

Title: Introduction to Pure Spinor Formalism of the Superstring

Date: Aug 24, 2011 10:30 AM

URL: <http://pirsa.org/11080000>

Abstract: d=10 super-Yang-Mills and the superparticle

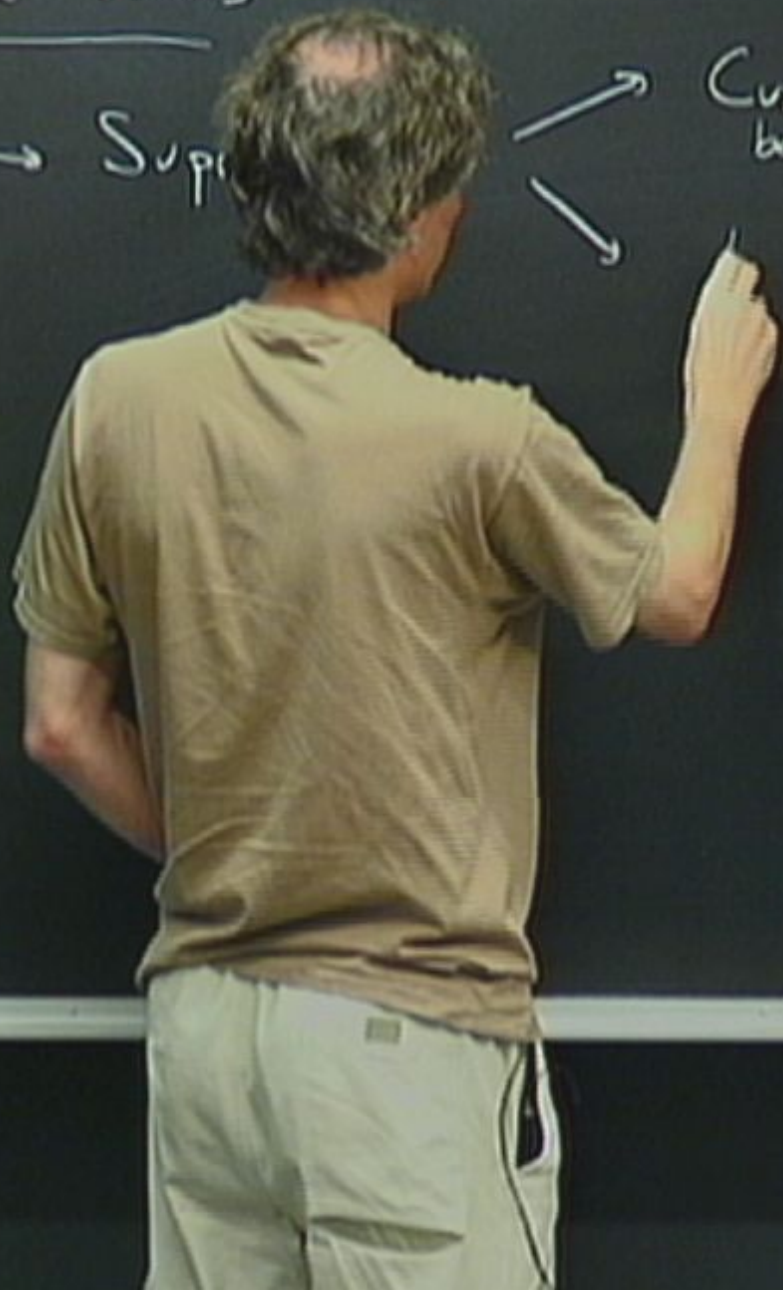
ICTP Lecture Notes

Superparticle \rightarrow Superstring

ICTP Lecture Notes

Superparticle \rightarrow Super

Curved backgrounds ($AdS_5 \times S^5$)



ICTP Lecture Notes

Superparticle

strings

Curved backgrounds ($AdS_5 \times S^5$)

Multiloop amplitudes

ICTP Lecture Notes

Superparticle

string

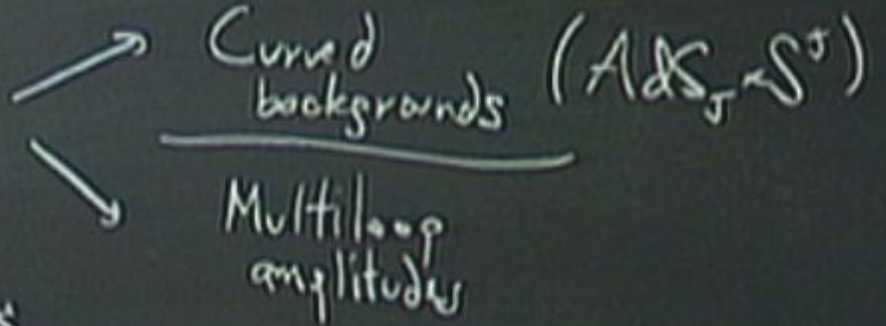


Curved backgrounds ($AdS_5 \times S^5$)

Multiloop amplitudes

TCTP Lecture Notes

particle \rightarrow Superstring



RNS

NS-NS backgrounds



why?

ICTP Lecture Notes

Superparticle \rightarrow Superstring

Curved backgrounds ($AdS_5 \times S^5$)
Multiloop amplitudes

Why?

RNS



NS-NS backgrounds

R-R backgrounds

ICTP Lecture Notes

Superpa

Superstring

Curved backgrounds ($AdS_5 \times S^5$)

Multiloop amplitudes

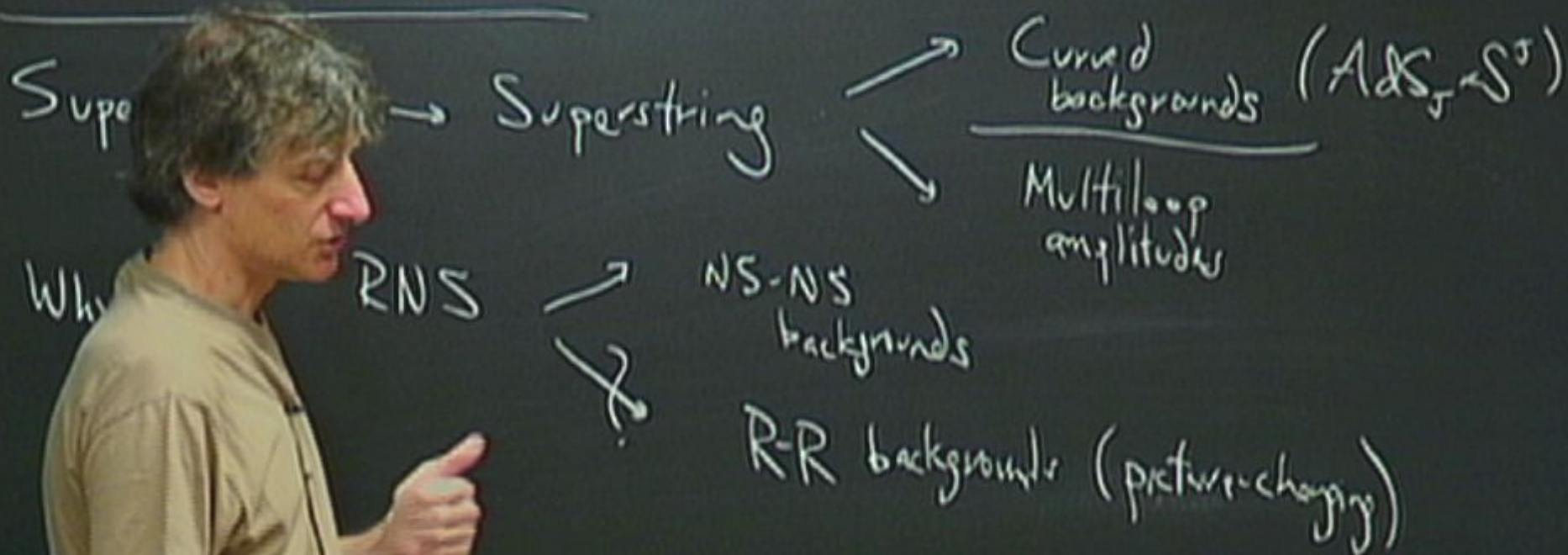
Why?

RNS

NS-NS backgrounds

R-R backgrounds (picture changing)

ICTP Lecture Notes



ICTP Lecture Notes

Superparticle \rightarrow Superstring

Curved backgrounds ($AdS_5 \times S^5$)

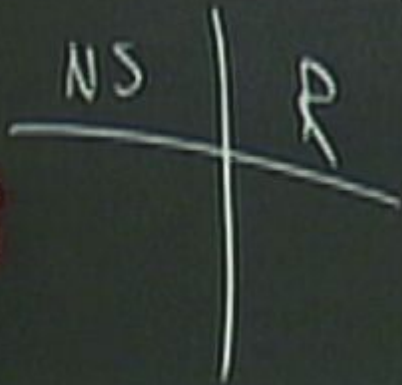
Multiloop amplitudes

Why?

RNS

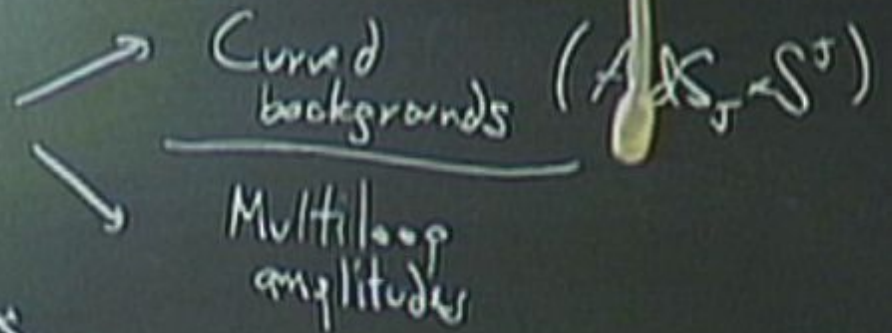
NS-NS backgrounds

R-R backgrounds (picture-changing)



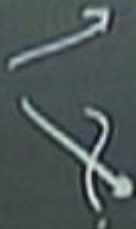
ICTP Lecture Notes

Superparticle \rightarrow Superstring



Why?

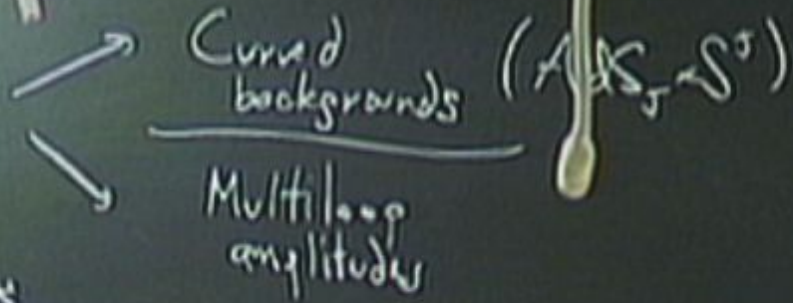
RNS



	NS	R
GSO(+)		
GSO(-)		

ICTP Lecture Notes

Superparticle \rightarrow Superstring



Why?

RNS



NS-NS backgrounds

R-R backgrounds (picture-changing)

Susy backgrounds

N
 $SO(1,9)$
 $SO(1,10)$

ICTP Lecture Notes

Superparticle \rightarrow Superstring

\rightarrow Curved backgrounds ($AdS_5 \times S^5$)
 \rightarrow Multiloop amplitudes

Why?

RNS \rightarrow NS-NS backgrounds

\rightarrow R-R backgrounds (picture-changing)

SUSY backgrounds

	NS	R
GSO(+)	✓	
GSO(-)		

ICTP Lecture Notes

Superparticle \rightarrow Superstring

Curved backgrounds ($AdS_5 \times S^5$)

Multiloop amplitudes

Why?

RNS

NS-NS backgrounds

R-R backgrounds (picture-changing)

susy in
if flat backg

	NS	R
GSO(+)	✓	✓
GSO(-)		

	NS	R
GSO(+)	✓	
GSO(-)	✓	

ICTP Lecture Notes

Superparticle \rightarrow Superstring $\begin{cases} \rightarrow \text{Curved backgrounds (AdS}_5\text{-S}^5) \\ \rightarrow \text{Multiloop amplitudes} \end{cases}$

Why?

RNS \rightarrow NS-NS backgrounds

R-R backgrounds (picture-changing)

	NS	R
GSO(+)	✓	✓
GSO(-)		

	NS	R
GSO(+)	✓	
GSO(-)	✓	

ICTP Lecture Notes

Superparticle \rightarrow Superstring $\begin{cases} \rightarrow \text{Curved backgrounds (AdS}_5 \times S^5) \\ \rightarrow \text{Multiloop amplitudes} \end{cases}$

Why?

RNS

\rightarrow
 ~~\rightarrow~~

NS-NS backgrounds

R-R backgrounds (picture-changing)

SUSY / Information
Hot backgrounds

	NS	R
GSO(+)	✓	✓
GSO(-)	?	?

	NS	R
GSO(+)	✓	?
GSO(-)	✓	?

RNS

ICTP Lecture Notes

Superparticle \rightarrow Superstring

Curved backgrounds ($AdS_5 \times S^5$)

Multiloop amplitudes

RNS

NS-NS backgrounds

R-R backgrounds (picture-changing)

	NS	R
GSO(+)	✓	✓
GSO(-)	?	?

	NS	R
GSO(+)	✓	?
GSO(-)	✓	?

RNS ψ^4

ICTP Lecture Notes

Superparticle \rightarrow Superstring $\begin{cases} \rightarrow \text{Curved backgrounds (AdS}_5 \times S^5) \\ \rightarrow \text{Multiloop amplitudes} \end{cases}$

Why?

RNS

NS-NS backgrounds

R-R backgrounds (picture-charging)

Θ^+

SUSY violation if flat backgrounds

GS / Pure Spinor

	NS	R
GSO(+)	✓	✓
GSO(-)	?	?

	NS	R
GSO(+)	✓	?
GSO(-)	✓	?

RNS ψ^{α}

Θ^+

SUSY deformation
of flat backgrounds

G_5 (Pure Spinor)

	NS	R
GSO(+)	✓	✓
GSO(-)	?	?

R-R backgrounds (picture-changes)

	NS	R
GSO(+)	✓	?
GSO(-)	✓	?

