Title: Boundary Theory and the Measure Problem

Date: Jul 13, 2011 11:20 AM

URL: http://pirsa.org/11070038

Abstract: TBA

Boundary theory and the measure problem

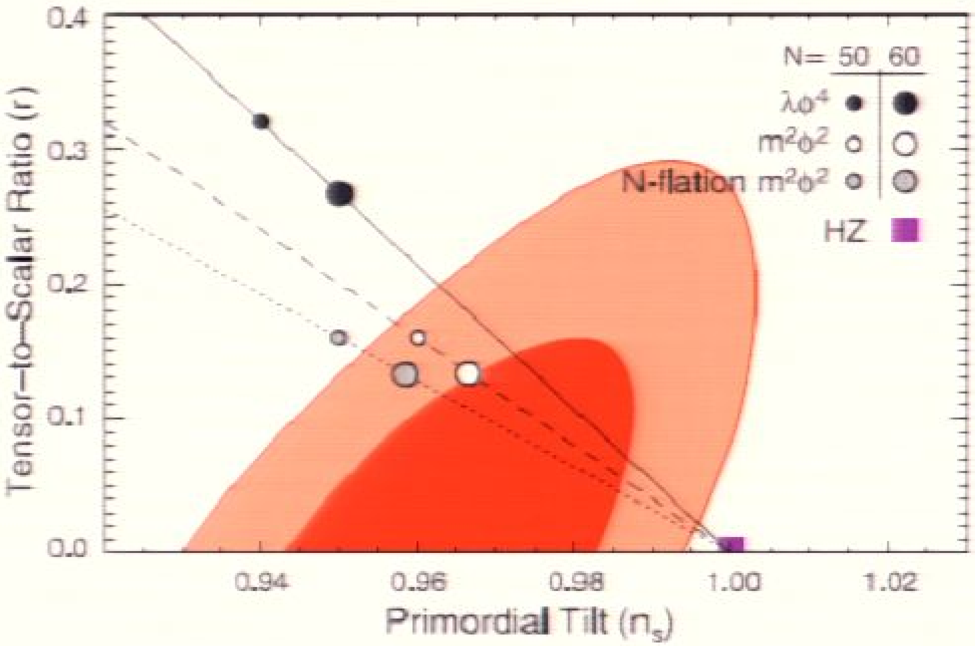
Jaume Garriga,
U. Barcelona

PI11

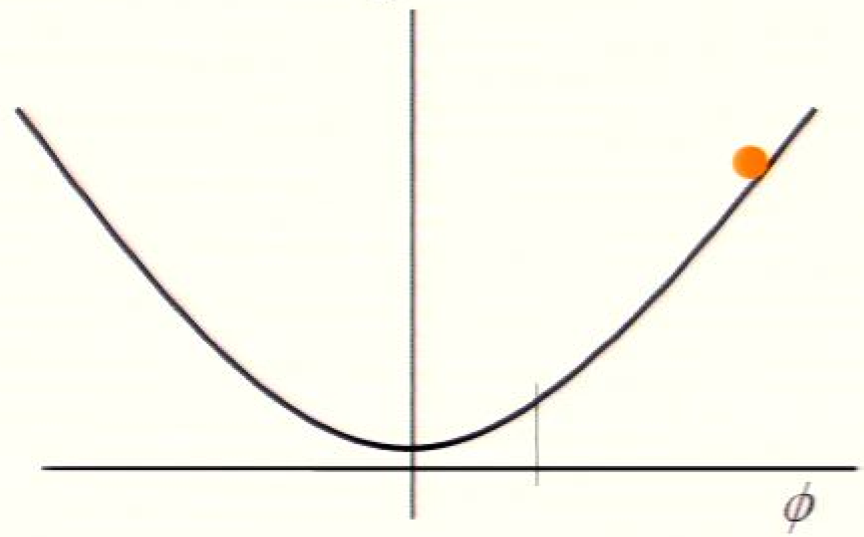
Inflation, in good agreement with observations:

Predictions (early 80's)	Observations (2011)
• Flatness	$\Omega_{TOT} \approx 1$
• Near-scale-invariance	$n_s \approx 0.97$
• Gaussianity	$f_{NL} < 10^2$
• Adiabaticity	$P_S < 10\% \, P_\zeta$
• Tensor modes (?)	$r < 0.24$

Simple models match all observations:



$$V = \frac{1}{2} m^2 \phi^2 + \Lambda$$



Pendulum in honey

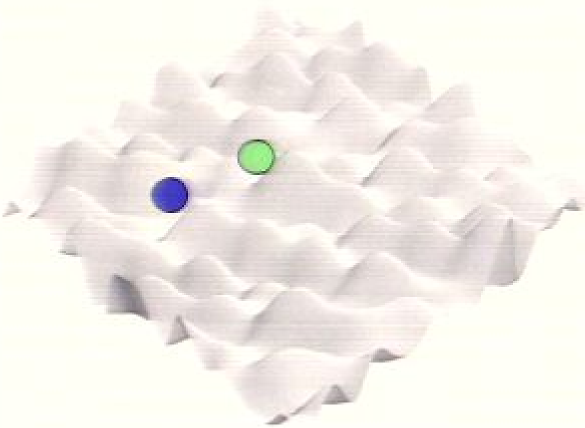
Inflation is generically eternal (to the future)

1- Models with metastable dS vacua

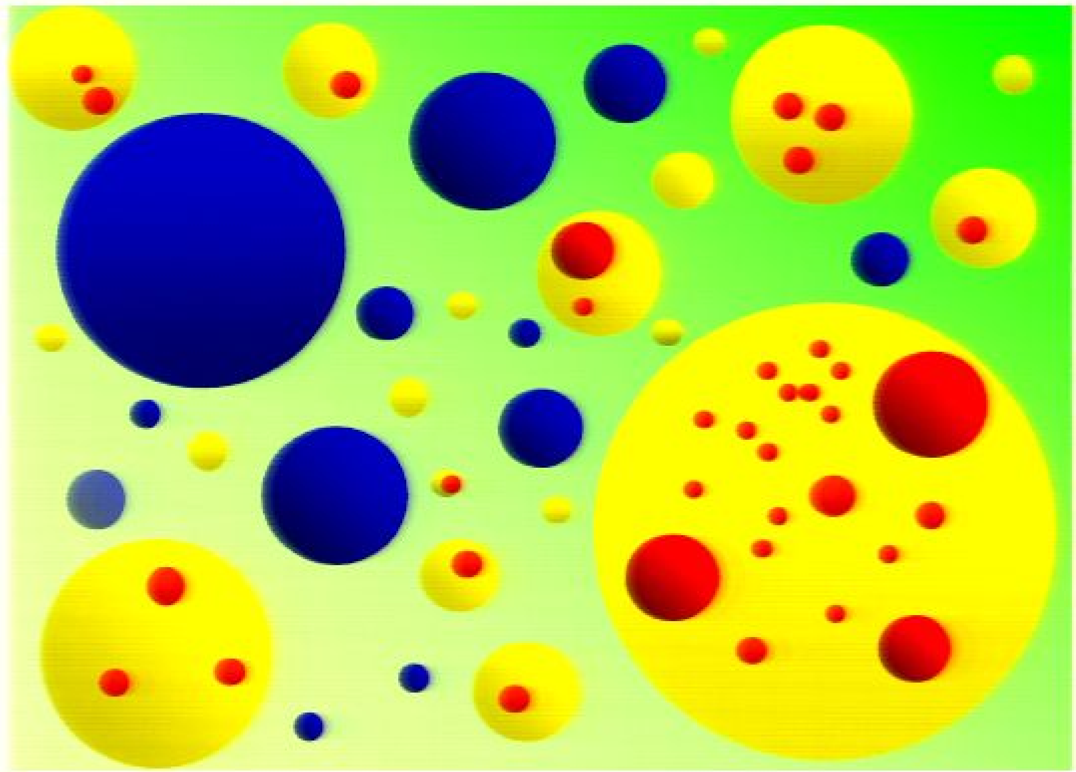
2- Models with a regime dominated
by quantum diffusion

Predictions can be problematic (Measure problem)

