Title: Intro to Mathematica

Date: Jul 31, 2011 11:15 AM

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Abstract:

DE NATURA ET PRÆCLARO USU

SIMPLICISSIMÆ SPECIEI NUMERORUM TRIGONALIUM.

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AUCTORE

E. DE JONCOURT,

A. L. M. PHILOSOPHIÆ ET MATHESEOS PROFESSORE.

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HAGÆ COMITUM,
APUD M. HUSSON.
M. D. CC. LXII.

Nat. N.	Trig. N.	Nat. N.	Trig. N.	Nat. N.	Trig. N.
1	1	31	496	61	1891
2	3	32	528	62	1953
3	6	33	561	63	2016
4	10	34	595	64	2080
5	15	35	630	65	2145
6	21	36	666	66	2211
7	28	37	703	67	2278
8	36	38	741	68	2346
9	45	39	780	69	2415
10	55	40	820	70	2485
11	66	41	861	71	2556
12	78	42	903	72	2628
13	91	43	946	73	2701
14	105	44	990	74	2775
15	120	45	1035	75	2850
16	136	46	1081	76	2926
17	153	47	1128	77	3003
18	171	48	1176	78	3081
19	190	49	1225	79	3160
20	210	50	1275	80	3240
21	231	51	1326	81	3321
22	253	52	1378	82	3403
23	276	53	1431	83	3486
24	300	54	1485	84	3570
25	325	55	1540	85	3655
26	351	56	1596	86	3741
27	378	57	1653	87	3828
28	406	58	1711	88	3916
29	435	59	1770	89	4005
30	465	60	1830	90	4095

Nat. N.	Trig. N.	Nat. N.	Trig. N.	Nat. N.	Trig. N.
91	4186	121	7381	151	11476
92	4278	22	7503	52	11628
93	4371	23	7626	53	11781
94	4465	24	7750	54	11935
95	4560	25	7875	55	12090
96	4656	26	8001	56	12246
97	4753	27	8128	57	12403
98	4851	28	8256	58	12561
99	4950	29	8385	59	12720
100	5050	30	8515	60	12880
101	5151	31	8646	61	13041
2	5253	32	8778	62	13203
3	5356	33	8911	63	13366
4	5460	34	9045	64	13530
5	5565	35	9180	65	13695
6	5671	36	9316	66	13861
7	5778	37	9453	67	14028
8	5886	38	9591	68	14196
9	5995	39	9730	69	14365
10	6105	40	9870	70	14535
11	6216	41	10011	71	14706
12	6328	42	10153	72	14878
13	6441	43	10296	73	15051
14	6555	44	10440	74	15225
15	6670	45	10585	75	15400
16	6786	46	10731	76	15576
17	6903	47	10878	77	15753
18	7021	48	11026	78	15931
19	7140	49	11175	79	16110
20	7260	50	11325	80	16290

Nat. N.	Trig. N.	Nat. N.	Trig. N.	Nat. N.	Trig. N.
19901	198034851	19931	198632346	19961	199230741
2	198054753	32	198652278	62	199250703
3	198074656	33	198672211	63	199270666
4	198094560	34	198692145	64	199290630
5	198114465	35	198712080	65	199310595
6	198134371	36	198732016	66	199330561
7	198154278	37	198751953	67	199350528
8	198174186	38	198771891	68	199370496
9	198194095	39	198791830	69	199390465
10	198214005	40	198811770	70	199410435
11	198233916	41	198831711	71	199430406
12	198253828	42	198851653	72	199450378
13	198273741	43	198871596	73	199470351
14	198293655	44	198891540	74	199490325
15	198313570	45	198911485	75	199510300
16	198333486	46	198931431	76	199530276
17	198353403	47	198951378	77	199550253
18	198373321	48	198971326	78	199570231
19	198393240	49	198991275	79	199590210
20	198413160	50	199011225	80	199610190
21	198433081	51	199031176	81	199630171
22	198453003	52	199051128	82	199650153
23	198472926	53	199071081	83	199670136
24	198492850	54	199091035	84	199690120
25	198512775	55	199110990	85	199710105
26	198532701	56	199130946	86	199730091
27	198552628	57	199150903	87	199750078
28	198572556	58	199170861	88	199770066
29	198592485	59	199190820	89	199790055
30	198612415	60	199210780	90	199810045

Nat. N.	Trig. N.	Nat. N.	Trig. N.	Nat. N.	Trig. N.
19991	199830036	19994	199890015	19997	199950003
92	199850028	95	199910010	98	199970001
93	199870021	96	199930006	99	199990000

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Nuclear Physics B269 (1986) 410–420

THE CROSS SECTION FOR FOUR-GLUON PRODUCTION BY GLUON–GLUON FUSION

Stephen J. PARKE and T.R. TAYLOR

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA

Received 13 September 1985

The cross section for two-gluon to four-gluon scattering is given in a form suitable for fast numerical calculations.

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gluons. The cross section for the scattering of two gluons with momenta p_1, p_2 into four gluons with momenta p_3, p_4, p_5, p_6 is obtained from eq. (5) by setting $I=2$ and replacing the momenta p_1, p_4, p_5, p_6 by $-p_1, -p_4, -p_5, -p_6$.

As the result of the computation of two hundred and forty Feynman diagrams, we obtain

$$A_{\binom{0}{2}}(p_1, p_2, p_3, p_4, p_5, p_6)$$

$$=(\mathcal{D}^t, \mathcal{D}^t_p, \mathcal{D}^t_r, \mathcal{D}^t_s)_{\binom{0}{2}} \cdot \begin{pmatrix} K & K_p & K_r & K_s \\ K_p & K & K_s & K_r \\ K_r & K_s & K & K_p \\ K_s & K_r & K_p & K \end{pmatrix} \cdot \begin{pmatrix} \mathcal{D} \\ \mathcal{D}_p \\ \mathcal{D}_r \\ \mathcal{D}_s \end{pmatrix}_{\binom{0}{2}}, \tag{6}$$

where $\mathcal{D}, \mathcal{D}_p, \mathcal{D}_r$ and $\mathcal{D}_s$ are 11-component complex vector functions of the momenta p_1, p_2, p_3, p_4, p_5 and p_6, and K, K_p, K_r and K_s are constant 11×11 symmetric matrices. The vectors $\mathcal{D}_p$, $\mathcal{D}_r$ and $\mathcal{D}_s$ are obtained from the vector $\mathcal{D}$ by the permutations $(p_2 \leftrightarrow p_3)$, $(p_5 \leftrightarrow p_6)$ and $(p_2 \leftrightarrow p_3, p_5 \leftrightarrow p_6)$, respectively, of the momentum variables in $\mathcal{D}$. The individual components of the vector $\mathcal{D}$ represent the sums of all contributions proportional to the appropriately chosen eleven basis color factors. The matrices K, which are the suitable sums over the color indices of products of the color bases, contain two independent structures, proportional to $N^4(N^2-1)$ and $N^2(N^2-1)$, respectively (N is the number of colors, $N=3$ for QCD):

$$K=\tfrac{1}{8}g^8N^4(N^2-1)K^{(4)}+\tfrac{1}{16}g^8N^2(N^2-1)K^{(2)}. \tag{7}$$

Here g denotes the gauge coupling constant. The matrices $K^{(4)}$ and $K^{(2)}$ are given in table 1. The vector $\mathcal{D}$ is related to the thirty-three diagrams $D^G(I=1-33)$ for two-gluon to four-scalar scattering, eleven diagrams $D^F(I=1-11)$ for two-fermion to four-scalar scattering and sixteen diagrams $D^S(I=1-16)$ for two-scalar to four-scalar scattering, in the following way:

$$\mathcal{D}_1=\frac{2s_{34}}{\sqrt{|s_{15}s_{45}s_{16}s_{46}|}\,s_{23}s_{56}}(t^2_{123}C^G\cdot D^G_6-4s_{14}t_{123}E(p_5+p_6,p_4)C^F\cdot D^F_1$$

$$-2s_{14}G(p_5+p_6,p_5+p_6)C^S\cdot D^S_{10},$$

$$\mathcal{D}_2=\frac{s_{56}}{s_{23}}C^G\cdot D^G_2, \tag{8}$$

where the constant matrices $C^G(11\times 33)$, $C^F(11\times 11)$ and $C^S(11\times 16)$ are given in table 2. The Lorentz invariants s_{ij} and t_{ijk} are defined as $s_{ij}=(p_i+p_j)^2$, $t_{ijk}=(p_i+p_j+p_k)^2$ and the complex functions E and G are given by

$$E(p_a,p_j)=\tfrac{1}{2}[(p_1p_4)(p_ap_j)-(p_1p_a)(p_jp_4)-(p_1p_j)(p_ap_4)+i\epsilon_{\mu\nu\rho\sigma}p_1^\mu p_a^\nu p_j^\rho p_4^\sigma]/(p_1p_4),$$

$$G(p_a,p_j)=E(p_a,p_4)E(p_j,p_4). \tag{9}$$

TABLE 1

Matrices $K(I,J)$ [$I=1$–11, $J=1$–11].

Matrix $K^{(4)}$

8	4	−2	2	−1	2	0	1	0	0	−1
4	8	−1	1	−4	0	2	1	0	1	−1
−2	−1	8	4	4	1	1	2	2	1	2
2	1	4	8	2	−1	−1	4	1	1	1
−1	−4	4	2	8	1	2	4	−2	−1	4
2	0	1	−1	1	8	4	−1	0	1	0
0	2	1	−1	2	4	8	−2	0	0	0
1	1	2	4	4	−1	−2	8	−1	−1	2
0	0	2	1	−2	0	0	−1	8	4	−2
0	1	1	1	−1	1	0	−1	4	8	−1
−1	−1	2	1	4	0	0	2	−2	−1	8

Matrix $K^{(2)}$

0	0	0	0	0	0	0	0	3	3	−3
0	0	0	0	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	0	0	0
3	3	0	0	0	3	3	0	0	0	0
3	3	0	0	0	3	3	0	0	0	0
−3	0	0	0	0	0	0	0	0	0	0

Matrix $K_p^{(4)}$

0	0	0	0	1	1	0	1	1	0	−1
0	0	0	0	2	0	1	1	2	1	−2
0	0	0	0	0	1	1	1	0	1	1
0	0	0	0	1	0	0	2	0	1	0
1	2	0	1	0	1	2	2	0	0	2
1	0	1	0	1	4	2	0	0	0	−1
0	1	1	0	2	2	4	0	0	0	−2
1	1	1	2	2	0	0	4	0	0	0
1	2	0	0	0	0	0	0	0	2	−1
0	1	1	1	0	0	0	0	2	4	0
−1	−2	1	0	2	−1	−2	0	−1	0	4

Matrix $K_p^{(2)}$

3	3	0	3	0	0	0	3	0	0	0
3	3	0	0	0	0	0	0	0	0	0
0	0	3	0	0	3	0	0	3	3	0
3	0	0	0	0	3	0	0	0	0	0
0	0	0	0	0	0	0	0	3	3	0
0	0	3	3	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	0	3	0
3	0	0	0	0	0	0	0	3	0	0
0	0	3	0	3	3	0	3	0	0	0
0	0	3	0	3	3	3	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

Matrix $K_r^{(4)}$

4	2	0	2	0	1	0	1	0	0	0
2	4	0	1	0	0	1	1	0	1	0
0	0	4	2	2	1	1	1	2	1	0
2	1	2	0	1	2	1	0	1	0	0
0	0	2	1	0	0	0	0	4	2	2
1	0	1	2	0	0	0	0	1	2	0
0	1	1	1	0	0	0	0	2	4	0
1	1	1	0	0	0	0	0	2	0	1
0	0	2	1	4	1	2	2	0	0	−4
0	1	1	0	2	2	4	0	0	0	−2
0	0	0	0	2	0	0	1	−4	−2	4

Matrix $K_r^{(2)}$

0	0	0	0	0	0	0	0	3	0	−3
0	0	0	0	3	0	0	3	0	0	0
0	0	0	0	0	0	3	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	3	0	0	0	0	3	0	0	0	0
0	0	0	0	0	3	3	0	0	0	−3
0	0	3	0	3	3	3	0	0	0	0
0	3	0	0	0	0	0	3	0	0	0
3	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	3	0
−3	0	0	0	0	−3	0	0	0	0	0

Matrix $K_s^{(4)}$

0	1	−1	−1	1	1	0	1	2	0	0
1	0	−2	−1	2	0	1	1	4	2	0
−1	−2	0	0	0	1	1	1	−1	1	0
−1	−1	0	1	0	2	1	0	1	−1	0
1	2	0	0	1	−1	−1	0	−2	2	1
1	0	1	2	−1	0	1	−2	2	4	−1
0	2	1	1	−1	1	0	−1	4	6	−1
1	1	1	0	0	−2	−1	0	2	−2	0
2	4	−1	1	−2	2	4	2	1	0	−2
0	2	1	−1	2	4	8	−2	0	0	0
0	0	0	0	1	−1	−1	0	−2	0	2

Matrix $K_s^{(2)}$

3	3	0	0	0	3	3	0	0	0	0
3	3	0	0	0	3	3	0	0	0	0
0	0	3	3	3	0	0	3	0	0	0
0	0	3	3	3	0	0	3	0	0	3
0	0	3	3	3	0	0	3	0	0	0
3	3	0	0	0	3	3	0	0	0	−3
3	3	0	0	0	3	3	0	0	0	0
0	0	3	3	3	0	0	3	0	0	0
0	0	0	0	0	0	0	0	3	3	0
0	0	0	0	0	0	0	0	3	3	0
0	0	0	3	0	−3	0	0	0	0	0

TABLE 2

Matrices $C^{-1}(I,J)$ $(I=1\text{–}11,\ J=1\text{–}11)$, $C^{0}(I,J)$ $(I=1\text{–}11,\ J=1\text{–}33)$, $C^{1}(I,J)$ $(I=1\text{–}11,\ J=1\text{–}11)$ and $C^{2}(I,J)$ $(I=1\text{–}11,\ J=1\text{–}16)$

Matrix C^{-1} / Matrix C^{0} / Matrix C^{1} / Matrix C^{2}

[illegible]	[illegible]

Indices I and J specify row numbers and column numbers, respectively.

where ε is the totally antisymmetric tensor, $\varepsilon_{xyzt}=1$. For the future use, we define one more function,

$$F(p_s, p_j)=[(p_1p_4)(p_s p_j)+(p_1p_s)(p_j p_4)-(p_1p_j)(p_s p_4)]/(p_1p_4). \tag{10}$$

Note that when evaluating A_6 and A_7 at crossed configurations of the momenta, care must be taken with the implicit dependence of the functions E, F and G on the momenta p_1, p_4, p_5, p_6.

The diagrams D_2^{G} are listed below:

$$D_2^{G}(1)=\frac{\delta_2}{s_{14}s_{25}s_{36}}\{[(p_2-p_5)(p_3-p_6)][(p_1-p_4)(p_3+p_6)]-[(p_2-p_5)(p_3+p_6)]$$
$$\times[(p_1-p_4)(p_3-p_6)]+[(p_2+p_5)(p_3-p_6)][(p_1-p_4)(p_2-p_5)]\},$$

$$D_2^{G}(2)=\frac{1}{s_{25}s_{36}}\{2E(p_2-p_5,p_3-p_6)-2E(p_3-p_6,p_2-p_5)+\delta_2[(p_2-p_5)(p_3-p_6)]\},$$

$$D_2^{G}(3)=\frac{4}{s_{25}s_{36}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_2,p_3)$$
$$-[(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_2,p_6)$$
$$-[(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3)$$
$$+[(p_1-p_2+p_5)(p_4-p_3+p_6)]E(p_5,p_6)$$
$$-[p_1(p_2-p_5)]E(p_3-p_6,p_3+p_6)-[p_4(p_3-p_6)]E(p_2+p_5,p_2-p_5)$$
$$+\delta_2[p_1(p_2-p_5)][p_4(p_3-p_6)]\},$$

$$D_2^{G}(4)=\frac{-2}{s_{36}t_{125}}\{E(p_3-p_6,p_3+p_6)-\delta_2[p_4(p_3-p_6)]\},$$

$$D_2^{G}(5)=\frac{-2}{s_{25}t_{125}}\{E(p_2+p_5,p_2-p_5)-\delta_2[p_1(p_2-p_5)]\},$$

$$D_2^{G}(6)=\frac{\delta_2}{t_{125}},$$

$$D_2^{G}(7)=\frac{4}{s_{12}s_{36}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_2,p_3)$$
$$-[(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_2,p_6)-[p_4(p_3-p_6)]E(p_2,p_1-p_5)\},$$

$$D_2^{G}(8)=\frac{4}{s_{34}s_{25}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_2,p_3)$$
$$-[(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3)-[p_1(p_2-p_5)]E(p_3-p_4,p_3)\},$$

$$D_6^E(4)=\frac{4}{s_{23}s_{34}t_{125}}\{F(p_2,p_3)E(p_4,p_5)-F(p_4,p_5)E(p_2,p_3)$$
$$+[F(p_5,p_2)-\tfrac{1}{2}s_{23}-\tfrac{1}{2}s_{12}+\tfrac{1}{2}s_{15}]E(p_3,p_5)\}\,,$$

$$D_6^E(5)=\frac{2}{s_{16}s_{25}t_{146}}[s_{13}-s_{23}+s_{25}]E(p_4,p_5)\,,$$

$$D_6^E(6)=\frac{2}{s_{25}s_{34}t_{134}}[s_{16}-s_{26}-s_{25}]E(p_3,p_5)\,,$$

$$D_6^E(7)=\frac{4}{s_{23}s_{56}t_{123}}\{[F(p_3,p_2)-\tfrac{1}{2}s_{23}-\tfrac{1}{2}s_{12}+\tfrac{1}{2}s_{13}]E(p_3,p_5)$$
$$+[F(p_2,p_3)+\tfrac{1}{4}t_{123}]E(p_3,p_5)-[F(p_3,p_5)+\tfrac{1}{4}t_{123}]E(p_2,p_5)\}\,,$$

$$D_6^E(8)=\frac{1}{s_{16}s_{36}}E(p_3-p_6,p_5)\,,$$

$$D_6^E(9)=\frac{2}{s_{16}s_{36}t_{126}}[s_{23}-s_{56}+s_{36}]E(p_2,p_5)\,,$$

$$D_6^E(10)=\frac{2}{s_{16}s_{36}t_{146}}[s_{23}-s_{26}-s_{36}]E(p_2,p_5)\,,$$

$$D_6^E(11)=\frac{1}{2s_{16}s_{25}s_{36}}\{[s_{23}+s_{35}-s_{26}-s_{36}]E(p_2-p_6,p_5)$$
$$-[s_{23}+s_{26}-s_{35}-s_{56}]E(p_3-p_6,p_5)-[s_{23}+s_{36}-s_{35}-s_{26}]E(p_2+p_6,p_5)\}\,. \tag{12}$$

The diagrams D_6^S are listed below:

$$D_6^S(1)=\frac{1}{s_{25}s_{36}t_{125}}[s_{34}-s_{46}+s_{36}][s_{12}-s_{15}-s_{25}]\,,$$

$$D_6^S(2)=\frac{1}{s_{14}s_{36}t_{126}}[s_{12}-s_{24}-s_{14}][s_{35}-s_{56}+s_{36}]\,,$$

$$D_6^S(3)=\frac{1}{s_{14}s_{36}t_{145}}[s_{15}-s_{45}+s_{14}][s_{23}-s_{26}-s_{36}]\,,$$

$$D_6^S(4)=\frac{1}{s_{15}s_{36}t_{125}}[s_{15}+s_{25}-s_{12}][s_{34}-s_{46}+s_{36}]\,,$$

$$D_6^S(5)=\frac{1}{s_{15}s_{34}t_{156}}[s_{16}-s_{15}-s_{56}][s_{23}-s_{24}-s_{34}]\,,$$

$$D_6^S(6)=\frac{1}{s_{15}s_{34}t_{125}}[s_{46}-s_{34}-s_{36}][s_{12}-s_{25}-s_{15}]\,,$$

$$D_6^S(7)=\frac{1}{s_{25}s_{36}t_{125}}[s_{36}-s_{46}+s_{34}][s_{12}-s_{15}-s_{25}]\,,$$

$$D_6^S(8)=\frac{1}{s_{16}s_{25}t_{146}}[s_{25}+s_{35}-s_{23}][s_{14}-s_{46}+s_{16}]\,,$$

$$D_6^S(9)=\frac{1}{s_{25}s_{34}t_{134}}[s_{14}+s_{34}-s_{13}][s_{26}-s_{56}+s_{25}]\,,$$

$$D_6^S(10)=\frac{1}{s_{25}s_{36}}(p_2-p_5)(p_3-p_6)\,,$$

$$D_6^S(11)=\frac{1}{s_{16}s_{36}}(p_1-p_6)(p_3-p_6)\,,$$

$$D_6^S(12)=\frac{1}{s_{16}s_{25}}(p_6-p_1)(p_2-p_5)\,,$$

$$D_6^S(13)=\frac{1}{s_{15}s_{34}}(p_5-p_1)(p_3-p_4)\,,$$

$$D_6^S(14)=\frac{1}{s_{25}s_{34}}(p_2-p_5)(p_3-p_4)\,,$$

$$D_6^S(15)=\frac{1}{s_{14}s_{25}s_{36}}\{[(p_2+p_5)(p_3-p_6)][(p_1-p_4)(p_2-p_5)]$$
$$+[(p_2-p_5)(p_3-p_6)][(p_1-p_4)(p_5+p_6)]$$
$$+[(p_1+p_4)(p_2-p_5)][(p_1-p_4)(p_3-p_6)]\}\,,$$

$$D_6^S(16)=\frac{2}{s_{16}s_{34}s_{25}}\{[(p_2-p_5)(p_3+p_4)][(p_1-p_6)(p_3-p_4)]$$
$$+[(p_1+p_6)(p_3-p_4)][(p_1-p_6)(p_2-p_5)]$$
$$+[(p_1-p_6)(p_2+p_5)][(p_3-p_4)(p_2-p_5)]\}\,. \tag{13}$$

The preceding list completes the result. Let us recapitulate now the numerical procedure of calculating the full cross section. First the diagrams D are calculated by using eqs. (11)–(13). The result is substituted to eq. (8) to obtain the vectors $\mathcal{D}_0$ and $\mathcal{D}_2$. After generating the vectors $\mathcal{D}_{0_1}, \mathcal{D}_{0_2}, \mathcal{D}_{0_3}, \mathcal{D}_{2_1}, \mathcal{D}_{2_2}$ and $\mathcal{D}_{2_3}$ by the appropriate permutations of momenta, eq. (6) is used to obtain the functions A_0 and A_2. Finally, the total cross section is calculated by using eq. (5). The FORTRAN 5 program based on such a scheme generates ten Monte Carlo points in less than a second on the heterotic CDC CYBER 175/875.

Given the complexity of the final result, it is very important to have some reliable testing procedures available for numerical calculations. Usually in QCD, the multi-gluon amplitudes are tested by checking the gauge invariance. Due to the specifics

of our calculation, the most powerful test does not rely on the gauge symmetry, but on the appropriate permutation symmetries. The function $A_6(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under arbitrary permutations of the momenta (p_1, p_2, p_3) and separately, (p_4, p_5, p_6), whereas the function $A_3(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under the permutations of (p_1, p_2, p_3, p_4) and separately, (p_5, p_6). This test is extremely powerful, because the required permutation symmetries are hidden in our supersymmetry relations, eqs. (1) and (3), and in the structure of amplitudes involving different species of particles. Another, very important test relies on the absence of the double poles of the form $(s_{ij})^{-2}$ in the cross section, as required by general arguments based on the helicity conservation. Further, in the leading $(s_{ij})^{-1}$ pole approximation, the answer should reduce to the two goes to three cross section [3, 4], convoluted with the appropriate Altarelli-Parisi probabilities [5]. Our result has succesfully passed both these numerical checks.

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References

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[5] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298

$$D_6^E(4)=\frac{4}{s_{23}s_{34}t_{123}}\{F(p_2,p_3)E(p_6,p_5)-F(p_6,p_5)E(p_2,p_3)$$

$$+[F(p_6,p_2)-\tfrac{1}{2}s_{23}-\tfrac{1}{2}s_{12}+\tfrac{1}{2}s_{13}]E(p_3,p_5)\},$$

$$D_6^E(5)=\frac{2}{s_{16}s_{23}t_{146}}[s_{15}-s_{23}+s_{25}]E(p_6,p_5),$$

$$D_6^E(6)=\frac{2}{s_{23}s_{34}t_{134}}[s_{56}-s_{26}-s_{25}]E(p_5,p_5),$$

$$D_6^E(7)=\frac{4}{s_{23}s_{56}t_{125}}\{[F(p_6,p_2)-\tfrac{1}{2}s_{25}-\tfrac{1}{2}s_{12}+\tfrac{1}{2}s_{15}]E(p_3,p_5)$$

$$+[F(p_2,p_5)+\tfrac{1}{2}t_{125}]E(p_3,p_5)-[F(p_3,p_5)+\tfrac{1}{2}t_{125}]E(p_2,p_5)\},$$

$$D_6^E(8)=\frac{1}{s_{14}s_{36}}E(p_3-p_6,p_5),$$

$$D_6^E(9)=\frac{2}{s_{14}s_{56}t_{126}}[s_{15}-s_{56}+s_{36}]E(p_2,p_5),$$

$$D_6^E(10)=\frac{2}{s_{14}s_{56}t_{145}}[s_{23}-s_{26}-s_{36}]E(p_5,p_5),$$

$$D_6^E(11)=\frac{1}{2s_{14}s_{25}s_{36}}\{[s_{23}+s_{35}-s_{26}-s_{36}]E(p_2-p_5,p_5)$$

$$-[s_{23}+s_{26}-s_{35}-s_{36}]E(p_3-p_6,p_5)-[s_{23}+s_{56}-s_{35}-s_{26}]E(p_2+p_5,p_5)\}. \tag{12}$$

The diagrams D_9^S are listed below:

$$D_9^S(1)=\frac{1}{s_{23}s_{56}t_{123}}[s_{34}-s_{46}+s_{36}][s_{12}-s_{15}-s_{25}],$$

$$D_9^S(2)=\frac{1}{s_{14}s_{36}t_{134}}[s_{12}-s_{24}-s_{14}][s_{35}-s_{56}+s_{36}],$$

$$D_9^S(3)=\frac{1}{s_{16}s_{36}t_{145}}[s_{15}-s_{45}+s_{14}][s_{23}-s_{36}-s_{26}],$$

$$D_9^S(4)=\frac{1}{s_{15}s_{36}t_{125}}[s_{15}+s_{25}-s_{12}][s_{36}-s_{46}+s_{36}],$$

$$D_9^S(5)=\frac{1}{s_{15}s_{34}t_{156}}[s_{16}-s_{15}-s_{56}][s_{23}-s_{24}-s_{34}],$$

$$D_9^S(6)=\frac{1}{s_{15}s_{34}t_{125}}[s_{46}-s_{34}-s_{36}][s_{15}-s_{25}-s_{15}],$$

$$D_9^S(7)=\frac{1}{s_{23}s_{56}t_{125}}[s_{56}-s_{46}+s_{34}][s_{12}-s_{15}-s_{25}],$$

$$D_9^S(8)=\frac{1}{s_{16}s_{23}s_{146}}[s_{25}+s_{35}-s_{23}][s_{34}-s_{46}+s_{16}],$$

$$D_9^S(9)=\frac{1}{s_{25}s_{34}t_{134}}[s_{14}+s_{34}-s_{13}][s_{26}-s_{56}+s_{25}],$$

$$D_9^S(10)=\frac{1}{s_{25}s_{36}}(p_2-p_5)(p_3-p_6),$$

$$D_9^S(11)=\frac{1}{s_{14}s_{36}}(p_1-p_4)(p_3-p_6),$$

$$D_9^S(12)=\frac{1}{s_{16}s_{25}}(p_6-p_1)(p_2-p_5),$$

$$D_9^S(13)=\frac{1}{s_{15}s_{34}}(p_5-p_1)(p_3-p_4),$$

$$D_9^S(14)=\frac{1}{s_{25}s_{34}}(p_2-p_5)(p_3-p_4),$$

$$D_9^S(15)=\frac{1}{s_{14}s_{25}s_{36}}\{[(p_2+p_5)(p_3-p_6)][(p_1-p_4)(p_2-p_5)]$$

$$+[(p_2-p_5)(p_3-p_6)][(p_1-p_4)(p_3+p_6)]$$

$$+[(p_1+p_4)(p_2-p_5)][(p_1-p_4)(p_3-p_6)]\},$$

$$D_9^S(16)=\frac{2}{s_{16}s_{34}s_{25}}\{[(p_2-p_5)(p_3+p_4)][(p_1-p_6)(p_3-p_4)]$$

$$+[(p_1+p_6)(p_3-p_4)][(p_1-p_6)(p_2-p_5)]$$

$$+[(p_1-p_6)(p_2+p_5)][(p_3-p_4)(p_2-p_5)]\}. \tag{13}$$

The preceding list completes the result. Let us recapitulate now the numerical procedure of calculating the full cross section. First the diagrams D are calculated by using eqs. (11)–(13). The result is substituted to eq. (8) to obtain the vectors $\mathscr{D}_6$ and $\mathscr{D}_9$. After generating the vectors $\mathscr{D}_{6_a}$, $\mathscr{D}_{6_b}$, $\mathscr{D}_{6_c}$, $\mathscr{D}_{7_a}$, $\mathscr{D}_{7_b}$ and $\mathscr{D}_{7_c}$ by the appropriate permutations of momenta, eq. (6) is used to obtain the functions A_6 and A_7. Finally, the total cross section is calculated by using eq. (5). The FORTRAN 5 program based on such a scheme generates ten Monte Carlo points in less than a second on the heterotic CDC CYBER 175/875.

Given the complexity of the final result, it is very important to have some reliable testing procedures available for numerical calculations. Usually in QCD, the multi-gluon amplitudes are tested by checking the gauge invariance. Due to the specifics

TABLE 2
Matrices $C^{(1)}(I,J)$ [$I=1$–11, $J=1$–11], $C^{(2)}(I,J)$ [$I=1$–11, $J=1$–33], $C^{(3)}(I,J)$ [$I=1$–11, $J=1$–11] and $C^{(4)}(I,J)$ [$I=1$–11, $J=1$–16]

Matrix $C^{(1)}$, Matrix $C^{(2)}$, Matrix $C^{(3)}$, Matrix $C^{(4)}$

[illegible]

Indices I and J specify row numbers and column numbers, respectively.

where ε is the totally antisymmetric tensor, $\varepsilon_{xyzw}=1$. For the future use, we define one more function,

$$F(p_x,p_y)=[(p_1p_4)(p_xp_y)+(p_1p_x)(p_yp_4)-(p_1p_y)(p_xp_4)]/(p_1p_4). \qquad (10)$$

Note that when evaluating A_6 and A_7 at crossed configurations of the momenta, care must be taken with the implicit dependence of the functions E, F and G on the momenta p_1, p_4, p_5, p_6.

