Title: Y-System for Amplitudes

Date: Aug 02, 2011 09:30 AM

URL: http://pirsa.org/11080007

Abstract:

The solution

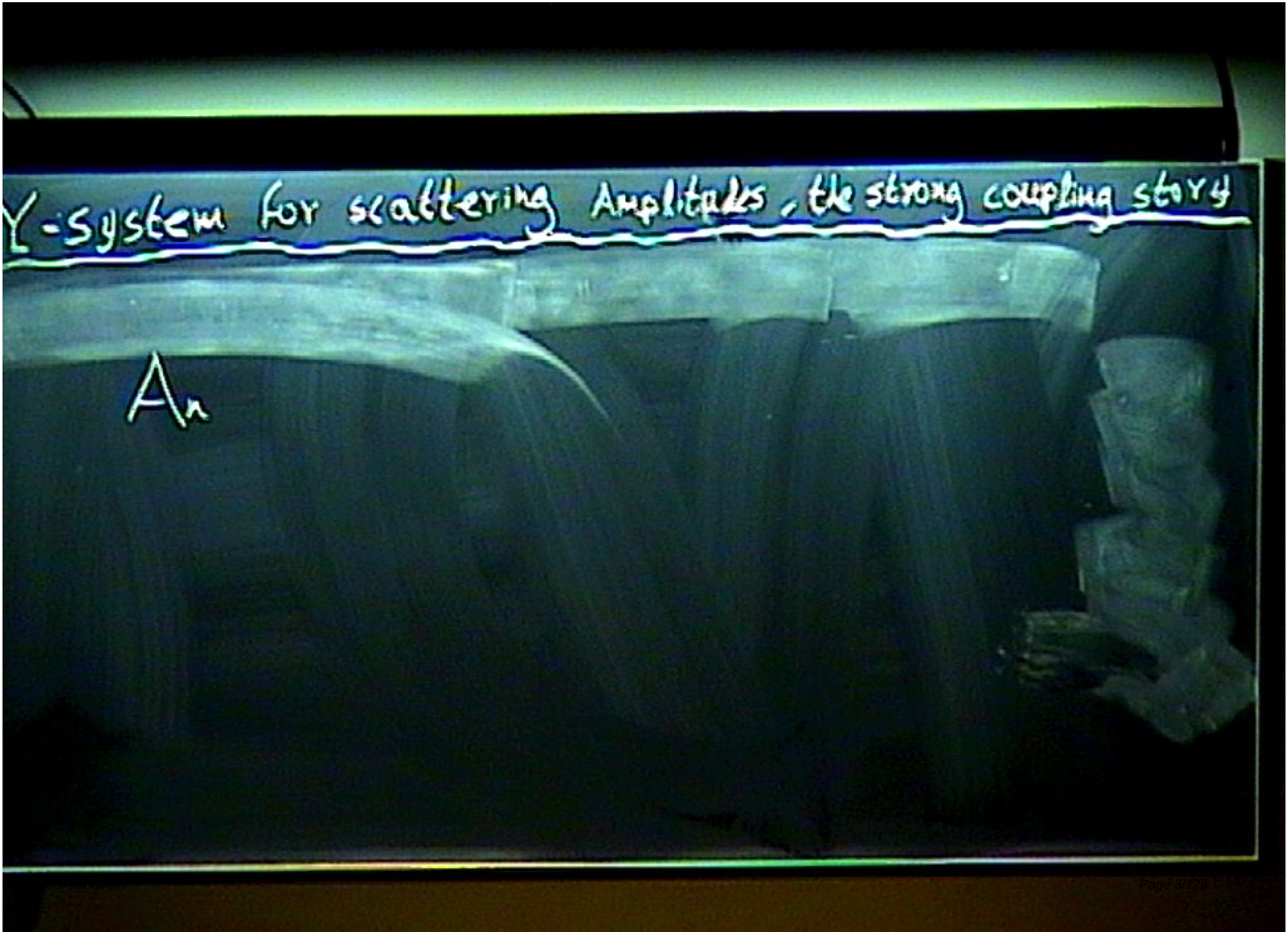
$$\text{Area} = \underbrace{A_{\text{BDS-like}} + A_{\text{period}}}_{\text{trivial}} + \underbrace{YY_c}_{\text{Non-trivial}}$$

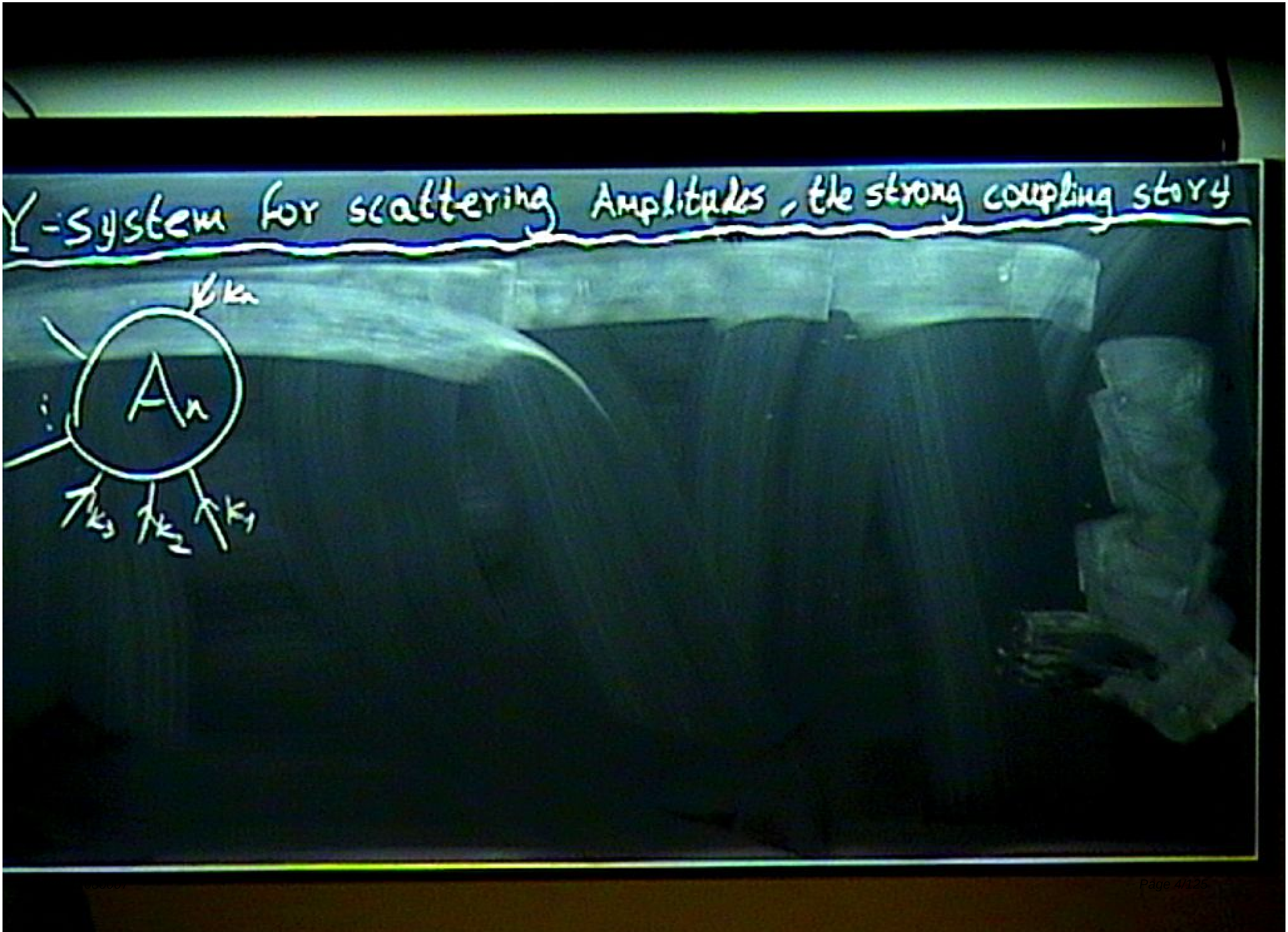
$$A_{\text{BDS-like}} = \sum \log(x^+_{i+1} - x^+_i) M_{ij} \log(x^-_{j+1} - x^-_j)$$

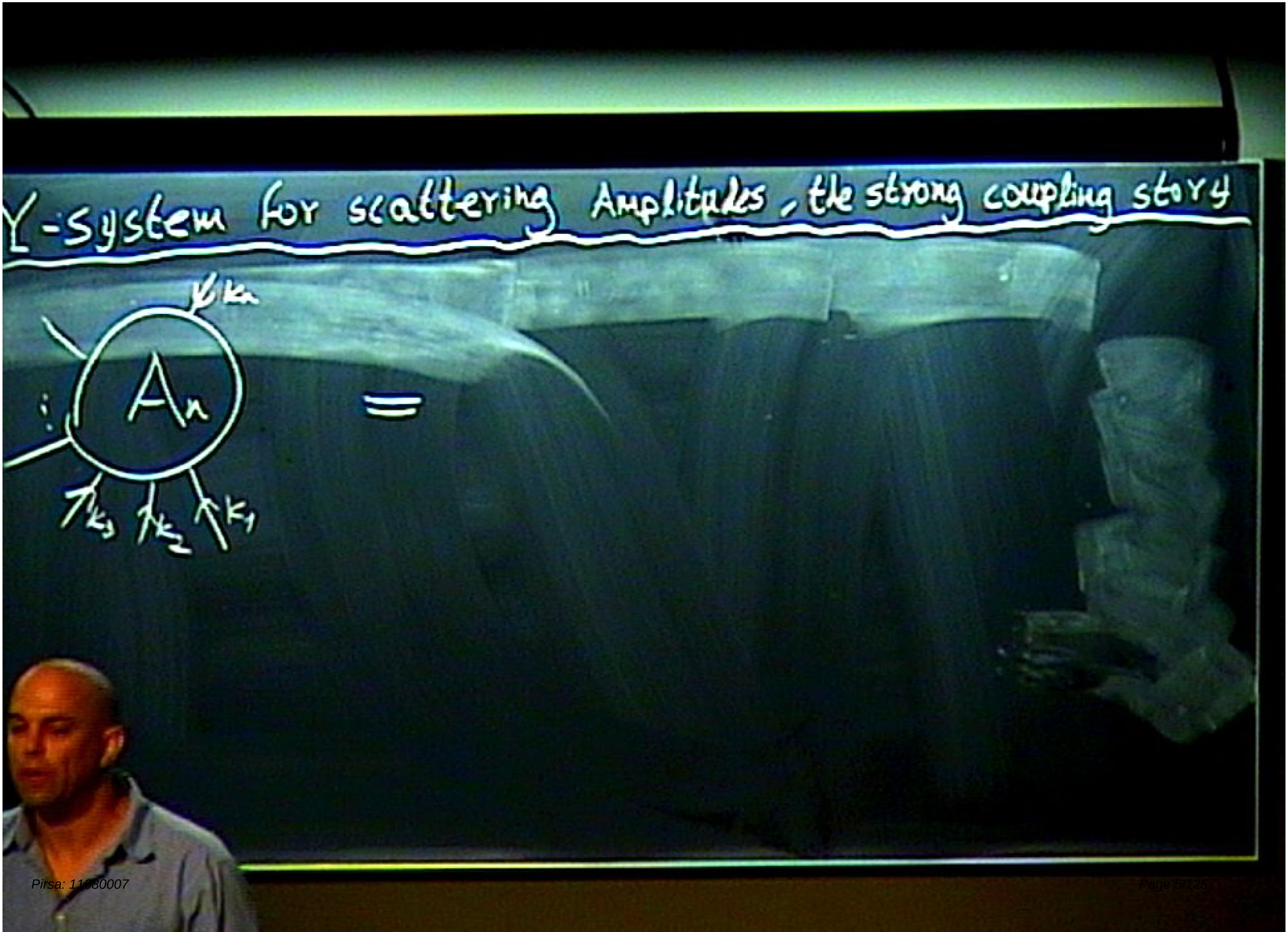
$$(A_{\text{period}} + YY_c) = \sum_{s,s'} W_{ss'} A_s B_{s'} \qquad W = \begin{pmatrix} 0 & 1 & & & \\ -1 & 0 & -1 & & \\ & 1 & 0 & 1 & \\ & & -1 & & \\ & & & & \ddots \end{pmatrix}^{-1}$$

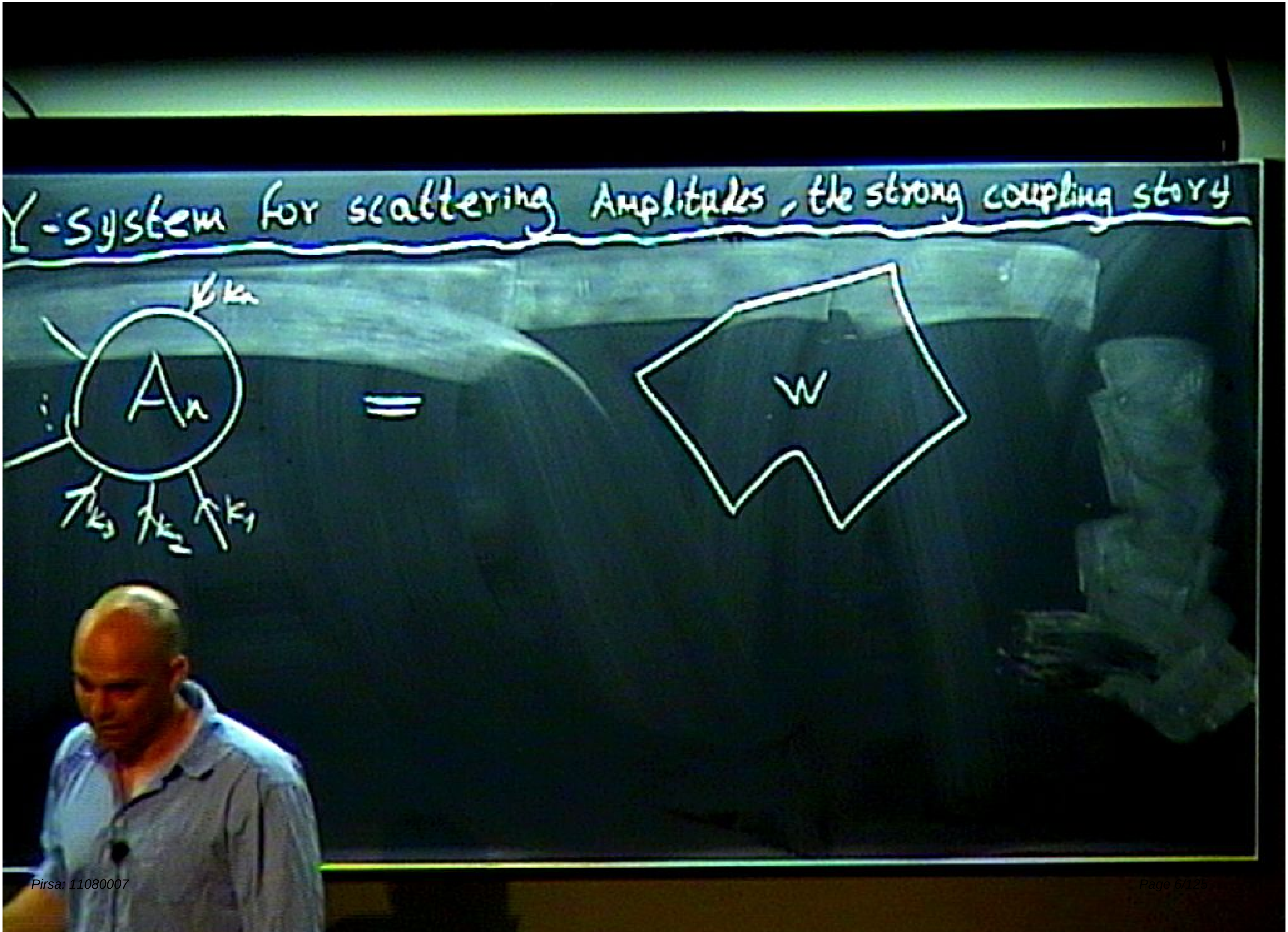
$$\log Y_s(\theta) = \left[\cosh\theta \log x_s^{\pm} \mp i \sinh\theta \log x_s^{\mp}\right]$$

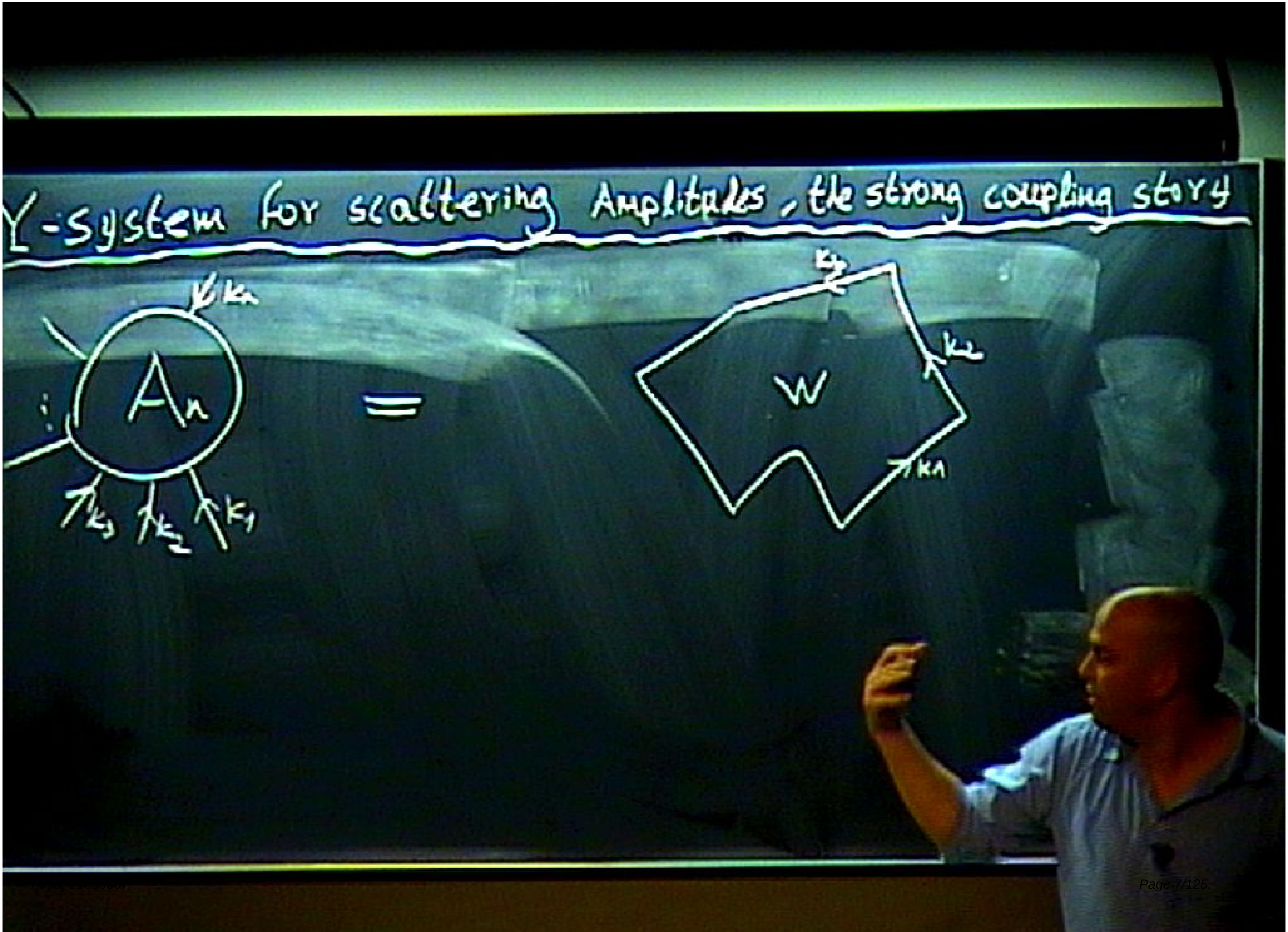
$$+ \int_{\mathbb{R} \pm i0} \frac{d\theta' \sinh(2\theta)}{2\pi \sinh(\theta-\theta') \cosh(\theta-\theta')} \log\left[(1+Y_{s+1})(1+Y_{s-1})\right] \xrightarrow[\theta \to -\infty]{} e^{-\theta} A_s + e^{\theta} B_s + O(e^{\dots})$$

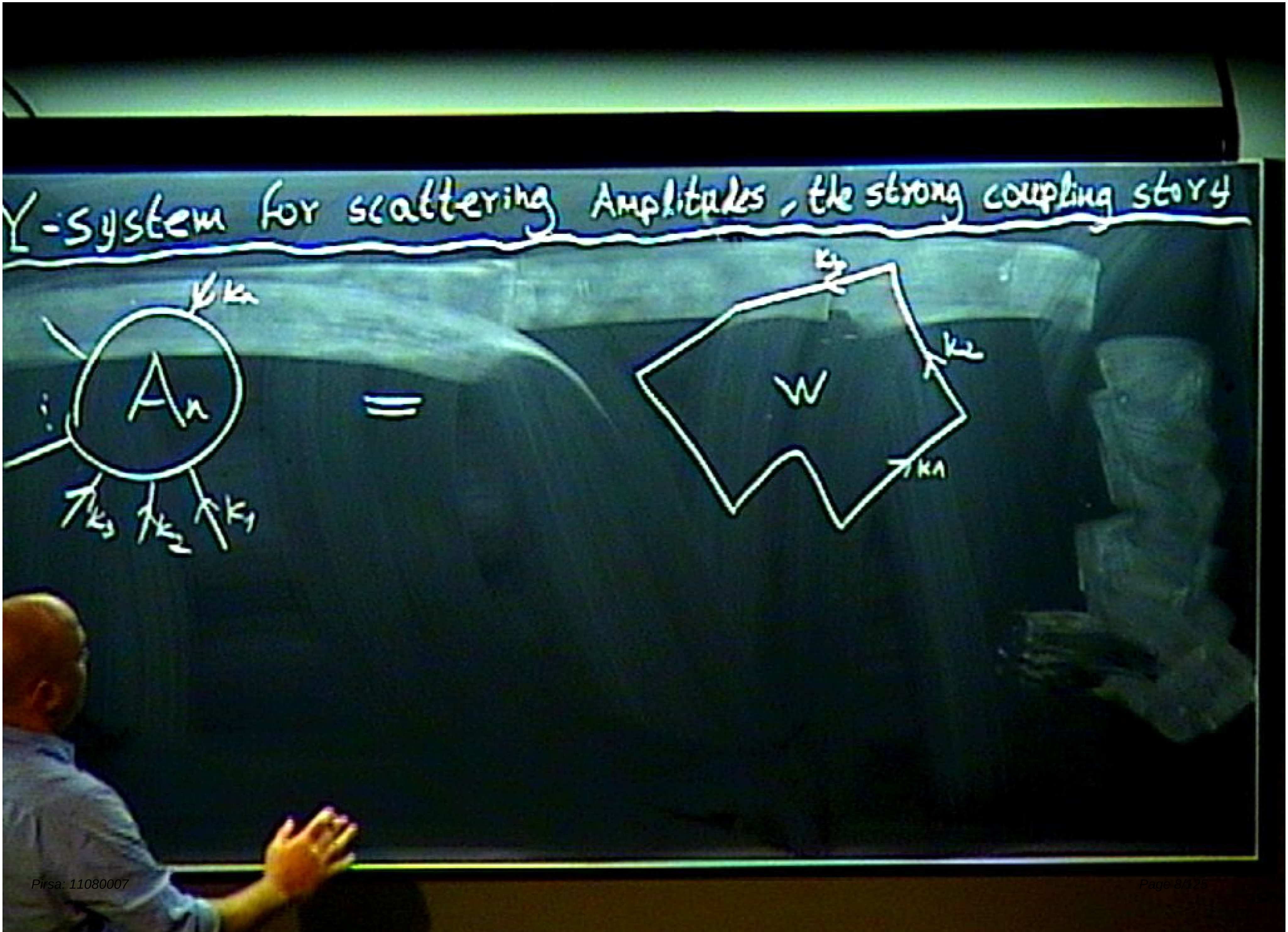

Y-system for scattering Amplitudes, the strong coupling story
A_n


Y-system for scattering Amplitudes, the strong coupling story
A_n
k_n
k_3
k_2
k_1

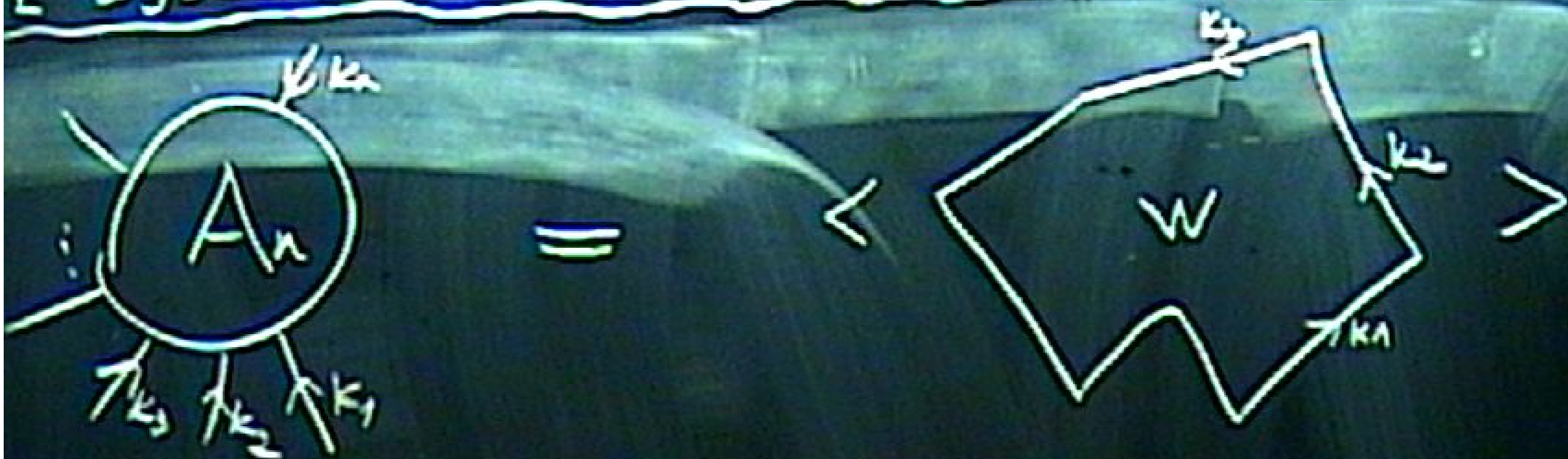

Y-system for scattering Amplitudes, the strong coupling story
A_n =
k_n
k_3 k_2 k_1


Y-system for scattering Amplitudes, the strong coupling story
k_n
A_n
=
W
k_3
k_2
k_1


Y-system for scattering Amplitudes, the strong coupling story
A_n
=
W
k_n
k_1
k_2
k_3


Y-system for scattering Amplitudes, the strong coupling story
kn
An
k3
k2
k1
=
kn
k2
W
k1

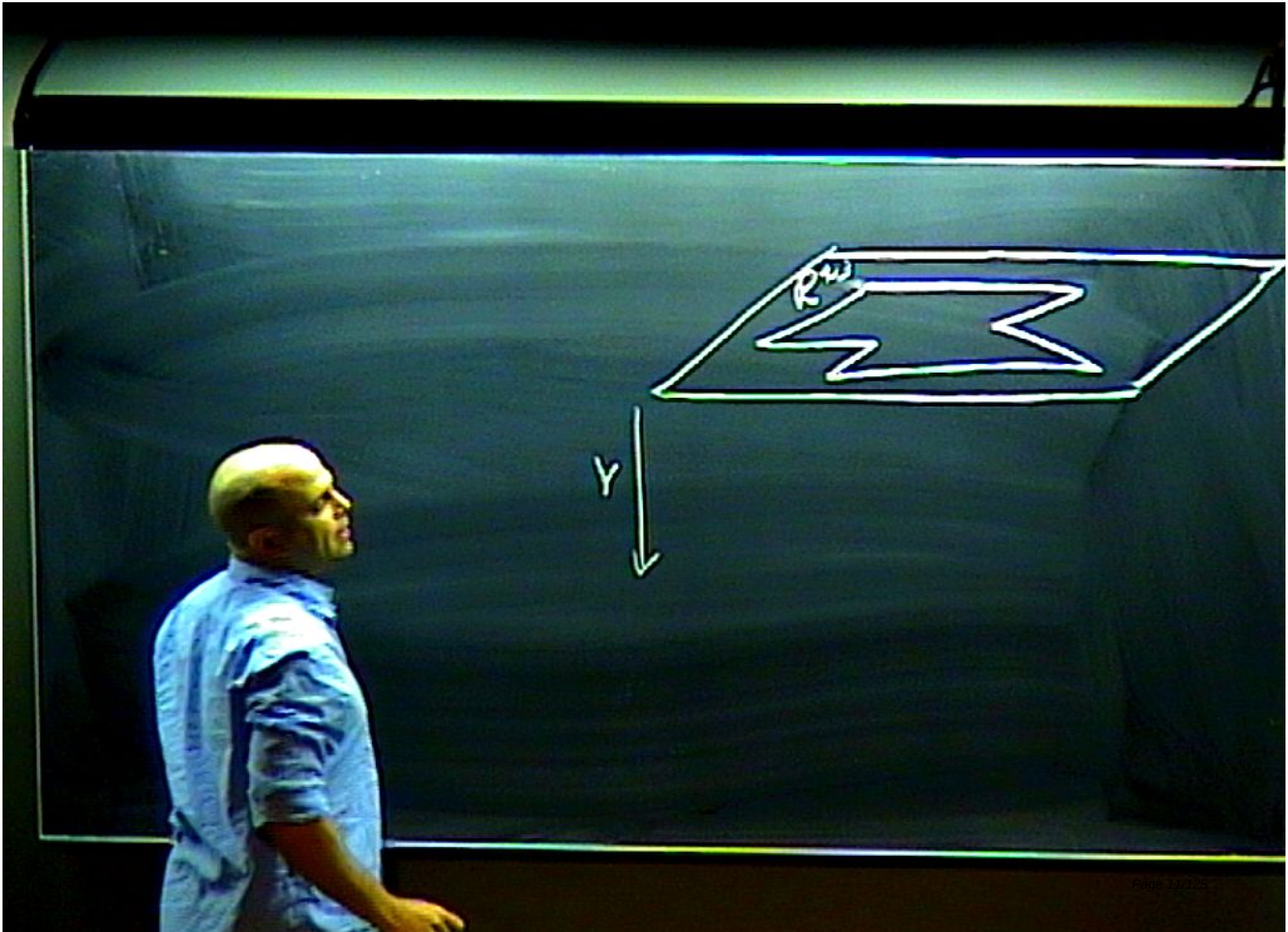
Y-system for scattering Amplitudes, the strong coupling story

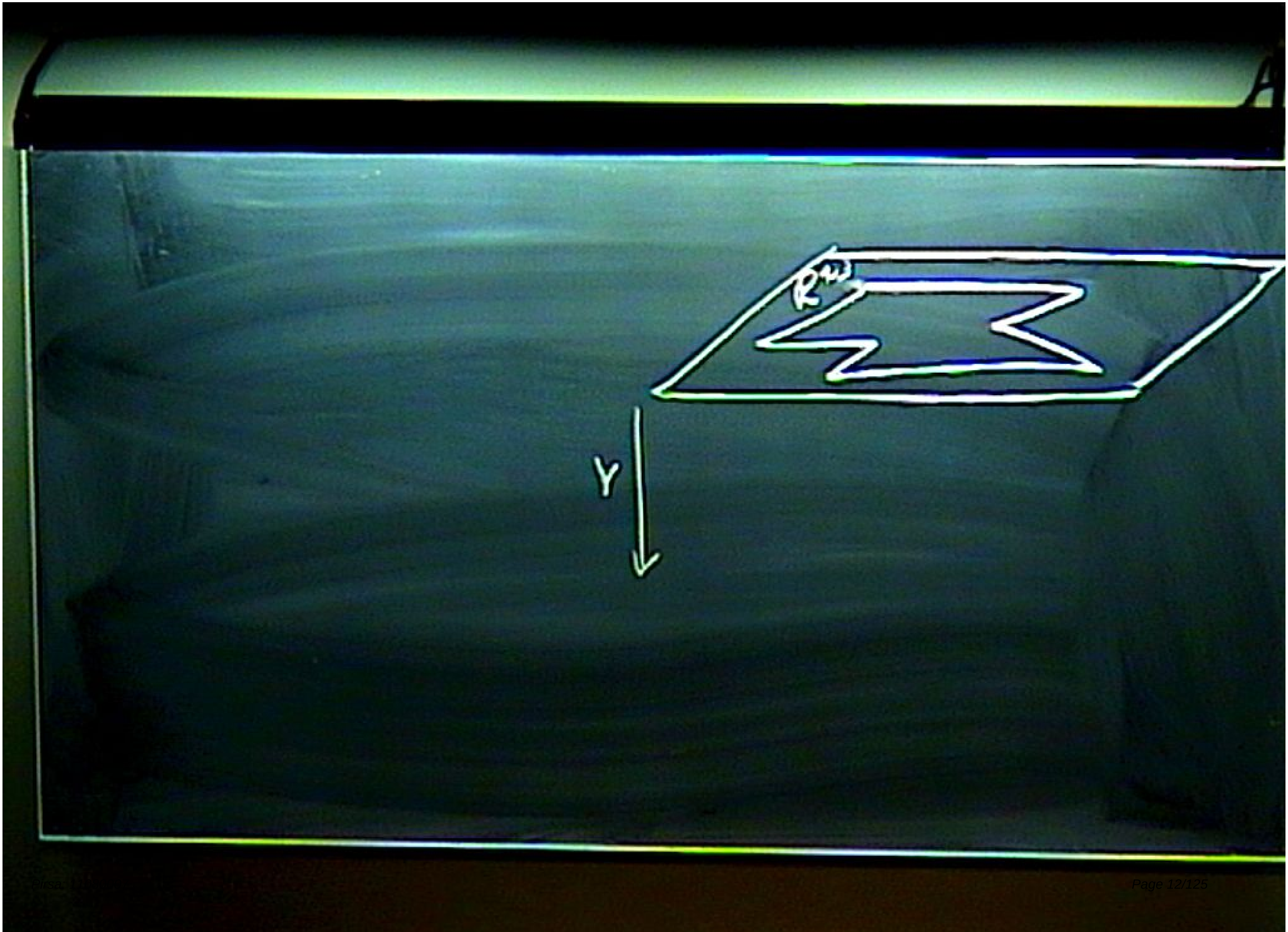


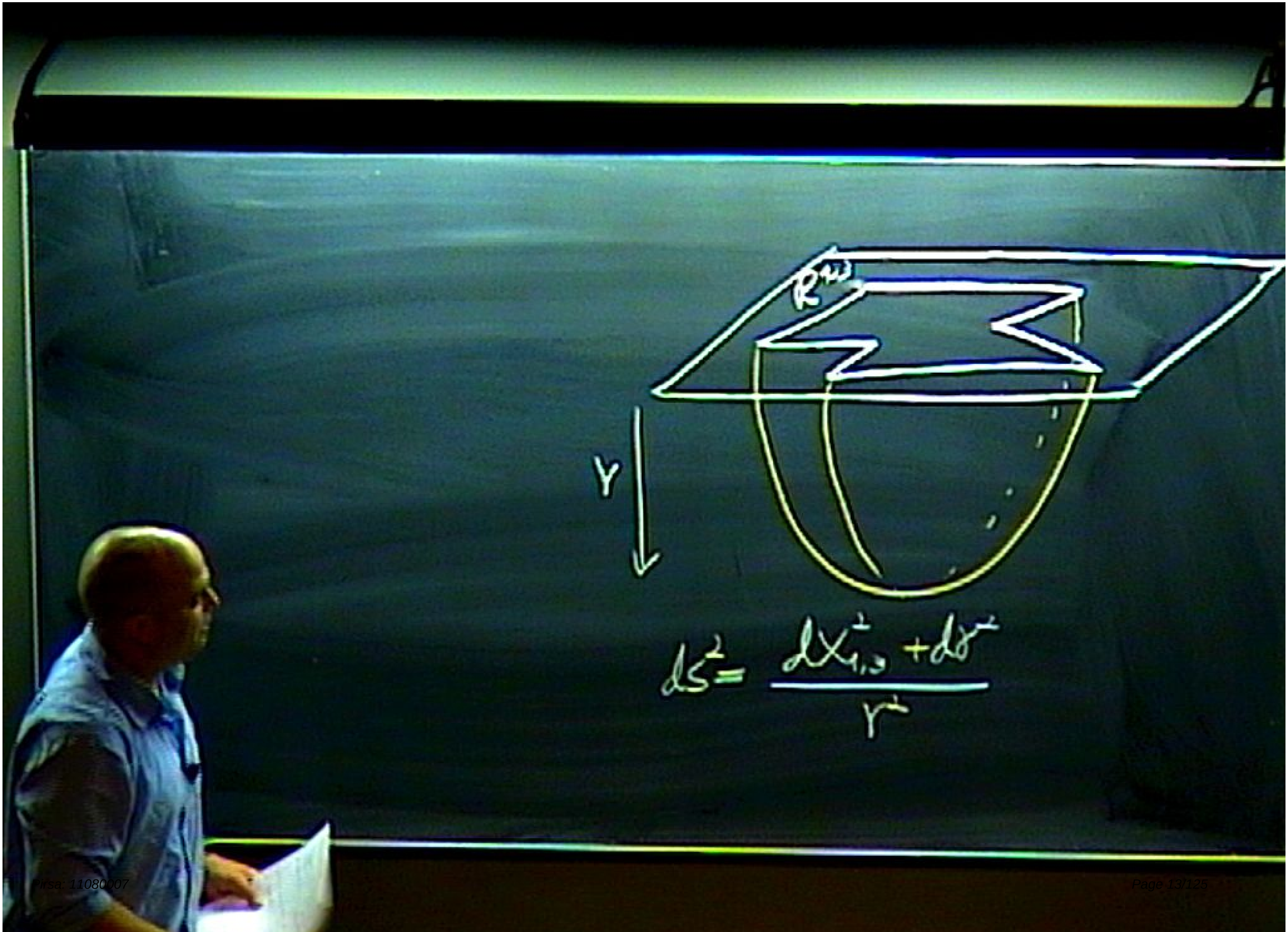
At strong coupling

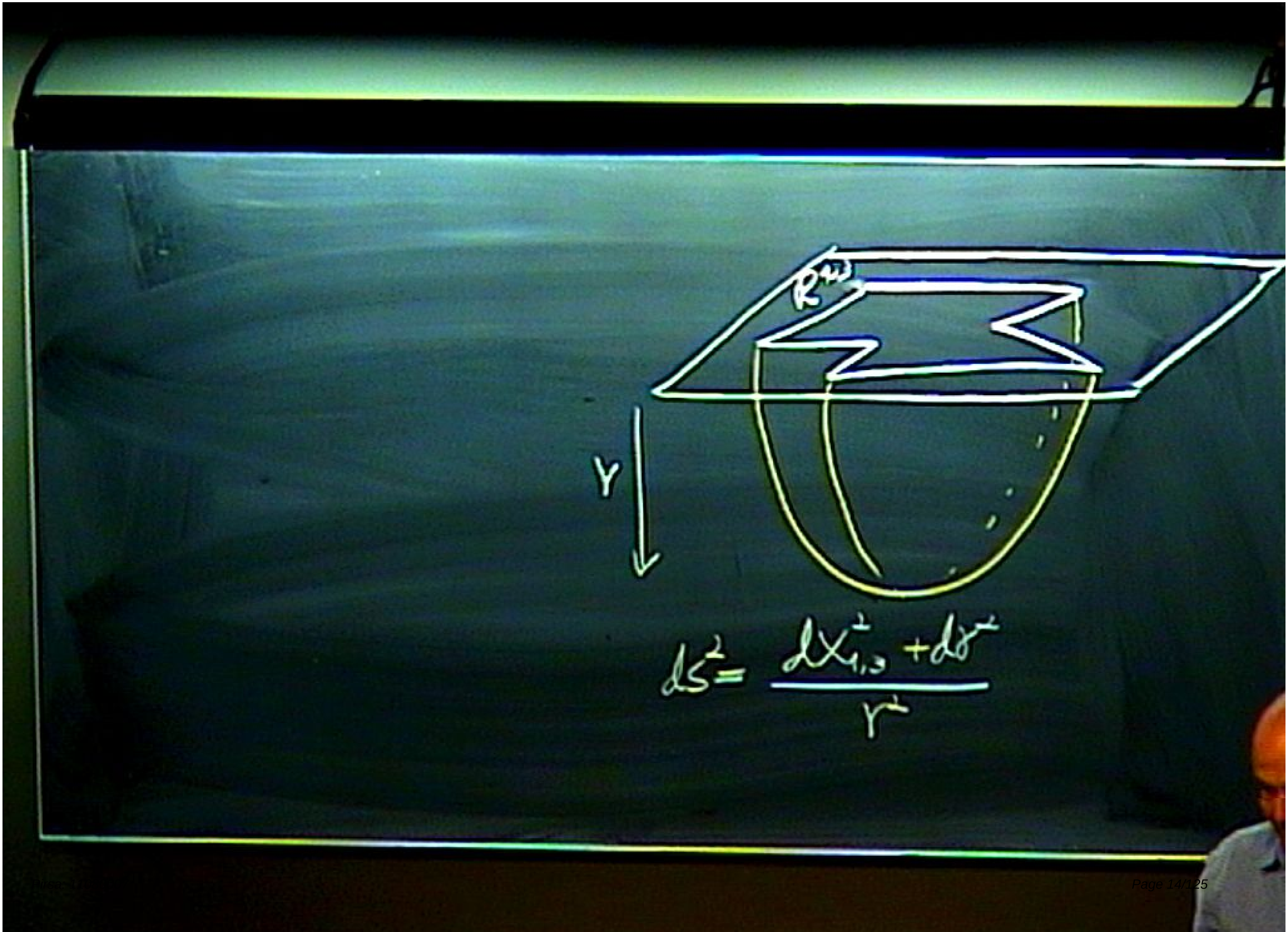
Y-system for scattering Amplitudes, the strong coupling story

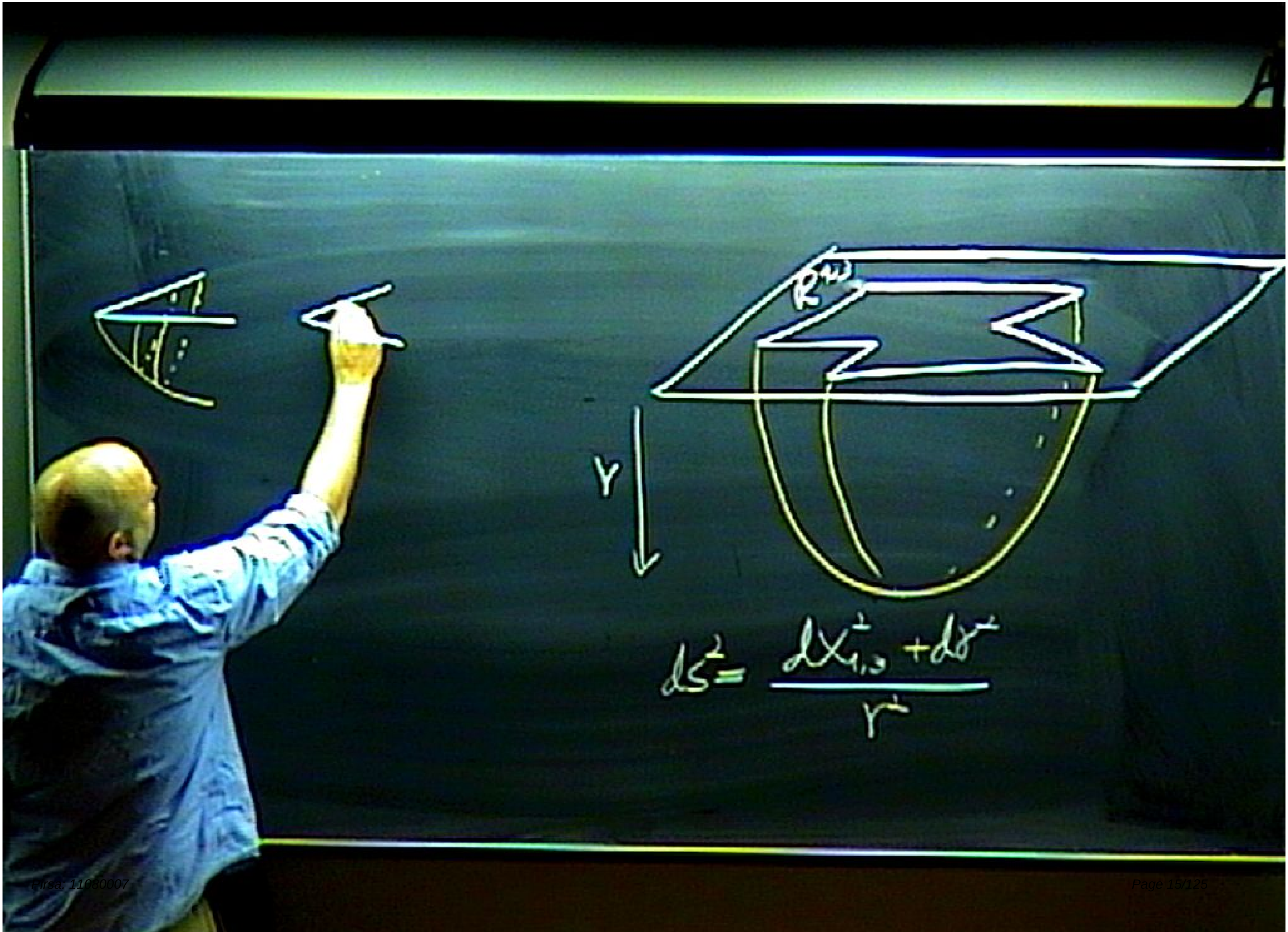
At strong coupling $\langle W \rangle = e^{-\sqrt{\lambda}\,\text{Area}}$

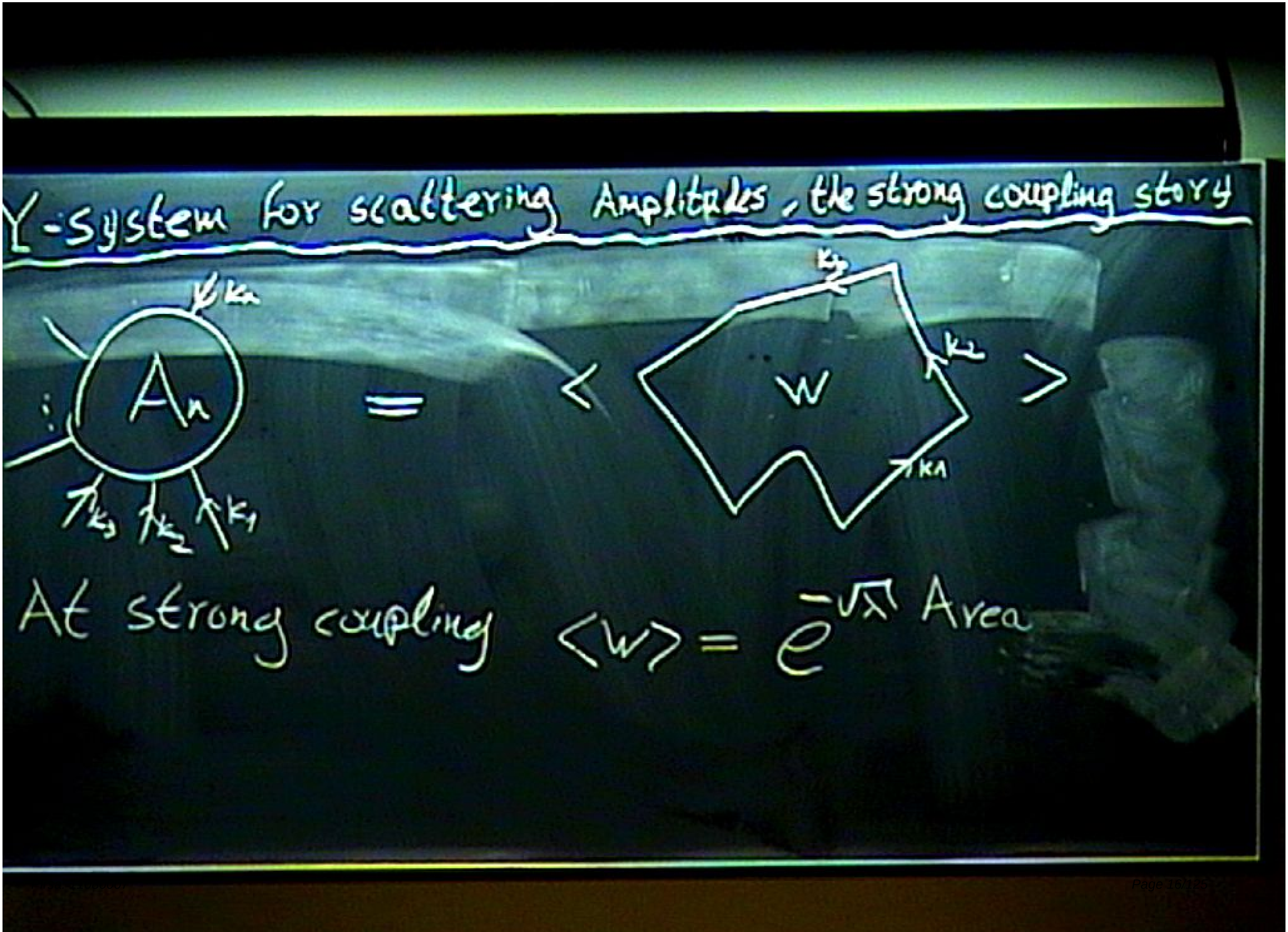

Y

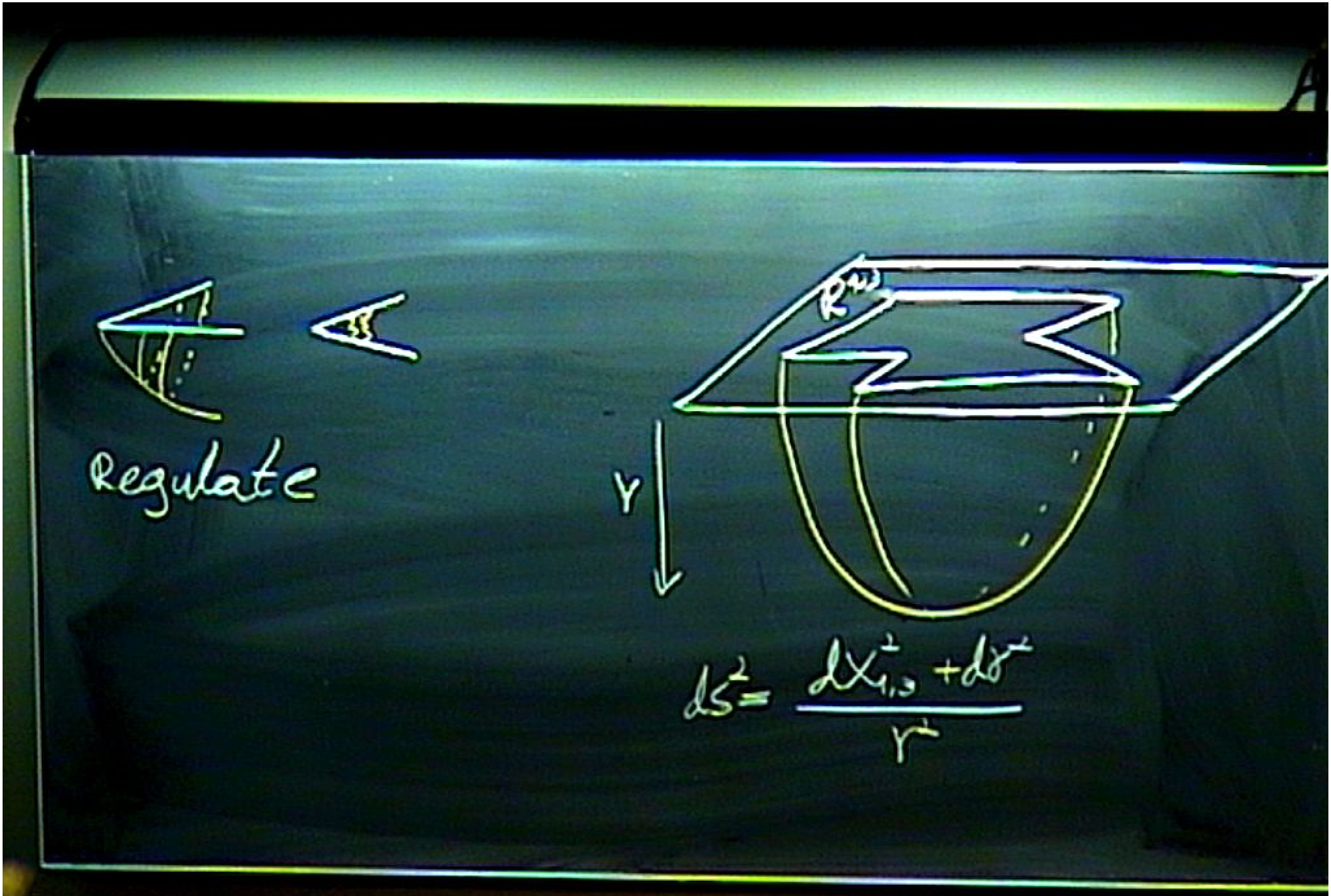

Y


R^{1,3}
Y
ds² = (dX²_{1,3} + dr²) / r²


Y
$ds^2 = \frac{dX_{1,3}^2 + dy^2}{y^2}$

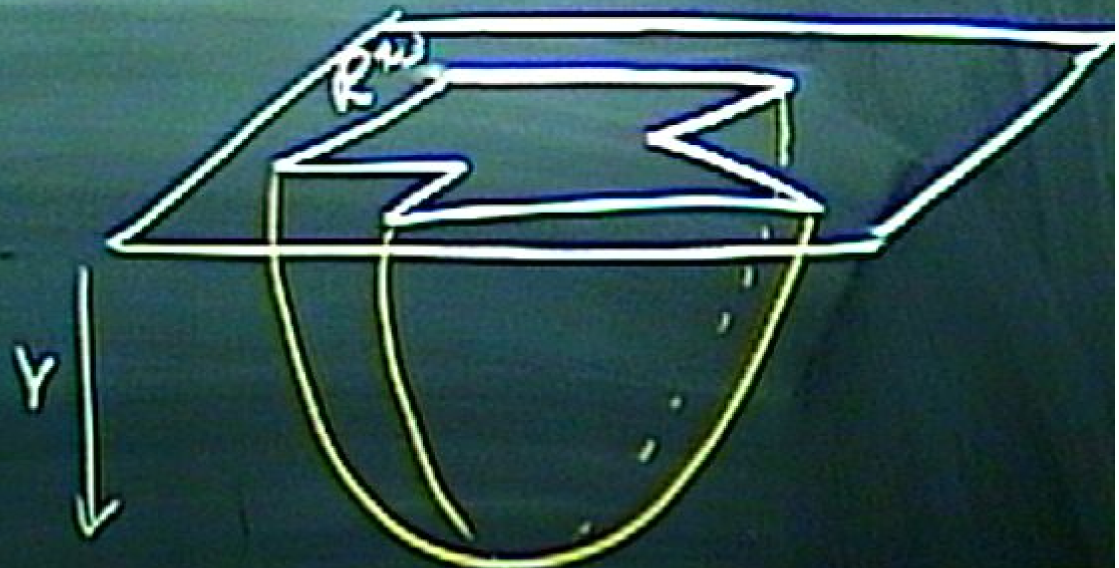

Y
$ds^2 = \frac{dX_{1,3}^2 + dY^2}{Y^2}$


Y-system for scattering Amplitudes, the strong coupling story
kn
An
k3
k2
k1
=
<
W
>
At strong coupling
<W> = e^{-√λ Area}

