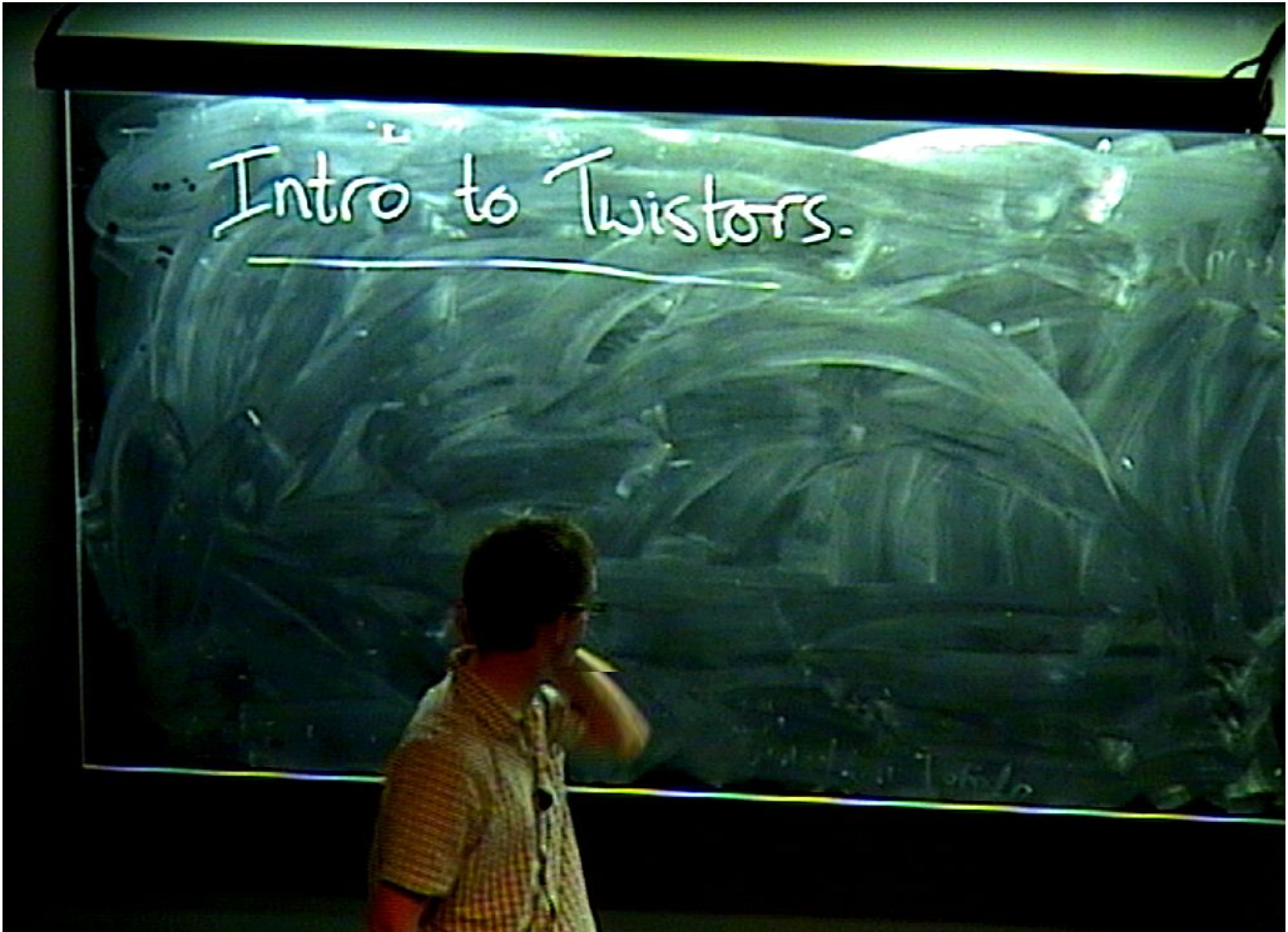
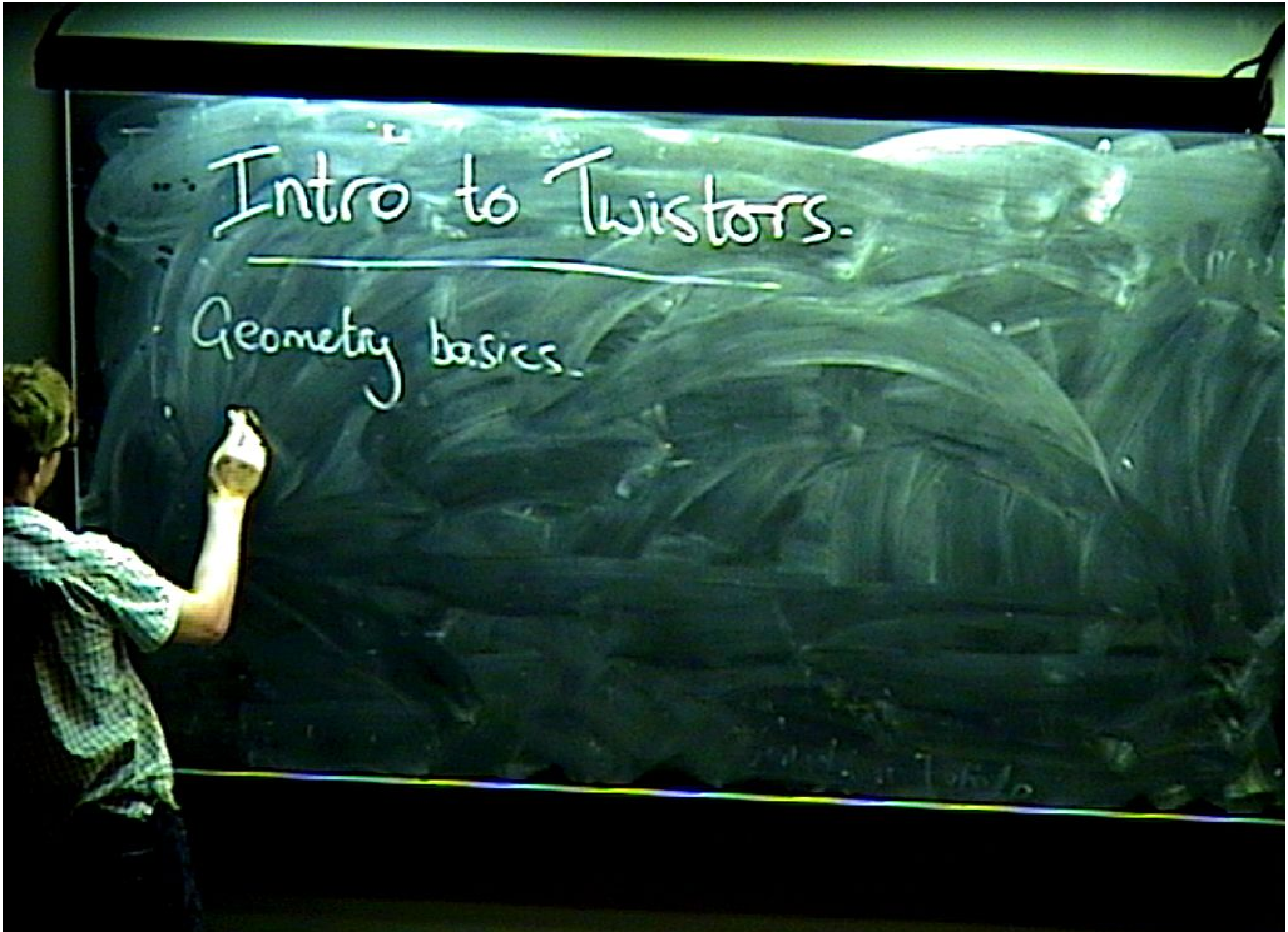
Title: Intro to Twistors

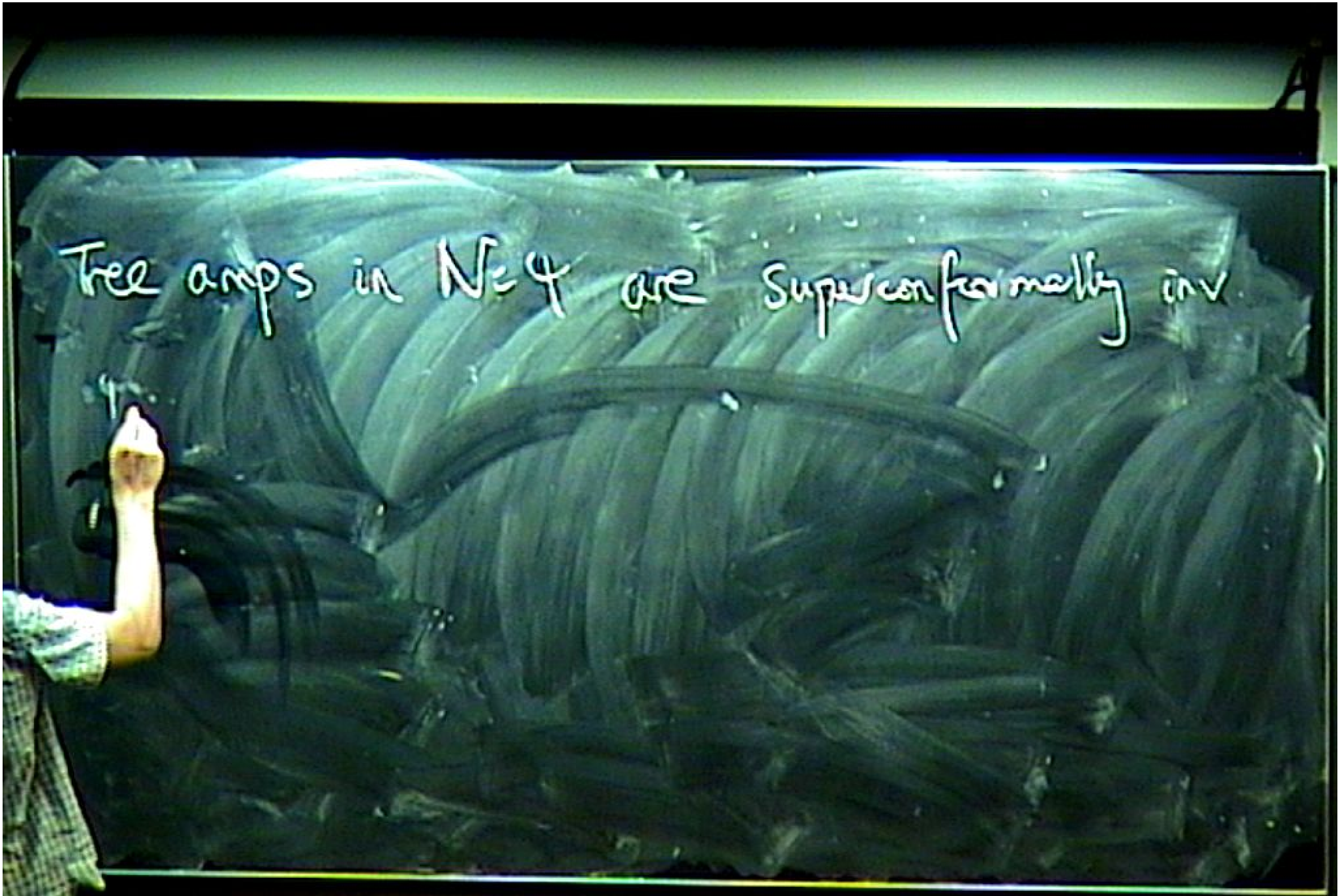
Date: Aug 01, 2011 07:45 PM

URL: http://pirsa.org/11080006

Abstract:

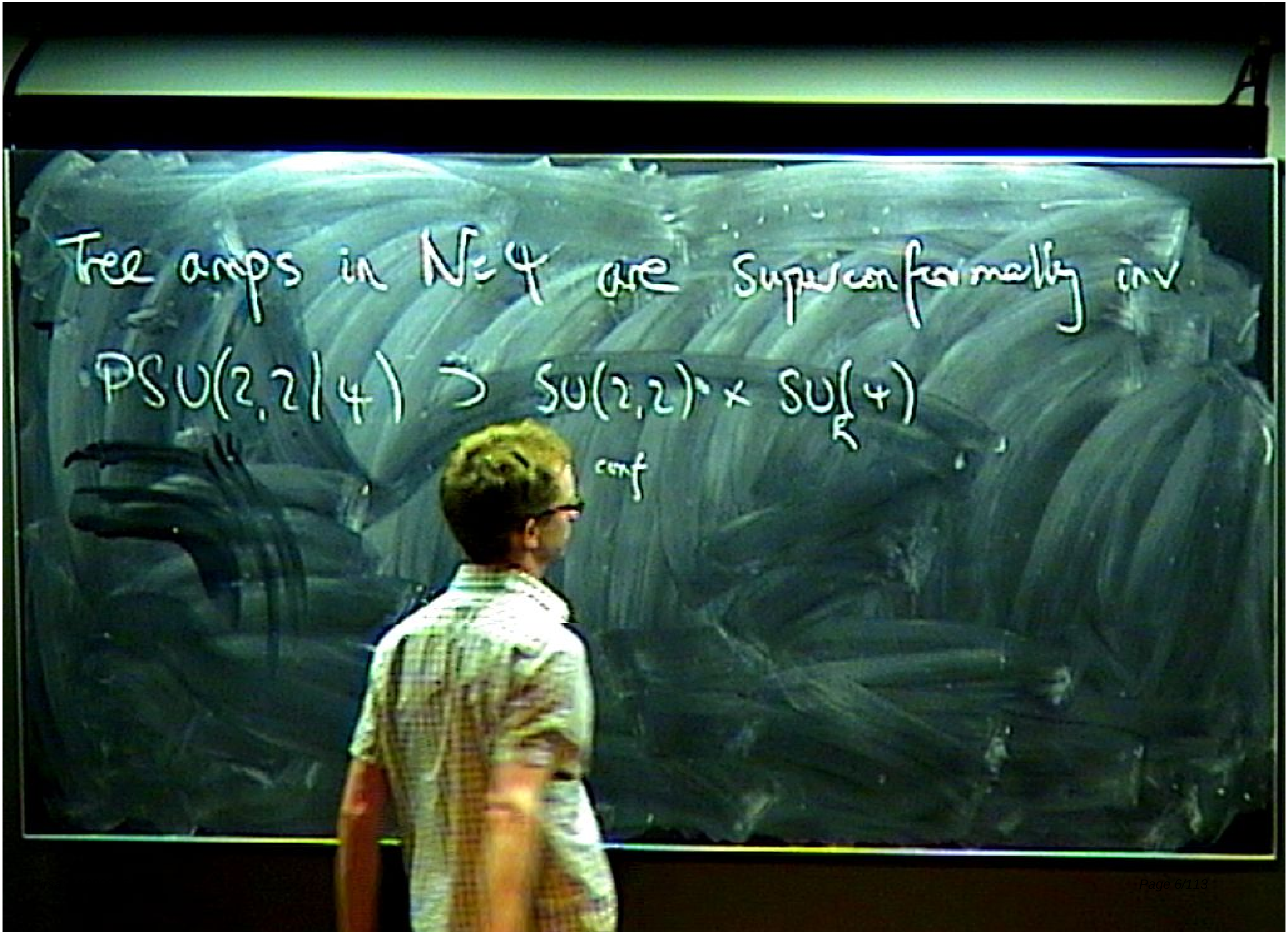

Intro to Twistors.

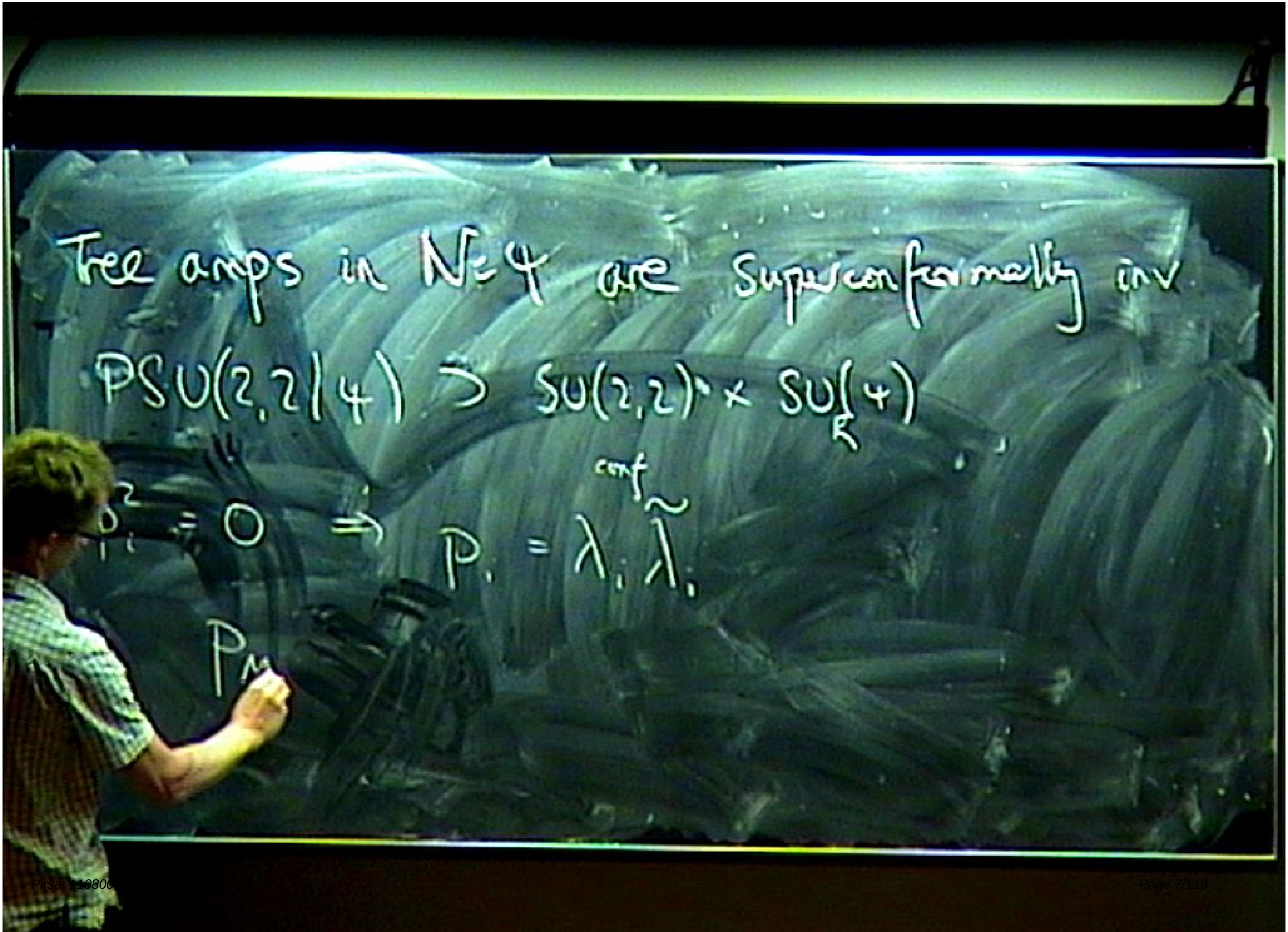

Intro to Twistors.
Geometry basics.


Tree amps in N=4 are superconformally inv

Intro to Twistors.

- Geometry basics.
- CSW/MHV diagrams in $n\mathbb{PT}$.


Tree amps in N=4 are superconformally inv
PSU(2,2|4) ⊃ SU(2,2) × SU(4)
conf
R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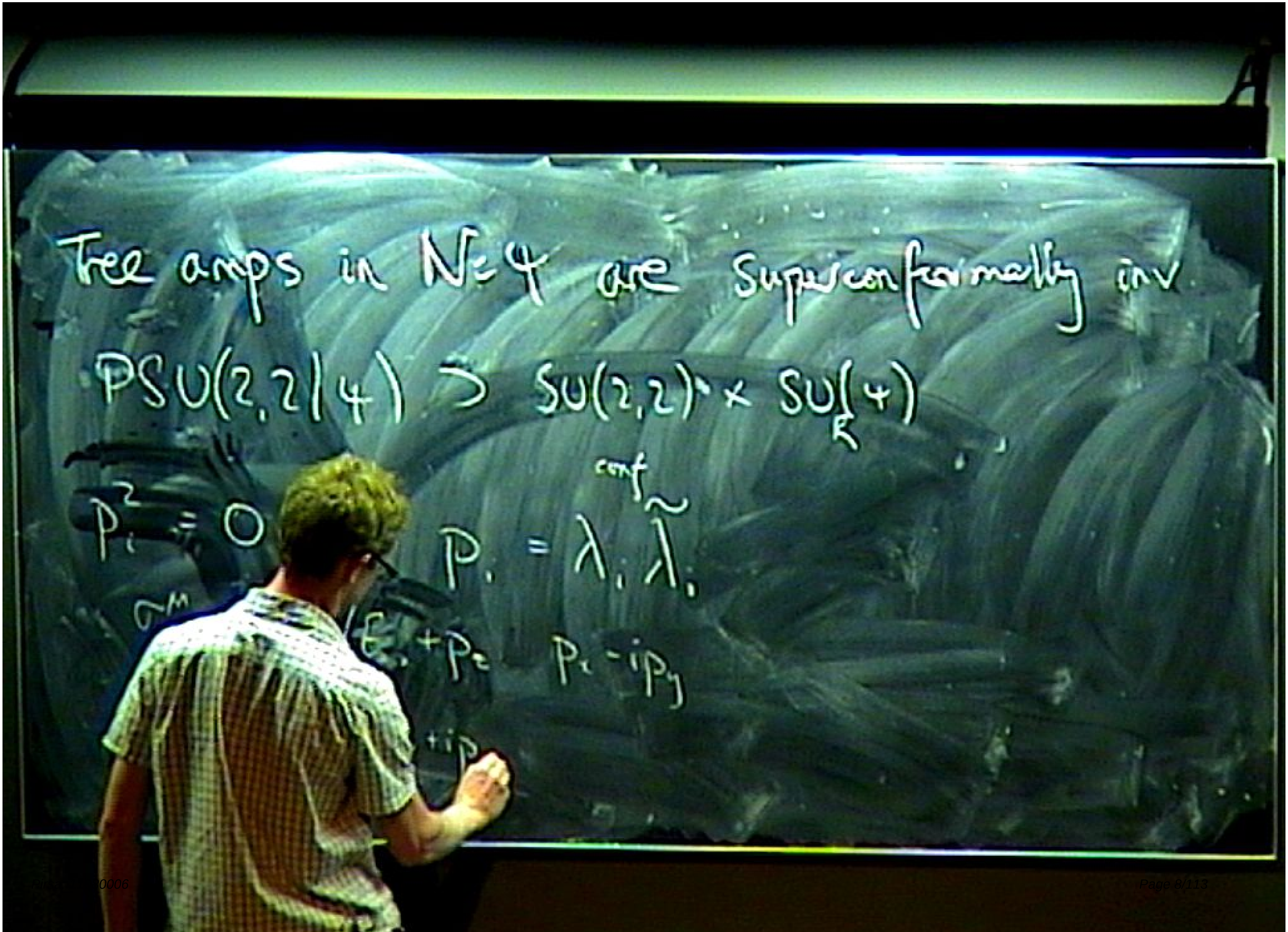
p_i^2 = 0 ⇒ p_i = λ_i λ̃_i
P_M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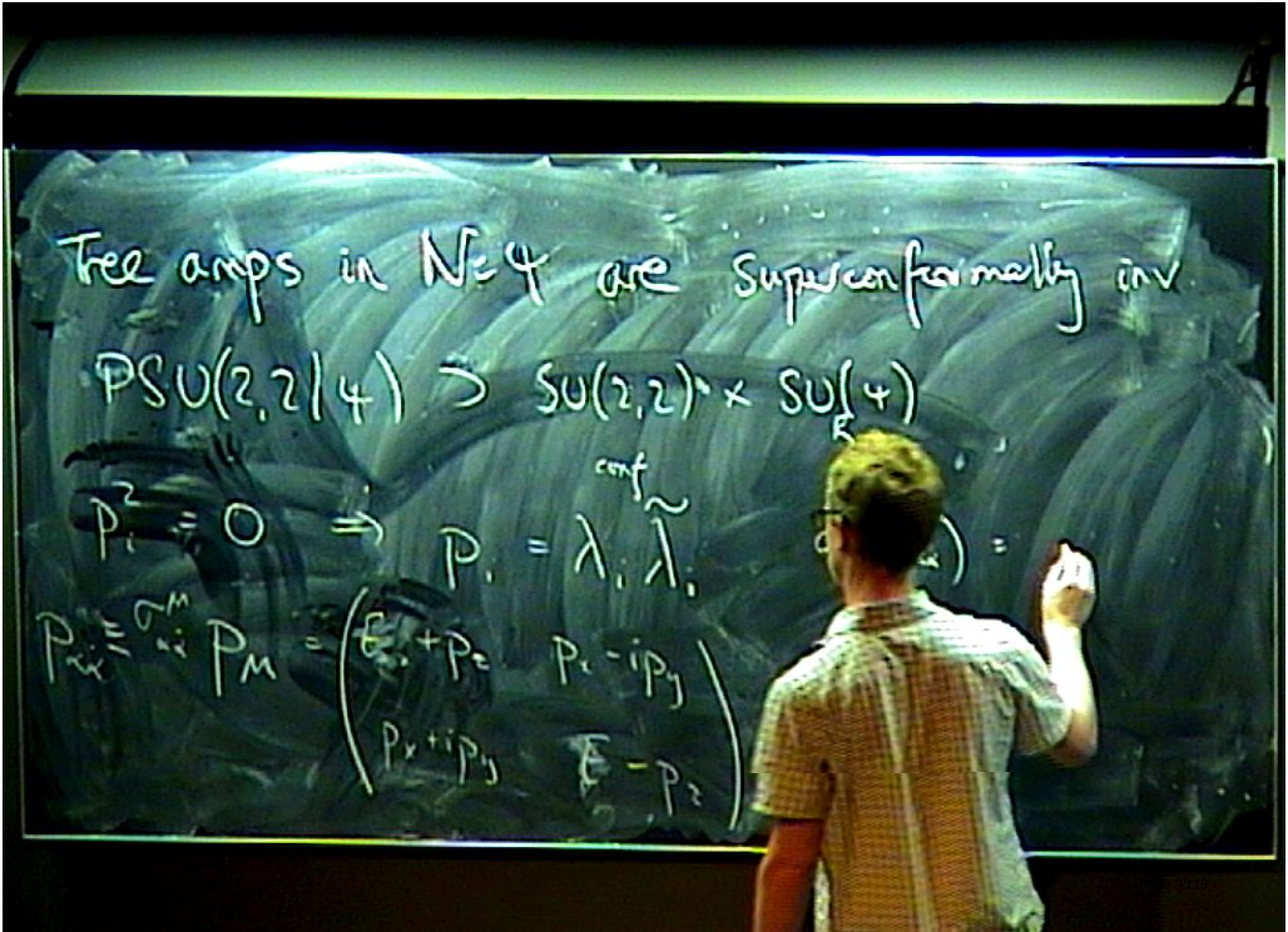
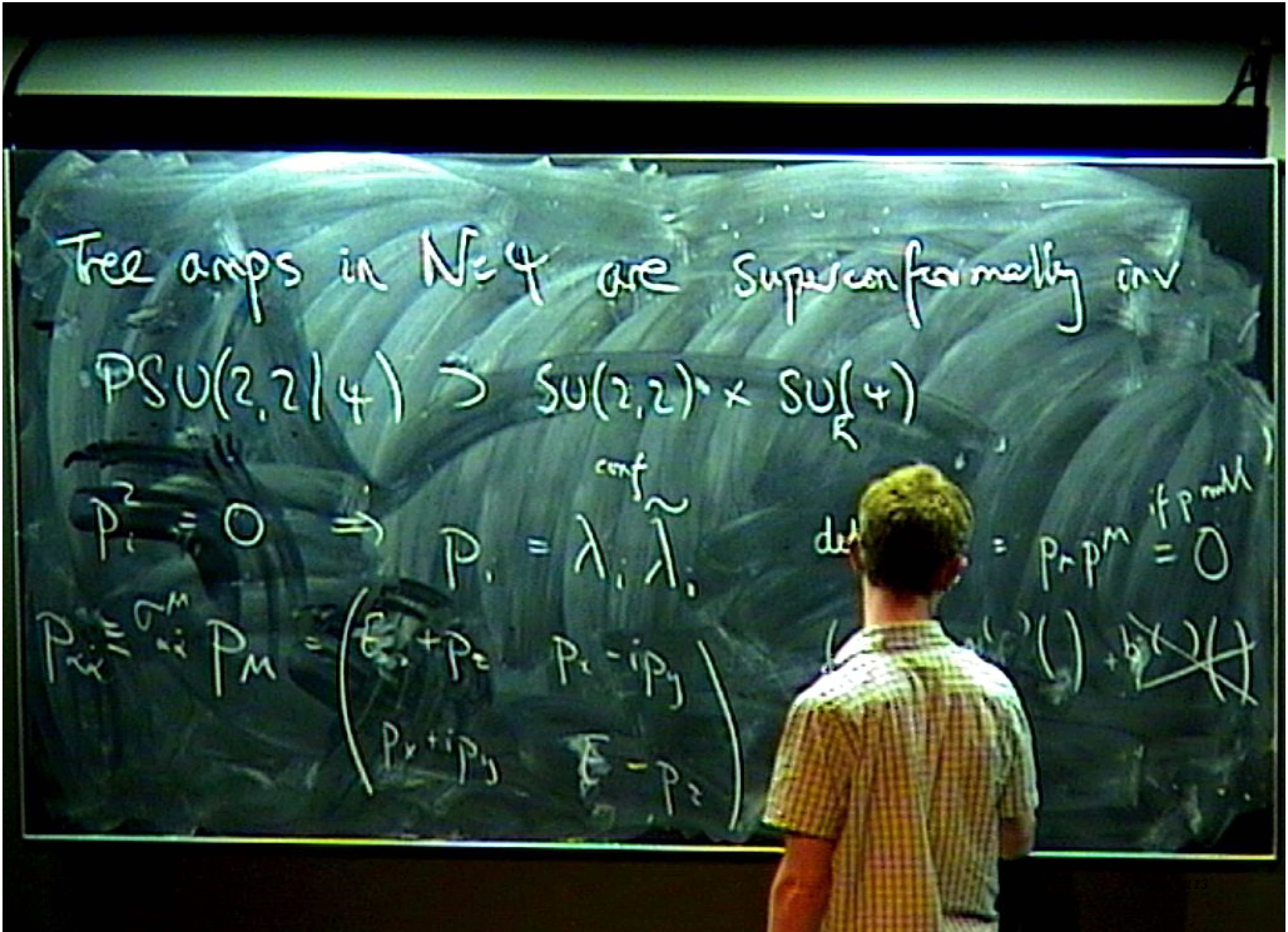
p_i = λ_i λ̃_i
E + p_z   p_x - ip_y
+ip


Tree amps in N=4 are superconformally inv
PSU(2,2|4) ⊃ SU(2,2) × SU_R(4)
conf
p_i^2 = 0 ⇒ p_i = λ_i λ̃_i
p_αα̇ = σ^μ_αα̇ p_μ = (E + p_z   p_x − ip_y
p_x + ip_y   E − p_z)



Tree amps in N=4 are superconformally inv
PSU(2,2|4) ⊃ SU(2,2) × SU(4)
conf
R
p_i^2 = 0 ⇒ p_i = λ_i λ̃_i
= p_μ p^μ = 0
if p null
p_{αα̇} ≡ σ^μ_{αα̇} p_μ =
E + p_z   p_x − ip_y
p_x + ip_y   E − p_z

Tree amps in $N=4$ are superconformally inv

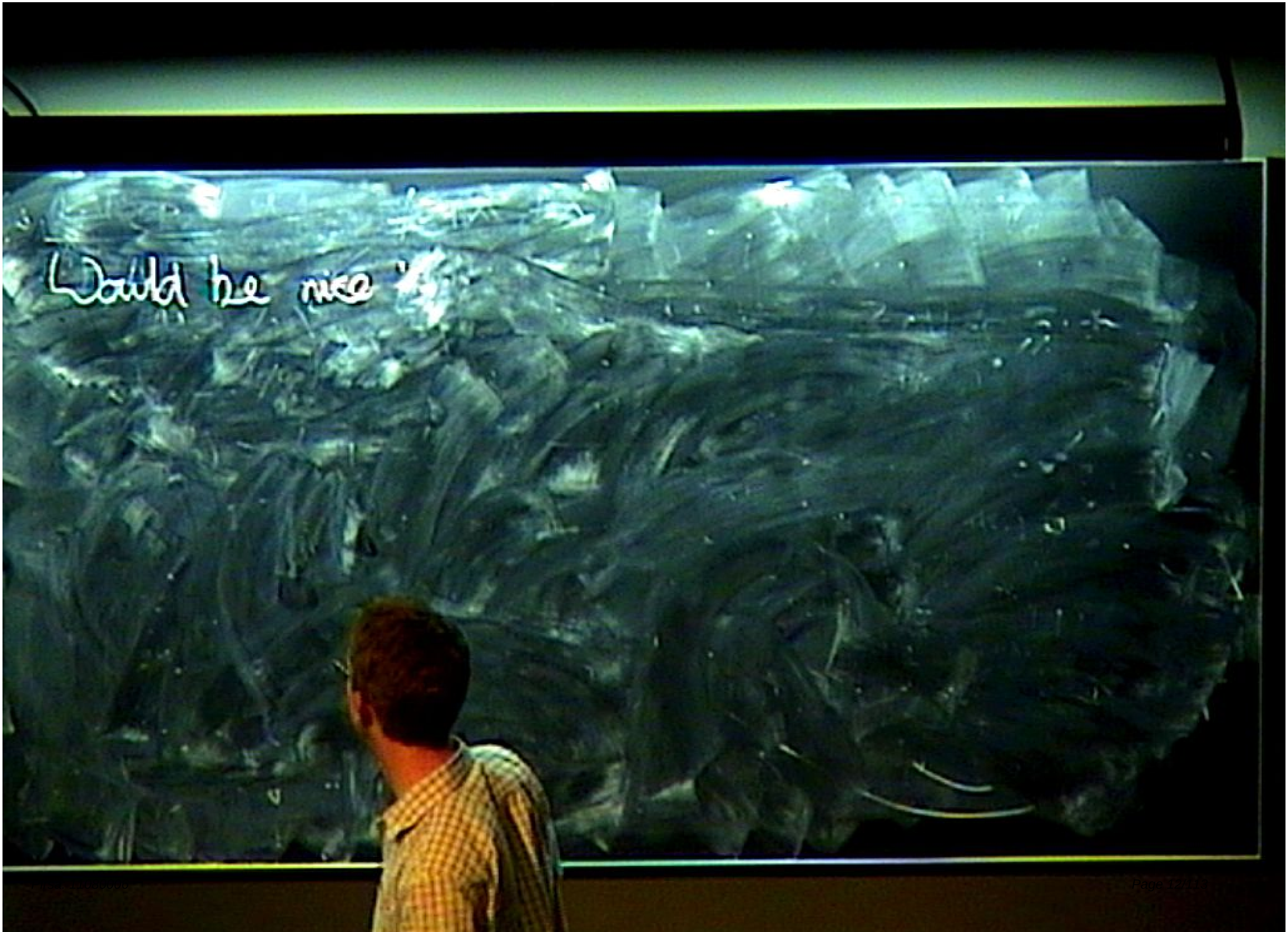
$PSU(2,2|4) \supset SU(2,2)_{conf} \times SU_R(4)$ $\quad SO(3,1) = SU(2,\mathbb{C})$

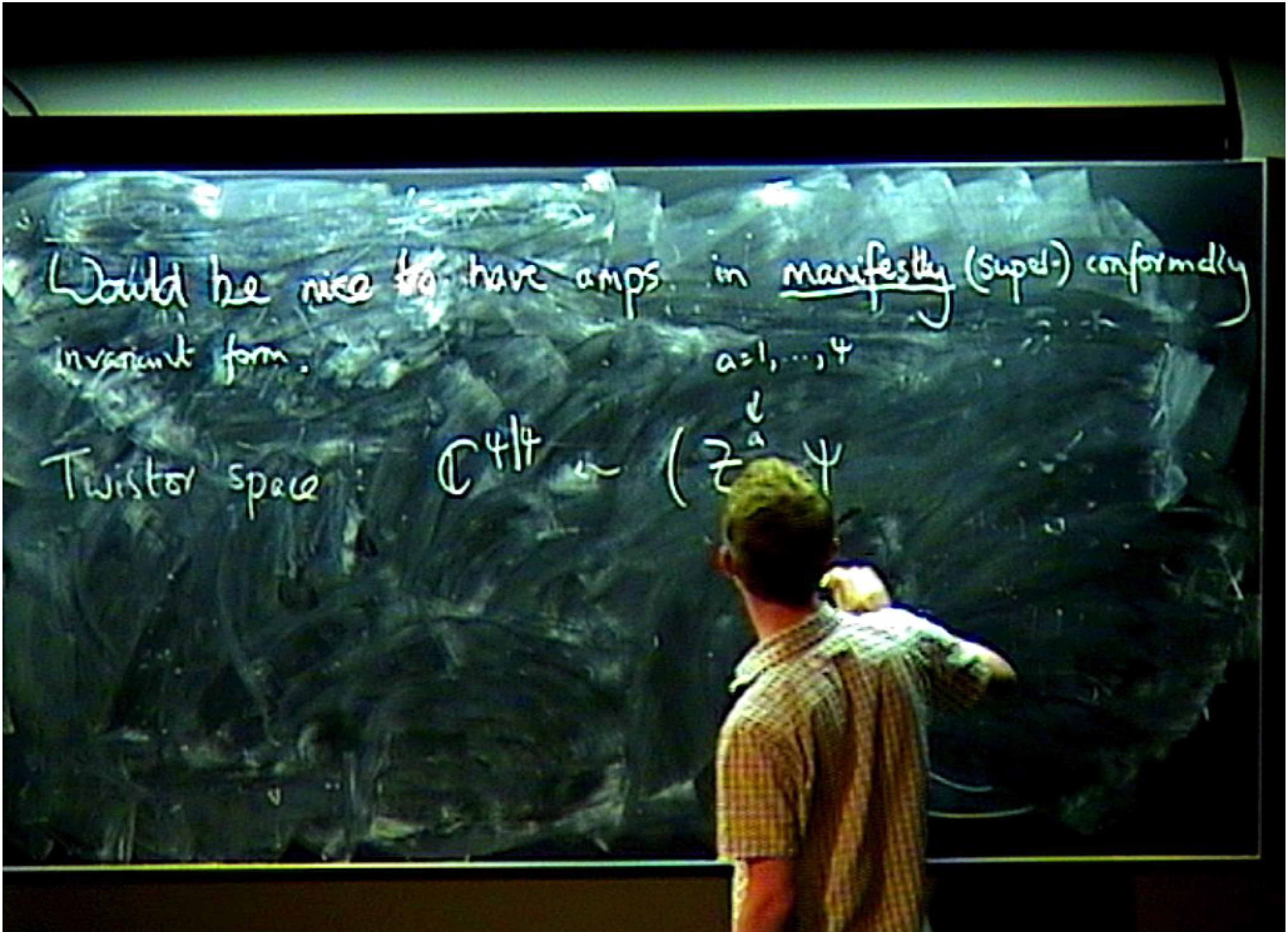
$p_i^2 = 0 \Rightarrow p_i = \lambda_i \tilde{\lambda}_i$ $\quad \det(p_{\alpha\dot\alpha}) = p_\mu p^\mu = 0$ if p null

$p_{\alpha\dot\alpha} = \sigma^\mu_{\alpha\dot\alpha} p_\mu = \begin{pmatrix} E + p_z & p_x - ip_y \\ p_x + ip_y & E - p_z \end{pmatrix}$ $\quad (\ ) = a(\ )(\ ) + b(\ )(\ )$

$p_{\alpha\dot\alpha} = \lambda_\alpha \tilde{\lambda}_{\dot\alpha}$

$\begin{pmatrix} 1 \\ 2, 0 \end{pmatrix} \quad \begin{pmatrix} 0, \frac{1}{2} \end{pmatrix}$


Would be nice


Would be nice to have amps in manifestly (super-) conformally invariant form.
a=1,...,4
Twistor space : $\mathbb{C}^{4|4}$ ~ (Z^a Ψ

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Twistor space : $\mathbb{C}^{4|4} \sim (Z^a, \psi^A)$ ($a = 1, \ldots, 4$; $A = 1, \ldots, 4$.)

All

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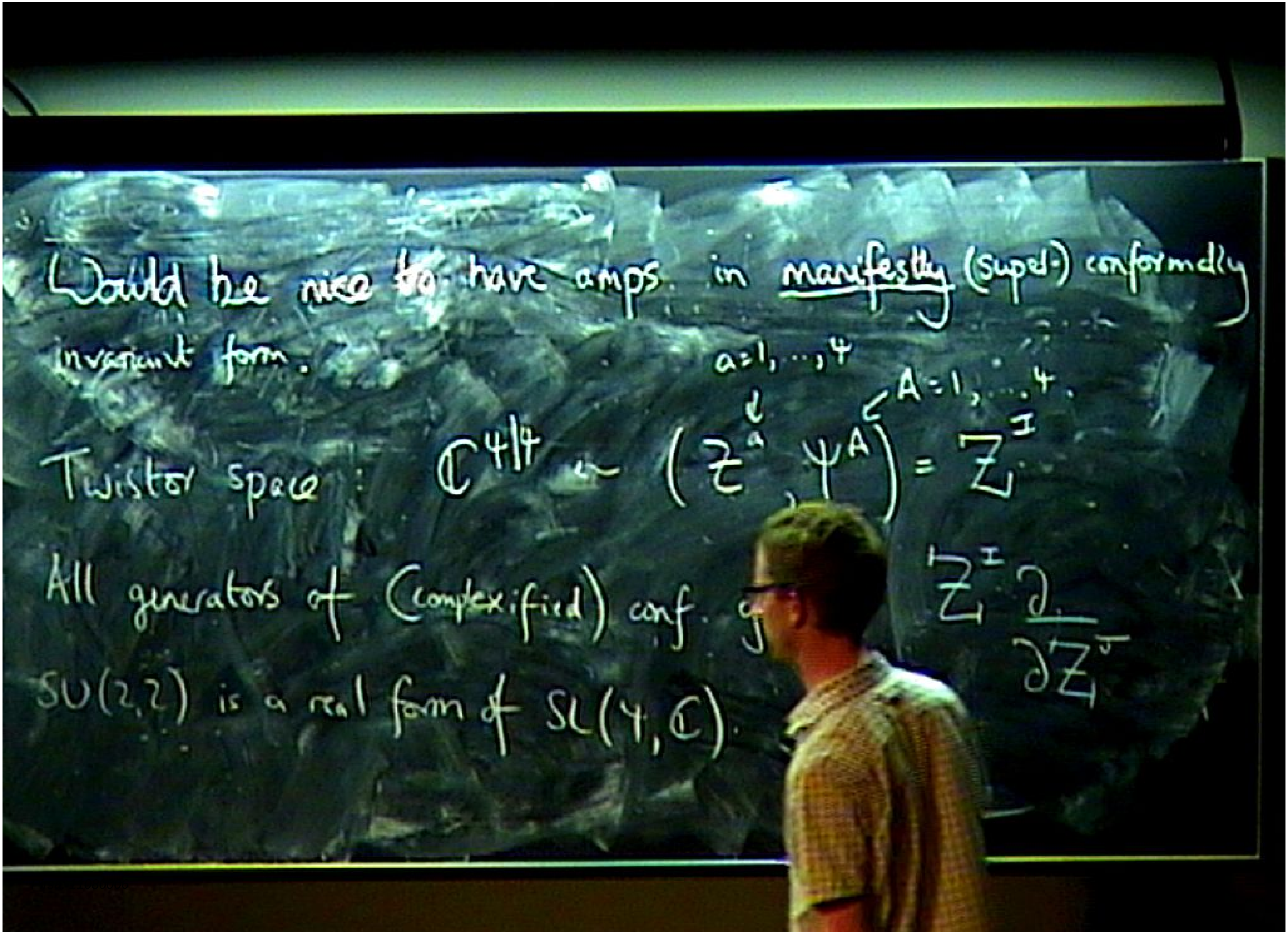
Would be nice to have amps in <u>manifestly</u> (super-) conformally invariant form.

$a = 1, \dots, 4$

$A = 1, \dots, 4.$

Twistor space: $\mathbb{C}^{4|4} \ni (Z^a, \psi^A) = Z^I$

All generators of (complexified) conf. group: $Z_i^I \dfrac{\partial}{\partial Z_i^J}$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Twistor space: ℂ^{4|4} ∋ (Z^a, ψ^A) = Z_i^I
All generators of (complexified) conf.
Z_i^I ∂/∂Z_i^J
SU(2,2) is a real form of SL(4,ℂ).

