Title: Large Volumes, Small Curvatures and Cosmology from Brane Back-Reaction

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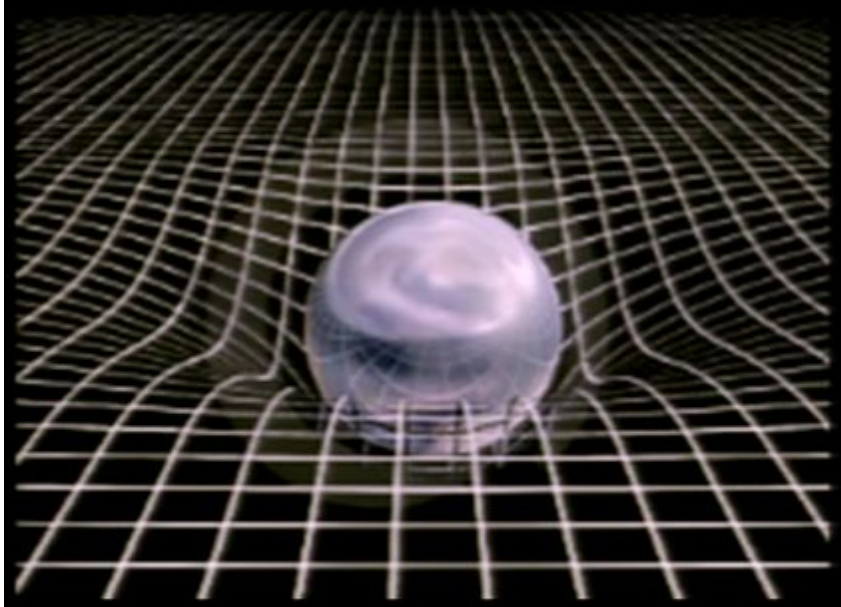
URL: http://pirsa.org/11070090

Abstract: TBA



Good Things from Brane Backreaction

Natural hierarchies and cosmology from higher codimension branes

w Leo van Nierop

Pirsa: 11070090

Outline

- Motivation
  - Extra-dimensional cosmology
  - Technical naturalness

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- Setup
  - A 6D example

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  - FRW geometries

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- Few honest-to-God extra dimensional cosmologies exist
  - work in 4D effective theory
  - work with moving branes in static backgrounds

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Naturalness

- The Standard Model

$$
\begin{aligned}
L = {} & \bar{E}DE + \bar{L}DL + \bar{Q}DQ + \bar{U}DU + \bar{D}DD \\
& + B_{\mu\nu}B^{\mu\nu} + W^{a}_{\mu\nu}W^{\mu\nu}_{a} + G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha} + G^{\alpha}_{\mu\nu}\tilde{G}^{\mu\nu}_{\alpha} \\
& + H\left(\bar{L}y_l E\right) + H\left(\bar{Q}y_d D\right) + H^{*}\left(\bar{Q}y_u U\right) \\
& + D_{\mu}H^{*}D^{\mu}H + \lambda\left(H^{*}H - m^{2}\right)^{2}
\end{aligned}
$$

A theory only a mother could love….

Naturalness

- Ideas for what lies beyond the Standard Model are largely driven by 'technical naturalness'.
  - Motivated by belief that SM is an effective field theory.

$$L_{SM} = m^2{}_0 H^* H + dimensionless$$

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BUT: effective theory can be defined at many scales

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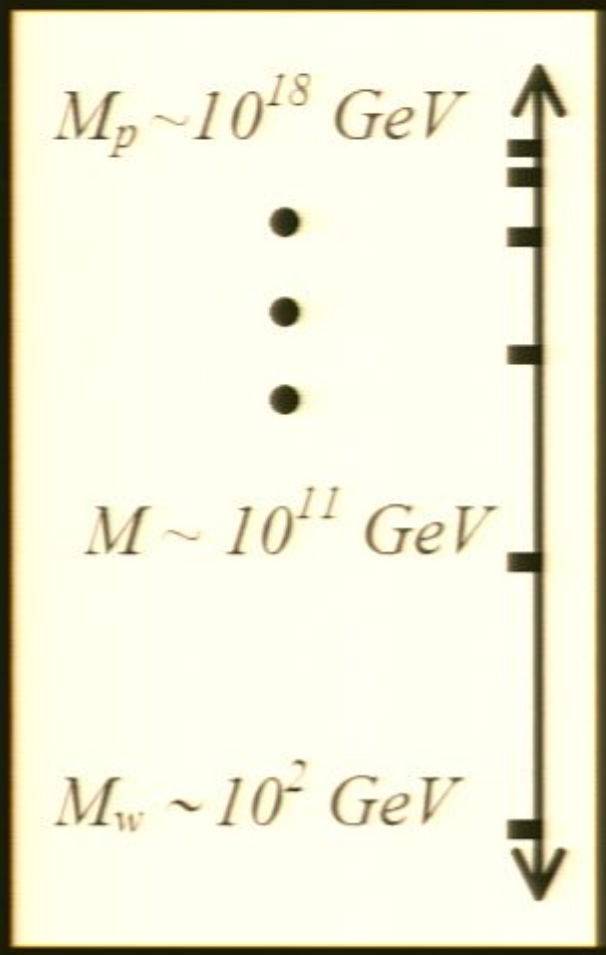

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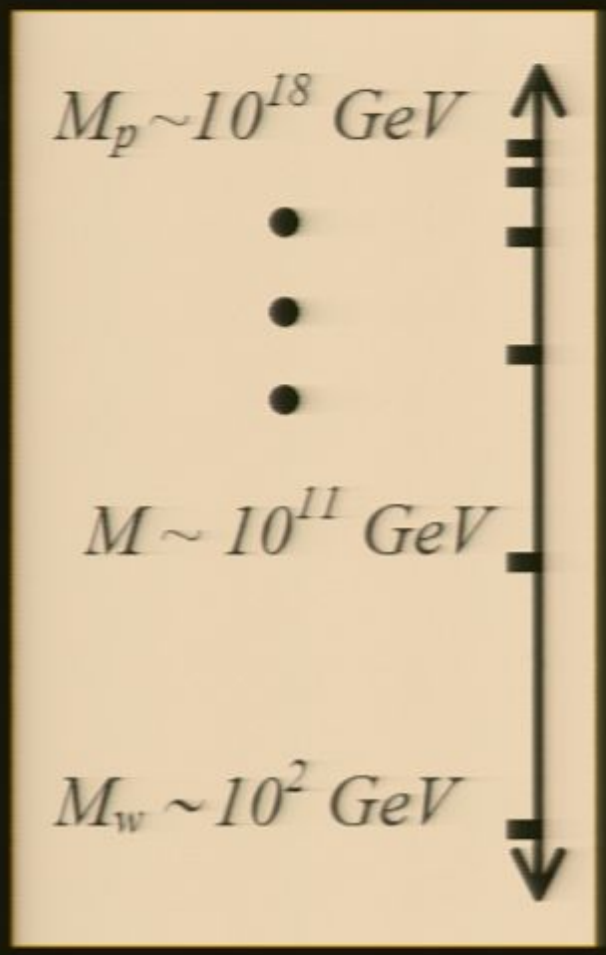

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$$m^2 \approx m_1^2 + kM^2 + \cdots$$

$$m^2 \approx m_0^2$$

Must cancel to 20 decimal places!!

Naturalness in Crisis

- Ideas for what lies beyond the Standard Model are largely driven by 'technical naturalness'.
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$$L_{SM} = \mu^4 + m^2 H^* H + \text{ dimensionless}$$

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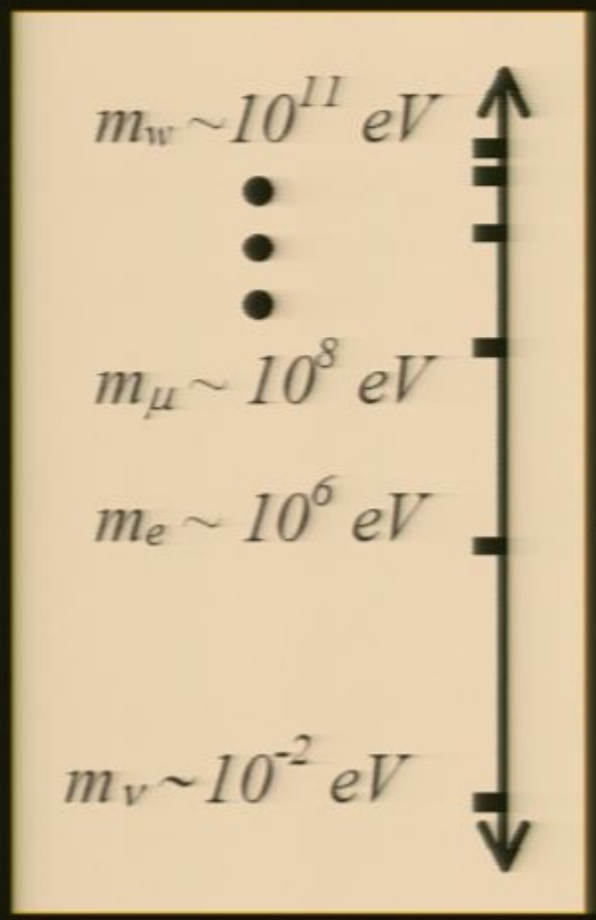

Can apply same argument to scales between TeV and sub-eV scales.

$$\mu^4 \approx \mu_1^4 + k_e m_e^4 + k_\nu m_\nu^4$$

$$\mu^4 \approx \mu_0^4 + k_\nu m_\nu^4$$

Must cancel to 32 decimal places!!

Naturalness in Crisis

- Id
  la
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L

Harder than the Hierarchy problem:

Integrating out the electron already gives too large a contribution!!

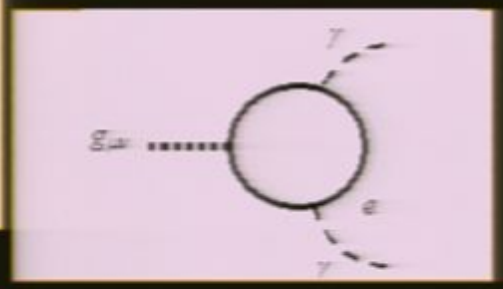
Cosmological constant problem: *Why is $\mu \sim 10^{-3}$ eV rather than m_e, M_w, M_{GUT} or M_p?*

Naturalness in Crisis

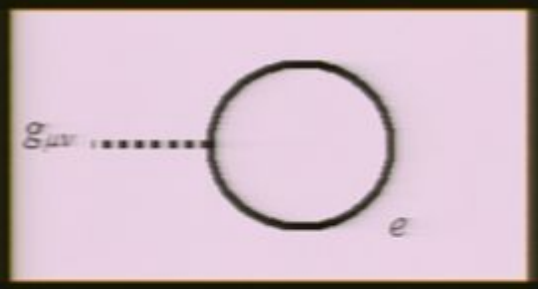
- $m_W \sim 10^{11}$ eV
- $m_\mu \sim 10^{8}$ eV
- $m_e \sim 10^{6}$ eV
- $m_\nu \sim 10^{-2}$ eV

Seek to change properties of *low-energy* particles (like the electron) so that their zero-point energy does not gravitate, *even though quantum effects do gravitate in atoms!*

Why this?



But not this?



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Setup

- 6D Einstein-Maxwell-scalar system

$$L = \frac{1}{2\kappa^2}[R + (\partial\phi)^2] + e^{-a\phi} F_{mn}F^{mn} + V(\phi)$$

- Two specific cases
  - 6D axion: $a = 0$ and $V = \Lambda$
  - 6D supergravity: $a = 1$ and $V = \frac{2g_R^2}{\kappa^4} e^{\phi}$

Setup

- Simple solution

$$ds^2 = \hat{g}_{mn} dx^m\, dx^n + [dr^2 + \alpha^2 L^2 \sin^2\left(\frac{r}{L}\right) d\theta^2] e^{-a\phi_0}$$

$$F_{r\theta} = Q\alpha L \sin\left(\frac{r}{L}\right) e^{-a\phi_0} \qquad \phi = \phi_0$$

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- Simple solution (including back-reaction)

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$$1 - \alpha = \frac{\kappa^2 T}{2\pi}$$

Setup

- Simple solution (non-SUSY case)

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Field equations

$$\frac{2}{L^2} = \kappa^2\left(\frac{3Q^2}{2} + \Lambda\right)$$

$$\hat{R} = \kappa^2(Q^2 - 2\Lambda)$$

Flux quantization

$$\frac{n}{g} = 2\alpha L^2 Q$$

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$$Q = \frac{n}{2\alpha g L^2} \qquad \hat{R} = \kappa^2 (Q^2 - 2\Lambda)$$

$$\frac{1}{L^2} = \frac{8\alpha^2 g^2}{3n^2\kappa^2}\left[1 \mp \sqrt{1 - \left(\frac{3n^2\kappa^4\Lambda}{8\alpha^2 g^2}\right)}\right]$$

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Tune $\Lambda = \frac{Q^2}{2}$ so $\hat{R} = 0$

If $T \to T + \delta T$ then $\hat{R} \to -\frac{\kappa^2 \rho}{\pi \alpha L^2}$ where $\rho = 2\,\delta T$

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$$\kappa^2 Q^2 L^2 = 1 \qquad \hat{R} = 0$$

Flux quantization

$$\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_R}$$

Setup

- In SUSY case, how does system respond to changes in brane tension?

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- Resolution: subdominant effects in the brane action are important for flux quantization

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$$(e^B \phi')_b = \frac{\kappa^2}{2\pi}\left(\frac{\partial L_b}{\partial \phi}\right)$$

$$(e^B W')_b = \frac{\kappa^2}{4\pi}\left(\frac{\partial L_b}{\partial g_{\theta\theta}}\right) = U_b$$

$$(e^B B' - 1)_b = -\frac{\kappa^2}{2\pi}\left[\left(\frac{\partial L_b}{\partial \phi} + \frac{3}{2}\frac{\partial L_b}{\partial g_{\theta\theta}}\right)\right]$$

Constraint: $4U_b[2 - 2L_b - 3U_b] - \left(\frac{\partial L_b}{\partial \phi}\right)^2 = 0$

Calculation

- Non-SUSY result:

$$V_{eff}(\phi) = \phi \int \frac{d\phi}{\phi^2}\left[\frac{\pi\alpha L^2 \hat{R}(\phi)}{\kappa^2}\right]$$

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$\delta T_b = A + B\,(\phi + v)^2$ with $v \sim 50$ then $r = Le^{-\phi/2} \gg L$

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Case 3: parametrically small vacuum energy:

δT_b and $\delta \Phi_b$ both independent of ϕ then $\rho = 0$

and ϕ_* adjusts to satisfy flux quantization condition

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$$\left[\delta T_b - 2Q\delta\Phi_b + \frac{1}{2}\frac{\partial}{\partial\phi}\sum_b \delta T_b - Q\delta\Phi_b\right]_{\phi_*} = 0$$

$$\rho = [\delta T_b - 2Q\delta\Phi_b] = \left[-\frac{1}{2}\frac{\partial}{\partial\phi}\sum_b \delta T_b - Q\delta\Phi_b\right]_{\phi_*}$$

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if δT independent of ϕ and $\delta\Phi = Ce^{-\phi}$ then $V(\phi) = Ae^{2\phi}$

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Case 2: exponentially large volume:

$\delta T_b = A + B\,(\phi + v)^2$ with $v \sim 50$ then $r = Le^{-\phi/2} \gg L$

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Outline

- Motivation
  - Extra-dimensional cosmology
  - Technical naturalness
- Setup
  - A 6D example
- Calculation
  - Maximally symmetric geometries
  - FRW geometries

Higher-dimensional inflation

- What about cosmological solutions?
  - Must generalize brane matching conditions to case where on-brane geometry is not maximally symmetric
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Higher-dimensional inflation

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  - Must generalize brane matching conditions to case where on-brane geometry is not maximally symmetric
  - Must solve the higher-dimensional field equations exactly
- For supersymmetric system exact time-dependent scaling solutions are known
  - Can these be matched to sensible brane physics to see how brane properties control bulk fields?

Higher-dimensional inflation

- 6D Einstein-Maxwell-scalar system

$$L = \frac{1}{2\kappa^2}[R + (\partial\phi)^2] + e^{-a\phi} F_{mn}F^{mn} + V(\phi)$$

- Brane-localized inflaton, χ

$$L_{b1} = T_1 + e^{-\phi}\left[(\partial\chi)^2 + V_0 + V_1 e^{\lambda\chi} + \cdots\right]$$

$$L_{b2} = T_2$$

Higher-dimensional inflation

Tolley, CB, de Rham

- Exact time-dependent solution $e^{-\phi} = (H_0\tau)^{c+2}$

$$ds^2 = (H_0\tau)^c\left[g_{mn}dx^m dx^n + \tau^2\left(g_{ij}dx^i dx^j\right)\right]$$

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Higher-dimensional inflation

- Connection to brane inflaton, and how does it end?
  - Add inflaton χ evolution to the equations

$$L_b = T + e^{-\phi}\left[\,(\partial\chi)^2 + V_0 + V_1 e^{\lambda\chi} + \cdots\right]$$

$$\chi = \chi_0 + \chi_1 \ln(H_0\tau)$$

Then $c + 2 = -\lambda\chi_1$ controls the slow roll

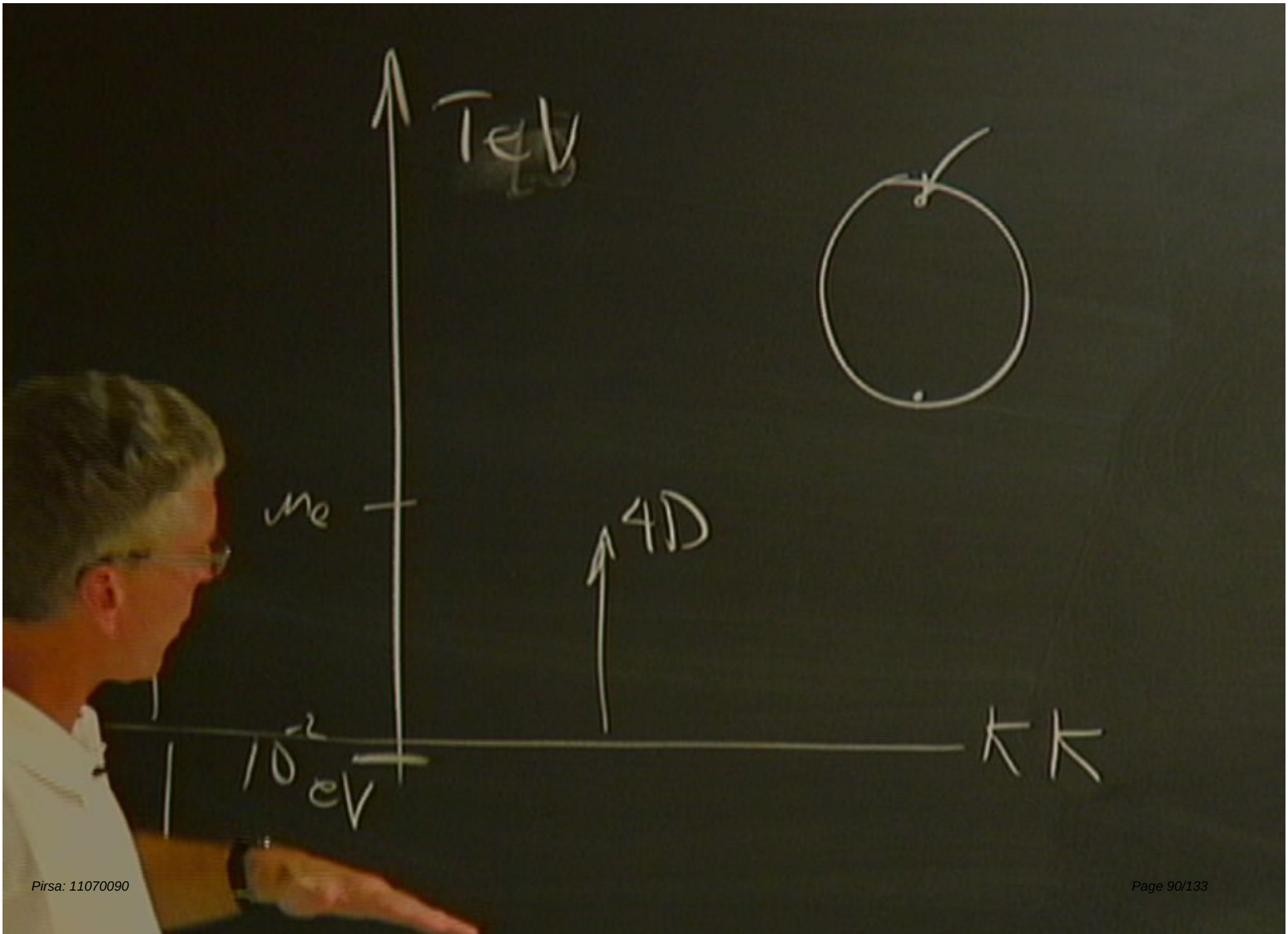
and $H_0{}^2 = \lambda V_1/[\chi_1(3 + 2\lambda\chi_1)]$

Conclusions

- Relatively little is known about explicitly higher-dimensional cosmology
  - Higher-dimensional inflation with evolving x-dims

