Title: On Slow Roll Eternal Inflation

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Abstract:

Leonardo Senatore
(Stanford)

# On Slow-Roll Eternal Inflation and the Universal Bound

with S. Dubovsky and G. Villadoro

**in progress**

**e also:**

**he Volume of the Universe after Inflation and de Sitter Entropy**

with S. Dubovsky and G. Villadoro

**JHEP 0904:118 2009**.

**he Phase Transition to Eternal Inflation**

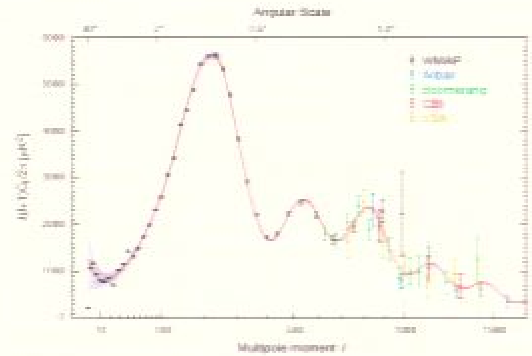
th P. Creminelli, S. Dubovsky, A. Nicolis and M. Zaldarriaga

**JHEP 0809:036 2008**.

# Outline

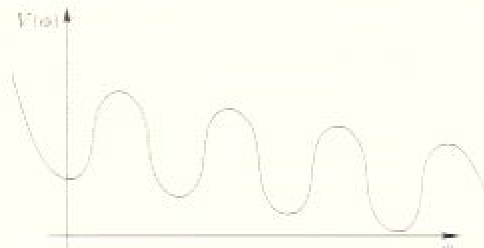
- The Landscape and de Sitter
- Slow Roll Eternal Inflation
- Bacteria Model
- Probability Distribution of the Volume
- A Bound on the Number of e-foldings
- Educated Speculations on dS Entropy and Holography

# de Sitter and us



- Quasi dS phase in our present and in our past
- Possibly both are eternal

- Landscape in String Theory



- Weinberg's solution to CC (and to other parameters)

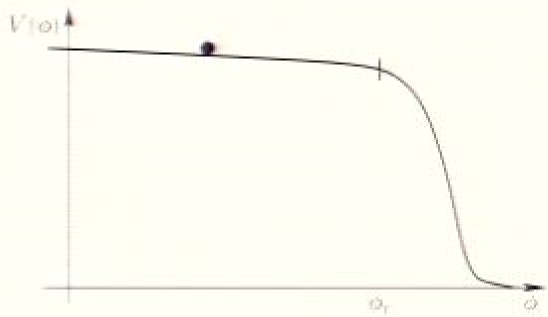

- Eternal Inflation has become extremely important
- It is also just beautiful (even better than Black Hole evaporation!)
- Difficult task
- Puzzles with de Sitter Entropy and Holography
- This motivates improved studies:
  - » Step by Step with calculations

# What is Eternal Inflation?

- What is Inflation?

$$a \sim e^{Ht}$$

$$\dot{\phi} \sim \frac{V'}{H}$$



- What is Eternal Inflation?

  Classical Motion Vs Quantum Motion

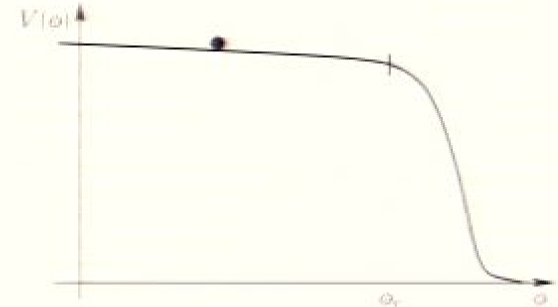
  $\Delta\phi_{\mathrm{Cl}} \sim \dot{\phi}\, H^{-1}$ Vs $\Delta\phi_{\mathrm{Q}} \sim H$

  Reproduction of space

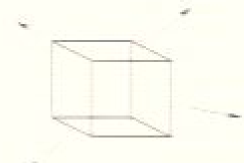
  Quantum dominates for $\dfrac{\dot{\phi}}{H^2} \lesssim 1 \Longrightarrow$ Slow Roll Eternal Inflation

- Make Eternal Inflation sharp? Calculable?
  - No Semiclassical
  - No FRW $\delta\rho/\rho \sim H^2/\dot{\phi} \sim 1$

# Perturbativity of the system

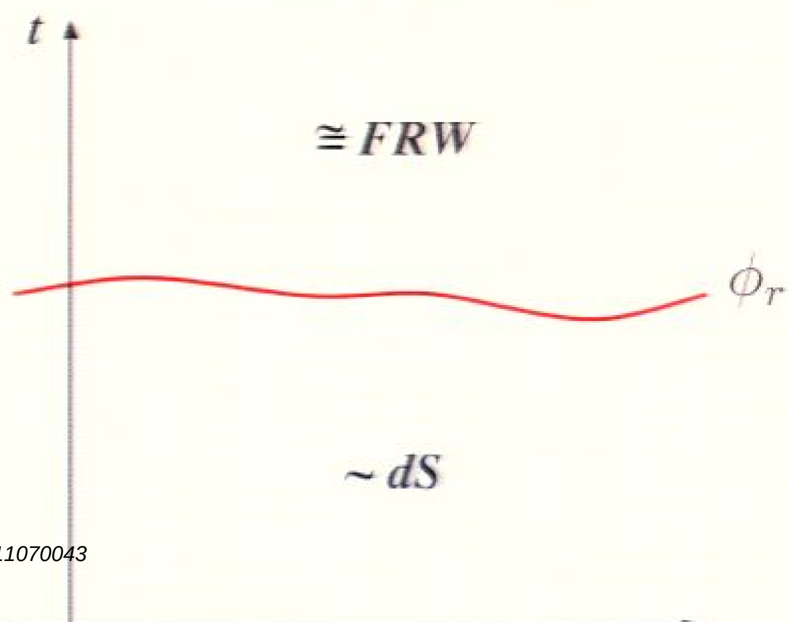


- Close to de-Sitter $\epsilon \simeq \frac{\dot{\phi}^2}{V(\phi)} \ll 1$
  - Still unperturbed before reheating: $\delta g \sim \sqrt{\epsilon}\frac{H}{M_{\rm Pl}}$
- No big interactions: $\frac{S_3}{S_2} \sim \sqrt{\epsilon}\frac{H}{M_{\rm Pl}}$

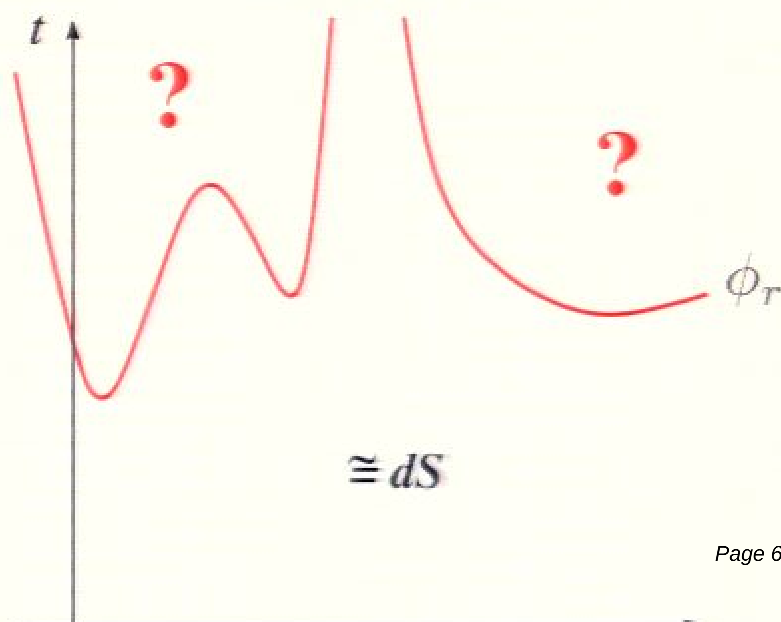


- $\Longrightarrow$ Study the volume of the Reheating surface $\phi = \phi_r$

Standard Infl.




Eternal Infl.



# A random walk

Smoothing the field: $\Lambda \ll H$ :

» $\Delta t \gtrsim H^{-1}$ for reheating

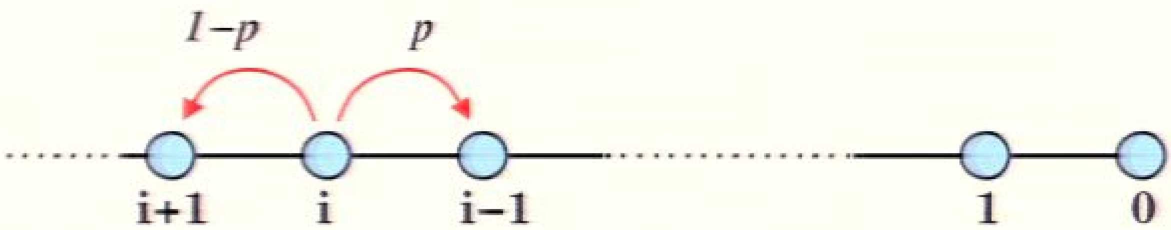
» . $[\delta\phi_k, \delta\dot{\phi}_{-k}] \to 0$

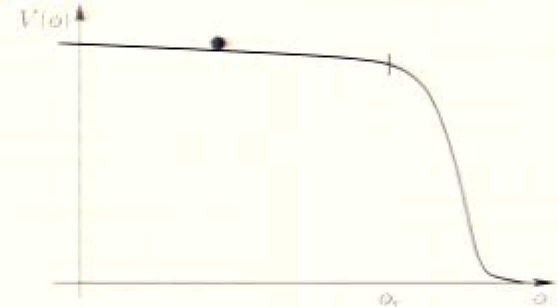
Inflaton ~ Classical stochastic system with Gaussian statistics

Probability distribution follows a diffusion equation see Starobinsky

$$P(\bar{\phi}, \phi, t) \sim e^{-\frac{(\bar{\phi}-\phi-\dot{\phi}t)^2}{H^3 t}}$$