Eternally inflating multiverse

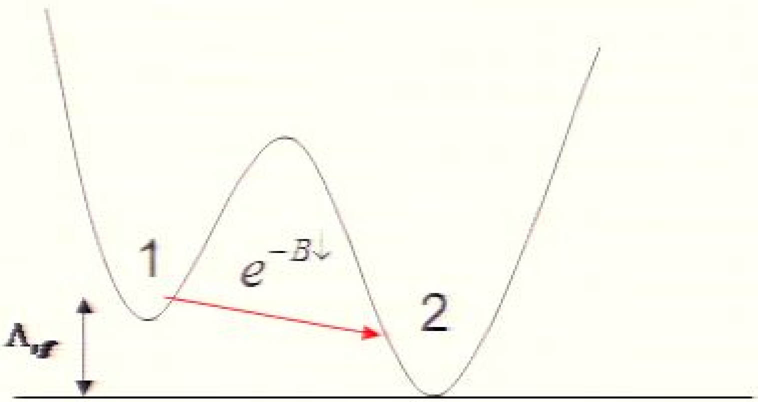


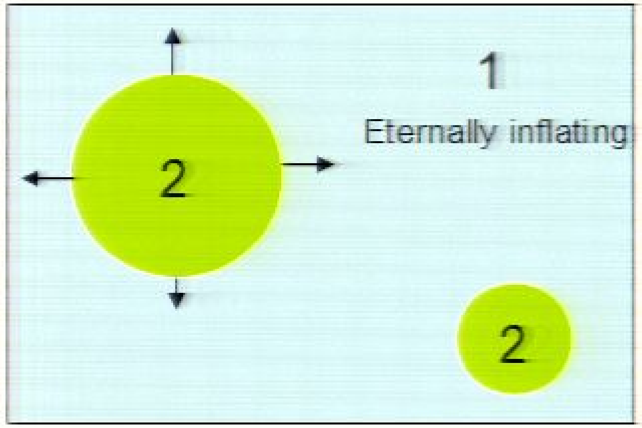
Field space
(Landscape of vacua)



Physical space

Metastable dS vacua





$H \equiv$ Hubble expansion rate $\sim V_1^{1/2}$

$\lambda \equiv$ dimensionless decay rate $\sim e^{-B} \ll 1$

$$\frac{dV_1}{dt} = 3HV_1 - \lambda H V_1$$

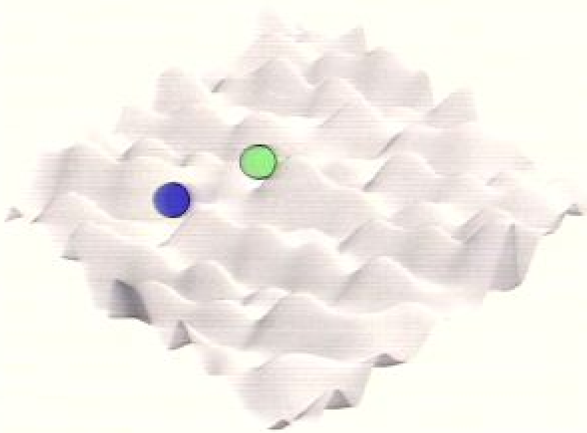
$$V_1 = C\, e^{(3-\lambda)Ht}$$

Average volume grows unbounded for $\lambda \ll 1$,

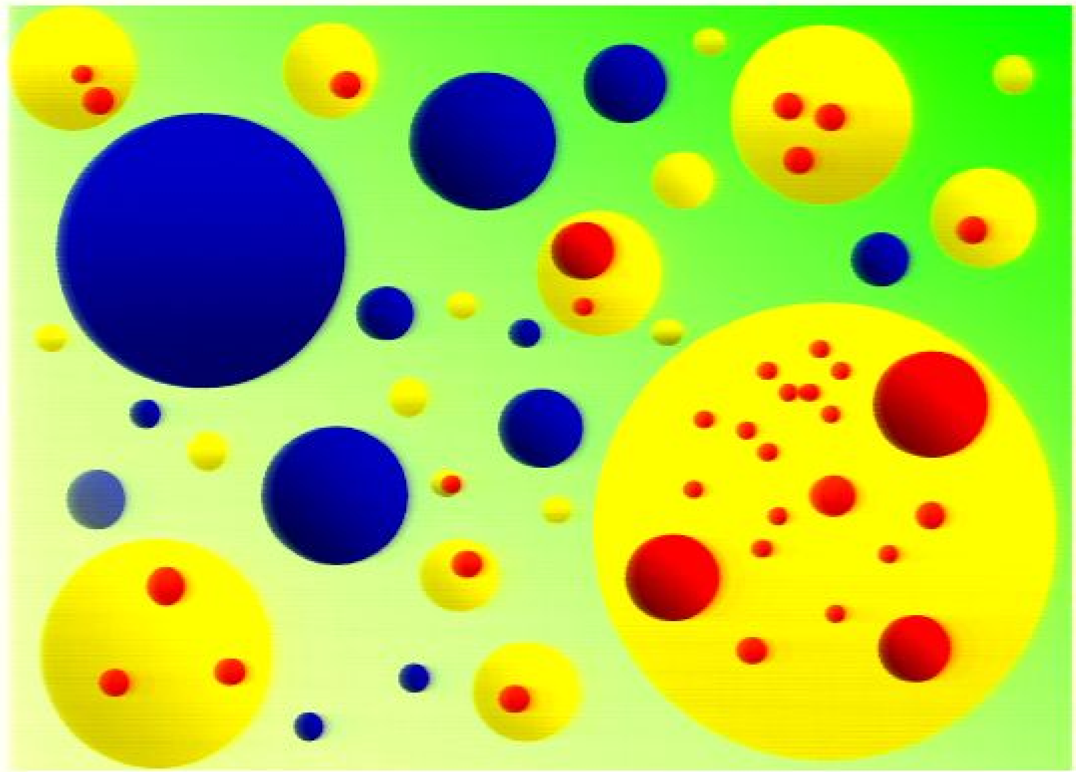
→ Eternal inflation

Finite probability that transition is never complete

Eternally inflating multiverse

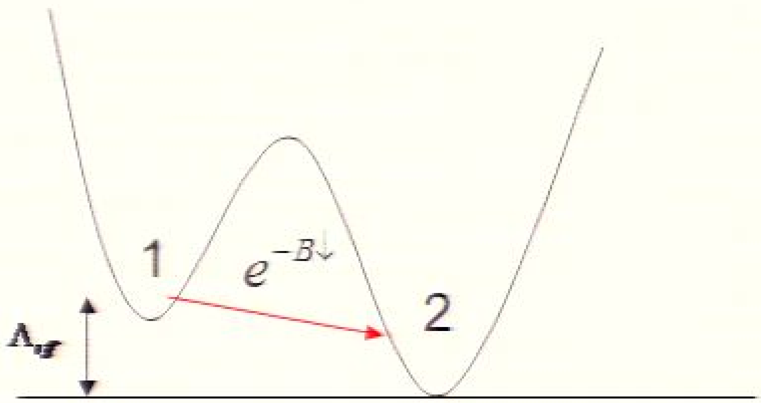


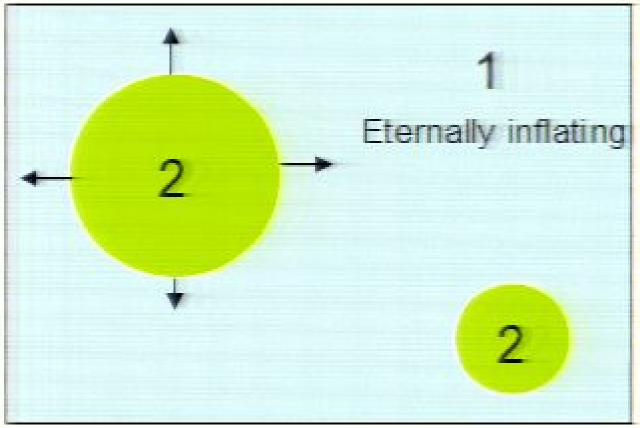
Field space
(Landscape of vacua)



