Title: What Can Gauge-Gravity Duality Teach us About Condensed Matter Physics?

Date: Jul 14, 2011 03:00 PM

URL: http://pirsa.org/11070047

Abstract: TBA

Outline

1. Conformal quantum matter

2. Compressible quantum matter

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1. Conformal quantum matter

   The AdS_4 - Schwarzschild black brane

2. Compressible quantum matter

   The AdS_4 - Reissner-Nordström black-brane and $AdS_2 \times R^2$

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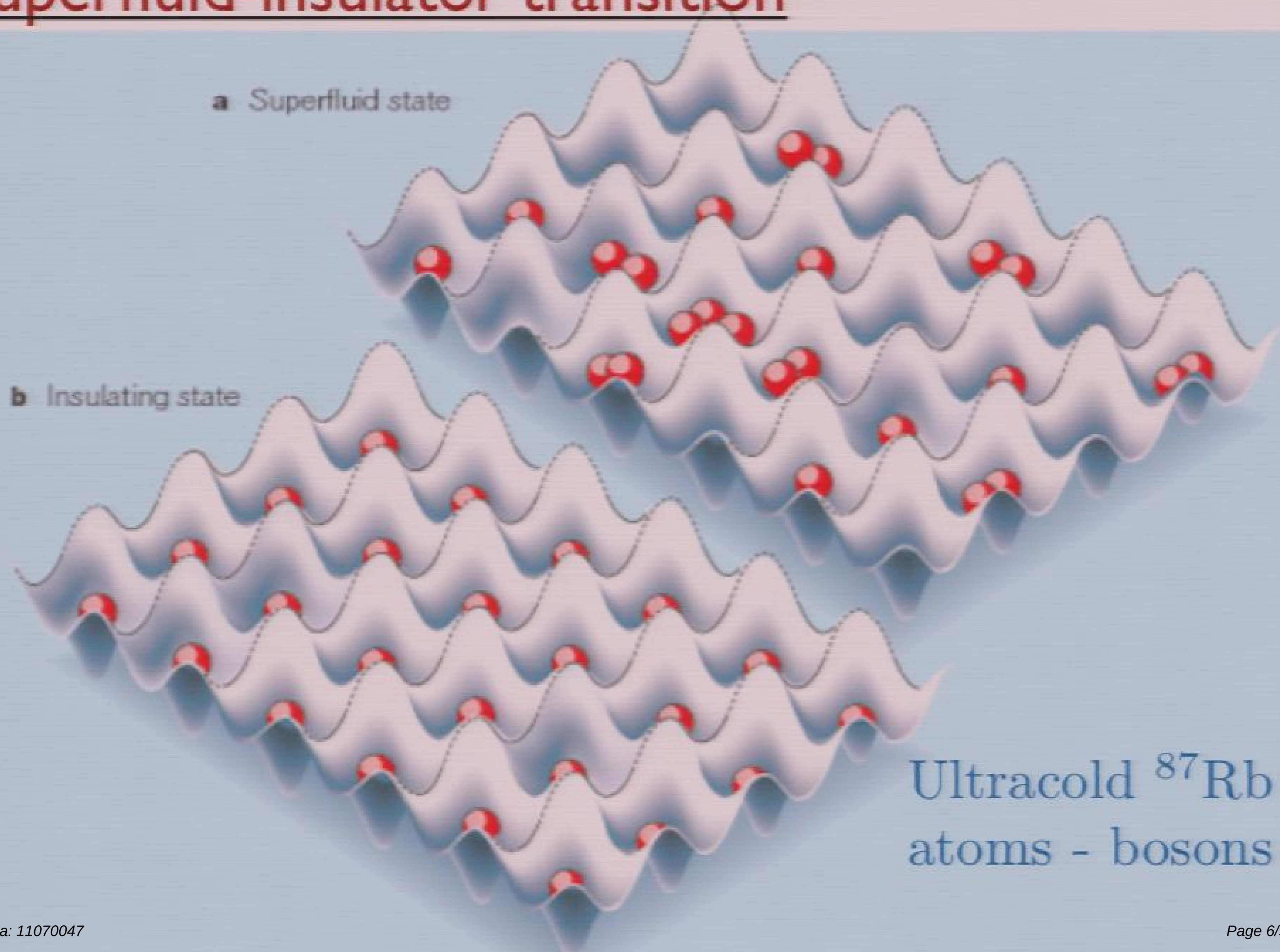
Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

$$H = -t\sum_{\langle ij \rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

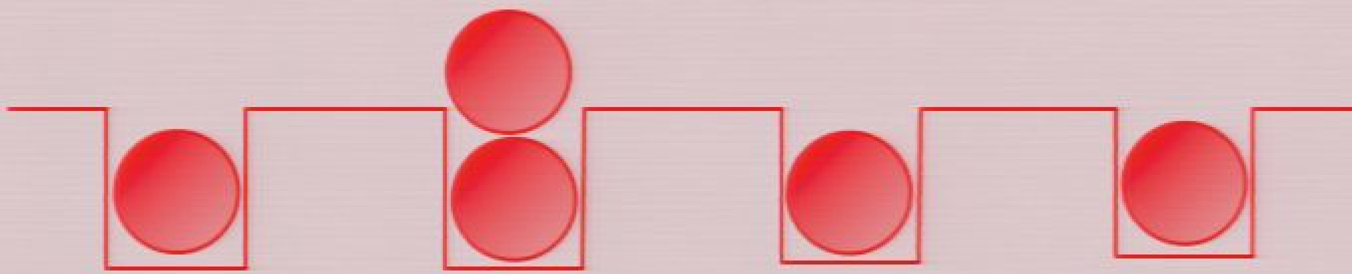
$$n_j \equiv b_j^\dagger b_j$$

$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

Insulator (the vacuum)
at large repulsion between bosons

Excitations of the insulator:



Particles $\sim \psi^\dagger$

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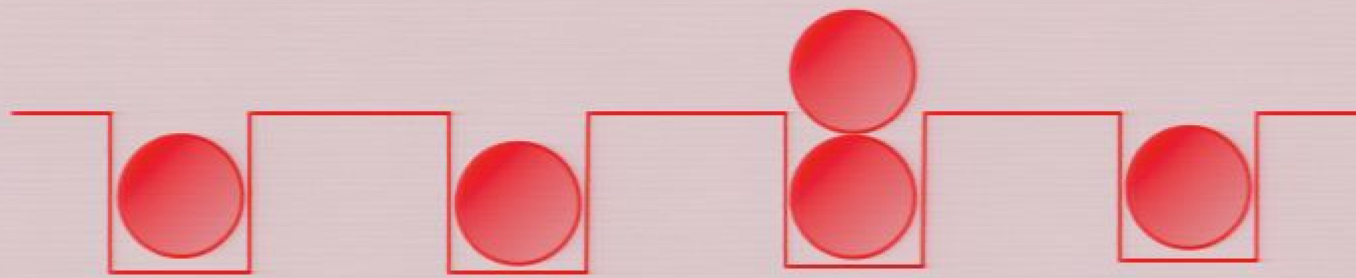
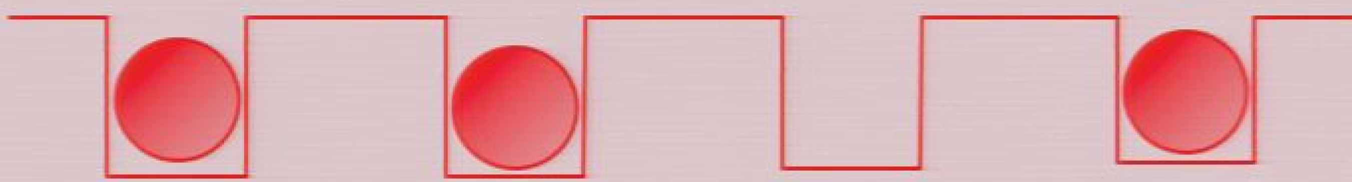


Particles ~ ψ †



Holes ~ ψ

Density of particles = density of holes ⇒
"relativistic" field theory for ψ:

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla} \psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

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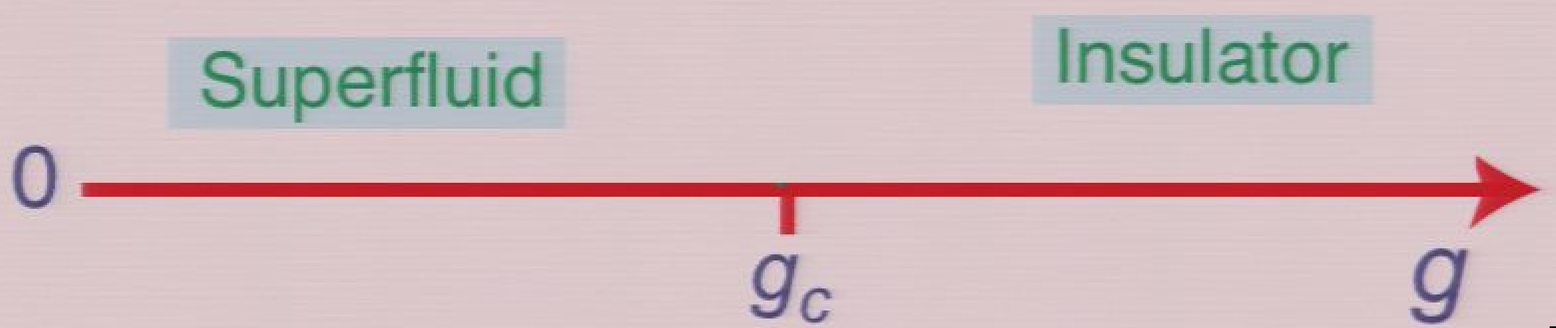
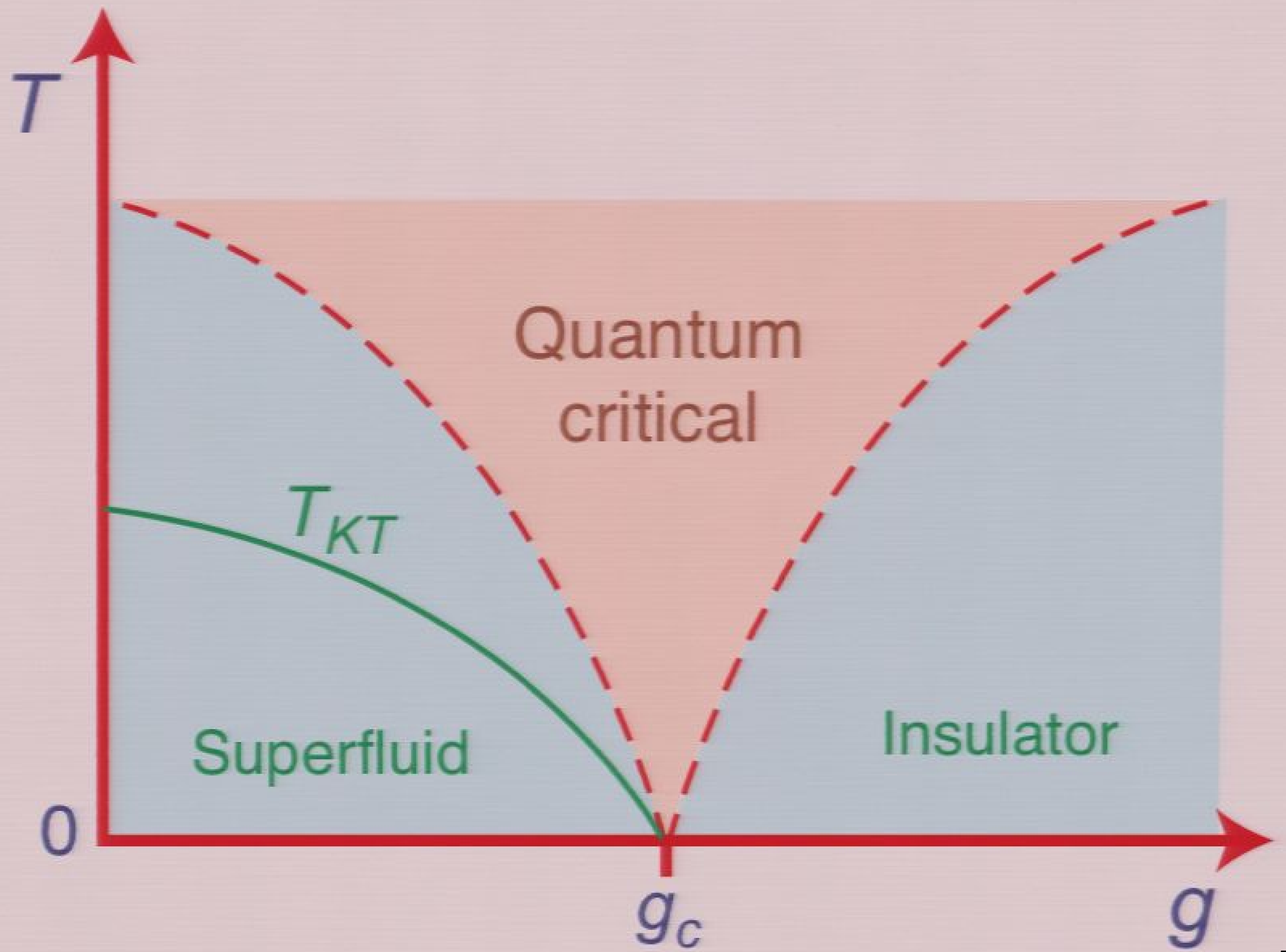
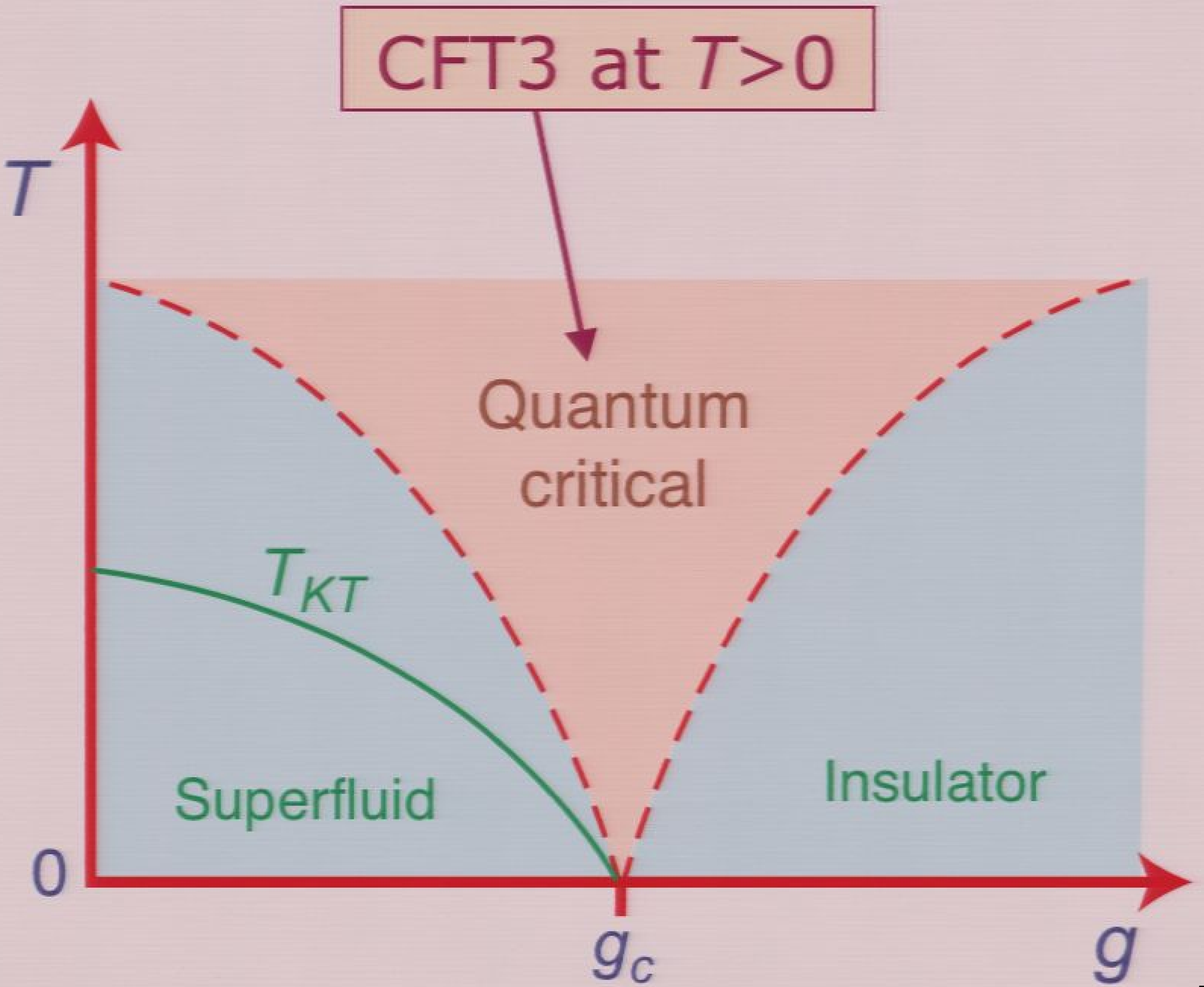
$\langle\psi\rangle \neq 0$ $\quad\quad\quad$ $\langle\psi\rangle = 0$





T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g


CFT3 at T>0
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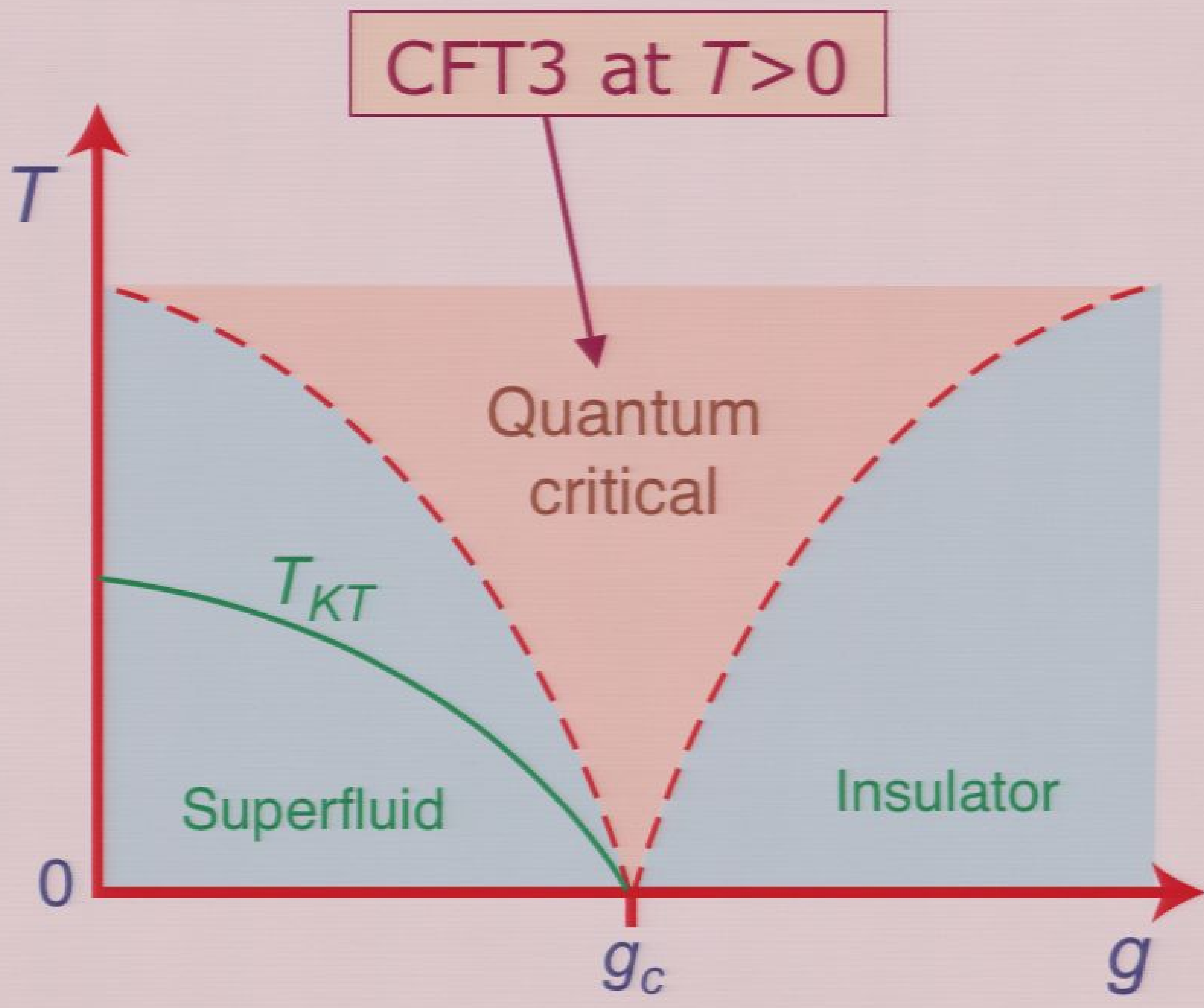
Quantum critical transport