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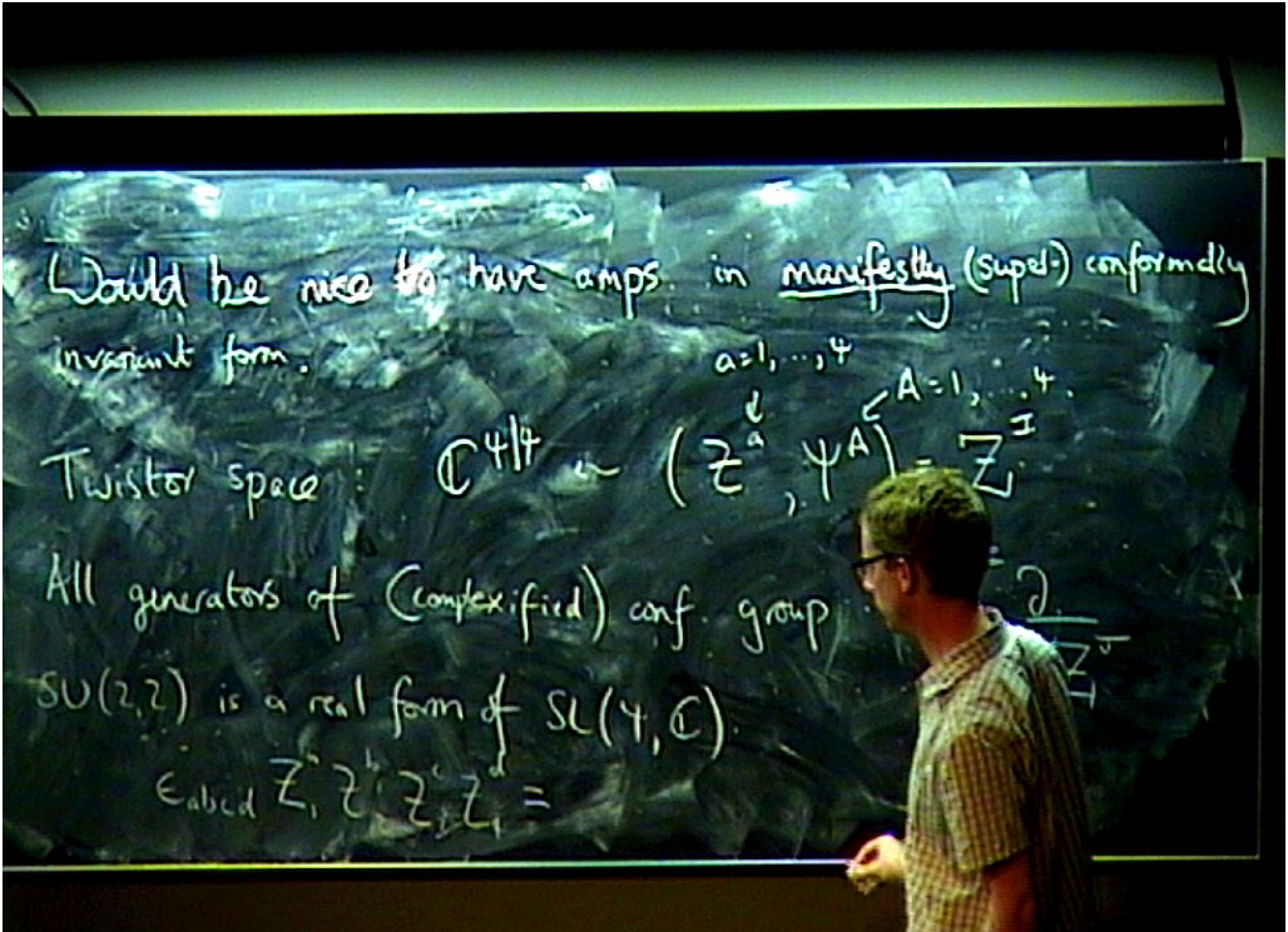
All generators of (complexified) conf. group: $Z^I \dfrac{\partial}{\partial Z^J}$

$SU(2,2)$ is a real form of $SL(4,\mathbb{C})$.

ϵ_{abcd}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All generators of (complexified) conf. group
$\frac{\partial}{\partial Z^J}$
SU(2,2) is a real form of SL(4,$\mathbb{C}$).
$\epsilon_{abcd} Z_1^a Z_1^b Z_1^c Z_1^d =$

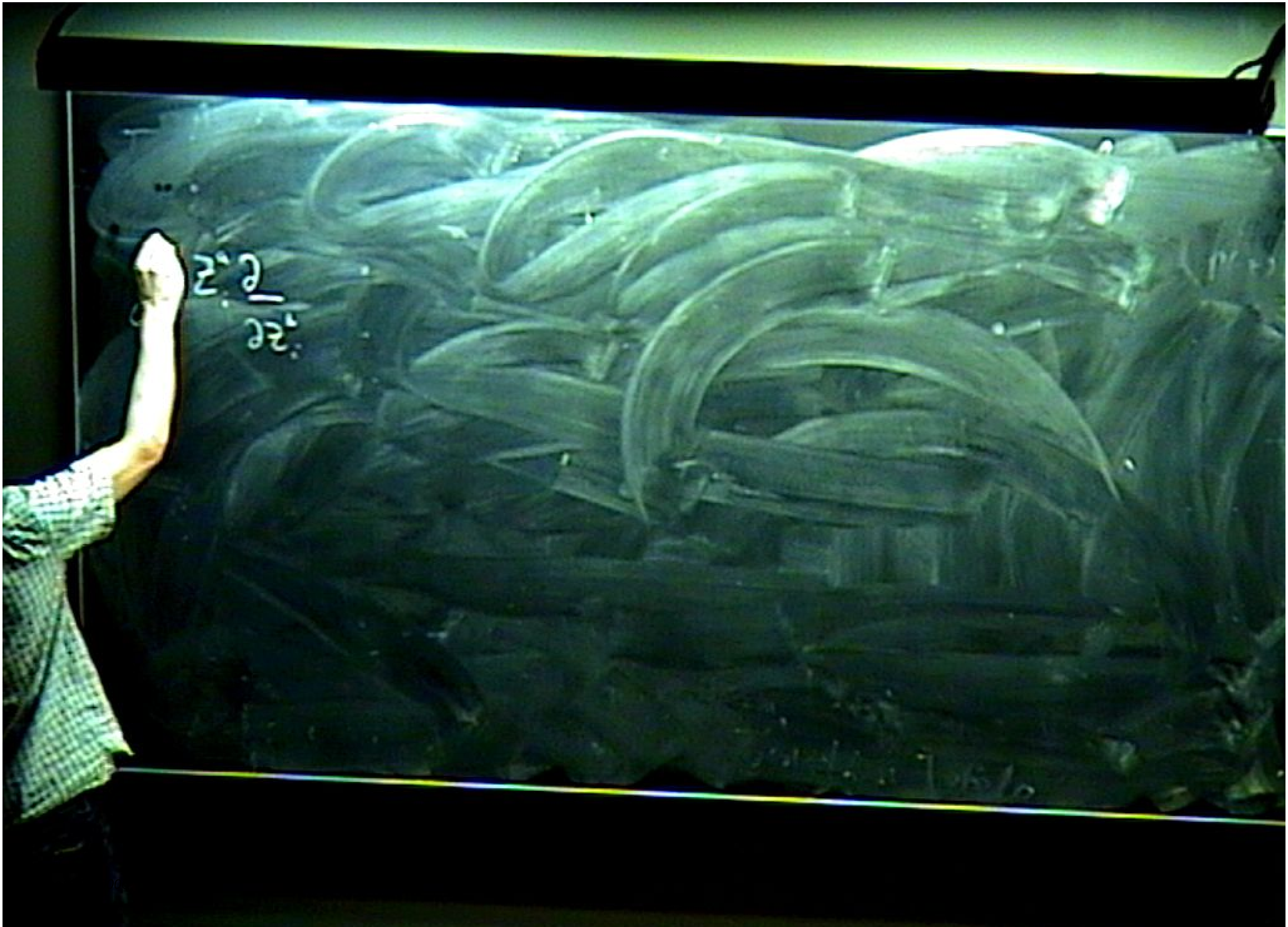
Would be nice to have amps in <u>manifestly</u> (super-) conformally invariant form.

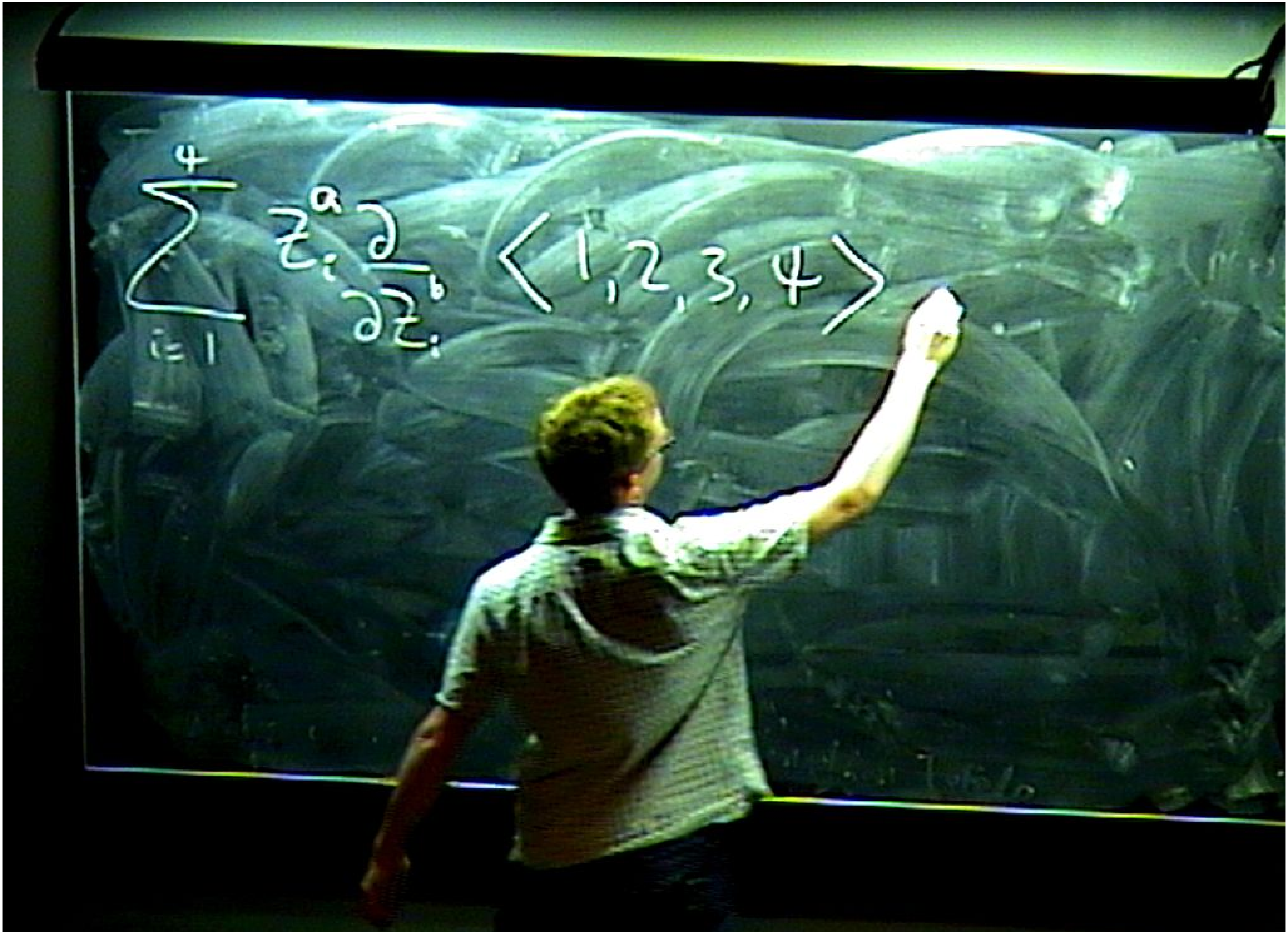
Twistor space : $\mathbb{C}^{4|4} \ni (Z^a, \psi^A) = Z^I$, $a = 1, \ldots, 4$, $A = 1, \ldots, 4$.

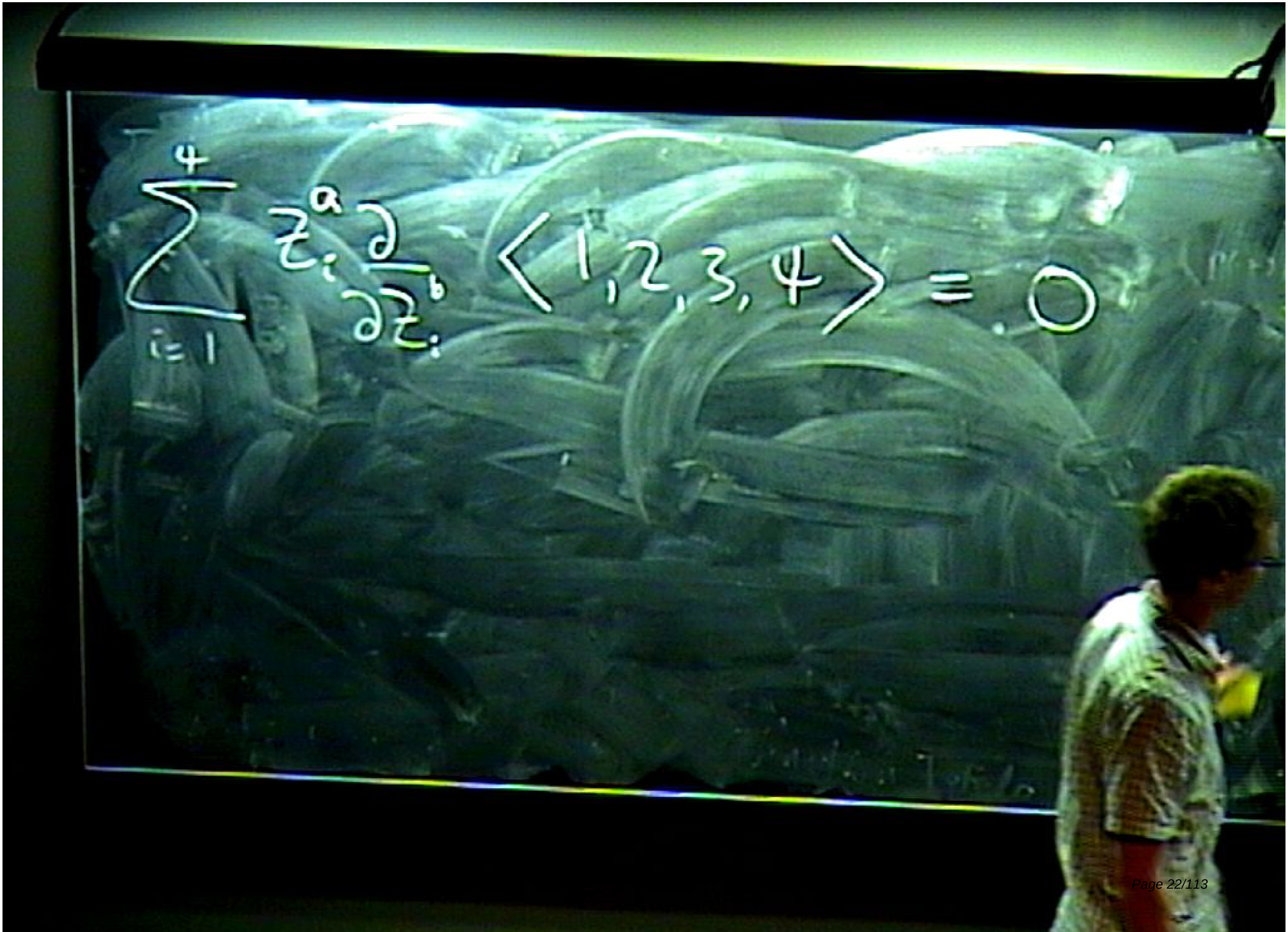
All generators of (complexified) conf. group : $Z^I \dfrac{\partial}{\partial Z^J}$

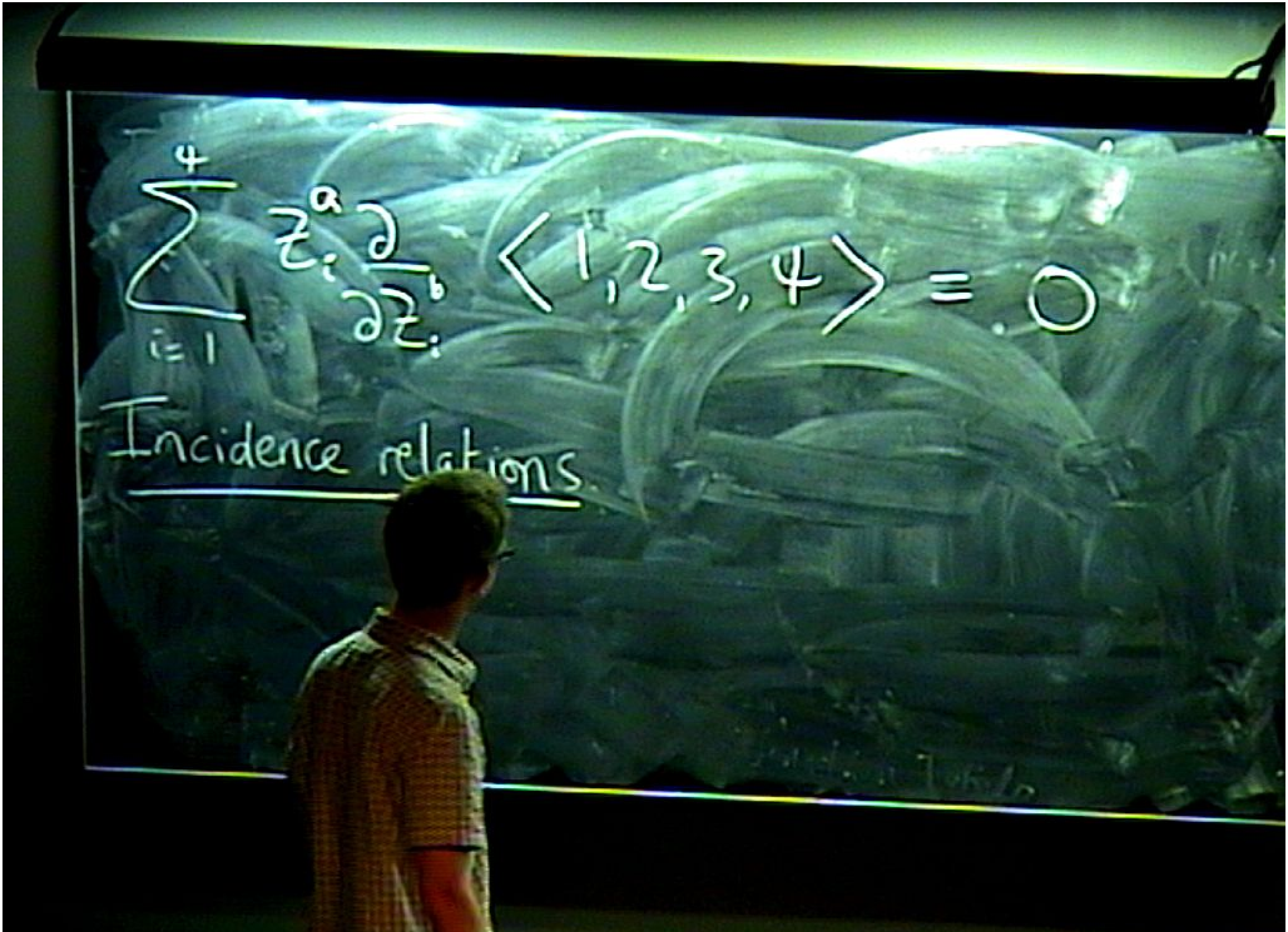
$SU(2,2)$ is a real form of $SL(4, \mathbb{C})$.

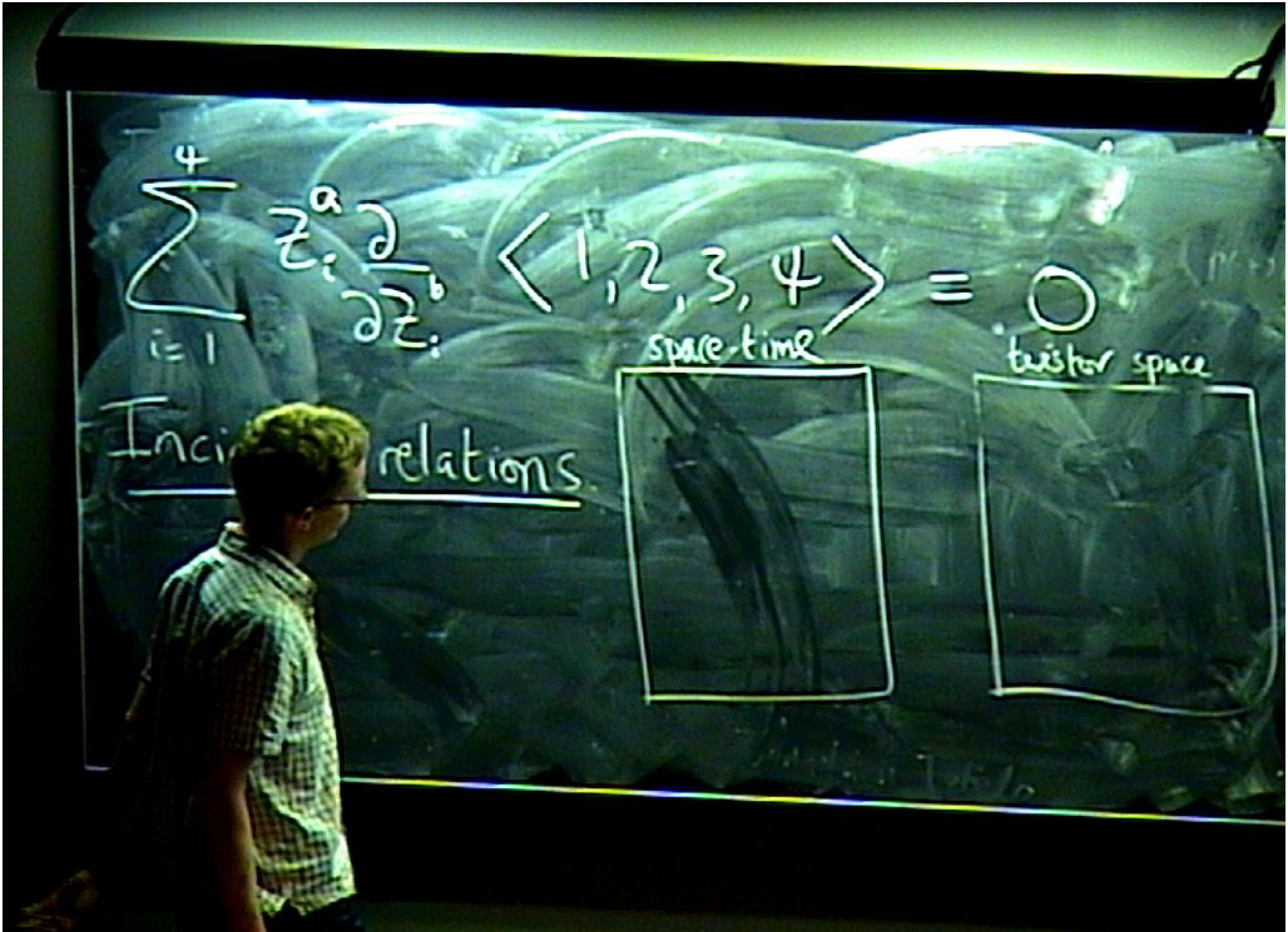
$\epsilon_{abcd} Z_1^a Z_2^b Z_3^c Z_4^d = \langle 1,2,3,4 \rangle$ is conformal invariant.



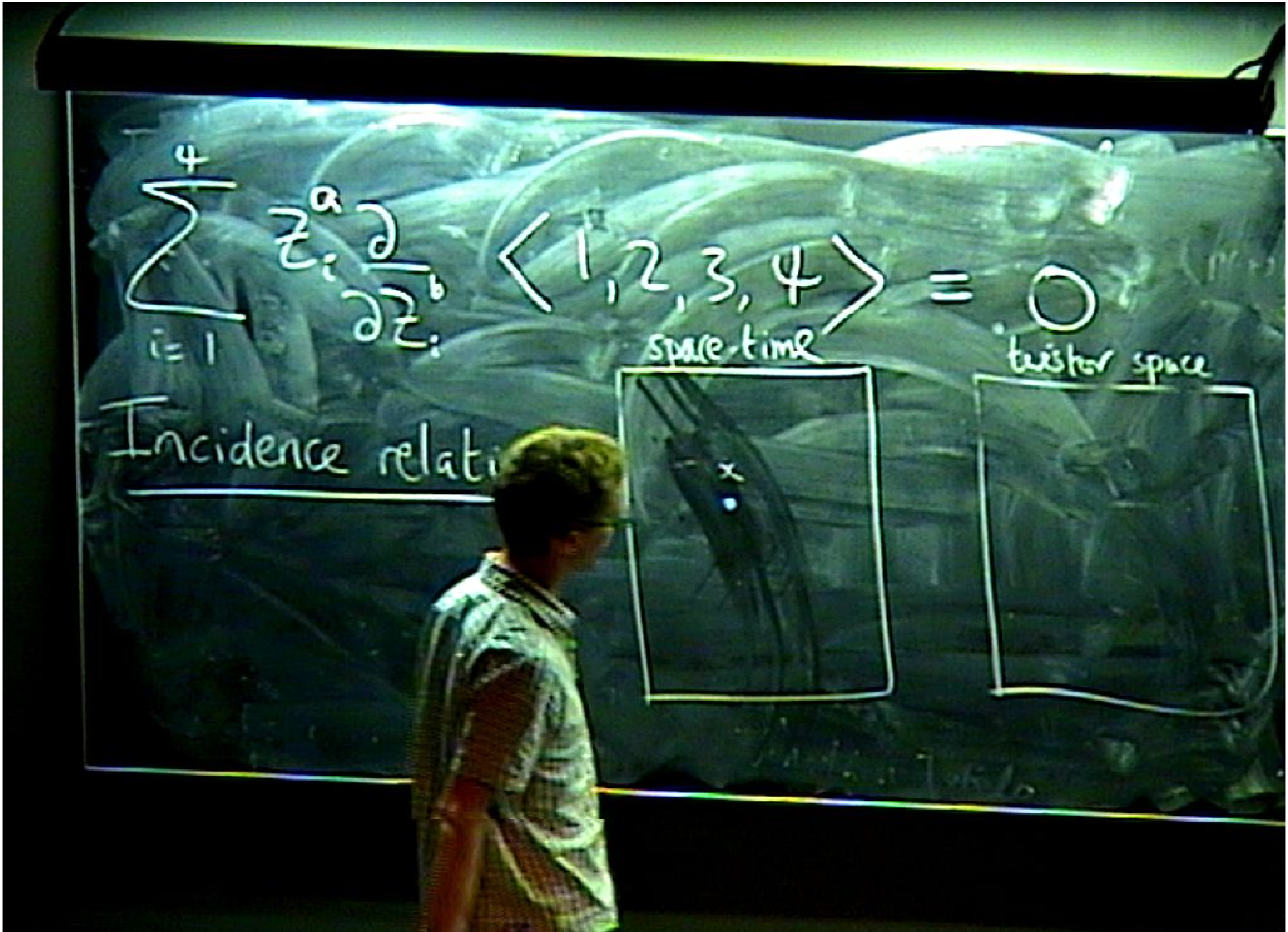

$\sum_{i=1}^{4} z_i^a \frac{\partial}{\partial z_i} \langle 1,2,3,4 \rangle$

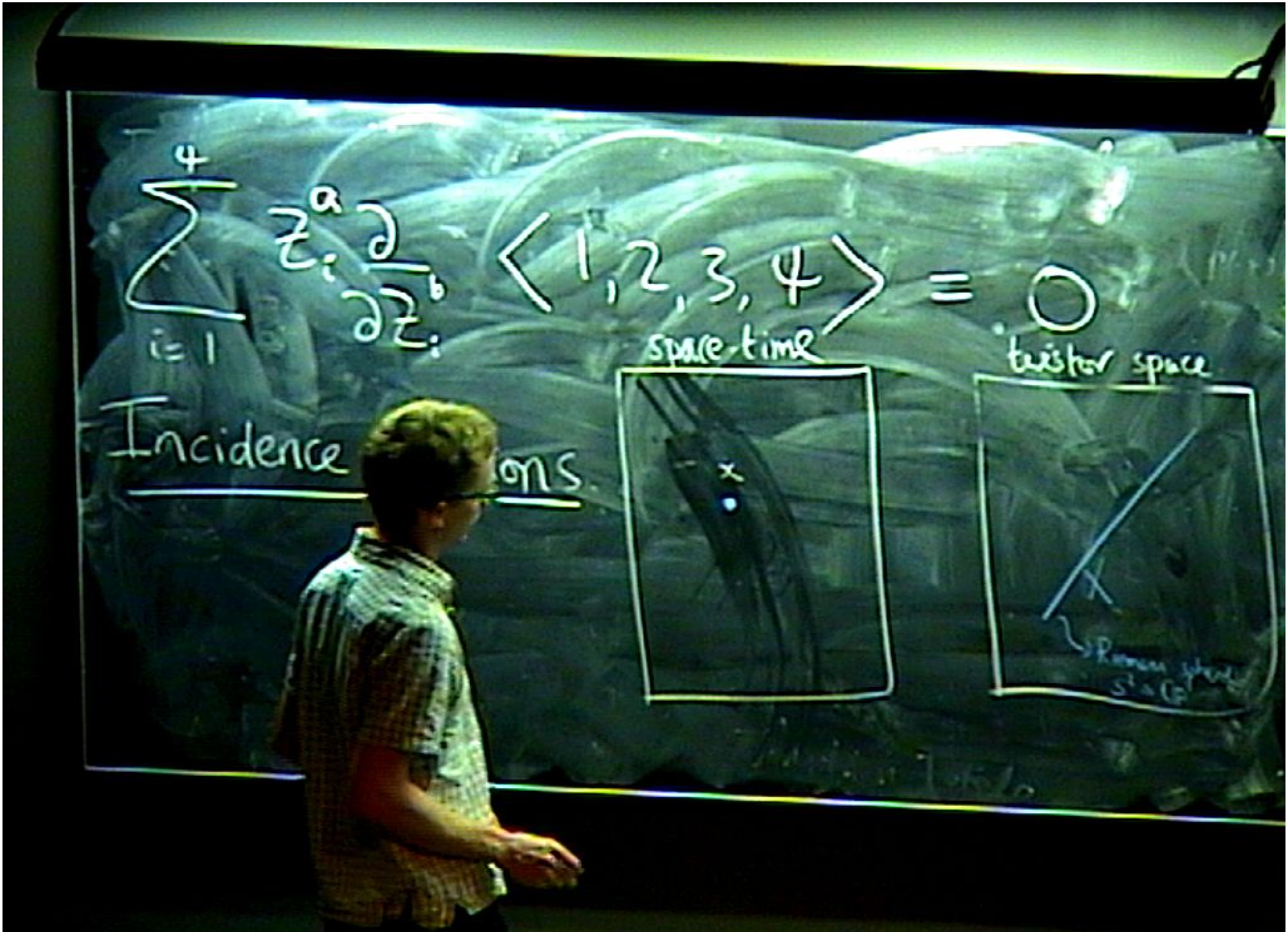

$\sum_{i=1}^{4} z_i^a \frac{\partial}{\partial z_i^b} \langle 1,2,3,4 \rangle = 0$


$\sum_{i=1}^{4} z_i^a \frac{\partial}{\partial z_i^b} \langle 1,2,3,4 \rangle = 0$
Incidence relations

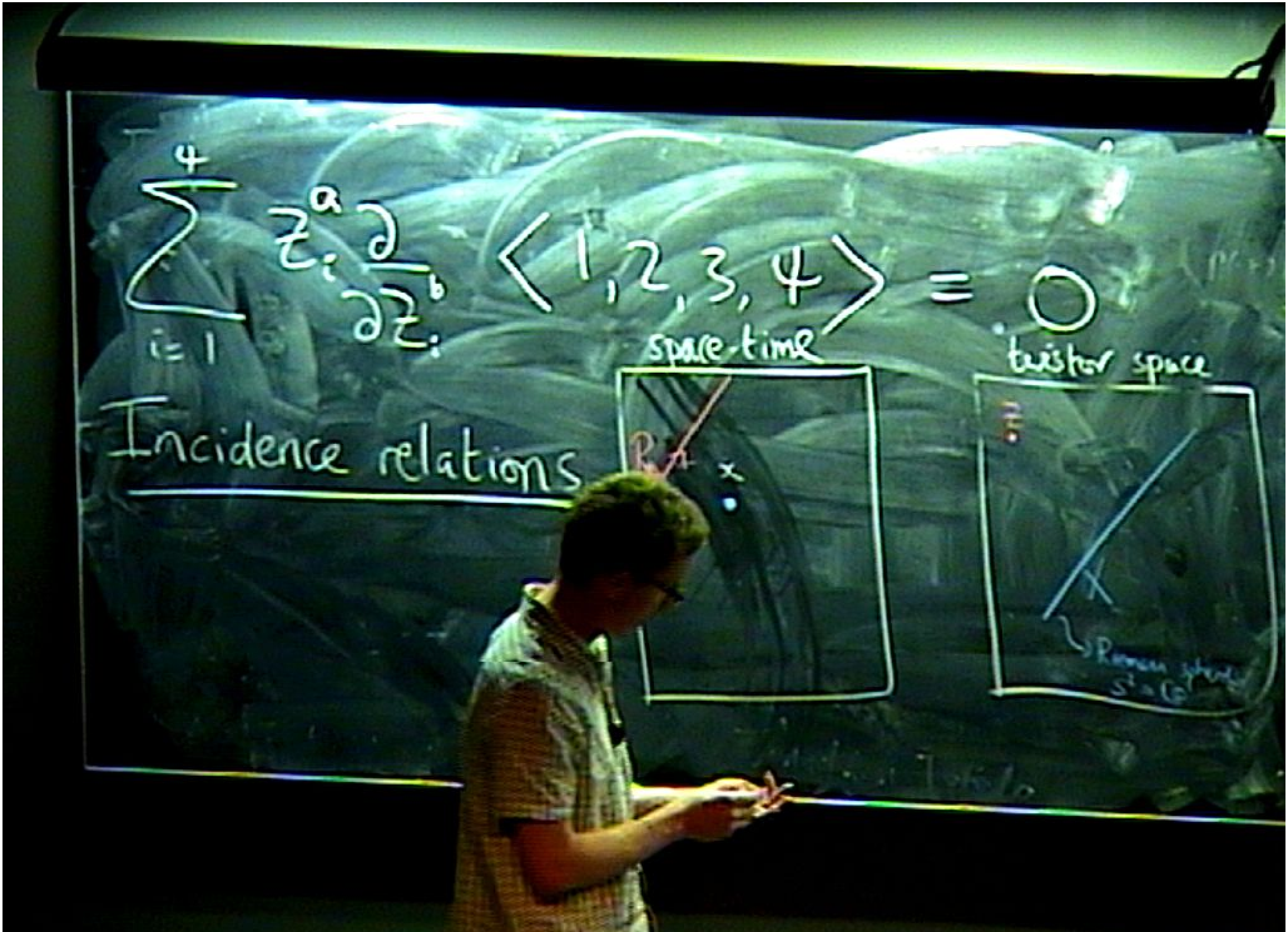


$\sum_{i=1}^{4} Z_i^a \frac{\partial}{\partial Z_i^b} \langle 1,2,3,4 \rangle = 0$
space-time
twistor space
relations


space-time
twistor space



$\sum_{i=1}^{4} Z_i^a \frac{\partial}{\partial Z_i^b} \langle 1,2,3,4 \rangle = 0$
Incidence
space-time
twistor space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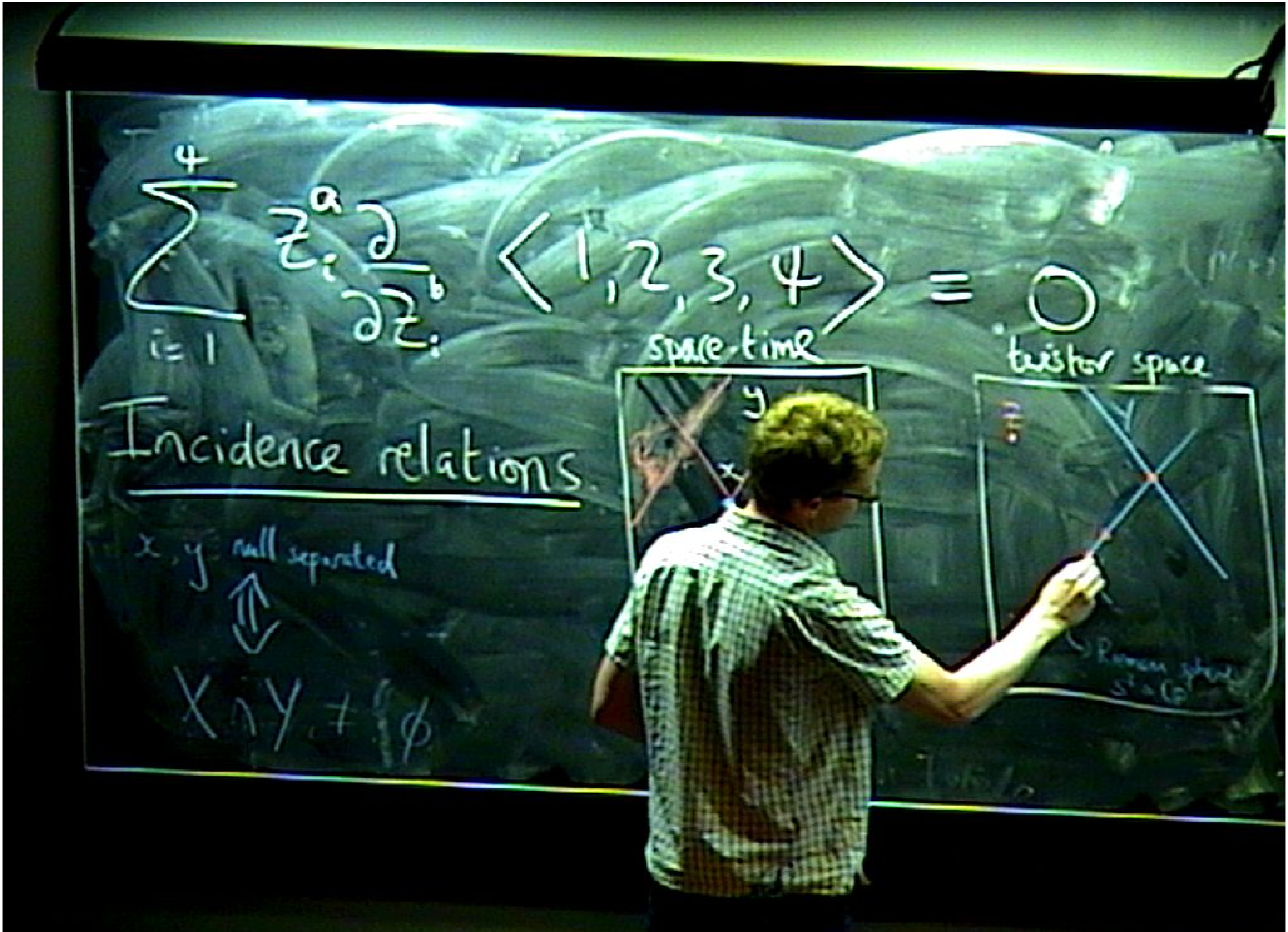
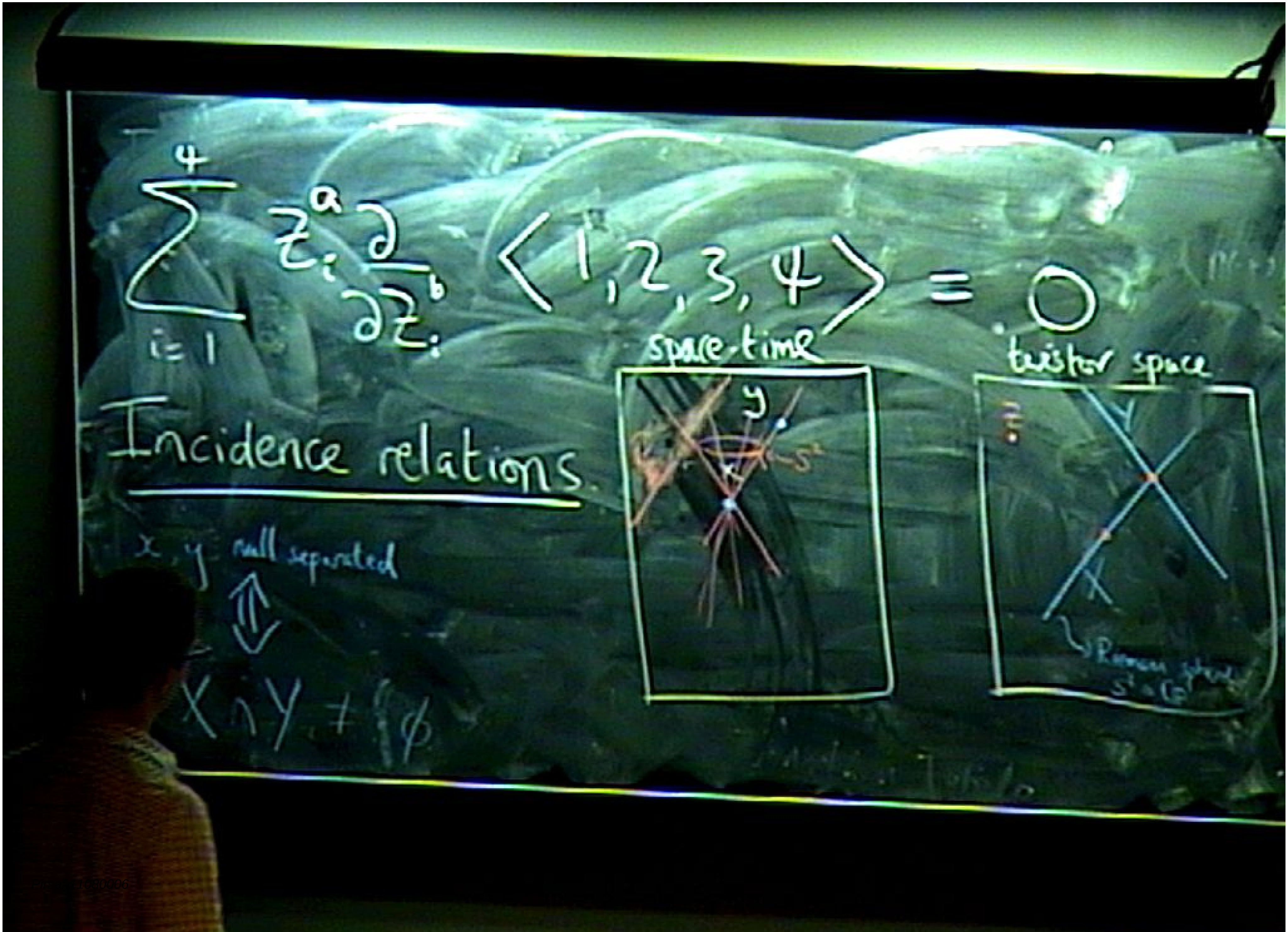
space-time
twistor space
Incidence relations



$\sum_{i=1}^{4} Z_i^a \frac{\partial}{\partial Z_i^b} \langle 1,2,3,4 \rangle = 0$
Incidence relations
space-time
twistor space
x, y null separated
$X \cap Y \neq \emptyset$



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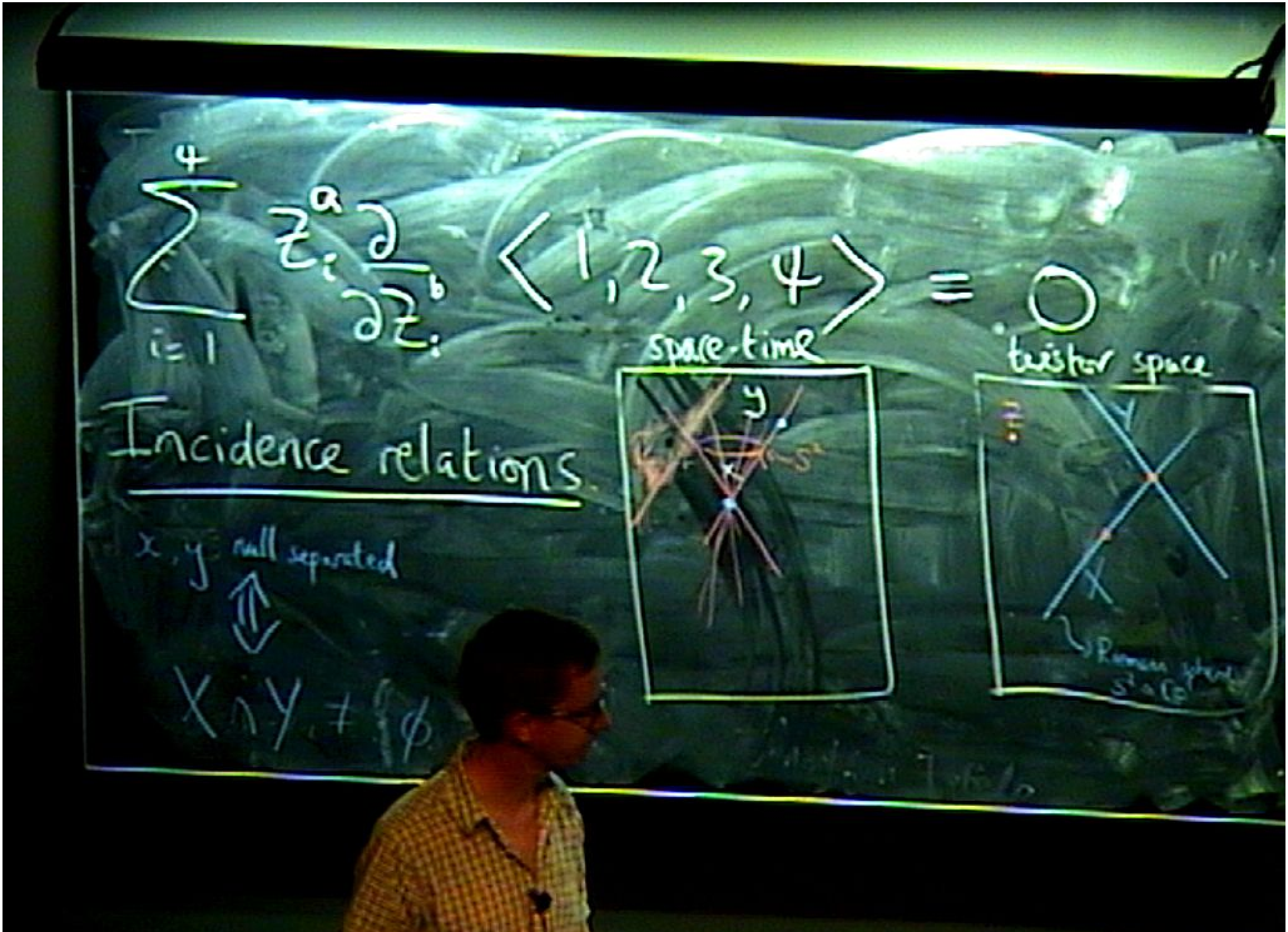
space-time
twistor space
Incidence relations
x, y null separated
X ∩ Y ≠ ∅

$$Z^a = (\lambda_\alpha, \mu^{\dot\alpha})$$

$\alpha, \dot\alpha$ 2-cmpt spinor indices.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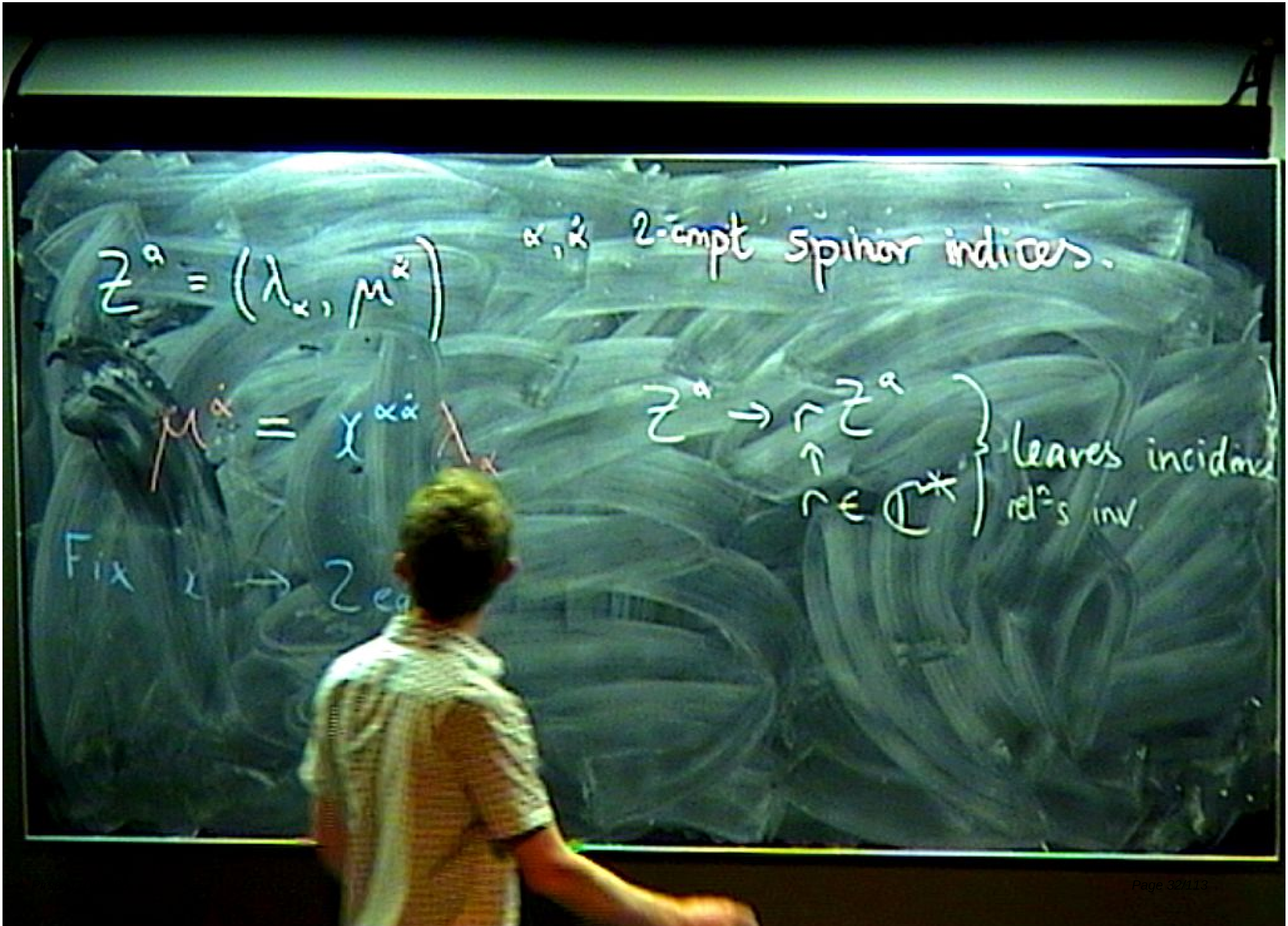

$Z^a = (\lambda_\alpha, \mu^{\dot\alpha})$
$\alpha, \dot\alpha$ 2-cmpt spinor indices.
$\mu^{\dot\alpha} = x^{\alpha\dot\alpha}\lambda_\alpha$
$Z^a \to r Z^a$
$r \in \mathbb{C}^*$
leaves incidence
rel^ns inv.
Fix $x \to$ 2 eq

$Z^a = (\lambda_\alpha, \mu^{\dot\alpha})$ $\alpha, \dot\alpha$ 2-cmpt spinor indices.