Conclusions

- Relatively little is known about explicitly higher-dimensional cosmology
  - Higher-dimensional inflation with evolving x-dims
- Branes and brane back-reaction can have important implications for low-energy theory
  - Little explored beyond codimension 1
  - Different parametric dependences in energy: unusual stability to quantum corrections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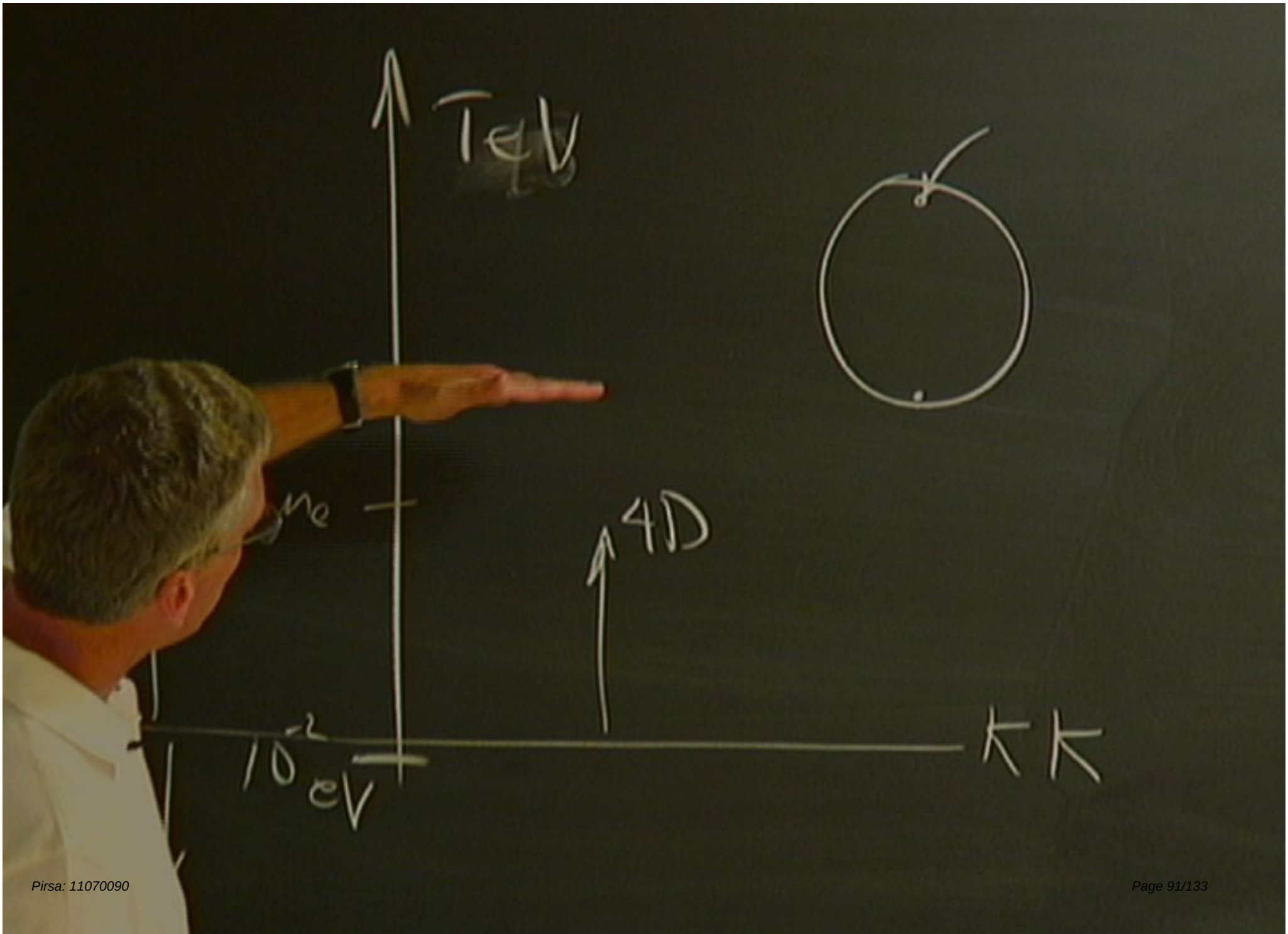
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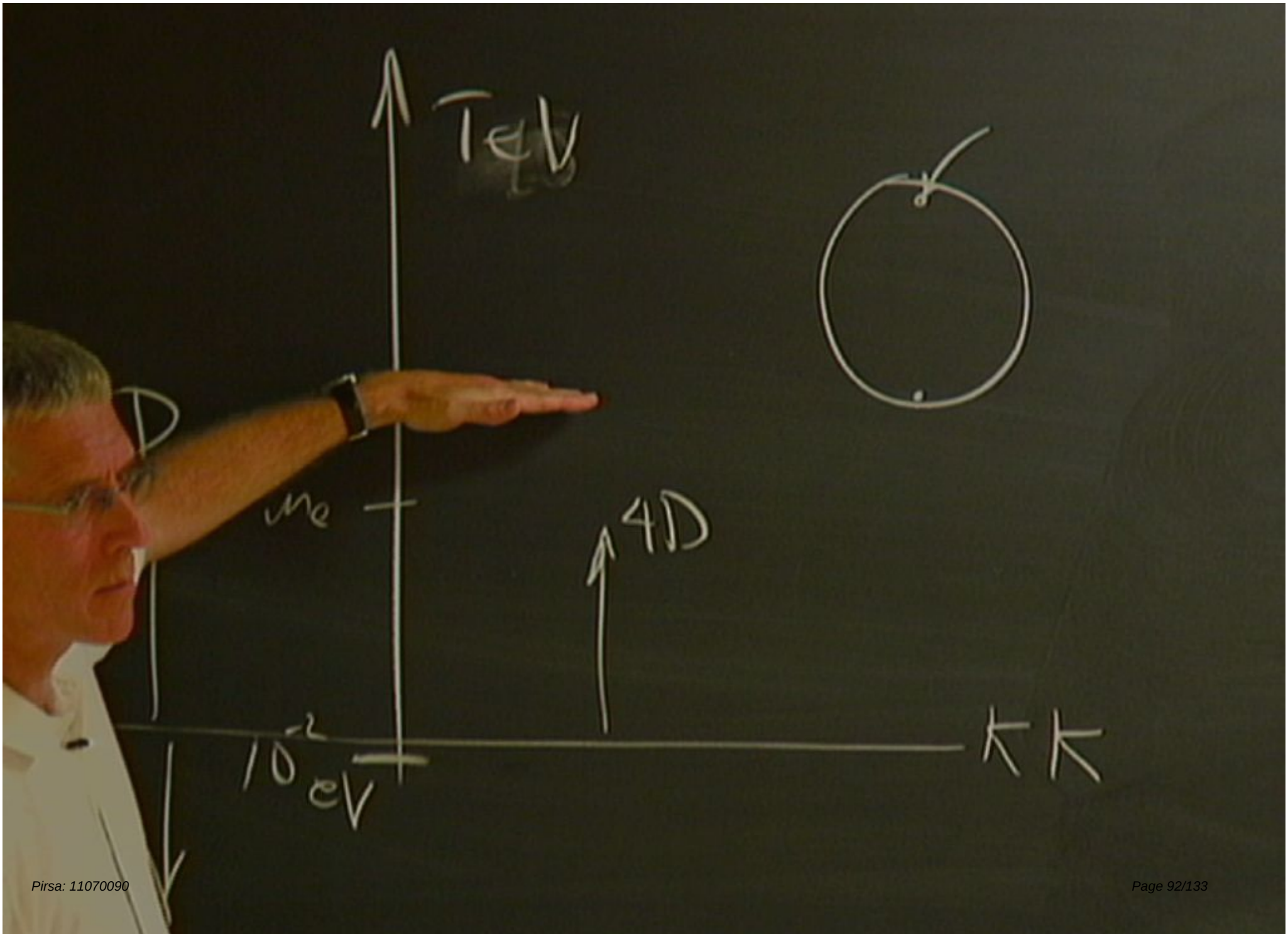
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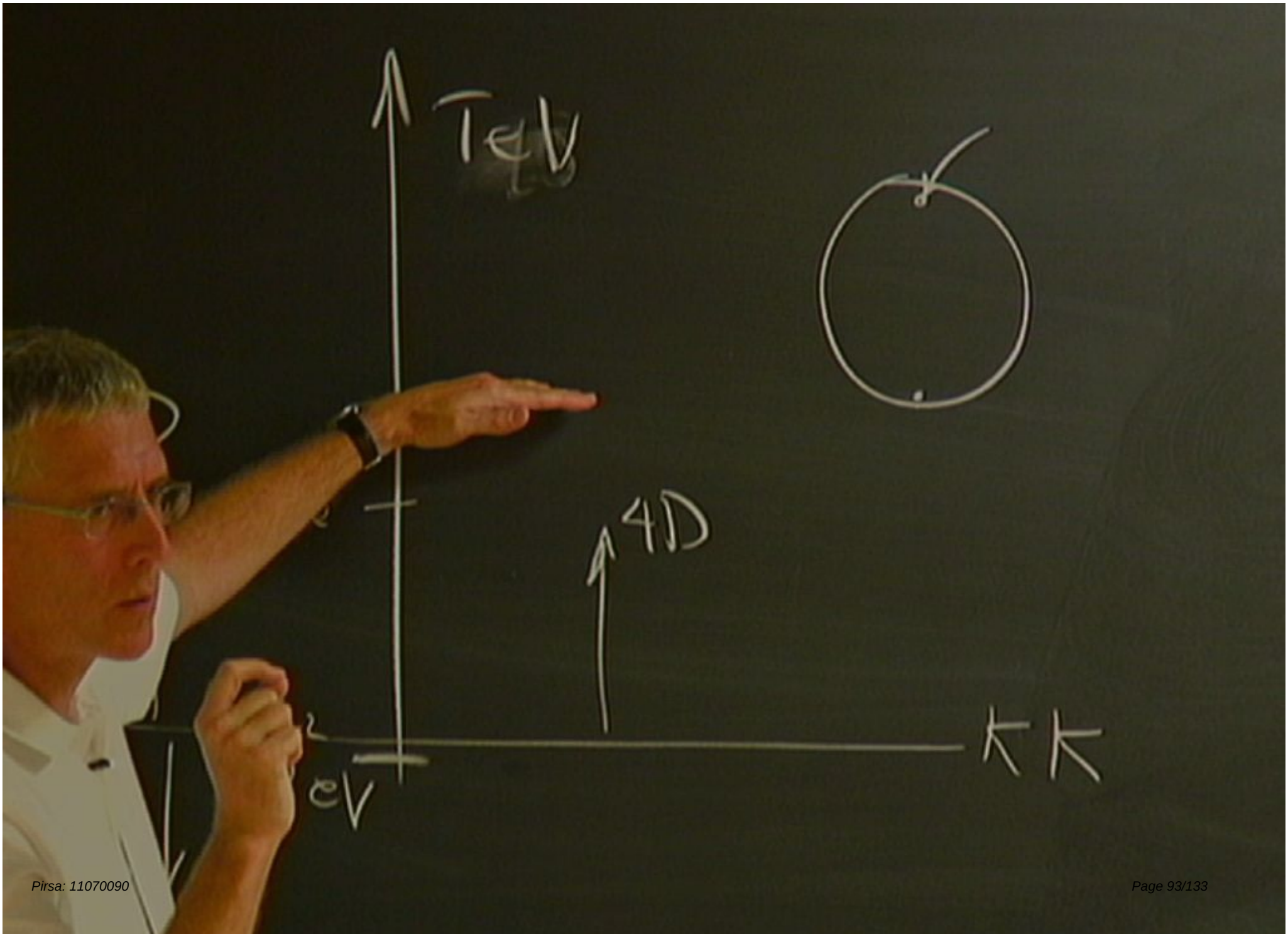
