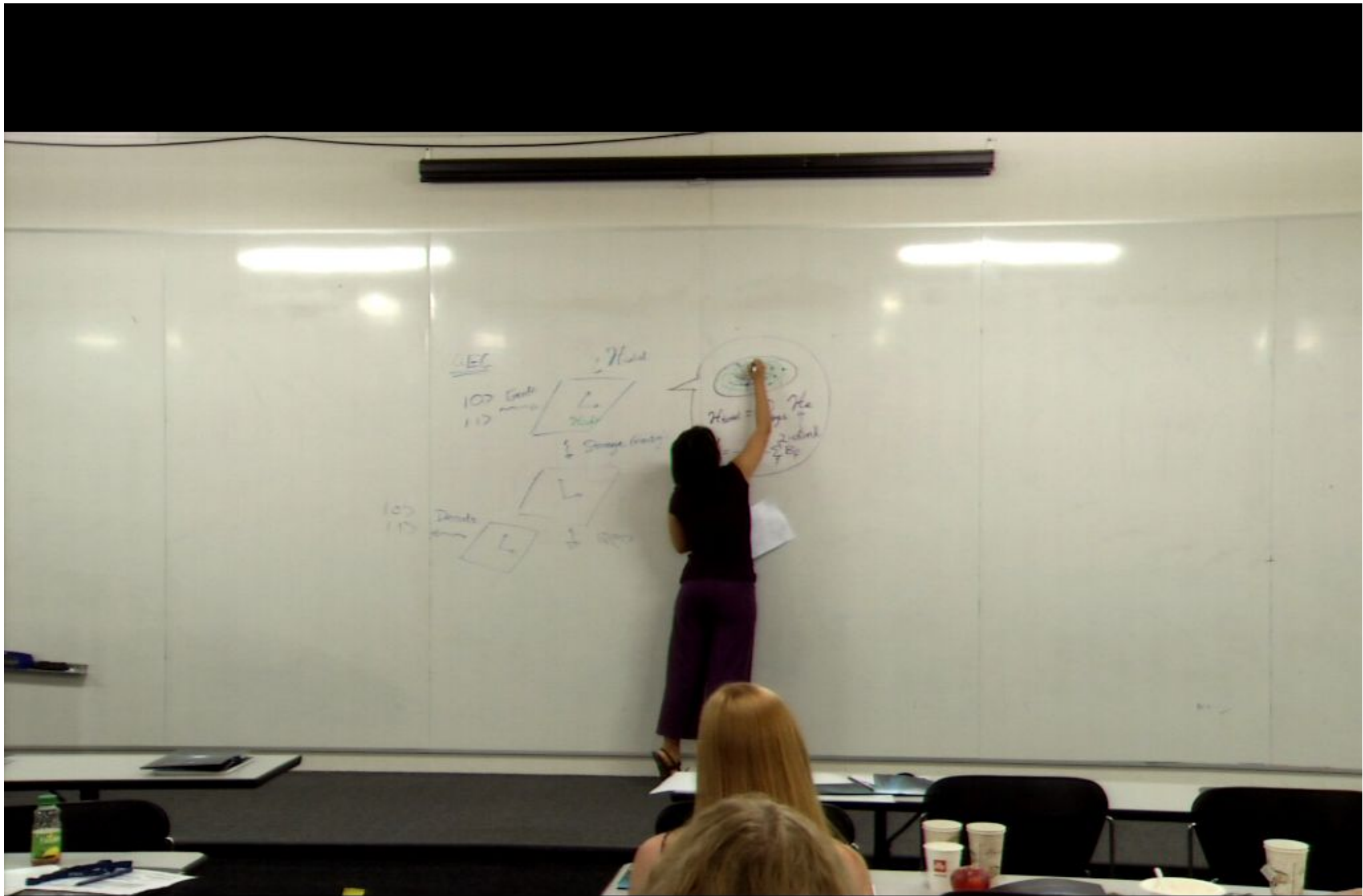


Title: Mathematics and Topological Quantum Computation

Date: Jul 20, 2011 11:10 AM

URL: <http://pirsa.org/11070066>

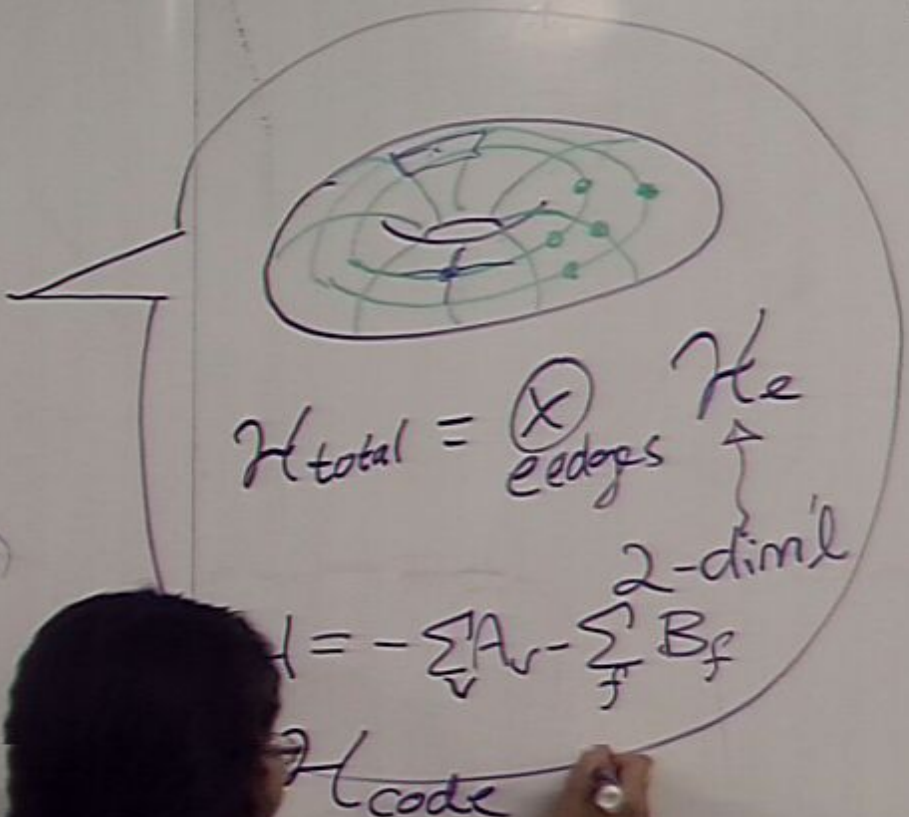
Abstract: TBA



$\mathcal{H}_{total}$

$\mathcal{H}_{code}$

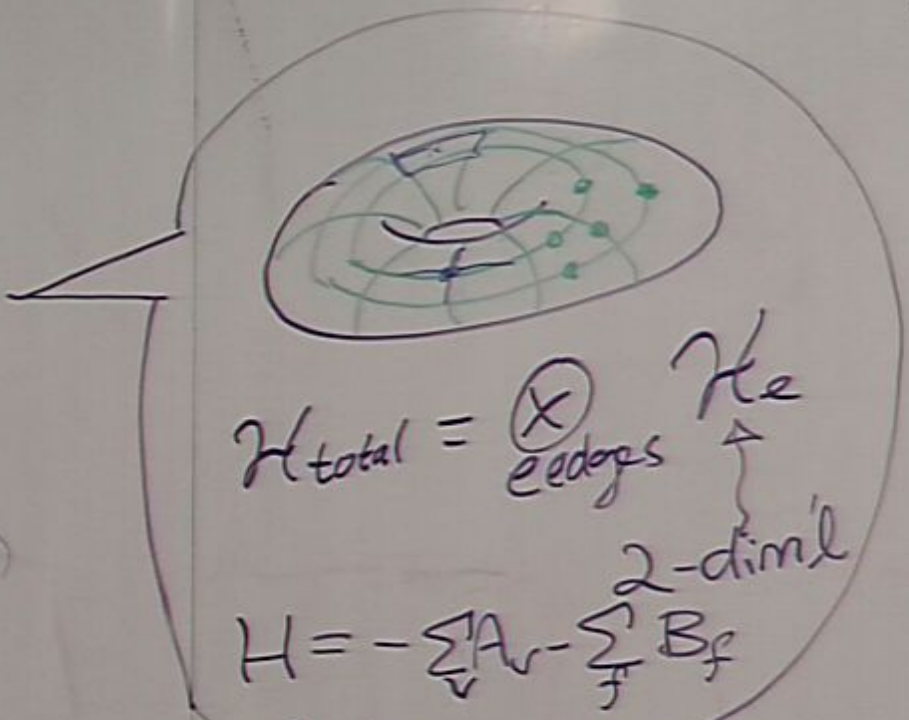
Storage (noisy)



$\mathcal{H}_{total}$

$\mathcal{H}_{code}$

Storage (noisy)



$$\mathcal{H}_{total} = \bigotimes_{\text{edges}} \mathcal{H}_e$$

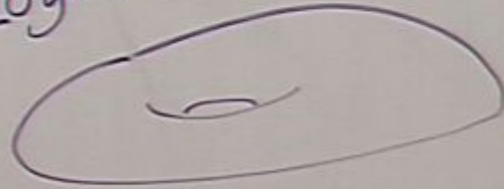
$$H = -\sum_v A_v - \sum_f B_f$$

2-dim'l

$$\mathcal{H}_{code} \subset \mathcal{H}_{total}$$

(ground space)

