Title: Intro to Mathematica, example

Date: Jul 31, 2011  12:15 PM

URL: http://pirsa.org/11070095

Abstract:

# Generating Functions

$$\frac{A_n^{N^kMHV}}{A_n^{MHV}} = P_{n,k}\left[z_i, \eta_i^{I=1\ldots4}\right]$$

$i = 1 \ldots n$

Pol of degree 4

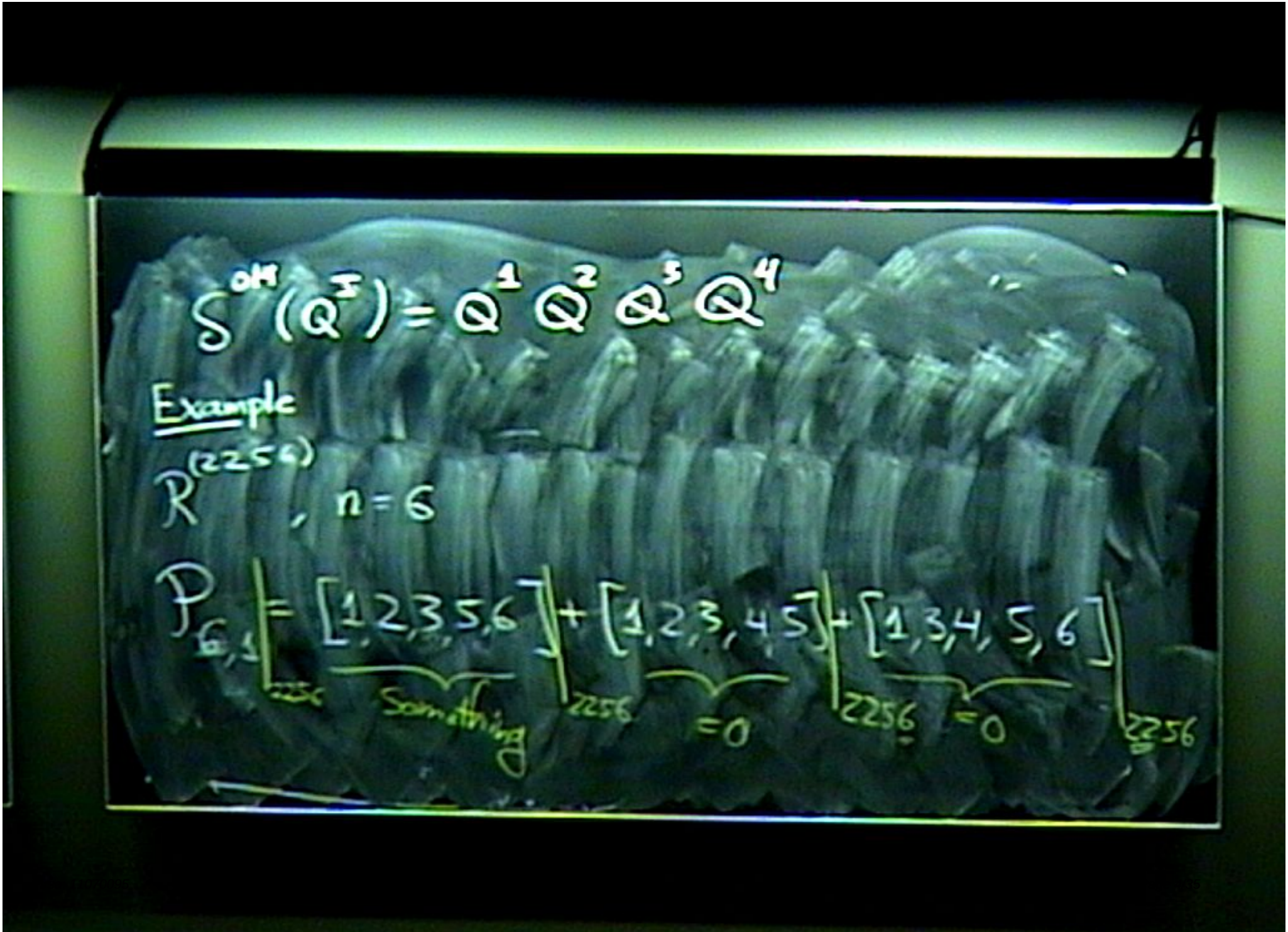
$$= \eta_2^{(1)}\eta_2^{(2)}\eta_5^{(3)}\eta_6^{(4)} R^{(2,5,6)}[z_i] + \ldots$$