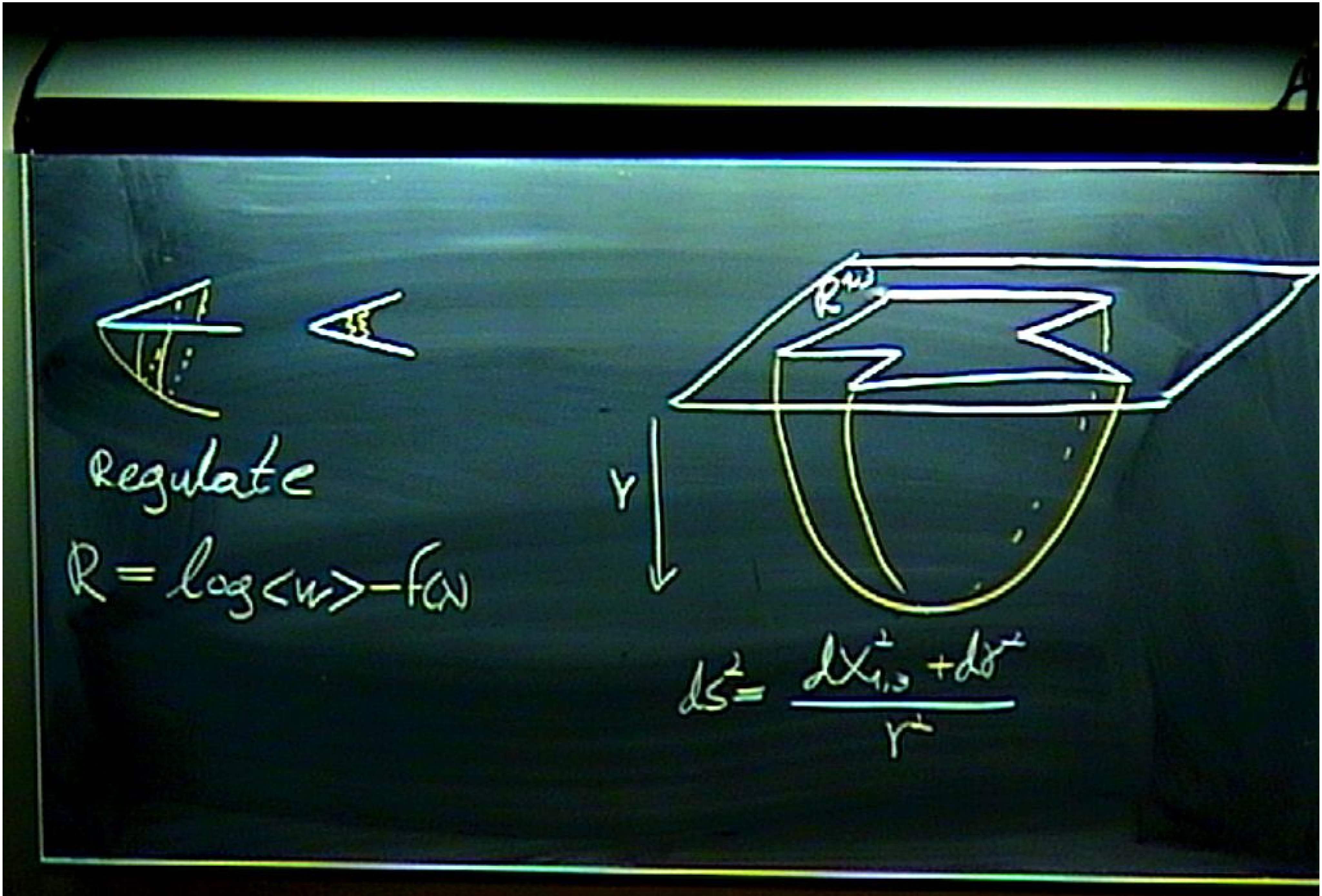

Regulate
r
$ds^2 = \frac{dx_{1,3}^2 + dr^2}{r^2}$

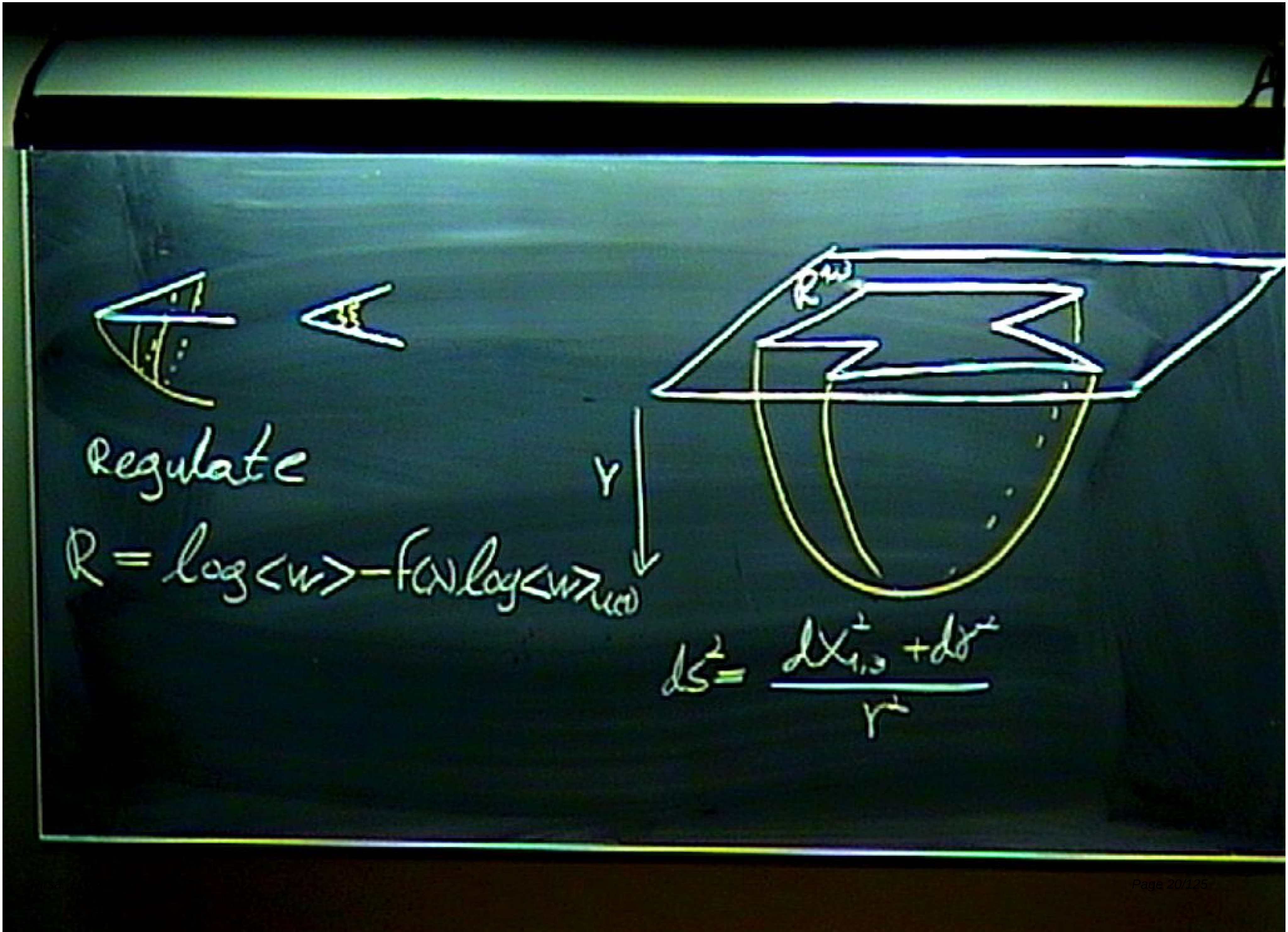
Regulate

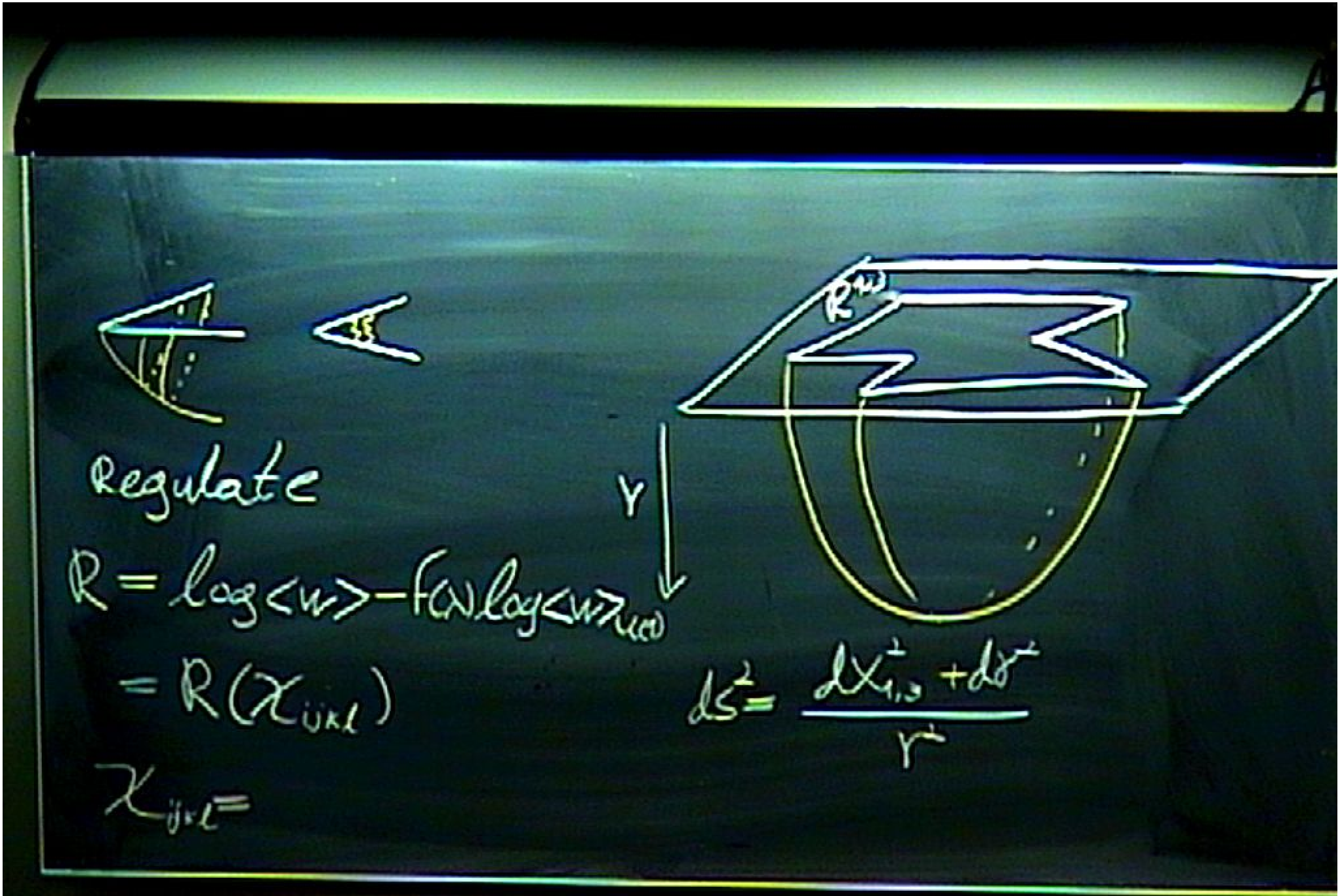
$R = \log\langle w \rangle$

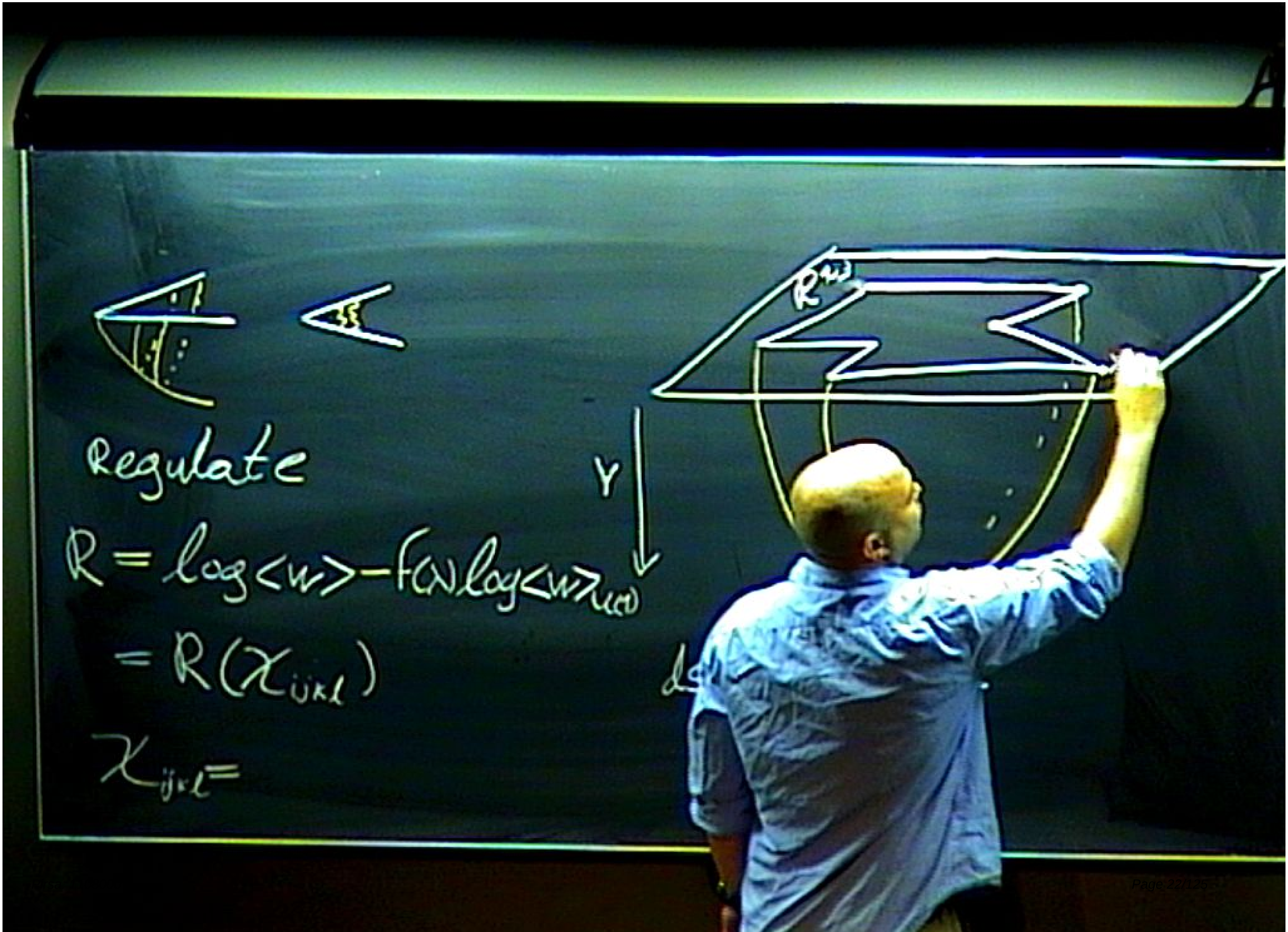


$$ds^2 = \frac{dX_{1,3}^2 + dr^2}{r^2}$$


Regulate
R = log<w> − f(N)
r
ds² = (dX²₁,₃ + dr²) / r²


Regulate
$R = \log\langle w\rangle - f(\lambda)\log\langle w\rangle_{U(1)}$
$R^{1,3}$
r
$ds^2 = \frac{dX_{1,3}^2 + dr^2}{r^2}$


Regulate
R = log<W> − f(N) log<W>_{UCD}
= R(X_{ijkl})
X_{ijkl} =
R^{(4)}
r
ds² = (dX²_{1,3} + dr²) / r²

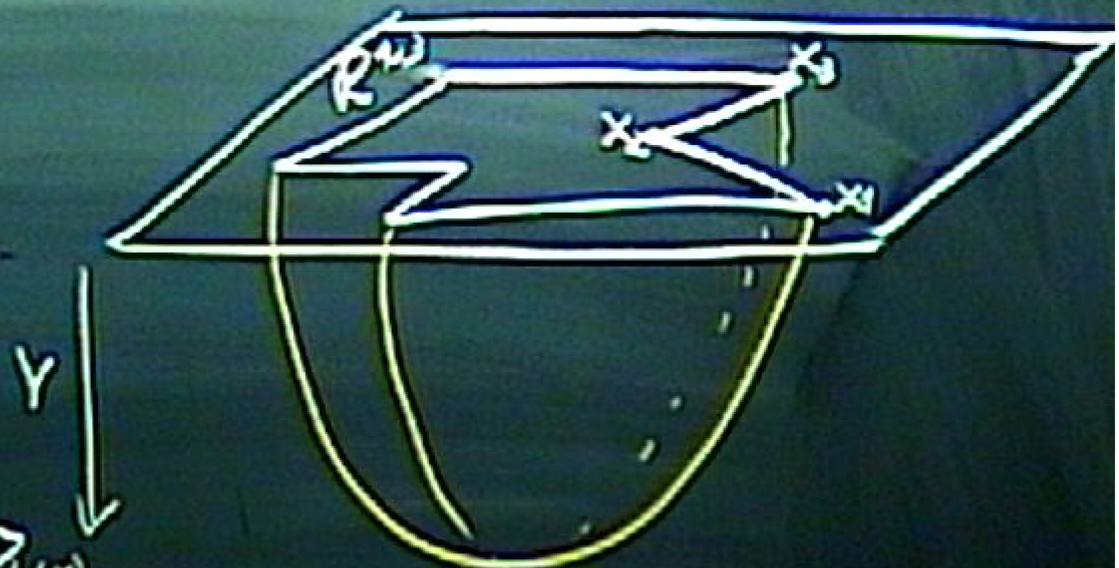

Regulate
R = log<w> − f(v) log<w>u(v)
= R(x ijkl)
x ijkl =
Y

Regulate

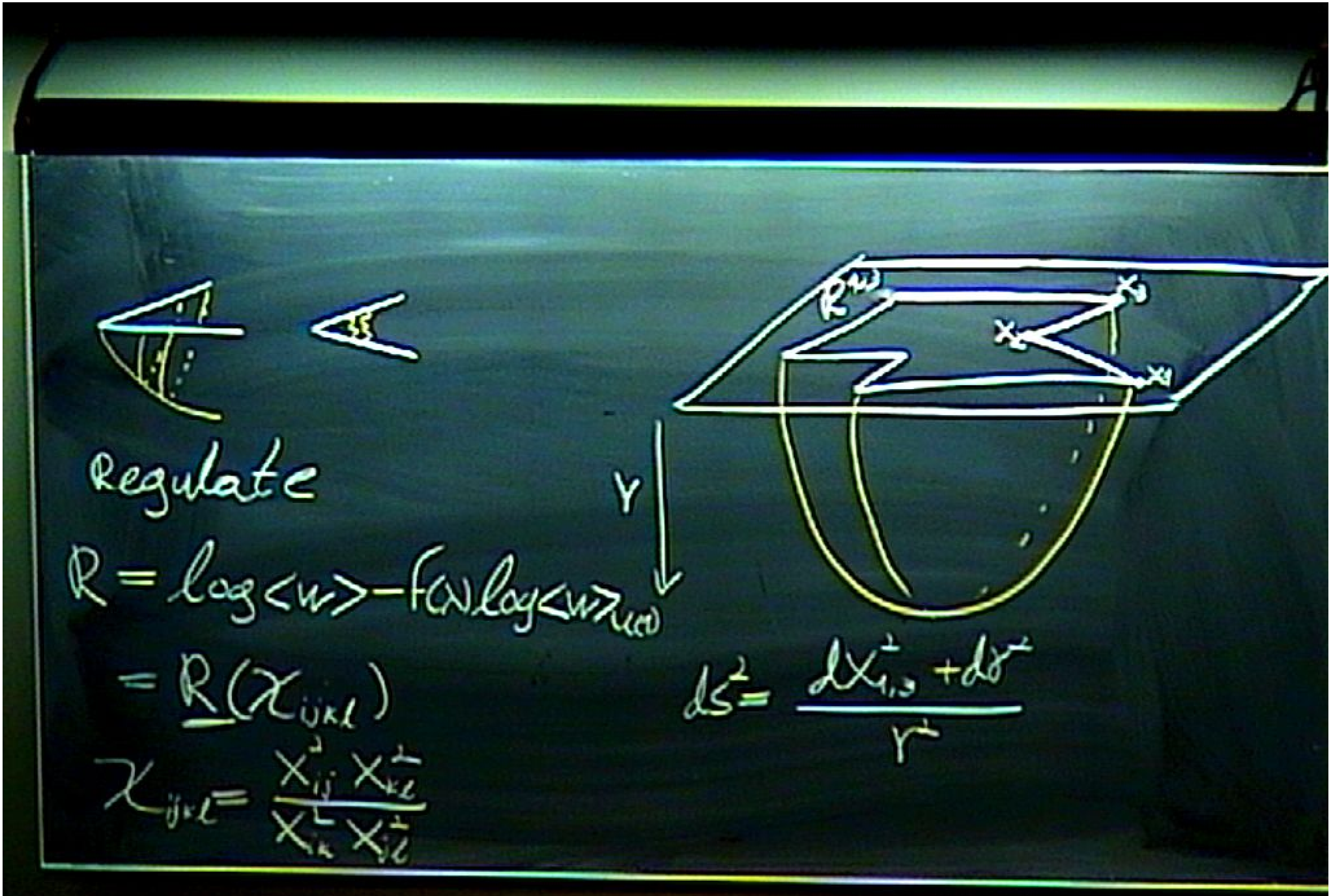
$$R = \log\langle W\rangle - f(N)\log\langle W\rangle_{U(1)}$$

$$= R(\chi_{ijkl})$$

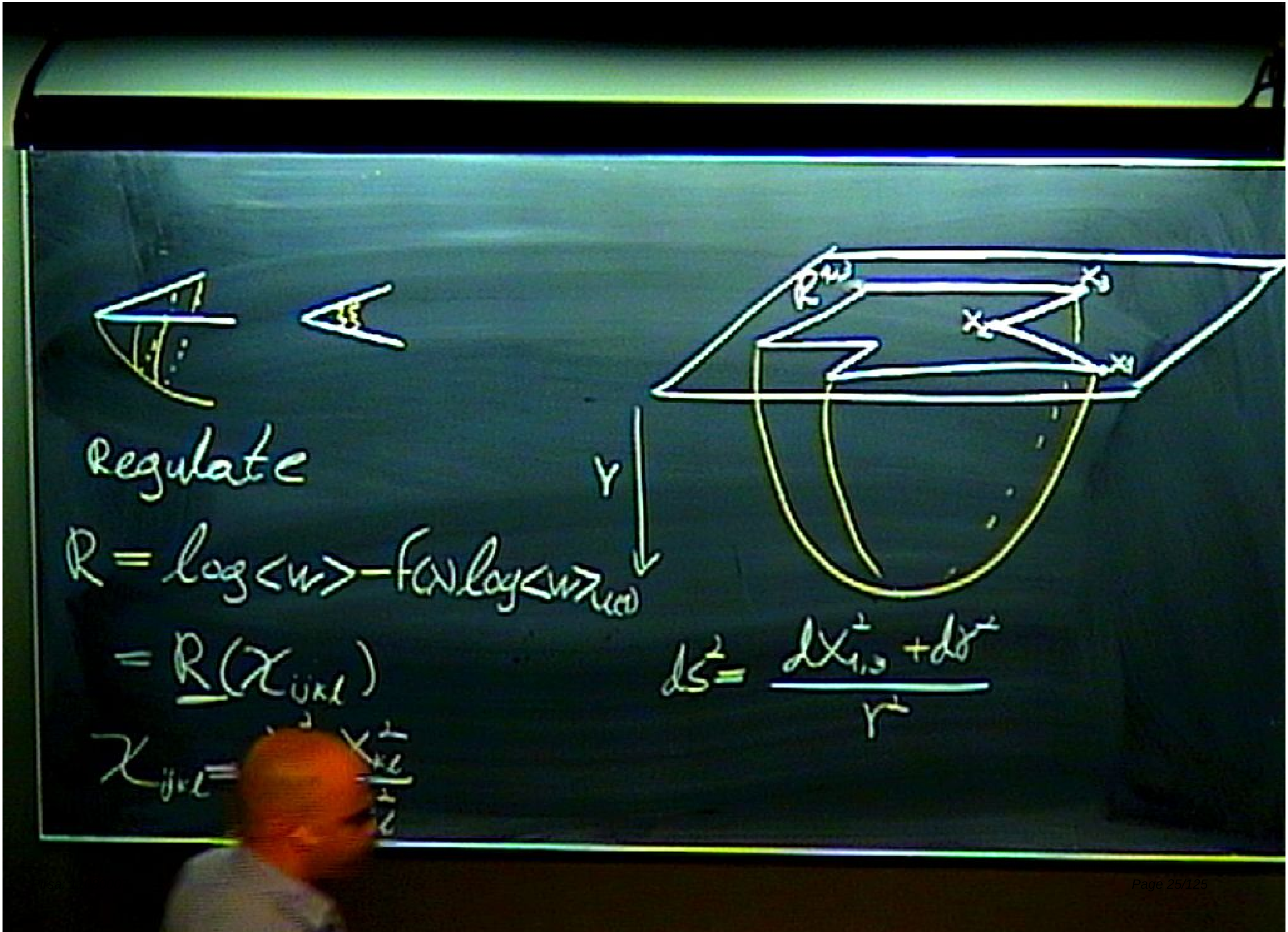
$$\chi_{ijkl} =$$

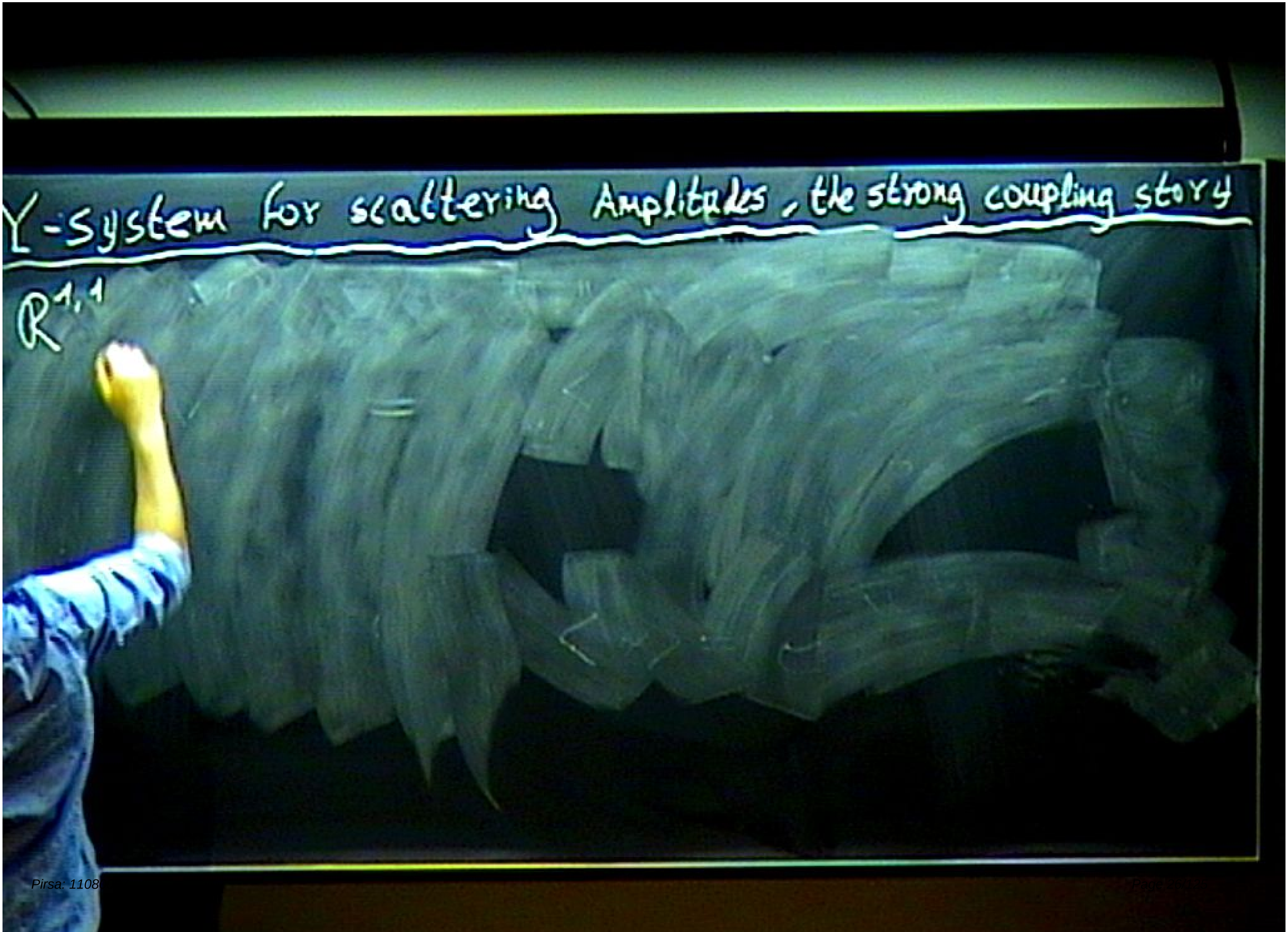


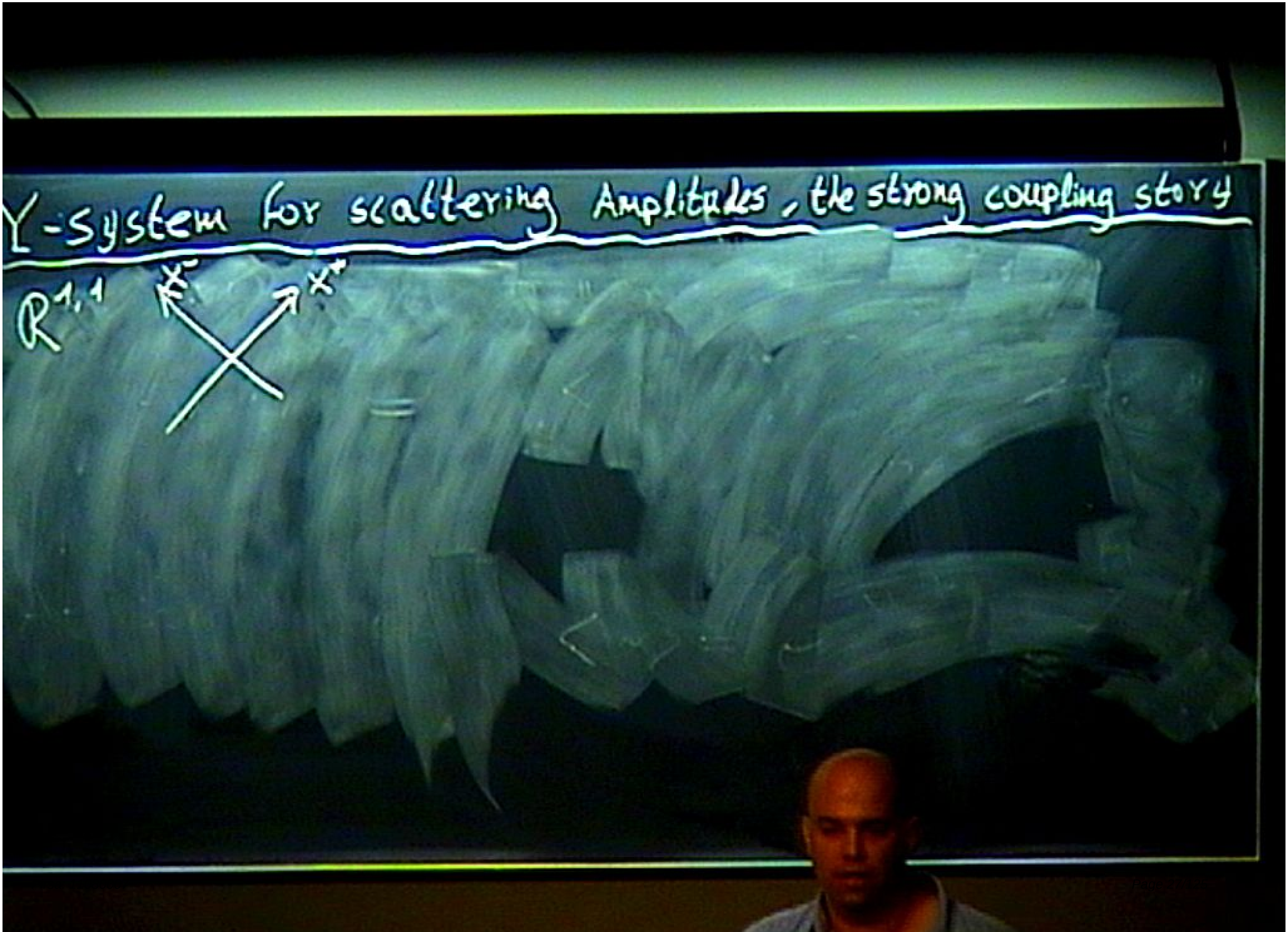
$$ds^2 = \frac{dx_{1,3}^2 + dr^2}{r^2}$$

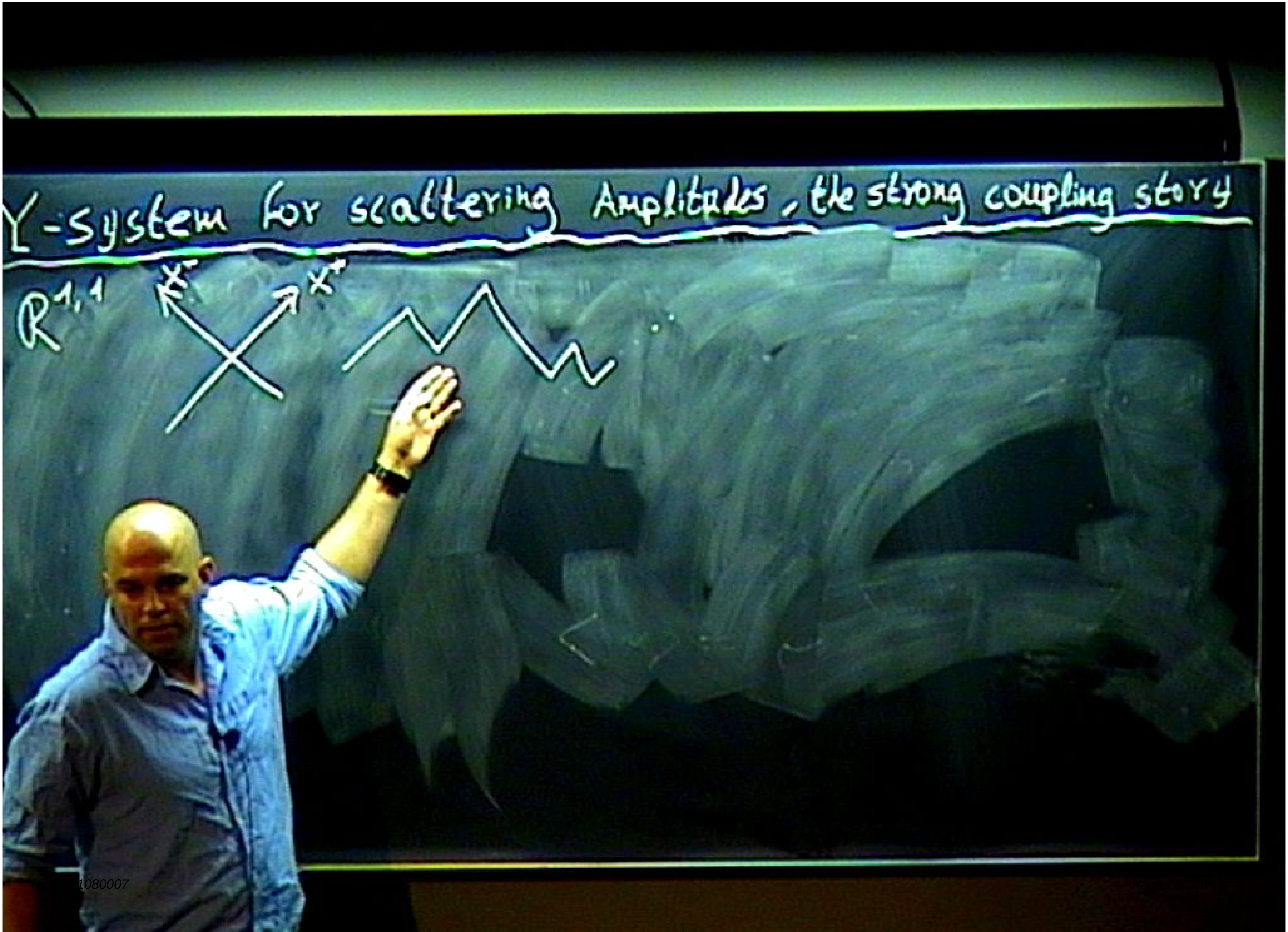


Regulate
R = log<w> − f(N) log<w>U(1)
= R(χijkl)
χijkl = X²ij X²kl / X²ik X²jl
ds² = (dX²1,3 + dr²) / r²
r

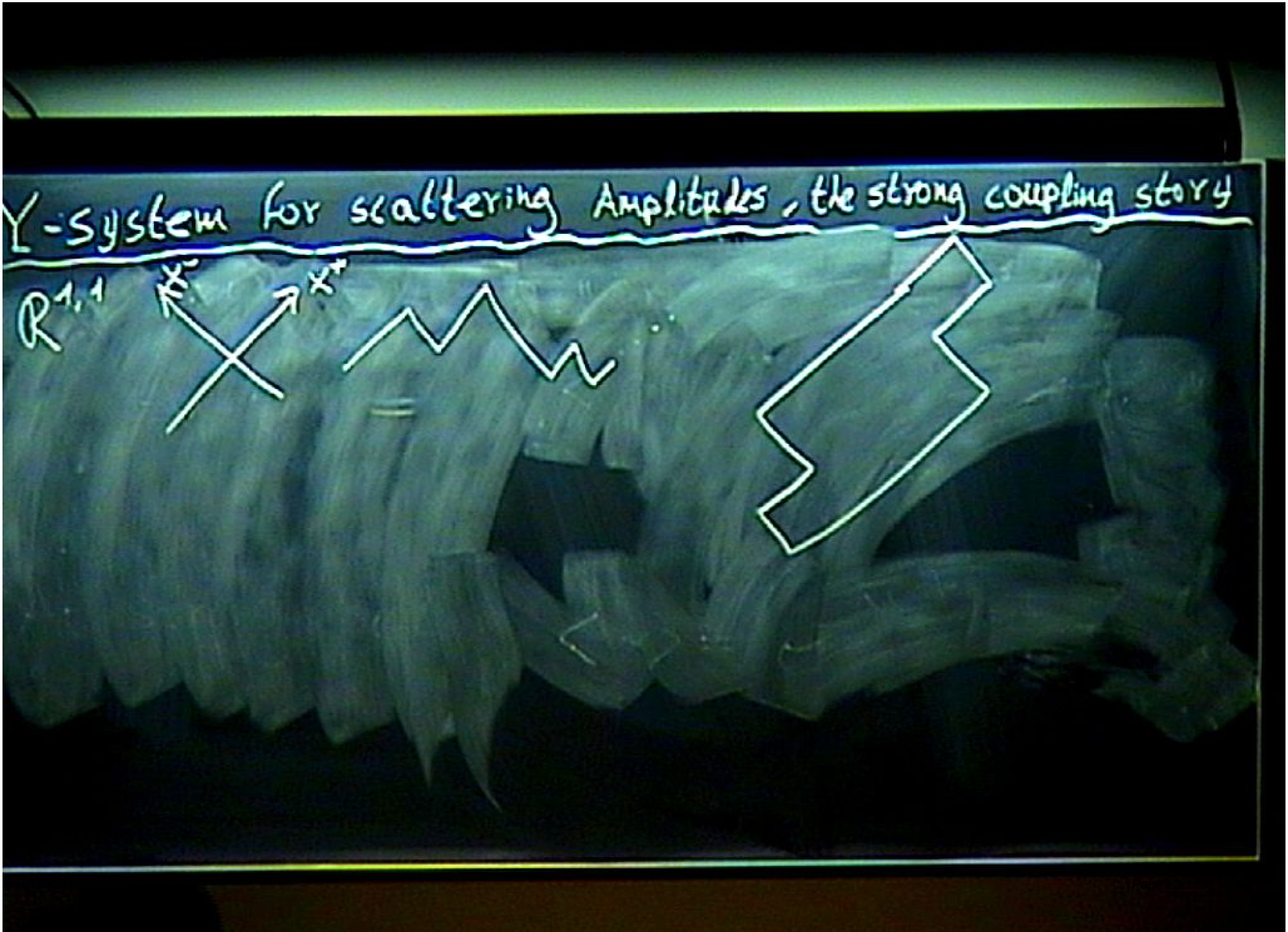

Regulate
R = log<w> − f(v) log<w>u(v)
= R(χijkl)
χijkl =
r
ds² = (dx²1,3 + dr²) / r²
R(4)

