Title: Deformations of Lifshitz Holography in Higher Dimensions

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Abstract: (n+1)-dimensional Lifshitz spacetime is deformed by logarithmic expansions in the way to admit a marginally relevant mode in which z is restricted by n=z+1. According to the holographic principle, the deformed spacetime is assumed to be dual for quantum critical theories, and then thermodynamics of generic black holes in the bulk describe the field theory with a dynamically generated momentum scale $Lambda$. This is a basically UV-expanded theory considered in higher dimensions of the Lifshitz holography from the previous works. By finding the proper counterterms, the renormalized action is obtained and by performing the numerical works, the free energy and energy density is expressed in terms of $T/Lambda^2$.

Deformation of Lifshitz Holography in (n+1)-dimensional spacetime

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Gauge Theory in d-dimensional spacetime

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Gravity theory on the boundary in d+1-dimensional spacetime

AdS/CFT $\rightarrow$ Generalized to many other backgrounds and their dual theories.

Introduction I : Holographic Correspondence

Gauge Theory in d-dimensional spacetime
$\Updownarrow$
Gravity theory on the boundary in d+1-dimensional spacetime

AdS/CFT $\rightarrow$ Generalized to many other backgrounds and their dual theories.

- (Asymptotically) AdS $\longleftrightarrow$ QCD
- (Asymptotically) dS $\longleftrightarrow$ CFT
- (Asymptotically) Minkowski $\longleftrightarrow$?
- Lifshitz spacetime $\longleftrightarrow$ Quantum Critical Theories

Introduction II

- The **quantum critical theories** arising in condensed matter systems are scale invariant, but in general space and time need not scale equally. The dynamical critical exponent z determines the relative scaling in which
$$t \to \lambda^z t, \quad \vec{x} \to \lambda \vec{x} \tag{1}$$
- The scaling symmetry is geometrically realized,
$$ds^2 = l^2\left(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{1}{r^2}(dx^2 + dy^2)\right) \tag{2}$$
To obtain this meric, **Lifshitz Spacetime**, it is clear that an anisotropic energy - momentum tensor is needed to source the gravitational field. The minimal way to achieve this is to include a vector field, i.e. Proca field.
- **Proca field** is hodge dual to the two and three form action and describes a massive spin - 1 field.

Introduction III

- **NOTE**
  In $(3+1)$-dim. the operator due to Proca field becomes marginal when $z=2$, and in $(n+1)$-dim., z is restricted by $n-1$ to have marginal operator.

$$z = n-1, \quad \text{for marginal} \tag{3}$$

- **GOAL**

Introduction III

- **NOTE**
  In $(3+1)$-dim. the operator due to Proca field becomes marginal when $z=2$, and in $(n+1)$-dim., z is restricted by $n-1$ to have marginal operator.

$$z = n-1, \quad \text{for marginal} \tag{3}$$

- **GOAL**
  To describe the thermodynamics of black holes in the quantum critical regime where $T \gg \Lambda^2$. This can be done by slightly deforming asymptotics of the spacetime.

Lifshitz spacetime with Proca field

- Action

- Equations of Motion

- Ansatz

Lifshitz spacetime with Proca field

- Action

$$S=\int d^{n+1}x\sqrt{-g}\left(\frac{1}{2\kappa^2}[R+2\tilde{\Lambda}]-\frac{1}{g^2}\left[\frac{1}{4}F^2+\frac{\alpha}{2}A^2\right]\right),$$

$$\text{where }\ \tilde{\Lambda}=\frac{(z-1)^2+n(z-2)+n^2}{2l^2},\quad \alpha=\frac{(n-1)z}{l^2}.$$

- Equations of Motion

$$\frac{1}{\kappa^2}\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\Lambda g_{\mu\nu}\right)=\frac{1}{g^2}\left(F_{\mu\rho}F_{\nu}^{\ \rho}-\frac{1}{4}g_{\mu\nu}F^2\right)+\frac{\alpha}{g^2}\left(A_\mu A_\nu-\frac{1}{2}g_{\mu\nu}A^2\right),$$

$$\nabla_\mu F^{\mu\nu}-\alpha A^\nu=0$$

- Ansatz

Lifshitz spacetime with Proca field

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$$\nabla_\mu F^{\mu\nu}-\alpha A^\nu=0$$

- Ansatz

$$ds^2=l^2\left(-f(r)dt^2+\frac{dr^2}{r^2}+p(r)(dx^2+dy^2+\cdots)\right),\quad A=\frac{l}{\kappa}gh(r)dt.$$

- In this coordinate, when $r\to 0$, we approach to boundary of the spacetime.

Equations of Motion : Type I (f,p,h)

$$
\begin{aligned}
& -\frac{2zh(r)^2}{f(r)} - \frac{rp'(r)}{p(r)} + \frac{r^2 f'(r)p'(r)}{2f(r)p(r)} + \frac{r^2 p'(r)^2}{2p(r)^2} - \frac{r^2 p''(r)}{p(r)} = 0, \\
& \beta + \frac{(n-1)zh(r)^2}{f(r)} - \frac{r^2 h'(r)^2}{f(r)} - \frac{(n-1)r^2 f'(r)p'(r)}{2f(r)p(r)} - \frac{(n-2)(n-1)r^2 p'(r)^2}{4p(r)^2} = 0 \\
& 2\beta + \frac{z(4n-6)h(r)^2}{f(r)} - \frac{rf'(r)}{f(r)} + \frac{r^2 f'(r)^2}{2f(r)^2} - \frac{(3n-5)r^2 f'(r)p'(r)}{2f(r)p(r)} \\
& -\frac{(n-2)^2 r^2 p'(r)^2}{2p(r)^2} - \frac{r^2 f''(r)}{f(r)} = 0, \qquad (4)
\end{aligned}
$$

where

$$\beta = (n-1)^2 + (n-2)z + z^2.$$

Lifshitz spacetime with Proca field

- **Action**

$$S = \int d^{n+1}x\sqrt{-g}\left(\frac{1}{2\kappa^2}[R+2\tilde{\Lambda}] - \frac{1}{g^2}\left[\frac{1}{4}F^2 + \frac{\alpha}{2}A^2\right]\right),$$

$$\text{where } \tilde{\Lambda} = \frac{(z-1)^2 + n(z-2) + n^2}{2l^2}, \quad \alpha = \frac{(n-1)z}{l^2}.$$

- **Equations of Motion**

$$\frac{1}{\kappa^2}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu}\right) = \frac{1}{g^2}\left(F_{\mu\rho}F^{\rho}_{\nu} - \frac{1}{4}g_{\mu\nu}F^2\right) + \frac{\alpha}{g^2}\left(A_\mu A_\nu - \frac{1}{2}g_{\mu\nu}A^2\right),$$

$$\nabla_\mu F^{\mu\nu} - \alpha A^\nu = 0$$

- **Ansatz**

$$ds^2 = l^2\left(-f(r)dt^2 + \frac{dr^2}{r^2} + p(r)(dx^2 + dy^2 + \cdots)\right), \quad A = \frac{l}{\kappa}gh(r)dt.$$

- **In this coordinate, when $r \to 0$, we approach to boundary of the spacetime.**

Equations of Motion : Type I (f,p,h)

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$$\beta+\frac{(n-1)zh(r)^2}{f(r)}-\frac{r^2h'(r)^2}{f(r)}-\frac{(n-1)r^2f'(r)p'(r)}{2f(r)p(r)}-\frac{(n-2)(n-1)r^2p'(r)^2}{4p(r)^2}=0$$

$$2\beta+\frac{z(4n-6)h(r)^2}{f(r)}-\frac{rf'(r)}{f(r)}+\frac{r^2f'(r)^2}{2f(r)^2}-\frac{(3n-5)r^2f'(r)p'(r)}{2f(r)p(r)}$$

$$-\frac{(n-2)^2r^2p'(r)^2}{2p(r)^2}-\frac{r^2f''(r)}{f(r)}=0, \qquad (4)$$

where

$$\beta=(n-1)^2+(n-2)z+z^2.$$

Equations of Motion : Type II (x,q,k)

Let us change Variables

$$p(r) = e^{\int^r \frac{q(s)}{s} ds}, \quad f(r) = e^{\int^r \frac{m(s)}{s} ds}, \quad h(r) = k(r)\sqrt{f(r)}. \tag{5}$$

For simplification, let us use

$$x(r) = \left(4\beta + 4(n-1)zk(r)^2 - 2(n-1)m(r)q(r) - (n-2)(n-1)q(r)^2\right)^{\frac{1}{2}}. \tag{6}$$

Tyep II equations of motions are

$$rx'(r) = -2(n-1)zk(r) - \frac{(n-1)}{2}q(r)x(r), \tag{7}$$

$$rq'(r) = \frac{\beta}{(n-1)} - zk(r)^2 - \frac{n}{4}q(r)^2 - \frac{1}{4(n-1)}x(r)^2, \tag{8}$$

$$rk'(r) = -\frac{\beta}{(n-1)}\frac{k(r)}{q(r)} - \frac{zk(r)^3}{q(r)} + \frac{(n-2)}{4}k(r)q(r) - \frac{x(r)}{2} + \frac{1}{4(n-1)}\frac{k(r)x(r)^2}{q(r)} \tag{9}$$