The diagrams D_2^G are listed below:

$$D_2^G(1)=\frac{\delta_2}{s_{14}t_{25}s_{36}}\{[(p_2-p_5)(p_3-p_6)][(p_1-p_4)(p_5+p_6)]-[(p_2-p_5)(p_3+p_6)]\times[(p_1-p_4)(p_3-p_6)]+[(p_2+p_5)(p_3-p_6)][(p_1-p_4)(p_2-p_5)]\},$$

$$D_2^G(2)=\frac{1}{s_{25}s_{36}}\{2E(p_2-p_5,p_3-p_6)-2E(p_3-p_6,p_2-p_5)+\delta_2[(p_2-p_5)(p_3-p_6)]\},$$

$$D_2^G(3)=\frac{4}{s_{23}s_{36}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_5-p_6)]E(p_2,p_3)-[(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_2,p_6)-[(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3)+[(p_1-p_2+p_5)(p_4-p_3+p_6)]E(p_5,p_6)-[p_1(p_2-p_5)]E(p_3-p_6,p_3+p_6)-[p_4(p_3-p_6)]E(p_2+p_5,p_2-p_5)+\delta_2[p_1(p_2-p_5)][p_4(p_3-p_6)]\},$$

$$D_2^G(4)=\frac{-2}{s_{36}t_{125}}\{E(p_3-p_6,p_3+p_6)-\delta_2[p_4(p_3-p_6)]\},$$

$$D_2^G(5)=\frac{-2}{s_{25}t_{125}}\{E(p_2+p_5,p_2-p_5)-\delta_2[p_1(p_2-p_5)]\},$$

$$D_2^G(6)=\frac{\delta_2}{t_{125}},$$

$$D_2^G(7)=\frac{4}{s_{12}s_{36}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_5,p_3)-[(p_1+p_2-p_5)(p_4-p_3+p_6)]E(p_5,p_6)-[p_4(p_3-p_6)]E(p_5,p_3-p_5)\},$$

$$D_2^G(8)=\frac{4}{s_{34}t_{25}t_{125}}\{[(p_1+p_2-p_5)(p_4+p_3-p_6)]E(p_5,p_3)-[(p_1-p_2+p_5)(p_4+p_3-p_6)]E(p_5,p_3)-[p_1(p_2-p_5)]E(p_3-p_6,p_3)\}.$$

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involving different species of particles. Another, very important test relies on the absence of the double poles of the form $(s_{ij})^{-2}$ in the cross section, as required by general arguments based on the helicity conservation. Further, in the leading $(s_{ij})^{-1}$ pole approximation, the answer should reduce to the two goes to three cross section [3, 4], convoluted with the appropriate Altarelli-Parisi probabilities [5]. Our result has succesfully passed both these numerical checks.

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$$D_0^{\varepsilon}(4)=\frac{4}{s_{23}s_{34}t_{125}}\{F(p_2,p_3)E(p_4,p_5)-F(p_4,p_3)E(p_2,p_5)$$

$$+[F(p_3,p_2)-\tfrac{1}{2}s_{23}-\tfrac{1}{2}s_{12}+\tfrac{1}{2}s_{13}]E(p_4,p_5)\}\,,$$

$$D_0^{\varepsilon}(5)=\frac{2}{s_{16}s_{23}t_{146}}[s_{15}-s_{23}+s_{25}]E(p_6,p_5)\,,$$

$$D_0^{\varepsilon}(6)=\frac{2}{s_{25}s_{34}t_{134}}[s_{56}-s_{26}-s_{25}]E(p_3,p_5)\,,$$

$$D_0^{\varepsilon}(7)=\frac{4}{s_{23}s_{36}t_{125}}\{[F(p_3,p_2)-\tfrac{1}{2}s_{23}-\tfrac{1}{2}s_{12}+\tfrac{1}{2}s_{13}]E(p_3,p_5)$$

$$+[F(p_2,p_3)+\tfrac{1}{4}t_{123}]E(p_3,p_5)-[F(p_3,p_2)+\tfrac{1}{4}t_{123}]E(p_2,p_5)\}\,,$$

$$D_0^{\varepsilon}(8)=\frac{1}{s_{14}s_{36}}E(p_3-p_6,p_5)\,,$$

$$D_0^{\varepsilon}(9)=\frac{2}{s_{14}s_{36}t_{134}}[s_{15}-s_{56}+s_{36}]E(p_2,p_5)\,,$$

$$D_0^{\varepsilon}(10)=\frac{2}{s_{14}s_{36}t_{145}}[s_{23}-s_{26}-s_{36}]E(p_3,p_5)\,,$$

$$D_0^{\varepsilon}(11)=\frac{1}{2s_{14}s_{23}s_{56}}\{[s_{23}+s_{35}-s_{26}-s_{36}]E(p_2-p_3,p_5)$$

$$-[s_{23}+s_{36}-s_{35}-s_{56}]E(p_3-p_6,p_5)-[s_{23}+s_{36}-s_{35}-s_{26}]E(p_2+p_3,p_5)\}\,. \qquad (12)$$

The diagrams D_0^{δ} are listed below:

$$D_0^{\delta}(1)=\frac{1}{s_{23}s_{36}t_{125}}[s_{34}-s_{46}+s_{36}][s_{12}-s_{15}-s_{25}]\,,$$

$$D_0^{\delta}(2)=\frac{1}{s_{14}s_{36}t_{136}}[s_{12}-s_{24}-s_{14}][s_{35}-s_{56}+s_{36}]\,,$$

$$D_0^{\delta}(3)=\frac{1}{s_{14}s_{36}t_{145}}[s_{15}-s_{45}+s_{14}][s_{23}-s_{26}-s_{36}]\,,$$

$$D_0^{\delta}(4)=\frac{1}{s_{15}s_{36}t_{125}}[s_{15}+s_{25}-s_{12}][s_{34}-s_{46}+s_{36}]\,,$$

$$D_0^{\delta}(5)=\frac{1}{s_{15}s_{34}t_{156}}[s_{56}-s_{15}-s_{16}][s_{23}-s_{24}-s_{34}]\,,$$

$$D_0^{\delta}(6)=\frac{1}{s_{15}s_{34}t_{125}}[s_{46}-s_{34}-s_{36}][s_{12}-s_{25}-s_{15}]\,,$$

$$D_0^{\delta}(7)=\frac{1}{s_{25}s_{36}t_{125}}[s_{36}-s_{46}+s_{34}][s_{12}-s_{15}-s_{25}]\,,$$

$$D_0^{\delta}(8)=\frac{1}{s_{16}s_{25}t_{146}}[s_{25}+s_{35}-s_{23}][s_{14}-s_{46}+s_{16}]\,,$$

$$D_0^{\delta}(9)=\frac{1}{s_{25}s_{34}t_{134}}[s_{14}+s_{36}-s_{13}][s_{26}-s_{56}+s_{25}]\,,$$

$$D_0^{\delta}(10)=\frac{1}{s_{25}s_{36}}(p_2-p_5)(p_3-p_6)\,,$$

$$D_0^{\delta}(11)=\frac{1}{s_{14}s_{36}}(p_1-p_4)(p_3-p_6)\,,$$

$$D_0^{\delta}(12)=\frac{1}{s_{16}s_{25}}(p_6-p_1)(p_2-p_5)\,,$$

$$D_0^{\delta}(13)=\frac{1}{s_{15}s_{34}}(p_5-p_1)(p_3-p_4)\,,$$

$$D_0^{\delta}(14)=\frac{1}{s_{25}s_{34}}(p_2-p_5)(p_3-p_4)\,,$$

$$D_0^{\delta}(15)=\frac{1}{s_{14}s_{25}s_{36}}\{[(p_2+p_5)(p_3-p_6)][(p_1-p_4)(p_2-p_5)]$$

$$+[(p_2-p_5)(p_3-p_6)][(p_1-p_4)(p_5+p_2)]$$

$$+[(p_1+p_4)(p_2-p_5)][(p_1-p_4)(p_3-p_6)]\}\,,$$

$$D_0^{\delta}(16)=\frac{2}{s_{16}s_{34}s_{25}}\{[(p_2-p_5)(p_3+p_4)][(p_1-p_6)(p_3-p_4)]$$

$$+[(p_1+p_6)(p_3-p_4)][(p_1-p_6)(p_2-p_5)]$$

$$+[(p_1-p_6)(p_2+p_5)][(p_3-p_4)(p_2-p_5)]\}\,. \qquad (13)$$

The preceding list completes the result. Let us recapitulate now the numerical procedure of calculating the full cross section. First the diagrams D are calculated by using eqs. (11)–(13). The result is substituted to eq. (8) to obtain the vectors $\mathscr{D}_0$ and $\mathscr{D}_2$. After generating the vectors $\mathscr{D}_{0_1}$, $\mathscr{D}_{0_2}$, $\mathscr{D}_{0_3}$, $\mathscr{D}_{2_1}$, $\mathscr{D}_{2_2}$ and $\mathscr{D}_{2_3}$ by the appropriate permutations of momenta, eq. (6) is used to obtain the functions A_0 and A_2. Finally, the total cross section is calculated by using eq. (5). The FORTRAN 5 program based on such a scheme generates ten Monte Carlo points in less than a second on the heterotic CDC CYBER 175/875.

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of our calculation, the most powerful test does not rely on the gauge symmetry, but on the appropriate permutation symmetries. The function $A_6(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under arbitrary permutations of the momenta (p_1, p_2, p_3) and separately, (p_4, p_5, p_6), whereas the function $A_2(p_1, p_2, p_3, p_4, p_5, p_6)$ must be symmetric under the permutations of (p_1, p_2, p_3, p_4) and separately, (p_5, p_6). This test is extremely powerful, because the required permutation symmetries are hidden in our supersymmetry relations, eqs. (1) and (3), and in the structure of amplitudes involving different species of particles. Another, very important test relies on the absence of the double poles of the form $(s_{ij})^{-2}$ in the cross section, as required by general arguments based on the helicity conservation. Further, in the leading $(s_{ij})^{-1}$ pole approximation, the answer should reduce to the two goes to three cross section [3, 4], convoluted with the appropriate Altarelli-Parisi probabilities [5]. Our result has succesfully passed both these numerical checks.

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Nat. N.	Trig. N.	Nat. N.	Trig. N.	Nat. N.	Trig. N.
1	1	31	496	61	1891
2	3	32	528	62	1953
3	6	33	561	63	2016
4	10	34	595	64	2080
5	15	35	630	65	2145
6	21	36	666	66	2211
7	28	37	703	67	2278
8	36	38	741	68	2346
9	45	39	780	69	2415
10	55	40	820	70	2485
11	66	41	861	71	2556
12	78	42	903	72	2628
13	91	43	946	73	2701
14	105	44	990	74	2775
15	120	45	1035	75	2850
16	136	46	1081	76	2926
17	153	47	1128	77	3003
18	171	48	1176	78	3081
19	190	49	1225	79	3160
20	210	50	1275	80	3240
21	231	51	1326	81	3321
22	253	52	1378	82	3403
23	276	53	1431	83	3486
24	300	54	1485	84	3570
25	325	55	1540	85	3655
26	351	56	1596	86	3741
27	378	57	1653	87	3828
28	406	58	1711	88	3916
29	435	59	1770	89	4005
30	465	60	1830	90	4095

Nat. N.	Trig. N.	Nat. N.	Trig. N.	Nat. N.	Trig. N.
91	4186	121	7381	151	11476
92	4278	22	7503	52	11628
93	4371	23	7626	53	11781
94	4465	24	7750	54	11935
95	4560	25	7875	55	12090
96	4656	26	8001	56	12246
97	4753	27	8128	57	12403
98	4851	28	8256	58	12561
99	4950	29	8385	59	12720
100	5050	30	8515	60	12880
101	5151	31	8646	61	13041
2	5253	32	8778	62	13203
3	5356	33	8911	63	13366
4	5460	34	9045	64	13530
5	5565	35	9180	65	13695
6	5671	36	9316	66	13861
7	5778	37	9453	67	14028
8	5886	38	9591	68	14196
9	5995	39	9730	69	14365
10	6105	40	9870	70	14535
11	6216	41	10011	71	14706
12	6328	42	10153	72	14878
13	6441	43	10296	73	15051
14	6555	44	10440	74	15225
15	6670	45	10585	75	15400
16	6786	46	10731	76	15576
17	6903	47	10878	77	15753
18	7021	48	11026	78	15931
19	7140	49	11175	79	16110
20	7260	50	11325	80	16290

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An Introduction to Mathematica

An Eager Pedestrian's Guide to *Mathematica*

Mathematica as a(n Expensive) Calculator

Mathematica Vernacular: The Anatomy of an Expression

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An Introduction to Mathematica

An Eager Pedestrian's Guide to *Mathematica*

Mathematica as a(n Expensive) Calculator

- SHIFT + ENTER [sic] :: **evaluates a command (or an arbitrarily compounded command...)**

◇ To evaluate an expression, simply type it in, and hit "SHIFT + ENTER"

◇ All the commands within the **active cell** are evaluated with SHIFT + ENTER.
The output of those ending in ";" are not printed.

- **Input and Output are stored *as functions* in memory**
- **Finding and learning about built-in functions in *Mathematica***
- **Interface Tips & Tricks**
- ALT + **. : Abort an evaluation**
- **Defining new functions**

100%

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SHIFT + ENTER [sic] :: evaluates a command (or an arbitrarily compounded command...)

◇ To evaluate an expression, simply type it in, and hit "SHIFT + ENTER"

```
19991 (19991 + 1) / 2
199830036
```

The first time you evaluate an expression, there should be a pause, reflecting the fact that *Mathematica*'s user interface is starting up the "Kernel" (which is the background-program doing all the computation).

◇ All the commands within the **active cell** are evaluated with SHIFT + ENTER.
The output of those ending in ";" are not printed.

```
Sum[n, {n, 1, 19991}]
13;
PolyLog[2, 1]
1 + 1;
Fibonacci[90]
```

Input and Output are stored *as functions* in memory

Finding and learning about built-in functions in *Mathematica*

Interface Tips & Tricks

ALT + . : Abort an evaluation

100%

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```

```
199830036
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Input and Output are stored *as functions* in memory

Finding and learning about built-in functions in *Mathematica*

Interface Tips & Tricks

ALT + . : Abort an evaluation

Defining new functions

Replacements

Using Packages: "<< "

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13;
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1 + 1;
Fibonacci[90]
```

```
199830036
```

$$\frac{\pi^2}{6}$$

```
2880067194370816120
```

Input and Output are stored *as functions* in memory

Finding and learning about built-in functions in *Mathematica*

Interface Tips & Tricks

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◇ All the commands within the **active cell** are evaluated with SHIFT + ENTER.
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```
Sum[n, {n, 1, 19991}]
13;
PolyLog[2, 1]
1 + 1;
Fibonacci[90]
```

199 830 036

$\frac{\pi^2}{6}$

2 880 067 194 370 816 120

Input and Output are stored *as functions* in memory

◇ Each "line" of input—even those whose output is not printed to screen—is stored in memory in the function "**In[]**"—as in "**In[1]**:=PolyLog[2,1]", and can be re-evaluated any time by calling:

```
In[1]
```

Notice that input which has not yet been generated, is as-yet undefined; thus:

```
In[100]
```

◇ The output of each line is similarly stored (even if not printed) in the function "**Out[]**"—as in "**Out[4]** = $\frac{\pi^2}{6}$"

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```
199 830 036
```

Notice that input which has not yet been generated, is as-yet undefined; thus:

```
In[100]
In[100]
```

◇ The output of each line is similarly stored (even if not printed) in the function "**Out[]**"—as in "**Out[4]** = $\frac{\pi^2}{6}$" (notice it's *not* "**Out[4]:=**$\frac{\pi^2}{6}$"—we'll see why later)

```
Out[1]
Out[4]
```

The function **Out[]** can also be called by the notation "%**x**"
(which although *highly* non-standard, is very commonly used).
For example,

```
%6
Out[6]
```

◇ % by itself (without any number following it) *always* means "the line above"
%% calls "two lines above",
%%% "three lines above",
etc.

Finding and learning about built-in functions in *Mathematica*

100%

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◇ The output of each line is similarly stored (even if not printed) in the function "**Out[]**"—as in "**Out[4]** = $\frac{\pi^2}{6}$"
(notice it's *not* "**Out[4]:=**$\frac{\pi^2}{6}$"—we'll see why later)

```
Out[1]
Out[4]
```

199 830 036

$$\frac{\pi^2}{6}$$

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etc.

Finding and learning about built-in functions in *Mathematica*

Interface Tips & Tricks

100%

```
%6
Out[6]
```

```
2 880 067 194 370 816 120
```

```
2 880 067 194 370 816 120
```

◊ % by itself (without any number following it) *always* means "the line above"
%% calls "two lines above",
%%% "three lines above",
etc.

```
Random[];
%
%%%%
```

```
0.865067
```

```
0.386983
```

Finding and learning about built-in functions in *Mathematica*

Interface Tips & Tricks

ALT + . : Abort an evaluation

Defining new functions

Replacements

Using Packages: "<< "

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etc.

```
Random[];
%
%%%%
```

```
0.0436885
```

```
0.137233
```

Finding and learning about built-in funct

★ *All* built-in functions in *Mathematica* begi
This suggests that *your* functions should b

◇ Documentation Center is a very useful res

◇ **?Function** :: looks-up the basic syntax/op
(for a user-defined **function**, **?function** by

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```
0.137233
```

Finding and learning about built-in functions in *Mathematica*

★ *All* built-in functions in *Mathematica* begin with a Capitalized first letter.
This suggests that *your* functions should be defined with lower-case first letters!

◊ Documentation Center is a very useful resource (F1 key)

◊ **?Function ::** looks-up the basic syntax/options for a built-in **Function**
(for a user-defined **function**, **?function** by default just returns its definition)

Let us breifly mention some *very* useful functions with easily-overlooked options

- **Range**

```
? Range
```

Range[i_{max}] generates the list {1, 2, ..., i_{max}}.
Range[i_{min}, i_{max}] generates the list {i_{min}, ..., i_{max}}.
Range[i_{min}, i_{max}, di] uses step di. »

And so,

```
Range[6]
Range[6, 10]
Range[6, 10, 2]
```

- **Mod**

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Range[i_{min}, i_{max}, di] uses step di. »

And so,

```
Range[6]
Range[6, 10]
Range[6, 10, 2]
```

```
{1, 2, 3, 4, 5, 6}
```

```
{6, 7, 8, 9, 10}
```

```
{6, 8, 10}
```

Range[i_{max}] generates the list {1, 2, ..., i_{max}}.
Range[i_{min}, i_{max}] generates the list {i_{min}, ..., i_{max}}.
Range[i_{min}, i_{max}, *di*] uses step *di*. »

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Range[6, 10, 2]
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```
{1, 2, 3, 4, 5, 6}
```

```
{6, 7, 8, 9, 10}
```

```
{6, 8, 10}
```

- **Mod**

```
? Mod
```

Thus,

```
Mod[Range[12], 6]
Mod[Range[12], 6, 1]
```

- **RandomInteger[]**

```
? RandomInteger
```



```
RandomInteger[]
```

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```
{6, 8, 10}
```

- **Mod**

```
?Mod
```

Mod[*m*, *n*] gives the remainder on division of *m* by *n*.
Mod[*m*, *n*, *d*] uses an offset *d*. »

Thus,

```
Mod[Range[12], 6]
Mod[Range[12], 6, 1]
```

```
{1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, 0}
```

```
{1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6}
```

- **RandomInteger[]**

```
?RandomInteger
```

```
RandomInteger[]
RandomInteger[50]
RandomInteger[50, 10]
RandomInteger[{-50, 50}, 10]
RandomInteger[{-50, 50}, {10, 4}]
```

- If you even encounter an unfamiliar built-in function, just look it up!

100%

```
{1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, 0}
```

```
{1, 2, 3, 4, 5, 6, 1, 2, 3, 4, 5, 6}
```

- **RandomInteger[]**

```
? RandomInteger
```

RandomInteger[{i_{min}, i_{max}}] gives a pseudorandom integer in the range {i_{min}, ..., i_{max}}.
RandomInteger[i_{max}] gives a pseudorandom integer in the range {0, ..., i_{max}}.
RandomInteger[] pseudorandomly gives 0 or 1.
RandomInteger[*range*, *n*] gives a list of *n* pseudorandom integers.
RandomInteger[*range*, {n_1, n_2, ...}] gives an $n_1 \times n_2 \times \ldots$ array of pseudorandom integers. »

```
RandomInteger[]
RandomInteger[50]
RandomInteger[50, 10]
RandomInteger[{-50, 50}, 10]
RandomInteger[{-50, 50}, {10, 4}]
```

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◊ **?Func*** :: searches for a Function whose name begins with "**Func**"

◊ **??Function** :: gives more detailed information about **Function** (including more subtle attributes such as the Function's "**Attributes**").

◊ **General Advice**: If you are ever frustrated that a built-in function **almost** does what you want ***but not quite***, and you think that the functionality you need is not too uncommon, check to see if there is an optional

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RandomInteger[i_{max}] gives a pseudorandom integer in the range {0, ..., i_{max}}.
RandomInteger[] pseudorandomly gives 0 or 1.
RandomInteger[*range*, *n*] gives a list of *n* pseudorandom integers.
RandomInteger[*range*, {n_1, n_2, ...}] gives an $n_1 \times n_2 \times \ldots$ array of pseudorandom integers. »

```
RandomInteger[]
RandomInteger[50]
RandomInteger[50, 10]
RandomInteger[{-50, 50}, 10]
RandomInteger[{-50, 50}, {10, 4}]
```

```
1
```

```
8
```

```
{43, 34, 46, 36, 28, 5, 38, 23, 47, 37}
```

```
{-17, -5, 39, -38, -27, 3, -11, -34, 26, -2}
```

```
{{-22, -50, 44, 42}, {-2, 4, 6, 20}, {-16, 7, 22, 10}, {-3, -16, 39, -14}, {31, -38, 25, -16},
 {36, 9, -42, 12}, {23, -15, 30, 5}, {19, 44, -48, -4}, {-20, 29, 7, -32}, {16, 38, 33, -50}}
```

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```
RandomInteger[{-50, 50}, {10, 4}]
```

```
1
```

```
8
```

```
{43, 34, 46, 36, 28, 5, 38, 23, 47, 37}
```

```
{-17, -5, 39, -38, -27, 3, -11, -34, 26, -2}
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```
{{-22, -50, 44, 42}, {-2, 4, 6, 20}, {-16, 7, 22, 10}, {-3, -16, 39, -14}, {31, -38, 25, -16},
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```

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Interface Tips & Tricks

ALT + . : Abort an evaluation

Defining new functions

```
RandomInteger[{-50, 50}, {10, 4}]
```

```
1
```

```
8
```

```
{43, 34, 46, 36, 28, 5, 38, 23, 47, 37}
```

```
{-17, -5, 39, -38, -27, 3, -11, -34, 26, -2}
```

```
{{-22, -50, 44, 42}, {-2, 4, 6, 20}, {-16, 7, 22, 10}, {-3, -16, 39, -14}, {31, -38, 25, -16},
 {36, 9, -42, 12}, {23, -15, 30, 5}, {19, 44, -48, -4}, {-20, 29, 7, -32}, {16, 38, 33, -50}}
```

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◇ **?Func*** :: searches for a Function whose name begins with "**Func**"

```
?Matrix*
```

▼ System

MatrixExp	MatrixForm	MatrixPlot	MatrixPower	MatrixQ	MatrixRank

◇ **??Function** :: gives more detailed information about **Function** (including more subtle attributes such as the Function's "**Attributes**").

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Interface Tips & Tricks

100%

MatrixQ[*expr*, *test*] gives True only if *test* yields True when applied to each of the matrix elements in *expr*. »

◇ **??Function** :: gives more detailed information about **Function** (including more subtle attributes such as the Function's "**Attributes**").

```
?? Partition
```

Partition[*list*, *n*] partitions *list* into non-overlapping sublists of length *n*.
Partition[*list*, *n*, *d*] generates sublists with offset *d*.
Partition[*list*, {n_1, n_2, ...}] partitions a nested list into blocks of size $n_1 \times n_2 \times \ldots$.
Partition[*list*, {n_1, n_2, ...}, {d_1, d_2, ...}] uses offset d_i at level *i* in *list*.
Partition[*list*, *n*, *d*, {k_L, k_R}] specifies that the first element of *list* should appear at position k_L in the first sublist, and the last element of *list* should appear at or after position k_R in the last sublist. If additional elements are needed, Partition fills them in by treating *list* as cyclic.
Partition[*list*, *n*, *d*, {k_L, k_R}, *x*] pads if necessary by repeating the element *x*.
Partition[*list*, *n*, *d*, {k_L, k_R}, {x_1, x_2, ...}] pads if necessary by cyclically repeating the elements x_i.
Partition[*list*, *n*, *d*, {k_L, k_R}, {}] uses no padding, and so can yield sublists of different lengths.
Partition[*list*, *nlist*, *dlist*, {$klist_L$, $klist_R$}, *padlist*] specifies alignments and padding in a nested list. »

```
Attributes[Partition] = {Protected}
```

Moreover, for Functions defined for **particular arguments**, these particular definitions will be listed explicitly: e.g.,

```
?? In
```

◇ **General Advice:** If you are ever frustrated that a built-in function **almost** does what you want ***but not quite***, and you think that the functionality you need is not too uncommon, check to see if there is an optional argument for the function which would make it do what you want before inventing it from scratch.

MatrixQ[*expr*, *test*] gives True only if *test* yields True when applied to each of the matrix elements in *expr*. »

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Partition[*list*, *n*, *d*, {k_L, k_R}, *x*] pads if necessary by repeating the element *x*.
Partition[*list*, *n*, *d*, {k_L, k_R}, {x_1, x_2, ...}] pads if necessary by cyclically repeating the elements x_i.
Partition[*list*, *n*, *d*, {k_L, k_R}, {}] uses no padding, and so can yield sublists of different lengths.
Partition[*list*, *nlist*, *dlist*, {$klist_L$, $klist_R$}, *padlist*] specifies alignments and padding in a nested list. »

```
Attributes[Partition] = {Protected}
```

```
Partition[Range[8], 2, 1, 1]
```

```
{{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 1}}
```

Moreover, for Functions defined for **particular arguments**, these particular definitions will be listed explicitly: e.g.,

```
?? In
```

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Partition[*list*, *n*, *d*, {k_L, k_R}, *x*] pads if necessary by repeating the element *x*.
Partition[*list*, *n*, *d*, {k_L, k_R}, {x_1, x_2, ...}] pads if necessary by cyclically repeating the elements x_i.
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Attributes[Partition] = {Protected}
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```

```
{{1, 2}, {2, 3}, {3, 4}, {4, 5}, {5, 6}, {6, 7}, {7, 8}, {8, 1}}
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Moreover, for Functions defined for **particular arguments**, these particular definitions will be listed explicitly: e.g.,

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Interface Tips & Tricks

ALT + . : Abort an evaluation

Defining new functions

Replacements

Using Packages: "<< "

Visualizing data with *Mathematica*:

100%

Moreover, for Functions defined for **particular arguments**, these particular definitions will be listed explicitly: e.g.,

```
?? In
```

In[n] is a global object that is assigned to have a delayed value of the n^{th} input line. »

```
Attributes[In] = {Listable, Protected}
```

In[1] = $\frac{19991\,(19991-1)}{2}$

In[2] = $\sum_{n=1}^{19991} n$

```
In[3] = (13;)

In[4] = PolyLog[2, 1]

In[5] = (1 + 1;)

In[6] = Fibonacci[90]

In[7] = In[1]

In[8] = In[100]

In[9] = %1

In[10] = %4
```

Moreover, for Functions defined for **particular arguments**, these particular definitions will be listed explicitly: e.g.,

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?? In
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In[*n*] is a global object that is assigned to have a delayed value of the n^{th} input line. >>

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Interface Tips & Tricks

◇ Highlighting (parts of) expressions:

◇ Æsthetical ways to input expressions

◇ Special characters: ESC __ ESC

ALT + . : Abort an evaluation

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***Visualizing* data with *Mathematica*:**

and you think that the functionality you need is not too uncommon, check to see if there is an optional argument for the function which would make it do what you want before inventing it from scratch

Interface Tips & Tricks

◇ Highlighting (parts of) expressions:

- double clicking highlights a single character, and successive mouse-clicks successively expands the highlighted region, stopping at levels dictated by commas, parthenses, etc.

```
Power[(1 + 2 + 3) * (5) + 16, x]
```

- CMD + SHIFT +b highlights the nearest contiguous region bounded by brackets, parentheses, etc. (not commas)

★ Because CMD + SHIFT +b locates matching brackets, it is very useful for debugging.

◇ Æsthetical ways to input expressions

◇ Special characters: ESC __ ESC

ALT + . : Abort an evaluation

Defining new functions

Replacements

Using Packages: "<< "

Visualizing data with *Mathematica*:

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◊ Highlighting (parts of) expressions:

- double clicking highlights a single character, and successive mouse-clicks successively expands the highlighted region, stopping at levels dictated by commas, parthenses, etc.

```
Power[(1+2+3)*(5)+16, x]
```

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◊ Æsthetical ways to input expressions

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ALT + . : Abort an evaluation

Defining new functions

Replacements

Using Packages: "<< "

***Visualizing* data with *Mathematica*:**

Mathematica Vernacular: The Anatomy of an Expression

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commas)

★ Because CMD+SHFT+b locates matching brackets, it is very useful for debugging.

◇ Æsthetical ways to input expressions

- Factions: CTRL+"/"

$\frac{\square}{\square}$

Useful to know: the sub-expression (or more, if highlighted) immediately preceeding the cursor, if not a space, will become the numerator; e.g.,

(1 + 2 + 3) * (5) + 16
(1 + 2 + 3) * (5) + 16

This pre-highlighting structure also goes for most of the other examples below.

- Exponentiation: CTRL+6

$\square^{\square}$

- Square-Roots: CTRL+2

$\sqrt{\square}$

A complete list of *Mathematica*'s Control Keys (copied from Documentation Center):

Ctrl+2 or Ctrl+@ square root

100%

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commas)

★ Because CMD + SHIFT +b locates matching brackets, it is very useful for debugging.

