

Title: Mathematica Presentations

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Abstract:

④ Conformal gauge

⑦ Further points

What do we mean by an effective (Bog) String?

- Solution, any theory, string-like profile

- IR Classical solution, Soliton,
→ fluctuation

- Quantum string-state

Examples

Vortex solution
in AHM₁₊₁

Any domain wall
3d

Conformal gauge

- ⑥ Comparisons
- ⑦ Further points

What do we mean by an effective (Dug) String?

Solution, any theory, string-like profile

- IR Classical solution, Soliton,
→ Fluctuation
- Quantum string-state

Examples

Vortex solution
in AHM/LS
Any domain wall
3d

QCD flux tubes
($\bar{q}q$ -potential)



(3) Open strings
(4) Conformal gauge

(6) Comparisons
(7) Further points

What do we mean by an effective (low) string?

- Solution, any theory, string-like profile

- IR Classical solution, Soliton,
→ Fluctuation

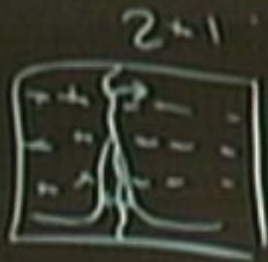
- Quantum string-state

Examples

→ Vortex solution
in AHM / in
any domain wall
3d

→ QCD flux tubes
($q\bar{q}$ -potential)
confining





- Approximate 1d string
- Defined at IR
- Stable, free
- among strings (large N)
- with bulk field (mass gap)
- Space-time symmetries
certain pattern

- Curiosity
- Applicative : Cosmic string, flux tubes
- Theoretical : help to better understand QFT

-
- Compare w/ LGT
 - AdS/CFT

The philosophy

bulk: IR field config. string-like
↕
~~thick~~ string config.

The philosophy

bulk: IR field config. string-like



~~thick~~ thick string config. (inner/structural dof)

W.S.:



1d string + Massive modes

The philosophy

bulk: IR field config. string-like



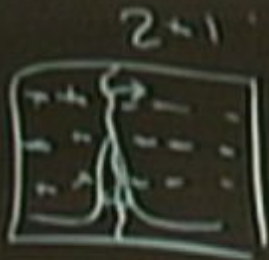
~~thick~~ thick string config. (inner/structural dof)



W.S.:

1d string + Massive modes

IR \rightarrow 1d string w/ eff. action



- Approximate 1d string
- Defined at IR
- Stable, free
 - among strings (charge N)
 - with bulk field (mass gap)
- Space-time symmetries
certain pattern

Method: EFT (Heinberg)
'guess' dof
+ symmetries

$$\chi_{SB} : SU(N_f) \times SU(N_f) \rightarrow SU(N_f)_{diag}$$

$$\rightarrow \pi \text{ 's } \Rightarrow S(\pi)$$

$$\frac{1}{2} \partial \pi \partial \pi + \frac{1}{f_\pi^2} \partial^2 \pi^4 + a_6 \partial^2 \pi^6 + \dots$$

$$\chi_{SB} : \underline{SU(N_f) \times SU(N_f)} \rightarrow SU(N_f)_{diag}$$

$$\rightarrow \pi^5 \Rightarrow S(\pi)$$

$$\frac{1}{2} \partial_\mu \pi \partial^\mu \pi + \frac{1}{f_\pi^2} \partial^2 \pi^4 + a_6 \partial^2 \pi^6 + \dots = \mathcal{L}_2 = \left(\frac{1}{f_\pi} \text{Tr}(\partial_\mu U) \partial^\mu U \right)$$

$S\pi$ - non-linear

$$U = e^{i \frac{\pi^a}{f_\pi} T^a}$$

on
 ns
 oints
 String?
 ralk
 ak
 v solution
 AHM/AA
 domain with
 3d
 Flux (with
 -potential)
 ref.