Equations of Motion : Type I (f,p,h)

$$-\frac{2zh(r)^2}{f(r)}-\frac{rp'(r)}{p(r)}+\frac{r^2f'(r)p'(r)}{2f(r)p(r)}+\frac{r^2p'(r)^2}{2p(r)^2}-\frac{r^2p''(r)}{p(r)}=0,$$

$$\beta+\frac{(n-1)zh(r)^2}{f(r)}-\frac{r^2h'(r)^2}{f(r)}-\frac{(n-1)r^2f'(r)p'(r)}{2f(r)p(r)}-\frac{(n-2)(n-1)r^2p'(r)^2}{4p(r)^2}=0$$

$$2\beta+\frac{z(4n-6)h(r)^2}{f(r)}-\frac{rf'(r)}{f(r)}+\frac{r^2f'(r)^2}{2f(r)^2}-\frac{(3n-5)r^2f'(r)p'(r)}{2f(r)p(r)}$$
$$-\frac{(n-2)^2r^2p'(r)^2}{2p(r)^2}-\frac{r^2f''(r)}{f(r)}=0, \quad (4)$$

where

$$\beta=(n-1)^2+(n-2)z+z^2.$$

Equations of Motion : Type II (x,q,k)

Let us change Variables

$$p(r) = e^{\int^r \frac{q(s)}{s} ds}, \quad f(r) = e^{\int^r \frac{m(s)}{s} ds}, \quad h(r) = k(r)\sqrt{f(r)}. \tag{5}$$

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Asymptotic Solutions : Type II

$$k(r) = \frac{\sqrt{z-1}}{\sqrt{z}}\left(1 + \frac{1}{(z-1)^2 log(r\Lambda)} + \frac{(z-1)(-3z+2(z-1)^3\lambda)+\cdots}{2z(z-1)^4 log^2(r\Lambda)} + \cdots\right)$$
$$+ (r\Lambda)^{2z} log^2(r\Lambda)\left(\beta\left(1 + \frac{2(3z-1)log(-log(r\Lambda))}{z(z-1)^2 log(r\Lambda)} + \cdots\right) + \alpha\left(\frac{1}{log(r\Lambda)}\right.\right.$$
$$\left.\left. + \frac{(2z^2-4z+1) - 2(z-1)^4(2z-1)\lambda + 2(6z^2-5z+1)log(-log(r\Lambda))}{2z(z-1)^2(2z-1)log^(r\Lambda)} + \cdots\right)\right) \tag{10}$$

$$q(r) = -2\left(1 - \frac{1}{(z-1)log(r\Lambda)} - \frac{z+2(z-1)^4\lambda - 2(3z-1)log(-log(r\Lambda))}{2z(z-1)^3 log^3(r\Lambda)} + \cdots\right)$$
$$+ \frac{2\sqrt{z-1}\sqrt{z}}{1-2z}(r\Lambda)^{2z} log^2(r\Lambda)\left(\beta\left(1 + \frac{-z(4z^2-7z+2) + 2(6z^2-5z+1)log(-log(r\right.\right.}{z(z-1)^2(2z-1)log(r\Lambda)}$$
$$\left.\left. + \alpha\left(\frac{1}{log(r\Lambda)} - \frac{(2z^2-4z+1) + 2(z-1)^4\lambda - 2(3z-1)log(-log(r\Lambda))}{2z(z-1)^2 log^2(r\Lambda)} + \cdots\right)\right) \tag{11}$$

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Asymptotic Solutions : Type II

$$
\begin{aligned}
x(r) = {} & 2\sqrt{z-1}\sqrt{z}\left(1+\frac{z}{(z-1)^2 log(r\Lambda)}+\frac{(z-1)^4\lambda+(1-3z)log(-log(r\Lambda))}{(z-1)^4 log^2(r\Lambda)}+\cdots\right) \\
& +\frac{2z^2}{1-2z}(r\Lambda)^{2z}log^2(r\Lambda)\left(\beta\left(1+\frac{-z(4z^2-5z+1)+2(6z^2-5z+1)log(-log(r\Lambda}{z(z-1)^2(2z-1)log(r\Lambda)}\right.\right. \\
& \left.\left.+\alpha\left(\frac{1}{log(r\Lambda)}-\frac{(2z-1)^2+2(z-1)^4\lambda-2(3z-1)log(-log(r\Lambda))}{2z(z-1)^2 log^2(r\Lambda)}+\cdots\right)\right)\right)
\end{aligned}
\tag{12}
$$

"Λ" is the scale dynamically generated by the marginally relevant mode.

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Let us change Variables

$$p(r) = e^{\int^r \frac{q(s)}{s} ds}, \quad f(r) = e^{\int^r \frac{m(s)}{s} ds}, \quad h(r) = k(r)\sqrt{f(r)}. \tag{5}$$

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$$+ (r\Lambda)^{2z} log^2(r\Lambda)\left(\beta\left(1 + \frac{2(3z-1)log(-log(r\Lambda))}{z(z-1)^2 log(r\Lambda)} + \cdots\right) + \alpha\left(\frac{1}{log(r\Lambda)}\right.\right.$$
$$\left.\left.+ \frac{(2z^2-4z+1) - 2(z-1)^4(2z-1)\lambda + 2(6z^2-5z+1)log(-log(r\Lambda))}{2z(z-1)^2(2z-1)log^(r\Lambda)} + \cdots\right)\right) \quad (10)$$

$$q(r) = -2\left(1 - \frac{1}{(z-1)log(r\Lambda)} - \frac{z+2(z-1)^4\lambda - 2(3z-1)log(-log(r\Lambda))}{2z(z-1)^3 log^3(r\Lambda)} + \cdots\right)$$
$$+ \frac{2\sqrt{z-1}\sqrt{z}}{1-2z}(r\Lambda)^{2z} log^2(r\Lambda)\left(\beta\left(1 + \frac{-z(4z^2-7z+2) + 2(6z^2-5z+1)log(-log(r\right.\right.}{z(z-1)^2(2z-1)log(r\Lambda)}$$
$$\left.\left.+ \alpha\left(\frac{1}{log(r\Lambda)} - \frac{(2z^2-4z+1) + 2(z-1)^4\lambda - 2(3z-1)log(-log(r\Lambda))}{2z(z-1)^2 log^2(r\Lambda)} + \cdots\right)\right) \quad (11)$$

Asymptotic Solutions : Type I

$$f(\rho) = \frac{1}{(r\Lambda)^{2z}}\left(1 - \frac{(7z-4)+(6z-2)log(-log(r\Lambda))}{(z-1)^3 log(r\Lambda)} - \frac{(23z^4 - 142z^3 + 152z^2 + \cdots}{4z(z-1)^6 log^2(r\Lambda)} + \frac{(3z-1)^2(5z-2)log(-log(r\Lambda)) + (3z-1)^3 log^2(-log(r\Lambda))}{z(z-1)^6 log^2(r\Lambda)} + \cdots\right) \quad (13)$$

$$p(\rho) = \frac{1}{(r\Lambda)^2}\left(1 + \frac{(5z-2)+2(3z-1)log(-log(r\Lambda))}{z(z-1)^3 log(r\Lambda)} + \frac{(31z^4 - 64z^3 + 106z^2 - \cdots}{4z^2(z-1)^6 log^2(r\Lambda)} + \frac{(3z^3+26z^2-21z+4)log(-log(r\Lambda)) + (3z-1)^2(z-3)log^2(-log(r\Lambda))}{z^2(z-1)^6 log^2(r\Lambda)} + \cdots\right) \quad (14)$$

Free Energy Density and Energy Density

- **Free Energy** $\quad F = -T\log(Z) = TS_E(g_*)$
- **Free Energy Density**

$$\begin{aligned}\mathscr{F} &= \mathscr{F}_E + \mathscr{F}_{G-H}\\ &= -\frac{l^{n-1}}{2\kappa^2}\lim_{r\to 0}\sqrt{f_\epsilon(r)}p(r)^{\frac{n-1}{2}}\left(\frac{(n-2)rp'(r)}{p(r)} + \frac{rf'_\epsilon(r)}{f_\epsilon(r)}\right)\end{aligned}$$

where C_n is not a constant, but a series of $\frac{1}{log(r\Lambda)}$.

- **Energy Density**

$$\begin{aligned}\mathscr{E} &= \sqrt{\sigma}k_a\xi_b\mathscr{T}^{ab}\\ &= -\frac{l^{n-1}}{2\kappa^2}\lim_{r\to 0}\sqrt{f_\epsilon(r)}p(r)^{\frac{n-1}{2}}\left(\frac{(n-1)rp'(r)}{p(r)} - x(r)k(r) + \sum_{n=0}^{2}C_n\left(k(r)^2 - \frac{1}{2}\right)^n\right)\end{aligned}$$

Free Energy Density and Energy Density

- **Free Energy** $F = -T log(Z) = TS_E(g_*)$
- **Free Energy Density**

$$
\begin{aligned}
\mathscr{F} &= \mathscr{F}_E + \mathscr{F}_{G-H} + \mathscr{F}_{C.T.} \\
&= -\frac{l^{n-1}}{2\kappa^2}\lim_{r\to 0}\sqrt{f_\epsilon(r)}p(r)^{\frac{n-1}{2}}\left(\frac{(n-2)rp'(r)}{p(r)} + \frac{rf'_\epsilon(r)}{f_\epsilon(r)} + \sum_{n=0}^{2} C_n\left(k(r)^2 - \frac{1}{2}\right)^n\right)
\end{aligned}
$$

where C_n is not a constant, but a series of $\frac{1}{log(r\Lambda)}$.