◊ Æsthetical ways to input expressions

◊ Special characters: ESC __ ESC

ALT + . : **Abort an evaluation**

Defining new functions

Replacements

Using Packages: "<< "

***Visualizing* data with *Mathematica*:**

Mathematica Vernacular: The Anatomy of an Expression

100%

commas)

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- Factions: CTRL+"/"

$\frac{\square}{\square}$

Useful to know: the sub-expression (or more, if highlighted) immediately preceeding the cursor, if not a space, will become the numerator; e.g.,

```
(1 + 2 + 3) * (5) + 16
(1 + 2 + 3) * (5) + 16
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$\sqrt{\square}$

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commas)

★ Because CMD+SHIFT+b locates matching brackets, it is very useful for debugging.

◇ Æsthetical ways to input expressions

- Factions: CTRL+"/"

$\frac{a}{b}$

Useful to know: the sub-expression (or more, if highlighted) immediately preceeding the cursor, if not a space, will become the numerator; e.g.,

$(1+2+3)*(5)+\frac{16}{\square}$

$(1+2+3)*(5)+16$

This pre-highlighting structure also goes for most of the other examples below.

- Exponentiation: CTRL+6

$\square^{\square}$

- Square-Roots: CTRL+2

$\sqrt{\square}$

A complete list of *Mathematica*'s Control Keys (copied from Documentation Center):

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$\sqrt{\sqrt{4}\ 2}$

2

A complete list of *Mathematica*'s Control Keys (copied from Documentation Center):

Ctrl+2 or Ctrl+@	square root
Ctrl[illegible]	switch to alternate position (e.g. subscript to superscript)
Ctrl[illegible]	superscript
Ctrl+7 or Ctrl+&	overscript
Ctrl+9 or Ctrl+(	begin a new cell within an existing cell
Ctrl+0 or Ctrl+)	end a new cell within an existing cell
Ctrl+- or Ctrl+_	subscript
Ctrl+4 or Ctrl+$	underscript
Ctrl+Enter	create a new row in a table
Ctrl+,	create a new column in a table
Ctrl+.	expand current selection
Ctrl+/	fraction
Ctrl+Space	return from current position or state
Ctrl+←, Ctrl+→, Ctrl+↑, Ctrl+↓	move an object by minimal increments on the screen

Double-click to edit.
CTRL-t for tools.

◊ Special characters: ESC __ ESC

ALT + . : **Abort an evaluation**

Defining new functions

100%

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Ctrl+/	fraction
Ctrl+Space	return from current position or state
Ctrl+←, Ctrl+→, Ctrl+↑, Ctrl+↓	move an object by minimal increments on the screen

Special characters: ESC __ ESC

Hitting ESC, prepares *Mathematica* to expect a special-character code—which is terminated by hitting ESC a second time.

α
η
g

A semi-complete list of special character codes (from Documentation Center):

Special Character Shortcuts

The \[AliasDelimiter] or (⋮) character is produced by typing Esc.

Greek Letters (α, Α, ...)	⋮a⋮ or ⋮A⋮, ...
Hebrew Letters (ℵ, ℶ, ...)	⋮al⋮, ...
Script Letters (𝒶, 𝒜, ...)	⋮sca⋮ or ⋮scA⋮, ...
Double-Struck Letters (𝕒, 𝔸, ...)	⋮dsa⋮ or ⋮dsA⋮, ...
Gothic Letters (𝔞, 𝔄, ...)	⋮goa⋮ or ⋮goA⋮, ...
Formal Symbols (a, A, ...)	⋮$a⋮ or ⋮$A⋮, ...
International Character Sets (á, Á, ...)	⋮a'⋮ or ⋮A'⋮, ...

A Selection of Notational Alphabet Shortcuts.

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α
η
γ

α

η

γ

A semi-complete list of special character codes (from Documentation Center):

Special Character Shortcuts

The \[AliasDelimiter] or ⋮ character is produced by typing Esc.

Greek Letters (α, Α, ...)	⋮a⋮ or ⋮A⋮, ...
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Formal Symbols (a, A, ...)	⋮$a⋮ or ⋮$A⋮, ...
International Character Sets (á, Á, ...)	⋮a'⋮ or ⋮A'⋮, ...

A Selection of Notational Alphabet Shortcuts

Exponential (ⅇ)	⋮ee⋮
Integral (∫)	⋮int⋮
Differential (ⅆ)	⋮dd⋮
Riemann Sum (∑)	⋮sum⋮

Writing Assistant

Text Cells | Math Cells
Start Slide | End Slide | Page Break

Cell Modifications
Merge Cells | Divide Cell
Group Together | Group Normally

Text Properties
B | I | U | A↑ | A↓ | Font...
Text Color | 12 | Clear

Cell Properties
Background | Frame

Paragraph Properties

Notebook Properties and Actions
Find... | Spelling...
Content | Appearance
Stylesheet Chooser...
Drawing Tools...

Typesetting

Help and Settings

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Transpose (^{T})	:tr:
Hbar (ℏ)	:hb:
Infinity (∞)	:inf:

A Selection of Mathematical Typesetting Shortcuts

Element (∈)	:el:
For All (∀)	:fa:
Union (⋃)	:un:
Intersection (⋂)	:inter:
Not (¬)	:not:
Implies (⇒)	:=>:
Therefore (∴)	:tf:

A Selection of Logical Connective Shortcuts

ALT + . : Abort an evaluation

Evaluations occassionally need to be **Aborted**, typically because of one or two reasons:

◇ Endless loops (typically accidental):

```
While[True, a]
```

◇ An expression is taking an *intollerably*-long time to evaluate (e.g. **FullSimplify**)

```
Pause[50]
```

◇ If *Mathematica* does not respond to ALT + . , try killing the Kernel directly

But beware of the red "Interrupted" of death!

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Transpose ($^{\top}$)	:tr:
Hbar ($\hbar$)	:hb:
Infinity (∞)	:inf:

A Selection of Mathematical Typesetting Shortcuts

Element ($\in$)	:el:
For All ($\forall$)	:fa:
Union ($\bigcup$)	:un:
Intersection ($\bigcap$)	:inter:
Not ($\neg$)	:not:
Implies ($\Rightarrow$)	:=>:
Therefore ($\therefore$)	:tf:

A Selection of Logical Connective Shortcuts

ALT + . : Abort an evaluation

Evaluations occassionally need to be **Aborted**, typically because of one or two reasons:

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◇ An expression is taking an *intollerably*-long time to evaluate (e.g. **FullSimplify**)

```
Pause[50]
$Aborted
```

◇ If *Mathematica* does not respond to ALT + . , try killing the Kernel directly

★ But beware of the red "Interrupted" of death!

Defining new functions

◇ f[x] = y :: "**Set** (or replace) all instances of "f[x]" to some expression y.

◇ f[x_] = (some function of the 'pattern' "x") :: Set[f[x_],...]

◇ f[x_] := (some function of the 'pattern' "x") :: SetDelayed[f[x_],...]

◇ Why choose one or the other?
f[x_] := f[x] = (some function of the 'pattern' "x")

Replacements

Using Packages: "<< "

Visualizing data with *Mathematica*:

Mathematica Vernacular: The Anatomy of an Expression

100%

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Defining new functions

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◇ Why choose one or the other?
f[x_] := f[x] = **(some function of the 'pattern' "x")**

Replacements

Using Packages: "<< "

Visualizing data with *Mathematica:*

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Defining new functions

◊ f[x] = y :: "Set (or replace) all instances of "f[x]" to some expression y.

```
myFunction[x] = y
```

```
y
```

Because *only* myFunction[*x*] is defined, myFunction with any other arguments is undefined, and thus remains purely symbolic:

```
myFunction[1]
myFunction[X]
myFunction[x]
```

```
myFunction[1]
```

```
myFunction[X]
```

```
y
```

```
?myFunction
```

Compare this behaviour with,

◊ f[x_] = (some function of the 'pattern' "x") :: Set[f[x_],...]

◊ f[x_] := (some function of the 'pattern' "x") :: SetDelayed[f[x_],...]

◊ Why choose one or the other?

100%

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```
myFunction[x]
```

```
y
```

```
?myFunction
```

Compare this behaviour with,

◊ f[x_] = (some function of the 'pattern' "x") :: Set[f[x_],...]

Here, the operative bit of notation is "_" (called a "**Blank**"), which is an important example of a **Pattern**, and means:

```
Blank[]
```

```
_
```

★ "_" :: "any *single* expression whatsoever", and

★ "x_" :: "any *single* expression whatsoever **which we will herein call x**"

```
triangularNumber[x_] = x (x + 1) / 2
```

```
triangularNumber[19991]
triangularNumber[π]
triangularNumber[√-2]
```

Here, the RHS of the definition is evaluated *immediately*, treating **x** symbolically.

Sometimes this is more efficient, but it can often be problematic.

100%

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```
Blank[]
```

```
_
```

★ "_" :: "any *single* expression whatsoever", and

★ "x_" :: "any *single* expression whatsoever **which we will herein call x**"

```
triangularNumber[x_] = x (x + 1) / 2
```

$\frac{1}{2} x (1 + x)$

```
triangularNumber[19991]
triangularNumber[π]
triangularNumber[√-2]
triangularNumber[I]
```

199 830 036

$\frac{1}{2} \pi (1 + \pi)$

$\frac{i \left(1 + i \sqrt{2}\right)}{\sqrt{2}}$

Here, the RHS of the definition is evaluated *immediately*, treating **x** symbolically. *Sometimes* this is more efficient, but it can often be problematic.

Alternatively, we can define functions as follows:

100%

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```
Blank[]

_
```

★ "_" :: "any *single* expression whatsoever", and

★ "x_" :: "any *single* expression whatsoever **which we will herein call x**"

```
triangularNumber[x_] = x (x + 1) / 2
```

$\frac{1}{2} x (1 + x)$

```
triangularNumber[19991]
triangularNumber[π]
triangularNumber[√-2]
triangularNumber[List[1, 2, 3]]
```

199 830 036

$\frac{1}{2} \pi (1 + \pi)$

$\frac{i (1 + i \sqrt{2})}{\sqrt{2}}$

{1, 3, 6}

Here, the RHS of the definition is evaluated *immediately*, treating **x** symbolically.

Sometimes this is more efficient, but it can often be problematic.

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```
triangularNumber[19991]
triangularNumber[π]
triangularNumber[√-2]
triangularNumber[f[10]]
```

199 830 036

$$\frac{1}{2}\pi\,(1+\pi)$$

$$\frac{i\left(1+i\sqrt{2}\right)}{\sqrt{2}}$$

$$\frac{1}{2}\,f[10]\,(1+f[10])$$

Here, the RHS of the definition is evaluated *immediately*, treating **x** symbolically.
Sometimes this is more efficient, but it can often be problematic.

Alternatively, we can define functions as follows:

◇ **f[x_] := (some function of the 'pattern' "x") :: SetDelayed[f[x_],...]**

◇ Why choose one or the other?
f[x_] := f[x] = (some function of the 'pattern' "x")

Replacements

100%

$\sqrt{2}$

$\frac{1}{2}$ f[10] (1 + f[10])

Here, the RHS of the definition is evaluated *immediately*, treating **x** symbolically.
Sometimes this is more efficient, but it can often be problematic.

Alternatively, we can define functions as follows:

◇ f[x_] := (some function of the 'pattern' "x") :: SetDelayed[f[x_],...]

The only difference between this definition and the one above, is that here, the RHS is left **Unevaluated** until the function is called.

```
triangularNumberD[x_] := (Print["The ", x, "th triangular number has been computed!"]; x (x + 1) / 2)
```

```
triangularNumberD[19991]
triangularNumberD[π]
triangularNumberD[√-2]
```

Compare this with:

```
triangularNumber[x_] = (Print["The ", x, "th triangular number has been computed!"]; x (x + 1) / 2)
```

```
triangularNumber[19991]
triangularNumber[π]
triangularNumber[√-2]
```

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Here, the RHS of the definition is evaluated *immediately*, treating **x** symbolically.
Sometimes this is more efficient, but it can often be problematic.

Alternatively, we can define functions as follows:

◊ **f[x_] := (some function of the 'pattern' "x") :: SetDelayed[f[x_],...]**

The only difference between this definition and the one above, is that here, the RHS is left **Unevaluated** until the function is called.

```
triangularNumberD[x_] := (Print["The ", x, "th triangular number has been computed!"]; x (x + 1) / 2)
```

```
triangularNumberD[19991]
triangularNumberD[π]
triangularNumberD[√-2]
```

Compare this with:

```
triangularNumber[x_] = (Print["The ", x, "th triangular number has been computed!"]; x (x + 1) / 2)
```

```
triangularNumber[19991]
triangularNumber[π]
triangularNumber[√-2]
```

★ **Rule of thumb:** Keep *Mathematica* on a "need to know basis," or

★ When in doubt, procrastinate any evaluation until its necessary.

100%

Compare this with:

```
triangularNumber[x_] = (Print["The ", x, "th triangular number has been computed!"]; x (x + 1) / 2)
The xth triangular number has been computed!
```

$\frac{1}{2} x (1 + x)$

```
triangularNumber[19991]
triangularNumber[π]
triangularNumber[√-2]
```

199 830 036

$\frac{1}{2} \pi (1 + \pi)$

$\frac{i \left(1 + i \sqrt{2}\right)}{\sqrt{2}}$

★ **Rule of thumb:** Keep *Mathematica* on a "need to know basis," or

★ When in doubt, procrastinate any evaluation until its necessary.

◊ Why choose one or the other?

• f[x_] := f[x] = (some function of the 'pattern' "x")

Replacements

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★ When in doubt, procrastinate any evaluation until its necessary.

◇ Why choose one or the other?

f[x_] := f[x] = (some function of the 'pattern' "x")

Here, **SetDelayed** *calls* **Set**.

```
triangularNumberStored[x_] :=
 triangularNumberStored[x] = (Print["The ", x, "th triangular number has been computed!"]; x (x + 1) / 2)
```

```
triangularNumberStored[19991]
triangularNumberStored[691]
triangularNumberStored[19991]
```

```
The 19991th triangular number has been computed!
199830036
The 691th triangular number has been computed!
239086
199830036
```

```
? triangularNumberStored
```

Replacements

Using Packages: "<< "

Visualizing data with *Mathematica:*

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```
triangularNumberStored[691]
triangularNumberStored[19991]
```

The 19991th triangular number has been computed!

199830036

The 691th triangular number has been computed!

239086

199830036

```
?triangularNumberStored
```

Global`triangularNumberStored

```
triangularNumberStored[691] = 239086

triangularNumberStored[19991] = 199830036

triangularNumberStored[x_] :=
 triangularNumberStored[x] = (Print[The , x, th triangular number has been computed!]; 1/2 x (x + 1))
```

Replacements

Using Packages: "<< "

Visualizing data with *Mathematica*:

Mathematica Vernacular: The Anatomy of an Expression

100%

```
triangularNumberStored[19991]
The 19991th triangular number has been computed!
199830036
The 691th triangular number has been computed!
239086
199830036
```

```
?triangularNumberStored
```

```
Global`triangularNumberStored

triangularNumberStored[691] = 239086

triangularNumberStored[19991] = 199830036

triangularNumberStored[x_] :=
 triangularNumberStored[x] = (Print[The , x, th triangular number has been computed!]; 1/2 x (x + 1))
```

```
{m, p, m, p, m, p, m, p} /. {p → m, m → p}
{p, m, p, m, p, m, p, m}
```

Replacements

Using Packages: "<< "

Visualizing data with *Mathematica:*

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Global`triangularNumberStored

```
triangularNumberStored[691] = 239086

triangularNumberStored[19991] = 199830036

triangularNumberStored[x_] :=
 triangularNumberStored[x] = (Print[The , x, th triangular number has been computed!]; 1/2 x (x + 1))
```

```
{m, p, m, p, m, p, m, p} /. {p → m, m → p}
{p, m, p, m, p, m, p, m}
```

Replacements

Using Packages: "<< "

Visualizing data with *Mathematica*:

Mathematica Vernacular: The Anatomy of an Expression

```
{m, p, m, p, m, p, m, p} /. {p → m, m → p}
{p, m, p, m, p, m, p, m}
```

Replacements

◇ (some expression where "x" appears)/.{x->(new expression possibly with x)}
Replace[LHS,Rule[x,RHS]]

```
(x (x + 1) / 2) /. {x → 19991}
199830036
```

◇ (some expression where "x" appears)/.{x :> (new expression possibly with x)}
Replace[LHS,RuleDelayed[x,RHS]]

★ "->" is to ":>" as "=" is to ":="

Using Packages: "<< "

Visualizing data with *Mathematica*:

Mathematica Vernacular: The Anatomy of an Expression

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Replacements

◇ (some expression where "x" appears)/.{x->(new expression possibly with x)}
Replace[LHS,Rule[x,RHS]]

```
(x (x + 1) / 2) /. {x → 19991}
(x (x + 1) / 2) /. {x :→ 19991}
```

199 830 036

◇ (some expression where "x" appears)/.{x :> (new expression possibly with x)}
Replace[LHS,RuleDelayed[x,RHS]]

★ "->" is to ":>" as "=" is to ":="

Using Packages: "<< "

Visualizing data with *Mathematica*:

Mathematica Vernacular: The Anatomy of an Expression

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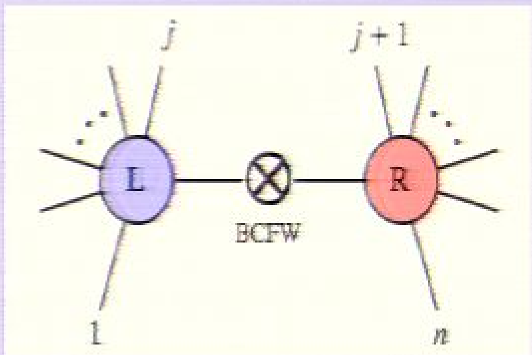
```
<< bcfw.m
```



Efficient Tree−Amplitudes in $\mathcal{N}$=4 SYM
via BCFW in the Momentum−Twistor Grassmannian
Jacob L. Bourjaily, 2010

This package defines all tree amplitudes analyticall in N=4 SYM, using a (*much* slower) recursion algorithm than the one each of you will develop during this school. But it can be useful as a reference:

```
nice /@ AmpTerms[m, p, m, p, m, p]
```

```
nAmp[m, p, m, p, m, p, m, p, m, p]
```

```
bcfwTerms1 = nAmpTerms[0, 0, 0][m, p, m, p, m, p, m, p, m, p] // withTiming
```

```
Total[bcfwTerms1]
```

```
bcfwTerms2 = nAmpTerms[1, 1, 1][m, p, m, p, m, p, m, p, m, p] // withTiming;
Intersection[bcfwTerms1, bcfwTerms2]
Total[bcfwTerms2]
```

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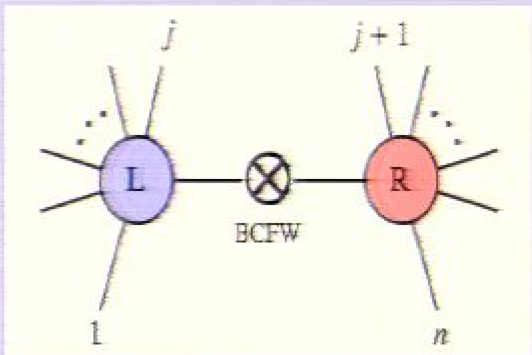
```
<< bcfw.m
```



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```
nice /@ AmpTerms[m, p, m, p, m, p]
```

$$\left\{\frac{\langle 15\rangle^4 (\langle 35\rangle \langle 1234\rangle + \langle 34\rangle \langle 5123\rangle)^4}{\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle \langle 56\rangle \langle 61\rangle \langle 1234\rangle \langle 2345\rangle \langle 3451\rangle \langle 4512\rangle \langle 5123\rangle}, \frac{(\langle 13\rangle \langle 56\rangle \langle 1345\rangle - \langle 15\rangle (\langle 36\rangle \langle 1345\rangle + \langle 34\rangle \langle 5613\rangle + \langle 35\rangle \langle 6134\rangle))^4}{\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle \langle 56\rangle \langle 61\rangle \langle 1345\rangle \langle 3456\rangle \langle 4561\rangle \langle 5613\rangle \langle 6134\rangle}, \frac{(\langle 13\rangle \langle 56\rangle \langle 1235\rangle - \langle 15\rangle (\langle 36\rangle \langle 1235\rangle + \langle 35\rangle \langle 6123\rangle))^4}{\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle \langle 56\rangle \langle 61\rangle \langle 1235\rangle \langle 2356\rangle \langle 3561\rangle \langle 5612\rangle \langle 6123\rangle}\right\}$$

```
nAmp[m, p, m, p, m, p, m, p, m, p]
```

100%

$$\left\{\frac{\langle 15\rangle^4 (\langle 35\rangle \langle 1234\rangle + \langle 34\rangle \langle 5123\rangle)^4}{\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle \langle 56\rangle \langle 61\rangle \langle 1234\rangle \langle 2345\rangle \langle 3451\rangle \langle 4512\rangle \langle 5123\rangle},\right.$$
$$\frac{(\langle 13\rangle \langle 56\rangle \langle 1345\rangle - \langle 15\rangle (\langle 36\rangle \langle 1345\rangle + \langle 34\rangle \langle 5613\rangle + \langle 35\rangle \langle 6134\rangle))^4}{\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle \langle 56\rangle \langle 61\rangle \langle 1345\rangle \langle 3456\rangle \langle 4561\rangle \langle 5613\rangle \langle 6134\rangle},$$
$$\left.\frac{(\langle 13\rangle \langle 56\rangle \langle 1235\rangle - \langle 15\rangle (\langle 36\rangle \langle 1235\rangle + \langle 35\rangle \langle 6123\rangle))^4}{\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle \langle 56\rangle \langle 61\rangle \langle 1235\rangle \langle 2356\rangle \langle 3561\rangle \langle 5612\rangle \langle 6123\rangle}\right\}$$

```
nAmp[m, p, m, p, m, p, m, p, m, p]
```

$$-\frac{56687187056}{1969120125}$$

```
bcfwTerms1 = nAmpTerms[0, 0, 0][m, p, m, p, m, p, m, p, m, p] // withTiming
```

Evaluation of the 10-point N^{3}MHV amplitude required 1 ms to complete.

$A_{10}^{(5)}(g-, g+, g-, g+, g-, g+, g-, g+, g-, g+)$:

$$\left\{0, -\frac{65536}{385875}, -\frac{16777216}{10418625}, -\frac{32768}{385875}, -\frac{65536}{40516875}, -\frac{8388608}{13589359875}, -\frac{819200}{3250611}, -\frac{16777216}{225736875}, -\frac{479756288}{17472034125},\right.$$
$$-\frac{32768}{2083725}, -\frac{6400}{9261}, -\frac{102400}{27783}, -\frac{12800}{83349}, -\frac{10000}{527877}, -\frac{3276800}{4750893}, -\frac{400}{250047}, -\frac{256}{694575}, -\frac{256}{33075}, 0, -\frac{128}{19845},$$
$$-\frac{1042568}{22920975}, -\frac{2097152}{1819125}, -\frac{8}{6125}, -\frac{256}{55125}, -\frac{512}{27783}, -\frac{512}{3675}, -\frac{256}{99225}, -\frac{128}{43659}, -\frac{6912}{67375}, -\frac{128}{496125},$$
$$-\frac{128}{7441875}, -\frac{6272}{109285605}, -\frac{236421376}{9464179725}, -\frac{3136}{3391875}, -\frac{2000000}{1125914699}, -\frac{64}{3075975}, -\frac{2085136}{3476347875},$$
$$-\frac{1483275763712}{14006205588375}, -\frac{322417936}{174765488625}, -\frac{512}{505110375}, -\frac{2048}{14586075}, -\frac{10648}{1879983}, -\frac{5451776}{45919125}, -\frac{8}{111375},$$

64 80000 64 7311616 128 128 4096 524288

$\langle 12\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle \langle 56\rangle \langle 61\rangle \langle 1235\rangle \langle 2356\rangle \langle 3561\rangle \langle 5612\rangle \langle 6123\rangle$

```
nAmp[m, p, m, p, m, p, m, p, m, p]
```

$$-\frac{56687187056}{1969120125}$$

```
bcfwTerms1 = nAmpTerms[0, 0, 0][m, p, m, p, m, p, m, p, m, p] // withTiming
```

Evaluation of the 10-point N^3MHV amplitude required 1 ms to complete.

$A_{10}^{(5)}(g_-, g_+, g_-, g_+, g_-, g_+, g_-, g_+, g_-, g_+)$:

$$\Big\{0, -\frac{65536}{385875}, -\frac{16777216}{10418625}, -\frac{32768}{385875}, -\frac{65536}{40516875}, -\frac{8388608}{13589359875}, -\frac{819200}{3250611}, -\frac{16777216}{225736875}, -\frac{479756288}{17472034125},$$
$$-\frac{32768}{2083725}, -\frac{6400}{9261}, -\frac{102400}{27783}, -\frac{12800}{83349}, -\frac{10000}{527877}, -\frac{3276800}{4750893}, -\frac{400}{250047}, -\frac{256}{694575}, -\frac{256}{33075}, 0, -\frac{128}{19845},$$
$$-\frac{1042568}{22920975}, -\frac{2097152}{1819125}, -\frac{8}{6125}, -\frac{256}{55125}, -\frac{512}{27783}, -\frac{512}{3675}, -\frac{256}{99225}, -\frac{128}{43659}, -\frac{6912}{67375}, -\frac{128}{496125},$$
$$-\frac{128}{7441875}, -\frac{6272}{109285605}, -\frac{236421376}{9464179725}, -\frac{3136}{3391875}, -\frac{2000000}{1125914699}, -\frac{64}{3075975}, -\frac{2085136}{3476347875},$$
$$-\frac{1483275763712}{14006205588375}, -\frac{322417936}{174765488625}, -\frac{512}{505110375}, -\frac{2048}{14586075}, -\frac{10648}{1879983}, -\frac{5451776}{45919125}, -\frac{8}{111375},$$
$$-\frac{64}{194481}, -\frac{80000}{14975037}, -\frac{64}{11344725}, -\frac{7311616}{170170875}, -\frac{128}{29712375}, -\frac{128}{540225}, -\frac{4096}{128625}, -\frac{524288}{3472875},$$
$$-\frac{4096}{694575}, -\frac{1024}{1466325}, -\frac{8388608}{329923125}, -\frac{1024}{17364375}, -\frac{2048}{1488375}, 0, -\frac{512}{496125}, -\frac{8192}{33075}, -\frac{8192}{13395375},$$
$$-\frac{1024}{1289925}, -\frac{131072}{522419625}, -\frac{4096}{535815}, -\frac{1024}{27088425}, -\frac{1048576}{72570225}, -\frac{1024}{2583819}, -\frac{4096}{72930375}, -\frac{65536}{1250235},$$

4096 32 8192 128 16 131072 16 131072 0 16384 262144

100%

```
Intersection[bcfwTerms1, bcfwTerms2]
Total[bcfwTerms2]
```

Evaluation of the 10-point N³MHV amplitude required 1 ms to complete.

$A_{10}^{(5)}(g-, g+, g-, g+, g-, g+, g-, g+, g-, g+)$:

{}

$-\frac{56\,687\,187\,056}{1\,969\,120\,125}$

For doing multiloop integrals, Czakon's Mellin-Barnes package may be useful:

```
<< MB.m
```

◊ It is generally a good idea to **create your own package** file to store the functions used most frequently in whatever project your currently working on.
For me, these days, it is:

◊ Packages can also be a useful repository of data:

◊ Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

***Visualizing* data with *Mathematica*:**

Mathematica Vernacular: The Anatomy of an Expression

100%

$$\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle\langle 1345\rangle\langle 3456\rangle\langle 4561\rangle\langle 5613\rangle\langle 6134\rangle,$$

$$\frac{(\langle 13\rangle\langle 56\rangle\langle 1235\rangle - \langle 15\rangle(\langle 36\rangle\langle 1235\rangle + \langle 35\rangle\langle 6123\rangle))^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle\langle 1235\rangle\langle 2356\rangle\langle 3561\rangle\langle 5612\rangle\langle 6123\rangle}\}$$

```
nAmp[m, p, m, p, m, p, m, p, m, p]
```

$$-\frac{56\,687\,187\,056}{1\,969\,120\,125}$$

```
useRandomTwistors[10]
```

```
bcfwTerms1 = nAmpTerms[0, 0, 0][m, p, m, p, m, p, m, p, m, p] // withTiming;
```

Evaluation of the 10-point N^3MHV amplitude required 86 ms to complete.

$A_{10}^{(5)}(g_-, g_+, g_-, g_+, g_-, g_+, g_-, g_+, g_-, g_+)$:

```
Total[bcfwTerms1]
```

505 100 528 098 978 777 551 112 943 289 015 108 003 049 258 450 110 958 307 222 163 304 814 316 722 738 244 732 153 054 \
426 428 331 007 952 014 337 121 488 353 946 748 255 982 999 804 351 911 030 092 797 214 991 415 574 530 783 686 884 332 \
104 960 069 048 473 998 793 593 763 823 022 903 268 300 846 001 039 785 271 203 /
506 970 385 676 627 763 576 445 461 126 311 368 402 719 897 773 014 282 574 537 116 152 744 671 667 362 511 755 486 636 \
945 098 251 023 959 442 500 168 689 250 128 166 839 956 377 345 203 743 176 671 677 392 579 444 204 610 406 591 799 264 \
359 486 535 891 707 358 804 434 055 456 769 818 526 051 188 899 840 000

```
bcfwTerms2 = nAmpTerms[1, 1, 1][m, p, m, p, m, p, m, p, m, p] // withTiming
Intersection[bcfwTerms1, bcfwTerms2]
Total[bcfwTerms2]
```

Evaluation of the 10-point N^3MHV amplitude required 1 ms to complete.

```
505 100 528 098 978 777 551 112 943 289 015 108 003 049 258 450 110 958 307 222 163 304 814 316 722 738 244 732 153 054 \
  426 428 331 007 952 014 337 121 488 353 946 748 255 982 999 804 351 911 030 092 797 214 991 415 574 530 783 686 884 332 \
  104 960 069 048 473 998 793 593 763 823 022 903 268 300 846 001 039 785 271 203 /
 506 970 385 676 627 763 576 445 461 126 311 368 402 719 897 773 014 282 574 537 116 152 744 671 667 362 511 755 486 636 \
  945 098 251 023 959 442 500 168 689 250 128 166 839 956 377 345 203 743 176 671 677 392 579 444 204 610 406 591 799 264 \
  359 486 535 891 707 358 804 434 055 456 769 818 526 051 188 899 840 000
```

For doing multiloop integrals, Czakon's Mellin-Barnes package may be useful:

```
<< MB.m
MB 1.2
by Michal Czakon
improvements by Alexander Smirnov
more info in hep-ph/0511200
last modified 2 Jan 09
```

◇ It is generally a good idea to **create your own package** file to store the functions used most frequently in whatever project your currently working on.
For me, these days, it is:

◇ Packages can also be a useful repository of data:

◇ Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

Visualizing data with *Mathematica*:

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505 100 528 098 978 777 551 112 943 289 015 108 003 049 258 450 110 958 307 222 163 304 814 316 722 738 244 732 153 054
426 428 331 007 952 014 337 121 488 353 946 748 255 982 999 804 351 911 030 092 797 214 991 415 574 530 783 686 884 332
104 9
506 97 … 4 671 667 362 511 755 486 636
945 0 … 79 444 204 610 406 591 799 264
359 4

Untitled-6

Functions Sections Update Debug Run Package

```
BeginPackage["myPackage`"]
Begin["Global`"]

End[]
EndPackage[]
```

For doi … :

```
<< MB.m
```

MB 1.2
by Micha
improvem
more inf
last mod

◇ It is gen … ns used most frequently in whatev … For me,

◇ Package

◇ Unless t … package is in the notebook's director

Visualiz

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```
threeLoopChain[{1, 2, 3}, {1, 2, 3}, {3, 4, 5}, {4, 5, 6}, {4, 5, 6}, {6, 1, 2}]}
```

Of course, these pure abstract objects "tripleHexagon" and "threeLoopChain" stand for completely explicit, rational functions:

```
nice /@ (threeLoopIntegrandSeeds[5] /. integrandRules)
threeLoopIntegrandSeeds[5] /. toFigures
```

$$\left\{\frac{(\langle 23(34(521)\cap(FE))\cap(42)3\rangle - \langle 23(34(523)\cap(FE))\cap(42)1\rangle)\,\langle 3451\rangle}{\langle AB12\rangle\langle AB23\rangle\langle ABCD\rangle\langle CD23\rangle\langle CD34\rangle\langle CDEF\rangle\langle EF34\rangle\langle EF45\rangle\langle EF51\rangle\langle EFAB\rangle},\right.$$

$$\frac{\langle EF(345)\cap(512)\rangle\,(\langle 23(34(121)\cap(FE))\cap(42)3\rangle - \langle 23(34(123)\cap(FE))\cap(42)1\rangle)}{\langle AB12\rangle\langle AB23\rangle\langle ABCD\rangle\langle CD23\rangle\langle CD34\rangle\langle CDEF\rangle\langle EF12\rangle\langle EF34\rangle\langle EF45\rangle\langle EF51\rangle\langle EFAB\rangle},$$

$$\frac{(\langle 24(45(121)\cap(FE))\cap(53)3\rangle - \langle 24(45(123)\cap(FE))\cap(53)1\rangle)\,\langle 4512\rangle}{\langle AB12\rangle\langle AB23\rangle\langle ABCD\rangle\langle CD34\rangle\langle CD45\rangle\langle CDEF\rangle\langle EF12\rangle\langle EF45\rangle\langle EF51\rangle\langle EFAB\rangle},$$

$$\frac{\langle 2345\rangle\,(\langle 23(45(121)\cap(FE))\cap(DC)3\rangle - \langle 23(45(123)\cap(FE))\cap(DC)1\rangle)\,\langle 4512\rangle}{\langle AB12\rangle\langle AB23\rangle\langle ABCD\rangle\langle CD23\rangle\langle CD34\rangle\langle CD45\rangle\langle CDEF\rangle\langle EF12\rangle\langle EF45\rangle\langle EF51\rangle\langle EFAB\rangle},$$

$$\frac{\langle AB(451)\cap(234)\rangle\,\langle 2345\rangle\,\langle 5123\rangle^2}{\langle AB23\rangle\langle AB45\rangle\langle AB51\rangle\langle ABCD\rangle\langle CD23\rangle\langle CD34\rangle\langle CD45\rangle\langle EF12\rangle\langle EF23\rangle\langle EF51\rangle\langle EFAB\rangle},$$

$$\frac{\langle EF(451)\cap(123)\rangle\,\langle 2345\rangle\,\langle 4523\rangle\,\langle 5234\rangle}{\langle AB23\rangle\langle AB45\rangle\langle ABCD\rangle\langle CD23\rangle\langle CD34\rangle\langle CD45\rangle\langle EF12\rangle\langle EF23\rangle\langle EF45\rangle\langle EF51\rangle\langle EFAB\rangle},$$

$$\left.\frac{\langle 2345\rangle\,\langle 4123\rangle\,\langle 5123\rangle\,\langle 5234\rangle}{\langle AB23\rangle\langle AB45\rangle\langle ABCD\rangle\langle CD23\rangle\langle CD34\rangle\langle CD45\rangle\langle EF12\rangle\langle EF23\rangle\langle EF51\rangle\langle EFAB\rangle}\right\}$$

```
{tripleHexagon[{1, 2, 3}, {1, 2, 3}, {2, 3, 4}, {2, 3, 4}, {3, 4, 5}, {4, 5, 1}],
 tripleHexagon[{1, 2, 3}, {1, 2, 3}, {2, 3, 4}, {2, 3, 4}, {3, 4, 5}, {5, 1, 2}],
 tripleHexagon[{1, 2, 3}, {1, 2, 3}, {3, 4, 5}, {3, 4, 5}, {4, 5, 1}, {5, 1, 2}],
 tripleHexagon[{1, 2, 3}, {1, 2, 3}, {2, 3, 4}, {3, 4, 5}, {4, 5, 1}, {5, 1, 2}],
```