RNS

Ψ^4

$$\dot{P} = -\dot{X} P_3 + c P_m P^m$$

$$S = \int dt \left(\dot{x}^m - i\theta\gamma^m\dot{\theta} \right) \epsilon P_m P^m$$

$$\Theta^\alpha \quad \alpha=1\dots 16$$

$$S = \int dt \left(P_m (\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e P_m P^m \right)$$

$$\Theta^\alpha \quad \alpha = 1 \dots 16$$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$A_B = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma & \end{pmatrix}$$

$$S = \int dt \left(P_m (\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e P_m P^m \right)$$

$$\Theta^\alpha \quad \alpha = 1 \dots 16$$

$$\gamma_m^{\alpha\beta} = \gamma_{m\alpha\beta}$$

$$e P_m P^m = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_{m\alpha\beta} & 0 \end{pmatrix}$$



$$S = \int dt \left(P_m (\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e P_m P^m \right)$$

$$\text{Why! } \Theta^\alpha \quad \alpha = 1 \dots 16$$

$$\gamma_m^{\alpha\beta} = \gamma_{m\alpha\beta}$$

$$P_m^{\alpha\beta} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_{m\alpha\beta} & 0 \end{pmatrix}$$

q_α

$$S = \int dt \left(P_m (\dot{x}^m - i \theta \gamma^m \dot{\theta}) + e \mathcal{P}^m \right) \quad \text{Weyl } \Theta^\alpha \quad \alpha = 1 \dots 16$$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$\Gamma_m^{\alpha\beta} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_m^{\beta\alpha} & 0 \end{pmatrix}$$

Anti-Weyl ψ_α

$$S = \int dt \left(P(\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e P_m P^m \right)$$

Wahl $\Theta^\alpha \quad \alpha=1\dots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$\Gamma_m^{\alpha\beta} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_m^{\beta\alpha} & 0 \end{pmatrix}$$

Anti-Wahl $\alpha = \beta$ $\Gamma_m \Gamma_n = \gamma_{mn} \int$
 $\Rightarrow \gamma_m^{\alpha\beta} \gamma_n^{\beta\alpha} = 2\gamma_{mn} \int$

$$S = \int dt \left(P_m (\dot{x}^m - i \theta \gamma^m \dot{\theta}) + e P_m P^m \right)$$

Wzgl $\theta^\alpha \quad \alpha = 1 \dots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$P_m^{\alpha\beta} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_m^{\alpha\beta} & 0 \end{pmatrix}$$

Susy:

$$\delta \theta^\alpha = \epsilon^\alpha$$

$$\delta x^m = i \epsilon \gamma^m \theta$$

Susy
K-sym

$$\delta \theta^\alpha = P^m (\gamma_m \kappa)^\alpha$$

$$\delta x^m = -i \delta \theta \gamma^m \theta, \quad \delta e = \kappa \cdot \dot{\theta}$$

$$\Rightarrow \gamma_m^{\alpha\beta} \gamma_n^{\gamma\delta} = 2 \eta^{mn} \delta_{\alpha\gamma} \delta_{\beta\delta}$$

$$S = \int dt \left(P_m (\dot{x}^m - i \theta \gamma^m \dot{\theta}) + e P_m P^m \right)$$

Wzgl $\theta^\alpha \quad \alpha = 1 \dots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

Susy:

$$\delta \theta^\alpha = \epsilon^\alpha$$

$$\delta x^m = i \epsilon \gamma^m \theta$$

$$P_m^{\alpha\beta} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_m^{\beta\alpha} & 0 \end{pmatrix}$$

Sicyl
K-syl

$$\delta \theta^\alpha = P^{\alpha\beta} (\gamma_m k)^\beta$$

$$\delta x^m = -i \delta \theta \gamma^m \theta, \quad \delta e = k_\nu \dot{\theta}^\nu$$

$$\Rightarrow \gamma_m^{\alpha\beta} \gamma_n^{\gamma\delta} = 2 \eta^{mn} \delta_{\alpha\gamma} \delta_{\beta\delta}$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\Phi \theta)_\alpha \rightarrow d_\alpha = p_\alpha - (\Phi \theta)_\alpha \quad \text{constraint}$$

$\{d_\alpha, d_\beta\}$

$$p_\alpha = \frac{\partial L}{\partial \dot{\alpha}} = (\mathcal{P}\theta)_\alpha$$

$$\rightarrow d_\alpha = p_\alpha - (\mathcal{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_p\} = \delta_{\alpha p} P_\alpha$$

$$p_{\alpha} = \frac{\partial L}{\partial \dot{q}^{\alpha}} = (\Phi \theta)_{\alpha} \rightarrow d_{\alpha} = p_{\alpha} - (\Phi \theta)_{\alpha} \quad \text{constraint}$$

$$\{d_{\alpha}, d_{\beta}\} = \gamma_{\alpha\beta} P_{\alpha} \rightarrow \begin{matrix} 8 & 1^{\text{st}} \text{ class} \\ 8 & 2^{\text{nd}} \text{ class} \end{matrix}$$

$$p_{\alpha} = \frac{\partial L}{\partial \dot{q}^{\alpha}} = (\Phi \Theta)_{\alpha} \rightarrow d_{\alpha} = p_{\alpha} - (\Phi \Theta)_{\alpha} \quad \text{constraint}$$

$$\{d_{\alpha}, d_{\beta}\} = \delta_{\alpha\beta} \quad P_n \rightarrow \begin{matrix} 8 & 1^{\text{st}} \text{ class} \\ 8 & 2^{\text{nd}} \text{ class} \end{matrix}$$

$$p_{\alpha} = \frac{\partial L}{\partial \dot{q}^{\alpha}} = (\mathcal{P}\theta)_{\alpha} \rightarrow d_{\alpha} = p_{\alpha} - (\mathcal{P}\theta)_{\alpha} \quad \text{constraint}$$

$$K_{\alpha} = (\mathcal{P}d)_{\alpha} \quad \{d_{\alpha}, d_{\beta}\} = \delta_{\alpha\beta} \quad P_n \rightarrow \begin{matrix} 8 & 1^{\text{st}} \text{ class} \\ 8 & 2^{\text{nd}} \text{ class} \end{matrix}$$

$$p_{\alpha} = \frac{\partial L}{\partial \dot{\theta}^{\alpha}} = (\mathcal{P}\theta)_{\alpha} \rightarrow d_{\alpha} = p_{\alpha} - (\mathcal{P}\theta)_{\alpha} \quad \text{constraint}$$

$$K: (\mathcal{P}d)^{\alpha} \quad \{d_{\alpha}, d_{\beta}\} = \delta_{\alpha\beta} \quad P_n \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = c^M G_M$$

$$p_{\alpha} = \frac{\partial L}{\partial \dot{q}^{\alpha}} = (\Phi \theta)_{\alpha} \rightarrow d_{\alpha} = p_{\alpha} - (\Phi \theta)_{\alpha} \quad \text{constraint}$$

$$\{d_{\alpha}, d_{\beta}\} = \gamma_{\alpha\beta} \quad P_n \approx \begin{matrix} 8 & 1^{\text{st}} \text{ class} \\ 8 & 2^{\text{nd}} \text{ class} \end{matrix}$$

$$Q = d_{\alpha}$$

$$p_{\alpha} = \frac{\partial L}{\partial \dot{q}^{\alpha}} = (\Phi \theta)_{\alpha} \rightarrow d_{\alpha} = p_{\alpha} - (\Phi \theta)_{\alpha} \quad \text{constraint}$$

$$K_{\alpha} (\Phi d)_{\alpha} \quad \{d_{\alpha}, d_p\} = \gamma_{\alpha p} P_{\alpha} \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^{\alpha} d_{\alpha}$$

$$p_{\alpha} = \frac{\partial L}{\partial \dot{q}^{\alpha}} = (\Phi \theta)_{\alpha}$$

ker: $(\Phi d)^{\alpha}$

$$\rightarrow d_{\alpha} = p_{\alpha} - (\Phi \theta)_{\alpha} \quad \text{constraint}$$

$$\{d_{\alpha}, d_{\beta}\} = \gamma_{\alpha\beta}^m P_m \approx \begin{matrix} 8 & 1^{\text{st}} \text{ class} \\ 8 & 2^{\text{nd}} \text{ class} \end{matrix}$$

$$Q = \lambda^{\alpha} d_{\alpha}$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$$\text{when } \boxed{|\lambda \gamma^m \lambda| = 0}$$

$$p_{\alpha} = \frac{\partial L}{\partial \dot{q}^{\alpha}} = (\Phi \theta)_{\alpha}$$

$$\rightarrow d_{\alpha} = p_{\alpha} - (\Phi \theta)_{\alpha} \quad \text{constraint}$$

$$K_{\alpha}: (\Phi d)_{\alpha}$$

$$\{d_{\alpha}, d_{\beta}\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 & 1^{st} \text{ class} \\ 8 & 2^{nd} \text{ class} \end{matrix}$$

$$Q = \lambda^{\alpha} d_{\alpha}$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$$\text{when } \boxed{|\lambda \gamma^m \lambda| = 0}$$

$$p_{\alpha} = \frac{\partial L}{\partial \dot{\theta}^{\alpha}} = (\Phi \theta)_{\alpha}$$

ker: $(\Phi d)^{\alpha}$

$$\rightarrow d_{\alpha} = p_{\alpha} - (\Phi \theta)_{\alpha} \quad \text{constraint}$$

$$\{d_{\alpha}, d_{\beta}\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 & 1^{\text{st}} \text{ class} \\ 8 & 2^{\text{nd}} \text{ class} \end{matrix}$$

$$Q = \lambda^{\alpha} d_{\alpha}$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$$\text{when } \boxed{|\lambda \gamma^m \lambda| = 0}$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha \quad \rightarrow \quad d_\alpha = p_\alpha - (\not{P} \not{\theta})_\alpha \quad \text{constraint}$$

Kr: $(\not{P} d)^\alpha$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \quad \rightarrow \quad \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_\alpha \quad Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $|\lambda \gamma^m \lambda| = 0$

$\lambda = 10$
pure spinor

λ

$$p_\alpha = \frac{\partial L}{\partial \dot{q}^\alpha} = (\Phi \Theta)_\alpha$$

Kri: $(\Phi d)^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\Phi \Theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta} P_n \approx 8 \text{ 1st class}$$

$$\approx 8 \text{ 2nd class}$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^{\alpha\beta} \lambda) P_n = 0$$

$$\text{when } |\lambda \gamma^{\alpha\beta} \lambda| = 0$$

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha\beta})^{\gamma\delta}$$

$$p_\alpha = \frac{\partial L}{\partial \dot{q}^\alpha} = (\mathcal{P}\theta)_\alpha$$

Kr: $(\mathcal{P}d)^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\mathcal{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta} P_n \approx \frac{8}{8} \text{ 1st class}$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^{\alpha\beta} \lambda) P_n = 0$$

when $|\lambda \gamma^{\alpha\beta} \lambda| = 0$

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha\beta})^{\gamma\delta} (\lambda \gamma^{\alpha\beta} \lambda)$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha$$

Kr: $(\not{P} d)^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\not{P} \theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta} P_n \rightarrow 8 \text{ 1st class}$$

$$8 \text{ 2nd class}$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_n = 0$$

$$\text{when } \boxed{|\lambda \gamma^m \lambda| = 0}$$

1st pure

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha\beta})^{\gamma\delta} (\lambda \gamma^m \lambda)$$

"pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha$$

Key: $(\not{P} d)^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\not{P} \not{\theta})_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_{\alpha 1}$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $\boxed{|\lambda \gamma^m \lambda| = 0}$

also pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha_1 \dots \alpha_p})^{\beta_1 \dots \beta_p} (\lambda \gamma^{\alpha_1 \dots \alpha_p})_{\beta_1 \dots \beta_p}$$

"pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \theta)_\alpha$$

Key: $(\not{P} d)^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\not{P} \theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_{\alpha i}$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $\boxed{|\lambda \gamma^m \lambda| = 0}$

also pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha_1 \dots \alpha_p})^{\beta_1 \dots \beta_p} (\lambda \gamma^{\alpha_1 \dots \alpha_p})_{\beta_1 \dots \beta_p}$$

"pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha$$

Ke: $(\not{P} \not{\theta})^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\not{P} \not{\theta})_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$= \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $|\lambda \gamma^m \lambda| = 0$

also pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha\beta}^m)^\rho (\lambda \gamma^m \lambda)^\sigma$$

"pure spinor"

$$\begin{matrix} \lambda^\alpha \lambda^\beta \\ \lambda^\alpha \lambda^\beta \\ (\lambda \sigma^{\mu\nu} \lambda) \end{matrix}$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \theta)_\alpha \quad \rightarrow \quad d_\alpha = p_\alpha - (\not{P} \theta)_\alpha \quad \text{constraint}$$

$$K: (\not{P} d)^\alpha \quad \{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$$\text{when } \boxed{|\lambda \gamma^m \lambda| = 0}$$

also pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha_1 \dots \alpha_p})^{\beta_1 \dots \beta_p} (\lambda \gamma^{\alpha_1 \dots \alpha_p})_{\beta_1 \dots \beta_p}$$

"pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha$$

Key: $(\not{P} d)^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\not{P} \theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $|\lambda \gamma^m \lambda| = 0$

also pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha_1 \dots \alpha_p})^{\beta_1 \dots \beta_p} (\lambda \gamma^{\alpha_1 \dots \alpha_p})_{\beta_1 \dots \beta_p}$$

"pure spinor"

$$\lambda^\alpha = \lambda^\alpha$$

$$p_\alpha = \frac{2L}{2\dot{\theta}^\alpha} = (\not{P}\not{\theta})_\alpha$$

ker: $(\not{P}\not{d})^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\not{P}\not{\theta})_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $\boxed{|\lambda \gamma^m \lambda| = 0}$

also pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha_1 \dots \alpha_p})^{\beta_1 \dots \beta_p} (\lambda \gamma^{\alpha_1 \dots \alpha_p})_{\beta_1 \dots \beta_p}$$

"pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha \rightarrow d_\alpha = p_\alpha - (\not{P} \not{\theta})_\alpha \quad \text{constraint}$$

$$K: (\not{P} d)^\alpha \quad \{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