$\mu^{\dot\alpha} = x^{\alpha\dot\alpha}\lambda_\alpha$

Fix x → 2 eq^ns

$Z^a \to r Z^a$, $r \in \mathbb{C}^*$ } leaves incidence rel^ns inv.

$p = \lambda\tilde\lambda$

$\lambda \to r\lambda$, $\tilde\lambda \to r^{-1}\tilde\lambda$

$$Z^a = (\lambda_\alpha, \mu^{\dot\alpha})$$

$\alpha, \dot\alpha$ 2-cmpt spinor indices.

$$\mu^{\dot\alpha} = x^{\alpha\dot\alpha}\lambda_\alpha$$

Fix x → 2 eq^ns

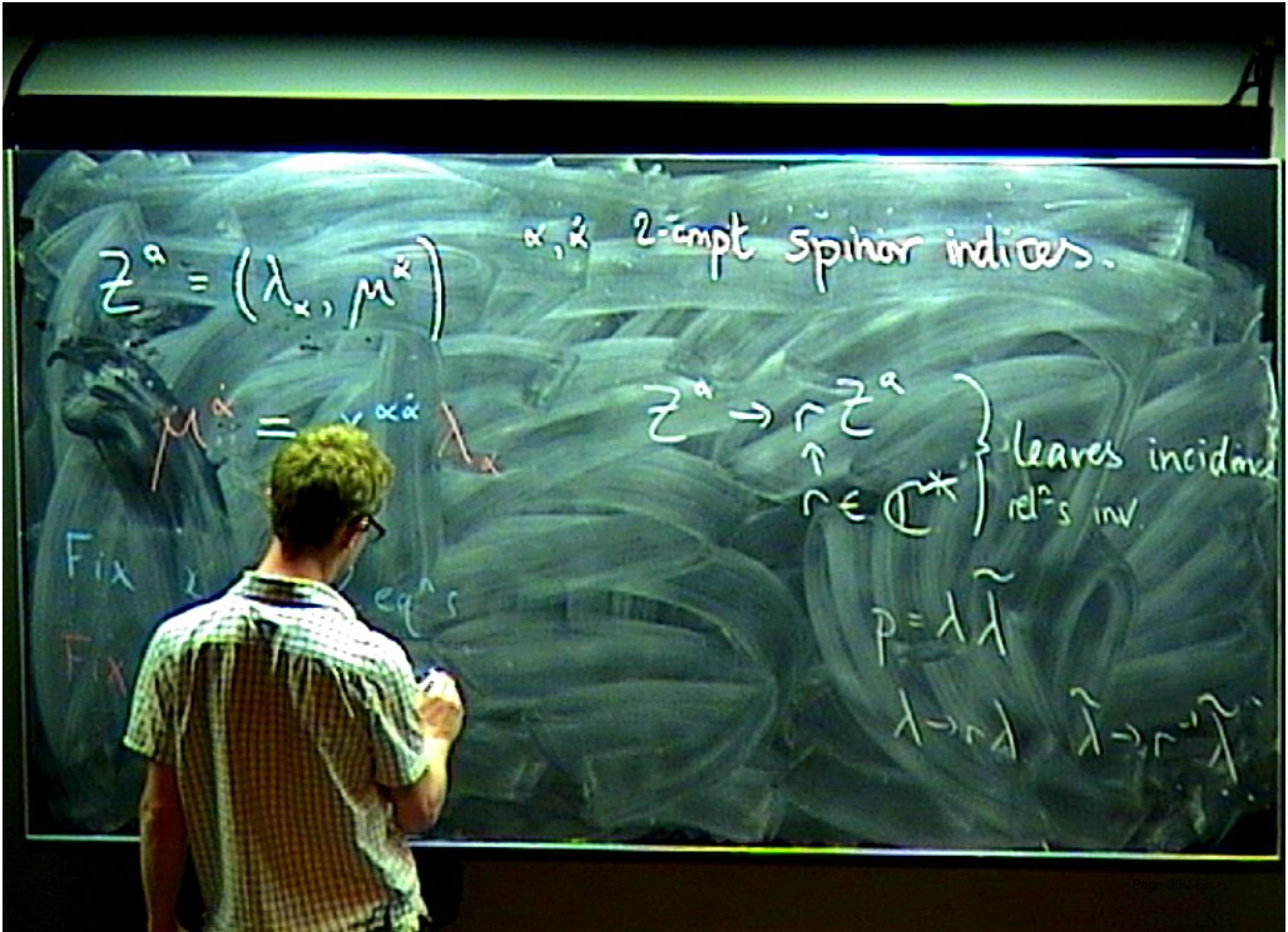
$$Z^a \to r Z^a$$

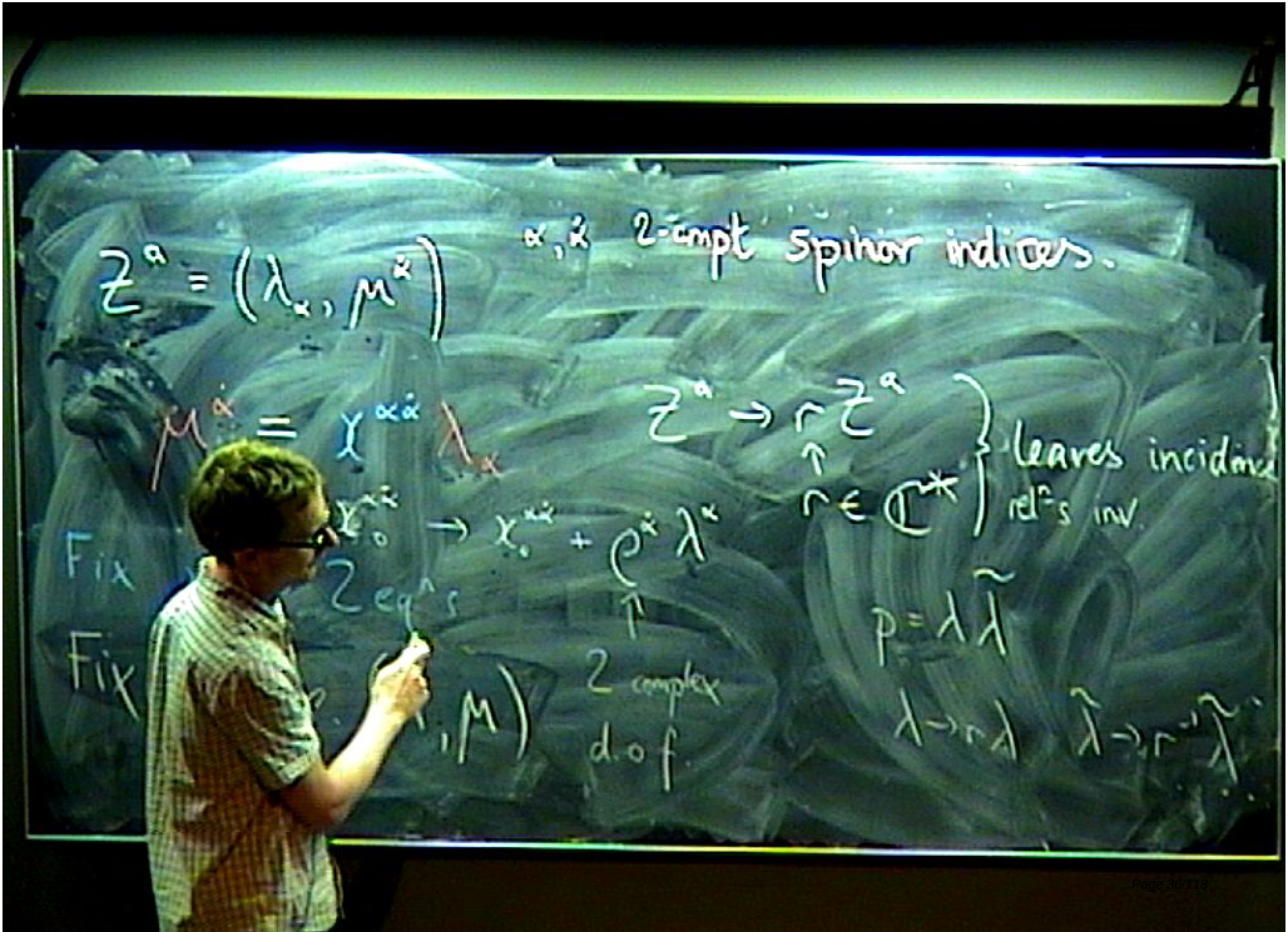
$\uparrow$

$r \in \mathbb{C}^*$ } leaves incidence rel^ns inv.

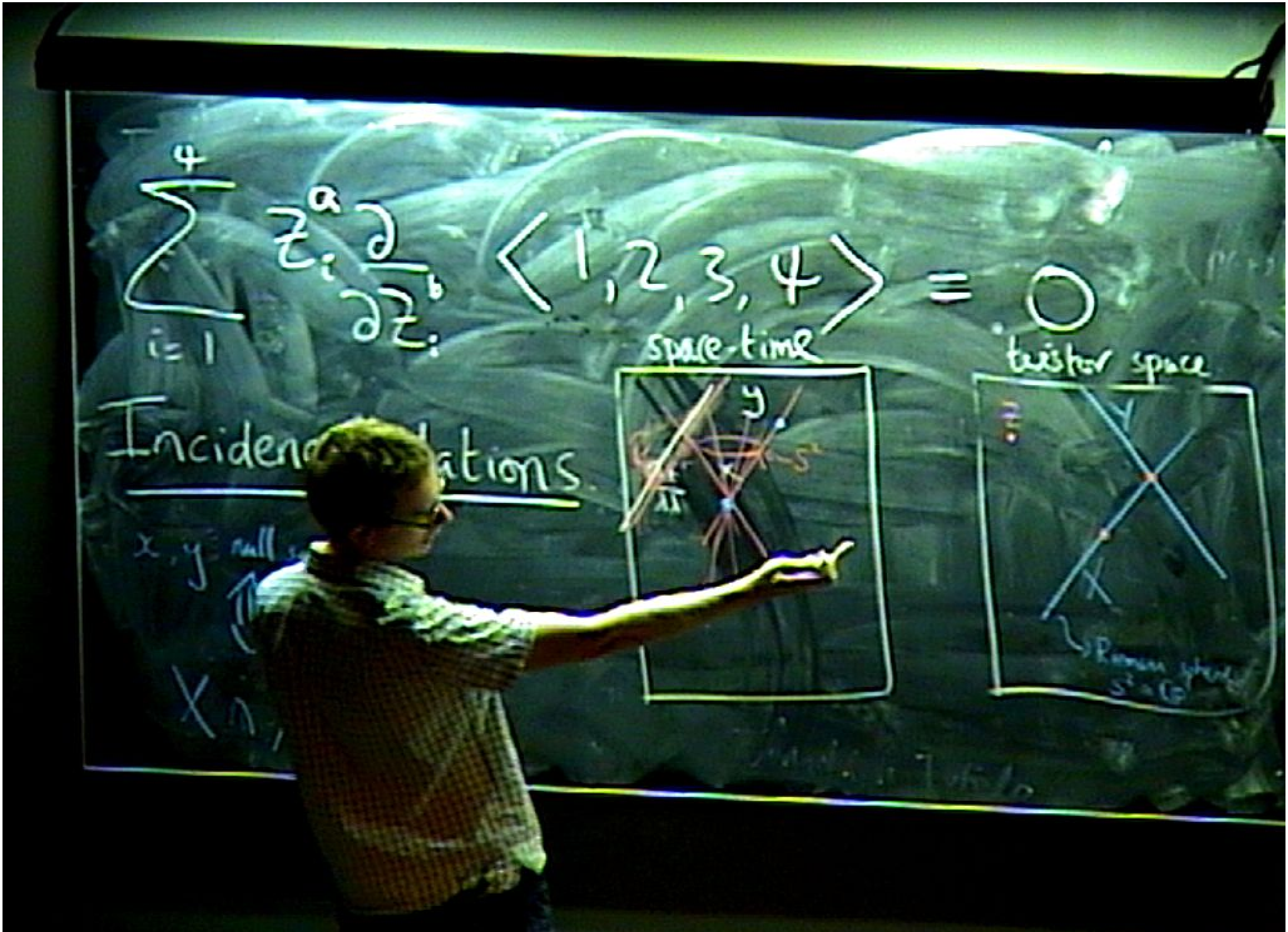
$$p = \lambda\tilde\lambda$$

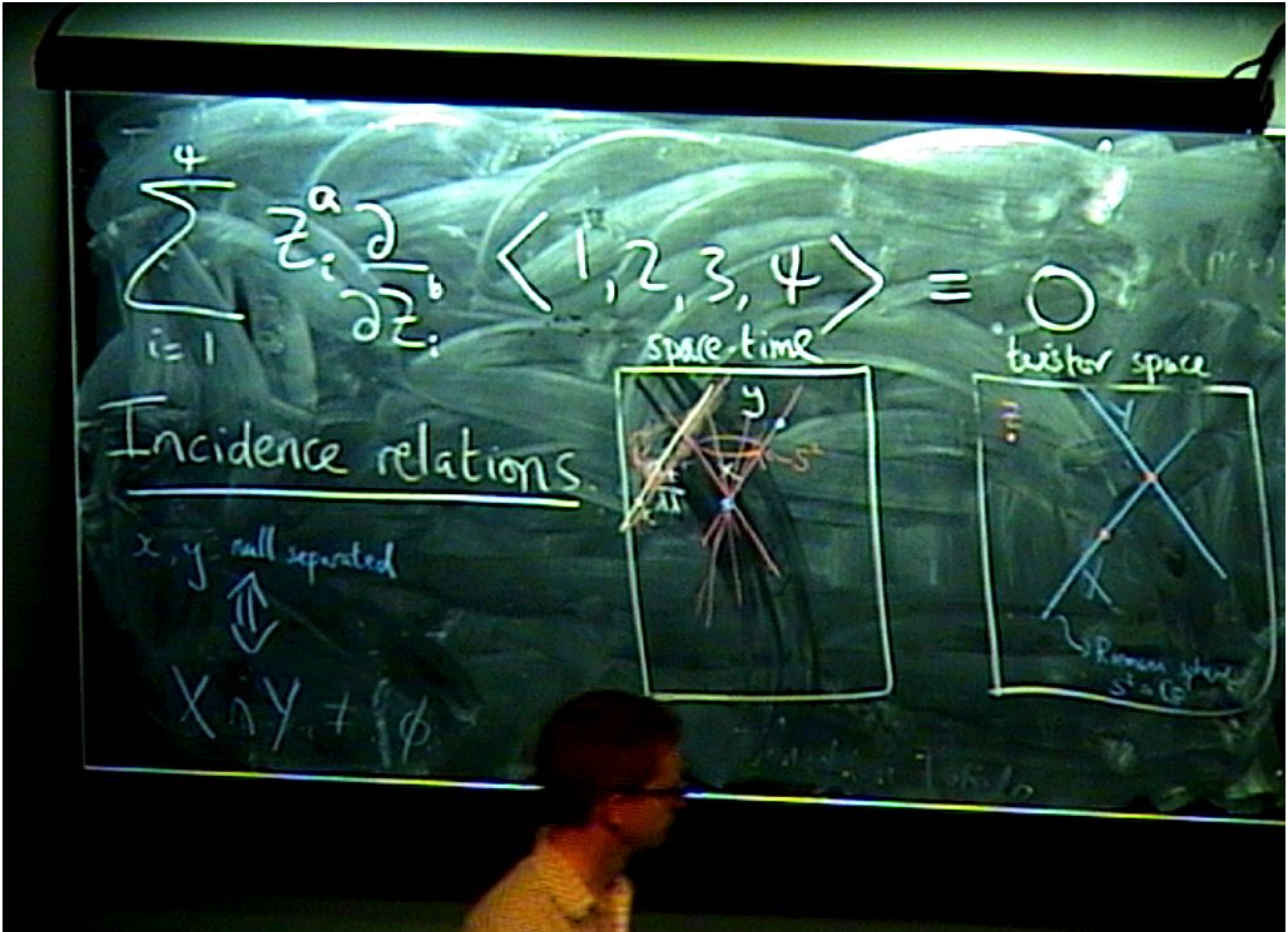
$$\lambda \to r\lambda \qquad \tilde\lambda \to r^{-1}\tilde\lambda$$


Z^a = (λ_α, μ^α̇)
α, α̇ 2-cmpt spinor indices.
μ^α̇ = x^αα̇ λ_α
Z^a → r Z^a
↑
r ∈ ℂ*
leaves incidence rel^ns inv.
Fix
eq^ns
Fix
p = λ λ̃
λ → r λ
λ̃ → r^-1 λ̃

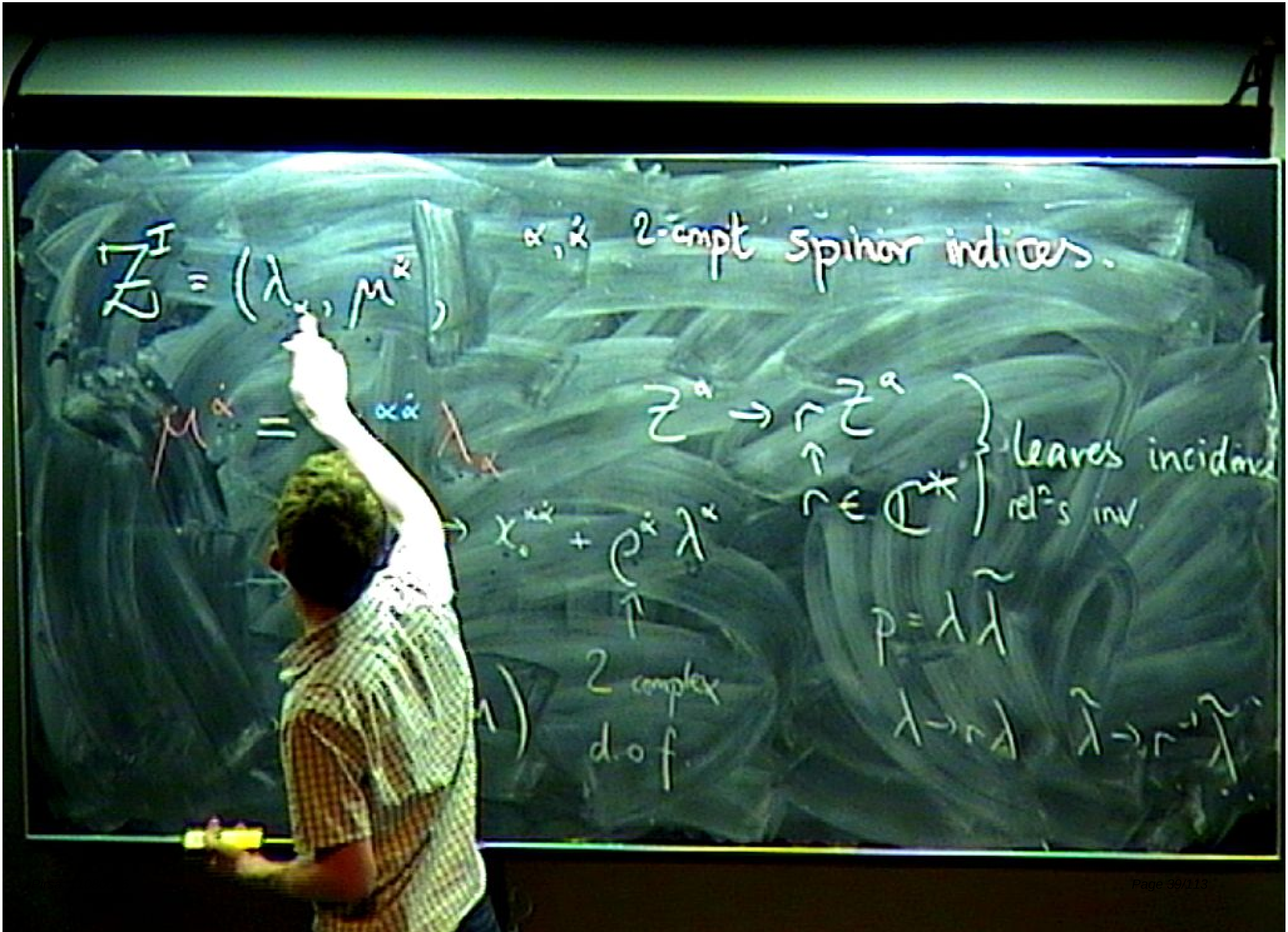


Z^a = (λ_α, μ^α̇)
α, α̇ 2-cmpt spinor indices.
μ^α̇ = x^αα̇ λ_α
Z^a → r Z^a
r ∈ ℂ*
leaves incidence
rel^ns inv.
x_0^αα̇ → x_0^αα̇ + ρ^α̇ λ^α
Fix
Z eq^ns
Fix
, μ)
2 complex
d.o.f
p = λ λ̃
λ → r λ
λ̃ → r⁻¹ λ̃


space-time
twistor space



$\sum_{i=1}^{4} Z_i^a \frac{\partial}{\partial Z_i^b} \langle 1,2,3,4 \rangle = 0$
space-time
twistor space
Incidence relations
x, y null separated
$X \cap Y \neq \emptyset$


$Z^I = (\lambda_\alpha, \mu^{\dot\alpha})$
$\alpha, \dot\alpha$ 2-cmpt spinor indices.
$\mu^{\dot\alpha} =$
λ_α
$x_0^{\alpha\dot\alpha} + \rho^{\dot\alpha}\lambda^\alpha$
2 complex d.o.f.
$Z^a \to r Z^a$
$r \in \mathbb{C}^*$
leaves incidence rel^ns inv.
$p = \lambda\tilde\lambda$
$\lambda \to r\lambda \quad \tilde\lambda \to r^{-1}\tilde\lambda$

$Z^I = (\lambda_\alpha, \mu^{\dot\alpha}, \psi^A)$ $\quad \alpha, \dot\alpha$ 2-cmpt spinor indices.

$\mu^{\dot\alpha} = x^{\alpha\dot\alpha}\lambda_\alpha$

$\psi^A = \theta^{A\alpha}\lambda_\alpha$

(x, θ) co-ords on <u>chiral</u> $N=4$ Minkowski

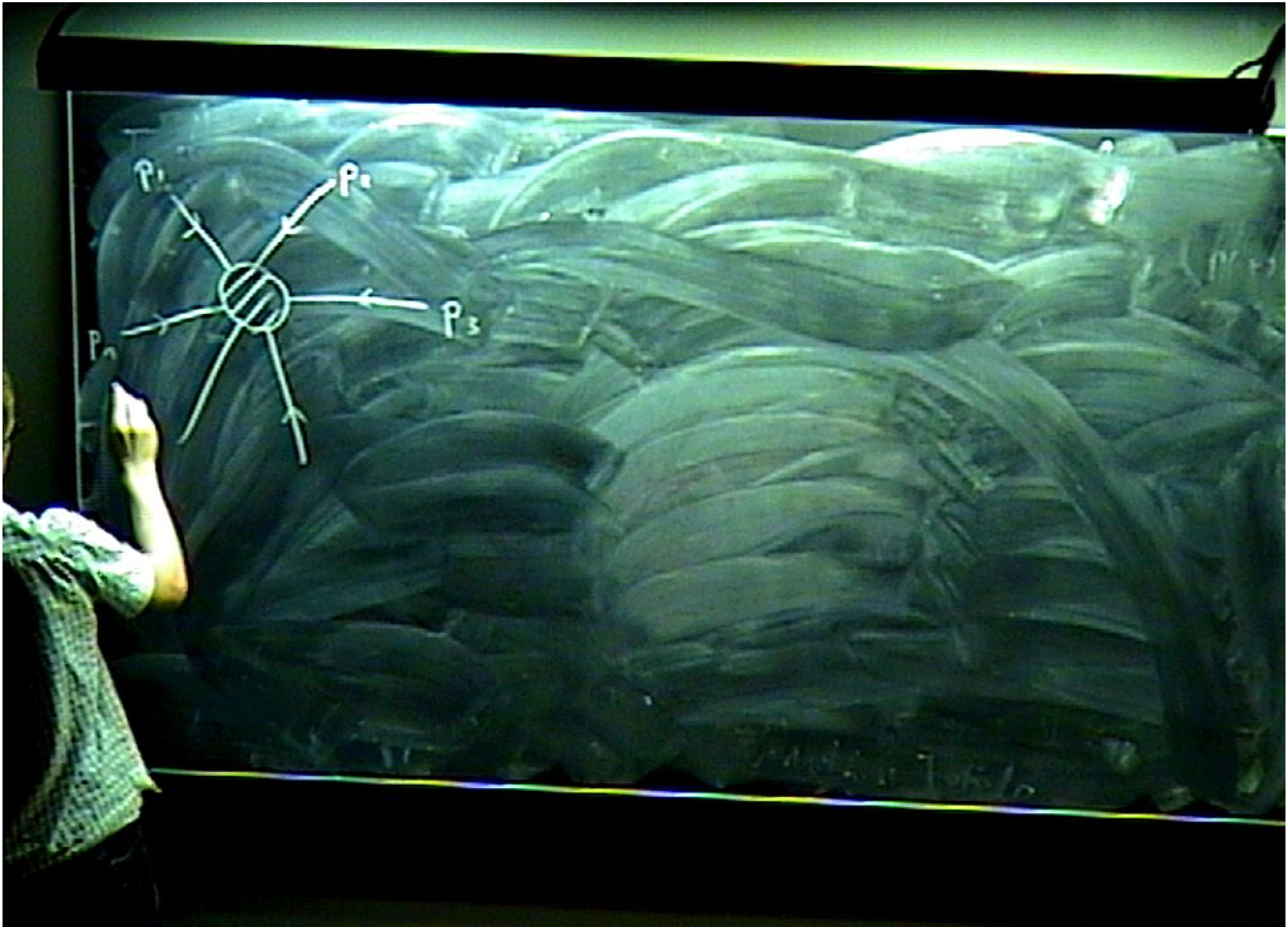
$Z^a \to r Z^a$, $\quad r \in \mathbb{C}^*$ } leaves incidence rel^{n}s inv.

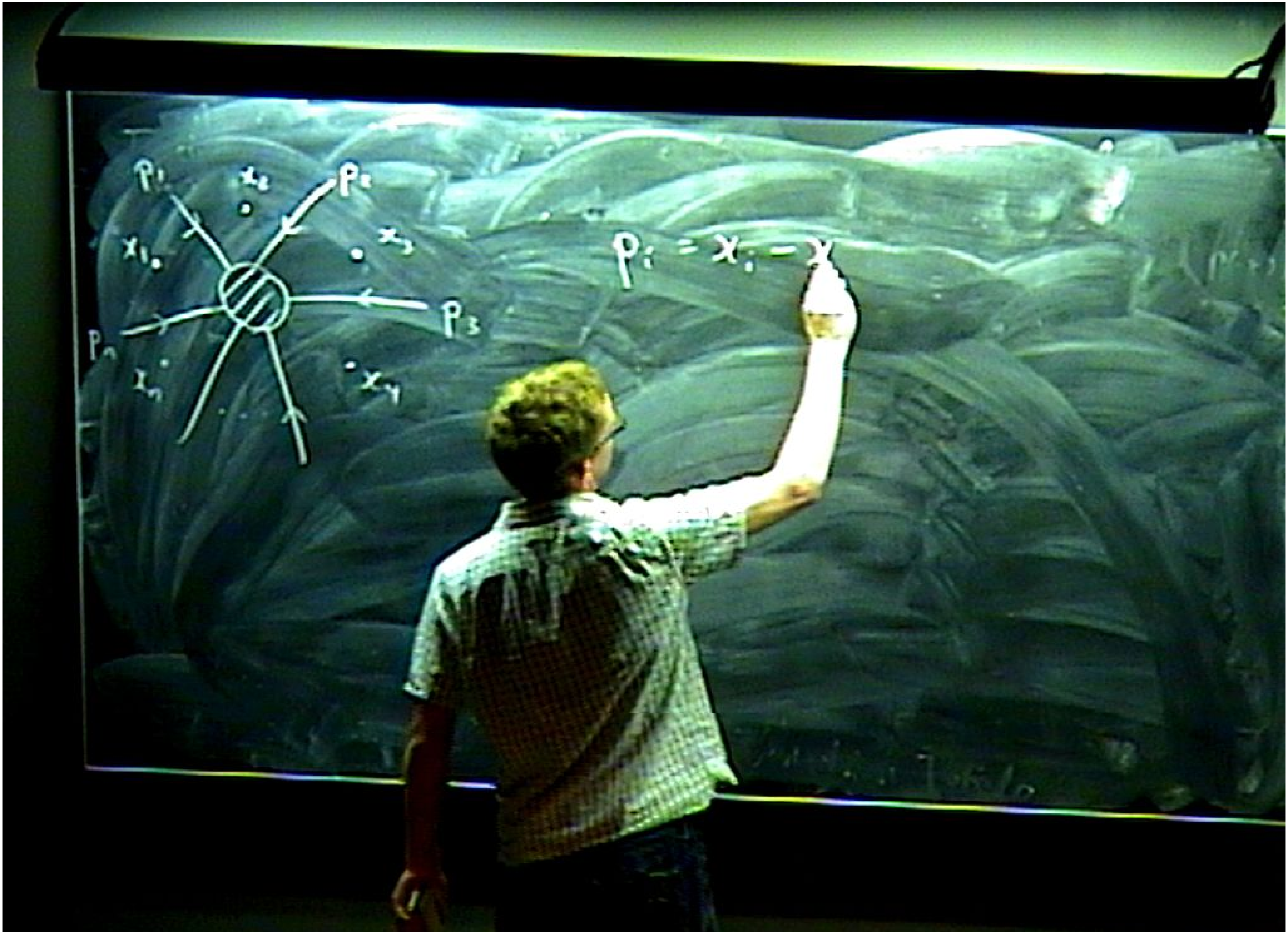
$p = \lambda\tilde\lambda$

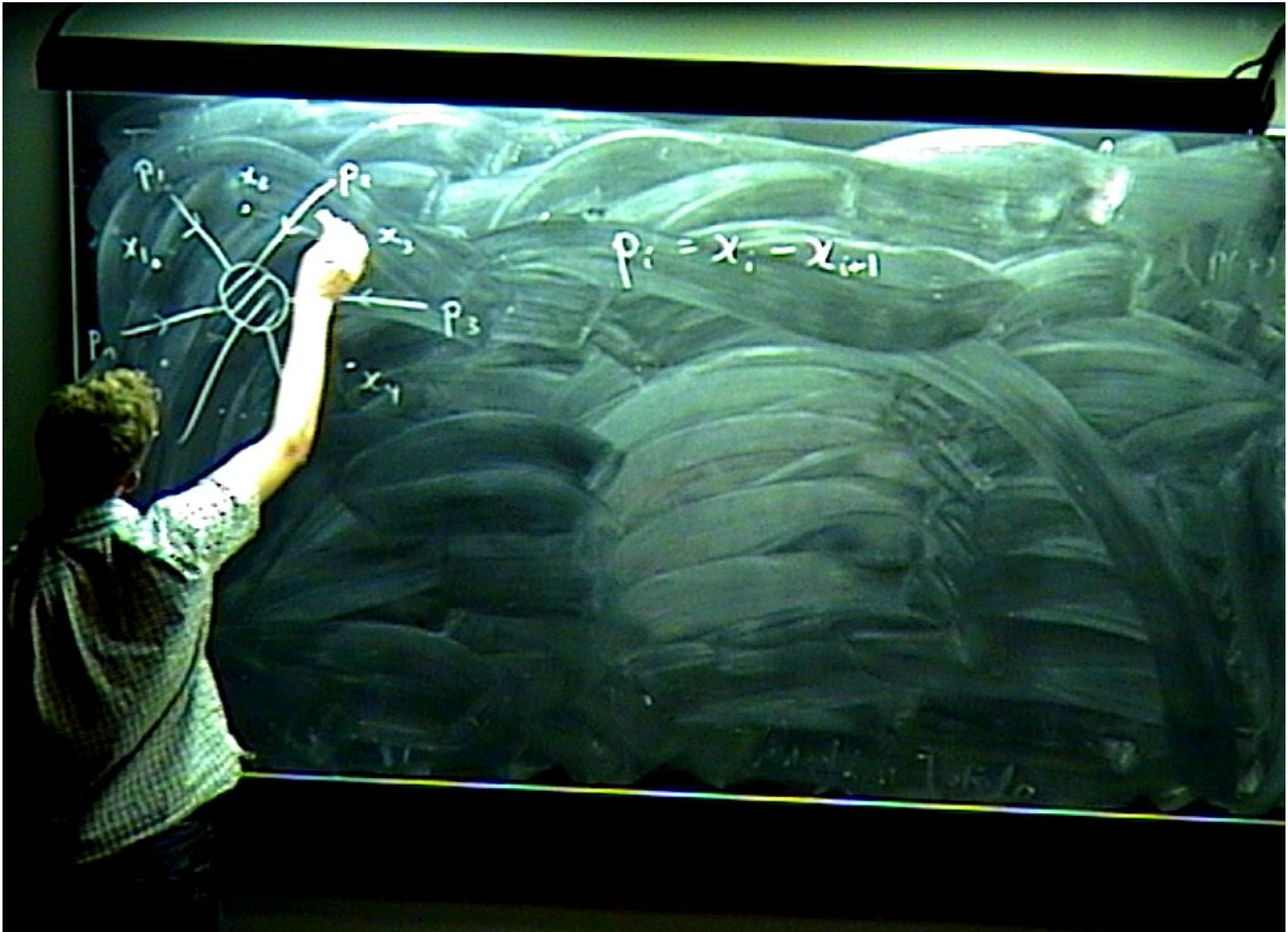
$\lambda \to r\lambda \quad \tilde\lambda \to r^{-1}\tilde\lambda$

$\mathcal{N}=4$ SYM (classically)

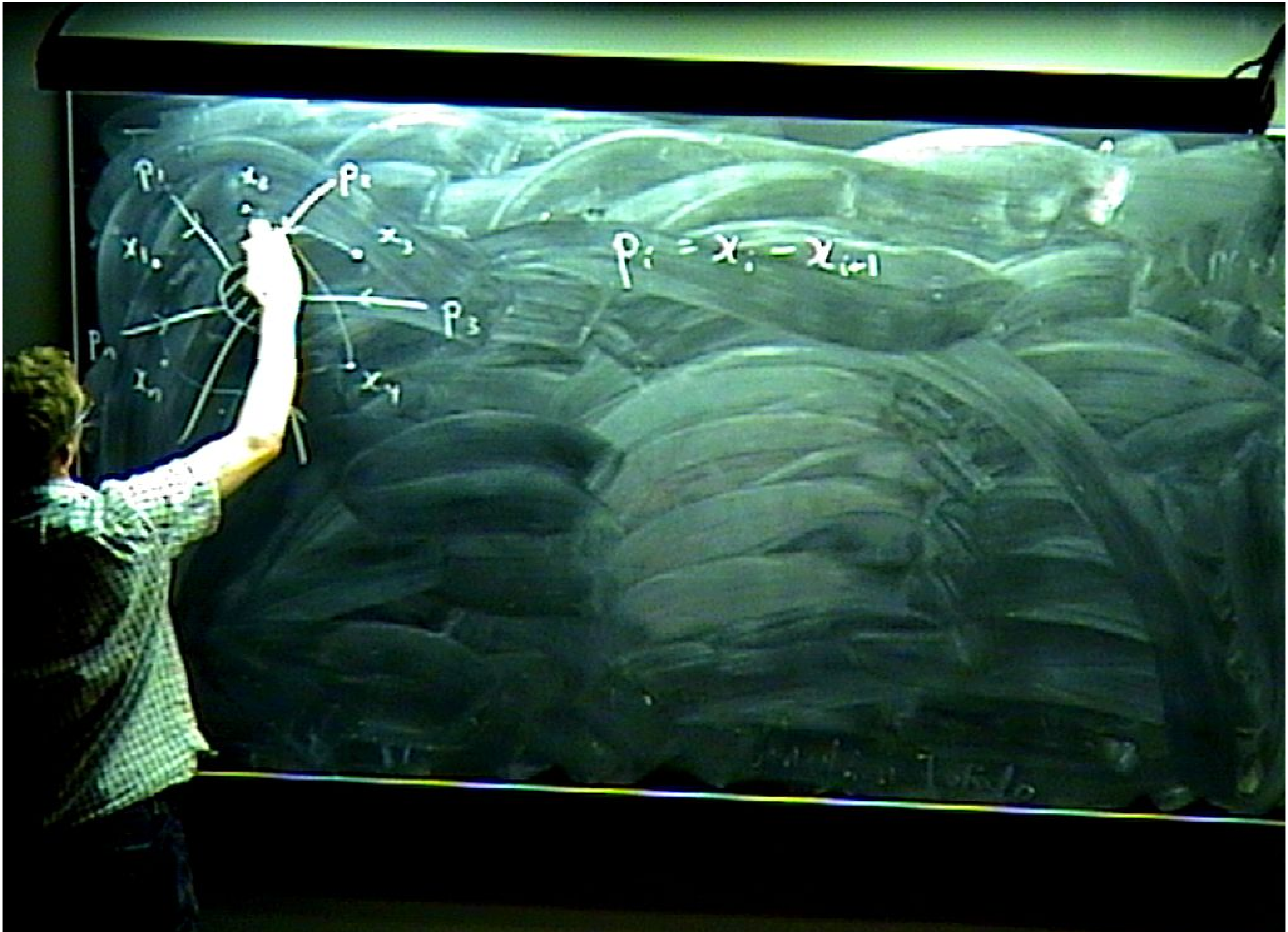
Both superconformal + dual superconformal } integrable

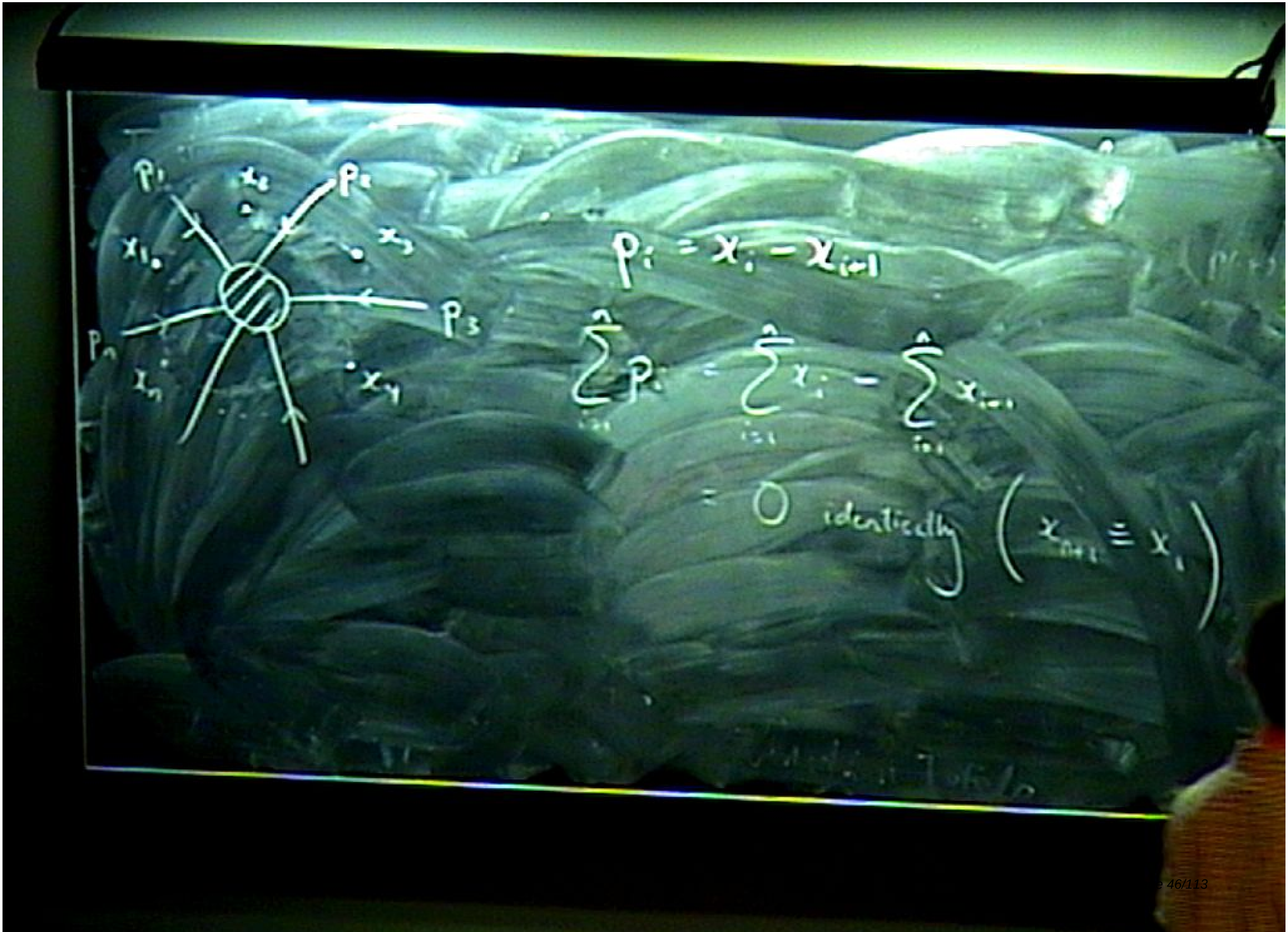

p1
p2
p3
p4

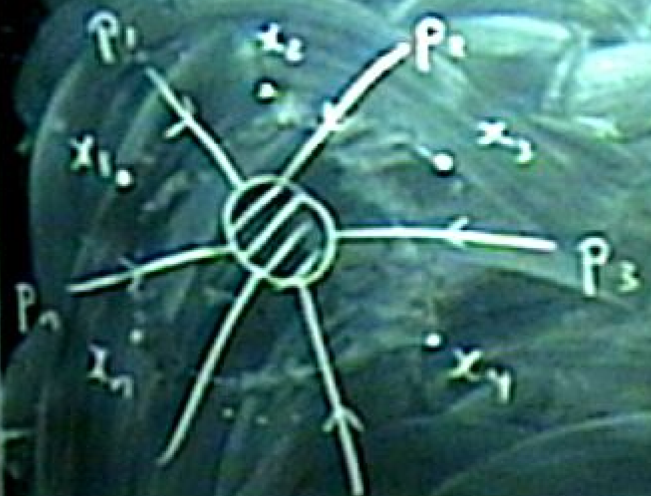

p1
x2
p2
x1
x3
p3
pn
xn
x4
pi = xi − x



p_1
x_2
p_2
x_1
x_3
p_3
p_n
x_4
$p_i = x_i - x_{i+1}$


p1
x2
p2
x1
x3
p3
pn
xn
x4
pi = xi − xi+1

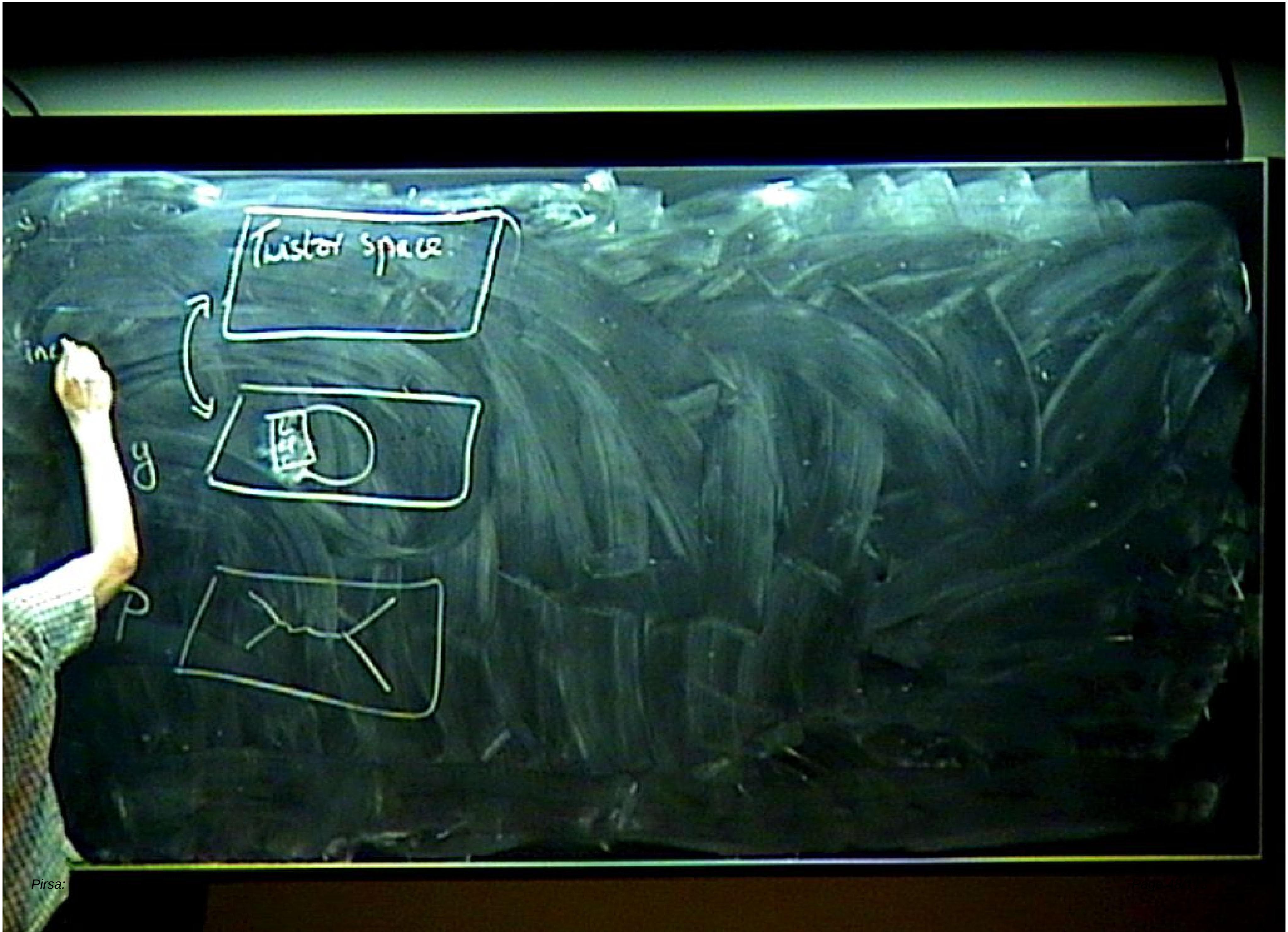

p1
x2
p2
x1
x3
p3
pn
xn
x4
pi = xi − x(i+1)
Σ(i=1 to n) pi = Σ(i=1 to n) xi − Σ(i=1 to n) x(i+1)
= 0 identically (x(n+1) ≡ x1)

