, 9, 1, 2])
      (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]) + ab[3, 10, 9, 1]
     ab[6, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])
      (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))),
 {{ab[2, 3, 9, 10], ab[3, 9, 10, 1], ab[9, 10, 1, 2], 0, 0, 0, 0, 0, ab[10, 1, 2, 3], ab[1, 2, 3, 9]},
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    ab[6, 2, 3, 4] ab[9, 3, 2, 1] (ab[3, 10, 9, 1] ab[8, 7, 2, 5] + ab[8, 7, 3, 5] ab[10, 9, 1, 2])
     (ab[3, 10, 9, 1] ab[10, 9, 2, 7] + ab[10, 9, 1, 2] ab[10, 9, 3, 7]))),
{{ab[2, 3, 9, 10], ab[3, 9, 10, 1], ab[9, 10, 1, 2], 0, 0, 0, 0, 0, ab[10, 1, 2, 3], ab[1, 2, 3, 9]},
 {0, ab[3, 4, 5, 6] ab[3, 10, 9, 1]
   ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}], 7, cap[{2, 3}, {10, 9, 1}], 5],
  ab[3, 10, 9, 1] ab[4, 5, 6, 2] ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}],
    7, cap[{2, 3}, {10, 9, 1}], 5], ab[3, 10, 9, 1] ab[5, 6, 2, 3]
   ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}], 7, cap[{2, 3}, {10, 9, 1}], 5],
  ab[3, 10, 9, 1] ab[6, 2, 3, 4] ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}],
    7, cap[{2, 3}, {10, 9, 1}], 5], ab[2, 3, 4, 5] ab[3, 10, 9, 1]
   ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}], 7, cap[{2, 3}, {10, 9, 1}], 5],
  0, 0, 0, 0}, {0, ab[3, 10, 9, 1] ab[5, 6, 7, 8] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[5, 6, 7, 8] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7], 0,
  ab[3, 10, 9, 1] ab[6, 7, 8, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
   ab[6, 7, 8, 3] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[3, 10, 9, 1] ab[7, 8, 2, 5] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
   ab[7, 8, 3, 5] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[3, 10, 9, 1] ab[8, 2, 5, 6] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
   ab[8, 3, 5, 6] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[2, 5, 6, 7] ab[3, 10, 9, 1] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
   ab[3, 5, 6, 7] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7], 0, 0},
 {0, ab[3, 10, 9, 1] ab[7, 8, 9, 10] ab[9, 3, 2, 1], ab[7, 8, 9, 10] ab[9, 3, 2, 1] ab[10, 9, 1, 2],
  0, 0, 0, ab[3, 10, 9, 1] ab[8, 9, 10, 2] ab[9, 3, 2, 1] + ab[8, 9, 10, 3] ab[9, 3, 2, 1] ab[10, 9, 1, 2],
  ab[3, 10, 9, 1] ab[9, 3, 2, 1] ab[9, 10, 2, 7] + ab[9, 3, 2, 1] ab[9, 10, 3, 7] ab[10, 9, 1, 2],
  ab[3, 10, 9, 1] ab[9, 3, 2, 1] ab[10, 2, 7, 8] + ab[9, 3, 2, 1] ab[10, 3, 7, 8] ab[10, 9, 1, 2],
  ab[2, 7, 8, 9] ab[3, 10, 9, 1] ab[9, 3, 2, 1] + ab[3, 7, 8, 9] ab[9, 3, 2, 1] ab[10, 9, 1, 2]}}}
```



```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{{ab[2, 3, 9, 10], ab[3, 9, 10, 1], ab[9, 10, 1, 2], 0, 0, 0, 0, 0, ab[10, 1, 2, 3], ab[1, 2, 3, 9]},
 {0, ab[3, 4, 5, 6] ab[3, 10, 9, 1]
    ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}], 7, cap[{2, 3}, {10, 9, 1}], 5],
  ab[3, 10, 9, 1] ab[4, 5, 6, 2] ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}],
     7, cap[{2, 3}, {10, 9, 1}], 5], ab[3, 10, 9, 1] ab[5, 6, 2, 3]
    ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}], 7, cap[{2, 3}, {10, 9, 1}], 5],
  ab[3, 10, 9, 1] ab[6, 2, 3, 4] ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}],
     7, cap[{2, 3}, {10, 9, 1}], 5], ab[2, 3, 4, 5] ab[3, 10, 9, 1]
    ab[cap[{7, 8}, {cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}]}], 7, cap[{2, 3}, {10, 9, 1}], 5],
  0, 0, 0, 0}, {0, ab[3, 10, 9, 1] ab[5, 6, 7, 8] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[5, 6, 7, 8] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7], 0,
  ab[3, 10, 9, 1] ab[6, 7, 8, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
   ab[6, 7, 8, 3] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[3, 10, 9, 1] ab[7, 8, 2, 5] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
   ab[7, 8, 3, 5] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[3, 10, 9, 1] ab[8, 2, 5, 6] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
   ab[8, 3, 5, 6] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7],
  ab[2, 5, 6, 7] ab[3, 10, 9, 1] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7] +
   ab[3, 5, 6, 7] ab[10, 9, 1, 2] ab[cap[{10, 9}, {3, 2, 1}], 9, cap[{2, 3}, {10, 9, 1}], 7], 0, 0},
 {0, ab[3, 10, 9, 1] ab[7, 8, 9, 10] ab[9, 3, 2, 1], ab[7, 8, 9, 10] ab[9, 3, 2, 1] ab[10, 9, 1, 2],
  0, 0, 0, ab[3, 10, 9, 1] ab[8, 9, 10, 2] ab[9, 3, 2, 1] + ab[8, 9, 10, 3] ab[9, 3, 2, 1] ab[10, 9, 1, 2],
  ab[3, 10, 9, 1] ab[9, 3, 2, 1] ab[9, 10, 2, 7] + ab[9, 3, 2, 1] ab[9, 10, 3, 7] ab[10, 9, 1, 2],
  ab[3, 10, 9, 1] ab[9, 3, 2, 1] ab[10, 2, 7, 8] + ab[9, 3, 2, 1] ab[10, 3, 7, 8] ab[10, 9, 1, 2],
  ab[2, 7, 8, 9] ab[3, 10, 9, 1] ab[9, 3, 2, 1] + ab[3, 7, 8, 9] ab[9, 3, 2, 1] ab[10, 9, 1, 2]}}
```

```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

```
{1, 2, 2, 2, 3, 1, 2, 1}
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
%[[1]]
```

```
componentAmp[10, 4][[1]]
```

```
{1/ (ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
```

```
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```



```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    ab[x__, y_, w__, y_, z__] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x__, cap[y_, z_], w__] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x__, y_, w__, y_, z__] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x__, cap[y_, z_], w__] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[198]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
Out[199]= {1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
  {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
   {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
   {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
    ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
    ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```



```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
```

```
   ab[x___, y_, w___, y_, z___] ⧴ 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] ⧴
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] ⧴ 0) /.
    {R[x__] ⧴ Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] ⧴
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[198]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %

Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]

Out[199]= {1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}

(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
  ((exprn /. {R[x__] :> (1/ (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))}) //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x___, cap[y_, z_], w___] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=

```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=

```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=

```
{1/ (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
  ((exprn /. {R[x__] :> (1/(Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))} //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
      R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[198]:= treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
Out[198]= R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
Out[199]= {1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
     ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
     (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
     (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
     ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    ab[x__, y_, w__, y_, z__] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
      {R[x__, cap[y_, z_], w__] :>
        (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
     R[x__, y_, w__, y_, z__] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x__, cap[y_, z_], w__] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 *Mathematica Summer School 2011*

In[225]:=
```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=
```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=
```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

Out[231]=
```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

In[235]:=
```
%[[1]]
```

In[234]:=
```
componentAmp[10, 4][[1]]
```

Out[234]=
```
{1/(ab[1, 2, 3, 9] ab[2, 3, 4, 5] ab[2, 3, 9, 10] ab[3, 4, 5, 6] ab[3, 9, 10, 1]
    ab[3, 10, 9, 1]^2 ab[5, 6, 2, 3] ab[5, 6, 7, 8] ab[7, 8, 9, 10] ab[9, 3, 2, 1]^9 ab[9, 10, 1, 2]
    ab[10, 1, 2, 3] (ab[2, 5, 6, 7] ab[3, 10, 9, 1] + ab[3, 5, 6, 7] ab[10, 9, 1, 2])
    (ab[2, 7, 8, 9] ab[3, 10, 9, 1] + ab[3, 7, 8, 9] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[6, 7, 8, 2] + ab[6, 7, 8, 3] ab[10, 9, 1, 2])
    (ab[3, 10, 9, 1] ab[7, 8, 2, 5] + ab[7, 8, 3, 5] ab[10, 9, 1, 2])
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
{0, 0, <<101>>, 0, 0}
```

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```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

```
{1, 2, 2, 2, 3, 1, 2, 1}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
{448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
{1, 2, 2, 2, 3, 1, 2, 1}
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
{0, 0, <<101>>, 0, 0}
```

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```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

```
{1, 2, 2, 2, 3, 1, 2, 1}
```

```
componentAmp[10, 1];
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

```
{1, 2, 2, 2, 3, 1, 2, 1}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

$$\frac{\text{ab}[4,5,1,2]^3}{\text{ab}[1,2,3,4]\,\text{ab}[2,3,4,5]\,\text{ab}[3,4,5,1]\,\text{ab}[5,1,2,3]} +$$

$$\frac{\text{ab}[5,6,1,2]^3}{\text{ab}[1,2,3,5]\,\text{ab}[2,3,5,6]\,\text{ab}[3,5,6,1]\,\text{ab}[6,1,2,3]} +$$

$$\frac{\text{ab}[4,5,6,1]^3}{\cdots} +$$

```
Out[200]= (-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
Out[231]= {0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
(R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
   R[x__, y_, w__, y_, z__] :> 0) /.
  {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]}]
capRules =
 {ab[x___, cap[y_, z_], w___] :>
   (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[198]:=
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

```
Out[198]=
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

```
Out[199]=
{1/(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
   ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
Out[200]=
(-ab[3, 5, 6, 1] ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1] ab[7, 8, 9, 10] +
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^4 /
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
  ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
```

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
rToResidue[exprn_] :=
  ((exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))}) //. capRules /.
    ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
        {R[x___, cap[y_, z_], w___] :>
          (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
      R[x___, y_, w___, y_, z___] :> 0) /.
     {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
     (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

```
In[198]:=
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %

Out[198]=
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]

Out[199]=
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

```
  ((exprn /. {R[x__] :> (1 / (Times @@ (ab @@@ Partition[{x}, 4, 1, 1])))}) //. capRules /.
     ab[x___, y_, w___, y_, z___] :> 0);
termToDMatrix[term_, n_] := Block[{rList = If[Head[term] === R, {term}, List @@ term]},
  ((rList //.
       {R[x___, cap[y_, z_], w___] :>
         (R[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + R[x, y[[2]], w] ab @@ Append[z, y[[1]]])} /.
      R[x___, y_, w___, y_, z___] :> 0) /.
    {R[x__] :> Normal[SparseArray[Thread[Rule[{x}, (ab @@@ RotateLeft[Partition[{x}, 4, 1, 1]])]], n]]})]
capRules =
  {ab[x___, cap[y_, z_], w___] :>
    (ab[x, y[[1]], w] ab @@ Prepend[z, y[[2]]] + ab[x, y[[2]], w] ab @@ Append[z, y[[1]]])};
```

In[198]:=
```
treeAmp[10, 3][[10]]
{rToResidue[%], termToDMatrix[%, 10]}
(#1 * Power[Det[#2[[All, {1, 2, 3}]]], 4]) & @@ %
```

Out[198]=
```
R[1, 2, 3, 5, 6] R[1, 7, 8, 9, 10] R[cap[{2, 3}, {6, 5, 1}], 3, 4, 5, cap[{6, 5}, {3, 2, 1}]]
```

Out[199]=
```
{1 / (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6]
    ab[3, 5, 6, 1] ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
    (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
    (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
    ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8]),
 {{ab[2, 3, 5, 6], ab[3, 5, 6, 1], ab[5, 6, 1, 2], 0, ab[6, 1, 2, 3], ab[1, 2, 3, 5], 0, 0, 0, 0},
  {ab[7, 8, 9, 10], 0, 0, 0, 0, 0, ab[8, 9, 10, 1], ab[9, 10, 1, 7], ab[10, 1, 7, 8], ab[1, 7, 8, 9]},
  {0, ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1], ab[3, 6, 5, 1] ab[4, 5, 6, 2] ab[5, 3, 2, 1],
   ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 2, 3], ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4],
   ab[2, 3, 4, 5] ab[3, 6, 5, 1] ab[5, 3, 2, 1], 0, 0, 0, 0}}}
```

Verifying the Recursion Relations

Outline for Today:

- **Organizing strategy for supersymmetric functions in *Mathematica***
  - ◇ (possibly also: "opt-in" human-friendly formatting for complex formulae.)
- **Evaluating tree amplitudes for particular η-components**
  - ◇ Expanding all 'hatted' momentum twistors in terms of external twistors
  - ◇ Picking out particular η-components
- ★ **Checking internal consistency of the recursed amplitudes.**
  - ◇ Generalizing our recursion algorithm for randomized 'schemes.'

Code Developed During Last Lecture:

```
In[1]:=
bcfwPartitions[n_,k_]:=Select[Flatten[Table[{{n-nR+2,k-kR-1},{nR,kR}},{nR,4,n-1},{kR,0,nR-4}],1],Function[{nL,kL},(0≤kL≤nL-4)||(nL==3&&kL==0)]@@#[[1]]&];
termsInBCFW[n_,k_]:=termsInBCFW[n,k]=If[(k==0)||(k==n-4),1,((Total[(Times@@(termsInBCFW@@@#))&/@bcfwPartitions[n,k]]+termsInBCFW[n-1,k]))];
bcfwBridge[{nL_,kL_},{nR_,kR_}][twistorLabels_]:=Block[{jHat,nHat,leftLabelRange,rightLabelRange},leftLabelRange=twistorLabels[[1;;nL]];rightLabelRange=twistorLab
bcfwRecurse=(A[k_][labelRange__]:>Which[k==0,1,k>Length[{labelRange}]-4,0,True,(A[k]@@{labelRange}[[1;;-2]]+Total[bcfwBridge[##][{labelRange}]&@@@bcfwPartitions[Leng
treeAmp[n_,k_]:=treeAmp[n,k]=If[Head[#]===Plus,List@@#,{#}]&@Expand[(A[k]@@Range[n])//.bcfwRecurse];
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 *Mathematica Summer School 2011*

Outline for Today:

- **Organizing strategy for supersymmetric functions in *Mathematica***
  - ◇ (possibly also: "opt-in" human-friendly formatting for complex formulae.)
- **Evaluating tree amplitudes for particular *η*-components**
  - ◇ Expanding all 'hatted' momentum twistors in terms of external twistors
  - ◇ Picking out particular *η*-components
- ★ **Checking internal consistency of the recursed amplitudes.**
  - ◇ Generalizing our recursion algorithm for randomized `schemes.'

Code Developed During Last Lecture:

In[1]:=

```
bcfwPartitions[n_,k_]:=Select[Flatten[Table[{{n-nR+2,k-kR-1},{nR,kR}},{nR,4,n-1},{kR,0,nR-4}],1],Function[{nL,kL},(0≤kL≤nL-4)||(nL==3&&kL==0)]@@#[[1]]&];
termsInBCFW[n_,k_]:=termsInBCFW[n,k]=If[(k==0)||(k==n-4),1,((Total[(Times@@(termsInBCFW@@@#))&/@bcfwPartitions[n,k]]+termsInBCFW[n-1,k]))];
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bcfwRecurse={A[k_][labelRange__]:>Which[k==0,1,k>Length[{labelRange}]-4,0,True,(A[k]@@{labelRange}[[1;;-2]]+Total[bcfwBridge[##][{labelRange}]&@@@bcfwPartitions[Leng
treeAmp[n_,k_]:=treeAmp[n,k]=If[Head[#]===Plus,List@@#,{#}]&@Expand[(A[k]@@Range[n])//.bcfwRecurse];
```

The Status Quo

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{0, 0, «101», 0, 0}
```

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```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

```
{1, 2, 2, 2, 3, 1, 2, 1}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

$$\frac{ab[4,5,1,2]^3}{ab[1,2,3,4]\,ab[2,3,4,5]\,ab[3,4,5,1]\,ab[5,1,2,3]} +$$

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
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```

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```
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  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 -> jHat}];
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   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
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       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

Out[231]=

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

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termsInBCFW[n_, k_] :=
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    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, ][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
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   (A[kL] @@ leftLabelRange) (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]]) (A[kR] @@ rightLabelRange)];
bcfwRecurse =
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    (A[k] @@ {labelRange}[[1 ;; -2]] +
      Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

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Show Less | Show More | Show Full Output | Set Size Limit...

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       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

{0, 0, <<101>>, 0, 0}

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  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{0, 0, <<101>>, 0, 0}
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```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

Out[231]= {0, 0, <<101>>, 0, 0}

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

{0, 0, <<101>>, 0, 0}

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

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```

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Tree-Level BCFW Recursion in N=4 *Mathematica Summer School 2011*

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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bcfwRecurse =
  {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
    (A[k] @@ {labelRange}[[1 ;; -2]] +
      Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
termsInBCFW[n_, k_] :=
    termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
      ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
   Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
    rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
    jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
    nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
    leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
    rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
    (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
     (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
     (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomiI]]];
bcfwRecurse =
  {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
    (A[k] @@ {labelRange}[[1 ;; -2]] +
      Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
termsInBCFW[n_, k_] :=
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    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
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  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
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       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
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  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 -> jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 -> jHat, -1 -> nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

Out[231]=
```
{0, 0, <<101>>, 0, 0}
```
Show Less | Show More | Show Full Output | Set Size Limit...

```
termsInBCFW[n_, k_] :=
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    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
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   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
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  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

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   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
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   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{0, 0, «101», 0, 0}
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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{0, 0, <<101>>, 0, 0}
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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

{0, 0, <<101>>, 0, 0}

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   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
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     (A[k] @@ {labelRange}[[1 ;; -2]] +
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treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
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 treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse]
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
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   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
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   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
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bcfwRecurse =
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     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
random[n_, k_] :=
 treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

Out[231]=

{0, 0, ≪101≫, 0, 0}

Show Less | Show More | Show Full Output | Set Size Limit...

