

Title: Tools for Evaluating Loop Integrals

Date: Aug 02, 2011 12:15 PM

URL: <http://pirsa.org/11080008>

Abstract:

MB Tools

This project is a collection of tools devoted to the evaluation of Mellin-Barnes integrals.

The project has been started by [Michael Czakon](#); currently the web-page is also being updated by [Alexander Smirnov](#).

The project is at the development stage, so expect more codes to appear here.

Currently the following codes can be downloaded:

- **MB.m** : version 1.2 of MB (last updated January 2nd, 2009) by [Michal Czakon](#),
the main collection of routines for the resolution of singularities and the numerical evaluation of Mellin-Barnes
integrals;
for details see [hep-ph/0511200](#);
the current version is documented in the [Manual](#);
the distribution contains two example notebooks, [MBexamples1.nb](#) and [MBexamples2.nb](#);
- **MBasymptotics.m** : a routine which expands Mellin-Barnes integrals in a small parameter by [Michal Czakon](#);
example usage is illustrated in [MBasymptotics.nb](#);
- **MBresolve.m** : a tool by [Alexander Smirnov](#) and [Vladimir Smirnov](#) realizing another strategy of resolving
singularities of Mellin-Barnes integrals. This code should be loaded together with **MB.m** since it uses some of its
routines. For details see [arXiv:0901.0386](#)
- **AMBRE.m** : a tool by Janusz Gluza, Krzysztof Kajda and Tord Riemann for constructing Mellin-Barnes
representations. It works both for planar multiloop scalar and one-loop tensor Feynman integrals. This is version
1.2, for previous versions and detailed description of the package with examples see the [home page](#). The
program is described in [arXiv:0704.2423](#) and Computer Physics Communications 177 (2007) 879.
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MB Tools :: HepForge

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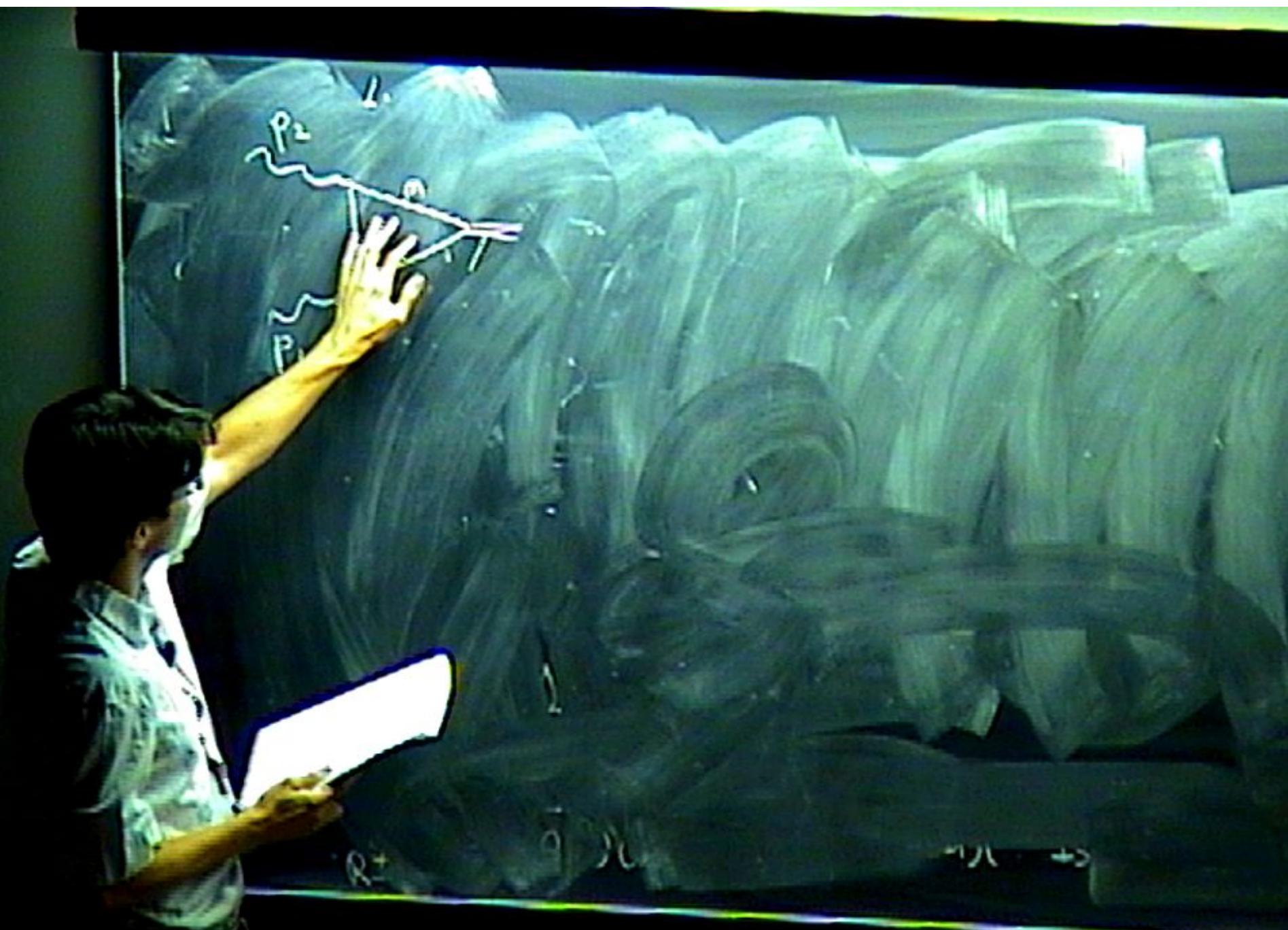
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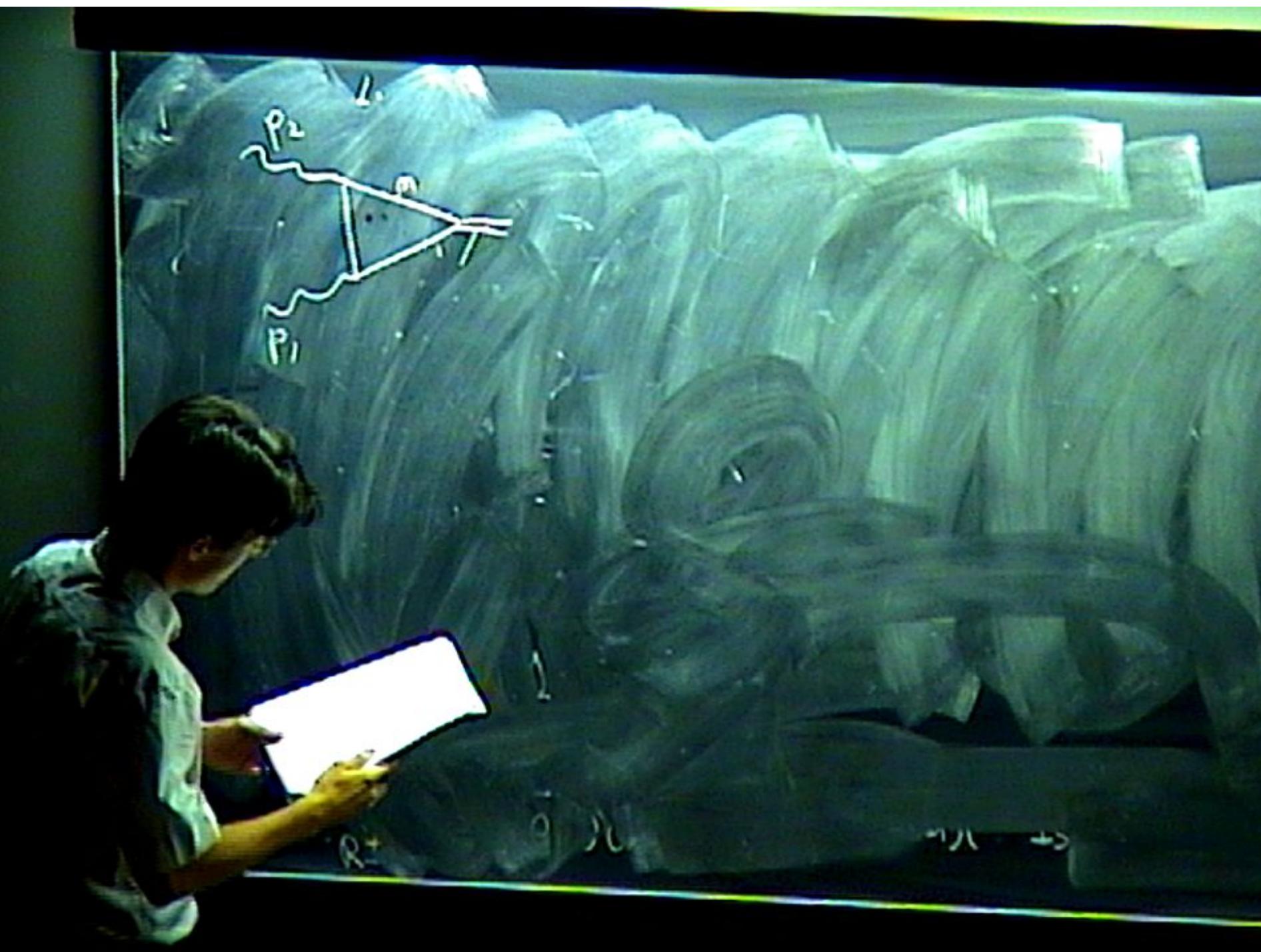
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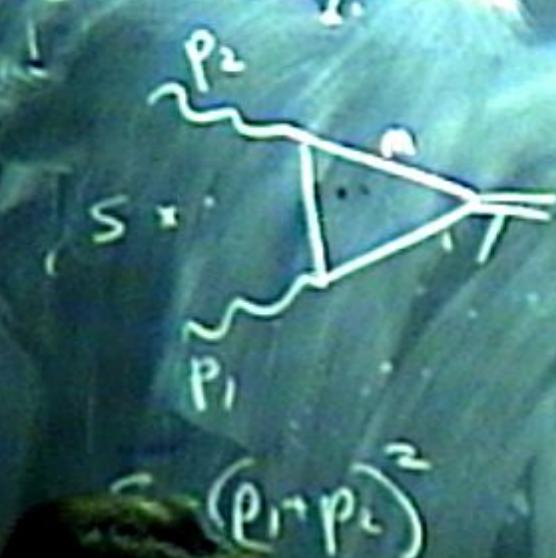
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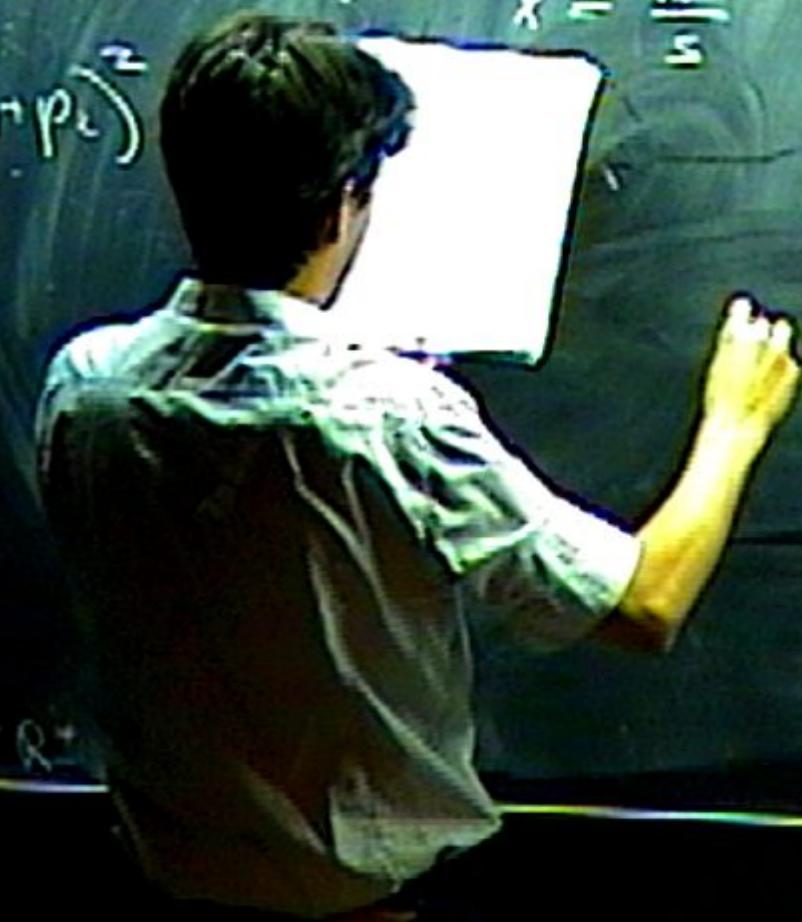
$$= \int \frac{dt}{2\pi i} \Gamma(-z) \Gamma(1-z) \sqrt{\gamma(3+2z)}$$

$$z = \frac{m^2}{s}$$

$$S = \langle \rho_1, \rho_2 \rangle^2$$

$$= \int_{-\infty}^{i\infty} \frac{dt}{2\pi i} \Gamma(-z) \Gamma(1-z) \sqrt{\Gamma(3+2z)} \times$$

$$x = \frac{m^2}{z} \quad Re z$$



$$\begin{array}{c} \rho_2 \\ \rho_1 \\ S - (\alpha + \rho_2) \end{array}$$

$$= \int_{-\infty}^{ib} \frac{dt}{2\pi i} \Gamma(-z) \Gamma(1-z) \sqrt{\gamma(\beta+2z)} x^{-z-2}$$
$$x = \frac{m^2}{s}, -1 < \operatorname{Re} z < 0$$



$$S = \langle p_1, p_2 \rangle$$

$$= \int_{-\infty}^{i\infty} \frac{dt}{2\pi i} \Gamma(-z) \Gamma(1-z) \sqrt{\gamma(\beta+2z)} \times$$

$$x = \frac{m^2}{s}; -1 < Re z < 0$$

$$S = \frac{1}{2} (P_1 + P_2) \cdot S$$

$$S = (P_1 + P_2)^2$$

$$= \int_{-\infty}^{i\infty} \frac{dt}{2\pi i} \cdot \Gamma(-z) \Gamma(1-z) \sqrt{\Gamma(3+2z)} \cdot x^{-z}$$

$$x = \frac{m^2}{c^2}; -1 < Re z < 0$$



P_1

$$S = (\rho_1 + \rho_2)^2$$

$$= \int_{-\infty}^{+\infty} \frac{1}{2\pi i} R(-z) R^*(1-z) \sqrt{\beta z^2} dz$$

$$x = \frac{m}{2} - i\kappa R_0 z < 0$$

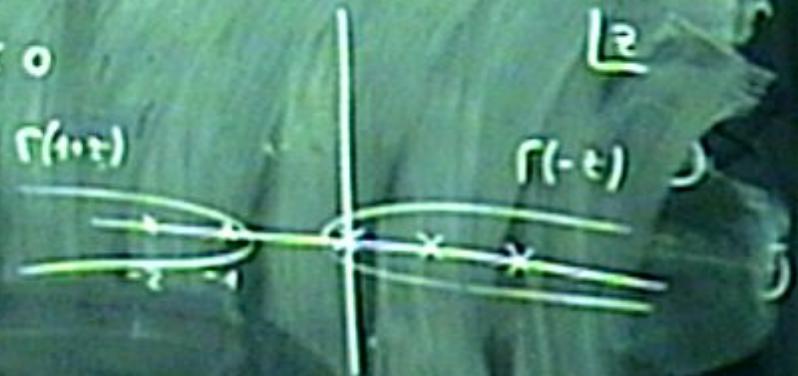
L_2
 $R(-z)$

$R(z)$

$$S = \langle p_1, p_2 \rangle^2$$

$$= \int_{-\infty}^{i\infty} \frac{dt}{2\pi i} R(-z) R(1-t) \sqrt{\beta + 2t}$$

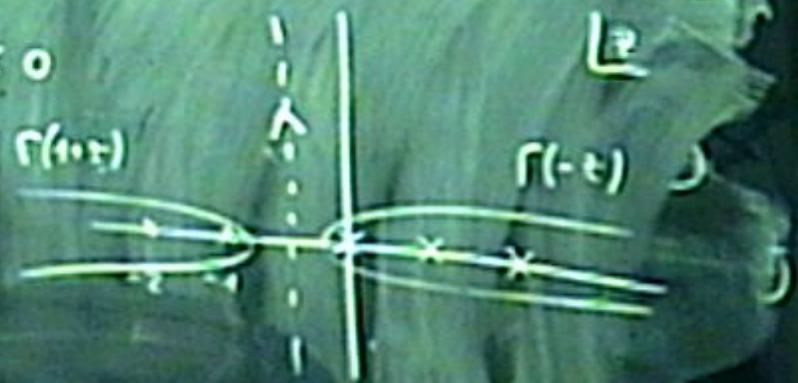
$$x = \frac{n^2}{s^2} - kR_0 z < 0$$



$$S = \begin{pmatrix} p_1 & \\ & p_2 \end{pmatrix}$$

$$= \int_{-\infty}^{i\infty} \frac{1}{2\pi i} R(-z) R(1-z) \sqrt{\gamma(3+2z)} \cdot e^{-sz} dz$$

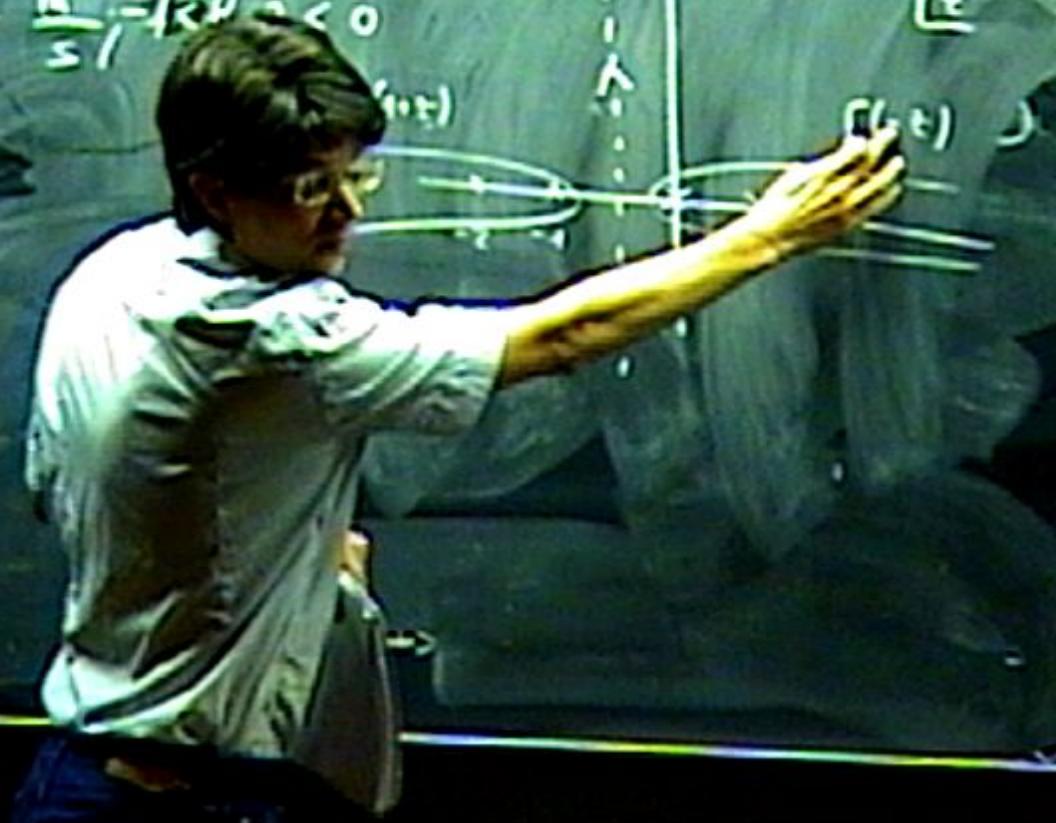
$$x = \frac{n^2}{s}, -1 < Re z < 0$$



$$S = \langle p_1, p_2 \rangle^2$$

$$= \int_{-\infty}^{1/2} \frac{dt}{2\pi i} P(-z) P(1-t) \sqrt{\gamma(3+2z)} * e^{-i z t}$$

$$x = \frac{m^2}{s} - k^2, x < 0$$



$$S = \langle p_1^2 + p_2^2 \rangle$$

$$= \int_{-\infty}^{i\infty} \frac{dt}{2\pi i} R(-z) R(1-t) \sqrt{\gamma(\beta+2z)} \times$$

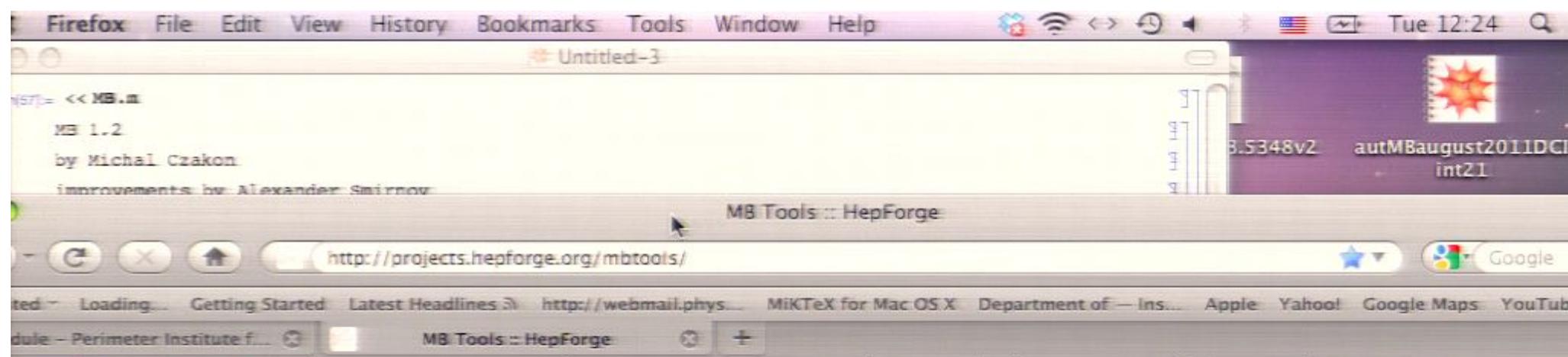
$x = \frac{t-i}{2} - iR$, $\Re z < 0$

$R(z)$

$R(-z)$

$\Gamma(c-z)\Gamma(b-z)$

$\Gamma(a-z)\Gamma(b-z)$



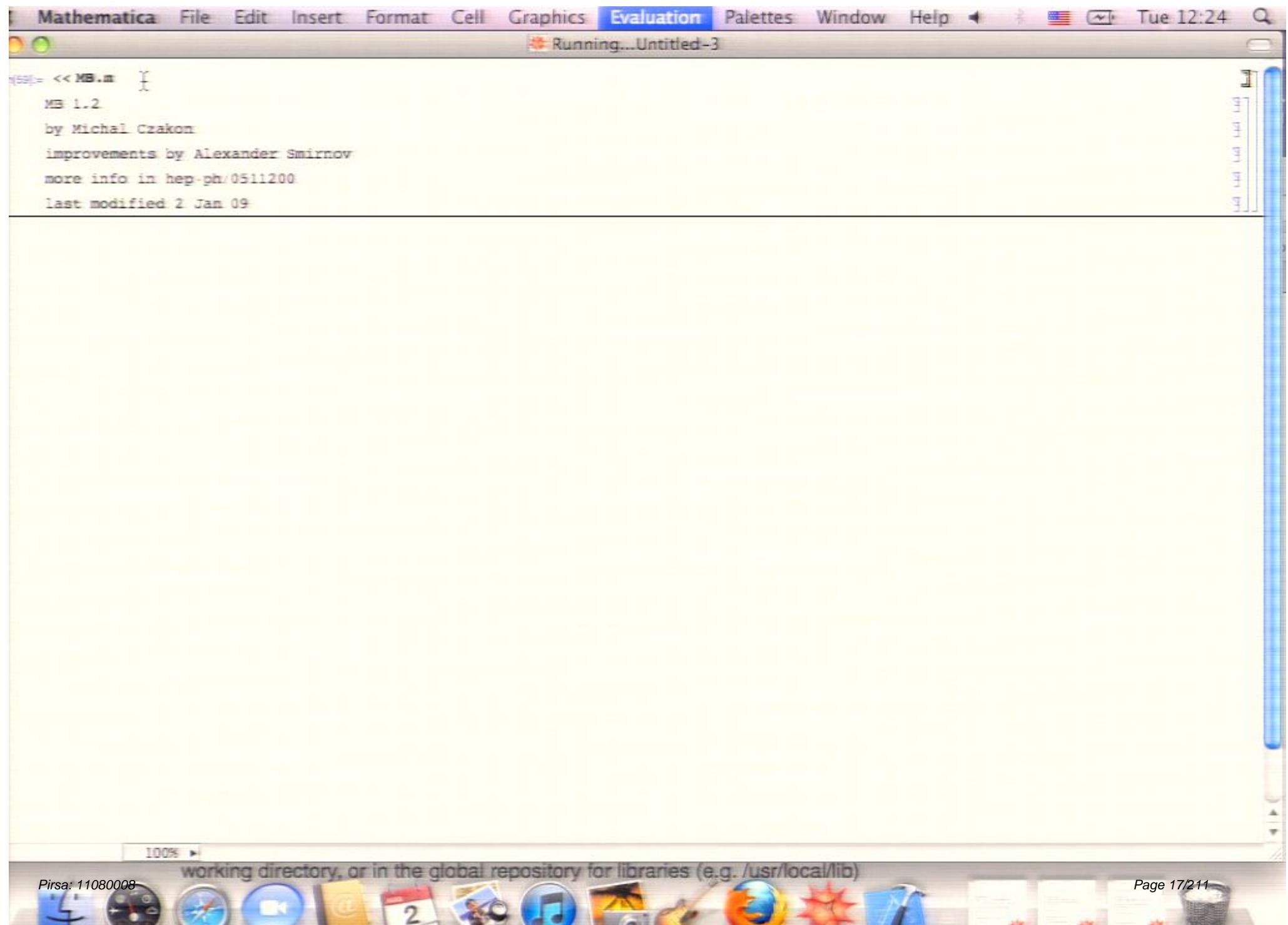
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Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Tue 12:25

