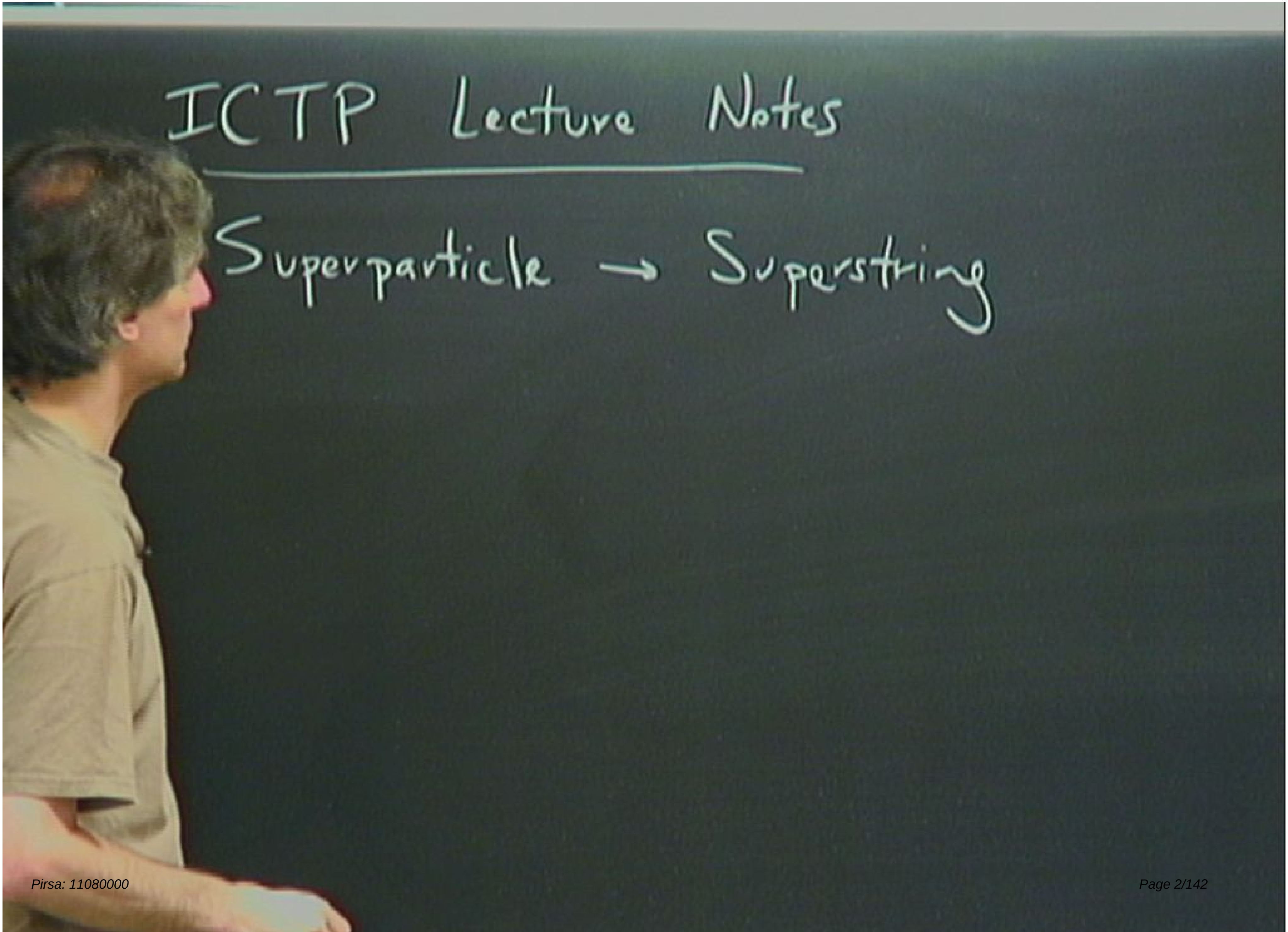
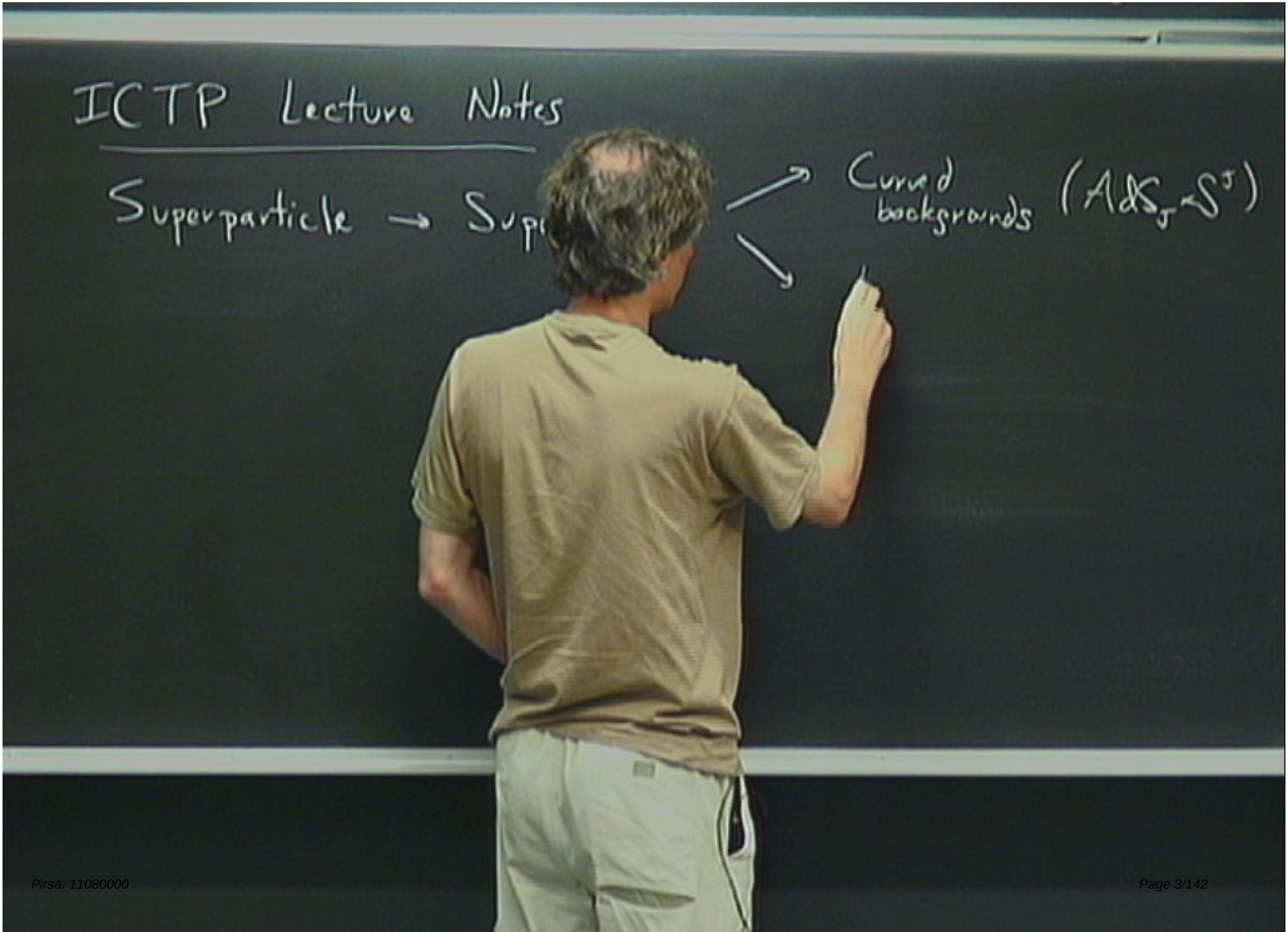
Title: Introduction to Pure Spinor Formalism of the Superstring

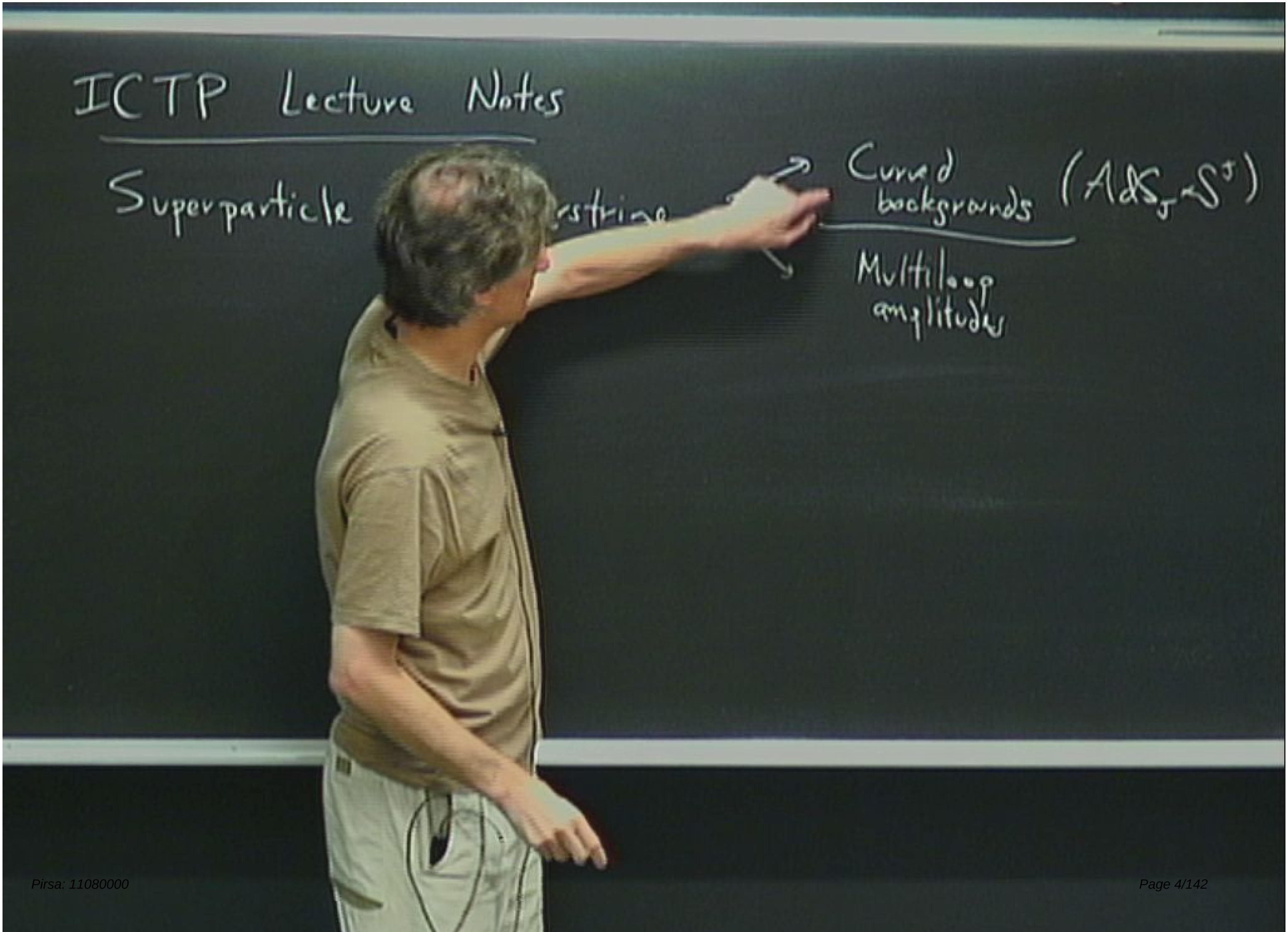
Date: Aug 24, 2011 10:30 AM

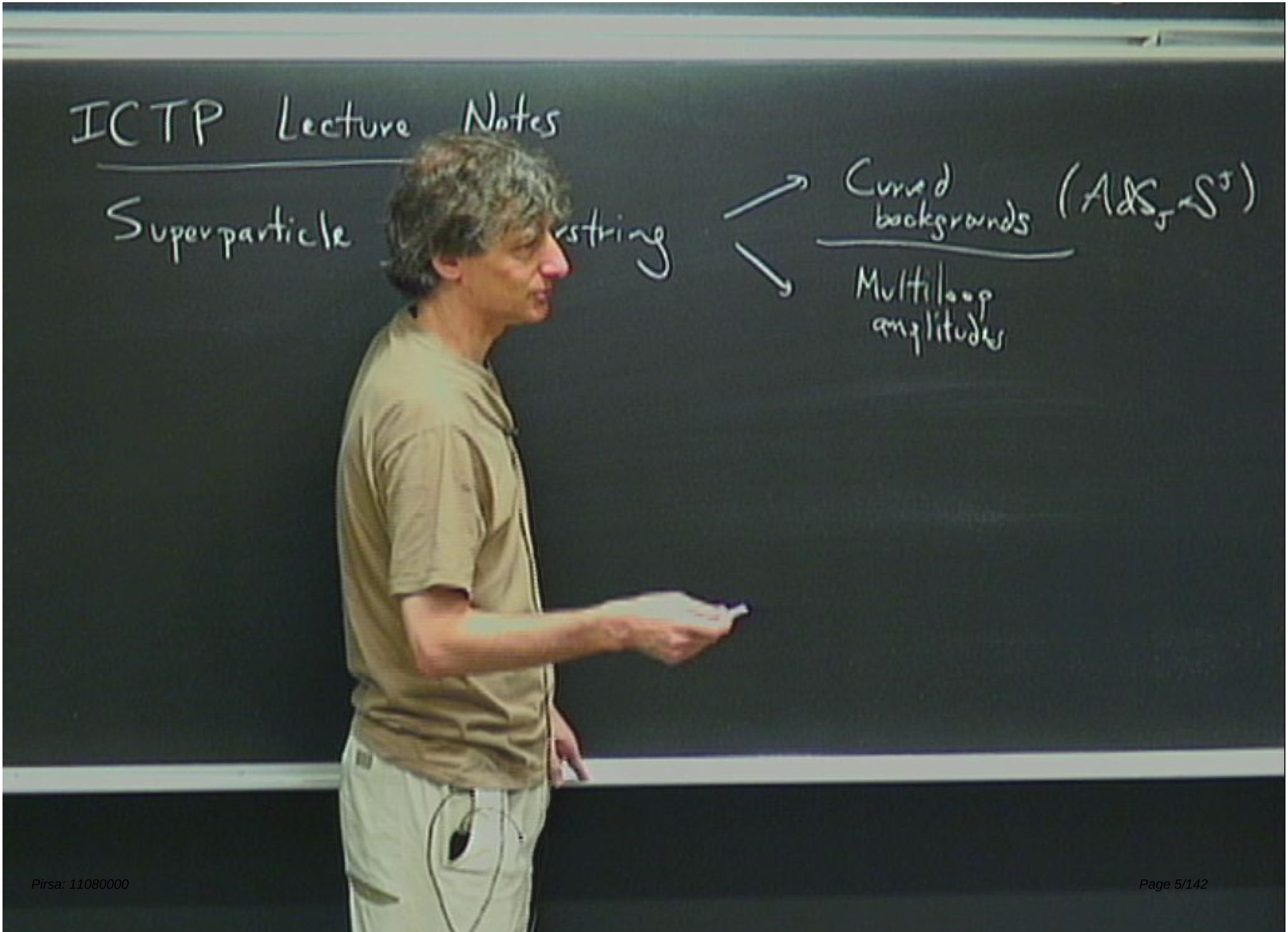
URL: http://pirsa.org/11080000

Abstract: d=10 super-Yang-Mills and the superparticle


ICTP Lecture Notes
Superparticle → Superstring


ICTP Lecture Notes
Superparticle → Sup
Curved backgrounds $(AdS_5 \times S^5)$


ICTP Lecture Notes
Superparticle
string
Curved backgrounds
(AdS5×S5)
Multiloop amplitudes


ICTP Lecture Notes
Superparticle
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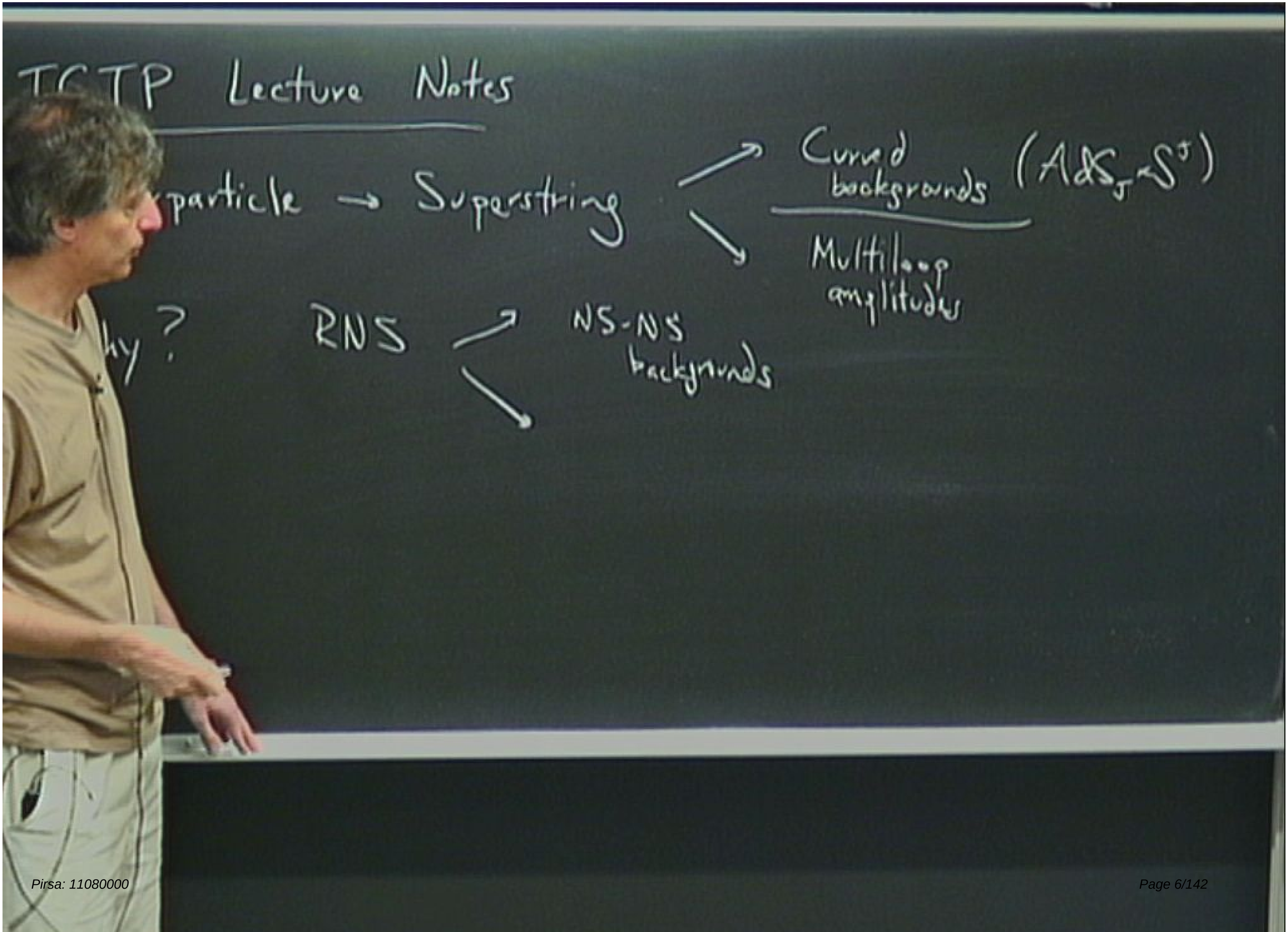
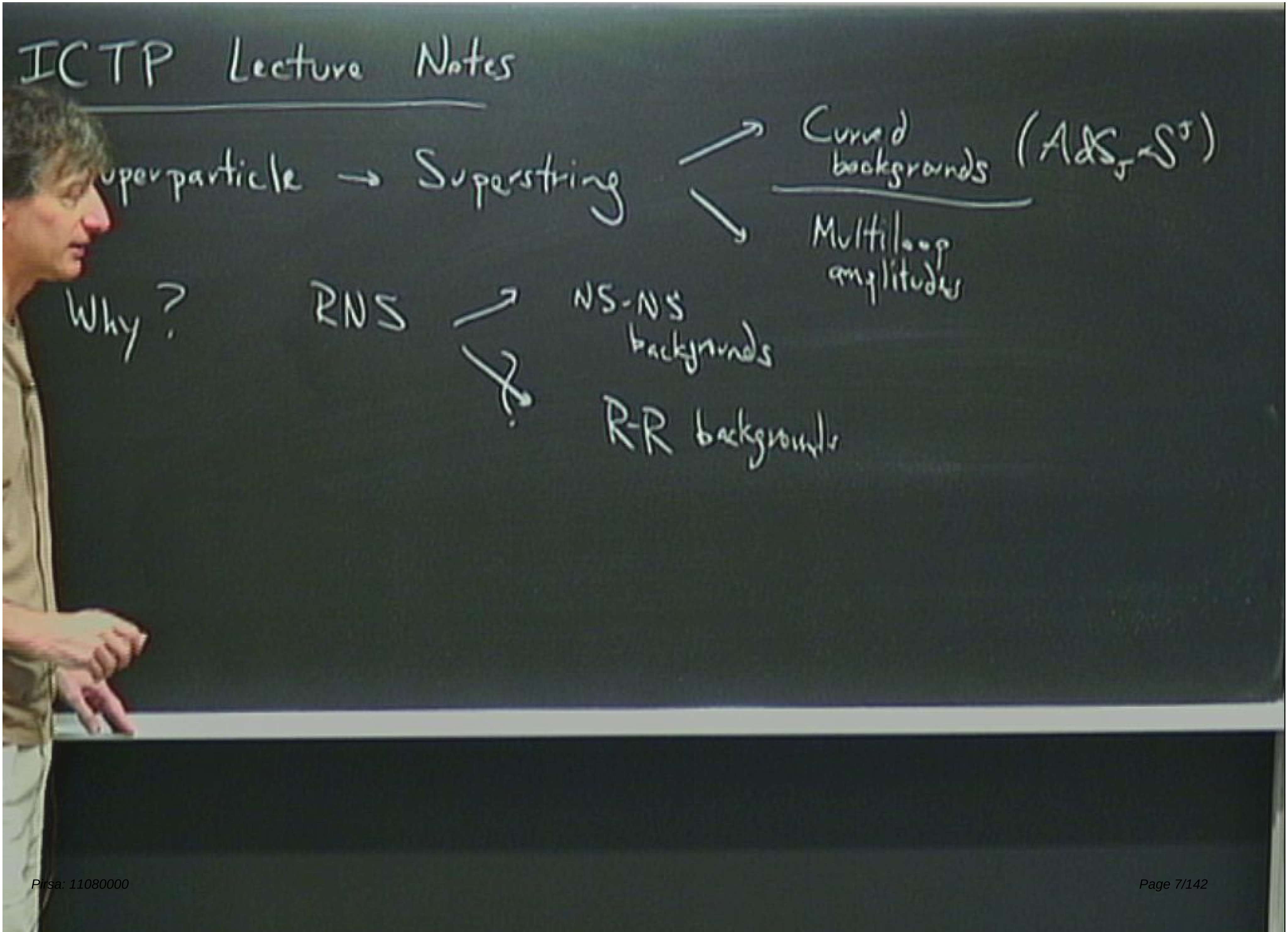
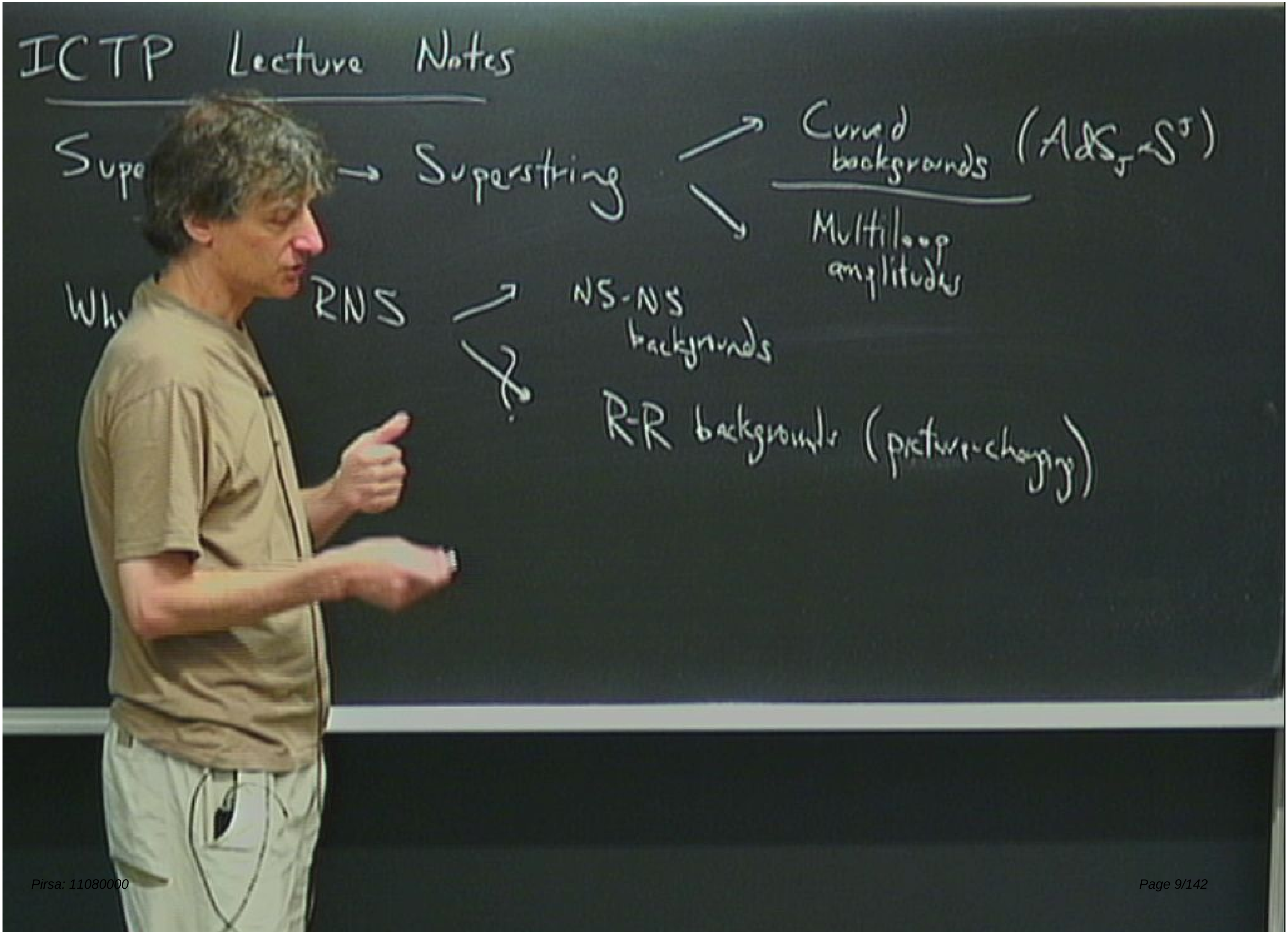

Lecture Notes
particle → Superstring
Curved backgrounds
Multiloop amplitudes
RNS
NS-NS backgrounds


ICTP Lecture Notes
uperparticle → Superstring
Curved backgrounds
$(AdS_5 \times S^5)$
Multiloop amplitudes
Why?
RNS
NS-NS backgrounds
R-R backgrounds


ICTP Lecture Notes
Superstring
Curved backgrounds ($AdS_5 \times S^5$)
Multiloop amplitudes
Why ?
RNS
NS-NS backgrounds
R-R backgrounds (picture-changing)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Curved backgrounds
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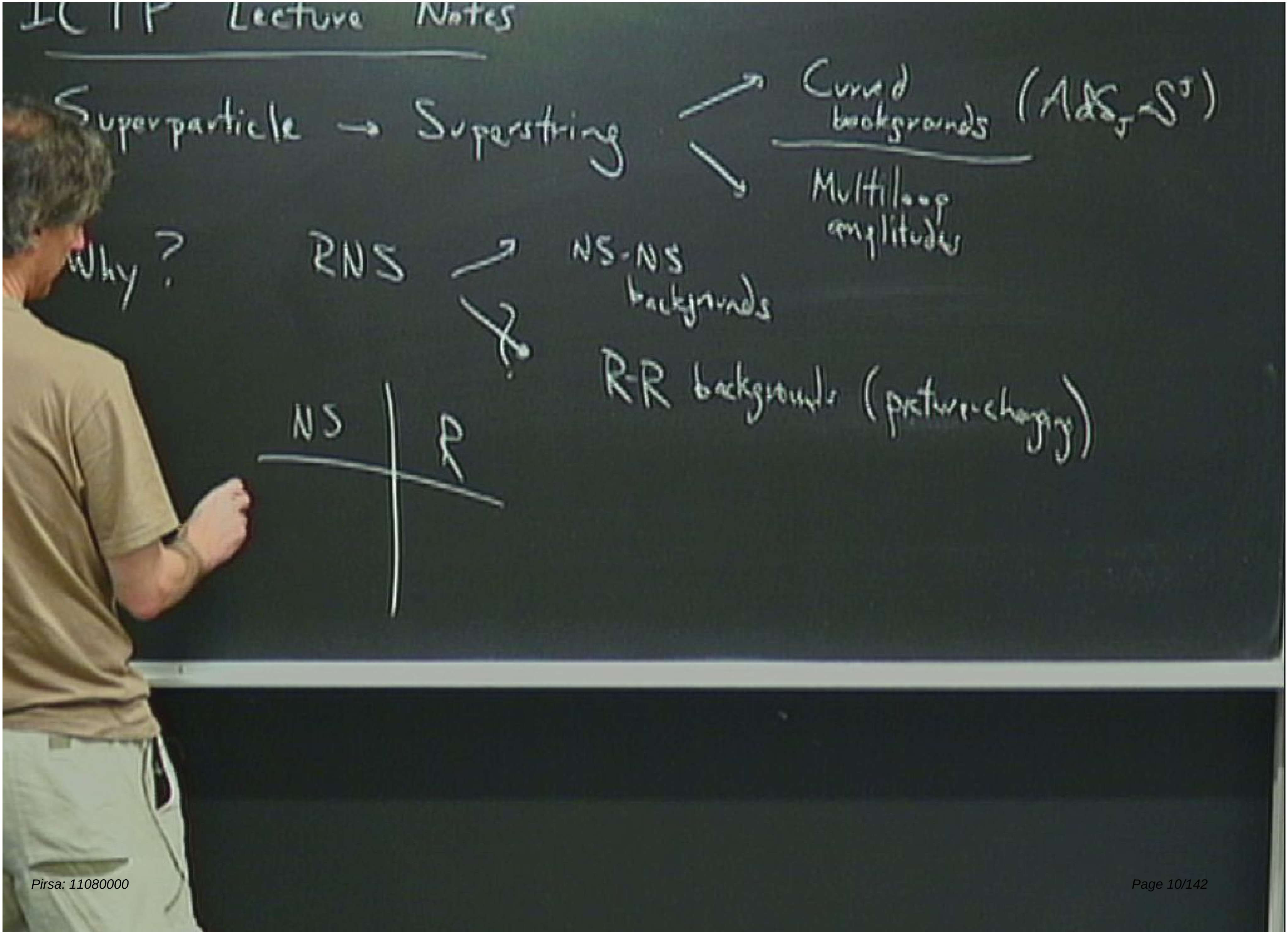
NS-NS backgrounds
R-R backgrounds (picture-changing)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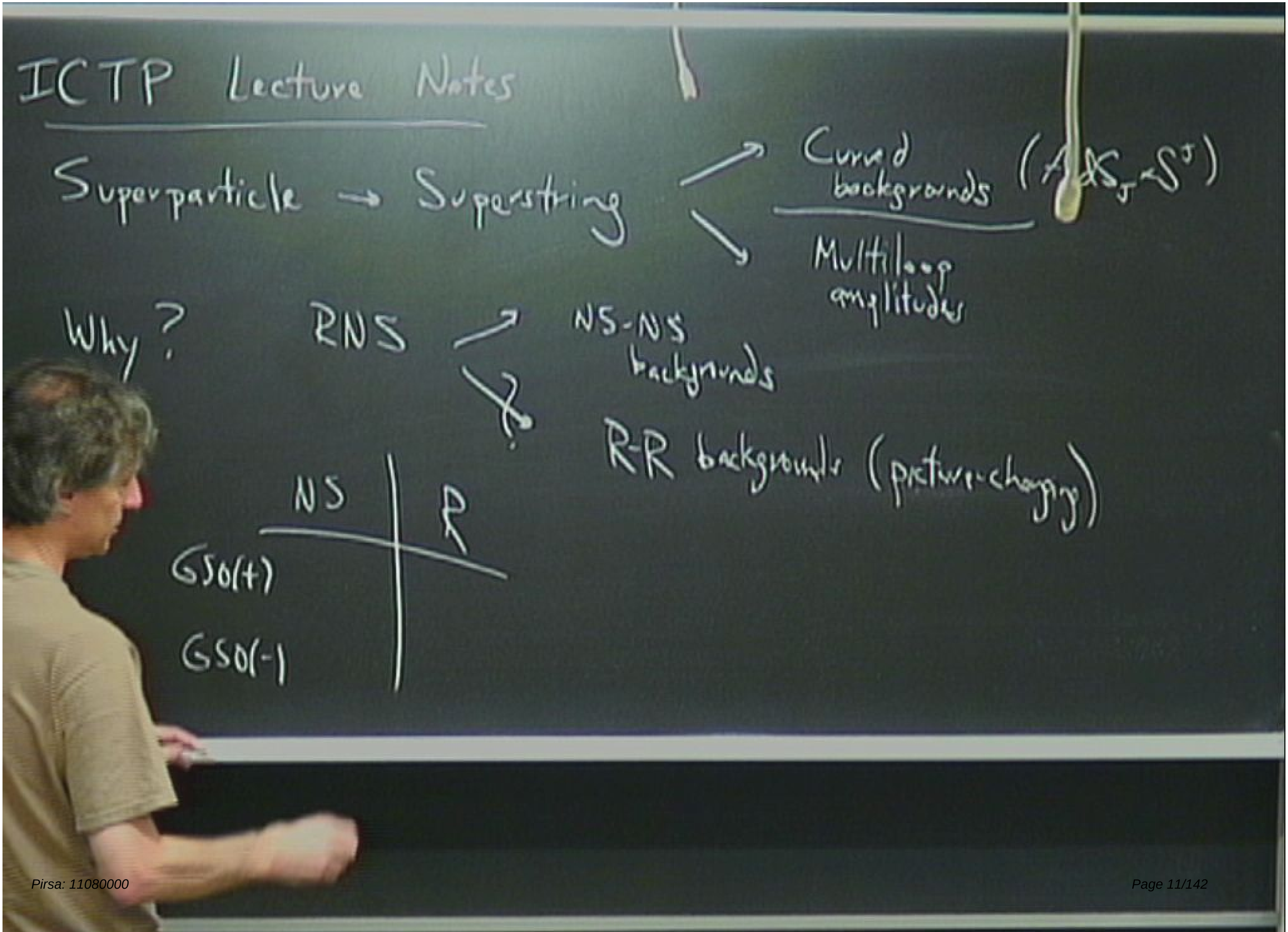
Why?
RNS
NS-NS backgrounds
R-R backgrounds (picture-changing)
NS
R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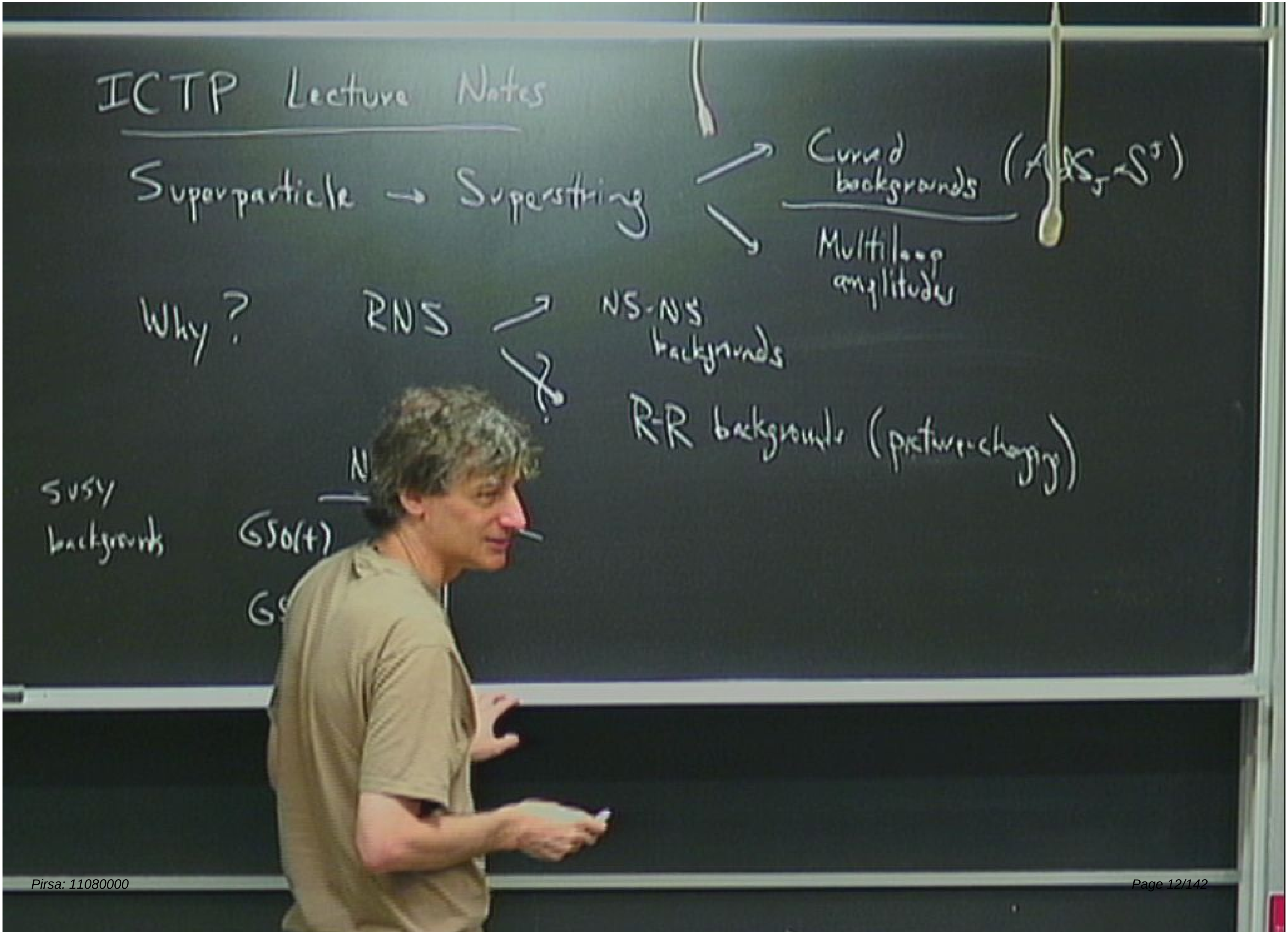
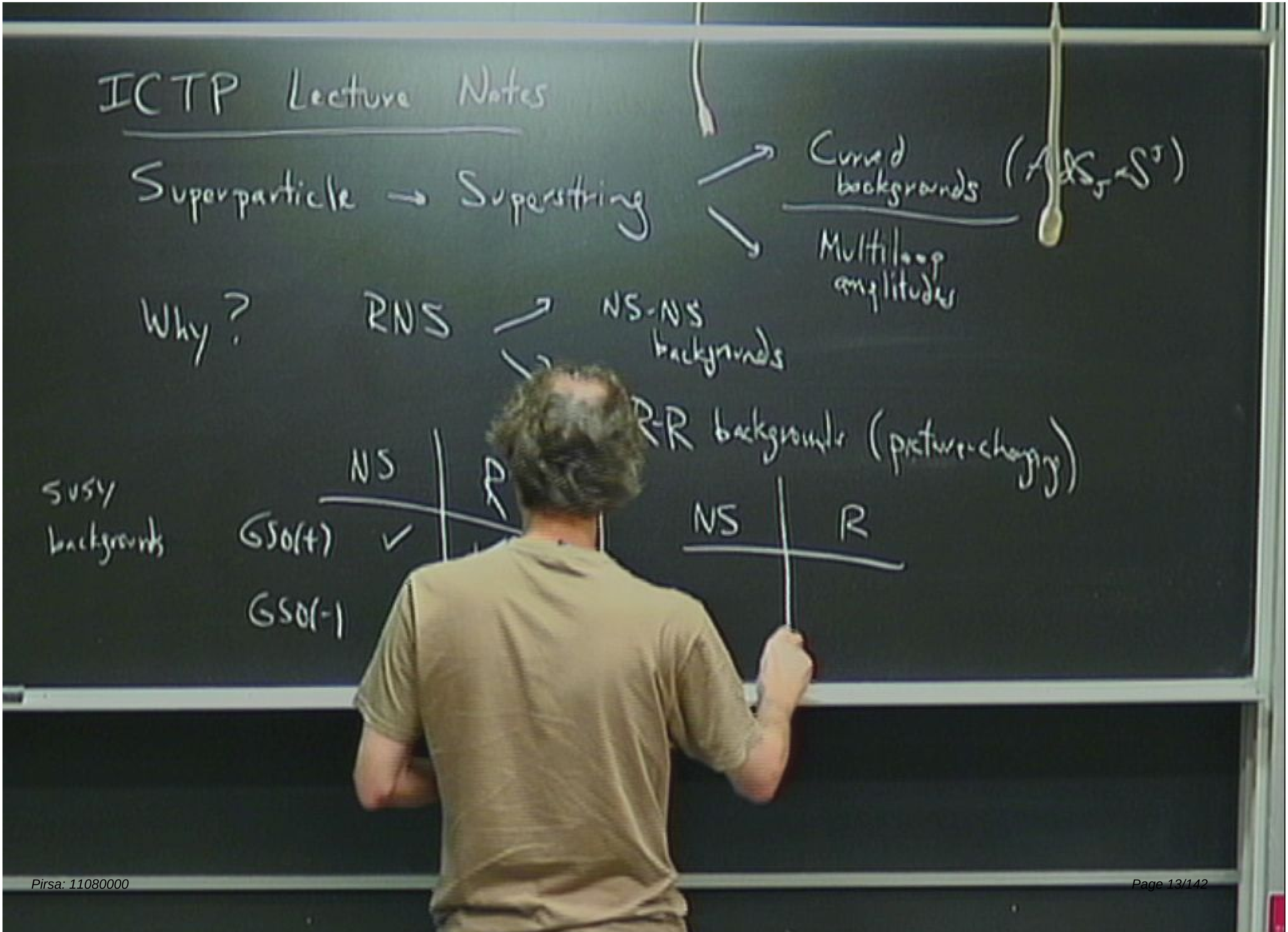
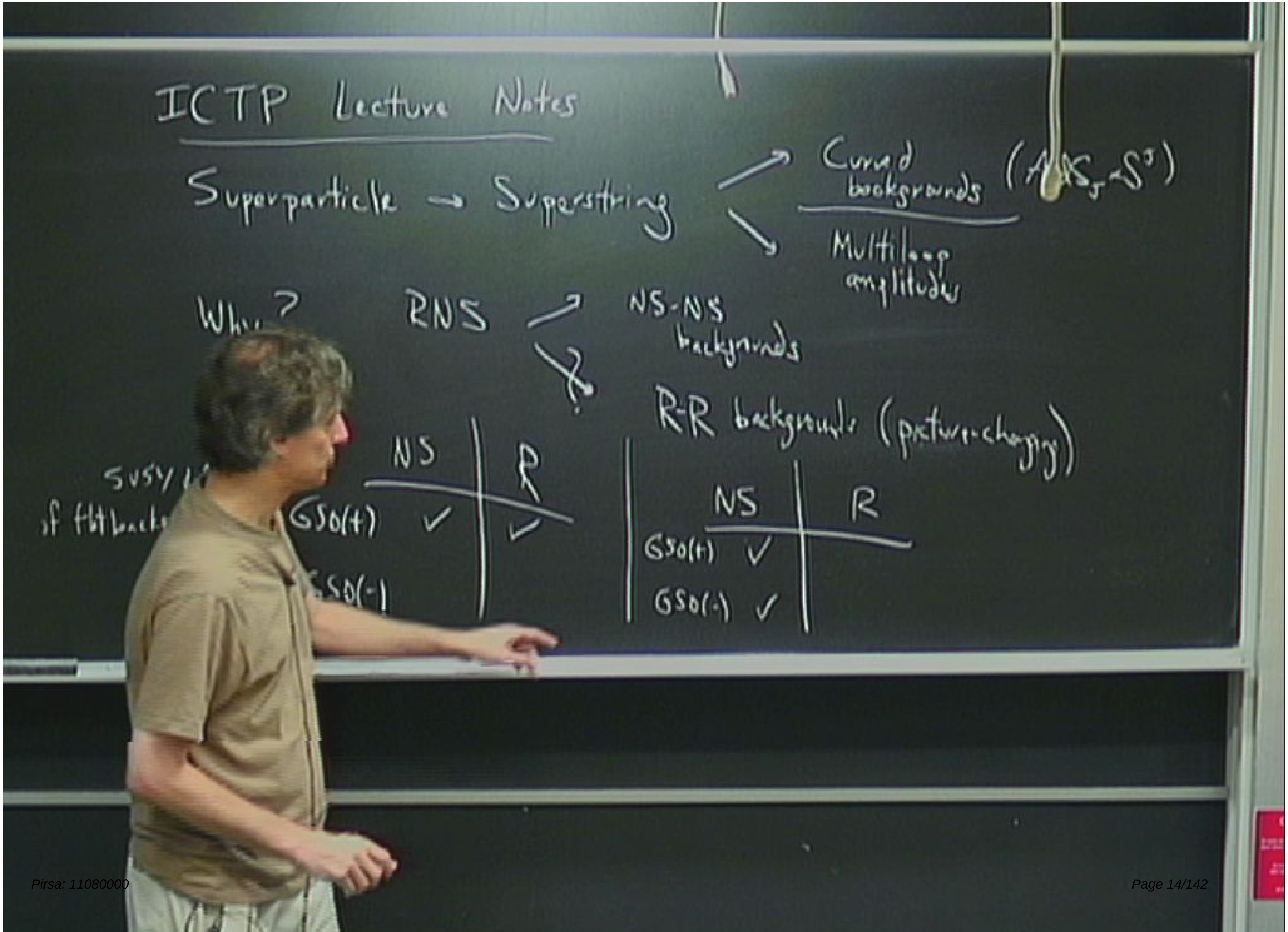
RNS
NS-NS backgrounds
R-R backgrounds (picture-changing)
NS
R
GSO(+)
GSO(-)


ICTP Lecture Notes
Superparticle → Superstring
Curved backgrounds
Multiloop amplitudes
Why?
RNS
NS-NS backgrounds
R-R backgrounds (picture-changing)
SUSY backgrounds
GSO(+)


ICTP Lecture Notes
Superparticle → Superstring
Curved backgrounds
Multiloop amplitudes
Why?
RNS
NS-NS backgrounds
R-R backgrounds (picture-changing)
NS
R
SUSY backgrounds
GSO(+)
GSO(-)
NS
R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Why?
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SUSY
NS
R
GSO(+)
GSO(-)
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GSO(+)
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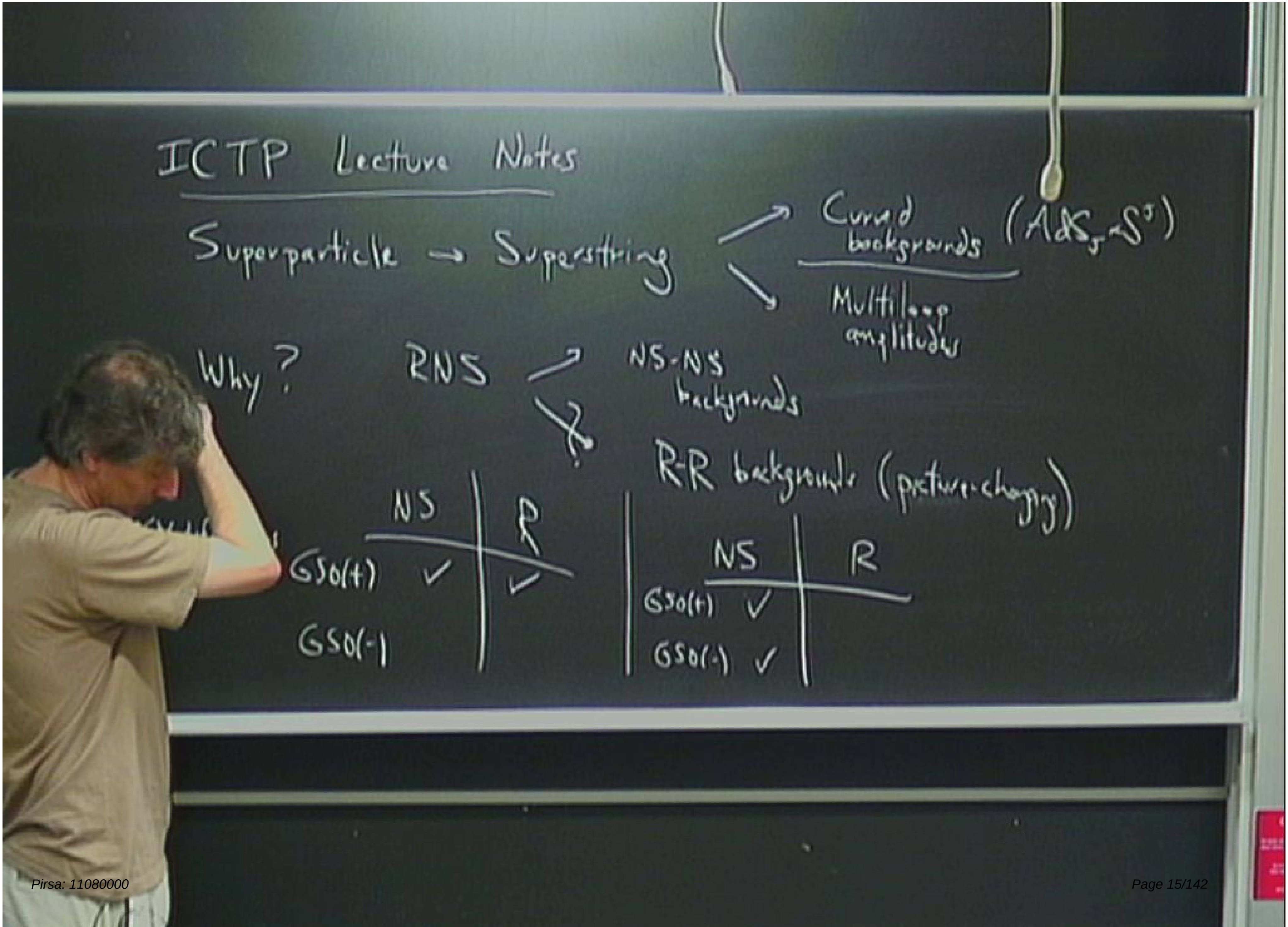
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Curved backgrounds (AdS5×S5)
Multiloop amplitudes
Why?
RNS
NS-NS backgrounds
R-R backgrounds (picture-changing)
NS
R
GSO(+) ✓ ✓
GSO(-)
NS
R
GSO(+) ✓
GSO(-) ✓

ICTP Lecture Notes

Superparticle → Superstring → Curved backgrounds ($AdS_5 \times S^5$)
→ Multiloop amplitudes

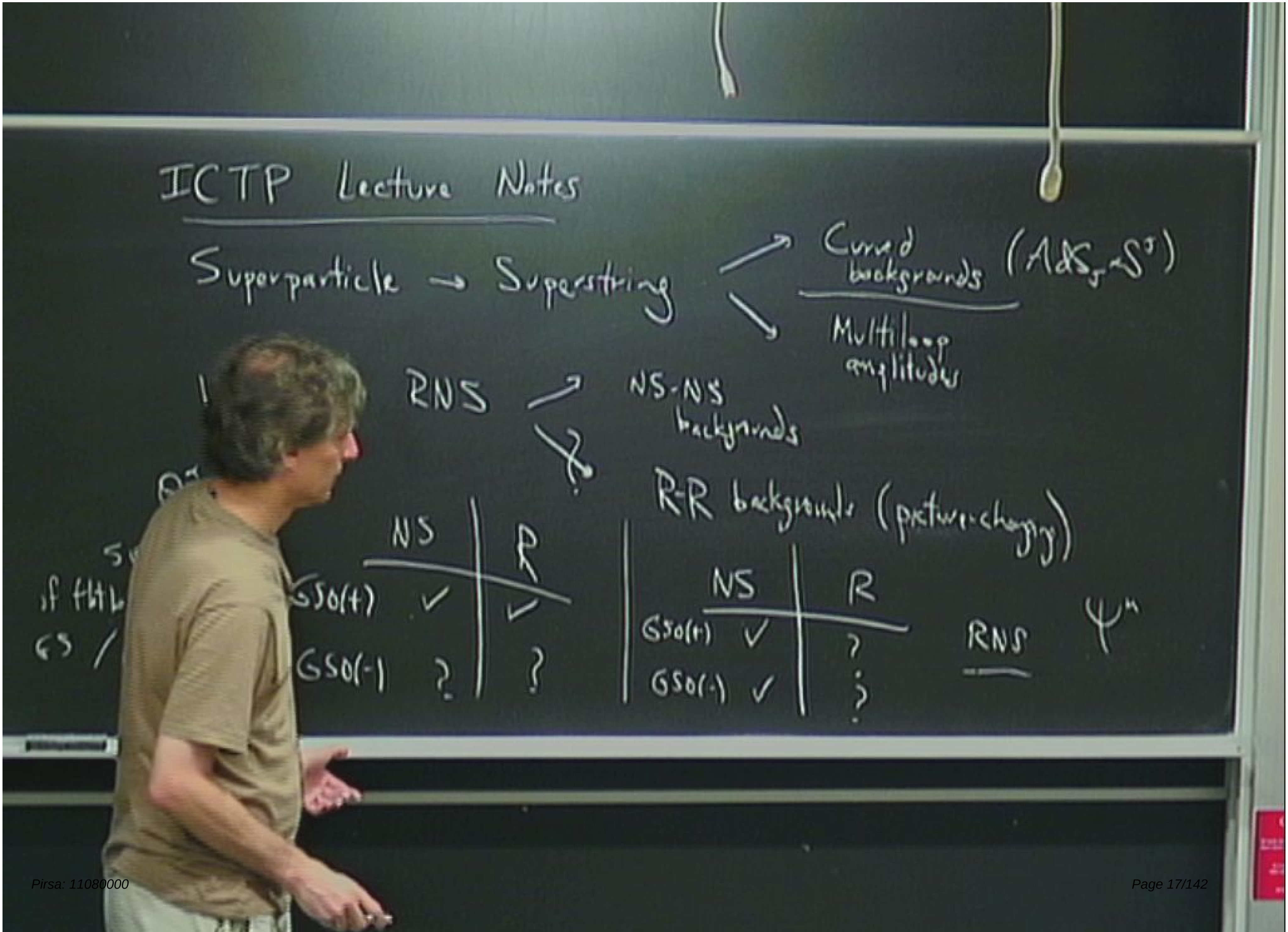
Why? RNS → NS-NS backgrounds
→ R-R backgrounds (picture-changing)