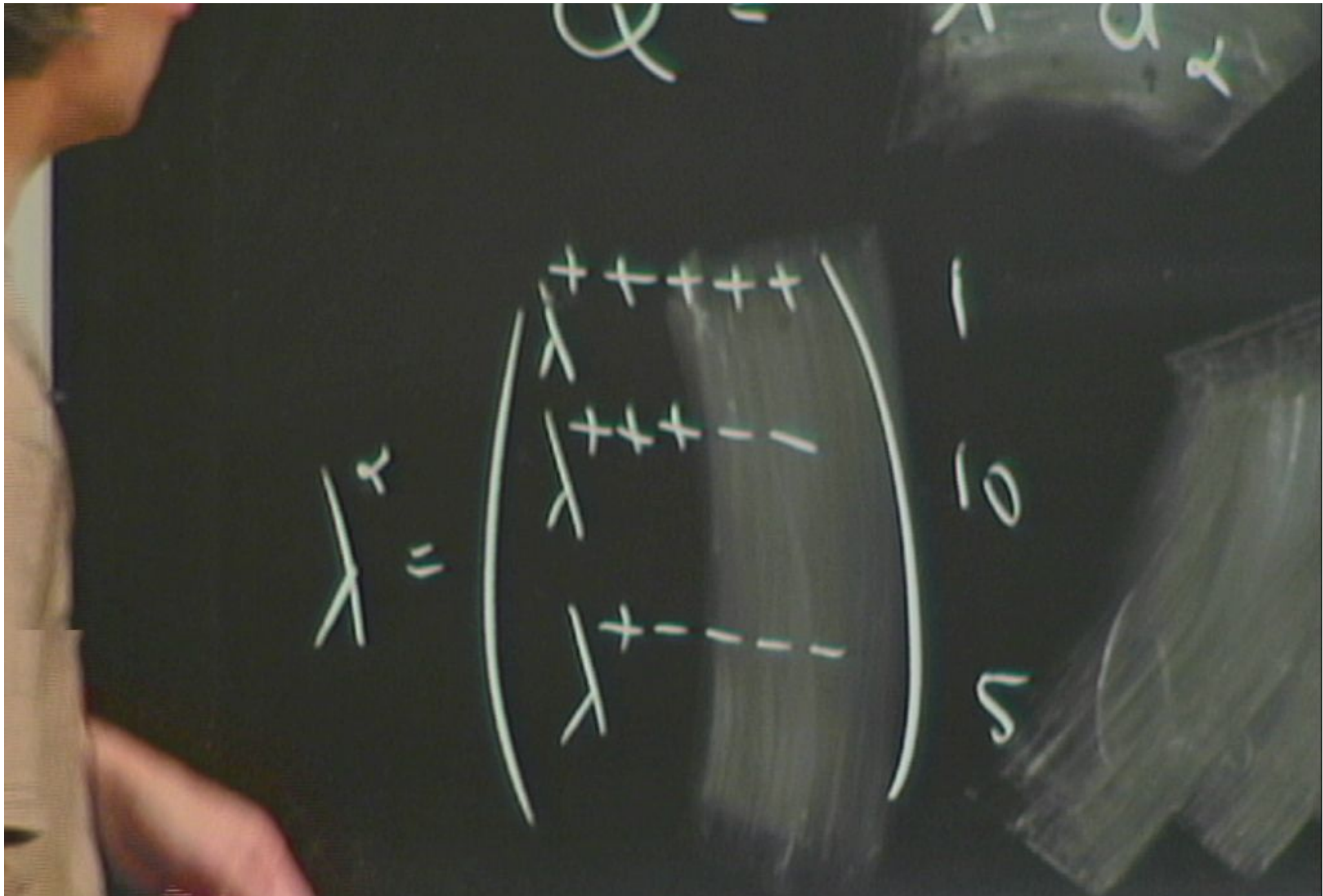
when $|\lambda \gamma^m \lambda| = 0$

also pure spinor

$$\lambda^\alpha = \begin{pmatrix} + \\ + \\ + \\ + \\ + \\ + \\ + \\ + \end{pmatrix}$$

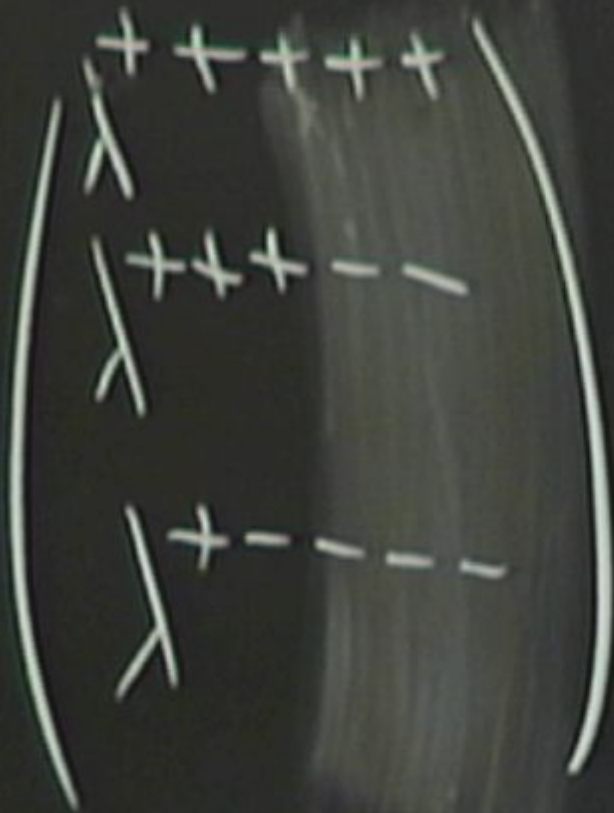
$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha_1 \dots \alpha_p})^{\beta_1 \dots \beta_p} (\lambda \gamma^{\alpha_1 \dots \alpha_p})_{\beta_1 \dots \beta_p}$$

"pure spinor"



$$Q =$$

$$x \quad d \quad x$$

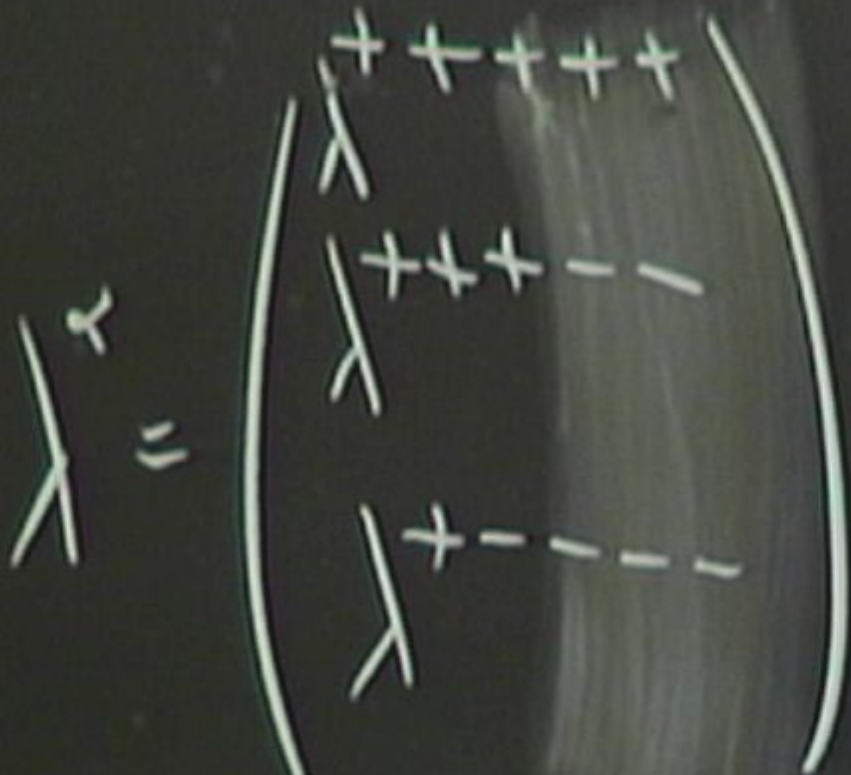


$$\begin{matrix} - \\ 0 \\ 5 \end{matrix}$$

$$\frac{SO(10)}{U(5)}$$

$Q =$

γd_x

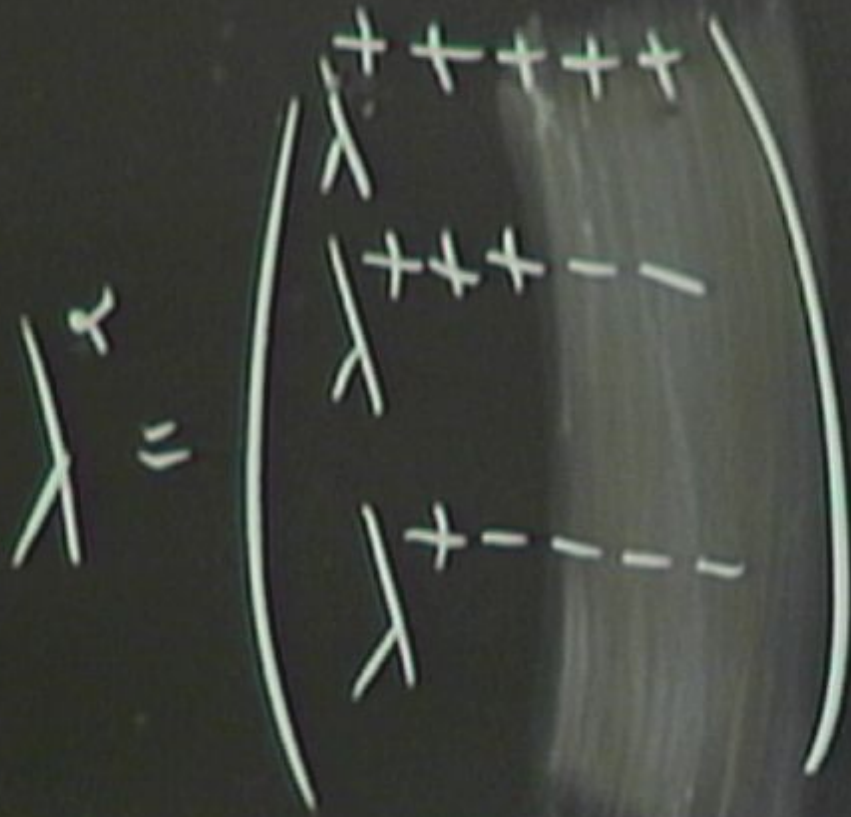


$\begin{matrix} - \\ 0 \\ 5 \end{matrix}$

$$\frac{SO(10)}{U(5)}$$

$Q =$

$x d x$



1
 0
 $1/5$

$SO(10)$

 $U(5)$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha$$

Kr: $(\not{P} d)^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\not{P} \theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \delta_{\alpha\beta} P_n \rightarrow 8 \text{ 1st class}$$

$$8 \text{ 2nd class}$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $\boxed{|\lambda \gamma^m \lambda| = 0}$

$\lambda \neq 0$
pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha_1 \dots \alpha_p})^\beta (\lambda \gamma^{\alpha_1 \dots \alpha_p} \lambda)$$

"pure spinor"



1
10
15

$\frac{SO(10)}{U(5)}$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha$$

cr: $(\not{P} \not{d})^\alpha$

$$\rightarrow d_\alpha = p_\alpha - (\not{P} \not{\theta})_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow 8 \text{ 1st class}$$

$$8 \text{ 2nd class}$$

$$Q = \lambda^\alpha d_\alpha$$

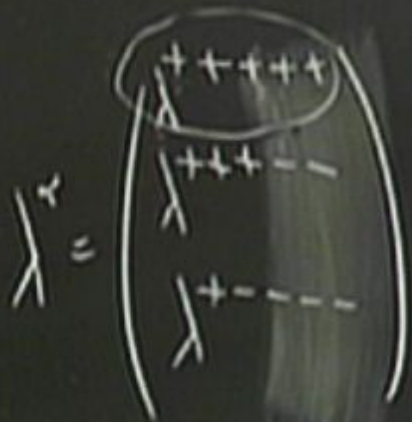
$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $|\lambda \gamma^m \lambda| = 0$

$\lambda = 10$
pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{\alpha\beta}^m)^\rho (\lambda \gamma^m \lambda)^\sigma$$

"pure spinor"



$\frac{SO(1,6)}{U(5)}$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha \rightarrow d_\alpha = p_\alpha - (\not{P} \not{\theta})_\alpha \quad \text{constraint}$$

$\{d_\alpha, d_\beta\} = \chi^m P_m \rightarrow 8 \text{ 1st class}$
 $\{d_\alpha, d_\beta\} = \chi^m P_m \rightarrow 8 \text{ 2nd class}$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m)$$

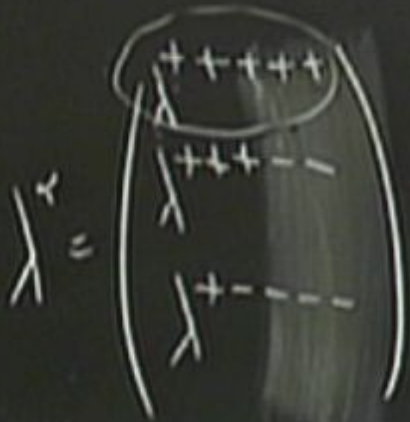
when

0

0

2+10 pure spinor

"pure spinor"



$$\gamma^1 \pm i \gamma^2$$

$$\gamma^3 \pm i \gamma^4$$

when

$$\gamma' \pm i\gamma^2$$

$$\gamma^3 \pm i\gamma^4$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\mathcal{P} \Theta)_\alpha$$

$$K_\alpha: (\mathcal{P} d)^\alpha$$

$$\Rightarrow d_\alpha = p_\alpha - (\mathcal{P} \Theta)_\alpha$$

$$\{d_\alpha, d_\beta\} = \delta_{\alpha\beta} F$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$$\text{when } |\lambda \gamma^m \lambda| =$$



1
0
1/5

$$\gamma^1 \pm i\gamma^2$$

$$\gamma^3 \pm i\gamma^4$$

$$(\lambda \gamma^m \dots)$$

$$p_\alpha = \frac{\partial L}{\partial \dot{x}^\alpha} = (\mathcal{P} \Theta)_\alpha$$

$$\rightarrow d_\alpha = p_\alpha - (\mathcal{P} \Theta)_\alpha$$

$$\{d_\alpha, d_\beta\} = \delta_{\alpha\beta} F$$

$$K_\alpha: (\mathcal{P} d)^\alpha$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$$\text{when } |\lambda \gamma^m \lambda| =$$



$\gamma^1 \pm i\gamma^2$
 $\gamma^3 \pm i\gamma^4$
 $2^{1/2}$
 \vdots

$(\lambda \gamma^m \lambda)$
 $\lambda \gamma^m \lambda$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (P \Theta)_\alpha$$

$$\Rightarrow d_\alpha = p_\alpha - (P \Theta)_\alpha$$

$$\{d_\alpha, d_\beta\} = \delta_{\alpha\beta} F$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $|\lambda \gamma^m \lambda| =$

$$(\lambda \gamma^m \dots)$$

$$\lambda \gamma^m \dots$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\mathcal{P} \theta)_\alpha$$

$$K_\alpha: (\mathcal{P} d)^\alpha$$

$$\rightarrow d_\alpha = p_\alpha - (\mathcal{P} \theta)_\alpha$$

$$\{d_\alpha, d_\beta\} = \delta_{\alpha\beta} F$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$$\text{when } |\lambda \gamma^m \lambda| =$$



1
0
15

$$2^{D/2 - 1}$$

$$\begin{pmatrix} \lambda \gamma^m \lambda \\ \lambda \gamma^m \lambda \end{pmatrix}$$

$$p_\alpha = \frac{\partial L}{\partial \dot{x}^\alpha} = (\mathcal{P} \Theta)_\alpha$$

$$K_\alpha: (\mathcal{P} d)^\alpha$$

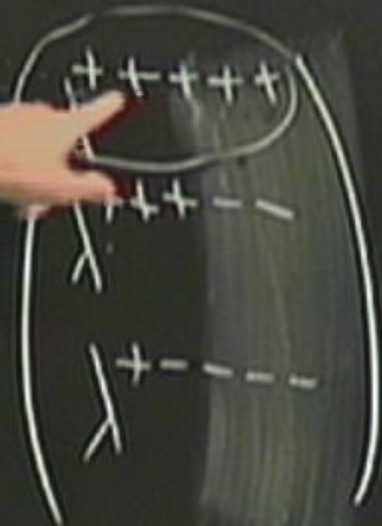
$$\rightarrow d_\alpha = p_\alpha - (\mathcal{P} \Theta)_\alpha$$

$$\{d_\alpha, d_\beta\} = \delta_{\alpha\beta} F$$

$$Q = \int d_\alpha \dot{x}^\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$$\text{when } |\lambda \gamma^m \lambda| =$$