- **Energy Density**

$$
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\mathscr{E} &= \sqrt{\sigma} k_a \xi_b \mathscr{T}^{ab} \\
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\end{aligned}
$$

$$C_0 = \frac{(z^2+52z-12)}{16} + \frac{(z-2)^2(12z^4-79z^3+68z^2-67z+18)}{16(z-1)(3z^4-23z^3+19z^2-21z+6)log(r\Lambda)}$$
$$+\frac{(45z^6-297z^5+115z^4+695z^3-602z^2+964z-296)+4(z-1)(z-2)^2(9z^2}{64(z-1)^2(3z^4-23z^3+19z^2-21z+6)log^2(r\Lambda)}$$
$$+\frac{(9z^5-81z^4+95z^3-159z^2+60z-4)}{8(z-1)^3(3z^4-23z^3+19z^2-21z+6)log^3(r\Lambda)}+\cdots \qquad (15)$$
$$C_1 = -\frac{z(z-6)}{4} + \frac{z(-12z^5+103z^4-226z^3+203z^2-152z+36)}{4(z-1)(3z^4-23z^3+19z^2-21z+6)log(r\Lambda)}$$
$$-\frac{z(45z^5-303z^4+333z^3-447z^2+128z+4)-4z(z-1)(-9z^3+33z^2-34z+}{16(z-1)^2(3z^4-23z^3+19z^2-21z+6)log^2(r\Lambda)}$$
$$-\frac{z(9z^4-87z^3+105z^2-101z+26)}{4(z-1)^3(3z^4-23z^3+19z^2-21z+6)log^3(r\Lambda)}+\cdots \qquad (16)$$
$$C_2 = \frac{z^2}{4} + \frac{z^2(12z^4-79z^3+68z^2-67z+18)}{4(z-1)(3z^4-23z^3+19z^2-21z+6)log(r\Lambda)}$$
$$+\frac{z^2(45z^4-309z^3+347z^2-297z+70)+4z^2(z-1)(9z^2-15z+4)log(-log(r\Lambda}{16(z-1)^2(3z^4-23z^3+19z^2-21z+6)log^2(r\Lambda)}$$
(17)

Near Horizon Solutions and Physical Quantites

Near horizon solutions

$$f(r) = f_0\left(\left(1-\frac{r}{r_+}\right)^2 + \left(1-\frac{r}{r_+}\right)^3 + \frac{(-6z^2+14z+7)z+8(3z-2)h_0^2}{12z}\left(1-\frac{r}{r_+}\right)^4 -$$

$$p(r) = p_0\left(1+\frac{(3z-1)z-4h_0^2}{2z}\left(1-\frac{r}{r_+}\right)^2 + \frac{(3z-1)z-4h_0^2}{2z}\left(1-\frac{r}{r_+}\right)^3 + \cdots\right),$$

$$h(r) = \sqrt{f_0}\left(h_0\left(1-\frac{r}{r_+}\right)^2 + h_0\left(1-\frac{r}{r_+}\right)^3 + h_0\left(\frac{z(-9z^2+10z+20)+8h_0^2(3z-1)}{24z}\right.\right.$$

Physical Quantities

$$T = \frac{r_+}{2\pi}\sqrt{\frac{1}{2}\frac{d^2f}{dr^2}\Big|_{r=r_+}} = \frac{\sqrt{f_0}}{2\pi}, \quad s = 2\pi\left(\frac{l}{\kappa}\right)^2 p(r_+) = 2\pi p_0\left(\frac{l}{\kappa}\right)^2,$$

$$\phi = \frac{lgr_+}{\kappa}\left(\frac{p}{\sqrt{f}\frac{dh}{dr}}\right)\Bigg|_{r=r_+} = 2h_0p_0\left(\frac{lg}{\kappa}\right)$$

Logistics

- GOAL : The characteristic energy scale in condensed matter systems is driven to zero at a quantum critical point, so **temperature itself seems to be the only energy scale** that remains in quantum critical matter. Express physical quantities in functions of T.

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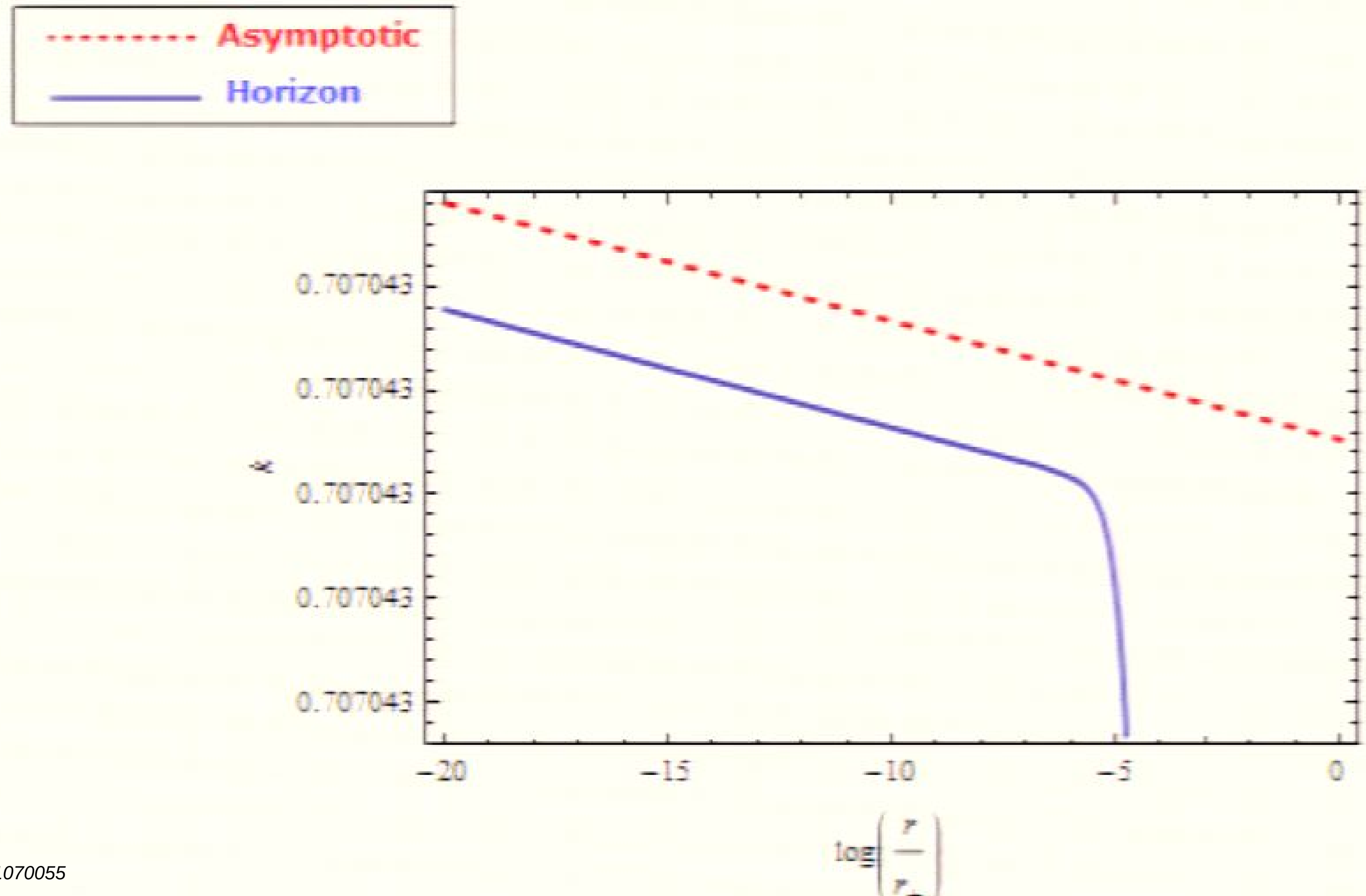
Results 1 : Extracting Λ