Key Properties  
(i) Logical error

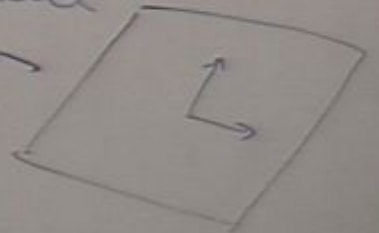


QEC

$|0\rangle$  Encode  
 $|1\rangle$

$|0\rangle$   
 $|1\rangle$

Decode



Key Properties  
(i) Logical error

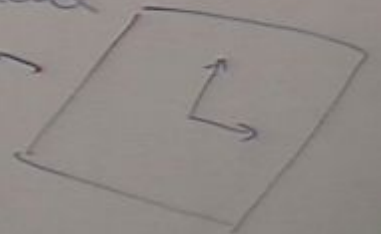


QEC

$|0\rangle$  Encode  
 $|1\rangle$

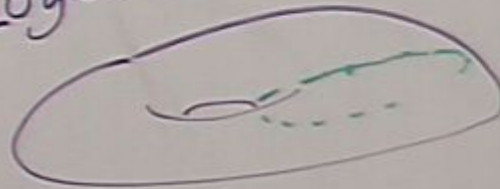
$|0\rangle$   
 $|1\rangle$

Decode





Key Properties  
(i) Logical error



QEC

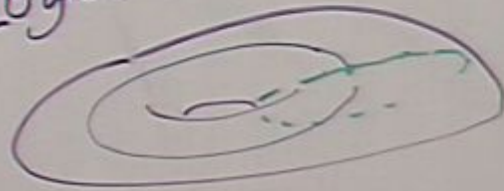
$|0\rangle$  Encode  
 $|1\rangle$  →

$|0\rangle$   
 $|1\rangle$

Decode ←



Key Properties  
(i) Logical error



QEC

$|0\rangle$  Encode  
 $|1\rangle$  →

$|0\rangle$   
 $|1\rangle$

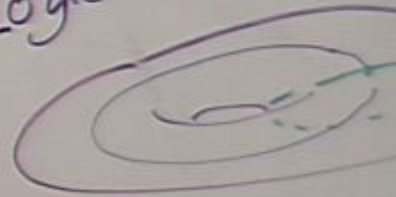
Decode ←



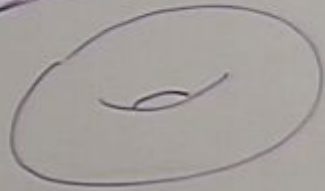


Homology

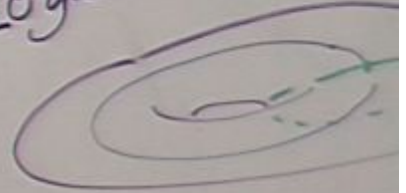
Key Properties  
(i) Logical error



Homology



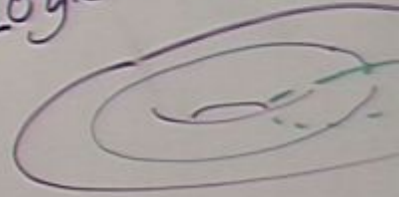
Key Properties  
(i) Logical error



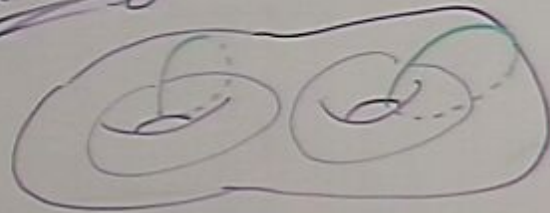
Homology



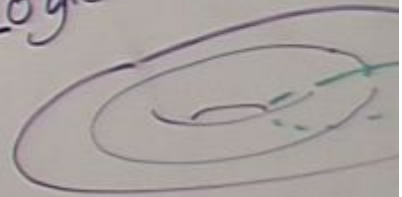
Key Properties  
(i) Logical error



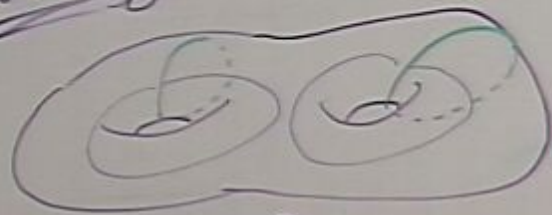
# Homology



Key Properties  
(i) Logical error

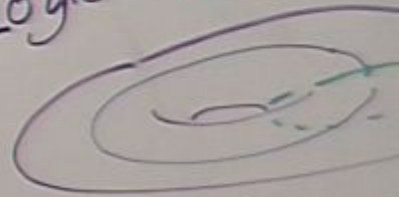


# Homology



$g=2$   
cycles/boundary