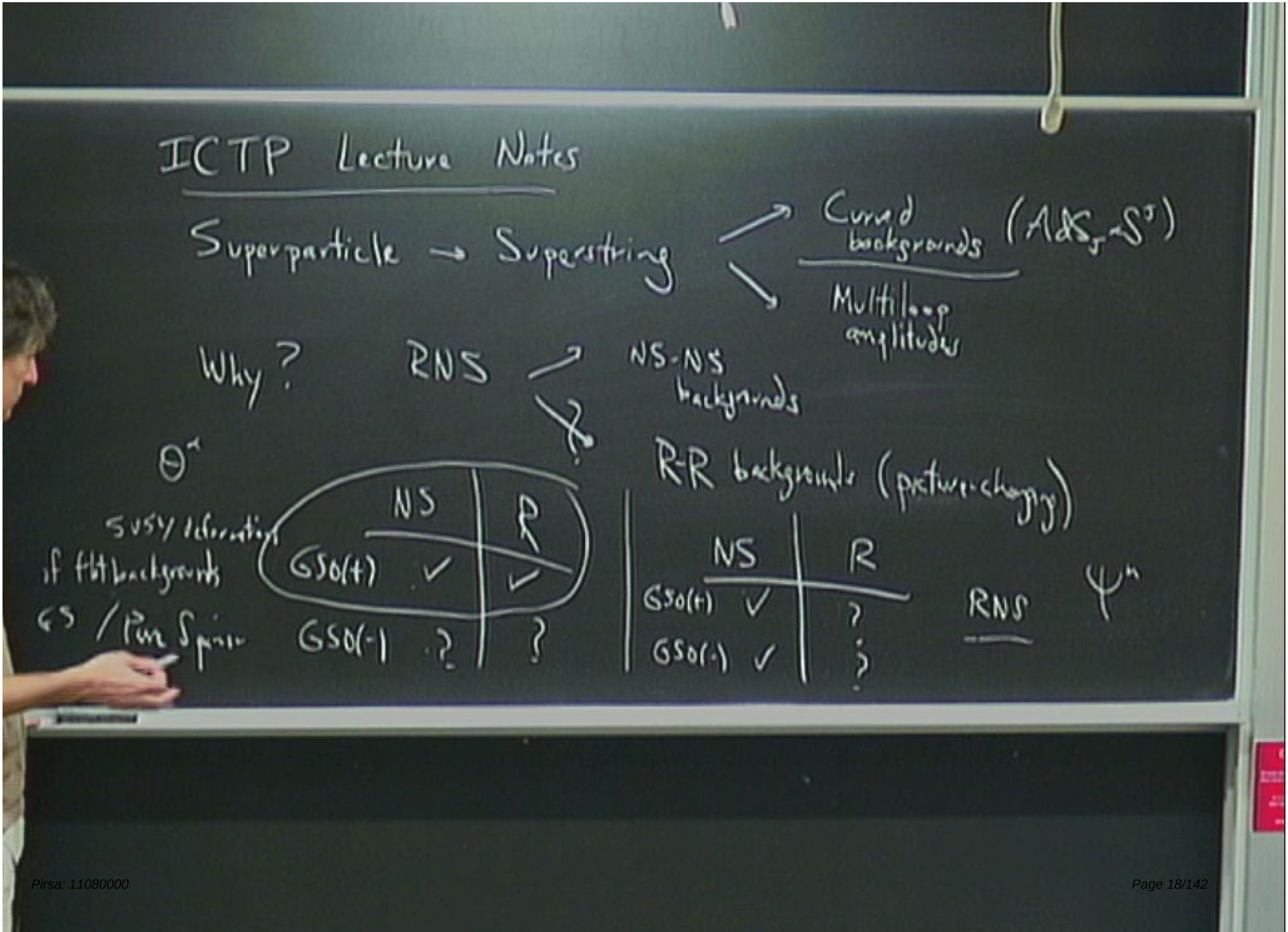
SUSY deformations
flat backgrounds

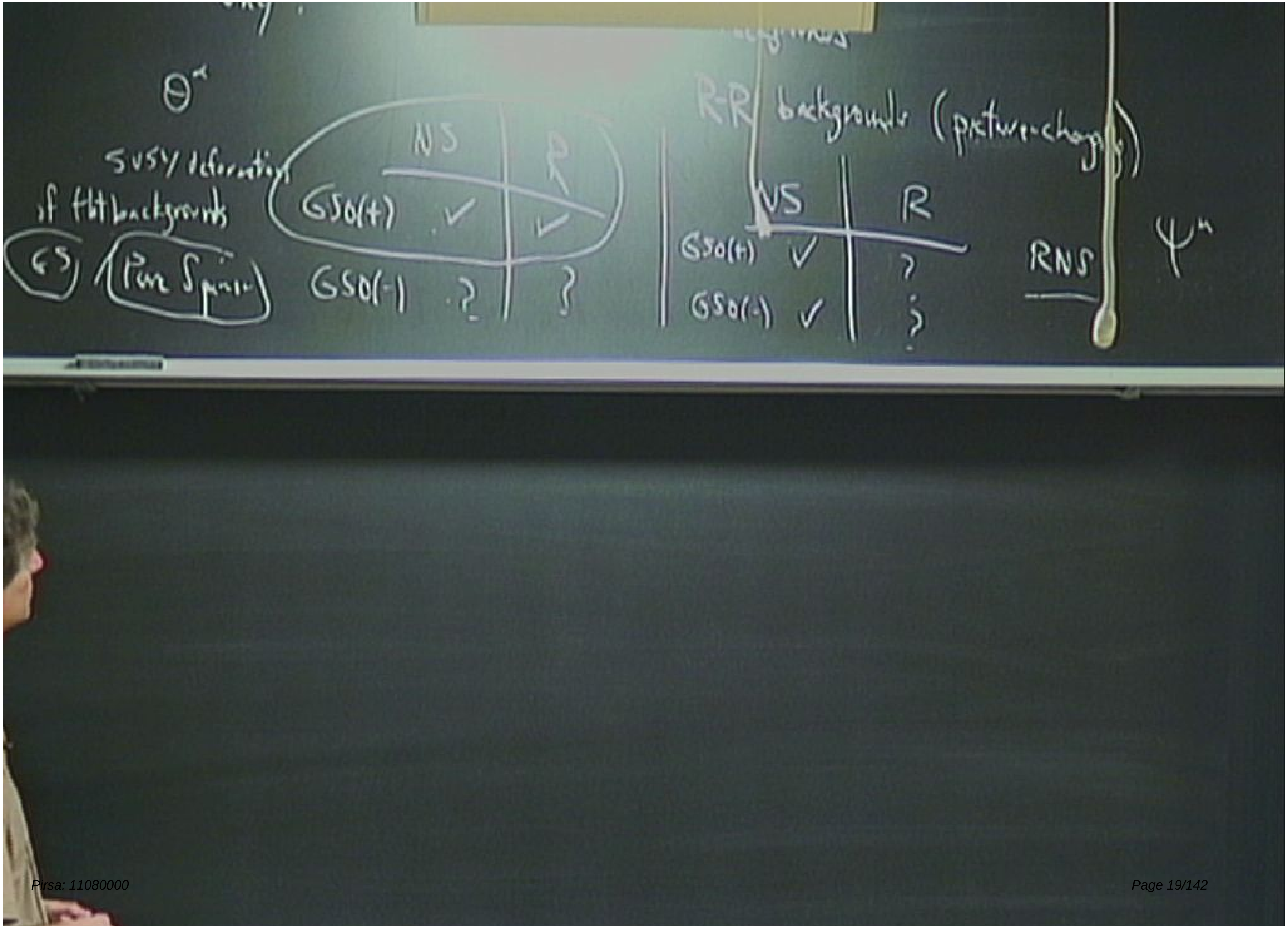
	NS	R
GSO(+)	✓	✓
GSO(−)	?	?

	NS	R
GSO(+)	✓	?
GSO(−)	✓	?

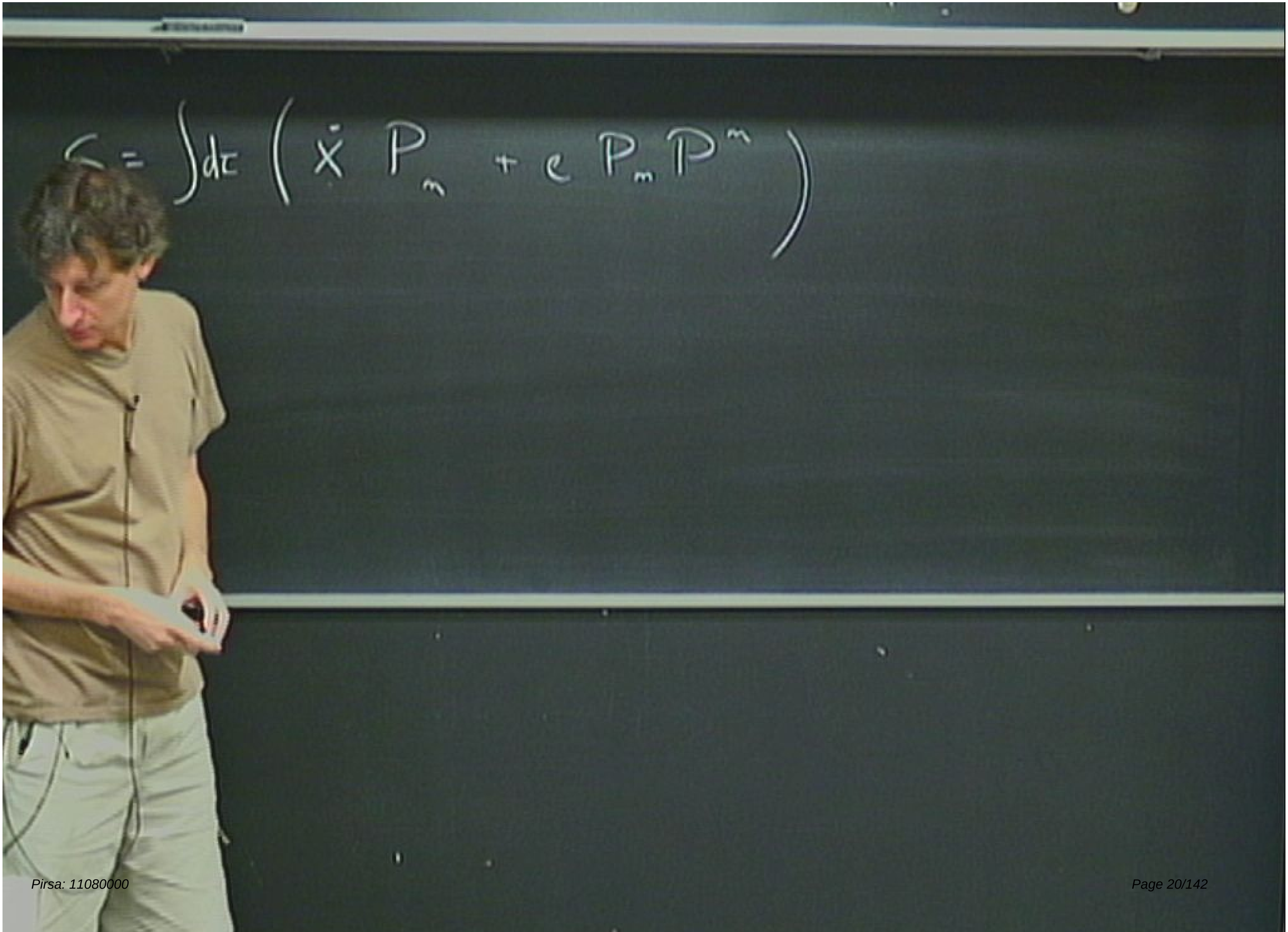
RNS

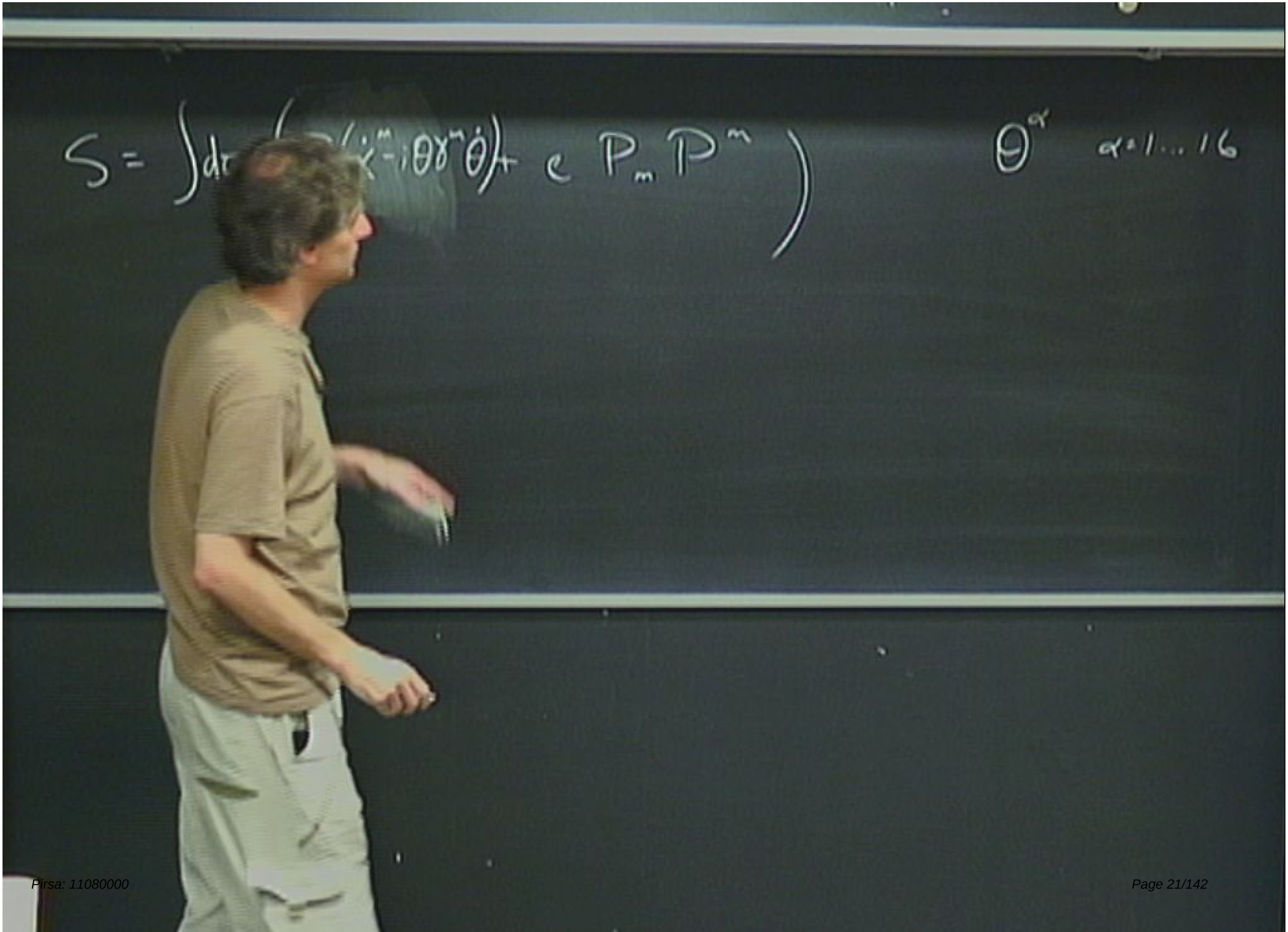

ICTP Lecture Notes
Superparticle → Superstring
Curved backgrounds (AdS5×S5)
Multiloop amplitudes
RNS
NS-NS backgrounds
R-R backgrounds (picture-changing)
NS
R
GSO(+)
GSO(-)
RNS

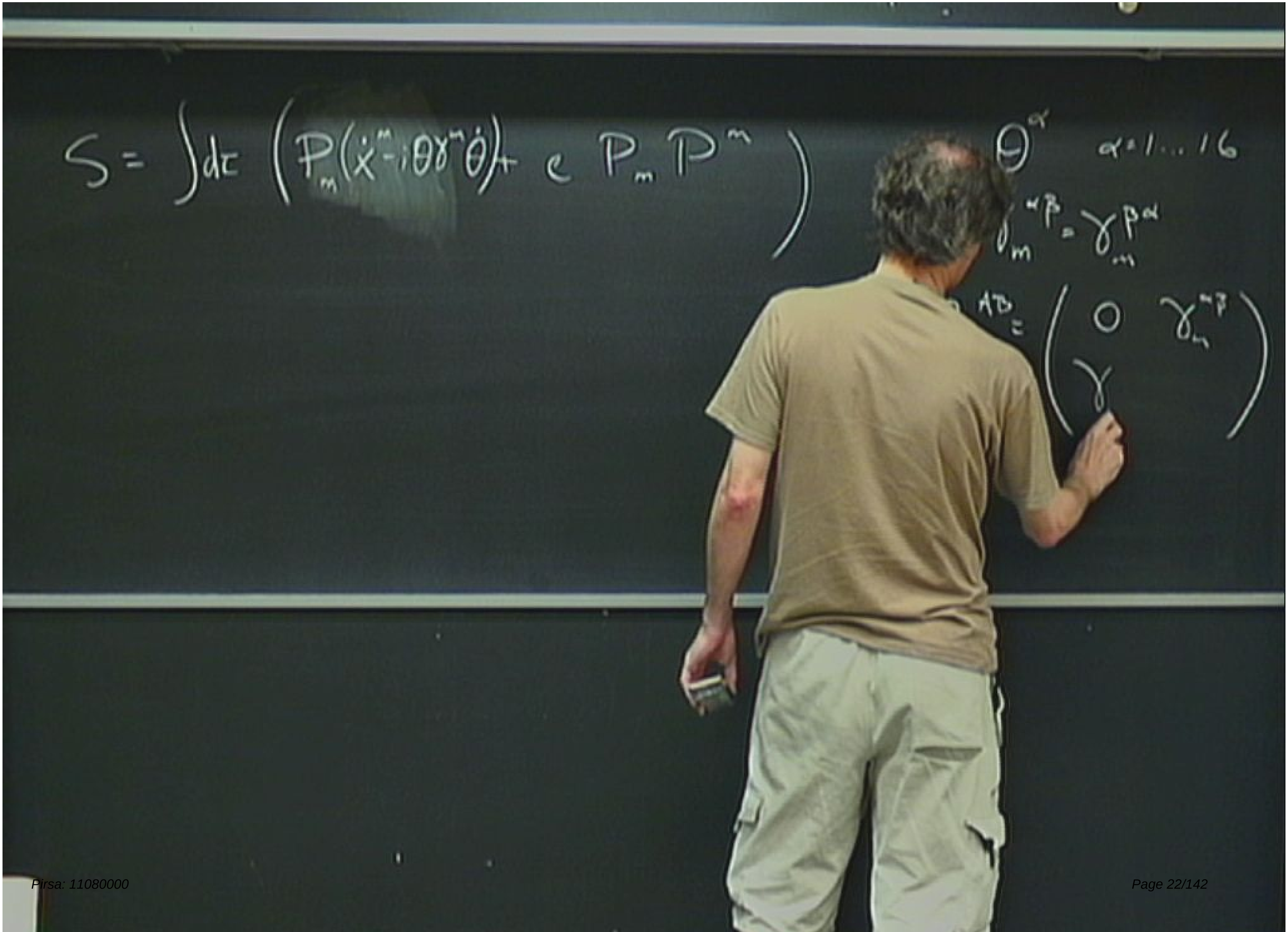

ICTP Lecture Notes
Superparticle → Superstring
Curved backgrounds
(AdS5×S5)
Multiloop amplitudes
Why?
RNS
NS-NS backgrounds
R-R backgrounds (picture-changing)
Θ^α
SUSY deformation
if flat backgrounds
GS / Pure Spinor
NS
R
GSO(+)
GSO(-)
RNS
Ψ^m

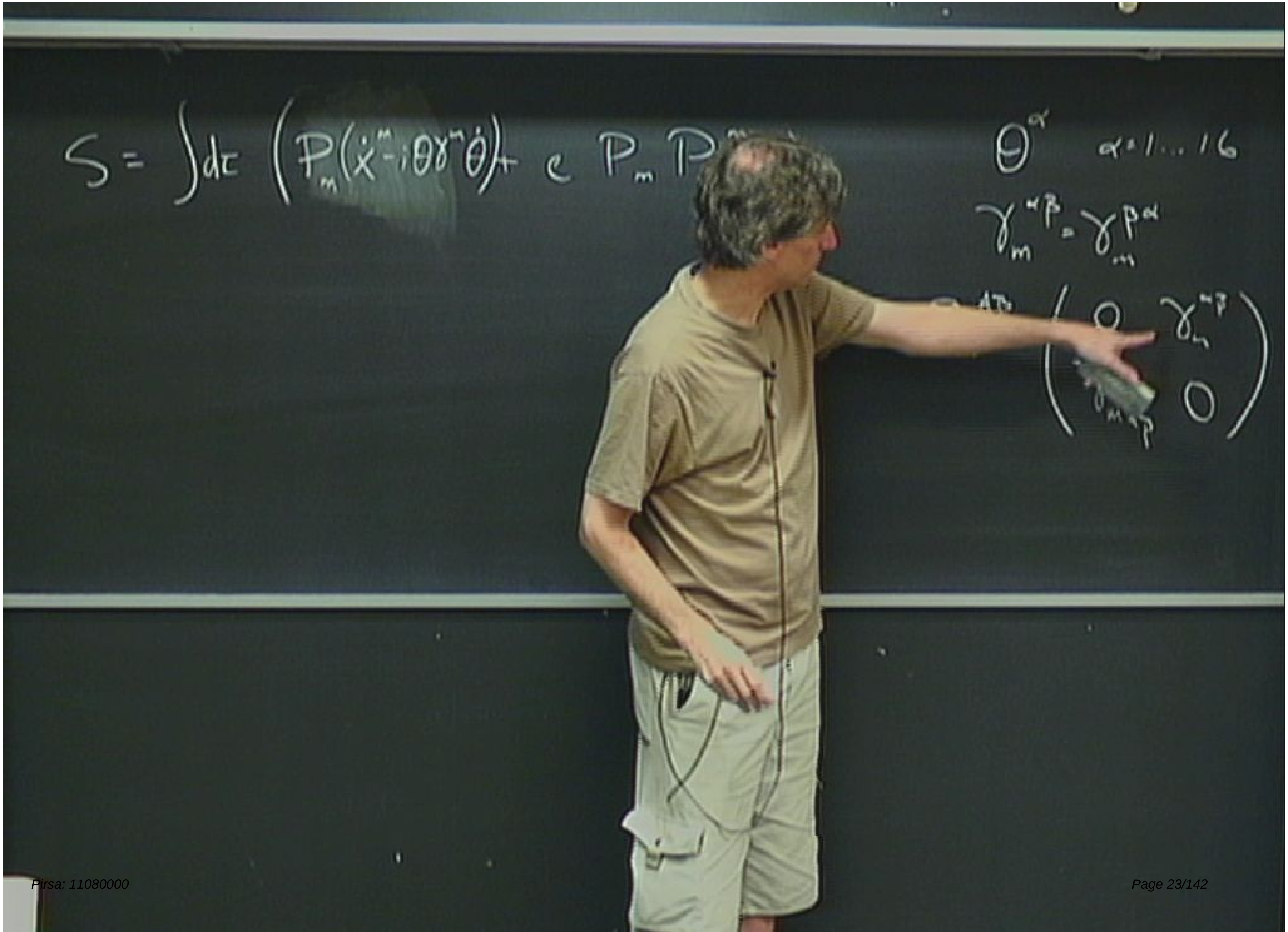


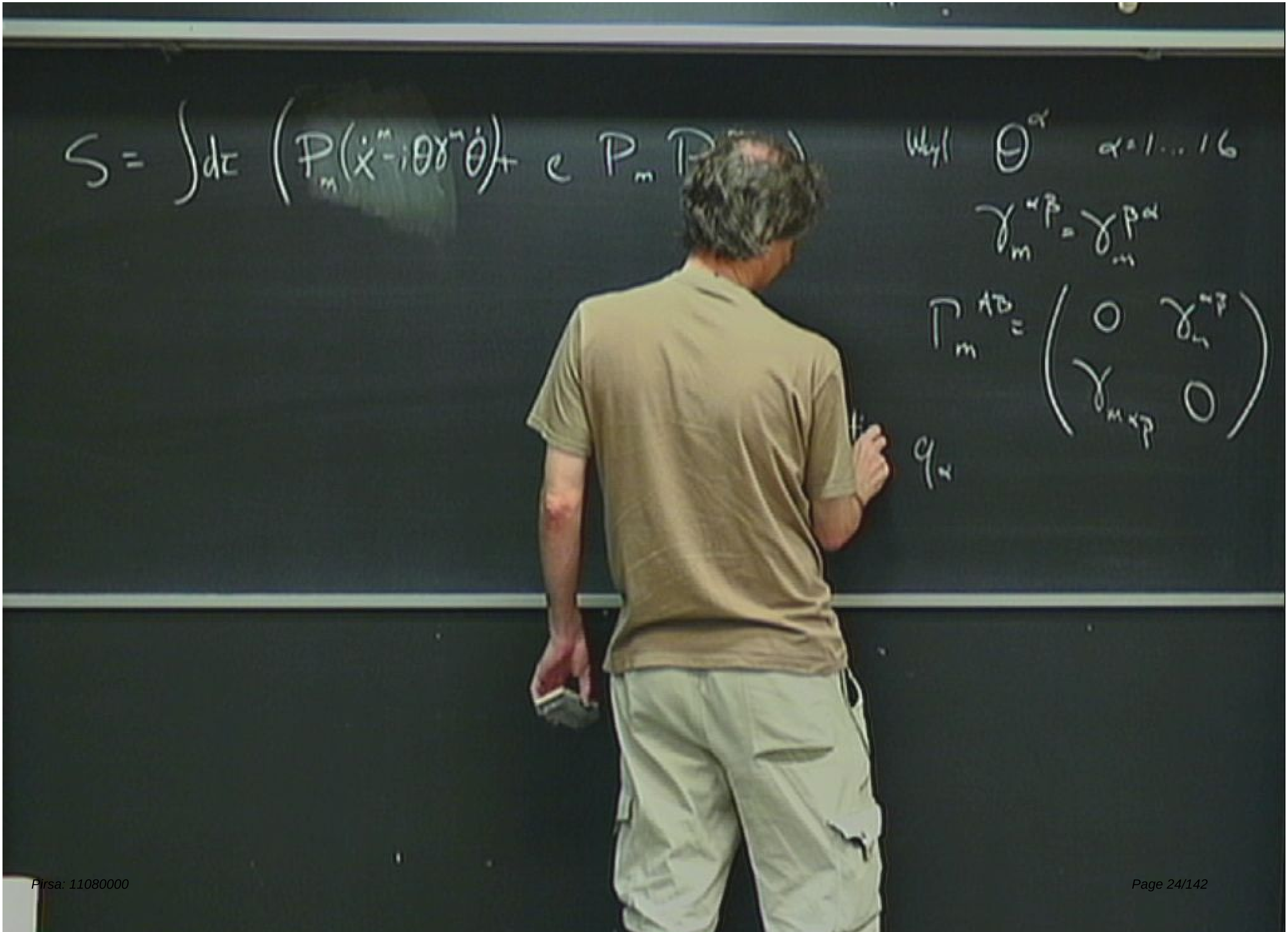
Θ^α
SUSY deformations of flat backgrounds
GS
Pure Spinor
NS
R
GSO(+)
GSO(-)
?
R-R backgrounds (picture changing)
NS
R
GSO(+)
GSO(-)
?
RNS
Ψ^μ


$S = \int d\tau \left(\dot{x} P_m + e P_m P^m \right)$

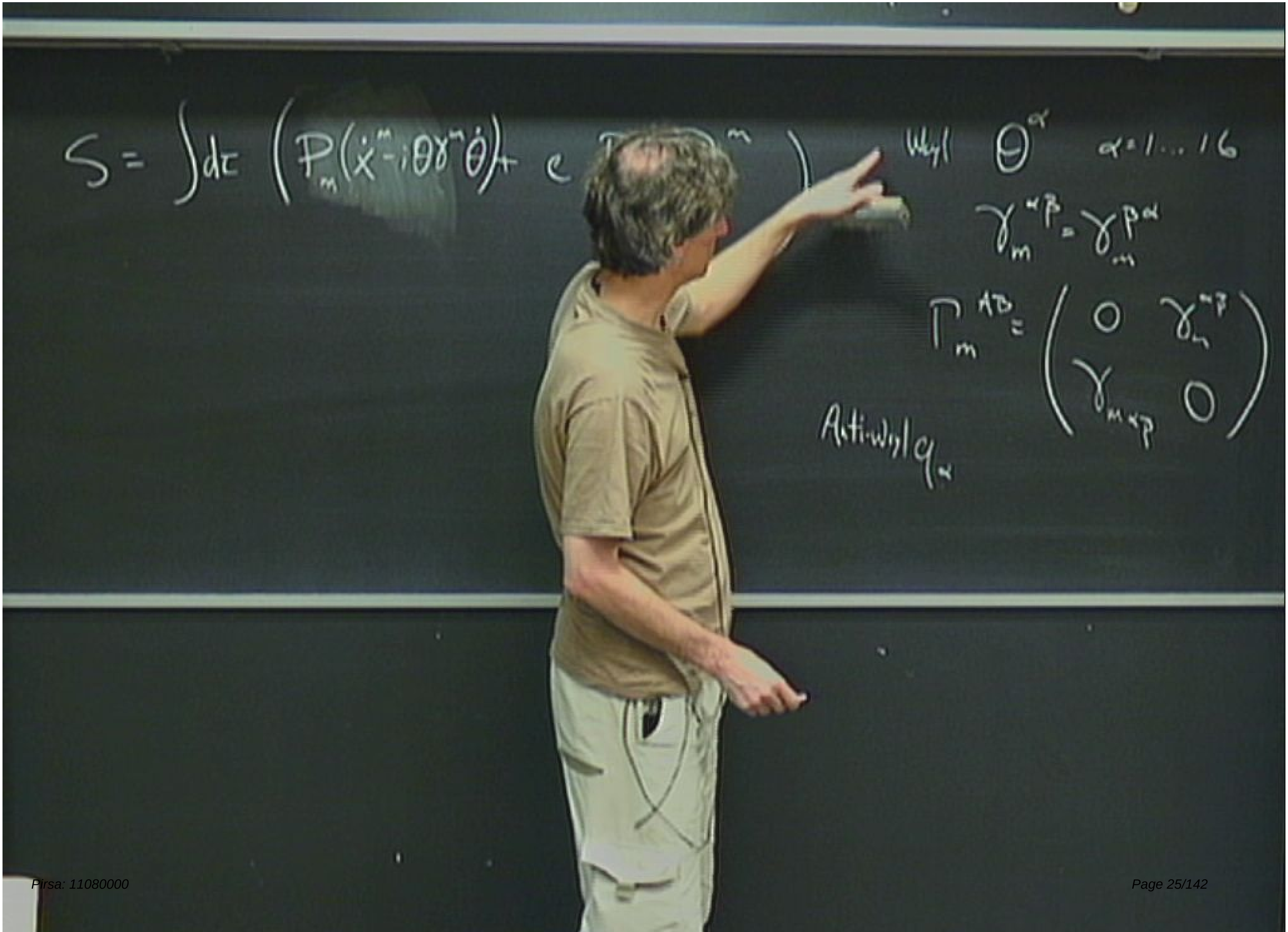

S = ∫dτ (
e P_m P^m)
θ^α α=1...16

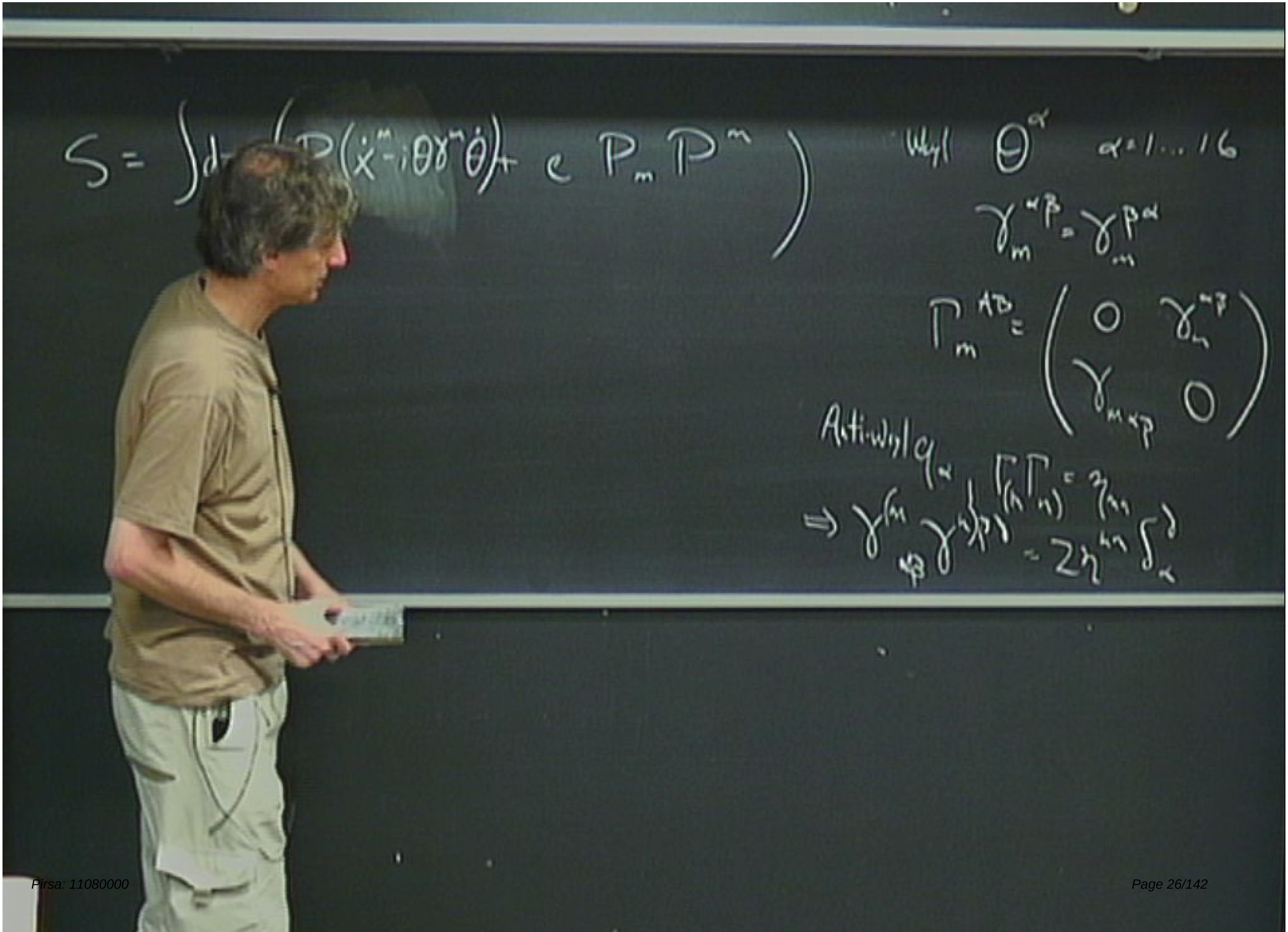

S = ∫dτ (P_m(ẋ^m - iθγ^m θ̇) + e P_m P^m)
θ^α
α = 1 ... 16
γ_m^{αβ} = γ_m^{βα}


$S = \int d\tau \left(P_m(\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e P_m P^m \right.$
$\theta^\alpha \quad \alpha = 1 \ldots 16$
$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$



$S = \int d\tau \left(P_m(\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e P_m P \right)$
Weyl θ^α $\alpha = 1 \ldots 16$
$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$
$\Gamma_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_{m\,\alpha\beta} & 0 \end{pmatrix}$
q_α


$S = \int d\tau \left(P_m(\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e \right)$
Weyl θ^α $\alpha = 1 \ldots 16$
$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$
$\Gamma_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_{m\alpha\beta} & 0 \end{pmatrix}$
Anti-Weyl q_α


$S = \int d\tau \left(P(\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e P_m P^m \right)$
Weyl θ^α $\alpha = 1 \ldots 16$
$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$
$\Gamma_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_{m\alpha\beta} & 0 \end{pmatrix}$
Anti-Weyl q_α
$\Gamma_{(m}\Gamma_{n)} = \eta_{mn}$
$\Rightarrow \gamma^{(m}_{\alpha\beta}\gamma^{n)\beta\gamma} = 2\eta^{mn}\delta_\alpha^\gamma$

$$S = \int d\tau \left(P_m(\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e\, P_m P^m \right)$$

Weyl θ^α $\alpha = 1 \ldots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

Susy: $\delta\theta^\alpha = \epsilon^\alpha$

$\delta x^m = i\,\epsilon\gamma^m\theta$

$$\Gamma_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_{m\,\alpha\beta} & 0 \end{pmatrix}$$

Siegel

K-sym. $\delta\theta^\alpha = P^m(\gamma_m k)^\alpha$

$\delta x^m = -i\,\delta\theta\gamma^m\theta$, $\delta e = k_\alpha\dot{\theta}^\alpha$

$$\Rightarrow \gamma^{(m}_{\alpha\beta}\gamma^{n)\beta\gamma} = 2\eta^{mn}\delta_\alpha^\gamma$$

$$S = \int d\tau \left(P_m(\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e\, P_m P^m \right)$$

Weyl θ^α $\quad \alpha = 1 \ldots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$\Gamma_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_{m\alpha\beta} & 0 \end{pmatrix}$$

Susy: $\delta\theta^\alpha = \epsilon^\alpha$

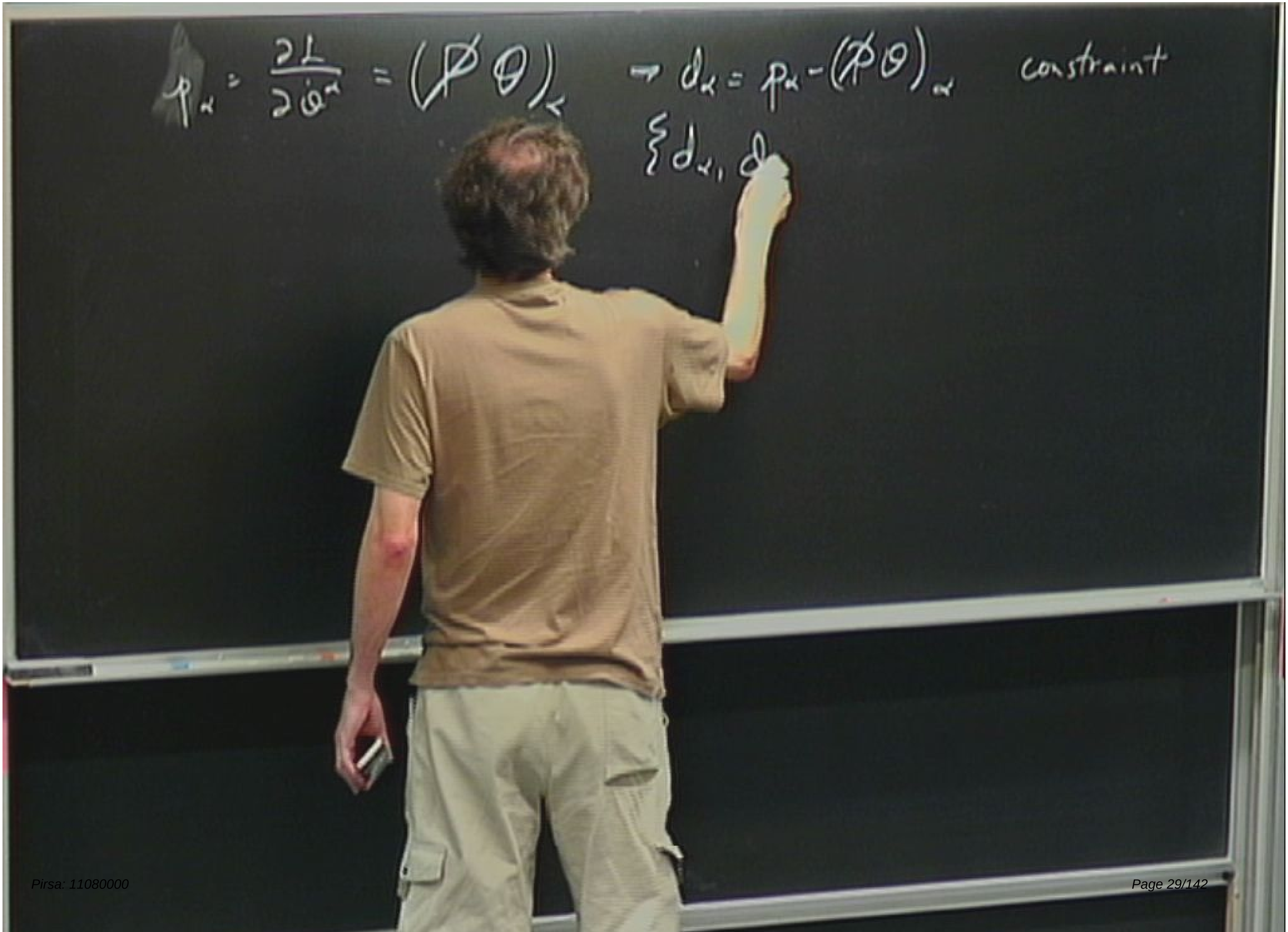
$\delta x^m = i\,\epsilon\gamma^m\theta$

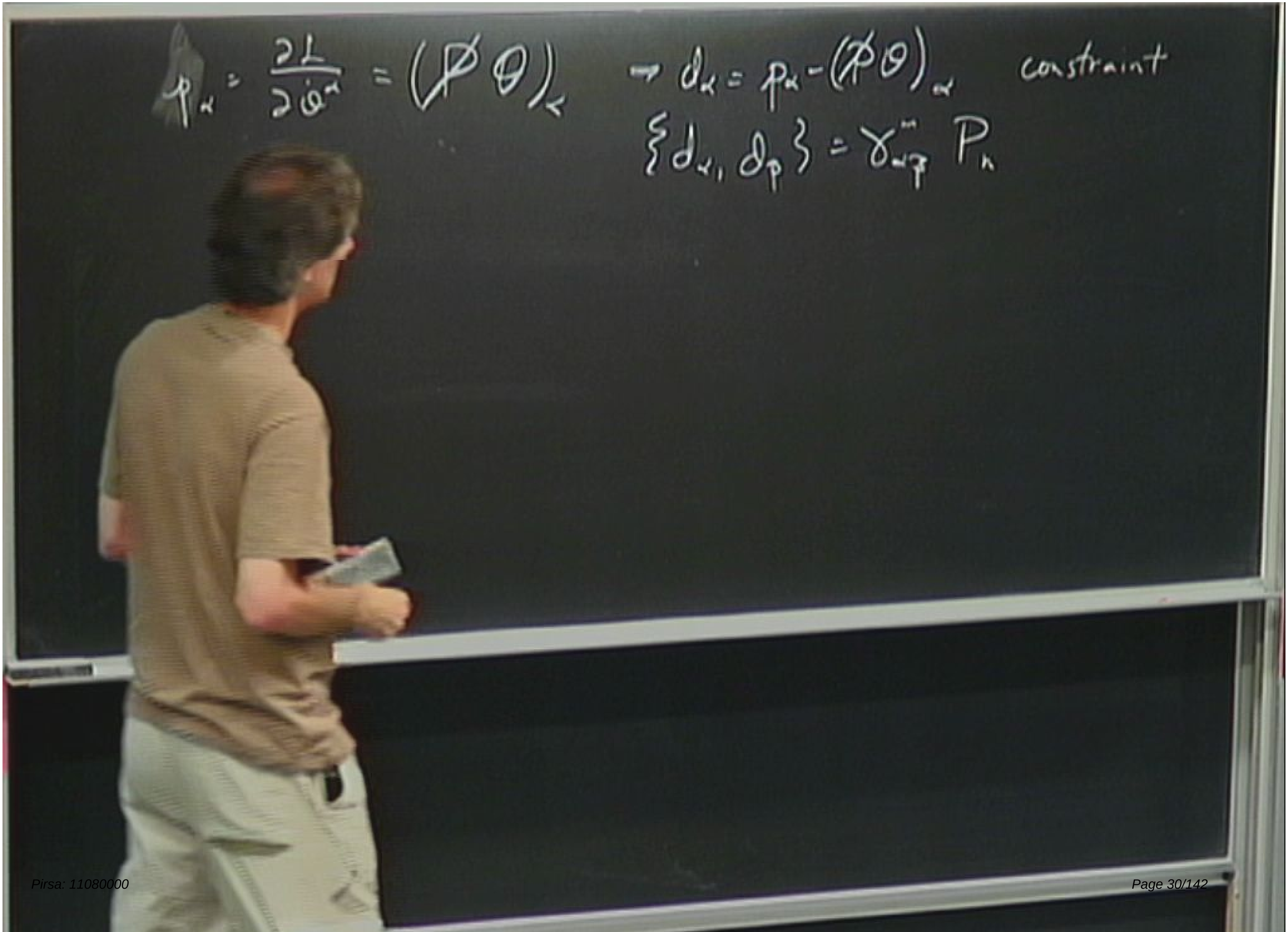
Siegel

K-sym. $\delta\theta^\alpha = P^m(\gamma_m \kappa)^\alpha$

$\delta x^m = -i\,\delta\theta\gamma^m\theta \,, \quad \delta e = \kappa_\alpha \dot{\theta}^\alpha$

$$\Rightarrow \gamma^{(m}_{\alpha\beta}\gamma^{n)\beta\gamma} = 2\eta^{mn}\delta_\alpha^\gamma$$


$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{p}\,\theta)_\alpha \Rightarrow d_\alpha = p_\alpha - (\not{p}\,\theta)_\alpha$ constraint
$\{d_\alpha, d$


$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P}\theta)_\alpha$
$\Rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha$ constraint
$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m$

$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P}\theta)_\alpha \rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha$ constraint

$\{d_\alpha, d_\beta\} = \gamma^{m}_{\alpha\beta} P_m \rightarrow$ 8 1^{st} class, 8 2^{nd} class

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{p}\,\theta)_\alpha \quad \Rightarrow d_\alpha = p_\alpha - (\not{p}\,\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \to \begin{matrix} 8 & 1^{st}\text{ class} \\ 8 & 2^{nd}\text{ class} \end{matrix}$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P}\theta)_\alpha \quad \to \quad d_\alpha = p_\alpha - (\not{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \to \begin{matrix} 8 \ 1^{st} \text{ class} \\ 8 \ 2^{nd} \text{ class} \end{matrix}$$

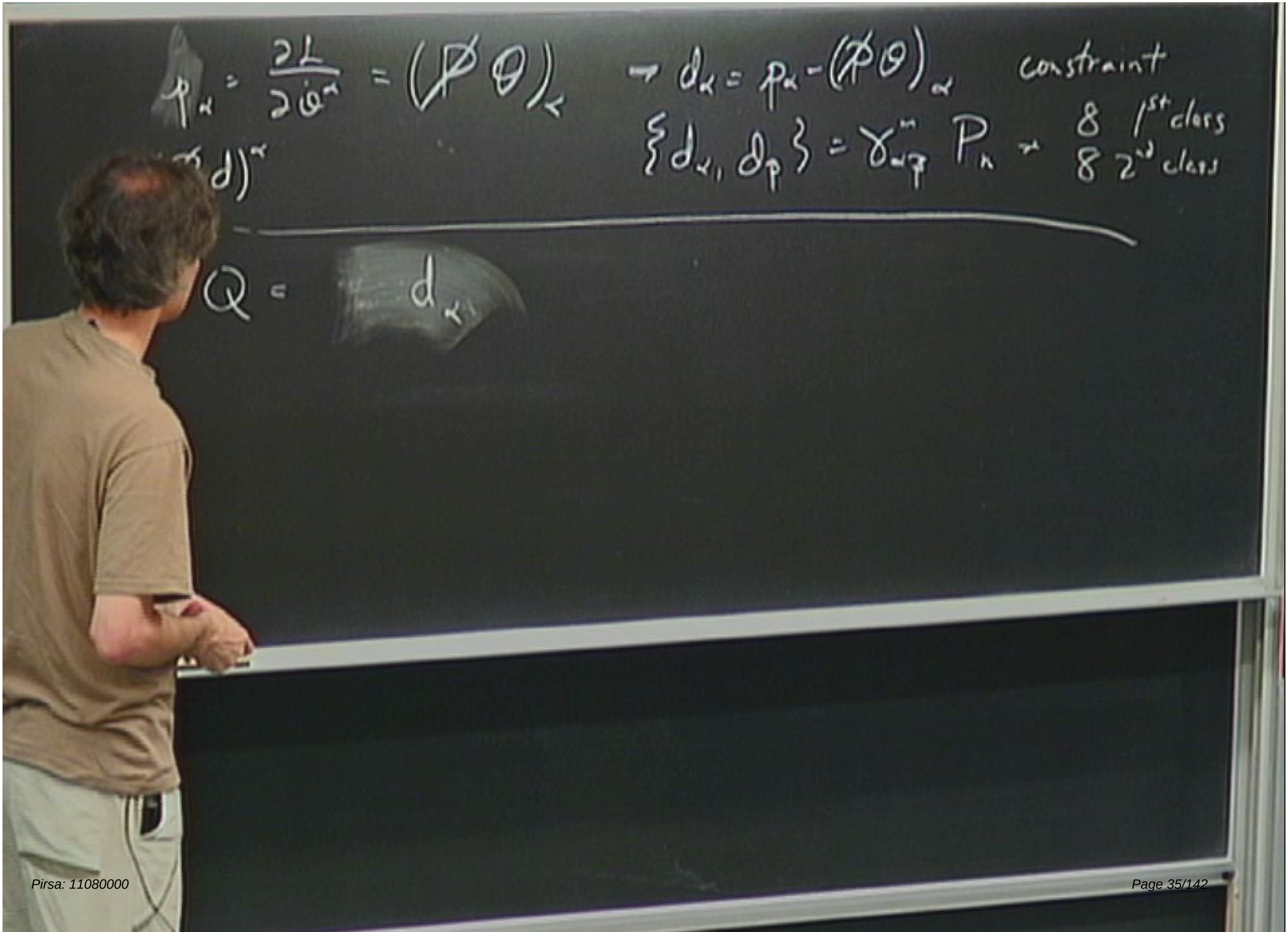
Ker: $(\not{P} d)^\alpha$

$$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not{P}\theta)_\alpha \quad \rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow \begin{matrix} 8 & 1^{st}\text{ class} \\ 8 & 2^{nd}\text{ class} \end{matrix}$$

