Title: What's the Entropy of Gravity?

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Abstract: I present a proposal, originally motivated by a result in graph theory: the entropy function of a density matrix naturally associated to a simple undirected graph, is maximized, among all graphs with a fixed number of links and nodes, by regular graphs.I recover this result starting from the Hamiltonian operator of a non-relativistic quantum particle interacting with the loop-quantized gravitational field and setting elementary area and volume eigenvalues to a fixed value. This operator provides a spectral characterization of the physical geometry, and can be interpreted as a state describing the spectral information about the geometry available when geometry is measured by its physical interactionwith matter. It is then tempting to interpret the associated entropy function as a genuine physical entropy: I discuss the difficulties of this interpretation and I present a possible viable definition of quantum-gravitational entropy.

things you know	Loop Quantum Gravity
Yang-Mills Theory electric field, vector potential	Gravity as a $SU(2)$ gauge theory $(E^{ai}(x), A^i_a(x))$
angular momentum $L^2\|j, m\rangle = \hbar^2 j(j+1)\|j, m\rangle$	area $E_{i\ell} E^i_\ell \mid s \rangle = (8\pi\gamma\, G\hbar)^2 j_\ell(j_\ell + 1) \mid s \rangle$
states	$\mid s \rangle := \mid \Gamma, j_\ell, \nu_n \rangle$
Hilbert space	$\mathcal{H}_{LQG} = \bigoplus_\Gamma \mathcal{H}_\Gamma$

What's the entropy of gravity?

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Perimeter Institute July 19th, 2011

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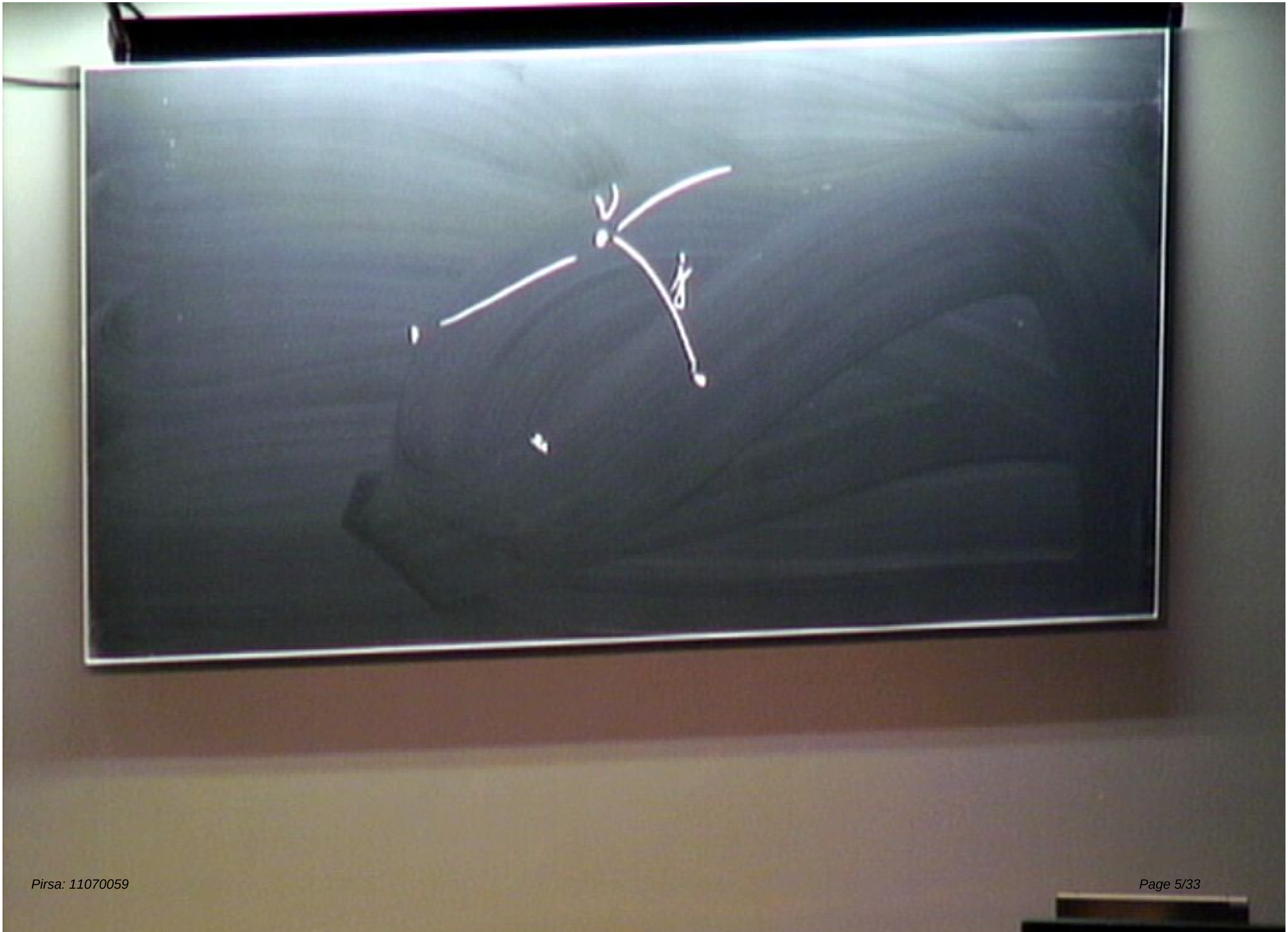