# Bacteria Model: a discretization



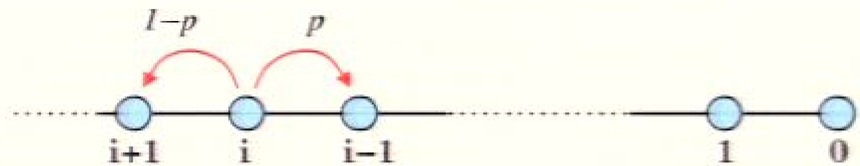
- Reproduction $N \Rightarrow \sim e^3$ New Hubble volumes
- Number of dead bacteria $\Longrightarrow$ Reheated Volume
- Classical motion $\dot{\phi} \Rightarrow p - \frac{1}{2}$ ,
- $j = -\frac{\phi}{\Delta\phi}$ , $n = \frac{t}{\Delta t}$

- Continuum Limit: $\bar{P}(j, n+1) = (1-p)\bar{P}(j-1, n) + p\ \bar{P}(j+1, n)$

$$\partial_{\sigma^2} P(\psi, \sigma^2) = \frac{4\pi^2}{H^3}\left((1-2p)\frac{\Delta\phi}{\Delta t} + \dot{\phi}\right)\partial_\psi\ P(\psi, \sigma^2) + \frac{1}{2}\frac{4\pi^2}{H^3}\frac{\Delta\phi^2}{\Delta t}\partial_\psi^2 P(\psi, \sigma^2)$$

- Identifications: $-(1-2p)\frac{\Delta\phi}{\Delta t} = \dot{\phi}$ $\qquad \frac{4\pi^2}{H^3}\frac{\Delta\phi^2}{\Delta t} = 1$ $\qquad N = 1 + 3H\Delta t$

# The Extinction Probability

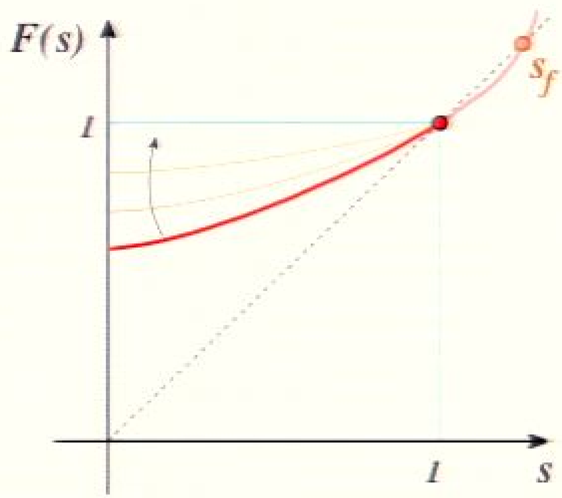


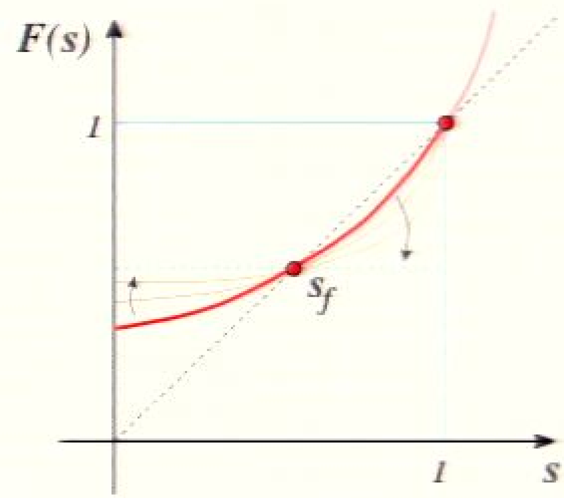

Generating function:

$$f_i^{(n)}(s_j) = \sum_{k_1 \ldots k_L} p^{(n)}_{i;k_0 \ldots k_L} s_0^{k_0} \ldots s_L^{k_L} \qquad 0 \leq s_i \leq 1$$

– Recursion: $F_{n+1} = F_1(F_n)$ , $F_1(F_\infty) = F_\infty$

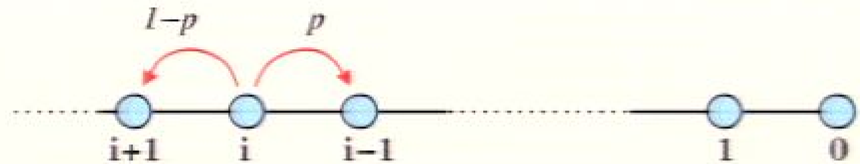
$F_\infty$ is only a function of $s_0$



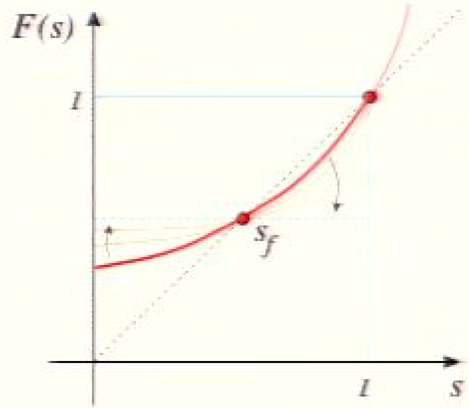




$$f_i^{(n)}(s_j) = \sum_{k_1 \ldots k_L} p^{(n)}_{i;k_0 \ldots k_L} s_0^{k_0} \ldots s_L^{k_L} \quad \Rightarrow \quad f_j^{(\infty)}(s_0) = \sum_{k=0}^{\infty} p_{j,k}\, s_0^k$$

## Concentrating on $F_\infty$ :



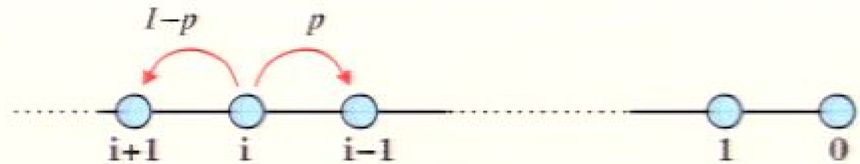

- $F_\infty$ is only a function of $s_0$




- $F_1(F_\infty) = F_\infty$ ,

$$
\begin{aligned}
f_0^{(1)}(s_0, \dots, s_L) &= s_0 \, , \\
f_1^{(1)}(s_0, \dots, s_L) &= ((1-p)s_2 + p\, s_0)^{N_r} \, , \\
&\vdots \\
f_i^{(1)}(s_0, \dots, s_L) &= ((1-p)s_{i+1} + p\, s_{i-1})^{N_r} \\
&\vdots \\
f_L^{(1)}(s_0, \dots, s_L) &= ((1-p)s_L + p\, s_{L-1})^{N_r}
\end{aligned}
\quad \Longrightarrow \quad
\begin{aligned}
f_0^{(\infty)}(s_0) &= s_0 \, , \\
&\vdots \\
f_i^{(\infty)}(s_0) &= \Big( (1-p) f_{i+1}^{(\infty)}(s_0) + p\, f_{i-1}^{(\infty)}(s_0) \Big)^{N_r} \\
&\vdots \\
f_L^{(\infty)}(s_0) &= \Big( (1-p) f_L^{(\infty)}(s_0) + p\, f_{L-1}^{(\infty)}(s_0) \Big)^{N_r}
\end{aligned}
$$

# Taking the continuum limit:




- In the sites: $i \Rightarrow \phi$
- A differential equation:

$$f^{(\infty)}(\phi; s_0) \;=\; \left((1-p) f^{(\infty)}(\phi + \Delta\phi; s_0) + p\, f^{(\infty)}(\phi - \Delta\phi; s_0)\right)^{N_r} \qquad \Rightarrow$$

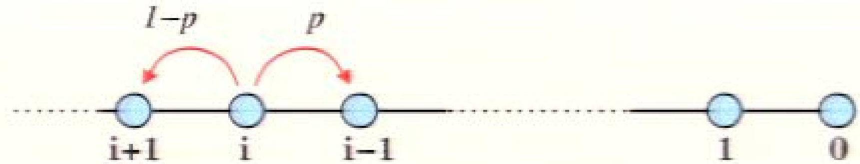
$$\frac{1}{2}\frac{\partial^2}{\partial\phi^2} f^{(\infty)}(\phi; s_0) - \frac{2\pi\sqrt{6\Omega}}{H}\frac{\partial}{\partial\phi} f^{(\infty)}(\phi; s_0) + \frac{12\pi^2}{H^2} f^{(\infty)}(\phi; s_0)\log\left[f^{(\infty)}(\phi; s_0)\right] = 0$$

$$\Omega \equiv \frac{2\pi^2}{3}\frac{\dot{\phi}^2}{H^4}$$

- with boundary conditions: $f^{(\infty)}(\phi = 0, s_0) = s_0 \;,\; \left.\frac{\partial}{\partial\phi} f^{(\infty)}(\phi; s_0)\right|_{\phi_b} \;=\; 0$
- For the quantity: $f_j^{(\infty)}(s_0) = \sum_{k=0}^{\infty} p_{j,k}\, s_0^k \;\Rightarrow f^{(\infty)}(\phi; s_0) = \int_0^{\infty} dV\, \rho(\phi, V)\, s_0^V$
- i.e. for the Laplace transform of $\rho(\phi, V)$

$$\rho(\phi, V) = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} d\left(-\log(s_0)\right) f^{(\infty)}(\phi; s_0) e^{-V\log(s_0)}$$

# Studying the differential eq.




Diff. Eq. :

$$\ddot{f}(\tau;z) - 2\sqrt{\Omega}\dot{f}(\tau;z) + f(\tau;z)\log\left[f(\tau;z)\right] = 0$$

where $\tau \propto \phi \propto N_c\,, \qquad z = -\log(s_0)\ , \qquad \Omega \equiv \frac{2\pi^2}{3}\frac{\dot{\phi}^2}{H^4}$

boundary conditions:

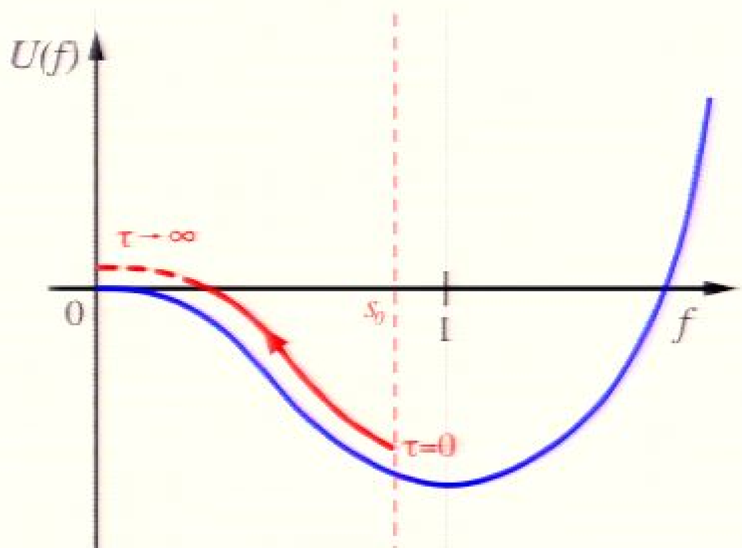
$$f(0;z) = s_0 = e^{-z}\,,$$

$$\dot{f}(\tau_b;z) = 0\,,$$

$$f(\tau;z) \in [0,1]$$

Mechanical Problem: Anti-friction and potential $\quad U(f) = \frac{f^2}{4}\left(\log f^2 - 1\right)$

Barrier at infinity:



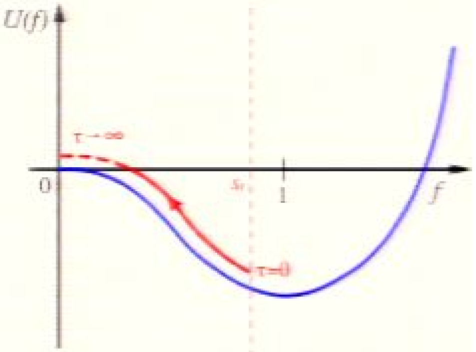
# The Phase Transition

$$\ddot{f}(\tau;z) - 2\sqrt{\Omega}\dot{f}(\tau;z) + f(\tau;z)\log\left[f(\tau;z)\right] = 0$$

- $P_{\rm ext} \equiv \int_0^\infty dV\,\rho(V,\tau) = f(\tau;0)$

$$s_0 \to 1 \quad \Rightarrow \quad z \to 0$$

$z = -\log(s_0)$




- $\Longrightarrow$ Linearized solution:
- .

$$\ddot{f} - 2\sqrt{\Omega}\dot{f} + f - 1 = 0\,, \qquad \Longrightarrow \qquad f = 1 - e^{\sqrt{\Omega}\tau}\left(Ae^{\sqrt{\Omega-1}\tau} + Be^{-\sqrt{\Omega-1}\tau}\right)$$

- Different behaviors for $\Omega \gtrless 1$

$$\Omega \equiv \frac{2\pi^2}{3}\frac{\dot{\phi}^2}{H^4}$$

# The Phase Transition

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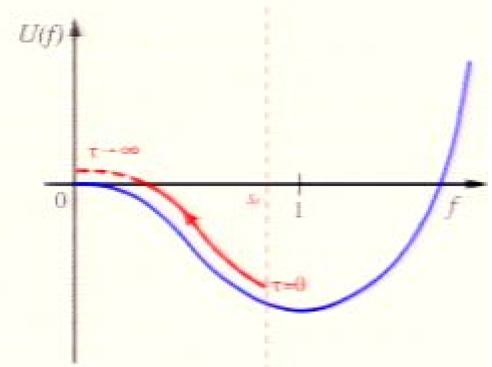
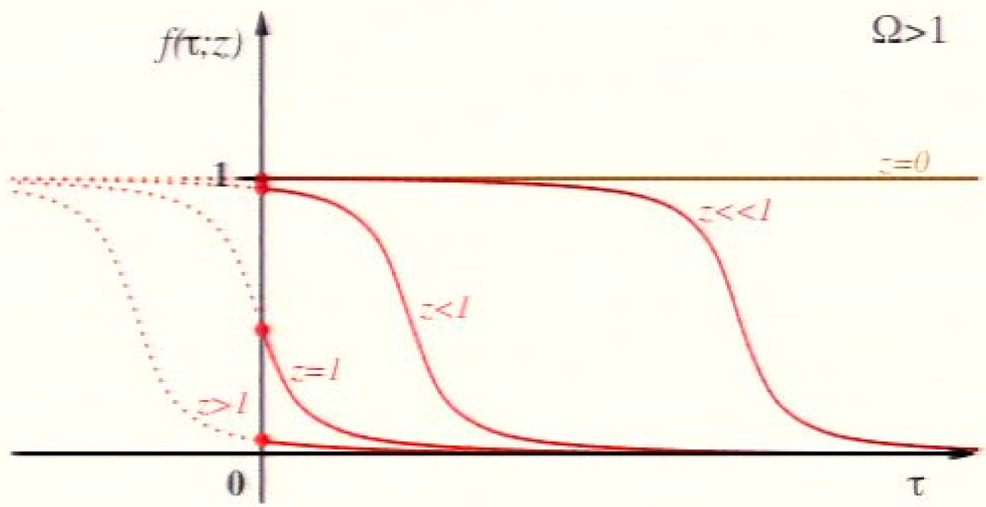


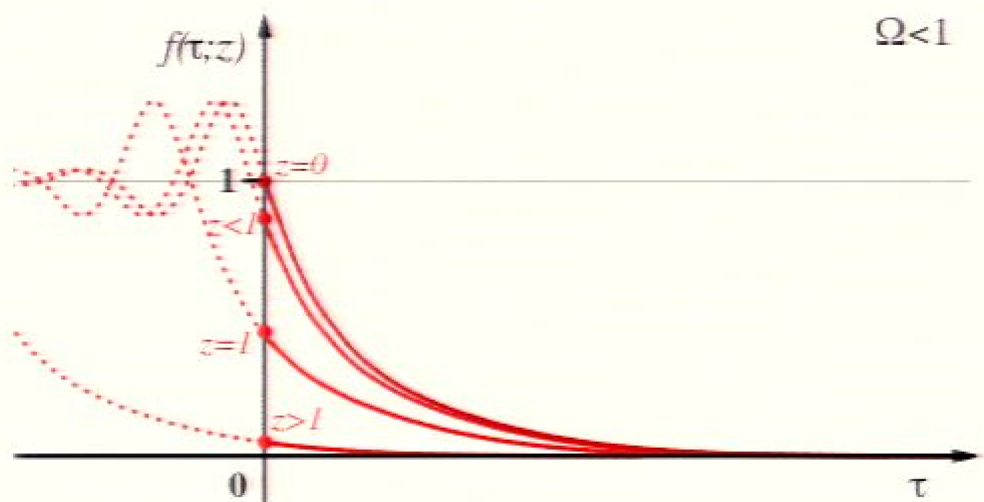

Total Extinction

$\Omega > 1$




No Extinction

$\Omega < 1$




Phase transition:

$$\Omega \equiv \frac{2\pi^2}{3}\frac{\dot\phi^2}{H^4} < 1$$

with P. Creminelli, S. Dubovsky
A. Nicolis, M. Zaldarriaga,
**JHEP 0809:036 2008**

# An Equation for the Average

- Take diff equation $\ddot{f}(\tau;z) - 2\sqrt{\Omega}\dot{f}(\tau;z) + f(\tau;z)\log\left[f(\tau;z)\right] = 0$
- Derivative wrt z:

$$\ddot{f}' - 2\sqrt{\Omega}\dot{f}' + f' + f'\log f = 0$$

- Since for $z = 0$ $f = 1$ , and $f'_0 \equiv f'(\tau;0) = -\langle V\rangle$
- Solve with $f'_0(0) = -1\,,$ $\dot{f}'_0(\tau_b) = 0$ , $f'_0 = Ae^{\omega_+\tau} + Be^{\omega_-\tau}$
- and find

$$\lim_{\tau_b\to\infty}\langle V\rangle = e^{\omega_-\tau} = e^{3N_c\frac{2}{1+\sqrt{1-1/\Omega}}}$$

$$\bar{V} \sim e^{3N_c} \quad \text{for} \quad \Omega \gg 1\,, \qquad \bar{V} \sim e^{6N_c} \quad \text{for} \quad \Omega \simeq 1$$