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 \quad (\text{derivative expansion})$$

$$\begin{aligned}
 \mathcal{L}_0 &= T \\
 \mathcal{L}_2 &= \frac{T}{2} \partial_\mu X \partial^\mu X \\
 \mathcal{L}_4 &= T \left[\begin{aligned} &C_2 (\partial_\mu X \partial^\mu X)^2 \\ &C_3 (\partial_\mu X \partial^\nu X)^2 \end{aligned} \right]
 \end{aligned}$$

$$\begin{aligned}
 \mathbb{R}^{1,D+1} &\propto SO(1, D+1) \\
 \rightarrow \mathbb{R}^{1,1} &\propto (SO(1,1) \times SO(D-2))
 \end{aligned}$$

$$\begin{aligned}
 \rightarrow \text{NGSBs} &\Rightarrow X^i, i=2, \dots, D-1 \\
 &C_2, C_3 \\
 &= ? \\
 &X^i \boxed{X^i} X^i
 \end{aligned}$$

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_4 \quad (\text{derivative expansion})$$

$$\mathcal{L}_0 = T$$

$$\mathcal{L}_2 = \frac{T}{2} \partial_\mu X^\mu \partial^\mu X^\mu$$

$$\mathcal{L}_4 = T \left[\begin{array}{l} C_2 (\partial_\mu X^\mu \partial^\mu X^\mu)^2 \\ C_3 (\partial_\mu X^\nu \partial_\nu X^\mu)^2 \end{array} \right]$$

$$\mathcal{L}_6 = T \left[\begin{array}{l} C_6 (\partial_\mu X^\nu \partial_\nu X^\mu)^3 \\ C_7 (\partial_\mu X^\nu \partial_\nu X^\mu \partial_\rho X^\sigma \partial_\sigma X^\rho)^2 + \end{array} \right]$$

$$\mathbb{R}^{1,D-1} \times S^1 \propto SO(1, D-1)$$

$$\rightarrow \mathbb{R}^{1,1} \times (S^1 \times S^1) \propto (SO(1,1) \times SO(2))$$

$$\rightarrow \text{NGSBs} \Rightarrow X^i, i=2, \dots, D-1$$

$$\dots = ?$$

$$\frac{C_1 (\partial_\mu X^\nu \partial_\nu X^\mu)^2 (\partial_\rho X^\sigma \partial_\sigma X^\rho)^2}{C_1 (\partial_\mu X^\nu \partial_\nu X^\mu)^2 (\partial_\rho X^\sigma \partial_\sigma X^\rho)^2} \quad \left[\begin{array}{l} X^i \\ X^j \end{array} \right]$$

— ADS/CFT

$$\text{E} \text{ O} \text{ L} \rightarrow \partial_\alpha \sim \frac{1}{R} \text{ , } \frac{1}{L}$$

\Rightarrow Scaling \equiv # of ∂_α - # of X_i

— ADS/CFT

$$\text{OL} \rightarrow \partial_\alpha \sim \frac{1}{R}, \frac{1}{L}$$

→ Scaling \equiv # of ∂_i - # of X_i

$S_D(1, D-1)$: Partitioning $X^+ - X^i, X^0 - X^i$

$$\delta G^+ = \delta X^+ = \epsilon X^+ \rightarrow$$
$$\delta X^+ = \epsilon X^+ = \epsilon G^+$$

$$\Rightarrow \delta(\partial_\alpha X^i) = \epsilon \delta_{\alpha 0} \delta^{i0}$$
$$+ \epsilon \partial_\alpha (X^+ \delta_i X^i)$$

— ADS/CFT

$$C_2 = \frac{1}{8}, C_3 = -\frac{1}{4}, C_4 = -\frac{1}{16}, C_5 = \frac{1}{8}$$

$SD(1, D-1) = \int d^D x \sqrt{g} \left[\frac{1}{2} R - \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \right]$

$$\delta S = \int d^D x \sqrt{g} \left[\delta R - \partial_\mu \delta \phi \partial^\mu \phi - \delta V \right]$$
$$\delta R = \delta g^{\mu\nu} R_{\mu\nu} = \delta g^{\mu\nu} \left(\partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\lambda\nu} + \Gamma^\lambda_{\mu\nu} \Gamma^\sigma_{\lambda\rho} - \Gamma^\sigma_{\mu\rho} \Gamma^\lambda_{\lambda\nu} \right)$$
$$\delta S = \int d^D x \sqrt{g} \left[\delta g^{\mu\nu} \left(\partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\lambda\nu} + \Gamma^\lambda_{\mu\nu} \Gamma^\sigma_{\lambda\rho} - \Gamma^\sigma_{\mu\rho} \Gamma^\lambda_{\lambda\nu} \right) - \partial_\mu \delta \phi \partial^\mu \phi - \delta V \right]$$
$$\Rightarrow \delta S(\partial_\mu X^i) = \int d^D x \sqrt{g} \left[\delta g^{\mu\nu} \delta_{ij} \partial_\mu \partial_\nu X^i + \delta g^{\mu\nu} \partial_\mu X^i \partial_\nu X^j \right]$$