Untitled-3

```
v59:= << MB.m
MB 1.2
by Michal Czakon
improvements by Alexander Smirnov
more info in hep-ph/0511200
last modified 2 Jan 09

v60:= int = Gamma[-z] Gamma[1+z]^3 / Gamma[3+2z] x^(-1-z)
      x^1-z Gamma[-z] Gamma[1+z]^2
d(60)=                                     Gamma[3+2z]

v61:= ?MBrules
      MBrules[integrand, constraints, fixedVars] determines real
parts of fixed and integration variables such that the arguments of all Gamma
and PolyGamma functions in the integrand be positive. It is only necessary to
specify the fixed variables, fixedVars, since the integration variables are
determined automatically. The user can also specify further constraints.

      MBrules[integrand, limit, constraints, fixedVars] determines the contours such
that during analytic continuation no contour starts or ends on a pole.
```

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working directory, or in the global repository for libraries (e.g. /usr/local/lib)

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more info in help-pr/0511200

last modified 2 Jan 09

In[60]:= int = Gamma[-z] Gamma[1 + z]^3 / Gamma[3 + 2 z] x^(-1 - z)

$$\frac{x^{-1-z} \Gamma(-z) \Gamma(1+z)^3}{\Gamma(3+2z)}$$

In[61]:= ? MBrules

MBrules[integrand, constraints, fixedVars] determines real parts of fixed and integration variables such that the arguments of all Gamma and PolyGamma functions in the integrand be positive. It is only necessary to specify the fixed variables, fixedVars, since the integration variables are determined automatically. The user can also specify further constraints.

MBrules[integrand, limit, constraints, fixedVars] determines the contours such that during analytic continuation no contour starts or ends on a pole.

In[62]:= MBrules[int, {}, {}]

$$\left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$$

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working directory, or in the global repository for libraries (e.g. /usr/local/lib)

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more info in help or in documentation center

last modified 2 Jan 09

In[60]:= int = Gamma[-z] Gamma[1 + z]^3 / Gamma[3 + 2 z] x^(-1 - z)

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more info in hep-ph/0511200

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Untitled-3

more info in help-primer.m

last modified 2 Jan 09

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In[63]:= rules = MBrules[int, {}, {}]

Out[63]= $\left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

MBI

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Out[60]= 
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In[63]:= rules = MBrules[int, {}, {}]
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```
Out[63]=  $\left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$ 
```

```
I1MB = MBint[
```

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working directory, or in the global repository for libraries (e.g. /usr/local/lib)

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Out[60]=

Gamma[3 + 2 z]

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Out[63]= $\left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

In[64]:= I1MB = MBint[int, rules]

Out[64]= MBint $\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]$

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In[63]:= rules = MBrules[int, {}, {}]

Out[63]= $\left\{ \{ \}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

In[64]:= I1MB = MBint[int, rules]

Out[64]= $\text{MBint}\left[\frac{x^{-1-z} \text{Gamma}[-z] \text{Gamma}[1+z]^3}{\text{Gamma}[3+2z]}, \left\{ \{ \}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]$

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working directory, or in the global repository for libraries (e.g. /usr/local/lib)



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working directory, or in the global repository for libraries (e.g. /usr/local/lib)

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option is set to true, Mersenne Twister pseudo-random numbers will be used.

Complex – by default, only the real part of the integrals is evaluated, with this option set to True, the imaginary part will also be given.

FixedContours – contours will not be shifted if this option is set to True

NoHigherDimensional – by default, the complete integration is performed within MBintegrate, however with this option, 1-dimensional integrals are evaluated and the Fortran programs are prepared, but not run. This may be used to run them in parallel for example.

Debug – with this option set to True, the Fortran programs are kept after evaluation and the value of every integral given in the output as described above.

ContourDebug – with this options set to True, MBshiftContours will print contour optimization information.

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= {{}, {z → -1/2}}
```

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In[64]:= I1MB = MBint[int, rules]
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Out[64]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {{}, {z → -1/2}}]
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In[65]:= ? MBintegrate
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MBintegrate[integrals, kinematics] numerically integrates a list of integrals expanded in some variable with the values of the parameters as given on the kinematics list. The input should be as produced by MBexpand. Multiple integrals are evaluated in Fortran with the help of the CERN library implementation of the Gamma and PolyGamma functions and of the Cuhre and Vegas integration routines from CUBA. The libraries libmathlib.a, libkernlib.a and libcuba.a should be available; the compilation is performed with f77. The output is a list containing the value of the integral and the error estimates on the real and imaginary parts. Optionally the contributions of all the numerical integrations can be given, if option Debug is set to True. In this case, the numerical results are contained in MBval[value, error, probability, part], where probability is the probability that the error is underestimated, and part is the part number of the integral.

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In[65]:= ? MBintegrate
```

MBintegrate[integrals, kinematics] numerically integrates a list of integrals expanded in some variable with the values of the parameters as given on the kinematics list. The input should be as produced by MBexpand. Multiple integrals are evaluated in Fortran with the help of the CERN library implementation of the Gamma and PolyGamma functions and of the Cuhre and Vegas integration routines from CUBA. The libraries libmathlib.a, libkernlib.a and libcuba.a should be available, the compilation is performed with f77. The output is a list containing the value of the integral and the error estimates on the real and imaginary parts. Optionally the contributions of all the numerical integrations can be given, if option Debug is set to True. In this case, the

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working directory, or in the global repository for libraries (e.g. /usr/local/lib)

Untitled-3

MBrules[integrand, limit, constraints, fixedVars] determines the contours such that during analytic continuation no contour starts or ends on a pole.

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= {{}, {z → -1/2}}
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {{}, {z → -1/2}}]
```

```
MBintegrate[{I1MB}, {x →}]
```

```
In[65]:= ? MBintegrate
```

MBintegrate[integrals, kinematics] numerically integrates a list of integrals expanded in some variable with the values of the parameters as given on the kinematics list. The input should be as produced by MBexpand. Multiple integrals are evaluated in Fortran with the help of the CERN library implementation of the Gamma and PolyGamma functions and of the Cuhre and Vegas integration routines from CUBA. The libraries libmathlib.a, libkernlib.a and libcuba.a should be available, the compilation is performed with f77. The output is a list containing the value of the integral and the error estimates on the real and imaginary parts. Optionally the contributions of all the numerical integrations can be given, if option Debug is set to True. In this case, the

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Out[64]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {{}, {z → -1/2}}]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

```
Out[66]= {0.3905710601515178, 0}
```

```
In[65]:= ? MBintegrate
```

MBintegrate[integrals, kinematics] numerically integrates a list of integrals expanded in some variable with the values of the parameters as given on the kinematics list. The input should be as produced by MBexpand. Multiple integrals are evaluated in Fortran with the help of the CERN library implementation of the Gamma and PolyGamma functions and of the Cubre and Vegas

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working directory, or in the global repository for libraries (e.g. /usr/local/lib)

Untitled-3

```
Out[64]= MBint[ $\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}$ , {{}, {z  $\rightarrow$  - $\frac{1}{2}$ }}]
```

```
In[66]:= MBintegrate[{I1MB}, {x  $\rightarrow$  1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

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Out[66]= {0.3905710601515178, 0}
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Untitled-3

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```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[x^-1-z Gamma[-z] Gamma[1+z]^3, {{}, {z → -1/2}}]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
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Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

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Out[63]= {(), {z → -1/2}}
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[x^-1-z Gamma[-z] Gamma[1+z]^3, {(), {z → -1/2}}]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

```
Out[66]= {0.3905710601515178, 0}
```

```
I1an[x_] := 1/2 Log[(1 + Sqrt[
```

Untitled-3

MBrules[integrand, limit, constraints, fixedVars] determines the contours such that during analytic continuation no contour starts or ends on a pole.

In[63]:= rules = MBrules[int, {}, {}]

Out[63]= $\left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

In[64]:= I1MB = MBint[int, rules]

Out[64]= MBint $\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]$

In[66]:= MBintegrate[{I1MB}, {x → 1.2}]

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

Out[66]= {0.3905710601515178, 0}

IIan[x_] := 1/2 Log[(1 - Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2

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working directory, or in the global repository for libraries (e.g. /usr/local/lib)

Untitled-3

that during analytic continuation no contour starts or ends on a pole.

```
In[63]:= rules = MBrules[int, {}, {}]
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```

```
In[64]:= I1MB = MBint[int, rules]
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```
Out[64]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {{}, {z → -1/2}}]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...I

Higher-dimensional integrals

```
Out[66]= {0.3905710601515178, 0}
```

```
In[67]:= Ilan[x_] := 1/2 Log[(1 - Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
```

```
In[68]:= Ilan[1.2]
```

```
Out[68]= -4.54423 - 2.77661 i
```

Untitled-3

that during analytic continuation no contour starts or ends on a pole.

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= {{}, {z → -1/2}}
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {{}, {z → -1/2}}]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

```
Out[66]= {0.3905710601515178, 0}
```

```
In[67]:= Ilan[x_] := 1/2 Log[(1 - Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
```

```
In[68]:= Ilan[1.2]
```

```
Out[68]= -4.54423 - 2.77661 i
```

150%

working directory, or in the global repository for libraries (e.g. /usr/local/lib)

that during analytic continuation no contour starts or ends on a pole.

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= {{}, {z → -1/2}}
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {{}, {z → -1/2}}]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

```
Out[66]= {0.3905710601515178, 0}
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```
In[67]:= IIan[x_] := 1/2 Log[(1 - Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
```

```
In[68]:= IIan[1.2]
```

```
Out[68]= -4.54423 - 2.77661 i
```

example-triangle

that during analytic continuation no

```
In[63]:= rules = MBrules[int, {}, {}]
```

$$\text{Out}[63]= \left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$$

```
In[64]:= II1MB = MBint[int, rules]
```

$$\text{Out}[64]= \text{MBint}\left[\frac{x^{-1-z} \Gamma(-z) \Gamma[1+z]^2}{\Gamma[3+2z]} \right]$$

```
In[66]:= MBintegrate[{II1MB}, {x \rightarrow 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional

Higher-dimensional integral:

$$\text{Out}[66]= \{0.3905710601515178, 0\}$$

```
In[67]:= IIlan[x_] := 1/2 Log[(1 - Sqr
```

```
In[68]:= IIlan[1.2]
```

$$\text{Out}[68]= -4.54423 - 2.77661 i$$

```
In[1]:= $Path = Union[$Path, {"Users/johannes/Desktop/mathschool"}];
```

```
In[2]:= << MB.m
```

MB 1.2

by Michal Czakon

improvements by Alexander Smirnov

more info in hep-ph/0511200

last modified 2 Jan 09

MB integrand $x=m^2/s$:

```
In[3]:= IIint = Gamma[-z] Gamma[1+z] Gamma[1+z]^2 / Gamma[3+2z] x^(-1-z);
```

find allowed set of real parts for integration variables:

```
In[4]:= IIrules = MBrules[IIint, {}, {}]
```

$$\text{Out}[4]= \left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$$

```
In[5]:= IIintMB = MBint[IIint, IIrules]
```

$$\text{Out}[5]= \text{MBint}\left[\frac{x^{-1-z} \Gamma(-z) \Gamma[1+z]^2}{\Gamma[3+2z]}, \left\{ \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]$$

asymptotic expansion $x=m^2 \rightarrow 0$

```
In[6]:= << MBasymptotics.m
```

100% ▶

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working directory, or in the global repository for libraries (e.g. /usr/local/lib)

example-triangle

that during analytic continuation no

asymptotic expansion $x=m^2 \rightarrow 0$

```
In[63]:= rules = MBrules[int, {}, {}]
```

$$\text{Out}[63]= \left\{ \{\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$$

```
In[64]:= I1MB = MBint[int, rules]
```

$$\text{Out}[64]= \text{MBint}\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[z]}{\Gamma[3+2z]} \right]$$

```
In[66]:= MBintegrate[{I1MB}, {x \rightarrow 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integral

Higher-dimensional integral:

$$\text{Out}[66]= \{0.3905710601515178, 0\}$$

```
In[67]:= Ilan[x_] := 1/2 Log[(1 - Sqr
```

```
In[6]:= << MBasymptotics.m
```

```
In[14]:= step1 = MBmerge[MBasymptotics[I1intMB /. m^2 \rightarrow x, {x, 0}]]
```

$$\text{Out}[14]= \text{MBint}\left[\frac{\log(x)^2}{2}, \{\}, \{\} \right]$$

```
In[16]:= step1 /. MBint[a___, {\{}_, {\}}] \rightarrow a
```

$$\text{Out}[16]= \left\{ \frac{\log(x)^2}{2} \right\}$$

numerical evaluation of I1intMB:

```
In[18]:= MBintegrate[{I1intMB}, {x \rightarrow 1.2}, Verbose \rightarrow False]
```

$$\text{Out}[18]= \{0.3905710601515178, 0\}$$

```
In[19]:= Head[z \rightarrow -\frac{1}{2}]
```

$$\text{Out}[19]= \text{Rule}$$

```
In[20]:= I1intMB /. Rule[z, b_] \rightarrow Rule[z, -1/10]
```

$$\text{Out}[20]= \text{MBint}\left[\frac{x^{1-z} \Gamma[-z] \Gamma[1+z]^2}{\Gamma[3+2z]}, \{\}, \left\{ z \rightarrow -\frac{1}{10} \right\} \right]$$

100% ▶

```
In[68]:= Ilan[1.2]
```

$$\text{Out}[68]= -4.54423 - 2.77661 i$$

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working directory, or in the global repository for libraries (e.g. /usr/local/lib)

example-triangle

that during analytic continuation no

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= { {}, { z → -1/2 } }
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[ x^-1-z Gamma[-z] Gamma[ ] / Gamma[3 + 2 z] ]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]

Shifting contours...
```

Performing 1 lower-dimensional integral

Higher-dimensional integral:

```
Out[66]= {0.3905710601515178, 0}
```

```
In[67]:= Ilan[x_] := 1/2 Log[(1 - Sqr
```

```
In[68]:= Ilan[1.2]
```

```
Out[68]= -4.54423 - 2.77661 i
```

approximate numerical evaluation of I1intMB:

```
In[25]:= NIntegrate[1/(2 Pi I) I I1int /. z → -1/2 + I a /. x → 1.2, {a, -5, 5}]
```

```
Out[25]= 0.390571 - 1.13798 × 10^-15 i
```

```
In[26]:= NIntegrate[1/(2 Pi I) I I1int /. z → -1/100 + I a /. x → 1.2, {a, -5, 5}]
```

NIntegrate::slwcon:

Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small. >>

NIntegrate::ncvib:

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in a near $|a| = 0.0193758$. NIntegrate obtained 0.390571 + 0. i and 5.916684588951808' -^6 for the integral and error estimates. >>

```
Out[26]= 0.390571 - 0. i
```

analytical result:

```
In[27]:= y[x] := (Sqrt[1 + 4 x] - 1) / (Sqrt[1 + 4 x] + 1)
Ilan[x_] := 1/2 Log[y[x]]^2
```

```
In[29]:= Series[y[x], {x, 0, 1}]
```

```
Out[29]= x + O[x]^2
```

```
In[30]:= Expand[Normal[Series[Ilan[x], {x, 0, 4}]]]
```

100% ▶

that during analytic continuation no poles are crossed.

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= {{}, {z → -1/2}}
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[ $\frac{x^{-1-z} \Gamma[-z] \Gamma[z]}{\Gamma[3+2 z]}$ ]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integral...

Higher-dimensional integral...

```
Out[66]= {0.3905710601515178, 0}
```

```
In[67]:= Ilan[x_] := (Sqrt[1 + 4 x] - 1) / (Sqrt[1 + 4 x] + 1)
```

```
In[68]:= Ilan[1.2]
```

```
Out[68]= -4.54423 - 2.77661 i
```

example-triangle

approximate numerical evaluation of I1intMB:

```
In[25]:= NIntegrate[1/(2 Pi I) I I1int /. z → -1/2 + I a /. x → 1.2, {a, -5, 5}]
```

```
Out[25]= 0.390571 + 1.13798 × 10-13 i
```

```
In[26]:= NIntegrate[1/(2 Pi I) I I1int /. z → -1/100 + I a /. x → 1.2, {a, -5, 5}]
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Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small. >>

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analytical result:

```
In[27]:= y[x_] := (Sqrt[1 + 4 x] - 1) / (Sqrt[1 + 4 x] + 1)
```

Iilan[x_] := 1/2 Log[y[x]]^2

```
In[29]:= Series[y[x], {x, 0, 1}]
```

```
Out[29]= x + O[x]^2
```

```
In[30]:= Expand[Normal[Series[Iilan[x], {x, 0, 4}]]]
```

100% ▶

that during analytic continuation no contour starts or ends on a pole.

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= {{}, {z → -1/2}}
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {{}, {z → -1/2}}]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

```
Out[66]= {0.3905710601515178, 0}
```

```
In[67]:= Ilan[x_] := 1/2 Log[(1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
```

```
In[68]:= Ilan[1.2]
```

```
Out[68]= -4.54423 - 2.77661 i
```

Untitled-3

that during analytic continuation no contour starts or ends on a pole.

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= {{}, {z → -1/2}}
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {{}, {z → -1/2}}]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

```
Out[66]= {0.3905710601515178, 0}
```

```
In[67]:= Ilan[x_] := 1/2 Log[(1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
```

```
In[68]:= Ilan[1.2]
```

```
Out[68]= -4.54423 - 2.77661 i
```

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= {[], {z → -1/2}}
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {[], {z → -1/2}}]
```

```
In[66]:= MBintegrate[{I1MB}, {x → 1.2}]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

```
Out[66]= {0.3905710601515178, 0}
```

```
In[69]:= I1an[x_] := 1/2 Log[(-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
```

```
In[70]:= I1an[1.2]
```

```
Out[70]= 0.390571
```

0.3905710601471226`

In[63]:= **rules** = MBrules[int, {}, {}]

Out[63]= $\left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

In[64]:= I1MB = MBint[int, rules]

Out[64]= MBint $\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \left\{ \left\{ \right\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]$

In[66]:= MBintegrate[{I1MB}, {x → 1.2}]

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

Out[66]= {0.3905710601515178, 0}

In[69]:= I1an[x_] := 1/2 Log[(-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2

In[70]:= I1an[1.2]

Out[70]= 0.390571

0.3905710601471226`

```
In[63]:= rules = MBrules[int, {}, {}]
```

```
Out[63]= { {}, { z → - 1/2 } }
```

```
In[64]:= I1MB = MBint[int, rules]
```

```
Out[64]= MBint[ x^-1-z Gamma[-z] Gamma[1+z]^3 , { {}, { z → - 1/2 } } ]
```

```
In[71]:= val1 = MBintegrate[{I1MB}, {x → 1.2}][[1]]
```

Shifting contours...

Performing 1 lower-dimensional integrations with NIntegrate...1

Higher-dimensional integrals

```
Out[71]= 0.3905710601515178
```

```
In[69]:= IIan[x_] := 1/2 Log[ (-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x]) ]^2
```

```
In[73]:= val2 = IIan[1.2]
```

```
Out[73]= 0.390571
```

val1 - val2

0.3905710601471226`

150%

working directory, or in the global repository for libraries (e.g. /usr/local/lib)

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Untitled-3 Tue 12:32

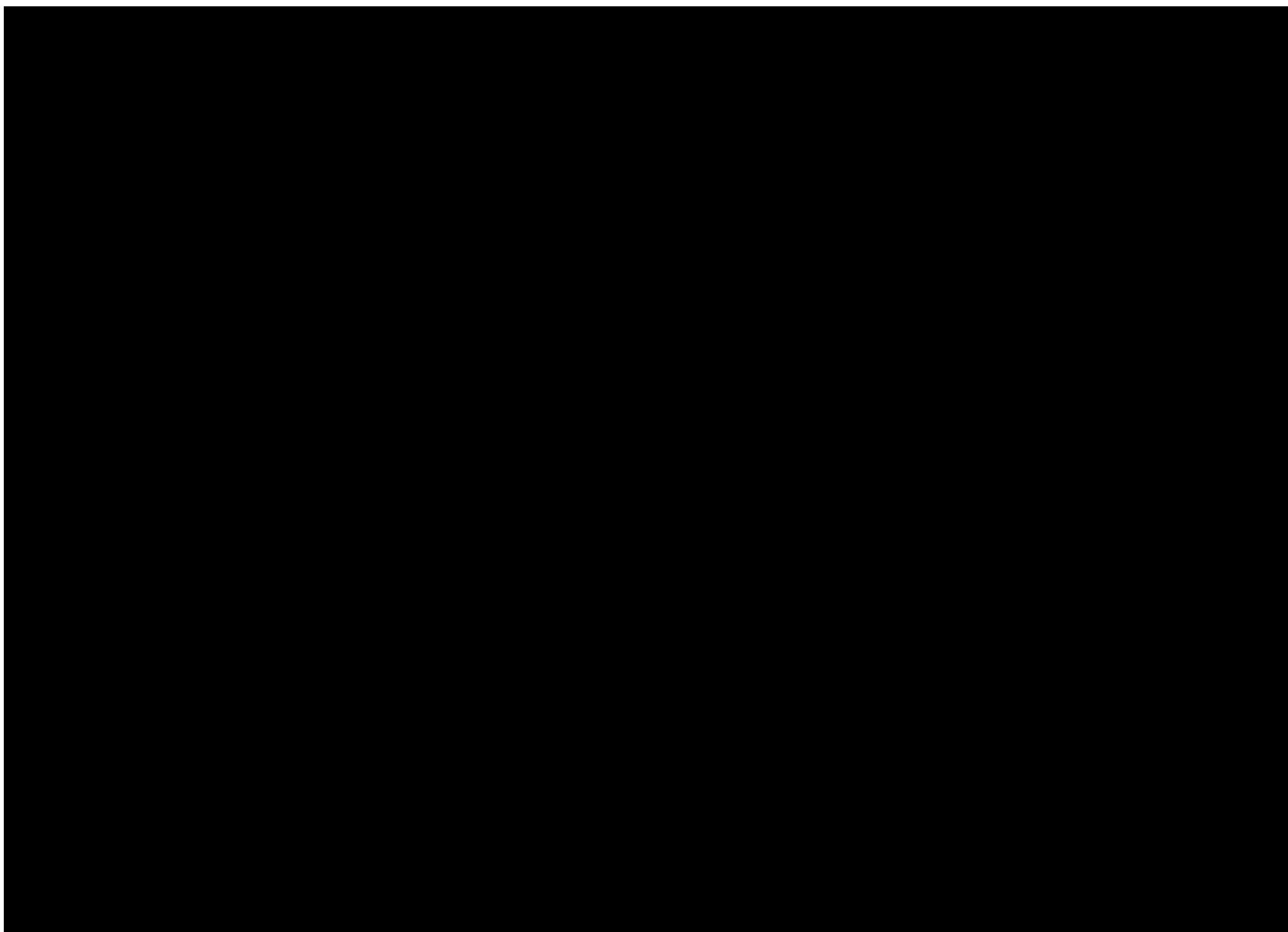
```
In[64]:= I1MB = MBint[int, rules]
Out[64]= MBint[ $\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}$ , {{}, {z → - $\frac{1}{2}$ }}]