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References:

2004 The Laplacian of a graph as a density matrix: a basic combinatorial approach to separability of mixed states. Braunstein. Ghosh. Severini. quant-ph/0406165

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$$\Gamma = (N, L) \qquad \text{undirected simple graph}$$

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Adjacency $[A(\Gamma)]_{n,m} = 1$ if $\{n, m\} \in L(\Gamma)$ and $[A(\Gamma)]_{u,v} = 0$ otherwise

Degree $[\Delta(\Gamma)]_{n,n} := d_n$ $= \#$ links adjacent to the node n

Laplacian $L(\Gamma) := \Delta(\Gamma) - A(\Gamma)$

Density $\rho_\Gamma := \frac{L(\Gamma)}{d_\Gamma} = \frac{L(\Gamma)}{\mathrm{Tr}(\Delta(\Gamma))}$ Hermitian, positive semi-definite, trace-1

Entropy $S(\Gamma) = -\mathrm{Tr}[\rho_\Gamma \log \rho_\Gamma]$

Plan of the talk

1. Interaction Hamiltonian
   single quantum particle on the quantum gravitational field
   - construction and proprieties -

2. comparison with Graph Theory
   our Hamiltonian provides (up to a certain approximation)
   the same objects already studied in Graph Theory

3. Loop Quantum Thermodynamics
   this Hamiltonian is a tool to construct statistical objects in LQG

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Hamiltonian constraint $C(x) = H_{ADM}(x) + \delta^3(x, X) P_0$

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$P^2 = q^{ab}(x) P_a P_b$

$$H = \int \mathrm{d}x \, \delta^3(x, X) \, N(x) \, q^{ab}(X) \, \frac{P_a \, P_b}{2m}$$

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connection variables

Phase Space $(q_{ab}(x), \pi^{ab}(x), X^a, P_a) \rightarrow (E^{ai}(x), A^i_a(x), X^a, P_a)$

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$$P^2 = q^{ab}(x)P_aP_b \qquad q^{ab}(X) = \frac{E^{ai}(X)E^{bi}(X)}{q(X)}$$

$$H = \int \mathrm{d}x\ f_R(x,X)\frac{E^{ai}(x)E^{bi}(X)}{\sqrt{q(X)}}\,\frac{P_a\,P_b}{2m}$$

pointlike nature of the particle $\rightarrow$ regularization by a smearing function

$$f_R(x,X) = \begin{cases} \frac{1}{V_R} = \frac{3}{4\pi R^3} & \text{if} \quad |x-X| \leq R \\ 0 & \text{if} \quad |x-X| \geq R \end{cases}$$

Quantum States

Spin network states $|\, s, x \,\rangle \equiv |\, s \,\rangle \otimes |\, x \,\rangle \subset \mathcal{H}_{\mathrm{LQG}} \otimes \mathcal{H}_{\mathrm{P}}$.

