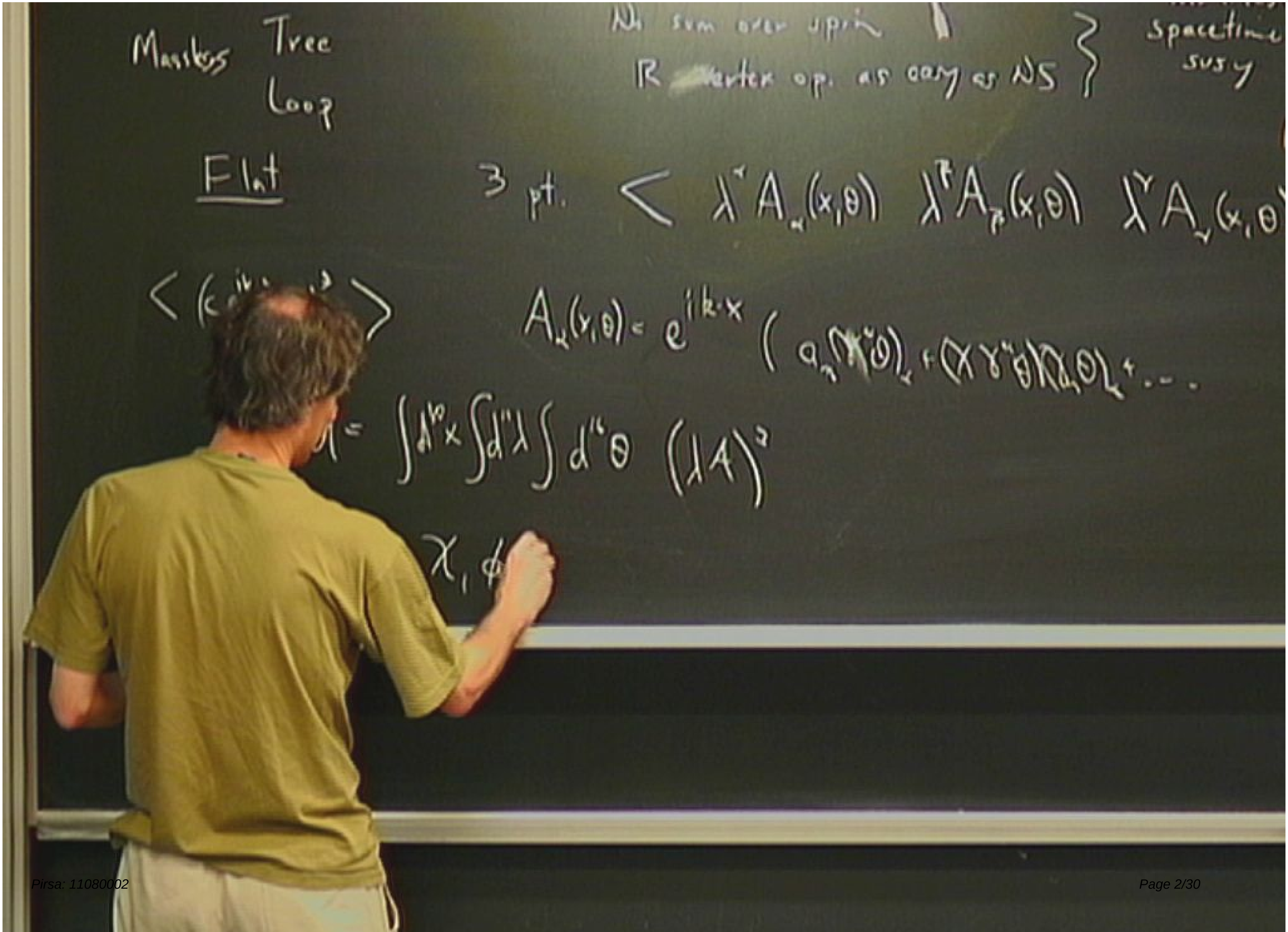
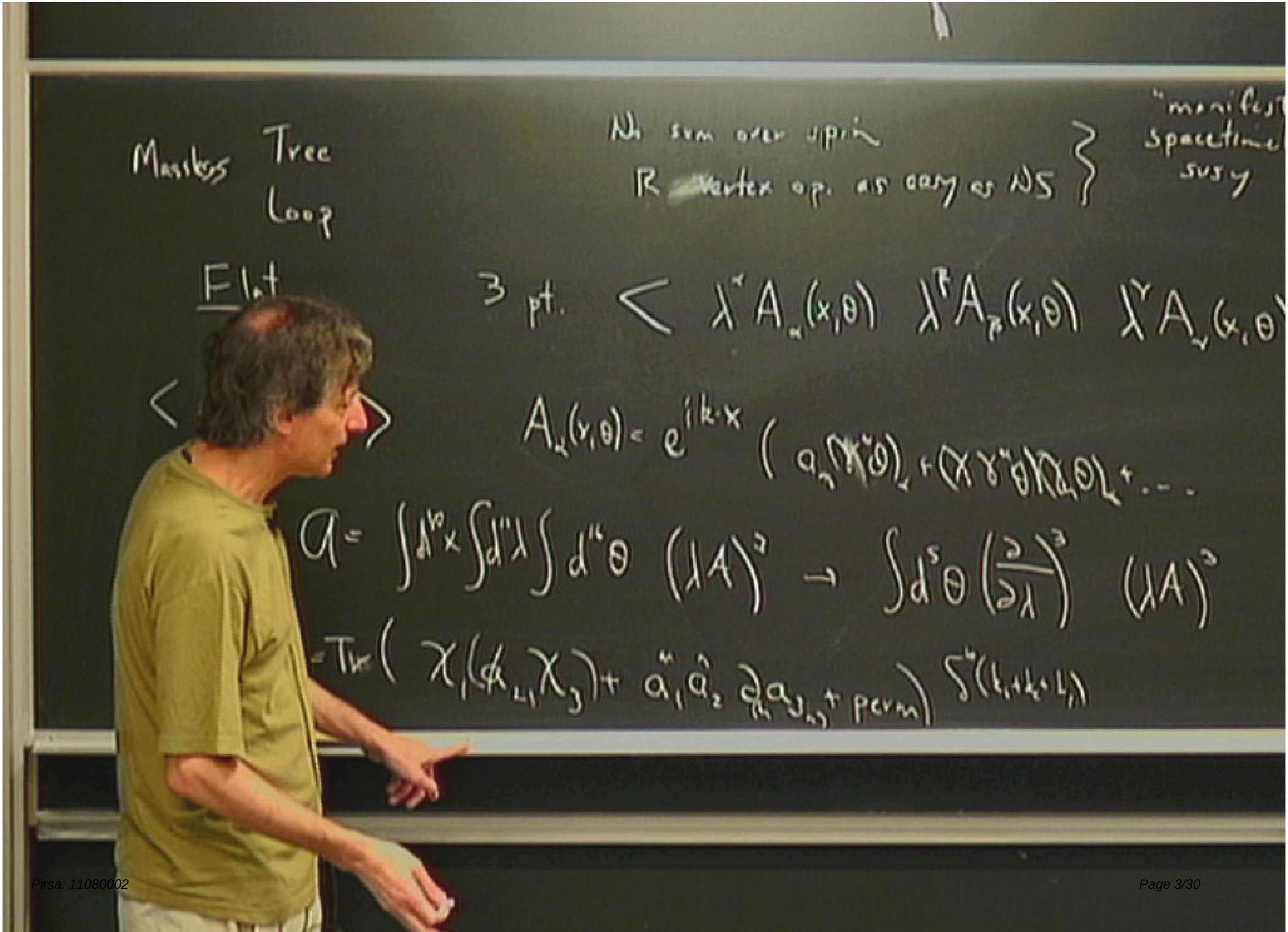
Title: Introduction to Pure Spinor Formalism of the Superstring - Lecture 3