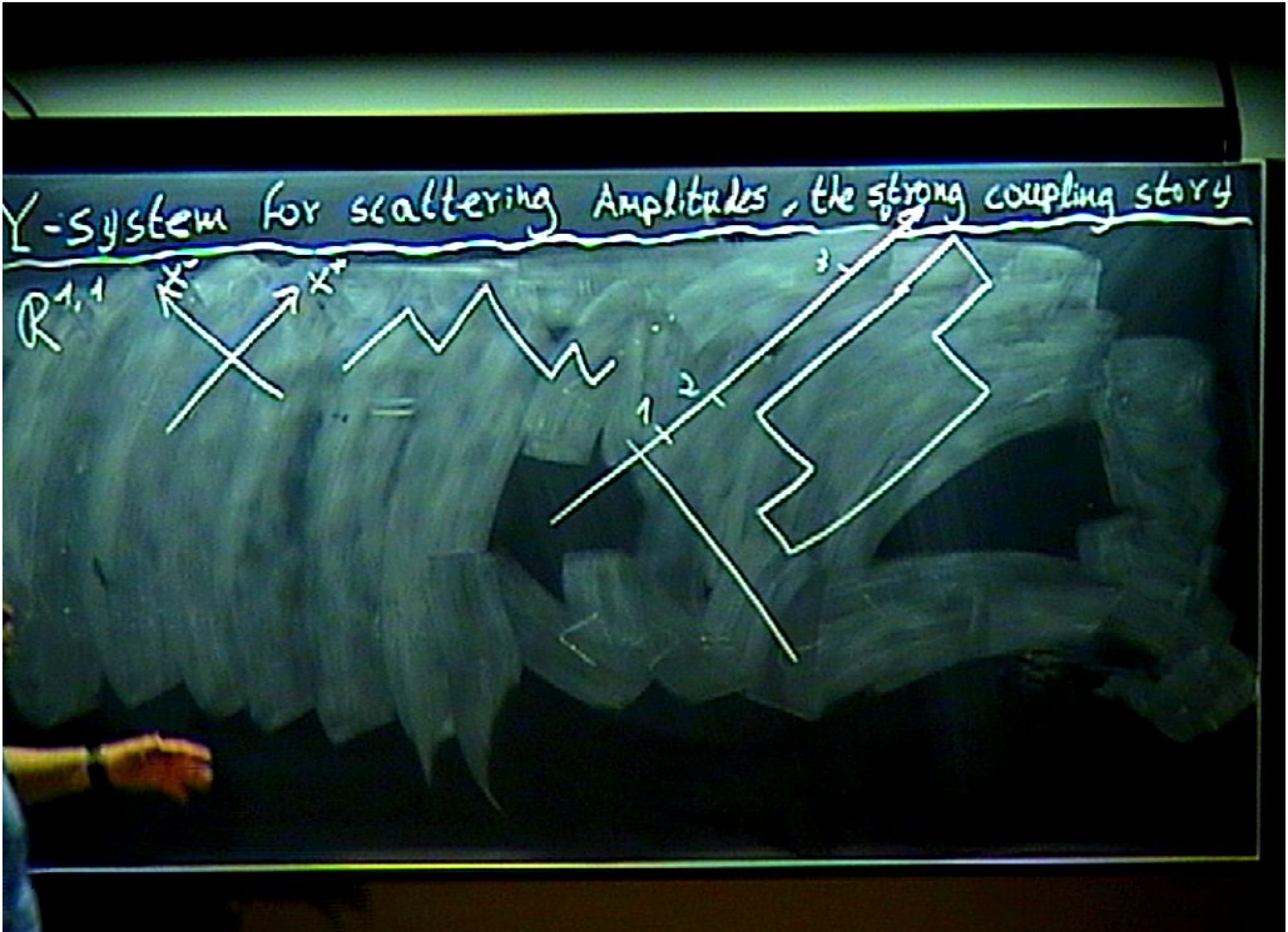

Y-system for scattering Amplitudes, the strong coupling story
$R^{1,1}$

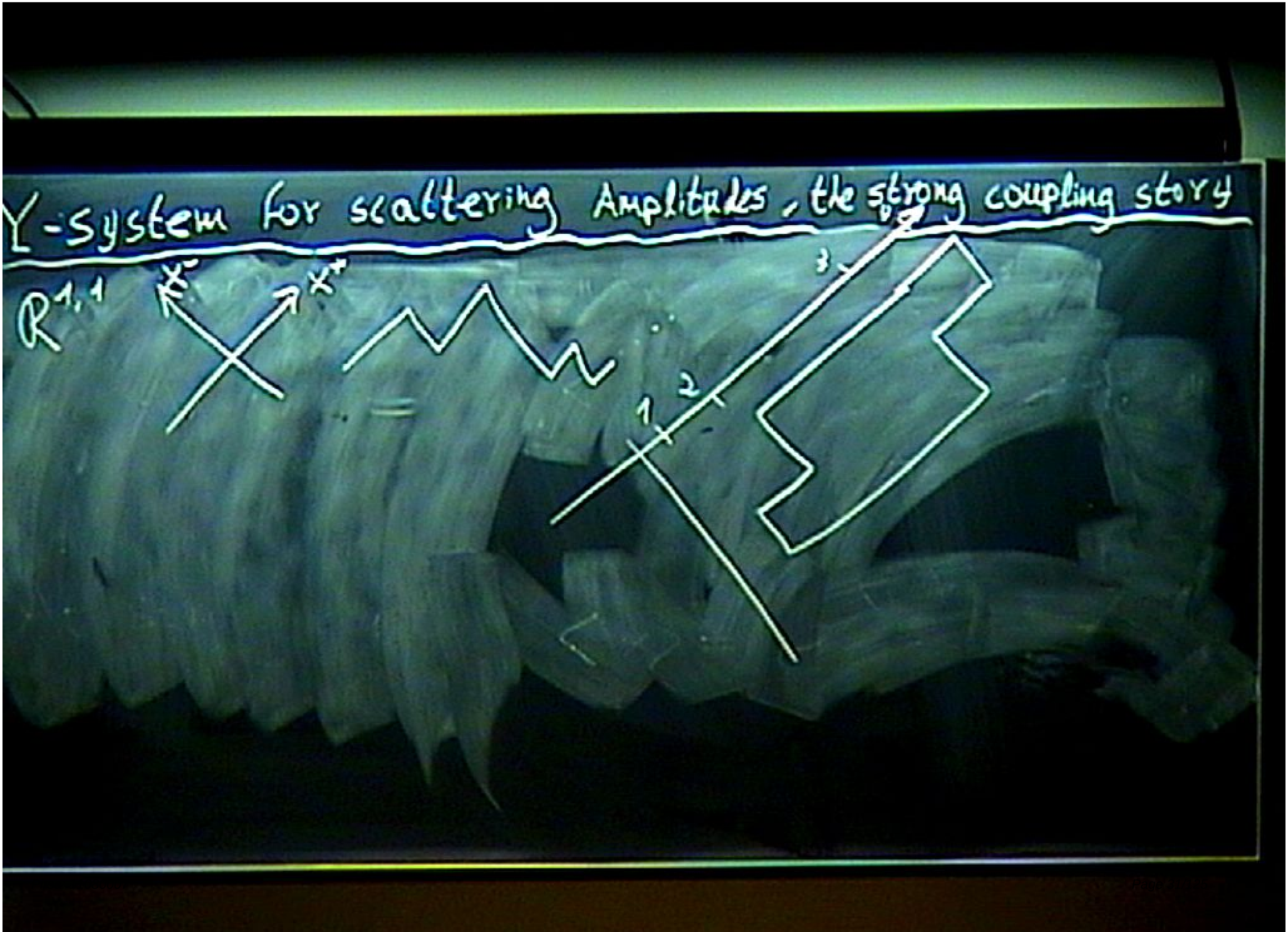

Y-system for scattering Amplitudes, the strong coupling story
R^{1,1}
x^-
x^+

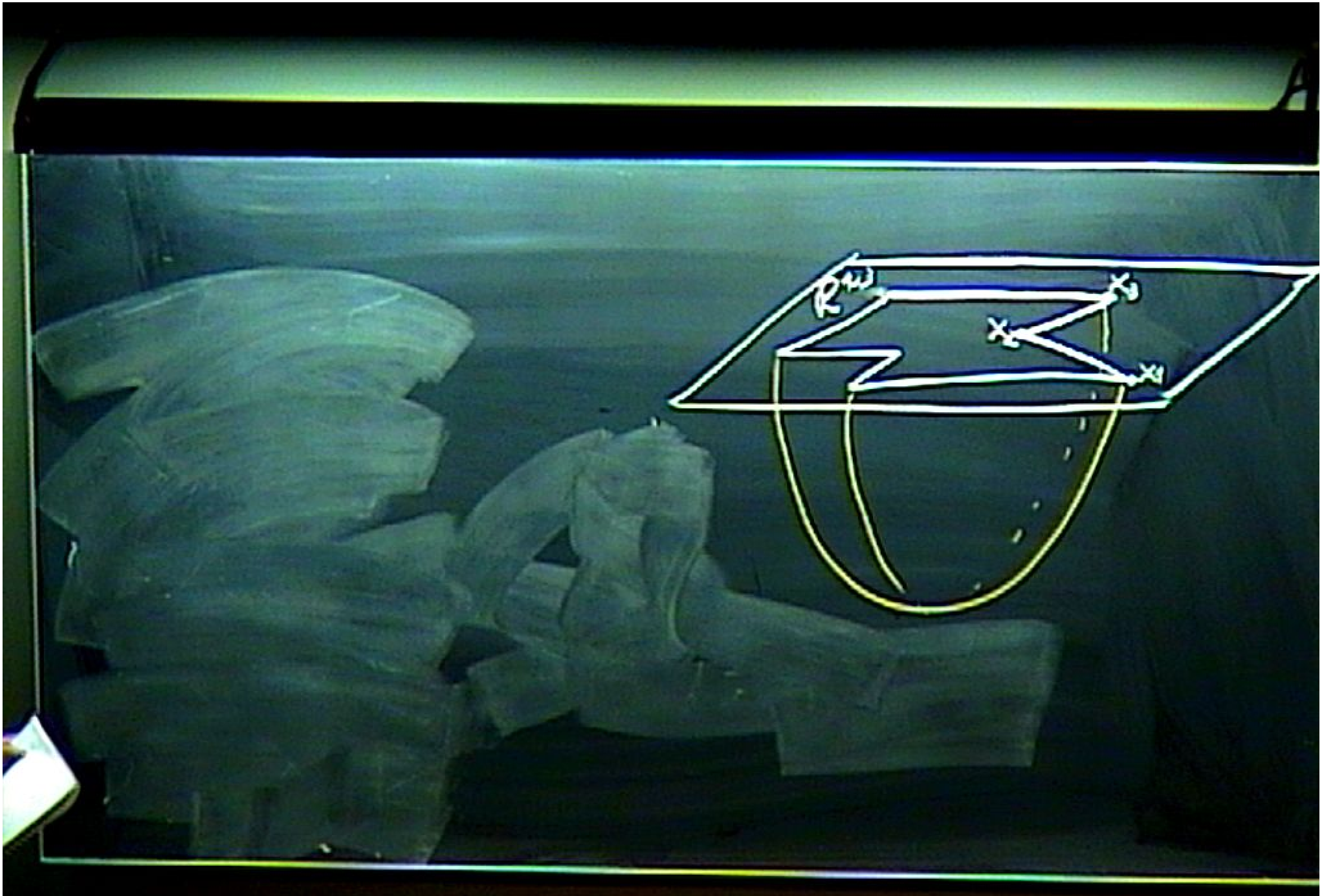

Y-system for scattering Amplitudes, the strong coupling story
R^{1,1}
x^-
x^+

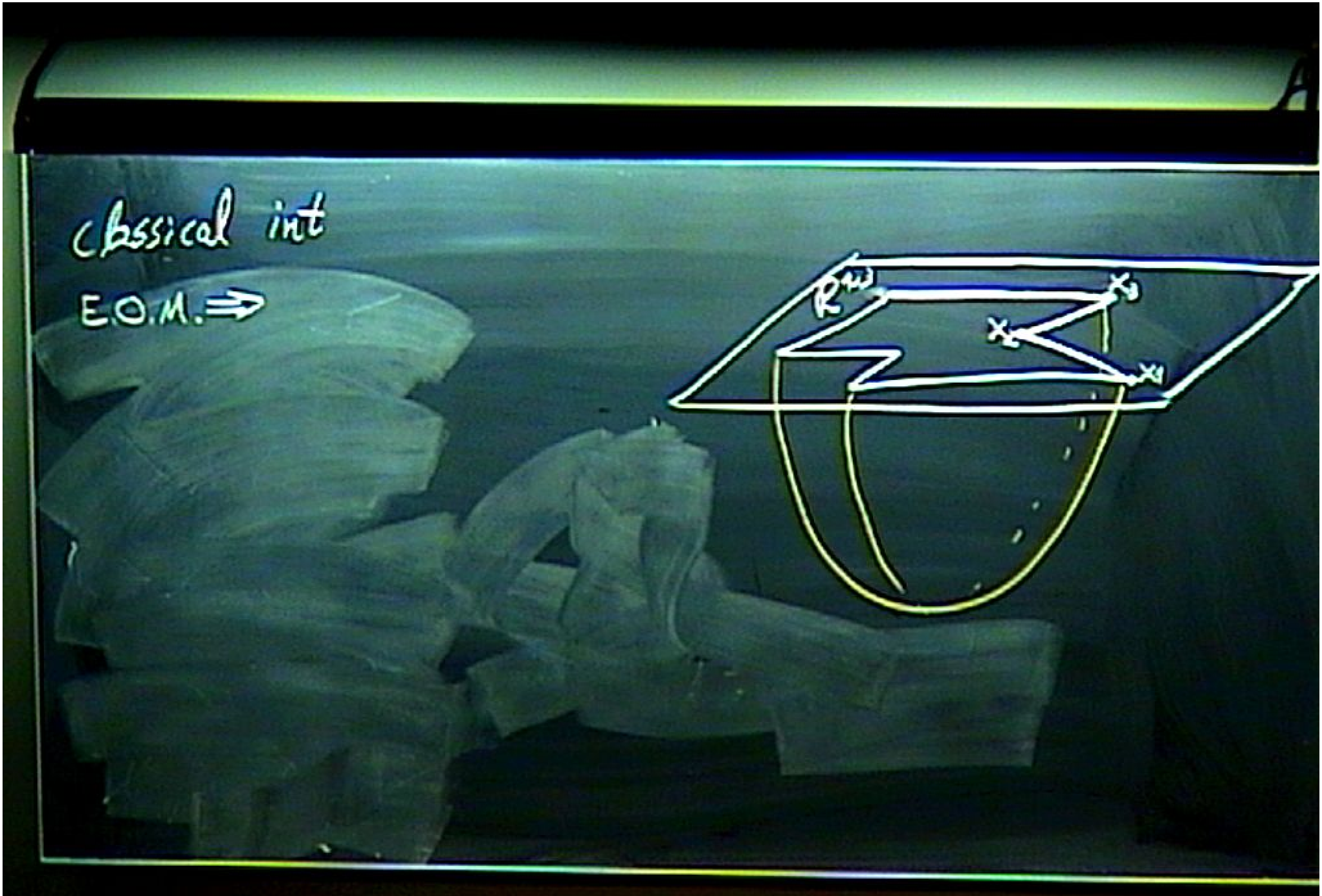
1080007

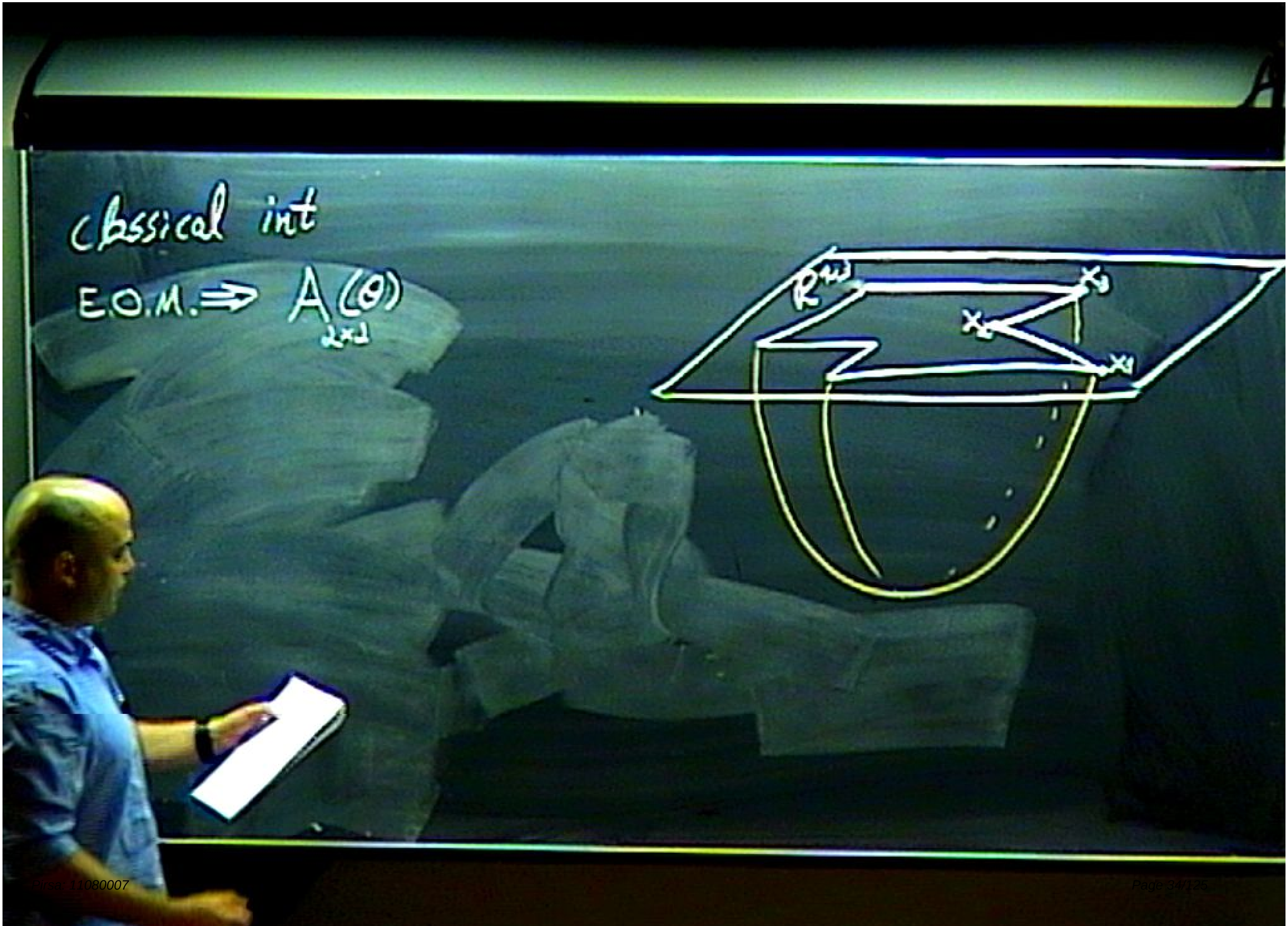

Y-System for scattering Amplitudes, the strong coupling story
R1,1
x-
x+

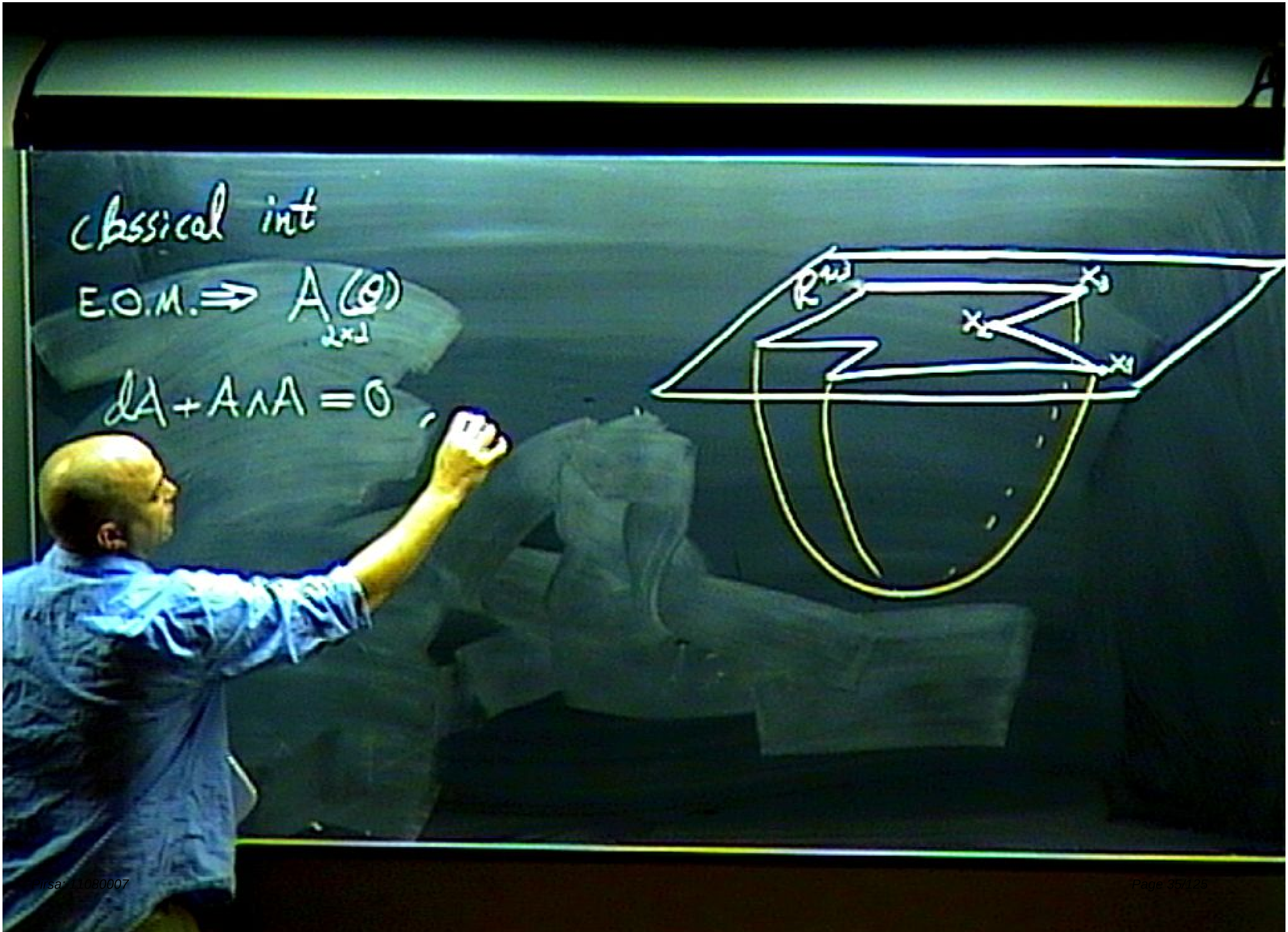

Y-system for scattering Amplitudes, the strong coupling story
R^{1,1}
x^-
x^+
1
2
3


Y-system for scattering Amplitudes, the strong coupling story
R1,1
x-
x+
1
2
3


R^{ω}
x_3
x_2
x_1


classical int
E.O.M. ⇒


classical int
E.O.M. ⇒ A(θ)
x1
x2
x3

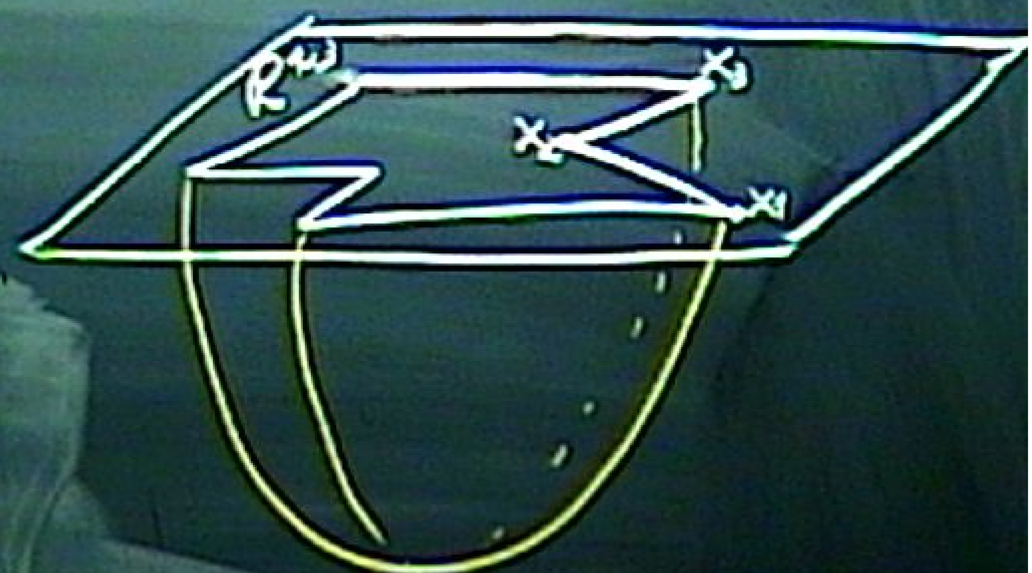


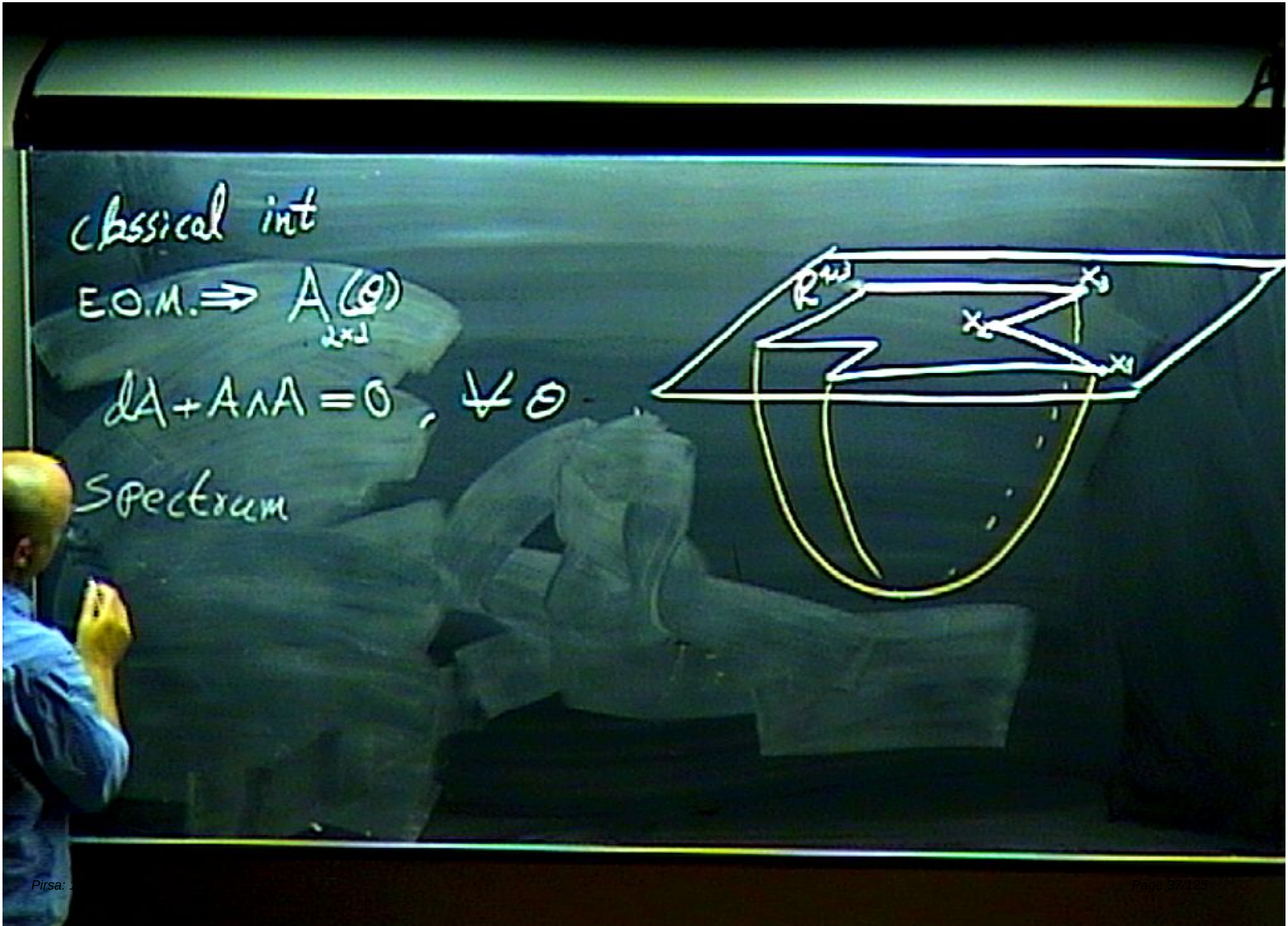
classical int
E.O.M. ⇒ A(θ)
dA + A∧A = 0
x1
x2
x3

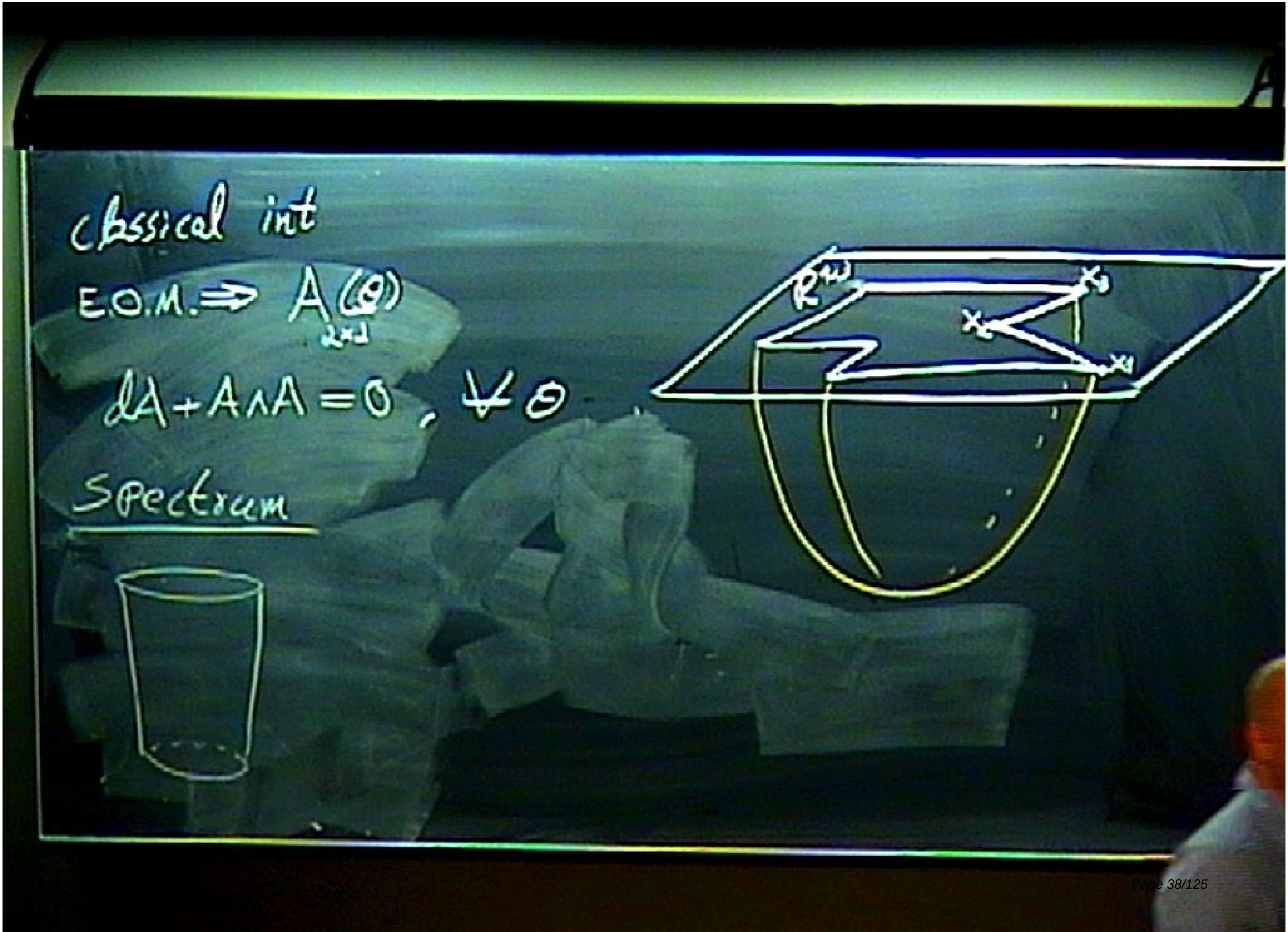
classical int

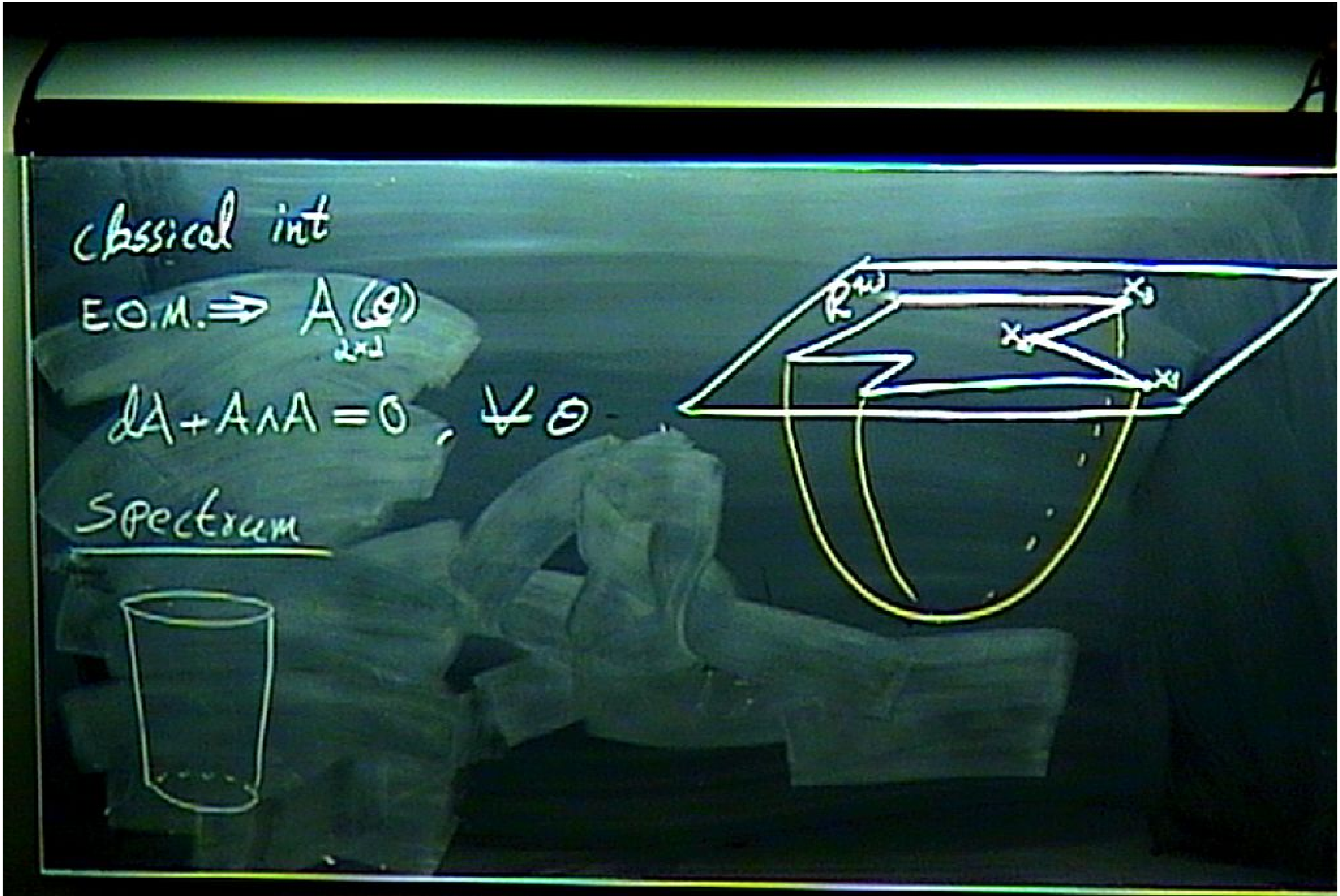
E.O.M. $\Rightarrow A_{2\times 2}(\theta)$

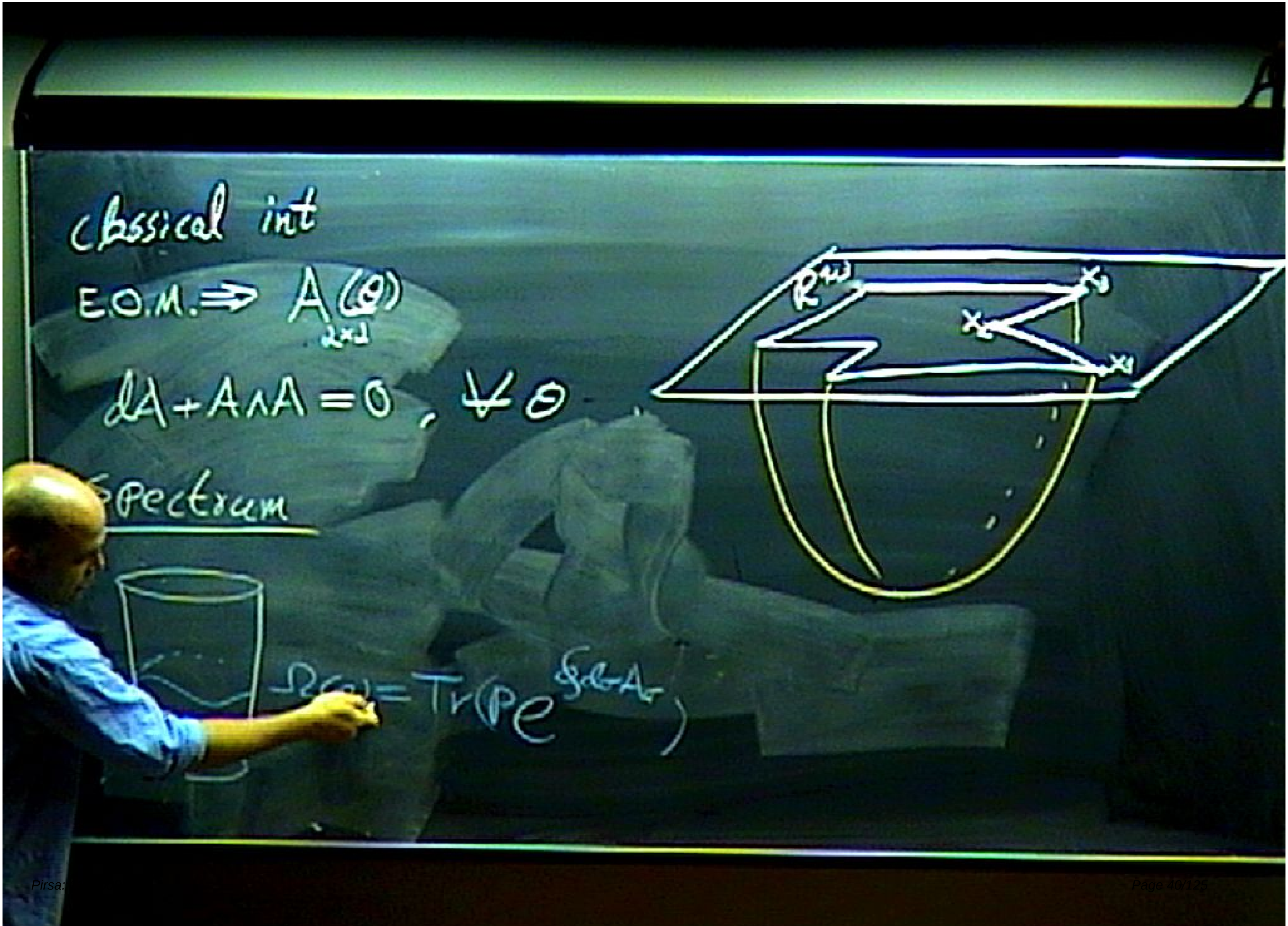
$dA + A \wedge A = 0 \,, \; \forall \theta$

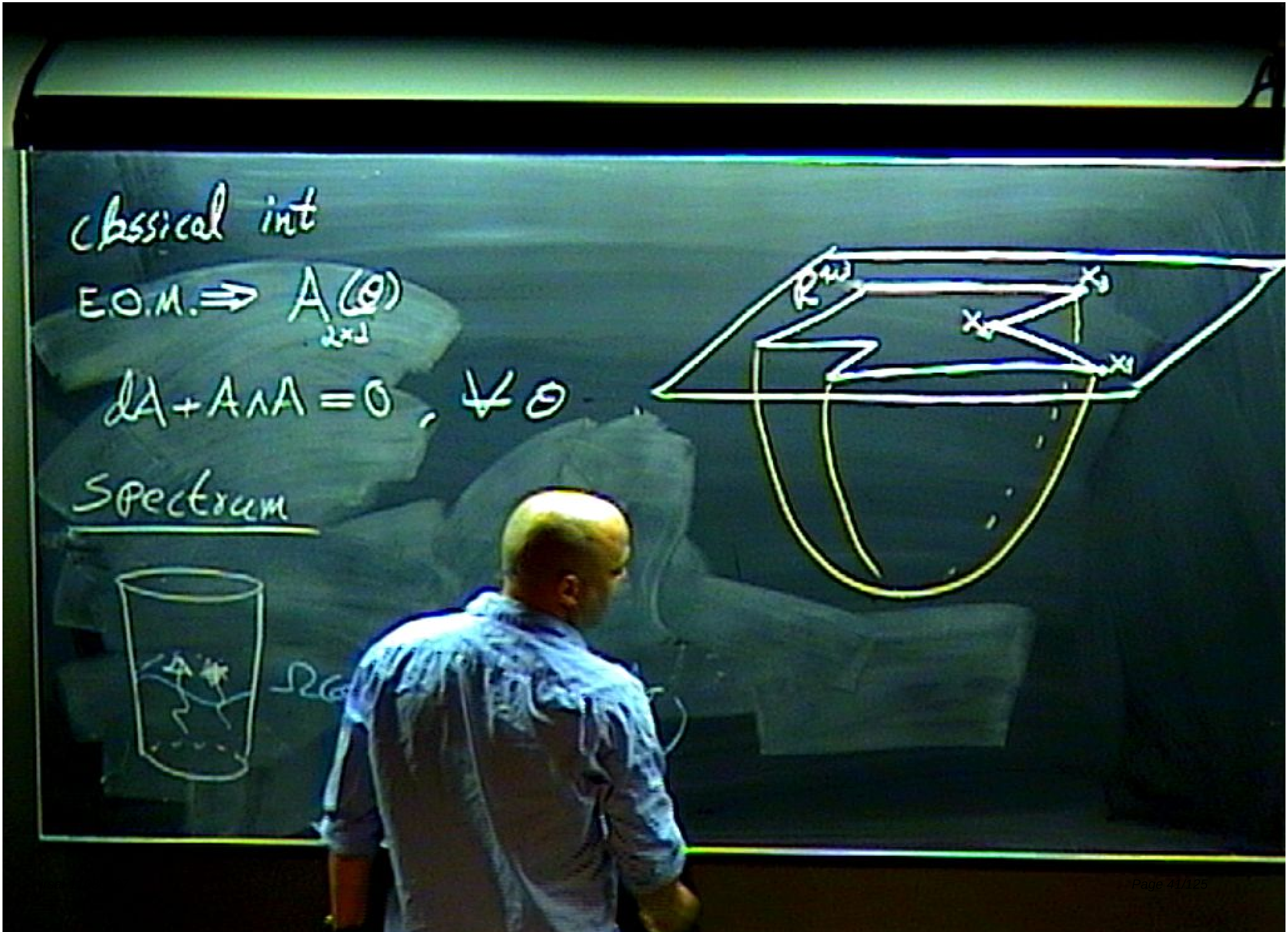


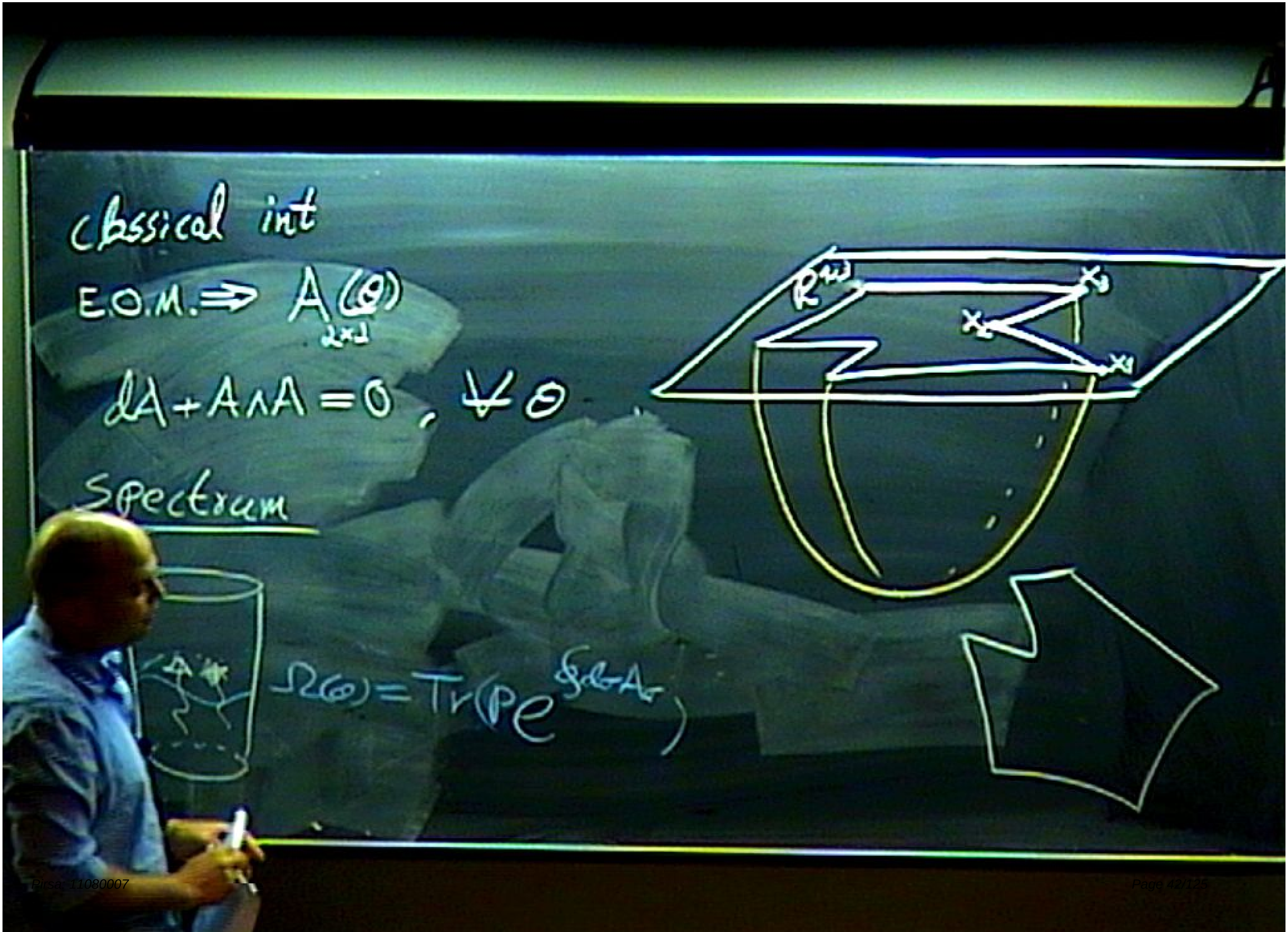

classical int
E.O.M. ⇒ A(θ)
2×2
dA + A∧A = 0 , ∀θ
Spectrum
R^{2,1}
x_1
x_2
x_3

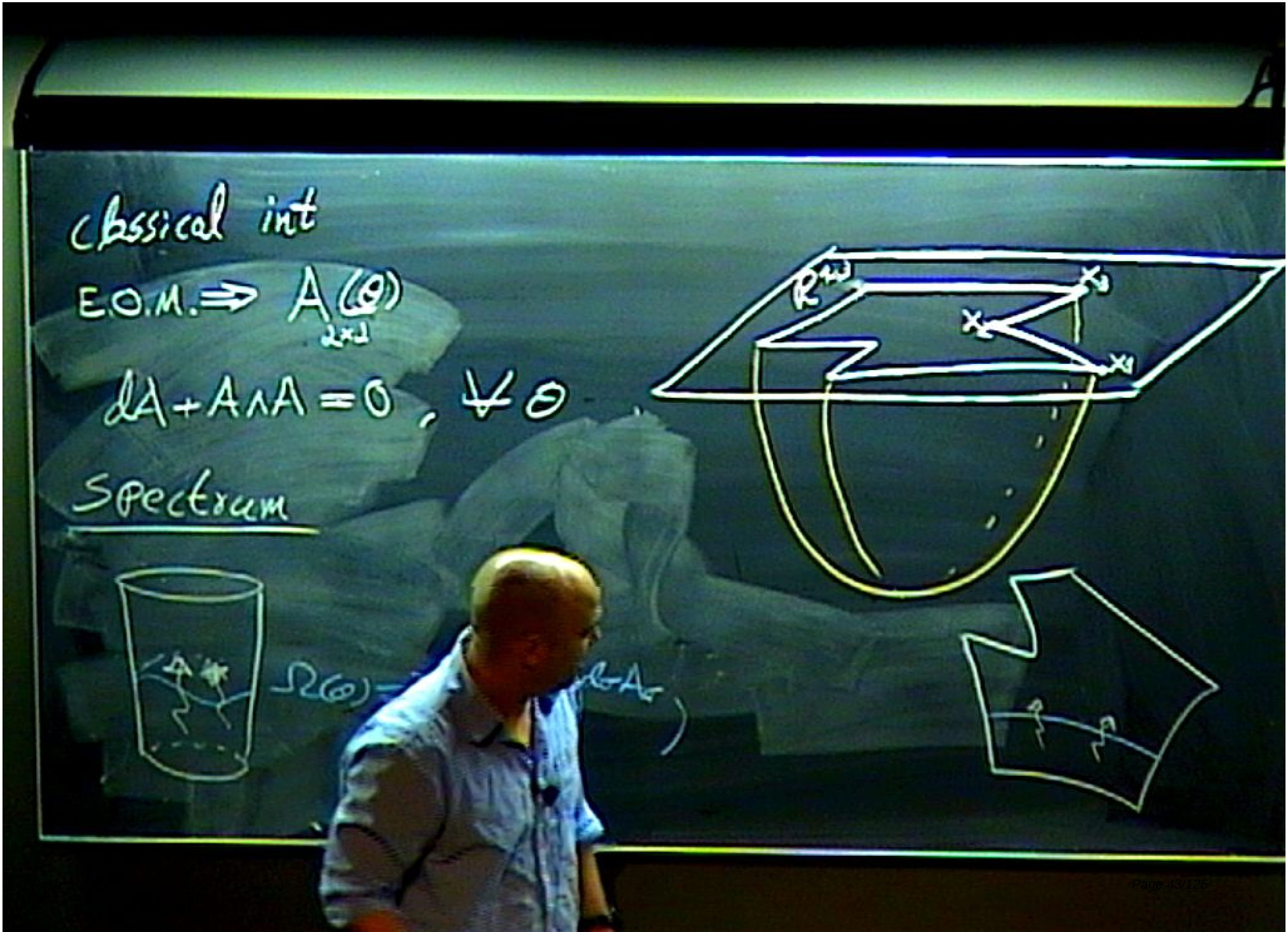

classical int
E.O.M. ⇒ A(θ)
dA + A∧A = 0 , ∀θ
Spectrum

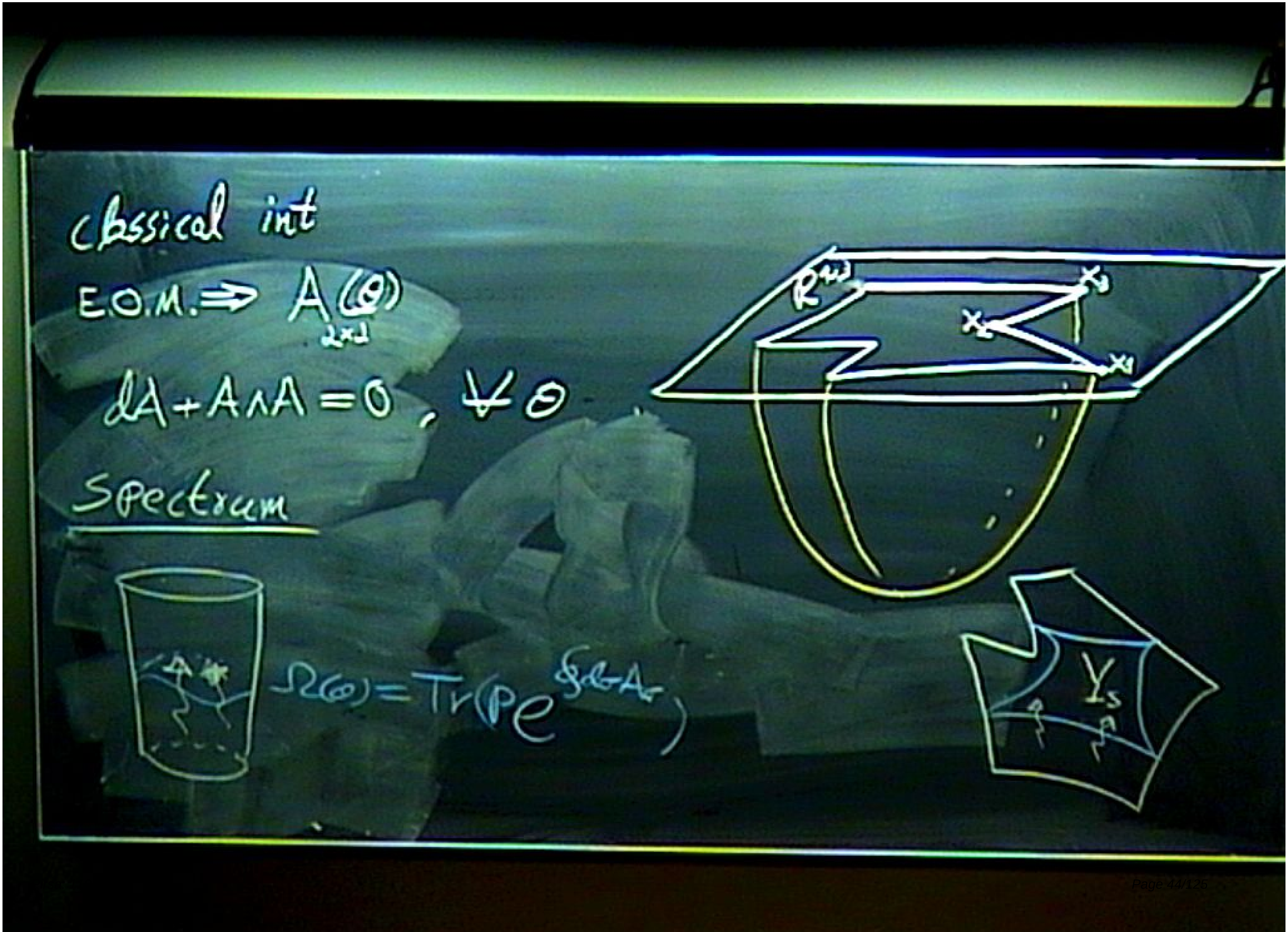

classical int
E.O.M. ⇒ A(θ)
dA + A∧A = 0 , ∀θ
Spectrum

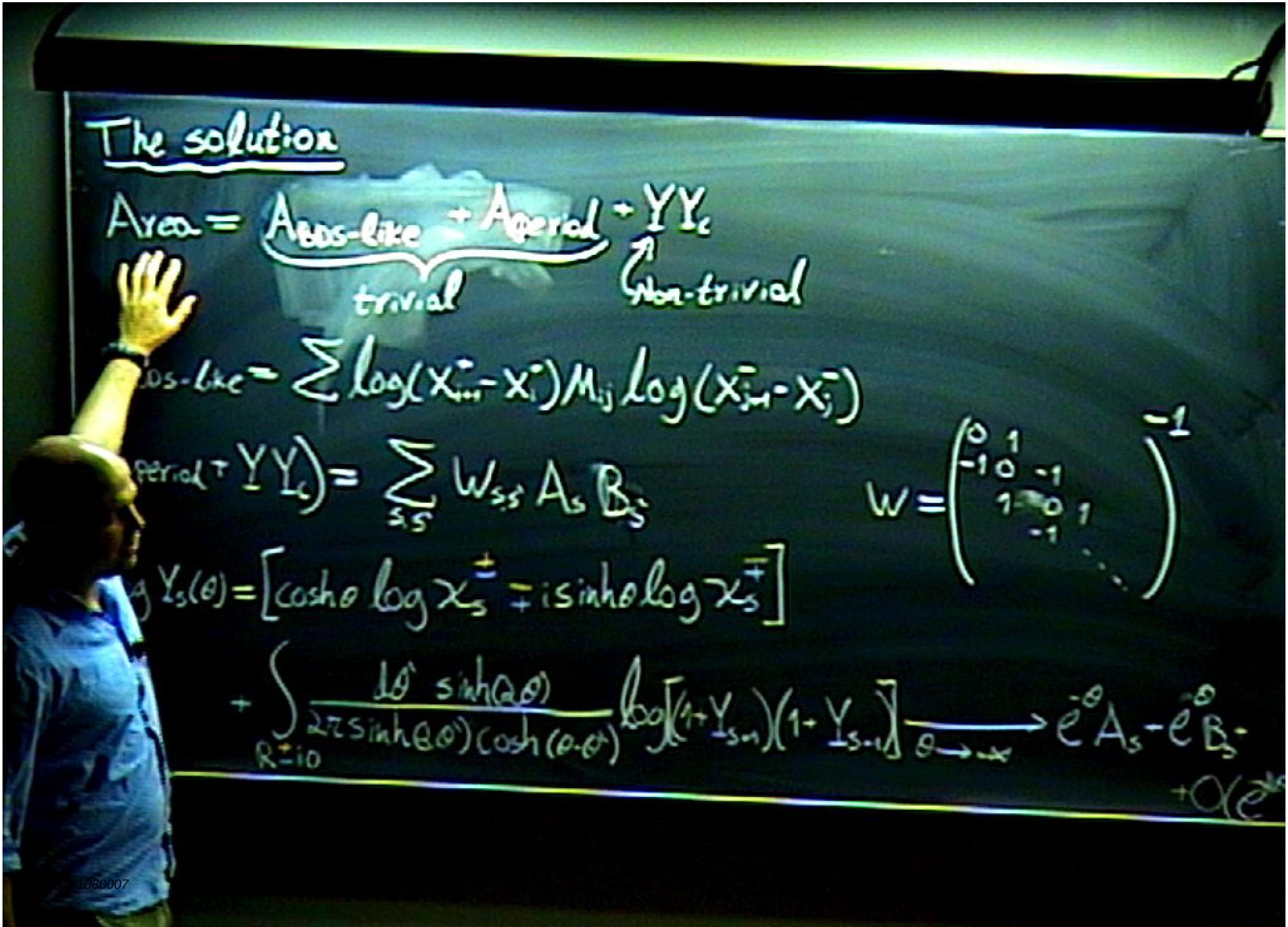

classical int
E.O.M. ⇒ A(θ)
dA + A∧A = 0, ∀θ
Spectrum
Ω(θ) = Tr(P e^{∮dσ A_σ})