Quantum *"nearly perfect fluid"* with shortest possible equilibration time, τ_{eq}

$$\tau_{\mathrm{eq}} = \mathcal{C} \frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions*, Cambridge (1999)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Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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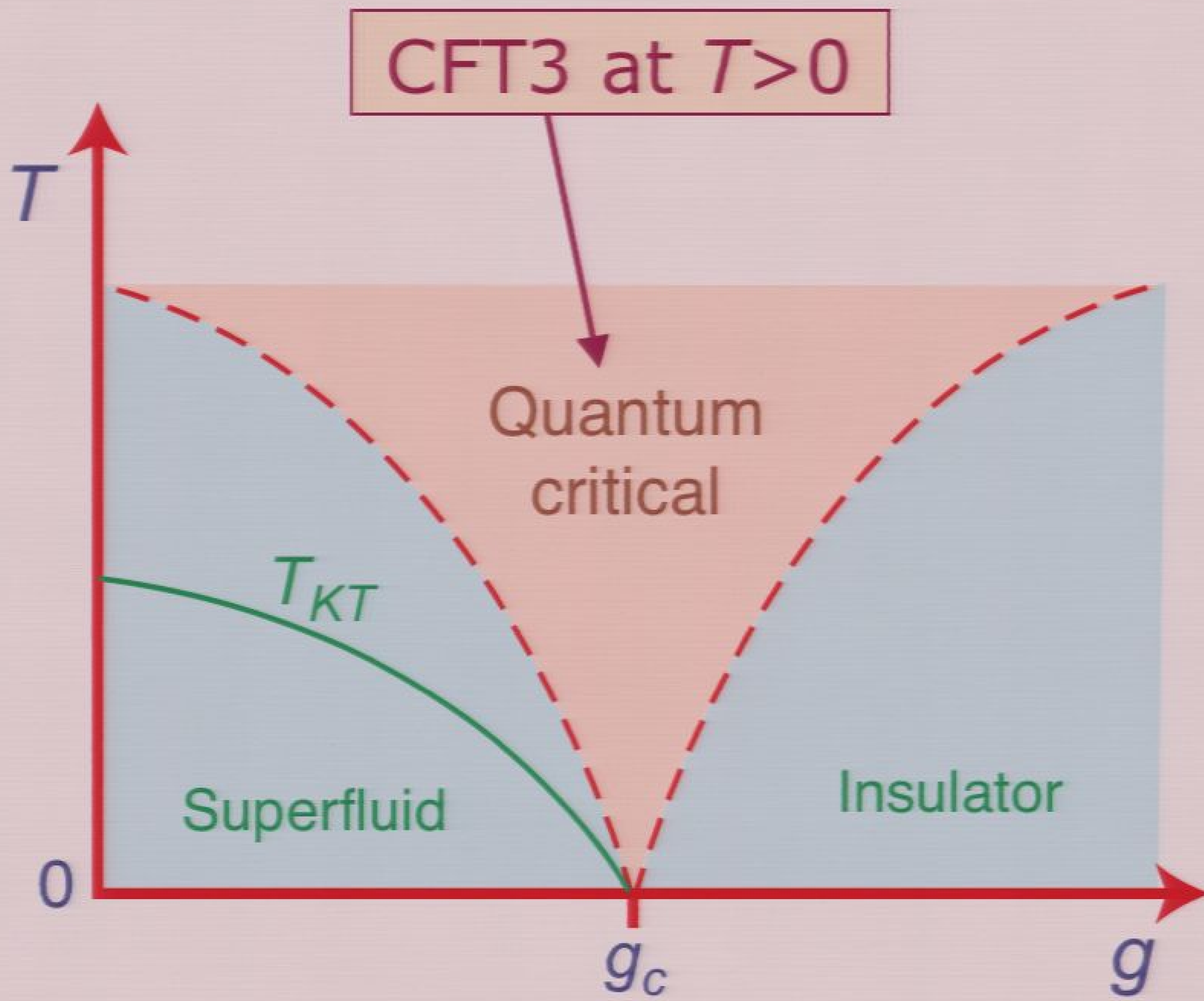
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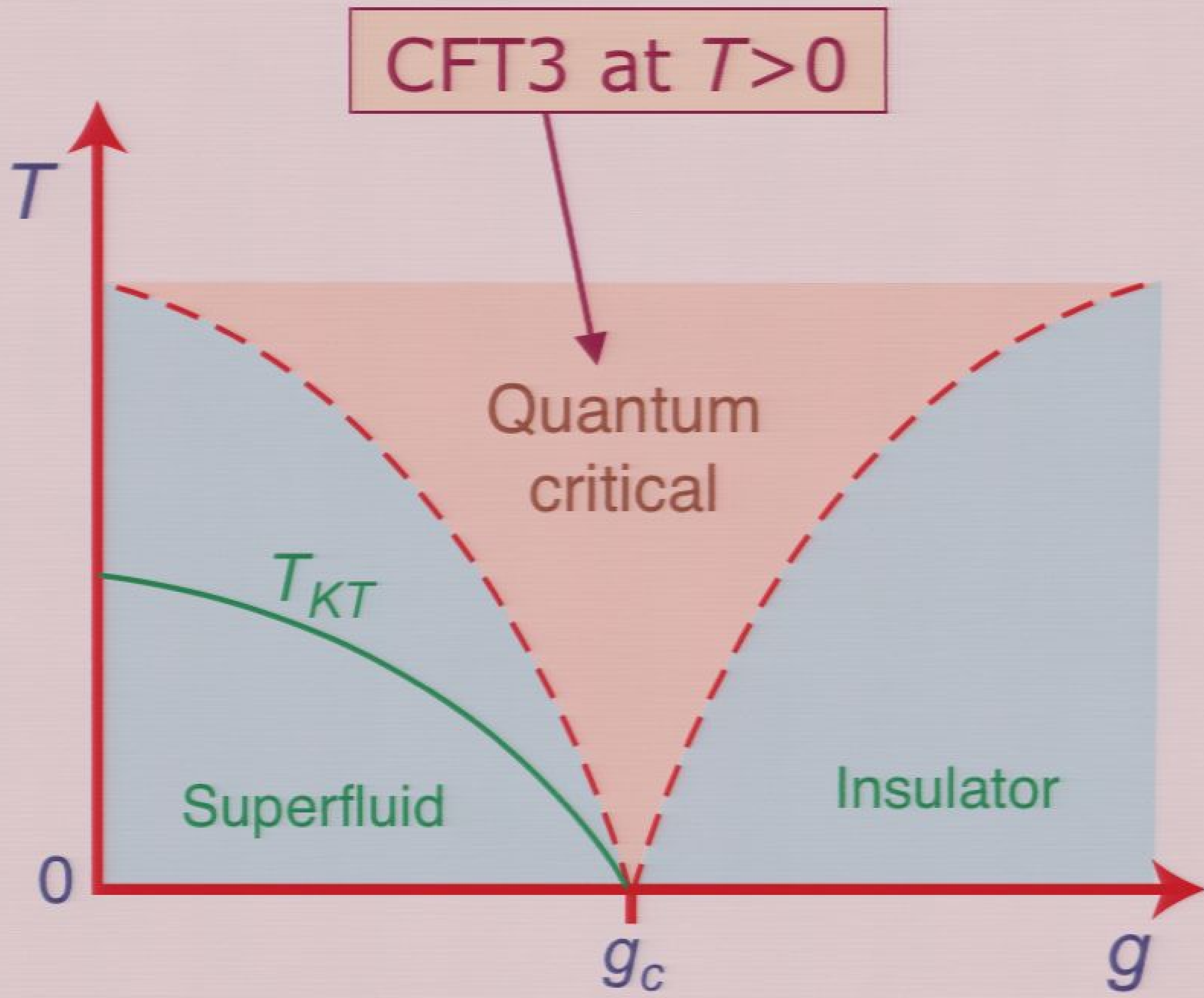
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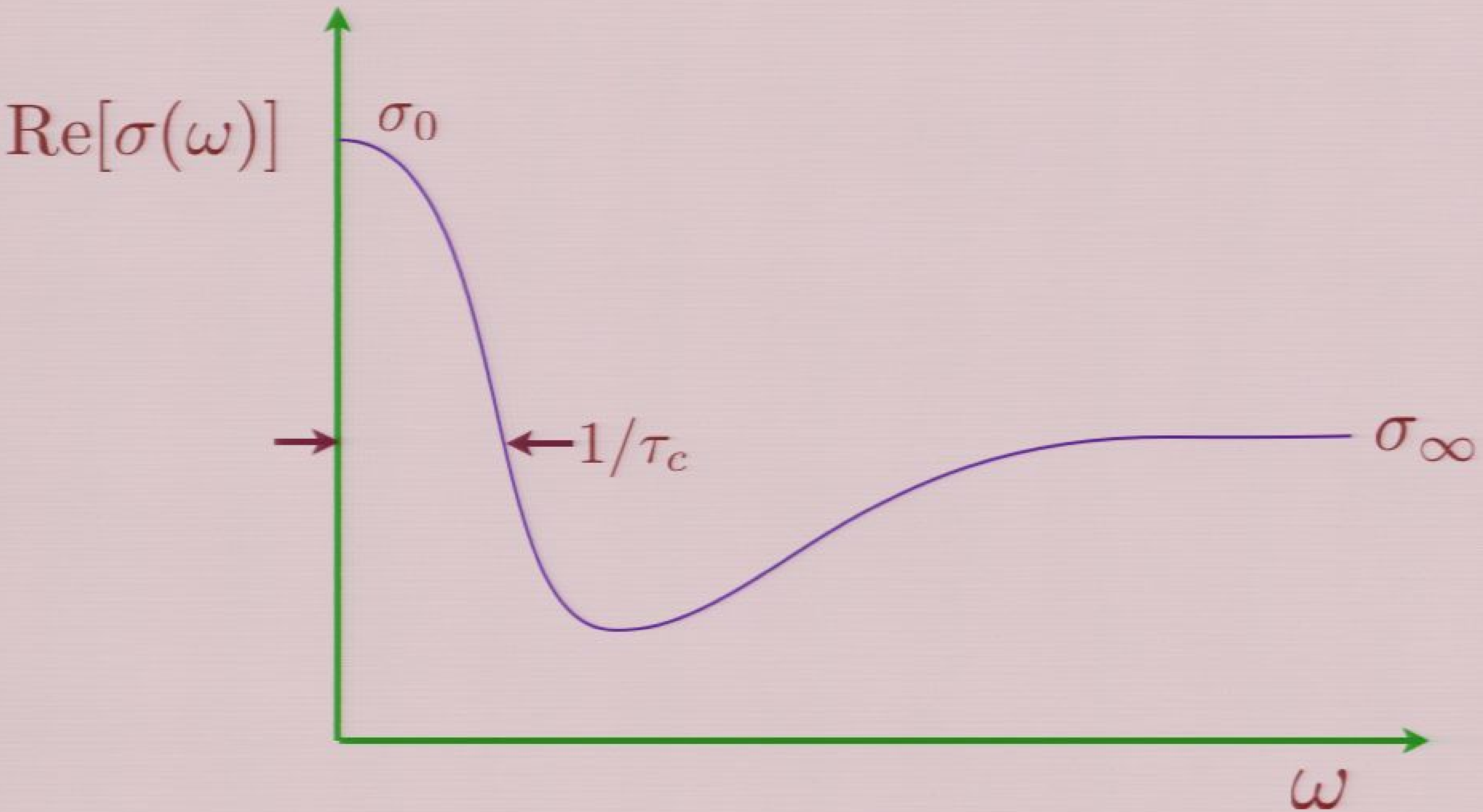
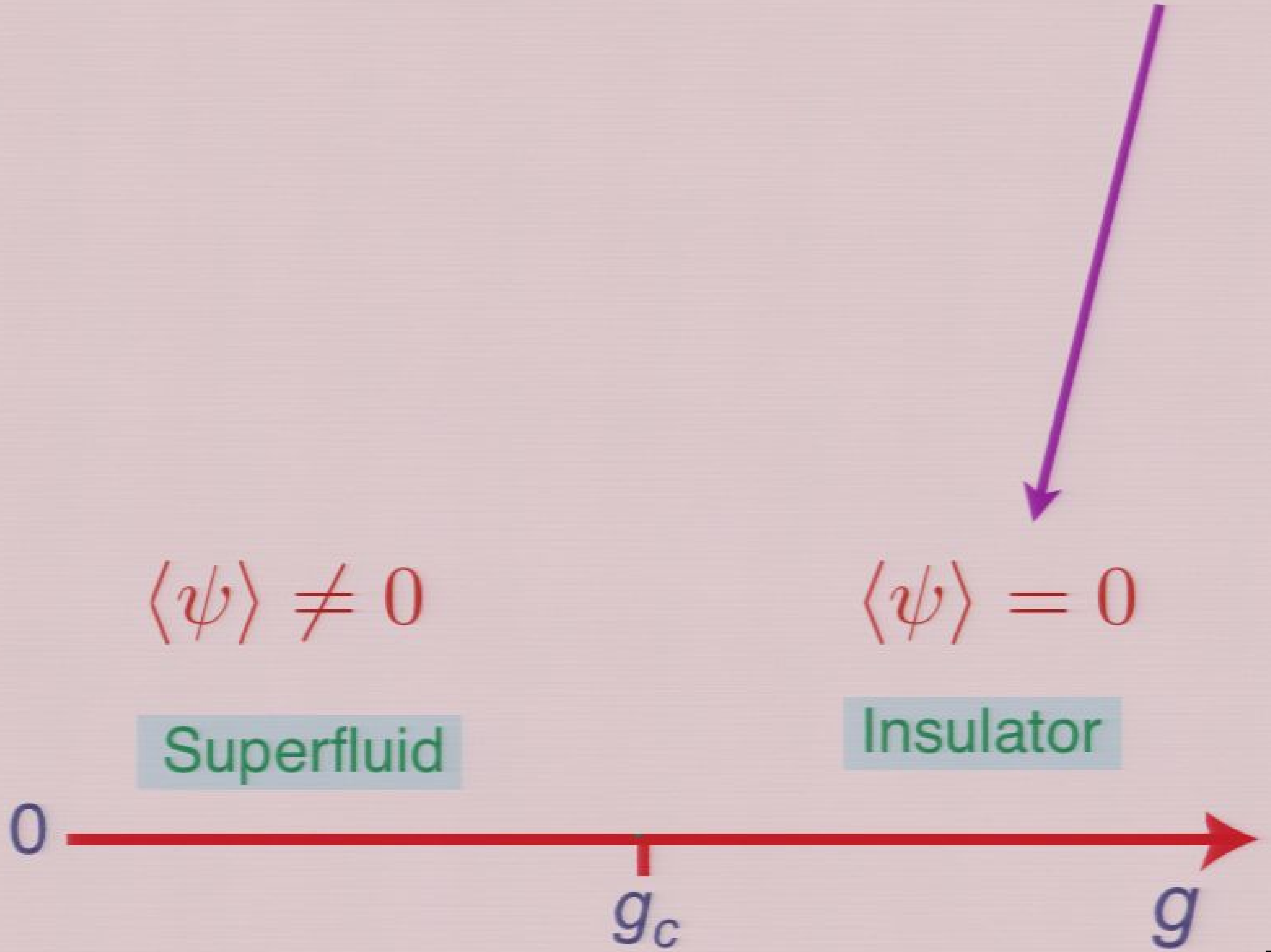
Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

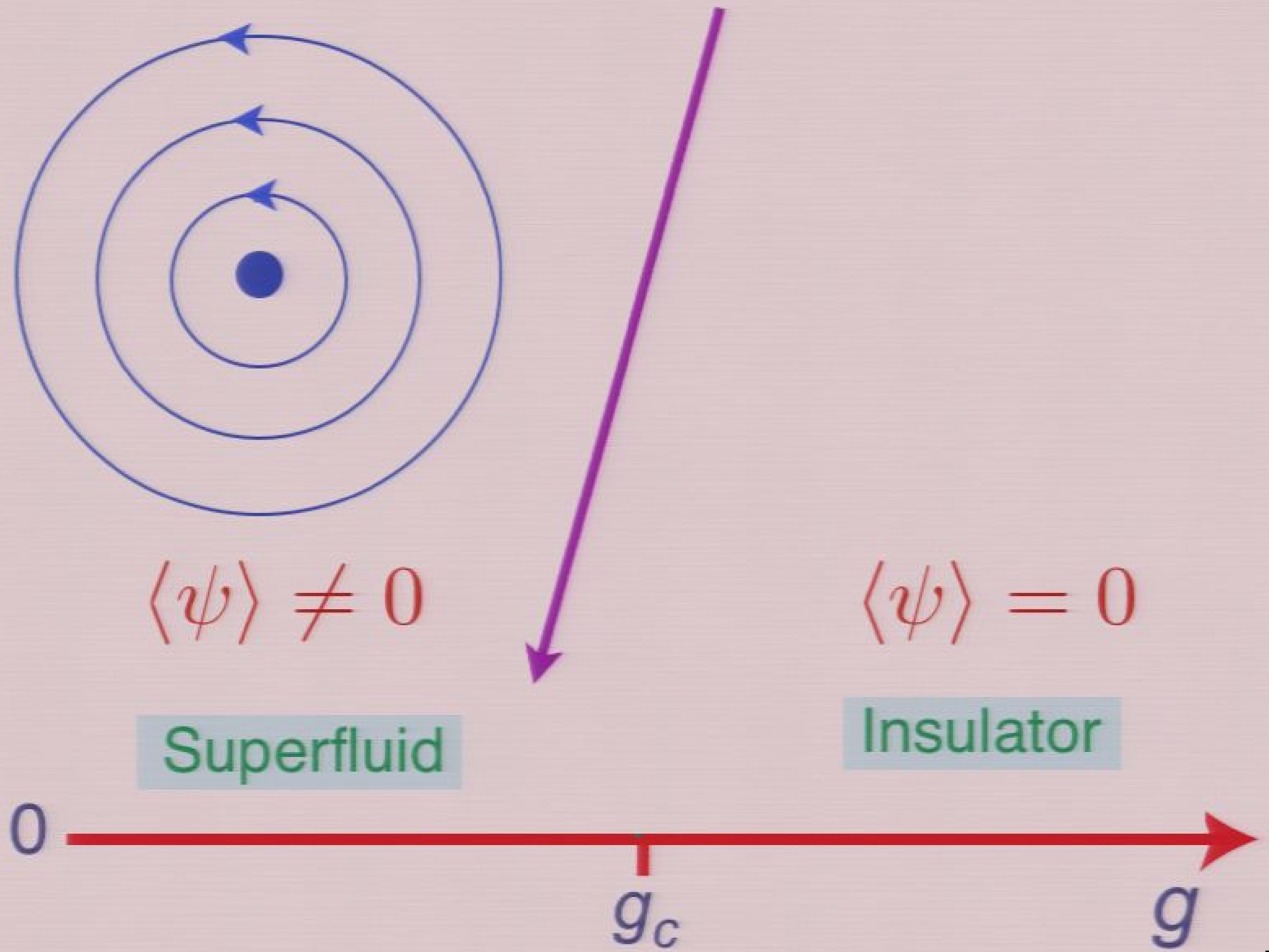
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



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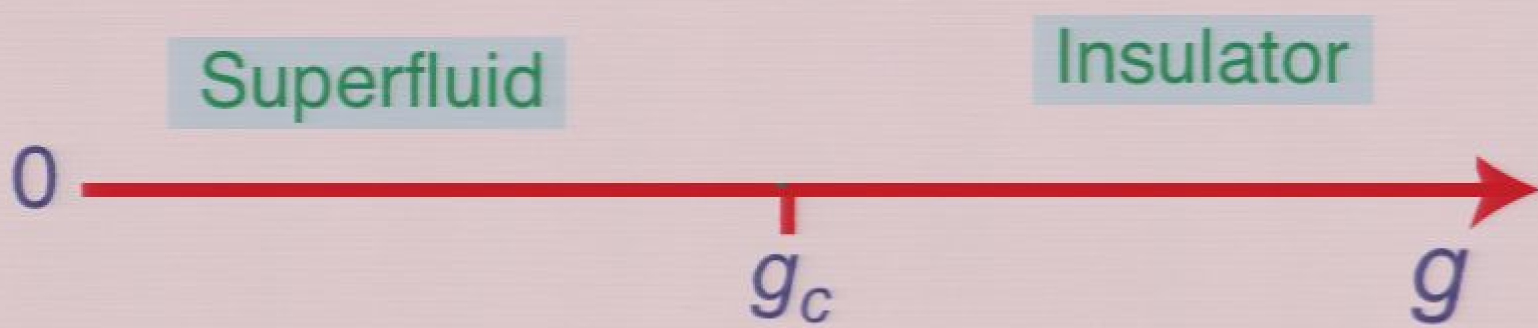
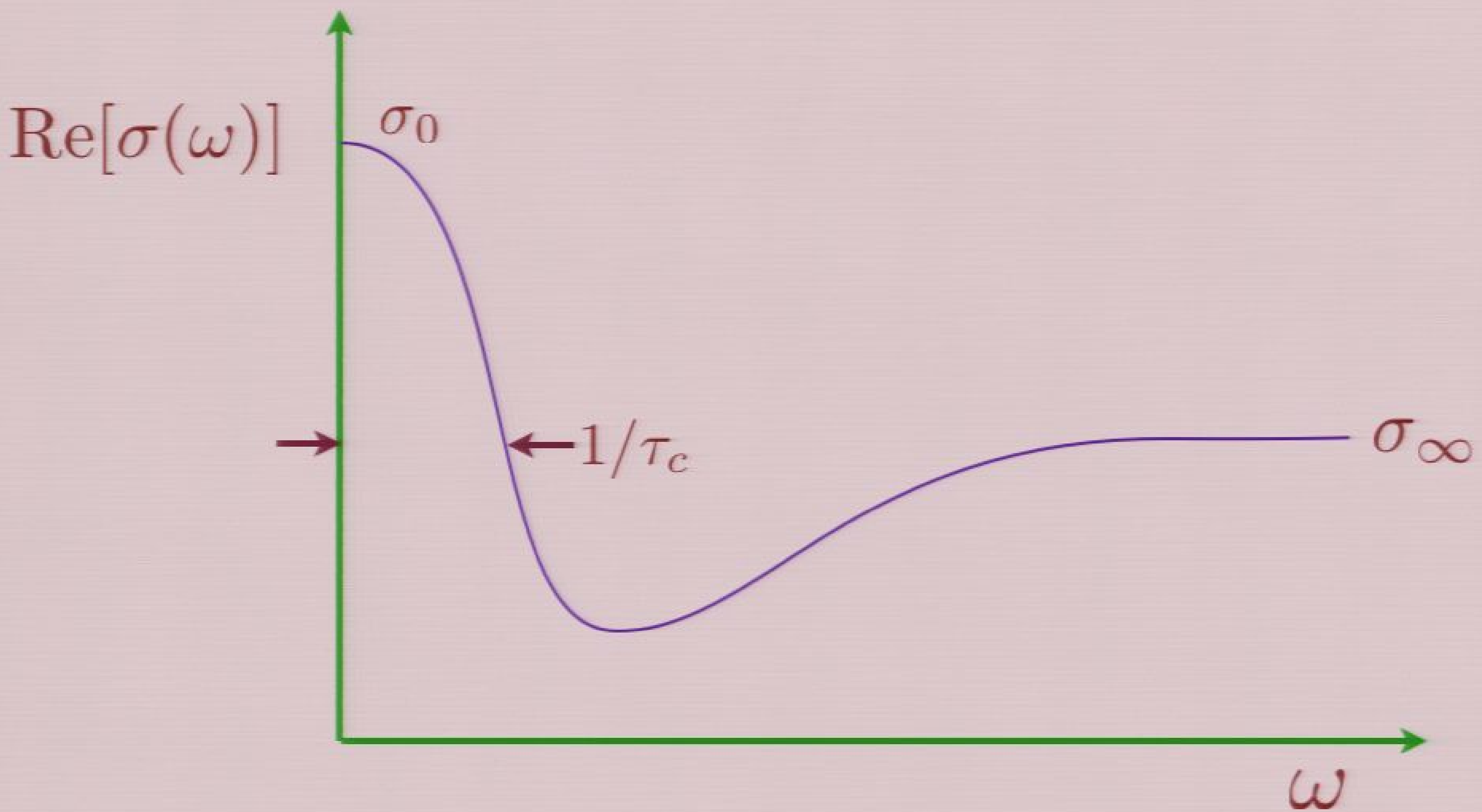
These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