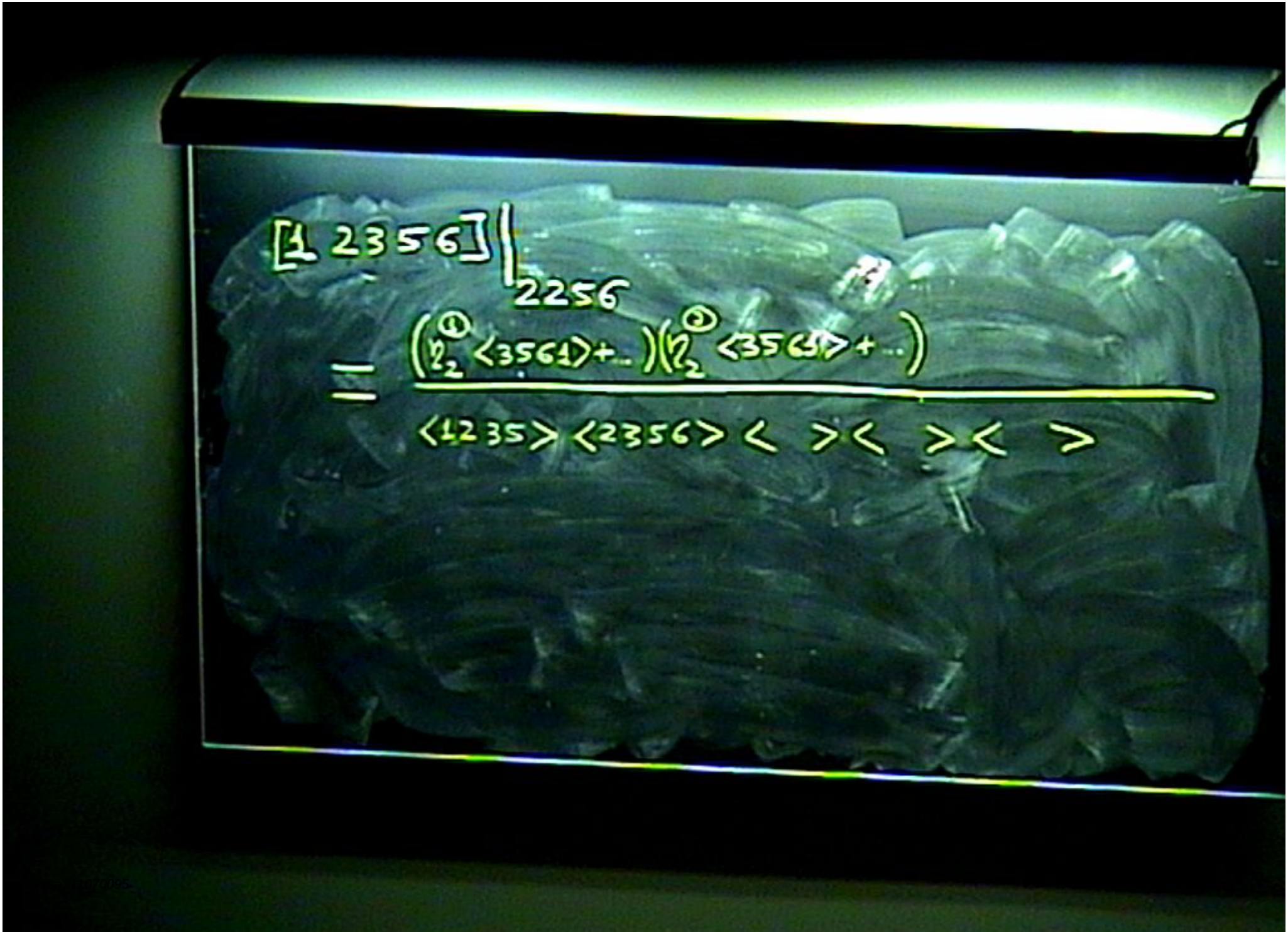
= Component

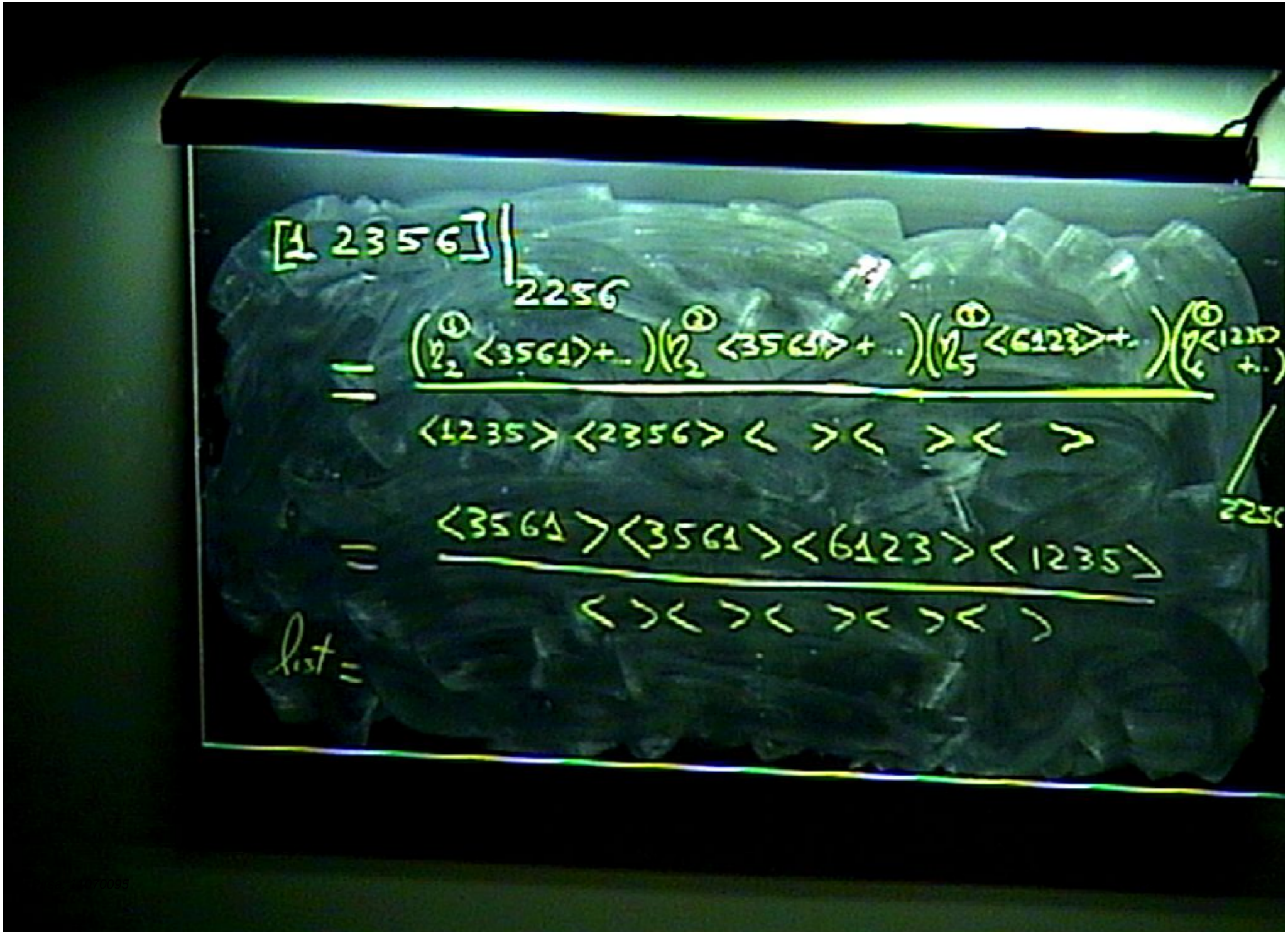
$$P_{n,1} = \sum_{i=2}^{n-2} \sum_{j=i+2}^{n-1} [1, i, i+1, j, j+1]$$

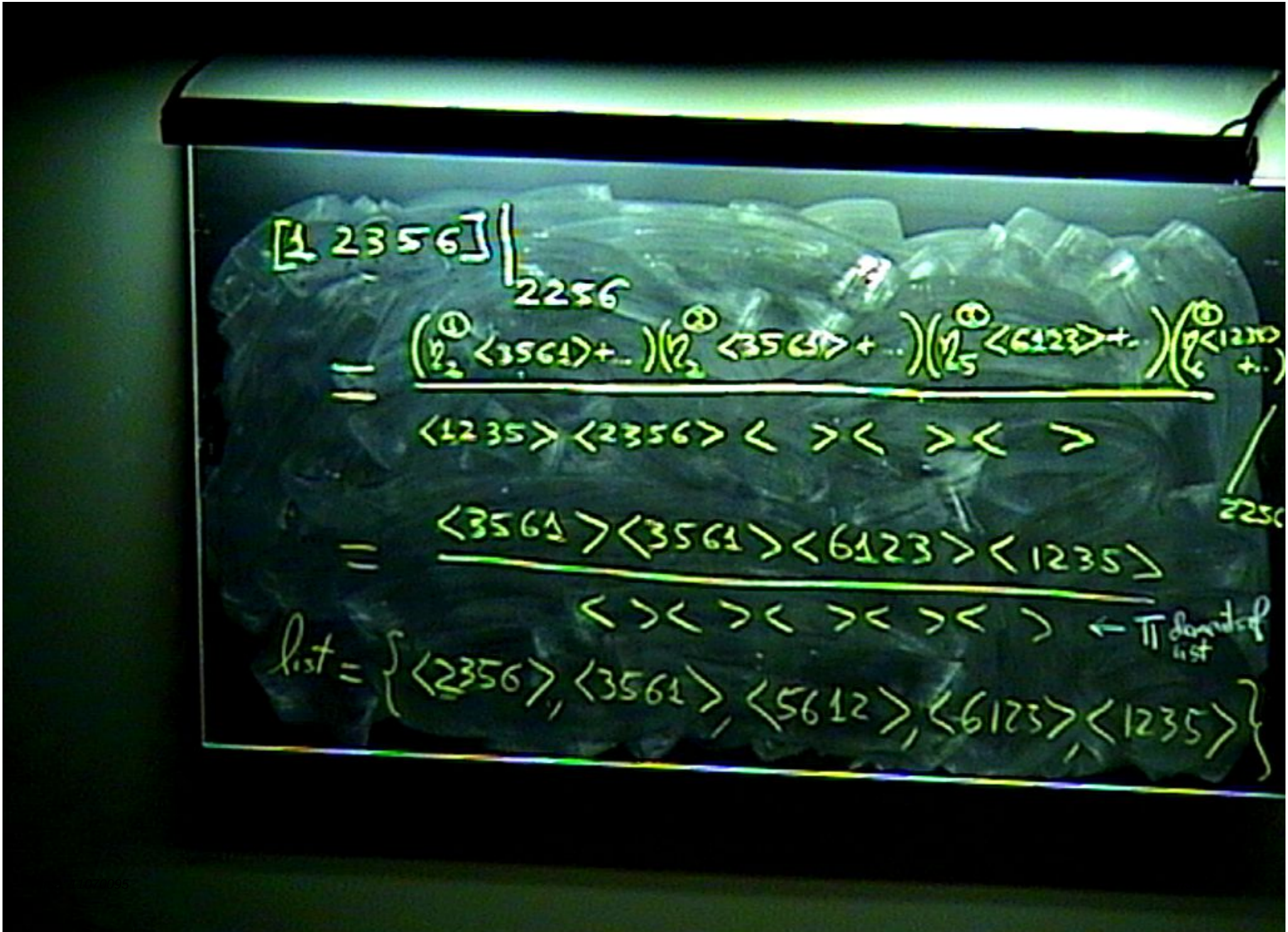
$$[a,b,c,d,e] = \frac{\delta^{0|4}(\langle abcd\rangle \eta_e^I + \text{cyclic})}{\langle abcd\rangle\langle bcde\rangle\langle cdea\rangle\langle deab\rangle\langle eabc\rangle}$$