1
0
15

$2^{D/2 - 1}$

$$(\lambda \gamma^m \dots)$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\mathcal{P} \theta)_\alpha$$

$$K_\alpha: (\mathcal{P} d)^\alpha$$

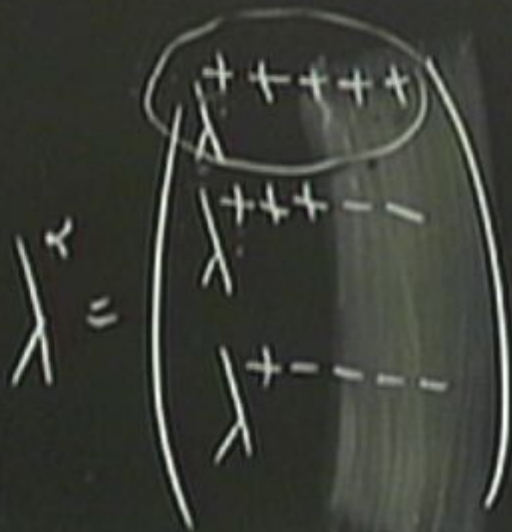
$$\Rightarrow d_\alpha = p_\alpha - (\mathcal{P} \theta)_\alpha$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m F$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$$\text{when } |\lambda \gamma^m \lambda| =$$



1
0
1/2

$2^{D/2 - 1}$

$$(\lambda \gamma^m \lambda)$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\phi}^\alpha} = (\mathcal{P} \Theta)_\alpha$$

$$\Rightarrow d_\alpha = p_\alpha - (\mathcal{P} \Theta)_\alpha$$

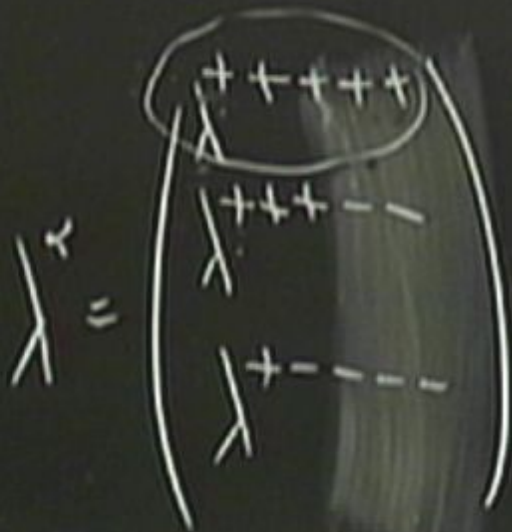
$$K_\alpha: (\mathcal{P} d)^\alpha$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta} F$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) F$$

when $|\lambda \gamma^m \lambda|$



1
0
NS

$2^{D/2 - 1}$

γ_{12}
 γ_{37}
 γ_{56}
 γ_{78}
 γ_{910}



if f(t) backgrounds

$GSO(+)$	✓	✓
$GSO(-)$?	?

	NS	R
$GSO(+)$	✓	?
$GSO(-)$	✓	?

RNS ψ^m

$$S = \int dt \left(P_m (\dot{x}^m - i \theta \gamma^m \dot{\theta}) + e P_m P^m \right)$$

Why $\theta^\alpha \quad \alpha = 1 \dots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$P_m^{\text{AD}} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_m^{\alpha\beta} & 0 \end{pmatrix}$$

Susy: $\delta \theta^\alpha = \epsilon^\alpha$

$$\delta x^m = i \epsilon \gamma^m \theta$$

Sixt
K-sym

$$\delta \theta^\alpha = P^\mu (\gamma_\mu k)^\alpha$$

$$\delta x^m = -i \delta \theta \gamma^m \theta, \quad \delta e = k_r \dot{\theta}^r$$

$$\Rightarrow \gamma_{\mu\nu}^{\alpha\beta} \gamma^{\mu\nu} = 2 \gamma^{\alpha\beta}$$

if fdt backgrounds

$GSO(+)$	✓	✓
$GSO(-)$?	?

	NS	R
$GSO(+)$	✓	?
$GSO(-)$	✓	?

RNS ψ^m

$$S = \int dt \left(P_m (\dot{x}^m - i \theta \gamma^m \dot{\theta}) + e P_m P^m \right)$$

Why $\theta^\alpha \quad \alpha=1 \dots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$P_m^{\text{AD}} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_m^{\alpha\beta} & 0 \end{pmatrix}$$

Susy:

$$\delta \theta^\alpha = \epsilon^\alpha$$

$$\delta x^m = i \epsilon \gamma^m \theta$$

Sixt
K-sym

$$\delta \theta^\alpha = P^\mu (\gamma_\mu \kappa)^\alpha$$

$$\delta x^m = -i \delta \theta \gamma^m \theta, \quad \delta e = \kappa_r \dot{\theta}^r$$

$$\Rightarrow \gamma_{\mu\nu}^{\alpha\beta} \gamma^{\mu\nu} = 2 \gamma^{\alpha\beta}$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha \quad \rightarrow \quad d_\alpha = p_\alpha - (\not{P} \not{\theta})_\alpha \quad \text{constraint}$$

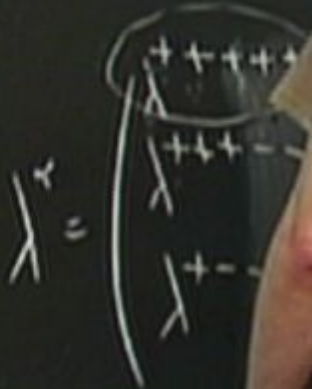
$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow 8 \text{ 1st class}$
 8 2nd class

$$Q = \lambda^\alpha d_\alpha \quad Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $|\lambda \gamma^m \lambda| = 0$

$\lambda = 0$
pure spinor

"pure spinor"



$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\theta})_\alpha$$

Kr: $(\not{P} \not{d})^\alpha$

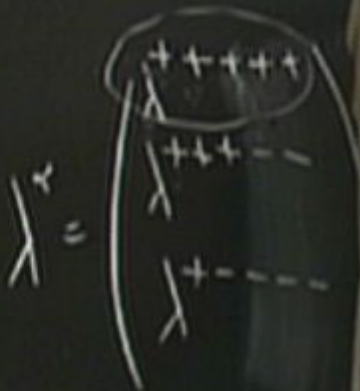
$$\rightarrow d_\alpha = p_\alpha - (\not{P} \not{\theta})_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_\alpha \quad Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $|\lambda \gamma^m \lambda| = 0$

$\lambda = 0$
pure spinor



$$u_{ab} = -u_{ba}$$

$$(\lambda \gamma^m \lambda) = 0$$

"pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot{x}^\alpha} = (\not{P} \not{D})_\alpha \rightarrow d_\alpha = p_\alpha - (\not{P} \not{D})_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_\alpha \quad Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$\lambda = 0$
 pure spinor

when $|\lambda \gamma^m \lambda| = 0$



$u_{ab} = -u_{ba}$

$u \in \frac{SO(1,5)}{U(5)}$

$(\lambda \gamma^m \lambda)$

"pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{\Theta})_\alpha \rightarrow d_\alpha = p_\alpha - (\not{P} \not{\Theta})_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_\alpha \quad Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $|\lambda \gamma^m \lambda| = 0$ $\lambda \neq 0$
pure spinor

$u \in \frac{SO(1,5)}{U(5)}$ "pure spinor"

$u_{ab} = -u_{ba}$
 $u_{bc} = u_{cb}$
 $u_{cd} = u_{dc}$

$\lambda_{ab} = \lambda_{ba}$

+++++
 +++--
 = λ_{ab}

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P} \not{D})_\alpha \quad \rightarrow \quad d_\alpha = p_\alpha - (\not{P} \not{D})_\alpha \quad \text{constraint}$$

$$K.T.: (\not{P} d)^\alpha \quad \{d_\alpha, d_\beta\} = \gamma_{\alpha\beta}^m P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$Q = \lambda^\alpha d_\alpha \quad Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$\lambda = 0$
pure spinor

when $|\lambda \gamma^m \lambda| = 0$

$\lambda^\alpha = \begin{pmatrix} + & + & + & + & + \\ + & + & + & - & - \\ + & + & - & - & - \\ + & - & - & - & - \end{pmatrix}$

$\lambda^\alpha = \begin{pmatrix} + \\ + \\ + \\ + \end{pmatrix}$

$\lambda^\alpha = \begin{pmatrix} + \\ + \\ - \\ - \end{pmatrix}$

$\lambda^\alpha = \begin{pmatrix} + \\ - \\ - \\ - \end{pmatrix}$

$u_{ab} = -u_{ba}$

$u \in \frac{SO(1,5)}{U(5)}$

$(\lambda \gamma^m \lambda) = 0$

"pure spinor"

(Pure Spinor)	GSO(-)	?	?	GSO(-)	✓	?	?	KNS
---------------	--------	---	---	--------	---	---	---	-----

$$\int dt \left(\dot{x}^m - i\theta\gamma^m\dot{\theta} \right) \in P_m P^m$$

Weyl $\Theta^\alpha \quad \alpha=1\dots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$\Gamma_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_m^{\mu\nu} & 0 \end{pmatrix}$$

\rightarrow d=10 SYM

$$i \epsilon \gamma^m \theta$$

$$P^m (\gamma_m k)^\alpha$$

$$-i \int \theta \gamma^m \theta, \quad \int e = k_\alpha \dot{\theta}^\alpha$$

$$\Rightarrow \gamma_m^{\alpha\beta} \gamma_m^{\mu\nu} = 2\eta^{\mu\nu} \delta_\alpha^\beta$$

$$A_x(x, \theta)$$

$$A_m(x, \theta)$$

$$A_k(x, \theta)$$

$$A_m(x, \theta)$$

$$\Delta_k =$$

$$D_k = \frac{\partial}{\partial \theta^k} + (\gamma^k \theta)_\alpha \frac{\partial}{\partial x^\alpha}$$

$$\{D_k, D_p\} = \gamma_{k\alpha}^p \frac{\partial}{\partial x^\alpha}$$

$$A_\alpha(x, \theta)$$

$$A_m(x, \theta)$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + (\gamma^\alpha \theta)_\alpha \frac{\partial}{\partial x^\alpha}$$

$$\{D_\alpha, D_\beta\} = \gamma_{\alpha\beta} \frac{\partial}{\partial x^\alpha}$$

$$\nabla_\alpha = D_\alpha + A_\alpha(x, \theta)$$

$$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x, \theta)$$

$$A_\alpha(x, \theta) =$$

$$A_m(x, \theta) =$$

$$\{\nabla_\alpha, \nabla_\beta\} = \gamma_{\alpha\beta}^m \nabla_m$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + (\gamma^\alpha \theta)_\mu \frac{\partial}{\partial x^\mu}$$

$$\{D_\alpha, D_\beta\} = \gamma_{\alpha\beta}^m \frac{\partial}{\partial x^m}$$

$$\nabla_\alpha = D_\alpha + A_\alpha(x, \theta)$$

$$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x, \theta)$$

$$A_\alpha(x, \theta) =$$

$$A_m(x, \theta) =$$

$$\{\nabla_\alpha, \nabla_\beta\} = \gamma_{\alpha\beta}^m \nabla_m$$

$$\Rightarrow D_\alpha A_\beta + D_\beta A_\alpha = \gamma_{\alpha\beta}^m A_m$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + (\gamma^\alpha \theta)_\alpha \frac{\partial}{\partial x^\alpha}$$

$$\{D_\alpha, D_\beta\} = \gamma_{\alpha\beta}^m \frac{\partial}{\partial x^m}$$

$$\nabla_\alpha = D_\alpha + A_\alpha(x, \theta)$$

$$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x, \theta)$$

$$A_m(x, \theta) =$$

$$\{\nabla_\alpha, \nabla_\beta\} = \gamma_{\alpha\beta}^m \nabla_m$$

$$\Rightarrow D_\alpha A_\beta + D_\beta A_\alpha = \gamma_{\alpha\beta}^m A_m$$

$$\Rightarrow A_m = \gamma_m^{\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma_m^{\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha) = 0$$

$$\{D_\alpha, D_\beta\} = \gamma_{\alpha\beta}^m \frac{\partial}{\partial x^m}$$

$$\nabla_\alpha = D_\alpha + A_\alpha(x, \theta)$$

$$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x, \theta)$$

$$A_m(x, \theta) =$$

$$\{D_\alpha, D_\beta\} = \gamma_{\alpha\beta} \Delta_m$$

$$A_\beta + D_\beta A_\alpha = \gamma_{\alpha\beta} A_m$$

$$(D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma_{m\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha) = 0$$

$$\{D_\alpha, D_\beta\} = \gamma_{\alpha\beta} \frac{\partial}{\partial x^m}$$

$$\Delta_\alpha = D_\alpha + A_\alpha(x, \theta)$$

$$\Delta_m = \frac{\partial}{\partial x^m} + A_m(x, \theta)$$

$$\left. \begin{array}{l} \delta A_\alpha = D_\alpha \Omega \\ \delta A_m = \partial_m \Omega \end{array} \right|$$

$$A_\alpha(x, \theta) =$$

$$A_m(x, \theta) =$$

$$\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta} \nabla_m$$

$$\Rightarrow D_\alpha A_\beta + D_\beta A_\alpha = \gamma^m_{\alpha\beta} A_m$$

$$\Rightarrow A_m = \gamma^m_{\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma^{\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha) = 0$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + (\gamma^\mu{}_\alpha) \frac{\partial}{\partial x^\mu}$$

$$\{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta} \frac{\partial}{\partial x^m}$$

$$\nabla_\alpha = D_\alpha + A_\alpha(x, \theta)$$

$$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x, \theta)$$

$$\delta A_\alpha = D_\alpha \Omega$$

$$\delta A_m = \partial_m \Omega$$

$$P_m(\dot{x}^m - i\theta\gamma^m\dot{\theta}) \in P_m P^m \quad \text{Weyl } \Theta^\alpha \quad \alpha=1\dots 16$$

$$\epsilon^r$$

$$= i\epsilon\gamma^m\theta$$

$$\rightarrow \boxed{d=}$$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$P_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_m^{\alpha\beta} & 0 \end{pmatrix}$$

$$P^m (\gamma_m k)^{\alpha\beta}$$

$$k^{\alpha\beta} = -i\int \theta\gamma^m\theta, \quad \text{se} = k$$

$$= 2\eta^{\alpha\beta} \int_k$$



$$P_m(\dot{x}^m - i\theta\gamma^m\dot{\theta}) \in P_m(\Gamma^m)$$

Weyl $\Theta^\alpha \quad \alpha=1\dots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$\Gamma_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_m^{\alpha\beta} & 0 \end{pmatrix}$$

$$\gamma_m^{\alpha\beta} \gamma_n^{\gamma\delta}$$

$$\gamma_m^{\alpha\beta} \gamma_n^{\gamma\delta} = 2\eta^{\alpha\gamma} \delta_m^n$$

$$\epsilon^{\alpha\beta\gamma\delta}$$

$$= i\epsilon\gamma^m\theta$$

$$P^m(\gamma_m k)^\alpha$$

$$k^\alpha = -i\delta\theta\gamma^\alpha\theta, \quad \text{se}$$

→ Γ^m



$$A_m = \cancel{f_x} + a_m(\gamma^m \theta)_x = \chi \gamma^m \theta$$

$$A_m = a_m(x) + (\chi \gamma^m \theta)$$

CAUTION

$$A_{\alpha} = \cancel{f_{\alpha}} + a_m(\gamma^m \theta)_{\alpha} = (\chi \gamma^m \theta)_{\alpha} + \partial_{\alpha} a(\theta \gamma^m \theta) (\gamma_m \theta)_{\alpha} \dots$$

$$A_m = a_m(x) + (\chi \gamma^m \theta)$$

CAUTION

$$A_\mu = \cancel{f}_\mu + a_m (\gamma^m \theta)_\mu + (\chi \gamma^m \theta) (\gamma_m \theta)_\mu + \partial_\nu a (\theta \gamma^{\nu\mu} \theta) (\gamma_{\mu\nu} \theta) + \dots$$

$$A_m = a_m(x) + (\chi \gamma^m \theta) + \dots$$

$$\partial^\mu \partial_\mu a_n = 0, (\not{\chi} \chi)_\mu = 0$$

$$\int d^{10}x \left(F^{mn} F_{mn} + \chi \not{\psi} \chi \right)$$



$$A_m = \cancel{f_m} + a_m(\gamma^m \theta)_\alpha + (\chi \gamma^m \theta)_\alpha (\gamma_m \theta)_\alpha + \partial_\alpha a_m (\theta \gamma^m \theta) (\gamma_m \theta)_\alpha \dots$$

$$A_m = a_m(x) + (\chi \gamma^m \theta)_\alpha \dots$$

$$\partial^\mu \partial_\mu a_m = 0, (\cancel{\psi} \chi)_\alpha = 0$$

$$\int d^{10}x \left(F^{mn} F_{mn} + \chi \cancel{\psi} \chi \right) \stackrel{?}{=} \int d^4x \int d^6\theta \left(\dots \right)$$

$$A_\alpha(x, \theta) =$$

$$A_m(x, \theta) =$$

$$\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta} \nabla_m$$

$$\Rightarrow D_\alpha A_\beta + D_\beta A_\alpha = \gamma^m_{\alpha\beta} A_m$$

$$\Rightarrow A_\alpha = \gamma^{\beta\gamma}_m (D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma^{\alpha\beta}_{mnp} (D_\alpha A_\beta + D_\beta A_\alpha) = 0$$