Ker: $(\not{P}d)^\alpha$

$$Q = c^m G_m$$

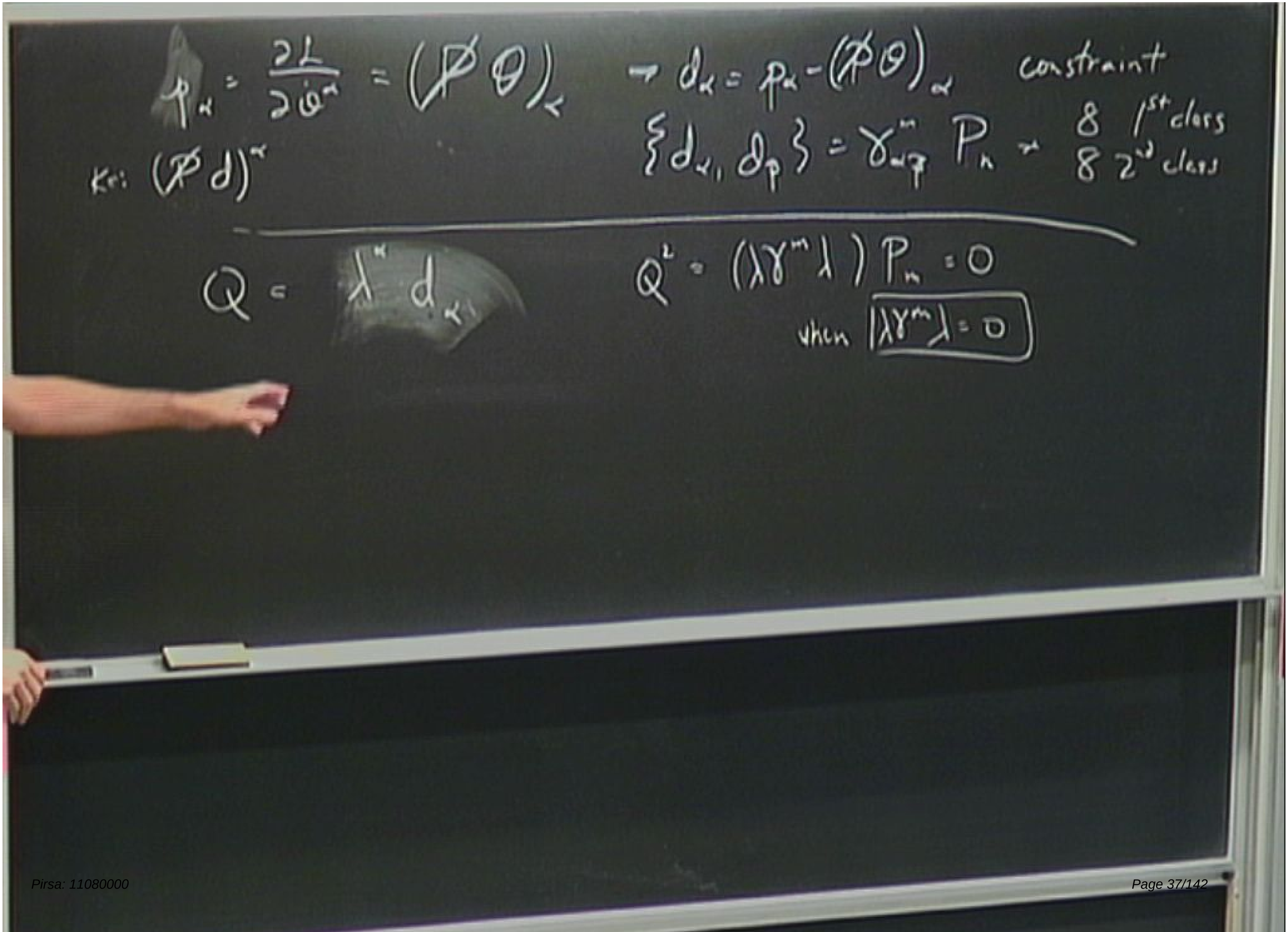

$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not{P}\theta)_\alpha$
$\Rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha$
constraint
$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m$
8 1st class
8 2nd class
$Q =$
d_α

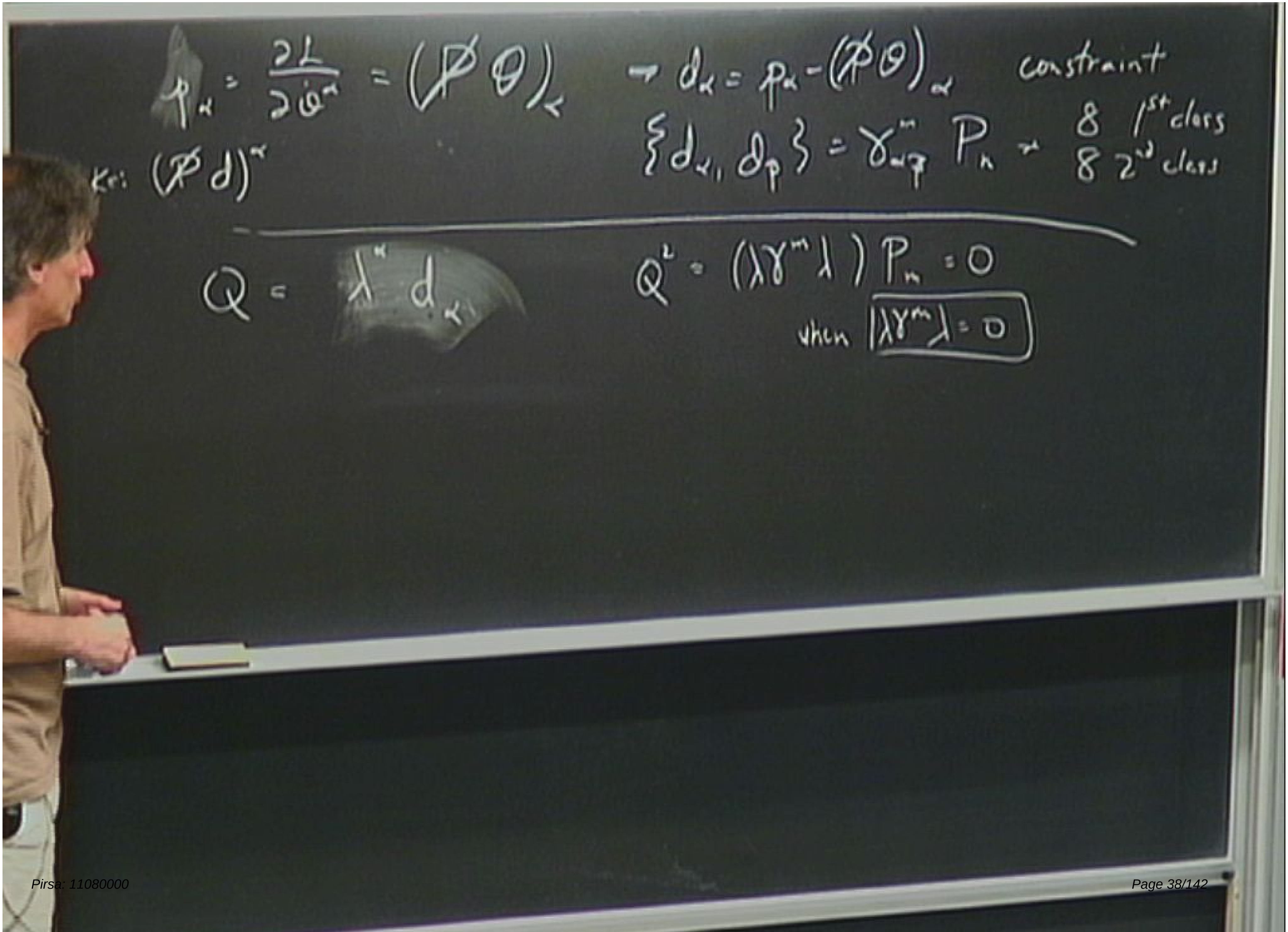
$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P}\theta)_\alpha \quad \Rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow \begin{matrix} 8 & 1^{st}\text{ class} \\ 8 & 2^{nd}\text{ class} \end{matrix}$$

Ker: $(\not{P} d)^\alpha$

$$Q = \lambda^\alpha d_\alpha$$


$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not p\,\theta)_\alpha$
$\rightarrow d_\alpha = p_\alpha - (\not p\,\theta)_\alpha$ constraint
$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow$ 8 1st class
8 2nd class
Ker: $(\not P d)^\alpha$
$Q = \lambda^\alpha d_\alpha$
$Q^2 = (\lambda\gamma^m\lambda) P_m = 0$
when $\lambda\gamma^m\lambda = 0$


$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not P \theta)_\alpha$
$\to d_\alpha = p_\alpha - (\not P \theta)_\alpha$ constraint
$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m$ → 8 1st class, 8 2nd class
Ker: $(\not P d)^\alpha$
$Q = \lambda^\alpha d_\alpha$
$Q^2 = (\lambda\gamma^m\lambda) P_m = 0$
when $\lambda\gamma^m\lambda = 0$

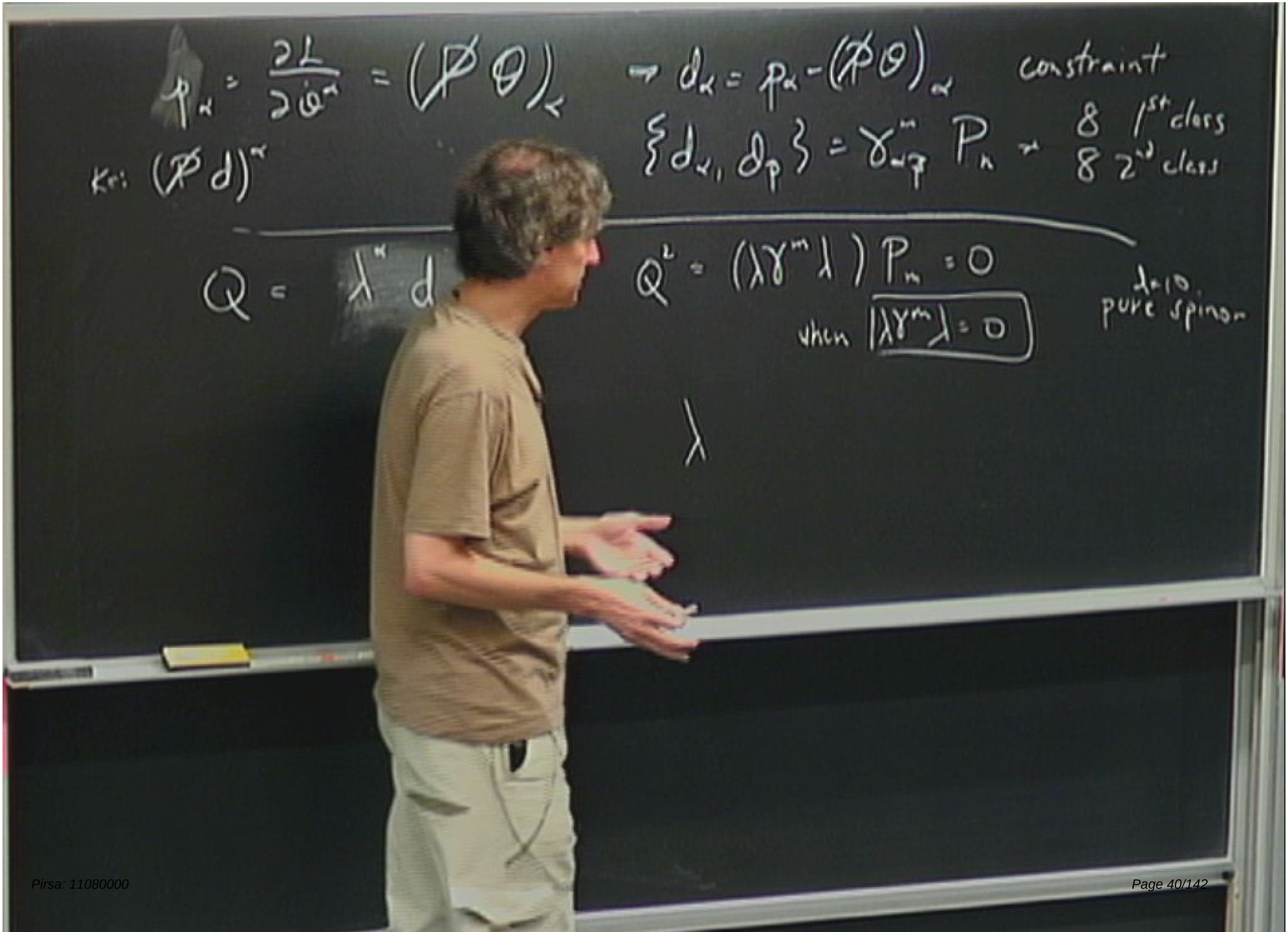
$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P}\theta)_\alpha \quad \Rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \Rightarrow \begin{matrix} 8 \ 1^{st} \text{ class} \\ 8 \ 2^{nd} \text{ class} \end{matrix}$$

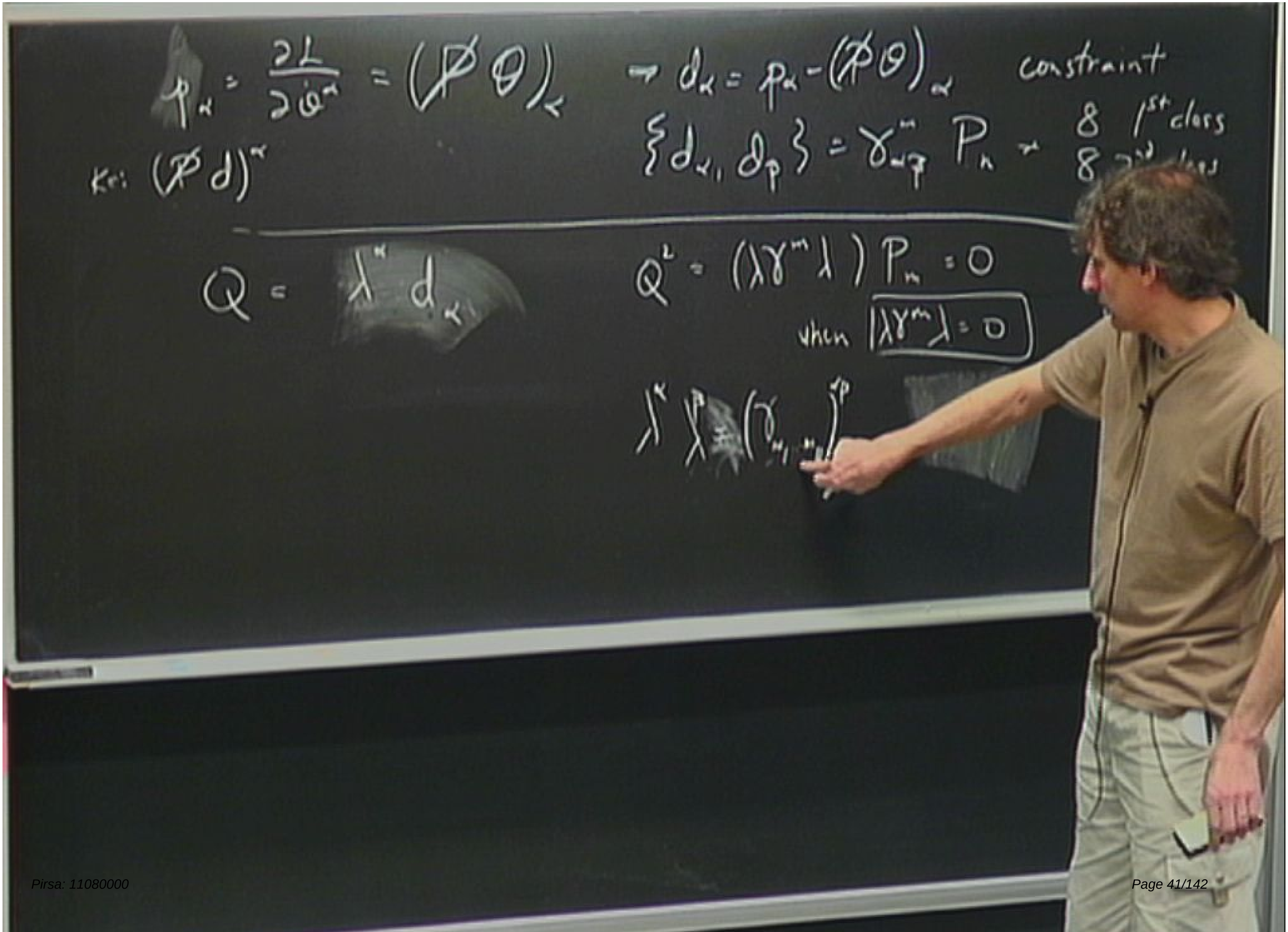
Ker: $(\not{P} d)^\alpha$

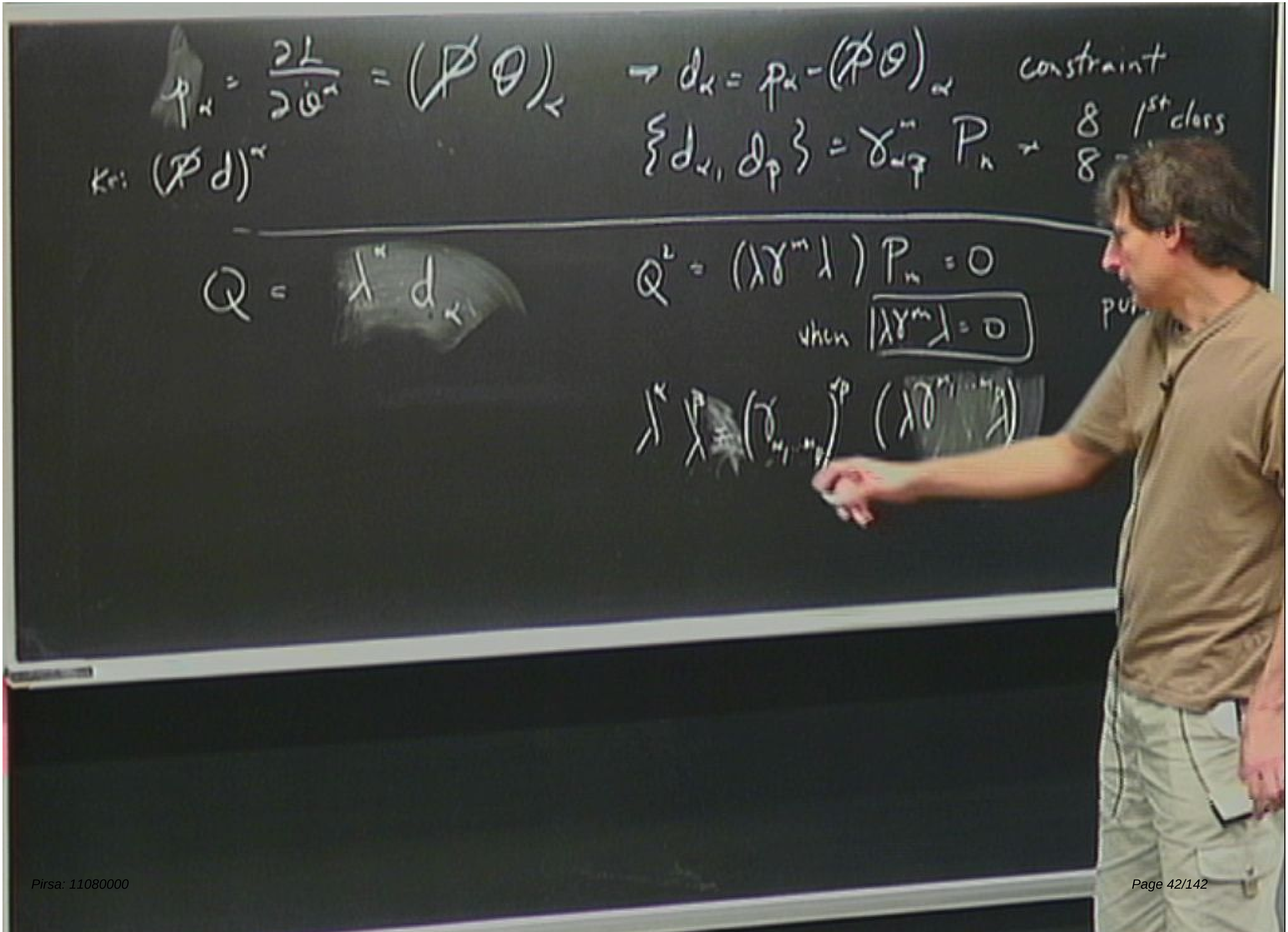
$$Q = \lambda^\alpha d_\alpha \qquad Q^2 = (\lambda\gamma^m\lambda) P_m = 0$$

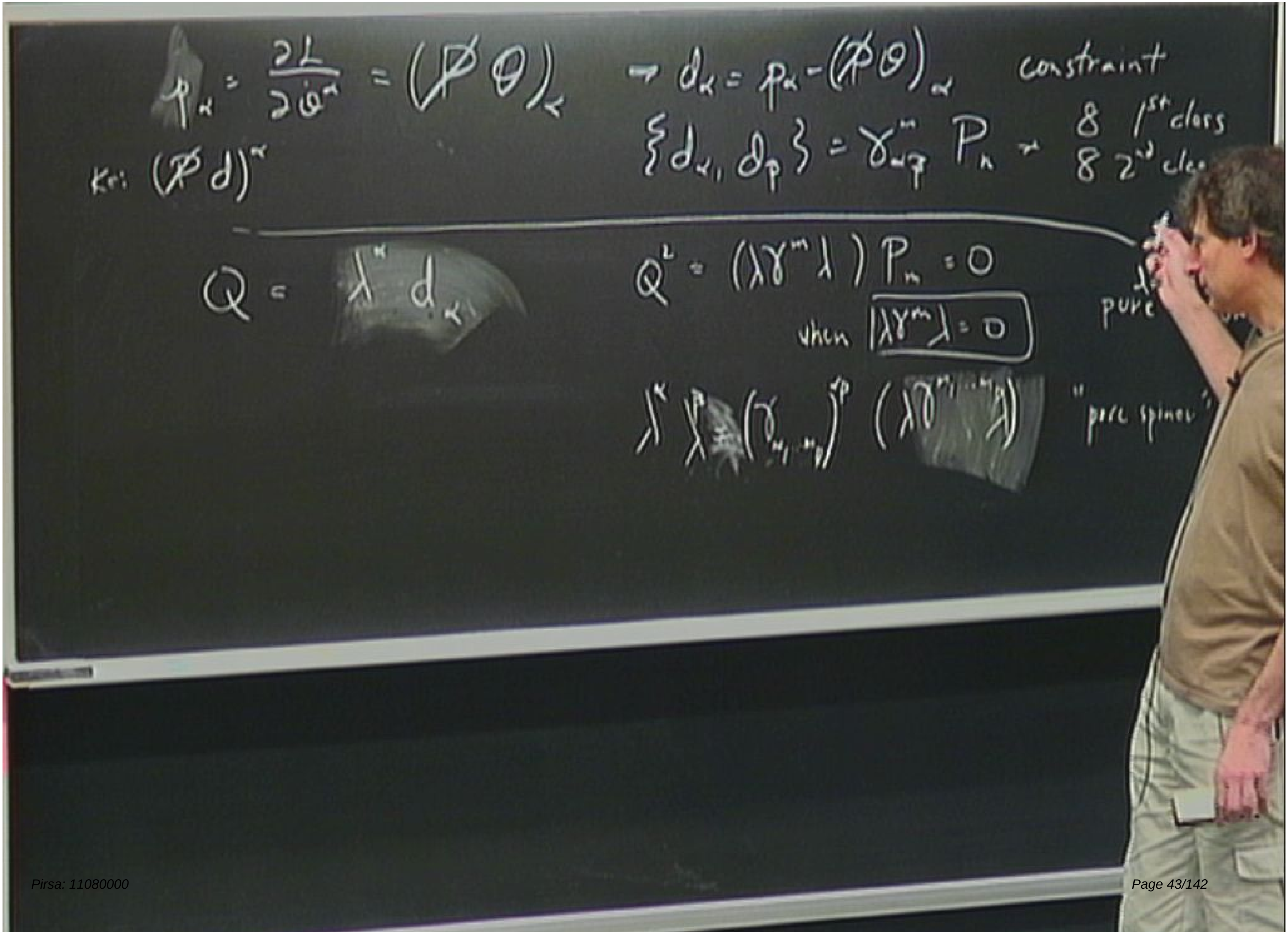
when $\boxed{\lambda\gamma^m\lambda = 0}$



p_α = ∂L/∂θ̇^α = (∂̸θ)_α
⇒ d_α = p_α − (∂̸θ)_α
constraint
{d_α, d_β} = γ^m_αβ P_m ⇒
8 1st class
8 2nd class
Ker: (P̸d)^α
Q = λ^α d_α
Q² = (λγ^m λ) P_m = 0
when (λγ^m λ = 0)
d=10
pure spinor
λ


constraint
1st class
Q = λ^α d_α
Q² = (λγ^m λ) P_m = 0
when (λγ^m λ) = 0


constraint
1st class
when


p_α = ∂L/∂θ̇^α = (p̸ θ)_α
→ d_α = p_α − (p̸θ)_α constraint
{d_α, d_β} = γ^m_αβ P_m → 8 1st class
8 2nd class
Q = λ^α d_α
Q² = (λγ^m λ) P_m = 0
when (λγ^m λ = 0)
pure
λ^α λ^β (γ_{m1…m5})^αβ (λγ^{m1…m5} λ)
"pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P}\theta)_\alpha \quad \rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha$$ constraint

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow$$ 8 1st class, 8 2nd class

Ker: $(\not{P} d)^\alpha$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda\gamma^m\lambda) P_m = 0$$

when $\boxed{(\lambda\gamma^m\lambda) = 0}$ — $d=10$ pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{m_1\dots m_p})_{\alpha\beta} (\lambda\gamma^{m_1\dots m_p}\lambda)$$ "pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P}\theta)_\alpha \quad \rightarrow \quad d_\alpha = p_\alpha - (\not{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow \begin{matrix} 8 & 1^{st}\text{ class} \\ 8 & 2^{nd}\text{ class} \end{matrix}$$

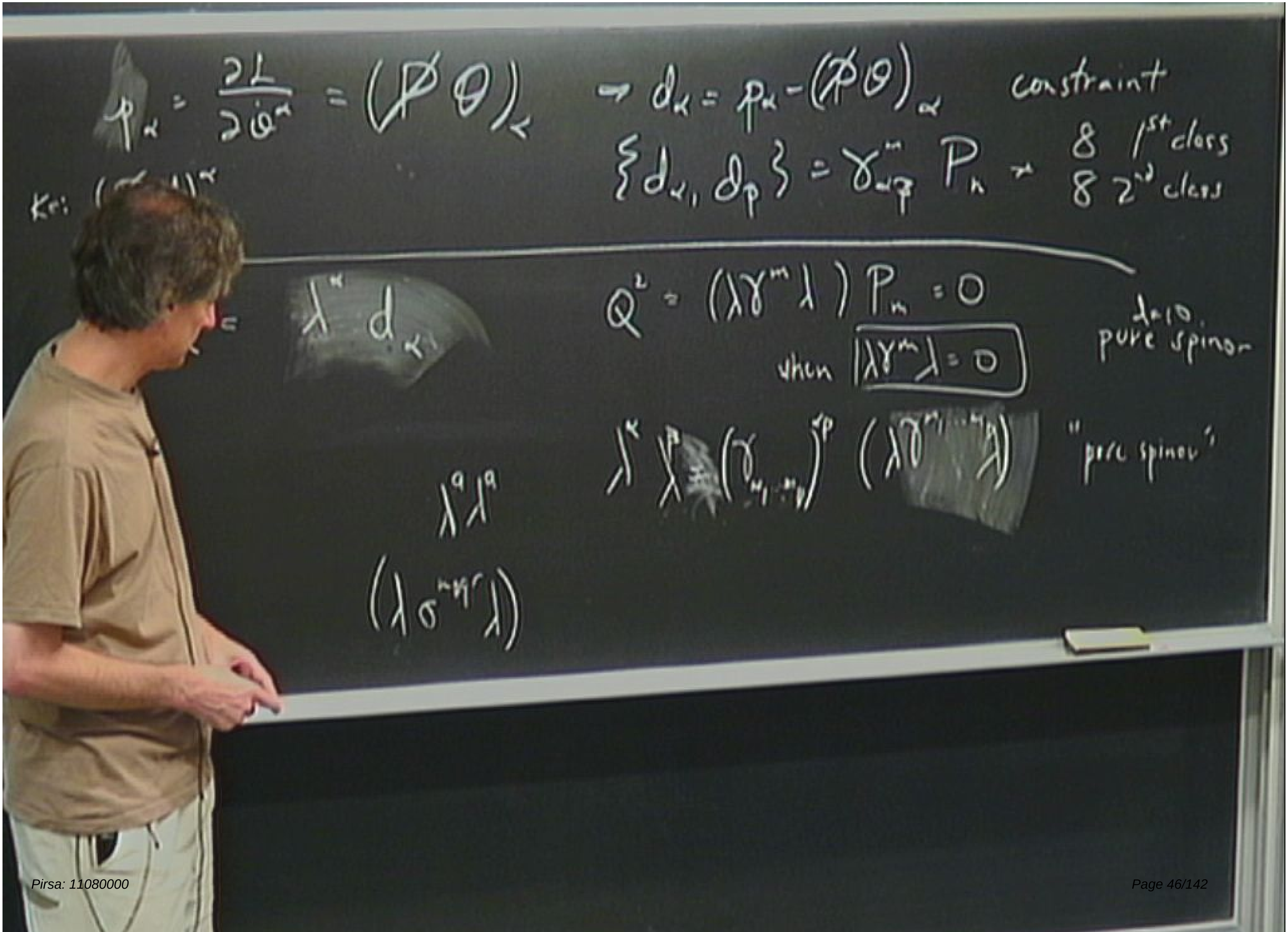
κ-sym: $(\not{P} d)^\alpha$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $\boxed{\lambda \gamma^m \lambda = 0}$ $d=10$ pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{m_1 \dots m_p})_{\alpha\beta} (\lambda \gamma^{m_1 \dots m_p} \lambda)$$ "pure spinor"


constraint
8 1st class
8 2nd class
$Q^2 = (\lambda\gamma^m\lambda) P_m = 0$
when $(\lambda\gamma^m\lambda) = 0$
d=10
pure spinor
"pure spinor"

$$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not{P}\theta)_\alpha \rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow \begin{matrix} 8 \ 1^{st} \text{ class} \\ 8 \ 2^{nd} \text{ class} \end{matrix}$$

Kr: $(\not{P} d)^\alpha$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $\boxed{\lambda \gamma^m \lambda = 0}$ d=10 pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{m_1 \ldots m_p})_{\alpha\beta} \quad (\lambda \gamma^{m_1 \ldots m_p} \lambda) \quad \text{"pure spinor"}$$

$$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not P \theta)_\alpha \rightarrow d_\alpha = p_\alpha - (\not P\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow \begin{matrix} 8 \text{ 1st class} \\ 8 \text{ 2nd class} \end{matrix}$$

$$\text{Ker: } (\not P d)^\alpha$$

$$Q = \lambda^\alpha d_\alpha$$

$$Q^2 = (\lambda\gamma^m\lambda) P_m = 0$$

when $\boxed{\lambda\gamma^m\lambda = 0}$

$d=10$ pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{m_1 \dots m_p})_{\alpha\beta} \quad (\lambda\gamma^{m_1 \dots m_p}\lambda) \quad \text{"pure spinor"}$$

$$\lambda^\alpha = \begin{pmatrix} \lambda \\ \\ \\ \end{pmatrix}$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{P}\theta)_\alpha \quad \rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow \begin{matrix} 8 \ 1^{st} \text{ class} \\ 8 \ 2^{nd} \text{ class} \end{matrix}$$

Ker: $(\not{P} d)^\alpha$

$$Q = \lambda^\alpha d_\alpha \qquad Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

when $\boxed{\lambda \gamma^m \lambda = 0}$ — d=10 pure spinor

$$\lambda^{+++++} \quad \lambda$$

$$\lambda^\alpha \lambda^\beta (\gamma_{m_1 \ldots m_p})_{\alpha\beta} \quad (\lambda \gamma^{m_1 \ldots m_p} \lambda) \quad \text{"pure spinor"}$$

$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = (\not{p}\,\theta)_\alpha \quad \rightarrow d_\alpha = p_\alpha - (\not{p}\,\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow \begin{array}{l} 8 \ 1^{st} \text{ class} \\ 8 \ 2^{nd} \text{ class} \end{array}$$

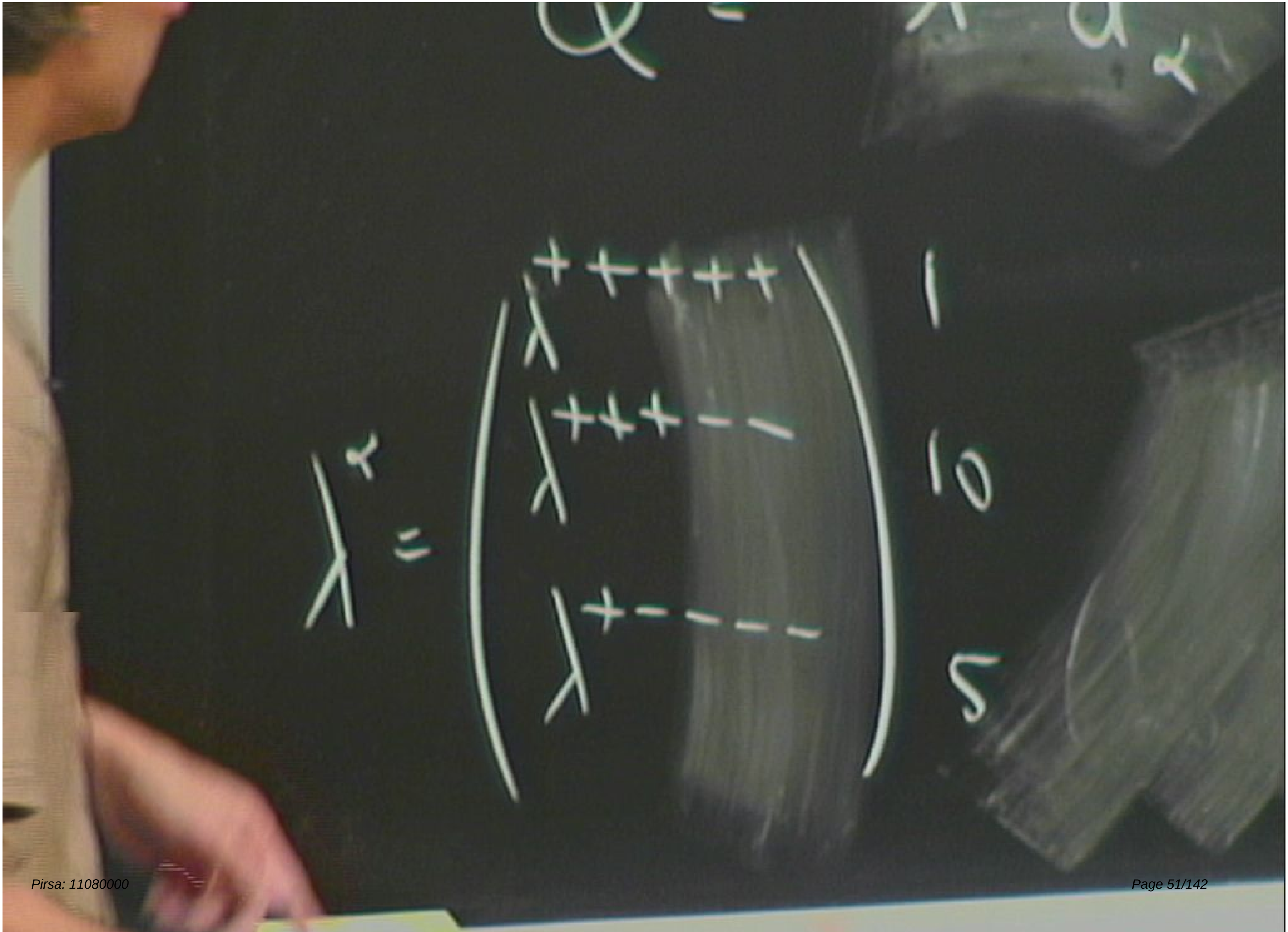
Ker: $(\not{p}\, d)^\alpha$

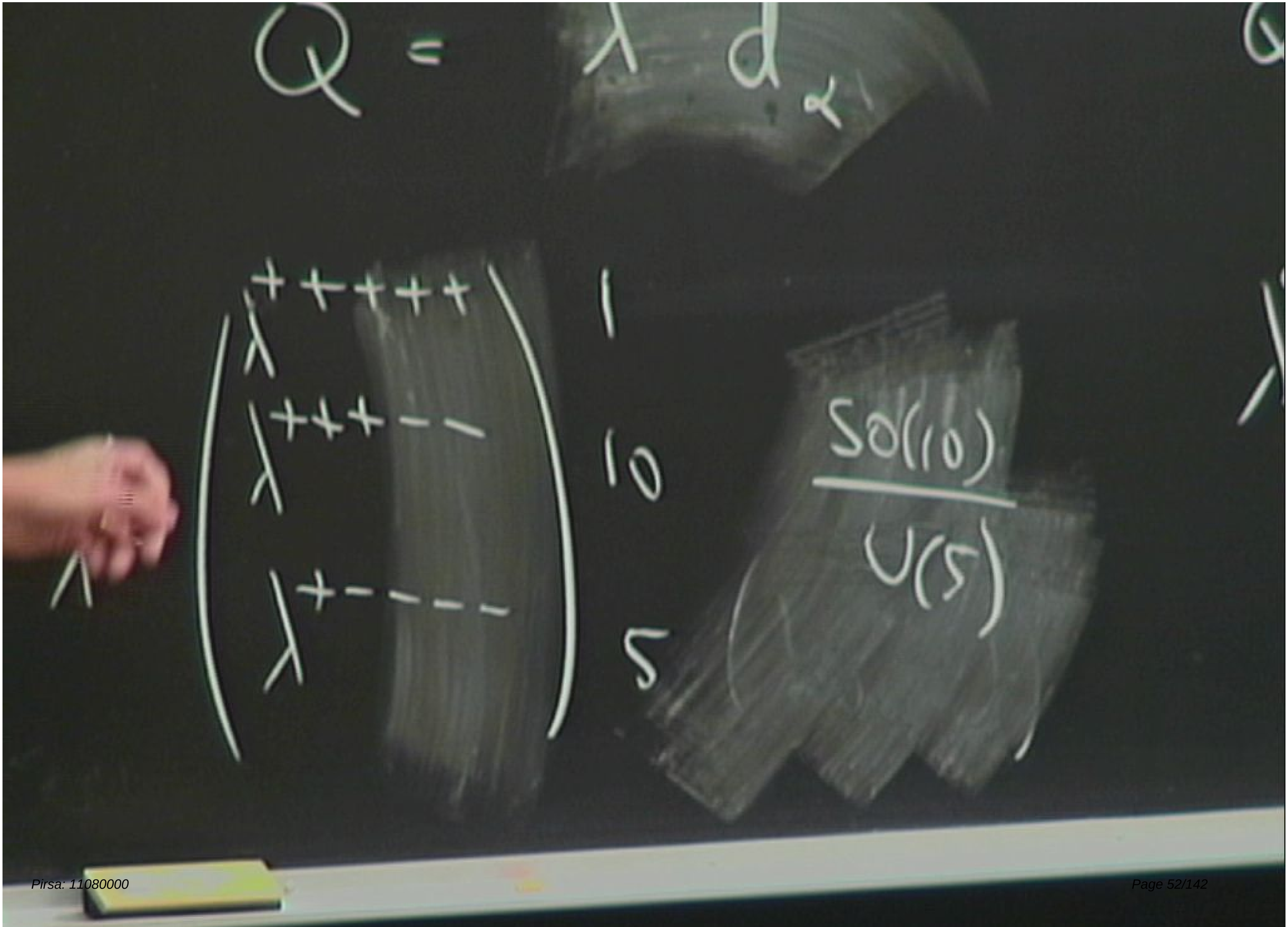
$$Q = \lambda^\alpha d_\alpha \qquad Q^2 = (\lambda \gamma^m \lambda) P_m = 0$$

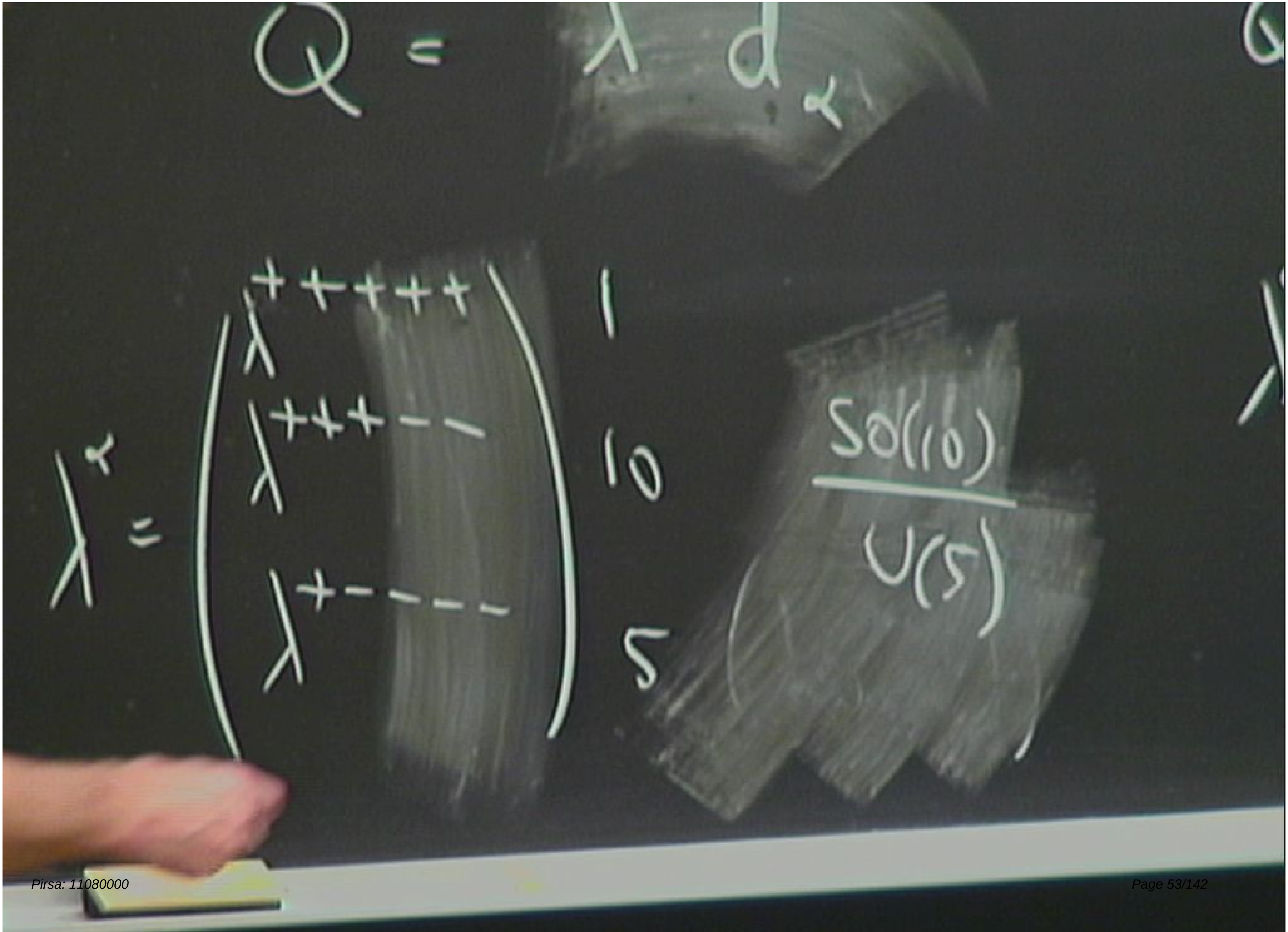
when $\boxed{\lambda \gamma^m \lambda = 0}$ $\quad d=10$ pure spinor

$$\lambda^\alpha \lambda^\beta (\gamma_{m_1 \dots m_p})^{\alpha\beta} \quad (\lambda \gamma^{m_1 \dots m_p} \lambda)$$ "pure spinor"

$$\lambda^\alpha = \begin{pmatrix} \lambda^{+++++} \\ \\ \\ \end{pmatrix}$$


λ^α =
λ^{+++++}   1
λ^{+++−−}   10
λ^{+−−−−}   5

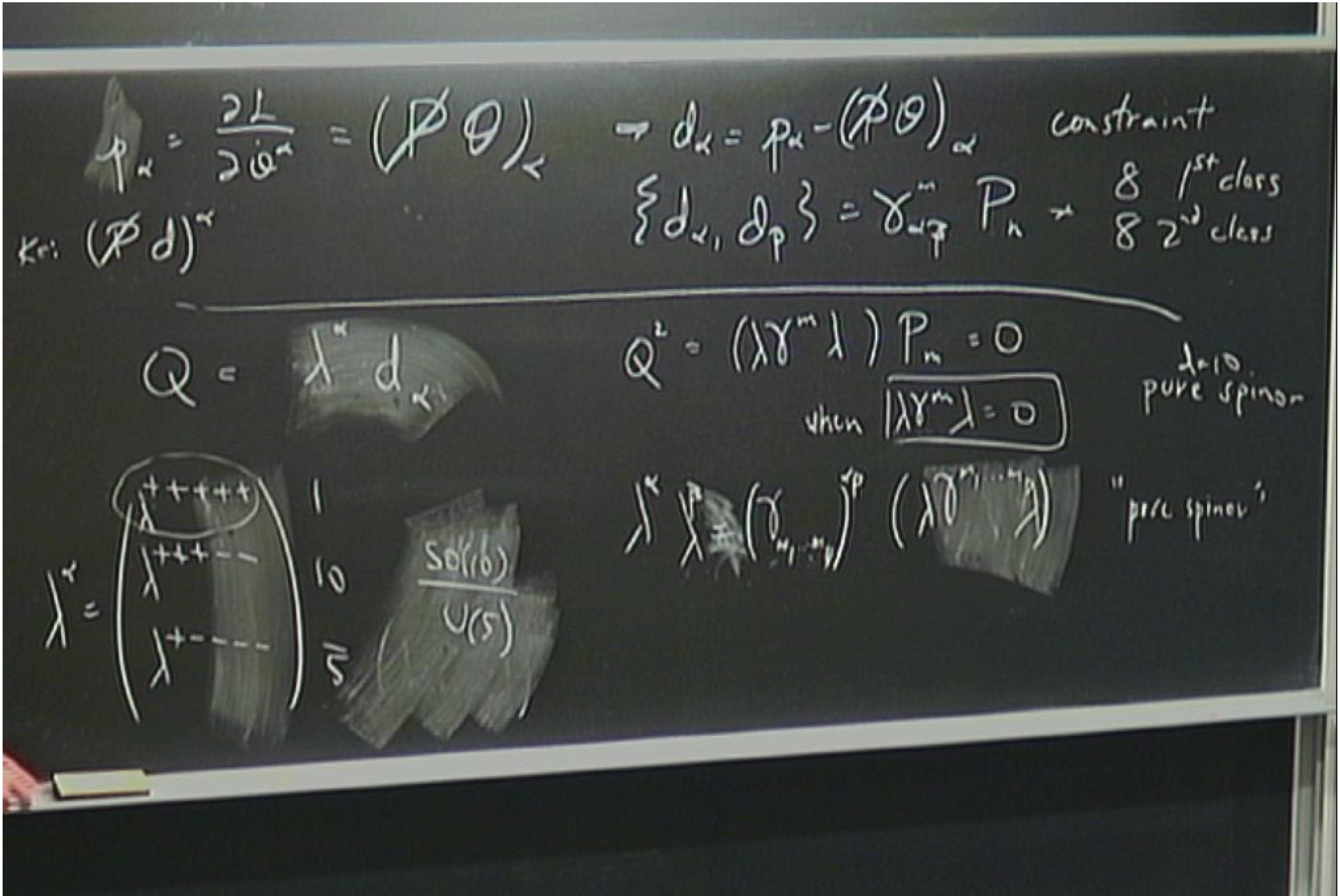

Q = λ d
+++++ 1
+++−− 10
+−−−− 5
SO(10)
U(5)


Q = λ d
λ = λ+++++ 1
λ+++-- 10
λ+---- 5
SO(10)
U(5)

$$Q = \lambda \, d_{r1}$$

$$\lambda^{r} = \begin{pmatrix} \lambda^{+++++} \\ \lambda^{+++--} \\ \lambda^{+----} \end{pmatrix} \begin{matrix} 1 \\ 10 \\ \bar{5} \end{matrix}$$

$$\frac{SO(10)}{U(5)}$$


p_α = ∂L/∂θ̇^α = (∂̸θ)_α
⇒ d_α = p_α − (∂̸θ)_α constraint
{d_α, d_β} = γ^m_αβ P_m ⇒ 8 1st class
8 2nd class
Kκ: (∂̸d)^α
Q = λ^α d_α
Q² = (λγ^m λ) P_m = 0
when λγ^m λ = 0
d=10
pure spinor
"pure spinor"
λ^α = (λ^{+++++}, λ^{+++−−}, λ^{+−−−−})
1
10
5̄
SO(10)/U(5)

$$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not{P}\theta)_\alpha \Rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha \quad \text{constraint}$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m \rightarrow \begin{matrix} 8 \ 1^{st} \text{ class} \\ 8 \ 2^{nd} \text{ class} \end{matrix}$$

Kri: $(\not{P} d)^\alpha$

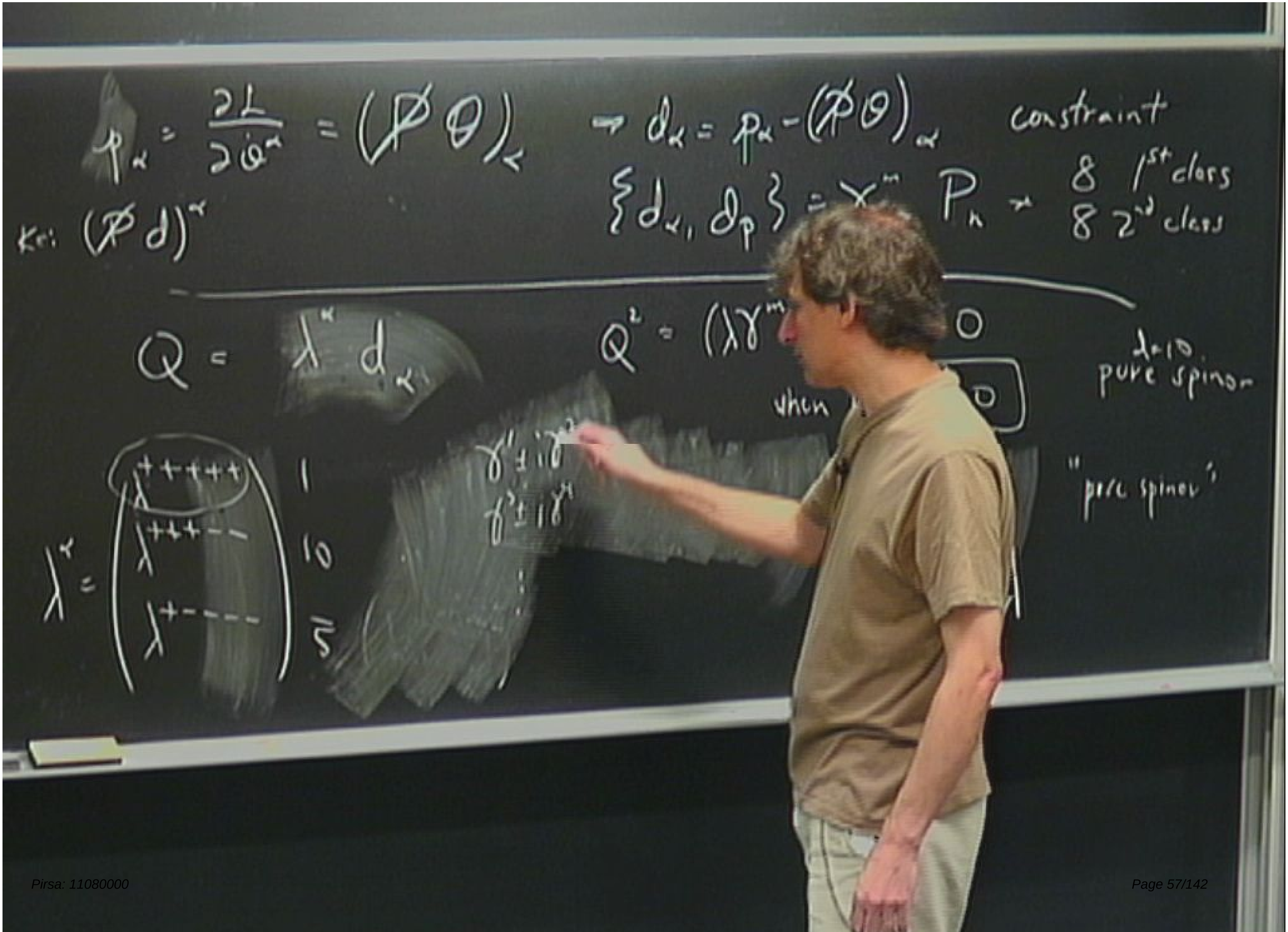
$$Q = \lambda^\alpha d_\alpha \qquad Q^2 = (\lambda\gamma^m\lambda)\, P_m = 0$$