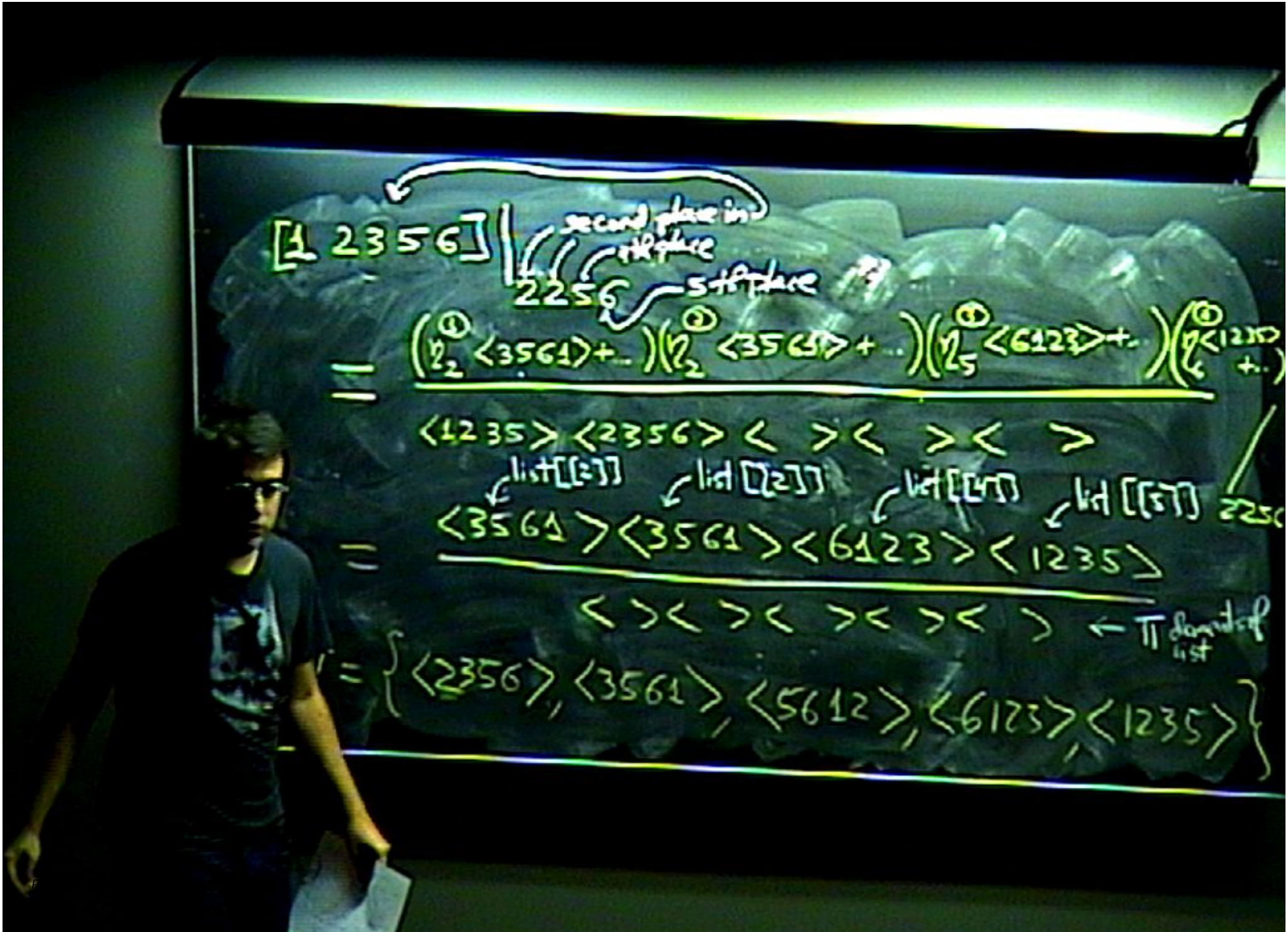
$$\det(Z_a, Z_b, Z_c, Z_d)$$

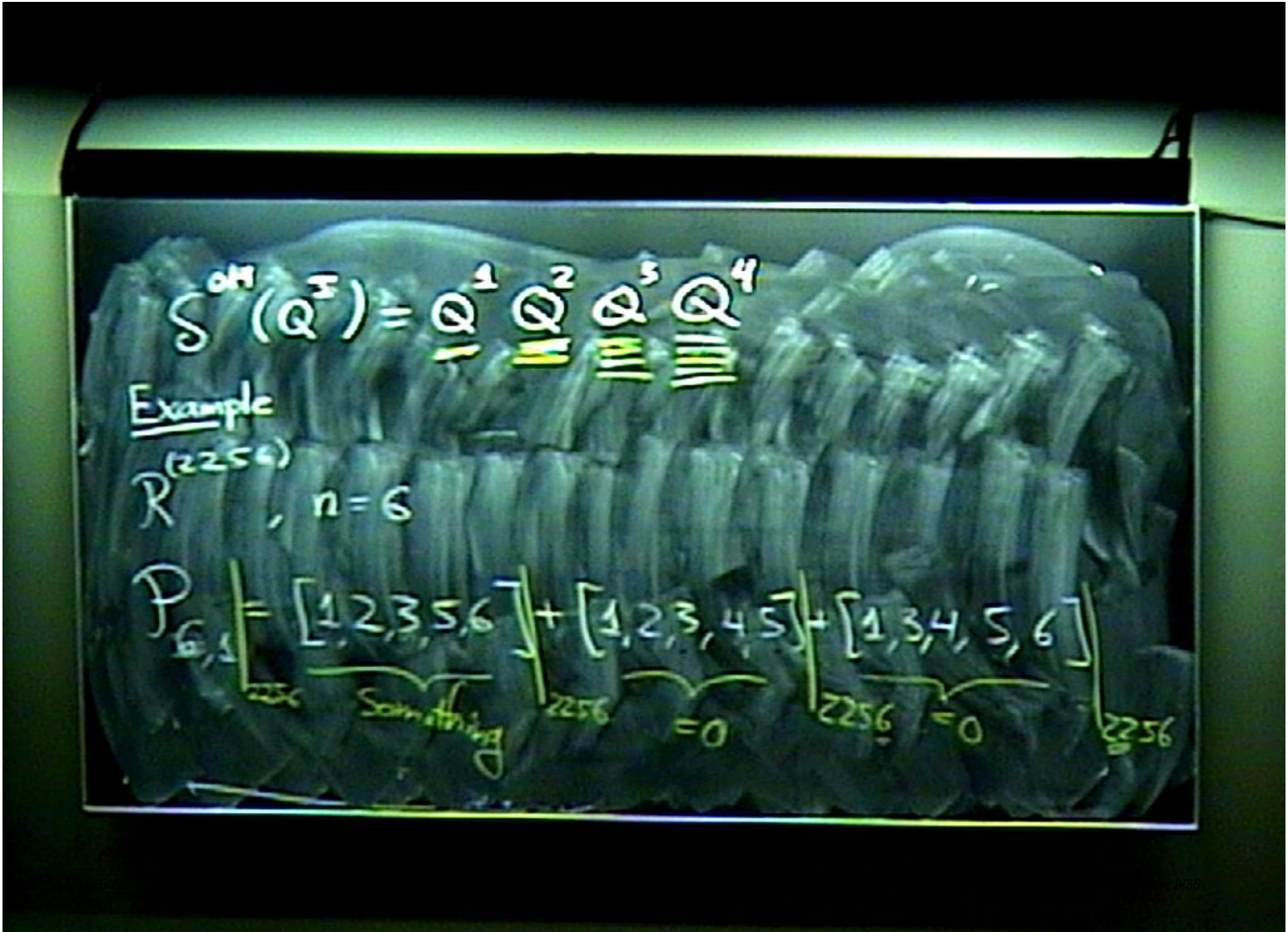

$S^{OM}(Q^3) = Q^1 Q^2 Q^3 Q^4$
Example
$R^{(2256)}$, $n = 6$
$P_{6,1}$
$= [1,2,3,5,6]$
Something
$+ [1,2,3,4,5]$
$= 0$
$+ [1,3,4,5,6]$
$= 0$
2256


[1 2 3 5 6]
2256
= (η₂^① ⟨3561⟩ + ..)(η₂^② ⟨3567⟩ + ..)
⟨1235⟩⟨2356⟩⟨ ⟩⟨ ⟩⟨ ⟩


[1 2 3 5 6]
2256
= (η₂^① ⟨3561⟩+..)(η₂^① ⟨3561⟩+..)(η₅^① ⟨6123⟩+..)(η₆^① ⟨1235⟩+..)
⟨1235⟩⟨2356⟩⟨ ⟩⟨ ⟩⟨ ⟩
= ⟨3561⟩⟨3561⟩⟨6123⟩⟨1235⟩
⟨ ⟩⟨ ⟩⟨ ⟩⟨ ⟩⟨ ⟩
list =


[1 2 3 5 6]
2256
⟨1235⟩ ⟨2356⟩
⟨3561⟩ ⟨3561⟩ ⟨6123⟩ ⟨1235⟩
list = {⟨2356⟩, ⟨3561⟩, ⟨5612⟩, ⟨6123⟩, ⟨1235⟩}

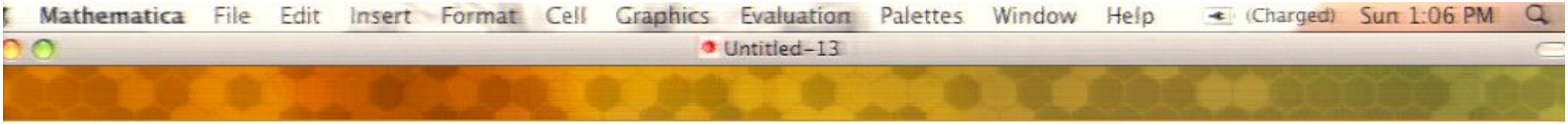

[1 2 3 5 6]
second place in
1st place
5th place
2256
= (η₂⁽¹⁾ <3561>+..)(η₂⁽²⁾ <3561>+..)(η₅⁽¹⁾ <6123>+..)(η₄⁽¹⁾ <1235>+..)
<1235> <2356> < > < > < >
list[[3]]
list[[2]]
list[[4]]
list[[5]]
= <3561><3561><6123><1235>
< >< >< >< >< > ← Π domain of list
= {<2356>, <3561>, <5612>, <6123>, <1235>}


$S^{OH}(Q^3) = Q^1 Q^2 Q^5 Q^4$
Example
$R^{(2256)}$, $n = 6$
$P_{6,1}\Big|_{2256} = [1,2,3,5,6]\Big|_{2256} + [1,2,3,4,5]\Big|_{2256} + [1,3,4,5,6]\Big|_{2256}$
Something