- A huge quantum effect
- Analogous for the Variance

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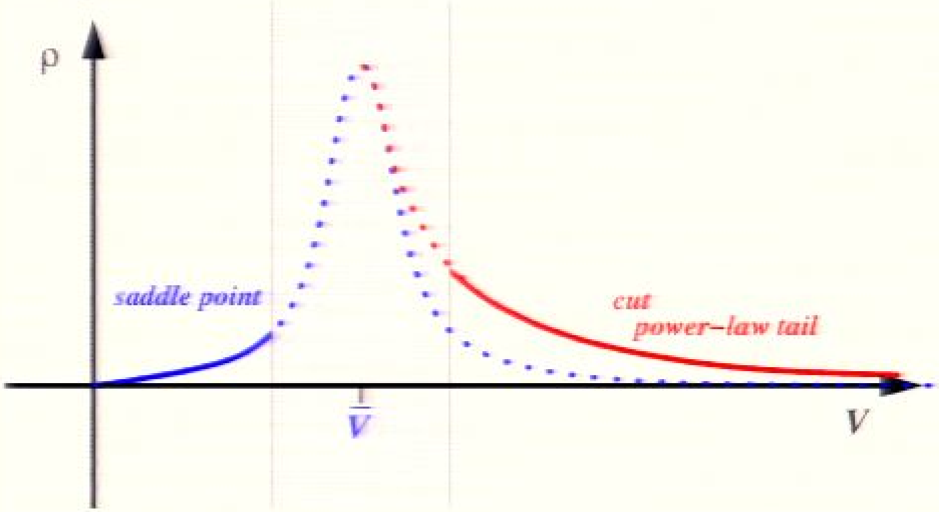
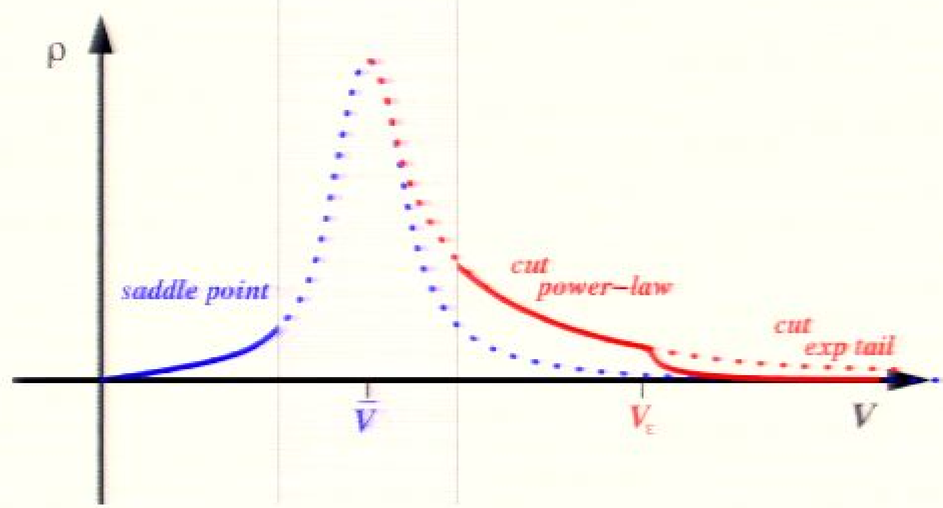
# Single Field Summary

$\Omega \gtrsim 1$ $\Omega \lesssim 1$




Gaussian behavior $N \ll N_c$

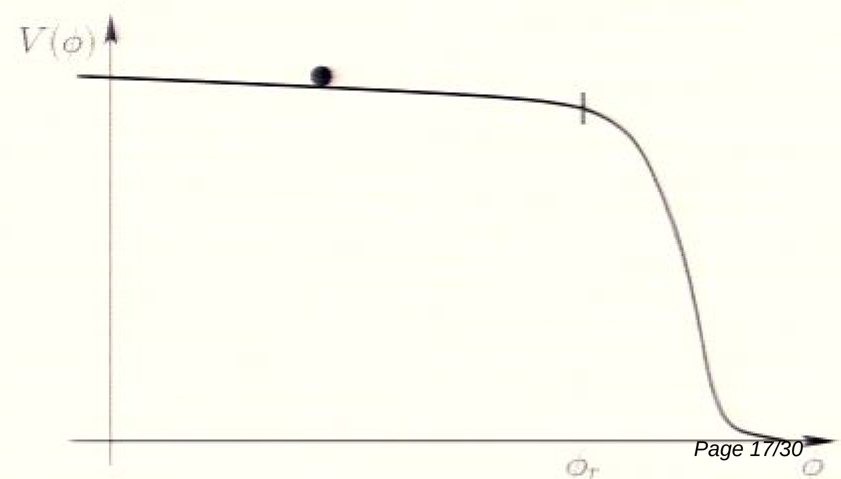
$$P(\bar{\phi}, \phi, t) \sim e^{-\frac{(\bar{\phi}-\phi-\dot{\phi}t)^2}{H^3 t}} \sim e^{-\Omega\frac{(N-N_c-Ht)^2}{Ht}} \quad \Rightarrow \quad p \propto e^{-c(N-N_c)^2}\,, \quad \text{for} \quad N \ll N_c\,.$$

Exponential $N \gg N_c$

$$p \propto e^{-c_1 \Delta\phi^2/t} \propto e^{-c_2 N}$$

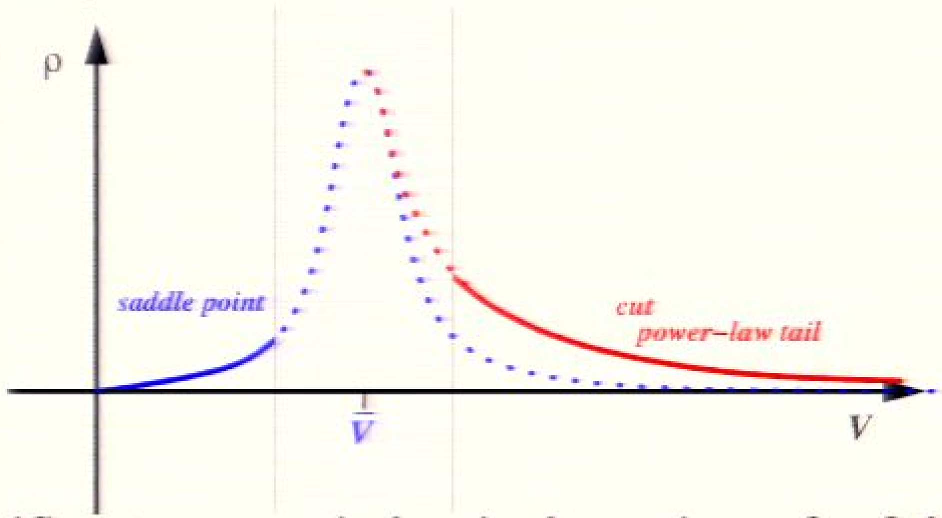
Super-Exponential: Eternal Inflation

$$p \propto \tilde{p}^{e^{3N}}$$

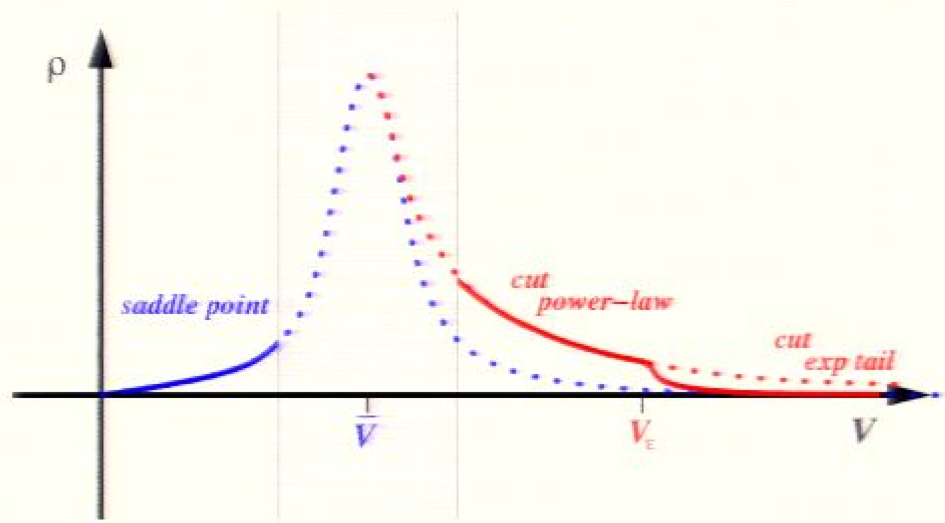


# Connection with de Sitter Entropy

$\Omega \gtrsim 1$




$\Omega \lesssim 1$




dS entropy and classical number of e-foldings

$$S_{\rm dS} = \pi \frac{M_{\rm Pl}^2}{H^2}$$

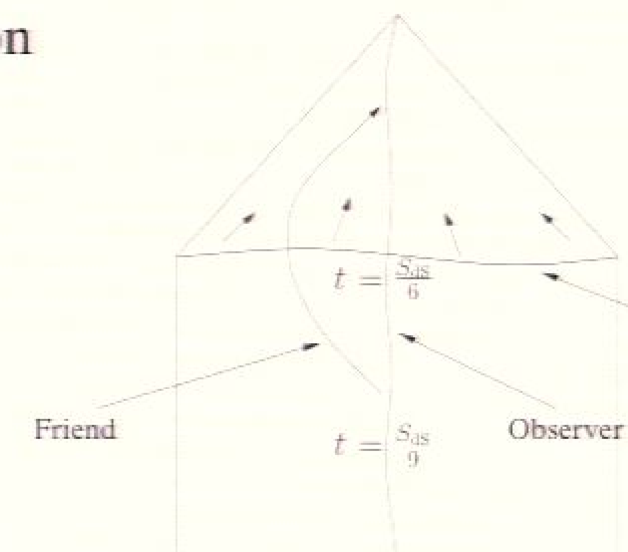
$$\frac{dS_{\rm ds}}{dN_c} = \pi M_{\rm Pl}^2 \frac{dH^{-2}}{H\,dt} = 8\pi \frac{\dot{\phi}^2}{H^4} \quad \Rightarrow \quad N_c \lesssim \frac{S_{ds}}{12}$$

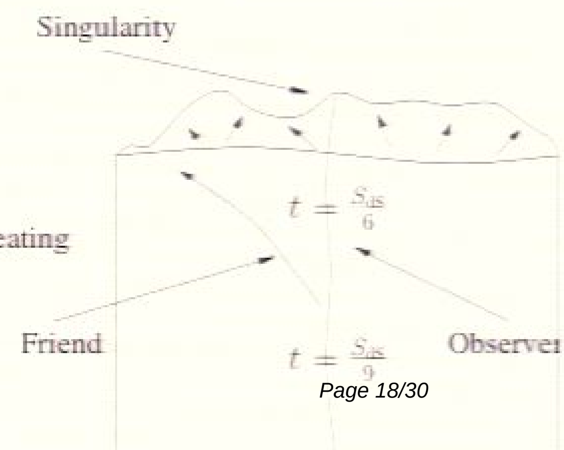
generalization of
N. Arkani-Hamed et al.
**JHEP 0705:055 2007**

In every finite volume realization

$$P(V > e^{\frac{S_{\rm ds}}{2}}) < e^{-\alpha S_{\rm ds}}$$

$$V_{\rm Finite\ \ Realization} < e^{\frac{S_{\rm ds}}{2}}$$




Check for de Sitter entropy

(Avoid Xerox Paradox).

Page Argument (?)