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + (\gamma^m \theta)_\alpha \frac{\partial}{\partial x^m}$$

$$\{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta} \frac{\partial}{\partial x^m}$$

$$\nabla_\alpha = D_\alpha + A_\alpha(x, \theta)$$

$$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x, \theta)$$

$$\delta A_\alpha = \partial_\alpha \Omega$$

$$\Omega = f_n \theta^n$$

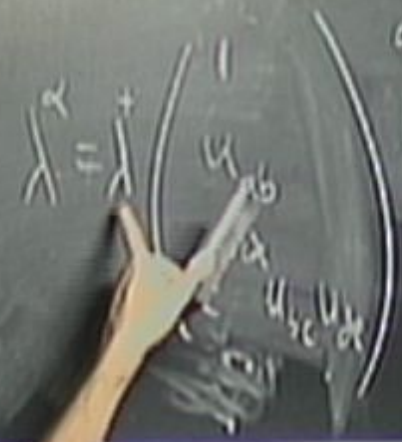
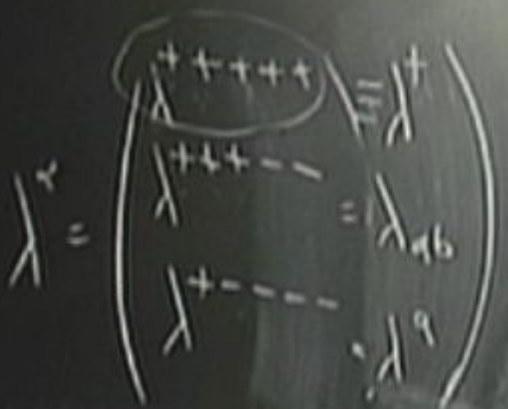
$$A_n = \cancel{f_n} + a_m (\gamma^m \theta)_n + (\chi \gamma^m \theta)_n (\gamma_m \theta)_n + \partial_n a (\theta \gamma^{m1} \theta) (\gamma_{m1} \theta)_n$$

$$A_m = a_m(x) + (\chi \gamma^m \theta)$$

$$\partial^m \partial_m a_n = 0$$

$$\int d^{10}x \left(F^{mn} F_{mn} + \chi \gamma^m \theta \right)$$

$$Q = \lambda d_x \quad Q^2$$



when $|\lambda \gamma^m| = 0$

cont. $U_{ab} = U_{ba}$

$u \in \frac{SO(m)}{U(1)}$

$(\lambda \gamma^m \dots \gamma^a)$

$\lambda \gamma^m \dots \gamma^a$

2-dim pure spinor

"pure spinor"

$$A_k(x, \theta)$$

$$A_m(x, \theta)$$

$$D_x = \frac{\partial}{\partial \theta^x} + (\gamma^x \theta)_\alpha \frac{\partial}{\partial x^\alpha}$$

$$\{D_x, D_\beta\} = \gamma_{x\beta}^a \frac{\partial}{\partial x^a}$$

$$\nabla = D + A(x, \theta)$$

$$d=4$$

$$A = \begin{pmatrix} 1 \\ u \end{pmatrix}$$

$$A=1, 2$$

$$d=4$$

$$\lambda = \begin{pmatrix} 1 \\ u \end{pmatrix}$$

$$A=1, 2$$

$$\rightarrow d=10$$

$$\lambda^* = \begin{pmatrix} 1 \\ u \\ uu \end{pmatrix}$$

$$k) + (\chi \gamma^m \theta) \rightarrow \dots$$

$$\partial^m \partial_n a_n = 0, (\not{\partial} \chi)_k = 0$$

(5) $(\text{Pin } \mathbb{S}^{n-1})$ GSO(-) ? | ?

GSO(+)	✓	?
GSO(-)	✓	?

RNS ψ

$$Q = \lambda^\alpha d_\alpha$$

$$Q \cdot V = 0, \quad \delta V = Q \Omega$$

$$g_n = 1 \quad V = \lambda^\alpha \tilde{A}_\alpha(x, \theta)$$

(5) (Pm Spin)	GSD(-1)	?	?	GSD(+)	✓	?	RNS	✓
				GSD(-)	✓	?		

$$Q = \lambda^\alpha d_\alpha = \lambda^\alpha \left(\frac{\partial}{\partial \theta^\alpha} - (\gamma^\mu \theta)_\alpha \frac{\partial}{\partial x^\mu} \right) = \lambda^\alpha D_\alpha$$

$$V = 0, \quad \delta V = Q\Omega$$

$$g_n = 1 \quad V = \lambda^\alpha \tilde{A}_\alpha(x, \theta) \quad \rightarrow \quad QV = \lambda^\alpha \lambda^\beta (D_\alpha \tilde{A}_\beta + D_\beta \tilde{A}_\alpha)$$

$\langle \psi \hat{p}_m \hat{p}_{m+1} \psi \rangle$	GSD(-1) ?	?	GSD(+1) ✓	?	RNS
			GSD(-1) ✓	?	

$$Q = \lambda^\alpha d_\alpha \left(\frac{\partial}{\partial \theta^\alpha} - (\gamma^m \theta)_\alpha \frac{\partial}{\partial x^m} \right) = \lambda^\alpha D_\alpha$$

$$Q V = 0, \quad \{ \dots \}$$

$$g_n = 1$$

$$V(x, \theta) \rightarrow QV = \lambda^\alpha \lambda^\rho (D_\alpha \tilde{A}_\rho) = 0$$

$$\Rightarrow \cancel{(\lambda^\alpha \lambda^\rho)} \gamma^{\alpha\rho} D_\alpha A_\rho + (\lambda^\alpha \lambda^\rho \gamma^{\alpha\rho}) \gamma^{\alpha\rho} D_\alpha A_\rho$$

(5) $(\text{Pin } \delta_{p-1})$	GSD(-1) ?	?	GSD(+)	✓	?	RNS	✓
			GSD(-)	✓	?		

$$Q = \lambda^\alpha d_\alpha = \lambda^\alpha \left(\frac{\partial}{\partial \theta^\alpha} - (\gamma^m \theta)_\alpha \frac{\partial}{\partial x^m} \right) = \lambda^\alpha D_\alpha$$

$$Q V = 0, \quad \delta V = Q \Omega$$

$$g^n = 1 \quad V = \lambda^\alpha \tilde{A}_\alpha(x, \theta) \quad \rightarrow \quad QV = \lambda^\alpha \lambda^\beta (D_\alpha \tilde{A}_\beta) = 0$$

$$\Rightarrow \cancel{(\lambda^\alpha \lambda^\beta)} \gamma^{\alpha\beta} D_\alpha A_\beta + (\lambda^\alpha \lambda^\beta \gamma^{\alpha\beta}) \gamma^{\alpha\beta} D_\alpha A_\beta = 0$$

65) (Pure Spin) GSO(-) ? | ?

GSO(+)	✓	?
GSO(-)	✓	?

RNS

$$Q = \lambda^\alpha d_\alpha = \lambda^\alpha \left(\frac{\partial}{\partial \theta^\alpha} - (\gamma^\mu \theta)_\alpha \frac{\partial}{\partial x^\mu} \right) = \lambda^\alpha D_\alpha$$

$$Q V = 0, \quad \delta V = Q \Omega(x, \theta)$$

$$g^{\alpha\beta} = 1 \quad V = \lambda^\alpha \tilde{A}_\alpha(x, \theta) \quad \rightarrow QV = \lambda^\alpha \lambda^\beta (D_\alpha \tilde{A}_\beta) = 0$$

$$\delta V = Q \Omega = \lambda^\alpha D_\alpha \Omega$$

$$\rightarrow \lambda^\alpha A_\alpha = D_\alpha \Omega$$

$$\Rightarrow (\cancel{\lambda^\alpha} \lambda^\beta) \gamma^{\alpha\beta} D_\alpha A_\beta$$

$$+ (\lambda^\alpha \gamma^{\alpha\beta}) \gamma^{\gamma\delta} D_\alpha A_\beta = 0$$

(5) (Pure Spin)	GSO(-)	?	?	RNS	?
	GSO(+)	?	?		?

$$Q = \lambda^\alpha d_\alpha = \lambda^\alpha \left(\frac{\partial}{\partial \theta^\alpha} - (\gamma^\mu \theta)_\alpha \frac{\partial}{\partial x^\mu} \right) = \lambda^\alpha D_\alpha$$

$Q V = 0$, $\delta V = Q \Omega(x, \theta) \Rightarrow \bar{A}_\alpha$ describes SYM on shell

$$g_n = 1 \quad V = \lambda^\alpha \bar{A}_\alpha(x, \theta) \quad \rightarrow \quad QV = \lambda^\alpha \lambda^\rho (D_\alpha \bar{A}_\rho) = 0$$

$$\delta V = Q \Omega = \lambda^\alpha D_\alpha \Omega$$

$$\rightarrow \bar{A}_\alpha = D_\alpha \Omega$$

$$\Rightarrow \cancel{(\lambda^\alpha \lambda^\rho)} \gamma^{\mu \alpha \rho} D_\mu \bar{A}_\rho + (\lambda^\mu \lambda^\nu \lambda^\rho) \gamma^{\mu \nu \rho} D_\mu \bar{A}_\rho = 0$$