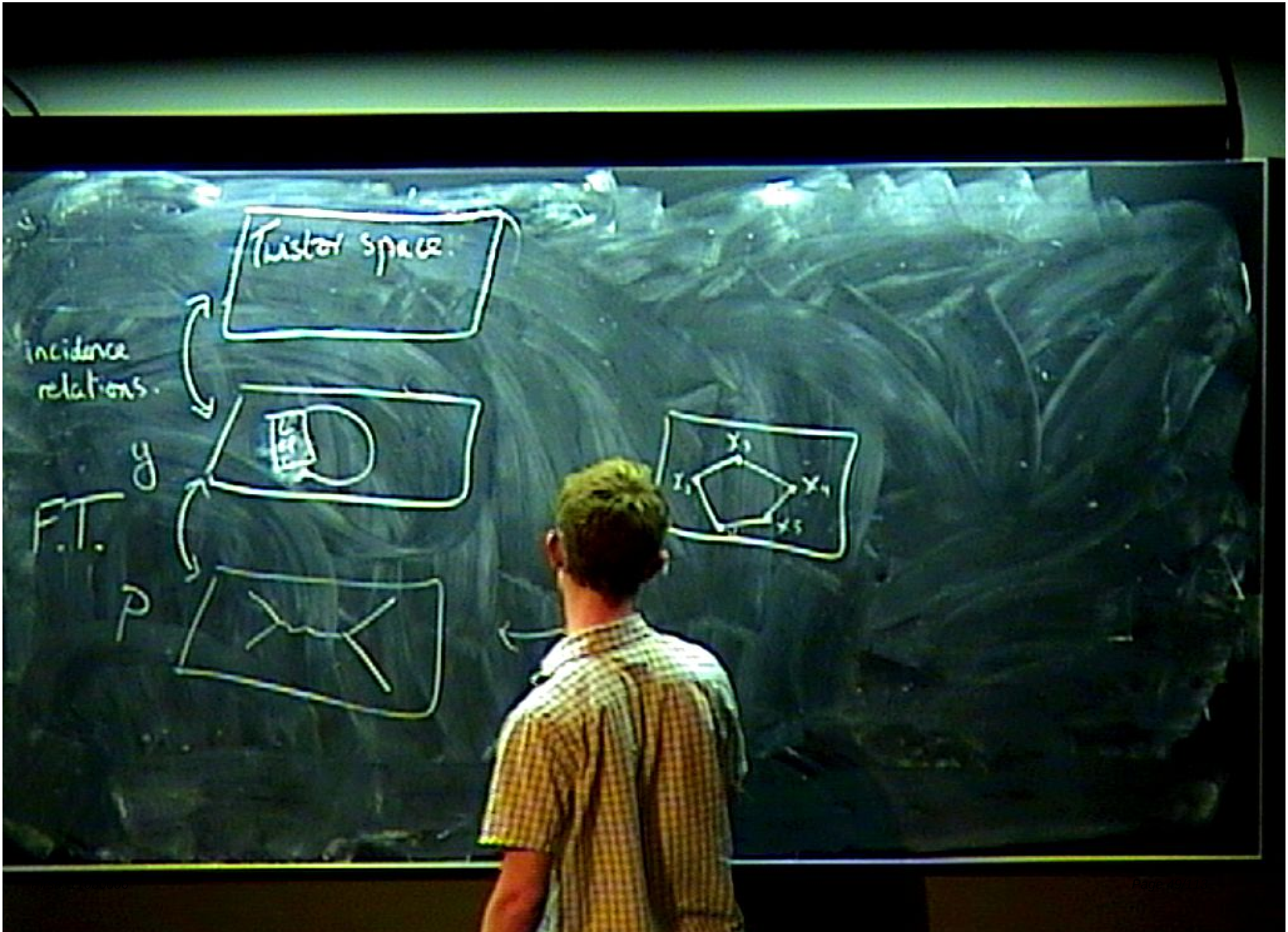


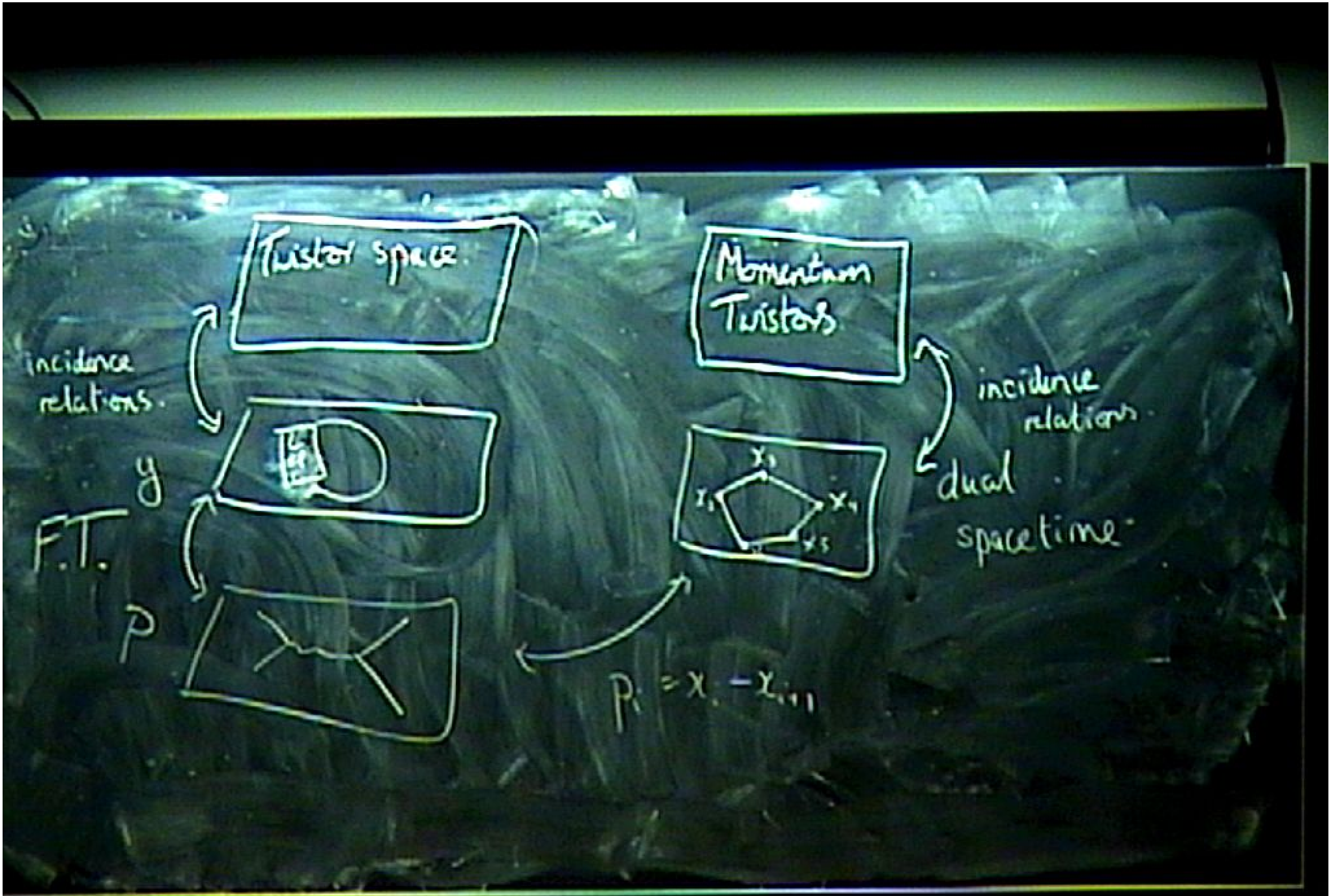
$$p_i = x_i - x_{i+1}$$

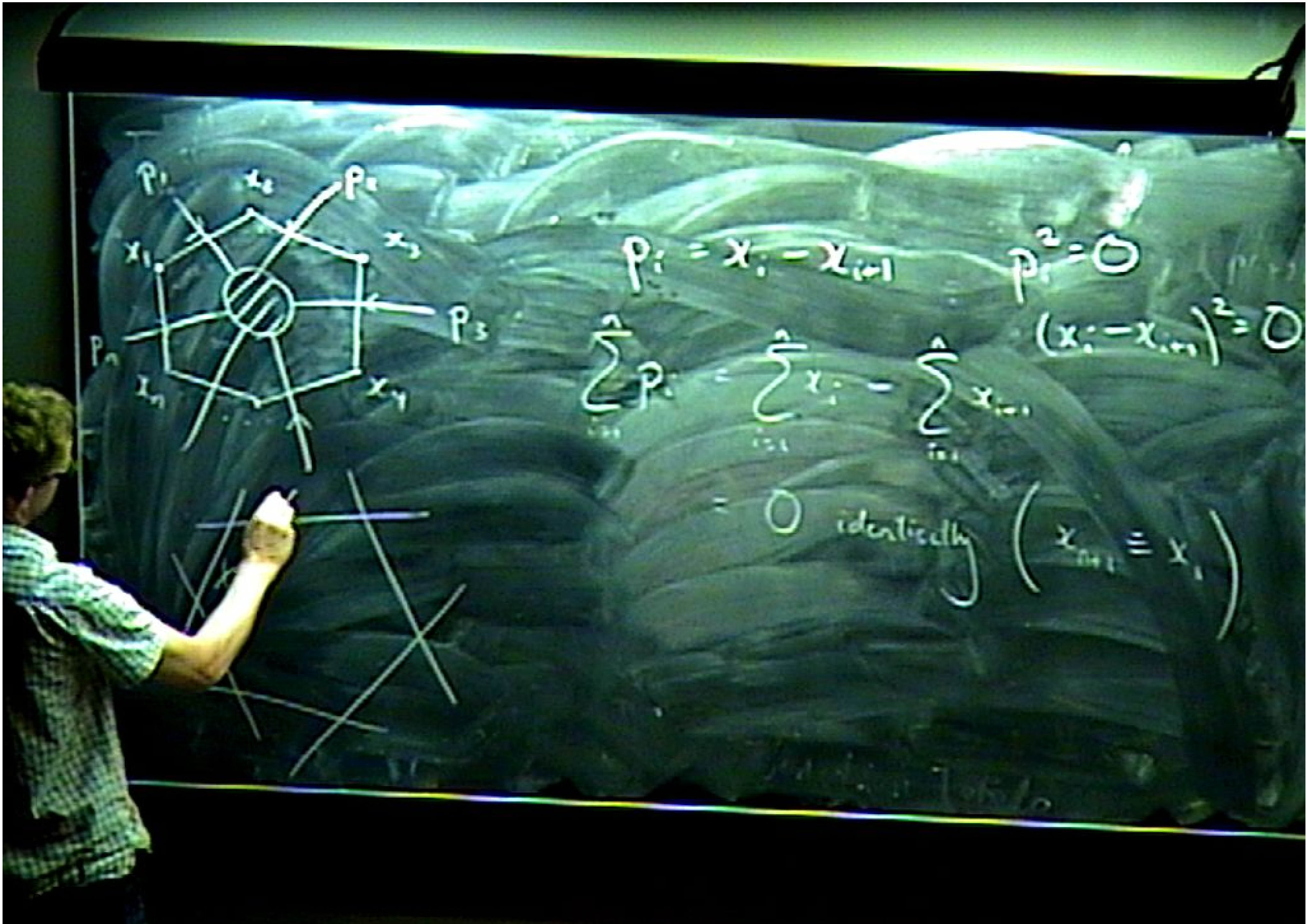
$$\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_{i+1}$$

$$= 0 \text{ identically } \left(x_{n+1} \equiv x_1 \right)$$

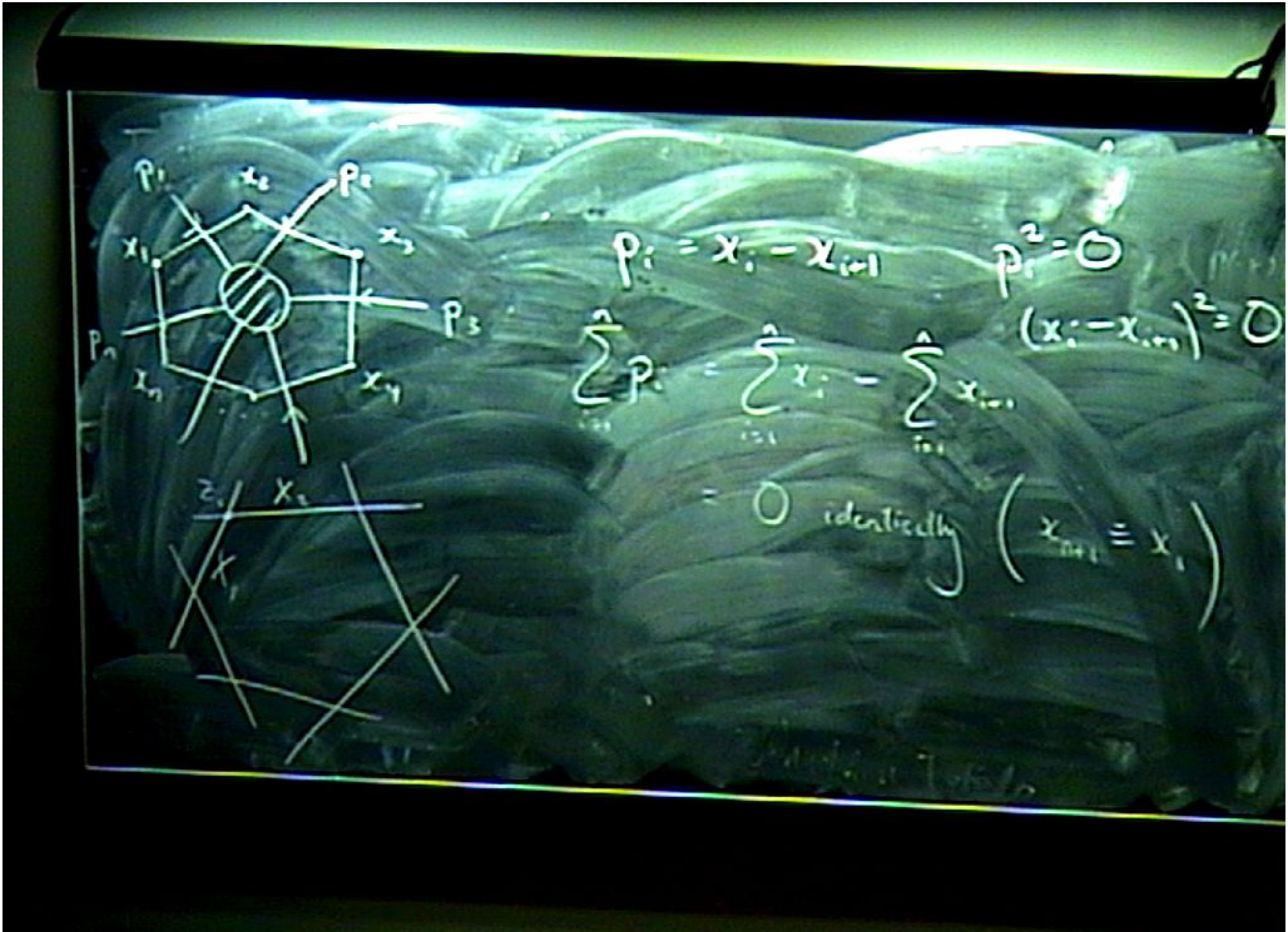

Twistor space
inc
y
p


Twistor space.
incidence
relations.
y
F.T.
p

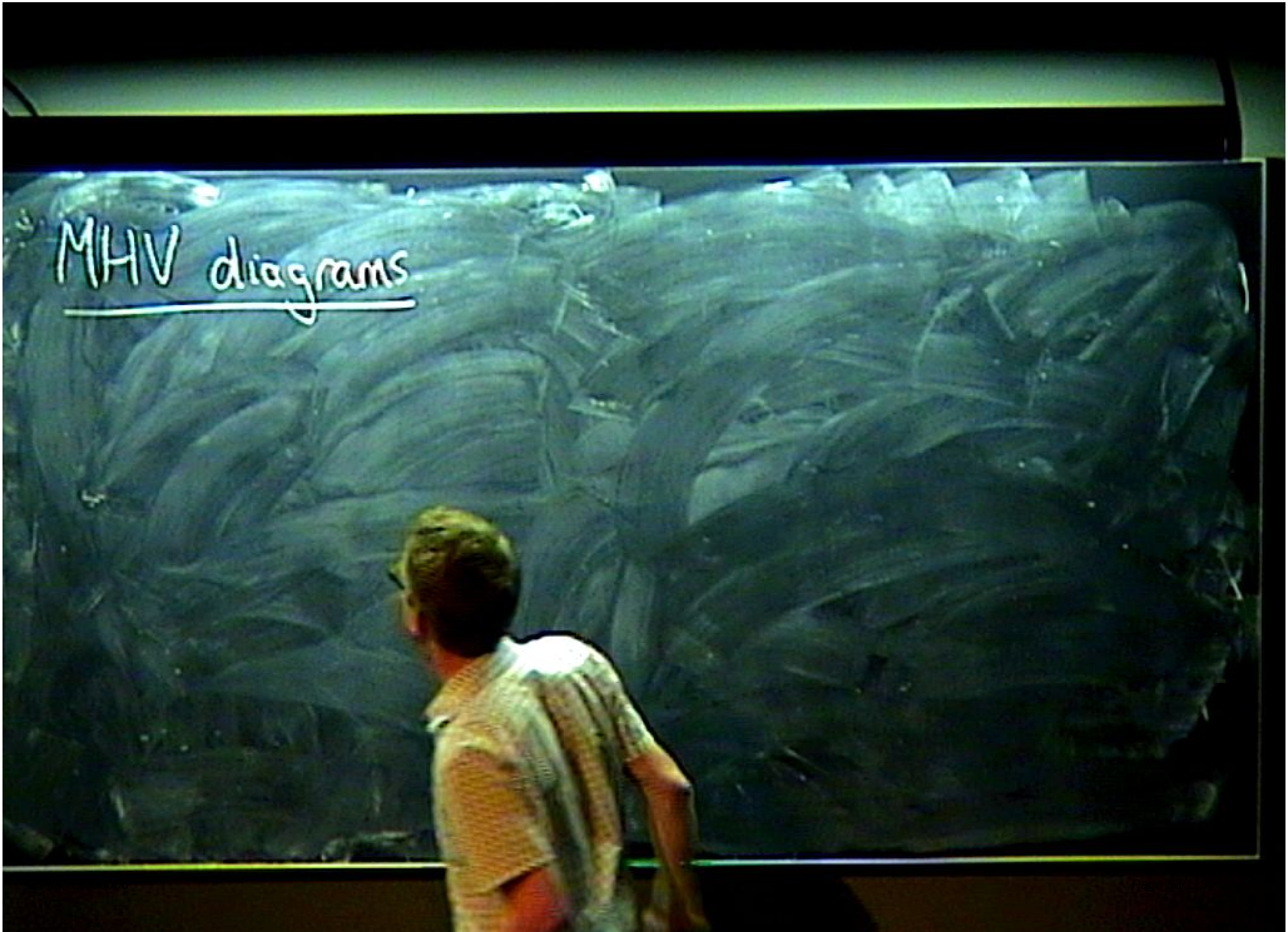

Twistor space
Momentum Twistors
incidence relations.
incidence relations
y
F.T.
p
dual spacetime
x_1
x_2
x_3
x_4
x_5
$p_i = x_i - x_{i+1}$

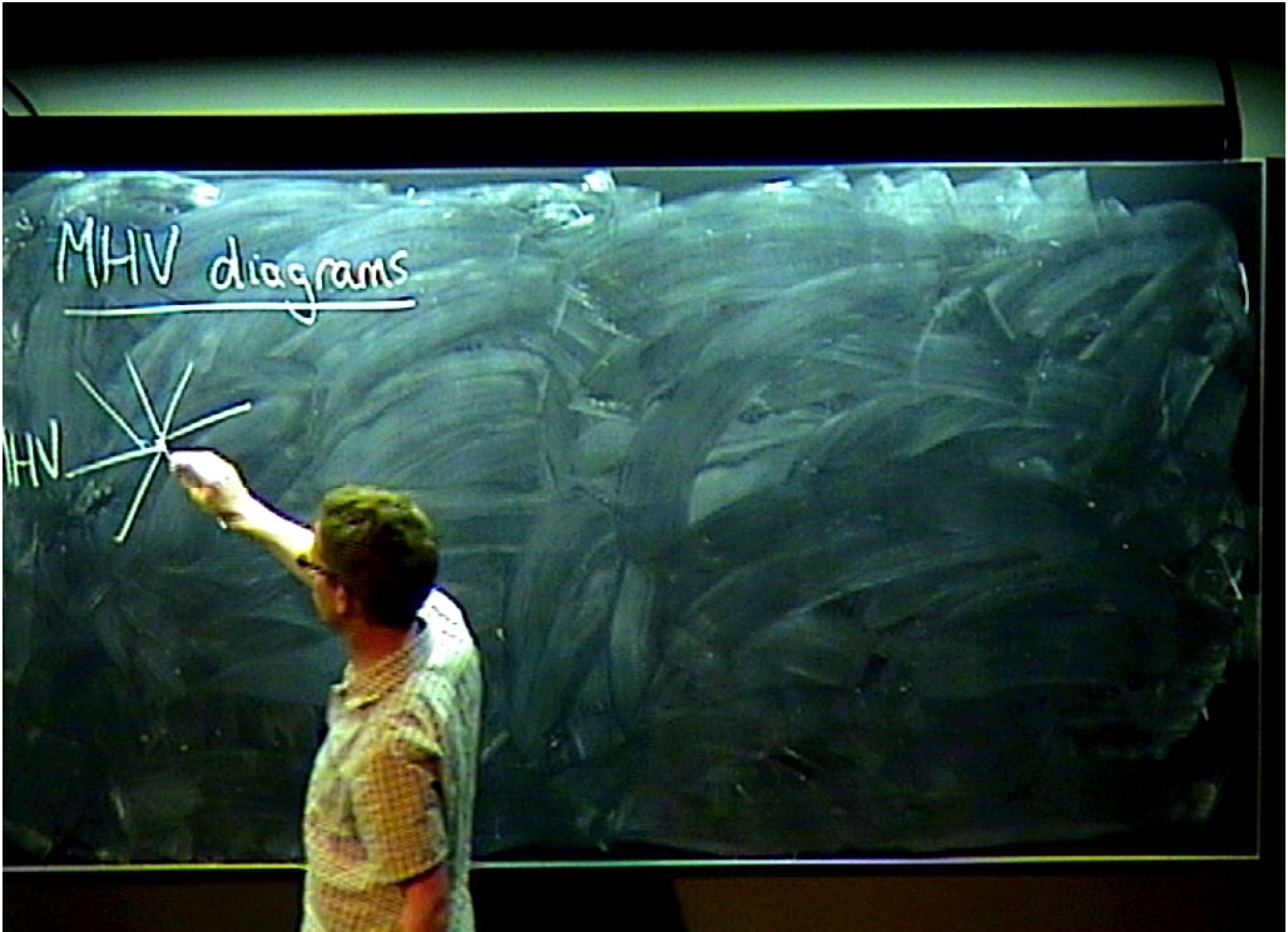


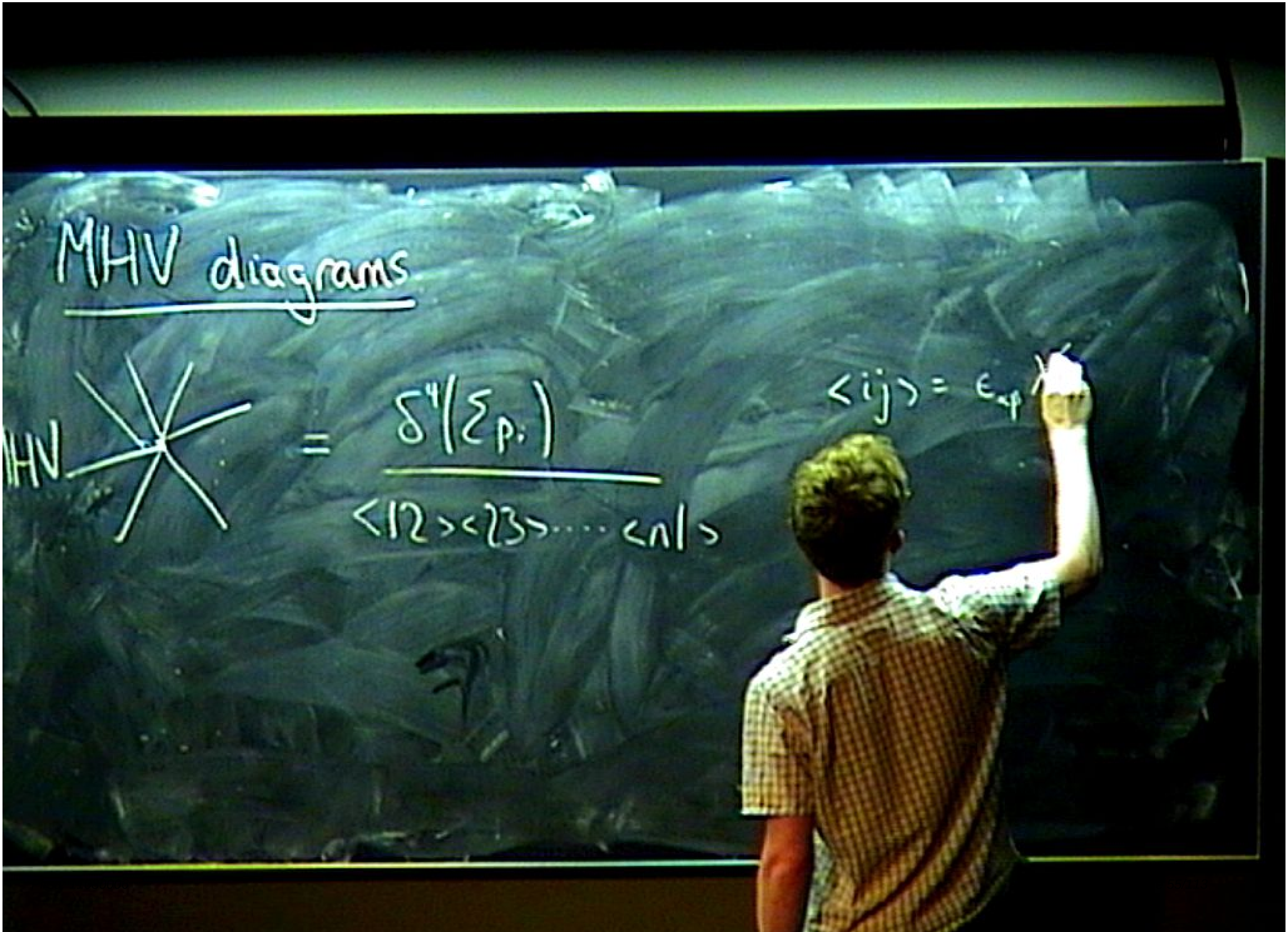
p1
x2
p2
x1
x3
p3
pn
xn
x4
$p_i = x_i - x_{i+1}$
$p_i^2 = 0$
$(x_i - x_{i+1})^2 = 0$
$\sum_{i=1}^{n} p_i = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_{i+1}$
$= 0$ identically $(x_{n+1} \equiv x_1)$

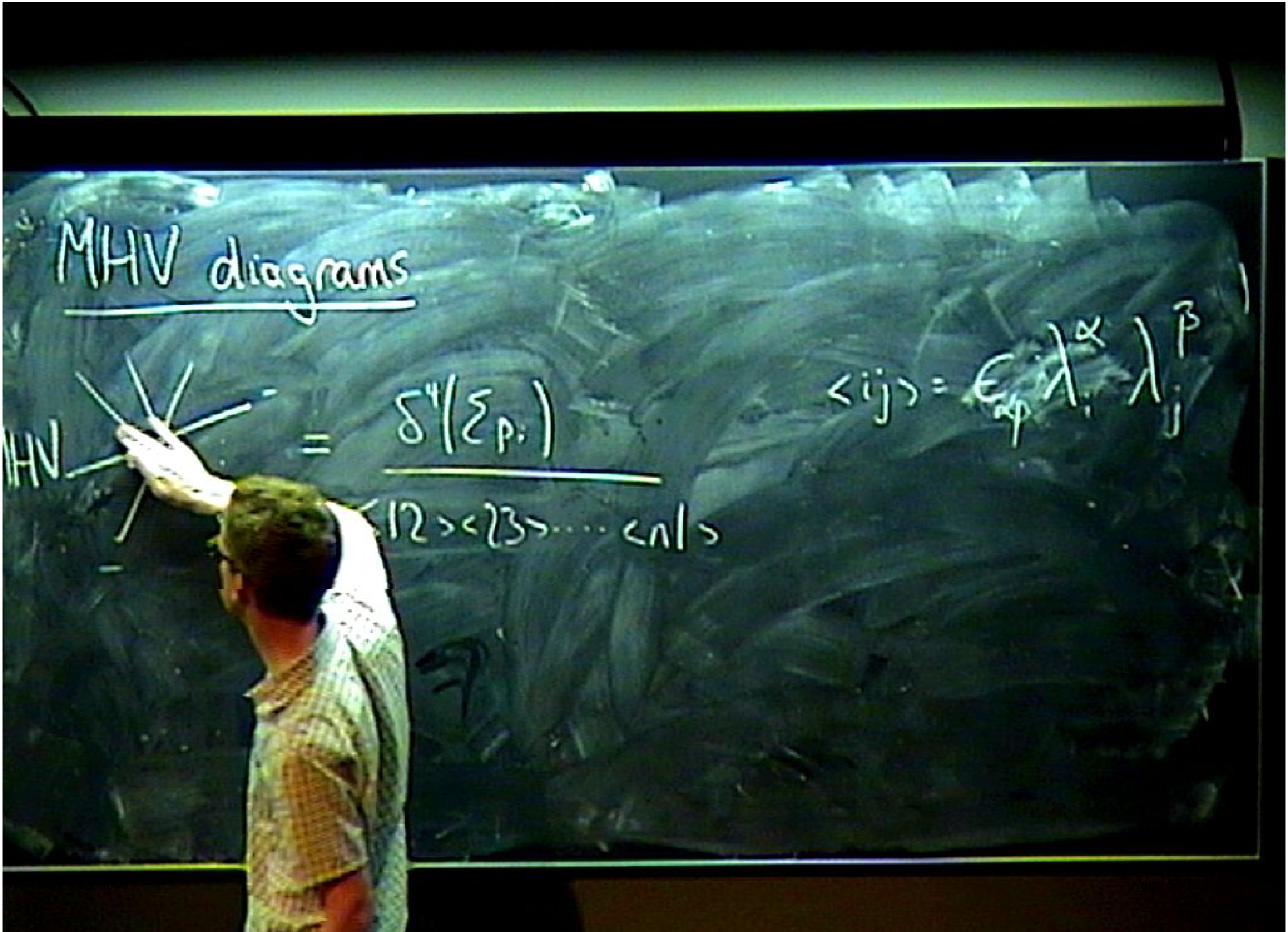


p1
x2
p2
x3
x1
p3
pn
xn
x4
pi = xi − xi+1
pi² = 0
(xi − xi+1)² = 0
Σ pi = Σ xi − Σ xi+1
= 0 identically (xn+1 ≡ x1)
x1
x2
x4


MHV diagrams


MHV diagrams
MHV

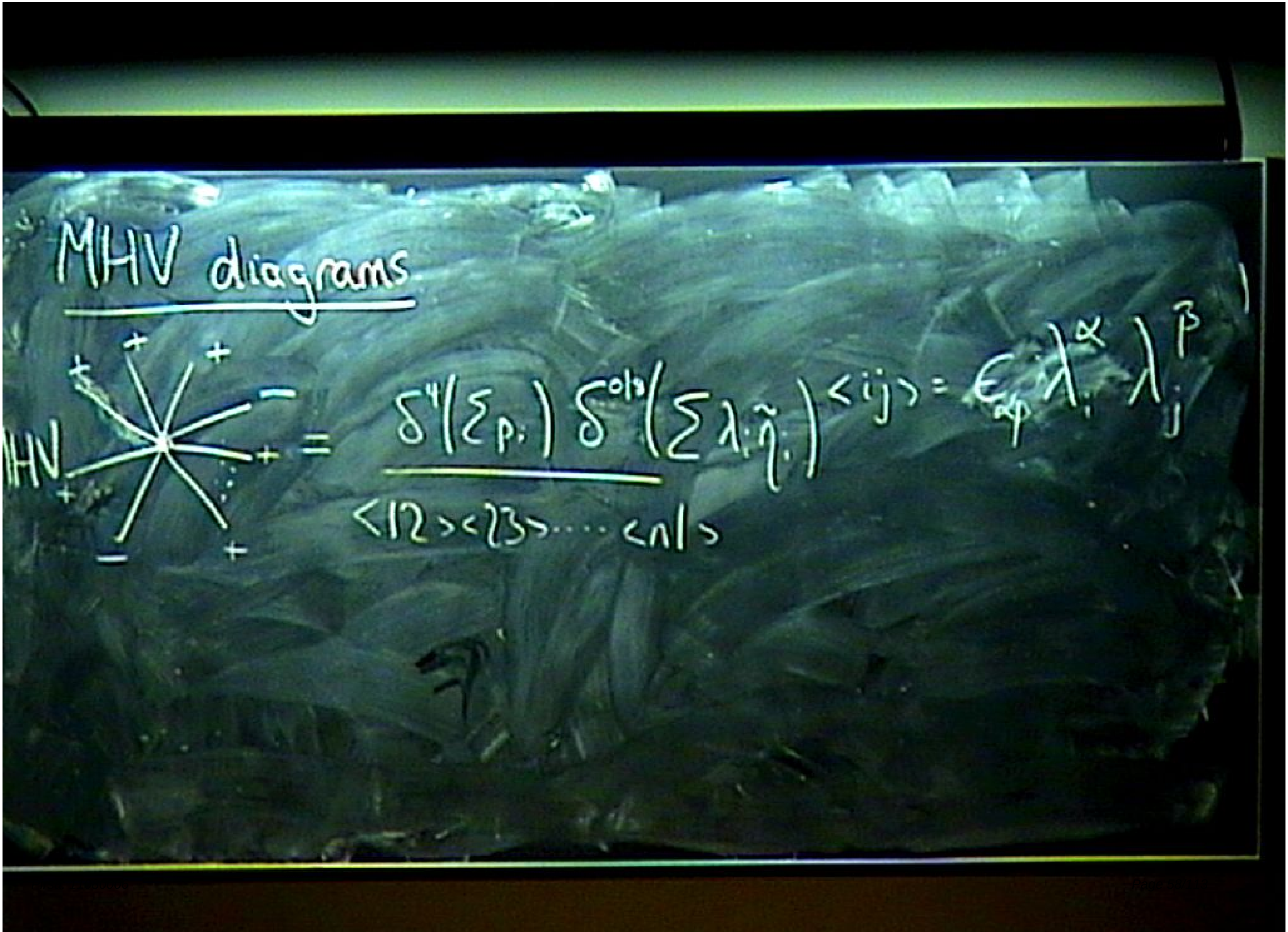

MHV diagrams
MHV
= δ⁴(Σ pᵢ) / <12><23>.....<n1>
<ij> = ε

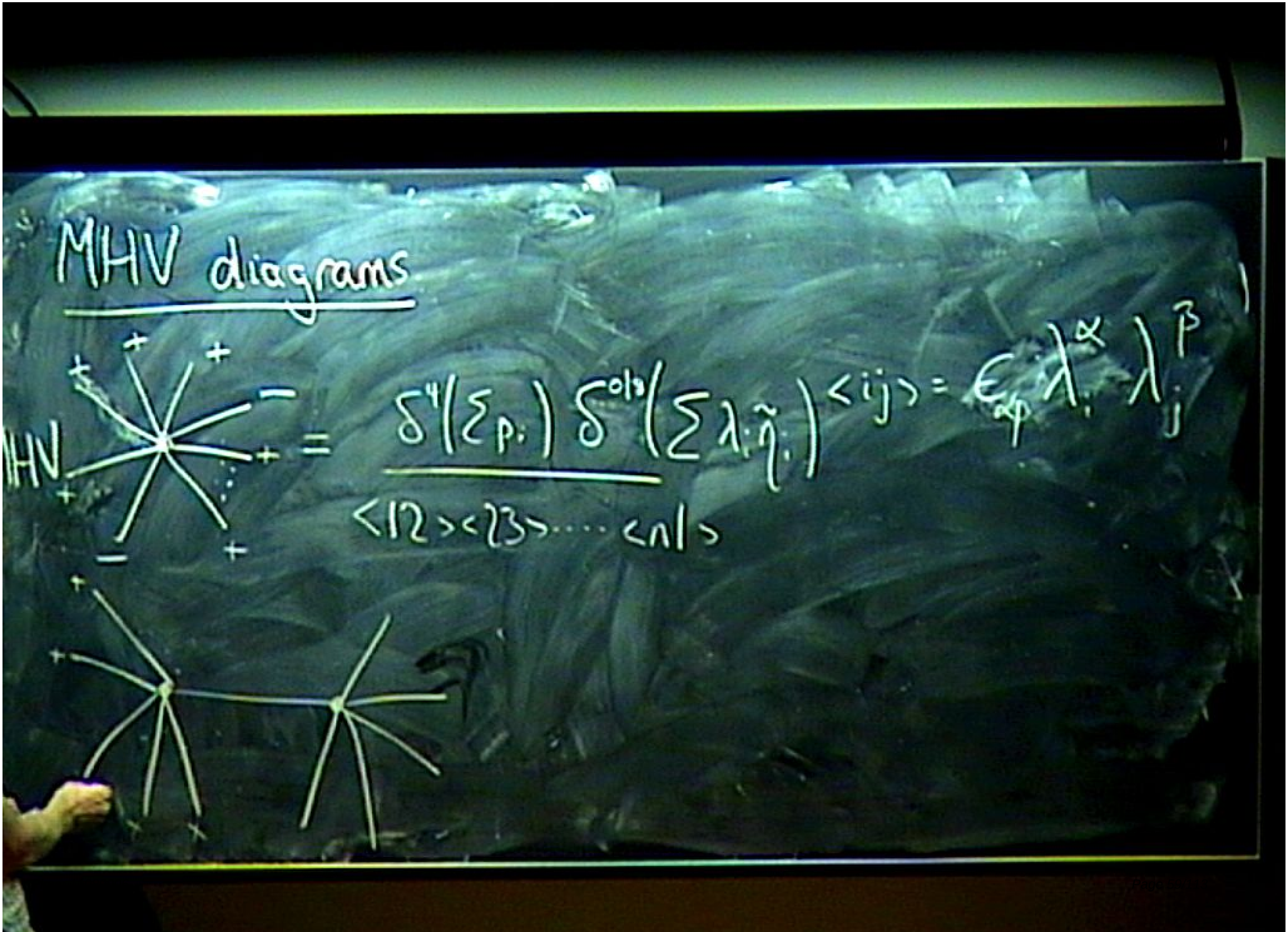

MHV diagrams
MHV = δ⁴(Σ p_i) / <12><23>.... <n1>
<ij> = ε_αβ λ_i^α λ_j^β

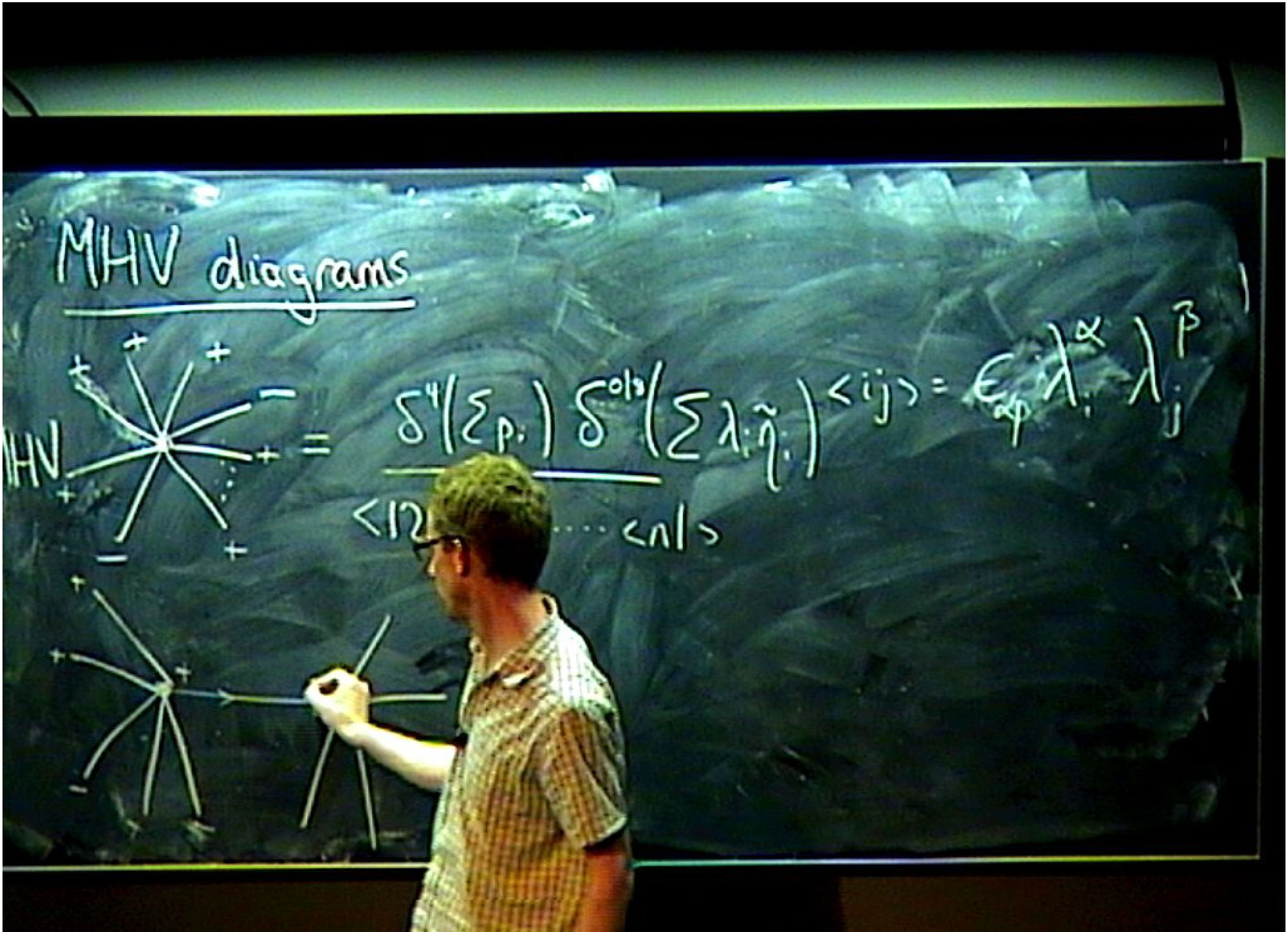
MHV diagrams

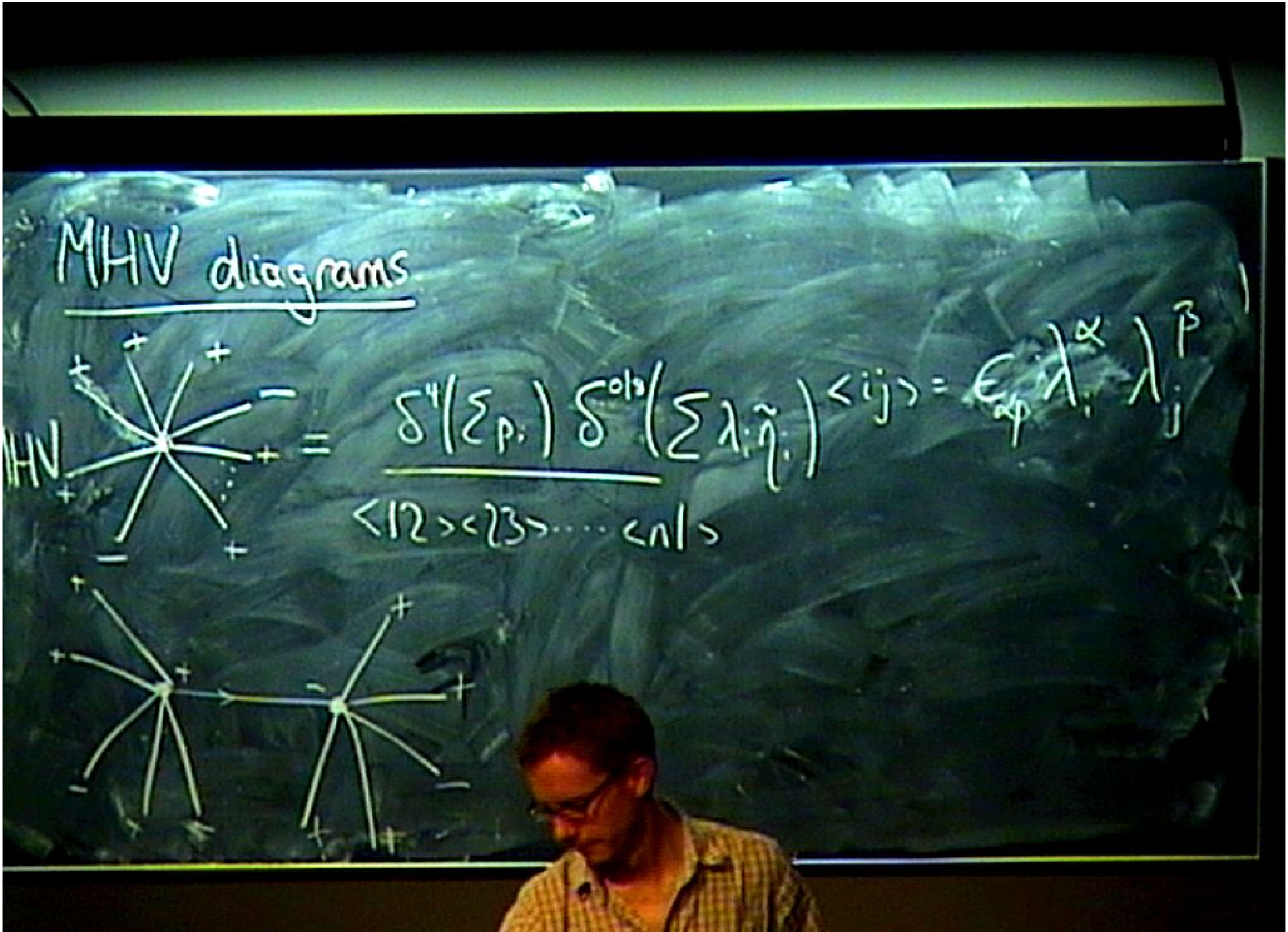
$$= \frac{\delta^4\left(\sum p_i\right)\,\delta^{0|8}\left(\sum \lambda_i \tilde{\eta}_i\right)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