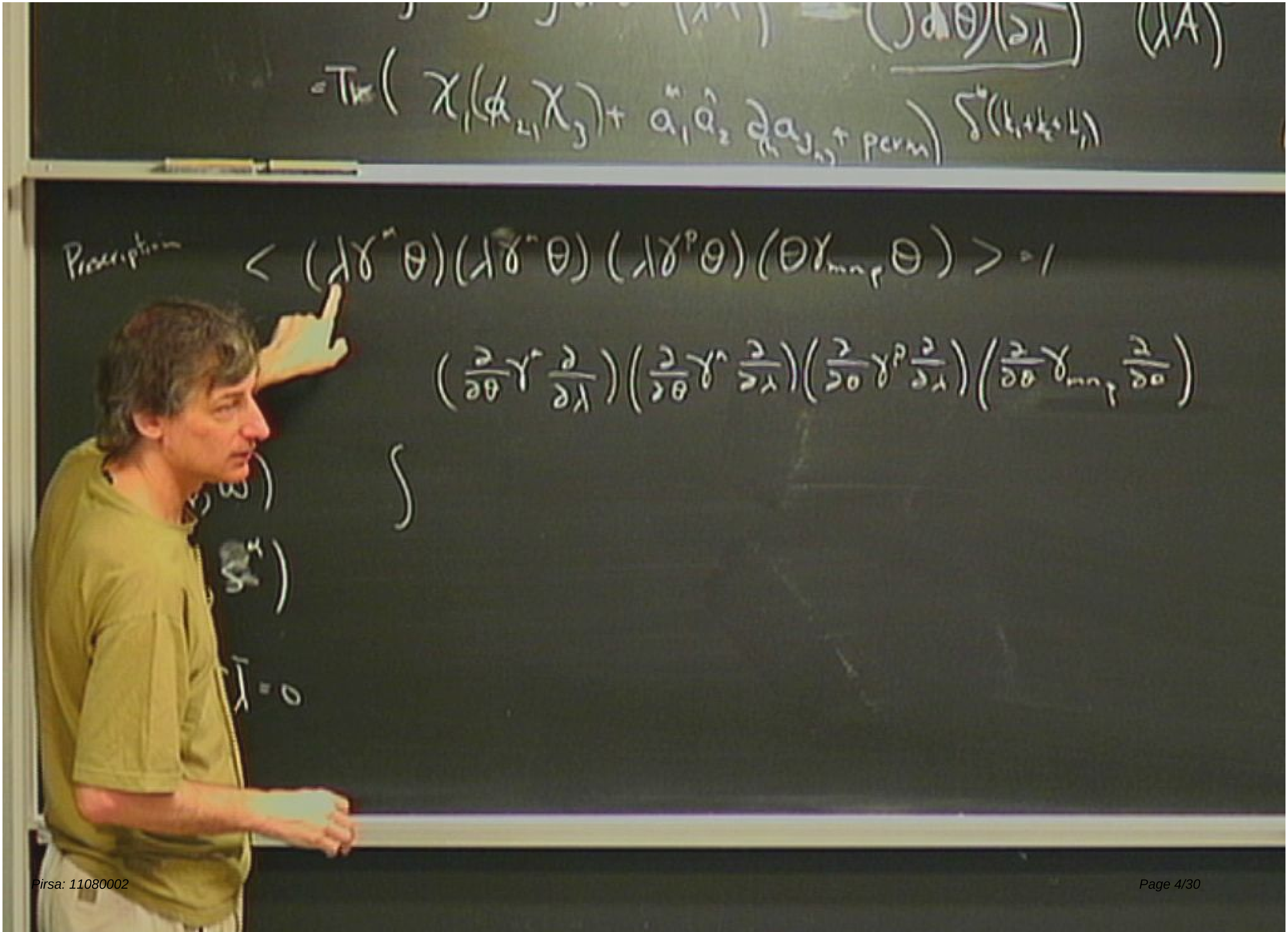
Date: Aug 26, 2011 10:30 AM

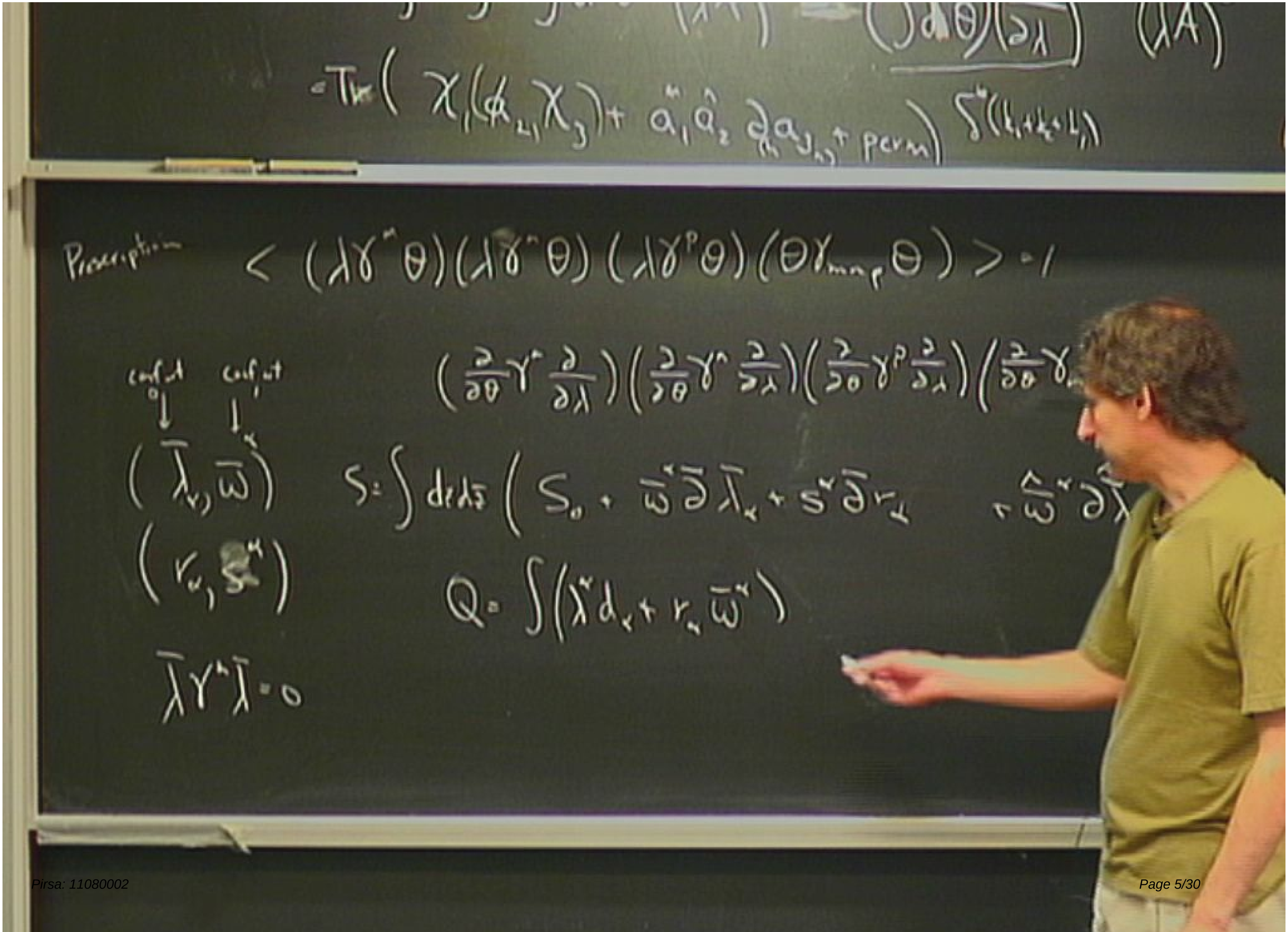
URL: http://pirsa.org/11080002

Abstract: Multiloop amplitude computations


Massless Tree
Loop
sum over spin
R vertex op. as easy as NS
spacetime susy
Flat
3 pt.


Massless Tree
Loop
Flat
No sum over spin
R vertex op. as easy as NS
"manifest spacetime susy"
3 pt.


Prescription
< (λγ^m θ)(λγ^n θ)(λγ^p θ)(θγ_mnp θ) > = 1
(∂/∂θ γ^m ∂/∂λ)(∂/∂θ γ^n ∂/∂λ)(∂/∂θ γ^p ∂/∂λ)(∂/∂θ γ_mnp ∂/∂θ)


Prescription
$< (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta) > = 1$
$(\frac{\partial}{\partial\theta}\gamma^m\frac{\partial}{\partial\lambda})(\frac{\partial}{\partial\theta}\gamma^n\frac{\partial}{\partial\lambda})(\frac{\partial}{\partial\theta}\gamma^p\frac{\partial}{\partial\lambda})$
conf wt 0
conf wt 1
$(\bar\lambda_\alpha, \bar\omega)$
(r_α, s^α)
$S = \int d\tau d\bar\tau (S_0 + \bar\omega^\alpha \bar\partial \bar\lambda_\alpha + s^\alpha \bar\partial r_\alpha$
$Q = \int (\bar\lambda^\alpha d_\alpha + r_\alpha \bar\omega^\alpha)$
$\bar\lambda\gamma^m\bar\lambda = 0$

$$= \mathrm{Tr}\left(\chi_1 (\phi_{2}, \chi_3) + a_1^m \hat{a}_2 \partial_m a_3 + \text{perm} \right) \delta^{10}(k_1+k_2+k_3)$$

Prescription $\langle (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta) \rangle = 1$

$$\left(\frac{\partial}{\partial\theta}\gamma^m\frac{\partial}{\partial\lambda}\right)\left(\frac{\partial}{\partial\theta}\gamma^n\frac{\partial}{\partial\lambda}\right)\left(\frac{\partial}{\partial\theta}\gamma^p\frac{\partial}{\partial\lambda}\right)\left(\frac{\partial}{\partial\theta}\gamma_{mnp}\frac{\partial}{\partial\theta}\right)$$

conf wt 0, conf wt 1

$(\bar\lambda_\alpha, \bar\omega^\alpha)$

(r_α, s^α)

$$S = \int dz\, d\bar z \left(S_0 + \bar\omega^\alpha \bar\partial \bar\lambda_\alpha + s^\alpha \bar\partial r_\alpha + \hat{\bar\omega}^\alpha \partial \hat{\bar\lambda}_\alpha + \hat s^\alpha \partial \hat r \ldots \right.$$

$$Q = \int \left(\lambda^\alpha d_\alpha + r_\alpha \bar\omega^\alpha \right) \qquad d_\alpha = p_\alpha + (\partial x\, \theta)_\alpha$$

$$\bar\lambda\gamma^m\bar\lambda = 0$$

$$\bar\lambda\gamma^m r = 0$$

$$\int d^{11}\lambda \int d^{16}\theta \int d^{11}\bar{\lambda} \int d^{11}r \; e^{-\{Q,\, \theta^\alpha \bar{\lambda}_\alpha\}} \qquad (\lambda A)^3$$

$$= e^{-(\lambda^\alpha \bar{\lambda}_\alpha + \theta^\alpha r_\alpha)}$$

$$\bar{\lambda} = \begin{pmatrix} 0 \\ 0 \\ \bar{\lambda}^{+++++} \end{pmatrix}$$

$$r = \begin{pmatrix} \cancel{r_1} \\ r^{ab} \\ r^{+} \end{pmatrix} \begin{matrix} 5 \\ \overline{10} \\ 1 \end{matrix}$$

$$\int d^{11}\lambda \int d^{16}\theta \int d^{11}\bar{\lambda} \int d^{11} r \; e^{-\{Q,\ \theta^\alpha \bar{\lambda}_\alpha\}} \qquad (\lambda A)^3$$

$$= e^{-(\lambda^\alpha \bar{\lambda}_\alpha + \theta^\alpha r_\alpha)}$$

$\mathcal{N}$

$CP(15)$

$$\bar{\lambda} = \begin{pmatrix} 0 \\ 0 \\ \bar{\lambda}^{+++++} \end{pmatrix}$$

$$r = \begin{pmatrix} \not{r_a} \\ r^{ab} \\ r^{+} \end{pmatrix} \begin{matrix} 5 \\ 10 \\ 1 \end{matrix}$$