In[71]:= val1 = MBintegrate[{I1MB}, {x → 1.2}] [[1]]
Shifting contours...
Performing 1 lower-dimensional integrations with NIntegrate...
Higher-dimensional integrals
Out[71]= 0.3905710601515178

In[69]:= I1an[x_] := 1/2 Log[(-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
In[73]:= val2 = I1an[1.2]
Out[73]= 0.390571
In[74]:= val1 - val2
Out[74]= 4.39521 × 10-12
```

150%

working directory, or in the global repository for libraries (e.g. /usr/local/lib)



$$S = (p_1 \cdot p_2)^2$$

$$= \int_{-\infty}^{i\infty} \frac{h}{2\pi i} \Gamma(-z) \bar{\Gamma}(1-z) \sqrt{\Gamma(3+2z)} \cdot x^{-z-2}$$

$$x = \frac{R^2}{s}, -1 < \text{Re } z < 0$$

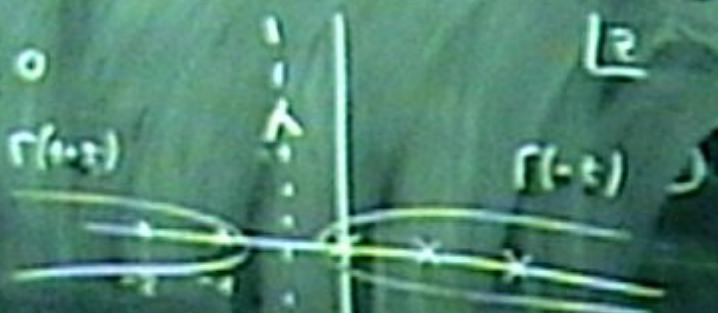
$$= \frac{1}{2} \log \left(\frac{1 - \sqrt{1-4x}}{1 + \sqrt{1-4x}} \right) \Gamma(c-z) \Gamma(bz)$$



$$S = (\rho_1 \rho_2)^2$$

$$= \int_{-\infty}^{i\infty} \frac{h}{2\pi i} \Gamma(-z) \bar{\Gamma}(1-z) \sqrt{\Gamma(3+2z)} \cdot x^{-z-2} dz$$

$$x = \frac{R^2}{z}, -1 < Re z < 0$$



$$= \frac{1}{2} \log \left(\frac{1 - \sqrt{1 - 4x}}{1 + \sqrt{1 - 4x}} \right) \frac{\Gamma(c-z)\Gamma(bz)}{\Gamma(a-z)\Gamma(bz)}$$

\rightarrow

$$S = \{(x,y) \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$$

$$= \int_{-\infty}^{+\infty} \frac{h}{2\pi i} (t-z) \bar{R}(t-z) \sqrt{V(3,2z)} z^{-\frac{d-1}{2}} dz$$

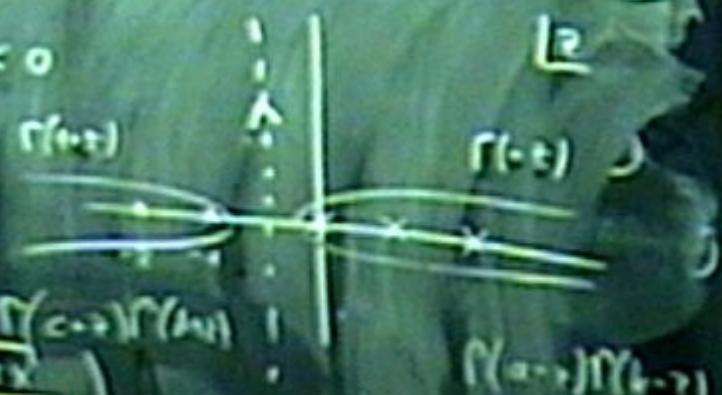
$$k = \frac{\pi^2}{4} - 4\Re z < 0$$

$$= \frac{1}{2} \log \left(\frac{1 - \sqrt{1-4k}}{1 + \sqrt{1-4k}} \right)$$

$$S = \langle \vec{p}_1 \cdot \vec{p}_2 \rangle$$

$$= - \int_{-\infty}^{i\infty} \frac{h}{2\pi i} \Gamma(-z) \Gamma(1-z) \sqrt{\Gamma(3+2z)} \cdot x^{-4-z}$$

$$x = \frac{R^2}{z} - 1 \quad (R > 0, z < 0)$$



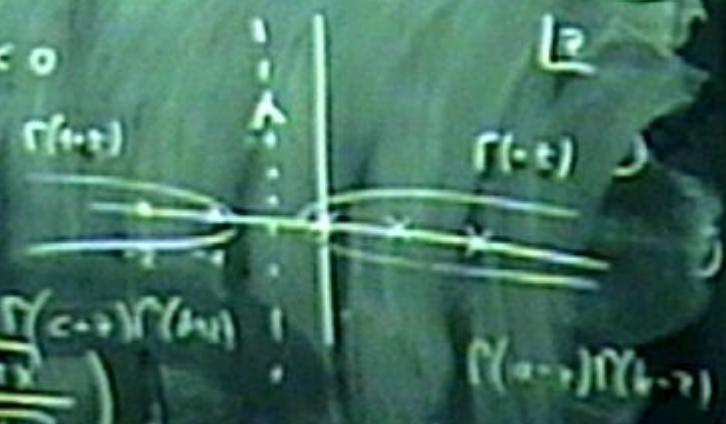
$$= \frac{1}{2} \lg \left(\frac{1 - \sqrt{1 - 4x}}{1 + \sqrt{1 - 4x}} \right)$$

$$\Gamma(-z) \Gamma(1-z)$$

$$S = \langle \vec{q}_0 - \vec{p}_0 \rangle$$

$$= \int_{-\infty}^{\infty} \frac{i\hbar}{2m} \Pi(x) \tilde{N}(x + \sqrt{3}x_0) x^{d-1} dx$$

$$x = \frac{x_0}{\sqrt{3}} - i k x_0, x < 0$$



$$= \frac{1}{2} \log \left(\frac{1 - \sqrt{1 - 4x}}{1 + \sqrt{1 - 4x}} \right)$$

$$\tilde{N}(x_0) \tilde{N}(b - x_0)$$

```

In[64]:= IIMB = MBint[int, rules]

Out[64]= MBint[ $\frac{x^{1-z} \Gamma(-z) \Gamma(1+z)^3}{\Gamma(3+2z)}, \{(), \{z \rightarrow -\frac{1}{2}\}\}]$ 

val1 = MBintegrate[{IIMB}, {x -> 1.2}, Verbose -> {{1}}]
Shifting contours...
Performing 1 lower-dimensional integrations with NIntegrate...
Higher-dimensional integrals

Out[71]= 0.3905710601515178

In[72]:= Ilam[x_] := 1/2 Log[ (-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2

In[73]:= val2 = Ilam[1.2]

Out[73]= 0.390571

In[74]:= val1 - val2

Out[74]= 4.39521 \times 10^{-12}

```

```

In[64]:= IIMB = MBint[int, rules]

Out[64]= MBint\left[\frac{x^{-1-z} \Gamma(-z) \Gamma[1+z]^3}{\Gamma[3+2 z]}, \left\{(), \left\{z \rightarrow -\frac{1}{2}\right\}\right\}\right]

val1 = MBintegrate[{IIMB}, {x \rightarrow 1.2}, Verbose \rightarrow {}[[1]]]
Shifting contours...
Performing 1 lower-dimensional integrations with NIntegrate...
Higher-dimensional integrals

Out[71]= 0.3905710601515178

In[69]:= Ilan[x_] := 1/2 Log[ (-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2

In[73]:= val2 = Ilan[1.2]
Out[73]= 0.390571

In[74]:= val1 - val2
Out[74]= 4.39521 \times 10^{-12}

```

```
In[64]:= I1MB = MBint[int, rules]
Out[64]= MBint[ $\frac{x^{1-z} \Gamma(-z) \Gamma(1+z)^3}{\Gamma(3+2z)}, \{(), \{z \rightarrow -\frac{1}{2}\}\}]$ 

In[75]:= val1 = MBintegrate[{I1MB}, {x \rightarrow 1.2}, Verbose \rightarrow False][[1]]
Out[75]= 0.3905710601515178

In[76]:= Ilan[x_] := 1/2 Log[ (-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])] ^ 2
In[77]:= val2 = Ilan[1.2]
Out[77]= 0.390571

In[78]:= val1 - val2
Out[78]= 4.39521 \times 10^{-12}
```

```

In[64]:= IIMB = MBint[int, rules]
Out[64]= MBint\left[\frac{x^{1-x} \Gamma(-x) \Gamma(1+x)^3}{\Gamma(3+2 x)}, \left\{(), \left\{x \rightarrow -\frac{1}{2}\right\}\right\}\right]

In[75]:= val1 = MBintegrate[{IIMB}, {x \rightarrow 1.2}, Verbose \rightarrow False][[1]]
Out[75]= 0.3905710601515178

In[69]:= Ilan[x_] := 1/2 Log[ (-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])] ^ 2
In[73]:= val2 = Ilan[1.2]
Out[73]= 0.390571

In[74]:= val1 - val2
Out[74]= 4.39521 \times 10^{-12}

In[77]:= val3 = NIntegrate[1 / (2 Pi I) \int int /. x \rightarrow -1/2 + I y /. x \rightarrow 1.2, {y, -5, 5}]
Out[77]= 0.390571 - 1.13798 \times 10^{-15} I

In[78]:= val2 - val3
Out[78]= -2.67586 \times 10^{-9} - 1.13798 \times 10^{-15} I

```

```

In[63]:= rules = MBrules[int, {}, {}]
Out[63]= {{}, {z → -1/2} }

In[64]:= IIMB = MBint[int, rules]
Out[64]= MBint[ $\frac{x^{1-z} \Gamma(-z) \Gamma(1+z)^3}{\Gamma(3+2z)}, \{ \}, \{ z \rightarrow -\frac{1}{2} \}]$ ]

In[65]:= val1 = MBintegrate[{IIMB}, {x → 1.2}, Verbose → False][[1]]
Out[65]= 0.3905710601515178

In[66]:= Ilan[x_] := 1/2 Log[ (-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
In[67]:= val2 = Ilan[1.2]
Out[67]= 0.390571

In[68]:= val1 - val2
Out[68]= 4.39521 × 10-12

In[69]:= val3 = NIntegrate[1 / (2 Pi I) I int /. z → -1/2 + I y /. x → 1.2, {y, -5, 5}]
Out[69]= 0.390571 + 1.13798 × 10-15 i

In[70]:= val2 - val3

```

```

In[63]:= rules = MBrules[int, {}, {}]
Out[63]= {(), {z -> -1/2} }

In[64]:= IIMB = MBint[int, rules]
Out[64]= MBint[ $\frac{x^{1-z} \Gamma(-z) \Gamma(1+z)^3}{\Gamma(3+2z)}, \{(), \{z \rightarrow -\frac{1}{2}\}\}]$ 

In[65]:= val1 = MBintegrate[{IIMB}, {x -> 1.2}, Verbose -> False][[1]]
Out[65]= 0.3905710601515178

In[66]:= Ilan[x_] := 1/2 Log[ (-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
In[67]:= val2 = Ilan[1.2]
Out[67]= 0.390571

In[68]:= val1 - val2
Out[68]= 4.39521  $\times 10^{-12}$ 

In[69]:= val3 = NIntegrate[1 / (2 Pi I) I int /. z -> -1/2 + I y /. x -> 1.2, {y, -5, 5}]
Out[69]= 0.390571 + 1.13798  $\times 10^{-15}$  I

In[70]:= val2 - val3

```

that during analytic continuation no contour starts or ends on a pole.

In[62]:= rules = MBrules[int, {}, {}]

Out[62]= $\left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

In[64]:= IIMB = MBint[int, rules]

Out[64]=
$$\text{MBint}\left[\frac{x^{1-z} \Gamma(-z) \Gamma(1+z)^3}{\Gamma(3+2z)}, \left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]$$

In[75]:= val1 = MBintegrate[{IIMB}, {x → 1.2}, Verbose → False][[1]]

Out[75]= 0.3905710601515178

In[69]:= Iian[x_] := 1/2 Log[(-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])] ^ 2

In[73]:= val2 = Iian[1.2]

Out[73]= 0.390571

In[74]:= val1 - val2

Out[74]= 4.39521×10^{-12}

In[77]:= val3 = NIntegrate[1 / (2 Pi I) I int /. z → -1/2 + I y /. x → 1.2, {y, -5, 5}]

Out[77]= $0.390571 - 1.13798 \times 10^{-15} i$

`MBrules[integrand, limit, constraints, fixedVars]` determines the contours such that during analytic continuation no contour starts or ends on a pole.

In[63]:= `rules = MBrules[int, (), ()]`

Out[63]= $\left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

In[64]:= `IIMB = MBint[int, rules]`

Out[64]=
$$\text{MBint}\left[\frac{x^{-1-z} \Gamma(-z) \Gamma(1+z)^3}{\Gamma(3+2z)}, \left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]$$

In[65]:= `vall = MBintegrate[{IIMB}, {x → 1.2}, Verbose → False][[1]]`

Out[65]= 0.3905710601515178

In[66]:= `Ilam[x_] := 1/2 Log[(-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])] ^ 2`

In[72]:= `val2 = Ilam[1.2]`

Out[72]= 0.390571

In[74]:= `vall - val2`

Out[74]= 4.39521×10^{-12}

In[77]:= `val3 = NIntegrate[1 / (2 Pi I) I int /. z → -1/2 + I y /. x → 1.2, {y, -5, 5}]`

psi-omega

symbols2

Upatorregge

travelat2011

july 2011 IAS photos

VISA stuff

Jp-Cut-09055348v2

autMRAugust2011
int21

M8 Tools HepForge

<http://projects.hepforge.org/m8rules/>

Smirnov.

The project is at the development stage, so expect more codes to appear here.

Currently the following codes can be downloaded:

Untitled-3

M8Rules[integrand, limit, constraints, fixedVars] determines the contours such that during analytic continuation no contour starts or ends on a pole.

In[63]:= rules = M8Rules[int, (), ()]

Out[63]= $\left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

In[64]:= I1M8 = M8int[int, rules]

Out[64]= M8int $\left| \frac{x^{1-z} \Gamma[-z] \Gamma[1-z]^2}{\Gamma[3+2z]}, \left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right|$

In[65]:= val1 = M8integrate[{I1M8}, {x -> 1.2}, Verbose -> False][[1]]

M& Tools :: HepForge

http://projects.hepforge.org/mbtools/

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Alexander Smirnov, Vladimir Smirnov.

The project is at the development stage, so expect more codes to appear here.

Currently the following codes can be downloaded:

- **MB.m** : version 1.2 of MB (last updated January 2nd, 2009) by Michal Czakon, the main collection of routines for the resolution of singularities and the numerical evaluation of Mellin-Barnes integrals; for details see [hep-ph/0511200](#); the current version is documented in the [Manual](#); the distribution contains two example notebooks, `MBexamples1.nb` and `MBexamples2.nb`;
- **MBasymptotics.m** : a routine which expands Mellin-Barnes integrals in a small parameter by Michal Czakon; example usage is illustrated in `MBasymptotics.nb`;
- **MBresolve.m** : a tool by Alexander Smirnov and Vladimir Smirnov realizing another strategy of resolving singularities of Mellin-Barnes integrals. This code should be loaded together with `MB.m` since it uses some of its routines. For details see [arXiv:0901.0386](#)
- **AMBRE.m** : a tool by Janusz Gluza, Krzysztof Kajda and Tord Riemann for constructing Mellin-Barnes representations. It works both for planar multiloop scalar and one-loop tensor Feynman integrals. This is version 1.2, for previous versions and detailed description of the package with examples see the [home page](#). The program is described in [arXiv:0704.2423](#) and Computer Physics Communications 177 (2007) 879.
- **barnesroutines.m** : a tool by David Kosower for automatic application of the first and second Barnes lemmas to lists of multiple Mellin-Barnes integrals. An example notebook is included.

The numerical integration routines used by MB require the following libraries to be installed, either in the current working directory, or in the global repository for libraries (e.g. `/usr/local/lib`)

- **CUBA**
- **CERNLIB** (only `libmathlib.a` and `libkernlib.a` are actually required)

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0.3905710601515178

Page 68/211

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for details see arXiv:0901.0386.

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- **CUBA**
- **CERNlib** (only `libmathlib.a` and `libkernlib.a` are actually required)

You will also need a Fortran 77 compiler named `f77`...

Contact

If you would like to be informed of updates and new software available on this web page, please send an e-mail to `mbtools-announce-request@projects.hepforge.org` with subject `subscribe`. You can also contact the developers at `mbtools@projects.hepforge.org`

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Page 69/211

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- **CERNlib** (only libmathlib.a and libkernlib.a are actually required)

You will also need a Fortran 77 compiler named f77...

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- **MBresolve.m** : a tool by Alexander Smirnov and Vladimir Smirnov realizing another strategy of resolving singularities of Mellin-Barnes integrals. This code should be loaded together with `MB.m` since it uses some of routines. For details see [arXiv:0901.0386](#)
- **AMBRIE.m** : a tool by Janusz Gluza, Krzysztof Kajda and Tord Riemann for constructing Mellin-Barnes representations. It works both for planar multiloop scalar and one-loop tensor Feynman integrals. This is version 1.2, for previous versions and detailed description of the package with examples see the [home page](#). The program is described in [arXiv:0704.2423](#) and Computer Physics Communications 177 (2007) 879.
- **barnesroutines.m** : a tool by David Kosower for automatic application of the first and second Barnes lemmas to lists of multiple Mellin-Barnes integrals. An example notebook is included.

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- **CERNlib** (only `libmathlib.a` and `libkernlib.a` are actually required)

You will also need a Fortran 77 compiler named f77...

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- **AMBRE.m** : a tool by Janusz Gluza, Krzysztof Kajda and Tord Riemann for constructing representations. It works both for planar multiloop scalar and one-loop tensor Feynman integrals; for previous versions and detailed description of the package with examples see the [AMBRE page](#); the program is described in [arXiv:0704.2423](#) and [Computer Physics Communications 177 \(2007\) 1–16](#)
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You will also need a Fortran 77 compiler named `f77`...

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Currently the following codes can be downloaded:

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- **MBasymptotics.m** : a routine which expands Mellin-Barnes integrals in a small parameter; example usage is illustrated in `MBasymptotics.nb`;

MBrules[integrand, limit, constraints, fixedVars] determines the contours such that during analytic continuation no contour starts or ends on a pole.

```

In[63]:= rules = MBrules[int, {}, {}]
Out[63]= {(), {z → -1/2}}
In[64]:= IIMB = MBint[int, rules]
Out[64]= MBint[(x^-1 z Gamma[-z] Gamma[1 + z]^3)/Gamma[3 + 2 z], {(), {z → -1/2}}]
In[75]:= val1 = MBintegrate[{IIMB}, {z → 1.2}, Verbose -> False][[1]]
Out[75]= 0.3905710601515178

```

`MBrules[integrand, limit, constraints, fixedVars]` determines the contours such that during analytic continuation no contour starts or ends on a pole.

```
In[53]:= rules = MBrules[int, {}, {}]
```

```
Out[53]= {{}, {z → -1/2}}
```

```
In[54]:= IIMB = MBint[int, rules]
```

```
Out[54]= MBint[(x^-1-z Gamma[-z] Gamma[1+z]^3)/Gamma[3+2 z], {{}, {z → -1/2}}]
```

```
In[55]:= val1 = MBintegrate[{IIMB}, {x → 1.2}, Verbose → False][[1]]
```

```
Out[55]= 0.3905710601515178
```

```
In[56]:= Iiam[x_] := 1/2 Log[(-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2
```

```
In[57]:= val2 = Iiam[1.2]
```

```
Out[57]= 0.390571
```

```
In[58]:= val1 - val2
```

```
Out[58]= 4.39521 × 10^-12
```

Out[74]= 4.39521×10^{-12}

In[77]:= val3 = NIntegrate[1 / (2 Pi I) I int /. z -> -1/2 + I y /. z -> 1.2, {y, -5, 5}]
Out[77]= 0.390571 - 1.13798 $\times 10^{-15}$ I

In[78]:= val2 - val3

Out[78]= -2.67586 $\times 10^{-9}$ - 1.13798 $\times 10^{-15}$ I

MBrules[integrand, limit, constraints, fixedVars] determines the contours such that during analytic continuation no contour starts or ends on a pole.