```
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
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bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
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   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] :=
  randomAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse]
```

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
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   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
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   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
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randomAmp[n_, k_] :=
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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```
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] :=
  randomAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse]
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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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```
termsInBCFW[n_, k_] := termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
      ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
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     rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
     jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
     nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
     leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
     rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
     (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
      (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
      (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
       (A[k] @@ {labelRange}[[1 ;; -2]] +
         Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
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          Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] :=
  randomAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse]
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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

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    leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
    rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
    (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
     (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
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bcfwRecurse = {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
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      (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]],
        RandomInteger[]+Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[
           Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
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```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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       leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
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       (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
        (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
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bcfwRecurse = {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
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             Length[{labelRange}], k]])]};
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Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

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        RandomInteger[Length[{labelRange} + Total[bcfwBridge[##][{labelRange}] & @@@
            bcfwPartitions[Length[{labelRange}], k]])]]};
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  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
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   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
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   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
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Tree-Level BCFW Recursion in N=4 *Mathematica Summer School 2011*

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bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] :=
 randomAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse]
```

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := randomAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. rand
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_ : False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] :=
 randomAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. randomBCFW]
```

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```
    rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
    jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
    nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
    leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
    rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
    (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
     (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
     (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] :=
 randomAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

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bcfw_part_deux.nb

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```
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] :=
 randomAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomA
```



```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
      Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] :=
 randomAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
In[242]:= randomAmp[6, 1]
Out[242]= {R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}
```

```
In[233]:= Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
Out[233]= {1, 2, 2, 2, 3, 1, 2, 1}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
     Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

In[244]:=

```
randomAmp[6, 1]
```

Out[244]=

```
{R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}
```

In[233]:=

```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

Out[233]=

```
{1, 2, 2, 2, 3, 1, 2, 1}
```

```
componentAmp[10, 1];
Total[(#1 * Power[Det[#2[[All, {3}]]], 4]) & @@@ %]
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 -> jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 -> jHat, -1 -> nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

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```
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

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```
ClearAll[rand
  termsInBCFW[n_, k_] :=
 termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
   ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
 Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
  rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
  jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
  nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
  leftLabelRange = ReplacePart[leftLabelRange, {-1 → jHat}];
  rightLabelRange = ReplacePart[rightLabelRange, {1 → jHat, -1 → nHat}];
  (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
   (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
   (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]])];
bcfwRecurse =
  {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
 treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
 randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
  {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :→ Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

In[244]:=

```
randomAmp[6, 1]
```

Out[244]=

```
{R[1, 2, 3, 4, 5], R[1, 2, 3, 5, 6], R[1, 3, 4, 5, 6]}
```

In[233]:=

```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

Out[233]=

```
{1, 2, 2, 2, 3, 1, 2, 1}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ {labelRange}[[1 ;; -2]] +
       Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
       Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomAmp[10, 4]
```

A very large output was generated. Here is a sample of it:

```
{R[1, 2, 3, 9, 10] R[3, 4, 6, 8, cap[{8, 9}, {cap[{2, 3}, {10, 9, 1}], cap[{10, 9}, {3, 2, 1}], 3}]]
  R[3, 8, 9, cap[{10, 9}, {3, 2, 1}], cap[{2, 3}, {10, 9, 1}]]
  R[6, 7, 8, cap[{cap[{8, 9}, {cap[{2, 3}, {10, 9, 1}], cap[{10, 9}, {3, 2, 1}], 3}], 8}, {6, 4, 3}],
   cap[{4, 6}, {cap[{8, 9}, {cap[{2, 3}, {10, 9, 1}], cap[{10, 9}, {3, 2, 1}], 3}], 8, 3}]],
  <<103>>, R[1, 3, 4, 9, 10] R[9, cap[{10, 9}, {4, 3, 1}], cap[{3, 4}, {10, 9, 1}], 7, 8]
  R[cap[{8, 7}, {<<1>>, cap[<<1>>], 9}], <<3>>, 7] R[<<1>>]}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
      Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
     (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
      Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

In[264]:=

```
randomAmp[10, 4]
```

A very large output was generated. Here is a sample of it:

Out[264]=

```
{R[1, 2, 3, 9, 10] R[3, 4, 6, 8, cap[{8, 9}, {cap[{2, 3}, {10, 9, 1}], cap[{10, 9}, {3, 2, 1}], 3}]]
  R[3, 8, 9, cap[{10, 9}, {3, 2, 1}], cap[{2, 3}, {10, 9, 1}]]
  R[6, 7, 8, cap[{cap[{8, 9}, {cap[{2, 3}, {10, 9, 1}], cap[{10, 9}, {3, 2, 1}], 3}], 8}, {6, 4, 3}],
   cap[{4, 6}, {cap[{8, 9}, {cap[{2, 3}, {10, 9, 1}], cap[{10, 9}, {3, 2, 1}], 3}], 8, 3}]],
 <<103>>, R[1, 3, 4, 9, 10] R[9, cap[{10, 9}, {4, 3, 1}], cap[{3, 4}, {10, 9, 1}], 7, 8]
  R[cap[{8, 7}, {<<1>>, cap[<<1>>], 9}], <<3>>, 7] R[<<1>>]}
```

Show Less Show More Show Full Output Set Size Limit...

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bcfw_part_deux.nb

Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
      Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
    (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
      Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomAmp[8, 4]
```

```
{R[1, 2, 3, 7, 8] R[7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 5, 6]
  R[cap[{2, 3}, {8, 7, 1}], 3, 4, cap[{6, 5}, {cap[{2, 3}, {8, 7, 1}], cap[{8, 7}, {3, 2, 1}], 7}],
   cap[{cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]}, {6, 5, 7}]]
  R[cap[{6, 5}, {cap[{2, 3}, {8, 7, 1}], cap[{8, 7}, {3, 2, 1}], 7}],
   cap[{cap[{cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]}, {6, 5, 7}],
     cap[{6, 5}, {cap[{2, 3}, {8, 7, 1}], cap[{8, 7}, {3, 2, 1}], 7}]}, {4, 3, cap[{2, 3}, {8, 7, 1}]}],
   cap[{3, 4}, {cap[{cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]}, {6, 5, 7}],
     cap[{6, 5}, {cap[{2, 3}, {8, 7, 1}], cap[{8, 7}, {3, 2, 1}], 7}], cap[{2, 3}, {8, 7, 1}]}], 4, 5]}
```



```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
randomAmp[8, 2]

{R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[1, 5, 6, 7, 8] R[2, 3, 4, cap[{5, 6}, {8, 7, 1}], 1],
 R[1, 5, 6, 7, 8] R[2, 4, 5, cap[{5, 6}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, 7, cap[{2, 3}, {8, 7, 1}]],
 R[1, 3, 4, 7, 8] R[4, 5, 6, 7, cap[{8, 7}, {4, 3, 1}]], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[4, 5, 6, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[4, 6, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}]],
 R[1, 2, 3, cap[{4, 3}, {7, 6, 5}], cap[{6, 7}, {4, 3, 5}]] R[5, 6, 7, 3, 4],
 R[1, 2, 3, cap[{6, 7}, {4, 3, 5}], 7] R[5, 6, 7, 3, 4],
 R[1, 3, cap[{4, 3}, {7, 6, 5}], cap[{6, 7}, {4, 3, 5}], 7] R[5, 6, 7, 3, 4],
 R[1, 5, 6, 7, 8] R[5, cap[{5, 6}, {8, 7, 1}], 2, 3, 4], R[5, 1, 2, 3, 4] R[6, 7, 1, cap[{1, 2}, {4, 3, 5}], 5],
 R[1, 4, 5, 7, 8] R[6, 7, cap[{8, 7}, {5, 4, 1}], cap[{4, 5}, {8, 7, 1}], 5],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 3, 5, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[5, 7, 1, 3, 4] R[cap[{7, 1}, {4, 3, 5}], 1, 2, 3, cap[{4, 3}, {1, 7, 5}]]}

Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]

{1, 2, 2, 2, 3, 1, 2, 1}
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomAmp[8, 2]

{R[1, 5, 6, 7, 8] R[2, 3, 5, cap[{5, 6}, {8, 7, 1}], 1], R[2, 3, 4, 5, cap[{5, 6}, {1, 7, 2}]] R[2, 5, 6, 7, 1],
 R[1, 4, 5, 7, 8] R[3, 4, cap[{4, 5}, {8, 7, 1}], 1, 2], R[1, 5, 6, 7, 8] R[5, 1, 2, 3, 4],
 R[1, 2, 3, 7, 8] R[5, 6, 7, 3, 4], R[2, 3, 4, 7, 1] R[5, 6, 7, cap[{1, 7}, {4, 3, 2}], cap[{3, 4}, {1, 7, 2}]],
 R[2, 3, 4, 7, 1] R[5, 6, 7, cap[{3, 4}, {1, 7, 2}], 4],
 R[2, 3, 4, 7, 1] R[5, 7, cap[{1, 7}, {4, 3, 2}], cap[{3, 4}, {1, 7, 2}], 4],
 R[1, 2, 3, 7, 8] R[5, 7, cap[{8, 7}, {3, 2, 1}], 3, 4], R[1, 5, 6, 7, 8] R[5, cap[{5, 6}, {8, 7, 1}], 1, 3, 4],
 R[1, 2, 3, 7, 8] R[5, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3, 4],
 R[1, 3, 4, 7, 8] R[6, 7, cap[{8, 7}, {4, 3, 1}], 4, 5],
 R[1, 3, 4, 7, 8] R[6, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 7, 8] R[6, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4, 5],
 R[1, 2, 3, 7, 8] R[7, cap[{2, 3}, {8, 7, 1}], 3, 5, 6],
 R[1, 2, 3, 7, 8] R[7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3, 5],
 R[1, 2, 3, 7, 8] R[7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 5, 6],
 R[2, 4, 5, 7, 1] R[cap[{1, 7}, {5, 4, 2}], cap[{4, 5}, {1, 7, 2}], 5, 6, 7],
 R[2, 3, 4, 6, 7] R[cap[{3, 4}, {7, 6, 2}], 4, 5, 6, cap[{7, 6}, {4, 3, 2}]],
 R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]]}

Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]

{1, 2, 2, 2, 3, 1, 2, 1}
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 2, 3, 7, 8] R[5, 6, 7, 3, 4],
 R[2, 4, 5, 7, 1] R[5, 6, 7, cap[{1, 7}, {5, 4, 2}], cap[{4, 5}, {1, 7, 2}]],
 R[1, 2, 3, 7, 8] R[5, 7, cap[{2, 3}, {8, 7, 1}], 3, 4],
 R[4, 6, 7, 2, 3] R[7, 1, 2, cap[{3, 2}, {7, 6, 4}], cap[{6, 7}, {3, 2, 4}]],
 R[1, 2, 3, 7, 8] R[7, cap[{2, 3}, {8, 7, 1}], 3, 5, 6],
 R[1, 3, 4, 7, 8] R[7, cap[{3, 4}, {8, 7, 1}], 4, 5, 6], R[4, 7, 1, 2, 3] R[7, cap[{7, 1}, {3, 2, 4}], 4, 5, 6],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[4, 5, 6, 2, 3] R[cap[{3, 2}, {6, 5, 4}], 6, 7, 1, 2],
 R[4, 5, 6, 2, 3] R[cap[{3, 2}, {6, 5, 4}], cap[{5, 6}, {3, 2, 4}], 6, 1, 2],
 R[4, 5, 6, 2, 3] R[cap[{3, 2}, {6, 5, 4}], cap[{5, 6}, {3, 2, 4}], 6, 7, 1],
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {4, 3, 1}]],
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {4, 3, 1}]],
 R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 1, 2, 3, 4],
 R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]],
 R[1, 5, 6, 7, 8] R[cap[{5, 6}, {8, 7, 1}], 1, 2, 4, 5], R[1, 5, 6, 7, 8] R[cap[{5, 6}, {8, 7, 1}], 2, 3, 4, 5]}
```

```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 *Mathematica Summer School 2011*

In[279]:=
```
randomAmp[8, 2]
```

Out[279]=
```
{R[1, 2, 3, 4, cap[{4, 5}, {8, 7, 1}]] R[1, 4, 5, 7, 8], R[2, 5, 6, 7, 1] R[4, 5, cap[{5, 6}, {1, 7, 2}], 2, 3],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3], R[2, 3, 4, 7, 1] R[5, 6, 7, cap[{3, 4}, {1, 7, 2}], 4],
 R[1, 4, 5, 7, 8] R[5, 6, 7, cap[{8, 7}, {5, 4, 1}], cap[{4, 5}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[6, 7, cap[{2, 3}, {8, 7, 1}], 4, 5], R[1, 3, 4, 7, 8] R[6, 7, cap[{8, 7}, {4, 3, 1}], 4, 5],
 R[1, 3, 4, 7, 8] R[6, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 2, 3, 7, 8] R[6, cap[{2, 3}, {8, 7, 1}], 3, 4, 5],
 R[1, 3, 4, 7, 8] R[6, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4, 5],
 R[2, 4, 5, 7, 1] R[cap[{1, 7}, {5, 4, 2}], cap[{4, 5}, {1, 7, 2}], 5, 6, 7],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[2, 3, 4, 7, 1] R[cap[{3, 4}, {1, 7, 2}], 4, 5, 7, cap[{1, 7}, {4, 3, 2}]],
 R[2, 3, 4, 7, 1] R[cap[{3, 4}, {1, 7, 2}], 5, 6, 7, cap[{1, 7}, {4, 3, 2}]],
 R[1, 5, 6, 7, 8] R[cap[{5, 6}, {8, 7, 1}], 1, 2, 3, 4],
 R[1, 5, 6, 7, 8] R[cap[{5, 6}, {8, 7, 1}], 1, 2, 4, 5], R[1, 5, 6, 7, 8] R[cap[{5, 6}, {8, 7, 1}], 2, 3, 4, 5],
 R[2, 3, 4, 6, 7] R[cap[{7, 6}, {4, 3, 2}], cap[{3, 4}, {7, 6, 2}], 4, 5, 6]}
```

```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

```
{1, 2, 2, 2, 3, 1, 2, 1}
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomAmp[8, 2]

{R[1, 2, 3, 4, cap[{4, 5}, {7, 6, 1}]] R[1, 4, 5, 6, 7], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8], R[1, 5, 6, 7, 8] R[3, 4, 5, 1, 2],
 R[1, 2, 3, 7, 8] R[3, 4, 5, 6, 7], R[1, 2, 3, 7, 8] R[3, 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[3, 4, 5, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[3, 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[3, 5, 6, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[3, 6, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 6, 7] R[4, 5, 6, cap[{2, 3}, {7, 6, 1}], 3],
 R[1, 2, 3, 6, 7] R[4, 6, cap[{7, 6}, {3, 2, 1}], cap[{2, 3}, {7, 6, 1}], 3],
 R[1, 4, 5, 7, 8] R[4, cap[{4, 5}, {8, 7, 1}], 1, 2, 3],
 R[3, cap[{4, 3}, {1, 6, 5}], cap[{6, 1}, {4, 3, 5}], 1, 2] R[5, 6, 1, 3, 4],
 R[1, 4, 5, 7, 8] R[5, 6, 7, cap[{8, 7}, {5, 4, 1}], cap[{4, 5}, {8, 7, 1}]],
 R[1, 3, 4, 7, 8] R[5, 6, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 6, 7] R[6, cap[{7, 6}, {4, 3, 1}], cap[{3, 4}, {7, 6, 1}], 4, 5],
 R[1, 2, 3, 6, 7] R[cap[{2, 3}, {7, 6, 1}], 4, 5, 6, cap[{7, 6}, {3, 2, 1}]],
 R[1, 3, 4, 7, 8] R[cap[{8, 7}, {4, 3, 1}], 4, 5, 6, 7],
 R[1, 3, 4, 7, 8] R[cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4, 6, 7]}

Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomAmp[8, 2]

{R[1, 5, 6, 7, 8] R[3, 4, 5, 1, 2], R[1, 5, 6, 7, 8] R[3, 5, cap[{5, 6}, {8, 7, 1}], 1, 2],
 R[5, 1, 2, 3, 4] R[5, 6, 7, 1, cap[{1, 2}, {4, 3, 5}]], R[1, 2, 3, 7, 8] R[5, 6, 7, 3, 4],
 R[2, 3, cap[{4, 3}, {7, 6, 5}], 7, 1] R[5, 6, 7, 3, 4], R[1, 2, 3, 7, 8] R[5, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[5, 7, cap[{2, 3}, {8, 7, 1}], 3, 4], R[1, 5, 6, 7, 8] R[5, cap[{5, 6}, {8, 7, 1}], 1, 3, 4],
 R[3, 5, cap[{6, 5}, {2, 1, 7}], cap[{1, 2}, {6, 5, 7}], 2] R[7, 1, 2, 5, 6],
 R[5, 6, 7, 3, 4] R[7, 1, 2, cap[{4, 3}, {7, 6, 5}], cap[{6, 7}, {4, 3, 5}]],
 R[5, 6, 7, 3, 4] R[7, 2, 3, cap[{4, 3}, {7, 6, 5}], cap[{6, 7}, {4, 3, 5}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 3, 4, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 4, 5, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 2, 3, 7, 8] R[cap[{2, 3}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {3, 2, 1}]],
 R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 1, 2, 3, 4],
 R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]],
 R[5, 7, 1, 3, 4] R[cap[{7, 1}, {4, 3, 5}], 1, 2, 3, cap[{4, 3}, {1, 7, 5}]],
 R[1, 3, 4, 7, 8] R[cap[{8, 7}, {4, 3, 1}], 4, 5, 6, 7],
 R[1, 3, 4, 7, 8] R[cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4, 5, 6],
 R[1, 3, 4, 7, 8] R[cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4, 6, 7]}

Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]

{1, 2, 2, 2, 3, 1, 2, 1}
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomAmp[8, 2]

{R[1, 5, 6, 7, 8] R[3, 4, 5, 1, 2], R[1, 5, 6, 7, 8] R[3, 4, 5, cap[{5, 6}, {8, 7, 1}], 1],
 R[1, 4, 5, 7, 8] R[3, 4, cap[{4, 5}, {8, 7, 1}], 1, 2],
 R[1, 5, 6, 7, 8] R[3, 5, cap[{5, 6}, {8, 7, 1}], 1, 2], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[4, 5, 6, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[4, 6, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{8, 7}, {4, 3, 1}], 4], R[1, 2, 3, 7, 8] R[6, 7, cap[{8, 7}, {3, 2, 1}], 4, 5],
 R[1, 2, 3, 7, 8] R[6, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 4, 5],
 R[2, 3, 5, cap[{6, 5}, {2, 1, 7}], cap[{1, 2}, {6, 5, 7}]] R[7, 1, 2, 5, 6],
 R[5, cap[{1, 2}, {6, 5, 7}], 2, 3, 4] R[7, 1, 2, 5, 6],
 R[5, cap[{6, 5}, {2, 1, 7}], cap[{1, 2}, {6, 5, 7}], 3, 4] R[7, 1, 2, 5, 6],
 R[4, 5, cap[{6, 5}, {3, 2, 7}], cap[{2, 3}, {6, 5, 7}], 3] R[7, 2, 3, 5, 6],
 R[3, cap[{3, 4}, {6, 5, 7}], 7, 1, 2] R[7, 3, 4, 5, 6],
 R[1, 2, 3, 7, 8] R[7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 4, 6],
 R[7, 1, 2, 4, 5] R[cap[{1, 2}, {5, 4, 7}], 2, 3, 4, cap[{5, 4}, {2, 1, 7}]],
 R[1, 4, 5, 7, 8] R[cap[{4, 5}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {5, 4, 1}]]}

Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];

Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
Out[231]= {0, 0, <<101>>, 0, 0}
```

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```
In[245]:= ClearAll[randomAmp]
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 -> jHat}];
   rightLabelRange = ReplacePart[rightLabelRange, {1 -> jHat, -1 -> nHat}];
   (A[kL] @@ RotateLeft[leftLabelRange, If[randomQ, RandomInteger[nL], 0]])
    (R @@ twistorLabels[[{1, nL - 1, nL, -2, -1}]])
    (A[kR] @@ RotateLeft[rightLabelRange, If[randomQ, RandomInteger[nR], 0]]);
```

```
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
Out[231]= {0, 0, <<101>>, 0, 0}
```

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```
In[246]:= ClearAll[randomAmp]
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
```

```
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[225]:=
```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=
```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=
```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

Out[231]=
```
{0, 0, <<101>>, 0, 0}
```

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In[246]:=
```
ClearAll[randomAmp]
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
```

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```
ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])*/
(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
 ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
 ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
Out[231]= {0, 0, <<101>>, 0, 0}
```

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```
ClearAll[randomAmp]
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
```

```
    ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^2/
 (ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
   ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
   (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
   (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
   ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

```
A very large output was generated. Here is a sample of it:

Out[231]= {0, 0, <<101>>, 0, 0}

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```

```
ClearAll[randomAmp]
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
```

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bcfw_part_deux.nb

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```
ab[3, 4, 5, 6] ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[5, 6, 1, 2] ab[7, 8, 9, 10])^*/
(ab[1, 2, 3, 5] ab[1, 7, 8, 9] ab[2, 3, 4, 5] ab[2, 3, 5, 6] ab[3, 4, 5, 6] ab[3, 5, 6, 1]
 ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
 (ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
 (ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
 ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

In[225]:=

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[214]:=

```
splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[230]=

```
{{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

Out[231]=

```
{0, 0, <<101>>, 0, 0}
```

Show Less | Show More | Show Full Output | Set Size Limit...

```
ClearAll[randomAmp]
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    (A[k] @@ {labelRange}[[1 ;; -2]] +
      Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
    (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
      Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

In[285]:=

```
randomAmp[8, 2]
```

Out[285]=

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 7, 8] R[5, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
 R[5, cap[{1, 2}, {6, 5, 7}], 2, 3, 4] R[7, 1, 2, 5, 6],
```

```
      (A[k] @@ {labelRange}[[1 ;; -2]] +
        Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
      (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
        Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

In[285]:=

```
randomAmp[8, 2]
```

Out[285]=

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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```
      (A[k] @@ {labelRange}[[1 ;; -2]] +
        Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
      (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
        Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] & @ Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomComponentAmp[n_, k_] :=
 Block[{residueList, dMatrixList, amp = {}}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[285]:= randomAmp[8, 2]

Out[285]= {R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3]
```

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bcfw_part_deux.nb

Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
      (A[k] @@ {labelRange}[[1 ;; -2]] +
        Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
      (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
        Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[285]:= randomAmp[8, 2]

Out[285]= {R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
```

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```
      (A[k] @@ {labelRange}[[1 ;; -2]] +
        Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
      (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
        Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
```

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```
    (A[k] @@ {labelRange}[[1 ;; -2]] +
      Total[bcfwBridge[##][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
    (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
      Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[]
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2]
```