$$\text{Conductivity} = \text{Resistivity of vortices}$$

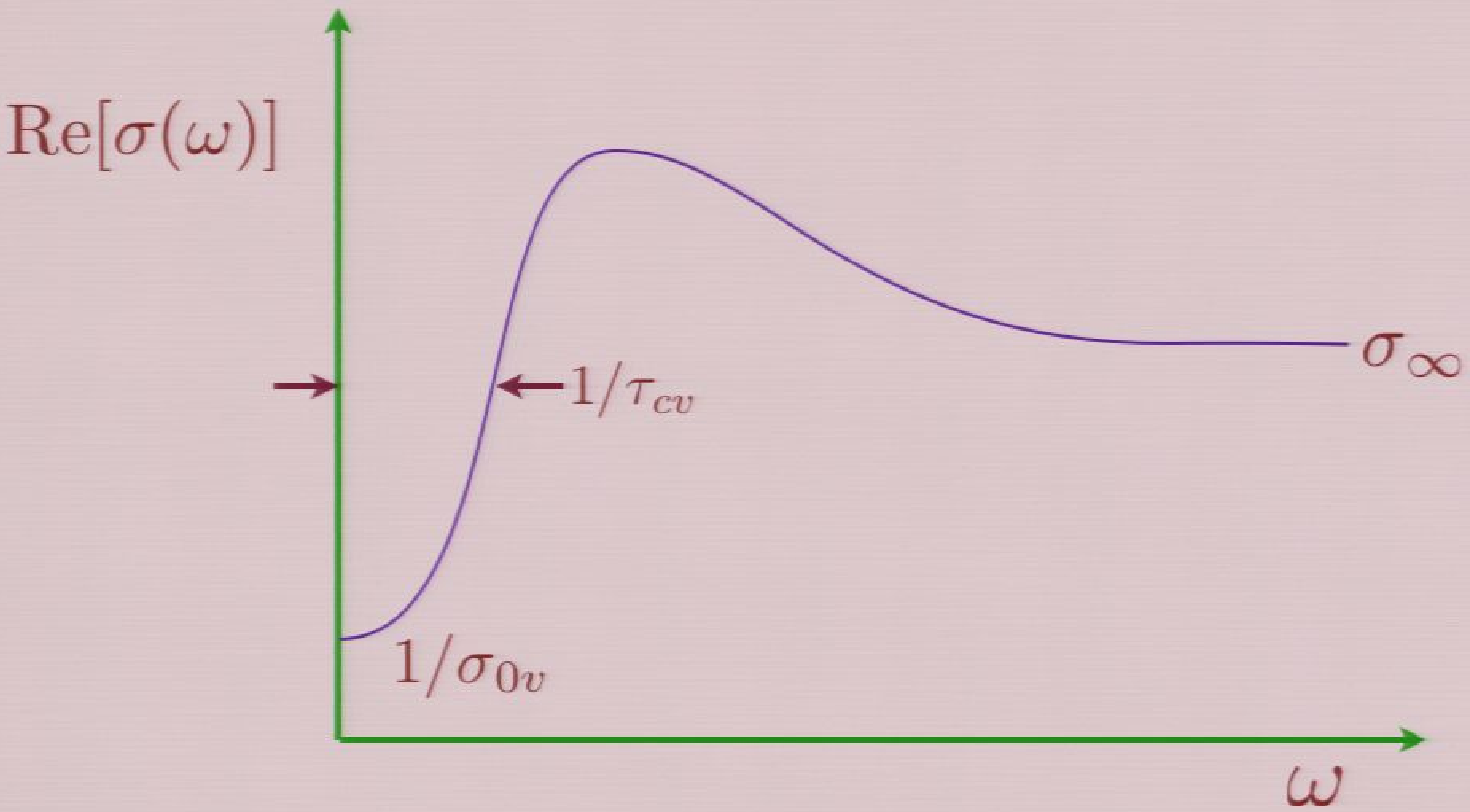
$$\langle \psi \rangle \neq 0 \qquad\qquad \langle \psi \rangle = 0$$



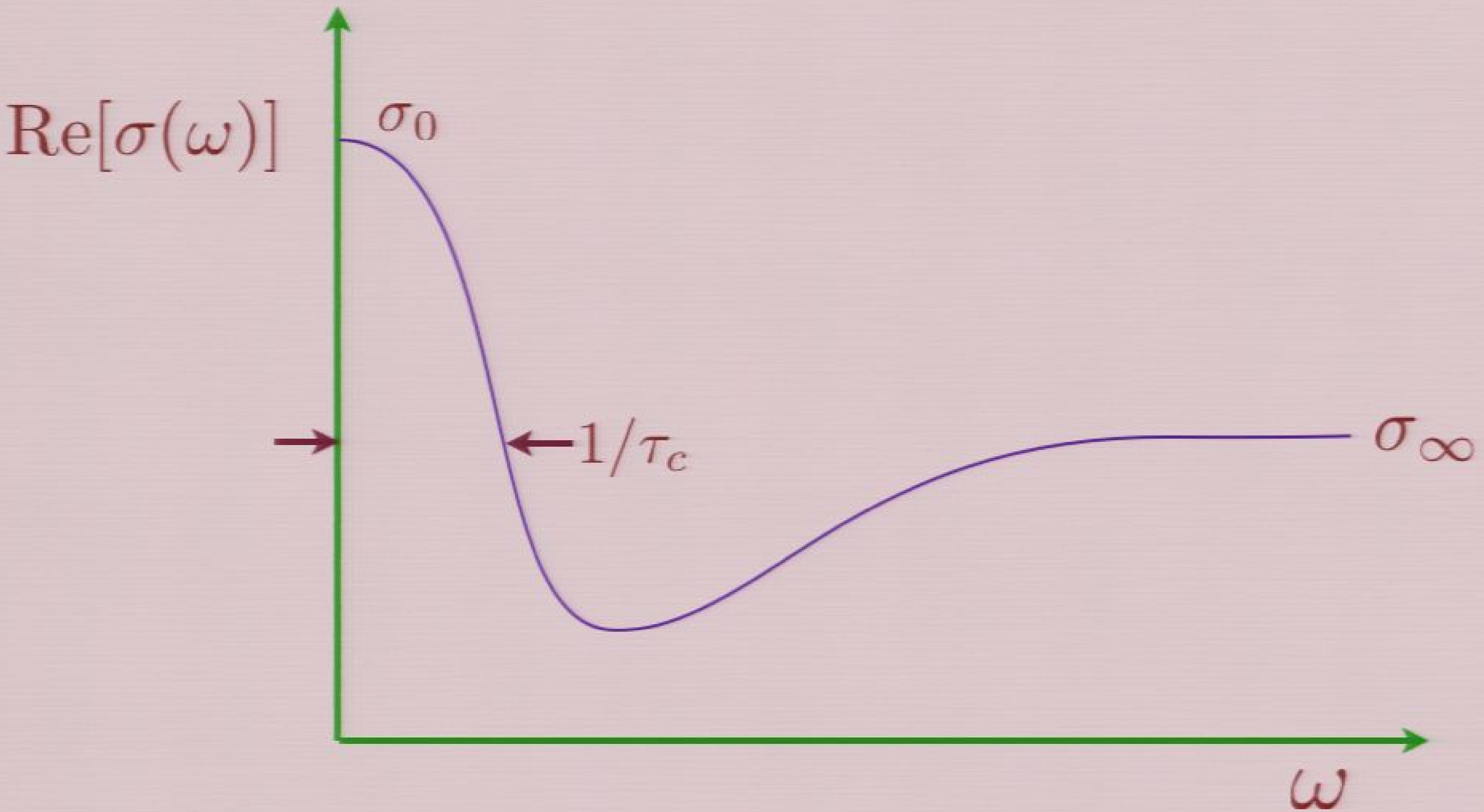
Boltzmann theory of bosons



Boltzmann theory of vortices

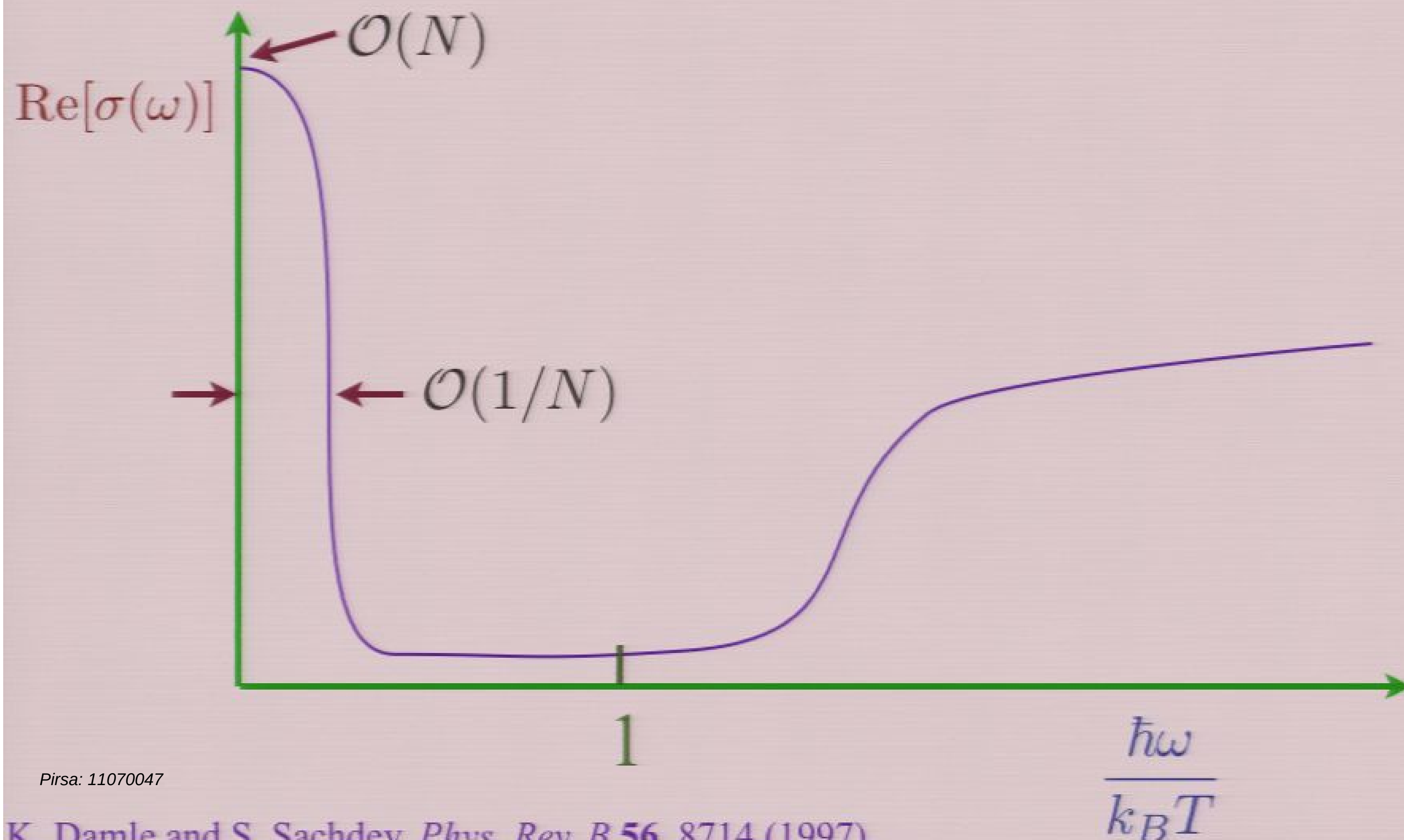


Boltzmann theory of bosons



Vector large N expansion for CFT3

$$\sigma = \frac{Q^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right); \quad \Sigma \to \text{a universal function}$$



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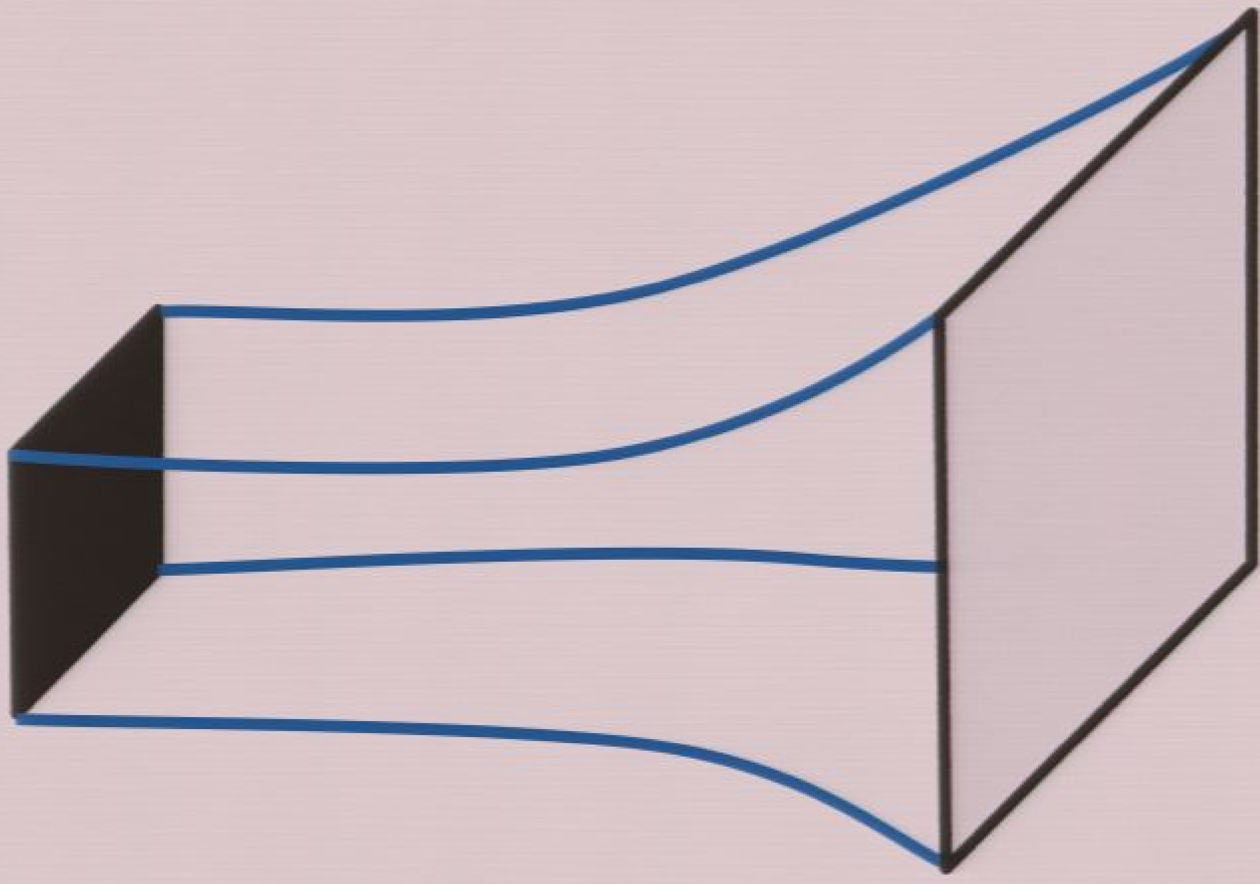
AdS/CFT correspondence

AdS_4-Schwarzschild black-brane



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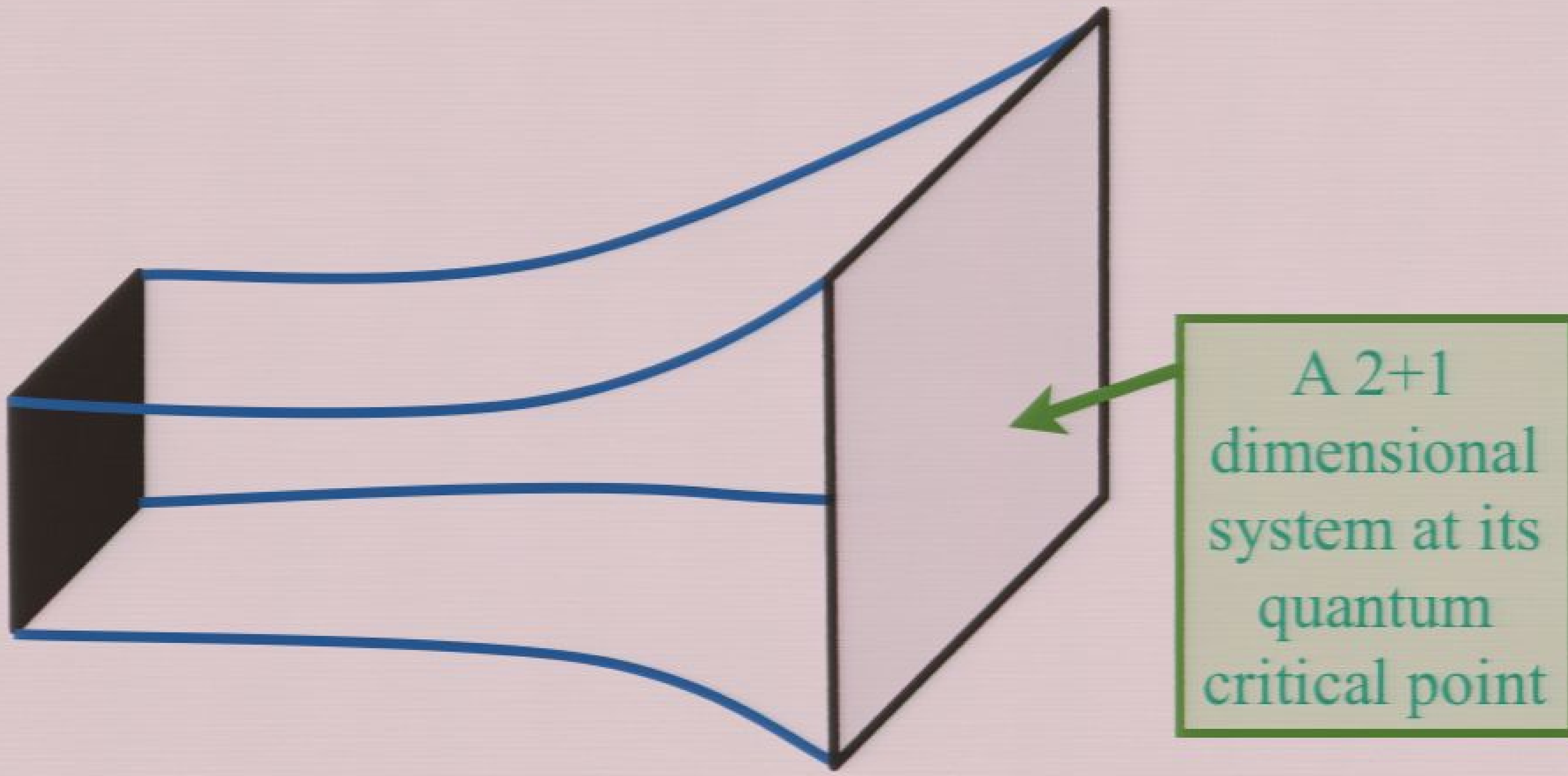

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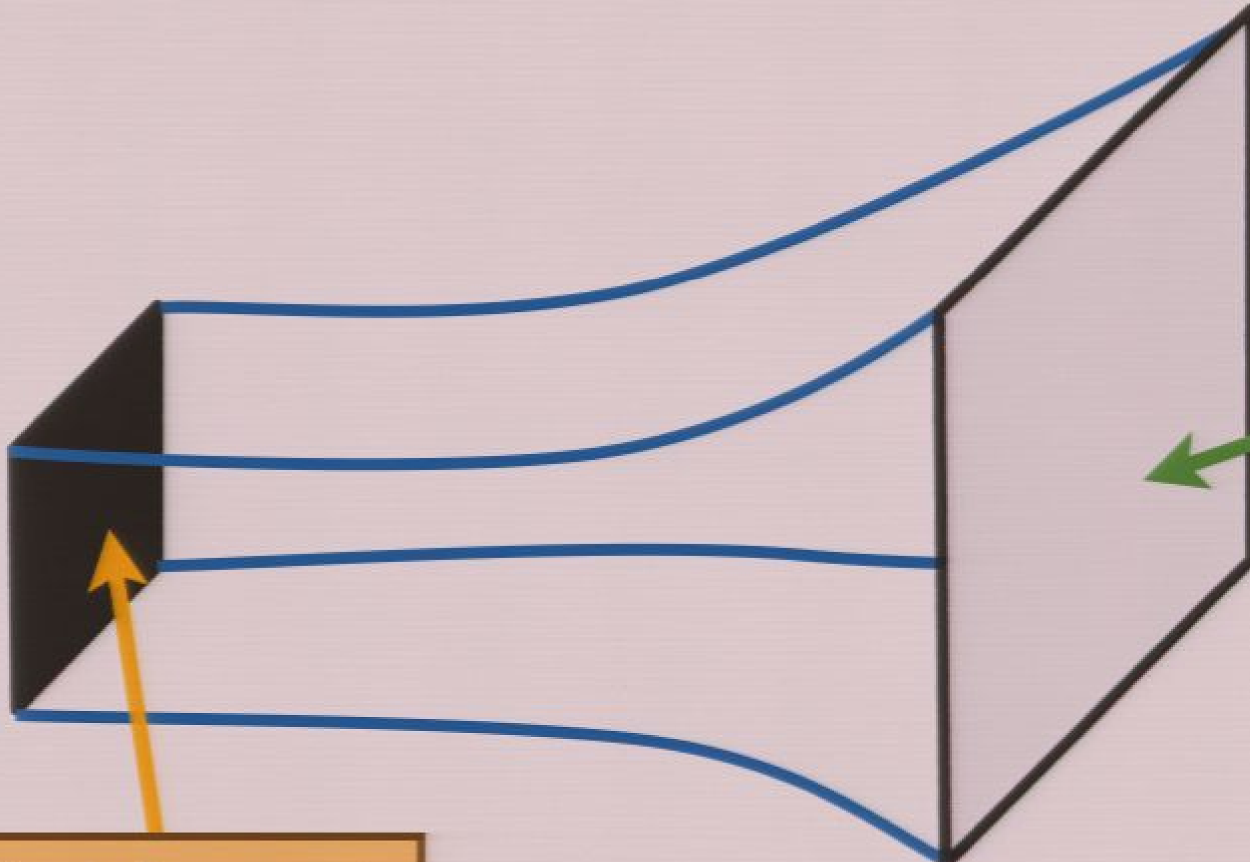

