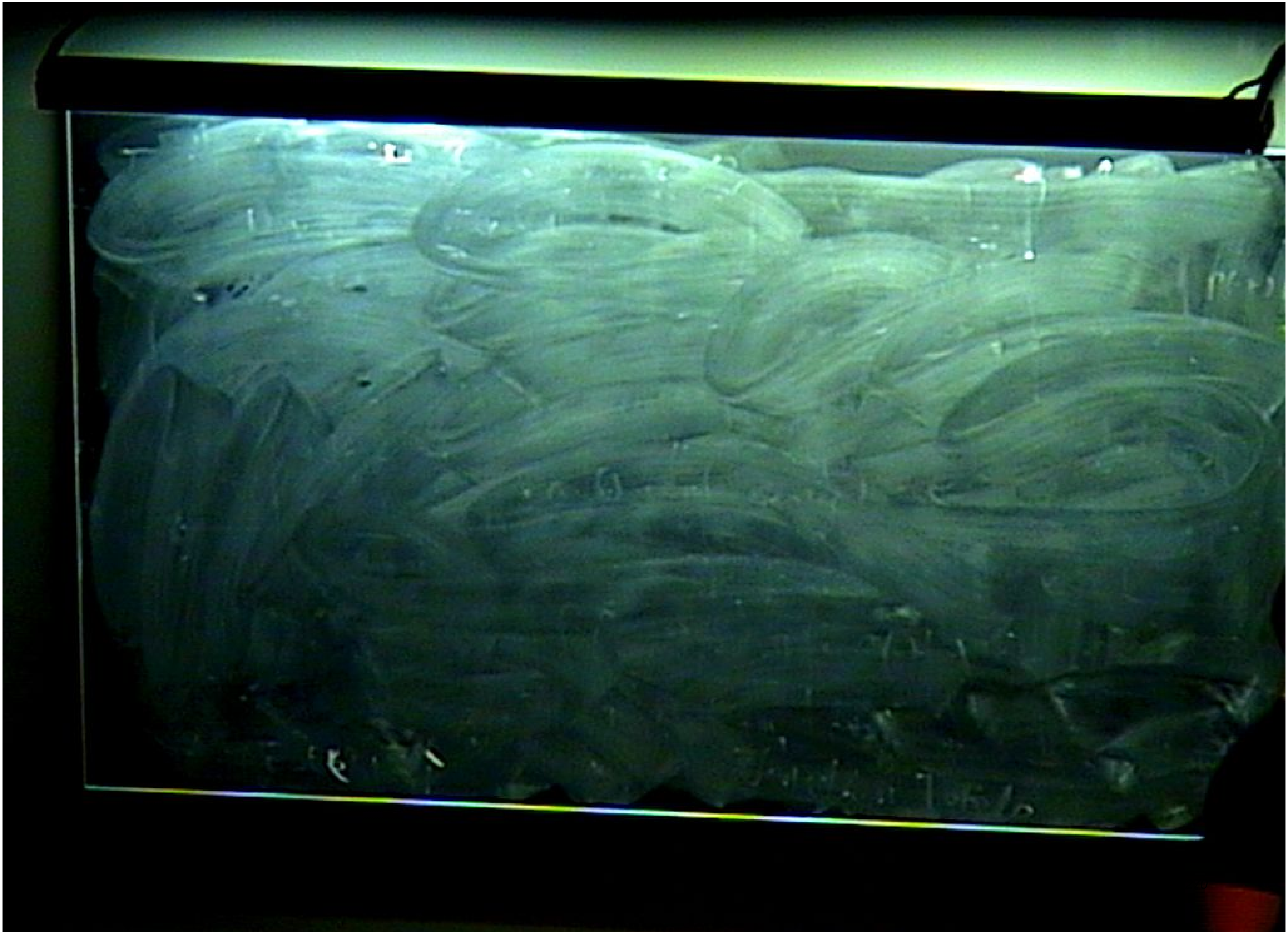
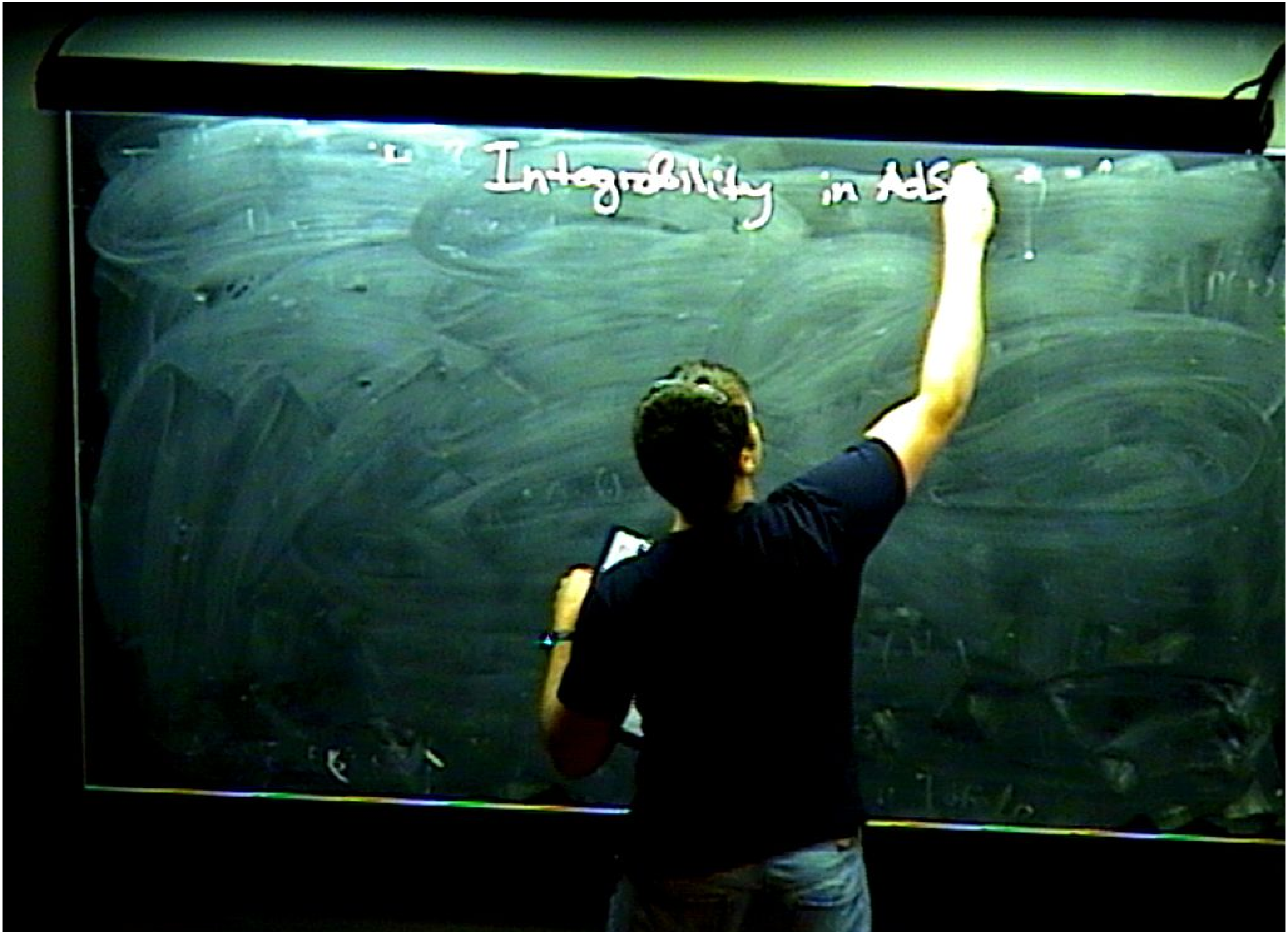
Title: Correlation Functions

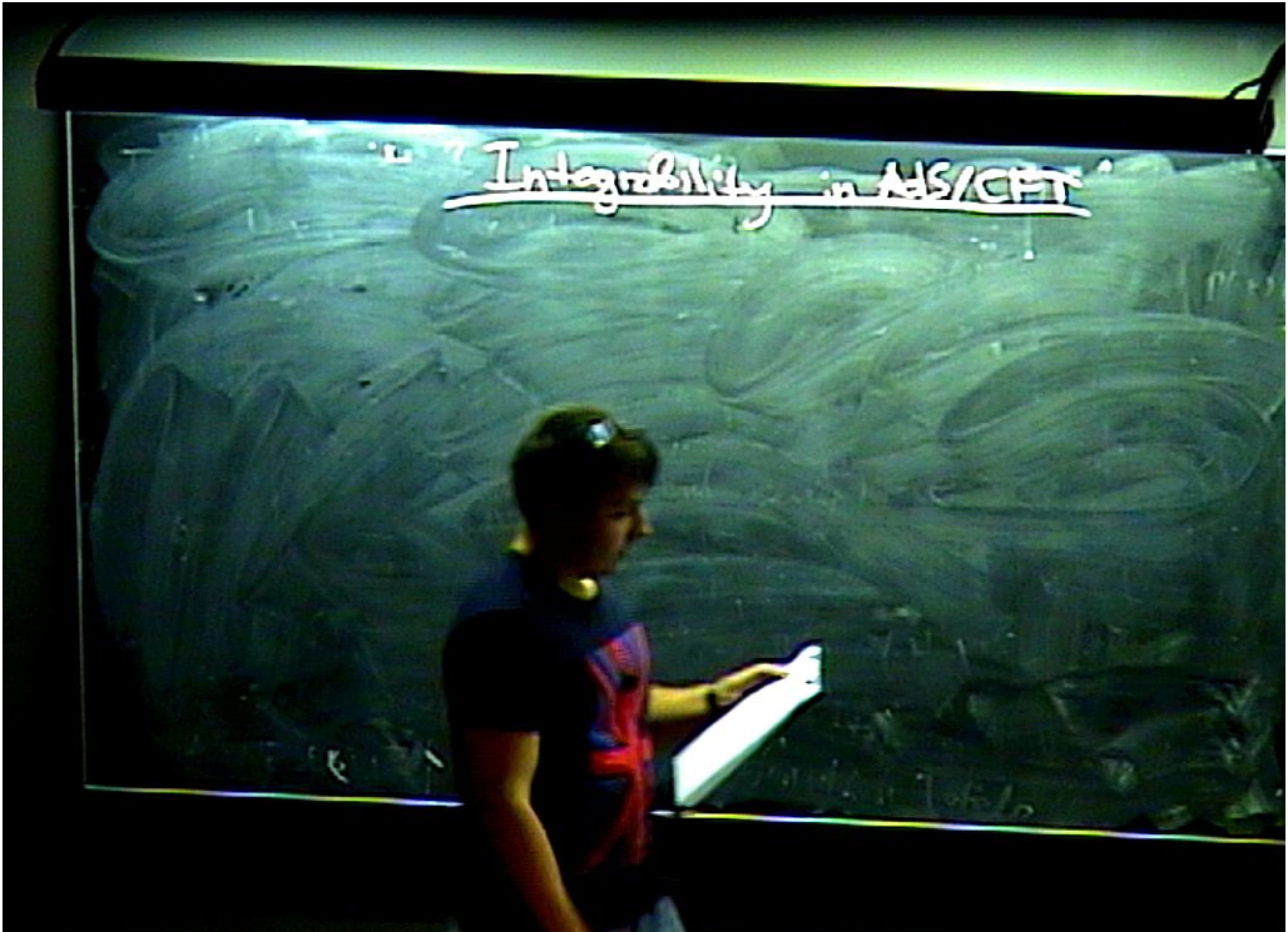
Date: Aug 01, 2011 12:15 PM

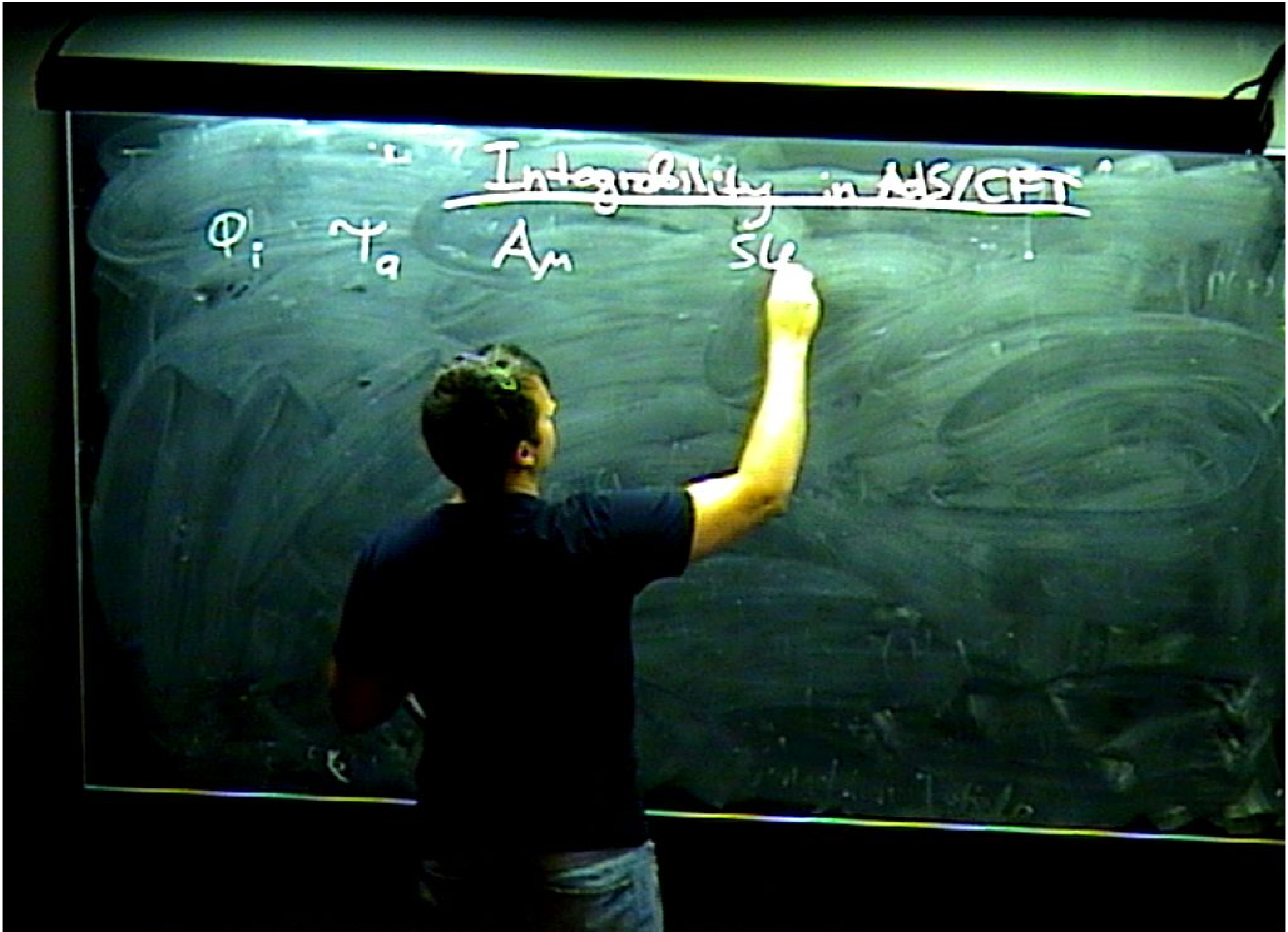
URL: http://pirsa.org/11080005

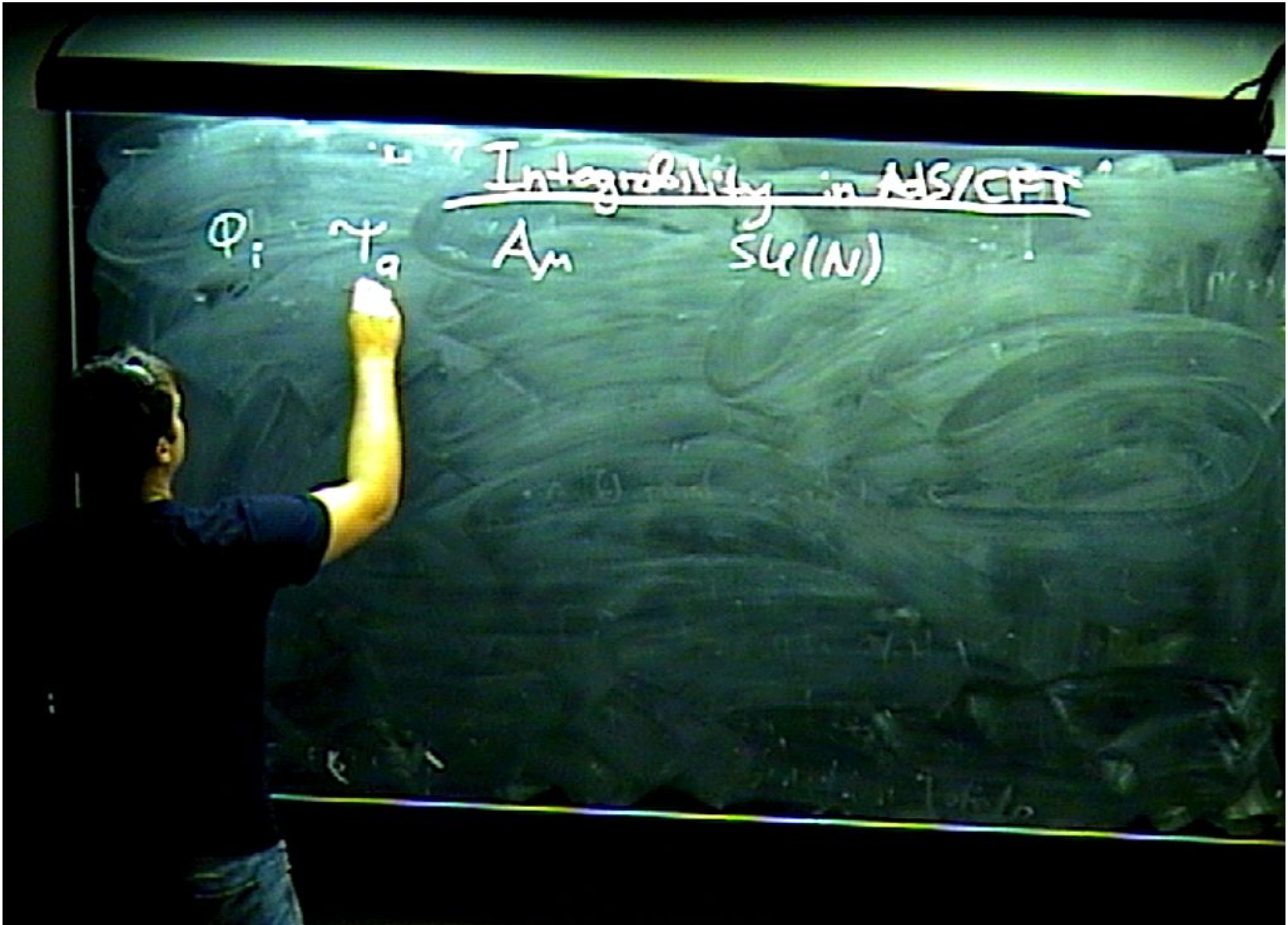
Abstract:

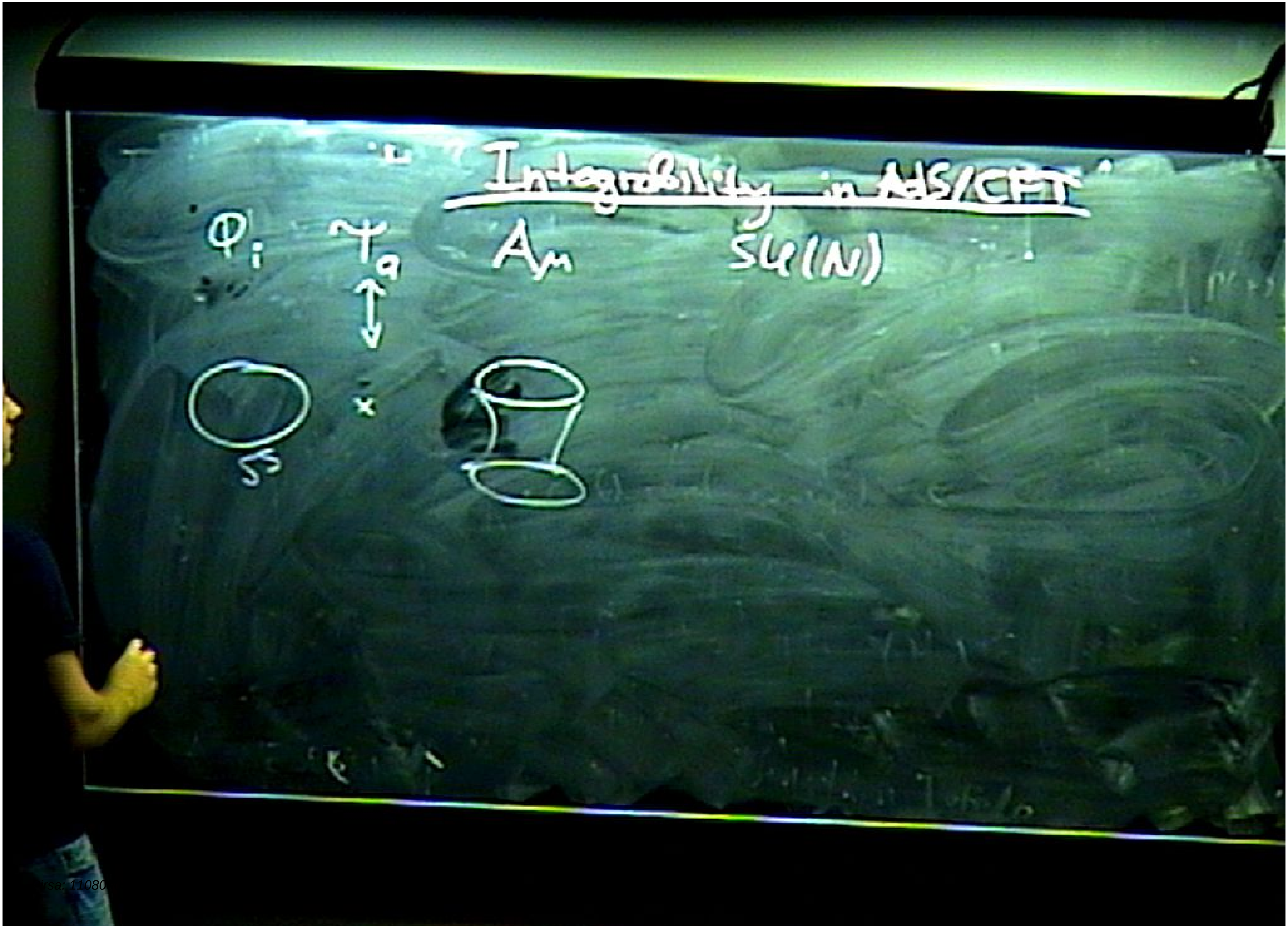


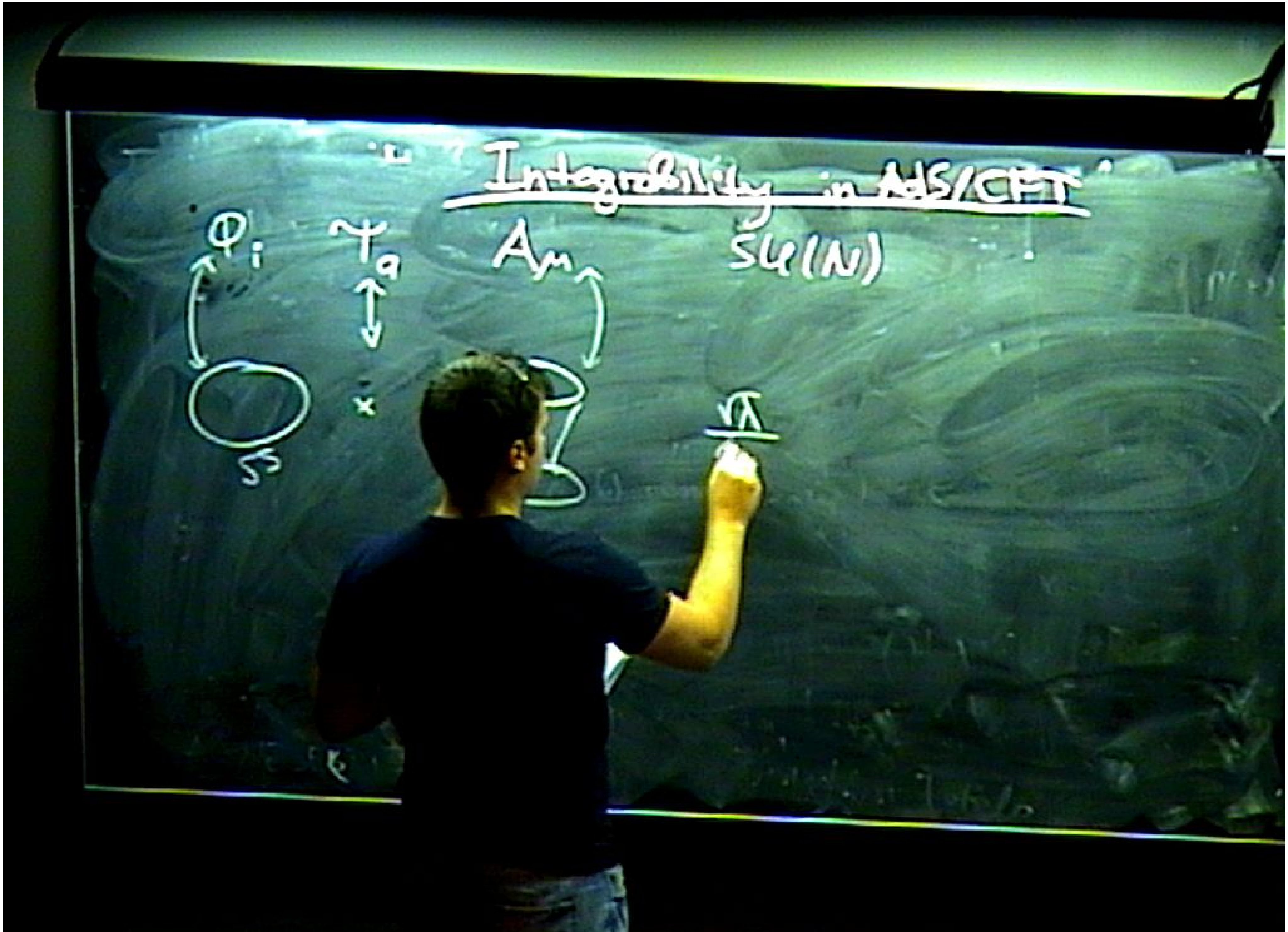

Integrability in AdS

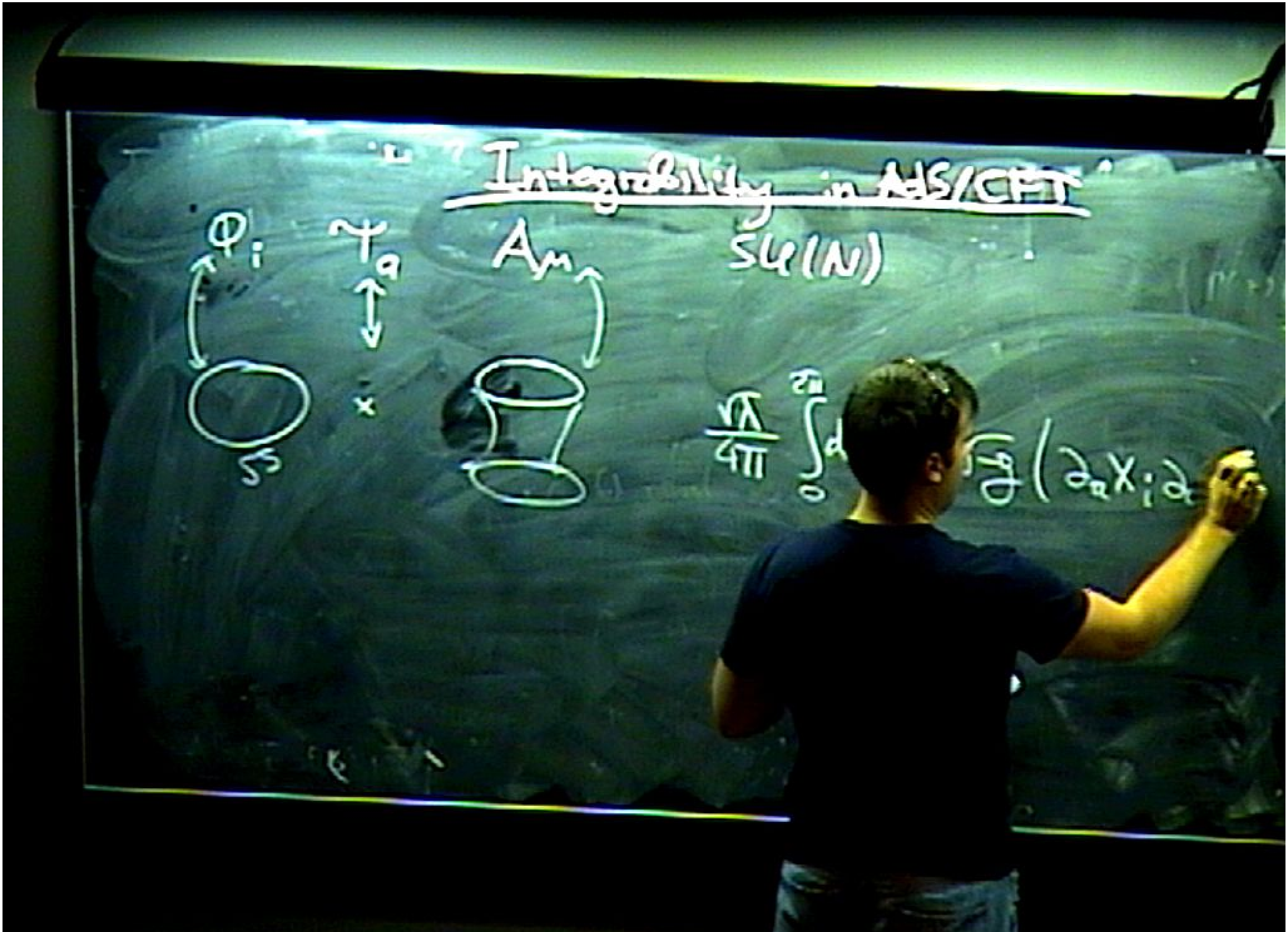

"Integrability in AdS/CFT"

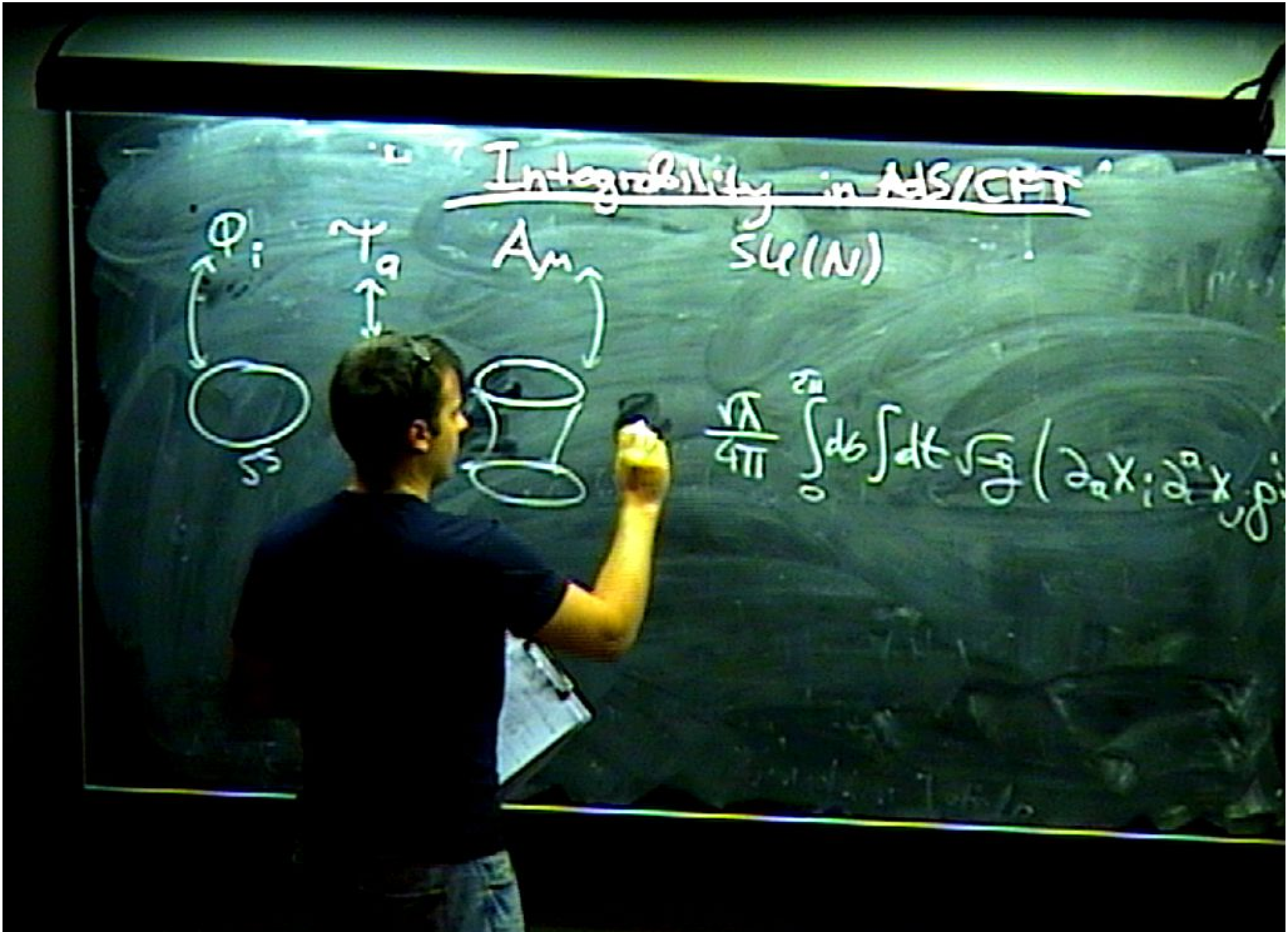

"Integrability in AdS/CFT"
Φ_i Ψ_a A_μ SU


"Integrability in AdS/CFT"
Φ_i Ψ_a A_μ SU(N)