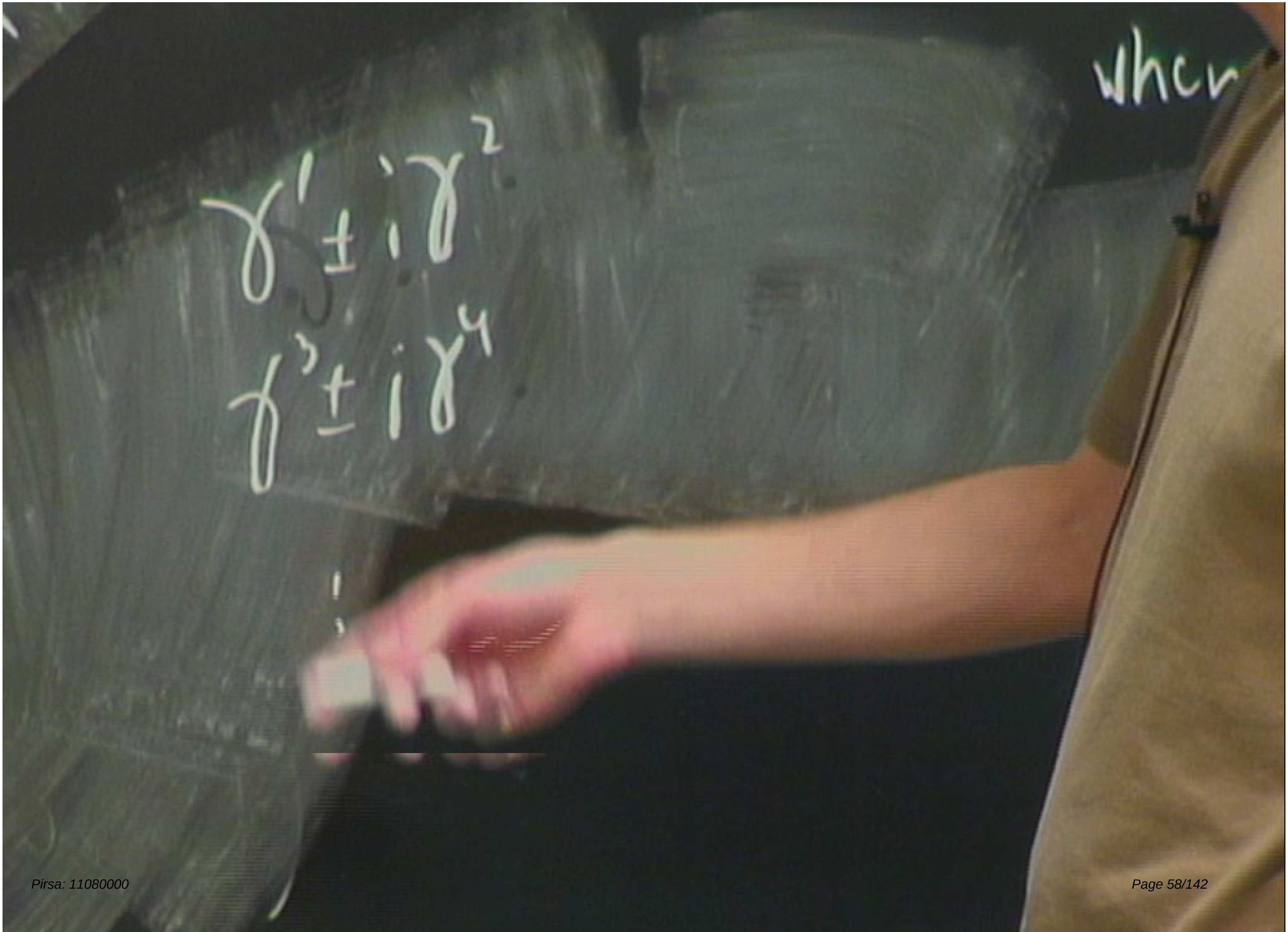
when $\boxed{\lambda\gamma^m\lambda = 0}$ $\quad$ d=10 pure spinor

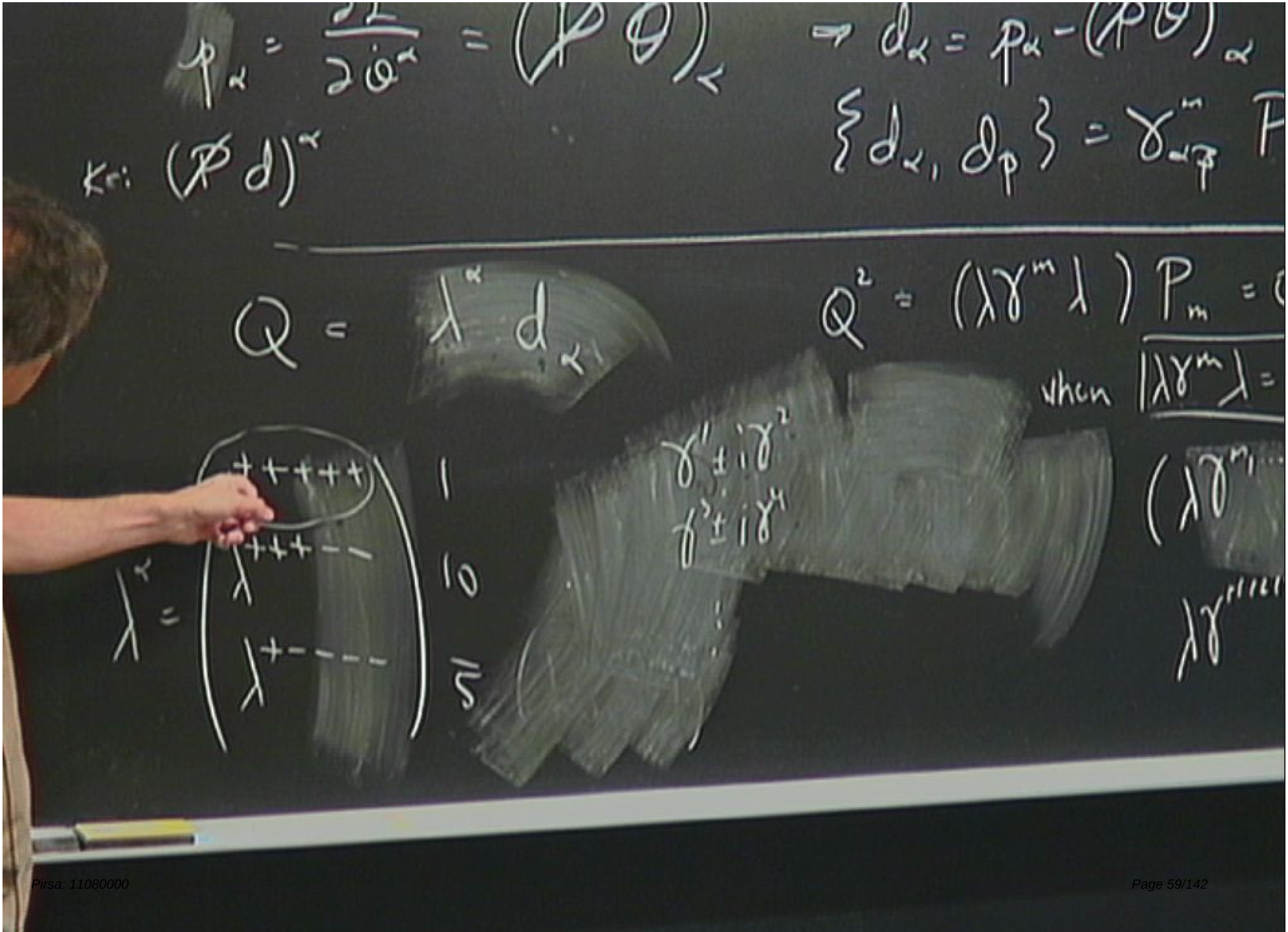
$$\lambda^\alpha = \begin{pmatrix} \lambda^{+++++} \\ \lambda^{+++--} \\ \lambda^{+----} \end{pmatrix} \begin{matrix} 1 \\ 10 \\ \bar{5} \end{matrix} \qquad \frac{SO(10)}{U(5)}$$

$$\lambda^\alpha \lambda^\beta \neq (\gamma_{m_1 \dots m_p})^{\alpha\beta} \quad (\lambda\gamma^{m_1 \dots m_5}\lambda) \quad \text{"pure spinor"}$$

$$\lambda\gamma^{m_1 m_2 m_3 m_4 m_5}\lambda$$


constraint
8 1st class
8 2nd class
pure spinor
when
"pure spinor"


$\gamma^1 \pm i\gamma^2$
$\gamma^3 \pm i\gamma^4$
when


$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not P \theta)_\alpha$
$\Rightarrow d_\alpha = p_\alpha - (\not P \theta)_\alpha$
$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P$
$\ker: (\not P d)^\alpha$
$Q = \lambda^\alpha d_\alpha$
$Q^2 = (\lambda\gamma^m\lambda) P_m$
when $\lambda\gamma^m\lambda =$
$\gamma^1 \pm i\gamma^2$
$\gamma^3 \pm i\gamma^4$
$\lambda^\alpha =$
+++++
+++--
+----
1
10
$\bar 5$

$$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not{P}\,\theta)_\alpha \quad \Rightarrow \quad d_\alpha = p_\alpha - (\not{P}\,\theta)_\alpha$$

$$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P$$

$$\text{Ker}: \ (\not{P}\, d)^\alpha$$

$$Q = \lambda^\alpha d_\alpha \qquad Q^2 = (\lambda \gamma^m \lambda)\, P_m = 0$$

when $\lambda\gamma^m\lambda = 0$

$$\lambda^\alpha = \begin{pmatrix} \lambda^{+++++} \\ \lambda^{+++--} \\ \lambda^{+----} \end{pmatrix} \begin{matrix} 1 \\ 10 \\ \overline{5} \end{matrix}$$

$$2^{D/2}$$

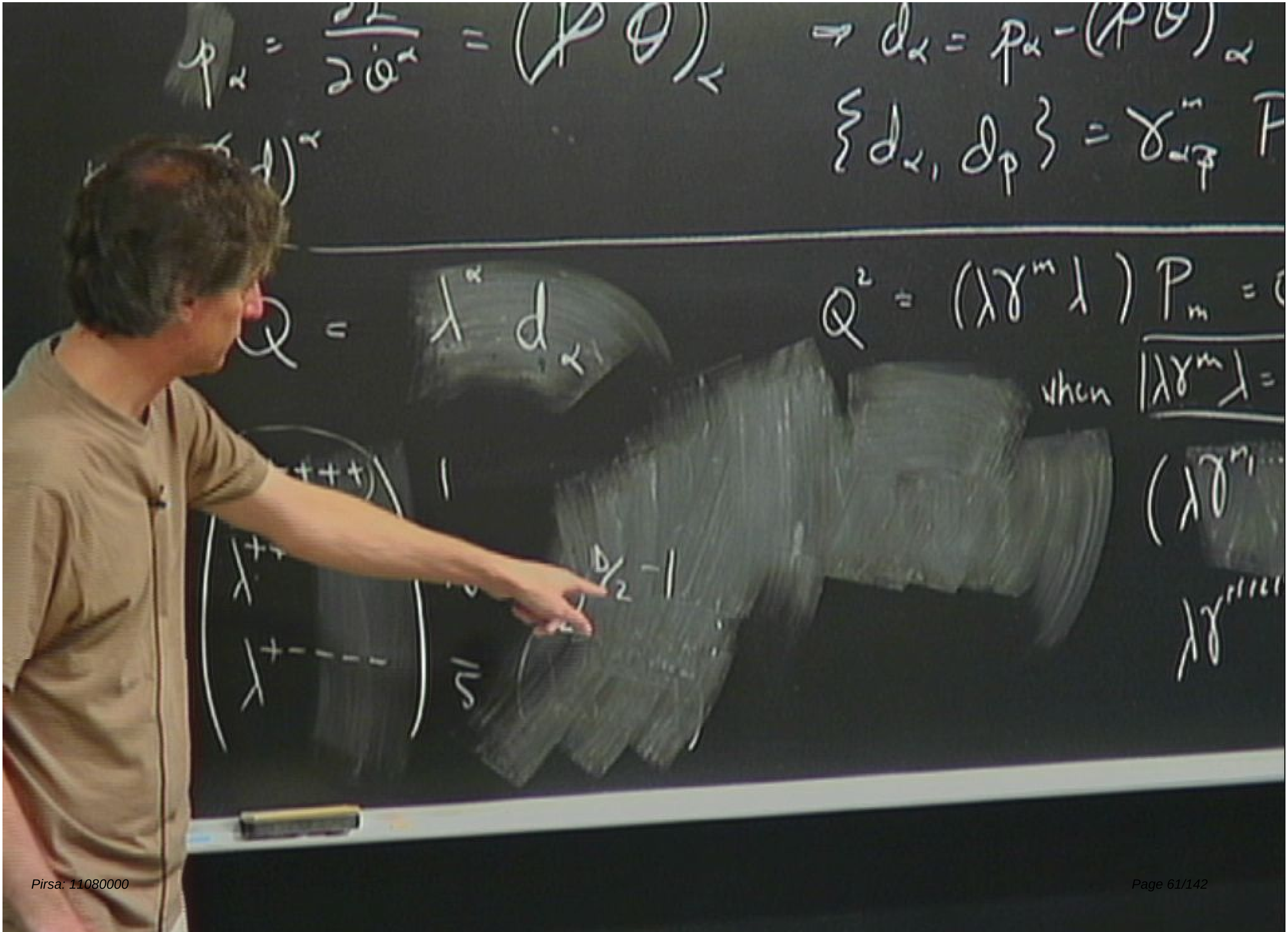
$\gamma^1 \pm i\gamma^2$

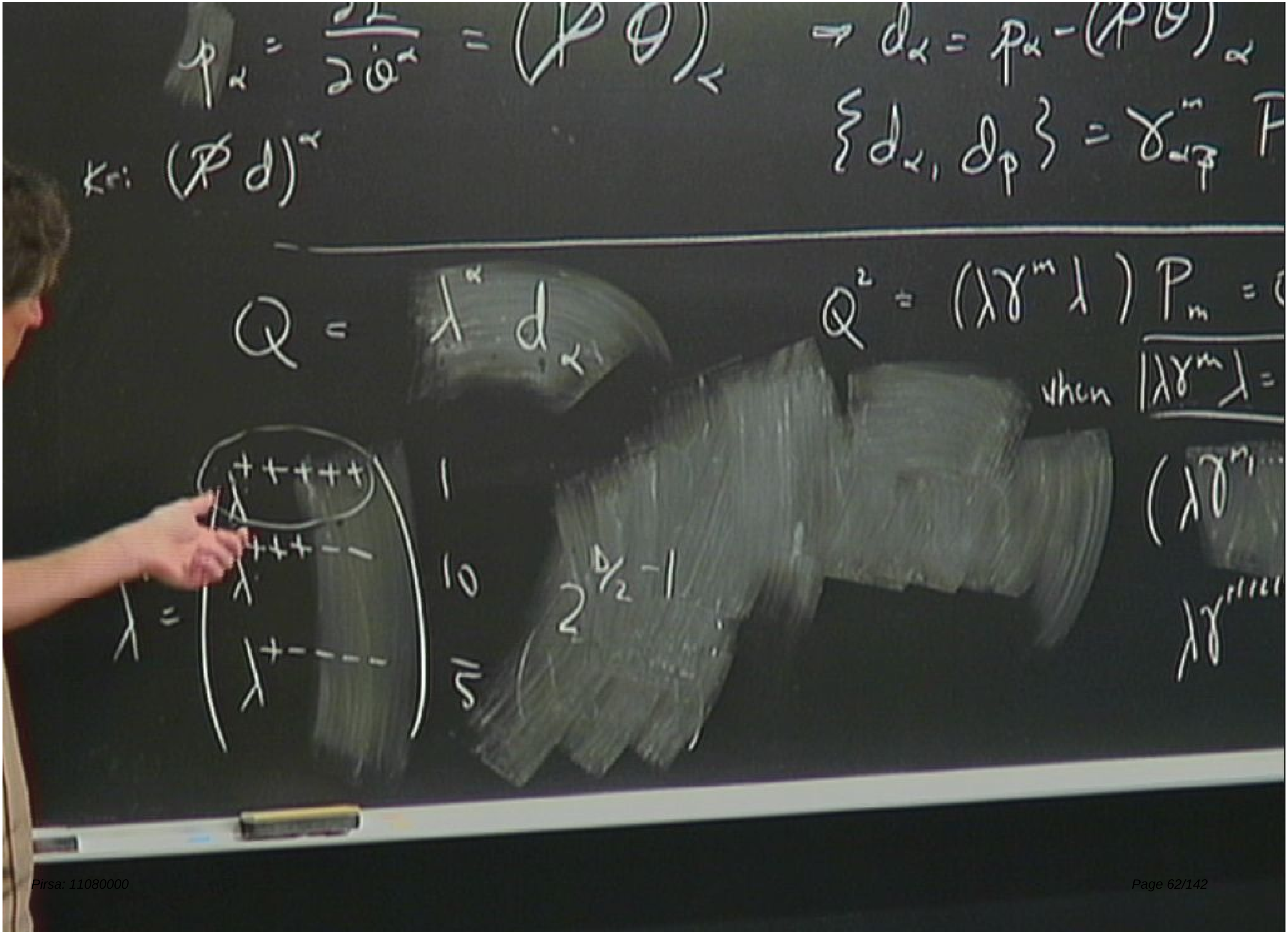
$\gamma^3 \pm i\gamma^4$

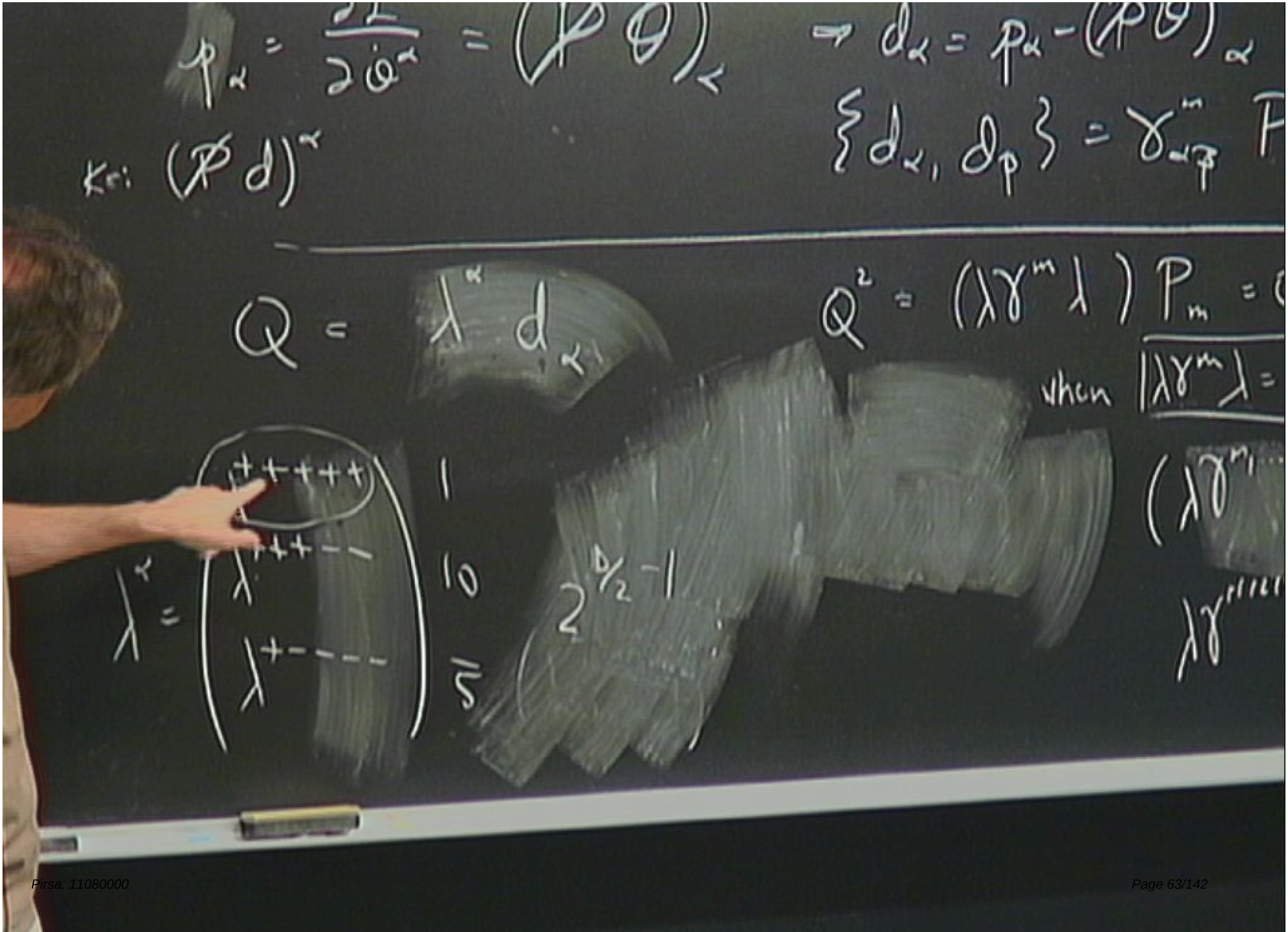
$\vdots$

$(\lambda\gamma^{m_1 \cdots}$

$\lambda\gamma^{mnpqr}$


when


Q = λ^α d_α
Q² = (λγ^m λ) P_m
when
λ =
1
10
5̄


p_α = ∂L/∂θ̇^α = (P̸θ)_α
⇒ d_α = p_α − (P̸θ)_α
{d_α, d_β} = γ^m_αβ P
Q = λ^α d_α
Q² = (λγ^m λ) P_m =
when (λγ^m λ) =
λ^α =
1
10
5̄
2^{D/2 − 1}

$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not{P}\theta)_\alpha \Rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha$

$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P$

Ker: $(\not{P} d)^\alpha$

$Q = \lambda^\alpha d_\alpha$

$Q^2 = (\lambda\gamma^m\lambda) P_m = 0$

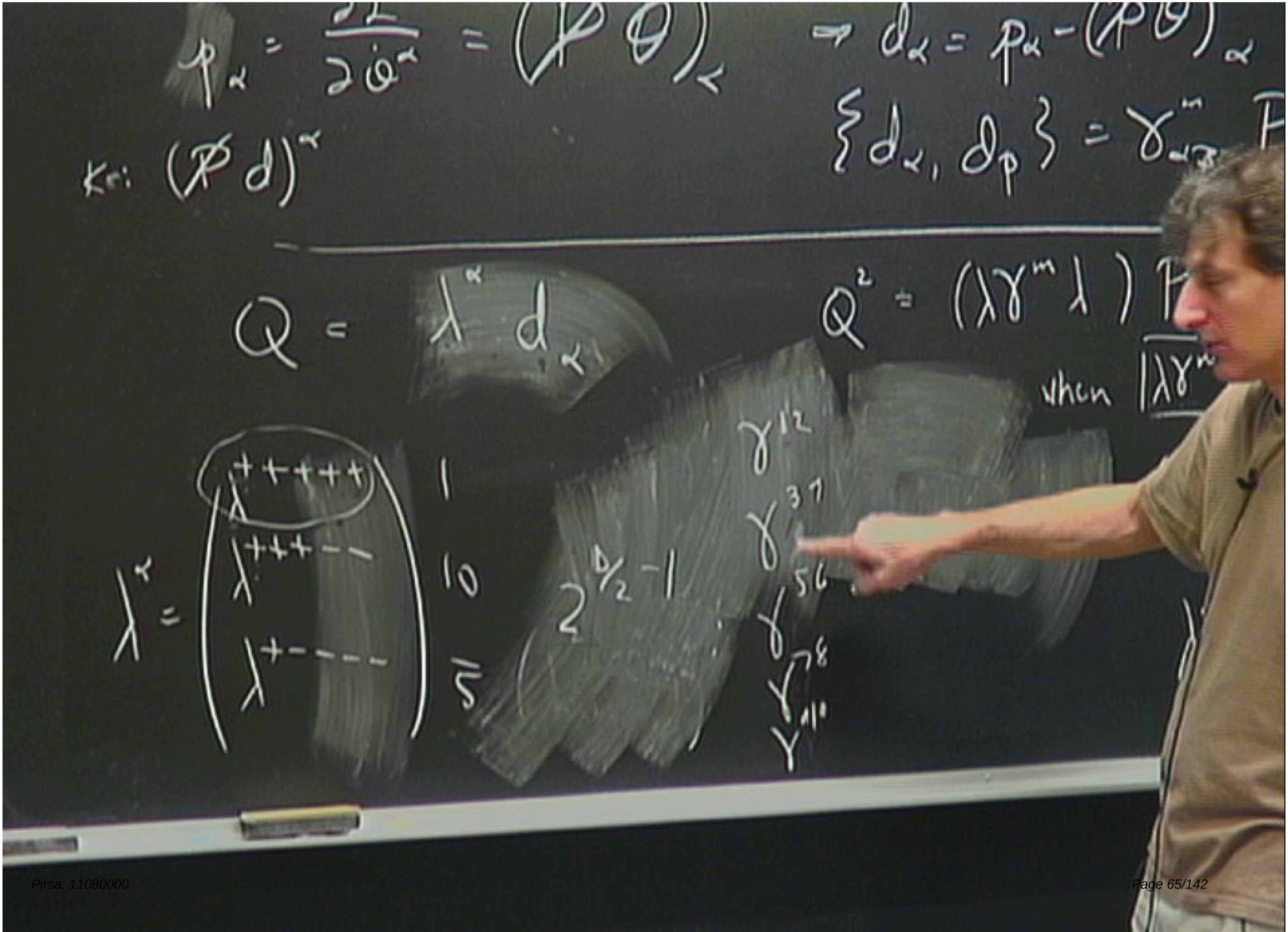
when $\lambda\gamma^m\lambda =$

$(\lambda\gamma^m$

$\lambda\gamma^m$

$$\lambda^\alpha = \begin{pmatrix} \lambda^{+++++} \\ \lambda^{+++--} \\ \lambda^{+----} \end{pmatrix} \quad \begin{matrix} 1 \\ 10 \\ \bar{5} \end{matrix}$$

$2^{D/2-1}$



$p_\alpha = \frac{\partial L}{\partial \dot\theta^\alpha} = (\not{P}\theta)_\alpha$
$\Rightarrow d_\alpha = p_\alpha - (\not{P}\theta)_\alpha$
$\{d_\alpha, d_\beta\} = \gamma^m_{\alpha\beta} P_m$
Ker: $(\not{P} d)^\alpha$
$Q = \lambda^\alpha d_\alpha$
$Q^2 = (\lambda\gamma^m\lambda) P_m$
when $(\lambda\gamma^m$
$2^{D/2-1}$
1
10
$\bar{5}$

if flat backgrounds

GSO(+)	✓	✓
GSO(-)	?	?

GS / Pure Spinor

	NS	R
GSO(+)	✓	?
GSO(-)	✓	?

RNS ψ^m

$$S = \int d\tau \left(P_m(\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e\, P_m P^m \right)$$

Weyl θ^α $\alpha = 1 \ldots 16$

$$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$$

$$\Gamma_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_{m\,\alpha\beta} & 0 \end{pmatrix}$$

Susy: $\delta\theta^\alpha = \epsilon^\alpha$

$\delta x^m = i\,\epsilon\gamma^m\theta$

Siegel K-sym. $\delta\theta^\alpha = P^m(\gamma_m K)^\alpha$

$\delta x^m = -i\,\delta\theta\gamma^m\theta$, $\delta e = K_\alpha\dot{\theta}^\alpha$

$$\Rightarrow \gamma^{(m}_{\alpha\beta}\gamma^{n)\beta\gamma} = 2\eta^{mn}\delta_\alpha^\gamma$$

if flat backgrounds

GS / (Pure Spinor)

GSO(+)	✓	✓
GSO(-)	?	?

	NS	R
GSO(+)	✓	?
GSO(-)	✓	?

RNS ψ^m

$$S = \int d\tau \left(P_m (\dot{x}^m - i\theta\gamma^m\dot{\theta}) + e P_m P^m \right)$$

Weyl θ^α $\alpha = 1 \ldots 16$

$\gamma_m^{\alpha\beta} = \gamma_m^{\beta\alpha}$

$$\Gamma_m^{AB} = \begin{pmatrix} 0 & \gamma_m^{\alpha\beta} \\ \gamma_{m\,\alpha\beta} & 0 \end{pmatrix}$$

Susy: $\delta\theta^\alpha = \epsilon^\alpha$

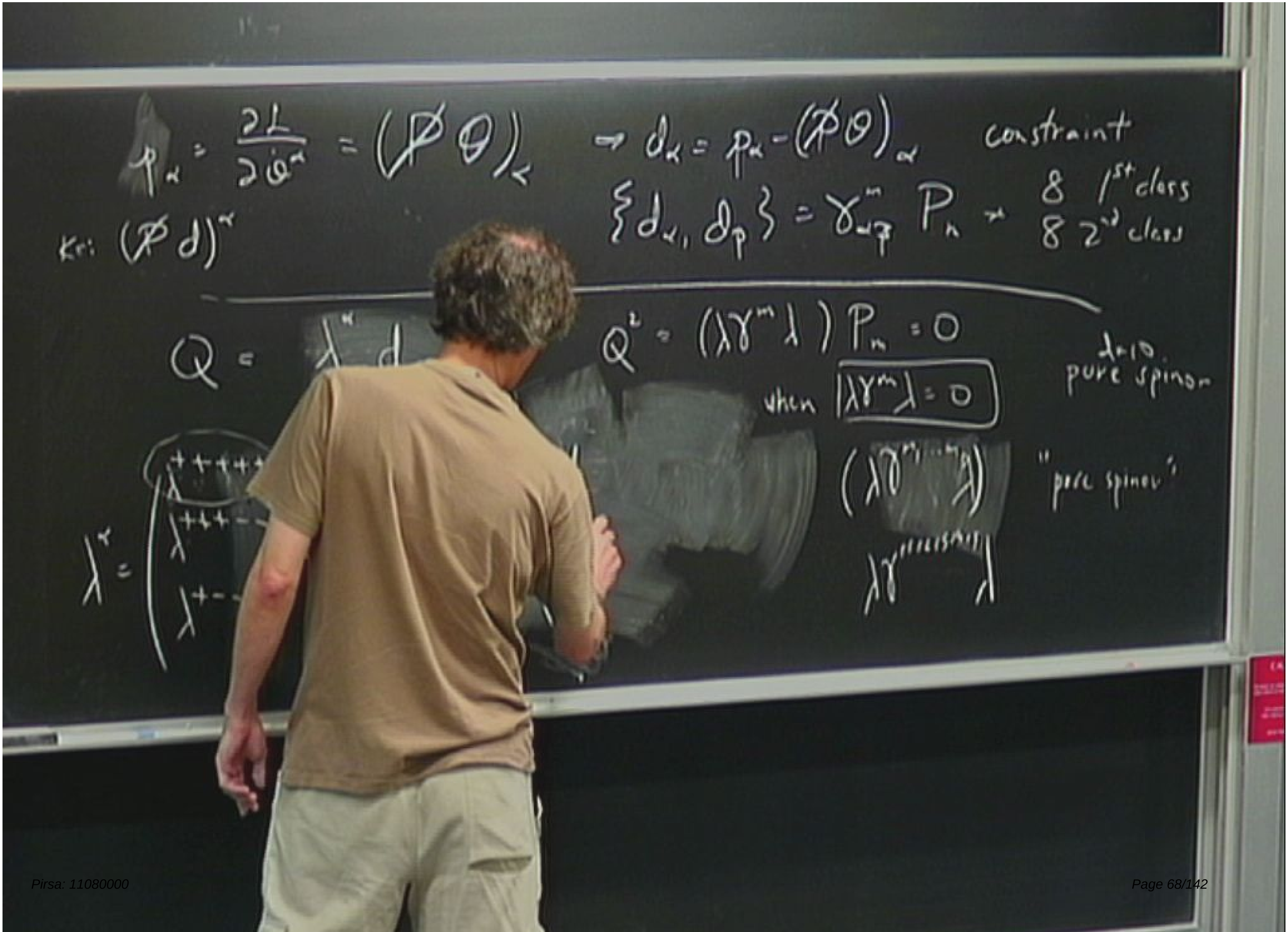
$\delta x^m = i\,\epsilon\gamma^m\theta$

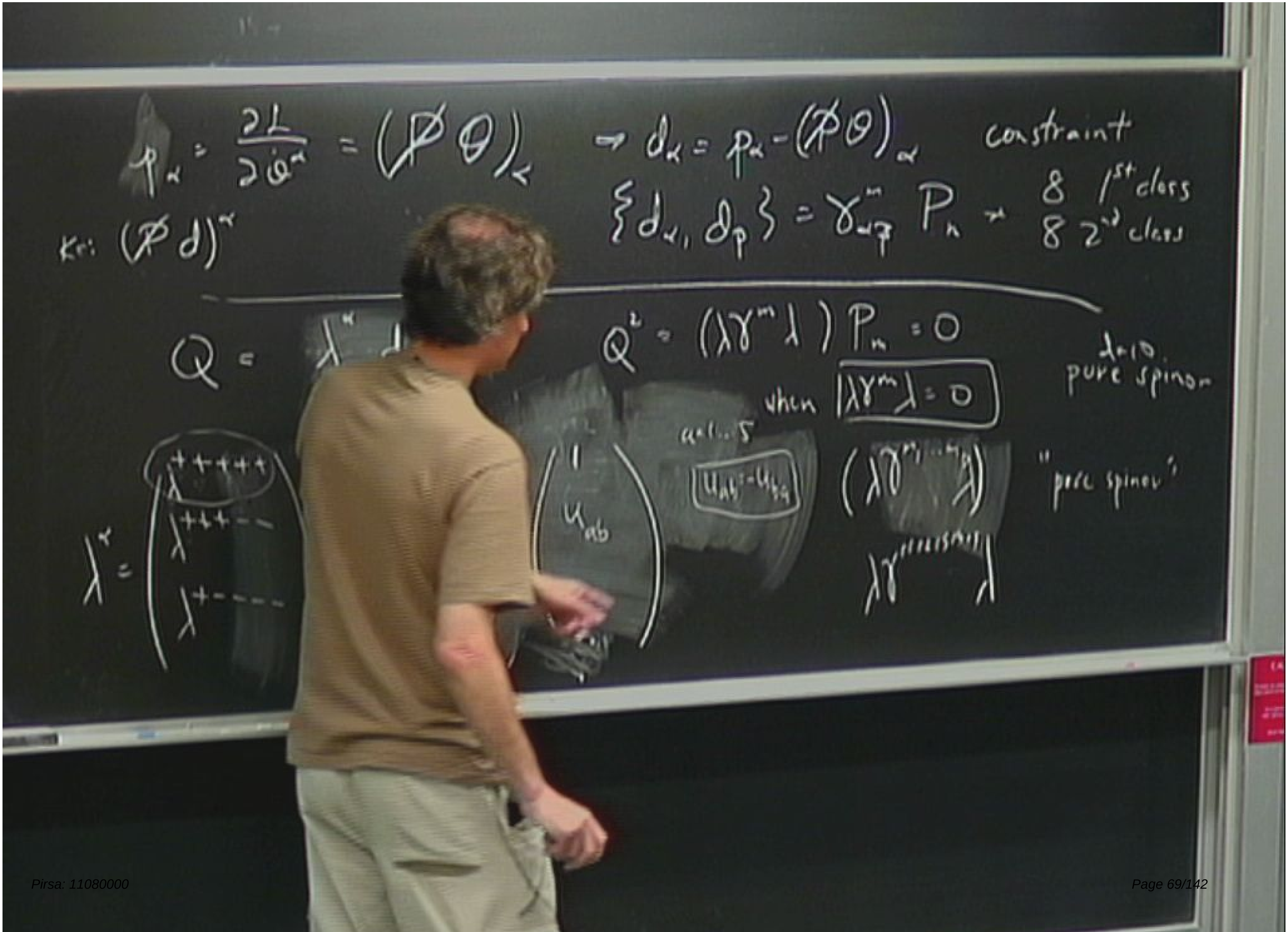
Siegel

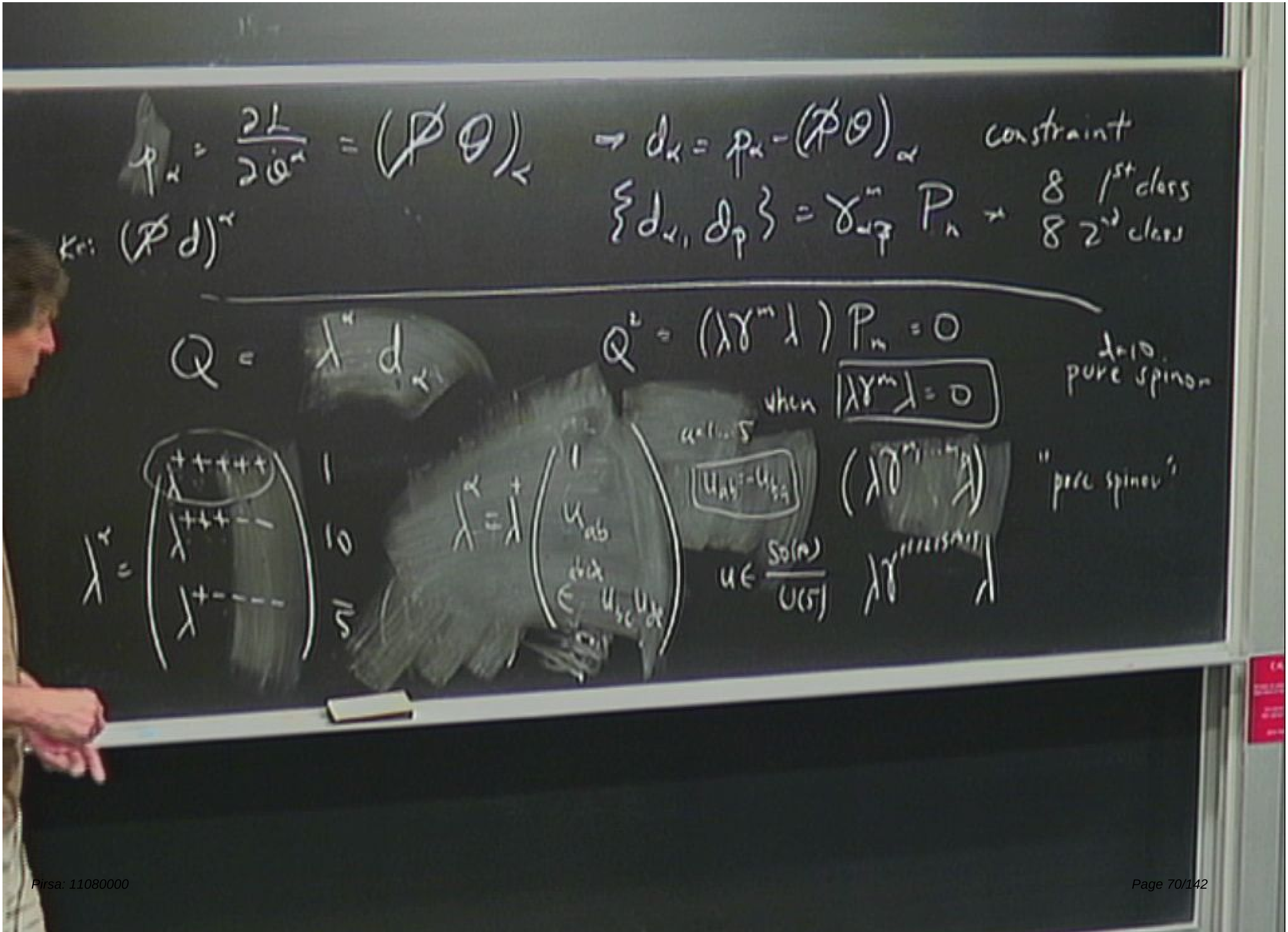
K-sym. $\delta\theta^\alpha = P^m(\gamma_m K)^\alpha$

$\delta x^m = -i\,\delta\theta\gamma^m\theta$, $\delta e = K_\alpha\dot{\theta}^\alpha$

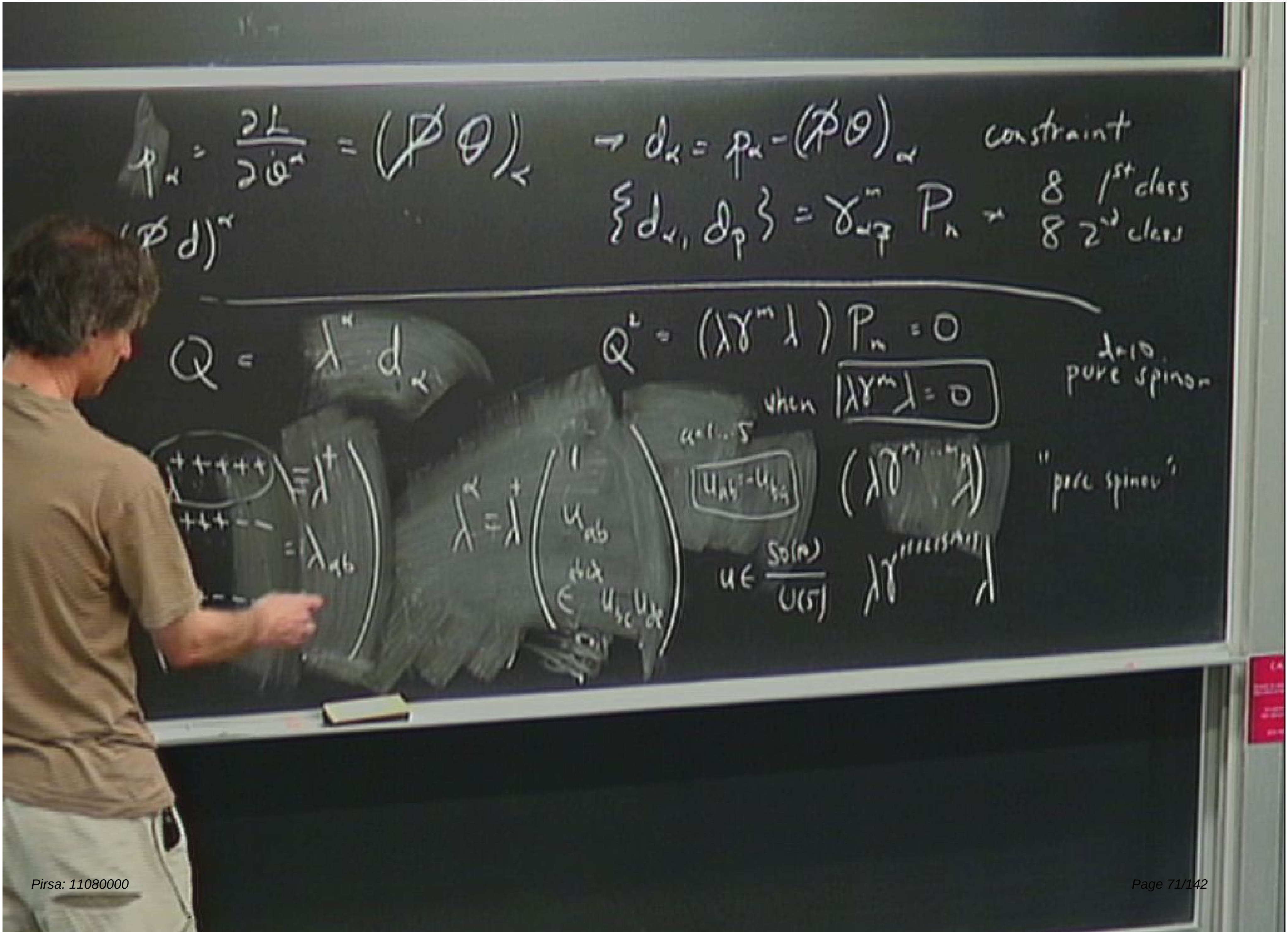
$\Rightarrow \gamma^{(m}_{\alpha\beta}\gamma^{n)\beta\gamma} = 2\eta^{mn}\delta_\alpha^\gamma$

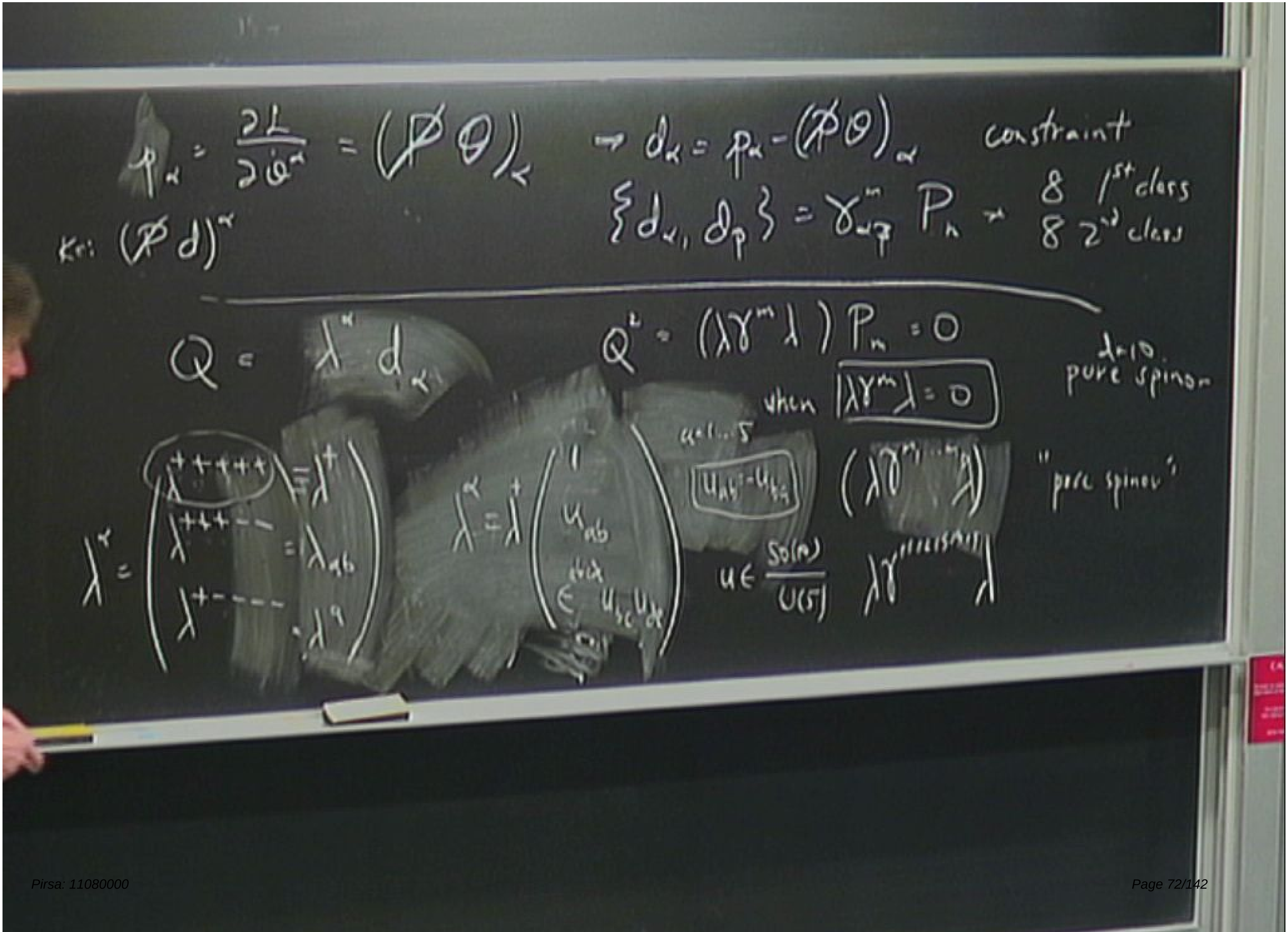

constraint
8 1st class
8 2nd class
when
pure spinor
"pure spinor"

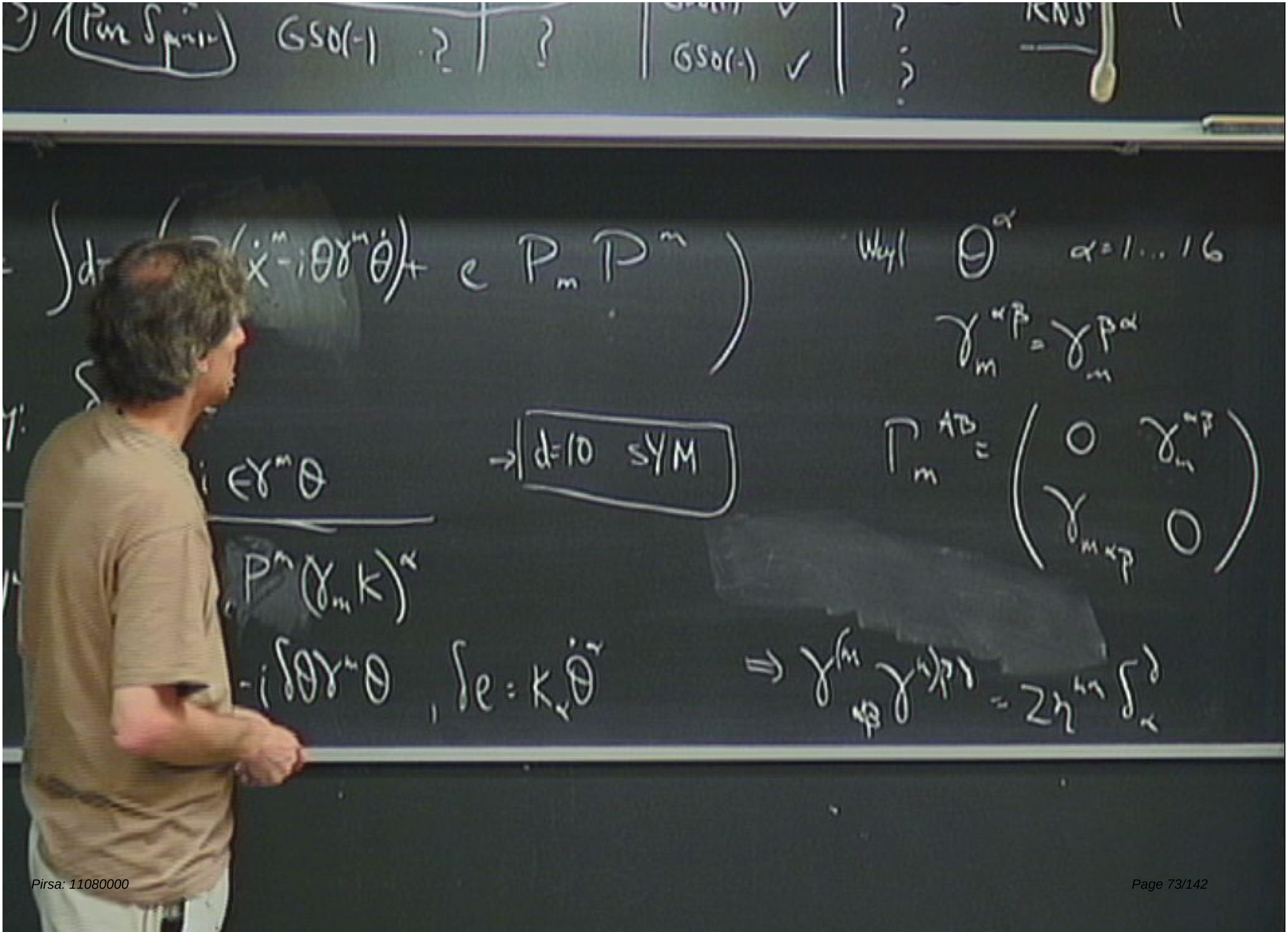

constraint
8 1st class
8 2nd class
pure spinor
when (λγ^m λ) = 0
"pure spinor"
Q^2 = (λγ^m λ) P_m = 0
d=10