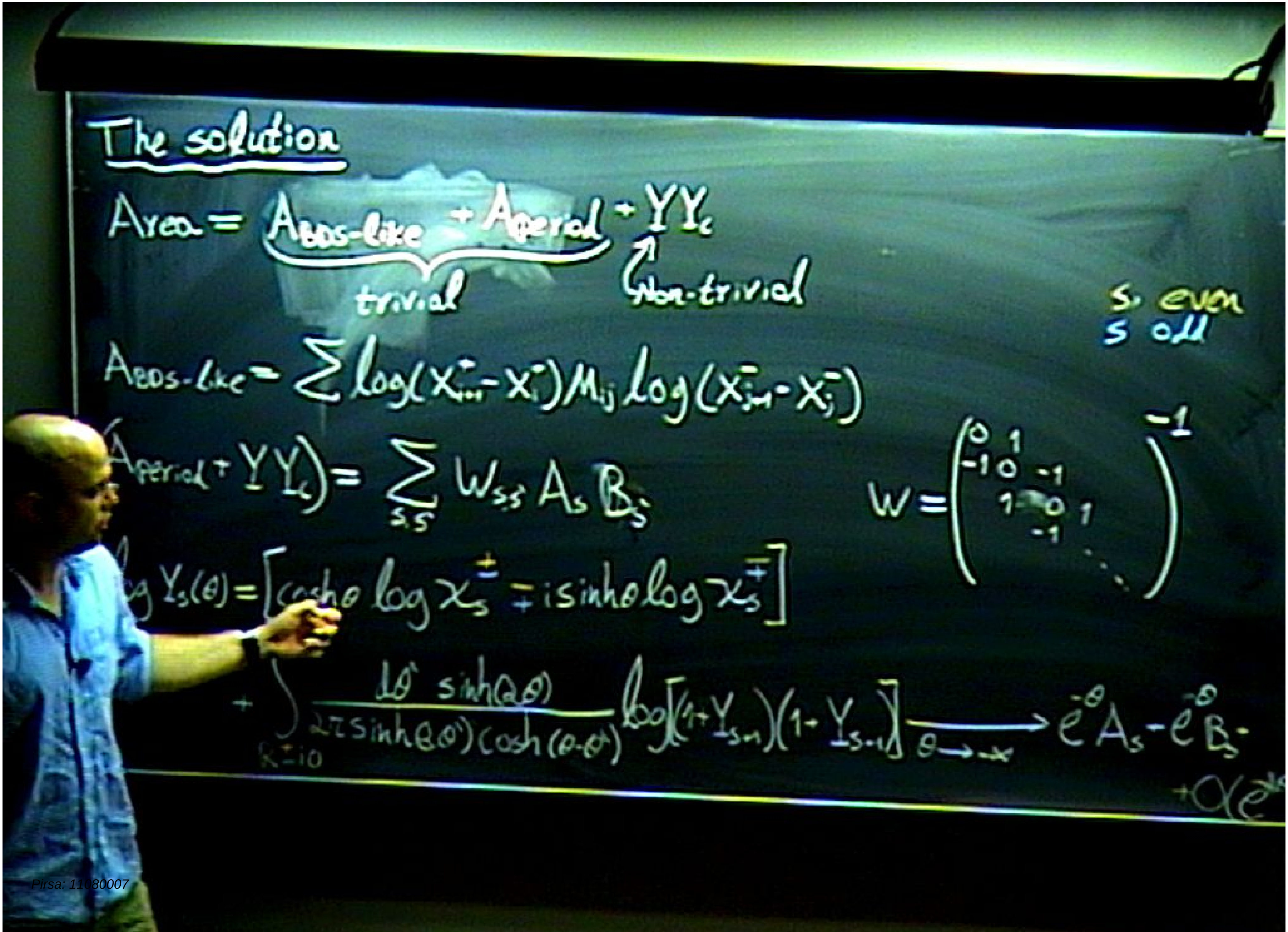

classical int
E.O.M. ⇒ A(θ) 2×2
dA + A∧A = 0 , ∀θ
Spectrum
X1
X2
X3

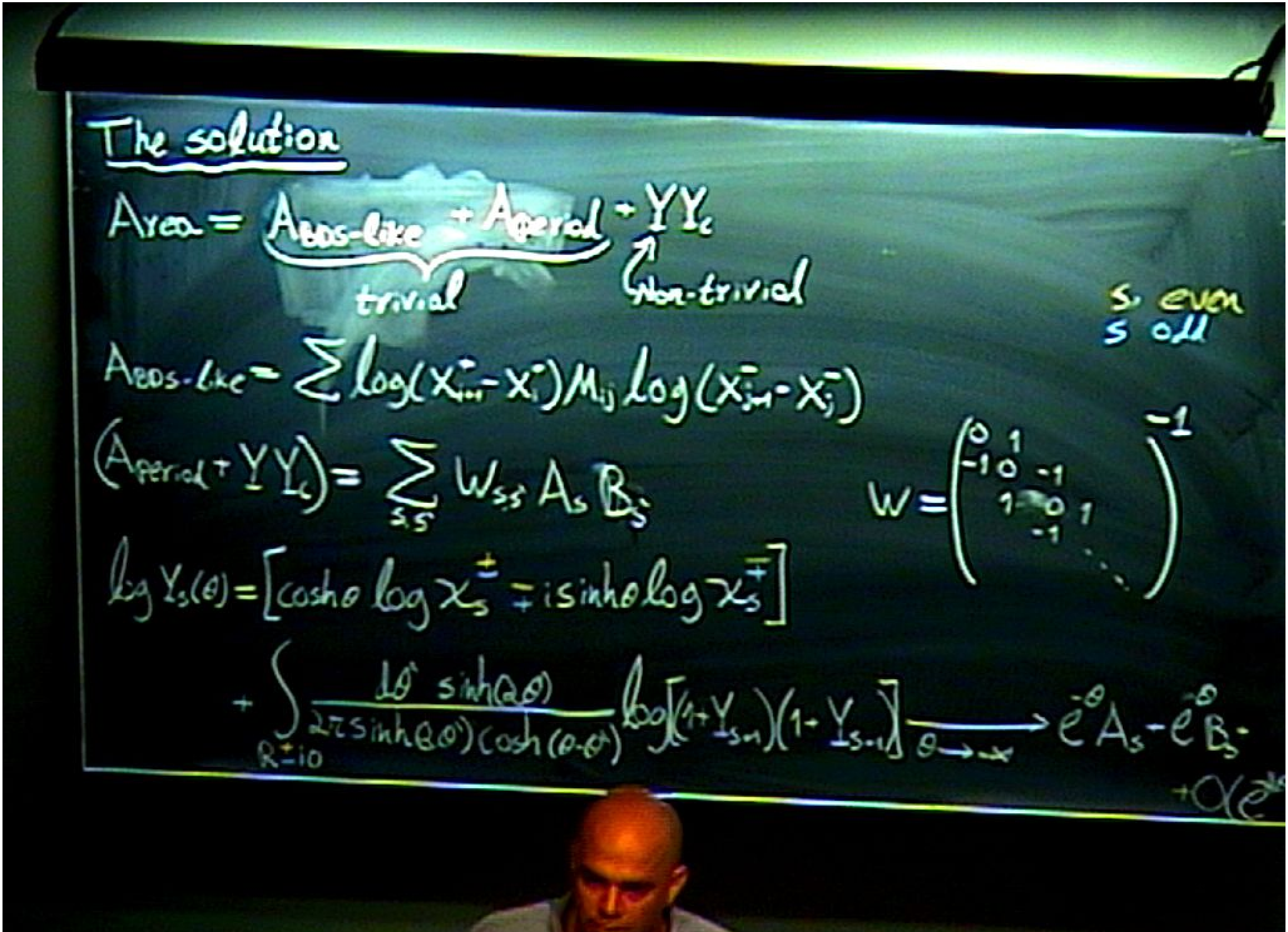

classical int
E.O.M. ⇒ A(θ)
dA + A∧A = 0 , ∀θ
Spectrum
Ω(θ) = Tr(P e^∮A)


classical int
E.O.M. ⇒ A(θ)
2×2
dA + A∧A = 0 , ∀θ
Spectrum

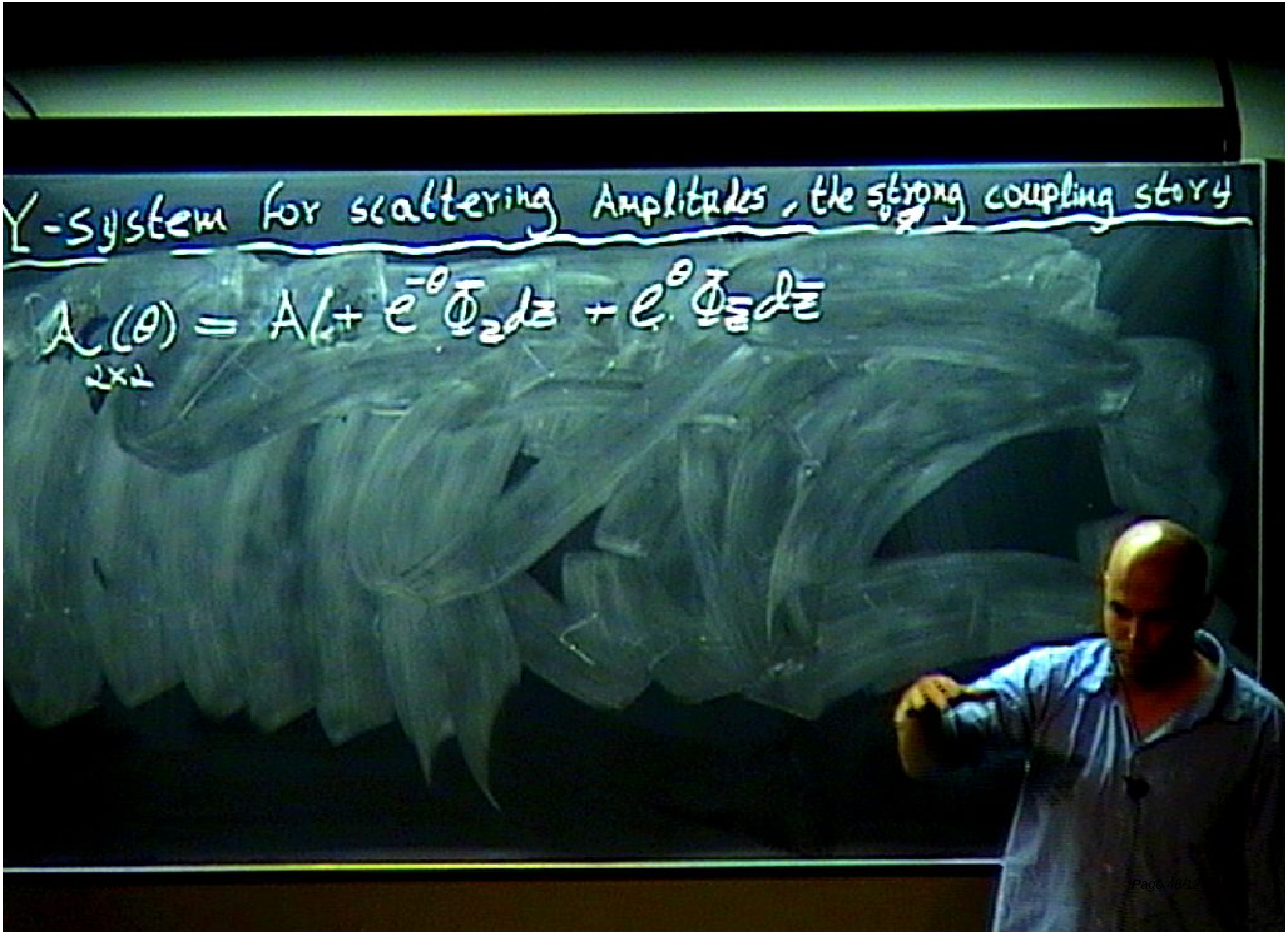

classical int
E.O.M. ⇒ A(θ)
dA + A∧A = 0 , ∀ θ
Spectrum
Ω(θ) = Tr(P e^∮dσ A_σ)
x3
x2
x1
Ys

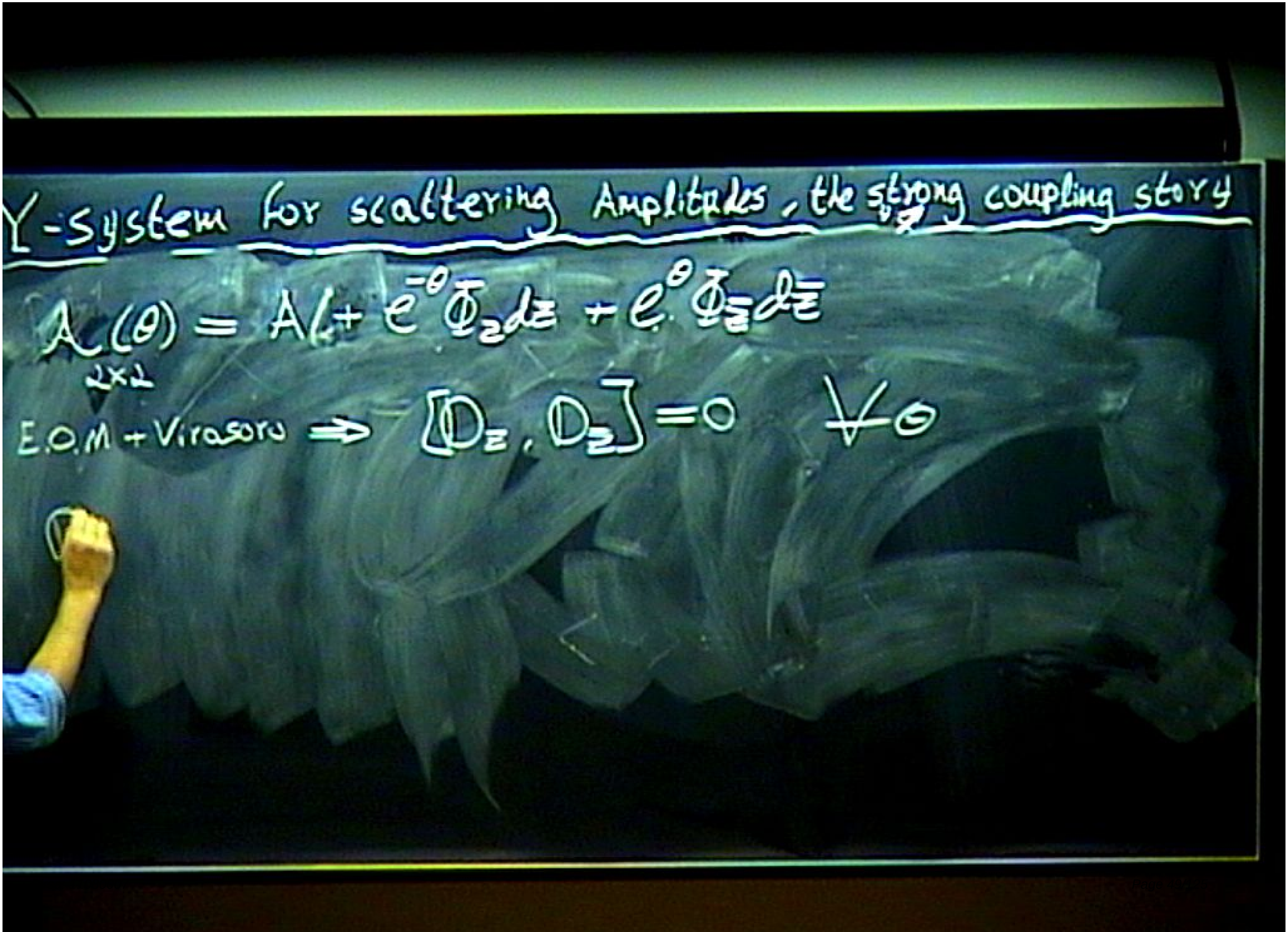

The solution
Area = A_BDS-like + A_periodic + YY_c
trivial
Non-trivial
W =
-1

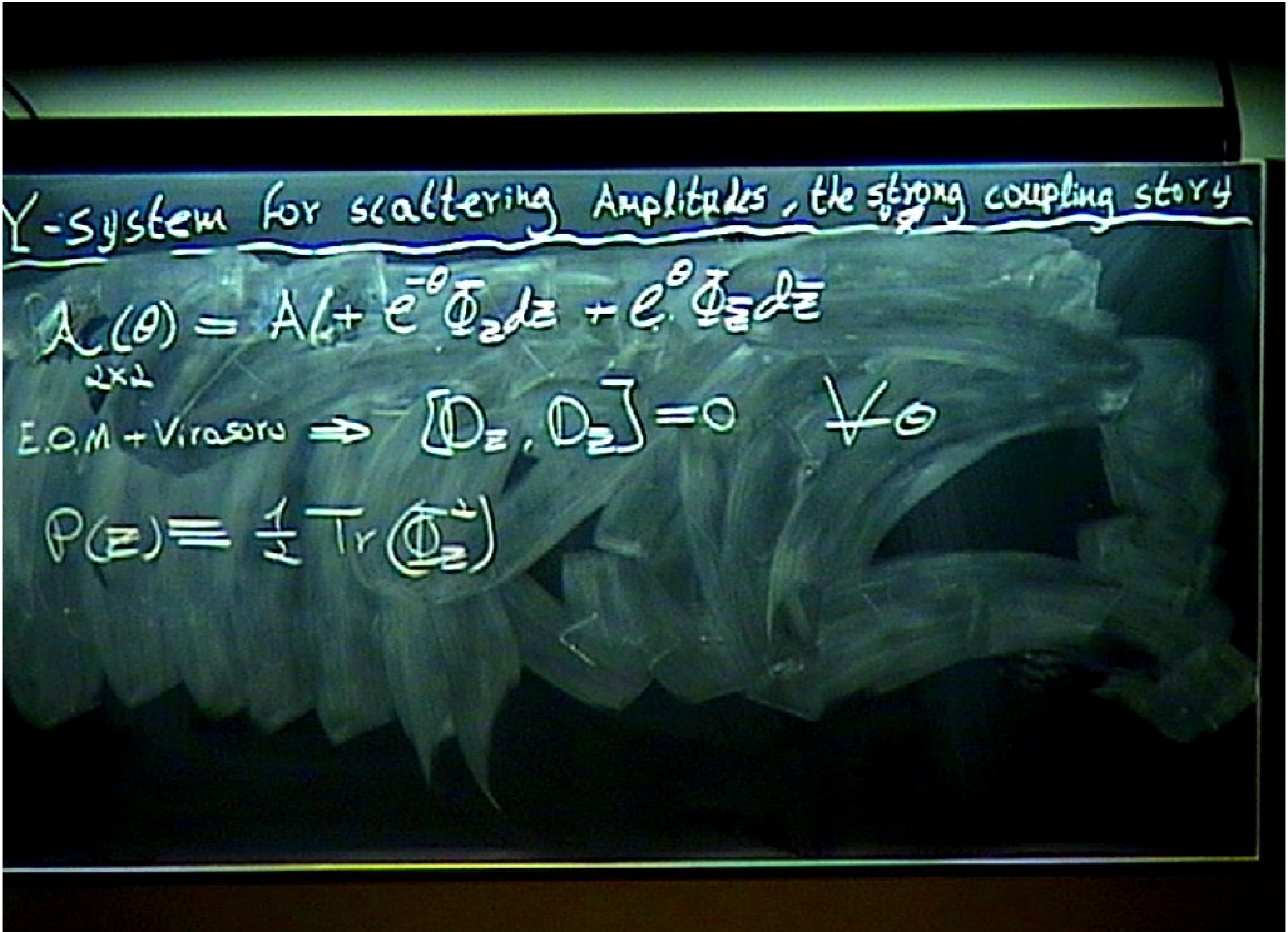

The solution
Area = A_BDS-like + A_period + YY_c
trivial
Non-trivial
s even
s odd
A_BDS-like = Σ log(x⁺_{i+1} − x⁺_i) M_ij log(x⁻_{j+1} − x⁻_j)
(A_period + YY_c) = Σ_{s,s'} W_{ss'} A_s B_s'
W = (0 1 / −1 0 −1 / 1 0 1 / −1 …)^−1
log Y_s(θ) = [cosh θ log x_s^± ∓ i sinh θ log x_s^∓]
+ ∫ dθ' sinh(2θ') / (2π sinh(2θ') cosh(θ−θ')) log[(1+Y_{s+1})(1+Y_{s−1})] → e^θ A_s − e^{−θ} B_s

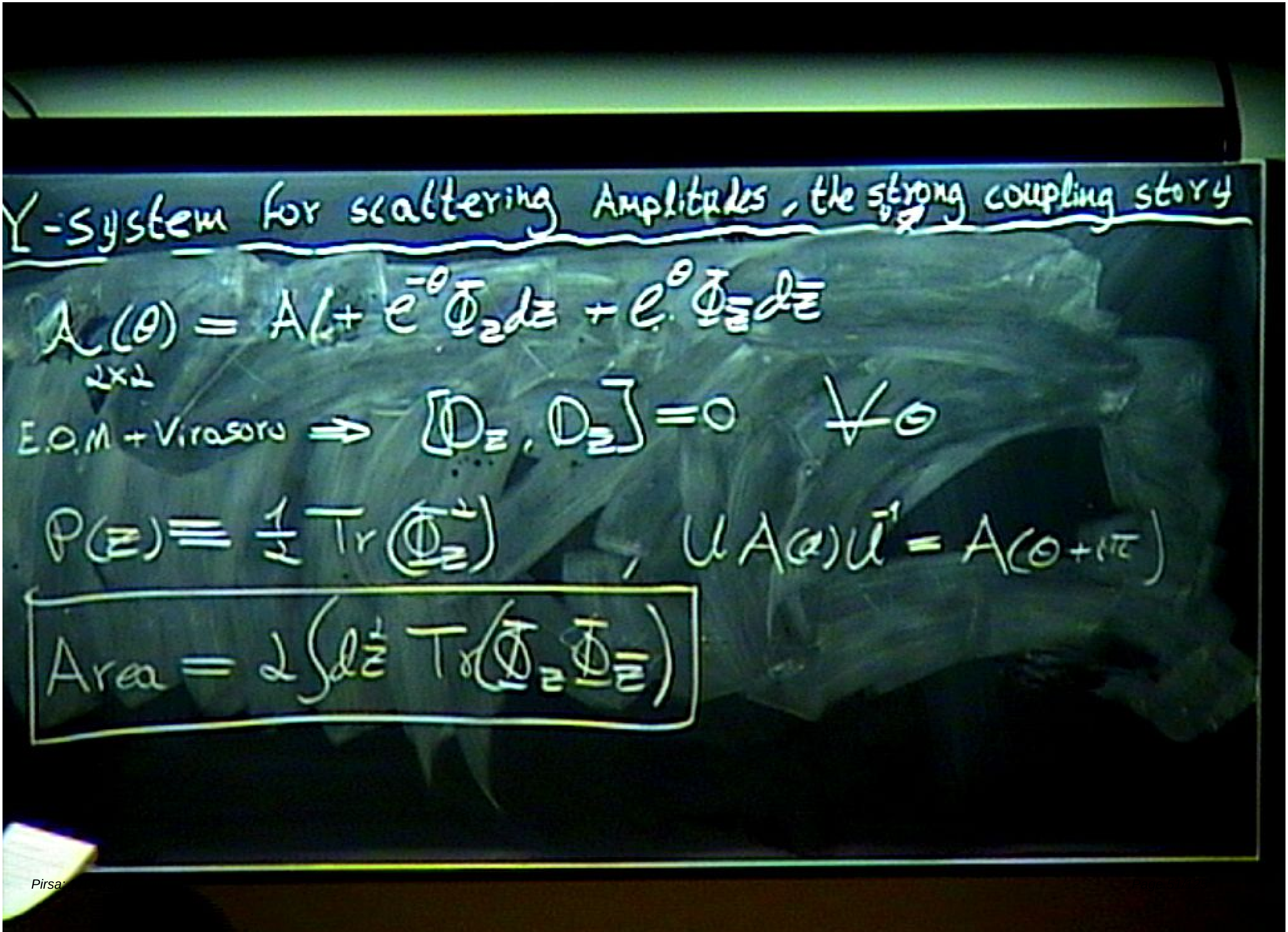


The solution
Area = A_BDS-like + A_period + YY_c
trivial
Non-trivial
s even
s odd
A_BDS-like = Σ log(x⁺_{i+1} − x⁺_i) M_ij log(x⁻_{j+1} − x⁻_j)
(A_period + YY_c) = Σ_{s,s'} W_{ss'} A_s B_{s'}
W = (0 1 / −1 0 −1 / 1 0 1 / −1 ...)^{−1}
log Y_s(θ) = [cosh θ log x_s^± ∓ i sinh θ log x_s^∓]
+ ∫_{R±i0} dθ' sinh(2θ') / (2π sinh(2θ') cosh(θ−θ')) log[(1+Y_{s+1})(1+Y_{s−1})] → (θ→∞) e^θ A_s + e^{−θ} B_s


Y-system for scattering Amplitudes, the strong coupling story
$\mathcal{A}_{2\times 2}(\theta) = A + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$


Y-system for scattering Amplitudes, the strong coupling story
$\mathcal{A}_{2\times 2}(\theta) = A + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$
E.O.M + Virasoro $\Rightarrow$ $[D_z, D_{\bar z}] = 0 \quad \forall \theta$


Y-system for scattering Amplitudes, the strong coupling story
$\mathcal{A}_{2\times2}(\theta) = A_z + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar{z}} d\bar{z}$
E.O.M + Virasoro $\Rightarrow$ $[D_z, D_{\bar{z}}] = 0 \quad \forall \theta$
$P(z) \equiv \frac{1}{2}\mathrm{Tr}(\Phi_z^2)$



Y-system for scattering Amplitudes, the strong coupling story
$\mathcal{A}_{2\times 2}(\theta) = A + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$
E.O.M + Virasoro $\Rightarrow$ $[D_z, D_{\bar z}] = 0 \quad \forall \theta$
$P(z) \equiv \frac{1}{2}\mathrm{Tr}(\Phi_z^2)$, $U A(\theta) U^{-1} = A(\theta + i\pi)$
$\text{Area} = 2\int d^2 z \, \mathrm{Tr}(\Phi_z \Phi_{\bar z})$

Y-system for scattering Amplitudes, the strong coupling story

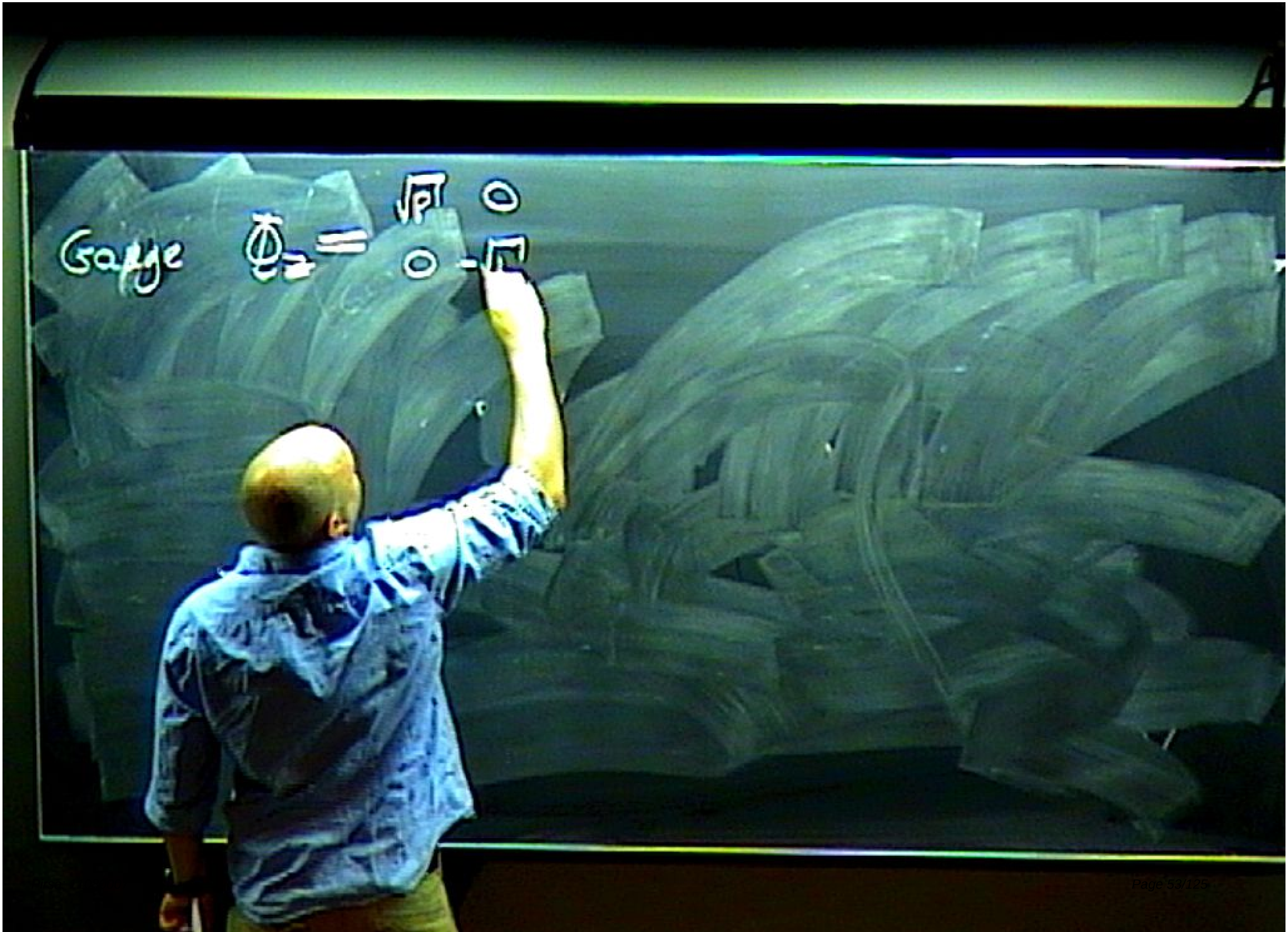
$$\underset{2\times 2}{\mathcal{A}}(\theta) = A + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$$

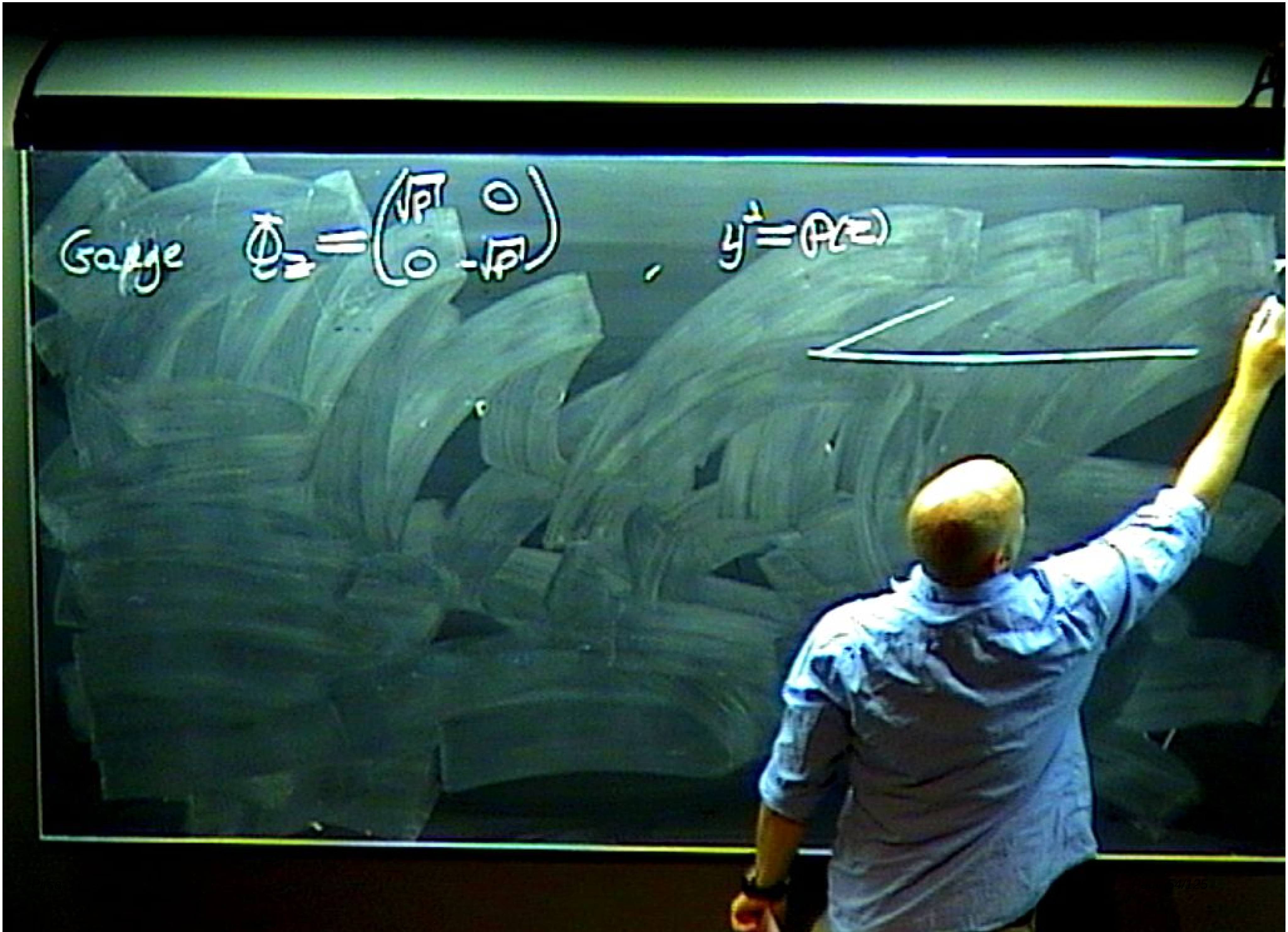
E.O.M + Virasoro $\Rightarrow$ $[D_z, D_{\bar z}] = 0 \quad \forall \theta$

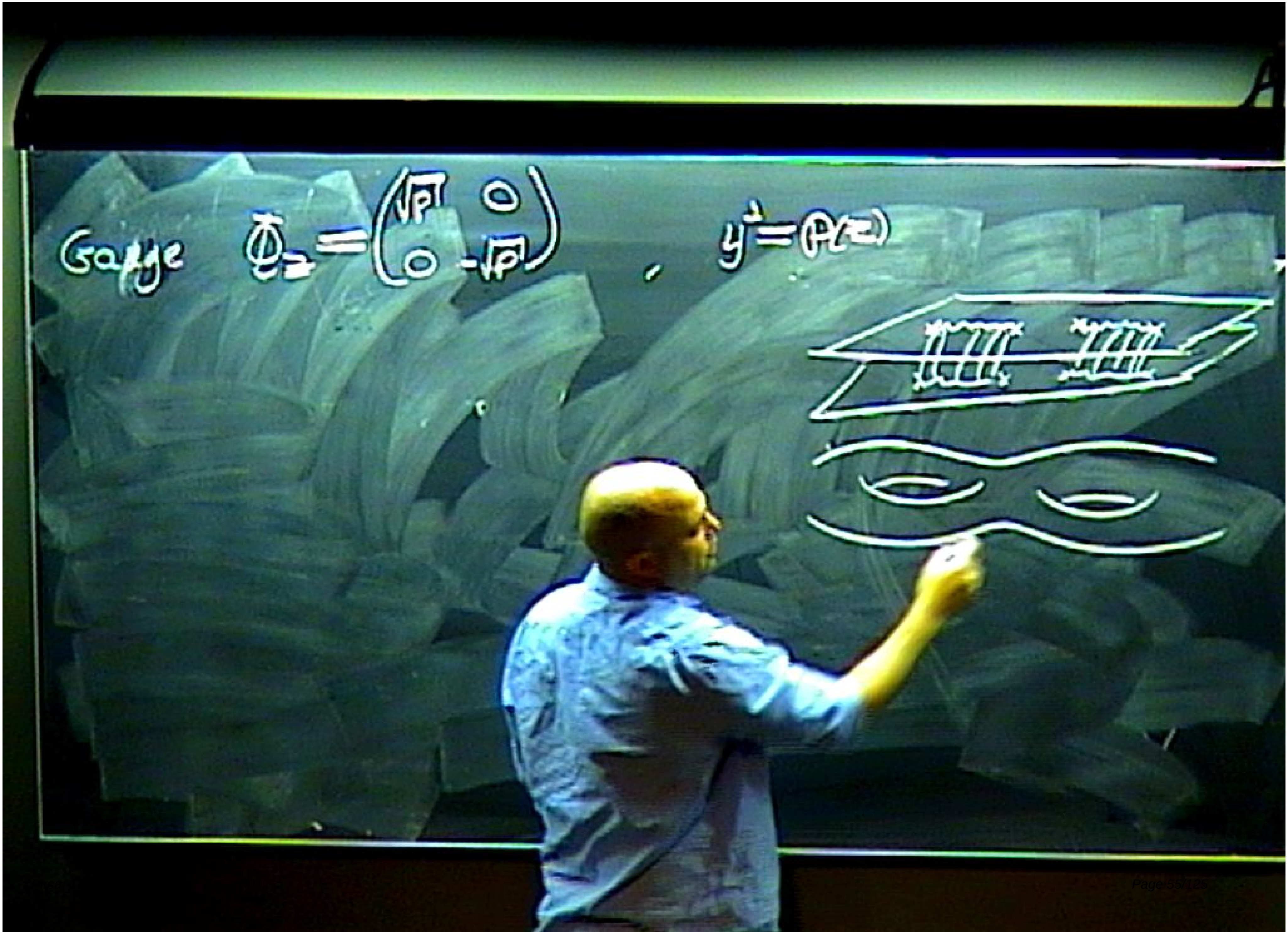
$$P(z) \equiv \frac{1}{2}\mathrm{Tr}(\Phi_z^2) \, , \quad U A(\theta) U^{-1} = A(\theta + i\pi)$$

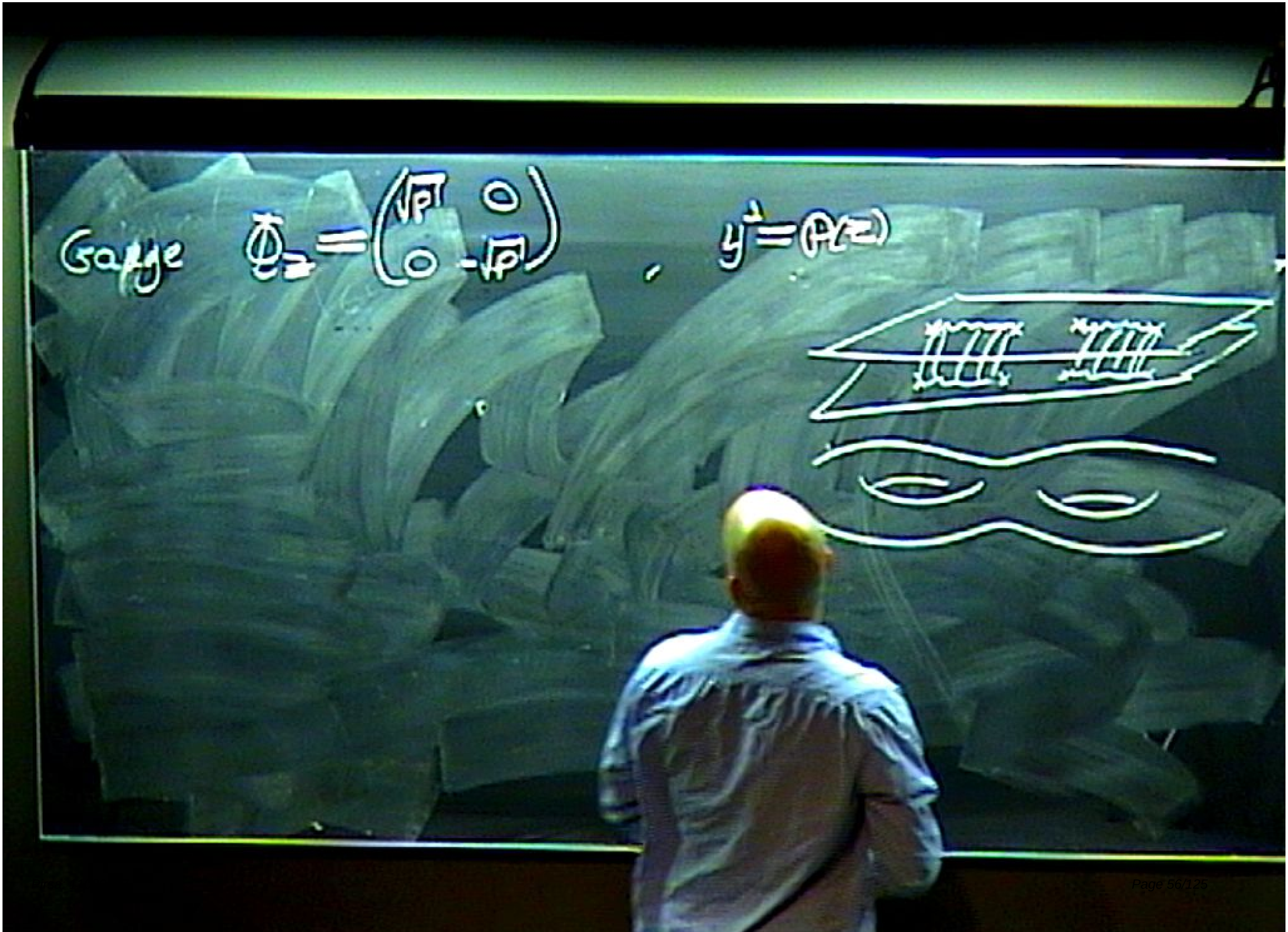
$$\boxed{\text{Area} = 2\int d^2z \, \mathrm{Tr}(\Phi_z \Phi_{\bar z})}$$

1) $P(z) = z^{n-2} + a_1 z^{n-3} + \dots \, , \quad A \xrightarrow[|z|\to\infty]{} 0$


Gauge $\Phi_z = \begin{pmatrix} \sqrt{\Gamma} & 0 \\ 0 & -\sqrt{\Gamma} \end{pmatrix}$


Gauge
$\Phi_z = \begin{pmatrix} \sqrt{P} & 0 \\ 0 & -\sqrt{P} \end{pmatrix}$, $y^2 = P(z)$

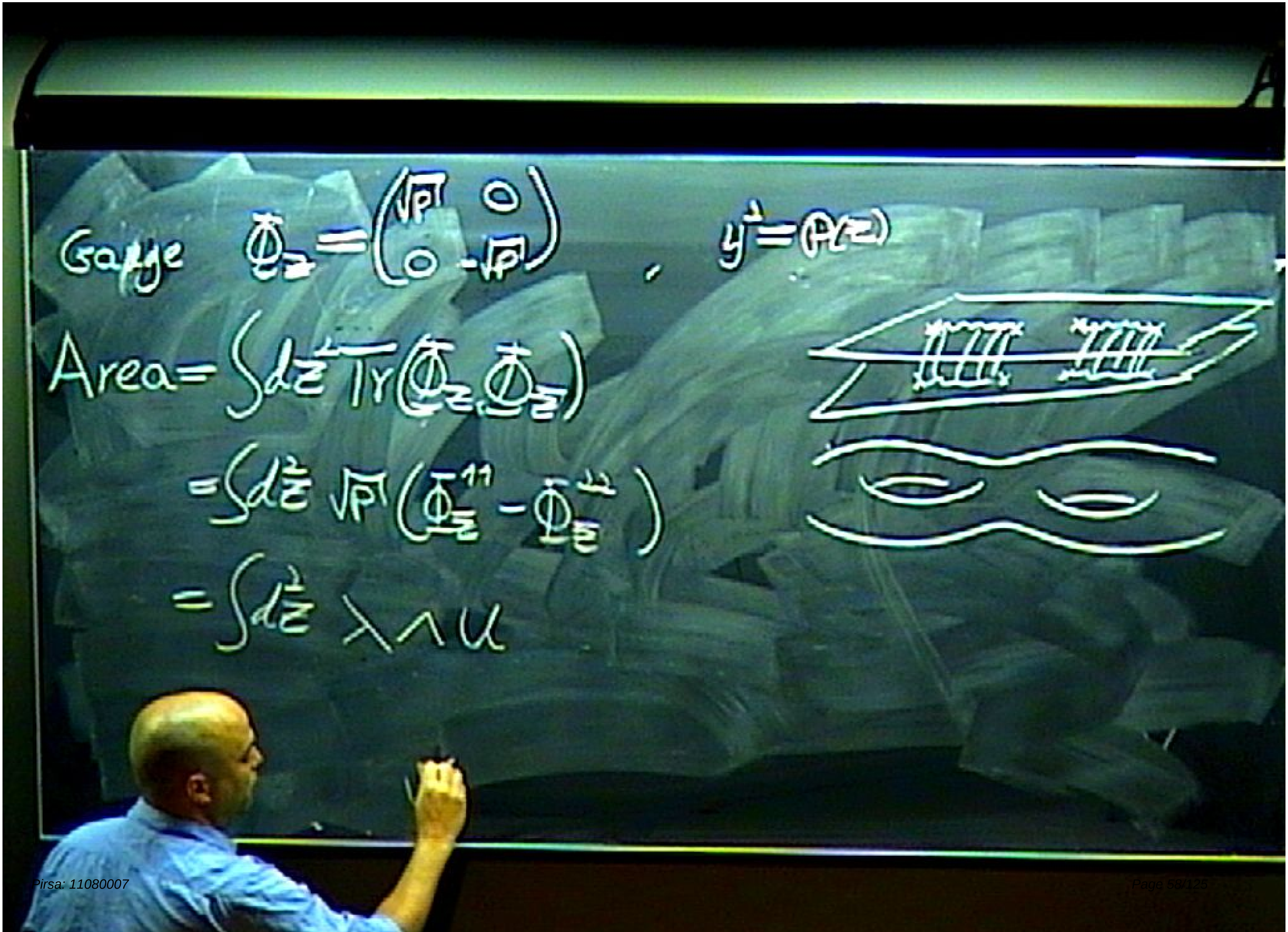

Gauge
$\Phi_z = \begin{pmatrix} \sqrt{P} & 0 \\ 0 & -\sqrt{P} \end{pmatrix}$
$y^2 = P(z)$


Gauge
Φz = (√P 0 / 0 -√P)
y² = P(z)

Gauge $\Phi_z = \begin{pmatrix} \sqrt{P} & 0 \\ 0 & -\sqrt{P} \end{pmatrix}$, $y^2 = P(z)$

$$\text{Area} = \int d^2z \, \mathrm{Tr}(\Phi_z \bar{\Phi}_{\bar{z}})$$

$$= \int d^2z \, \sqrt{P} \left(\bar{\Phi}_{\bar{z}}^{11} - \bar{\Phi}_{\bar{z}}^{22} \right)$$


Gauge $\Phi_z = \begin{pmatrix} \sqrt{P} & 0 \\ 0 & -\sqrt{P} \end{pmatrix}$, $y^2 = P(z)$
Area $= \int d^2z \, \mathrm{Tr}(\Phi_z \bar{\Phi}_{\bar{z}})$
$= \int d^2z \, \sqrt{P} \, (\bar{\Phi}^{11}_{\bar{z}} - \bar{\Phi}^{22}_{\bar{z}})$
$= \int d^2z \, \lambda \wedge u$

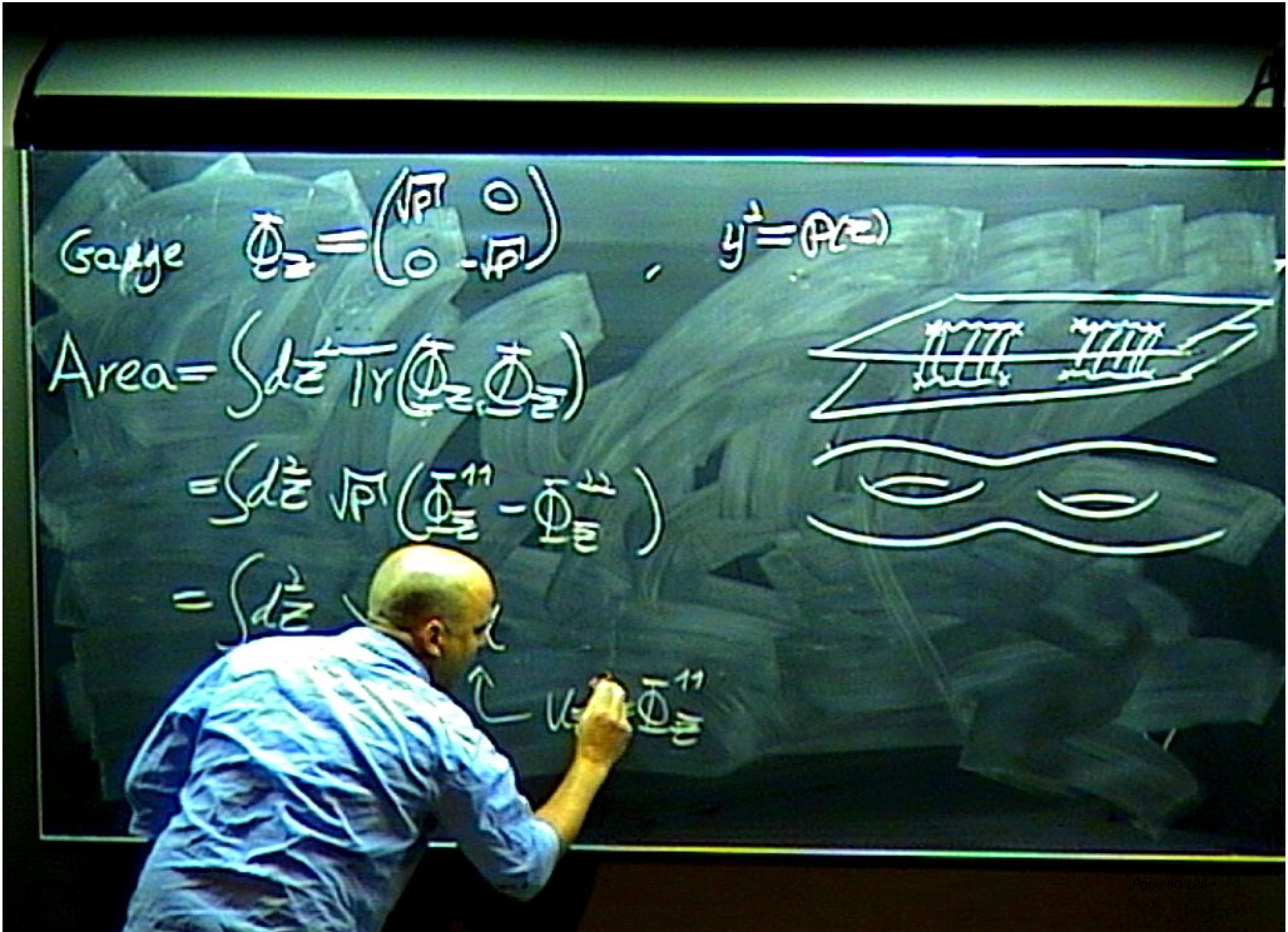
Gauge $\Phi_z = \begin{pmatrix} \sqrt{P} & 0 \\ 0 & -\sqrt{P} \end{pmatrix}$, $y^2 = P(z)$