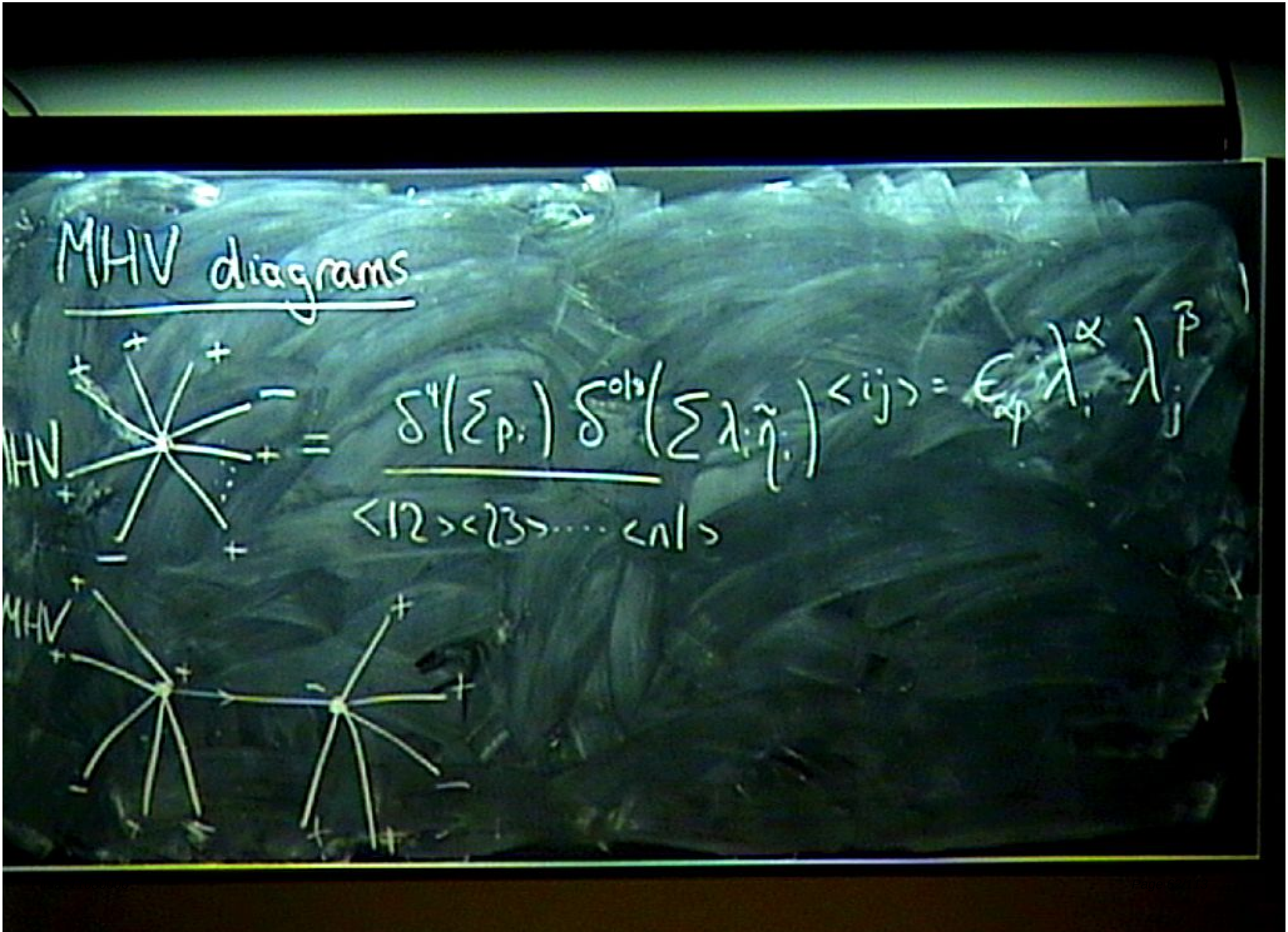
$$\langle ij\rangle = \epsilon_{\alpha\beta}\,\lambda_i^{\alpha}\,\lambda_j^{\beta}$$

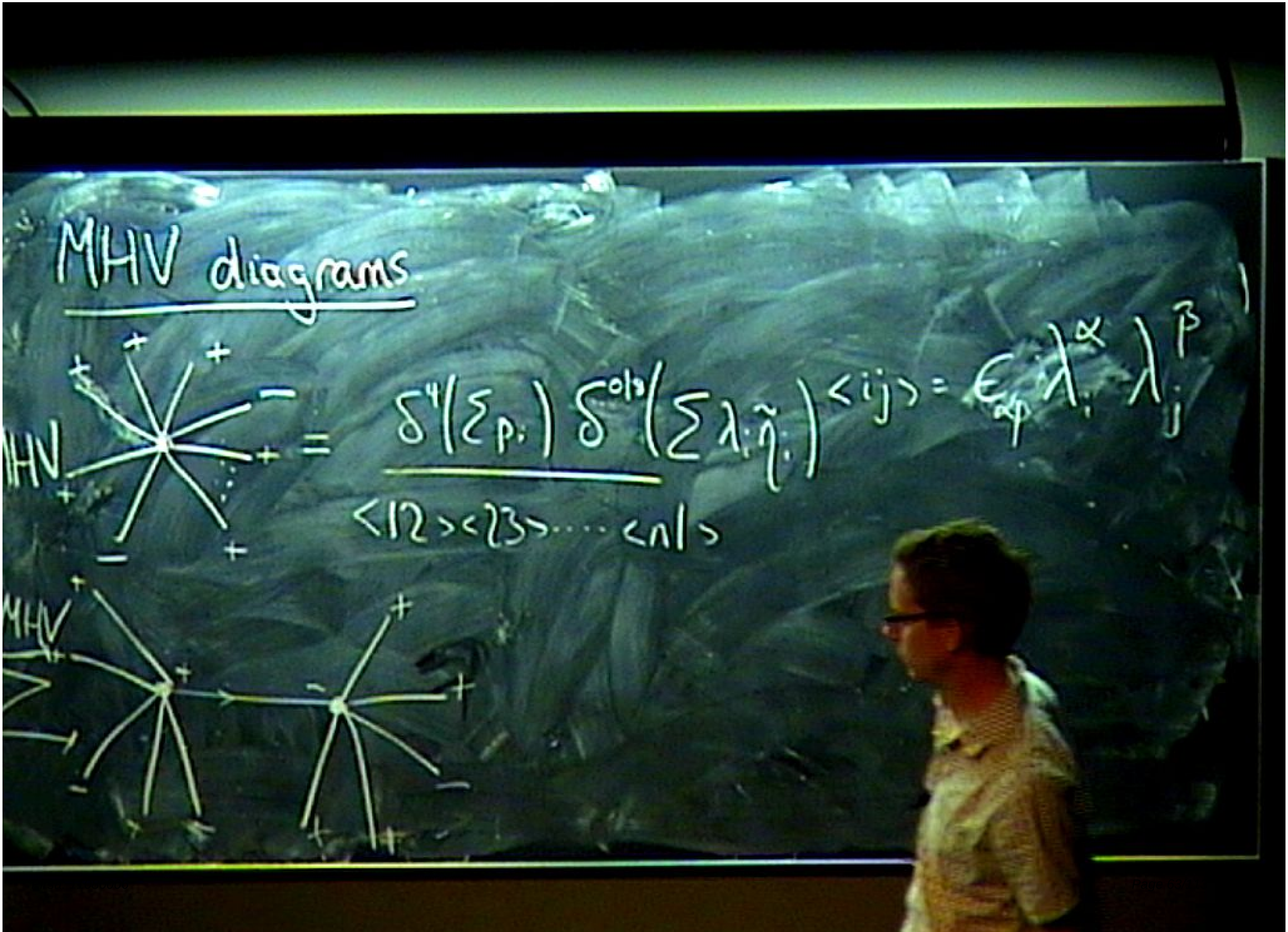

MHV diagrams
MHV
$= \frac{\delta^4(\sum p_i)\,\delta^{0|8}(\sum \lambda_i \tilde{\eta}_i)}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$
$\langle ij\rangle = \epsilon_{\alpha\beta}\lambda_i^\alpha \lambda_j^\beta$


MHV diagrams
MHV = δ⁴(Σ pᵢ) δ^{0|8}(Σ λᵢ η̃ᵢ) / <12><23>.....<n1>
<ij> = ε_{αβ} λᵢ^α λⱼ^β

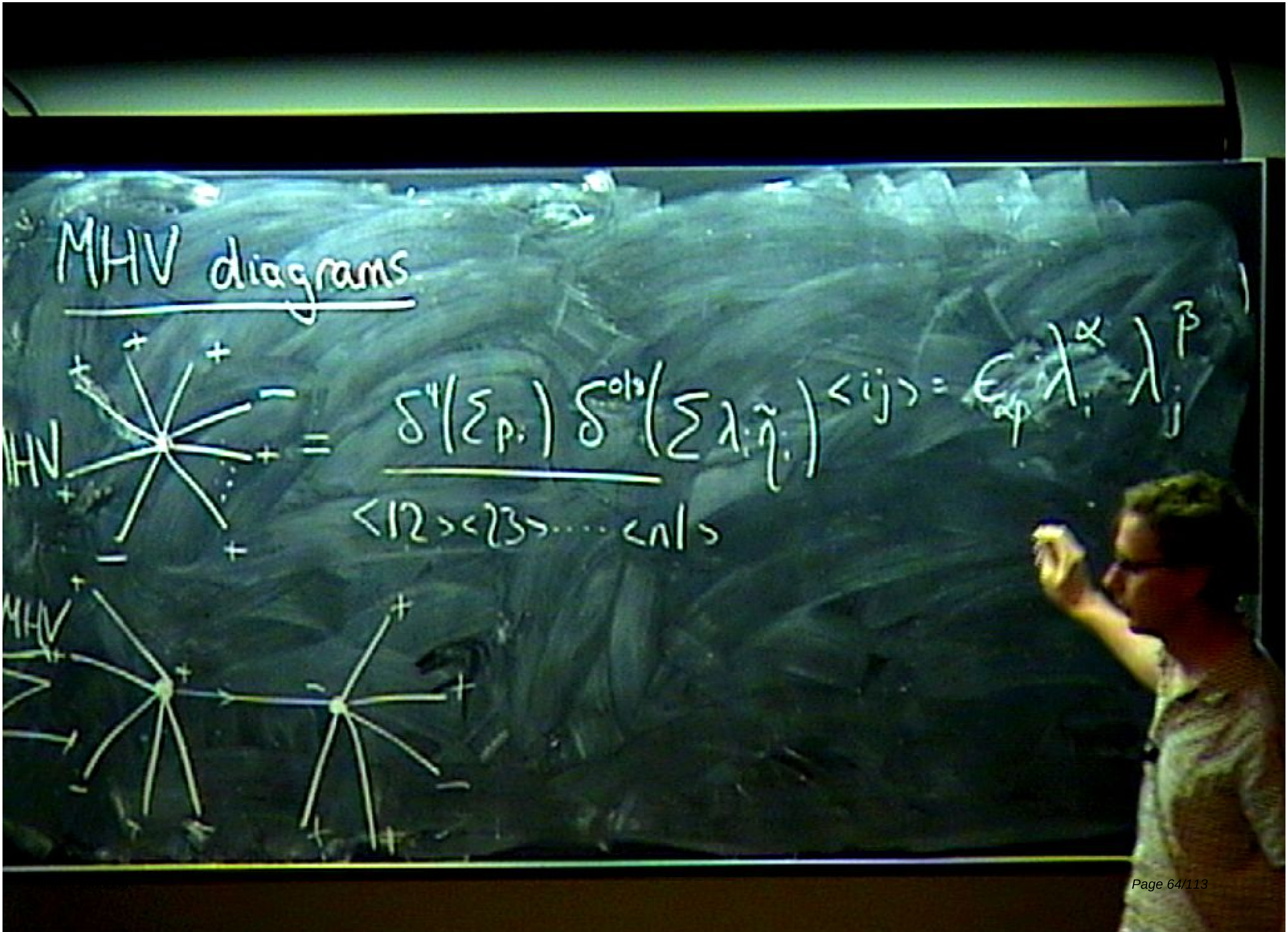

MHV diagrams
MHV
<ij> =
<n1>

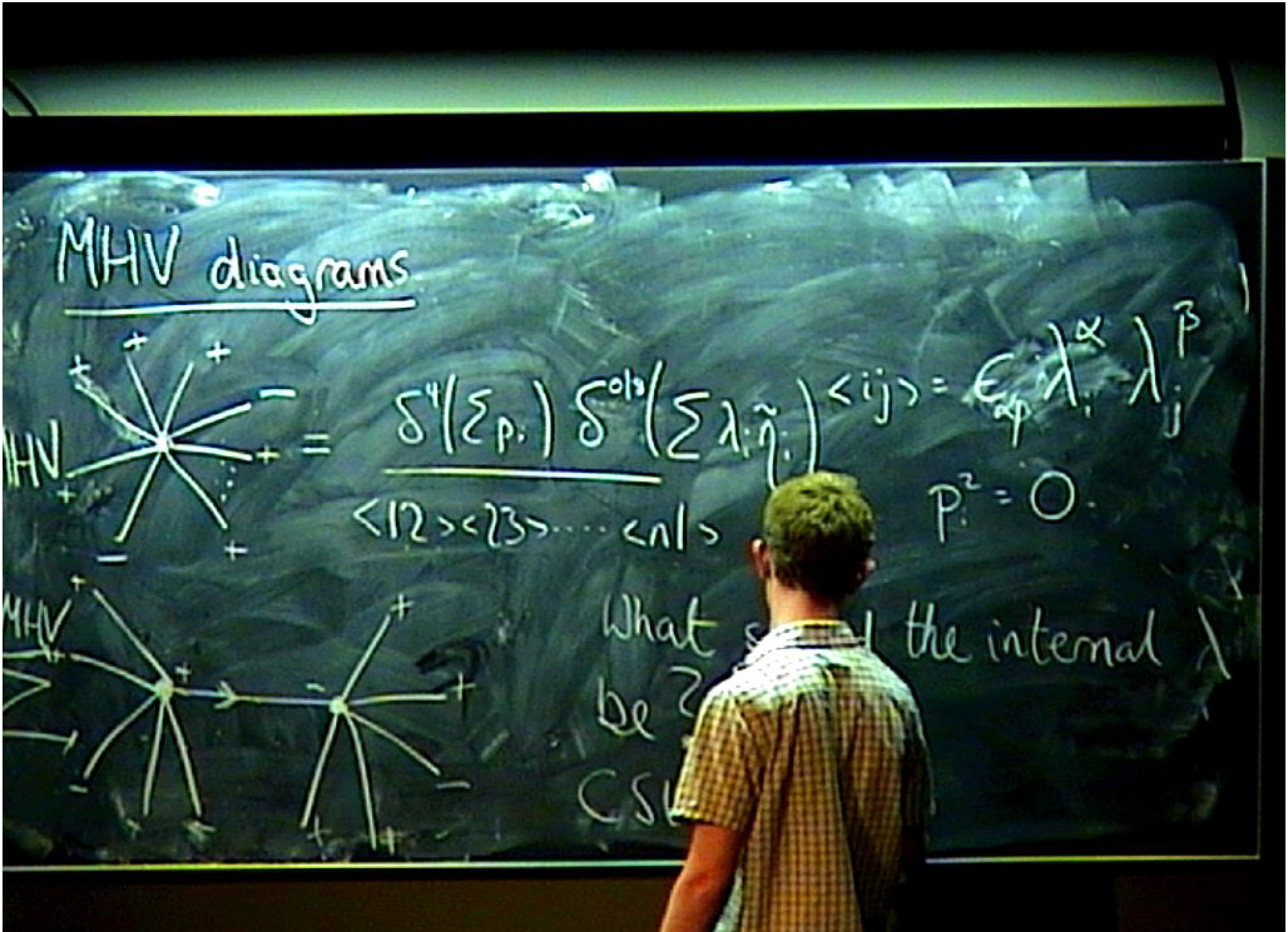

MHV diagrams
MHV
$= \frac{\delta^4(\sum p_i)\,\delta^{0|8}(\sum \lambda_i \tilde{\eta}_i)}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$
$\langle ij\rangle = \epsilon_{\alpha\beta}\lambda_i^\alpha \lambda_j^\beta$

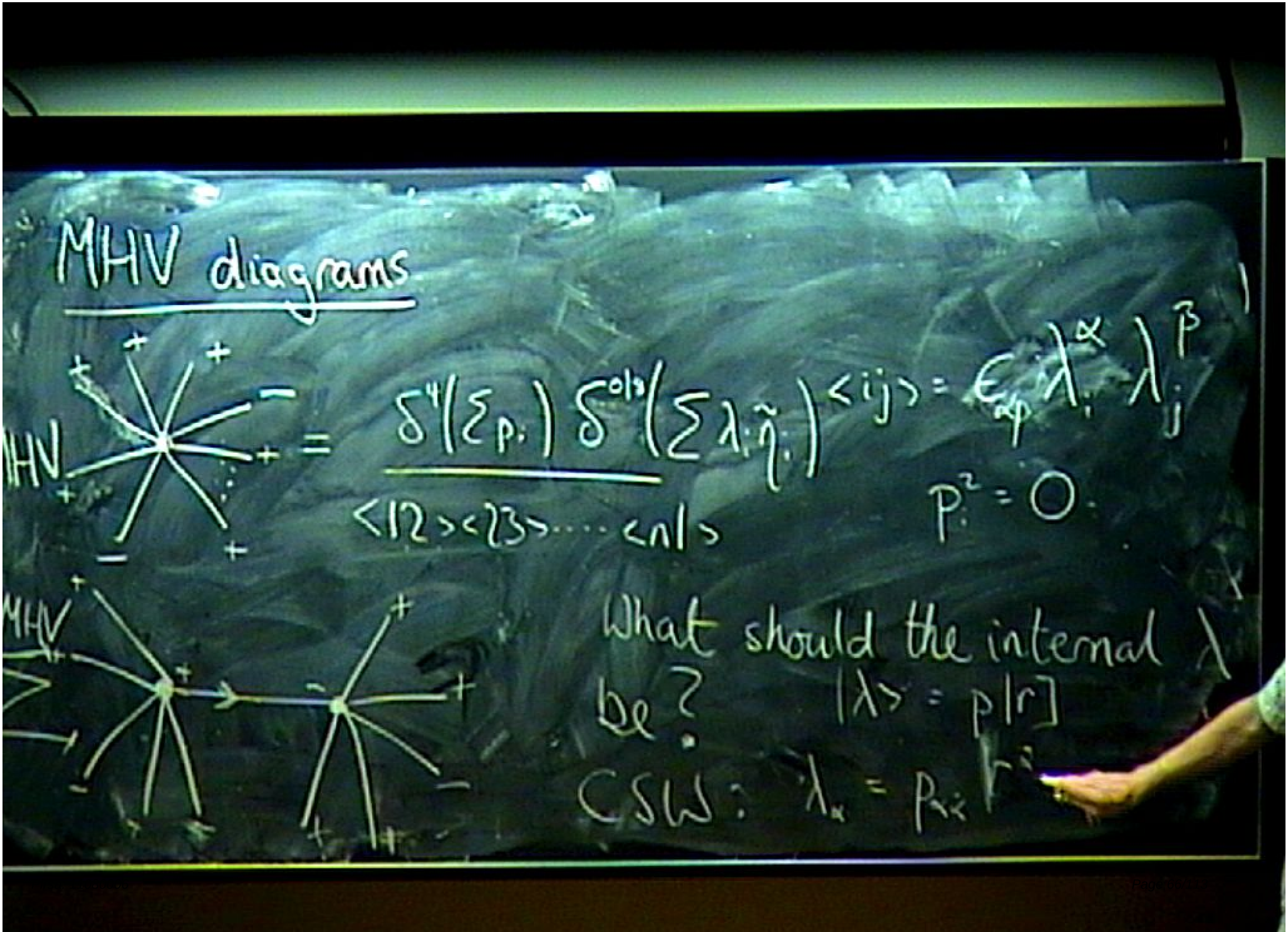

MHV diagrams
MHV
= δ⁴(Σ pᵢ) δ^{0|8}(Σ λᵢ η̃ᵢ) / ⟨12⟩⟨23⟩ …… ⟨n1⟩
⟨ij⟩ = ε_{αβ} λᵢ^α λⱼ^β
MHV



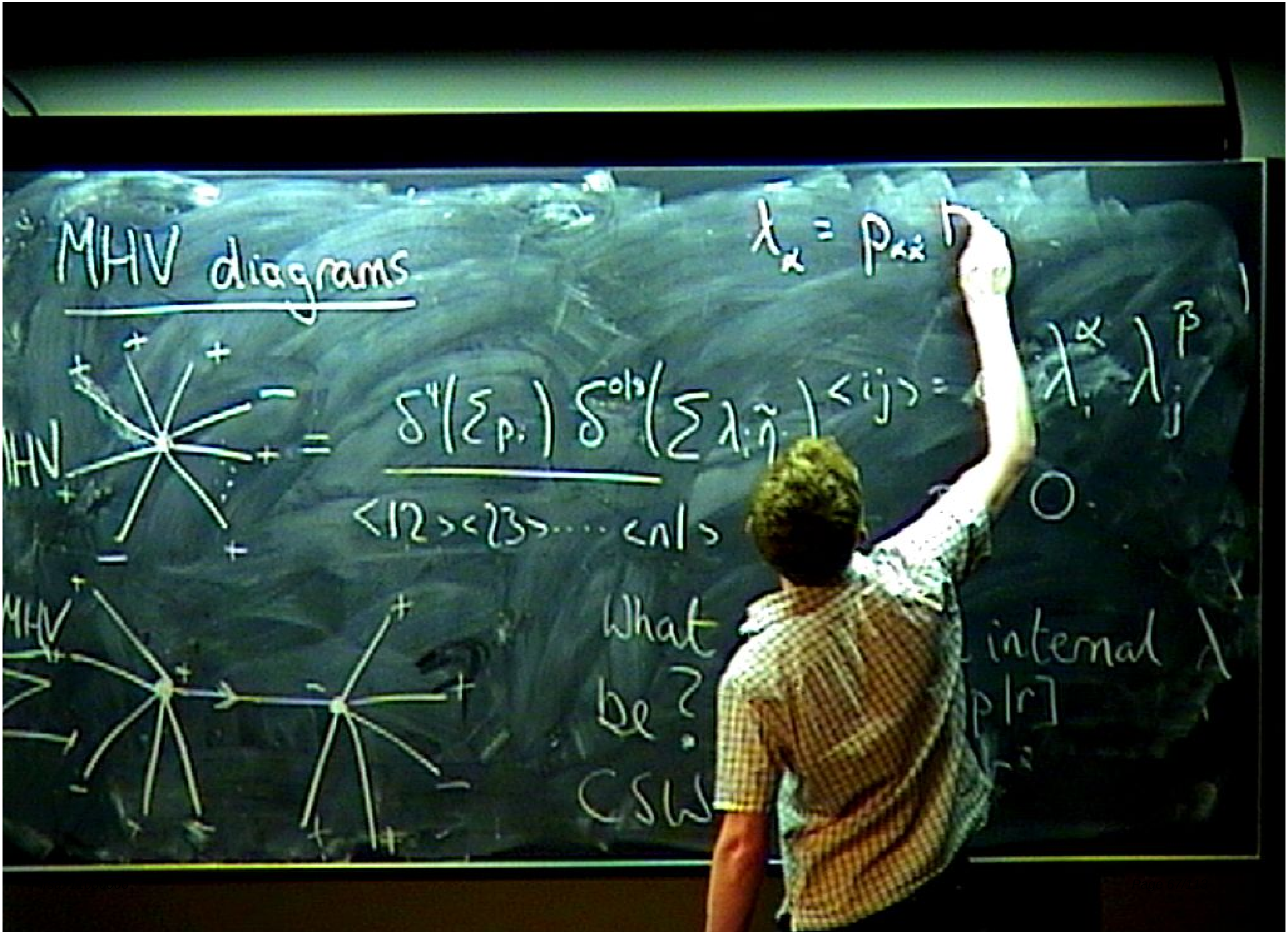
MHV diagrams
MHV
$= \frac{\delta^4(\sum p_i)\, \delta^{0|8}(\sum \lambda_i \tilde{\eta}_i)}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$
$\langle ij\rangle = \epsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta$
MHV

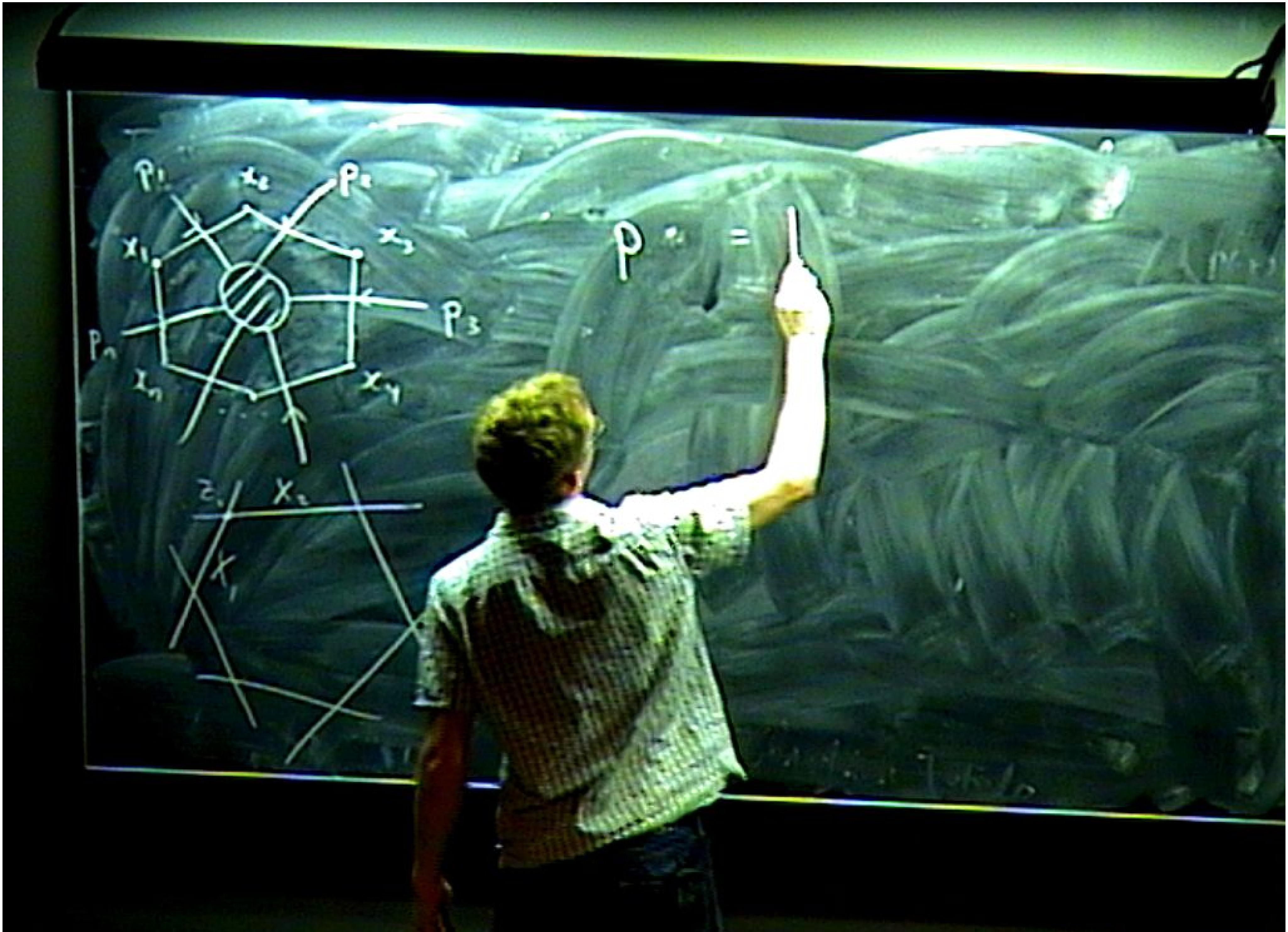

MHV diagrams
MHV
δ⁴(Σp_i) δ^{0|8}(Σλ_i η̃_i) / <12><23>.....<n1>
<ij> = ε_αβ λ_i^α λ_j^β
MHV


MHV diagrams
MHV
=
δ⁴(Σp_i) δ^{0|8}(Σλ_i η̃_i)
<12><23>....<n1>
<ij> = ε_{αβ} λ_i^α λ_j^β
p_i² = 0.
MHV
What
the internal λ
be ?

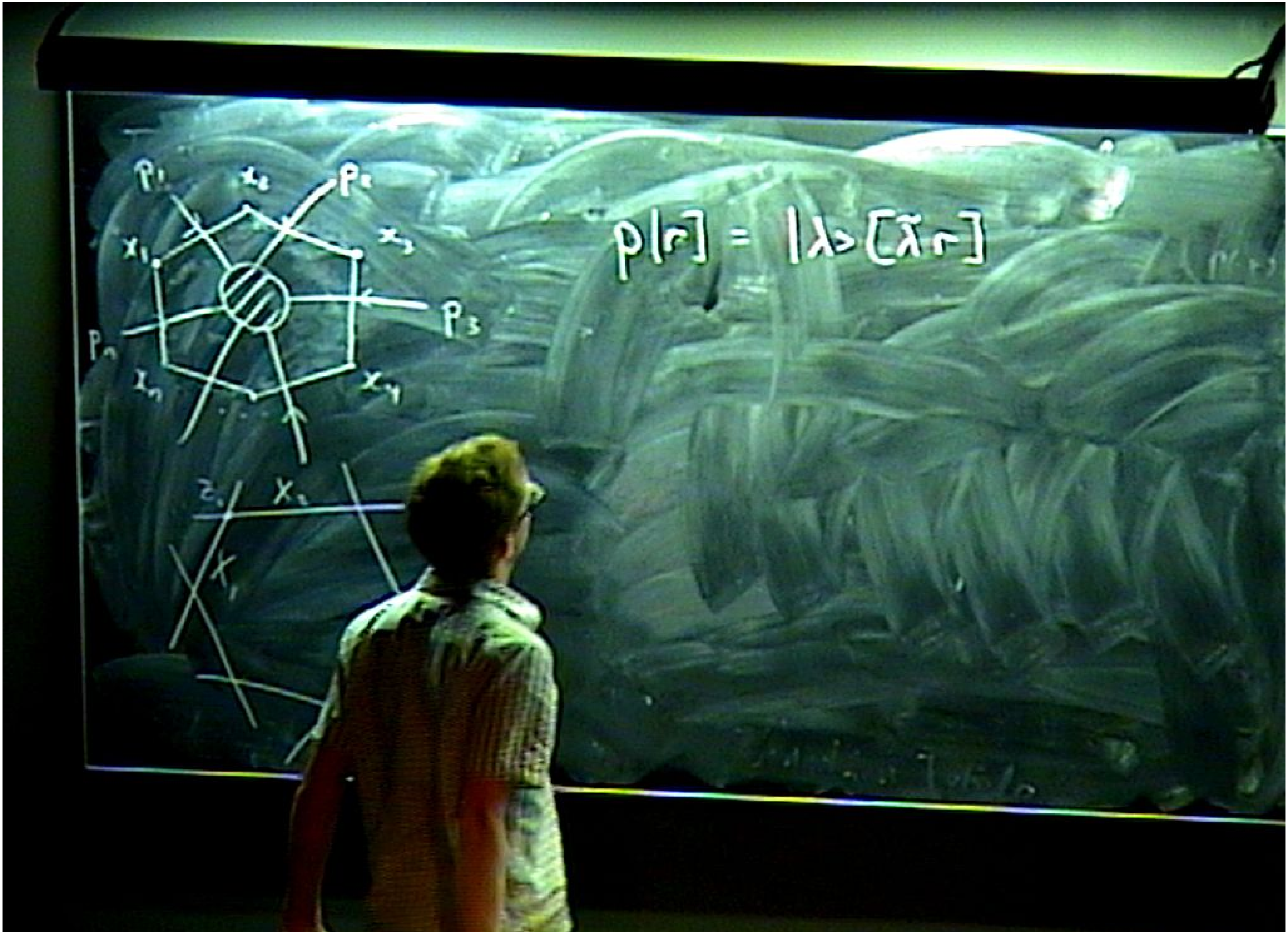


MHV diagrams
MHV
MHV
δ⁴(Σpᵢ) δ^{0|8}(Σλᵢη̃ᵢ)
<12><23>.....<n1>
<ij> = ε_{αβ} λᵢ^α λⱼ^β
pᵢ² = 0
What should the internal λ be?
|λ> = p|r]
CSW: λ_α = p_{αα̇} r^{α̇}