$= 0$
$= 0$

# Extracting NMHV components

In[91]:=
```
NMHVTree[6][2, 2, 5, 6]
```

Out[91]=
$$\frac{\langle 3,4,5,6\rangle}{\langle 4,5,6,2\rangle\langle 5,6,2,3\rangle} + \frac{\langle 4,5,6,7\rangle}{\langle 2,4,5,6\rangle\langle 5,6,7,2\rangle}$$




# Extracting NMHV components

In[91]:=
```
NMHVTree[6][2, 2, 5, 6]
```

In[1]:=
```
NMHVTree[n_][b_] := Sum[R[1, i, i + 1, j, j + 1][b], {i, 2, n - 2}, {j, i + 2, n - 1}]
```

In[3]:=
```
NMHVTree[6][2, 2, 5, 6]
```

Out[3]=
```
NMHVTree[6][2, 2, 5, 6]
```

In[91]:=
```
NMHVTree[6][2, 2, 5, 6]
```

In[4]:=
```
NMHVTree[n_][b__] :=
 Sum[R[1, i, i + 1, j, j + 1][b], {i, 2, n - 2},
  {j, i + 2, n - 1}]
```

In[5]:=
```
NMHVTree[6][2, 2, 5, 6]
```

Out[5]=
```
R[1, 2, 3, 4, 5][2, 2, 5, 6] +
 R[1, 2, 3, 5, 6][2, 2, 5, 6] + R[1, 3, 4, 5, 6][2, 2, 5, 6]
```

```
R[a__][b__] := If["a does contain all elements in b",
  "extrac it", 0]
```

In[6]:=
```
MemberQ[{1, 2, 3, 5, 6}, 2]
```

Out[6]=
```
True
```

```
  "extrac it", 0]
```

```
MemberQ[{1, 2, 3, 5, 6}, 2]
MemberQ[{1, 2, 3, 5, 6}, 2]
MemberQ[{1, 2, 3, 5, 6}, 5]
MemberQ[{1, 2, 3, 5, 6}, 6]
```