- $z = 2$ and $h_0 = 0.97128$



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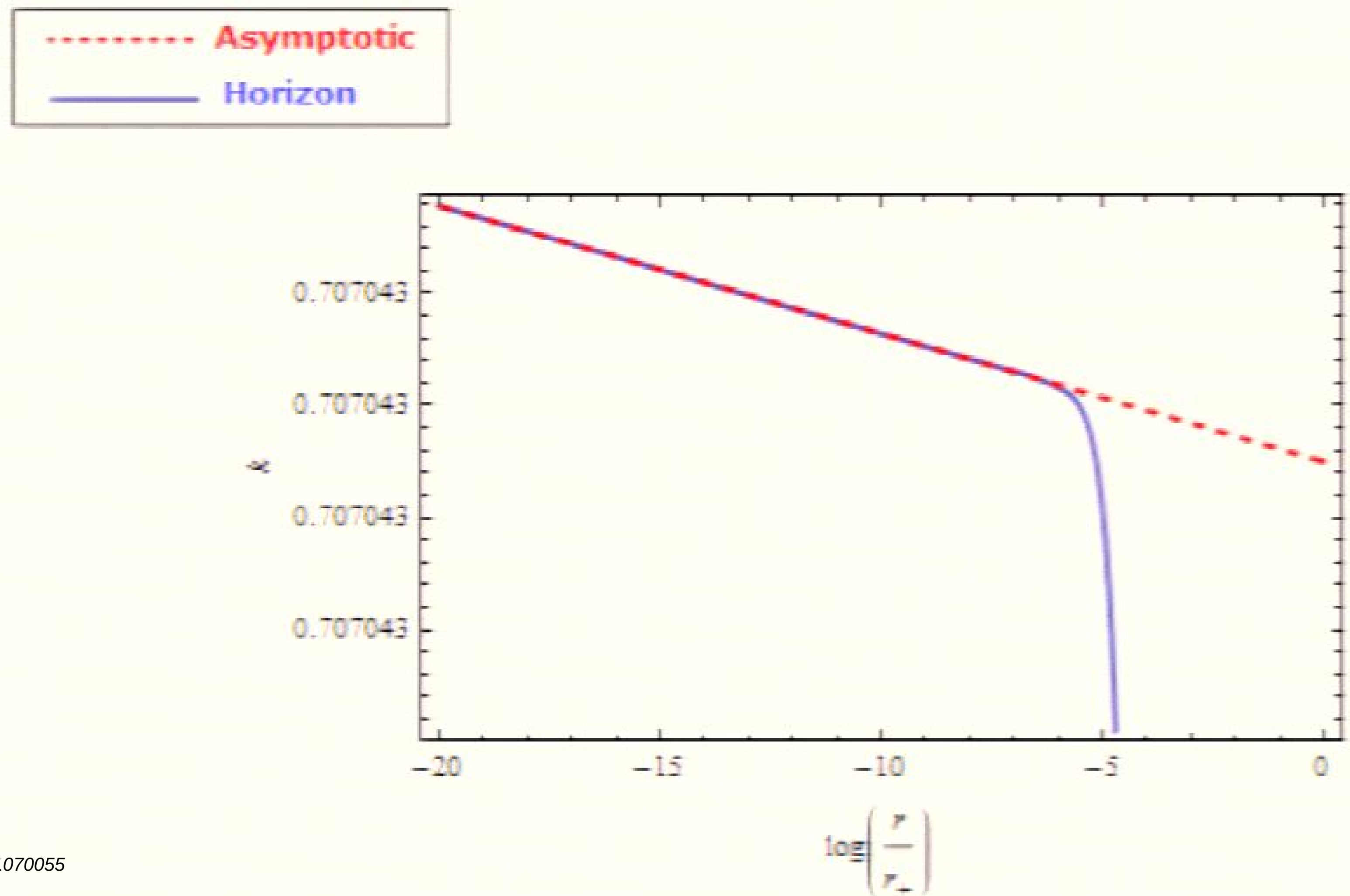
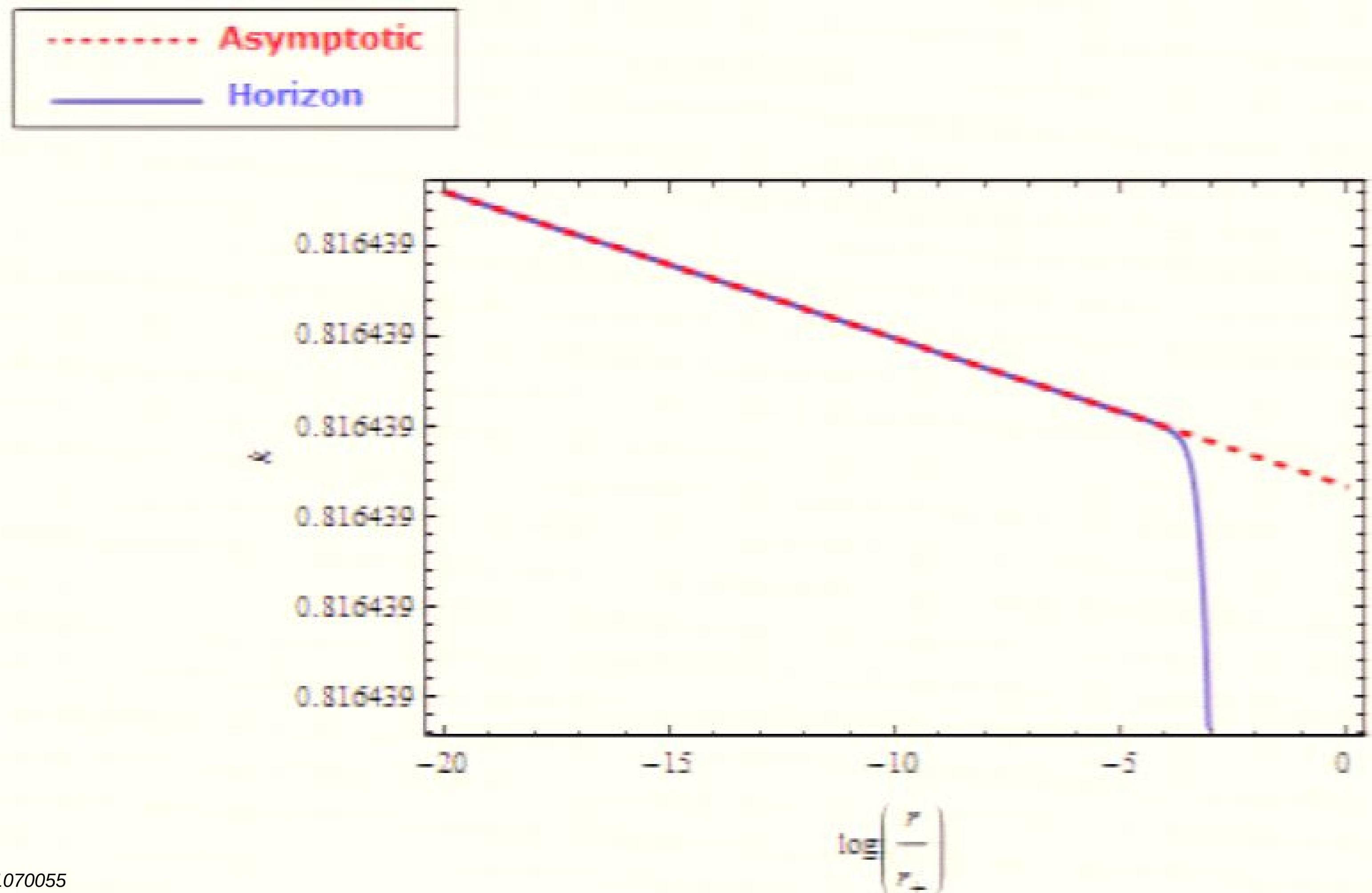