$\bigcirc \; R^4$ Humberto Gomes

$\bigcirc \; \partial^4 R^4$ Carlos Mafra

$$\int d^{11}\lambda \int d^{16}\theta \int d^{11}\bar{\lambda} \int d^{11} r \; e^{-\{Q,\, \theta^\alpha \bar{\lambda}_\alpha\}}$$

$$= e^{-(\lambda^\alpha \bar{\lambda}_\alpha + \theta^\alpha r_\alpha)}$$

$\dot{\mathcal{N}}$

$$(\lambda A)^3 \to \langle (\lambda A)^3 \rangle$$

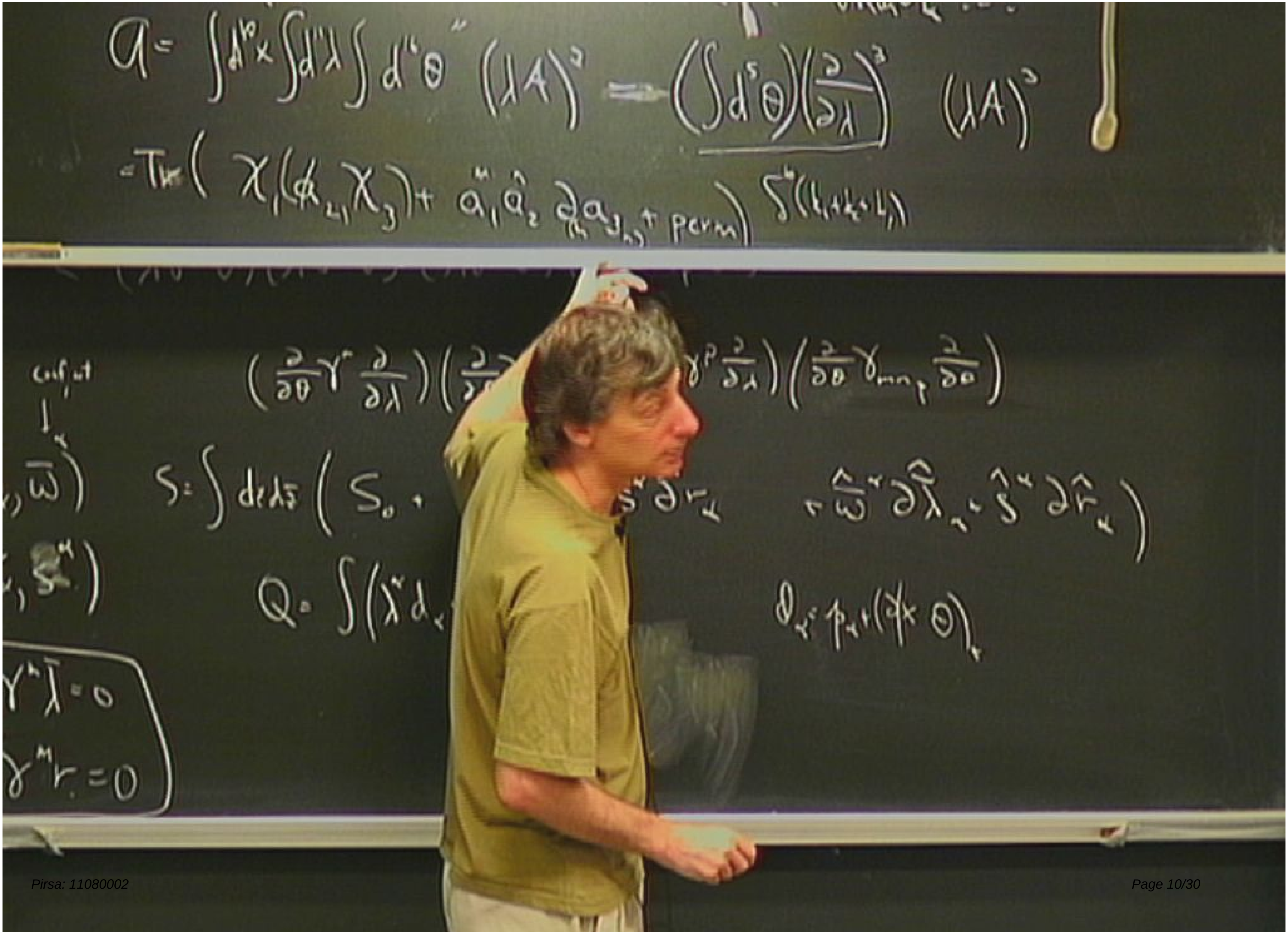
$CP(15)$

$$\bar{\lambda} = \begin{pmatrix} 0 \\ 0 \\ \bar{\lambda}^{+++++} \end{pmatrix}$$

$$r = \begin{pmatrix} \cancel{r_a} \\ r^{ab} \\ r^{+} \end{pmatrix} \begin{matrix} 5 \\ \overline{10} \\ 1 \end{matrix}$$

○ R^4 Humberto Gomez

○ $\partial^4 R^4$ Carlos Mafra



$$= \mathrm{Tr}\left(\chi_1 (\phi_{2}, \chi_3) + \hat{a}_1 \hat{a}_2 \partial a_3 + \text{perm} \right) \delta(k_1+k_2+k_3)$$

$$< (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta) > = 1$$

$$\left(\frac{\partial}{\partial\theta}\gamma^m\frac{\partial}{\partial\lambda}\right)\left(\frac{\partial}{\partial\theta}\gamma^n\frac{\partial}{\partial\lambda}\right)\left(\frac{\partial}{\partial\theta}\gamma^p\frac{\partial}{\partial\lambda}\right)\left(\frac{\partial}{\partial\theta}\gamma_{mnp}\frac{\partial}{\partial\theta}\right)$$

$$S = \int d^2z \left(S_0 + \bar{\omega}^\alpha \bar{\partial} \bar{\lambda}_\alpha + s^\alpha \bar{\partial} r_\alpha + \hat{\bar{\omega}}^\alpha \partial \hat{\bar{\lambda}}_\alpha + \hat{s}^\alpha \partial \hat{r}_\alpha \right)$$

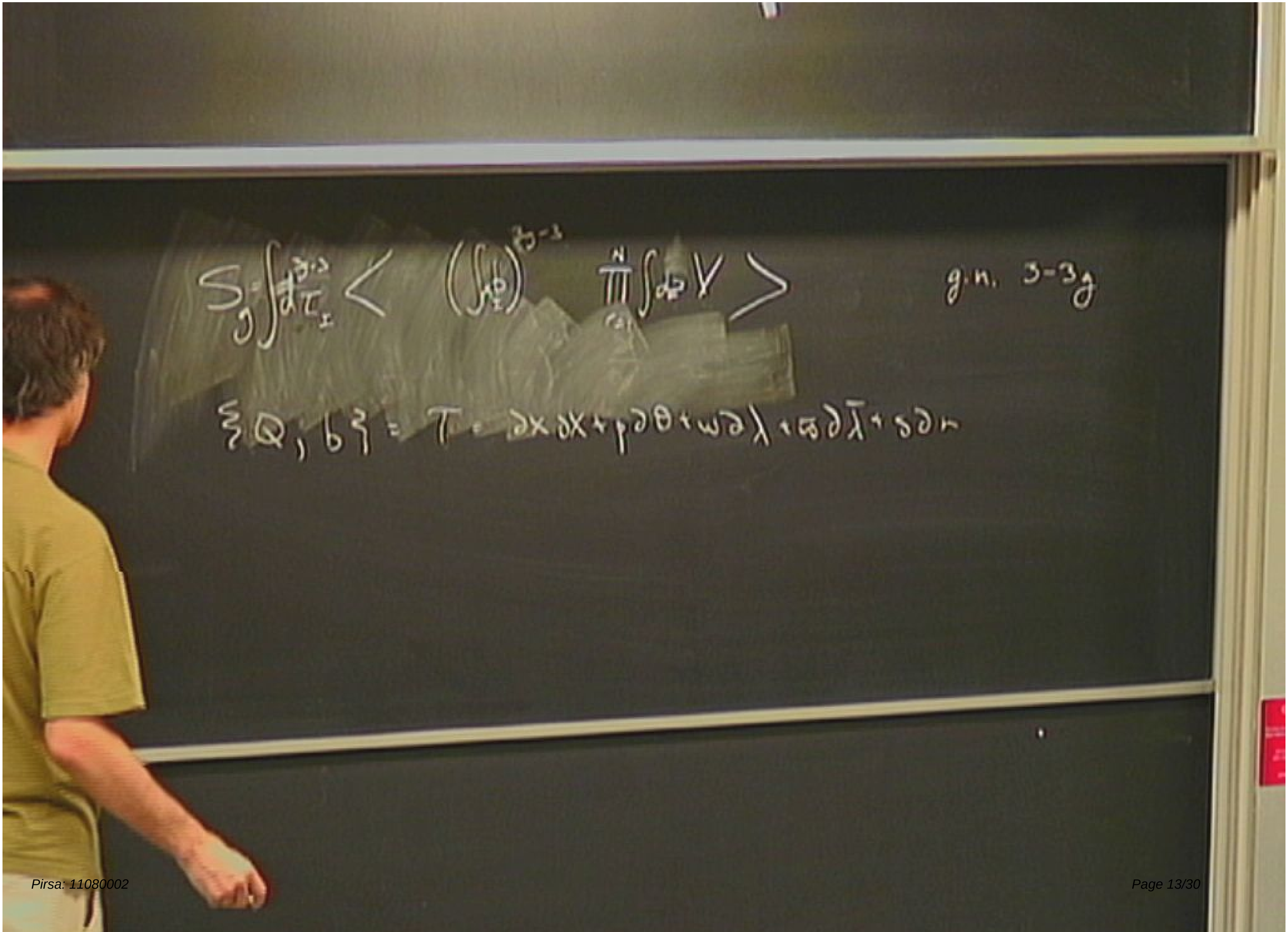
$$Q = \int \left(\lambda^\alpha d_\alpha + r_\alpha \bar{\omega}^\alpha \right) \qquad d_\alpha = p_\alpha + (\partial x\, \theta)_\alpha$$