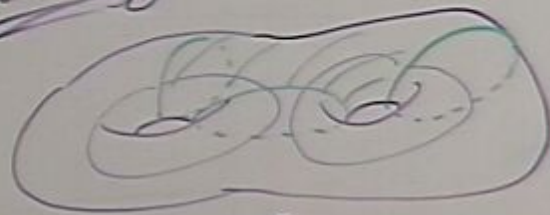
Key Properties  
(i) Logical error



$g=1$



# Homology

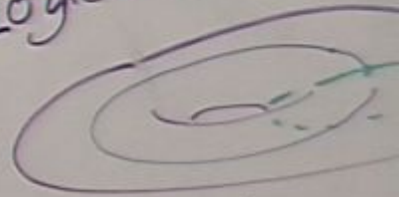


$g=2$

Cycles/boundary

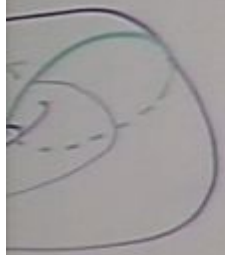
$$\text{New} = \text{Old}_1 + \text{Old}_2$$

Key Properties  
(i) Logical error



$g=1$

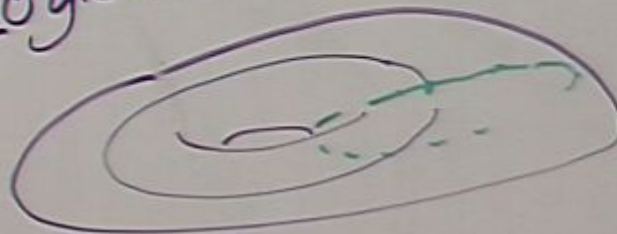




boundary

Old 1 + Old 2

Key Properties  
(i) Logical error

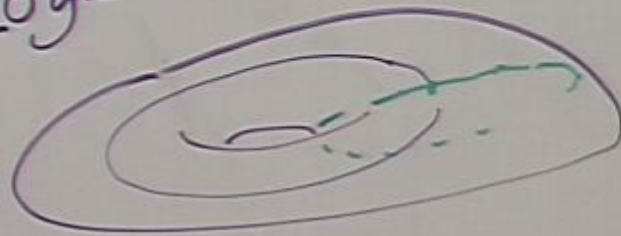


$g=1$

(ii) encode qubits

$|0\rangle$   
 $|1\rangle$

Key Properties  
(i) Logical error



$g=1$

(i) 2 #encode qubits  
( $:=$  Ground state degeneracy)

QEC

$|0\rangle$

$|1\rangle$

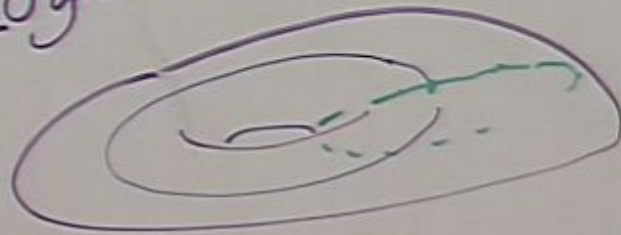
$|0\rangle$

$|1\rangle$

Decode



Key Properties  
(i) Logical error



$g=1$

(ii)  $2^{\# \text{encode qubits}}$  (:= Ground state degeneracy)

$$= \text{rank } H.(\Sigma; \mathbb{Z}/2\mathbb{Z})$$

$|0\rangle$

$|1\rangle$

QEC

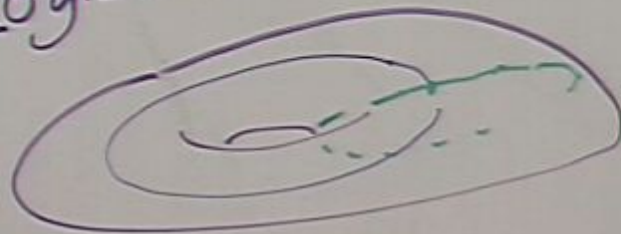
$|0\rangle$

$|1\rangle$

Decode



Key Properties  
 (i) Logical error



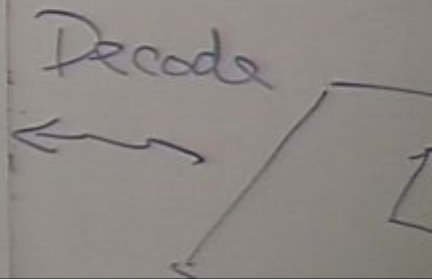
(ii)  $2^{\text{\#encode qubits}}$  (:= Ground state degeneracy)

$= 2^{\text{rank } H(\Sigma; \mathbb{Z}/2\mathbb{Z})}$   
 # indep generator  $|0\rangle$   
 $|1\rangle$