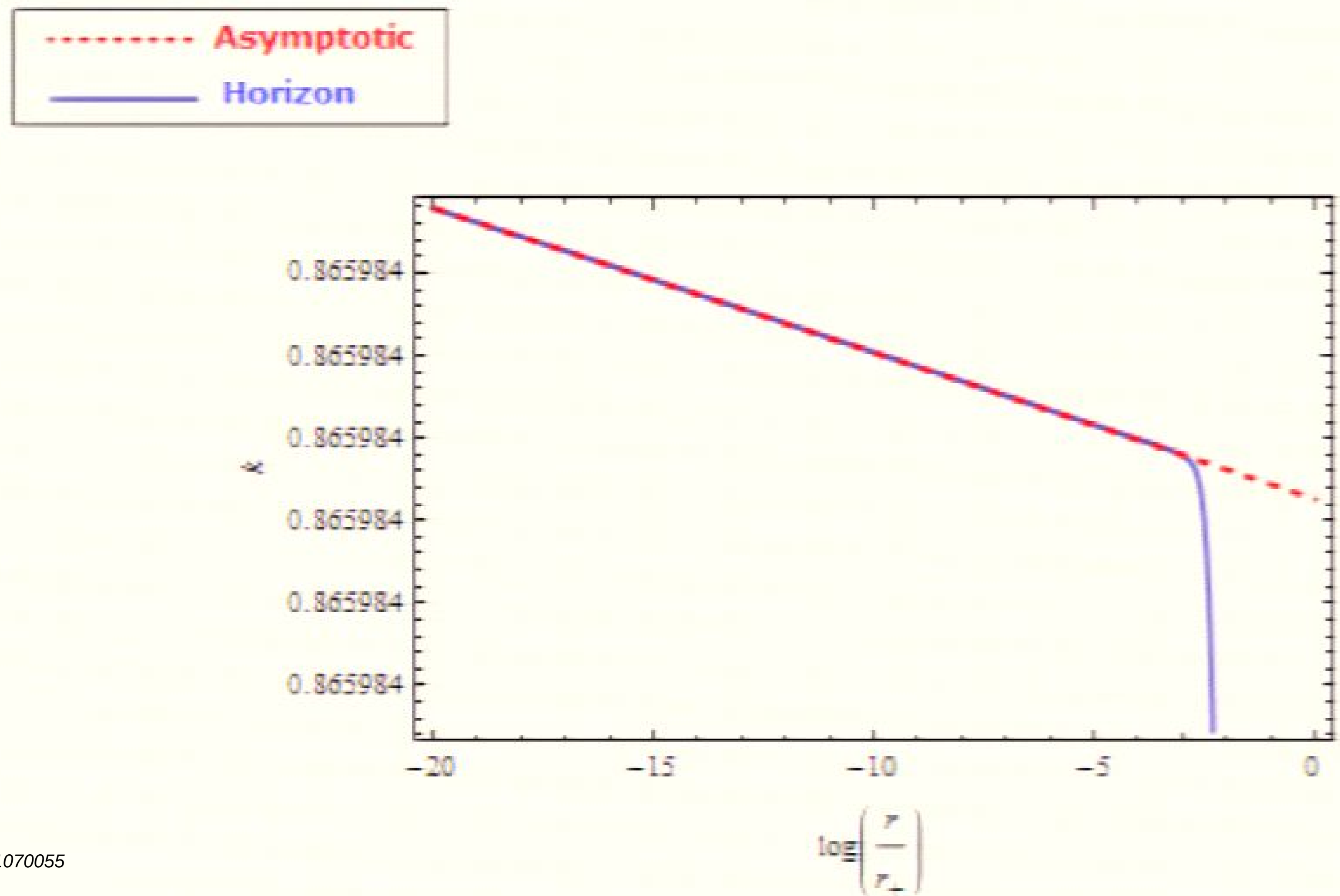
Results 1 : Extracting Λ

- $z = 3$ and $h_0 = 1.63426$



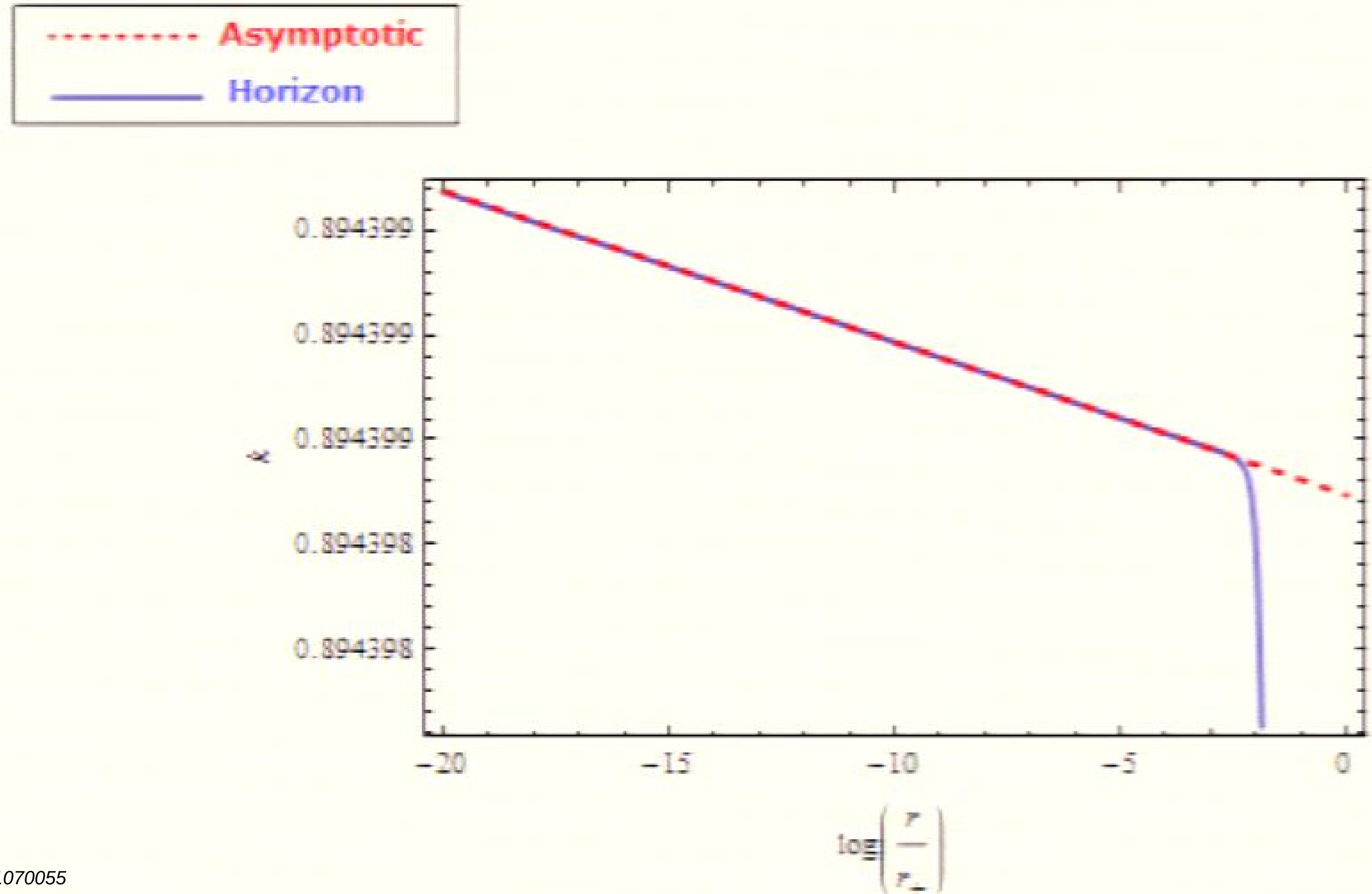
Results 1 : Extracting Λ

- $z = 4$ and $h_0 = 2.28216$



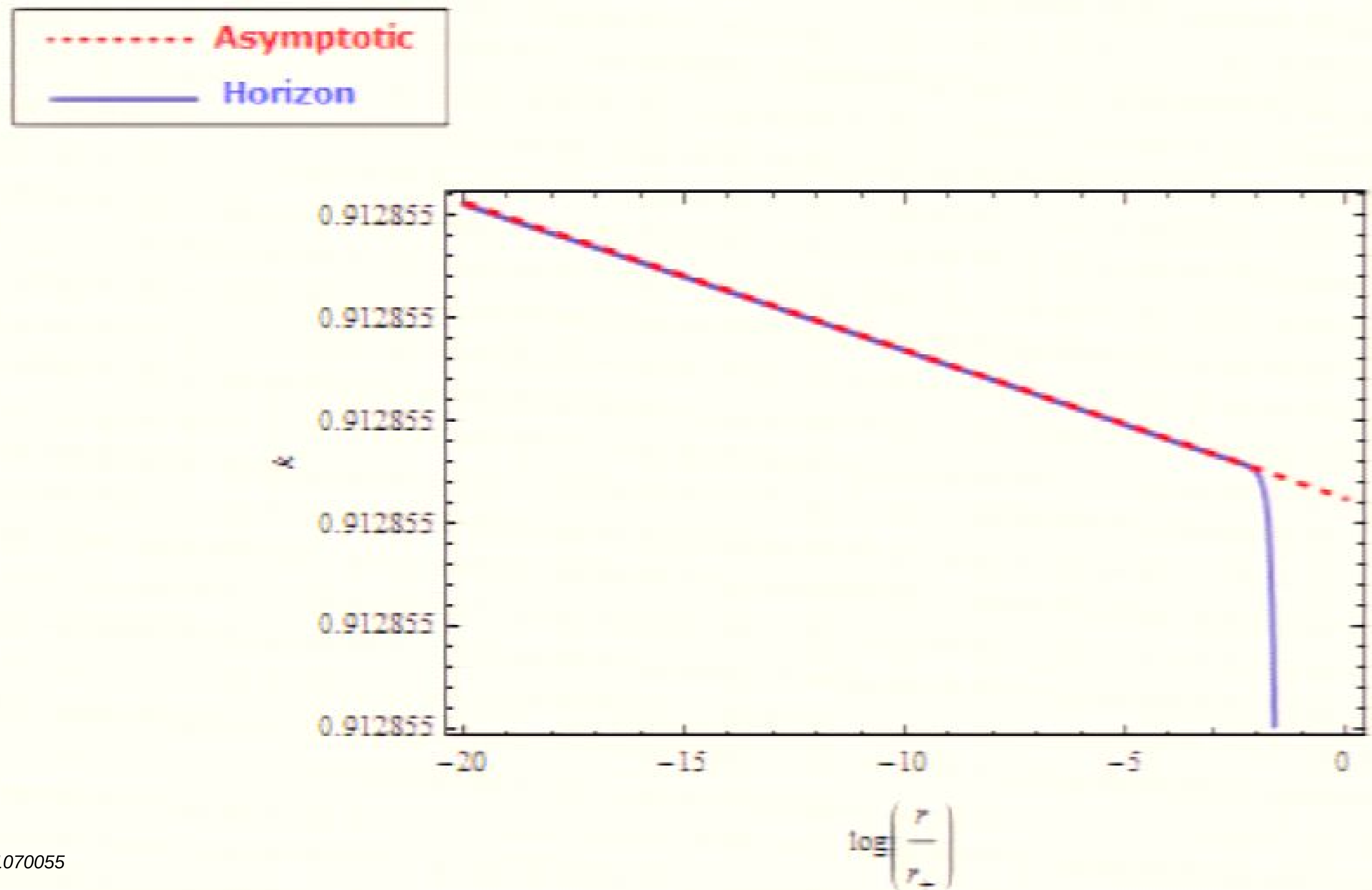
Results 1 : Extracting Λ

- $z = 5$ and $h_0 = 2.92546$



Results 1 : Extracting Λ

- $z = 6$ and $h_0 = 3.56676$

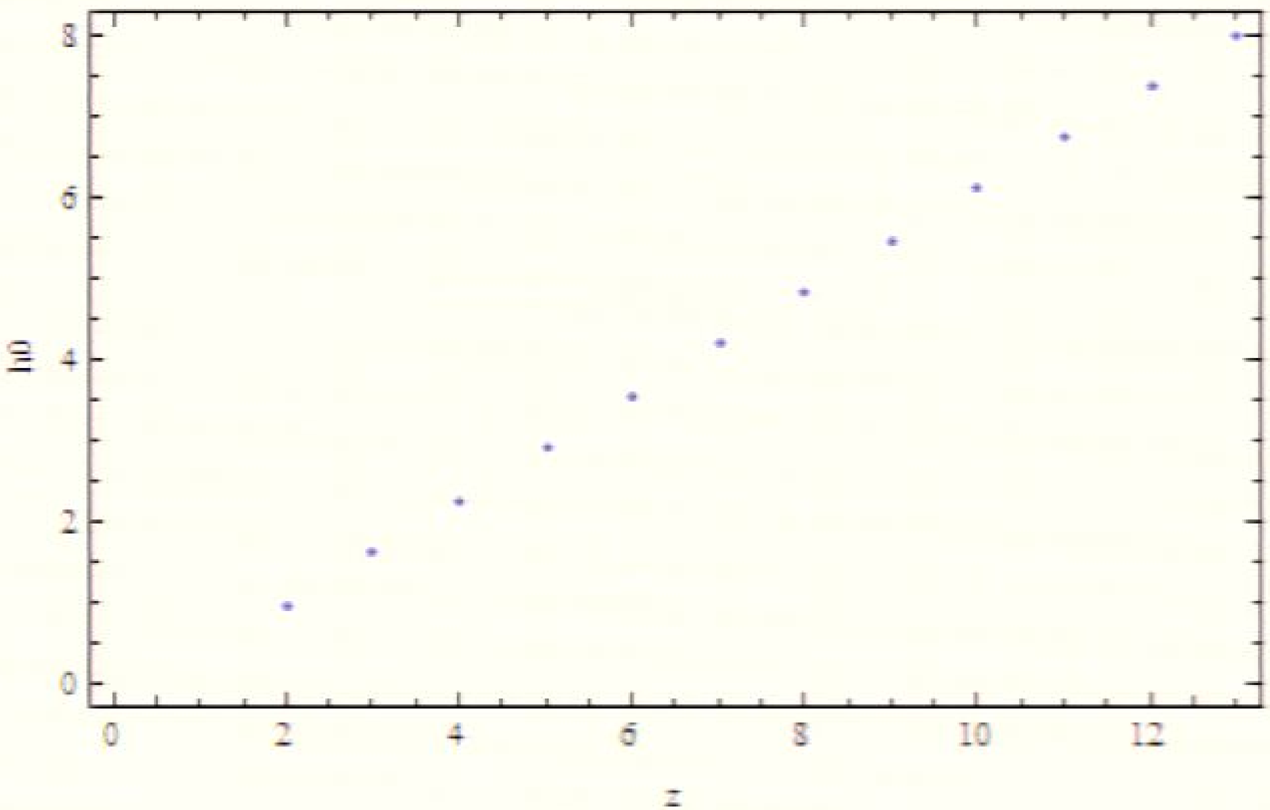


NOTE

- A value of h_0 cannot be bigger than h_{max}.