— ADS/CFT

$$c_2 = \frac{1}{8}, c_7 = -\frac{1}{4}, c_8 = -\frac{1}{16}, c_9 = \frac{1}{8}$$

$$\mathcal{L} = T \sqrt{1 + \frac{1}{2}(\partial_\mu X \partial^\mu X) + \frac{1}{2}(\partial_\mu X \partial^\mu X)^2 - \frac{1}{2}(\partial_\mu X \partial_\nu X)^2}$$

SD(1, D-1) = ...

$$\delta G^1 = \delta X^1 = \epsilon X^1 \Rightarrow \delta(\partial_\mu X^1) = \epsilon \partial_\mu \delta X^1$$
$$\delta X^2 = \epsilon X^2 \Rightarrow \delta(\partial_\mu X^2) = \epsilon \partial_\mu (X^2 \delta X^1) + \epsilon \partial_\mu (X^1 \delta X^2)$$

- ③ Open Strings
- ④ Conformal gauge

- ⑤ Quantization
- ⑥ Comparisons
- ⑦ Further points

Open Strings



Dirichlet
Neumann

Sol^o → $\mathcal{L}_\sigma = \mu + b_1 \partial_\tau X \partial_\tau X + b_2 (\partial_\tau X \partial_\sigma X)^2 + b_3 \partial_\sigma X \partial_\sigma X$

$b_1 = b_2 = 0$

$\left(\begin{array}{l} \partial_\sigma X|_0 = 0 \\ \partial_\sigma X|_1 = 0 \end{array} \right)$

$\mathcal{L}_{\partial X} = \mu + a_1 \partial_\tau X \partial_\tau X + a_2 \partial_\sigma^2 X \partial_\sigma^2 X - a_3 (\partial_\tau X \partial_\sigma X)^2$

$a_1 = \frac{\mu}{2}, a_2 = -\frac{1}{4}$

→ $\mu \sqrt{1 + \partial_\sigma X \partial_\sigma X}$



- AdS/CFT

$X^M(\sigma, \tau)$ \rightarrow re-parametrization invariance

- diff inv. formalism ($g_{\mu\nu} = \partial_\mu X^\alpha \partial_\nu X^\beta g'_{\alpha\beta}$)

$R \int D R \frac{e^{-S}}{\sqrt{g}}$ $\stackrel{?}{=} \int D R \sqrt{g}$

- $X^\alpha = G^\alpha \Rightarrow \checkmark$

- Conformal gauge

④ Conformal gauge

⑥ Comparisons
⑦ Further points

Polyakov: $g_{ab} = e^{\phi} \delta_{ab}$ (CG)

$\leftarrow g_{+-} = g_{-+} = 0$

NG: $h_{+-} = h_{-+} = 0, h_{ab} = 2\partial_a X \cdot \partial_b X$

$\partial_{\pm} = \frac{1}{2}(\partial_0 \pm \partial_1)$

Most general of $\mathcal{L}_{NG} = T \partial_a X \cdot \partial_a X$

$\mathcal{L}_{CG} = \frac{1}{2} \partial_a X \cdot \partial_a X$

$C = 0$
 $\mathcal{L}_{NG} = \mathcal{L}_{CG}$

Singular Operators ✓

if they are regular upon expansion

$\left(\begin{array}{c} \text{---} \\ \text{---} \end{array} \right) \rightarrow \text{large } \forall \epsilon$

$$\mathcal{L} = T \sqrt{1 + \frac{1}{2}(\partial_\mu X \cdot \partial^\mu X) + \frac{1}{2}(\partial_\mu X \cdot \partial^\mu X)^2 - \frac{1}{2}(\partial_\mu X \cdot \partial_\nu X)^2}$$