```

In[63]:= rules = MBrules[int, {}, {}]

Out[63]= {(), {z -> -1/2} }

In[64]:= I1MB = MBint[int, rules]

Out[64]= MBint[ $\frac{x^{-1-z} \Gamma(-z) \Gamma(1+z)^3}{\Gamma(3+2z)}, \{(), \{z \rightarrow -\frac{1}{2}\}\}]$ 

In[65]:= val1 = MBintegrate[{I1MB}, {x -> 1.2}, Verbose -> False][[1]]

Out[65]= 0.3905710601515178

In[66]:= I1am[x_] := 1/2 Log[(-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2

In[67]:= val2 = I1am[1.2]

Out[67]= 0.390571

In[68]:= val1 - val2

Out[68]= 4.39521 \times 10^{-12}

```

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MB-Tools HepForge

Untitled-1

http://projects.hepforge.org/mbtools/

```
Out[74]= 4.39521 \times 10^{-12}

In[77]= val3 = NIntegrate[1 / (2 Pi I) \! \int_{}^{} x \rightarrow -1/2 + I y / . x \rightarrow 1.2, {y, -5, 5}]

Out[77]= 0.390571 - 1.13798 \times 10^{-15} I

In[78]= val2 - val3

Out[78]= -2.67586 \times 10^{-9} - 1.13798 \times 10^{-15} I
```

<http://projects.hepforge.org/motools/>

Untitled-1

MBrules(integrand, limit, constraints, fixedVars) determines the contours such that during analytic continuation no contour starts or ends on a pole.

In[53]:= rules = MBrules[int, {}, {}]

Out[53]= $\left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

In[54]:= I1MB = MBint[int, rules]

Out[54]= MBint \left[\frac{x^{1-z} \Gamma(-z) \Gamma(1+z)^3}{\Gamma(3+2z)}, \left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]

In[55]:= val1 = MBintegrate[(I1MB), {x → 1.2}, Verbose → False][[1]]

Out[55]= 0.3905710601515178

In[56]:= Ilam[x_] := 1/2 Log[(-1 + Sqrt[1 + 4 x]) / (1 + Sqrt[1 + 4 x])]^2

In[57]:= val2 = Ilam[1.2]

Out[57]= 0.390571

In[58]:= val1 - val2

Out[58]= 4.39521×10^{-12}

```
In[78]:= val2 - val3
```

```
Out[78]= -2.67586 × 10-9 - 1.13798 × 10-13 i
```

http://projects.hepforge.org/mbtools/

Untitled-3

In[78]:= val2 - val3

Out[78]= $-2.67586 \times 10^{-9} - 1.13798 \times 10^{-15} i$

In[79]:= I1MB

Out[79]= MBint [$\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}$, $\{ \}, \{ z \rightarrow -\frac{1}{2} \} \}]$

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Tue 12:38

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Untitled-3

In[78]:= val2 - val3

Out[78]= $-2.67586 \times 10^{-9} - 1.13798 \times 10^{-15} i$

In[79]:= I1MB

Out[79]= MBint $\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \left\{ \{ \}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]$

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Currently the following codes can be downloaded:

- **MB.m**: version 1.2 of MB (last updated January 2nd, 2009) by [Michał Czakon](#), the main collection of routines for the resolution of singularities and the numerical evaluation of integrals; for details see [hep-ph/0511200](#); the current version is documented in the [Manual](#); the distribution contains two example notebooks, **MBexamplest.nb** and **MBexamples2.nb**;
- **MBasymptotics.m**: a routine which expands Mellin-Barnes integrals in a small parameter ϵ ; example usage is illustrated in [MBasymptotics.nb](#);
- **MBresolve.m**: a tool by [Alexander Smirnov](#) and [Vladimir Smirnov](#) realizing another strategy for the resolution of singularities of Mellin-Barnes integrals. This code should be loaded together with **MB.m** since it depends on some of its routines. For details see [arXiv:0901.0386](#)
- **AMBRE.m**: a tool by [Janusz Gluza](#), [Krzysztof Kaida](#) and [Tord Riemann](#) for constructing Mellin-Barnes representations of Feynman diagrams.

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In[80]:= << mbasymptotics.m

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Untitled-3

In[78]:= **val2 - val3**Out[78]= $-2.67586 \times 10^{-9} - 1.13798 \times 10^{-15} i$ In[79]:= **I1MB**Out[79]=
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- **AMBRE.m**: a tool by Janusz Gluza, Krzysztof Kajda and Tord Riemann for constructing Mellin representations. It works both for planar multiloop scalar and one-loop tensor Feynman integrals. Version 1.2, for previous versions and detailed description of the package with examples see the [help page](#); the program is described in [arXiv:0704.2423](#) and Computer Physics Communications 177 (2007) 1–16.
- **barnesroutines.m**: a tool by David Kosower for automatic application of the first and second order differential equations for the resolution of singularities of Mellin-Barnes integrals. An example notebook is included.

The numerical integration routines used by MB require the following libraries to be installed, either in the working directory, or in the global repository for libraries (e.g. /usr/local/lib)

- **CUBA**
- **CERNlib** (only libmathlib.a and libkernlib.a are actually required)

You will also need a Fortran 77 compiler named f77...

Contact

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Untitled-3

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Out[78]= $-2.67586 \times 10^{-9} - 1.13798 \times 10^{-15} i$

In[79]:= II1MB

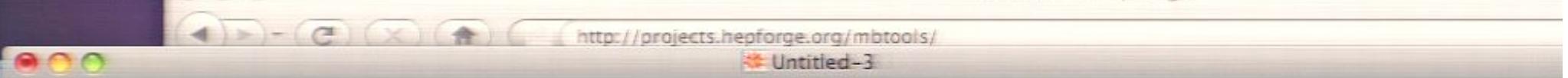
Out[79]=
$$\text{MBint}\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2 z]}, \left\{\{z\}, \left\{z \rightarrow -\frac{1}{2}\right\}\right\}\right]$$

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In[80]:= << mbasymptotics.m

http://projects.hepforge.org/mbtools/

Untitled-3

Out[74]= 4.39521×10^{-12}

In[77]:= val3 = NIntegrate[1 / (2 Pi I) I int /. z → -1/2 + I y /. x → 1.2, {y, -5, 5}]

Out[77]= $0.390571 + 1.13798 \times 10^{-15} i$

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In[80]:= << mbasympototics.m

approximate numerical evaluation of II1MB:

In[25]:= NIntegrate[1 / (2 Pi I) II1MB /. z -> -1/2 + I a /. x -> 1.2, {a, -5, 5}]

Out[25]= $0.390571 + 1.13798 \times 10^{-15} i$

In[26]:= NIntegrate[1 / (2 Pi I) II1MB /. z -> -1/100 + I a /. x -> 1.2, {a, -5, 5}]

NIntegrate::slwcon :

Numerical integration converging too slowly; suspect one of the following: singularity, value of the integration is 0, highly oscillatory integrand, or WorkingPrecision too small. >>

NIntegrate::ncvib :

NIntegrate failed to converge to prescribed accuracy after 9 recursive bisections in a near $|a| = |0.0193758|$. NIntegrate obtained $0.390571 + 0. i$ and $5.916684588951808` \times 10^{-6}$ for the integral and error estimates. >>

Out[26]= $0.390571 + 0. i$

analytical result:

In[27]:= y[x_]:= (Sqrt[1+4x]-1)/(Sqrt[1+4x]+1)
II1an[x_]:= 1/2 Log[y[x]]^2

In[28]:= Series[y[x], {x, 0, 1}]

Out[28]= $x + O(x)^2$

In[29]:= Expand[Normal[Series[II1an[x], {x, 0, 4}]]]

100% ▶

Out[74]= 4.39521×10^{-12}

In[77]:= val3 = NIntegrate[1 / (2 P:

Out[77]= $0.390571 + 1.13798 \times 10^{-15}$

In[78]:= val2 - val3

Out[78]= $-2.67586 \times 10^{-9} - 1.13798 \times$

In[79]:= IIIMB

Out[79]= MBint $\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^2}{\Gamma[3+2z]} \right]$

In[80]:= << mbasympototics.m

example-triangle

In[1]:= \$Path = Union[\$Path, {"Users/johannes/Desktop/mathschool"}];

In[2]:= << MB.m

MB 1.2

by Michal Czakon

improvements by Alexander Smirnov

more info in hep-ph/0511200

last modified 2 Jan 09

MB integrand $x=m^2/s$:

In[3]:= IIint = Gamma[-z] $\overset{\rightarrow}{\Gamma}[1+z] \Gamma[1+z]^2 / \Gamma[3+2z] x^{(-1-z)}$;

find allowed set of real parts for integration variables:

In[4]:= IIRules = MBRules[IIint, {}, {}]

Out[4]= $\left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\}$

In[5]:= IIintMB = MBint[IIint, IIRules]

Out[5]= MBint $\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^2}{\Gamma[3+2z]}, \left\{ \left(\right), \left\{ z \rightarrow -\frac{1}{2} \right\} \right\} \right]$

asymptotic expansion $x=m^2 \rightarrow 0$

In[6]:= << MBasympototics.m

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Out[74]= 4.39521×10^{-12}

In[77]:= val3 = NIntegrate[1 / (2 P)

Out[77]= $0.390571 + 1.13798 \times 10^{-15}$

In[78]:= val2 - val3

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In[79]:= IIintMB

Out[79]= MBint $\left[\frac{x^{-1-z} \Gamma[-z] \Gamma}{\Gamma[3+z]} \right]$

In[80]:= << mbasymptotics.m

asymptotic expansion $x=m^2 \rightarrow 0$

```
In[8]:= << MBasymptotics.m
In[14]:= step1 = MBmerge[MBasymptotics[IIintMB /. m^2 + x, {x, 0}]];
Out[14]= MBint[ $\frac{\log(x)^2}{2}, \{ \}, \{ \} \}]$ 
In[16]:= step1 /. MBint[a___, { {} }, { {} }] :> a
Out[16]=  $\left\{ \frac{\log(x)^2}{2} \right\}$ 
```

numerical evaluation of IIintMB:

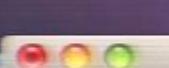
```
In[15]:= MBintegrate[{IIintMB}, {x → 1.2}, Verbose → False]
Out[15]= {0.3905710601515178, 0}
In[19]:= Head[x → - $\frac{1}{2}$ ]
Out[19]= Rule
In[20]:= IIintMB /. Rule[x, b_] :> Rule[x, -1/10]
Out[20]= MBint[ $\frac{x^{-1-z} \Gamma[-z] \Gamma(1+z)^2}{\Gamma[3+2z]}, \{ \}, \{ z \rightarrow -\frac{1}{10} \}]$ 
```

100% ▶

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Untitled-3

Out[74]:= 4.39521×10^{-12}

In[77]:= val3 = NIntegrate[1 / (2 Pi I) I int /. z → -1/2 + I y /. x → 1.2, {y, -5, 5}]

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In[78]:= val2 - val3

Out[78]:= $-2.67586 \times 10^{-9} - 1.13798 \times 10^{-15} i$

In[79]:= II1MB

Out[79]:= MBint[$\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}$, $\{ \}, \{ z \rightarrow -\frac{1}{2} \}]$

In[80]:= << mbasympotitics.m

MB|

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Out[79]= MBint[$\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}$, $\{ \}, \left\{ z \rightarrow -\frac{1}{2} \right\}]$

In[80]:= << mbasymptotics.m

In[81]:= ? MBasymptotics

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

MBasymptotics[



```
In[77]:= val3 = NIntegrate[1 / (2 Pi I) I int /. z → -1/2 + I y /. x → 1.2, {y, -5, 5}]
```

```
Out[77]= 0.390571 + 1.13798×10-15 i
```

```
In[78]:= val2 - val3
```

```
Out[78]= -2.67586×10-9 - 1.13798×10-15 i
```

```
In[79]:= II1MB
```

```
Out[79]= MBint[ $\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \{ \}, \{ z \rightarrow -\frac{1}{2} \}]$ ]
```

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In[82]:= MBasymptotics[{I1MB}, {x, 0}]

MBasymptotics::exp : more than one exponent with integration variables

Throw::nocatch : Uncaught Throw[{}] returned to top level. >>

Out[82]= Hold[Throw[{}]]

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Untitled-3

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In[79]:= I1MB

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In[83]:= MBasymptotics[I1MB, {x, 0}]

Out[83]= {MBInt[$\frac{\log[x]^2}{2}$, {{}, {}}]}

$S \cdot$
 P_1
 $S = (P_1 \cdot P_2)^2$

$$- \int_{-\infty}^{\infty} \frac{h}{2\pi i} N(z) N(z + \sqrt{3}i\omega_z) e^{-z^2/2} dz$$

$$z = \frac{h}{2\pi i} - i\omega_z \quad \text{for } z < 0$$

$N(z)$

$$= \frac{1}{2} \int_0^2 \log \left(\frac{1 - \sqrt{1 - 4x}}{1 + \sqrt{1 - 4x}} \right) dx$$



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Untitled-3

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In[85]:= MBasymptotics[II1MB, {x, 3}]

Out[85]= $\left\{ MBInt \left[-6x^3 - \frac{20}{3}x^3 \text{Log}[x], \{ \}, \{ \} \right], MBInt \left[2x^2 + 3x^2 \text{Log}[x], \{ \}, \{ \} \right], MBInt \left[-2x \text{Log}[x], \{ \}, \{ \} \right], MBInt \left[\frac{\text{Log}[x]^2}{2}, \{ \}, \{ \} \right] \right\}$

<http://projects.hepforge.org/mbtools/>

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In[85]:= MBasymptotics[I1MB, {x, 3}]

Out[85]= $\left\{ MBInt \left[-6x^3 - \frac{20}{3}x^3 \log[x], \{\{\}, \{\}\} \right], MBInt \left[2x^2 + 3x^2 \log[x], \{\{\}, \{\}\} \right], MBInt \left[-2x \log[x], \{\{\}, \{\}\} \right], MBInt \left[\frac{\log[x]^2}{2}, \{\{\}, \{\}\} \right] \right\}$



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MBasymptotics[II1MB, {x, 3}] /. MBint[a_, {

Out[85]= {MBint[-6 x³ - $\frac{20}{3} x^3 \text{Log}[x]$, {{}, {}}], MBint[2 x² + 3 x² Log[x], {{}, {}}], MBint[-2 x Log[x], {{}, {}}], MBint[$\frac{\text{Log}[x]^2}{2}$, {{}, {}}]}

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Untitled-3

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In[86]:= MBasymptotics[II1MB, {x, 3}] /. MBint[a_, {}, {}] :> a

Out[86]= $\left\{ -6x^3 - \frac{20}{3}x^3 \log[x], 2x^2 + 3x^2 \log[x], -2x \log[x], \frac{\log[x]^2}{2} \right\}$

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In[87]:= (MBasymptotics[II1MB, {x, 3}] /. MBInt[a_, {}, {}] :> a)[[1]]

Out[87]= $-6 x^3 - \frac{20}{3} x^3 \log[x]$



http://projects.hepforge.org/mbtools/

Untitled-3

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In[88]:= (MBasymptotics[I1MB, {x, 3}] /. MBInt[a_, {}, {}] :> a)

Out[88]= $\left\{ -6 x^3 - \frac{20}{3} x^3 \log[x], 2 x^2 + 3 x^2 \log[x], -2 x \log[x], \frac{\log[x]^2}{2} \right\}$

A set of standard Mac OS X window controls: close, minimize, maximize, and scroll.

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Untitled-3

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http://projects.hepforge.org/mbtools/ Untitled-3

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In[79]:= I1MB

Out[79]= MBInt $\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \{ \}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right]$

In[80]:= << mbasymptotics.m

In[81]:= ? MBasymptotics

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

In[89]:= MBasymptotics[I1MB, {x, 3}]

Out[89]= $\left\{ MBInt \left[-6x^3 - \frac{20}{3}x^3 \text{Log}[x], \{ \}, \{ \} \right], MBInt \left[2x^2 + 3x^2 \text{Log}[x], \{ \}, \{ \} \right], MBInt \left[-2x \text{Log}[x], \{ \}, \{ \} \right], MBInt \left[\frac{\text{Log}[x]^2}{2}, \{ \}, \{ \} \right] \right\}$

In[88]:= (MBasymptotics[I1MB, {x, 3}] /. MBInt[a_, { }, { }] :> a)



In[78]:= val2 - val3

Out[78]= $-2.67586 \times 10^{-9} - 1.13798 \times 10^{-15} i$

In[79]:= I1MB

Out[79]= MBint $\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \{ \}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right]$

In[80]:= << mbasymptotics.m

In[81]:= ? MBasymptotics

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

Out[90]= $\left\{ MBint \left[2(1-3x)x^2 - \frac{1}{3}x(6-9x+20x^2) \text{Log}[x] + \frac{\text{Log}[x]^2}{2}, \{ \}, \{ \} \right] \right\}$

In[88]:= (MBasymptotics[I1MB, {x, 3}] /. MBint[a_, {[], {}}] :> a)

Out[88]= $\left\{ -6x^3 - \frac{20}{3}x^3 \text{Log}[x], 2x^2 + 3x^2 \text{Log}[x], -2x \text{Log}[x], \frac{\text{Log}[x]^2}{2} \right\}$

In[79]:= II1MB

$$\text{Out}[79]= \text{MBInt}\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2 z]}, \left\{\{\}, \left\{z \rightarrow -\frac{1}{2}\right\}\right\}\right]$$

In[80]:= << mbasymptotics.m

In[81]:= ? MBasymptotics

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

In[90]:= MBmerge[MBasymptotics[II1MB, {x, 3}]]

$$\text{Out}[90]= \left\{\text{MBInt}\left[2 (1-3 x) x^2-\frac{1}{3} x (6-9 x+20 x^2) \log [x]+\frac{\log [x]^2}{2}, \{\{\}, \{\}\}\right]\right\}$$

In[91]:= (MBmerge[MBasymptotics[II1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)

$$\text{Out}[91]= \left\{2 (1-3 x) x^2-\frac{1}{3} x (6-9 x+20 x^2) \log [x]+\frac{\log [x]^2}{2}\right\}$$

In[79]:= II1MB

$$\text{Out}[79]= \text{MBInt}\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2 z]}, \left\{\{\}, \left\{z \rightarrow -\frac{1}{2}\right\}\right\}\right]$$

In[80]:= << mbasymptotics.m

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In[90]:= MBmerge[MBasymptotics[II1MB, {x, 3}]]

$$\text{Out}[90]= \left\{\text{MBInt}\left[2 (1-3 x) x^2-\frac{1}{3} x (6-9 x+20 x^2) \log [x]+\frac{\log [x]^2}{2}, \{\{\}, \{\}\}\right]\right\}$$

In[92]:= (MBmerge[MBasymptotics[II1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]

$$\text{Out}[92]= 2 (1-3 x) x^2-\frac{1}{3} x (6-9 x+20 x^2) \log [x]+\frac{\log [x]^2}{2}$$

http://projects.hepforge.org/mbtools/

Untitled-3

In[79]:= I1MB

$$\text{Out}[79]= \text{MBint}\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2 z]}, \left\{\{\}, \left\{z \rightarrow -\frac{1}{2}\right\}\right\}\right]$$

In[80]:= << mbasymptotics.m

In[81]:= ? MBasymptotics

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

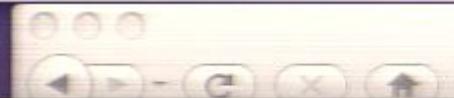
In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

$$\text{Out}[90]= \left\{\text{MBint}\left[2 (1-3 x) x^2-\frac{1}{3} x (6-9 x+20 x^2) \log [x]+\frac{\log [x]^2}{2}, \{\{\}, \{\}\}\right]\right\}$$

In[92]:= (MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]

$$\text{Out}[92]= 2 (1-3 x) x^2-\frac{1}{3} x (6-9 x+20 x^2) \log [x]+\frac{\log [x]^2}{2}$$

II



http://projects.hepforge.org/mbtools/

Untitled-3

In[79]:=

$$\text{Out}[79]= \text{MBint}\left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2 z]}, \{\}, \left\{z \rightarrow -\frac{1}{2}\right\}\right]$$

In[80]:= << mbasymptotics.m

In[81]:= ? MBasymptotics

`MBasymptotics[integrals, {var, order}]` expands a list of integrals in variable `var`, around `var = 0`, up to the given order by closing contours and taking residues. The integrands may contain polynomials in `var`, but denominators which aren't products of `Gamma` functions are forbidden.

In[90]:= `MBmerge[MBasymptotics[I1MB, {x, 3}]]`

$$\text{Out}[90]= \left\{ \text{MBint}\left[2 (1-3 x) x^2-\frac{1}{3} x (6-9 x+20 x^2) \log[x]+\frac{\log[x]^2}{2}, \{\}, \{\}\right]\right\}$$

In[92]:= `(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] \rightarrow a)[[1]]`

$$\text{Out}[92]= 2 (1-3 x) x^2-\frac{1}{3} x (6-9 x+20 x^2) \log[x]+\frac{\log[x]^2}{2}$$

`I1an[x]`

```
In[80]:= << mbasymptotics.m
```

```
In[81]:= ? MBasymptotics
```

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

```
In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]
```

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\}\right]\right\}$$

```
In[92]:= (MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]
```

$$\text{Out}[92]= 2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}$$

```
In[93]:= I1an[x]
```

$$\text{Out}[93]= \frac{1}{2}\log\left[\frac{-1 + \sqrt{1 + 4x}}{1 + \sqrt{1 + 4x}}\right]^2$$

```
In[81]:= ? MBasymptotics
```

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

```
In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]
```

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$$

```
In[92]:= (MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]
```

$$\text{Out}[92]= 2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}$$

```
In[94]:= Series[I1an[x], {x, 3}]
```

Series::sspec : Series specification {x, 3} is not a list with three elements. >>

$$\text{Out}[94]= \text{Series}\left[\frac{1}{2}\log\left[\frac{-1 + \sqrt{1 + 4x}}{1 + \sqrt{1 + 4x}}\right]^2, \{x, 3\}\right]$$

```
In[81]:= ? MBasymptotics
```

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

```
In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]
```

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$$

```
In[92]:= (MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]
```

$$\text{Out}[92]= 2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}$$

```
In[95]:= Series[I1an[x], {x, 0, 3}]
```

$$\text{Out}[95]= \frac{\log[x]^2}{2} - 2\log[x]x + (2 + 3\log[x])x^2 + \left(-6 - \frac{20\log[x]}{3}\right)x^3 + O[x]^4$$

```
In[81]:= ? MBasymptotics
```

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

```
In[90]:= MBmerge[MBasymptotics[IIMB, {x, 3}]]
```

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2) \log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$$

```
In[92]:= (MBmerge[MBasymptotics[IIMB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]
```

$$\text{Out}[92]= 2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2) \log[x] + \frac{\log[x]^2}{2}$$

```
Expand[Normal[Series[I1an[x], {x, 0, 3}]]]
```

$$\text{Out}[96]= x^3 \left(-6 - \frac{20 \log[x]}{3} \right) - 2x \log[x] + \frac{\log[x]^2}{2} + x^2 (2 + 3 \log[x])$$

```
In[81]:= ? MBasymptotics
```

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

```
In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]
```

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$$

```
Expand[MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a][[1]]
```

$$\text{Out}[92]= 2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}$$

```
In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]
```

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

<http://projects.hepforge.org/mbtools/>

Untitled-3

In[81]:= ? MBasymptotics

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\}\right]\right\}$$

In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]]

$$\text{Out}[98]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

```
In[81]:= ? MBasymptotics
```

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

```
In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]
```

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$$

```
In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] \rightarrow a)[[1]]]
```

$$\text{Out}[98]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

```
In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]
```

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[81]:= ? MBasymptotics

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In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\{\}, \{\}\}\right]\right\}$$

In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {\{\}, \{\}}] :> a)[[1]]]

$$\text{Out}[98]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[80]:= MBasymptotics.m

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MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

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In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {{}, {}}] \rightarrow a)[[1]]]

$$\text{Out}[98]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

Untitled-3

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$$

In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {[], {}}] :> a)[[1]]]

$$\text{Out}[98]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[99]:= I1MB

$$\text{Out}[99]= \text{MBint}\left[\frac{x^{-1-z}\Gamma[-z]\Gamma[1+z]^3}{\Gamma[3+2z]}, \{\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right]$$

MBasymptotics

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

Out[90]= $\left\{ MBint \left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2) \log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$

In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {[], {}}] :> a)[[1]]]

Out[98]= $2x^2 - 6x^3 - 2x \log[x] + 3x^2 \log[x] - \frac{20}{3}x^3 \log[x] + \frac{\log[x]^2}{2}$

In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]

Out[97]= $2x^2 - 6x^3 - 2x \log[x] + 3x^2 \log[x] - \frac{20}{3}x^3 \log[x] + \frac{\log[x]^2}{2}$

I1MB /. x → 1/x

Out[99]= $MBint \left[\frac{x^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \{\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right]$

Untitled-3

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$$

In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]]

$$\text{Out}[98]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[100]:= I1MB /. x → 1/x

$$\text{Out}[100]= \text{MBint}\left[\frac{\left(\frac{1}{x}\right)^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \{\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right]$$

Untitled-3

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$$

In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]]

$$\text{Out}[98]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[100]:= I1MB /. x → 1/x

$$\text{Out}[100]= \text{MBint}\left[\frac{\left(\frac{1}{x}\right)^{-1-z} \Gamma[-z] \Gamma[1+z]^3}{\Gamma[3+2z]}, \{\}, \left\{ z \rightarrow -\frac{1}{2} \right\} \right]$$

Untitled-3

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In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\}\right]\right\}$$

In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]]

$$\text{Out}[98]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[101]:= PowerExpand[I1MB /. x → 1/x]

$$\text{Out}[101]= \text{MBint}\left[\frac{x^{1-z}\Gamma[-z]\Gamma[1+z]^3}{\Gamma[3+2z]}, \{\}, \left\{z \rightarrow -\frac{1}{2}\right\}\right]$$

Untitled-3

MBasymptotics[integrals, {var, order}] expands a list of integrals in variable var, around var = 0, up to the given order by closing contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]

$$\text{Out}[90]= \left\{ \text{MBint}\left[2(1 - 3x)x^2 - \frac{1}{3}x(6 - 9x + 20x^2)\log[x] + \frac{\log[x]^2}{2}, \{\}, \{\} \right] \right\}$$

In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]]

$$\text{Out}[98]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]

$$\text{Out}[97]= 2x^2 - 6x^3 - 2x\log[x] + 3x^2\log[x] - \frac{20}{3}x^3\log[x] + \frac{\log[x]^2}{2}$$

In[102]:= MBasymptotics[PowerExpand[I1MB /. x :> 1/x], {x, 0}]

$$\text{Out}[102]= \{\}$$

150%



Untitled-3

contours and taking residues. The integrands may contain polynomials in var, but denominators which aren't products of Gamma functions are forbidden.