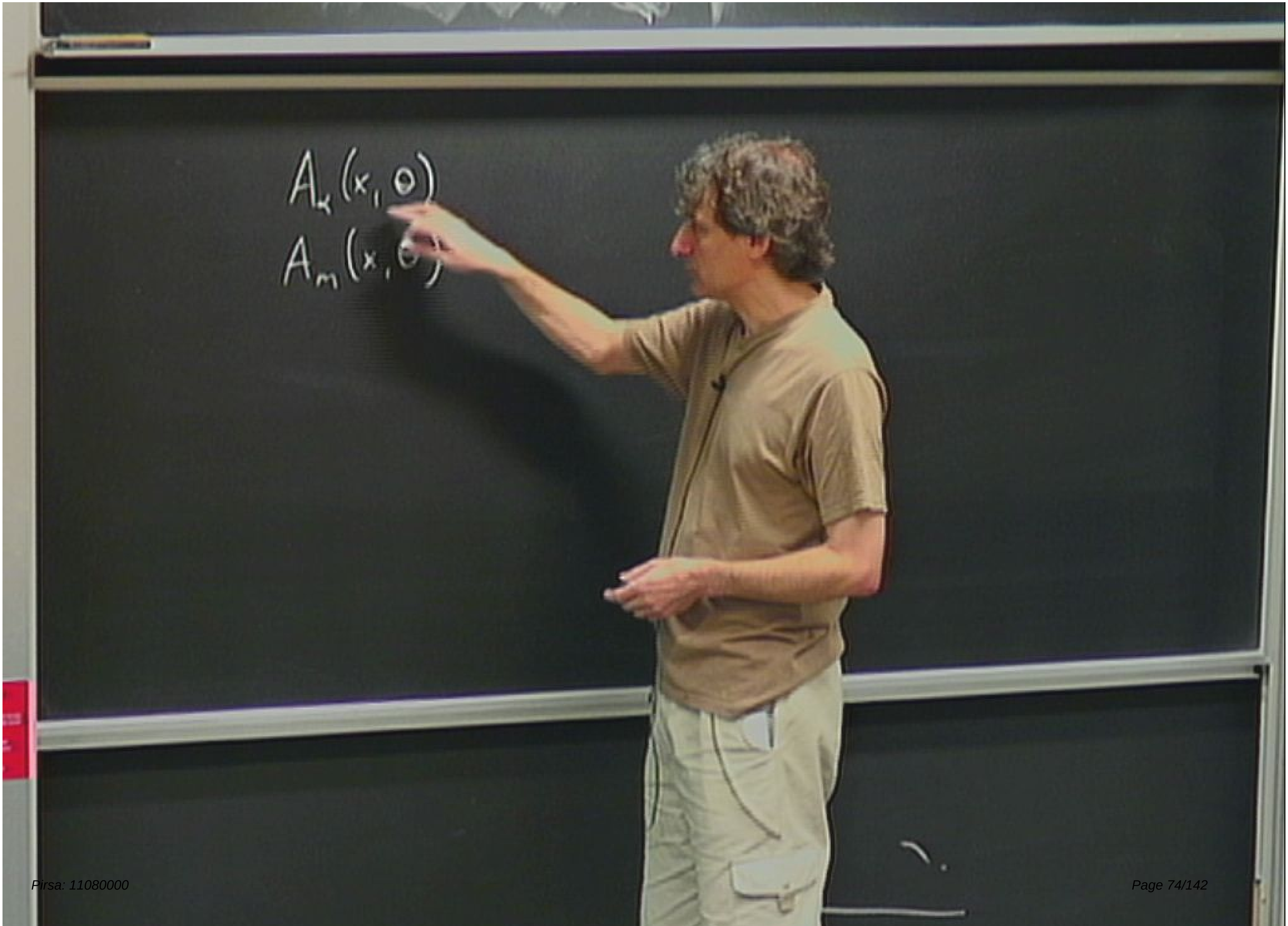


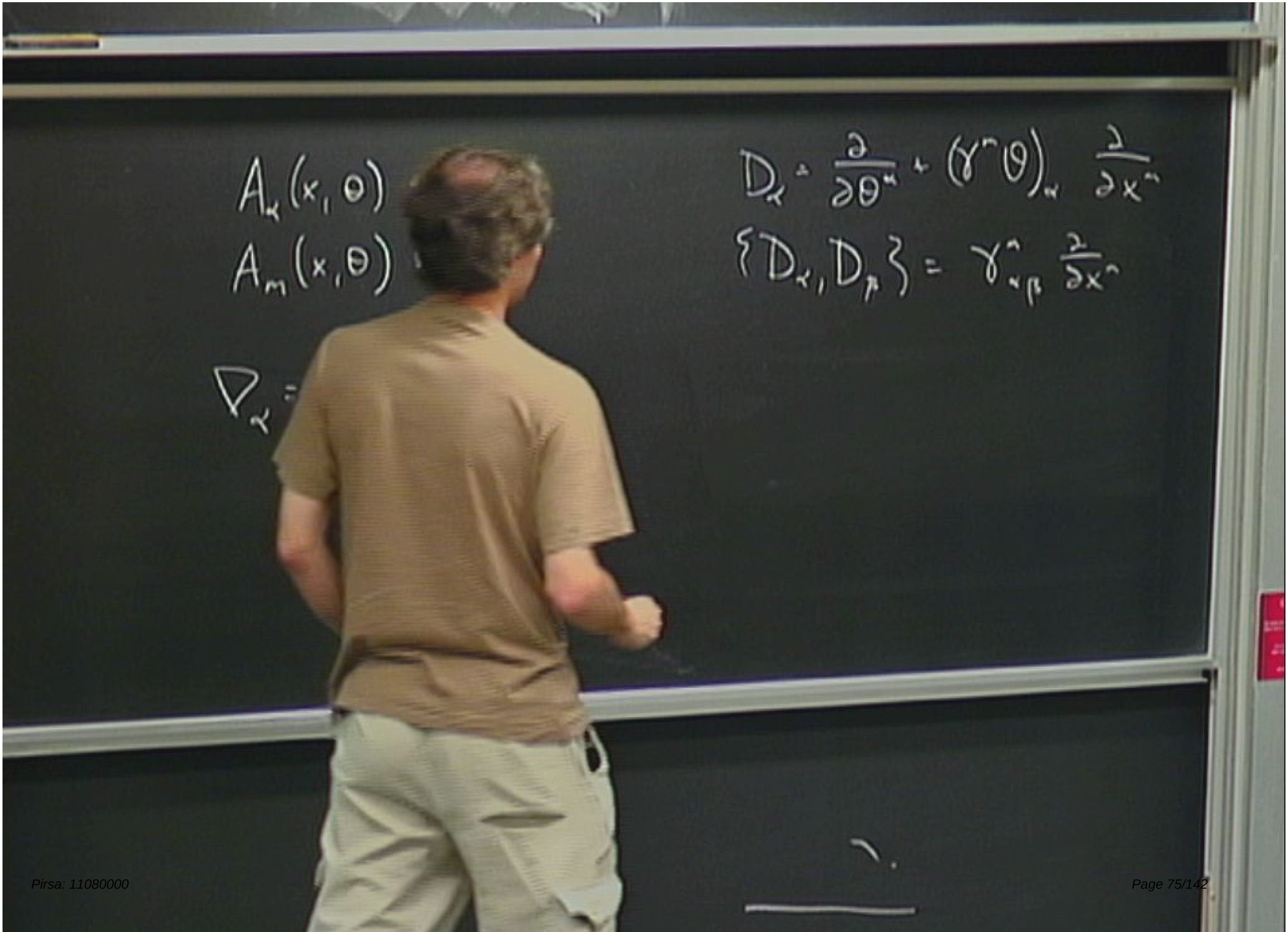
constraint
8 1st class
8 2nd class
d=10
pure spinor
when
"pure spinor"

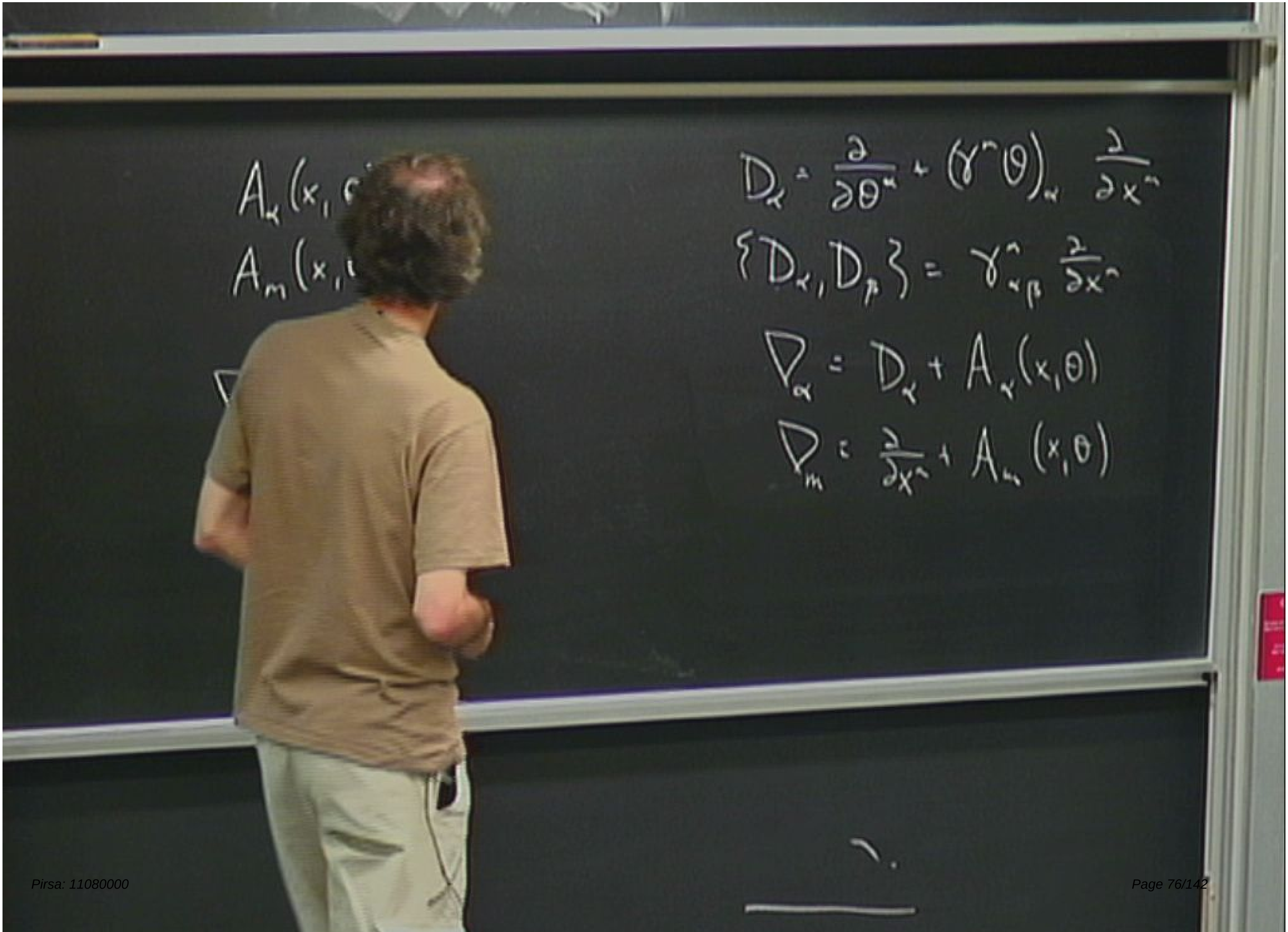

constraint
8 1st class
8 2nd class
when
pure spinor
"pure spinor"


constraint
1st class
2nd class
pure spinor
when
"pure spinor"


GSO(-1)
GSO(-1)
Weyl
α=1...16
d=10 SYM


$A_k(x, \theta)$
$A_m(x, \theta)$


$A_\alpha(x,\theta)$
$A_m(x,\theta)$
$\nabla_\alpha =$
$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\gamma^m\theta)_\alpha \frac{\partial}{\partial x^m}$
$\{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta} \frac{\partial}{\partial x^m}$


$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\gamma^m\theta)_\alpha \frac{\partial}{\partial x^m}$
$\{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta}\frac{\partial}{\partial x^m}$
$\nabla_\alpha = D_\alpha + A_\alpha(x,\theta)$
$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x,\theta)$

$$A_\alpha(x,\theta) =$$

$$A_m(x,\theta) =$$

$$\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta}\nabla_m$$

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\gamma^m\theta)_\alpha \frac{\partial}{\partial x^m}$$

$$\{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta}\frac{\partial}{\partial x^m}$$

$$\nabla_\alpha = D_\alpha + A_\alpha(x,\theta)$$

$$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x,\theta)$$

$$A_\alpha(x,\theta) =$$

$$A_m(x,\theta) =$$

$$\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta}\nabla_m$$

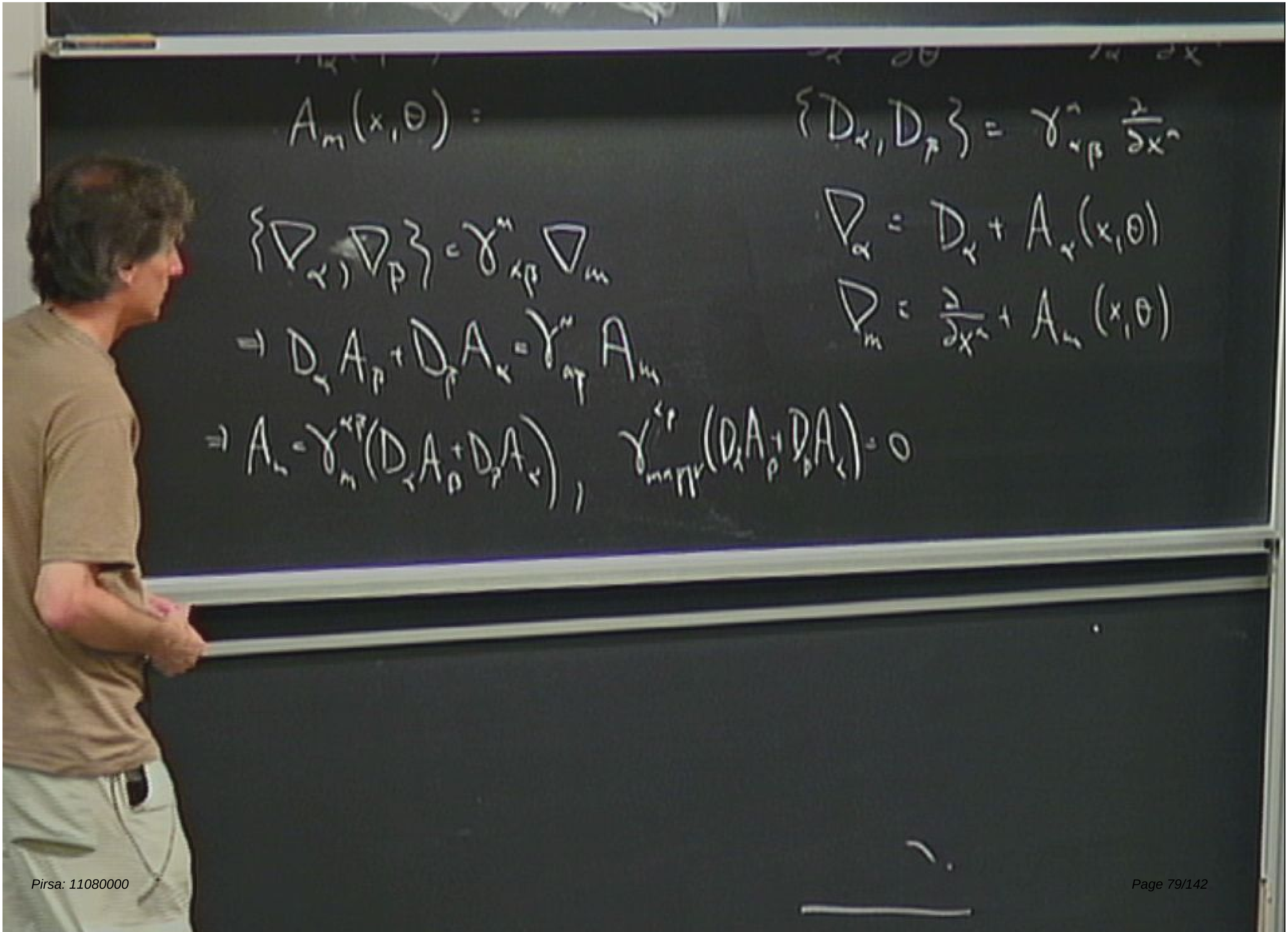
$$\Rightarrow D_\alpha A_\beta + D_\beta A_\alpha = \gamma^m_{\alpha\beta} A_m$$

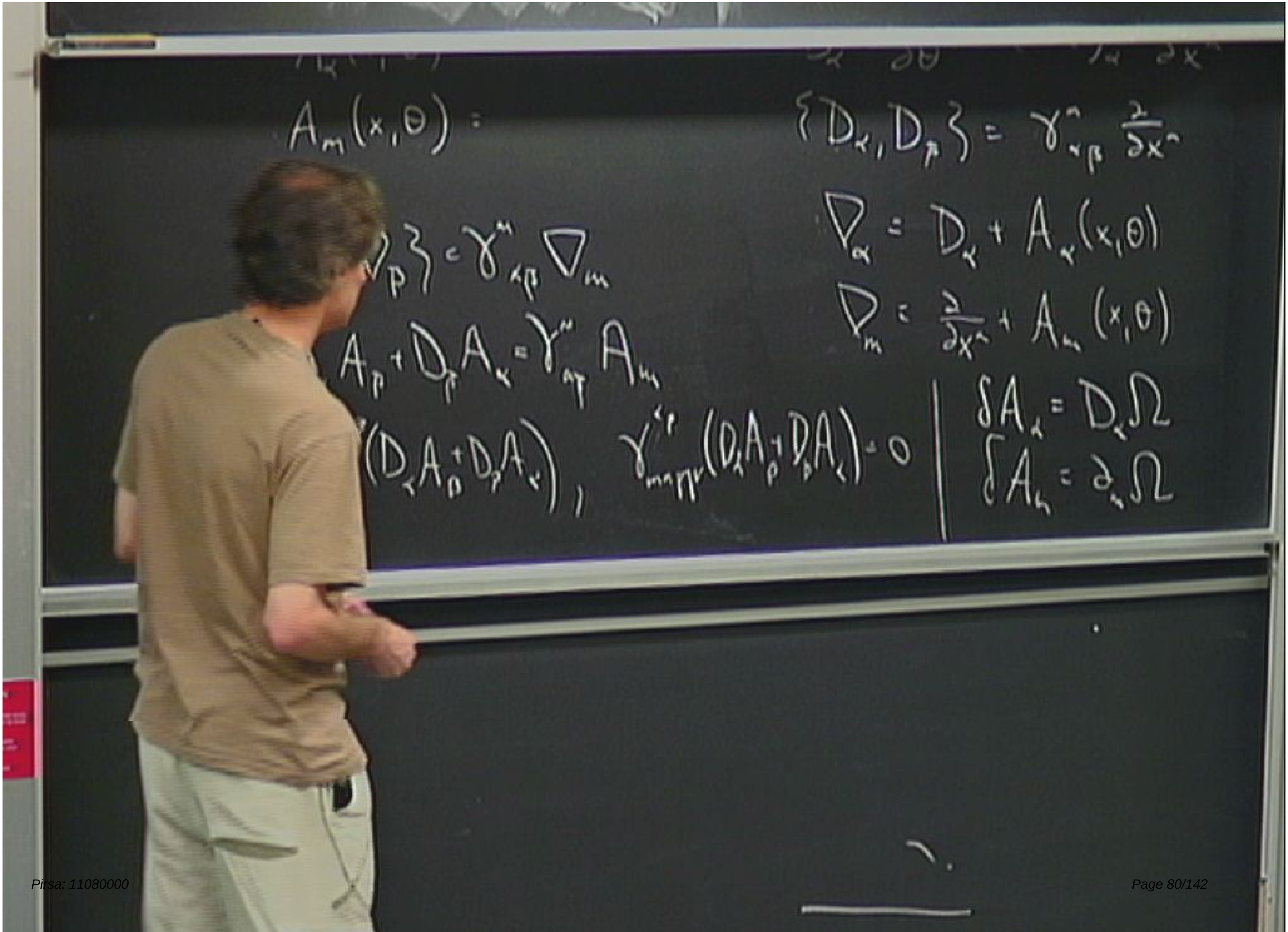
$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\gamma^m\theta)_\alpha \frac{\partial}{\partial x^m}$$

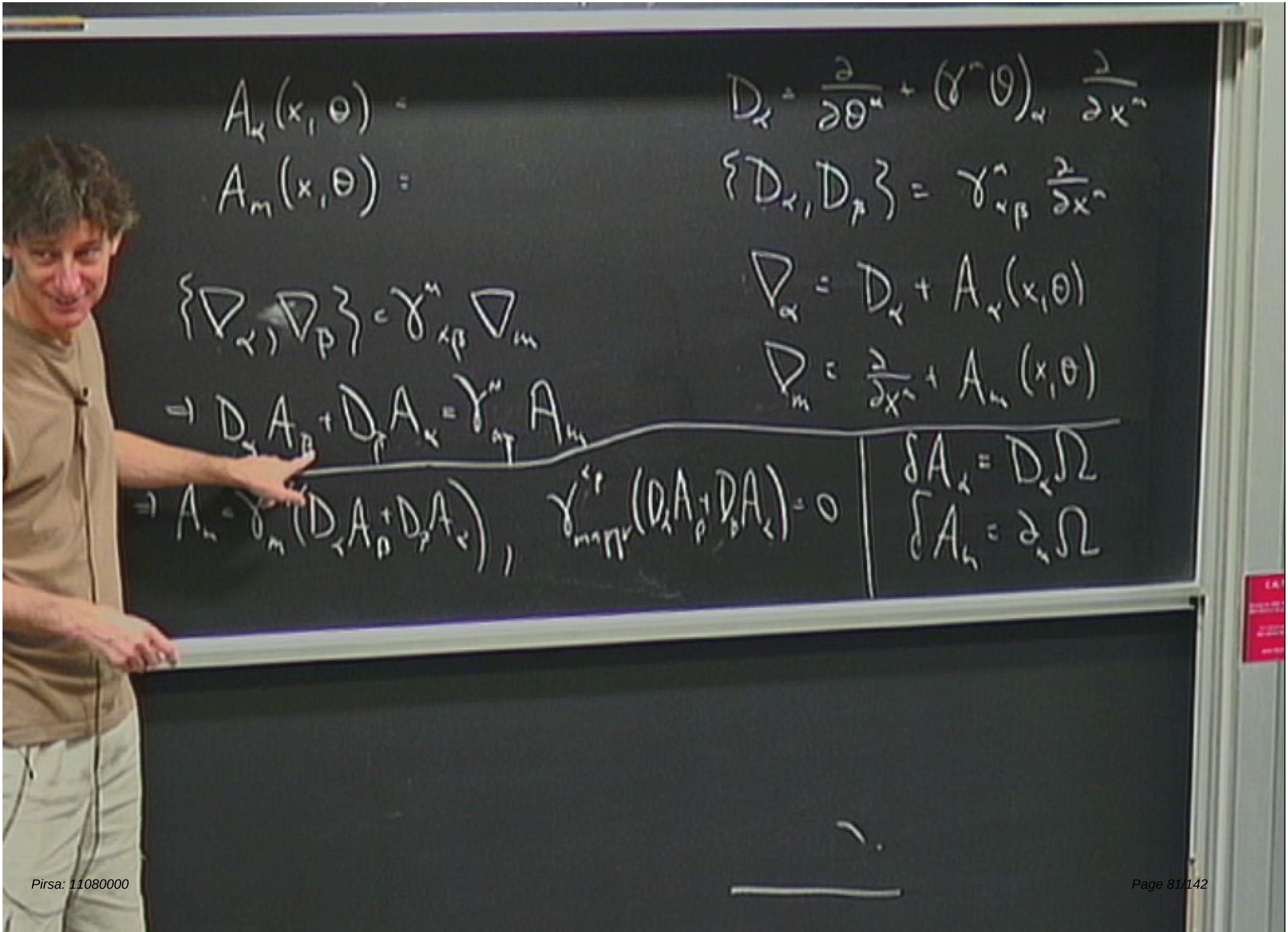
$$\{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta}\frac{\partial}{\partial x^m}$$

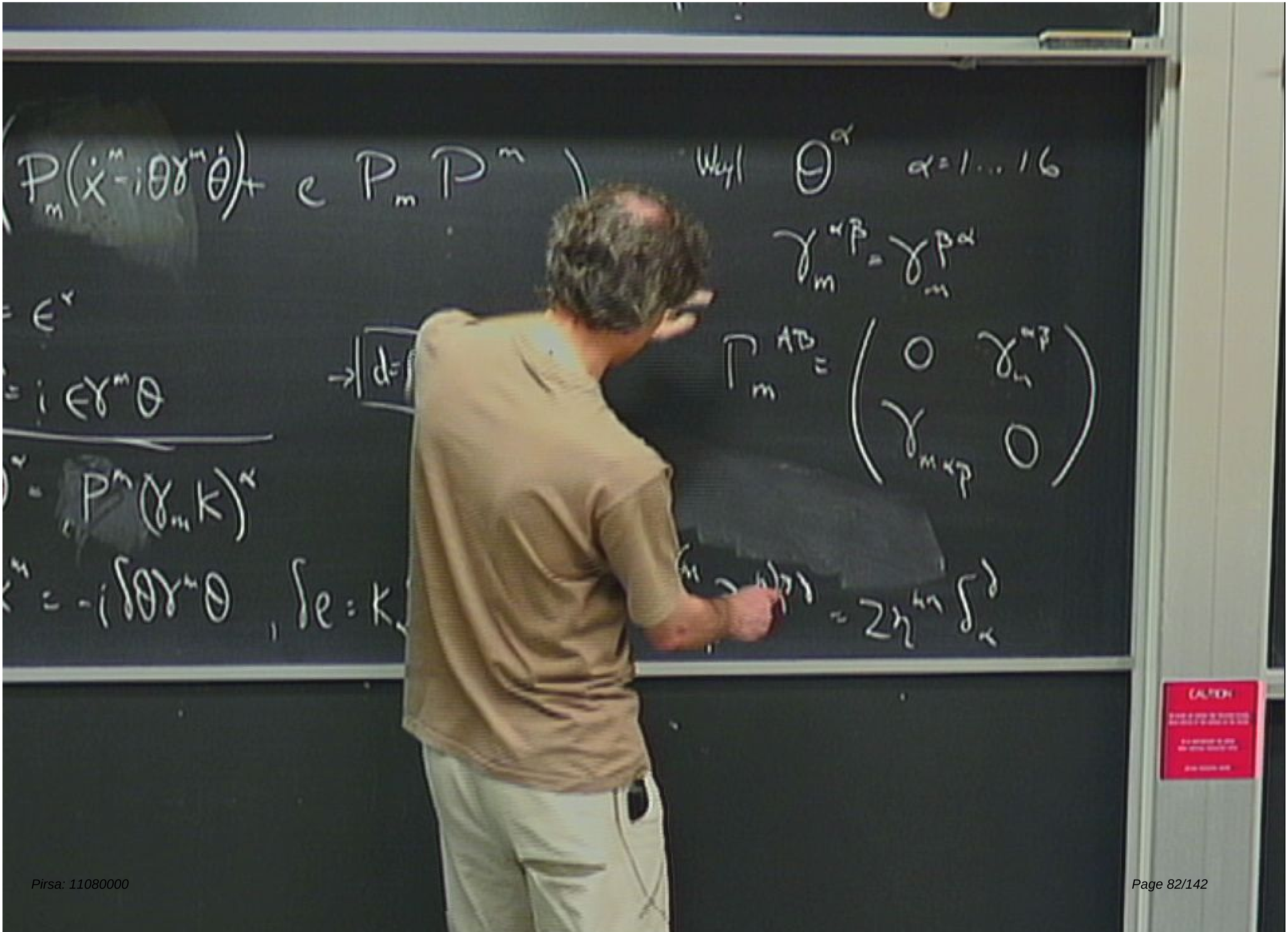
$$\nabla_\alpha = D_\alpha + A_\alpha(x,\theta)$$

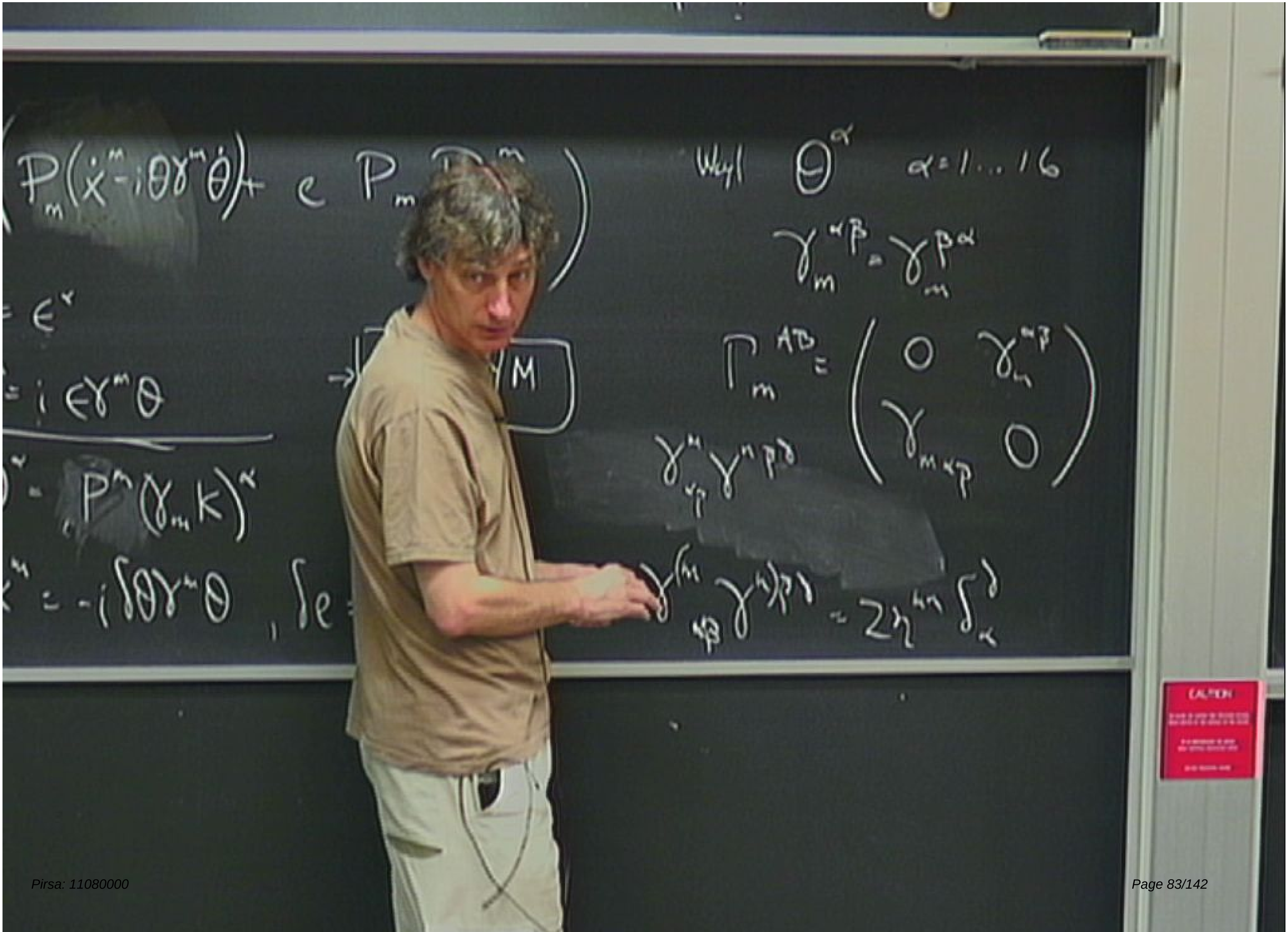
$$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x,\theta)$$

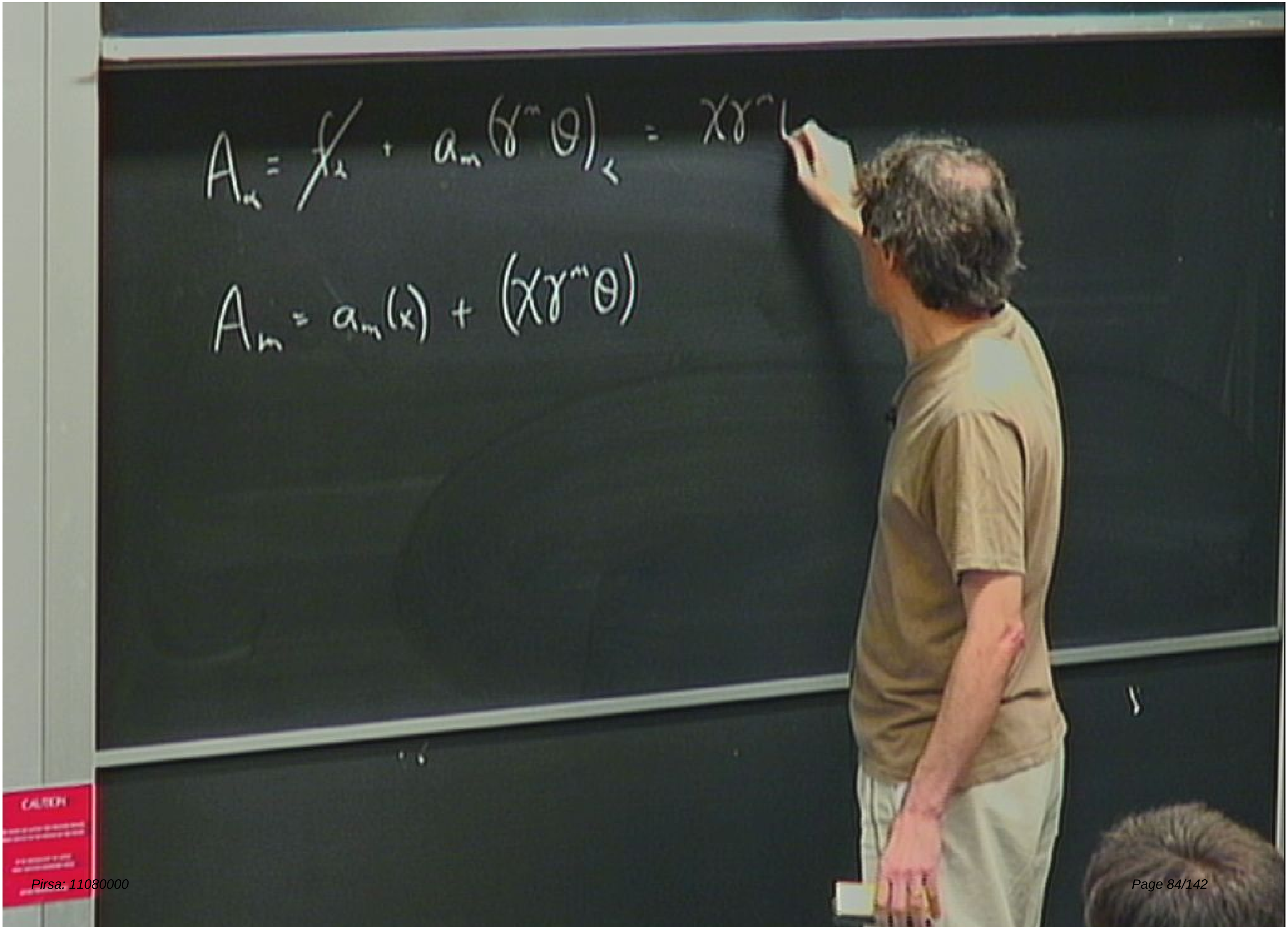

$A_m(x,\theta)$:
$\{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta} \frac{\partial}{\partial x^m}$
$\nabla_\alpha = D_\alpha + A_\alpha(x,\theta)$
$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x,\theta)$
$\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta} \nabla_m$
$\Rightarrow D_\alpha A_\beta + D_\beta A_\alpha = \gamma^m_{\alpha\beta} A_m$
$\Rightarrow A_m = \gamma_m^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha)$, $\gamma_{mnpqr}^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha) = 0$

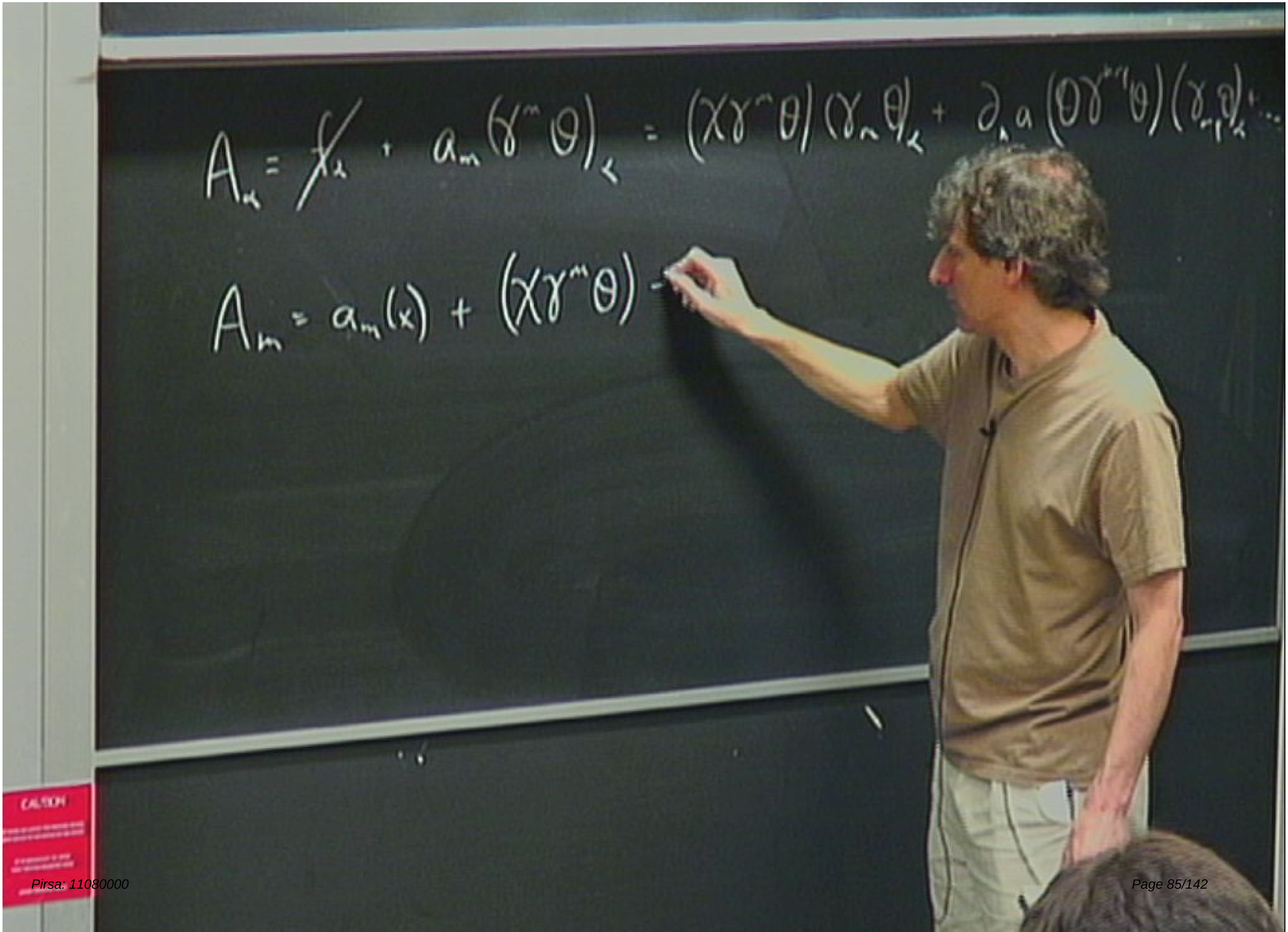

Am(x,θ) :
{Dα, Dβ} = γ^m αβ ∂/∂x^m
∇α = Dα + Aα(x,θ)
∇m = ∂/∂x^m + Am(x,θ)
δAα = DαΩ
δAm = ∂mΩ


$A_\alpha(x,\theta) =$
$A_m(x,\theta) =$
$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\gamma^m\theta)_\alpha \frac{\partial}{\partial x^m}$
$\{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta}\frac{\partial}{\partial x^m}$
$\nabla_\alpha = D_\alpha + A_\alpha(x,\theta)$
$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x,\theta)$
$\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta}\nabla_m$
$\Rightarrow D_\alpha A_\beta + D_\beta A_\alpha = \gamma^m_{\alpha\beta} A_m$
$\Rightarrow A_m = \gamma_m(D_\alpha A_\beta + D_\beta A_\alpha)$, $\gamma^{\alpha\beta}_{mnpqr}(D_\alpha A_\beta + D_\beta A_\alpha) = 0$
$\delta A_\alpha = D_\alpha\Omega$
$\delta A_m = \partial_m\Omega$

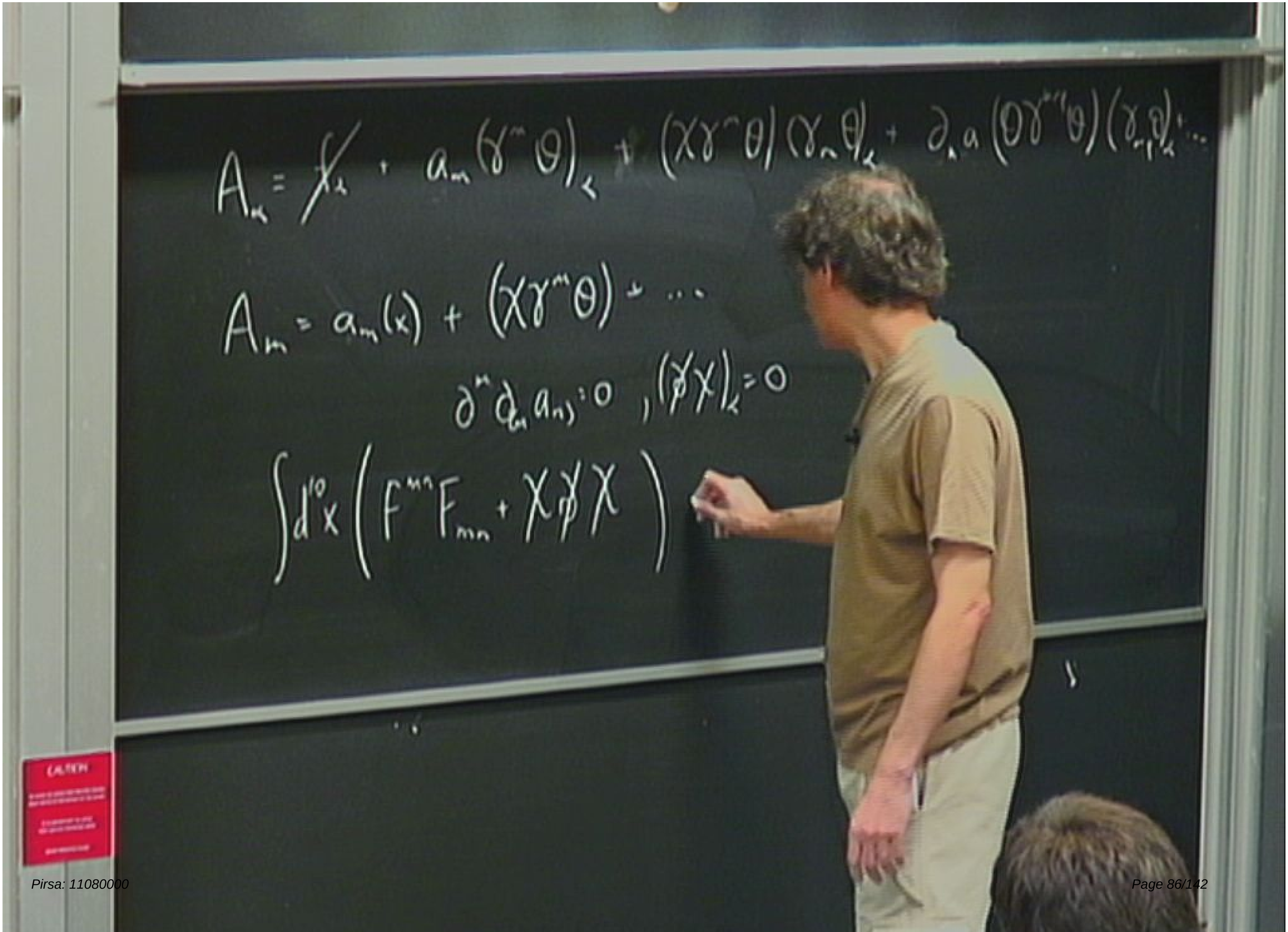

Weyl
α = 1 ... 16
CAUTION


Weyl
α=1...16
YM
CAUTION


$A_\alpha = f_\alpha + a_m(\gamma^m\theta)_\alpha = \chi\gamma^m$
$A_m = a_m(x) + (\chi\gamma^m\theta)$
CAUTION



Aα = fα + am (γ^m θ)α = (χγ^m θ)(γm θ)α + ∂n a (θγ^mn θ)(γm θ)α + ...
Am = am(x) + (χγ^m θ) -

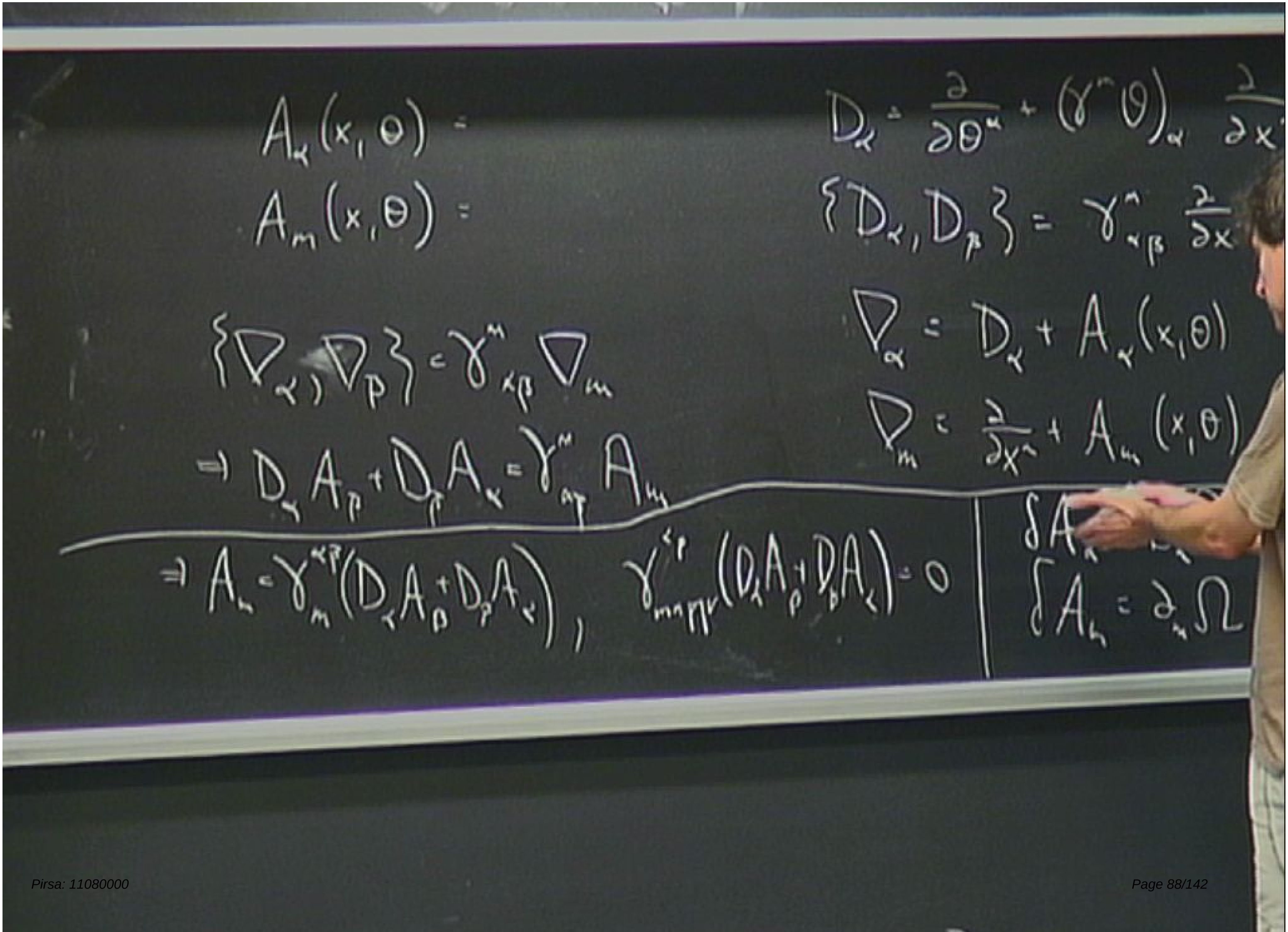

$A_m = a_m(x) + (\chi\gamma^m\theta) + \ldots$
$\partial^m \partial_{[m} a_{n]} = 0 \;,\; (\not\partial\chi)_\alpha = 0$
$\int d^{10}x \left(F^{mn}F_{mn} + \chi\not\partial\chi \right)$
CAUTION

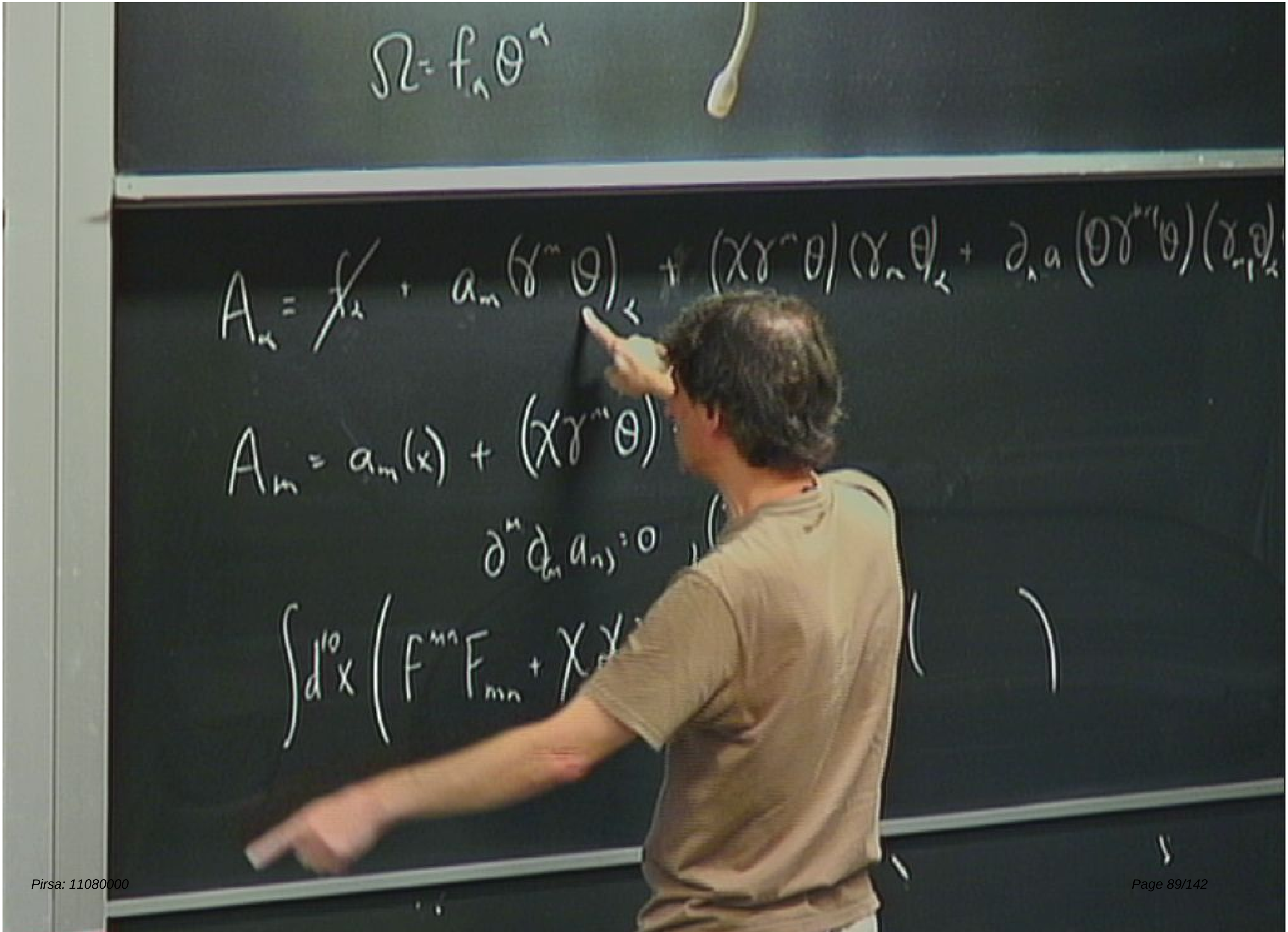
$$A_\alpha = \not{\chi}_\alpha + a_m(\gamma^m\theta)_\alpha + (\chi\gamma^m\theta)(\gamma_m\theta)_\alpha + \partial_n a_m(\theta\gamma^{mnp}\theta)(\gamma_p\theta)_\alpha + \dots$$

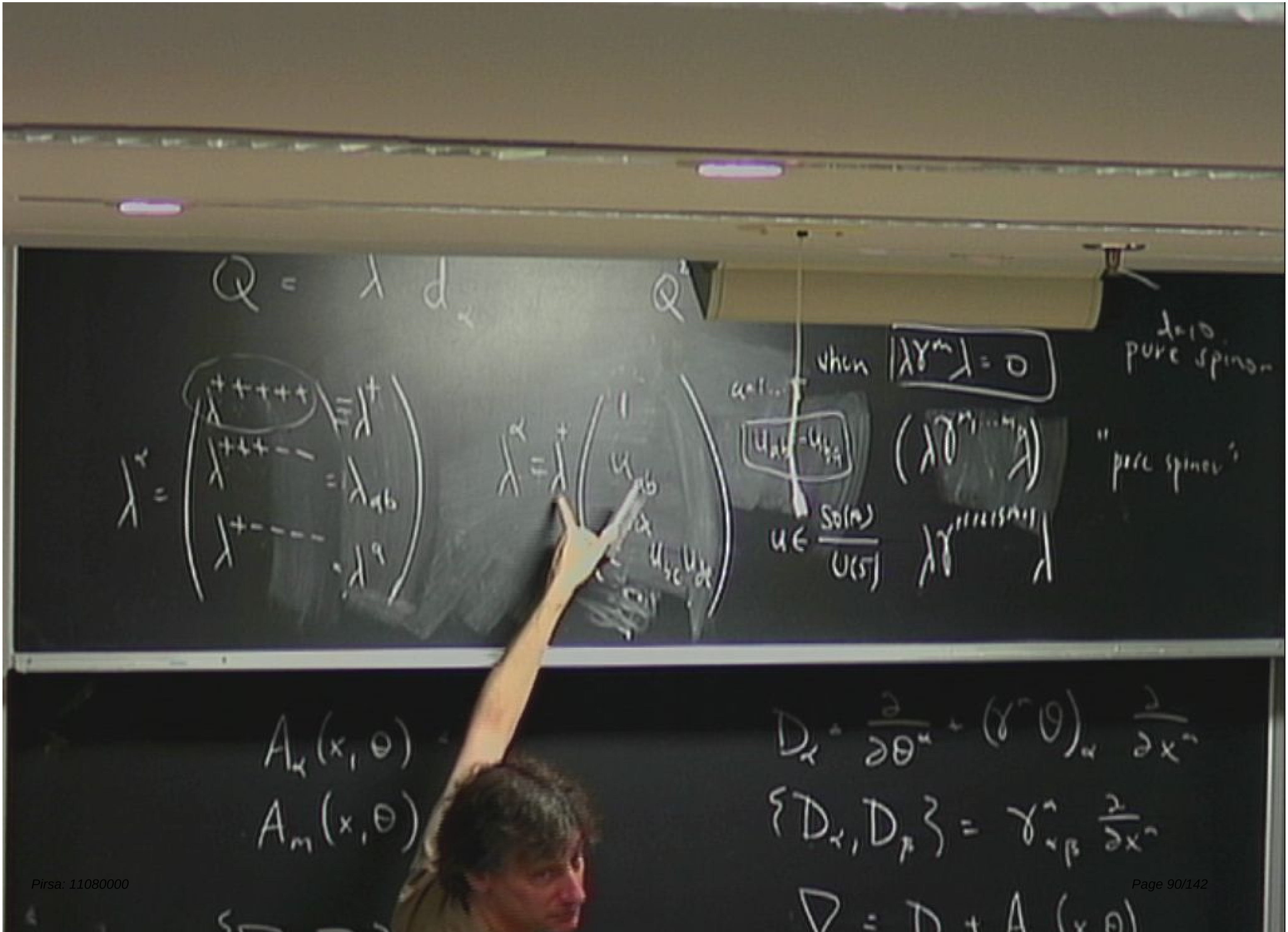
$$A_m = a_m(x) + (\chi\gamma^m\theta) + \dots$$