# Exploring The Bound in Any Dimensions

- Volume in any dimensions

$$\langle\phi^2\rangle = \frac{H^{D-1}}{\pi\Theta_{(D-1)}} t + \ldots$$

$$\Omega = \frac{\dot{\phi}^2}{2(D-1)H}\frac{\Delta t}{\Delta\phi^2} = \frac{\pi\Theta_{(D-1)}}{2(D-1)}\frac{\dot{\phi}^2}{H^D},$$

$$\tau = \phi\sqrt{2(D-1)H}\sqrt{\frac{\Delta t}{\Delta\phi^2}} = 2(D-1)\sqrt{\Omega}N_c$$

- Equation (in $\tau$) the same, so same solution

$$\langle V\rangle = e^{(D-1)N_c\frac{2}{1+\sqrt{1-1/\Omega}}}$$

- dS Entropy

$$\Delta S = \int_0^{N_c} 4(D-1)\Omega dN_c = 4(D-1)\Omega N_c',$$

- Combining

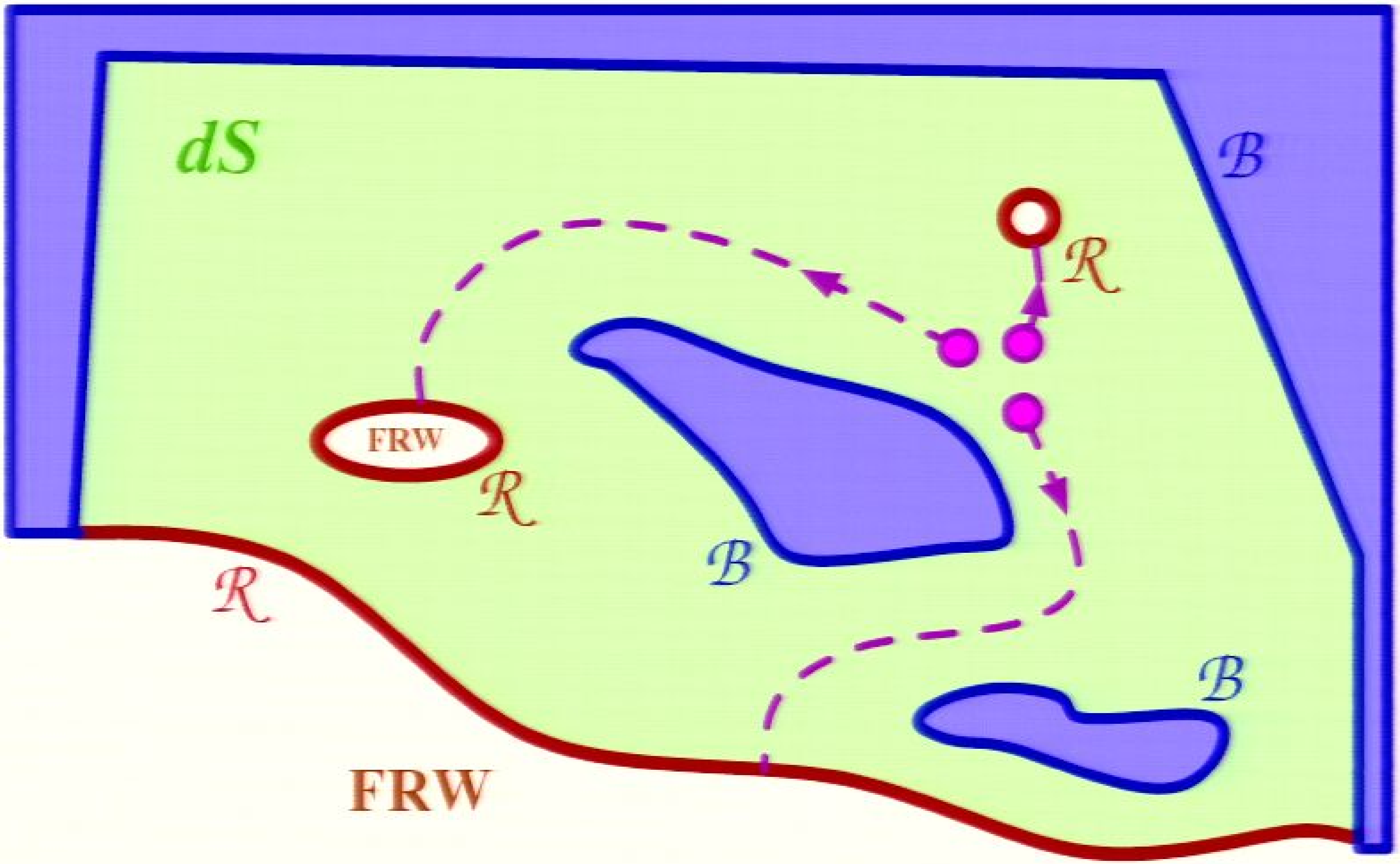
$$\langle V\rangle = e^{(D-1)N_c\frac{2}{1+\sqrt{1-1/\Omega}}} \leq e^{\frac{S_{end}}{2}\frac{1}{\Omega(1+\sqrt{1-1/\Omega})}} \leq e^{\frac{S_{end}}{2}}$$





- Bound holds in any dimensions

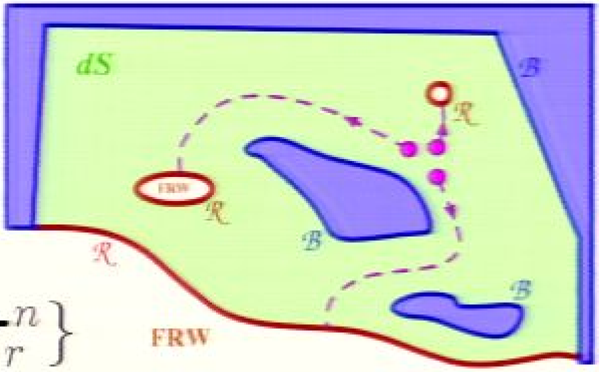
# Studying Multifield Slow Roll Eternal Inflation




- Equations trivially generalized

$$\nabla^2 f(\boldsymbol{\tau}, z) - \sqrt{\Omega} \cdot \nabla f(\boldsymbol{\tau}, z) + f(\boldsymbol{\tau}, z) \log[f(\boldsymbol{\tau}, z)] = 0$$

# New Questions




- Can keep truck of the kind of volume produced
- Suppose Reheating surface $\mathcal{R}$ is discrete
- Keep truck of $\vec{V} = \{V_1, \ldots, V_n\}$ $\quad \vec{\boldsymbol{\tau}}_r = \{\boldsymbol{\tau}_r^1, \ldots, \boldsymbol{\tau}_r^n\}$

$$\rho(\vec{V}, \vec{\boldsymbol{\tau}}) = \int_{\mathcal{C}} d\vec{z}\, e^{\vec{V}\cdot\vec{z}} f(\vec{\boldsymbol{\tau}}, \vec{z}) \qquad f(\boldsymbol{\tau}_r^i, \vec{z}) = e^{-z_i}$$

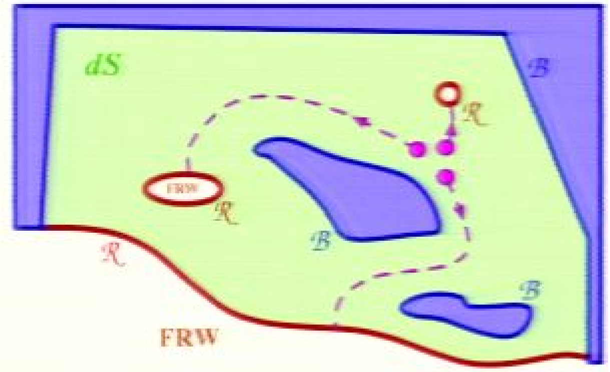
- Generalized Bound

$$P\left(\int_{\mathcal{I}} d\boldsymbol{\tau}_r\, V(\boldsymbol{\tau}_r) > \mathrm{Sup}_{\mathcal{I}}\left[e^{S(\boldsymbol{\tau}_r)/2}\right]; V < +\infty\right) =$$

$$= \int_{\mathcal{I}} d\boldsymbol{\tau}_r \int_{e^{S(\boldsymbol{\tau}_r)/2}\delta(\boldsymbol{\tau}_r' - \boldsymbol{\tau}_r)}^{+\infty} \mathcal{D}V(\boldsymbol{\tau}_r')\ \rho\left(V(\boldsymbol{\tau}_r', \boldsymbol{\tau})\right) \lesssim \mathrm{Sup}_{\mathcal{I}}\left[e^{-k\, e^{S(\boldsymbol{\tau}_r)/2}}\right]$$

- ... difficult to check ....