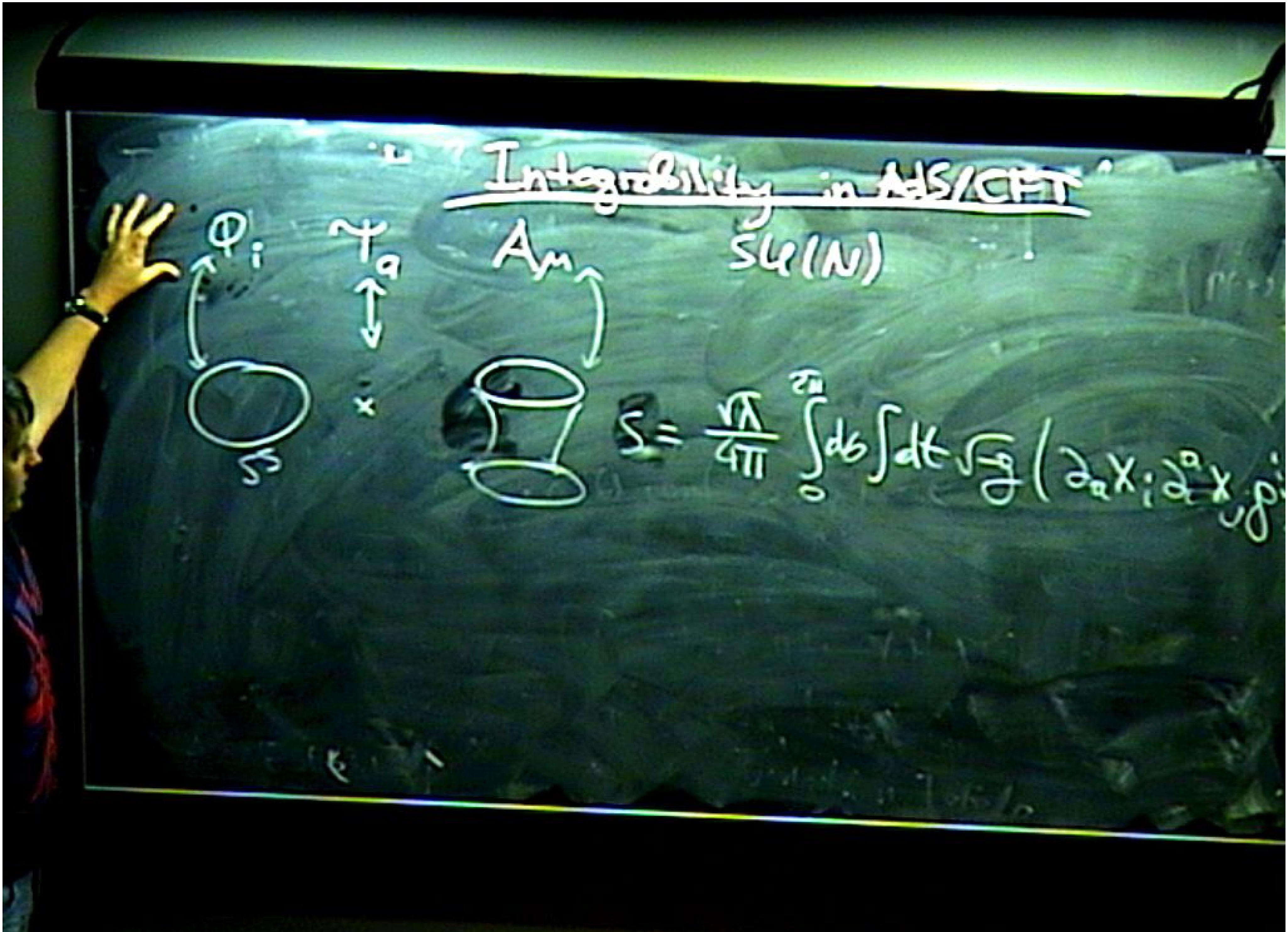

Integrability in AdS/CFT
Φi
Ψa
Aμ
SU(N)
S5


Integrability in AdS/CFT
SU(N)
S5

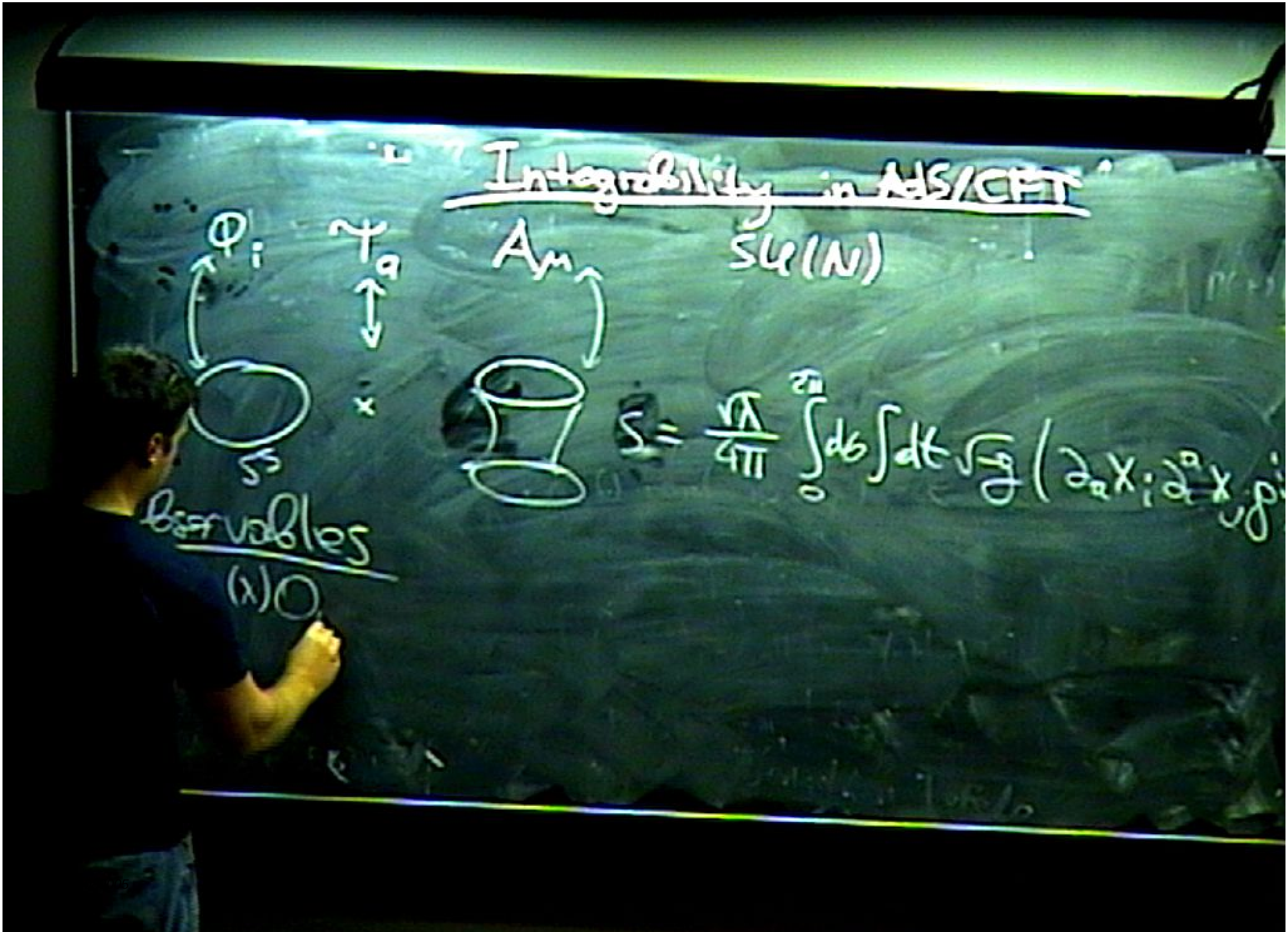

"Integrability in AdS/CFT"
Φ_i
Ψ_a
A_μ
SU(N)
S^5
x

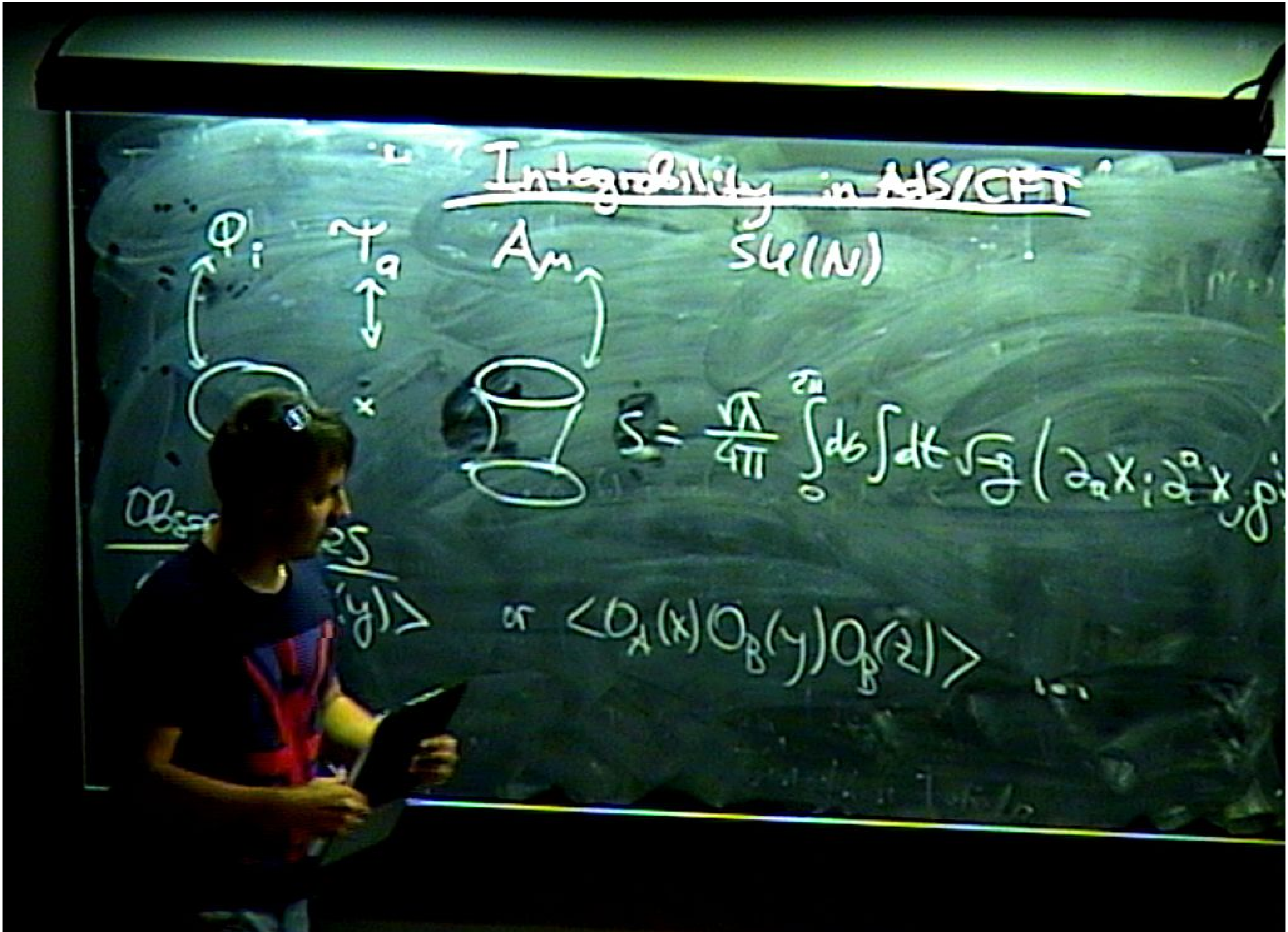


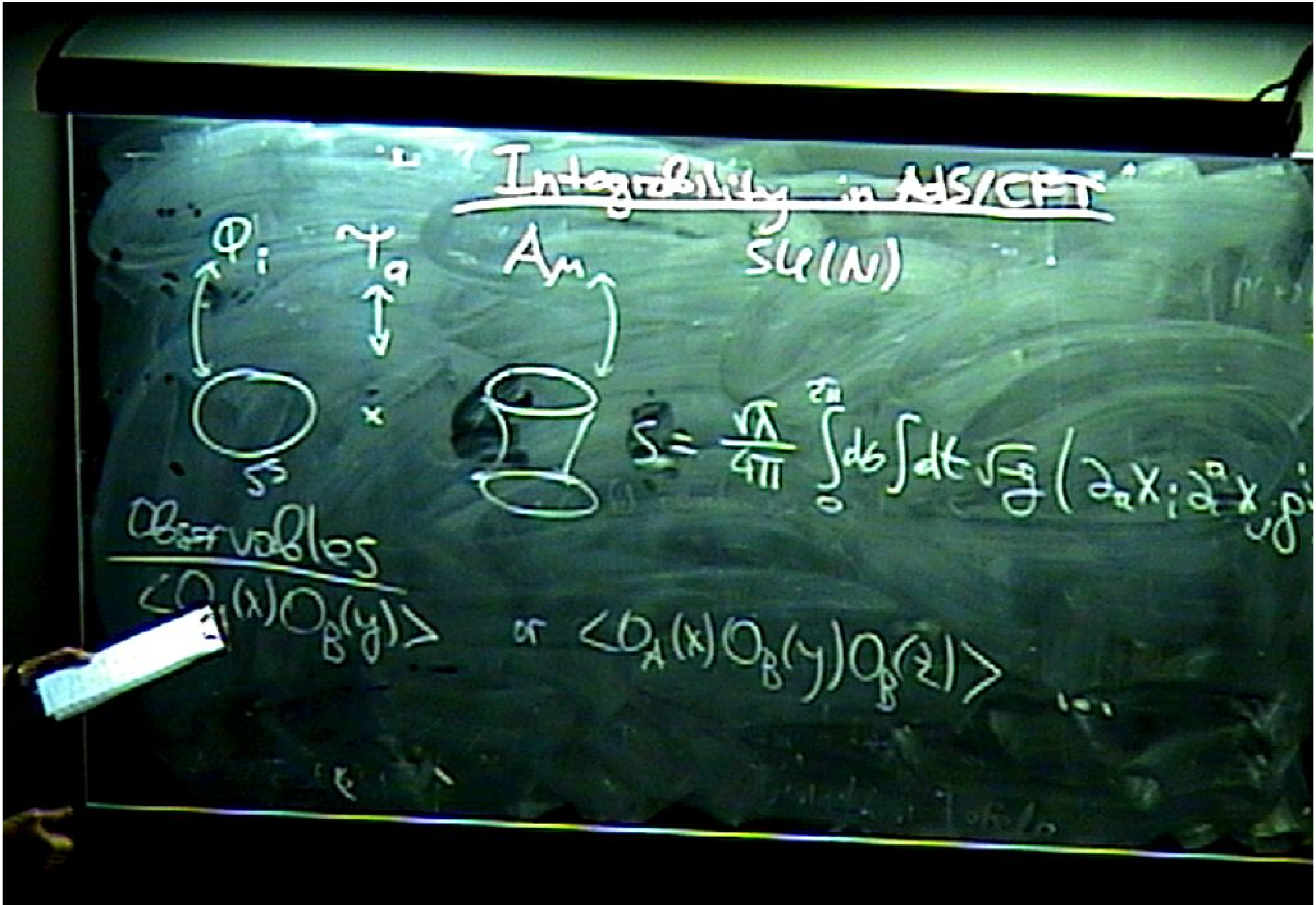
"Integrability in AdS/CFT"
SU(N)

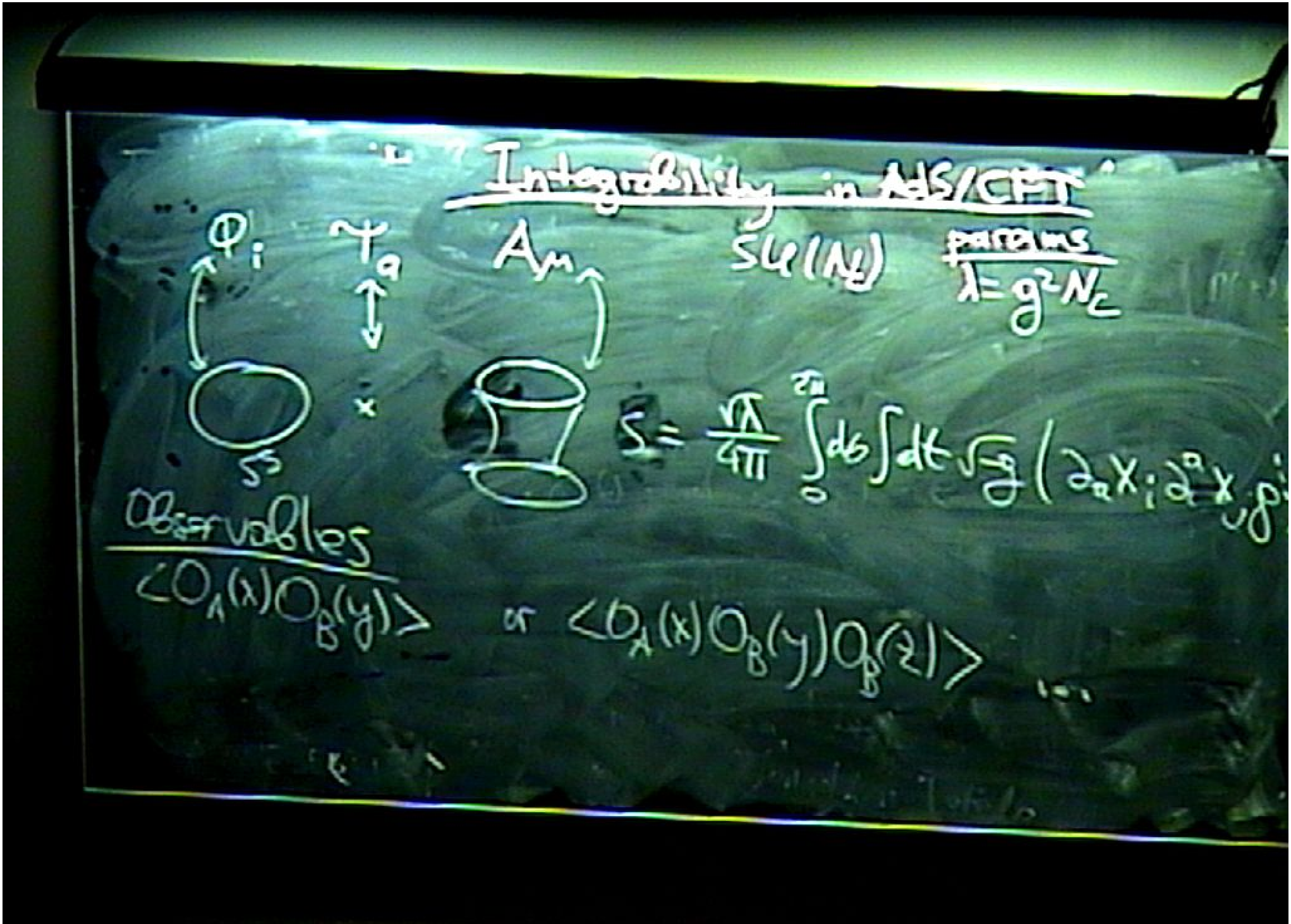


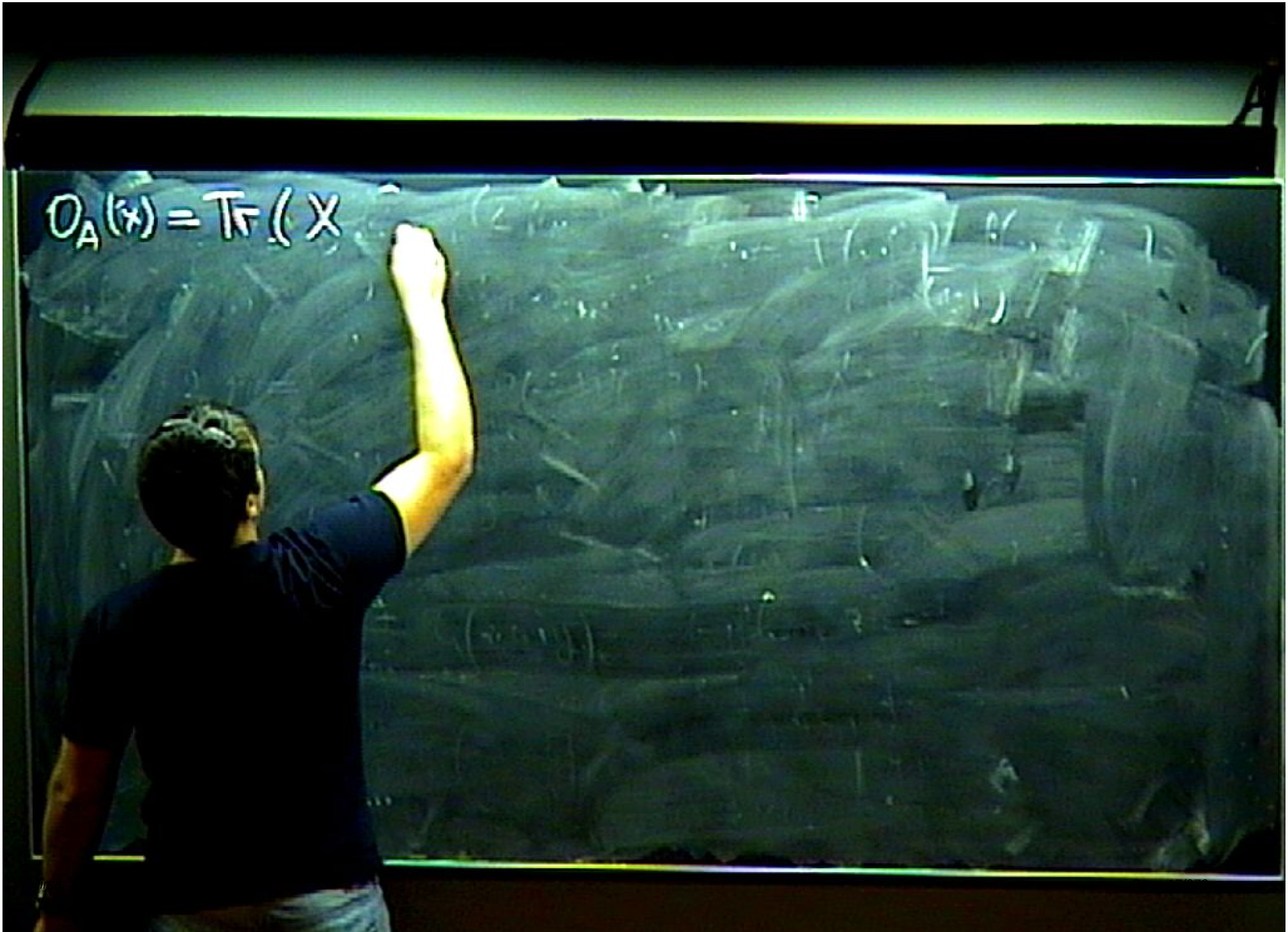
"Integrability in AdS/CFT"
SU(N)
Φ_i
Ψ_a
A_μ
S^5
$S = \frac{\sqrt{\lambda}}{4\pi} \int_0^{2\pi} d\sigma \int dt \sqrt{-g} \left(\partial_a X_i \partial^a X_j g^{ij}\right.$

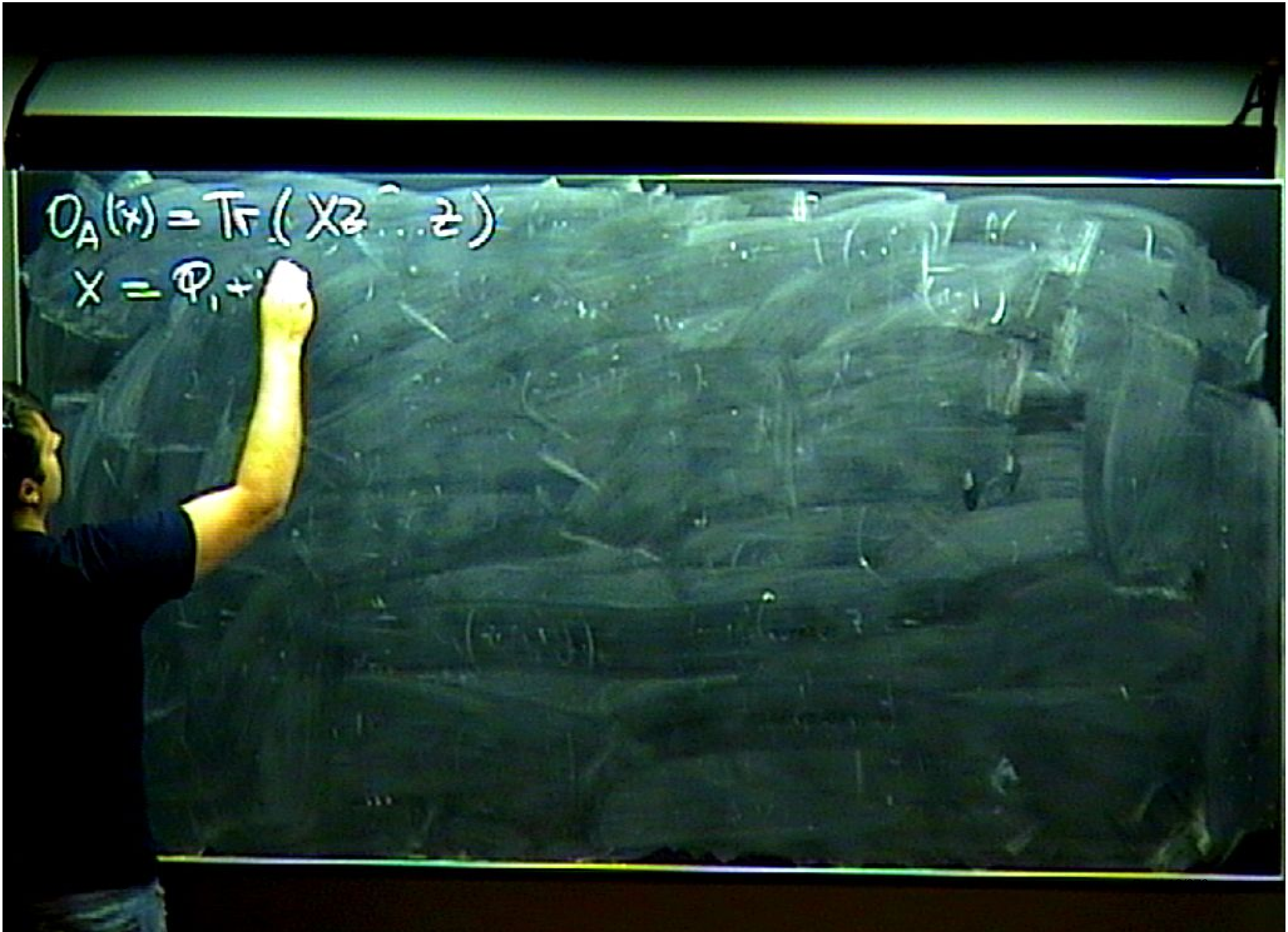

"Integrability in AdS/CFT"
SU(N)
S5


"Integrability in AdS/CFT"
SU(N)
Observables
or <O_A(x) O_B(y) O_B(z)> ...


"Integrability in AdS/CFT"
Φi Ψa Aμ
SU(N)
S5
x
S = √λ/4π ∫dσ ∫dt √-g (∂aXi ∂aXj gij
Observables
<OA(x)OB(y)> or <OA(x)OB(y)OB(z)> ...


Integrability in AdS/CFT
Φ_i Ψ_a A_μ
$SU(N_c)$
params
$\lambda = g^2 N_c$
S^5
$S = \frac{\sqrt{\lambda}}{4\pi} \int_0^{2\pi} d\sigma \int dt \sqrt{-g} (\partial_a X_i \partial^a X_j g^{ij}$
Observables
$\langle O_A(x) O_B(y) \rangle$ or $\langle O_A(x) O_B(y) O_B(z) \rangle$ ...


O_A(x) = Tr(X


$O_A(x) = \mathrm{Tr}(XZ \ldots Z)$
$X = \Phi_1 +$

$$O_A(x) = \mathrm{Tr}(XZ \ldots Z)$$

$$X = \varphi_1 + i\varphi_2$$

$$Z = \varphi_3 + i\varphi_4$$

$$O_A(x) = \mathrm{Tr}(X Z \ldots Z)$$

$$X = \varphi_1 + i\varphi_2$$

$$Z = \varphi_3 + i\varphi_4$$

$$\langle O_A(x)\, O_B(z) \rangle$$

$$O_A(x) = \mathrm{Tr}(XZ \ldots Z)$$

$$X = \varphi_1 + i\varphi_2$$

$$Z = \varphi_3 + i\varphi_4$$

$$\langle O_A(x)\, \overline{O}_B(z) \rangle_{\text{free}}$$

$$\langle X(x)\, \overline{X}(y) \rangle = \frac{1}{(x-y)^2}$$

$$O_A(x) = \mathrm{Tr}(XZ \ldots Z)$$

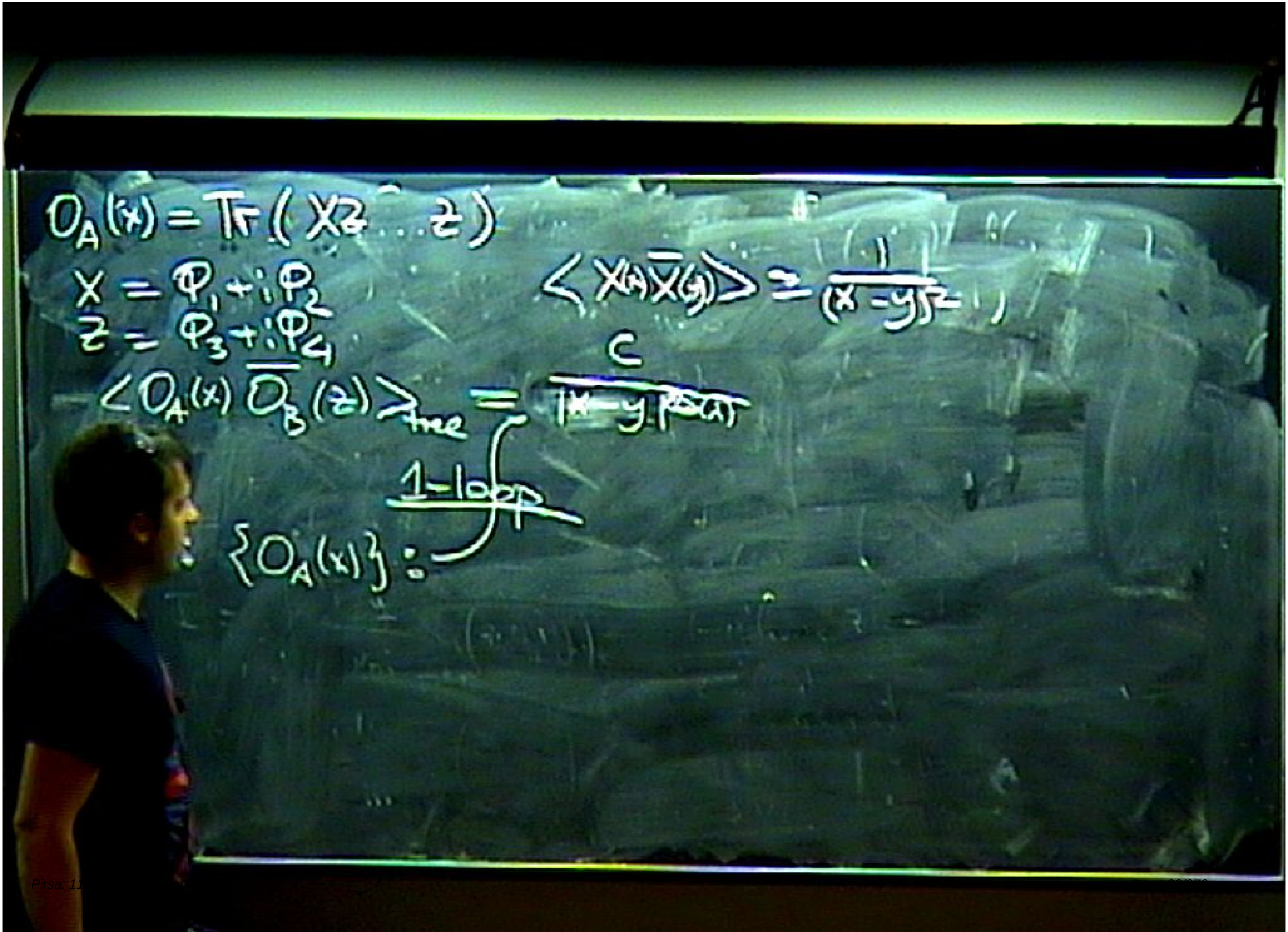
$$X = \varphi_1 + i\varphi_2$$

$$Z = \varphi_3 + i\varphi_4$$

$$\langle X(x)\bar{X}(y)\rangle = \frac{1}{(x-y)^2}$$

$$\langle O_A(x)\,\overline{O}_B(z)\rangle_{free}$$

1-loop



O_A(x) = Tr(XZ...Z)
X = φ1 + iφ2
Z = φ3 + iφ4
<X(x) X̄(y)> = 1/(x-y)^2
<O_A(x) Ō_B(z)>free = C/|x-y|^2Δ(A)
1-loop
{O_A(x)}:

$$O_A(x) = \mathrm{Tr}(XZ \ldots Z)$$

$$X = \varphi_1 + i\varphi_2$$

$$Z = \varphi_3 + i\varphi_4$$

$$\langle X(x)\bar{X}(y)\rangle = \frac{1}{(x-y)^2}$$

$$\langle O_A(x)\,\bar{O}_B(z)\rangle_{free} = \frac{\delta_{AB}}{|x-y|^{2\Delta(A)}}$$

1-loop

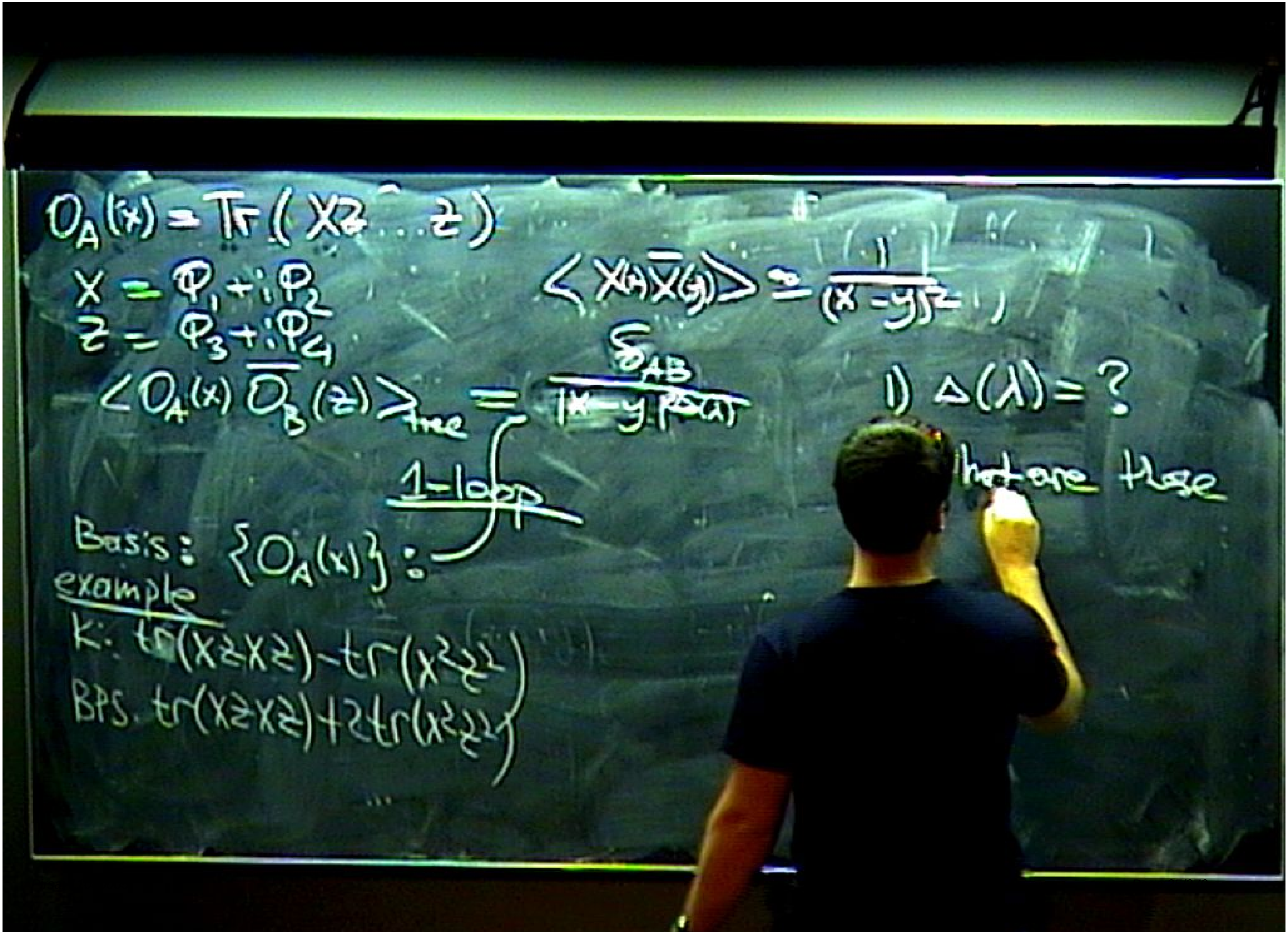
Basis: $\{O_A(x)\}$:

example

K: $\mathrm{tr}(XZXZ) - \mathrm{tr}(X^2Z^2)$

BPS: $\mathrm{tr}(XZXZ) + 2\,\mathrm{tr}(X^2Z^2)$

...


$O_A(x) = \mathrm{Tr}(XZ \ldots Z)$
$X = \varphi_1 + i\varphi_2$
$Z = \varphi_3 + i\varphi_4$
$\langle O_A(x)\, \overline{O}_B(z) \rangle_{free} = \frac{\delta_{AB}}{|x-y|^{\Delta(\lambda)}}$
1-loop
$\langle X(x) \overline{X}(y) \rangle = \frac{1}{(x-y)^2}$
1) $\Delta(\lambda) = ?$
hat are these
Basis: $\{O_A(x)\}$:
example
K: $tr(XZXZ) - tr(X^2Z^2)$
BPS. $tr(XZXZ) + 2tr(X^2Z^2)$
...

$O_A(x) = \mathrm{Tr}(XZ \ldots Z)$

$X = \varphi_1 + i\varphi_2$

$Z = \varphi_3 + i\varphi_4$

$\langle X(x)\bar{X}(y)\rangle = \frac{1}{(x-y)^2}$

$\langle O_A(x)\,\bar{O}_B(z)\rangle_{\text{free}} = \frac{\delta_{AB}}{|x-y|^{2\Delta(\lambda)}}$

1-loop

1) $\Delta(\lambda) = ?$

2) What are these coeff.