Out[12]=

```
True
```

Out[13]=

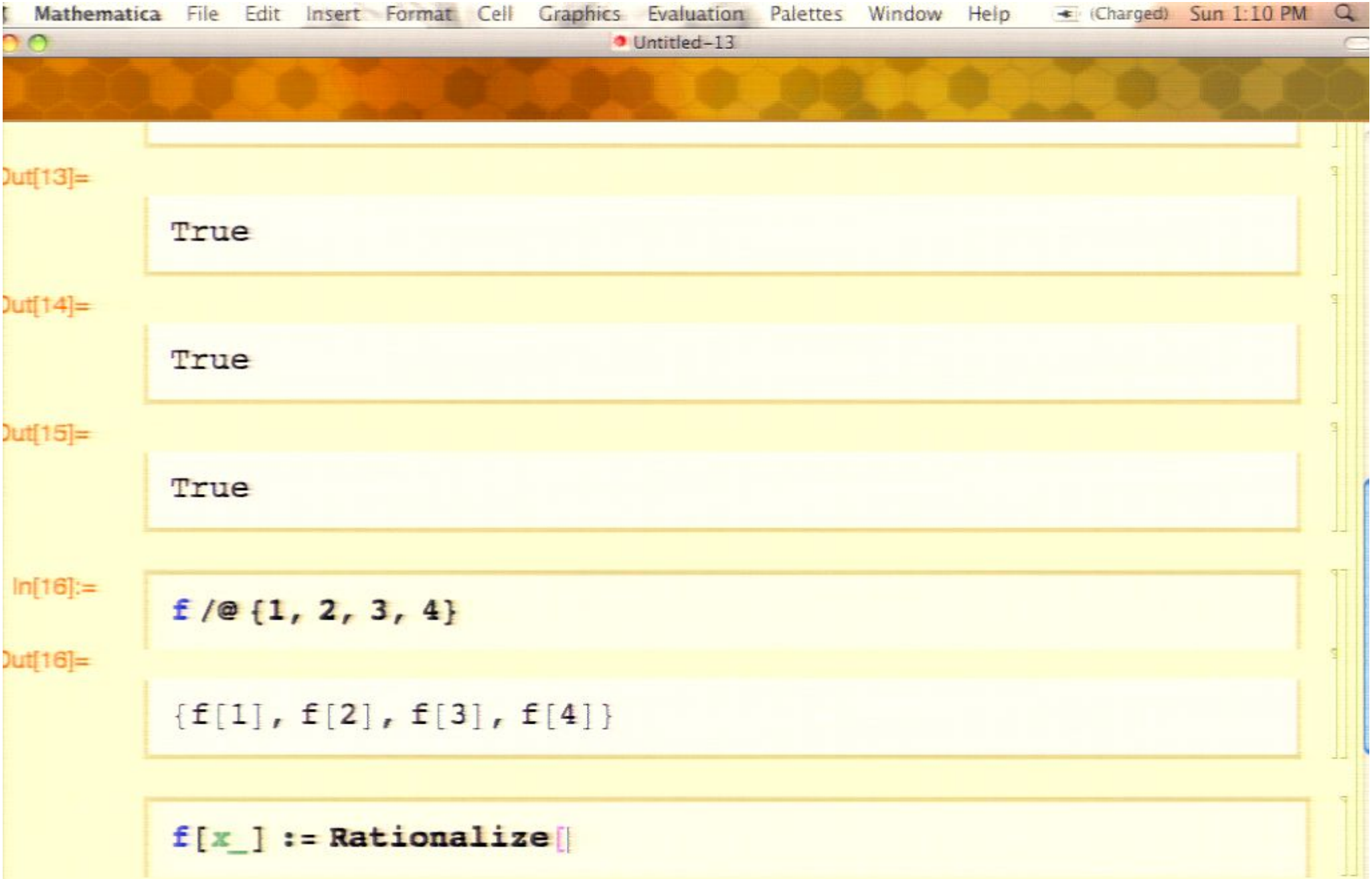
```
True
```

Out[14]=

```
True
```

Out[15]=

```
True
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (Charged) Sun 1:10 PM
Untitled-13
Out[13]=
True
Out[14]=
True
Out[15]=
True
In[16]:=
f /@ {1, 2, 3, 4}
Out[16]=
{f[1], f[2], f[3], f[4]}
f[x_] := Rationalize[

```
x
```

In[24]:=
```
f = Rationalize;
```

In[27]:=
```
ClearAll[f]
```

In[34]:=
```
f1[x_] := {Rationalize[x], x}
f2 := x ↦ {Rationalize[x], x}
f3 = {Rationalize[#], #} &;
```