A 2+1 dimensional system at its quantum critical point

Black-brane at temperature of 2+1 dimensional quantum critical system

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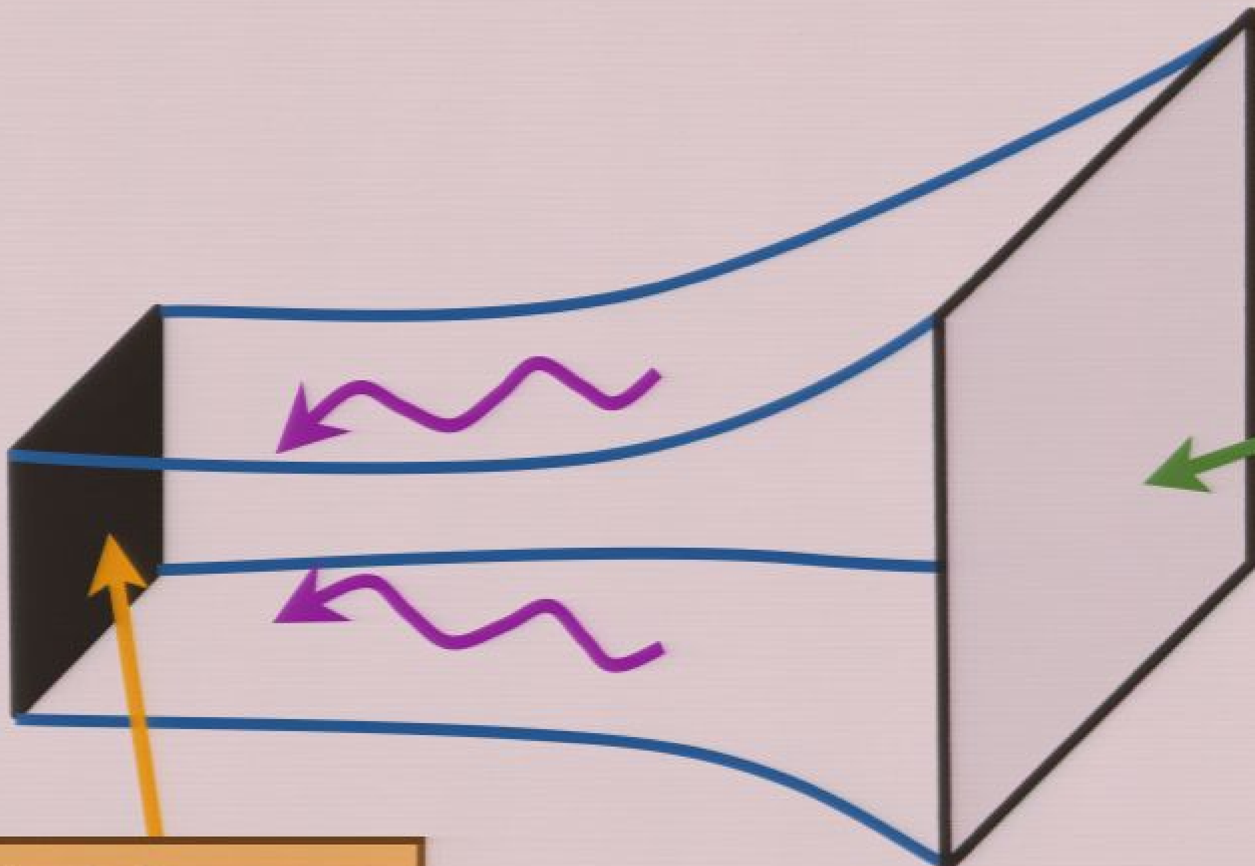

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Friction of quantum criticality = waves falling into black brane

AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4e^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

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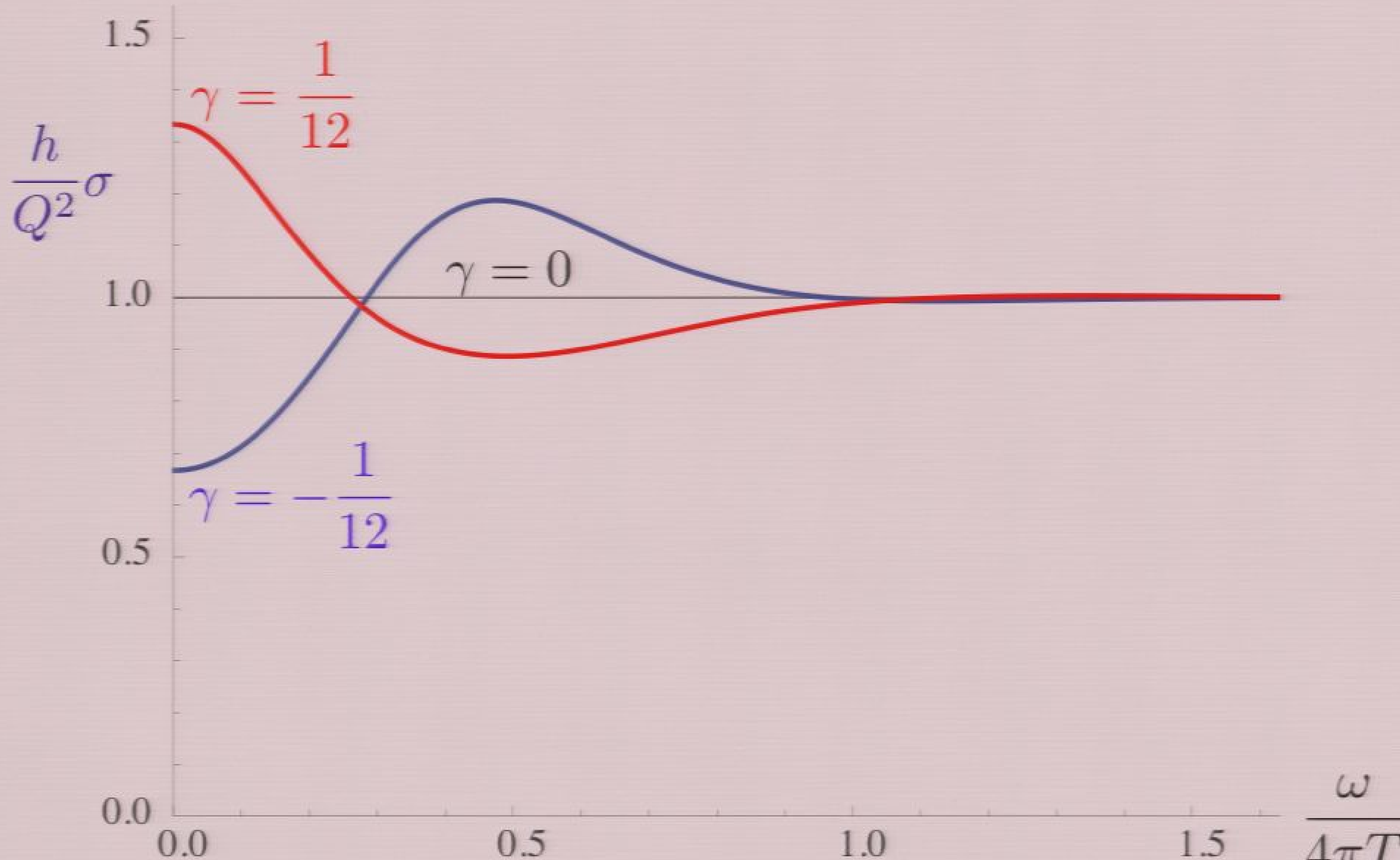

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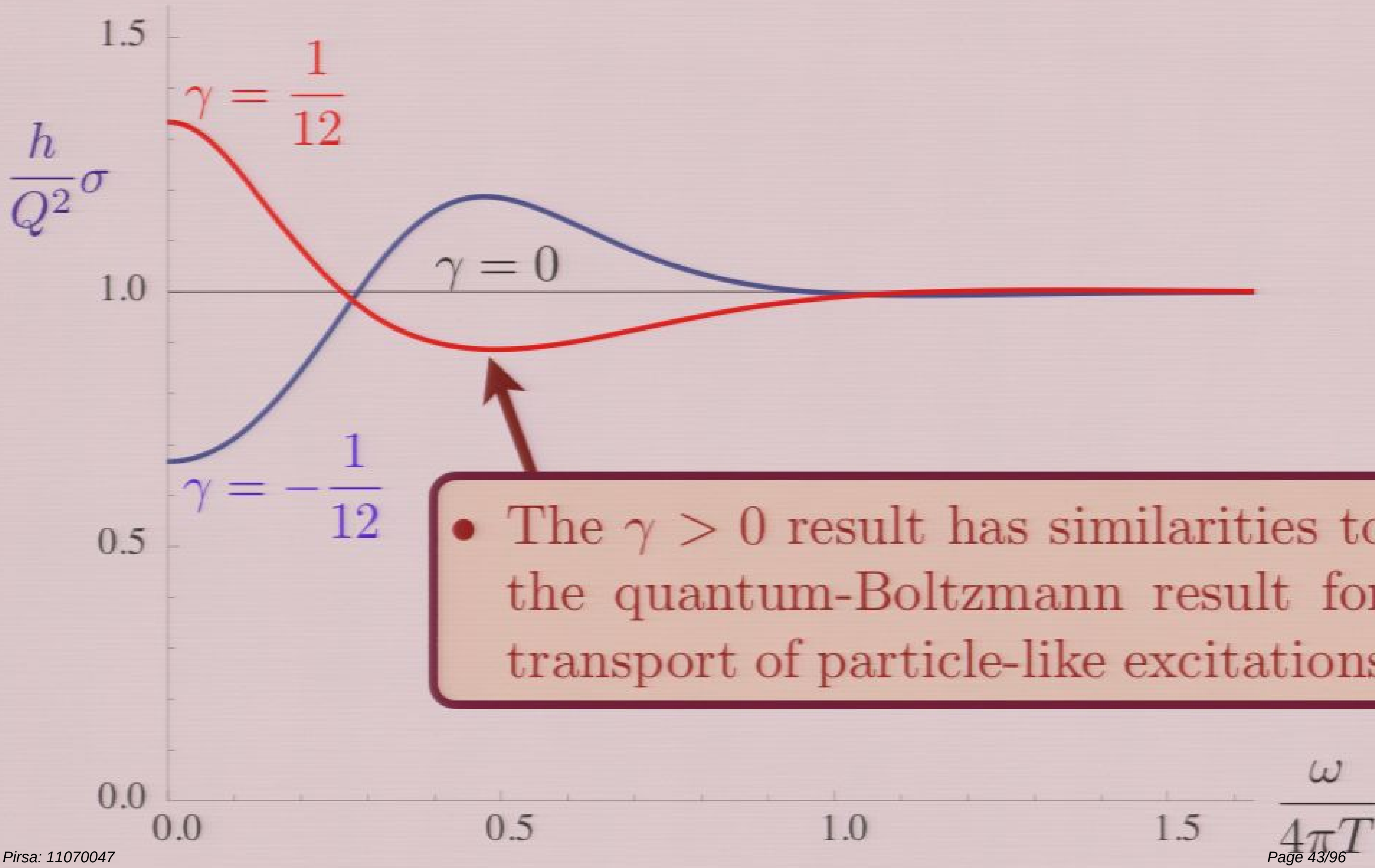

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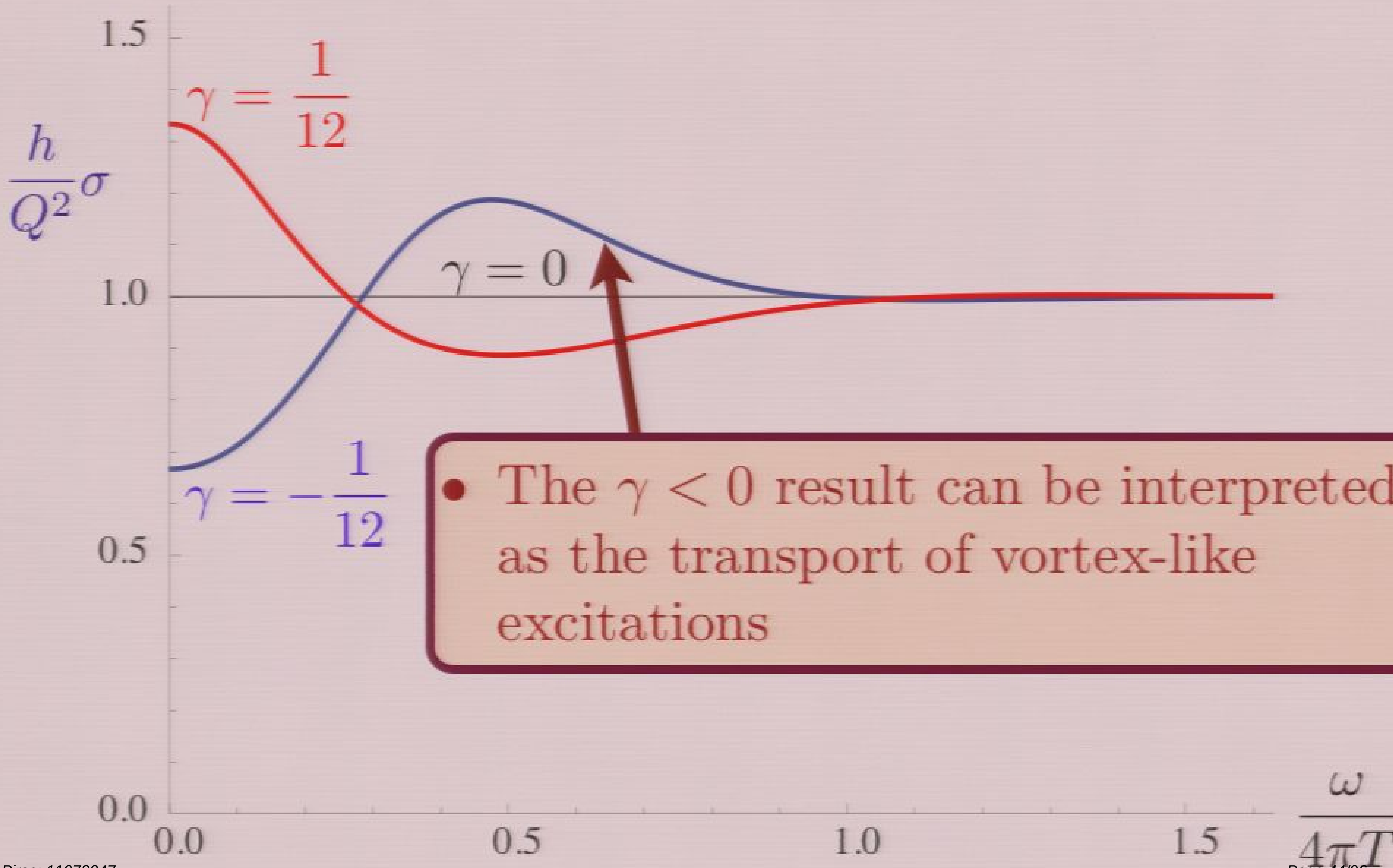

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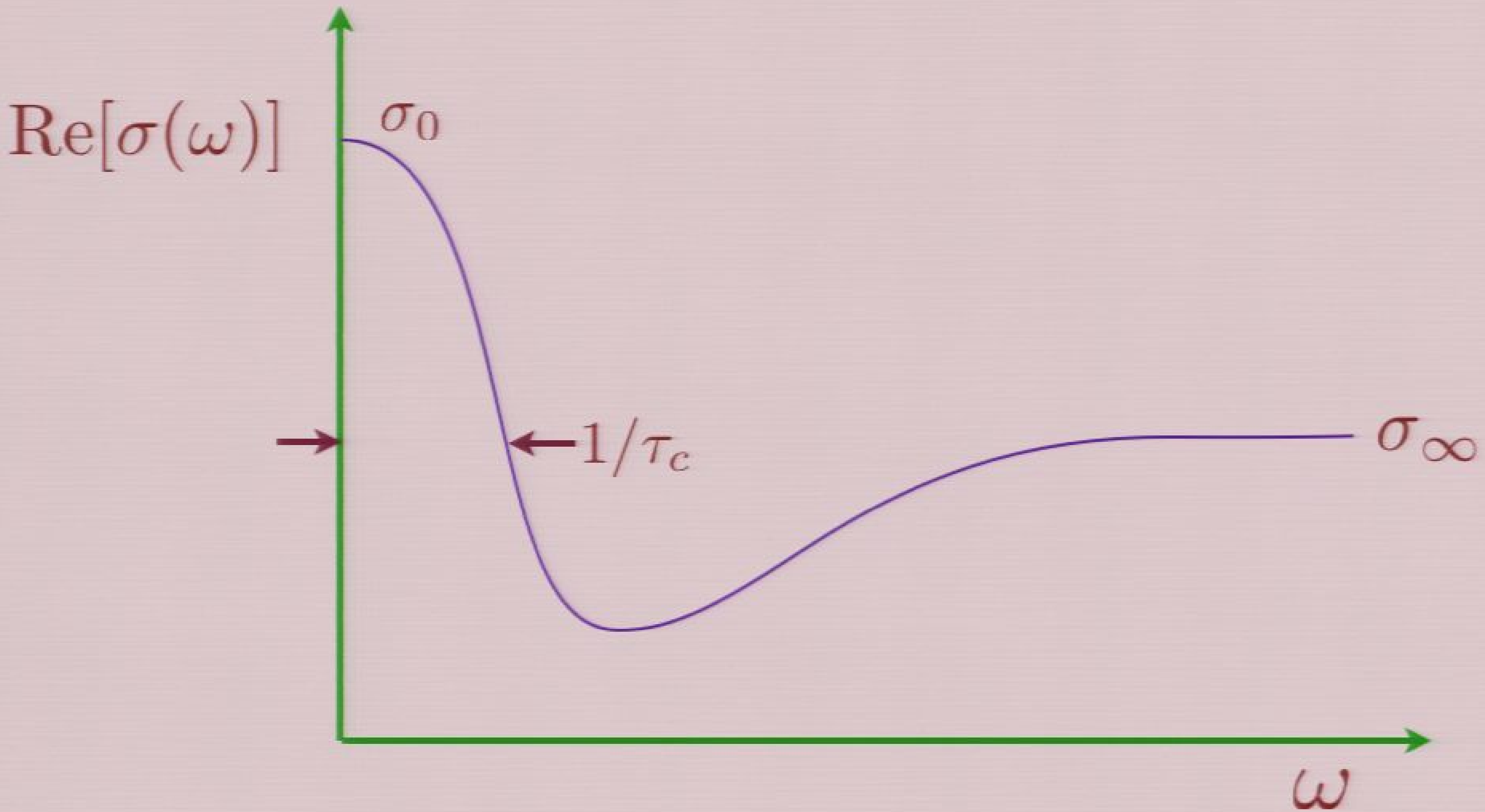

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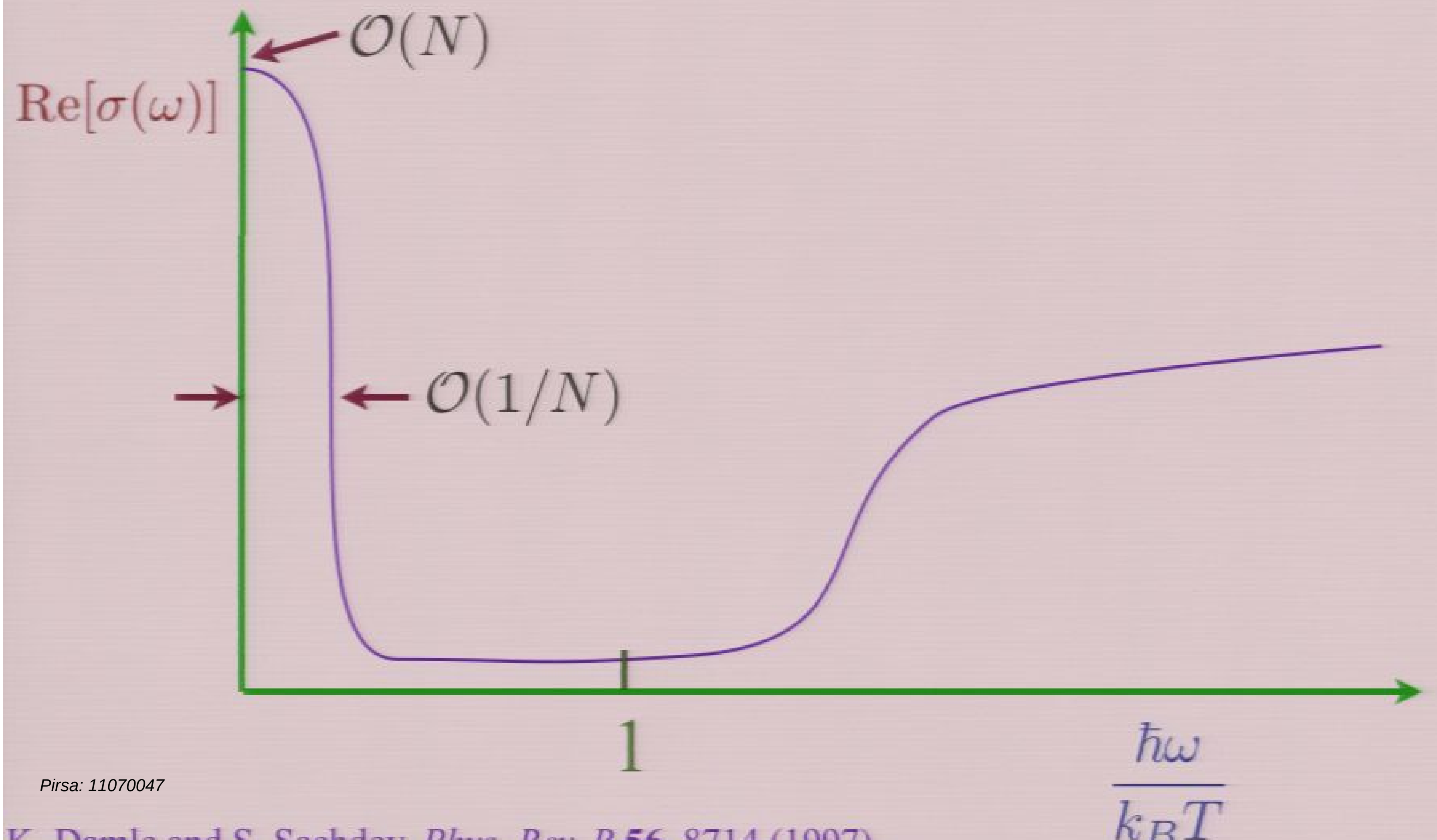