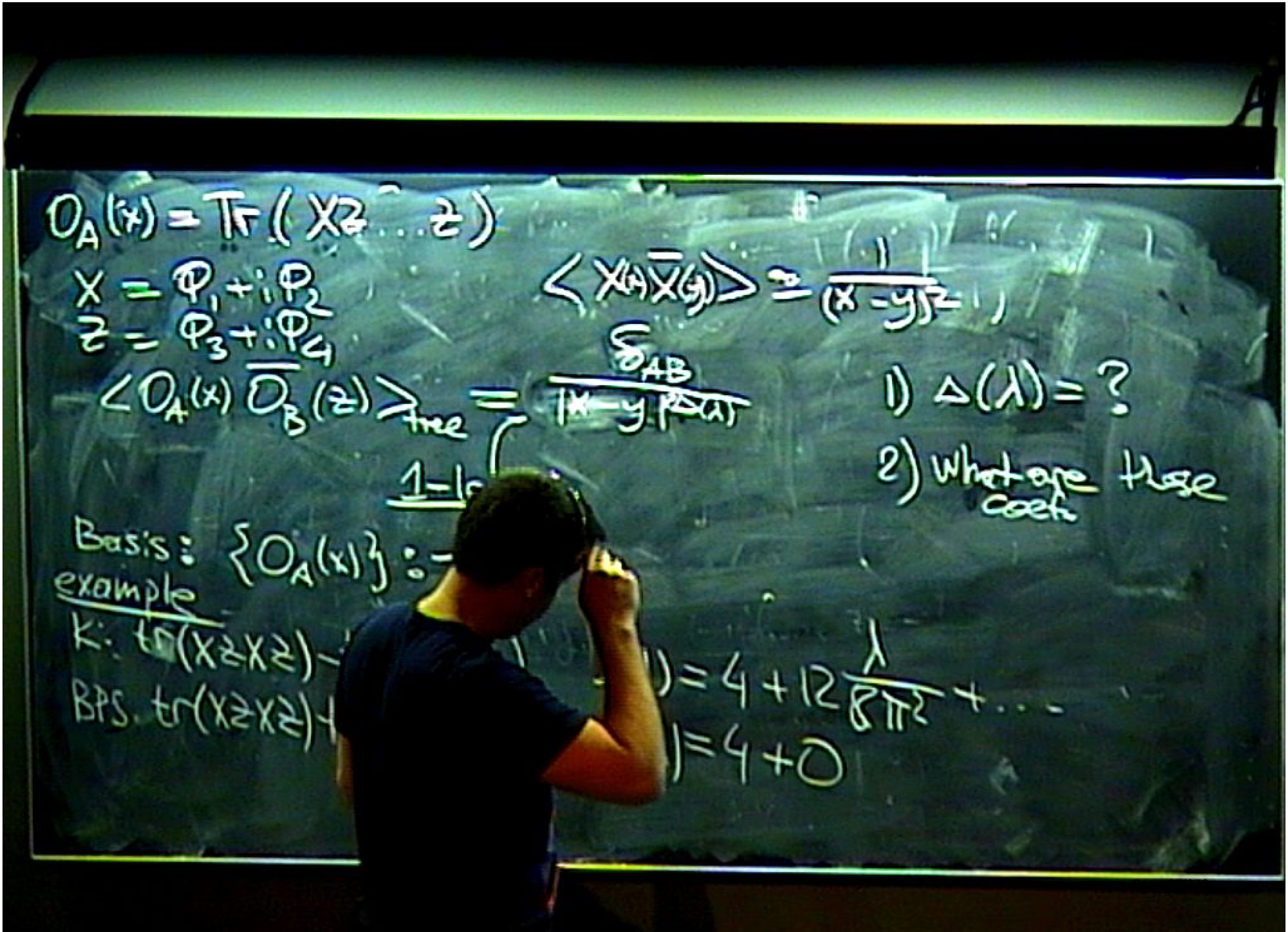
Basis: $\{O_A(x)\}$:

example

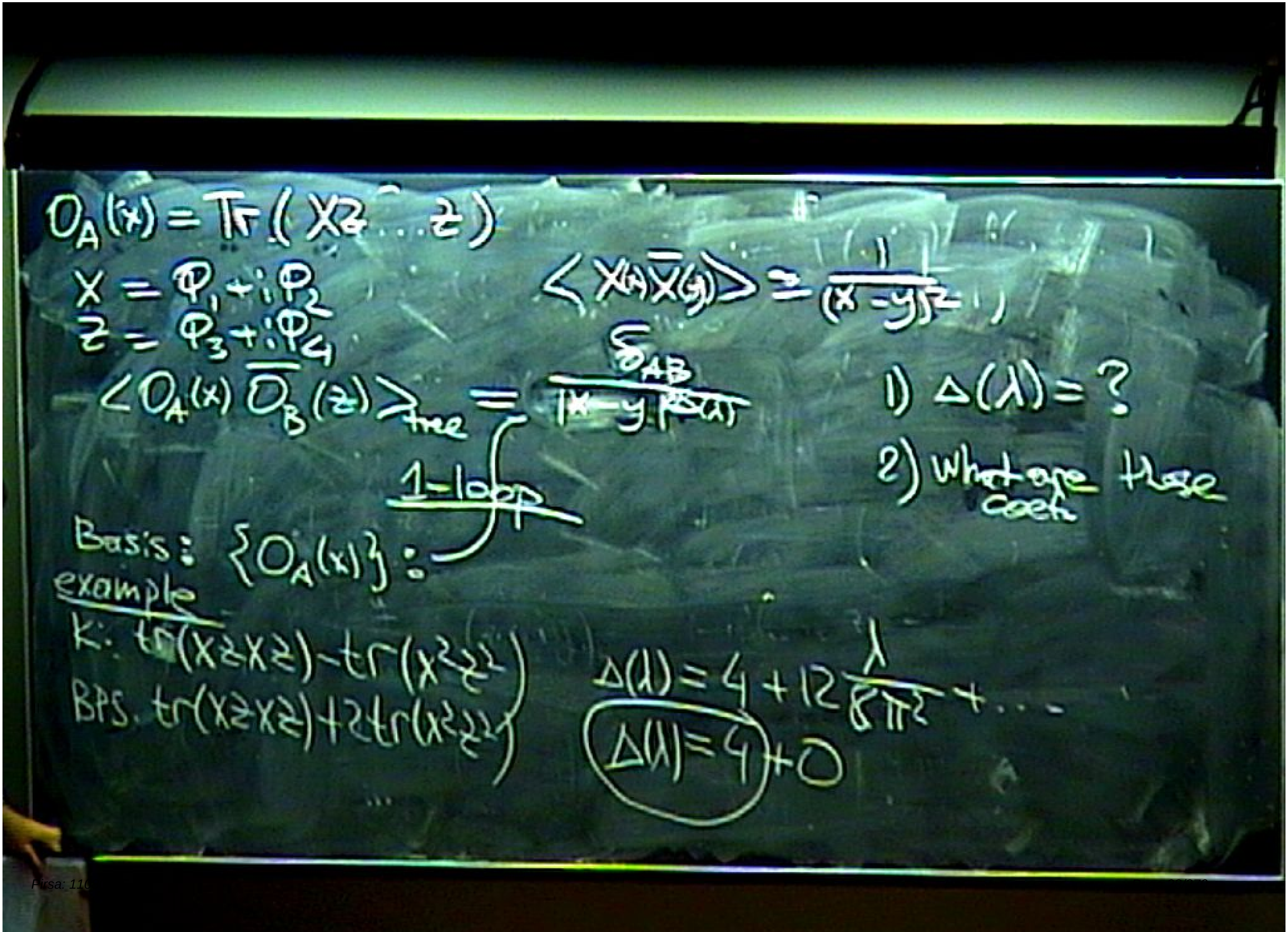
K: $\mathrm{tr}(XZXZ) - \mathrm{tr}(X^2Z^2)$ $\Delta(\lambda) =$

BPS: $\mathrm{tr}(XZXZ) + 2\,\mathrm{tr}(X^2Z^2)$ $\Delta(\lambda)$

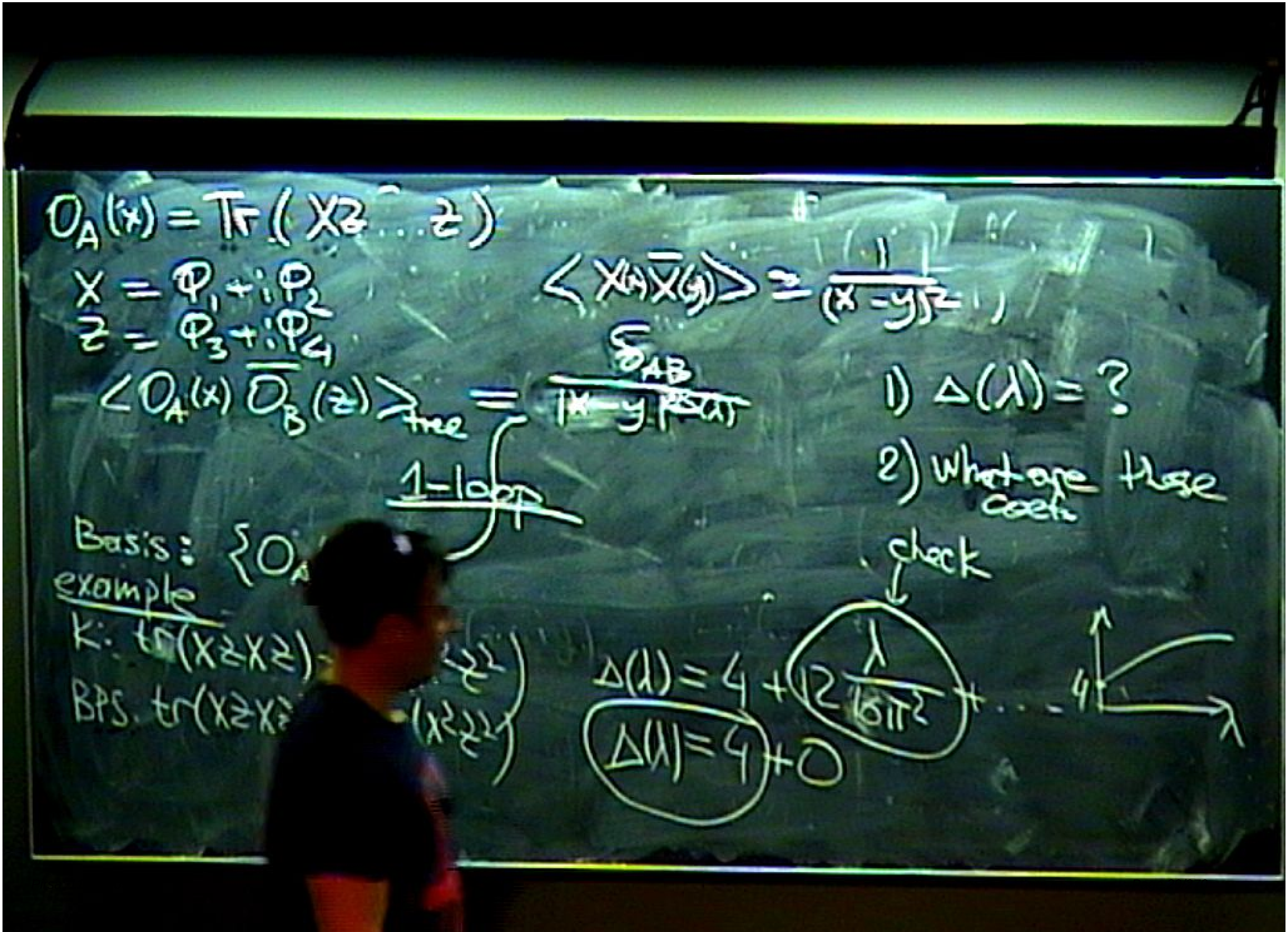
...

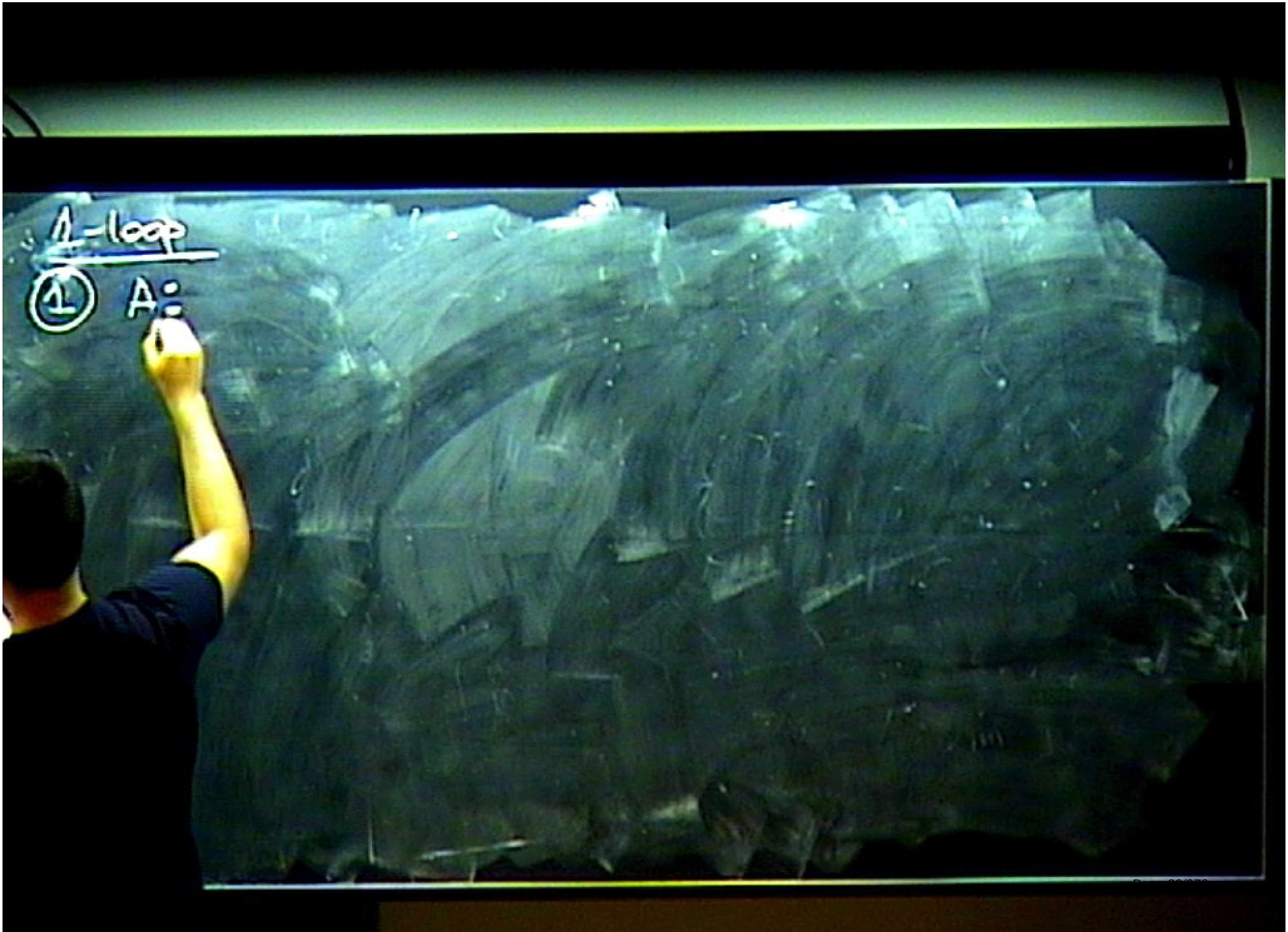


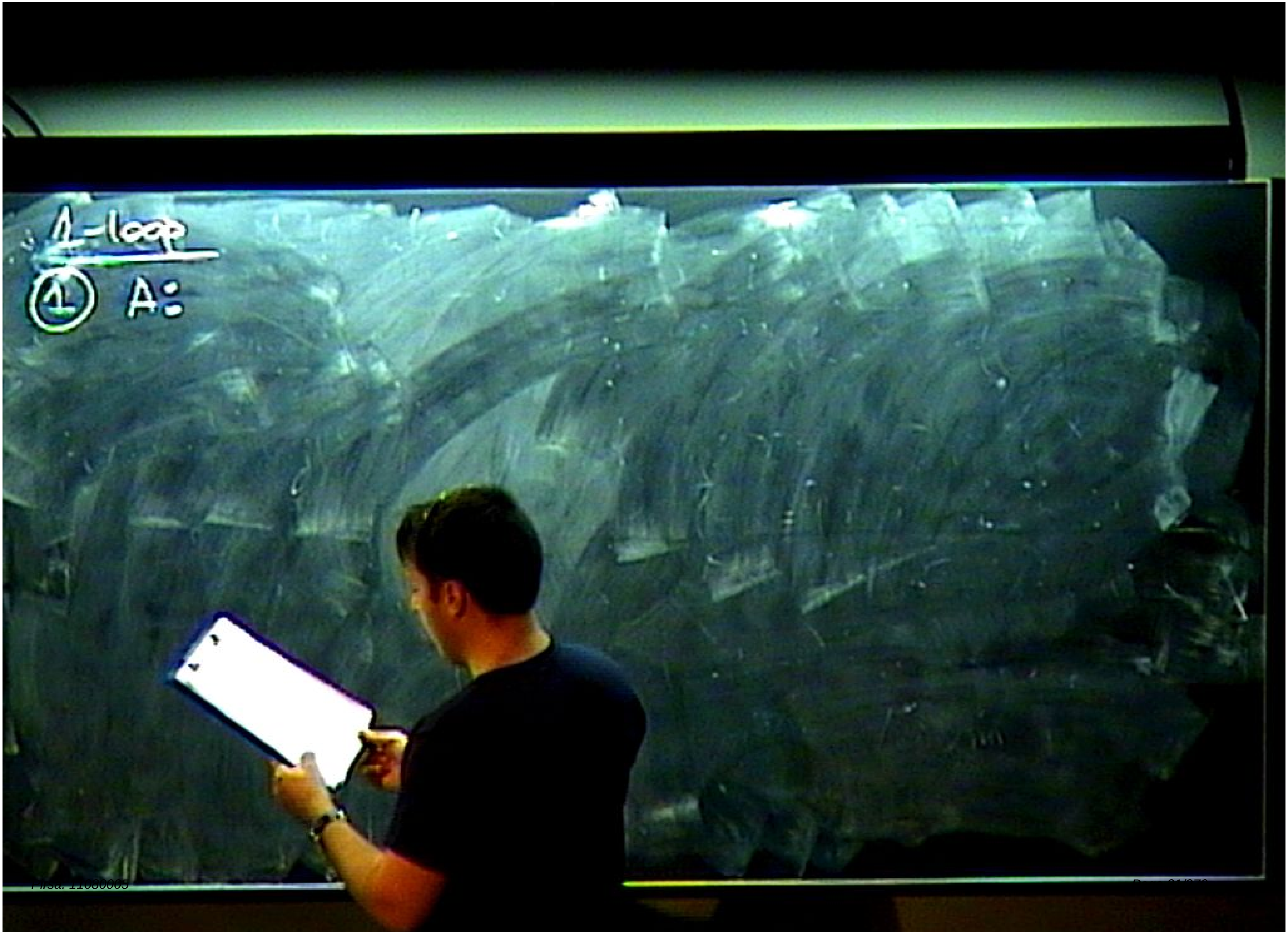
O_A(x) = Tr (XZ ... Z)
X = φ_1 + i φ_2
Z = φ_3 + i φ_4
< X(x) X̄(y) > = 1 / (x - y)^2
< O_A(x) Ō_B(z) >_free = δ_AB / |x - y|^{2Δ(λ)}
1) Δ(λ) = ?
2) What are these coeffs
Basis: { O_A(x) } :
example
K: tr(XZXZ)
BPS. tr(XZXZ)
= 4 + 12 λ/8π^2 + ...
= 4 + 0

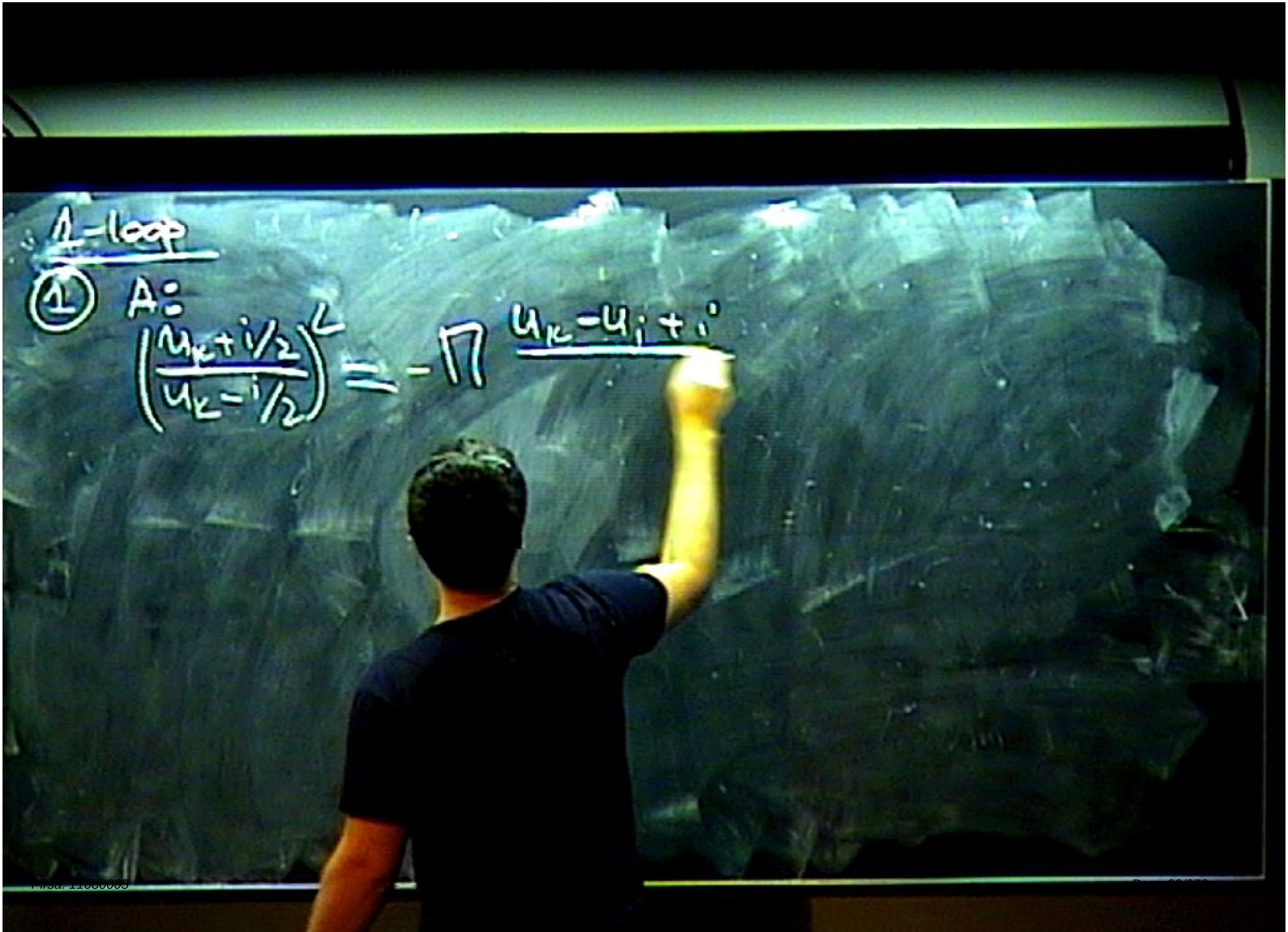


O_A(x) = Tr (XZ ... Z)
X = φ1 + i φ2
Z = φ3 + i φ4
<X(x) X̄(y)> = 1 / (x − y)^2
<O_A(x) Ō_B(z)>_free = δ_AB / |x − y|^{2Δ(λ)}
1-loop
1) Δ(λ) = ?
2) What are these coeff.
Basis: {O_A(x)} :
example
K: tr(XZXZ) − tr(X^2 Z^2)
BPS. tr(XZXZ) + 2 tr(X^2 Z^2)
Δ(λ) = 4 + 12 λ / 8π^2 + ...
Δ(λ) = 4 + 0


O_A(x) = Tr(XZ ... Z)
X = φ1 + i φ2
Z = φ3 + i φ4
< X(x) X̄(y) > = 1 / (x − y)²
< O_A(x) Ō_B(z) >free = δAB / |x − y|^2Δ(λ)
1-loop
1) Δ(λ) = ?
2) What are these coefs
check
Basis: {O_A
example
K: tr(XZXZ)
BPS. tr(XZXZ
Δ(λ) = 4 + 12 λ/16π² + ...
Δ(λ) = 4 + 0
λ


1-loop
① A:

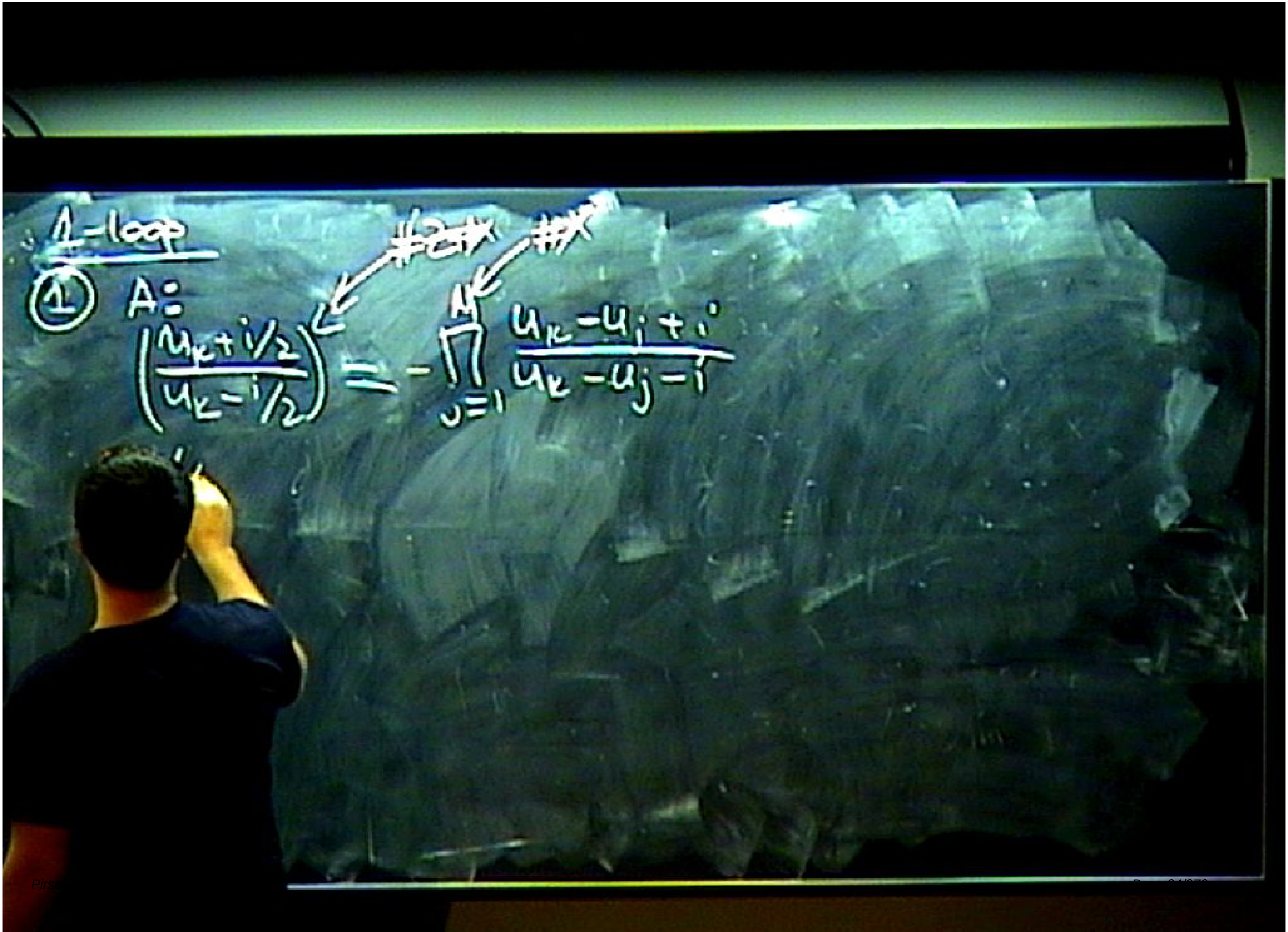

1-loop
① A:


1-loop
① A:
(uk + i/2 / uk − i/2)^L = − ∏ uk − uj + i

1-loop

① A:

$$\left(\frac{u_k + i/2}{u_k - i/2}\right)^L = -\prod_{j=1}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$$


1-loop
① A:
$\left(\frac{u_k + i/2}{u_k - i/2}\right) = - \prod_{j=1}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$

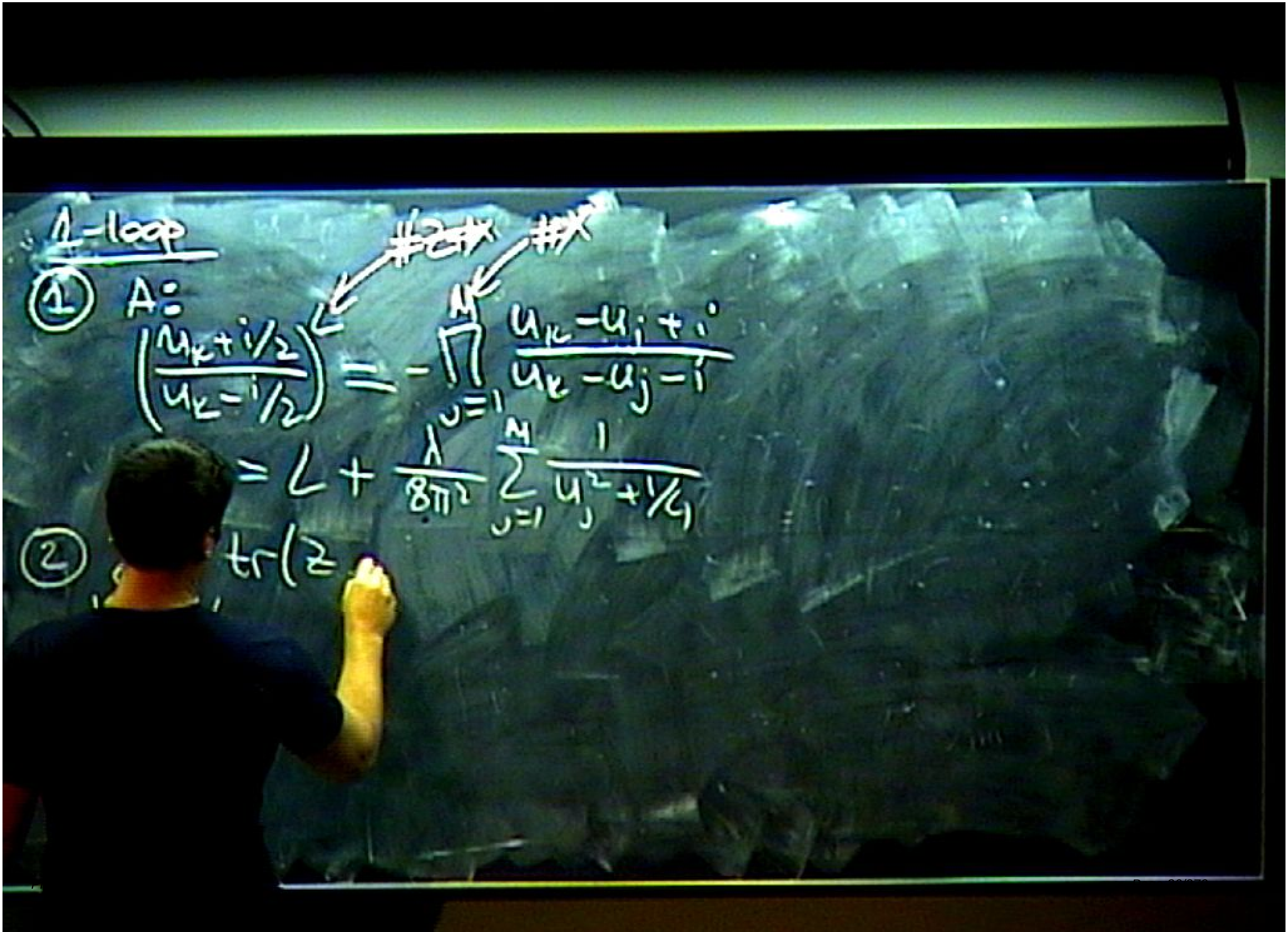
1-loop

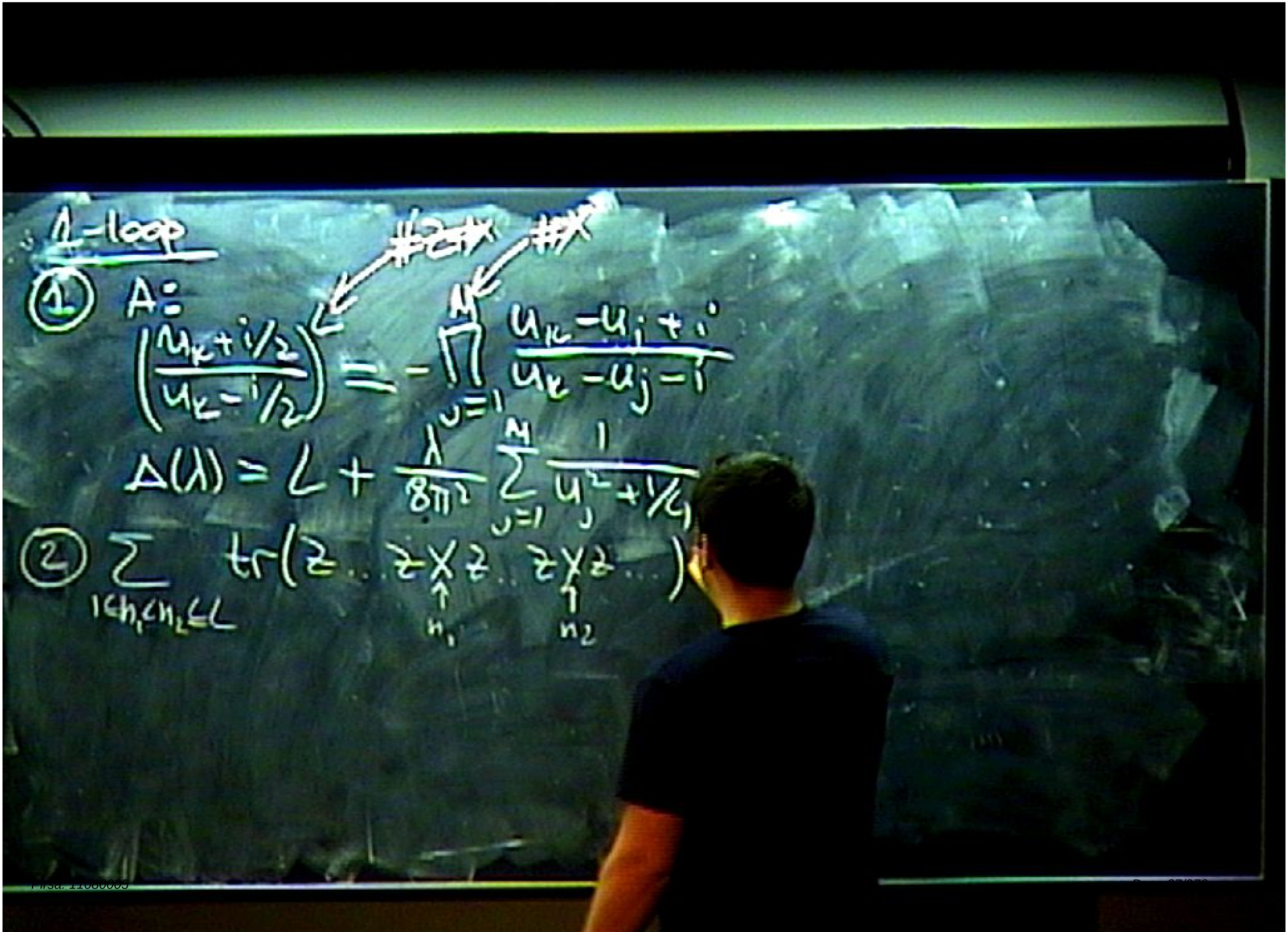
① A:

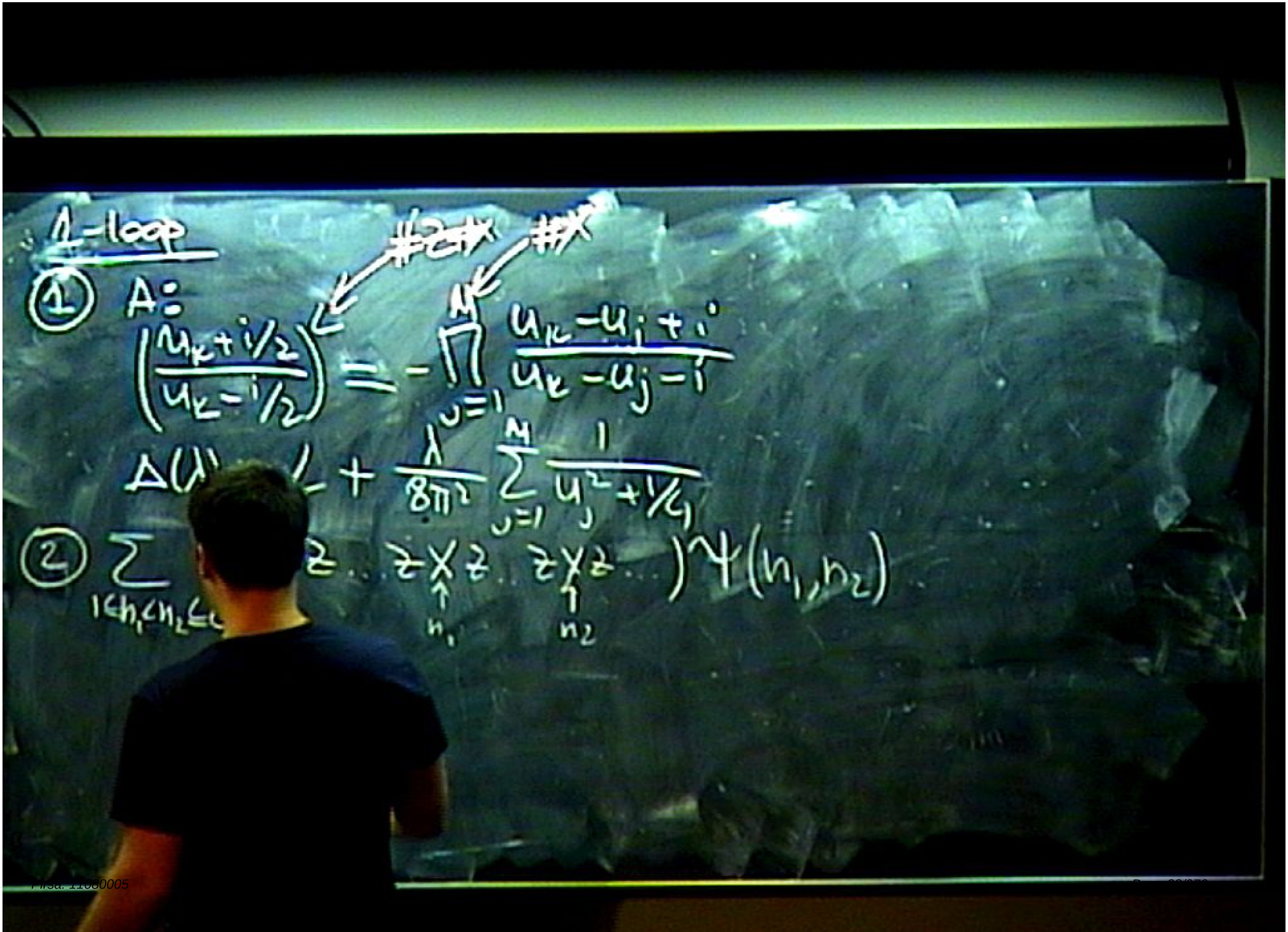
#2#X → exponent; #X → M

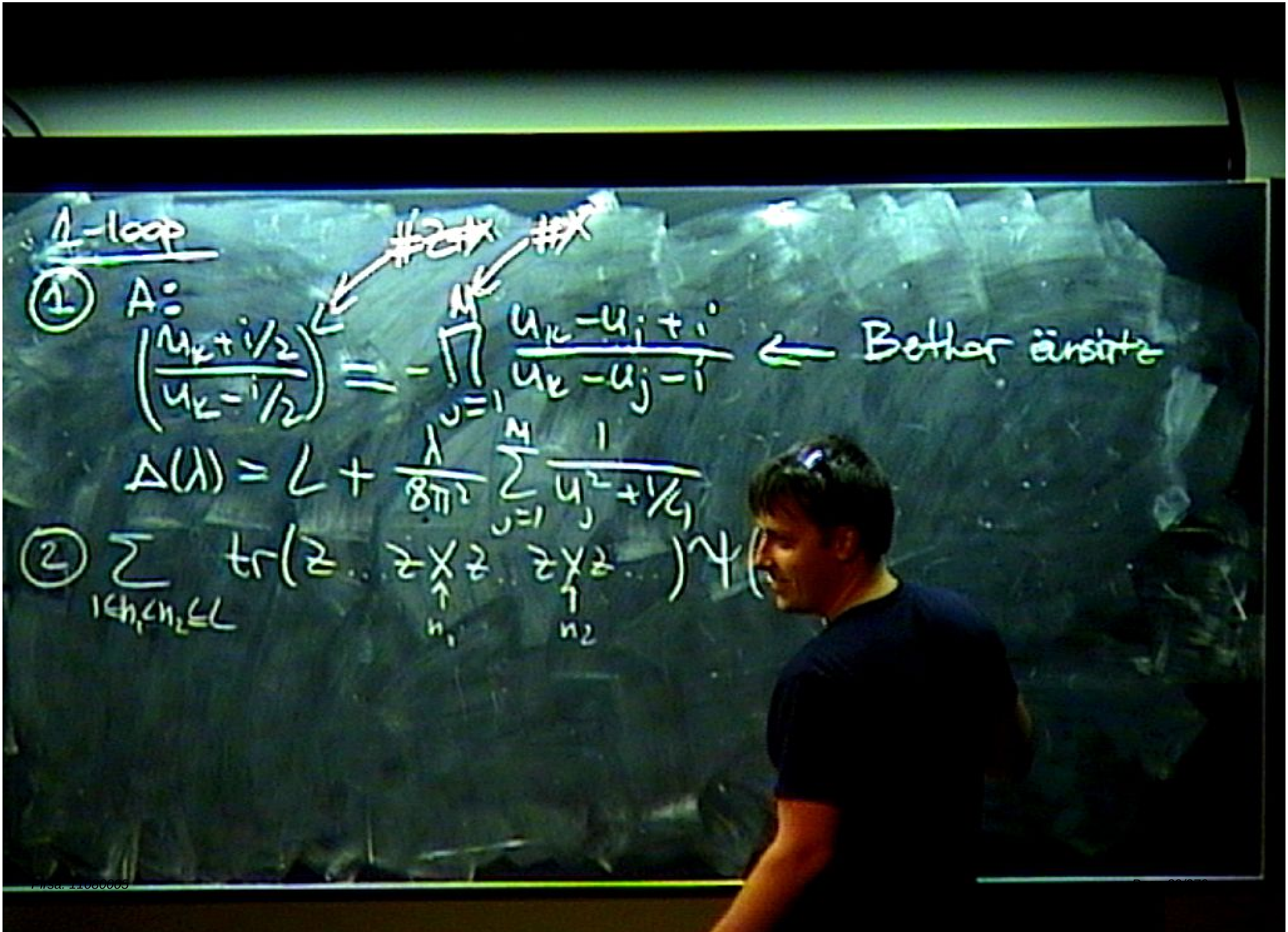
$$\left(\frac{u_k+i/2}{u_k-i/2}\right) = -\prod_{j=1}^{M}\frac{u_k-u_j+i}{u_k-u_j-i}$$