100%

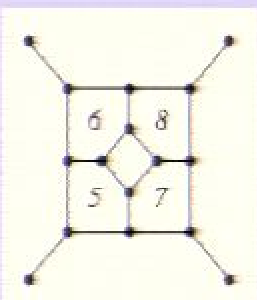
$$\frac{}{\langle AB23\rangle\langle AB45\rangle\langle AB51\rangle\langle ABCD\rangle\langle CD23\rangle\langle CD34\rangle\langle CD45\rangle\langle EF12\rangle\langle EF23\rangle\langle EF51\rangle\langle EFAB\rangle},$$

$$\frac{\langle EF(451)\bigcap(123)\rangle\langle 2345\rangle\langle 4523\rangle\langle 5234\rangle}{\langle AB23\rangle\langle AB45\rangle\langle ABCD\rangle\langle CD23\rangle\langle CD34\rangle\langle CD45\rangle\langle EF12\rangle\langle EF23\rangle\langle EF45\rangle\langle EF51\rangle\langle EFAB\rangle},$$

$$\left.\frac{\langle 2345\rangle\langle 4123\rangle\langle 5123\rangle\langle 5234\rangle}{\langle AB23\rangle\langle AB45\rangle\langle ABCD\rangle\langle CD23\rangle\langle CD34\rangle\langle CD45\rangle\langle EF12\rangle\langle EF23\rangle\langle EF51\rangle\langle EFAB\rangle}\right\}$$

◇ Packages can also be a useful repository of data:

```
<< four_point_multiloops.m
```



Four–Point Multi–Loop Integrands Through L=7

```
integrandCoefficients[6]
Length@%
integrandCoefficients[7]
Length@%
```

```
{2, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, -1, -1, -1, -1, -1, 1, -1, -1,
 -1, -1, -1, -1, -1, -1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1,
 -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, 1, 1, 1, 1, -1, -1, -1, -1, -1, -1, -1,
```

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```
Length@%
```

```
{2, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, -1, -1, -1, -1, -1, 1, -1, -1,
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 1, 1, 1, 1, 1, 1, 1, -1, -1, -1, -1, -1, -1, -1, -1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1,
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```

```
245
```

```
{-1, -1, -1, -1, -1, -1, 2, -1, 1, -1, -1, 1, 1, -1, -1, 1, 1, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, -1, -1, -1, -1,
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```

100%

```
1, 1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1,
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-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1,
-1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, 1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1,
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245
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```
fourPointGraphs[6][[-5 ;; -1]]
```

◊ Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

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```
fourPointGraphs[6][[-5 ;; -1]]
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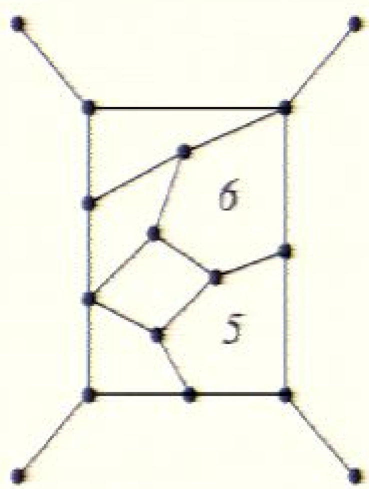
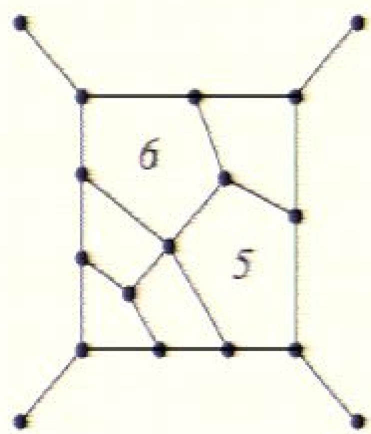


$s^2 t\, x_{45}^2 x_{36}^2$,



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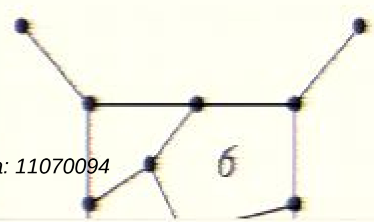

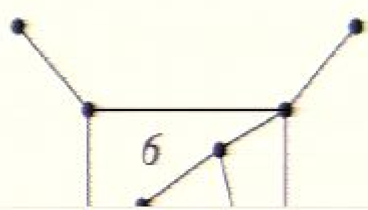

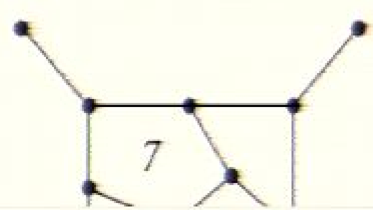

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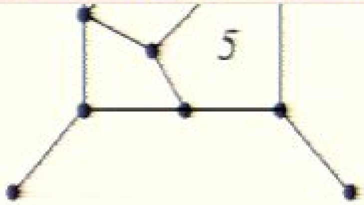
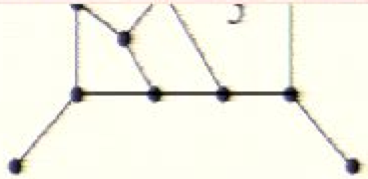
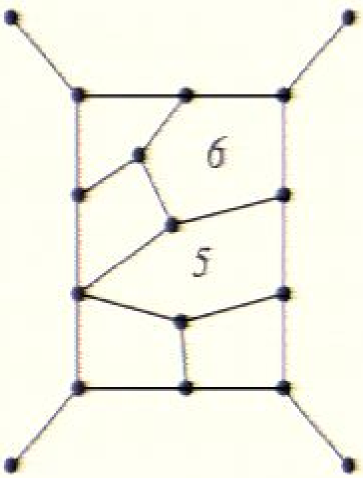


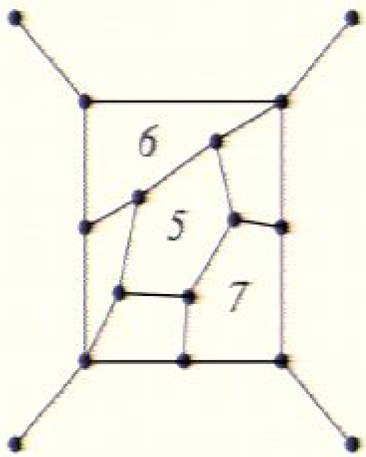
$s^2 t\, x_{45}^2 x_{36}^2$



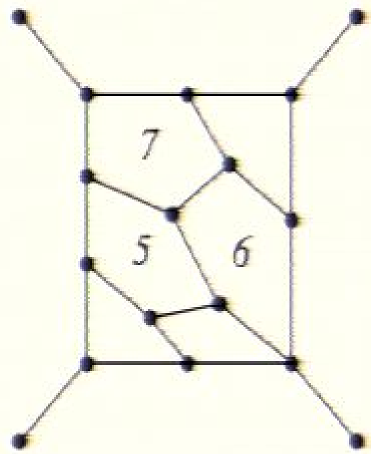
$s^2 t^2\, x_{45}^2 x_{36}^2$



$s^3 t\, x_{15}^2 x_{36}^2$



$s^2 t\, x_{35}^2 x_{67}^2$



$s^2 t\, x_{15}^2 x_{46}^2 x_{37}^2$

◇ Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

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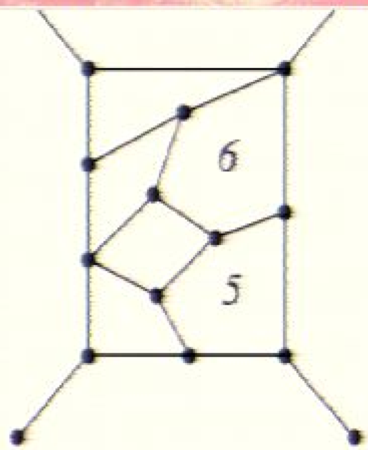
{



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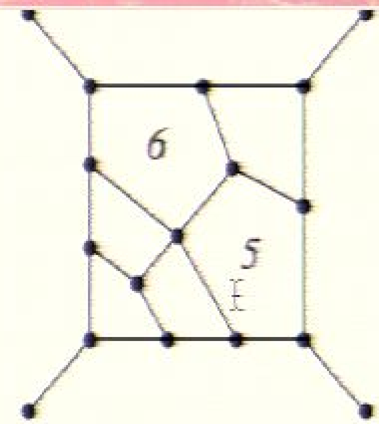

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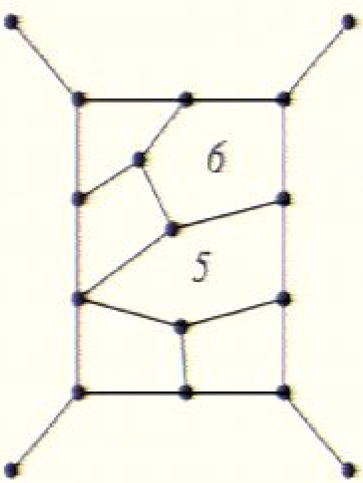

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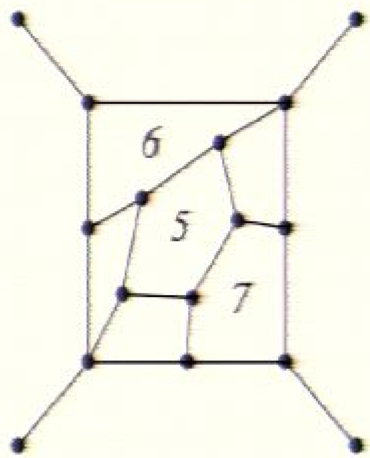
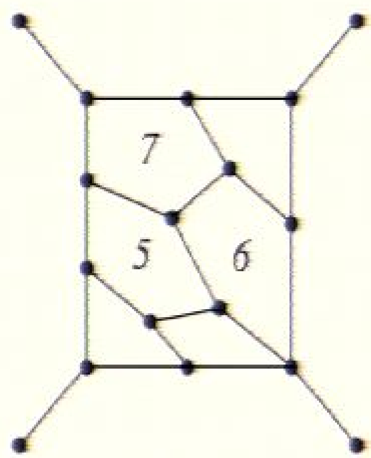


$s^2 t\, x_{35}^2 x_{67}^2$,



$s^2 t\, x_{15}^2 x_{46}^2 x_{37}^2$ }

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introduction_to_mathematica.nb


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```
fourPointGraphs[6][[-5 ;; -1]]
```

◇ Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

```
SetDirectory[NotebookDirectory[]]
```

Visualizing data with *Mathematica*:

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myPackage.m

Functions Sections Update Debug Run Package

```
BeginPackage["myPackage`"]
Begin["Global`"]
```

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```
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```

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- *Visualizing* data with *Mathematica:*

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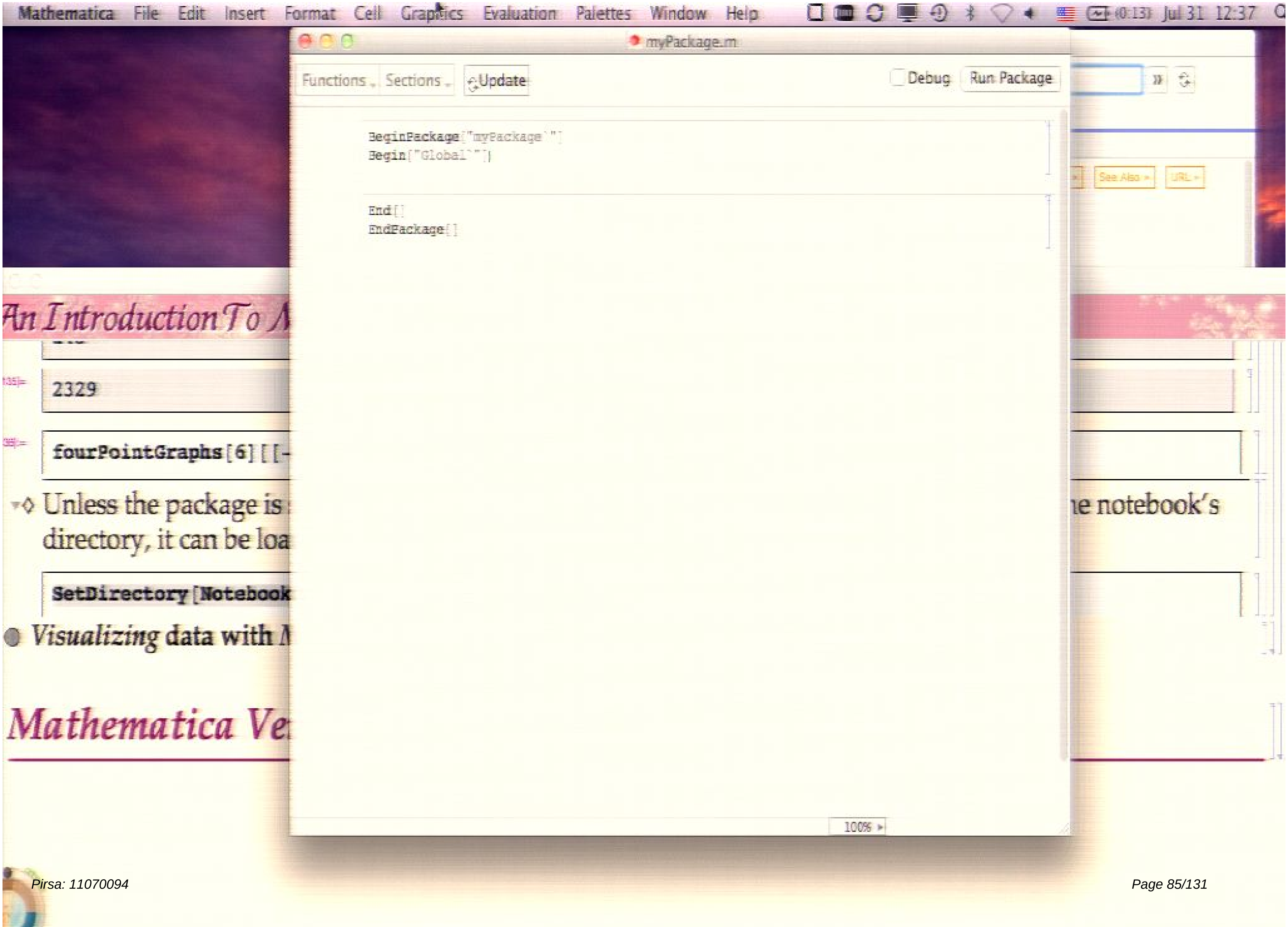

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myPackage.m

Functions Sections Update Debug Run Package

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Begin["Global`"]

End[]
EndPackage[]
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⟨EF(451)⋂(123)⟩ ⟨2345⟩ ⟨4523⟩ ⟨5234⟩ / ⟨AB23⟩ ⟨AB45⟩ ⟨ABCD⟩ ⟨CD23⟩ ⟨CD34⟩ ⟨CD45⟩ ⟨EF12⟩ ⟨EF23⟩ ⟨EF45⟩ ⟨EF51⟩ ⟨EFAB⟩,

⟨2345⟩ ⟨4123⟩ ⟨5123⟩ ⟨5234⟩ / ⟨AB23⟩ ⟨AB45⟩ ⟨ABCD⟩ ⟨CD23⟩ ⟨CD34⟩ ⟨CD45⟩ ⟨EF12⟩ ⟨EF23⟩ ⟨EF51⟩ ⟨EFAB⟩ }

Packages can also be a useful repository of data:

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<< four_point_multiloops.m
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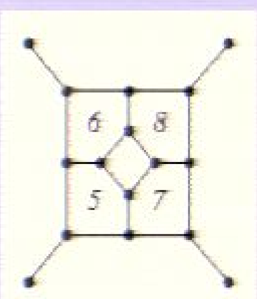

Four-Point Multi-Loop Integrands Through L=7

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myPackage.m

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Packages can also be a useful repository of data:

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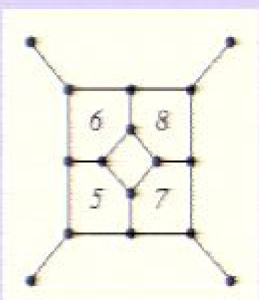
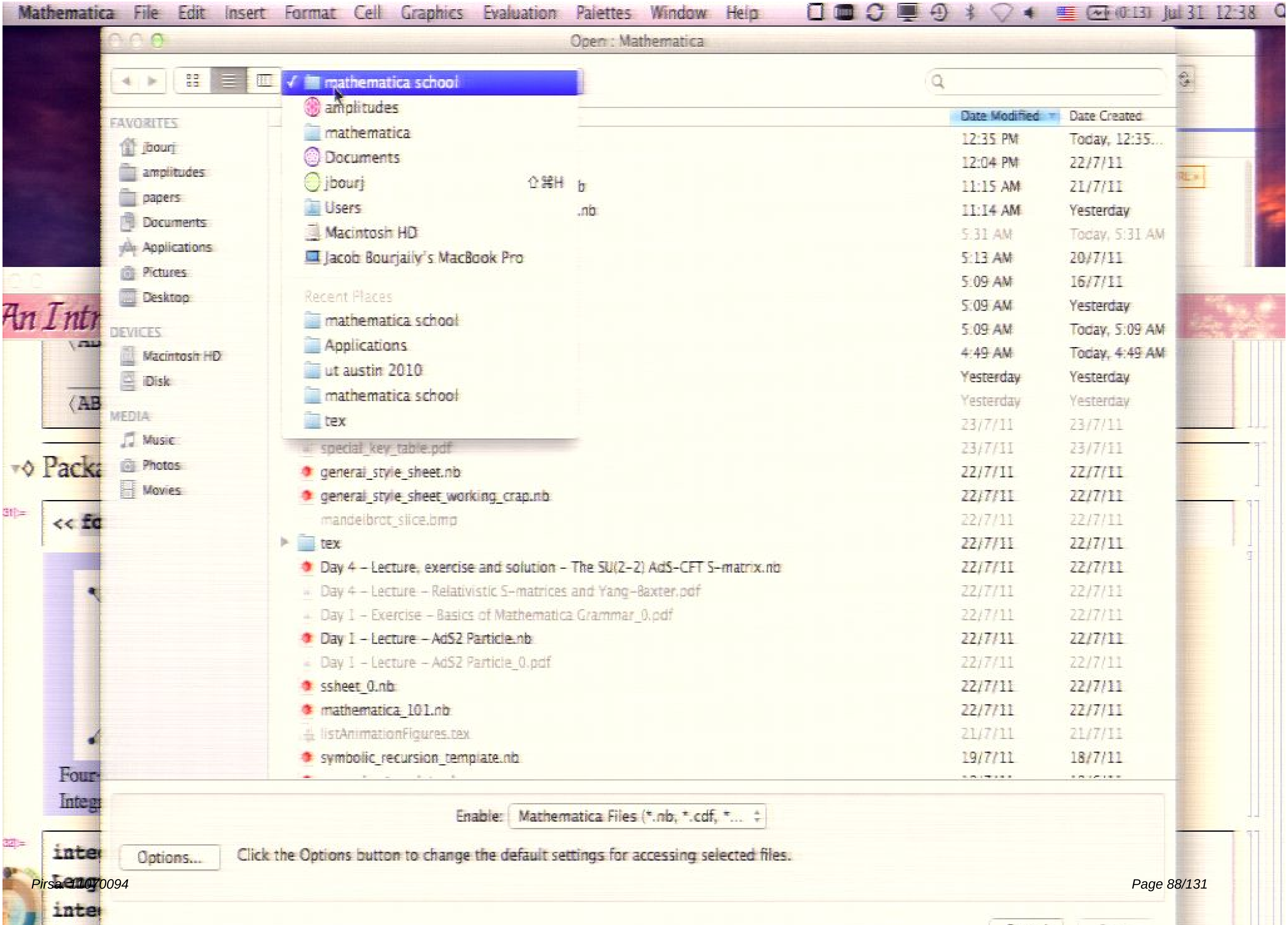