```
f1[.2]
f2[.2]
f2[.2]
```

Out[32]=
$$\left\{\frac{1}{5}, 0.2\right\}$$

Out[33]=

In[27]:=
```
ClearAll[f]
```

In[34]:=
```
f1[x_] := {Rationalize[x], x}
f2 := x ↦ {Rationalize[x], x}
f3 = {Rationalize[#], #} &;
```

In[40]:=
```
f1[.2]
f2[.2]
f3[.2]
```

Out[40]=
$$\left\{\frac{1}{5}, 0.2\right\}$$

Out[41]=
$$\left\{\frac{1}{5}, 0.2\right\}$$

Out[42]=
$$\left\{\frac{1}{5}, 0.2\right\}$$

In[27]:=
```
ClearAll[f]
```

In[34]:=
```
f1[x_] := {Rationalize[x], x}
f2 := x ↦ {Rationalize[x], x}
f3 = {Rationalize[#], #} &;
```

In[40]:=
```
f1[.2]
f2[.2]
f3[.2]
```

Out[40]=

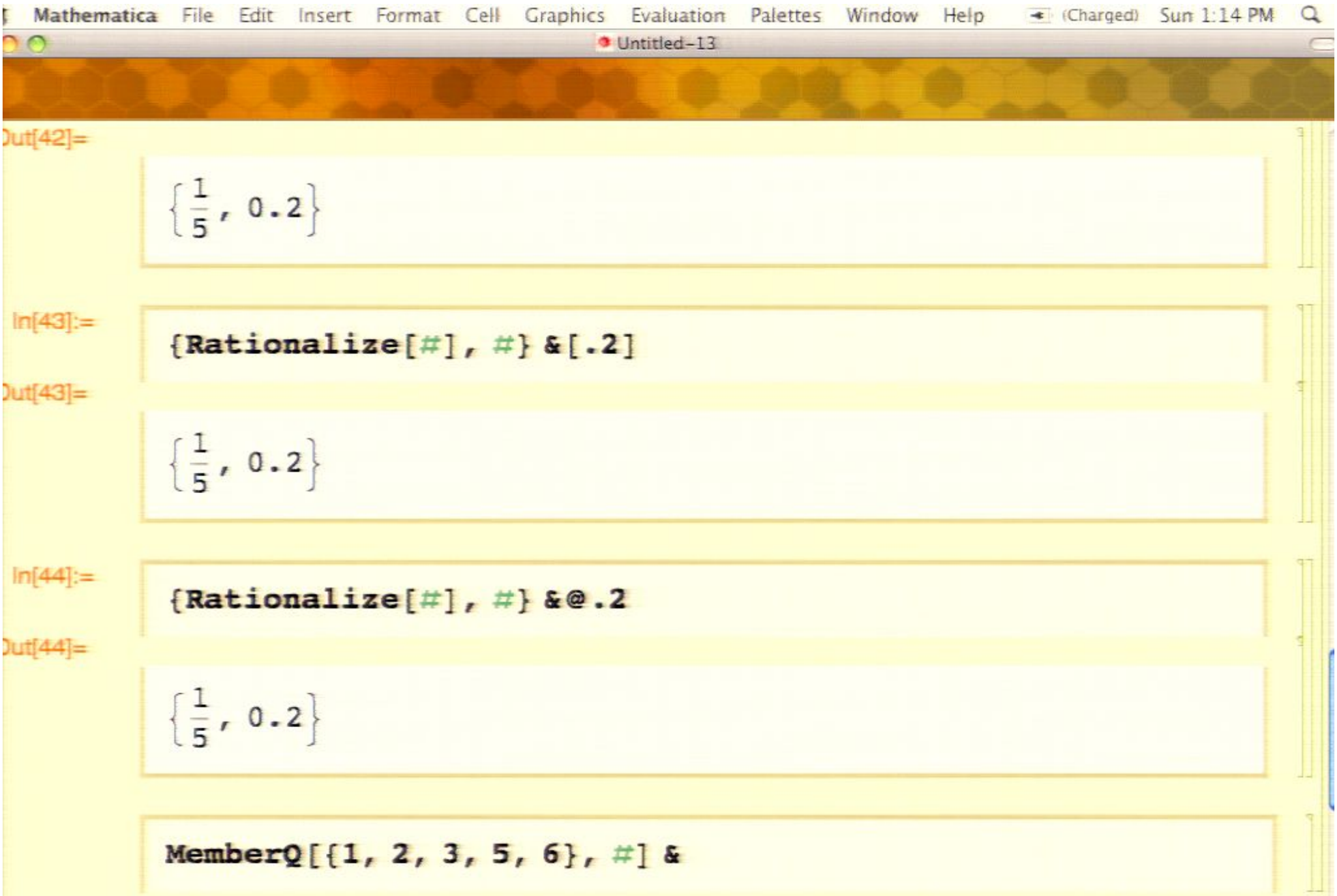
$$\left\{\frac{1}{5}, 0.2\right\}$$

Out[41]=

$$\left\{\frac{1}{5}, 0.2\right\}$$

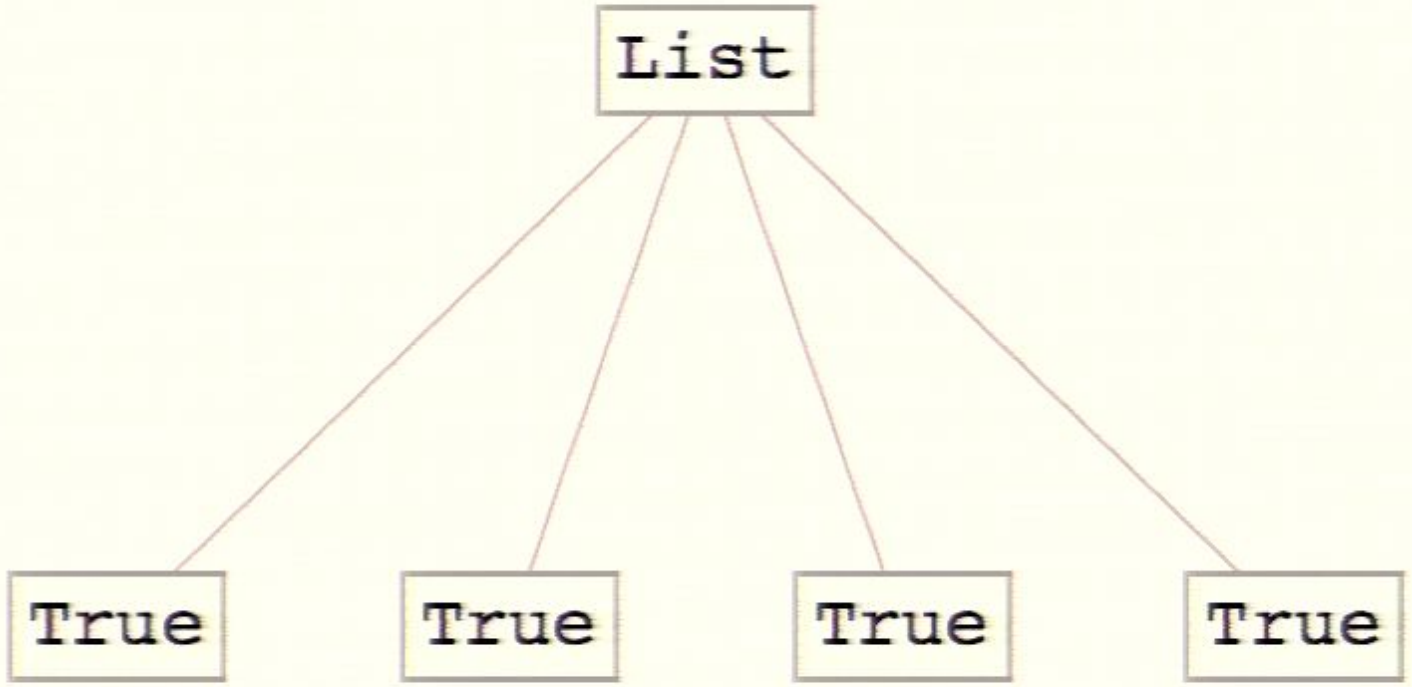
Out[42]=

$$\left\{\frac{1}{5}, 0.2\right\}$$

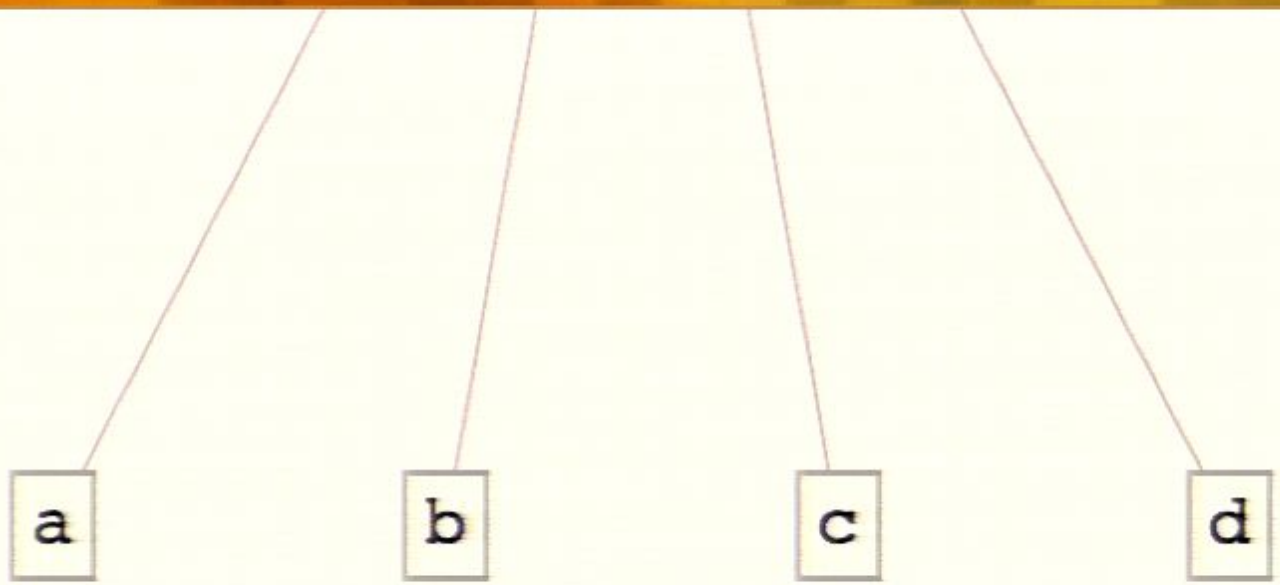

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (Charged) Sun 1:14 PM
Untitled-13
Out[42]=
{1/5, 0.2}
In[43]:=
{Rationalize[#], #} &[.2]
Out[43]=
{1/5, 0.2}
In[44]:=
{Rationalize[#], #} &@.2
Out[44]=
{1/5, 0.2}
MemberQ[{1, 2, 3, 5, 6}, #] &

```
MemberQ[{1, 2, 3, 5, 6}, #] & /@ {2, 2, 5, 6} //
 TreeForm
```

Out[52]//TreeForm=




```
And[a, b, c
```




```
MemberQ[{1, 2, 3, 5, 6}, #] & /@ {2, 2, 5, 6}
```

```
Sum[R[1, i, i + 1, j, j + 1][b], {i, 2, n - 2},
  {j, i + 2, n - 1}]
```

In[60]:=

```
R[a__][b__] := If[And @@ (MemberQ[{a}, #] & /@ {b}),
  R[a][b], 0]
```

In[61]:=

```
NMHVTree[6][2, 2, 5, 6]
```

Out[61]=

```
R[1, 2, 3, 5, 6][2, 2, 5, 6]
```

In[62]:=

```
list = {1, 2, 3, 5, 6}
```

Out[62]=

```
{1, 2, 3, 5, 6}
```

```
i
```

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Untitled-13

In[64]:=