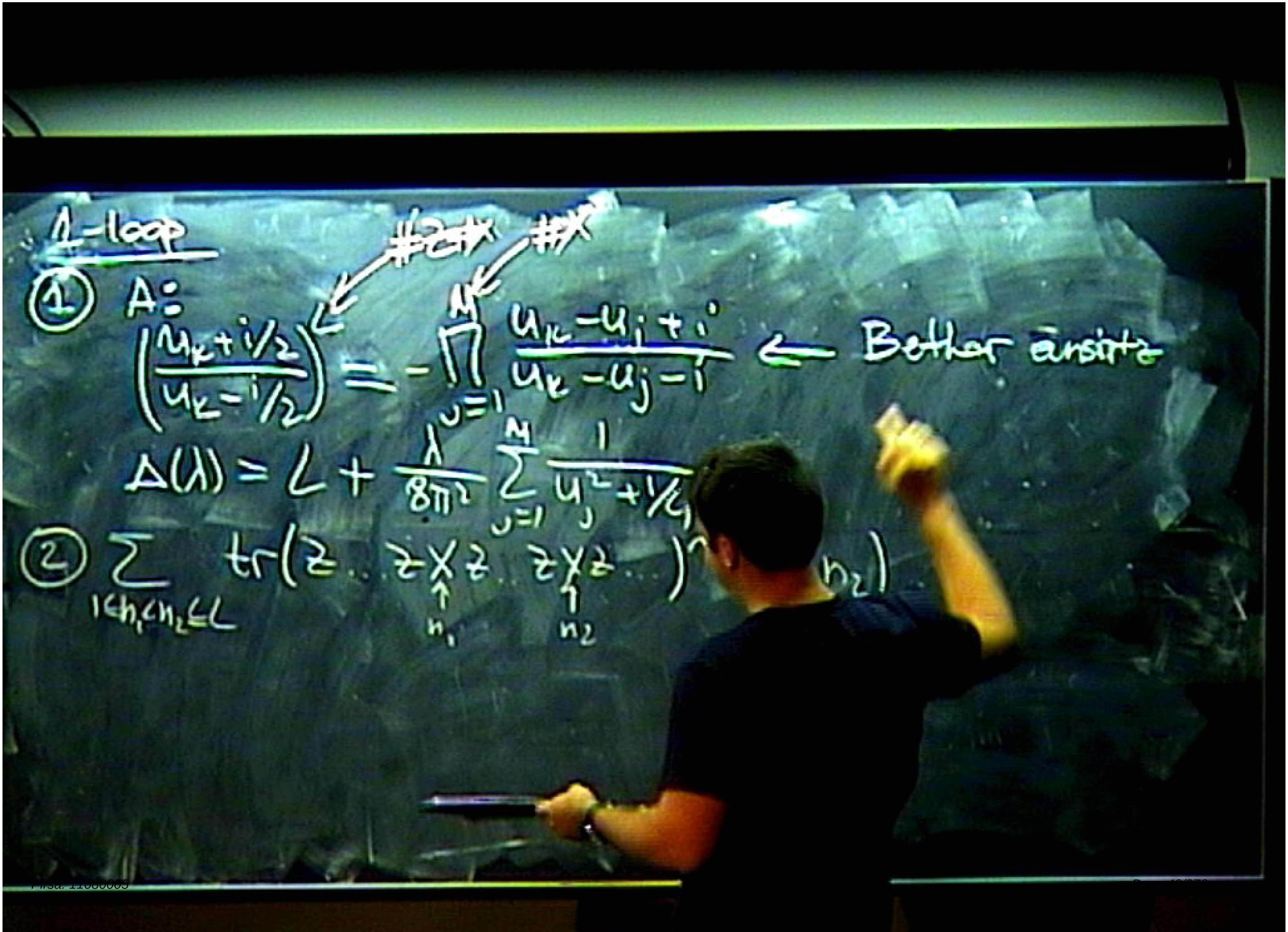
$$\Delta(\lambda) = L + \frac{\lambda}{8\pi^2}\sum_{j=1}^{M}\frac{1}{u_j^2+1/4}$$

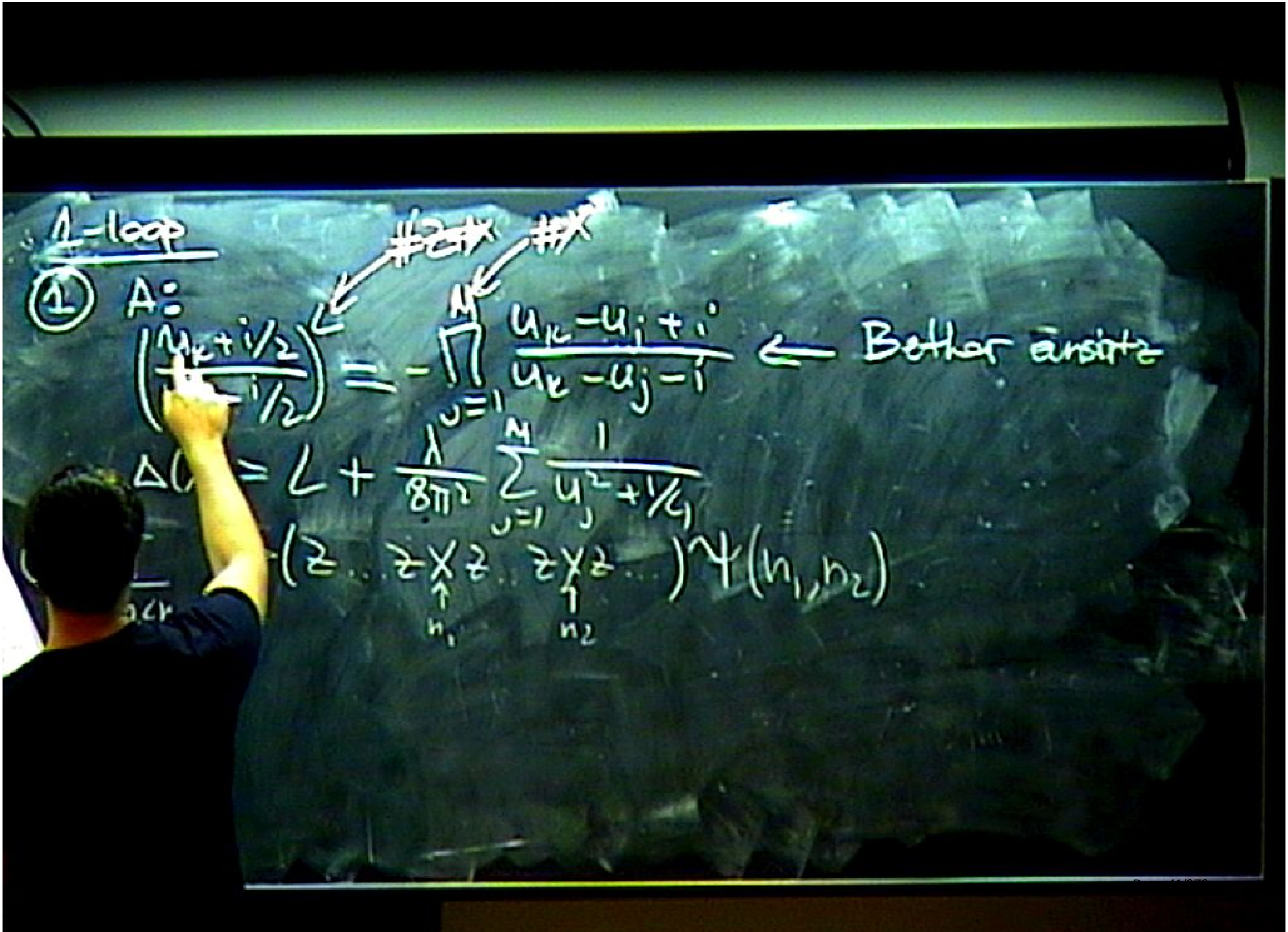

1-loop
#2#X
#X
① A:
$\left(\frac{u_k + i/2}{u_k - i/2}\right) = -\prod_{j=1}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$
$= L + \frac{\lambda}{8\pi^2} \sum_{j=1}^{M} \frac{1}{u_j^2 + 1/4}$
② tr(Z

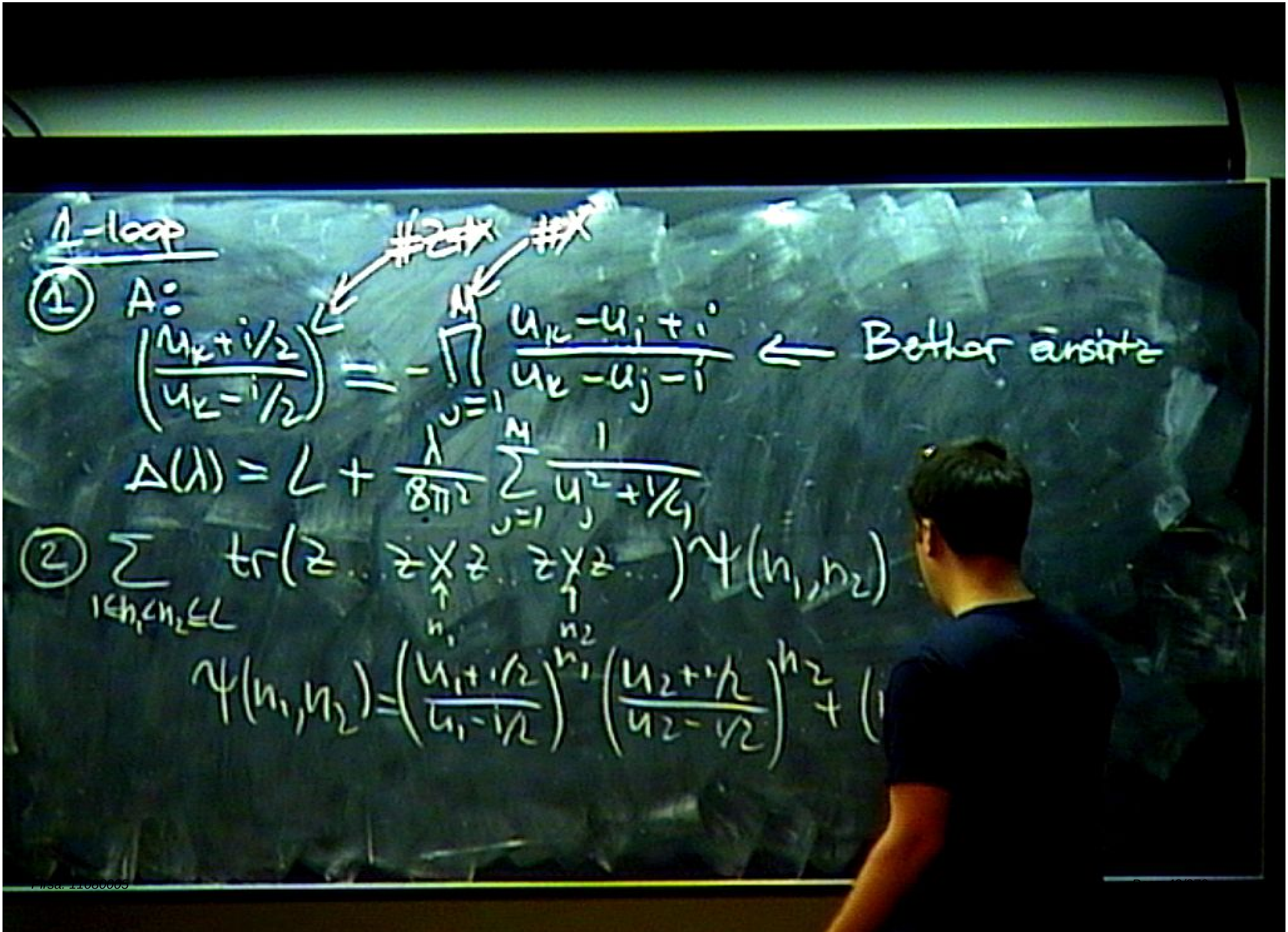

1-loop
#Z+#X
#X
① A:
$\left(\frac{u_k+i/2}{u_k-i/2}\right)^L = -\prod_{j=1}^{M} \frac{u_k-u_j+i}{u_k-u_j-i}$
$\Delta(\lambda) = L + \frac{\lambda}{8\pi^2}\sum_{j=1}^{M} \frac{1}{u_j^2+1/4}$
② $\sum_{1\le n_1<n_2\le L} tr(Z \ldots ZXZ \ldots ZXZ \ldots)$
n_1
n_2

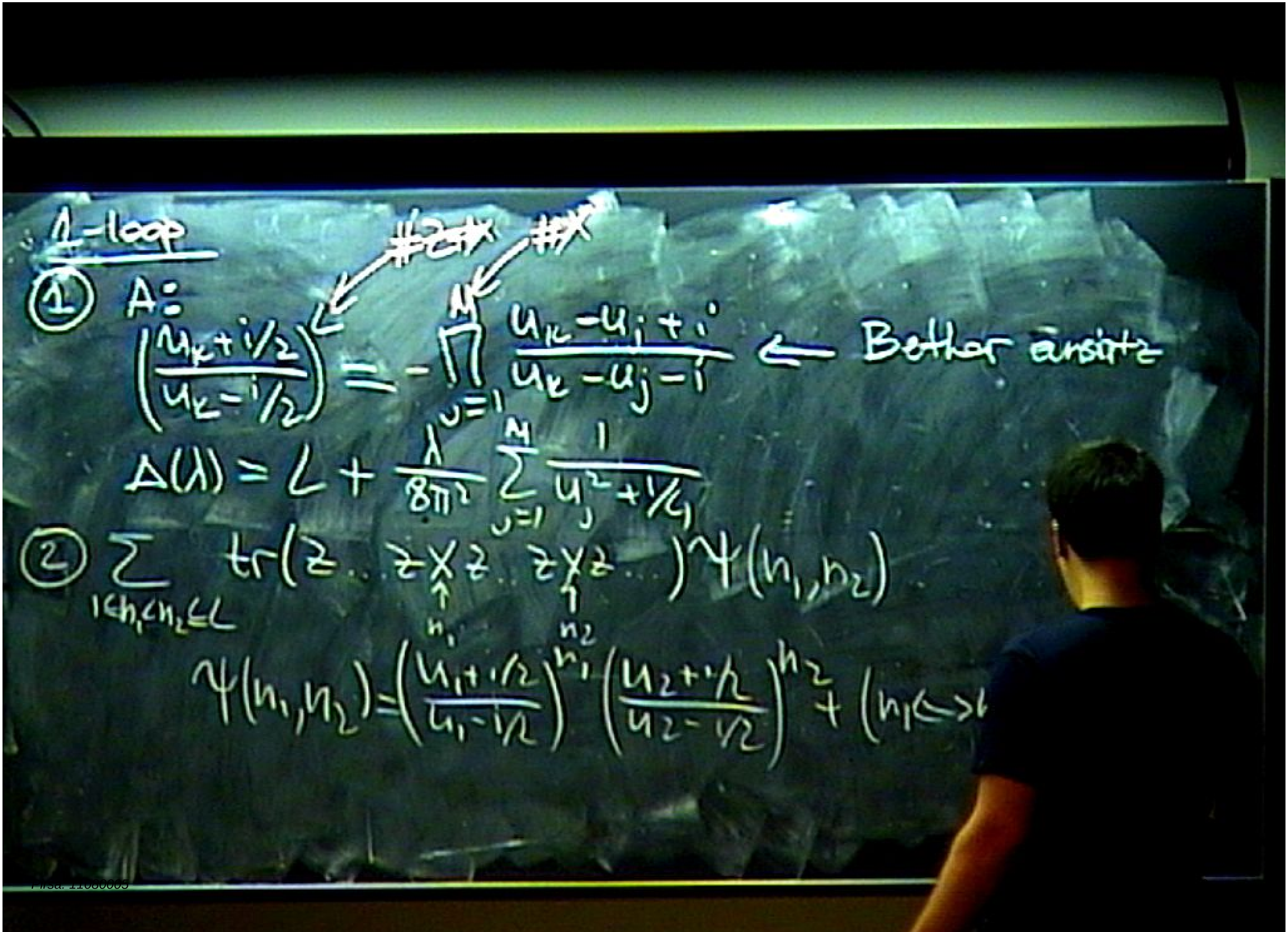

1-loop
#Z+#X
#X
① A:
2 Σ
Ψ(n1, n2)
n1
n2


1-loop
#Z+X
#X
① A:
(u_k + i/2 / u_k − i/2)^L = − ∏_{j=1}^{M} (u_k − u_j + i) / (u_k − u_j − i) ← Bethe ansatz
Δ(λ) = L + λ/8π² ∑_{j=1}^{M} 1/(u_j² + 1/4)
② ∑_{1≤n₁<n₂≤L} tr(Z … Z X Z … Z X Z …) ψ(
n₁
n₂


1-loop
#2#X
#X
① A:
$\left(\frac{u_k + i/2}{u_k - i/2}\right)^{L} = -\prod_{j=1}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$
← Bethe ansatz
$\Delta(\lambda) = L + \frac{\lambda}{8\pi^2} \sum_{j=1}^{M} \frac{1}{u_j^2 + 1/4}$
② $\sum_{1 \le n_1 < n_2 \le L}$ tr(Z … Z X Z … Z X Z …)
n_1
n_2


1-loop
A:
#Z+X
#X
Bethe ansatz


1-loop
#Z+X
#X
① A:
(u_k + i/2 / u_k − i/2)^L = − ∏_{j=1}^{M} (u_k − u_j + i)/(u_k − u_j − i)
← Bethe ansatz
Δ(λ) = L + λ/8π² Σ_{j=1}^{M} 1/(u_j² + 1/4)
② Σ_{1≤n_1<n_2≤L} tr(Z … Z X Z … Z X Z …) ψ(n_1, n_2)
n_1
n_2
ψ(n_1, n_2) = (u_1 + i/2 / u_1 − i/2)^{n_1} (u_2 + i/2 / u_2 − i/2)^{n_2} + (


1-loop
#Z+#X
#X
① A:
$\left(\frac{u_k + i/2}{u_k - i/2}\right) = -\prod_{j=1}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$ ← Bethe ansatz
$\Delta(\lambda) = L + \frac{\lambda}{8\pi^2} \sum_{j=1}^{M} \frac{1}{u_j^2 + 1/4}$
② $\sum_{1 \le n_1 < n_2 \le L} \mathrm{tr}(Z \ldots Z X Z \ldots Z X Z \ldots)\, \psi(n_1, n_2)$
n_1
n_2
$\psi(n_1, n_2) = \left(\frac{u_1 + i/2}{u_1 - i/2}\right)^{n_1} \left(\frac{u_2 + i/2}{u_2 - i/2}\right)^{n_2} + (n_1 \leftrightarrow$

1-loop

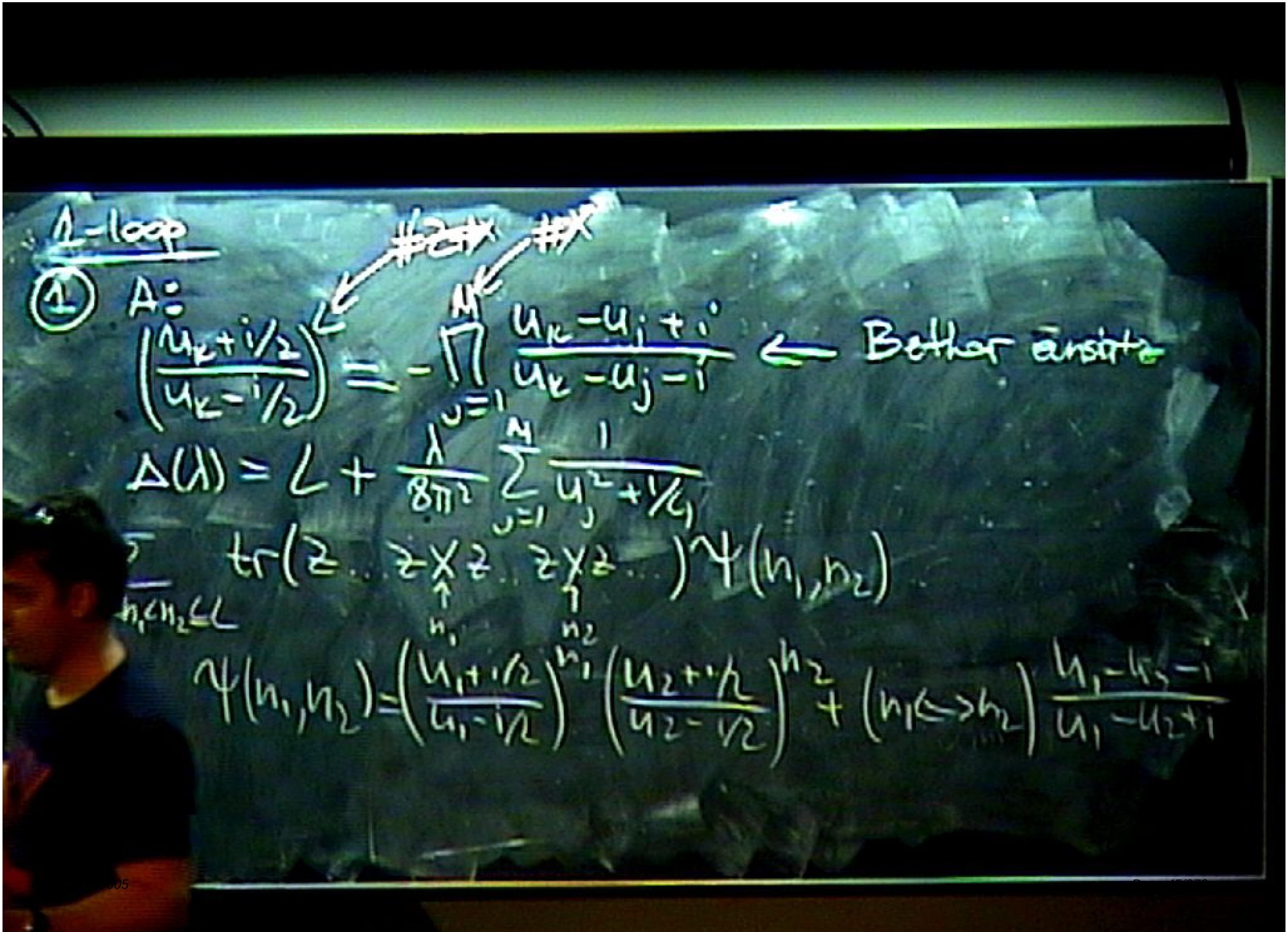
① A: ($\#Z + \#X$ = L, $\#X$ = M)

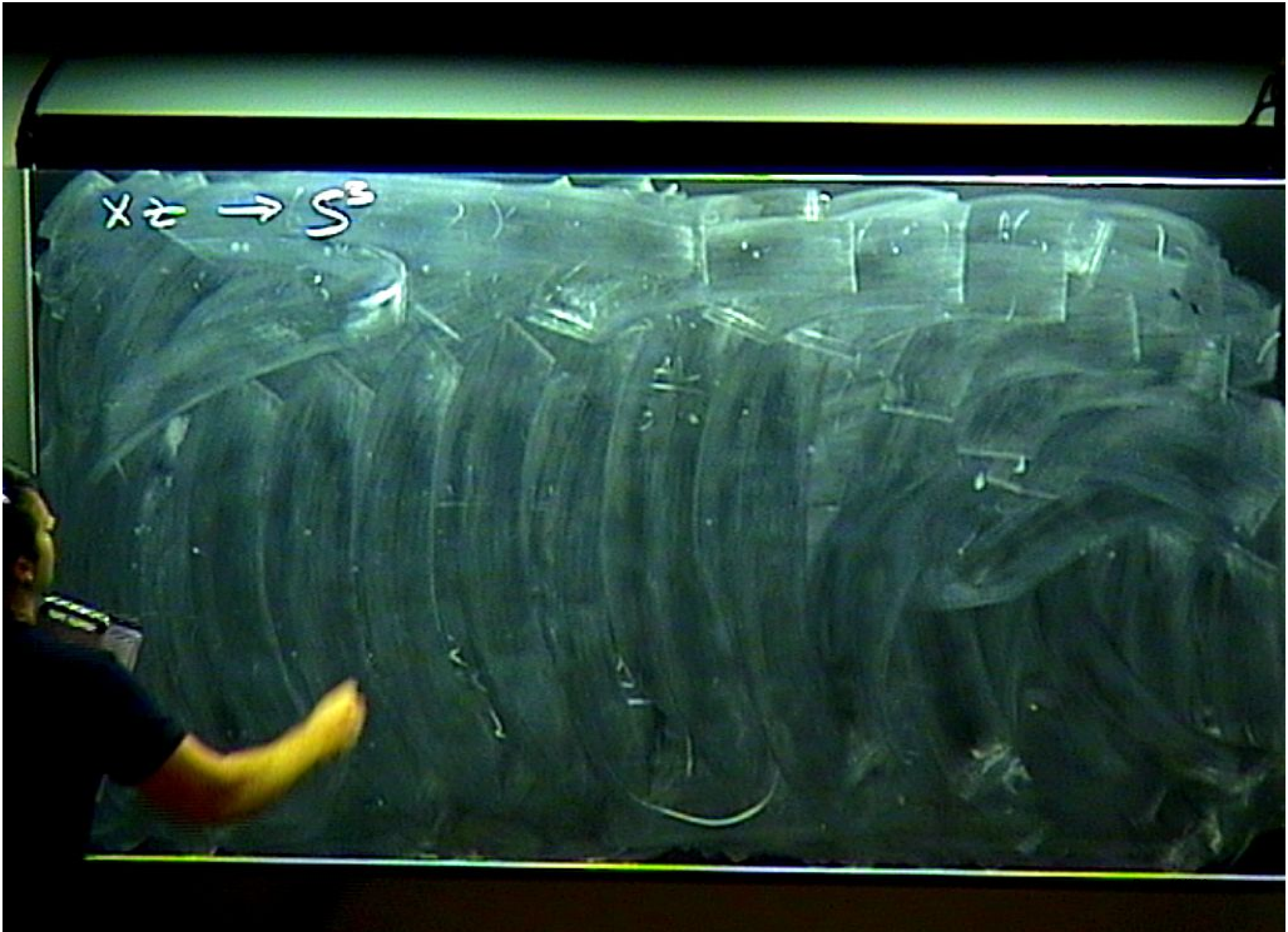
$$\left(\frac{u_k+i/2}{u_k-i/2}\right)^{L} = -\prod_{j=1}^{M}\frac{u_k-u_j+i}{u_k-u_j-i} \quad \leftarrow \text{Bethe ansatz}$$

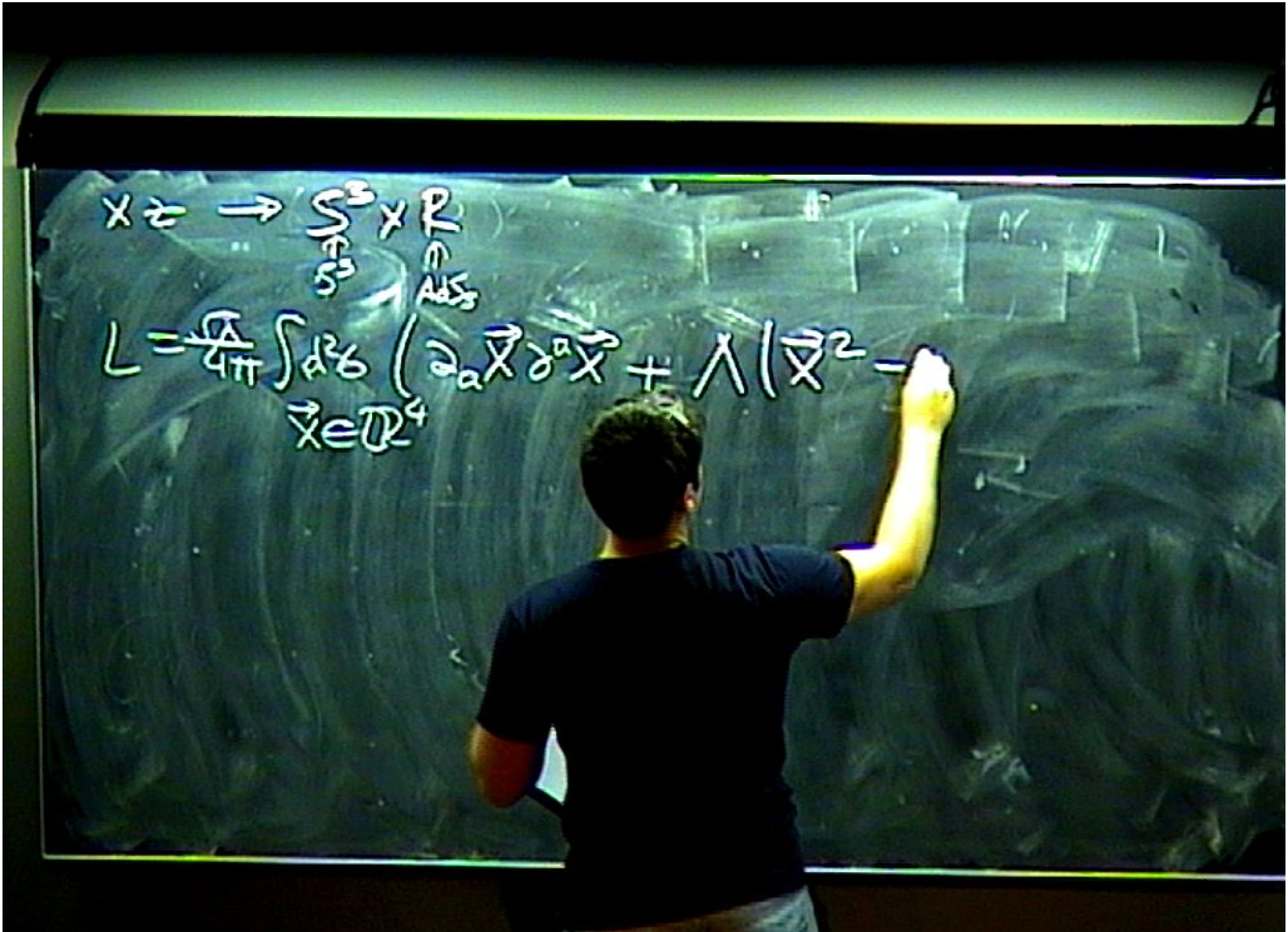
$$\Delta(\lambda) = L + \frac{\lambda}{8\pi^2}\sum_{j=1}^{M}\frac{1}{u_j^2+1/4}$$

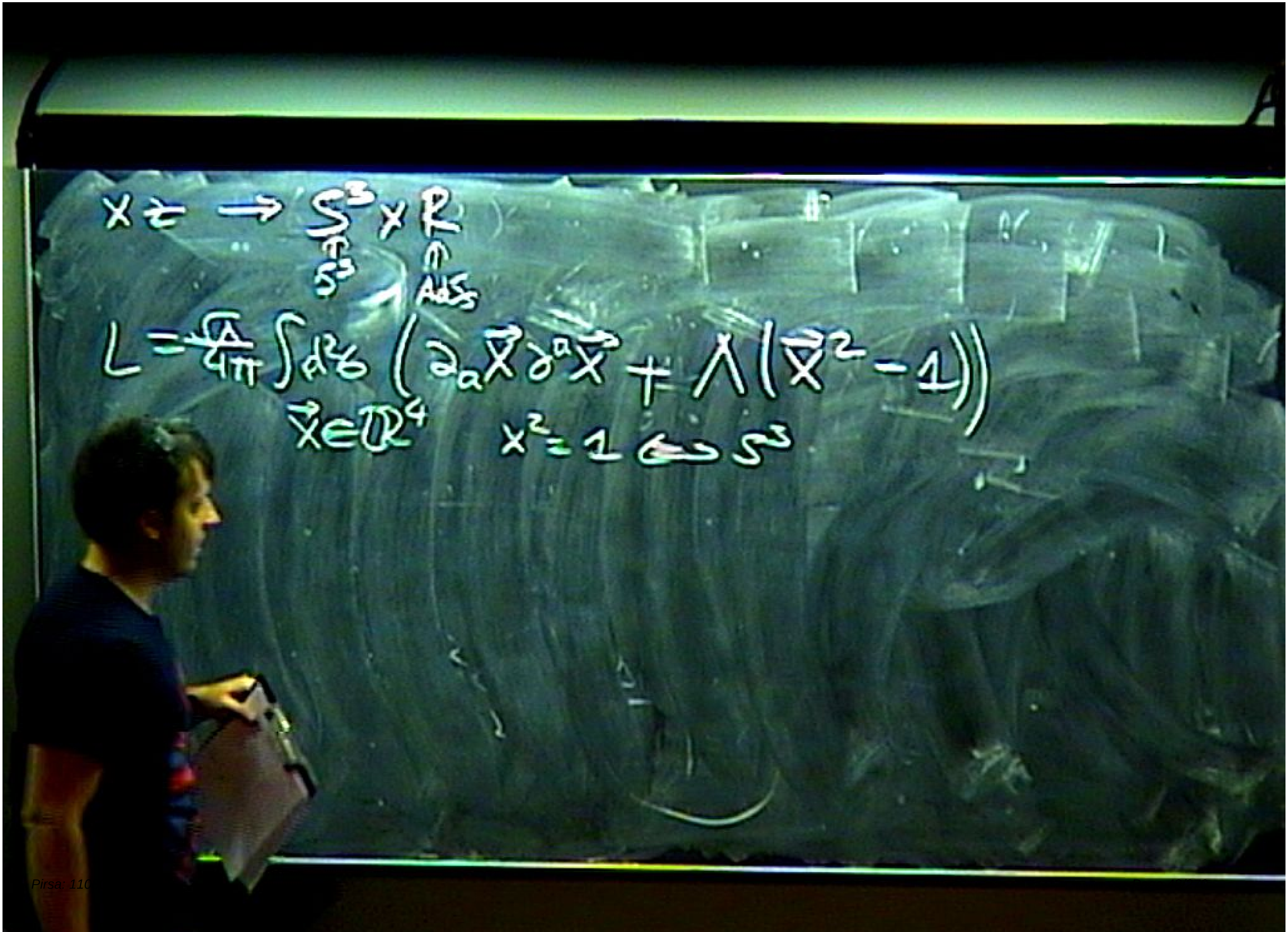
② $$\sum_{1\le n_1<n_2\le L} \mathrm{tr}(Z\ldots Z\underset{\substack{\uparrow\\ n_1}}{X}Z\ldots Z\underset{\substack{\uparrow\\ n_2}}{X}Z\ldots)\,\psi(n_1,n_2)$$

$$\psi(n_1,n_2) = \left(\frac{u_1+i/2}{u_1-i/2}\right)^{n_1}\left(\frac{u_2+i/2}{u_2-i/2}\right)^{n_2} + (n_1 \leftrightarrow n_2)\,\frac{u_1-u_2-i}{u_1-u_2+i}$$


1-loop
#Z+X
#X
① A:
$\left(\frac{u_k + i/2}{u_k - i/2}\right)^L = -\prod_{j=1}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$ ← Bethe ansatz
$\Delta(\lambda) = L + \frac{\lambda}{8\pi^2} \sum_{j=1}^{M} \frac{1}{u_j^2 + 1/4}$
$\sum_{n_1 < n_2 \le L} \mathrm{tr}(Z \ldots Z X Z \ldots Z X Z \ldots) \psi(n_1, n_2)$
n_1
n_2
$\psi(n_1, n_2) = \left(\frac{u_1 + i/2}{u_1 - i/2}\right)^{n_1} \left(\frac{u_2 + i/2}{u_2 - i/2}\right)^{n_2} + (n_1 \leftrightarrow n_2) \frac{u_1 - u_2 - i}{u_1 - u_2 + i}$