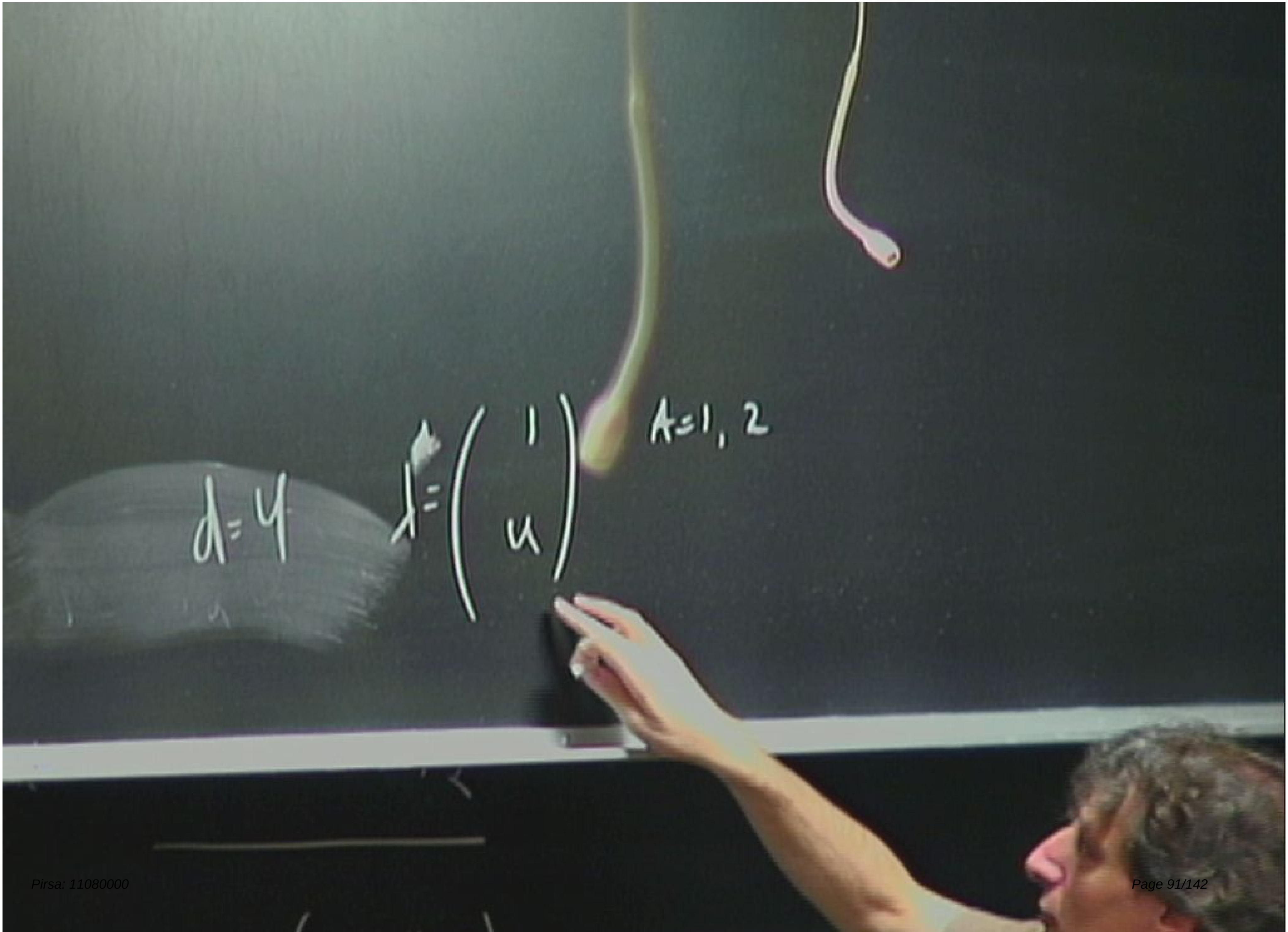
$$\partial^m \partial_{[m} a_{n]} = 0 \quad , \quad (\not{\partial}\chi)_\alpha = 0$$

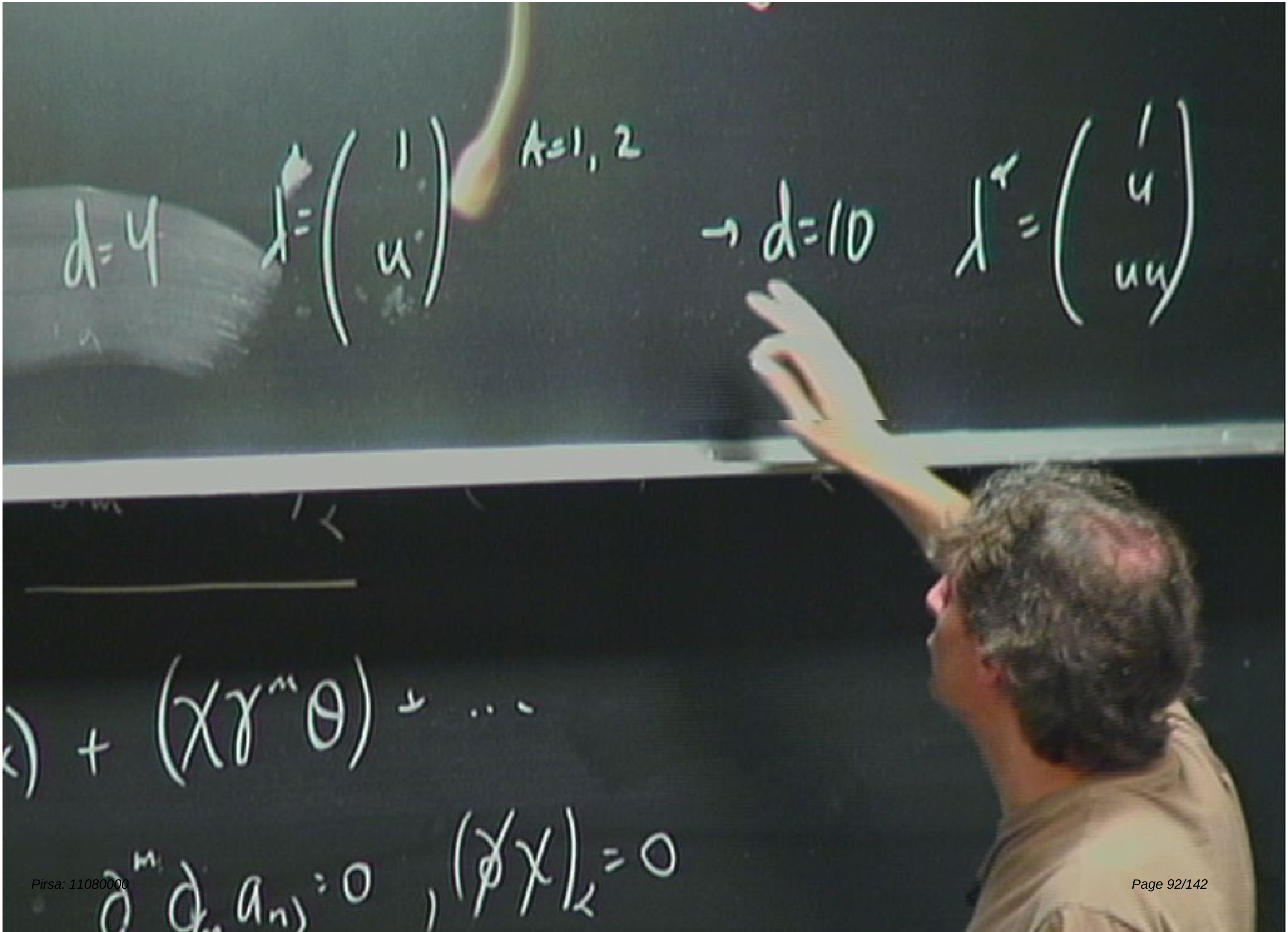
$$\int d^{10}x \left(F^{mn}F_{mn} + \chi\not{\partial}\chi \right) \stackrel{?}{=} \int d^{10}x \int d^{16}\theta \left(\qquad \right)$$

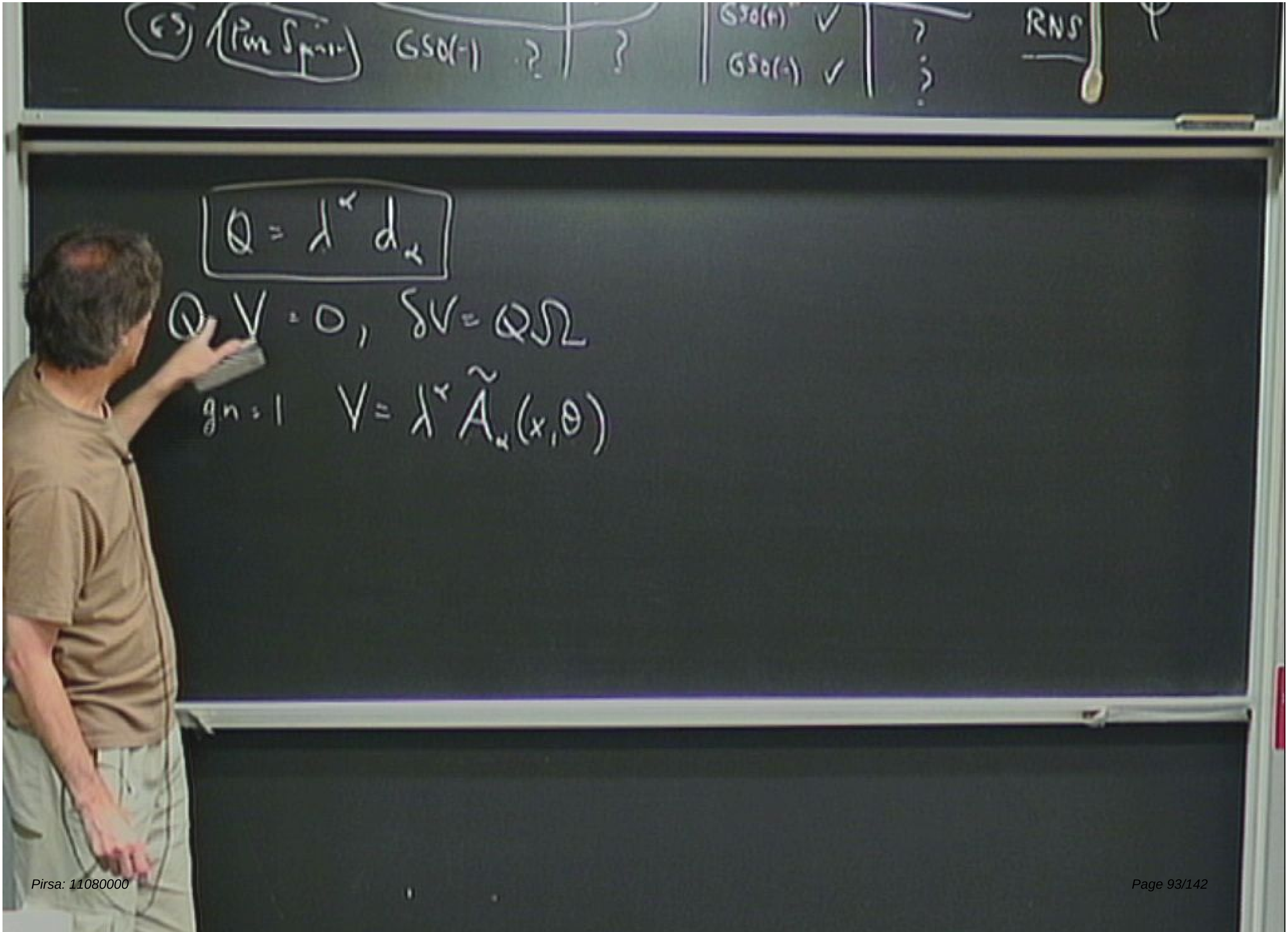

$A_\alpha(x,\theta) =$
$A_m(x,\theta) =$
$\{\nabla_\alpha, \nabla_\beta\} = \gamma^m_{\alpha\beta}\nabla_m$
$\Rightarrow D_\alpha A_\beta + D_\beta A_\alpha = \gamma^m_{\alpha\beta} A_m$
$\Rightarrow A_m = \gamma_m^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha)$, $\gamma^{\alpha\beta}_{mnpqr}(D_\alpha A_\beta + D_\beta A_\alpha) = 0$
$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\gamma^m\theta)_\alpha \frac{\partial}{\partial x^m}$
$\{D_\alpha, D_\beta\} = \gamma^m_{\alpha\beta}\frac{\partial}{\partial x^m}$
$\nabla_\alpha = D_\alpha + A_\alpha(x,\theta)$
$\nabla_m = \frac{\partial}{\partial x^m} + A_m(x,\theta)$
δA_α
$\delta A_m = \partial_m \Omega$

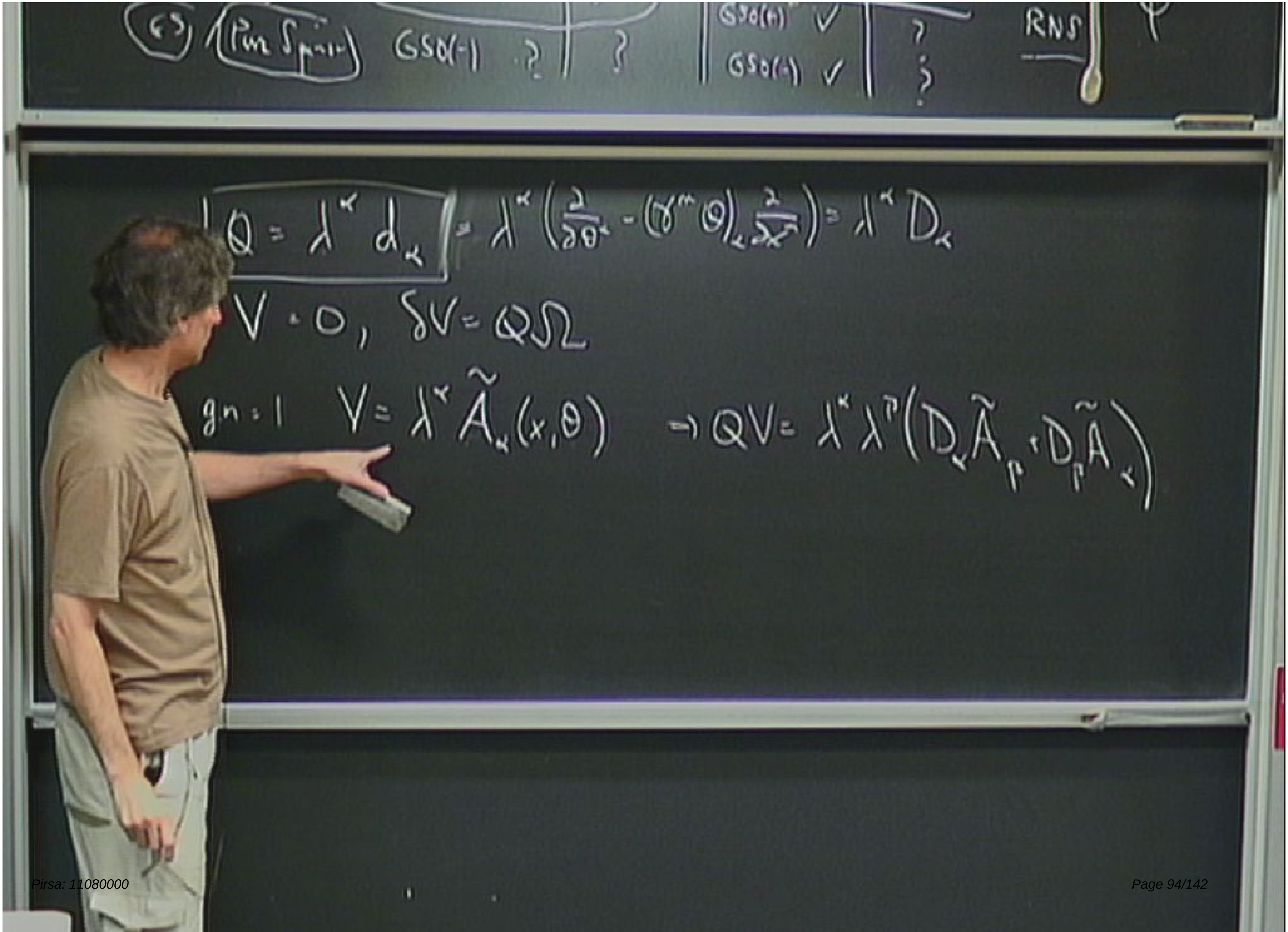

Ω = f_α θ^α
A_α = f_α + a_m(γ^m θ)_α + (χγ^m θ)(γ_m θ)_α + ∂_n a_m(θγ^{mnp}θ)(γ_p θ)_α
A_m = a_m(x) + (χγ_m θ)
∂^m ∂_{[m} a_{n]} = 0
∫d^10 x (F^{mn} F_{mn} + χ

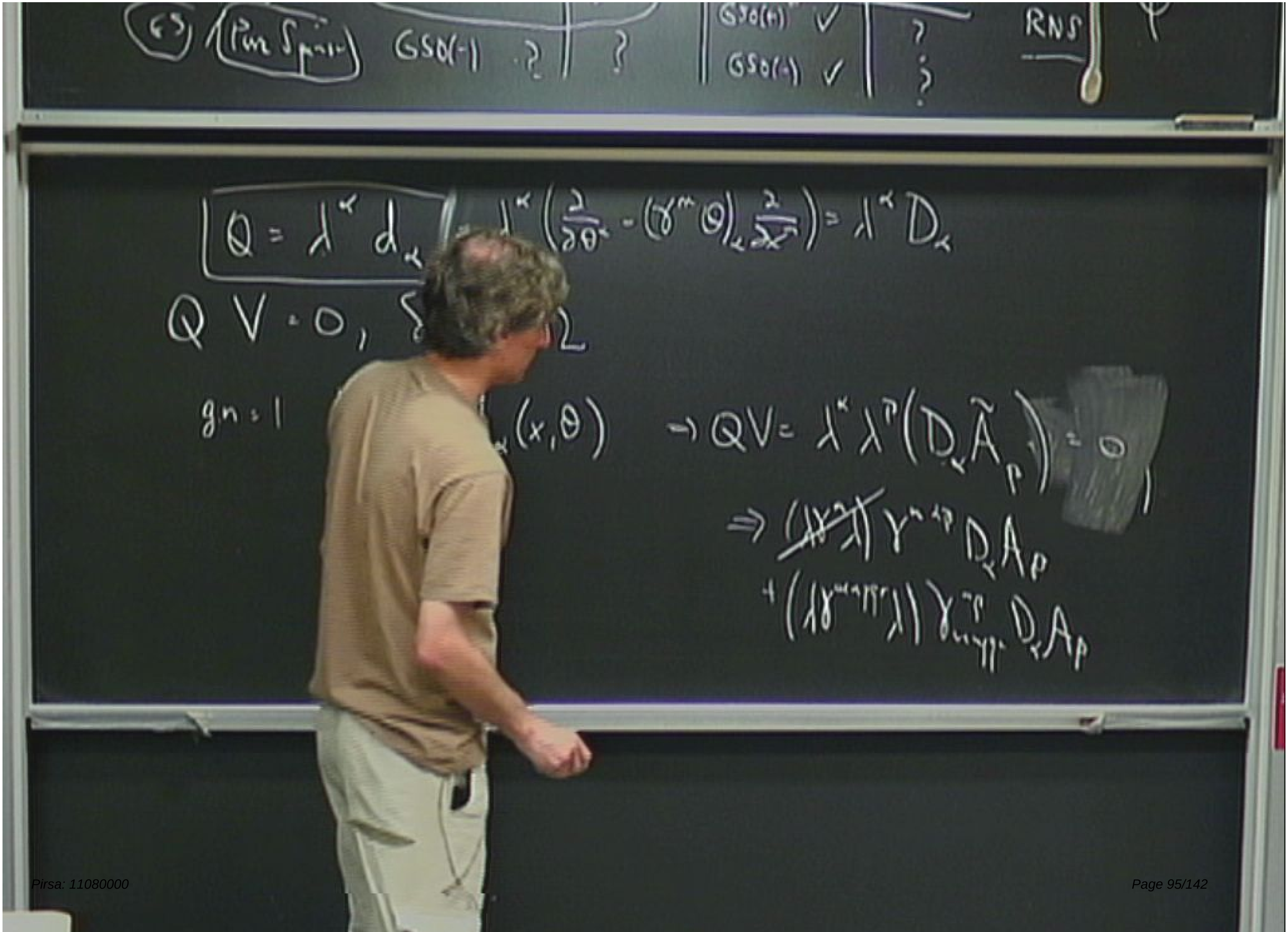

Q = λ d
when λγ^m λ = 0
pure spinor
"pure spinor"
u ∈ SO(10)/U(5)
A_α(x, θ)
A_m(x, θ)
D_α = ∂/∂θ^α + (γ^m θ)_α ∂/∂x^m
{D_α, D_β} = γ^m_αβ ∂/∂x^m


d=4
A=1, 2


d=4
A=1, 2
→ d=10


RNS
GSO(+) ✓
GSO(-) ✓
GSO(-) ?
Q = λ^α d_α
Q V = 0, δV = QΩ
gh = 1 V = λ^α Ã_α(x, θ)


GS Pure Spinor GSO(-) ? ?
GSO(+) ✓ ?
GSO(-) ✓ ?
RNS
Q = λ^α d_α = λ^α (∂/∂θ^α − (γ^m θ)_α ∂/∂x^m) = λ^α D_α
V ≈ 0, δV = QΩ
gh = 1 V = λ^α Ã_α(x,θ) → QV = λ^α λ^β (D_α Ã_β + D_β Ã_α)


GSO(-)
GSO(+)
GSO(-)
RNS
Q = λ^α d_α
= λ^α D_α
Q V = 0,
gh = 1

(5) (Pure Spinor) GSO(-) ? | ? | GSO(+) ✓ | ? GSO(-) ✓ | ? RNS

$$\boxed{Q = \lambda^\alpha d_\alpha} = \lambda^\alpha\left(\frac{\partial}{\partial\theta^\alpha} - (\gamma^m\theta)_\alpha \frac{\partial}{\partial x^m}\right) = \lambda^\alpha D_\alpha$$

$$Q\,V = 0, \quad \delta V = Q\Omega$$

$$gh = 1 \quad V = \lambda^\alpha \tilde{A}_\alpha(x,\theta) \quad \to QV = \lambda^\alpha\lambda^\beta\left(D_\alpha \tilde{A}_\beta\right) = 0$$

$$\Rightarrow (\lambda\gamma^m\lambda)\gamma_m^{\alpha\beta} D_\alpha A_\beta + \left((\lambda\gamma^{mnpqr}\lambda)\right)\gamma_{mnpqr}^{\alpha\beta} D_\alpha A_\beta = 0$$

(GS) / (Pure Spinor) GSO(-) ? | ? | GSO(+) ✓ | ? RNS
GSO(-) ✓ | ?

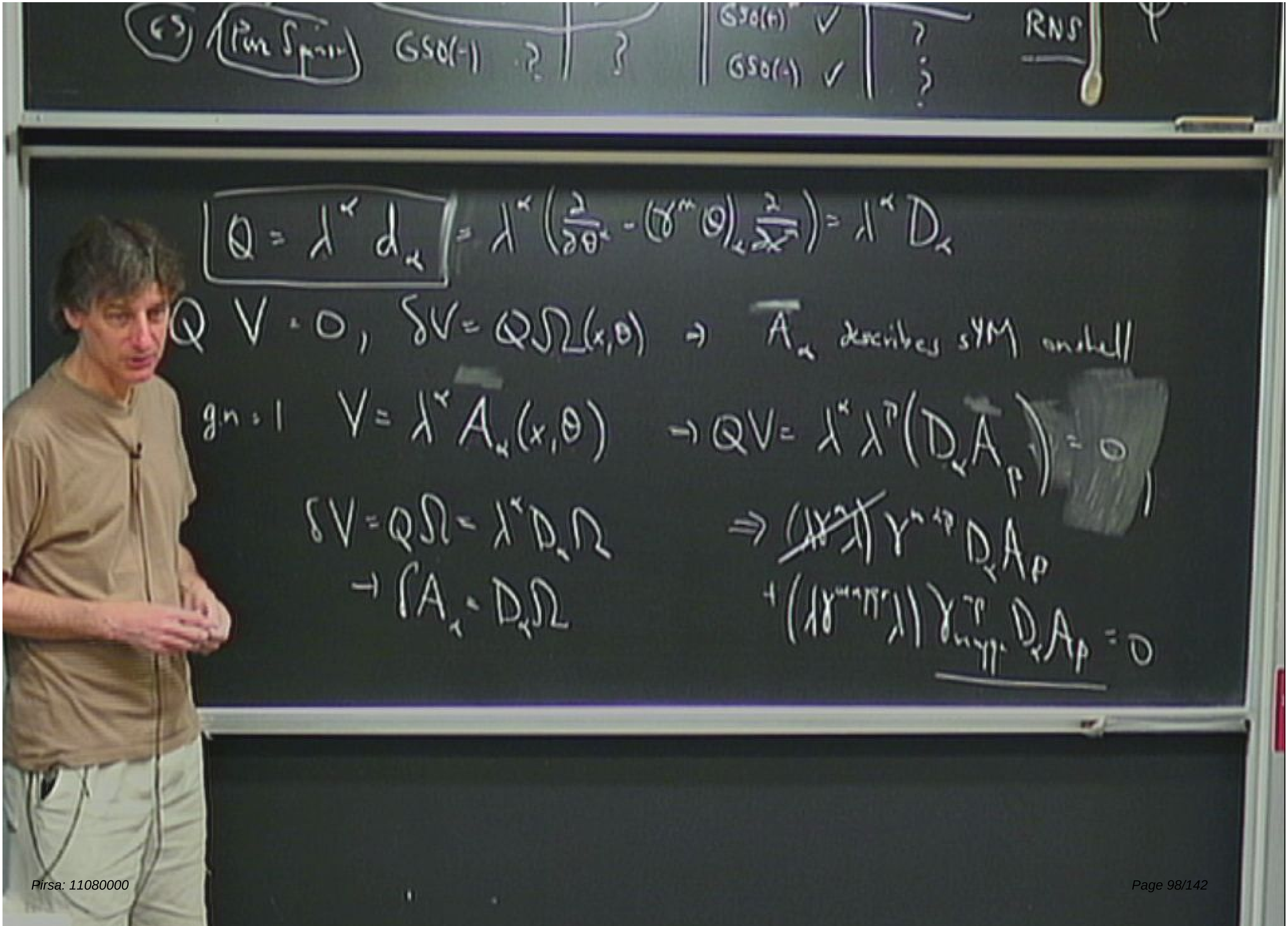
$$\boxed{Q = \lambda^\alpha d_\alpha} = \lambda^\alpha\left(\frac{\partial}{\partial\theta^\alpha} - (\gamma^m\theta)_\alpha \frac{\partial}{\partial x^m}\right) = \lambda^\alpha D_\alpha$$

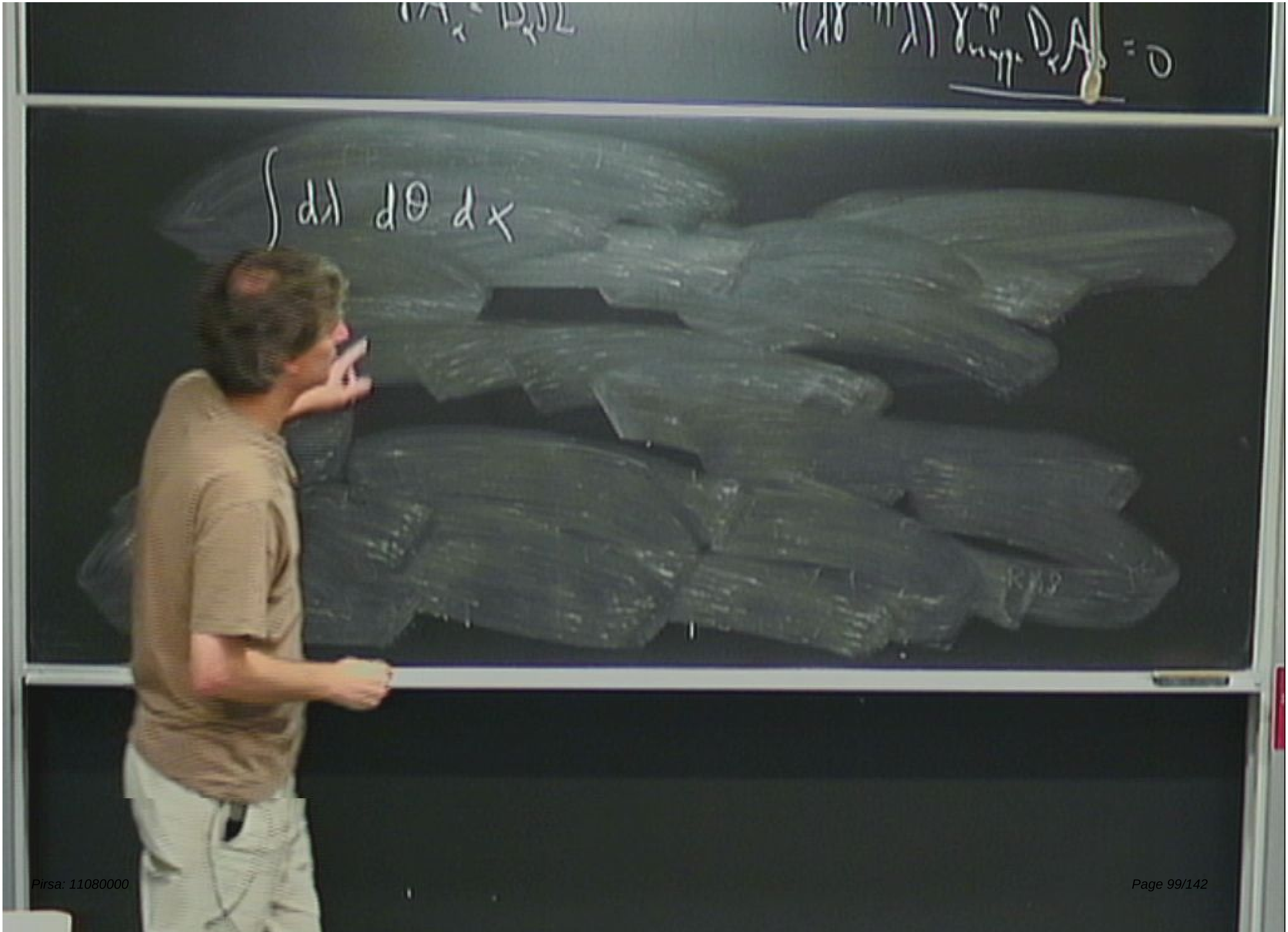
$$QV = 0, \quad \delta V = Q\Omega(x,\theta)$$

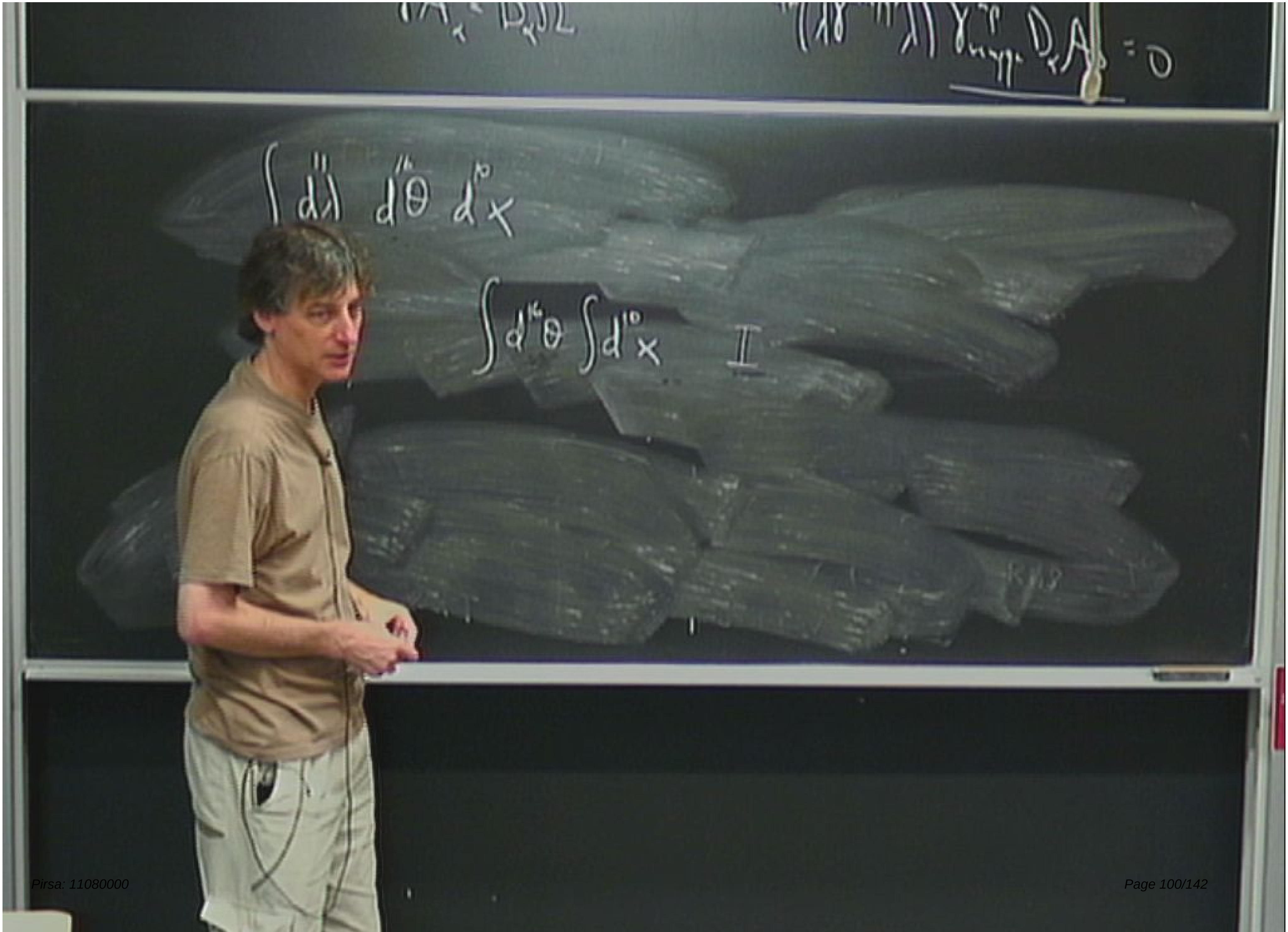
$$gh = 1 \quad V = \lambda^\alpha \tilde{A}_\alpha(x,\theta) \quad \rightarrow QV = \lambda^\alpha\lambda^\beta(D_\alpha \tilde{A}_\beta) = 0$$

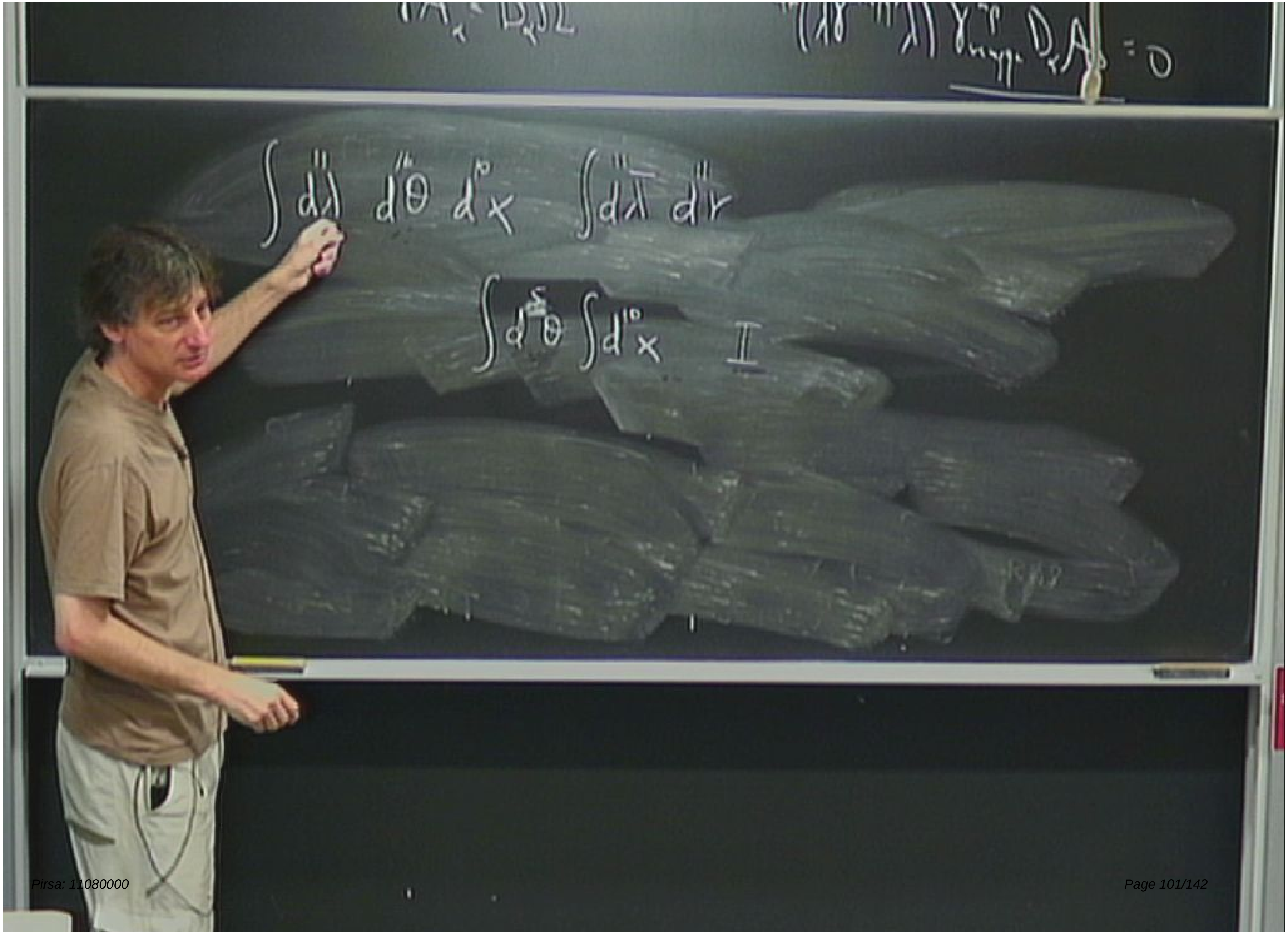
$$\delta V = Q\Omega = \lambda^\alpha D_\alpha \Omega \qquad \Rightarrow (\lambda\gamma^m\lambda)\gamma_m^{\alpha\beta} D_\alpha A_\beta$$

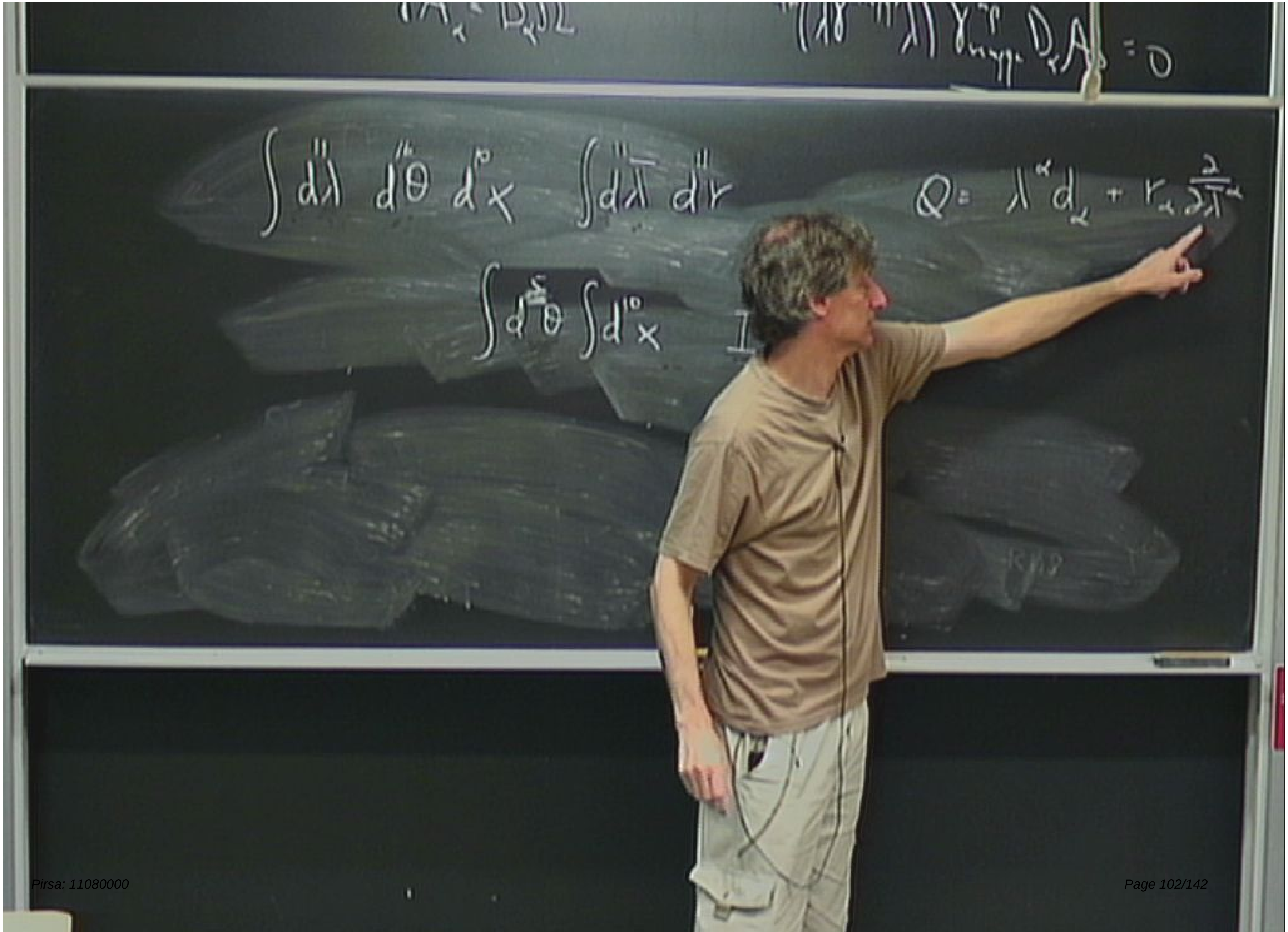
$$\rightarrow \delta A_\alpha = D_\alpha \Omega \qquad + (\lambda\gamma^{mnpqr}\lambda)\gamma_{mnpqr}^{\alpha\beta} D_\alpha A_\beta = 0$$


GSO(-) ?
GSO(+) ✓
GSO(-) ✓
RNS
Q = λ^α d_α = λ^α (∂/∂θ^α − (γ^m θ)_α ∂/∂x^m) = λ^α D_α
QV = 0, δV = QΩ(x,θ) ⇒ A_α describes SYM on shell
gh = 1 V = λ^α A_α(x,θ) → QV = λ^α λ^β (D_α A_β) = 0
δV = QΩ = λ^α D_α Ω
→ δA_α = D_α Ω


∫ dλ dθ dx


∫d¹¹λ d¹⁶θ d¹⁰x
∫d¹⁶θ ∫d¹⁰x I



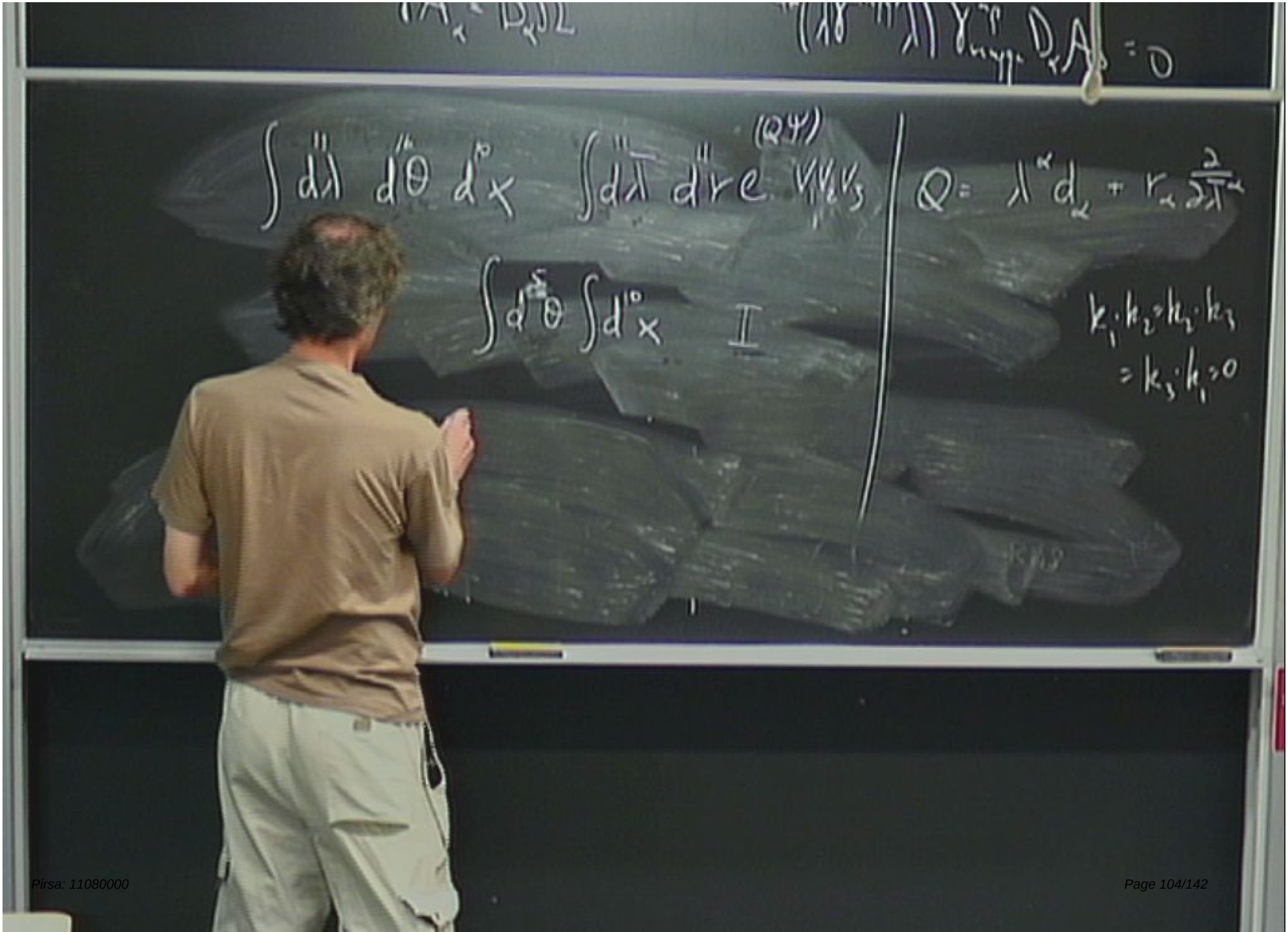

∫d¹¹λ d¹¹θ d¹⁰x ∫d¹¹λ̄ d¹¹r
Q = λ^α d_α + r_α ∂/∂λ̄_α
∫d¹⁶θ ∫d¹⁰x

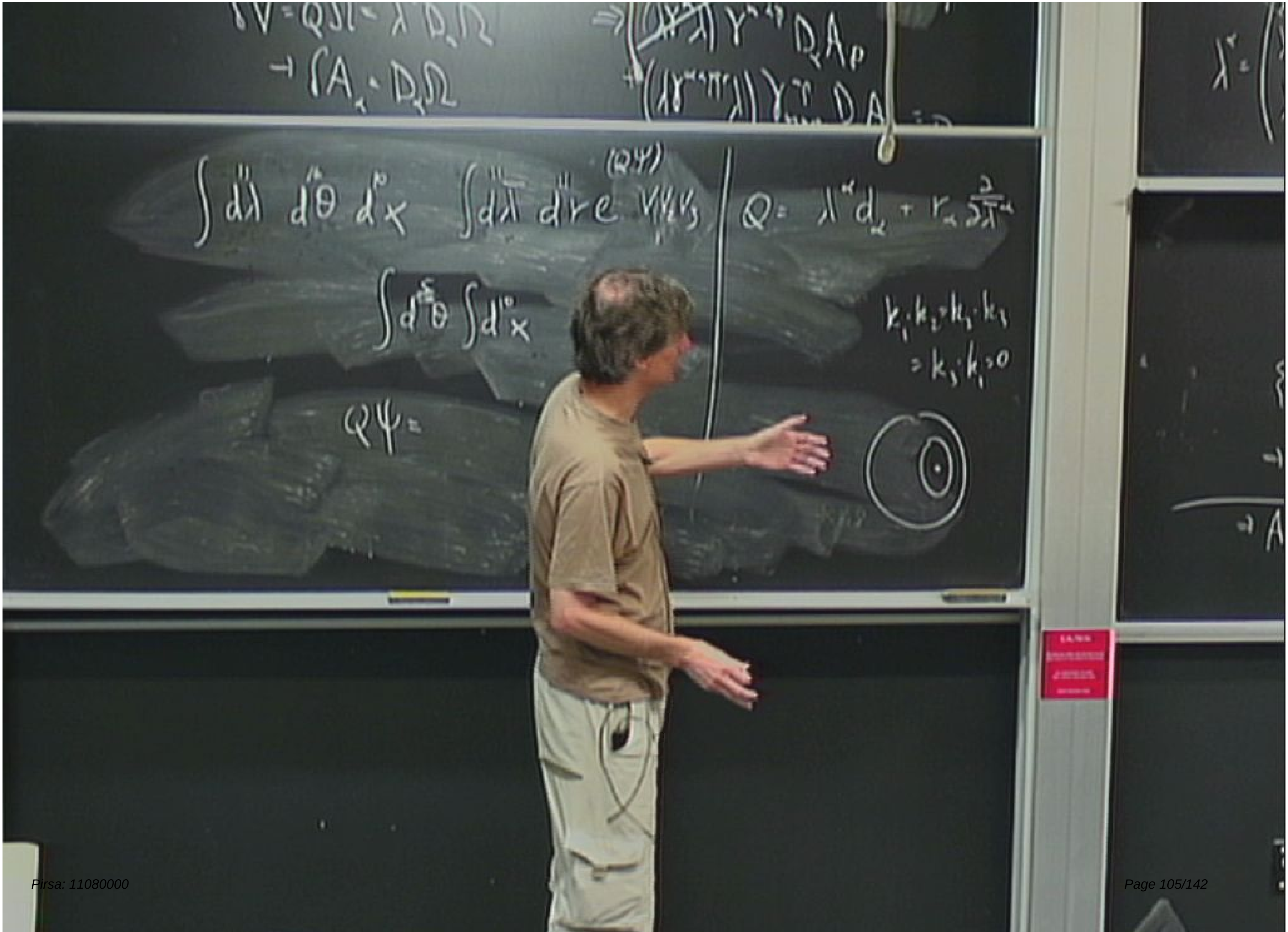
$$(A_\alpha - D_\alpha)\Omega \qquad (\lambda\gamma^{mnpqr}\lambda)\gamma_{mnpqr}^{\alpha\beta} D_\alpha A_\beta = 0$$