$$\bar{\lambda}\gamma^m\bar{\lambda} = 0$$

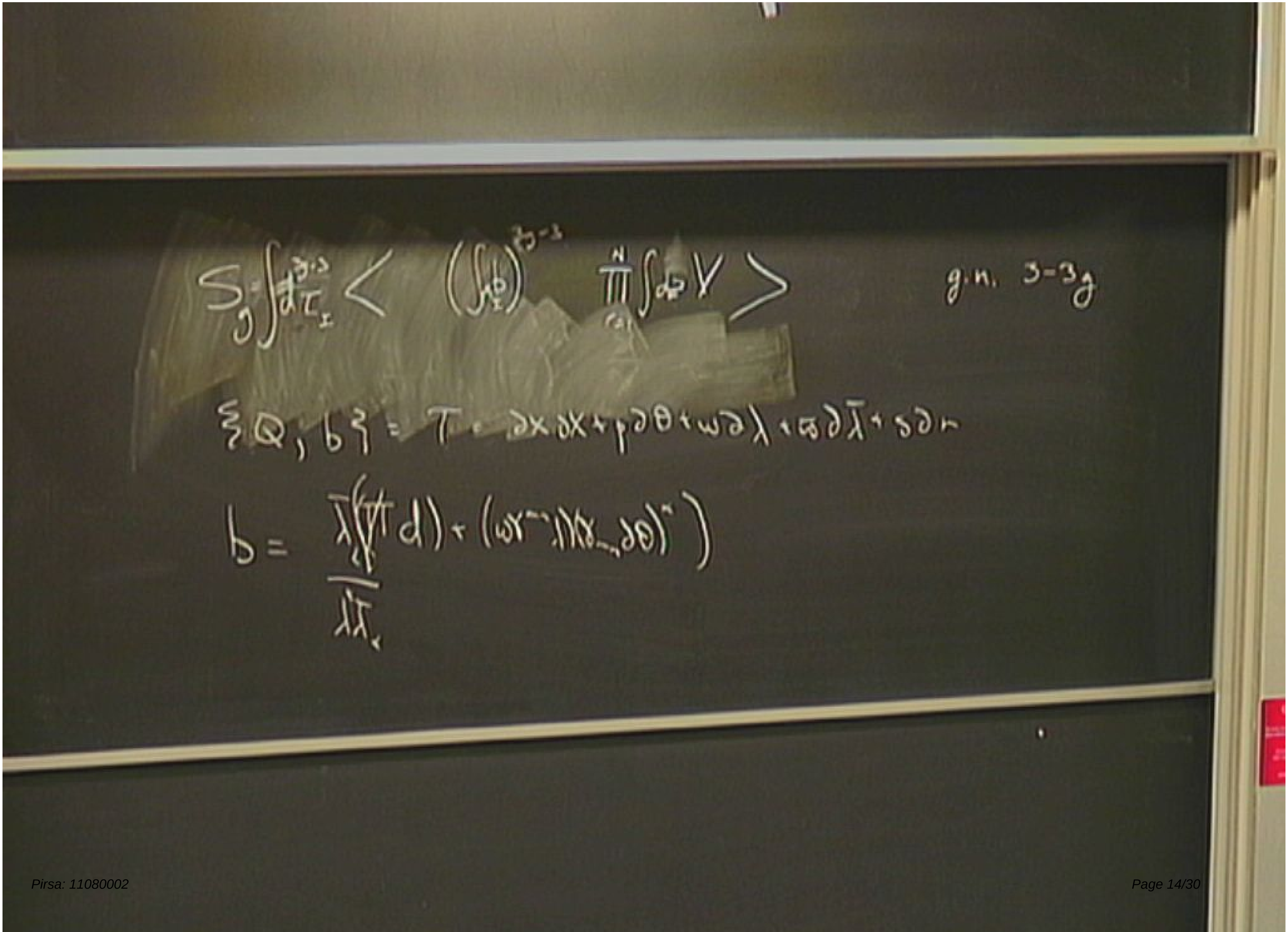
$$\bar{\lambda}\gamma^m r = 0$$

$$\int d^5\theta \; \frac{\partial}{\partial\theta} f \overset{?}{=} 0$$

$$\int d^{16}\theta \; \frac{\partial}{\partial\theta} f = \left(\frac{\partial}{\partial\theta}\right)^{16} \frac{\partial}{\partial\theta} f$$



g.n. 3-3g
{Q, b} = T = ∂x∂x + p∂θ + w∂λ + w̄∂λ̄ + s∂r

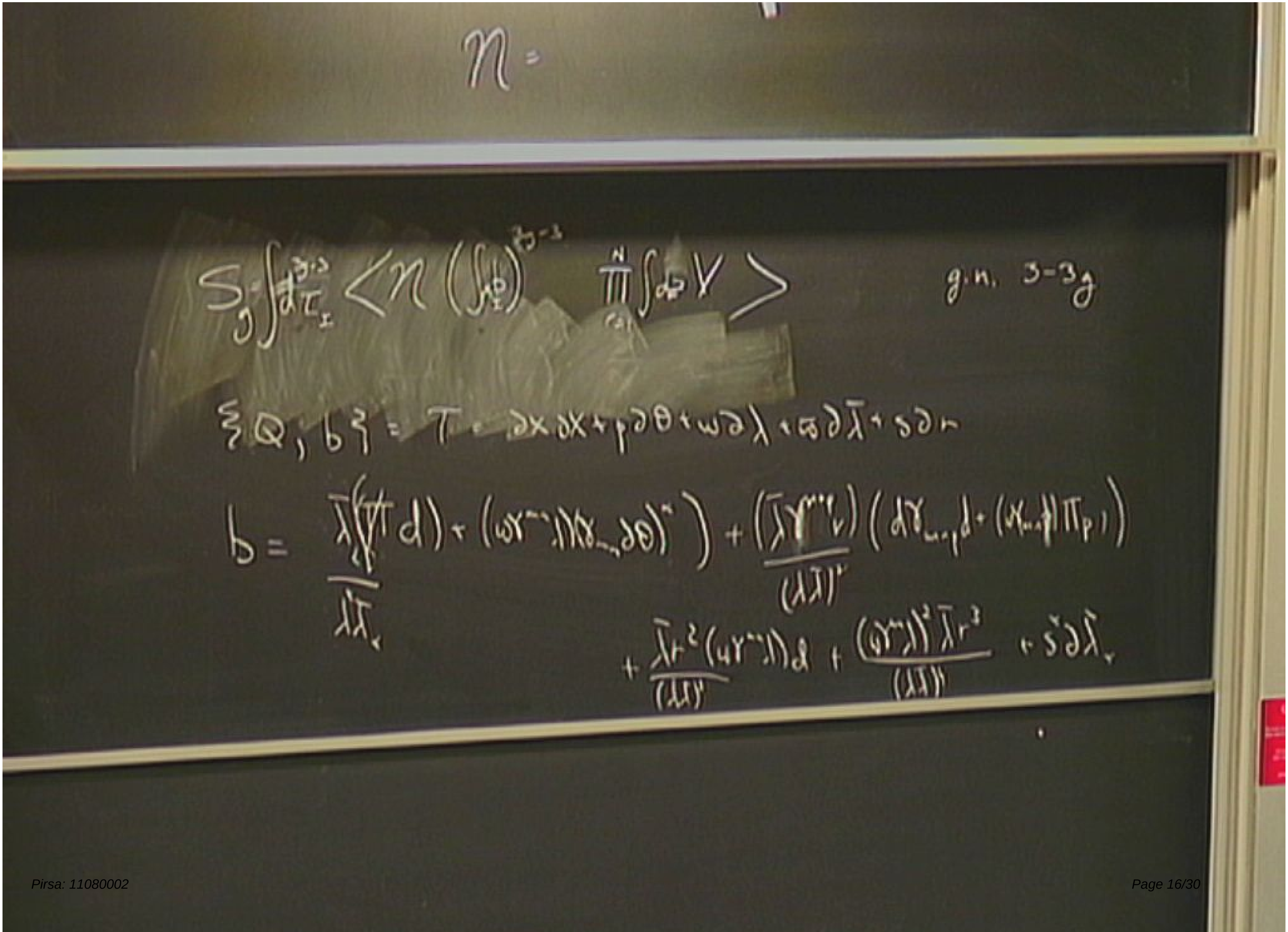

g.n. 3-3g
{Q, b} = T = ∂x∂x + p∂θ + ω∂λ + ω̄∂λ̄ + s∂r
b =

$$S_g = \int d^{3g-3}\tau_z \left\langle \left(\int \mu b\right)^{3g-3} \prod_{r=1}^{N} \int d\sigma V \right\rangle \qquad g.n.\ 3-3g$$

$$\{Q, b\} = T = \partial x \partial x + p \partial \theta + w \partial \lambda + \bar{w} \partial \bar{\lambda} + s \partial r$$

$$b = \frac{\bar{\lambda}\left((\gamma^{m} d) + ((w \gamma^{mn} \lambda)(\gamma_{mn} \partial \theta))^{\alpha} \right)}{\lambda \bar{\lambda}_{\alpha}} + \frac{(\bar{\lambda} \gamma^{mnp} r)}{(\lambda \bar{\lambda})^{2}} \left(d \gamma_{mnp} d + (\gamma_{mnp} d) \Pi_{p} \right)$$

$$+ \frac{\bar{\lambda} r^{2} (w \gamma^{mn} \lambda) d}{(\lambda \bar{\lambda})^{3}} + \frac{((\gamma^{m} \lambda))^{2} \bar{\lambda} r^{3}}{(\lambda \bar{\lambda})^{4}} + s^{\alpha} \partial \bar{\lambda}_{\alpha}$$


n =
g.n. 3-3g
{Q, b} = T = ∂x∂x + p∂θ + w∂λ + w̄∂λ̄ + s∂r
b =

$$\mathcal{N} = e^{-\{Q,\, \theta^\alpha \lambda \ldots s\, \omega_\alpha\}} = e^{-(\ldots)}$$

$$S_g = \int d^{3g-3}\tau_i \left\langle \mathcal{N} \left(\int \mu b\right)^{3g-3} \prod_{r=1}^{n} \int dz\, V \right\rangle \qquad \text{g.n. } 3-3g$$

$$\{Q, b\} = T = \partial x \partial x + p\partial\theta + w\partial\lambda + \bar{w}\partial\bar{\lambda} + s\partial r$$

$$b = \frac{\bar{\lambda}_\alpha\left(\gamma^\alpha d\right) + \left((w\gamma^{mn}\lambda)(\gamma_{mn}\partial\theta)^\alpha\right)}{\lambda\bar{\lambda}_+} + \frac{(\bar{\lambda}\gamma^{mnp} r)}{(\lambda\bar{\lambda})^2}\left(d\gamma_{mnp} d + (w\gamma_{mn}\lambda)\Pi_p\right) + \frac{\bar{\lambda} r^2 (w\gamma^{mn}\lambda) d}{(\lambda\bar{\lambda})^3} + \frac{((w\gamma\lambda))^2 \bar{\lambda} r^3}{(\lambda\bar{\lambda})^4} + s\partial\bar{\lambda}_+$$