```
{{1 / (ab[1, 2, 3, 4] ab[1, 5, 6, 7] ab[5, 6, 7, 8] ab[6, 7, 8, 1]
     ab[7, 8, 1, 5] ab[8, 1, 5, 6] (ab[2, 3, 4, 5] ab[6, 8, 7, 1] + ab[2, 3, 4, 6] ab[8, 7, 1, 5])
     (ab[3, 4, 5, 1] ab[6, 8, 7, 1] + ab[3, 4, 6, 1] ab[8, 7, 1, 5])
     (ab[4, 5, 1, 2] ab[6, 8, 7, 1] + ab[4, 6, 1, 2] ab[8, 7, 1, 5])
     (ab[5, 1, 2, 3] ab[6, 8, 7, 1] + ab[6, 1, 2, 3] ab[8, 7, 1, 5])),
  {{ab[2, 3, 4, 5] ab[6, 8, 7, 1] + ab[2, 3, 4, 6] ab[8, 7, 1, 5], ab[3, 4, 5, 1] ab[6, 8, 7, 1] +
     ab[3, 4, 6, 1] ab[8, 7, 1, 5], ab[4, 5, 1, 2] ab[6, 8, 7, 1] + ab[4, 6, 1, 2] ab[8, 7, 1, 5],
    ab[5, 1, 2, 3] ab[6, 8, 7, 1] + ab[6, 1, 2, 3] ab[8, 7, 1, 5],
    ab[1, 2, 3, 4] ab[6, 8, 7, 1], ab[1, 2, 3, 4] ab[8, 7, 1, 5], 0, 0},
   {ab[5, 6, 7, 8], 0, 0, 0, ab[6, 7, 8, 1], ab[7, 8, 1, 5], ab[8, 1, 5, 6], ab[1, 5, 6, 7]}}},
 {1 / (ab[1, 2, 3, 4] ab[1, 4, 5, 7] ab[2, 3, 4, 5] ab[3, 4, 5, 1] ab[4, 5, 1, 2] ab[4, 5, 7, 8] ab[5, 7, 8, 1]
     ab[7, 8, 1, 4] ab[8, 1, 4, 5] ab[8, 7, 1, 4]^3 (ab[4, 1, 2, 3] ab[5, 8, 7, 1] + ab[5, 1, 2, 3] ab[8, 7, 1, 4])),
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
ab[3, 6, 5, 1]^2 ab[5, 3, 2, 1]^3 ab[5, 6, 1, 2] ab[5, 6, 2, 3] ab[6, 1, 2, 3]
(ab[3, 2, 1, 6] ab[3, 6, 5, 1] ab[5, 2, 3, 4] + ab[3, 6, 5, 1] ab[5, 3, 2, 1] ab[6, 2, 3, 4])
(ab[3, 6, 5, 1] ab[4, 5, 6, 2] + ab[4, 5, 6, 3] ab[6, 5, 1, 2])
ab[7, 8, 9, 10] ab[8, 9, 10, 1] ab[9, 10, 1, 7] ab[10, 1, 7, 8])
```

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]
```

```
In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];
```

```
Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] -> Det[Zs[[{x}]]]
```

```
Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}
```

A very large output was generated. Here is a sample of it:

```
Out[231]= {0, 0, <<101>>, 0, 0}
```

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```
In[245]:= ClearAll[randomAmp]
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
```

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```
      Total[bcfwBridge[##][{labelRange}] &@@@bcfwPartitions[Length[{labelRange}], k]])]};
randomBCFWRecurse =
  {A[k_][labelRange__] :> Which[k == 0, 1, k > Length[{labelRange}] - 4, 0, True,
    (A[k] @@ RotateLeft[{labelRange}[[1 ;; -2]], RandomInteger[Length[{labelRange}] - 1]] +
      Total[bcfwBridge[##, True][{labelRange}] &@@@bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

In[286]:=

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /.
```

Out[287]=

```
{{1/(ab[1, 2, 3, 4] ab[1, 5, 6, 7] ab[5, 6, 7, 8] ab[6, 7, 8, 1]
    ab[7, 8, 1, 5] ab[8, 1, 5, 6] (ab[2, 3, 4, 5] ab[6, 8, 7, 1] + ab[2, 3, 4, 6] ab[8, 7, 1, 5])
    (ab[3, 4, 5, 1] ab[6, 8, 7, 1] + ab[3, 4, 6, 1] ab[8, 7, 1, 5])
    (ab[4, 5, 1, 2] ab[6, 8, 7, 1] + ab[4, 6, 1, 2] ab[8, 7, 1, 5])
    (ab[5, 1, 2, 3] ab[6, 8, 7, 1] + ab[6, 1, 2, 3] ab[8, 7, 1, 5])),
  {{ab[2, 3, 4, 5] ab[6, 8, 7, 1] + ab[2, 3, 4, 6] ab[8, 7, 1, 5], ab[3, 4, 5, 1] ab[6, 8, 7, 1] +
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[286]:=

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

In[288]:=

```
randomComponentAmp[8, 2] /. ab[x__] :> Det[Zs[[{x}]]]
```

Out[288]=

```
{{1/
   1 668 344 340 427 089 614 641 303 343 174 898 910 112 670 049 209 375 530 934 395 571 231 242 341 342 199 216 686 503 299
    982 323 531 375 601 409 631 053 299 122 699 853 892 676 296 846 226 227 200,
  {{-22 425 075 541, 0, 0, -51 956 359 251, -10 124 555 826, 0, -64 586 929 164, 37 857 623 540},
   {-693 076 518 689 278 103 226, -294 044 819 912 492 859 672, -436 324 969 867 995 745 152,
    -132 297 399 255 750 635 214, -1 145 817 083 759 451 864 684, 0, 0, 0}}}, {1/
   90 095 789 031 535 102 534 750 149 714 819 337 780 979 393 977 621 171 570 681 033 867 751 041 334 144 081 711 079 747 022
    390 208 424 473 867 518 942 401 978 046 688 847 573 700 074 902 819 595 878 400,
  {{-127 852 198 423, 110 543 205 402, -33 873 308 403, 0, 0, 0, -28 691 875 161, 105 809 617 490},
   {0, 6 887 897 732 588 372 527 104, 356 761 021 110 934 129 074, 7 857 876 159 685 974 963 366,
    6 780 900 392 402 646 254 976, 0, 7 567 235 667 566 159 675 202, 0}}}, {1/
   1 009 681 773 975 057 063 261 015 988 876 213 366 334 655 655 011 019 752 693 542 076 555 544 373 251 958 603 320 650 887
    956 184 341 741 632 855 802 229 701 267 695 936 692 287 092 046 830 614 784,
  {{-43 128 971 807, 0, 0, 0, -28 459 249 812, 51 956 359 251, -8 815 253 832, 37 986 938 749},
   {404 780 501 842 866 157 311, -4 605 750 534 555 172 529 478, 1 425 717 336 613 156 685 934,
    -157 177 868 293 684 791 909, -3 220 792 614 114 187 682 808, 5 880 009 460 450 501 223 634, 0, 0}}}, {-1/
    253 799 391 650 274 889 286 854 690 719 101 614 491 904 632 535 263 007 225 774 452 269 590 899 292 779 410 232 745 811
     862 268 089 427 334 330 186 905 748 789 199 931 108 915 738 123 601 338 362 003 381 294 016 006 471 035 868 865 853 474
     817 966 080, {{47 633 634 147 606 068 555 007 049 563 894,
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
In[225]:= componentAmp[n_, k_] := Block[{residueList, dMatrixList}, residueList = rToResidue /@ treeAmp[n, k];
  dMatrixList = termToDMatrix[#, n] & /@ treeAmp[n, k]; Transpose[{residueList, dMatrixList}]]

In[214]:= splitHelicityAmplitude[n_, k_] := Total[(#1 * Power[Det[#2[[All, Range[k]]]], 4]) & @@@ componentAmp[n, k]];

Zs = RandomInteger[{-500, 500}, {10, 4}]
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ componentAmp[10, 4] /. ab[x__] :> Det[Zs[[{x}]]]

Out[230]= {{-183, 455, -404, -292}, {-487, 489, -281, -98}, {-191, -397, 451, -449},
 {-291, -403, -366, -236}, {342, -203, 187, 400}, {-143, 326, -209, 265},
 {448, 174, 447, -3}, {348, -41, 71, -395}, {427, -366, -488, 429}, {409, 243, -376, -430}}

A very large output was generated. Here is a sample of it:

Out[231]= {0, 0, <<101>>, 0, 0}

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In[245]:= ClearAll[randomAmp]
termsInBCFW[n_, k_] :=
  termsInBCFW[n, k] = If[(k == 0) || (k == n - 4), 1,
    ((Total[(Times @@ (termsInBCFW @@@ #)) & /@ bcfwPartitions[n, k]] + termsInBCFW[n - 1, k]))];
bcfwBridge[{nL_, kL_}, {nR_, kR_}, randomQ_: False][twistorLabels_] :=
  Block[{jHat, nHat, leftLabelRange, rightLabelRange}, leftLabelRange = twistorLabels[[1 ;; nL]];
   rightLabelRange = twistorLabels[[nL - 1 ;; -1]];
   jHat = cap[twistorLabels[[{nL - 1, nL}]], twistorLabels[[{-1, -2, 1}]]];
   nHat = cap[twistorLabels[[{-1, -2}]], twistorLabels[[{nL, nL - 1, 1}]]];
   leftLabelRange = ReplacePart[leftLabelRange, {-1 -> jHat}];
```

```
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] :> Det[Zs[[{x}]]]
```

```
{{1/
  1 668 344 340 427 089 614 641 303 343 174 898 910 112 670 049 209 375 530 934 395 571 231 242 341 342 199 216 686 503 299
   982 323 531 375 601 409 631 053 299 122 699 853 892 676 296 846 226 227 200,
 {{-22 425 075 541, 0, 0, -51 956 359 251, -10 124 555 826, 0, -64 586 929 164, 37 857 623 540},
  {-693 076 518 689 278 103 226, -294 044 819 912 492 859 672, -436 324 969 867 995 745 152,
   -132 297 399 255 750 635 214, -1 145 817 083 759 451 864 684, 0, 0, 0}}}, {1/
  90 095 789 031 535 102 534 750 149 714 819 337 780 979 393 977 621 171 570 681 033 867 751 041 334 144 081 711 079 747 022
   390 208 424 473 867 518 942 401 978 046 688 847 573 700 074 902 819 595 878 400,
 {{-127 852 198 423, 110 543 205 402, -33 873 308 403, 0, 0, 0, -28 691 875 161, 105 809 617 490},
  {0, 6 887 897 732 588 372 527 104, 356 761 021 110 934 129 074, 7 857 876 159 685 974 963 366,
   6 780 900 392 402 646 254 976, 0, 7 567 235 667 566 159 675 202, 0}}}, {1/
  1 009 681 773 975 057 063 261 015 988 876 213 366 334 655 655 011 019 752 693 542 076 555 544 373 251 958 603 320 650 887
   956 184 341 741 632 855 802 229 701 267 695 936 692 287 092 046 830 614 784,
 {{-43 128 971 807, 0, 0, 0, -28 459 249 812, 51 956 359 251, -8 815 253 832, 37 986 938 749},
  {404 780 501 842 866 157 311, -4 605 750 534 555 172 529 478, 1 425 717 336 613 156 685 934,
   -157 177 868 293 684 791 909, -3 220 792 614 114 187 682 808, 5 880 009 460 450 501 223 634, 0, 0}}}, {-1/
   253 799 391 650 274 889 286 854 690 719 101 614 491 904 632 535 263 007 225 774 452 269 590 899 292 779 410 232 745 811
    862 268 089 427 334 330 186 905 748 789 199 931 108 915 738 123 601 338 362 003 381 294 016 006 471 035 868 865 853 474
    817 966 080, {{47 633 634 147 606 068 555 007 049 563 894,
   20 209 057 725 976 420 649 893 902 460 968, 29 987 661 425 123 499 373 025 640 233 088,
   9 092 511 064 645 431 194 719 013 510 466, 78 749 503 548 458 585 555 484 651 417 396, 0, 0, 0},
  {13 631 471 643, 40 798 377 556, 0, 4 320 319 041, 51 046 476 058, -43 095 714 752, 0, 0}}}, {1/
  2 232 134 567 017 266 063 809 369 765 153 458 101 784 819 595 646 280 658 358 701 936 178 692 389 712 350 68[illegible] 309
   018 832 377 066 966 764 471 676 605 413 861 786 491 756 595 486 776 050,
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@
```

```
{{1 /
  1 668 344 340 427 089 614 641 303 343 174 898 910 112 670 049 209 375 530 934 395 571 231 242 341 342 199 216 686 503 299
   982 323 531 375 601 409 631 053 299 122 699 853 892 676 296 846 226 227 200,
 {{-22 425 075 541, 0, 0, -51 956 359 251, -10 124 555 826, 0, -64 586 929 164, 37 857 623 540},
  {-693 076 518 689 278 103 226, -294 044 819 912 492 859 672, -436 324 969 867 995 745 152,
   -132 297 399 255 750 635 214, -1 145 817 083 759 451 864 684, 0, 0, 0}}}, {1 /
  90 095 789 031 535 102 534 750 149 714 819 337 780 979 393 977 621 171 570 681 033 867 751 041 334 144 081 711 079 747 022
   390 208 424 473 867 518 942 401 978 046 688 847 573 700 074 902 819 595 878 400,
 {{-127 852 198 423, 110 543 205 402, -33 873 308 403, 0, 0, 0, -28 691 875 161, 105 809 617 490},
  {0, 6 887 897 732 588 372 527 104, 356 761 021 110 934 129 074, 7 857 876 159 685 974 963 366,
   6 780 900 392 402 646 254 976, 0, 7 567 235 667 566 159 675 202, 0}}}, {1 /
  1 009 681 773 975 057 063 261 015 988 876 213 366 334 655 655 011 019 752 693 542 076 555 544 373 251 958 603 320 650 887
   956 184 341 741 632 855 802 229 701 267 695 936 692 287 092 046 830 614 784,
 {{-43 128 971 807, 0, 0, 0, -28 459 249 812, 51 956 359 251, -8 815 253 832, 37 986 938 749},
  {404 780 501 842 866 157 311, -4 605 750 534 555 172 529 478, 1 425 717 336 613 156 685 934,
   -157 177 868 293 684 791 909, -3 220 792 614 114 187 682 808, 5 880 009 460 450 501 223 634, 0, 0}}}, {-1 /
   253 799 391 650 274 889 286 854 690 719 101 614 491 904 632 535 263 007 225 774 452 269 590 899 292 779 410 232 745 811
    862 268 089 427 334 330 186 905 748 789 199 931 108 915 738 123 601 338 362 003 381 294 016 006 471 035 868 865 853 474
    817 966 080, {{47 633 634 147 606 068 555 007 049 563 894,
   20 209 057 725 976 420 649 893 902 460 968, 29 987 661 425 123 499 373 025 640 233 088,
   9 092 511 064 645 431 194 719 013 510 466, 78 749 503 548 458 585 555 484 651 417 396, 0, 0, 0},
  {13 631 471 643, 40 798 377 556, 0, 4 320 319 041, 51 046 476 058, -43 095 714 752, 0, 0}}}, {1 /
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[4]]]], 4]) & @@@ %
```

```
{Det[{{912 795 622 408 351 541 706, 387 262 902 681 466 582 232, 574 648 702 853 426 698 112, 174 238 318 050 418 247 934},
    {15 029 929 321, 0, 0, -37 986 938 749}}]^4 /
  1 209 219 019 179 530 956 265 439 053 114 373 808 629 967 700 036 449 284 049 354 656 150 662 710 704 720 748 214 539 054
   070 297 143 205 796 752 230 860 689 383 413 430 693 217 624 648 122 368 000,
 -Det[{{-43 128 971 807, 0, 0, 0}, {0, -2 889 781 626 701 933 773 155,
      708 241 637 803 527 219 393, -141 809 456 097 575 247 807}}]^4 /
   171 742 130 058 186 761 183 316 651 763 224 469 307 400 804 033 150 322 743 714 148 080 037 210 060 841 304 326 130 555
    655 120 885 473 252 534 175 270 355 193 181 898 412 805 920 808 548 420 960,
 Det[{{-43 128 971 807, 0, 0, 0}, {404 780 501 842 866 157 311,
     -4 605 750 534 555 172 529 478, 1 425 717 336 613 156 685 934, -157 177 868 293 684 791 909}}]^4 /
  1 009 681 773 975 057 063 261 015 988 876 213 366 334 655 655 011 019 752 693 542 076 555 544 373 251 958 603 320 650 887
   956 184 341 741 632 855 802 229 701 267 695 936 692 287 092 046 830 614 784,
 Det[{{-43 128 971 807, 0, 0, 0}, {-708 241 637 803 527 219 393,
     -2 119 735 161 157 471 370 556, 0, -224 468 048 173 131 798 291}}]^4 /
  442 166 148 234 380 061 950 945 864 748 064 851 653 674 320 845 668 009 433 135 626 450 861 314 179 128 590 148 809 509 500
   232 991 703 032 996 978 465 343 456 057 851 938 817 008 889 382 453 248,
 Det[{{-127 852 198 423, 110 543 205 402, -33 873 308 403, 0}, {0, 338 495 284 895 649 174 912 591 095 397 120,
     -240 688 656 711 253 636 571 450 885 204 700, 1 495 427 621 590 152 746 887 591 265 094 540}}]^4 /
  9 886 536 434 197 475 700 037 199 487 243 617 649 879 382 577 684 215 318 821 238 420 279 672 604 823 818 887 909 784 420
   662 601 137 586 999 758 274 227 306 799 582 534 310 527 335 451 866 332 182 354 696 292 883 808 174 228 261 136 413 995
   941 077 975 040 000,
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2]]]], 4]) & @@@ %
```

$$\Big\{\frac{392\,411\,517\,513\,285\,444\,280\,743\,744\,710\,900\,339\,871\,880\,568\,595\,855\,414\,235\,477}{346\,288\,163\,727\,498\,428\,142\,404\,641\,499\,997\,657\,353\,813\,630\,208\,269\,638\,945\,238\,515\,232\,259\,027\,255\,425\,558\,400},$$
$$\frac{98\,652\,026\,530\,457\,206\,288\,630\,877\,150\,537\,739\,657\,364\,588\,062\,066\,816\,122\,891\,075}{90\,032\,974\,329\,026\,116\,303\,104\,557\,422\,897\,975\,801\,678\,639\,627\,325\,227\,969\,703\,815\,609\,309\,598\,131\,373\,574},$$
$$\frac{85\,124\,783\,072\,911\,313\,494\,998\,832\,646\,036\,815\,124\,006\,285\,652\,084\,977\,035\,260\,467}{538\,813\,755\,988\,541\,319\,567\,526\,906\,067\,091\,652\,389\,562\,196\,284\,791\,398\,048\,854\,479\,873\,269\,884\,473\,967\,296},$$
$$-\frac{18\,934\,713\,102\,040\,383\,920\,087\,843\,204\,803\,562\,500\,925\,378\,741\,927\,773\,177\,718\,375}{13\,477\,180\,760\,575\,747\,680\,620\,685\,937\,748\,582\,413\,430\,266\,700\,175\,116\,243\,708\,294\,424\,568\,112\,174\,038\,944},$$
$$\frac{123\,822\,101\,845\,561\,949\,853\,837\,529\,536\,311\,932\,220\,566\,274\,048}{489\,658\,696\,282\,788\,201\,308\,076\,521\,518\,998\,458\,897\,393\,539\,385\,353\,164\,203\,484\,986\,748\,825},$$
$$27\,428\,051\,252\,064\,994\,559\,778\,396\,981\,830\,980\,523\,444\,494\,914\,656\,855\,259\,607\,859\,974\,756\,912\,745\,347\,967\,398\,284\,383\,677\,/$$
$$17\,786\,946\,776\,878\,329\,879\,037\,149\,307\,829\,062\,936\,023\,429\,978\,669\,990\,454\,998\,439\,134\,964\,877\,776\,057\,933\,573\,124\,989\,121$$
$$220\,896\,816\,449\,282\,128,\ 0,\ 0,\ \frac{308\,658\,420\,716\,010\,399\,150\,932\,576\,651}{3\,304\,446\,886\,866\,235\,380\,946\,599\,073\,562\,853\,958\,338\,330\,024\,049\,536},$$
$$-\frac{785\,432\,938\,444\,970\,332\,684\,288}{434\,249\,555\,229\,917\,880\,765\,751\,738\,422\,768\,738\,058\,793\,549\,005},$$
$$-\frac{2\,967\,538\,772\,568\,126\,751\,276\,914\,755\,895\,103\,679\,548\,886\,053\,186\,098\,176}{36\,568\,159\,860\,786\,586\,039\,890\,324\,024\,337\,478\,313\,621\,895\,559\,604\,403\,727\,693\,404\,455\,222\,761\,399\,115},$$
$$\frac{1\,451\,684\,286\,181\,254\,592\,173\,056}{4\,091\,410\,471\,824\,703\,029\,588\,727\,501\,290\,131\,621\,312\,094\,549},$$
$$25\,199\,964\,284\,375\,802\,561\,540\,764\,012\,286\,173\,481\,712\,926\,358\,468\,408\,092\,670\,125\,652\,150\,209\,440\,521\,372\,953\,976\,337\,/$$
$$95\,499\,720\,334\,083\,366\,703\,978\,486\,695\,261\,684\,472\,291\,550\,671\,253\,122\,645\,042\,630\,769\,142\,597\,988\,688\,001\,468\,065\,839\,983$$

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] :> Det[Zs[[{x}]]];
Total[(#1 * Power[Det[#2[[All, Range[2]]]], 4]) & @@@ %]
```

```
{392 411 517 513 285 444 280 743 744 710 900 339 871 880 568 595 855 414 235 477 /
 346 288 163 727 498 428 142 404 641 499 997 657 353 813 630 208 269 638 945 238 515 232 259 027 255 425 558 400,
 98 652 026 530 457 206 288 630 877 150 537 739 657 364 588 062 066 816 122 891 075 /
 90 032 974 329 026 116 303 104 557 422 897 975 801 678 639 627 325 227 969 703 815 609 309 598 131 373 574,
 85 124 783 072 911 313 494 998 832 646 036 815 124 006 285 652 084 977 035 260 467 /
 538 813 755 988 541 319 567 526 906 067 091 652 389 562 196 284 791 398 048 854 479 873 269 884 473 967 296,
 -18 934 713 102 040 383 920 087 843 204 803 562 500 925 378 741 927 773 177 718 375 /
 13 477 180 760 575 747 680 620 685 937 748 582 413 430 266 700 175 116 243 708 294 424 568 112 174 038 944,
 123 822 101 845 561 949 853 837 529 536 311 932 220 566 274 048 /
 489 658 696 282 788 201 308 076 521 518 998 458 897 393 539 385 353 164 203 484 986 748 825,
 27 428 051 252 064 994 559 778 396 981 830 980 523 444 494 914 656 855 259 607 859 974 756 912 745 347 967 398 284 383 677 /
 17 786 946 776 878 329 879 037 149 307 829 062 936 023 429 978 669 990 454 998 439 134 964 877 776 057 933 573 124 989 121
 220 896 816 449 282 128, 0, 0,
 308 658 420 716 010 399 150 932 576 651 /
 3 304 446 886 866 235 380 946 599 073 562 853 958 338 330 024 049 536,
 -785 432 938 444 970 332 684 288 /
 434 249 555 229 917 880 765 751 738 422 768 738 058 793 549 005,
 -2 967 538 772 568 126 751 276 914 755 895 103 679 548 886 053 186 098 176 /
 36 568 159 860 786 586 039 890 324 024 337 478 313 621 895 559 604 403 727 693 404 455 222 761 399 115,
 1 451 684 286 181 254 592 173 056 /
 4 091 410 471 824 703 029 588 727 501 290 131 621 312 094 549,
 25 199 964 284 375 802 561 540 764 012 286 173 481 712 926 358 468 408 092 670 125 652 150 209 440 521 372 953 976 337 /
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2]]]], 4]) & @@@ %
Tot
```