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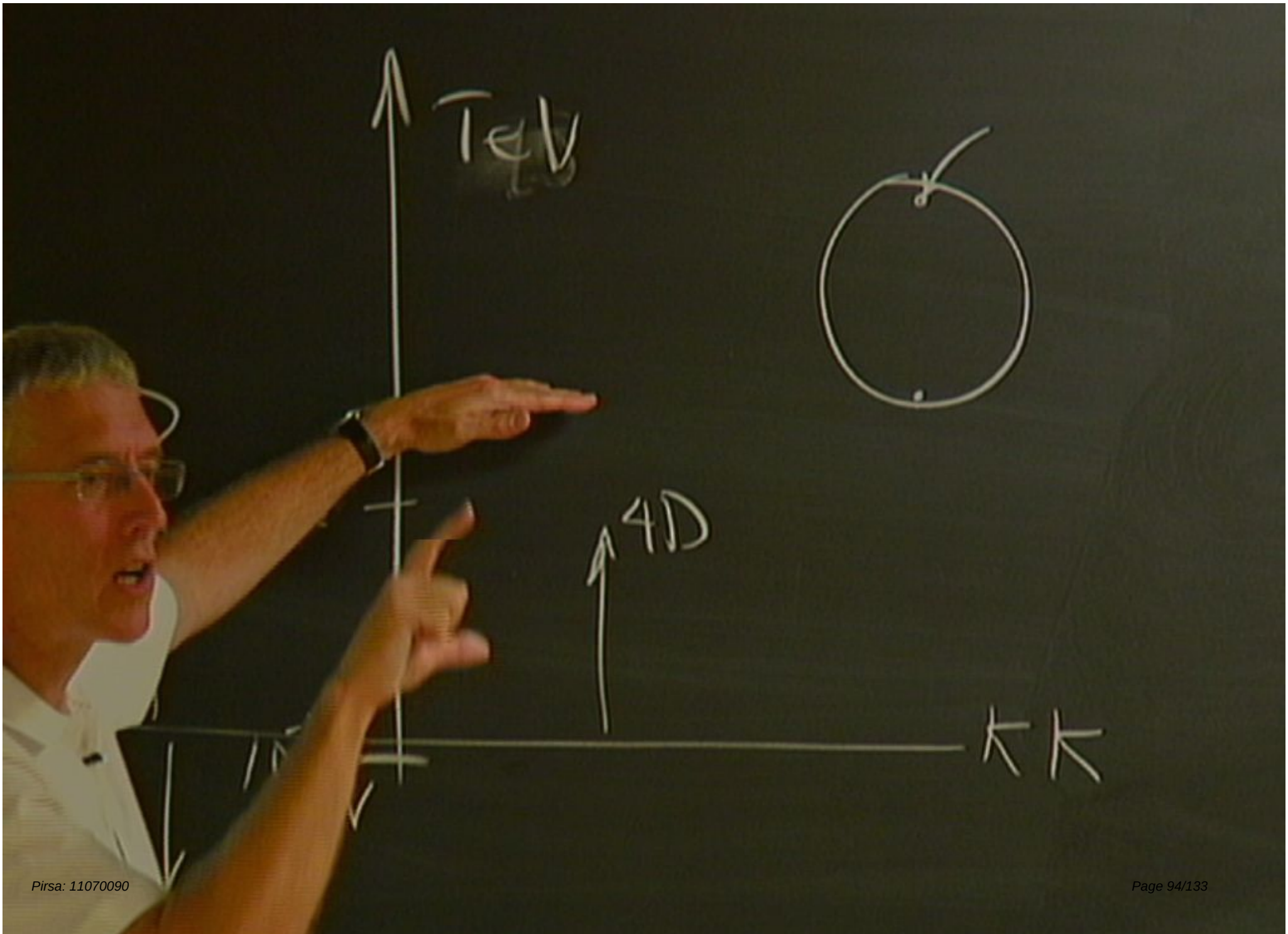
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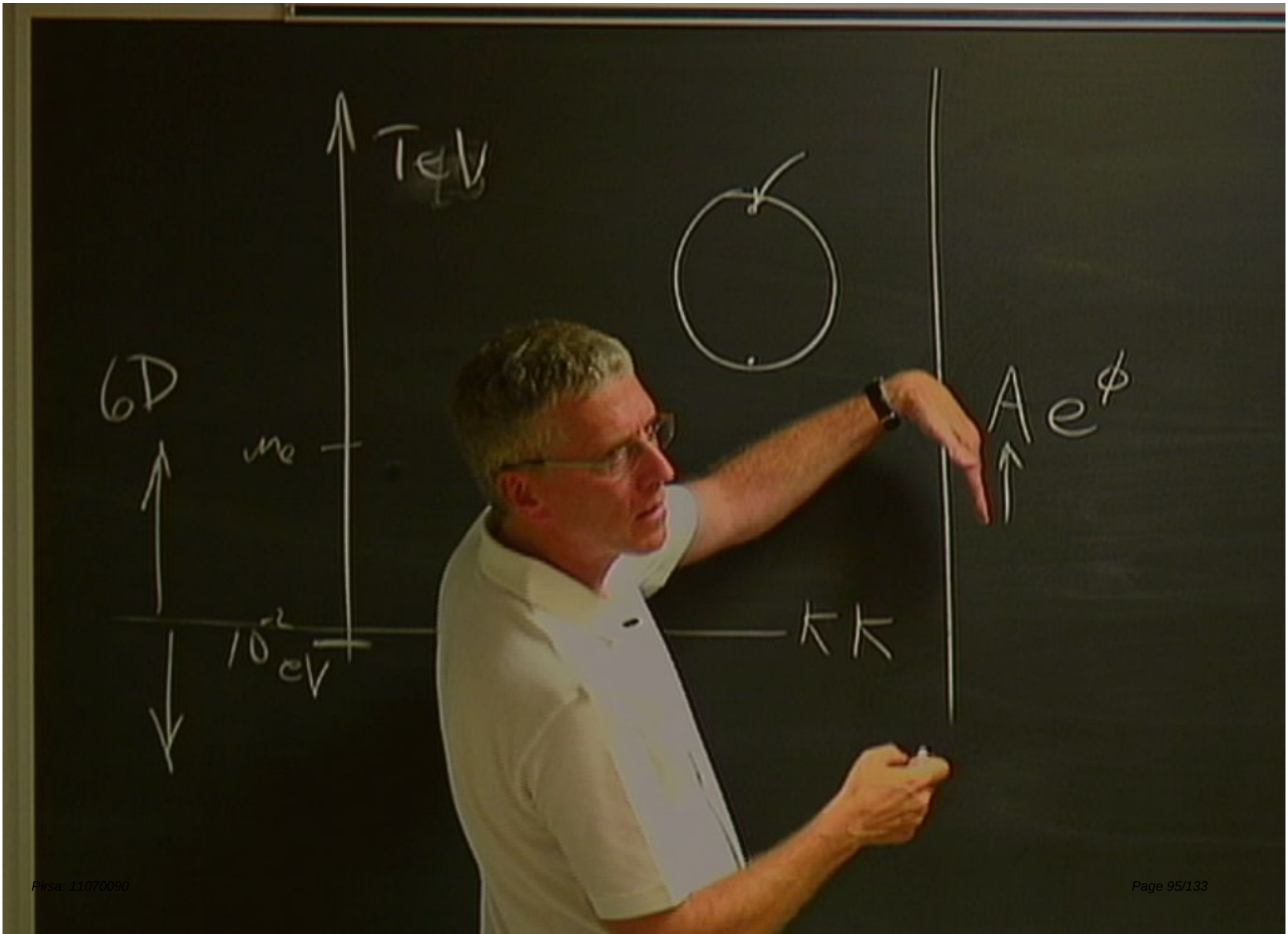
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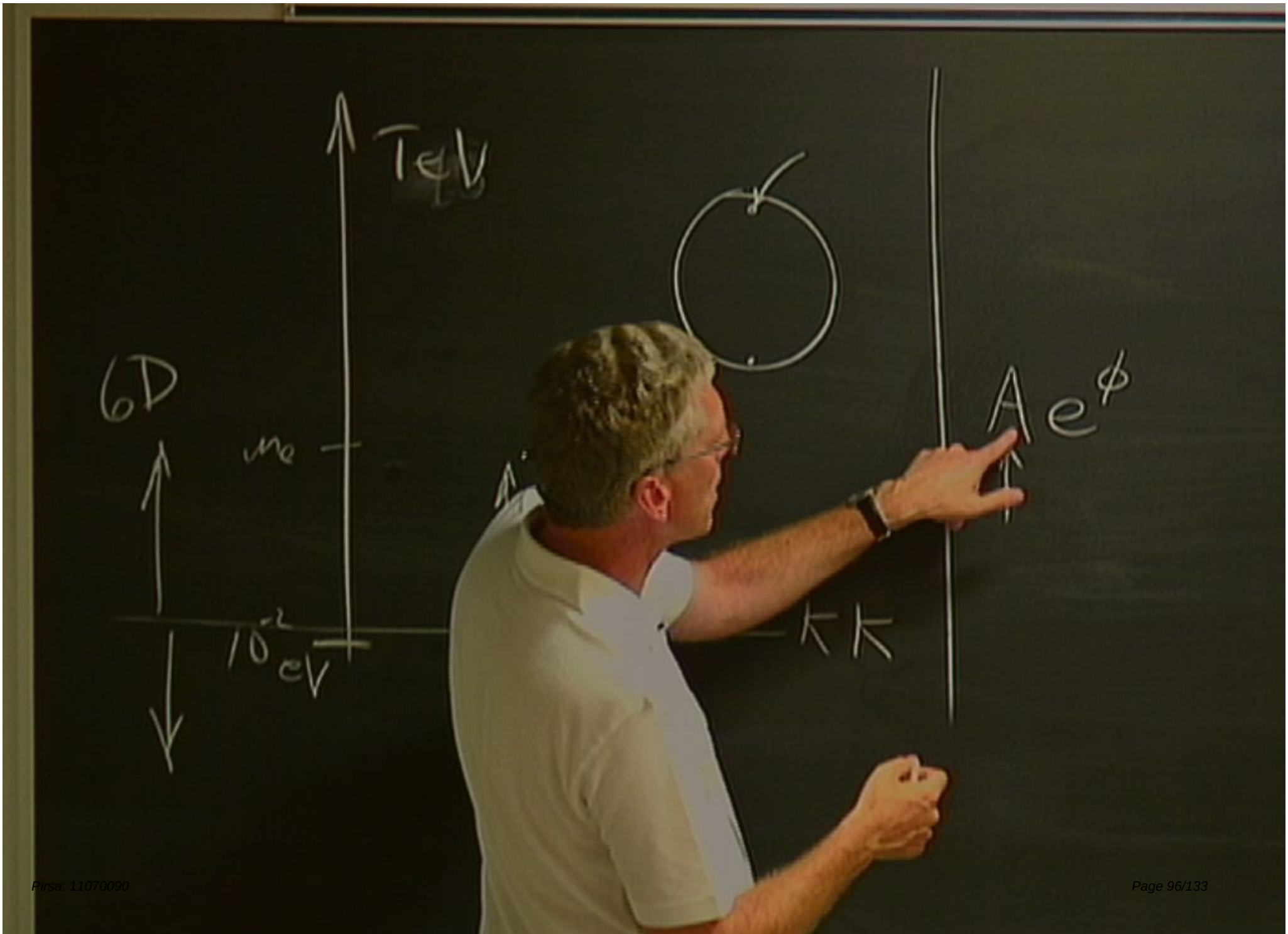
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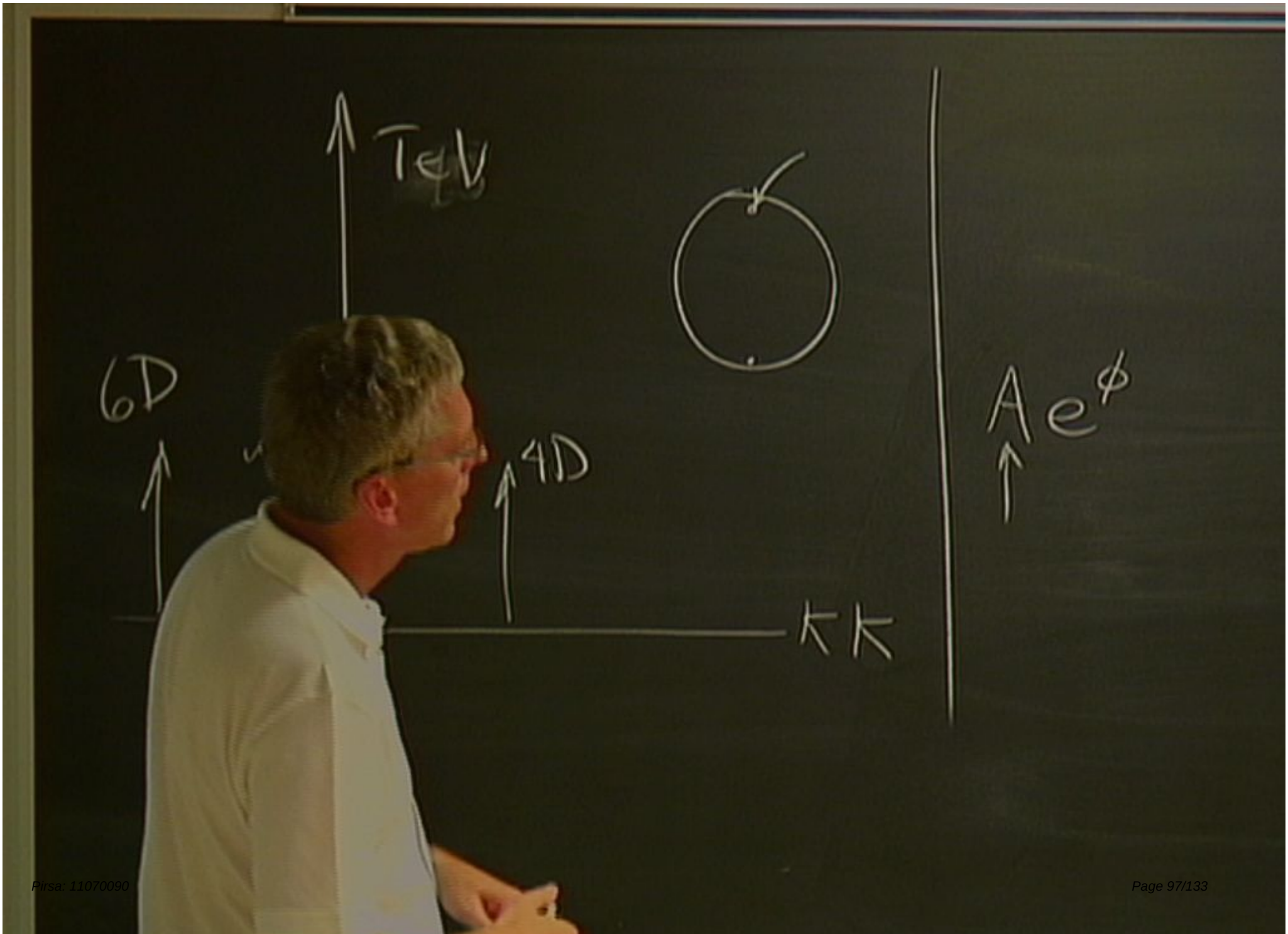
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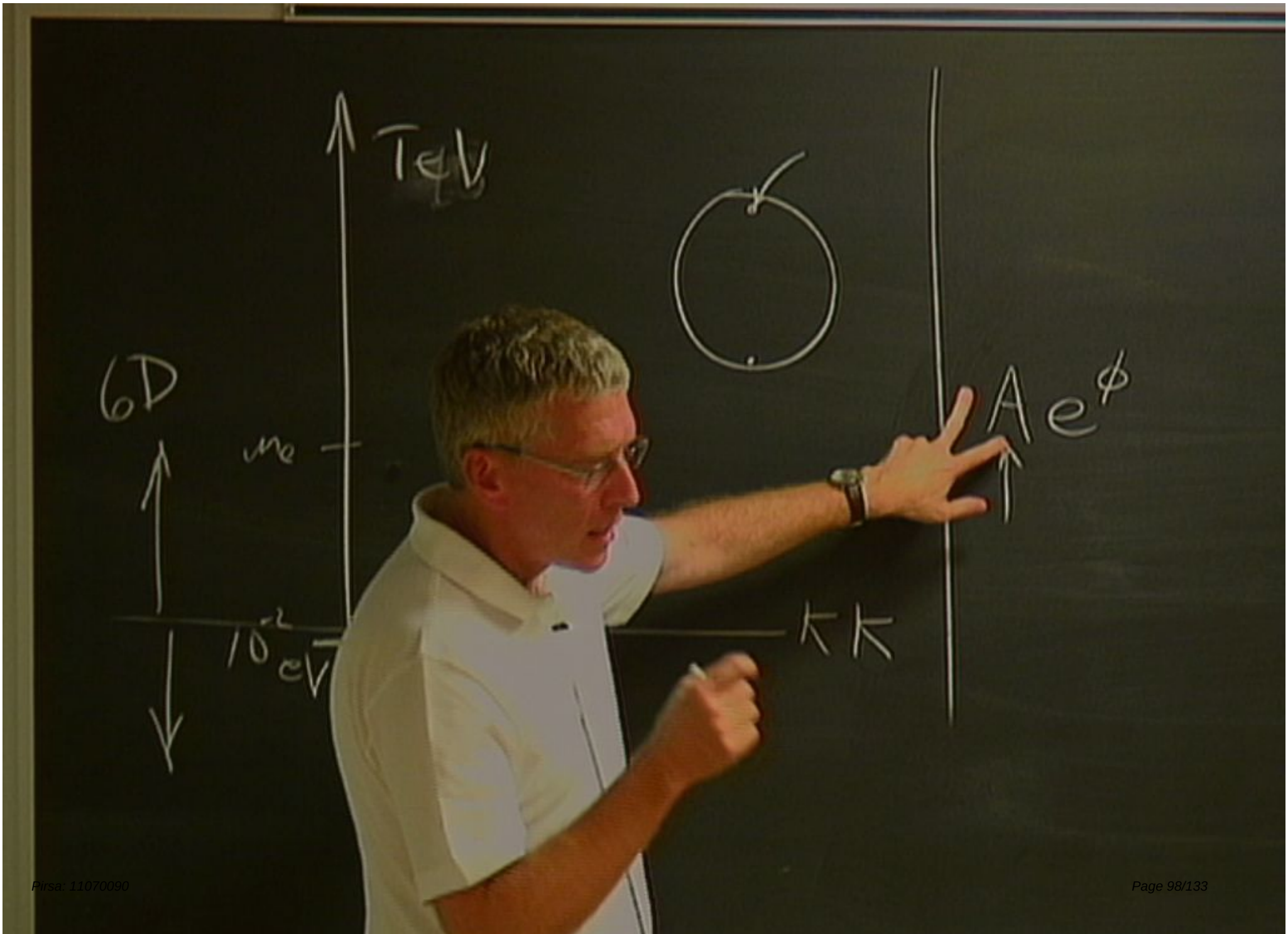
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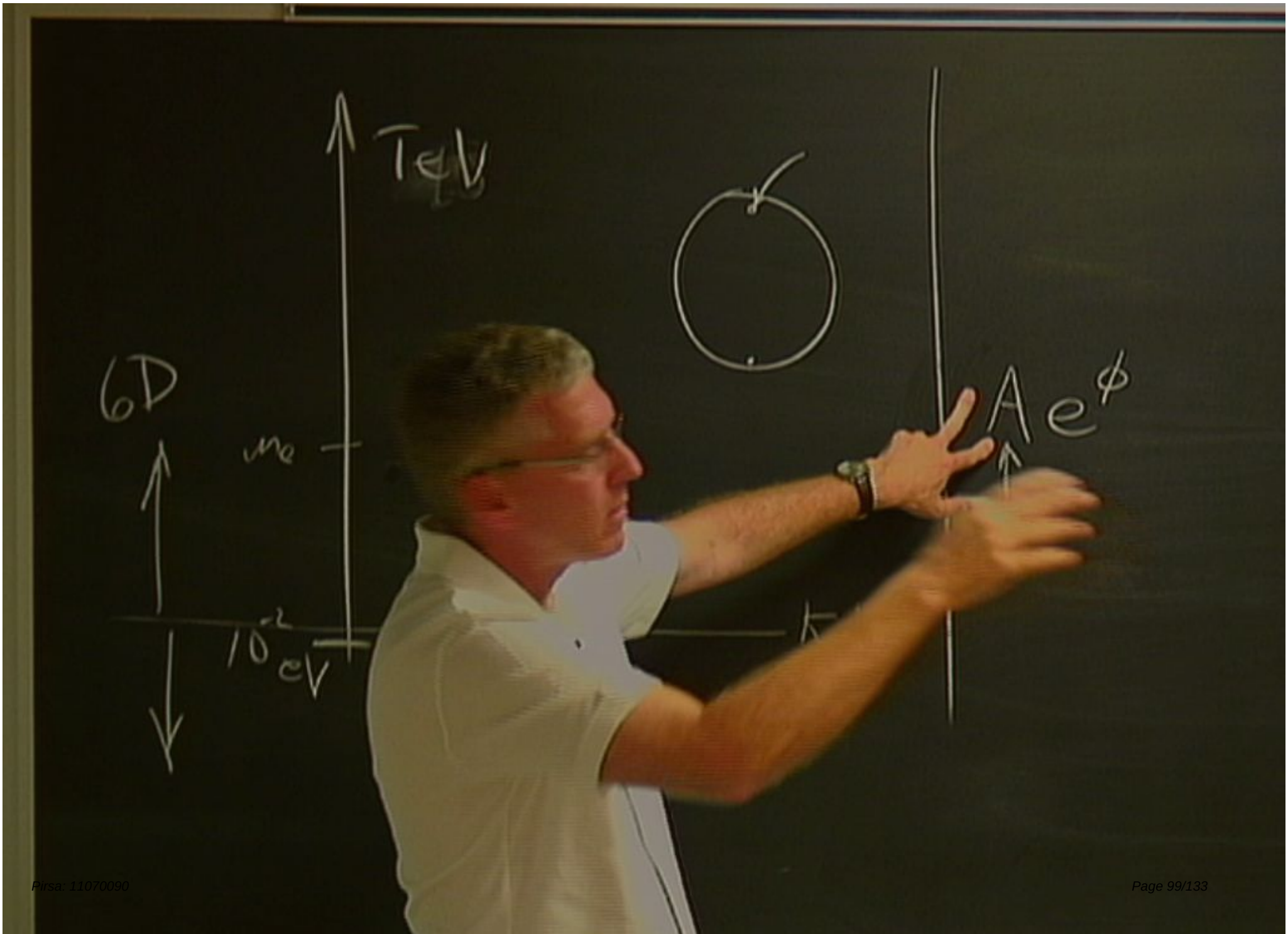
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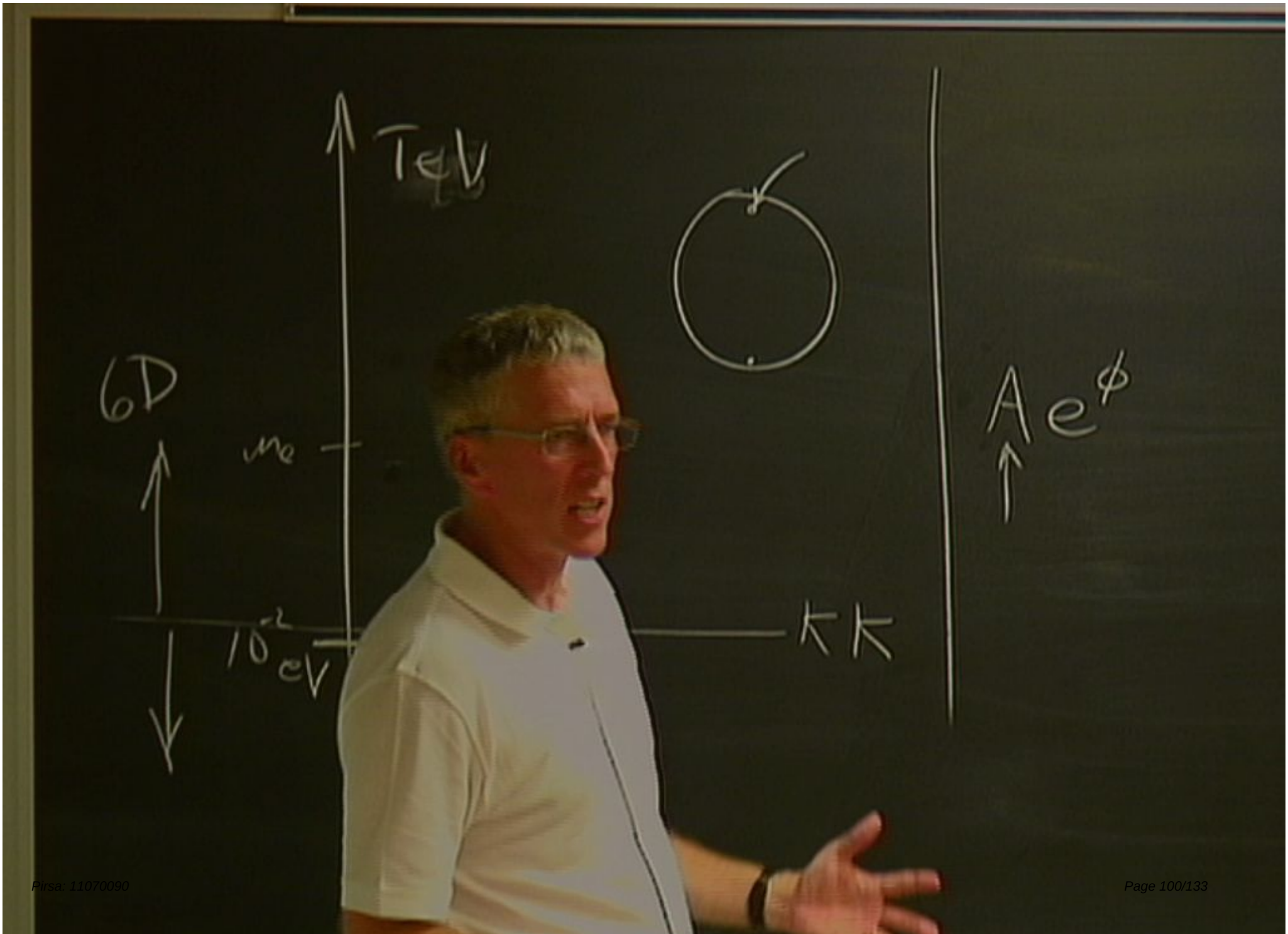
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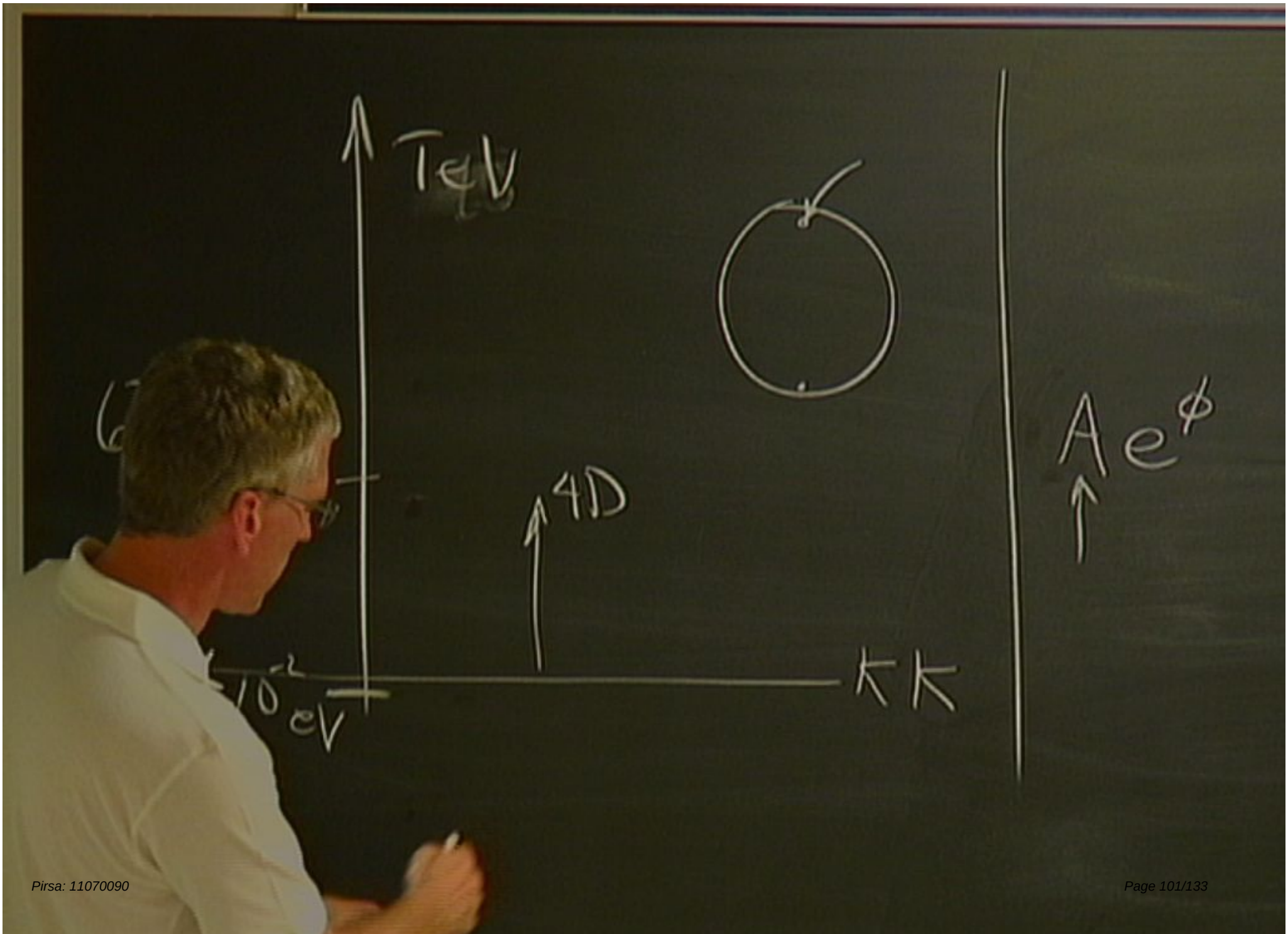