Four-Point Multi-Loop Integrands Through L=7

```
integrandCoefficients[6]
Length@%
integrandCoefficients[7]
```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Day 4 – Lecture – Relativistic S-matrices and Yang-Baxter.pdf
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Day 1 – Lecture – AdS2 Particle.nb
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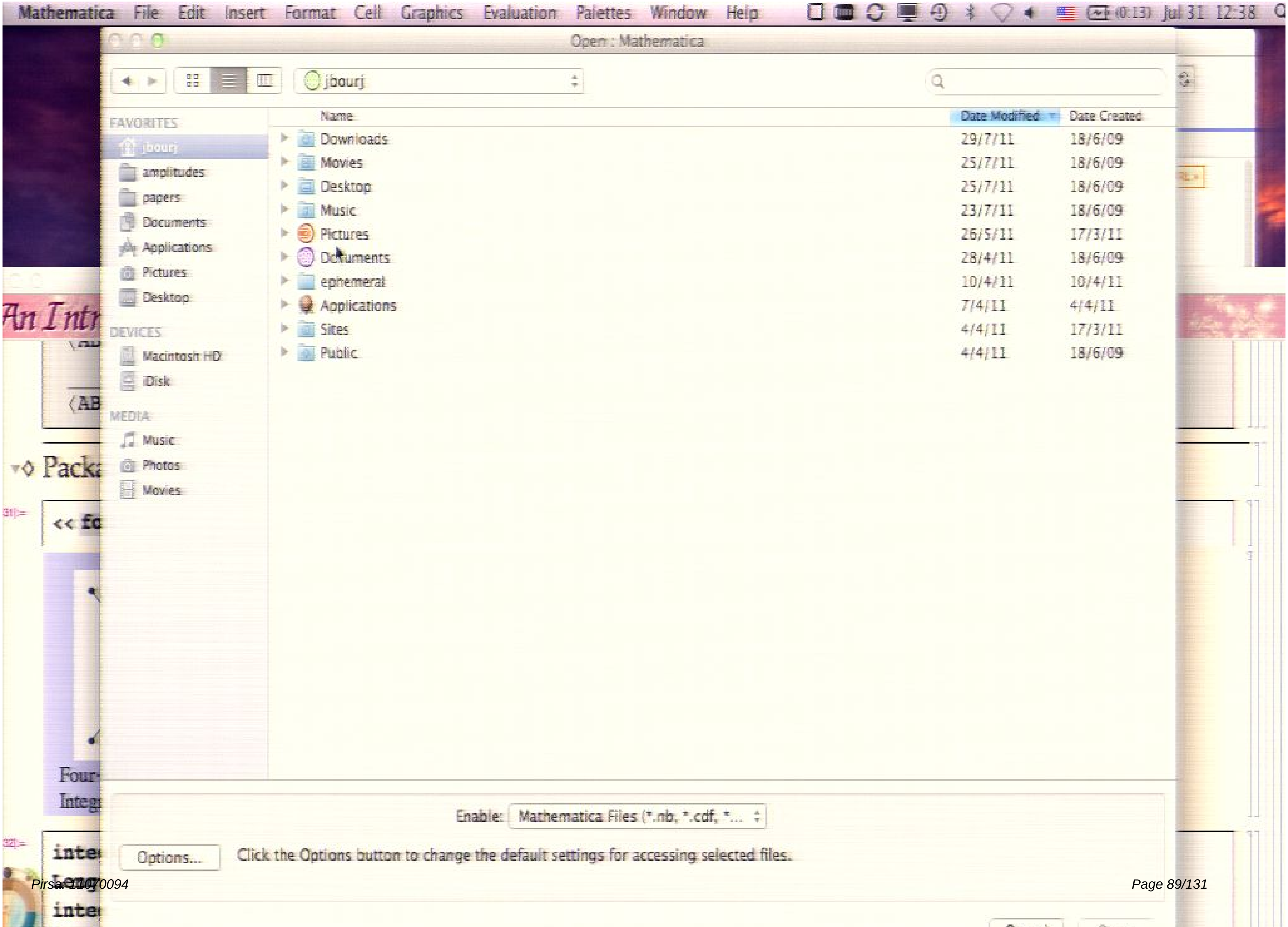
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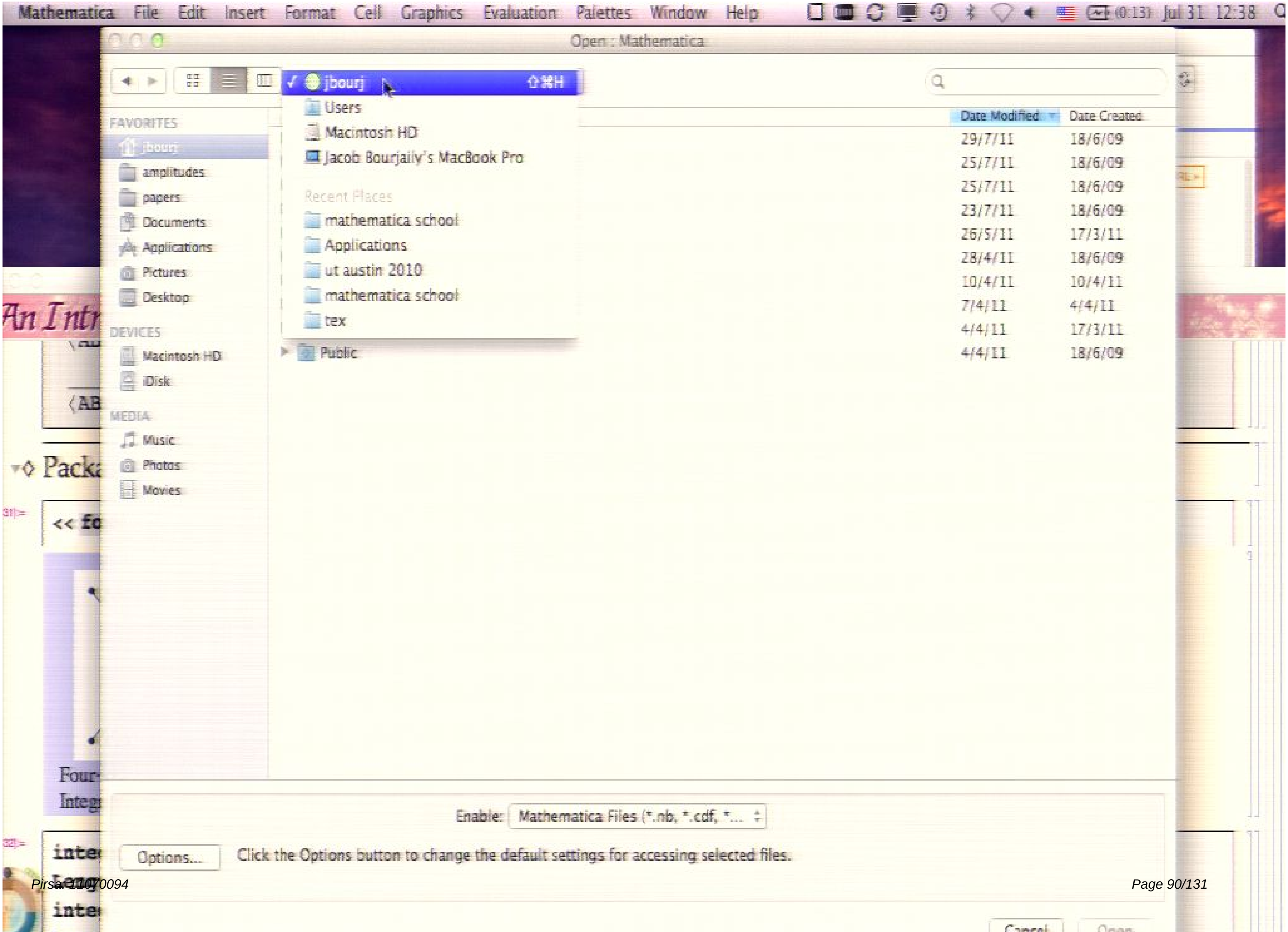
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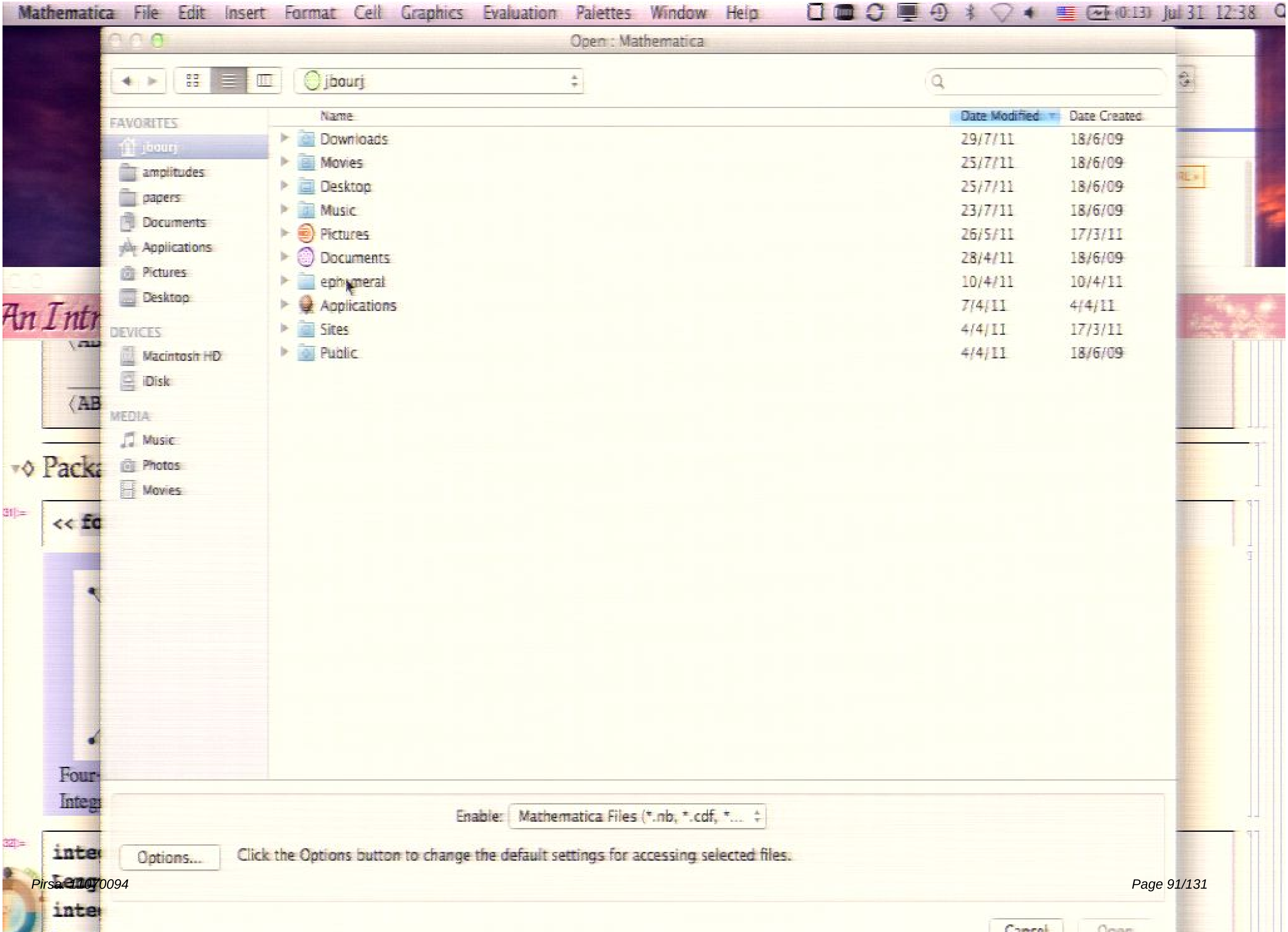
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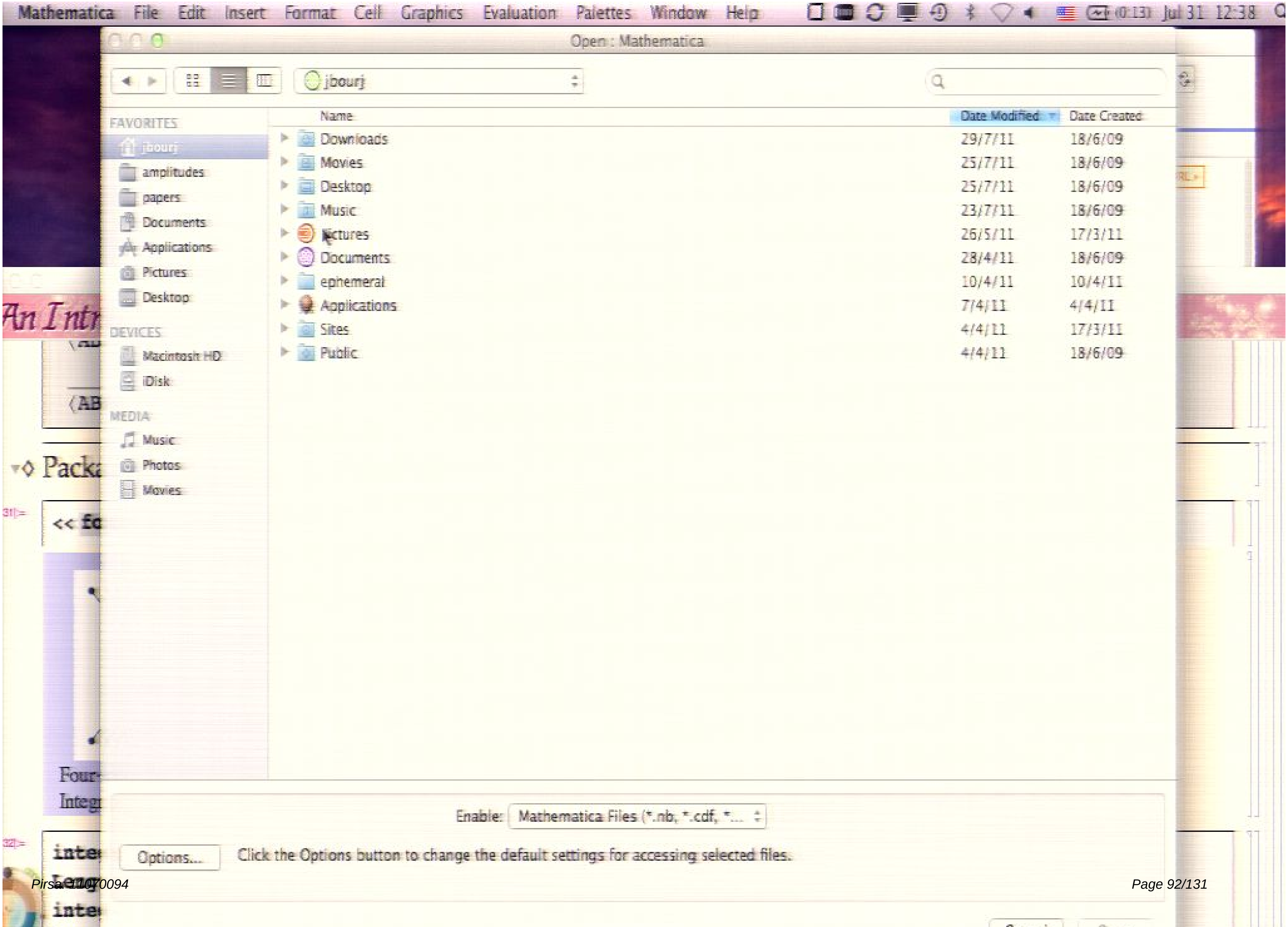
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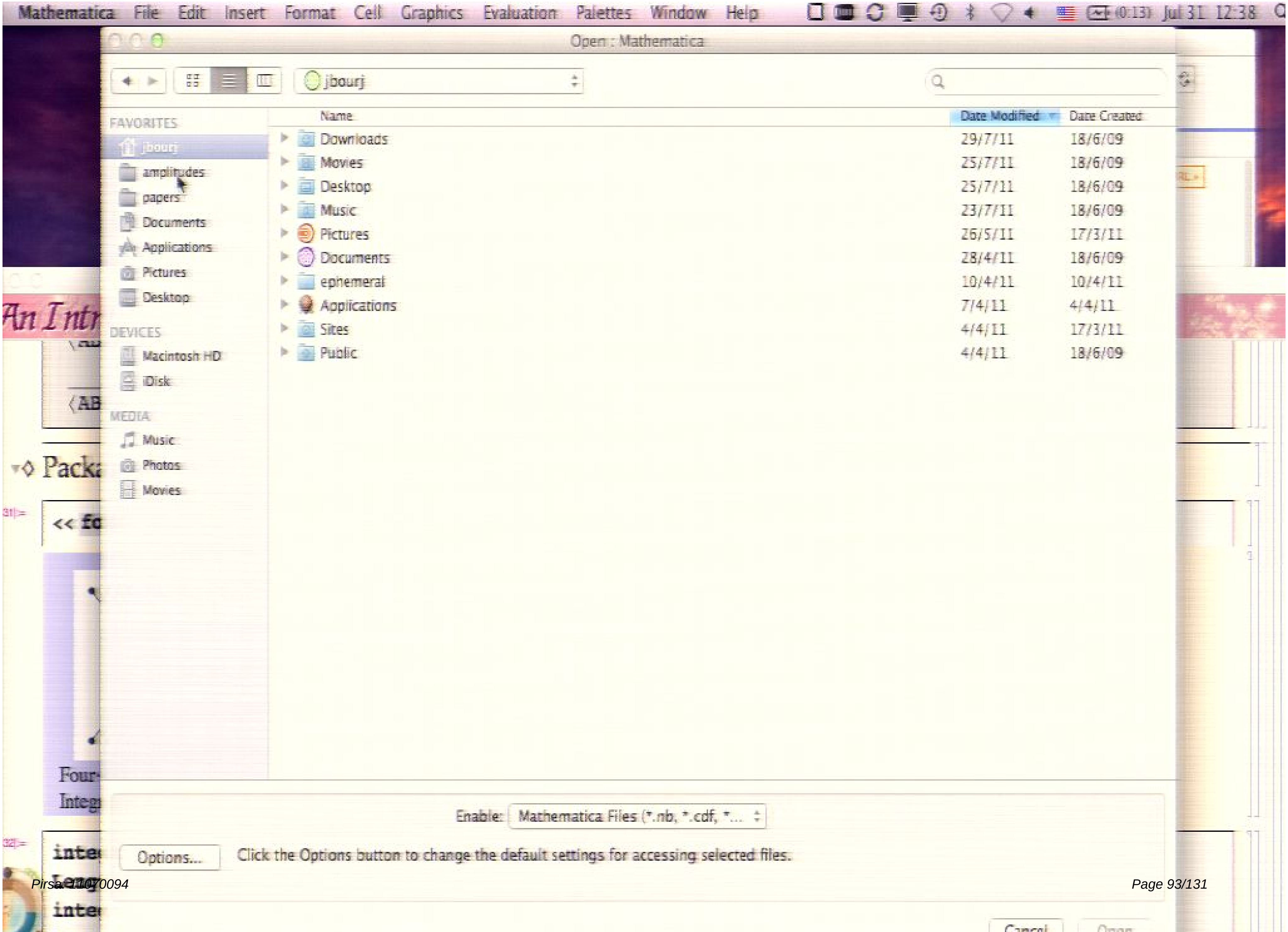
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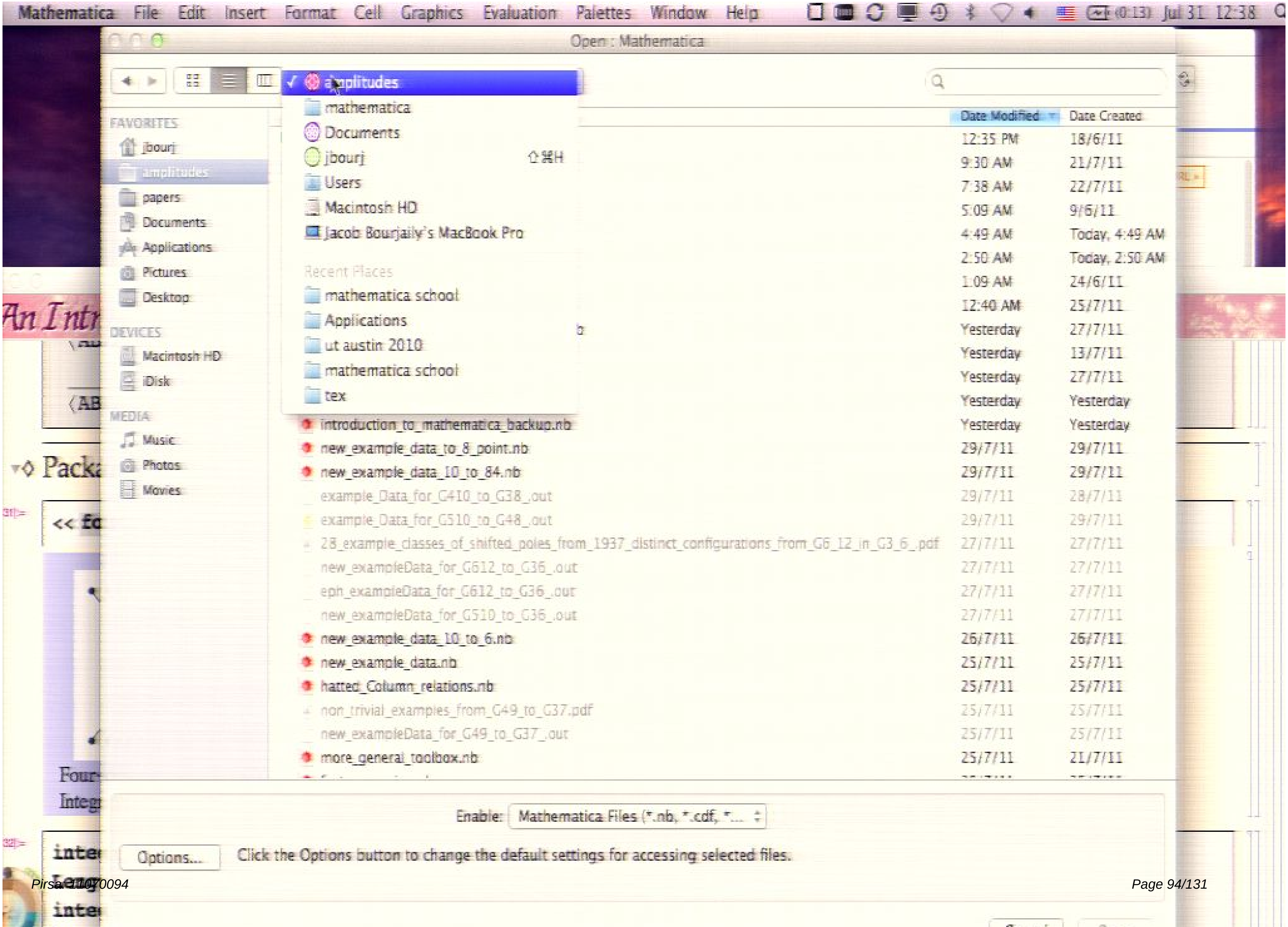

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eph_exampleData_for_G612_to_G36_.out
new_exampleData_for_G510_to_G36_.out
new_example_data_10_to_6.nb
new_example_data.nb
hatted_Column_relations.nb
non_trivial_examples_from_G49_to_G37.pdf
new_exampleData_for_G49_to_G37_.out
more_general_toolbox.nb
Enable: Mathematica Files (*.nb, *.cdf, *...
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Click the Options button to change the default settings for accessing selected files.
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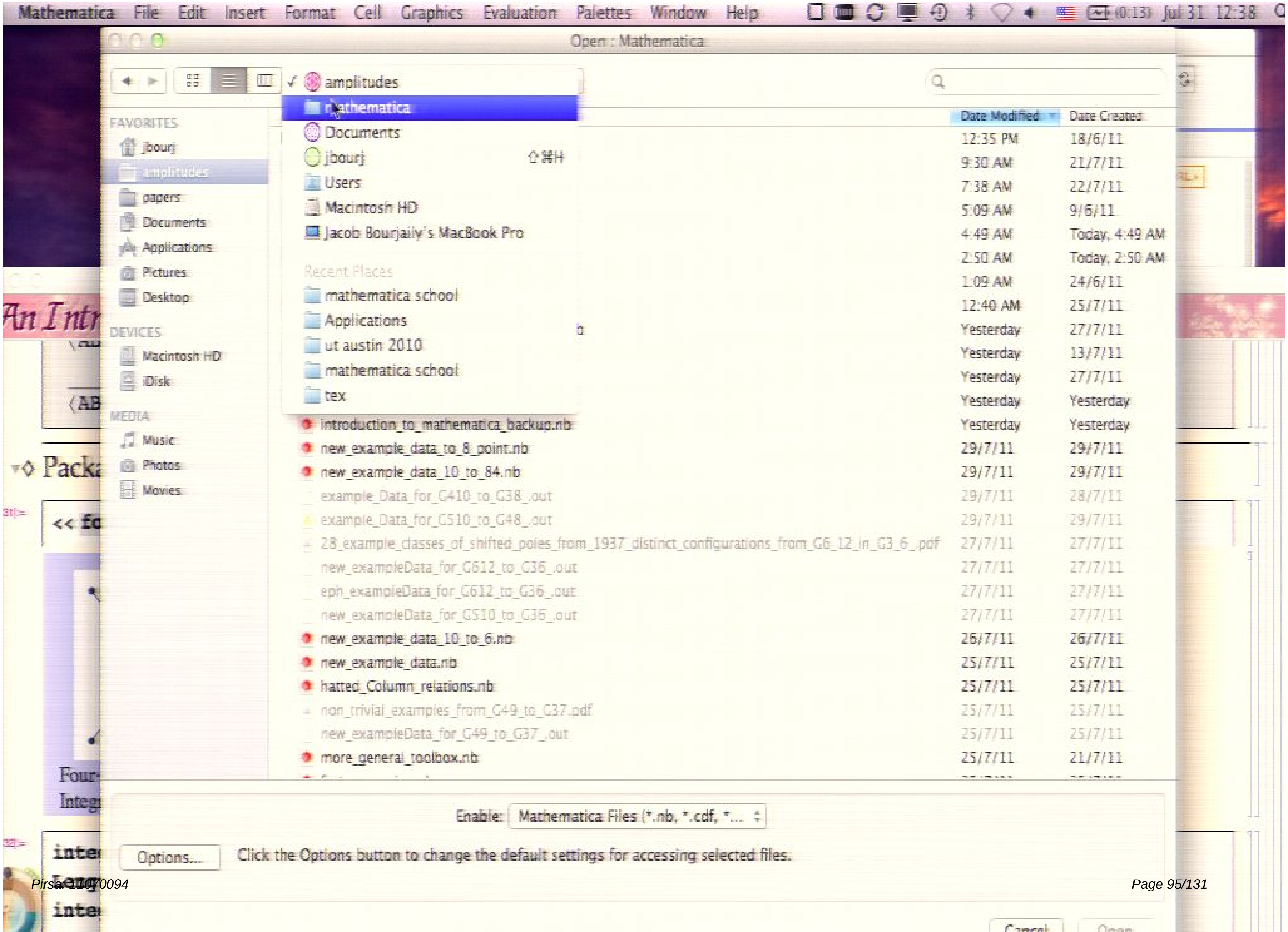
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new_exampleData_for_G510_to_G36_.out
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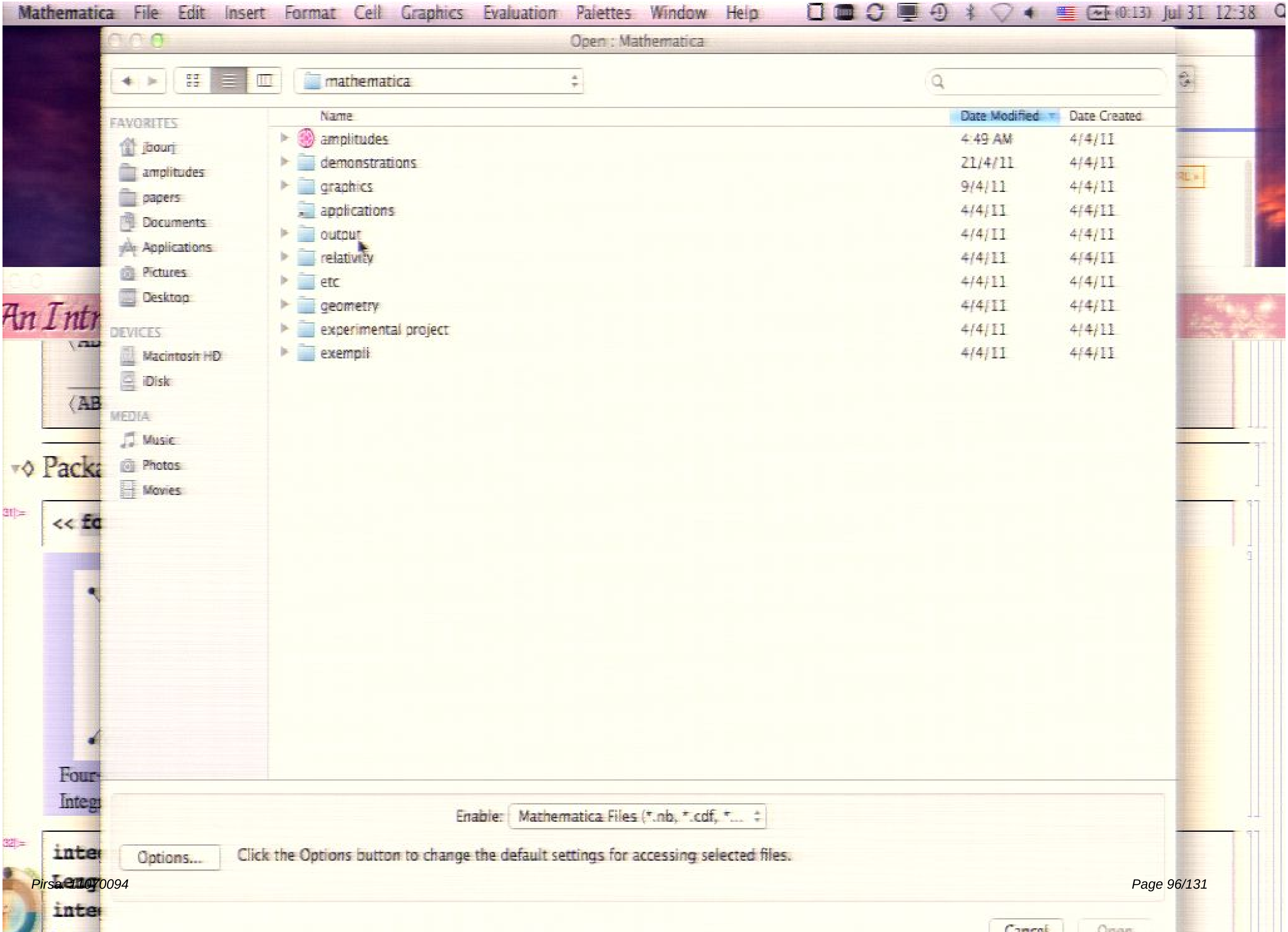
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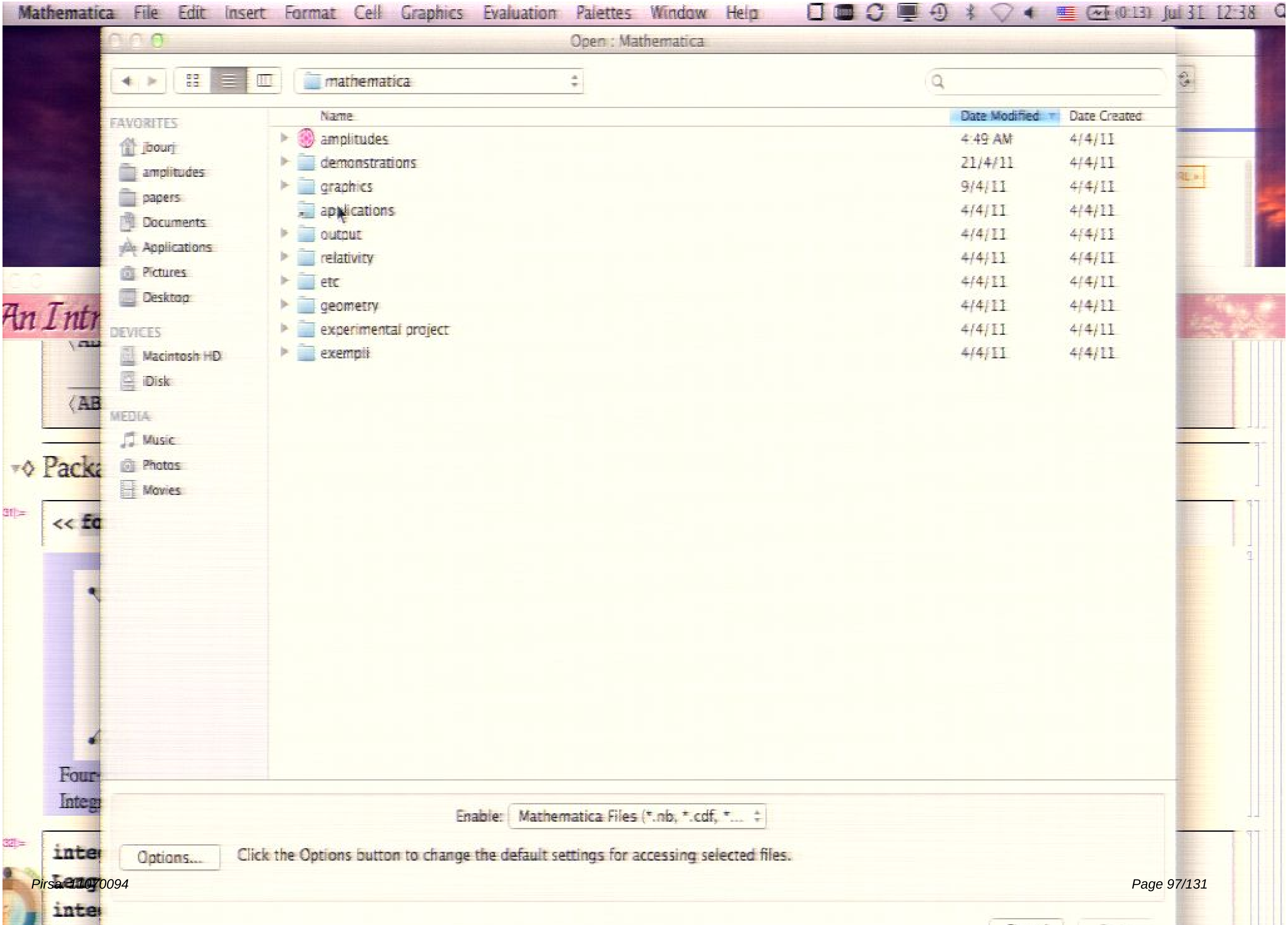
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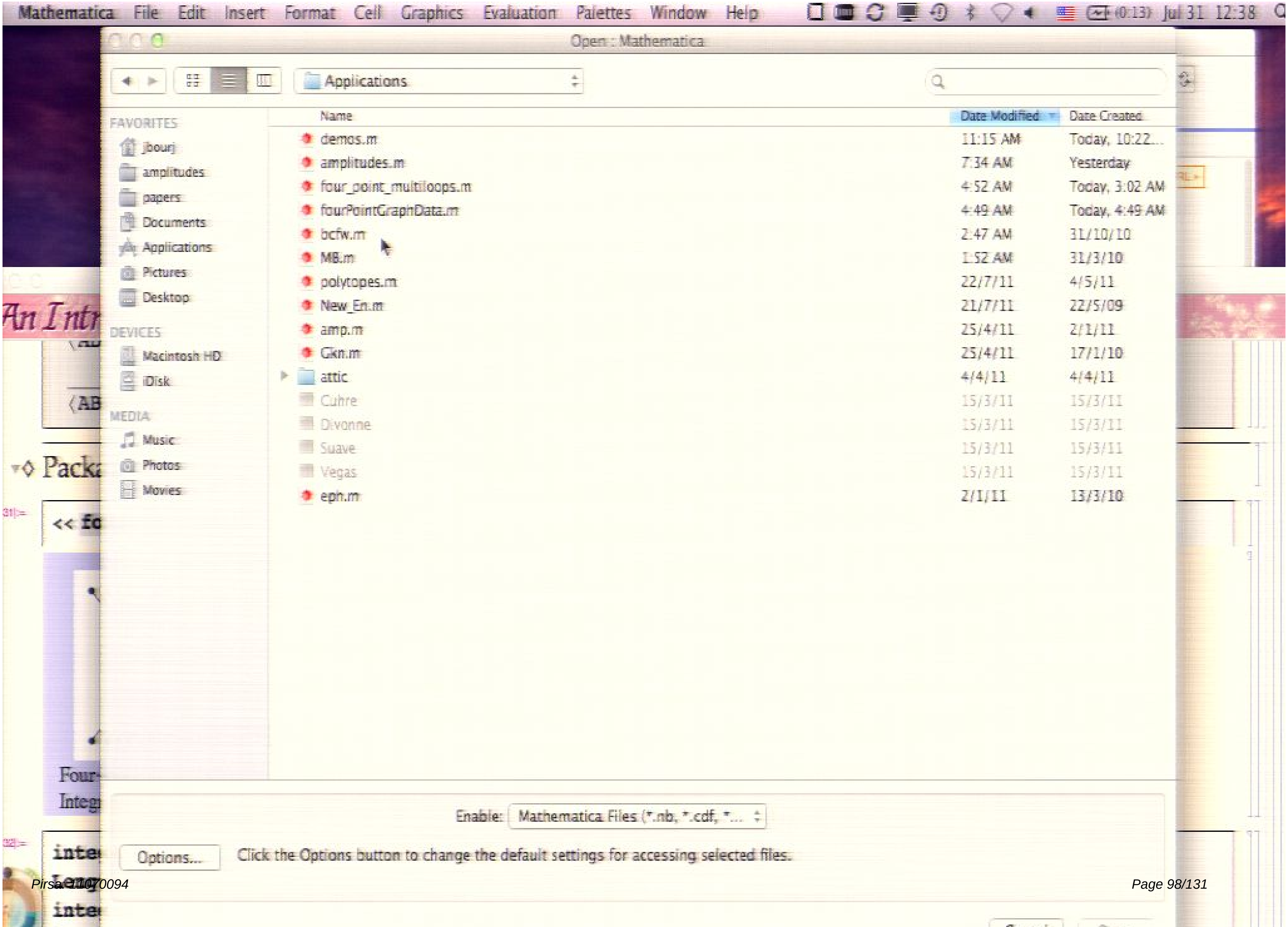
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four_point_multiloops.m 4:52 AM Today, 3:02 AM
fourPointGraphData.m 4:49 AM Today, 4:49 AM
bcfw.m 2:47 AM 31/10/10
MB.m 1:52 AM 31/3/10
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New_En.m 21/7/11 22/5/09
amp.m 25/4/11 2/1/11
Gkn.m 25/4/11 17/1/10
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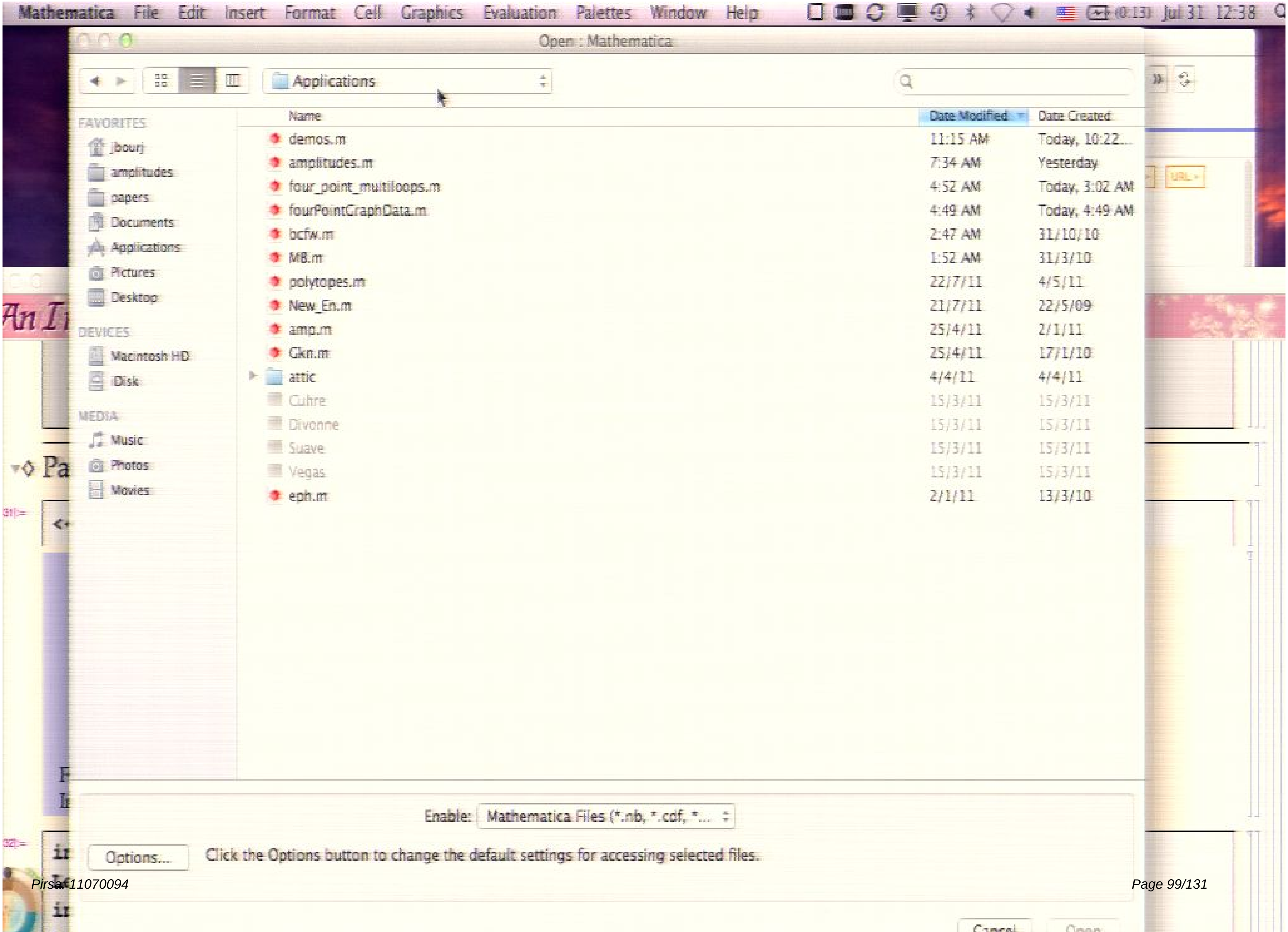
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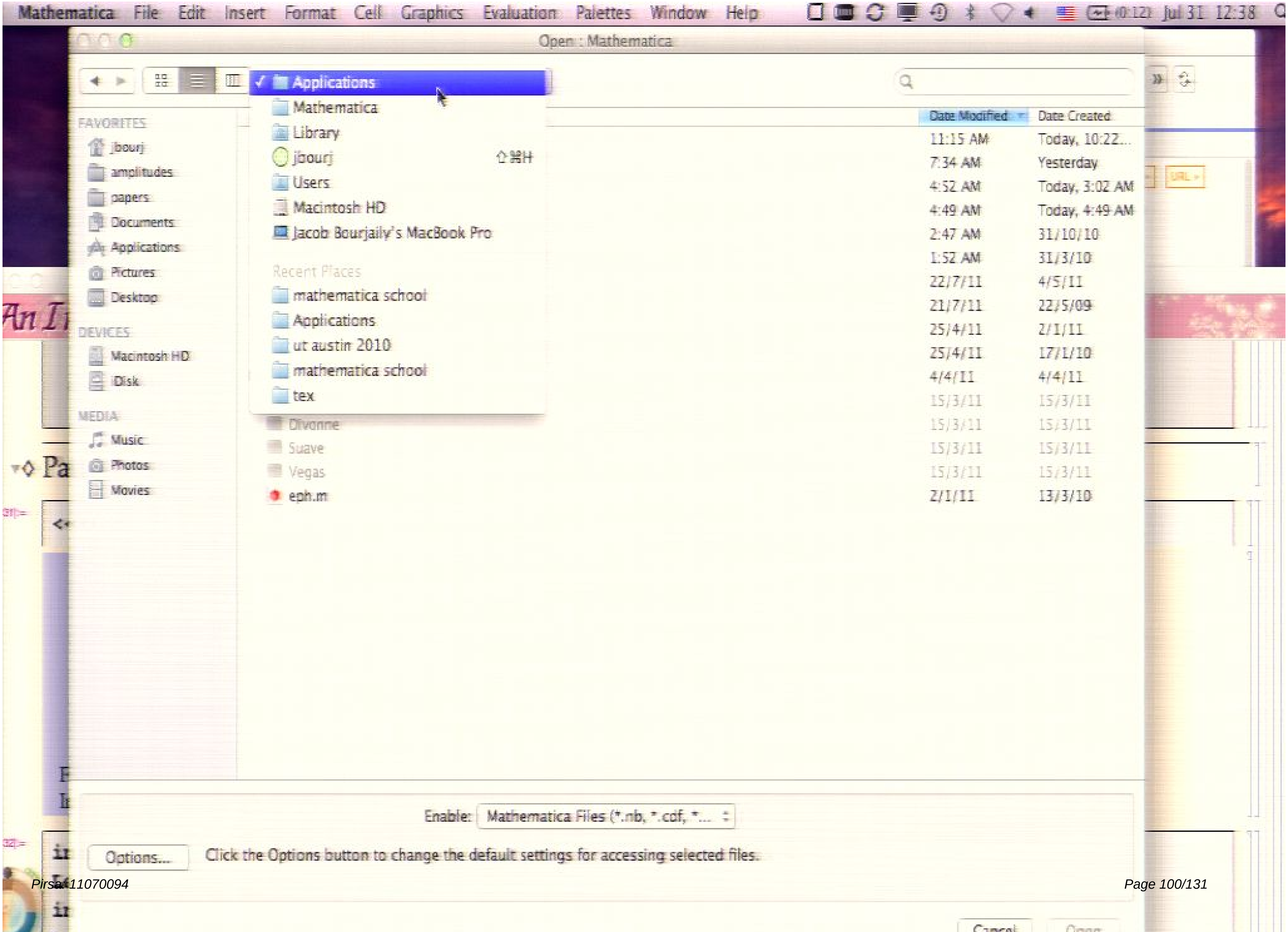
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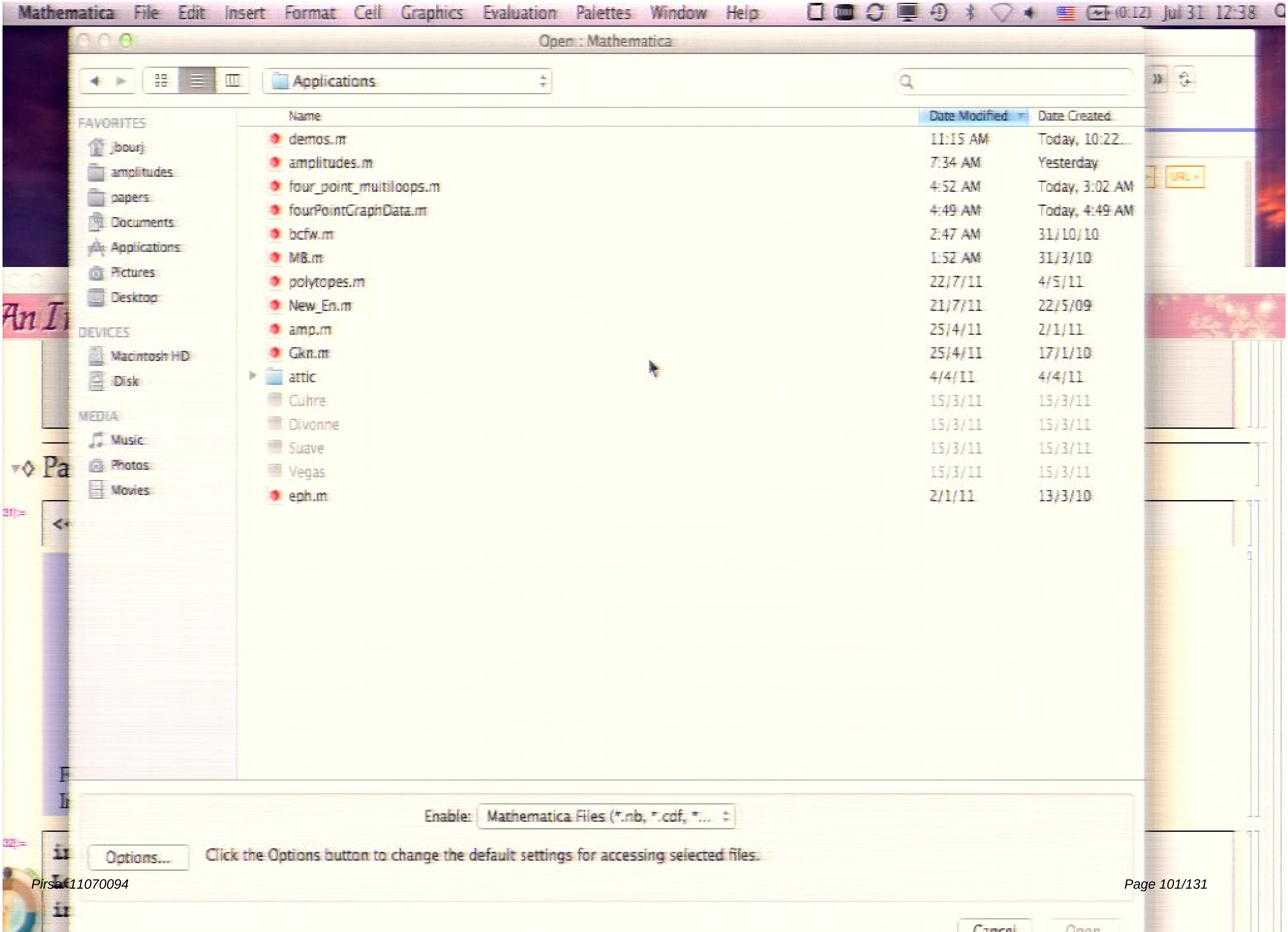
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fourPointGraphData.m 4:49 AM Today, 4:49 AM
bcfw.m 2:47 AM 31/10/10
MB.m 1:52 AM 31/3/10
polytopes.m 22/7/11 4/5/11
New_En.m 21/7/11 22/5/09
amp.m 25/4/11 2/1/11
Gkn.m 25/4/11 17/1/10
attic 4/4/11 4/4/11
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Vegas 15/3/11 15/3/11
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myPackage.m