Divergences when $\lambda\bar{\lambda} \to \infty$ are regularized by η

$\boxed{\lambda\bar{\lambda} \to 0}$? $\tilde{\eta}$

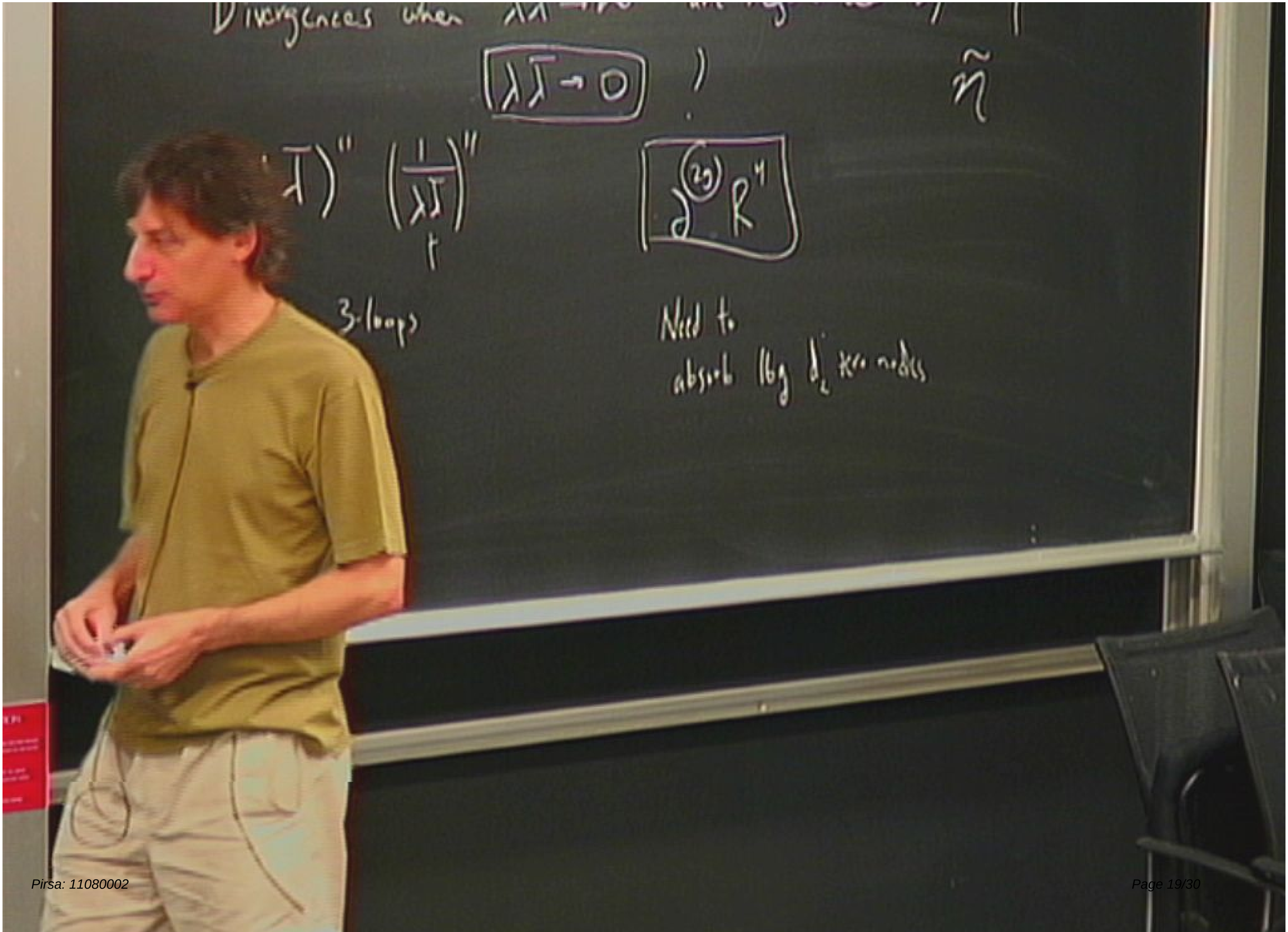
$(\lambda\bar{\lambda})''$ $\left(\frac{1}{\lambda\bar{\lambda}}\right)''$ $\uparrow$