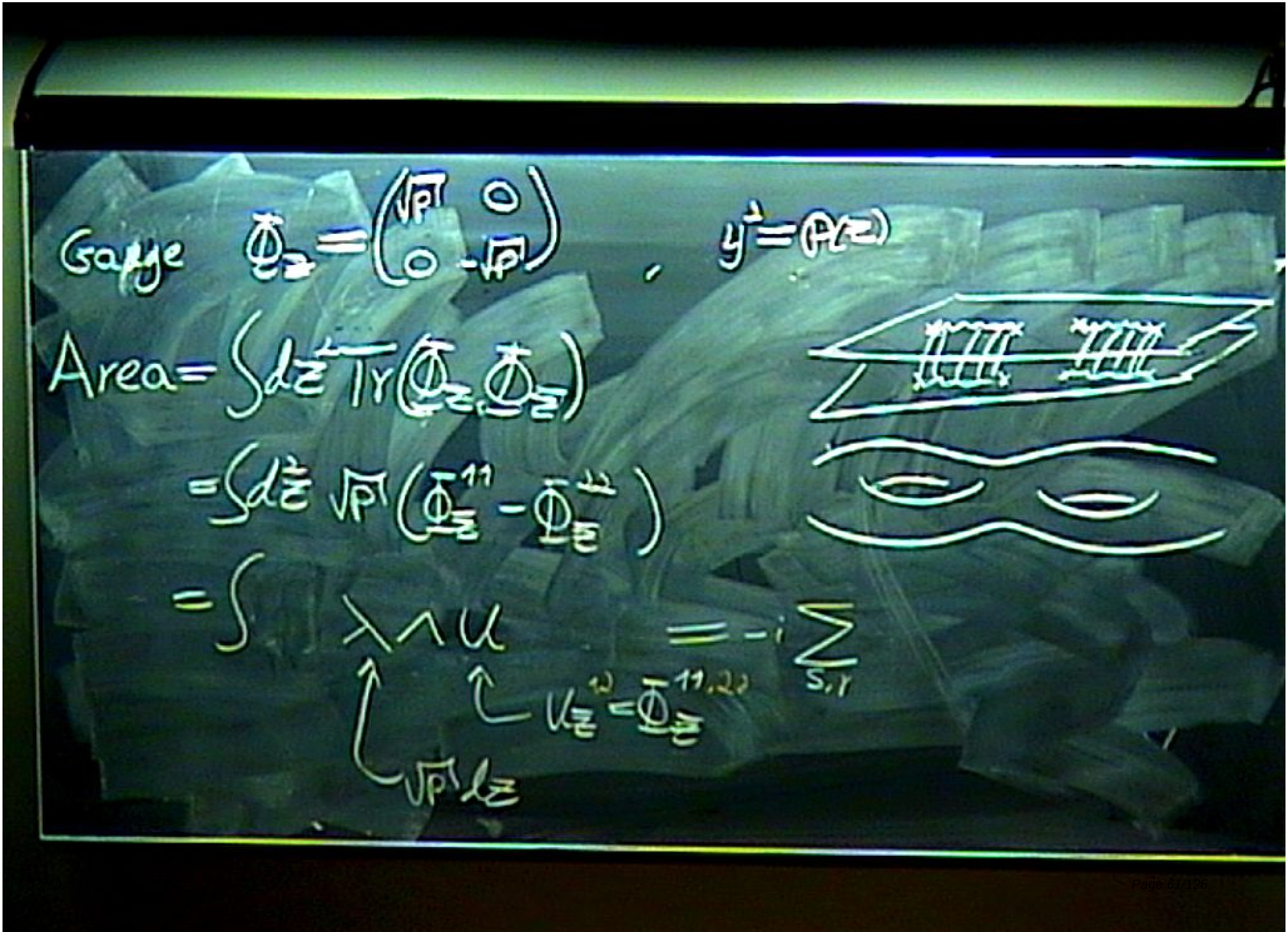
$$\text{Area} = \int d^2z \, \mathrm{Tr}(\Phi_z \bar{\Phi}_{\bar{z}})$$

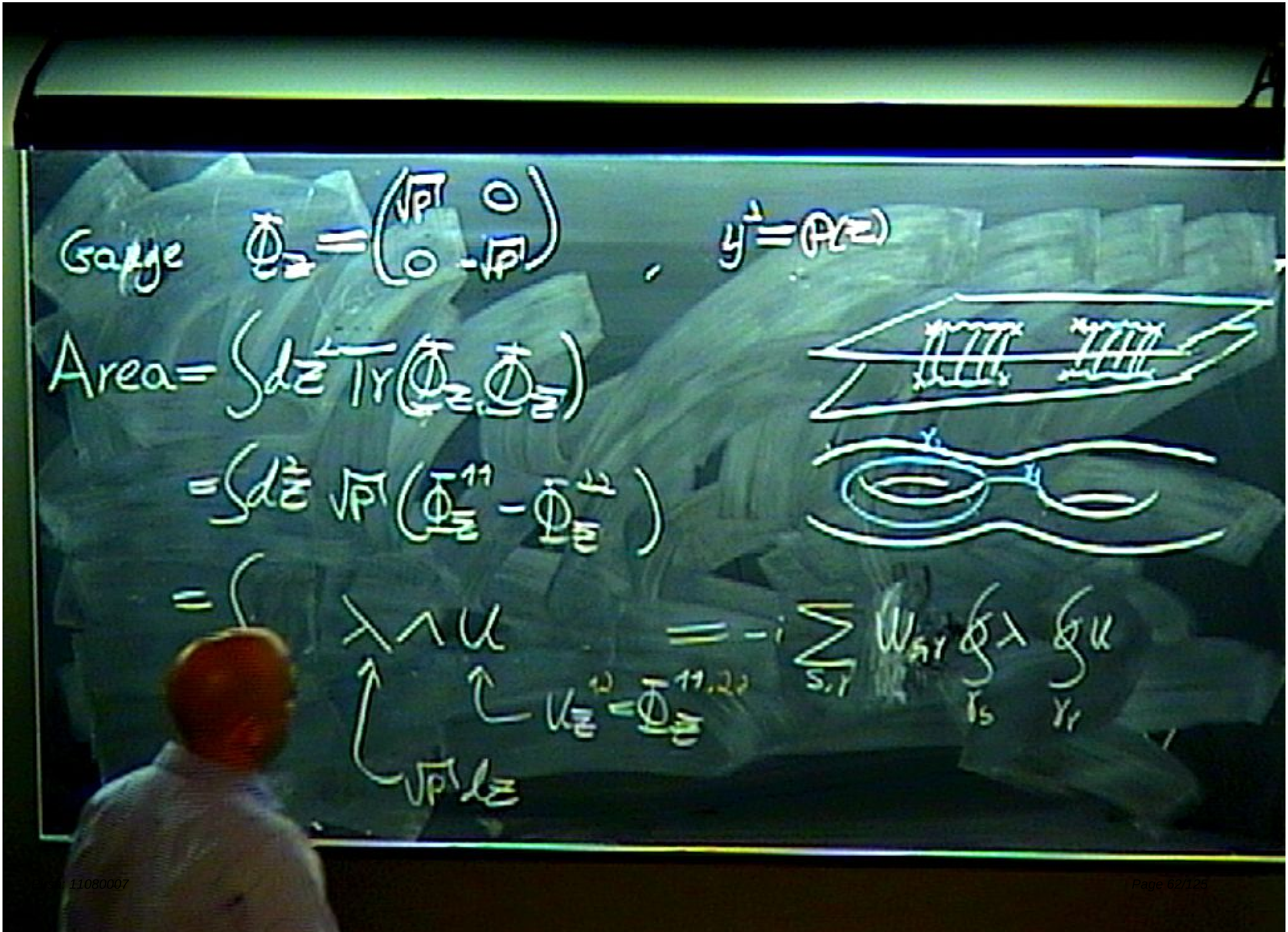
$$= \int d^2z \, \sqrt{P} \left(\bar{\Phi}_{\bar{z}}^{11} - \bar{\Phi}_{\bar{z}}^{22} \right)$$

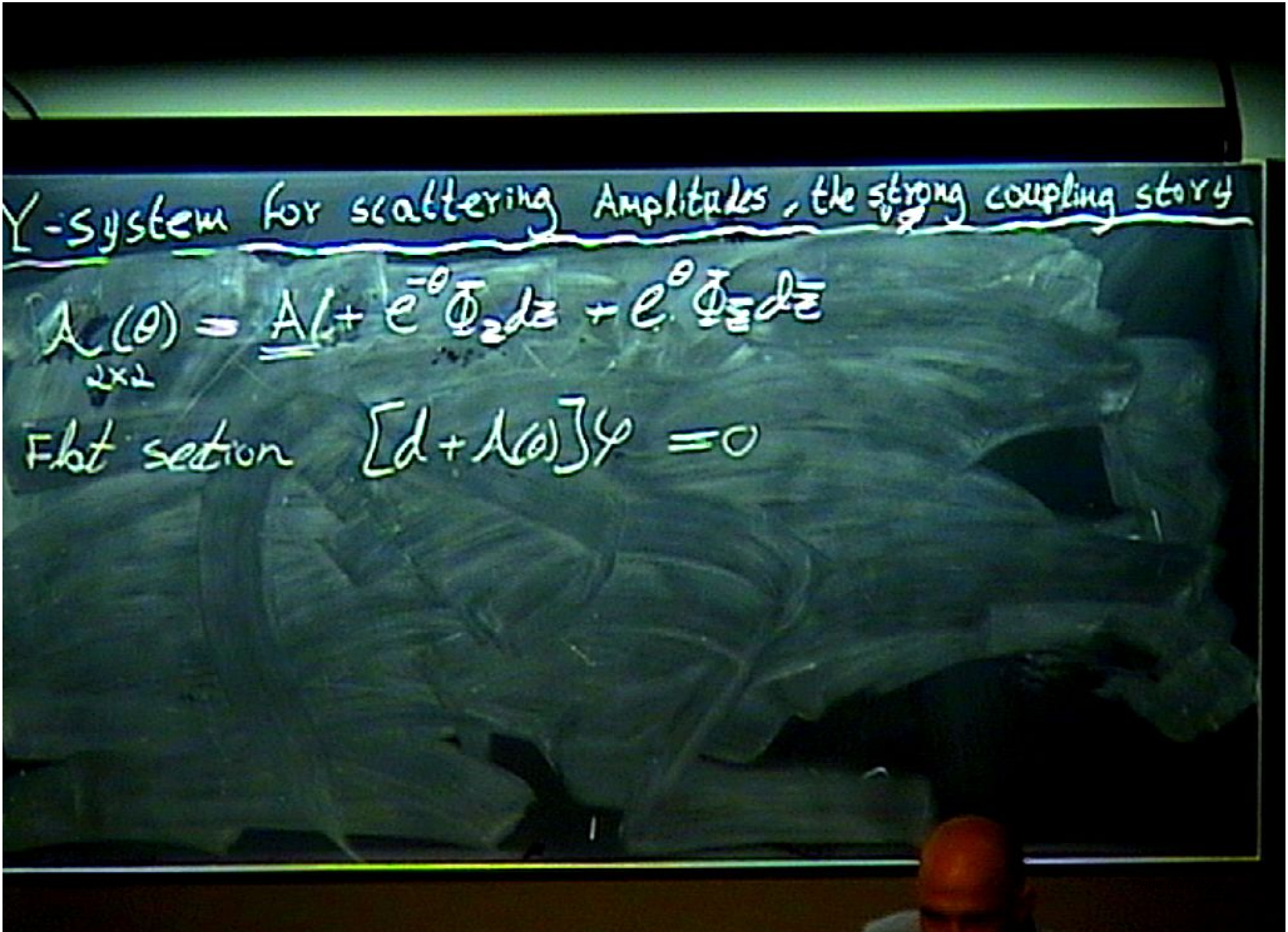
$$= \int d^2z \; \lambda \wedge u$$

$\lambda \leftarrow \sqrt{P}\, dz$


Gauge Φ_z = (√P 0 / 0 -√P)
y² = P(z)
Area = ∫dz dz̄ Tr(Φ_z Φ̄_z̄)
= ∫dz dz̄ √P (Φ̄^11_z̄ - Φ̄^22_z̄)
= ∫dz dz̄
v = Φ̄^11_z̄


Gauge Φ_z = (√P 0 / 0 −√P) , y² = P(z)
Area = ∫ d²z Tr(Φ_z Φ̄_z̄)
= ∫ d²z √P (Φ̄_z̄^11 − Φ̄_z̄^22)
= ∫ λ ∧ u = −i Σ_{s,r}
u_z̄ = Φ̄_z̄^{11,22}
√P dz


Gauge
Area=


Y-system for scattering Amplitudes, the strong coupling story
$\Lambda_{2\times2}(\theta) = A + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$
Flat section $[d + \Lambda(\theta)]\psi = 0$

Y-system for scattering Amplitudes, the strong coupling story

$$\underset{2\times 2}{\Lambda}(\theta) = \underline{A} + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$$

Flat section $\quad [d + \Lambda(\theta)]\psi = 0$

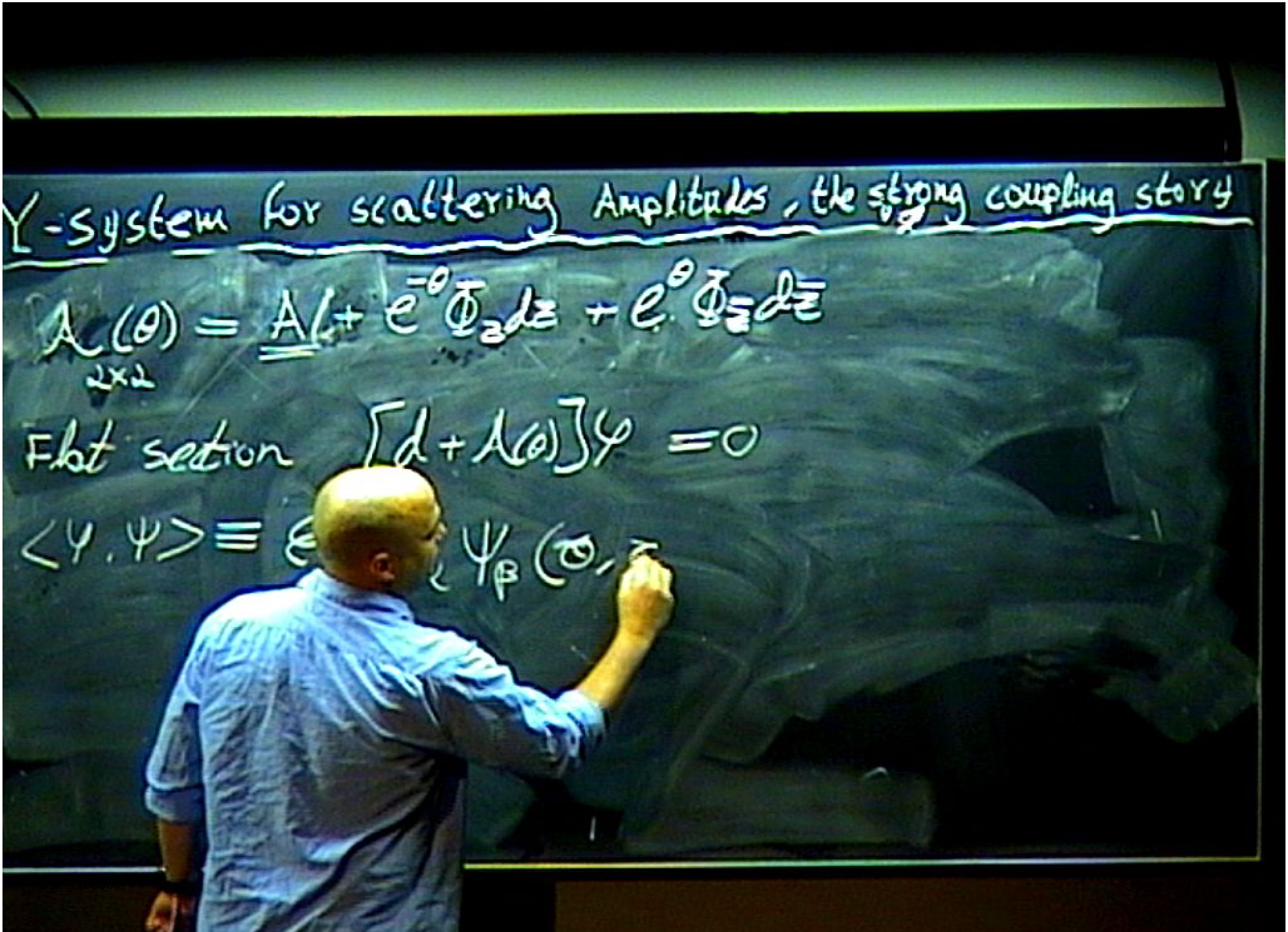
$\psi . \Psi$

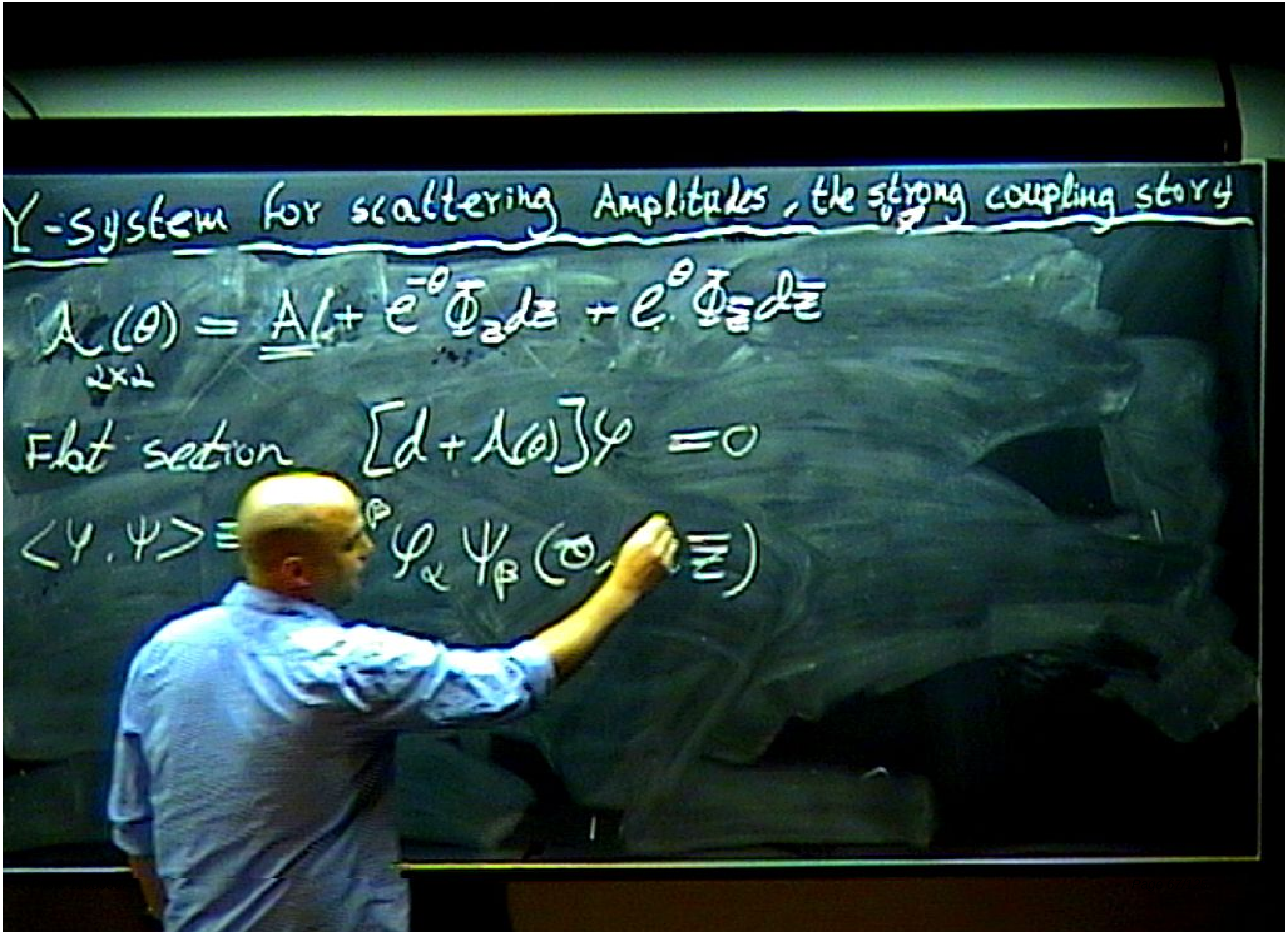
Y-system for scattering Amplitudes, the strong coupling story

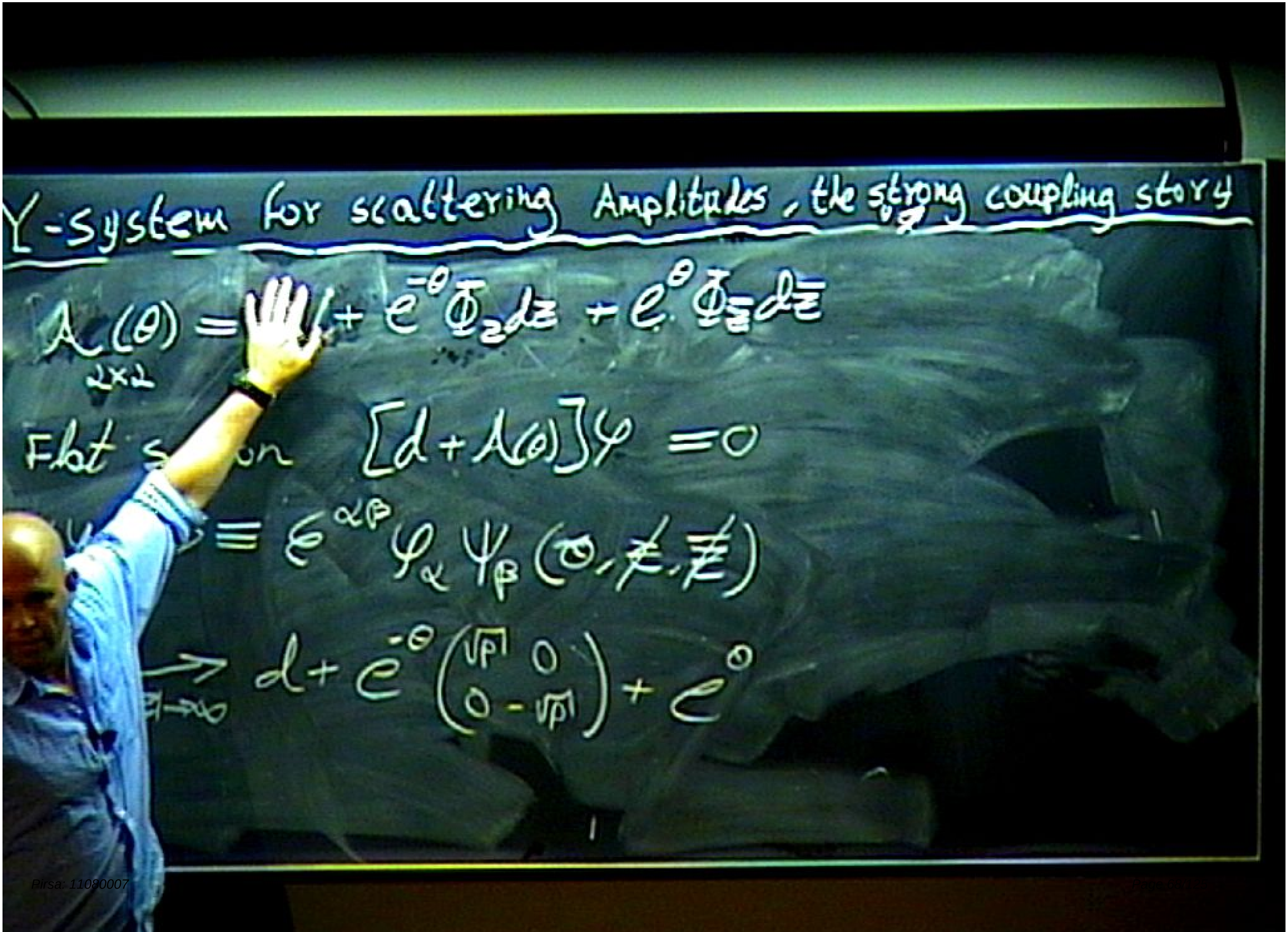
$$\mathcal{A}_{2\times 2}(\theta) = \underline{\underline{A}} + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$$

Flat section $\left[d + \mathcal{A}(\theta)\right]\psi = 0$

$$\langle \psi , \Psi \rangle \equiv \epsilon^{\alpha\beta}\psi_\alpha \Psi_\beta$$


Y-system for scattering Amplitudes, the strong coupling story
Flat section


Y-system for scattering Amplitudes, the strong coupling story
Flat section


Y-System for scattering Amplitudes, the strong coupling story
(θ) =
+ e^{-θ} Φ_z dz + e^{θ} Φ_z̄ dz̄
2x2
Flat
on
[d + λ(θ)]ψ = 0
≡ ε^{αβ} ψ_α ψ_β (θ, z, z̄)
d + e^{-θ} (√P 0 ; 0 -√P) + e^{θ}

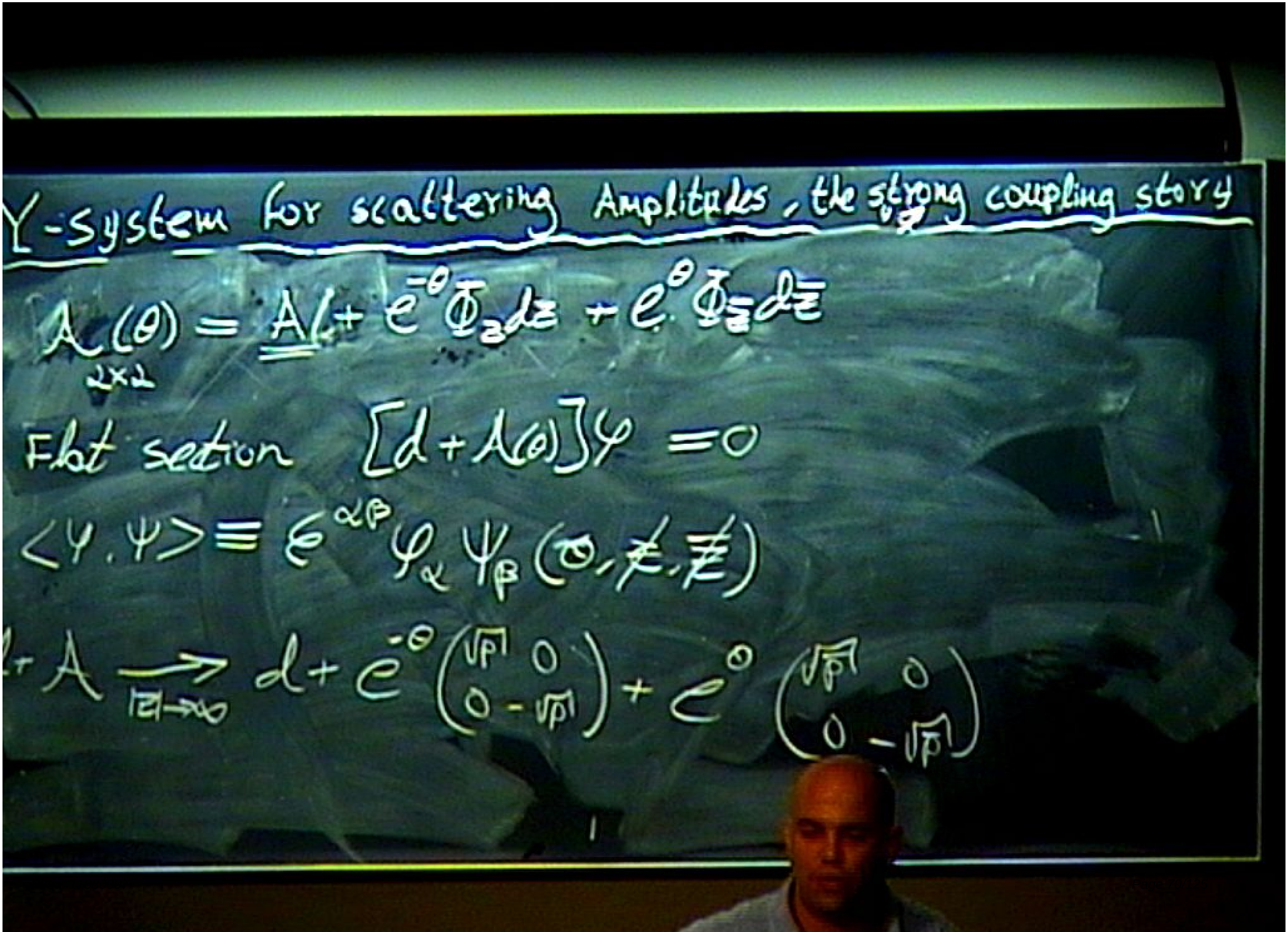
Y-system for scattering Amplitudes, the strong coupling story

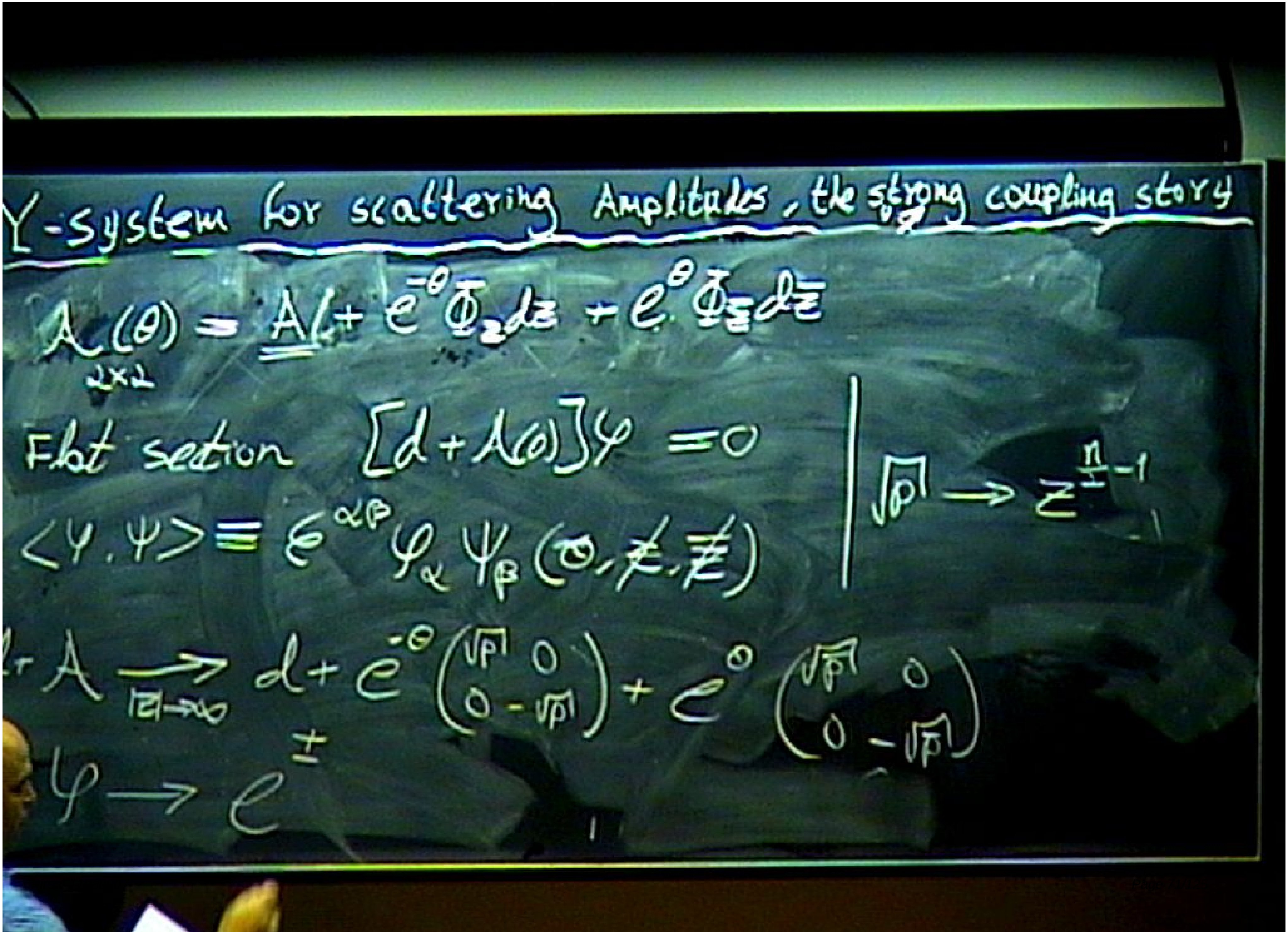
$$\underset{2\times 2}{\mathcal{A}}(\theta) = \underline{\underline{A}} + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$$

Flat section $[d+\mathcal{A}(\theta)]\psi = 0$

$$\langle \psi, \Psi \rangle \equiv \epsilon^{\alpha\beta}\psi_\alpha \Psi_\beta(\theta, z, \bar z)$$

$$\mathcal{A} \xrightarrow[|z|\to\infty]{} d + e^{-\theta}\begin{pmatrix}\sqrt{p} & 0 \\ 0 & -\sqrt{p}\end{pmatrix} + e^{\theta}$$


Y-system for scattering Amplitudes, the strong coupling story
$\mathcal{A}_{2\times2}(\theta) = \underline{\underline{A}} + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$
Flat section $[d+\mathcal{A}(\theta)]\psi = 0$
$\langle \psi,\psi \rangle \equiv \epsilon^{\alpha\beta}\psi_\alpha\psi_\beta(\theta, z, \bar z)$
$d+A \xrightarrow[|z|\to\infty]{} d + e^{-\theta}\begin{pmatrix}\sqrt{p} & 0\\ 0 & -\sqrt{p}\end{pmatrix} + e^{\theta}\begin{pmatrix}\sqrt{\bar p} & 0\\ 0 & -\sqrt{\bar p}\end{pmatrix}$


Y-system for scattering Amplitudes, the strong coupling story
$\underset{2\times2}{A}(\theta) = A + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$
Flat section $[d + A(\theta)]\psi = 0$
$\langle \psi . \psi \rangle \equiv \epsilon^{\alpha\beta}\psi_\alpha \psi_\beta (\theta, z, \bar z)$
$\sqrt{P} \to z^{\frac{n}{2}-1}$
$d + A \xrightarrow[|z|\to\infty]{} d + e^{-\theta}\begin{pmatrix}\sqrt{P} & 0\\ 0 & -\sqrt{P}\end{pmatrix} + e^{\theta}\begin{pmatrix}\sqrt{\bar P} & 0\\ 0 & -\sqrt{\bar P}\end{pmatrix}$
$\psi \to e^{\pm}$

Y-system for scattering Amplitudes, the strong coupling story

$$\underset{2\times 2}{\mathcal{A}}(\theta) = \underline{\underline{A}} + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$$

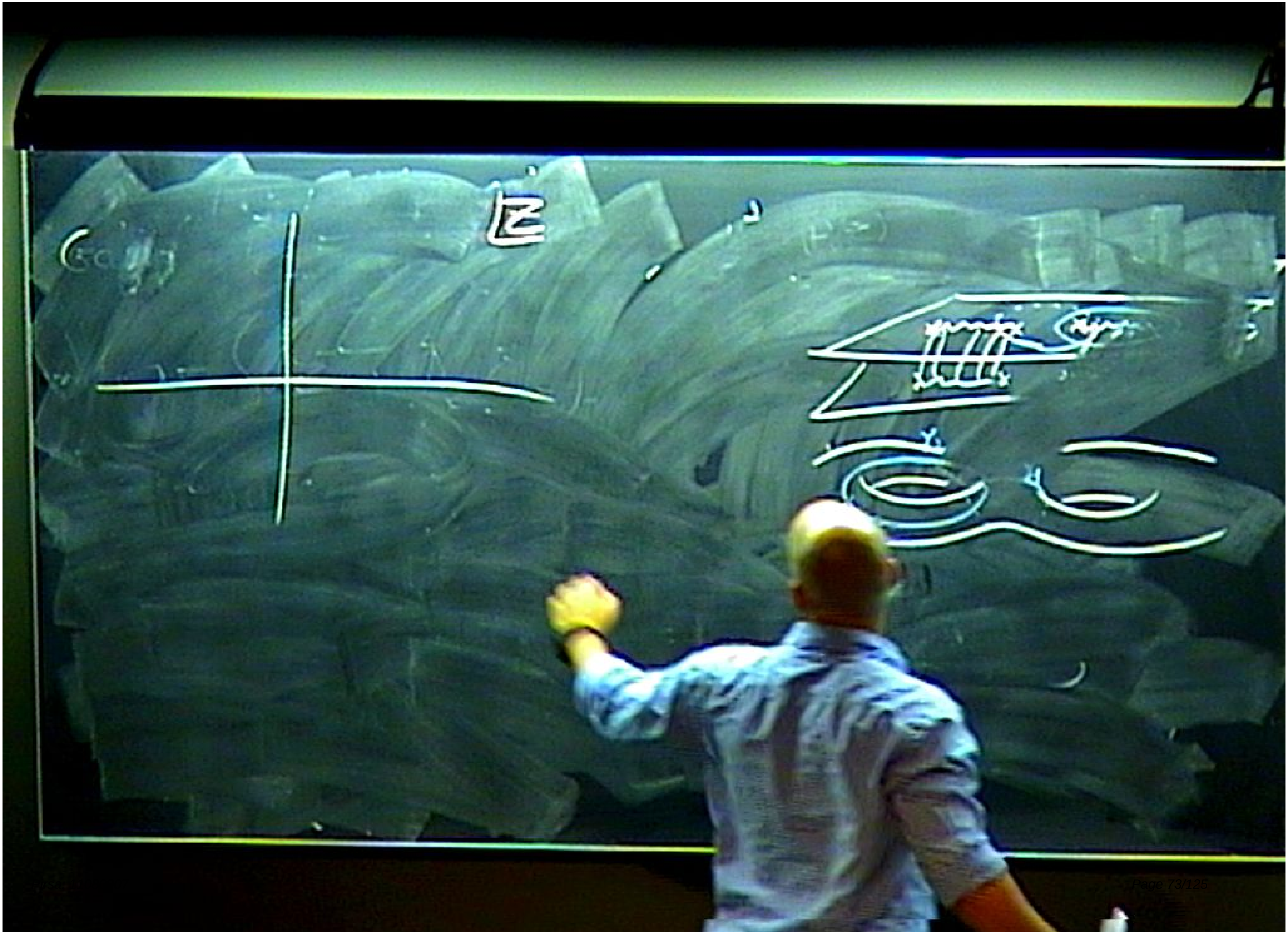
Flat section $[d + \mathcal{A}(\theta)]\psi = 0$

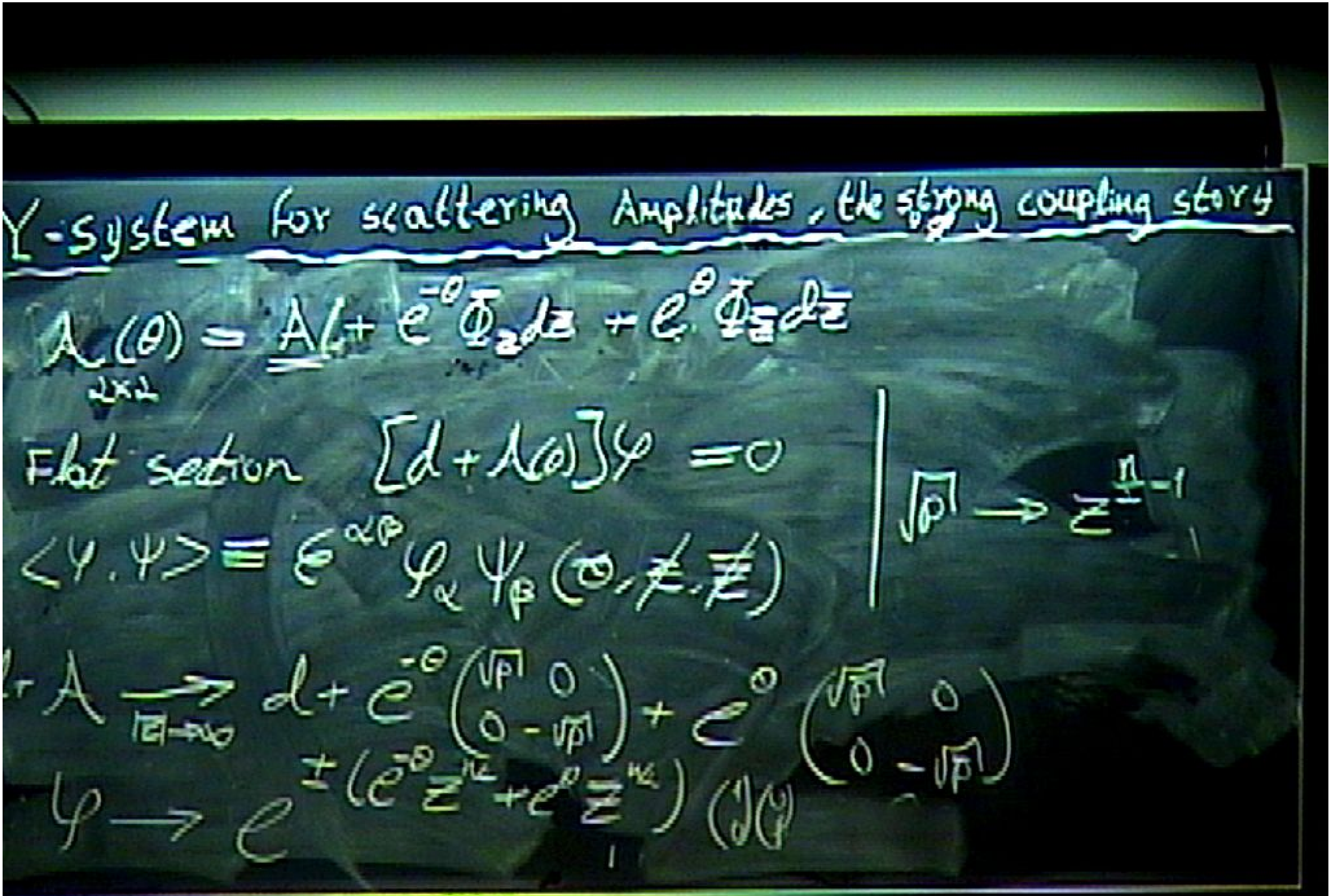
$$\langle \psi , \psi \rangle \equiv \epsilon^{\alpha\beta}\psi_\alpha \psi_\beta (\theta, z, \bar z)$$

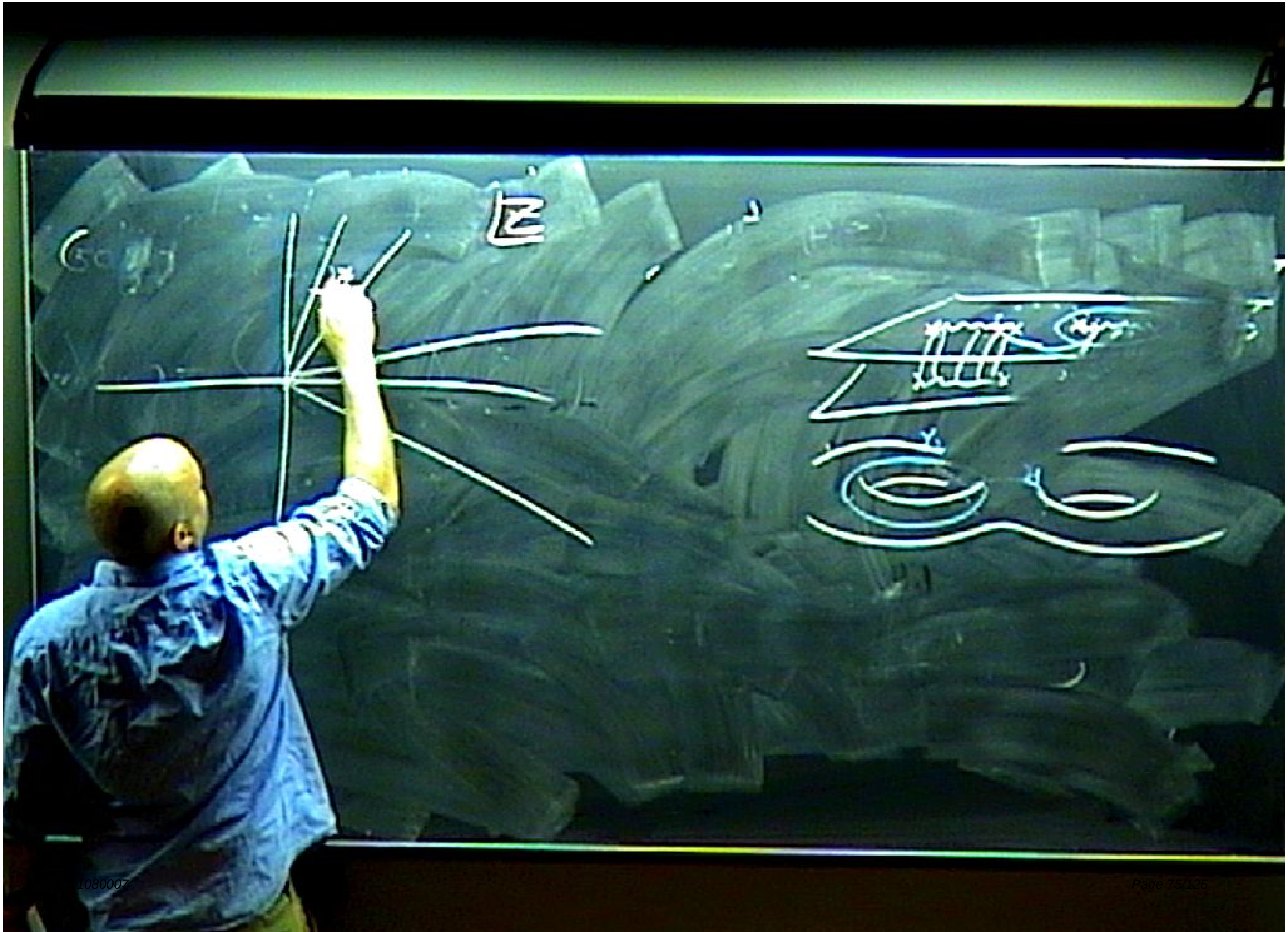
$$\sqrt{p} \longrightarrow z^{\frac{n}{2}-1}$$

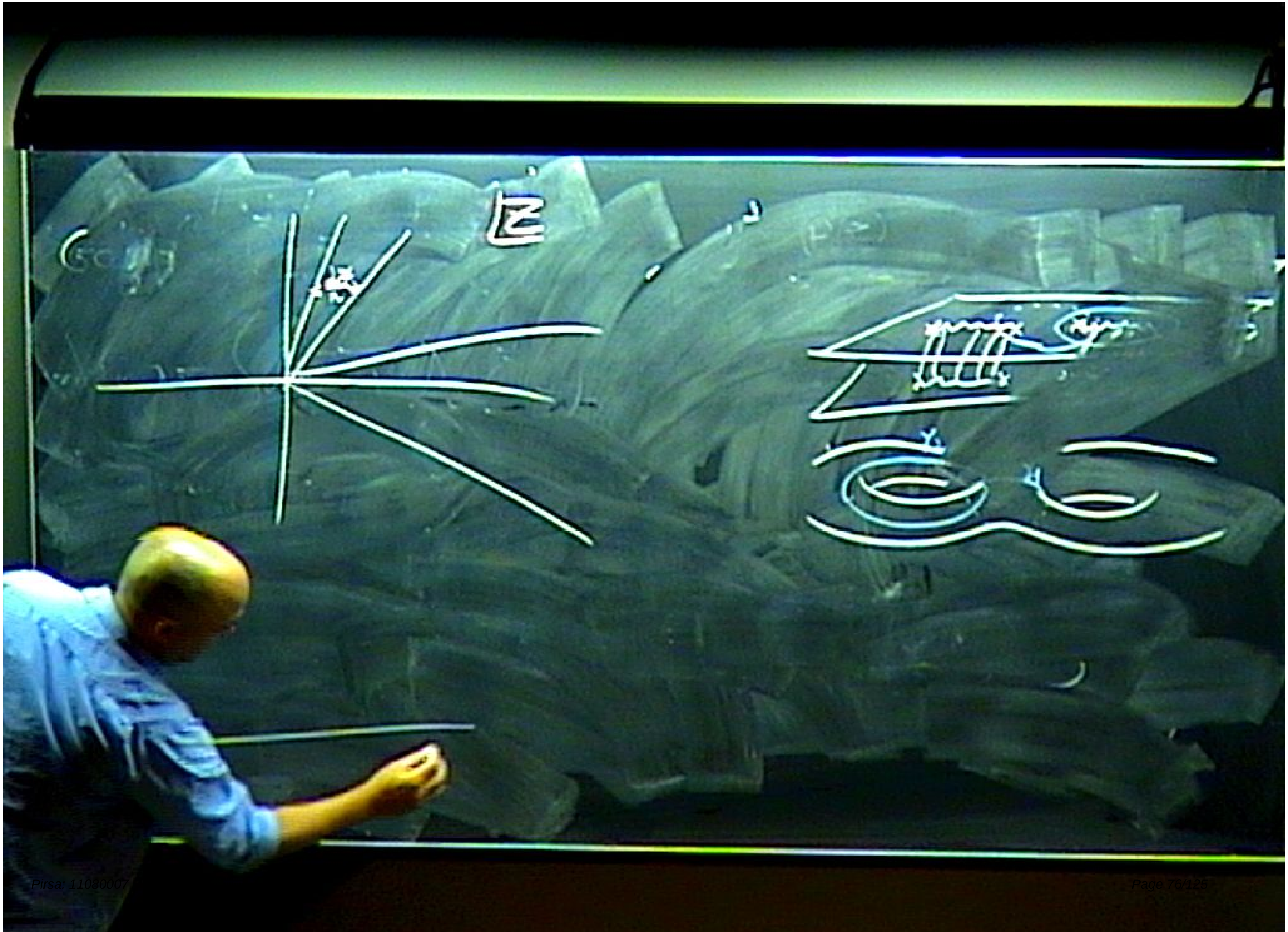
$$d + \mathcal{A} \xrightarrow[|z|\to\infty]{} d + e^{-\theta}\begin{pmatrix}\sqrt{p} & 0 \\ 0 & -\sqrt{p}\end{pmatrix} + e^{\theta}\begin{pmatrix}\sqrt{\bar p} & 0 \\ 0 & -\sqrt{\bar p}\end{pmatrix}$$