$$\nabla_{\mathbf{x}} \cdot \mathbf{D} \nabla_{\mathbf{x}} \psi$$

$$\nabla_{\mathbf{x}} \cdot (\mathbf{K} \nabla_{\mathbf{x}} \psi) = 0$$

$$\int d\lambda \, d\theta \, dx$$



$$(A_x^{\mu\nu} D_\mu D_\nu) L$$

$$\frac{\delta}{\delta A_\mu^{\nu\rho}} \int d^4x \mathcal{L} = 0$$

$$\int d^4x d^4\theta d^4x$$

$$\int d^4\theta \int d^4x \quad \text{I}$$

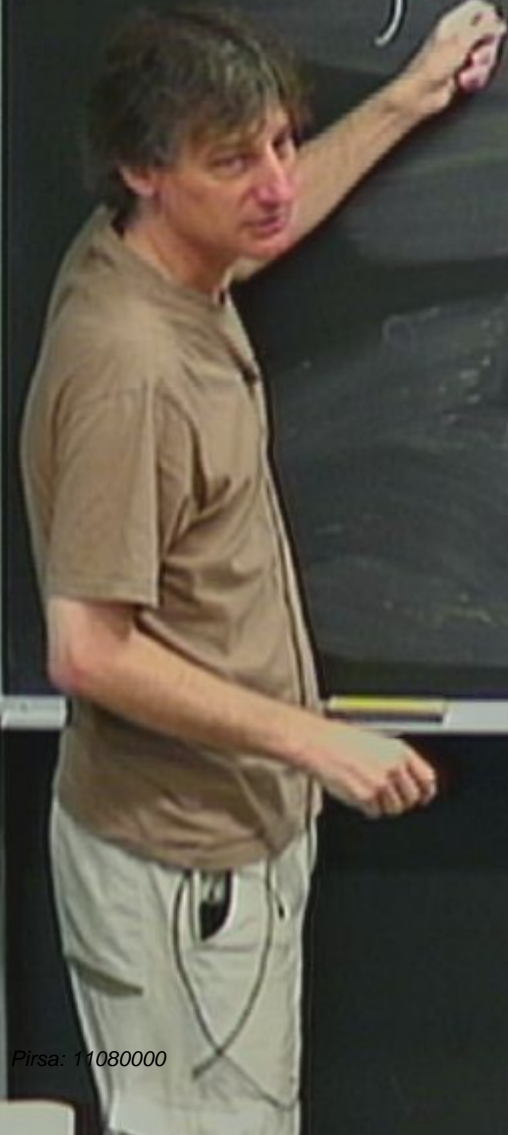


$$(A_x^{-1} D_x) L$$

$$\int (A_x^{-1} D_x) L = 0$$

$$\int d\lambda \int d\theta \int d^p x \int d\bar{\lambda} d\bar{r}$$

$$\int d^p \theta \int d^p x \quad \text{I}$$



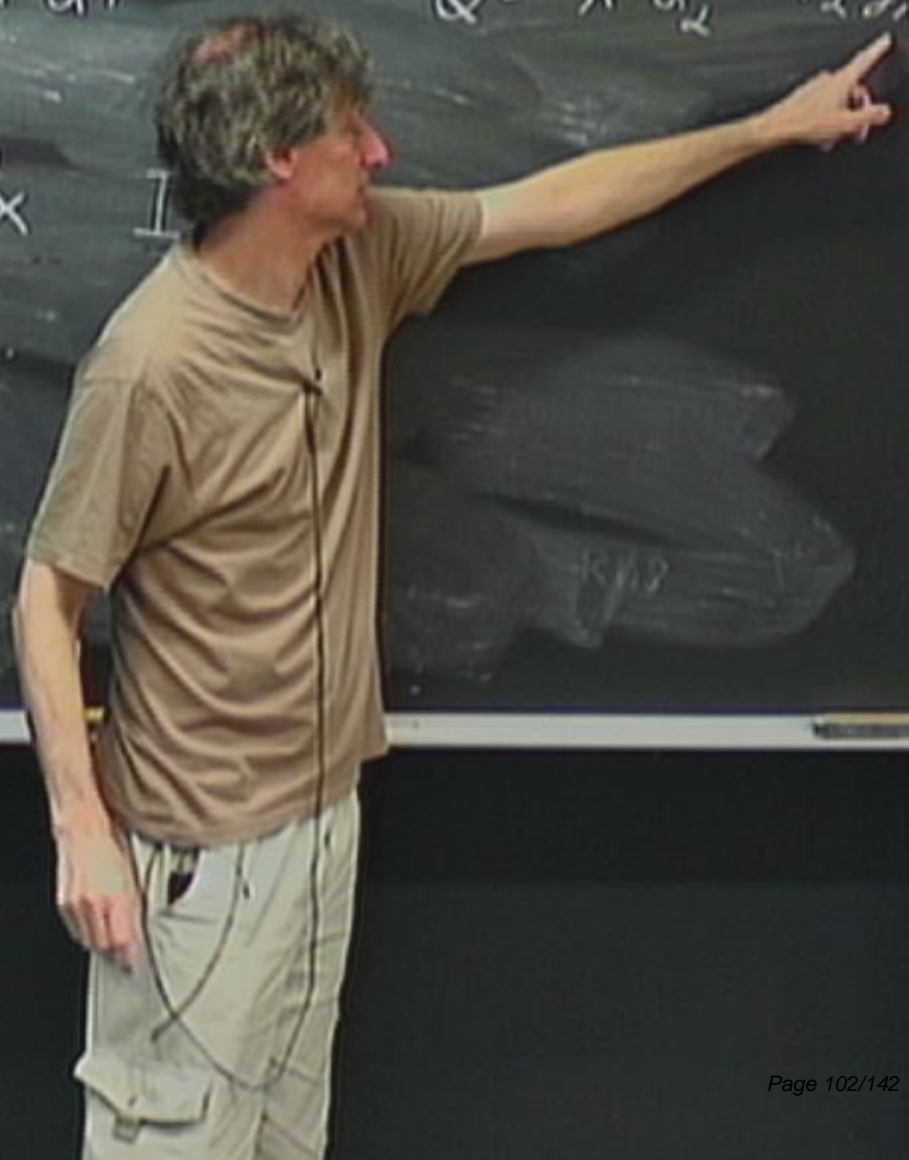
$$(A_x - D_x) L$$

$$(A_x - D_x) \gamma^{ij} D_j A_i = 0$$

$$\int d\lambda d\theta d^D x \int d\bar{\lambda} d\bar{r}$$

$$Q = \lambda^x d_x + r_x \frac{\partial}{\partial \lambda^x}$$

$$\int d^D \theta \int d^D x \quad I$$



$$(A_x^{-1} D_x) L$$

$$(A_x^{-1} D_x) \gamma_{\mu\nu} D_x A_\mu = 0$$

$$\int d\lambda d\theta d^p x \int d\tilde{\lambda} d\tilde{r} \gamma_{123} Q = \lambda^2 d_2 + r_2 \frac{2}{\lambda^2}$$

$$\int d^5 \theta \int d^p x \quad I$$

$$k_1 \cdot k_2 = k_2 \cdot k_3 \\ = k_3 \cdot k_1 = 0$$

$$(A_x - D_x) L$$

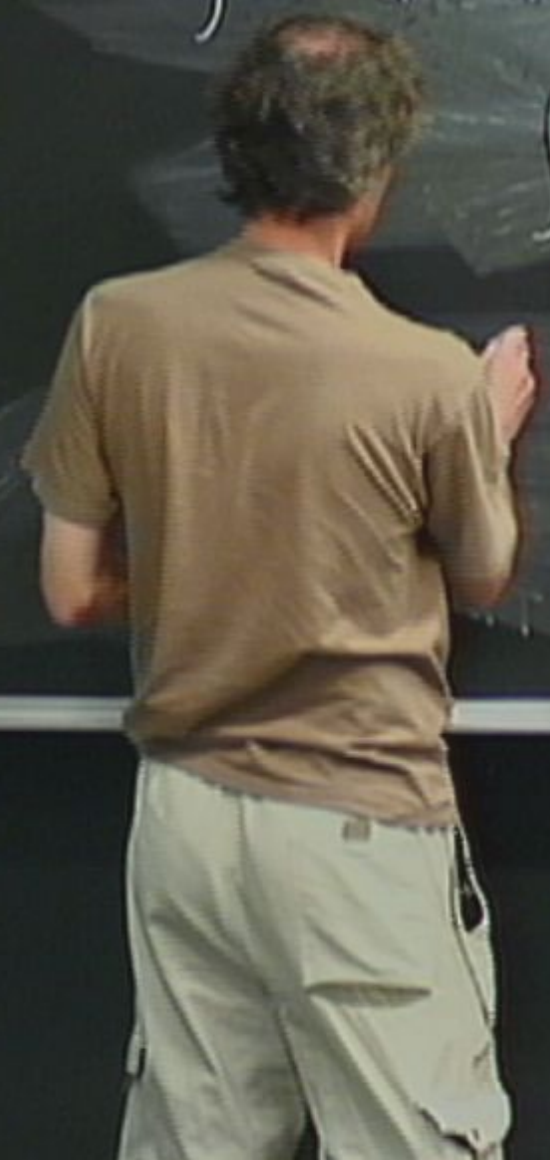
$$(18 \dots) \gamma_{\mu\nu} D_\nu A_\mu = 0$$

$$\int d\lambda d\theta d^D x \int d\vec{\lambda} d^D r e^{i\vec{\lambda}\cdot\vec{r}} \dots^{(Q4)}$$

$$Q = \lambda^2 d_2 + r_2 \frac{2}{\lambda^2}$$

$$\int d^5 \theta \int d^D x \quad I$$

$$k_1 \cdot k_2 = k_2 \cdot k_3 \\ = k_3 \cdot k_1 = 0$$



$$\delta V = Q \int \Omega - \lambda \int D_2 \Omega$$

$$\rightarrow \int A_2 \cdot D_2 \Omega$$

$$\Rightarrow \left(\frac{\partial \psi}{\partial \lambda} \right) \gamma^{2,2} D_2 A_2$$

$$+ \left(\frac{\partial \psi}{\partial \lambda} \right) \gamma^{2,2} D_2 A_2$$

$$\int d\lambda \, d^4\theta \, d^p x \quad \int d\lambda \, d^3r \, d^3k \quad (Q\psi)$$

$$Q = \lambda^2 d_2 + r_2 \frac{\partial^2}{\partial \lambda^2}$$

$$\int d^5\theta \int d^{10}x$$

$$k_1 \cdot k_2 = k_3 \cdot k_4$$

$$= k_5 \cdot k_6 = 0$$

$$Q\psi =$$



$$(17) \quad D_2 \psi = 0$$

$$(18) \quad \gamma_{\mu\nu} D_\mu A_\nu = 0$$

$$\int d^4x \int d^3\theta \int d^3x \int d^3\theta \int d^3x \int d^3\theta \quad (24)$$

$$\int d^4x \int d^3\theta \quad I$$

$$V_1, V_2, V_3$$

$$Q = \lambda^2 d_2 + r_2 \frac{2}{\lambda^2}$$

$$k_1 \cdot k_2 = k_2 \cdot k_3 \\ = k_3 \cdot k_1 = 0$$



$$(17) \quad D_4 A = 0$$

$$(18) \quad \gamma_{\mu\nu} D_4 A = 0$$

$$\int d^4\theta \int d^4x \int d^4p \int d^4q \int d^4r \int d^4s \quad (24)$$

$$Q = \lambda^2 d_2 + r_2 \frac{2}{\lambda^2}$$

$$\int d^4\theta \int d^4x \quad I$$

$$k_1 \cdot k_2 = k_2 \cdot k_3 \\ = k_3 \cdot k_1 = 0$$

$$V_1 V_2 V_3$$

$$V_1 = e^{ik_1 x}$$

$$V_2 = e^{ik_2 x}$$



$$g_n = 1 \quad V = \lambda^x A_x(x, \theta) \rightarrow QV = \lambda \lambda^p (D_x A_p) = 0$$

$$\delta V = Q\Omega - \lambda^x D_x \Omega$$

$$\rightarrow \int A_x - D_x \Omega$$

$$\Rightarrow \left(\frac{\delta V}{\delta \lambda} \right) \gamma^{x\mu} D_x A_\mu + \left(\frac{\delta V}{\delta \theta} \right) \gamma^{x\mu} D_x A_\mu = 0$$

$$\int d\lambda d\theta d^3x \int d\vec{\lambda} d\vec{r} e^{iQ} \left(\frac{\delta V}{\delta \lambda} \right) \gamma^{x\mu} D_x A_\mu \Big|_{Q = \lambda^x d_x + r_x \frac{\partial}{\partial \lambda^x}}$$

Q

$$g_n = 1 \quad V = \lambda^x A_x(x, \theta) \Rightarrow QV = \lambda \lambda^p (D_p A_p) = 0$$

$$\delta V = Q\Omega - \lambda^x D_x \Omega$$

$$\rightarrow (A_x - D_x) \Omega$$

$$\Rightarrow \left(\cancel{\lambda^x} \lambda^p \right) \gamma^{x,p} D_p A_p + \left(\lambda^x \lambda^p \right) \gamma^{x,p} D_p A_p = 0$$

$$\int d\lambda \, d\theta \, d^p x \int d\bar{\lambda} \, d\bar{r} e^{i(QY)} \Big| Q = \lambda^x d_x + r_x \frac{\partial}{\partial \bar{\lambda}^x}$$

$g_n = 0 \Rightarrow$ YM ghost et zero Ω

$$g_n = 1 \quad V = \lambda^x A_x \quad \Rightarrow \quad QV = \lambda \lambda^p (D_x A_p) = 0$$

$$\delta V = Q\Omega - \lambda^x D_x \Omega$$

$$\rightarrow (A_x - D_x) \Omega$$

$$\Rightarrow \begin{aligned} & \cancel{(\lambda^x \lambda^p)} \gamma^{x,p} D_x A_p \\ & + (\lambda^x \lambda^p) \gamma^{x,p} D_x A_p = 0 \end{aligned}$$

$$\int d\lambda \, d\theta \, d^3x \int d\bar{\lambda} \, d\bar{\theta} \, d^3\bar{x} \, \mathcal{L} / Q = \lambda^x d_x + r_x \frac{\partial}{\partial \lambda^x}$$

$g_n = 0 \rightarrow$ YM ghost at zero mass Ω

$$g_n = 1 \quad V = \lambda^x A_x \quad \Rightarrow \quad QV = \lambda \lambda^p (D_p A_p) = 0$$

$$\delta V = Q\Omega - \lambda^x D_x \Omega$$

$$\rightarrow (A_x - D_x)\Omega$$

$$\Rightarrow \left(\cancel{\lambda^x} \lambda^p \right) \gamma^{x,p} D_p A_p$$

$$+ \left(\lambda^x \lambda^p \right) \gamma^{x,p} D_p A_p = 0$$

$$\int d\lambda \, d\theta \, d^p x \int d\bar{\lambda} \, d\bar{\theta} \, d^p \bar{x} \, \mathcal{L} / Q = \lambda^x d_x + r_x \frac{2}{\lambda^x}$$

ghost $g_n = 0 \rightarrow$ YM ghost at zero mass Ω

field
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ω

$$K: (\not{D})^2$$

$$\{d_{\alpha}, d_{\dot{\beta}}\} = \gamma_{\alpha\dot{\beta}}^m P_m \rightarrow \begin{matrix} 8 & 1^{\text{st}} \text{ class} \\ 8 & 2^{\text{nd}} \text{ class} \end{matrix}$$

$$Q = \lambda^{\alpha} d_{\alpha}$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

$\lambda = 0$
pure spinor

$$\lambda^{\alpha} = \begin{pmatrix} + & + & + & + \\ + & + & + & - \\ - & + & + & + \\ + & - & + & - \\ - & + & - & + \end{pmatrix}$$

$$\lambda^{\alpha} = \begin{pmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{pmatrix}$$

when $\lambda \gamma^m \lambda = 0$
cont. 5
 $u_{ab} = -u_{ba}$
 $(\lambda \gamma^m \lambda)$

"pure spinor"

$$\not{D} \neq 0$$

$$\Rightarrow D_{\alpha} A_{\beta} + D_{\beta} A_{\alpha} = \gamma_{\alpha\beta}^m A_m$$

$$\Rightarrow A_m = \gamma_m^{\alpha\beta} (D_{\alpha} A_{\beta} + D_{\beta} A_{\alpha}), \quad \gamma_m^{\alpha\beta} = \gamma_{\alpha\beta}^m$$

$$\frac{\partial}{\partial x^{\alpha}} + A_{\alpha}(x, \theta)$$

$$\delta A_{\alpha} = D_{\alpha} \Omega$$

$$\delta A_m = \partial_m \Omega$$

$$\int dt \left((\dot{x}^m - i\theta\gamma^m \dot{\theta}) P_m + e P^m P_m + \dots \right)$$

$$\Rightarrow A_m = \gamma_m^{\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma_{mnpq} (D_\alpha A_\beta + D_\beta A_\alpha) = 0$$

$$\frac{\delta A_\alpha}{\delta A_\beta} = \dots$$

$$D_\mu \gamma^\mu = 0$$

$$D_\mu \gamma^\mu = 0$$

$$\int d\lambda d\theta d^p x \int d\tilde{\lambda} d\tilde{\theta} d^p \tilde{x} \int d^4 y \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi + \dots \right]$$

ghost $g_n = 0 \rightarrow$ YM ghost at zero mass Ω

field
antifield
antighost

$$QV_1 = 0, \quad \psi_1$$

$$QV_2 = Q\Omega_2, \quad \psi_2$$

$$QV_2 = Q\Omega_1$$

$$\int d\vec{a} \, d\vec{\theta} \, d^p x \int d\vec{a} \, d\vec{r} e^{iQY} / Q = \lambda^2 d_2 + r_2 \frac{2}{\lambda^2}$$

ghost $n=0$ \rightarrow YM ghost at zero mass Ω

$$QV_1 = 0 \quad \text{or}$$

$$\delta V_2 = Q\Omega_1$$

$$\delta V_1 = Q\Omega_2 = 0$$

$$\int A^{*n} \cdot \partial^p \Omega$$

$$\int d\lambda d\theta d^4x \int d\bar{\lambda} d^4re \stackrel{(QY)}{V_2 V_3} \quad | \quad Q = \lambda^\alpha d_\alpha + r_\alpha \frac{\partial}{\partial \bar{\lambda}^\alpha}$$

ghost $g_n = 0 \rightarrow$ YM ghost at zero mass Ω

field
antifield
antighost

$$\partial_n A^{*n} = 0$$

$$QV_2 = 0 \quad \psi$$

$$\delta V_2 = Q\Omega_1$$

$$\delta V_3 = Q\Omega_2 + \partial V_2$$

$$\int d^4x (F_{mn} F^{mn} + \partial_n C A^{*n})$$

$$\int A^{*n} \partial_n \Omega_{mp}$$

$$\int d\bar{\lambda} d\theta d^p x \int d\bar{\lambda} d^3 r e^{i\lambda_1 \lambda_2 \lambda_3} / Q = \lambda^\alpha d_\alpha + r_\alpha \frac{\partial}{\partial \lambda^\alpha}$$