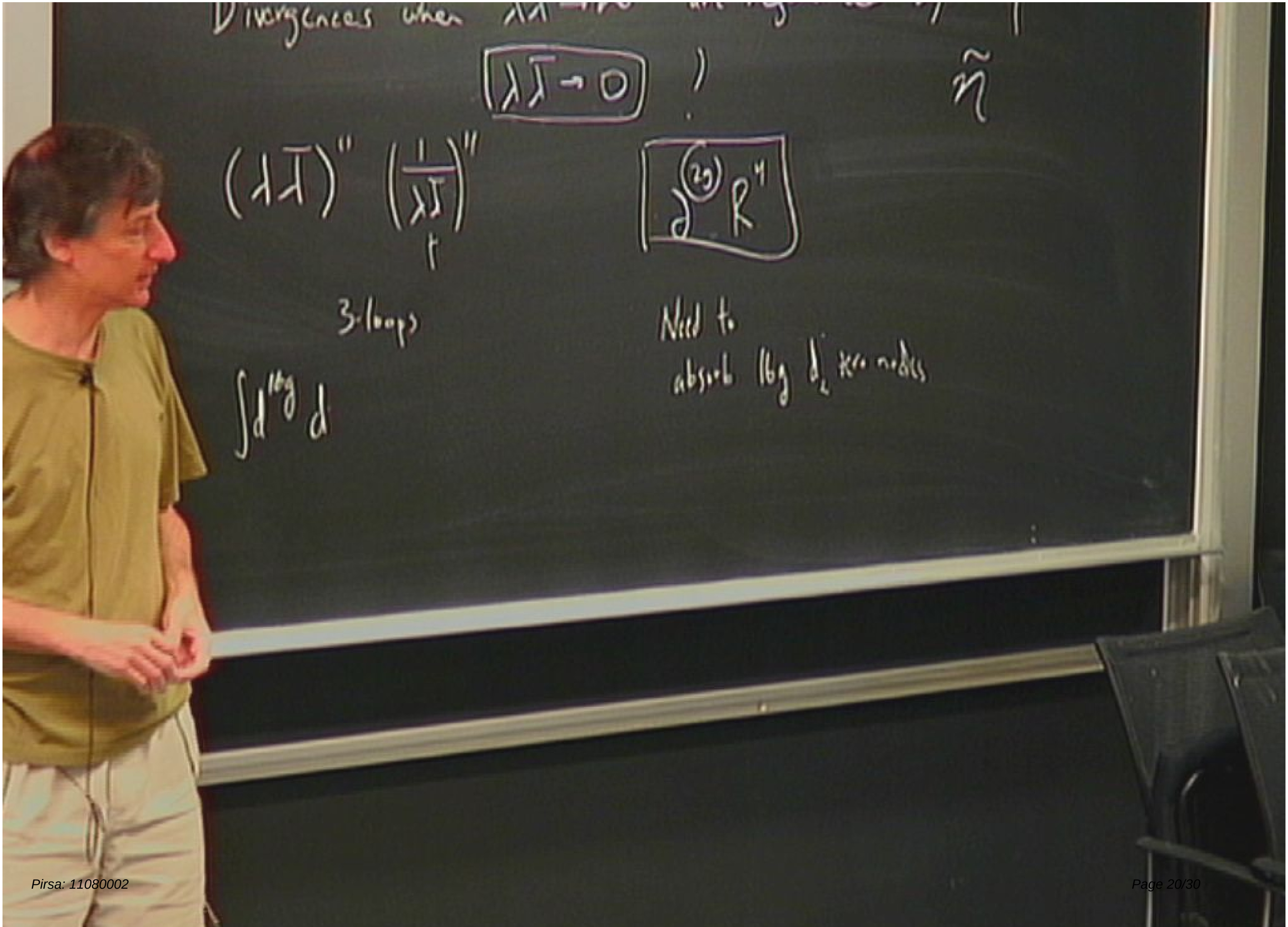
$\boxed{\partial^{(23)} R^4}$

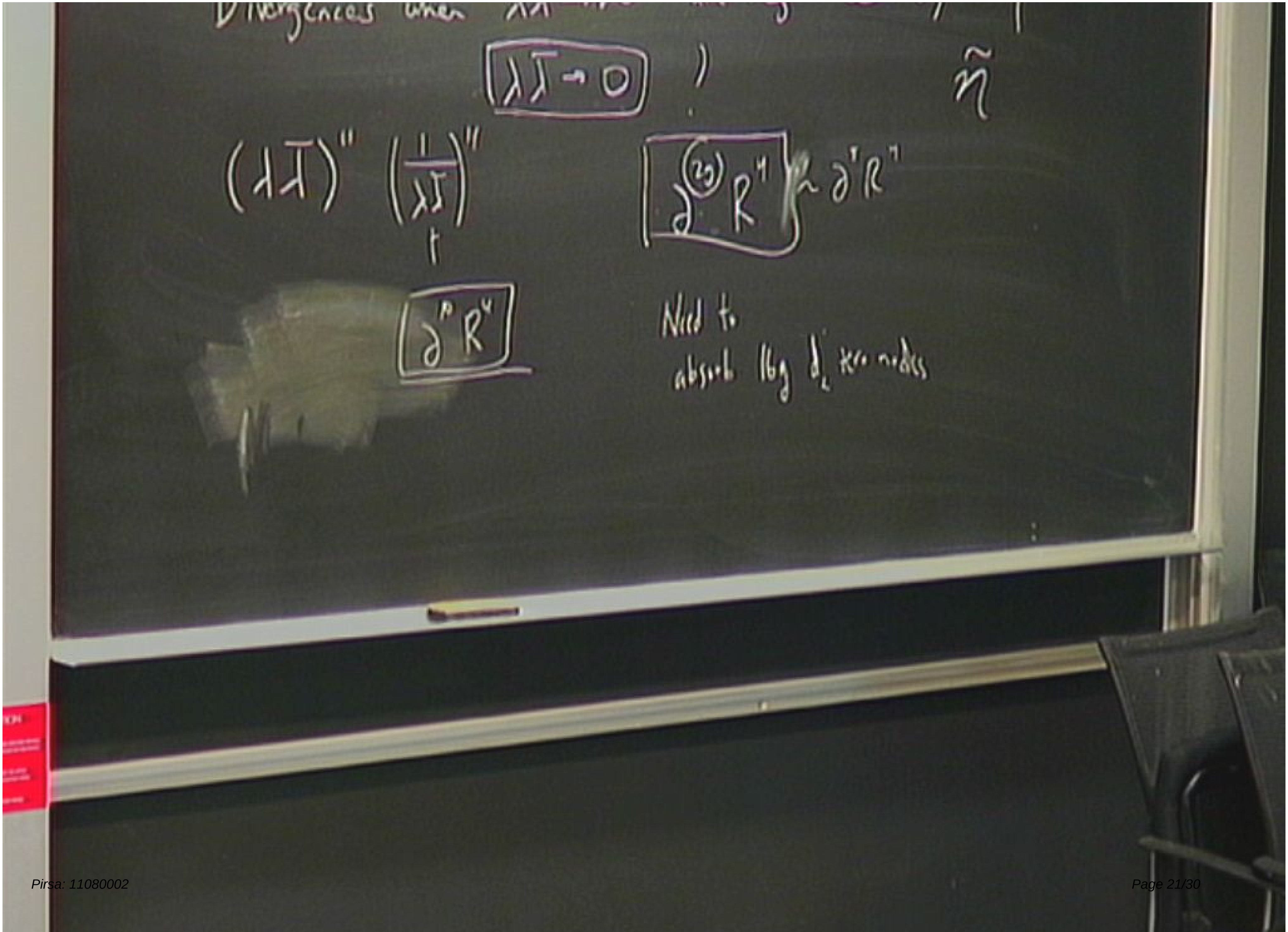
3-loops

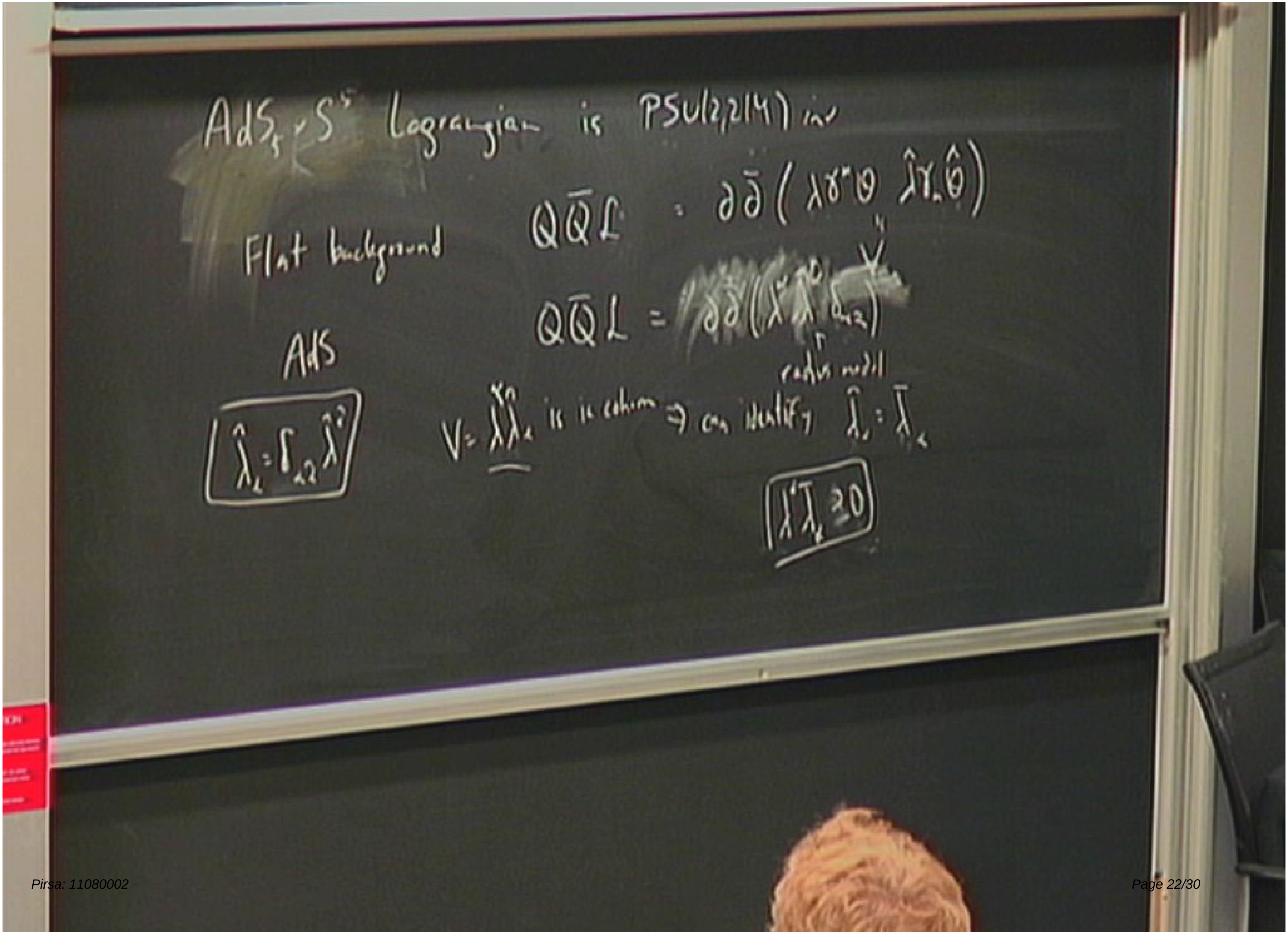
Need to absorb log ∂_i zero modes

$\int d^{\log} d$

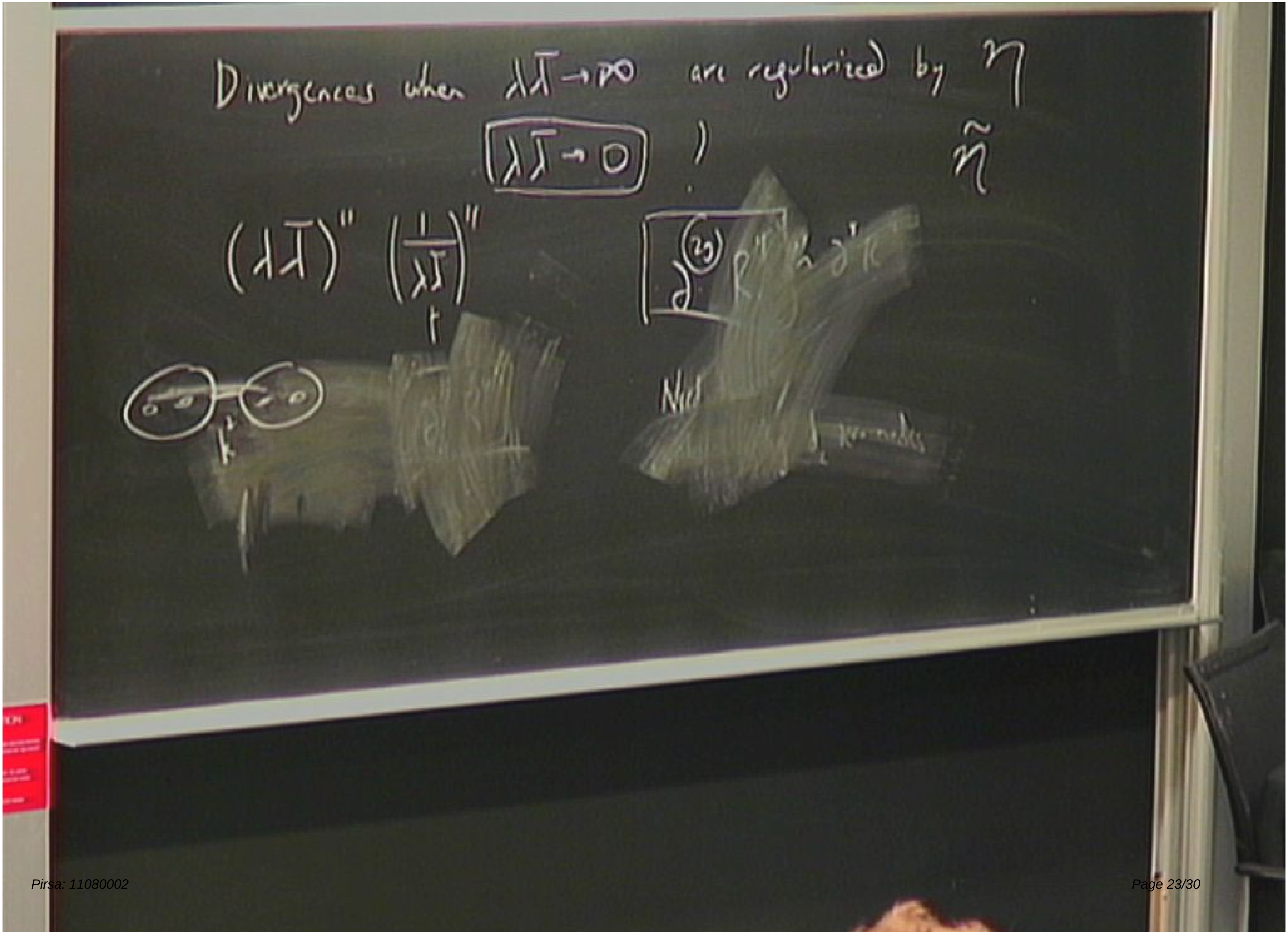

Divergences when
λλ̄ → 0
3-loops
Need to
absorb

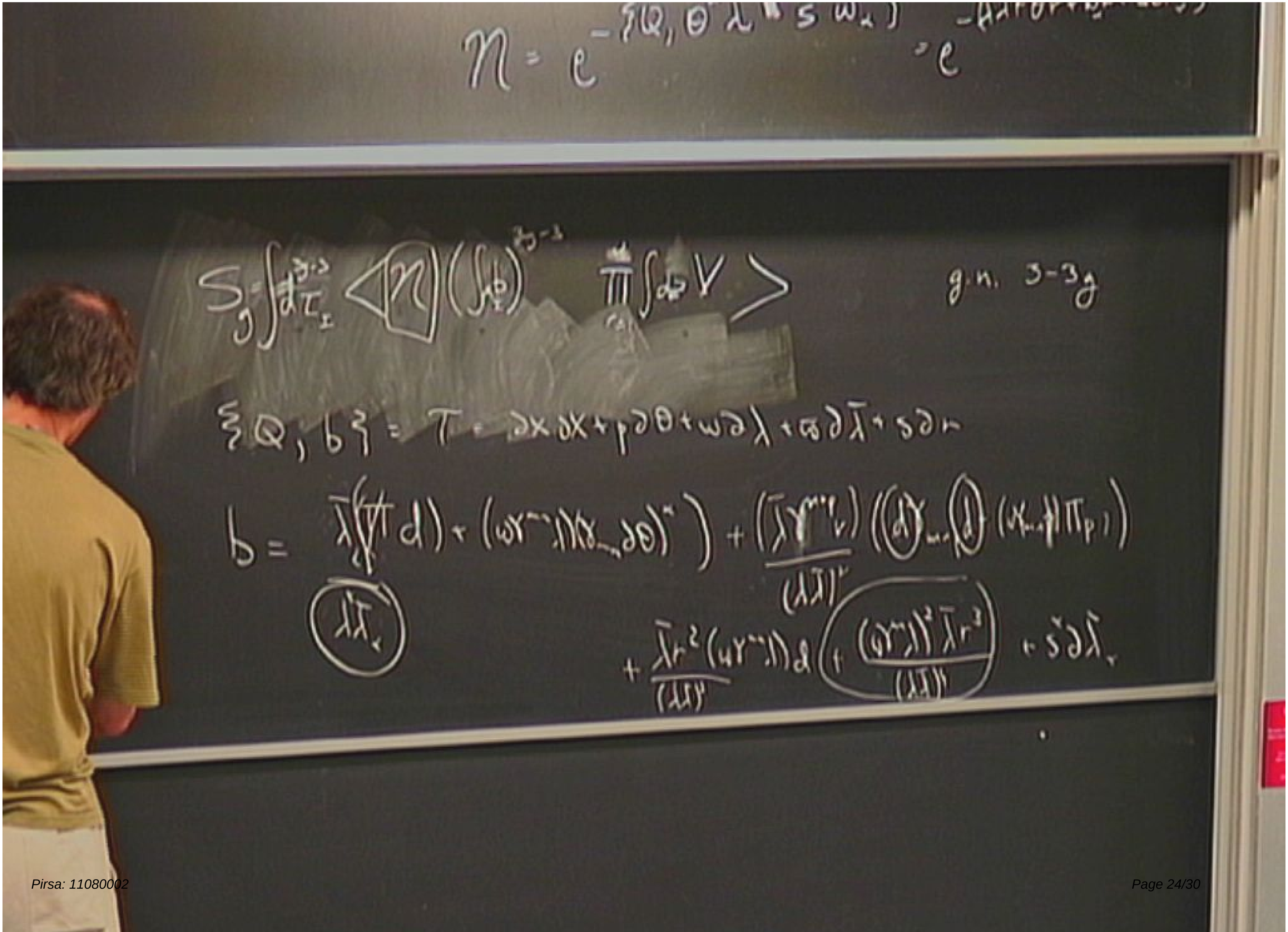

Divergences when
λλ̄ → 0
3-loops
Need to

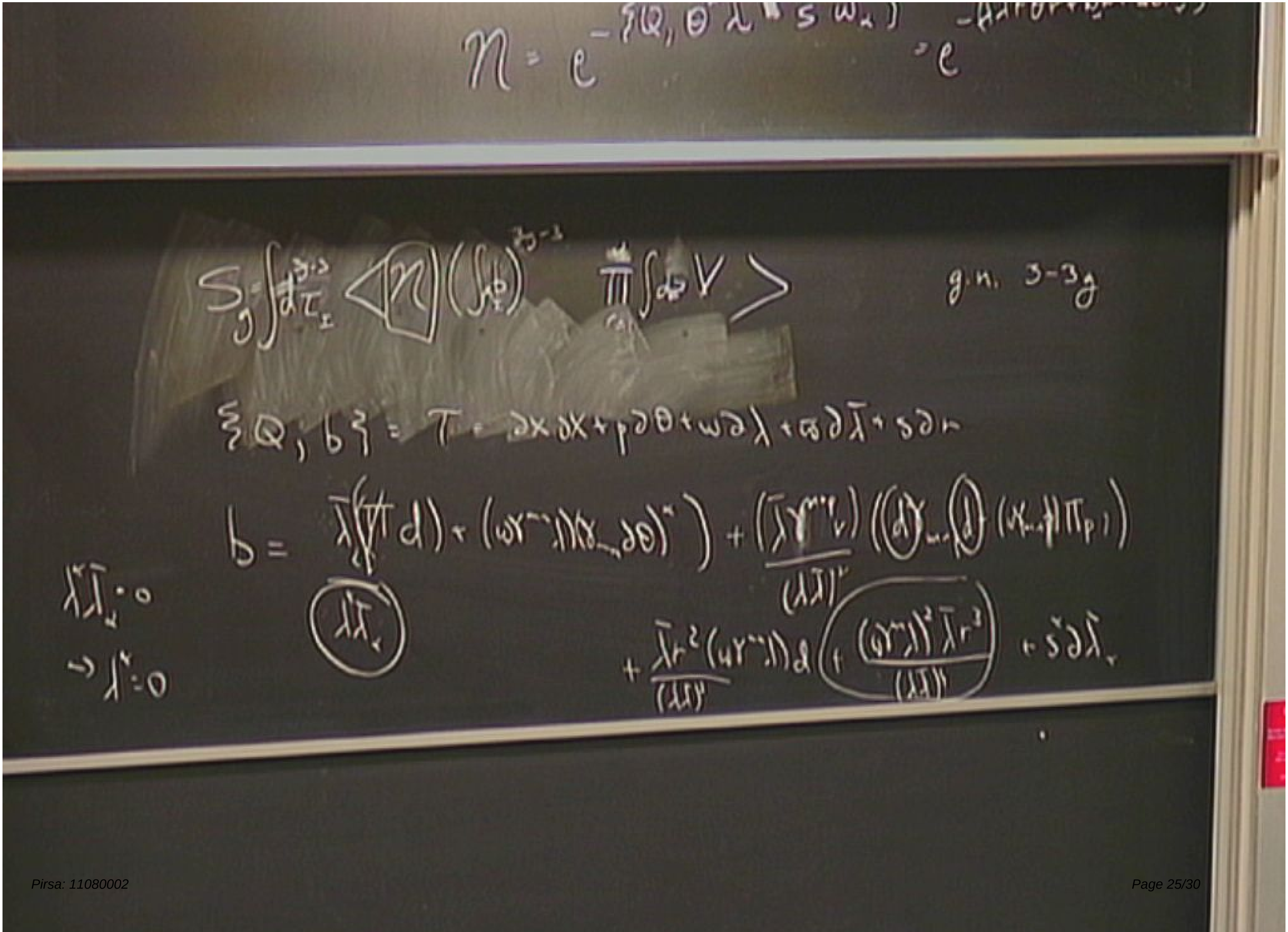

λλ̄ → 0
Need to



AdS5 × S5 Lagrangian is PSU(2,2|4) inv
Flat background
AdS
is in cohom ⇒ can identify


Divergences when λλ̄ → ∞ are regularized by η
λλ̄ → 0 !
η̃
(λλ̄)" (1/λλ̄)"


g.n. 3-3g


g.n. 3-3g
{Q, b} = T
b =
λ^α λ̄_α = 0
→ λ^α = 0

$\lambda^\alpha = \frac{S v(t)}{v(t)}$

$\lambda^\alpha \bar{\lambda}_\alpha \geq 0$

Divergences when $\lambda\bar{\lambda} \to \infty$ are regularized by η

$\boxed{\lambda\bar{\lambda} \to 0}$! $\tilde{\eta}$

$(\lambda\bar{\lambda})^{11} \left(\frac{1}{\lambda\bar{\lambda}}\right)^{11}$

$Q\left(\frac{\theta^\alpha \bar{\lambda}_\alpha}{\lambda^\alpha \bar{\lambda}_\alpha + r_\alpha \theta^\alpha}\right) = 1$