QEC

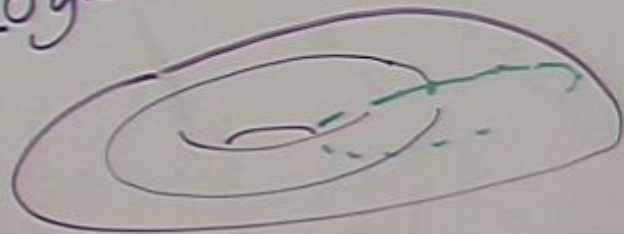
$|0\rangle$

$|1\rangle$





Key Properties  
 (i) Logical error



$g=1$

(ii)  $2^{\text{\#encode qubits}}$  (:= Ground state degeneracy)

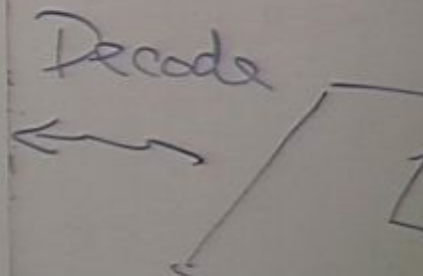
$$= 2^{\text{rank } H_1(\Sigma; \mathbb{Z}/2\mathbb{Z})}$$

$$= 2^{2g} \quad \text{\# indep generator} \quad \begin{matrix} |0\rangle \\ |1\rangle \end{matrix}$$

QEC

$|0\rangle$

$|1\rangle$



$$g=1$$

(2i)  $2^{\text{\#encode qubits}}$   
( $\equiv$  Ground state degeneracy)

$$= 2^{\text{rank } H(\Sigma; \mathbb{Z}/2\mathbb{Z})}$$

$$\equiv 2^{2g} \quad \text{\# indep. generator}$$

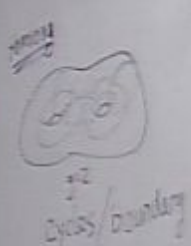
~~$|0\rangle$~~   
 ~~$|1\rangle$~~   
 $|0\rangle$   
 $|1\rangle$

Decode



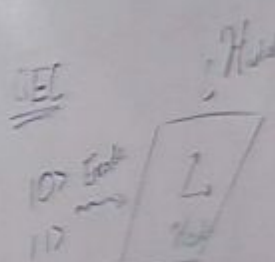
$|1\rangle$





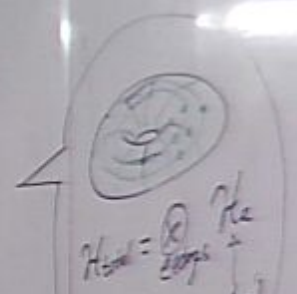
(1)  $g$  handles  
 (i)  $g$  holes  
 $J=1$

(2)  $2$  handles  
 (= Grand state  
 deficiency)  
 $= \text{rank } H(\Sigma, \mathbb{Z}/2\mathbb{Z})$