```
list = ⟨1, 2, 3, 5, 6⟩
```

Out[64]=

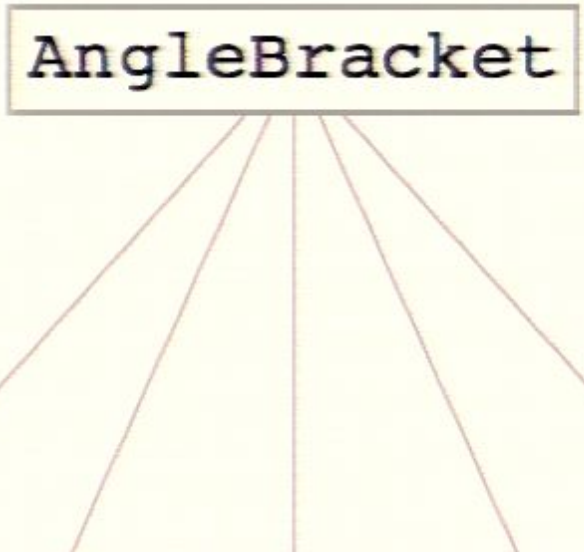
```
⟨1, 2, 3, 5, 6⟩
```

```
list
```

In[65]:=

```
list // TreeForm
```

Out[65]//TreeForm=



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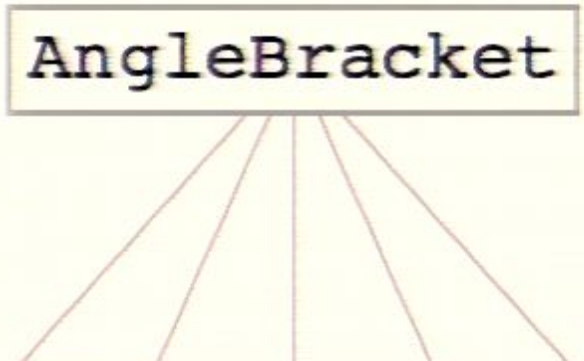
Untitled-13

In[64]:=
```
list = ⟨1, 2, 3, 5, 6⟩
```
Out[64]=
```
⟨1, 2, 3, 5, 6⟩
```

In[66]:=
```
RotateLeft[list, 1]
```
Out[66]=
```
⟨2, 3, 5, 6, 1⟩
```

In[65]:=
```
list // TreeForm
```
Out[65]//TreeForm=



In[67]:=
```
RotateLeft[list, 1]
RotateLeft[list, 2]
```

Out[67]=
```
⟨2, 3, 5, 6, 1⟩
```

Out[68]=
```
⟨3, 5, 6, 1, 2⟩
```

```
⟨1, 2, 3, 5, 6⟩[[4]]
⟨1, 2, 3, 5, 6⟩[[4]]
⟨1, 2, 3, 5, 6⟩[[1 ;; 4]]
```

Out[70]=
```
5
```

Out[71]=
```
5
```



In[65]:=
```
list // TreeForm
```

```
⟨1, 2, 3, 5, 6⟩
```

```
Table[RotateLeft[list, j][[1 ;; 4]], {j, 5
   RotateLeft[list, 2]
```

Out[75]=

```
⟨2, 3, 5, 6⟩
```

Out[76]=

```
⟨3, 5, 6, 1, 2⟩
```

In[72]:=

```
⟨1, 2, 3, 5, 6⟩[[4]]
⟨1, 2, 3, 5, 6⟩[[4]]
⟨1, 2, 3, 5, 6⟩[[1 ;; 4]]
```

Out[72]=

```
5
```

Out[73]=

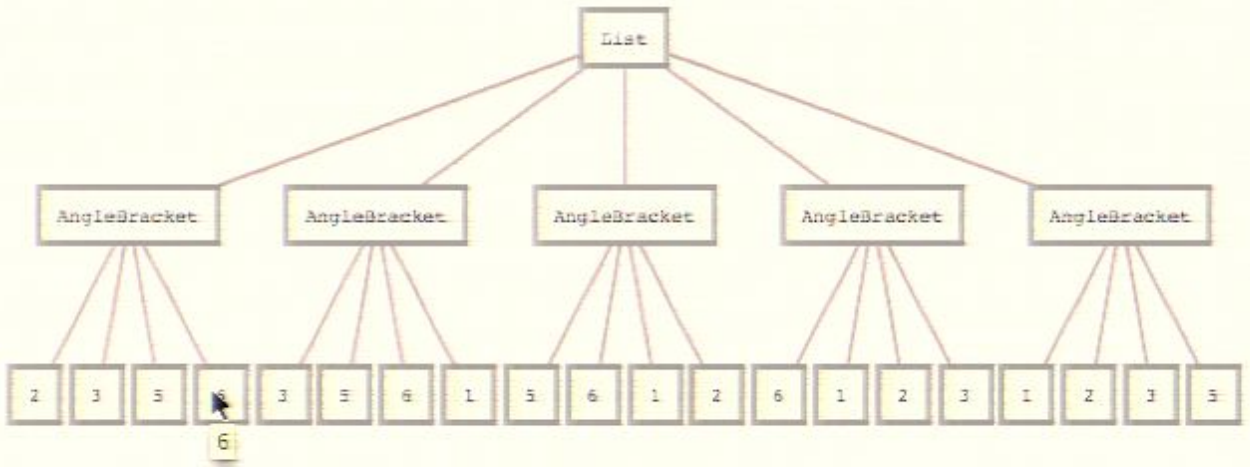
In[80]:=
```
list = Table[RotateLeft[aa, j][[1 ;; 4]], {j, 5}]
```

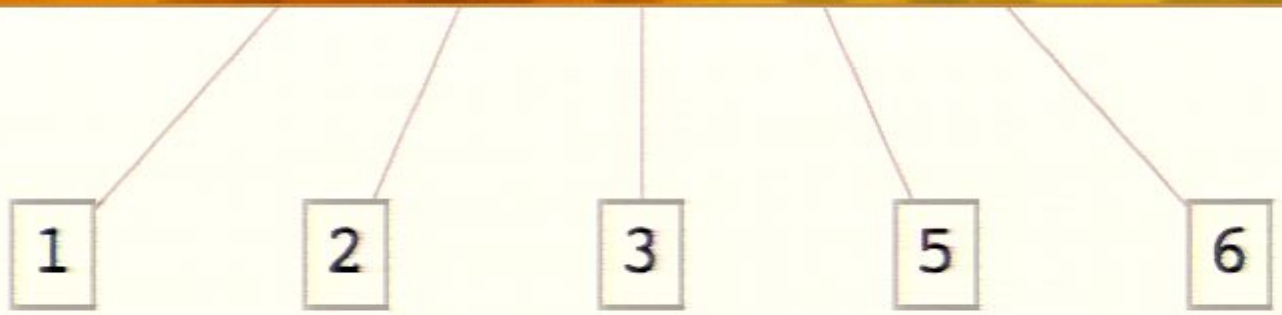
Out[80]=
```
{⟨2, 3, 5, 6⟩, ⟨3, 5, 6, 1⟩,
 ⟨5, 6, 1, 2⟩, ⟨6, 1, 2, 3⟩, ⟨1, 2, 3, 5⟩}
```

## Denominator

In[81]:=
```
list // TreeForm
```

Out[81]//TreeForm=






```
Position[aa, 2] :[[1]]
   Position[aa,2]
   Position[aa,5]
   Position[aa,6]
```

Out[93]=