- This is because when r goes from the horizon to the boundary, the metric function grow exponentially, so it can never reach the boundary.
- Physically, this is the situation in which the deformation is turned off and we recover a black hole in the pure Lifshitz spacetime.

Results 1 : Extracting Λ

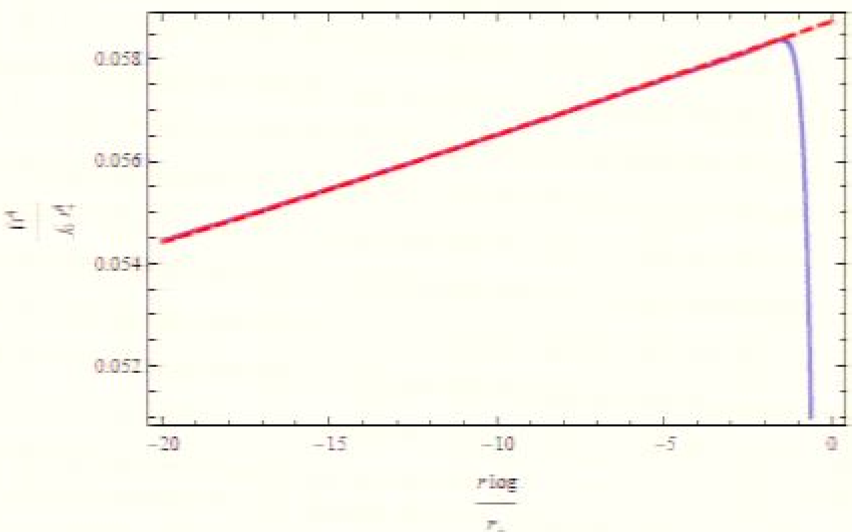
- Maximum value of h_0 on z

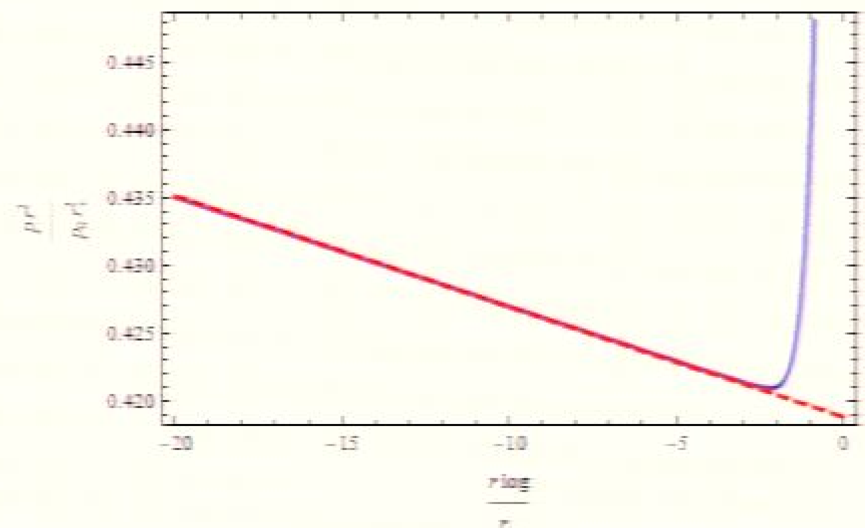


- Maximum value of h_0 is linearly increased on z.