$$\mathbb{I}_{\mathrm{P}} = \int \mathrm{d}x \quad |\, x \,\rangle\langle\, x \,|$$

$$\langle\, x \,|\, y \,\rangle = \delta(x, y)$$

Quantum States

restriction to the nodes

Spin network states $|\, s, x \,\rangle \equiv |\, s \,\rangle \otimes |\, x \,\rangle \subset \mathcal{H}_{\rm LQG} \otimes \mathcal{H}_{\rm P}$.

$$\mathbb{I}_{\rm P} = \int \mathrm{d}x \,\sqrt{q}\, \;|\, x \,\rangle\langle\, x \,|$$

$$\langle\, x \,|\, y \,\rangle = \frac{1}{\sqrt{q(x)}} \;\delta(x, y)$$

$$\langle\, s, x \,|\, s, x \,\rangle = \int \mathrm{d}x \,\langle\, s \,|\sqrt{q}|\, s \,\rangle \,|\, x \,\rangle\langle\, x \,|$$

Quantum States

restriction to the nodes

Spin network states $| s . x \rangle \equiv | s \rangle \otimes | x \rangle \subset \mathcal{H}_{LQG} \otimes \mathcal{H}_P$.

$$\mathbb{I}_P = \int \mathrm{d}x \sqrt{q} \; | x \rangle \langle x |$$

$$\langle x | y \rangle = \frac{1}{\sqrt{q(x)}} \; \delta(x . y)$$

$$\langle s . x | s . x \rangle = \int \mathrm{d}x \langle s | \sqrt{q} | s \rangle | x \rangle \langle x |$$

the volume operator vanishes everywhere except at the nodes

$$\sqrt{q(x)} | s \rangle = \sum_{n \in N(s)} \nu_n \, \delta(x . x_n) | s \rangle$$

Quantum States

restriction to the nodes

Spin network states $| s, x \rangle \equiv | s \rangle \otimes | x \rangle \subset \mathcal{H}_{\mathrm{LQG}} \otimes \mathcal{H}_{\mathrm{P}}$.

$$\mathbb{I}_{\mathrm{P}} = \int \mathrm{d}x \, \sqrt{q} \;\; | x \rangle \langle x | \rightarrow \langle s | \mathbb{I}_{\mathrm{P}} | s \rangle = \sum_{n \in N(s)} \nu_n \, | x_n \rangle \langle x_n |$$

$$\langle x | y \rangle = \frac{1}{\sqrt{q(x)}} \;\; \delta(x, y)$$

$$\langle s, x_n | s', x_{n'} \rangle = \nu_n^{-1} \, \delta_{ss'} \, \delta_{nn'} \qquad \mathbb{I} = \sum_s \sum_{n \in N(s)} \nu_n \, | s, x_n \rangle \langle s, x_n |$$

$$\sqrt{q(x)} \, | s \rangle = \sum_{n \in N(s)} \nu_n \, \delta(x, x_n) \, | s \rangle$$