$\sim \frac{(r\theta)^{11}(\theta\lambda)}{(\lambda\bar{\lambda})^{11}}$

$$\lambda^\alpha = \frac{S\nu(t)}{\nu(t)}$$

$$\lambda^\alpha \bar\lambda_\alpha \geq 0$$

$PSL(4|4)$

$$\int d^{10}x \int d^{32}\theta \,(\text{sdet}\, E) \left\langle \left(\lambda^\alpha \hat\lambda^\beta A_{\alpha\beta}(x,\theta,\bar\theta)\right)^3 \int V \right\rangle$$

$$\to \int d^4x \int d^8\theta \int d^4 u \,(\tilde V(x,\theta,u))^2$$

$$\langle (\lambda^\alpha \hat\lambda_\alpha)^3 \rangle = 1$$

$$b = \bar\lambda_\alpha \left(J^q (\gamma^m J)^\alpha + (w\gamma^{mn}\lambda)(\gamma_{mn} J)^\alpha \right)$$

$$\{Q, b\} = T, \quad \{\bar Q, b\} = 0, \quad \bar\partial b = Q \bar\Lambda$$

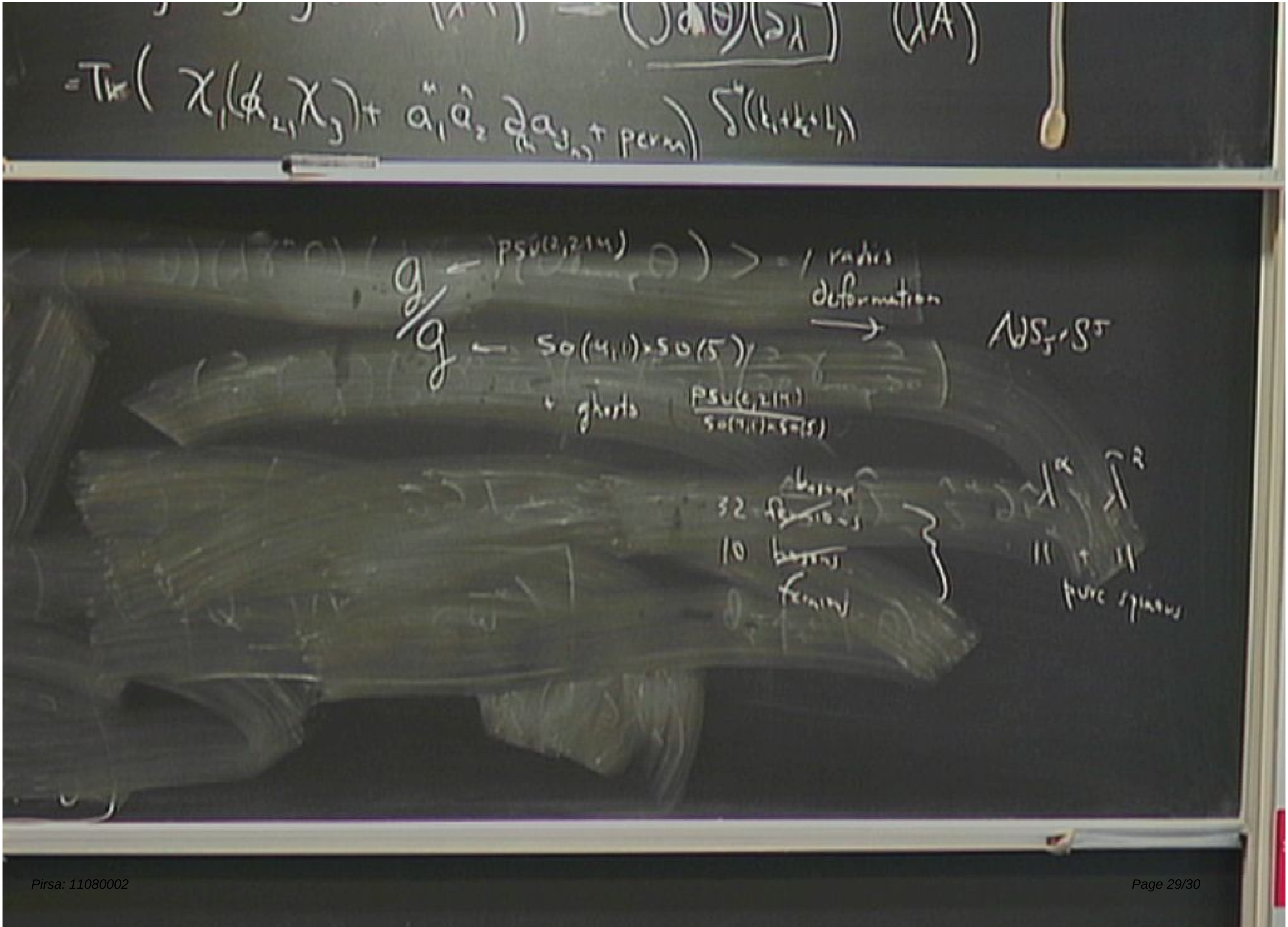
$$= \mathrm{Tr}\left(X_1 (\phi_{L_1} X_3) + \hat{a}_1^m \hat{a}_2 \, \partial_m a_{3\,n} + \text{perm} \right) \delta^{(n)}(k_1+k_2+k_3)$$

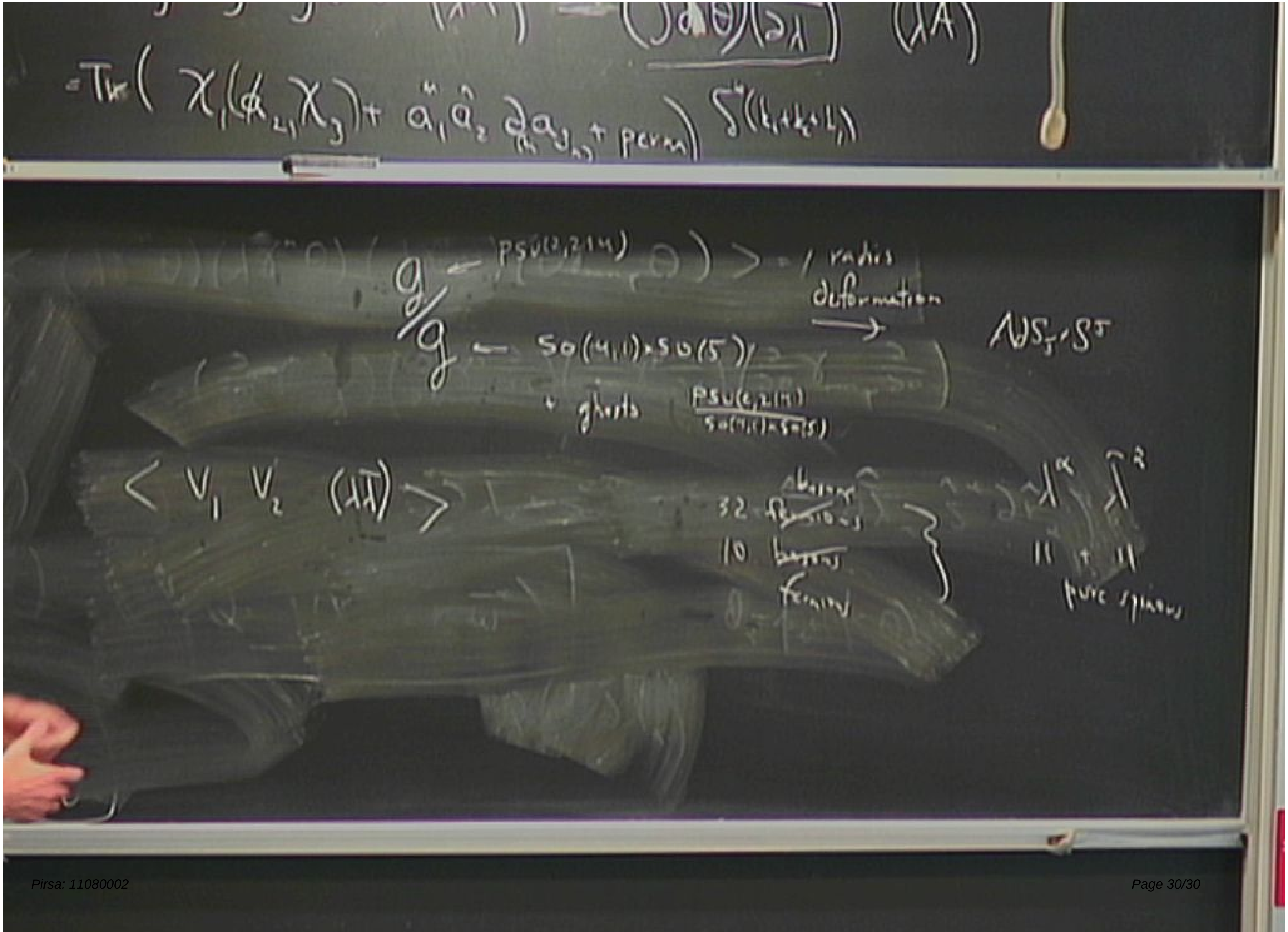
$g \leftarrow PSU(2,2|4)$

$g/g_0 \leftarrow SO(4,1)\times SO(5)$

radius deformation $\longrightarrow$ $AdS_5 \times S^5$

$+$ ghosts $\quad \dfrac{PSU(2,2|4)}{SO(4,1)\times SO(5)}$


= Tr(χ₁(φ₂,₁ χ₃) + a₁ a₂ ∂a₃ + perm) δ(k₁+k₂+k₁)
PSU(2,2|4)
radius deformation
AdS₅ × S⁵
SO(4,1)×SO(5)
+ ghosts
PSU(2,2|4) / SO(4,1)×SO(5)
32 fermions
10 bosons
fermions
11 + 11
pure spinors


Tr(X1(ϕ2,1 X3) + a1 a2 ∂a3 + perm) δ(k1+k2+k3)
PSU(2,2|4)
radius
deformation
AdS5×S5
SO(4,1)×SO(5)
+ ghosts
PSU(2,2|4) / SO(4,1)×SO(5)
< V1 V2 (λλ̄) >
32 fermions
10 bosons
fermions
11 + 11
pure spinors