```
{{2}}
```

Out[94]=

```
{{2}}
```

Out[95]=

```
{{4}}
```

$\langle 3, 5, 6, 1\rangle \langle 5, 6, 1, 2\rangle \langle 6, 1, 2, 3\rangle$

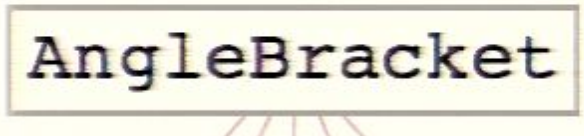
In[83]:=
```
list[[2]] list[[2]] list[[4]] list[[5]]
```
Out[83]=

$\langle 1, 2, 3, 5\rangle \langle 3, 5, 6, 1\rangle^2 \langle 6, 1, 2, 3\rangle$

In[105]:=
```
Product[list[[ Position[aa, {2, 2, 5, 6}[[j]]][[1, 1]] ]],
 {j, 4}]
```
Out[105]=

$\langle 1, 2, 3, 5\rangle \langle 3, 5, 6, 1\rangle^2 \langle 6, 1, 2, 3\rangle$

In[87]:=
```
aa // TreeForm
```
Out[87]//TreeForm=



# Extracting NMHV components

In[91]:=
```
NMHVTree[6][2, 2, 5, 6]
```

In[4]:=
```
NMHVTree[n_][b__] :=
 Sum[R[1, i, i + 1, j, j + 1][b], {i, 2, n - 2},
  {j, i + 2, n - 1}]
```

In[60]:=
```
R[a__][b__] := If[And @@ (MemberQ[{a}, #] & /@ {b}),
  R[a][b], 0]
```

In[61]:=
```
NMHVTree[6][2, 2, 5, 6]
```

Out[61]=
```
R[1, 2, 3, 5, 6][2, 2, 5, 6]
```



In[79]:=
```
aa = {1, 2, 3, 5, 6}
```

$$\text{NMHVTree2}[n\_][b\_\_] := \sum_{i=3} \sum_{j=i+2} \text{R}[2, i, i+1, j, j+1][b]$$

In[120]:=

```
temp1 = NMHVTree[6][2, 2, 2, 3]
temp2 = NMHVTree2[6][2, 2, 2, 3]
```

Out[120]=

$$\frac{\langle 3,4,5,1\rangle^2}{\langle 1,2,3,4\rangle\langle 2,3,4,5\rangle\langle 5,1,2,3\rangle} + \frac{\langle 3,5,6,1\rangle^2}{\langle 1,2,3,5\rangle\langle 2,3,5,6\rangle\langle 6,1,2,3\rangle}$$

Out[121]=

$$\frac{\langle 3,4,5,6\rangle^2}{\langle 2,3,4,5\rangle\langle 5,6,2,3\rangle\langle 6,2,3,4\rangle} + \frac{\langle 3,4,6,7\rangle^2}{\langle 2,3,4,6\rangle\langle 6,7,2,3\rangle\langle 7,2,3,4\rangle}$$



In[122]:=

```
temp1 - temp2
```

Out[126]=

$$\frac{\langle 3,\,4,\,5,\,1\rangle^2}{\langle 1,\,2,\,3,\,4\rangle\,\langle 2,\,3,\,4,\,5\rangle\,\langle 5,\,1,\,2,\,3\rangle} - \frac{\langle 3,\,4,\,6,\,1\rangle^2}{\langle 1,\,2,\,3,\,4\rangle\,\langle 2,\,3,\,4,\,6\rangle\,\langle 6,\,1,\,2,\,3\rangle} + \frac{\langle 3,\,5,\,6,\,1\rangle^2}{\langle 1,\,2,\,3,\,5\rangle\,\langle 2,\,3,\,5,\,6\rangle\,\langle 6,\,1,\,2,\,3\rangle} - \frac{\langle 3,\,4,\,5,\,6\rangle^2}{\langle 2,\,3,\,4,\,5\rangle\,\langle 5,\,6,\,2,\,3\rangle\,\langle 6,\,2,\,3,\,4\rangle}$$

```
Random[] & /@ Range[
```

Out[127]=

```
0.417928
```

```
z[a] =
```

zero = temp1 - temp2

Out[132]=

$$\frac{\langle 3,4,5,1\rangle^2}{\langle 1,2,3,4\rangle\langle 2,3,4,5\rangle\langle 5,1,2,3\rangle} - \frac{\langle 3,4,6,1\rangle^2}{\langle 1,2,3,4\rangle\langle 2,3,4,6\rangle\langle 6,1,2,3\rangle} + \frac{\langle 3,5,6,1\rangle^2}{\langle 1,2,3,5\rangle\langle 2,3,5,6\rangle\langle 6,1,2,3\rangle} - \frac{\langle 3,4,5,6\rangle^2}{\langle 2,3,4,5\rangle\langle 5,6,2,3\rangle\langle 6,2,3,4\rangle}$$

In[129]:=

```
Do[Z[a] = Random[] & /@ Range[4] // Rationalize[#, 10^-10] &,
 {a, 6}]
```

```
zero
```

In[129]:=

```
Do[Z[a] = Random[] & /@ Range[4] // Rationalize[#, 10^-10] &,
 {a, 6}]
```

In[137]:=

```
Det@(List @@ (Z /@ ⟨1, 2, 3, 4⟩))
```

Out[137]=

$$-\frac{1\,877\,423\,200\,727\,936\,696\,556\,615\,161\,273\,035\,956\,499\,155\,660}{194\,840\,461\,762\,515\,037\,667} \Big/ \frac{26\,019\,590\,214\,060\,354\,346\,547\,047\,455\,232\,902\,135\,082\,439}{739\,940\,763\,347\,273\,193\,648\,640}$$

```
zero /. ⟨a__⟩ → Det
```

Out[133]=

$$\frac{\langle 3, 4, 5, 1\rangle^2}{\langle 1, 2, 3, 4\rangle \langle 2, 3, 4, 5\rangle \langle 5, 1, 2, 3\rangle} - \frac{\langle 3, 4, 6, 1\rangle^2}{\langle 1, 2, 3, 4\rangle \langle 2, 3, 4, 6\rangle \langle 6, 1, 2, 3\rangle} +$$

# Extracting NMHV components

In[115]:=

$$\text{NMHVTree}[n\_][b\_\_] := \sum_{i=2}^{n-2} \sum_{j=i+2}^{n-1} R[1, i, i+1, j, j+1][b]$$

In[116]:=

```
R[a__][b__] := If[And @@ (MemberQ[{a}, #] & /@ {b}),
  Block[
   {list = Table[RotateLeft[⟨a⟩, j][[1 ;; 4]], {j, 5}]},
   ∏_{j=1}^{4} list[[Position[{a}, {b}[[j]]][[1, 1]]]]
   ------------------------------------------------------
                    Times @@ list
  ], 0]
```

In[117]:=

```
NMHVTree[6][2, 2, 2, 3]
```

```
NMHVTree[n_][b__] := Sum_{i=2} Sum_{j=i+2} R[1, i, i + 1, j, j + 1][b]
```

```
Table[RotateLeft[⟨a⟩, j][[1 ;; 4]], {j, 5}]
```

In[145]:=

```
Partition[⟨1, 2, 3, 5, 6⟩, 4, 1, 1]
```

Out[145]=

```
⟨⟨1, 2, 3, 5⟩, ⟨2, 3, 5, 6⟩, ⟨3, 5, 6, ⟨1, 2, 3, 5, 6⟩⟩,
 ⟨5, 6, ⟨1, 2, 3, 5, 6⟩, ⟨1, 2, 3, 5, 6⟩⟩,
 ⟨6, ⟨1, 2, 3, 5, 6⟩, ⟨1, 2, 3, 5, 6⟩, ⟨1, 2, 3, 5, 6⟩⟩⟩
```

```
R[a__][b__] := If[Complement[{b}, {a}] == {},
  Block[
   {list = Table[RotateLeft[⟨a⟩, j][[1 ;; 4]], {j, 5}]},
   (Product_{j=1}^{4} list[[Position[{a}, {b}[[j]]][[1, 1]]]]) / Times @@ list
  ], 0]
```