$X_t \to S^3$

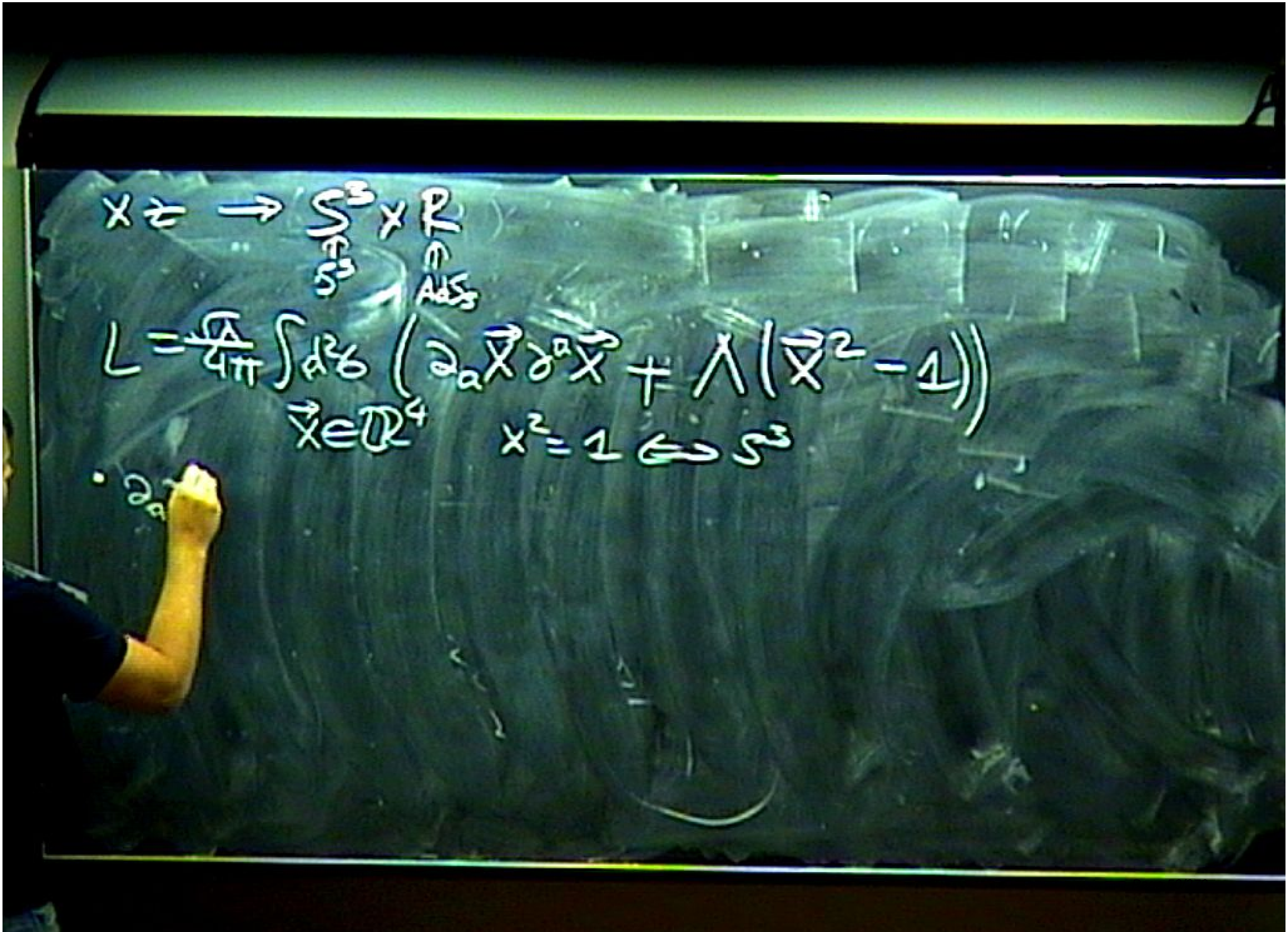

X ∈ → S³ × R
S³
AdS₅
L = √λ/4π ∫ d²σ (∂ₐX ∂ᵃX + Λ(X² −
X ∈ R⁴

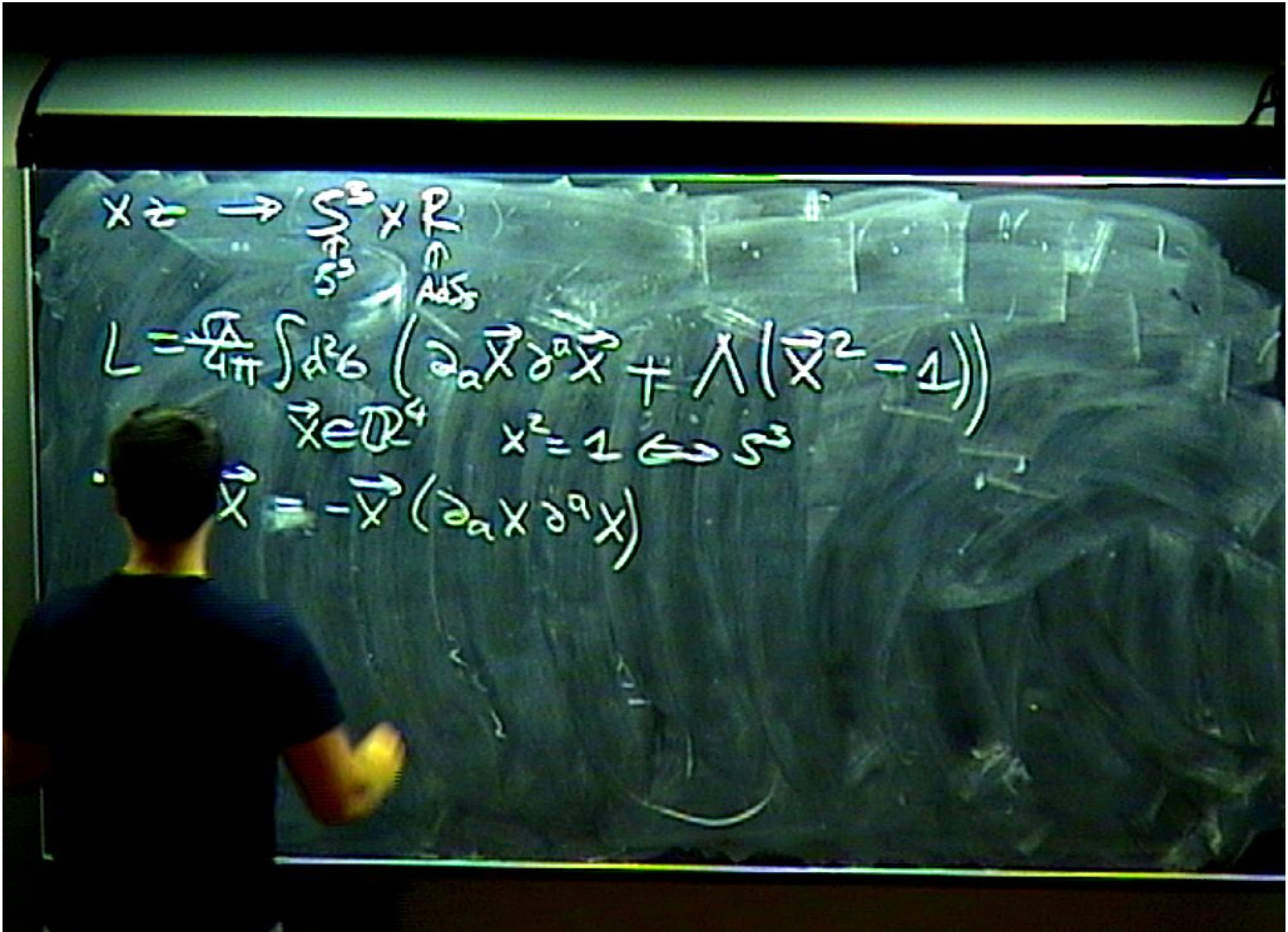

x ⊂ → S³ × R
S³ ⊂ S⁵ , R ⊂ AdS₅
L = √λ/4π ∫ d²σ (∂ₐX⃗ ∂ᵃX⃗ + Λ(X⃗² − 1))
X⃗ ∈ ℝ⁴
x² = 1 ⟺ S³

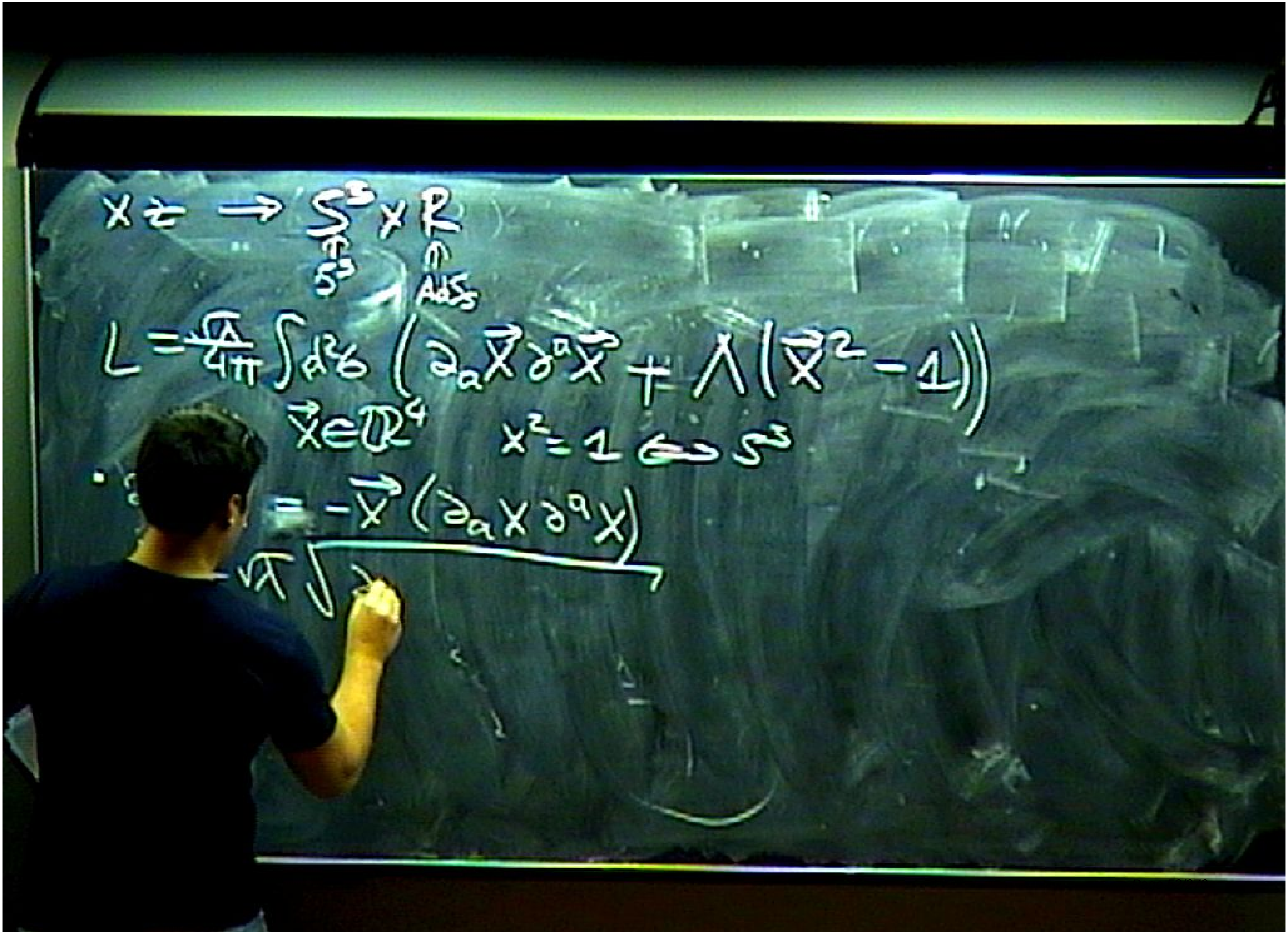
$$x\, z \rightarrow \underset{\underset{S^5}{\cap}}{S^3} \times \underset{\underset{AdS_5}{\cap}}{R}$$

$$L = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \left(\partial_a \vec{X}\, \partial^a \vec{X} + \Lambda \left(\vec{X}^2 - 1 \right) \right)$$

$$\vec{X} \in \mathbb{R}^4 \qquad x^2 = 1 \Leftrightarrow S^3$$


x z → S³ × R
S³ ⊂ S³   R ⊂ AdS₅
L = √λ/4π ∫ d²σ (∂ₐX⃗ ∂ᵃX⃗ + Λ(X⃗² − 1))
X⃗ ∈ ℝ⁴   x² = 1 ⟺ S³
· ∂ₐ


X ∈ → S^3 × R
S^5   AdS_5
L = √λ/4π ∫ d²σ (∂_a X ∂^a X + Λ(X² − 1))
X ∈ ℝ^4   x² = 1 ⟺ S^3
X = −X (∂_a X ∂^a X)


X z → S³ × R
S³ AdS₅
L = √λ/4π ∫ d²σ (∂ₐX ∂ᵃX + Λ(X² − 1))
X ∈ ℝ⁴
x² = 1 ⟺ S³
= −X (∂ₐX ∂ᵃX)
√λ ∫

$$x\, z \rightarrow S^3 \times R$$

$$S^3 \subset S^5, \qquad R \subset AdS_5$$

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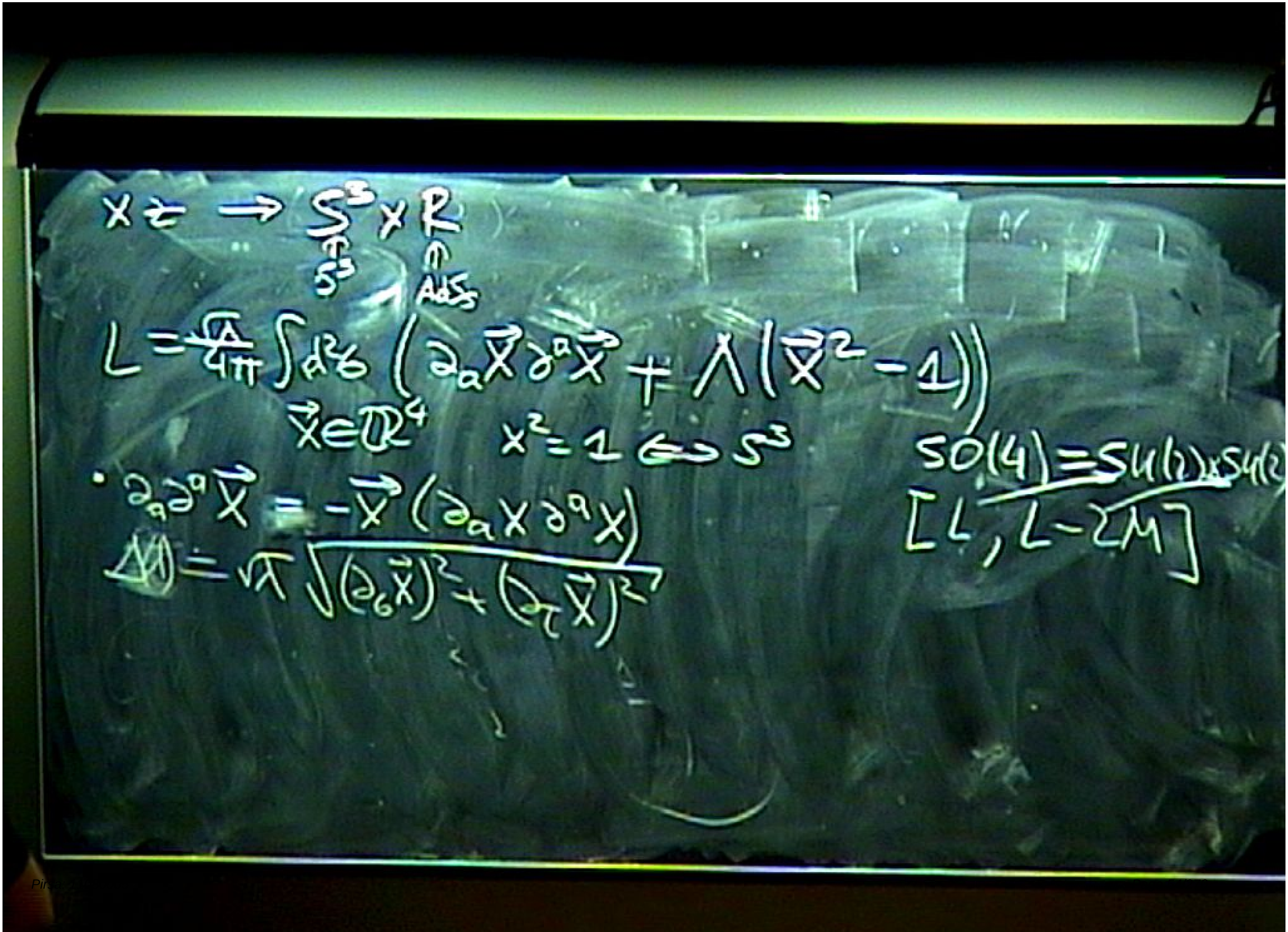
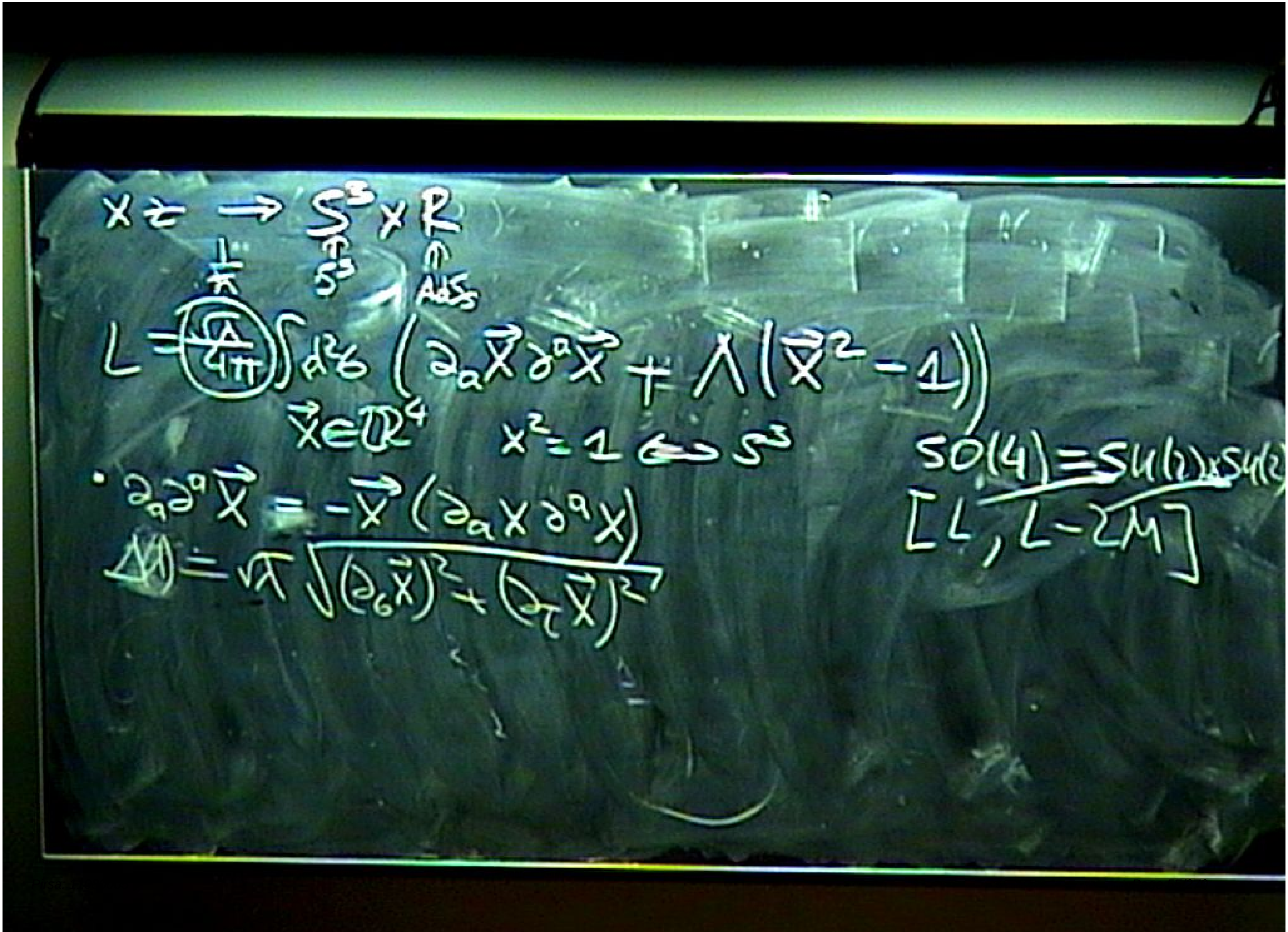
$$\vec{X} \in \mathbb{R}^4 \qquad x^2 = 1 \Leftrightarrow S^3$$

- $\partial_a \partial^a \vec{X} = -\vec{X} (\partial_a X \partial^a X)$

$$E = \sqrt{\lambda} \sqrt{(\partial_\sigma \vec{X})^2 + (\partial_\tau \vec{X})^2}$$


X τ → S^3 × R
S^3 ⊂ S^5, R ⊂ AdS_5
L = -√λ/4π ∫ d²σ (∂_a X ∂^a X + Λ(X² − 1))
X ∈ R^4
x² = 1 ⟺ S^3
• ∂_a ∂^a X = −X (∂_a X ∂^a X)
Δ(J) = √λ √((∂_σ X)² + (∂_τ X)²)
SO(4) = SU(2) × SU(2)
[L, L − ΣM]


$\times \mathbb{Z} \to S^3 \times R$
$\frac{1}{k}$ S^3 AdS_5
$L = \frac{\sqrt{\lambda}}{4\pi} \int d^2\sigma \left(\partial_a \vec{X} \partial^a \vec{X} + \Lambda (\vec{X}^2 - 1) \right)$
$\vec{X} \in \mathbb{R}^4$ $x^2 = 1 \Leftrightarrow S^3$
$SO(4) = SU(2) \times SU(2)$
$\cdot\ \partial_a \partial^a \vec{X} = -\vec{X} (\partial_a X \partial^a X)$
$[L, L - 2M]$
$\Delta(\lambda) = \sqrt{\lambda} \sqrt{(\partial_\sigma \vec{X})^2 + (\partial_\tau \vec{X})^2}$

$$x \geq \;\rightarrow\; S^3 \times \mathbb{R}$$

$$S^3 \subset S^5 \qquad \mathbb{R} \subset AdS_5$$

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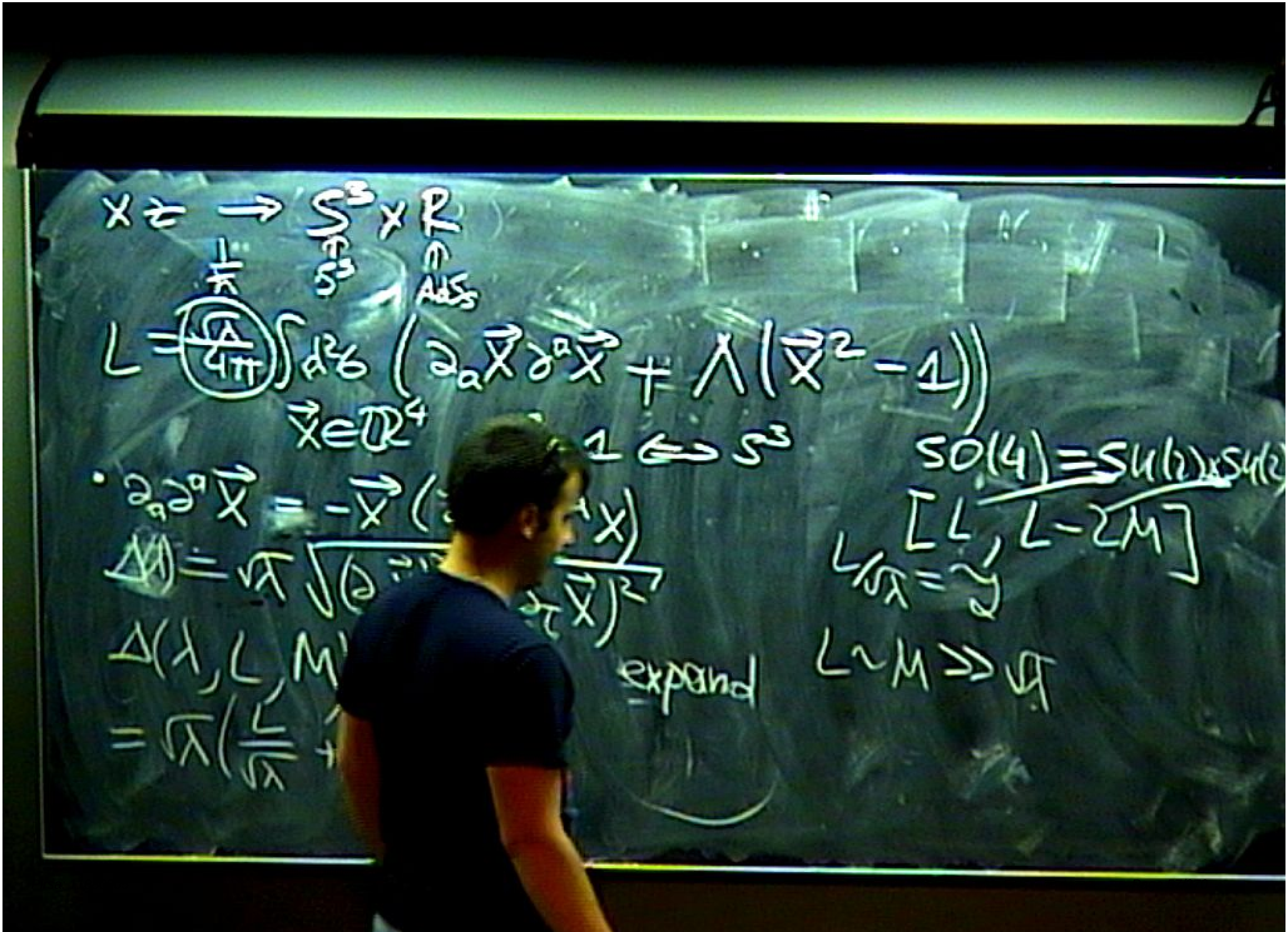
$$[L, L-2M]$$

$$\partial_a \partial^a \vec{X} = -\vec{X} (\partial_a X \partial^a X)$$

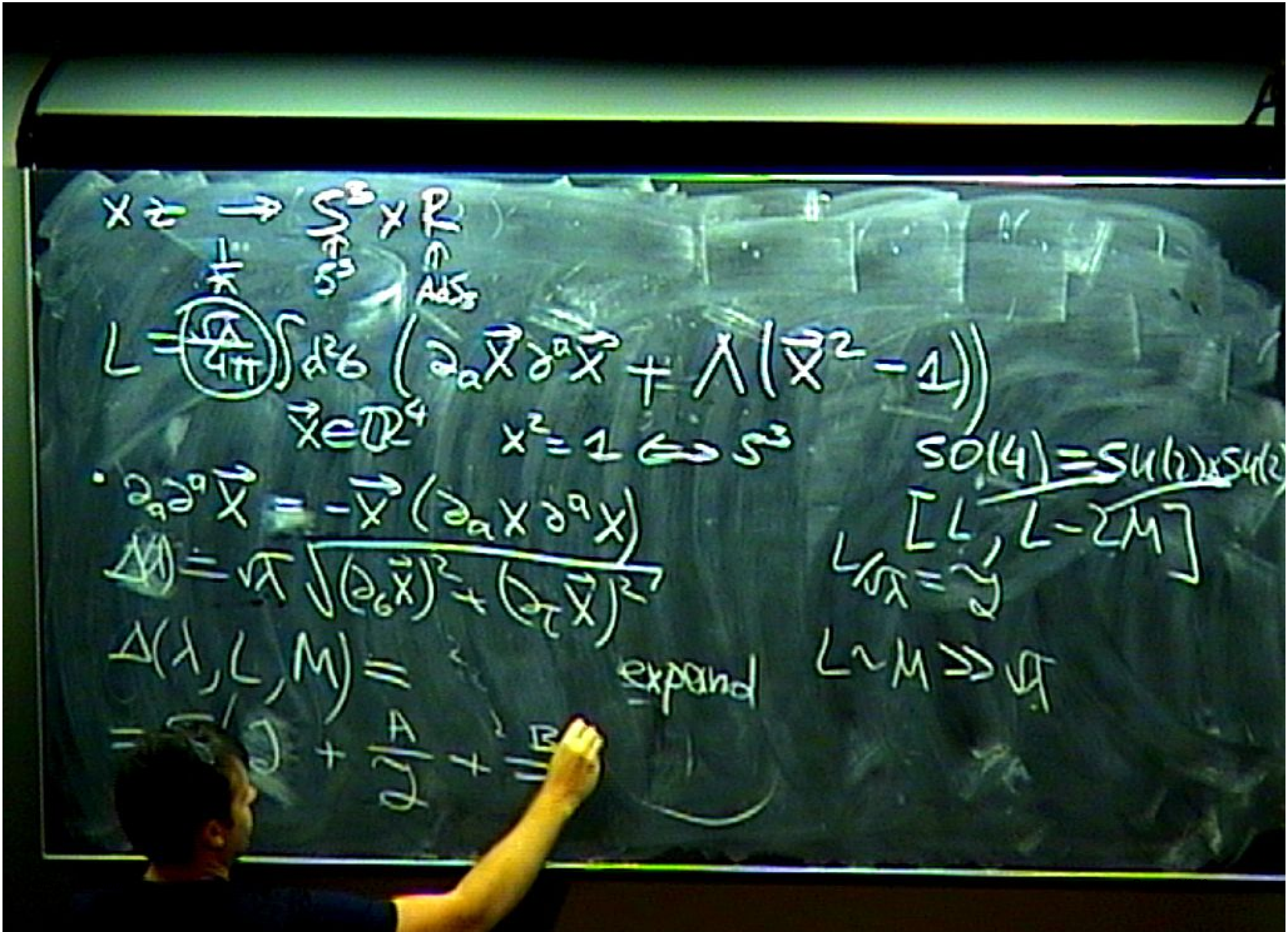
$$\Delta(\lambda) = \sqrt{\lambda} \int \sqrt{(\partial_\sigma \vec{X})^2 + (\partial_\tau \vec{X})^2}$$

$$\Delta(\lambda, L, M)$$

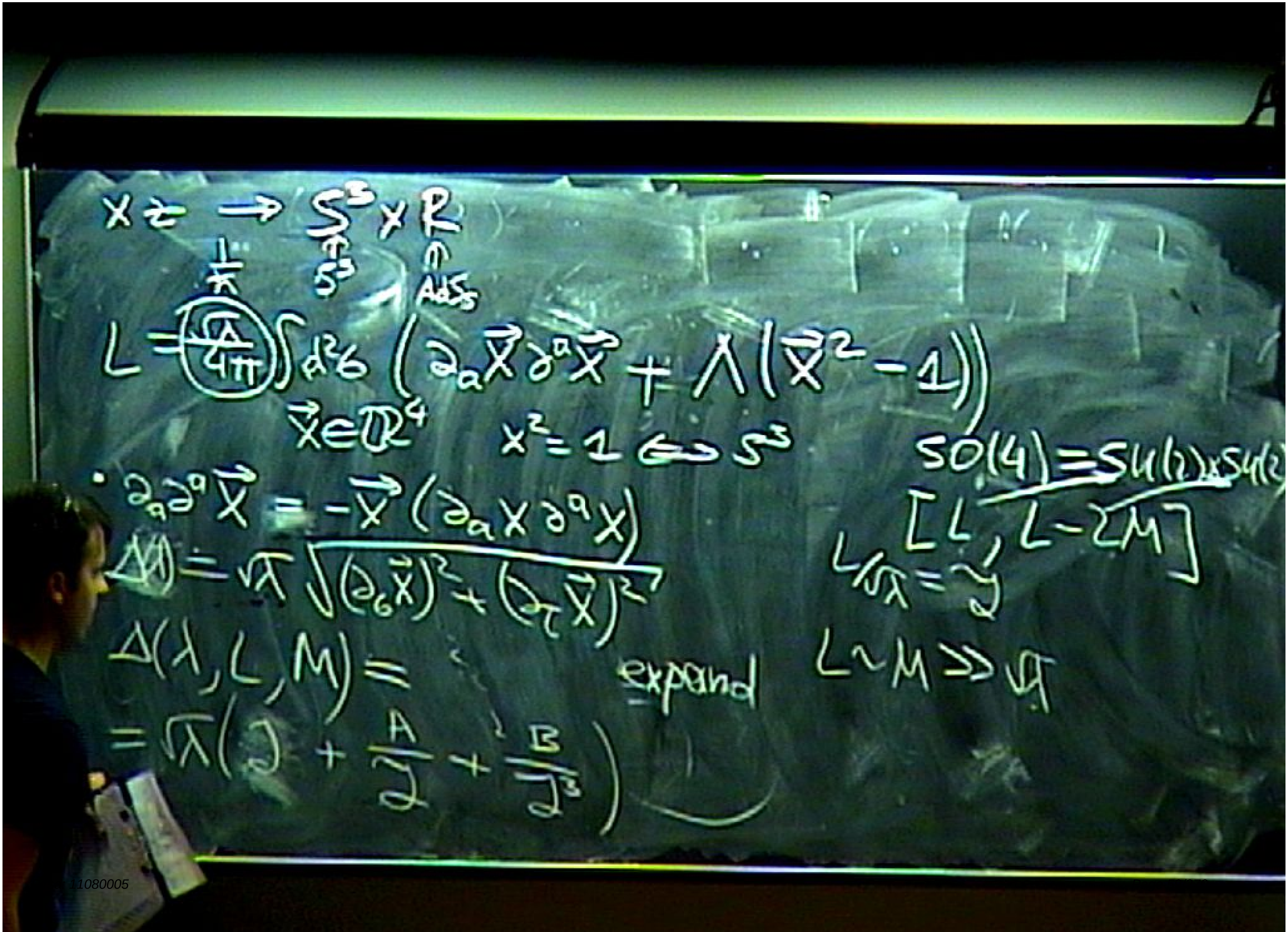
expand $\quad L \sim M \gg \sqrt{\lambda}$



S³ × R
AdS₅
L = √λ/4π ∫ d²σ (∂ₐX ∂ᵃX + Λ(X² − 1))
X ∈ R⁴
SO(4) = SU(2) × SU(2)
expand
L ~ M ≫ √λ



x z → S^3 × R
S^5   AdS_5
L = (√λ / 4π) ∫ d²σ (∂_a X ∂^a X + Λ(X² − 1))
X ∈ R^4   x² = 1 ⟺ S^3
SO(4) = SU(2) × SU(2)
∂_a ∂^a X = −X (∂_a X ∂^a X)
[L, L − M]
Δ(M) = √λ ∫ √((∂_σ X)² + (∂_τ X)²)
L/√λ = 𝒥
Δ(λ, L, M) =
expand
L ~ M ≫ √λ
+ A/𝒥 +



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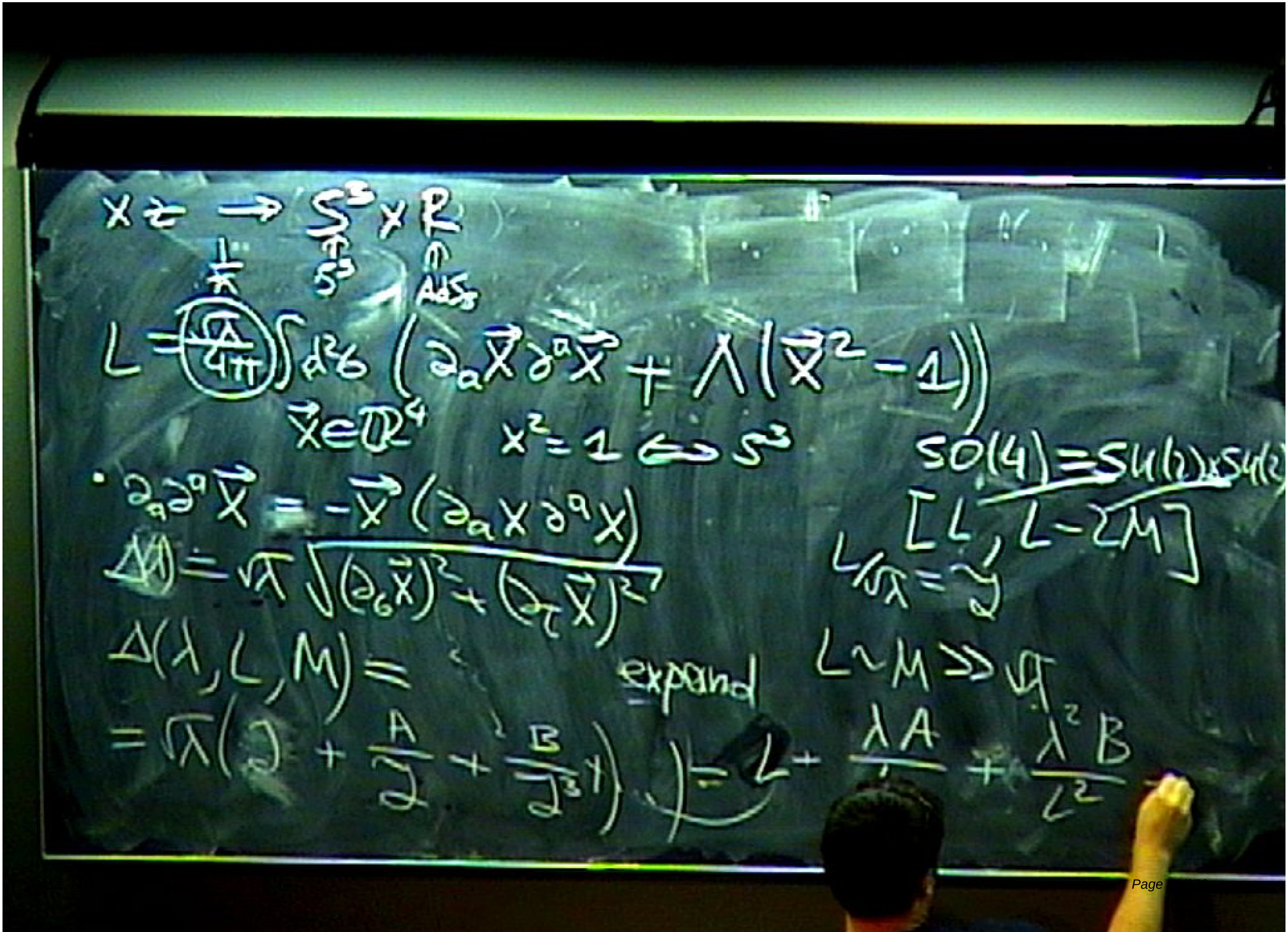
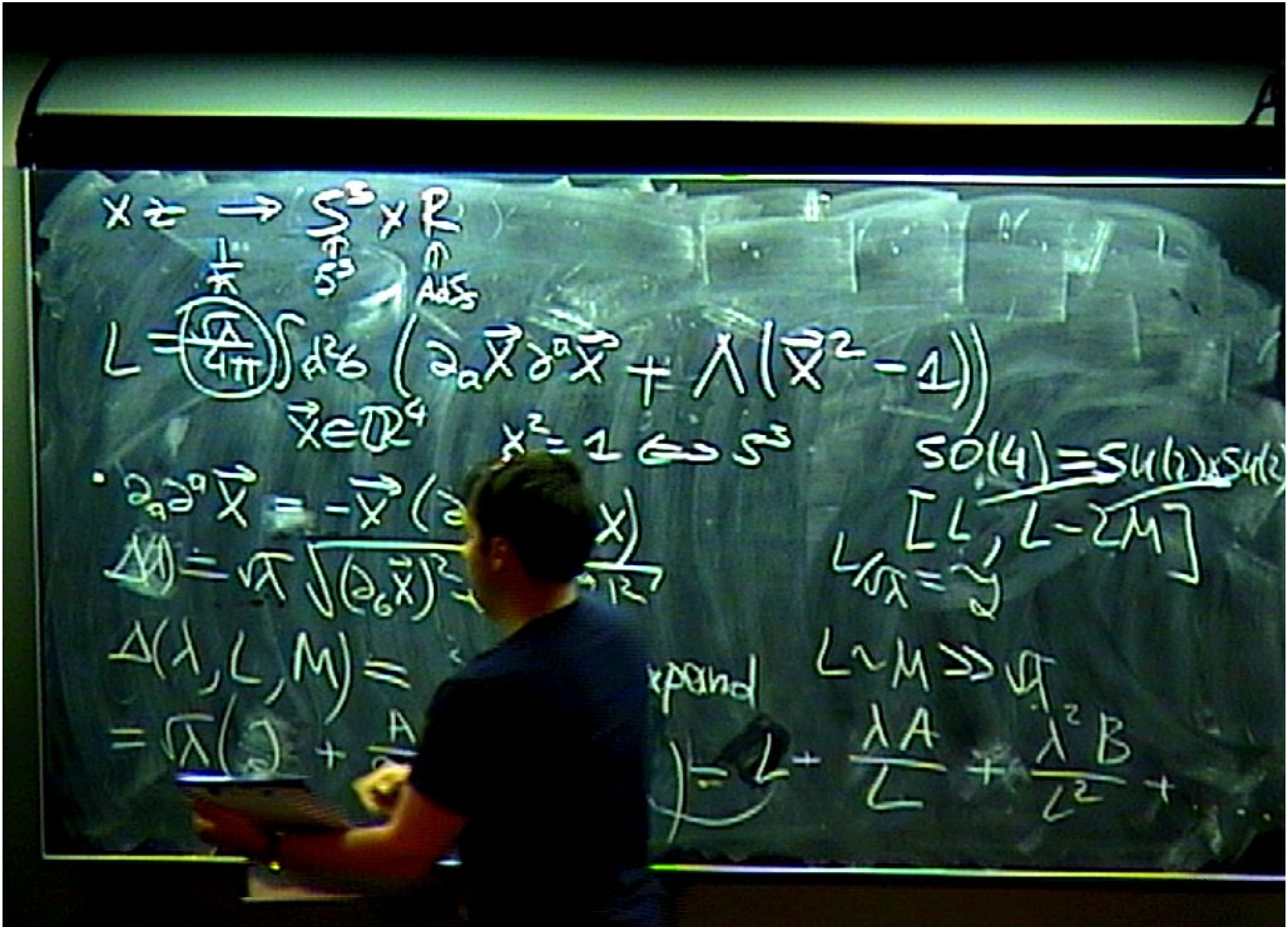

x z → S^3 × R
1/K   S^5   AdS_5
L = (√λ / 4π) ∫ d^2σ (∂_a X ∂^a X + Λ(X^2 − 1))
X ∈ R^4   x^2 = 1 ⟺ S^3
SO(4) = SU(2) × SU(2)
∂_a ∂^a X = −X (∂_a X ∂^a X)
[L, L − 2M]
Δ(M) = √λ ∫ √((∂_σ X)^2 + (∂_τ X)^2)
L/√λ = J
Δ(λ, L, M) =
expand   L ~ M ≫ √λ
= √λ (J + A/J + B/J^3 +) = L + λA/L + λ^2 B/L^2