$\mathcal{L}_{NG} \sim \mathbb{R}^0$ unique

$$\rightarrow \mathbb{R}^2 \quad \mathbb{P} \frac{\partial_\mu \partial_\nu Z}{Z} \quad \leftrightarrow \quad \mathbb{P} \frac{\partial_\mu Z \partial_\nu Z}{Z^2} \quad (\text{i.b.p.})$$

$$\mathcal{L} = T \sqrt{1 + \frac{1}{2}(\partial_\mu X \cdot \partial^\mu X) + \frac{1}{2}(\partial_\mu X \cdot \partial^\mu X)^2 - \frac{1}{2}(\partial_\mu X \cdot \partial_\nu X)^2}$$

$\mathcal{L}_{NG} \sim \mathbb{R}^0$ unique

$$\rightarrow \mathbb{R}^{-2} \quad \mathbb{P} \frac{\partial_\mu \partial_\nu \mathcal{L}}{\mathcal{L}^2} \leftrightarrow \mathbb{P} \frac{\partial_\mu \mathcal{L} \partial_\nu \mathcal{L}}{\mathcal{L}^2} \quad (\text{i.b.p.})$$

- Conf. at the g -level $\Rightarrow \beta = \frac{26-D}{12}$

$L_{NG} \sim R^0$ unique

$$\rightarrow R^{-2} \quad \mathbb{P} \frac{\partial_+ \partial_- Z}{Z} \leftrightarrow \mathbb{P} \frac{\partial_+ Z \partial_- Z}{Z^2} \quad (\text{i.b.p.})$$

- Conf. at the g -level $\Rightarrow R = \frac{26-D}{12}$

$$\frac{26-D}{12} \int d^2x (\partial_\mu \phi \partial_\mu \phi + m^2 e^\phi) \Rightarrow \text{Only physical dof}$$

$g_{2,2} = e^\phi \delta_{2,2} \quad e^\phi \rightarrow Z$

(conformal)

$X^M(\sigma, \tau)$ re-parametrization invariance

- diffeomorphism formalism ($\mathcal{L}_{\text{diff}} = \int d\sigma d\tau \sqrt{-g} \mathcal{L}$)

$$R \int d\sigma d\tau \sqrt{-g} \mathcal{L} = \int d^4x \sqrt{-g} \mathcal{L}$$

- $X^\alpha = G^\alpha \rightarrow \checkmark$ $m^2 \rightarrow \frac{T}{\alpha}$

- Conformal gauge

$X^M(\sigma, \tau)$ re-parametrization invariance

- diffeomorphism formalism ($g_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$)

$$R \int D R \frac{e^{\frac{c_0}{5g}}}{5g} \checkmark \quad = \int \sqrt{5} R \sqrt{5} K K$$

- $X^\alpha = G^\alpha \Rightarrow \checkmark$

- Conformal gauge $m^2 \rightarrow \frac{T}{\alpha}$



④ Conformal gauge

⑥ Comparisons
⑦ Further points

- Weakly coupled QFT

- LGT's $(\sim \frac{1}{R^D})$

- AdS/CFT



$$C_n \sim \frac{26-D}{112 \pi^2}$$

④ Conformal gauge

⑥ Comparisons
⑦ Further points

- Anomaly? Non-critical string in $N=6$
- Cosmic strings, High-spin mesons

$X^\mu(\sigma, \tau)$ re-parametrization invariance

R^{-2}

- dirac inv. formalism ($g_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$)

$$R \int \sqrt{-g} \mathcal{L} = \int \sqrt{-g} \mathcal{L} \quad \cdot = \int \sqrt{-g} \mathcal{L}$$

- $X^\alpha = \sigma^\alpha \Rightarrow \checkmark$

- Conformal gauge

$$m^2 \rightarrow \frac{T}{\alpha'}$$

④ Conformal field

⑦ Further points

- Anomaly? Non-critical string in $N=6$
- Cosmic string, High-spin mesons

Symmetry $d=6$

L6

NO OPEN FLAMES



④ Conformal field

⑦ Further points

- Anomaly? Non-critical string in NG
- Cosmic strings, High-spin mesons

Sanomaly $d=6$
 $S \rightarrow S_{NG}$
 ϕ, g_{ab}