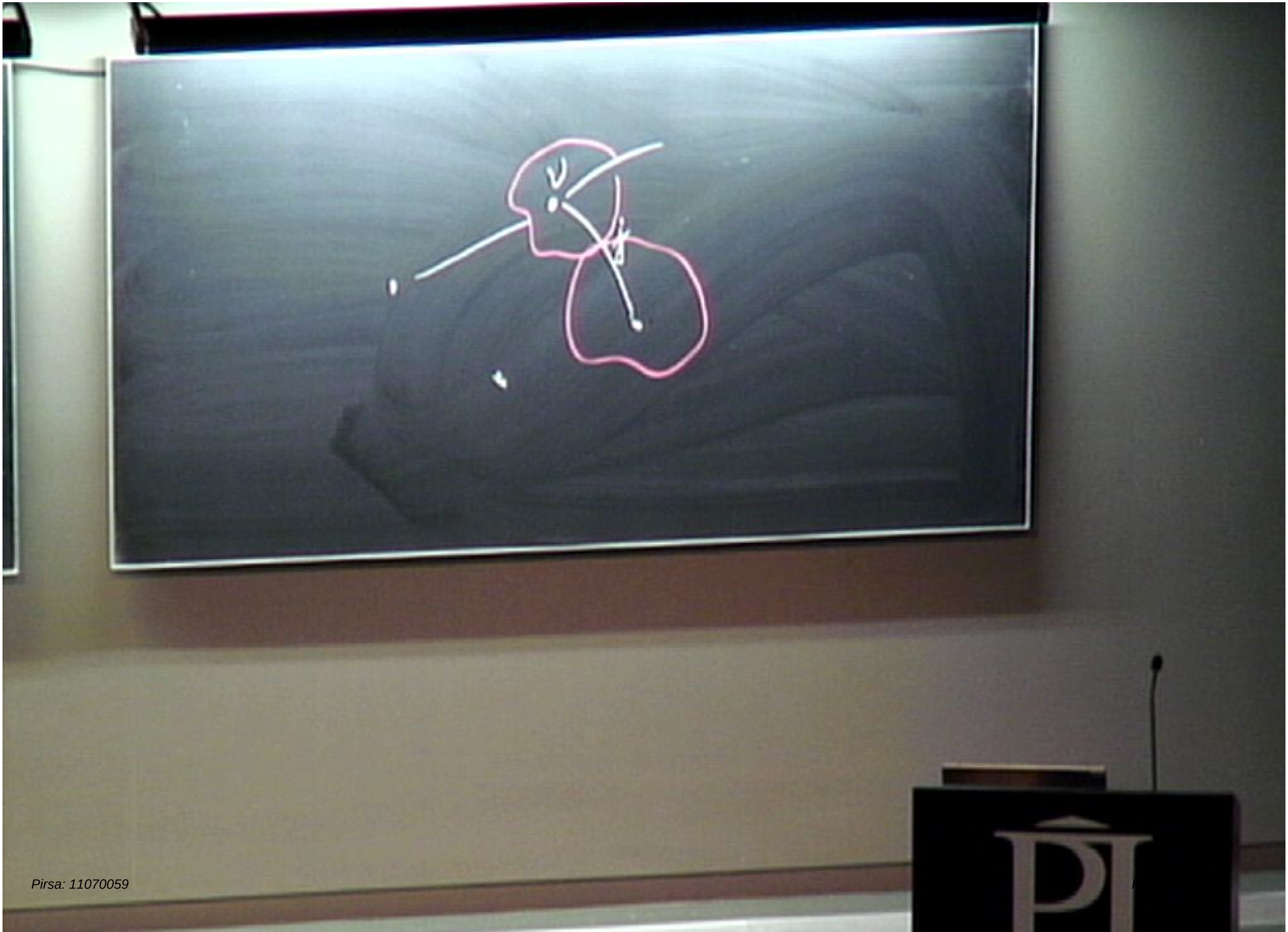
Quantum Operators

$$E^{ai}(x)\,|\,s\,\rangle \;=\; \kappa\hbar \sum_{\ell} \int_{\ell} \mathrm{d}t\, \dot{\ell}^{a}(t)\, \delta^{3}(x, \ell(t))\,|\,s, \tau^{i}\,\rangle$$

$$E^{ai}(x)E^{bi}(x)\,|\,s\,\rangle \;=\; (\kappa\hbar)^{2} \sum_{\ell,\ell'} \int_{\ell}\mathrm{d}t \int_{\ell'}\mathrm{d}t' \;\; \dot{\ell}^{a}(t)\, \dot{\ell}^{b}(t')\, \delta^{3}(x, \ell(t))\delta^{3}(x, \ell(t'))j_{\ell}(j_{\ell}+1)\,|\,s\,\rangle$$

$$P_a = -i\hbar\, D_a$$ covariant derivative

$$\langle s, \psi | H | s, \phi \rangle = \frac{\kappa^2 \hbar^4}{2m} \sum_{\ell} j_\ell (j_\ell + 1) \int_\ell \mathrm{d}s \int_\ell \mathrm{d}t \, \overline{\partial_s \psi(\ell(s))} \, \partial_t \phi(\ell(t)) \, f_R(\ell(s), \ell(t))$$



$$\langle s, \iota | H | s, o \rangle = \frac{\kappa^2 \hbar^4}{2m} \sum_{\ell} j_\ell (j_\ell + 1) \; \overline{\Delta_\ell o} \; \Delta_\ell \iota \; \frac{1}{V_R}$$