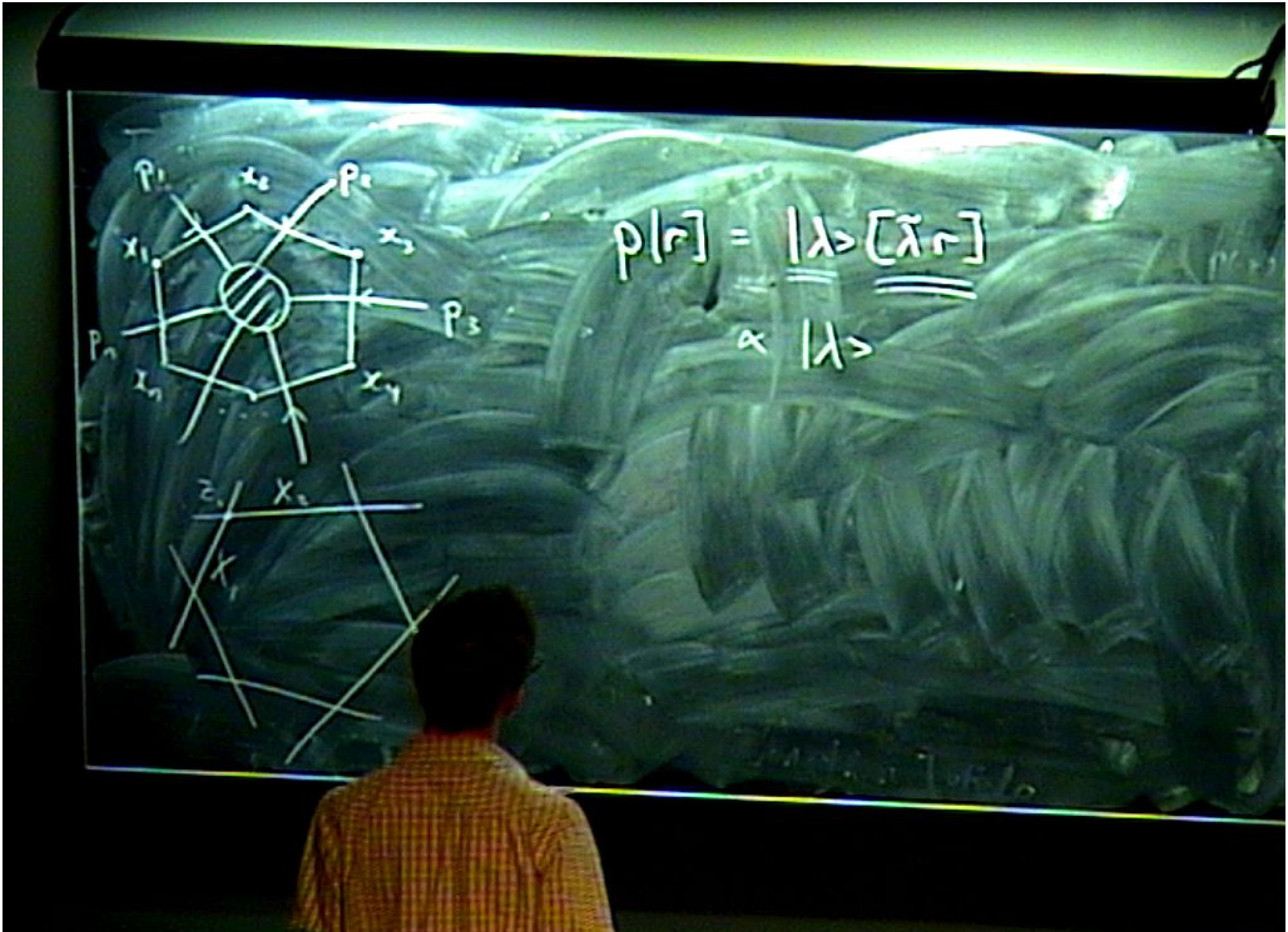

MHV diagrams
MHV
What
internal
be ?
CSW

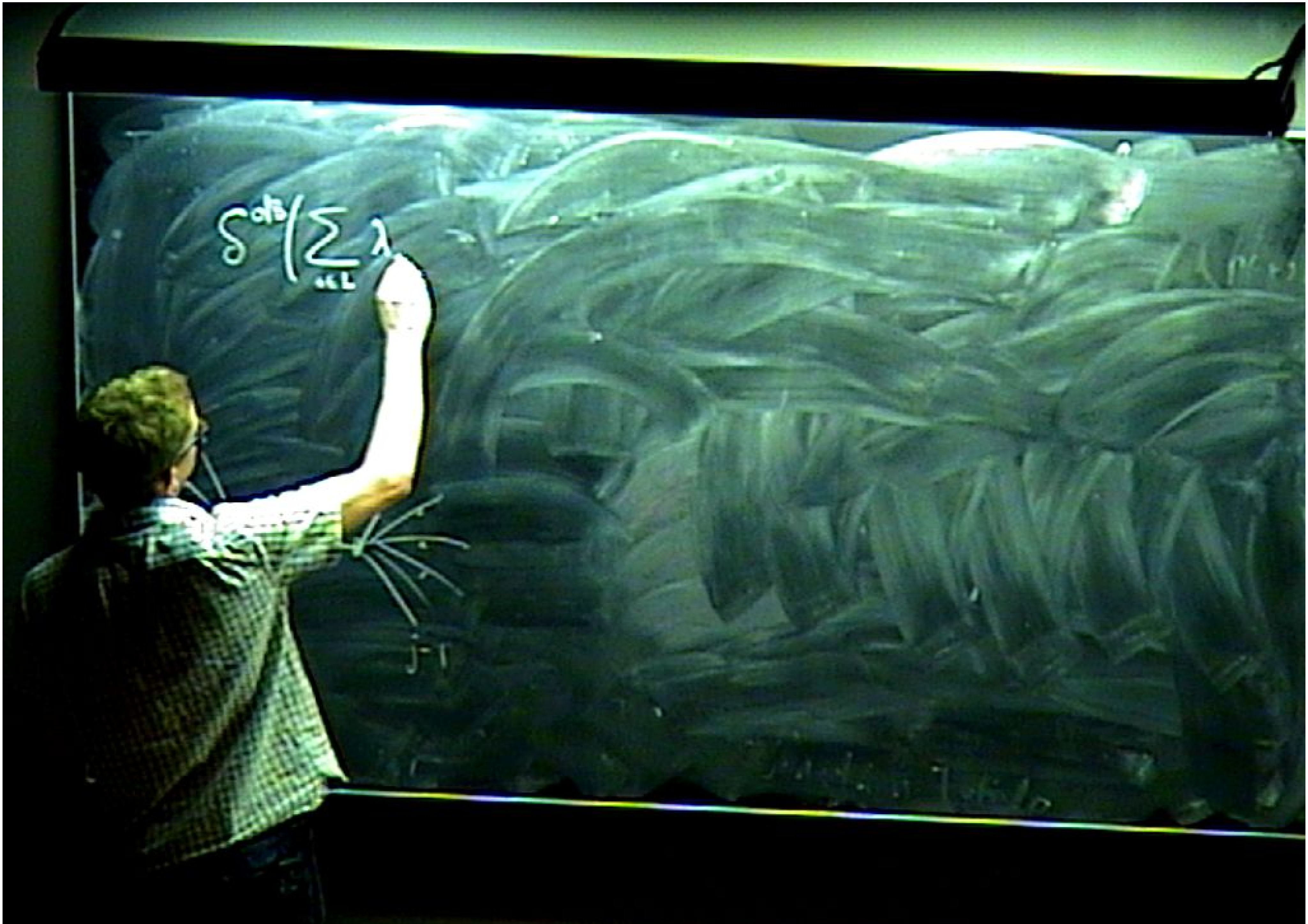


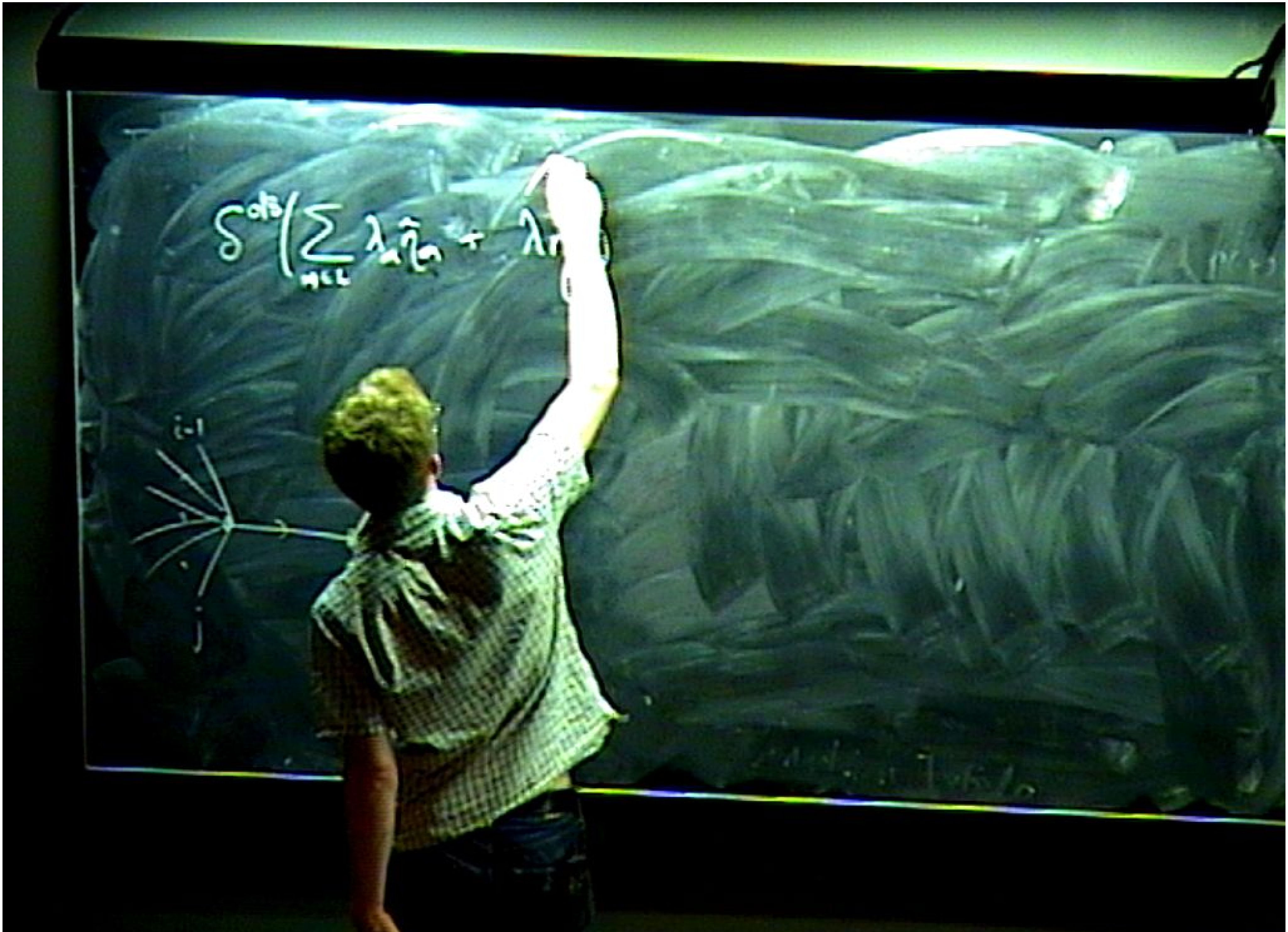
p1
x2
p2
x3
x1
p3
pn
xn
x4
x2
x1
p =

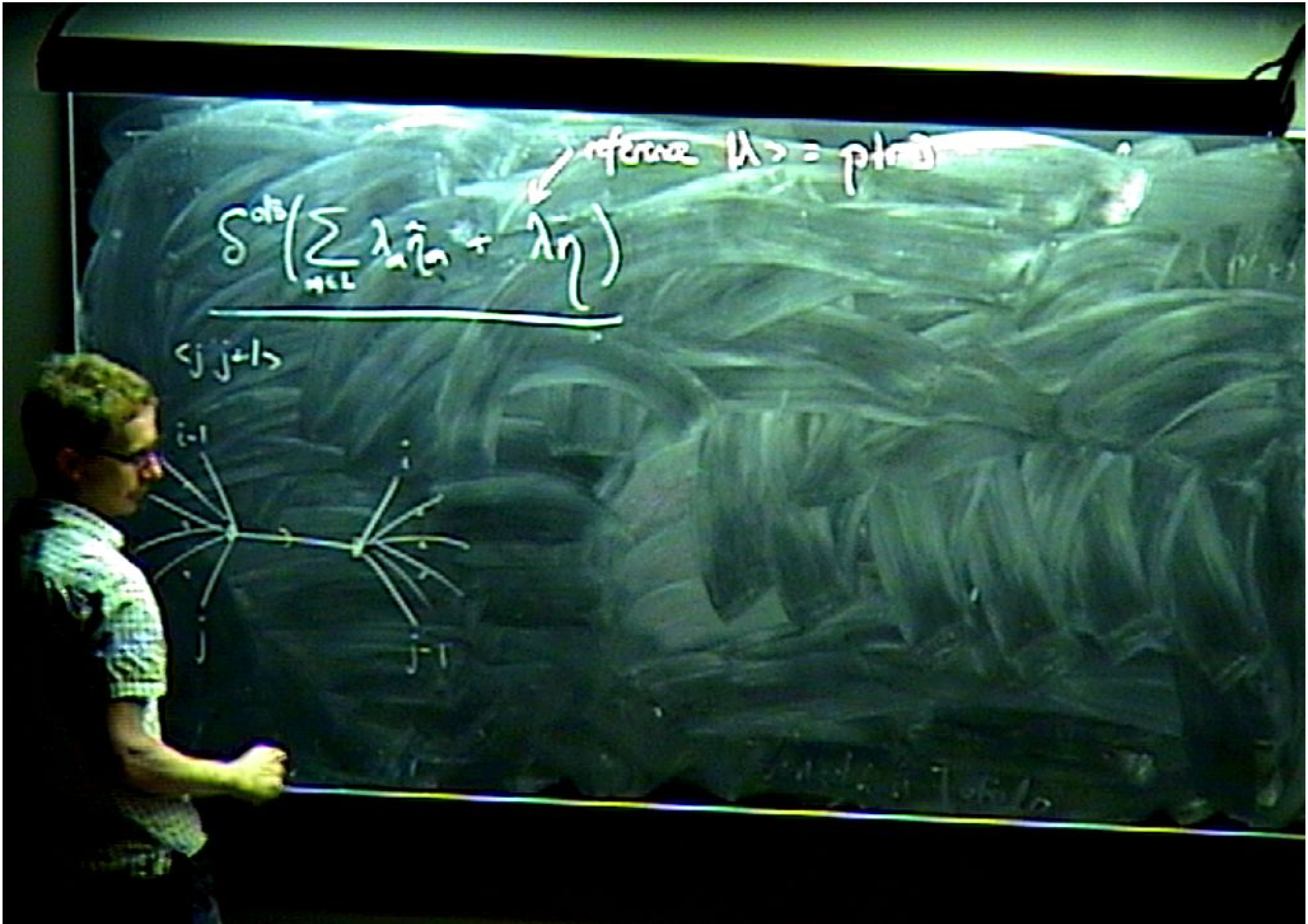


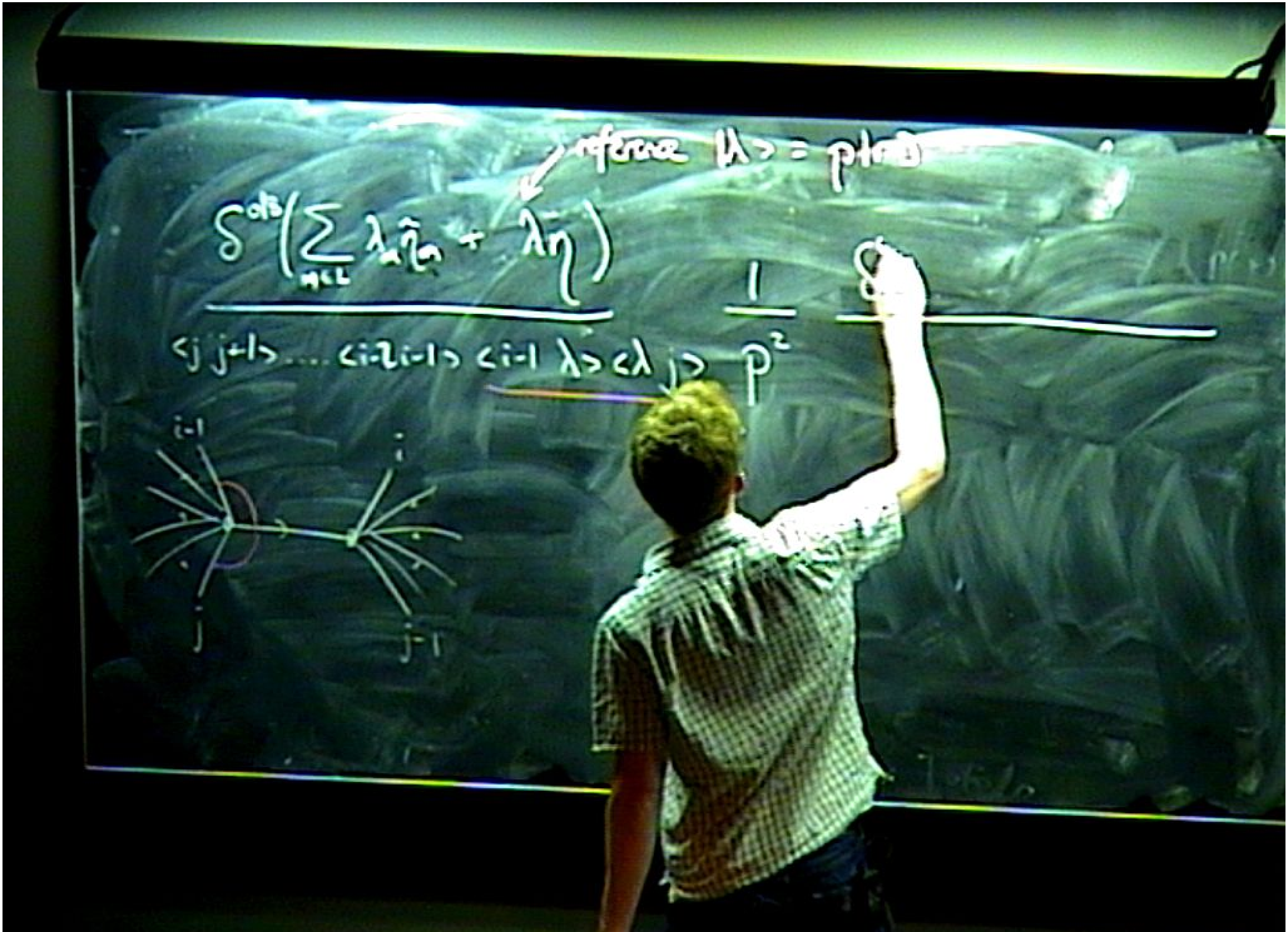
p[r] = |λ⟩[λ̃ r]


p|r] = |λ>[λ̃ r]
∝ |λ>
p1
x2
p2
x1
x3
p3
pn
xn
x4

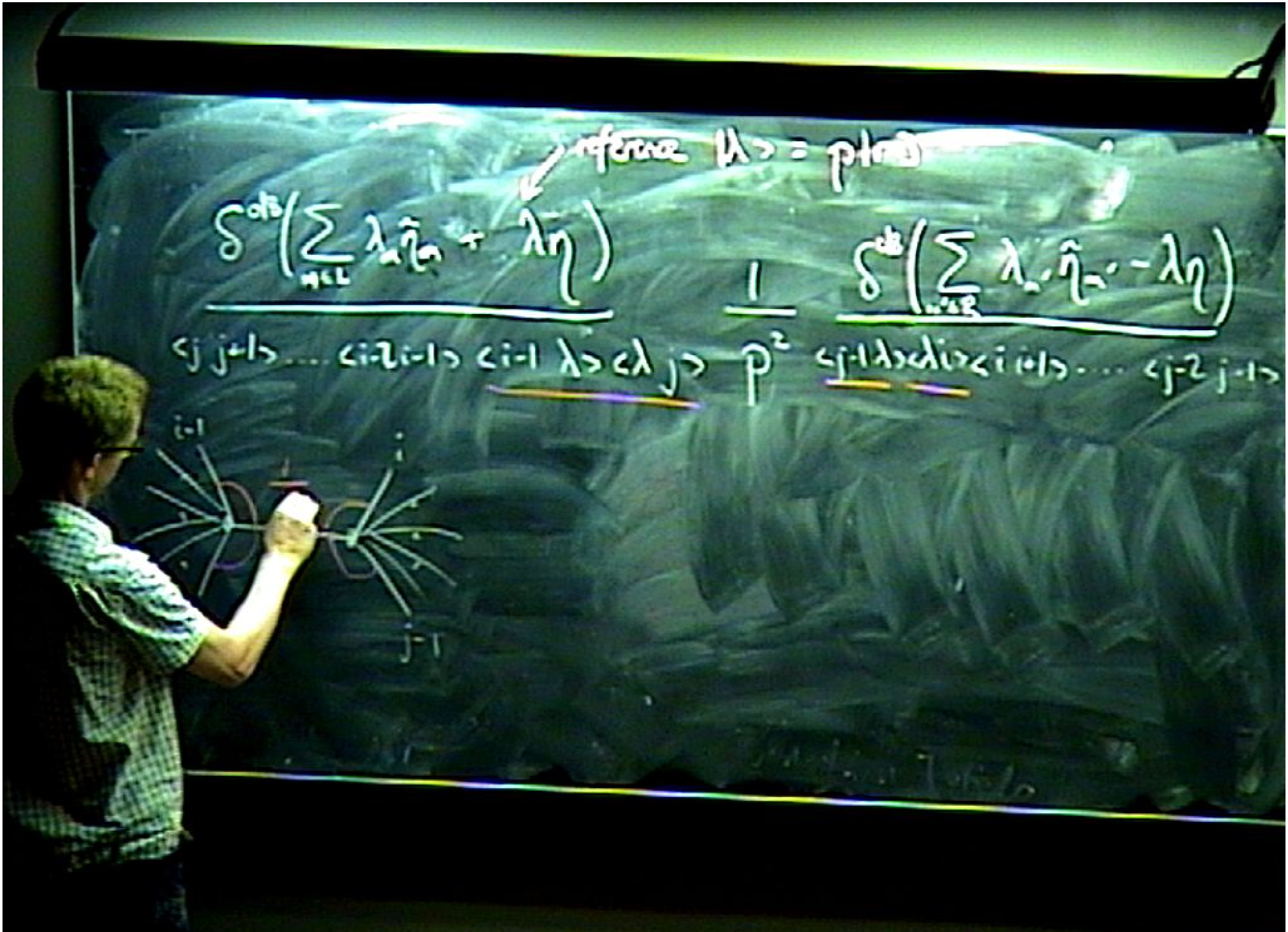


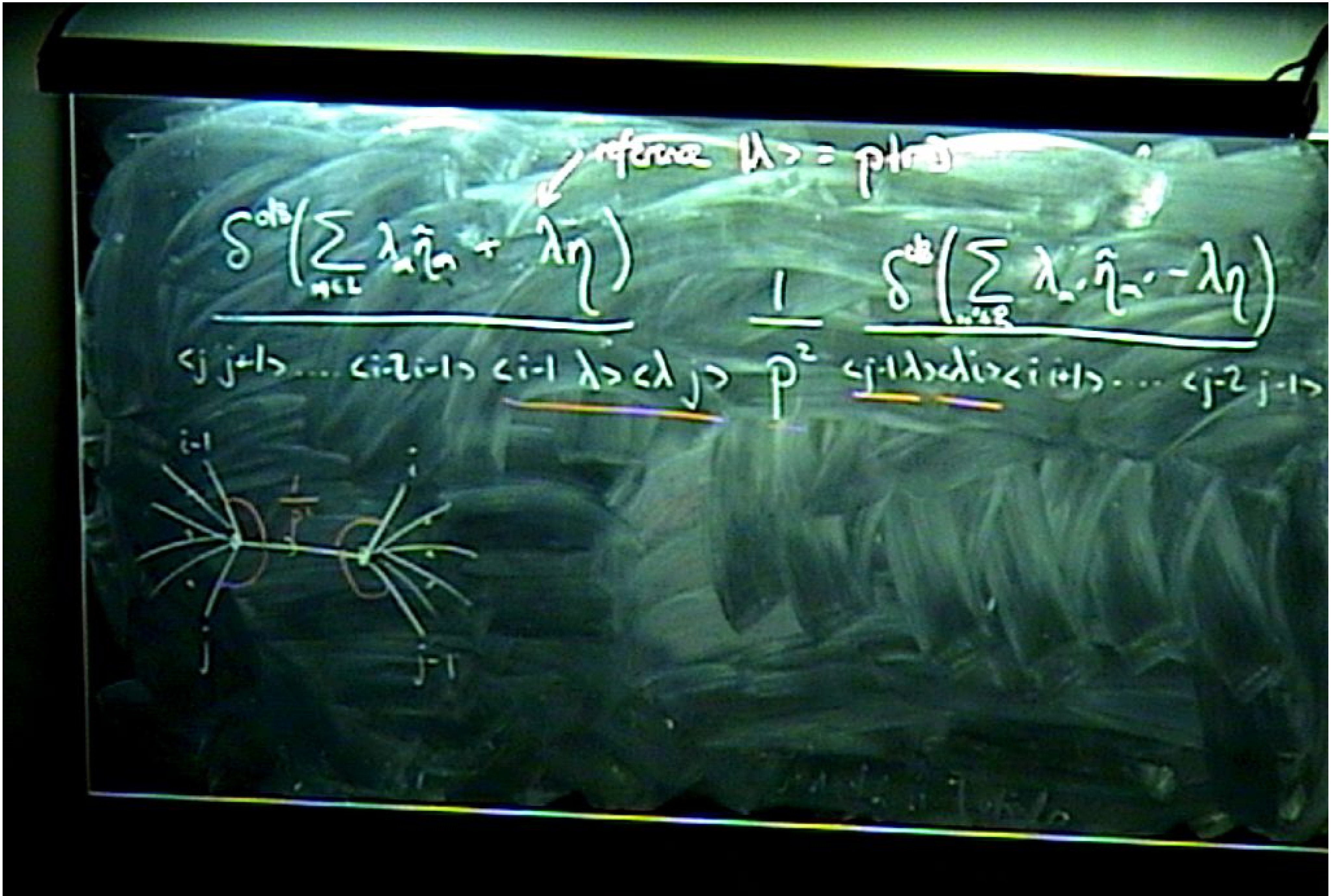



reference
<j j+1>
i-1
i
j
j-1

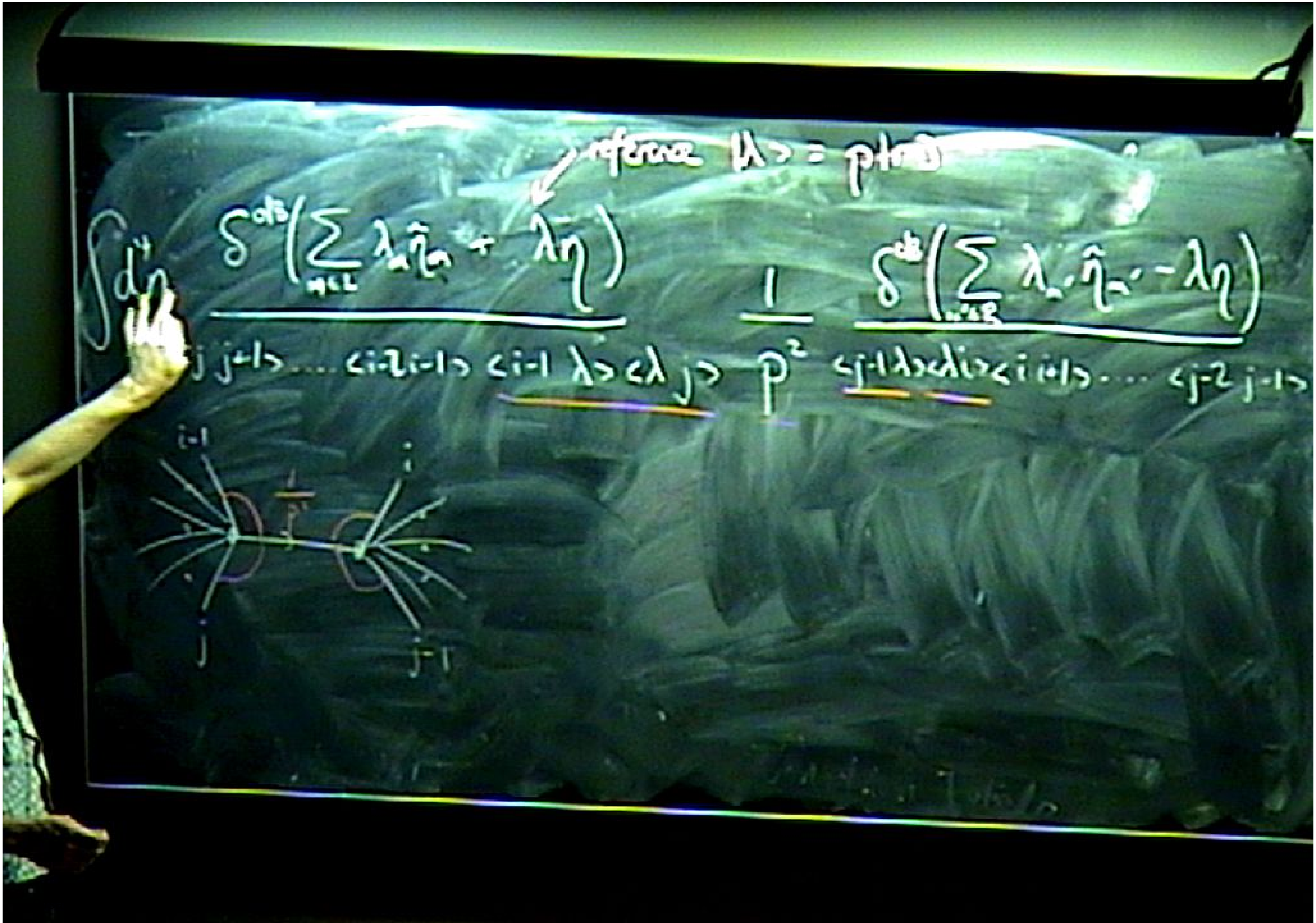


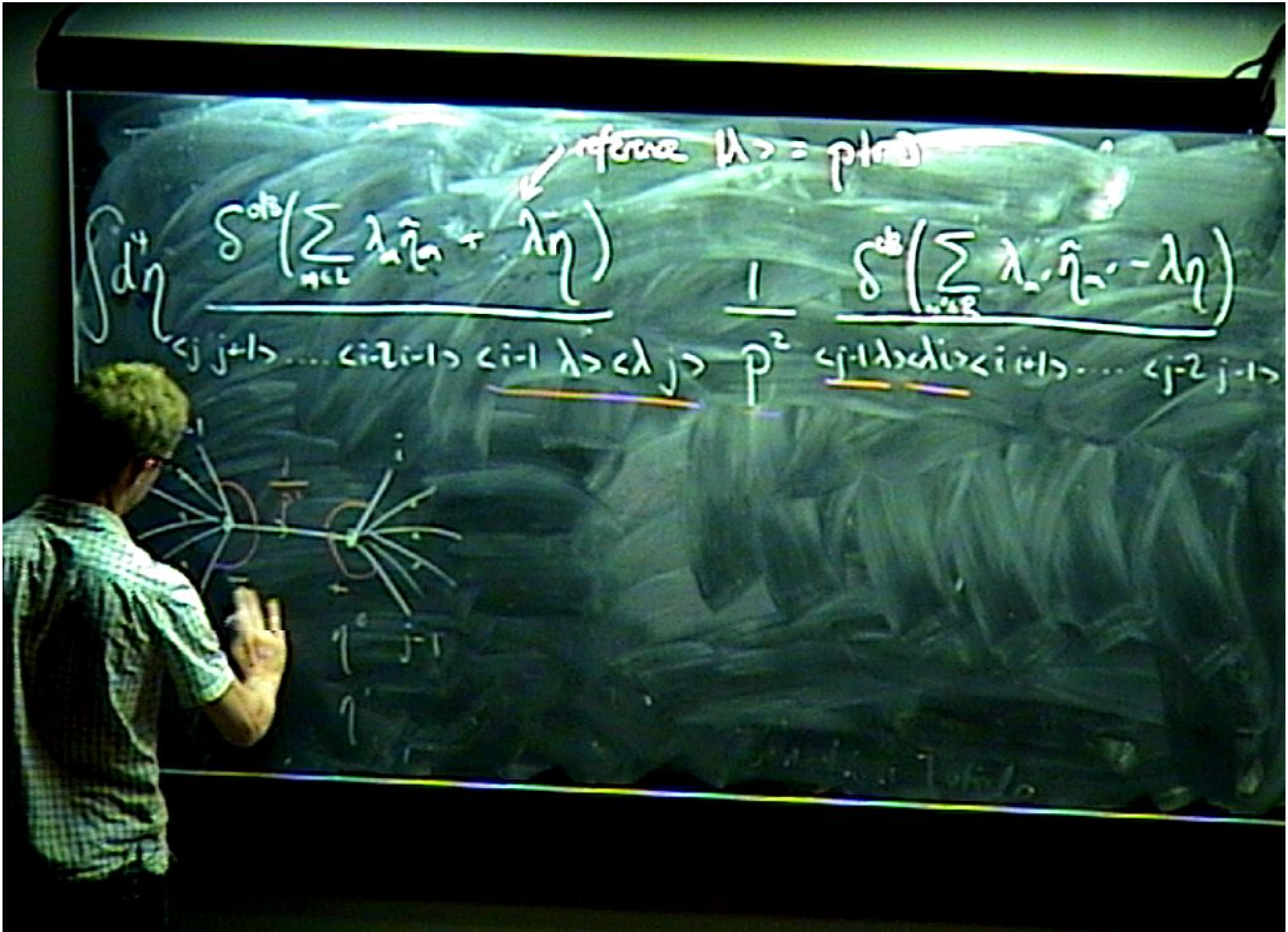
reference |λ> = p|η]
<j j+1> .... <i-2 i-1> <i-1 λ> <λ j>
1
p²
i-1
i
j
j-1

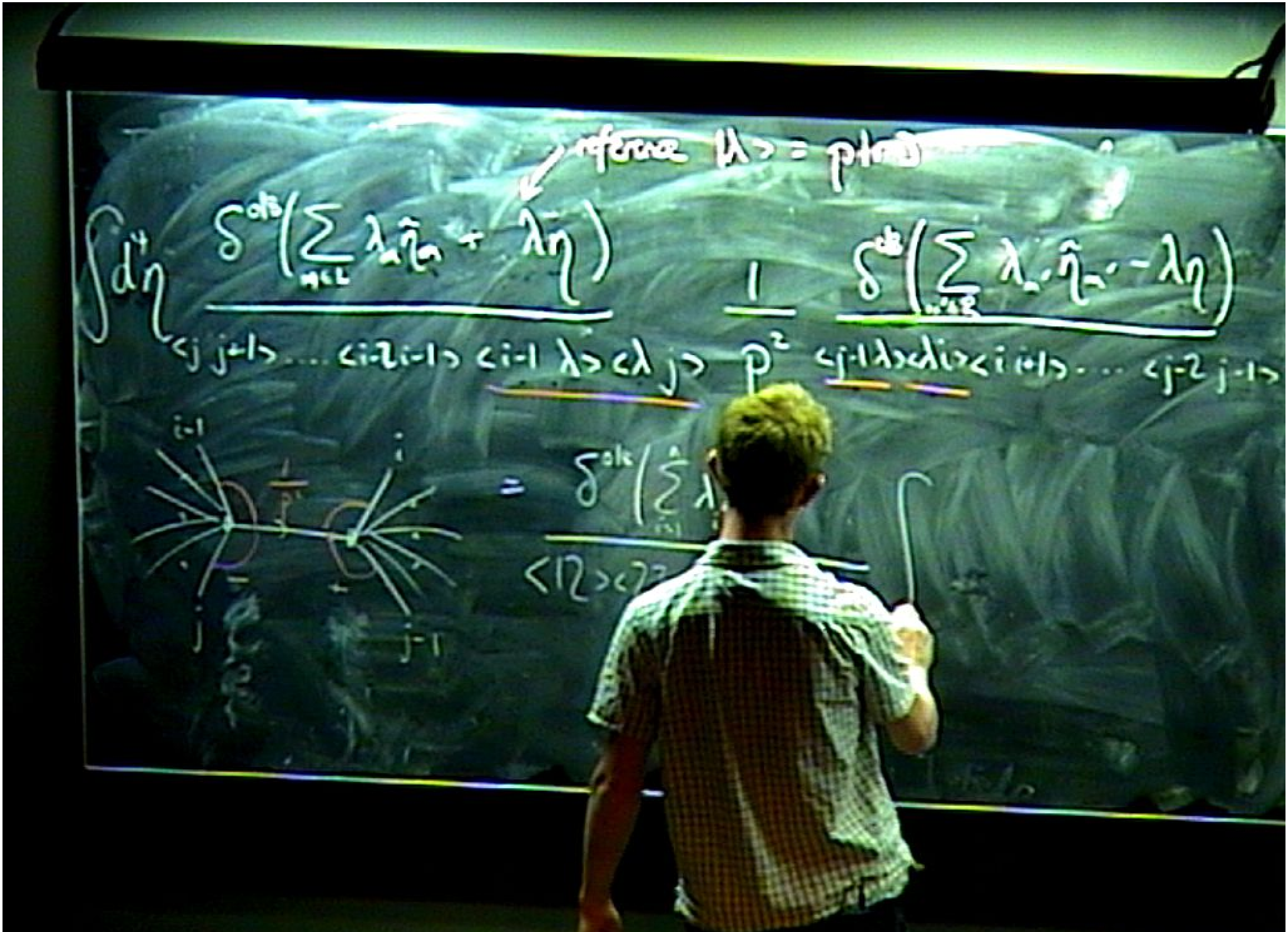

reference
<j j+1> … <i-2 i-1> <i-1 λ> <λ j>
1
p²
<j-1 λ> <λ i> <i i+1> … <j-2 j-1>
i-1
i
j-1



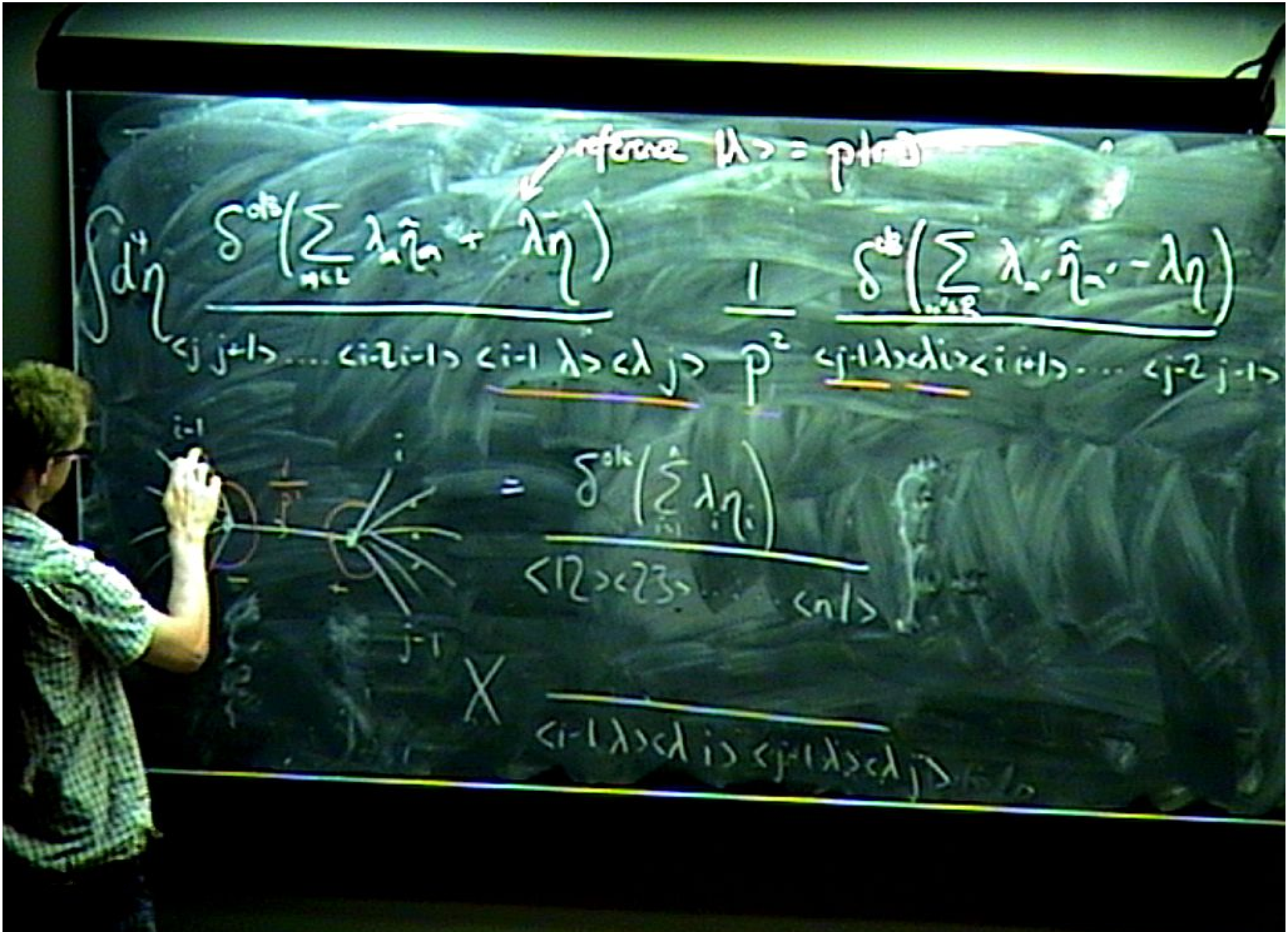
reference $|\lambda\rangle = p|\eta]$
$\frac{\delta^{0|8}\left(\sum_{n\in L} \lambda_n \hat{\eta}_n + \lambda\eta\right)}{\langle j\, j{+}1\rangle \cdots \langle i{-}2\, i{-}1\rangle \langle i{-}1\, \lambda\rangle \langle \lambda\, j\rangle}$
$\frac{1}{p^2}$
$\frac{\delta^{0|8}\left(\sum_{n'\in R} \lambda_{n'} \hat{\eta}_{n'} - \lambda\eta\right)}{\langle j{-}1\, \lambda\rangle \langle \lambda\, i\rangle \langle i\, i{+}1\rangle \cdots \langle j{-}2\, j{-}1\rangle}$
i-1
i
j
j-1

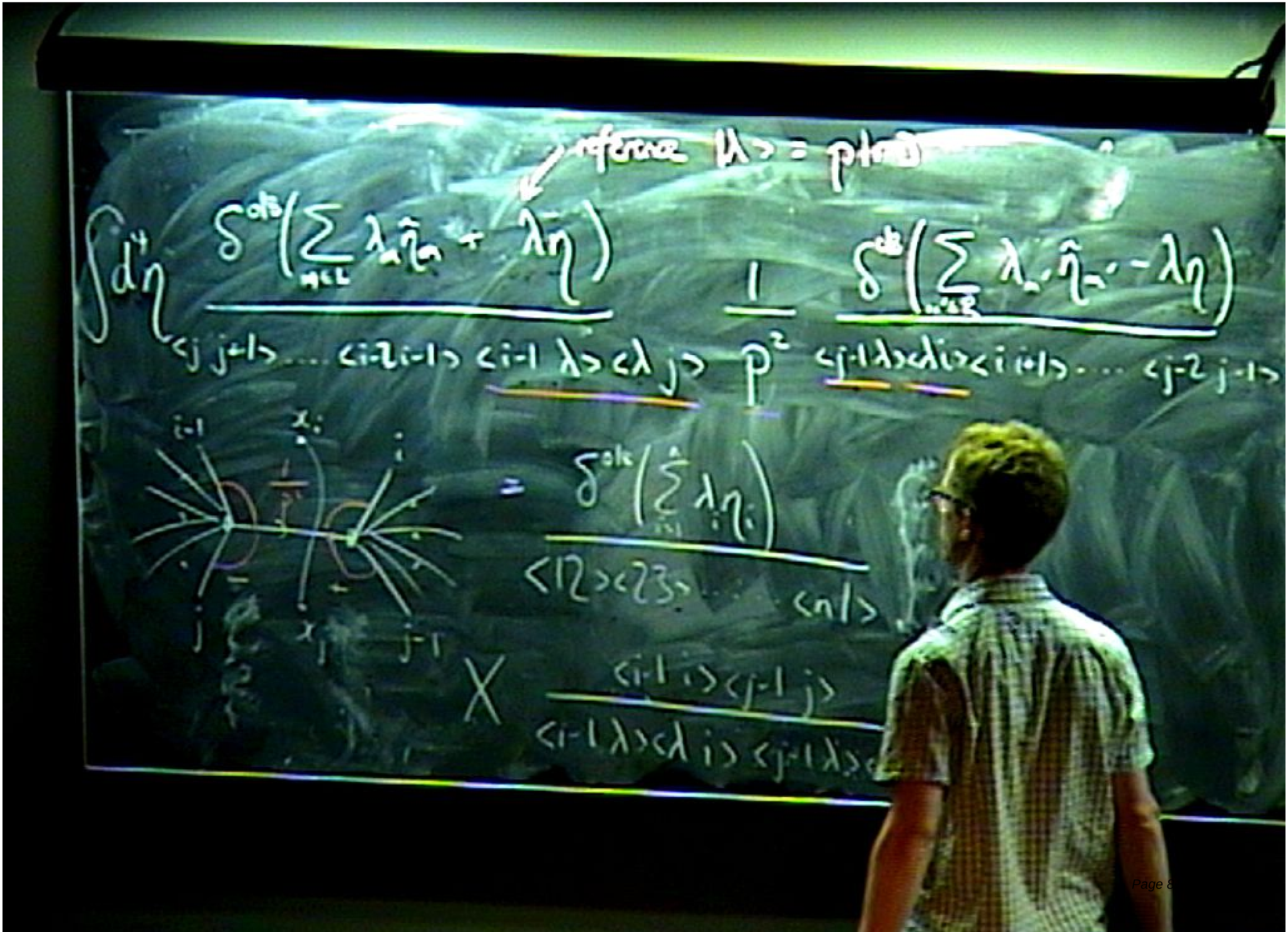

reference |λ> =
<j j+1> … <i-2 i-1> <i-1 λ> <λ j>
1
p²
<j-1 λ><λ i><i i+1> … <j-2 j-1>
i-1
i
j
j-1


reference |λ> = p|...>
∫d⁴η δ⁰ˡ³(Σ λₙ η̃ₙ + λη) / <j j+1> … <i-2 i-1> <i-1 λ> <λ j>
1/p²
δ⁰ˡ³(Σ λₙ' η̃ₙ' − λη) / <j-1 λ> <λ i> <i i+1> … <j-2 j-1>
i
j-1