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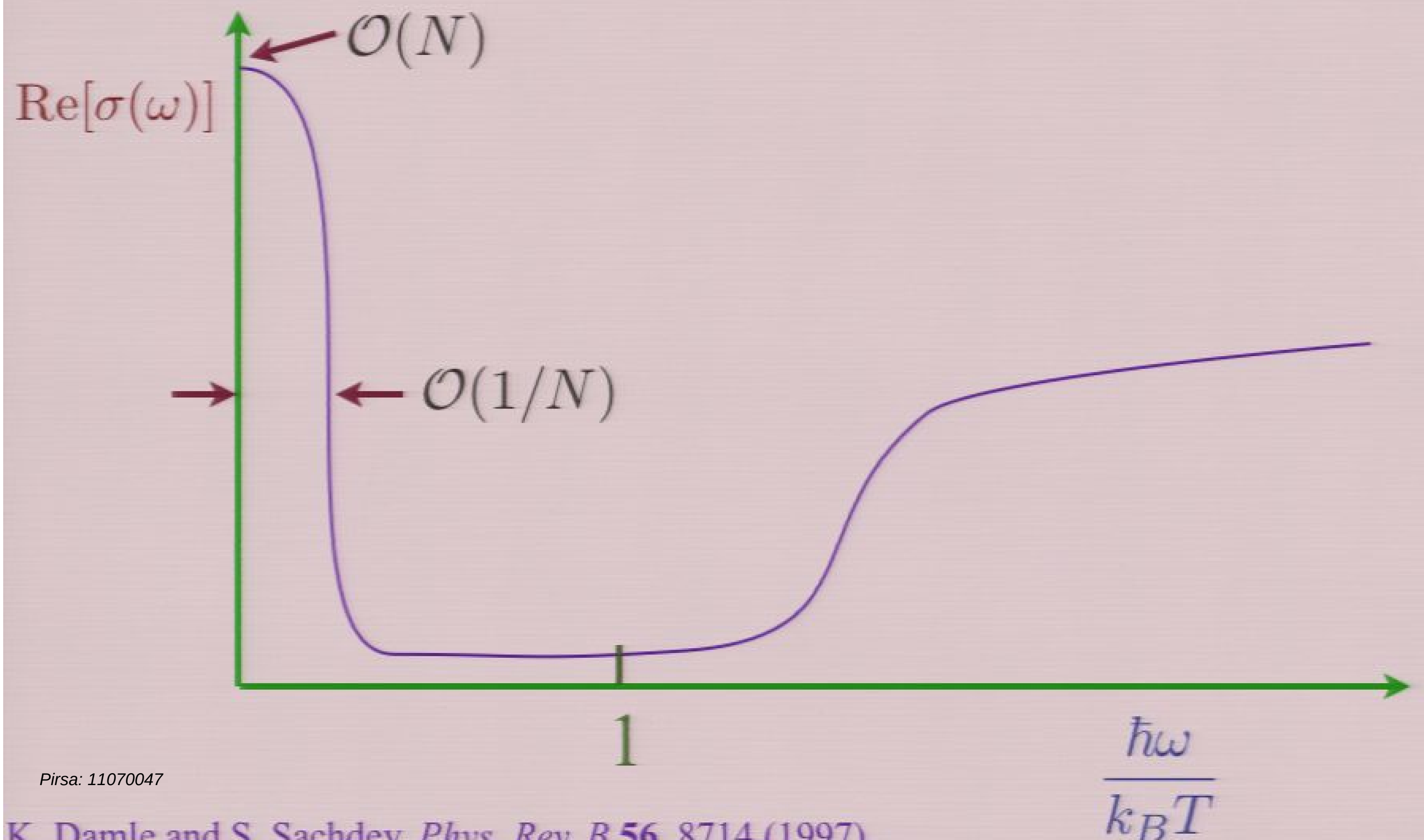

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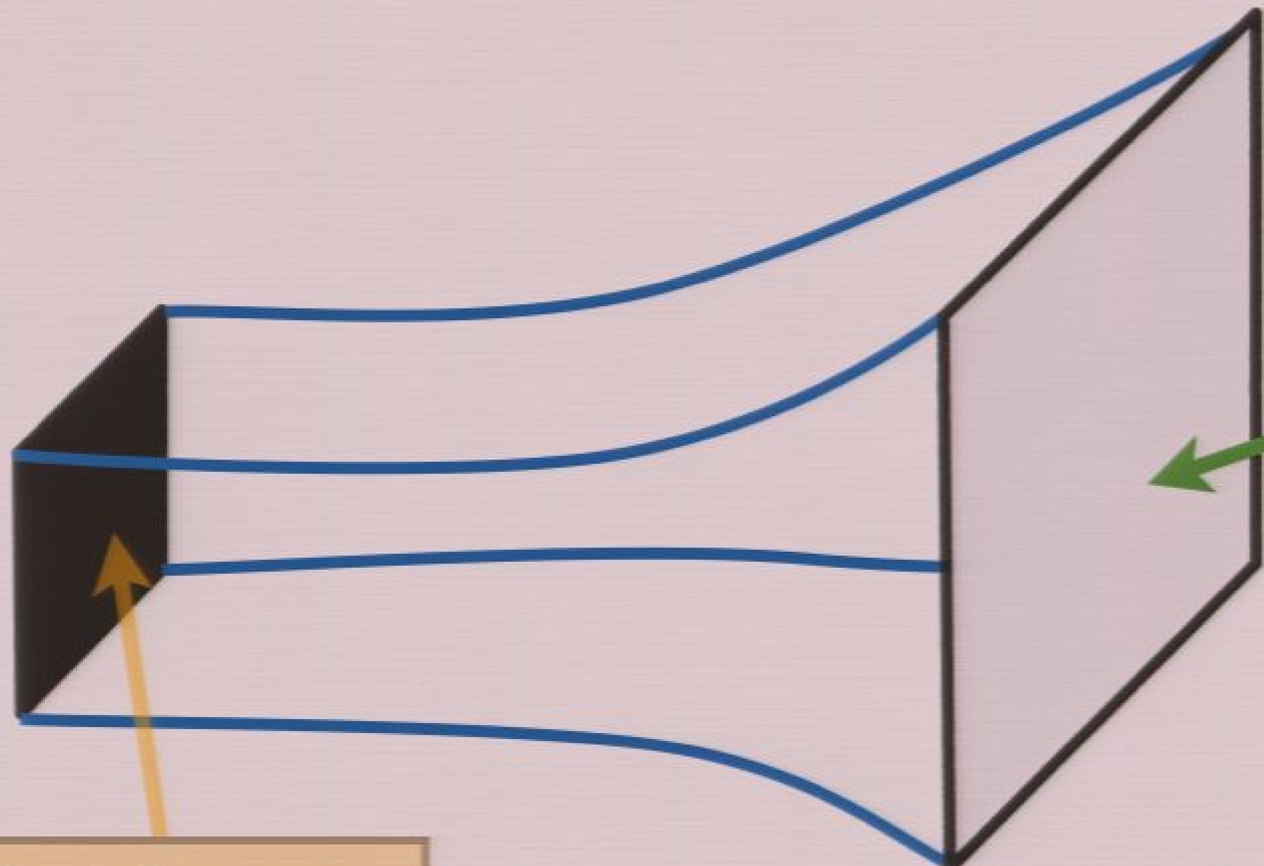

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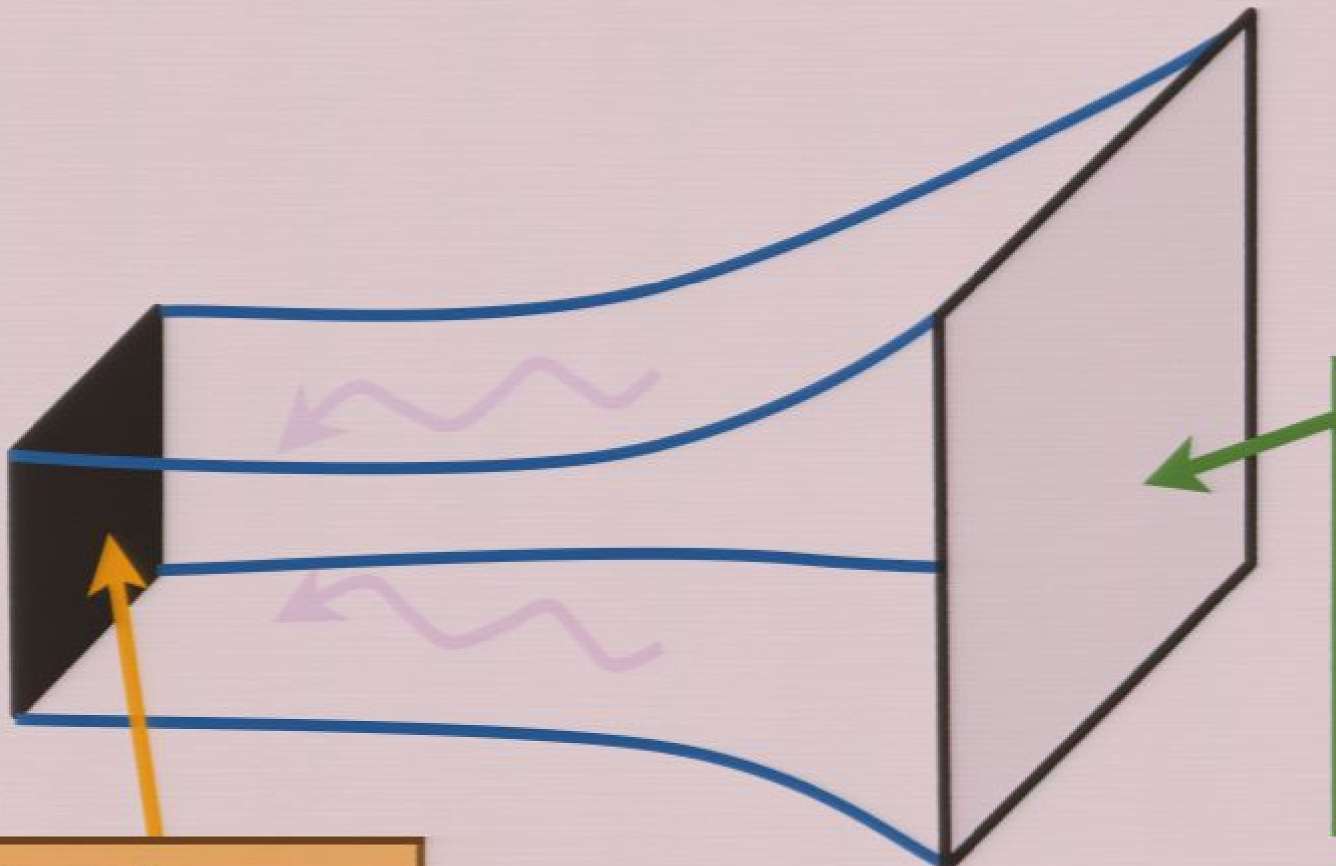

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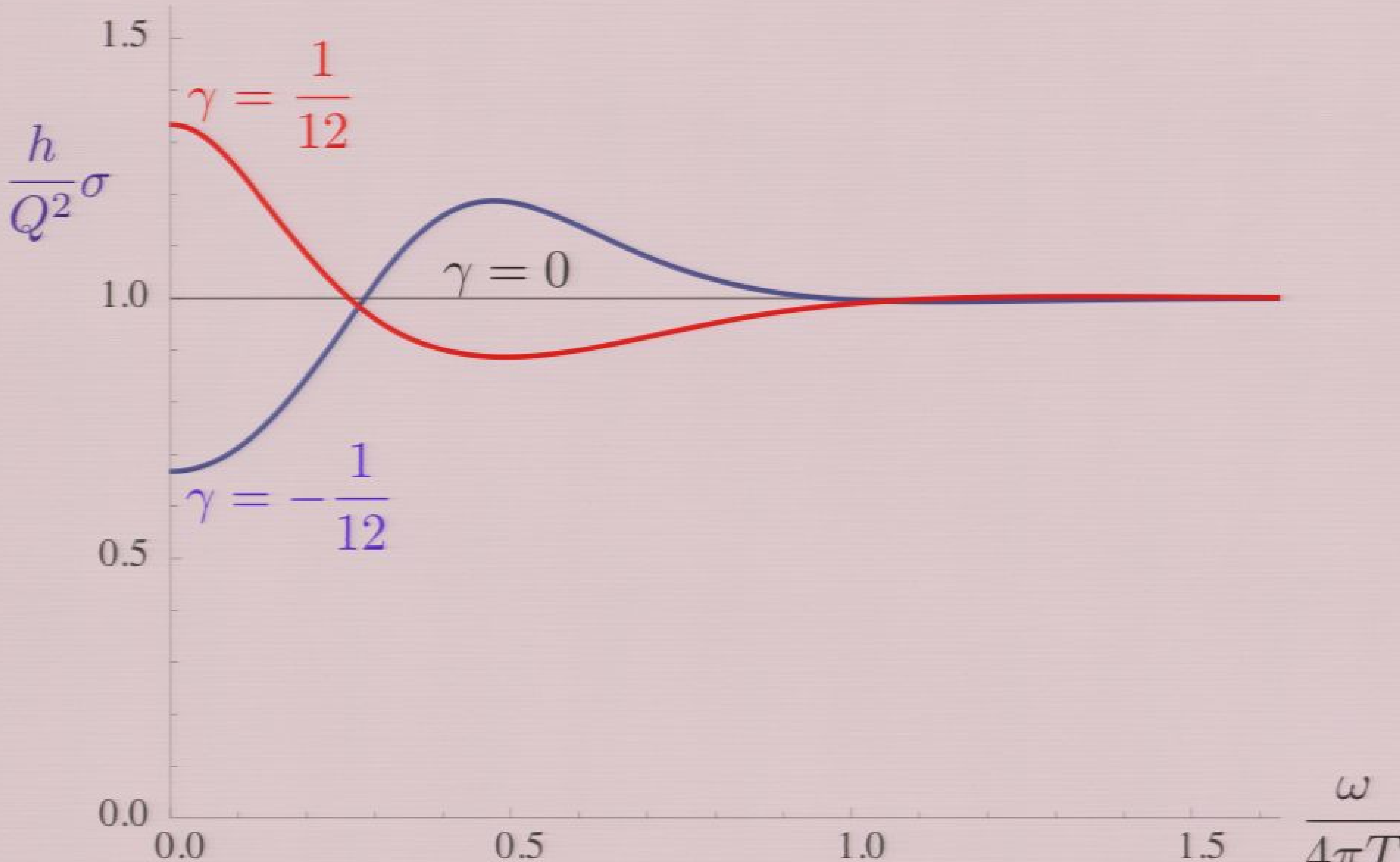

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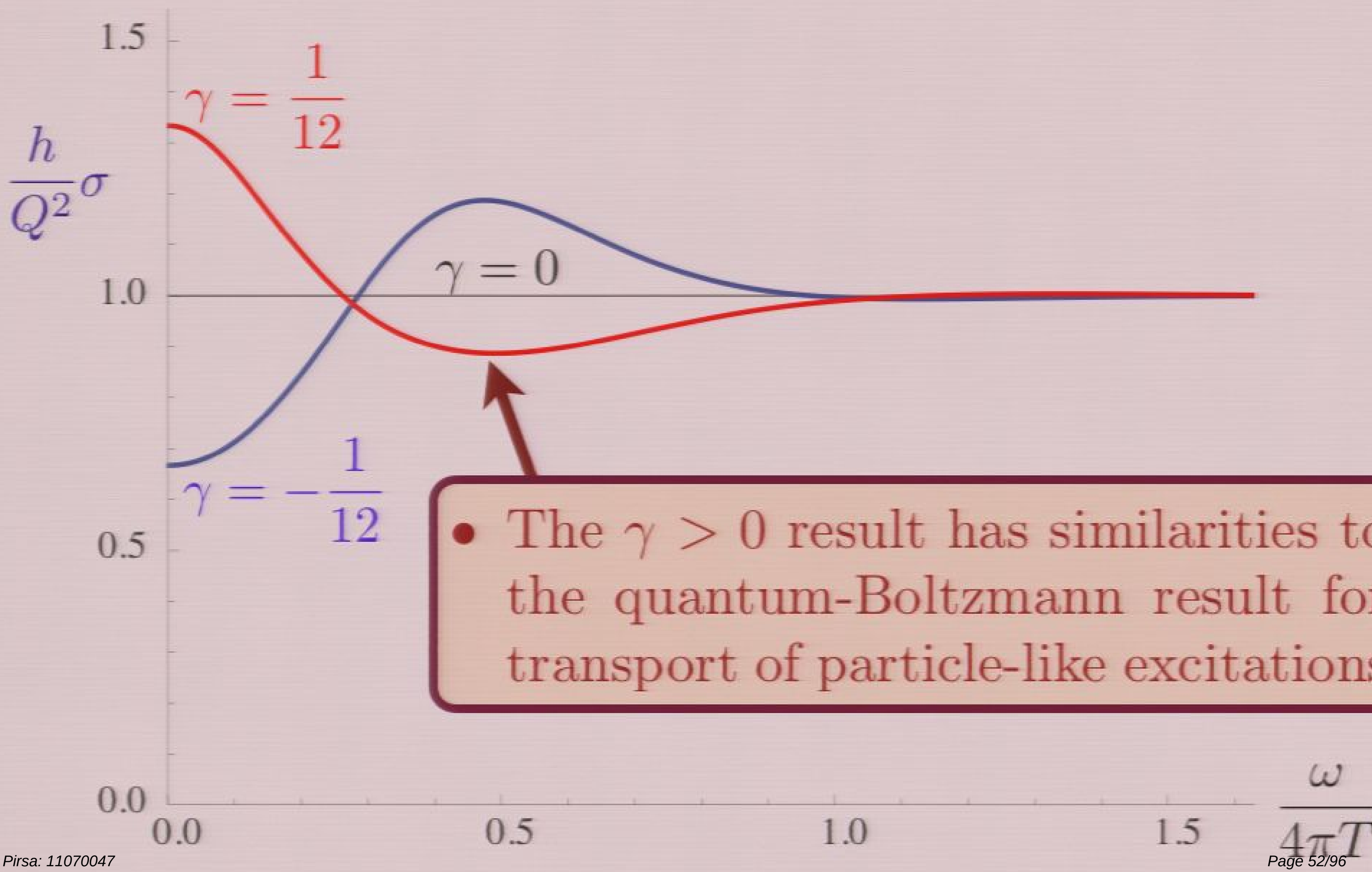

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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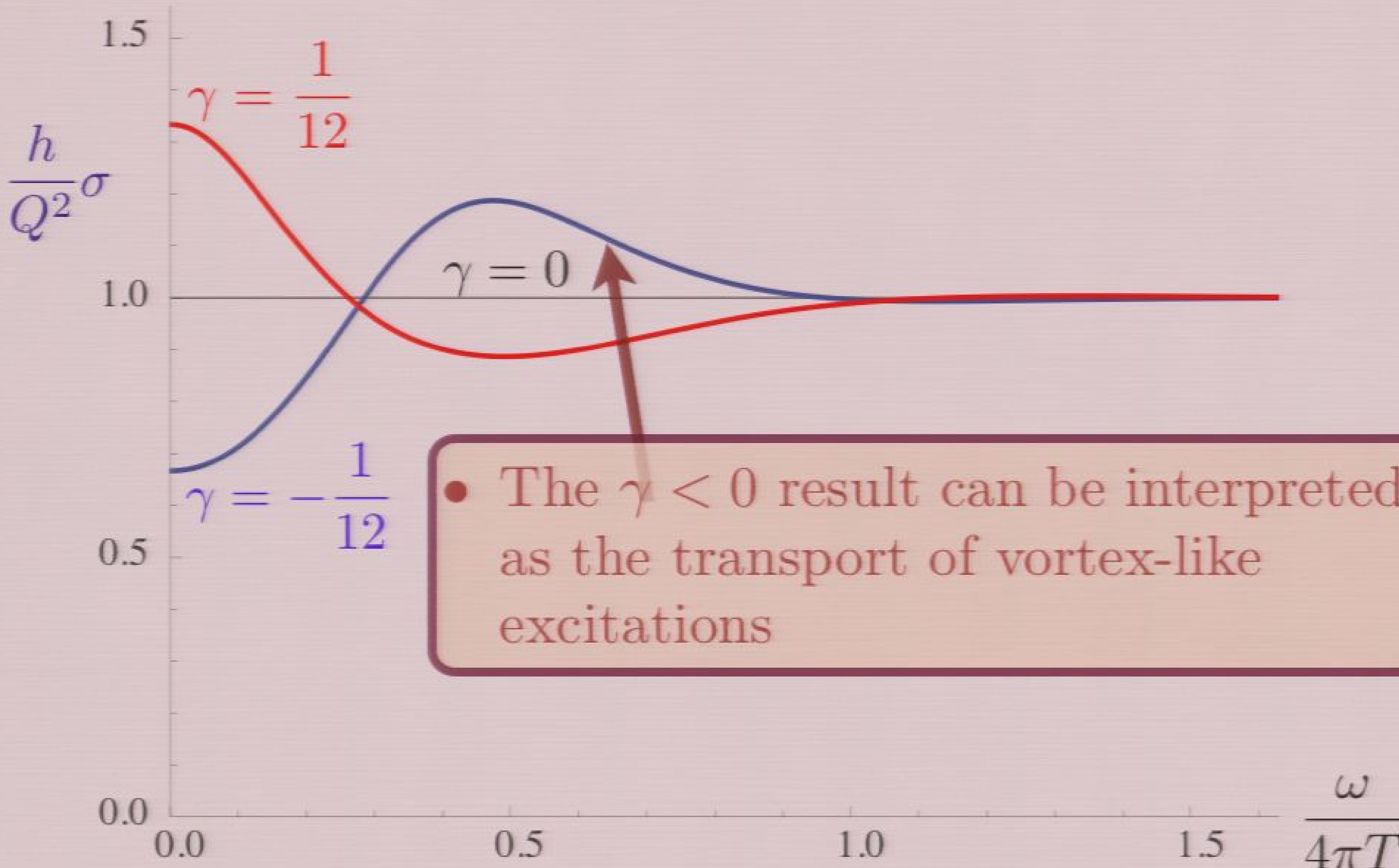

- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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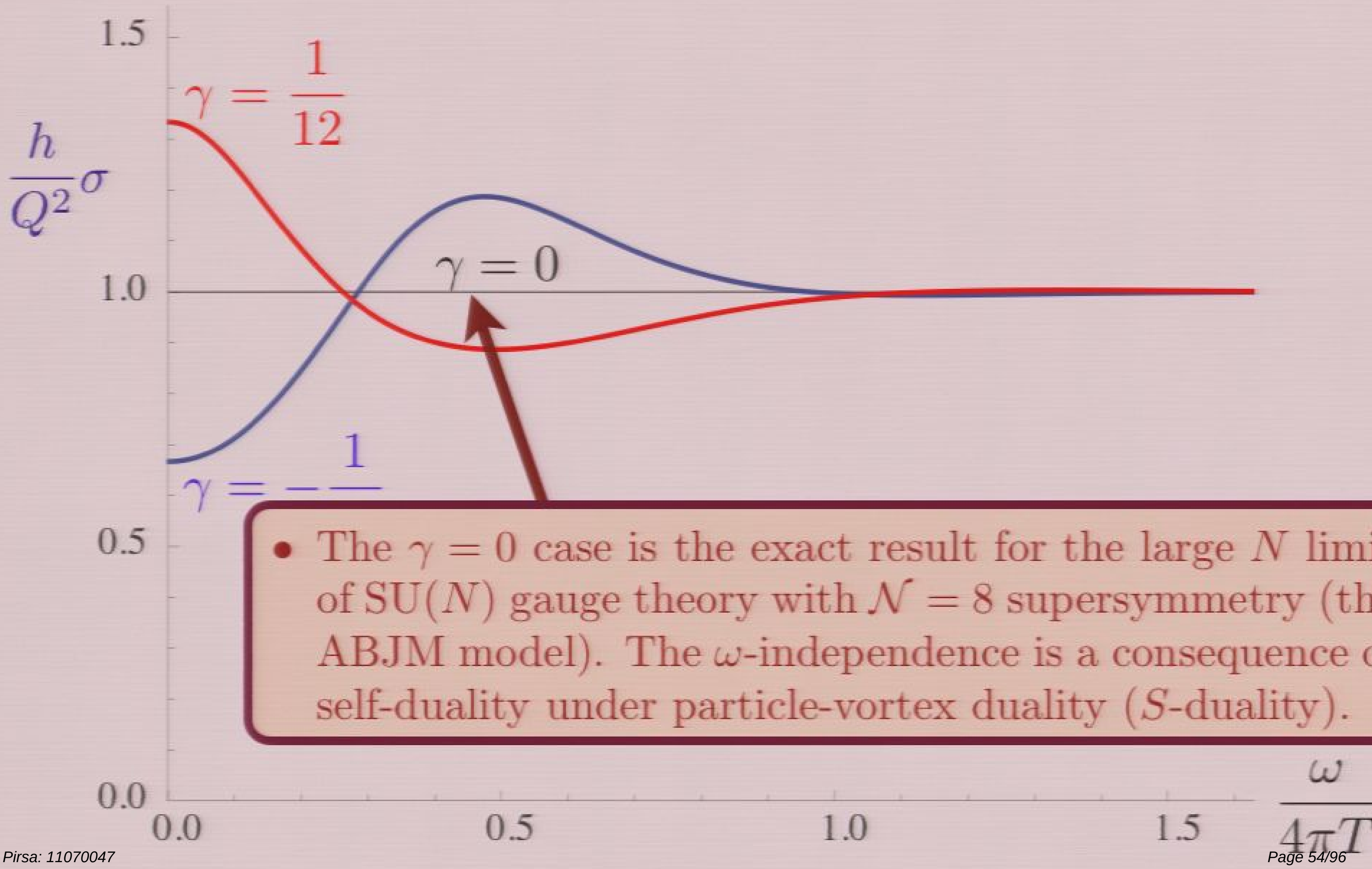

- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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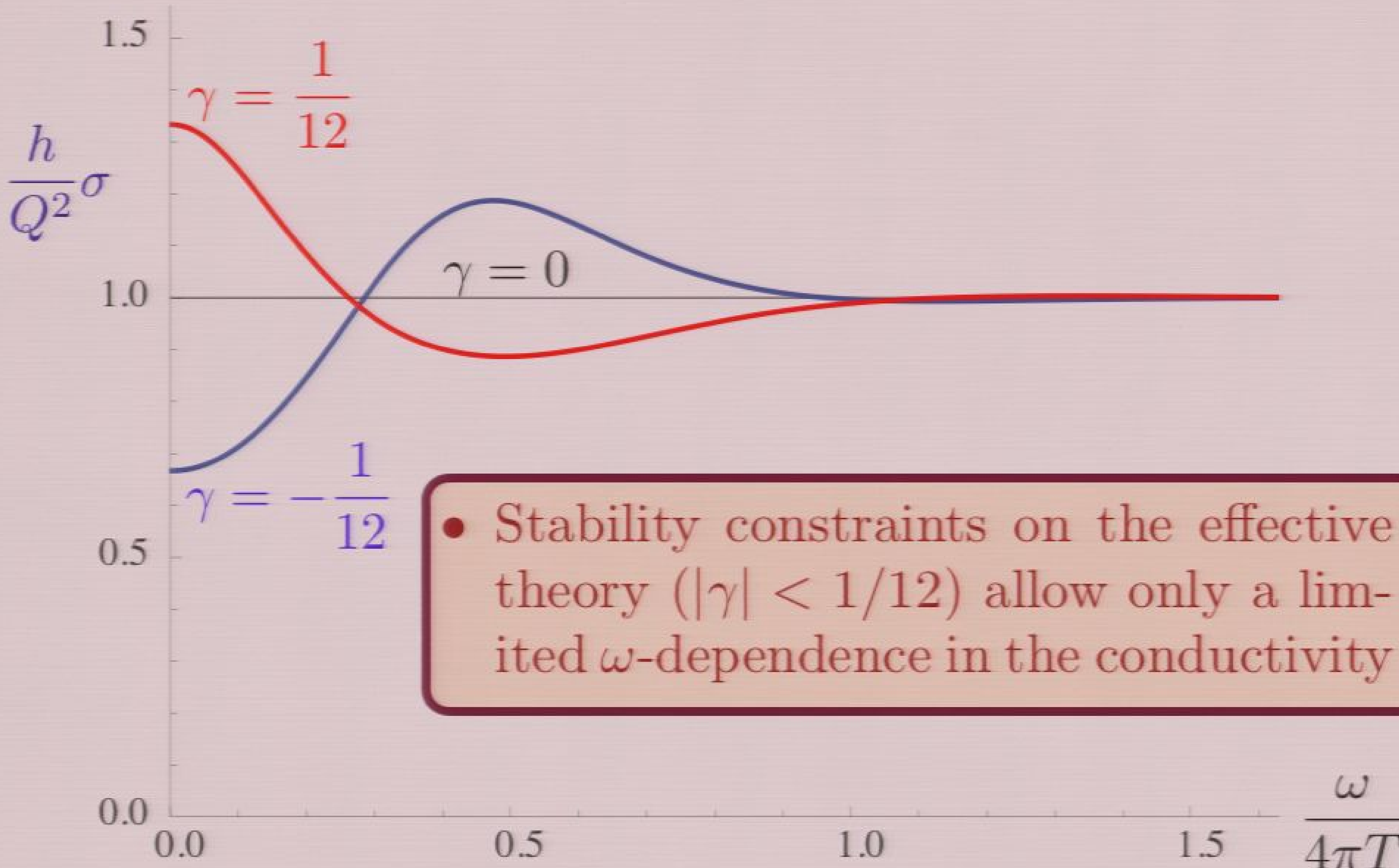

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Frequency dependency of integer quantum Hall effect

Little frequency dependence, and conductivity is close to self-dual value

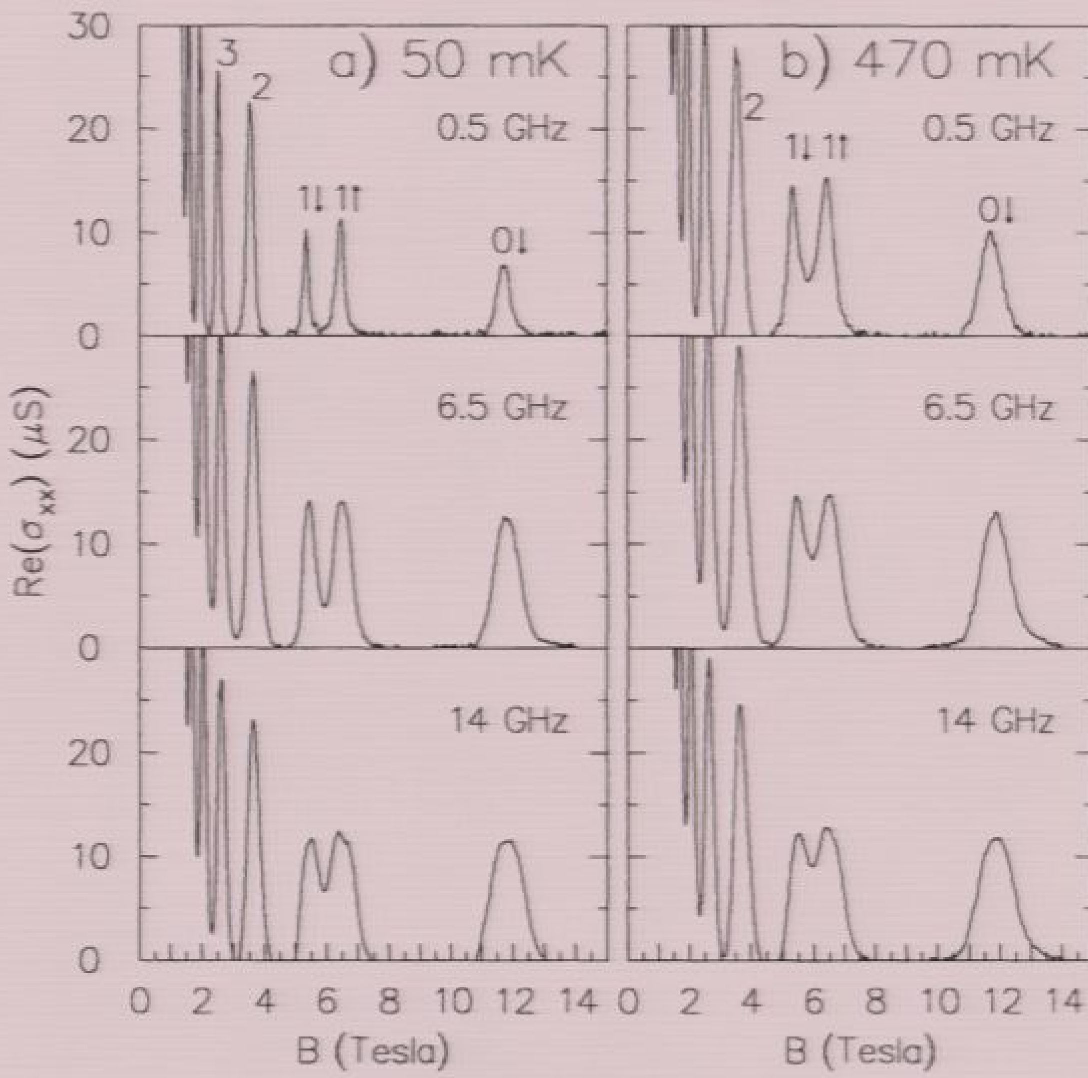

FIG. 3. Re(σ_{xx}) vs B at three frequencies and two temperatures. Peaks are marked with Landau level index N and spin.

L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui,

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2. Compressible quantum matter

A. Condensed matter overview

B. The AdS_4 - Reissner-Nordström black-brane and $AdS_2 \times R^2$

C. Beyond $AdS_2 \times R^2$

Compressible quantum matter

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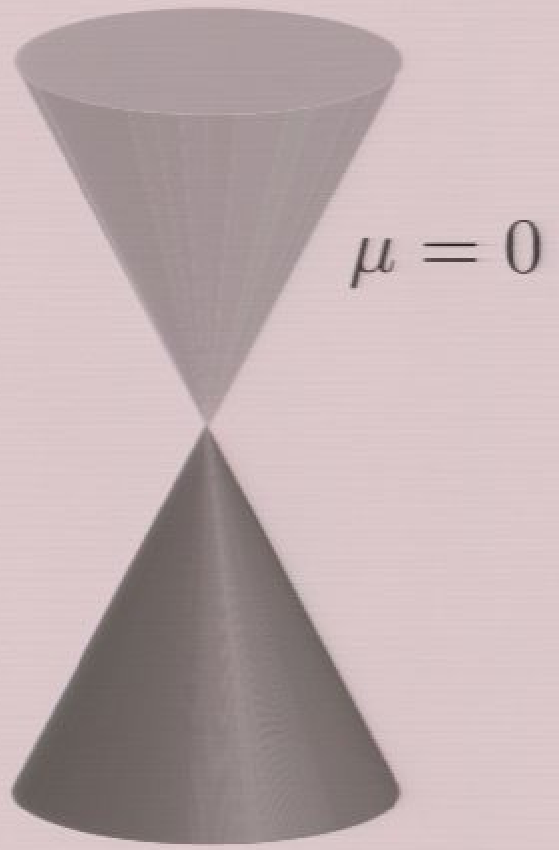
Turning on a chemical potential on a CFT

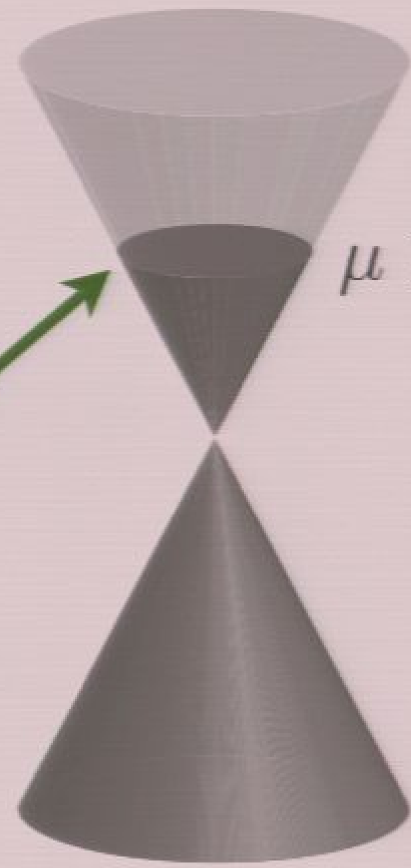


Massless Dirac fermions

(e.g. graphene)

Turning on a chemical potential on a CFT

Compressible phase is a **Fermi Liquid** with a **Fermi surface**



Massless Dirac fermions
(e.g. graphene)

The Fermi surface

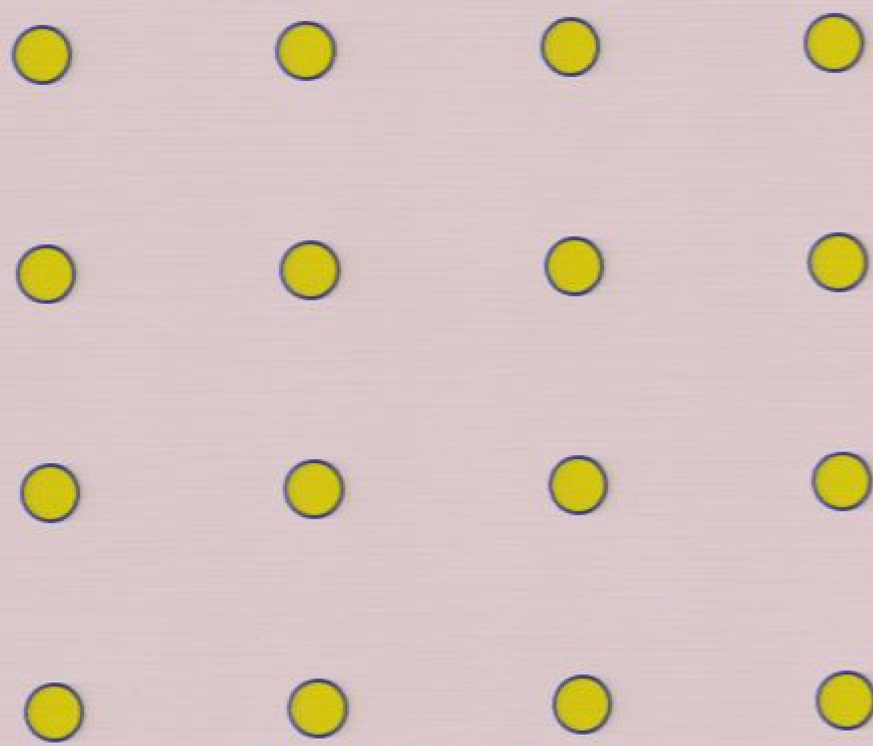
This is the locus of zero energy singularities in momentum space in the two-point correlator of fermions carrying charge Q.

$$G^{-1}_{\text{fermion}}(k = k_F, \omega = 0) = 0.$$

Luttinger relation: The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle Q \rangle$. This is a *key* constraint which allows extrapolation from weak to strong coupling.

Compressible quantum matter

Another compressible state is the **solid**
(or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

The only other familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs

Compressible quantum matter

Conjecture: *All compressible states which preserve translational and global* $U(1)$ *symmetries must have* FERMI SURFACES, *but they are not necessarily Fermi liquids.*

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- Non-Fermi liquids have quasiparticles coupled to deconfined gauge fields (or gapless bosonic modes at quantum critical points).

Theory similar to the ABJM model in a chemical potential

- U(1) gauge invariance and U(1) global symmetry
- Fermions, f_+ and f_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- Bosons, b_+ and b_-, carry U(1) gauge charges ± 1, and global U(1) charge 1.
- No supersymmetry

L. Huijse and S. Sachdev, arXiv:1104.5022

Theory similar to the ABJM model in a chemical potential

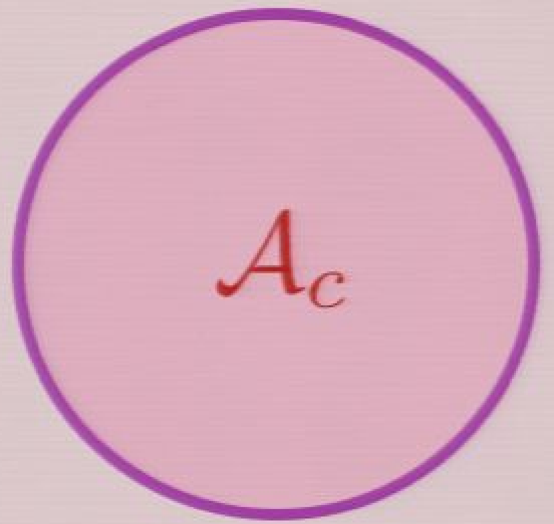
$$\begin{aligned}\mathcal{L} &= f_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m} - \mu \right] f_\sigma \\ &+ b_\sigma^\dagger \left[(\partial_\tau - i\sigma A_\tau) - \frac{(\nabla - i\sigma \mathbf{A})^2}{2m_b} + \epsilon_1 - \mu \right] b_\sigma \\ &\quad + \frac{u}{2} \left(b_\sigma^\dagger b_\sigma \right)^2 - g_1 \left(b_+^\dagger b_-^\dagger f_- f_+ + \text{H.c.} \right)\end{aligned}$$

The index $\sigma = \pm 1$

L. Huijse and S. Sachdev, arXiv:1104.5022

Phases of ABJM-like theories

$$\langle b_{\pm} \rangle = 0$$



$$2\mathcal{A}_c = \langle \mathcal{Q} \rangle$$

Fermi liquid (FL) of gauge-neutral particles

U(1) gauge theory is in *confining* phase

Phases of ABJM-like theories

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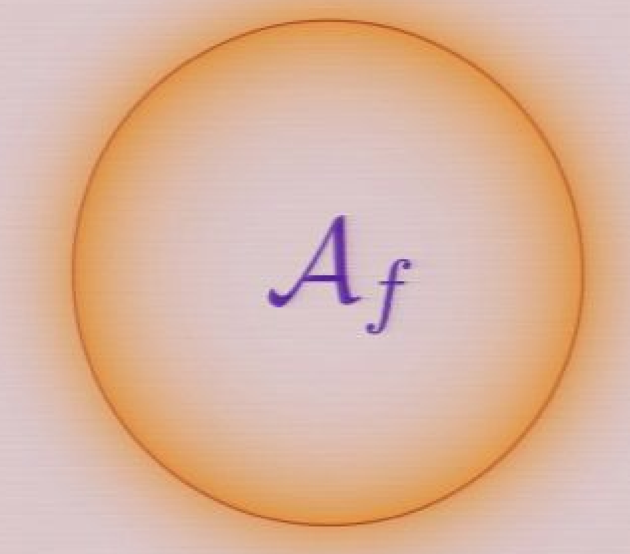

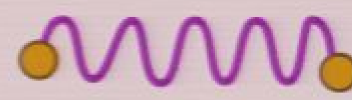

$$2\mathcal{A}_f = \langle \mathcal{Q} \rangle$$

non-Fermi liquid (NFL)