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1/K
S³
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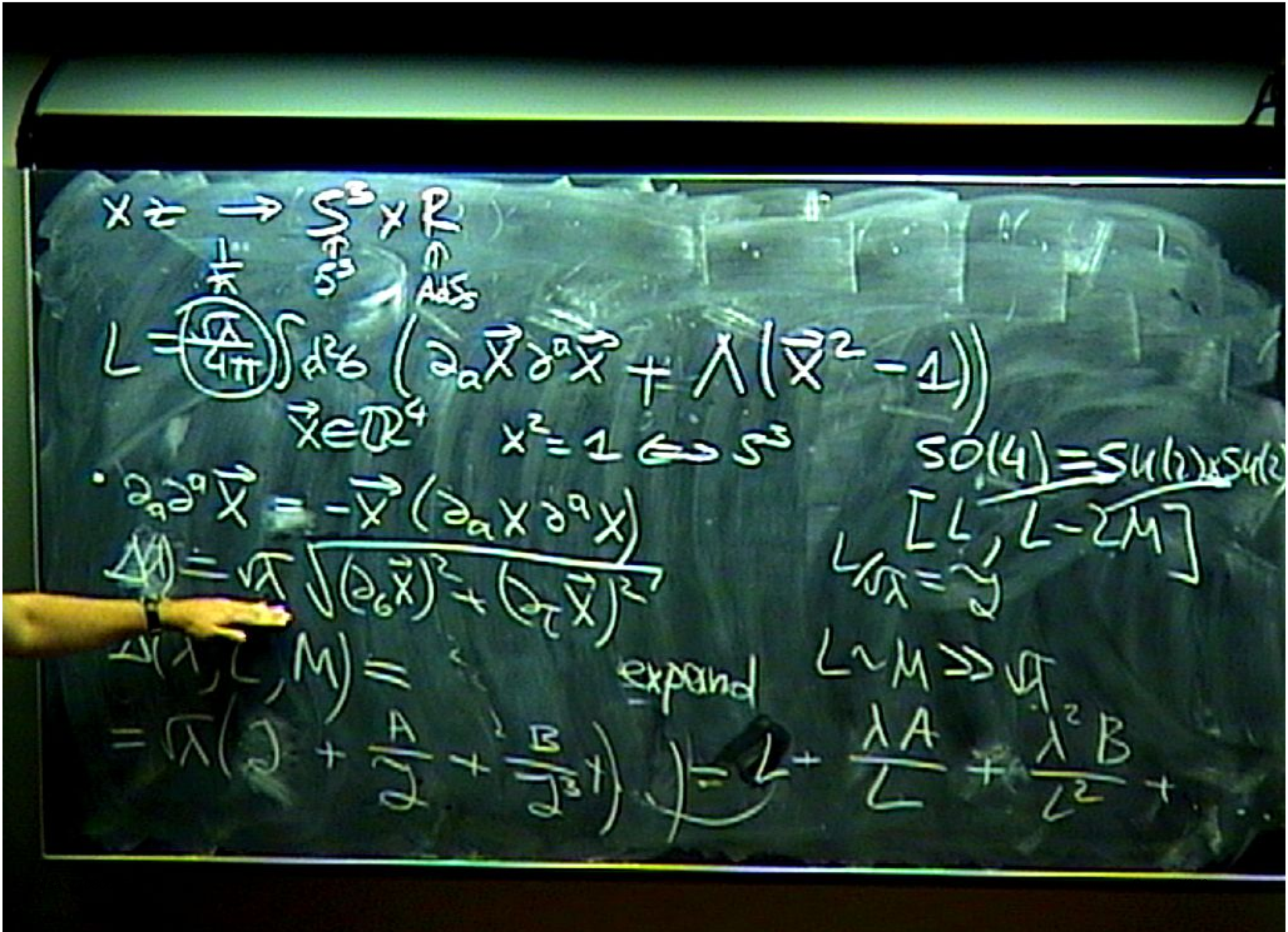
x z → S³ x R
1/κ S³ AdS₅
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x2v3.nb

ircular string vs Bethe ansatz (Simple)

xercises for the lecture of Nikolay Gromov

```
ToPDF["D:\\Links\\My Dropbox\\MSTP\\2011\\Kolya\\ex2.pdf"];
prcount = 0;
```

Theory

ı this exercise we check the AdS/CFT duality. Namely we compare energy of classical string moving ı $AdS^5 \times S_5$ with a dual quantity in Super-Yang-Mills theory. We will check numerical that two very ifferent quantities match.

Strong coupling (Classical string theory)

Check (without Mathematica) that the equations of motion are

$$\partial^\mu \partial_\alpha \vec{X} = -\vec{X}(\partial_\alpha \vec{X} \partial^\mu \vec{X})$$

he Polyakov action for the string moving inside S^3 is given by

$$L = \int d\tau \int_0^{2\pi} d\sigma [\partial_\alpha \vec{X}_i \partial^\mu \vec{X} - \Lambda(X^2 - 1)]$$

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a solution of the equations of motion. Derive the relation between w_1, w_2 and $\mathcal{J}$, α, n

```
X[1] = Sqrt[J (1 - α) / ...] Cos[w1 τ + α n σ];
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```
X[1] = Sqrt[J (1 - α)/w1] Cos[w1 τ + α n σ];
X[2] = Sqrt[J (1 - α)/w1] Sin[w1 τ + α n σ];
X[3] = Sqrt[J α/w2] Cos[w2 τ + (α - 1) n σ];
X[4] = Sqrt[J α/w2] Sin[w2 τ + (α - 1) n σ];
Xl = Table[X[i], {i, 4}];
```

One can try to find w_1, w_2 as functions of the other parameters which is very complicated. Instead
nd w_1, w_2 perturbatively for large $\mathcal{J}$ i.e. find a and b in the expansion

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$$w_1 = \mathcal{J} + \frac{a}{\mathcal{J}} + \ldots \ , \ w_2 = \mathcal{J} + \frac{b}{\mathcal{J}} + \ldots$$

ich that the equations of motion are satisfied up to a terms $\sim 1/\mathcal{J}$
int: you should find $a = \alpha(2\alpha - 1)n^2$ up to a simple multiplier

Compute the energy Δ also as an expansion in $\mathcal{J}$. Denote $L = \sqrt{\lambda}\mathcal{J}$. The result you should get is

$$L - \frac{(\alpha - 1)\alpha\lambda n^2}{2L} - O\left(\left(\frac{1}{L}\right)^2\right)$$

n the second part of the exercises we compare this result with the Yang-Mills prediction.

Weak coupling

For given L and M the Bethe ansatz equation has many different solutions. They all correspond to
ifferent operators and different classical string. To find the solution which correspond to the circular
ring from the part 1 of this excessive we have to specify a good set of starting points.
efine the function `StartingPoints[L_,M_]` as follows

Find numerically a solution of the Bethe ansatz equation using `FindRoot` function. Take L and M

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x2v3.nb

ccording to the AdS/CFT duality the objects dual to the classical string energies at weak coupling re the anomalous dimensions. They are defined by two point functions:

$$\langle O_1(x)\bar{O}_1(x)\rangle \sim \frac{1}{(x-y)^{2\Delta}} .$$

here Δ can be found order by order in perturbation theory. The operators dual to the string moving S^3 subspace are the scalar single trace operators of the form $O_1 = \mathrm{tr}(XZXXZ\dots) +$permutations. o the leading order in λ it is very easy to compute $\Delta_0 = \#X + \#Z \equiv L$ (see another exercises here this quantity is computed to the leading order). To the next order the problem can be solved y means of the Bethe ansatz

$$\left(\frac{u_k + i/2}{u_k - i/2}\right)^L = -\prod_{j=1}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$$

here $M = \#X$. When the Bethe ansatz is solved the anomalous dimension of the operator is given y

$$\Delta = L + \frac{\lambda}{8\pi^2}\sum_{j=1}^{M} \frac{1}{u_j^2 + 1/4} + \mathcal{O}(\lambda^2) .$$

```
StartingPoints[L_, M_, n_] :=
  L/(2 π n) + I Sqrt[2 L]/(2 π n) zk /. NSolve[HermiteH[M, zk] == 0, zk];
```

Define a function SolveBAE[L_,M_,n_] which solve the Bethe ansatz using these starting points. heck

```
SolveBAE[10, 2, 1]
```


150%

x2v3.nb

$$\langle O(x) O(y) \rangle \sim \frac{1}{(x-y)^{2\Delta}} .$$

here Δ can be found order by order in perturbation theory. The operators dual to the string moving S^3 subspace are the scalar single trace operators of the form $O_1 = \mathrm{tr}(XZXXZ\dots) +$ permutations. the leading order in λ it is very easy to compute $\Delta_0 = \#X + \#Z \equiv L$ (see another exercises here this quantity is computed to the leading order). To the next order the problem can be solved means of the Bethe ansatz

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```
StartingPoints[L_, M_, n_] :=
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```

Define a function SolveBAE[L_,M_,n_] which solve the Bethe ansatz using these starting points. heck

```
SolveBAE[10, 2, 1]
```

{u(1) → 1.3088741844046434659894738897З − 0.58168645590651625751431799321 8 i,
u(2) → 1.30887418440464346598947388973 + 0.58168645590651625751431799321 8 i}

x2v3.nb

S^2 subspace are the scalar single trace operators of the form $O_1 = \text{tr}(XZXXZ\ldots) +$ permutations. To the leading order in λ it is very easy to compute $\Delta_0 = \#X + \#Z \equiv L$ (see another exercises where this quantity is computed to the leading order). To the next order the problem can be solved by means of the Bethe ansatz

$$\left(\frac{u_k + i/2}{u_k - i/2}\right)^L = -\prod_{j=1}^{M} \frac{u_k - u_j + i}{u_k - u_j - i}$$

where $M = \#X$. When the Bethe ansatz is solved the anomalous dimension of the operator is given by

$$\Delta = L + \frac{\lambda}{8\pi^2}\sum_{j=1}^{M} \frac{1}{u_j^2 + 1/4} + \mathcal{O}(\lambda^2)\ .$$

```
StartingPoints[L_, M_, n_] :=
  L/(2 π n) + I Sqrt[2 n]/(2 π n) zk /. NSolve[HermiteH[M, zk] == 0, zk];
```

Define a function SolveBAE[L_,M_,n_] which solve the Bethe ansatz using these starting points. Check

```
SolveBAE[10, 2, 1]
```

$\{u(1) \to 1.30887418440464346598947388973 - 0.58168645590651625751431799321 8\, i,$
$u(2) \to 1.30887418440464346598947388973 + 0.58168645590651625751431799321 8\, i\}$

Plot the points generated by SolveBAE[210,30,1] on the complex plain (use ListPlot). You should see that the Bethe roots u_k are distributed along nice cuts.

x2v3.nb

```
SolveBAE[10, 2, 1]
```

$\{u(1) \to 1.30887418440464346598947388973 - 0.58168645590651625751431799321 8\,i,$
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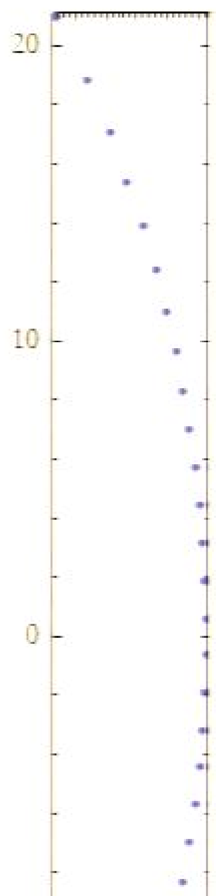

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x2v3.nb

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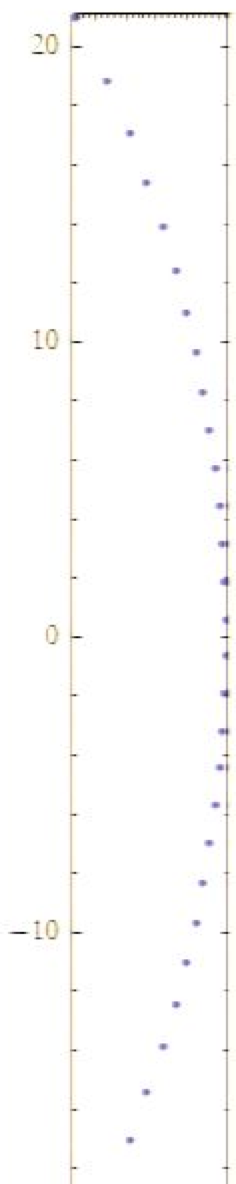

x2v3.nb

here $M = \#X$. When the Bethe ansatz is solved the anomalous dimension of the operator is given
y

$$\Delta = L + \frac{\lambda}{8\pi^2}\sum_{j=1}^{M}\frac{1}{u_j^2+1/4} + \mathcal{O}(\lambda^2)\,.$$

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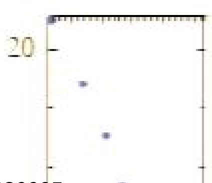

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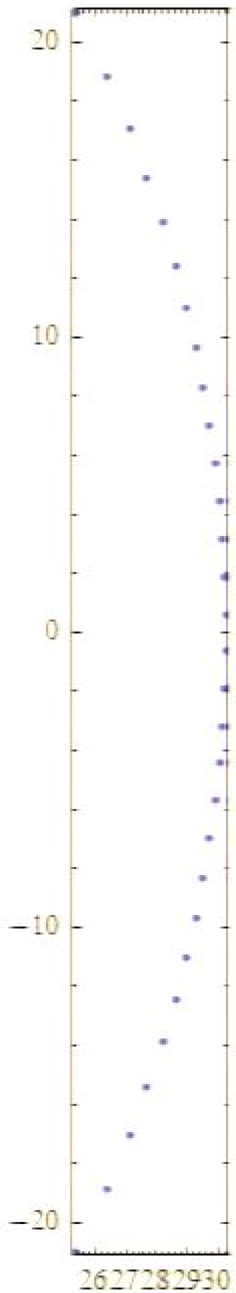
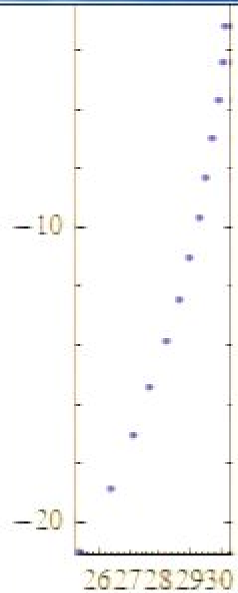
x2v3.nb



x2v3.nb



Compute energy for the same solution SolveBAE[210,30,1]. You should get:

0.00029224974357119224624961305060 λ − 210

ompare this with the energy of the circular string from part1 of the exercises under identification
$= M/L$.

Make a list of coefficients in front of λ for $L = 7M$ and $M = 10 \ldots 30$ and call it deltas. it should
e of the form {{L,coefficient},...} . Plot deltas to get

ou should see that the results are very similar!!!
o make the test of AdS/CFT even more convincing we should extrapolate the result to large L.

150%

x2v3.nb

0.0002922497435711922462496130506 0 λ − 210

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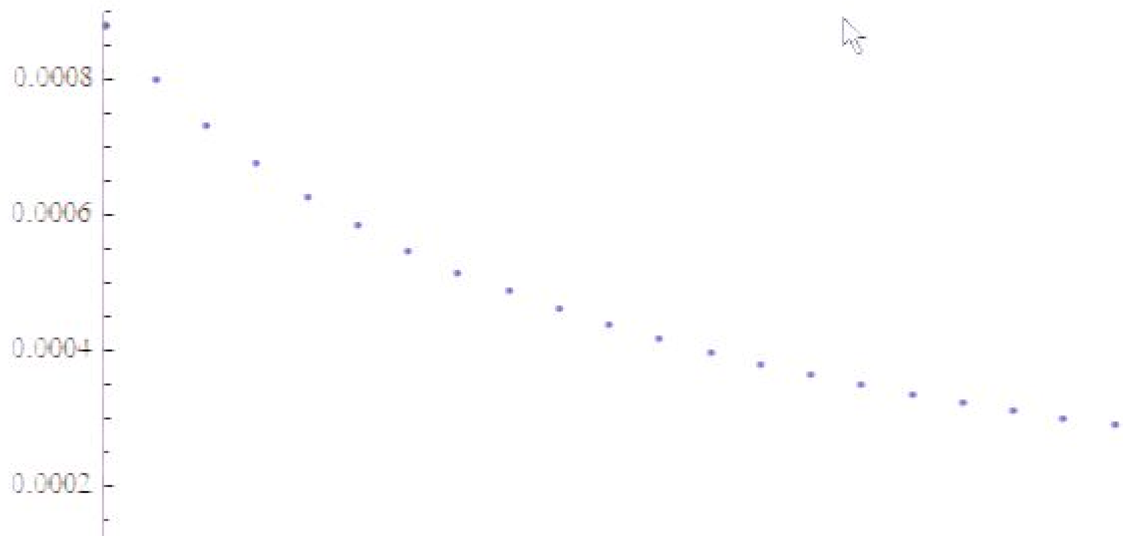
```
ListPlot[deltas]
```



x2v3.nb

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```
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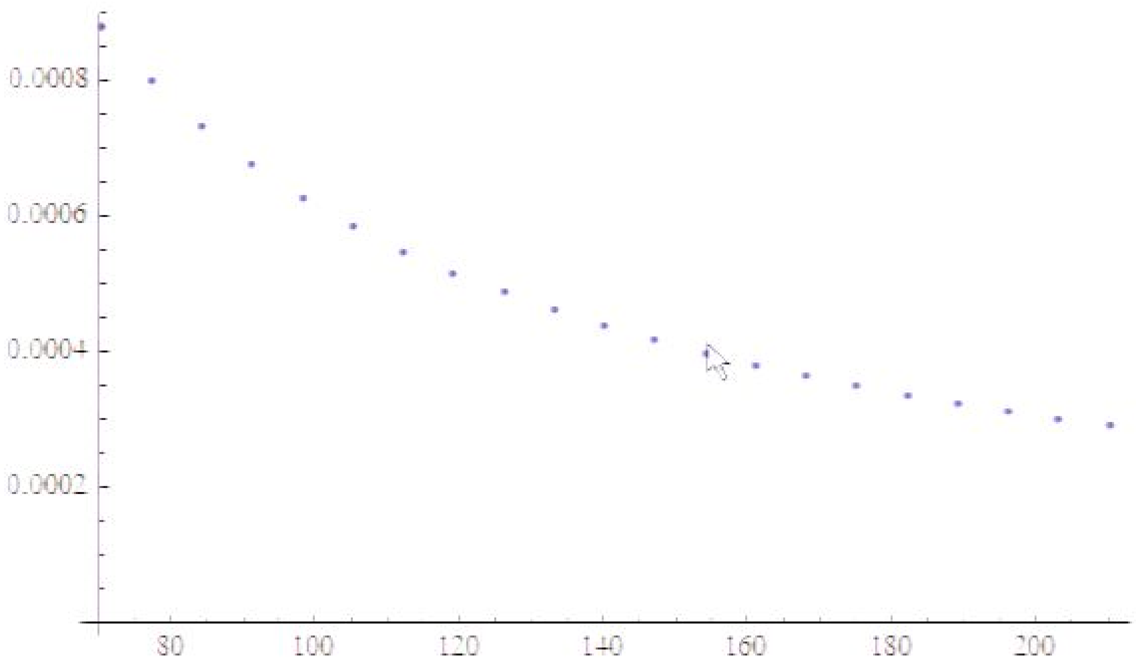
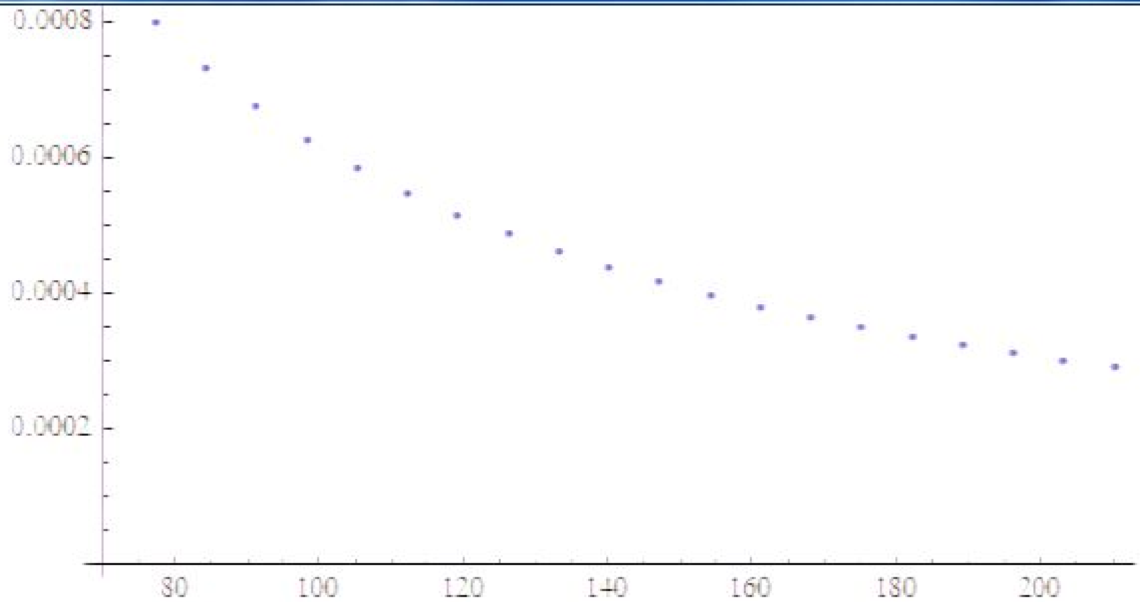
```



Using Fit function make of fit of this data by many inverse powers of L. Try to match the coefficient /L with the AdS/CFT prediction $\alpha(1-\alpha)/2$ with $\alpha = 1/7$. It is possible to get at least 10 digits atch!

x2v3.nb



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ay 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}}
```

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2\,k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

3

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Day 1 - Solution - Basics of Mathematica Grammar.nb

tivate Style

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a1___} → a1
```

Sequence[P, o, r, t, o, W, i, n, e]

```
Integrate
```

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a1___, {a2___}, a3___} → {a1, a2, a3}
```

{P, o, r, t, o, W, i, n, e}

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a1___, {a2___}, a3___} → {a1, a2, a3}
```

{P, o, r, t, o, W, i, n, e}

```
KSqr[aa_] :=
  Block[{tmp}, (tmp = Factor[aa]; Collect[Numerator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]), Sqrt[_], Factor] /
     Collect[Denominator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]), Sqrt[_], Factor])];
```

$$\frac{a+\sqrt{a1}}{\left(a1+a2\sqrt{b}\right)\left(a2+a3\sqrt{b1}\right)} \text{ // KSqr}$$

$$\frac{\sqrt{b}\left(-a\,a2^2-a\,a2\,a3\sqrt{b1}-\sqrt{a1}\left(a2\,a3\sqrt{b1}-a2^2\right)\right)-a1\left(a\,a2-a\,a3\sqrt{b1}\right)-a1^{3/2}\left(a2-a3\sqrt{b1}\right)}{\left(a1^2-a2^2\,b\right)\left(a2^2-a3^2\,b1\right)}$$

$$\text{Sym[G\_[a1\_\_]] := }\frac{\text{Plus @@ G /@ Permutations[\{a1\}] /. List -> Sequence}}{\text{Length[Permutations[\{a1\}]]}}$$

100%

ay 1 - Solution - Basics of Mathematica Grammar.nb

tivate Style

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Sequence[P, o, r, t, o, W, i, n, e]

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Integrate
```

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{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a1___, {a2___}, a3___} → {a1, a2, a3}
```

{P, o, r, t, o, W, i, n, e}

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```

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ay 1 - Solution - Basics of Mathematica Grammar.nb

ivate Style

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a1___} → a1
```

Sequence[P, o, r, t, o, W, i, n, e]

Integrate

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a1___, {a2___}, a3___} → {a1, a2, a3}
```

{P, o, r, t, o, W, i, n, e}

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```

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$$\frac{\sqrt{b}\left(-a\,a2^2-a\,a2\,a3\sqrt{b1}-\sqrt{a1}\left(a2\,a3\sqrt{b1}-a2^2\right)\right)-a1\left(a\,a2-a\,a3\sqrt{b1}\right)-a1^{3/2}\left(a2-a3\sqrt{b1}\right)}{\left(a1^2-a2^2\,b\right)\left(a2^2-a3^2\,b1\right)}$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}}
```

2

$$eqn = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2\,k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

3

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

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3

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} /. {a___} → a
```

b

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2\,k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} /. {a___} :> a
```

```
Sequence[{P, o, {r, t, {}, {{o}, W}}}, i, {5, {e}}]
```

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2\,k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
n =.
```

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a___} :> a
```

Sequence[P, o, r, t, o, W, i, 5, e]

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2\,k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a___} :> a
```

Sequence[*P*, *o*, *r*, *t*, *o*, *W*, *i*, 5, *e*]

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2\,k}\, y[k-1] = 0,\ y[0] = 0,\ y[1] = 1\right\};$$

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a___} :> a
```

Sequence[*P*, *o*, *r*, *t*, *o*, *W*, *i*, *n*, *e*]

$$\equiv \int_0^1 dx_1 \int_{x_1}^1 dx_2 \ldots \int_{x_n}^1 dx_{n-1}$$

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

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```
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```

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```
In
```

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2k}\, y[k-1] = 0,\ y[0] = 0,\ y[1] = 1\right\};$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

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```
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```

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```
Inte
```

- Integer
- IntegerDigits
- IntegerExponent
- IntegerLength
- IntegerPart
- IntegerPartitions
- IntegerQ
- Integers
- IntegerString
- Integral
- Integrate
- Integrate1
- Interactive
- InteractiveTradingChart
- Interlaced
- Interleaving
- InternallyBalancedDecomposition
- InterpolatingFunction
- InterpolatingPolynomial
- Interpolation
- InterpolationOrder
- InterpolationPoints
- InterpolationPrecision
- Interpretation
- InterpretationBox

2

$$y[k] + \frac{k+2}{2k}\, y[k-1] = 0,\ y[0] = 0,\ y[1] = 1\};$$

150%

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a___} :> a
```

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$$\equiv \int_0^1 dx_1 \int_{x_1}^1 dx_2 \ldots \int_{x_n}^1 dx_{n-1}$$

```
Integrate
```

Integrate
Integrate1

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2k}\, y[k-1] = 0,\ y[0] = 0,\ y[1] = 1\right\};$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
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```

Sequence[$P, o, r, t, o, W, i, n, e$]

$$\equiv \int_0^1 dx_1 \int_{x_1}^1 dx_2 \ldots \int_{x_n}^1 dx_{n-1}$$

```
Integrate[1, {x1, 0, 1}]
```

1

2

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a___} :> a
```

Sequence[P, o, r, t, o, W, i, n, e]

$$\equiv \int_0^1 dx_1 \int_{x_1}^1 dx_2 \ldots \int_{x_n}^1 dx_{n-1}$$

```
Integrate[1, {x1, 0, 1}, {x2, x1, 1}]
```

$$\frac{1}{2}$$

2

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a___} :> a
```

Sequence[P, o, r, t, o, W, i, n, e]

$$\equiv \int_0^1 dx_1 \int_{x_1}^1 dx_2 \ldots \int_{x_n}^1 dx_{n-1}$$

```
Integrate[1, {x1, 0, 1}, {x2, x1, 1}]
```

$\frac{1}{2}$

```
n = 5;
```

```
Int[1, ]
```


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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

Sequence[$P, o, r, t, o, W, i, n, e$]

$$\equiv \int_0^1 dx_1 \int_{x_1}^1 dx_2 \ldots \int_{x_n}^1 dx_{n-1}$$

```
Integrate[1, {x1, 0, 1}, {x2, x1, 1}]
```

$\frac{1}{2}$

```
n = 5;
```

```
Integrate[1, Table[{x[i], x[i - 1], 1}, {i, 1, n}]]
```

Integrate::ilim : Invalid integration variable or limit(s) in $\begin{pmatrix} x(1) & 0 & 1 \\ x(2) & x(1) & 1 \\ x(3) & x(2) & 1 \\ x(4) & x(3) & 1 \\ x(5) & x(4) & 1 \end{pmatrix}$. >>

$$\int 1\, d\begin{pmatrix} x(1) & 0 & 1 \\ x(2) & x(1) & 1 \\ x(3) & x(2) & 1 \\ x(4) & x(3) & 1 \\ x(5) & x(4) & 1 \end{pmatrix}$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

Sequence[P, o, r, t, o, W, i, n, e]

$$\equiv \int_0^1 dx_1 \int_{x_1}^1 dx_2 \ldots \int_{x_n}^1 dx_{n-1}$$

```
Integrate[1, {x1, 0, 1}, {x2, x1, 1}]
```

$\frac{1}{2}$

```
n = 5;
```

```
Integrate1[1, Table[{x[i], x[i - 1], 1}, {i, 1, n}]]
```

```
Integrate1[1, {{x[1], 0, 1},
   {x[2], x[1], 1}, {x[3], x[2], 1},
   {x[4], x[3], 1}, {x[5], x[4], 1}}]
```

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a___} :> a
```

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Integrate1[1, {{x[1], 0, 1},
   {x[2], x[1], 1}, {x[3], x[2], 1},
   {x[4], x[3], 1}, {x[5], x[4], 1}}]
```

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

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$$\frac{1}{2}$$

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n = 5;
```

```
Integrate1[1, Sequence @@ Table[{x[i], x[i - 1], 1}, {i, 1, n}]]
```

Integrate1(1, {x(1), 0, 1}, {x(2), x(1), 1}, {x(3), x(2), 1}, {x(4), x(3), 1}, {x(5), x(4), 1})

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
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```
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```

```
Integrate[1, Sequence @@ Table[{x[i], x[i - 1], 1}, {i, 1, n}]]
```

$\frac{1}{120}$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

$$\equiv \int_0^1 dx_1 \int_{x_1}^1 dx_2 \ldots \int_{x_n}^1 dx_{n-1}$$

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Integrate[1, {x1, 0, 1}, {x2, x1, 1}]
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```
n = 5;
```

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```

$\frac{1}{120}$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

$$\frac{1}{120}$$

2

```
eqn = {k/(k+2) y[k+1] - y[k] + (k+2)/(2 k) y[k-1] == 0, y[0] == 0, y[1] == 1};
```

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2\,k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

3

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a___} :> a
```

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```

```
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```

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

1

```
{{P, o, {r, t, {}, {{o}, W}}}, i, {n, {e}}} //. {a___, {b___}, c___} :> {a, b, c}
```

$\{P, o, r, t, o, W, i, 5, e\}$

$$\equiv \int_0^1 dx_1 \int_{x_1}^1 dx_2 \ldots \int_{x_n}^1 dx_{n-1}$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

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```

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2\,k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

3

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

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In
IncidenceGraph
IncidenceMatrix
IncludeConstantBasis
IncludeFileExtension
IncludePods
IncludeSingularTerm
Increment
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IndentingNewlineSpacings
IndentMaxFraction
IndependentEdgeSetQ
IndependentVertexSetQ
Indeterminate
IndexCreationOptions
IndexGraph
IndexTag
Inequality
InexactNumberQ
InexactNumbers
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Information
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[k] + $\frac{k+2}{2k}$ y[k - 1] == 0, y[0] == 0, y[1] == 1};

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

2

$$\text{eqn} = \left\{\frac{k}{k+2}\, y[k+1] - y[k] + \frac{k+2}{2\,k}\, y[k-1] == 0,\ y[0] == 0,\ y[1] == 1\right\};$$

3

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RSolve - Wolfram Mathematica

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RSolve

RSolve[*eqn*, *a*[*n*], *n*]
solves a recurrence equation for *a*[*n*].

RSolve[{*eqn*$_1$, *eqn*$_2$, …}, {*a*$_1$[*n*], *a*$_2$[*n*], …}, *n*]
solves a system of recurrence equations.

RSolve[*eqn*, *a*[*n*$_1$, *n*$_2$, …], {*n*$_1$, *n*$_2$, …}]
solves a partial recurrence equation.

MORE INFORMATION

EXAMPLES

Basic Examples

Solve a difference equation:

In[1]:= RSolve[a[n + 1] - 2 a[n] == 1, a[n], n]

Out[1]= {{a[n] → -1 + 2^n + 2^(-1+n) C[1]}}

Include a boundary condition:

In[1]:= RSolve[{a[n + 1] - 2 a[n] == 1, a[0] == 1}, a[n], n]

2

$$eqn = \{\frac{k}{k+2}$$

RSolve

3

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x0sol.nb *

2

```
eqn = {k/(k + 2) y[k + 1] - y[k] + (k + 2)/(2 k) y[k - 1] == 0, y[0] == 0, y[1] == 1};
```

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```
RSolve
```

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eqn = {k/(k + 2) y[k + 1] - y[k] + (k + 2)/(2 k) y[k - 1] == 0, y[0] == 0, y[1] == 1};
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? RS*
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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

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RSolve[*eqn*, $a[n_1, n_2, \ldots]$, {n_1, n_2, …}] solves a partial recurrence equation. >>

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

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```
RSolve[eqn, y[k], k]
```

3

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

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RSolve[*eqn*, *a*[*n*$_1$, *n*$_2$, …], {*n*$_1$, *n*$_2$, …}] solves a partial recurrence equation. >>

```
RSolve[eqn, y[k], k]
```

$$\left\{\left\{y(k) \to (1+i)\, 2^{-k-2} (k+1) \left(i (1-i)^k k - (1+i)^k k + 2 (1-i)^{k-1} - (1+i)^{k+1}\right)\right\}\right\}$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

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```
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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

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RSolve[eqn, y[k], k]
```

$$\{\{y(k) \to (1+i)\, 2^{-k-2}\, (k+1)\, (i\,(1-i)^k\, k - (1+i)^k\, k + 2\,(1-i)^{k-1} - (1+i)^{k+1})\}\}$$

3

```
Sym[G @@ Range[4]]
```

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
? RS*
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3

```
G[1, 2
```

```
Sym[G @@ Range[4]]
```

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

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? RS*
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3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Sym[G @@ Range[4]]
```


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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

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3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Permutations[{1, 2, 3}]
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

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3

```
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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[*eqn*, *a*[n_1, n_2, …], {n_1, n_2, …}] solves a partial recurrence equation.

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RSolve[eqn, y[k], k]
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$$\left\{\left\{y(k)\to(1+i)\,2^{-k-2}\,(k+1)\left(i\,(1-i)^{k}\,k-(1+i)^{k}\,k+2\,(1-i)^{k-1}-(1+i)^{k+1}\right)\right\}\right\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
G @@@ Permutations[{1, 2, 3}]
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$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

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3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
G @@@ Permutations[{1, 2, 3}]
```

$\{G(1, 3, 2), G(1, 3, 2), G(2, 1, 3), G(2, 3, 1), G(3, 1, 2), G(3, 2, 1)\}$

```
Sym[G @@ Range[4]]
```

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[*eqn*, *a*[*n*₁, *n*₂, …], {*n*₁, *n*₂, …}] solves a partial recurrence equation. »

```
RSolve[eqn, y[k], k]
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$$\left\{\left\{y(k)\to(1+i)\,2^{-k-2}\,(k+1)\left(i\,(1-i)^{k}\,k-(1+i)^{k}\,k+2\,(1-i)^{k-1}-(1+i)^{k+1}\right)\right\}\right\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
G @@@ Permutations[{1, 2, 3}]
```

$\{G(1, 2, 3), G(1, 3, 2), G(2, 1, 3), G(2, 3, 1), G(3, 1, 2), G(3, 2, 1)\}$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

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RSolve[*eqn*, *a*[*n*₁, *n*₂, ...], {*n*₁, *n*₂, ...}] solves a partial recurrence equation.

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3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Permutations[{1, 2, 3}]
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$$

```
G @@@ Permutations[{1, 2, 3}]
```

$\{G(1, 2, 3), G(1, 3, 2), G(2, 1, 3), G(2, 3, 1), G(3, 1, 2), G(3, 2, 1)\}$

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

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```
RSolve[eqn, y[k], k]
```

$$\{\{y(k)\to(1+i)\,2^{-k-2}\,(k+1)\,(i\,(1-i)^{k}\,k-(1+i)^{k}\,k+2\,(1-i)^{k-1}-(1+i)^{k+1})\}\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Permutations[{1, 2, 3}]
```

{{1, 2, 3}, {1, 3, 2}, {2, 1, 3}, {2, 3, 1}, {3, 1, 2}, {3, 2, 1}}

```
G @@@ Permutations[{1, 2, 3}]
```

$\{G(1, 2, 3), G(1, 3, 2), G(2, 1, 3), G(2, 3, 1), G(3, 1, 2), G(3, 2, 1)\}$

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[*eqn*, *a*[*n*₁, *n*₂, …], {*n*₁, *n*₂, …}] solves a partial recurrence equation. »

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RSolve[eqn, y[k], k]
```

$$\{\{y(k)\to(1+i)\,2^{-k-2}\,(k+1)\,(i\,(1-i)^{k}\,k-(1+i)^{k}\,k+2\,(1-i)^{k-1}-(1+i)^{k+1})\}\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
G @@@ Permutations[{1, 2, 3}]
```

$\{G(1, 2, 3), G(1, 3, 2), G(2, 1, 3), G(2, 3, 1), G(3, 1, 2), G(3, 2, 1)\}$

```
Sym[G @@ Range[4]]
```

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x0sol.nb *

RSolve[eqn, a[n1, n2, ...], {n1, n2, ...}] solves a partial recurrence equation.