Functions Sections Update Debug Run Package

```
BeginPackage["myPackage`"]
Begin["Global`"]

End[]
EndPackage[]
```

See Also URL

introduction_to_mathematica.nb

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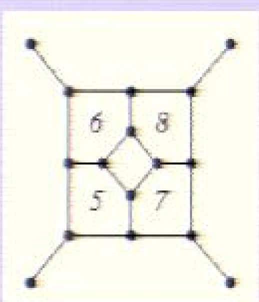
⟨2345⟩ ⟨4123⟩ ⟨5123⟩ ⟨5234⟩

⟨AB23⟩ ⟨AB45⟩ ⟨ABCD⟩ ⟨CD23⟩ ⟨CD34⟩ ⟨CD45⟩ ⟨EF12⟩ ⟨EF23⟩ ⟨EF51⟩ ⟨EFAB⟩

◇ Packages can also be a useful repository of data:

In[31]:=

```
<< four_point_multiloops.m
```



Four-Point Multi-Loop Integrands Through L=7

In[32]:=

```
integrandCoefficients[6]
Length@%
integrandCoefficients[7]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:12) Jul 31 12:38

myPackage.m

Functions Sections Update Debug Run Package

```
BeginPackage["myPackage`"]
Begin["Global`"]

End[]
EndPackage[]
```

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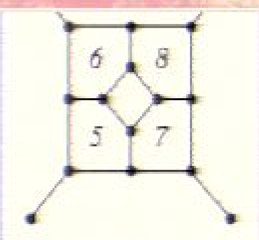

Four−Point Multi−Loop Integrands Through L=7

```
integrandCoefficients[6]
Length@%
integrandCoefficients[7]
Length@%
```

```
{2, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, -1, -1, -1, -1, -1, 1, -1, -1,
 -1, -1, -1, -1, -1, -1, 1, 1, 1, 1, 1, -1, -1, -1, 1, 1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1,
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 1, 1, 1, 1, -1, 1, 1, 1, 1, 1, 1, -1, 1, -1, -1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}
```

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myPackage.m

Functions Sections Update Debug Run Package

```
BeginPackage["myPackage`"]
Begin["Global`"]

End[]
EndPackage[]
```

introduction_to_mathematica.nb

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```
{2, 1, 1, 1, -1, -1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, -1, -1, -1, -1, -1, -1, -1, 1, -1, -1,
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```

```
245
```

```
2329
```

```
fourPointGraphs[6][[-5 ;; -1]]
```

- Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

```
SetDirectory[NotebookDirectory[]]
```


Visualizing data with *Mathematica*:

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Jul 31 12:38

myPackage.m

Functions Sections Update Debug Run Package

```
BeginPackage["myPackage`"]
Begin["Global`"]

End[]
EndPackage[]
```

See Also » URL »

introduction_to_mathematica.nb

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```
243
```

Out[135]= 2329

In[136]:=
```
fourPointGraphs[6][[-5 ;; -1]]
```

◇ Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

```
SetDirectory[NotebookDirectory[]]
```

● *Visualizing* data with *Mathematica*:

Mathematica Vernacular: The Anatomy of an Expression

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myPackage.m

Functions Sections Update Debug Run Package

```
BeginPackage["myPackage`"]
Begin["Global`"]

End[]
EndPackage[]
```

introduction_to_mathematica.nb

An Introduction To Mathematica *Mathematica Summer School 2011*

```
2329
```

```
fourPointGraphs[6][[-5 ;; -1]]
```

Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

```
SetDirectory[NotebookDirectory[]]
```

Visualizing data with *Mathematica*:

```
<< demos.m
```

```
ADEfibrations[]
```

```
unfoldingAnimation[]
```

```
patiSalamModel3D[]
```

```
matterTableDemo[0.75]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Jul 31 12:38

myPackage.m

Functions Sections Update Debug Run Package

```
BeginPackage["myPackage`"]
Begin["Global`"]

End[]
EndPackage[]
```

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```
2329
```

```
fourPointGraphs[6][[-5 ;; -1]]
```

Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

```
SetDirectory[NotebookDirectory[]]
```

Visualizing data with *Mathematica*:

```
<< demos.m
```

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ADEfibrations[]
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unfoldingAnimation[]
```

```
patiSalamModel3D[]
```

```
matterTableDemo[0.75]
```

```
guillochePlot
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Jul 31 12:38

myPackage.m

Functions Sections Update Debug Run Package

```
BeginPackage["myPackage`"]
Begin["Global`"]

End[]
EndPackage[]
```

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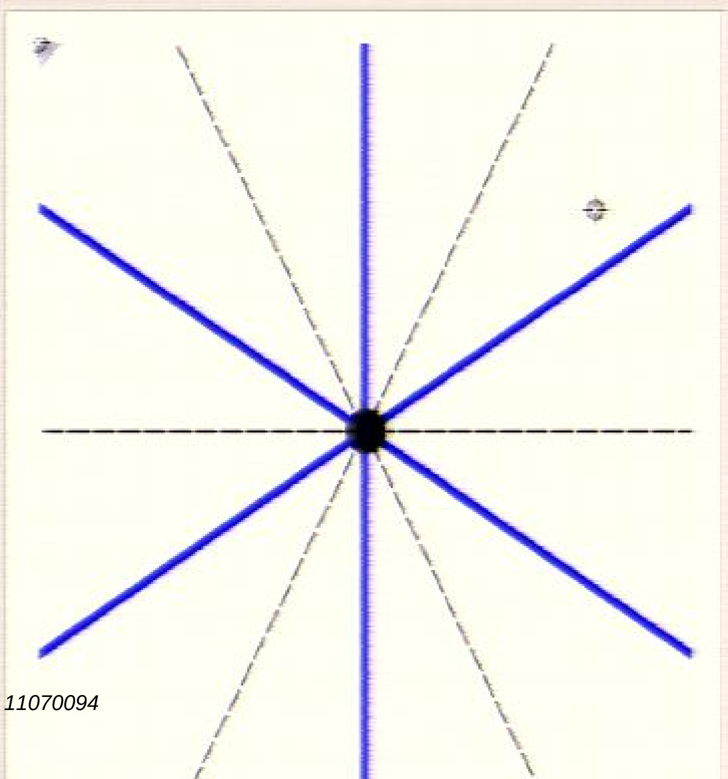
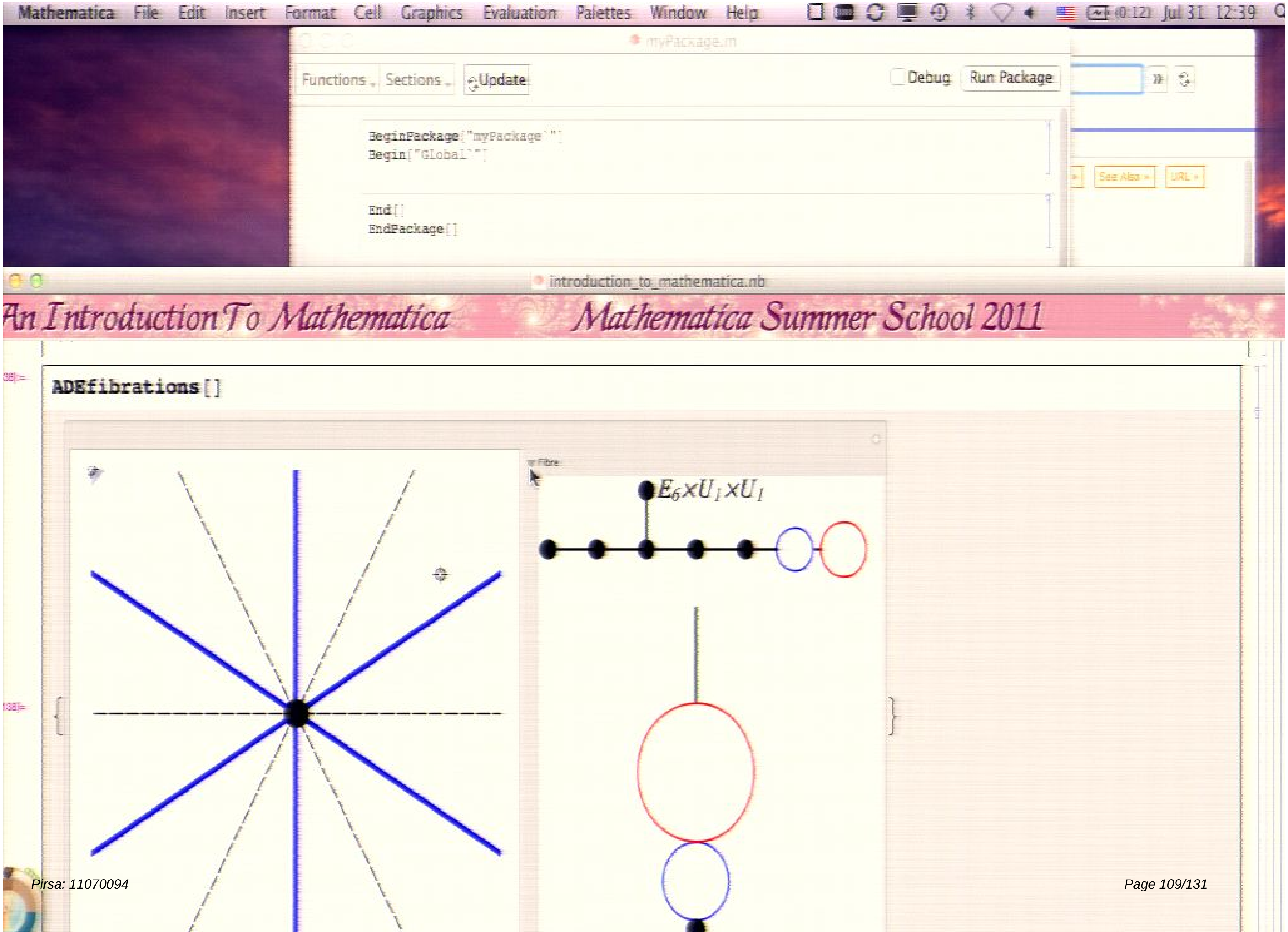
```
ADEfibrations[]
```

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Functions Sections Update
Debug Run Package
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EndPackage[]
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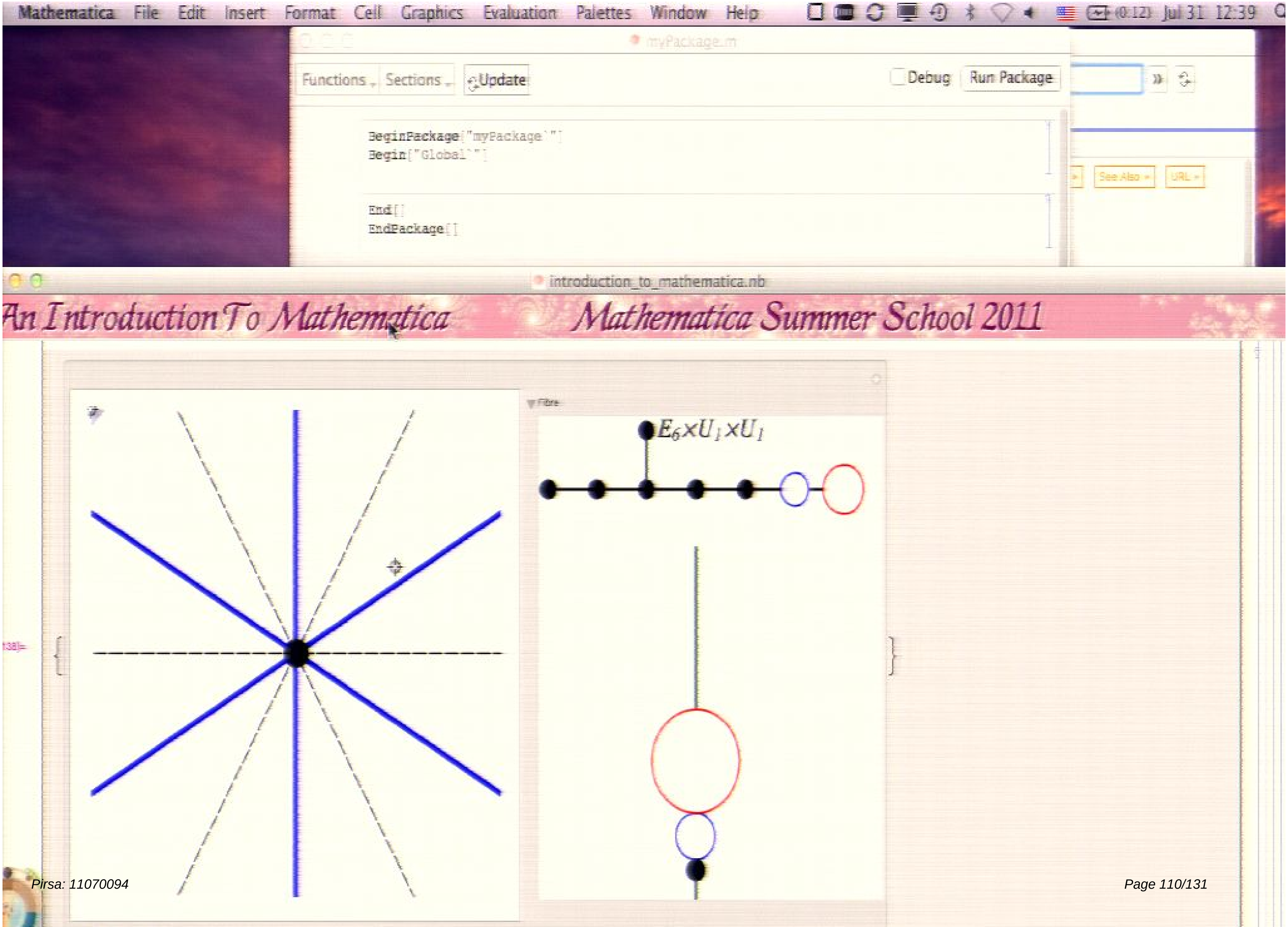
An Introduction To Mathematica
Mathematica Summer School 2011
ADEfibrations[]
$E_6 \times U_1 \times U_1$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Functions Sections Update
Debug Run Package
BeginPackage["myPackage`"]
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$E_6 \times U_1 \times U_1$

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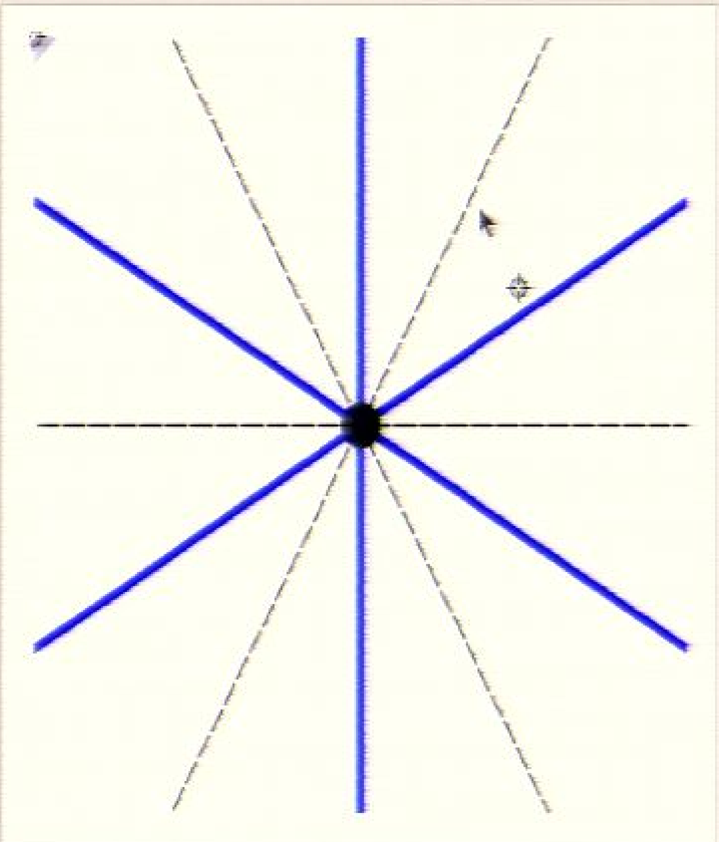

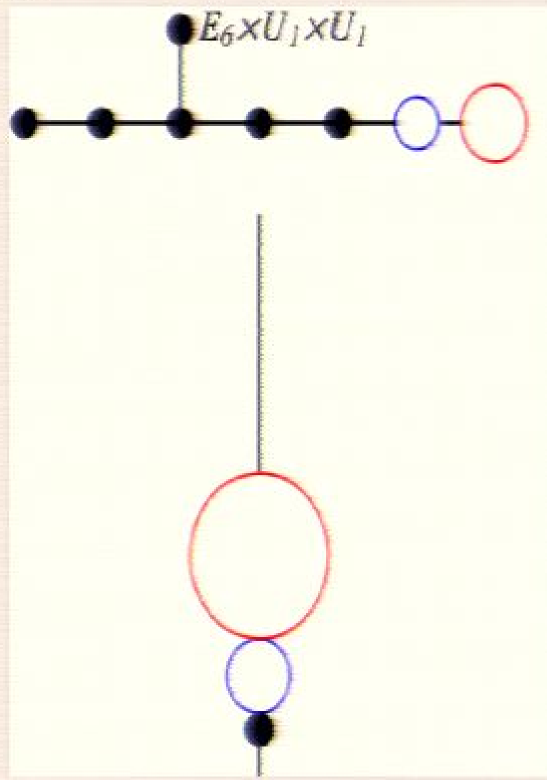


```
unfoldingAnimation[]
```

```
patiSalamModel3D[]
```

```
matterTableDemo[0.75]
```

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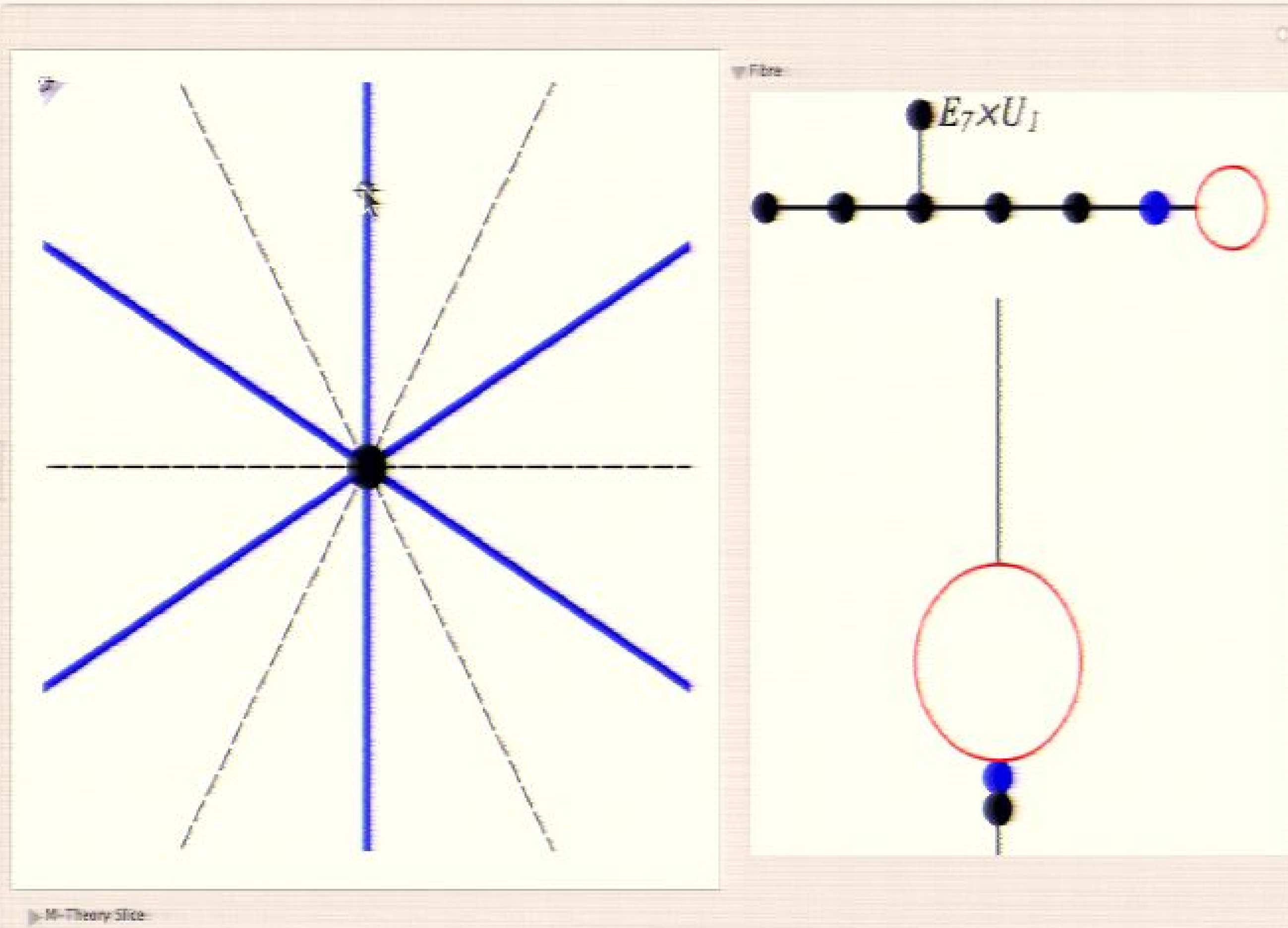


```
unfoldingAnimation[]
patiSalamModel3D[]
matterTableDemo[0.75]
```

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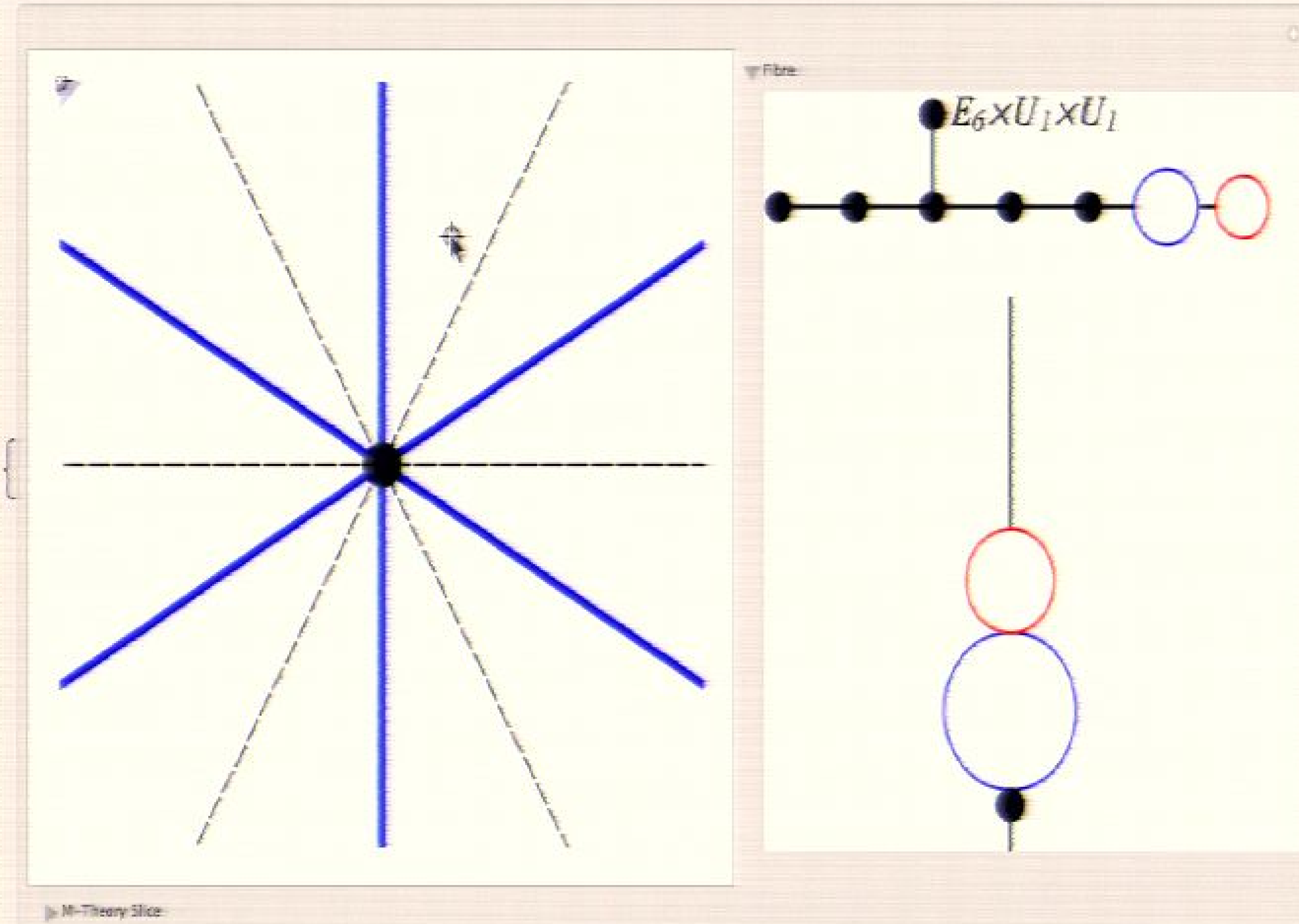