# Concentrate on the Average



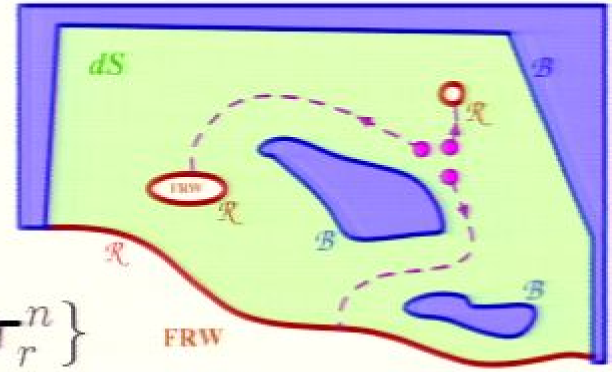

- Eq. for average is linear

$$\boldsymbol{\nabla}^2\langle V(\tau'_{\mathbf{r}})\rangle - \sqrt{\boldsymbol{\Omega}}\cdot\boldsymbol{\nabla}\langle V(\tau'_{\mathbf{r}})\rangle + \langle \mathbf{V}(\tau'_{\mathbf{r}})\rangle = 0$$

- Bound for average

$$\int_{\mathcal{I}} d\boldsymbol{\tau}_r\ \langle V(\boldsymbol{\tau}_r)\rangle \lesssim \mathrm{Sup}_{\mathcal{I}}\left[e^{S(\boldsymbol{\tau}_r)/2}\right]$$

# New Questions




- Can keep truck of the kind of volume produced
- Suppose Reheating surface $\mathcal{R}$ is discrete
- Keep truck of $\vec{V} = \{V_1, \ldots, V_n\}$ $\vec{\boldsymbol{\tau}}_r = \{\boldsymbol{\tau}_r^1, \ldots, \boldsymbol{\tau}_r^n\}$

$$\rho(\vec{V}, \vec{\boldsymbol{\tau}}) = \int_{\mathcal{C}} d\vec{z}\, e^{\vec{V}\cdot\vec{z}}\, f(\vec{\boldsymbol{\tau}}, \vec{z}) \qquad f(\boldsymbol{\tau}_r^i, \vec{z}) = e^{-z_i}$$

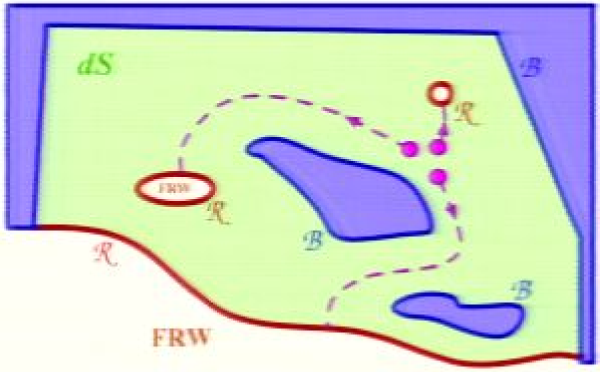
- Generalized Bound

$$P\left(\int_{\mathcal{I}} d\boldsymbol{\tau}_r\, V(\boldsymbol{\tau}_r) > \mathrm{Sup}_{\mathcal{I}}\left[e^{S(\boldsymbol{\tau}_r)/2}\right]; V < +\infty\right) =$$

$$= \int_{\mathcal{I}} d\boldsymbol{\tau}_r \int_{e^{S(\boldsymbol{\tau}_r)/2}\delta(\boldsymbol{\tau}_r'-\boldsymbol{\tau}_r)}^{+\infty} \mathcal{D}V(\boldsymbol{\tau}_r')\ \rho\left(V(\boldsymbol{\tau}_r', \boldsymbol{\tau})\right) \lesssim \mathrm{Sup}_{\mathcal{I}}\left[e^{-k\, e^{S(\boldsymbol{\tau}_r)/2}}\right]$$

- ... difficult to check ....

# Concentrate on the Average



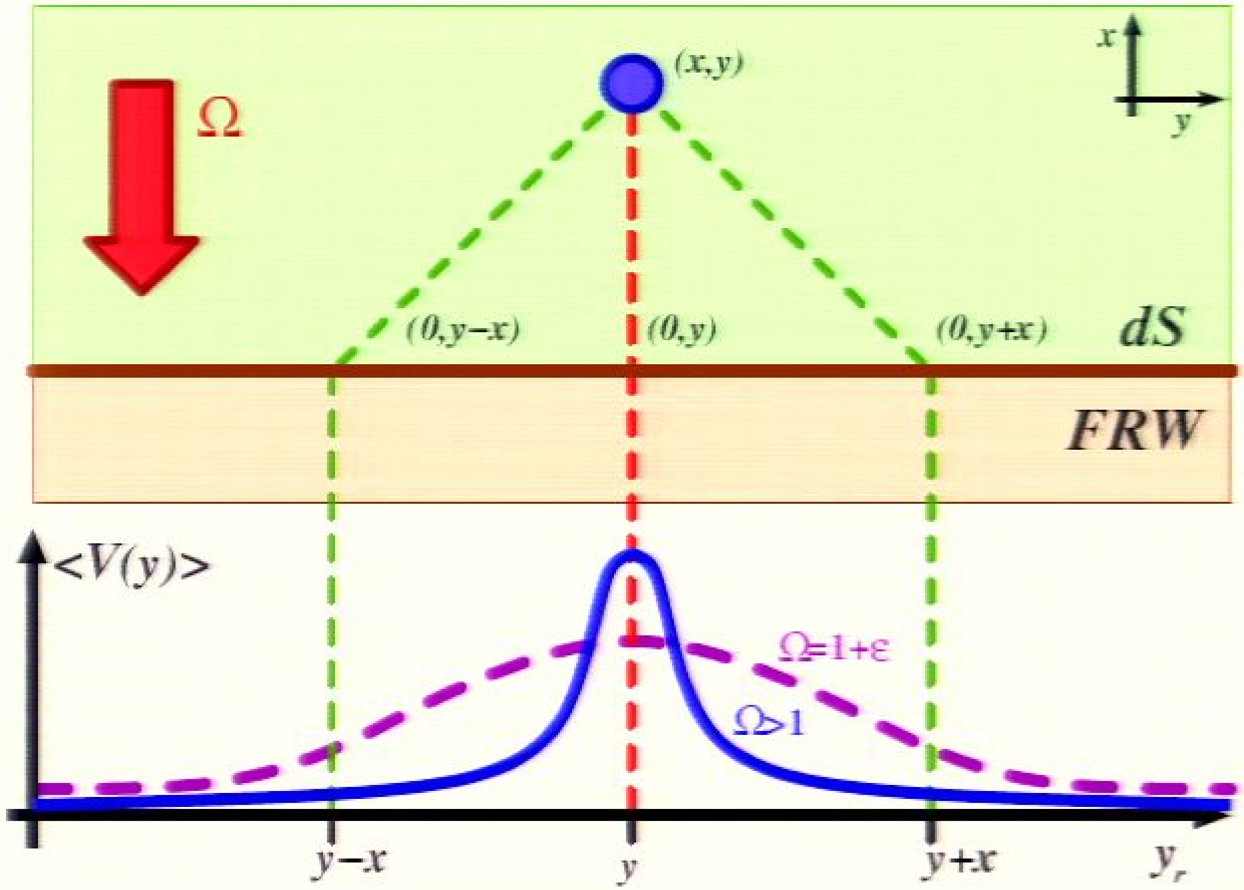

- Eq. for average is linear

$$\nabla^2\langle V(\tau'_{\mathbf{r}})\rangle - \sqrt{\boldsymbol{\Omega}}\cdot\nabla\langle V(\tau'_{\mathbf{r}})\rangle + \langle \mathbf{V}(\tau'_{\mathbf{r}})\rangle = 0$$

- Bound for average

$$\int_{\mathcal{I}} d\boldsymbol{\tau}_r \, \langle V(\boldsymbol{\tau}_r)\rangle \lesssim \mathrm{Sup}_{\mathcal{I}}\left[e^{S(\boldsymbol{\tau}_r)/2}\right]$$

# xample 1: the Waterfall



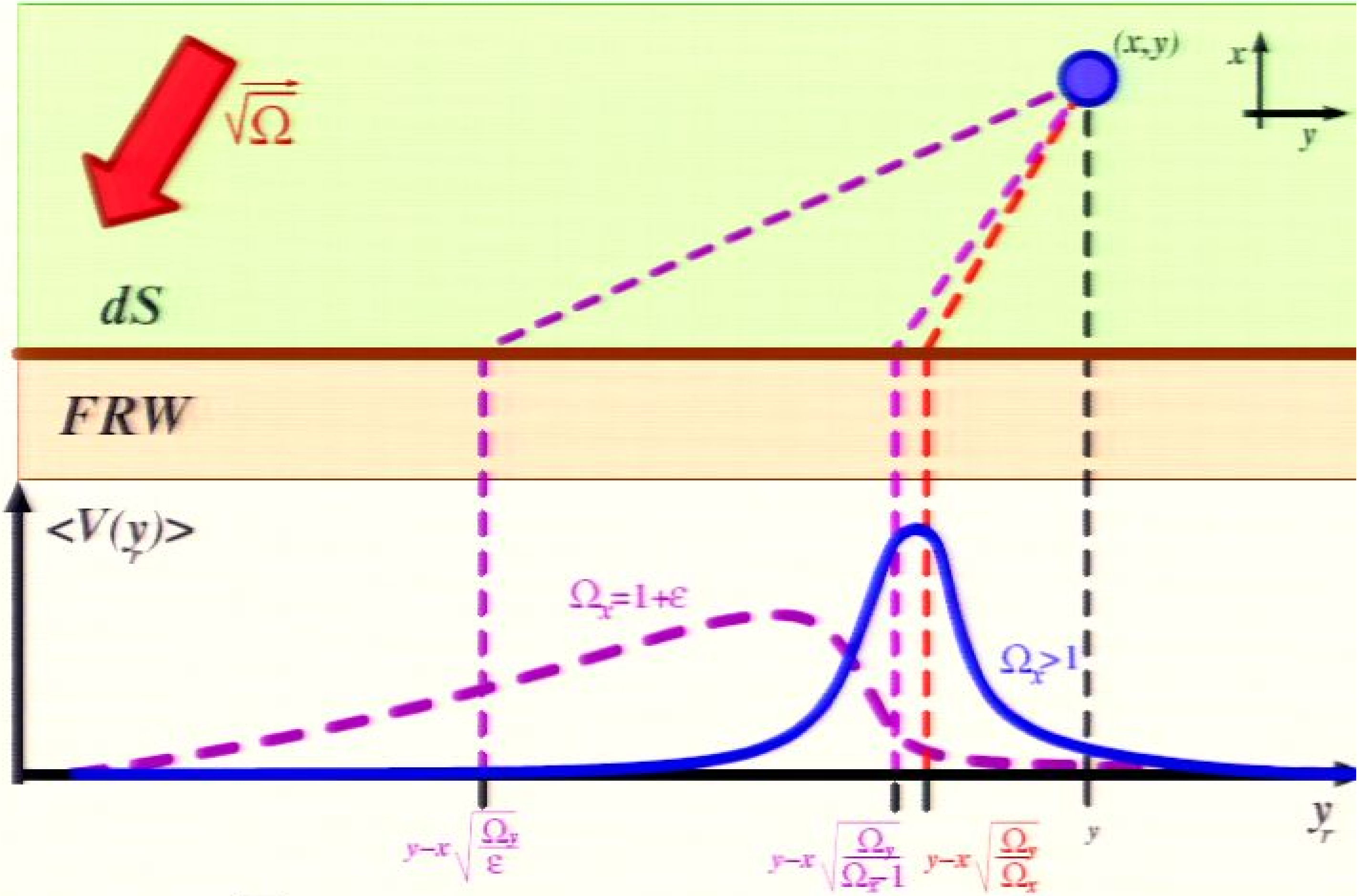

- Average overall volume $\langle V \rangle = e^{3N_c \frac{2}{1+\sqrt{1-1/\Omega}}}$ $\Rightarrow$ Transition at $\Omega = 1$
- Away from Eternal Inflation

$$\langle V(y_r) \rangle \quad \to \quad e^{-\frac{\sqrt{\Omega-1}}{x}\frac{\Delta y^2}{1+\sqrt{1+\Delta y^2/x^2}}}\,, \qquad \text{for} \quad (\Omega-1)(x^2+\Delta y^2) \gg 1\,,$$