107  $E$  hole  
 117  $m$

$\mathbb{Z}/2\mathbb{Z}$  (rank)  
 $AC$

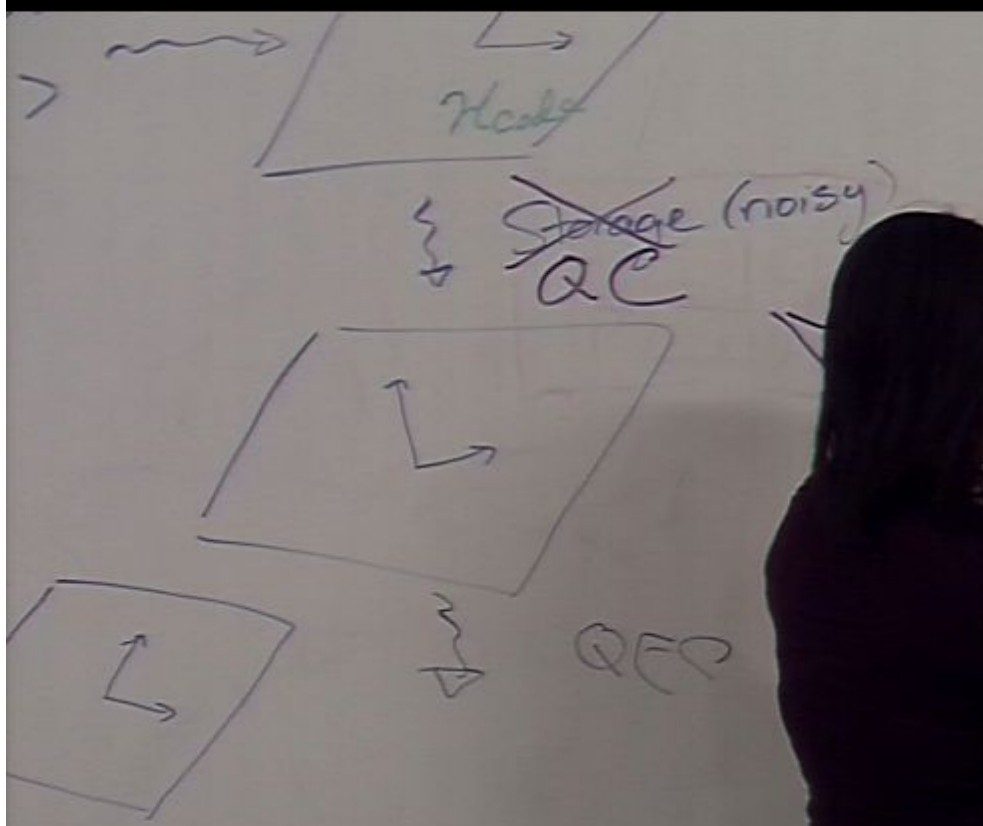


$H_{total} = \mathbb{R} \oplus \mathbb{R}^2$

$H = \sum \mathbb{R} \oplus \sum \mathbb{R}^2$

$H_{total} \subset H_{total}$   
 (Grand space)





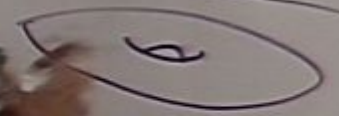
$$H_{total} = \sum_{e \text{ edges}} H_e$$

$\uparrow$   
 2-dim'l

$$H = -\sum_v A_v - \sum_f B_f$$

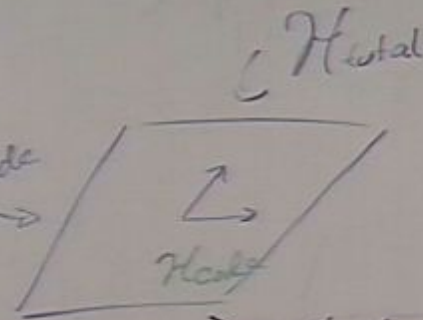
$$H_{code} \subset H_{total}$$

(ground space)