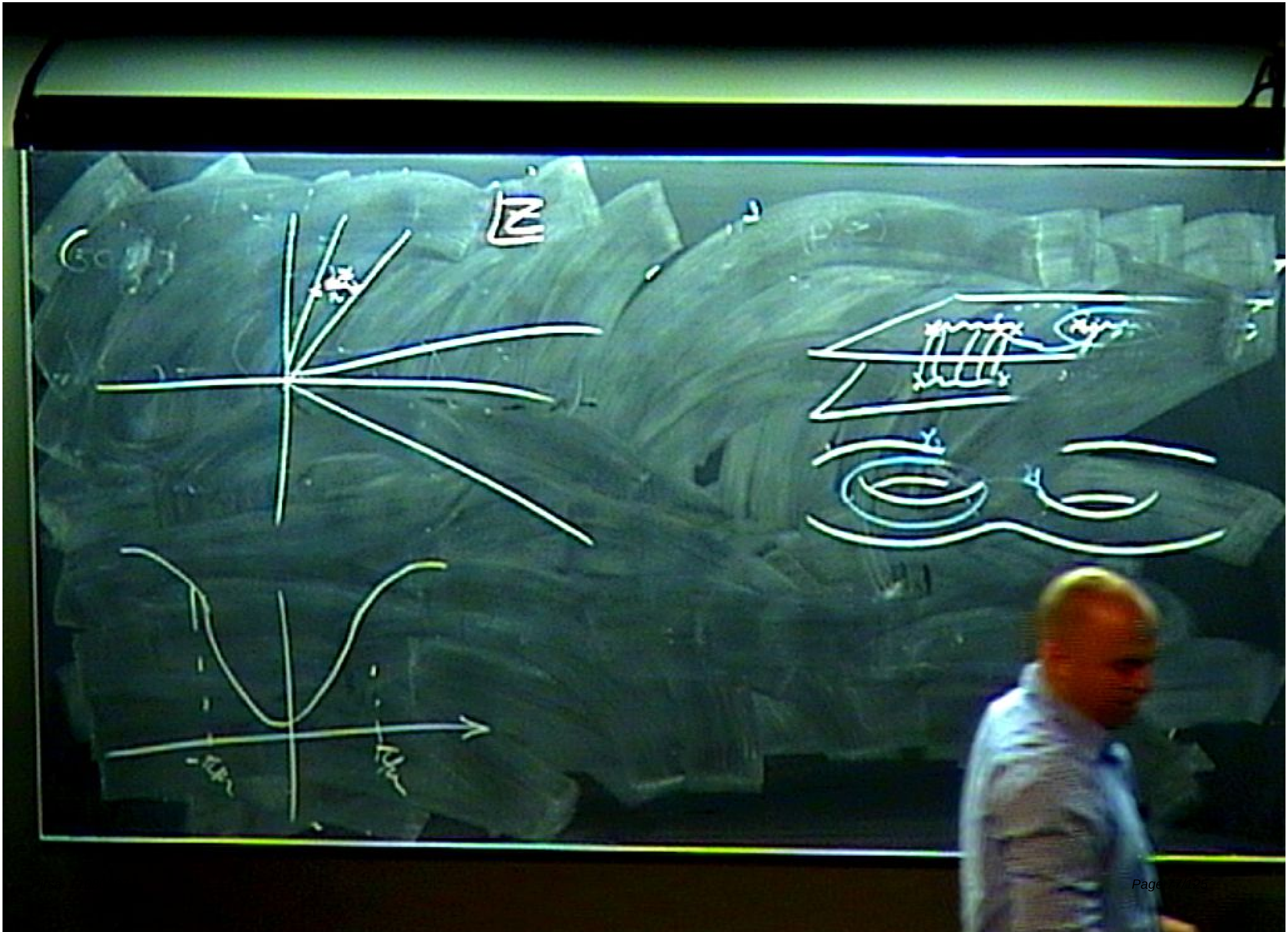
$$\psi \longrightarrow e^{\pm\left(e^{-\theta} z^{n/2} + e^{\theta} \bar z^{n/2}\right)}$$

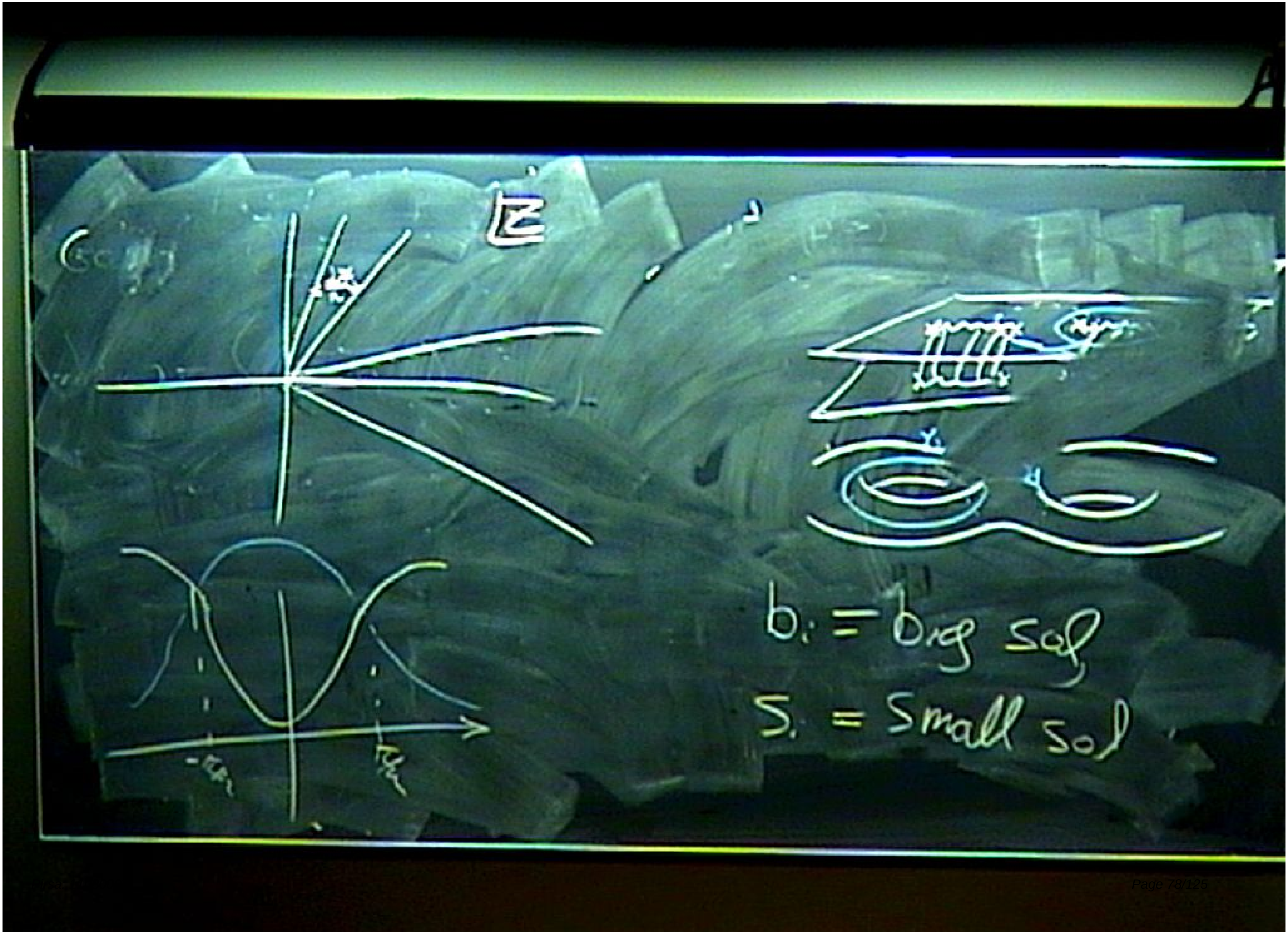


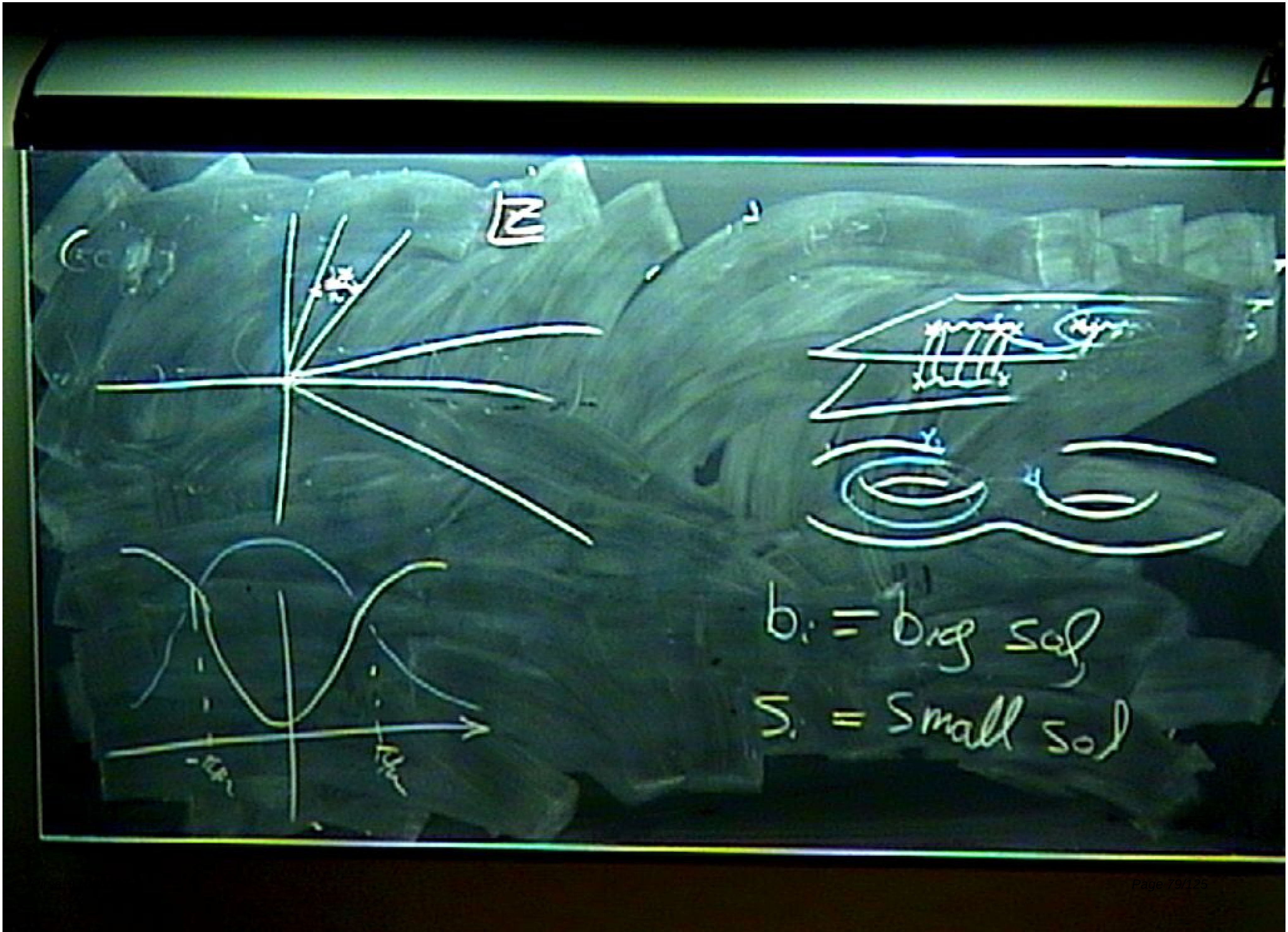

Y-system for scattering Amplitudes, the strong coupling story
$\lambda_{2\times 2}(\theta) = A + e^{-\theta}\Phi_z dz + e^{\theta}\Phi_{\bar z} d\bar z$
Flat section $[d + \lambda(\theta)]\psi = 0$
$\langle \psi, \psi \rangle \equiv \epsilon^{\alpha\beta}\psi_\alpha \psi_\beta(\theta, z, \bar z)$
$\sqrt{p} \to z^{\frac{n}{2}-1}$
$d + A \xrightarrow[|z|\to\infty]{} d + e^{-\theta}\begin{pmatrix}\sqrt{p} & 0\\ 0 & -\sqrt{p}\end{pmatrix} + e^{\theta}\begin{pmatrix}\sqrt{\bar p} & 0\\ 0 & -\sqrt{\bar p}\end{pmatrix}$
$\psi \to e^{\pm(e^{-\theta} z^{n/2} + e^{\theta}\bar z^{n/2})}\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$

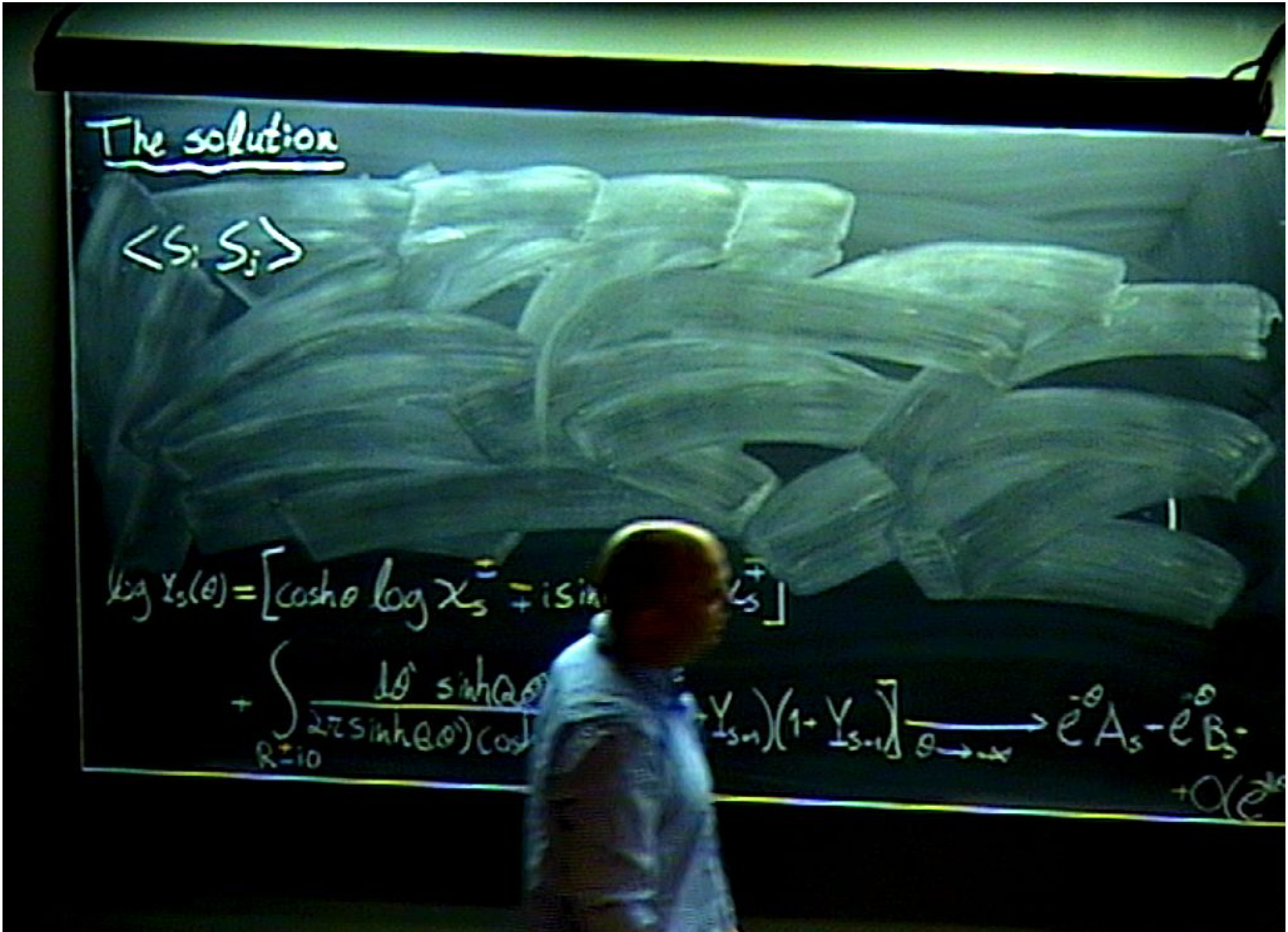


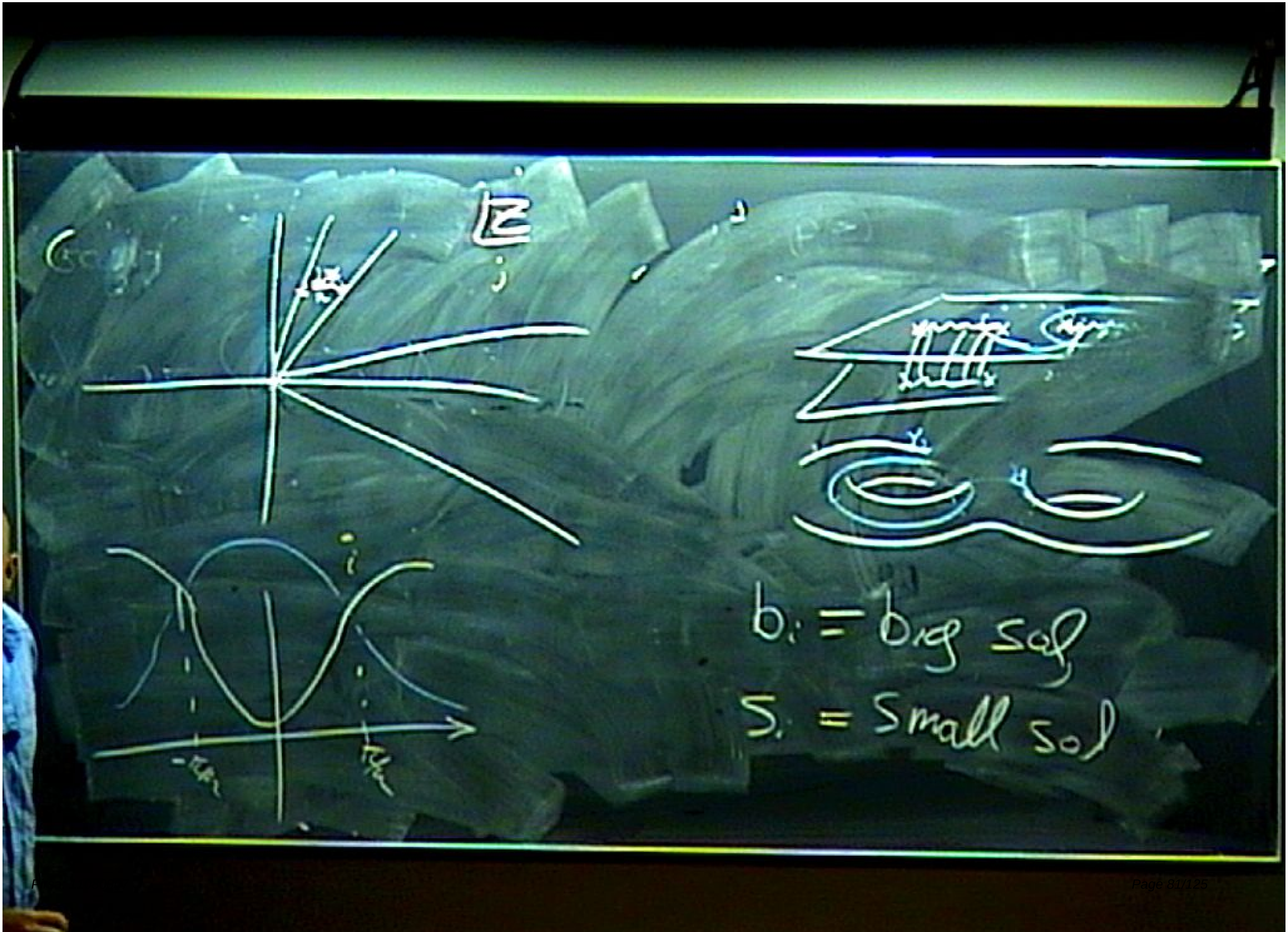


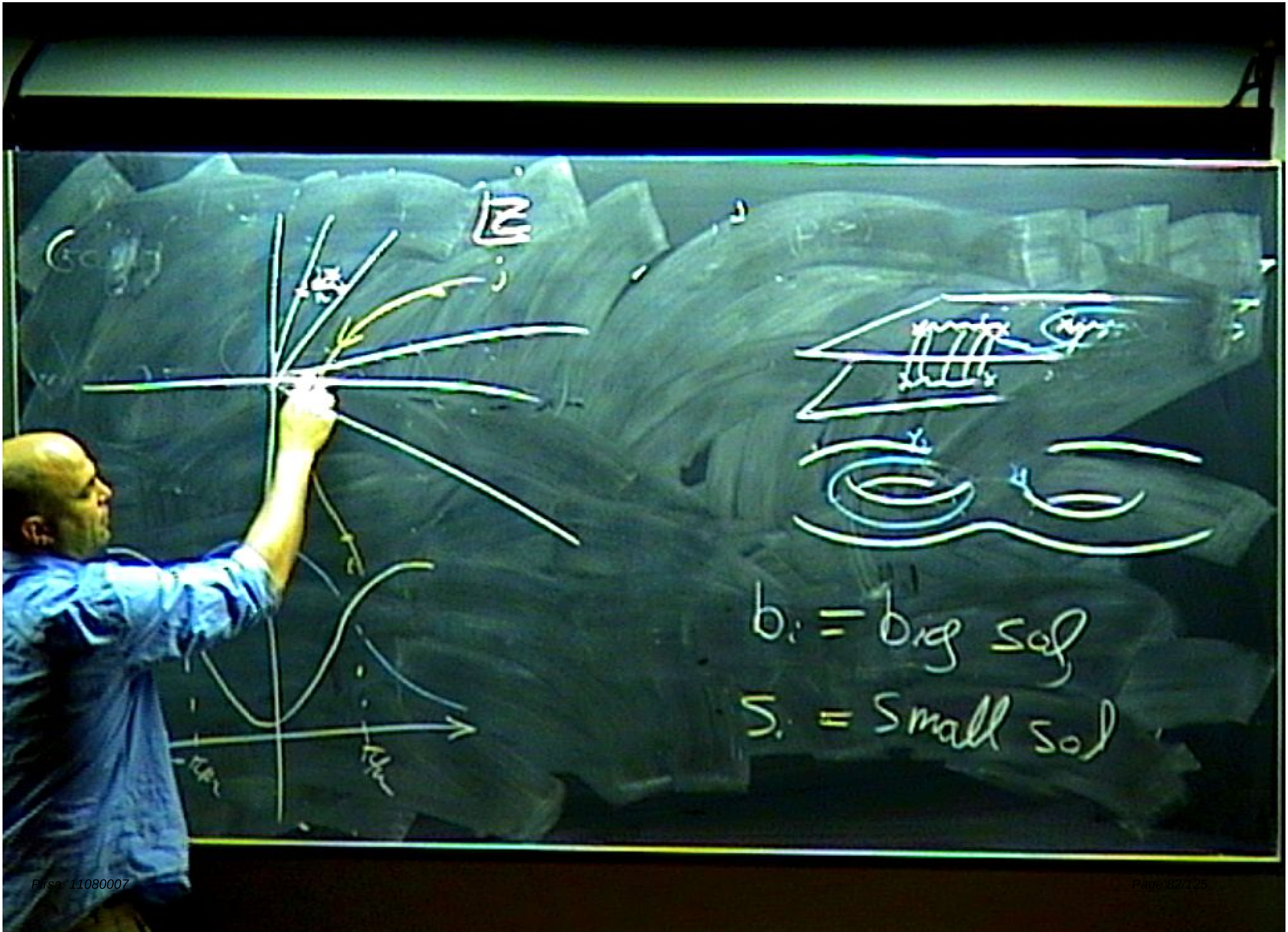



b. = big sol
S. = Small sol


b_i = big sol
S_i = small sol


The solution
<S_i S_j>
log Y_s(θ) = [cosh θ log x_s^∓ ∓ i sin
R±i0
e^{-θ} A_s − e^{θ} B_s


b_i = big sol
s_i = small sol


b_i = big sol
s_i = small sol

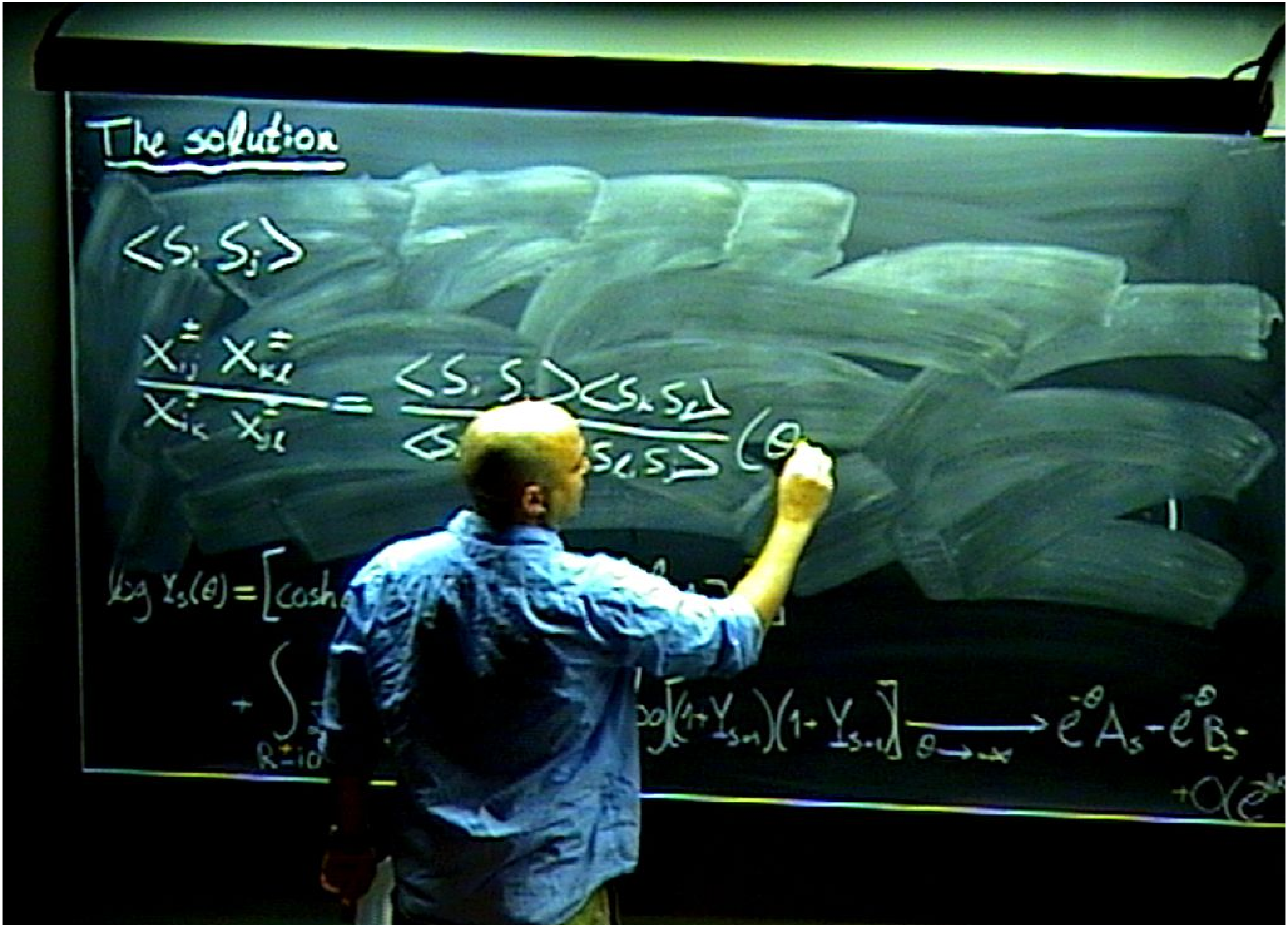
The solution

$\langle S_i S_j \rangle$

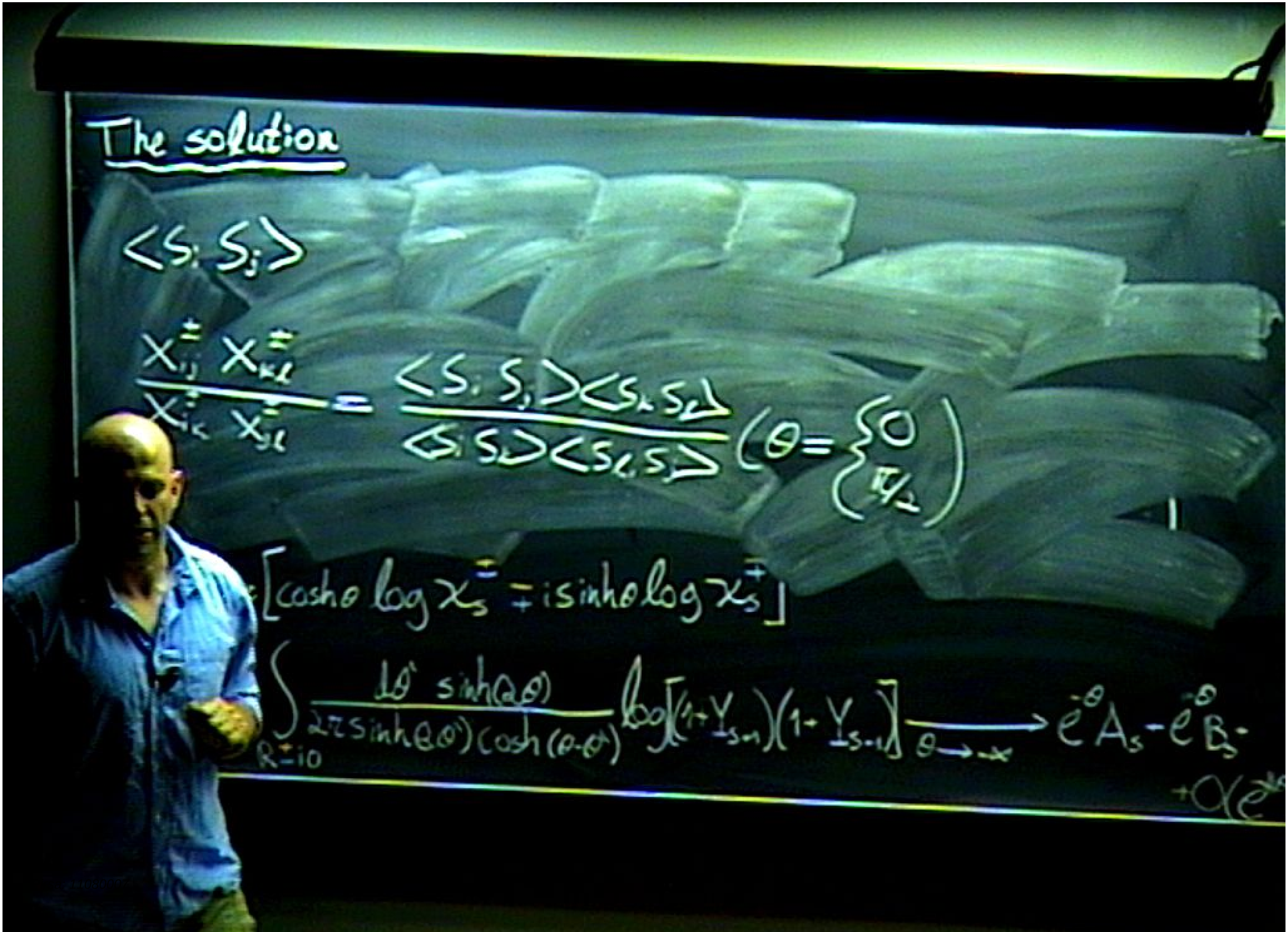
$$\frac{X^{\pm}_{ij}\, X^{\pm}_{k\ell}}{X^{\pm}_{ik}\, X^{\pm}_{j\ell}} =$$

$$\log Y_s(\theta) = \left[\cosh\theta \log x_s^{\mp} \mp i \sinh\theta \log x_s^{\mp}\right]$$

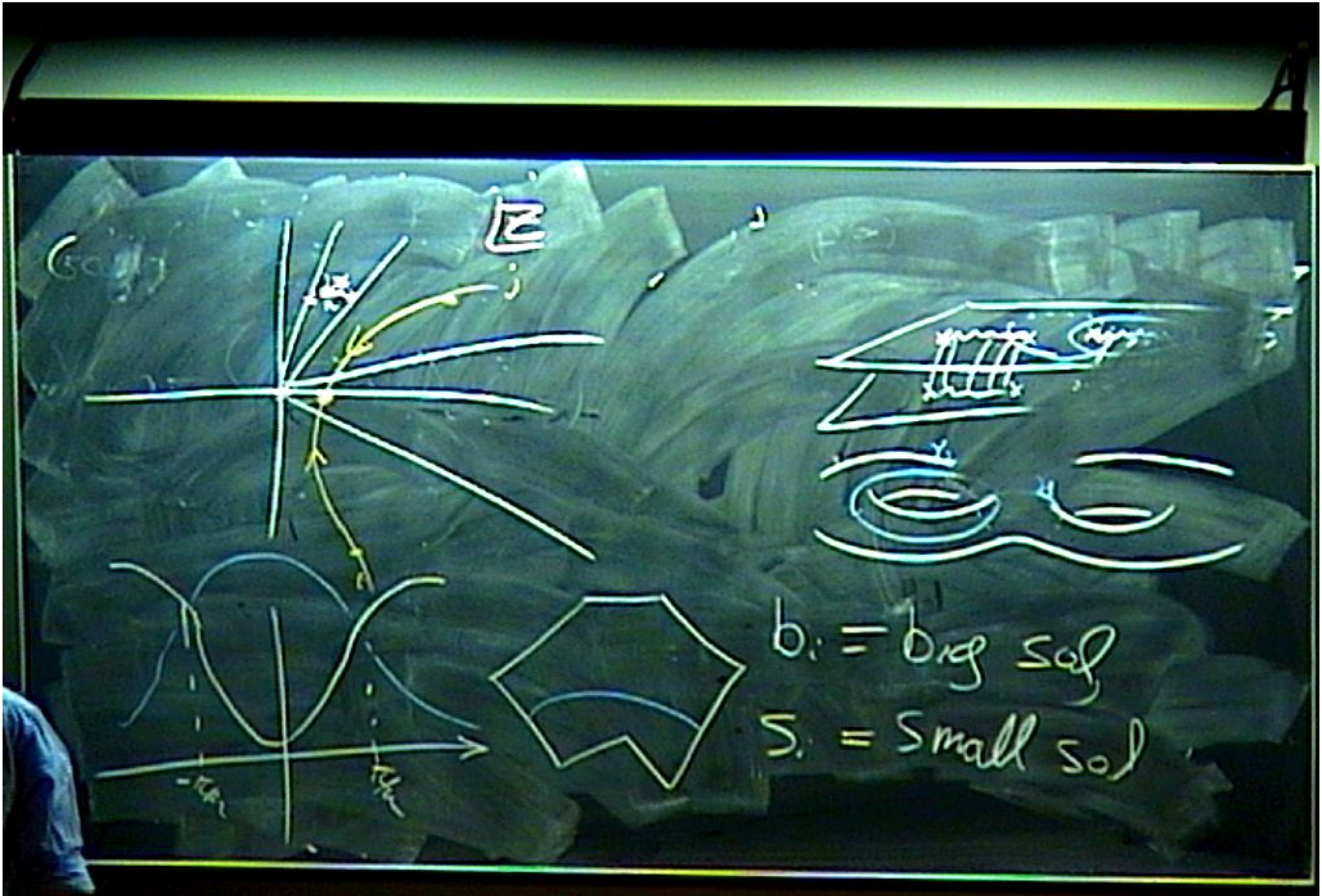
$$+ \int_{\mathbb{R} \pm i0} \frac{d\theta' \sinh(2\theta')}{2\pi \sinh(2\theta')\cosh(\theta-\theta')} \log\left[(1+Y_{s+1})(1+Y_{s-1})\right] \xrightarrow[\theta \to -\infty]{} e^{-\theta} A_s - e^{-\theta} B_s - + O(e^{\ldots})$$



The solution



The solution
<S_i S_j>
X_ij X_kl / X_ik X_jl = <S_i S_j><S_k S_l> / <S_i S_l><S_k S_j> (θ = {0, π/2)
[cosh θ log x_s^± ∓ i sinh θ log x_s^∓]
∫ dθ' sinh(2θ) / 2π sinh(θ-θ') cosh(θ-θ') log[(1+Y_{s-1})(1+Y_{s+1})]
R±i0
θ → ∞
e^θ A_s − e^{-θ} B_s

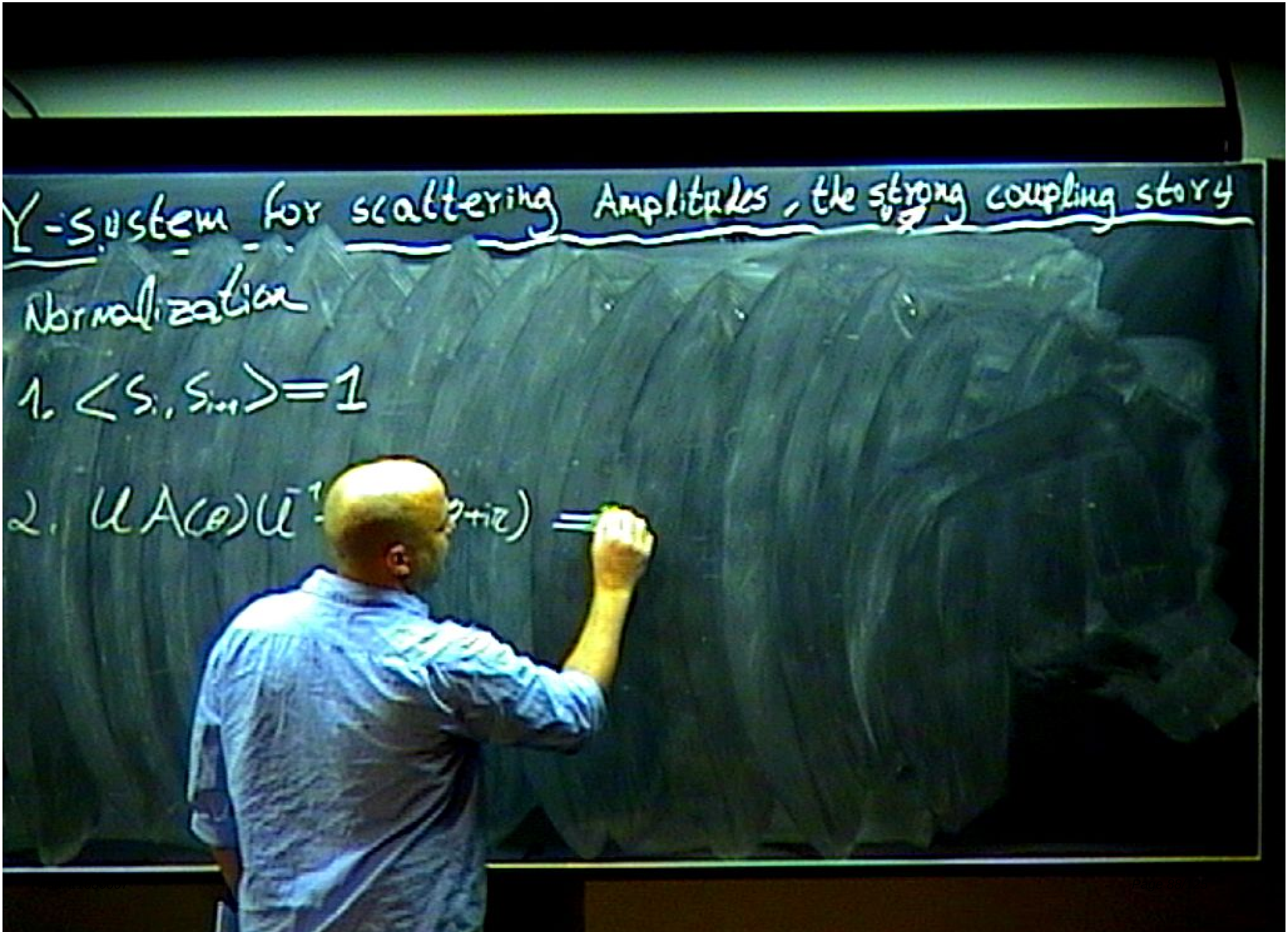


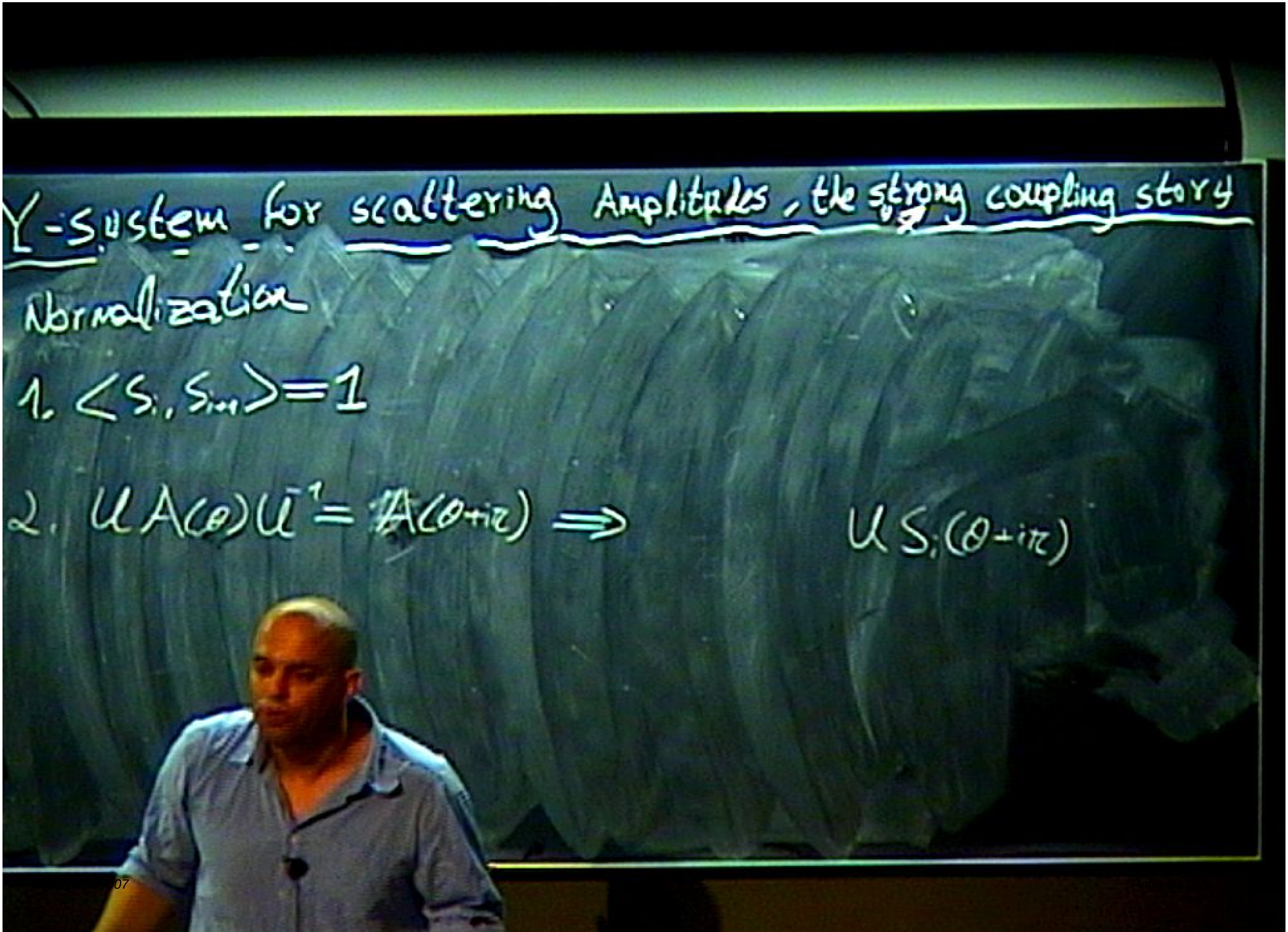
b_i = big sol
s_i = small sol

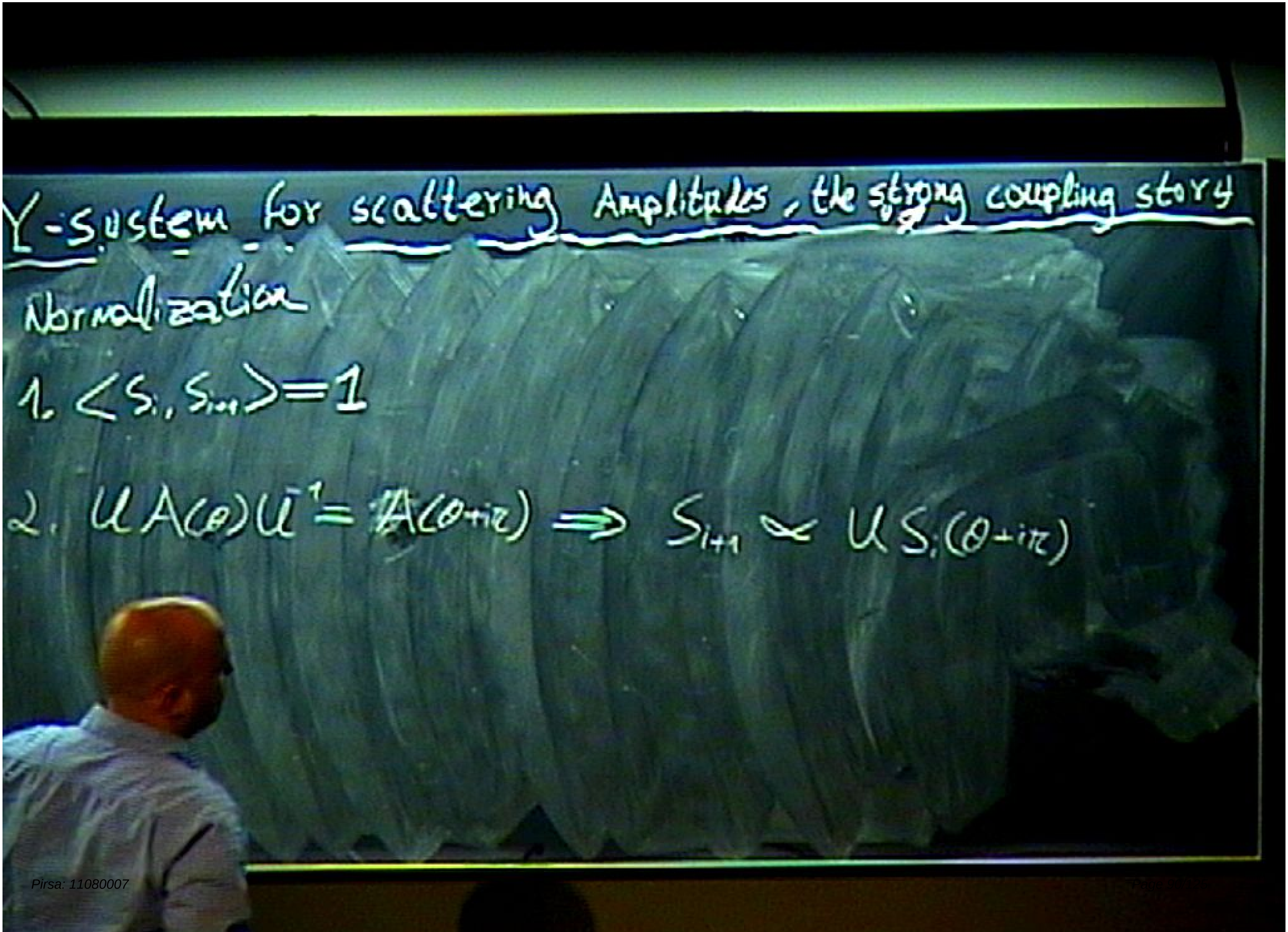
Y-system for scattering Amplitudes, the strong coupling story

Normalization

1. $\langle s_i , s_{i+1} \rangle = 1$


Y-system for scattering Amplitudes, the strong coupling story
Normalization
1. <s_i, s_{i+1}>=1


Y-system for scattering Amplitudes, the strong coupling story
Normalization
1. <S_i, S_{i+1}> = 1
2. U A(θ) U^{-1} = A(θ+iπ) ⟹
U S_i(θ+iπ)


Y-system for scattering Amplitudes, the strong coupling story
Normalization
1. $\langle S_i, S_{i+1} \rangle = 1$
2. $U A(\theta) U^{-1} = A(\theta + i\pi) \Longrightarrow S_{i+1} \propto U S_i(\theta + i\pi)$

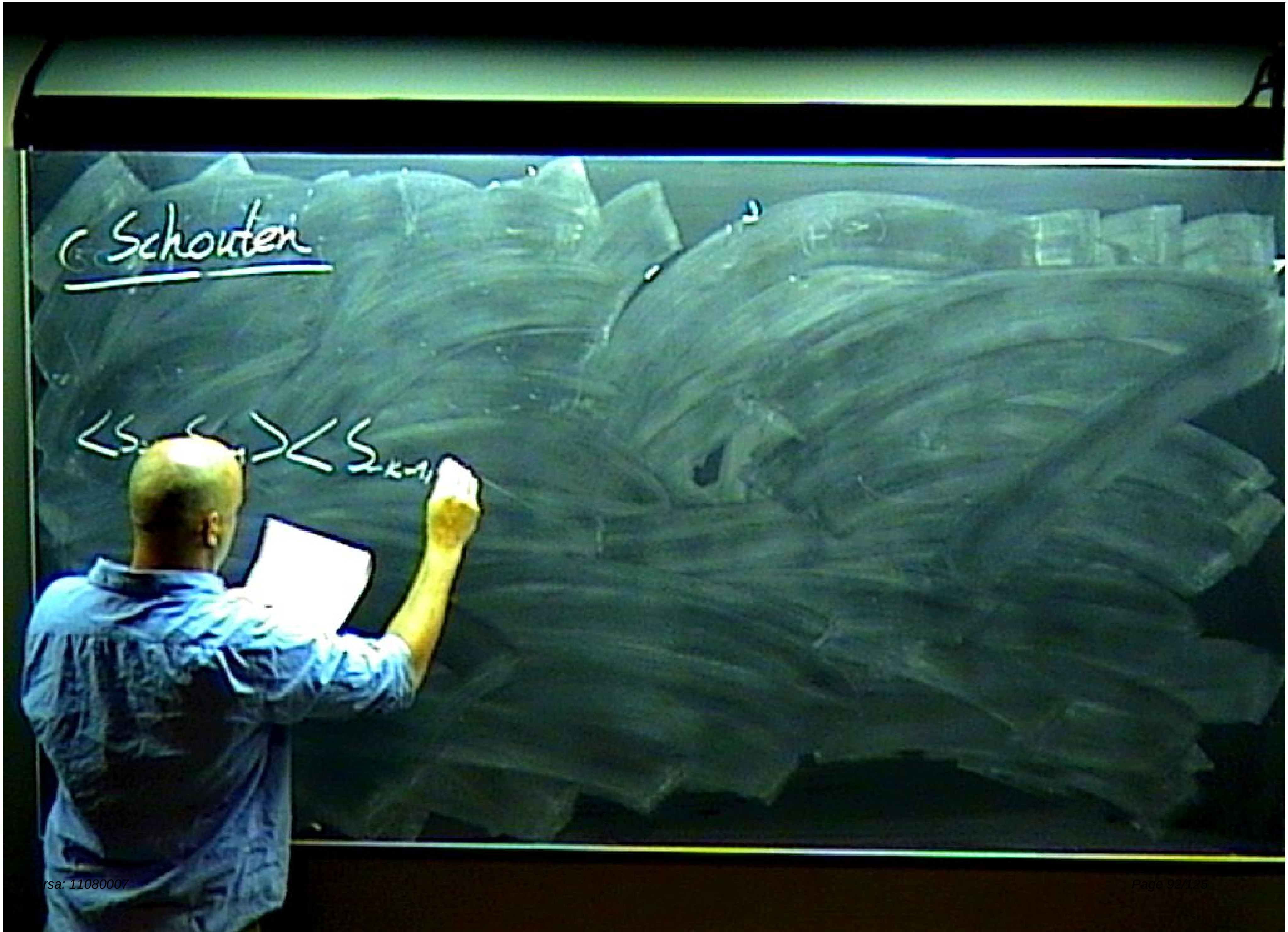
Y-system for scattering Amplitudes, the strong coupling story