Physical space

Metastable dS vacua





$H \equiv$ Hubble expansion rate $\sim V_1^{1/2}$

$\lambda \equiv$ dimensionless decay rate $\sim e^{-B} \ll 1$

$$\frac{dV_1}{dt} = 3HV_1 - \lambda H V_1$$

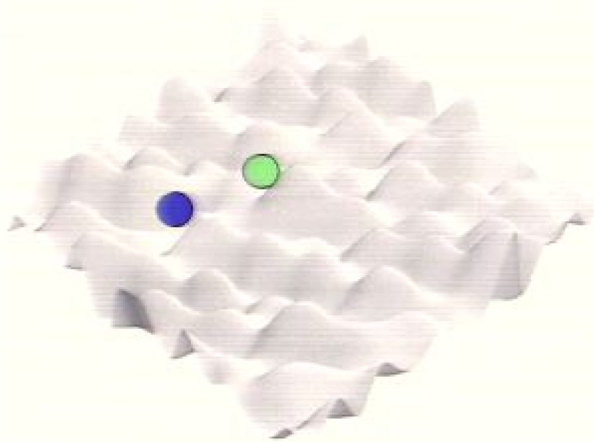
$$V_1 = C\, e^{(3-\lambda)Ht}$$

Average volume grows unbounded for $\lambda \ll 1$,

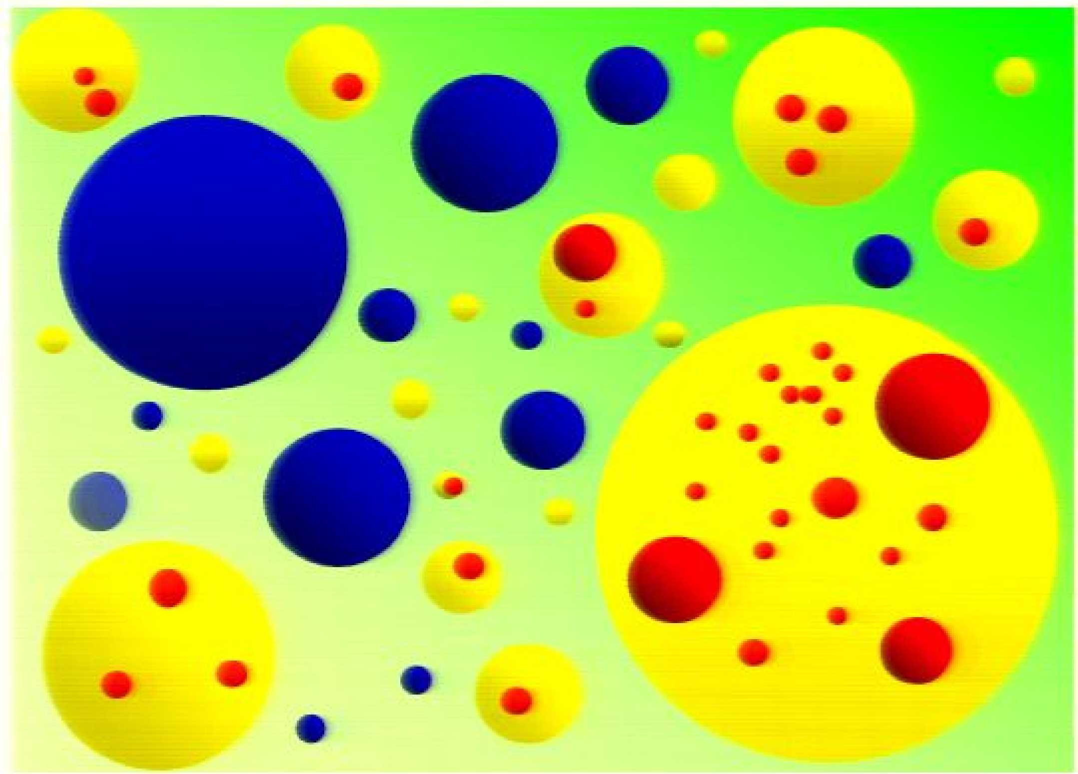
Eternal inflation

Finite probability that transition is never complete

Eternally inflating multiverse



Field space
(Landscape of vacua)

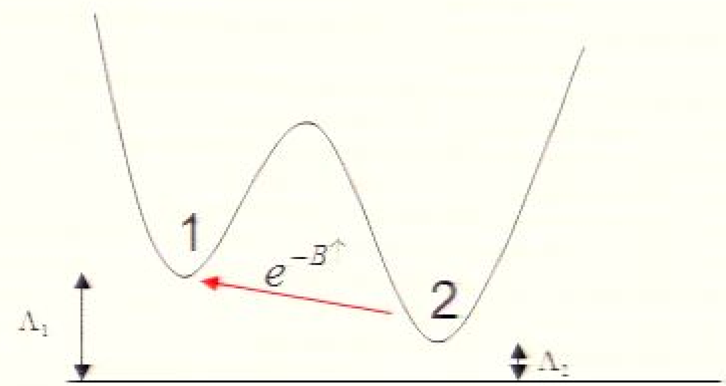


Physical space

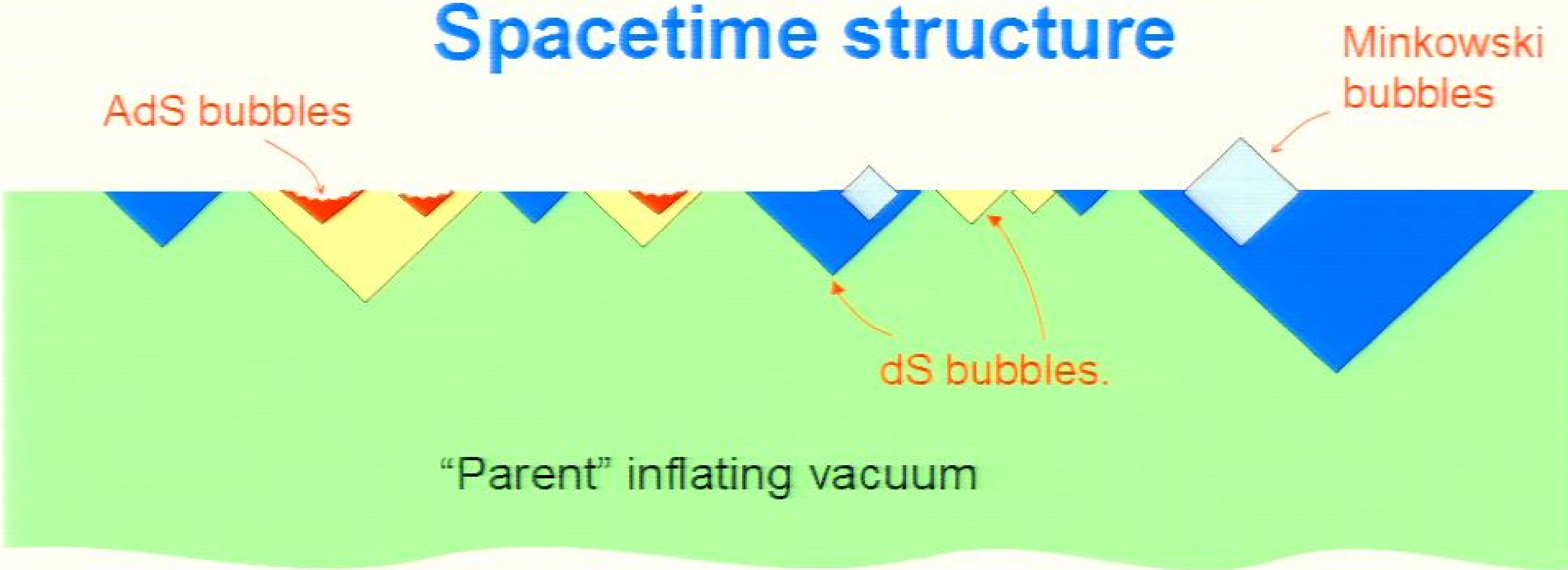
Tunneling uphill is also possible

$$\Gamma^{\uparrow} / \Gamma^{\downarrow} \sim e^{-S(2)+S(1)}$$

Entropy difference



Spacetime structure



- Bubbles nucleate and expand at nearly the speed of light.
- dS (Inflating)
  AdS
  Minkowski } (Terminal bubbles)

Attractor behaviour of volume distribution:

Fraction of volume $V_i(t)$ in inflating vacuum of type

Scale factor gauge $t = \log a$

$$\frac{dV_i}{dt} = 3V_i + M_{ij}V_j$$

rate equation for inflating vacua

$$M_{ij} = \underbrace{\lambda_{ij}}_{\text{Gained from other vacua}} - \underbrace{\delta_{ij}\sum_r \lambda_{ri}}_{\text{Lost to other vacua (Including terminal ones)}}$$

Gained from other vacua

Lost to other vacua (Including terminal ones)

$$\lambda_{ij} = \frac{4\pi}{3} H_j^{-4}\,\Gamma_{ij}$$

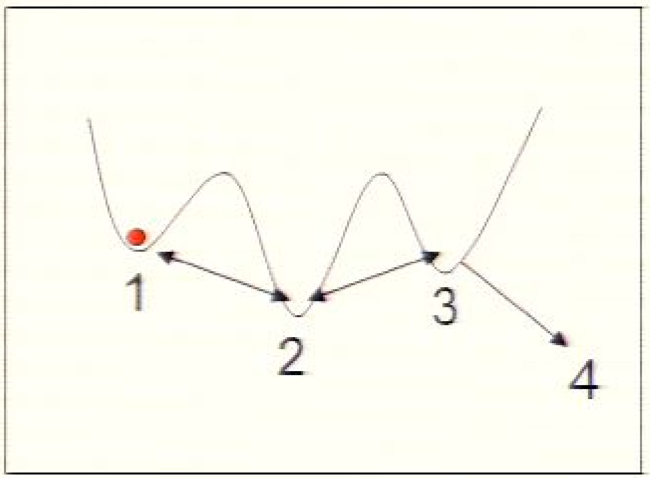
From bubbles of type "i" in vacuum "j".