U(1) gauge theory is in *deconfined* phase

Phases of ABJM-like theories

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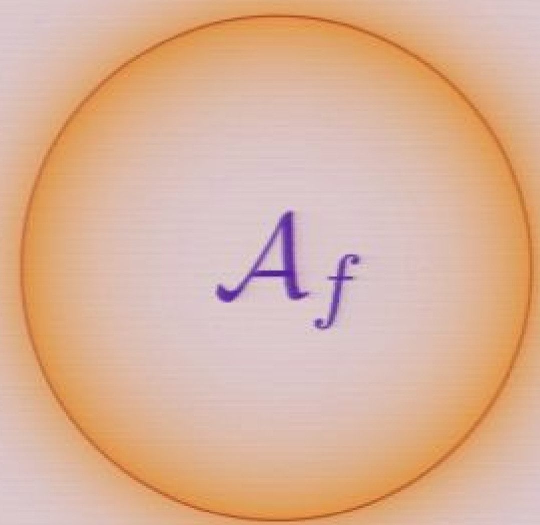

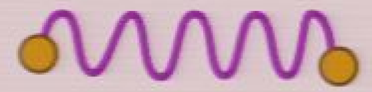

Fermi surface coupled to Abelian or non-Abelian gauge fields:

- Longitudinal gauge fluctuations are screened by the fermions.
- Transverse gauge fluctuations are unscreened, and Landau-damped. They are IR fluctuations with dynamic critical exponent $z > 1$.
- Theory is *strongly coupled in two spatial dimensions*.
- "Non-Fermi liquid" broadening of the fermion quasiparticle pole.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)

Phases of ABJM-like theories

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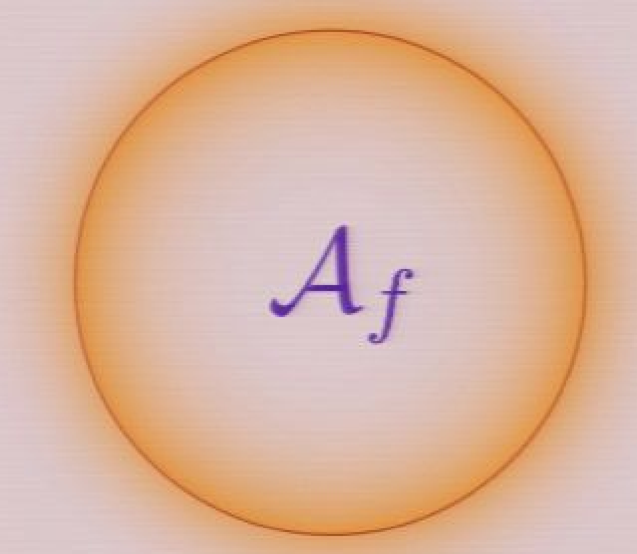

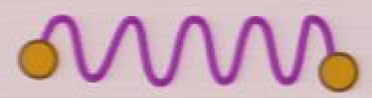

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Outline

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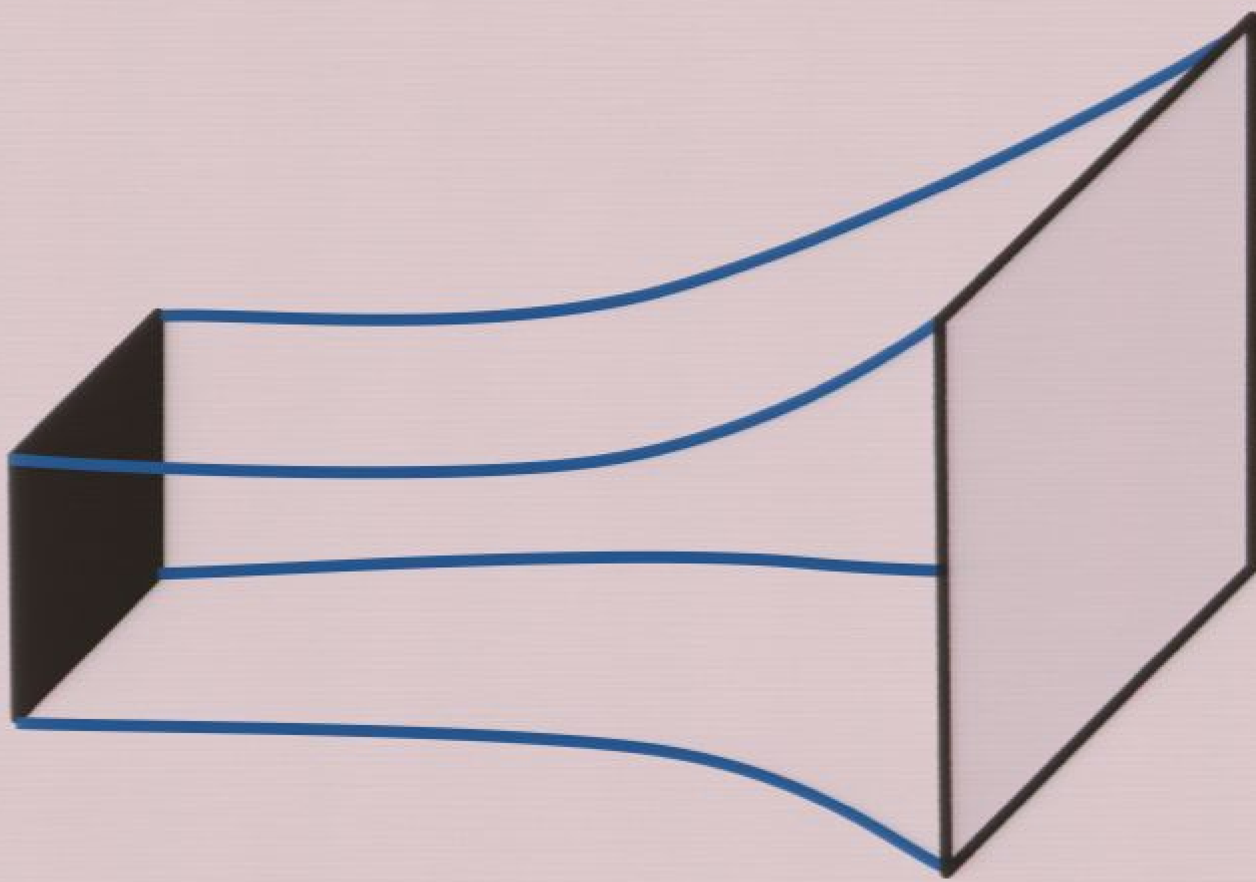
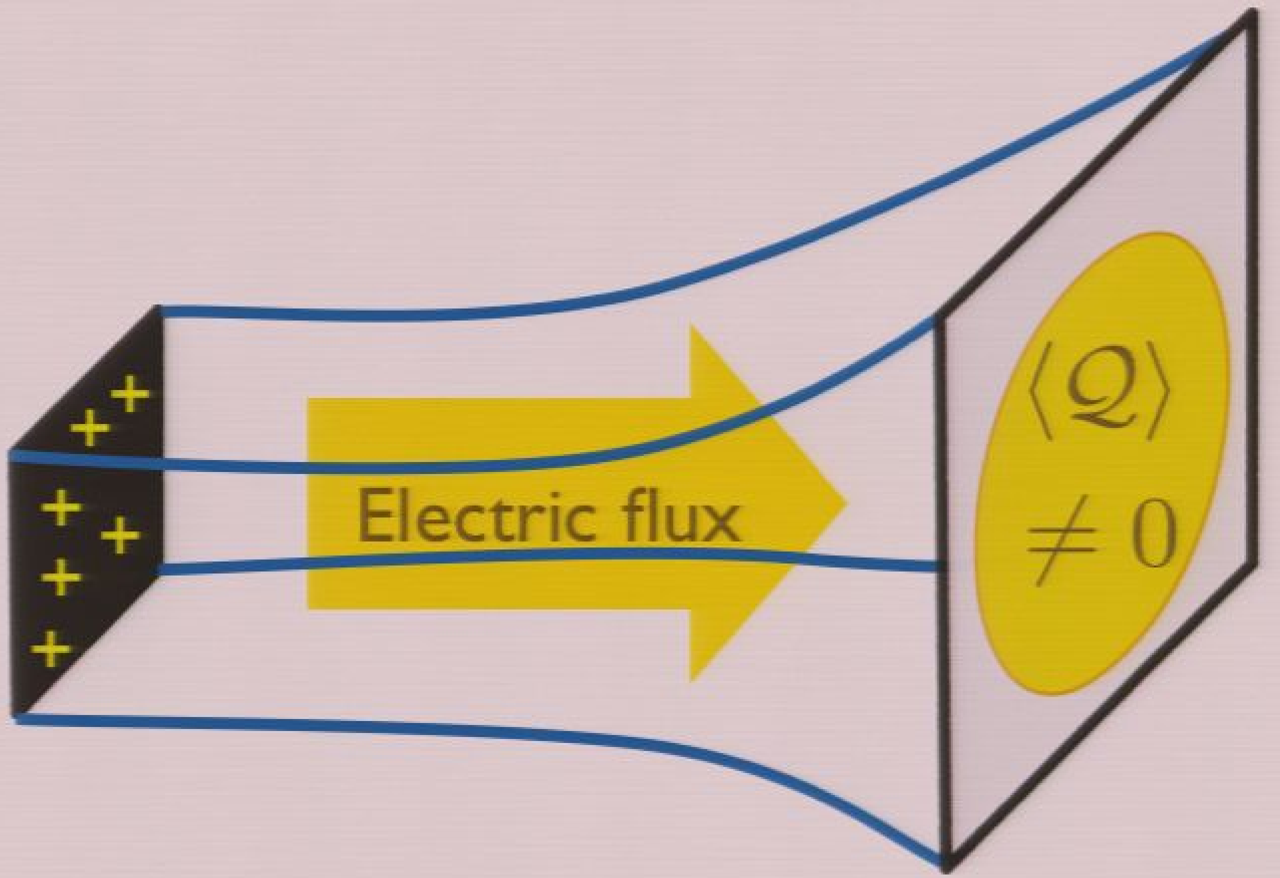
AdS$_4$-Schwarzschild black-brane



$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

AdS4-Reissner-Nordtröm black-brane



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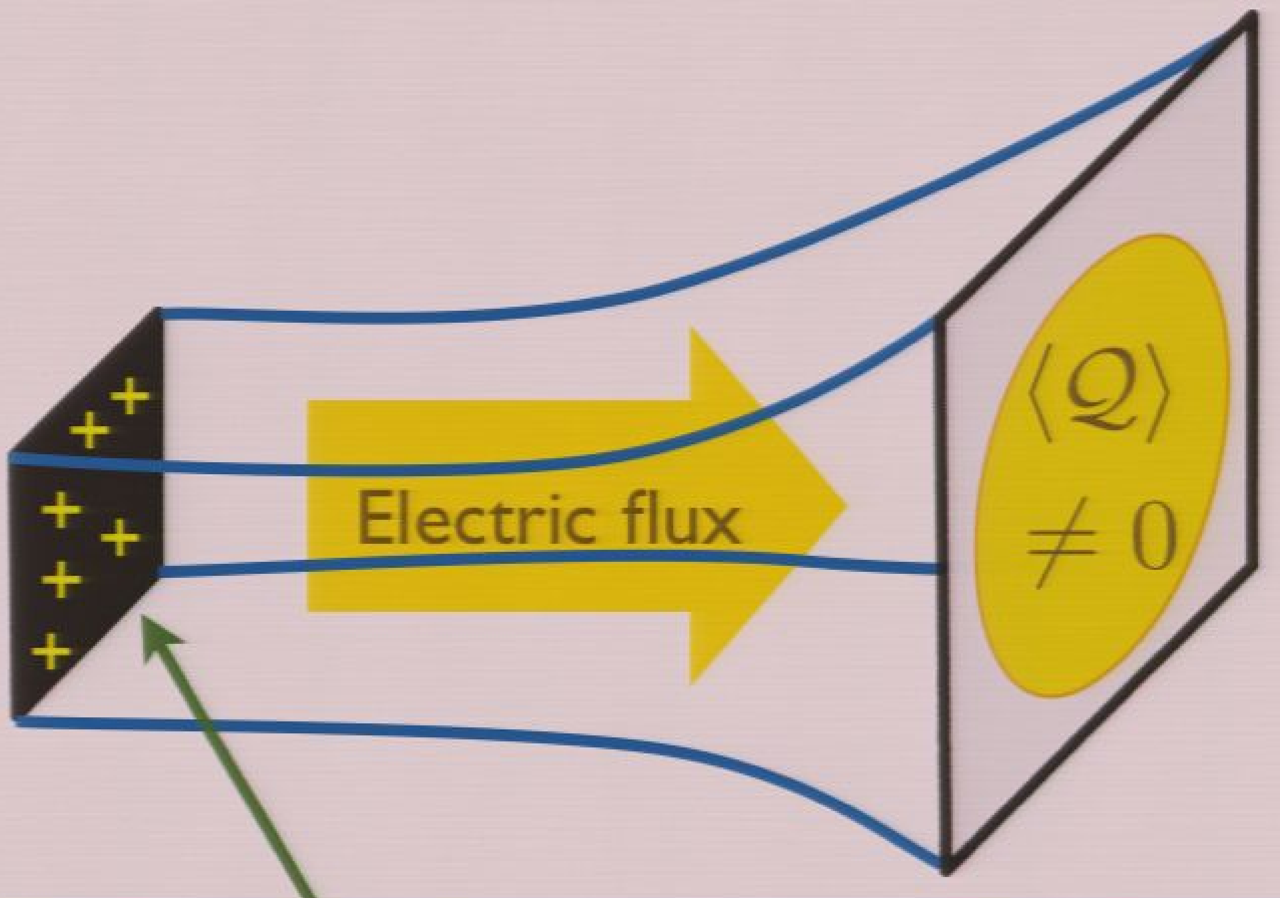

At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $\text{AdS}_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

Properties of $AdS_2 \times R^2$

This state appears stable in the presence of matter fields (with large enough bulk mass). The single-particle Green's function of the boundary theory has the IR (small ω) limit

$$G^{-1}(k,\omega) = A(k) + B(k)\omega^{\nu_k}$$

where $A(k)$, $B(k)$, and ν_k are smooth functions of k.

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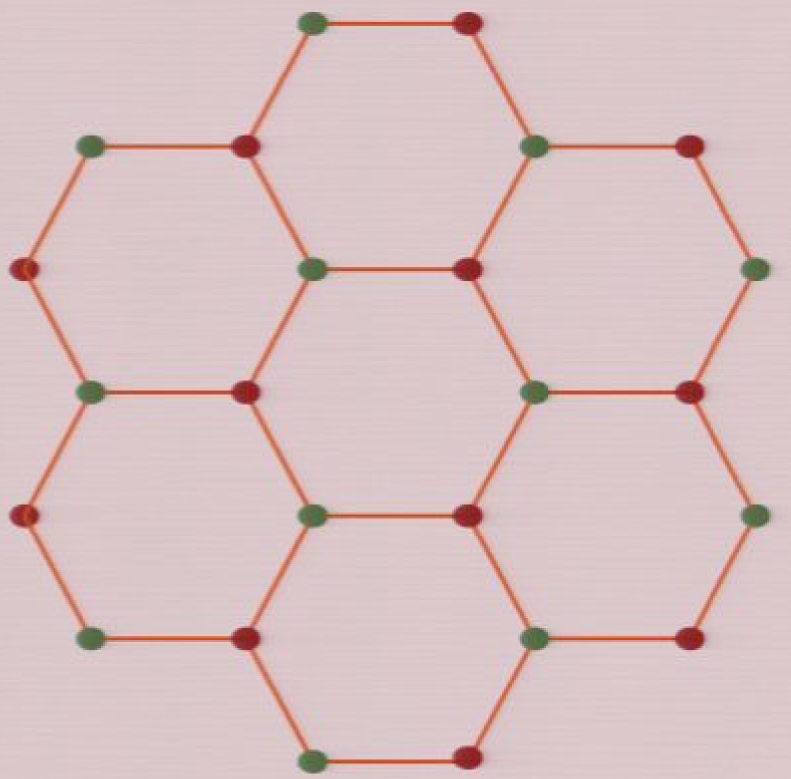
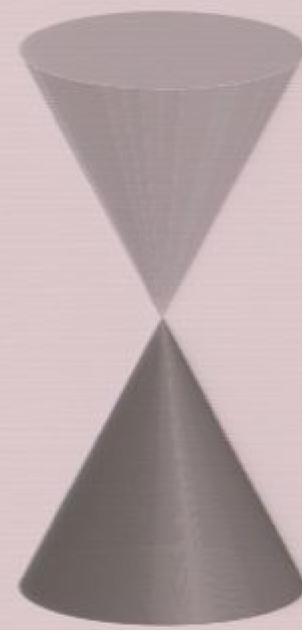
For fermions, if $A(k)$ changes sign at a $k = k_F$, we have a *Fermi surface* at $k = k_F$. This Fermi surface is non-Fermi liquid like.

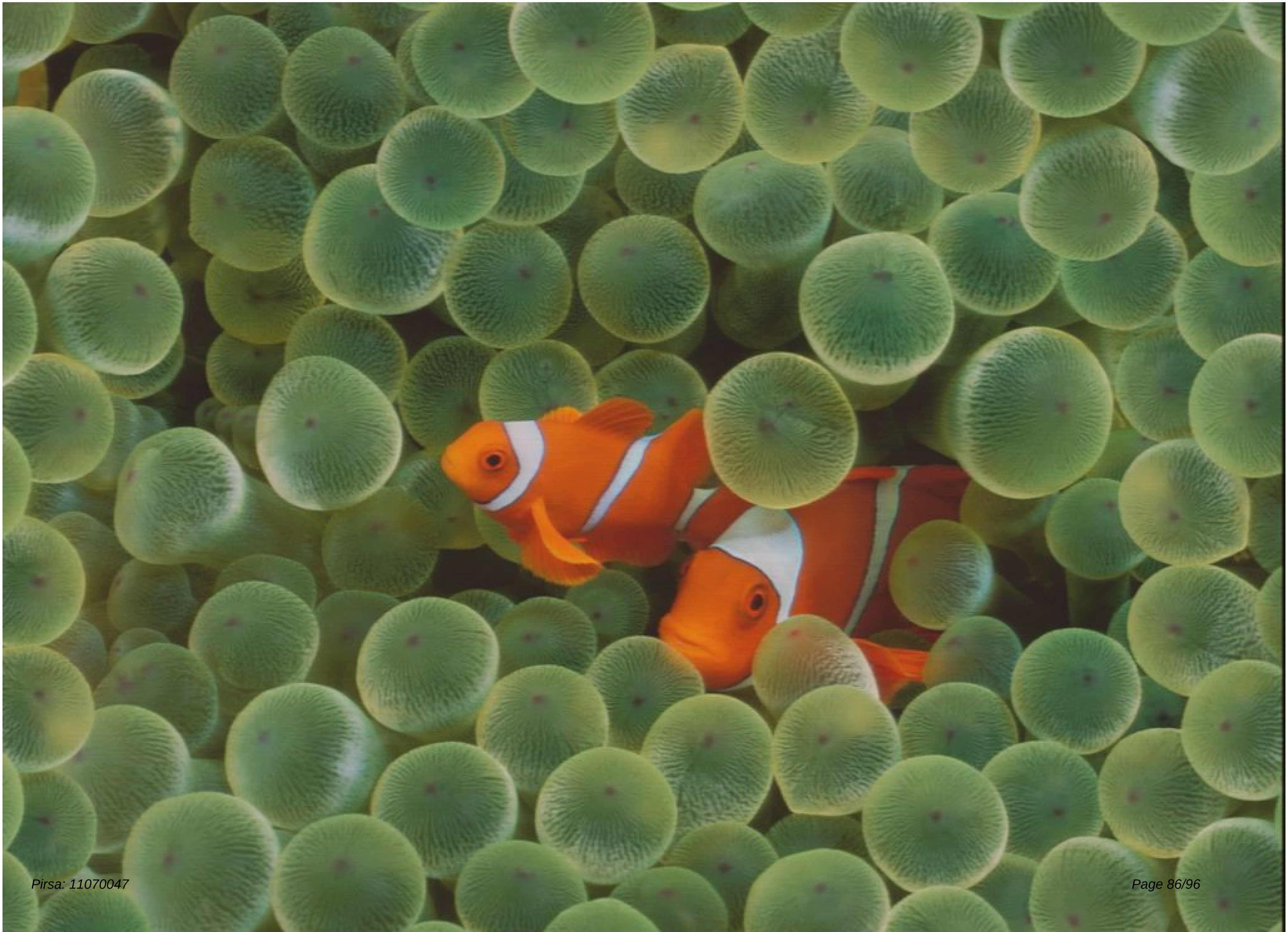
T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Interpretation of AdS_2



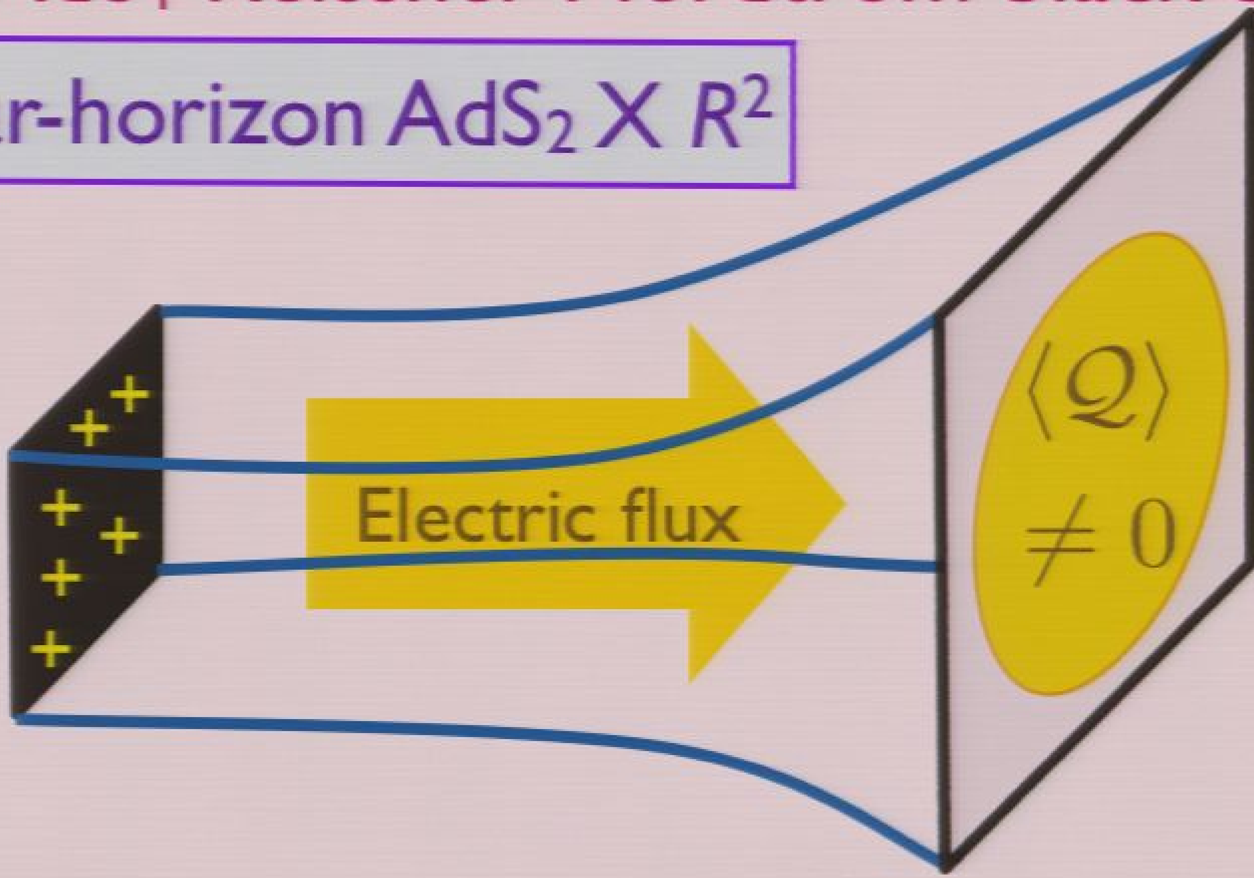


CFT on graphene



AdS4-Reissner-Nordtröm black-brane

Near-horizon AdS$_2$ X R^2



$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right) - \frac{1}{4e^2}F_{ab}F^{ab}\right]$$