```
RSolve[eqn, y[k], k]
```

$$\{\{y(k) \to (1+i)\, 2^{-k-2} (k+1) (i (1-i)^k k - (1+i)^k k + 2 (1-i)^{k-1} - (1+i)^{k+1})\}\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
```

$G(1, 2, 3) + G(1, 3, 2) + G(2, 1, 3) + G(2, 3, 1) + G(3, 1, 2) + G(3, 2, 1)$

```
Sym[G @@ Range[4]]
```


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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[*eqn*, *a*[*n*₁, *n*₂, ...], {*n*₁, *n*₂, ...}] solves a partial recurrence equation.

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RSolve[eqn, y[k], k]
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3

```
G[1, 2, 3]
```

$G(1,2,3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
            Length[{1
```

$G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1)$

```
Sym[G @@ Range[4]]
```


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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[*eqn*, *a*[*n*₁, *n*₂, …], {*n*₁, *n*₂, …}] solves a partial recurrence equation. »

```
RSolve[eqn, y[k], k]
```

$$\left\{\left\{y(k)\to(1+i)\,2^{-k-2}\,(k+1)\left(i\,(1-i)^{k}\,k-(1+i)^{k}\,k+2\,(1-i)^{k-1}-(1+i)^{k+1}\right)\right\}\right\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
G[Plus @@ (G @@@ Permutations[{1, 2, 3}]) / Length[{1, 2, 3}]!]
```

$$\frac{1}{6}\left(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1)\right)$$

```
Sum[G @@ Range[4]]
```


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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[*eqn*, *a*[*n*₁, *n*₂, ...], {*n*₁, *n*₂, ...}] solves a partial recurrence equation.

```
RSolve[eqn, y[k], k]
```

$$\{\{y(k) \to (1+i)\, 2^{-k-2}\, (k+1)\left(i\,(1-i)^{k}\, k-(1+i)^{k}\, k+2\,(1-i)^{k-1}-(1+i)^{k+1}\right)\}\}$$

3

```
G[1, 2, 3]
```

$$G(1, 2, 3)$$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
         Length[{1, 2, 3}]!
```

$$\frac{1}{6}\,(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[eqn, a[n1, n2, ...], {n1, n2, ...}] solves a partial recurrence equation.

```
RSolve[eqn, y[k], k]
```

$$\{\{y(k) \to (1+i)\, 2^{-k-2} (k+1) (i (1-i)^k k - (1+i)^k k + 2 (1-i)^{k-1} - (1+i)^{k+1})\}\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
          Length[{1, 2, 3}]!
```

$$\frac{1}{6} (G(1, 2, 3) + G(1, 3, 2) + G(2, 1, 3) + G(2, 3, 1) + G(3, 1, 2) + G(3, 2, 1))$$

```
Total@(G @@@ Permutations[{1, 2, 3}])
```

$$G(1, 2, 3) + G(1, 3, 2) + G(2, 1, 3) + G(2, 3, 1) + G(3, 1, 2) + G(3, 2, 1)$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[*eqn*, *a*[*n*₁, *n*₂, …], {*n*₁, *n*₂, …}] solves a partial recurrence equation. »

```
RSolve[eqn, y[k], k]
```

$$\{\{y(k) \to (1+i)\, 2^{-k-2}\, (k+1) \left(i\,(1-i)^k\, k - (1+i)^k\, k + 2\,(1-i)^{k-1} - (1+i)^{k+1}\right)\}\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
          Length[{1, 2, 3}] !
```

$$\frac{1}{6}\,(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$$

```
Sym[G @@ Range[4]]
```

3*

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[eqn, a[n1, n2, ...], {n1, n2, ...}] solves a partial recurrence equation. »

```
RSolve[eqn, y[k], k]
```

$\{\{y(k) \to (1+i)\, 2^{-k-2}\, (k+1)\, (i\, (1-i)^k\, k - (1+i)^k\, k + 2\, (1-i)^{k-1} - (1+i)^{k+1})\}\}$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
          Length[{1, 2, 3}]!
```

$\frac{1}{6}\,(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
```

$G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1)$

```
Sym[G @@ Range[4]]
```

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[*eqn*, *a*[*n*1, *n*2, ...], {*n*1, *n*2, ...}] solves a partial recurrence equation. »

```
RSolve[eqn, y[k], k]
```

$$\left\{\left\{y(k)\to(1+i)\,2^{-k-2}\,(k+1)\left(i\,(1-i)^{k}\,k-(1+i)^{k}\,k+2\,(1-i)^{k-1}-(1+i)^{k+1}\right)\right\}\right\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
          Length[{1, 2, 3}]!
```

$$\frac{1}{6}\left(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1)\right)$$

```
Sym[G @@ Range[4]]
```

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

RSolve[*eqn*, *a*[*n*₁, *n*₂, ...], {*n*₁, *n*₂, ...}] solves a partial recurrence equation.

```
RSolve[eqn, y[k], k]
```

$$\{\{y(k) \to (1+i)\,2^{-k-2}\,(k+1)\,(i\,(1-i)^k\,k - (1+i)^k\,k + 2\,(1-i)^{k-1} - (1+i)^{k+1})\}\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
        Length[{1, 2, 3}]!
```

$$\frac{1}{6}\,(G(1,2,3) + G(1,3,2) + G(2,1,3) + G(2,3,1) + G(3,1,2) + G(3,2,1))$$

```
                  Total@(G @@@ Permutations[{a1}])
Sym[G_[a1__]] := ----------------------------------
                  Length[Permutations[{a1}]]
```

```
Sym[G @@ Range[4]]
```

3*

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
RSolve[eqn, y[k], k]
```

$$\{\{y(k)\to(1+i)\,2^{-k-2}(k+1)\left(i(1-i)^k k-(1+i)^k k+2(1-i)^{k-1}-(1+i)^{k+1}\right)\}\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

$$\frac{\text{Plus @@ (G @@@ Permutations[\{1, 2, 3\}])}}{\text{Length[\{1, 2, 3\}]!}}$$

$$\frac{1}{6}(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$$

$$\text{Sym[G\_[a1\_\_]] := }\frac{\text{Total@(G @@@ Permutations[\{a1\}])}}{\text{Length[Permutations[\{a1\}]]!}}$$

```
Sym[G @@ Range[4]]
```

3*

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

$$\{\{y(k) \to (1+i)\,2^{-k-2}\,(k+1)\,(i\,(1-i)^k\,k-(1+i)^k\,k+2\,(1-i)^{k-1}-(1+i)^{k+1})\}\}$$

3

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
        Length[{1, 2, 3}]!
```

$$\frac{1}{6}\,(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$$

```
                   Total@(G @@@ Permutations[{a1}])
Sym[G_[a1__]] := -----------------------------------
                   Length[Permutations[{a1}]]!
```

```
Sym[G @@ Range[4]]
```

$$(G(1,2,3,4)+G(1,2,4,3)+G(1,3,2,4)+G(1,3,4,2)+G(1,4,2,3)+G(1,4,3,2)+G(2,1,3,4)+ G(2,1,4,3)+G(2,3,1,4)+G(2,3,4,1)+G(2,4,1,3)+G(2,4,3,1)+G(3,1,2,4)+G(3,1,4,2)+ G(3,2,1,4)+G(3,2,4,1)+G(3,4,1,2)+G(3,4,2,1)+G(4,1,2,3)+G(4,1,3,2)+ G(4,2,1,3)+G(4,2,3,1)+G(4,3,1,2)+G(4,3,2,1))\,/\,620\,448\,401\,733\,239\,439\,360\,000$$

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
G[1, 2, 3]
```

G(1, 2, 3)

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
          Length[{1, 2, 3}]!
```

$\frac{1}{6}(G(1, 2, 3) + G(1, 3, 2) + G(2, 1, 3) + G(2, 3, 1) + G(3, 1, 2) + G(3, 2, 1))$

```
Sym[G_[a1__]] := Total@(G @@@ Permutations[{a1}])
                 --------------------------------
                 Length[Permutations[{a1}]]!
```

```
Sym[G @@ Range[4]]
```

(G(1, 2, 3, 4) + G(1, 2, 4, 3) + G(1, 3, 2, 4) + G(1, 3, 4, 2) + G(1, 4, 2, 3) + G(1, 4, 3, 2) + G(2, 1, 3, 4) + G(2, 1, 4, 3) + G(2, 3, 1, 4) + G(2, 3, 4, 1) + G(2, 4, 1, 3) + G(2, 4, 3, 1) + G(3, 1, 2, 4) + G(3, 1, 4, 2) + G(3, 2, 1, 4) + G(3, 2, 4, 1) + G(3, 4, 1, 2) + G(3, 4, 2, 1) + G(4, 1, 2, 3) + G(4, 1, 3, 2) + G(4, 2, 1, 3) + G(4, 2, 3, 1) + G(4, 3, 1, 2) + G(4, 3, 2, 1)) / 620 448 401 733 239 439 360 000

3*

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
         Length[{1, 2, 3}]!
```

$$\frac{1}{6}(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$$

```
                Total@(G @@@ Permutations[{a1}])
Sym[G_[a1__]] := --------------------------------
                Length[Permutations[{a1}]]
```

```
Sym[G @@ Range[4]]
```

$$\frac{1}{24}(G(1,2,3,4)+G(1,2,4,3)+G(1,3,2,4)+G(1,3,4,2)+G(1,4,2,3)+G(1,4,3,2)+G(2,1,3,4)+G(2,1,4,3)+G(2,3,1,4)+G(2,3,4,1)+G(2,4,1,3)+G(2,4,3,1)+G(3,1,2,4)+G(3,1,4,2)+G(3,2,1,4)+G(3,2,4,1)+G(3,4,1,2)+G(3,4,2,1)+G(4,1,2,3)+G(4,1,3,2)+G(4,2,1,3)+G(4,2,3,1)+G(4,3,1,2)+G(4,3,2,1))$$

3*

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
          Length[{1, 2, 3}]!
```

$$\frac{1}{6}(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$$

```
                   Total@(G @@@ Permutations[{a1}])
Sym[G_[a1__]] := ---------------------------------
                   Length[Permutations[{a1}]]
```

```
Sym[1, 2, 2, 3]
```

Sym(1, 2, 2, 3)

3*

```
ASym[G[2, 1, 3, 4]]
```

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
          Length[{1, 2, 3}]!
```

$$\frac{1}{6}(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$$

```
                  Total@(G @@@ Permutations[{a1}])
Sym[G_[a1__]] := ----------------------------------
                  Length[Permutations[{a1}]]
```

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
G[1, 2, 3]
```

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
         Length[{1, 2, 3}]!
```

$$\frac{1}{6}(G(1, 2, 3) + G(1, 3, 2) + G(2, 1, 3) + G(2, 3, 1) + G(3, 1, 2) + G(3, 2, 1))$$

```
                 Total@(G @@@ Permutations[{a1}])
Sym[G_[a1__]] := --------------------------------
                   Length[Permutations[{a1}]]
```

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1, 2, 2, 3) + G(1, 2, 3, 2) + G(1, 3, 2, 2) + G(2, 1, 2, 3) + G(2, 1, 3, 2) + G(2, 2, 1, 3) + G(2, 2, 3, 1) + G(2, 3, 1, 2) + G(2, 3, 2, 1) + G(3, 1, 2, 2) + G(3, 2, 1, 2) + G(3, 2, 2, 1))$$

```
3!
```

6

3*

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
        Length[{1, 2, 3}] !
```

$\frac{1}{6}(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$

```
                 Total@(G @@@ Permutations[{a1}])
Sym[G_[a1__]] := --------------------------------
                 Length[Permutations[{a1}]]
```

```
Sym[G[1, 2, 2, 3]]
```

$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$

```
3 !
```

6

3*

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
        Length[{1, 2, 3}]!
```

$$\frac{1}{6}(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$$

```
                 Total@(G @@@ Permutations[{a1}])
Sym[G_[a1__]] := --------------------------------
                   Length[Permutations[{a1}]]
```

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$$

3*

```
ASym[G[2, 1, 3, 4]]
```


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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

$G(1, 2, 3)$

```
Plus @@ (G @@@ Permutations[{1, 2, 3}])
---------------------------------------
        Length[{1, 2, 3}]!
```

$$\frac{1}{6}(G(1,2,3)+G(1,3,2)+G(2,1,3)+G(2,3,1)+G(3,1,2)+G(3,2,1))$$

```
                 Total@(G @@@ Permutations[{a1}])
Sym[G_[a1__]] := ---------------------------------
                 Length[Permutations[{a1}]]
```

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$$

3*

```
ASym[G[2, 1, 3, 4]]
```


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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$$

3*

```
ASym[G[2, 1, 3, 4]]
```

4

```
Y[j, u+1] Y[j, u-1] == (1+Y[j-1, u]) (1+Y[j+1, u])
```

$${}_{n-j+1}(u) = Y_j(u+n+3)$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$$

3*

```
Plus @@ ((G @@@ Permutations[{1, 2, 3}]) /. G[a__] :> Signature[{a}] G[a])
```

$$G(1,2,3)-G(1,3,2)-G(2,1,3)+G(2,3,1)+G(3,1,2)-G(3,2,1)$$

```
ASym[G[2, 1, 3, 4]]
```

4

```
Y[j, u+1] Y[j, u-1] == (1+Y[j-1, u]) (1+Y[j+1, u])
```

$$_{n-j+1}(u) = Y_j(u+n+3)$$

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$$

3*

```
ASym[G[2, 1, 3, 4]]
```

4

```
Y[j, u+1] Y[j, u-1] = (1+Y[j-1, u]) (1+Y[j+1, u])
```

$_{n-j+1}(u) = Y_j(u+n+3)$

5

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$$

3*

```
Plus @@ ((G @@@ Permutations[{1, 2, 3}]) /. G[a__] :> Signature[{a}] G[a])
```

$$G(1,2,3)-G(1,3,2)-G(2,1,3)+G(2,3,1)+G(3,1,2)-G(3,2,1)$$

```
ASym[G[2, 1, 3, 4]]
```

4

```
Y[j, u+1] Y[j, u-1] == (1 + Y[j-1, u]) (1 + Y[j+1, u])
```

$${}_{n-j+1}(u) = Y_j(u+n+3)$$

Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}\left(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1)\right)$$

3*

```
Plus @@ ((G @@@ Permutations[{1, 2, 3}]) /. G[a__] :> Signature[{a}] G[a])
```

$$G(1,2,3)-G(1,3,2)-G(2,1,3)+G(2,3,1)+G(3,1,2)-G(3,2,1)$$

$$G(1,2,3)-G(1,3,2)-G(2,1,3)+G(2,3,1)+G(3,1,2)-G(3,2,1)$$

```
ASym[G[2, 1, 3, 4]]
```

4

```
Y[j, u+1] Y[j, u-1] == (1+Y[j-1, u]) (1+Y[j+1, u])
```

$${}_{n-j+1}(u)=Y_j(u+n+3)$$

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Style Definitions for Day 1 - Solution - Basics of Mathematica Grammar.nb

x0sol.nb *

```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$$

3*

```
Plus @@ ((G @@@ Permutations[{1, 2, 3}]) /. G[a__] :> Signature[{a}] G[a])
```

$$G(1,2,3)-G(1,3,2)-G(2,1,3)+G(2,3,1)+G(3,1,2)-G(3,2,1)$$

$$G(1,2,3)-G(1,3,2)-G(2,1,3)+G(2,3,1)+G(3,1,2)-G(3,2,1)$$

```
ASym[G_[a1__]] := Sym[G[a1]] /. G[a__] :> Signature[{a}]/Signature[{a1}] G[a]
```

```
ASym[G[2, 1, 3, 4]]
```

4

```
Y[j, u+1] Y[j, u-1] == (1 + Y[j-1, u]) (1 + Y[j+1, u])
```

$Y_{n-j-1}(u) = Y_j(u+n+3)$

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```
Sym[G[1, 2, 2, 3]]
```

$$\frac{1}{12}(G(1,2,2,3)+G(1,2,3,2)+G(1,3,2,2)+G(2,1,2,3)+G(2,1,3,2)+G(2,2,1,3)+G(2,2,3,1)+G(2,3,1,2)+G(2,3,2,1)+G(3,1,2,2)+G(3,2,1,2)+G(3,2,2,1))$$

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```
Plus @@ ((G @@@ Permutations[{1, 2, 3}]) /. G[a__] :> Signature[{a}] G[a])
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```
ASym[G_[a1__]] := Sym[G[a1]] /. G[a__] :> Signature[{a}]/Signature[{a1}] G[a]
```

```
ASym[G[2, 1, 3, 4]]
```

$$\frac{1}{24}(-G(1,2,3,4)+G(1,2,4,3)+G(1,3,2,4)-G(1,3,4,2)-G(1,4,2,3)+G(1,4,3,2)+G(2,1,3,4)-G(2,1,4,3)-G(2,3,1,4)+G(2,3,4,1)+G(2,4,1,3)-G(2,4,3,1)-G(3,1,2,4)+G(3,1,4,2)+G(3,2,1,4)-G(3,2,4,1)-G(3,4,1,2)+G(3,4,2,1)+G(4,1,2,3)-G(4,1,3,2)-G(4,2,1,3)+G(4,2,3,1)+G(4,3,1,2)-G(4,3,2,1))$$

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$$G(4,1,2,3) - G(4,1,3,2) - G(4,2,1,3) + G(4,2,3,1) + G(4,3,1,2) - G(4,3,2,1))$$

4

```
Y[j, u + 1] Y[j, u - 1] == (1 + Y[j - 1, u]) (1 + Y[j + 1, u])
```

$${}_{n-j+1}(u) = Y_j(u + n + 3)$$

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]]
```

$$\sin(3(u+\pi)) + \cos(3(u+\pi))$$

```
ee = -Sin[3 u] - Cos[3 u];
```

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```
Y[j, u+1] Y[j, u-1] == (1 + Y[j-1, u]) (1 + Y[j+1, u])
```

$Y(j, u-1)\,Y(j, u+1) = (Y(j-1, u)+1)(Y(j+1, u)+1)$

$_{n-j+1}(u) = Y_j(u+n+3)$

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```
Solve[Y[j, u + 1] Y[j, u - 1] == (1 + Y[j - 1, u]) (1 + Y[j + 1, u]), Y[j, u + 1]]
```

$$\left\{\left\{Y(j, u+1) \to \frac{(Y(j-1, u)+1)(Y(j+1, u)+1)}{Y(j, u-1)}\right\}\right\}$$

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 u → u - 1
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```

$$\left\{Y[j, u] \to \frac{(Y[j-1, u-1]-1)\,(Y[j-1, u-1]-1)}{Y[j, u-2]}\right\}$$

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```
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```

$$\left\{Y(j,u)\to\frac{(Y(j-1,u-1)+1)(Y(j+1,u-1)+1)}{Y(j,u-2)}\right\}$$

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```


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```
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 u -> u-1
```

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$Y_{n-j+1}(u) = Y_j(u+n+3)$

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✓ TraditionalForm Shift+Ctrl+T
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Merge Cells Shift+Ctrl+M
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Notebook History...
Text Display
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Show Expression Shift+Ctrl+E
✓ TraditionalForm Display

$G(4.$ … $(4, 3, 2, 1))$

4

```
Solve[Y… , Y[j, u+1]][[1]] /.
 u → u - …
```

$\{Y(j, u) \to$ … $Y(j, u-2)$

$\ldots_{n-j+1}(u) = Y_j(u+n+3)$

$$\left\{Y(j, u) \to \frac{(Y(j-1, u-1)+1)(Y(j+1, u-1)+1)}{Y(j, u-2)}\right\}$$

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```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]]
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```
Solve[Y[j, u+1] Y[j, u-1] == (1+Y[j-1, u]) (1+Y[j+1, u]), Y[j, u+1]][[1]] /.
 u → u-1
```

```
{Y[j, u] -> ((Y[j - 1, u - 1] + 1)*
    (Y[j + 1, u - 1] + 1))/Y[j, u - 2]}
```

$Y_{n-j+1}(u)=Y_j(u+n+3)$

$$\left\{Y(j,u)\to\frac{(Y(j-1,u-1)+1)(Y(j+1,u-1)+1)}{Y(j,u-2)}\right\}$$

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```
FullSimplify[Sin[3 (u+π)] + Cos[3 (u+π)]]
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```
n
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```
n = 5;
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$$Y\left[\left\{Y[j, u] \to \frac{(Y[j-1, u-1]+1)(Y[j+1, u-1]+1)}{Y[j, u-2]}\right\}\right.$$

$$\text{rule} = \left\{Y[j\_, u\_] :> \frac{(Y[j-1, u-1]+1)\,(Y[j+1, u-1]+1)}{Y[j, u-2]} \;/;\; u > 0\right\};$$

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$G(4,1,2,3) - G(4,1,3,2) - G(4,2,1,3) + G(4,2,3,1) + G(4,3,1,2) - G(4,3,2,1))$

4

```
Solve[Y[j, u + 1] Y[j, u - 1] == (1 + Y[j - 1, u]) (1 + Y[j + 1, u]), Y[j, u + 1]][[1]] /.
 u → u - 1
```

$$\left\{Y[j,u] \to \frac{(Y[j-1,u-1]+1)\,(Y[j+1,u-1]+1)}{Y[j,u-2]}\right\}$$

$Y_{n-j+1}(u) = Y_j(u+n+3)$

```
n = 5;
```

$$\text{rule} = \left\{Y[j\_,u\_] :> \frac{(Y[j-1,u-1]+1)\,(Y[j+1,u-1]+1)}{Y[j,u-2]} \;/;\; u > 0\right\};$$

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]]
```


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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

$$G(4,1,2,3) - G(4,1,3,2) - G(4,2,1,3) + G(4,2,3,1) + G(4,3,1,2) - G(4,3,2,1))$$

4

```
Solve[Y[j, u+1] Y[j, u-1] == (1+Y[j-1, u]) (1+Y[j+1, u]), Y[j, u+1]][[1]] /.
 u → u-1
```

$$\left\{Y[j,u] \to \frac{(Y[j-1,u-1]+1)(Y[j+1,u-1]+1)}{Y[j,u-2]}\right\}$$

$$_{n-j+1}(u) = Y_j(u+n+3)$$

```
n = 5;
```

```
rule = {Y[j_, u_] :> (Y[j-1, u-1]+1) (Y[j+1, u-1]+1) / Y[j, u-2] /; u > 0};
```

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n+1
```

5

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u → u - 1

$$\left\{Y[j, u] \to \frac{(Y[j-1, u-1]+1)\,(Y[j+1, u-1]+1)}{Y[j, u-2]}\right\}$$

$Y_{n-j+1}(u) = Y_j(u+n+3)$

```
n = 5;
```

$$\text{rule} = \left\{Y[j\_, u\_] :> \frac{(Y[j-1, u-1]+1)\,(Y[j+1, u-1]+1)}{Y[j, u-2]} \;/;\; u > 0\right\};$$

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]]
```

$\sin(3(u+\pi)) + \cos(3(u+\pi))$

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

u → u - 1

$$\left\{Y[j,u] \to \frac{(Y[j-1,u-1]+1)\,(Y[j+1,u-1]+1)}{Y[j,u-2]}\right\}$$

$$_{n-j+1}(u) = Y_j(u+n+3)$$

```
n = 5;
```

$$\text{rule} = \left\{Y[j\_,u\_] :> \frac{(Y[j-1,u-1]+1)\,(Y[j+1,u-1]+1)}{Y[j,u-2]} \;/;\; u > 0\right\};$$

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

$$\text{Table}\left[\frac{Y[n-j+1,1]}{Y[j,1+n+3]} \;//.\; \text{rule} \;//\; \text{Factor},\ \{j,1,n\}\right]$$

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]]
```

$\sin(3(u+\pi)) + \cos(3(u+\pi))$

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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```
u → u - 1
```

$$\left\{Y[j,u] \to \frac{(Y[j-1,u-1]+1)\,(Y[j+1,u-1]+1)}{Y[j,u-2]}\right\}$$

$Y_{n-j+1}(u) = Y_j(u+n+3)$

```
n = 5;
```

$$\text{rule} = \left\{Y[j\_, u\_] :> \frac{(Y[j-1,u-1]+1)\,(Y[j+1,u-1]+1)}{Y[j,u-2]} \ /;\ u > 0\right\};$$

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

$$\frac{Y[n-j+1, 1]}{Y[j, 1+n+3]}$$

$$\text{Table}\left[\frac{Y[n-j+1, 1]}{Y[j, 1+n+3]} \text{ //. rule // Factor}, \{j, 1, n\}\right]$$

5

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$$\left\{Y[j,u] \to \frac{(Y[j-1,u-1]-1)\,(Y[j-1,u-1]-1)}{Y[j,u-2]}\right\}$$

$$Y_{n-j+1}(u) = Y_j(u+n+3)$$

```
n = 5;
```

$$\text{rule} = \left\{Y[j\_, u\_] :> \frac{(Y[j-1, u-1]+1)\,(Y[j+1, u-1]+1)}{Y[j, u-2]} \;/;\; u > 0\right\};$$

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

$$\frac{Y[n-j+1, 1]}{Y[j, 1+n+3]}$$

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]]
```

$$\sin(3(u+\pi)) + \cos(3(u+\pi))$$

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x0sol.nb *

```
Solve[Y[j, u+1] Y[j, u-1] == (1+Y[j-1, u]) (1+Y[j+1, u]), Y[j, u+1]][[1]] /.
 u → u-1
```

$$\left\{Y[j, u] \to \frac{(Y[j-1, u-1]+1)\,(Y[j+1, u-1]+1)}{Y[j, u-2]}\right\}$$

$$Y_{n-j+1}(u) = Y_j(u+n+3)$$

```
n = 5;
```

$$rule = \left\{Y[j\_, u\_] :> \frac{(Y[j-1, u-1]+1)\,(Y[j+1, u-1]+1)}{Y[j, u-2]} \;/;\; u > 0\right\};$$

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n+1
```

$$\frac{Y[n-j+1, 1]}{Y[j, 1+n+3]}$$

5

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```
u → u - 1
```

$$\left\{Y[j, u] \to \frac{(Y[j-1, u-1]+1)\,(Y[j+1, u-1]+1)}{Y[j, u-2]}\right\}$$

$$Y_{n-j+1}(u) = Y_j(u+n+3)$$

```
n = 5;
```

$$\texttt{rule} = \left\{Y[j\_, u\_] :> \frac{(Y[j-1, u-1]+1)\,(Y[j+1, u-1]+1)}{Y[j, u-2]} \;/;\; u > 0\right\};$$

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

$$\frac{Y[n-j+1, 1]}{Y[j, 1+n+3]} \;/.\; \texttt{rule}$$

$$\frac{(Y(5-j, 0)+1)\,(Y(7-j, 0)+1)\,Y(j, 7)}{Y(6-j, -1)\,(Y(j-1, 8)+1)\,(Y(j+1, 8)+1)}$$

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```
u → u - 1
```

$$\left\{Y[j,u] \to \frac{(Y[j-1,u-1]+1)\,(Y[j+1,u-1]+1)}{Y[j,u-2]}\right\}$$

$Y_{n-j+1}(u) = Y_j(u+n+3)$

```
n = 5;
```

$$rule = \left\{Y[j\_, u\_] :> \frac{(Y[j-1,u-1]+1)\,(Y[j+1,u-1]+1)}{Y[j,u-2]} /; u > 0\right\};$$

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

$$\frac{Y[n-j+1,1]}{Y[j,1+n+3]} \;/.\; j \to 1 \;//.\; rule \;//\; Simplify$$

1

5

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x0sol.nb *

```
Table[Y[n - j + 1, 1]/Y[j, 1 + n + 3] //. rule // Simplify, {j, n}]
```

{1, 1, 1, 1, 1}

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]]
```

$\sin(3(u+\pi)) + \cos(3(u+\pi))$

```
ee = -Sin[3 u] - Cos[3 u];
```

6

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x0sol.nb *

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

```
Table[Y[n - j + 1, 1] / Y[j, 1 + n + 3] //. rule // Simplify, {j, n}]
```

{1, 1, 1, 1, 1}

5

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$\sin(3(u+\pi)) + \cos(3(u+\pi))$

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```


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ex0sol.nb *

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

```
Table[Y[n - j + 1, 1] / Y[j, 1 + n + 3] //. rule // Simplify, {j, n}]
```

$$\text{Table}\left[\frac{Y[n-j+1,1]}{Y[j,1+n+3]} \text{ //. rule // Simplify}, \{j, n\}\right]$$

{1, 1, 1, 1, 1}

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]]
```

```
ee = -Sin[3 u] - Cos[3 u];
```

6

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

```
Table[Y[n - j + 1, 1] / Y[j, 1 + n + 3] //. rule // Simplify, {j, n}]
```

{1, 1, 1, 1, 1}

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]]
```

$\sin(3(u+\pi)) + \cos(3(u+\pi))$

```
ee = -Sin[3 u] - Cos[3 u];
```


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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
n = 5;
Clear[Y];
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1
```

$$\text{Table}\left[\frac{Y[n - j + 1, 1]}{Y[j, 1 + n + 3]} \text{ //. rule // Simplify}, \{j, n\}\right]$$

```
{1, 1, 1, 1, 1}
```

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]] // ByteCount
```

```
424
```

```
ee = -Sin[3 u] - Cos[3 u];
```

```
ee // Byt
```

Byte
ByteCount
ByteOrdering

150%

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ex0sol.nb *

```
Y[j_, u_] := 0 /; j ≤ 0 || j ≥ n + 1

Table[Y[n - j + 1, 1] / Y[j, 1 + n + 3] //. rule // Simplify, {j, n}]

{1, 1, 1, 1, 1}
```

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)]] // ByteCount

424

ee = -Sin[3 u] - Cos[3 u];

ee // ByteCount

472
```

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ex0sol.nb *

$-\sin(3u) - \cos(3u)$

```
ee = -Sin[3 u] - Cos[3 u];
```

```
ee // ByteCount
```

472

6

```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

7

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ex0sol.nb *

{1, 1, 1, 1, 1}

5

```
FullSimplify[Sin[3 (u + π)] + Cos[3 (u + π)] // TrigToExp]
```

$-\sin(3u) - \cos(3u)$

```
ee = -Sin[3 u] - Cos[3 u];
```

```
ee // ByteCount
```

472

6

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

```
ee // ByteCount
```

472

6

```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

$$\frac{3+\sqrt{m^2+1}}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)}\ //\ \text{KSqr}$$

7

```
Clear[F];
```


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$$\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```

8

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x0sol.nb *

```
ee // ByteCount
```

472

6

$$e1 = \frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)};$$

$$\frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)} \text{ // KSqr}$$

7

```
Clear[F];
```

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

472

6

$$e1 = \frac{3+\sqrt{m^2+1}}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)};$$

$$\frac{3+\sqrt{m^2+1}}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)} \text{ // KSqr}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```


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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

472

6

```
KSqr[aa_] :=
  Block[{tmp}, (tmp = Factor[aa];
    Collect[Numerator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor] /
     Collect[Denominator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor])];
```

$$e1 = \frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)};$$

$$\frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)} \text{ // KSqr}$$

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6

```
KSqr[aa_] :=
  Block[{tmp}, (tmp = Factor[aa];
    Collect[Numerator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor] /
     Collect[Denominator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor])];
```

$$e1 = \frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)};$$

$$\frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)} \text{ // KSqr}$$

7

```
Clear[F];
F[1] = 1;
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F[n_] := F[n] = F[n - 1] + F[n - 2];
```


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6

```
KSqr[aa_] :=
  Block[{tmp}, (tmp = Factor[aa];
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      Sqrt[_], Factor] /
     Collect[Denominator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor])];
```

$$e1 = \frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)};$$

$$\frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)} \text{ // KSqr}$$

$$\frac{-2\,(m^2 - 2) + \sqrt{m^2 - 1}\left(m^2 + 2\sqrt{m^2 + 1} - 2\right) - 4\sqrt{m^2 + 1}}{m^2\,(m^2 - 5)}$$

7

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6

```
KSqr[aa_] :=
  Block[{tmp}, (tmp = Factor[aa];
    Collect[Numerator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor] /
     Collect[Denominator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor])];
```

$$e1 = \frac{3 + \sqrt{m\,\hat{}\,2 + 1}}{\left(1 + \sqrt{m\,\hat{}\,2 + 1}\right)\left(2 + \sqrt{m\,\hat{}\,2 - 1}\right)};$$

$$\frac{3 + \sqrt{m\,\hat{}\,2 + 1}}{\left(1 + \sqrt{m\,\hat{}\,2 + 1}\right)\left(2 + \sqrt{m\,\hat{}\,2 - 1}\right)} \;//\; \text{KSqr}$$

$$\frac{-2(m^2-2) + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1}}{m^2(m^2-5)}$$

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6

```
KSqr[aa_] :=
  Block[{tmp}, (tmp = Factor[aa];
    Collect[Numerator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor] /
     Collect[Denominator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor])];
```

$$e1 = \frac{3 + \sqrt{m^2+1}}{\left(1 + \sqrt{m^2+1}\right)\left(2 + \sqrt{m^2-1}\right)};$$

$$\frac{3 + \sqrt{m^2+1}}{\left(1 + \sqrt{m^2+1}\right)\left(2 + \sqrt{m^2-1}\right)} \; // \; KSqr$$

$$\frac{-2(m^2-2) + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1}}{m^2(m^2-5)}$$

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ex0sol.nb *

6

```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2(m^2-2)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2(m^2-5)}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
```


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x0sol.nb *

6

```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
e1
```

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

7

```
Clear[F];
```


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In[19]:=

e1 = $\frac{3+\sqrt{m^2+1}}{(1+\sqrt{m^2+1})(2+\sqrt{m^2-1})}$;

In[20]:=

e1

Out[20]=

$$\frac{\sqrt{m^2+1}+3}{\left(\sqrt{m^2-1}+2\right)\left(\sqrt{m^2+1}+1\right)}$$

In[19]:=

$\frac{3+\sqrt{m^2+1}}{(1+\sqrt{m^2+1})(2+\sqrt{m^2-1})}$ // KSqr

Out[19]=

$$\frac{-2(m^2-2)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2(m^2-5)}$$

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6