$$\left\{\frac{392\,411\,517\,513\,285\,444\,280\,743\,744\,710\,900\,339\,871\,880\,568\,595\,855\,414\,235\,477}{346\,288\,163\,727\,498\,428\,142\,404\,641\,499\,997\,657\,353\,813\,630\,208\,269\,638\,945\,238\,515\,232\,259\,027\,255\,425\,558\,400},\right.$$
$$\frac{98\,652\,026\,530\,457\,206\,288\,630\,877\,150\,537\,739\,657\,364\,588\,062\,066\,816\,122\,891\,075}{90\,032\,974\,329\,026\,116\,303\,104\,557\,422\,897\,975\,801\,678\,639\,627\,325\,227\,969\,703\,815\,609\,309\,598\,131\,373\,574},$$
$$\frac{85\,124\,783\,072\,911\,313\,494\,998\,832\,646\,036\,815\,124\,006\,285\,652\,084\,977\,035\,260\,467}{538\,813\,755\,988\,541\,319\,567\,526\,906\,067\,091\,652\,389\,562\,196\,284\,791\,398\,048\,854\,479\,873\,269\,884\,473\,967\,296},$$
$$-\frac{18\,934\,713\,102\,040\,383\,920\,087\,843\,204\,803\,562\,500\,925\,378\,741\,927\,773\,177\,718\,375}{13\,477\,180\,760\,575\,747\,680\,620\,685\,937\,748\,582\,413\,430\,266\,700\,175\,116\,243\,708\,294\,424\,568\,112\,174\,038\,944},$$
$$\frac{123\,822\,101\,845\,561\,949\,853\,837\,529\,536\,311\,932\,220\,566\,274\,048}{489\,658\,696\,282\,788\,201\,308\,076\,521\,518\,998\,458\,897\,393\,539\,385\,353\,164\,203\,484\,986\,748\,825},$$
$$27\,428\,051\,252\,064\,994\,559\,778\,396\,981\,830\,980\,523\,444\,494\,914\,656\,855\,259\,607\,859\,974\,756\,912\,745\,347\,967\,398\,284\,383\,677\,/$$
$$17\,786\,946\,776\,878\,329\,879\,037\,149\,307\,829\,062\,936\,023\,429\,978\,669\,990\,454\,998\,439\,134\,964\,877\,776\,057\,933\,573\,124\,989\,121\,\cdot$$
$$220\,896\,816\,449\,282\,128,\ 0,\ 0,\ \frac{308\,658\,420\,716\,010\,399\,150\,932\,576\,651}{3\,304\,446\,886\,866\,235\,380\,946\,599\,073\,562\,853\,958\,338\,330\,024\,049\,536},$$
$$-\frac{785\,432\,938\,444\,970\,332\,684\,288}{434\,249\,555\,229\,917\,880\,765\,751\,738\,422\,768\,738\,058\,793\,549\,005},$$
$$-\frac{2\,967\,538\,772\,568\,126\,751\,276\,914\,755\,895\,103\,679\,548\,886\,053\,186\,098\,176}{36\,568\,159\,860\,786\,586\,039\,890\,324\,024\,337\,478\,313\,621\,895\,559\,604\,403\,727\,693\,404\,455\,222\,761\,399\,115},$$
$$\frac{1\,451\,684\,286\,181\,254\,592\,173\,056}{4\,091\,410\,471\,824\,703\,029\,588\,727\,501\,290\,131\,621\,312\,094\,549},$$

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13 069 491 697 346 622 635 742 344 507 718 292 838 806 193 123 548 086 514 325 956 871 / 9 660 707 273 434 613 955 393 405 410 755 797 417 075 341 877 713 245 262 151 455 484 071 756 766 176 523 210,

0, 0, 6 253 657 566 957 647 158 160 891 254 913 / 259 841 820 113 010 069 107 180 236 445 899 300 138 338 934 347 770,

0, 0, 1 451 684 286 181 254 592 173 056 / 4 091 410 471 824 703 029 588 727 501 290 131 621 312 094 549,

− 40 050 344 306 541 743 152 872 547 480 087 084 566 546 273 894 771 480 371 / 19 062 185 737 982 015 100 212 635 184 649 286 800 641 364 173 300 176 364 981 938 121 681 249 677 350,

− 5 309 063 443 733 826 416 681 494 214 934 703 326 718 784 587 426 637 785 666 762 / 3 245 098 812 172 285 976 749 734 002 221 144 501 436 441 643 539 118 668 312 157 321 894 444 598 951 659,

− 2 124 192 760 686 461 674 444 273 682 625 / 69 664 070 715 653 273 297 627 629 166 381 137 494 726 899 864 696,

1 072 981 099 873 010 747 391 284 696 650 303 847 247 345 480 689 231 343 654 779 627 / 44 862 286 299 193 122 615 066 707 200 133 900 232 144 333 846 015 211 701 999 796 179 215 991 934 280 430 250,

392 411 517 513 285 444 280 743 744 710 900 339 871 880 568 595 855 414 235 477 / 346 288 163 727 498 428 142 404 641 499 997 657 353 813 630 208 269 638 945 238 515 232 259 027 255 425 558 400,

85 124 783 072 911 313 494 998 832 646 036 815 124 006 285 652 084 977 035 260 467 / 538 813 755 988 541 319 567 526 906 067 091 652 389 562 196 284 791 398 048 854 479 873 269 884 473 967 296,

− 18 934 713 102 040 383 920 087 843 204 803 562 500 925 378 741 927 773 177 718 375 / 13 477 180 760 575 747 680 620 685 937 748 582 413 430 266 700 175 116 243 708 294 424 568 112 174 038 944, 0, 0, 0}

251 278 395 999 653 103 770 719 478 827 429 763 111 561 039 304 189 366 752 819 688 584 596 391 263 368 553 765 235 994 631
593 451 /
7 321 452 401 165 780 304 847 586 733 098 666 683 194 940 877 771 044 369 101 069 611 872 122 787 297 171 023 537 164 281 181
200 921 105 980 881 868 105 792

randomAmp[8, 2]

4 536 274 648 196 345 215 631 264 208 070 323 457 643 606 647 428 045 074 322 731 752 228 946 392 252 725,

13 069 491 697 346 622 635 742 344 507 718 292 838 806 193 123 548 086 514 325 956 871 / 9 660 707 273 434 613 955 393 405 410 755 797 417 075 341 877 713 245 262 151 455 484 071 756 766 176 523 210,

0, 0, 6 253 657 566 957 647 158 160 891 254 913 / 259 841 820 113 010 069 107 180 236 445 899 300 138 338 934 347 770,

0, 0, 1 451 684 286 181 254 592 173 056 / 4 091 410 471 824 703 029 588 727 501 290 131 621 312 094 549,

− 40 050 344 306 541 743 152 872 547 480 087 084 566 546 273 894 771 480 371 / 19 062 185 737 982 015 100 212 635 184 649 286 800 641 364 173 300 176 364 981 938 121 681 249 677 350,

− 5 309 063 443 733 826 416 681 494 214 934 703 326 718 784 587 426 637 785 666 762 / 3 245 098 812 172 285 976 749 734 002 221 144 501 436 441 643 539 118 668 312 157 321 894 444 598 951 659,

− 2 124 192 760 686 461 674 444 273 682 625 / 69 664 070 715 653 273 297 627 629 166 381 137 494 726 899 864 696,

1 072 981 099 873 010 747 391 284 696 650 303 847 247 345 480 689 231 343 654 779 627 / 44 862 286 299 193 122 615 066 707 200 133 900 232 144 333 846 015 211 701 999 796 179 215 991 934 280 430 250,

392 411 517 513 285 444 280 743 744 710 900 339 871 880 568 595 855 414 235 477 / 346 288 163 727 498 428 142 404 641 499 997 657 353 813 630 208 269 638 945 238 515 232 259 027 255 425 558 400,

85 124 783 072 911 313 494 998 832 646 036 815 124 006 285 652 084 977 035 260 467 / 538 813 755 988 541 319 567 526 906 067 091 652 389 562 196 284 791 398 048 854 479 873 269 884 473 967 296,

− 18 934 713 102 040 383 920 087 843 204 803 562 500 925 378 741 927 773 177 718 375 / 13 477 180 760 575 747 680 620 685 937 748 582 413 430 266 700 175 116 243 708 294 424 568 112 174 038 944, 0, 0, 0}

251 278 395 999 653 103 770 719 478 827 429 763 111 561 039 304 189 366 752 819 688 584 596 391 263 368 553 765 235 994 631 \
593 451 /
7 321 452 401 165 780 304 847 586 733 098 666 683 194 940 877 771 044 369 101 069 611 872 122 787 297 171 023 537 164 281 181 \
200 921 105 980 881 868 105 792

randomAmp[8, 2]

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

$$\frac{30\,281\,292\,611\,361\,523\,607\,364\,951\,338\,256\,486\,562\,644\,566\,340\,339\,157\,172\,224}{4\,536\,274\,648\,196\,345\,215\,631\,264\,208\,070\,323\,457\,643\,606\,647\,428\,045\,074\,322\,731\,752\,228\,946\,392\,252\,725},$$

$$\frac{13\,069\,491\,697\,346\,622\,635\,742\,344\,507\,718\,292\,838\,806\,193\,123\,548\,086\,514\,325\,956\,871}{9\,660\,707\,273\,434\,613\,955\,393\,405\,410\,755\,797\,417\,075\,341\,877\,713\,245\,262\,151\,455\,484\,071\,756\,766\,176\,523\,210},$$

$$0,\ 0,\ \frac{6\,253\,657\,566\,957\,647\,158\,160\,891\,254\,913}{259\,841\,820\,113\,010\,069\,107\,180\,236\,445\,899\,300\,138\,338\,934\,347\,770},$$

$$0,\ 0,\ \frac{1\,451\,684\,286\,181\,254\,592\,173\,056}{4\,091\,410\,471\,824\,703\,029\,588\,727\,501\,290\,131\,621\,312\,094\,549},$$

$$-\frac{40\,050\,344\,306\,541\,743\,152\,872\,547\,480\,087\,084\,566\,546\,273\,894\,771\,480\,371}{19\,062\,185\,737\,982\,015\,100\,212\,635\,184\,649\,286\,800\,641\,364\,173\,300\,176\,364\,981\,938\,121\,681\,249\,677\,350},$$

$$-\frac{5\,309\,063\,443\,733\,826\,416\,681\,494\,214\,934\,703\,326\,718\,784\,587\,426\,637\,785\,666\,762}{3\,245\,098\,812\,172\,285\,976\,749\,734\,002\,221\,144\,501\,436\,441\,643\,539\,118\,668\,312\,157\,321\,894\,444\,598\,951\,659},$$

$$-\frac{2\,124\,192\,760\,686\,461\,674\,444\,273\,682\,625}{69\,664\,070\,715\,653\,273\,297\,627\,629\,166\,381\,137\,494\,726\,899\,864\,696},$$

$$\frac{1\,072\,981\,099\,873\,010\,747\,391\,284\,696\,650\,303\,847\,247\,345\,480\,689\,231\,343\,654\,779\,627}{44\,862\,286\,299\,193\,122\,615\,066\,707\,200\,133\,900\,232\,144\,333\,846\,015\,211\,701\,999\,796\,179\,215\,991\,934\,280\,430\,250},$$

$$\frac{392\,411\,517\,513\,285\,444\,280\,743\,744\,710\,900\,339\,871\,880\,568\,595\,855\,414\,235\,477}{346\,288\,163\,727\,498\,428\,142\,404\,641\,499\,997\,657\,353\,813\,630\,208\,269\,638\,945\,238\,515\,232\,259\,027\,255\,425\,558\,400},$$

$$\frac{85\,124\,783\,072\,911\,313\,494\,998\,832\,646\,036\,815\,124\,006\,285\,652\,084\,977\,035\,260\,467}{538\,813\,755\,988\,541\,319\,567\,526\,906\,067\,091\,652\,389\,562\,196\,284\,791\,398\,048\,854\,479\,873\,269\,884\,473\,967\,296},$$

$$-\frac{18\,934\,713\,102\,040\,383\,920\,087\,843\,204\,803\,562\,500\,925\,378\,741\,927\,773\,177\,718\,375}{13\,477\,180\,760\,575\,747\,680\,620\,685\,937\,748\,582\,413\,430\,266\,700\,175\,116\,243\,708\,294\,424\,568\,112\,174\,038\,944},\ 0,\ 0,\ 0\}$$

[295]=

251 278 395 999 653 103 770 719 478 827 429 763 111 561 039 304 189 366 752 819 688 584 596 391 263 368 553 765 235 994 631
593 451 /
7 321 452 401 165 780 304 847 586 733 098 666 683 194 940 877 771 044 369 101 069 611 872 122 787 297 171 023 537 164 281 181
200 921 105 980 881 868 105 792

$$\frac{118\,143\,767\,648\,880\,387\,261\,685\,685\,296\,294\,312\,260\,851\,455\,000\,829\,296\,817\,857}{124\,469\,930\,591\,072\,138\,448\,623\,710\,032\,056\,335\,742\,249\,503\,279\,692\,307\,769\,940\,700\,566\,865\,718\,031\,812\,496\,000},$$

$$\frac{30\,281\,292\,611\,361\,523\,607\,364\,951\,338\,256\,486\,562\,644\,566\,340\,339\,157\,172\,224}{4\,536\,274\,648\,196\,345\,215\,631\,264\,208\,070\,323\,457\,643\,606\,647\,428\,045\,074\,322\,731\,752\,228\,946\,392\,252\,725},$$

$$\frac{13\,069\,491\,697\,346\,622\,635\,742\,344\,507\,718\,292\,838\,806\,193\,123\,548\,086\,514\,325\,956\,871}{9\,660\,707\,273\,434\,613\,955\,393\,405\,410\,755\,797\,417\,075\,341\,877\,713\,245\,262\,151\,455\,484\,071\,756\,766\,176\,523\,210},$$

$$0,\ 0,\ \frac{6\,253\,657\,566\,957\,647\,158\,160\,891\,254\,913}{259\,841\,820\,113\,010\,069\,107\,180\,236\,445\,899\,300\,138\,338\,934\,347\,770},$$

$$0,\ 0,\ \frac{1\,451\,684\,286\,181\,254\,592\,173\,056}{4\,091\,410\,471\,824\,703\,029\,588\,727\,501\,290\,131\,621\,312\,094\,549},$$

$$-\frac{40\,050\,344\,306\,541\,743\,152\,872\,547\,480\,087\,084\,566\,546\,273\,894\,771\,480\,371}{19\,062\,185\,737\,982\,015\,100\,212\,635\,184\,649\,286\,800\,641\,364\,173\,300\,176\,364\,981\,938\,121\,681\,249\,677\,350},$$

$$-\frac{5\,309\,063\,443\,733\,826\,416\,681\,494\,214\,934\,703\,326\,718\,784\,587\,426\,637\,785\,666\,762}{3\,245\,098\,812\,172\,285\,976\,749\,734\,002\,221\,144\,501\,436\,441\,643\,539\,118\,668\,312\,157\,321\,894\,444\,598\,951\,659},$$

$$-\frac{2\,124\,192\,760\,686\,461\,674\,444\,273\,682\,625}{69\,664\,070\,715\,653\,273\,297\,627\,629\,166\,381\,137\,494\,726\,899\,864\,696},$$

$$\frac{1\,072\,981\,099\,873\,010\,747\,391\,284\,696\,650\,303\,847\,247\,345\,480\,689\,231\,343\,654\,779\,627}{44\,862\,286\,299\,193\,122\,615\,066\,707\,200\,133\,900\,232\,144\,333\,846\,015\,211\,701\,999\,796\,179\,215\,991\,934\,280\,430\,250},$$

$$\frac{392\,411\,517\,513\,285\,444\,280\,743\,744\,710\,900\,339\,871\,880\,568\,595\,855\,414\,235\,477}{346\,288\,163\,727\,498\,428\,142\,404\,641\,499\,997\,657\,353\,813\,630\,208\,269\,638\,945\,238\,515\,232\,259\,027\,255\,425\,558\,400},$$

$$\frac{85\,124\,783\,072\,911\,313\,494\,998\,832\,646\,036\,815\,124\,006\,285\,652\,084\,977\,035\,260\,467}{538\,813\,755\,988\,541\,319\,567\,526\,906\,067\,091\,652\,389\,562\,196\,284\,791\,398\,048\,854\,479\,873\,269\,884\,473\,967\,296},$$

$$-\frac{18\,934\,713\,102\,040\,383\,920\,087\,843\,204\,803\,562\,500\,925\,378\,741\,927\,773\,177\,718\,375}{13\,477\,180\,760\,575\,747\,680\,620\,685\,937\,748\,582\,413\,430\,266\,700\,175\,116\,243\,708\,294\,424\,568\,112\,174\,038\,944},\ 0,\ 0,\ 0\}$$