Results 2 : Extracting f_0 and p_0

- $z = 2$ and $h_0 = 0.9705$

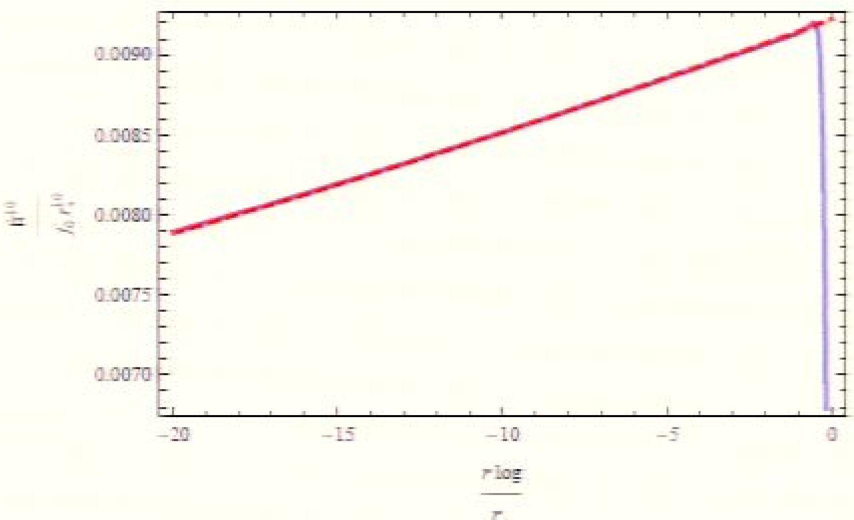


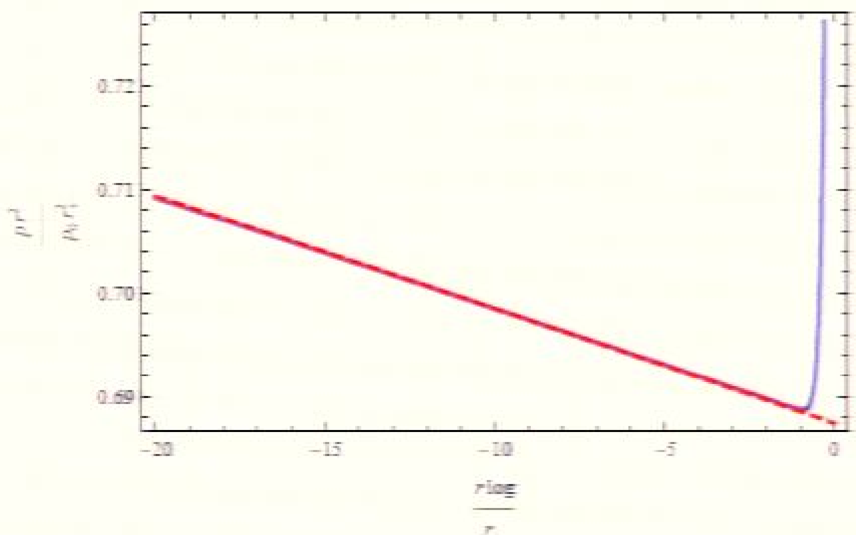


- $f_0 r_+^4 = 18.37$ and $p_0 r_+^2 = 2.3009 \;\Rightarrow\; log(\frac{\Lambda^2}{T}) \sim -2104.02$

Results 2 : Extracting f_0 and p_0

- $z = 5$ and $h_0 = 2.9247$

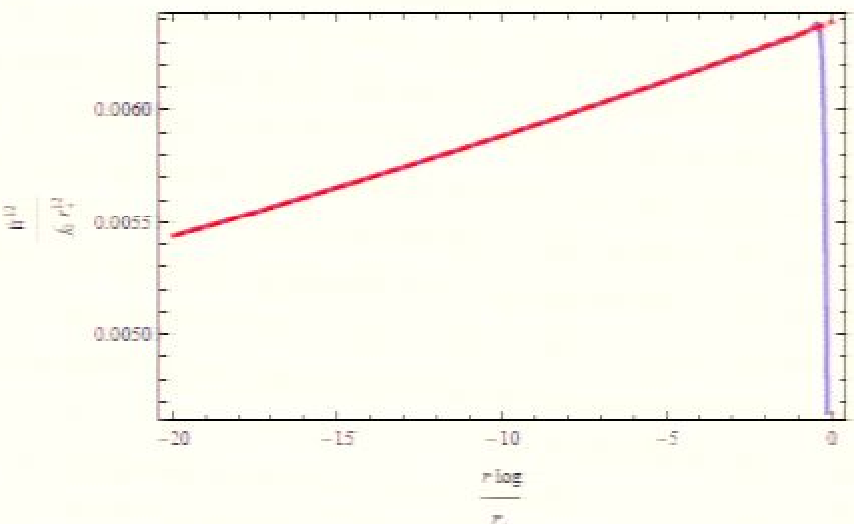


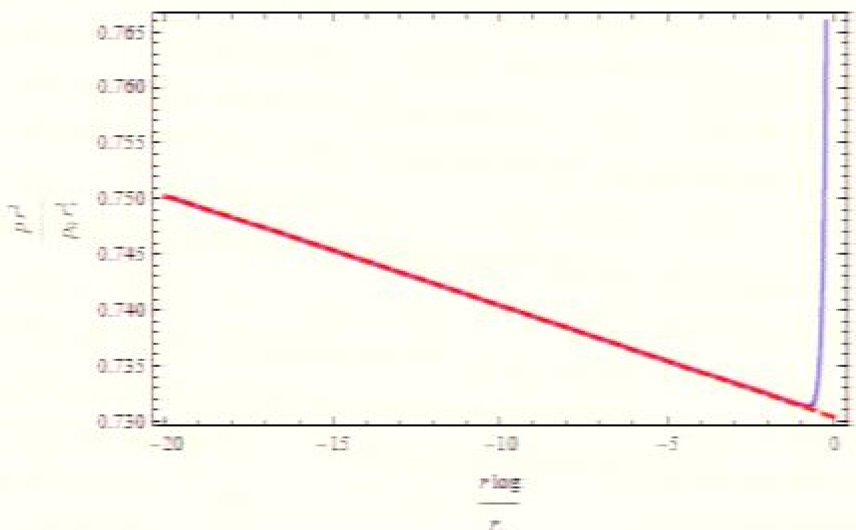


- $f_0 r_+^{10} = 109.46$ and $p_0 r_+^2 = 1.4520 \;\Rightarrow\; log(\frac{\Lambda^2}{T}) \sim -620.51$

Results 2 : Extracting f_0 and p_0

- $z = 6$ and $h_0 = 3.566$

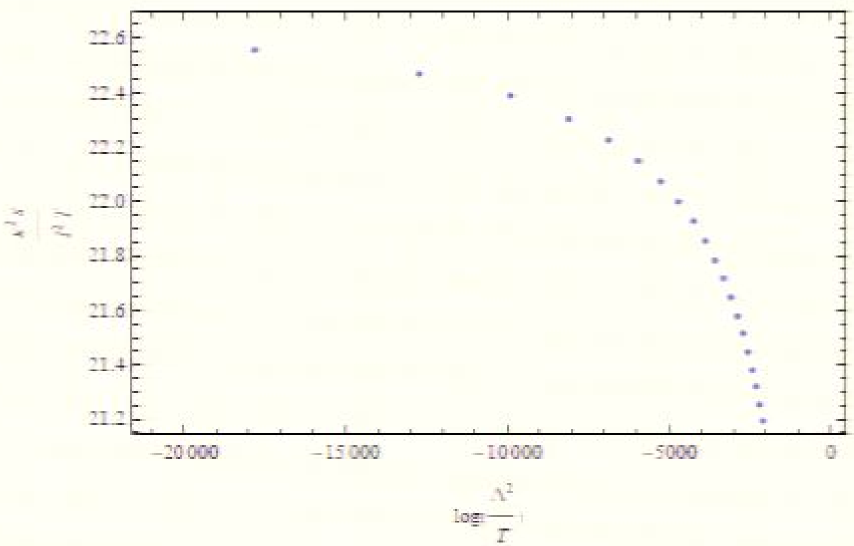


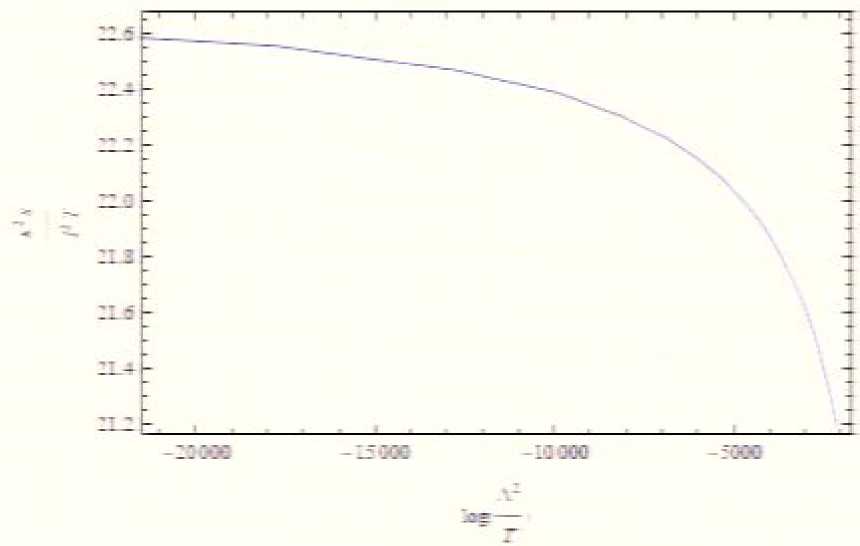


- $f_0 r_+^{12} = 157.46$ and $p_0 r_+^2 = 1.3679 \;\Rightarrow\; log(\frac{\Lambda^2}{T}) \sim -576.292$