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unfoldingAnimation[]
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matterTableDemo[0.75]
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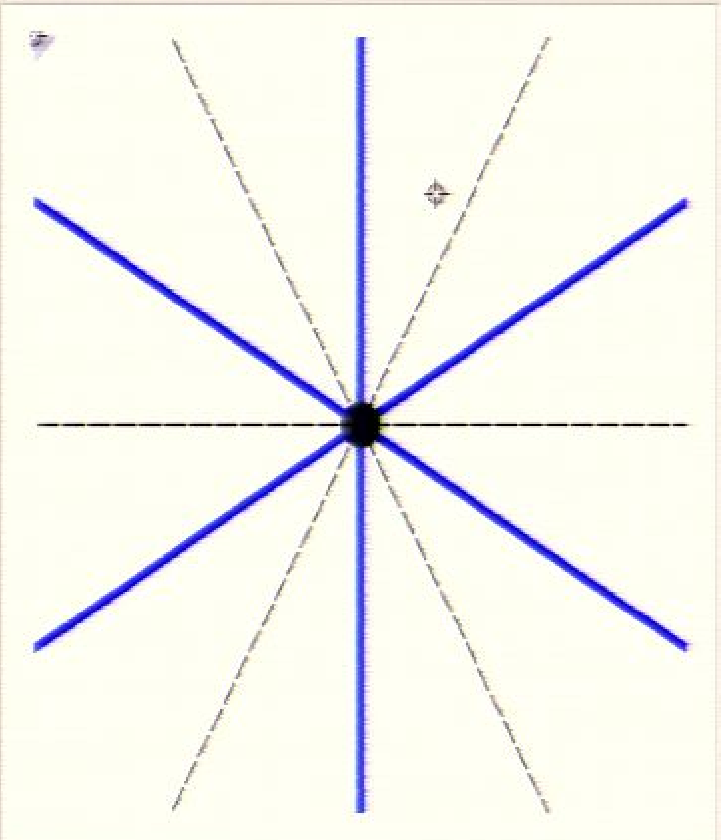

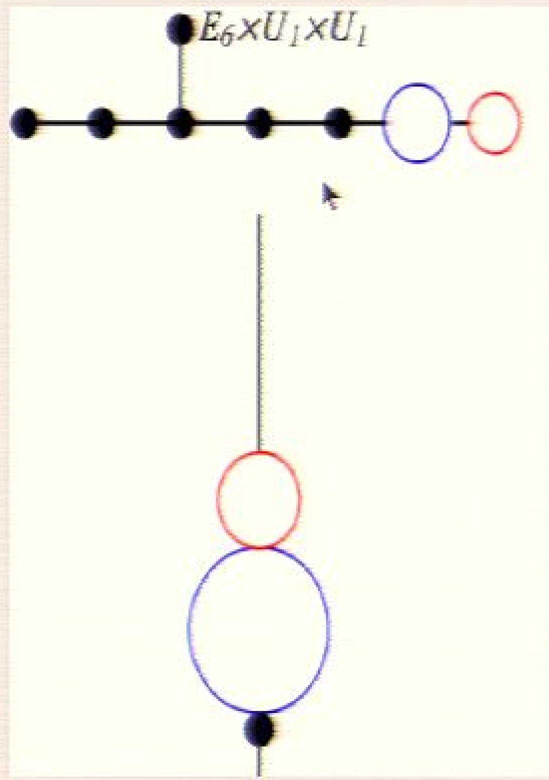

M-Theory Slice

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unfoldingAnimation[]
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patiSalamModel3D[]
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matterTableDemo[0.75]
```


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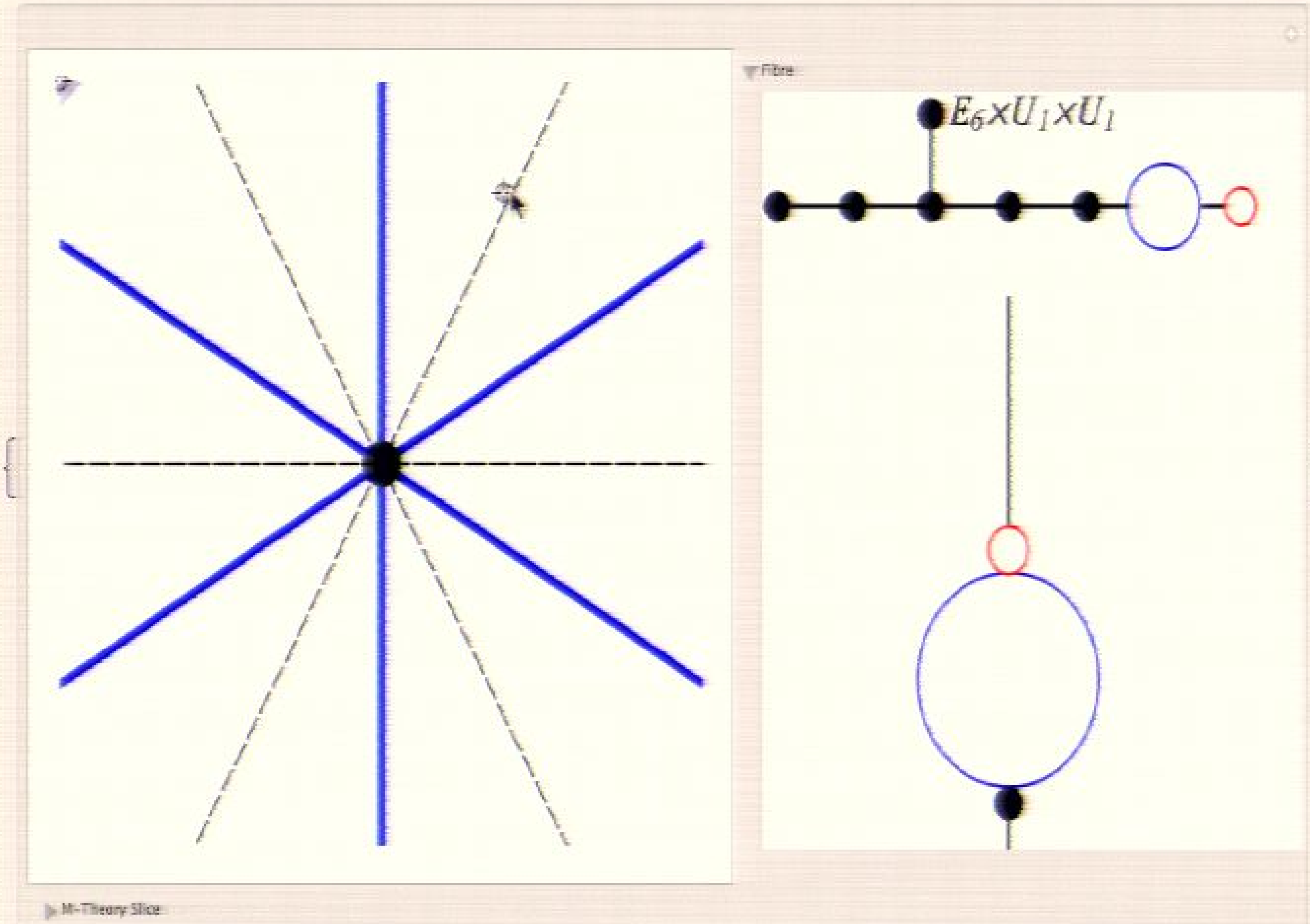


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unfoldingAnimation[]
```

```
patiSalamModel3D[]
```

```
matterTableDemo[0.75]
```

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (0:12) Jul 31 12:39

introduction_to_mathematica.nb

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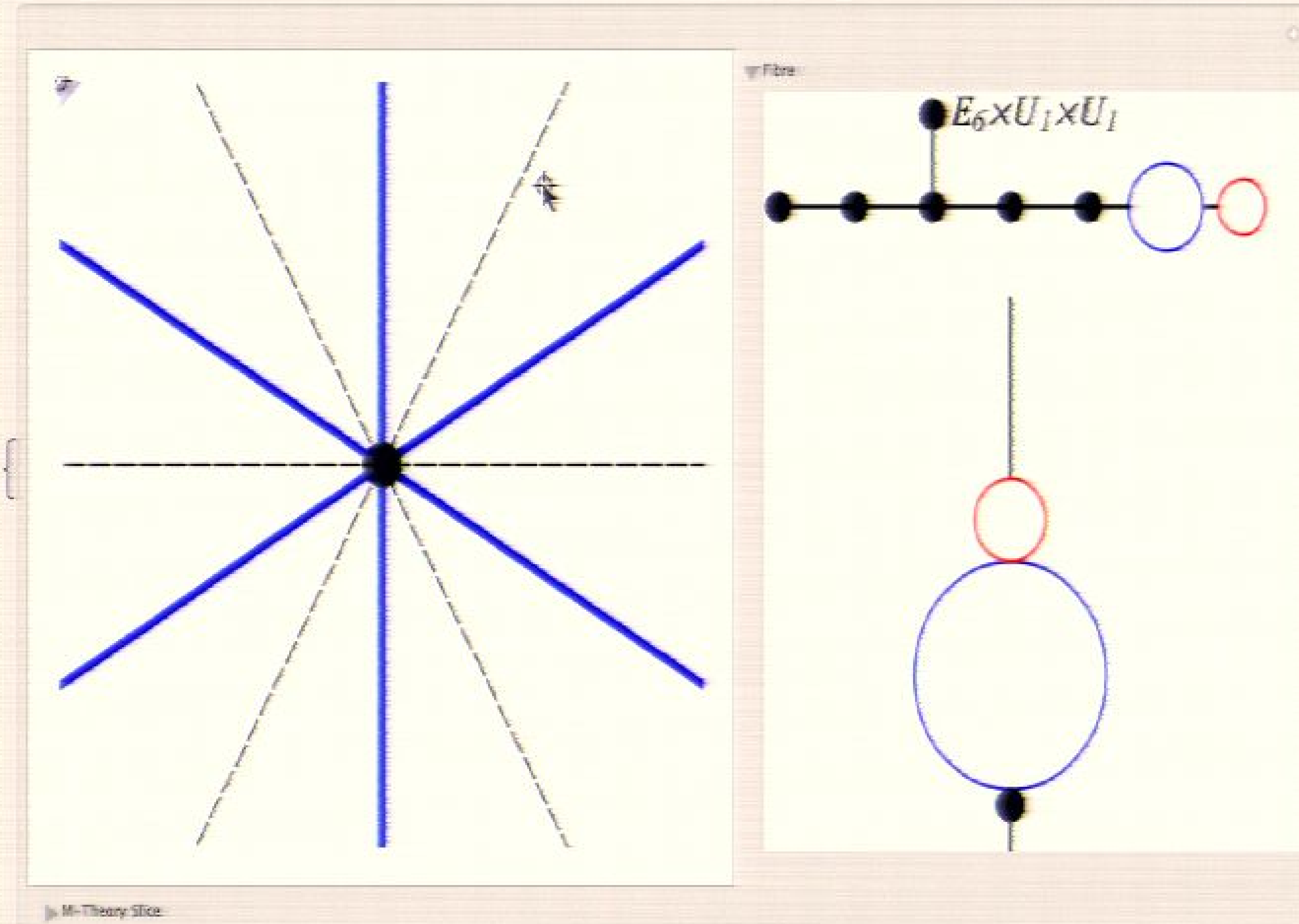


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matterTableDemo[0.75]
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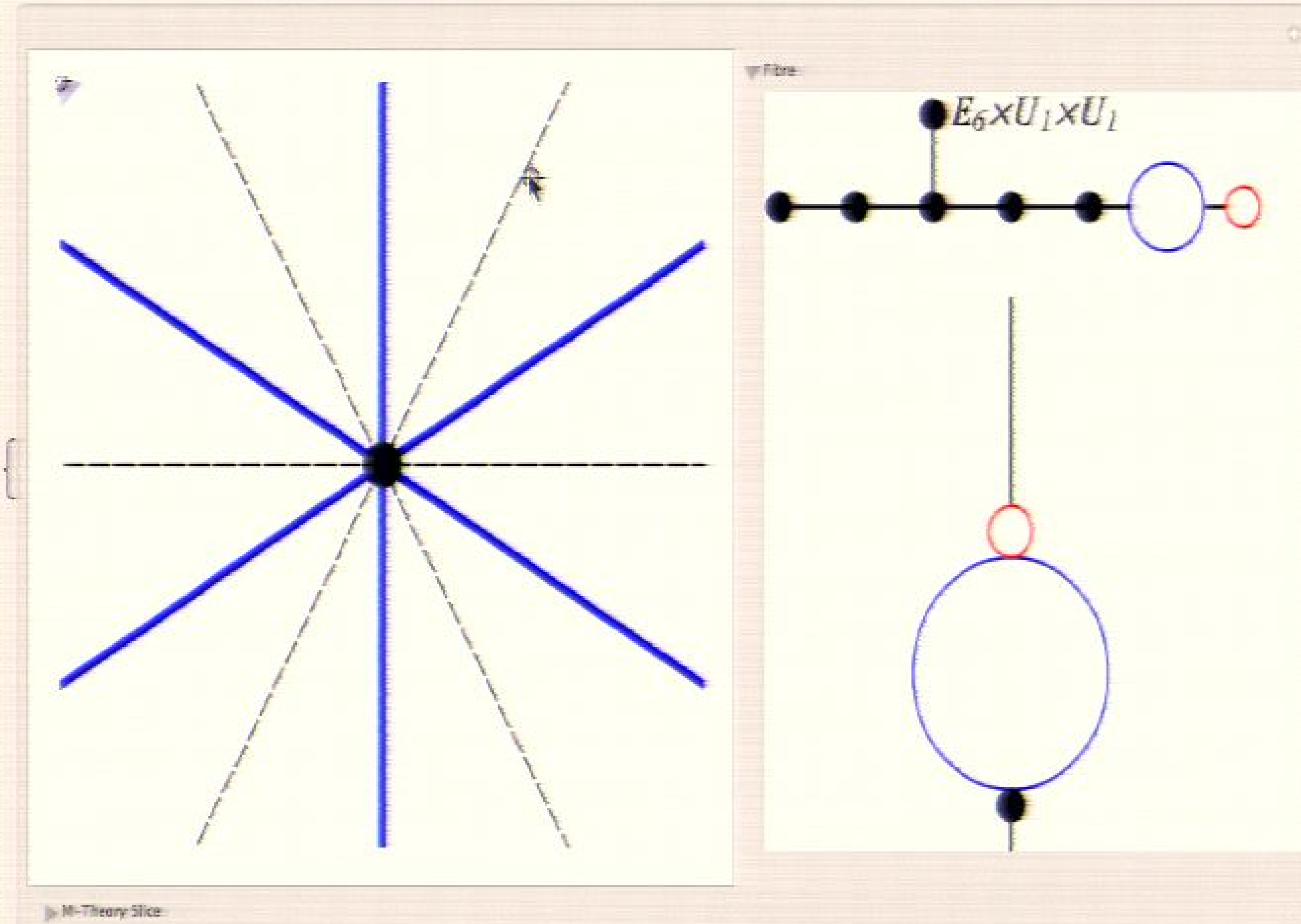


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unfoldingAnimation[]
patiSalamModel3D[]
matterTableDemo[0.75]
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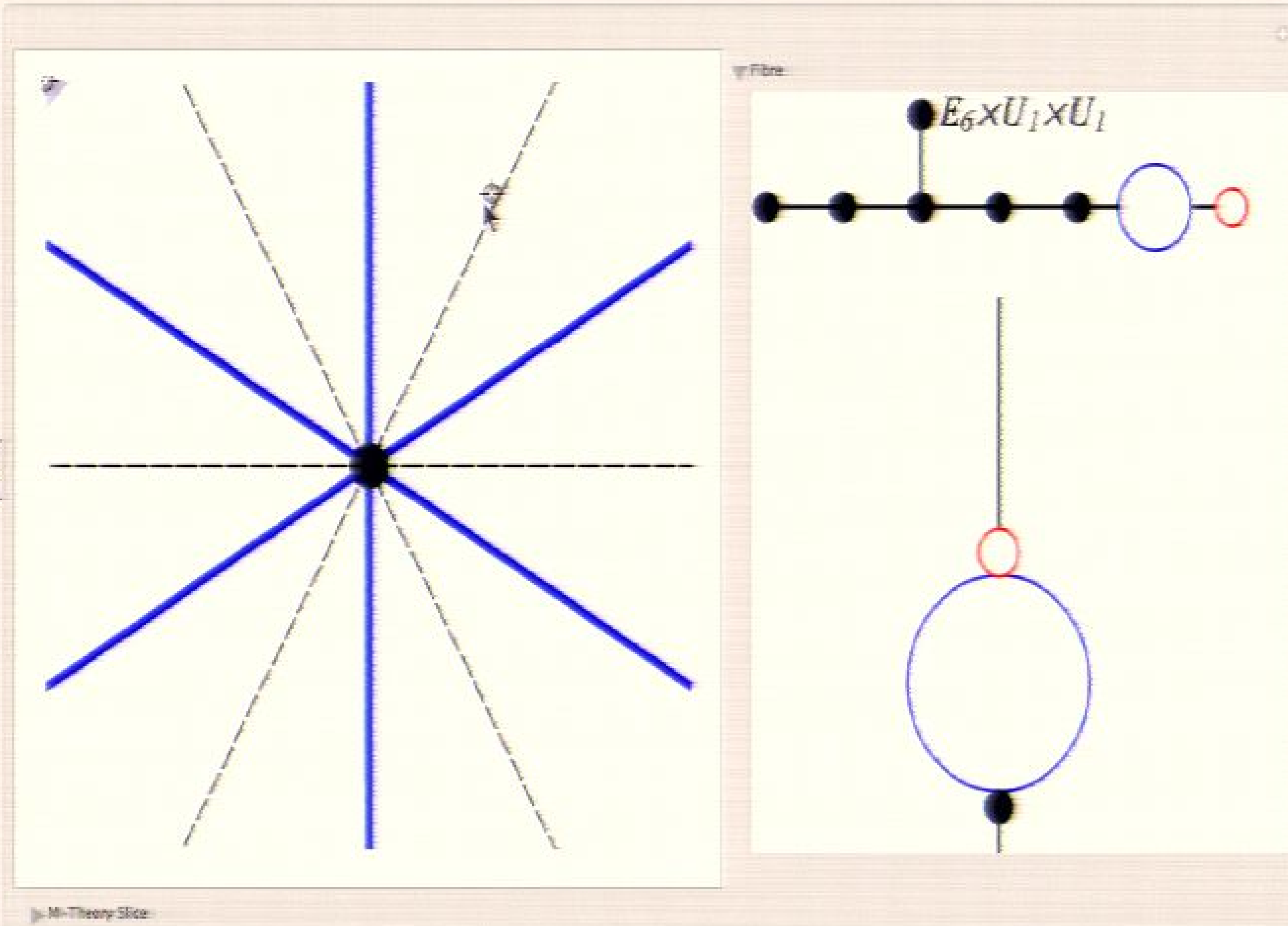


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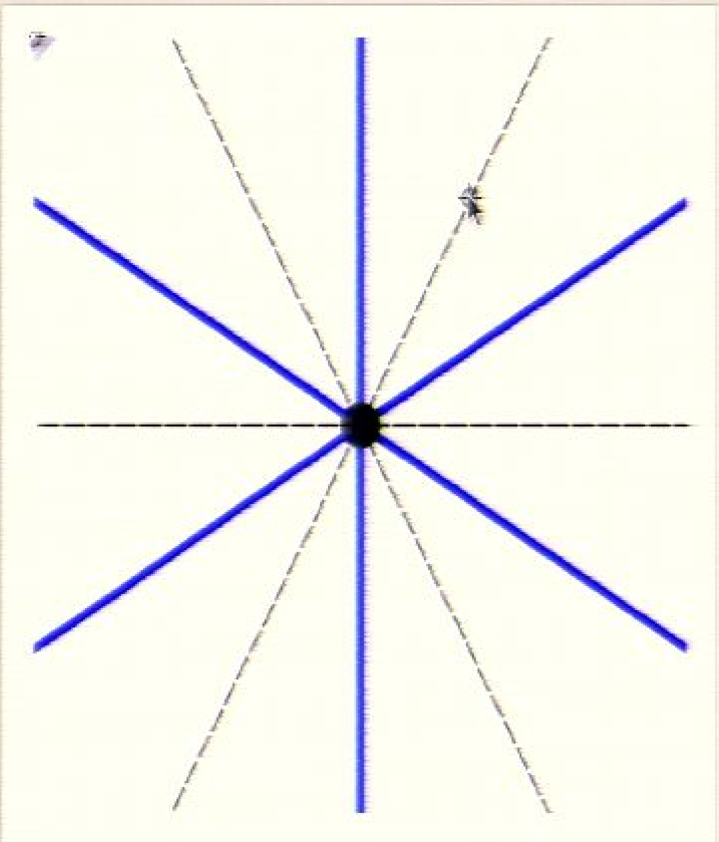

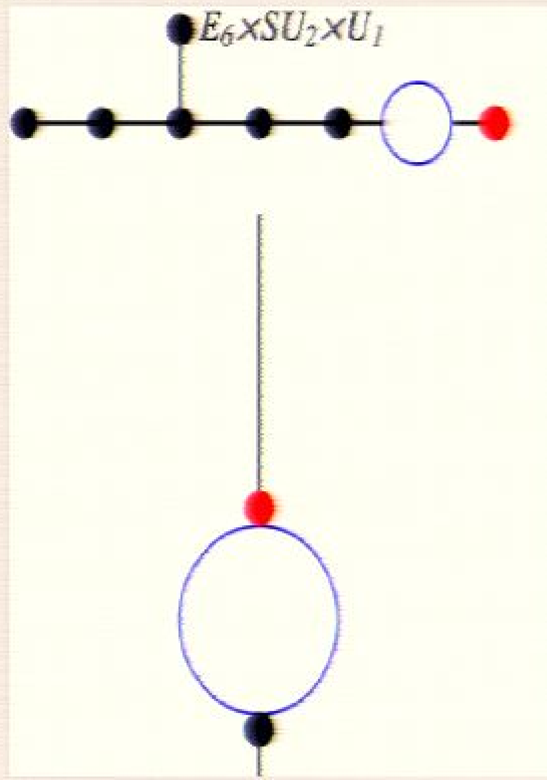

M-Theory Slice

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unfoldingAnimation[]
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patiSalamModel3D[]
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matterTableDemo[0.75]
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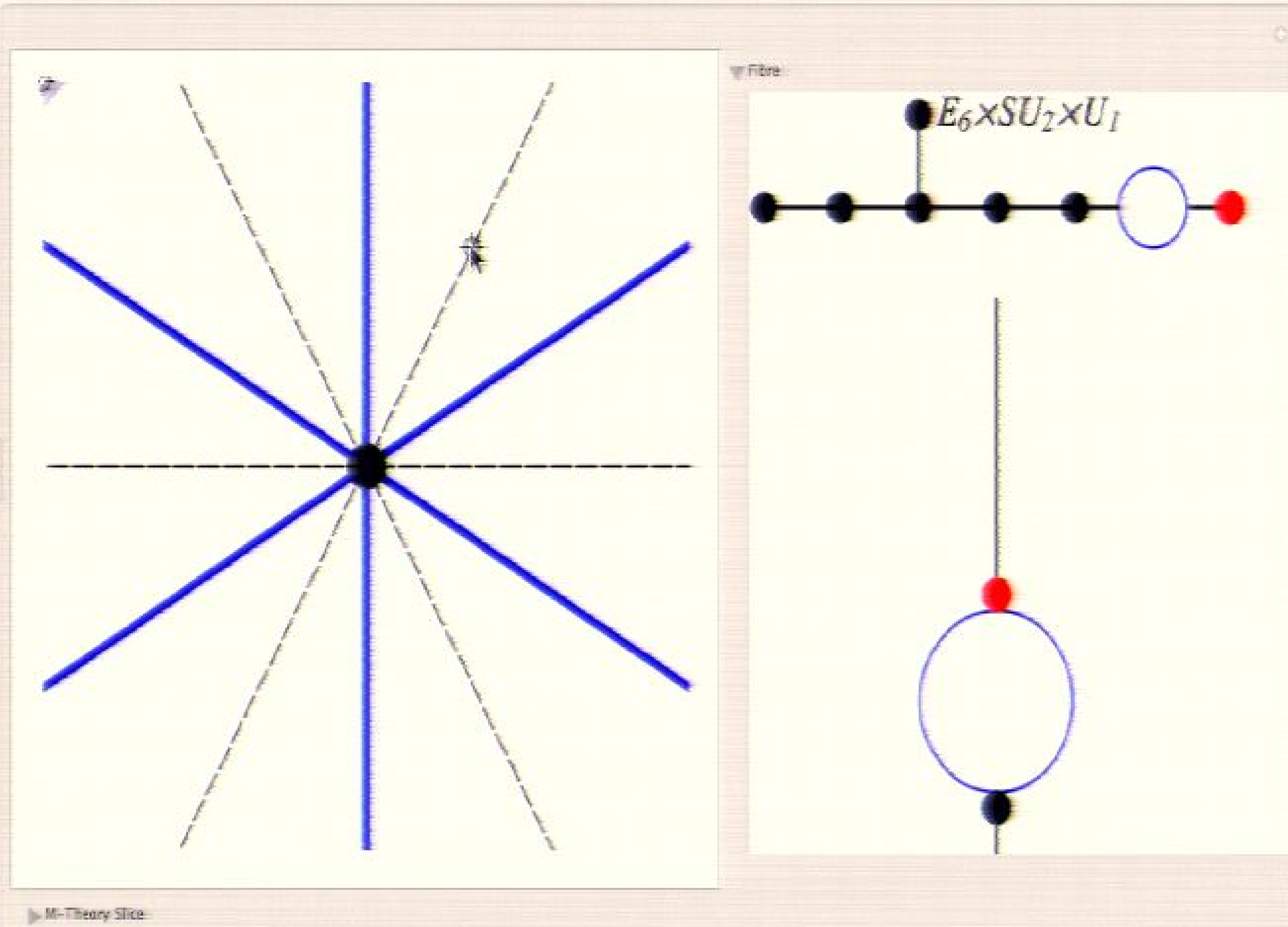


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unfoldingAnimation[]

patiSalamModel3D[]

matterTableDemo[0.75]
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Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help Jul 31 12:39

introduction_to_mathematica.nb

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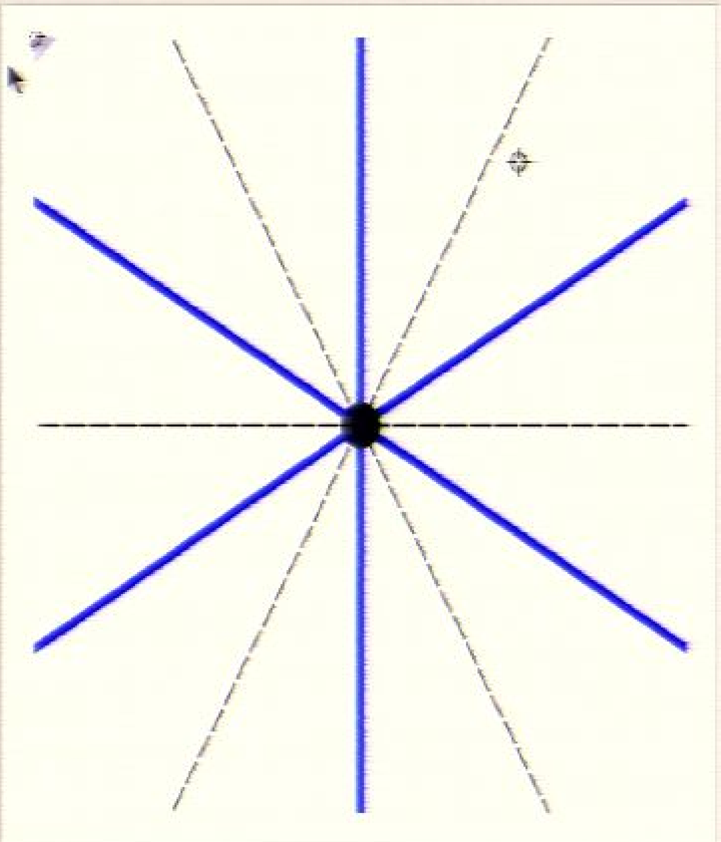
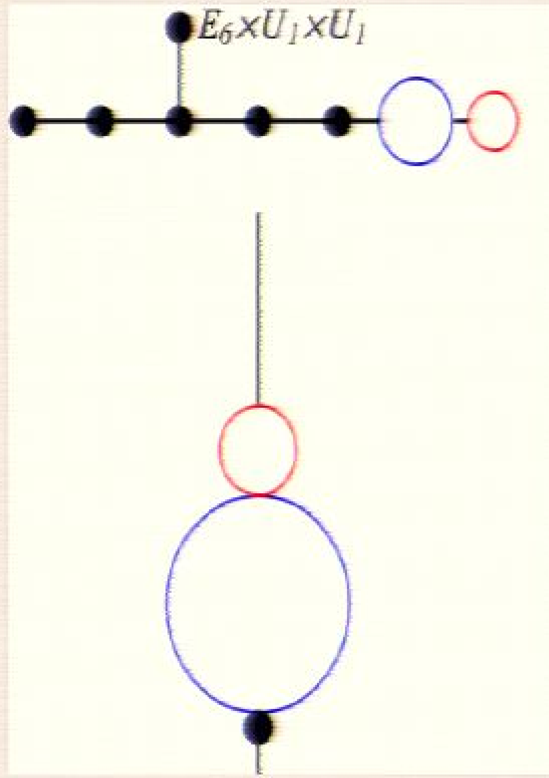
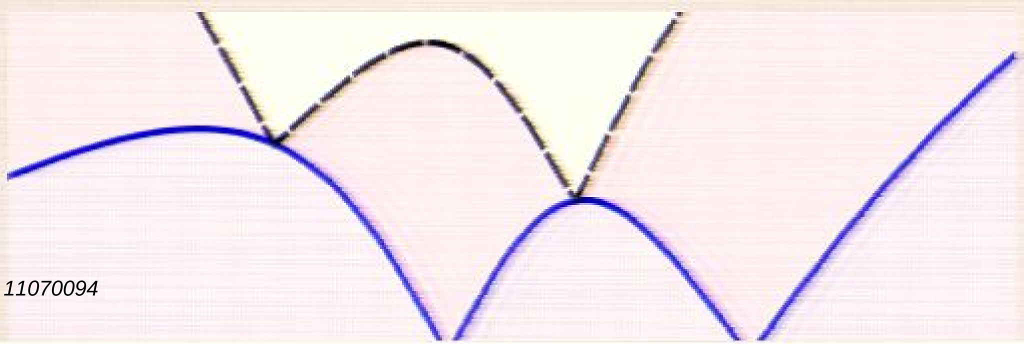