251 278 395 999 653 103 770 719 478 827 429 763 111 561 039 304 189 366 752 819 688 584 596 391 263 368 553 765 235 994 631
593 451 /
7 321 452 401 165 780 304 847 586 733 098 666 683 194 940 877 771 044 369 101 069 611 872 122 787 297 171 023 537 164 281 181

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```
residueList = rToResidue /@ amp;
dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

In[293]:=

```
randomComponentAmp[8, 2] /. ab[x__] ⧴ Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2]]]], 4]) & @@@ %
Total[%]
```

Out[294]=

{27 428 051 252 064 994 559 778 396 981 830 980 523 444 494 914 656 855 259 607 859 974 756 912 745 347 967 398 284 383 677 / 17 786 946 776 878 329 879 037 149 307 829 062 936 023 429 978 669 990 454 998 439 134 964 877 776 057 933 573 124 989 121 220 896 816 449 282 128,

$$\frac{118\,143\,767\,648\,880\,387\,261\,685\,685\,296\,294\,312\,260\,851\,455\,000\,829\,296\,817\,857}{124\,469\,930\,591\,072\,138\,448\,623\,710\,032\,056\,335\,742\,249\,503\,279\,692\,307\,769\,940\,700\,566\,865\,718\,031\,812\,496\,000},$$

$$\frac{30\,281\,292\,611\,361\,523\,607\,364\,951\,338\,256\,486\,562\,644\,566\,340\,339\,157\,172\,224}{4\,536\,274\,648\,196\,345\,215\,631\,264\,208\,070\,323\,457\,643\,606\,647\,428\,045\,074\,322\,731\,752\,228\,946\,392\,252\,725},$$

$$\frac{13\,069\,491\,697\,346\,622\,635\,742\,344\,507\,718\,292\,838\,806\,193\,123\,548\,086\,514\,325\,956\,871}{9\,660\,707\,273\,434\,613\,955\,393\,405\,410\,755\,797\,417\,075\,341\,877\,713\,245\,262\,151\,455\,484\,071\,756\,766\,176\,523\,210},$$

$$0,\ 0,\ \frac{6\,253\,657\,566\,957\,647\,158\,160\,891\,254\,913}{259\,841\,820\,113\,010\,069\,107\,180\,236\,445\,899\,300\,138\,338\,934\,347\,770},$$

$$0,\ 0,\ \frac{1\,451\,684\,286\,181\,254\,592\,173\,056}{4\,091\,410\,471\,824\,703\,029\,588\,727\,501\,290\,131\,621\,312\,094\,549},$$

$$-\frac{40\,050\,344\,306\,541\,743\,152\,872\,547\,480\,087\,084\,566\,546\,273\,894\,771\,480\,371}{19\,062\,185\,737\,982\,015\,100\,212\,635\,184\,649\,286\,800\,641\,364\,173\,300\,176\,364\,981\,938\,121\,681\,249\,677\,350},$$

$$-\frac{5\,309\,063\,443\,733\,826\,416\,681\,494\,214\,934\,703\,326\,718\,784\,587\,426\,637\,785\,666\,762}{3\,245\,098\,812\,172\,285\,976\,749\,734\,002\,221\,144\,501\,436\,441\,643\,539\,118\,668\,312\,157\,321\,894\,444\,598\,951\,659},$$

$$-\frac{2\,124\,192\,760\,686\,461\,674\,444\,273\,682\,625}{69\,664\,070\,715\,653\,273\,297\,627\,629\,166\,381\,137\,494\,726\,899\,864\,696},$$

$$\frac{1\,072\,981\,099\,873\,010\,747\,391\,284\,696\,650\,303\,847\,247\,345\,480\,689\,231\,343\,654\,779\,627}{44\,862\,286\,299\,193\,122\,615\,066\,707\,200\,133\,900\,232\,144\,333\,846\,015\,211\,701\,999\,796\,179\,215\,991\,934\,280\,430\,250},$$

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

13 477 180 760 575 747 680 620 685 937 748 582 413 430 266 700 175 116 243 708 294 424 568 112 174 038 944

179 425 876 533 471 793 185 249 341 845 285 066 675 465 746 462 436 730 328 237 031 939 / 7 479 396 498 062 561 224 738 817 563 164 533 876 567 722 116 826 949 273 489 192 033 737 306 968 319 561 784 550,

− 27 692 017 220 799 271 774 626 964 739 47[illegible] 757 808 849 758 338 333 612 580 528 157 / 88 227 051 707 316 888 434 606 756 486 028 755 315 834 856 642 154 777 235 426 683 861 543 968 448 348 230,

9 781 015 528 710 284 470 516 058 / 840 497 889 712 713 271 379 114 837 355 427 370 307 314 743,

− 2 124 192 760 686 461 674 444 273 682 625 / 69 664 070 715 653 273 297 627 629 166 381 137 494 726 899 864 696,

154 903 652 766 418 348 912 864 728 240 502 991 668 255 616 180 499 440 594 725 / 3 232 027 826 620 810 240 601 430 055 086 857 418 208 229 732 375 529 566 250 280 329 709 912 565 373 178,

− 785 432 938 444 970 332 684 288 / 434 249 555 229 917 880 765 751 738 422 768 738 058 793 549 005, 0, 0,

4 438 844 353 122 869 502 924 901 512 848 854 806 616 173 733 832 038 605 663 / 75 876 937 341 346 673 932 176 517 101 073 249 849 208 343 446 904 250 333 206 092 938 678 750 388 112 250, 0, 0, 0, 0,

0, 392 411 517 513 285 444 280 743 744 710 900 339 871 880 568 595 855 414 235 477 / 346 288 163 727 498 428 142 404 641 499 997 657 353 813 630 208 269 638 945 238 515 232 259 027 255 425 558 400, 0}

Out[298]=

251 278 395 999 653 103 770 719 478 827 429 763 111 561 039 304 189 366 752 819 688 584 596 391 263 368 553 765 235 994 631 593 451 /
7 321 452 401 165 780 304 847 586 733 098 666 683 194 940 877 771 044 369 101 069 611 872 122 787 297 171 023 537 164 281 181 200 921 105 980 881 868 105 792

In[285]:=

```
randomAmp[8, 2]
```

Out[285]=

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
```

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Tree-Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

In[296]:=

```
randomComponentAmp[8, 2] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2]]]], 4]) & @@@ %
Total[%]
```

Out[297]=

$$\Big\{\frac{118143767648880387261685685296294312260851455000829296817857}{124469930591072138448623710032056335742249503279692307769940700566865718031812496000},$$

27428051252064994559778396981830980523444494914656855259607859974756912745347967398284383677 / 17786946776878329879037149307829062936023429978669990454998439134964877776057933573124989121 220896816449282128,

$$\frac{85124783072911313494998832646036815124006285652084977035260467}{538813755988541319567526906067091652389562196284791398048854479873269884473967296},$$

$$-\frac{18934713102040383920087843204803562500925378741927773177718375}{13477180760575747680620685937748582413430266700175116243708294424568112174038944},$$

$$\frac{179425876533471793185249341845285066675465746462436730328237031939}{7479396498062561224738817563164533876567722116826949273489192033737306968319561784550},$$

$$-\frac{27692017220799271774626964739472757808849758338333612580528157}{88227051707316888434606756486028755315834856642154777235426683861543968448348230},$$

$$\frac{9781015528710284470516058}{840497889712713271379114837355427370307314743},$$

$$-\frac{2124192760686461674444273682625}{69664070715653273297627629166381137494726899864696},$$

$$\frac{154903652766418348912864728240502991668255616180499440594725}{3232027826620810240601430055086857418208229732375529566250280329709912565373178},$$

$$-\frac{785432938444970332684288}{434249555229917880765751738422768738058793549005}, 0, 0,$$

$$\frac{4438844353122869502924901512848854806616173733832038605663}{75876937341346673932176517101073249849208343446904250333206092938678750388112250}, 0, 0, 0, 0,$$

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8,] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2]]]], 4]) & @@@ %
Total[%]
```

$\left\{\frac{118143767648880387261685685296294312260851455000829296817857}{124469930591072138448623710032056335742249503279692307769940700566865718031812496000},\right.$
$27428051252064994559778396981830980523444494914656855259607859974756912745347967398284383677 /$
$17786946776878329879037149307829062936023429978669990454998439134964877776057933573124989121 \cdot$
$220896816449282128,$
$\frac{85124783072911313494998832646036815124006285652084977035260467}{538813755988541319567526906067091652389562196284791398048854479873269884473967296},$
$-\frac{18934713102040383920087843204803562500925378741927773177718375}{13477180760575747680620685937748582413430266700175116243708294424568112174038944},$
$\frac{179425876533471793185249341845285066675465746462436730328237031939}{7479396498062561224738817563164533876567722116826949273489192033737306968319561784550},$
$-\frac{27692017220799271774626964739472757808849758338333612580528157}{88227051707316888434606756486028755315834856642154777235426683861543968448348230},$
$\frac{9781015528710284470516058}{840497889712713271379114837355427370307314743},$
$-\frac{2124192760686461674444273682625}{69664070715653273297627629166381137494726899864696},$
$\frac{154903652766418348912864728240502991668255616180499440594725}{3232027826620810240601430055086857418208229732375529566250280329709912565373178},$
$-\frac{785432938444970332684288}{434249555229917880765751738422768738058793549005}, 0, 0,$

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{\text{Det}[\{\{-44580815043, 70817316822\}\}]^4}{67557694037546450970244888951330282474279782192225780},\right.$$
$$-\frac{\text{Det}[\{\{-127852198423, 110543205402\}\}]^4}{1453389260910182904147176607840249753978836771386058820},$$
$$\frac{\text{Det}[\{\{8618199744, 0\}\}]^4}{45572939926359388962261272814969953391569366895427584},$$
$$-\frac{\text{Det}[\{\{-28939793344, 0\}\}]^4}{346885124011646243917330037773474829366877651025133568},$$
$$-\frac{\text{Det}[\{\{15029929321, 0\}\}]^4}{11758605456552595701828798297130598370182356344217760},$$
$$\frac{\text{Det}[\{\{-22425075541, 0\}\}]^4}{28843409081676347862026611434491460901930920691644960},$$
$$-\frac{\text{Det}[\{\{-43128971807, 0\}\}]^4}{21355039658643964545541618149081585819488795933207712},$$
$$\frac{\text{Det}[\{\{2729395557, -9458894391\}\}]^4}{6022664056770143041547166279392873222030351648217},$$
$$-\frac{\text{Det}[\{\{-68455004901, -29042737772\}\}]^4}{126704339299220881023337403783762805868625586392492544},$$

Det[{{55619401905, 0}}]^4

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```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] → Det[Zs[[{x}]]];
   (#1 * Power[Det[#2[[All, Range12]]]], 4]) & @@@ %
Total[%]
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 7, 8] R[5, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
 R[5, cap[{1, 2}, {6, 5, 7}], 2, 3, 4] R[7, 1, 2, 5, 6],
 R[5, cap[{6, 5}, {2, 1, 7}], cap[{1, 2}, {6, 5, 7}], 3, 4] R[7, 1, 2, 5, 6],
 R[1, 2, 3, cap[{3, 4}, {6, 5, 7}], 7] R[7, 3, 4, 5, 6], R[1, 2, 3, 7, 8] R[7, cap[{2, 3}, {8, 7, 1}], 4, 5, 6],
 R[1, 2, 3, 7, 8] R[7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 5, 6],
 R[1, 4, 5, 7, 8] R[7, cap[{8, 7}, {5, 4, 1}], cap[{4, 5}, {8, 7, 1}], 5, 6],
 R[7, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 7}], 3, 4, 5, cap[{6, 5}, {3, 2, 7}]],
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {4, 3, 1}]],
 R[5, 7, 1, 3, 4] R[cap[{4, 3}, {1, 7, 5}], cap[{7, 1}, {4, 3, 5}], 1, 2, 3],
 R[7, 1, 2, 5, 6] R[cap[{6, 5}, {2, 1, 7}], cap[{1, 2}, {6, 5, 7}], 2, 3, 5]}
```



```
Position[componentAmp[10, 4], _cap, {0, ∞}][[1]]
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{3281559236759737874553696514241}{56125865213366151018870459576590312908980}, -\frac{2089895642256092154739987759512967}{11367729916552808497240998653048634515765340}, \frac{278917946890969578368}{2304184674000177182626359858396297}, -\frac{285152270499889294891136}{141020031562167247833971785445700363}, \frac{93200099010948601135823649301}{10629854121602518464652269180296657139360}, -\frac{80224554509493937480823290894943}{495143722744115093565314545572458650648416}, \frac{10320107186279720718552268787913}{192937306114982582194854769408702803554008}, -\frac{281439936375928282250523920523}{43084003925014836379134141493736641416704}, 0, -\frac{28059856428728218104630234241}{6465669626030186999294422722555734195360}\right\}$$

-6762027049495337690860528538881935487482145687250413431872800667779831626018117599929559 /
19055950470021039981194189659660974347347923502812076243282444380104452381875798168029552374 \
212096

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{2089895642256092154739987759512967}{11367729916552808497240998653048634515765340}, \frac{24421745729124374496801186625}{9162818220527311518803886119192651847432}, -\frac{285152270499889294891136}{141020031562167247833971785445700363}, -\frac{28059856428728218104630234241}{6465669626030186999294422722555734195360}, \frac{93200099010948601135823649301}{10629854121602518464652269180296657139360}, -\frac{80224554509493937480823290894943}{495143722744115093565314545572458650648416}, 0, 0, -\frac{133061547892557232777155712}{17688032921885919900057721696360455735}, -\frac{281439936375928282250523920523}{43084003925014836379134141493736641416704}\right\}$$

$-6762027049495337690860528538881935487482145687250413431872800667779831626018117599929559 / 19055950470021039981194189659660974347347923502812076243282444380104452381875798168029552374212096$

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{2089895642256092154739987759512967}{11367729916552808497240998653048634515765340}, -\frac{285152270499889294891136}{141020031562167247833971785445700363}, \frac{93200099010948601135823649301}{10629854121602518464652269180296657139360}, -\frac{80224554509493937480823290894943}{495143722744115093565314545572458650648416}, 0, -\frac{133061547892557232777155712}{17688032921885919900057721696360455735}, -\frac{281439936375928282250523920523}{43084003925014836379134141493736641416704}, 0, \frac{24421745729124374496801186625}{9162818220527311518803886119192651847432}, -\frac{28059856428728218104630234241}{6465669626030186999294422722555734195360}\right\}$$

-6762027049495337690860528538881935487482145687250413431872800667779831626018117599929559 /
 19055950470021039981194189659660974347347923502812076243282444380104452381875798168029552374 ⋯
 212096

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
In[335]:= randomComponentAmp[8, 1] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

Out[336]=

$$\left\{-\frac{2089895642256092154739987759512967}{11367729916552808497240998653048634515765340}, -\frac{285152270499889294891136}{141020031562167247833971785445700363}, \frac{93200099010948601135823649301}{10629854121602518464652269180296657139360}, -\frac{80224554509493937480823290894943}{495143722744115093565314545572458650648416}, -\frac{133061547892557232777155712}{17688032921885919900057721696360455735}, \frac{277811784256862427510719693}{19805815643088200899891760094251745246720}, -\frac{323802397070075492282991935928}{39387740793219072417786143691108545358629}, 0, 0, 0\right\}$$

Out[337]=

-6762027049495337690860528538881935487482145687250413431872800667779831626018117599929559 / 19055950470021039981194189659660974347347923502812076243282444380104452381875798168029552374212096

```
In[285]:= randomAmp[8, 2]
```

```
Out[285]= {R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{2089895642256092154739987759512967}{11367729916552808497240998653048634515765340}, -\frac{285152270499889294891136}{141020031562167247833971785445700363}, \frac{93200099010948601135823649301}{10629854121602518464652269180296657139360}, -\frac{80224554509493937480823290894943}{495143722744115093565314545572458650648416}, 0, -\frac{133061547892557232777155712}{17688032921885919900057721696360455735}, 0, \frac{277811784256862427510719693}{19805815643088200899891760094251745246720}, -\frac{323802397070075492282991935928}{39387740793219072417786143691108545358629}, 0\right\}$$

```
-6762027049495337690860528538881935487482145687250413431872800667779831626018117599929559 /
  19055950470021039981194189659660974347347923502812076243282444380104452381875798168029552374
  212096
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{2089895642256092154739987759512967}{11367729916552808497240998653048634515765340}, -\frac{285152270499889294891136}{141020031562167247833971785445700363}, \frac{93200099010948601135823649301}{10629854121602518464652269180296657139360}, -\frac{80224554509493937480823290894943}{495143722744115093565314545572458650648416}, -\frac{133061547892557232777155712}{17688032921885919900057721696360455735}, \frac{24421745729124374496801186625}{9162818220527311518803886119192651847432}, -\frac{281439936375928282250523920523}{43084003925014836379134141493736641416704}, 0, -\frac{28059856428728218104630234241}{6465669626030186999294422722555734195360}, 0\right\}$$

-6 762 027 049 495 337 690 860 528 538 881 935 487 482 145 687 250 413 431 872 800 667 779 831 626 018 117 599 929 559 /
19 055 950 470 021 039 981 194 189 659 660 974 347 347 923 502 812 076 243 282 444 380 104 452 381 875 798 168 029 552 374 ·
212 096

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{3281559236759737874553696514241}{56125865213366151018870459576590312908980}, -\frac{2089895642256092154739987759512967}{11367729916552808497240998653048634515765340}, -\frac{281439936375928282250523920523}{43084003925014836379134141493736641416704}, \frac{278917946890969578368}{2304184674000177182626359858396297}, -\frac{285152270499889294891136}{141020031562167247833971785445700363}, -\frac{28059856428728218104630234241}{6465669626030186999294422722555734195360}, \frac{93200099010948601135823649301}{10629854121602518464652269180296657139360}, -\frac{80224554509493937480823290894943}{495143722744115093565314545572458650648416}, 0, \frac{10320107186279720718552268787913}{192937306114982582194854769408702803554008}\right\}$$

−6762027049495337690860528538881935487482145687250413431872800667779831626018117599929559 /
19055950470021039981194189659660974347347923502812076243282444380104452381875798168029552374
212096

```
randomAmp[8, 2]
```

{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{2089895642256092154739987759512967}{11367729916552808497240998653048634515765340}, \frac{24421745729124374496801186625}{9162818220527311518803886119192651847432}, -\frac{285152270499889294891136}{141020031562167247833971785445700363}, -\frac{28059856428728218104630234241}{6465669626030186999294422722555734195360}, \frac{93200099010948601135823649301}{10629854121602518464652269180296657139360}, -\frac{80224554509493937480823290894943}{495143722744115093565314545572458650648416}, 0, 0, -\frac{133061547892557232777155712}{17688032921885919900057721696360455735}, -\frac{281439936375928282250523920523}{43084003925014836379134141493736641416704}\right\}$$

-6762027049495337690860528538881935487482145687250413431872800667779831626018117599929559 / 19055950470021039981194189659660974347347923502812076243282444380104452381875798168029552374 212096

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{2089895642256092154739987759512967}{11367729916552808497240998653048634515765340}, -\frac{285152270499889294891136}{141020031562167247833971785445700363}, \frac{93200099010948601135823649301}{10629854121602518464652269180296657139360}, -\frac{80224554509493937480823290894943}{495143722744115093565314545572458650648416}, -\frac{133061547892557232777155712}{17688032921885919900057721696360455735}, \frac{277811784256862427510719693}{19805815643088200899891760094251745246720}, -\frac{323802397070075492282991935928}{39387740793219072417786143691108545358629}, 0, 0, 0\right\}$$

-6762027049495337690860528538881935487482145687250413431872800667779831626018117599929559 / 19055950470021039981194189659660974347347923502812076243282444380104452381875798168029552374212096