To each irreducible ``landscape” there corresponds a unique attractor volume distribution.

$$V_i(t) \to V_i^{(0)}\ e^{(3-q)t}$$

$$q \le \min_j \sum_i \lambda_{ij} \qquad 0 < q \ll 1$$

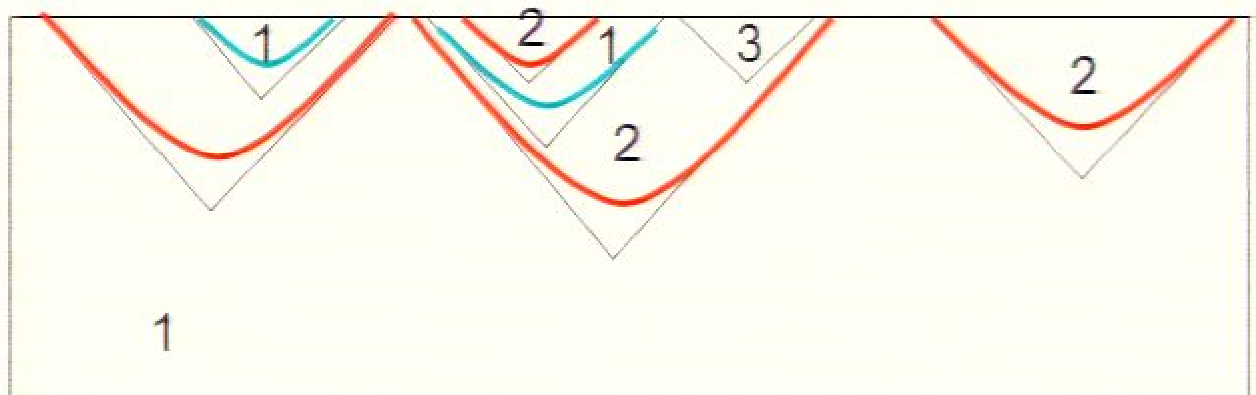
THEORY



J.G., Schwartz-Perlov, Vilenkin & Winitzki (2005)

In this sense, initial conditions do not play a role.

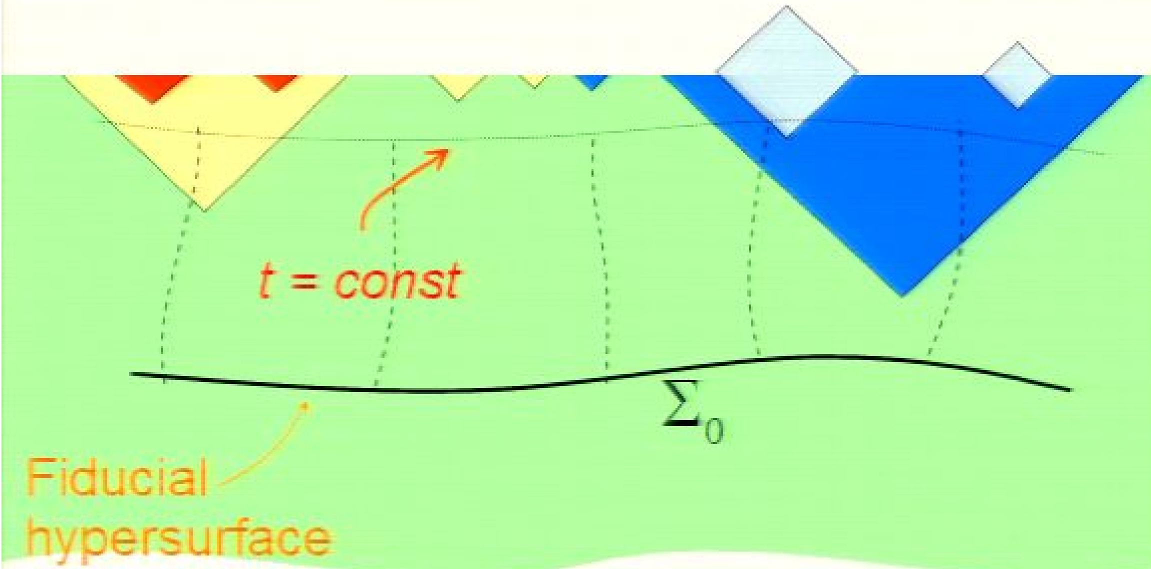
(Self-similar fractal)



MULTIVERSE

Global time cutoff measures:

Count events that happened before some time *t*.



Garcia-Bellido, Linde & Linde (1994); Vilenkin (1995)

$t \to \infty$ ⟹ attractor distribution.

The distribution does not depend on the choice of Σ_0
-- but depends on what we use as *t* (*e.g. proper time vs scale factor time*).

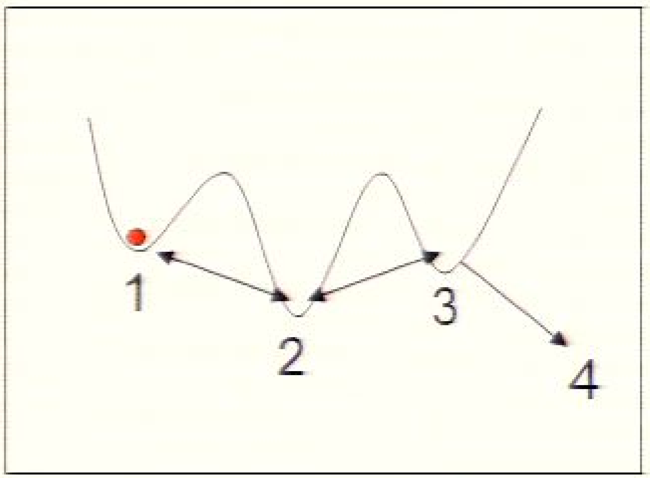
To each irreducible ``landscape" there corresponds a unique attractor volume distribution.

$$V_i(t) \rightarrow \; V_i^{(0)} \; e^{(3-q)t}$$

$$q \leq \min_j \sum_i \lambda_{ij} \qquad 0 < q \ll 1$$

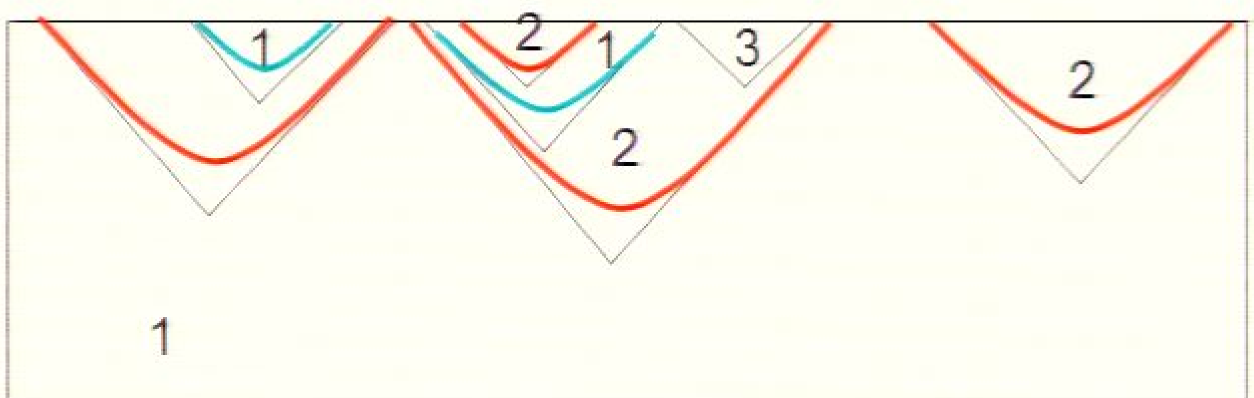
J.G., Schwartz-Perlov, Vilenkin & Winitzki (2005)

THEORY



In this sense, initial conditions do not play a role.