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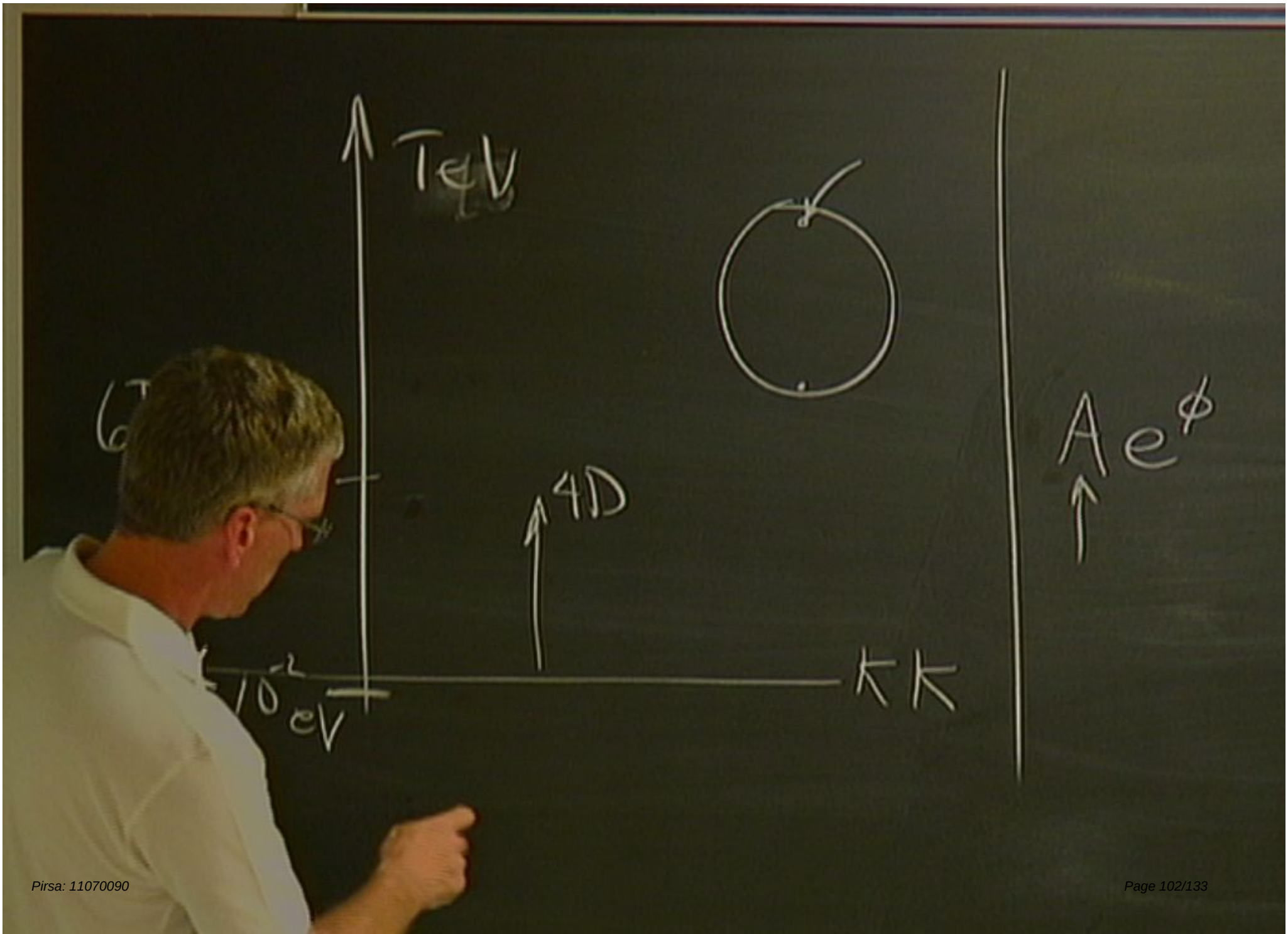
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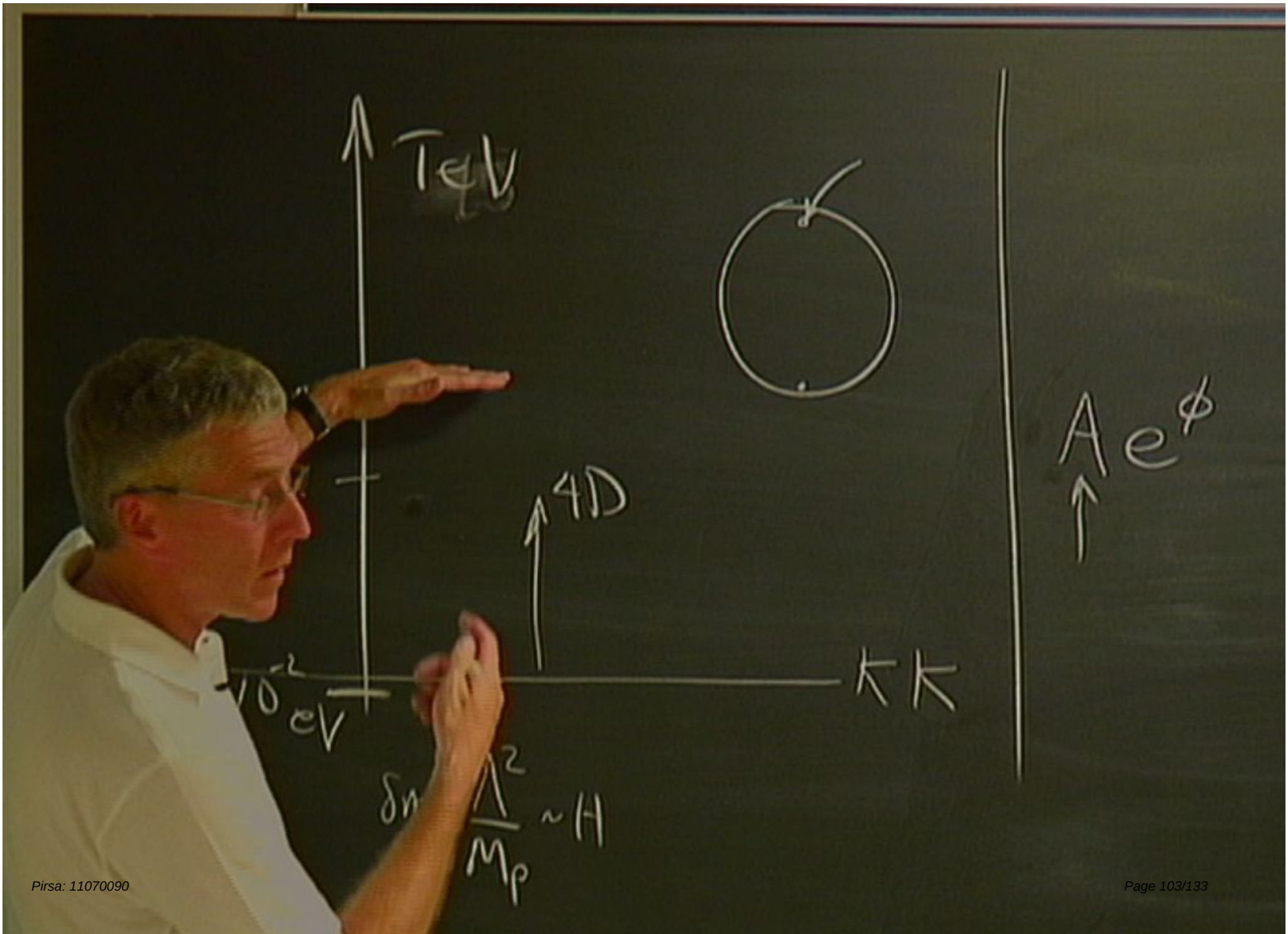
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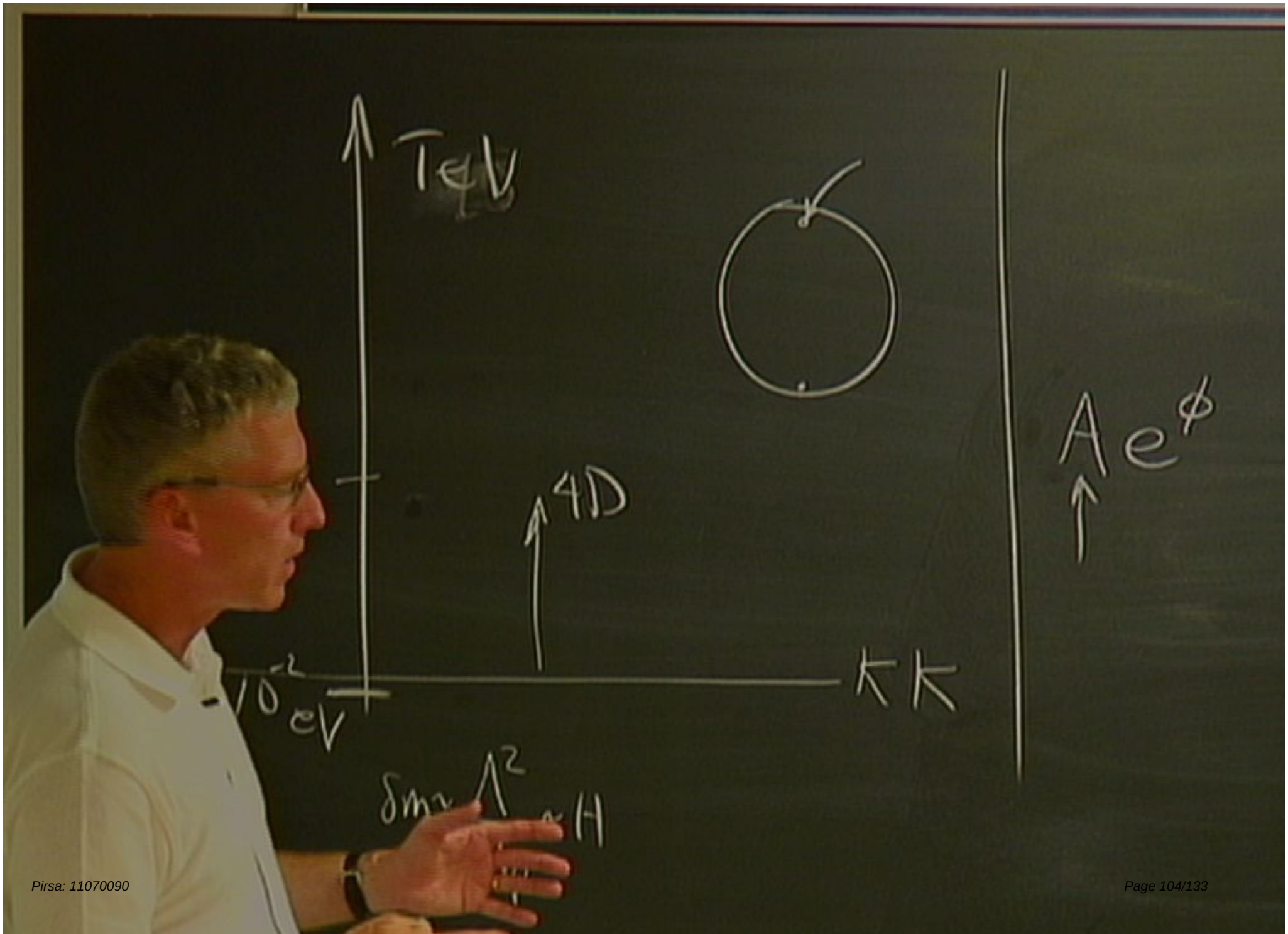
$\delta n \frac{\Lambda^2}{M_p} \sim 1$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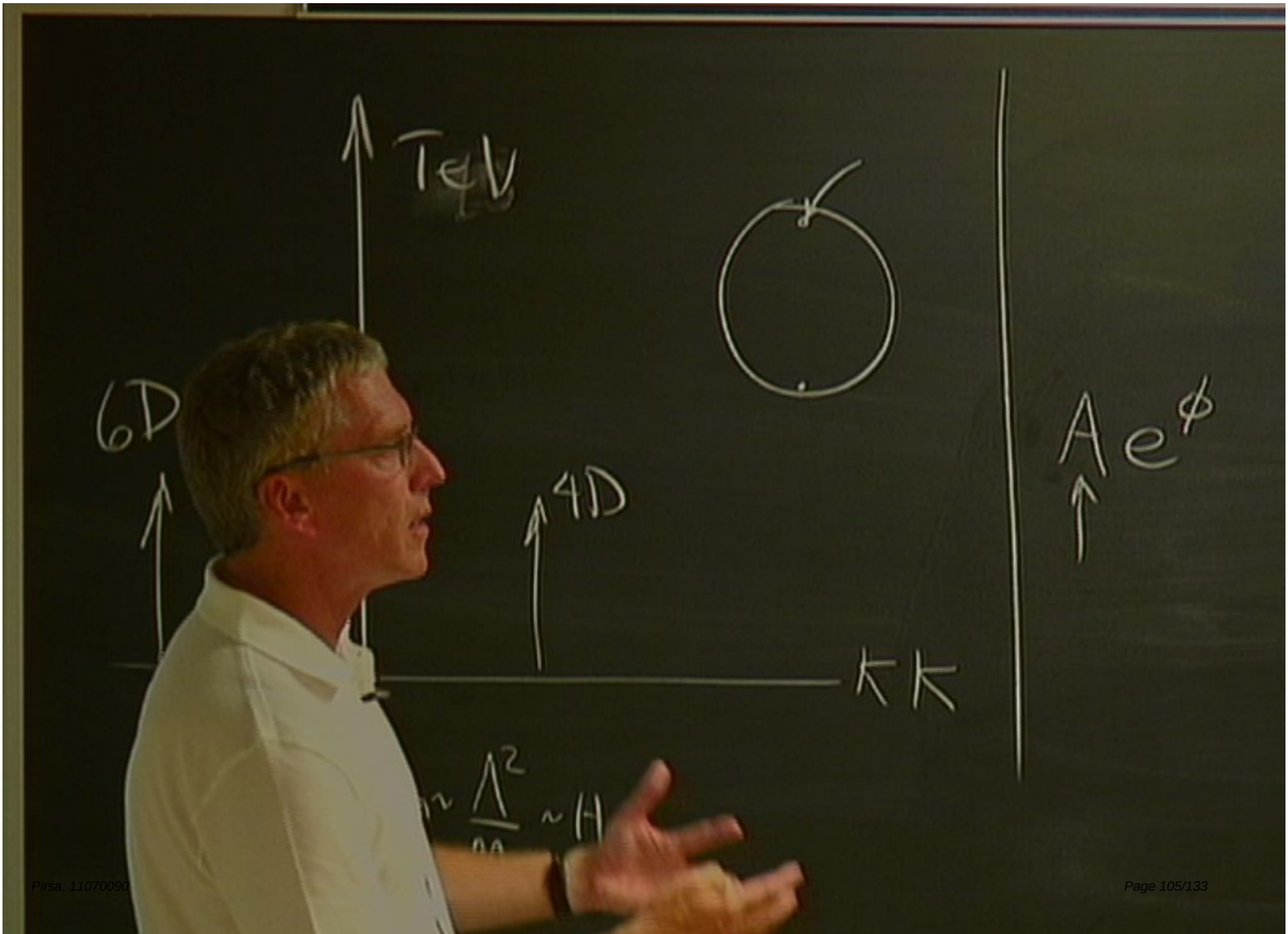
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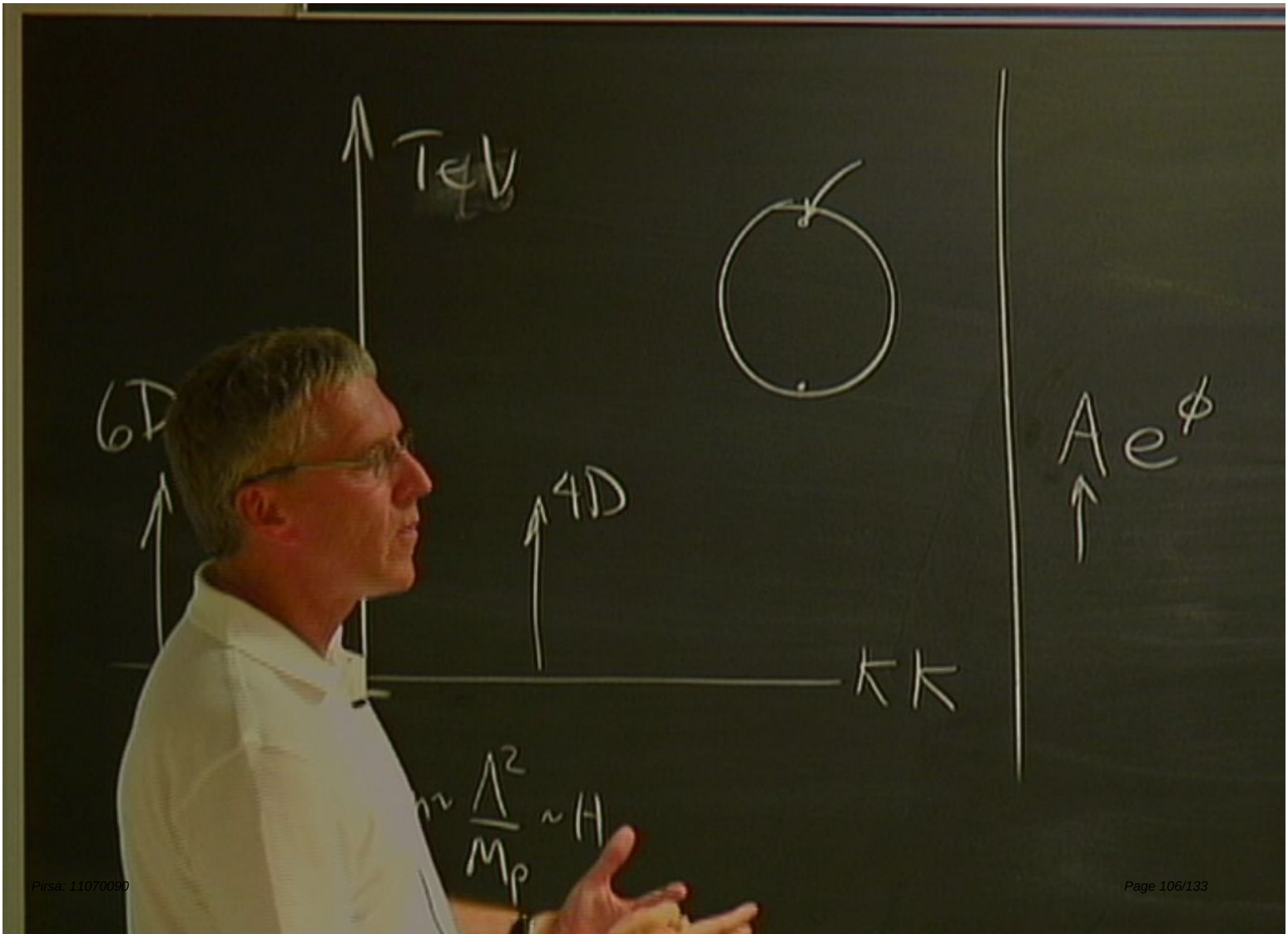
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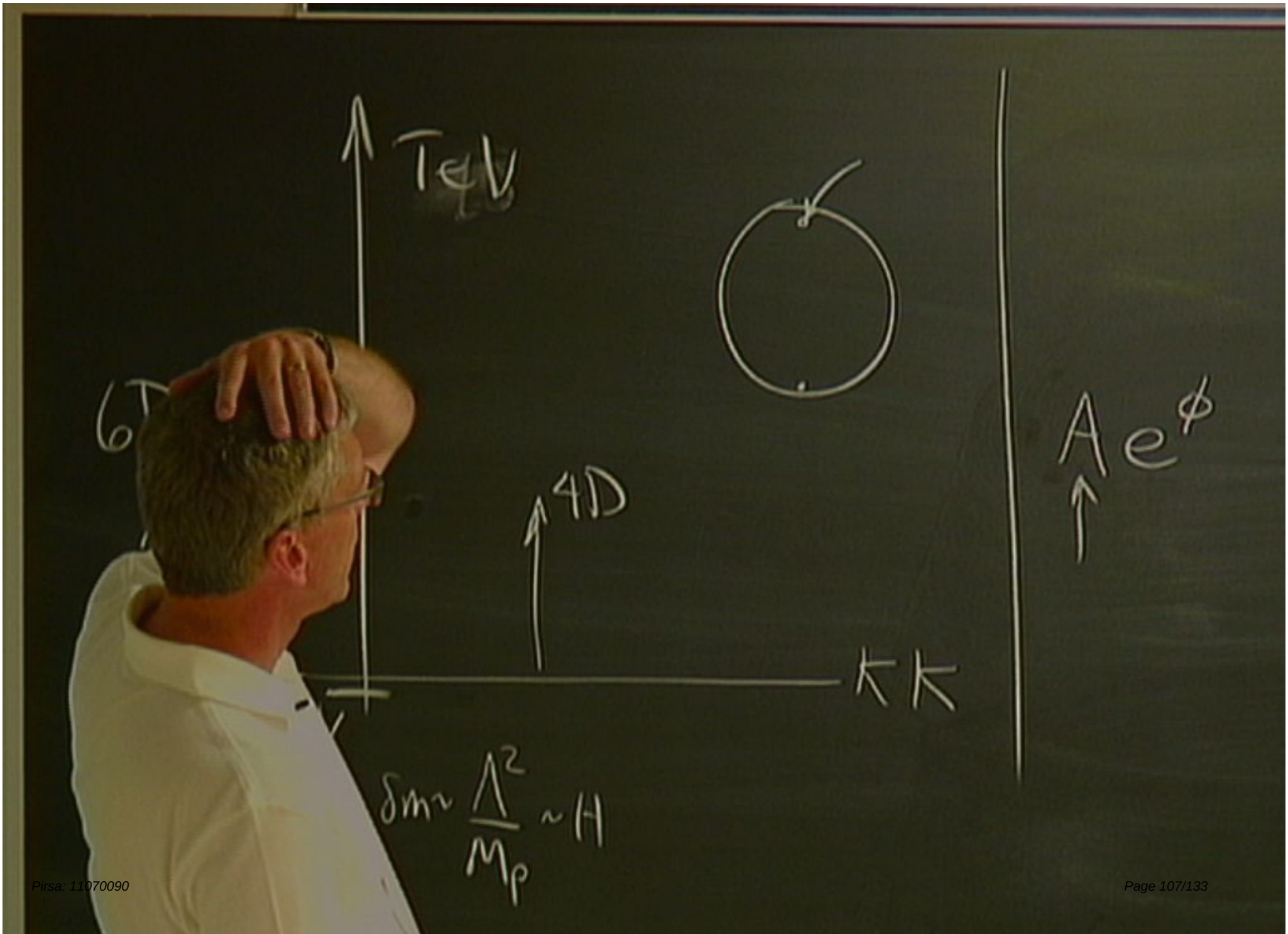