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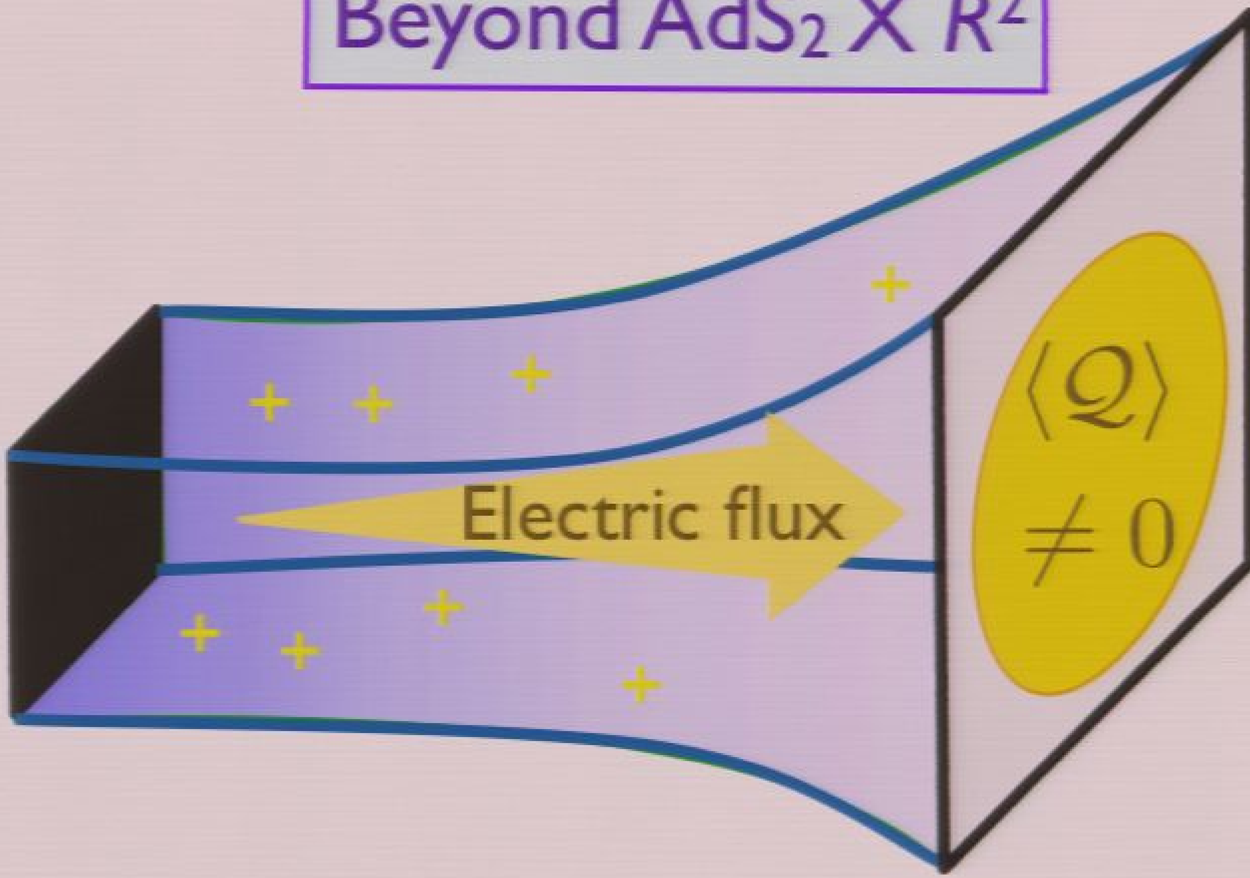

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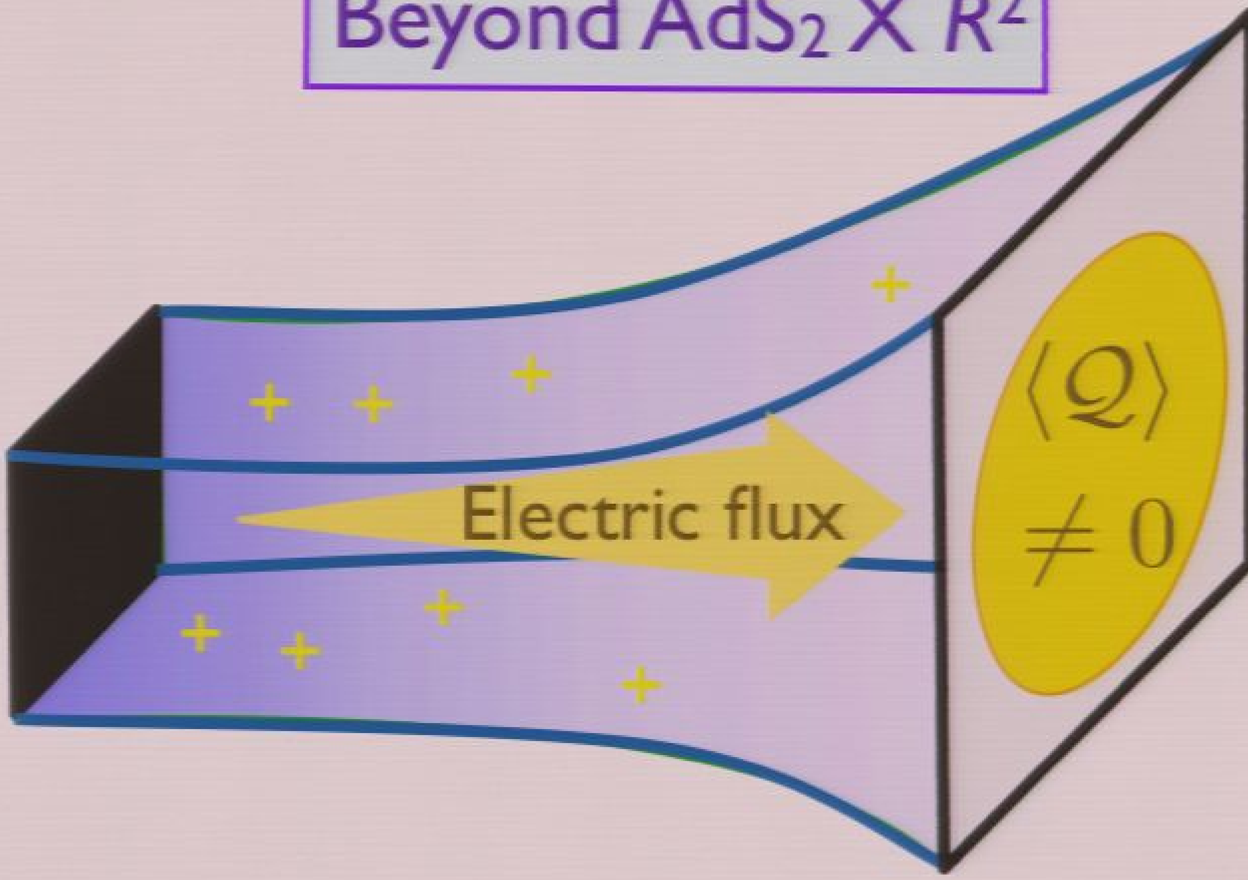
Sufficiently light matter undergoes Schwinger pair-creation, back-reacts on the metric, the horizon may disappear, and the charge density is delocalized in the bulk spacetime

Beyond AdS$_2$ X R^2



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Conclusions

Quantum criticality and conformal field theories

- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

- The description is far removed from, and complementary to, that of the quantum Boltzmann equation which builds on the quasiparticle/vortex picture.

- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

Conclusions

Compressible quantum matter

- The Reissner-Nordström solution provides the simplest holographic theory of a compressible state
- The RN solutions has many problems: finite ground-state entropy density, violation of Luttinger relation.
- Condensation of a scalar leads to the holographic theory of a superfluid. The IR metric has a Lifshitz form, indicating the presence of neutral gapless excitations not found in a superfluid.

Conclusions

Compressible quantum matter

- Fermion back-reaction leads to a Fermi liquid with many Fermi surfaces which do obey the Luttinger relation. However, the IR Lifshitz metric, and the very small Fermi wavevectors appear to be unwanted artifacts.

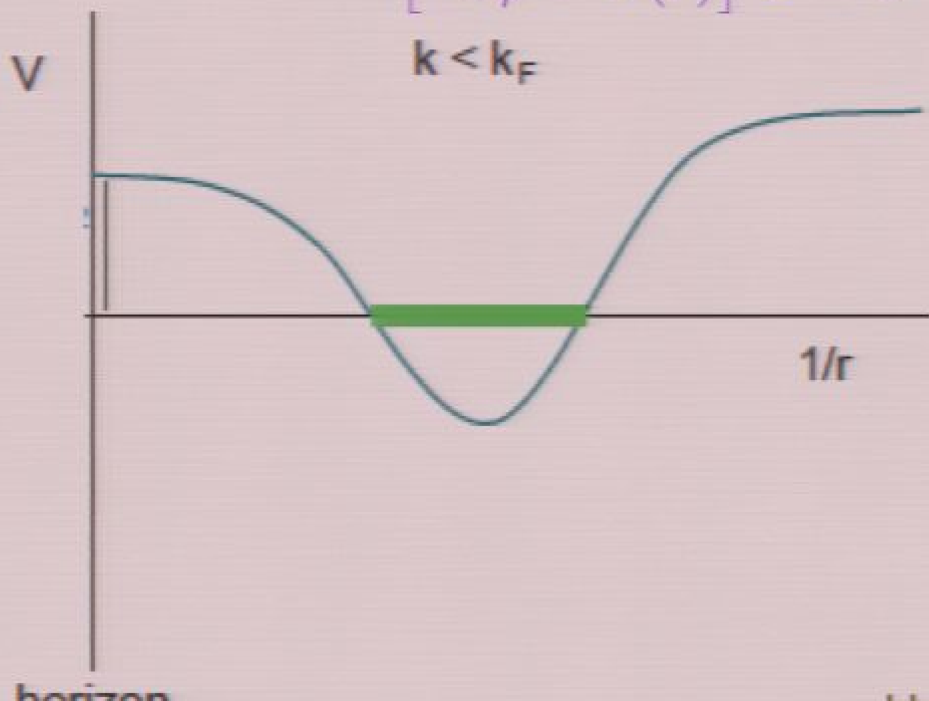
- Needed: a complete holographic theory of non-Fermi liquids and "fractionalized" Fermi liquids, obeying the Luttinger relations, to describe experiments on "strange metals".

Beyond $AdS_2 \times R^2$

- Account for the matter in a Thomas-Fermi approximation: the local chemical potential determines the local density and pressure, using the equation of state of a free Fermi gas: so determine the density, electric field, and metric as a function of r, the "extra" dimension.

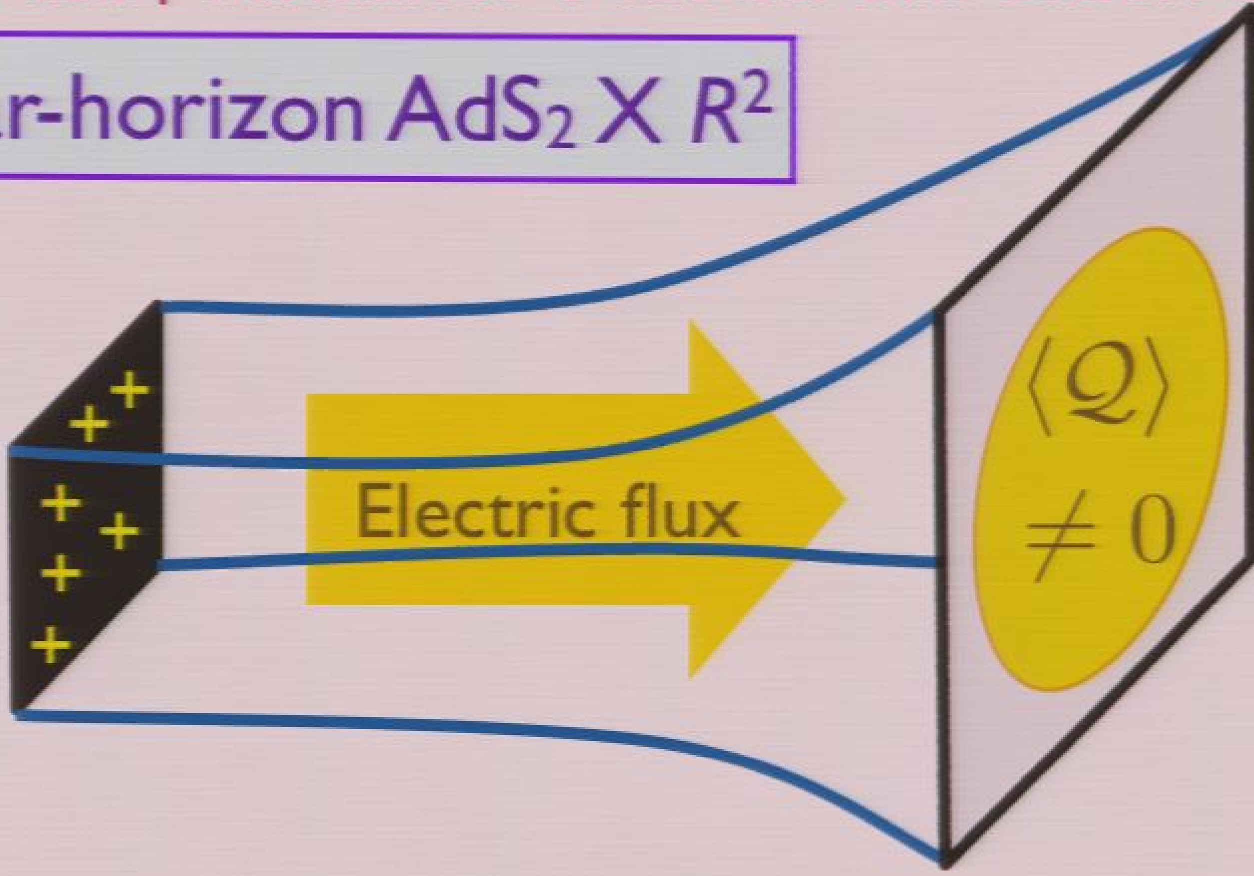
- Then compute the fermion Green's function in the background. The bulk equation for the fermion field leads to poles in Green's function at many $k = k_F^{(n)}$.

$$\left[-\partial_r^2 + V(r)\right] \Psi = 0$$



AdS_4-Reissner-Nordtröm black-brane

Near-horizon $AdS_2 \times R^2$



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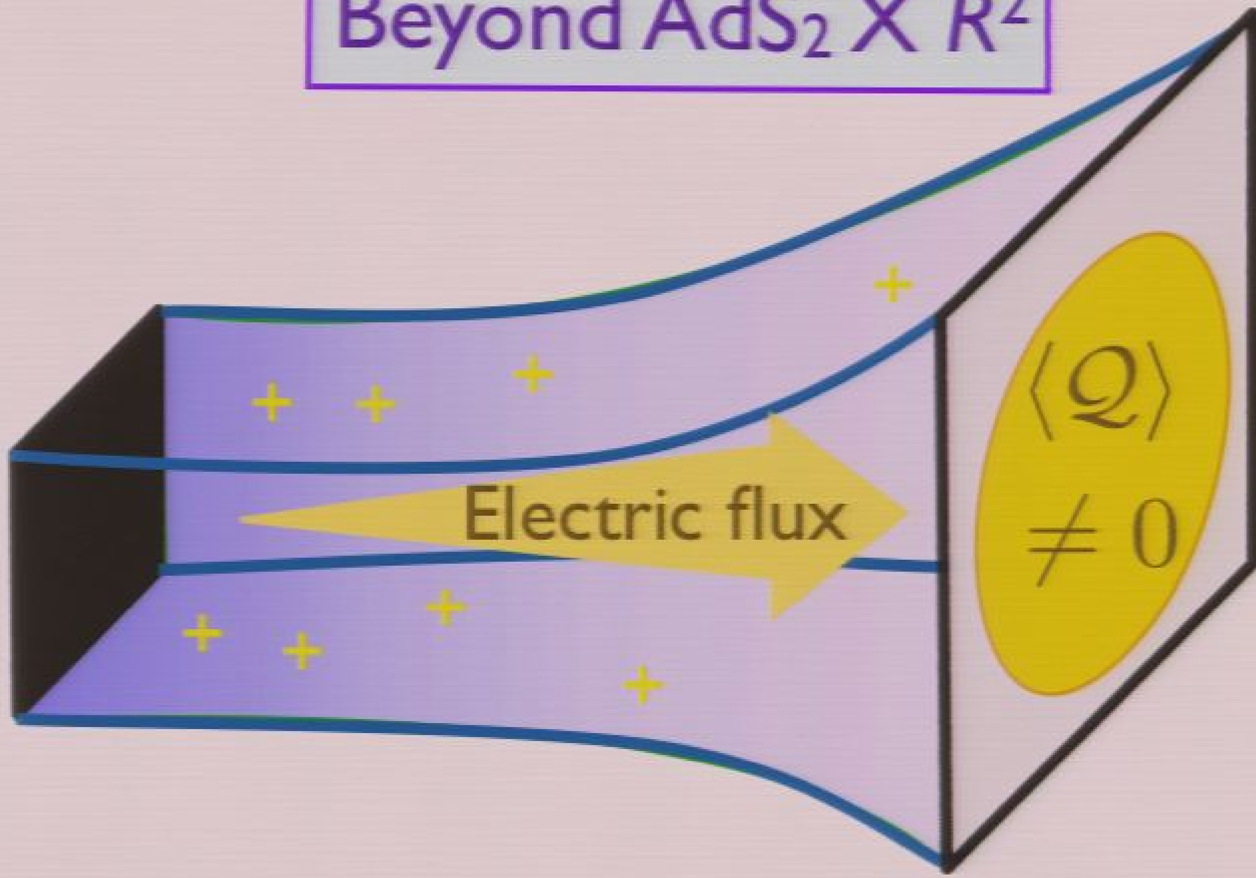
Beyond AdS$_2$ X R^2



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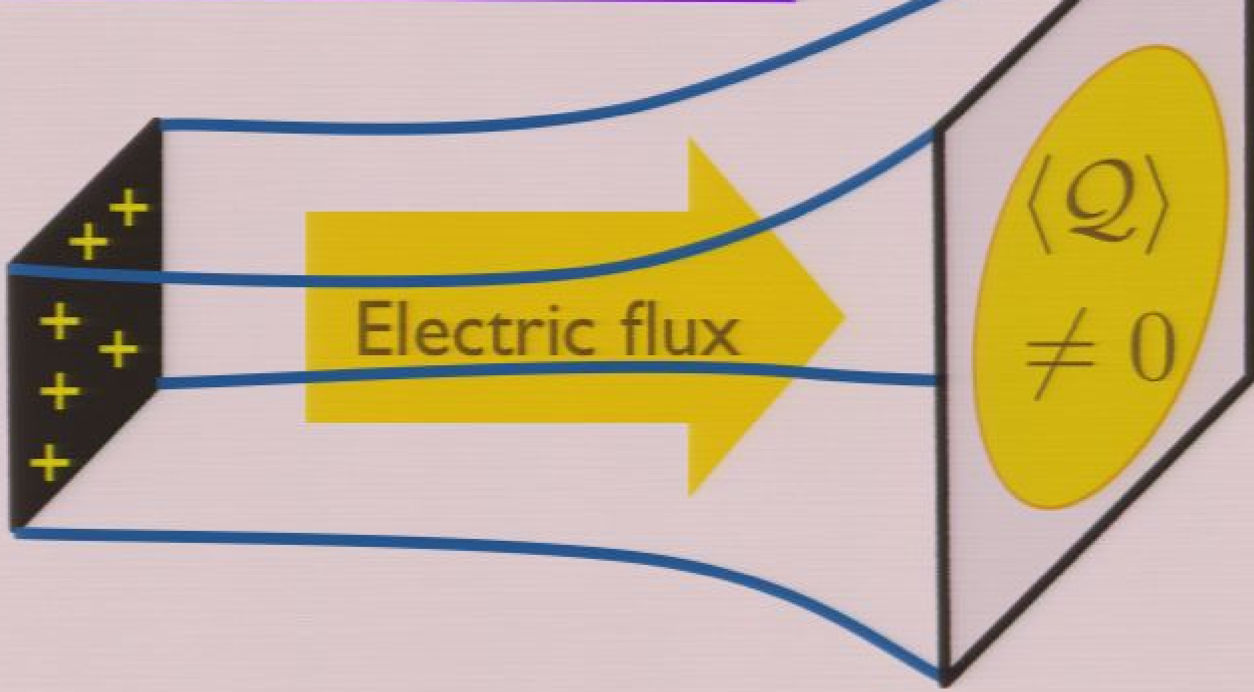

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