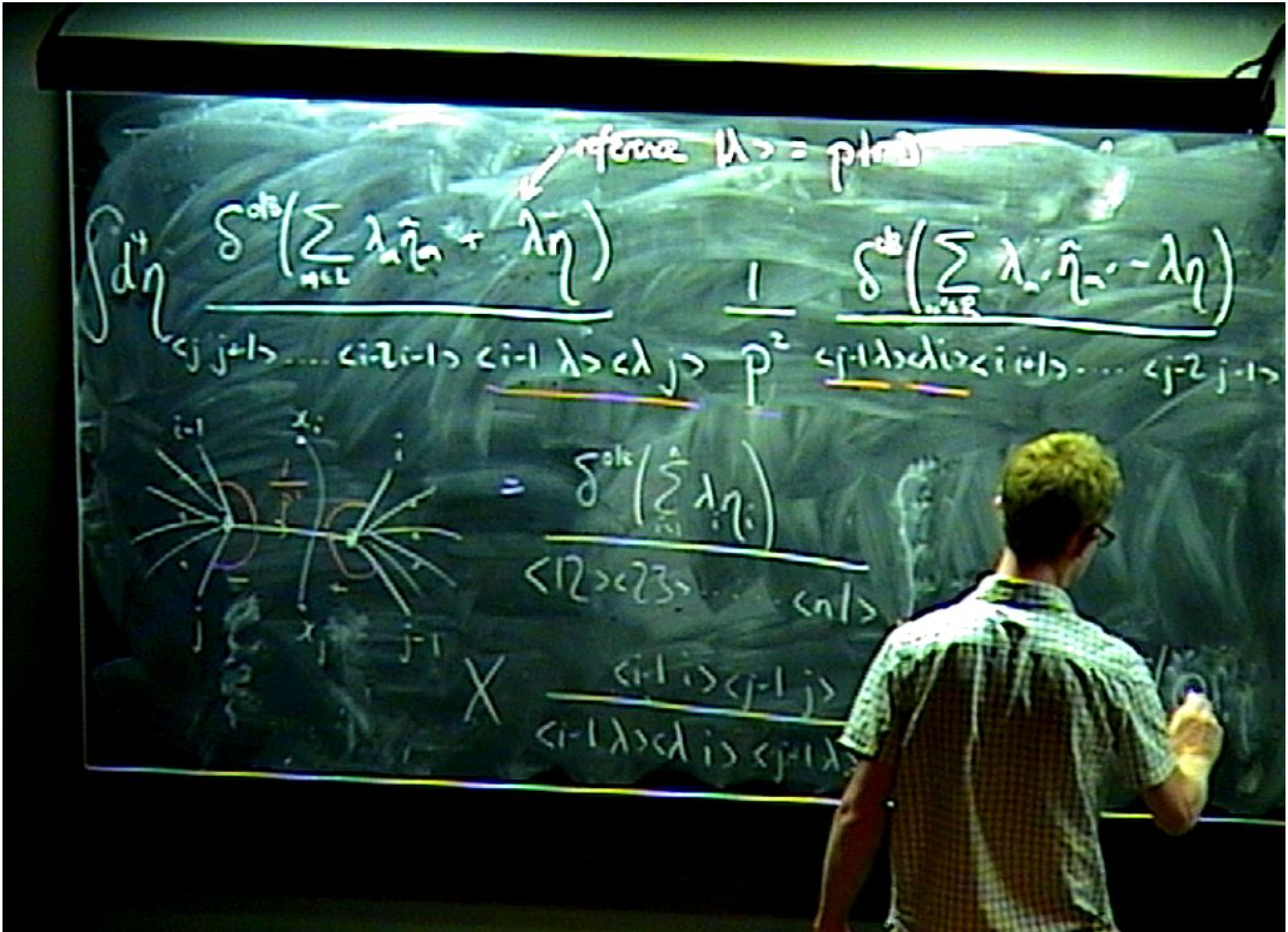


reference
i-1
i
j
j-1

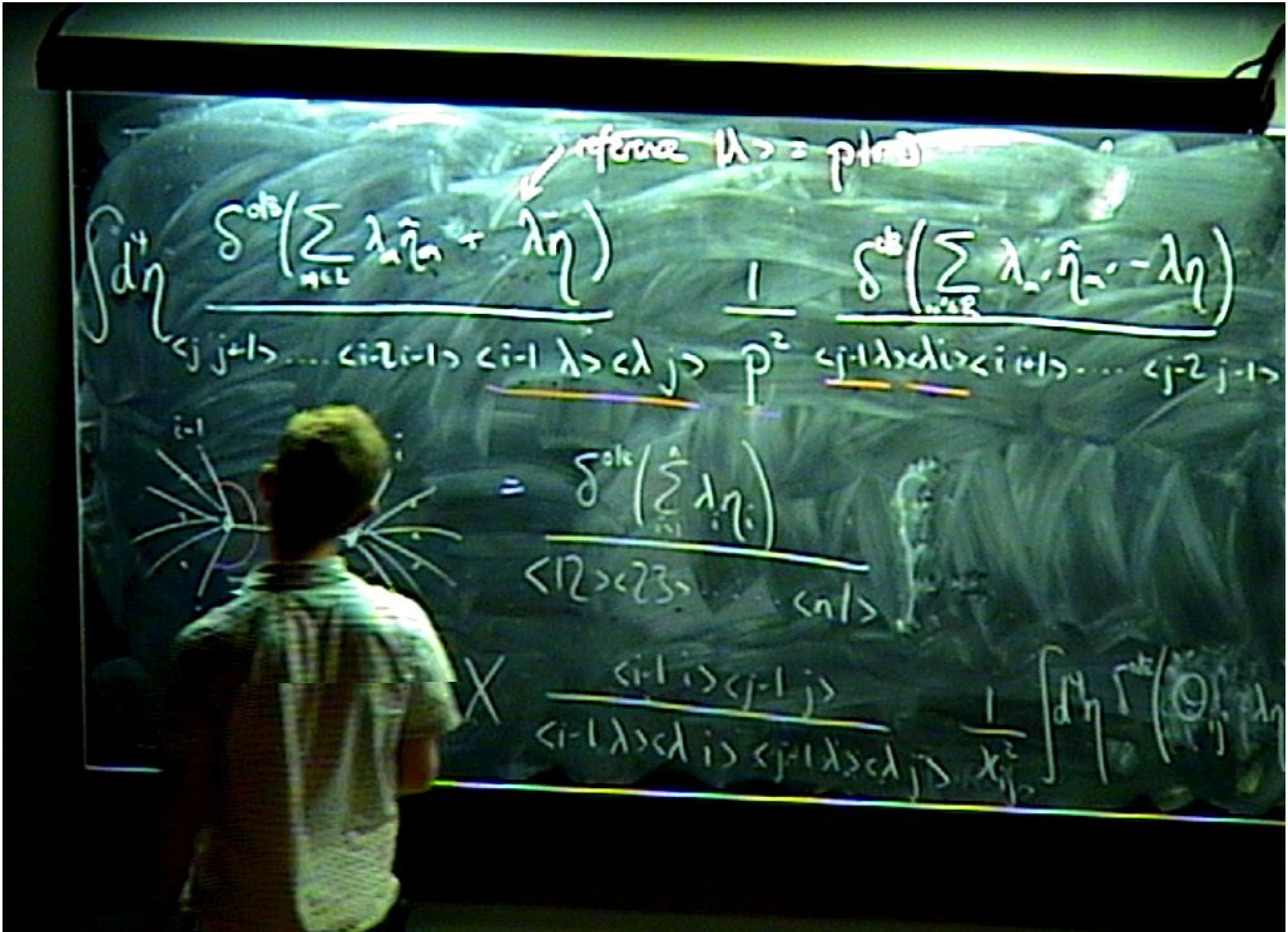

reference
i-1
i
j-1
<12><23> ... <n1>
<i-1 λ><λ i> <j-1 λ><λ j>

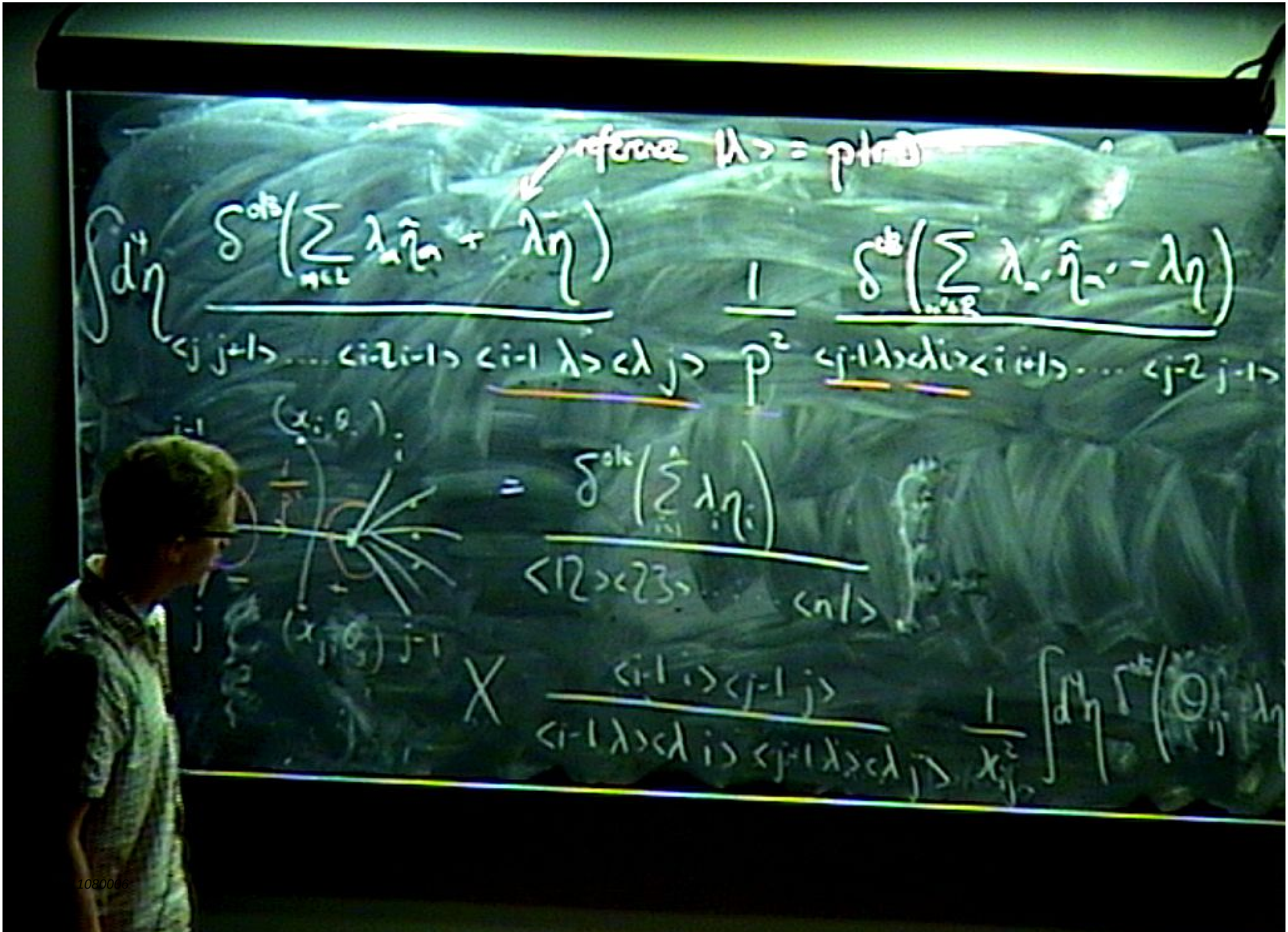


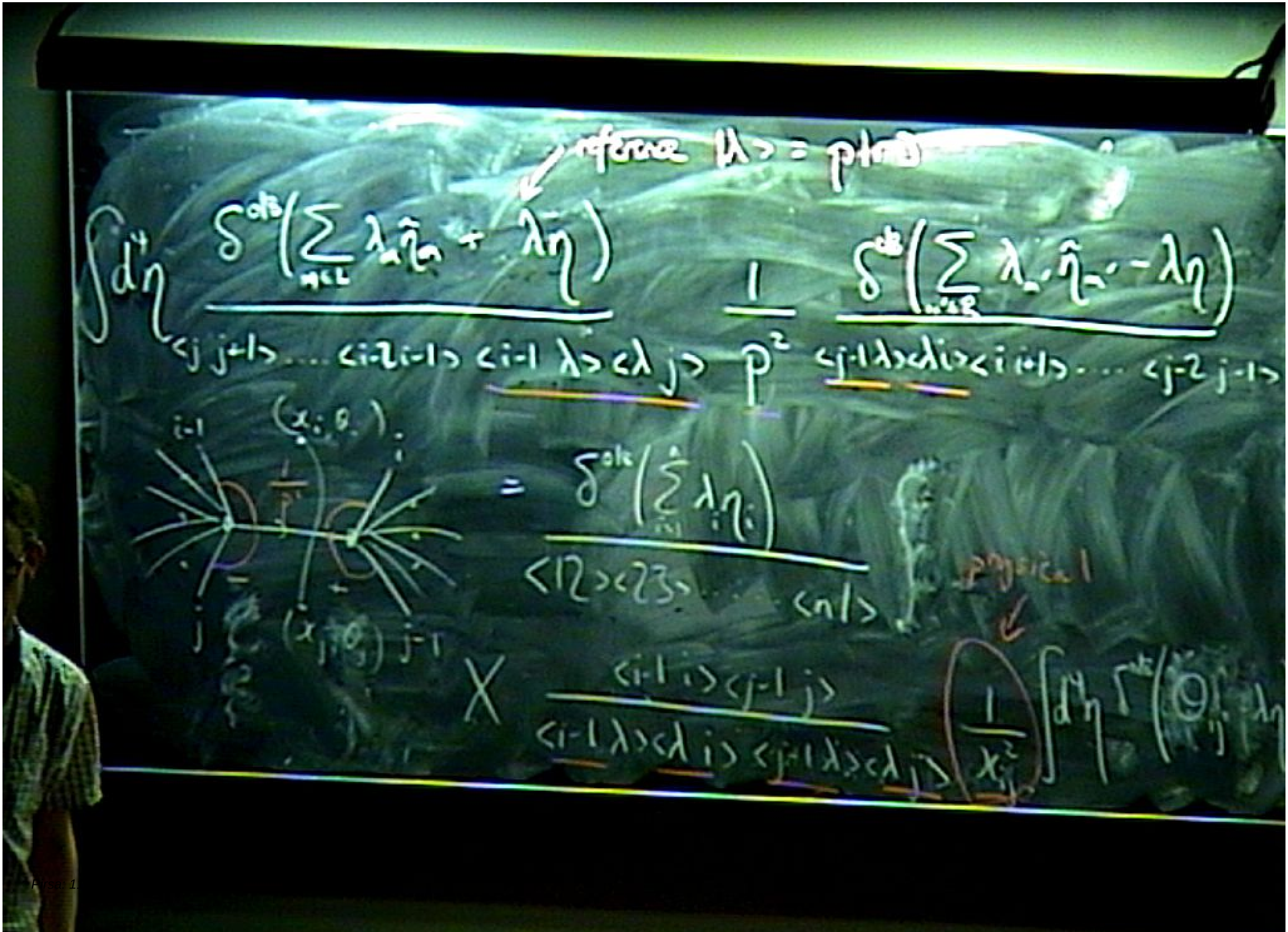
<j j+1> ... <i-2 i-1> <i-1 λ> <λ j>
<j-1 λ><λ i><i i+1> ... <j-2 j-1>
<12><23> ... <n1>
<i-1 i><j-1 j>



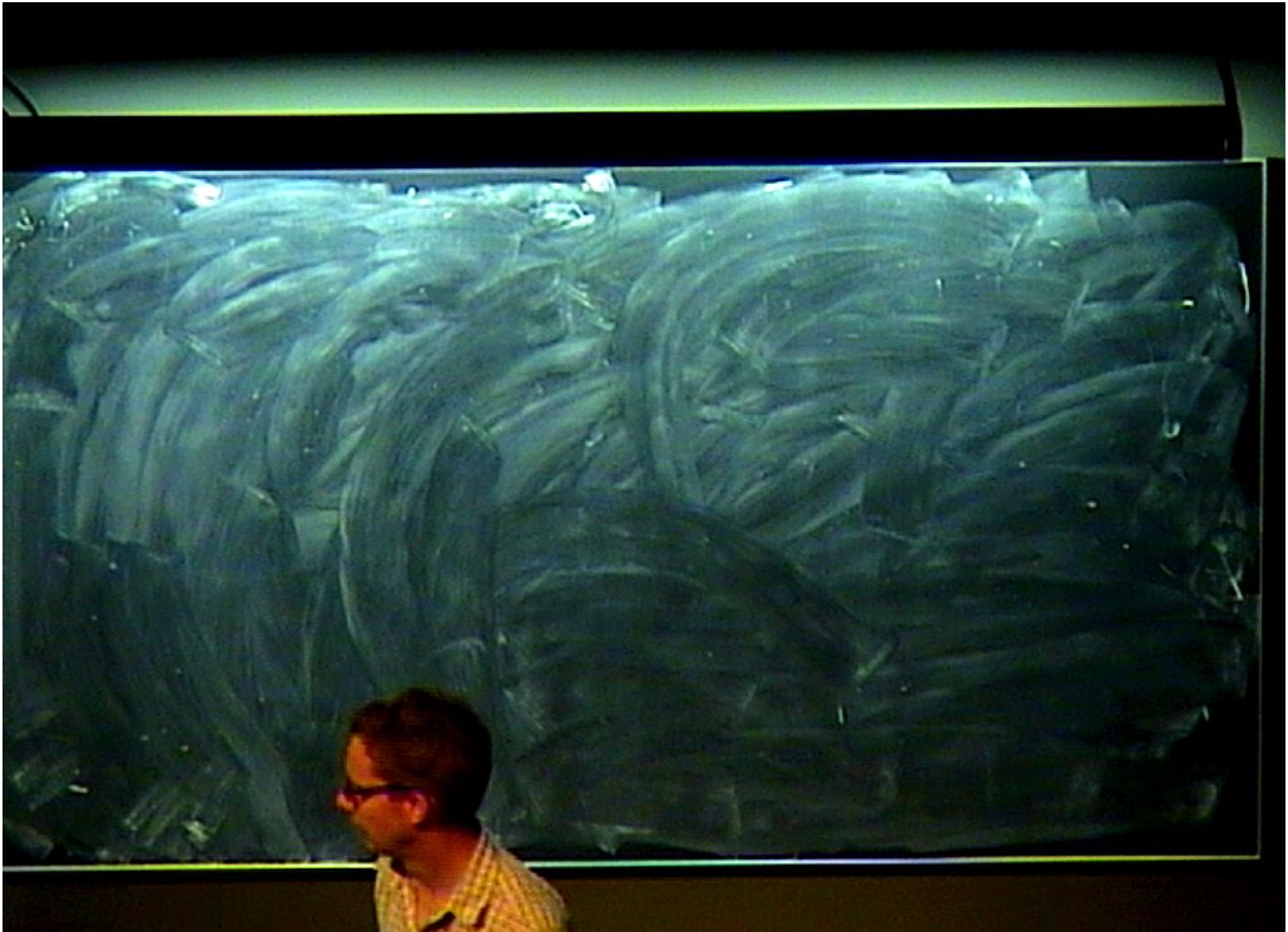
reference
1
p^2
<j j+1> .... <i-2 i-1> <i-1 λ><λ j>
<j-1 λ><λ i><i i+1> .... <j-2 j-1>
<12><23> .... <n1>
<i-1 i><j-1 j>
<i-1 λ><λ i> <j-1 λ>

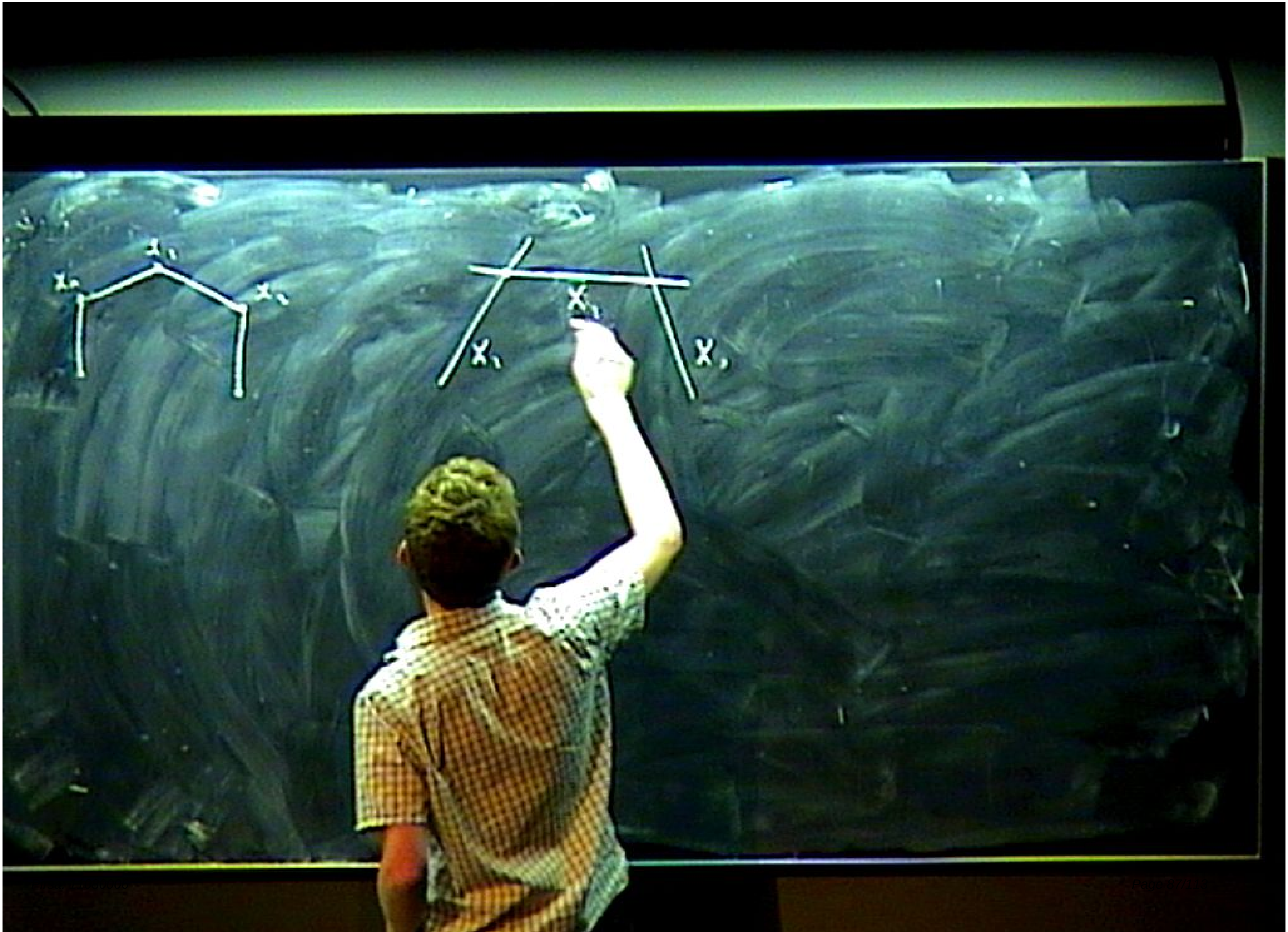


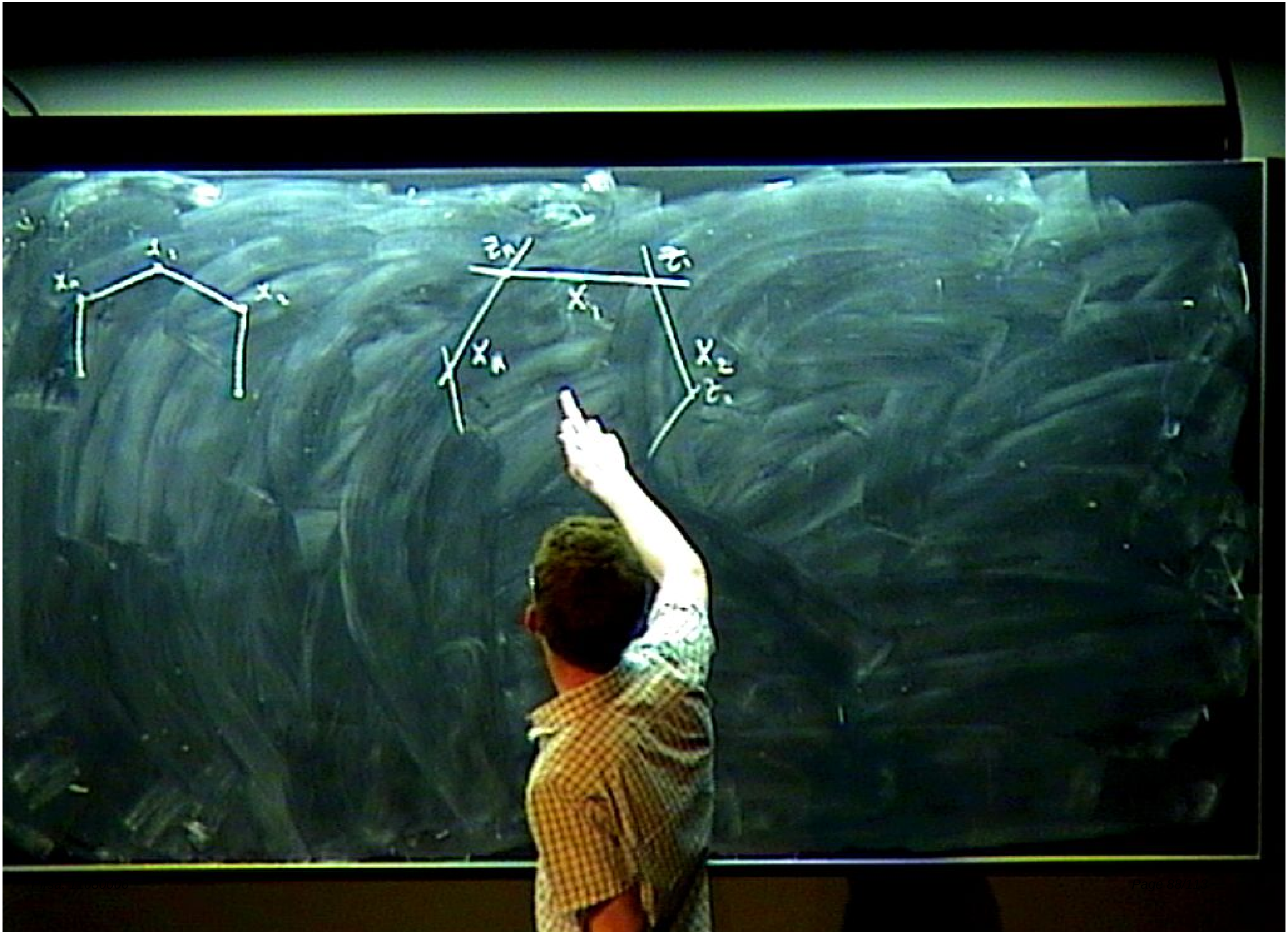

reference



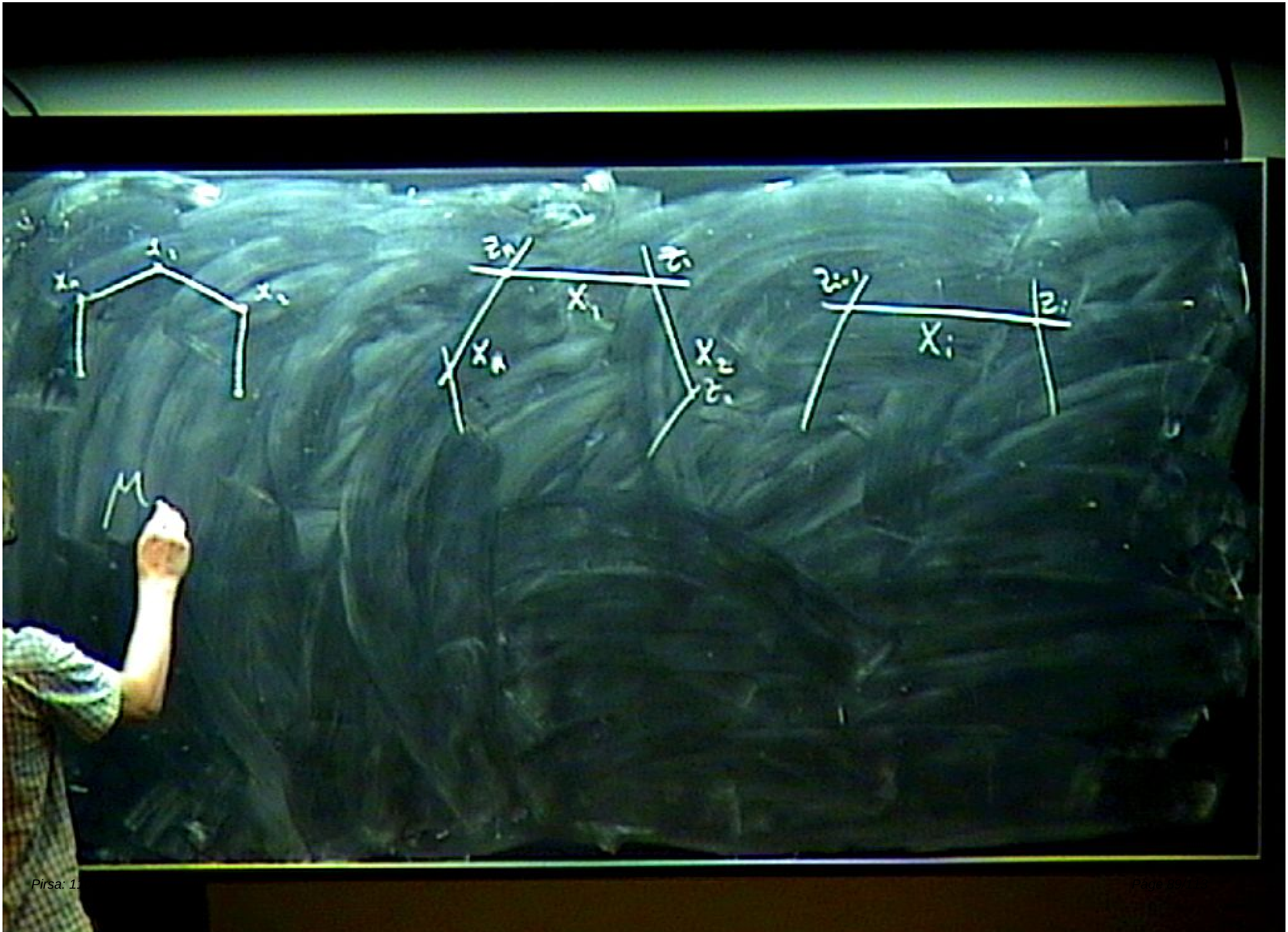
reference |λ> = p|n>
<j j+1> ... <i-2 i-1> <i-1 λ> <λ j>
<j-1 λ><λ i><i i+1> ... <j-2 j-1>
<12><23> ... <n1>
physical
<i-1 i><j-1 j>
<i-1 λ><λ i> <j-1 λ><λ j>



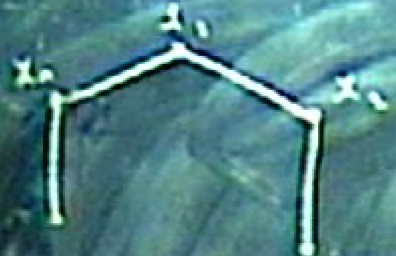




x_1
x_n
x_2
z_n
z_1
X_1
X_n
X_2
z_2

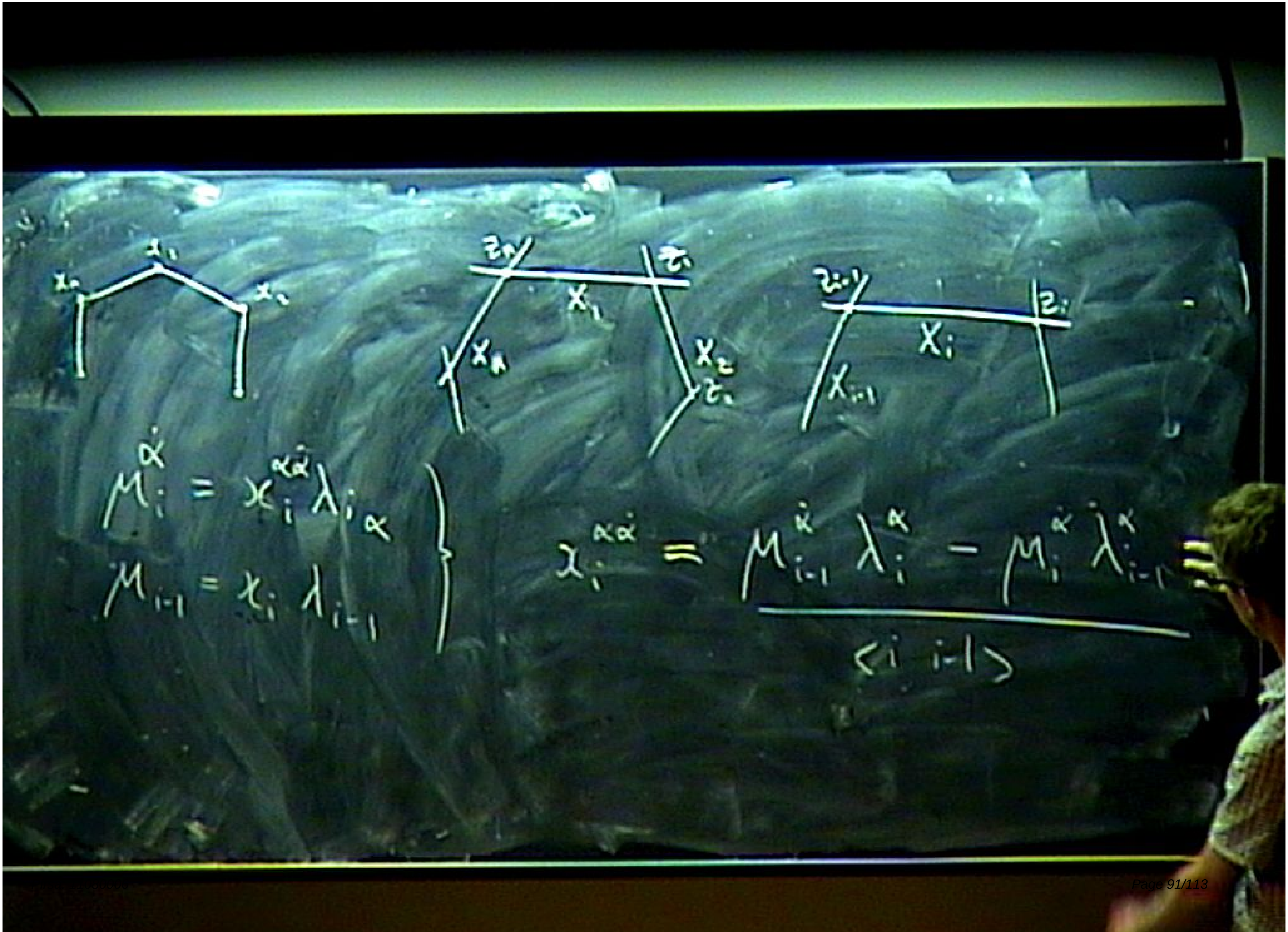


X1
X2
Xi
M

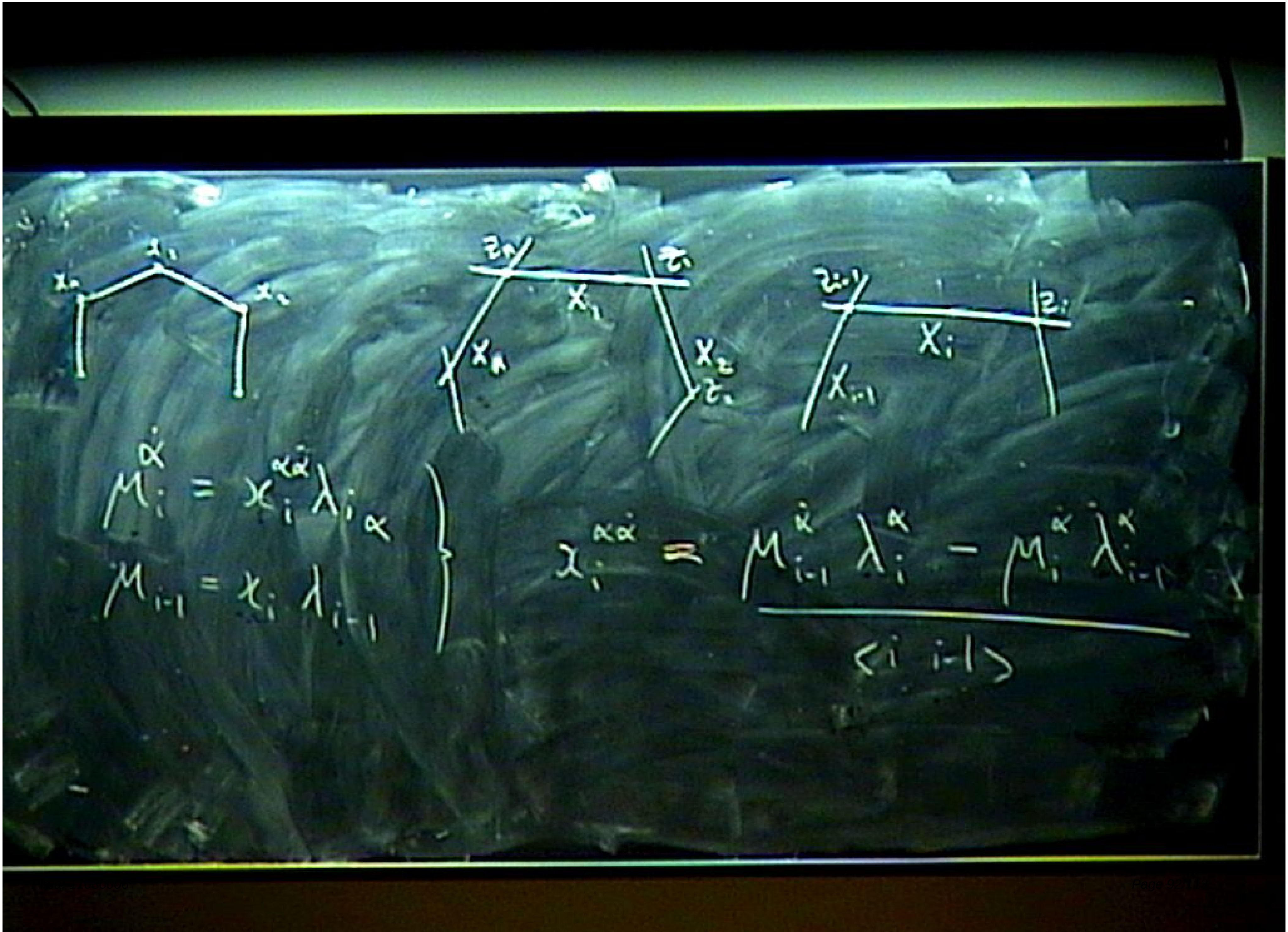


$$\mu_i = x_i \lambda_i$$

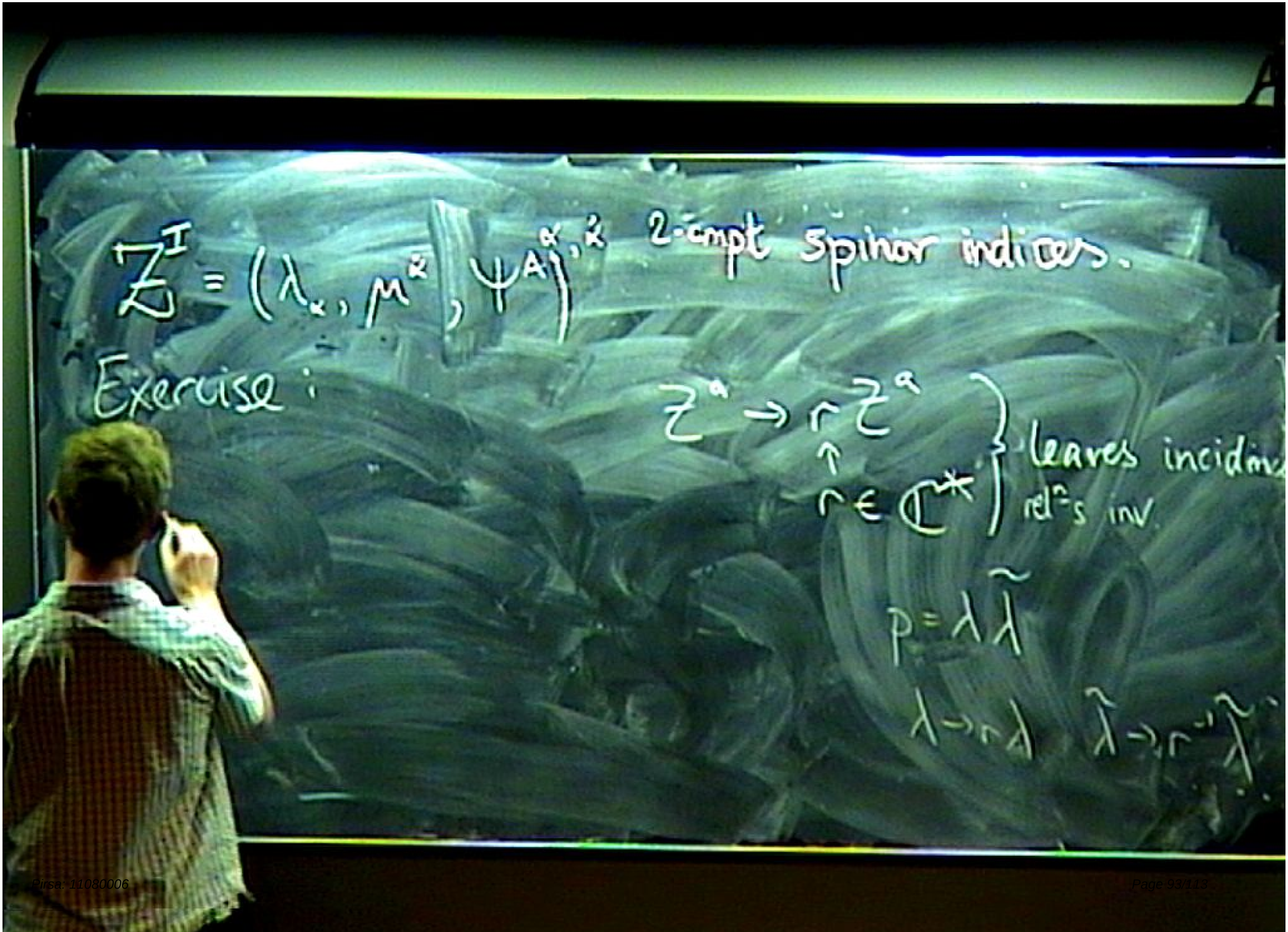
$$\mu_{i-1} = x_i \lambda_{i-1}$$

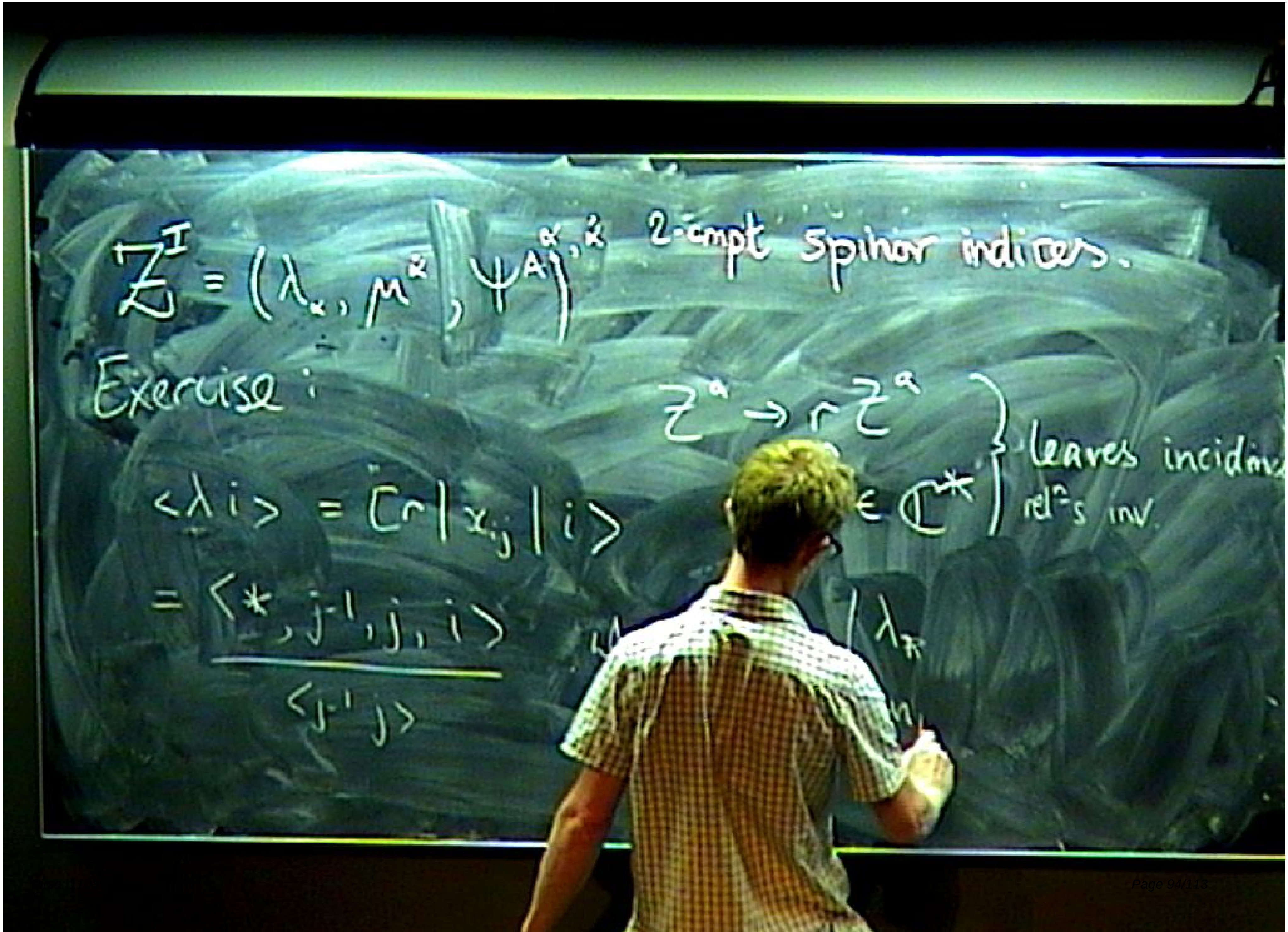


x_0 x_1 x_2
z_n z_1 X_1 X_n X_2 z_2
z_{i-1} z_i X_i X_{i-1}
$\mu_i^{\dot\alpha} = x_i^{\alpha\dot\alpha} \lambda_{i\alpha}$
$\mu_{i-1} = x_i \lambda_{i-1}$
$x_i^{\alpha\dot\alpha} = \frac{\mu_{i-1}^{\dot\alpha} \lambda_i^{\alpha} - \mu_i^{\dot\alpha} \lambda_{i-1}^{\alpha}}{\langle i \; i-1 \rangle}$



z_n
z_1
X_1
X_n
X_2
z_2
z_{i-1}
z_i
X_i
X_{i-1}
$\mu_i^{\dot\alpha} = x_i^{\alpha\dot\alpha} \lambda_{i\alpha}$
$\mu_{i-1} = x_i \lambda_{i-1}$
$x_i^{\alpha\dot\alpha} = \frac{\mu_{i-1}^{\dot\alpha} \lambda_i^{\alpha} - \mu_i^{\dot\alpha} \lambda_{i-1}^{\alpha}}{\langle i \; i-1 \rangle}$


Z^I = (λ_α, μ^α̇, ψ^A)
α, α̇ 2-cmpt spinor indices.
Exercise:
Z^a → r Z^a
r ∈ ℂ*
leaves incidence rel^ns inv.
p = λ λ̃
λ → r λ
λ̃ → r^-1 λ̃


Z^I = (λ_α, μ^α̇, ψ_A)
2-cmpt spinor indices.
Exercise:
Z^a → r Z^a
leaves incidence
∈ ℂ*
rel^ns inv
<λ i> = [r | x_ij | i>
= <*, j-1, j, i>
<j-1 j>

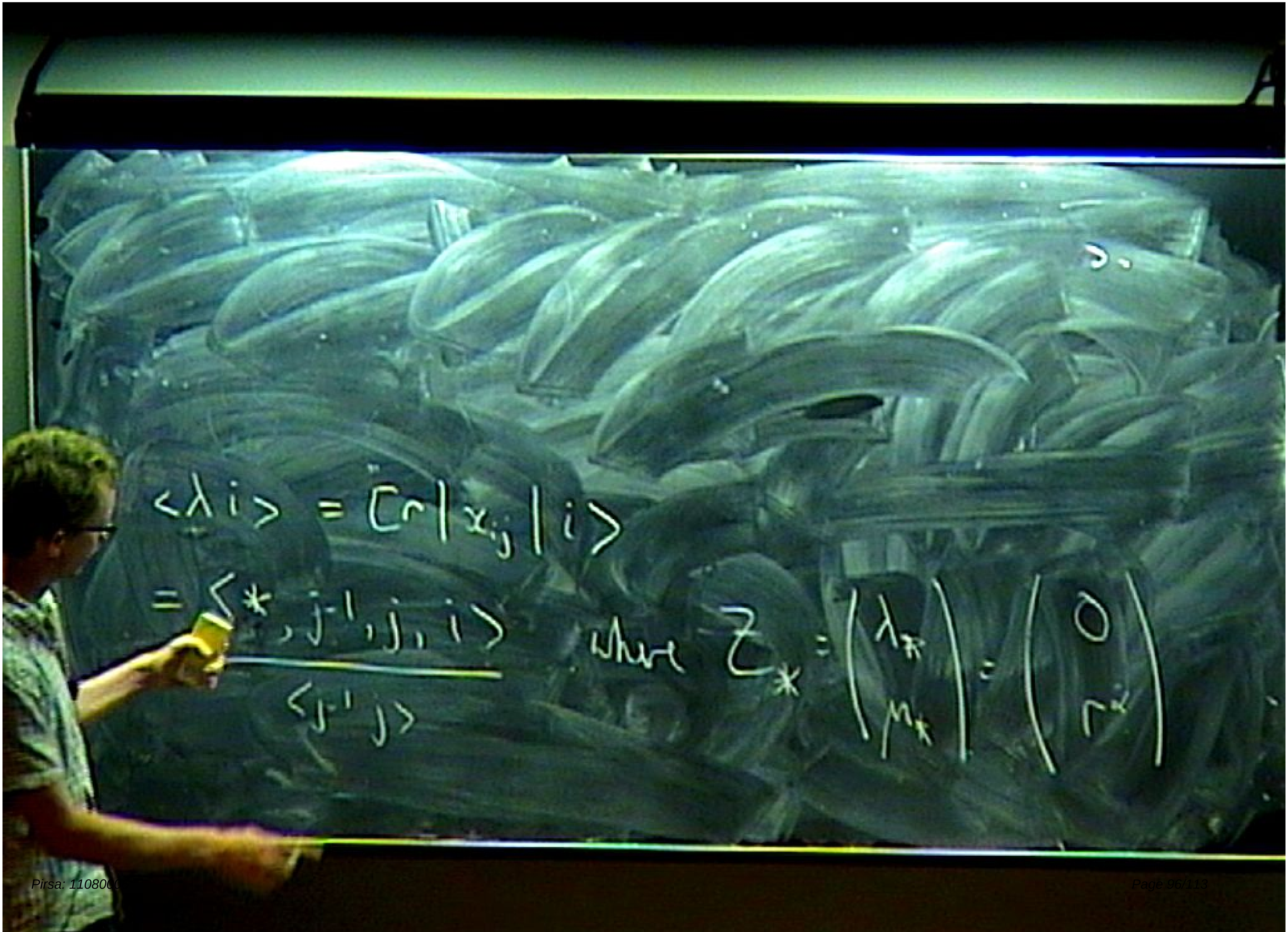
$Z^I = (\lambda_\alpha, \mu^{\dot\alpha}, \psi^A)$ — $\alpha, \dot\alpha$ 2-cmpt spinor indices.