$\sim \frac{\Lambda^2}{M_p} \sim 1$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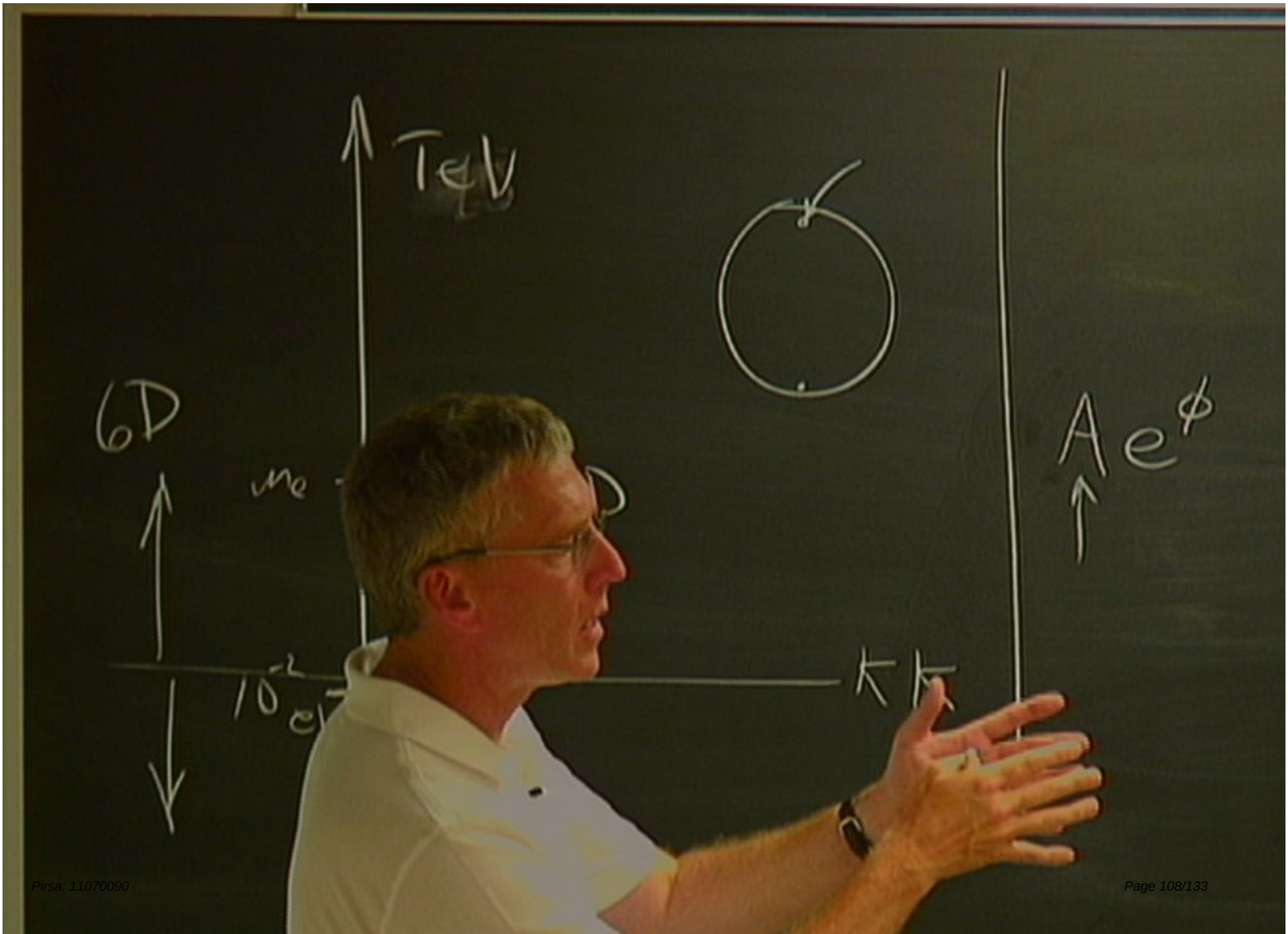
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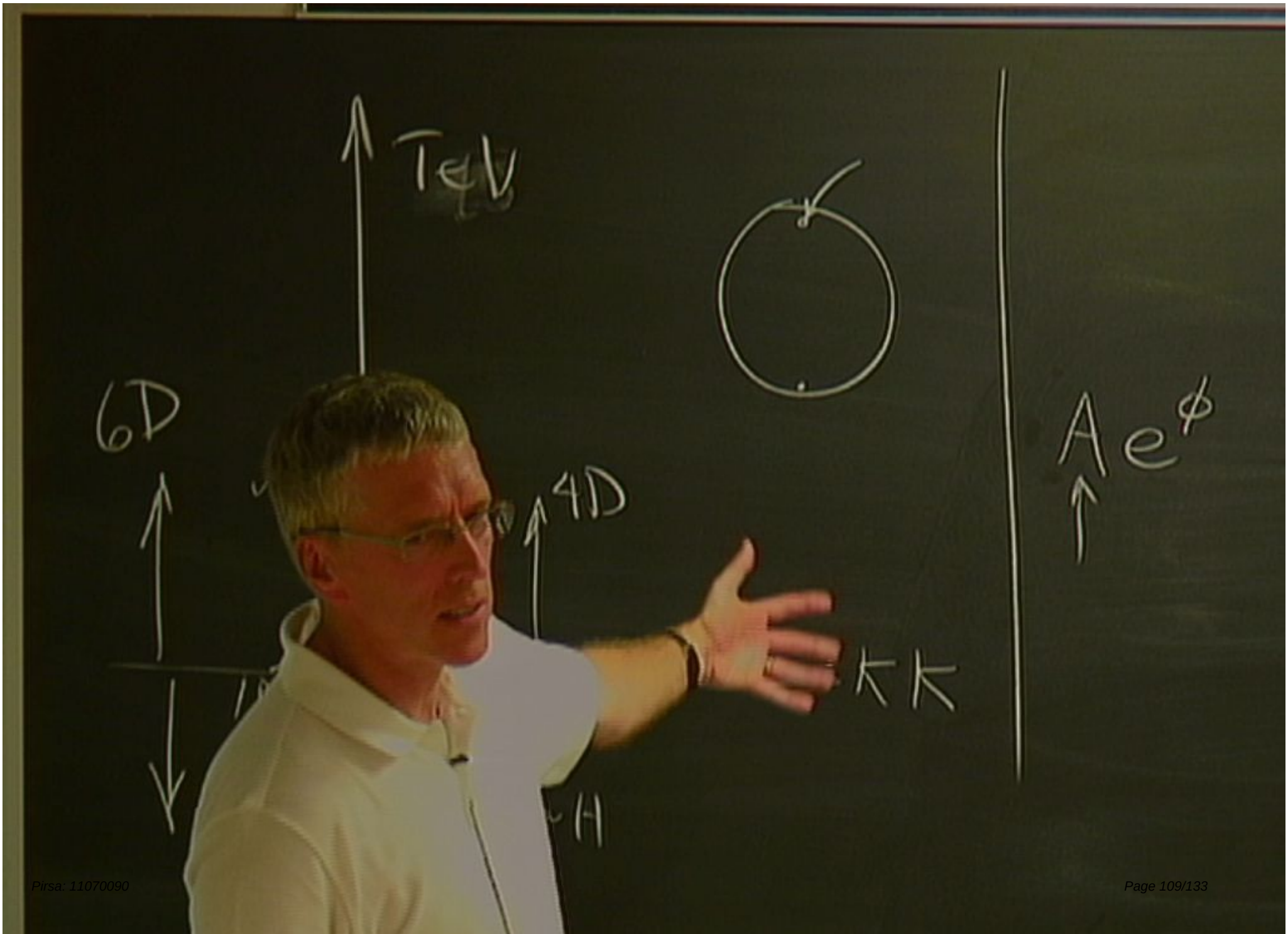
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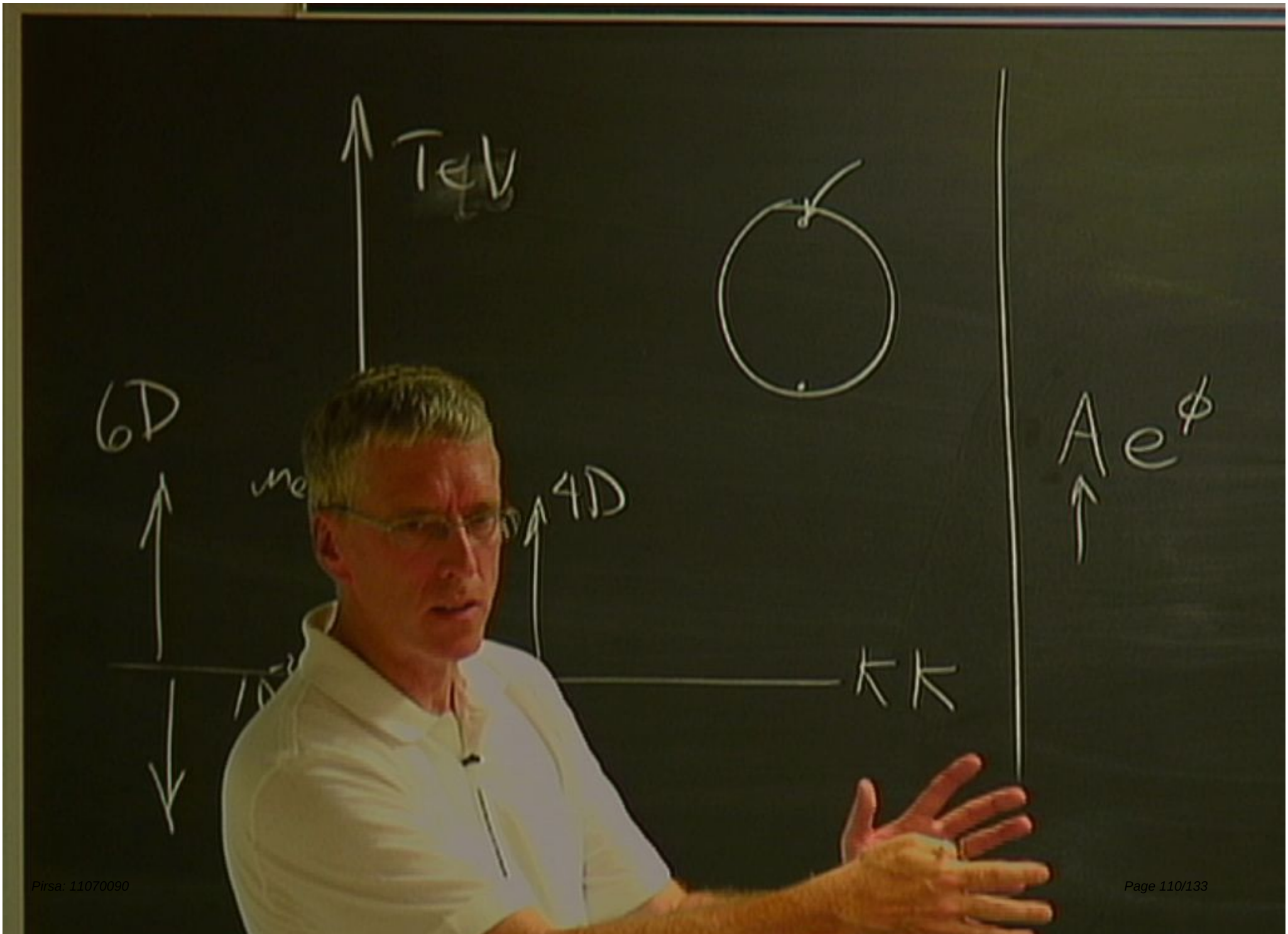
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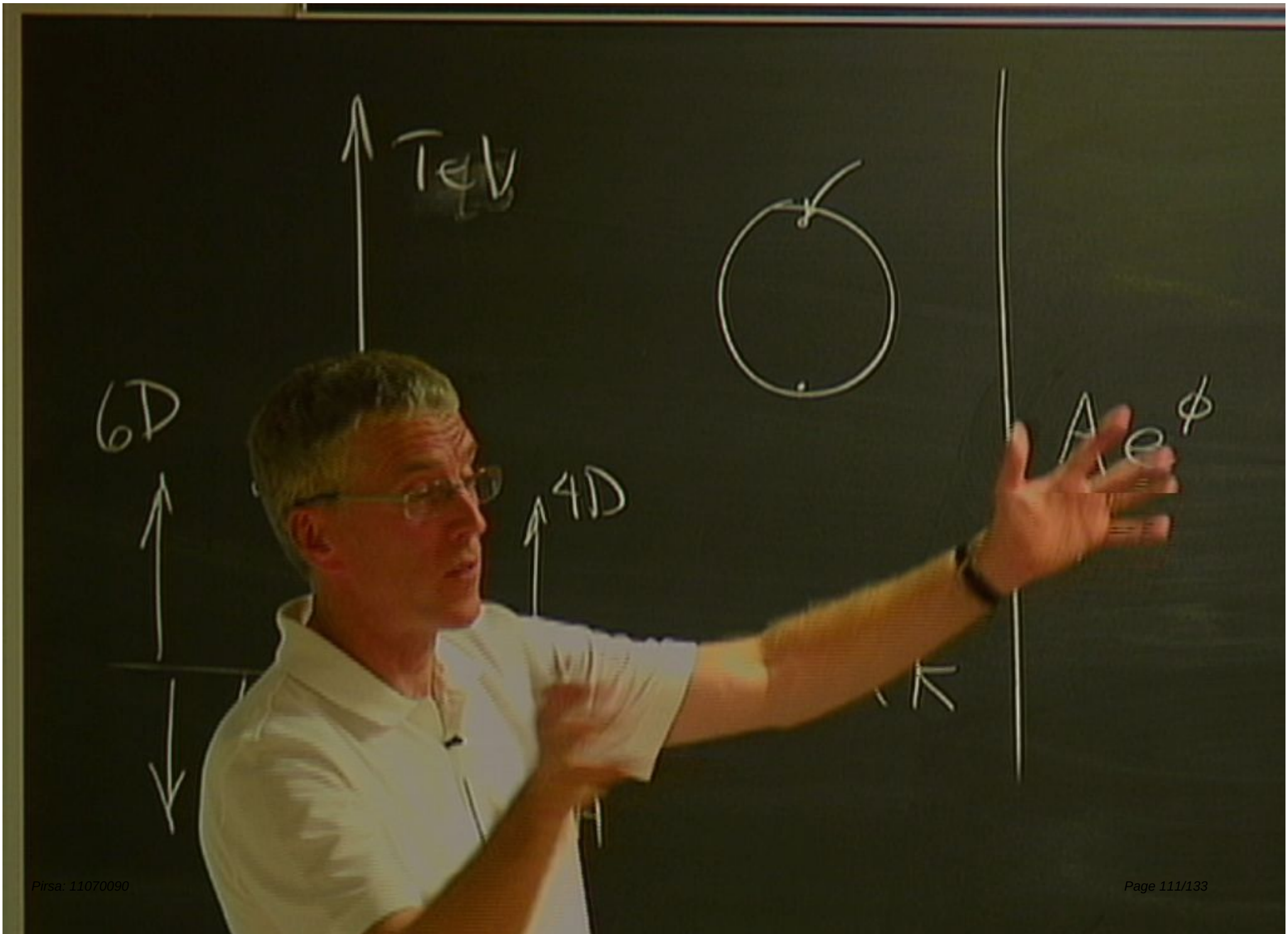
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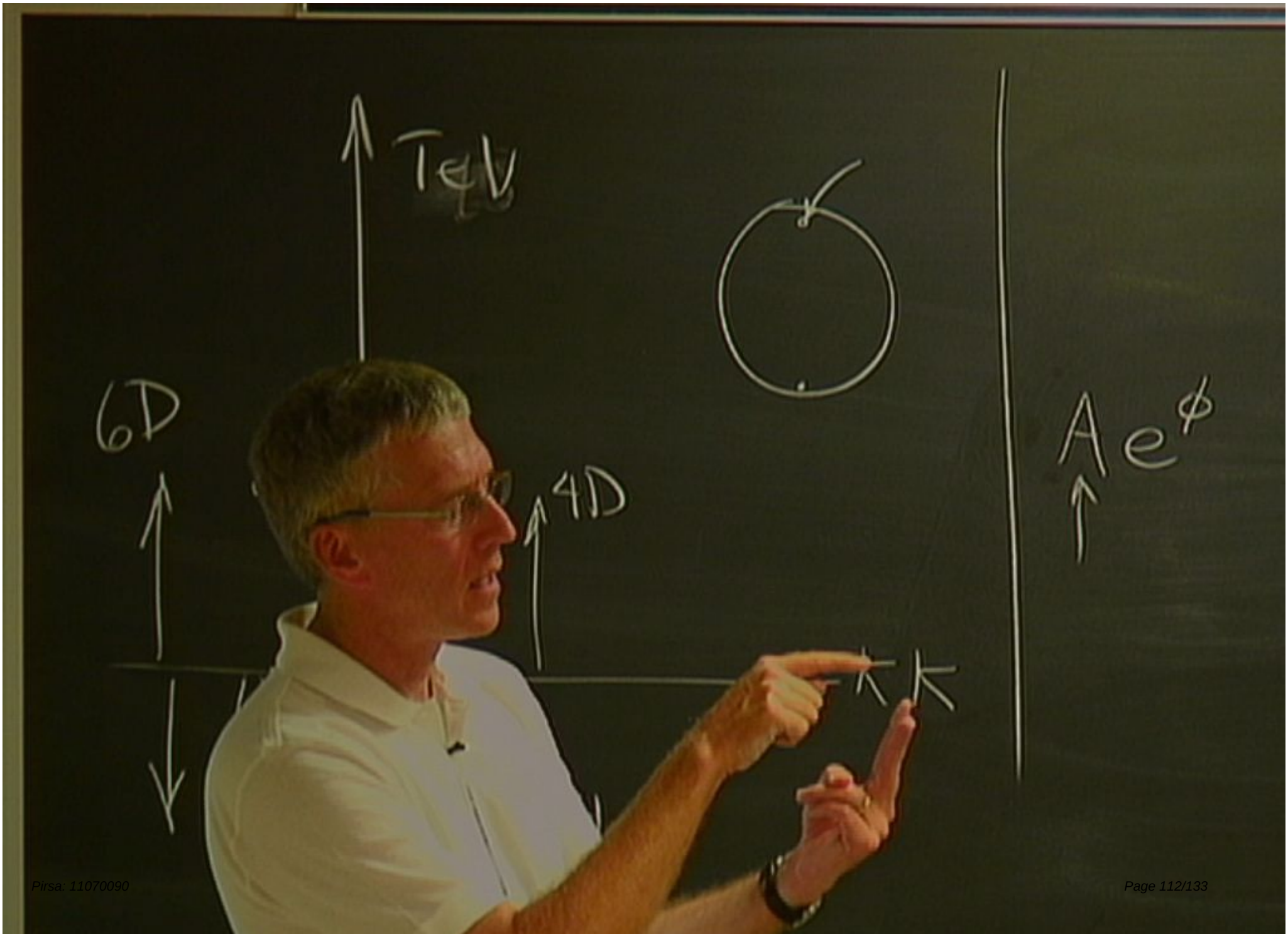
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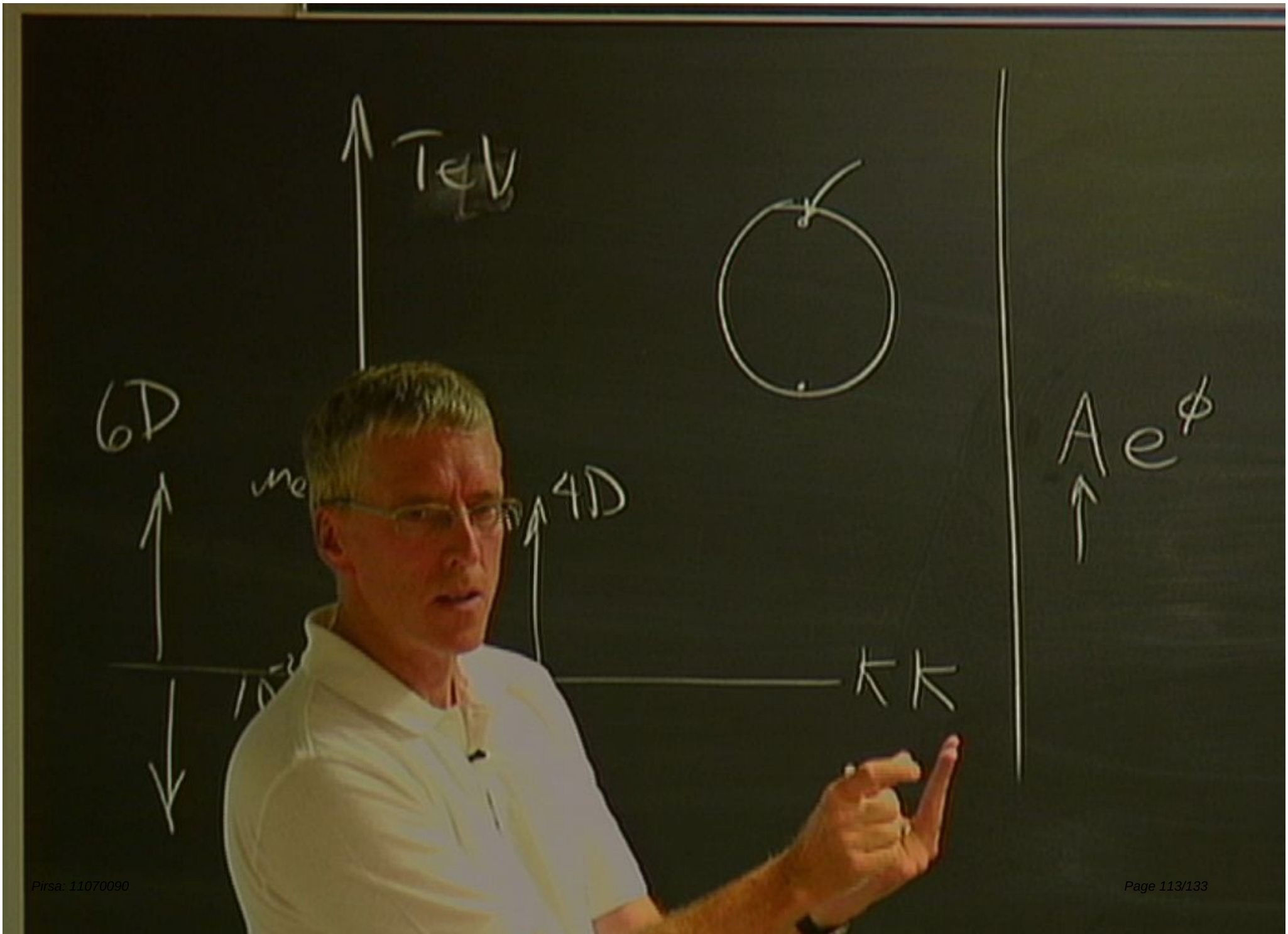
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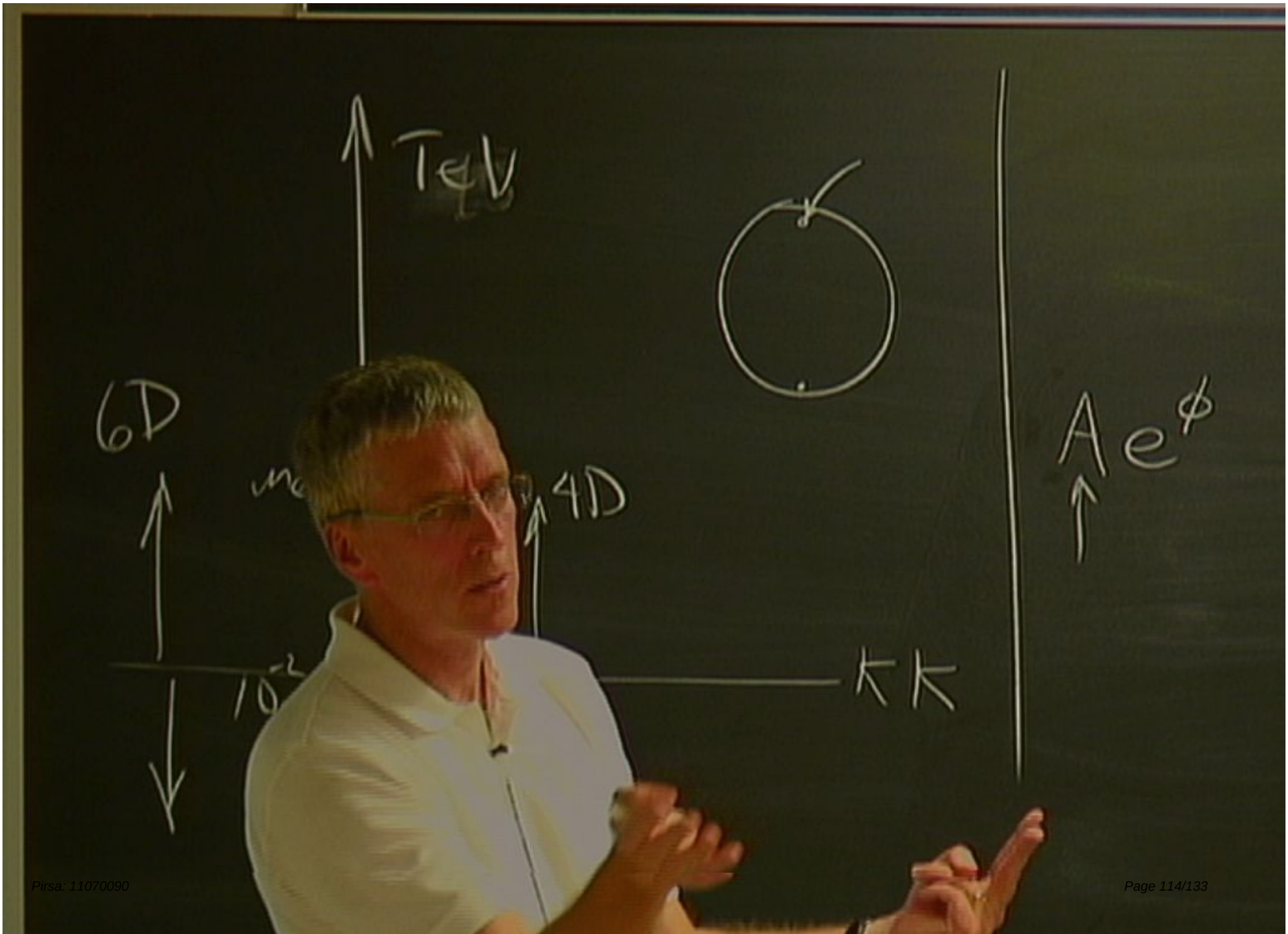