Fibre



M-Theory Slice



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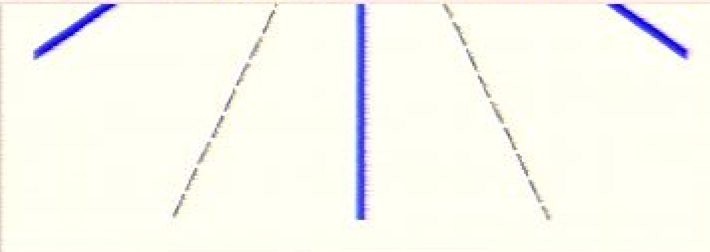
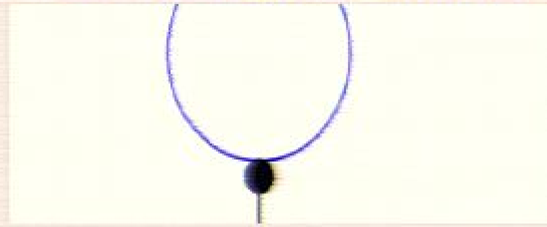
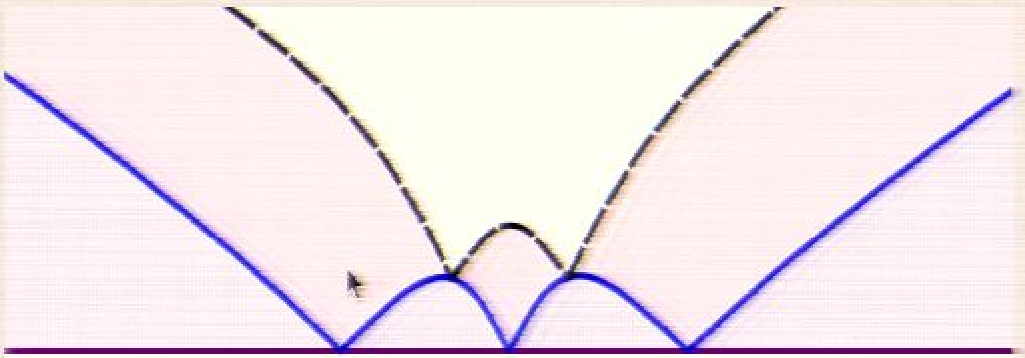




M-Theory Slice



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unfoldingAnimation[]
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patiSalamModel3D[]
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matterTableDemo[0.75]
```

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guillochePlot
calabiYauPlot
gyroidPlot
```

Mathematica Vernacular: The Anatomy of an Expression

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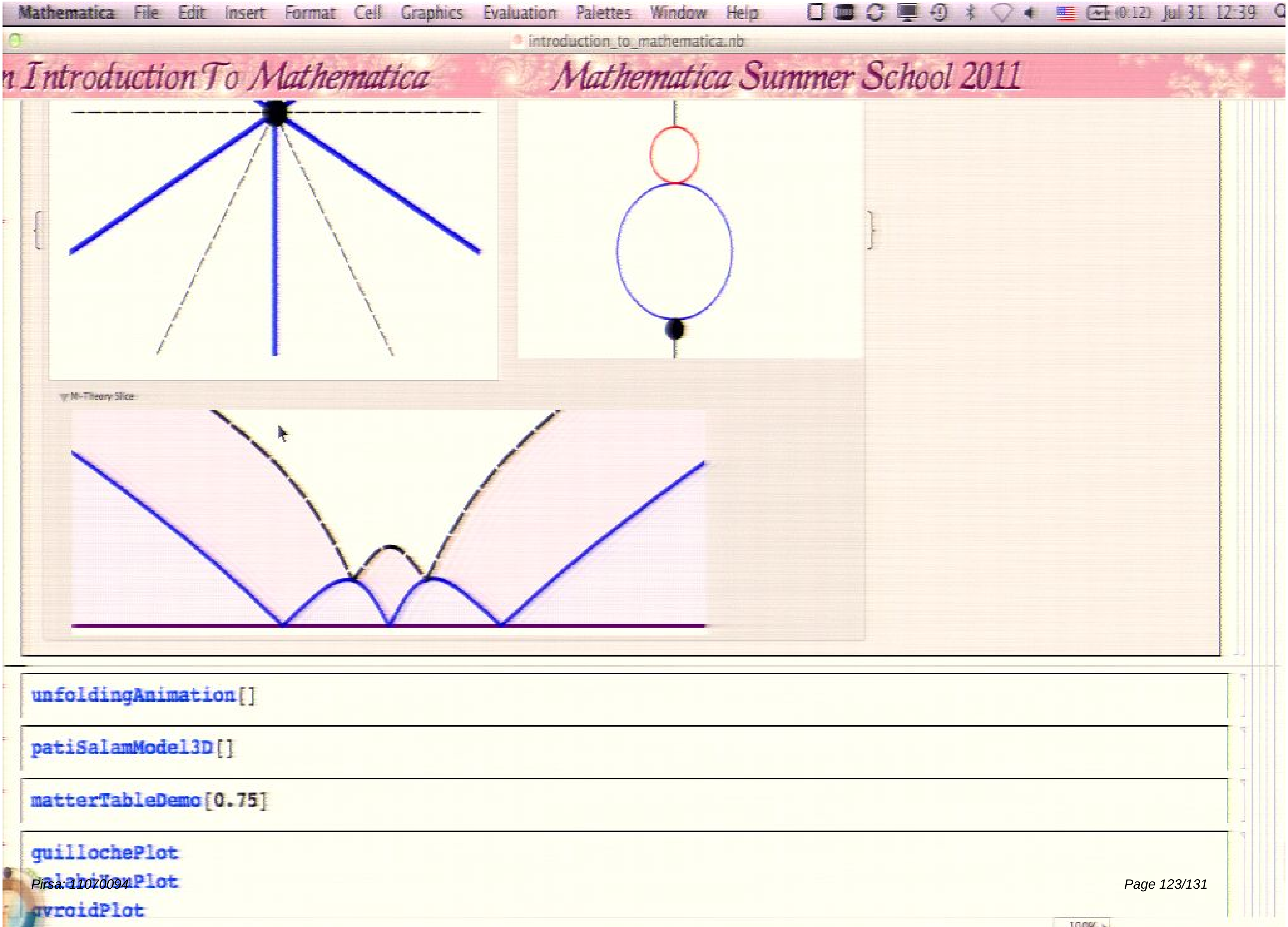
Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
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introduction_to_mathematica.nb
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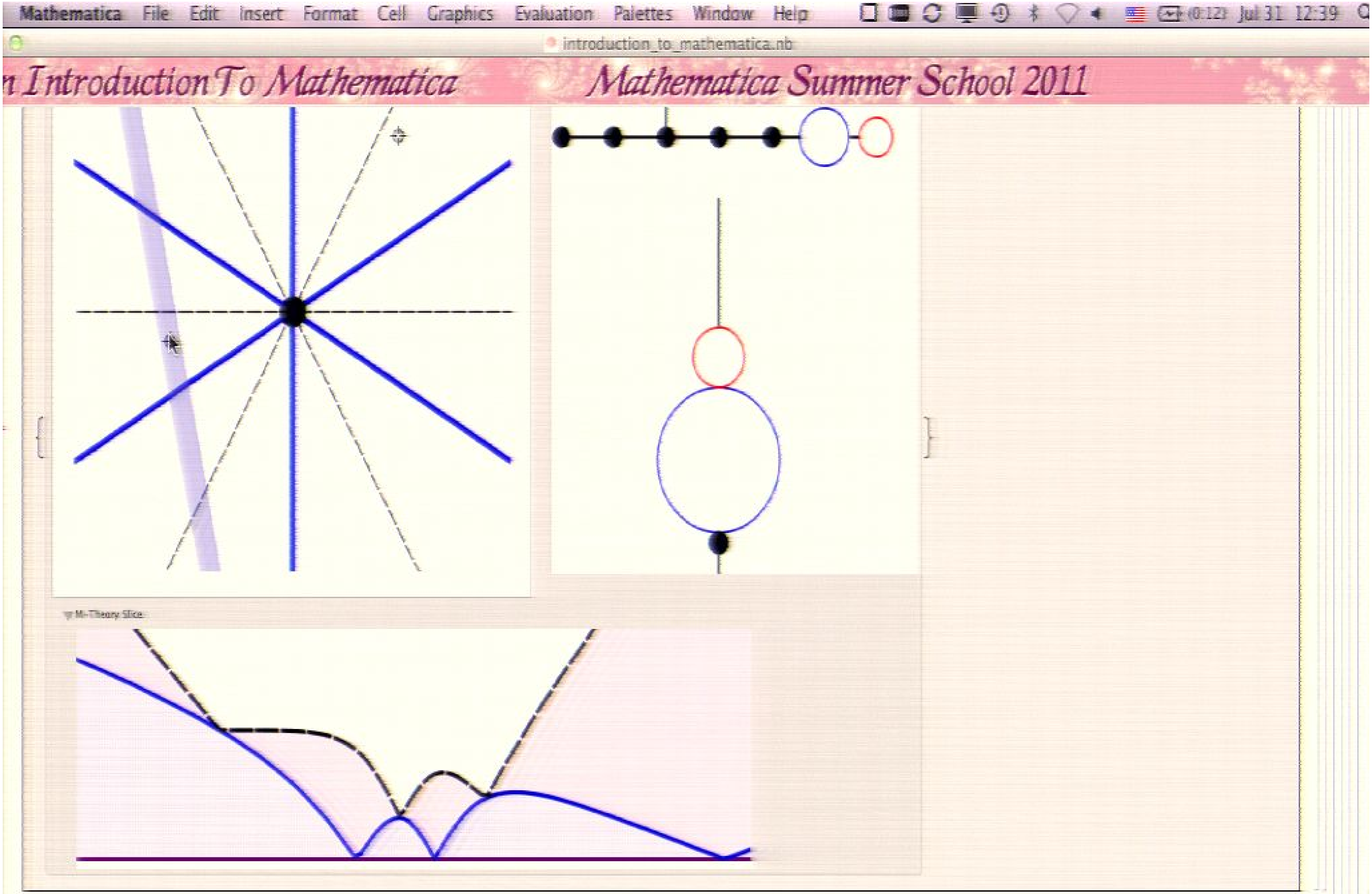
M-Theory Slice
unfoldingAnimation[]
patiSalamModel3D[]
matterTableDemo[0.75]
guillochePlot
gyroidPlot
100%

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unfoldingAnimation[]
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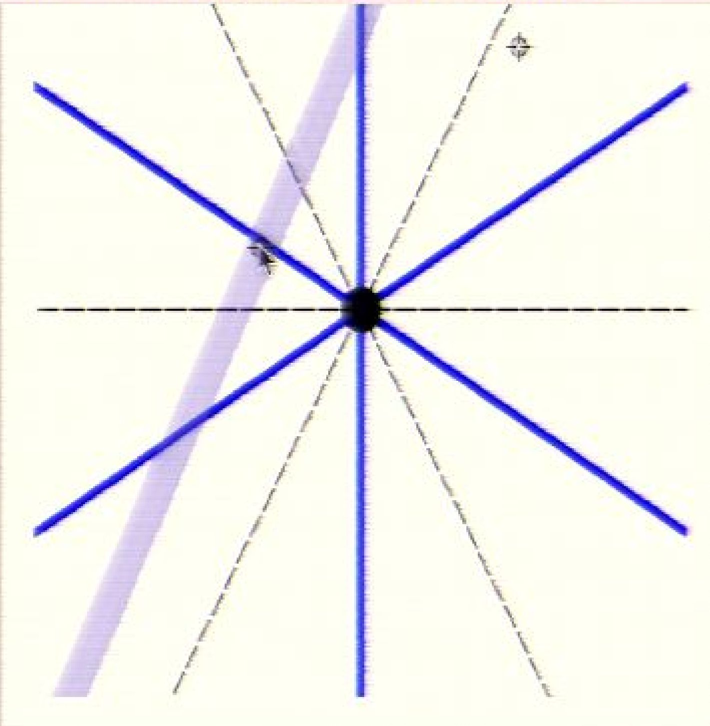
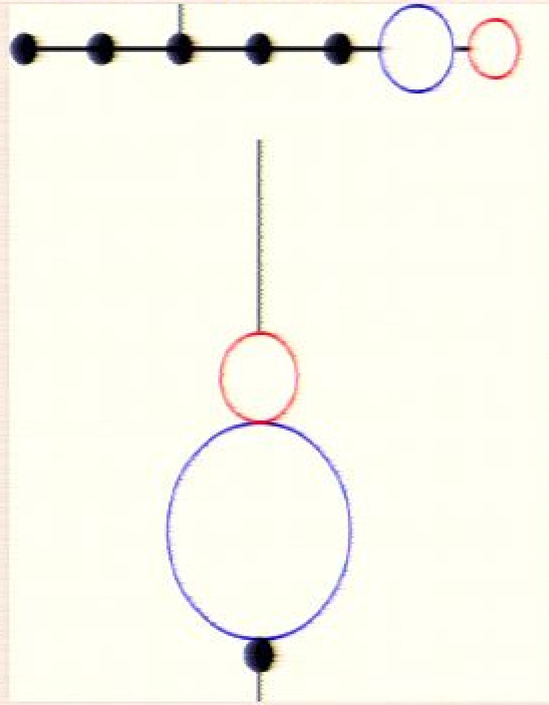
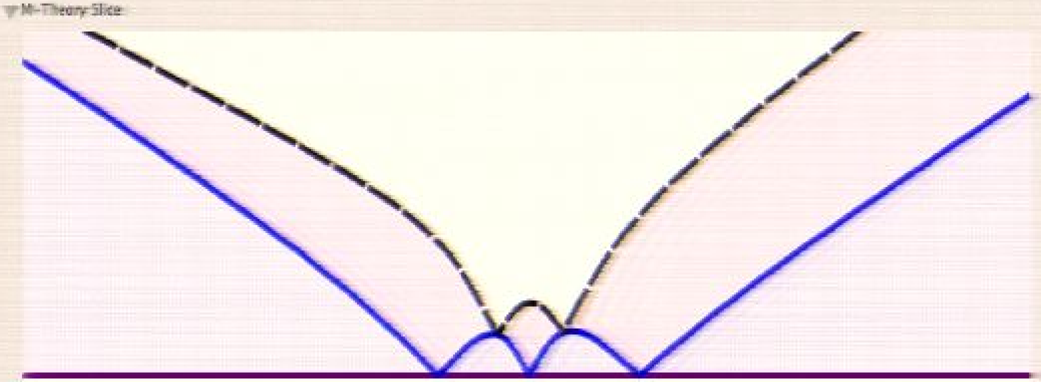




M-Theory Slice



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unfoldingAnimation[]
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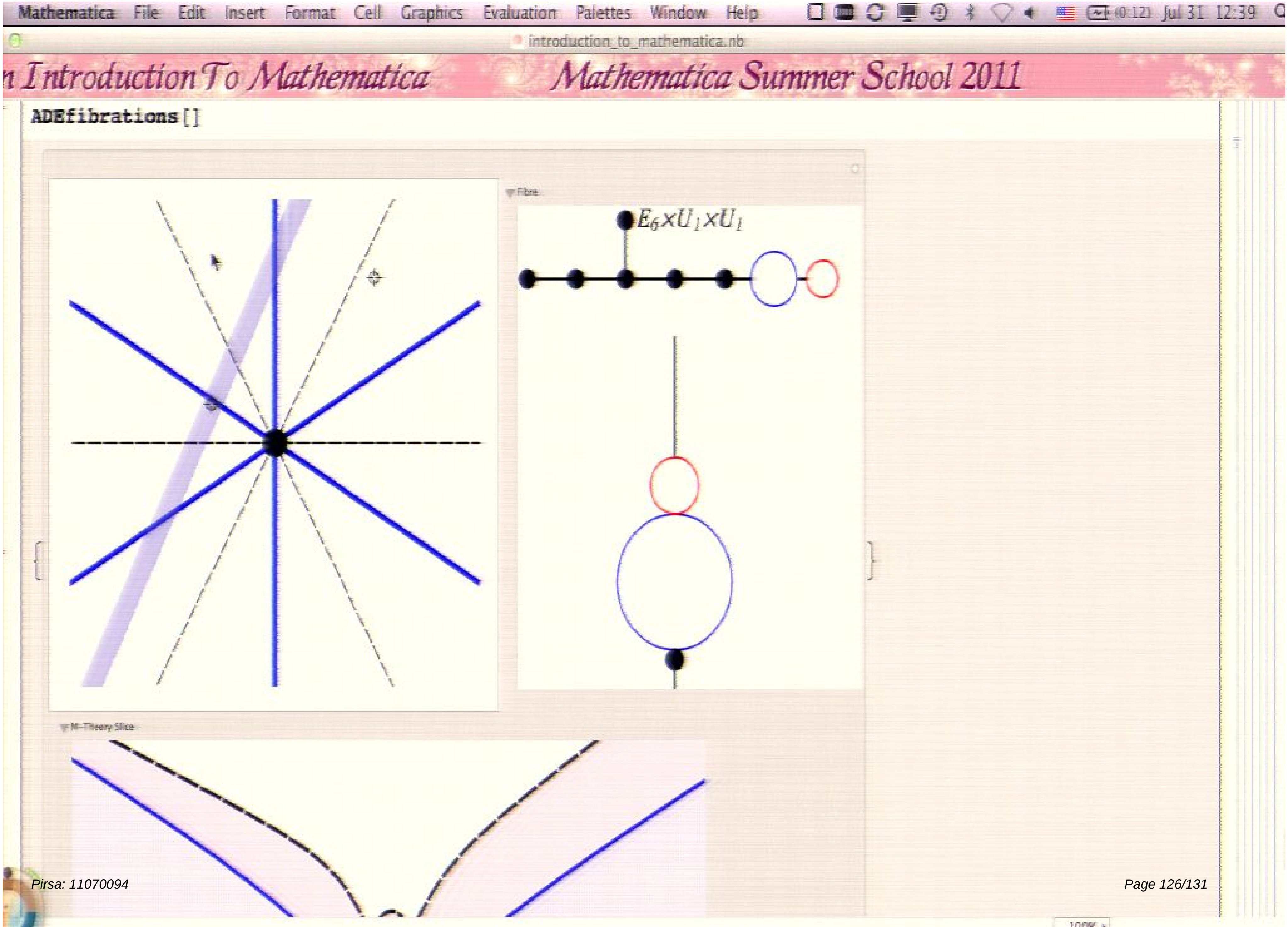
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Jul 31 12:39
introduction_to_mathematica.nb
n Introduction To Mathematica
Mathematica Summer School 2011
ADEfibrations[]
Fibre
$E_6 \times U_1 \times U_1$
M-Theory Slice
100%

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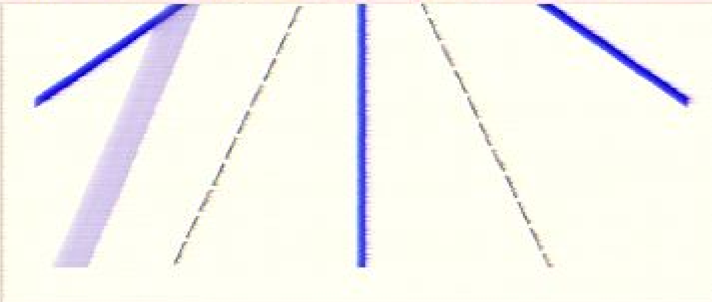

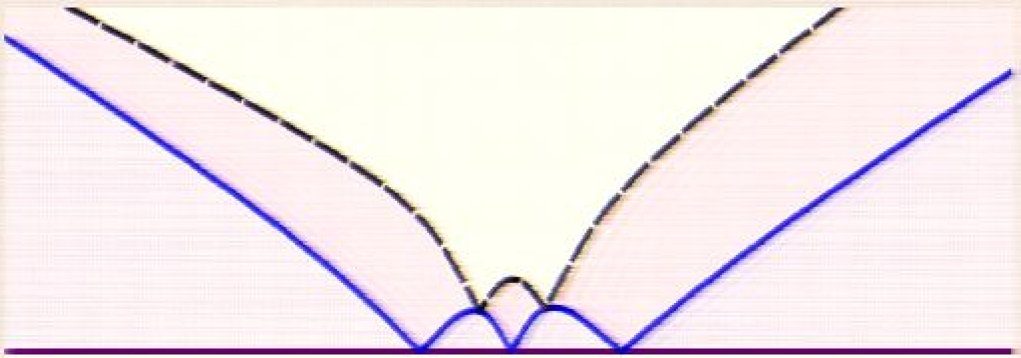


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unfoldingAnimation[]
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patiSalamModel3D[]
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matterTableDemo[0.75]
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guillochePlot
calabiYauPlot
gyroidPlot
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Mathematica Vernacular: The Anatomy of an Expression

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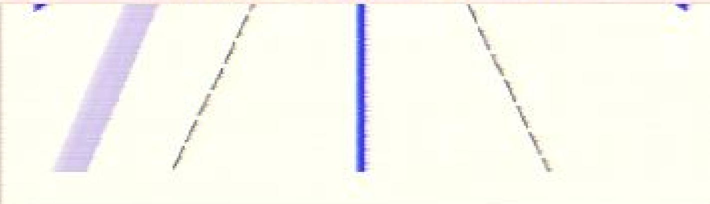
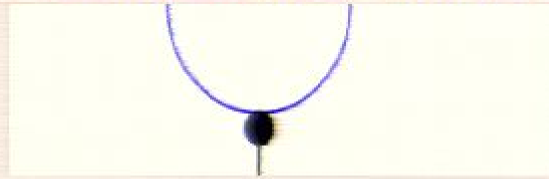
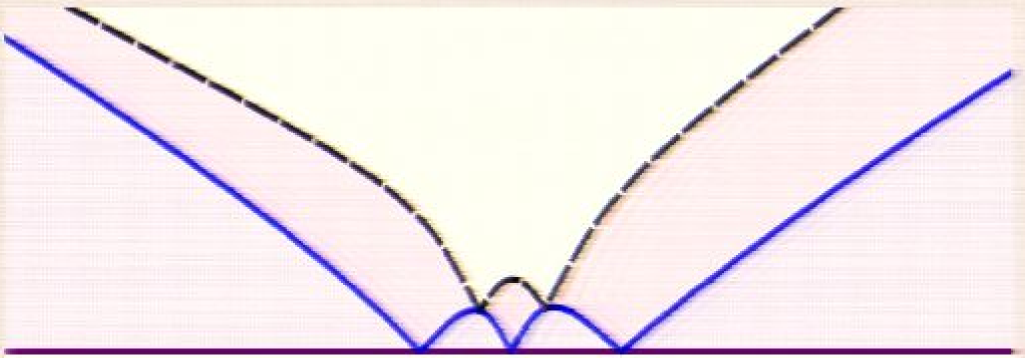




M-Theory Slice



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unfoldingAnimation[]
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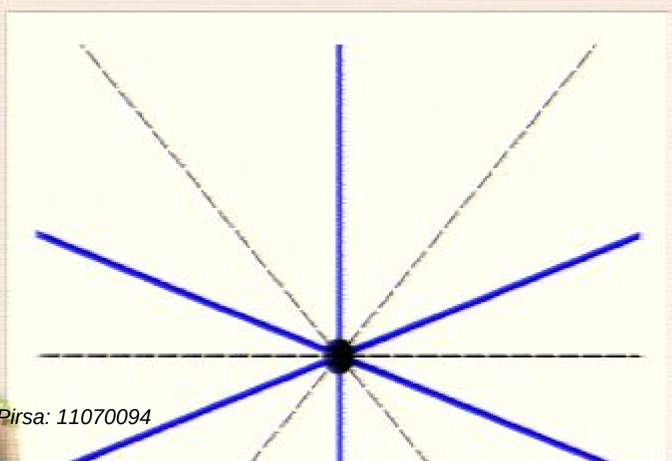

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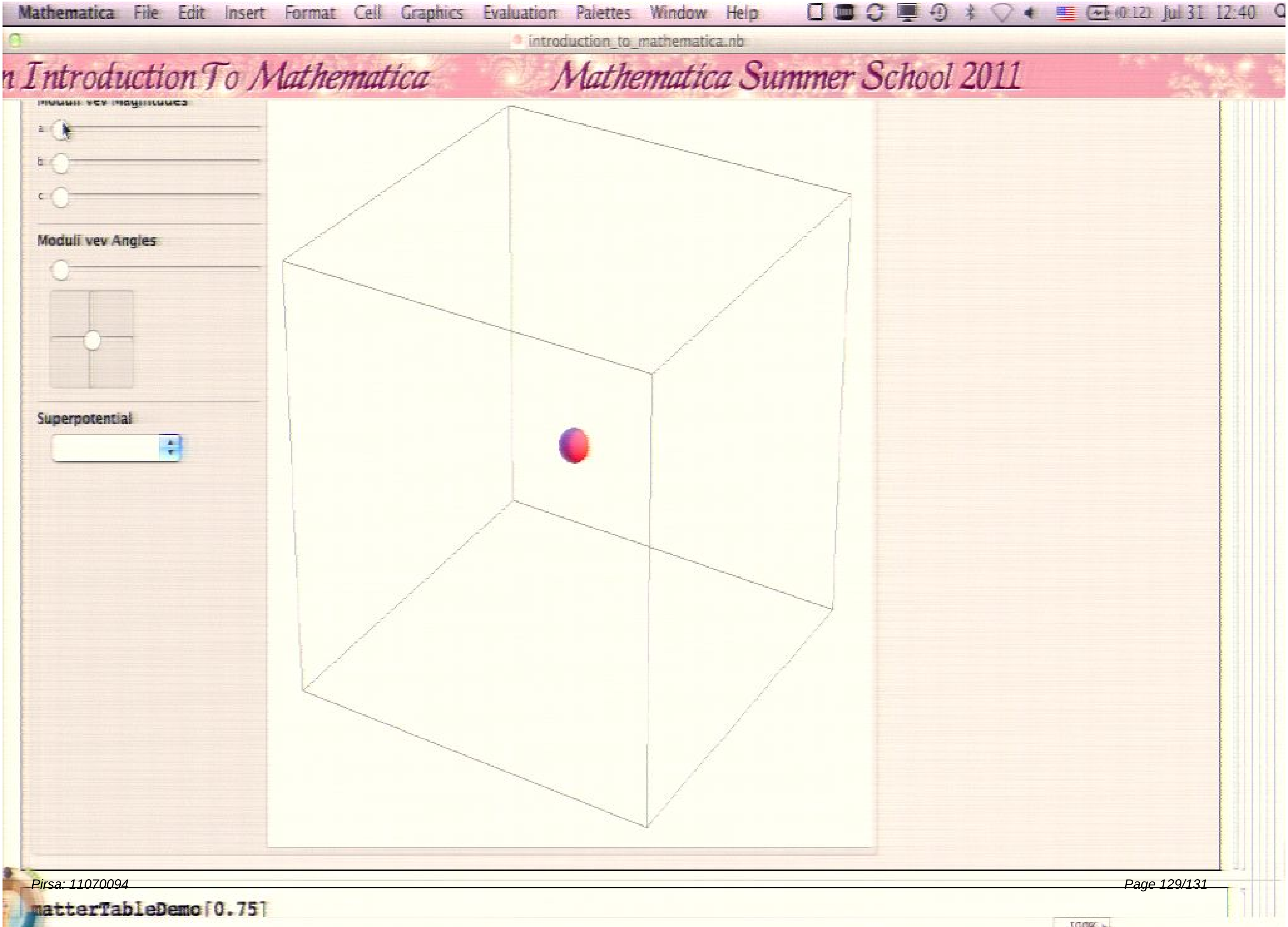
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n Introduction To Mathematica
Mathematica Summer School 2011
Moduli vev Angles
Superpotential
matterTableDemo[0.75]
100%

		SU_5	U_1^a	U_1^b	U_1^c	U_1^d
1	V	10	0	0	0	-4
2	V^c	$\overline{10}$	0	0	3	-1
3	T_1	$\overline{10}$	1	1	-1	-1
4	T_2	10	0	1	1	1
5	T_3	10	1	0	1	1
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	H_1	5	1	1	-1	3
8	H_2	5	0	1	1	-3
9	H_3	5	1	0	1	-3
10	Y_1	$\overline{5}$	1	1	2	2
11	Y_2	5	1	1	2	-2
12	Y_2^c	$\overline{5}$	0	-1	2	2
13	Y_1^c	5	0	-1	2	-2
14	E^u	5	1	0	-2	-2
15	E^d	$\overline{5}$	1	0	-2	2
16	(X)	1	0	0	-3	5
17	v_1	1	1	1	-1	-5
18	v_2	1	0	-1	-1	-5
19	v_3	1	-1	0	-1	-5
20	S_1	1	-1	-1	4	0
21	(S_2)	1	0	1	4	0
22	(S_3)	1	-1	0	-4	0
23	N_1	1	1	2	0	0
24	N_2	1	-2	-1	0	0
25	(N_3)	1	1	-1	0	0
		0	48	38	63	-65
			42	36	399	-1025

V	(X)	V^c
V^c	(S_3)	T_3
T_1	T_2	N_2
H_1	H_3	H_2
H_2	(X)	Y_1^c
Y_1	E^u	N_2
Y_2	E^d	N_2
Y_2^c	v_3	H_1
Y_2^c	(S_3)	Y_2
Y_1^c	(S_3)	Y_1
(X)	v_1	S_1
(X)	(S_2)	v_2
(S_2)	(N_3)	(S_3)
N_1	(N_3)	N_2

(N_3)	T_2	V^c	(S_3)
N_1	(X)	v_2	S_1
(S_3)	S_1	T_1	T_3
(S_3)	S_1	N_1	N_1
(S_3)	S_1	Y_1	E^u
(S_3)	S_1	Y_2	E^d
(S_3)	Y_1^c	T_2	T_3
(S_3)	Y_1^c	E^d	(S_2)
(S_3)	Y_2^c	V	Y_1
(S_3)	Y_2^c	T_2	H_3
(S_3)	Y_2^c	T_3	H_2
(S_3)	Y_2^c	E^u	(S_2)
v_3	Y_1^c	Y_1	(X)
v_3	Y_2^c	Y_2	(X)
v_3	V^c	T_3	(X)
v_2	V^c	T_2	(X)
(N_3)	(S_2)	(X)	v_3
(N_3)	H_2	H_1	N_2
(N_3)	T_2	T_1	N_2
(S_3)	Y_1^c	Y_2^c	N_1
(S_3)	Y_1^c	V^c	H_2
(S_3)	Y_2^c	V^c	T_1
(S_3)	V^c	V^c	E^d
(S_2)	E^d	E^u	N_2
(X)	E^u	H_2	S_1
(X)	T_2	H_3	N_2
(X)	H_2	V	Y_2^c
(X)	T_2	V	Y_1^c
(X)	T_1	V	S_1

```
Massless Modes
9

Massive Modes
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$-\epsilon$
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$-\epsilon$
$-\epsilon$
$-\epsilon$
$-\epsilon$
$-\eta^2\epsilon$
$-\epsilon$
$-\epsilon$

```
Massless Field Combinations
```

$E^d - \eta N_2$
$E^u - \eta Y_2$
$T_1 - H_2$
H_3
H_1
$T_3 - V$
$T_2 - \eta V$
T_1

```
guillochePlot
calabiYauPlot
gyroidPlot
```

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```
245
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2329
```

```
fourPointGraphs[6][[-5 ;; -1]]
```

◊ Unless the package is stored in the *appropriate, global* directory, so long as the package is in the notebook's directory, it can be loaded by first evaluating:

```
SetDirectory[NotebookDirectory[]]
```

Visualizing data with *Mathematica*:

```
<< demos.m
```

```
ADEfibrations[]
```

```
unfoldingAnimation[]
```

```
patiSalamModel3D[]
```

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matterTableDemo[0.75]
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Mathematica Vernacular: The Anatomy of an Expression

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