ghost $g, n=0$

field
antifield
antighost

ghost of zero mass

C^I $I=1 \dots 3$

$$QV_i = 0 \quad \psi$$

$$\delta V_i(Q) = 0 \quad \psi$$

$$\partial^p \Omega$$

$$\int d^4 x (F_{\mu\nu} F^{\mu\nu} + \partial_\mu C A^{\mu\nu})$$



$$C^H \quad I = 1 \dots 3$$

$$Q = C^H \frac{\partial}{\partial x^H}$$

$$V_F = \Omega(x)$$

$$C^H A_H(x)$$

$$C^H C^H D^{*K}$$

$$C^H C^H D^{*K}$$

$$C^H \quad I=1 \dots N$$

$$Q = C^H \frac{\partial}{\partial x^H}$$

$$V_H \Omega(x)$$

$$C^H A_H(x)$$

$$C^H C^H D^{*K}$$

$$\sum_{J,K} C^H \left(\sum_{L,K} \Omega^* \right) C^H_{JK}$$

eg

o gey

$$(\nabla_\mu - D_\mu) \psi = 0$$

$$(\nabla_\mu - D_\mu) \psi = 0$$

$$\int d\lambda d\theta d^p x \int d\bar{\lambda} d\bar{\theta} d^p x \quad (QY) \quad | \quad Q = \lambda^2 d_\lambda + r_\lambda \frac{\partial}{\partial \lambda^2}$$

ghost $g_n = 0 \rightarrow$ YM ghost at zero mass

field
antifield
antighost

$$C^I \quad I=1 \dots 3$$

$$Q = C^I \frac{\partial}{\partial x^I}$$

$$\int d\lambda (F_{\mu\nu} F^{\mu\nu} + \partial_\mu C A^{\mu\nu})$$

$$\partial_\mu A^{\mu\nu} = 0$$

$$QV = 0 \quad \forall$$

$$\delta V = Q\Omega = 0$$

$$\int A^{\mu\nu} \partial_\mu \Omega$$

$$V = \Omega(\omega)$$

$$C^I A_I(\omega)$$

$$C^I C^J A^{JK}$$

$$\epsilon_{IJK} C^I C^J C^K \Omega^*$$

$$\int dt \left((\dot{x}^m - i\theta\gamma^m\dot{\theta}) P_m + e P^m P_m + A_m(x,\theta) \dot{x}^m + A_\alpha(x,\theta) \dot{\theta}^\alpha \right)$$

$$\Rightarrow A_m = \gamma_m^{\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma_m^{\alpha\beta} (D_\alpha A_\beta - D_\beta A_\alpha) = 0 \quad \left| \begin{array}{l} \delta A_m = D_\alpha \Omega \\ \delta A_\alpha = \partial_\alpha \Omega \end{array} \right.$$

$$S = \int dt \left((\dot{x}^m - i\theta\gamma^m\dot{\theta}) P_m + e P^m P_m + \left(A_m(x,\theta) \dot{x}^m + A_\alpha(x,\theta) \dot{\theta}^\alpha + W^\alpha(x,\theta) d_\alpha + F^m(x,\theta) (\omega\gamma_{m\alpha}\lambda) \right) \right)$$

$$Q = \int d^4x d^4\theta$$

$$\frac{\delta}{\delta e} Q = 0$$

$$\Rightarrow A_m = \gamma_{m\alpha}^{\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma_{m\alpha\beta\gamma} (D_\alpha A_\beta + D_\beta A_\alpha) = 0 \quad \left| \begin{array}{l} \delta A_m = D_\alpha \lambda^\alpha \\ \delta A_\alpha = \partial_\alpha \Omega \end{array} \right.$$

$$S = \int dt \left((\dot{x}^m - i\theta\gamma^m\dot{\theta}) P_m + e P^m P_m + \left(A_m(x,\theta) \dot{x}^m + A_\alpha(x,\theta) \dot{\theta}^\alpha + W^\alpha(x,\theta) d_\alpha + F^m(x,\theta) (\omega\gamma_m\lambda) \right) \right)$$

$$Q = \int d^4x$$

$$\frac{\delta}{\delta z} Q = 0$$

$$\Rightarrow A_m = \gamma_m^{\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma_{m\alpha\beta\gamma} (D_\alpha A_\beta + D_\beta A_\alpha) = 0 \quad \left| \begin{array}{l} \delta A_m = D_m \Omega \\ \delta A_\alpha = \partial_\alpha \Omega \end{array} \right.$$

$$A_n = \cancel{f_n} + \underline{a_m(x) (\gamma^m \theta)_n} + (\chi \gamma^m \theta)_n (\gamma_m \theta)_n + \partial_n a (\theta \gamma^{m1} \theta) (\gamma_{m1} \theta)_n \dots$$

$$A_m = a_m(x) + (\chi \gamma^m \theta)_m \dots$$

$$D_{\mu\nu} A_m - \partial_\mu A_\nu = \gamma_{\mu\nu\rho} W^\rho$$

$$A_n = \cancel{f_n} + \underline{a_m(x) (\gamma^m \theta)_n} + (\chi \gamma^m \theta)_n (\gamma_m \theta)_n + \partial_n a (\theta \gamma^{m_1} \theta) (\gamma_{m_1} \theta)_n \dots$$

$$A_m = a_m(x) + (\chi \gamma^m \theta)_n \dots, \quad W^A = \chi^B$$

$$D_n A_m - \partial_n A_m = \gamma_{mnp} W^B \quad F_{mn} = \partial_m a_n - \dots$$

$$D_n W^B = (\gamma^{mn})_n^B F_{mn}$$

$$\begin{aligned}
 & \left((\dot{x}^m - i\theta\gamma^m \dot{\theta}^x) P_m + c P^m P_m + \left(A_m(x, \theta) \dot{x}^m \right. \right. \\
 & \left. \left. + A_x(x, \theta) \dot{\theta}^x \right) \right. \\
 & \left. + W^x(x, \theta) d_x \right. \\
 & \left. + F^m(x, \theta) \left(\omega \gamma_{m, n} \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & x^m = \gamma^m d_x \\
 & Q = 0
 \end{aligned}$$

$$\begin{array}{l|l}
 \gamma^{x\rho} (D_x A_\rho + D_\rho A_x) & \gamma^{m\rho} (D_x A_\rho + D_\rho A_x) = 0 \\
 \hline
 \delta A_x = D_x \Omega & \\
 \delta A_m = \partial_m \Omega &
 \end{array}$$

$$= \int d\tau \left(\dot{x}^m P_m + \dot{\theta}^{\alpha} P_{\alpha} + \lambda \omega_{\alpha} + \left(A_m(x, \theta) \dot{x}^m + A_{\alpha}(x, \theta) \dot{\theta}^{\alpha} + W^{\alpha}(x, \theta) d_{\alpha} + F^m(x, \theta) (\omega \gamma_{mn}) \right) \right)$$

$$Q = \lambda^{\alpha} d_{\alpha}$$

$$\Rightarrow A_m = \gamma^{\alpha\beta} (D_{\alpha} A_{\beta} + D_{\beta} A_{\alpha}), \quad \gamma^{\alpha\beta} (D_{\alpha} A_{\beta} + D_{\beta} A_{\alpha}) = 0 \quad \left| \begin{array}{l} \delta A_m = D_m \Omega \\ \delta A_n = \partial_n \Omega \end{array} \right.$$

$$S = \int dt \left(\dot{x}^m P_m + \dot{\theta}^x P_x + \lambda \omega_x + \left(A_m(x, \theta) \dot{x}^m + A_x(x, \theta) \dot{\theta}^x + W^x(x, \theta) d_x + F^m(x, \theta) (\omega \delta_{mn} + 1) \right) \right)$$

$$\Rightarrow A_m = \gamma^{xp} (D_p A_m + D_m A_p), \quad \gamma^{xp} (D_x A_p + D_p A_x) = 0 \quad \left| \begin{array}{l} \delta A_m = D_x \Omega \\ \delta A_x = \partial_m \Omega \end{array} \right.$$

$$S = \int dt \left(\dot{x}^m P_m + \dot{\theta}^x P_x + \lambda \omega_x + \left(A_m(x, \theta) \dot{x}^m + A_x(x, \theta) \dot{\theta}^x + W^x(x, \theta) d_x + F^m(x, \theta) (\omega \delta_{m, x}) \right) \right)$$

$$\Rightarrow A_m = \gamma_m^{x\rho} (D_\rho A_\rho + D_x A_m), \quad \gamma_m^{x\rho} (D_\rho A_\rho + D_x A_m) = 0 \quad \left| \begin{array}{l} \delta A_m = D_x \Omega \\ \delta A_x = \partial_m \Omega \end{array} \right.$$

$$S = \int dt \left(\dot{x}^m P_m + \dot{\theta}^a P_a + \lambda \omega_a \right) + \left(A_m(x, \theta) \dot{x}^m + A_a(x, \theta) \dot{\theta}^a + W^x(x, \theta) d_x + F^a(x, \theta) (\omega_a \delta_{ab}) \right)$$

$$Q = \lambda^a d_a$$

$$\Rightarrow A_m = \gamma_m^{xp} (D_x A_p + D_t A_x), \quad \gamma_{mnp} (D_x A_p + D_t A_x) = 0 \quad \left| \begin{array}{l} \delta A_x = D_x \Omega \\ \delta A_a = \partial_a \Omega \end{array} \right.$$

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$$(A_{\mu}^{\nu})^2$$

$$(18) \quad \gamma_{\mu\nu}^{\rho} D_{\rho} A_{\mu}^{\nu} = 0$$

$$\int d\lambda d\theta d^D x \int d\bar{\lambda} d\bar{\theta} d^D y \mathcal{L} / Q = \lambda^2 d_x + r_x \frac{2}{\lambda^2}$$