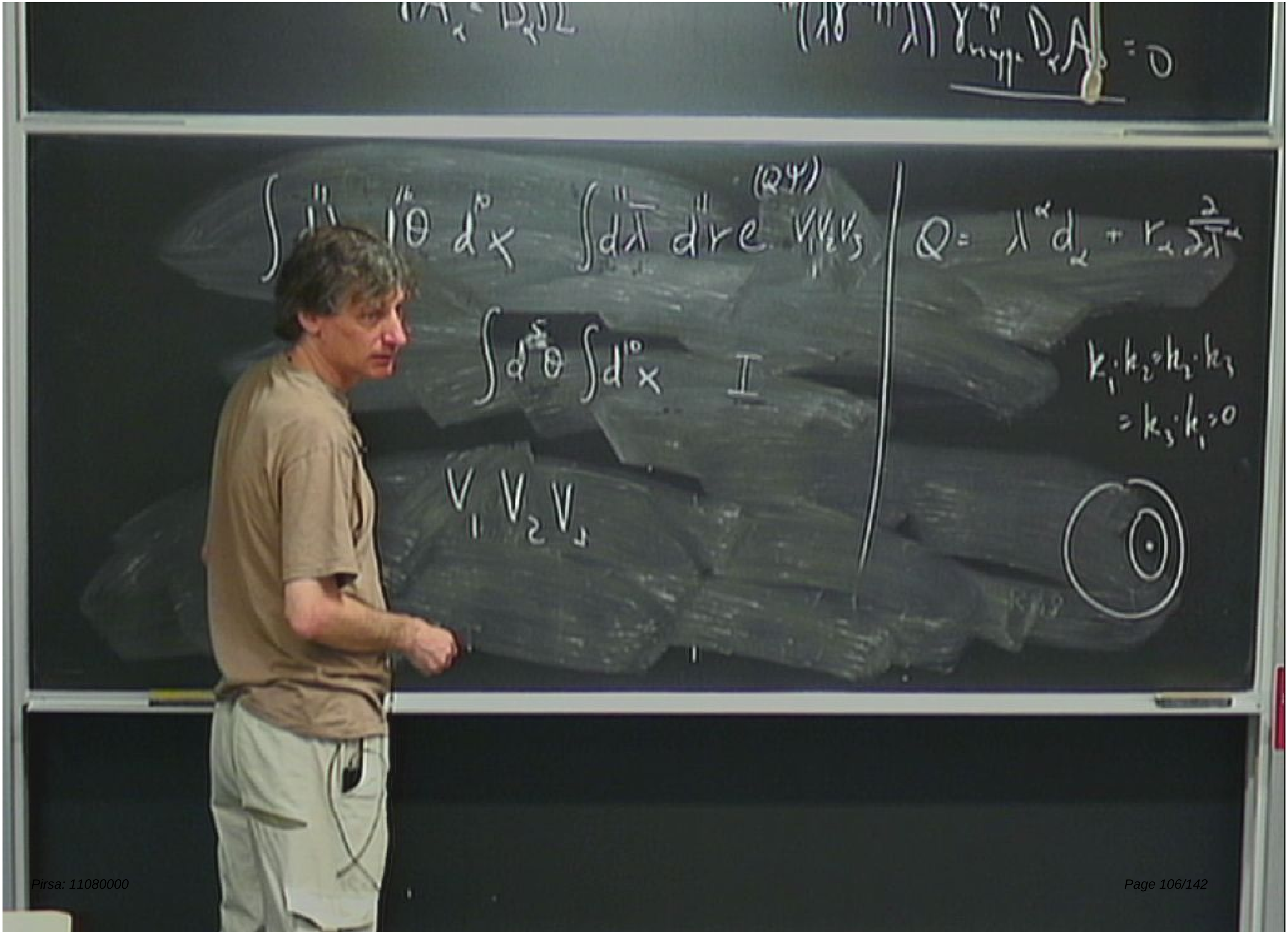
$$\int d^{11}\lambda \, d^{16}\theta \, d^{10}x \quad \int d^{11}\bar\lambda \, d^{11}r \quad V_1 V_2 V_3 \quad Q = \lambda^\alpha d_\alpha + r_\alpha \frac{\partial}{\partial \bar\lambda_\alpha}$$

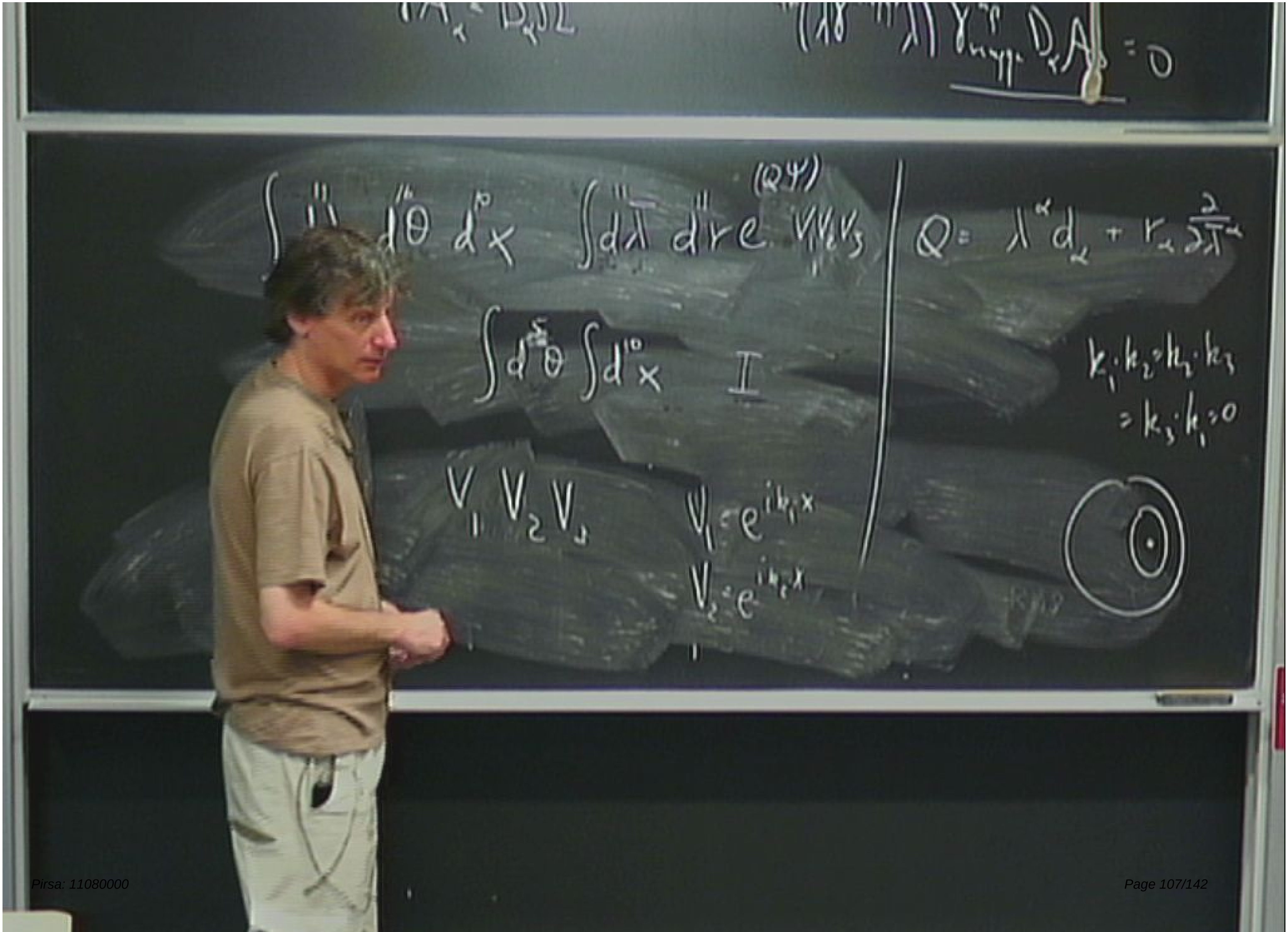
$$\int d^{5}\theta \int d^{10}x \quad I$$

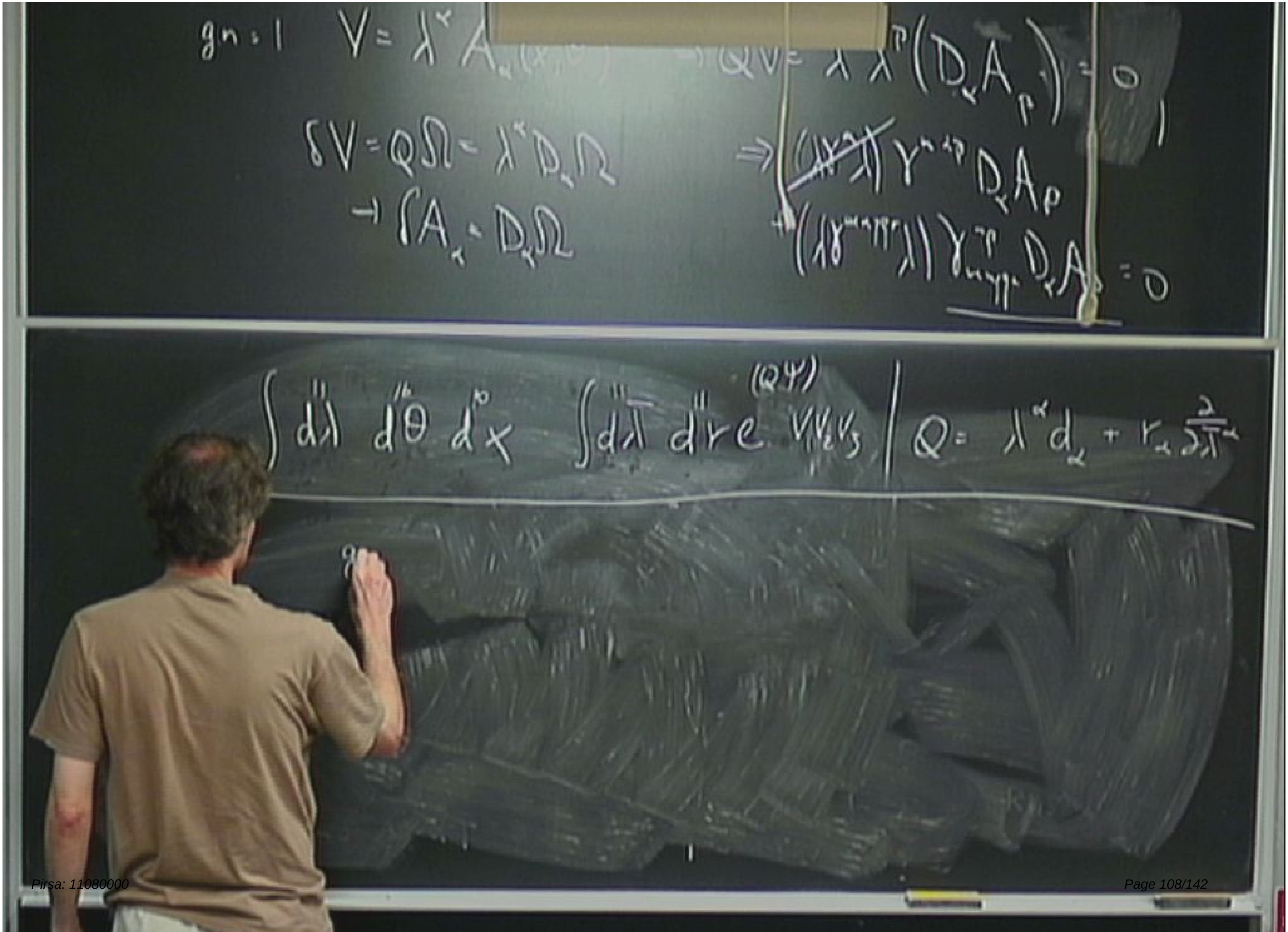
$$k_1 \cdot k_2 = k_2 \cdot k_3 = k_3 \cdot k_1 = 0$$

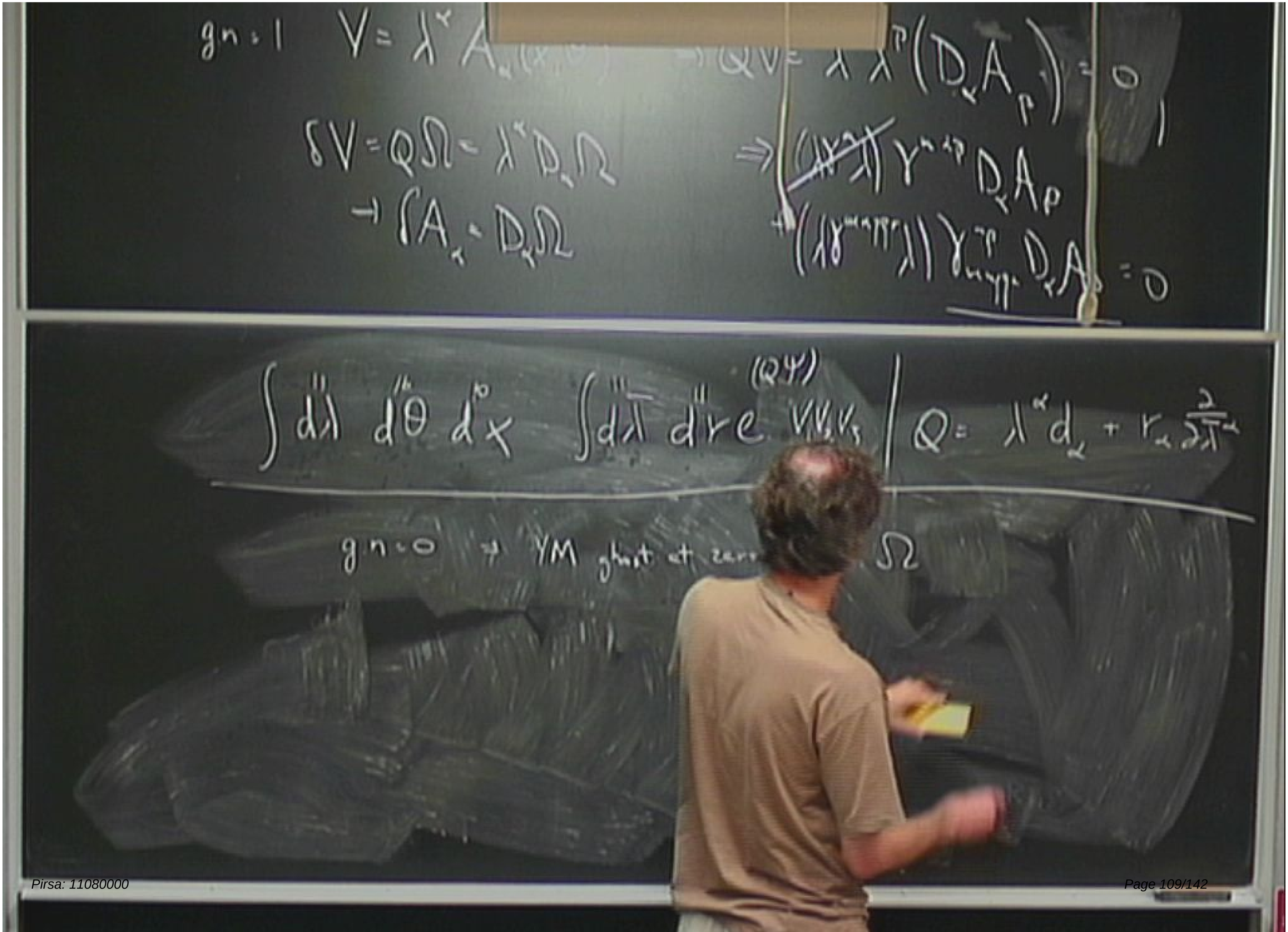

Q = λ^α d_α + r_α ∂/∂λ̄^α
k₁·k₂ = k₂·k₃
= k₃·k₁ = 0

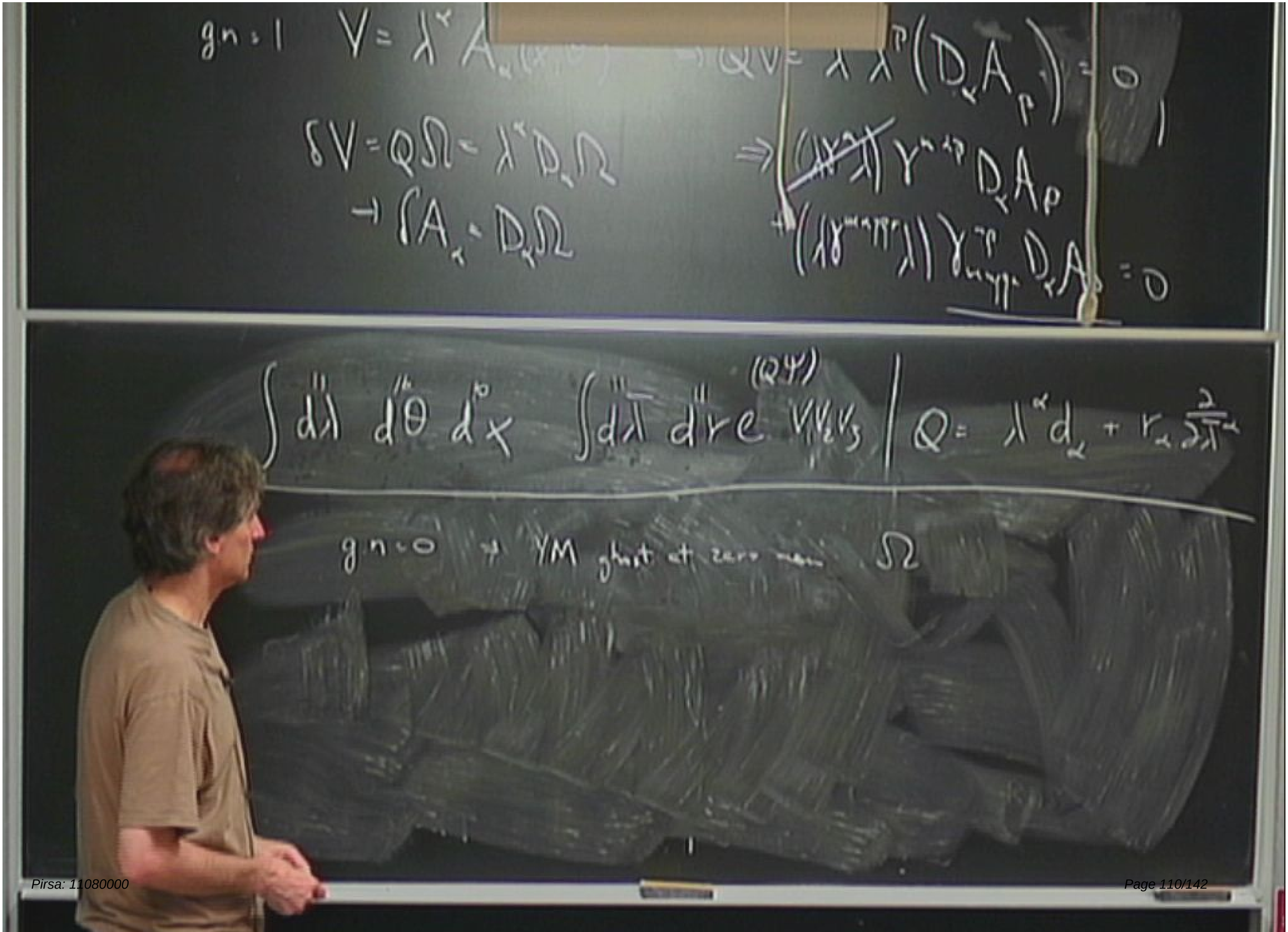

(QΨ)
QΨ =


k1·k2 = k2·k3
= k3·k1 = 0
V1 V2 V3


Q = λ^α d_α + r_α ∂/∂λ̄^α
k_1 · k_2 = k_2 · k_3
= k_3 · k_1 = 0
V_1 V_2 V_3
V_1 = e^{ik_1·x}
V_2 = e^{ik_2·x}


gh = 1 V = λ^α A_α
δV = QΩ = λ^α D_α Ω
→ ∫A_α ∧ D_α Ω
= 0
Q = λ^α d_α + r_α ∂/∂λ̄_α


gn = 1
gn = 0
YM ghost at zero


gh = 1
$\delta V = Q\Omega = \lambda^\alpha D_\alpha \Omega$
$\to \int A_\alpha \cdot D_\alpha \Omega$
$Q = \lambda^\alpha d_\alpha + r_\alpha \frac{\partial}{\partial \bar\lambda_\alpha}$
gh = 0
YM ghost at zero mom
Ω

$gh = 1 \quad V = \lambda^\alpha A_\alpha(x,\theta) \quad \Rightarrow QV = \lambda \lambda^\beta (D_\alpha A_\beta) = 0$

$\delta V = Q\Omega = \lambda^\alpha D_\alpha \Omega \quad \Rightarrow (\lambda \gamma^{\dots} \lambda)\gamma^{\alpha\beta} D_\alpha A_\beta$

$\to \delta A_\alpha = D_\alpha \Omega \quad + ((\lambda \gamma^{mnpqr} \lambda))\gamma_{mnpqr}^{\alpha\beta} D_\alpha A_\beta = 0$

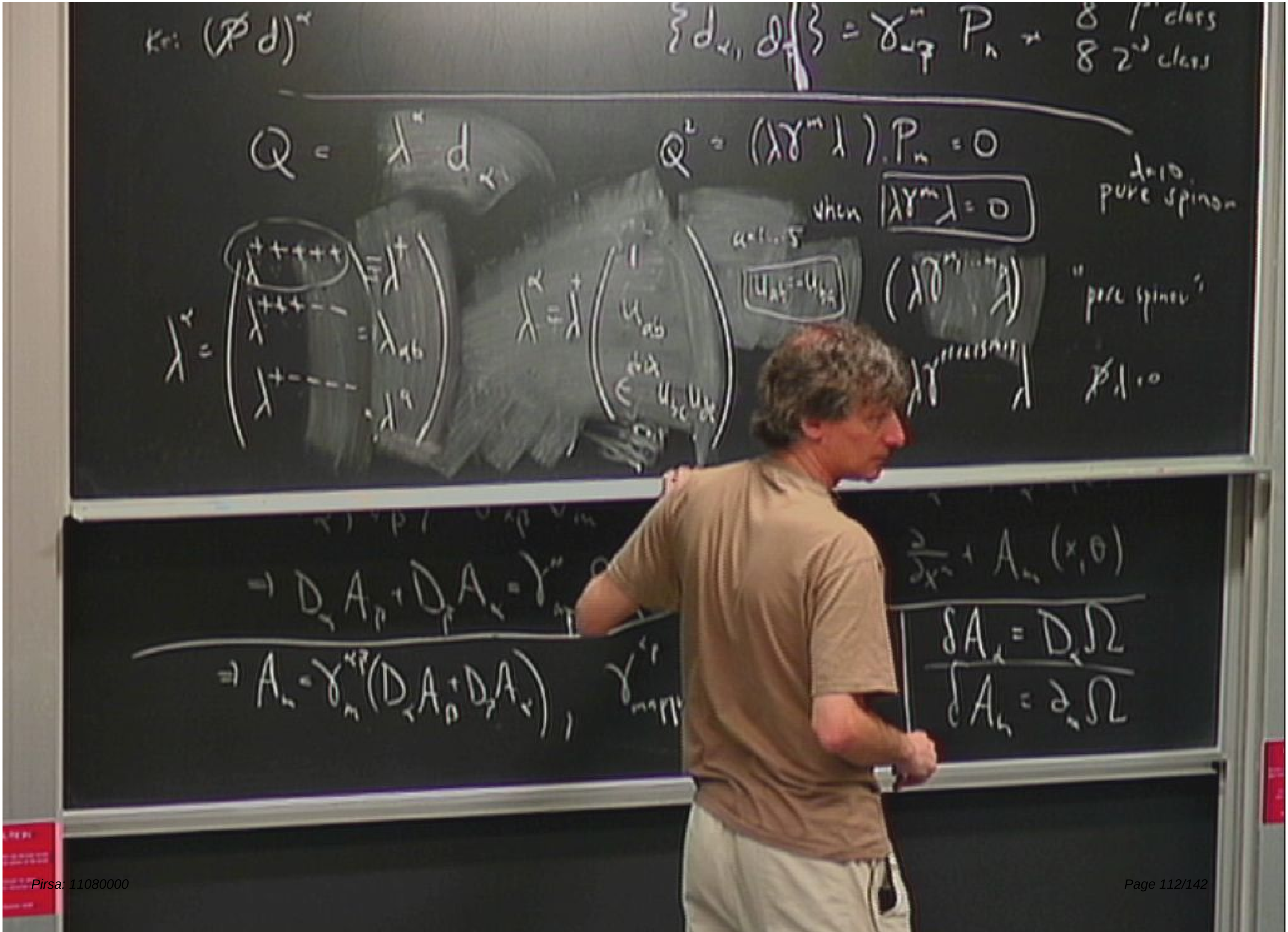
$\int d^{11}\lambda \, d^{16}\theta \, d^{10}x \int d^{11}\bar{\lambda} \, d^{11}r \, e^{(Q\psi)} V_1 V_2 V_3 \quad \Big| \quad Q = \lambda^\alpha d_\alpha + r_\alpha \frac{\partial}{\partial \bar{\lambda}_\alpha}$

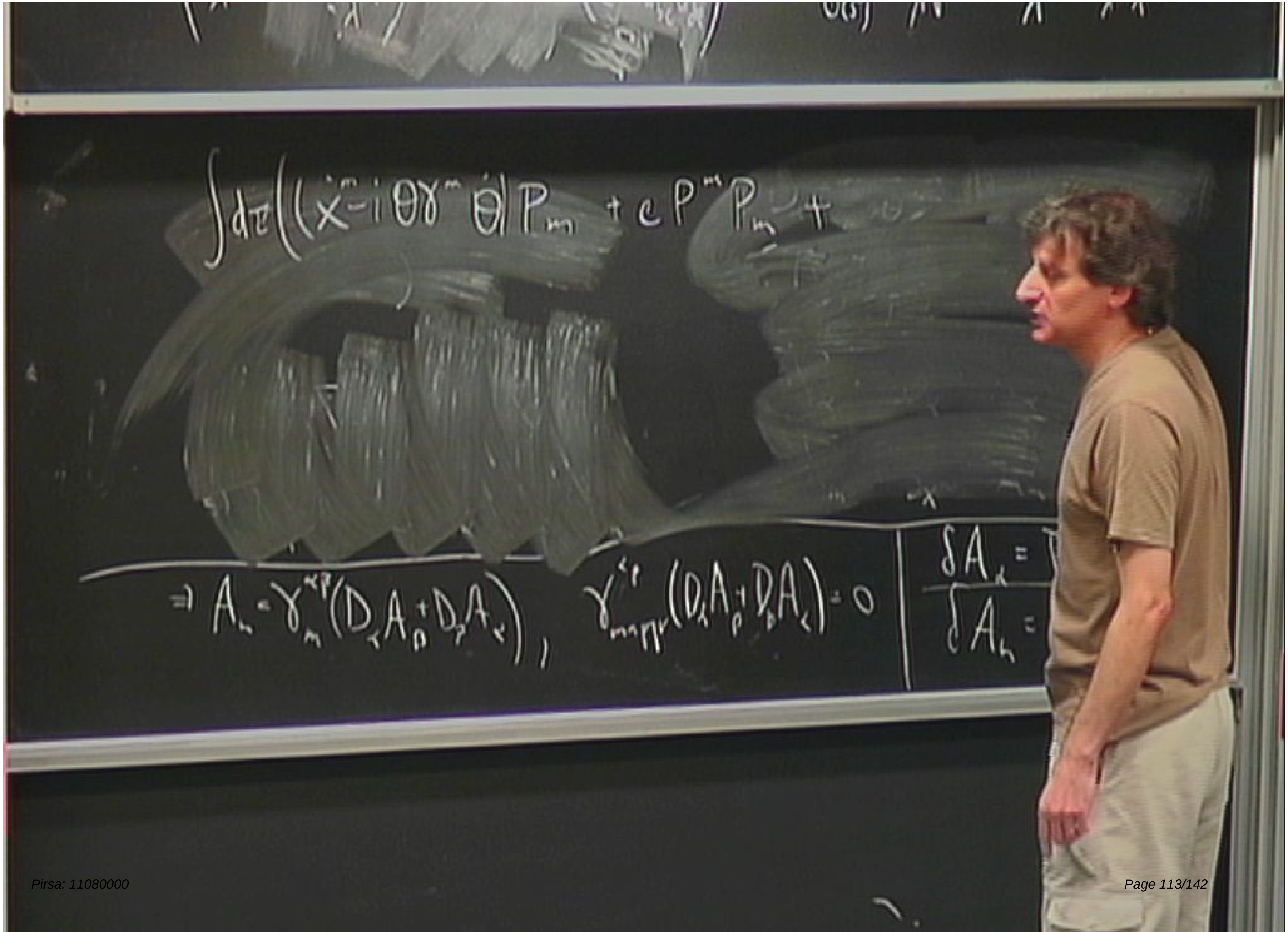
ghost $\quad gh = 0 \quad \to$ YM ghost at zero mom. $\quad \Omega$

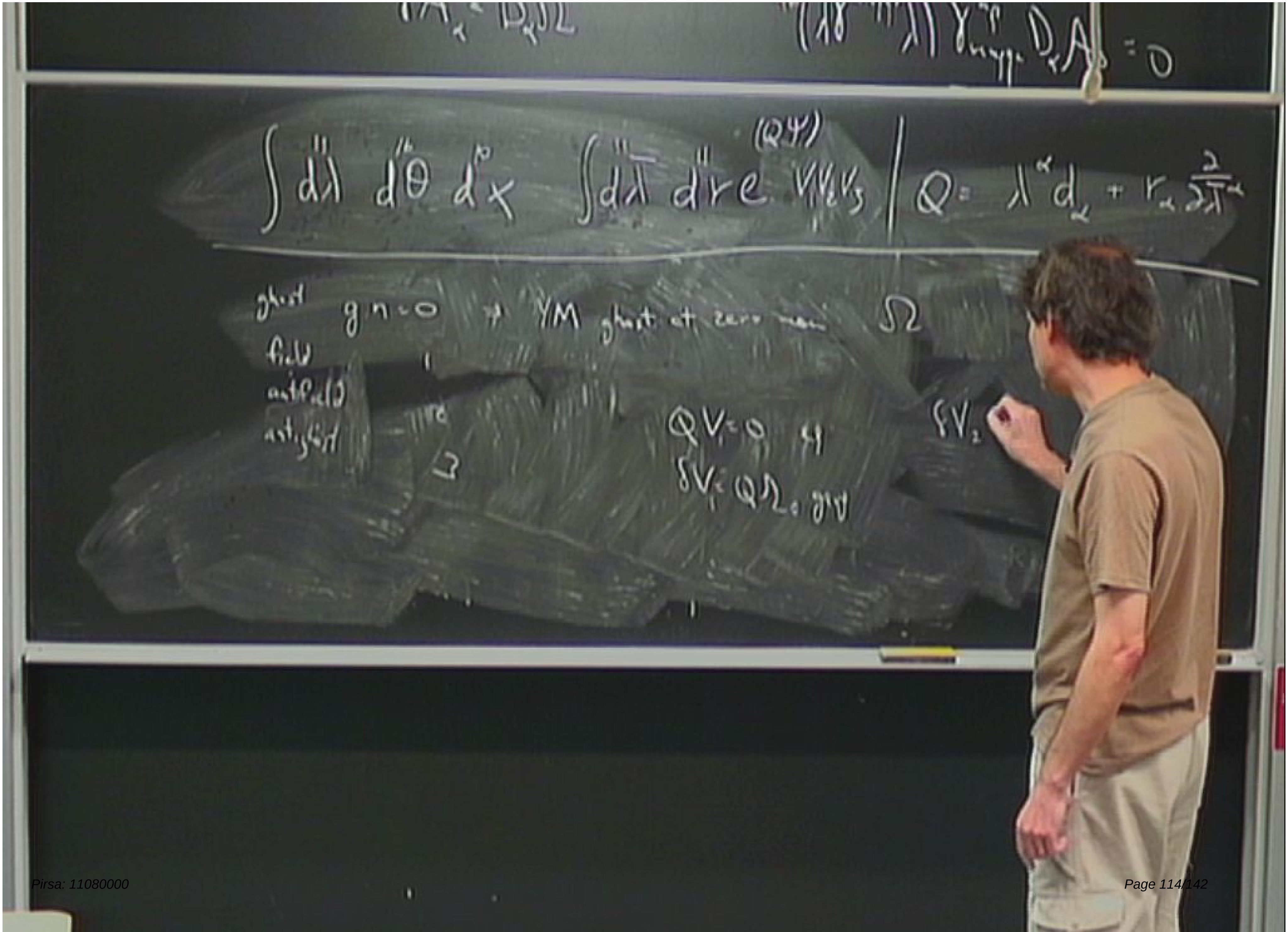
field $\quad 1$

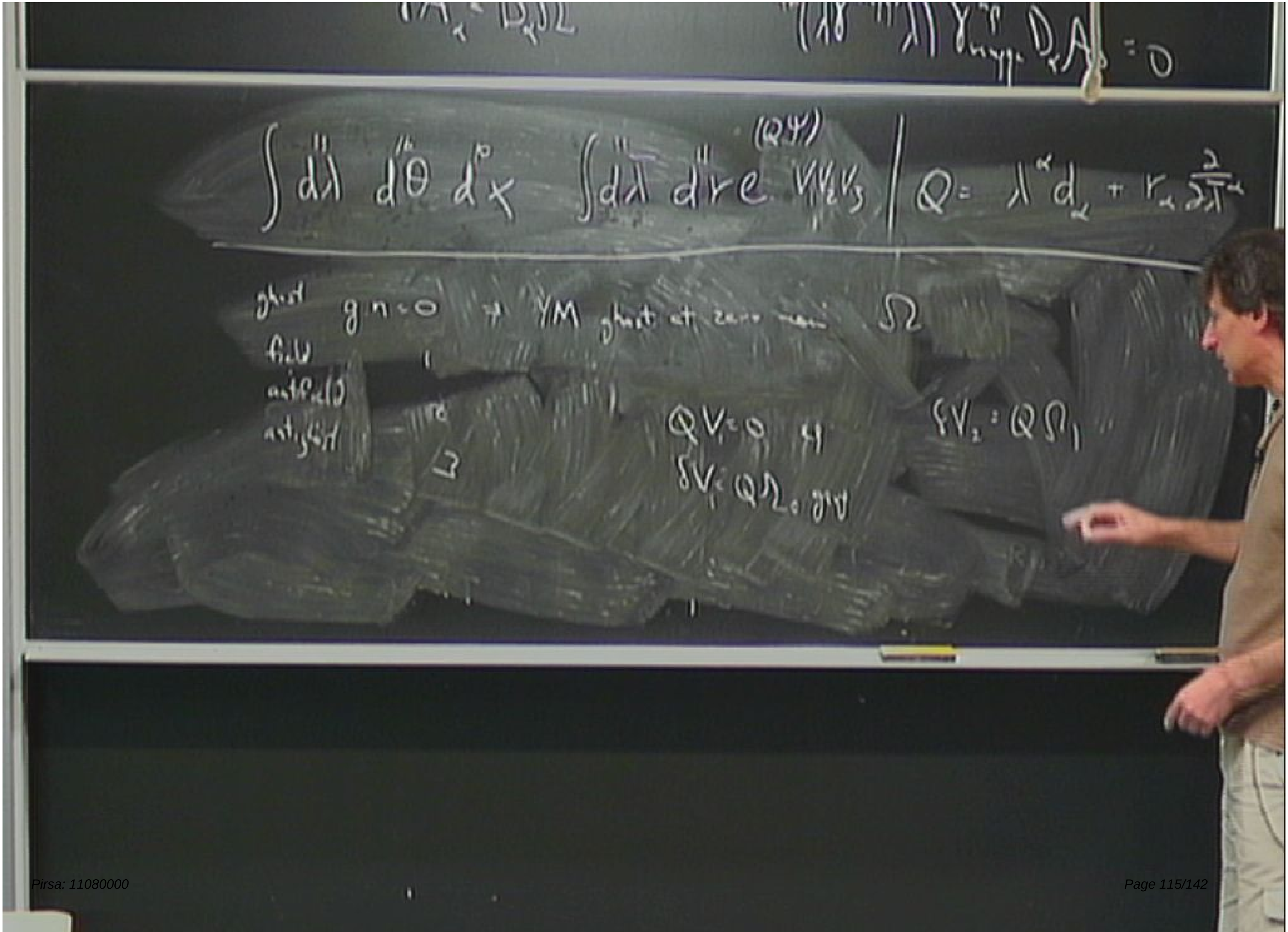
antifield $\quad 2$

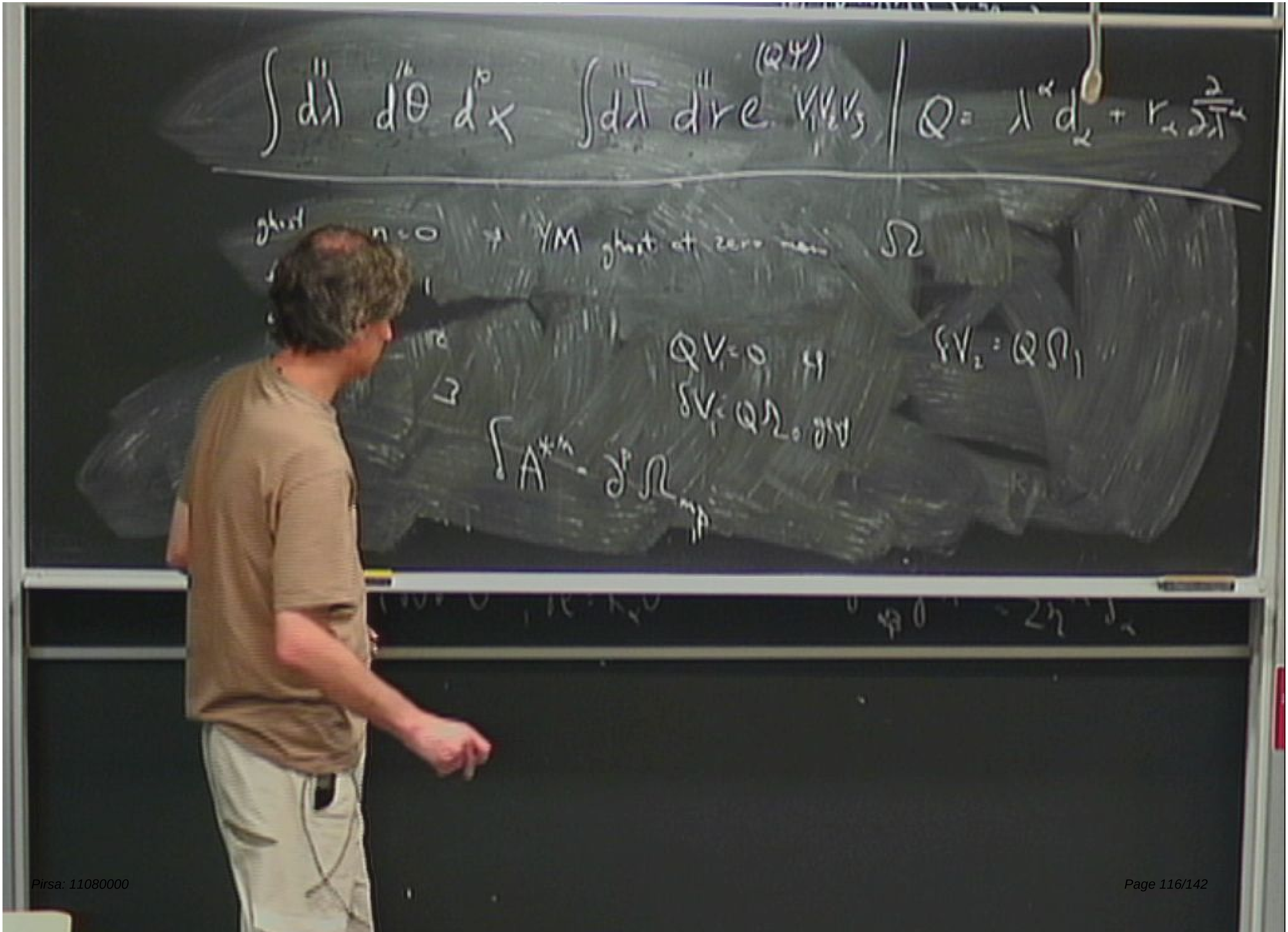
antighost $\quad 3$


8 1st class
8 2nd class
when
pure spinor
"pure spinor"

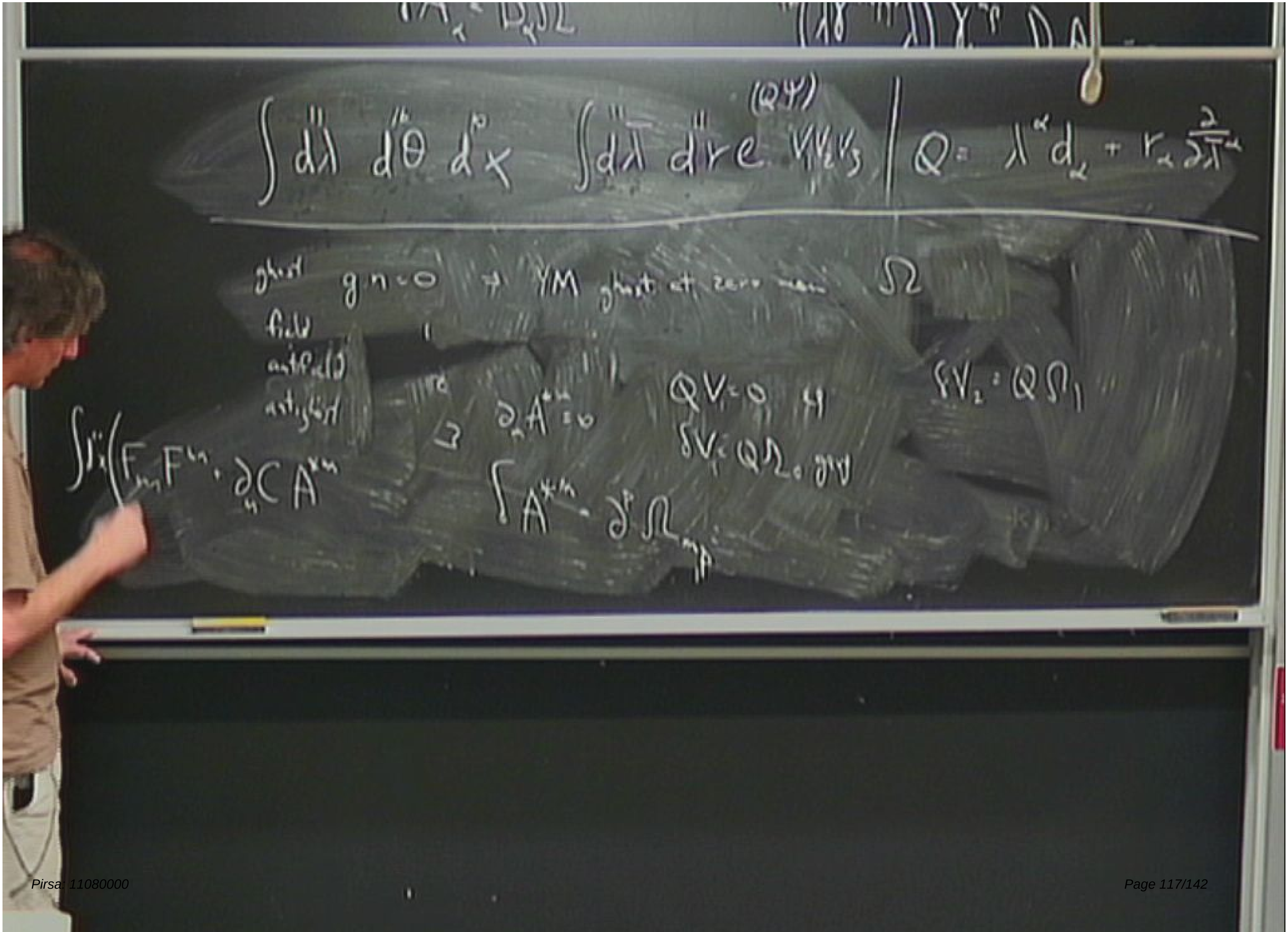

∫dτ((ẋ^m − iθγ^m θ̇)P_m + cP^m P_m +
⇒ A_m = γ_m^{αβ}(D_α A_β + D_β A_α), γ_{mnpqr}^{αβ}(D_α A_β + D_β A_α) = 0
δA_α =
δA_m =

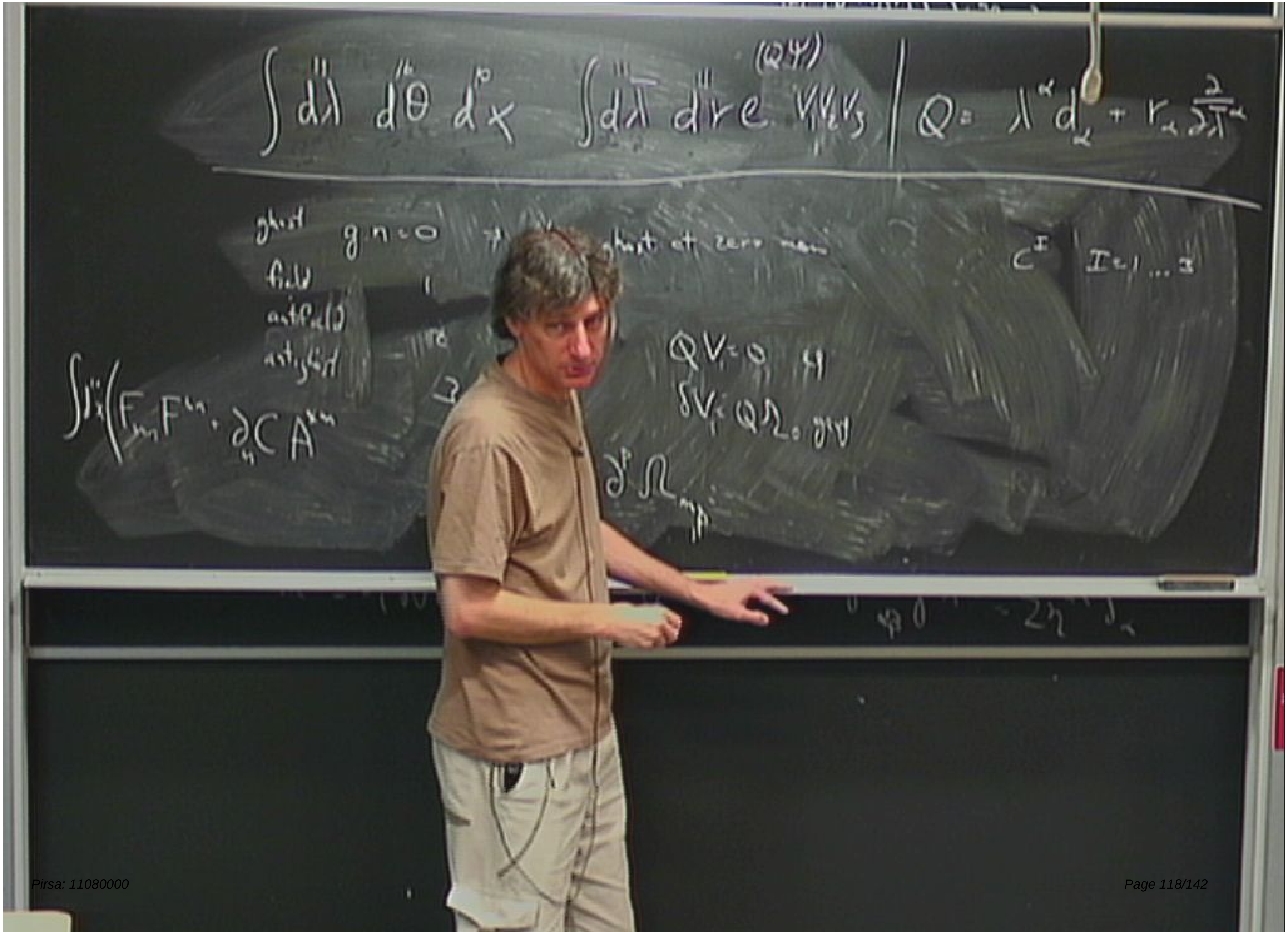

ghost
field
antifield
antighost
YM ghost at zero mom


∫ d¹¹λ d¹⁶θ d¹⁰x ∫ d¹¹λ̄ d¹¹r e^(QΨ) V₁V₂V₃
Q = λ^α d_α + r_α ∂/∂λ̄_α
ghost
gh = 0 ⇔ YM ghost at zero mom.
Ω
field
antifield
antighost
QV₁ = 0 ⇔
δV₁ = QΩ₀ gh0
δV₂ = QΩ₁



∫ d¹¹λ d¹⁶θ d¹⁰x ∫ d¹¹λ̄ d¹¹r e^(QΨ) V₁V₂V₃
Q = λ^α d_α + r_α ∂/∂λ̄_α
ghost
YM ghost at zero mom
Ω
QV₁ = 0
δV₁ = QΩ₁
δV₂ = QΩ₁


ghost
field
antifield
antighost
YM ghost at zero mom
QV₁ = 0
δV₁ = QΩ₂ + gV
δV₂ = QΩ₁
∂ₘA*ᵐ = 0


ghost
field
antifield
antighost
ghost at zero mom

$C^I \qquad I = 1 \ldots 3$

$$Q = C^I \frac{\partial}{\partial x^I}$$

$$V = \Omega(x)$$

$$C^I A_I(x)$$

$$C^I C^J \quad A^{*K} \quad \epsilon_{IJK}$$

$$\epsilon \; C^I C^J C^K D^*$$

$$C^I \qquad I = 1 \ldots 3$$

$$Q = C^I \frac{\partial}{\partial x^I}$$

$$V = \Omega(x)$$

$$C^I A_I(x)$$

$$C^I C^J A^{*K} \quad \epsilon_{IJK}$$

$$\epsilon_{IJK} C^I C^J C^K \Omega^*$$

$$\int d^{11}\lambda \, d^{16}\theta \, d^{10}x \int d^{11}\bar{\lambda} \, d^{11}r \, e^{(Q\Psi)} V_1 V_2 V_3 \quad \Big| \quad Q = \lambda^\alpha d_\alpha + r_\alpha \frac{\partial}{\partial \bar{\lambda}_\alpha}$$

ghost $g\,n = 0$ → YM ghost at zero mom

field 1

antifield

antighost

$$\int d^{10}x \left(F_{mn} F^{mn} + \partial_n C A^{*n} \right)$$

$\exists\ \partial_n A^{*n} = 0$

$\int A^{*m} = \partial^p \Omega_{mp}$

$QV_1 = 0$

$\delta V_1 = Q\Omega_1$ gh 0

$C^I \quad I = 1 \ldots 3$

$Q = C^I \frac{\partial}{\partial x^I}$

$V_1 = \Omega(x)$

$C^I A_I(x)$

$C^I C^J \ A^{*K} \epsilon_{IJK}$

$\epsilon_{IJK} C^I C^J C^K \Omega^*$

$$\int d\tau \Big((\dot{x}^m - i\theta\gamma^m \dot{\theta}) P_m + c P^m P_m + A_m(x,\theta)\dot{x}^m$$

$$+ A_\alpha(x,\theta)\dot{\theta}^\alpha$$

$$\Rightarrow A_m = \gamma_m^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma^{\alpha\beta}_{mnpqr}(D_\alpha A_\beta + D_\beta A_\alpha) = 0$$

$$\delta A_\alpha = D_\alpha \Omega$$

$$\delta A_m = \partial_m \Omega$$

$$S = \int d\tau \left((\dot{x}^m - i\theta\gamma^m\dot{\theta}) P_m + c P^m P_m + \left(A_m(x,\theta)\dot{x}^m + A_\alpha(x,\theta)\dot{\theta}^\alpha \right) + W^\alpha(x,\theta) d_\alpha + F^{mn}(x,\theta)(\omega\gamma_{mn}\lambda) \right)$$

$$Q = \lambda^\alpha d_\alpha$$

$$\frac{\partial}{\partial\tau} Q = 0$$

$$\Rightarrow A_m = \gamma_m^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma_{mnpqr}^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha) = 0$$

$$\delta A_\alpha = D_\alpha \Omega$$

$$\delta A_m = \partial_m \Omega$$

$$S = \int d\tau \Big((\dot{x}^m - i\theta\gamma^m \dot{\theta}) P_m + c P^m P_m + \big(A_m(x,\theta)\dot{x}^m + A_\alpha(x,\theta)\dot{\theta}^\alpha \big) + W^\alpha(x,\theta)\, d_\alpha + F^{mn}(x,\theta)\,(\omega\gamma_{mn}\lambda) \Big)$$