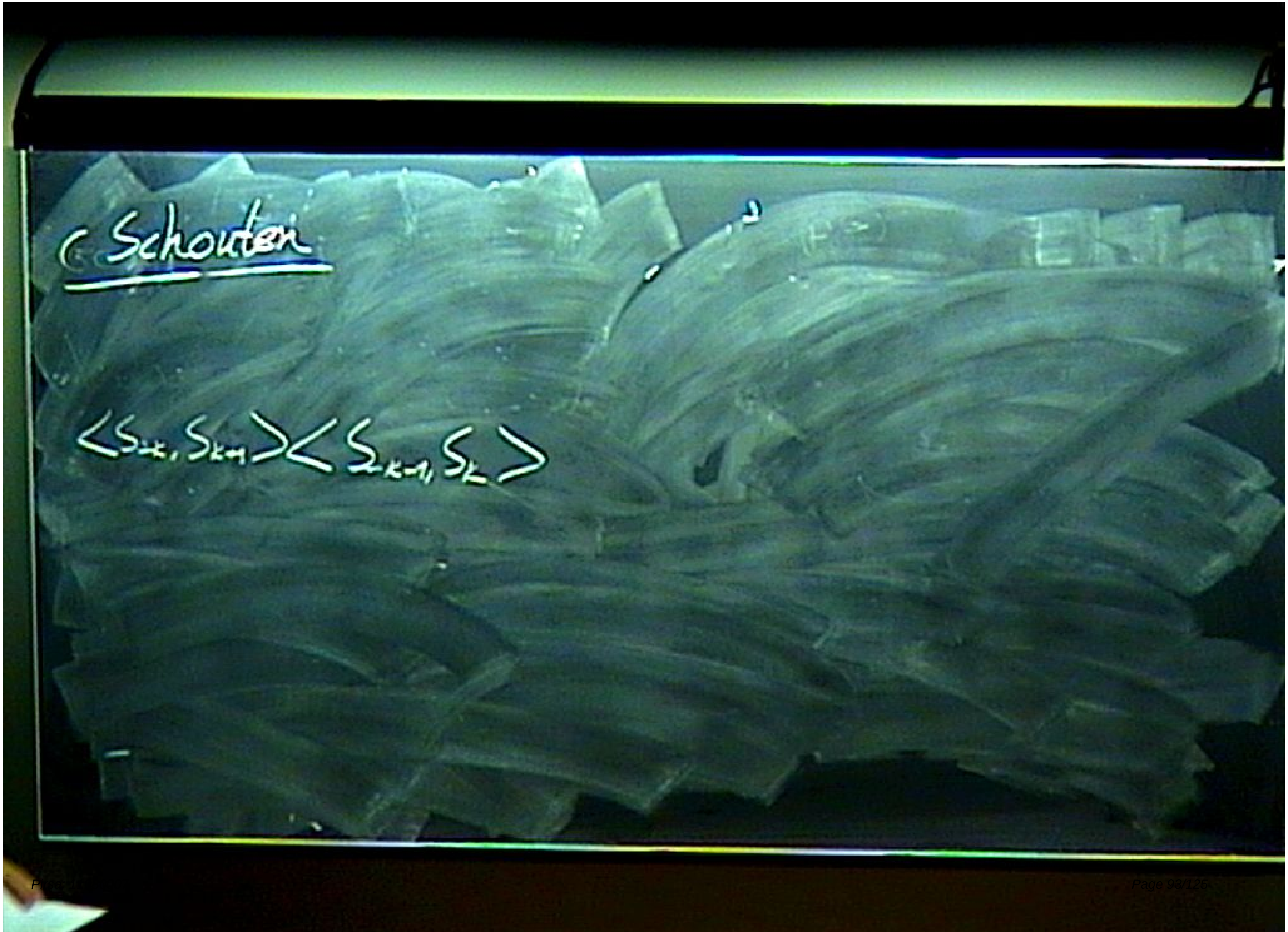
Normalization

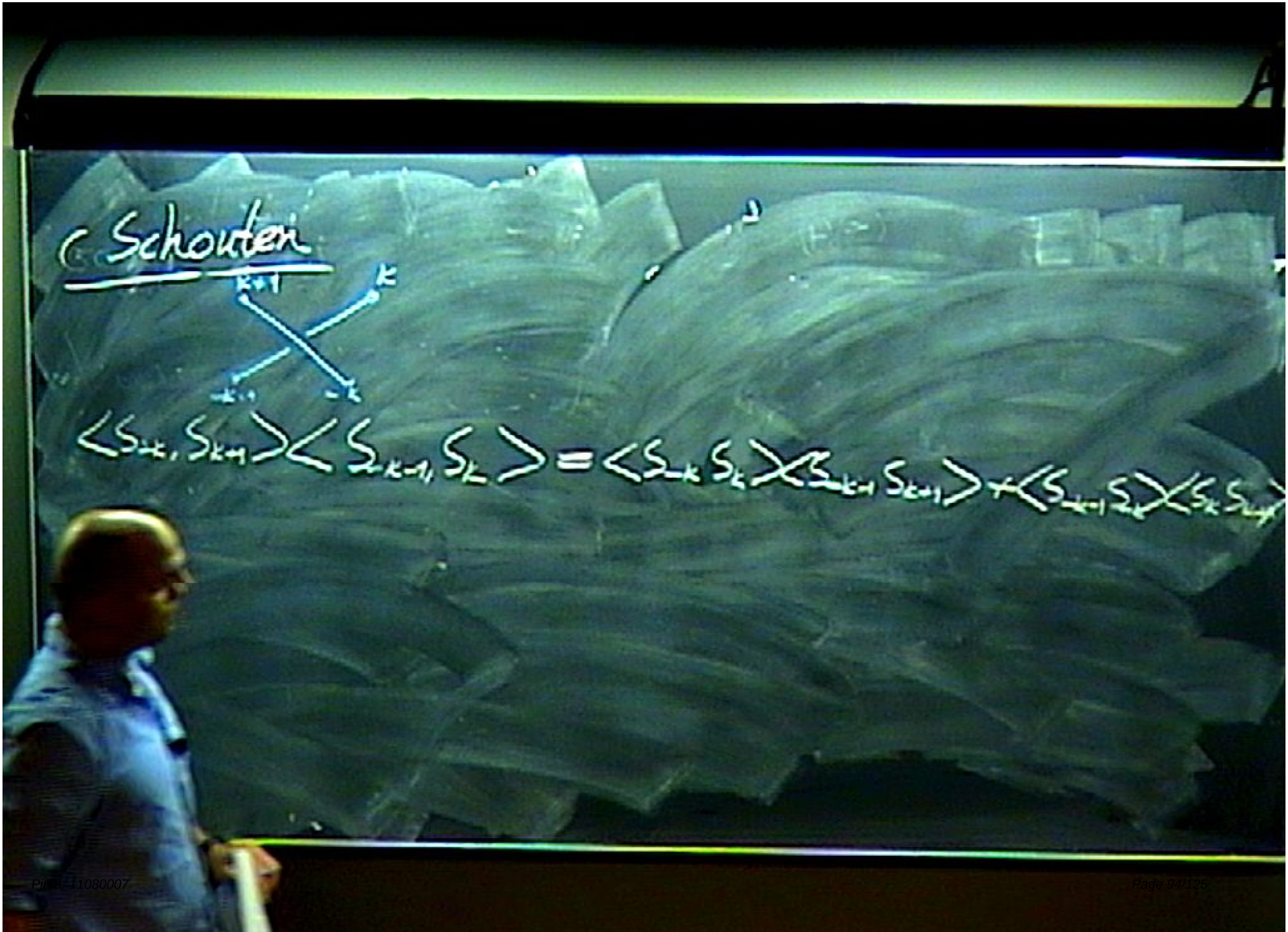
1. $\langle s_i , s_{i+1} \rangle = 1$

2. $U A(\theta) U^{-1} = A(\theta + i\pi) \Longrightarrow s_{i+1} = U s_i(\theta + i\pi)$

$$\langle s_i , s_j \rangle \equiv \det\left((s_i)(s_j) \right)$$

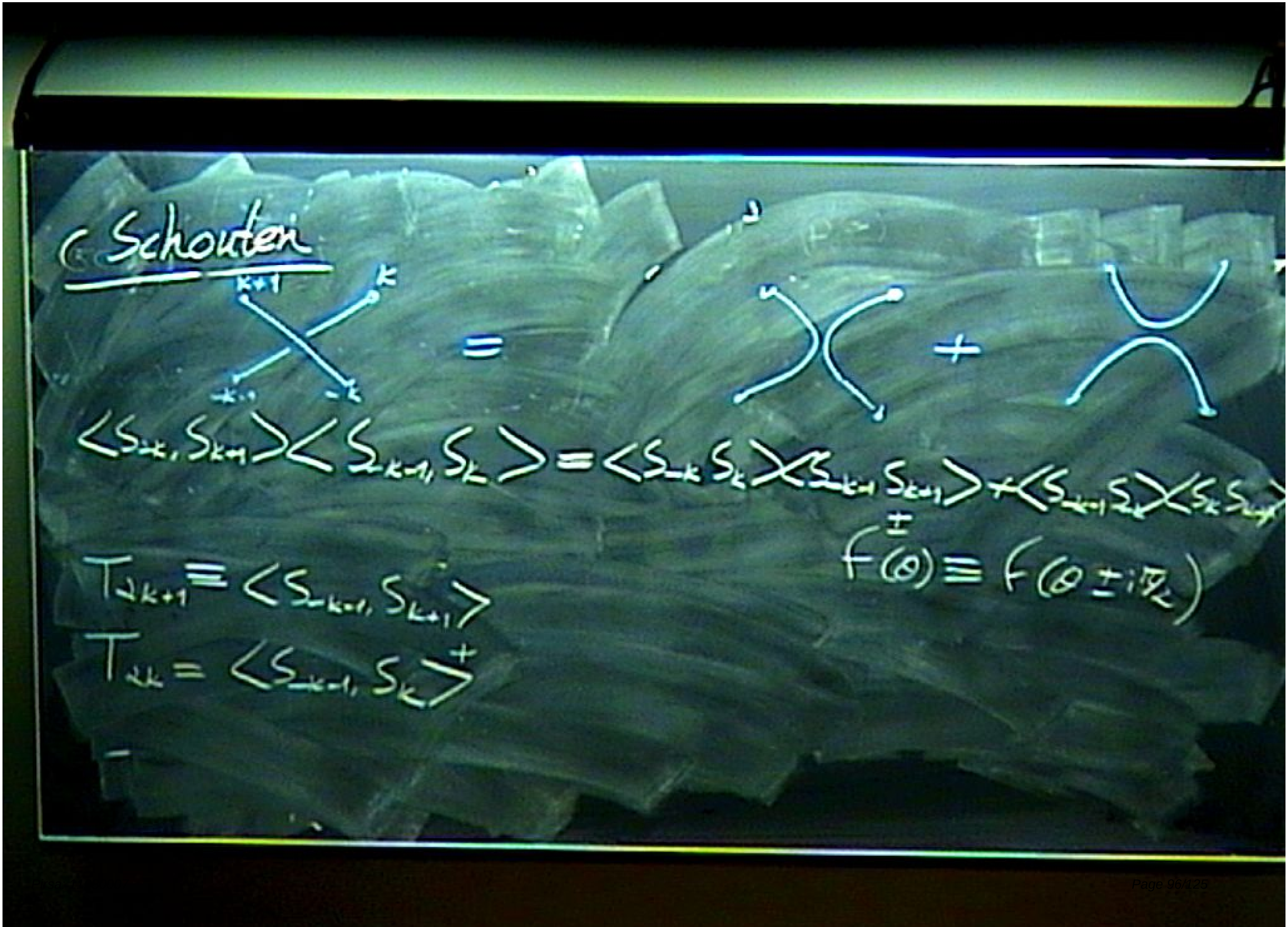

Schouten

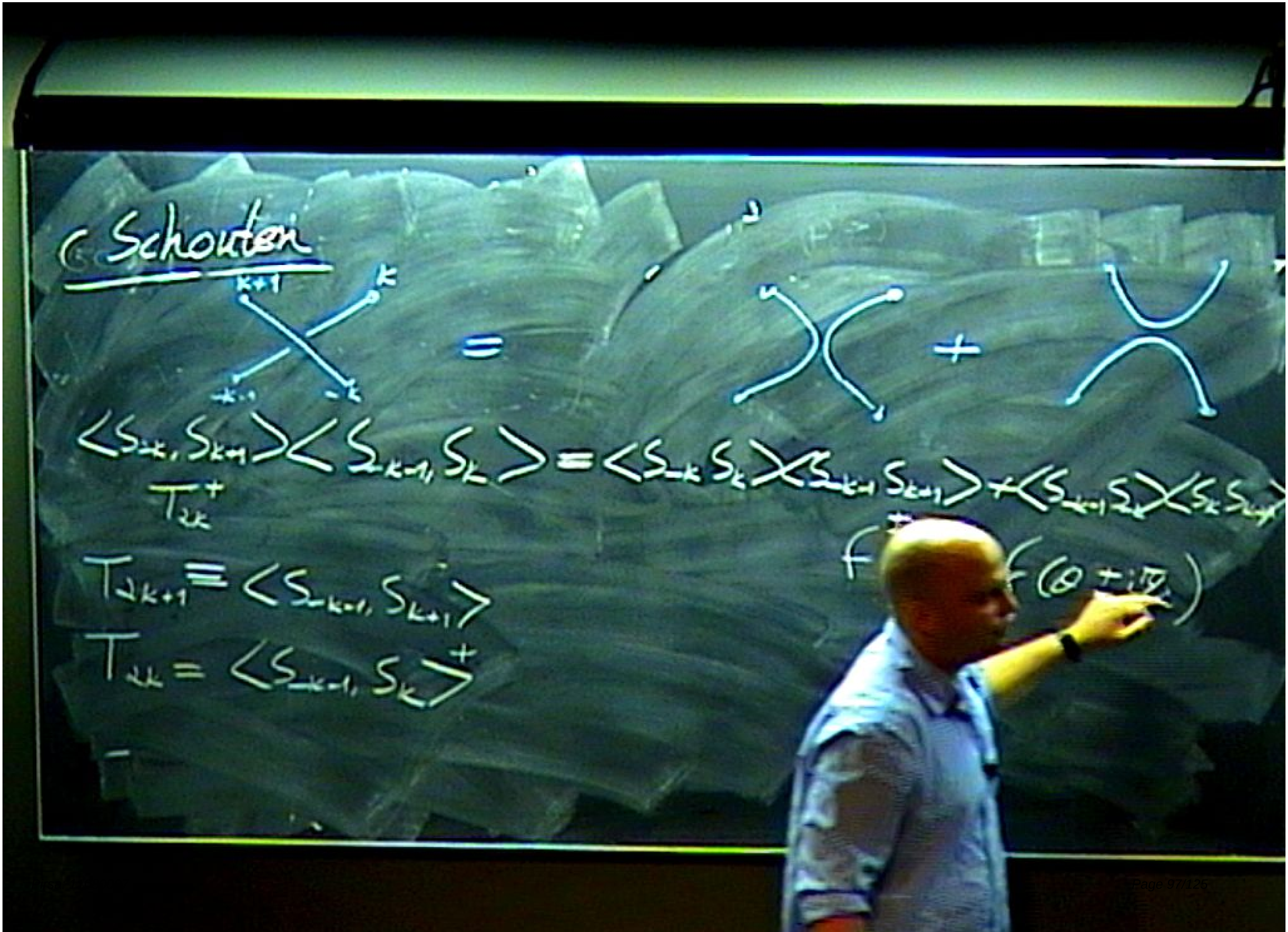

Schouten

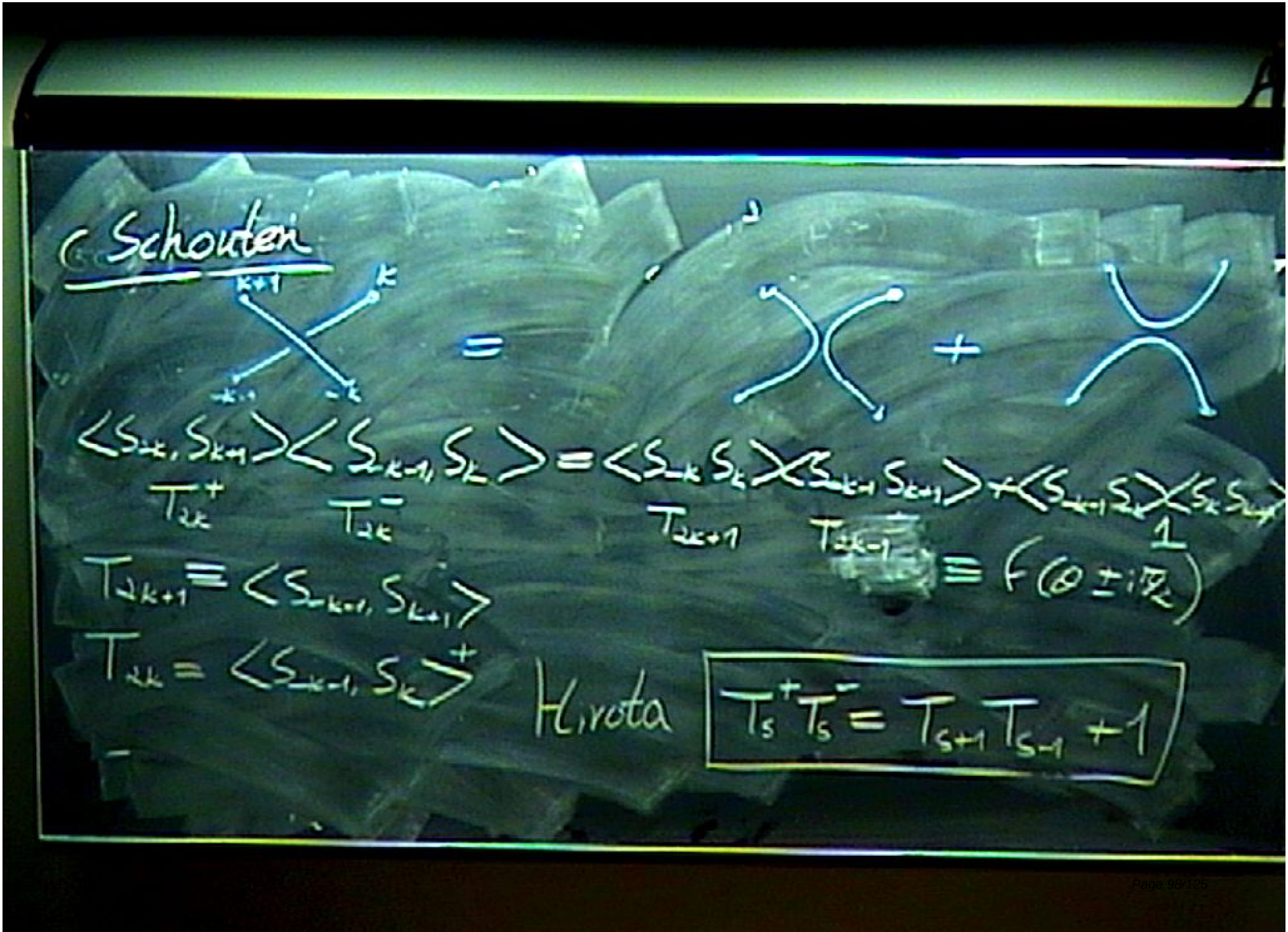

Schouten
⟨s-k, sk+1⟩⟨s-k-1, sk⟩ = ⟨s-k sk⟩⟨s-k-1 sk+1⟩ + ⟨s-k-1 s-k⟩⟨sk sk+1⟩

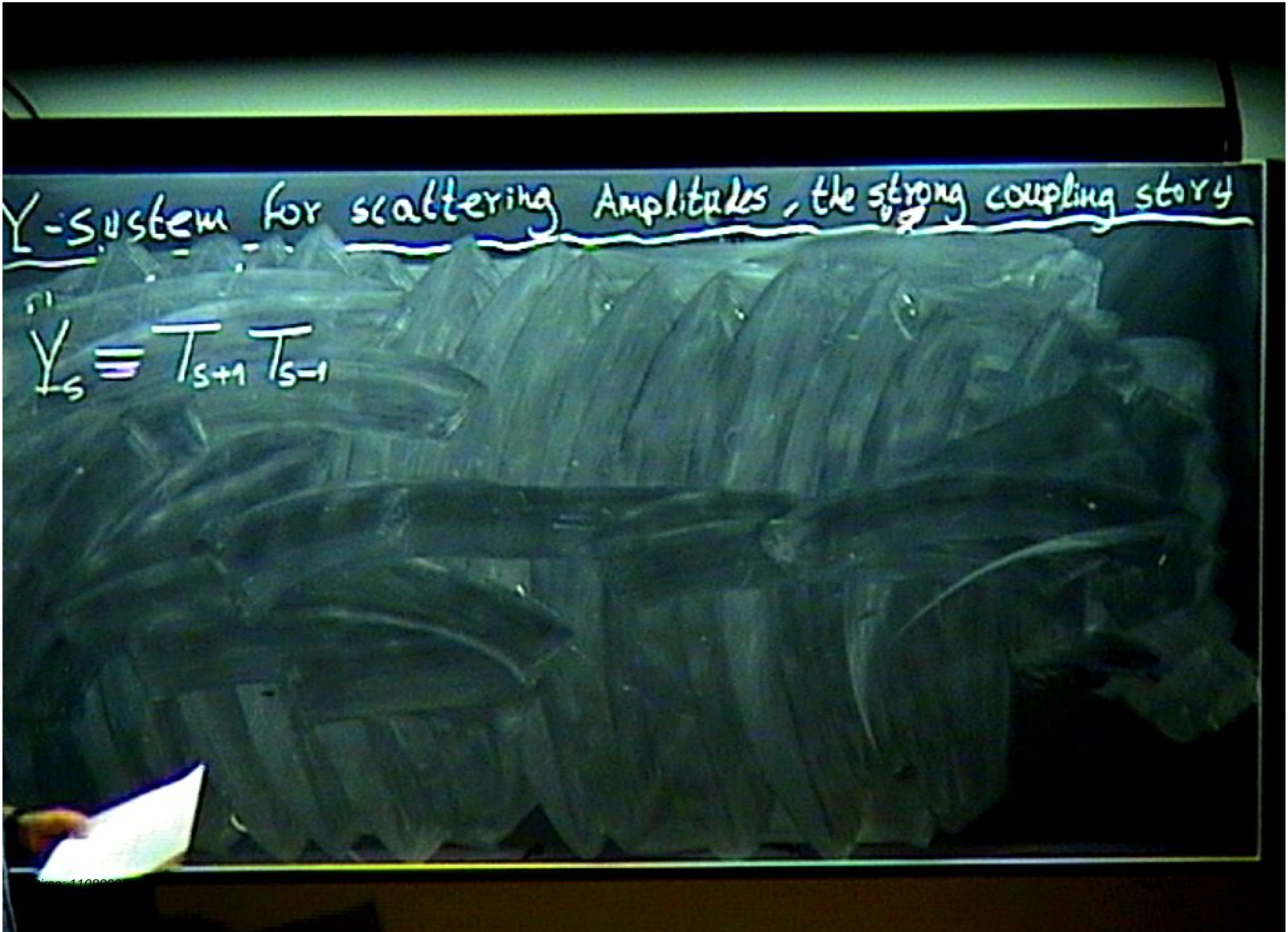
Schouten

$$\langle s_{-k}, s_{k+1}\rangle\langle s_{-k-1}, s_k\rangle = \langle s_{-k}\, s_k\rangle\langle s_{-k-1}\, s_{k+1}\rangle + \langle s_{-k-1}\, s_{-k}\rangle\langle s_k\, s_{k+1}\rangle$$


Schouten
= +
<S2k, Sk+1><S2k+1, Sk> = <S2k Sk><S2k+1 Sk+1> + <S2k+1 S2k><Sk Sk+1>
f±(θ) ≡ f(θ ± iπ/2)
T2k+1 ≡ <S2k+1, Sk+1>
T2k = <S2k+1, Sk>+

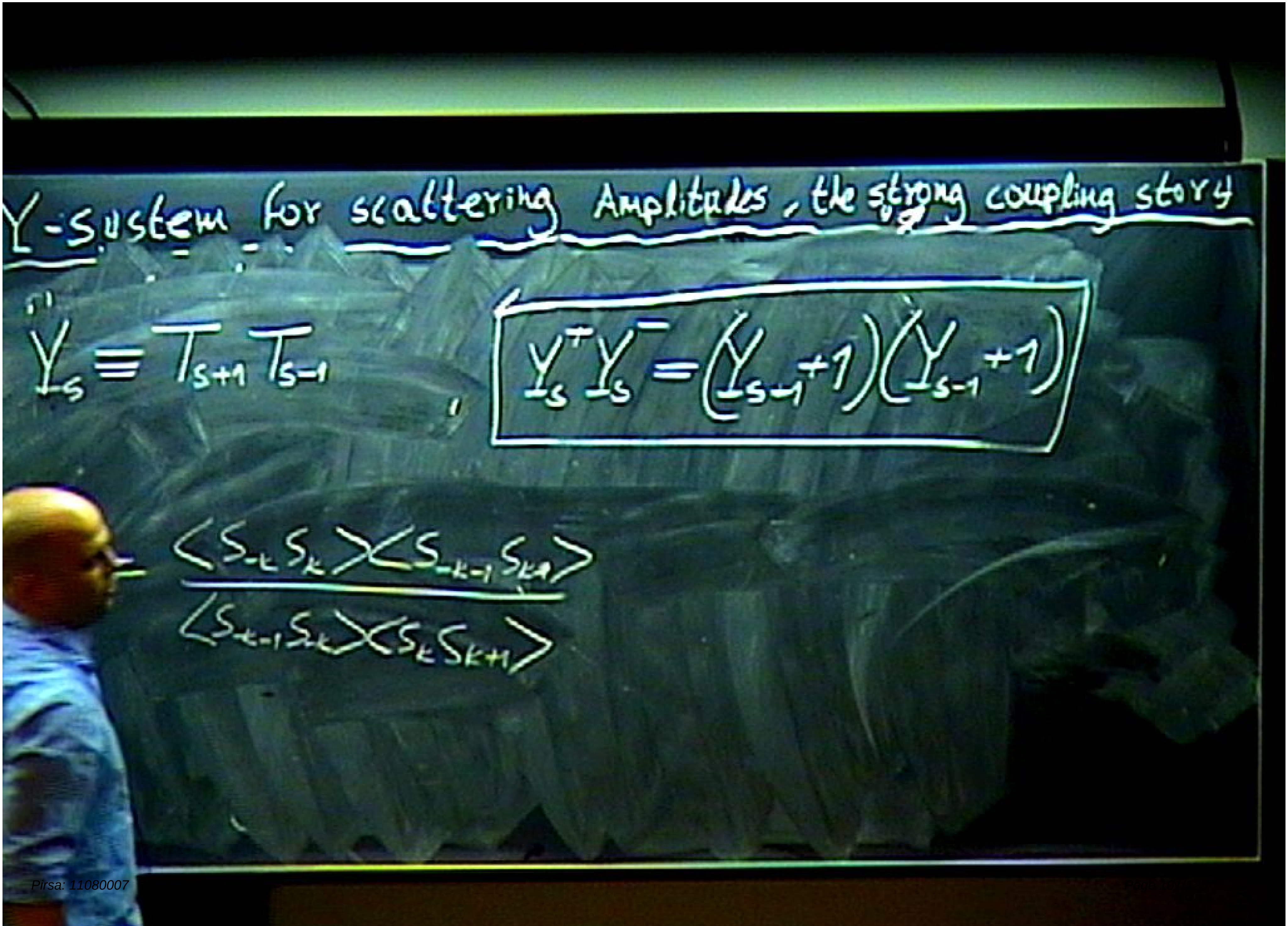

Schouten


Schouten
<S-k, Sk+1><S-k-1, Sk> = <S-k Sk><S-k-1 Sk+1> + <S-k-1 S-k><Sk Sk+1>
T+2k T-2k T2k+1 T2k-1 1
T2k+1 = <S-k-1, Sk+1>
T2k = <S-k-1, Sk>+
Hirota
Ts+ Ts- = Ts+1 Ts-1 + 1


Y-system for scattering Amplitudes, the strong coupling story
Y_s = T_{s+1} T_{s-1}

Y-system for scattering Amplitudes, the strong coupling story

$$Y_s \equiv T_{s+1} T_{s-1}\,,\qquad \boxed{Y_s^+ Y_s^- = (Y_{s+1}+1)(Y_{s-1}+1)}$$

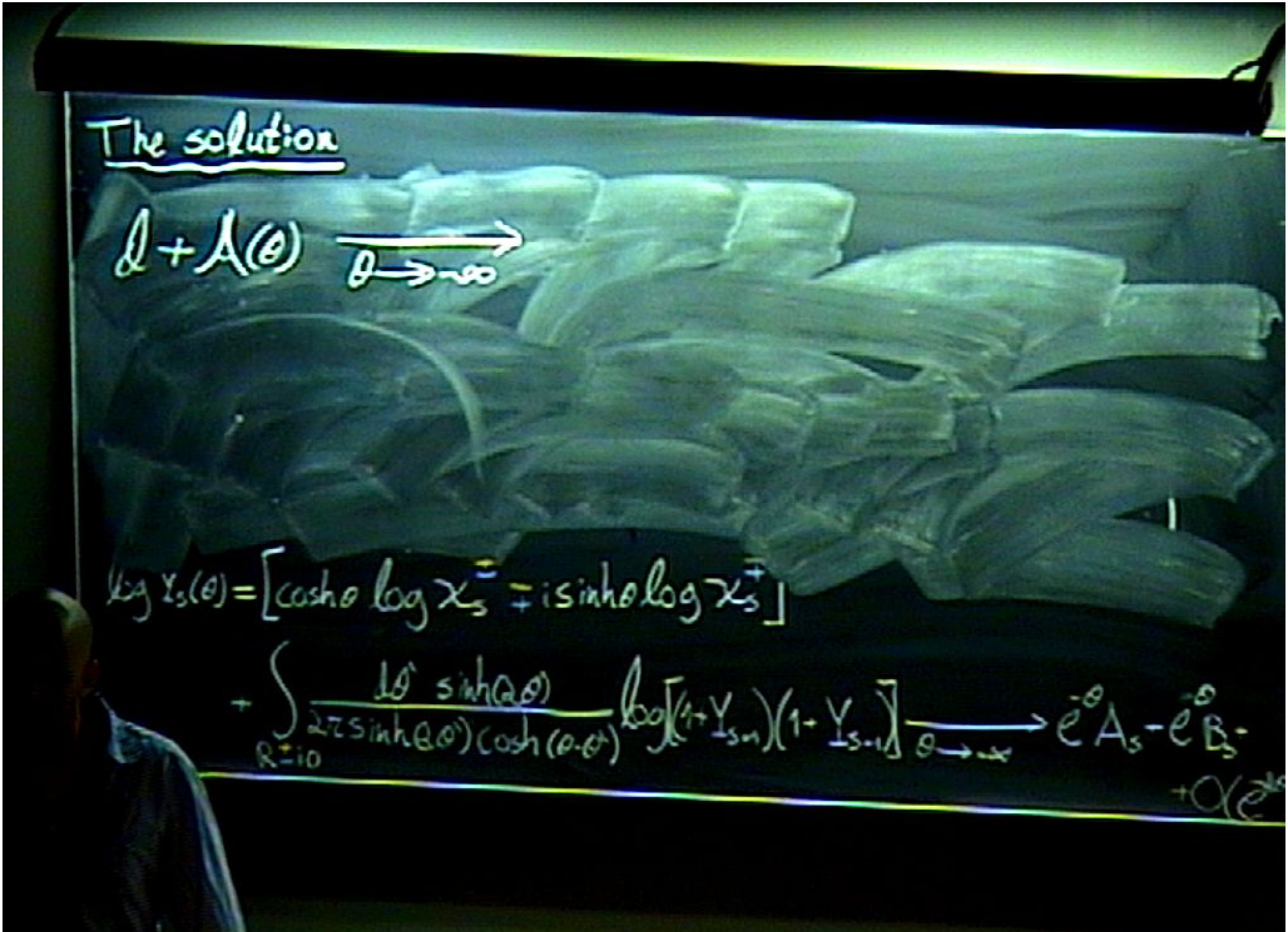


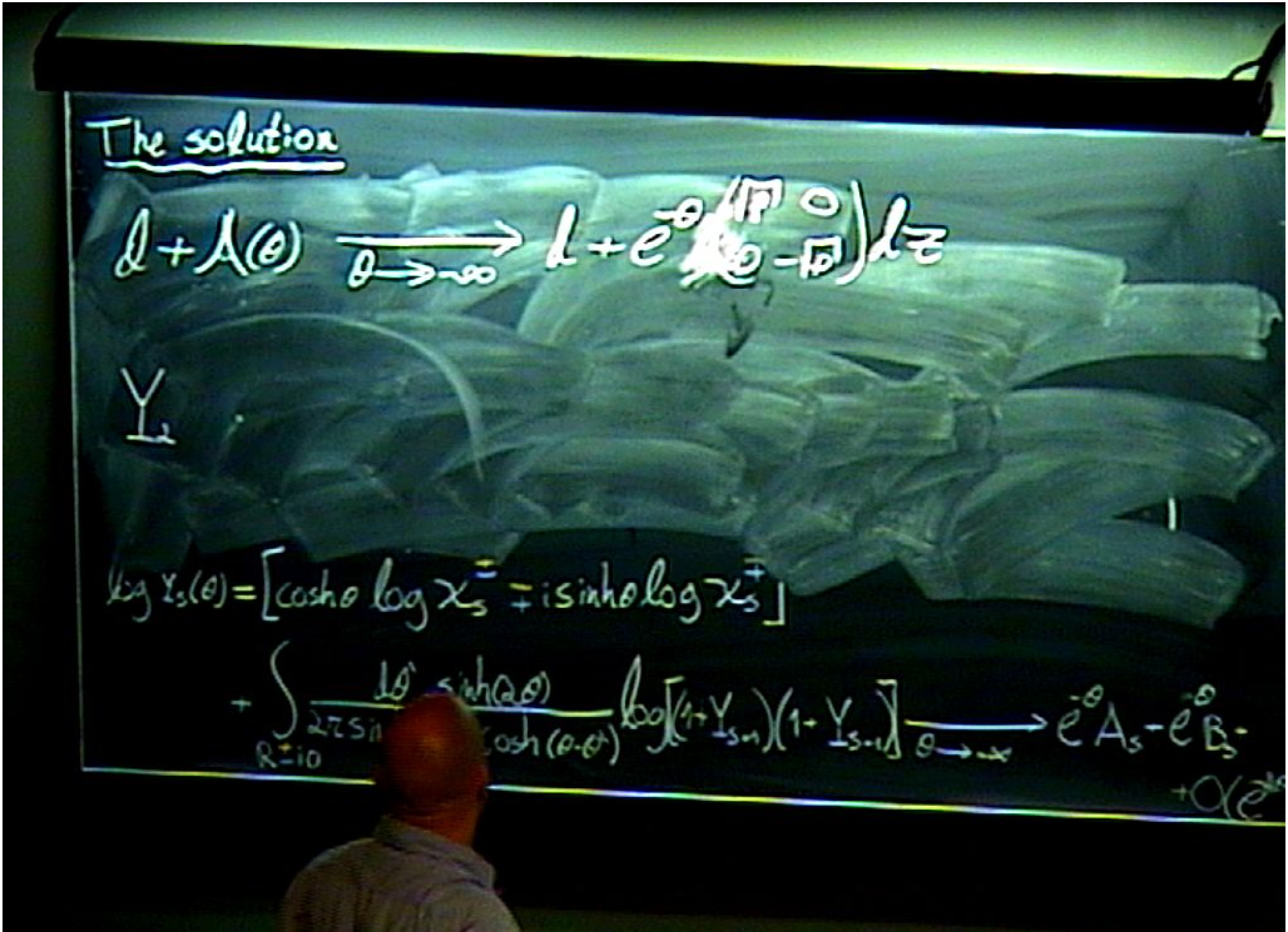
Y-system for scattering Amplitudes, the strong coupling story
$Y_s \equiv T_{s+1} T_{s-1}$
$Y_s^+ Y_s^- = (Y_{s+1}+1)(Y_{s-1}+1)$
$\frac{\langle S_{-k} S_k \rangle \langle S_{-k-1} S_{k+1} \rangle}{\langle S_{-k-1} S_{-k} \rangle \langle S_k S_{k+1} \rangle}$

Y-system for scattering Amplitudes, the strong coupling story

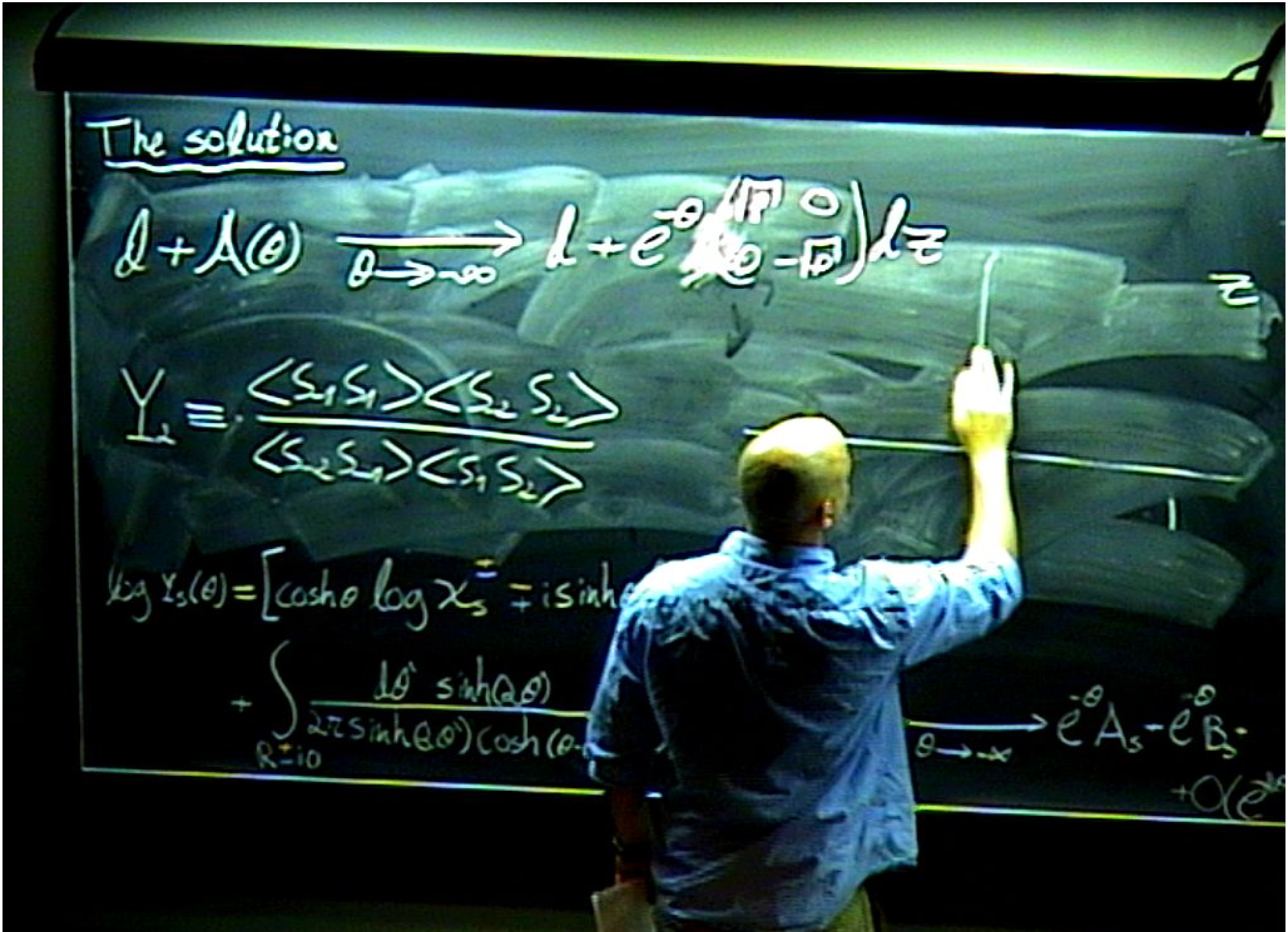
$$Y_s \equiv T_{s+1} T_{s-1} \,, \qquad \boxed{Y_s^+ Y_s^- = (Y_{s+1}+1)(Y_{s-1}+1)}$$

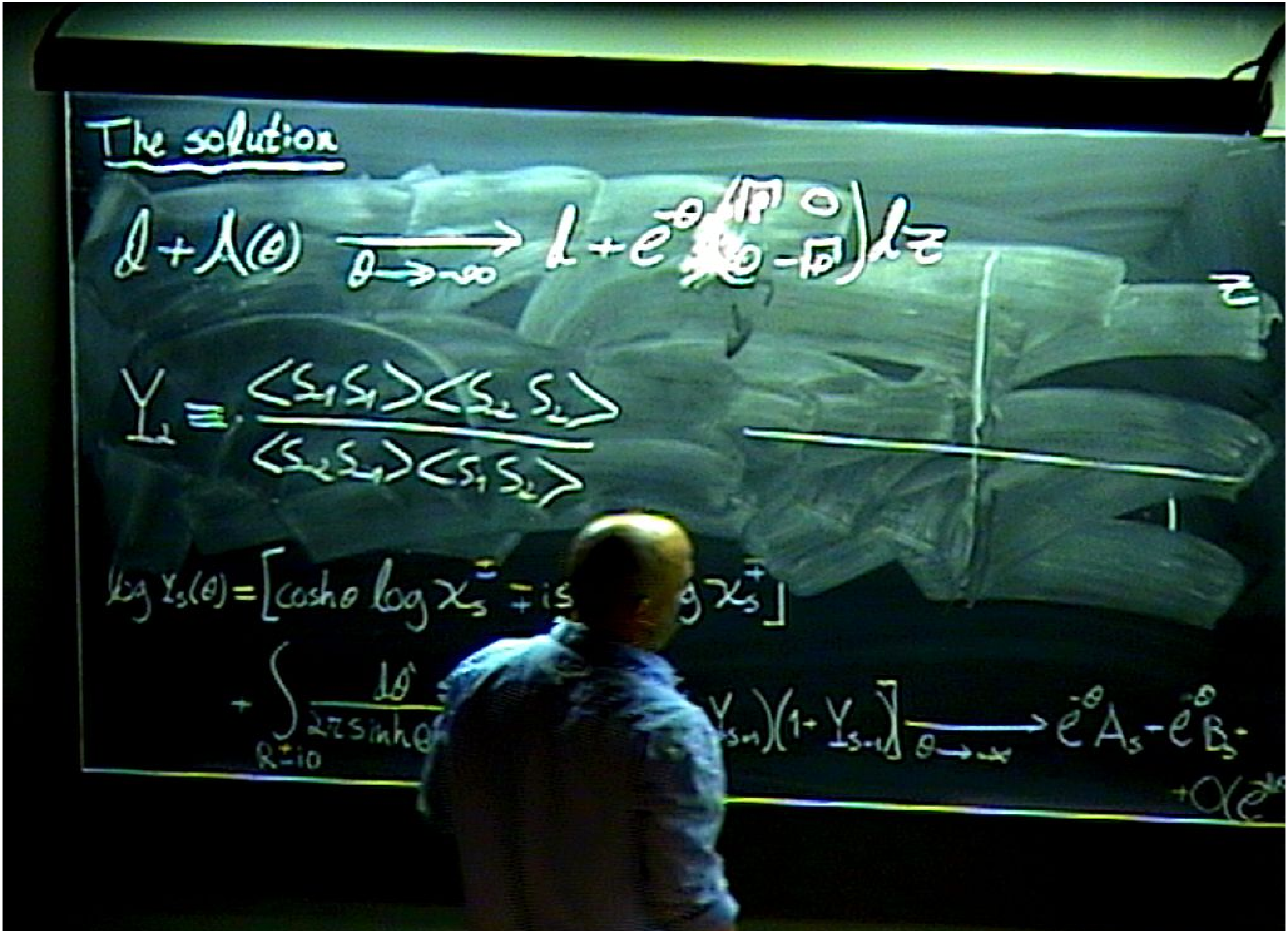
$$Y_{2k} = \frac{\langle s_{-k} s_k \rangle \langle s_{-k-1} s_{k+1} \rangle}{\langle s_{-k-1} s_{-k} \rangle \langle s_k s_{k+1} \rangle}$$


The solution
$1+\lambda(\theta) \xrightarrow[\theta\to-\infty]{}$
$\log Y_s(\theta) = [\cosh\theta \log x_s^{\mp} \mp i \sinh\theta \log x_s^{\mp}]$
$+ \int_{R\pm i0} \frac{d\theta' \sinh(2\theta')}{2\pi \sinh(2\theta')\cosh(\theta-\theta')} \log[(1+Y_{s+1})(1+Y_{s-1})] \xrightarrow[\theta\to-\infty]{} e^{-\theta}A_s - e^{-\theta}B_s -$



The solution
log Y_s(θ) = [cosh θ log x_s^∓ ∓ i sinh θ log x_s^∓]
R±i0


The solution
d + A(θ) → d + e^{-θ}(...) dz   (θ → -∞)
Y_2 ≡ ⟨s_1 s_1⟩⟨s_2 s_2⟩ / ⟨s_2 s_1⟩⟨s_1 s_2⟩
log Y_s(θ) = [cosh θ log x_s^∓ ∓ i sinh
+ ∫_{R∓i0} dθ' sinh(2θ) / 2π sinh(2θ') cosh(θ-
→ e^{-θ} A_s - e^{-θ} B_s


The solution

The solution

$$d + A(\theta) \xrightarrow[\theta \to -\infty]{} d + e^{-\theta}\begin{pmatrix}\sqrt{i} & 0 \\ 0 & -\sqrt{i}\end{pmatrix} dz$$

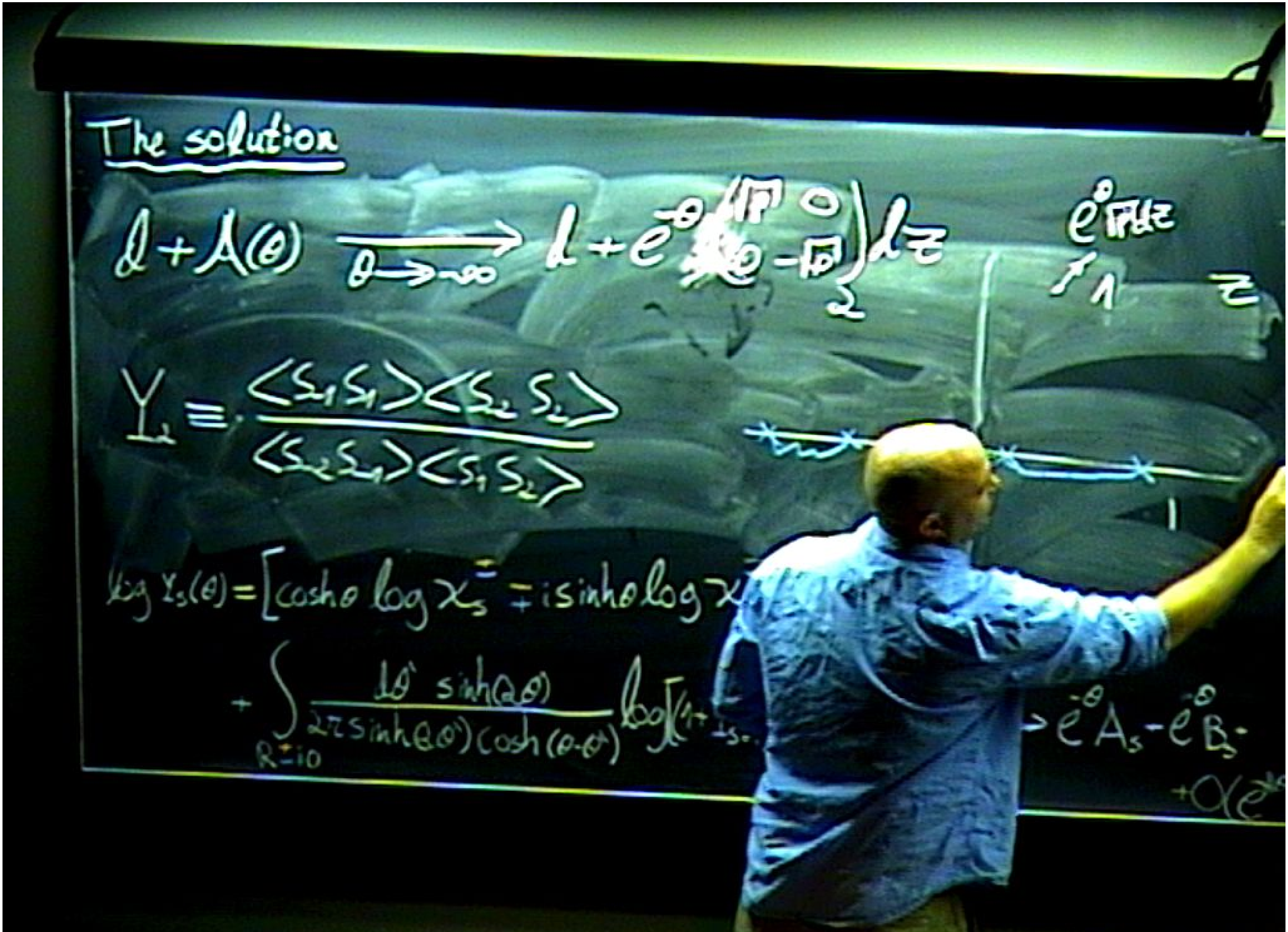
$e^{\sqrt{i}\, z}$

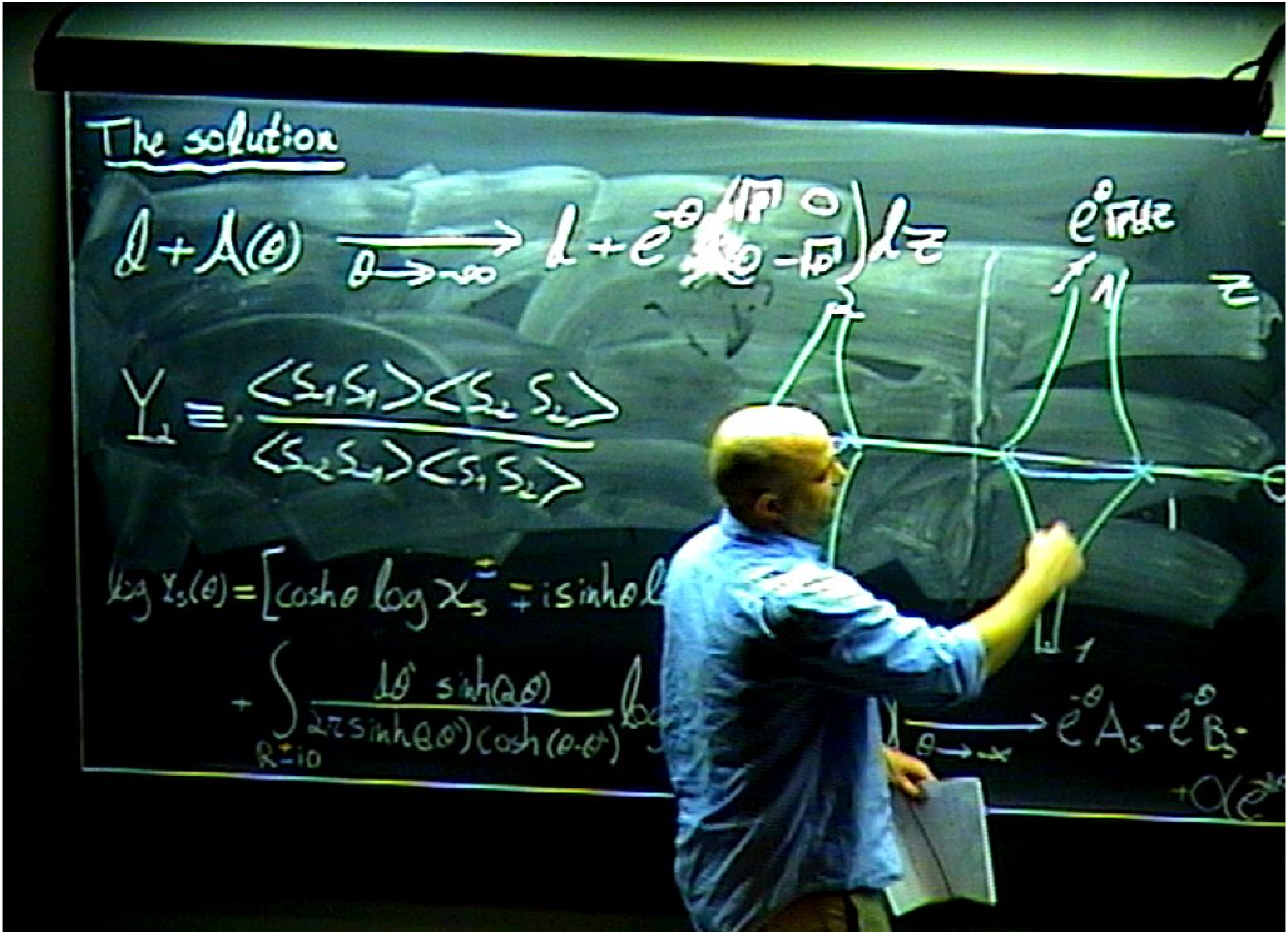
z

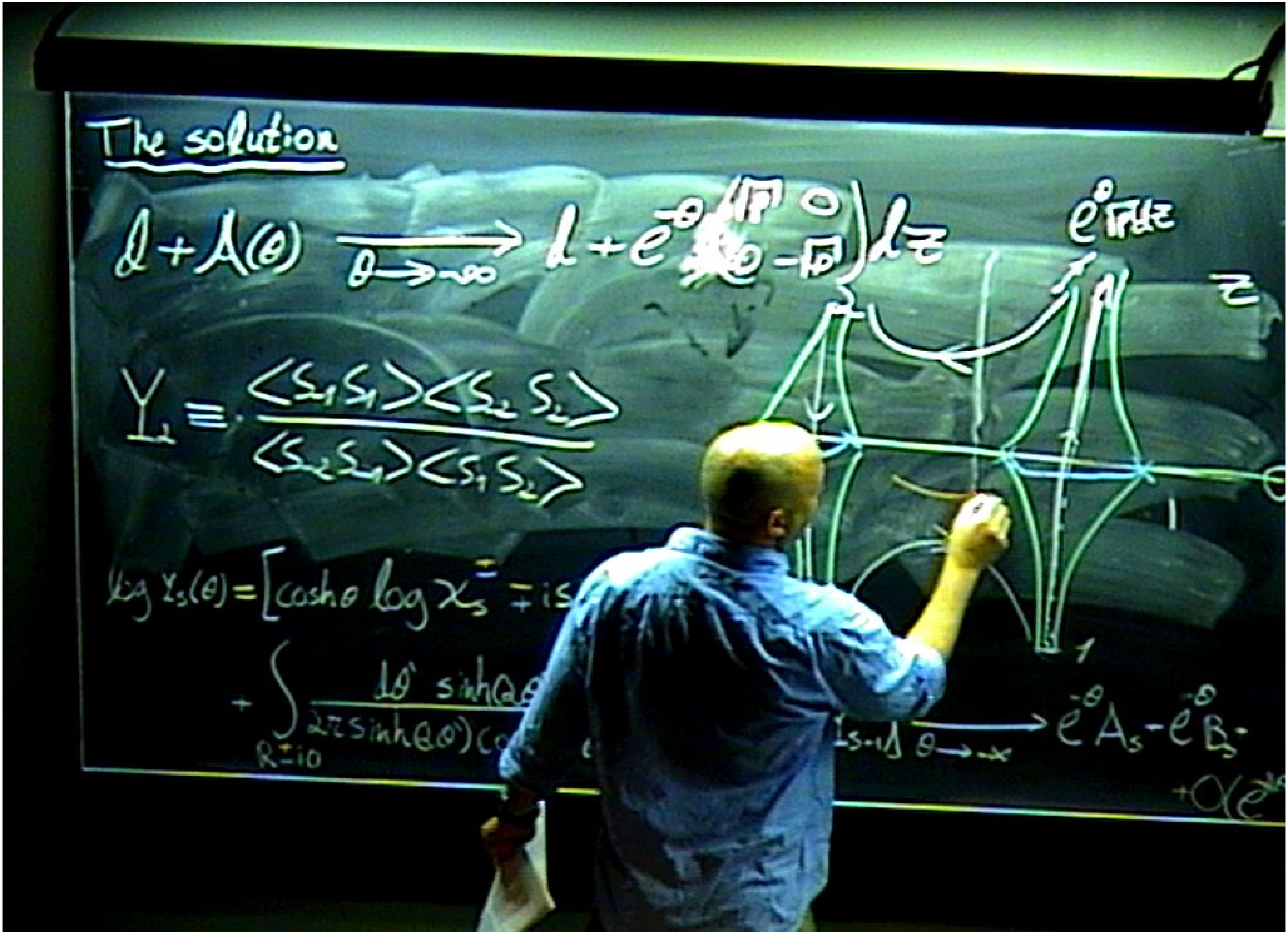
$$Y_1 \equiv \frac{\langle s_1 s_1 \rangle \langle s_2 s_2 \rangle}{\langle s_{-2} s_{-1} \rangle \langle s_1 s_2 \rangle}$$