```
In[90]:= MBmerge[MBasymptotics[I1MB, {x, 3}]]
```

```
Out[90]= {MBint[2 (1 - 3 x) x2 -  $\frac{1}{3}$  x (6 - 9 x + 20 x2) Log[x] +  $\frac{\text{Log}[x]^2}{2}$ , {}, {}]}
```

```
In[98]:= Expand[(MBmerge[MBasymptotics[I1MB, {x, 3}]] /. MBint[a_, {}, {}] :> a)[[1]]]
```

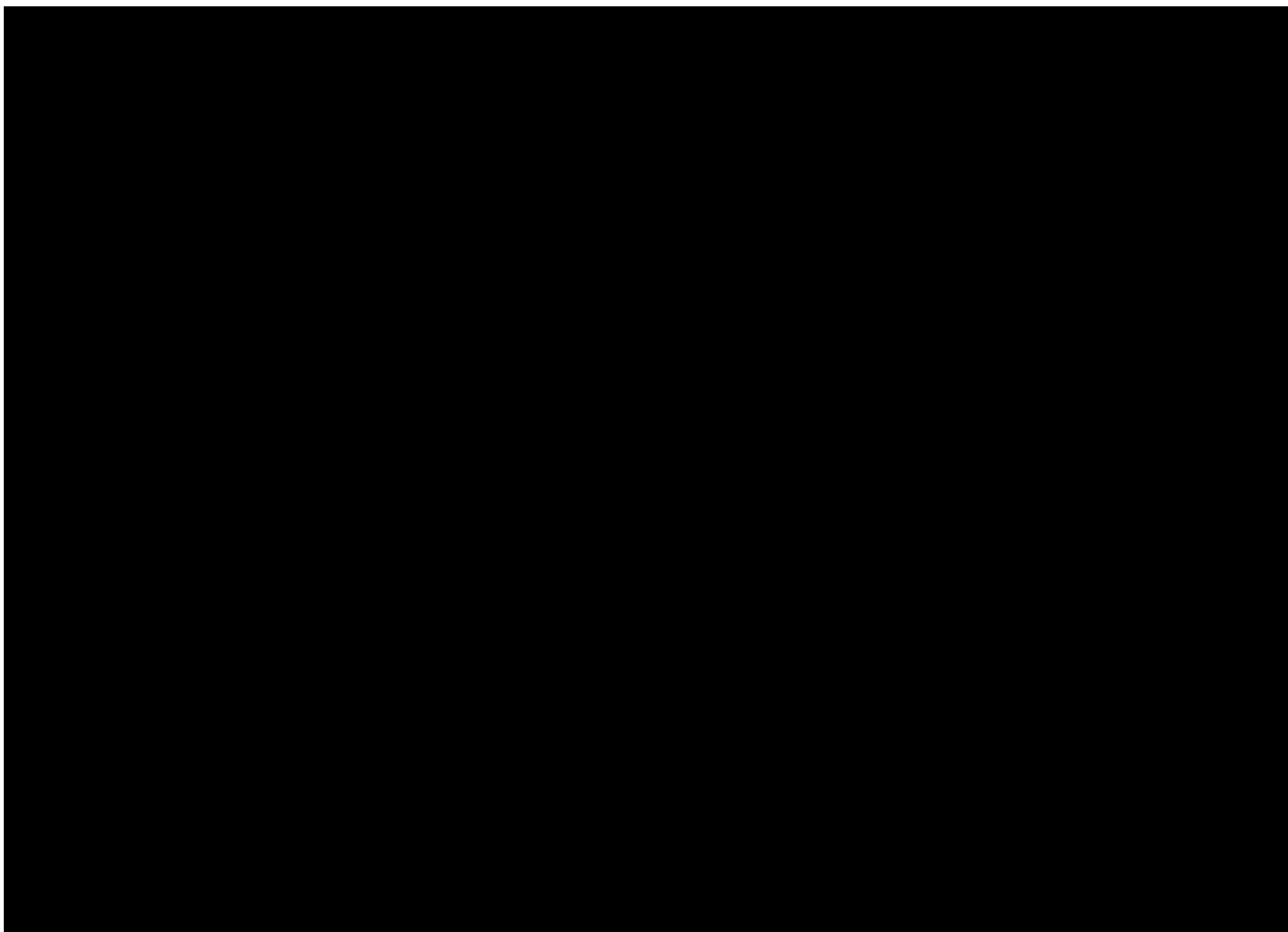
```
Out[98]= 2 x2 - 6 x3 - 2 x Log[x] + 3 x2 Log[x] -  $\frac{20}{3}$  x3 Log[x] +  $\frac{\text{Log}[x]^2}{2}$ 
```

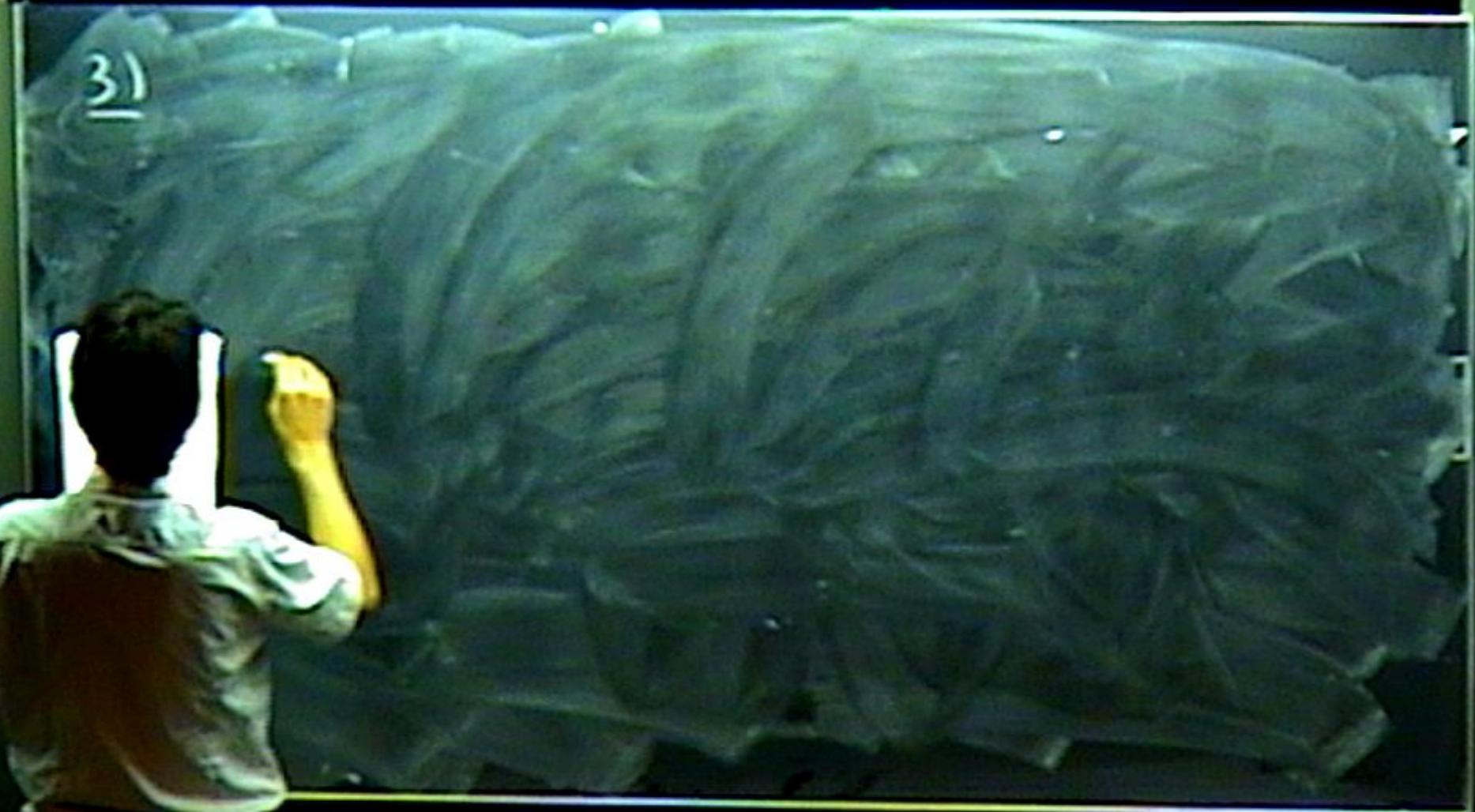
```
In[97]:= Expand[Normal[Series[I1an[x], {x, 0, 3}]]]
```

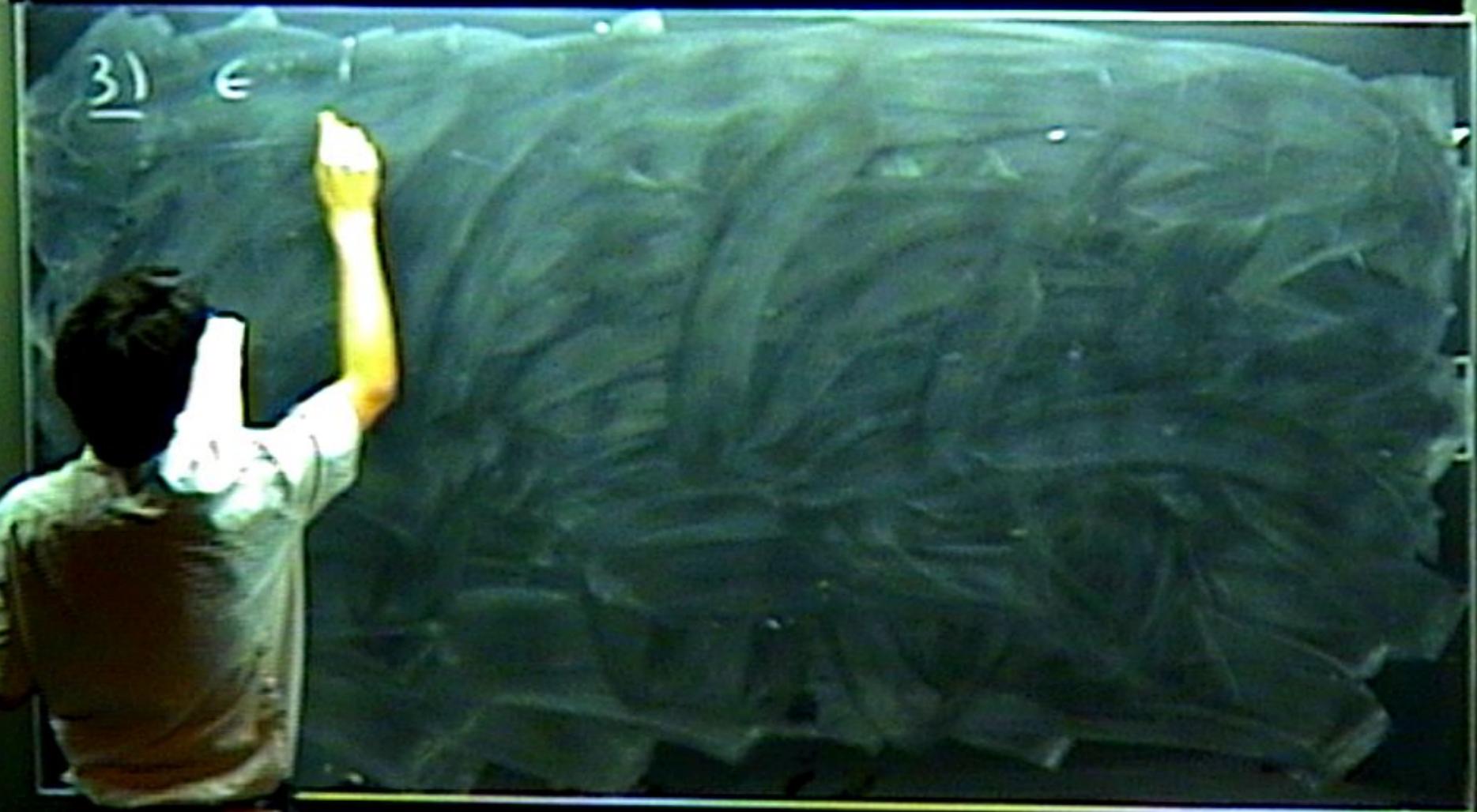
```
Out[97]= 2 x2 - 6 x3 - 2 x Log[x] + 3 x2 Log[x] -  $\frac{20}{3}$  x3 Log[x] +  $\frac{\text{Log}[x]^2}{2}$ 
```