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
In[386]:= randomComponentAmp[8, 1] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

```
Out[387]= {-2089895642256092154739987759512967/11367729916552808497240998653048634515765340,
 -285152270499889294891136/141020031562167247833971785445700363, 93200099010948601135823649301/10629854121602518464652269180296657139360,
 -80224554509493937480823290894943/495143722744115093565314545572458650648416, 0, -133061547892557232777155712/17688032921885919900057721696360455735,
 277811784256862427510719693/19805815643088200899891760094251745246720, -323802397070075492282991935928/39387740793219072417786143691108545358629, 0, 0}
```

```
Out[388]= -6762027049495337690860528538881935487482145687250413431872800667779831626018117599929559/
 19055950470021039981194189659660974347347923502812076243282444380104452381875798168029552374
 212096
```

```
In[285]:= randomAmp[8, 2]

Out[285]= {R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{2\,089\,895\,642\,256\,092\,154\,739\,987\,759\,512\,967}{11\,367\,729\,916\,552\,808\,497\,240\,998\,653\,048\,634\,515\,765\,340}, -\frac{285\,152\,270\,499\,889\,294\,891\,136}{141\,020\,031\,562\,167\,247\,833\,971\,785\,445\,700\,363}, \frac{93\,200\,099\,010\,948\,601\,135\,823\,649\,301}{10\,629\,854\,121\,602\,518\,464\,652\,269\,180\,296\,657\,139\,360}, -\frac{80\,224\,554\,509\,493\,937\,480\,823\,290\,894\,943}{495\,143\,722\,744\,115\,093\,565\,314\,545\,572\,458\,650\,648\,416}, 0, 0, -\frac{133\,061\,547\,892\,557\,232\,777\,155\,712}{17\,688\,032\,921\,885\,919\,900\,057\,721\,696\,360\,455\,735}, -\frac{281\,439\,936\,375\,928\,282\,250\,523\,920\,523}{43\,084\,003\,925\,014\,836\,379\,134\,141\,493\,736\,641\,416\,704}, -\frac{28\,059\,856\,428\,728\,218\,104\,630\,234\,241}{6\,465\,669\,626\,030\,186\,999\,294\,422\,722\,555\,734\,195\,360}, \frac{24\,421\,745\,729\,124\,374\,496\,801\,186\,625}{9\,162\,818\,220\,527\,311\,518\,803\,886\,119\,192\,651\,847\,432}\right\}$$

```
-6 762 027 049 495 337 690 860 528 538 881 935 487 482 145 687 250 413 431 872 800 667 779 831 626 018 117 599 929 559 /
  19 055 950 470 021 039 981 194 189 659 660 974 347 347 923 502 812 076 243 282 444 380 104 452 381 875 798 168 029 552 374 
   212 096
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2]]]], 4]) & @@@ %
Total[%]
```

$$\left\{-\frac{\text{Det}[\{\{-127852198423, 110543205402\}\}]^4}{1453389260910182904147176607840249753978836771386058820}, -\frac{\text{Det}[\{\{-28939793344, 0\}\}]^4}{346885124011646243917330037773474829366877651025133568}, \frac{\text{Det}[\{\{-22425075541, 0\}\}]^4}{28843409081676347862026611434491460901930920691644960}, -\frac{\text{Det}[\{\{-43128971807, 0\}\}]^4}{21355039658643964545541618149081585819488795933207712}, \frac{\text{Det}[\{\{-71084207583, -18264290948\}\}]^4}{79224282500695618369613175751661562764345317636448520}, -\frac{\text{Det}[\{\{-68455004901, -29042737772\}\}]^4}{126704339299220881023337403783762805868625586392492544}, \frac{\text{Det}[\{\{0, 0\}\}]^4}{24811540635969114676914131193847425241953924622909440}, -\frac{\text{Det}[\{\{-62309553152, 0\}\}]^4}{96331435296669122850820863197761198815205901320847360}, \frac{\text{Det}[\{\{-14283704478, -37986938749\}\}]^4}{5063425645917658830560444047383347203108036469165958}, \ldots\right.$$

Det[{{0, -55271250051}}]^4

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
In[404]:= randomComponentAmp[8, 1] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1] + 2]]], 4]) & @@@ %
Total[%]
```

Out[405]=
$$\left\{-\frac{117223503167785040342554805895817}{1380394600875387319669821196987336230883860}, -\frac{1439491432803852938048328828401}{1589134171517622626689629625114318464545220}, \frac{3792147582537872007373}{5466620251148597337051588673953792}, -\frac{430370422650056385673}{14207712973867686612443505816920064}, 0, 0, 0, \frac{33253495816157595936142314996778}{385131050814567309530219697725293836471183}, 0, \frac{93813312125309427416841306841}{17430498543340279820617291333265619945915}\right\}$$

Out[406]=
$$\frac{216167251224053621483045351956347636453288786182384670392322349070562534709237003361}{32938552505448153867943488765017061254412215934355567158635833691494422859747274320236057507840}$$

```
In[285]:= randomAmp[8, 2]
```

```
Out[285]= {R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 7, 8] R[5, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
 R[5, cap[{1, 2}, {6, 5, 7}], 2, 3, 4] R[7, 1, 2, 5, 6],
 R[5, cap[{6, 5}, {2, 1, 7}], cap[{1, 2}, {6, 5, 7}], 3, 4] R[7, 1, 2, 5, 6],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
In[416]:= randomComponentAmp[8, 1] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1] + 2]]], 4]) & @@@ %
Total[%]
```

Out[417]= $\left\{-\frac{1439491432803852938048328828401}{1589134171517622626689629625114318464545220}, -\frac{430370422650056385673}{14207712973867686612443505816920064}, 0, 0, -\frac{10004889051961334748725018853376}{367508521521100728115316245375410979645659}, \frac{331849512475182155320871547}{5736758214571237264874702900759649859}, -\frac{117223503167785040342554805895817}{1380394600875387319669821196987336230883860}, -\frac{26046083015983612064514625}{1221510089637195039340903314399494144}, \frac{54815438106415455287604503003749}{676210559392303506662295456699424532275896}, \frac{28059856428728218104630234241}{13643048510961121904769787789671895203840}\right\}$

Out[418]= $\frac{216167251224053621483045351956347636453288786182384670392322349070562534709237003361}{32938552505448153867943488765017061254412215934355567158635833691494422859747274320236057507840}$

```
In[285]:= randomAmp[8, 2]
```

```
Out[285]= {R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 7, 8] R[5, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 1] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1] + 2]]], 4]) & @@@ %
Total[%]
```

$$\left\{\frac{85969756958464433435226069576617}{1795049441709462130950067636587169502823240}, -\frac{1439491432803852938048328828401}{1589134171517622626689629625114318464545220}, -\frac{430370422650056385673}{14207712973867686612443505816920064}, 0, 0, -\frac{10004889051961334748725018853376}{367508521521100728115316245375410979645659}, \frac{28059856428728218104630234241}{13643048510961121904769787789671895203840}, -\frac{26046083015983612064514625}{1221510089637195039340903314399494144}, 0, \frac{107934132356691830760997311976}{17706109009780416841152830842855536781929}\right\}$$

$$\frac{216167251224053621483045351956347636453288786182384670392322349070562534709237003361}{32938552505448153867943488765017061254412215934355567158635833691494422859747274320236057507840}$$

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 7, 8] R[5, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[1] + 2]]], 4]) & @@@ %;
Total[%]
```

```
-Det[{{-2 119 735 161 157 471 370 556}, {0}}]^4 /
   8 599 319 081 631 814 440 480 742 515 545 616 327 914 872 987 460 056 171 865 507 682 416 203 217 365 567 307 135 828 774
    191 955 186 884 378 903 017 621 203 867 427 565 266 769 465 693 298 856 960 +
 Det[{{-436 324 969 867 995 745 152}, {0}}]^4 /
  1 668 344 340 427 089 614 641 303 343 174 898 910 112 670 049 209 375 530 934 395 571 231 242 341 342 199 216 686 503 299
   982 323 531 375 601 409 631 053 299 122 699 853 892 676 296 846 226 227 200 +
 Det[{{-224 468 048 173 131 798 291}, {0}}]^4 /
  686 129 217 433 574 565 042 932 685 176 679 363 115 599 123 142 089 175 082 258 090 891 012 870 825 001 668 964 967 241 078
   099 252 352 941 732 280 500 702 984 418 167 219 172 307 763 975 904 -
 Det[{{-33 873 308 403}, {-154 579 488 291 146 201 877 074 188 525 290}}]^4 /
  10 576 647 283 640 037 110 942 788 886 111 755 721 059 508 604 938 983 130 139 028 390 200 114 328 852 755 231 794 240 867
   135 041 497 224 762 685 148 517 518 740 364 209 056 391 184 065 823 480 649 962 356 005 745 709 752 546 234 187 640 161
   863 007 920 000 + Det[{{-33 873 308 403}, {-2 372 788 665 177 564 812 090}}]^4 /
  75 656 023 305 086 437 554 203 949 528 289 984 008 418 207 568 405 750 378 657 576 267 913 032 334 967 706 544 908 531 234
   067 041 822 057 827 593 451 554 764 494 982 896 016 170 966 847 662 325 760 000 +
 Det[{{-33 873 308 403}, {-1 578 966 478 013 021 190 156}}]^4 /
  275 237 996 887 537 926 296 305 270 969 677 714 225 376 337 641 895 099 595 312 056 921 425 197 512 150 153 665 710 714 523
   608 351 245 201 638 583 976 125 091 852 279 209 087 737 127 633 965 438 560 -
 Det[{{-33 873 308 403}, {15 029 929 321}}]^4 /
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2] + 2]]], 4]) & @@@ %;
Total[%]
```

```
-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
  666 677 /
 399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
  777 071 100 419 203 147 671 429 120
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 7, 8] R[5, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
 R[5, cap[{1, 2}, {6, 5, 7}], 2, 3, 4] R[7, 1, 2, 5, 6],
 R[5, cap[{6, 5}, {2, 1, 7}], cap[{1, 2}, {6, 5, 7}], 3, 4] R[7, 1, 2, 5, 6],
 R[1, 2, 3, cap[{3, 4}, {6, 5, 7}], 7] R[7, 3, 4, 5, 6], R[1, 2, 3, 7, 8] R[7, cap[{2, 3}, {8, 7, 1}], 4, 5, 6],
 R[1, 2, 3, 7, 8] R[7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 5, 6],
 R[1, 4, 5, 7, 8] R[7, cap[{8, 7}, {5, 4, 1}], cap[{4, 5}, {8, 7, 1}], 5, 6],
 R[7, 2, 3, 5, 6] R[cap[{2, 3}, {6, 5, 7}], 3, 4, 5, cap[{6, 5}, {3, 2, 7}]],
 R[1, 3, 4, 7, 8] R[cap[{3, 4}, {8, 7, 1}], 5, 6, 7, cap[{8, 7}, {4, 3, 1}]],
```

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

In[286]:=
```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

In[452]:=
```
randomComponentAmp[8, 2] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2] + 2]]], 4]) & @@@ %
Total[%]
```

Out[452]=

$$\left\{\frac{8\,569\,099\,634\,208\,439\,795\,037\,766\,767\,123\,999\,289\,579\,332\,828\,809\,444\,497\,020\,416}{45\,634\,495\,425\,305\,225\,187\,793\,686\,302\,931\,906\,936\,781\,652\,122\,416\,281\,354\,362\,753\,380\,314\,457\,202\,360\,842\,375},\right.$$

$$0,\ 0,\ 0,\ -\frac{995\,548\,537\,425\,546\,465\,962\,614\,641}{896\,849\,031\,449\,170\,788\,720\,359\,179\,982\,433\,292\,877\,335\,042\,578\,680},$$

$$-\frac{6\,763\,400\,819\,928\,264\,763\,184\,688\,333\,597\,359\,740\,434\,098\,425\,696\,402\,648\,523}{19\,850\,579\,599\,167\,722\,758\,652\,351\,710\,660\,318\,020\,407\,758\,979\,804\,817\,891\,556\,845\,131\,406\,978\,573\,926\,400},$$

$$\frac{14\,387\,543\,317\,971\,104\,492\,989\,071\,156\,778\,926\,884\,517\,832\,719\,823\,404\,327\,951}{258\,249\,775\,250\,607\,230\,910\,945\,826\,028\,266\,296\,738\,806\,710\,626\,278\,099\,481\,531\,237\,019\,330\,947\,704\,422\,400},$$

$$\frac{3\,269\,539\,649\,319\,328\,882\,270\,974\,169\,718\,489\,070\,879\,090\,374\,680\,277\,323\,789}{8\,099\,256\,120\,500\,300\,653\,922\,672\,758\,891\,030\,603\,799\,048\,304\,612\,776\,013\,391\,960\,575\,802\,763\,846\,285\,685\,450},$$

$$\frac{6\,425\,657\,682\,510\,916\,936\,232\,605\,503\,359\,729\,806\,617\,499\,467\,907\,156\,031\,254\,417\,933}{11\,801\,022\,514\,125\,483\,150\,551\,440\,896\,695\,579\,938\,132\,580\,843\,202\,305\,824\,943\,072\,269\,820\,736\,102\,459\,443\,916\,000},$$

$$-\frac{54\,815\,438\,106\,415\,455\,287\,604\,503\,003\,749}{132\,367\,068\,249\,053\,312\,378\,419\,363\,016\,225\,953\,651\,626\,120\,701\,245\,440},$$

$$-\frac{125\,584\,889\,331\,825\,247\,552\,511\,080\,801\,226\,333\,790\,860\,285\,998\,229}{176\,288\,716\,398\,507\,232\,373\,693\,350\,561\,851\,847\,256\,608\,210\,563\,082\,056\,929\,984\,423\,853\,834\,240},$$

$$-\frac{117\,223\,503\,167\,785\,040\,342\,554\,805\,895\,817}{62\,319\,638\,990\,360\,538\,149\,423\,339\,406\,334\,360\,087\,347\,486\,032\,171\,520},$$

$$\frac{1\,081\,511\,219\,236\,553\,672\,463\,056\,971}{1\,623\,989\,223\,058\,476\,081\,535\,916\,246\,777\,755\,467\,917\,460\,946\,944},\ 0,$$

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[8, 2] /. ab[x__] :> Det[zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2] + 2]]], 4]) & @@@ %
Total[%]
```

$$\Big\{\frac{812\,053\,555\,053\,250\,981\,916\,891\,927\,374\,576\,162\,270\,382\,102\,098\,597\,337\,595\,392}{5\,129\,502\,557\,558\,451\,419\,319\,069\,019\,490\,027\,986\,908\,019\,364\,738\,058\,384\,843\,910\,741\,140\,557\,650\,480\,174\,525},$$

$$-\frac{48\,841\,568\,020\,743\,401\,496\,319\,727\,816\,727\,010\,025\,917\,160\,805\,205\,708\,769\,477}{121\,263\,682\,383\,725\,894\,149\,744\,237\,338\,540\,647\,716\,490\,316\,663\,520\,666\,988\,474\,574\,983\,740\,516\,049\,848\,320},$$

$$\frac{85\,969\,756\,958\,464\,433\,435\,226\,069\,576\,617}{1\,933\,649\,027\,499\,669\,488\,439\,285\,025\,095\,286\,237\,540\,840\,852\,415\,692\,800},$$

$$-\frac{3\,549\,043\,131\,144\,998\,607\,689\,619\,654\,387\,023\,116\,341\,121\,527\,689\,695\,824\,011}{12\,396\,369\,798\,966\,017\,859\,012\,954\,928\,870\,311\,511\,259\,691\,337\,573\,136\,788\,660\,581\,826\,082\,424\,731\,811\,840},$$

$$-\frac{3\,115\,514\,278\,166\,483\,807\,751\,729\,289\,714\,885\,729\,083\,137\,908\,677\,122}{1\,782\,456\,717\,519\,593\,950\,034\,747\,416\,713\,762\,113\,449\,646\,428\,306\,177\,158\,231\,830\,630\,201\,143\,905},$$

$$-\frac{1\,440\,352\,269\,468\,710\,231\,560\,291}{2\,336\,647\,662\,866\,925\,004\,079\,289\,565\,583\,441\,711\,056\,590\,320},$$

$$\frac{1\,081\,511\,219\,236\,553\,672\,463\,056\,971}{1\,623\,989\,223\,058\,476\,081\,535\,916\,246\,777\,755\,467\,917\,460\,946\,944}, 0, 0, 0,$$

$$\frac{86\,763\,755\,760\,272\,148\,388\,928\,475\,443\,199\,237\,342\,573\,336\,548\,760\,184\,797\,653}{71\,388\,120\,433\,567\,462\,914\,153\,120\,530\,632\,681\,997\,723\,269\,048\,064\,845\,865\,199\,429\,483\,382\,385\,427\,106\,020},$$

$$\frac{844\,324\,371\,907\,541\,151\,853\,712\,525\,793\,656\,272\,285\,838\,783\,624\,690\,081}{10\,289\,088\,205\,421\,874\,847\,584\,599\,590\,000\,684\,752\,229\,391\,088\,293\,430\,327\,806\,583\,261\,280\,452\,405\,120},$$

$$\frac{40\,149\,579\,113\,594\,556\,715\,123\,849\,695\,752\,750\,763\,647\,435\,134\,623\,082\,517}{1\,282\,384\,487\,834\,804\,510\,556\,274\,795\,284\,268\,703\,121\,579\,071\,932\,574\,635\,561\,543\,774\,538\,324\,753\,235\,244\,650},$$

−7 667 534 511 423 072 809 707 375 286 031 554 587 389 886 571 372 264 944 540 816 767 111 306 002 484 716 299 175 701 /
568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456
654 648 438 397 150 167 720 800,

$$-\frac{13\,491\,766\,544\,586\,478\,845\,124\,663\,997}{724\,499\,387\,936\,917\,900\,890\,932\,769\,042\,661\,470\,794\,620\,012\,644\,120},$$

337 141 169 695 098 128 214 491 204 592 977 690 431 967 537 217 174 031

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Tree−Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
randomComponentAmp[8, 2] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2] + 2]]], 4]) & @@@ %
Total[%]
```

$$\Big\{0,\ 0,\ \frac{118\,951\,877\,854\,863\,047\,464\,271\,388\,997\,485\,232\,840\,783\,105\,998\,646\,481\,923\,997}{2\,091\,804\,499\,876\,123\,229\,069\,495\,157\,683\,580\,388\,377\,490\,923\,872\,049\,822\,424\,854\,991\,569\,566\,733\,707\,008\,938\,400},\ 0,$$

$$0,\ 0,\ -\frac{48\,841\,568\,020\,743\,401\,496\,319\,727\,816\,727\,010\,025\,917\,160\,805\,205\,708\,769\,477}{121\,263\,682\,383\,725\,894\,149\,744\,237\,338\,540\,647\,716\,490\,316\,663\,520\,666\,988\,474\,574\,983\,740\,516\,049\,848\,320},$$

$$-\frac{117\,223\,503\,167\,785\,040\,342\,554\,805\,895\,817}{62\,319\,638\,990\,360\,538\,149\,423\,339\,406\,334\,360\,087\,347\,486\,032\,171\,520},$$

$$-\frac{1\,440\,352\,269\,468\,710\,231\,560\,291}{2\,336\,647\,662\,866\,925\,004\,079\,289\,565\,583\,441\,711\,056\,590\,320},$$

$$\frac{3\,249\,997\,690\,008\,699\,936\,705\,984\,337\,015\,350\,757\,145\,393\,006\,075\,733}{9\,018\,048\,277\,401\,366\,691\,481\,548\,658\,214\,192\,659\,519\,200\,900\,642\,582\,763\,470\,642\,799\,020\,083\,200},$$

$$\frac{812\,053\,555\,053\,250\,981\,916\,891\,927\,374\,576\,162\,270\,382\,102\,098\,597\,337\,595\,392}{5\,129\,502\,557\,558\,451\,419\,319\,069\,019\,490\,027\,986\,908\,019\,364\,738\,058\,384\,843\,910\,741\,140\,557\,650\,480\,174\,525},$$

$$-\frac{6\,763\,400\,819\,928\,264\,763\,184\,688\,333\,597\,359\,740\,434\,098\,425\,696\,402\,648\,523}{19\,850\,579\,599\,167\,722\,758\,652\,351\,710\,660\,318\,020\,407\,758\,979\,804\,817\,891\,556\,845\,131\,406\,978\,573\,926\,400},$$

$$\frac{844\,324\,371\,907\,541\,151\,853\,712\,525\,793\,656\,272\,285\,838\,783\,624\,690\,081}{10\,289\,088\,205\,421\,874\,847\,584\,599\,590\,000\,684\,752\,229\,391\,088\,293\,430\,327\,806\,583\,261\,280\,452\,405\,120},$$

$$\frac{966\,857\,396\,487\,978\,869\,452\,913\,885\,424\,359\,920\,465\,741\,547\,376\,403\,886\,063\,587}{5\,147\,144\,548\,159\,282\,275\,430\,167\,345\,295\,084\,548\,702\,049\,074\,855\,148\,242\,311\,766\,202\,354\,580\,899\,942\,367\,232},$$

$$0,\ 0,\ -\frac{13\,491\,766\,544\,586\,478\,845\,124\,663\,997}{724\,499\,387\,936\,917\,900\,890\,932\,769\,042\,661\,470\,794\,620\,012\,644\,120},$$

$$-\frac{1\,428\,583\,278\,272\,854\,159\,093\,751\,749\,829\,301\,545\,214\,551\,415\,635\,432\,958\,973\,103}{58\,759\,630\,217\,938\,289\,589\,649\,625\,582\,298\,095\,387\,945\,933\,991\,403\,209\,361\,865\,498\,332\,680\,150\,070\,291\,812\,600},\ 0,\ 0\Big\}$$