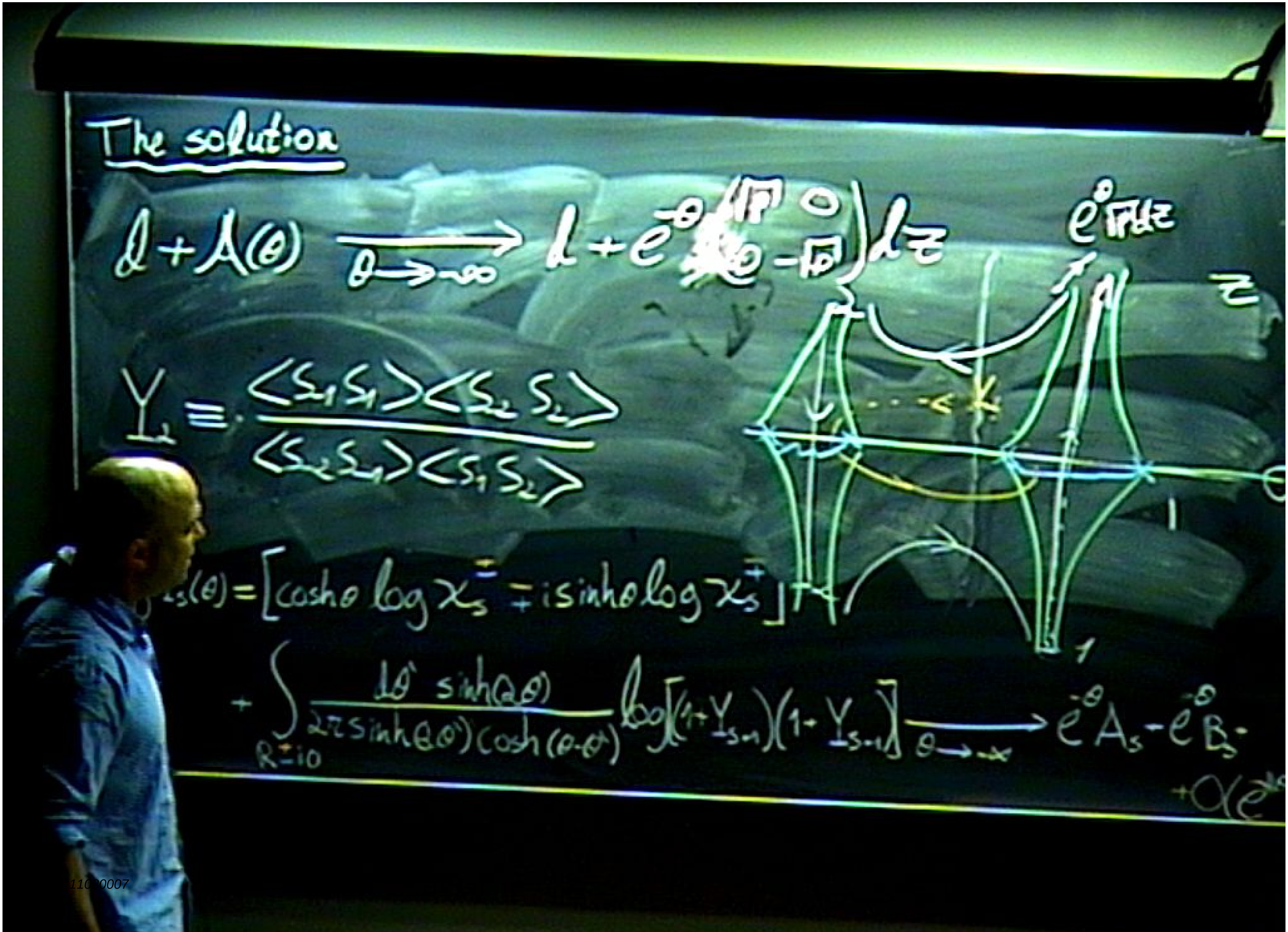
$$\log Y_s(\theta) = \left[\cosh\theta \log x_s^{\mp} \mp i \sinh\theta \log x_s^{\pm}\right]$$

$$+ \int_{\mathbb{R}\pm i0} \frac{d\theta' \sinh(2\theta')}{2\pi \sinh(\theta-\theta')\cosh(\theta-\theta')} \log\left[(1+Y_{s+1})(1+Y_{s-1})\right] \xrightarrow[\theta \to \infty]{} e^{-\theta} A_s - e^{\theta} B_s + O(e^{\dots})$$


The solution
Y ≡ ⟨s₁s₁⟩⟨s₂s₂⟩ / ⟨s₂s₁⟩⟨s₁s₂⟩
θ → −∞
log Y_s(θ) = [cosh θ log x_s ∓ i sinh θ log x

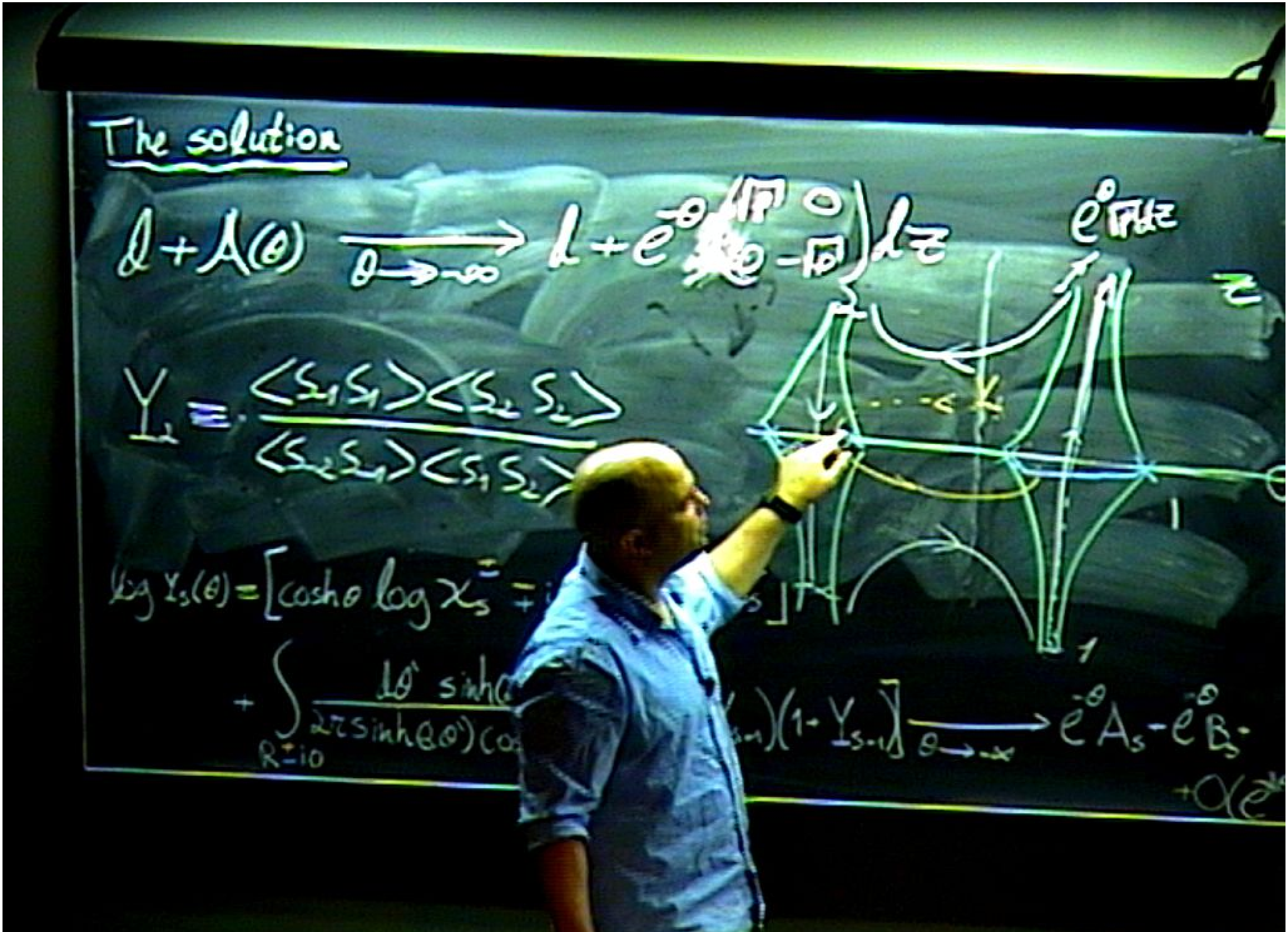

The solution

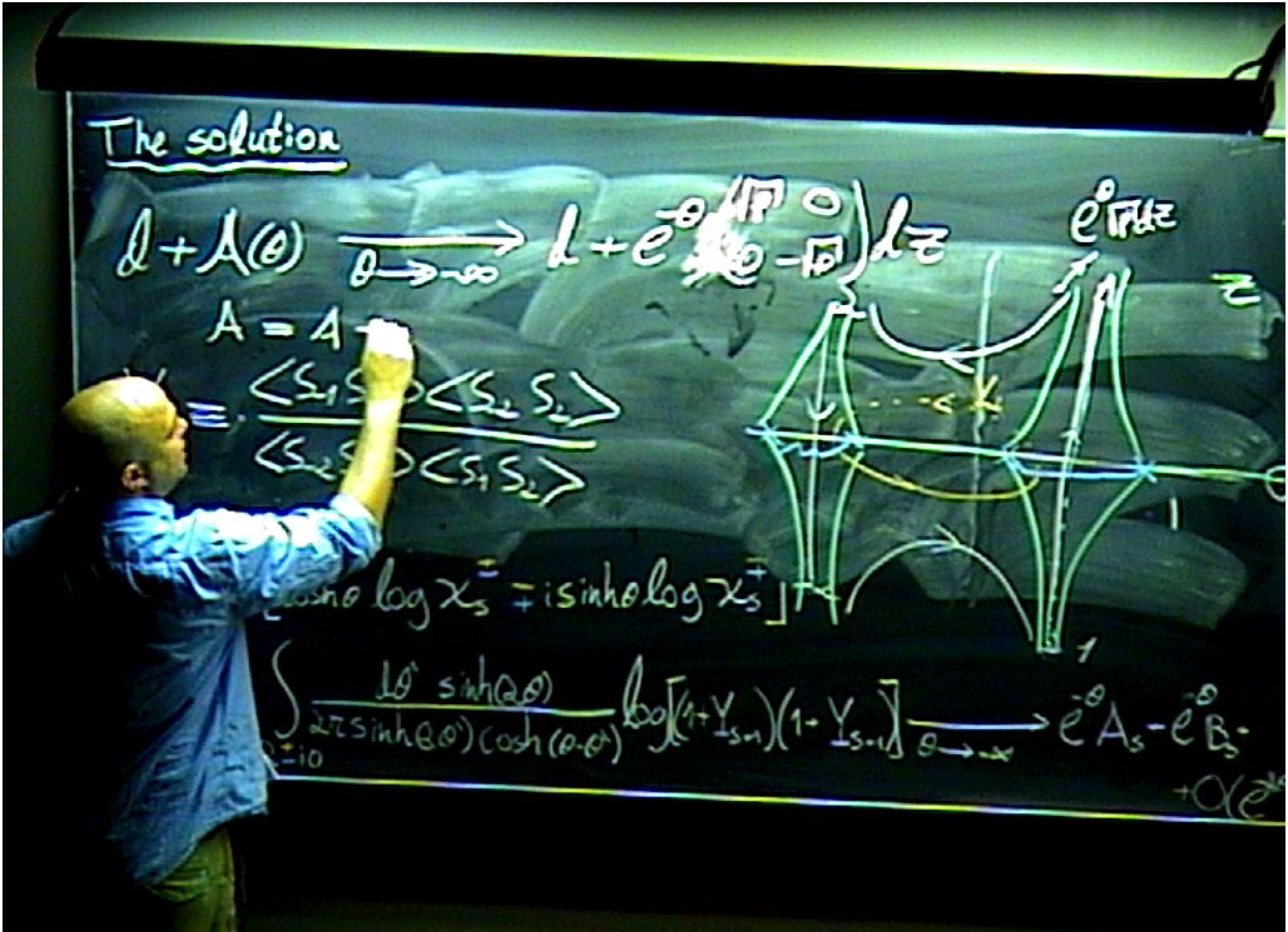

The solution
θ→−∞
z
R±i0


The solution
θ → −∞
Y₁ ≡ ⟨S₁S₁⟩⟨S₂S₂⟩ / ⟨S₋₂S₋₁⟩⟨S₁S₂⟩
= [cosh θ log x_s^∓ ∓ i sinh θ log x_s^±]
dθ′ sinh(2θ′) / 2π sinh(2θ′) cosh(θ−θ′)
R ± i0
θ → −∞
e^{−θ} A_s − e^{θ} B_s

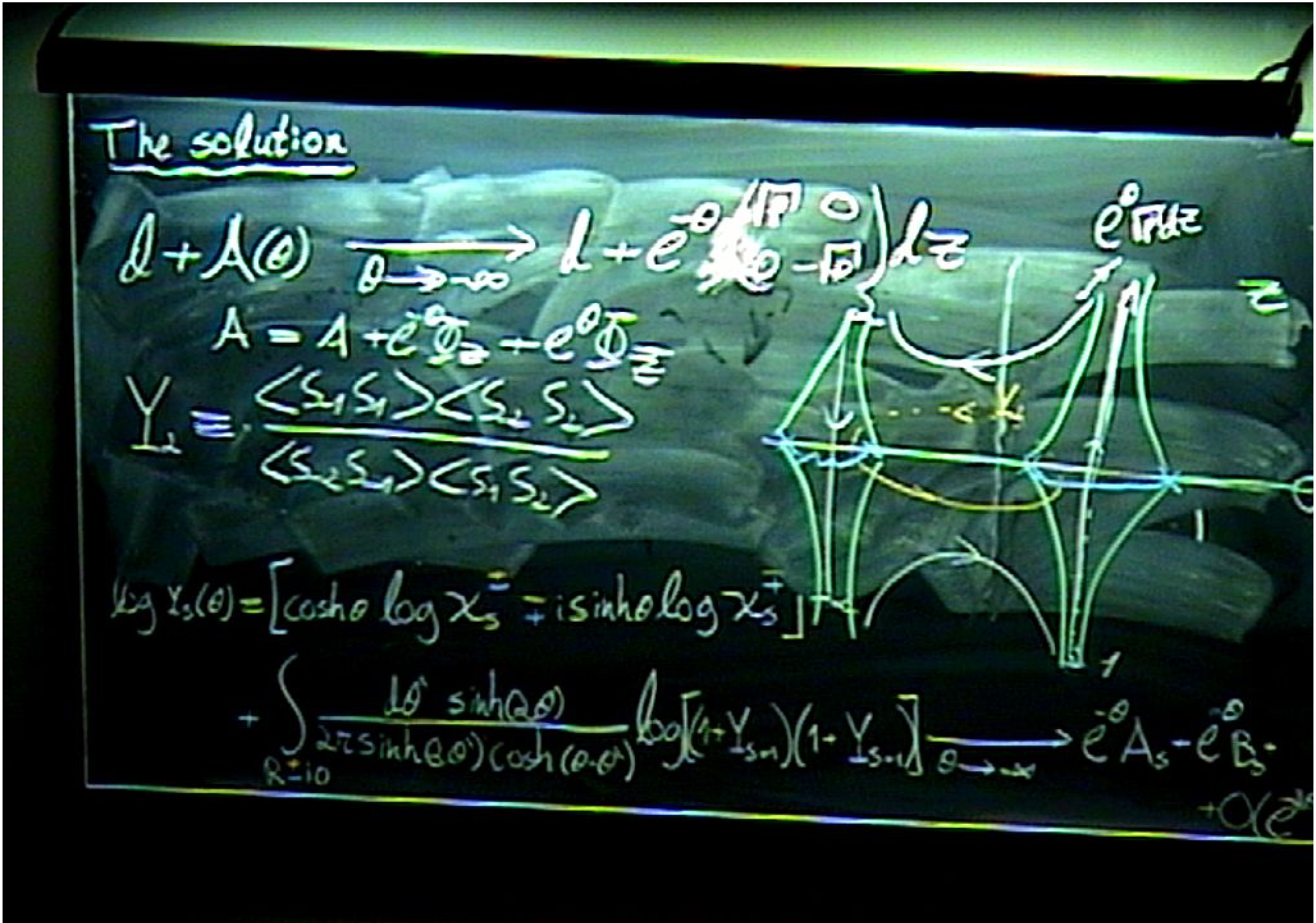
Schouten

$$\log Y_s = -e^{-\theta} \oint_{\gamma_s} \lambda^{\sqrt{2}\lambda z}$$

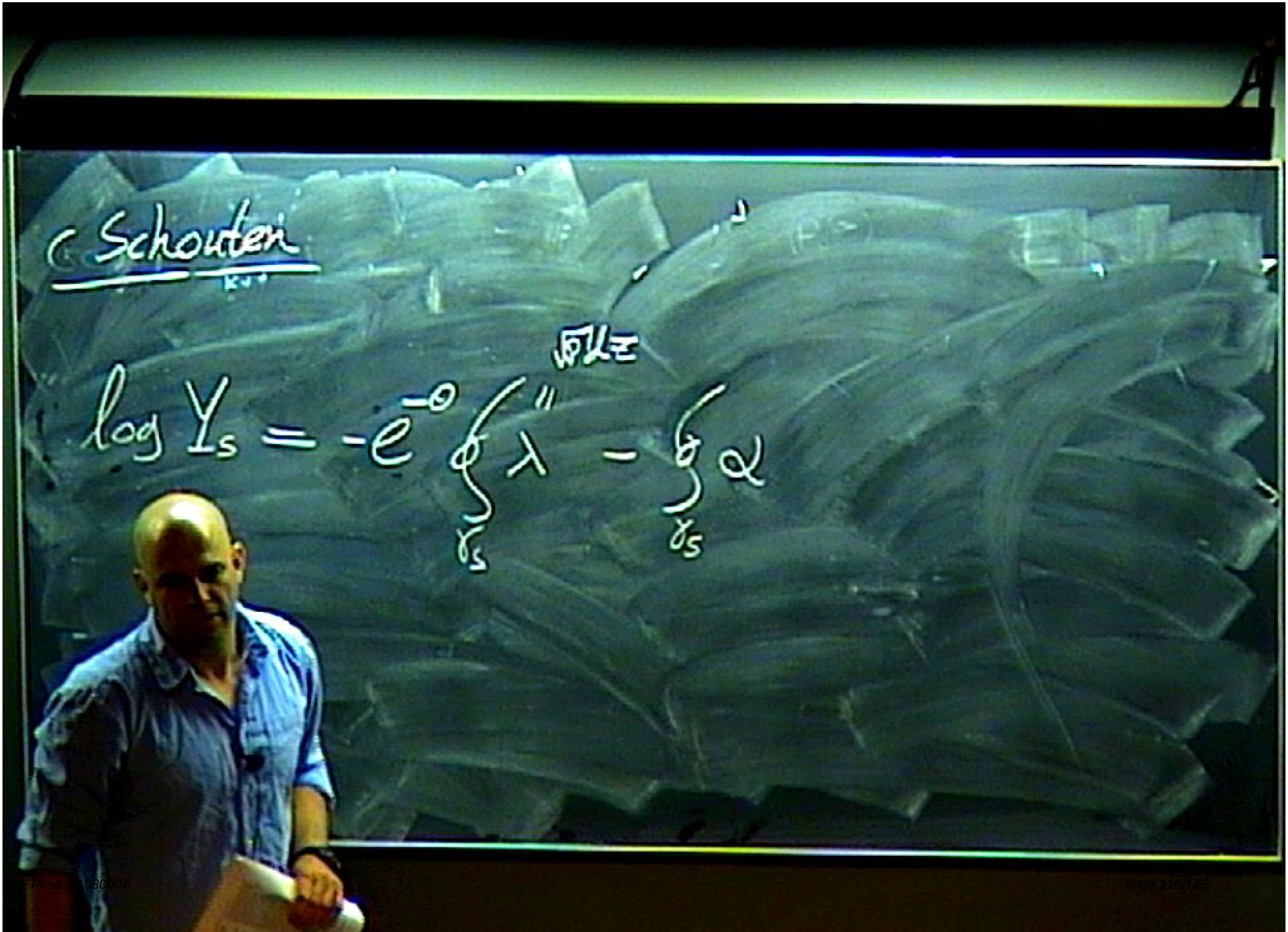

The solution



The solution
A = A
θ→-∞
isinhθ log
θ→-∞

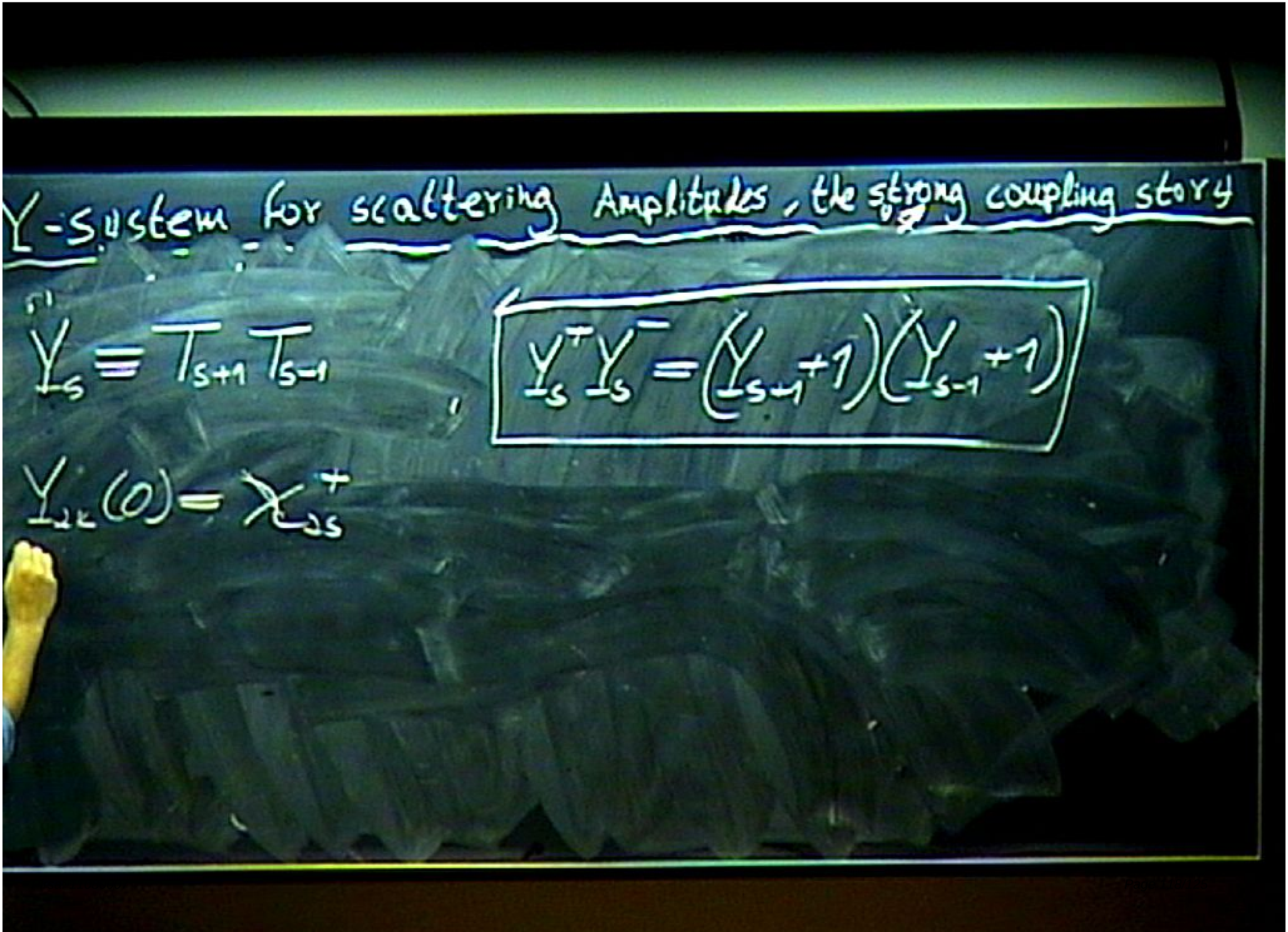


The solution
$d + A(\theta) \xrightarrow[\theta \to -\infty]{} d + e^{-\theta}\begin{pmatrix}\sqrt{P} & 0 \\ 0 & -\sqrt{P}\end{pmatrix} dz$
$e^{\theta} \sqrt{P} dz$
$A = A + e^{-\theta}\Phi_z - e^{\theta}\bar{\Phi}_{\bar z}$
$Y_1 = \frac{\langle s_1 s_1 \rangle \langle s_2 s_2 \rangle}{\langle s_2 s_1 \rangle \langle s_1 s_2 \rangle}$
$\log Y_s(\theta) = [\cosh\theta \log x_s^{\mp} \mp i \sinh\theta \log x_s^{\pm}]$
$+ \int_{R \pm i0} \frac{d\theta' \sinh(2\theta)}{2\pi \sinh(\theta-\theta') \cosh(\theta-\theta')} \log[(1+Y_{s+1})(1+Y_{s-1})] \xrightarrow[\theta \to -\infty]{} e^{-\theta}A_s - e^{\theta}B_s + O(e^{\dots})$


Schouten
$\log Y_s = -e^{-\theta} \oint_{\gamma_s} \lambda'' - \oint_{\gamma_s} \alpha$

Schouten

$$\log Y_s = -e^{-\theta}\oint_{\gamma_s}\lambda'' - \oint_{\gamma_s}\alpha'' - e^{\theta}\oint_{\gamma_s} u$$


Y-system for scattering Amplitudes, the strong coupling story
$Y_s = T_{s+1} T_{s-1}$,
$Y_s^+ Y_s^- = (Y_{s+1}+1)(Y_{s-1}+1)$
$Y_{2k}(0) = \chi_{2s}^+$

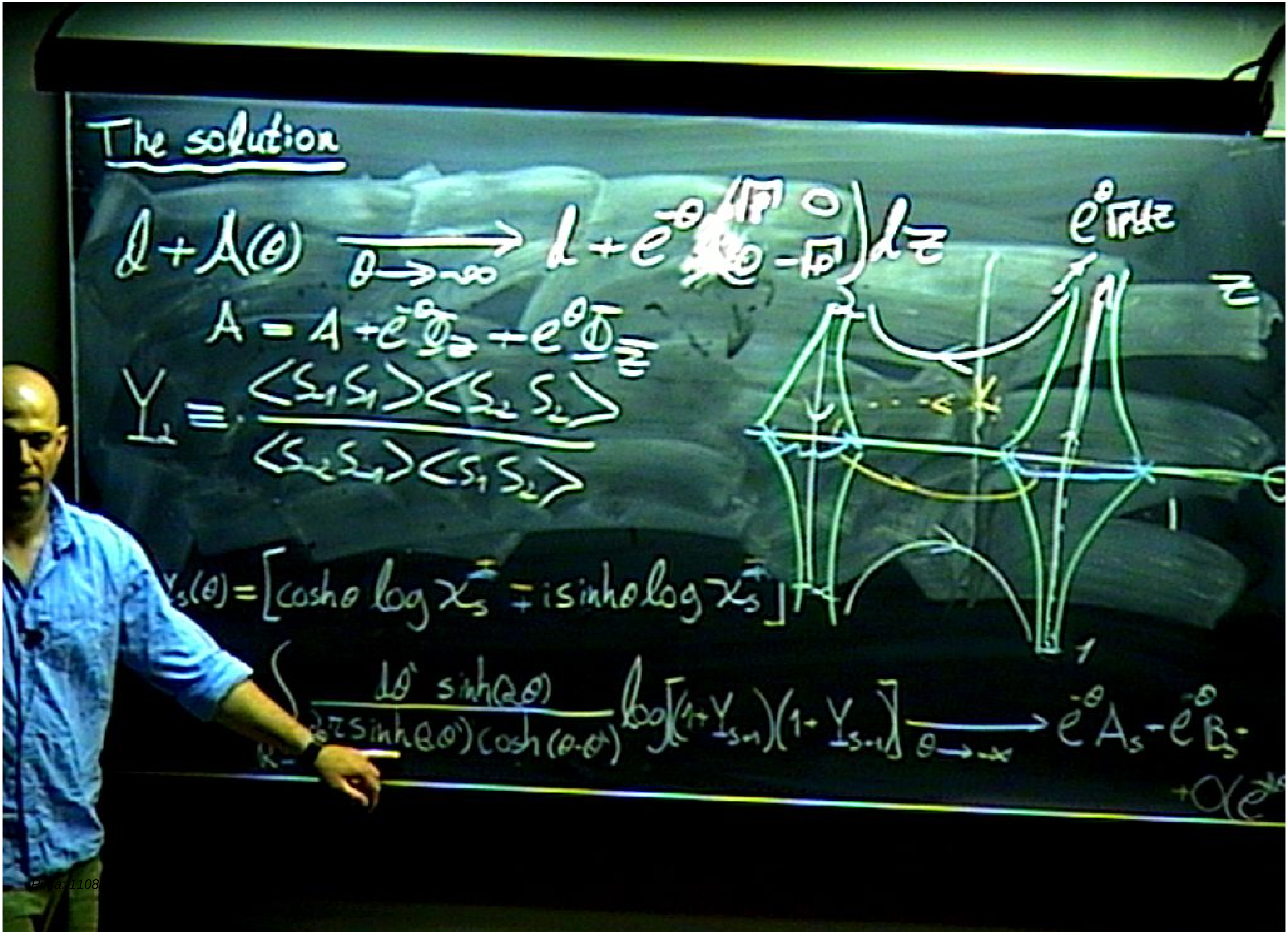
Y-system for scattering Amplitudes, the strong coupling story

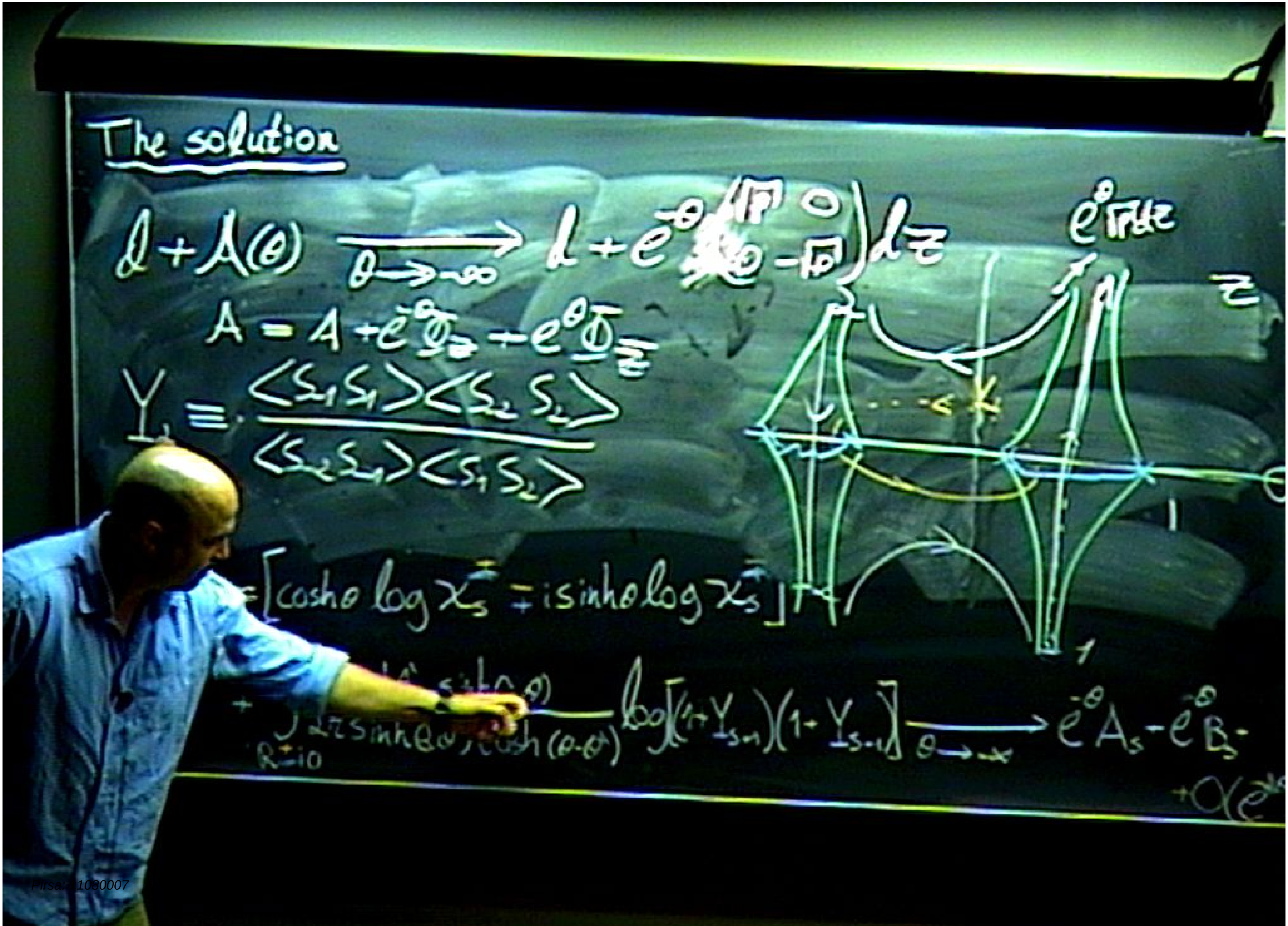
$$Y_s \equiv T_{s+1} T_{s-1}$$

$$Y_s^+ Y_s^- = (Y_{s+1}+1)(Y_{s-1}+1)$$

$$Y_{2k}(0) = \chi_{2k}^+$$

$$Y_{2k}(\pi/2) = \chi_{2k}^-$$

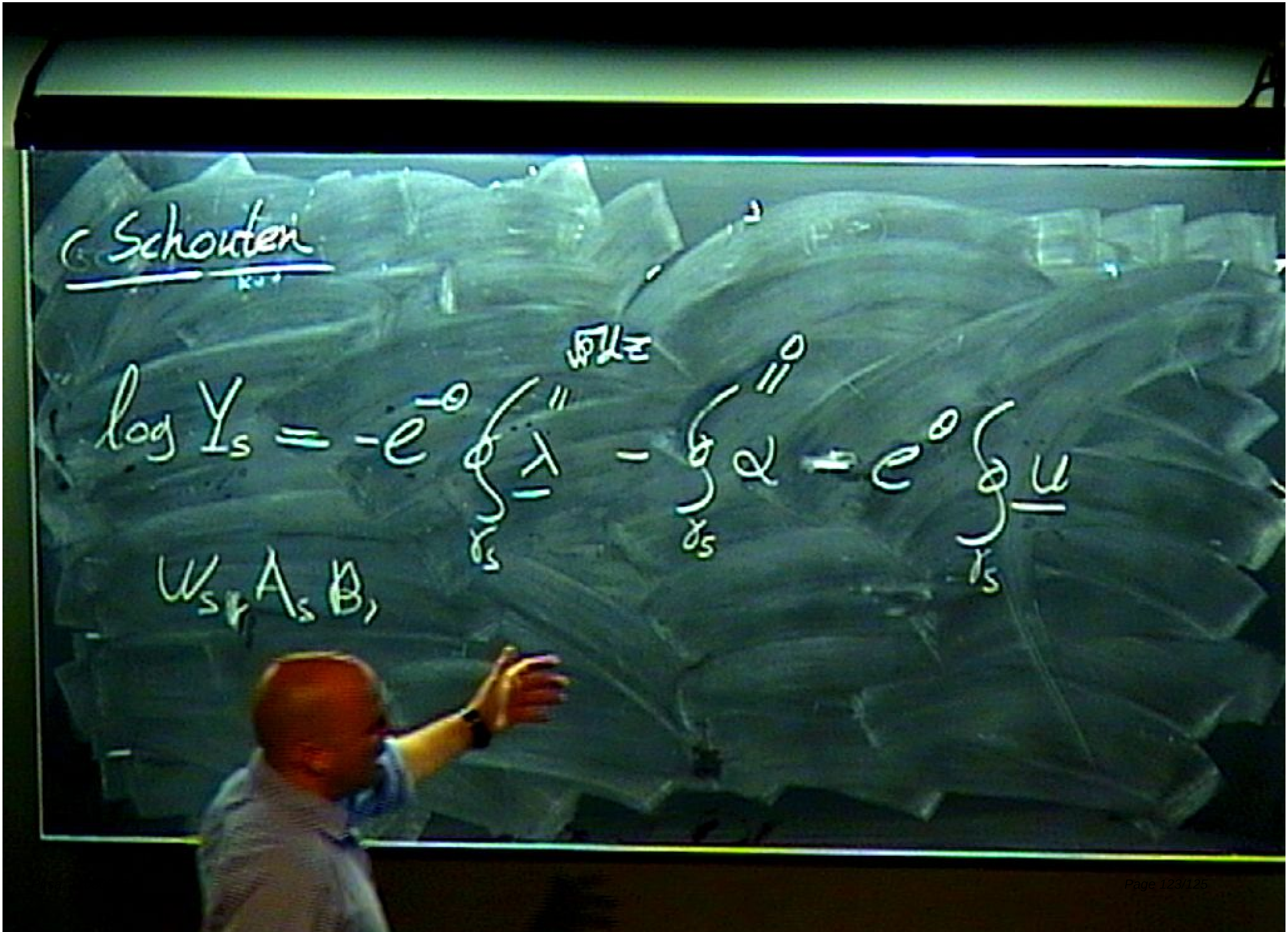

The solution

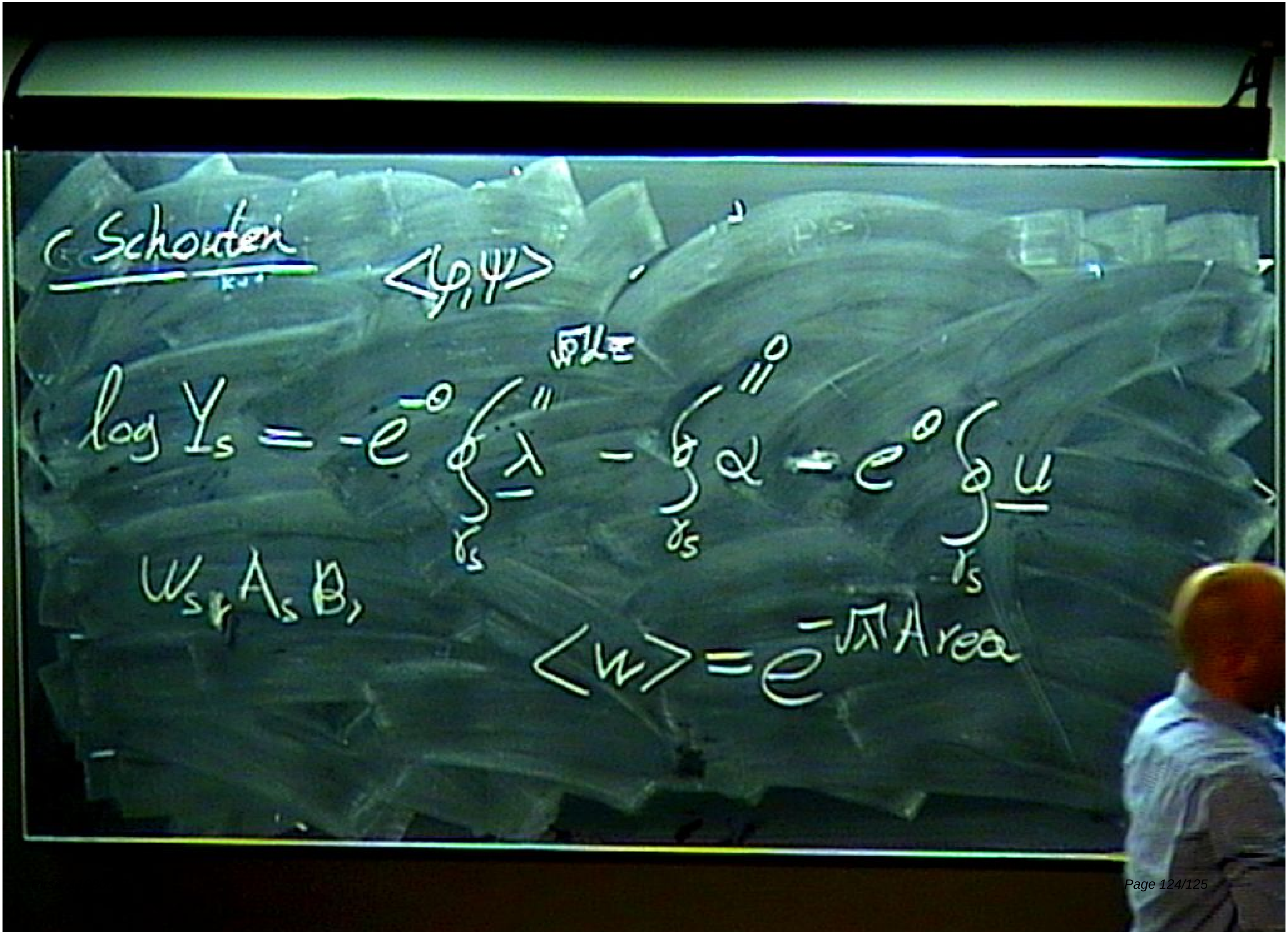

The solution
A = A + e^{-θ} Φ_z + e^θ Φ̄_z̄
Y_1 ≡ ⟨S_1 S_1⟩⟨S_2 S_2⟩ / ⟨S_2 S_2⟩⟨S_1 S_2⟩
[cosh θ log x_s ∓ i sinh θ log x_s]
log[(1+Y_{s+1})(1−Y_{s−1})]
e^{-θ} A_s − e^{-θ} B_s
Pirsa: 11080007

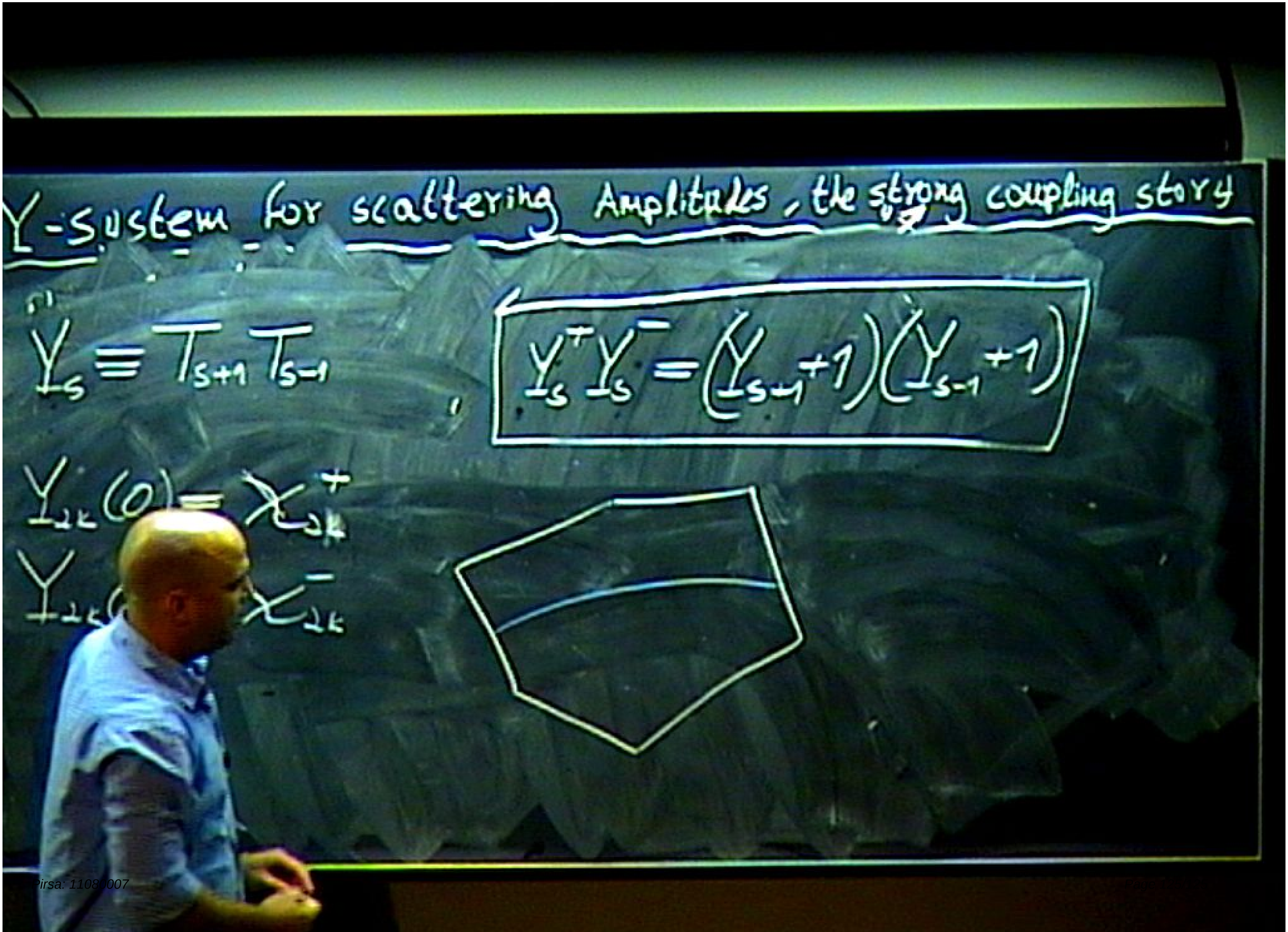
Schouten

$$\log Y_s = -e^{-\theta} \oint_{\gamma_s} \underline{\lambda}'' - \oint_{\gamma_s} \alpha'' - e^{\theta} \oint_{\gamma_s} \underline{u}$$

W_{s, r_1}


Schouten
$\log Y_s = -e^{-\theta} \oint_{\gamma_s} \underline{\lambda}'' - \oint_{\gamma_s} \alpha - e^{\theta} \oint_{\gamma_s} \underline{u}$
$W_{s,r}, A_s, B,$


Schouten
<φ,ψ>
log Y_s = -e^{-θ} ∮_{γ_s} λ − ∮_{γ_s} α − e^{θ} ∮_{γ_s} u
W_{s,r}, A_s, B_r
<W> = e^{-√λ Area}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χ^-_{2k}