```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Numerator[e1]
```

$$\sqrt{m^2+1}+3$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2(m^2-2)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2(m^2-5)}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Numerator[e1]
DeNumerator[e1]
```

$$\sqrt{m^2+1}+3$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Numerator[e1]
Denominator[e1]
```

$$\sqrt{m^2+1}+3$$

$$\left(\sqrt{m^2-1}+2\right)\left(\sqrt{m^2+1}+1\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}$$

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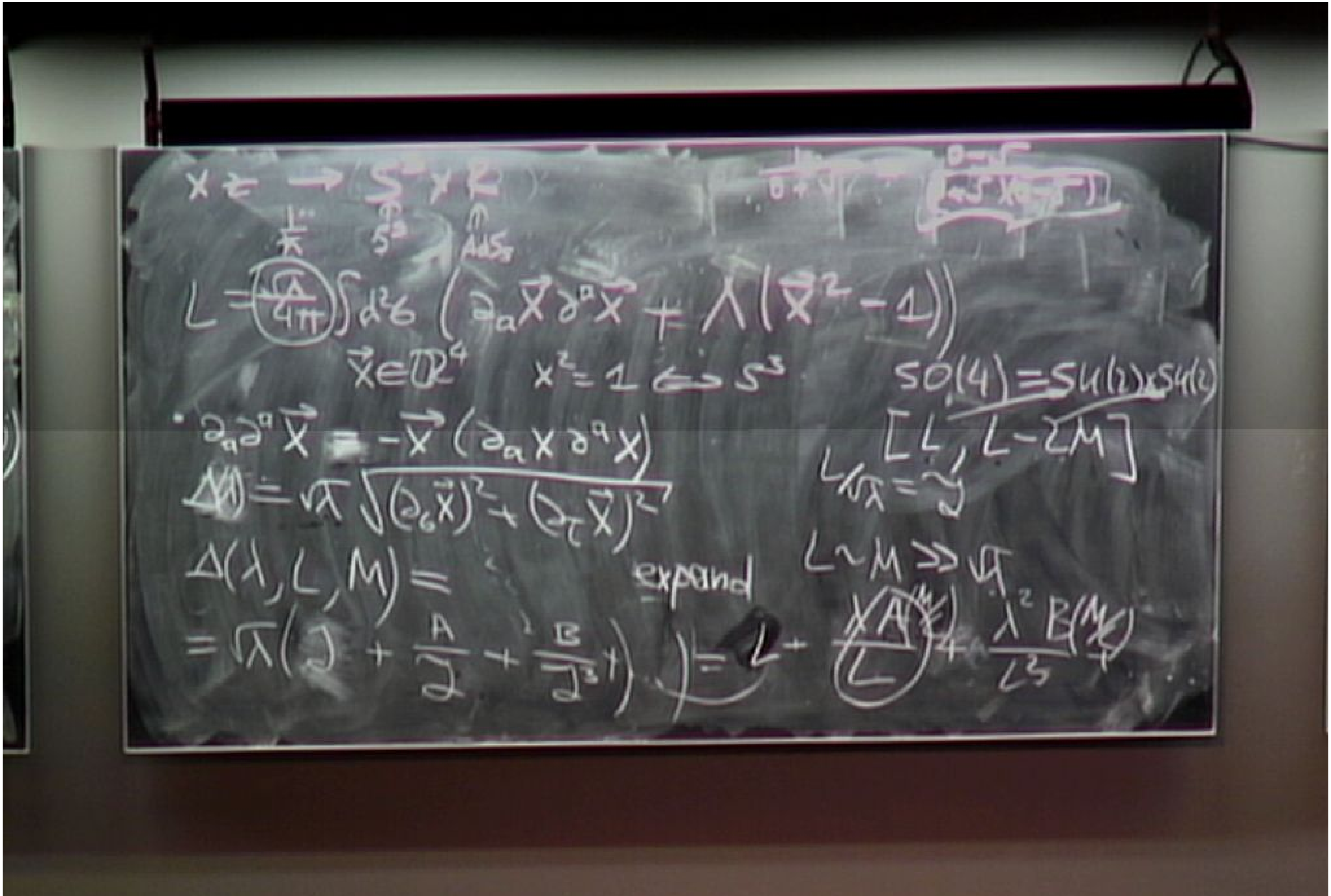
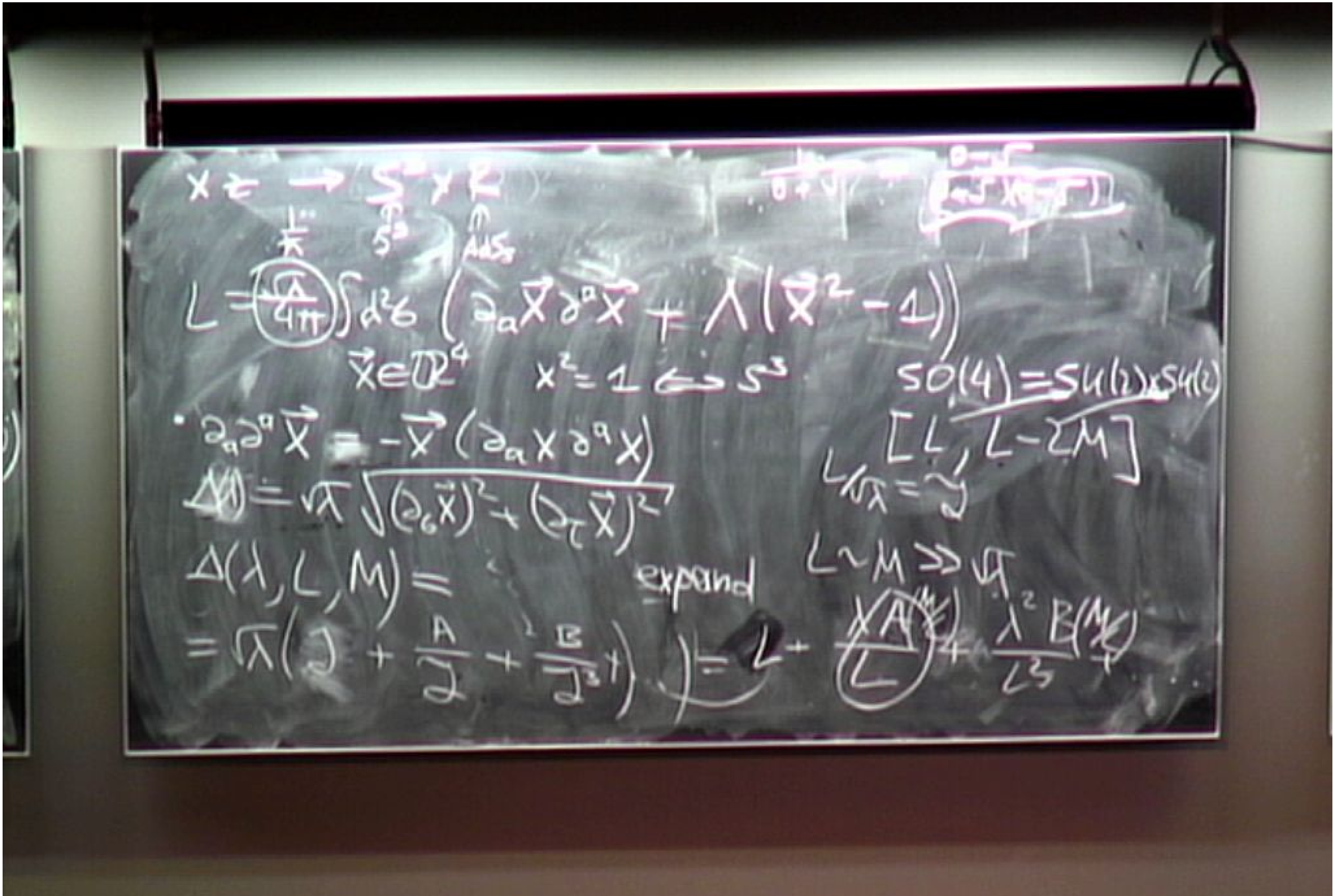

L = √λ/4π ∫ d²σ (∂ₐX ∂ᵃX + Λ(X² − 1))
X ∈ ℝ⁴   x² = 1 ⟺ S³
SO(4) = SU(2) x SU(2)
[L, L − ½M]
∂ₐ∂ᵃX = −X (∂ₐX ∂ᵃX)
Δ(λ) = √λ √((∂σX)² + (∂τX)²)
L/√λ = J
Δ(λ, L, M) =
expand
L ~ M ≫ √λ
= √λ (J + A/J + B/J³ + …)
= L + λA/L + λ²B/L³ + …


L = (√λ / 4π) ∫ d²σ (∂ₐX⃗ ∂ᵃX⃗ + Λ(X⃗² − 1))
X⃗ ∈ ℝ⁴   x² = 1 ⟺ S³
SO(4) = SU(2) × SU(2)
∂ₐ∂ᵃX⃗ = −X⃗ (∂ₐX ∂ᵃX)
Δ(λ, L, M) =
expand
L ~ M ≫ √λ

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Numerator[e1]
Denominator[e1]
```

$$\sqrt{m^2+1}+3$$

$$\left(\sqrt{m^2-1}+2\right)\left(\sqrt{m^2+1}+1\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$2(m^2-2)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Numerator[e1]
Denominator[e1]
```

$$\sqrt{m^2+1}+3$$

$$\left(\sqrt{m^2-1}+2\right)\left(\sqrt{m^2+1}+1\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$2(m^2-2)+\sqrt{m^2-1}\,(m^2+2\sqrt{m^2+1}-2)-4\sqrt{m^2+1}$$

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6

```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Numerator[e1]
Denominator[e1]
```

$$\sqrt{m^2+1}+3$$

$$\left(\sqrt{m^2-1}+2\right)\left(\sqrt{m^2+1}+1\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Numerator[e1]
Denominator[e1] /. Sqrt[_]
```

$$\sqrt{m^2+1}+3$$

$$\left(\sqrt{m^2-1}+2\right)\left(\sqrt{m^2+1}+1\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2\,(m^2-2)+\sqrt{m^2-1}\,\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\,(m^2-5)}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Numerator[e1]
Denominator[e1] /. Sqrt[a_] -> -
```

$$\sqrt{m^2+1}+3$$

$$\left(\sqrt{m^2-1}+2\right)\left(\sqrt{m^2+1}+1\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2(m^2-2)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2(m^2-5)}$$

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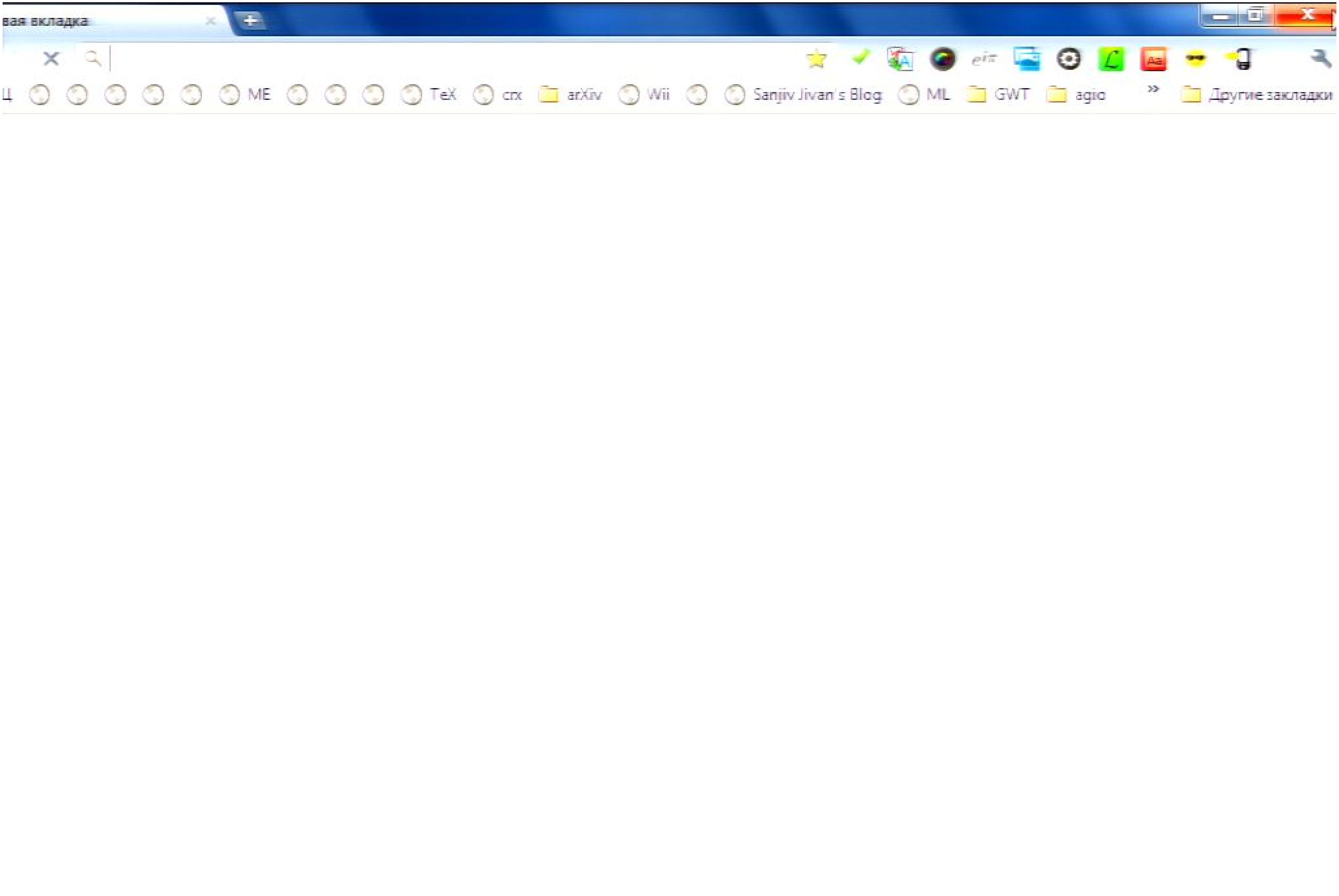
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$$e1 = \frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)};$$

Numerator[e1]

Denominator[e1] /. $\sqrt{a\_} \to -\sqrt{a}$

$$\sqrt{m^2 + 1} + 3$$

$$\left(\sqrt{m^2 - 1} + 2\right)\left(\sqrt{m^2 + 1} + 1\right)$$

$$\frac{3 + \sqrt{m^2 + 1}}{\left(1 + \sqrt{m^2 + 1}\right)\left(2 + \sqrt{m^2 - 1}\right)} \; // \; \text{KSqr}$$

$$\frac{-2(m^2 - 2) + \sqrt{m^2 - 1}\left(m^2 + 2\sqrt{m^2 + 1} - 2\right) - 4\sqrt{m^2 + 1}}{m^2 (m^2 - 5)}$$

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6

$$e1 = \frac{3+\sqrt{m^2+1}}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)};$$

```
Numerator[e1]
(Denominator[e1] /. Sqrt[a_] -> -Sqrt[a])
```

$$\sqrt{m^2+1}+3$$

$$\left(2-\sqrt{m^2-1}\right)\left(1-\sqrt{m^2+1}\right)$$

$$\frac{3+\sqrt{m^2+1}}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)} \text{ // KSqr}$$

$$\frac{-2(m^2-2)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2(m^2-5)}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Numerator[e1]
Collect[Denominator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_],
 Simplify]
```

$$\sqrt{m^2+1}+3$$

$$\left(2-\sqrt{m^2-1}\right)\left(1-\sqrt{m^2+1}\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}$$

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6

```
e1 = (3 + √(m^2 + 1)) / ((1 + √(m^2 + 1)) (2 + √(m^2 - 1)));
```

```
Numerator[e1]
Collect[Denominator[e1] (Denominator[e1] /. √a_ → -√a), √_,
 Simplify]
```

$$\sqrt{m^2+1}+3$$

$$m^2\,(m^2-5)$$

```
(3 + √(m^2 + 1)) / ((1 + √(m^2 + 1)) (2 + √(m^2 - 1))) // KSqr
```

$$-2\,(m^2-2)+\sqrt{m^2-1}\,\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));

Numerator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify]

Collect[Denominator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_],
 Simplify]
```

$$\sqrt{m^2+1}+3$$

$$m^2\left(m^2-5\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));

Collect[Numerator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify]
Collect[Denominator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_],
 Simplify]
```

$$-2 m^2 + \sqrt{m^2 - 1}\left(m^2 + 2\sqrt{m^2 + 1} - 2\right) - 4\sqrt{m^2 + 1} + 4$$

$$m^2 (m^2 - 5)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```


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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Collect[Numerator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify]
```

$$-2m^2+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}+4$$

$$m^2\left(m^2-5\right)$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Collect[Numerator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify] /
Collect[Denominator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify]
```

$$\frac{-2m^2 + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1} + 4}{m^2\left(m^2-5\right)}$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$-2\left(m^2-2\right) + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1}$$

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$$e1 = \frac{3 + \sqrt{m\verb|^|2 + 1}}{\left(1 + \sqrt{m\verb|^|2 + 1}\right)\left(2 + \sqrt{m\verb|^|2 - 1}\right)};$$

$$\frac{\text{Collect}\left[\text{Numerator[e1]}\left(\text{Denominator[e1]} \,/.\, \sqrt{a\_} \to -\sqrt{a}\right), \sqrt{\_}, \text{Simplify}\right]}{\text{Collect}\left[\text{Denominator[e1]}\left(\text{Denominator[e1]} \,/.\, \sqrt{a\_} \to -\sqrt{a}\right), \sqrt{\_}, \text{Simplify}\right]}$$

$$\frac{-2m^2 + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1} + 4}{m^2\left(m^2-5\right)}$$

$$\frac{3 + \sqrt{m\verb|^|2 + 1}}{\left(1 + \sqrt{m\verb|^|2 + 1}\right)\left(2 + \sqrt{m\verb|^|2 - 1}\right)} \; // \; \text{KSqr}$$

$$-2\left(m^2-2\right) + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Collect[Numerator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify] /
Collect[Denominator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify]
```

$$\frac{-2 m^2 + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1} + 4}{m^2\left(m^2-5\right)}$$

```
e1 // P
```

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```


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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Collect[Numerator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify] /
Collect[Denominator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify]
```

$$\frac{-2m^2+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}+4}{m^2\left(m^2-5\right)}$$

```
e1 // PowerExpand
```

$$\frac{\sqrt{m^2+1}+3}{\left(\sqrt{m^2-1}+2\right)\left(\sqrt{m^2+1}+1\right)}$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Collect[Numerator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify] /
Collect[Denominator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify]
```

$$\frac{-2m^2+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}+4}{m^2\left(m^2-5\right)}$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Collect[Numerator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify] /
Collect[Denominator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify]
```

$$\frac{-2m^2+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}+4}{m^2\left(m^2-5\right)}$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

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$$\frac{-2m^2+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}+4}{m^2\left(m^2-5\right)}$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```


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```
e1 = (3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1]));
```

```
Collect[Numerator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify] /
Collect[Denominator[e1] (Denominator[e1] /. Sqrt[a_] -> -Sqrt[a]), Sqrt[_], Simplify]
```

$$\frac{-2m^2 + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1} + 4}{m^2\left(m^2-5\right)}$$

```
(3 + Sqrt[m^2 + 1]) / ((1 + Sqrt[m^2 + 1]) (2 + Sqrt[m^2 - 1])) // KSqr
```

$$-2\left(m^2-2\right) + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1}$$

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```
KSqr[aa_] :=
  Block[{tmp}, (tmp = Factor[aa];
    Collect[Numerator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor] /
     Collect[Denominator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor])];
```

$$e1 = \frac{3 + \sqrt{m^2+1}}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)};$$

$$\frac{\text{Collect}\left[\text{Numerator}[e1]\left(\text{Denominator}[e1] /. \sqrt{a\_} \to -\sqrt{a}\right), \sqrt{\_}, \text{Simplify}\right]}{\text{Collect}\left[\text{Denominator}[e1]\left(\text{Denominator}[e1] /. \sqrt{a\_} \to -\sqrt{a}\right), \sqrt{\_}, \text{Simplify}\right]}$$

$$\frac{-2m^2 + \sqrt{m^2-1}\left(m^2 + 2\sqrt{m^2+1} - 2\right) - 4\sqrt{m^2+1} + 4}{m^2\left(m^2-5\right)}$$

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```
KSqr[aa_] :=
  Block[{tmp}, (tmp = Factor[aa];
    Collect[Numerator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor] /
     Collect[Denominator[tmp] (Denominator[tmp] /. Sqrt[aaa_] :> -Sqrt[aaa]),
      Sqrt[_], Factor])];
```

$$e1 = \frac{3+\sqrt{m\verb|^|2+1}}{\left(1+\sqrt{m\verb|^|2+1}\right)\left(2+\sqrt{m\verb|^|2-1}\right)};$$

$$\frac{\text{Collect}\left[\text{Numerator}[e1]\left(\text{Denominator}[e1]\,/.\,\sqrt{a\_}\to-\sqrt{a}\right),\sqrt{\_},\text{Simplify}\right]}{\text{Collect}\left[\text{Denominator}[e1]\left(\text{Denominator}[e1]\,/.\,\sqrt{a\_}\to-\sqrt{a}\right),\sqrt{\_},\text{Simplify}\right]}$$

$$\frac{-2m^2+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}+4}{m^2\left(m^2-5\right)}$$

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ex0sol.nb *

```
(1 + √(m^2 + 1)) (2 + √(m^2 - 1)) // KSqr
```

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```


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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

$$\frac{3+\sqrt{m^2+1}}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)} \text{ // KSqr}$$

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

$$\frac{3+\sqrt{m^2+1}}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)} \text{ // KSqr}$$

$$\frac{-2\,(m^2-2)+\sqrt{m^2-1}\,\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\,(m^2-5)}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] = F[n-1] + F[n-2];
```

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n-1] + F[n-2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

$$\frac{3+\sqrt{m^2+1}}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)} \text{ // KSqr}$$

$$\frac{-2(m^2-2)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2(m^2-5)}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] = F[n - 1] + F[n - 2];
```

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
  3 + √m^2 + 1
------------------------------------ // KSqr
(1 + √m^2 + 1) (2 + √m^2 - 1)
```

$$\frac{-2(m^2-2)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2(m^2-5)}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] = F[n - 1] + F[n - 2];
```

```
A
```

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

$$\frac{\ldots}{\left(1+\sqrt{m^2+1}\right)\left(2+\sqrt{m^2-1}\right)} \text{ // KSqr}$$

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] = F[n - 1] + F[n - 2];
```

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```


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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

$$\frac{\ldots}{m^2\,(m^2-5)}$$

7

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] = F[n - 1] + F[n - 2];
```

```
F[2400]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

((F(
2147 − 2) + Hold[F(2146 − 2) + F(2146 − 1)]) + F(
2148 − 2)) + F(2149 − 2)) + F(2150 − 2)) + F(2151 −
2)) + F(2152 − 2)) + F(2153 − 2)) + F(
2154 − 2)) + F(2155 − 2)) + F(2156 − 2)) +
F(2157 − 2)) + F(2158 − 2)) + F(2159 −
2)) + F(2160 − 2)) + F(2161 − 2)) + F(2162 −
2)) + F(2163 − 2)) + F(2164 − 2)) + F(2165 −
2)) + F(2166 − 2)) + F(2167 − 2)) + F(

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

2)) + F(2372 − 2)) + F(2373 − 2)) +
F(2374 − 2)) + F(2375 − 2)) + F(
2376 − 2)) + F(2377 − 2)) +
F(2378 − 2)) + F(2379 − 2)) +
F(2380 − 2)) + F(2381 − 2)) +
F(2382 − 2)) + F(2383 −
2)) +
F(2384 − 2)) + F(2385 −
2)) +
F(2386 − 2)) + F(
2387 −
2)) + F(
2388 − 2)) + F(2389 −
2)) + F(2390 −
2)) + F(2391 −
2)) + F(2392 − 2)) + F(
2393 − 2)) + F(2394 − 2)) + F(2395 − 2)) +
F(2396 − 2)) + F(2397 − 2)) + F(2398 − 2)) + F(
2399 − 2)) + F(2400 − 2)

```
Clear[F];
F[1] = 1;
```


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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
F[n_] = F[n - 1] + F[n - 2];
```

```
F[2400]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. »

((F(
2147 – 2) + Hold[F(2146 – 2) + F(2146 – 1)]) + F(
2148 – 2)) + F(2149 – 2)) + F(2150 – 2)) + F(2151 –
2)) + F(2152 – 2)) + F(2153 – 2)) + F(
2154 – 2)) + F(2155 – 2)) + F(2156 – 2)) +
F(2157 – 2)) + F(2158 – 2)) + F(2159 –
2)) + F(2160 – 2)) + F(2161 – 2)) + F(2162 –
2)) + F(2163 – 2)) + F(2164 – 2)) + F(2165 –
2)) + F(2166 – 2)) + F(2167 – 2)) + F(
2168 – 2)) + F(2169 – 2)) + F(2170 – 2)) +
F(2171 – 2)) + F(2172 – 2)) + F(2173 –
2)) + F(2174 – 2)) + F(2175 – 2)) + F(2176 –
2)) + F(2177 – 2)) + F(2178 – 2)) + F(2179 –
2)) + F(2180 – 2)) + F(2181 – 2)) + F(2182 –
2)) + F(2183 – 2)) + F(2184 – 2)) + F(2185 –
2)) + F(2186 – 2)) + F(2187 – 2)) + F(2188 –
2)) + F(2189 – 2)) + F(2190 – 2)) + F(2191 –
2)) + F(2192 – 2)) + F(2193 – 2)) + F(2194 –

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
F[2400]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

$$\frac{-2\left(m^2-2\right)+\sqrt{m^2-1}\left(m^2+2\sqrt{m^2+1}-2\right)-4\sqrt{m^2+1}}{m^2\left(m^2-5\right)}$$

7

```
Clear[F];
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```
F[2400]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

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F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

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```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```


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Volfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

```
Clear[F];
F[1] = 1;
F[2] = 1;
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```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```


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Volfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

```
Clear[F];
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F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

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```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```


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Wolfram Mathematica 8.0 - [Running...ex0sol.nb *]

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Running...ex0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
? F
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

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```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```


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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. »

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
?F
```

```
Global`F

F[1] = 1

F[2] = 1

F[n_] := F[n] = F[n - 1] + F[n - 2]
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
?F
```

```
Global`F

F[1] = 1

F[2] = 1

F[n_] := F[n] = F[n - 1] + F[n - 2]
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

```
Clear[F];
F[1] = 1;
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```

```
?F
```

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Global`F

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F[n_] := F[n] = F[n - 1] + F[n - 2]
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. »

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
F[3]
```

2

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```

Crimson Editor

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. »

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
F[3]
```

2

```
? F
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```


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Crimson Editor

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
F[3]
```

2

```
?F
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```


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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. »

```
In[44]:=
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];

In[49]:=
F[3]

Out[49]=
2

In[50]:=
?F
```

```
Global`F

F[1] = 1

F[2] = 1

F[3] = 2

F[n_] := F[n] = F[n - 1] + F[n - 2]
```

150%

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

In[44]:=

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

In[49]:=

```
F[3]
```

Out[49]=

```
2
```

In[50]:=

```
?F
```

```
Global`F

F[1] = 1

F[2] = 1

F[3] = 2

F[n_] := F[n] = F[n - 1] - F[n - 2]
```

In[…]:=

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```


150%

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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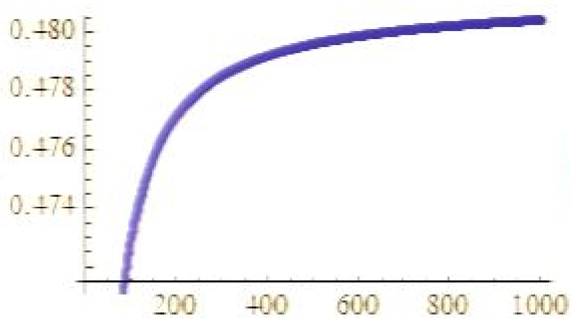
x0sol.nb *

In[44]:=

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

In[51]:=

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

Out[51]= {0.016, }

8

In[59]:=

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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ex0sol.nb *

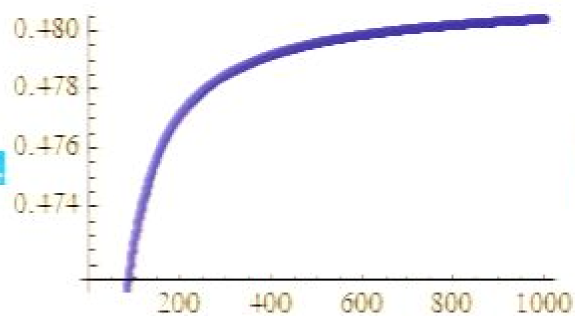
```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] = F[n - 1] + F[n - 2];
```

```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. >>

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

{0.016,  }

150%

Wolfram Mathematica 8.0 - [ex0sol.nb *]

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x0sol.nb *

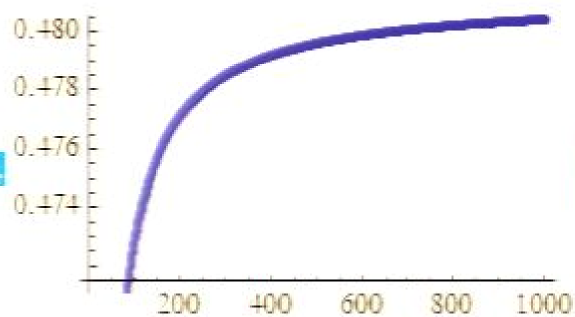
```
F[100]
```

$RecursionLimit::reclim : Recursion depth of 256 exceeded. »

```
Clear[F];
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

{0.016,



}

8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```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Wolfram Mathematica 8.0 - [ex0sol.nb *]

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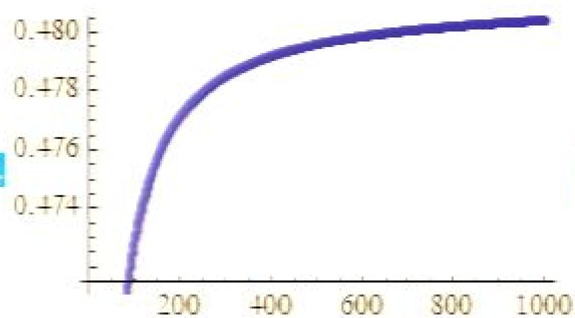
ex0sol.nb *

```
F[1] = 1;
F[2] = 1;
F[n_] := F[n] = F[n - 1] + F[n - 2];
```

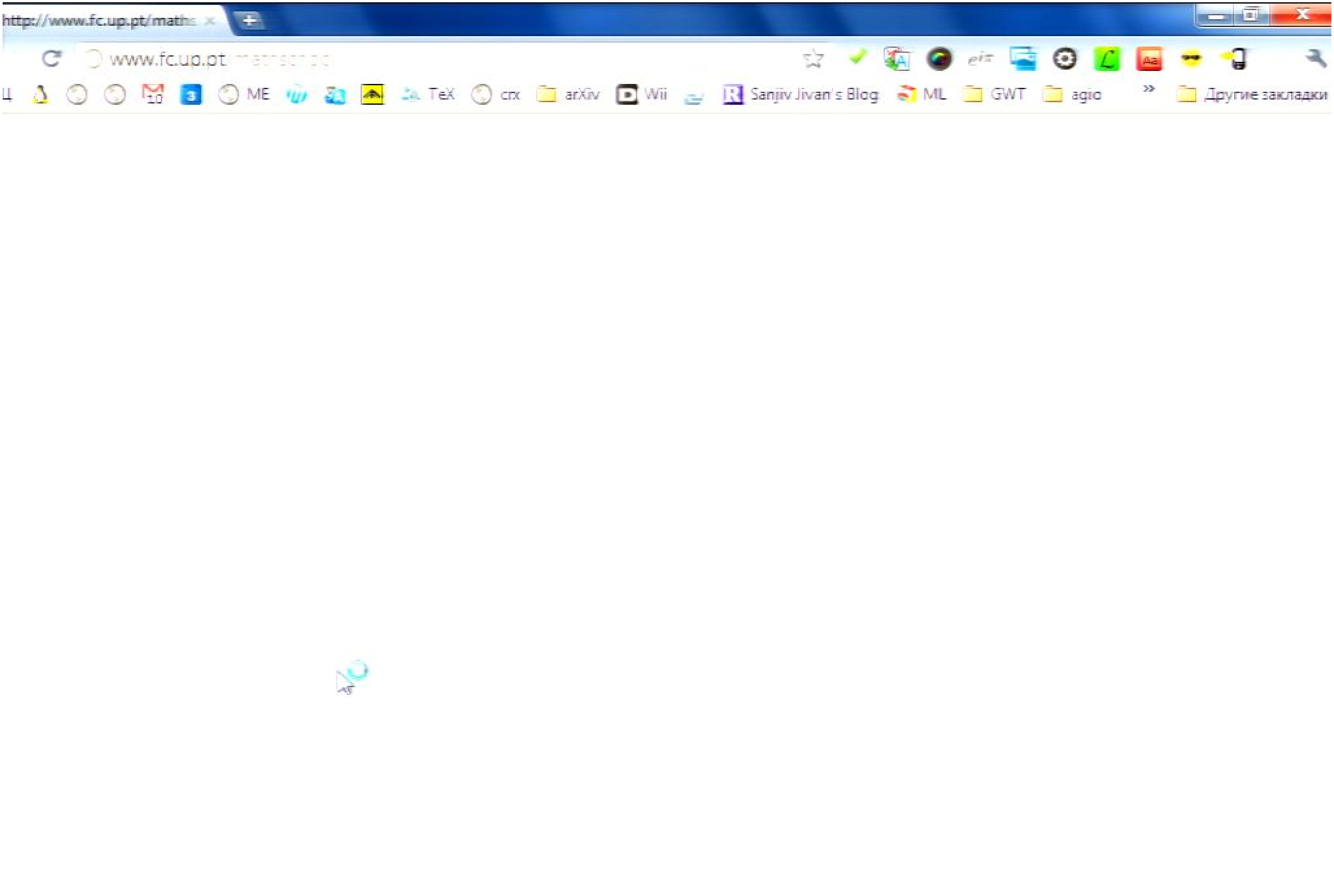
```
ListPlot@Table[(F[i] // Log) / i, {i, 1000}] // Timing
```

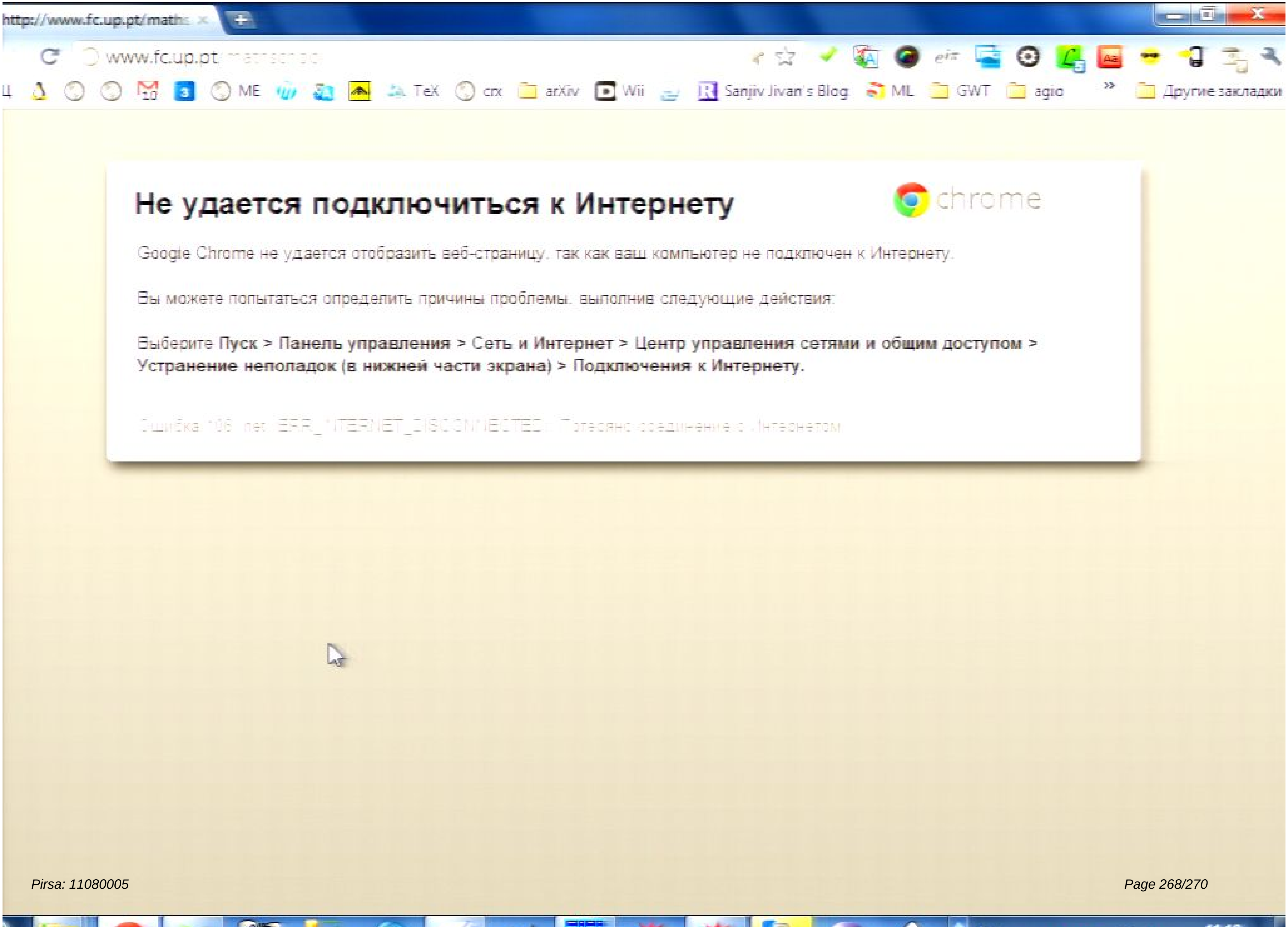
{0.016, }

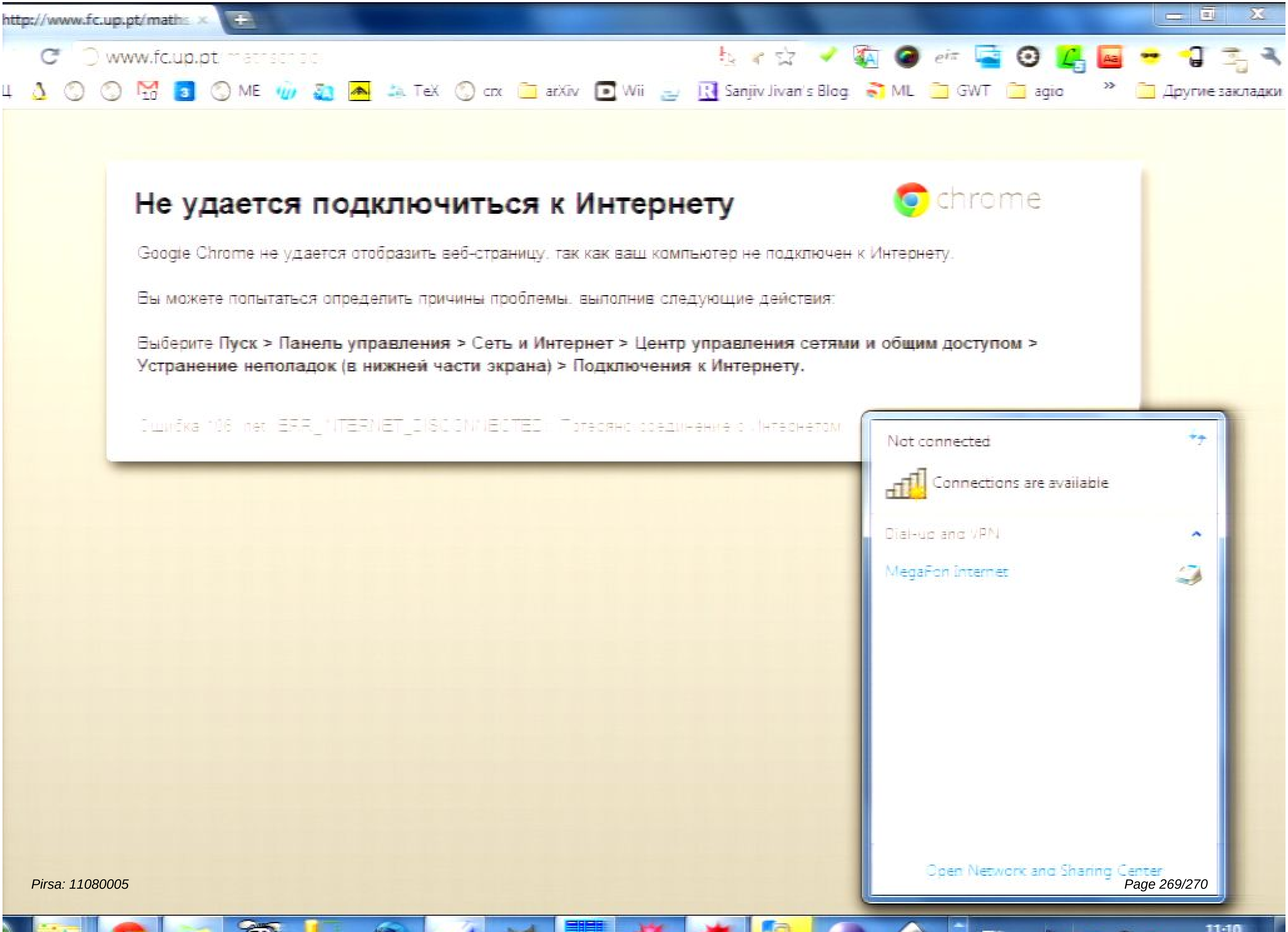
8

```
Run["explorer \"http://www.fc.up.pt/mathschool/\""];
```


150%


http://www.fc.up.pt/maths
www.fc.up.pt
ME
TeX
crx
arXiv
Wii
Sanjiv Jivan's Blog
ML
GWT
agio
Другие закладки


http://www.fc.up.pt/maths
www.fc.up.pt
ME
TeX
crx
arXiv
Wii
Sanjiv Jivan's Blog
ML
GWT
agio
Другие закладки
Не удается подключиться к Интернету
chrome
Google Chrome не удается отобразить веб-страницу, так как ваш компьютер не подключен к Интернету.
Вы можете попытаться определить причины проблемы, выполнив следующие действия:
Выберите Пуск > Панель управления > Сеть и Интернет > Центр управления сетями и общим доступом > Устранение неполадок (в нижней части экрана) > Подключения к Интернету.


http://www.fc.up.pt/maths
www.fc.up.pt
ME
TeX
crx
arXiv
Wii
Sanjiv Jivan's Blog
ML
GWT
agio
Другие закладки
Не удается подключиться к Интернету
chrome
Google Chrome не удается отобразить веб-страницу, так как ваш компьютер не подключен к Интернету.
Вы можете попытаться определить причины проблемы, выполнив следующие действия:
Выберите Пуск > Панель управления > Сеть и Интернет > Центр управления сетями и общим доступом > Устранение неполадок (в нижней части экрана) > Подключения к Интернету.
Not connected
Connections are available
Dial-up and VPN
MegaFon Internet
Open Network and Sharing Center

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Ошибка 106 (net::ERR_INTERNET_DISCONNECTED): Потеряно соединение с Интернетом