Results 3 : Entropy density over T versus $log(\frac{\Lambda^2}{T})$

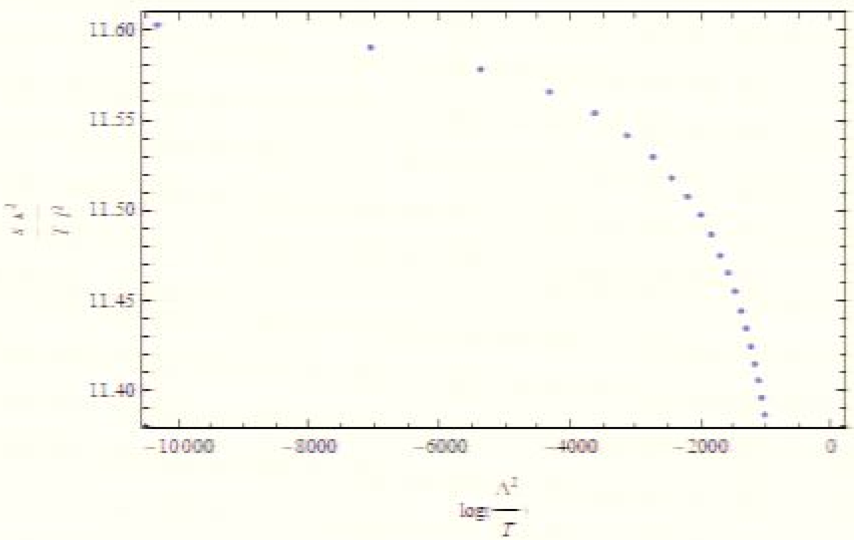
- $z = 2$ and $h_0 = 0.9705 \sim 0.9713$ $(log\frac{\Lambda^2}{T} = -2104.02 \sim -29694.2)$

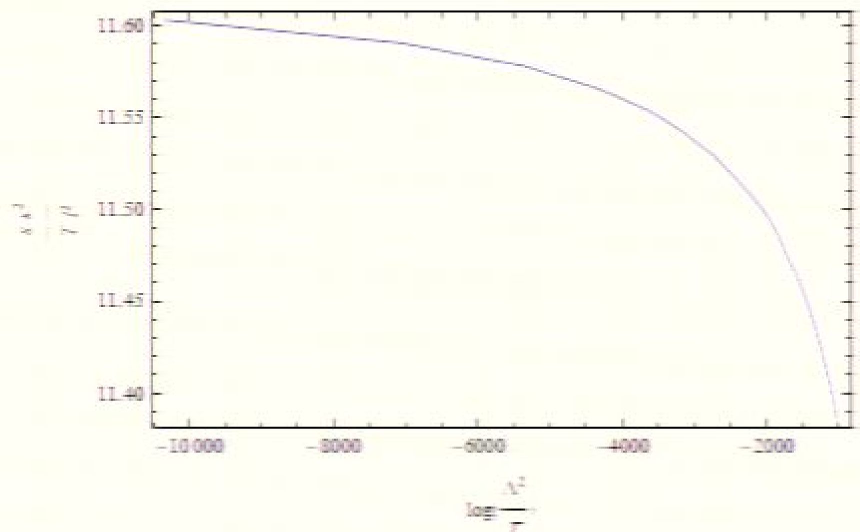




Results 3 : Entropy density over T versus $log(\frac{\Lambda^2}{T})$

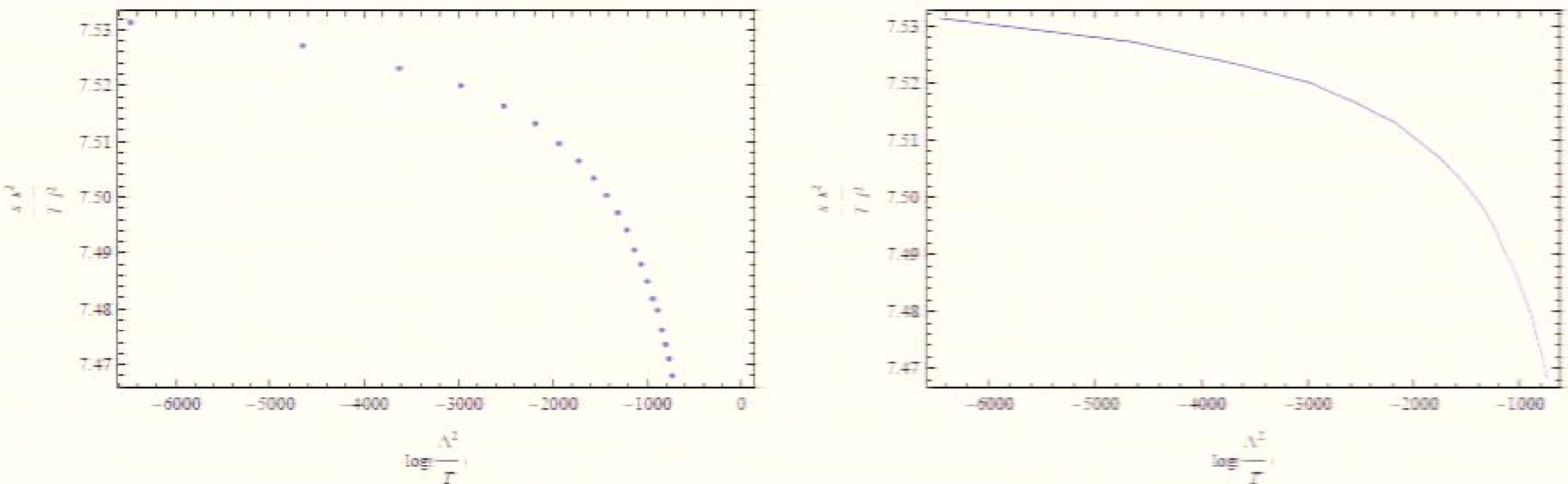
- $z = 3$ and $h_0 = 1.6335 \sim 1.6343$ ($log\frac{\Lambda^2}{T} = -1015 \sim -10337$)





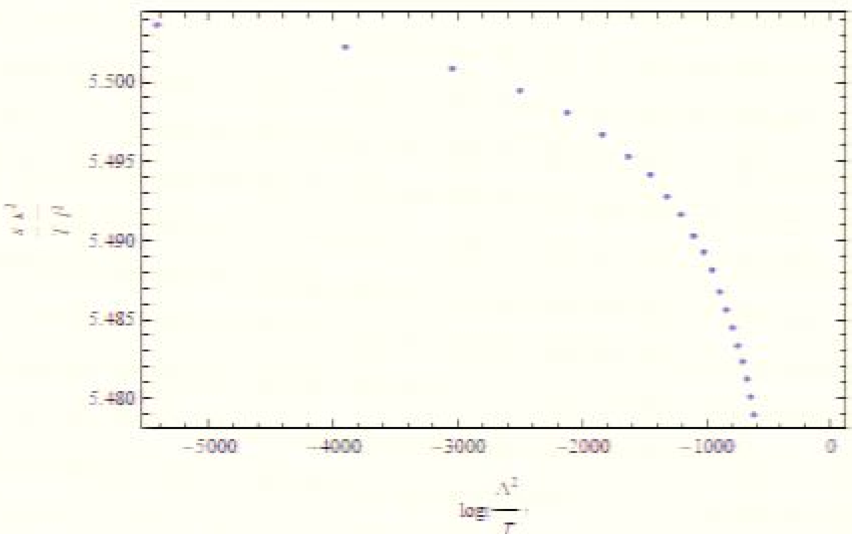
Results 3 : Entropy density over T versus $log(\frac{\Lambda^2}{T})$

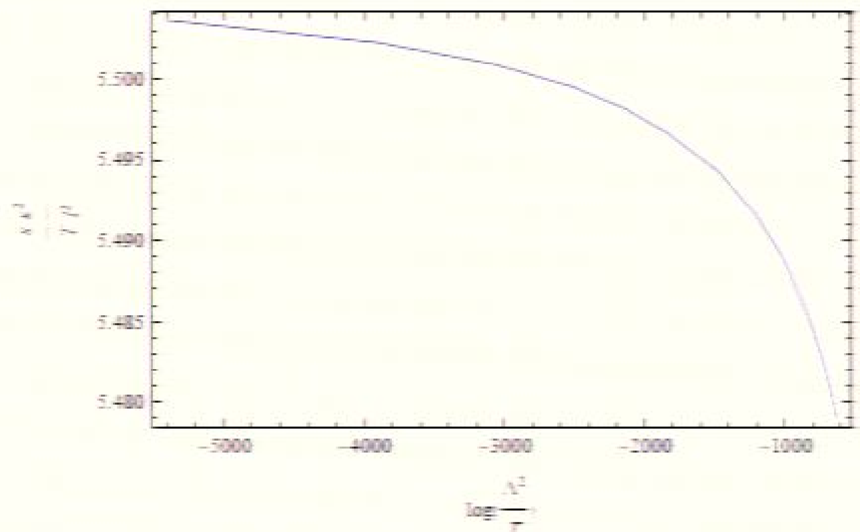
- $z = 4$ and $h_0 = 2.2822 \sim 2.2814$ $(log\frac{\Lambda^2}{T} = -736.488 \sim -6479.28)$



Results 3 : Entropy density over T versus $log(\frac{\Lambda^2}{T})$

- $z = 5$ and $h_0 = 2.9255 \sim 2.9247$ ($log\frac{\Lambda^2}{T} = -620.51 \sim -5422.71$)





Next things to do

- find that Free energy density vs $log(\frac{\Lambda^2}{T})$
- apply this method to Lovelock gravity