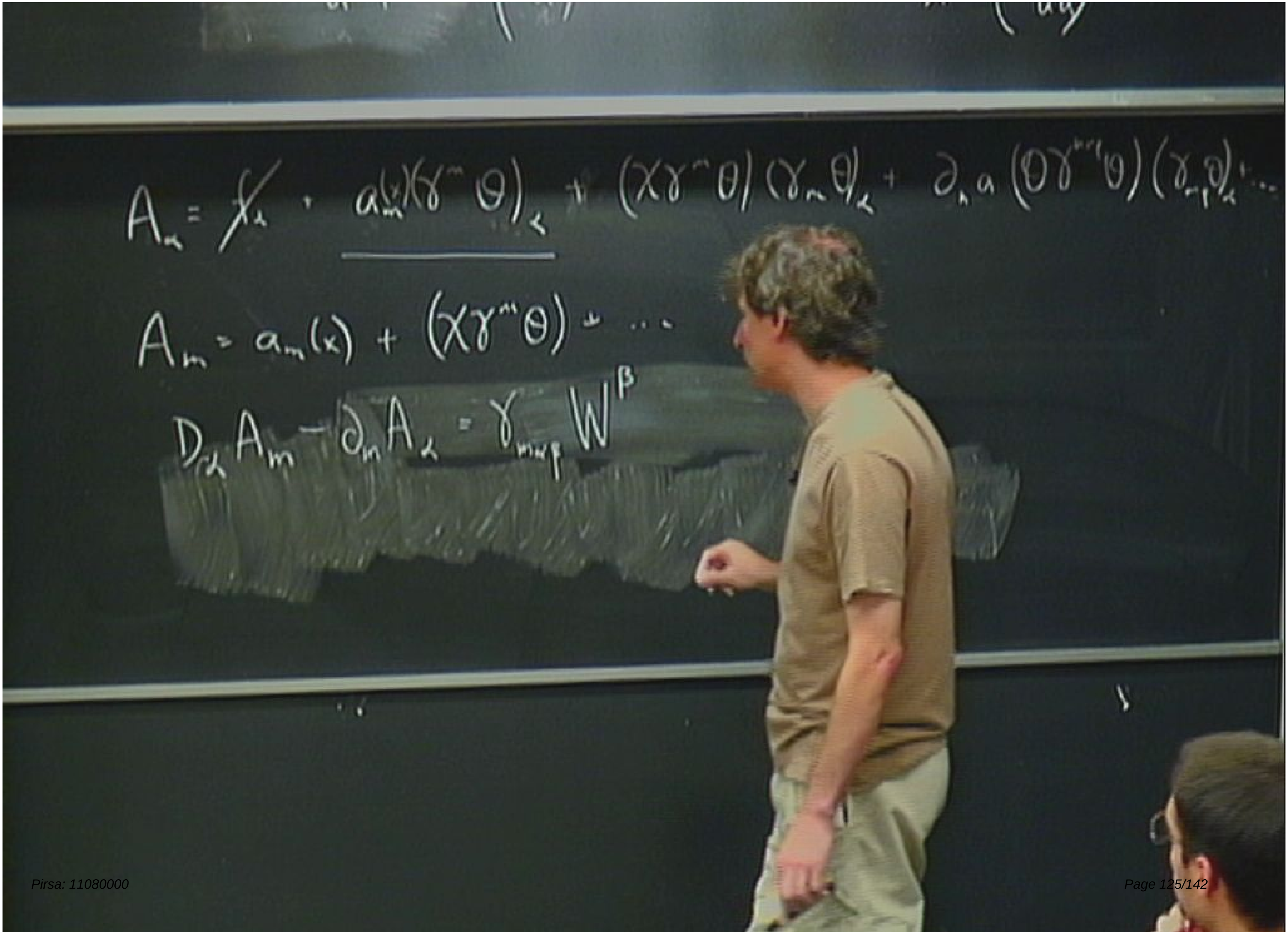
$$Q = \lambda^\alpha d_\alpha$$

$$\frac{\partial}{\partial\tau} Q = 0$$

$$\Rightarrow A_m = \gamma_m^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma_{mnpqr}^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha) = 0$$

$$\delta A_\alpha = D_\alpha \Omega$$

$$\delta A_m = \partial_m \Omega$$


$A_m = a_m(x) + (\chi\gamma^m\theta) + \dots$
$D_\alpha A_m - \partial_m A_\alpha = \gamma_{m\alpha\beta} W^\beta$

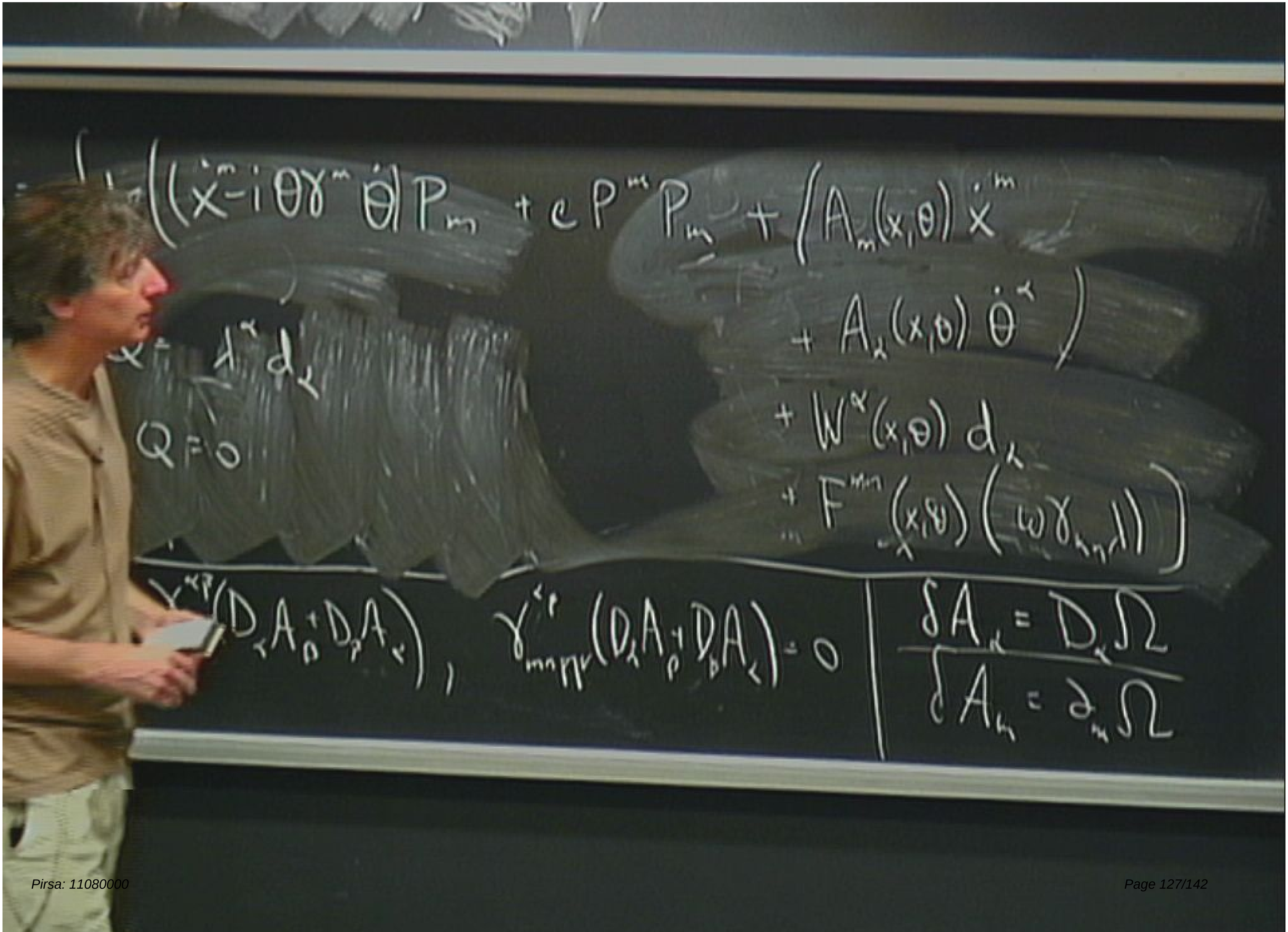
$$A_\alpha = \chi_\alpha + a_m(x)(\gamma^m\theta)_\alpha + (\chi\gamma^m\theta)(\gamma_m\theta)_\alpha + \partial_n a_p(\theta\gamma^{mnp}\theta)(\gamma_m\theta)_\alpha + \ldots$$

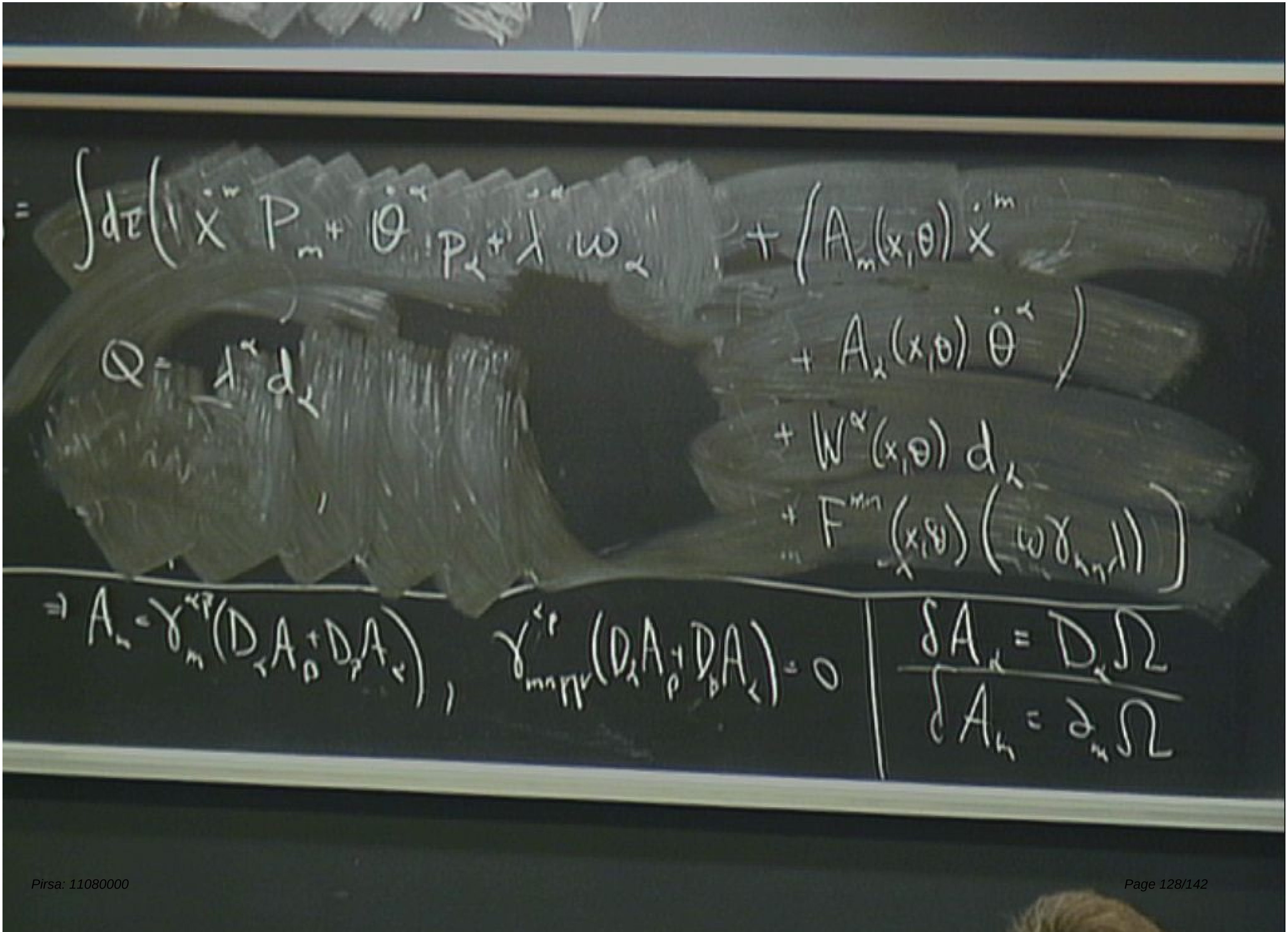
$$A_m = a_m(x) + (\chi\gamma^m\theta) + \ldots, \quad W^\beta = \chi^\beta + \ldots$$

$$F_{mn} = \partial_{[m} a_{n]} + \ldots$$

$$D_{(\alpha} A_{m)} - \partial_m A_\alpha = \gamma_{m\alpha\beta} W^\beta$$

$$D_\alpha W^\beta = (\gamma^{mn})_\alpha{}^\beta F_{mn}$$


∫ dτ ((ẋ^m − i θ γ^m θ̇) P_m + e P^m P_m + (A_m(x,θ) ẋ^m
+ A_α(x,θ) θ̇^α)
+ W^α(x,θ) d_α
+ F^mn(x,θ) (ω γ_mn λ))
Q = λ^α d_α
Q² = 0
γ^αβ_mnpqr (D_α A_β + D_β A_α) = 0
δA_α = D_α Ω
δA_m = ∂_m Ω


$= \int d\tau \left(\dot{x}^m P_m + \dot{\theta}^\alpha p_\alpha + \dot{\lambda}^\alpha \omega_\alpha \right. + \left(A_m(x,\theta)\, \dot{x}^m \right.$
$+ A_\alpha(x,\theta)\, \dot{\theta}^\alpha \Big)$
$Q = \lambda^\alpha d_\alpha$
$+ W^\alpha(x,\theta)\, d_\alpha$
$+ F^{mn}(x,\theta) \left(\omega \gamma_{mn} \lambda \right) \Big)$
$\Rightarrow A_m = \gamma_m^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma^{\alpha\beta}_{mnpqr}(D_\alpha A_\beta + D_\beta A_\alpha) = 0$
$\delta A_\alpha = D_\alpha \Omega$
$\delta A_m = \partial_m \Omega$

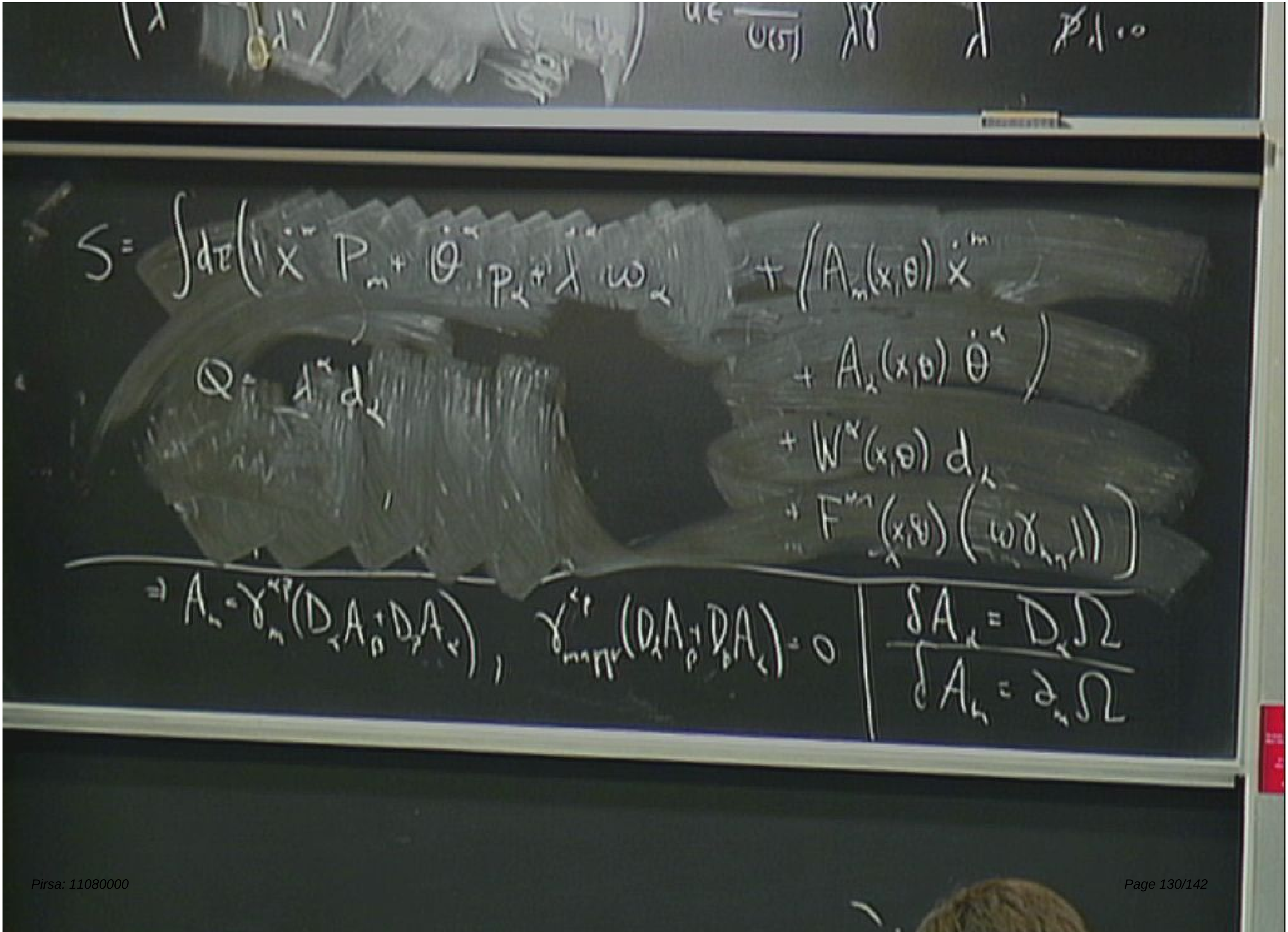
$$S = \int d\tau \left(\dot{x}^m P_m + \dot{\theta}^\alpha p_\alpha + \dot{\lambda}^\alpha \omega_\alpha + \left(A_m(x,\theta)\, \dot{x}^m + A_\alpha(x,\theta)\, \dot{\theta}^\alpha \right) + W^\alpha(x,\theta)\, d_\alpha + F^{mn}(x,\theta)\left(\omega \gamma_{mn} \lambda \right) \right)$$

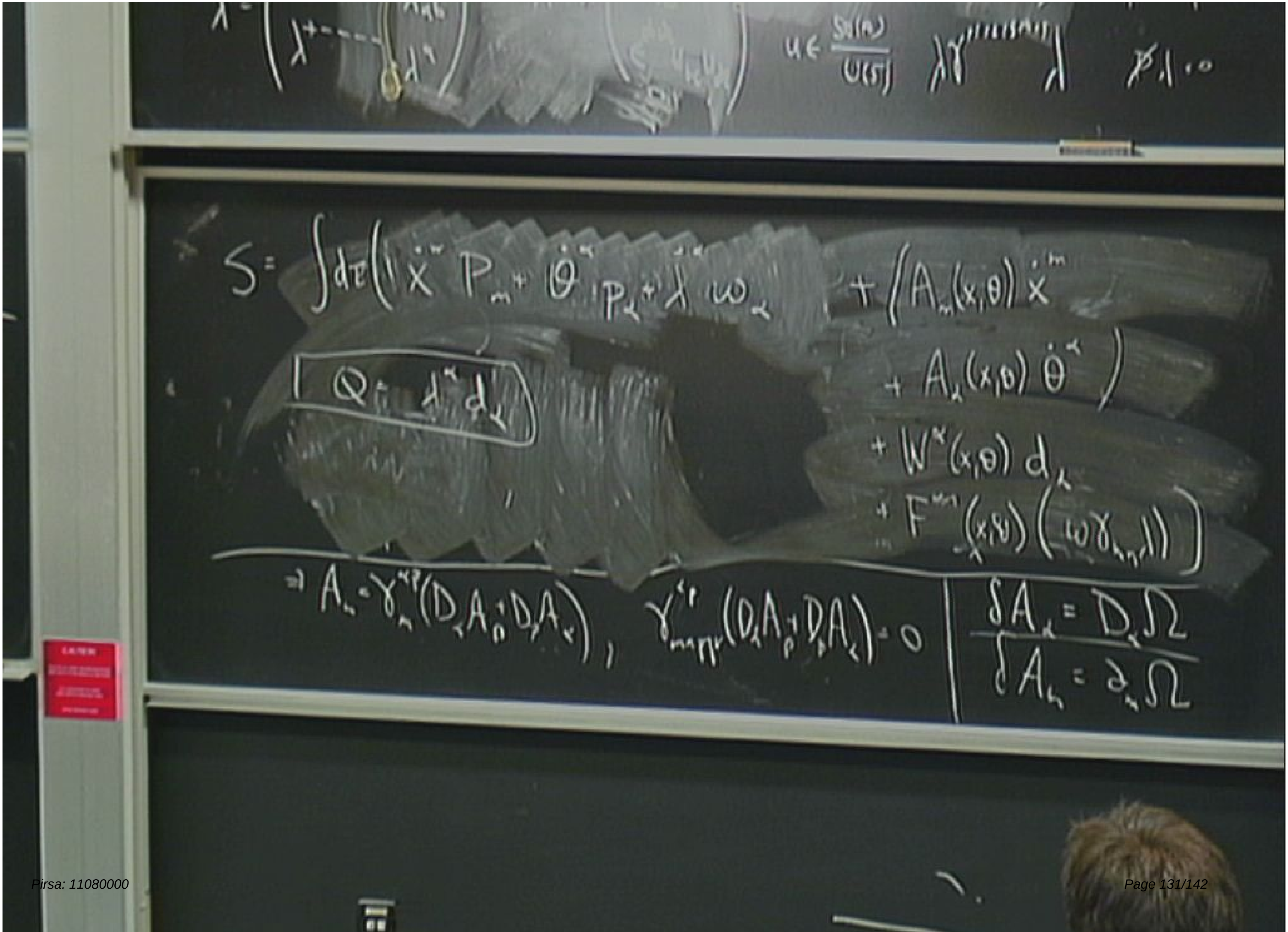
$$Q = \lambda^\alpha d_\alpha$$

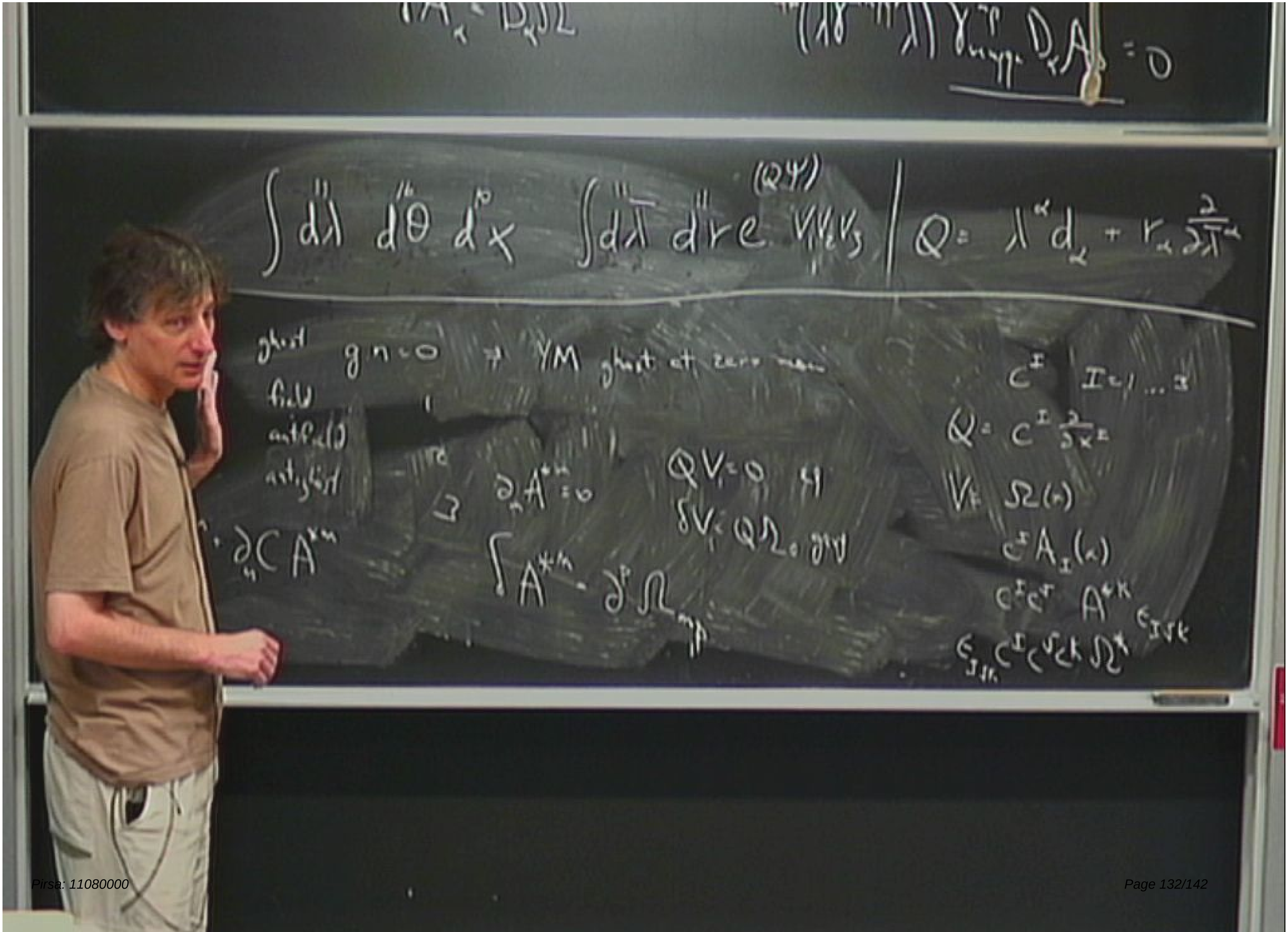
$$\Rightarrow A_m = \gamma_m^{\alpha\beta}(D_\alpha A_\beta + D_\beta A_\alpha), \quad \gamma^{\alpha\beta}_{mnpqr}(D_\alpha A_\beta + D_\beta A_\alpha) = 0$$

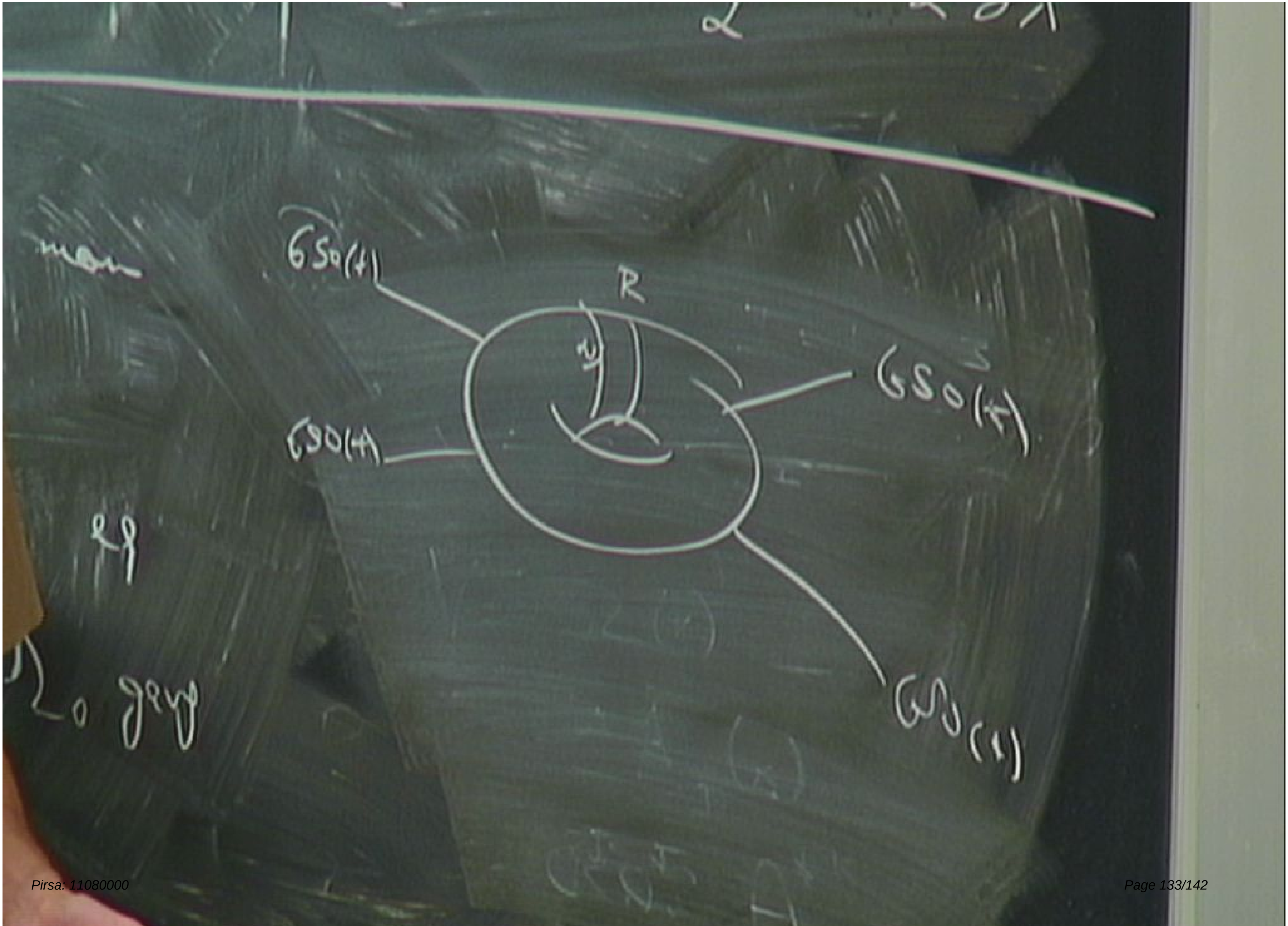
$$\delta A_\alpha = D_\alpha \Omega$$

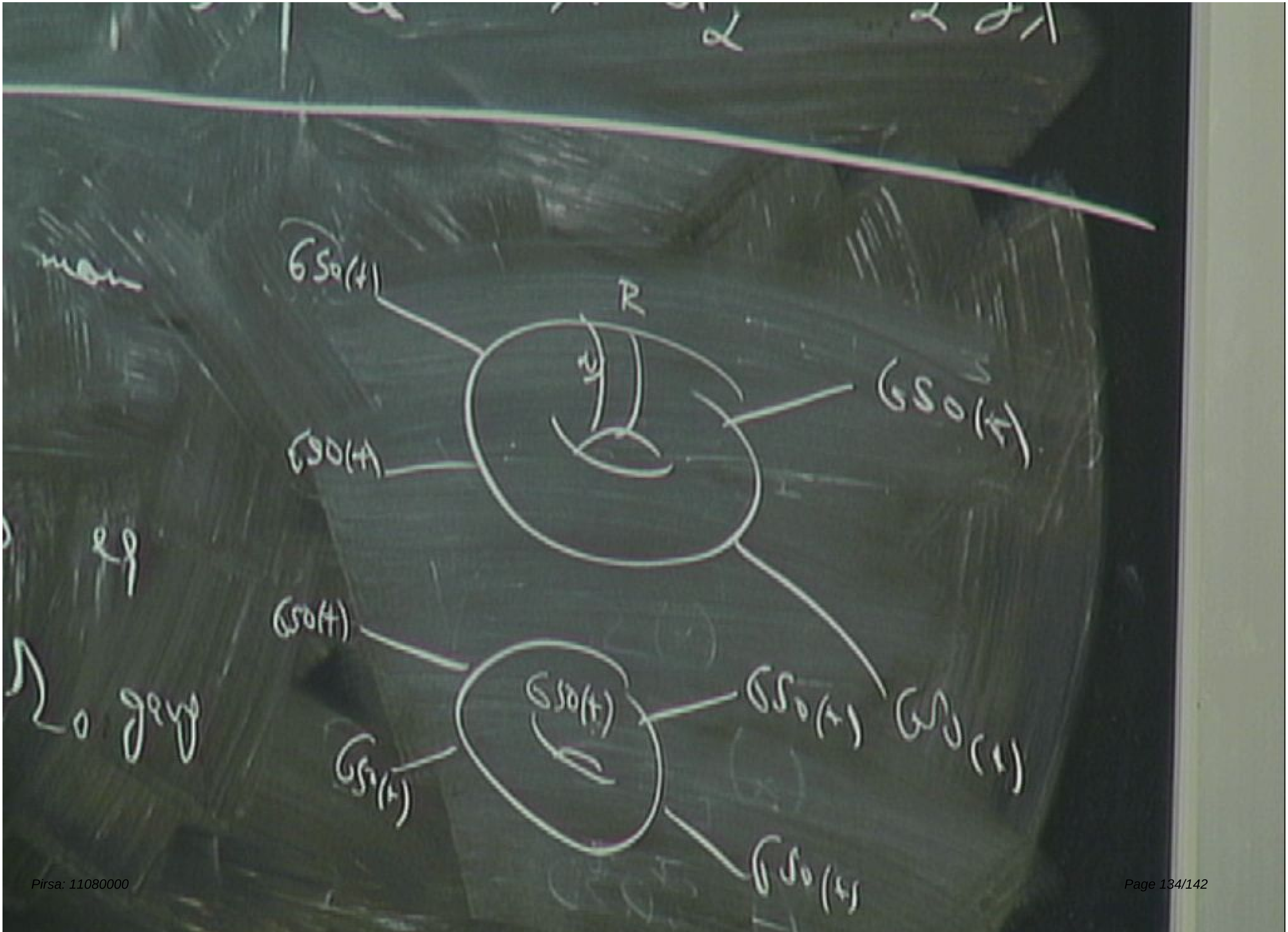
$$\delta A_m = \partial_m \Omega$$


S = ∫dτ (ẋ^m P_m + θ̇^α p_α + λ̇^α ω_α + (A_m(x,θ) ẋ^m
+ A_α(x,θ) θ̇^α)
Q = λ^α d_α
+ W^α(x,θ) d_α
+ F^mn(x,θ) (ωγ_mn λ))
⇒ A_m = γ_m^αβ (D_α A_β + D_β A_α) , γ^αβ_mnpqr (D_α A_β + D_β A_α) = 0
δA_α = D_α Ω
δA_m = ∂_m Ω


S = ∫dτ (i ẋ^m P_m + θ̇^α p_α + λ̇^α ω_α + (A_m(x,θ) ẋ^m
+ A_α(x,θ) θ̇^α)
+ W^α(x,θ) d_α
+ F^{mn}(x,θ) (λ ω γ_{mn} λ))
Q = λ^α d_α
⇒ A_m = γ_m^{αβ}(D_α A_β + D_β A_α) , γ^{αβ}_{mnpqr}(D_α A_β + D_β A_α) = 0
δA_α = D_α Ω
δA_m = ∂_m Ω


ghost
field
antifield
antighost
YM


R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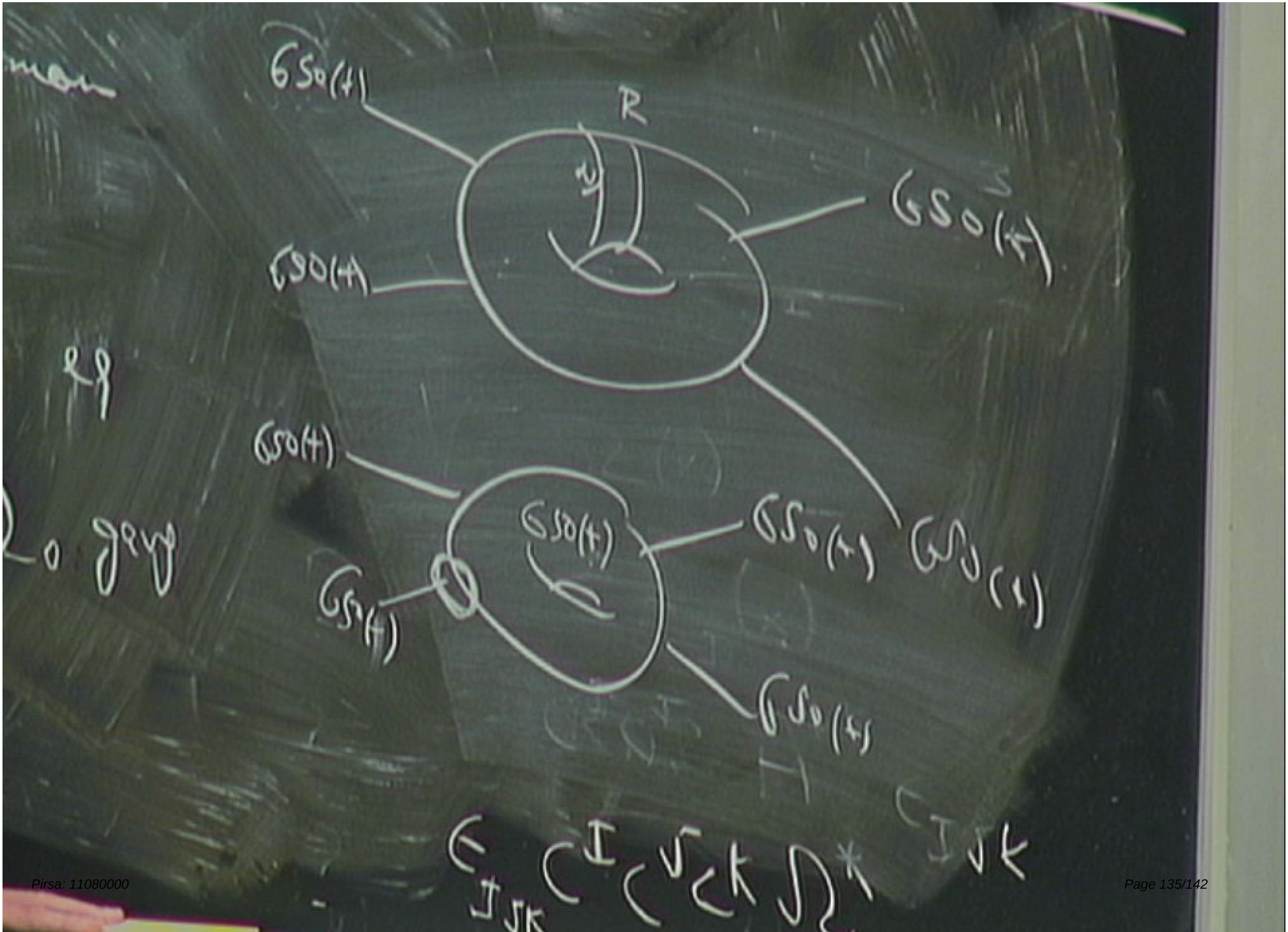
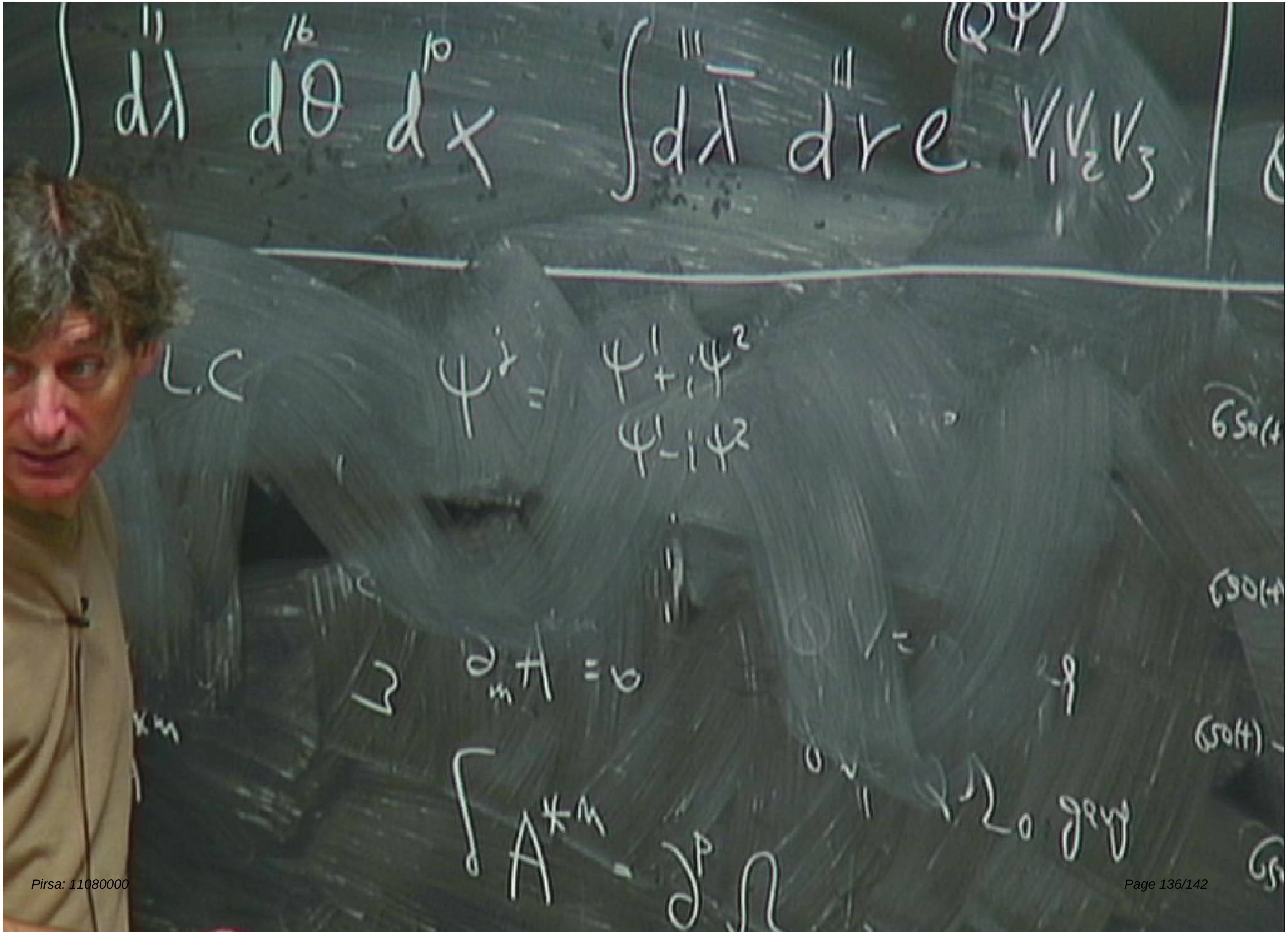
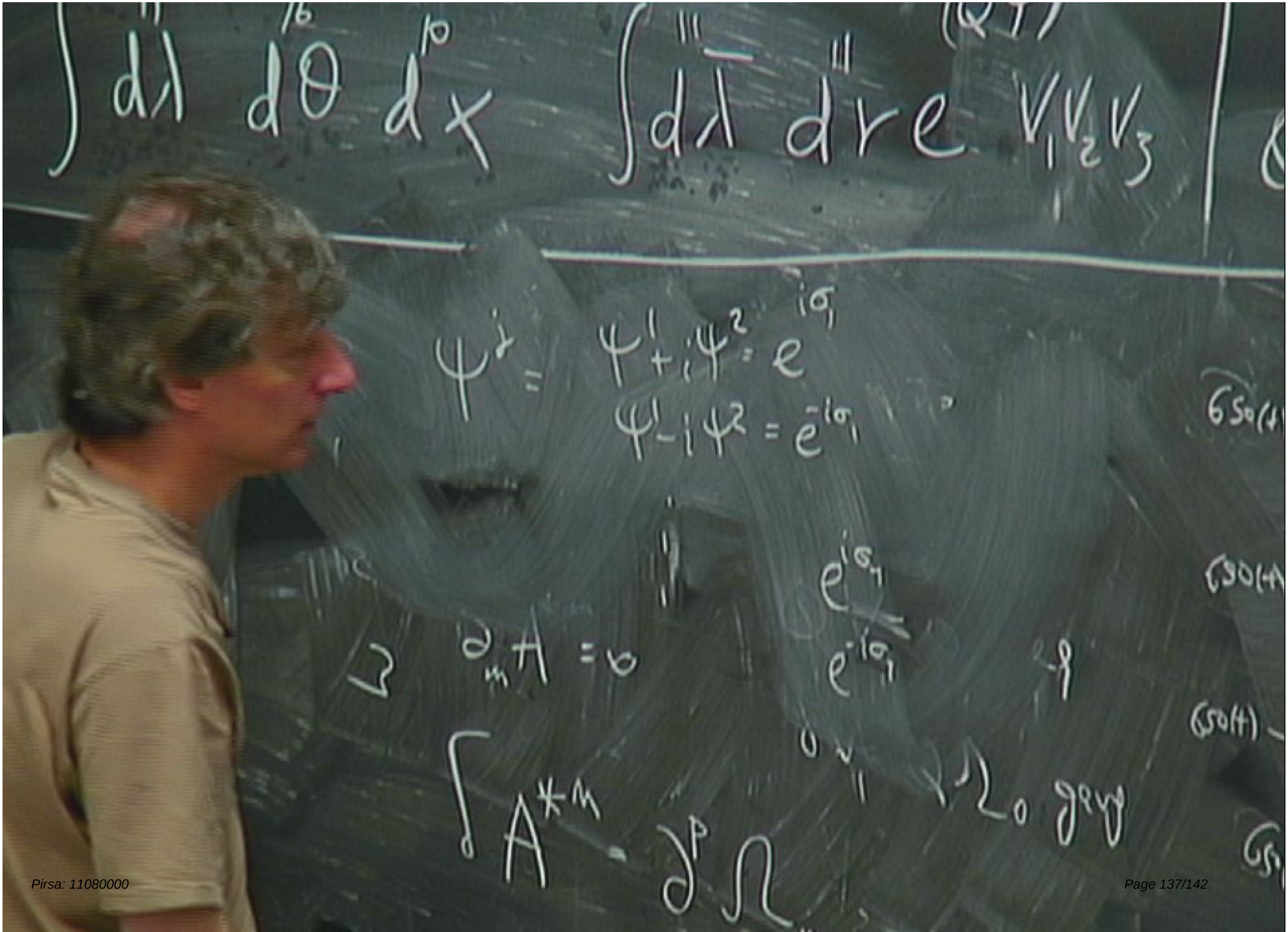
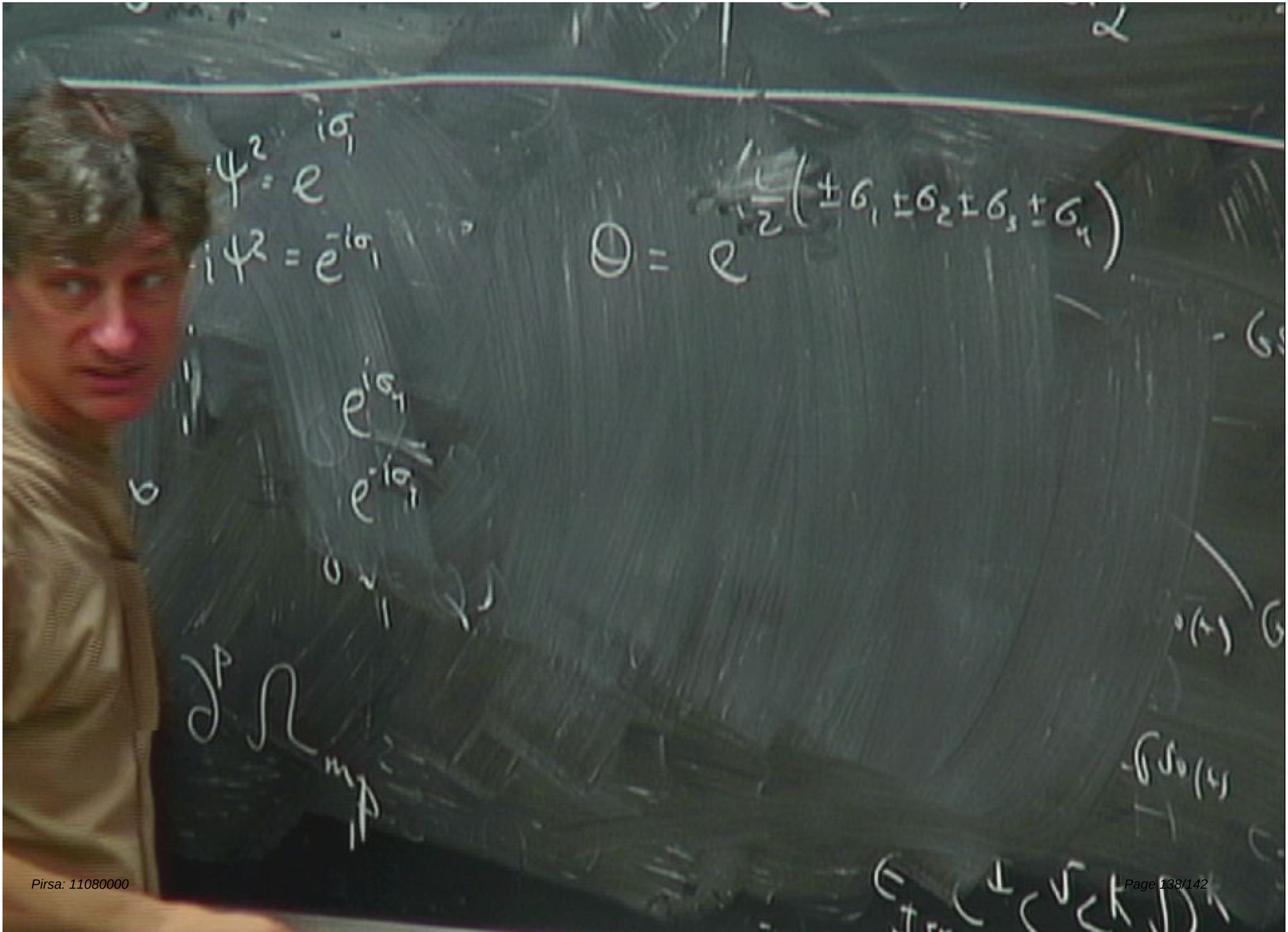
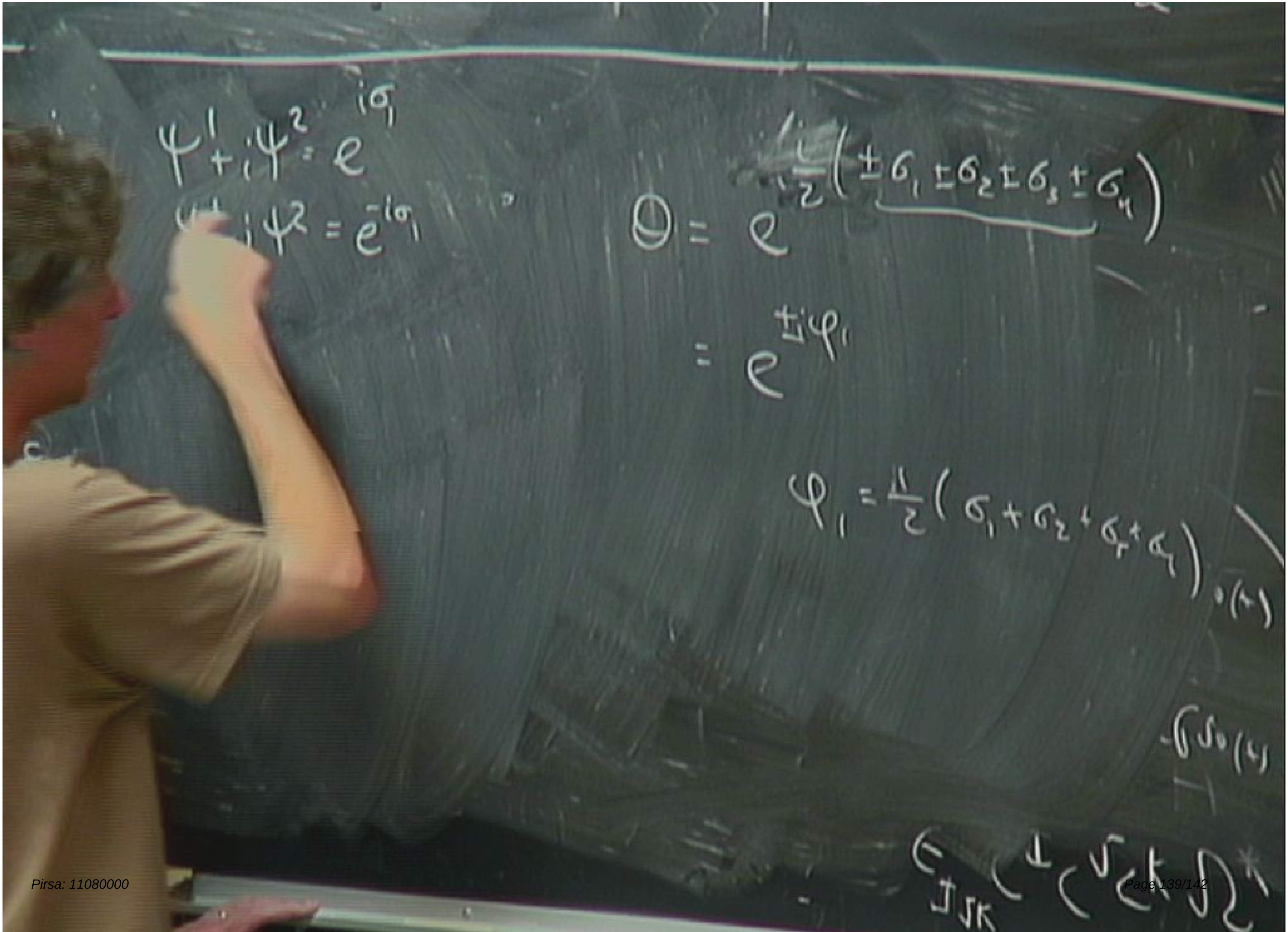

GSO(+)
R
GSO(+)
GSO(+)
GSO(+)
GSO(+)
GSO(+)
GSO(+)
GSO(+)
GSO(+)


L.C
ψ¹ + iψ²
ψ¹ − iψ²
∂ₘA = 0






$\psi^1 + i\psi^2 = e^{i\sigma_1}$
$\psi^1 - i\psi^2 = e^{-i\sigma_1}$
$\Theta = e^{\frac{i}{2}(\pm\sigma_1 \pm\sigma_2 \pm\sigma_3 \pm\sigma_4)}$
$= e^{\pm i\varphi_1}$
$\varphi_1 = \frac{1}{2}(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)$

$$\psi^{j} = \psi^1 + i\psi^2 = e^{i\sigma_1}$$

$$\psi^1 - i\psi^2 = e^{-i\sigma_1}$$

$$\Theta = e^{\frac{i}{2}(\pm\sigma_1 \pm \sigma_2 \pm \sigma_3 \pm \sigma_4)}$$

$$= e^{\pm i\varphi_1}$$

$$\varphi_1 = \frac{1}{2}(\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)$$

$$\int A^*$$