Planck scale! $\Delta_\ell \iota := \int_\ell \mathrm{d}s \, \partial_s \iota(\underset{\sim}{\ell(s)}) = \iota(\underset{\sim}{\ell_f}) - \iota(\underset{\sim}{\ell_i})$

particle states

$$| \underset{\sim}{\ell} \rangle := | \underset{\sim}{\ell_f} \rangle - | \underset{\sim}{\ell_i} \rangle$$

$$H = \frac{\hbar^2 \ell_{\mathrm{Pl}}^4}{2m^*} \sum_{s, \ell \in s} j_\ell (j_\ell + 1) \; | s, \underset{\sim}{\ell} \rangle \langle s, \underset{\sim}{\ell} |$$

$< E >$ mean energy of the particle on a gravitational field

from a measure of the particle (geometry known)

from a measure of the geometry (particle position known)

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$< E > \; = \mathrm{Tr}_{\mathrm{LQG}}[H' \tilde{\rho}]$ mean energy of the particle on a gravitational field

$|\, \iota \cdot \rangle \in \mathcal{H}_{\mathrm{P}}$ $H' = \langle \iota \cdot | H | \iota \cdot \rangle$ operator in $\mathcal{H}_{\mathrm{LQG}}$ $\tilde{\rho} = \frac{1}{Z(\mu)} e^{-\mu H'}$

$< E > \; = \mathrm{Tr}_{\mathrm{LQG}}[H' \tilde{\rho}] = -d(\ln Z)/d\mu$ mean energy of the particle on a gravitational field

$|\,\iota\cdot\,\rangle \in \mathcal{H}_{\mathrm{P}}$ $H' = \langle\,\iota\cdot\,|H|\,\iota\cdot\,\rangle$ operator in $\mathcal{H}_{\mathrm{LQG}}$ $\tilde{\rho} = \frac{1}{Z(\mu)}\, e^{-\mu H'}$

Jayne's principle of maximum entropy $S = -\mathrm{Tr}_{\mathrm{LQG}}[\tilde{\rho} \log \tilde{\rho}] = \log Z + \mu\, \mathrm{Tr}_{\mathrm{LQG}}[H'\; e^{-\mu H'}]$

Partition function: $Z = \mathrm{Tr}_{\mathrm{LQG}}[e^{-\mu H'}]$

$Z = \sum_s e^{-\frac{E_s}{kT}}$ where $E_s = E_0 d_s$ and take $\frac{E_0}{kT} := \mu$ $\rightarrow$ $Z(\mu) = \sum_s e^{-\mu d_s}$

Partition Function $Z(\mu) = \sum_s e^{-\mu \bar{d}_s}$ $\bar{d}_s(\mu) = \frac{\sum_n^N d_n}{N} = \frac{2\ell}{N}$

Partition Function $$Z(\mu) = \sum_s e^{-\mu \bar{d}_s} = \left(1 + e^{-\mu \frac{2}{N}}\right)^L$$

Energy Density $$\rho_s(\mu) = \frac{1}{Z(\mu)} e^{-\mu \bar{d}_s} = e^{-\frac{2}{N}\ell} \left(1 + e^{-\mu \frac{2}{N}}\right)^{-L}$$

$$< d > = \frac{1}{Z(\mu)} \sum_s \bar{d}_s e^{-\mu \bar{d}_s} = -\frac{1}{Z(\mu)} \frac{d}{d\mu} Z(\mu) = \frac{2}{N} L \left(1 + e^{+\mu \frac{2}{N}}\right)^{-1}$$

$$\Delta d = < d^2 > - < d >^2 = -\frac{4L}{N^2} e^{+\mu \frac{2}{N}} \left(1 + e^{+\mu \frac{2}{N}}\right)^{-2}$$

Entropy

$$S = \mu < d > - \ln Z(\mu) = \mu \frac{2}{N} L \left(1 + e^{-\mu \frac{2}{N}}\right)^{-1} - L \ln \left(1 + e^{-\mu \frac{2}{N}}\right)$$

Summary

- Hamiltonian of a non-relativistic particle on a gravitational field described by a spinnetwork:
- we have calculated a partition function. and from this other statistical quantities... is this a first step through a viable thermodynamics of the gravitational field?

"Single particle in quantum gravity and BGS entropy of a spin network" by C. Rovelli and FV. Phys.Rev.D81:044038.2010 (arXiv:0905.2983)