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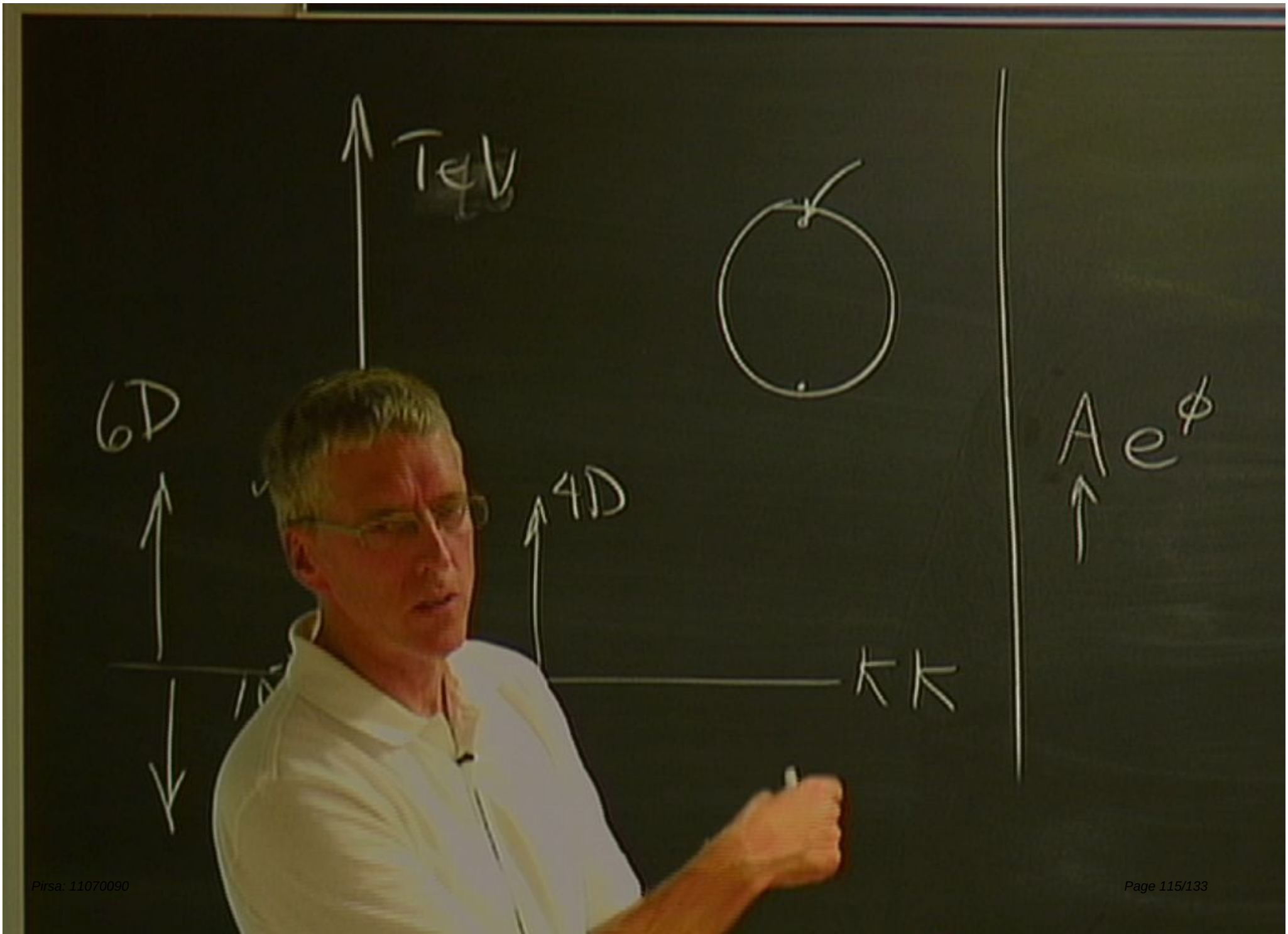
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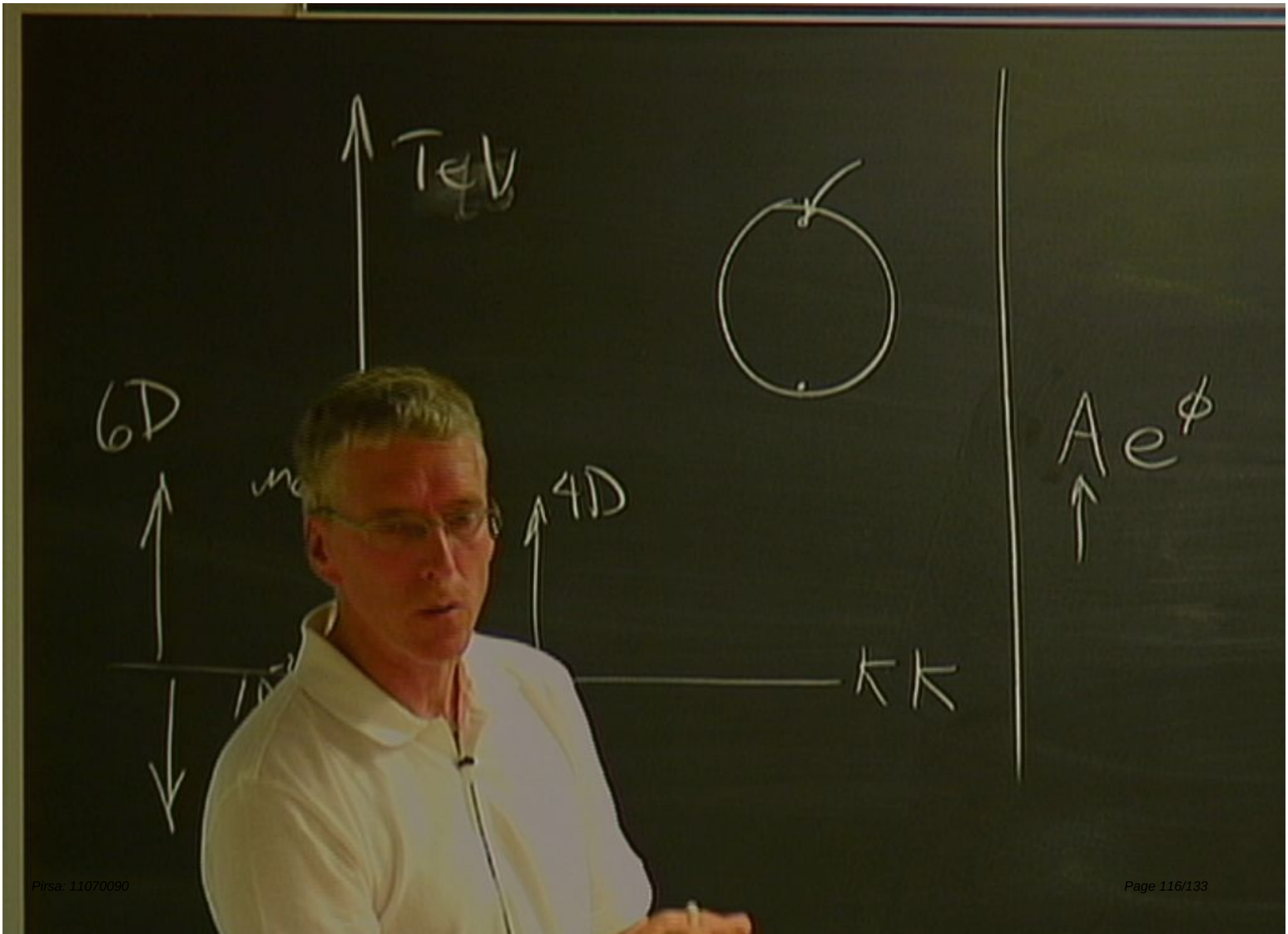
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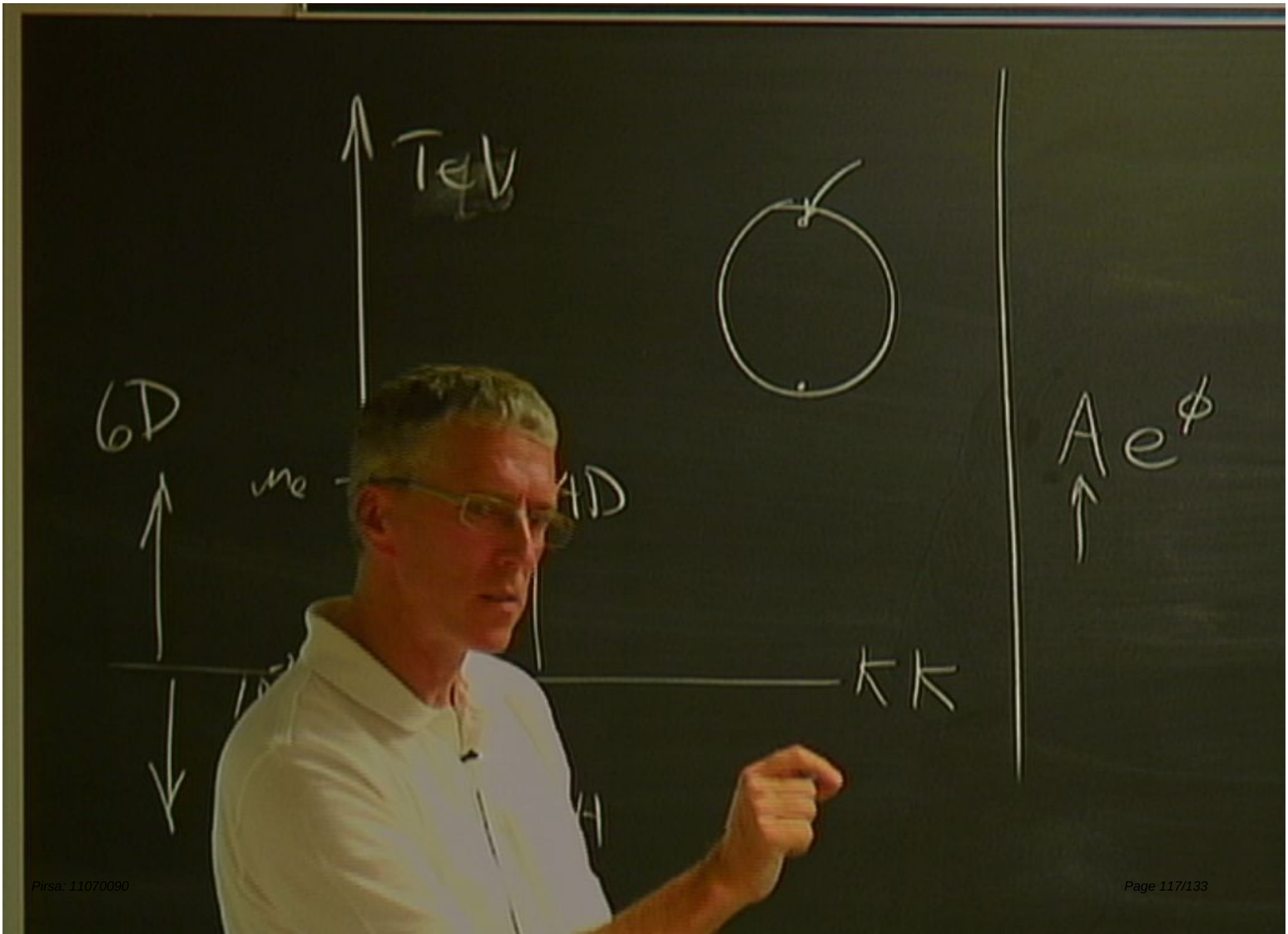
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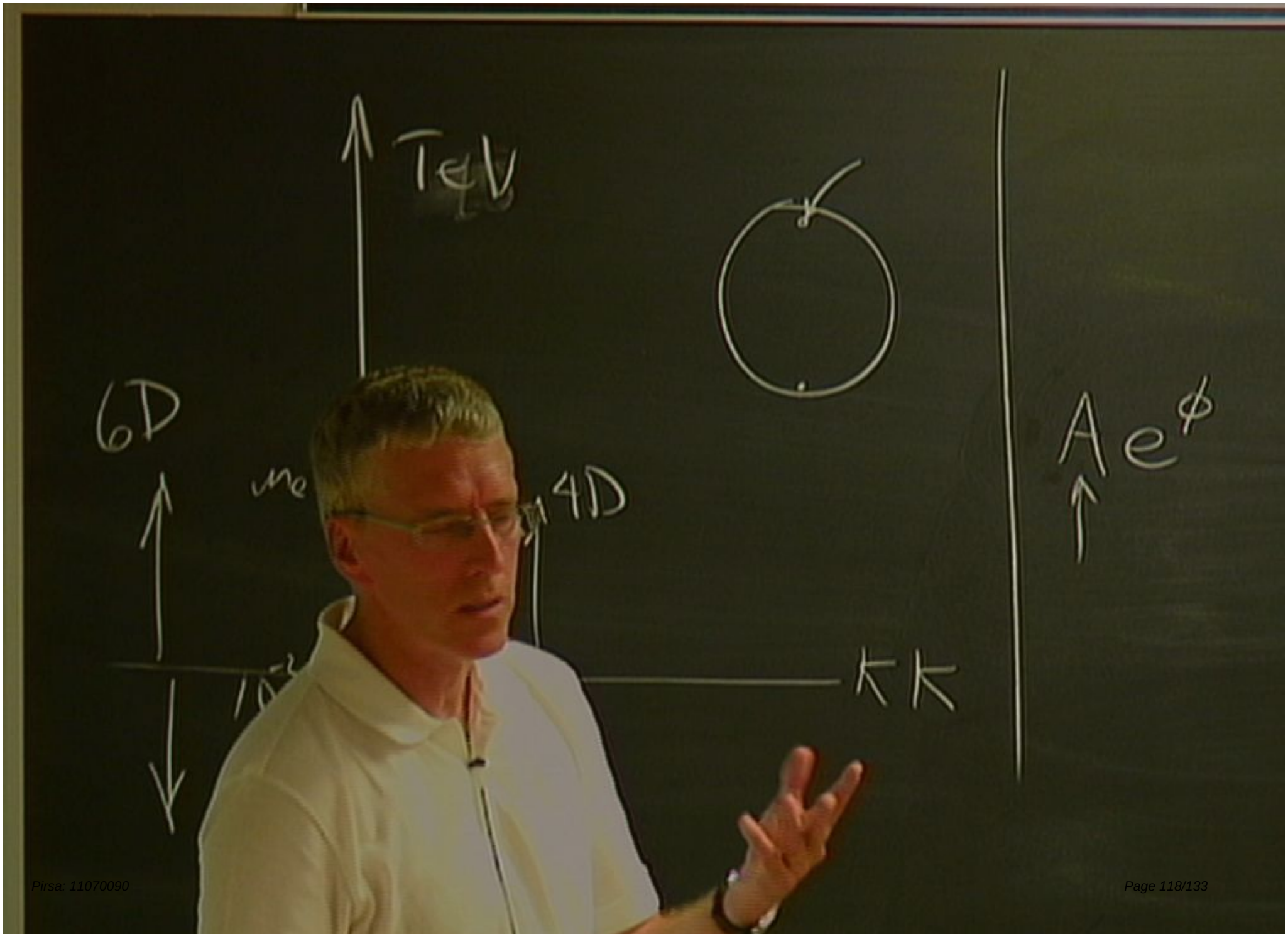
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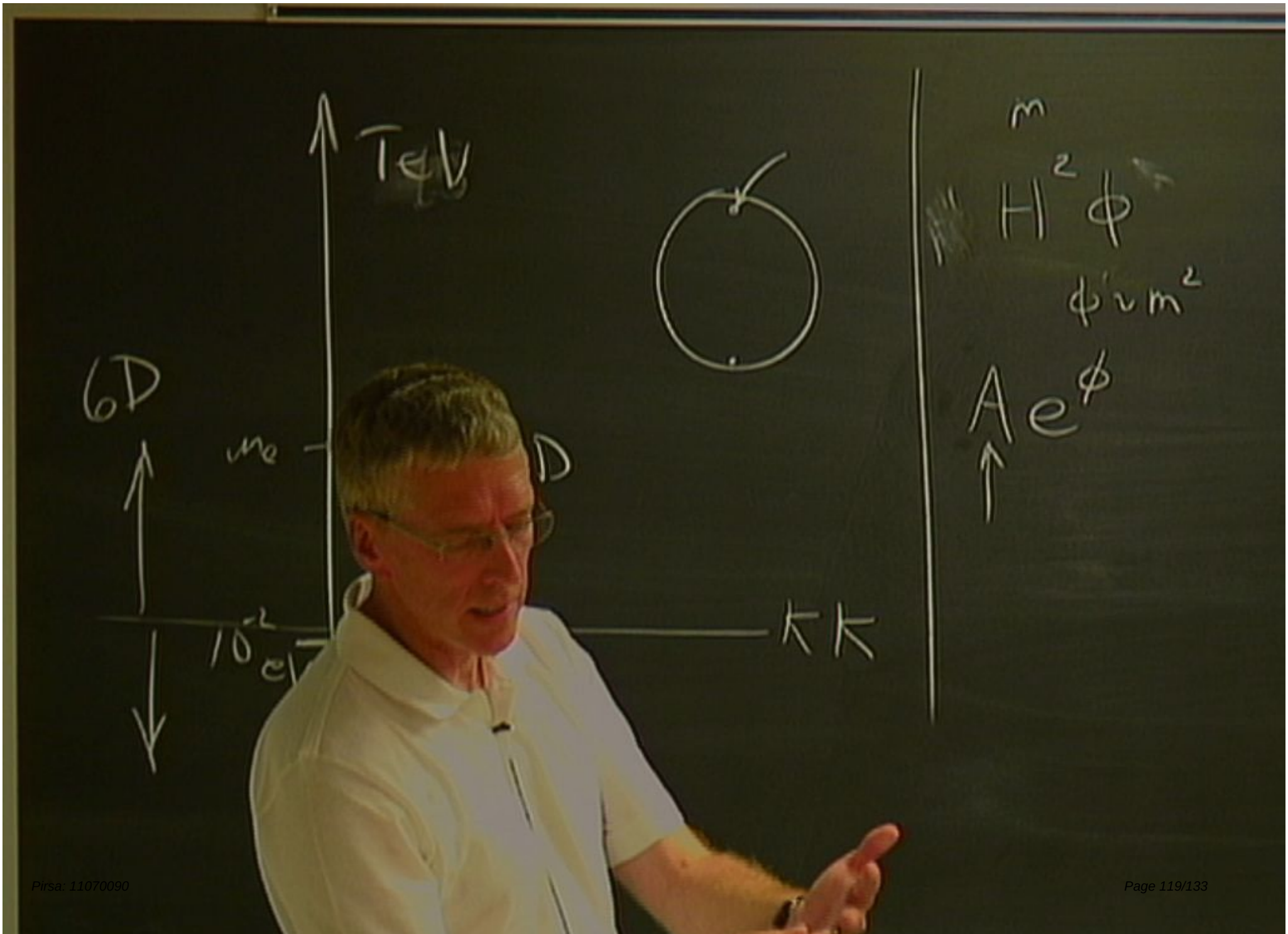
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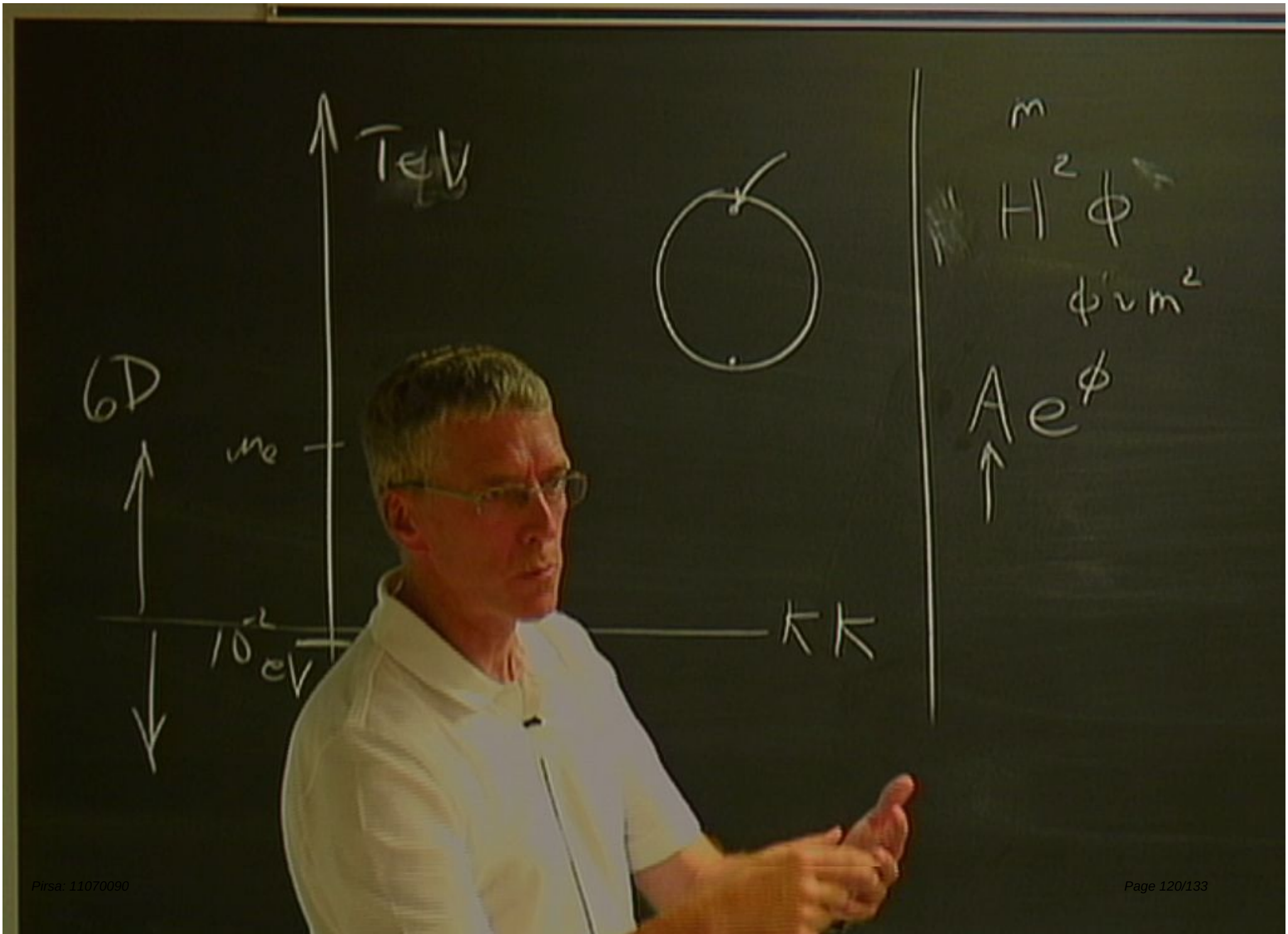
$H^2\phi$
$\phi \sim m^2$
Ae^{ϕ}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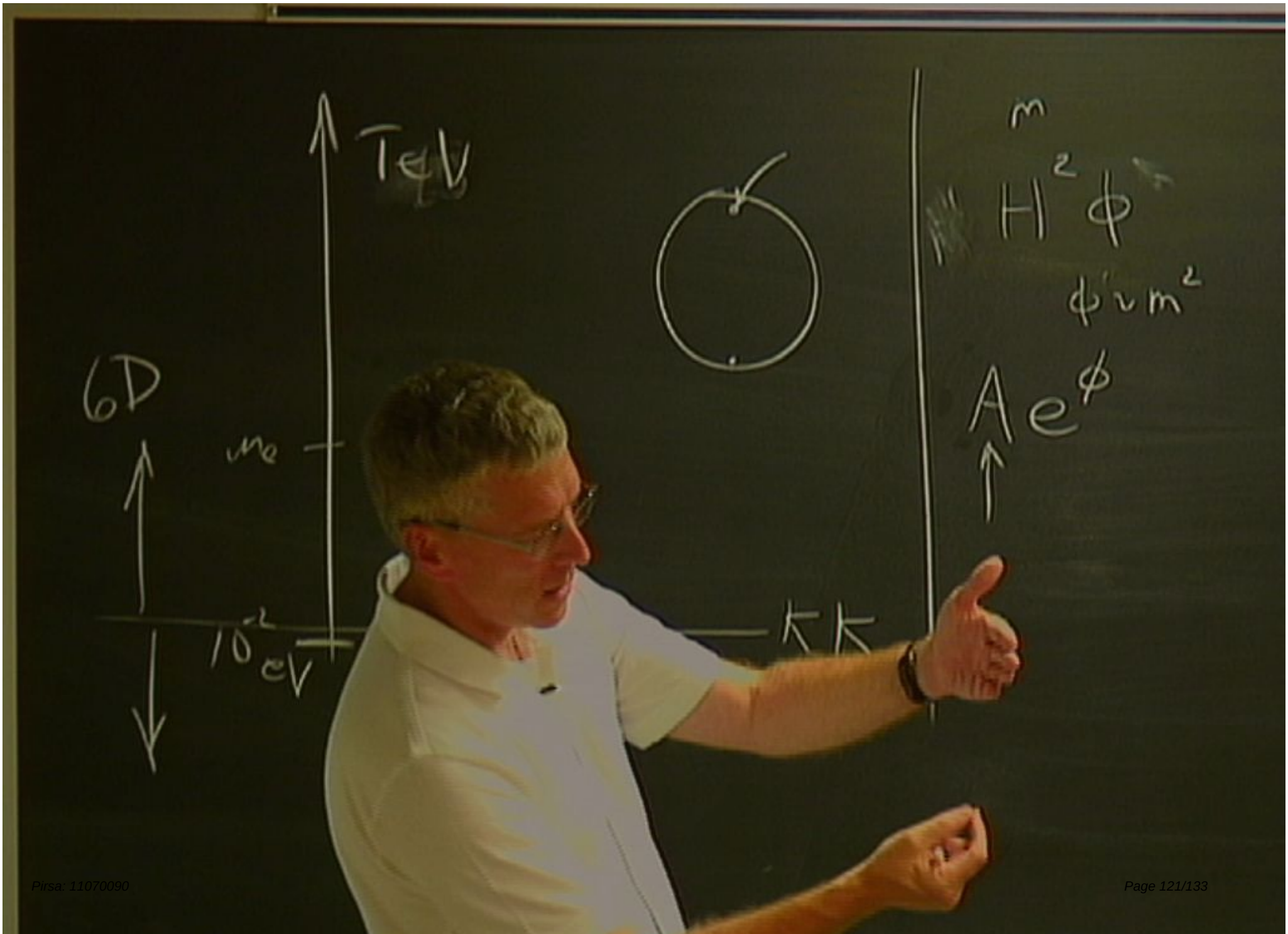