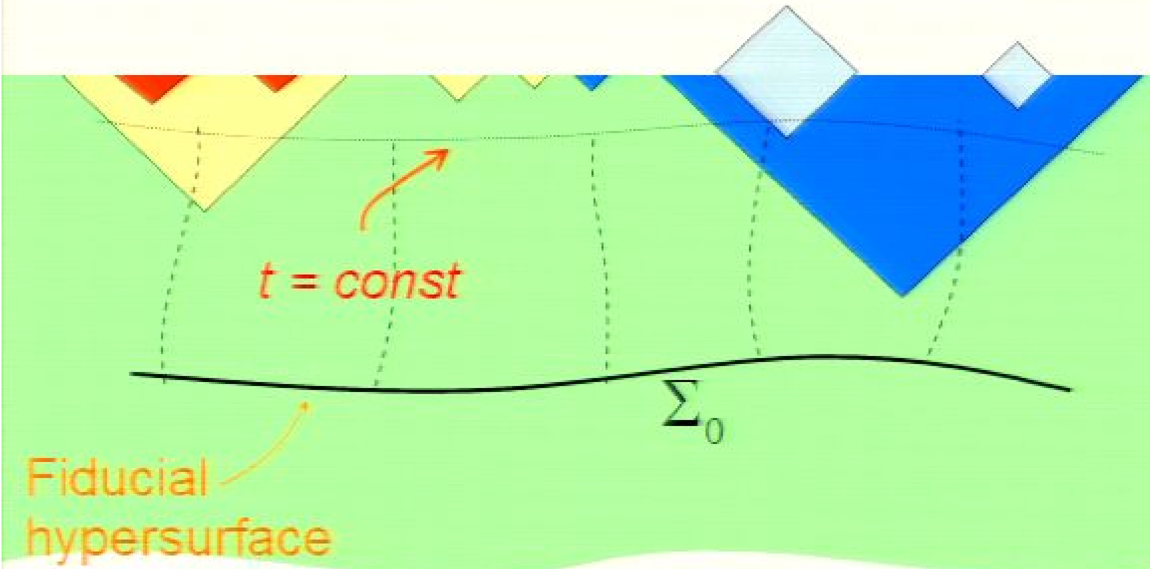
(Self-similar fractal)



MULTIVERSE

Global time cutoff measures:

Count events that happened before some time *t*.



Garcia-Bellido, Linde & Linde (1994); Vilenkin (1995)

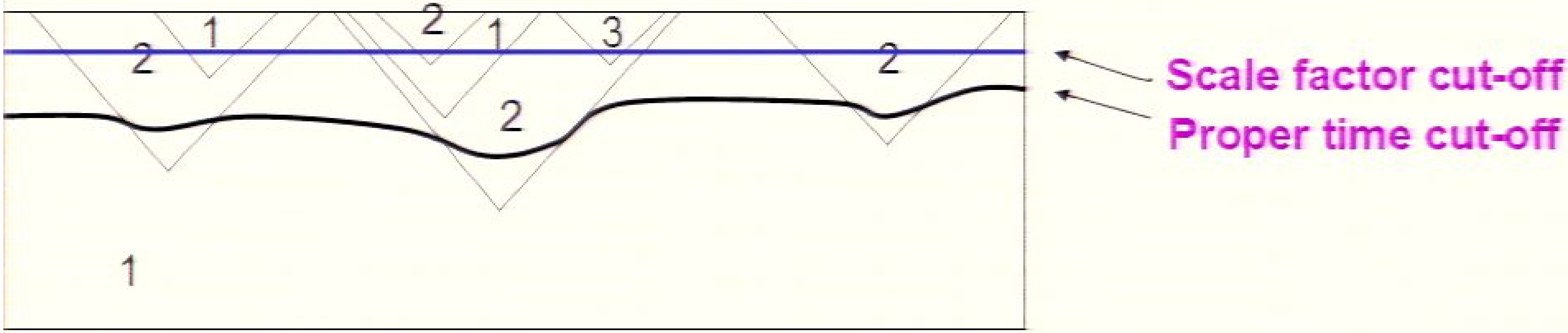
$t \to \infty$ ⟹ attractor distribution.

The distribution does not depend on the choice of Σ_0
-- but depends on what we use as *t (e.g. proper time vs scale factor time)*.

Different choices of time variable
give rise to diferent
results for the dominant eigenvalue

$$dt = H^{\alpha}\, d\tau$$

$$V_i^{(0)}(\alpha)$$



The regularized probability distributions will be different

	Youngness paradox	Q catastrophe	Dependence on initial state	Boltzmann Brain Parado[x]
Proper time cutoff				
Scale factor cutoff				OK (some restrictions apply)
Pocket-based measure				
Etc…				

e.g. De Simone, Guth, Linde, Noorbala, Salem, Vilenkin '10

In the causal diagram, the problem is reminiscent of a UV problem in field theory:

-The number of events diverges as we approach the future boundary

- The divergence is due to the smaller “UV” bubbles (i.e. later bubbles)

-The relative number of events is regulator dependent

Proposal:

The dynamics of eternal inflation may admit a holographic description in terms of a more fundamental theory at the future boundary.

The wave function and its dual interpretation

$$\Psi[\bar{h},\bar{\varphi}] = e^{iW[\bar{h},\bar{\varphi}]}$$

→ Related to
CFT effective action
with prescribed sources

- Gravitons in de Sitter
- Bubble fluctuations

Maldacena 02
J.G., Vilenkin 08,09
Vilenkin 11

1- Linearized tensor modes in de Sitter

$$ds^2 = a^2(\eta)[-d\eta^2 + d\mathbf{x}^2], \qquad a(\eta) = -1/H\eta \qquad h(\mathbf{x}) = \int d^d\mathbf{k} \frac{e^{i\mathbf{kx}}}{(2\pi)^{d/2}} h_{\mathbf{k}}.$$

Gaussian wave functional

$$\Psi[h] = e^{iW[h]} \qquad \boxed{W = \int d^d\mathbf{k} \left(\frac{a^{d-1}}{2} \frac{v'_{\mathbf{k}}}{v_{\mathbf{k}}} |h_{\mathbf{k}}|^2 + i \ln v_{\mathbf{k}} \right)} \qquad v^*_{\mathbf{k}} v'_{\mathbf{k}} - v_{\mathbf{k}} v'^*_{\mathbf{k}} = i a^{1-d}$$

Bunch-Davies vacuum: $v_{\mathbf{k}}(\eta) = \frac{\pi^{1/2}}{2} a^{-d/2} H^{(1)}_{d/2}(k\eta)$

(d+1=5)

$$W[\bar{h}(\mathbf{x})] = \frac{1}{2} \int d^4\mathbf{k} \left(\frac{-k^2 a^2}{2H} + \frac{k^4}{8H^3} [\ln(k^2/H^2 a^2) + \boxed{i\pi} + 2\gamma] + O(a^{-2}) \right) |h_{\mathbf{k}}|^2 + \ldots$$

$Im[W]$ determines the power spectrum.

$$|\Psi|^2 \propto \exp\left[-\int d^4\mathbf{k}\left(\frac{\pi}{8H^3}k^4\right)|h_\mathbf{k}|^2\right]$$

$$\langle h^*_\mathbf{k} h_{\mathbf{k}'}\rangle = (8H^3/\pi k^4)\delta(\mathbf{k}'-\mathbf{k})$$

There is also

$$Re[W] = \frac{H^{-3}}{16}\int d^4\mathbf{k}\; k^4 \ln(k^2/\mu^2)\; |h_\mathbf{k}|^2 + \text{analytic.}$$

which has the form of an effective action in a 4D CFT, with effective cutoff sale = mode freezing scale $\boxed{\mu \equiv aH}$

$$\left\langle T(k)T^*(k')\right\rangle \sim c\; k^4 \ln k^2$$ (As in Gubser, Klebanov, Polyakov 98)

$Im[W]$ determines the power spectrum.