QEC

$|0\rangle$   
 $|1\rangle$  Encode



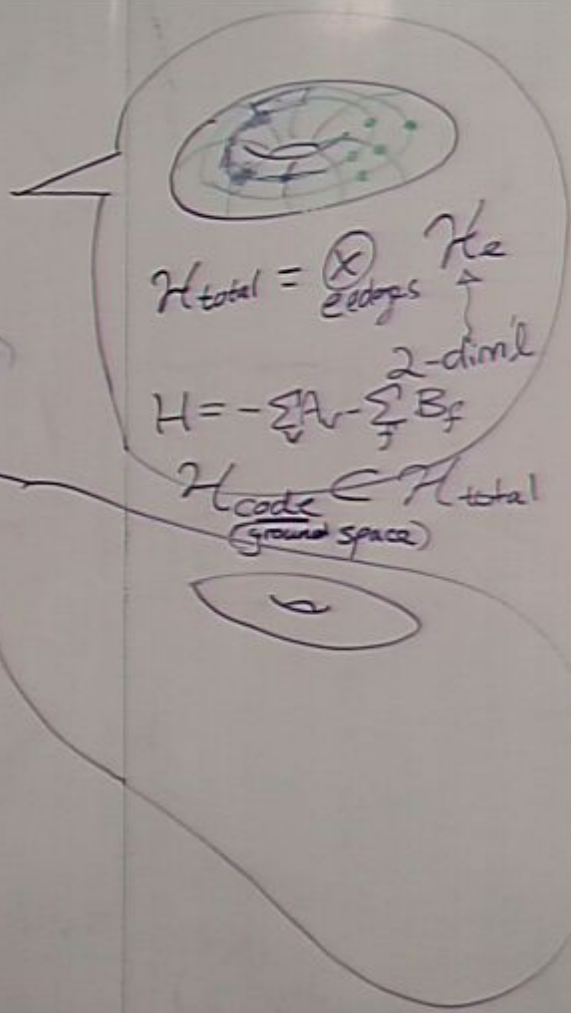
~~Storage (noisy)~~  
QC



QEC

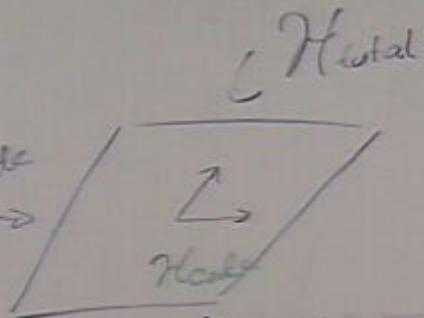
Dec

$|0\rangle$   
 $|1\rangle$



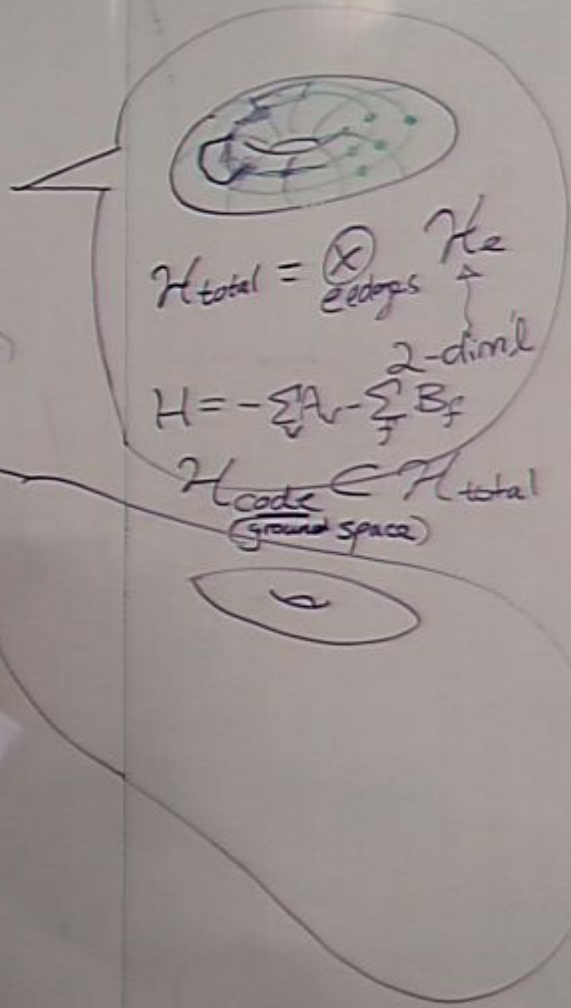
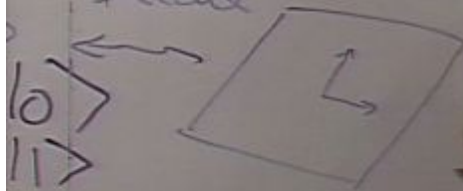
# QEC

$|0\rangle$   
 $|1\rangle$  Encode



noisy channel

Decode

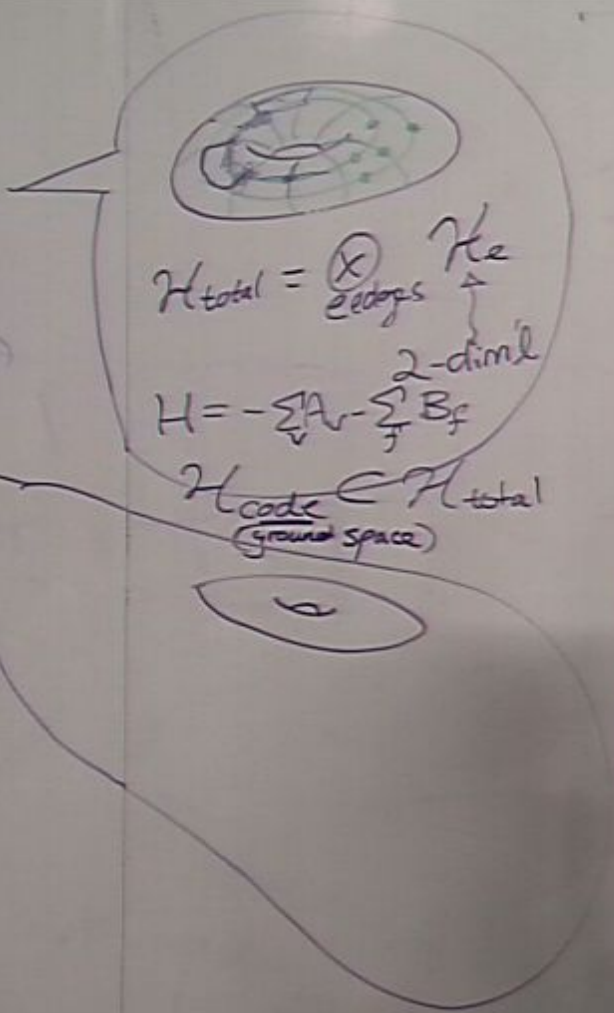




$\subset \mathcal{H}_{total}$

~~Storage (noisy)~~  
QC

QED



$$\mathcal{H}_{total} = \sum_{edges} K_e$$

$$H = -\sum_v A_v - \sum_f B_f$$

$\mathcal{H}_{code} \subset \mathcal{H}_{total}$   
(ground space)