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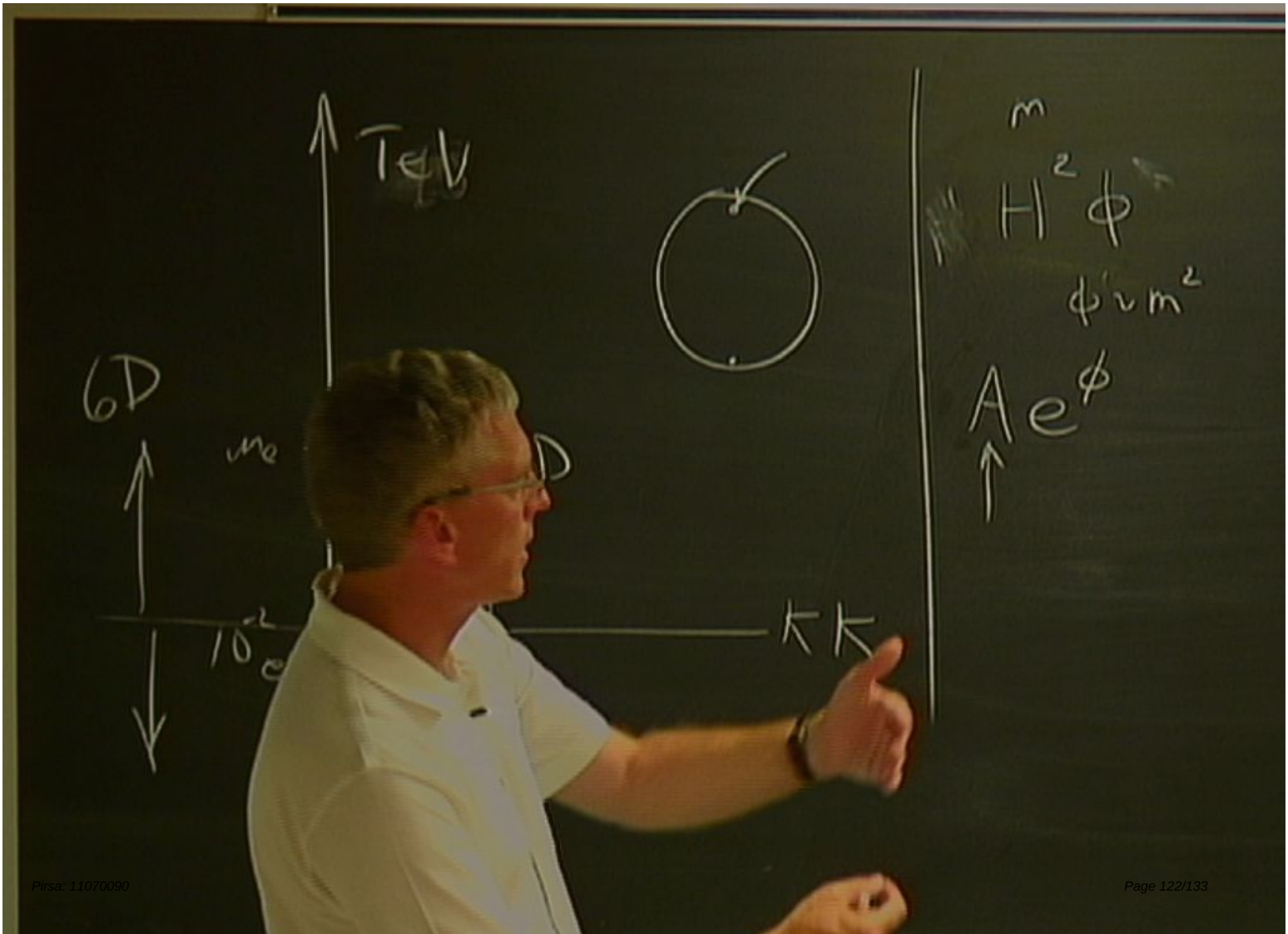
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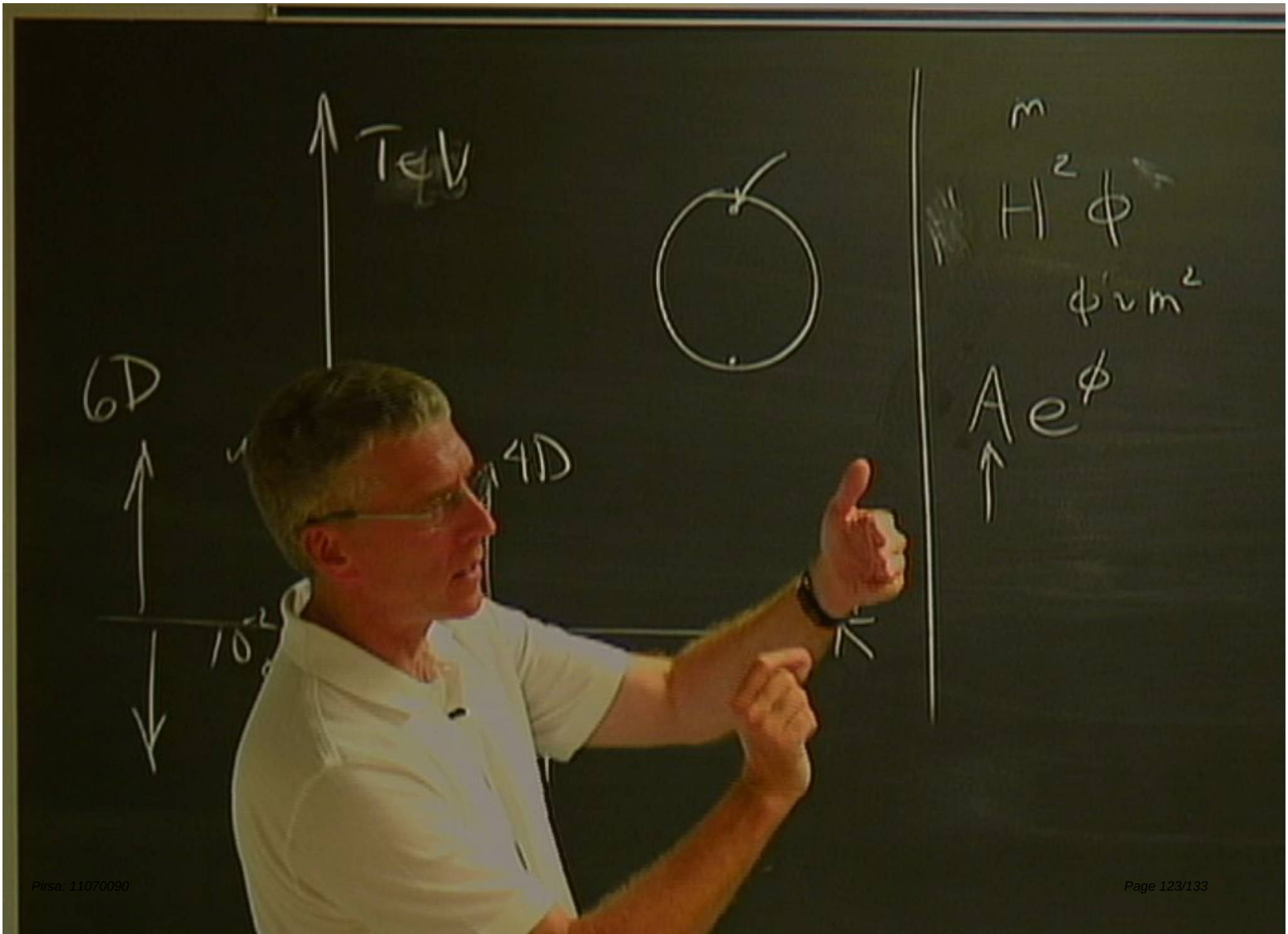
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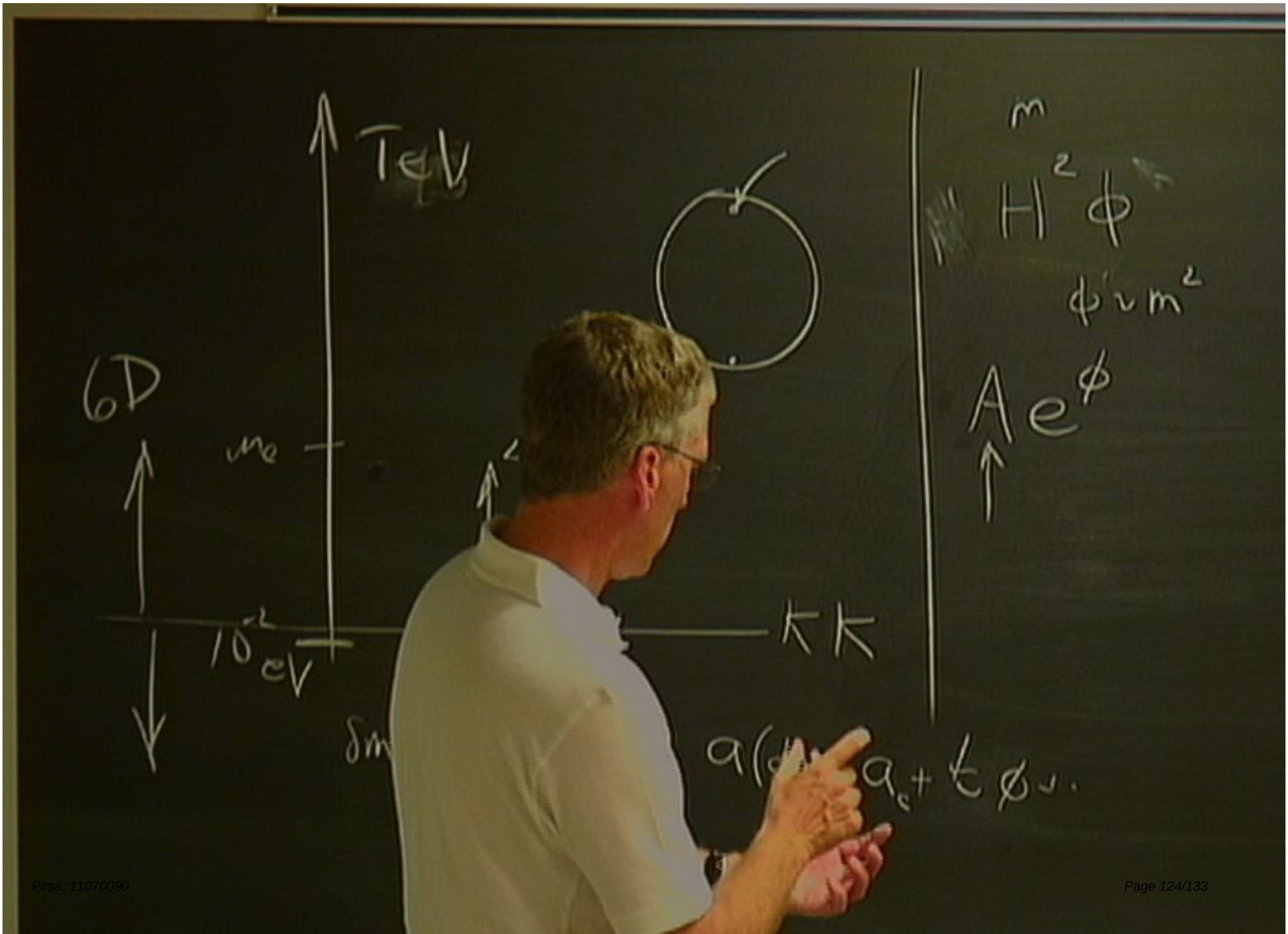
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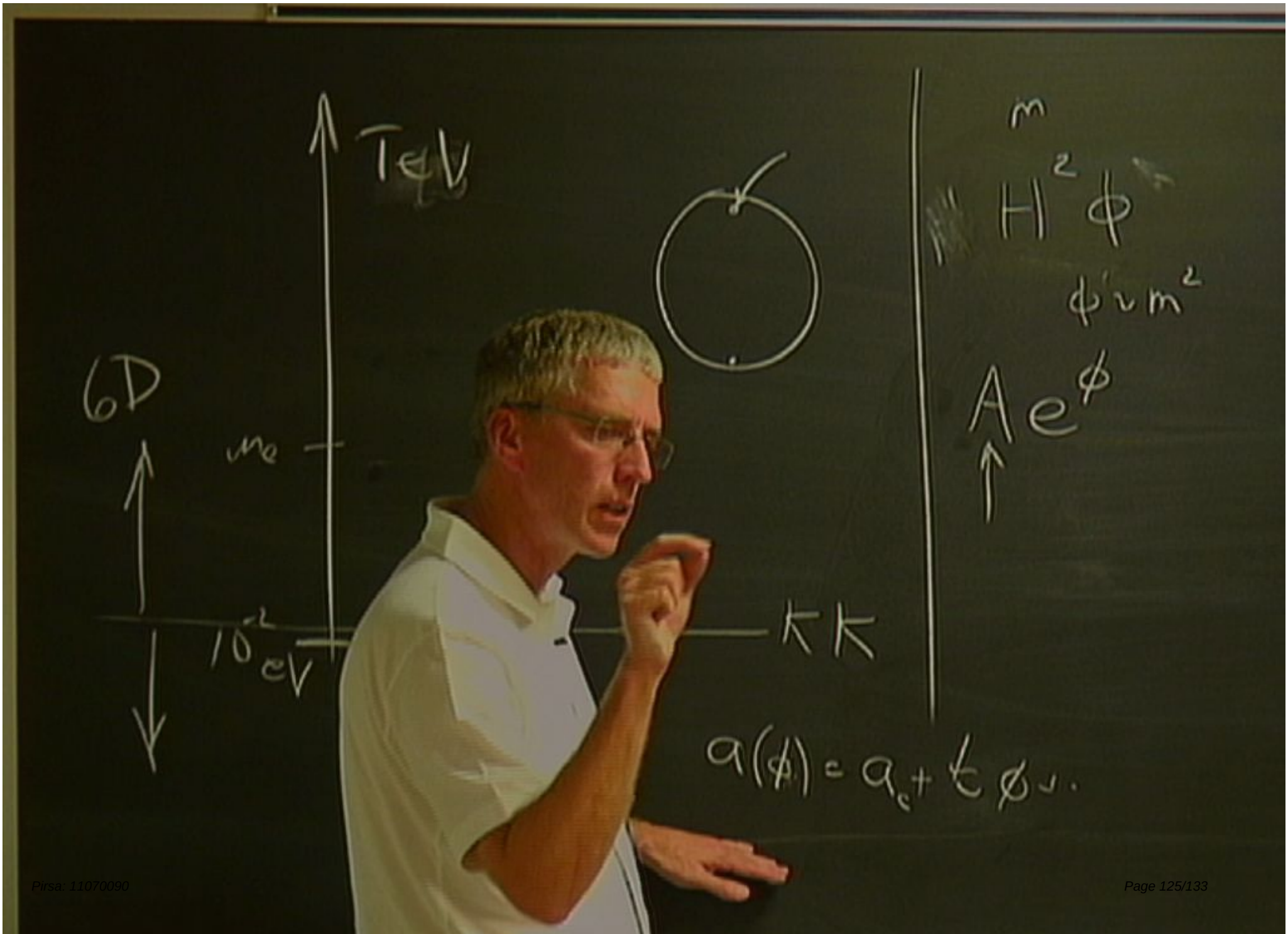
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H^2 φ
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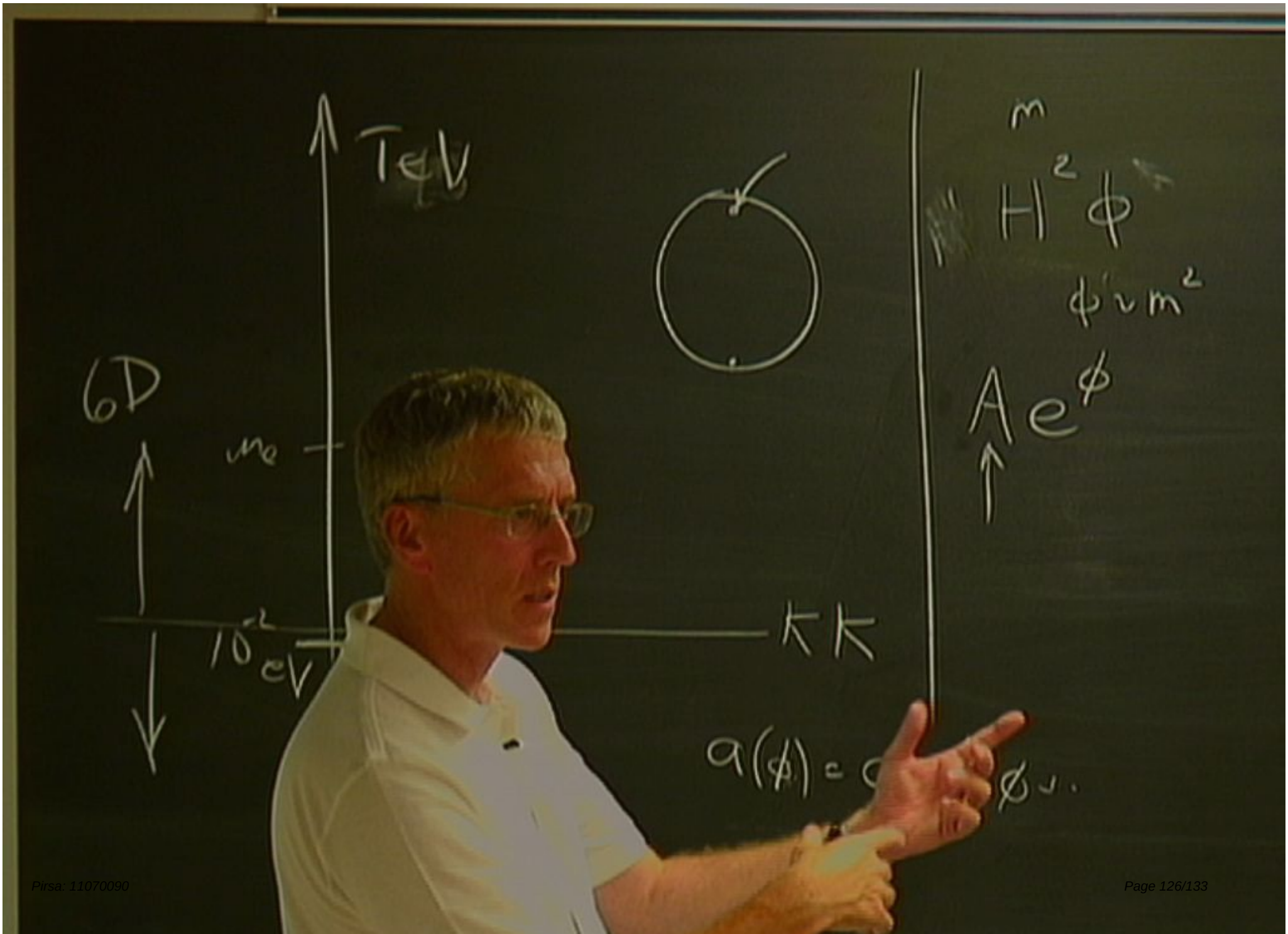
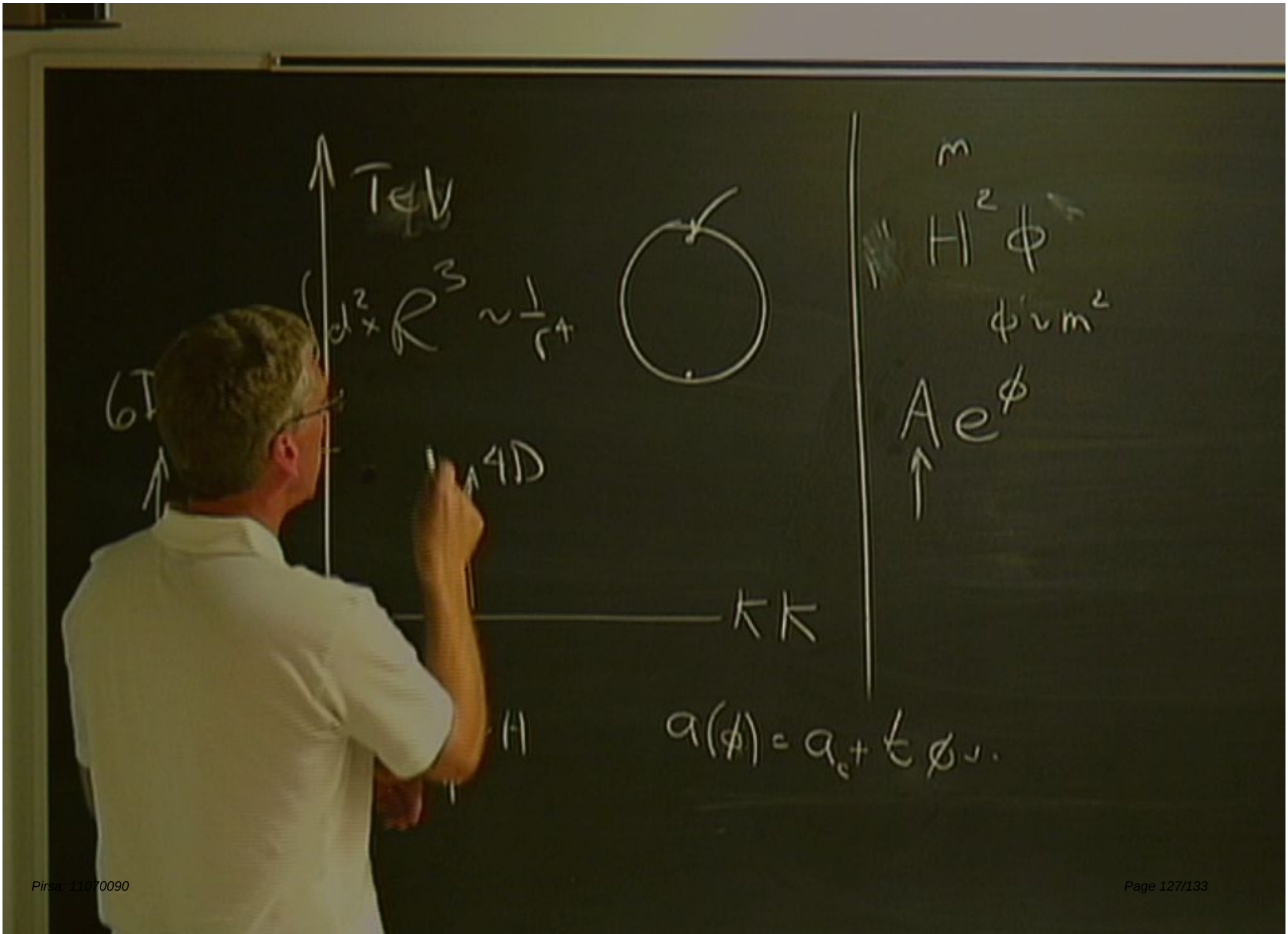
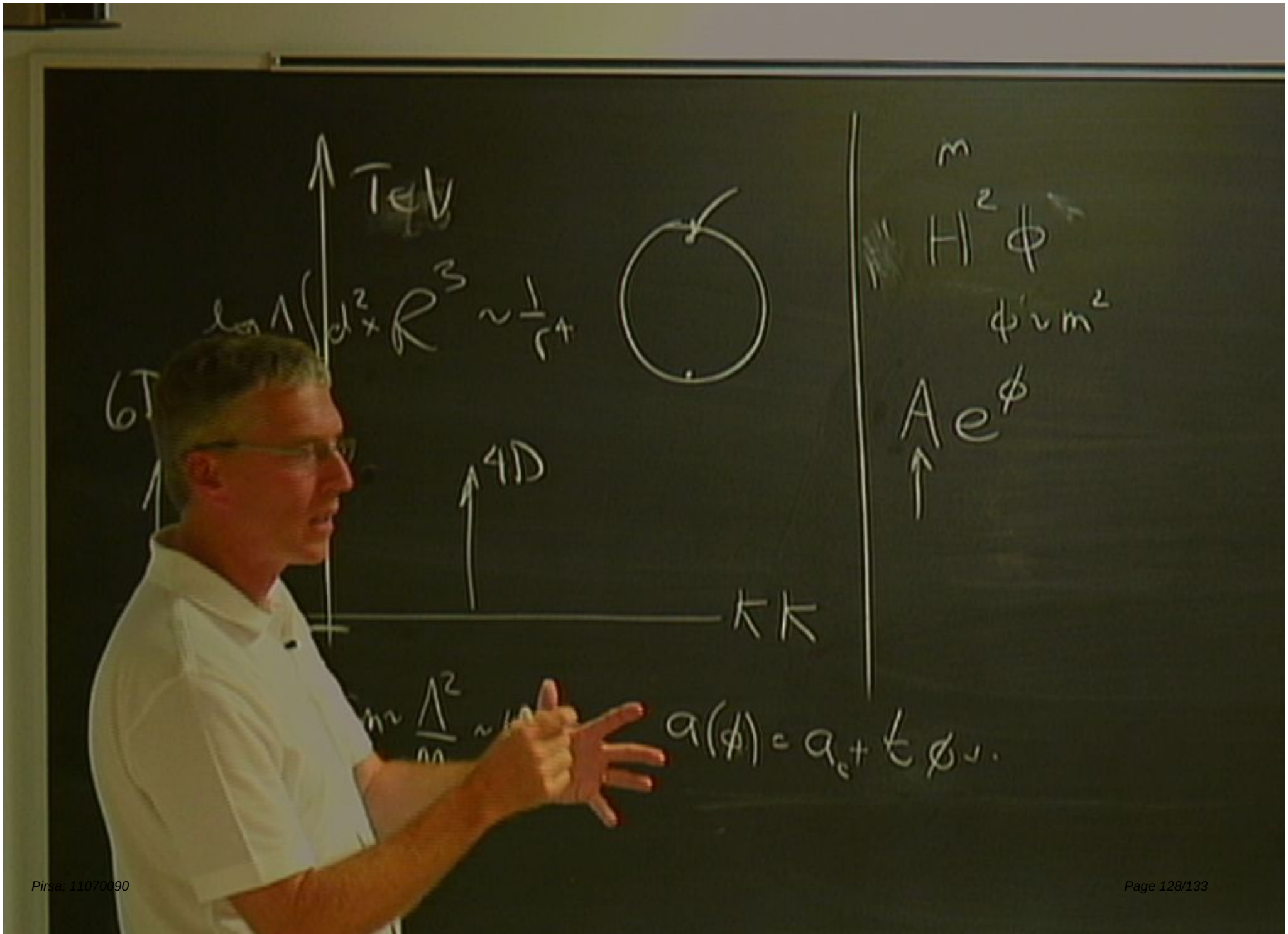
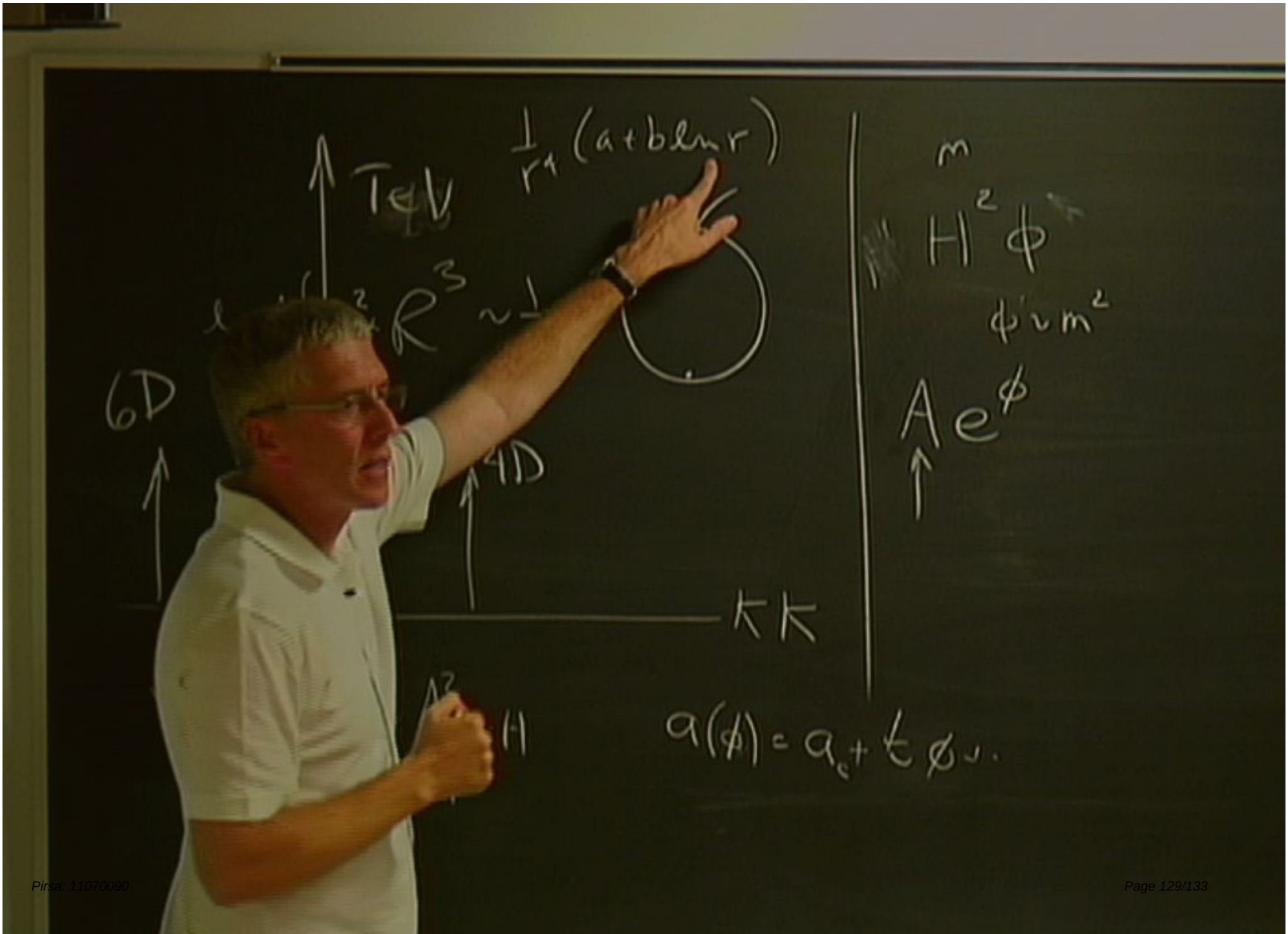
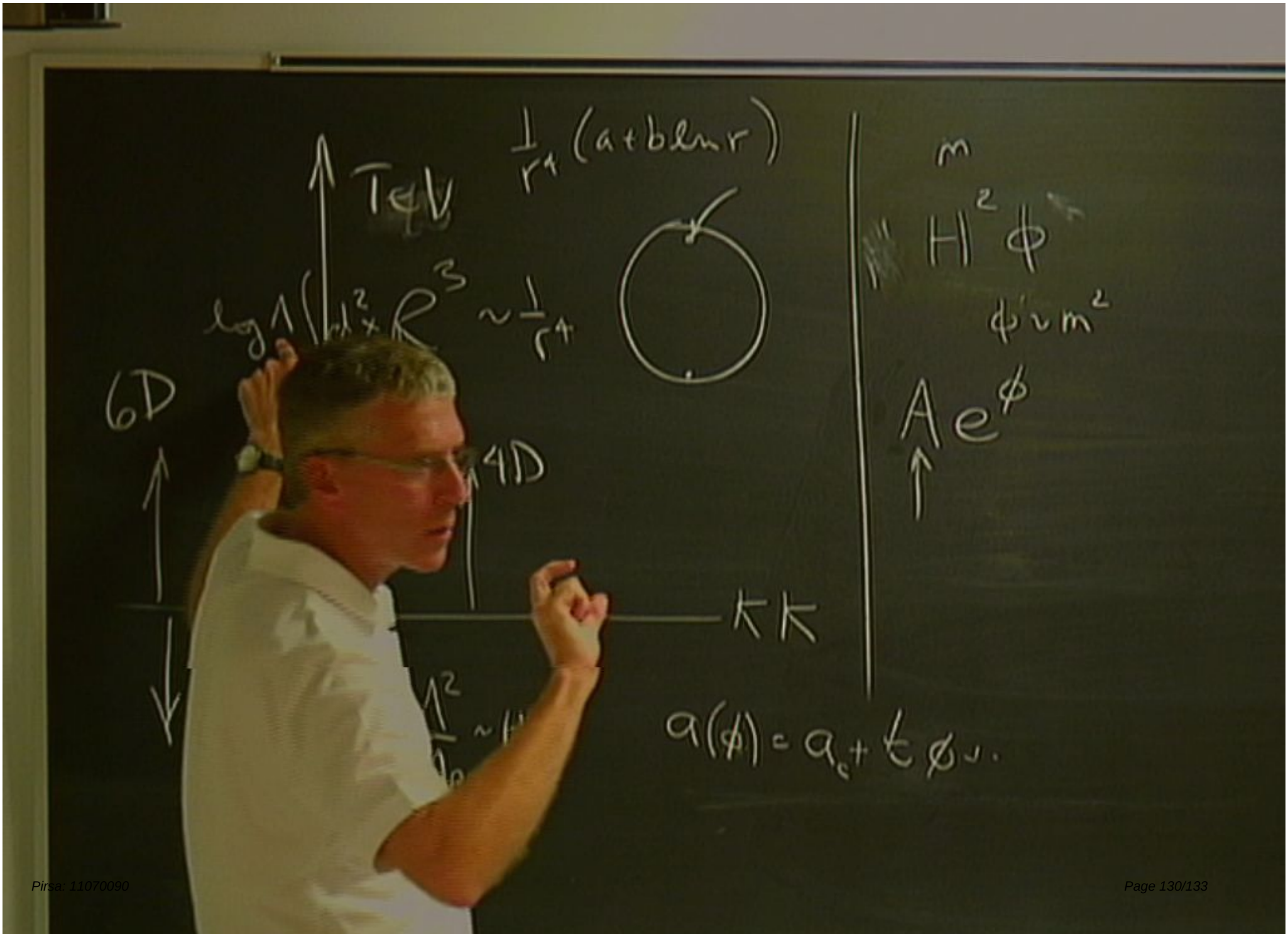
Ae^φ
a(φ) = a_c + t φ ...