```
In[104]:= MBasymptotics[PowerExpand[I1MB /. x :> 1/x], {x, 4}]
```

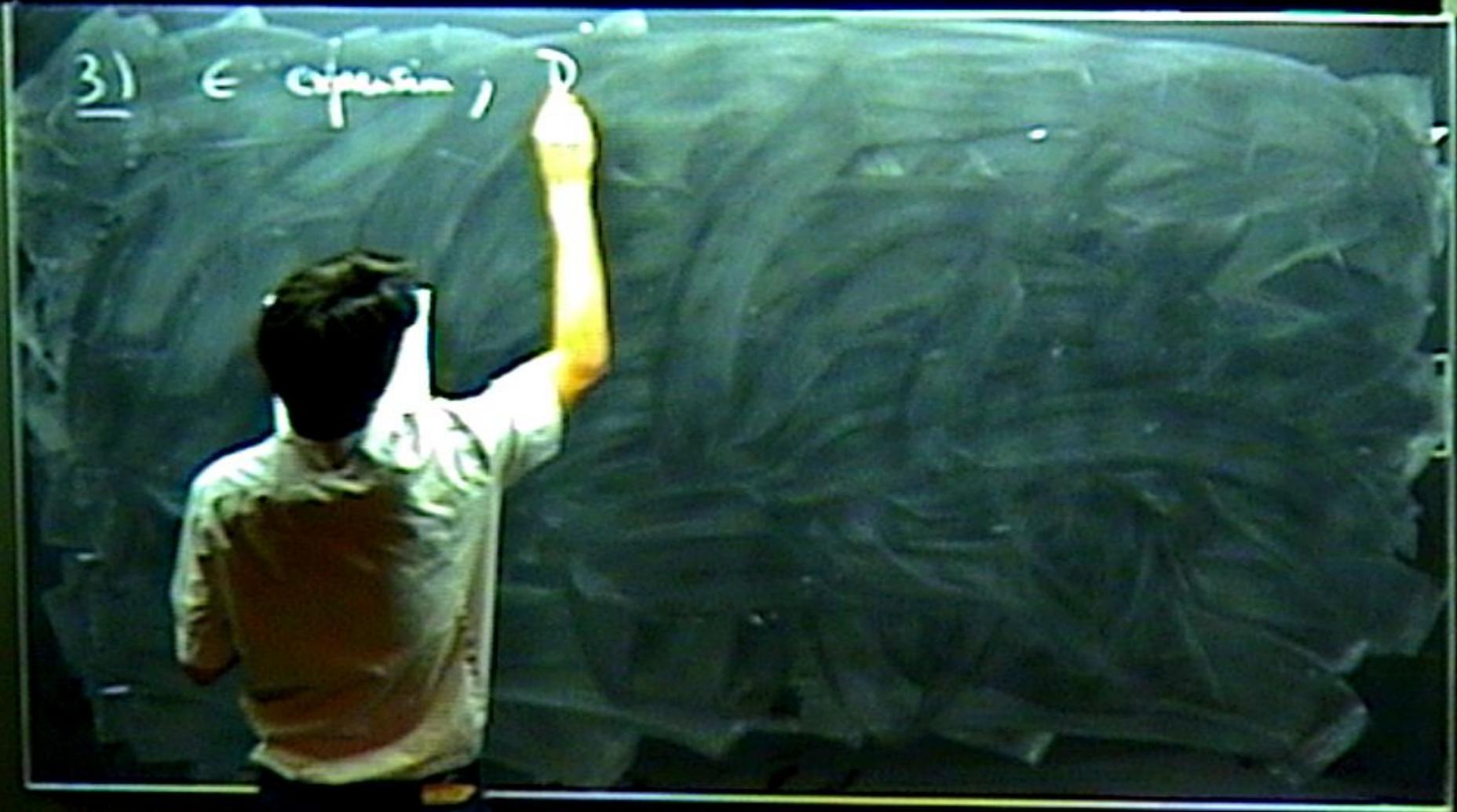
```
Out[104]= {MBint[x/2, {}, {}], MBint[-x2/24, {}, {}],  
MBint[x3/180, {}, {}], MBint[-x4/1120, {}, {}]}
```











31 \in Opusum , $D=4-2\epsilon$



3) ϵ expansion, $D=4-2\epsilon$

3) ϵ expansion, $D=4-2\epsilon$



3) ϵ open, $D=4-2\epsilon$

31 ϵ openin, $D=4-2\epsilon$

3) ϵ expansion, $D=4-2\epsilon$



3) \in open, $D=4-2\epsilon$



3.1 ϵ open, $D=4-2\epsilon$

3) ϵ expansion, $D=4-2\epsilon$

3) ϵ capacitor, $D=4-2\epsilon$

length

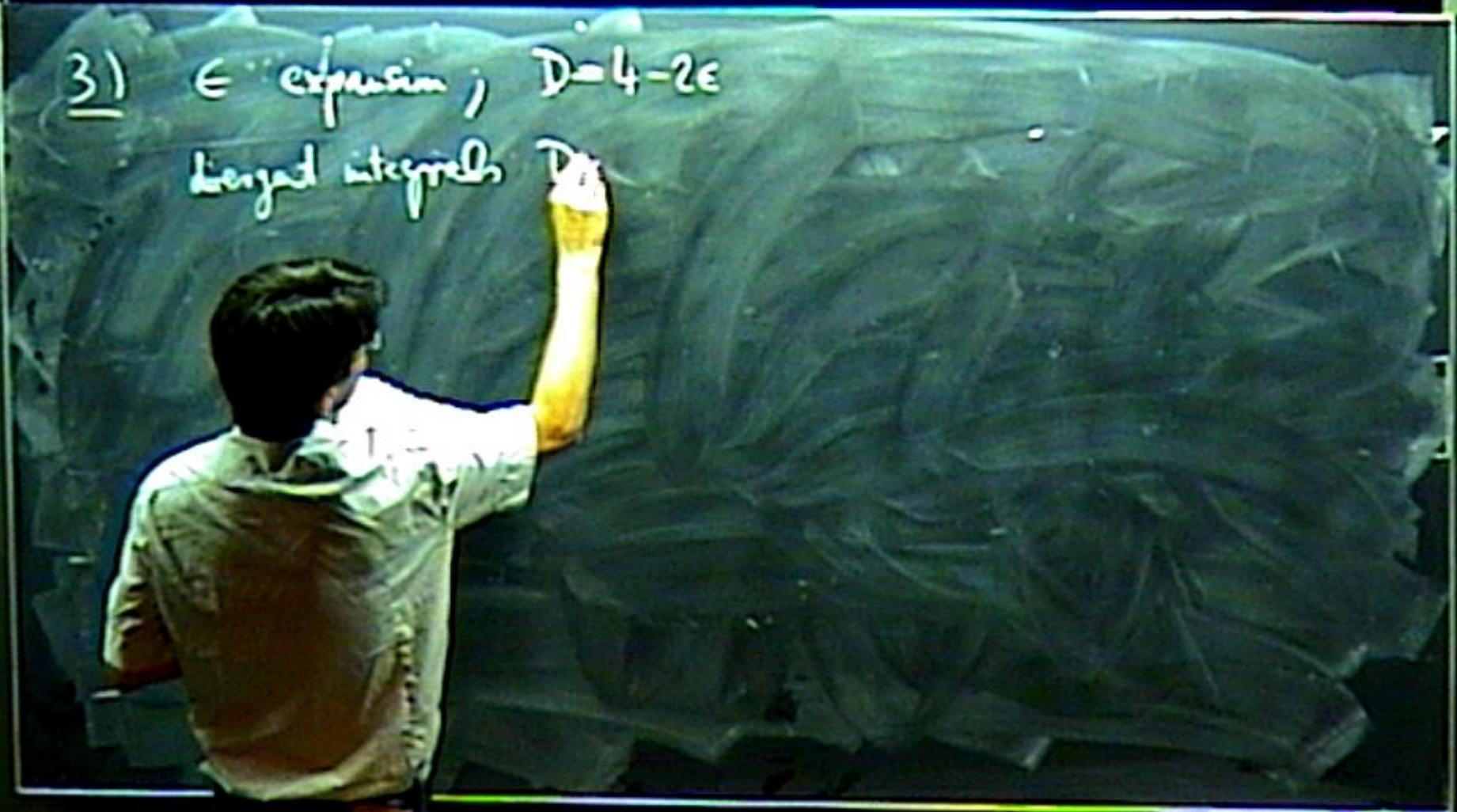
3) ϵ expansion; $D=4-2\epsilon$

Lengad integr.

3) ϵ expansion; $D=4-2\epsilon$
Lengat integrals D



3) ϵ expansion ; $D=4-2\epsilon$
Legend integrals



3) ϵ expansion, $D=4-2\epsilon$
Legend integrals $D=4$

3) ϵ expansion, $D=4-2\epsilon$

Lenged integrals $D=4$

3) ϵ expansion; $D=4-2\epsilon$
Legend integrals $D=4$

3) ϵ expansion; $D=4-2\epsilon$

Lenged integrals $D=4$



3) ϵ expansion; $D=4-2\epsilon$
Large integrals $D=4$
 \rightarrow no loop of contours allowed



3) ϵ expansion; $D=4-2\epsilon$

Lenged integrals $D=4$

\rightarrow no change of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$\begin{aligned}
 S &= \int_{-\infty}^{i\infty} \frac{h}{2\pi i} R(-z) \tilde{M}(1-z) \sqrt{3+2z} \times e^{-4-z} dz \\
 &\quad x = \frac{h^2}{2} - 1 \quad \operatorname{Re} z < 0 \\
 &= \frac{1}{2} R(-s) \tilde{M}(1-s) e^{-4-s} \\
 S &= (\rho_1, \rho_2)
 \end{aligned}$$



3) ϵ expansion; $D=4-2\epsilon$

längst integrals $D=4$

→ no choice of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

3) ϵ expansion; $D=4-2\epsilon$

längst integrals $D=4$

→ no choice of contours allowed

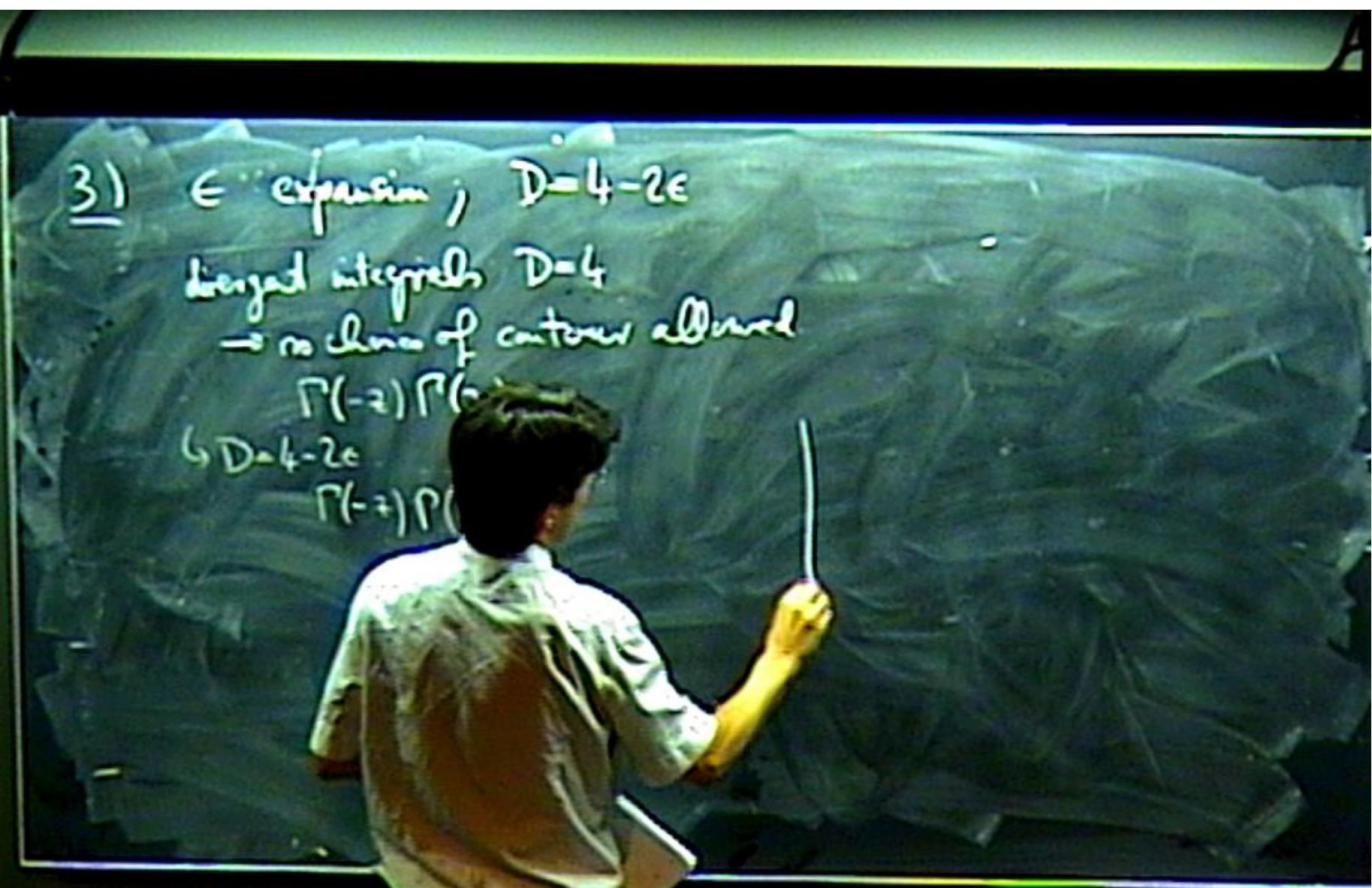
$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z)$$



3) ϵ expansion; $D=4-2\epsilon$

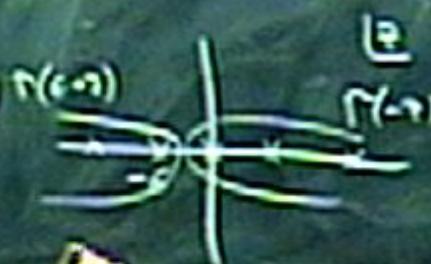
längst integrals $D=4$

→ no line of contour allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z)$$



3) ϵ expansion; $D=4-2\epsilon$

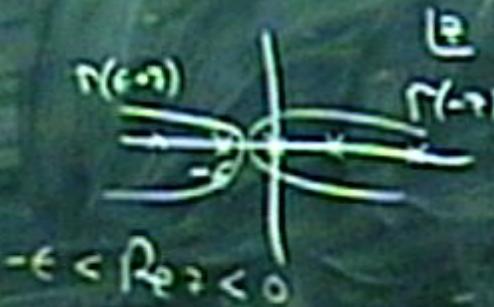
längst integrals $D=4$

\rightarrow no choice of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$-2\epsilon$$

$$\Gamma(-z) \Gamma(z-\epsilon)$$



3) ϵ expansion; $D=4-2\epsilon$

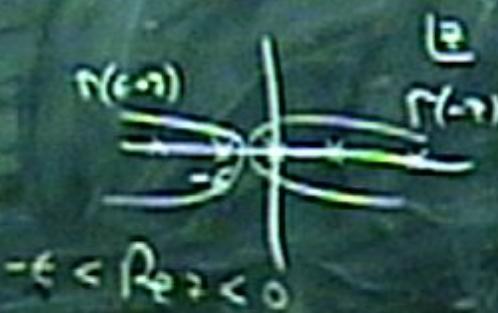
längst integrals $D=4$

\rightarrow no choice of contours allowed

$$\Gamma(z) \Gamma(z)$$

$\zeta \Gamma$

$$\Gamma(z-\epsilon)$$



3) ϵ expansion; $D=4-2\epsilon$

längst integrals $D=4$

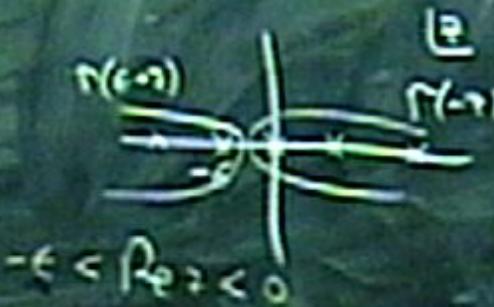
→ no choice of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z-\epsilon)$$

$$\epsilon > 0$$



3) ϵ expansion; $D=4-2\epsilon$

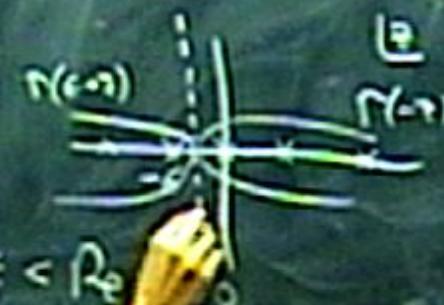
längst integrals $D=4$

\rightarrow no choice of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma$$



3) ϵ expansion; $D=4-2\epsilon$

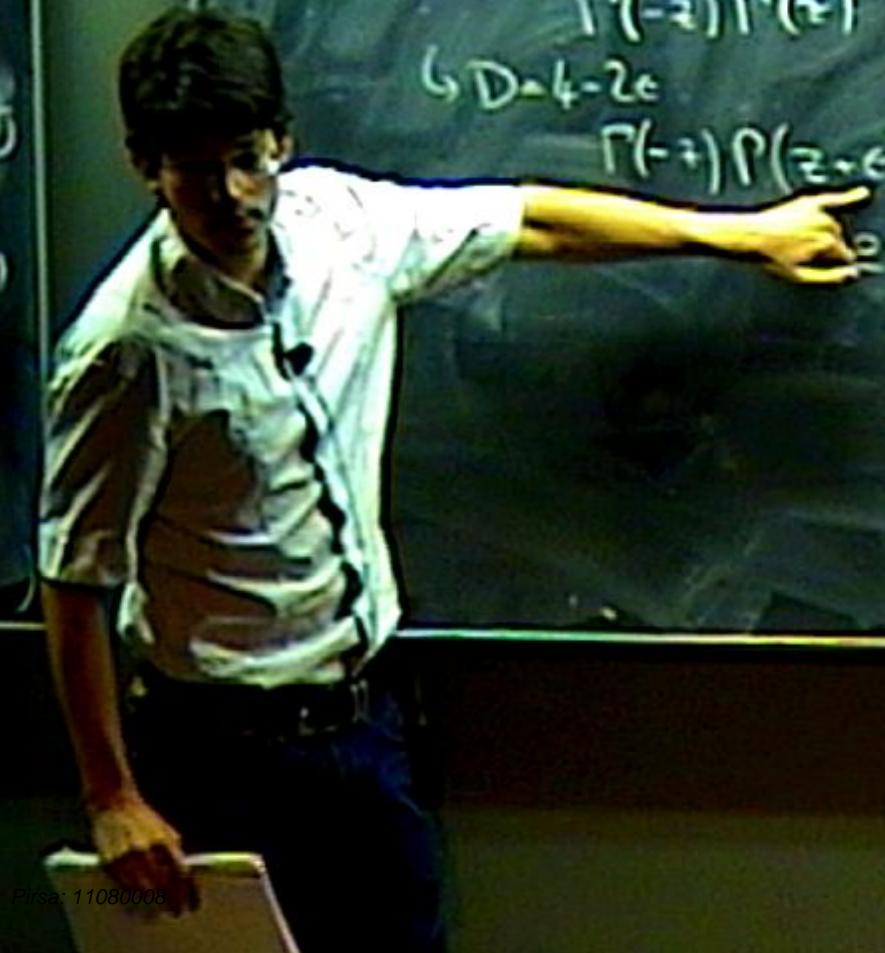
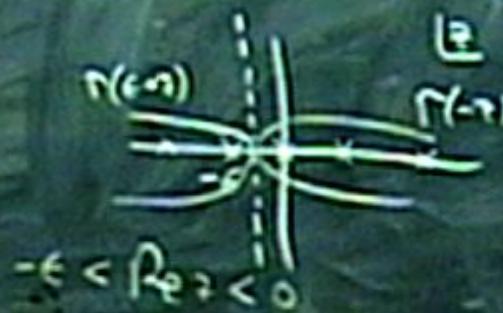
längst integrals $D=4$

\rightarrow no choice of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z-\epsilon)$$



3) ϵ expansion; $D=4-2\epsilon$

längst integrals, $D=4$

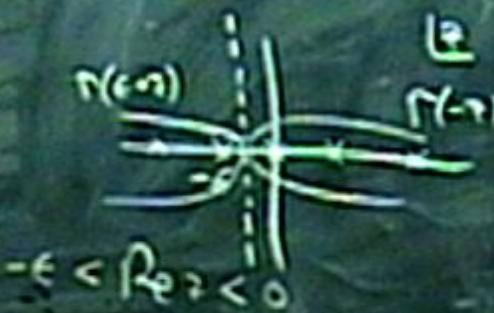
\rightarrow no choice of contour allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(-z)$$

$\epsilon > 0$



3) ϵ expansion; $D=4-2\epsilon$

längst integrals $D=4$

→ no choice of contours allowed

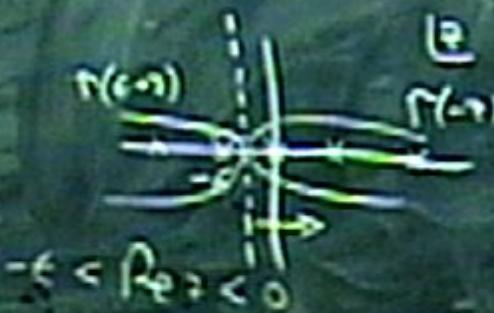
$$\Gamma(-z) \Gamma(z)$$

$$\therefore D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z-\epsilon)$$

$$\epsilon > 0$$

deform contour
pick up



3) ϵ expansion; $D=4-2\epsilon$

Deformed integrals $D=4$

\rightarrow no choice of contours allowed

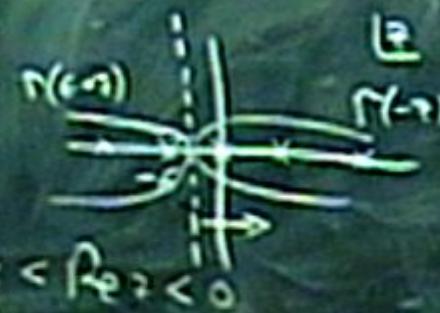
$$\Gamma(-z) \Gamma(z)$$

$$D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z-\epsilon)$$

$$\epsilon > 0$$

Deform contour
pole at $z=0$ $\Gamma(-z)$



3) ϵ expansion; $D=4-2\epsilon$

degenerate integrals $D=4$

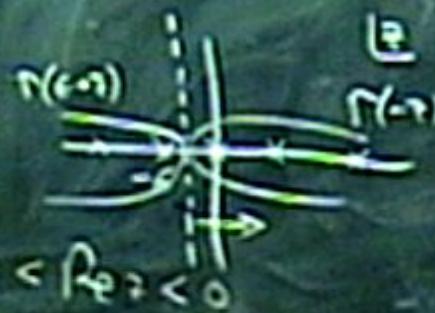
\rightarrow no choice of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z-\epsilon)$$

$$\epsilon > 0$$



\rightarrow deform contour

\rightarrow pick up pole at $z=0$ $\Gamma(-z)$.

$$\sim \text{Residue} + \int_{R>z>0} dz$$

3) ϵ expansion; $D=4-2\epsilon$

larged integrals $D=4$

\rightarrow no choice of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z-\epsilon)$$

$$\epsilon > 0$$



\rightarrow deform contour

\rightarrow pick up pole at $z=0$ $\int N(-z) dz$

$$\sim \text{Residue} + \int_{R_0}^{\infty} dz$$

↳ safe to carry out

3) ϵ expansion; $D=4-2\epsilon$

Deformed integrals $D=4$

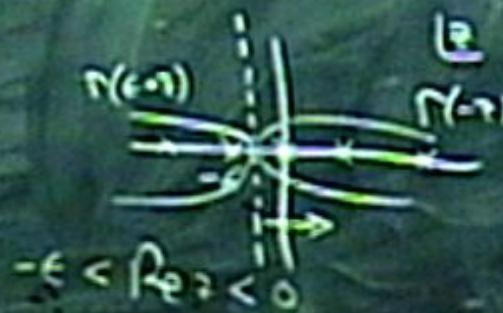
\rightarrow no choice of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z-\epsilon)$$

$$\epsilon > 0$$



\rightarrow deform contour

\rightarrow pick up pole at $z=0$ $\Gamma(-z)$.

$$\sim \underset{\text{Residue}}{\overset{\uparrow}{\Gamma(\epsilon)}} + \int_{R>0} dr$$

\hookrightarrow safe to carry out Laurent exp. in ϵ

\hookrightarrow list of int. techn.

3) ϵ expansion; $D=4-2\epsilon$

längst integriert $D=4$

→ no choice of contours allowed

$$\Gamma(-z) \Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z) \Gamma(z-\epsilon)$$

$$\epsilon > 0$$



→ deform contour

→ pick up pole at $z=0$ $\Gamma(-z)$

$$\underset{\substack{\uparrow \\ \text{Residue}}}{\sim \Gamma(\epsilon)} + \int_{R>z>0} dz$$

↳ safe to carry out Laurent exp. in ϵ

↳ list of MB-integrals

MBcontour
MBexpanded

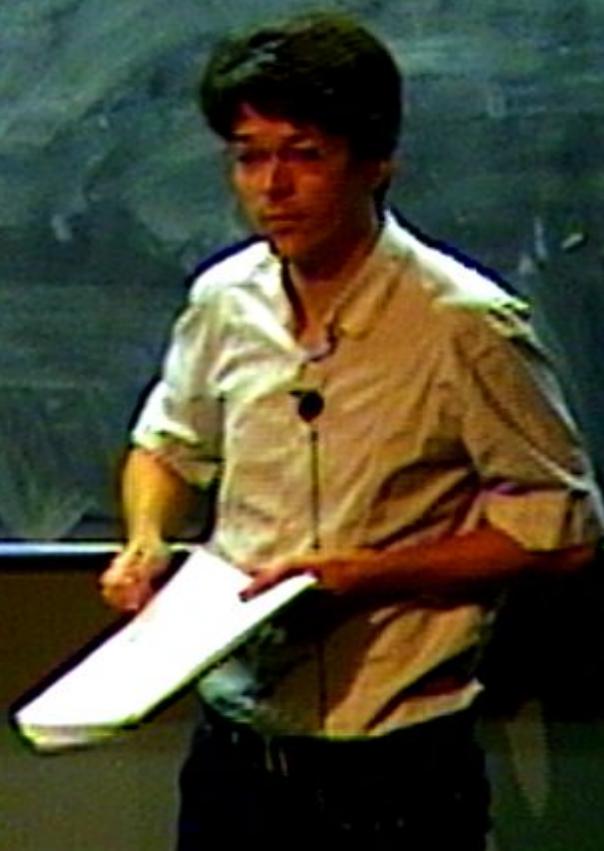
Comment: how to deal with numerators?



Comment: how to deal with numerators?

$$\int_{t_0}^{\infty} \frac{(t+q)^2}{t^{q+1}} dt \quad \text{play with power } q=-1$$

$$\zeta \frac{1}{\Gamma(x)} - \frac{1}{\Gamma(-1)} = 0$$



Comment: how to deal with numerators?

$$\int \frac{(z+1)^2}{\Gamma(z)} dz \quad \text{map with power } q=1$$

$$\oint \frac{1}{\Gamma(z)} dz \rightarrow \frac{1}{\Gamma(-1)} = 0$$
$$z \rightarrow -1 - \delta - \frac{1}{\Gamma(-1-\delta)}$$

3) ϵ expansion; $D=4-2\epsilon$

Deformed integrals $D=4$

\rightarrow no choice of contours allowed

$$\Gamma(-z)\Gamma(z)$$

$$\hookrightarrow D=4-2\epsilon$$

$$\Gamma(-z)\Gamma(z-\epsilon)$$

$$\epsilon \gg 0$$

\rightarrow deform contour

\rightarrow pick up pole at $-z$

$$\sim \Gamma(\epsilon)$$

$\frac{1}{z}$
tension

$$R_0 z > 0$$



\hookrightarrow safe to carry out Laurent exp. in ϵ .

\hookrightarrow list of MB integrals
MBontine
MBexpanded

Ques: how to deal with numerators?

$$\int \frac{(b+q)^2}{\Gamma(q)} \leftarrow \text{play with power } q=-1$$

$$\frac{1}{\Gamma(\alpha)} - \frac{1}{\Gamma(-1)} = 0$$

$$\alpha = -1 - \delta - \frac{1}{\Gamma(-1-\delta)}$$



$$S = \begin{pmatrix} p_1 & p_2 \end{pmatrix}$$

$$= \int_{-\infty}^{\infty} \frac{h}{2\pi i} R(-z) \Gamma(1-z) \sqrt{\Gamma(3+2z)} \cdot e^{-4z} dz$$

$$x = \frac{h^2}{S}, -1 \leq R \leq 0$$

$$= \frac{1}{2} \log \left(\frac{1 - \sqrt{1 - h^2}}{1 + \sqrt{1 - h^2}} \right)$$

$$R(z)$$

$$R(-z)$$

$$R(z)$$

$$R(-z)$$

Symptom

• muscle problem



Summation

- no scale problem

- type of fans log, bin, ...



Summary

• muscle problem

type of fans log, bin, ...

Summary

- multi problem

- type of fans \log , \sin ,

- Set up series expansion

- XSummer (Moch, Uwer)

- Sigma (Schneider)



Summary

- residue problem

- type of fans \log , \ln , ...

- set up series expansion

- Smirnov (Möck, Varen)

- Sigma (Schneider)

- Smirnov "Evaluating Feynman integrals" (Springer)

Summation

- multi problem

- type of fans

log, lin,

- Set up series expansion

- XSummer (Moch, Uwer)

- Sigma (Schneider)

- Smirnov "Evaluating Feynman integrals" (Springer)

$\frac{dx}{x}$:



R?

0.00

-0.00 ->

Summation

• residue problem

types of fans \log , \sin , \dots

→ Sum up series expansion

- Sommer (Möck, Uwe)

- Sigma (Schneider)

- Smirnov "Evaluating Feynman integrals" (Springer)

$$\frac{dx}{x}$$

$$\Rightarrow \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} x^z \Gamma(-z) \Gamma(z) \Gamma(1-z) dz$$

$$S_{\text{Feynman}}$$

$$R^2$$

Summation

• residue problem

types of functions

$\log x, \text{Li}_n, \dots$

→ Sum up series expansion

- Sommer (Möch, Varen)

- Sigma (Schneider)

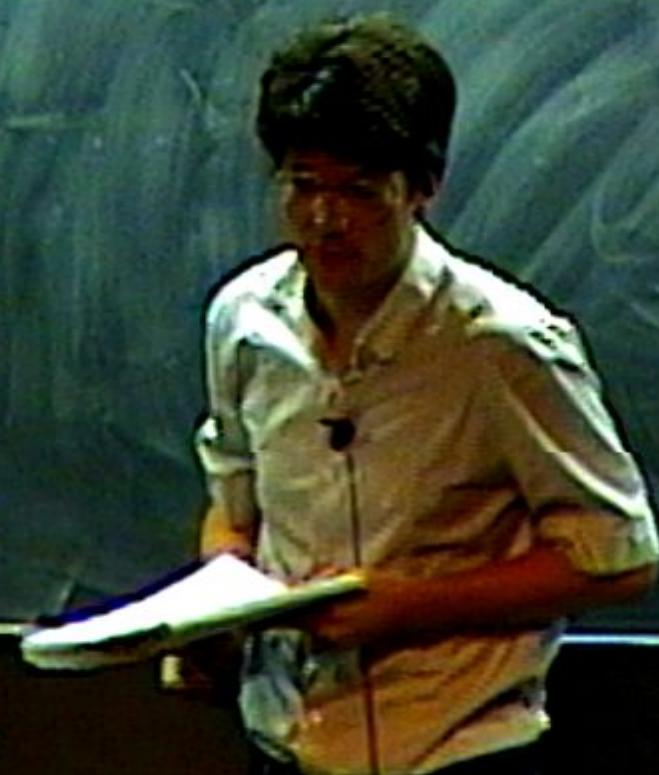
- Smirnov "Evaluating Feynman integrals" (Springer)

$$\frac{dx}{x}$$

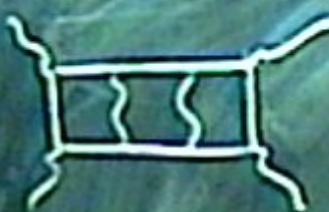
$$\Rightarrow \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} x^z \Gamma(-z) \Gamma(z) \Gamma(1-z) dz =$$

$$x = \frac{1}{t}, z = \frac{1}{2} - i\theta \quad \frac{1}{2} \left\{ \pi^2 \text{Li}_2(-x) + \log x \text{Li}_2(-x) - 4 \log x \text{Li}_3(-x) + 6 \text{Li}_4(-x) \right\}.$$

higher loops



higher loops



higher loops



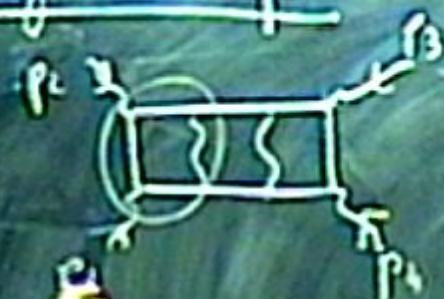
higher loops



$$= \int \frac{dz_1 dz_2 dz_3}{(2\pi i)^3} G$$

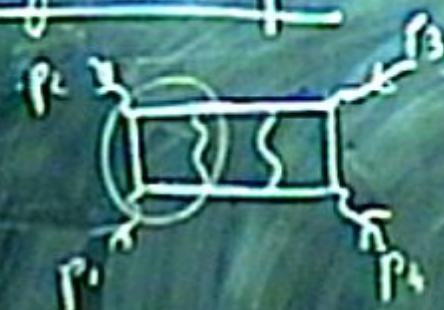


higher loops

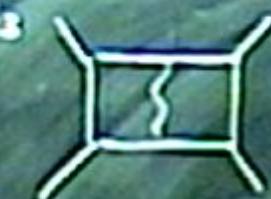


$$= \int \frac{dz_1 dz_2 dz_3}{(2\pi i)^3} G(z)$$

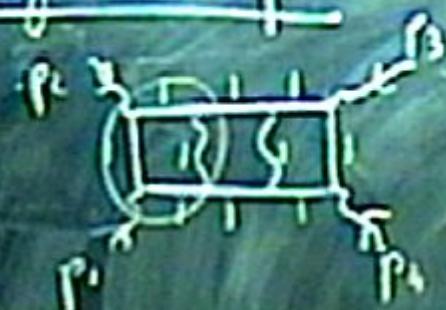
higher loops



$$= \int \frac{dz_1 dz_2 dz_3}{(2\pi i)^3} G(z) \left(\sum_{n=1}^{\infty} z_1^{1+\eta_1 z_3} \right)$$



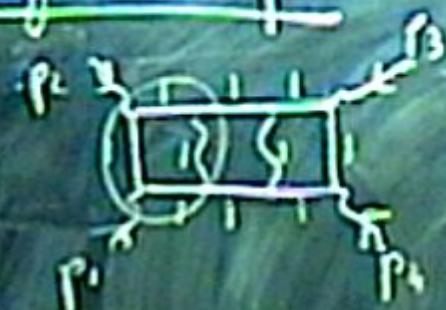
higher loops



$$= \int \frac{dz_1 dz_2 dz_3}{(2\pi i)^3} G(z) \left(\frac{\Sigma}{\pi^2} \right)$$



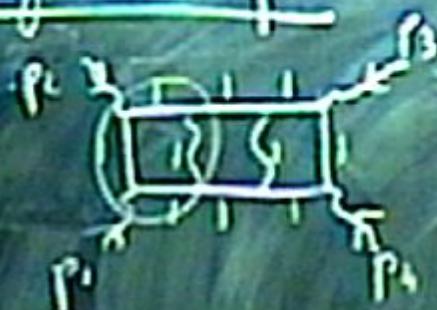
higher loops



$$= \int \frac{dz_1 dz_2 dz_3}{(2\pi i)^3} G(z) \left(\sum_{n=1}^{\infty} \right)$$



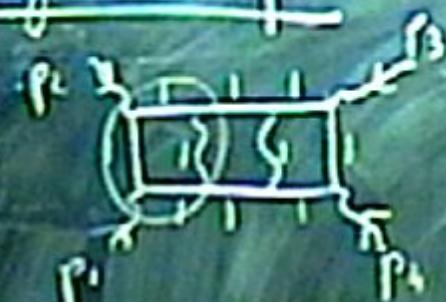
higher loops



$$= \int \frac{dz_1 dz_2 dz_3}{(2\pi i)^3} G(z) \left(\sum_{n=1}^{\infty} \right)^{z_1 z_2 z_3}$$



higher loops



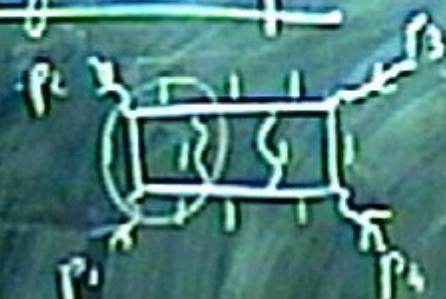
$$= \int \frac{dz_1 dz_2 dz_3}{(2\pi i)^3} G(z) \left(\sum_{n=1}^{\infty}\right)^3$$



↳ Mathematica impl.

AMBRE

higher loops



$$= \int \frac{dz_1 dz_2 dz_3}{(2\pi i)^3} G(z) \left(\sum_{n=1}^{\infty} \right)^3$$



↳ Mathematica impl.

AMBRE

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help  Tue 13:13 

pset8