595 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
666 677 /

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Tree−Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[8, 2] /. ab[x__] → Det[zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2] + 2]]], 4]) & @@@ %
Total[%]
```

Out[482]=

{812 053 555 053 250 981 916 891 927 374 576 162 270 382 102 098 597 337 595 392 / 5 129 502 557 558 451 419 319 069 019 490 027 986 908 019 364 738 058 384 843 910 741 140 557 650 480 174 525,

0, 0, − 125 584 889 331 825 247 552 511 080 801 226 333 790 860 285 998 229 / 176 288 716 398 507 232 373 693 350 561 851 847 256 608 210 563 082 056 929 984 423 853 834 240,

− 3 549 043 131 144 998 607 689 619 654 387 023 116 341 121 527 689 695 824 011 / 12 396 369 798 966 017 859 012 954 928 870 311 511 259 691 337 573 136 788 660 581 826 082 424 731 811 840,

1 081 511 219 236 553 672 463 056 971 / 1 623 989 223 058 476 081 535 916 246 777 755 467 917 460 946 944, 0, 0, 0,

86 763 755 760 272 148 388 928 475 443 199 237 342 573 336 548 760 184 797 653 / 71 388 120 433 567 462 914 153 120 530 632 681 997 723 269 048 064 845 865 199 429 483 382 385 427 106 020,

40 149 579 113 594 556 715 123 849 695 752 750 763 647 435 134 623 082 517 / 1 282 384 487 834 804 510 556 274 795 284 268 703 121 579 071 932 574 635 561 543 774 538 324 753 235 244 650,

−7 667 534 511 423 072 809 707 375 286 031 554 587 389 886 571 372 264 944 540 816 767 111 306 002 484 716 299 175 701 / 568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456 654 648 438 397 150 167 720 800,

− 54 815 438 106 415 455 287 604 503 003 749 / 132 367 068 249 053 312 378 419 363 016 225 953 651 626 120 701 245 440,

0, 85 969 756 958 464 433 435 226 069 576 617 / 1 933 649 027 499 669 488 439 285 025 095 286 237 540 840 852 415 692 800,

128 884 919 857 124 053 027 621 192 433 302 276 013 274 915 538 643 072 983 414 377 / 888 758 058 597 069 544 625 502 682 470 954 048 934 331 300 082 836 912 004 726 933 826 400 414 404 309 565 440,

− 5 194 603 083 901 235 997 054 004 773 713 / 23 113 032 818 882 500 433 655 165 030 795 420 150 093 642 110 779 392, 0,

733 075 647 610 502 639 678 126 671 228 963 312 062 134 768 255 143 128 267 398 921 / 547 516 571 351 815 672 046 608 080 095 888 653 149 520 093 528 475 456 795 426 050 319 189 042 288 669 299 200

3 115 514 278 166 483 807 751 729 289 714 885 729 083 137 908 677 122

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] ⧴ Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2] + 2]]], 4]) & @@@ %
Total[%]
```

$\Big\{\frac{812053555053250981916891927374576162270382102098597337595392}{5129502557558451419319069019490027986908019364738058384843910741140557650480174525},$
$0, 0, -\frac{125584889331825247552511080801226333790860285998229}{176288716398507232373693350561851847256608210563082056929984423853834240},$
$-\frac{3549043131144998607689619654387023116341121527689695824011}{12396369798966017859012954928870311511259691337573136788660581826082424731811840},$
$\frac{1081511219236553672463056971}{1623989223058476081535916246777755467917460946944}, 0, 0, 0,$
$\frac{86763755760272148388928475443199237342573336548760184797653}{71388120433567462914153120530632681997723269048064845865199429483382385427106020},$
$\frac{40149579113594556715123849695752750763647435134623082517}{1282384487834804510556274795284268703121579071932574635561543774538324753235244650},$
$-7667534511423072809707375286031554587389886571372264944540816767111306002484716299175701 /$
$568553625865801160034210216990435224421705980788524790331360238841149119317541419486834456 \cdot$
$654648438397150167720800, -\frac{54815438106415455287604503003749}{132367068249053312378419363016225953651626120701245440},$
$0, \frac{85969756958464433435226069576617}{1933649027499669488439285025095286237540840852415692800},$
$\frac{128884919857124053027621192433302276013274915538643072983414377}{888758058597069544625502682470954048934331300082836912004726933826400414404309565440},$

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Tree–Level BCFW Recursion in N=4   Mathematica Summer School 2011

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2] + 2]]], 4]) & @@@ %
Total[%]
```

```
{812 053 555 053 250 981 916 891 927 374 576 162 270 382 102 098 597 337 595 392 / 5 129 502 557 558 451 419 319 069 019 490 027 986 908 019 364 738 058 384 843 910 741 140 557 650 480 174 525,
 0, 0, -125 584 889 331 825 247 552 511 080 801 226 333 790 860 285 998 229 / 176 288 716 398 507 232 373 693 350 561 851 847 256 608 210 563 082 056 929 984 423 853 834 240,
 -3 549 043 131 144 998 607 689 619 654 387 023 116 341 121 527 689 695 824 011 / 12 396 369 798 966 017 859 012 954 928 870 311 511 259 691 337 573 136 788 660 581 826 082 424 731 811 840,
 1 081 511 219 236 553 672 463 056 971 / 1 623 989 223 058 476 081 535 916 246 777 755 467 917 460 946 944, 0, 0, 0,
 86 763 755 760 272 148 388 928 475 443 199 237 342 573 336 548 760 184 797 653 / 71 388 120 433 567 462 914 153 120 530 632 681 997 723 269 048 064 845 865 199 429 483 382 385 427 106 020,
 40 149 579 113 594 556 715 123 849 695 752 750 763 647 435 134 623 082 517 / 1 282 384 487 834 804 510 556 274 795 284 268 703 121 579 071 932 574 635 561 543 774 538 324 753 235 244 650,
 -7 667 534 511 423 072 809 707 375 286 031 554 587 389 886 571 372 264 944 540 816 767 111 306 002 484 716 299 175 701 /
  568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456 
  654 648 438 397 150 167 720 800, -54 815 438 106 415 455 287 604 503 003 749 / 132 367 068 249 053 312 378 419 363 016 225 953 651 626 120 701 245 440,
```

568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456 654 648 438 397 150 167 720 800, $-\frac{54\,815\,438\,106\,415\,455\,287\,604\,503\,003\,749}{132\,367\,068\,249\,053\,312\,378\,419\,363\,016\,225\,953\,651\,626\,120\,701\,245\,440}$,

0, $\frac{85\,969\,756\,958\,464\,433\,435\,226\,069\,576\,617}{1\,933\,649\,027\,499\,669\,488\,439\,285\,025\,095\,286\,237\,540\,840\,852\,415\,692\,800}$,

$\frac{128\,884\,919\,857\,124\,053\,027\,621\,192\,433\,302\,276\,013\,274\,915\,538\,643\,072\,983\,414\,377}{888\,758\,058\,597\,069\,544\,625\,502\,682\,470\,954\,048\,934\,331\,300\,082\,836\,912\,004\,726\,933\,826\,400\,414\,404\,309\,565\,440}$,

$-\frac{5\,194\,603\,083\,901\,235\,997\,054\,004\,773\,713}{23\,113\,032\,818\,882\,500\,433\,655\,165\,030\,795\,420\,150\,093\,642\,110\,779\,392}$, 0,

$-\frac{733\,075\,647\,610\,502\,639\,678\,126\,671\,228\,963\,312\,062\,134\,768\,255\,143\,128\,267\,398\,921}{547\,516\,571\,351\,815\,672\,046\,608\,080\,095\,888\,653\,149\,520\,093\,528\,475\,456\,795\,426\,050\,319\,189\,042\,288\,669\,299\,200}$,

$-\frac{3\,115\,514\,278\,166\,483\,807\,751\,729\,289\,714\,885\,729\,083\,137\,908\,677\,122}{1\,782\,456\,717\,519\,593\,950\,034\,747\,416\,713\,762\,113\,449\,646\,428\,306\,177\,158\,231\,830\,630\,201\,143\,905}$}

Out[482]=

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108 666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902 777 071 100 419 203 147 671 429 120

In[285]:=

```
randomAmp[8, 2]
```

Out[285]=

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456 \
654 648 438 397 150 167 720 800, $-\frac{54\,815\,438\,106\,415\,455\,287\,604\,503\,003\,749}{132\,367\,068\,249\,053\,312\,378\,419\,363\,016\,225\,953\,651\,626\,120\,701\,245\,440}$,

0, $\frac{85\,969\,756\,958\,464\,433\,435\,226\,069\,576\,617}{1\,933\,649\,027\,499\,669\,488\,439\,285\,025\,095\,286\,237\,540\,840\,852\,415\,692\,800}$,

$\frac{128\,884\,919\,857\,124\,053\,027\,621\,192\,433\,302\,276\,013\,274\,915\,538\,643\,072\,983\,414\,377}{888\,758\,058\,597\,069\,544\,625\,502\,682\,470\,954\,048\,934\,331\,300\,082\,836\,912\,004\,726\,933\,826\,400\,414\,404\,309\,565\,440}$,

$-\frac{5\,194\,603\,083\,901\,235\,997\,054\,004\,773\,713}{23\,113\,032\,818\,882\,500\,433\,655\,165\,030\,795\,420\,150\,093\,642\,110\,779\,392}$, 0,

$-\frac{733\,075\,647\,610\,502\,639\,678\,126\,671\,228\,963\,312\,062\,134\,768\,255\,143\,128\,267\,398\,921}{547\,516\,571\,351\,815\,672\,046\,608\,080\,095\,888\,653\,149\,520\,093\,528\,475\,456\,795\,426\,050\,319\,189\,042\,288\,669\,299\,200}$,

$-\frac{3\,115\,514\,278\,166\,483\,807\,751\,729\,289\,714\,885\,729\,083\,137\,908\,677\,122}{1\,782\,456\,717\,519\,593\,950\,034\,747\,416\,713\,762\,113\,449\,646\,428\,306\,177\,158\,231\,830\,630\,201\,143\,905}$}

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108 \
666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902 \
777 071 100 419 203 147 671 429 120

In[285]:=
```
randomAmp[8, 2]
```

Out[285]=
```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
```

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568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456
654 648 438 397 150 167 720 800, $-\frac{54\,815\,438\,106\,415\,455\,287\,604\,503\,003\,749}{132\,367\,068\,249\,053\,312\,378\,419\,363\,016\,225\,953\,651\,626\,120\,701\,245\,440}$,

0, $\frac{85\,969\,756\,958\,464\,433\,435\,226\,069\,576\,617}{1\,933\,649\,027\,499\,669\,488\,439\,285\,025\,095\,286\,237\,540\,840\,852\,415\,692\,800}$,

$\frac{128\,884\,919\,857\,124\,053\,027\,621\,192\,433\,302\,276\,013\,274\,915\,538\,643\,072\,983\,414\,377}{888\,758\,058\,597\,069\,544\,625\,502\,682\,470\,954\,048\,934\,331\,300\,082\,836\,912\,004\,726\,933\,826\,400\,414\,404\,309\,565\,440}$,

$-\frac{5\,194\,603\,083\,901\,235\,997\,054\,004\,773\,713}{23\,113\,032\,818\,882\,500\,433\,655\,165\,030\,795\,420\,150\,093\,642\,110\,779\,392}$, 0,

$-\frac{733\,075\,647\,610\,502\,639\,678\,126\,671\,228\,963\,312\,062\,134\,768\,255\,143\,128\,267\,398\,921}{547\,516\,571\,351\,815\,672\,046\,608\,080\,095\,888\,653\,149\,520\,093\,528\,475\,456\,795\,426\,050\,319\,189\,042\,288\,669\,299\,200}$,

$-\frac{3\,115\,514\,278\,166\,483\,807\,751\,729\,289\,714\,885\,729\,083\,137\,908\,677\,122}{1\,782\,456\,717\,519\,593\,950\,034\,747\,416\,713\,762\,113\,449\,646\,428\,306\,177\,158\,231\,830\,630\,201\,143\,905}$}

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
777 071 100 419 203 147 671 429 120

```
physicalPoles = ab @@@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}], Length[DeleteDuplicates[
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456
654 648 438 397 150 167 720 800, $-\frac{54815438106415455287604503003749}{132367068249053312378419363016225953651626120701245440}$,

0, $\frac{85969756958464433435226069576617}{1933649027499669488439285025095286237540840852415692800}$,

$\frac{128884919857124053027621192433302276013274915538643072983414377}{888758058597069544625502682470954048934331300082836912004726933826400414404309565440}$,

$-\frac{5194603083901235997054004773713}{23113032818882500433655165030795420150093642110779392}$, 0,

$-\frac{733075647610502639678126671228963312062134768255143128267398921}{547516571351815672046608080095888653149520093528475456795426050319189042288669299200}$,

$-\frac{3115514278166483807751729289714885729083137908677122}{1782456717519593950034747416713762113449646428306177158231830630201143905}$}

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
777 071 100 419 203 147 671 429 120

```
physicalPoles = ab @@@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}], Length[DeleteDuplicates[#]] == 4 &]
{}
```

```
randomAmp[8, 2]
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
```

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456
654 648 438 397 150 167 720 800, $-\frac{54815438106415455287604503003749}{132367068249053312378419363016225953651626120701245440}$,

0, $\frac{85969756958464433435226069576617}{1933649027499669488439285025095286237540840852415692800}$,

$\frac{128884919857124053027621192433302276013274915538643072983414377}{888758058597069544625502682470954048934331300082836912004726933826400414404309565440}$,

$-\frac{5194603083901235997054004773713}{23113032818882500433655165030795420150093642110779392}$, 0,

$-\frac{733075647610502639678126671228963312062134768255143128267398921}{547516571351815672046608080095888653149520093528475456795426050319189042288669299200}$,

$-\frac{3115514278166483807751729289714885729083137908677122}{1782456717519593950034747416713762113449646428306177158231830630201143905}$}

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
777 071 100 419 203 147 671 429 120

```
physicalPoles = ab @@@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}], Length[DeleteDuplicates[#]] = 4 &]
{}
randomAmp[8, 2]
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
```

568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456 654 648 438 397 150 167 720 800, $-\frac{54\,815\,438\,106\,415\,455\,287\,604\,503\,003\,749}{132\,367\,068\,249\,053\,312\,378\,419\,363\,016\,225\,953\,651\,626\,120\,701\,245\,440}$,

0, $\frac{85\,969\,756\,958\,464\,433\,435\,226\,069\,576\,617}{1\,933\,649\,027\,499\,669\,488\,439\,285\,025\,095\,286\,237\,540\,840\,852\,415\,692\,800}$,

$\frac{128\,884\,919\,857\,124\,053\,027\,621\,192\,433\,302\,276\,013\,274\,915\,538\,643\,072\,983\,414\,377}{888\,758\,058\,597\,069\,544\,625\,502\,682\,470\,954\,048\,934\,331\,300\,082\,836\,912\,004\,726\,933\,826\,400\,414\,404\,309\,565\,440}$,

$-\frac{5\,194\,603\,083\,901\,235\,997\,054\,004\,773\,713}{23\,113\,032\,818\,882\,500\,433\,655\,165\,030\,795\,420\,150\,093\,642\,110\,779\,392}$, 0,

$-\frac{733\,075\,647\,610\,502\,639\,678\,126\,671\,228\,963\,312\,062\,134\,768\,255\,143\,128\,267\,398\,921}{547\,516\,571\,351\,815\,672\,046\,608\,080\,095\,888\,653\,149\,520\,093\,528\,475\,456\,795\,426\,050\,319\,189\,042\,288\,669\,299\,200}$,

$-\frac{3\,115\,514\,278\,166\,483\,807\,751\,729\,289\,714\,885\,729\,083\,137\,908\,677\,122}{1\,782\,456\,717\,519\,593\,950\,034\,747\,416\,713\,762\,113\,449\,646\,428\,306\,177\,158\,231\,830\,630\,201\,143\,905}$}

Out[463]=

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108 666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902 777 071 100 419 203 147 671 429 120

```
physicalPoles =
 ab @@@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}], Length[DeleteDuplicates[Flatten[#]]] == 4 &]
```

Out[464]=

{}

In[285]:=

```
randomAmp[8, 2]
```

{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456
654 648 438 397 150 167 720 800, $-\frac{54\,815\,438\,106\,415\,455\,287\,604\,503\,003\,749}{132\,367\,068\,249\,053\,312\,378\,419\,363\,016\,225\,953\,651\,626\,120\,701\,245\,440}$,

0, $\frac{85\,969\,756\,958\,464\,433\,435\,226\,069\,576\,617}{1\,933\,649\,027\,499\,669\,488\,439\,285\,025\,095\,286\,237\,540\,840\,852\,415\,692\,800}$,

$\frac{128\,884\,919\,857\,124\,053\,027\,621\,192\,433\,302\,276\,013\,274\,915\,538\,643\,072\,983\,414\,377}{888\,758\,058\,597\,069\,544\,625\,502\,682\,470\,954\,048\,934\,331\,300\,082\,836\,912\,004\,726\,933\,826\,400\,414\,404\,309\,565\,440}$,

$-\frac{5\,194\,603\,083\,901\,235\,997\,054\,004\,773\,713}{23\,113\,032\,818\,882\,500\,433\,655\,165\,030\,795\,420\,150\,093\,642\,110\,779\,392}$, 0,

$-\frac{733\,075\,647\,610\,502\,639\,678\,126\,671\,228\,963\,312\,062\,134\,768\,255\,143\,128\,267\,398\,921}{547\,516\,571\,351\,815\,672\,046\,608\,080\,095\,888\,653\,149\,520\,093\,528\,475\,456\,795\,426\,050\,319\,189\,042\,288\,669\,299\,200}$,

$-\frac{3\,115\,514\,278\,166\,483\,807\,751\,729\,289\,714\,885\,729\,083\,137\,908\,677\,122}{1\,782\,456\,717\,519\,593\,950\,034\,747\,416\,713\,762\,113\,449\,646\,428\,306\,177\,158\,231\,830\,630\,201\,143\,905}$}

Out[463]= -995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
777 071 100 419 203 147 671 429 120

In[465]:=
```
physicalPoles =
 ab @@@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}], Length[DeleteDuplicates[Flatten@#]] == 4 &]
```

Out[465]=
```
{ab[{1, 2}, {3, 4}], ab[{1, 2}, {4, 5}], ab[{1, 2}, {5, 6}], ab[{1, 2}, {6, 7}], ab[{1, 2}, {7, 8}],
 ab[{2, 3}, {4, 5}], ab[{2, 3}, {5, 6}], ab[{2, 3}, {6, 7}], ab[{2, 3}, {7, 8}], ab[{2, 3}, {8, 1}],
 ab[{3, 4}, {5, 6}], ab[{3, 4}, {6, 7}], ab[{3, 4}, {7, 8}], ab[{3, 4}, {8, 1}], ab[{4, 5}, {6, 7}],
 ab[{4, 5}, {7, 8}], ab[{4, 5}, {8, 1}], ab[{5, 6}, {7, 8}], ab[{5, 6}, {8, 1}], ab[{6, 7}, {8, 1}]}
```

```
randomAmp[8, 2]
```

568 553 625 865 801 160 034 210 216 990 435 224 421 705 980 788 524 790 331 360 238 841 149 119 317 541 419 486 834 456

$$654648438397150167720800, -\frac{54815438106415455287604503003749}{132367068249053312378419363016225953651626120701245440},$$

$$0, \frac{85969756958464433435226069576617}{1933649027499669488439285025095286237540840852415692800},$$

$$\frac{128884919857124053027621192433302276013274915538643072983414377}{888758058597069544625502682470954048934331300082836912004726933826400414404309565440},$$

$$-\frac{5194603083901235997054004773713}{23113032818882500433655165030795420150093642110779392}, 0,$$

$$-\frac{733075647610502639678126671228963312062134768255143128267398921}{547516571351815672046608080095888653149520093528475456795426050319189042288669299200},$$

$$-\frac{3115514278166483807751729289714885729083137908677122}{1782456717519593950034747416713762113449646428306177158231830630201143905}\}$$

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
777 071 100 419 203 147 671 429 120

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] == 4 &]
```

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