- At the phase transition

$$\langle V(y_r) \rangle \quad \to \quad \frac{x}{x^2+\Delta y^2}\,, \qquad \text{for} \quad (\Omega-1)(x^2+\Delta y^2) \ll 1$$

# xample 2: ıe Tilted Waterfall




Average overall volume $\langle V \rangle = e^{(\sqrt{\Omega_x} - \sqrt{\Omega_x - 1})x}$

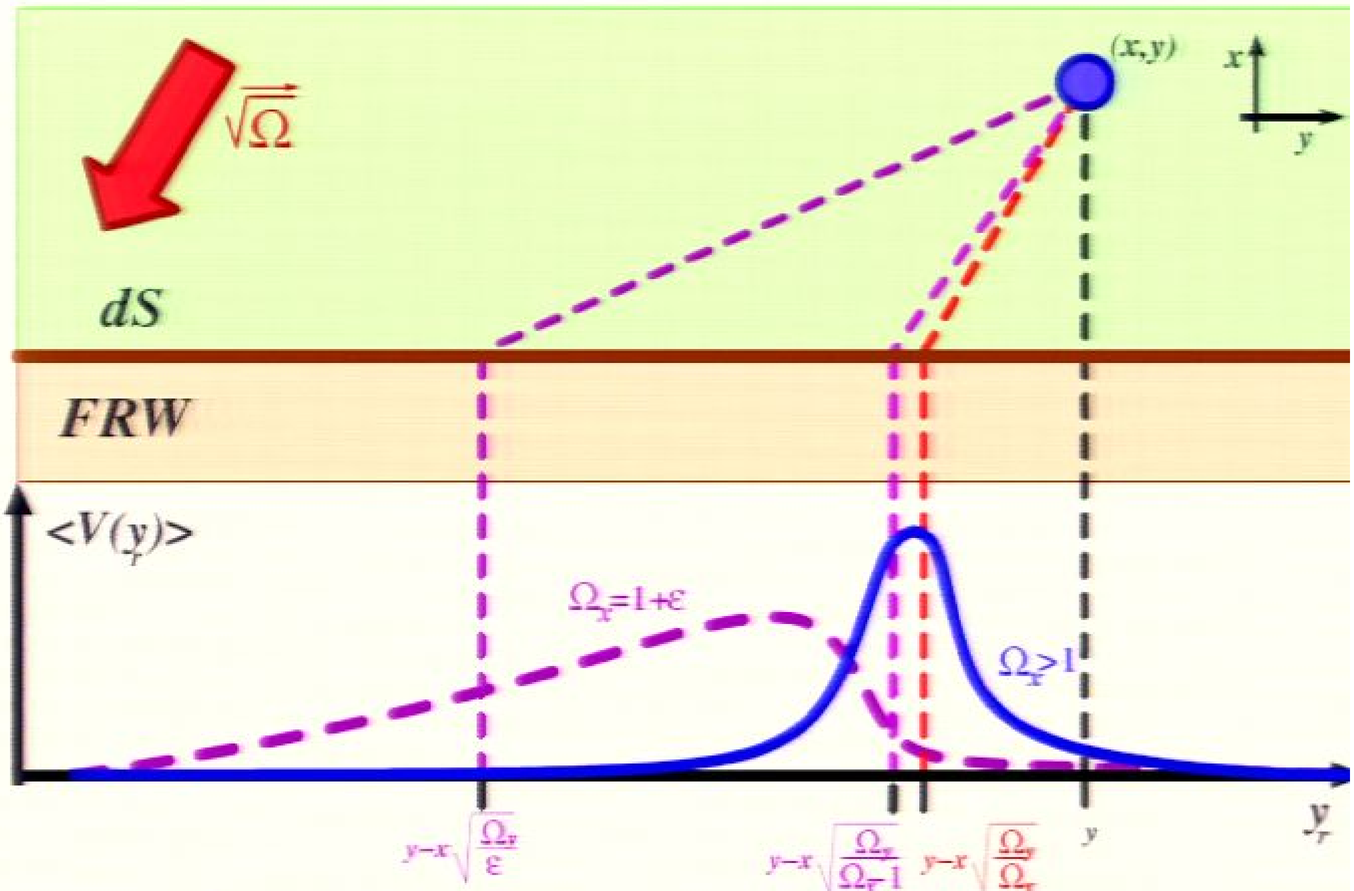
Average Exit point $\Rightarrow$ Transition at $\Omega_x = 1 \Rightarrow \Omega > 1$

$$\langle y_r \rangle = y - x\sqrt{\frac{\Omega_y}{\Omega_x - 1}} \, ,$$

Notice that

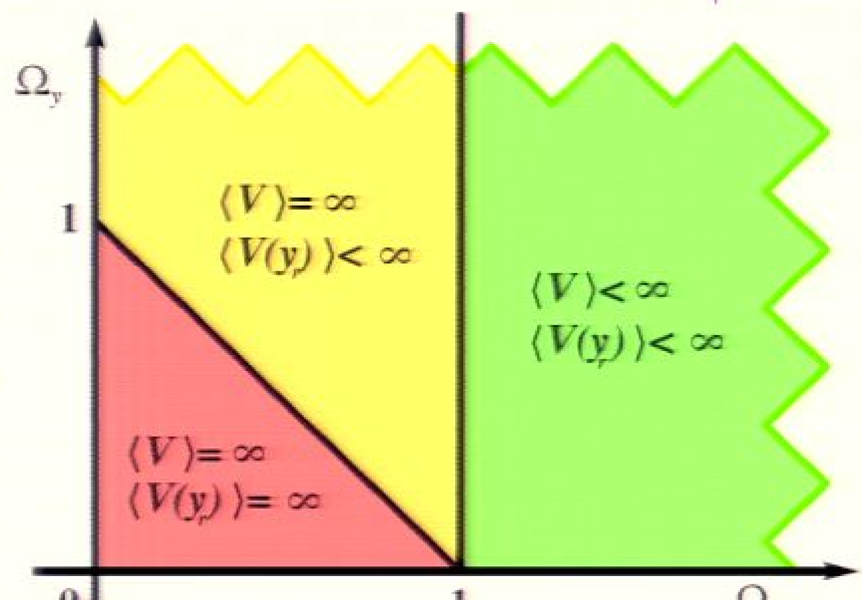
$$\langle V(y_r) \rangle \to \infty \quad \text{at} \quad \Omega = 1$$