```

In[46]:= $Path = Union[$Path, {"Users/johannes/Desktop/mathschool"}];
In[47]:= << MB.m

```

MB 1.2
 by Michal Czakon
 improvements by Alexander Smirnov
 more info in hep-ph/0511200
 last modified 2 Jan 09

MB integrand for I2:

```

In[48]:= I2int = s^(1+z) t^(-1-eps-z) Gamma[-z] Gamma[2+eps+z] Gamma[1+z]^2 Gamma[-1-eps-z]^2 / Gamma[-2 eps]
          s^{1-z} t^{1-eps-z} Gamma[-1-eps-z]^2 Gamma[-z] Gamma[1+z]^2 Gamma[2+eps+z]
Out[48]= -----
          Gamma[-2 eps]

```

find allowed set of real parts for integration variables:

```

In[49]:= I2rules = MBoptimizedRules[I2int, eps -> 0, {}, {eps}]

```

MBrules::norules: no rules could be found to regulate this integral

```

Out[49]= {{(eps -> -1), {z -> -1/2}}

```

```

In[50]:= I2cont = MBcontinue[I2int, eps -> 0, I2rules]

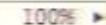
```

Level 1
 Taking residue in $z = -1 - \text{eps}$
 Level 2
 Integral (1)
 2 integral(s) found

```

Out[50]= {{MBint[
    EulerGamma s^eps Gamma[-eps]^2 Gamma[1-eps] - s^eps Gamma[-eps]^2 Gamma[1+eps] Log[s] + s^eps Gamma[-eps]^2 Gamma[1+eps] Log[t]
    -----
    Gamma[-2 eps] + Gamma[-2 eps] + Gamma[-2 eps]
    2 s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, -eps] - s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, 1+eps]
    -----
    Gamma[-2 eps] + Gamma[-2 eps], {(eps -> 0), {}}}]}

```

100%  Done



Tue 13:13



pset8

In[46]:= \$Path = Union[\$Path, {"Users/johannes/Desktop/mathschool"}];

In[47]:= << MB.m

MB 1.2
 by Michal Czakon
 improvements by Alexander Smirnov
 more info in hep-ph/0511200
 last modified 2 Jan 09

MB integrand for I2:

In[48]:= I2int = $s^{(1+z)} t^{(-1-\text{eps}-z)} \Gamma[-z] \Gamma[2+\text{eps}+z] \Gamma[1+z]^2 \Gamma[-1-\text{eps}-z]^2 / \Gamma[-2 \text{eps}]$
 $s^{1+z} t^{1-\text{eps}-z} \Gamma[-1-\text{eps}-z]^2 \Gamma[-z] \Gamma[1+z]^2 \Gamma[2+\text{eps}+z]$

In[49]:= $\frac{s^{1+z} t^{1-\text{eps}-z} \Gamma[-1-\text{eps}-z]^2 \Gamma[-z] \Gamma[1+z]^2 \Gamma[2+\text{eps}+z]}{\Gamma[-2 \text{eps}]}$

find allowed set of real parts for integration variables:

In[50]:= I2rules = MBoptimizedRules[I2int, $\text{eps} \rightarrow 0$, {}, {}];
 MBrules=norules : no rules could be found to regulate this integral

Out[50]= $\{\{\text{eps} \rightarrow -1\}, \{z \rightarrow -\frac{1}{2}\}\}$ In[51]:= I2cont = MBcontinue[I2int, $\text{eps} \rightarrow 0$, I2rules]

Level 1

Taking residue in $z = -1 - \text{eps}$

Level 2

Integral[1]

2 integral(s) found

Out[51]= $\left\{ \left\{ \text{MBint}\left[\frac{\text{EulerGamma} s^{\text{eps}} \Gamma[-\text{eps}]^2 \Gamma[1+\text{eps}] - s^{-\text{eps}} \Gamma[-\text{eps}]^2 \Gamma[1+\text{eps}] \log[s] + s^{-\text{eps}} \Gamma[-\text{eps}]^2 \Gamma[1+\text{eps}] \log[t]}{\Gamma[-2 \text{eps}]} \right], 2 s^{\text{eps}} \Gamma[-\text{eps}]^2 \Gamma[1+\text{eps}] \text{PolyGamma}[0, -\text{eps}] - s^{-\text{eps}} \Gamma[-\text{eps}]^2 \Gamma[1+\text{eps}] \text{PolyGamma}[0, 1+\text{eps}]\right\}, \{\{\text{eps} \rightarrow 0\}, \{\}\}\right\}$

100% ▶

Done

pset8

In[46]:= \$Path = Union[\$Path, {"Users/johannes/Desktop/mathschool"}];

In[105]:= << MB.m

```
MB 1.2
by Michal Czakon
improvements by Alexander Smirnov
more info in hep-ph/0511200
last modified 2 Jan 09
```

MB integrand for I2:

```
In[106]:= I2int = s^(1+z) t^(-1-eps-z) Gamma[-z] Gamma[2+eps+z] Gamma[1+z]^2 Gamma[-1-eps-z]^2 / Gamma[-2 eps]
          s^z t^1-eps-z Gamma[-1-eps-z]^2 Gamma[-z] Gamma[1+z]^2 Gamma[2+eps+z]
In[106]:= -----
                  Gamma[-2 eps]
```

find allowed set of real parts for integration variables:

```
In[48]:= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]
MBrules=norules : no rules could be found to regulate this integral
```

Out[49]= {{eps → -1}, {z → -1/2}}

In[50]:= I2cont = MBcontinue[I2int, eps → 0, I2rules]

Level 1

Taking residue in z = -1-eps

Level 2

Integral[1]

2 integral(s) found

```
Out[50]= {{MBint[
          EulerGamma s^-eps Gamma[-eps]^2 Gamma[1-eps] - s^-eps Gamma[-eps]^2 Gamma[1-eps] Log[s] + s^-eps Gamma[-eps]^2 Gamma[1-eps] Log[t]
          Gamma[-2 eps] + Gamma[-2 eps] + Gamma[-2 eps] + Gamma[-2 eps],
          2 s^-eps Gamma[-eps]^2 Gamma[1-eps] PolyGamma[0, -eps] - s^-eps Gamma[-eps]^2 Gamma[1-eps] PolyGamma[0, 1+eps]
          Gamma[-2 eps] + Gamma[-2 eps], {{eps → 0}, {}}],
```

100%

Done

 pset8

In[46]:= \$Path = Union[\$Path, {"Users/johannes/Desktop/mathschool"}];

In[47]:= << MB.m

MB 1.2
by Michal Czakon
improvements by Alexander Smirnov
more info in hep-ph/0511200
last modified 2 Jan 09

MB integrand for I2:

In[48]:= I2int = s^(1+z) t^(-1-eps-z) Gamma[-z] Gamma[2+eps+z] Gamma[1+z]^2 Gamma[-1-eps-z]^2 / Gamma[-2 eps]

$$\frac{s^{1+z} t^{-1-eps-z} \Gamma(-1-\epsilon-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\epsilon+z)}{\Gamma(-2 \epsilon)}$$

find allowed set of real parts for integration variables:

In[49]:= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]
 MBrules=norules : no rules could be found to regulate this integral

In[49]:= {{eps → -1}, {z → -1/2}}

In[50]:= I2cont = MBcontinue[I2int, eps → 0, I2rules]

Level 1

Taking residue in z = -1 - eps

Level 2

Integral[1]

2 integral(s) found

In[50]:= {{MBint[

$$\frac{\text{EulerGamma} s^{\epsilon z} \Gamma(-\epsilon z)^2 \Gamma(1-\epsilon z) - s^{\epsilon z} \Gamma(-\epsilon z)^2 \Gamma(1+\epsilon z) \log(s) + s^{\epsilon z} \Gamma(-\epsilon z)^2 \Gamma(1+\epsilon z) \log(t)}{\Gamma(-2 \epsilon)}$$

$$2 s^{\epsilon z} \Gamma(-\epsilon z)^2 \Gamma(1-\epsilon z) \text{PolyGamma}[0, -\epsilon z] + s^{\epsilon z} \Gamma(-\epsilon z)^2 \Gamma(1+\epsilon z) \text{PolyGamma}[0, 1+\epsilon z]}, ((\epsilon z \rightarrow 0), {})]},$$

100% ▶

Done

pset8

last modified 2 Jan 09

MB integrand for I2:

```
(106)= I2int = s^(1+z) t^(-1-eps-z) Gamma[-z] Gamma[2+eps+z] Gamma[1+z]^2 Gamma[-1-eps-z]^2 / Gamma[-2 eps]
d(106)= 
$$\frac{s^{1+z} t^{-1-eps-z} \Gamma(-1-\text{eps}-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\text{eps}+z)}{\Gamma(-2 \text{eps})}$$

```

find allowed set of real parts for integration variables:

```
(107)= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]
```

MBrules:norules : no rules could be found to regulate this integral

```
d(107)= {{eps → -1}, {z → - $\frac{1}{2}$ }}
```

```
(108)= I2cont = MBcontinue[I2int, eps → 0, I2rules]
```

Level 1

Taking residue in z = -1-eps

Level 2

Integral[1]

2 integral(s) found

```
d(108)= {{MBint[
```

$$\frac{\text{EulerGamma} s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps})}{\Gamma(-2 \text{eps})} + \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log(s)}{\Gamma(-2 \text{eps})} + \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps}) \log(t)}{\Gamma(-2 \text{eps})}$$

$$2 s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, -\text{eps}] + \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, 1+\text{eps}]}{\Gamma(-2 \text{eps})}, \{(\text{eps} \rightarrow 0), \{\}\}],$$

$$\text{MBint}\left[\frac{s^{1+z} t^{-1-eps-z} \Gamma(-1-\text{eps}-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\text{eps}+z)}{\Gamma(-2 \text{eps})}, \{(\text{eps} \rightarrow 0), \{z \rightarrow -\frac{1}{2}\}\}\right]$$

```
I(51)= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]
```

```
d(51)= [MBint[- $\frac{1}{\Gamma(-2 \text{eps})} s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps}) (\text{EulerGamma} + \log(s) - \log(t) + 2 \text{PolyGamma}[0, -\text{eps}] - \text{PolyGamma}[0, 1+\text{eps}])$ ],
```

100%
Done

pset8

last modified 2 Jan 09

MB integrand for I2:

```
(106)= I2int = s^(1+z) t^(-1-eps-z) Gamma[-z] Gamma[2+eps+z] Gamma[1+z]^2 Gamma[-1-eps-z]^2 / Gamma[-2 eps]
      s^(1+z) t^(-1-eps-z) Gamma[-1-eps-z]^2 Gamma[-z] Gamma[1+z]^2 Gamma[2+eps+z]
d(106)= -----
      Gamma[-2 eps]
```

find allowed set of real parts for integration variables:

```
(107)= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]
```

M8rules::norules : no rules could be found to regulate this integral

```
d(107)= {{eps → -1}, {z → -1/2}}
```

```
(108)= I2cont = MBcontinue[I2int, eps → 0, I2rules]
```

Level 1

Taking residue in z = -1-eps

Level 2

Integral [1]

2 integral(s) found

```
d(108)= [MBint]
      EulerGamma s^eps Gamma[-eps]^2 Gamma[1+eps] - s^-eps Gamma[-eps]^2 Gamma[1+eps] Log[s] + s^-eps Gamma[-eps]^2 Gamma[1+eps] Log[t]
      -----
      Gamma[-2 eps] + Gamma[-2 eps] + Gamma[-2 eps]
      2 s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, -eps] + s^-eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, 1+eps], {{eps → 0}, {}}], ,
      -----
      Gamma[-2 eps] + Gamma[-2 eps]
      MBint[
      s^(1+z) t^(-1-eps-z) Gamma[-1-eps-z]^2 Gamma[-z] Gamma[1+z]^2 Gamma[2+eps+z],
      -----
      Gamma[-2 eps], {{eps → 0}, {z → -1/2}}]]
```

```
In[51]= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]
```

```
d(51)= [MBint[-1/Gamma[-2 eps] s^eps Gamma[-eps]^2 Gamma[1+eps] (EulerGamma + Log[s] - Log[t] + 2 PolyGamma[0, -eps] - PolyGamma[0, 1+eps]) ,
```

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Done

pset8

last modified 2 Jan 09

MB integrand for I2:

```
(106)= I2int = s^(1+z) t^(-1-eps-z) Gamma[-z] Gamma[2+eps+z] Gamma[1+z]^2 Gamma[-1-eps-z]^2 / Gamma[-2 eps]
      s^(1+z) t^(-1-eps-z) Gamma[-1-eps-z]^2 Gamma[-z] Gamma[1+z]^2 Gamma[2+eps+z]
(106)= -----
      Gamma[-2 eps]
```

find allowed set of real parts for integration variables:

```
(107)= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]
```

```
M8rules: norules : no rules could be found to regulate this integral
```

```
(107)= {(eps → -1), {z → -1/2}}
```

```
(108)= I2cont = MBcontinue[I2int, eps → 0, I2rules]
```

Level 1

Taking residue in z = -1-eps

Level 2

Integral [1]

2 integral(s) found

```
(108)= MBint[
```

$$\frac{\text{EulerGamma} s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps})}{\Gamma(-2 \text{eps})} - \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log(s)}{\Gamma(-2 \text{eps})} + \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log(t)}{\Gamma(-2 \text{eps})}$$

$$2 s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, -\text{eps}] - \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, 1+\text{eps}]}{\Gamma(-2 \text{eps})}, \{(\text{eps} \rightarrow 0), (\text{})\}],$$

$$\text{MBint}\left[\frac{s^{1+z} t^{-1-\text{eps}-z} \Gamma(-1-\text{eps}-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\text{eps}+z)}{\Gamma(-2 \text{eps})}, \{(\text{eps} \rightarrow 0), \left\{z \rightarrow -\frac{1}{2}\right\}\}\right]$$

```
(109)= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]
```

```
(110)= MBint[-\frac{1}{\Gamma(-2 \text{eps})} s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) (\text{EulerGamma} + \log(s) - \log(t) + 2 \text{PolyGamma}[0, -\text{eps}] - \text{PolyGamma}[0, 1+\text{eps}]),
```

100%

Done

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help pset8

```

In[106]:= 
$$\frac{s^{1/2} t^{1/\text{eps}} \Gamma(-1-\text{eps}-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\text{eps}+z)}{\Gamma(-2 \text{eps})}$$


find allowed set of real parts for integration variables:
```

```

In[107]:= I2rules = MBoptimizedRules[I2int, eps -> 0, {}, {eps}]
M8rules: norules: no rules could be found to regulate this integral

In[107]:= {{eps -> -1}, {z -> - $\frac{1}{2}$ }}
```

```

In[108]:= I2cont = MBcontinue[I2int, eps -> 0, I2rules]
Level 1
Taking residue in z = -1-eps
Level 2
Integral (1)
2 integral(s) found

In[108]:= {{MBint[
$$\frac{\text{EulerGamma} s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps}) - s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log[s] + s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log[t]}{\Gamma(-2 \text{eps})}$$
, {{eps -> 0}, {}}],
$$\frac{2 s^{\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, -\text{eps}] - s^{\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, 1+\text{eps}]}{\Gamma(-2 \text{eps})}, {{(eps -> 0), {}}}],
$$\frac{s^{1/2} t^{1/\text{eps}} \Gamma(-1-\text{eps}-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\text{eps}+z)}{\Gamma(-2 \text{eps})}, {{(eps -> 0), {z -> -\frac{1}{2}}}}]}$$$$

```

```

In[51]:= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]
Out[51]:= {{MBint[- $\frac{1}{\Gamma(-2 \text{eps})} s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps}) (\text{EulerGamma} + \log[s] - \log[t] + 2 \text{PolyGamma}[0, -\text{eps}] - \text{PolyGamma}[0, 1+\text{eps}])$ , {{eps -> 0}, {}}}]}
```

```

In[52]:= I2exp = MBmerge[MBexpand[I2select, Exp[eps EulerGamma] m12^eps, {eps, 0, 0}]];
In[53]:= Expand[Normal[Series[I2exp /. MBint[a___, {{___}, {}}] -> a, {eps, 0, 0}]]][[1]]
```

100% Done

Pisa: 11080008 Page 198/211

i(106)= $\frac{s^{1-\epsilon} t^{1-\epsilon} \Gamma(-1-\epsilon-s-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\epsilon s+z)}{\Gamma(-2 \epsilon s)}$

find allowed set of real parts for integration variables:

i(107)= I2rules = MBoptimizedRules[I2int, $\epsilon s \rightarrow 0$, {}], { ϵs }]

M8rules::norules : no rules could be found to regulate this integral

i(107)= $\{(\epsilon s \rightarrow -1), \{z \rightarrow -\frac{1}{2}\}\}$

i(108)= I2cont = MBcontinue[I2int, $\epsilon s \rightarrow 0$, I2rules]

Level 1

Taking residue in $z = -1 - \epsilon s$

Level 2

Integral[1]

2 integral(s) found

i(108)= {{MBint[

$$\frac{\text{EulerGamma} s^{-\epsilon s} \Gamma(-\epsilon s)^2 \Gamma(1-\epsilon s)}{\Gamma(-2 \epsilon s)} - \frac{s^{-\epsilon s} \Gamma(-\epsilon s)^2 \Gamma(1+\epsilon s) \log[s]}{\Gamma(-2 \epsilon s)} + \frac{s^{-\epsilon s} \Gamma(-\epsilon s)^2 \Gamma(1+\epsilon s) \log[t]}{\Gamma(-2 \epsilon s)}$$

$$2 s^{-\epsilon s} \Gamma(-\epsilon s)^2 \Gamma(1+\epsilon s) \text{PolyGamma}[0, -\epsilon s] - \frac{s^{-\epsilon s} \Gamma(-\epsilon s)^2 \Gamma(1+\epsilon s) \text{PolyGamma}[0, 1+\epsilon s]}{\Gamma(-2 \epsilon s)}, \{(\epsilon s \rightarrow 0), \{z\}\}],$$

$$\text{MBint}\left[\frac{s^{1-\epsilon} t^{1-\epsilon} \Gamma(-1-\epsilon-s-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\epsilon s+z)}{\Gamma(-2 \epsilon s)}, \{(\epsilon s \rightarrow 0), \{z \rightarrow -\frac{1}{2}\}\}\right]$$

i(109)= I2select = MBmerge[MBpreselect[I2cont, { ϵs , 0, 0}]]

i(109)= [MBint[- $\frac{s^{-\epsilon s} \Gamma(-\epsilon s)^2 \Gamma(1-\epsilon s) (\text{EulerGamma} + \log[s] - \log[t] + 2 \text{PolyGamma}[0, -\epsilon s] - \text{PolyGamma}[0, 1+\epsilon s])}{\Gamma(-2 \epsilon s)}, \{(\epsilon s \rightarrow 0), \{z\}\}]$]

i(52)= I2exp = MBmerge[MBexpand[I2select, Exp[$\epsilon s \text{ EulerGamma}$] $\mu^2 \epsilon s$, { ϵs , 0, 0}]];

i(55)= Expand[Normal[Series[I2exp /. MBint[a_ \[Rule] {{}, {}}, {}] \[Rule] a, { ϵs , 0, 0}]]][[1]]

$$\frac{4}{\mu^2} - \frac{4 \pi^2}{\mu^2} - \frac{4 \log[\mu^2]}{\mu^2} - 2 \log[\mu^2]^2 - \frac{2 \log[s]}{\mu^2} - 2 \log[\mu^2] \log[s] - \frac{2 \log[t]}{\mu^2} - 2 \log[\mu^2] \log[t] + 2 \log[s] \log[t]$$

100% ►

Done

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i(106)= Gamma[-2 eps]

find allowed set of real parts for integration variables:

i(107)= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]

M8rules: norules : no rules could be found to regulate this integral

i(107)= {{eps → -1}, {z → -1/2}}

i(108)= I2cont = MBcontinue[I2int, eps → 0, I2rules]

Level 1

Taking residue in z = -1 - eps

Level 2

Integral[1]

2 integral(s) found

i(108)= {{MBint[

$$\frac{\text{EulerGamma} s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps})}{\Gamma(-2 \text{eps})} + \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log(s)}{\Gamma(-2 \text{eps})} + \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log(t)}{\Gamma(-2 \text{eps})}$$

$$+ \frac{2 s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, -\text{eps}]}{\Gamma(-2 \text{eps})} + \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, 1+\text{eps}]}{\Gamma(-2 \text{eps})}, \{(\text{eps} \rightarrow 0), \{z \rightarrow -\frac{1}{2}\}\}],$$

$$\text{MBint}\left[\frac{s^{1-\text{eps}} z^{\text{eps}} \Gamma(-1-\text{eps}-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\text{eps}+z)}{\Gamma(-2 \text{eps})}, \{(\text{eps} \rightarrow 0), \{z \rightarrow -\frac{1}{2}\}\}\right]\}$$

i(109)= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]

i(109)= {{MBint[$\frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps}) (\text{EulerGamma} + \log(s) - \log(t) + 2 \text{PolyGamma}[0, -\text{eps}] - \text{PolyGamma}[0, 1+\text{eps}])}{\Gamma(-2 \text{eps})}$], {(\text{eps} \rightarrow 0), \{z \rightarrow -\frac{1}{2}\}}]}

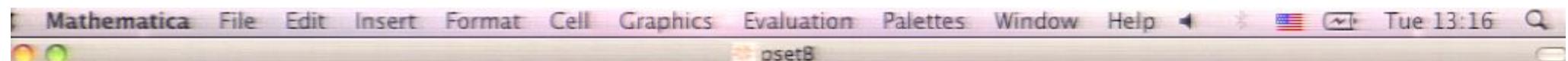
i(52)= I2exp = MBmerge[MBexpand[I2select, Exp[eps EulerGamma] μ2^eps, {eps, 0, 0}]];

i(55)= Expand[Normal[Series[I2exp /. MBint[a___, {(_____, {})}] → a, {eps, 0, 0}]]][[1]]

$$\frac{4}{\text{eps}^2} - \frac{4 \pi^2}{3} + \frac{4 \log(\mu2)}{\text{eps}} - 2 \log(\mu2)^2 - \frac{2 \log(s)}{\text{eps}} - 2 \log(\mu2) \log(s) - \frac{2 \log(t)}{\text{eps}} - 2 \log(\mu2) \log(t) + 2 \log(s) \log(t)$$

100% ►

Done



find allowed set of real parts for integration variables:

```
(107)= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]
MBrules::norules : no rules could be found to regulate this integral

(107)= {{(eps → -1), {z → -1/2}}}

(108)= I2cont = MBcontinue[I2int, eps → 0, I2rules]
Level 1
Taking residue in z = -1 - eps
Level 2
Integral[1]
2 integral(s) found

(108)= {{MBint[
    EulerGamma s^eps Gamma[-eps]^2 Gamma[1 - eps] - s^eps Gamma[-eps]^2 Gamma[1 + eps] Log[s], s^eps Gamma[-eps]^2 Gamma[1 + eps] Log[t]
    Gamma[-2 eps]                                     Gamma[-2 eps]                                     Gamma[-2 eps]
    2 s^eps Gamma[-eps]^2 Gamma[1 + eps] PolyGamma[0, -eps] - s^eps Gamma[-eps]^2 Gamma[1 + eps] PolyGamma[0, 1 + eps],
    Gamma[-2 eps]                                     Gamma[-2 eps], ((eps → 0), {})],

    MBint[ s^(z - 1)^eps z Gamma[-1 - eps - z]^2 Gamma[-z] Gamma[1 + z]^2 Gamma[2 + eps + z]
    Gamma[-2 eps], ((eps → 0), {z → -1/2})]]},

(109)= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]
(109)= {MBint[ s^eps Gamma[-eps]^2 Gamma[1 - eps] (EulerGamma + Log[s] - Log[t] + 2 PolyGamma[0, -eps] - PolyGamma[0, 1 + eps]),
    Gamma[-2 eps], ((eps → 0), {})]}

(110)= I2exp = MBmerge[MBexpand[I2select, Exp[eps EulerGamma] mu2^eps, {eps, 0, 0}]];
I2

(111)= Expand[Normal[Series[I2exp /. MBint[a___, {}] :> a, {eps, 0, 0}]]][[1]]
(111)= 
$$\frac{4}{\text{eps}^2} - \frac{4 \pi^2}{3} + \frac{4 \log[\mu_2]}{\text{eps}} - 2 \log[\mu_2]^2 - \frac{2 \log[s]}{\text{eps}} - 2 \log[\mu_2] \log[s] - \frac{2 \log[t]}{\text{eps}} - 2 \log[\mu_2] \log[t] + 2 \log[s] \log[t]$$

```

100% ►

Done

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pset8

```

In[107]= {{eps → -1}, {z → -1/2}}
Out[107]= {{eps → -1}, {z → -1/2}]

In[108]= I2cont = MBcontinue[I2int, eps → 0, I2rules]
Level 1
Taking residue in z = -1 - eps
Level 2
Integral (1)
2 integral(s) found

```

```

In[108]= {{MBint[
      EulerGamma s^eps Gamma[-eps]^2 Gamma[1+eps] - s^eps Gamma[-eps]^2 Gamma[1+eps] Log[s] - s^eps Gamma[-eps]^2 Gamma[1+eps] Log[t]
      Gamma[-2 eps] Gamma[-2 eps] Gamma[-2 eps] ,
      2 s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, -eps] - s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, 1+eps],
      Gamma[-2 eps] Gamma[-2 eps] , ((eps → 0), {})],
      MBint[s^(1+eps) z^eps Gamma[-1-eps-z]^2 Gamma[1+z]^2 Gamma[2+eps+z]
      Gamma[-2 eps], ((eps → 0), {z → -1/2})]},
      I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]}

```

```

In[109]= MBint[-s^eps Gamma[-eps]^2 Gamma[1+eps] (EulerGamma + Log[s] - Log[t] + 2 PolyGamma[0, -eps] - PolyGamma[0, 1+eps]),
      Gamma[-2 eps], ((eps → 0), {})]

```

```

In[110]= I2exp = MBmerge[MBexpand[I2select, Exp[eps EulerGamma] mu2^eps, {eps, 0, 0}]];

```

```

In[111]= I2exp

```

```

In[111]= MBint[-2 (-6 + 2 eps^2 π^2 - 3 eps^2 Log[mu2]^2 + 3 eps Log[t] - 3 eps Log[s] (-1 + eps Log[t])) + 3 eps Log[mu2] (-2 + eps Log[s] + eps Log[t]),
      3 eps^2, ((eps → 0), {})]

```

```

In[55]= Expand[Normal[Series[I2exp /. MBint[a __, {{__}, {}}], {a, {eps, 0, 0}}]][[1]]

```

```

Out[55]= 
$$\frac{4}{\text{eps}^2} - \frac{4 \pi^2}{3} + \frac{4 \log[\mu_2]}{\text{eps}} - 2 \log[\mu_2]^2 - \frac{2 \log[s]}{\text{eps}} - 2 \log[\mu_2] \log[s] - \frac{2 \log[t]}{\text{eps}} - 2 \log[\mu_2] \log[t] + 2 \log[s] \log[t]$$


```

100% Done

```

Taking residue in z = -1 - eps
Level 2
Integral[1]
2 integral(s) found
a[108]= {{MBint[
  EulerGamma s^eps Gamma[-eps]^2 Gamma[1+eps] - s^eps Gamma[-eps]^2 Gamma[1+eps] Log[s] - s^eps Gamma[-eps]^2 Gamma[1+eps] Log[t]
  Gamma[-2 eps] Gamma[-2 eps] Gamma[-2 eps] + 2 s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, -eps] - s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, 1+eps]
  Gamma[-2 eps] Gamma[-2 eps], {{eps → 0}, {}}], MBint[
  s^(1+eps) z Gamma[-1-eps - z]^2 Gamma[-z] Gamma[1+z]^2 Gamma[2+eps + z]
  Gamma[-2 eps], {{eps → 0}, {z → -1/2}}]}, {I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]}
a[109]= MBint[ s^eps Gamma[-eps]^2 Gamma[1+eps] (EulerGamma + Log[s] - Log[t] + 2 PolyGamma[0, -eps] - PolyGamma[0, 1+eps])
  Gamma[-2 eps], {{eps → 0}, {}}]
a[110]= I2exp = MBmerge[MBexpand[I2select, Exp[eps EulerGamma] mu2^eps, {eps, 0, 0}]];
a[111]= I2exp
a[111]= MBint[ 2 (-6 - 2 eps^2 π^2 - 3 eps^2 Log[mu2]^2 - 3 eps Log[t] - 3 eps Log[s] (-1 + eps Log[t]) + 3 eps Log[mu2] (-2 + eps Log[s] + eps Log[t]))
  3 eps^2, {{eps → 0}, {}}]
a[112]= Expand[Normal[Series[I2exp /. MBint[a___, {{___}, {}}] → a, {eps, 0, 0}]]][[1]]
a[112]= 4/eps^2 - 4 π^2/3 + 4 Log[mu2]/eps - 2 Log[mu2]^2 - 2 Log[s]/eps - 2 Log[mu2] Log[s] - 2 Log[t]/eps - 2 Log[mu2] Log[t] + 2 Log[s] Log[t]
a[113]= Expand[PowerExpand[Normal[Series[2/eps^2 ((mu2/s)^eps + (mu2/t)^eps) - Log[s/t]^2 - 4 Pi^2/3, {eps, 0, 0}]]]]
a[113]= 4/eps^2 - 4 π^2/3 + 4 Log[mu2]/eps - 2 Log[mu2]^2 - 2 Log[s]/eps - 2 Log[mu2] Log[s] - 2 Log[t]/eps - 2 Log[mu2] Log[t] + 2 Log[s] Log[t]

```

100% ▶

Done

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In[108]:= I2cont = MBcontinue[I2int, eps → 0, I2rules]

Level 1

Taking residue in $z = -1 - \text{eps}$

Level 2

Integral (1)

2 integral(s) found

In[108]:= {{MBint[

$$\frac{\text{EulerGamma} s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps})}{\Gamma(-2 \text{eps})} - \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log[s]}{\Gamma(-2 \text{eps})} + \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log[t]}{\Gamma(-2 \text{eps})}$$

$$2 s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, -\text{eps}] - \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, 1+\text{eps}]}{\Gamma(-2 \text{eps})}, \{(\text{eps} \rightarrow 0), \{\}\}],$$

$$\text{MBint}\left[\frac{s^{1+z} t^{1-\text{eps}} z \Gamma(-1-\text{eps}-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\text{eps}+z)}{\Gamma(-2 \text{eps})}, \{(\text{eps} \rightarrow 0), \{z \rightarrow -\frac{1}{2}\}\}\right]\right]$$

In[109]:= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]

In[109]:= {MBint[- $\frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps}) (\text{EulerGamma} + \log[s] - \log[t] + 2 \text{PolyGamma}[0, -\text{eps}] - \text{PolyGamma}[0, 1+\text{eps}])}{\Gamma(-2 \text{eps})}, \{(\text{eps} \rightarrow 0), \{\}\}]}$

In[110]:= I2exp = MBmerge[MBexpand[I2select, Exp[eps EulerGamma] μ2^eps, {eps, 0, 0}]];

In[111]:= I2exp

In[111]:= {MBint[- $\frac{2 (-6 - 2 \text{eps}^2 \pi^2 - 3 \text{eps}^2 \log[\mu2]^2 - 3 \text{eps} \log[t] - 3 \text{eps} \log[s] (-1 - \text{eps} \log[t]) + 3 \text{eps} \log[\mu2] (-2 - \text{eps} \log[s] - \text{eps} \log[t]))}{3 \text{eps}^2}, \{(\text{eps} \rightarrow 0), \{\}\}]}$

In[112]:= Expand[Normal[Series[I2exp /. MBint[a__r, {{___}, r, {}}] → a, {eps, 0, 0}]]][[1]]

In[112]:= $\frac{4}{\text{eps}^2} - \frac{4 \pi^2}{3} + \frac{4 \log[\mu2]}{\text{eps}} - 2 \log[\mu2]^2 - \frac{2 \log[s]}{\text{eps}} - 2 \log[\mu2] \log[s] - \frac{2 \log[t]}{\text{eps}} - 2 \log[\mu2] \log[t] + 2 \log[s] \log[t]$

In[56]:= Expand[PowerExpand[Normal[Series[2/eps^2 ((μ2/s)^eps + (μ2/t)^eps) - Log[s/t]^2 - 4 Pi^2/3, {eps, 0, 0}]]]]

100% ►
Done

pset8

Find allowed set of real parts for integration variables.

```
(107)= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]
```

MBrules:norules : no rules could be found to regulate this integral

```
(107)= {{eps → -1}, {z → -1/2}}
```

```
(108)= I2cont = MBcontinue[I2int, eps → 0, I2rules]
```

Level 1

Taking residue in $z = -1 - \text{eps}$

Level 2

Integral[1]

2 integral(s) found

```
(108)= {{MBint[
```

$$\frac{\text{EulerGamma} s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps})}{\Gamma(-2 \text{eps})} - \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \log[s]}{\Gamma(-2 \text{eps})} + \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps}) \log[t]}{\Gamma(-2 \text{eps})}$$

$$2 s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, -\text{eps}] - \frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1+\text{eps}) \text{PolyGamma}[0, 1+\text{eps}]}{\Gamma(-2 \text{eps})}, \{\{\text{eps} \rightarrow 0\}, \{\}\}],$$

$$\text{MBint}\left[\frac{s^{1-\text{eps}} t^{1-\text{eps}} \Gamma(-1-\text{eps}-z)^2 \Gamma(-z) \Gamma(1+z)^2 \Gamma(2+\text{eps}+z)}{\Gamma(-2 \text{eps})}, \{\{\text{eps} \rightarrow 0\}, \{z \rightarrow -\frac{1}{2}\}\}\right]$$

```
(109)= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]
```

$$\text{MBint}\left[\frac{s^{-\text{eps}} \Gamma(-\text{eps})^2 \Gamma(1-\text{eps}) (\text{EulerGamma} + \log[s] - \log[t] + 2 \text{PolyGamma}[0, -\text{eps}] - \text{PolyGamma}[0, 1+\text{eps}])}{\Gamma(-2 \text{eps})}, \{\{\text{eps} \rightarrow 0\}, \{\}\}\right]$$

```
(110)= I2exp = MBmerge[MBexpand[I2select, Exp[eps EulerGamma] μu2^eps, {eps, 0, 0}]];
```

```
(111)= I2exp
```

$$\text{MBint}\left[-\frac{2 (-6 - 2 \text{eps}^2 \pi^2 - 3 \text{eps}^2 \log[\mu u2]^2 + 3 \text{eps} \log[t] - 3 \text{eps} \log[s] (-1 - \text{eps} \log[t]) + 3 \text{eps} \log[\mu u2] (-2 + \text{eps} \log[s] - \text{eps} \log[t]))}{3 \text{eps}^2}, \{\{\text{eps} \rightarrow 0\}, \{\}\}\right]$$

100%

Done

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```

In[46]:= $Path = Union[$Path, {"Users/johannes/Desktop/mathschool"}];
In[105]:= << MB.m
MB 1.2
by Michal Czakon
improvements by Alexander Smirnov
more info in hep-ph/0511200
last modified 2 Jan 09

MB integrand for I2:

In[106]:= I2int = s^(1+z) t^(-1-eps-z) Gamma[-z] Gamma[2+eps+z] Gamma[1+z]^2 Gamma[-1-eps-z]^2 / Gamma[-2 eps]
           s^z t^1-eps-z Gamma[-1-eps-z]^2 Gamma[-z] Gamma[1+z]^2 Gamma[2+eps+z]
In[106]:= -----
           Gamma[-2 eps]

find allowed set of real parts for integration variables:

In[107]:= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]
MBrules: norules : no rules could be found to regulate this integral
In[107]:= {{eps → -1}, {z → -1/2} }

In[108]:= I2cont = MBcontinue[I2int, eps → 0, I2rules]
Level 1
Taking residue in z = -1-eps
Level 2
Integral (1)
2 integral(s) found
In[108]:= {{MBint[
           EulerGamma s^eps Gamma[-eps]^2 Gamma[1-eps] - s^eps Gamma[-eps]^2 Gamma[1+eps] Log[s], s^eps Gamma[-eps]^2 Gamma[1+eps] Log[t]
           ]/Gamma[-2 eps], Gamma[-2 eps]}, {Gamma[-2 eps], Gamma[-2 eps]}},
           2 s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, -eps], s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, 1+eps],
           Gamma[-2 eps], Gamma[-2 eps]}, {{eps → 0}, {}}], 100% -->
Done.

```

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more info in help/guide/Integrals

last modified 2 Jan 09

MB integrand for I2:

```
I105:= I2int = s^(1+z) t^(-1-eps-z) Gamma[-z] Gamma[2+eps+z] Gamma[1+z]^2 Gamma[-1-eps-z]^2 / Gamma[-2 eps]
s^(1+z) t^(-1-eps-z) Gamma[-1-eps-z]^2 Gamma[-z] Gamma[1+z]^2 Gamma[2+eps+z]
I106= -----
Gamma[-2 eps]
```

find allowed set of real parts for integration variables:

```
I107:= I2rules = MBoptimizedRules[I2int, eps → 0, {}, {eps}]
MBrules::norules : no rules could be found to regulate this integral
I107= {{eps → -1}, {z → -1/2}}
```

```
I108:= I2cont = MBcontinue[I2int, eps → 0, I2rules]
Level 1
Taking -residue in z = -1-eps
Level 2
Integral[1]
2 integral(s) found
I108= {{MBint[
EulerGamma s^eps Gamma[-eps]^2 Gamma[1+eps] - s^eps Gamma[-eps]^2 Gamma[1+eps] Log[s] - s^eps Gamma[-eps]^2 Gamma[1+eps] Log[t]
Gamma[-2 eps] Gamma[-2 eps] Gamma[-2 eps] Gamma[-2 eps],
2 s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, -eps] - s^eps Gamma[-eps]^2 Gamma[1+eps] PolyGamma[0, 1+eps],
{{eps → 0}, {}}], {eps → 0}, {z → -1/2}],
MBint[
s^(1+z)^2 Gamma[-1-eps-z]^2 Gamma[-z] Gamma[1+z]^2 Gamma[2+eps+z]
Gamma[-2 eps], {eps → 0}, {z → -1/2}]}}
```

```
I109:= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]
I109= {{MBint[
s^eps Gamma[-eps]^2 Gamma[1+eps] (EulerGamma + Log[s] - Log[t] + 2 PolyGamma[0, -eps] - PolyGamma[0, 1+eps])
Gamma[-2 eps], {eps → 0}, {}]}}
```

100% Done

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 pset8

```

In[107]:= {{(eps → -1), {z → -1/2}}}

In[108]:= I2cont = MBcontinue[I2int, eps → 0, I2rules]
Level 1
Taking residue in z = -1 - eps
Level 2
Integral (1)
2 integral(s) found

```

```

In[108]:= {{MBint[
      EulerGamma s^eps Gamma[-eps]^2 Gamma[1 + eps] - s^eps Gamma[-eps]^2 Gamma[1 + eps] Log[s] - s^eps Gamma[-eps]^2 Gamma[1 + eps] Log[t]
      Gamma[-2 eps] - Gamma[-2 eps] - Gamma[-2 eps] - Gamma[-2 eps],
      2 s^eps Gamma[-eps]^2 Gamma[1 + eps] PolyGamma[0, -eps] - s^eps Gamma[-eps]^2 Gamma[1 + eps] PolyGamma[0, 1 + eps], {{eps → 0}, {}}},

      MBint[s^(1 + z)^(1 + eps) Gamma[-1 - eps - z]^2 Gamma[-z] Gamma[1 + z]^2 Gamma[2 + eps + z]
      Gamma[-2 eps], {{eps → 0}, {z → -1/2}}]}}

In[109]:= I2select = MBmerge[MBpreselect[I2cont, {eps, 0, 0}]]

```

```

In[109]:= MBint[s^eps Gamma[-eps]^2 Gamma[1 + eps] (EulerGamma + Log[s] - Log[t] + 2 PolyGamma[0, -eps] - PolyGamma[0, 1 + eps])
      Gamma[-2 eps], {{eps → 0}, {}}]

```

```

In[110]:= I2exp = MBmerge[MBexpand[I2select, Exp[eps EulerGamma] mu2^eps, {eps, 0, 0}]];

```

```

In[111]:= I2exp

```

```

In[111]:= MBint[2 (-6 - 2 eps^2 π^2 - 3 eps^2 Log[mu2]^2 - 3 eps Log[t] - 3 eps Log[s] (-1 + eps Log[t])) + 3 eps Log[mu2] (-2 + eps Log[s] + eps Log[t])
      3 eps^2, {{eps → 0}, {}}]

```

```

In[112]:= Expand[Normal[Series[I2exp /. MBint[a___, {{___}, {}}] → a, {eps, 0, 0}]]][[1]]

```

```

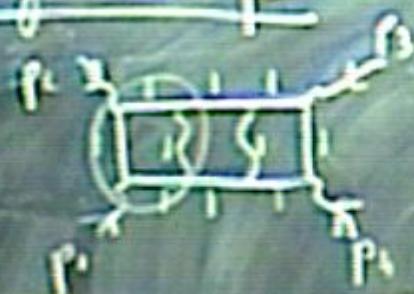
In[112]:= 4/eps^2 - 4 π^2/3 + 4 Log[mu2]/eps - 2 Log[mu2]^2 - 2 Log[s]/eps - 2 Log[mu2] Log[s] - 2 Log[t]/eps - 2 Log[mu2] Log[t] + 2 Log[s] Log[t]

```

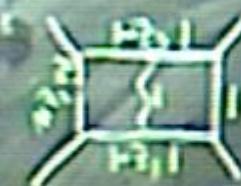
100%

Done

higher loops



$$= \int \frac{d\alpha_1 d\alpha_2 d\alpha_3}{(2\pi i)^3} G(\alpha) \left(\sum_{n=1}^{\infty} \right)^3$$

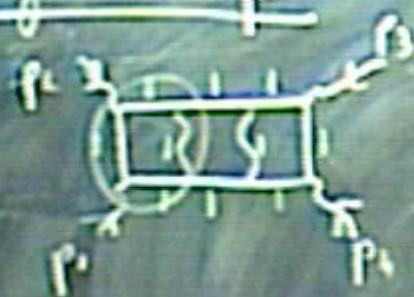


Mathematica

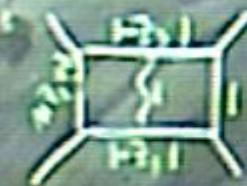
AMBRE

barnes, hep-th
DoA

higher loops



$$= \int \frac{dz_1 dz_2 dz_3}{(2\pi i)^3} G(z) \left(\sum_{n=1}^{\infty}\right)^{z_1 z_2 z_3}$$

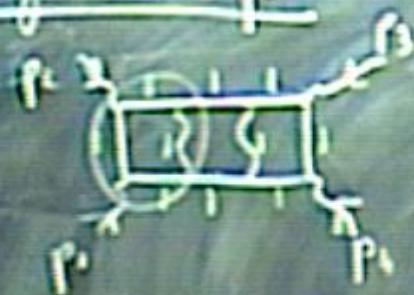


↳ Mathematica impl. AMBRE

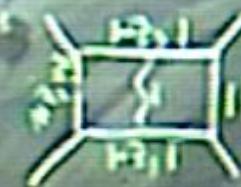
barnerroutines.m

DoAllBarnes[{ }]

higher loops



$$= \int \frac{dz_2 dz_3 dz_4}{(2\pi i)^3} G(z) \left(\sum_{n=1}^{\infty} \right)^{z_2 z_3 z_4}$$



↳ Mathematica impl. AMBRE

barnesroutines.m

DoAllBarnes[{ l }]

MBnrgs