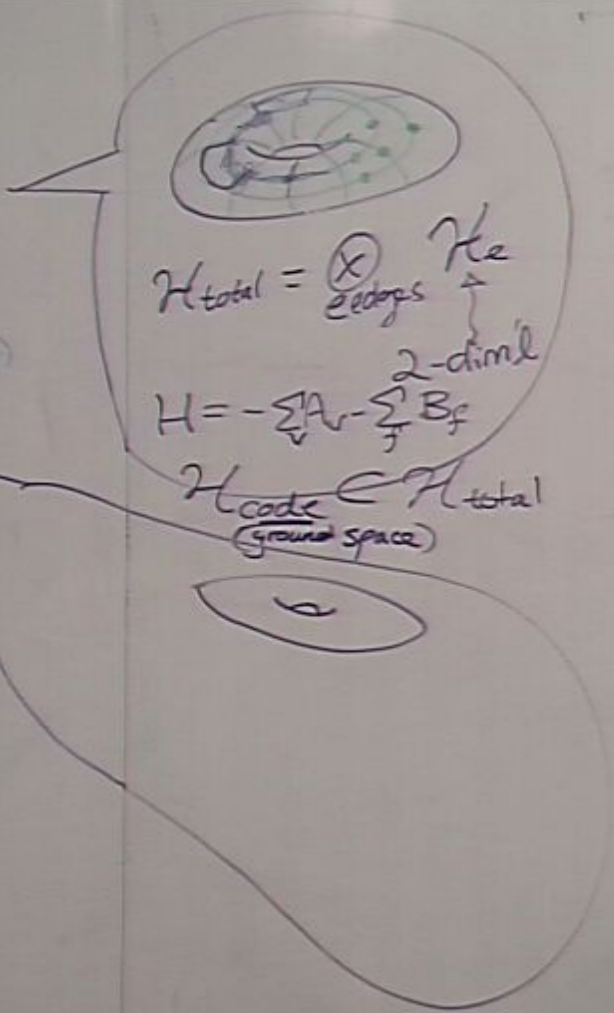
Kitaev



$\subset H_{total}$

~~Storage (noisy)~~  
QC

QEC



$$H_{total} = \sum_{edges} H_e$$

$$H = -\sum_v A_v - \sum_f B_f$$

$$H_{code} \subset H_{total}$$

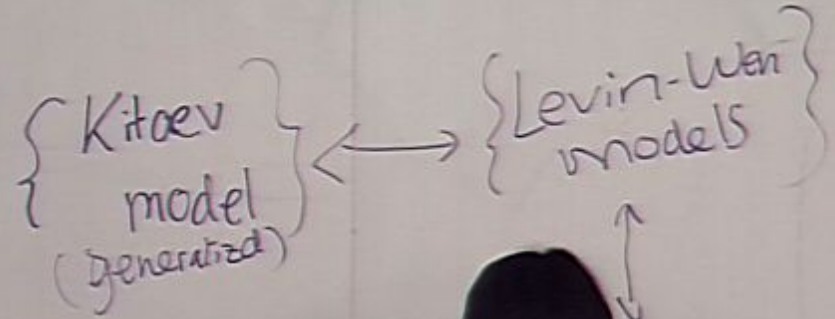
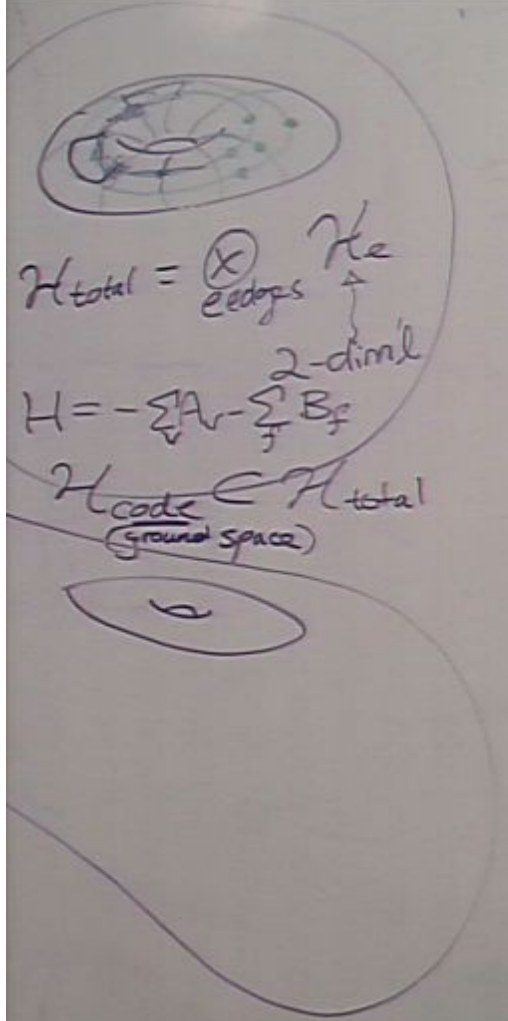
(ground space)

{ Kitaev  
model  
(generalized)

Levin-Wen









$$\mathcal{H}_{total} = \sum_{\text{edges}} \mathcal{H}_e$$

$$H = -\sum_v A_v - \sum_f B_f$$

2-dim'l

$$\mathcal{H}_{code} \subset \mathcal{H}_{total}$$

(ground space)