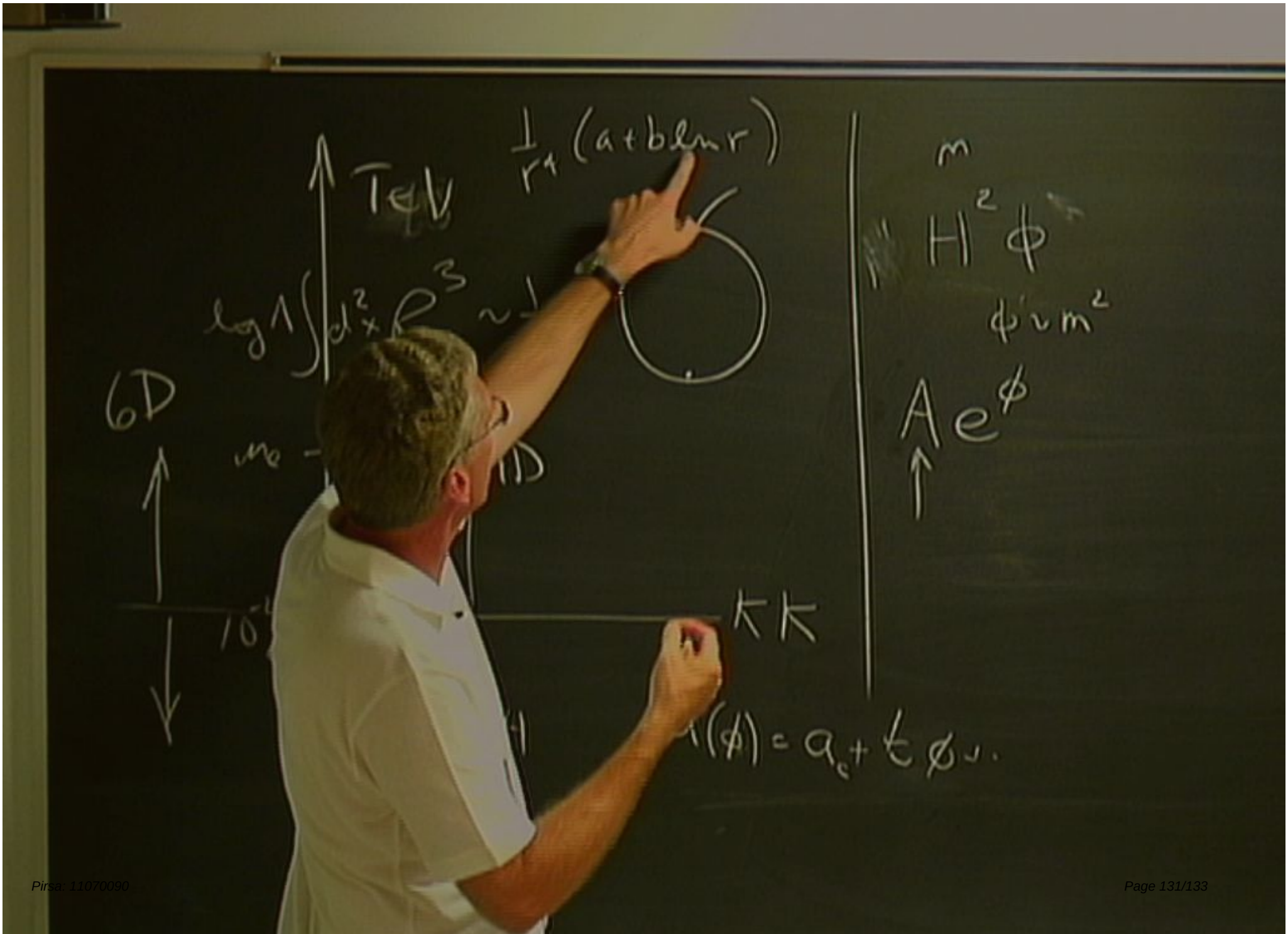

TeV
6D
KK
10^{-2} eV
$H^2 \phi$
$\phi \sim m^2$
Ae^{ϕ}

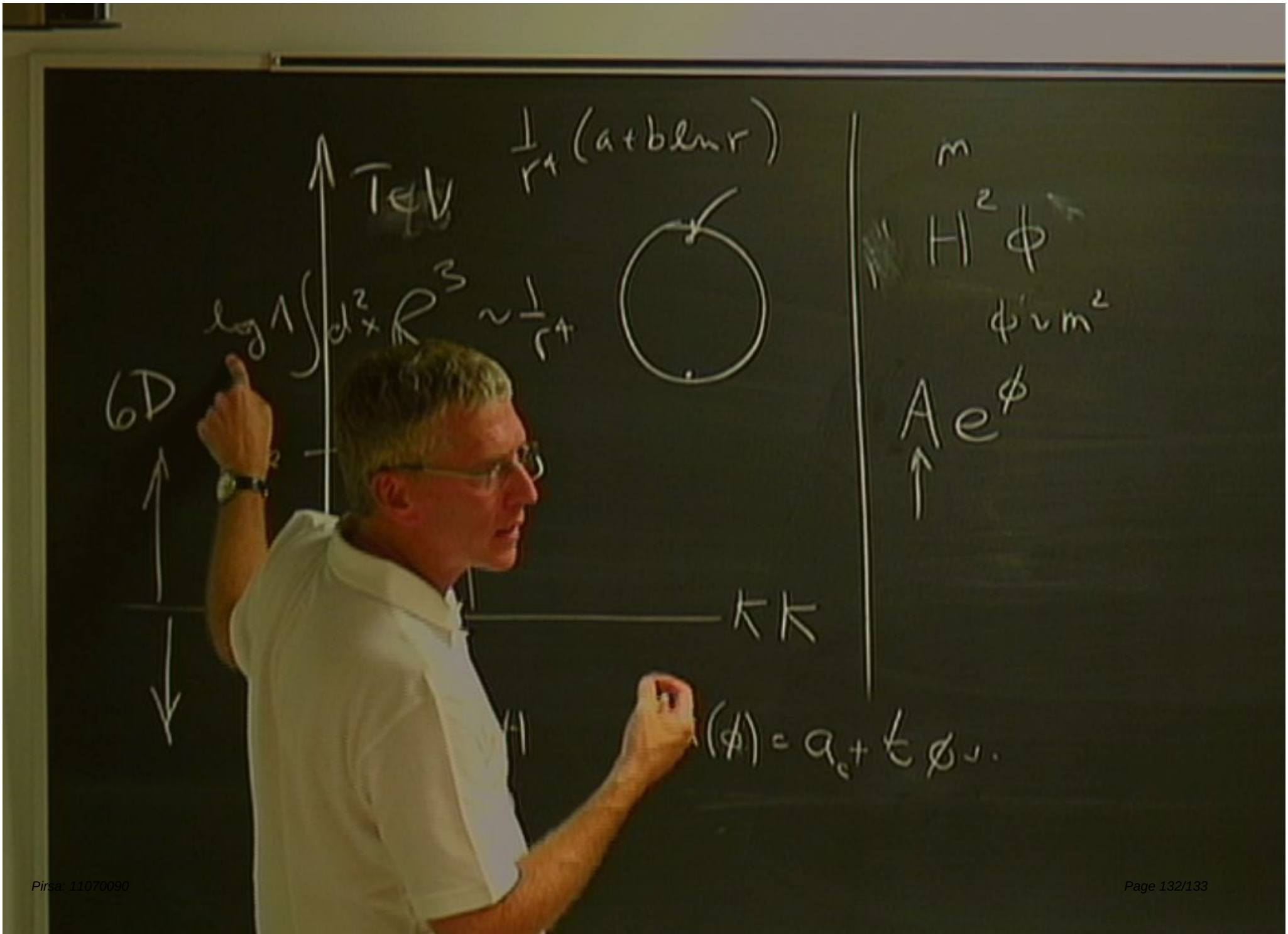

TeV
$\int d^2x\, R^3 \sim \frac{1}{r^4}$
6D
4D
KK
4
$a(\phi) = a_c + \epsilon\phi$
m
$H^2\phi$
$\phi' \sim m^2$
Ae^{ϕ}

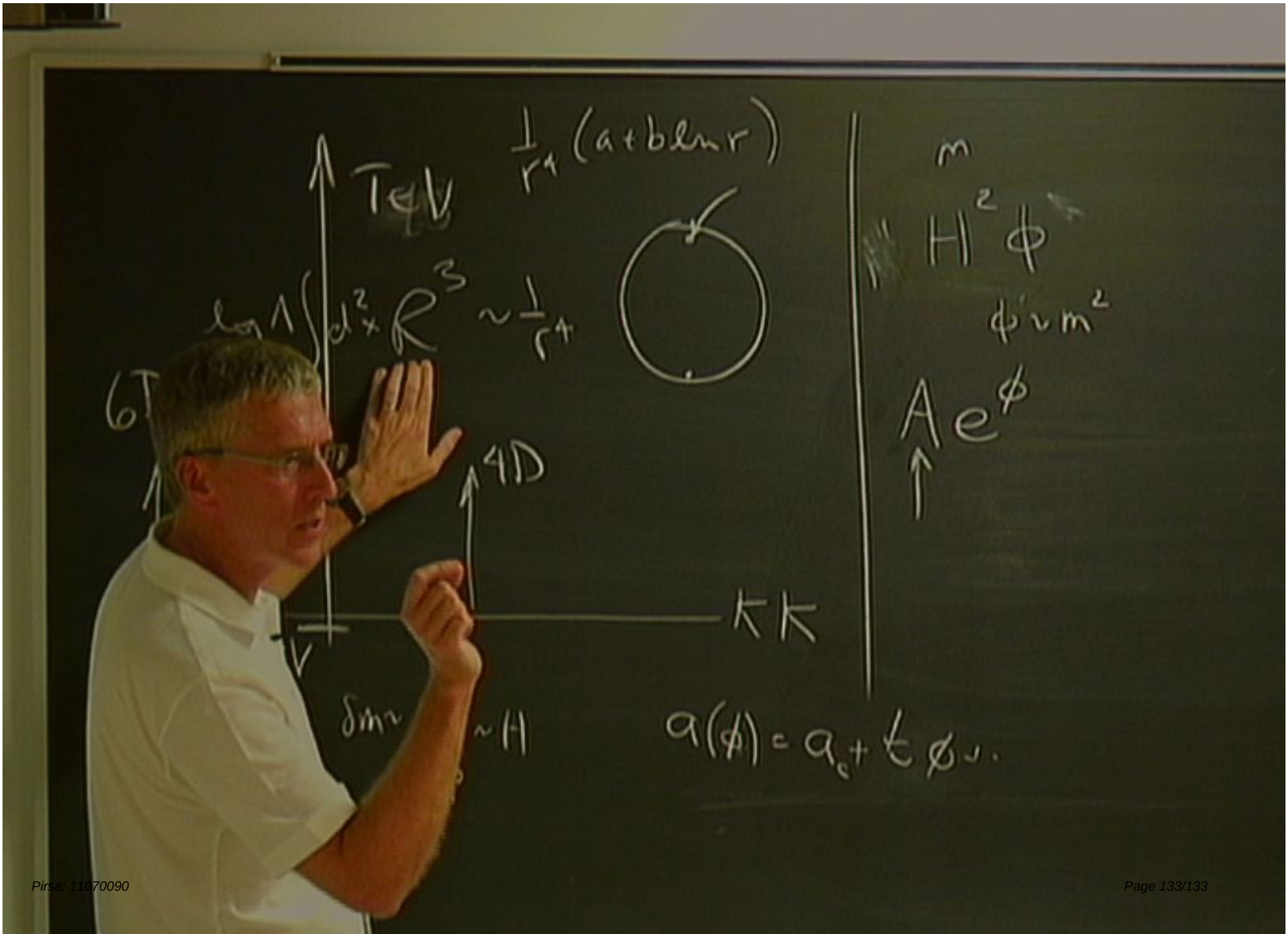

TeV
4D
KK
m
$H^2\phi$
$\phi' \sim m^2$
Ae^{ϕ}
$a(\phi) = a_c + \phi$


6D
4D
KK
m
$H^2 \phi$
$\phi \sim m^2$
$A e^{\phi}$
$a(\phi) = a_c + \phi$


6D
4D
KK
m
$H^2\phi$
$\phi' \sim m^2$
Ae^{ϕ}
$a(\phi) = a_c + t\phi \ldots$


TeV
6D
KK


6D
KK
m


TeV
4D
KK
m
$H^2\phi$
$\phi \sim m^2$
Ae^{ϕ}
$a(\phi) = a_c + t\phi$