ghost field $g_n = 0 \Rightarrow$ YM ghost at zero mass

antifield
antighost

$$C^I \quad I=1 \dots 3$$

$$Q = C^I \frac{\partial}{\partial x^I}$$

$$V_{\mu} \Omega(\lambda)$$

$$C^I A_I(\lambda)$$

$$C^I C^J A^{JK}$$

$$\epsilon_{IJK} C^I C^J C^K \Omega^* \epsilon_{IJK}$$

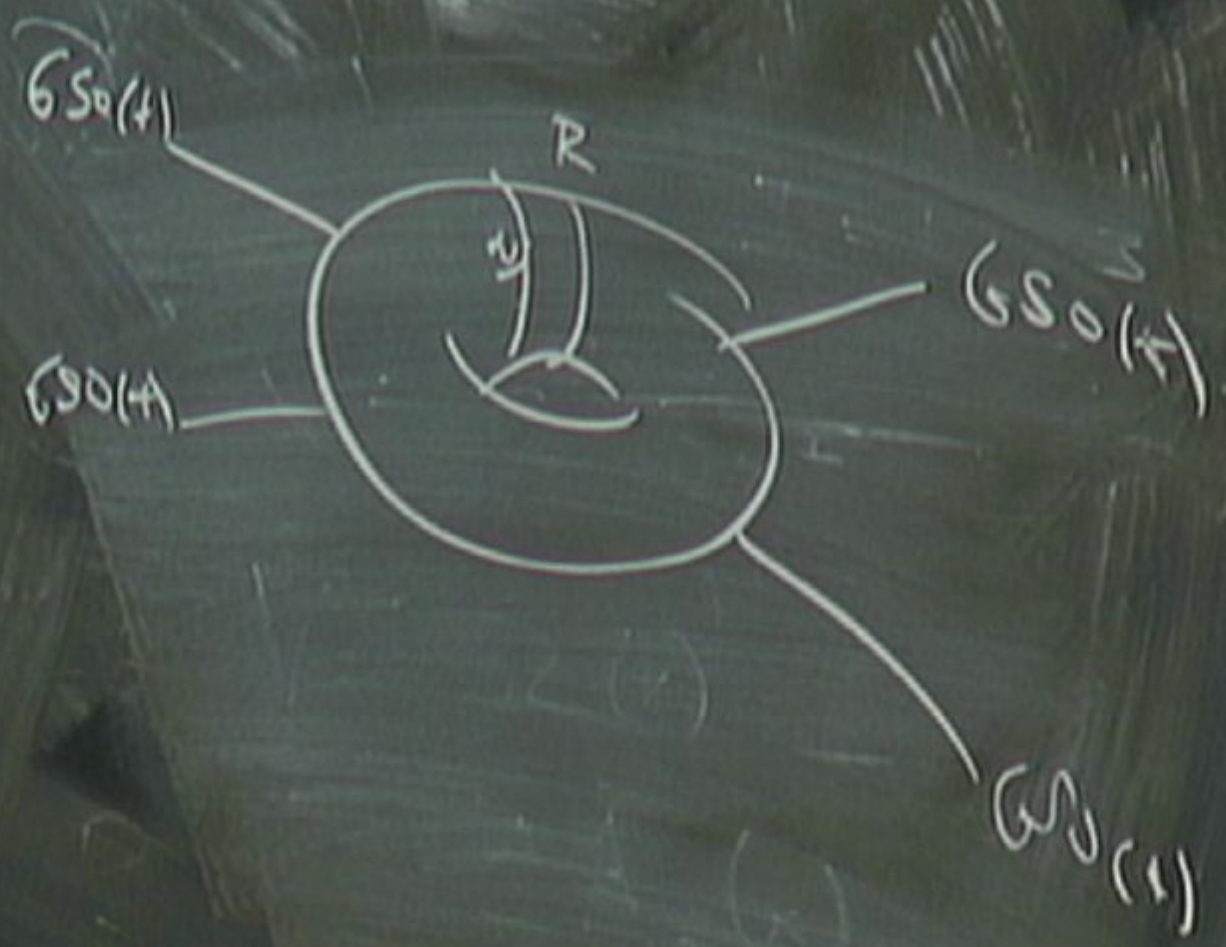
$$\partial_{\nu} A^{\mu\nu} = 0$$

$$QV_{\mu} = 0 \quad \forall \mu$$

$$\delta V_{\mu} = Q \Omega_{\mu} \delta \lambda$$

$$\int A^{\mu\nu} \partial^{\rho} \Omega$$

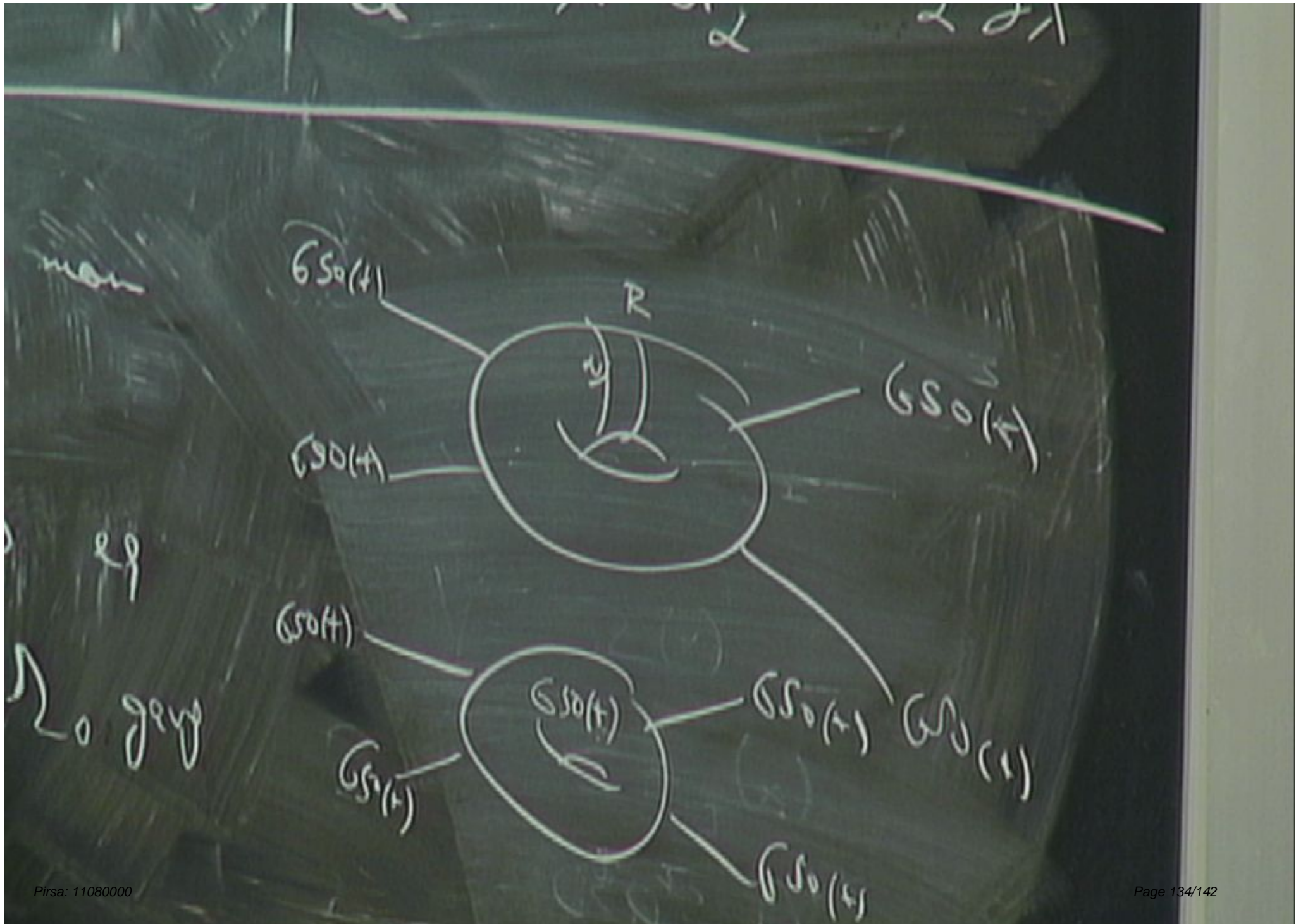
$$\partial_{\mu} C A^{\mu\nu}$$



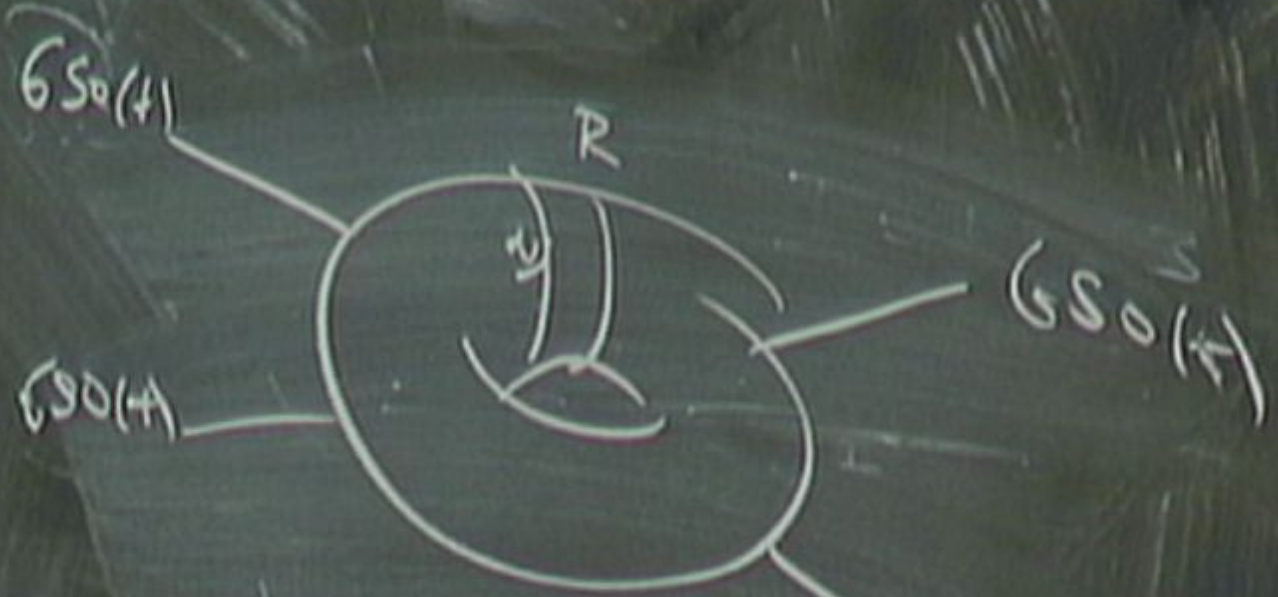
man

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for 0

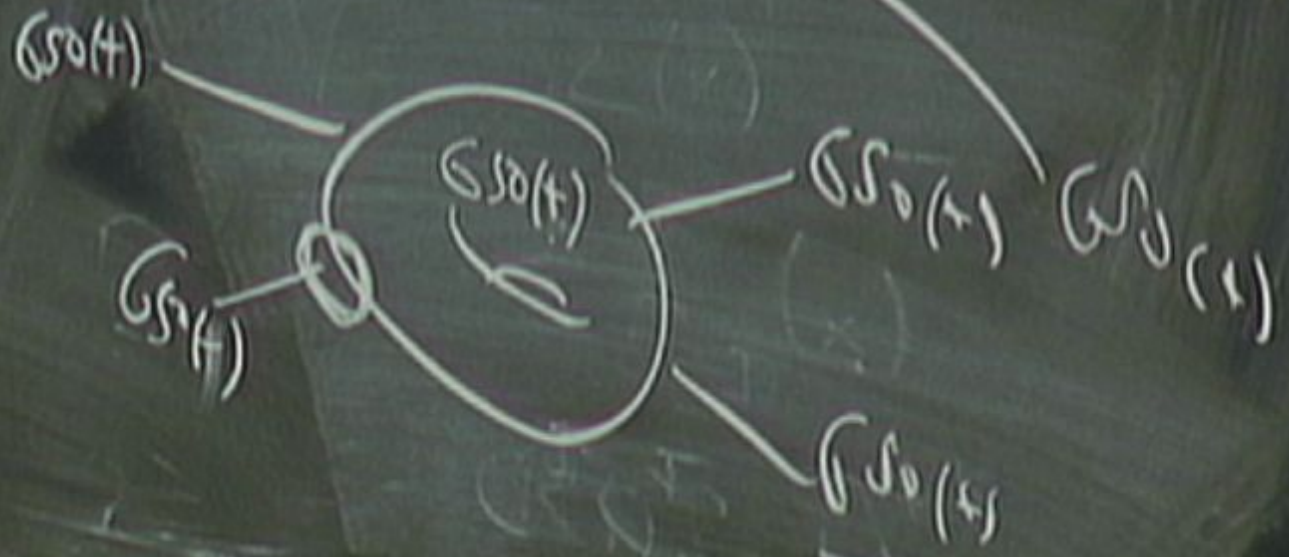


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$$\int d^1 \overset{11}{d} \overset{16}{d\theta} \overset{10}{dx} \int d^1 \overset{11}{d} \overset{11}{d^3x} \quad (Q\psi)$$

L.C

$$\psi_i = \psi_+^1 \psi_+^2$$

$$\psi_i = \psi_-^1 \psi_-^2$$

$$\partial_\mu H = 0$$

$$\int A^* \rightarrow \partial \Omega$$

$$\int d\lambda d\theta dx \quad \int d\lambda dre^{i\lambda} v_1 v_2 v_3$$

$$\psi^2 = \psi^1 + i\psi^2 = e^{i\theta}$$

$$\psi^1 - i\psi^2 = e^{-i\theta}$$

$$\omega_{\text{in}} H = 0$$

$$\int A^* \mathcal{H} \psi$$

$$\int \mathcal{H} \psi$$

$$\psi^2 = e^{i\theta_1}$$
$$i\psi^2 = e^{-i\theta_1}$$

$$\Theta = e^{i\frac{\theta}{2}(\pm\sigma_1 \pm \sigma_2 \pm \sigma_3 \pm \sigma_4)}$$

$$e^{i\theta_1}$$
$$e^{-i\theta_1}$$

$\partial^A \Omega$
 m_{AB}

$$\psi^1; \psi^2 = e^{i\sigma_1}$$

$$\psi^1; \psi^2 = e^{-i\sigma_1}$$

$$\Theta = e^{\frac{i}{2}(\pm\sigma_1 \pm \sigma_2 \pm \sigma_3 \pm \sigma_4)}$$

$$= e^{\pm i\varphi_1}$$

$$\varphi_1 = \frac{1}{2}(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)$$

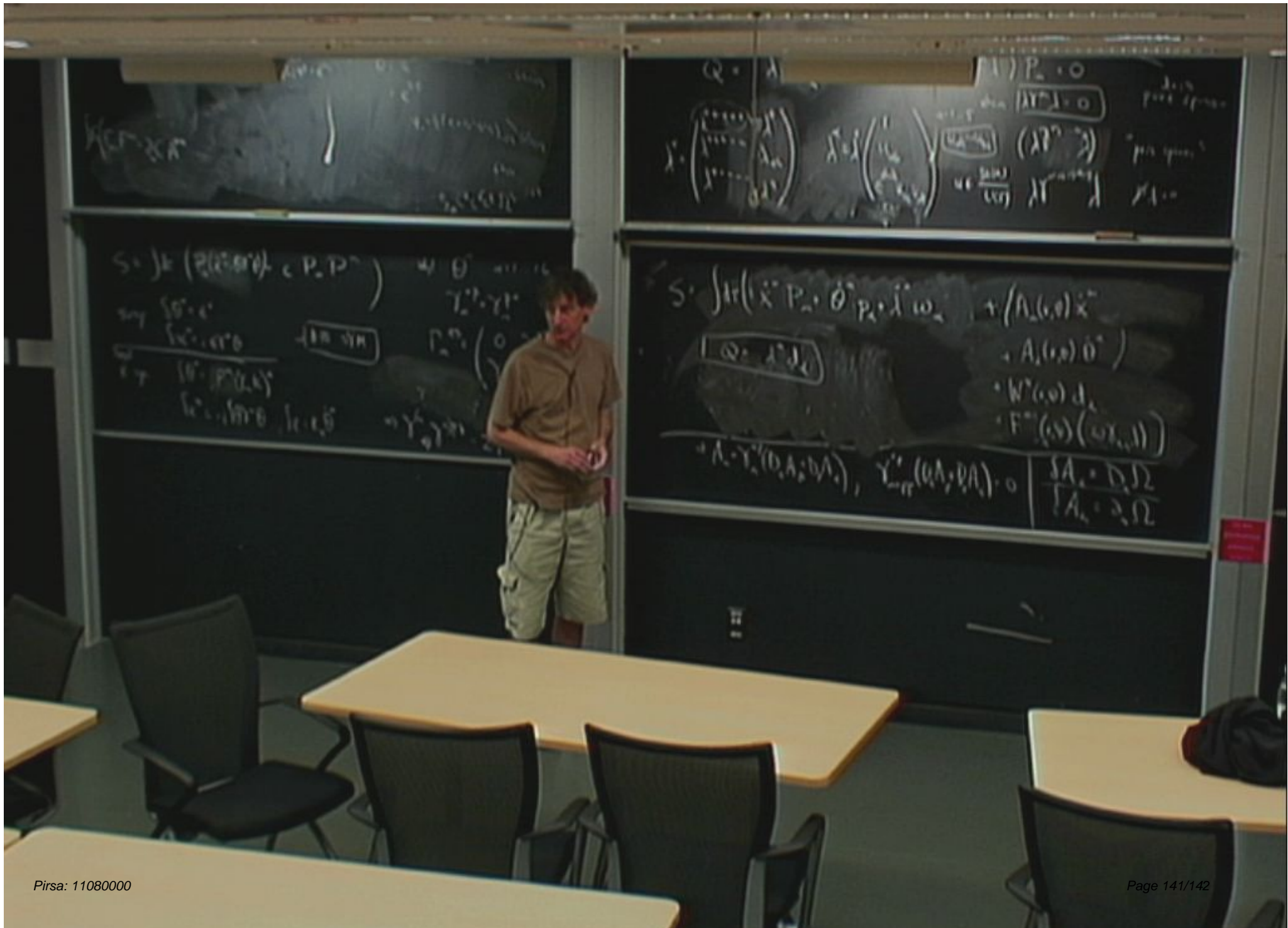
$$\psi^{\pm} = \begin{cases} \psi^1 + i\psi^2 = e^{i\sigma_1} \\ \psi^1 - i\psi^2 = e^{-i\sigma_1} \end{cases}$$

$$\begin{aligned} \Theta &= e^{\frac{i}{2}(\pm\sigma_1 \pm \sigma_2 \pm \sigma_3 \pm \sigma_4)} \\ &= e^{\pm i\varphi_1} \end{aligned}$$

$$\varphi_1 = \frac{1}{2}(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)$$

$\int A^*$

$$\int_{JK} (\dots)$$



$$\begin{aligned}
 & \text{Avec } \dots \\
 & \dots
 \end{aligned}$$

$$\begin{aligned}
 & Q = \lambda \\
 & \dots \\
 & \dots
 \end{aligned}$$

$$\begin{aligned}
 & S = \int dt (\dots) \in P, D \\
 & \dots \\
 & \dots
 \end{aligned}$$

$$\begin{aligned}
 & S = \int dt (\dot{x}^i P_i - \dot{\theta}^a p_a + \lambda \omega_a + \dots) \\
 & \dots \\
 & \dots
 \end{aligned}$$

$$S = \int dt \left(\dot{x}^m P_m + \dot{\theta}^r P_r + \lambda \omega_\alpha + \left(A_m(x, \theta) \dot{x}^m + A_r(x, \theta) \dot{\theta}^r + W^\alpha(x, \theta) d_\alpha + F^\alpha(x, \theta) (\omega \delta_{\alpha\beta} \lambda) \right) \right)$$

$$Q = \lambda d_\alpha$$

$$\Rightarrow A_m = \gamma_m^{\alpha\beta} (D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma_{m\alpha\beta\gamma} (D_\alpha A_\beta + D_\beta A_\alpha) = 0 \quad \left| \begin{array}{l} \delta A_m = D_\alpha \Omega \\ \delta A_n = \partial_n \Omega \end{array} \right.$$