```
randomAmp[8, 2]
```

654 648 438 397 150 167 720 800, $-\frac{54\,815\,438\,106\,415\,455\,287\,604\,503\,003\,749}{132\,367\,068\,249\,053\,312\,378\,419\,363\,016\,225\,953\,651\,626\,120\,701\,245\,440}$,

0, $\frac{85\,969\,756\,958\,464\,433\,435\,226\,069\,576\,617}{1\,933\,649\,027\,499\,669\,488\,439\,285\,025\,095\,286\,237\,540\,840\,852\,415\,692\,800}$,

$\frac{128\,884\,919\,857\,124\,053\,027\,621\,192\,433\,302\,276\,013\,274\,915\,538\,643\,072\,983\,414\,377}{888\,758\,058\,597\,069\,544\,625\,502\,682\,470\,954\,048\,934\,331\,300\,082\,836\,912\,004\,726\,933\,826\,400\,414\,404\,309\,565\,440}$,

$-\frac{5\,194\,603\,083\,901\,235\,997\,054\,004\,773\,713}{23\,113\,032\,818\,882\,500\,433\,655\,165\,030\,795\,420\,150\,093\,642\,110\,779\,392}$, 0,

$-\frac{733\,075\,647\,610\,502\,639\,678\,126\,671\,228\,963\,312\,062\,134\,768\,255\,143\,128\,267\,398\,921}{547\,516\,571\,351\,815\,672\,046\,608\,080\,095\,888\,653\,149\,520\,093\,528\,475\,456\,795\,426\,050\,319\,189\,042\,288\,669\,299\,200}$,

$-\frac{3\,115\,514\,278\,166\,483\,807\,751\,729\,289\,714\,885\,729\,083\,137\,908\,677\,122}{1\,782\,456\,717\,519\,593\,950\,034\,747\,416\,713\,762\,113\,449\,646\,428\,306\,177\,158\,231\,830\,630\,201\,143\,905}$}

Out[462]=

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108 666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902 777 071 100 419 203 147 671 429 120

In[466]:=

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] == 4 &]
```

Out[466]=

{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6], ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1], ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}

```
...Amp[8, 2]
```

{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
  5 194 603 083 901 235 997 054 004 773 713
- ------------------------------------------------------ , 0,
  23 113 032 818 882 500 433 655 165 030 795 420 150 093 642 110 779 392
  733 075 647 610 502 639 678 126 671 228 963 312 062 134 768 255 143 128 267 398 921
- ------------------------------------------------------ ,
  547 516 571 351 815 672 046 608 080 095 888 653 149 520 093 528 475 456 795 426 050 319 189 042 288 669 299 200
  3 115 514 278 166 483 807 751 729 289 714 885 729 083 137 908 677 122
- ------------------------------------------------------ }
  1 782 456 717 519 593 950 034 747 416 713 762 113 449 646 428 306 177 158 231 830 630 201 143 905
```

```
-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
 666 677 /
 399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
 777 071 100 419 203 147 671 429 120
```

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] = 4 &]
```

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
```

```
  5 194 603 083 901 235 997 054 004 773 713
- ---------------------------------------------------------------- , 0,
  23 113 032 818 882 500 433 655 165 030 795 420 150 093 642 110 779 392
  733 075 647 610 502 639 678 126 671 228 963 312 062 134 768 255 143 128 267 398 921
- ---------------------------------------------------------------- ,
  547 516 571 351 815 672 046 608 080 095 888 653 149 520 093 528 475 456 795 426 050 319 189 042 288 669 299 200
  3 115 514 278 166 483 807 751 729 289 714 885 729 083 137 908 677 122
- ---------------------------------------------------------------- }
  1 782 456 717 519 593 950 034 747 416 713 762 113 449 646 428 306 177 158 231 830 630 201 143 905
```

```
-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108 \
  666 677 /
 399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902 \
  777 071 100 419 203 147 671 429 120
```

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
    Length[DeleteDuplicates[Flatten@#]] = 4 &]
%
```

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
```

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```
      Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] → Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2] + 2]]], 4]) & @@@ %
Total[%]
```

$$\left\{\frac{812053555053250981916891927374576162270382102098597337595392}{5129502557558451419319069019490027986908019364738058384843910741140557650480174525},\right.$$

$$0, 0, -\frac{125584889331825247552511080801226333790860285998229}{176288716398507232373693350561851847256608210563082056929984423853834240},$$

$$\frac{3549043131144998607689619654387023116341121527689695824011}{12396369798966017859012954928870311511259691337573136788660581826082424731811840},$$

$$\frac{1081511219236553672463056971}{623989223058476081535916246777755467917460946944}, 0, 0, 0,$$

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bcfw_part_deux.nb


```
      Total[bcfwBridge[##, True][{labelRange}] & @@@ bcfwPartitions[Length[{labelRange}], k]])]};
treeAmp[n_, k_] :=
  treeAmp[n, k] = If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. bcfwRecurse];
randomAmp[n_, k_] := If[Head[#] === Plus, List @@ #, {#}] &@Expand[(A[k] @@ Range[n]) //. randomBCFWRecurse]
```

```
randomComponentAmp[n_, k_] := Block[{residueList, dMatrixList, amp = randomAmp[n, k]},
  residueList = rToResidue /@ amp;
  dMatrixList = termToDMatrix[#, n] & /@ amp; Transpose[{residueList, dMatrixList}]]
```

```
randomComponentAmp[8, 2] /. ab[x__] :> Det[Zs[[{x}]]];
(#1 * Power[Det[#2[[All, Range[2] + 2]]], 4]) & @@@ %
Total[%]
```

$$\left\{\frac{812053555053250981916891927374576162270382102098597337595392}{5129502557558451419319069019490027986908019364738058384843910741140557650480174525}, 0, 0, -\frac{125584889331825247552511080801226333790860285998229}{176288716398507232373693350561851847256608210563082056929984423853834240}, -\frac{3549043131144998607689619654387023116341121527689695824011}{12396369798966017859012954928870311511259691337573136788660581826082424731811840}, \frac{1081511219236553672463056971}{1623989223058476081535916246777755467917460946944}, 0, 0, 0,\right.$$

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

$$654648438397150167720800, -\frac{54815438106415455287604503003749}{132367068249053312378419363016225953651626120701245440},$$

$$0, \frac{85969756958464433435226069576617}{1933649027499669488439285025095286237540840852415692800},$$

$$\frac{128884919857124053027621192433302276013274915538643072983414377}{888758058597069544625502682470954048934331300082836912004726933826400414404309565440},$$

$$-\frac{5194603083901235997054004773713}{23113032818882500433655165030795420150093642110779392}, 0,$$

$$-\frac{733075647610502639678126671228963312062134768255143128267398921}{547516571351815672046608080095888653149520093528475456795426050319189042288669299200},$$

$$-\frac{3115514278166483807751729289714885729083137908677122}{1782456717519593950034747416713762113449646428306177158231830630201143905}\}$$

```
-995712729659696679751942221260775604435855713417461828417583126089988722637398812023201792108\
  666677 /
 399053654875607366013806663014810125774864966464042343546892719480308431908038856326422243902\
  777071100419203147671429120
```

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] == 4 &]
%
```

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```



```
randomAmp[8, 2]
```

```
  733 075 647 610 502 639 678 126 671 228 963 312 062 134 768 255 143 128 267 398 921
- ------------------------------------------------------------------------------------------------ ,
  547 516 571 351 815 672 046 608 080 095 888 653 149 520 093 528 475 456 795 426 050 319 189 042 288 669 299 200
  3 115 514 278 166 483 807 751 729 289 714 885 729 083 137 908 677 122
- ------------------------------------------------------------------------------------------------ }
  1 782 456 717 519 593 950 034 747 416 713 762 113 449 646 428 306 177 158 231 830 630 201 143 905
```

```
-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
  666 677 /
 399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
  777 071 100 419 203 147 671 429 120
```

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] = 4 &]
% /. ab[x__] → Det[Zs[[{x}]]]
```

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

```
{-113 172 084 134, -43 095 714 752, 4 320 319 041, -48 940 856 473, -33 873 308 403, -68 455 004 901, 2 729 395 557,
 -44 580 815 043, -127 852 198 423, -28 691 875 161, 55 619 401 905, 8 618 199 744, -28 939 793 344, -139 111 342 398,
 15 029 929 321, -22 425 075 541, -64 586 929 164, -43 128 971 807, -8 815 253 832, -28 459 249 812}
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 1],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
                   733 075 647 610 502 639 678 126 671 228 963 312 062 134 768 255 143 128 267 398 921
 - ----------------------------------------------------------------------------------------------------------------------- ,
   547 516 571 351 815 672 046 608 080 095 888 653 149 520 093 528 475 456 795 426 050 319 189 042 288 669 299 200

                   3 115 514 278 166 483 807 751 729 289 714 885 729 083 137 908 677 122
 - ----------------------------------------------------------------------------------------- }
   1 782 456 717 519 593 950 034 747 416 713 762 113 449 646 428 306 177 158 231 830 630 201 143 905
```

```
-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108 \
  666 677 /
 399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902 \
  777 071 100 419 203 147 671 429 120
```

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] == 4 &]
Times @@ (% /. ab[x__] :> Det[Zs[[{x}]]])
```

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

```
-485 142 058 120 126 059 146 455 864 790 487 664 847 672 365 635 260 066 179 127 791 115 733 996 832 323 520 071 064 265 418 \
 415 860 464 571 813 817 670 588 410 017 259 122 243 781 232 021 809 061 143 933 763 425 108 084 018 534 904 667 945 567 607 \
 658 424 697 181 715 698 810 880
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
```

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bcfw_part_deux.nb

$$-\frac{733\,075\,647\,610\,502\,639\,678\,126\,671\,228\,963\,312\,062\,134\,768\,255\,143\,128\,267\,398\,921}{547\,516\,571\,351\,815\,672\,046\,608\,080\,095\,888\,653\,149\,520\,093\,528\,475\,456\,795\,426\,050\,319\,189\,042\,288\,669\,299\,200},$$

$$-\frac{3\,115\,514\,278\,166\,483\,807\,751\,729\,289\,714\,885\,729\,083\,137\,908\,677\,122}{1\,782\,456\,717\,519\,593\,950\,034\,747\,416\,713\,762\,113\,449\,646\,428\,306\,177\,158\,231\,830\,630\,201\,143\,905}\}$$

```
-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108 \
  666 677 /
 399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902 \
  777 071 100 419 203 147 671 429 120
```

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
    Length[DeleteDuplicates[Flatten@#]] == 4 &]
Times @@ (% /. ab[x__] :> Det[Zs[[{x}]]])
*
```

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

```
-485 142 058 120 126 059 146 455 864 790 487 664 847 672 365 635 260 066 179 127 791 115 733 996 832 323 520 071 064 265 418 \
 415 860 464 571 813 817 670 588 410 017 259 122 243 781 232 021 809 061 143 933 763 425 108 084 018 534 904 667 945 567 607 \
 658 424 697 181 715 698 810 880
```

```
randomAmp[8, 2]
```

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

$$-\frac{733\,075\,647\,610\,502\,639\,678\,126\,671\,228\,963\,312\,062\,134\,768\,255\,143\,128\,267\,398\,921}{547\,516\,571\,351\,815\,672\,046\,608\,080\,095\,888\,653\,149\,520\,093\,528\,475\,456\,795\,426\,050\,319\,189\,042\,288\,669\,299\,200},$$

$$-\frac{3\,115\,514\,278\,166\,483\,807\,751\,729\,289\,714\,885\,729\,083\,137\,908\,677\,122}{1\,782\,456\,717\,519\,593\,950\,034\,747\,416\,713\,762\,113\,449\,646\,428\,306\,177\,158\,231\,830\,630\,201\,143\,905}\}$$

Out[462]=

```
-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108 \
  666 677 /
 399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902 \
  777 071 100 419 203 147 671 429 120
```

In[471]:=

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] == 4 &]
Times @@ (% /. ab[x__] :> Det[Zs[[{x}]]])
% * Out[463]
```

Out[471]=

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

Out[472]=

```
-485 142 058 120 126 059 146 455 864 790 487 664 847 672 365 635 260 066 179 127 791 115 733 996 832 323 520 071 064 265 418 \
 415 860 464 571 813 817 670 588 410 017 259 122 243 781 232 021 809 061 143 933 763 425 108 084 018 534 904 667 945 567 607 \
 658 424 697 181 715 698 810 880
```

Out[473]=

```
1 210 519 229 836 633 393 614 502 559 420 243 253 719 442 405 395 817 810 694 097 320 263 143 217 027 613 276 329 205 475 403 \
 004 495 213 552 225 286 463 919 870 230 190 572 624 503 943 534 903 718 770 835 654 820 262 351 877 691 775 147 952 916 840 \
 448
```



```
randomAmp[8, 2]
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
1 782 456 717 519 593 950 034 747 416 713 762 113 449 646 428 306 177 158 231 830 630 201 143 905
```

```
-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
  666 677 /
 399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
  777 071 100 419 203 147 671 429 120
```

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] == 4 &]
Times @@ (% /. ab[x__] :> Det[Zs[[{x}]]])
% * Out[463]
```

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

```
-485 142 058 120 126 059 146 455 864 790 487 664 847 672 365 635 260 066 179 127 791 115 733 996 832 323 520 071 064 265 418
 415 860 464 571 813 817 670 588 410 017 259 122 243 781 232 021 809 061 143 933 763 425 108 084 018 534 904 667 945 567 607
 658 424 697 181 715 698 810 880
```

```
1 210 519 229 836 633 393 614 502 559 420 243 253 719 442 405 395 817 810 694 097 320 263 143 217 027 613 276 329 205 475 403
 004 495 213 552 225 286 463 919 870 230 190 572 624 503 943 534 903 718 770 835 654 820 262 351 877 691 775 147 952 916 840
 448
```

```
randomAmp[8, 2]
```

```
R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
```

1 782 456 717 519 593 950 034 747 416 713 762 113 449 646 428 306 177 158 231 830 630 201 143 905

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108 666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902 777 071 100 419 203 147 671 429 120

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] == 4 &]
Times @@ (% /. ab[x__] :> Det[Zs[[{x}]]])
% * Out[463]
Int
```

{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6], ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1], ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}

-485 142 058 120 126 059 146 455 864 790 487 664 847 672 365 635 260 066 179 127 791 115 733 996 832 323 520 071 064 265 418 415 860 464 571 813 817 670 588 410 017 259 122 243 781 232 021 809 061 143 933 763 425 108 084 018 534 904 667 945 567 607 658 424 697 181 715 698 810 880

1 210 519 229 836 633 393 614 502 559 420 243 253 719 442 405 395 817 810 694 097 320 263 143 217 027 613 276 329 205 475 403 004 495 213 552 225 286 463 919 870 230 190 572 624 503 943 534 903 718 770 835 654 820 262 351 877 691 775 147 952 916 840 448

```
randomAmp[8, 2]
```

{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],

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Tree-Level BCFW Recursion in N=4 Mathematica Summer School 2011

1 782 456 717 519 593 950 034 747 416 713 762 113 449 646 428 306 177 158 231 830 630 201 143 905

-995 712 729 659 696 679 751 942 221 260 775 604 435 855 713 417 461 828 417 583 126 089 988 722 637 398 812 023 201 792 108
666 677 /
399 053 654 875 607 366 013 806 663 014 810 125 774 864 966 464 042 343 546 892 719 480 308 431 908 038 856 326 422 243 902
777 071 100 419 203 147 671 429 120

```
physicalPoles =
 ab @@@ Flatten /@ Select[Subsets[Partition[Range[8], 2, 1, 1], {2}],
   Length[DeleteDuplicates[Flatten@#]] == 4 &]
Times @@ (% /. ab[x__] :> Det[Zs[[{x}]]])
% * Out[463]
IntegerQ@%
```

{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}

-485 142 058 120 126 059 146 455 864 790 487 664 847 672 365 635 260 066 179 127 791 115 733 996 832 323 520 071 064 265 418
415 860 464 571 813 817 670 588 410 017 259 122 243 781 232 021 809 061 143 933 763 425 108 084 018 534 904 667 945 567 607
658 424 697 181 715 698 810 880

1 210 519 229 836 633 393 614 502 559 420 243 253 719 442 405 395 817 810 694 097 320 263 143 217 027 613 276 329 205 475 403
004 495 213 552 225 286 463 919 870 230 190 572 624 503 943 534 903 718 770 835 654 820 262 351 877 691 775 147 952 916 840
448

True


```
randomAmp[8, 2]
```

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bcfw_part_deux.nb

Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    Length[DeleteDuplicates[Flatten@#]] = 4 &]
Times @@ (% /. ab[x__] → Det[Zs[[{x}]]])
% * Out[463]
IntegerQ@%
```

Out[474]=

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

Out[475]=

```
-485 142 058 120 126 059 146 455 864 790 487 664 847 672 365 635 260 066 179 127 791 115 733 996 832 323 520 071 064 265 418 \
  415 860 464 571 813 817 670 588 410 017 259 122 243 781 232 021 809 061 143 933 763 425 108 084 018 534 904 667 945 567 607 \
  658 424 697 181 715 698 810 880
```

Out[476]=

```
1 210 519 229 836 633 393 614 502 559 420 243 253 719 442 405 395 817 810 694 097 320 263 143 217 027 613 276 329 205 475 403 \
 004 495 213 552 225 286 463 919 870 230 190 572 624 503 943 534 903 718 770 835 654 820 262 351 877 691 775 147 952 916 840 \
 448
```

Out[477]=

```
True
```

In[285]:=

```
randomAmp[8, 2]
```

Out[285]=

```
{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 7, 8] R[5, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
 R[5, cap[{1, 2}, {6, 5, 7}], 2, 3, 4] R[7, 1, 2, 5, 6],
```

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Tree–Level BCFW Recursion in N=4 Mathematica Summer School 2011

```
    Length[DeleteDuplicates[Flatten@#]] == 4 &]
Times @@ (% /. ab[x__] :> Det[Zs[[{x}]]])
% * Out[463]
IntegerQ@%
```

```
{ab[1, 2, 3, 4], ab[1, 2, 4, 5], ab[1, 2, 5, 6], ab[1, 2, 6, 7], ab[1, 2, 7, 8], ab[2, 3, 4, 5], ab[2, 3, 5, 6],
 ab[2, 3, 6, 7], ab[2, 3, 7, 8], ab[2, 3, 8, 1], ab[3, 4, 5, 6], ab[3, 4, 6, 7], ab[3, 4, 7, 8], ab[3, 4, 8, 1],
 ab[4, 5, 6, 7], ab[4, 5, 7, 8], ab[4, 5, 8, 1], ab[5, 6, 7, 8], ab[5, 6, 8, 1], ab[6, 7, 8, 1]}
```

```
-485 142 058 120 126 059 146 455 864 790 487 664 847 672 365 635 260 066 179 127 791 115 733 996 832 323 520 071 064 265 418 \
  415 860 464 571 813 817 670 588 410 017 259 122 243 781 232 021 809 061 143 933 763 425 108 084 018 534 904 667 945 567 607 \
  658 424 697 181 715 698 810 880
```

```
1 210 519 229 836 633 393 614 502 559 420 243 253 719 442 405 395 817 810 694 097 320 263 143 217 027 613 276 329 205 475 403 \
 004 495 213 552 225 286 463 919 870 230 190 572 624 503 943 534 903 718 770 835 654 820 262 351 877 691 775 147 952 916 840 \
 448
```

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True
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randomAmp[8, 2]
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{R[1, 2, 3, 4, 5] R[1, 5, 6, 7, 8], R[1, 2, 3, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 3, 4, 5, cap[{5, 6}, {8, 7, 1}]] R[1, 5, 6, 7, 8],
 R[1, 4, 5, 7, 8] R[2, 3, 4, cap[{4, 5}, {8, 7, 1}], 1], R[1, 2, 3, 7, 8] R[4, 5, 6, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 5, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}]],
 R[1, 2, 3, 7, 8] R[4, 6, 7, cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 2, 3, 7, 8] R[4, 7, cap[{8, 7}, {3, 2, 1}], cap[{2, 3}, {8, 7, 1}], 3],
 R[1, 3, 4, 7, 8] R[5, 6, 7, cap[{3, 4}, {8, 7, 1}], 4],
 R[1, 3, 4, 7, 8] R[5, 7, cap[{8, 7}, {4, 3, 1}], cap[{3, 4}, {8, 7, 1}], 4],
 R[5, cap[{1, 2}, {6, 5, 7}], 2, 3, 4] R[7, 1, 2, 5, 6],
```