xample 2:
he Tilted Waterfall




Phase diagram

Bound holds

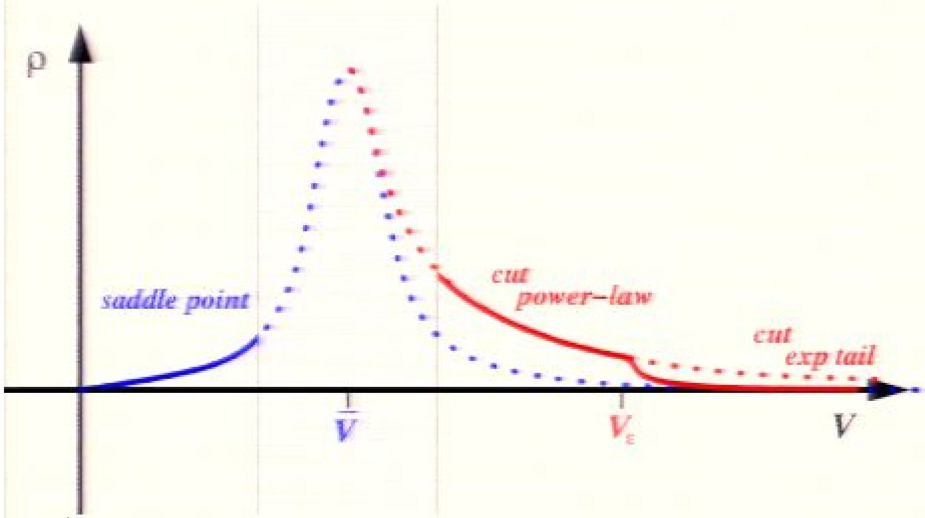


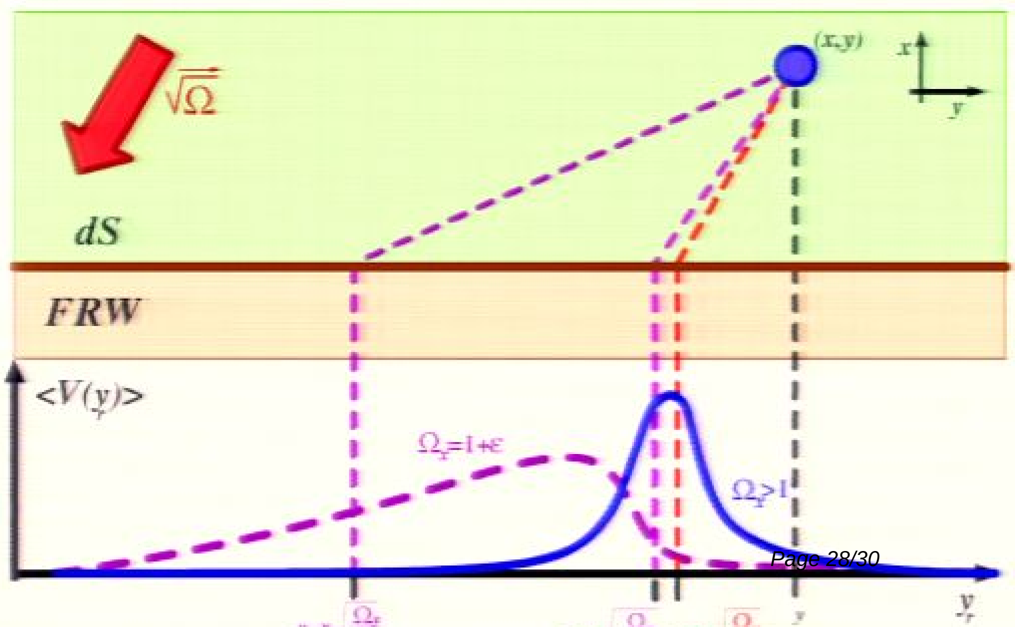
# Conclusions

Eternal Inflation, the Landscape and Holography: detailed calculations

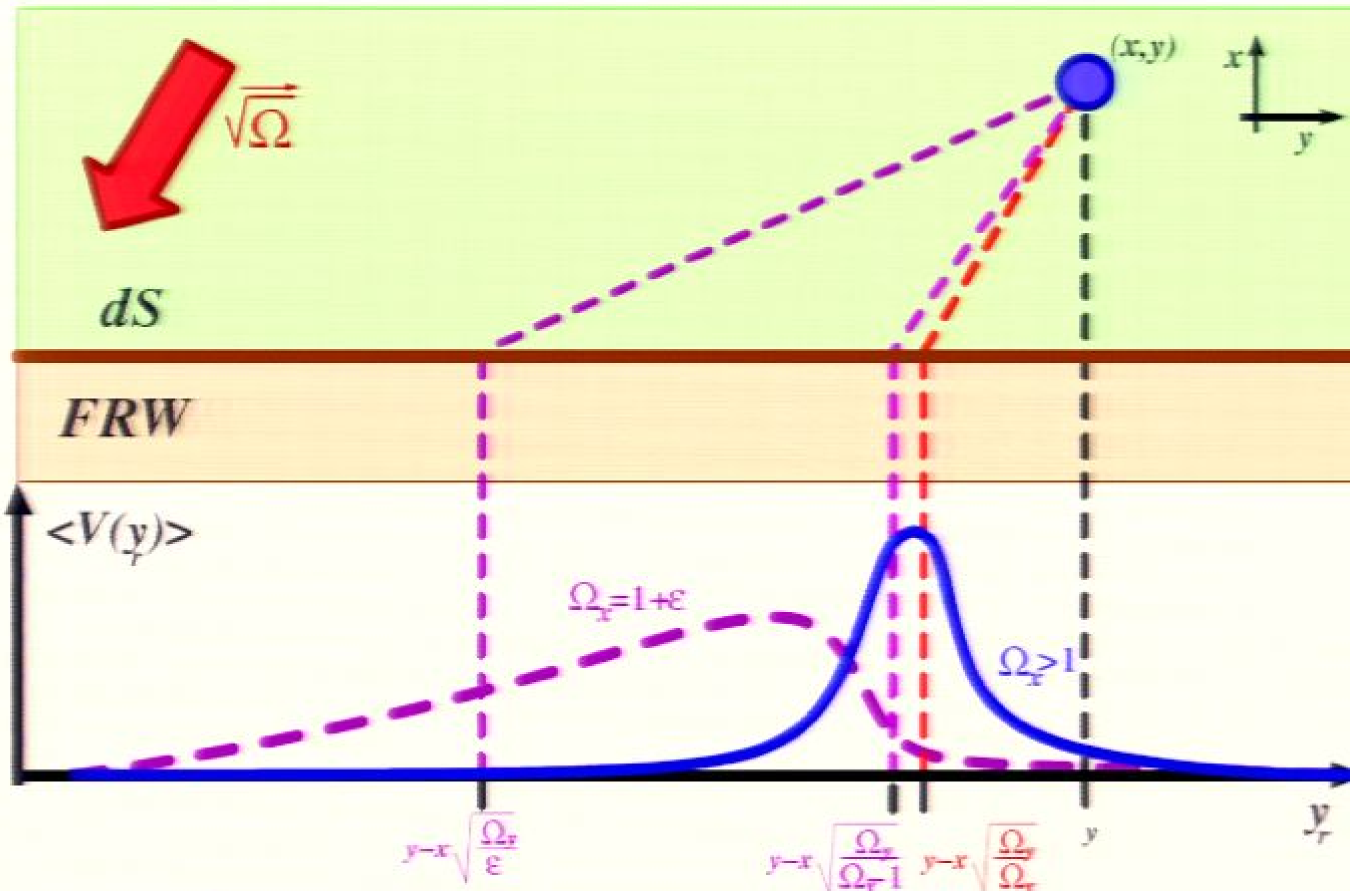
Volume of the Universe after Slow Roll Inflation

- Detailed study of the volume in Eternal and non-Eternal Inflation
- A bound of the finite volume produce by Inflation: $V_{\text{Finite Realization}} < e^{\frac{S_{\text{ds}}}{2}}$
  - Holds in any number of dimensions and for multifiled inflation
  - Educated speculations about de Sitter (and a check for Holography)

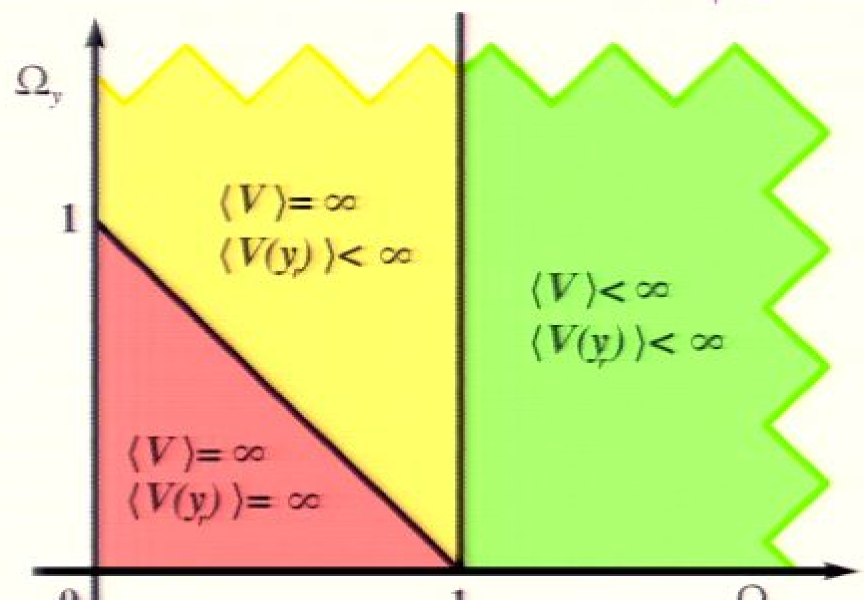


# xample 2:
# ne Tilted Waterfall



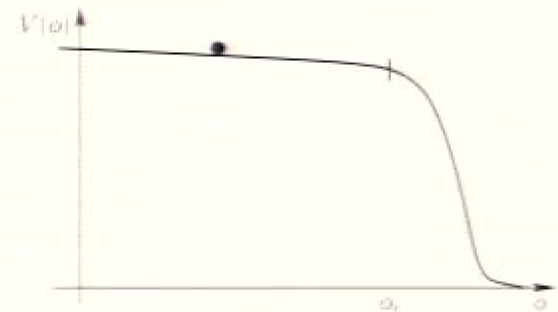

Phase diagram



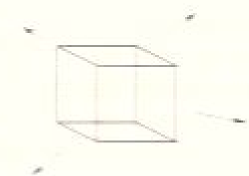

Bound holds

# Perturbativity of the system

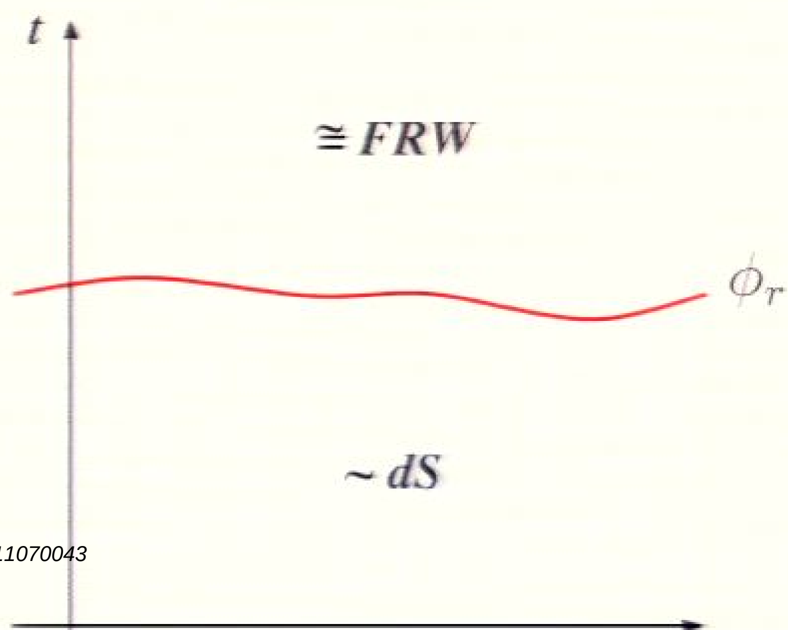
- Close to de-Sitter $\epsilon \simeq \frac{\dot{\phi}^2}{V(\phi)} \ll 1$
  - Still unperturbed before reheating: $\delta g \sim \sqrt{\epsilon}\frac{H}{M_{\rm Pl}}$
- No big interactions: $\frac{S_3}{S_2} \sim \sqrt{\epsilon}\frac{H}{M_{\rm Pl}}$



- $\Longrightarrow$ Study the volume of the Reheating surface $\phi = \phi_r$



Standard Infl.




Eternal Infl.