$$|\Psi|^2 \propto \exp\left[-\int d^4\mathbf{k}\left(\frac{\pi}{8H^3}k^4\right)|h_\mathbf{k}|^2\right]$$

$$\langle h^*_\mathbf{k} h_{\mathbf{k}'}\rangle = (8H^3/\pi k^4)\delta(\mathbf{k}'-\mathbf{k})$$

There is also

$$Re[W] = \frac{H^{-3}}{16}\int d^4\mathbf{k}\ k^4 \ln(k^2/\mu^2)\ |h_\mathbf{k}|^2 + \text{analytic.}$$

which has the form of an effective action in a 4D CFT, with effective cutoff sale = mode freezing scale $\boxed{\mu \equiv aH}$

$$\left\langle T(k)T^*(k')\right\rangle \sim c\ k^4 \ln k^2$$ (As in Gubser, Klebanov, Polyakov 98)

The coefficient of the logarithmically divergent term in the effective action is the trace anomaly

$$a_2 = c_1 \int \sqrt{g}\, W^2 d^4x + c_2\, \chi \qquad \sim H^{-3} \int d^4k\ k^4\ |h_k|^2$$

(Weyl invariant)

$$c_1 \sim H^{-3}$$

Number of fields in the CFT
(central charge)

Hartle-Hawking wave function from analytic continuation of Euclidean AdS partition function

$$Z \sim e^{-S}$$

Maldacena 11

Classical action of gravity with prescribed boundary metric

$$-S(\hat{g}) = \frac{M_{pl}^3 R_{AdS}^3}{2}\left[\int d^5x\sqrt{g}(R+12) + 2\int d^4x K\right] =$$

$$= \frac{M_{pl}^3 R_{AdS}^3}{2}\left[\frac{a_0}{\epsilon^4}\int d^4x\sqrt{\hat{g}} + \frac{a_2}{\epsilon^2}\int d^4x\sqrt{\hat{g}}\hat{R} + \frac{\log\epsilon}{8}\int\sqrt{\hat{g}}(\hat{W}^2 - \hat{e})\right] - S_R[\hat{g}]$$

$$R_{AdS} \to -iH^{-1} \qquad \varepsilon \to i/\mu \qquad Z \to \Psi$$

$$|\Psi|^2 \sim \exp\left[-\frac{\pi}{16H^{-3}}\left(\int d^4x\sqrt{g}W^2 - \chi\right)\right]$$

The coefficient of the logarithmically divergent term in the effective action is the trace anomaly

$$a_2 = c_1 \int \sqrt{g}\, W^2 d^4x + c_2\, \chi \qquad \sim H^{-3} \int d^4k\ k^4\, |h_k|^2$$

(Weyl invariant)

$$c_1 \sim H^{-3}$$

Number of fields in the CFT (central charge)

Hartle-Hawking wave function from analytic continuation of Euclidean AdS partition function

$$Z \sim e^{-S}$$

Maldacena 11

Classical action of gravity with prescribed boundary metric

$$-S(\hat{g}) = \frac{M_{pl}^3 R_{AdS}^3}{2}\left[\int d^5x\sqrt{g}(R+12) + 2\int d^4x K\right] =$$

$$= \frac{M_{pl}^3 R_{AdS}^3}{2}\left[\frac{a_0}{\epsilon^4}\int d^4x\sqrt{\hat{g}} + \frac{a_2}{\epsilon^2}\int d^4x\sqrt{\hat{g}}\hat{R} + \frac{\log\epsilon}{8}\int\sqrt{\hat{g}}(\hat{W}^2 - \hat{e})\right] - S_R[\hat{g}]$$

$$R_{AdS} \to -iH^{-1} \qquad \varepsilon \to i/\mu \qquad Z \to \Psi$$

$$|\Psi|^2 \sim \exp\left[-\frac{\pi}{16H^{-3}}\left(\int d^4x\sqrt{g}W^2 - \chi\right)\right]$$

$Im[W]$ determines the power spectrum.

$$|\Psi|^2 \propto \exp\left[-\int d^4\mathbf{k}\left(\frac{\pi}{8H^3}k^4\right)|h_\mathbf{k}|^2\right]$$

$$\langle h^*_\mathbf{k} h_{\mathbf{k}'}\rangle = (8H^3/\pi k^4)\delta(\mathbf{k}'-\mathbf{k})$$

There is also

$$Re[W] = \frac{H^{-3}}{16}\int d^4\mathbf{k}\ k^4 \ln(k^2/\mu^2)\ |h_\mathbf{k}|^2 + \text{analytic}.$$

which has the form of an effective action in a 4D CFT, with effective cutoff sale = mode freezing scale $\boxed{\mu \equiv aH}$

$$\left\langle T(k)T^*(k')\right\rangle \sim c\ k^4 \ln k^2$$ (As in Gubser, Klebanov, Polyakov 98)

1- Linearized tensor modes in de Sitter

$$ds^2 = a^2(\eta)[-d\eta^2 + d\mathbf{x}^2], \qquad a(\eta) = -1/H\eta \qquad h(\mathbf{x}) = \int d^d\mathbf{k} \frac{e^{i\mathbf{k}\mathbf{x}}}{(2\pi)^{d/2}} h_{\mathbf{k}}.$$

Gaussian wave functional

$$\Psi[h] = e^{iW[h]} \qquad \boxed{W = \int d^d\mathbf{k} \left(\frac{a^{d-1}}{2} \frac{v'_{\mathbf{k}}}{v_{\mathbf{k}}} |h_{\mathbf{k}}|^2 + i \ln v_{\mathbf{k}} \right)} \qquad v^*_{\mathbf{k}} v'_{\mathbf{k}} - v_{\mathbf{k}} v'^*_{\mathbf{k}} = i a^{1-d}$$

Bunch-Davies vacuum:

$$v_{\mathbf{k}}(\eta) = \frac{\pi^{1/2}}{2} a^{-d/2} H^{(1)}_{d/2}(k\eta)$$

(d+1=5)

$$W[\bar{h}(\mathbf{x})] = \frac{1}{2} \int d^4\mathbf{k} \left(\frac{-k^2 a^2}{2H} + \frac{k^4}{8H^3} [\ln(k^2/H^2 a^2) + i\pi + 2\gamma] + O(a^{-2}) \right) |h_{\mathbf{k}}|^2 + \ldots$$

The coefficient of the logarithmically divergent term in the effective action is the trace anomaly

$$a_2 = c_1 \int \sqrt{g}\, W^2 d^4x + c_2\, \chi \qquad \sim H^{-3} \int d^4k\ k^4\ |h_k|^2$$

(Weyl invariant)

$$c_1 \sim H^{-3}$$

Number of fields in the CFT (central charge)

Hartle-Hawking wave function from analytic continuation of Euclidean AdS partition function

$$Z \sim e^{-S}$$

Maldacena 11

Classical action of gravity with prescribed boundary metric

$$-S(\hat{g}) = \frac{M_{pl}^3 R_{AdS}^3}{2}\left[\int d^5x\sqrt{g}(R+12) + 2\int d^4x K\right] =$$

$$= \frac{M_{pl}^3 R_{AdS}^3}{2}\left[\frac{a_0}{\epsilon^4}\int d^4x\sqrt{\hat{g}} + \frac{a_2}{\epsilon^2}\int d^4x\sqrt{\hat{g}}\hat{R} + \frac{\log\epsilon}{8}\int\sqrt{\hat{g}}(\hat{W}^2 - \hat{e})\right] - S_R[\hat{g}]$$

$$R_{AdS} \to -iH^{-1} \qquad \varepsilon \to i/\mu \qquad Z \to \Psi$$

$$|\Psi|^2 \sim \exp\left[-\frac{\pi}{16H^{-3}}\left(\int d^4x\sqrt{g}W^2 - \chi\right)\right]$$

(d+1=4)