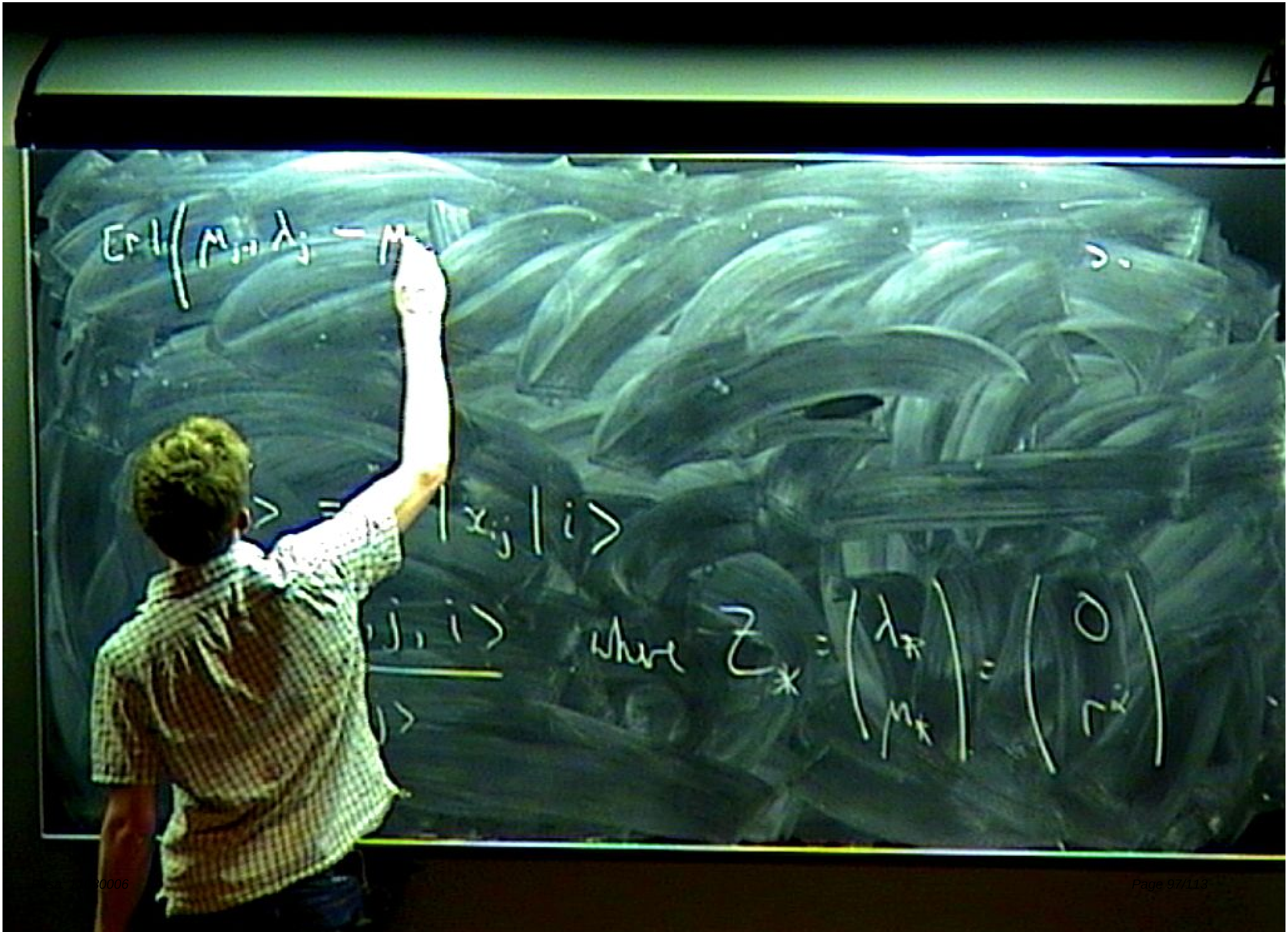
Exercise:

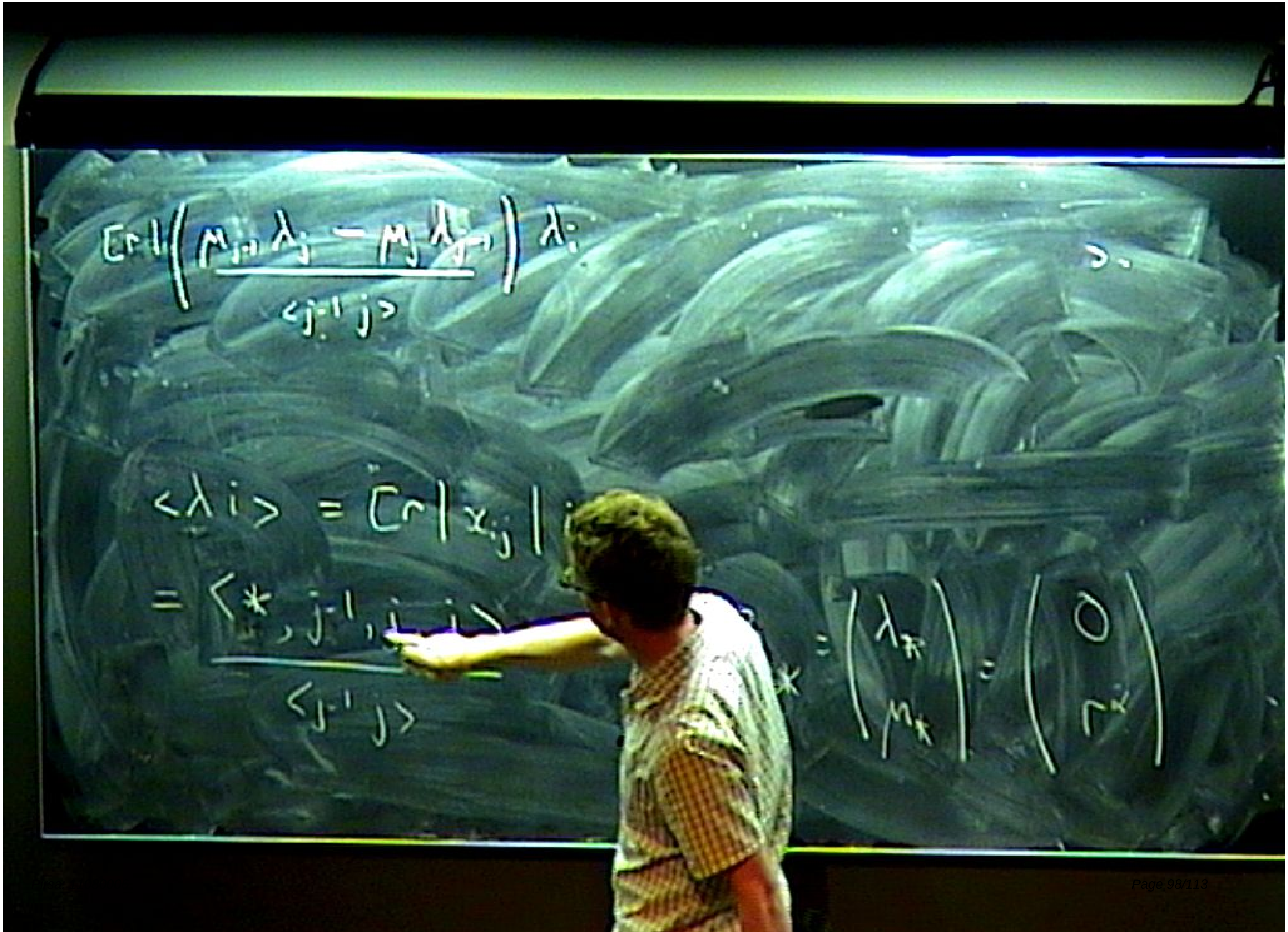
$Z^a \to r Z^a$, $r \in \mathbb{C}^*$ } leaves incidence rel^{n}s inv.

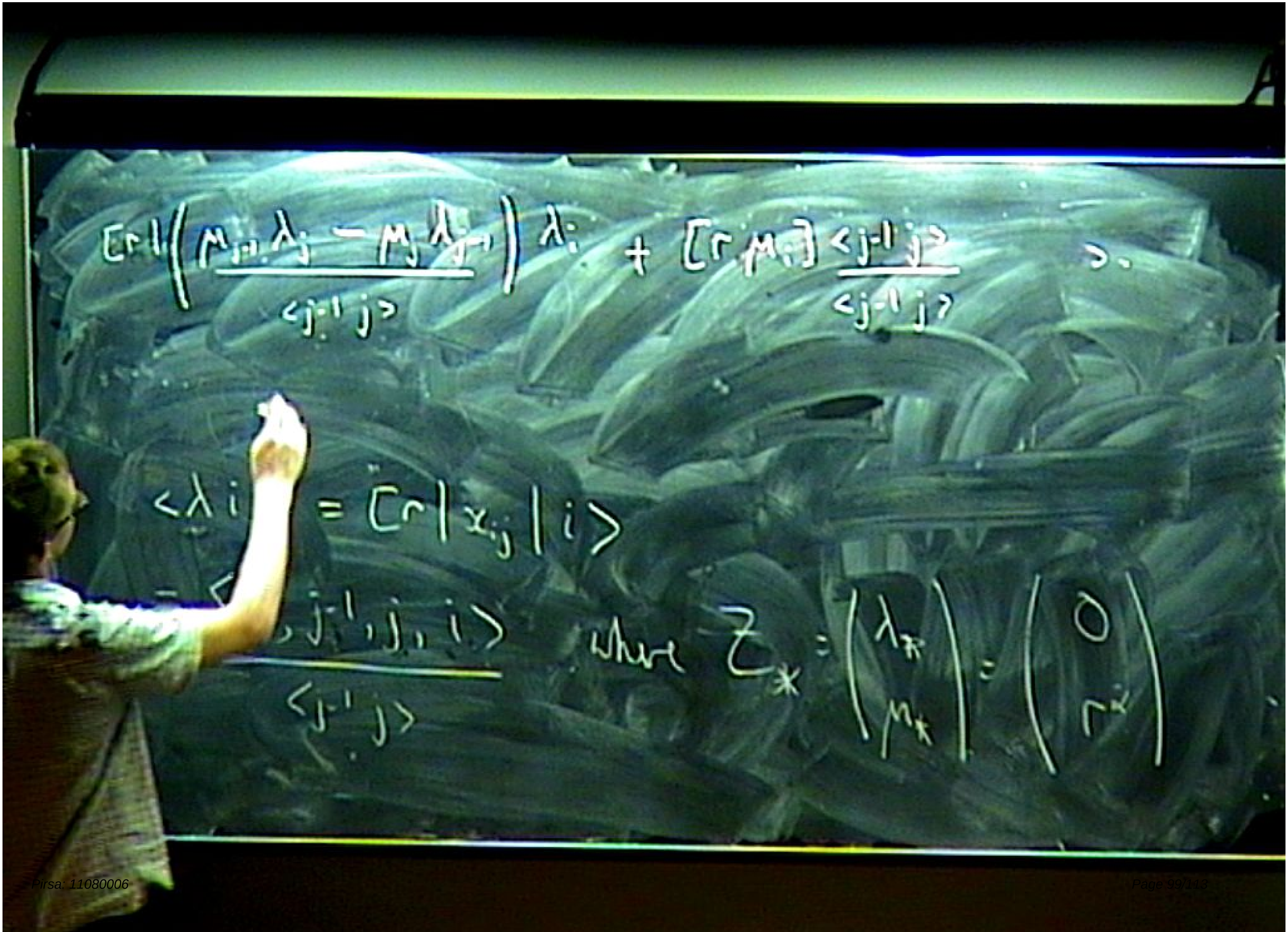
$$\langle \lambda\, i\rangle = [r|x_{ij}|i\rangle = \frac{\langle *, j-1, j, i\rangle}{\langle j-1\; j\rangle}$$

where $Z_* = \begin{pmatrix}\lambda_*\\ \mu_*\end{pmatrix} = \begin{pmatrix}0\\ r^{\dot\alpha}\end{pmatrix}$


$\langle \lambda i \rangle = [r | x_{ij} | i \rangle$
$= \frac{\langle *, j-1, j, i \rangle}{\langle j-1\, j \rangle}$
where $Z_* = \begin{pmatrix} \lambda_* \\ \mu_* \end{pmatrix} = \begin{pmatrix} 0 \\ r^* \end{pmatrix}$


> = | x_ij | i>
j, i>
where Z_* = (λ_* μ_*) = (0 r^*)


[r|(μ_{j+1} λ_j − μ_j λ_{j+1} / <j-1 j>) λ_i
<λ i> = [r | x_ij |
= <*, j-1, j, i> / <j-1 j>
= (λ_* μ_*) = (0 r^*)


[r|((μ_{j+1} λ_j − μ_j λ_{j+1}) / <j-1 j>) λ_i + [r μ_i] <j-1 j> / <j-1 j> >.
<λ i = [r | x_{ij} | i>
= < , j-1, j, i> / <j-1 j>
where Z_* = (λ_*, μ_*) = (0, r^*)

$$[r|\left(\frac{\mu_{j-1}\lambda_j - \mu_j\lambda_{j-1}}{\langle j\text{-}1\; j\rangle}\right)\lambda_i + [r\,\mu_i]\frac{\langle j\text{-}1\; j\rangle}{\langle j\text{-}1\; j\rangle} \;\; >.$$

$$\langle \lambda\, i\rangle = [r|\,x_{ij}\,|i\rangle$$

$$= \frac{\langle *, j\text{-}1, j, i\rangle}{\langle j\text{-}1\; j\rangle} \quad \text{where } Z_* = \begin{pmatrix}\lambda_* \\ \mu_*\end{pmatrix} = \begin{pmatrix}0 \\ r^{\dot\alpha}\end{pmatrix}$$

$$[r|\left(\frac{\mu_{j-1}\lambda_j - \mu_j \lambda_{j-1}}{\langle j\text{-}1\; j\rangle}\right)\lambda_i\rangle + [r\,\mu_i]\frac{\langle j\text{-}1\; j\rangle}{\langle j\text{-}1\; j\rangle}\rangle.$$

$$= [r|x_j|\lambda_i\rangle$$

$$\langle \lambda\, i\rangle = [r|x_{ij}|i\rangle$$

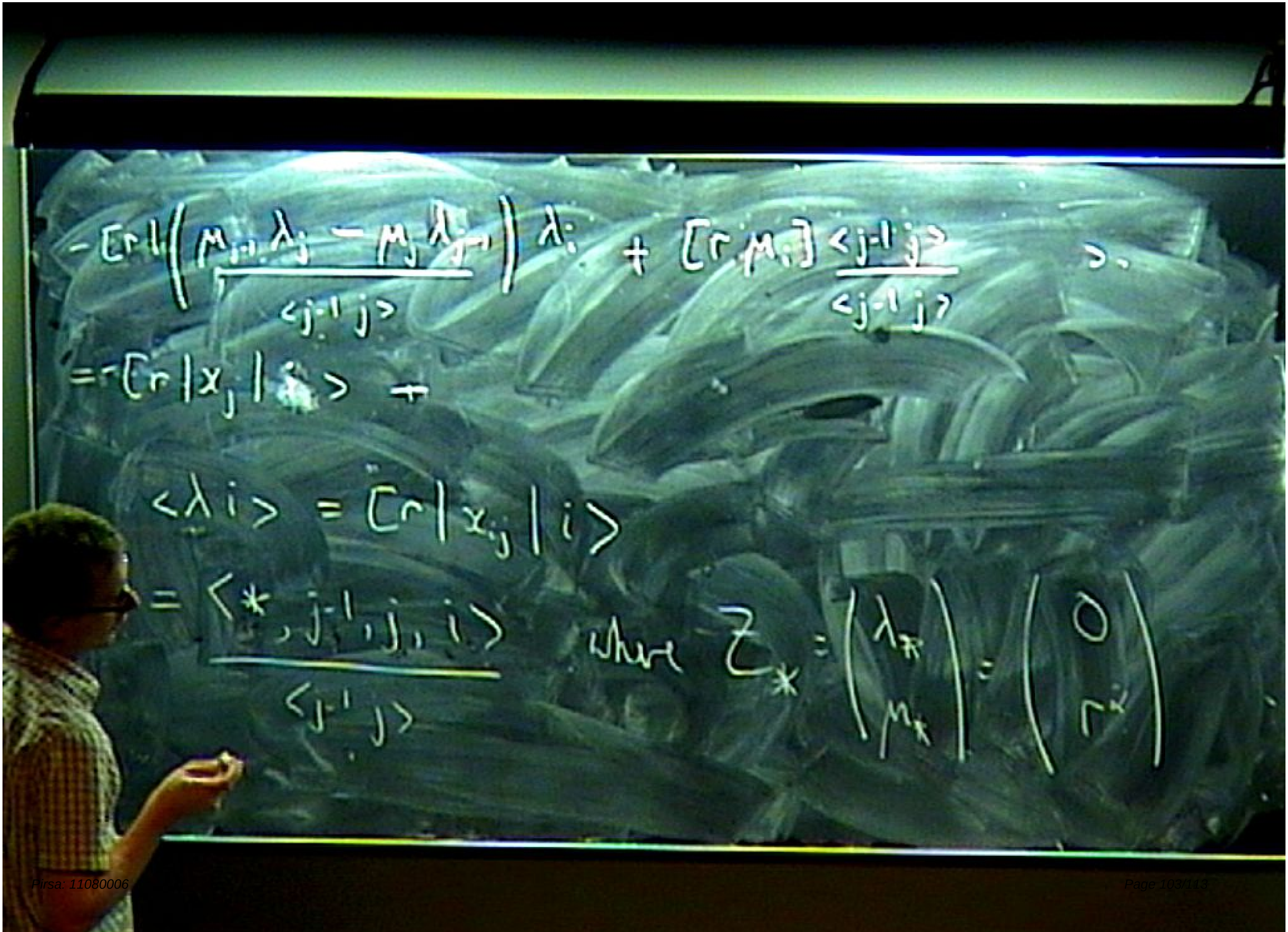
$$= \frac{\langle *, j\text{-}1, j, i\rangle}{\langle j\text{-}1\; j\rangle} \quad \text{where } Z_* = \begin{pmatrix}\lambda_* \\ \mu_*\end{pmatrix} = \begin{pmatrix} 0 \\ r^* \end{pmatrix}$$

$$[r|\left(\frac{\mu_{j-1}\lambda_j - \mu_j\lambda_{j-1}}{\langle j-1\; j\rangle}\right)\lambda_i + [r\,\mu_i]\frac{\langle j-1\; j\rangle}{\langle j-1\; j\rangle} \quad \rangle.$$

$$= [r|x_j|i\rangle$$

$$\langle \lambda\, i\rangle = [r|x_{ij}|i\rangle$$

$$= \frac{\langle *, j-1, j, i\rangle}{\langle j-1\; j\rangle} \quad \text{where } Z_* = \begin{pmatrix}\lambda_* \\ \mu_*\end{pmatrix} = \begin{pmatrix}0 \\ r^*\end{pmatrix}$$

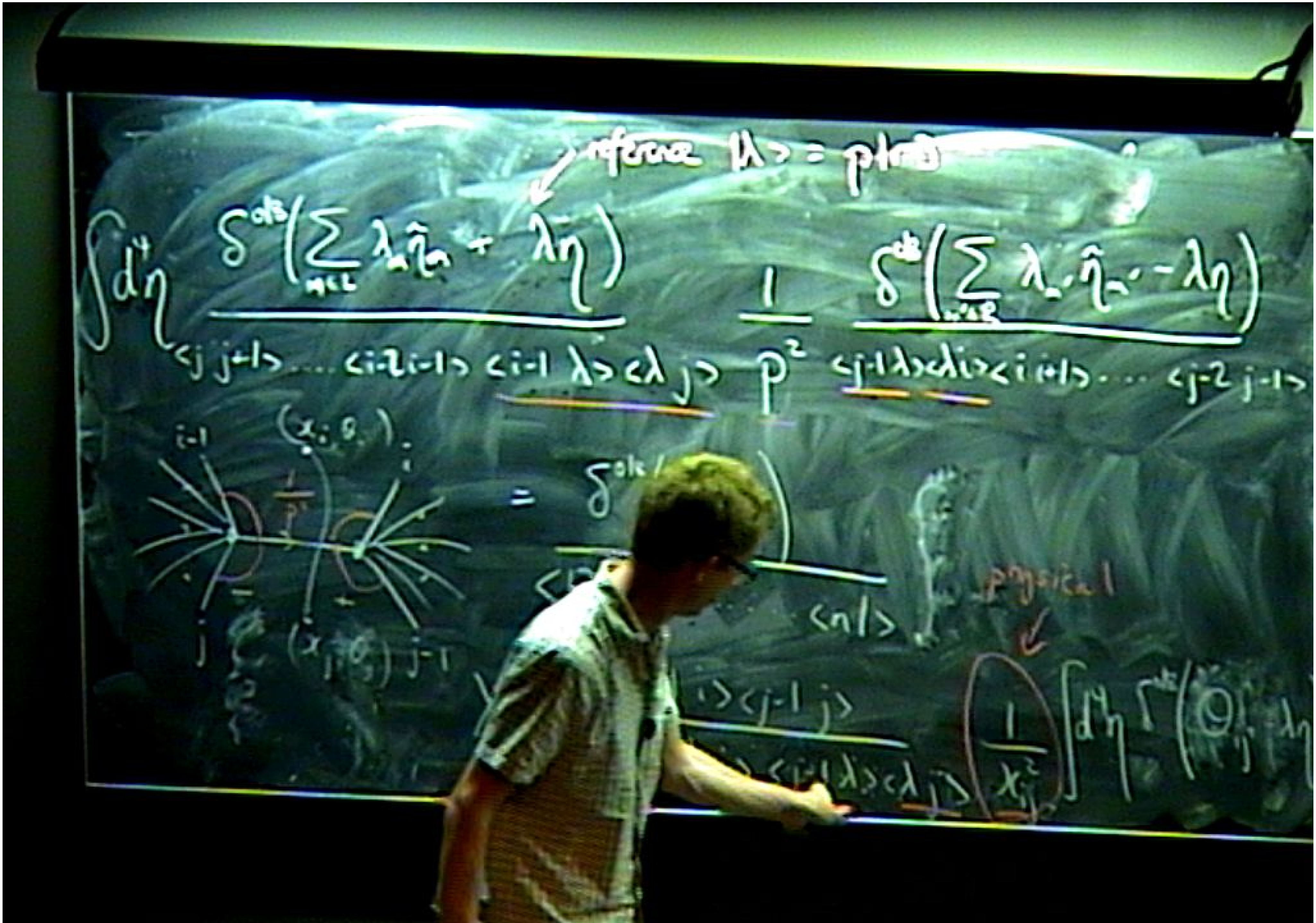

- [r|(μj+1 λj − μj λj+1 / <j-1 j>) λi + [r μi] <j-1 j> / <j-1 j> >.
= -[r|xj| i> +
<λ i> = [r| xij | i>
= <*, j-1, j, i> / <j-1 j>
where Z* = (λ* μ*) = (0 r*)

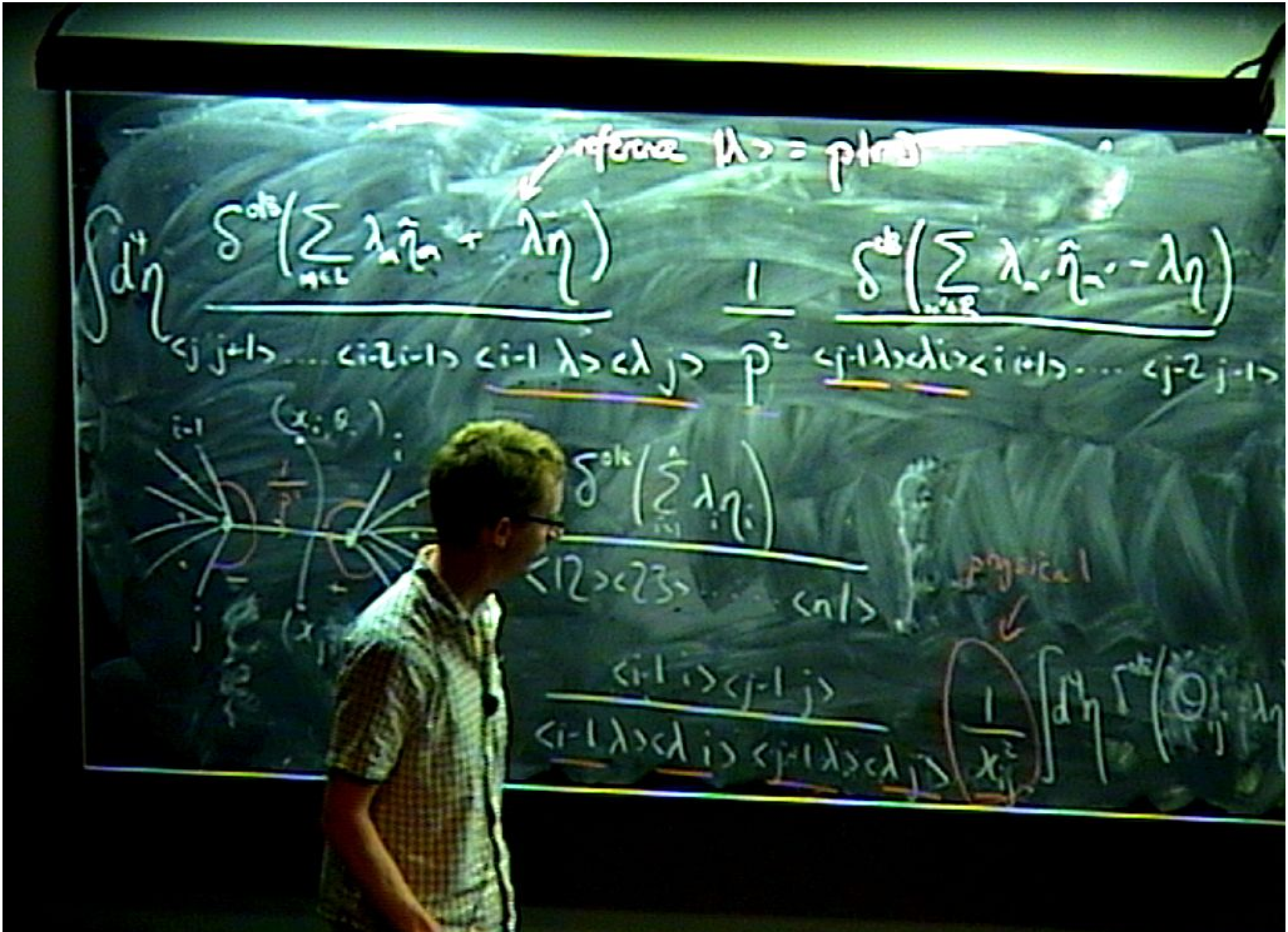
$$-[r|\left(\frac{\mu_{j-1}\lambda_j - \mu_j\lambda_{j-1}}{\langle j\!-\!1\; j\rangle}\right)\lambda_i + [r\,\mu_i]\,\frac{\langle j\!-\!1\; j\rangle}{\langle j\!-\!1\; j\rangle}\;\rangle.$$

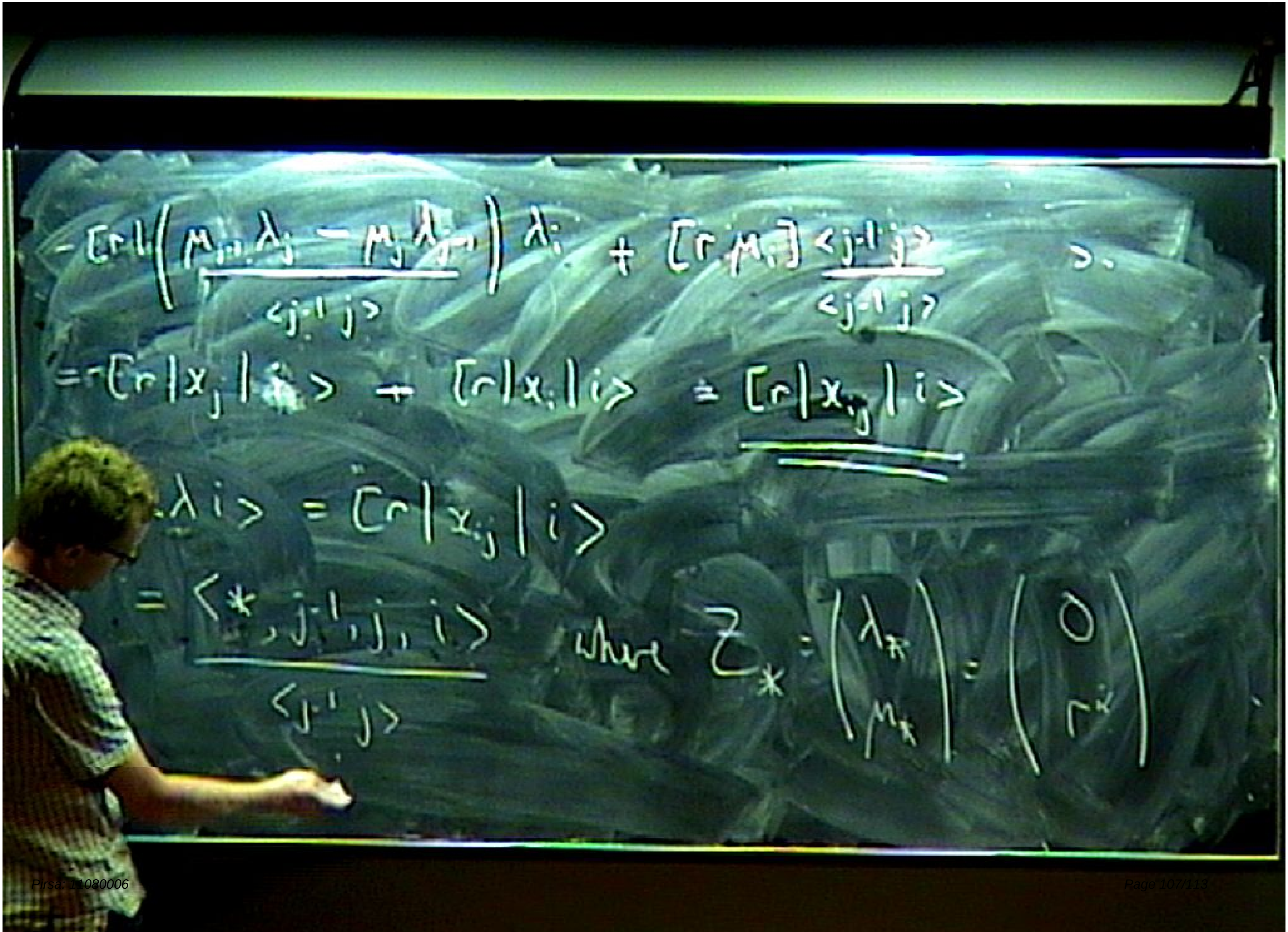
$$= -[r|x_j|i\rangle + [r|x_i|i\rangle = [r|x_{ij}|i\rangle$$

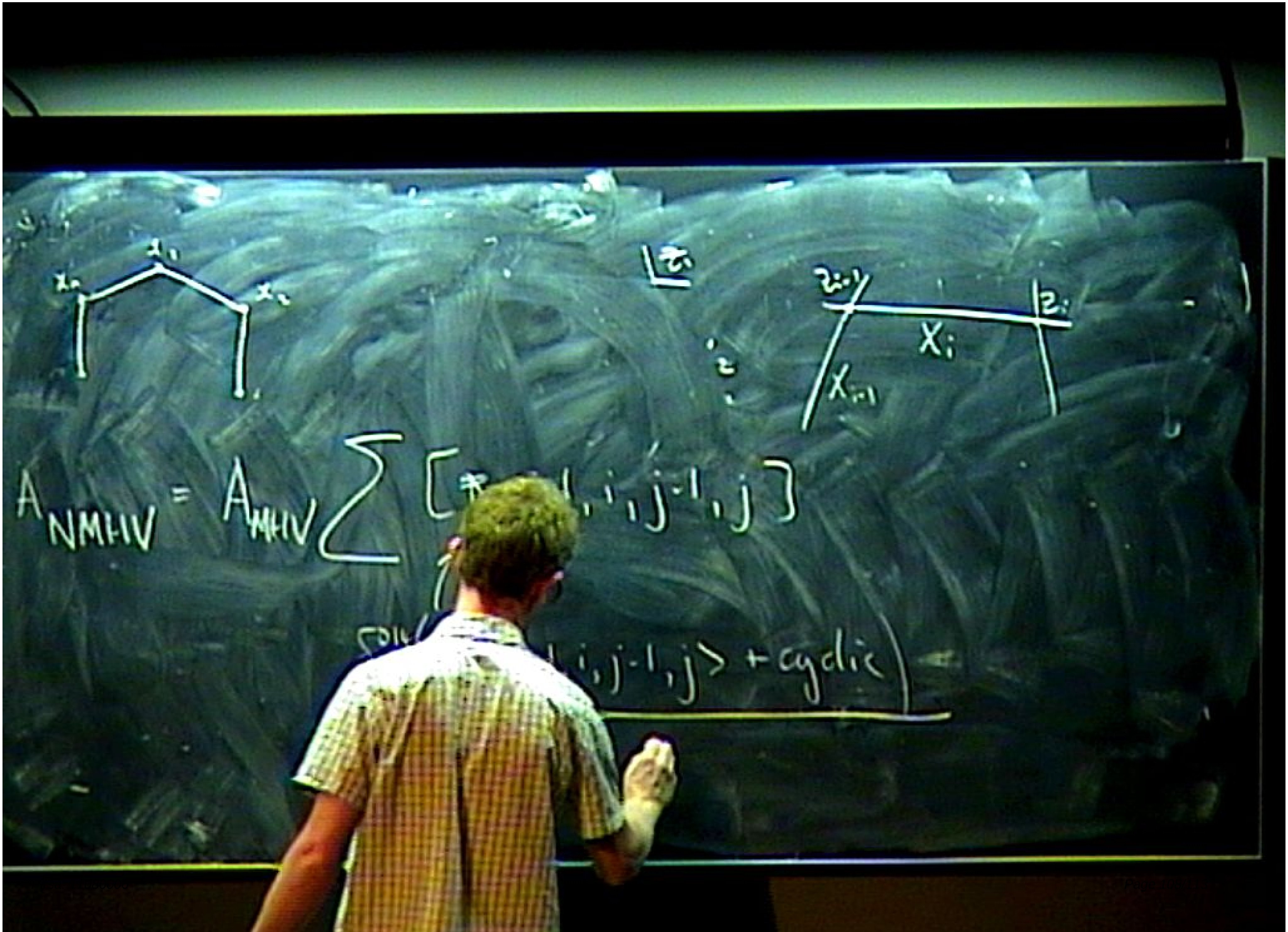
$$\langle \lambda\, i\rangle = [r|x_{ij}|i\rangle$$

$$= \frac{\langle *, j\!-\!1, j, i\rangle}{\langle j\!-\!1\; j\rangle} \quad \text{where } Z_* = \begin{pmatrix}\lambda_* \\ \mu_*\end{pmatrix} = \begin{pmatrix}0 \\ r^*\end{pmatrix}$$

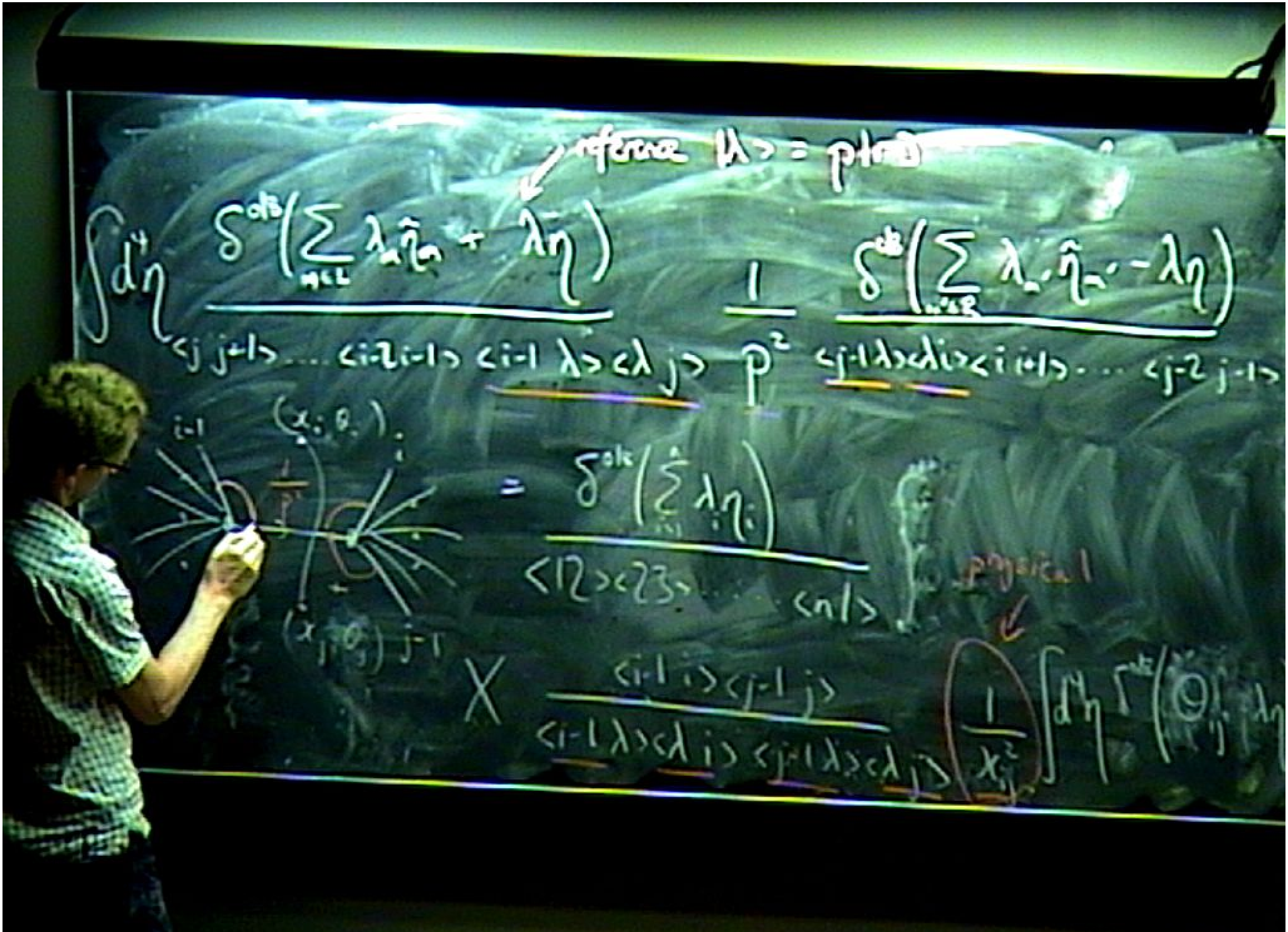

reference
physical


reference
physical

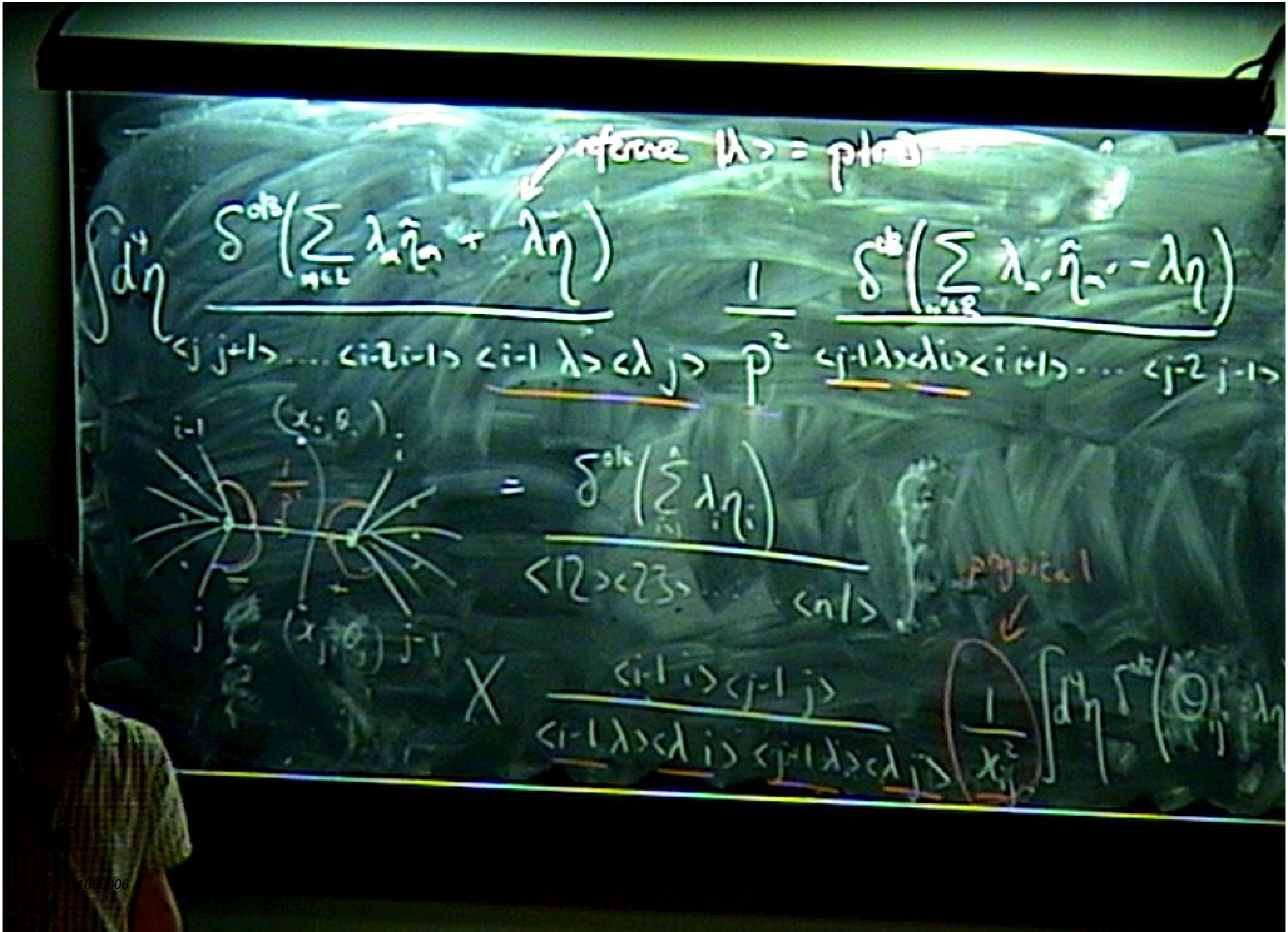

where

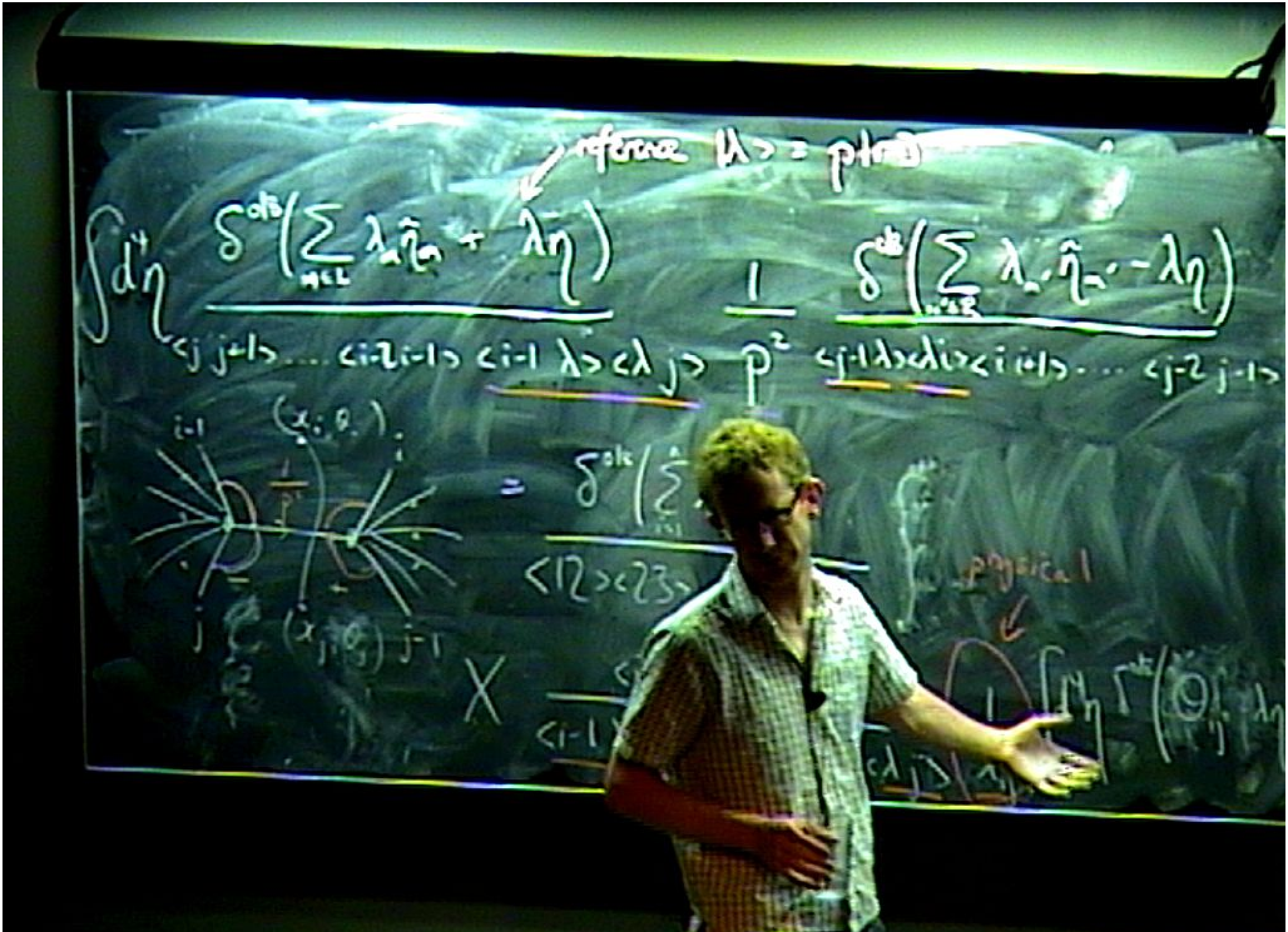


A_NMHV = A_MHV ∑ [
,j-1,j]
,j-1,j> + cyclic
X_i
X_{i-1}



reference |λ> = p|ref)
physical


reference
physical


physical

$$-[r|\left(\frac{\mu_{j-1}\lambda_j - \mu_j\lambda_{j-1}}{\langle j-1\; j\rangle}\right)\lambda_i + [r\,\mu_i]\frac{\langle j-1\; j\rangle}{\langle j-1\; j\rangle}\rangle.$$

$$= -[r|x_j|i\rangle + [r|x_i|i\rangle = [r|x_{ij}|i\rangle$$

$$\langle\lambda\, i\rangle = [r|x_{ij}|i\rangle$$

$$= \frac{\langle *, j-1, j, i\rangle}{\langle j-1\; j\rangle} \quad \text{where } Z_* = \begin{pmatrix}\lambda_*\\ \mu_*\end{pmatrix} = \begin{pmatrix}0\\ r^*\end{pmatrix}$$

$$-[r|\left(\frac{\mu_{j-1}\lambda_j - \mu_j \lambda_{j-1}}{\langle j\!-\!1\; j\rangle}\right)\lambda_i + [r\,\mu_i]\,\frac{\langle j\!-\!1\; j\rangle}{\langle j\!-\!1\; j\rangle}\;\rangle .$$

$$= -[r|x_j|i\rangle + [r|x_i|i\rangle = [r|x_{ij}|i\rangle$$

$$\langle \lambda\, i\rangle = [r|x_{ij}|i\rangle$$

$$= \frac{\langle *, j\!-\!1, j, i\rangle}{\langle j\!-\!1\; j\rangle} \quad \text{where} \quad Z_{1*} = \begin{pmatrix} \lambda_* \\ \mu_* \\ \eta_a \end{pmatrix} = \begin{pmatrix} 0 \\ r^{\alpha} \\ 0 \end{pmatrix}$$