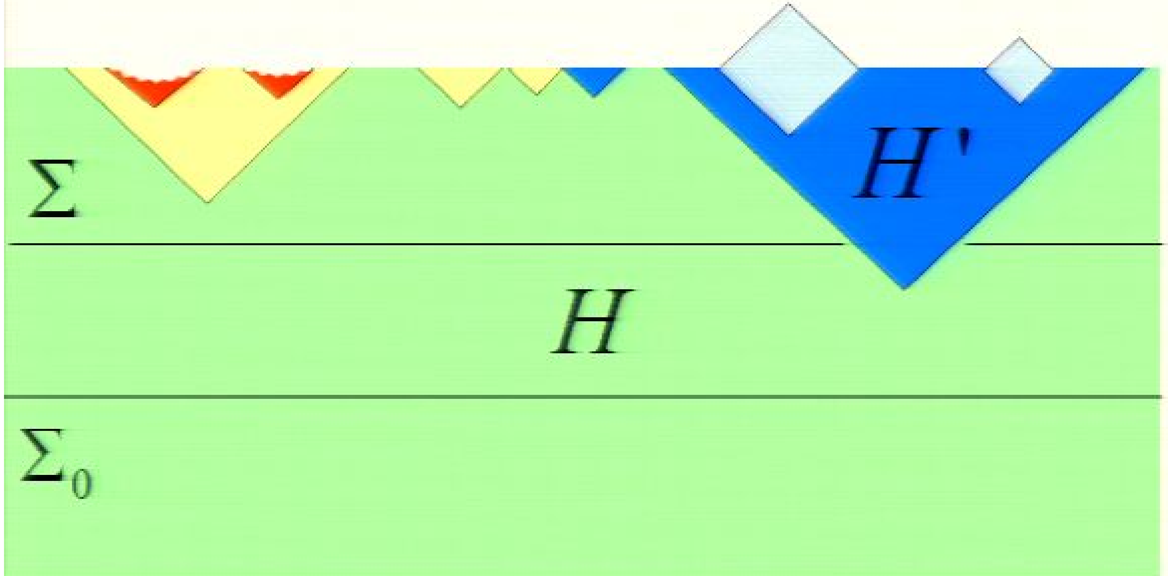
$$W[\bar{h}(\mathbf{x})] = \frac{1}{2}\int d^3\mathbf{k}\left(\frac{-k^2 a}{H} + \boxed{i\frac{k^3}{H^2}} + O(a^{-1})\right)|h_{\mathbf{k}}|^2 + \ldots,$$

Non-analytic part

$$\langle h^*_{\mathbf{k}} h_{\mathbf{k}'}\rangle = H^2 k^{-3}\delta(\mathbf{k}' - \mathbf{k}).$$

$c \sim M_P^2 H^{-2}$ Number of fields in the CFT

Simple model: dS bubbles separated by thin walls

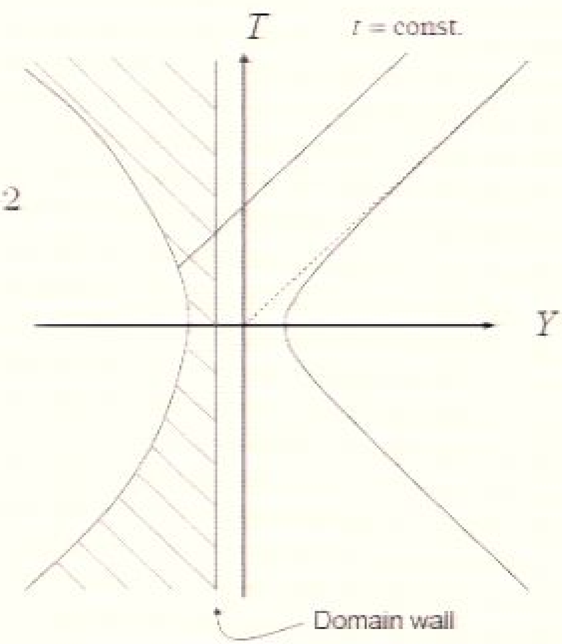


- Nested bubbles
  plus linearized fluctuations.

- Inflating part of spacetime can be foliated by flat surfaces. (They are very close to constant-a surfaces.)

  - $\ln a$ – scale factor time

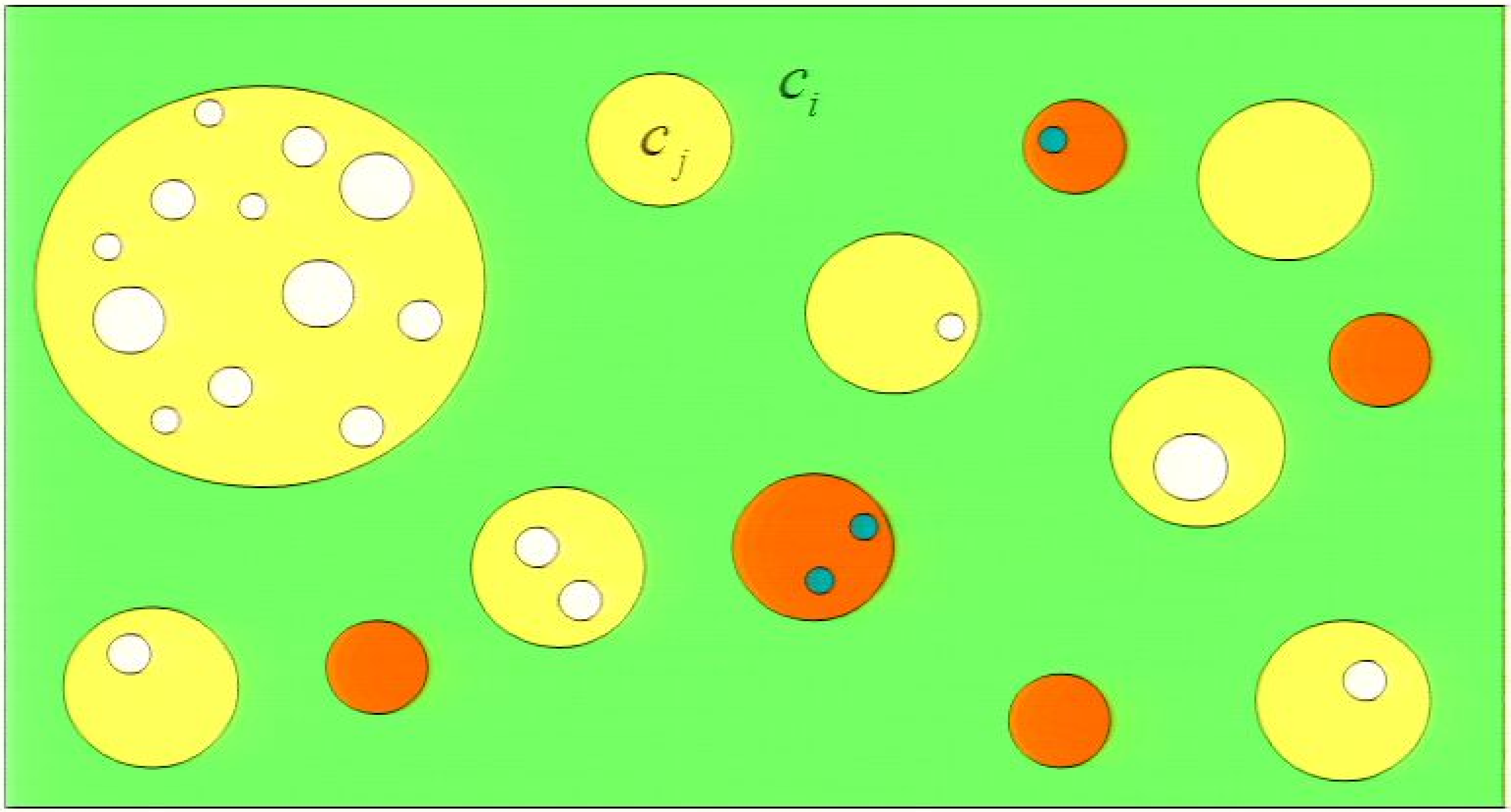
$$\mathbf{X}^2 + (Y - Y_0)^2 - T^2 = H'^{-2}$$



$$\mathbf{X}^2 + Y^2 - T^2 = H^{-2}$$

$$ds^2 = -H^{-2}dt^2 + e^{2t}d\mathbf{x}^2$$

2- Linearized bubble fluctuations



- The boundary effective action should depend on the shape of the surfaces which separate regions with different central charge.

Bubble fluctuations

Restrict attention to the case where gravity of the bubble is unimportant

$$TR_0 \ll 1, \quad (\Delta\rho_V)R_0^2 \ll 1.$$

Bubble wall tension

$$R_0^2 \approx \frac{(p+1)^2T^2}{(p+1)^2H^2T^2 + (\Delta\rho_V)^2}.$$

Intrinsic curvature radius of the worldsheet of bubble wall (which is a p+1 dS space)

$$\delta x^\mu = T^{-1/2}\phi\, n^\mu.$$

Normal displacement of the worldsheet

ϕ - Canonical world-sheet scalar field with a tachyonic mass

$$m_\phi^2 = -(p+1)R_0^{-2}.$$

Bubble fluctuations

Restrict attention to the case where gravity of the bubble is unimportant

$$TR_0 \ll 1, \quad (\Delta\rho_V)R_0^2 \ll 1.$$

Bubble wall tension

$$R_0^2 \approx \frac{(p+1)^2T^2}{(p+1)^2H^2T^2 + (\Delta\rho_V)^2}.$$

Intrinsic curvature radius of the worldsheet of bubble wall (which is a p+1 dS space)

$$\delta x^\mu = T^{-1/2}\phi\, n^\mu.$$ Normal displacement of the worldsheet

ϕ - Canonical world-sheet scalar field with a tachyonic mass

$$m_\phi^2 = -(p+1)R_0^{-2}.$$

$$ds_w^2 = \tilde{a}^2 \left(-d\tilde{\eta}^2 + d\Omega_p^2\right) .$$

Unperturbed w.s. metric

$$\tilde{a} = R_0 / \cos\tilde{\eta},$$

Bubble radius

$$\phi = \sum_{LM} \phi_{LM} Y_{LM}(\Omega),$$

Worldsheet normal displacement

$$\Psi[\phi] = e^{iW[\phi]}$$

Gaussian wave function

$$W = \sum_{LM} \left(\frac{\tilde{a}^{d-1}}{2} \frac{v_L'}{v_L} |\phi_{LM}|^2 + i \ln v_L \right) .$$

Bunch-Davies vacuum

$$v_L = A_L (\cos\tilde{\eta})^{p/2} \left(P^{\nu}_{L-1+p/2}(\sin\tilde{\eta}) + \frac{2i}{\pi} Q^{\nu}_{L-1+p/2}(\sin\tilde{\eta}) \right)$$

$$\nu = \left(\frac{p^2}{4} - m_\sigma^2 R_0^2\right)^{1/2} = \frac{p+2}{2}$$

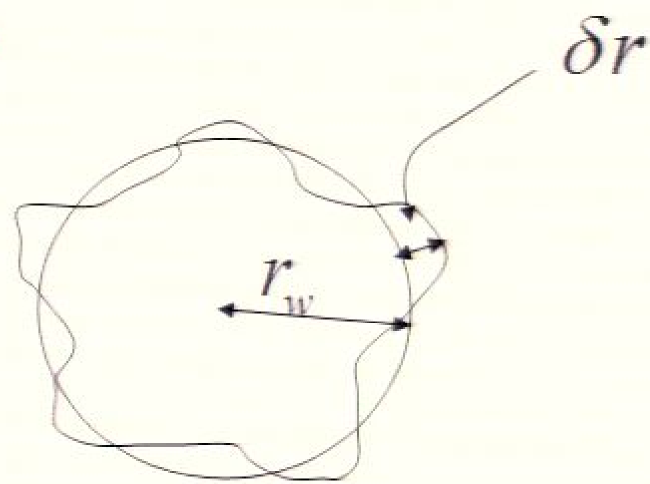
p=2

$$W = \sum_{LM} \left(\frac{\tilde{a}^2}{2R_0} + \frac{R_0 \Delta}{4} + \frac{R_0^3 \Delta(\Delta+2)}{16\ \tilde{a}^2} \left[\ln\left(\frac{R_0^2}{4\tilde{a}^2}\right) + 2\left(\psi(L) + \frac{1}{L}\right) + i\pi + 2\gamma \right] \right) |\phi_{LM}|^2.$$

$0 \ (a \to \infty)$

Relative displacement at the future boundary

$$\delta \equiv \frac{\delta r}{r_w} = \frac{1}{\gamma} \frac{T^{-1/2}\phi}{\tilde{a}(t)}.$$



$\gamma \sim 1/(R_0 H)$ Lorentz factor

Effective action for the deformations of the defect in the boundary theory

We just saw that bulk calculation leads to:

$$W[\bar{\delta}] = \frac{TR_0}{H^2}\sum_{LM}\frac{\Delta(\Delta+2)}{16}\left[\ln\left(\frac{R_0^2}{4\tilde{a}^2}\right)+2\left(\psi(L)+\frac{1}{L}\right)+i\pi+2\gamma\right]|\delta_{LM}|^2+\ldots$$

For $L \gg 1$ $\qquad W \sim d\int d^2k\ \ k^4\log(k^2/\mu^2) \qquad d\sim\frac{TR_0}{H^2}\ll c$

What do we expect from the CFT side?

$$W \sim a_{d/2}\ \ln\mu^2+\ldots$$

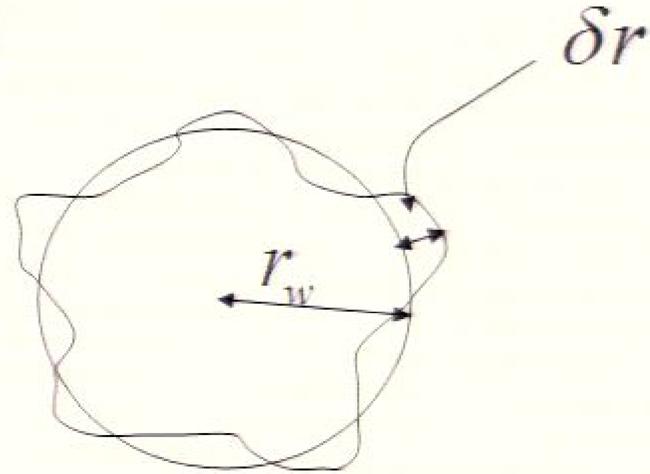
$$a_{3/2} = \int d\Sigma_2\left[d_1\left(K_{ab}K^{ab}-\frac{1}{2}K^2\right)+d_2\hat{R}\right]\propto\int d\Omega\ \delta\Delta(\Delta+2)\delta \qquad !$$

p=2

$$W = \sum_{LM} \left(\frac{\tilde{a}^2}{2R_0} + \frac{R_0 \Delta}{4} + \frac{R_0^3 \Delta(\Delta+2)}{16\ \tilde{a}^2} \left[\ln\left(\frac{R_0^2}{4\tilde{a}^2} \right) + 2\left(\psi(L) + \frac{1}{L} \right) + i\pi + 2\gamma \right] \right) |\phi_{LM}|^2.$$

$$0 \ \ (a \to \infty)$$

Relative displacement at the future boundary



$$\delta \equiv \frac{\delta r}{r_w} = \frac{1}{\gamma} \frac{T^{-1/2}\phi}{\tilde{a}(t)}.$$

$\gamma \sim 1/(R_0 H)$ Lorentz factor

Effective action for the deformations of the defect in the boundary theory

We just saw that bulk calculation leads to:

$$W[\delta] = \frac{TR_0}{H^2} \sum_{LM} \frac{\Delta(\Delta+2)}{16} \left[\ln\left(\frac{R_0^2}{4\tilde{a}^2}\right) + 2\left(\psi(L) + \frac{1}{L}\right) + i\pi + 2\gamma \right] |\delta_{LM}|^2 + \dots$$

For $L \gg 1$ $\qquad W \sim d \int d^2k \;\, k^4 \log(k^2/\mu^2) \qquad d \sim \frac{TR_0}{H^2} \ll c$

What do we expect from the CFT side?

$$W \sim a_{d/2} \ln \mu^2 + \dots$$

$$a_{3/2} = \int d\Sigma_2 \left[d_1 \left(K_{ab}K^{ab} - \frac{1}{2}K^2 \right) + d_2 \hat{R} \right] \propto \int d\Omega \;\delta\Delta(\Delta+2)\delta \qquad !$$

Possible implications for the measure problem

- Holography may suggest an educated guess for a global cut-off measure
  - V1 : Scale factor cut-off — JG + Vilenkin 09
  - V2: Co-moving horizon cut-off — GSVW 06, Bousso 09, Vilenkin 11

- Virtual BB ?

Open questions:

- Is the dynamics of the multiverse encoded in its future boundary (perhaps in terms of a UV complete theory?).
- Can the "measure" can be obtained by imposing a Wilsonian UV cutoff in the boundary theory?
- Heuristically, this kind of a measure seems to be closely related to the scale factor cutoff measure, or a CAH cutoff measure, both of which work well phenomenologically.
- Many open questions: "terminal" vacua, transdimensional transitions, etc.